ECE 340 Lab 3

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Section D21

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1 Z-Transform

A causal linear filter has the impulse response given by:

$$h[n] = n(0.5)^n \sin(\frac{\pi n}{6})u[n] \tag{1}$$

To get the impulse response $h_1[n]$ for $0 \leq n \leq 10$ the following MATLAB script was used:

```
figure(1);

% Plot the impulse response h_1[n]
n = 0:10;
h1 = n .* (0.5).^n .* sin((pi .* n)/6);
stem(n, h1, 'LineWidth', 1, 'Color', 'r');
title('Plot of h_1[n] for 0 \leq n \leq 10.');
xlabel('n');
ylabel('h_1[n]');
```

After running the script the following output was produced:

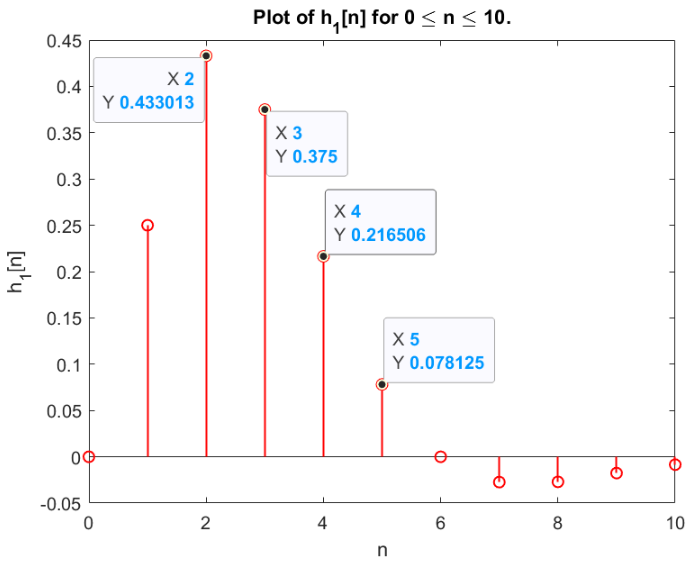

Figure 1: Plot of $h_1[n]$ for $0 \leq n \leq 10$.

Using the $\mathcal{Z}$-Transform pair:

$$\mathcal{Z}\{a^n \sin(\omega_0 n)u[n]\} = \frac{az^{-1}\sin(\omega_0)}{1 - 2az^{-1}\cos(\omega_0) + a^2z^{-2}}, \text{ ROC: } |z| > |a| \tag{2}$$

and the property:

$$\mathcal{Z}\{nx[n]\} = -z\frac{dX(z)}{dz} \tag{3}$$

the $H(z)$ of the system was determined through the following calculations:

$$\begin{aligned}\mathcal{Z}\{h[n]\} &= \mathcal{Z}\{n(0.5)^n \sin(\tfrac{\pi n}{6})u[n]\} \\ H(e^{j\omega}) &= -z\tfrac{d}{dz}\mathcal{Z}\{(0.5)^n \sin(\tfrac{\pi n}{6})u[n]\} \\ &= -z\tfrac{d}{dz}\left(\tfrac{(0.5)z^{-1}\sin(\pi/6)}{1-2(0.5)z^{-1}\cos(\pi/6)+(0.5)^2z^{-2}}\right) \\ &= \tfrac{4z^{-1}-z^{-3}}{16-16\sqrt{3}z^{-1}+20z^{-2}-4\sqrt{3}z^{-3}+z^{-4}}\end{aligned}$$

To get the impulse response of $H(z)$, $h_2[n]$ for $0 \leq n \leq 10$ the following script is used:

```
figure(2);

% Get the frequency response of H(z) h2_[n]
N = [0 4 0 -1];
D = [16 -16*sqrt(3) 20 -4*sqrt(3) 1];

x = zeros(1, 11);
x(1) = 1; % Creates the impulse response

h2 = filter(N, D, x);
stem(n, h2, 'LineWidth' , 1, 'Color', 'b');
title('Plot of Impulse Reponse, h_2[n], of H(z) for 0 \leq n \leq 10.');
xlabel('n');
ylabel('h_2[n]');
```

