Midterm Test – Marking Scheme

1. Find $\inf, \sup, \min, \max$, the interior and the closure of the set $\{0.1, 0.11, 0.111, \ldots\}$.
   1.5p: each item **0.25p**

2. Study the convergence of the following series:

   (a) $\sum_{n\geq 1} \frac{(n+1)^{n-1}}{n^{n+1}}$. **1p**

   (b) $\sum_{n\geq 1} \frac{a^n (n!)^2}{(2n)!}$, $a > 0$. **1p**

   (c) $\sum_{n\geq 1} \frac{\ln n}{n^2}$. **1p**

   (d) $\sum_{n\geq 1} n! \sin x \sin \frac{x}{2} \ldots \sin \frac{x}{n}$, $x \in (0, \pi)$. **1p**

   For (b) and (d): **0.75p** for the ratio test + **0.25p** for the Raabe-Duhamel test.

3. Study the convergence and the absolute convergence of the series $\sum_{n\geq 1} (-1)^n (\sqrt{n} - \sqrt{n+1})$.

   1.5p: 1p for convergence using the Leibniz test + **0.5p** for not absolutely convergent.

4. Using power series, find the sum of the following series:

   (a) $\sum_{n\geq 0} \frac{n+1}{4^n}$. **1p**

   (b) $\sum_{n\geq 2} \frac{n(n-1)}{2^n}$. **1p**

   (c) $\sum_{n\geq 0} \frac{(-1)^n}{2n+1}$. **1p**

5. Find the radius of convergence and the convergence set for the power series

$$\sum_{n\geq 1} \frac{x^n}{n^p},\ p \in \mathbb{R}.$$

   1.5p: 0.5p for the radius of convergence + **1p** for the convergence set.

6. **1p** Given the data points (x_i, y_i), $i \in \{1, \ldots, n\}$, the line of best fit f minimizes $\sum_{i=1}^{n} (y_i - f(x_i))^2$.

   Find the line of best fit that passes through the origin (and explain its uniqueness).