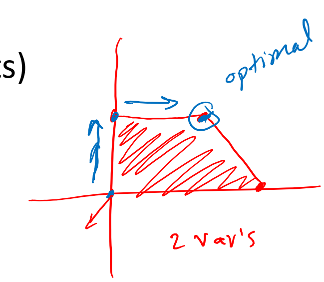
IEE 222 - Tutorial

Tutorial 2: Simplex Method

Steps

$\leq \rightarrow =$

- Convert to standard form (equality constraints)
  - Add slack variables
- Find a starting **BFS** $x's = 0$
- Build the initial tableau (tab0)
- Select entering and leaving variables
- Build the next tableau
  - $\boldsymbol{a'} = \boldsymbol{a} - \frac{\boldsymbol{b.c}}{\boldsymbol{pivot}}$
- Repeat until $\bar{c}(x_i) \leq 0$ for all its non-basic variables (if maximizing)



Problem 1

$$Max\ Z = 3x_1 + 2x_2$$

$$s.t.\ 2x_1 + x_2 \leq 18 \quad + S_1$$

$$2x_1 + 3x_2 \leq 42 \quad + S_2$$

$$3x_1 + x_2 \leq 24 \quad + S_3$$

$$x_1, x_2 \geq 0$$

$$Max\ Z = 3x_1 + 2x_2$$

$$s.t.\ 2x_1 + x_2 \leq 18 \quad ①$$

$$2x_1 + 3x_2 \leq 42 \quad ②$$

$$3x_1 + x_2 \leq 24 \quad ③$$

$$x_1, x_2 \geq 0$$

$$Max\quad Z = 3X_1 + 2X_2 + (0)S_1 + (0)S_2 + (0)S_3$$

Row 1 $\quad 2X_1 + X_2 + S_1 = 18$

Row 2 $\quad 2X_1 + 3X_2 + S_2 = 42$

Row 3 $\quad 3X_1 + X_2 + S_3 = 24$

BFS

Basics: S_1, S_2, S_3

non-b : $x_1, x_2 = 0$

$\div$

Basis	X_1	X_2	S_1	S_2	S_3	RHS	Ratio
S_1	2	1	1	0	0	18	9
S_2	2	3	0	1	0	42	21
S_3	3	1	0	0	1	24	8
	3	2	0	0	0		

pivot

c coef. in obj.

$\bar{c} \leq 0$

x_1 enters, S_3 leaves

$a' = a - \frac{b \cdot c}{\text{pivot}}$

3 ← pivot

→ $b = 2$

$b = 2$

Basis	X_1	X_2	S_1	S_2	S_3	RHS	Ratio
S_1	$2 - \left(\frac{2\times3}{3}\right)$ 0	$1 - \left(\frac{2\times1}{3}\right)$ 1/3	$1 - \left(\frac{2\times0}{3}\right)$ 1	$0 - \left(\frac{2\times0}{3}\right)$ 0	$0 - \left(\frac{2\times1}{3}\right)$ -2/3	$18 - \left(\frac{2\times24}{3}\right)$ 2	6
S_2	$2 - \left(\frac{2\times3}{3}\right)$ 0	$3 - \left(\frac{2\times1}{3}\right)$ 7/3	$0 - \left(\frac{2\times0}{3}\right)$ 0	$1 - \left(\frac{2\times0}{3}\right)$ 1	$0 - \left(\frac{2\times1}{3}\right)$ -2/3	$42 - \left(\frac{2\times24}{3}\right)$ 26	11.14
X_1	1	1/3	0	0	1/3	8	24
	0	1	0	0	-1	-24	

