

Edge Adjacency Relationships and Molecular Topographic Descriptors. Definition and QSAR Applications

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Edge adjacency relationships in molecular graphs have been used to define a new topographic index. The novel index is calculated considering molecules as weighted graphs, where the elements of edges set are substituted by the bond orders between connected atoms in the molecule. Good linear correlations were found between molar refractivity of alkanes and the proposed ϵ (ρ) index. The applicability of the novel index in QSAR studies was evaluated by using a data of pharmacokinetic properties for a series of amphetamine derivatives. The results obtained were statistical and pharmacologically significant. Special interest is dedicated to discrimination of isomers, including heteroatom's differentiation and conformational isomerism.

I. INTRODUCTION

One of the most prolific applications of graph theory to chemistry is in the generation of molecular descriptors. This kind of descriptor is based on invariants obtained from the representation of molecular structures as graphs. Most of these invariants are generated from the adjacency or distance matrices of the graphs.¹ Important classes of these descriptors are the so called topological indices, which are simple numbers obtained by mathematical manipulation of graphs associated to molecules. There are more than 120 topological indices described in the literature,² but only at most a dozen have been widely applied in structure–property and structure–activity studies.³ Topological indices such as the molecular connectivity of Randić,⁴ based on vertex adjacency matrix, and the Wiener number,⁵ based on the distance matrix,⁶ have received great attention due to their applications in chemistry and drug research.^{7–11} These indices have also shown interesting mathematical features.^{12–15} A new source of graph theoretical invariants was recently proposed by one of the present authors.¹⁶ The main idea is to represent the edge adjacency relationships in a matrix form. Some topological indices related to molecular volume of alkanes¹⁶ and other series of organic compounds¹⁷ have been derived from this graph theoretical matrix.

The main drawback of simple topological indices is its lack of information about the three-dimensional features of molecules, which generates some criticism to QSAR applications of these descriptors.¹⁸ This criticism is in part justified by the fact that specific drug actions strongly depend of spatial features of molecules such as configuration and conformation. Clearly, global three-dimensional properties of molecules are of profound importance in QSAR, where stereospecific interactions are abundant in the formation of drug–receptor complexes.¹⁹

Recent attention of graph theoretical chemists has been focused to resolve this problem. The inclusion of spatial molecular features into graph theoretical descriptors seems to be first introduced by Randić²⁰ in 1987, who named it topographic descriptors. Randić's approach is based on the

consideration of real distance matrices as the adjacency matrix of the weighted complete graph K_n . Distance matrix is obtained by embedding the molecular graph into a graphite-like lattice^{20,21} or any other three-dimensional lattice.²² Another important step in the search of three-dimensional descriptors was given by Bogdanov *et al.*,²³ who calculated a three-dimensional Wiener number. This topographic index is based on a 3D-distance matrix in which the nondiagonal entries of the matrix are geometric distances rather than topological ones. The 3D-Wiener number has been successfully applied in quantitative structure–property relationship studies of thermodynamic²³ and chromatographic properties.²⁴

Recent approaches to topographic descriptors have appeared in this journal that are also defined by considering three-dimensional distance matrices. In one of these papers, Diudea *et al.*²⁵ proposed two types of topographic indices of centrality and centrocomplexity on the basis of 3D-Metric distances supplied by molecular mechanic calculations. The other approach was introduced by Randić and Razinger.²⁶ They calculated a topographic index D^2 for benzenoid shapes, which is based on squared distances between atoms on the molecular periphery. Both sets of indices appears to be prominent in QSPR and QSAR studies.

All the afore-mentioned topographic descriptors have been defined by using some kind of geometric distances between atoms in the molecule. A different approach to topographic descriptors was given by Estrada and Montero in 1993.²⁷ In this approach the authors considered a three-dimensional adjacency matrix based on graphs weighted with bond orders or valency indices between bonded atoms, calculated from semiempirical quantum chemical methods. The topographic Ω index defined in that work was well correlated with thermodynamic properties of alkenes^{27,28} and biological properties of alcohols,²⁸ permits one to differentiate among geometrical isomers, and has been used in QSAR studies in combination with quantum chemical descriptors.²⁹ The use of charge densities as diagonal entries in topographic adjacency matrices for molecular graphs was recently introduced by Estrada.³⁰ Topographic indices derived from this approach have been used to describe boiling points³⁰ and chromatographic³¹ properties of alkenes as well as in

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the differentiation of geometrical and conformational isomers.³⁰

In the present work, we extend the edge adjacency matrix **E** in molecular graphs in a 3D-**E** matrix in order to generate topographic indices which can be useful in QSPR and QSAR studies of molecules with spatial diversity.

