Description of Organic Reactions Based on Imaginary Transition Structures. 3. Classification of One-String Reactions Having an Odd-Membered Cyclic Reaction Graph

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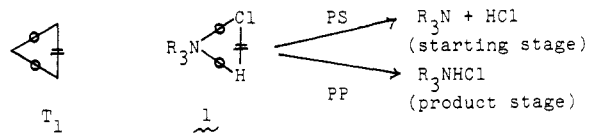
One-string reactions having a trigonal, a pentagonal, and a heptagonal reaction graph are classified and enumerated.

The preceding papers of this series[1] have proposed several concepts for the description of organic reactions: imaginary transition structures (ITS's), graphs of reaction centers (RC graphs) of various levels, reaction graphs, reaction strings, etc. The imaginary transition structures, wherein in-bonds (—o—) and out-bonds (—//—) appear alternately and par-bonds (—) are present as invariant bonds, describe the whole structural information of individual reactions. The RC graphs of level 2, of level 1, and of level 0 (the reaction graphs) indicate the reaction types corresponding to various classification levels.

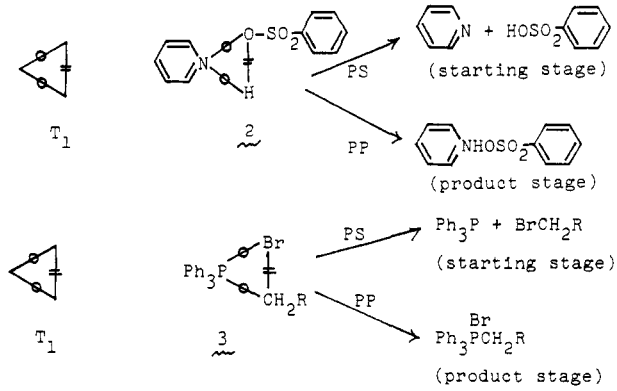
Enumeration of even-membered cyclic reaction graphs has been reported previously.[1] The present article will deal with enumeration of trigonal, pentagonal, and heptagonal reaction graphs and several examples of organic reactions having such graphs.

EXAMPLES OF ONE-STRING REACTIONS HAVING A TRIGONAL REACTION GRAPH

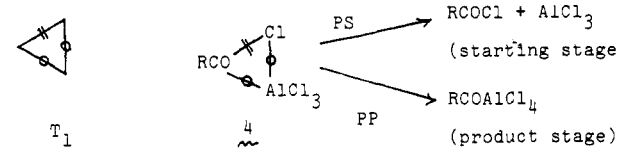
Change between ionic and covalent character of a bond is rather continuous. In order to avoid such confusion as comes from the duality of chemical bonds, representation as covalent bonds rather than as ionic bonds is preferred. This convention makes it possible to denote the formation of ammonium salts by an imaginary transition structure (**1**), from which the



corresponding trigonal reaction graph (T_1) can be extracted.[2] Other onium salt formations can be written in the form of trigonal reaction graphs as follows:

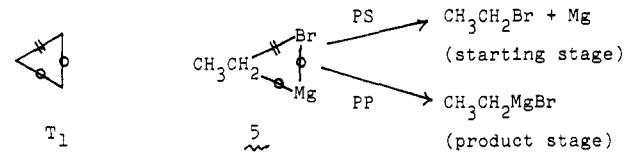


Acylinium salt formation in the Friedel–Crafts reaction[3] is represented by ITS **4** in the light of the present convention.



The same trigonal reaction graph (T_1) as above can be picked up easily.

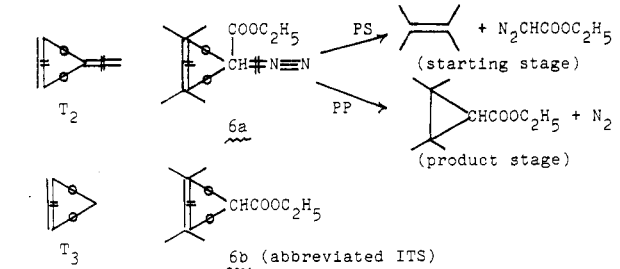
The formation of Grignard reagents[4] produces ITS **5**, which contains the same trigonal reaction graph (T_1). In the ITS



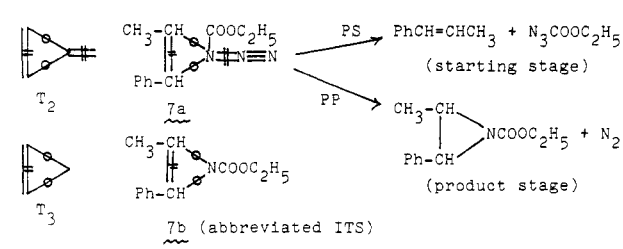
approach, attention is focused only on formation and cleavage of bonds. Thus, the same graph (T_1) is extracted from ITS's **4** and **5**, although the polarities of carbon atoms in ITS's **4** and **5** are opposite to one another. This treatment makes it possible to manipulate information of organic reactions without consideration of their reaction mechanisms.

