Pergamon

PII: S0005-1098(96)00043-X

Automatica, Vol. 32, No. 8, pp. 1127–1143, 1996

0005-1098/96 $15.00 + 0.00

Adaptive Control of Induction Motor Systems Despite Rotor Resistance Uncertainty*

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Using the full-order, nonlinear dynamic model, a position tracking controller is designed to compensate for uncertainty in the form of the rotor resistance parameter and the mechanical subsystem parameters.

Key Words—Adaptive tracking control; induction motors; partial-state feedback; unknown rotor resistance.

Abstract—We present an adaptive, partial-state feedback, position tracking controller for the full-order, nonlinear dynamic model representing an induction motor actuating a mechanical subsystem. The proposed controller compensates for uncertainty in the form of the rotor resistance parameter and all of the mechanical subsystem parameters, while yielding asymptotic rotor position tracking. The proposed controller does not require measurement of rotor flux or rotor current; however, it does exhibit a singularity when the magnitude of the estimated rotor flux is zero. Simulation results are provided to verify the effectiveness of the approach. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Because of the simplistic construction and durability of the induction motor, its use in industrial applications (e.g., web-handling and overhead cranes), will probably increase over the next decade. Hence several control researchers have proposed position tracking controllers for the *full-order* model used to describe the induction motor driving a mechanical load. For example, Marino *et al.* (1993) applied adaptive, nonlinear techniques for the design of a full-state feedback, velocity tracking controller for an induction motor driving a load. (For earlier work and other related full-state feedback work, see the reviews in Marino *et al.* (1993), Taylor (1994), Dawson *et al.* (1994a).) Recent work that involves the development of nonlinear controllers along with some corresponding experimental validation can be found in Raumer *et al.* (1994), Yang *et al.* (1994) and Bodson *et al.* (1994a). For recent induction motor control work involving other interesting aspects such as magnetic optimization for the torque tracking problem, removal of the linear magnetic circuit assumption, and the design of a general torque tracking controller for a class of electric machines, the reader is referred to Seleme *et al.* (1994), Bodson *et al.* (1994b) and Nickasson *et al.* (1994).

Some of the earlier control techniques suffered from the requirement of full-state feedback. That is, for the induction motor control problem, full-state feedback means that rotor flux (or rotor current) must be measured for actual implementation. Since measurement of rotor flux would tend to counter the argument that promulgates the use of the induction motor because of its simplistic construction, many control researchers have further constrained this control problem to state that rotor flux measurements are not available; moreover, any auxiliary signals that are generated as observers or surrogates for the rotor flux must be utilized in a stable closed-loop fashion. For example, Kanellakopoulos *et al.* (1992) and Kanellakopoulos and Krein (1993) developed a velocity tracking controller that utilized a rotor flux observer in a stable closed-loop fashion. Hu *et al.* (1995a) developed a position tracking controller that only required position and stator current measurements; however, roughly speaking, control singularities rendered a local exponential tracking result. Some recent control strategies for robotic manipulators actuated by induction motors can also be found in Canudas de Wit *et al.* (1993) and Hu *et al.* (1993). To address some of the shortcomings in previous work, some

* Received 10 March 1995; revised 17 August 1995; received in final form 5 January 1996. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor M. Tomizuka under the direction of Editor Yaman Arkun.
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researchers have proposed global position tracking controllers that do not exhibit control singularities, while also avoiding rotor flux (or current) measurements. For example, Espinosa-Pereza and Ortega (1994) presented a singularity-free, velocity tracking controller that did not require rotor current measurements; however, the convergence rate of the speed tracking error was determined by the natural mechanical damping of the motor. Ortega *et al.* (1994) used an ingenious filtering procedure and a two-part Lyapunov/Gronwall-inequality-type argument to develop a controller that retained the desirable characteristics of the algorithm proposed in Espinosa-Pereza and Ortega (1994) without the mechanical damping restriction. Dawson *et al.* (1994b) proposed a new set of desired electrical trajectories to illustrate how standard Lyapunov-type techniques can be used to design a singularity-free, adaptive rotor position tracking controller for a general, *uncertain* mechanical subsystem. Since almost all of the previous algorithms assumed exact knowledge of the electrical parameters, the next logical step in the evolution of induction motor control design would be the development of a controller that exhibits some robustness to electrical parameter variation. However, the design of a controller that enjoys this additional, desirable robustness feature is stymied by the fact that rotor flux measurements are not practical. Specifically, as pointed out in Marino *et al.* (1994), it is not obvious how to extend existing nonlinear techniques such that rotor flux can be observed while simultaneously adapting for parametric uncertainty in the electrical dynamic equations.

Since the design of an adaptive observer for all of the electrical problems seems somewhat intractable at this point in time, Marino *et al.* (1994) proposed an adaptive observer for the rotor flux that compensates for rotor resistance variation. More specifically, the observer ensures bounded rotor flux estimates provided the rotor resistance changes slowly with respect to time. In this paper, we modify the adaptive observer structure proposed in Marino *et al.* (1994) to facilitate the design of a new adaptive position tracking control strategy for induction motors. That is, by systematically injecting additional terms into the rotor flux observer, we illustrate how the rotor flux estimates can be utilized in a stable, closed-loop fashion. Specifically, the proposed controller† provides asymptotic rotor position tracking while compensating for parametric uncertainty in the form of the rotor resistance and a general, *uncertain* mechanical subsystem. While the observer-controller does not require measurement of rotor flux or rotor current, it does exhibit a singularity when the magnitude of estimated rotor flux is zero; however, we note that the singularity is avoided asymptotically, since the estimated flux is moved away from the singularity in the limit.