2. THEORETICAL APPROACH

The edge adjacency matrix in simple molecular graphs was defined as a square and symmetric matrix **E**, the rows and columns of which correspond to the edges (bonds) of the graph. The nondiagonal entries of the matrix are either ones or zeroes depending on whether the corresponding bonds are adjacent or not, respectively.¹⁶

In the present approach we will use molecular graphs with edges weighted by bond orders calculated from quantum chemical methods. Bond orders defined by Armstrong et al.³² will be used as edge weights. These bond orders are more appropriately called valence indexes and their calculation is as follows

$$\rho_{AB} = \sum_{\lambda}^A 2p_{\lambda\lambda} - \sum_{\lambda}^A \sum_{\sigma}^B p_{\lambda\sigma}^2$$

where

$$p_{\lambda\sigma} = 2 \sum_i^{\text{occ}} C_{i\lambda} C_{i\sigma}$$

are the elements of density matrix and eigenvectors $C_{i\lambda}$ sum is over all occupied orbitals.

Elements of the edge adjacency matrix for the weighted molecular graph are now defined in a more complex way. For instance, let e_i and e_j be two adjacent edges in G . If e_i is incident with vertices v_A and v_B , and e_j is incident with v_B and v_C , then the elements e_{ij} and e_{ji} of the **E** matrix are ρ_{BC} and ρ_{AB} , respectively. As can be appreciated the **E** matrix for bond orders weighted molecular graphs is nonsymmetric. Thence, edge degrees, $\delta(e_i)$, are defined as the sum of elements of i th row in **E** matrix

$$\delta(e_i) = \sum_j e_{ij}$$

In other words $\delta(e_i)$ is the sum of bond orders associated to all bonds adjacent with e_i . The topographic edge connectivity index $\epsilon(\rho)$ is introduced here using the Randić-type invariant as follows

$$\epsilon(\rho) = \sum_s [\delta(e_i)\delta(e_j)]_s^{-1/2}$$

where summation is over all adjacent bonds in the molecule. In Figure 1 we illustrate the calculation of $\epsilon(\rho)$ index for a molecule of 2-methylbutane.

3. APPLICATION OF THE NOVEL INDEX IN QSPR STUDIES

The topographic descriptor $\epsilon(\rho)$ was used in QSPR models for predicting the molar refractivity (MR) of a series of 69 C_5 – C_9 alkanes. Molar refractivity was calculated as follows

$$MR = \frac{\eta_0^2 - 1}{\eta_0^2 + 2} \frac{MW}{d}$$

where η_0 is the index of refraction, MW is the molecular weight, and d is the density (g/cm^3) at 20 °C; all values were taken from ref 33. Calculation of $\epsilon(\rho)$ was performed by using the bond orders calculated from quantum chemical semiempirical method PM3.³⁴ Full geometry optimization with the Broyden–Fletcher–Goldfarb–Shanno algorithm³⁵ was carried out using the package MOPAC version 6.0,^{36,37} and computation of the index was implemented in the system MODEST version 1.0.³⁸

Molar refractivity was selected as a property to prove the possibilities of $\epsilon(\rho)$ in QSPR studies because it was successfully correlated with the topological ϵ index in a previous paper.¹⁶ On the other hand, MR is an important bulkiness parameter used in QSAR studies to model the steric and hydrophobic³⁹ interactions between drugs and biological receptors.

Regression equation and statistical parameters obtained for the correlation are shown below

$$MR = 7.7314 + 9.2009\epsilon(\rho) \quad (1)$$

$$n = 69 \quad r = 0.9936 \quad s = 0.598 \quad F = 5169$$

where r is the correlation coefficient, s is the standard deviation, and F is the Fisher ratio. Analysis of the model showed that 3,3-diethylpentane was a statistical outlier. This compound was identified as outlier because it exceeded the diagnostic cut-off values of several standard statistical tests known as residuals, standardized residuals, studentized residuals, leverage, DFITS statistics, and Cook's distance.⁴⁰ Residuals are the difference between observed and calculated MR. A standardized residual is a residual divided by the standard deviation of the regression equation. The studentized residual is the residual for one observation divided by its own standardized deviation. The leverage is Mahalanobis' distance divided by $N - 1$. It is a measure of the influence that one observation has on the regression's fit. The DFITS statistic is used to describe the difference in the fit of the equation caused by the removal of an observation. Finally, the Cook's distance is a measure of how much omission of a case from the computation of the regression equation will change the residuals of all the cases. It serves as a measure of a case's influence on the regression results. Since the outlier significantly influenced the model in a way not accounted for by the $\epsilon(\rho)$ index, it was eliminated from consideration in subsequent work, leaving a revised series of 68 alkanes.

The regression model was rebuilt using the revised series of 68 alkanes yielding the following equation

$$MR = 7.7471 + 9.1860\epsilon(\rho) \quad (2)$$

$$n = 68 \quad r = 0.9951 \quad s = 0.523 \quad F = 6726$$

In this model no compounds were identified as outliers using the criteria previously discussed.

