Effect Of Cyclic Frequency Estimation Error and Timing Mismatch On Cyclostationary Detection Of Frequency Hopping Signal

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CERTIFICATE

It is certified that the work contained in the thesis entitled "*Effect of Cyclic Frequency Estimation Error and Timing Mismatch on Cyclostationary Detection of Frequency Hopping Signal*" by *Onkar Arun Pandit* has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

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Abstract

Cyclostationarity is property of every modulated signal. This inherent property of signal is preserved even at low SNR conditions. Frequency hopping signal is a spread spectrum signal where carrier frequency of the signal is changed periodically. It is difficult to detect the frequency hopping signal with the use of energy detection technique at low SNR condition.

In our research, we do the detection of frequency hopping signal with the use of cyclostationarity property. The decision statistic of the cyclostationary detection test depends on the cyclic frequency of the signal. Cyclic frequency of frequency hopping signal depends on the hopping period which is estimated from the algorithm. We compare the performance of cyclostationarity and energy detection technique. It proves effectiveness of cyclostationarity detection at low SNR scenarios.

We have shown the effect of error in estimation of hoping period on the detection of signal. It is seen that, if the estimated value of cyclic frequency deviates from actual cyclic frequency by ten to fifteen percent then detection of signal becomes difficult. This effect is sever at low SNR where tolerable error in estimation drops to two to three percent. We also investigate the effect of receiver timing mismatch on the detection of signal through simulations.

Dedicated To My Parents

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Chapter 1

Introduction

In military environment, communication between two parties is very confidential so data security is the integral part of it [1]. In these scenarios, we intentionally try to make signal difficult for interception or detection. In World War II, this necessity aroused and thus for very secretive communication spread spectrum was used [2]. Another advantage of using this technique is jamming. In jamming, communication between two parties can be blocked. In earlier days, spread spectrum techniques were used only in military. From past few years, it gained popularity in general communication as Federal Communications Commission made it available for civil use. Increase in dimensions of signal by increasing bandwidth of signal is basic idea behind the spread spectrum signals. Nowadays, it is used heavily in commercial systems because it causes very less interference to other signals and it provides robustness in fading environment.

1.1 Spread Spectrum Signal

We will see the basic generation process of Spread Spectrum Signal. Let $d(t)$ be data signal and $c(t)$ be a PN sequence (in Figure 1.2). We consider here that $d(t)$ is narrowband signal and $c(t)$ be a wideband signal. As we know, if we multiply low frequency signal with high frequency signal eventually it becomes high frequency signal [3]. This data signal after multiplication with wideband PN sequence will spread over the spectrum and will become wideband. The idealized spread spectrum transmitter is shown in Figure 1.1 and waveforms related to it are shown in Figure 1.2.

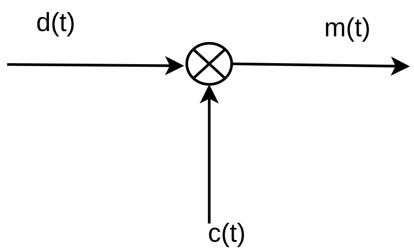
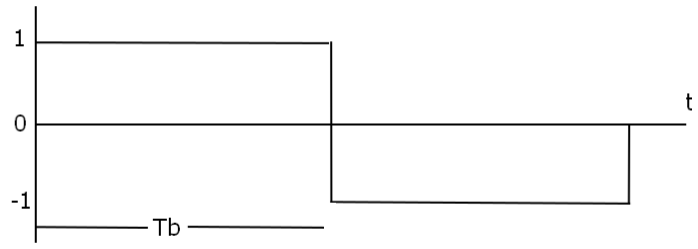


Figure 1.1: Idealized Spread Spectrum Transmitter

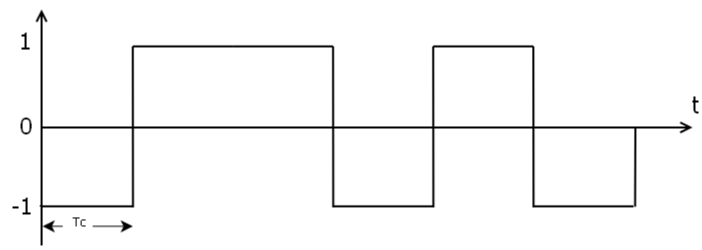
In general there are two spread spectrum techniques:

1.1.1 Direct Sequence Spread Spectrum (DSSS)

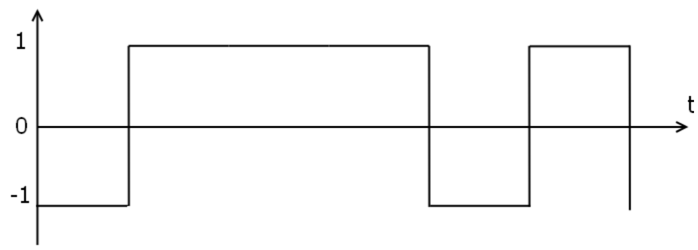
In direct sequence spread spectrum, data signal is directly multiplied with pseudo random noise (PN) signal. This PN sequence is high frequency signal compared to data signal. To generate DSSS signal, we first modulate given data and transfer it into some RF frequency range with simple modulation techniques like QPSK or DSBSC (AM). After getting this modulated signal, it is multiplied with pseudo random noise signal which is of higher frequency compared to modulated signal. In Figure 1.3, we have shown this basic generation technique.



(a) data signal d(t)



(b) PN sequence



(c) spread signal

Figure 1.2: Spread Spectrum Transmitter Waveforms

In this way, we spread energy of signal on large bandwidth. As we increase bandwidth of the signal, robustness of signal increases. It is a measure of goodness of DSSS signal. It is termed as Processing Gain, which is the ratio of DSSS bandwidth to message signal bandwidth. It is obvious that as processing gain increases system becomes more robust.

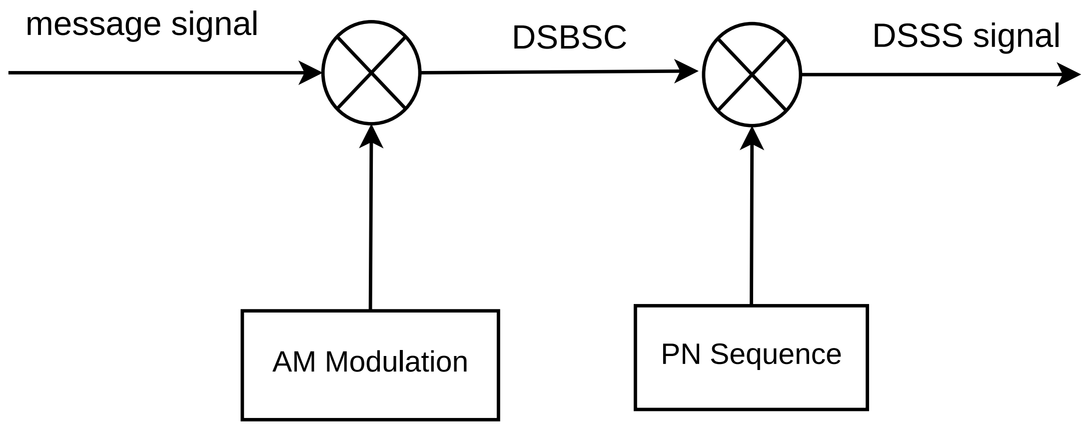


Figure 1.3: Generation Of DSSS Signal

1.1.2 Frequency Hopping Signal

In Direct Spread Spectrum Signal, we increase dimensionality of signal by directly multiplying it with high rate random signal, whereas in frequency hopping signal, it is achieved by periodically changing carrier frequency over some bandwidth which is very large compared to signal bandwidth. Here, frequency is hopped randomly and in non overlapping condition it becomes difficult for jamming and eavesdropping in military environment. Thus for commercial purpose it comes handy in tackling the impact of co-channel interference. Frequency Hopping signal generation can be simply explained as follows. We consider wide frequency band which is large compared to signal bandwidth is divided in such way that it gives N carrier frequencies with non overlapping bands. Data signal hopped on these frequencies is decided by pseudo random sequence. Now we define hopping signal as,

$$h(t) = \sum_{i=1}^{N} p(t - iT_{hopp})cos(2\pi f_i t + \phi_i) \tag{1.1}$$

Here $p(t)$ is pulse shape, generally square waveform. f_i is i_{th} carrier frequency randomly chosen from N available frequencies. T_{hopp} is hopping period and can be defined as dwelling time. Signal dwells in one frequency band for this much time. The phase of i_{th}oscillator is denoted by ϕ_i.

