

the reported data virtually useless for testing the validity of the author's own or someone else's mathematical model of the studied process or property. It should be noted in this context that the thought expended on the nature and consequences of error magnitude for testing the proposed interpretation of the data should also lead to better planning of the experiment as a whole, including the proper spacing of points in variable space. Far more than an editor's pedantry is obviously involved here.

Qualitative analysis is equally bedevilled by inadequate characterization of method uncertainty. Typical examples are optical spectroscopy data without measure of resolution, or even of slit width used. Subsequent investigators may then be unable to tell whether their compound truly differs from that reported by an author who gave only a portion of the spectrum without error bounds.

The enumerated examples for mounting numbers of inaccuracies in the reported chemical literature perhaps do not retard the development of chemistry very much because the leading chemists' unmatched intuitive judgment guides the phalanx of their followers nearly unharmed through a morass of inadequate data. However, the lack of emphasis on experimental rigor and thoroughness leaves many of today's graduate students unprepared to deal with the extreme sensitivity of product quality, reaction yield, and selectivity to reactant purity in most modern industrially important chemical processes. This alone seems to me to be sufficient reason for unbending editorial insistence on the indicated criteria of publication-worthiness.

LITERATURE CITED

- (1) Allen, T. J., "Managing the Flow of Scientific and Technological Information," Ph.D. thesis, M.I.T., Cambridge, Mass., 1966.
- (2) Allen, T. J., and P. J. Gerstberger, "Criteria for the Selection of an Information Source," M.I.T., Cambridge, Mass., 1967.
- (3) Case Institute of Technology, Operations Research Group, "An Operations Research Study of the Scientific Activity of Chemists," 1958.
- (4) Case Institute of Technology, Operations Research Group, "An Operations Research Study of the Dissemination and Use of Recorded Scientific Information," USOPB No. 171,503, Cleveland, 1960.
- (5) Bernal, J. D., "The Use of Scientific Literature," The Royal Society Scientific Information Conference, London, 1948, p. 589; *Proc. Intern. Conf. Scientific Information, 1958*, NAS-NRC, p. 77, 1959.
- (6) Menzel, H., "Planned and Unplanned Scientific Communication," *Proc. Intern. Conf. Scientific Information, 1958*, NAS-NRC, p. 199, 1959.
- (7) Scott, C., "The Use of Technical Literature by Industrial Technologists," *Ibid.*, p. 245.
- (8) Allen, T. J., and S. I. Cohen, "Information Flow in an R and D Laboratory," USOPB No. 173,524, M.I.T., 1966.
- (9) Heumann, K. F., *et al.*, Symposium on Error Control in the Chemical Literature, *J. CHEM. DOC.* 6, 125-42 (1966).
- (10) Handbook for Authors, Am. Chem. Soc., p. 24, 1967.
- (11) Bondi, A., "On Error Prevention," *J. CHEM. DOC.* 6, 137 (1966).
- (12) Royal Society Scientific Information Conference, Working Party I/B, p. 110-15, The Royal Society, London, 1948.

Experiments with Programmed Learning as a New Literary Form*

MANFRED KOCHEN

Mental Health Research Institute, University of Michigan, Ann Arbor, Mich. 48104

Received September 11, 1968

The artful use of computer aids can help us create a wealth of new forms for representing, transmitting, storing, transforming, and displaying knowledge. One such form is a programmed text. A novel kind of programmed text, used as an experimental instrument in behavioral engineering, is described. Its aim is to overcome blocks against mathematics on the part of people who are allergic to the manipulation of abstract symbols, or who are rusty in mathematics, or who have never been exposed to the excitement of mathematical thinking. Experimental findings resulting from its use in class are presented and used to evaluate and improve the instrument in a systematic way.

Some of the more recent new methods of teaching that have been developed or proposed are called programmed learning. There are now almost 300 computer-aided instruction programs in operation and many more book-

form programmed texts. In 1961, there were 82, and in 1962, 86 published programmed texts on mathematics alone.^{2a} By then, over 100 companies were selling programs. Harcourt, Brace, and World alone commissioned the writing of 50 programmed texts, and Encyclopaedia Britannica Films has planned an entire high school curriculum in the newer form. Eight semesters of mathematics materials have been tested on over 1000 students in 1965-67. As early as 1960, Grolier, Inc., added a machine

* Presented before the Division of Chemical Literature, Symposium on Redesign of the Technical Literature, 156th Meeting, ACS, Atlantic City, N. J., September 1968.