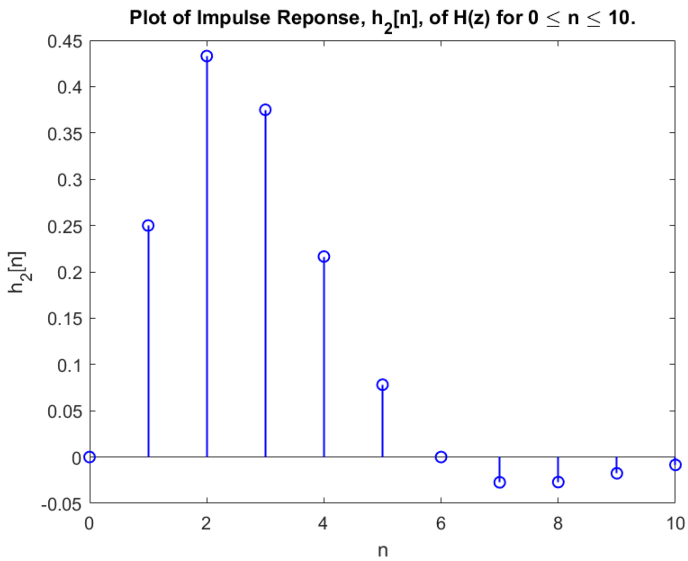

Figure 2: Plot of Impulse Reponse, $h_2[n]$, of $H(z)$ for $0 \leq n \leq 10$.

To compare $h_1[n]$ and $h_2[n]$ on the same figure, the following script was ran:

```
figure(3);

% Compare h_1[n] and h_2[n] on the same figure
stem(n, h1, 'LineWidth', 1, 'Color', 'r');
hold on;
stem(n, h2, 'LineWidth' , 1, 'Color', 'b', 'LineStyle', '--');
title('Plot of h_1[n] and h_2[n] for 0 \leq n \leq 10.');
xlabel('n');
ylabel('h_1[n] and h_2[n]');
legend('h_1[n]', 'h_2[n]');
```

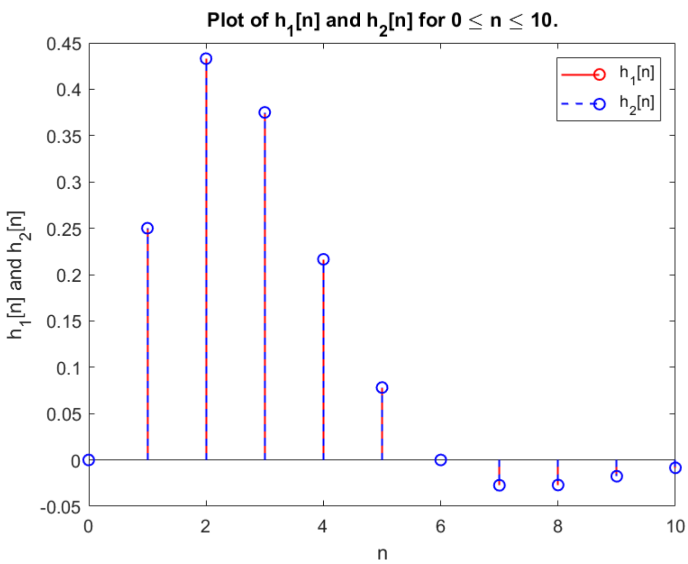

Figure 3: Plot of $h_1[n]$ (solid red lines) and $h_2[n]$ (dotted blue lines) $0 \leq n \leq 10$.

It is evident from the plot above that $h_2[n]$ is identical to $h_1[n]$. This shows that the filtered function is the same as the orginal signal, confirming our calculations for $H(z)$.

2 Inverse Z-Transform & Pole-Zero Diagrams

Given the transfer functions for two causal LTI systems:

$$H_1(z) = \frac{2 + 2z^{-1}}{1 - 1.25z^{-1}} \tag{4}$$

$$H_2(z) = \frac{2 + 2z^{-1}}{1 - 0.8z^{-1}} \tag{5}$$

Their plot-zero diagrams can be plotted using the following MATLAB script:

```
figure(1);

% Plot pole-zero diagram for H1(z)
subplot(2, 1, 1);
N1 = [2 2];
D1 = [1 -1.25];
[z1, p1] = tf2zpk(N1, D1);

zplane(z1, p1);
title('Pole-Zero Diagram of H_1(z).');
xlabel('Re{H_1(z)}');
ylabel('Im{H_1(z)}');

% Plot pole-zero diagram for H1(z)
subplot(2, 1, 2);
N2 = [2 2];
D2 = [1 -0.8];
[z2, p2] = tf2zpk(N2, D2);

zplane(z2, p2);
title('Pole-Zero Diagram of H_2(z).');
xlabel('Re{H_2(z)}');
ylabel('Im{H_2(z)}');
```

