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Simple Analytic Proportional-Integral-Derivative (PID) Controller Tuning Rules for Unstable Processes

Wonhui Cho,[†] Jietae Lee,[*,‡] and Thomas F. Edgar[§]

[†]enGibbs, #A-503 Shin Young Gwell Estates, Bansong-dong, 93-10, Hwasung, Gyeonggi 445-160, Korea

[‡]Department of Chemical Engineering, Kyungpook National University, Taegu 702-701, Korea

[§]Department of Chemical Engineering, University of Texas, Austin, Texas 78712, United States

ABSTRACT: Very simple proportional-integral-derivative (PID) controller tuning rules for a wide range of stable processes are available. However, for unstable processes, the design trend is for controllers to be more complex for better performances. Here, the design concept of "simplicity" is extended to unstable processes. Simple desired closed-loop transfer functions for the direct synthesis method and simple approximations of the process time delay are utilized for unstable processes. Very simple tuning rules for PID controllers and set-point filters are obtained, yielding similar or even improved performances over previous more complicated PID controller tuning methods.

1. INTRODUCTION

Proportional-integral-derivative (PID) controllers are widely used in industry because of their simplicity, robustness, and performance for various processes.[1−3] Well-tuned PID controllers have generally proved to be satisfactory for industrial control loops of stable processes, and they can also be applied to stabilize unstable processes that occur in some applications.

The direct synthesis (DS) method[4,5] and equivalently the internal model control (IMC) method[6] provide simple analytic tuning rules for stable processes. Pole-zero cancellations can occur in the resulting control systems, but such approaches cannot be used for unstable processes because cancellations of unstable poles cause instability in the presence of model errors. To address this difficulty, desired closed-loop transfer functions include additional terms to avoid the cancellation of unstable poles. The resulting controllers are usually fairly complicated and are not in the form of PID controllers. Approximate PID controllers can be obtained by the Taylor series expansion[7,8] and the frequency domain fitting.[9] Chen and Seborg[10] obtained PID controllers by assuming the desired closed-loop transfer functions for load changes with Pade approximations of time delays. Shamsuzzoha et al.[11] used desired closed-loop transfer functions with damping factors less than 1 for better performances and Lee[12] investigated stability regions of PID control systems for unstable processes. Several researchers used higher order controllers[13] and PID controllers with lead/lag modules.[14,15] Alcantara et al.[18] proposed rigorous tuning methods based on various control performance trade-offs and provided simple correlation rules for first and second-order (with one double pole) processes.

For stable and integrating processes, very simple tuning rules are shown to work well.[16] Here similar methods are extended to unstable processes. Simple desired closed-loop transfer functions for the direct synthesis method and simple approximations of the process time delays are used and shown to be an improvement over previous more complex methods. They have been used already for stable processes by Chen and Seborg[10] but not extended to unstable processes. For unstable processes, two degree of freedom controllers are necessary to achieve satisfactory closed-loop performance for both load and set-point changes.[17] The proposed method provides a systematic procedure to design the set-point filter for a two degree of freedom control system.

2. SIMPLE UNIFIED TUNING RULES

Consider a control system for a single-input single-output process. Here the poles of process transfer function $G(s)$ are in or near the right half plane. For such processes, two degree of freedom controllers are required for better responses of both set-point and load changes. The PID control system $C(s)$ with the set-point filter $F_R(s)$ (Figure 1) is considered.

$$\frac{Y(s)}{R(s)} = \frac{G(s)\,C(s)}{1 + G(s)\,C(s)} F_R(s)$$
$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + G(s)\,C(s)} \tag{1}$$

Here $Y(s)$, $R(s)$, and $D(s)$ are the Laplace transformations of the output variable $y(t)$, the set-point variable $r(t)$, and the disturbance variable $d(t)$ entering at the process input. The controller $C(s)$ is restricted to the ideal form of the parallel-form proportional-integral-derivative controller;

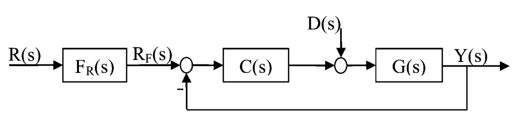

Figure 1. Control system with the set-point filter.

