4.04.2023

Seminar 6

1. Det. param. eq. for plane π

a) $\pi \ni M(1,0,2)$ $\quad \pi \parallel a_1(3,-1,1)$

$\pi \parallel a_2(0,3,1)$

b) $\pi \ni A(-2,1,1)$, $B(0,2,3)$, $C(1,0,-1)$

c) $\pi \ni A(1,2,1)$ $\quad \pi \parallel i, j$

d) $\pi \ni M(1,-2,1)$ $\quad \pi \parallel Oyz$

e) $\pi \ni M_1(5,3,4)$, $M_2(1,0,1)$

$\pi \parallel a(1,3,-3)$

f) $\pi \ni A(1,5,7)$ $\quad \pi \supset Ox$

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a) $\pi \ni M(1,0,2)$

$\pi \parallel a_1(3,-1,1)$

$\pi \parallel a_2(0,3,1)$

$$\boxed{\pi: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} + t \begin{bmatrix} a_{1x} \\ a_{1y} \\ a_{1z} \end{bmatrix} + s \begin{bmatrix} a_{2x} \\ a_{2y} \\ a_{2z} \end{bmatrix}}$$

$$\pi: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

b) $\pi \ni A(-2,1,1),\ B(0,2,3),\ C(1,0,-1)$

$\overrightarrow{AB}(\underset{0-(-2)}{2}, \underset{2-1}{1}, \underset{3-1}{2}) \parallel \pi$

$\overrightarrow{BC}(1,-2,-4) \parallel \pi$

$$\pi: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + s \begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix}$$

c) $\pi \ni A(1,2,1)$ $\quad \pi \parallel \vec{i}, \vec{j}$

$$\pi: \begin{cases} x = 1 + t \cdot 1 \\ y = 2 + s \\ z = 1 \end{cases} \qquad t, s \in \mathbb{R}$$

d) $\pi \ni M(1,2,1)$ $\quad \pi \parallel Oyz$

$\Downarrow$

$$\pi : \begin{cases} x = 1 \\ y = 2+s \\ z = 1+t \end{cases}$$

$j \parallel \pi$
$k \parallel \pi$

e) $\pi \ni M_1(5,3,4)$, $M_2(1,0,1)$

$\pi \parallel a(1,3,-3)$

$\overrightarrow{M_1M_2}(-4,-3,-3) \parallel \pi$

$$\pi : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} + t\begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix} + s\begin{bmatrix} -4 \\ -3 \\ -3 \end{bmatrix}$$

f) $\pi \ni A(1,5,7)$ $\qquad \pi \supseteq Ox$

let $O(0,0,0)$

$\overrightarrow{OA}(1,5,7) \parallel \pi$

$i(1,0,0) \parallel \pi$

$$\pi : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} + t \cdot \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} + s \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$t, s \in \mathbb{R}$

2. Det. ec. g. for:

$$\pi: \begin{cases} x = 2+3u-4v \\ y = 4-v \\ z = 2+3u \end{cases} \qquad u, v \in \mathbb{R}$$

$$\pi: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix} + u \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} + v \begin{bmatrix} -4 \\ -1 \\ 0 \end{bmatrix}$$

$$P \quad = \quad Q \quad + u \cdot a \quad + v \cdot b$$

$$\overrightarrow{QP} = u \cdot a + v \cdot b$$

$$\begin{array}{l} \\ a \rightarrow \\ b \rightarrow \end{array} \det = \begin{vmatrix} x-2 & y-4 & z-2 \\ 3 & 0 & 3 \\ -4 & -1 & 0 \end{vmatrix} = 0$$

$$\det = (x-2) \cdot \underbrace{\begin{vmatrix} 0 & 3 \\ -1 & 0 \end{vmatrix}}_{3} - (y-4) \cdot \underbrace{\begin{vmatrix} 3 & 3 \\ -4 & 0 \end{vmatrix}}_{12}$$

$$+ (z-2) \cdot \underbrace{\begin{vmatrix} 3 & 0 \\ -4 & -1 \end{vmatrix}}_{-3}$$

$$a\times b \quad \begin{array}{l} \to 3 \\ \to -12 \\ \to -3 \end{array}$$

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3. Det. param. eq. for:

$\pi: \quad 2x - y - z - 3 = 0$

$$\pi: \begin{cases} x = \lambda \\ y = 2\lambda - \mu - 3 \\ z = \mu \end{cases} \qquad \lambda, \mu \in \mathbb{R}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