The paper is organized as follows. In Section 2, the system model and problem statement are mathematically formulated. In Section 3, the observer is designed and a preliminary stability analysis is given. In Section 4, we use the observer to form a voltage input control relationship for the position tracking control objective. In Section 5, to ensure that all signals remain bounded during closed-loop operation, we design a second voltage input control relationship for a rotor flux tracking control objective. We then perform the composite observer–controller stability analysis. In Section 6, we state the main result of the paper, while in Section 7, we present some simulation results. Finally, in Section 8, we state some conclusions.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1. *Mechanical subsystem dynamics*

The mechanical subsystem model for the electromechanical system is assumed to be of the form

$$M_m\ddot{q} + W_m(q, \dot{q})\theta_m = \tau. \quad (1)$$

Here $q(t)$, $\dot{q}(t)$ and $\ddot{q}(t)$ are the rotor position, velocity and acceleration respectively, M_m is an unknown constant representing the mechanical inertia of the system (including the rotor inertia), $W_m(q, \dot{q})\theta_m$ denotes the mechanical load dynamics, where $W_m(q, \dot{q}) \in \mathbb{R}^{1\times p}$ is a known, differentiable regression vector dependent on rotor position and velocity and $\theta_m \in \mathbb{R}^p$ is a vector containing the unknown parameters associated with the mechanical load dynamics, and τ is the torque produced by the electrical subsystem.

2.2. *Electrical subsystem dynamics*

The standard induction motor model can be found in Krause (1986). We shall utilize the transformed nonlinear model in the stator fixed a–b reference frame (Marino *et al.*, 1993). That is, under the assumptions of equal mutual inductance and a linear magnetic circuit, the

† The early backstepping work given in Kanellakopoulos *et al.* (1991) and Krstic *et al.* (1992) heavily influenced the design procedure developed in this paper.

electrical dynamics of an induction motor can be described by the following differential equations:

$$L_O\dot{I}_a = R_r\psi_a - MR_rI_a + \beta_2\psi_b\dot{q} - \beta_1 I_a + \beta_3 V_a, \quad (2)$$

$$L_O\dot{I}_b = R_r\psi_b - MR_rI_b - \beta_2\psi_2\dot{q} - \beta_1 I_b + \beta_3 V_b, \quad (3)$$

$$L_r\dot{\psi}_a = -R_r\psi_a - \beta_2\psi_b\dot{q} + MR_rI_a, \quad (4)$$

$$L_r\dot{\psi}_b = -R_r\psi_b + \beta_2\psi_a\dot{q} + MR_rI_b \quad (5)$$

where (ψ_a, ψ_b), (I_a, I_b) and (V_a, V_b) are the transformed rotor flux, transformed stator current and transformed stator voltage respectively, R_r is the unknown constant rotor resistance and R_s is the known constant stator resistance. L_Q, β_1, β_2, and β_3 are known positive constants related to the motor parameters, and are given explicitly by

$$\beta_1 = \frac{R_sL_r^2}{M}, \quad \beta_2 = n_pL_r, \quad \beta_3 = \frac{L_r^2}{M}, \quad L_O = \frac{L_r^2}{M}\left(L_s - \frac{M^2}{L_r}\right), \quad (6)$$

where L_r, L_s and M are the rotor inductance, stator inductance and mutual inductance of the motor respectively, and n_p is the known pole pair number. The electromechanical coupling torque τ defined in (1) is given explicitly by

$$\tau = \alpha_1(\psi_aI_b - \psi_bI_a), \quad (7)$$

with $\alpha_1 = n_pM/L_r$.

Remark 2.1. To facilitate the control design, the parameters associated with the mechanical load (i.e. M_m and θ_m) and the rotor resistance R_r are assumed to be unknown constants. All of the other electrical parameters are assumed to be known exactly. In addition, we assume that lower bounds for R_τ and M_m are known (i.e. $\underline{R}_r < R_r$ and $\underline{M}_m < M_m$, where $\underline{R}_r$ and $\underline{M}_m$ are known positive constants). Furthermore, we shall assume that $W_m^{\mathrm{T}}(q, \dot{q}) \in L_\infty^p$ if $q, \dot{q} \in L_\infty$ (Sastry and Bodson, 1989) and $\dot{W}_m^{\mathrm{T}}(q, \dot{q}) \in L_\infty^p$ if $q, \dot{q}, \ddot{q} \in L_\infty$.

Remark 2.2. In contrast with nonlinear controllers developed for other electric machines such as the permanent magnet stepper motor or the switched reluctance motor (Krause, 1986), the unique dynamic characteristics of the induction motor demand a different control solution. That is, owing to the additional set of dynamic equations (i.e. the rotor flux electrical dynamics), it is not obvious how to separate the torque tracking objective into several, distinct electrical state variable tracking problems and then design the corresponding voltage control input. That is, since there are two stator current variables and two rotor flux variables, it is not obvious how to use the *two* voltage control inputs to force these *four* independent variables to track four independent signals. To remedy this problem, it is common to view the induction motor as a torque source and thus design a desired torque signal to ensure that the load follows the desired position trajectory. One can then develop a voltage input control *relationship*† to force the entire torque transmission term to track the desired torque. That is, the electrical dynamics are taken into account through the torque tracking objective, and hence the position tracking control objective is embedded inside the torque tracking objective. Therefore if the voltage input control relationship can be designed to guarantee that the actual torque tracks the desired torque signal then the load position will follow the desired position trajectory. In addition, to ensure that all of the electromechanical state variables remain bounded during closed-loop operation, a desired rotor flux tracking control objective is often formulated. Roughly speaking, this secondary control objective requires the magnitude of the rotor flux to track a positive function; hence the second voltage input control relationship can then be designed to guarantee that the rotor flux tracking goes to zero.