Editor's note: Other papers from this symposium were published in the November 1968 issue.

called "Minimum Time...Maximum Learning" to its reference books and encyclopedias.¹⁰ In 1965-67, over 26,000 school boards in the United States spent close to \$200 million on mechanical teaching aids. The use of the programmed text called "Exploring Mathematical Ideas,"¹¹ which is described in this paper, demonstrates that the creation of this form is still very much an art in that there is as yet no simple set of rules for generating, evaluating, and interconnecting frames. This paper presents ideas and describes experimentation toward providing a rationale for the design of this form.

LITERARY FORMS AS TEACHING DEVICES

A literary form, in its most abstract sense, is any visual device by which an author aims to create a specified effect on one or more readers. The restriction to the visual modality is purely arbitrary to conform to the conventional interpretation of literary as pertaining to reading and writing.

To specify the function of a literary form is to explicate a statement like: "Author A intends, with form D, that reader R does (or can do) C." The paragraph has long been recognized as a convenient unit for literary forms.²⁴ Its parts (main idea = MI, detail = D) can be arranged into patterns, such as: MI-D or MI-D-MI or D-MI or D-MI-D, etc. It is not impossible that paragraphs of specified structure can be computer-generated according to a grammar (production system) analogous to that for sentence formation. This type of analysis is especially useful in journalism and essays.

How well a form performs its function depends on: how precisely the author has defined the class of readers and the effect to be produced in them, and the extent to which the intended effect is actually produced. The reader should know, without doubt, whether or not he is doing (or can do) what the author intended him to do (or know how to do).

To what extent, however, can evaluation and, later, the design of newer literary forms eventually become like modern engineering, with a scientific foundation? What roles can the innovations in information technology play in the design, production, and embodiment of the newer literary forms?

One particular class of future literary forms is what shall be called "works of synthesis." The author of such a form intends that his audience be able to better "understand" so as to act more effectively. In behavioral terms, the author intends that the user: make better relevance and quality judgments, more rapidly, about literature and detailed facts; better recall and utilize valuable documents and facts; and ask more and better questions.

More concretely, imagine that the successful user of a good work of synthesis is a manager in a company making information storage devices or a doctor who services the health needs of a community. In either case, there is an increasingly large amount of literature he vaguely or sharply feels he probably ought to scrutinize, ranging from advances in organic, molecular chemistry to behavioral science. Moreover, with the faster and better document and information retrieval methods now under vigorous investigation, his screening problem would be vastly aggravated because of the vastly enlarged bibliographies pressing for his attention. He must either redefine

his interests much more precisely or as an increasingly narrow specialty. Yet, might he not reasonably ask for some guidance that helps him decide how little he needs to know about a variety of several specialties? Of course, his needs or objectives must be defined, but this means he must ask precisely formulated, perceptive questions. Good questions, however, reflect a rather exact knowledge of what he does not know (and needs to know). And that is a great deal of knowledge. It is precisely this kind of knowledge a work of synthesis aims to teach.

The work of synthesis should enable this practitioner to see the logical structure of various specialties, including his interests, and the over-all relationship of one special topic to the others. After all, detailed facts will be learned only if they fit readily into the total, logical structure. Details that do not readily fit are more likely to be rejected. Details that fit are more easily recalled when needed. Recognition of missing facts—detailed or not—shows up in questions the user asks, and the better the internalization of a subject's over-all structure, the better these questions. In short, a good work of synthesis has theoretical content: it enables the user to realize a coherent picture, or model, of what he has learned.

This new literary form resembles a network of "frames," a frame being a unit form, teaching the least that can be taught in one step. Physically, a unit form may be prose displayed on a screen or in print. Each frame points not to a unique next frame, but to a number of alternate next frames giving the user options about which trail in this network to follow.

Some pointers may lead to a published, technical article, or, rather, to a unit frame representing an article which, in its turn, would point to further frames. Indeed, one possible form for the technical article of the future may well be a micro-network of such frames. The entire collection of frames thus connected to represent an article would point, through special frames in the collection, to the references in the bibliography of the paper.