Now FH signal is generated by multiplying hopping signal with data signal.

$$S(t) = d(t)h(t)$$

$$= d(t)[\sum_{i=-\infty}^{\infty} p(t - iT_{hopp}) cos(2\pi f_i t + \phi_i)]$$

Frequency hopping signal generation is shown in Figure 1.4. As time advances, data signal will jump from one frequency to other from N frequencies, determined by pseudo random sequence. Unlike DSSS, in frequency hopping signal bandwidth will not expand with hop period. Power spectral density of signal is spread over the spectrum equally if signal dwells on an average $1/N$ of time.

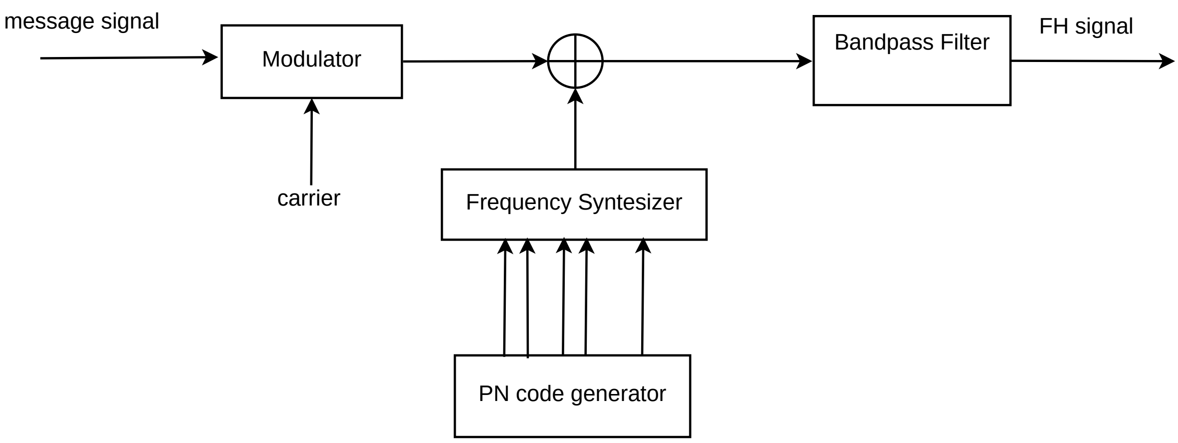


Figure 1.4: Generation of Frequency Hopping Signal

Frequency hopping signal is characterized by the rate at which carrier frequency of signal has been changed. Based on this we differentiate into two types:

- *slow frequency hopping signal*, here modulated signal's symbol rate R_b is integer multiple of hop rate R_h. In this hopping type, in every frequency hop many symbols are transmitted.
- *fast frequency hopping signal* ,here hop rate R_h of signal is integer multiple of symbol rate R_b. In this hopping type, carrier frequency of symbol is changed many times while transmitting.

1.2 Literature Survey

In the modern day communication, with the advent of new technologies, need of more robust and secure communication techniques are increasing. Inevitably, the use of spread spectrum techniques, especially frequency hopping signal, is increasing as it is easy to generate. In recent years more time and brainpower is devoted in doing research in the area of frequency hopping signal [4].

In literature there are few generally used techniques which are applied in detection [5] of frequency hopping signal like energy detection [6], matched filter detection [7], wavelet transform based detection [8] and cyclostationarity detection. We will discuss some of these techniques in next chapter.

We consider cyclostationarity detection as most prominent. As compared to other techniques, it gives very good results but with the expense of computational complexity.In most of the cases, while doing cyclostationarity detection on frequency hopping signal it is assumed that we know the cycle frequency. Otherwise we randomly search all the frequencies, which adds on to computation complexity and detection time. In cyclostationarity detection, it will be better to find frequency, where we get significant feature. This is cycle frequency of frequency hopping signal which depends on hopping rate.

In our work, we will estimate the cycle frequency first and then apply cyclostationarity detection algorithm. For this, we will first find hopping period of frequency hopping signal with the use of algorithm given in [9]. Form this we will calculate cycle frequency. Then we will look deeply into effect of wrong estimation of hopping rate on detection of signal through theoretical and simulation analysis. We will also look at another parameter, which can induce error in detection of signal i.e. the receiver timing mismatch.

1.3 Organisation of thesis

Thesis is organised as follows: In Chapter 2, we will see general techniques used to detect signal. We will compare in those techniques on various characteristics. In Chapter 3, we will discuss about cyclostationarity property of frequency hopping signal. Further, in this chapter we will look into hopping period estimation of frequency hopping signal. We will discuss effect on detection of signal of error in estimation of hopping period. We will also investigate the timing mismatch at the receiver end. Finally in Chapter 4, we present all simulation results.

Chapter 2

Detection Techniques

Detection of signal is nothing but judging whether signal is present or absent. We wish to determine between two hypothesis, whether signal is present with noise or only noise is present. It is done by hypothesis testing. We term it as binary hypothesis testing problem because it decides between two conditions. There is another hypothesis testing problem, where we are going to decide between more than two conditions, it is termed as multiple hypothesis. This multiple hypothesis [10] testing problem is used in pattern recognition, for us it is out of context. Here, we will be dealing only with binary hypothesis testing [11]. We will define two hypothesis as, H_0 null hypothesis where only noise is present and H_1 alternate hypothesis where signal is present with noise.

$$\begin{aligned} H_0 &: \quad y(n) = w(n) \\ H_1 &: \quad y(n) = x(n) + w(n) \end{aligned}$$

Here $y(n)$ is received signal, $x(n)$ is transmitted signal and $w(n)$ is unwanted noise. After making hypothesis we have to decide between these two, for us it will be calculated threshold. If decision statistic is below threshold level then we will go with null hypothesis otherwise with alternate hypothesis.

$$d < \lambda_{th} \quad accept \quad H_0$$

$$d > \lambda_{th} \quad accept \quad H_1$$

Here d is decision statistic and λ_{th} is threshold. In this process of decision making, we can end up doing two errors as follows:

- **False alarm error:** It occurs when we decide in favour of alternate hypothesis when null hypothesis was correct. It is measured as false alarm probability given by,

  $$P_f = P(H_1|H_0)$$

- **Miss detection error:** It occurs when we decide in favour of null hypothesis when alternate hypothesis was correct. It is measured as miss detection probability given by,

  $$P_m = P(H_0|H_1)$$

In hypothesis testing, the real challenge is to minimise both the errors, which is quite impossible, so we try to maintain one of the error at minimum level while decreasing other. There is another measure known as probability of detection, which is correctly deciding in favour of alternate hypothesis stated as,

$$P_d = P(H_1|H_1)$$

In other way, it can be written as,

$$P_d = 1 - P(H_0|H_1)$$
$$P_d = 1 - P_m$$

As stated earlier, we have to decrease probability of miss detection, which is nothing but increasing probability of detection while keeping constraint on probability of false alarm. So in any hypothesis testing problem threshold is determined, keeping in mind that we have to increase probability of detection while keeping probability of false alarm in control.

Now we will see few general techniques used for signal detection.