Special Issue: David Himmelblau and Gary Powers Memorial

Received: March 31, 2013
Revised: October 12, 2013
Accepted: October 29, 2013
Published: October 29, 2013

dx.doi.org/10.1021/ie401018g | Ind. Eng. Chem. Res. 2014, 53, 5048−5054

Table 1. PID Controller Parameters ($C(s) = k_c(1 + (1/\tau_I s) + \tau_D s)$), the Peak Value of Sensitivity Function (M_s), and the Set-Point Filter ($F_R(s)$)

example			method	λ	k_c	τ_I	τ_D	M_S	$F_R(s)$
[1]	$\frac{e^{-\theta s}}{s-1}$	$\theta = 0.2$	DS[1/2][7]	0.8	2.5957	3.0041		2.0	[a]
			DS[2/3][8]	0.9	2.6047	2.3151		2.1	[a]
			proposed	0.6	2.7500	2.2000		2.1	$(0.6s + 1)/(2.2s + 1)$
		$\theta = 0.4$	DS[1/2][7]	1.6	1.6773	9.1994		3.3	[a]
			DS[2/3][8]	1.8	1.6481	6.0078		3.7	[a]
			proposed	1.2	1.6563	7.0667		3.5	$(1.2s + 1)/(7.0667s + 1)$
		$\theta = 0.6$	DS[1/2][7]	2.4	1.3820	20.265		7.3	[a]
			DS[2/3][8]	2.7	1.3241	11.274		11	[a]
			proposed	1.8	1.2917	18.600		7.8	$(1.8s + 1)/(18.6s + 1)$
		$\theta = 0.8$	DS[1/2][7]	3.2	1.2416	38.563	0.3026	5.1	[a]
			proposed	4.0	1.2551	41.333	1.9116	4.3	$(4.0s + 1)/(41.333s + 1)$
		$\theta = 1.2$	DS[1/2][7]	4.8	1.1137	111.24	0.5510	10	[a]
			proposed	6.0	1.1141	123.00	4.7938	9.3	$(6.0s + 1)/(123.0s + 1)$
[2]	$\frac{e^{-0.4s}}{s^2 + 0.2s + 1}$		DCLR[5]	0.6	0.0156	0.0250	39.558	1.6	not applicable
			proposed	1.2	0.2109	0.6968	5.6519	1.6	$(1.44s^2 + 2.4s + 1)/(3.9381s^2 + 0.6968s + 1)$
[3]	$\frac{e^{-0.2s}}{(3s-1)(s-1)}$		DS[2/4][7]	0.45	2.3153	1.7843	1.8859	2.9	[a]
			proposed	0.8	2.4120	2.1536	1.9207	2.8	$(0.64s^2 + 1.6s + 1)/(4.1364s^2 + 2.1536s + 1)$
[4]	$\frac{e^{-0.2s}}{s(s-1)}$		DS[2/4][7]	0.8	0.8421	3.3066	2.8113	1.9	[a]
			proposed	1.4	0.8594	4.4000	2.7000	1.9	$(1.96s^2 + 2.8s + 1)/(11.88s^2 + 4.4s + 1)$
[5]	$\frac{e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)}$		DS[2/4][7]		7.1440	6.6840	1.6550	4.3	[a]
			Yang[9]		2.9890	12.799	1.5620	2.2	[a]
			proposed 1	2.16	4.7408	9.1627	1.6975	4.3	$(4.6656s^2 + 4.32s + 1)/(15.5539s^2 + 9.1627s + 1)$
			proposed 2	2.12	4.7317	9.1865	1.6989	4.3	$(2.1225s + 1)/(6.9365s + 1)$
[6]	$\frac{1}{2\theta}\frac{e^{-\theta s}}{s+1}$	$\theta = 0.02$	SIMC[15]	0.02	1	0.16		1.7	not applicable
			proposed	0.04	1.0933	0.0965		1.9	$(0.04s + 1)/(0.0965s + 1)$
		$\theta = 0.2$	SIMC[15]	0.2	1	1		1.6	not applicable
			proposed	0.4	0.9333	0.7		1.6	$(0.4s + 1)/(0.7s + 1)$

[a]Refer to the original paper.

Table 2. Proposed PID Controller Tuning Rules

process	$C(s)$	$F_R(s)$
$\dfrac{b_0 e^{-\theta s}}{s+a_0}$	$C(s)=\dfrac{\beta}{b_0(\lambda^2+\beta\theta)}\left(1+\dfrac{1}{\beta s}\right)$ $\beta=\dfrac{2\lambda+\theta-a_0\lambda^2}{a_0\theta+1}$	$\dfrac{\lambda s+1}{\beta s+1}$
$\dfrac{b_0 e^{-\theta s}}{s^2+a_1 s+a_0}$	$C(s)=\dfrac{\beta_1}{b_0(\lambda^3+\theta\beta_2)}\left(1+\dfrac{1}{\beta_1 s}+\dfrac{\beta_2}{\beta_1}s\right)$ $\begin{pmatrix}\beta_2\\ \beta_1\end{pmatrix}=\begin{pmatrix}-1-a_1\theta & \theta\\ a_0\theta & 1\end{pmatrix}^{-1}\begin{pmatrix}a_1\lambda^3-3\lambda^2\\ 3\lambda+\theta-a_0\lambda^3\end{pmatrix}$	$\dfrac{\lambda s+1}{\beta_2 s^2+\beta_1 s+1}$