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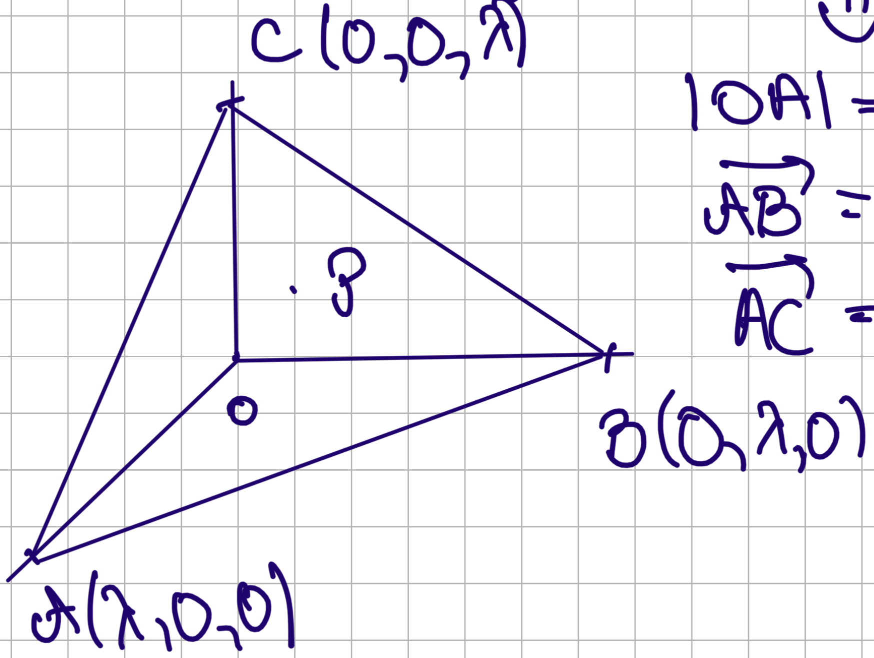
4. Det. an eq. for a plane π:

$\pi \ni P(3, 5, -7)$

π intersects the coord. axes in

Congruent segments $\Rightarrow$



$$|OA| = |OB| = |OC| = \lambda$$

$$\overrightarrow{AB} = (-\lambda, \lambda, 0)$$

$$\overrightarrow{AC} = (-\lambda, 0, \lambda)$$

$$\begin{array}{r} \\ \overrightarrow{AB} \rightarrow \\ \overrightarrow{AC} \rightarrow \end{array} \begin{vmatrix} x-3 & y-5 & z+7 \\ -\lambda & \lambda & 0 \\ -\lambda & 0 & \lambda \end{vmatrix} = 0$$

$$(x-3)\underbrace{\begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix}}_{\lambda^2} - (y-5)\underbrace{\begin{vmatrix} \lambda & 0 \\ -\lambda & \lambda \end{vmatrix}}_{-\lambda^2} + (z+7)\underbrace{\begin{vmatrix} -\lambda & \lambda \\ -\lambda & 0 \end{vmatrix}}_{-\lambda^2} = 0$$

$$x - 3 + y - 5 + z + 7 = 0$$

$$\pi: \boxed{x + y + z - 1 = 0}$$

$$\pi \cap Ox = (1, 0, 0)$$

$$\pi \cap Oy = (0, 1, 0)$$

$$\pi \cap Oz = (0, 0, 1)$$

$$\boxed{\pi : \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$$ $\rightarrow$ form of a plane where you can intersect all the axes

$\pi \cap Ox = (a, 0, 0)$

$\pi \cap Oy = (0, b, 0)$

$\pi \cap Oz = (0, 0, c)$

$|OA| = |a|$; $|OB| = |b|$; $|OC| = |c|$

$(*) \Rightarrow |a| = |b| = |c| = \lambda$

$C\,\underline{I}$. $a = b = c = \lambda$

$C\,\underline{II}$. $a = b = -c = \lambda$ .... $\Rightarrow$ 8 cases

$\downarrow$

4 overlap

$\Downarrow$

4 solutions

4 planes $\leftarrow$

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6. $\pi_1 : 2x + y - 2z + 6 = 0 \rightarrow n_1(2, 1, -2)$

$\pi_2 : 2x - 2y + z - 8 = 0 \rightarrow n_2(2, -2, 1)$

$\pi_3 : x + 2y + 2z + 1 = 0 \to n_3(1,2,2)$

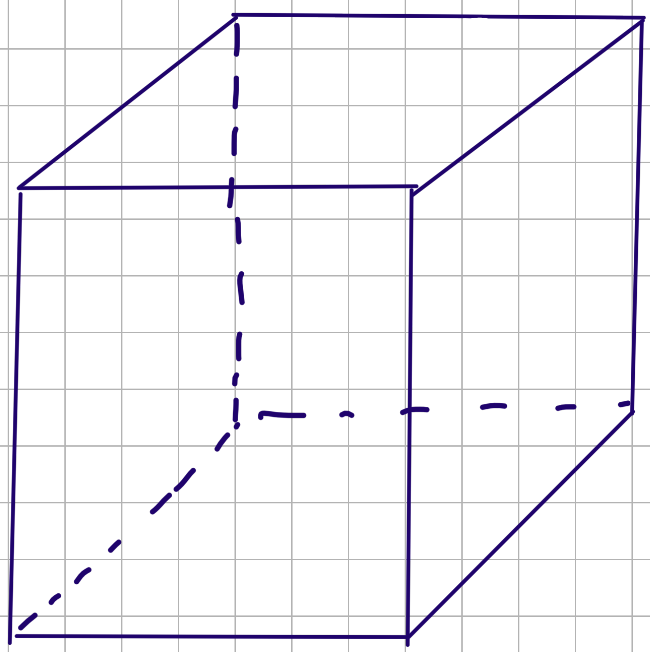
Show that the parallelipiped with faces in π_1, π_2, π_3 is rectangular