2.0.1 Energy Detection

Energy detection [6] is the most common way in spectrum sensing and most effective method in terms of computation and implementation complexities but its performance degrades under low SNR condition. Moreover it cannot be used for the detection of spread spectrum signals because we can't distinguish between signal and noise.

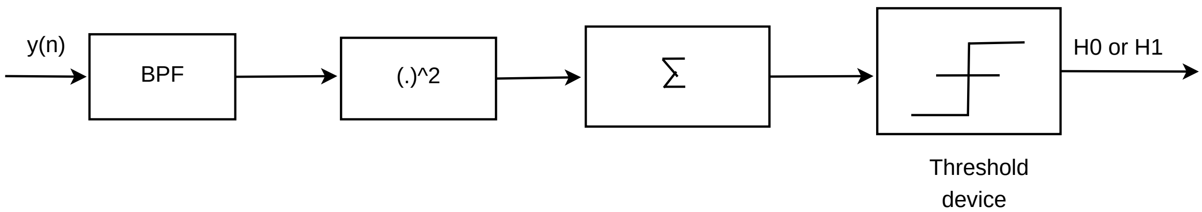


Figure 2.1: Energy Detector

Let us assume received signal is

$$y(n) = x(n) + w(n)$$

$x(n)$ is signal transmitted and $w(n)$ is additive white Gaussian noise. Threshold for detection can be found out by $x(n) = 0$ condition. Decision statistics for it can be written as

$$S = \sum_{n=0}^{N} |y(n)|^2 \tag{2.1}$$

It is used to distinguish between presence of signal and only noise condition. This metric is compared with pre-defined threshold value which depends on probability of false alarm.

$$S < \lambda_{th} \quad accept \quad H_0$$

$$S > \lambda_{th} \quad accept \quad H_1$$

We show this basic working of energy detection in Figure 2.1. In process of finding decision threshold, knowledge of noise power and signal power is required. Noise power can be found out easily but its difficult to find signal power as it changes with transmission characteristics and distance from primary user.

2.0.2 Matched Filter Detection

Matched filter detection technique [7] is used when we have some information about signal at the receiver. It requires less time to attain certain probability of detection and false alarm compared to energy detection. The basic working of matched filter is shown in Figure 2.2.

Matched filter is a linear filter which maximizes signal to noise ratio. We will rewrite the underlined theory behind matched filter and will present known results.

Received signal can be written as,

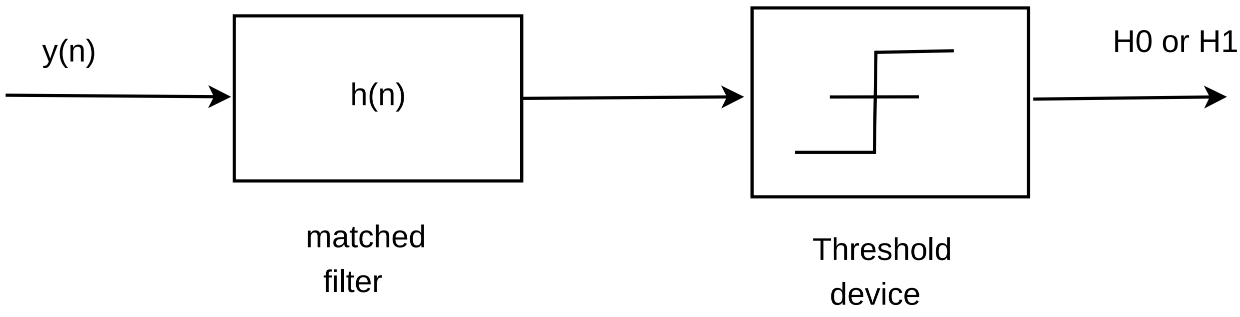


Figure 2.2: Matched Filter Detection

$$y(n) = x(n) + w(n)$$

where $x(n)$ is desired signal and $w(n)$ is noise. We have to pass this received signal through filter $h(n)$ to get output high SNR. Output of this filter can be written as,

$$r[n] = \sum_{k=-\infty}^{\infty} h[n-k]y[k] \tag{2.2}$$

For mathematical simplicity we write it in geometric form as,

$$r = \sum_{k=-\infty}^{\infty} h^*[k]y[k] \tag{2.3}$$

It can be modelled as

$$r = h^H y \tag{2.4}$$

which can be expanded as,

$$r = h^H x + h^H w \tag{2.5}$$

From the above equation, SNR can be given as,

$$SNR = \frac{|h^H x|^2}{E\{|h^H w|^2\}} \tag{2.6}$$

Our aim is to maximize the SNR, so we further investigate and find out the condition where SNR is maximum.

$$h = \frac{R_w^{-1} x}{\sqrt{x^H R_w^{-1} x}} \tag{2.7}$$

Here R_w is covariance of noise given by,

$$R_w = E\{ww^H\} \tag{2.8}$$

One simple way to maximize the SNR of output of matched filter is that we can keep linear filter $h(n)$ as $x(n)$, which ultimately leads to priori knowledge of signal. In detection of signal, we compare output of matched filter with threshold and take decision about it.

$$\Lambda_{MF} = \sum_{n=0}^{N-1} y(n)h^*(n) \lessgtr \lambda_{TH} \tag{2.9}$$

Here λ_{TH} is threshold for the test and Λ_{MF} is decision statistic for detection.

2.0.3 Cyclostationarity Detection

In communication applications, modern signal processing techniques consider signal as stationary random process but man-made signals are produced after processing through fixed pattern of either amplitude, phase or frequency [12] [13]. We can use this inherent property of all modulated signals to estimate some parameters and can be used as good tool in detection of signal. Cyclostationary process in simple words can be stated as process with periodically varying statistic.

Formally we define cyclostationary [14] process as the process in which mean and autocorrelation vary periodically with some period T [12].

$$R_x(t+T, u+T) = R_x(t, u) \tag{2.10}$$

where

$$R_x(t, \tau) = E\{x(t + \frac{\tau}{2}).x^*(t - \frac{\tau}{2})\} \tag{2.11}$$

Here we define new term α cycle frequency which is the frquency at which autocorrelation produces spectral lines, where $\alpha = m/T$. Here T is time with which autocorrelation is periodic. If cycle frequencies are not integer multiple of fundamental cycle frequency then it is known as *polycyclostationary process* . Now we will see some terms in connection with cyclostationarity.

- **Cyclic Autocorrelation:** A simple autocorrelation can be given as,

$$R_{xx}(\tau) = \langle x(t)x(t-\tau)\rangle \tag{2.12}$$

If the given signal $x(t)$ is cyclostationary then we can extend it as,

$$R_x^\alpha = \langle x(t)x(t-\tau)e^{-j2\pi\alpha t}\rangle \tag{2.13}$$

We can express autocorrelation as Fourier series given by,

$$R_x(t+\frac{\tau}{2}, t-\frac{\tau}{2}) = \sum_\alpha R_x^\alpha(\tau)e^{j2\pi\alpha t} \tag{2.14}$$

The cyclic autocorrelation [15] can be written as,

$$R_x^\alpha(\tau) = \lim_{z\to\infty}\frac{1}{z}\int_{-\frac{z}{2}}^{\frac{z}{2}} R_x(t,\tau)e^{-j2\pi\alpha t}dt \tag{2.15}$$

It can be interpreted that it produces spectral lines at α in cyclostationary environment.

And for stationary signal at $\alpha = 0$ it reduces to traditional autocorrelation.

- **Spectral Correlation Function:** If we take Fourier transform of cyclic autocorrelation then we get its equivalent in frequency domain as spectral correlation function as, From Equation 2.15, we can write

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau) e^{-j2\pi f\tau} d\tau \tag{2.16}$$

It can be called as Cyclic Spectral Density of signal which reduces to Power Spectral Density (PSD) at $\alpha = 0$. SCF can be termed as cross correlation between signal which are separated by α. With this background we look forward in detection of signal [16].

