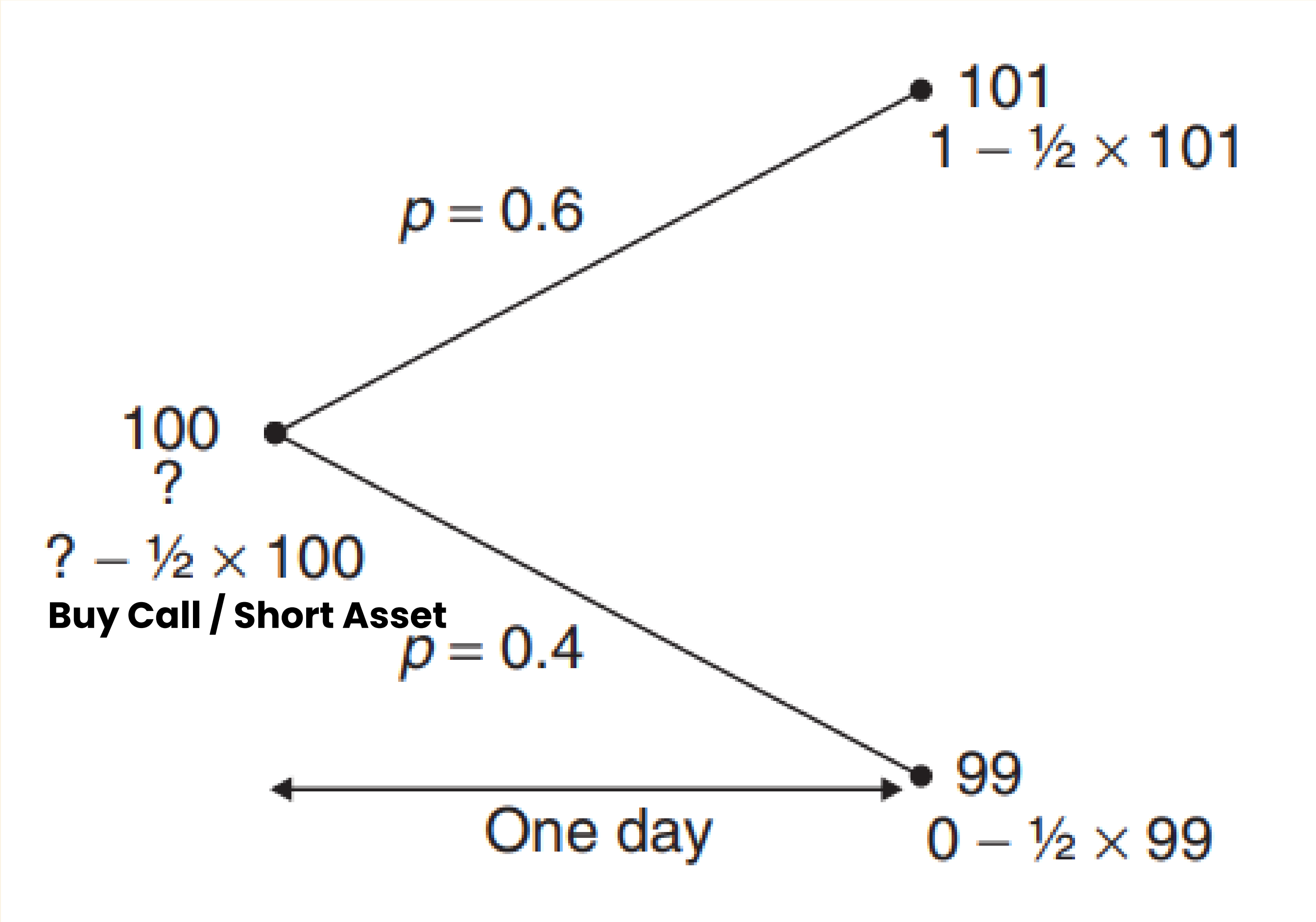
# CONTENTS

# BINOMIAL MODEL

Asset prices can either go up or down by a known amount



# BINOMIAL MODEL

Now let's say we buy a call and short delta no. of assets

Delta Hedging - choosing the delta such that the portfolio value does not depend on the direction of the stock

$$\Delta = \frac{\text{Range of option payoffs}}{\text{Range of stock prices}}.$$

Example: Stock price is 100, can rise to 103 or fall to 98. Value a call option with a strike price of 100. Interest rates are zero.

