IEE 222 - Tutorial

Tutorial 3: LP Modeling (Simple Form)

$11 $9

An oil refinery can buy two types of oil: light crude oil and heavy crude oil. The cost per barrel of these types is respectively $11 and $9. The following quantities of gasoline, kerosene, and jet fuel are produced per barrel of each type of oil.

	Gasoline	Kerosene	Jet Fuel
Light crude oil x_1	0.4 +	0.2 +	0.35
Heavy crude oil x_2	0.32 +	0.4 +	0.2

Note that 5% and 8% of the crude are lost respectively during the refining process. The refinery has contracted to deliver 1 million barrels of gasoline, 400,000 barrels of kerosene, and 250,000 barrels of jet fuel. Formulate the problem of finding the number of barrels of each crude oil that satisfy the demand and minimize the total cost as a linear program.

Let x_1 : # barrels of light crude oil

x_2 : " " " heavy " "

$$\boxed{\min \quad \boxed{11x_1 + 9x_2}}$$

gasoline demand $\quad 0.4x_1 + 0.32x_2 \geq 1{,}000{,}000$

kerosene $\quad 0.2x_1 + 0.4x_2 \geq 400{,}000$

jet fuel $\quad 0.35x_1 + 0.2x_2 \geq 250{,}000$

$$x_1, x_2 \geq 0$$

A(1)

F(1) S(1)

A company manufactures an assembly consisting of a frame, a shaft, and a ball bearing. The company manufactures the shafts and frames but purchases the ball bearings from a ball bearing manufacturer. Each shaft must be processed on a forging machine, a lathe, and grinder. These operations require 0.5 hours, 0.2 hours, and 0.3 hours per shaft, respectively. Each frame requires 0.8 hours on a forging machine, 0.1 hours on a drilling machine, 0.3 hours on a milling machine, and 0.5 hours on a grinder. The company has 5 lathes, 10 grinders, 20 forging machines, 3 drillers, and 6 millers. Assume that each machine operates a maximum of 2400 hours per year. Formulate the problem of finding the maximum number of assembled components that can be produced as a linear program.

0.5 0.2 0.3

Let x_1 : # shafts

x_2 : # frames

z : # assemblies

Max z

→ lathe hrs $\quad 5 \times 2400 = 12{,}000$ hrs

grinders $\quad 10 \times 2400 = 24{,}000$

forging $\quad 20 \times 2400 = 48{,}000$

drilling $\quad 3 \times 2400 = 7200$

milling $\quad 6 \times 2400 = 14{,}200$

s.t.

Lathe	$0.2X_1 \le 12{,}000$
grind.	$0.3X_1 + 0.5X_2 \le 24{,}000$
forg.	$0.5X_1 + 0.8X_2 \le 48{,}000$
drill.	$0.1X_2 \le 7{,}200$
mill.	$0.3X_2 \le 14{,}200$

$$Z \le X_1$$
$$Z \le X_2$$
$$X_1, X_2 \ge 0$$

Comp.	lathe	grind	Forg.	drill	mill
x_1	0.2	0.3	0.5	–	–
x_2		0.5	0.8	0.1	0.3

A manufacturer of plastics is planning to blend a new product from four chemical compounds. These compounds are mainly composed of three elements A, B, and C. The composition and unit cost of these chemicals are shown below.

Chemical Compound	1	2	3	4
% of A	30	20	40	20
% of B	20	60	30	40
% of C	40	15	25	30
Cost/kilogram	20	30	20	15
	x_1	x_2	x_3	x_4

The new product consists of 20% element A, at least 30% element B, and at least 20% element C. Owing to side effects of compounds 1 and 2, they must not exceed 30% and 40% of the content of the new product. Formulate the problem of finding the least costly way of blending as a linear program.