- **Spectral Coherence:** We have already found out spectral correlation function which is cross correlation between separated frequency components. Spectral coherence is coefficient for cross correlation and it ranges from 0 to 1. It can be mathematically presented as,

$$C_x^\alpha = \frac{S_x^\alpha}{[S(f + \frac{\alpha}{2})S(f - \frac{\alpha}{2})]} \tag{2.17}$$

In detection of unknown signal, spectral coherence can be used as decision statistic but in most of the cases in literature SCF is used to detect the signal.

Now we define dimensionless quantity crest factor.

$$CF = \frac{\max C_x^{\alpha}}{\sqrt{\frac{\sum_{\alpha=0}^{N}(C_x^{\alpha})^2}{N}}} \tag{2.18}$$

Detection threshold for test is found out by evaluating crest factor from no signal condition i.e. where signal is absent.

$$\lambda_{TH} = \frac{\max C_x^{\alpha}}{\sqrt{\frac{\sum_{\alpha=0}^{N}(C_x^{\alpha})^2}{N}}} \tag{2.19}$$

Decision is taken from the following conditions as

$$CF < \lambda_{th} \quad accept \quad H_0$$
$$CF > \lambda_{th} \quad accept \quad H_1$$

We judge whether signal is present or not by accepting one hypothesis between two hypotheses. Using cyclostationary property, the signal can be sensed at very low SNR condition as additive White Gaussian noise is stationary and cannot affect cyclostationary property of signal.

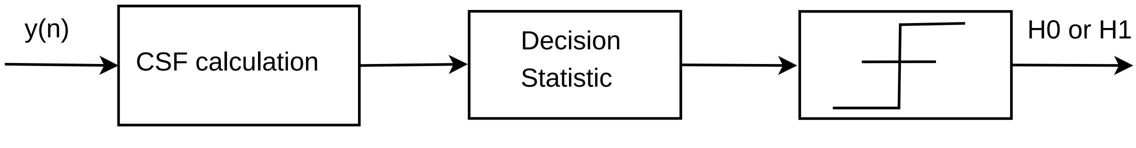


Figure 2.3: Cyclostationarity Detection

In general we do not require knowledge about the signal when we are using cyclostationarity detection. In figure 2.3, we tried to summarize working of cyclostationarity detection. Here we are considering that we do not have any knowledge about the received signal like modulation type or carrier frequency. There is another concept in cyclostationarity because of which we observe distinct features at some frequency. This frequency is known as significant cycle frequency. The significant cycle frequency [17] depends on modulation type. For example in BPSK modulation, it is twice the carrier frequency. If we know the significant cycle frequency, our detection process is simplified. We can calculate decision statistic only for this frequency which reduces computational complexity. If we don't know about cycle frequency then we have to search for every frequency which makes detection process slow.

2.1 Comparison between detection techniques

In previous sections we presented some known and generally used detection techniques. Now, we will compare these techniques by considering some metrics. In general, following are some metrics from which we can easily determine their better performance.

- **Sensing Time** It is the time taken by a detector to decide in between two hypothesis i.e. to judge between signal present or only noise present. Cyclostationarity detection, because of its large computational complexity takes more time time as compared to those other two techniques. Matched filter is the fastest detection technique. In every receiver we have to reduce this sensing time to get greater accuracy.

- **Detection Sensitivity** It is related to accuracy of detection in low SNR condition. Energy detection technique fails drastically in low SNR condition as it cannot differentiate between noise and signal power. Cyclostationarity detection is the better compared to energy detection. In Figure 4.1 we have shown the detection performance of these two techniques.

No.	Detection Technique	Sensing time	Implementation Complexity	Performance Under low SNR
1	Energy detection	More	No	Very Bad
2	Matched filter	Less	Yes	Good
3	Cyclostationarity detection	Most	Yes	Very Good

Table 2.1: Summary of detection techniques and metrics

- **Implementation Complexity**

  Energy detector is very simple to implement compared to other techniques. Cyclostationarity detection requires lot of computation power which increases complexity of hardware and cost of implementation. In case of matched filter, it generates carrier signal which increases difficulty in implementation.

Chapter 3

Detection of Frequency Hopping Signal

Frequency hopping signal is very difficult to detect in the presence of background noise. It is tedious job to distinguish between FH signal and noise. As energy of signal is spread over wide band which makes it difficult to judge between signal energy and noise energy. In case of matched filter detection, it can give good result if we apply dedicated system for it. But in low SNR condition the performance degrades. Thus we use inherent property of FH signal i.e. cyclostationarity. Cyclostationary detection is fairly robust technique in low SNR environment and additive white Gaussian noise environment [18].

3.1 Cyclostationarity of FH signal

FH signal possesses cyclostationarity property as mean and autocorrelation are periodic. Cyclic autocorrelation is given as

$$\Re_x(t,\tau) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t} \tag{3.1}$$

It follows from definition of cyclostationarity. Here we are considering $x(t)$ as frequency hopping signal and obviously second order cyclostationary signal. Now we go one step further in defining cyclic spectral density of FH signal as ,

$$S_x^{\alpha} = \int_{-\infty}^{\infty} \Re_x^{\alpha}(\tau) e^{-2\pi jft} d\tau \tag{3.2}$$

In [19], it is shown that cyclic spectral density of FH is periodic with frequency α , where $\alpha = N/T_H$. It clearly shows that we will have spectral lines at cyclic frequencies which is depending on hopping period of FH signal. If we want to derive test statistic based on spectral density then it is inevitable to know hopping period of signal. In literature, most of the time it is assumed that receiver knows hopping period of signal but its very impractical assumption. Here we will first estimate the period of hopping and then will apply detection strategy.

3.2 Hopping Period Estimation

In literature, often time frequency domain analysis is done to estimate the hopping period where mostly Wigner Ville Distribution is used. Disadvantage of these techniques is degradation of performance in low SNR condition. We apply autocorrelation technique which is relatively easy to understand and apply [9]. This method does not require any prior knowledge of parameters like signal power, hop transition time but we should know range of hopping period crudely, for better results.

Received signal can be written as

$$r(t) = s(t) + n(t) \tag{3.3}$$

where s(t) is desired signal and n(t) is AWGN noise .

Frequency hopping signal is modelled as

$$s(t) = \sqrt{2S}sin(\sum_{i=-\infty}^{\infty} 2\pi f_i + \theta_i)1_{\alpha T_{hop}+(i-1)T_{hop} \leq t < \alpha T_{hop}+iT_{hop}} \tag{3.4}$$

Here S is signal power, f_i and θ_i are respectively frequency and phase of the i^{th} signal. T_{hop} is hop period. At receiver, signal will be observed for certain time which is predetermined. We assume observation time $[0, T)$.

We find the autocorrelation of the signal as

$$y(\tau) = \int_{\tau}^{T} r(t)r(t-\tau)d\tau \tag{3.5}$$

Autocorrelation of this received signal will have terms like signal times signal ,noise times noise and signal times noise term. In low SNR case we can neglect signal times noise term. In the derivation of autocorrelation it is assumed that signal times signal term will be zero after one hop time which is obvious assumption as frequency will not be same in two consecutive hops.

Now power sampling is defined on this autocorrelation as

$$w = y^2(\tau_k) \tag{3.6}$$

where $\tau_k = kB^{-1}$

The main motive behind the calculation of power sampling on the autocorrelation is to suppress the frequency and phase property of the signal while preserving timing property of the signal.

From power sampling, we find power sum statistic as

$$Y = \sum_{\lambda G}^{k=1} (T-\tau_k)^{-2} W_k \tag{3.7}$$

where G is time bandwidth product as $G = TB$ and λ is predefined receiver parameter.