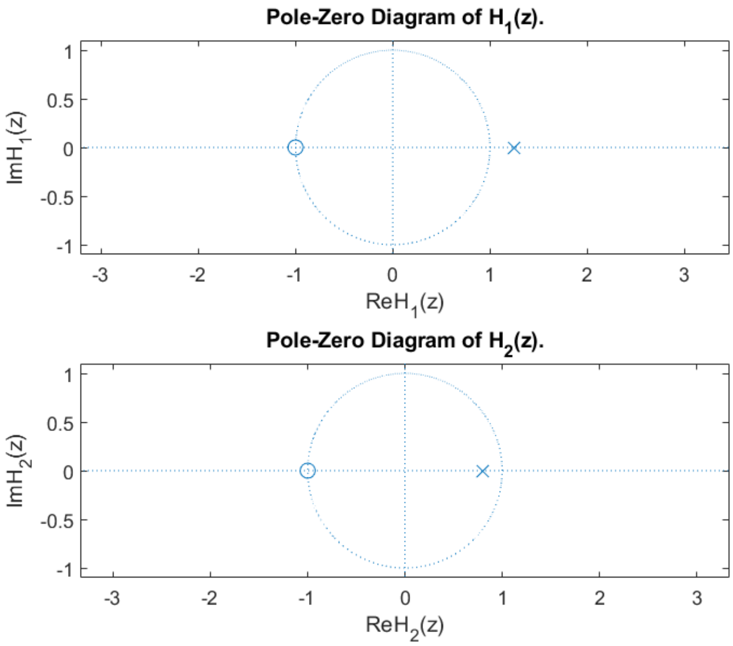

Figure 4: Pole-Zero diagrams of $H_1(z)$ (top) and $H_2(z)$ (bottom).

From the plots above, it is clear that $H_1(z)$ has a pole outside of the ROC (region of convergence). This indicates that $H_1(z)$ is not BIBO stable. While for $H_2(z)$ it's pole is inside the ROC indicating that is it is BIBO stable.

To plot the magnitude and phase of the frequency responses of the two systems in range $0 \leq \omega \leq 2\pi$, the following script was used:

```
figure(2);

% Plot the magnitude and phase of H1(z)
[H1, w1] = freqz(N1, D1, 'whole');
mag_H1 = abs(H1);
phase_H1 = angle(H1);

subplot(2, 2, 1);
plot(w1/pi, mag_H1, 'LineWidth' , 1, 'Color', 'r');
title('Plot of |H_1(z)|.');
xlabel('\omega');
ylabel('|H_1(z)|');

subplot(2, 2, 2);
plot(w1/pi, phase_H1, 'LineWidth' , 1, 'Color', 'r');
title('Plot of \angle H_1(z).');
xlabel('\omega');
ylabel('\angle H_1(z)');

% Plot the magnitude and phase of H2(z)
[H2, w2] = freqz(N2, D2, 'whole');
mag_H2 = abs(H2);
phase_H2 = angle(H2);

subplot(2, 2, 3);
plot(w2/pi, mag_H2, 'LineWidth' , 1, 'Color', 'b');
title('Plot of |H_2(z)|.');
xlabel('\omega');
ylabel('|H_2(z)|');
```

```
subplot(2, 2, 4);
plot(w2/pi, phase_H2, 'LineWidth' , 1, 'Color', 'b');
title('Plot of \angle H_1(z).');
xlabel('\omega');
ylabel('\angle H_2(z)');
```