2.3. *Control objective*

Our objective in this paper is to design a rotor position tracking controller for the electromechanical system given by (1)–(7). To facilitate the control development, we define the rotor position tracking error as

$$e = q_d - q, \quad (8)$$

where $q_d(t)$ is the desired rotor position trajectory. We shall assume that q_d and its first, second and third time derivatives are all bounded functions of time. In addition, we define the filtered tracking error (Soltine and Li, 1991) as

$$r = \dot{e} + \alpha e, \quad (9)$$

where α is a positive scalar constant control gain.

As stated previously, the above position

† The terminology 'voltage input control relationship' is used to highlight the fact that the control input is a nonlinear combination of the transformed stator input voltages.

tracking control problem is constrained by the fact that the mechanical subsystem parameters and rotor resistance are assumed to be unknown constants while all the other electrical system parameters are assumed to be known. To facilitate the design of adaptation update laws used to compensate for the parametric uncertainty, we define the parameter estimation error as follows:

$$\tilde{M}_m = M_m - \hat{M}_m,$$
$$\tilde{\theta} = \theta_m - \hat{\theta}_m, \quad \tilde{R}_r = R_r - \hat{R}_r, \tag{10}$$

where $\hat{M}_m$, $\hat{\theta}_m \in \mathbb{R}^p$ and $\hat{R}_r$ are dynamic estimates of the unknown parameters.

3. OBSERVER DESIGN

Since the rotor flux is assumed unmeasurable, we shall first build rotor flux observers to estimate the rotor flux. In addition, since the rotor resistance is assumed to be an unknown constant, we shall design current observers to aid in the construction of a dynamic estimate for the rotor resistance. To facilitate the observer design, we define the rotor flux and stator current observation errors as follows:

$$\tilde{\psi}_a = \psi_a - \hat{\psi}_a, \quad \tilde{\psi}_b = \psi_b - \hat{\psi}_b,$$
$$\tilde{I}_a = I_a - \hat{I}_a, \quad \tilde{I}_b = I_b - \hat{I}_b, \tag{11}$$

where $\hat{\psi}_a$ and $\hat{\psi}_b$ represent the estimated rotor flux, and $\hat{I}_a$, and $\hat{I}_b$ represent the estimated stator current. In addition to the above observation error quantities, we also define the following auxiliary *weighted* observation quantities (Marino *et al.*, 1994):

$$\tilde{Z}_a = L_O\tilde{I}_a + L_r\tilde{\psi}_a, \quad \tilde{Z}_b = L_O\tilde{I}_b + L_r\tilde{\psi}_b. \tag{12}$$

To provide for further design freedom during the observer design procedure, we also define the following auxiliary observation-error-related quantities (Marino *et al.*, 1994):

$$\eta_a = \tilde{Z}_a - \zeta_a, \quad \eta_b = \tilde{Z}_b - \zeta_b, \tag{13}$$

where ζ_a and ζ_b are the measurable states of two first-order auxiliary filters that will be designed to compensate for the unmeasurable quantities $R_r\psi_a$ and $R_r\psi_b$ in (2) and (3) respectively. It should be noted that these two terms require special attention in the design procedure given below, because each term represents an unknown parameter multiplied by an unmeasurable electrical state.

3.1. *Observers and auxiliary filters*

Motivated by the form of the subsequent stability analysis and the structure of (2) and (3), we propose the following stator current observer:

$$L_O\dot{\hat{I}}_a = \hat{R}_r\hat{\psi}_a - \hat{R}_rMI_a + \beta_2\dot{q}\hat{\psi}_b - \beta_1 I_a + K_O\tilde{I}_a + \beta_3 V_a + u_{O1} + u_{c1}, \tag{14}$$

$$L_O\dot{\hat{I}}_b = \hat{R}_r\hat{\psi}_b - \hat{R}_rMI_b - \beta_2\dot{q}\hat{\psi}_a - \beta_1 I_b + K_O\tilde{I}_b + \beta_3 V_b + u_{O2} + u_{c2}, \tag{15}$$

where K_O is a positive observer feedback gain, the u_{ci} are auxiliary observer inputs that will be designed later during the construction of the subsequent position tracking control algorithm, and the u_{Oi} are auxiliary observer inputs designed to facilitate the stability of the stator current observation error systems, and are given explicitly by

$$u_{O1} = -\frac{\hat{R}_r}{L_r}(L_O\tilde{I}_a - \zeta_a) - \frac{\beta_2\dot{q}}{L_r}L_O\tilde{I}_b,$$
$$u_{O2} = -\frac{\hat{R}_r}{L_r}(L_O\tilde{I}_b - \zeta_b) + \frac{\beta_2\dot{q}}{L_r}L_O\tilde{I}_a. \tag{16}$$

Motivated by the form of the subsequent stability analysis and the structure of (4) and (5), we propose the following rotor flux observers:

$$L_r\dot{\hat{\psi}}_a = -\hat{R}_r\hat{\psi}_a - \beta_2\dot{q}\hat{\psi}_b + \hat{R}_rMI_a + u_{O3} + u_{c3}, \tag{17}$$

$$L_r\dot{\hat{\psi}}_b = -\hat{R}_r\hat{\psi}_b + \beta_2\dot{q}\hat{\psi}_a + \hat{R}_rMI_b + \mu_{O4} + u_{c4}, \tag{18}$$

where the u_{ci} are auxiliary observer inputs that will be designed later during the construction of the subsequent position tracking control algorithm, and the u_{Oi} are auxiliary observer inputs designed to facilitate the stability of the rotor flux observation error system, and are given explicitly by

$$u_{O3} = -\frac{\beta_2\dot{q}}{L_r}\tilde{I}_b - K_O\tilde{I}_a - u_{O1} - u_{c1},$$
$$u_{O4} = \frac{\beta_2\dot{q}}{L_r}\tilde{I}_a - K_O\tilde{I}_b - u_{O2} - u_{c2}. \tag{19}$$