Last terms of the power sampling are of larger variance so to eliminate these terms, we keep condition on the value of λ as less than 0.1. Now after having enough prerequisites, we define new statistic as

$$Z = \frac{1}{GS^2}\frac{1-\lambda}{\lambda}[Y - f(\lambda, G, N_0, B)] + \lambda \tag{3.8}$$

Here,

$$f(\lambda, G, N_0, B) = \sum_{k=1}^{\lambda G} 2N_0^2 B^2 \int_0^1 (1-\rho) Sinc(\pi(G-k)\rho) d\rho \tag{3.9}$$

and

$$sincx = \frac{\sin x}{x} \tag{3.10}$$

Then based on this statistics , a new decision test is derived

$$T_{hop}^{est} = \begin{cases} \rho & Z^2 - \frac{4}{3\lambda^2} \geq (\rho - \frac{\lambda^2}{3\rho}) \\ \xi & Z^2 - \frac{4}{3\lambda^2} \leq (\xi - \frac{\lambda^2}{3\xi}) \\ \frac{1}{2}(\sqrt{Z^2 - \frac{4}{3}\lambda^2}) & \text{otherwise} \end{cases}$$

Here we assume that hop period lies in range $[\xi, \rho]$. If the hopping period of signal does not lie in the assumed range, then our estimation will fail. We estimated hopping period, where we considered different ranges of hopping period. In following table 3.2 we have shown that as we accurately guess the range of hopping period it improves hopping period estimation.

Range/SNR in dB	-30	-25	-20	-15	-10	-5	0	10
0.00045-0.00055	0.00053	0.00048	0.00049	0.00051	0.0005	0.0005	0.0005	0.0005
0.0004-0.0006	0.00058	0.00054	0.00047	0.00049	0.00052	0.0005	0.0005	0.0005
0.0003-0.0007	0.0003	0.00037	0.00061	0.00057	0.00054	0.00053	0.00048	0.0005

Table 3.1: Estimated values of hopping period at different ranges

We can also show the effect of this different ranges on the detection of signal. It is plotted in figure 4.7.

Now, after defining algorithm for the estimation of hopping time, we summarize the whole procedure for the cyclostationarity detection as,

- Step 1 : Estimate the hopping time by using the algorithm given above.
- Step 2 : Calculate cycle frequency as inverse of hopping time.
- Step 3 : Calculate SCF for the calculated cycle frequency and two three frequencies around it.
- Step 4 : Calculate threshold for this from the formula given in Chapter 2 and take the decision based on it.

3.3 Effects of wrong estimation of hopping period

In previous sections, we have seen importance of estimation of hopping period on the detection of signal. In this section we will see how wrong estimation of hopping period affects detection of frequency hopping signal. From Equation 2.15 and 2.16, we can write

$$S_x^{\alpha}(f) = \int_{-\infty}^{\infty} \lim_{z \to \infty} \frac{1}{z} \int_{-\frac{z}{2}}^{\frac{z}{2}} R_x(t,\tau) e^{-j2\pi\alpha t} dt e^{-j2\pi f\tau} d\tau \tag{3.11}$$

If there is error in estimation of cyclic frequency, we consider this as small change $\Delta\alpha$. Now rewriting 3.11

$$S_x^{\alpha+\Delta\alpha}(f) = \int_{-\infty}^{\infty} \lim_{z \to \infty} \frac{1}{z} \int_{-\frac{z}{2}}^{\frac{z}{2}} R_x(t,\tau) e^{-j2\pi(\alpha+\Delta\alpha)t} dt e^{-j2\pi f\tau} d\tau \tag{3.12}$$

We have plotted this in Figure (**??**). We can say from the simulation results that for the value of SCF for error in α estimation i.e. $\Delta\alpha = 0$ we get maximum value for the spectral density. It is very obvious that we will get maximum value of SCF at zero error in estimation of cycle frequency. As the error in estimation of cycle frequency increases, the SCF decreases until it reaches the next harmonic of the cycle frequency. Now we will see the effect of change in SCF on test statistic which affect detection. In the process of detection we find out maximum SCF value from different frequencies. If we wrongly estimate the cycle frequency because of error in estimation of hopping period of signal, the value of S_x^{α} will decrease. The value of Crest factor will decrease which is test statistic. We compare this crest factor with detection threshold and decide whether signal is present or not but as this crest factor value is decreasing. For certain change error in α, crest factor value will go below threshold and it will lead to wrong decision. The effect will be severe in low SNR condition. We simulated this and shown the effect of estimation error of hopping period on the probability of detection.

3.4 Effect of timing mismatch

We try to see effect of another receiver error on detection of signal. At the receiver we generally consider that receiver is completely time matched with the transmitter. If the transmission of symbol started at T and it lasts upto $2T$ unit time but receiver did not start detection at the start of the transmission. It starts sensing the environment at $t + T$ unit time. It produces loss of information at the receiver. We will see from the simulation how much timing mismatch error we can tolerate without hurting the performance of system. This timing mismatch error can be used for the better efficiency as we can deliberately neglect some data samples and can still achieve the same detection performance. We plotted probability of detection against percentage of time mismatch at some SNR value in Figure 4.9 on page 36.

Chapter 4

Simulation Results

For simulations we have considered FH/BPSK signal. Frequency of the signal is varied between 10KHz to 100KHz. We assume sampling frequency of 1MHz. Symbol frequency of signal is 100Hz. Hopping time of signal is 0.5ms considered as fast frequency hopping signal. In FH/BPSK signal generation, first we do BPSK modulation and after that we change carrier frequency of every signal to get FH signal. In simulations, we have combined this process and did it in one step where phase of the signal and carrier frequency is changed at a time. We generated random symbols with symbol frequency and then sampled them at sampling rate which is given by sampling frequency. Then we generate frequency hopping signal as stated earlier. Here in all simulations wherever we generated frequency hopping signal all these parameters are kept same.

In the figure 4.1, we have compared the different detection techniques at different SNR conditions. From figure we can say that cyclostationarity detection is very robust technique compared to energy detection technique as it gives very good results compared to energy detection. Here we tried to detect frequency hopping signal at different SNR values. At -30dB SNR, we can see that cyclostationarity detection outperforms the other technique. Energy detection fail miserably at low SNR values.