X_2 enters, S_1 leaves

$a' = a - \frac{b \cdot c}{\text{pivot}}$

1/3

$b = 7/3$

$b = 1/3$

$b = 1$

-ve coef, no ratio

Basis	X_1	X_2	S_1	S_2	S_3	RHS	Ratio
X_2	0	1	3	0	-2	6	-
S_2	$0 - \left(\frac{7/3 \times 0}{1/3}\right)$ 0	$\frac{7}{3} - \left(\frac{7/3 \times 1/3}{1/3}\right)$ 0	$0 - \left(\frac{7/3 \times 1}{1/3}\right)$ -7	$1 - \left(\frac{7/3 \times 0}{1/3}\right)$ 1	$\frac{-2}{3} - \left(\frac{7/3 \times -2/3}{1/3}\right)$ 4	$26 - \left(\frac{7/3 \times 2}{1/3}\right)$ 12	3
X_1	$1 - \left(\frac{1/3 \times 0}{1/3}\right)$ 1	$1/3 - \left(\frac{1/3 \times 1/3}{1/3}\right)$ 0	$0 - \left(\frac{1/3 \times 1}{1/3}\right)$ -1	$0 - \left(\frac{1/3 \times 0}{1/3}\right)$ 0	$1/3 - \left(\frac{1/3 \times -2/3}{1/3}\right)$ 1	$8 - \left(\frac{1/3 \times 2}{1/3}\right)$ 6	6
	$0 - \left(\frac{1 \times 0}{1/3}\right)$ 0	$1 - \left(\frac{1 \times 1/3}{1/3}\right)$ 0	$0 - \left(\frac{1 \times 1}{1/3}\right)$ -3	$0 - \left(\frac{1 \times 0}{1/3}\right)$ 0	$-1 - \left(\frac{1 \times -2/3}{1/3}\right)$ 1	$-24 - \left(\frac{1 \times 2}{1/3}\right)$ -30	

S_3 enters, S_2 leaves

pivot = 4

$b = -2$

$b = 1$

$b = 1$

Basis	X_1	X_2	S_1	S_2	S_3	RHS	Ratio
X_2	$0 - \left(\frac{-2\times0}{4}\right)$ 0	$1 - \left(\frac{-2\times0}{4}\right)$ 1	$3 - \left(\frac{-2\times-7}{4}\right)$ -1/2	$0 - \left(\frac{-2\times1}{4}\right)$ 1/2	$-2 - \left(\frac{-2\times4}{4}\right)$ 0	$6 - \left(\frac{-2\times12}{4}\right)$ 12	
S_3	0	0	-7/4	1/4	1	3	
X_1	$1 - \left(\frac{1\times0}{4}\right)$ 1	$0 - \left(\frac{1\times0}{4}\right)$ 0	$-1 - \left(\frac{1\times-7}{4}\right)$ 3/4	$0 - \left(\frac{1\times1}{4}\right)$ -1/4	$1 - \left(\frac{1\times4}{4}\right)$ 0	$6 - \left(\frac{1\times12}{4}\right)$ 3	
	$0 - \left(\frac{1\times0}{4}\right)$ 0	$0 - \left(\frac{1\times0}{4}\right)$ 0	$-3 - \left(\frac{1\times-7}{4}\right)$ -5/4	$0 - \left(\frac{1\times1}{4}\right)$ -1/4	$1 - \left(\frac{1\times4}{4}\right)$ 0	$-30 - \left(\frac{1\times12}{4}\right)$ -33	

$c = 0$ $c = 0$ $c = -7$ $c = 1$ $c = 4$ $c = 12$

all $\bar{c} \leq 0$

$\therefore$ optimal solution

$x_1^* = 3$

$x_2^* = 12$

$obj^* = 33 \rightarrow$ NOT -ve !

Homework

- Provide the optimal solution (value of variables and objective function)

$$
\begin{gathered}
Max\ z = 2x_1 - x_2 + 2x_3 \\
x_1 - x_2 + 2x_3 \leq 20 \\
x_1 + x_2 - 2x_3 \leq 40 \\
x_1, x_2, x_3 \geq 0
\end{gathered}
$$

- Are there multiple optimal solutions?