# RANDOM BEHAVIOUR OF ASSETS

Jensen's Inequality: E(f(S)) >= f(E(S))

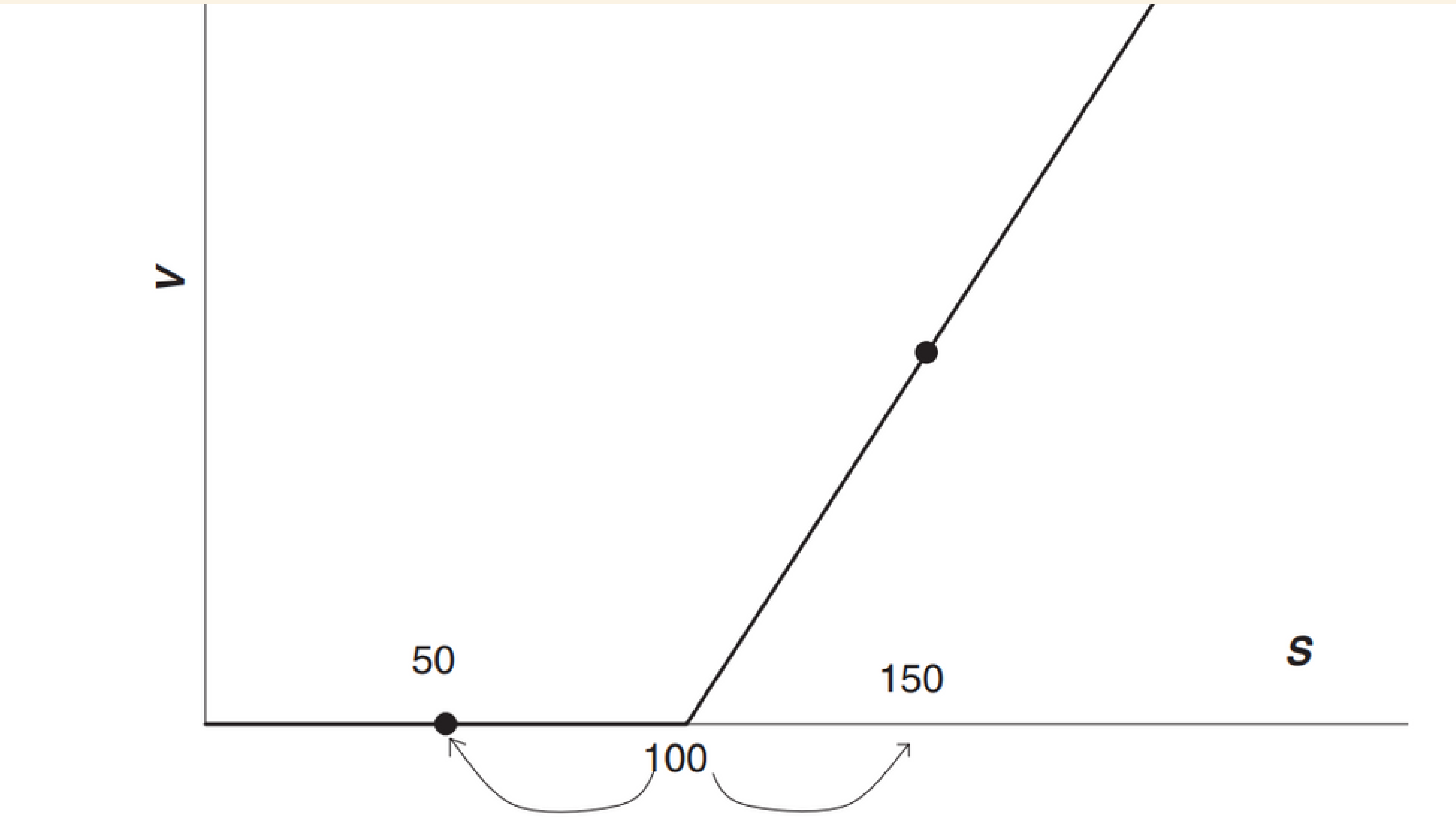



**Figure 4.1** Future scenarios.

Payoff(Expected [Stock price]) = 0

Expected [Payoff(Stock price)] = 25.