This concept of a network of frames, which underlies current development of programmed literature, is neither very novel nor profound and far-reaching. Generally, the network is specialized to a tree with little branching though there is very little rationale for this widespread practice. The key limitation of current literary forms of the program type is that the designer has to anticipate, by the content of the unit frames, alternative ways for users to delve into a subject. Not only does this require a great deal of effort and inefficiently used storage but, like a classification, it may simply not match the actual needs of many users at the time they arise. Even if it were practical to keep a program closely adjusted and attuned to users' needs when they become clear through a very effective feedback, data gathering, and analysis system,¹¹ such an anticipatory system is inefficient. Moreover, there are fundamental, theoretical limits on its teaching capacity; these are inherent in the concept of anticipatory where applied to an unpredictably and constantly changing user community.

Consequently, there is emerging a new literary form which we shall call responsive, or, even better, active. But this is still very much at an exploratory and conceptual stage, despite some pioneering first steps by Uhr³⁰ and others. It is in this direction, however, that information

technology has most to offer, for it is here possible to use the generative capability and general-purpose nature of a computer. The efficient use of the computer is often best preceded by experimentation toward good methods and the artful invention of useful forms. For example, the computer's role in CAI (computer-aided instruction) is commonly that of a switch; as such, ideas about how it should function could be tried by using games with cards. Even when the computer's role is to generate frames, the form and quality of the frames can be evaluated by testing them for their effect on a sample of users, under controlled experimental conditions.

USE OF A PROGRAMMED TEXT AS AN EXPERIMENTAL TOOL

The textbook, like the survey or review paper, has traditionally been the form of a work of synthesis in scientific and technical literature. The program—embodied in a text, a special machine, or a general-purpose computer—is a newer literary form. Its central idea is inherent in the ancient tutorial system. It was effectively restated in more modern terms by Thorndike,²⁰ and implemented mechanically by early pioneers like Pressey¹⁸ and Skinner.^{20, 21} While Pressey's were mainly testing devices, Skinner's applied the principles of operant reinforcement. Most recently, the main theme underlying programmed learning is being restated in very pragmatic, operational terms by a modern school of behavioral engineers.¹ Although a great number and variety of programs have now been produced and tested, none appear to be works of synthesis. Existing programs mostly aim to attain narrowly defined behavioral objectives, often the teaching of specialized skills.

Perhaps the greatest need and opportunity to experiment with a work of synthesis using newer forms is in nonscientific fields likely to be affected by automation. Librarianship, journalism, education, entertainment, administration—even the health and legal professions—come to mind. These fields, on which the knowledge industry is built, might most appropriately be called the knowledge arts. To practitioners in these arts, the amount of specialized technical knowledge and know-how, of which they cannot afford to be illiterate, must seem overwhelming. They may need nothing more than a way to put all these fragmentary facts in order, to sort them out mentally, so that they can take defensible positions on major issues, based on understanding.

Fortunately, there is one age-old tool that can be brought to bear on structuring, ordering, and synthesizing all these technical concepts: it is mathematical thinking. For this reason, a program called "Exploring Mathematical Ideas" was produced. Its major instructional objective was to produce the following effects on each person in a class U of people:

He should voluntarily seek out or at least select, from material he encounters, important articles in his field containing mathematics;

He should be able to write a critical review of that article of quality comparable to that of the reviewed article.

To write a good critical review does not, of course, require that the reviewer be able to do as well or better than the author. The review is a metaform; it talks about

the literary form and content being criticized. This requires appreciation rather than the skill of the author; a generalist's (bird's eye or broad perspective) view rather than the deeply specialized understanding the author displayed; the ability to see relations between the author's results, results of others, and possible, practical applications.

The class U is characterized by a fear of or block against mathematical thinking; no prior exposure even to high school mathematics; and no memory of mathematics, even arithmetic, learned decades ago. The intended audience excludes students of engineering or science, who presumably already appreciate the over-all structure of mathematics and its unifying potential.

This program differs from the large number of other programs in mathematical specialties in that: it has the above stated objectives; it de-emphasizes manipulative skills; it does not specify the grade of the learner, for a professional practitioner in law, medicine, journalism, etc., with 20 years of experience, a first-year graduate student of English, a college freshman, a high school freshman, and a housewife-mother can all have the same level of literacy in mathematical thinking. That is, their level of educational attainment does not accurately reflect their level of mathematical attainment. There are, of course, great individual differences, and it is precisely these that the newer literary form of a program is to take account of.

