A

1. Check if the following function is a natural cubic spline on the interval $[-1,1]$:

$$S(x) = \begin{cases} x^3 + x - 1, & x \in [-1,0] \\ x^3 - x - 1, & x \in [0,1] \end{cases}.$$

2. A clamped cubic spline S for a function f is defined by

$$S(x) = \begin{cases} 3(x-1) + 2(x-1)^2 - (x-1)^3, & x \in [1,2] \\ a + b(x-2) + c(x-2)^2 + d(x-2)^3, & x \in [2,3] \end{cases}.$$

   Knowing that $f'(1) = f'(3)$, find $a,\ b,\ c,\ d$.

3. Determine a constant function, a line and a quadratic polynomial that best fit the data:

x	-2	-1	0	1	2
y	2	1	1	1	2

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B

1. Check if the following function is a natural cubic spline on the interval $[-1,2]$:

$$S(x) = \begin{cases} 1 + 2(x+1) + (x+1)^3, & x \in [-1,0] \\ 3 + 5x + 3x^2, & x \in [0,1] \\ 11 + (x-1) + 3(x-1)^2 + (x-1)^3, & x \in [1,2] \end{cases}.$$

2. A natural cubic spline S is defined by

$$S(x) = \begin{cases} 1 + 2x - x^3, & x \in [0,1] \\ 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & x \in [1,2] \end{cases}.$$

   Determine $b,\ c,\ d$.

3. Determine a constant function, a line and a quadratic polynomial that best fit the data:

x	0	1	2	3
y	5	-6	7	4

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