Problem Set

<u>Problem 1</u>

A small foundry needs to schedule the production of four different castings during the next week. The production requirements of each casting are summarized in the following table:

Product	Unit production times (minutes)				
	Pouring	Cleaning	Grinding	Inspection	Packing
A	3	8	10	1	3
B	1	12	6	1	5
C	2	6	9	1	3
D	1	7	7	1	2

The unit profit for products A, B, C, and D are $18, $15, $13, and $14, respectively. Current demands indicate that all castings that are made can be sold; however, contracts dictate that at least 200 units of Product A and 300 units of Product D must be produced. The estimated times available for each of the operations during the next week are:

Pouring	40 hours
Cleaning	80 hours
Grinding	80 hours
Inspection	20 hours
Packing	40 hours

Determine how many of each of the products should be produced next week to maximize the total profit. Formulate the linear model for this problem; clearly define the variables, and state the objective function and constraints.

Solution:

First of all, note that unit production times were given in (minutes), while operations available times were given in hours.

Decision variables:

- X_1: amount of Product A to be manufactured next week
- X_2: amount of Product B to be manufactured next week
- X_3: amount of Product C to be manufactured next week
- X_4: amount of Product D to be manufactured next week

LP model:

$$\text{Max} \quad 18x_1 + 15x_2 + 13x_3 + 14x_4$$

Subject to:

$$3x_1 + x_2 + 2x_3 + x_4 \leq 2400$$

$$8x_1 + 12x_2 + 6x_3 + 7x_4 \leq 4800$$

$$10x_1 + 6x_2 + 9x_3 + 7x_4 \leq 4800$$

$$x_1 + x_2 + x_3 + x_4 \leq 1200$$

$$3x_1 + 5x_2 + 3x_3 + 2x_4 \leq 2400$$

$$x_1 \geq 200$$

$$x_4 \geq 300$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Problem 2

An agriculture mill manufactures feed for cattle, sheep, and chickens. This is done by mixing the following main ingredients: corn, limestone, soybeans, and fish meal. These ingredients contain the following nutrients: vitamins, protein, calcium, and crude fat. The contents of the nutrients in each kilogram of the ingredients is summarized below:

Ingredient	Nutrient			
	Vitamins	Protein	Calcium	Crude fat
Corn	3	8	10	1
Limestone	1	12	6	1
Soybeans	2	6	9	1
Fish meal	1	7	7	1

The mill contracted to produce 10, 6, and 8 tons of cattle feed, sheep feed, and chicken feed. Because of shortages, a limited amount of the ingredients is available-namely, 6 tons of corn, 10 tons of limestone, 4 tons of soybean, and 5 tons of fish meal. The price per kilogram of these ingredients is respectively \$0.2, \$0.12, \$0.24, and \$0.12. The minimal and maximal units of the various nutrients that are permitted is summarized below for a kilogram of the cattle feed, the sheep feed, and the chicken feed.

Product	Nutrient							
	Vitamins		Protein		Calcium		Crude fat	
	Min	Max	Min	Max	Min	Max	Min	Max
Cattle feed	6	∞	6	∞	7	∞	4	8
Sheep feed	6	∞	6	∞	6	∞	4	6
Chicken feed	4	6	6	∞	6	∞	4	6

Formulate this feed-mix problem so that the total cost is minimized.

Solution:

Let x_{ij}: amount of ingredient i for product j.

$i = 1,2,3,4$ corn, limestone, soybean, fish-meal

$j = 1,2,3$ cattle, sheep, chicken

$$\min \quad 0.2\left(\sum_{j=1}^{3} x_{1j}\right) + 0.12\left(\sum_{j=1}^{3} x_{2j}\right) + .24\left(\sum_{j=1}^{3} x_{3j}\right) + .12\left(\sum_{j=1}^{3} x_{4j}\right)$$

S.T.