Several trigonal reaction graphs appear in abbreviated forms (or substructures) of ITS's. The following examples are representative. It is noted that the abbreviated ITS's can be explained also in terms of intermediacy of carbenes or nitrenes. Formal expansion of valency is presumed in such cases.

Cyclopropane formation via carbene:[5]



Aziridine formation via nitrene:[6]



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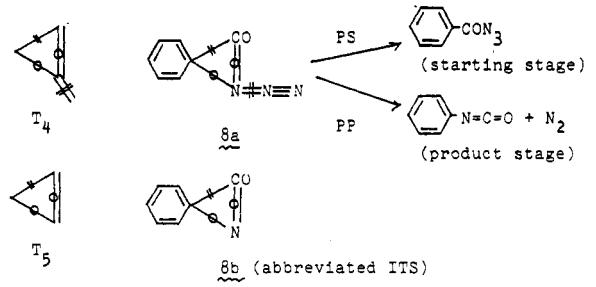
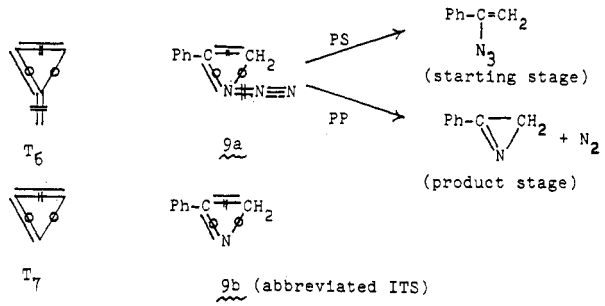
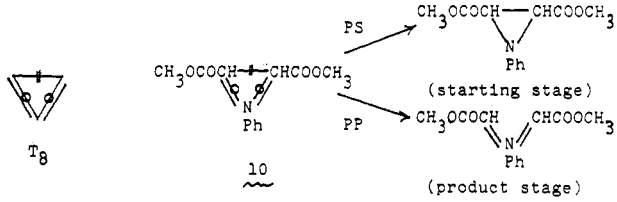
Curtius rearrangement:[7]



Azirine formation from a vinyl azide:[8]



Ring opening of an aziridine:[9]



In the present ITS approach, the perception of ring-opening reactions is translated into finding bridges of ring opening of order p (BOp).[10] For example, ITS **10** has a bridge of ring opening of order 1 (BO1), which corresponds to the ring opening of an aziridine. Ring-closure reactions are related to the presence of bridges of ring closure of order p (BCp).[11] Thus, BC1 appears in ITS **9a** or **9b**, which represents formation of an azirine ring by decomposition of a vinyl azide. The perception of BC2 in ITS's **6a** (or **6b**) and **7a** (or **7b**) reveals that these reactions are ring formations. Rearrangement reactions correspond to bridges of rearrangement (BR).[12] The ITS **8a** or **8b** of the Curtius rearrangement contains a BR.

ENUMERATION OF TRIGONAL REACTION GRAPHS

Enumeration of trigonal reaction graphs is equivalent to counting isomers of basic graphs T_1 and T_1' with m double

par-bonds and n single par-bonds. Since an even-membered basic reaction graph is identical with the corresponding reverse basic graph, there are several self-reaction pairs, each of which is invariant with respect to the transformation to the reverse reaction.[1] On the other hand, an odd-membered basic graph of the present case is different from the corresponding reverse basic graph, e.g., T_1 vs. T_1', and thus there are no self-reaction pairs. Therefore, it is sufficient for the present purpose to count trigonal reaction graphs based upon either T_1 or T_1'. Then, replacing in- and out-bonds by each other will produce the reverse reaction graphs.

This enumeration can be achieved by use of Polya's theorem.[13] The basic graph (T_1) is superposed on itself by two permutation operations, (1)(2)(3) and (2)(13), when the edges of T_1 are numbered as above. Since these operations construct a permutation group denoted by $\mathbf{C}_2$, the cycle index $Z(\mathbf{C}_2)$ is represented by the following equation:

$$Z(\mathbf{C}_2) = (1/2)(s_1^3 + s_1 s_2) \quad (1)$$

If each variable s_k in the cycle index is substituted by figure-counting series

$$s_k = 1 + x^k + y^k \quad (2)$$

then

$$G(x,y) = Z(\mathbf{C}_2, 1 + x^k + y^k) = 1 + 2x + 2y + 2x^2 + 3xy + 2y^2 + x^3 + 2x^2y + 2xy^2 + y^3 \quad (3)$$

The resulting $G(x,y)$ is the series of counting reaction graphs of trigonal class and has the coefficients of $x^m y^n$ indicating the numbers of the above-described isomers.

