Lecture 6: Scientific computing in C++

- A review of the work you completed in ACSE-3, specifically lecture 3 would be helpful over the next few weeks
- We're going to be writing versions of algorithms you should know
- But we're going to spend a lot of time thinking about how we might write them in C++
- C++ standard is big!
- There are lots of different ways to write C++ programs
- We're going to talk about some of things important for scientific computing/computational mathematics

Time to write some code!

- Name your C++ file "Matrix.cpp" and "Matrix.h"
- What are some things a "Matrix" class might need to do?

Column/row major ordering

How to store a dense matrix in a 1D array (remember 2D array are just 1D arrays under the hood)

Row-major order

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Column-major order

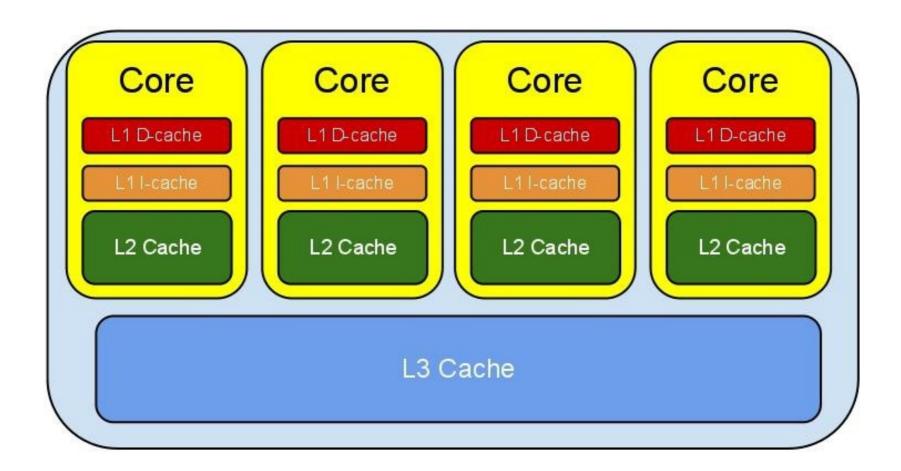
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Time to write some code!

Continue writing the new Matrix class

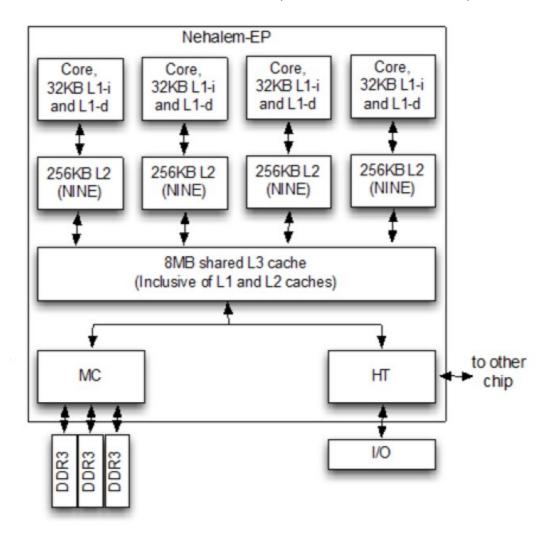
Column/row major ordering

- Why do we care?
- Modern CPUs have a hierarchy of caches



Caches

• Intel Nehalem architecture (2008 - 2011)



Caches

- Caches near to the "cores" are small but close (and quick)
- The further away, the more expensive accessing memory is

Core i7 Xeon 5500 Series Data Source Latency (approximate)

```
local L1 CACHE hit, ~4 cycles ( 2.1 - 1.2 ns )
local L2 CACHE hit, ~10 cycles ( 5.3 - 3.0 ns )
local L3 CACHE hit, line unshared ~40 cycles ( 21.4 - 12.0 ns )
local L3 CACHE hit, shared line in another core ~65 cycles ( 34.8 - 19.5 ns )
local L3 CACHE hit, modified in another core ~75 cycles ( 40.2 - 22.5 ns )
```

local DRAM

~60 ns

Time to write some code!

