Seminar 1

1. In how many ways can 10 students be seated in a classroom with
   a) 15 chairs? b) 10 chairs?

Solution:

Order matters => we use ARRANGEMENTS

a) $A_{15}^{10} = 6 \cdot 7 \cdot . . . . \cdot 15;$
b) A_{10}^{10} = P10 = 10!.

2. Find the number of possible outcomes for the following events:
   a) three dice are rolled; b) two letters and three digits are randomly selected.

Solution:

Actions are independent of each other => we multiply

a) 6 possibilities for each dice => 6*6*6=6^3
b) 26 letters, 10 digits => 26^2*10^3

3. A firm offers a choice of 10 free software packages to buyers of their new home computer. There are 25 packages available, five of which are computer games, and three of which are antivirus programs.
   a) How many selections are possible?
   b) How many selections are possible, if exactly three computer games are selected?
   c) How many selections are possible, if exactly three computer games and exactly two anti-virus programs are selected?

Solution:

Order does not matter => we use COMBINATIONS

a) 10(free software packages) out of 25(total of packages) => C_{25}^{10}
b) -3(computer games) are selected out of 5 => C_5^3
   -from 25(total of packages) subtract 5(the number of computer games)=20
   -from 10(free software packages) subtract 3(the number of computer games selected)=7
   => C_{20}^7
   actions are independent of each other=> multiply
   =>$C_5^3 * C_{20}^7$
c) -3(computer games) are selected out of 5 => C_5^3
   -2(anti-virus programs) are selected out of 3 =>C_3^2
   -from 25(total of packages) subtract 5(the number of computer games) and 3(the number of anti-virus programs)=5
   -from 10(free software packages) subtract 3(the number of computer games selected) and 2(the number of anti-virus programs selected)=17
   => C_{17}^5

actions are independent of each other=> multiply

=>$C_5^3 * C_3^2 * C_{17}^5$

4. A person buys n lottery tickets. For i = 1, n, let Ai denote the event: the i th ticket is a winning one. Express the following events in terms of A1, ..., An.
   a) A: all tickets are winning;
   b) B: all tickets are losing;
   c) C: at least one is winning;
   d) D: exactly one is winning;
   e) E: exactly two are winning;
   f) F: at least two are winning;
   g) G: at most two are winning.

Solution:

a) We know that ALL of the tickets are winning => AND=INTERSECTION
   A=$\bigcap_{i=1}^{n} A(i)$
b) We know that ALL of the tickets are losing => NOT AND
   B=$\bigcap_{i=1}^{n} not(A(i))$
c) At least one is winning => opposite to B
   C=not(B)
d) Exactly one is winning => OR=UNION (only the first, or the second,...)
   D=$\bigcup_{i=1}^{n}(A(i) \setminus \bigcup_{j!=i} A(j))$
e) Exactly two => OR=UNION
   E=$\bigcup_{1 \le i < j \le n}((A(i) \cap A(j))(\bigcup_{k!=i,j} A(k))$
f) At least two
   F=C\D
g) At most two => 0 or 1 or 2
   G=B$\cup D \cup E$

5. Three shooters aim at a target. For i = 1, 3, let Ai denote the event: the i th shooter hits the target. Express the following events in terms of A1, A2 and A3.
   a) A: the target is hit;
   b) B: the target is not hit;
   c) C: the target is hit exactly three times;
   d) D: the target is hit exactly once;
   e) E: the target is hit exactly twice.

Solution:

a) A = A1 ∪ A2 ∪ A3;

b) B = A = A1 ∩ A2 ∩ A3;

c) C = A1 ∩ A2 ∩ A3;

d) D = (A1 ∩ not(A2) ∩ not(A3))$\cup$(not(A1) ∩ A2 ∩ not(A3)) $\cup$(not(A1) ∩ not(A2) ∩ A3);

e) $E = (A1 \cap A2 \cap \text{not}(A3)) \cup (A1 \cap \text{not}(A2) \cap A3) \cup (\text{not}(A1) \cap A2 \cap A3)$;