$$C(s)=k_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right) \tag{2}$$

The direct synthesis method (equivalently the internal model control method) starts with assuming the desired closed-loop transfer function

$$\begin{aligned}\frac{Y(s)}{R_F(s)}&=Q(s)\\ &=\frac{G(s)\,C(s)}{1+G(s)\,C(s)}\\ &=\frac{(\beta_m s^m+\cdots+\beta s+1)G^+(s)}{(\lambda s+1)^r}\end{aligned} \tag{3}$$

where $G^+(s)$ is the noninvertible part of $G(s)$, λ is the design parameter representing the speed of closed-loop response, and other parameters r, m, and b_i are additional parameters to prevent the pole-zero cancellations of unstable process poles. We call this method DS[m/r]. Then

$$\begin{aligned}C(s)&=\frac{1}{G(s)}\frac{Q(s)}{1-Q(s)}\\ &=\frac{1}{G(s)}\frac{(\beta_m s^m+\cdots+\beta s+1)G^+(s)}{(\lambda s+1)^r-(\beta_m s^m+\cdots+\beta s+1)G^+(s)}\end{aligned} \tag{4}$$

To obtain a PID controller structure, $C(s)$ of eq 4 should be approximated. For this, methods based on the Taylor series expansion,[7,8] frequency domain fitting,[9] and Pade approximation of time delay are available. Although the frequency domain fitting will be the best, it is not considered here because analytic rules cannot be obtained.

PI Controller. Consider a first-order process with delay

$$G(s)=\frac{b_0 e^{-\theta s}}{s+a_0} \tag{5}$$

Let the desired closed-loop transfer function be

$$Q(s)=\frac{G(s)\,C(s)}{1+G(s)\,C(s)}=\frac{(\beta s+1)e^{-\theta s}}{(\lambda s+1)^2} \tag{6}$$

Then

$$\begin{aligned}C(s)&=\frac{1}{G(s)}\frac{Q(s)}{1-Q(s)}\\ &=\frac{s+a_0}{b_0 e^{-\theta s}}\frac{(\beta s+1)e^{-\theta s}}{(\lambda s+1)^2-(\beta s+1)e^{-\theta s}}\end{aligned} \tag{7}$$

The parameter β is such that $C(s)$ does not have the term $s+a_0$.

Here a simpler method is derived with approximation of $e^{-\theta s}\approx-\theta s+1$. Equation 7 becomes

$$\begin{aligned}C(s)&=\frac{s+a_0}{b_0}\frac{\beta s+1}{(\lambda s+1)^2-(\beta s+1)(-\theta s+1)}\\ &=\frac{\beta s+1}{b_0}\frac{s+a_0}{(\lambda^2+\theta\beta)s^2+(2\lambda+\theta-\beta)s}\\ &=\frac{\beta s+1}{b_0(\lambda^2+\theta\beta)s}\frac{s+a_0}{s+\frac{2\lambda+\theta-\beta}{\lambda^2+\theta\beta}}\end{aligned} \tag{8}$$

For $C(s)$ without the zero term $s+a_0$, we have $(2\lambda+\theta-\beta)/(\lambda^2+\theta\beta)=a_0$ and hence

$$\beta=\frac{2\lambda+\theta-a_0\lambda^2}{a_0\theta+1} \tag{9}$$

Then

$$C(s)=\frac{\beta s+1}{b_0(\lambda^2+\beta\theta)s}=\frac{\beta}{b_0(\lambda^2+\beta\theta)}\left(1+\frac{1}{\beta s}\right) \tag{10}$$

Hence the PI controller parameters (Table 1) in the form $C(s)=k_c(1+(1/\tau_I s))$ are

$$k_c=\frac{\beta}{b_0(\lambda^2+\beta\theta)}\qquad \tau_I=\beta \tag{11}$$

The desired closed-loop transfer function is

$$\frac{Y(s)}{R_F(s)}\approx\frac{(\beta s+1)e^{-\theta s}}{(\lambda s+1)^2} \tag{12}$$

It has a zero term $(\beta s+1)$ and will show a large overshoot for the step set-point change. To reduce this overshoot, we use a set-point filter

$$F_R(s)=\frac{\lambda s+1}{\beta s+1} \tag{13}$$

This will result in the first-order response with the time constant of λ for the set-point change. Tan et al.[19] used this set-point filter for some example processes.

