Seminar 9

09.05.2023

1. $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\det(A - \lambda I_n) = 0$

$A \cdot v = \lambda_1 \cdot v$

$(A - \lambda_1 I_n) \cdot v = 0$

$$\det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix}$$

$$= (1-\lambda)(-1-\lambda)$$

$$= -1-\lambda^2$$

$\Rightarrow \lambda = \pm 1$

$\lambda = 1$ $\quad \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$y = 0$

$V_1 = \left\{ \begin{bmatrix} t \\ 0 \end{bmatrix} : t \in R \right\}$

$\lambda = -1$ $\quad \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$x = 0$$

$$V = \left\{ \begin{bmatrix} 0 \\ y \end{bmatrix} : y \in \mathbb{R} \right\}$$

$$\det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2$$

$$\Rightarrow \lambda = 1$$

$\lambda = 1$ $\quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow y = 0$

$$V_1 = \left\{ \begin{bmatrix} t \\ 0 \end{bmatrix} : t \in \mathbb{R} \right\}$$

geometric mult. of an E.V. λ_1 is $\dim V_{\lambda_1}$

alg. mult of an E.V λ_1 is the mult of λ_1 in $\det(A - \lambda I_n)$

$h_A(\lambda_1)$

$$\dim V_{\lambda_1} \leq h_A(\lambda_1)$$

$$\det(A - \lambda I_2) = \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 1$$
$$= \lambda^2 - 2\lambda$$

$$\Rightarrow \lambda_1 = 0 \qquad \lambda_2 = 2$$

$$\lambda = 0 \Rightarrow A\begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} x+y \\ x+y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x + y = 0 \Rightarrow x = -y$$

$$\underbrace{V_0}_{\text{vector space}} = \left\{ \begin{bmatrix} t \\ -t \end{bmatrix} : t \in \mathbb{R} \right\}$$

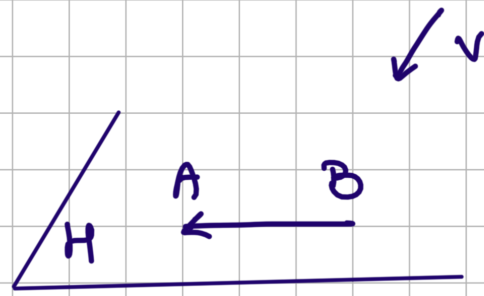
$$\phi_A(v) = A \cdot v = 0 = 0 \cdot v$$
↳ linear map

$$\phi_A^{-1}(0) = V_0$$

2. $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \in Mat_{2\times 2}(\mathbb{R}) \subseteq Mat_{2\times 2}(\mathbb{C})$

$\det(A - \lambda I_2) = \lambda^2 - 1$

3. Give the eigenval. for $lin(Pr_{H,v})$
$lin(Ref_{H,v})$



ϕ aff. map

$lin\,\phi : V^n \to V^n$

$lin\,\phi(\overrightarrow{AB}) = \overrightarrow{\phi(A)\phi(B)}$

for $A, B \in H$

$(lin\,Pr_{H,v})(\overrightarrow{AB}) = \overrightarrow{AB}$

$v = \overrightarrow{PQ}, \ (lin\,Pr_{H,v})(\overrightarrow{PQ}) = 0 = 0 \cdot \overrightarrow{PQ}$

with respect to the basis

$\{v_1, \underbrace{\dots\dots, v_n}_{v_2 \dots v_n \parallel H}\} = B$

$$[\text{lin } Pr_{H,V}]_B = \begin{bmatrix} 0 & \cdot & - & - & - & - \\ 0 & 1 & \cdot & - & - & - \\ 0 & 0 & 1 & - & - & - \\ & & & & & \\ 0 & 0 & 0 & \cdot & - & 1 \end{bmatrix}$$