The program has the physical form of a mimeographed, 580-page book.²² It is used to simulate a computer-aided instruction program. [Computers would have been used had enough terminals and software been available, and had these been sufficiently competent and reliable to administer the instruction sequence to 70 students within existing constraints on resources and time (four man-months).] The text is organized by five chapters: I. Sets, II. Algebraic Structures, III. Languages, Logics, and Programming, IV. Functional Analysis and Probability, and V. Graph Theory and Topology. Each chapter is subdivided into about a dozen sections, and each section consists of a very brief introduction to a major mathematical concept. It aims to enable the student to decide whether to go further, and, if so, where. It prepares him to ask questions and to begin learning by doing problems. Following this introductory frame is a sequence of 30 to 50 question-answer frames scattered over the next four to six pages.

These frames cover at least nine questions pertaining to the key and subsidiary concepts of that section. Each question is, of necessity, multiple-choice. The number of multiple-choices is never less than three, the simplest kind being: (a) True, (b) False, (c) Not sure. The questions are usually mathematical problems leading the student, by active practice, to discover the main concepts for himself. In addition, they stress a perspective for relating all these mathematical concepts both to one another and to the real world of his special interests.

The first problem-question in each section, called Q3, was designed to divide learners into two equal groups: Those who solved the problem "correctly" and those who didn't or admitted they weren't sure. Surprisingly, an overwhelming percentage of students preferred to guess incorrectly rather than admit they weren't sure. In

revising the text, there is a reinforcement schedule which rewards the person who knows what he doesn't know, as much as the person who knows what he knows. Least rewarded is the learner who doesn't know how much or how little he knows.

Possibly students were motivated due to time pressure from competing activities, to get out of each problem-sequence as rapidly as they could, though many students requested more problems. The tutorial frames require considerably more care and stylistic force if they are to be effectively remedial. People in the first group advanced to a more difficult problem, called Q4, designed to split this group into two equal parts. The quarter of the original class which got Q4 "correct" advanced to Q5, and half again might exit from this section to the next. At any level, the learners may exit to the next section or advance to a problem of difficulty greater, lesser, or equal to that of the problem just worked. In all cases, whether they answered correctly or incorrectly, they encounter an explanatory passage before proceeding to another problem.

The reason for dividing learners into groups this way is to allow each person to seek out the level or group into which he fits best at any stage. He can thus move from one group to another at his own rate, via a sequence best for him, as he progresses through each section.

The text attempts to anticipate common errors and misconceptions through appropriate, multiple-choice alternatives. Using this device to assess difficulty, the program then directs the learner to simpler problems appropriate for his error pattern and administers remedial help, which analyzes difficulty in more detail.

The procedure used in writing this program was:

- Creation of all the introductory frames, one for each section, according to an over-all logical structure;

- Creation of Q1 to Q5 for each section on cards, according to anticipated level of abstraction and mental effort by the learner;

- Writing of remedial frames and alternate Q's;

- Testing, data evaluation, and systematic rewrite of all frames.

On the whole, the first draft of all frames was based on prior teaching experience, intuition, some knowledge of and assistance from the students, and some scientific understanding of the relation between the content and form of a frame and its effect on learners of various types.

The experimental subjects were 70 first-year graduate students in Library Science, with backgrounds in English, History, and other humanities. Most had a strong dislike for mathematics, but began participating in this experiment as part of a required course. Shortly after the start, they were told that this program was not required and further participation was entirely voluntary. If they participated, they would earn two credits toward their degrees; otherwise, they could withdraw at any point, with no adverse record. About half who started continued; several new students also joined at this time.

Each participant did three things:

- He submitted a feedback sheet for each section that posed all questions which occurred to him while studying the section and working the problem-sequence, indicating the order of the questions he encountered, and estimating the time it took to work the section.

- He completed a questionnaire every two weeks to assess any changes in his behavior resulting from participation.

- He occasionally attempted to create either: questions that could be added to the problem sequences; critical reviews of selected papers; or original research papers showing his use of mathematical concepts for structuring, integrating, and synthesizing.