$$\tfrac{1}{2} f''(E[S])\, E\left[\epsilon^2\right].$$

# RANDOM BEHAVIOUR OF ASSETS

## Modelling returns

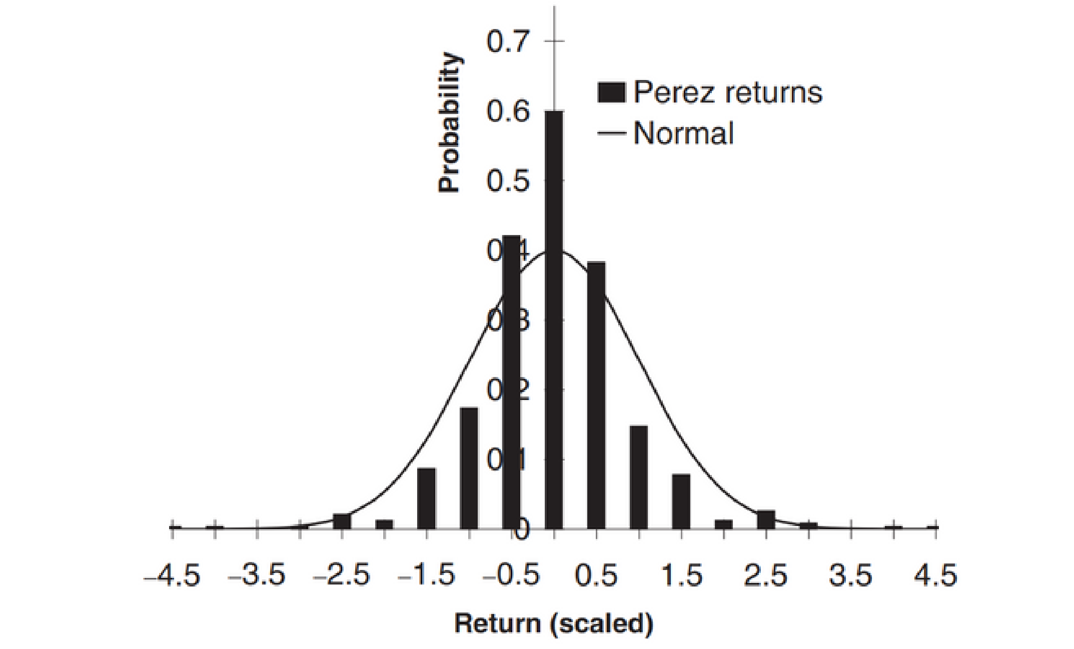
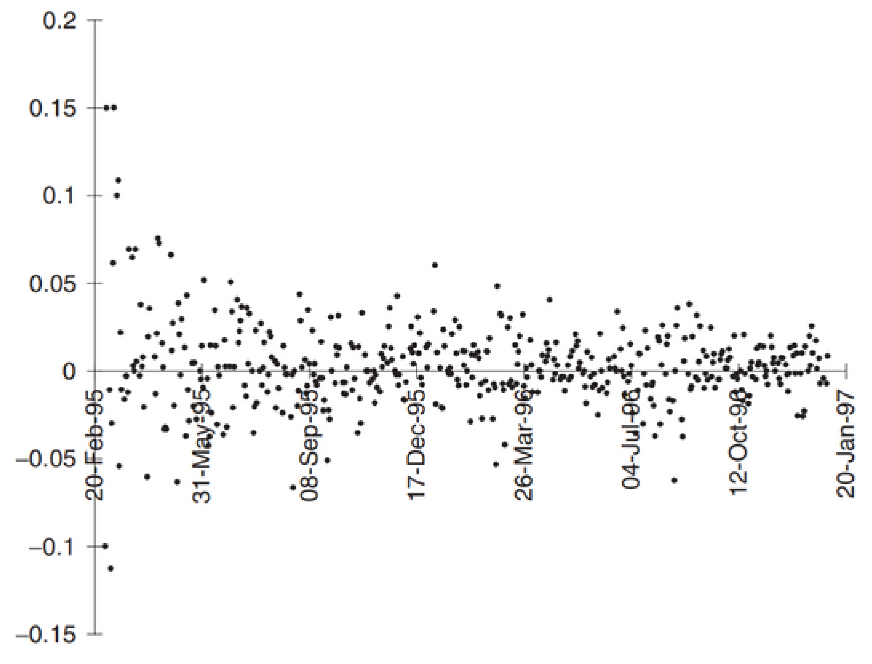