 Now that we are "cache aware" let's use that in our Matrix class

Caches

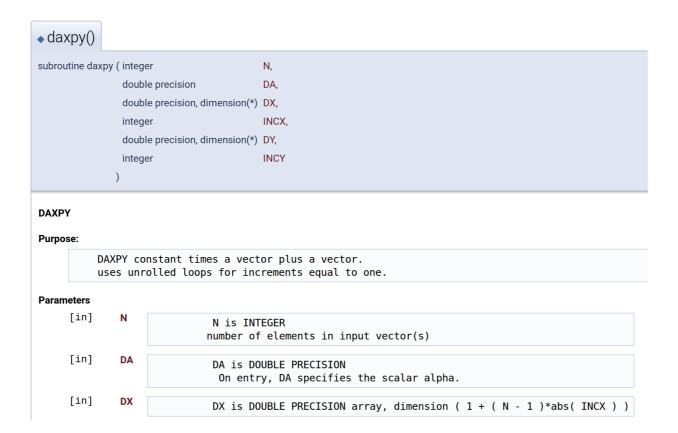
- Again why do we care? Caches are completely transparent to the user, so how can we benefit from thinking about them?
- When you access memory, the CPU automatically grabs neighbouring memory and stores it in the cache
- If you try to order your loops (etc) in your algorithm to use data close in memory, you will be rewarded!

- Thankfully a lot of these algorithms are commonly used in scientific computing
- BLAS Basic Linear Algebra Subroutines (originally 1979)
- LAPACK Linear Algebra PACKage (originally 1992)
- These are standard API that define common operations
- BLAS level 1 vector operations, like ax + y
- BLAS level 2 Matrix vector operations, like A * x
- BLAS level 3 Matrix matrix operations, like A*B
- LAPACK includes things as diverse as eigenvalue decompositions, etc

Here are descriptions of the BLAS/LAPACK standards:

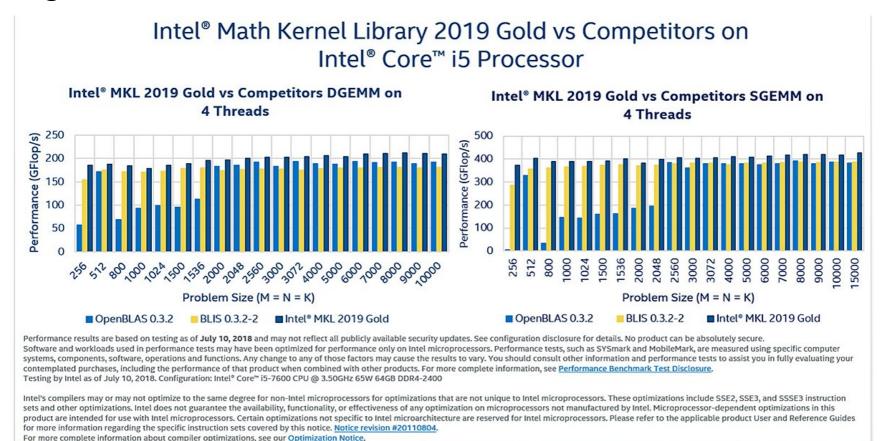
http://www.netlib.org/blas/

http://www.netlib.org/lapack/



- You should always use BLAS/LAPACK libraries to do basic linear algebra
- Many different people have produced their own BLAS/LAPACK libraries, including Intel, most supercomputer vendors, GPU vendors, etc, which are very high performance/threaded/thread safe/deterministic on different hardware
- You can pay a lot of money for BLAS/LAPACK implementations
- MATLAB for example started as basically a nice interface for BLAS/LAPACK under the hood

- For example, Intel sell a BLAS/LAPACK library they call they Intel MKL and they market it aggressively
- Dgemm: BLAS routine does dense double matmatmult



Homework

- Implement a "cache aware" matrix-vector product
- Change the Matrix class to use column-major storage (and change your matmatmult routines, etc to respect this)
- Overload the + operator for our Matrix class
- ADVANCED Build an LU factorisation for your Matrix class
 VERY ADVANCED Build an LU factorisation with partial pivoting

SUPER ADVANCED - Read

http://www.ncic.ac.cn/~tgm/dgemm/sc11_dgemm.pdf for an overview of the complexity of building a performant dgemm routine on modern hardware