The full list of trigonal reaction graphs based on T_1 as well as on T_1' is found in Table I. The two reaction graphs of each reaction pair have the same par-bond skeleton and reverse in- or out-bonds with respect to one another. Table I contains several reaction graphs unreasonable from an organic chemistry viewpoint in order to guarantee completeness of the list.

Table I. Reaction Graphs of the Trigonal Class[a]

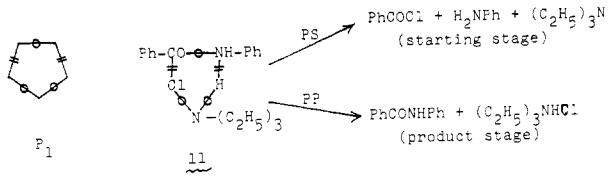
m	n	no. of reacn graphs[b]	reaction graphs based on T_1	reaction graphs based on T_1'
0	0	1		
0	1	2	BR, BC2	BR, BO2
0	2	2	BO1, BC1	BC1, BO1
0	3	1		
1	0	2	BR, BC2	BR, BO2
1	1	3	BC1, BO1, BC1	BO1, BC1, BO1
1	2	2		
2	0	2	BC1, BO1	BO1, BC1
2	1	2		
3	0	1		

[a] This table may contain several reaction graphs that are not reasonable chemically, but they are not omitted for completeness of the list. [b] The numbers of reaction graphs (based on either T_1 or T_1') are the coefficients of $x^m y^n$ in $G(x,y)$ (eq 3).

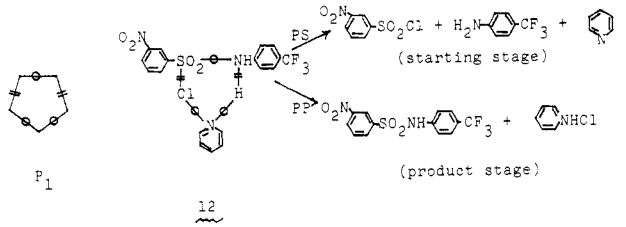
EXAMPLES OF ONE-STRING REACTIONS HAVING A PENTAGONAL REACTION GRAPH

When triethylamine is used as a base in amide formation from benzoyl chloride and aniline, the resulting ammonium

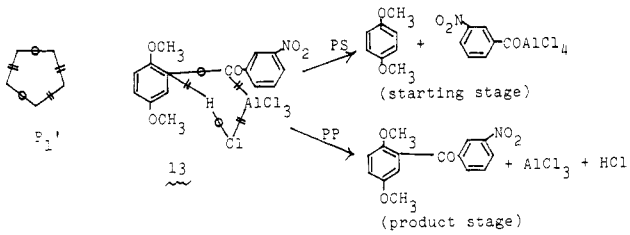
salt is stable enough not to evolve hydrogen chloride gas.[14] In this case, the ITS (**11**) possesses a pentagonal reaction graph



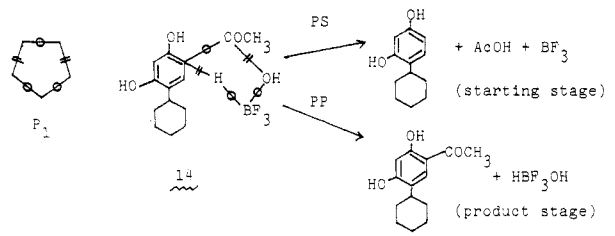
(P_1), which is a basic graph of this class. The formation of sulfonamide from a sulfonyl chloride and an aniline derivative in the presence of pyridine[15] gives ITS **12**, which consists of the same tetragonal reaction graph.



The Friedel–Crafts reaction involves formation of the acylinium ion (see ITS **4**) and the subsequent electrophilic substitution, the latter of which affords an ITS (**13**) of pentagonal class.[16] It should be noted here that the total process of the Friedel–Crafts acylation is represented by an ITS of tetragonal class, which is produced by superposition of ITS's **4** and **13**. In this case, hydrogen chloride gas evolves because of the instability of $HAlCl_4$.