$\Downarrow$

the box is made out of rectangles

$\downarrow$

tip: check the angles between planes



$\pi_1 \perp \pi_2 \Leftrightarrow n_1 \perp n_2 \Rightarrow n_1 \cdot n_2 = 0$

$4 - 2 - 2 = 0 \Rightarrow$ True

$\pi_2 \overset{?}{\perp} \pi_3$

$\pi_3 \overset{?}{\perp} \pi_1$

if all of the cond. are True $\Rightarrow$ parallelipiped = rectangular

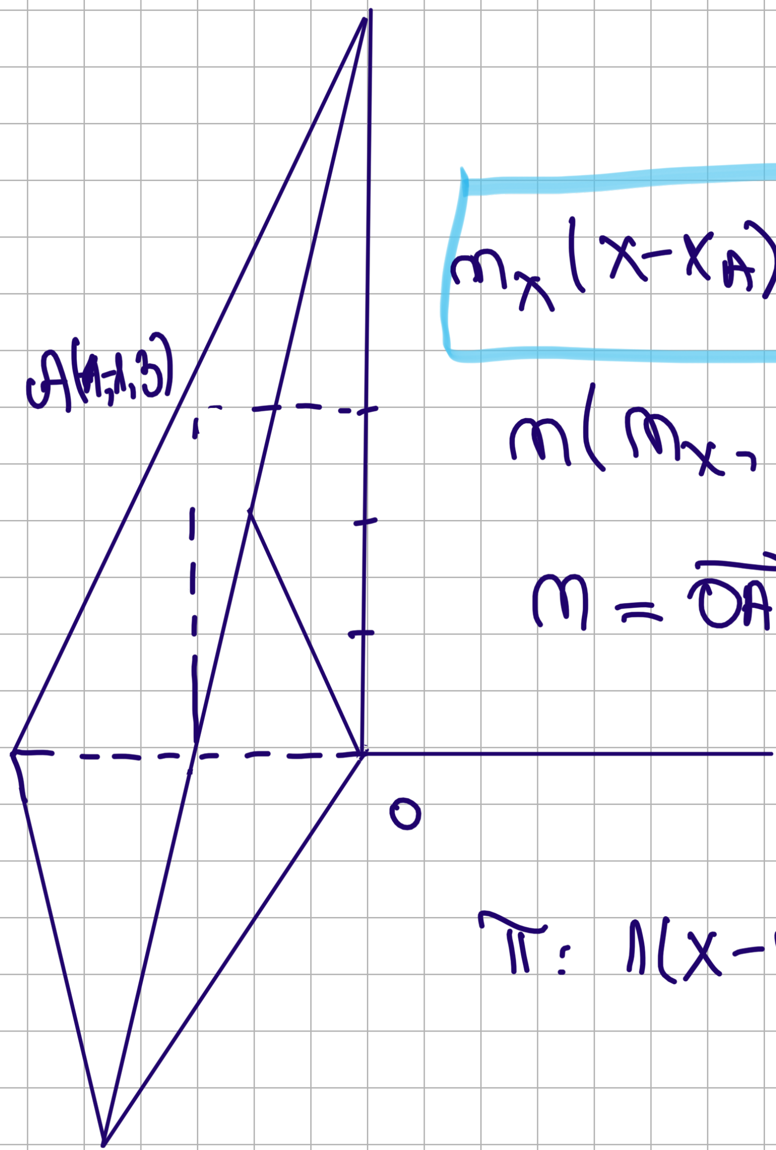
7. $A(1,0,-1)$ → Show that
$B(0,2,3)$ A, B, C, D are
$C(-2,1,1)$ coplanar
$D(4,2,3)$

A, B, C, D coplanar $\Leftrightarrow \left|\vec{AB}, \vec{AC}, \vec{AD}\right| = 0$

$$\Leftrightarrow \begin{array}{l} \vec{AB} \to \\ \vec{AC} \to \\ \vec{AD} \to \end{array} \begin{vmatrix} x_B - x_A & y_B - y_A & z_B - z_A \\ - \ - & - \ - & - \\ - & - \ - & - \end{vmatrix} = 0$$

$$\begin{vmatrix} x_A & y_A & z_A & 1 \\ x_B & y_B & z_B & 1 \\ x_C & y_C & z_C & 1 \\ x_D & y_D & z_D & 1 \end{vmatrix} = 0$$

8. Det. an eq. of π if the orthogonal proj of O on π is $A(1,-1,3)$



$$\boxed{m_x(x-x_A)+m_y(y-y_A)+m_z(z-z_A)=0}$$

$$m(m_x, m_y, m_z)$$

$$m=\overrightarrow{OA}(1,-1,3)$$

$$\Downarrow$$

$$\pi: 1(x-1)-1(y+1)+3(z-3)=0$$

9. $\pi_1: x-2y-2z+7=0 \rightarrow (1,-2,-2)$

$\pi_2: 2x-4y-4z+17=0 \rightarrow (2,-4,-4)$

π_1 :

π_2 :