The PI controller tuning rule of eq 11 with eq 9 can be applied to stable (positive a_0), integral ($a_0 = 0$), and unstable (negative a_0) processes. Chen and Seborg[10] obtained the same PI controller tuning rules for stable and integrating processes but suggested different set-point filters (Table 2).

PID Controller. Consider a second-order process

$$G(s) = \frac{b_0 e^{-\theta s}}{(s + a_0)(\tau_2 s + 1)} \tag{14}$$

Let the controller form be $C(s) = C_1(s)(1 + \tau_2 s)$ where $C_1(s)$ is the PI controller for the process $b_0 \exp(-\theta s)/(s + a_0)$. Then the controller becomes the series-form PID controller as

$$C(s) = \frac{\beta}{b_0(\lambda^2 + \beta\theta)}\left(1 + \frac{1}{\beta s}\right)(1 + \tau_2 s)$$
$$\beta = \frac{2\lambda + \theta - a_0\lambda^2}{a_0\theta + 1} \tag{15}$$

Here one pole $(s + a_0)$ is not canceled and the other pole $(\tau_2 s + 1)$ is canceled.

For the first-order process with large delay where the PI controller does not work well, this PID controller can be used. First, it is approximated as

$$G(s) = \frac{b_0 e^{-\theta s}}{s + a_0} = \frac{b_0 e^{-\theta s/2}}{(s + a_0)e^{\theta s/2}} \approx \frac{b_0 e^{-\theta s/2}}{(s + a_0)(\theta s/2 + 1)} \tag{16}$$

Then, applying the previous PI controller tuning rule with the pole-zero cancellation technique for the stable pole $(\theta s/2 + 1)$, we have the PID controller

$$C(s) = \frac{\beta}{b_0(\lambda^2 + \beta\theta/2)}\left(1 + \frac{1}{\beta s}\right)(1 + \theta s/2)$$
$$\beta = \frac{2\lambda + \theta/2 - a_0\lambda^2}{a_0\theta/2 + 1} \tag{17}$$

Consider a general second-order process

$$G(s) = \frac{b_0 e^{-\theta s}}{s^2 + a_1 s + a_0} \tag{18}$$

Let the desired closed-loop transfer function be

$$Q(s) = \frac{G(s)\,C(s)}{1 + G(s)\,C(s)} = \frac{(\beta_2 s^2 + \beta_1 s + 1)e^{-\theta s}}{(\lambda s + 1)^3} \tag{19}$$

Then

$$C(s) = \frac{1}{G(s)}\frac{Q(s)}{1 - Q(s)}$$
$$= \frac{s^2 + a_1 s + a_0}{b_0 e^{-\theta s}}\frac{(\beta_2 s^2 + \beta_1 s + 1)e^{-\theta s}}{(\lambda s + 1)^3 - (\beta_2 s^2 + \beta_1 s + 1)e^{-\theta s}} \tag{20}$$

With approximation $e^{-\theta s} \approx -\theta s + 1$, we have

$$C(s) = \frac{\beta_2 s^2 + \beta_1 s + 1}{b_0}\frac{s^2 + a_1 s + a_0}{(\lambda s + 1)^3 - (\beta_2 s^2 + \beta_1 s + 1)(-\theta s + 1)}$$
$$= \frac{\beta_2 s^2 + \beta_1 s + 1}{b_0} \times \frac{s^2 + a_1 s + a_0}{(\lambda^3 + \theta\beta_2)s^3 + (3\lambda^2 + \theta\beta_1 - \beta_2)s^2 + (3\lambda + \theta - \beta_1)s}$$
$$= \frac{\beta_2 s^2 + \beta_1 s + 1}{b_0(\lambda^3 + \theta\beta_2)s}\frac{s^2 + a_1 s + a_0}{s^2 + \frac{3\lambda^2 + \theta\beta_1 - \beta_2}{\lambda^3 + \theta\beta_2}s + \frac{3\lambda + \theta - \beta_1}{\lambda^3 + \theta\beta_2}} \tag{21}$$

The parameters β_1 and β_2 satisfying

$$\frac{3\lambda^2 + \theta\beta_1 - \beta_2}{\lambda^3 + \theta\beta_2} = a_1$$
$$\frac{3\lambda + \theta - \beta_1}{\lambda^3 + \theta\beta_2} = a_0 \tag{22}$$

remove the last term in eq 21. Rearranging eq 22 for the unknowns, β_1 and β_2, we have

$$\begin{pmatrix}\beta_2\\ \beta_1\end{pmatrix} = \begin{pmatrix}-1 - a_1\theta & \theta\\ a_0\theta & 1\end{pmatrix}^{-1}\begin{pmatrix}a_1\lambda^3 - 3\lambda^2\\ 3\lambda + \theta - a_0\lambda^3\end{pmatrix}$$
$$= \frac{1}{a_0\theta^2 + a_1\theta + 1}\begin{pmatrix}-1 & \theta\\ a_0\theta & 1 + a_1\theta\end{pmatrix}\begin{pmatrix}a_1\lambda^3 - 3\lambda^2\\ -a_0\lambda^3 + 3\lambda + \theta\end{pmatrix} \tag{23}$$