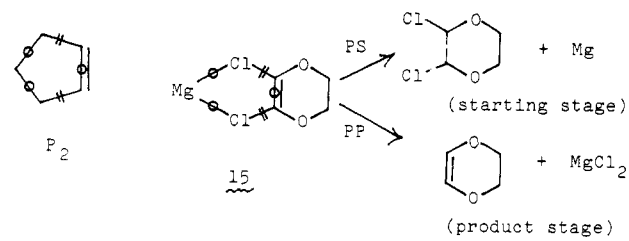


On the other hand, the total process of BF_3-promoted acylation of a resorcinol derivative[17] affords an ITS (**14**) of pentagonal class. This result comes from the fact that the resulting HBF_3OH is presumed to be stable.

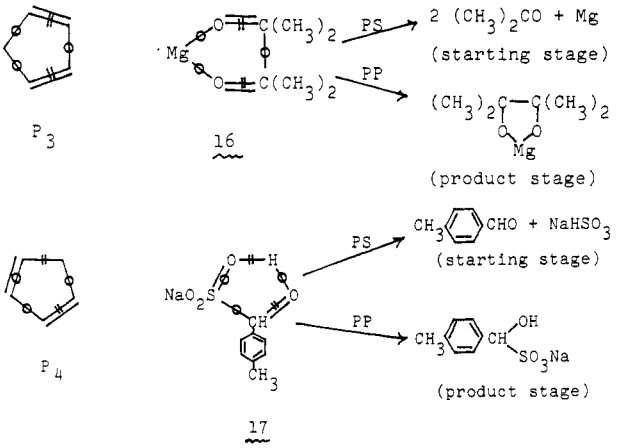


Information about par-bonds modifying the basic graphs such as P_1 and P_1' is essential for the retrieval of organic reactions. From this viewpoint, examples cited in Arens's article[18] are reexamined in the light of the present method. First, let us examine reaction graphs based upon the basic graph (P_1).

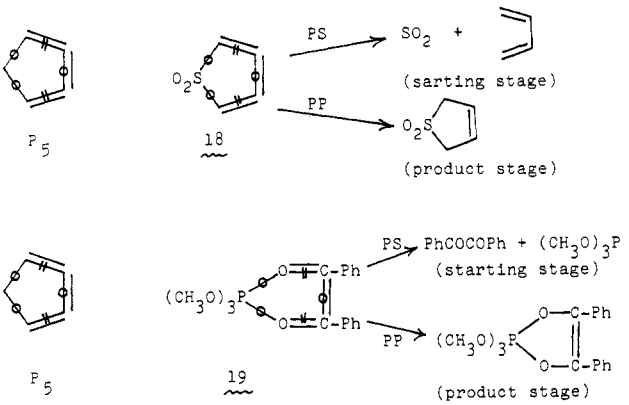
Reductive dechlorination of 2,3-dichloro-1,4-dioxane with magnesium[19] is represented by ITS **15**. The corresponding reaction graph is a pentagonal one modified by a single par-bond ($m = 0$, $n = 1$).



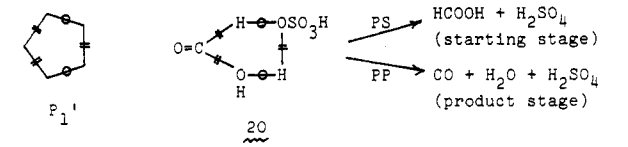
Modification by two single par-bonds ($m = 0$, $n = 2$) can be illustrated by formation of pinacol (ITS **16**)[20] and addition of sodium hydrogen sulfite to an aldehyde (ITS **17**).[21]



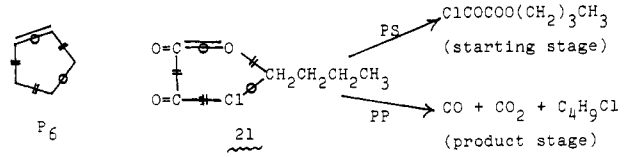
The reaction graphs of formation of "butadiene sulfone" (ITS **18**)[22] and of formation of a pentacoordinate phosphorus compound (ITS **19**)[23] are characterized by par-bond modification of $m = 0$ and $n = 3$.



Decomposition of formic acid by sulfuric acid[24] is represented by ITS **20**, which belongs to the pentagonal class based on the reverse basic graph (P_1'). In the present paper, carbon monoxide is regarded tentatively to have a divalent carbon atom.

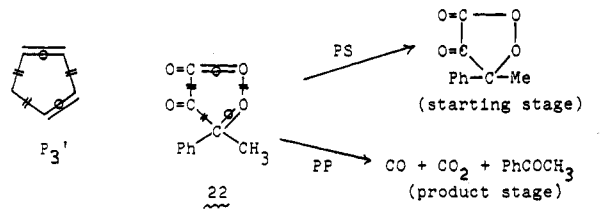


Decomposition of a derivative of oxalic acid[25] is expressed in terms of ITS **21**, which has a pentagonal reaction graph (P_6) modified by a single par-bond ($m = 0$, $n = 1$).

