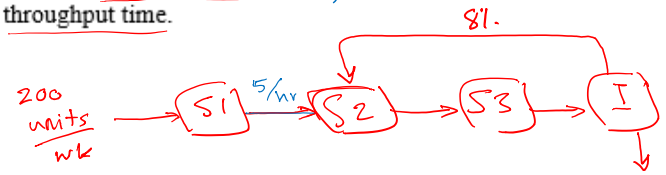
IE 469 Manufacturing Systems

Chapter 11 Tutorial (Askin's Book)

Question 1

A production line is used to produce a product. Number of products at the start of the line is 200 units per week. The line consists of four stations. It is arranged such that an inspection station (I) is placed after three process stations (S1, S2, S3). The processing time of stations (S1, S2, S3, I) are (8, 10, 13, 8) minute respectively. Inspection station (I) has defect rate of 10%. 2% of the defects are scraped and the reminders 8% are returned to station (S2). **Find**

(a) Determine the effective arrival rate at each station.
(b) Determine the number of machines at each station.
(c) Find the average work in process. WIP, L
(d) Find the throughput time.



H_w = 40 hrs

		S1	S2	S3	I
a)	$\lambda' = \frac{\lambda}{1-p}$ units/hr	5	$\frac{5}{1-0.08} = 5.43$	5.43	5.43
	$\mu = \frac{1}{T_p}$ units/hr	$\frac{1}{8} \times 60 = 7.5$	6	4.6154	7.5
b)	$M(c) = int(\frac{\lambda}{\mu}) + 1$	$int(\frac{5}{7.5}) + 1 = 1$	1	1+1 = 2	1
	$\rho = \frac{\lambda}{c*\mu}$	0.6667	0.9058	0.5888	0.7246
	P_0	0.3333	0.0942	0.2588	0.2754
	L_q	1.3333	8.7059	0.6292	1.9065
	L_j	2	9.6117	1.217	2.6311
c)	$WIP = \sum_{j=1}^{4} L_j =$	15.4598			
	$v_j = \frac{\lambda'}{\lambda}$	1	1.0869	1.0869	1.0869
	W_j	0.4	1.7686	0.2240	0.4841
d)	Throughput time $= \sum_j v_j W_j =$	3.092			

M/M/1: $P_0 = 1-\rho$, $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$, $L = \frac{\rho}{1-\rho}$, $W = \frac{L}{\lambda}$

M/M/c: $c = 2$, $P_0 = \frac{1}{1+2\rho+\frac{2\rho^2}{(1-\rho)}}$, $L_q = \frac{P_0(\frac{\lambda}{\mu})^c \rho}{c!(1-\rho)^2}$, $L = L_q + \frac{\lambda}{\mu}$, $W = \frac{L}{\lambda}$

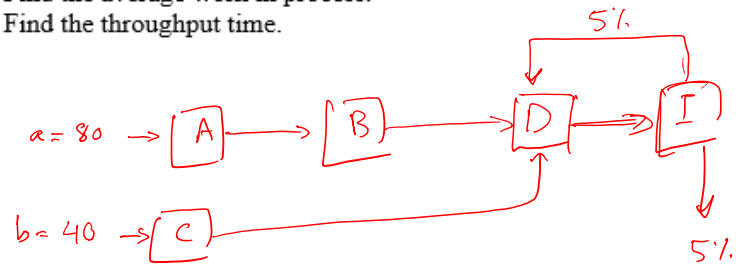
Question 4

A production line is used to produce 40 units per week of a product. The product is assembled from two pieces of part (a) and one piece of part (b). The part (a) is produced through stations (A); station (B) while the part (b) is produced at station (c). After that, the two parts are assembled on station (D). The processing time of stations (A, B, C.D) are (40, 20,40, 15,) minutes respectively. The product is tested on Inspection station (I) with processing time of 10 minutes and has defect rate of 10%. 5% of the defects are scraped and the reminders 5% are returned to station (D).

A) Find

(a) Determine the effective arrival rate at each station.
(b) Determine the number of machines at each station.
(c) Find the average work in process.
(d) Find the throughput time.