All data were then used to do three things:

- Analyze and evaluate how well the program performed in its functions;

- Formulate and try out new and hopefully improved designs of the program—i.e., to revise the program into an improved version;

- Learn how to accomplish the above revision in a systematic way, with a view towards finding design principles.

RESULTS AND GENERAL CONCLUSIONS

The simplest question to ask of a newer literary form like a program is whether it meets given objectives more effectively and more efficiently than conventional forms with the same objectives. One form is more effective than another if it can perform functions the other cannot; it is more efficient in a function if it can do the same thing at lower cost. Although not enough data about the use of the programmed text, "Exploring Mathematical Ideas," and its revisions, have as yet been compiled and analyzed, a qualified positive answer to the question, "Does a programmed work of synthesis meet its objectives better than a traditional form," may be tentatively ventured. Our experience leads us to believe that the design of newer literary forms, however, is still as much a fine art as has been the creation of traditional forms. The most artfully executed books⁴ of the past are more effective and efficient than the average program, and the few artfully designed programs^{8, 16, 25, 27} are more effective and efficient than the average artless textbook of the past. In other words, the design of newer forms is likely to develop into a finer art before it will be based on sound principles of scientific design.

Perhaps the most important factor in the success or failure of a program to meet the objectives which make it a work of synthesis is its ability to communicate the topic's over-all, logical structure. This is reflected in the design of the program, from the phrasing of the unit frames to how they are interconnected into a network.

Of the 67 students who tried the program at all, 42, after three and one-half months, successfully completed chapters I, II, III, and V. The amount of time spent on the problem sequence was about 15 minutes, averaged over all students and sections, although the distribution, being skewed toward the high end, had considerable variance. Many students spent much time on supplementary reading and tutoring from mathematically literate acquaintances. The average number of frames (which included questions and remedial passages) per section that the students were led through was about 10.

Although working the problem sequence was entirely self-paced, the average rate was about four sections per school week. The rate was slightly lower during the middle phase of the term than it was during the beginning and end. Students who wanted to attend a 1-hour class per week devoted to answering questions raised in their feedback sheets, questions raised orally in class, and

lecture-form presentations of some of the material covered in the program. Over 90% of the students taking the program attended this weekly session. Such high attendance, coupled with other student responses, suggests considerable student enthusiasm for the subject matter presented in this game-like manner. There are enough data to support the claim that these newer forms have considerable potential for creating a rewarding experience.

Many student questions concerned clarification of material in the program. Some were quite perceptive. Some questions revealed not only that student's grasp of the over-all structure, but his ability to anticipate—discover on his own—major theorems, concepts, and problems in mathematics.

The distribution of choices over the multiple-choice alternatives were not as uniform as intended in the program's first draft. The students' actual questions, as well as some of the multiple-choice frames they created, however, are expected, when incorporated into the second draft, to provide a more uniform distribution. Designing multiple-choice questions to get an equal number of people selecting each option is, incidentally, a departure from customary practice. It is common to expect close to 98% of the responses to be the "correct" answer, the other alternatives being considered as "distractors." Consequently, few programs make good use of branching, which would make the term program more appropriate than it is for most existing forms of that name.

The general conclusion emerging from the attitude test they took every two weeks throughout the course is that, although students found the material difficult as they proceeded, their liking and appreciation of mathematical thinking remained at about the same level. This level was reached shortly after the start, probably because those who disliked it had dropped out. The following is a typical comment by one of the students:

"After four years of high school math I thought 'never again' and literally shut my mind off when mathematics as a science was concerned. But by making this fun to do—and not being afraid of making a mistake—I realize that one today simply can't and shouldn't disregard this field, especially when being cognizant of computers is essential for me."

About 85% of the students said they liked or greatly liked the material and were definitely developing an appreciation for mathematical thinking. Those who said they were neutral, disliked, or greatly disliked it also said they found mastery very difficult. Less than 5% of those who pursued the entire course gave this response.

Like any other form for programmed learning, this text succeeds, where it does, because:

The material to be communicated is decomposed into small enough steps to be presented in one unit. The central idea is that learning can be a continuous process. As the student, by practice, exposure, and immediate reinforcement, associates some positive affect—i.e., a feeling of pleasure, or at least freedom from background anxiety, with a small increment to his over-all picture of the subject—he is not only ready, but eager for the next small bite. In this way, he can gradually climb a hierarchy of small steps, each of which would, by itself, have been more difficult than its predecessor. But the student imperceptibly finds himself solving fairly complex mathematical problems.

