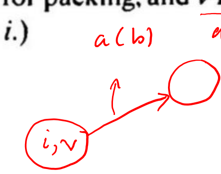
IE 222 - Tutorial

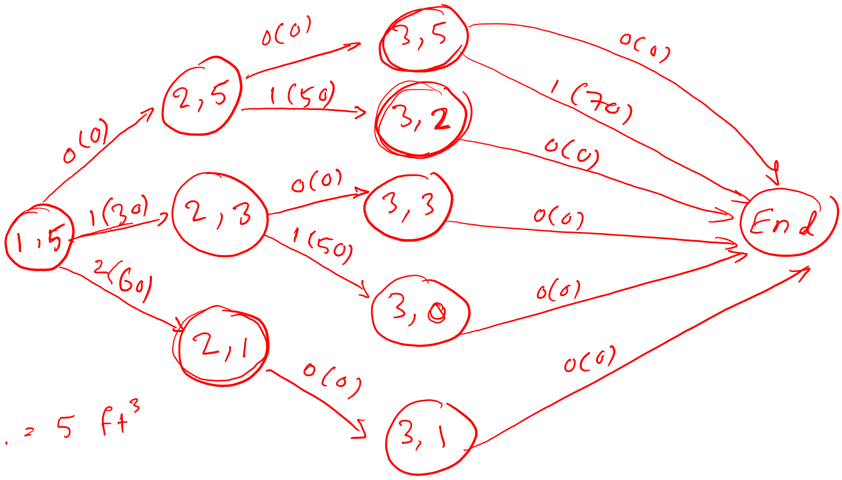
Tutorial 6 - Network Models

***4.** *Knapsack Problem.* A hiker has a 5-ft^3 backpack and needs to decide on the most valuable items to take on the hiking trip. There are three items from which to choose. Their volumes are 2, 3, and 4 ft^3, and the hiker estimates their associated values on a scale from 0 to 100 as 30, 50, and 70, respectively. Express the problem as longest-route network, and find the optimal solution. (*Hint*: A node in the network may be defined as $[i, v]$, where i is the item number considered for packing, and v is the volume remaining immediately before a decision is made on i.)



a: # units of item i
b: value of item i

node (i,v) : i : is the item number
v : volume remaining before item i is selected

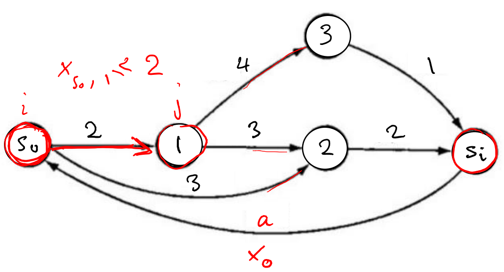


total available vol. = 5 ft^3

i	1	2	3
volume/unit	2	3	4
value/unit	30	50	70

Maximum Flow

Sunco Oil wants to ship the maximum possible amount of oil (per hour) via pipeline from node *so* to node *si* in Figure 6. On its way from node *so* to node *si*, oil must pass through some or all of stations 1, 2, and 3. The various arcs represent pipelines of different diameters. The maximum number of barrels of oil (millions of barrels per hour) that can be pumped through each arc is shown in Table 8. Each number is called an **arc capacity.** Formulate an LP that can be used to determine the maximum number of barrels of oil per hour that can be sent from *so* to *si*.



Arc Capacities for Sunco Oil

Arc	Capacity
(*so*, 1)	2
(*so*, 2)	3
(1, 2)	3
(1, 3)	4
(3, *si*)	1
(2, *si*)	2

x_{ij} = millions of barrels of oil per hour pass through arc (i,j)

① arc capacity : $0 \leq \overset{\text{arc}}{\text{flow}} \leq$ capacity

② conservation-of-flow   flow into node i = Flow out of node i

(arc capacity)

$x_{so,1} \leq 2$

$x_{so,2} \leq 3$

$x_{12} \leq 3$

$x_{2,si} \leq 2$

$x_{13} \leq 4$

$x_{3,si} \leq 1$

$x_0 = x_{so,1} + x_{so,2}$ (Node s_o flow constraint)

$x_{so,1} = x_{12} + x_{13}$ " 1 " "

$x_{12} + x_{so,2} = x_{2,si}$ " 2 " "

$x_{13} = x_{3,si}$ " 3 " "

$x_{3,si} + x_{2,si} = x_0$ " s_i " "

objective : max $z = x_0$

$x_{ij} \geq 0$