In order to test the advantage of $\epsilon(\rho)$ index, we contrast the correlations obtained to describe MR with other topological and topographic indices. These correlations were obtained by using the topographic Ω index²⁷ and the topological ϵ index¹⁶

$$MR = 5.4121 + 9.0743\Omega \quad (3)$$

$$n = 68 \quad r = 0.9680 \quad s = 1.333 \quad F = 981$$

and

$$MR = 7.6513 + 9.3624\epsilon \quad (4)$$

$$n = 68 \quad r = 0.9932 \quad s = 0.621 \quad F = 4768$$

Values of the three descriptors as well as the experimental molar refractivity are depicted in Table 1. The predicted values of MR and the residuals calculated by the three different models are illustrated in Table 2.

The Ω index can be considered as a topographic Randić's branching index, which is defined by using bond order weighted graphs. The use of this index to describe MR of alkanes is not successful, as can be appreciated in statistical parameters of eq 3. On the other hand, the comparison of eqs 2 and 4 demonstrate that the use of bond orders in the definitions of $\epsilon(\rho)$ index introduce some features that are important for the description of MR in alkanes. The improvement in the description of MR by $\epsilon(\rho)$ compared to topological ϵ index can be understood considering the fact that molar refractivity has the units of molar volumes, but it is considered to possess a polarizability component.⁴¹

The quality of QSPR models can be conveniently evaluated by the correlation coefficient r and the standard deviation. A good model must have, as suggested by Mihalić and Trinajstić² in a recent strategy to design QSPR models, $r > 0.99$ and standard deviation dependent on the property under study. According to this statement, eq 2 represents a good model to describe the molar refractivity of alkanes.

The good correlation between $\epsilon(\rho)$ index and molar refractivity, considered as a molecular bulkiness parameter, permits us to interpret the novel index as a good measurement of molecular volume. This index contains also information about polarizability or polarity of molecules present in the pure liquid. These interactions are important in processes such as partition of solutes between two phases and aqueous solubility,⁴² giving to $\epsilon(\rho)$ index possibilities to be used in several types of QSAR studies.

There are several useful properties fulfilled by the $\epsilon(\rho)$ index that are important for molecular descriptors in order to have a practical use in QSPR and QSAR studies. A list of attributes for topological indices has been proposed by Randić¹ that represents the very high level of sophistication that is desirable for topological indices. Among these attributes we can find the direct structural interpretation, to have a good correlation with at least one property, good discrimination of isomers, to be simple, and so forth.

Considering that several topological indexes are inter-related with each other, it is desirable that the new descriptors contain structural information not contained in other indices previously defined. The $\epsilon(\rho)$ index satisfies this demand, which can be proved by the fact that molar refractivity is a property not successfully described with the existing topological indexes.¹⁶

4. APPLICATION IN QSAR STUDIES

Quantitative structure–activity analysis is one of the main applications of molecular descriptors. In order to prove the applicability of the novel topographic index in QSAR studies,

we select a series of 15 N-alkyl-substituted amphetamine derivatives for which pharmacokinetic properties in humans have been used in QSAR studies with physico-chemical parameters.⁴³ The cumulative urinary excretion of unchanged drug expressed as the percent of the administered dose (percent excreted unchanged, PEU) will be correlated with the topographic $\epsilon(\rho)$ index. Three subjects were used in the study of Testa and Salvesen.⁴³ The first two subjects were kept under acidic urinary control by ingestion of ammonium chloride (pH values of 5.0–5.2 and 4.9–5.1, respectively), while urinary pH of the third subject was not controlled. The values of log PEU for the subjects 1–3 are depicted in Table 3.

Stepwise regression was used to correlate the $\epsilon(\rho)$ index against the experimental values of log PEU. Other three graph-theoretical descriptors were regressed with log PEU in order to compare the models found. These descriptors were the topological indices of edge connectivity¹⁶ ϵ and valence connectivity⁹ ${}^1\chi^v$ as well as the topographic Ω index.²⁷ Values of all molecular descriptors are reported in Table 4.

In the previous QSAR study for the same series of amphetamine derivatives, Testa and Salvesen⁴³ reported good quadratic models with the apparent *n*-heptane–water partition coefficient (log P_H) in order to describe the values of log PEU. In Table 5 we give the best models obtained by using the graph theoretical descriptors and the best models found by Testa and Salvesen to describe the percent of amphetamine derivative excreted unchanged for subjects 1–3.