$H_w = 40 \frac{hr}{Week}$



	A	B	C	D	I
a) λ'	2	2	1	$\frac{40}{1-0.05} = 1.053$	1.053
μ (units/hr)	1.5	3	1.5	4	6
✓ b) M(c)	2	1	1	1	1
ρ	0.6667	0.6667	0.6667	0.2633	0.1755
P_0	0.2264	0.3333	0.3333	0.7367	0.8245
L_q	1.2078	1.3333	1.3333	0.09406	0.03786
L	2.5403	2.0003	2.0003	0.3574	0.21286
✓ c) $\sum_{j=A}^{I} L_j$ =	7.1112				
v_j	1	1	1	1.053	1.053
W_j	1.2702	1.0015	2.0003	0.3394	0.2021
d) $\sum_j v_j W_j$ =	4.8422				

Question 6

Three parts are processed using three machining centers (C1, C2, and C3) as indicated in Table (1).

Determine:

(a) The effective arrival rate at each center.

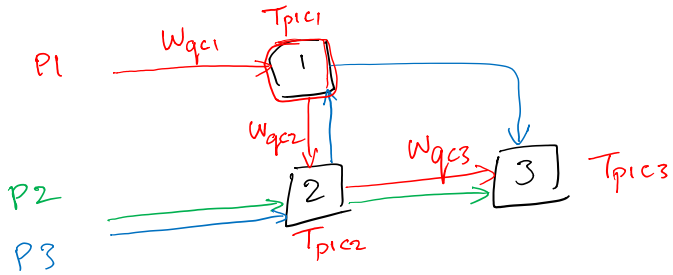
(b) The average processing time at each center.

(c) The average number of parts in process.

(d) The throughput time for each part.

TABLE (1)

Product	Demand / Week	Routing Data [Centers, Process Time in hr]		
		1	2	3
P1	9	C1, 2	C2, 1	C3, 1
P2	10	C2, 1.2	C3, 1.5	-
P3	8	C2, 1.2	C1, 1.5	C3, 1.2



a) effective arrival rate λ' (units / week)

Product	λ'		
	C1	C2	C3
P1	9	9	9
P2	0	10	10
P3	8	8	8
$\Sigma\lambda'$	17	27	27

b) avg. processing time

Product	Processing time, hr		
	C1	C2	C3
P1	2	1	1
P2	0	1.2	1.5
P3	1.5	1.2	1.2

$\mu^{-1} = \left(\frac{\lambda_i}{\Sigma\lambda'}\right)\frac{T_P}{\text{period time}}$ period time = 40 hr / wk

Product	Avg. Process time, wk		
	C1	C2	C3
P1	$(\frac{9}{17})(\frac{2}{40})$	0.0083	0.0083
P2	0	0.0111	0.0139
P3	0.01765	0.0089	0.0089
b) $\Sigma\mu_j^{-1}$	0.0442	0.0283	0.0311
$\mu = \frac{1}{\mu^{-1}}$	22.62	35.33	32.15

(units / week)

$\rho = \frac{\lambda}{c\mu}$ ✓ ✓ $1-\rho$ $L = \frac{\rho}{1-\rho}$ $W = \frac{L}{\lambda}$

Station	λ	μ	ρ	M(c)	P_0	L	L_q	W	W_q
C1				1		3.0225			
C2				1		3.2391			
C3				1		5.2383			

$int(\frac{\lambda}{\mu}) + 1$ $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$ $W_q = \frac{L_q}{\lambda}$

c) avg. # parts = $\sum_{j=1}^{c_3} L_j = 11.49$

d)

Part type 1

$$W_1 = W_{qc1} + \frac{T_{p1c1}}{\text{period time}} + W_{qc2} + \frac{T_{p1c2}}{\text{period time}} + W_{qc3} + \frac{T_{p1c3}}{\text{period time}}$$

$= 0.4882$ week

Part type 2

$$W_2 = W_{qc2} + \frac{T_{p2c2}}{\text{period time}} + W_{qc3} + \frac{T_{p2c3}}{\text{period time}}$$

$= 0.3221$ weeks

Part type 3

$$W_3 = W_{qc1} + \frac{T_{p3c1}}{\text{period time}} + W_{qc2} + \frac{T_{p3c2}}{\text{period time}} + W_{qc3} + \frac{T_{p3c3}}{\text{period time}}$$

$= 0.4857$

Question 9

Five parts are processed using four machining centers (C1, C2, C3, C4) and consiste of 3,1,3,3 machines respectively. The data is indicated in Table (1).