Then eq 21 becomes

$$C(s) = \frac{\beta_2 s^2 + \beta_1 s + 1}{b_0(\lambda^3 + \theta\beta_2)s} = \frac{\beta_1}{b_0(\lambda^3 + \theta\beta_2)}\left(1 + \frac{1}{\beta_1 s} + \frac{\beta_2}{\beta_1}s\right) \tag{24}$$

Hence the PID controller parameters in the parallel-form form are

$$k_c = \frac{\beta_1}{b_0(\lambda^3 + \theta\beta_2)} \qquad \tau_I = \beta_1 \qquad \tau_D = \frac{\beta_2}{\beta_1} \tag{25}$$

The desired closed-loop transfer function is

$$\frac{Y(s)}{R_F(s)} \approx \frac{(\beta_2 s^2 + \beta_1 s + 1)e^{-\theta s}}{(\lambda s + 1)^3} \tag{26}$$

As in the above PI controller, we use the set-point filer

$$F_R(s) = \frac{(\lambda s + 1)^2}{\beta_2 s^2 + \beta_1 s + 1} \tag{27}$$

This will result in a first-order response with a time constant λ for the set-point change.

3. EXAMPLES

Example 1 (First-Order Unstable Process). The proposed method is applied to a first-order unstable process,[7−9,17]

$$G(s) = \frac{e^{-\theta s}}{s - 1} \tag{28}$$

Load responses are compared with previous direct synthesis methods based on the second-order desired closed-loop transfer function[7] (DS[1/2]; $Q(s) = (\beta s + 1)\exp(-\theta s)/(\lambda s + 1)^2$) and the third-order desired closed-loop transfer function[8] (DS[2/3]; $Q(s) = (\beta s + 1)^2 \exp(-\theta s)/(\lambda s + 1)^3$). The previous methods of DS[1/2] and DS[2/3] use Taylor series expansion and can be better than our approximation of $e^{-\theta s} \approx -\theta s + 1$. The effects of different approximations of the delay term and the order of desired closed-loop transfer functions are investigated for load responses in Figure 2. The design parameter λ is usually chosen to be a constant multiple of the process time delay. Here, λ's are set to 4θ, 4.5θ, and 3θ for DS[1/2], DS[2/3], and the proposed method, respectively, which provide similar control performances for $\theta = 0.4$. As the process time delay increases, control performances are affected by the choice of λ, with the DS[2/3] method being the worst. High order $Q(s)$ may not be preferable in designing PI controllers. However, by adjusting the constant multiplied to the process time delay for λ, its performance can be improved. The proposed method will be useful for λ with a fixed constant multiple of time delay. The proposed tuning rule with $\lambda = 3\theta$ is the same as the correlation rule in Alcantara et al.[18] for the first-order unstable processes. Their correlation rule has been obtained from solutions optimizing cost functions for balanced robustness and performance. Due to the robustness concern, they suggested that the correlation rule should be limited to processes with $\theta < 0.5$. Figure 3 shows step set-point responses where process parameters have errors. Robustness of the proposed method is similar to the DS[1/2] method. When the time delay grows, the PI controller does not work well and the PID controller should be used. The PID controller of eq 17 is used for the proposed method. Responses for $\theta = 0.8$ and 1.2 are given in Figure 4.

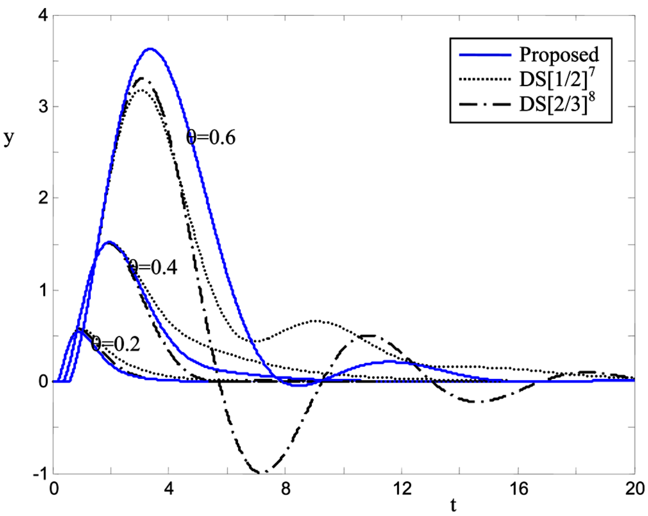

Figure 2. Step load responses for the process $G(s) = e^{-\theta s}/(s - 1)$.