The best models found for subjects 1 and 2 by using graph theoretical descriptors are those obtained with the topographic $\epsilon(\rho)$ index, while the subject 3 the best model is obtained with the topological ϵ index. There are some resemblances in the statistical features of equations obtained with $\epsilon(\rho)$ and log P_H for subjects 1 and 2, for instance they are quadratic models with similar correlation coefficients and standard deviations of regressions. However, there are significant differences in the physical interpretation of the results obtained with physico-chemical parameters and with graph theoretical ones. The differences obtained with topological and topographic edge adjacency indices to describe the values of log PEU in dependence of whether the subjects have urinary pH control or not are very important. Urinary pH has an important influence on the passive tubular reabsorption of drugs with moderated acid–base character. As a consequence, acids will be more reabsorbed (less excreted) at acid urinary pH, and bases will be preferably reabsorbed at alkaline urinary pH. The excretion process for subjects 1 and 2 is controlled by the dissociation of amphetamines at the acidic pH used. Thence, the values of log PEU will be dependent of the bulkiness (topological) and electronic properties of molecules, two features that appear to be contained in the topographic index. For subject 3, where urinary pH was not controlled, the excretion process is not dependent of the electronic features of amphetamines and the best model found is by using the topological ϵ index. This index does not discriminate among heteroatoms in molecules. This fact can be understood as a noninfluence of heteroatom differentiation in the excretion of amphetamines when no pH control is kept. Of course, this differentiation of heteroatom can be related with the protonation of nitrogen in such molecules, because nitrogen is the only heteroatom in amphetamines and is the protonation

Table 1. Values for the Molecular Descriptors (Topographic and Topological Indices) and Observed Molar Refractivities for a Series of Alkanes

no.	alkane	MR obsd	Ω	ϵ	$\epsilon(\rho)$
1	pentane	25.2656	2.4297	1.9142	1.9316
2	2-methylbutane	25.2923	2.2962	1.8937	1.9227
3	2,2-dimethylpropane	25.7243	2.0425	1.9998	2.0425
4	hexane	29.9066	2.9359	2.4142	2.4378
5	2-methylpentane	29.9459	2.8019	2.4317	2.4669
6	3-methylpentane	29.8016	2.392	2.3043	2.3442
7	2,2-dimethylbutane	29.9347	2.6081	2.3660	2.4219
8	2,3-dimethylbutane	29.8104	2.8842	2.4142	2.4622
9	heptane	34.5503	3.4422	2.9142	2.9441
10	2-methylhexane	34.5908	3.3082	2.9317	2.9722
11	3-methylhexane	34.4597	3.3451	2.8423	2.8878
12	3-ethylhexane	34.2827	3.3823	2.7318	2.7836
13	2,2-dimethylpentane	34.6166	3.1135	2.9267	2.9880
14	2,3-dimethylpentane	34.3237	3.2265	2.8350	2.8919
15	2,4-dimethylpentane	34.6192	3.1736	2.9661	3.0163
16	3,3-dimethylpentane	34.3323	3.1733	2.7381	2.8081
17	2,2,3-trimethylbutane	34.3736	3.0059	2.9069	2.9828
18	octane	39.1922	3.9484	3.4142	3.4503
19	2-methylheptane	39.2316	3.8143	3.4317	3.4784
20	3-methylheptane	39.1001	3.8515	3.3423	3.3935
21	4-methylheptane	39.1174	3.8508	3.3803	3.4313
22	3-ethylhexane	38.9441	3.8879	3.2698	3.3265
23	2,2-dimethylhexane	39.2525	3.6197	3.4267	3.4931
24	2,3-dimethylhexane	38.9808	3.7337	3.3730	3.4368
25	2,4-dimethylhexane	39.1300	3.7171	3.3767	3.4382
26	2,5-dimethylhexane	39.2596	3.6803	3.4492	3.5063
27	3,3-dimethylhexane	39.0087	3.6789	3.2988	3.3741
28	3,4-dimethylhexane	43.6780	3.7712	3.2556	3.3241
29	2-methyl-3-ethylpentane	38.8362	3.7707	3.2725	3.3427
30	3-methyl-3-ethylpentane	38.7171	3.7390	3.3154	3.3929
31	2,2,3-trimethylpentane	38.9449	3.5490	3.3344	3.4198
32	2,2,4-trimethylpentane	39.2617	3.4858	3.4711	3.5476
33	2,3,3-trimethylpentane	38.7617	3.5710	3.2116	3.3719
34	2,3,4-trimethylpentane	38.8681	3.6153	3.3716	3.4469
35	nonane	43.8423	4.4547	3.9142	3.9566
36	2-methyloctane	43.8795	4.3205	3.9317	3.9846
37	3-methyloctane	43.7296	4.3574	3.8423	3.8995
38	4-methyloctane	43.7687	4.3570	3.8803	3.9369
39	3-ethylheptane	43.6420	4.3943	3.7698	3.8221
40	4-ethylheptane	43.4907	4.3938	3.8078	3.8697
41	2,2-dimethylheptane	43.9138	4.1260	3.9267	3.9994
42	2,3-dimethylheptane	43.6269	4.2397	3.8730	3.9422
43	2,4-dimethylheptane	43.7393	4.2229	3.9147	3.9817
44	2,5-dimethylheptane	43.8484	4.2233	3.8598	3.9272
45	2,6-dimethylheptane	43.9258	4.1865	3.9492	4.0128
46	3,3-dimethylheptane	43.6870	4.1851	3.7988	3.8792
47	3,4-dimethylheptane	43.5473	4.2769	3.7936	3.8676
48	3,5-dimethylheptane	43.6378	4.2599	3.7873	3.8586
49	4,4-dimethylheptane	43.6022	4.1844	3.8595	3.9400
50	2-methyl-3-ethylhexane	43.6550	4.2761	3.8105	3.8848
51	2-methyl-4-ethylhexane	43.6472	4.2595	3.8042	3.8758
52	3-methyl-3-ethylhexane	43.2680	4.2445	3.8761	3.7665
53	3-methyl-4-ethylhexane	43.3746	4.3136	3.6931	3.7729
54	2,2,3-trimethylhexane	43.6226	4.0547	3.8724	3.9630
55	2,2,4-trimethylhexane	43.7638	4.0289	3.8817	3.9688
56	2,2,5-trimethylhexane	43.9356	3.9917	3.9442	4.0268
57	2,3,3-trimethylhexane	43.4347	4.0769	3.8438	3.9389
58	2,3,4-trimethylhexane	43.3917	4.1595	3.6231	3.8786
59	2,3,5-trimethylhexane	43.6474	4.1058	3.9074	3.9868
60	2,4,4-trimethylhexane	43.6598	4.0505	3.7695	3.9330
61	3,3,4-trimethylhexane	43.3407	4.1144	3.6231	3.8097
62	3,3-diethylpentane	43.1134	4.3049	3.5000	3.6007
63	2,2-dimethyl-3-ethylpentane	43.4571	4.0932	3.7788	3.8768
64	2,3-dimethyl-3-ethylpentane	42.9542	4.1368	3.6652	3.7693
65	2,4-dimethyl-3-ethylpentane	43.4037	4.1576	3.8191	3.9054
66	2,2,3,3-tetramethylpentane	43.2147	3.8993	3.7932	3.9109
67	2,2,3,4-tetramethylpentane	43.4359	3.9373	3.8751	3.9782
68	2,2,4,4-tetramethylpentane	43.8747	3.7980	3.9820	4.0844
69	2,3,3,4-tetramethylpentane	43.2016	3.9693	3.8309	3.9399