Motivated by the form of the subsequent stability analysis and the structure of the rotor flux observers and stator current observers given above, we proposed the following dynamic filter structure for ζ_a and ζ_b:

$$\dot{\zeta}_a = \frac{1}{L_r}\tilde{I}_a + \frac{\beta_2\dot{q}}{L_r}\tilde{I}_b - u_{c3},$$
$$\dot{\zeta}_b = \frac{1}{L_r}\tilde{I}_b - \frac{\beta_2\dot{q}}{L_r}\tilde{I}_a - u_{c4}. \tag{20}$$

Remark 3.1. At this point, we note the specific forms for the auxiliary inputs and the filters given by (16), (19), and (20) are not obvious; however, the structure of the auxiliary terms will become apparent during the subsequent observation error system development and the corresponding stability analysis. The auxiliary inputs denoted by u_{ci}, whose definitions will be explicitly given later, will be designed to cancel out unmeasurable quantities which arise during the position tracking control design. One of the differences between the above observer structure and the one proposed in Marino (1994) is that only two first-order auxiliary filters (i.e., (20)) are required as opposed to the four first-order filters used in Marino (1994).

3.2. *Observation error systems*

After taking the time derivative of the stator current observation error $\tilde{I}_a$ defined in (11), we have

$$L_O\dot{\tilde{I}}_a = L_O\dot{I}_a - L_O\dot{\hat{I}}_a. \quad (21)$$

After substituting the acutual current dynamics from (2) for $L_O\dot{I}_a$, the current observer dynamics from (14) for $L_O\dot{\hat{I}}_a$ kand then simplifying the resulting expression, we have

$$L_O\dot{\tilde{I}}_a = R_r\tilde{\psi}_a + \hat{\psi}_a\tilde{R}_r + \beta_2\dot{q}\tilde{\psi}_b - MI_a\tilde{R}_r - K_O\tilde{I}_a - u_{O1} - u_{c1}, \quad (22)$$

where (11) and (10) have been utilized. From the form of (10), (12) and (13), it is easy to see that u_{O1} of (16) can also be rewritten as

$$u_{O1} = -\frac{R_r}{L_r}\eta_a + \frac{\tilde{R}_r}{L_r}(L_O\tilde{I}_a - \zeta_a) - \frac{\beta_2\dot{q}}{L_r}\tilde{Z}_b + R_r\tilde{\psi}_a + \beta_2\dot{q}\tilde{\psi}_b. \quad (23)$$

After substituting (23) into (22) and then simplifying the resulting expression, we have the final form for the phase a stator current observation error dynamics as follows:

$$L_O\dot{\tilde{I}}_a = -K_O\tilde{I}_a + \frac{R_r}{L_r}\eta_a + \frac{\beta_2\dot{q}}{L_r}\tilde{Z}_b + \tilde{R}_r \times \left(-MI_a + \hat{\psi}_a - \frac{L_O}{L_r}\tilde{I}_a + \frac{\zeta_a}{L_r}\right) - u_{c1}. \quad (24)$$

Similarly to the above procedure, we can construct the phase b stator current observation error dynamics as follows:

$$L_O\dot{\tilde{I}}_b = -K_O\tilde{I}_b + \frac{R_r}{L_r}\eta_b - \frac{\beta_2\dot{q}}{L_r}\tilde{Z}_a + \tilde{R}_r \times \left(-MI_b + \hat{\psi}_b - \frac{L_O}{L_r}\tilde{I}_b + \frac{\zeta_b}{L_r}\right) - u_{c2}. \quad (25)$$

After taking the time derivative of the *weighted* observation error term $\tilde{z}_a$, defined in (12) and (11), we have

$$\dot{\tilde{Z}}_a = L_O\dot{I}_a - L_O\dot{\hat{I}}_a + L_r\dot{\psi}_a - L_r\dot{\hat{\psi}}_a. \quad (26)$$

After substituting the right-hand sides of (2), (4), (14) and (17) for $L_O\dot{I}_a$, $L_r\dot{\psi}_a$, $L_O\dot{\hat{I}}_a$ and $L_r\dot{\hat{\psi}}_a$ respectively into (26), we obtain

$$\dot{\tilde{Z}}_a = -K_O\tilde{I}_a - u_{O1} - u_{c1} - u_{O3} - u_{c3}. \quad (27)$$

After substituting (19) for u_{O3} into (27), the dynamics for $\tilde{Z}_a$ can be simplified into the following form:

$$\dot{\tilde{Z}}_a = \frac{\beta_2\tilde{I}_b\dot{q}}{L_r} - u_{c3}. \quad (28)$$

Similarly to the above procedure, the *weighted* observation error dynamics for $\tilde{Z}_b$, defined in (12) and (11), can be obtained as follows:

$$\dot{\tilde{Z}}_b = -\frac{\beta_2\tilde{I}_a\dot{q}}{L_r} - u_{c4}. \quad (29)$$