Example 2 (Underdamped Process). Consider the underdamped process

$$G(s) = \frac{e^{-0.4s}}{s^2 + 0.2s + 1} \tag{29}$$

Its damping factor is 0.1 and the process shows a considerable oscillatory response for the open-loop step change. Figure 5 shows closed-loop responses designed by the desired closed-

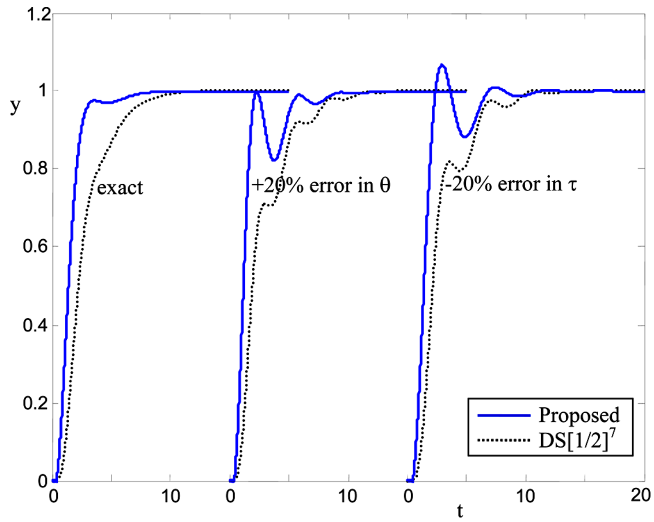

Figure 3. Step set-point responses of PI control systems designed for $G(s) = e^{-0.4s}/(s - 1)$ when there are model mismatches.

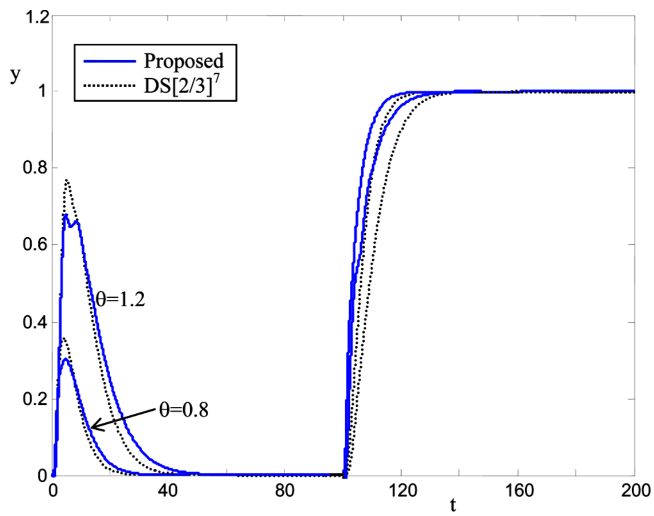

Figure 4. Responses of PID control systems for the process $G(s) = e^{-\theta s}/(s - 1)$.

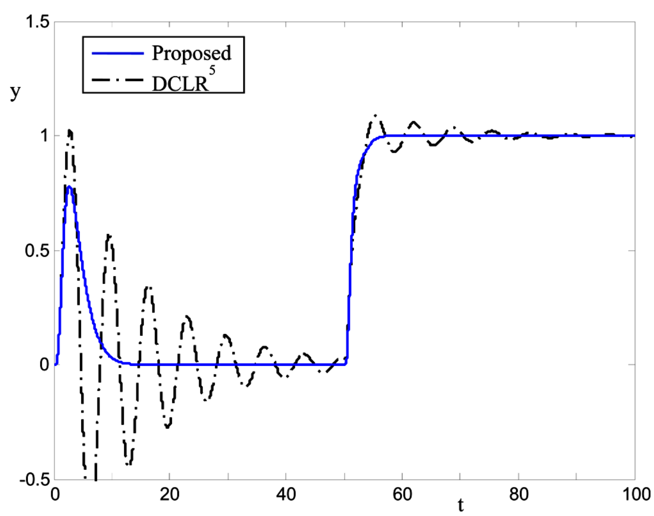

Figure 5. Responses of PID control systems for the process $G(s) = e^{-0.4s}/(s^2 + 0.2s + 1)$.

loop response (DCLR) method[5] and the proposed method. The PID controller of the DCLR method shows oscillatory responses. Incrementing λ is not a remedy to reduce the problem of oscillatory responses in the DCLR method. Even for λ greater than 1.7, the controller gain becomes negative. The proposed method shows excellent results, showing less oscillatory responses.