Decision Variables:

- X1: amount of compound 1 used to form 1 kg of the product
- X2: amount of compound 2 used to form 1 kg of the product
- X3: amount of compound 3 used to form 1 kg of the product
- X4: amount of compound 4 used to form 1 kg of the product

Chemical Compound	1	2	3	4
% of A	30	20	40	20
% of B	20	60	30	40
% of C	40	15	25	30
Cost/kilogram	20	30	20	15

min $\quad 20x_1 + 30x_2 + 20x_3 + 15x_4$

% A $\quad 0.3x_1 + 0.2x_2 + 0.4x_3 + 0.2x_4 = 0.2$

at least 30% % B $\quad 0.2x_1 + 0.6x_2 + 0.3x_3 + 0.4x_4 \geq 0.3$

20% % C $\quad 0.4x_1 + 0.15x_2 + 0.25x_3 + 0.3x_4 \geq 0.2$

$x \leq 30\%$

$x_1 \leq 0.3$

$x_2 \leq 0.4$

$x_1, x_2, x_3, x_4 \geq 0$

A steel manufacturer produces four sizes of I beams: small, medium, large, and extra large  . These beams can be produced on any one of three machine types: A, B, and C. The lengths in feet of the I beams that can be produced on the machines per hour are summarized below.

	Machine		
Beam	**A**	**B**	**C**
Small 1	300 x_{11}	600	800
Medium 2	250	400	700
Large 3	200	350	600
Extra Large 4	100	200	300

Assume that each machine can be used up to 50 hours per week and that the hourly operating costs of these machines are respectively $30, $50, and $80. Further suppose that 10,000, 8,000, 6,000, and 6,000 feet of the different-size I beams are required weekly. Formulate the machine scheduling problem as a linear program.

x_{11} : # hrs ...

x_2

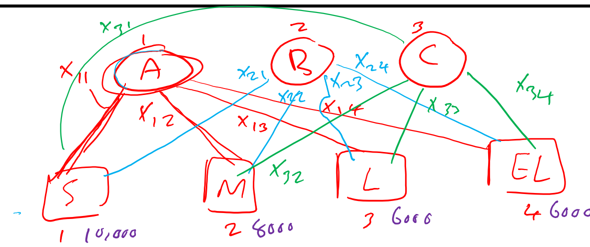
x_3

machine

X_{11}: number of hours of M/C 1 on beam size 1
X_{12}: number of hours of M/C 1 on beam size 2
X_{13}: number of hours of M/C 1 on beam size 3
X_{14}: number of hours of M/C 1 on beam size 4
\$30

X_{21}: number of hours of M/C 2 on beam size 1
X_{22}: number of hours of M/C 2 on beam size 2
X_{23}: number of hours of M/C 2 on beam size 3
X_{24}: number of hours of M/C 2 on beam size 4
\$50

X_{31}: number of hours of M/C 3 on beam size 1
X_{32}: number of hours of M/C 3 on beam size 2
X_{33}: number of hours of M/C 3 on beam size 3
X_{34}: number of hours of M/C 3 on beam size 4
\$80



min $30(x_{11}+x_{12}+x_{13}+x_{14}) + 50(x_{21}+x_{22}+x_{23}+x_{24}) + 80(x_{31}+x_{32}+x_{33}+x_{34})$

M/C availability constraints

$$\begin{cases} x_{11}+x_{12}+x_{13}+x_{14} \le 50 \\ x_{21}+x_{22}+x_{23}+x_{24} \le 50 \\ x_{31}+x_{32}+x_{33}+x_{34} \le 50 \end{cases}$$

demand constraints

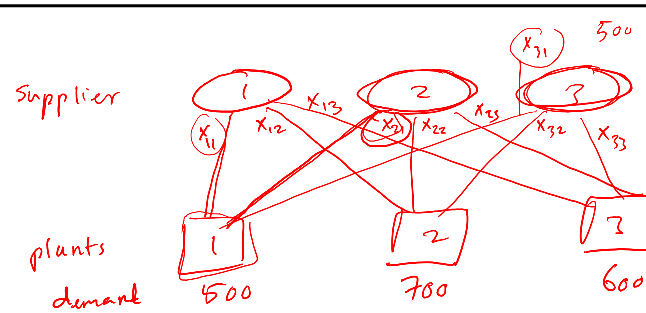
Size	
1	$300x_{11} + 600x_{21} + 800x_{31} \ge 10{,}000$
2	$250x_{12} + 400x_{22} + 700x_{32} \ge 8000$
3	$200x_{13} + 350x_{23} + 600x_{33} \ge 6000$
4	$100x_{14} + 200x_{24} + 300x_{34} \ge 6000$