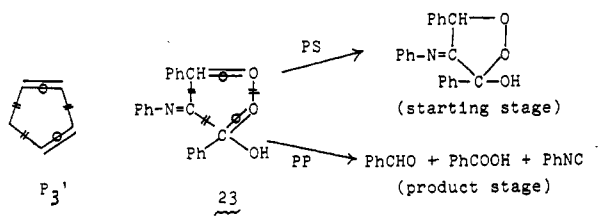


A variety of reactions are characterized by reaction graphs of the pentagonal class, which is based on the basic graph (P_1'), modified by two single bonds. The following examples are representative.

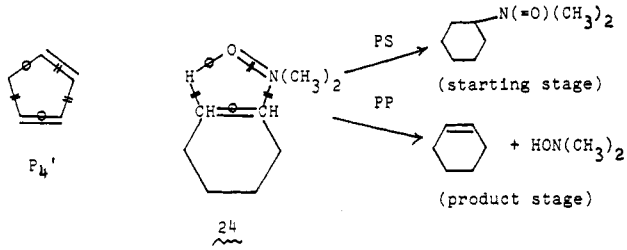
Fragmentation of a peroxylactone:[26]



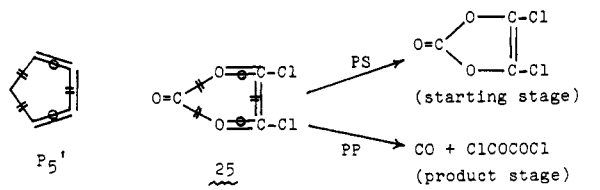
Fragmentation of a cyclic peroxide:[27]



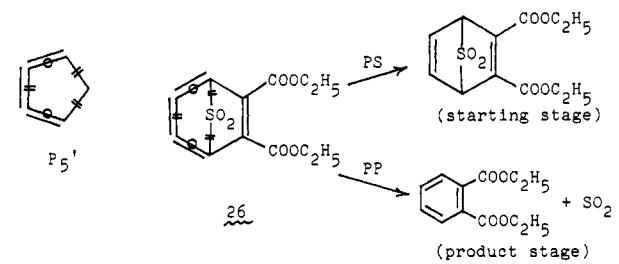
Cope elimination:[28]



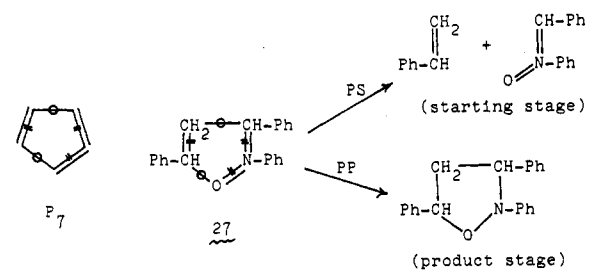
Fragmentation of a cyclic carbonate[29] is represented by ITS **25**. The reaction graph (P_5') abstracted from **25** is characterized by par-bond modification of $m = 0$ and $n = 3$.



Formation of a benzene derivative via SO_2 elimination[30] is a reverse reaction of butadiene sulfone formation (see ITS **18**). Thus, the corresponding ITS (**26**) gives reaction graph P_5', which constructs a reaction pair along with the graph P_5.

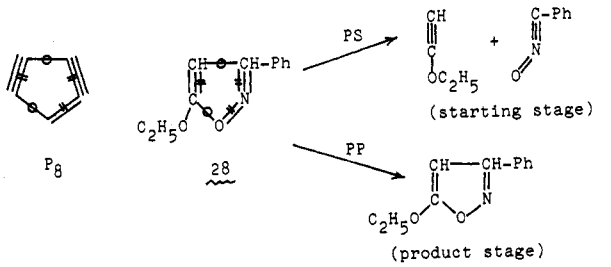


1,3-Dipolar cycloaddition of a nitrone[31] claims an ITS (**27**)



that affords a reaction graph P_7 ($m = 0$, $n = 3$).

1,3-Dipolar addition of a nitrile oxide to an acetylene[32] is represented by ITS **28**. The abstracted reaction graph (P_8)

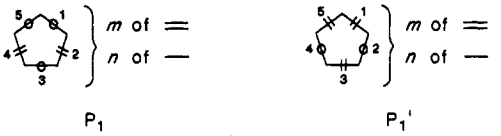


is characterized by modification of two double par-bonds ($m = 2$) and a single par-bond ($n = 1$). In this paper, the functional group $\equiv N^+—O^-$ is represented by a covalent form such as $\equiv N{=}O$, which has a pentavalent nitrogen atom formally.