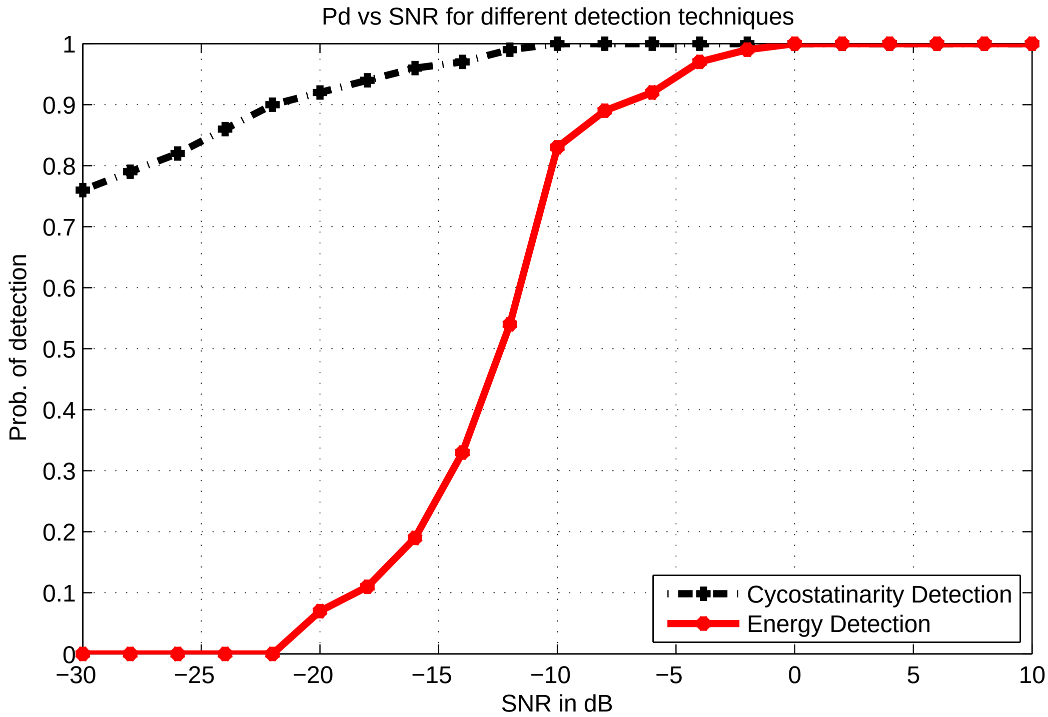


Figure 4.1: PD vs SNR for different detection techniques

In the following figure 4.2 we have shown Cyclic Spectral Density of frequency hopping signal which is periodic with inverse hopping period. It shows very significant magnitude of CSD around the cyclic frequency(inverse of hopping period). At zero cyclic frequency ($\alpha = 0$)we observe peak because it is nothing but power spectral density of the signal. From the figure we can say that it is having high peaks at 2000Hz and it's integer multiple. We get peaks at this frequency because it is inverse of hopping period. We use this property of FH signal in detection of signal.

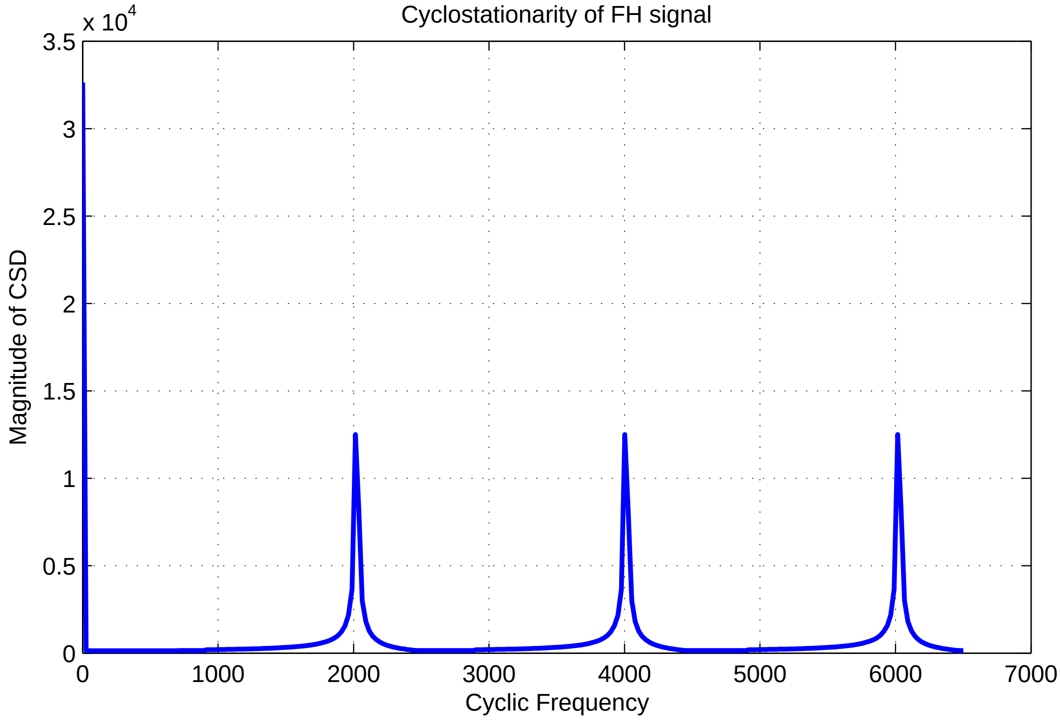


Figure 4.2: Cyclic Spectral Density of Frequency Hopping Signal

In the following Figure 4.3, we plotted the graph of probability of detection (PD) against SNR. Here, while generating FH signal we have kept every parameter same as mentioned above. In detection, we applied algorithm which is illustrated in the thesis. In one plot, we correctly estimated the hopping period so we know accurately the cyclic frequency of signal. In second plot, we first apply hopping time estimation algorithm, from that cyclic frequency is found out and then we apply detection algorithm. We plot this at different SNR values from -30dB to 10dB. In the first plot, where we assumed we know hopping time accurately, PD at -30 dB is around 0.74 and it increases and almost tend to 1 from -15dB, it is 1 at

-10 dB. In the second plot, where we apply hopping time estimation algorithm to find cycle frequency, we can see its almost similar to the first from -18dB SNR. It differs slightly at low SNR conditions as estimation algorithm does get affected in low SNR.

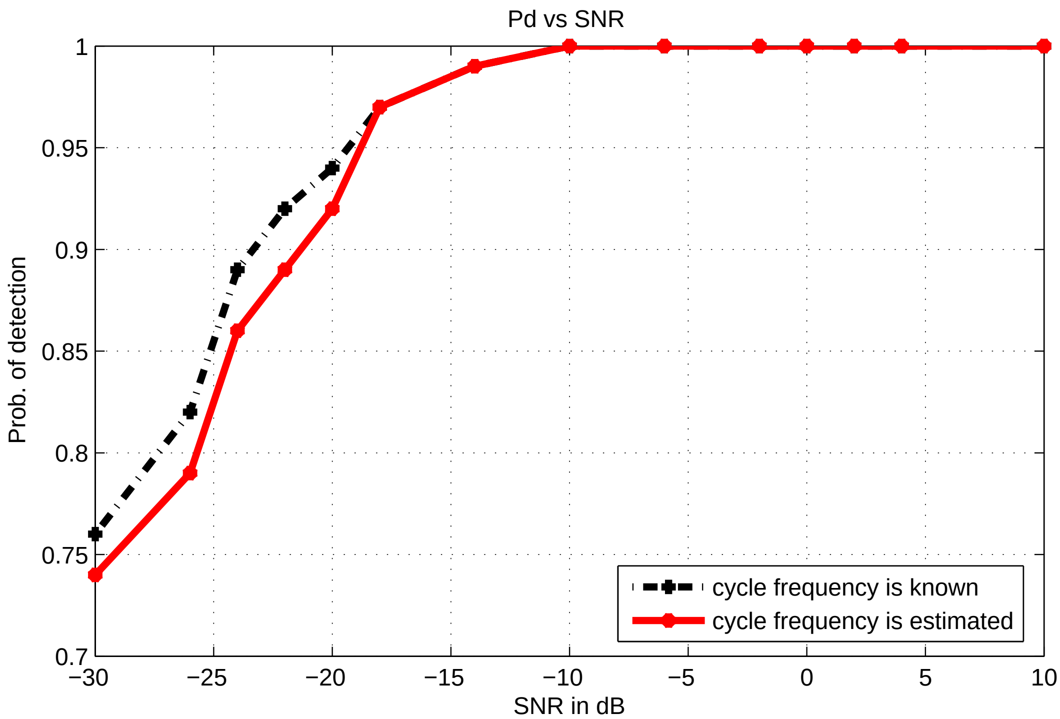


Figure 4.3: Probability of detection versus SNR

In Figures 4.4 and 4.5 , we plotted the effect of percentage change in cycle frequency on the magnitude of spectral correlation function. In figure 4.4, we plotted SCF against error in α, where the signal is not affected by noise. Initially, magnitude of SCF decreases as percentage change in cycle frequency increases but after that it again increases with increase in error. It is because SCF is periodic at cycle frequency, when percentage error in cycle frequency reaches 100 i.e. we are at the second harmonic. Form the plot, we can say if we do error below 10 percent or above 90 percent, there will not be any effect on the detection of signal. In Figure 4.5 we show the effect of noise on the magnitude of SCF. We plotted the magnitude of SCF against percentage error in estimation of α for different SNR. We plotted only upto 50 percent error in α since SCF can increase beyond that. In this figure, we can see magnitude of SCF decreases at low SNR condition very fast compared to high SNR. It is going to affect the detection of signal.