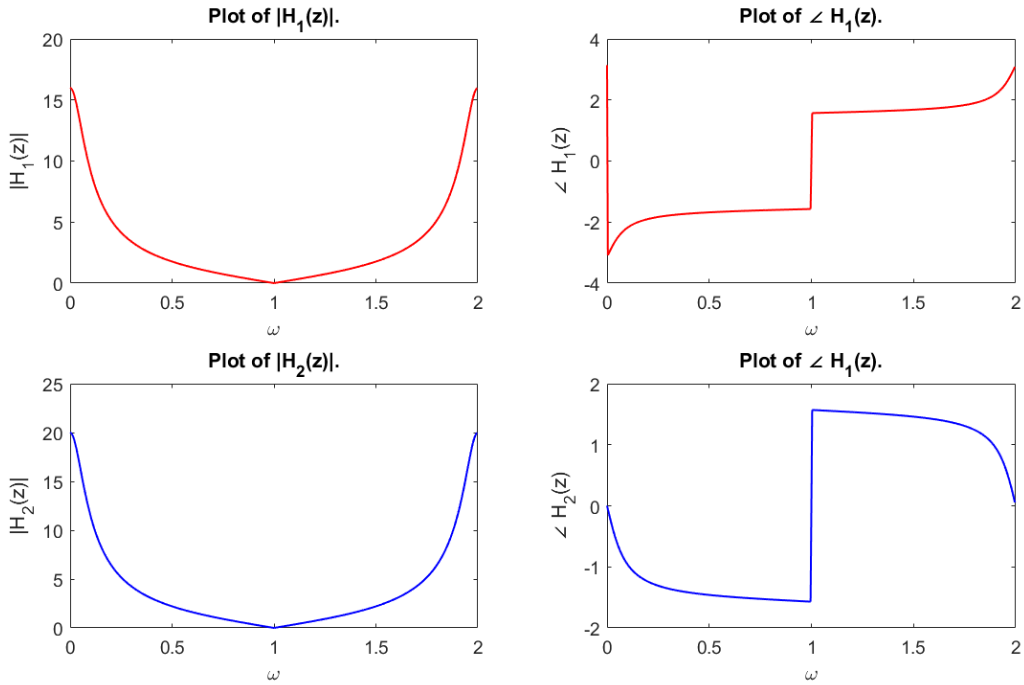

Figure 5: Magnitude and phase plots of $H_1(z)$ (top) and $H_2(z)$.

To obtain the impulse responses of the two systems $h_1[n]$ and $h_2[n]$ the transfer functions were split and the inverse Z-transform of each part was taken:

$$\begin{aligned} h_1[n] &= \mathcal{Z}\left\{\frac{2}{1-1.25z^{-1}}\right\} + \mathcal{Z}\left\{\frac{2z^{-1}}{1-1.25z^{-1}}\right\} \\ &= 2(1.25)^n u[n] + 2(1.25)^{n-1}u[n-1] \end{aligned} \tag{6}$$

$$\begin{aligned} h_1[n] &= \mathcal{Z}\left\{\frac{2}{1-0.8z^{-1}}\right\} + \mathcal{Z}\left\{\frac{2z^{-1}}{1-0.8z^{-1}}\right\} \\ &= 2(0.8)^n u[n] + 2(0.8)^{n-1}u[n-1] \end{aligned} \tag{7}$$

To plot $h_1[n]$ and $h_2[n]$ for $0 \le n \le 25$ in MATLAB the following script was ran:

```
figure(3);

n = 0:25;
u1_n = ones(1, 26); % u[n]
u2_n = ones(1, 26); % u[n-1]
u2_n(1) = 0;

% Plot h1
h1_n = (2.*(1.25).^n).*u1_n + (2.*(1.25).^(n-1)).*u2_n;
subplot(2, 1, 1);
stem(n, h1_n, 'LineWidth' , 1, 'Color', 'r');
title('Plot of h_1[n].');
xlabel('n');
ylabel('h_1[n]');
```

```
% Plot h2
h2_n = (2.*(0.8).^n).*u1_n + (2.*(0.8).^(n-1)).*u2_n;
subplot(2, 1, 2);
stem(n, h2_n, 'LineWidth' , 1, 'Color', 'b');
title('Plot of h_2[n].');
xlabel('n');
ylabel('h_2[n]');
```

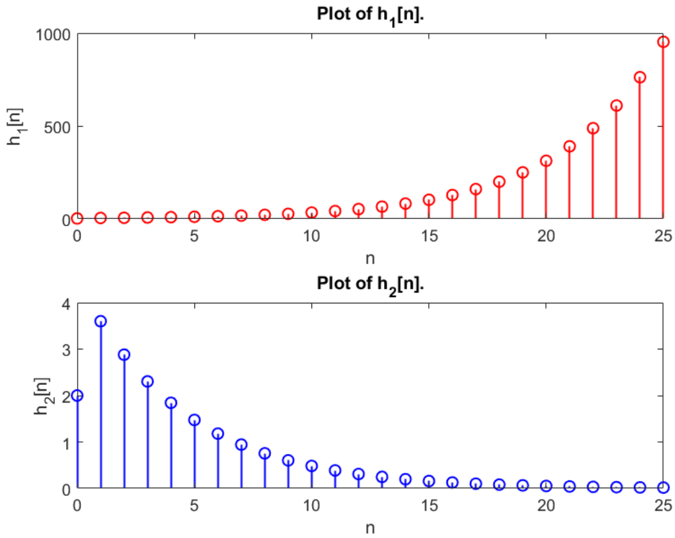

Figure 6: Plots of $h1[n]$ (top) and $h_2[n]$ (bottom).

Previously, based on the plot-zero diagrams of $H_1(z)$ and $H_2(z)$ the conclusion was made that $H_1(z)$ was not BIBO stable while $H_2(z)$ was. Seeing the plots above, it is evident that $h_1[n]$ is exponentially growing, which means it will continue to approach infinity and the output will not be bound when subjected to a bound input signal, hence it is not BIBO stable. On the other hand $h_2[n]$ displays exponential decay approaching zero, hence it is BIBO stable. This data supports both conclusions made earlier.