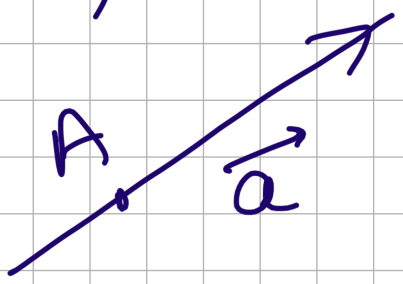
Sem 5

1. Det. param eq. for l

a) l contains $A(1,2)$

$l \parallel a(3,-1)$



$$\begin{cases} x = x_0 + a \cdot t \\ y = y_0 + b \cdot t \end{cases}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} x_A \\ y_A \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{cases} x = 1 + t v_x \\ y = 2 + t v_y \end{cases} \qquad \begin{cases} x = 1 + 3t \\ y = 2 - t \end{cases}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

b) $l \ni$ origin, $l \parallel b(4,5)$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

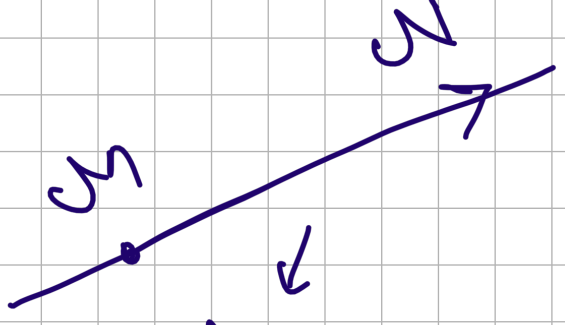
c) $l \ni M(1,7)$ $\quad l \parallel Oy$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

↓

vector axă Oy

d) $l \ni M(2,4)$ $\quad N(2,-5)$



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + t \begin{bmatrix} 0 \\ -7 \end{bmatrix}$$

$\overrightarrow{MN} = (2-2)\vec{i} + (-5-4)\vec{j}$

$\overrightarrow{MN} = 0\vec{i} - 9\vec{j}$

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2. $l = \{(1+3t, 2-t) : t \in \mathbb{R}\}$

↓ has dir. vector

$\{t(3,-1) : t \in \mathbb{R} \setminus \{0\}\}$

$(3t, -t)$

$3ti - tj = t(3i - j)$

b) l has dir. vector $\{t\begin{bmatrix}4\\5\end{bmatrix} : t \in \mathbb{R}, t \neq 0\}$

$l = \{t\begin{bmatrix}4\\5\end{bmatrix} : t \in \mathbb{R}\}$

a) cartesian eq. = linear eq in x and y

$(t =) \quad \frac{x-1}{3} = \frac{y-2}{-1}$

$\frac{m - x_A}{a_x} = \frac{y - y_A}{a_y}$

1.c) $\begin{cases} \boxed{x = 1} \leftarrow \text{cartesian eq.} \\ y = 7 + t \leftarrow \text{whatever value} \end{cases}$

3. Det. cart. eq for l

c) $l \ni A(-2,3)$ $60°$ with Ox

d) $l \ni B(1,4)$ $l \perp m(4,3)$

4. with l as above:

a) give param eq for l

b) describe all norm. vect. for l

3.c) $y - y_A = \tan 60° (x - x_A)$

$y - 3 = \sqrt{3}(x+2) \Leftrightarrow -\sqrt{3}x + y + 2\sqrt{3} = 0$

$m(-\sqrt{3}, 1)$

all normal vectors of l:

$= \{\lambda(-\sqrt{3}, 1) : \lambda \in \mathbb{R} \setminus \{0\}\}$

d) $l \perp m(4,3) \Rightarrow$

$m_l = -(m_{m(4,3)})^{-1} = -\left(\frac{3}{4}\right)^{-1}$

$$m_\ell = -\frac{4}{3}$$

$$y - 7 = \left(-\frac{4}{3}\right)(x-1)$$

Slope $= \frac{b}{a}$

$$\ell: \quad m_x(x - x_B) + m_y(y - y_B) = 0$$

4. $\ell: 4(x-1) + 3(y-7) = 0$

$\ell: 4x + 3y - 25 = 0$

$$\begin{cases} x = -\frac{3}{4}y + \frac{25}{4} \\ y = y \end{cases}$$

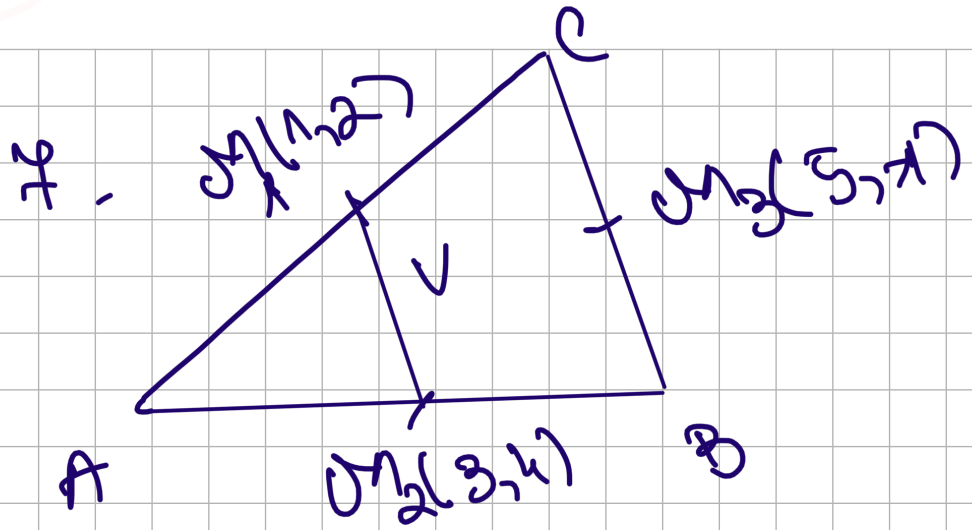
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{25}{4} \\ 0 \end{bmatrix} + a \begin{bmatrix} -\frac{3}{4} \\ 1 \end{bmatrix}$$

5. $\ell, \; v(v_1, v_2) \Rightarrow m(-v_2, v_1)$

product MUST BE $= 0$!!! ← is a $v \cdot v$?

since $v \cdot m = -v_1 \cdot v_2 + v_1 v_2 = 0$

$\Downarrow$ $m \perp v$

7. 

AB cont. eg?

∃ ↓ „contains"

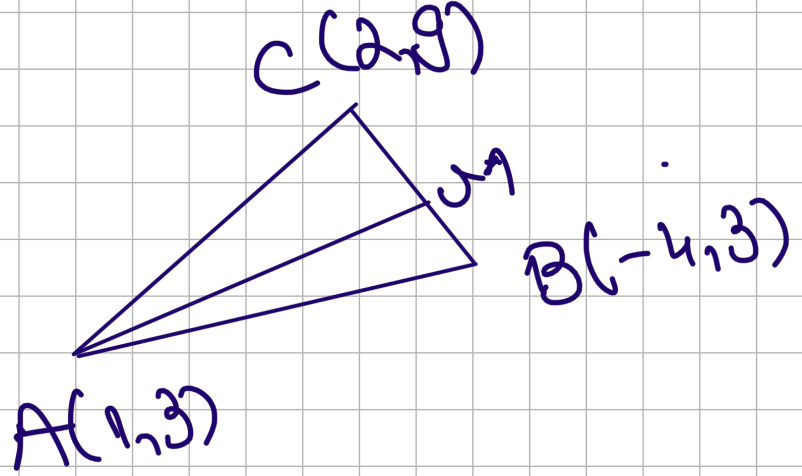
$\overrightarrow{M_1M_2}(2,2) = v \rightsquigarrow m(-2,2)$

$BC \ni M_3(5,-1)$, $BC \parallel \overrightarrow{M_1M_2} \Rightarrow$

$BC: 2(y+1)+2(x-5)$

$\Leftrightarrow (x-x_A)+m(y-y_A)=0$

$$\frac{x-x_A}{v_x} = \frac{y-y_A}{v_y}$$

8. 