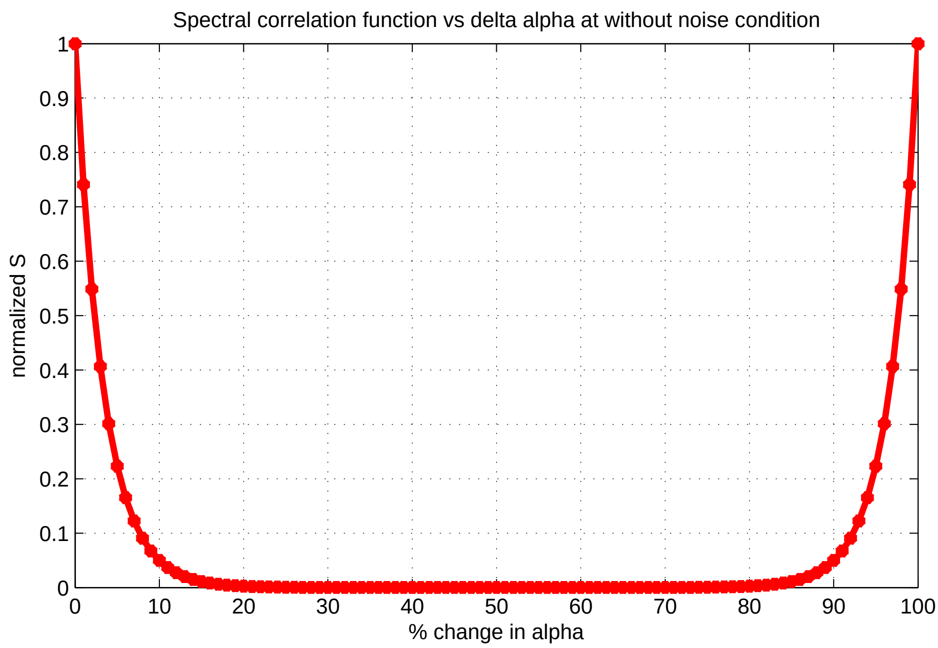


Figure 4.4: Spectral correlation function versus percentage error in cyclic frequency at without noise

In figure 4.6, we plotted the Probability of detection against percentage error in α value. From this plot, we can see that at high SNR values, if there is error in cycle frequency below

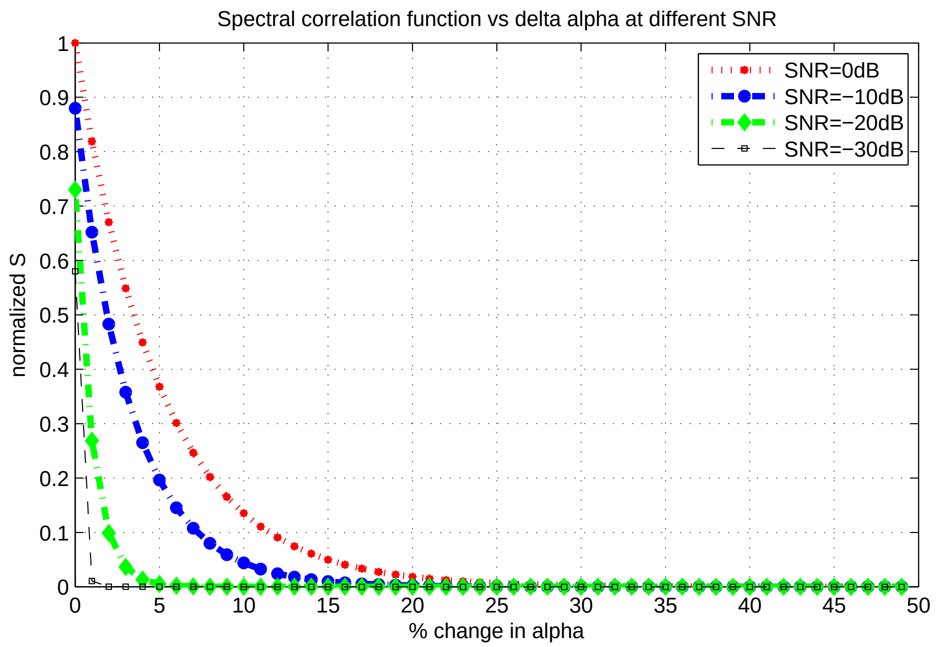


Figure 4.5: Spectral correlation function versus percentage error in cyclic frequency at different SNR

13 percent, we can tolerate this without affecting the detection. If the error in estimation of cycle frequency increase further then the SCF will fall below the threshold and we can't detect the signal. This effect is severe in low SNR condition as in low SNR condition the margin for error in cycle frequency is decreased to 2-3 percent. From this, we can infer that cyclostationarity detection is very sensitive to correct estimation of hopping time.

In the hopping time estimation algorithm, we have to guess the range of hopping time beforehand, in which hopping time will lie. The accuracy of the algorithm depends on how accurately we guess the range of it. In following figure 4.7, we plot the probability of detection against SNR for different ranges of hopping time. In simulation actual hopping time is 5ms. We plot for different ranges like 0.3-0.7ms,0.35-0.65 ms,0.4-0.6ms and 0.45-0.55ms. It is

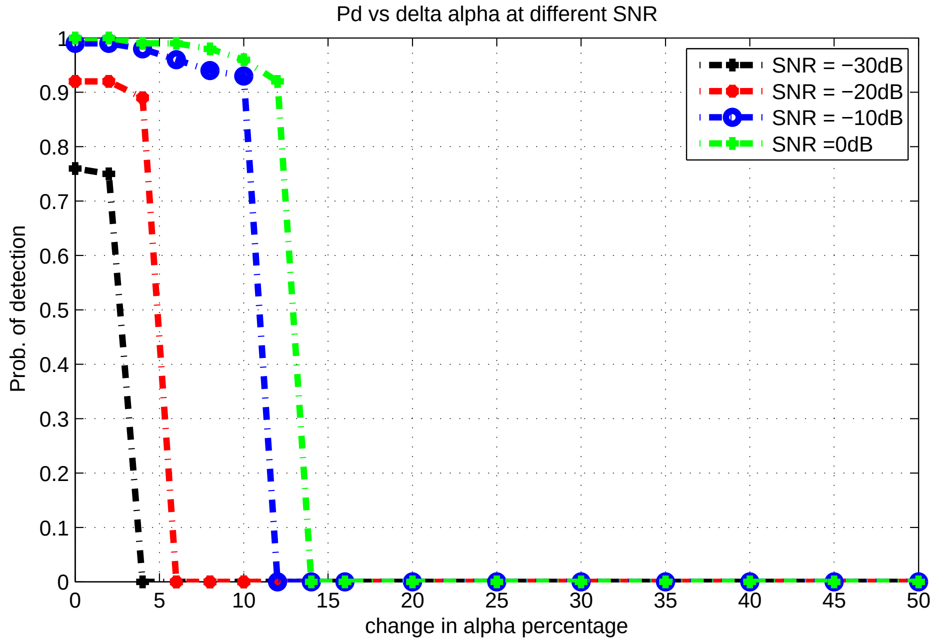


Figure 4.6: Pd vs percentage change in cycle frequency at different SNR

obvious that, if our guess is near to actual hopping time, then we will get best result. It can be also seen from the figure, where for the range 0.45ms-0.55ms we get best result. As we go away from this guess, our result degrades.