**Figure 4.5** Normalized frequency distribution of Perez Companc and the standardized Norma distribution.




gure 4.4 Daily returns of Perez Companc.

$$\frac{S_{i+1} - S_i}{S_i} = \text{mean} + \text{standard deviation} \times \phi$$

Returns

$$R_i = \frac{S_{i+1} - S_i}{S_i}$$

Returns' Mean

$$\mu = \frac{1}{M\,\delta t} \sum_{i=1}^{M} R_i.$$

Returns' std dev

$$\sqrt{\frac{1}{(M-1)\,\delta t} \sum_{i=1}^{M} (R_i - \overline{R})^2}.$$

# RANDOM BEHAVIOUR OF ASSETS

Modelling using timescales

$$S_{i+1} = S_i(1 + \mu\ \delta t).$$

$$S_M = S_0(1 + \mu\,\delta t)^M.$$

$$S_M = S_0\left(1 + \mu\ \delta t\right)^M = S_0 e^{M \log(1+\mu\ \delta t)} \approx S_0 e^{\mu M\ \delta t} = S_0 e^{\mu T}$$

$$S_{i+1} = S_i\left(1 + \mu\,\delta t + \sigma\phi\ \delta t^{1/2}\right)$$

Coin Example: Toss a coin. Every time you throw a head I give you $1, every time you throw a tail you give me $1, E[Ri] = 0, E[Ri^2 ] = 1 and E[RiRj] = 0

$$S_i = \sum_{j=1}^{i} R_j. \quad E[S_i] = 0 \quad \text{and} \quad E[S_i^2] = E[R_1^2 + 2R_1R_2 + \cdots] = i.$$

Now let's change the rules of the coin-tossing experiment. First of all let's restrict the time allowed for the six tosses to a period t, so each toss will take a time t/6. Second, the size of the bet will not be $1 but (t/6)^0.5.

$$\sum_{j=1}^{6} (S_j - S_{j-1})^2 = 6 \times \left(\sqrt{\frac{t}{6}}\right)^2 = t.$$

## Properties of Brownian Motion:

**Markov Property:** The expected value of the random variable Si conditional upon all of the past events only depends on the previous value Si-1. This is the Markov property.

**Martingale Property:** The conditional expectation of your winnings at any time in the future is just the amount you already hold.

**Quadratic Variation:**

$$\sum_{j=1}^{i} (S_j - S_{j-1})^2 = i.$$

**Normaility**

**Finiteness**

**Continuity**