a) det. length of altitude from A $|AM|$

b) det. the line containing the altitude from A AM

a) $|AM| = d(A, BC) = \frac{|1-3+2|}{\sqrt{1^2+(-1)^2}} = \frac{5}{\sqrt{2}}$

BC: $\frac{x+4}{1} = \frac{y-3}{1} \Leftrightarrow$

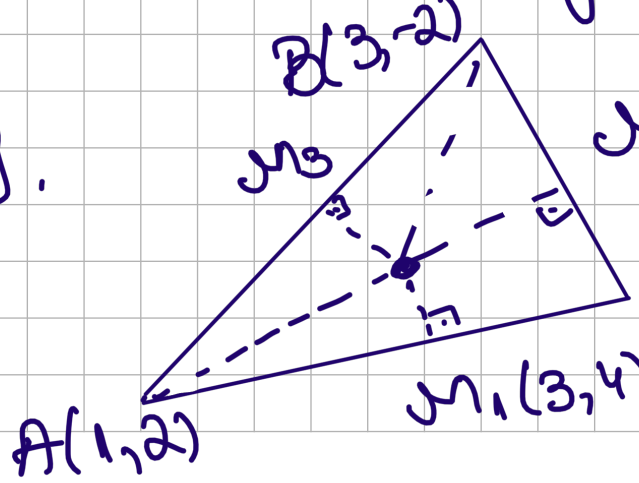
BC: $x - y + 7 = 0$

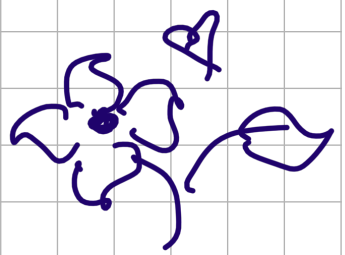
$\overrightarrow{BC}(-6,-6) = -6(1,1) \rightarrow m(1,1)$

b) AM: $1(x-1) + 1(y-3) = 0$

$\Leftrightarrow x + y - 4 = 0$

9.





? circumcenter

circumcentrul = $\cap$ mediatoare

$m_{M_1} \cdot m_{AC} = -1$

$m_{AC} = \frac{6-2}{5-1} = 1 \Rightarrow m_{M_1} = -1$

$l_1: \frac{y-4}{x-3} = -1$

$3 - x = y - 4$

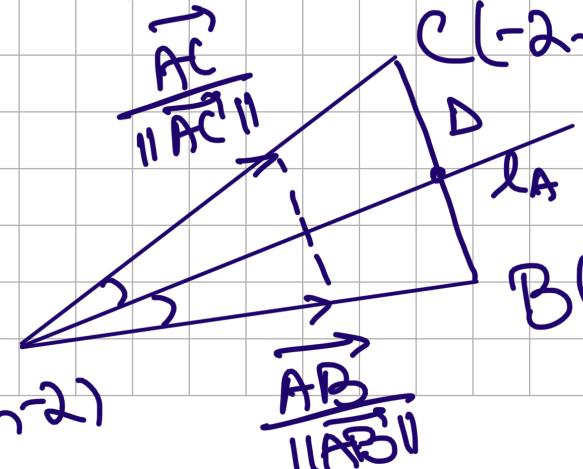
$m_{M_2} \cdot m_{BC} = -1$

$m_{BC} = \frac{6-(-2)}{5-3} = \frac{8}{2} = 4 \Rightarrow m_{M_2} = -\frac{1}{4}$

$l_2: \frac{y-2}{x-4} = -\frac{1}{4} \Rightarrow 4(y-2) = 4-x$

$\begin{cases} y = 4-x \\ 4(4-x-2) = 4-x \end{cases} \Rightarrow \dots\dots$

11.



? det l_A

I. thm of bisect.

II. $\frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} + \frac{\overrightarrow{AC}}{\|\overrightarrow{AC}\|}$ → dir. vect. for l_A

III. $d(P(x,y), AC) = d(P(x,y), AB)$

$$\frac{7 \quad 1}{\sqrt{\quad}} = \frac{1 \quad 1}{\sqrt{\quad}}$$

⇙

2 cases (bc. absolute value)

↙

- 1 for int angle bisect
- 7 for ext angle bisect.