EV. 0 mult. 1

EV 1 mult. n-1

$$A \cdot \alpha \cdot v = \alpha \cdot A \cdot v = \alpha \cdot 0 = 0$$

$$(\text{lin } Ref_{H,v})(\overrightarrow{PQ}) = -\overrightarrow{PQ}$$

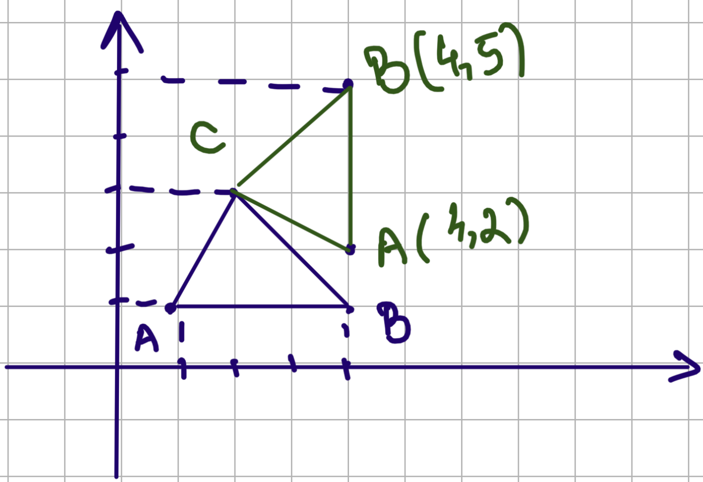
$$[\text{lin } Ref_{H,v}]_B = \begin{bmatrix} -1 & 0 & \cdot & - & - \\ 0 & 1 & - & - & - \\ & & & & \\ 0 & \cdot & - & - & 1 \end{bmatrix}$$

EV -1 mult. 1

EV 1 mult. n-1

7. $A(1,1)$ $B(4,1)$ $C(2,3)$

Det. the image of ΔABC under a rotate around C by $90°$ ($\underline{I}$) followed by an orth. refl. in the side AB ($\underline{II}$)



I. $Rot_{c,90°}(P)$

$$= \underset{\text{shift}}{T_{\vec{OC}}} \circ \underset{\text{rot.}}{Rot_{o,90°}} \circ \underset{\text{shift}}{T_{-\vec{OC}}}(P)$$

$$\underbrace{T_{-\vec{OC}}(P)}_{\begin{bmatrix} x-2 \\ y-3 \end{bmatrix}}$$

order of doing it :)

$$Rot_{o,\theta} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$Rot_{o,90°} \circ T_{-\vec{OC}}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x-2 \\ y-3 \end{bmatrix}$$

$$= \begin{bmatrix} -y+3 \\ x-2 \end{bmatrix}$$

$$Rot_{c,90°}(P) = \begin{bmatrix} -y+5 \\ x+1 \end{bmatrix}$$

II. $\mathrm{Ref}^{\perp}_{AB}(P) = T_{(4,0)} \circ \mathrm{Ref}^{\perp}_{Oy} \circ T_{-(4,0)}(P)$

$$\underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \circ \begin{bmatrix} x-4 \\ y \end{bmatrix}}_{\begin{bmatrix} -x+4 \\ y \end{bmatrix}}$$

$$= \begin{bmatrix} -x+8 \\ y \end{bmatrix}$$

$$\mathrm{Ref}^{\perp}_{AB}(C) = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

12. $A = \frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ -2 & -2 & -1 \\ -2 & 1 & 2 \end{bmatrix}$

Show that $A \in SO(3)$

Deduce the Rot. axes & Rot angle

$A \in SO(3) = \{ A \in Mat_{3\times3}(\mathbb{R}) : A \cdot A^t = I_3,$

$\det A = 1 \}$

↓

they do not change the orient. of the space

easy to check

Det the rot. axis:

$A \cdot v = v$

↓

eigenspace corresp. to the eigenvalue 1

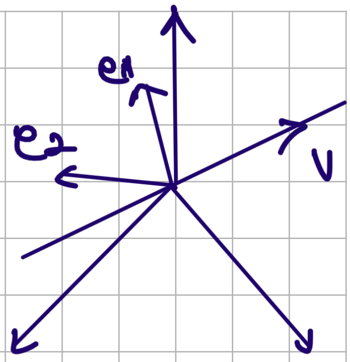
$$V_1 = \left\{ \begin{bmatrix} t \\ 0 \\ -2t \end{bmatrix} : t \in \mathbb{R} \right\}$$

$$(A - I_3) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

how to calculate

Det the rot. angle:

$\{v, e_1, e\}$



$$[\phi_A]_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

trace : $\longrightarrow$ sum of elem on the main diag.

$$\operatorname{tr}[\phi_A]_B = 1 + 2\cos\theta$$

$$\parallel$$

$$\operatorname{tr} A = -\frac{1}{3} \qquad \Rightarrow \cos\theta = \frac{-\frac{1}{3} - 1}{2}$$