In Figure 4.8, we plot the probability of detection against SNR but this time with different estimated hopping period. In this graph we will show that if there is error in hopping time estimation, detection performance degrades. From the graph, we can easily see if the hopping period deviates more from actual hopping period probability of detection lowers. In figure, when we estimate hopping period as 0.42ms or 0.57ms instead of correct estimation of 0.5ms, then probability of detection decreases significantly in low SNR condition while its effect is not so severe at higher SNR values. It can also be judged, if we estimate the hopping

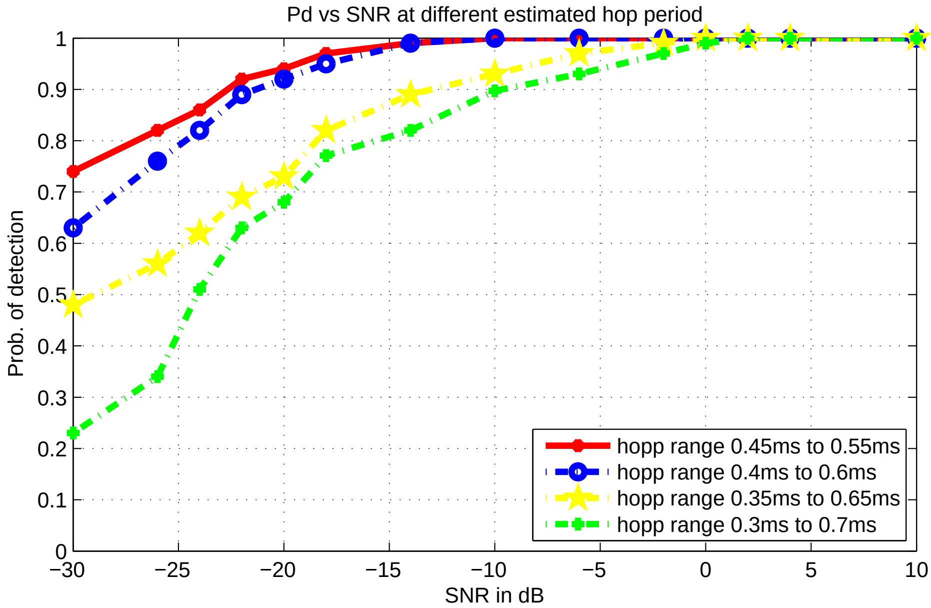


Figure 4.7: Pd vs SNR at different ranges of hopping period

period correctly or with less error then it attains maximum probability of detection very fast.

In the next Figure 4.9 we show the effects of timing mismatch at receiver. We plot probability of detection against percentage of timing mismatch at different SNR values. We can see from the graph that for the low timing mismatch we can detect the signal very easily as probability of detection is almost equal to the perfect timing detection of signal. The effect of timing mismatch is very severe in low SNR condition. In the simulation, we have considered one symbol time and then plotted percentage of mismatch in timing. From the graph we observe that we can tolerate the timing mismatch of upto 40 percent where probability of detection is close to, what we would have got with perfect timing.

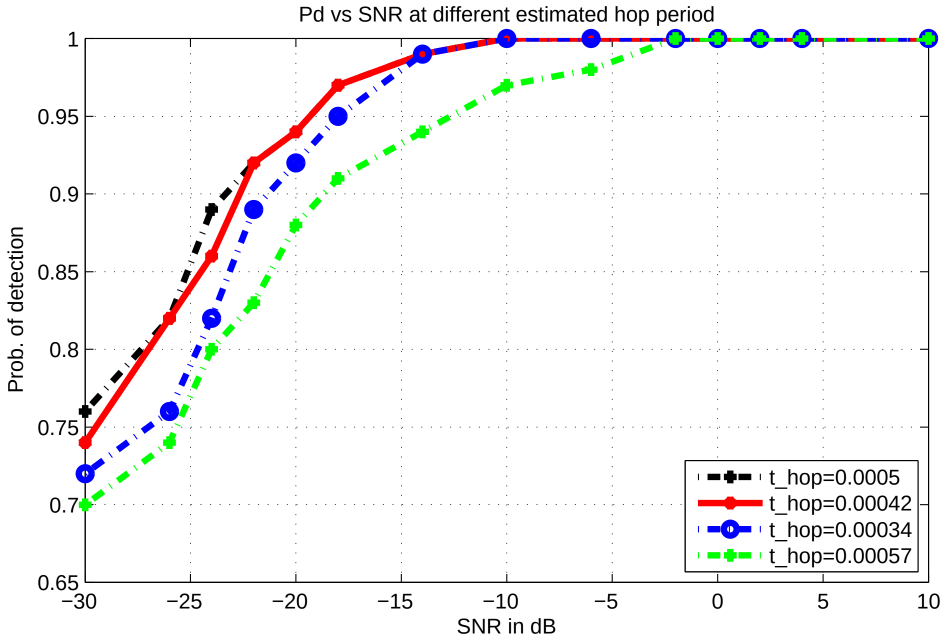


Figure 4.8: Pd vs SNR at different T_{hop}^{est} values

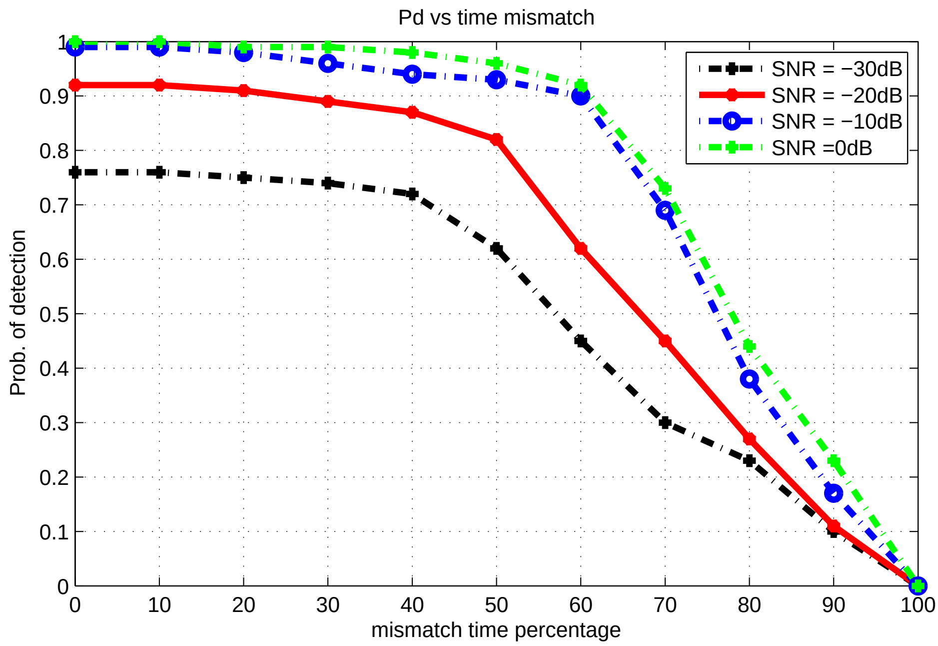


Figure 4.9: Probability of detection vs percentage of timing mismatch

Chapter 5

Conclusion and Future work

5.1 Summary

In this thesis, we presented cyclostationarity detection of frequency hopping signal. We used the hopping period estimation algorithm from the literature [9] to estimate the hopping period. From this hopping period we calculate cyclic frequency and we use cyclic frequency to detect the signal more efficiently. Here we also discussed about effect of error in estimation of hopping period on detection of signal. And we observed that for more accurate detection we should have accurate knowledge of hopping period otherwise we have to search for every frequency which takes more sensing time. We have also shown the effects of timing mismatch on the detection of signal. To summarize, we have shown the detection of frequency hopping signal and effects of various parameters on detection of frequency hopping signal when we are using cyclostationarity property.

5.2 Future work

In the process of estimating hopping period, we assumed that we know the range of hopping period and success of the estimation depends on how accurately we guess the range of hopping period. This is very important issue and can be worked on to reduce the dependence of this range.

Theoretical analysis of the effects of cyclic frequency estimation error and timing mismatch on the detection can be given to verify simulation result.

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