Example 3 (Process with Two Unstable Poles). The proposed method is applied to the process with two unstable poles,[7,15]

$$G(s) = \frac{e^{-0.2s}}{(3s-1)(s-1)} \tag{30}$$

Figure 6 shows control responses for the DS[2/4] method[7] and the proposed method. The controller parameters given in the

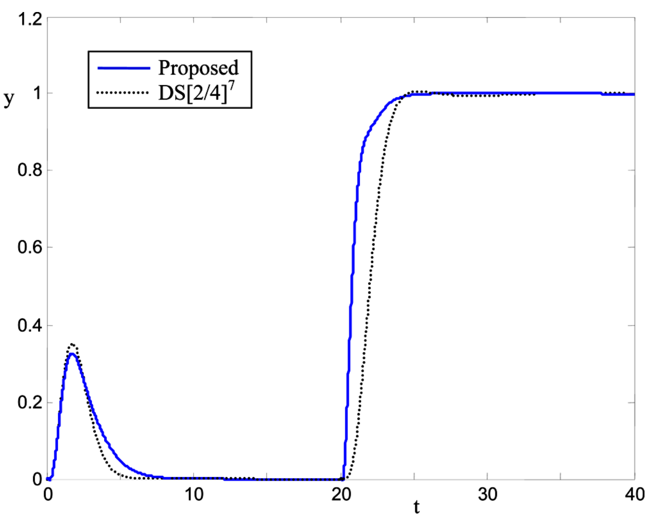

Figure 6. Responses of PID control systems for the process $G(s) = 2e^{-0.2s}/((3s-1)(s-1))$.

DS[2/4] method[7] are based on $\theta = 0.2$ and used here. Very similar load responses are obtained. The set-point response of the proposed method is better because of the set-point filter of the proposed method.

Example 4 (Integral Process with an Unstable Pole). Consider the integrating process with an unstable pole,[7]

$$G(s) = \frac{e^{-0.2s}}{s(s-1)} = \frac{e^{-0.2s}}{s^2 - s} \tag{31}$$

Figure 7 shows control responses for the DS[2/4] method[7] and the proposed method. For this integrating process ($a_0 = 0$), eq 23 becomes, for $G(s) = b_0 \exp(-\theta s)/(s^2 + a_1 s)$,

$$\begin{pmatrix} \beta_2 \\ \beta_1 \end{pmatrix} = \frac{1}{1 + a_1\theta}\begin{pmatrix} -a_1\lambda^3 + 3\lambda^2 + 3\theta\lambda + \theta^2 \\ (1 + a_1\theta)(3\lambda + \theta) \end{pmatrix} \tag{32}$$

The proposed PID controller settings can be obtained easily and show better responses.

Example 5 (Third-Order Unstable Process). Consider the unstable process,[7−9,17]

$$G(s) = \frac{e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)} \tag{33}$$

Applying the model reduction rule for stable processes,[16] we obtain the second-order model

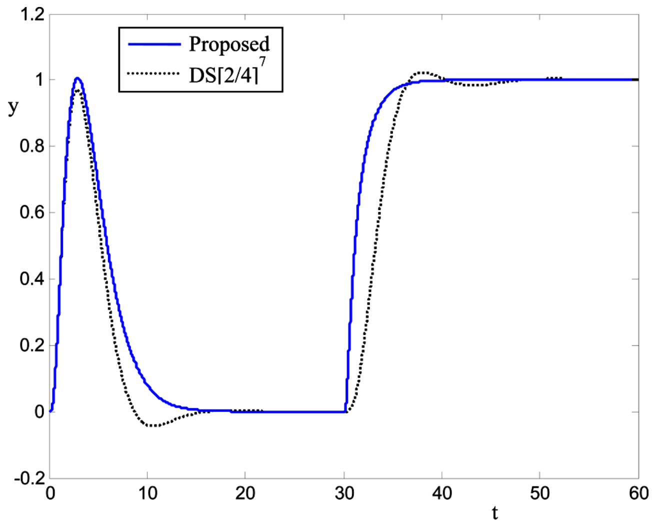

Figure 7. Responses of PID control systems for the process $G(s) = e^{-0.2s}/(s(s-1))$.

$$G(s) = \frac{e^{-0.75s}}{(5s-1)(2.25s+1)} \tag{34}$$

Then PID controllers are designed. The first controller (proposed 1) is based on eq 17, which cancels the pole $(2.25s + 1)$ and the second controller (proposed 2) is based on eqs 23 and 24. Figure 8 shows similar control performances for both proposed controllers. The previous controllers are somewhat worse. They can be improved by adjusting the design parameter λ.