The reaction graph P_5' is a bridge of ring opening of order 2 (BO2). This fact implies that the reaction represented by ITS's **25** and **26** are cleavages of five-membered rings. A bridge of ring opening of order 3 (BO3) appears in the reaction graph P_3', which is related to fragmentation of a peroxylactone (ITS **22**) and of a cyclic peroxide (ITS **23**). The reaction graph P_5 is a bridge of ring closure of order 2 (BC2), which corresponds to formation of five-membered rings by 1 + 4 cycloaddition reactions. Other types of BC2 appear in the reaction graphs P_7 and P_8, which are related to 2 + 3 cycloadditions (ITS's **27** and **28**). The reaction graph P_3 is a bridge of ring closure of order 3 (BC3).

ENUMERATION OF PENTAGONAL REACTION GRAPHS

The basic graphs for enumeration are P_1 and P_1', which have C_2 symmetry. The problem is the counting of isomers of P_1 or P_1', the edges of which are substituted with m double par-bonds and n single par-bonds. The basic graph (P_1) can



be superposed upon itself by two permutations, (1)(2)(3)(4)(5) and (3)(15)(24), when the edges of P_1 are numbered as shown.

According to Polya's theorem,[13] the cycle index $Z(C_2)$ and the reaction graph counting series $G(x,y)$ are obtained as follows:

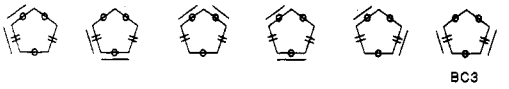
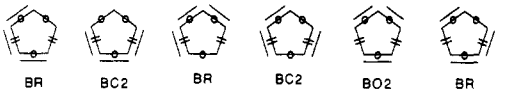
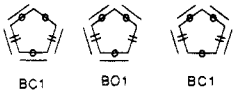
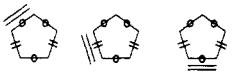
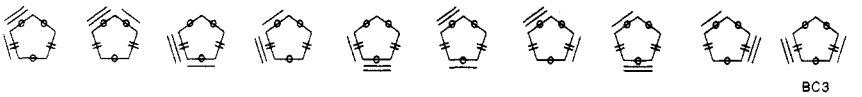
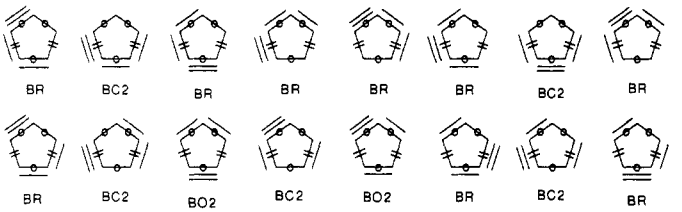
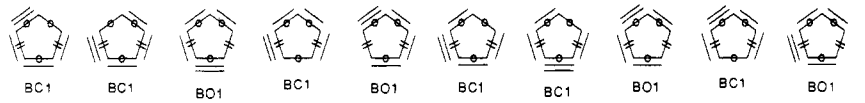
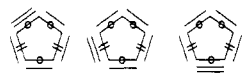
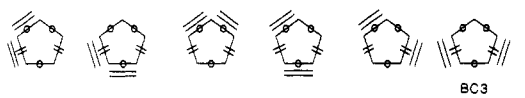
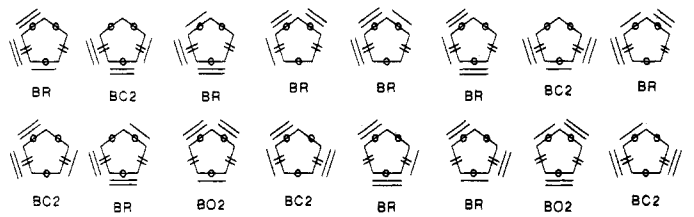
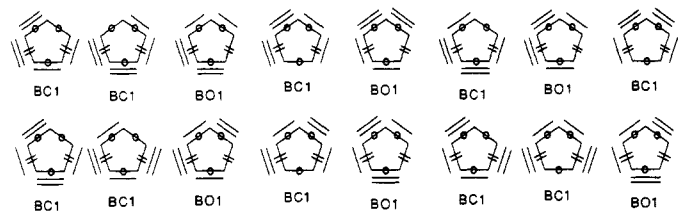
$$Z(C_2) = (1/2)(s_1^5 + s_1 s_2^2) \tag{4}$$

$$\begin{aligned} G(x,y) &= Z(C_2, 1 + x^k + y^k) \\ &= 1 + 3x + 3y + 6x^2 + 10xy + 6y^2 + 6x^3 + \\ &\quad 16x^2y + 16xy^2 + 6y^3 + 3x^4 + 10x^3y + \\ &\quad 16x^2y^2 + 10xy^3 + 3y^4 + x^5 + 3x^4y + \\ &\quad 6x^3y^2 + 6x^2y^3 + 3xy^4 + y^5 \end{aligned} \tag{5}$$

The coefficients of $x^m y^n$ indicate the numbers of the isomers of reaction graphs.