It attempts to match the capabilities not of one class or level of students but of at least three. Each class is characterized by a different level of aspiration, reflected by the question level—Q3, Q4, Q5—at which they choose to exit. Different individuals learning at different rates and at varying amounts of total time, because of dissimilarities in prior experiences with mathematical thinking, can follow the program at the best pace for them. This, of course, requires considerable branching. The degree of branching is another point of dispute between Skinner and Crowder, the latter favoring a high degree.

The learner is active rather than passive and, as in the Socratic tutorial system, is led to discover main concepts and results for himself. He does this by the practice of mathematical thinking, without the diversion of manipulation.

It cannot be claimed, on the basis of the objective data collected so far, that the first draft of this programmed text has entirely met its objectives. Changing the attitudes to mathematical thinking of an average person who has been conditioned negatively or by misconceptions is quite difficult, though, in principle, amenable to techniques of behavior engineering. There is little doubt that the newer literary forms for programmed teaching are effective in developing manipulative skills and aiding a reader to recall and perhaps apply, analyze, or interpret the kind of technical detail that appears in a technical paper. But it is still an open question whether these newer literary forms can be equally effective in producing the more profound attitude changes necessary to cope with and assimilate the growing amount of newly generated knowledge to which readers are or might be exposed.

WHAT NEXT

The revised version of the program contains several features designed to attain the instructional objectives more effectively. The most numerous changes are in the style and content of unit frames and in some restructuring of connections among frames. These changes stem from specific difficulties and suggestions made by students.

The most important new feature in the revision, however, is a tutorial review unit following every four or five sections of the programmed text. Each review unit has five parts which train the student in a five-step progression to want to write good critical reviews. In the last of these five steps, the student is presented with two passages on the same topic, such as the population explosion, which necessitates mathematical thinking. One passage avoids mathematics; the other doesn't. The student is asked to pick one of the two passages and write a critical review. He then enters a programmed question-answer sequence which ensures that he has seen all the weaknesses of the passage he chose. He is to discover, during this process, the inherent advantage of mathematical thinking for this particular topic.

The use of this programmed text, as a tool for experimentation with newer literary forms, raises more research questions than it has so far answered. Those it answers are, in general, systems problems^{22,23} such as:

How small in content and word count, and how pithy should a unit frame be? What should be the mix of concrete examples and general definitions? To what extent should the examples and their wording evoke associations with the learners' prior experiences?^{14, 26}

To what extent should the program be individualized? How many different classes and levels of learners should a simple program try to service? Just as unit frames are hierarchically organized, so entire programs can be embedded in a hierarchy of programs, but what is the optimal scope of a program?^{7,8,9}

Assuming that multiple-choice or fill-in questions in which all the acceptable, anticipated answers are prerecorded in a dictionary will be used until there exist efficient ways of linguistic analysis by computer, how many alternate answers should be included? Too few make a question of low instructional value; too many impose enormous loads on preparing and storing the program.^{9,17}

How can the probability that a learner will pick an article and write a review which calls the article useful, given that it is useful for this learner, be computed from the values of the variables characterizing D and the class R of learners?²

Much might have been gained if this program could have been administered by computer.^{6,19,21} The result of numerous comparisons¹⁰ between programmed texts and teaching machines is that there is no marked superiority of one over the other. Such comparisons, however, have been mostly with special teaching machines of the Skinner variety rather than computer-based programs. The latter can be fundamentally richer and faster, though there are not yet as many in operation as there are other programs. One obstacle to the use of computers has been the development of higher level programming languages to facilitate the scriptwriter's task. Considerable effort on this problem has resulted in programs like the "Coursewriter III," FOIL,⁷ PILOT or LYRIC, MENTOR, but this is merely the beginning. Adherents for the use of computers have offered its ability to prevent cheating as a major advantage of its use over programmed texts. Studies have shown, however, that learners also gain considerably from peeking.

It would then not have been all multiple-choice, which is a serious limitation. Secondly, there would have been fewer opportunities to skip harder frames and to disregard instructions displayed to the learner in the remedial and directive frames. Spending 1½ minutes per frame and 1 hour per week on a subject as deep as mathematics seems far too little, and a good computer tutor could, to some extent, impose itself on the learner's attention if its presentation were sufficiently artistic and delightful.