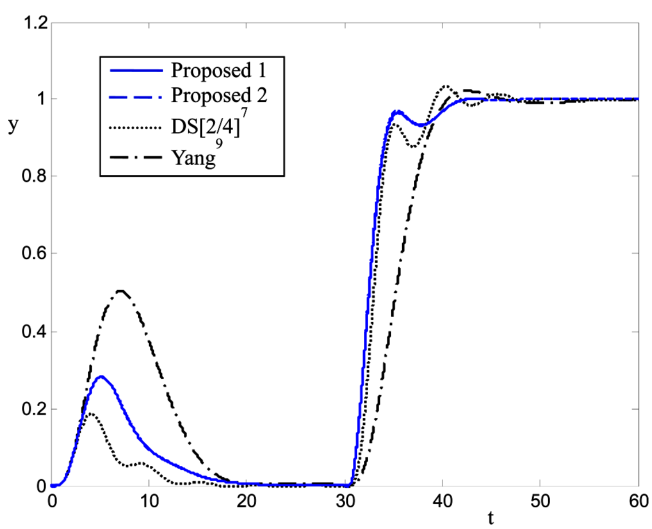

Figure 8. Responses of PID control systems for the process $G(s) = e^{-0.5s}/((5s-1)(2s+1)(0.5s+1))$.

Example 6 (Process with a Pole near Origin). Consider the first-order plus time delay process,

$$G(s) = \frac{1}{2\theta}\frac{e^{-\theta s}}{s+1} \tag{35}$$

The proposed method is compared with the well-known SIMC method.[16] Except the overshoot of the SIMC method for a small time delay process, both methods show excellent responses (Figure 9). The SIMC method, when it is applicable and an appropriate set-point filter is given, will be preferable.

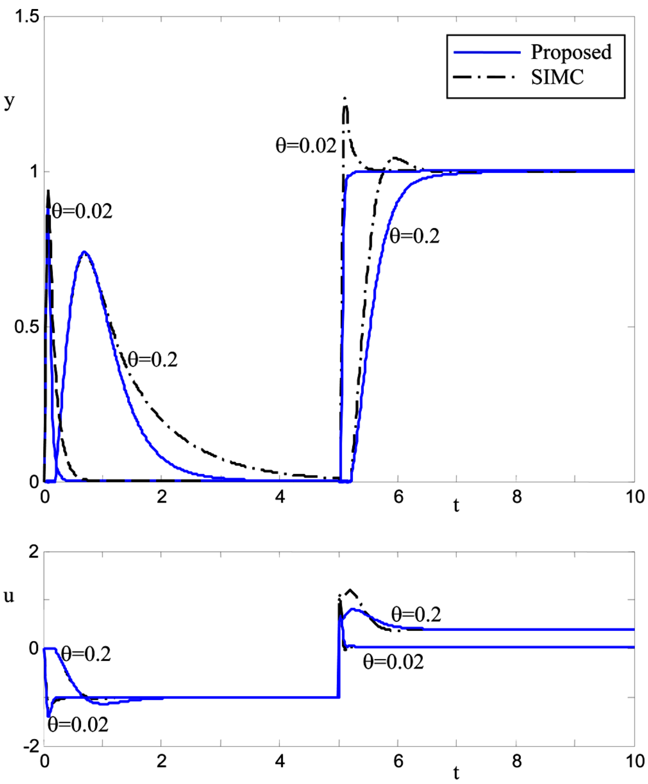

Figure 9. Responses of PID control systems for the process $G(s) = e^{-\theta s}/(2\theta(s+1))$.

4. CONCLUSION

Simple tuning rules for PI and PID controllers are proposed for unstable processes. For analytic expressions of tuning rules, the direct synthesis method (equivalently the internal model control method) is used. Instead of investigating better desired closed-loop transfer functions and approximations of process time delay, we use a simple desired closed-loop transfer function and the first-order Taylor series approximation of process time delay ($e^{-\theta s} \approx -\theta s + 1$). Each method based on the direct synthesis technique has one design parameter of the closed-loop time constant, and by adjustment of the design parameter, it can realize similar performances and robustness. The proposed tuning rules retain the underlying concept of "simplicity" in the SIMC method for stable processes and can be used preferably, replacing more complicated methods.

■ AUTHOR INFORMATION

Corresponding Author

*J. Lee: tel, +82-53-950-5620; fax, +82-53-950-6615; e-mail, jtlee@knu.ac.kr.

Notes

The authors declare no competing financial interest.

■ ACKNOWLEDGMENTS

This work (2011-0013841) was supported by Midcareer Research Program through NRF grant funded by the MEST.

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