After taking the time derivative of the auxiliary observation-error-related quantity η_a defined in (13), we have

$$\dot{\eta}_a = \dot{\tilde{Z}}_a - \dot{\zeta}_a. \quad (30)$$

After substituting for $\dot{\tilde{Z}}_a$ and $\dot{\zeta}_a$ from (28) and (20) respectively into (30), the dynamics for η_a can be simplified into the following form:

$$\dot{\eta}_a = -\frac{1}{L_r}\tilde{I}_a. \quad (31)$$

Similarly to the above procedure, the dynamics for the auxiliary observation-error-related quantity η_b, defined in (13), can be obtained as follows:

$$\dot{\eta}_b = -\frac{1}{L_r}\tilde{I}_b. \quad (32)$$

Remark 3.2. It is important to emphasize that the form of the observation error systems given by (24), (25), (28), (29), (31) and (32) (and hence the form of the observers and the auxiliary filters given by (14), (15), (17), (18) and (20)) have been sculpted to mesh with the subsequent, quadratic Lyapunov-like function that is used to analyze the stability of the observation error systems.

3.3. *Analysis of the observer error systems*

To analyze the stability of the proposed observer structure, we define the following non-negative function:

$$V_O = \tfrac{1}{2}L_O\tilde{I}_a^2 + \tfrac{1}{2}L_O\tilde{I}_b^2 + \tfrac{1}{2}\tilde{Z}_a^2 + \tfrac{1}{2}\tilde{Z}_b^2 + \tfrac{1}{2}R_r\eta_a^2 + \tfrac{1}{2}R_r\eta_b^2 + \tfrac{1}{2}\Gamma_r^{-1}\tilde{R}_r^2, \quad (33)$$

where Γ_r is a positive scalar adaptation gain. After taking the time derivative of (33) and substituting the observer error systems (i.e. (24) for $L_O\dot{\tilde{I}}_a$, (25) for $L_O\dot{\tilde{I}}_b$, (28) for $\dot{\tilde{Z}}_a$, (29) for $\dot{\tilde{Z}}_b$, (31) for $\dot{\eta}_a$ and (32) for $\dot{\eta}_b$), we have

$$\dot{V}_O = -K_O\tilde{I}_a^2 - K_O\tilde{I}_b^2 - \tilde{I}_a u_{c1} - \tilde{I}_b u_{c2} - \tilde{Z}_a u_{c3} - \tilde{Z}_b u_{c4} + \tilde{R}_r(\Omega_r - \Gamma_r^{-1}\dot{\hat{R}}_r), \quad (34)$$

where the auxiliary measurable term Ω_r is given by

$$\Omega_r = \tilde{I}_a\left(-MI_a + \hat{\psi}_a - \frac{L_O}{L_r}\tilde{I}_a + \frac{\zeta_a}{L_r}\right) + \tilde{I}_b \times \left(-MI_b + \hat{\psi}_b - \frac{L_O}{L_r}\tilde{I}_b + \frac{\zeta_b}{L_r}\right). \quad (35)$$

Based on the structure of (34), we define the following projection-type update law (Sastry and Bodson, 1989) for the rotor resistance:

$$\dot{\hat{R}}_r = \begin{cases} \Gamma_r\Omega_r & \text{if } \hat{R}_r > \underline{R}_r, \\ \Gamma_r\Omega_r & \text{if } \hat{R}_r = \underline{R}_r \text{ and } \Omega_r \geq 0, \\ 0 & \text{if } \hat{R}_r = \underline{R}_r \text{ and } \Omega_r < 0, \end{cases} \quad (36)$$

where $\hat{R}_r(0) = \underline{R}_r$, and $\underline{R}_r$ was defined in Remark 2.1. After substituting the adaptation law of (36) into (34), we can form the following upper bound for $\dot{V}_O$:

$$\dot{V}_O \leq -K_O\tilde{I}_a^2 - K_O\tilde{I}_b^2 - \tilde{I}_a u_{c1} - \tilde{I}_b u_{c2} - \tilde{Z}_a u_{c3} - \tilde{Z}_b u_{c4}. \quad (37)$$

From (37), we can see that if the auxiliary observer inputs, denoted by u_{ci}, were set equal to zero then $\dot{V}_O$ would be negative-semidefinite. Hence it would be easy to show from (33) and (37) that $\tilde{I}_a$, $\tilde{I}_b$, $\tilde{Z}_a$, $\tilde{Z}_b$, η_a, η_b, $\tilde{R}_r$, $\hat{R}_r$, $\hat{\psi}_a$, $\hat{\psi}_b$, ζ_a, $\zeta_b \in L_\infty$ and that $\tilde{I}_a$ and $\tilde{I}_b$ go to zero asymptotically fast. However, the auxiliary observer inputs, denoted by u_{ci}, will be designed to directly cancel terms during the subsequent composite observer–controller stability analysis.

Remark 3.3. A drawback of the adaptive observer proposed above is that the rotor flux observation error does not converge to zero; furthermore, even if the rotor resistance were known exactly, the proposed observer structure with $\hat{R}_r = R_r$ would not provide for convergence of the rotor flux observation error (note that this phenomenon is in contrast to the exponential rotor flux observation error result provided by the rotor-resistant-dependent, open-loop observers used in Kanellakopoulos *et al.* (1992). In addition, we note that in Marino *et al.* (1994), a projection algorithm for the rotor resistance was required during the observer analysis. The projection update law, defined in (36), was not required in the above observer analysis; however, the subsequent control design procedure will require that $\hat{R}_r$ always remain positive.