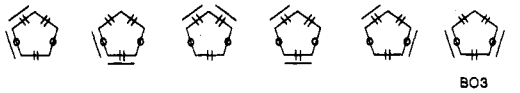
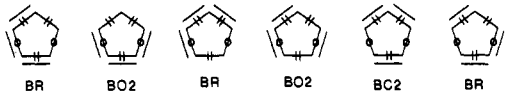
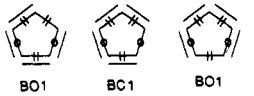
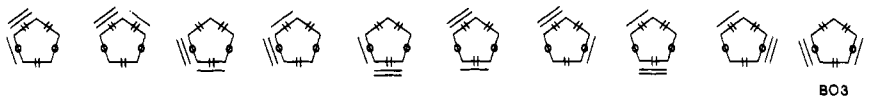
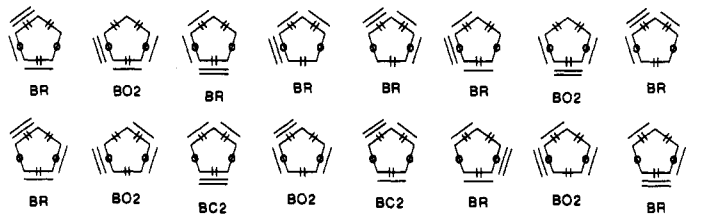
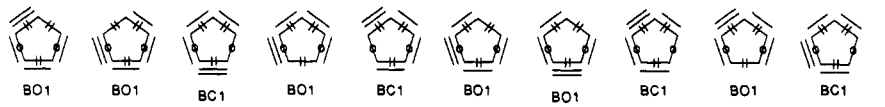
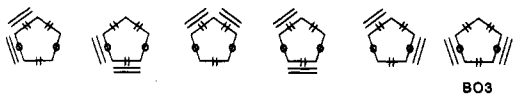
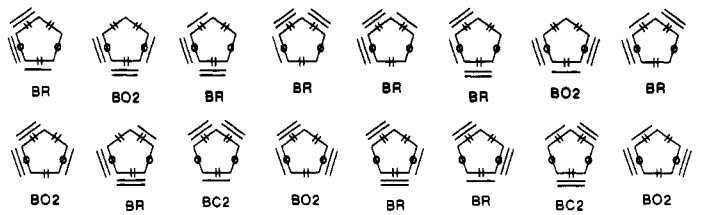
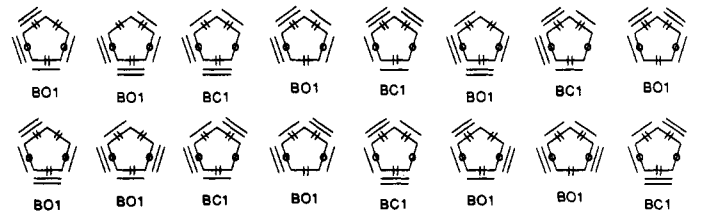
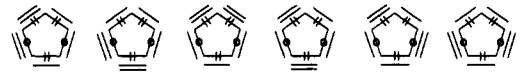
The full lists of reaction graphs ($m = 0$–2) based on P_1 and P_1' are shown in Table II and III, respectively. Reaction graphs appearing at corresponding positions of Tables II and III construct a reaction pair, indicating a pair of a given reaction and the corresponding reverse one.

Table II. Reaction Graphs of Pentagonal Class Based on P_1 (m = 0–2)

m	n	no. of reacn graphs[a]	reaction graphs
0	0	1	
0	1	3	
0	2	6	
0	3	6	
0	4	3	
0	5	1	
1	0	3	
1	1	10	
1	2	16	
1	3	10	
1	4	3	
2	0	6	
2	1	16	
2	2	16	
2	3	6	