Mathematical thinking is actually a set of attitudes, for example, that of persevering with confidence of success in working out a problem involving some unfamiliar ideas and terms. In following a mathematical proof, as in writing or reading a computer program, it is possible to appreciate fully the first step only after the last step has been studied. This means that the student must have sufficient faith and patience that he will later suddenly understand the first step if he will only persevere, and tentatively accept certain definitions, assumptions, etc., no matter how absurd or irrelevant they seem at first. Nothing enables a learner to develop such an attitude (or its opposite) as effectively as a well designed (or poorly designed) conversational computer³¹ terminal.

The way a computer terminal can "trap" an on-line, real-time user and fully absorb him has been stressed by Licklider,¹⁵ as has the importance of man-machine symbiosis. A human teacher closely coupled with a computer-administered program—possibly a well synchronized, prerecorded audio-video tape might do—can greatly⁴ amplify the effect produced by either alone. This

is proved by comments from most users of our mathematics program: the 1-hour lecture per week was an indispensable supplement to their pursuit of the course.

The users must also be able to ask questions. An active program—one which can answer totally unanticipated questions—is a concept far beyond the passive networks of frames considered so far. All these programs might best be relabeled "passive programs." An innovation to change passive to active programs is one that analyzes student responses to computer-produced questions to: assess his difficulty more precisely to compose next a more individualized remedy; test the effect of the last remedial step. Both questions and remedial steps would still be prerecorded frames, though the user could respond to the questions in lightly constrained English—i.e., a very English-like but formal language. Of course, the computer must be able to analyze such linguistic productions.

Next, the computer must be able to generate, rather than retrieve, prerecorded questions. The simpler question would probe into the user's unique difficulties, so that the best possible mix of remedial steps could be composed and prescribed. The remedial steps would still be prerecorded frames. Next are computer-generated remedial frames. The program designer would supply only general definitions and theorems; the computer would specialize in examples, depending on data supplied by and about a particular user.

Exploring ways to create such computer programs can be done as well by simulating with the revision of a programmed text what the computer would do. The diversions arising from working with an imperfect, large, time-sharing system, or with small computers, are thereby minimized.

Though we focused on a particular literary form, the textbook, most technical literature could be included. The technical paper, too, has a teaching function. When considering optimal, new forms for scientific papers, it is well to begin when the idea or question of the potential paper is first conceived. Presumably, a prospective author knows something is unknown, but it takes him time and study to formulate his question precisely. Could he learn more, faster, and with less effort than he does? Indeed, the recognition that something was not known probably occurred to him while he studied a new finding. Could this contagious process¹² be improved in speed, quality, and efficiency? This is of particular importance if the person to be "infected" is an applied scientist, technologist, or practitioner who will utilize what is known to be known and recognize what else needs to be known to create new products and services that increase economic productivity.

The next phase in the production of a literary form like the technical article is the interaction between author, referees, and editors. This process, too, has some instructional aspects, both for author and readers. It could resemble the very effective tutorial system still in use at Oxford and Cambridge, and this system could be amplified by behavior engineering techniques.

LITERATURE CITED

- (1) Brethower, D. M., *et al.*, "Programmed Learning: A Practicum," Ann Arbor Publishers, Ann Arbor, Mich., 1965.