Remark 3.4. It is interesting to note that the above observation strategy (as done similarly in Marino *et al.*, 1994) estimates I_a and I_b, despite the fact that these currents are assumed to be measurable. It seems that the observation of the stator currents is necessary for the development of the rotor flux observation error dynamics, which are not dependent on the rotor resistance. That is, we can see from (28) and (29) that the observation dynamics that contain the information related to the rotor flux observation error do not depend on the rotor resistance. Indeed, it is the structure of the electrical dynamics along with the selection of (12) that facilitates the advantageous form for (28) and (29). Moreover, this advantageous form along with filter structure given by (20) facilitates the construction of a rotor resistance update law that does not depend on unmeasurable quantities (i.e. rotor flux).

4. POSITION TRACKING CONTROL OBJECTIVE

By utilizing the proposed observer structure presented in the previous section, we now design an adaptive position controller that does not require rotor flux measurements. First, we rewrite the dynamics of (1) in terms of the filtered tracking error defined in (9) as follows:

$$M_m\dot{r} = M_m(\ddot{q}_d + \alpha\dot{e}) + W_m\theta_m - \alpha_1(\psi_a I_b - \psi_b I_a), \quad (38)$$

where (7) has been utilized. Since the above error system lacks a control input, and rotor flux is an unmeasurable quantity, we add and subtract the terms $\alpha_1\tau_d$ and $\alpha_1(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a)$ to the right-hand side of (38) to yield

$$M_m\dot{r} = M_m(\ddot{q}_d + \alpha\dot{e}) + W_m\theta_m - \alpha_1\tau_d + \alpha_1\eta_\tau + \alpha_1(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a) - \alpha_1(\psi_a I_b - \psi_b I_a), \quad (39)$$

where τ_d is the desired torque trajectory and η_τ is the measurable, *estimated* torque tracking error defined by

$$\eta_\tau = \tau_d - (\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a). \quad (40)$$

Based on the form of (39), we now design the desired torque trajectory as follows:

$$\tau_d = \frac{1}{\alpha_1}[\hat{M}_m(\ddot{q}_d + \alpha\dot{e}) + W_m\hat{\theta}_m + K_1 r], \quad (41)$$

where K_1 is a positive control gain (note that, to eliminate overparameterization, the update laws for $\hat{\theta}_m$ and $\hat{M}_m$ will be explicitly defined later in the design procedure). After substituting (41) into (39), we can simplify the resulting expression as follows:

$$
\begin{aligned}
M_m\dot{r} = {} & \tilde{M}_m(\ddot{q}_d + \alpha\dot{e}) + W_m\tilde{\theta}_m - K_1 r + \alpha_1\eta_\tau \\
& + \alpha_1\left(-\hat{\psi}_a\tilde{I}_b - \frac{1}{L_r}\tilde{Z}_a I_b + \frac{L_O}{L_r}\tilde{I}_a I_b\right) \\
& + \alpha_1\left(\hat{\psi}_b\tilde{I}_a + \frac{1}{L_r}\tilde{Z}_b I_a - \frac{L_O}{L_r}\tilde{I}_b I_a\right), \qquad (42)
\end{aligned}
$$

where $\tilde{M}_m$ and $\tilde{\theta}_m$ were defined in (10) and the definitions of $\tilde{Z}_a$, $\tilde{Z}_b$, $\tilde{I}_a$ and $\tilde{I}_b$ given in (12) and (11) respectively have been used to rewrite the second line in (39) in the *advantageous* form given by the second and third lines in (42). The advantageous term is used here because the form of (42), the form of (37) and some knowledge of recent control design methodology indicates how the auxiliary observer inputs denoted by u_{ci} in (37) might be designed to cancel out measurable and *unmeasurable* terms during the subsequent *composite* observer–controller stability analysis.

If we assume for now that the second and third lines of (42) can be handled by injecting the appropriate terms into the observation error dynamics via the auxiliary observer inputs, the form of the first-line of (42) indicates that the estimated torque tracking error should be driven to zero. The dynamics for η_τ can be obtained by taking the time derivative of (40) and multiplying the resulting expression by the constant $L_O L_r M_m$ to yield

$$
\begin{aligned}
L_O L_r M_m\dot{\eta}_\tau = {} & L_O L_r M_m(\dot{\tau}_d - \dot{\hat{\psi}}_a\hat{I}_b - \hat{\psi}_a\dot{\hat{I}}_b \\
& + \dot{\hat{\psi}}_b\hat{I}_a + \hat{\psi}_b\dot{\hat{I}}_a). \qquad (43)
\end{aligned}
$$

Given the definition of the desired torque trajectory τ_d defined in (41), we know that τ_d is function of the $\hat{\theta}_m$, $\hat{M}_m$ q, $\dot{q}$ and t; therefore the time derivative of τ_d can be expressed as

$$
\begin{aligned}
\dot{\tau}_d = {} & \frac{\partial\tau_d}{\partial\hat{\theta}_m}\dot{\hat{\theta}}_m + \frac{\partial\tau_d}{\partial\hat{M}_m}\dot{\hat{M}}_m + \frac{\partial\tau_d}{\partial q}\dot{q} \\
& + \frac{\partial\tau_d}{\partial\dot{q}}\ddot{q} + \frac{\partial\tau_d}{\partial t} \qquad (44)
\end{aligned}
$$

where the above partial derivative terms are given explicitly in Appendix A. Since the subsequent parameter update laws for $\hat{\theta}_m$ and $\hat{M}_m$ will allow direct substitution for $\dot{\hat{\theta}}_m$ and $\dot{\hat{M}}_m$ respectively into the right-hand side of (44), the only unmeasurable quantity in (44) is the rotor acceleration $\ddot{q}$. However, from (1) and (7), the rotor acceleration can be written as