Table 2. Calculated Values of Molar Refractivities of Alkanes by Using the Three Different Models Found in This Paper

no.	eq 3		eq 4		eq 2	
	MR	residual	MR	residual	MR	residual
1	27.4598	-2.1942	25.5728	-0.3072	25.4300	-0.1644
2	26.2484	-0.9561	25.3808	-0.0885	25.3400	-0.0477
3	23.9462	1.7781	26.3742	-0.6499	26.4203	-0.6960
4	32.0532	-2.1466	30.2542	-0.3474	30.0599	-0.1533
5	30.8372	-0.8913	30.4179	-0.4720	30.3143	-0.3684
6	31.1757	-1.3741	29.2532	0.5484	29.1881	0.6135
7	29.0786	0.8561	29.8027	0.1320	29.8890	0.0457
8	29.7692	0.041	30.2540	-0.4436	30.2730	-0.4626
9	36.6475	-2.0971	34.9353	-0.3849	34.6888	-0.1384
10	35.4315	-0.8407	35.0991	-0.5083	34.9405	-0.3497
11	35.7663	-1.3066	34.2621	0.1976	34.1596	0.3001
12	36.1039	-1.8212	33.2275	1.0552	33.1969	1.0858
13	33.6648	0.9518	35.0523	-0.4347	35.0663	-0.4497
14	34.6901	-0.3664	34.1938	0.1299	34.2092	0.1145
15	34.2101	0.4091	35.4212	-0.8020	35.3391	-0.7199
16	34.2074	0.1249	33.2865	1.0458	33.4082	0.9241
17	32.6884	1.6852	34.8669	-0.4933	35.0094	-0.6358
18	41.2408	-2.0486	39.6165	-0.4243	39.3204	-0.1282
19	40.0240	-0.7924	39.7804	-0.5488	39.5694	-0.3378
20	40.3615	-1.2614	38.9434	0.1567	38.7839	0.3162
21	40.3552	-1.2378	39.2991	-0.1817	19.1312	-0.0138
22	40.6918	-1.7477	38.2646	0.6795	38.1703	0.7738
23	38.2581	0.9944	39.7335	-0.4810	39.6860	-0.4335
24	39.2926	-0.3118	39.2308	-0.2500	39.1642	-0.1834
25	39.1420	-0.0120	39.2654	-0.1354	39.1835	-0.0553
26	38.8080	0.4516	39.9442	-0.6846	39.8128	-0.5532
27	38.7953	0.2134	38.5361	0.4726	38.5782	0.4305
28	39.6329	-0.7876	38.1316	0.7137	38.1336	0.7117
29	39.6283	-0.7921	38.2898	0.5464	38.3099	0.5263
30	39.3407	-0.6236	38.6915	0.0256	38.9144	-0.1973
31	37.6166	1.3083	38.8694	0.0555	39.0062	-0.0813
32	37.0431	2.2186	40.1492	-0.8875	40.1848	-0.9231
33	37.8162	0.9455	37.7197	1.0420	38.5671	0.1946
34	38.2182	0.6499	39.2177	-0.3496	39.2680	-0.3999
35	45.8351	-1.9928	44.2978	-0.4555	43.9511	-0.1088
36	44.6173	-0.7378	44.4616	-0.5821	44.2001	-0.3206
37	44.9522	-1.2226	43.6246	0.1050	43.4156	0.3140
38	44.9486	-1.1799	43.9804	-0.2117	43.7546	0.0141
39	45.2870	-1.6450	42.9458	0.6962	42.7946	0.8474
40	45.2825	-1.7918	43.3016	0.1891	43.1290	0.3617
41	42.8433	1.0705	44.4148	-0.5010	44.3177	-0.4039
42	43.8842	-0.2573	43.9120	-0.2851	43.7849	-0.1580
43	43.7317	0.0076	44.3024	-0.5631	44.1532	-0.4139