(a) Determine the effective arrival rate at each center.
(b) Determine the average processing time at each center.
(c) Determine number of machines in each center
(d) Find the average number of jobs in process.
(e) Find the throughput time for each parts.

Table (1)

Product	Demand/ Week	Routing Data [Centers, Process Time in hr]			
		1	2	3	4
1	15	C3, 1	C2, 2	C1, 1	C4, 2
2	10	C4, 3	C1, 3	C2, 2	C3, 1
3	6	C4, 4	C3, 2	C1, 2	-
4	10	C3, 2	C4, 3	C2, 2	C1, 1
5	8	C3, 3	C2, 1	C1, 2	-

a)

	Effective arrival rate			
	$\lambda_{i,c1}$	$\lambda_{i,c2}$	$\lambda_{i,c2}$	$\lambda_{i,c4}$
P1	15	15	15	15
P2	10	10	10	10
P3	6	0	6	6
P4	10	10	10	10
P5	8	8	8	0
$\sum\lambda$	49	43	49	41

$\mu^{-1} = \left(\frac{\lambda_i}{\sum\lambda}\right) \propto \frac{T_p}{\text{period time}}$, period time = 40 hr/week

b)

	processing time, hr			
	c1	c2	c3	c4
P1	1	2	1	2
P2	3	2	1	3
P3	2	0	2	4
P4	1	2	2	3
P5	2	1	3	0

$\left(\frac{15}{49}\right) \times \frac{1}{40}$

	avg. processing time, week			
	μ^{-1}, c1	μ^{-1}, c2	μ^{-1}, c3	μ^{-1}, c4
P1	0.0077	0.0174	---	-
P2	"	"		
P3	"	"		
P4				
P5				
$\sum\mu_j^{-1}$	0.0423	0.0453	0.0413	0.0695
$\mu_j = \frac{1}{\sum\mu_j^{-1}}$	23.61	22.05	24.19	14.38

c) # machines in each center

	λ	μ	M(c)	ρ	P_0	L	L_q	W	W_q
c1	49	23.6145	3	0.6917	0.0994	3.1522	1.0772	0.0643	0.0220
→ c2	43	22.0513	1	>1	-ve	NO	SOLUTION		
c3	49	24.1975	3	0.6750	0.1071	2.9726	0.9476	0.0607	0.0193
c4	41	14.386	3	0.9500	0.0118	20.0822	17.2322	0.4898	0.4203

when M(c) = 2 for center 2:

c2	43	22.0513	2	0.9750	0.0127	39.4924	37.5424	0.9184	0.8731

d) avg. number of jobs in process: $\sum_{j=1}^{4} L_j = 65.6994$

e) Throughput time for each part
(Calculated similarly as in the previous problem)

$$W_1 = W_{qc1} + \frac{T_{pic1}}{\text{period time}} + W_{qc2} + \frac{T_{pic2}}{\text{period time}} + W_{qc3} + \frac{T_{pic3}}{\text{period time}} + W_{qc4} + \frac{T_{pic4}}{\text{period time}}$$

$= 1.4846$ weeks

$W_2 = 1.5596$ weeks

$W_3 = 1.5346$ weeks

$W_4 = 1.5346$ weeks

$W_5 = 1.4846$ weeks

Question 12

A five department manufacturing system is used to produce five parts according the data given in table (2). **Find**

(a) Determine the effective arrival rate at each department.
(b) Determine the number of machines at each station.
(c) Find the average work in process.
(d) Find the throughput time.

TABLE (2)

Part	Weekly Demand	Process Sequence	Operation Time, hr					
			Load	Process Station				Unload
			A	B	C	D	E	A
1	6	A → C → B → D → A	0.2	2	1.5	2.1	0	0.2
2	7	A → B → D → E → A	0.2	2.8	0	2.2	1.5	0.2
3	3	A → C → D → A	0.2	0	1.5	1.8	0	0.2
4	5	A → E → B → A	0.2	1.5	0	0	2.2	0.2
5	4	A → B → E → C → A	0.2	2.5	2	0	1.8	0.2

DIY