$$
\begin{aligned}
\ddot{q} = {} & \frac{1}{M_m}\{-W_m\theta_m + [\alpha_1(\psi_a I_b - \psi_b I_a) \\
& - \alpha_1(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a)] + \alpha_1(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a)\} \qquad (45)
\end{aligned}
$$

where the term $\alpha_1(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a)$ has been added and subtracted to facilitate the analysis. After writing the square-bracketed term in (45) in terms of the observation quantities defined in (12) and (11), we can write the rotor acceleration in terms of unknown parameters, measurable quantities, and the observation error as follows:

$$
\begin{aligned}
\ddot{q} = {} & \frac{1}{M_m}\Big[-W_m\theta_m + \alpha_1(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a) \\
& + \alpha_1\left(\hat{\psi}_a\tilde{I}_b - \frac{L_O}{L_r}\tilde{I}_a I_b - \hat{\psi}_b\tilde{I}_a + \frac{L_O}{L_r}\tilde{I}_b I_a\right)\Big] \\
& + \frac{1}{M_m}\left[\frac{\alpha_1}{L_r}I_b\tilde{Z}_a - \frac{\alpha_1}{L_r}I_a\tilde{Z}_b\right]. \qquad (46)
\end{aligned}
$$

After substituting (46) into (44) and then substituting the resulting expression for $\dot{\tau}_d$, the right-hand sides of (17) and (18) for $L_r\dot{\hat{\psi}}_a$ and $L_r\dot{\hat{\psi}}_b$ respectively, and the right-hand sides of (14) and (15) for $L_O\dot{\hat{I}}_a$ and $L_O\dot{\hat{I}}_b$ respectively into (43), and then rearranging the resulting expression in a manner to highlight the unmeasurable quantities and the unknown parameters, we have

$$
\begin{aligned}
L_O L_r M_m\dot{\eta}_\tau = {} & \Omega_1 + M_m\Omega_2 + \Omega_3\tilde{Z}_a + \Omega_4\tilde{Z}_b + Y_1\theta_m \\
& - \alpha_1 r + L_r M_m\beta_3(\hat{\psi}_b V_a - \hat{\psi}_a V_b) \qquad (47)
\end{aligned}
$$

where the measurable auxiliary variables Ω_1, Ω_2, Ω_3, Ω_4, and $Y_1 \in \mathbb{R}^{1\times p}$ are defined explicitly below:

$$
\begin{aligned}
\Omega_1 = {} & \alpha_1 r + L_r L_O\alpha_1\frac{\partial\tau_d}{\partial\dot{q}}(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a) + L_r L_O\alpha_1\frac{\partial\tau_d}{\partial\dot{q}} \\
& \times\left(\hat{\psi}_a\tilde{I}_b - \frac{L_O}{L_r}\tilde{I}_a I_b - \hat{\psi}_b\tilde{I}_a + \frac{L_O}{L_r}\tilde{I}_b I_a\right), \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
\Omega_2 = {} & L_O L_r\left(\frac{\partial\tau_d}{\partial t} + \frac{\partial\tau_d}{\partial q}\dot{q} + \frac{\partial\tau_d}{\partial\hat{\theta}_m}\dot{\hat{\theta}}_m + \frac{\partial\tau_d}{\partial\hat{M}_m}\dot{\hat{M}}_m\right) \\
& + L_O\hat{I}_a(-\hat{R}_r\hat{\psi}_b + \beta_2\dot{q}\hat{\psi}_a + \hat{R}_r M I_b + u_{O4} \\
& + u_{c4}) - L_O\hat{I}_b(-\hat{R}_r\hat{\psi}_a - \beta_2\dot{q}\hat{\psi}_b + \hat{R}_r M I_a \\
& + u_{O3} + u_{c3}) - L_r\hat{\psi}_a(\hat{R}_r\hat{\psi}_b - \hat{R}_r M I_b - \beta_2\dot{q}\hat{\psi}_a \\
& - \beta_1 I_b + K_O\tilde{I}_b + u_{O2} + u_{c2}) \\
& + L_O\hat{\psi}_b(\hat{R}_r\hat{\psi}_a - \hat{R}_r M I_a + \beta_2\dot{q}\hat{\psi}_b \\
& - \beta_1 I_a + K_O\tilde{I}_a + u_{O1} + u_{c1}), \qquad (49)
\end{aligned}
$$

$$
\begin{aligned}
& \Omega_3 = L_O\alpha_1\frac{\partial\tau_d}{\partial\dot{q}}I_b, \quad \Omega_4 = -L_O\alpha_1\frac{\partial\tau_d}{\partial\dot{q}}I_a, \\
& Y_1 = -L_O L_r\frac{\partial\tau_d}{\partial\dot{q}}W_m. \qquad (50)
\end{aligned}
$$

Note that the term $-\alpha_1 r$ has been added and subtracted to the right-hand side of (47) to cancel the $\alpha_1\eta_\tau$ in (42) during the ensuing stability analysis.