44	43.7353	0.1131	43.7884	0.6000	43.6618	0.1866
45	43.4014	0.5244	44.6254	-0.6996	44.4481	-0.5223
46	43.3887	0.2983	43.2173	0.4697	43.1988	0.4882
47	44.2217	-0.6744	43.1686	0.3787	43.1014	0.4459
48	44.0675	-0.4296	43.1097	0.5282	43.0151	0.6228
49	43.3823	0.2199	43.7856	-0.1834	43.7555	-0.1533
50	44.2145	-0.5592	43.3269	0.3281	43.2649	0.3901
51	44.0638	-0.4166	43.2669	0.3793	43.1960	0.4512
52	43.9277	-0.6596	43.9411	-0.6731	42.1699	1.0981
53	44.5547	-1.1801	42.2277	1.1469	42.2223	1.1523
54	42.2054	1.4172	43.9064	-0.2838	43.9723	-0.3497
55	41.9713	1.7925	43.9935	-0.2297	44.0191	-0.2553
56	41.6337	2.3019	44.5786	-0.6430	44.5602	-0.6246
57	42.4069	1.0278	43.6386	-0.2039	43.7417	-0.3070
58	43.1564	0.2353	41.5723	1.8194	43.2015	0.1902
59	42.6691	0.9783	44.2341	-0.5867	44.1918	-0.5444
60	42.1673	1.4929	42.9430	0.7168	43.6912	-0.0314
61	42.7472	0.5935	41.5723	1.7698	42.5613	0.7794
63	42.5548	0.9032	43.0301	0.4270	43.1795	0.2776
64	42.9504	0.0039	41.9665	0.9877	42.1911	0.7631
65	43.1392	0.2645	43.4074	0.0037	43.4597	-0.0560
66	40.7953	2.4194	43.1649	0.0498	43.4918	-0.2771
67	41.1401	2.2958	43.9317	-0.4958	44.1055	-0.6696
68	39.8761	3.9986	44.9325	-1.0578	45.0774	-1.2027
69	41.4305	1.7711	43.5179	-0.3163	43.7316	-0.5300

Table 3. Percent of Amphetamine Derivative Excreted Unchanged (PEU) for Three Subjects

no.	compd	log PEU					
		subject 1		subject 2		subject 3	
		obsd	calcd	obsd	calcd	obsd	calcd
I	amphetamine	1.78	1.77	1.76	1.79	1.79	1.95
II	<i>N</i> -methyl	1.83	1.79	1.79	1.78	1.69	1.72
III	<i>N</i> -ethyl	1.68	1.72	1.66	1.68	1.16	1.40
IV	<i>N</i> - <i>n</i> -propyl	1.51	1.52	1.49	1.48	1.08	1.07
V	<i>N</i> -2-propyl	1.59	1.50	1.60	1.47	1.35	1.04
VI	<i>N</i> - <i>n</i> -butyl	0.87	1.20	0.76	1.19	0.29	0.71
VII	<i>N</i> -2-butyl	1.28	1.24	1.34	1.22	1.10	0.75
VIII	<i>N</i> -benzyl	-0.23	-0.30	-0.27	-0.28	-0.78	-1.00
IX	<i>N,N</i> -dimethyl	1.59	1.72	1.61	1.69	1.33	1.40
X	<i>N,N</i> -diethyl	1.60	1.31	1.44	1.28	1.29	0.82
XI	<i>N,N</i> -di- <i>n</i> -propyl	0.25	0.29	0.17	0.36	-0.28	-0.02
XII	<i>N,N</i> -di- <i>n</i> -butyl	-1.16	-1.16	-0.82	-0.90	-1.12	-0.92
XIII	<i>N</i> -ethyl- <i>N</i> -methyl	1.54	1.56	1.58	1.53	1.18	1.13
XIV	<i>N</i> -methyl- <i>N</i> - <i>n</i> -propyl	1.38	1.24	1.40	1.22	0.87	0.75
XV	<i>N</i> - <i>n</i> -butyl- <i>N</i> -methyl	0.47	0.58	0.57	0.57	-0.21	-0.06

