The Exponential Distribution: A Statistical Investigation

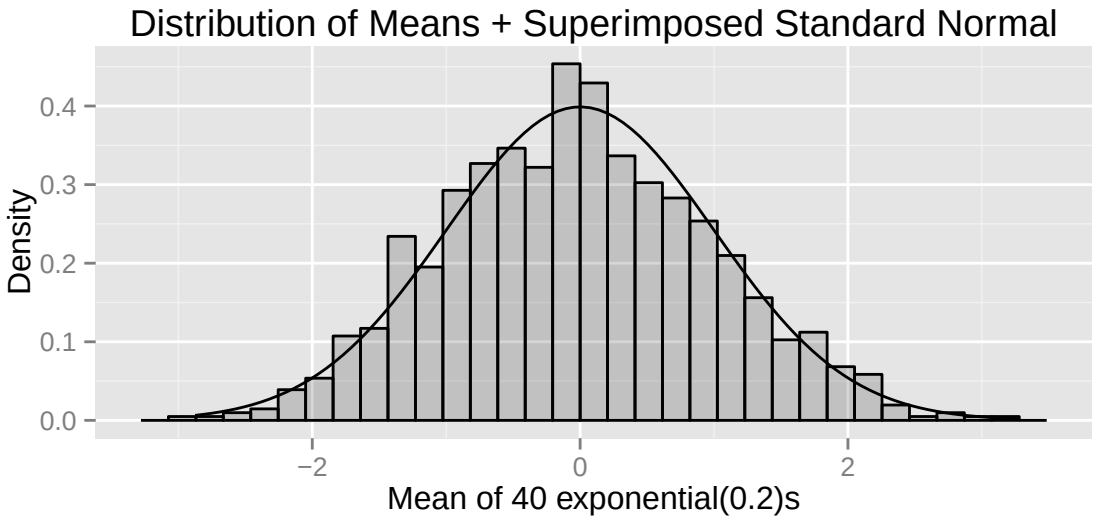
Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials with a rate of 0.2.

The Central Limit Theory states that the distribution of averages of i.i.d. random variables, properly normalized, becomes that of a standard normal as the sample size increases. Let's test that.

Let's draw 40 i.i.d. observations from the exponential(0.2) distribution. Take their mean. Normalize the mean. Repeat this process 1000 times. We now have 1000 normalized means of 40 exponential(0.2)s each.

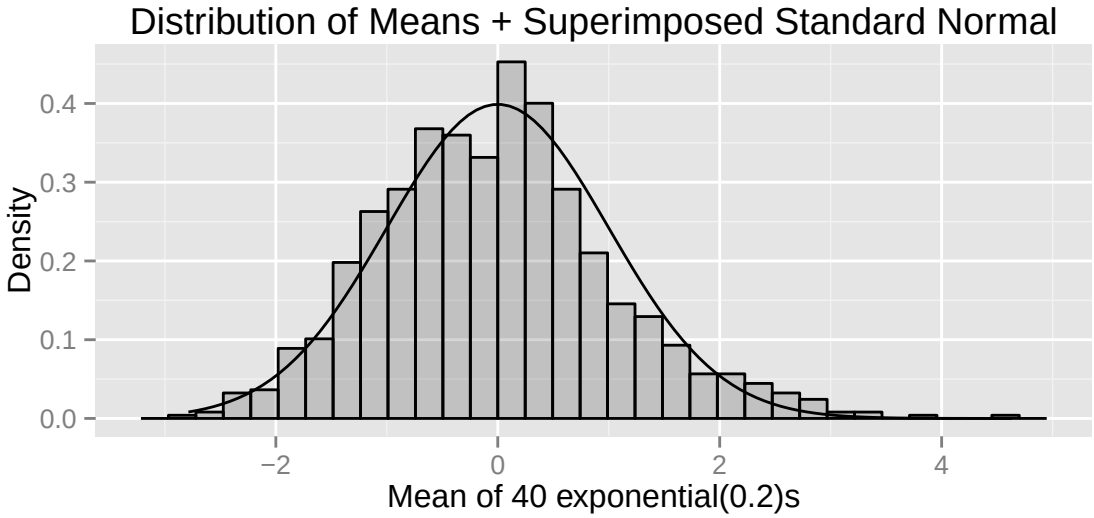
The exponential distribution can be simulated in R with rexp(n, lambda) where n is the number of observations - 40 in our case - and lambda is the rate parameter – 0.2 in our case.

1. Show where the distribution of the mean of 40 exponential(0.2)s is centered at and compare it to the theoretical center of the distribution.



Our distribution and the superimposed standard normal distribution look very similar. Our distribution looks centered at 0, the center of the standard normal.

2. Show how variable the distribution of the mean of 40 exponential(0.2)s is and compare it to the theoretical variance of the distribution.



Our distribution and the superimposed standard normal distribution again look very, very similar! Thus we conclude the variance of our distribution is that of the standard normal, i.e. variance = 1.

3. Show that the distribution is approximately normal.

Please see #1 and #2. The superimposed normal is a standard normal.