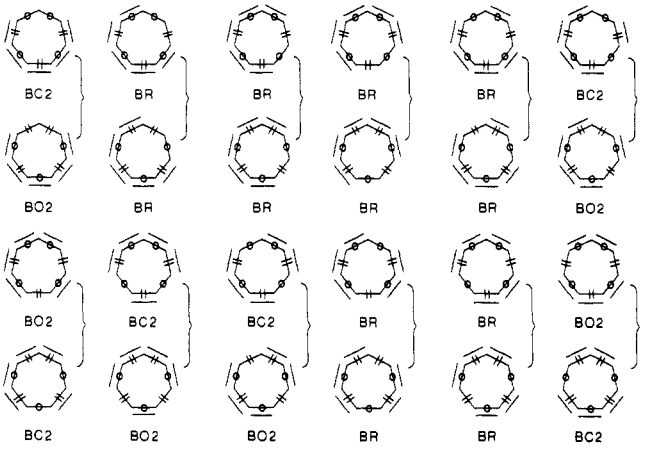
[a] See eq 5.

Table III. Reaction Graphs of Pentagonal Class Based on P'_1 (m = 0–2)

m	n	no. of reacn graphs[a]	reaction graphs
0	0	1	
0	1	3	
0	2	6	
0	3	6	
0	4	3	
0	5	1	
1	0	3	
1	1	10	
1	2	16	
1	3	10	
1	4	3	
2	0	6	
2	1	16	
2	2	16	
2	3	6	

[a] See eq 5.

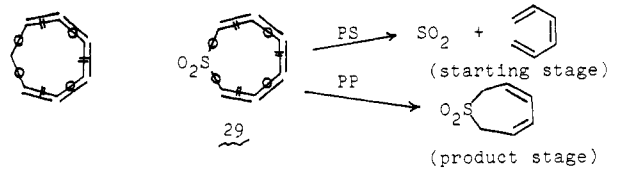
Table IV. Reaction Graphs of Heptagonal Class ($m = 0$, $n = 5$)[a]



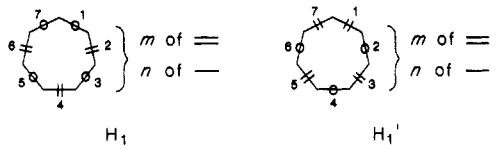
[a] Each couple of reaction graphs linked with a brace is a reaction pair.

EXAMPLES AND ENUMERATION OF ONE-STRING REACTIONS HAVING A HEPTAGONAL REACTION GRAPH

Addition of SO_2 to a hexatriene[33] is an example of this class. The sulfur atom enlarges its valency during this reaction.



The procedure of enumeration is similar to the above. Isomers of reaction graphs based on either basic graph H_1 or H_1' can be counted by the following cycle index $Z(\mathbf{C}_2)$ and



reaction graph counting series $G(x,y)$. The coefficients of $x^m y^n$ in $G(x,y)$ are the numbers of isomers with m double par-bonds and n single par-bonds.

$$Z(\mathbf{C}_2) = (1/2)(s_1^7 + s_1 s_2^3) \tag{6}$$

$$\begin{aligned} G(x,y) &= Z(\mathbf{C}_2, 1 + x^k + y^k) \\ &= 1 + 4x + 4y + 12x^2 + 21xy + 12y^2 + 19x^3 + \\ &\quad 54x^2y + 54xy^2 + 19y^3 + 19x^4 + 70x^3y + \\ &\quad 108x^2y^2 + 70xy^3 + 19y^4 + 12x^5 + 54x^4y + \\ &\quad 108x^3y^2 + 108x^2y^3 + 54xy^4 + \\ &\quad 12y^5 + 4x^6 + 21x^5y + 54x^4y^2 + 70x^3y^3 + \\ &\quad 54x^2y^4 + 21xy^5 + 4y^6 + x^7 + 4x^6y + 12x^5y^2 + \\ &\quad 19x^4y^3 + 19x^3y^4 + 12x^2y^5 + 4xy^6 + y^7 \end{aligned} \tag{7}$$

Since the number of heptagonal reaction graphs is too enormous to show the full list, only the graphs of $m = 0$ and $n = 5$ are collected in Table IV. Each pair linked by a bracket is a reaction pair, where the upper graph is based on H_1 and the lower one on H_1'.

CONCLUSION

Reactions having trigonal, pentagonal, and heptagonal reaction graphs have been discussed. Enumeration of these reaction graphs has been conducted by the use of Polya's theorem.

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(11) A ring structure wherein p of ITS bonds have complex bond numbers $(a\ b)$ of $a = 0$ and all other ITS bonds have $(a\ b)$ of $a + b \neq 0$ and $a \neq 0$ is called a bridge of ring closure of order p (BCp).
(12) A ring structure that has one ITS bond of $a + b = 0$, another ITS bond of $a = 0$, and all other bonds of $a + b \neq 0$ and $a \neq 0$ is defined as a bridge of rearrangement (BR).
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