$x_{11}, x_{12}, x_{13}, \ldots, x_{34} \ge 0$

A furniture manufacturer has three plants, which need 500, 700, and 600 tons of lumber weekly.  The manufacturer may purchase the lumber from three lumber companies. The first two lumber manufacturers virtually have unlimited supply, and because of other commitments the third manufacturer cannot ship more than 500 tons weekly. The first lumber manufacturer uses rail for transportation and there is no limit on tonnage that can be shipped to the furniture facilities. On the other hand, the last two lumber companies use trucks that limit the maximum tonnage that can be shipped to any of the furniture companies to 200 tons. The following table gives the transportation cost from the lumber companies to the furniture manufacturers ($ per ton).

Lumber	Furniture Facility		
Company	**1**	**2**	**3**
1	2	3	5
2	2.5	4	4.8
3	3	3.6	3.2

Formulate the problem as a linear program.



X_{11}: number of tons to be shipped from lumber company 1 to plant 1
X_{12}: number of tons to be shipped from lumber company 1 to plant 2
X_{13}: number of tons to be shipped from lumber company 1 to plant 3

X_{21}: number of tons to be shipped from lumber company 2 to plant 1
X_{22}: number of tons to be shipped from lumber company 2 to plant 2
X_{23}: number of tons to be shipped from lumber company 2 to plant 3

X_{31}: number of tons to be shipped from lumber company 3 to plant 1
X_{32}: number of tons to be shipped from lumber company 3 to plant 2
X_{33}: number of tons to be shipped from lumber company 3 to plant 3

min

transprt. costs

Lumber Company	Furniture Facility 1	2	3
1	2 x_{11} +	3 x_{12} +	5 x_{13}
2	+ 2.5 x_{21} +	4 x_{22} +	4.8 x_{23}
3	+ 3 x_{31} +	3.6 x_{32} +	3.2 x_{33}

demand const

$$x_{11} + x_{21} + x_{31} \geq 500$$
$$x_{12} + x_{22} + x_{32} \geq 700$$
$$x_{13} + x_{23} + x_{33} \geq 600$$

$$x_{31} + x_{32} + x_{33} \leq 500$$

$x_{21} \leq 200$, $x_{31} \leq 200$

$x_{22} \leq 200$, $x_{32} \leq 200$

$x_{23} \leq 200$, $x_{33} \leq 200$

all x's ≥ 0 $\quad x_{11}, x_{12}, x_{13}, x_{21}, \ldots x_{33} \geq 0$

Homework

The technical staff of a hospital wishes to develop a computerized menu-planning system. To start with, a lunch menu is sought. The menu is divided into three major categories: vegetables, meat, and dessert. At least one equivalent serving of each category is desired. The cost per serving of some suggested items as well as their content of carbohydrates, vitamins, protein, and fats is summarized in the next slide.

Suppose that the minimal requirements of carbohydrates, vitamins, protein, and fats per meal are respectively 5, 10, 10, and 2. Formulate the menu-planning problem as a linear program.

	Carbs	Vitamins	Protein	Fats	Cost in $/serving
Vegetables					
Peas	1	3	1	0	0.1
Green beans	1	5	2	0	0.12
Okra	1	5	1	0	0.13
Corn	2	6	1	2	0.09
Macaroni	4	2	1	1	0.1
Rice	5	1	1	1	0.07
Meat					
Chicken	2	1	3	1	0.7
Beef	3	8	5	2	1.2
Fish	3	6	6	1	0.63
Dessert					
Orange	1	3	1	0	0.28
Apple	1	2	0	0	0.42
Pudding	1	0	0	0	0.15
Jello	1	0	0	0	0.12