IE 222 - Tutorial

Tutorial 5: Goal Programming

Camyo Manufacturing produces four parts that require the use of a lathe and a drill press. The two machines operate 10 hours a day. The following table provides the time in minutes required by each part:

Part	Lathe	Drill Press
1	5	3
2	6	2
3	4	6
4	7	4

It is desired to balance the use of the two machines by requiring the difference between their total operation times not to exceed 30 minutes. The market demand limits the number of units produced of each part to at least 10 units. Additionally, the number of units of part 1 may not exceed that of part 2.

DV's: x_i = represent the number of units of part i produced, $i = 1, 2, 3, 4$

Lathe hrs: $T_L = 5x_1 + 6x_2 + 4x_3 + 7x_4 \le 600$

drill press hrs: $T_D = 3x_1 + 2x_2 + 6x_3 + 4x_4 \le 600$

$$2x_1 + 4x_2 - 2x_3 + 3x_4 \le 30$$

$$-2x_1 - 4x_2 + 2x_3 - 3x_4 \le 30$$

$$x_1 \ge 10$$

$$x_2 \ge 10$$

$$x_3 \ge 10$$

$$x_4 \ge 10$$

$$x_1 - x_2 \le 0$$

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T_L, T_D

$|f(x)| \le z$

① $f(x) \le z$

② $-f(x) \le z$

$|T_L - T_D| \le 30$

→ ① $T_L - T_D \le 30$ ✓

② $T_D - T_L \le 30$

deviational variables:

$$5x_1 + 6x_2 + 4x_3 + 7x_4 + S_1^- - S_1^+ = 600 \qquad G_1 = \min S_1^+$$

$$3x_1 + 2x_2 + 6x_3 + 4x_4 + S_2^- - S_2^+ = 600 \qquad G_2 = \min S_2^+$$

$$2x_1 + 4x_2 - 2x_3 + 3x_4 + S_3^- - S_3^+ = 30 \qquad G_3 = \min S_3^+$$

$$-2x_1 - 4x_2 + 2x_3 - 3x_4 + S_4^- - S_4^+ = 30 \qquad G_4 = \min S_4^+$$

$$x_1 + S_5^- - S_5^+ = 10 \qquad G_5 = \min S_5^-$$

$$x_2 + S_6^- - S_6^+ = 10 \qquad G_6 = \min S_6^-$$

$$x_3 + S_7^- - S_7^+ = 10 \qquad G_7 = \min S_7^-$$

$$x_4 + S_8^- + S_8^+ = 10 \qquad G_8 = \min S_8^-$$

$$x_1 - x_2 + S_9^- - S_9^+ = 0 \qquad G_9 = \min S_9^+$$

all variables (x's, S's) ≥ 0

① weights method:

$$\min\ z = w_1 S_1^+ + w_2 S_2^+ + \cdots + w_8 S_8^- + w_9 S_9^+$$

wher w_i : weight of goal i

② preemptive method:

1- rank the goals according to their priorities

2- Solve in steps

Remarks

- Goal programming is not suitable for some problems where constraints **must** be met
  - Remember: in GP we allow constraints to be violated
- If the original i*th* inequality is of the type ≤ and its $s_i^- > 0$, then the i*th* goal is satisfied; otherwise, if $s_i^+ > 0$, goal i is not satisfied.
- In the weights method we write the combined objective function for a problem with n goals as follows:
  - $Minimize \quad z = w_1 G_1 + w_2 G_2 + \cdots + w_n G_n$
  - If the optimum value of z is not zero, then at least one of the goals is not met

Mantel produces a toy carriage, whose final assembly must include four wheels and two seats. The factory producing the parts operates three shifts a day. The following table provides the amounts produced of each part in the three shifts:

	Units produced per run	
Shift	*Wheels*	*Seats*
1	500	300
2	600	280
3	640	360

Ideally, the number of wheels produced is exactly twice that of the number of seats. However, because production rates vary from shift to shift, exact balance in production may not be possible. Mantel is interested in determining the number of production runs in each shift that minimizes the imbalance in the production of the parts. The capacity limitations restrict the number of runs to between 4 and 5 for shift 1, 10 and 20 for shift 2, and 3 and 5 for shift 3. Formulate the problem as a GP model.

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Let x_j : number of production runs in shift j , $j=1,2,3$

each toy carriage has $\underline{\underline{4}}$ wheels & $\underline{\underline{2}}$ seats

wheels-to-seats ratio = $\frac{4}{2}$

$$\frac{500x_1 + 600x_2 + 640x_3}{300x_1 + 280x_2 + 360x_3} = \frac{4}{2} = 2$$

simplify:

$$-100x_1 + 40x_2 - 80x_3 = 0$$

GP model:

$$\min \quad z = s_1^- + s_1^+$$

s.t.

$$-100x_1 + 40x_2 - 80x_3 + s_1^- - s_1^+ = 0$$

$$4 \leq x_1 \leq 5, \quad 10 \leq x_2 \leq 20, \quad 3 \leq x_3 \leq 5$$