- (2) Briggs, L. J., "Learner Variables and Educational Media," *Rev. Educ. Res.* **38**, 160-76 (1968).
- (3) Bruner, J. S., "The Process of Education," Harvard University Press, Cambridge, Mass., 1960.
- (4) Courant, R., and H. Robbins, "What is Mathematics," **II**, p. 16, Oxford University Press, New York, 1941.
- (5) Deterline, W. A., "Practical Problems in Program Production," in "Programmed Instruction Yearbook Nat. Soc. Stud. Educ.," Part **II**, P. C. Large, Ed., 178-216.
- (6) Fry, E. B., "Teaching Machines and Programmed Instruction," McGraw-Hill, New York, 1963.
- (7) Hesselbart, J. C., "FOIL—a File-Oriented Interpretive Language," draft, Center for Research on Learning and Teaching, University of Michigan, February 12, 1968.
- (8) Horn, R. E., "Programmed Instruction: a Survey of Excellence," *Training Bus. Ind.* **2**, No. 4 (July/Aug. 1965). [From *Programmed Instruction* (June 1965).]
- (9) Karraker, R. J., "Knowledge of Results and Incorrect Recall of Plausible Multiple-Choice Alternatives," *J. Educ. Psychol.* **58**, No. 1 (1967).
- (10) Klaw, D., "What Can We Learn from the Teaching Machines," *Reporter*, p. 19-26 (July 19, 1962).
- (11) Kochen, M., "Some Problems in Information Science," Scarecrow Press, New York, 1965.
- (12) Kochen, M., Ed., "The Growth of Knowledge: Readings on Organization and Retrieval of Information," Wiley, New York, 1967.
- (13) Kochen, M., "Exploring Mathematical Ideas," mimeographed by Department of Library Science, University of Michigan, 1968. Copyrighted by author; over 100 copies distributed to students for experimental purposes by the Library Science Department, Univ. of Michigan, Ann Arbor, Mich., 1968. Though it is continually growing, the form in which it was distributed resembled a 480-page text.
- (14) Krumboltz, J. F., and C. A. Kiesler, "The Partial Reinforcement Paradigm and Programmed Instruction," *J. Progr. Instr.* **3**, No. 2, 9-14 (1965).
- (15) Licklider, J. C. R., "Man-Computer Partnership," *Intern. Sci. Technol.* 18-26 (May 1965).
- (16) May, K. O., "Programmed Learning and Mathematical Education," The Mathematical Association of America, G. Banta Co., printer, 1965.
- (17) Osin, L., "Measuring the Influence of Wrong Alternatives in Multiple-Choice Textbooks," MIT Education Research Center, 1968.
- (18) Pressey, J. C. R., "Certain Major Psycho-Educational Issues Appearing in the Conference on Teaching Machines," in "Automatic Teaching: The State of the Art," E. Galanter, Ed., pp. 187-98, Wiley, New York, 1959. For an early reference, see "A Simple Device for Teaching, Testing, and Research in Learning," *Sch. Soc.* **23**, 373-6 (1926).
- (19) Scandura, J. M., "Teaching—Technology or Theory," *Am. Educ. Res. J.* **3**, 139-46 (1966).
- (20) Skinner, B. F., "The Technology of Teaching," Appleton-Century Crofts, New York, 1968.
- (21) Skinner, B. F., "Teaching Machines," *Sci. Am.* 2-13 (Nov. 1961).
- (22) Smallwood, R. D., *et al.*, "Quantitative Methods in Computer-Directed Teaching Systems," AD 657190, Clearinghouse, Stanford U. Report, 1-150 (March 15, 1967).
- (23) Stolorow, L. M., "Systems Approach to Instruction," University of Illinois, TR. No. 7, ONR3985(04), July 1965.
- (24) Strunk, W., Jr., and E. B. White, "The Elements of Style," MacMillan, pp. 1-71, New York, 1959.
- (25) Suppes, P., "On Using Computers to Individualize Instruction," "The Computer in American Education: Issues and Applications," Fund for Advancement of Education, Palo Alto, Calif. (Nov. 1-3, 1965).
- (26) Suppes, P., "The Development of Mathematical Concepts in Children," Inst. for Math. Studies in the Social Sciences, Tech. Report Nos. **63** (Feb. 1964) and **64** (April 1964).
- (27) Swets, J. A., and W. Feurzeig, "Computer-Aided Instruction," *Science* **150**, No. 3696, 572-6 (Oct. 29, 1965).
- (28) Thorndike, E. L., "Education," pp. 164-7, MacMillan, New York, 1912.
- (29) U. S. Department of Health, Education and Welfare, Office of Education, "Teaching Machines and Programmed Learning," OE-34019, pp. 60-71, U. S. Government Printing Office, Washington, D. C., 1962.
- (30) Uhr, L., "The Compilation of Natural Language Text into Teaching Machine Programs," *Am. Federation Inform. Proc. Soc. Conf. Proc.* **26**, 35-44 (1964).
- (31) Uttal, W., "Computer Teaching Machines: Real Time Simulation of the Tutorial Dialogue," Proc. IVth International Congress of Medical Cybernetics, Sept. 1966.

This work was carried on with partial support of NSF grant GN 716.