Table 4. Values of Topological and Topographic Descriptors for the Series of Amphetamine Derivatives

no.	${}^1\chi^v$	Ω	ϵ	$\epsilon(q)$
I	3.583	3.961	4.966	4.117
II	4.038	4.487	5.377	4.522
III	4.599	4.997	5.915	5.062
IV	5.099	5.502	6.415	5.569
V	4.982	5.376	6.449	5.617
VI	5.599	6.007	6.915	6.074
VII	5.520	5.918	6.860	6.040
VIII	6.156	6.674	8.933	7.173
IX	4.402	4.875	5.907	5.065
X	5.554	5.973	6.766	5.943
XI	6.554	6.981	7.842	7.032
XII	7.554	7.991	8.842	8.041
XIII	4.978	5.422	6.328	5.494
XIV	5.478	5.927	6.866	6.039
XV	5.978	6.432	7.884	6.544

Table 5. Statistical Characteristics of the Models Found to Describe the log PEU of Amphetamines with Single Topological or Topographic Indices^a

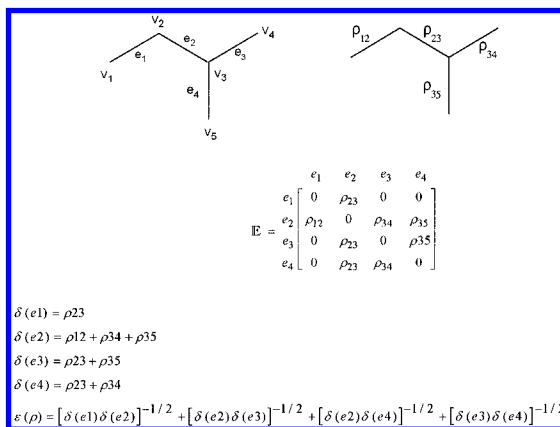
	index	<i>a</i>	<i>b</i>	<i>c</i>	r^2	<i>s</i>	<i>F</i>
log PEU ₁	${}^1\chi^v$	-0.843	1.490	-0.205	0.893	0.306	50.2
	Ω	-1.595	1.691	-0.207	0.908	0.284	59.4
	ϵ	-2.385	1.684	-0.168	0.920	0.266	68.4
	$\epsilon(q)$	-2.626	2.033	-0.232	0.959	0.189	141.3
	log (P_H)	1.43	-0.345	-0.077	0.947	0.220	88.7
log PEU ₂	${}^1\chi^v$	0.170	1.061	-0.162	0.868	0.318	39.3
	Ω	-0.429	1.239	-0.165	0.883	0.299	45.4
	ϵ	-1.605	1.422	-0.147	0.931	0.230	80.9
	$\epsilon(q)$	-1.457	1.588	-0.191	0.940	0.214	94.2
	log (P_H)	1.48	-0.341	-0.064	0.949	0.201	92.5
log PEU ₃	${}^1\chi^v$	2.903		-7.42×10^{-2}	0.828	0.391	62.8
	Ω	3.073		-6.88×10^{-2}	0.843	0.374	69.8
	ϵ	3.274		-5.36×10^{-2}	0.922	0.263	153.9
	$\epsilon(q)$	3.162		-6.86×10^{-2}	0.888	0.315	103.5
	log (P_H)	0.943	-0.517		0.896	0.307	94.4

^a Equations are of the form $\log \text{PEU} = a + bI + cP$.

in the main chemical process that can occur with these molecules in the studied conditions.

The normal urinary pH has a mean value of 6.3; however, it has big oscillations during the day producing a wide range of urinary pH values among 4.5–8.5, which can alter the dissociation process of amphetamine derivatives.

The validity of the stated hypothesis can be tested extracting the electronic information contained in the topo-

**Figure 1.** Computation of the $\epsilon(\rho)$ for a graph representing skeleton of 2-methylbutane.

graphic $\epsilon(\rho)$ index and using it as an independent variable in novel QSAR models. Both graph theoretical descriptors, the topological ϵ index and the topographic $\epsilon(\rho)$ index, are based on the same invariant, and their main difference is in the use of bond orders (an electronic feature) as edge weights in the definition of $\epsilon(\rho)$. Thence, we can consider the topographic index as a corrected ϵ index, where the correction factor is C . Mathematically, this definition of $\epsilon(\rho)$ can be expressed as follows

$$\epsilon(\rho) = \epsilon + C$$

We have no analytical expression for the correction factor C , but we can obtain it from the calculated values of $\epsilon(\rho)$ and ϵ indices. Using C as a novel descriptor, the following models for log PEU of subjects 1 and 2 were obtained