$$\sum_{i=1}^{4} x_{i1} \geq 10$$

$$\sum_{i=1}^{4} x_{i2} \geq 6$$

$$\sum_{i=1}^{4} x_{i3} \geq 8$$

$$\sum_{j=1}^{3} x_{1j} \leq 6$$

$$\sum_{j=1}^{3} x_{2j} \leq 10$$

$$\sum_{j=1}^{3} x_{3j} \leq 4$$

$$\sum_{j=1}^{3} x_{4j} \leq 5$$

$$8x_{11} + 6x_{21} + 10x_{31} + 4x_{41} \geq 6(x_{11} + x_{21} + x_{31} + x_{41})$$

$$10x_{11} + 5x_{21} + 12x_{31} + 8x_{41} \geq 6(x_{11} + x_{21} + x_{31} + x_{41})$$

$$6x_{11} + 10x_{21} + 6x_{31} + 6x_{41} \geq 7(x_{11} + x_{21} + x_{31} + x_{41})$$

$$8(x_{11}+x_{21}+x_{31}+x_{41}) \geq 8x_{11}+6x_{21}+6x_{31}+9x_{41} \geq 4(x_{11}+x_{21}+x_{31}+x_{41})$$

$$8x_{12}+6x_{22}+10x_{32}+4x_{42} \geq 6(x_{12}+x_{22}+x_{32}+x_{42})$$

$$10x_{12}+5x_{22}+12x_{32}+8x_{42} \geq 6(x_{12}+x_{22}+x_{32}+x_{42})$$

$$6x_{12}+10x_{22}+6x_{32}+6x_{42} \geq 6(x_{12}+x_{22}+x_{32}+x_{42})$$

$$6(x_{12}+x_{22}+x_{32}+x_{42}) \geq 8x_{12}+6x_{22}+6x_{32}+6x_{42} \geq 4(x_{12}+x_{22}+x_{32}+x_{42})$$

$$6(x_{13}+x_{23}+x_{33}+x_{43}) \geq 8x_{13}+6x_{23}+10x_{33}+4x_{43} \geq 4(x_{13}+x_{23}+x_{33}+x_{43})$$

$$10x_{13}+5x_{23}+12x_{33}+8x_{43} \geq 6(x_{13}+x_{23}+x_{33}+x_{43})$$

$$6x_{13}+10x_{23}+6x_{33}+6x_{43} \geq 6(x_{13}+x_{23}+x_{33}+x_{43})$$

$$6(x_{13}+x_{23}+x_{33}+x_{43}) \geq 8x_{13}+6x_{23}+6x_{33}+9x_{43} \geq 4(x_{13}+x_{23}+x_{33}+x_{43})$$

$$x_{ij} \geq 0 \qquad i=1,2,3,4, \quad j=1,2,3.$$

Problem 3

A corporation has $30 million available for the coming year to allocate to its three subsidiaries. Because of commitments to stability of personnel employment and for other reasons, the corporation has established a minimal level of funding for each subsidiary. These funding levels are $3 million, $5 million, and $8 million, respectively. Owing to the nature of its operation, subsidiary 2 cannot utilize more than $17 million without major new capital expansion. The corporation is unwilling to undertake such expansion at this time. Each subsidiary has the opportunity to conduct various projects with the funds it receives. A rate of return (as a percent of the investment) has been established for each project. In addition, certain of the projects permit only limited investment. The data for each project are given below.

Subsidiary	Project	Rate of return	Upper limit of investment
1	1	8%	$6 million
	2	6%	$5 million
	3	7%	$9 million
2	4	5%	$7 million
	5	8%	$10 million
	6	9%	$4 million
3	7	10%	$6 million
	8	6%	$3 million

Formulate this problem as a linear program.

Solution:

Decision variables:

- X_1: investment in project 1 (in millions of \$)
- X_2: investment in project 2 (in millions of \$)
- X_3: investment in project 3 (in millions of \$)
- X_4: investment in project 4 (in millions of \$)
- X_5: investment in project 5 (in millions of \$)
- X_6: investment in project 6 (in millions of \$)
- X_7: investment in project 7 (in millions of \$)
- X_8: investment in project 8 (in millions of \$)

LP model:

$$\text{Max} \quad 0.08x_1 + 0.06x_2 + 0.07x_3 + 0.05x_4 + 0.08x_5 + 0.09x_6 + 0.1x_7 + 0.06x_8$$

Subject to:

$$x_1 + x_2 + x_3 \geq 3 \text{ (subsidiary 1 min funding)}$$

$$17 \geq x_4 + x_5 + x_6 \geq 5 \text{ (subsidiary 2 min and max funding)}$$

$$x_7 + x_8 \geq 8 \text{ (subsidiary 3 min funding)}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \leq 30 \text{ (redundant)}$$

$$0 \leq x_1 \leq 6$$

$$0 \leq x_2 \leq 5$$

$$0 \leq x_3 \leq 9$$

$$0 \leq x_4 \leq 7$$

$$0 \leq x_5 \leq 10$$

$$0 \leq x_6 \leq 4$$

$$0 \leq x_7 \leq 6$$

$$0 \leq x_8 \leq 3$$