Seminar 2

1. The faces of a cube are painted each in a different color (the cube is transparent on the inside). Then the cube is broken into 1000 smaller, equally-sized cubes and one such cube is randomly picked. Find the probability of the following events:
   a) A: the cube picked has exactly three colored faces;
   b) B: the cube picked has exactly two colored faces;
   c) C: the cube picked has exactly one colored face;
   d) D: the cube picked has no colored faces

Solution:

a)total nr of possible outcomes = 1000

exactly 3 colored faces => cubes on corners => 8 corners

- ⇨ Probability:8/1000

b)exactly 2 colored faces => cubes on the edges => 12 edges, and on each edge-8 cubes(without corners) => 12*8=96

- ⇨ Probability:96/1000

c)exactly 1 colored face => cubes on faces, not edges => 6 faces, and 8*8=64 cubes on each face

- ⇨ Probability:384/1000 (6*64=384)

d)no colored faces => only possible nr of colored faces are 0,1,2 or 3. In other words, events A, B, C and D cover all possibilities, S = A ∪ B ∪ C ∪ D. They are also mutually exclusive, obviously. Then they form a partition of the sample space. Recall that the sum of probabilities of all events in a partition is equal to 1 (the probability of the sure event).

- ⇨ Probability:1-P(A)-P(B)-P(C)=512/1000

2. (Pigeonhole Principle) A postman distributes n letters in N mailboxes. What is the probability of the event A: there are m letters in a given (fixed) mailbox (0 ≤ m ≤ n)?

Solution:

Nr of possible cases = N^n
->m letters can be chosen in C_n^m ways
->the other n-m should be distributed in N-1 mailboxes => this can be done in $(\mathrm{N}-1)^{n-m}$ ways
=> this 2 cases are independent => multiply
Nr of favorable cases = $C_n^m*(\mathrm{N}-1)^{n-m}$ -> use combinations because we do not care about the order
Probability: $C_n^m*(\mathrm{N}-1)^{n-m}/N^n$

3. (Breaking Passwords) An account uses 8-character passwords, consisting of letters (distinguishing between lower-case and capital letters) and digits. A spy program can check about 1 million passwords per second. a) On the average, how long will it take the spy program to guess your

password? b) What is the probability that the spy program will break your password within a week (event A)? c) Same questions, if capital letters are not used.

4. (System Reliability) Compute the reliability of the system in Figure 1 if each of the five components is operable with probability 0.92, independently of each other.

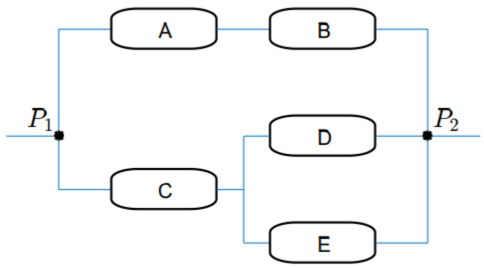

Figure 1: System Reliability

Solution:

A, B events - independent => P(A $\cap B$)=P(A)*P(B)

A, B, C, D, E components =>

A, B -> C_{AB}

C, D, E -> C_{CDE}

P($C_{AB} \cup C_{CDE}$)=P($C_{AB}) + P(C_{CDE})$ – P($C_{AB} \cap C_{CDE})$

P($C_{AB}) =$ P(A$\cap B$)= P(A)*P(B)

P($C_{CDE}) =$ P(C$\cap$($D \cup E$)= P(C)*P($D \cup E$)

5. Among employees of a certain firm, 70% know C/C++, 60% know Fortran and 50% know both. What portion of programmers
a) does not know Fortran (event A1)?
b) does not know C/C++ and does not know Fortran (event A2)?
c) knows C/C++, but not Fortran (event A3)?
d) Are “knowing C/C++” and “knowing Fortran” independent of each other?
e) What is the probability that someone who knows Fortran, also knows C/C++ (event A4)?
f) What is the probability that someone who knows C/C++, does not also know Fortran (event A5)?

Solution:

C:knows C/C++

F:knows Fortan

P(C) = 0.7

P(F) = 0.6

$P(C\bigcap F)$ = 0.5

P(A GIVEN B) = P(A|B) = P(A ∩ B)/ P(B)

a) P(not(F))= 1-P(F) = 0.4
b) P(not(F)∩not(C)) = 1-P(C∪F) = 1- (P(C) + P(F) − P(C ∩ F))= 1 − (0.7 + 0.6 − 0.5) = 0.2;
c) P(C ∩ not(F)) = P(C \ F) = P(C) − P(C ∩ F) = 0.7 − 0.5 = 0.2;
d) P(C ∩ F) = 0.5 6= 0.42 = P(C) · P(F), the answer is "NO", they are not;
e) P(C|F) = P(C ∩ F)/ P(F) = 0.5/ 0.6 = 5 /6 = 0.833;
f) P(not(F)|C) = P(not(F) ∩ C)/ P(C) = 0.2/ 0.7 = 2/ 7 = 0.286.

6. Three shooters aim at a target. The probabilities that they hit the target are 0.4, 0.5 and 0.7, respectively. Find the probability that the target is hit exactly once.

Solution:

A: the target is hit exactly once

Ai : the ith shooter hits the target, i = 1, 2, 3.

Then P(A1) = 0.4, P(A2) = 0.5, P(A3) = 0.7

A = (A1 ∩ not(A2) ∩ not(A3)) ∪ (not(A1) ∩ A2 ∩ not(A3)) ∪ (not(A1) ∩ not(A2) ∩ A3)

union is disjoint and the events A1, A2 and A3 are independent =>

P(A) = 0.4 · 0.5 · 0.3 + 0.6 · 0.5 · 0.3 + 0.6 · 0.5 · 0.7 = 0.36.

!!!This is a classical example of a Poisson probabilistic model

7. Under good weather conditions, 80% of flights arrive on time. During bad weather, only 50% of flights arrive on time. Tomorrow, the chance of good weather is 60%. What is the probability that your flight will arrive on time?

Solution:

A: the flight arrives on time

G: good weather

B: bad weather

B = not(G)

P(G) = 0.6

P(not(G)) = 0.4

{G, not(G)} form a partition

$P(A|G) = 0.8$

$P(A|G) = 0.5$

Total Probability Rule => $P(A) = P(A|G)P(G) + P(A|not(G))P(not(G)) = 0.8 \cdot 0.6 + 0.5 \cdot 0.4 = 0.68$