Seminar 3 DS

1.2.5.

a) $x_p = a \cdot e^t$ → particular solution for $x' - 2x = e^t$
x_{pa}

b) $x_p = b \cdot e^{-t}$ → particular solution for $x' - 2x = e^{-t}$
x_{pb}

c) $x_p = 2$ for $x' - 2x = 5e^t - 3e^{-t}$
(Superpos. Princ.)
$x_p = 5x_{pa} - 3x_{pb}$

d) gen. sol $x' - 2x = 5e^t - 3 \cdot e^{-t}$

$$\underbrace{x' - 2x}_{\downarrow \, x_h} = \underbrace{5e^t - 3 \cdot e^{-t}}_{\downarrow \, x_p}$$

$$x = x_h + x_p$$

EXAM!!!

Superposition Principle:

$\mathcal{L}(x') = x^2 - 2x$

$$\boxed{x' - 2x = 5e^t - 3 \cdot e^{-t}}$$

$\mathcal{L}[x] = 5e^{t} \longrightarrow x_{p_1}$

$\mathcal{L}[x] = -3e^{-t} \longrightarrow x_{p_2}$

$\Big\} \; x_p = x_{p_1} + x_{p_2}$

a) $x_p = ae^{t} \Rightarrow x_p' = a\cdot e^{t}$

$ae^{t} - 2ae^{t} = e^{t}$

$-a\,e^{t} = e^{t}$

$$\boxed{a = -1}$$

b) $x_p = be^{-t} \Rightarrow x_p' = -b\cdot e^{-t}$

$-b\cdot e^{-t} - 2be^{-t} = e^{-t}$

$$\boxed{b = -\frac{1}{3}}$$

c) $x' - 2x = 5e^{t}$

$x_{p_1} = Ae^{t}$

$\Rightarrow Ae^{t} - 2Ae^{t} = 5e^{t} \Rightarrow A = -5 = 5a$

$5\cdot(e^{t})$

$\downarrow$

$f(a)$

$$\Rightarrow \underline{x_{p1} = -5 \cdot e^{t}}$$

$$x' - 2x = -3e^{-t}$$

$$x_{p2} = -3\left(-\frac{1}{3}\right)e^{-t} = e^{-t}$$

$$\boxed{x_p = x_{p1} + x_{p2} = -5 \cdot e^{t} + e^{-t}}$$

d) $x' - 2x = 0$

$$r - 2 = 0 \Rightarrow r = 2$$

$$\Rightarrow x_h = e^{2t} \cdot c \qquad c \in \mathbb{R}$$

$\Rightarrow$ gen sol: $x = x_h + x_c$

$$x = c \cdot e^{2t} - 5e^{t} + e^{-t} \qquad c \in \mathbb{R}$$

$te^{t}\cos 2t$; $te^{t}\sin 2t$; $e^{t}\cos 2t$;

$e^{t}\sin 2t$;

first sol. also sol.