IE 222 - Tutorial

Tutorial 4: Modeling with binary variables

References:
- Ravindran, A. R., & Warsing Jr, D. (2016). *Supply chain engineering: Models and applications*. CRC Press.
- Chopra, S., Meindl, P., & Kalra, D. V. (2013). *Supply chain management: Strategy, planning, and operation* (Vol. 232). Boston, MA: Pearson.

Capital Budgeting Problem

- A company is planning its capital spending for the next T periods
- N: number of projects j
- B_i: capital available in period i
- aij: required investment in project j in period i
- vj: net present value of project j
- **Problem**: select projects that maximize the total value

DV's : Let $x_j = 1$, if project j is selected
$x_j = 0$ otherwise

$$\max \quad z = \sum_{j=1}^{N} v_j x_j$$

s.t.

$$\sum_{j=1}^{N} a_{ij} x_j \le B_i \qquad \forall\ i = 1, 2, \ldots, T$$

$$x_j \in \{0, 1\} \qquad \forall\ j = 1, 2, \ldots, N$$

Cost

Fixed Charge Problem

- Consider a production planning problem with N products.
- jth product requires a *fixed* production/setup cost Kj
  - Cj: variable cost of product j (per unit)
- aij: amount of resource i required by every unit of product j
- bi: availability limit for resource i
- dj: demand for product j
- pj: selling price of product j (per unit)
- **Problem**: determine optimal product mix that maximizes the net profit

profit = revenues − costs

DV's: x_j: quantity of product j produced

$$y_j = \begin{cases} 1, & \text{if product } j \text{ is produced} \\ 0 & \text{otherwise} \end{cases}$$

max $z = \sum_{j=1}^{N} p_j x_j - \sum_{j=1}^{N} (C_j x_j + K_j y_j)$

resource constraints $\sum_{j=1}^{N} a_{ij} x_j \le b_i \quad \forall i$

demand Constraints $x_j \le d_j y_j \quad \forall j = 1, 2, \dots, N$

$x_j \ge 0, \quad y_j \in \{0, 1\} \quad \forall j$

Constraint with Multiple Right-Hand-Side Constants

- Consider a problem where the constraint $(a_1x_1 + a_2x_2 + \ldots + a_nx_n)$ must be less than or equal to *one* of the RHS values b_1, b_2, b_3
  - $a_1x_1 + a_2x_2 + \ldots + a_nx_n \le b_1, b_2,$ **or** b_3
- Example: choosing among three warehouses with different capacities
  - xi: quantity of product i to be stored
  - ai: square footage occupied by one unit of product i
  - b_1, b_2, b_3: capacities of the three potential warehouses

$$y_i = \begin{cases} 1, & \text{if } W_j \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

$a_1x_1 + a_2x_2 + \dots + a_nx_n \le y_1b_1 + y_2b_2 + y_3b_3$

$y_1 + y_2 + y_3 = 1 \rightarrow$ only one is selected

$y_1 + y_2 + y_3 \le 1 \rightarrow$ either one is selected or nothing at all

Logical Constraints

- Refer to:
  - https://github.com/oalotaik/supply_chains_optimization/blob/master/03_logical_constraints.ipynb

$x \in \{0,1\}$

Other logical constraints

Logical Constraint	Constraint
If item *i* is selected, then item *j* is also selected.	$x_i - x_j \leq 0$
Either item *i* is selected or item *j* is selected, but not both.	$x_i + x_j = 1$
If item *i* is selected, then item *j* is not selected.	$x_i - x_j \leq 1$
If item *i* is not selected, then item *j* is not selected.	$-x_i + x_j \leq 0$
At most one of items *i*, *j*, and *k* are selected.	$x_i + x_j + x_k \leq 1$

Problem

Assume the data is from a car manufacture optimizing its Supply Chain network across five regions (i.e. USA, Germany, Japan, Brazil, and India). You are given the demand, manufacturing capacity (thousands of cars) for each region, and the variable and fixed costs (thousands of $US dollars). The variable costs represents the costs of producing in location i and shipping to location j.

Context

Multiple options to meet regional product demand

Option	Pro	Con
Small manufacturing facilities within region	Low transportation costs, few to no tariffs or duties	Overall network may have excess capacity, cannot take advantage economies of scale
A few large manufacturing plants and ship product to region	Economies of scale	Higher transportation, higher tariffs and duties

Capacitated plant location model

- Capacitated Plant Location Model[1]
- The goal is to optimize global Supply Chain network
  - Meet regional demand at the lowest cost
  - Determine regional production of a product

Capacitated plant location model

Modeling

- Production at regional facilities
  - Two plant sizes (low / high)
- Exporting production to other regions
- Production facilities open / close

Demand

	Dmd
USA	2719.6
Germany	84.1
Japan	1676.8
Brazil	145.4
India	156.4

Fixed Costs

	Low_Cap	High_Cap
USA	6500	9500
Germany	4980	7270
Japan	6230	9100
Brazil	3230	4730
India	2110	3080

Variable Costs

	USA	Germany	Japan	Brazil	India
USA	6	13	20	12	17
Germany	13	6	14	14	13
Japan	20	14	3	21	9
Brazil	12	14	21	8	21
India	22	13	10	23	8

Capacities

	Low_Cap	High_Cap
USA	500	1500
Germany	500	1500
Japan	500	1500
Brazil	500	1500
India	500	1500

Decision variables

What we can control:

- x_{ij} = quantity produced at location *i* and shipped to *j*
- y_{is} = 1 if the plant at location *i* of capacity *s* is open, 0 if closed
  - s = *low* or *high* capacity plant

Let:

- c_{ij} = cost of producing and shipping from plant *i* to region *j* (see previous slide: variable costs)
- f_{is} = fixed cost of keeping plant *i* of capacity *s* open (see previos slide: fixed costs)
- n = number of production facilities $i = 1, 2, \dots, n$
- m = number of markets or regional demand points $j = 1, 2, \dots, m$
- d_j = demand in market j
- K_{is} = capacity of plant i
  s = low or high capacity

Objective function:

two capacities in each location

$$\min \quad z = \sum_i \sum_s f_{is} y_{is} + \sum_i \sum_j c_{ij} x_{ij}$$

s.t. :

(demand) $$\sum_i x_{ij} \geq d_j \qquad \forall \; j = 1, 2, \dots, m$$

(capacity) $$\sum_j x_{ij} \leq \sum_s K_{is} y_{is} \qquad \forall \; i = 1, 2, \dots, n$$

two capacities

$$y_{is} \in \{0, 1\}, \qquad x_{ij} \geq 0$$