$$\log \text{PEU}_1 = 2.104\epsilon(\rho) - 0.235[\epsilon(\rho)]^2 - 0.435C - 2.547 \quad (5)$$

$$r^2 = 0.975 \quad s = 0.156 \quad F = 141.0$$

$$\log \text{PEU}_2 = 1.672\epsilon(\rho) - 0.194[\epsilon(\rho)]^2 - 0.511C - 1.364 \quad (6)$$

$$r^2 = 0.964 \quad s = 0.172 \quad F = 99.7$$

As can be appreciated from the statistical parameters of the eqs 5 and 6, both models are very significant. No analogue model can be found for log PEU₃. Values of log PEU for subjects 1–3 calculated with the best models found here are depicted in Table 3. The C index can be associated in some way with the electronic characteristics responsible of the dissociation of amphetamine derivatives. Indices analogous to C , obtained from the differences between other topographic and topological indices based on the same graph-theoretical invariant, have been successfully applied in QSPR and QSAR studies by one of the present authors.²⁸ The results obtained here shown the power of using topographic descriptors in QSAR analysis.

5. CHARACTERISTIC FEATURES OF $\epsilon(\rho)$ INDEX

The differentiation of a heteroatom's positions in organic compounds is one of the desirable properties of topographic indices. In Figure 2 we can appreciate that the $\epsilon(\rho)$ index shows a regular variation with the position of the heteroatom in a series of aliphatic ethers. The greater values of $\epsilon(\rho)$

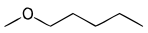
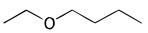
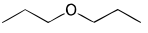
Ether	${}^1\chi$	${}^1\chi^v$	$\epsilon(\rho)$
	3.414	2.905	2.963
	3.414	2.992	2.971
	3.414	2.992	2.976

Figure 2. Numerical values of $\epsilon(\rho)$ index for isomeric ethers with degenerated values of connectivity indexes.





		
	<i>anti</i>	<i>gauche</i>
$\epsilon(\rho)$	1.4122	1.4132
$\chi\chi$	1.9814	1.9918
		
	<i>chair</i>	<i>boat</i>
$\epsilon(\rho)$	3.0264	3.0298
$\chi\chi$	3.0000	2.9997

Figure 3. Values of topographic $\epsilon(\rho)$ and $\chi\chi$ indexes for *anti* and *gauche* conformers of *n*-butane as well as *chair* and *boat* conformers of cyclohexane.

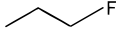
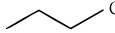
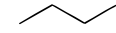
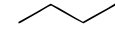
	
1.4266	1.4311
	
1.4532	1.4573

Figure 4. Values of $\epsilon(\rho)$ index for *n*-propyl halides.

are obtained when a heteroatom is positioned at the center of the molecule.

In Figure 3 we illustrate the *anti* and *gauche* conformations of *n*-butane and the *chair* and *boat* conformations of cyclohexane. In both cases we also give the values of the topographic index $\chi\chi$ introduced by Randić,^{20–22} which is based on graphs embedded into three-dimensional grids. As can be appreciated both indices produce discrete differentiation of conformational isomers, showing regular variation of the indices' values according to the "compactness" of the chemical structure.

In Figure 4 we depicted another characteristic of the topographic edge connectivity index $\epsilon(\rho)$: differentiation of heteroatoms in chemical structures. In this example we can appreciate that $\epsilon(\rho)$ index changes regularly with the volume variation of heteroatom from fluorine to iodine. The importance of heteroatom differentiation by molecular descriptors to be used in QSPR and QSAR studies is evident and has been mentioned elsewhere.^{44,45}

6. CONCLUDING REMARKS

A new topographic index based on edge adjacency relationships in molecular graphs is defined considering molecules as weighted graphs. Elements of edge set are substituted by the bond orders between the connected atoms in the molecule. The importance of using weighted graphs to generate structural invariants has been emphasized by Randić. This author remarked the importance of considering any matrix as a weighted graph in order for the further development of chemical combinatorics and topology.⁴⁴

When we are concerned with the generation of one descriptor to be used in QSAR studies, the consideration of global spatial features of molecules is a desirable condition. As a consequence, the use of topographic rather than topological indices is necessary.

Among the most important features of the topographic $\epsilon(\rho)$ index we can mention the isomer differentiation that includes compounds with different branching, position of heteroatoms, and chain conformations. The novel index permits differentiation among compounds with different heteroatoms and has been proven to be useful in QSPR and QSAR studies.

The $\epsilon(\rho)$ index has been obtained in this work by using the semiempirical quantum chemical method PM3, but the use of any other quantum chemical approach (semiempirical or *ab initio*) is valid too. On the other hand, the use of geometry optimization is not a necessary condition to generate the new index. We can use a frozen geometry or molecular mechanics models in the sake of simplicity to be used in non-time-consuming calculations of larger molecules of biological interest, by simple computers.

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