Based on the structure of (47) and the subsequent stability analysis, we now design the following voltage input control relationship to force η_τ to zero:

$$\hat{\psi}_b V_a - \hat{\psi}_a V_b = -K_2 \frac{1}{L_r\beta_3}\eta_\tau - \frac{1}{L_r\beta_3}\Omega_2 - \frac{1}{L_r\beta_3\hat{M}_m}(Y_1\hat{\theta}_m + \Omega_1 + u_{c5}), \tag{51}$$

where K_2 is the positive control gain and u_{c5} is an auxiliary control input designed to facilitate the subsequent flux tracking control objective (note that it will be defined explicitly during the composite observer–controller stability analysis). To eliminate overparameterization, the update law for $\hat{M}_m$ and $\hat{\theta}_m$ will be defined explicitly later; furthermore, the update law for $\hat{M}_m$ will be designed to ensure $\hat{M}_m > 0$. After substituting (51) into (47), we can write the closed-loop error system for η_τ in the following form:

$$\begin{aligned} L_O L_r M_m \dot{\eta}_\tau = {} & -K_2 M_m \eta_\tau + \Omega_3 \tilde{Z}_a + \Omega_4 \tilde{Z}_b + Y_1 \tilde{\theta}_m \\ & - \tilde{M}_m \hat{M}_m^{-1}(Y_1\hat{\theta}_m + \Omega_1 + u_{c5}) \\ & - \alpha_1 r - u_{c5}, \end{aligned} \tag{52}$$

where $\tilde{\theta}_m$ and $\tilde{M}_m$ were defined in (10).

To analyze the closed-loop tracking error systems for r and η_τ, we define the following non-negative function:

$$V_p = V_O + \tfrac{1}{2}M_m r^2 + \tfrac{1}{2}L_O L_r M_m \eta_\tau^2, \tag{53}$$

where V_O was defined in (33). After taking the time derivative of (53) and then substituting the closed-loop error systems from (42) and (52) for $M_m\dot{r}$ and $L_O L_r M_m \dot{\eta}_\tau$ respectively, we can simplify the resulting expression as follows:

$$\begin{aligned} \dot{V}_p = {} & \dot{V}_O - K_1 r^2 - K_2 M_m \eta_\tau^2 + \tilde{Z}_a\left(\Omega_3\eta_\tau - \frac{\alpha_1 r}{L_r} I_b\right) \\ & + \tilde{Z}_b\left(\Omega_4\eta_\tau + \frac{\alpha_1 r}{L_r} I_a\right) + \tilde{I}_a\left(\alpha_1 r\hat{\psi}_b + \frac{\alpha_1 r L_o}{L_r} I_b\right) \\ & + \tilde{I}_b\left(-\alpha_1 r\hat{\psi}_a - \frac{\alpha_1 r L_O}{L_r} I_a\right) - \eta_\tau u_{c5} \\ & + \tilde{M}_m[(\ddot{q}_d + \alpha\dot{e})r - \hat{M}_m^{-1}(Y_1\hat{\theta}_m + \Omega_1 + u_{c5})\eta_\tau] \\ & + \tilde{\theta}_m^T(W_m^T r + Y_1^T\eta_\tau). \end{aligned} \tag{54}$$

From the forms of (37) and (54), we are motivated to select the auxiliary observer inputs as follows:

$$u_{c1} = \alpha_1 r\hat{\psi}_b + \frac{\alpha_1 r L_O}{L_r} I_b, \qquad u_{c2} = -\alpha_1 r\hat{\psi}_a - \frac{\alpha_1 r L_O}{L_r} I_a, \tag{55}$$

$$u_{c3} = \Omega_3\eta_\tau - \frac{\alpha_1 r}{L_r} I_b + \bar{u}_{c3}, \qquad u_{c4} = \Omega_4\eta_\tau + \frac{\alpha_1 r}{L_r} I + \bar{u}_{c4}, \tag{56}$$

where the auxiliary observer inputs $\bar{u}_{c3}$ and $\bar{u}_{c4}$ will be defined later during the design procedure. After substituting (55) and (56) into (37) and then substituting the resulitng expression into (54) for $\dot{V}_O$, we have

$$\begin{aligned} \dot{V}_p = {} & -K_O\tilde{I}_a^2 - K_O\tilde{I}_b^2 - K_1 r^2 - K_2 M_m \eta_\tau^2 \\ & - \tilde{Z}_a\bar{u}_{c3} - \tilde{Z}_b\bar{u}_{c4} - \eta_\tau u_{c5} \\ & + \tilde{M}_m[(\ddot{q}_d + \alpha\dot{e})r \\ & - \hat{M}_m^{-1}(Y_1\hat{\theta}_m + \Omega_1 + u_{c5})\eta_\tau] \\ & + \tilde{\theta}_m^T(W_m^T r + Y_1^T\eta_\tau). \end{aligned} \tag{57}$$

From the form of (57), we can make the following observations: (i) the adaptive update laws must be designed to compensate for the last three lines, (ii) the u_{c5} input in the first two lines can be used to cancel torque tracking terms during the ensuing flux tracking control objective flux while the auxiliary observer inputs $\bar{u}_{c3}$ and $\bar{u}_{c4}$ can be used to cancel unmeasurable observation error terms.

5. FLUX TRACKING CONTROL OBJECTIVE

Now that we have designed one voltage control relationship to provide for rotor position tracking, we design a second voltage control relationship to ensure that all of the signals in the observer, controller and electromechanical system remain bounded during closed-loop operation. Roughly speaking, one method for accomplishing this task is to force the magnitude of the estimated flux to track a positive function. Specifically, we define the flux tracking error as in Kanellakopoulos *et al.* (1992) as follows:

$$\eta_\psi = \psi_d - \tfrac{1}{2}(\hat{\psi}_a^2 + \hat{\psi}_b^2), \tag{58}$$

where $\psi_d(t)$ is a positive scalar function used to represent the desired *pseudo*-magnitude of the estimated rotor flux. We shall assume that ψ_d is constructed such that ψ_d, $\dot{\psi}_d$ and $\ddot{\psi}_d$ are all bounded functions of time.

To obtain the dynamics for η_ψ, we take the time derivative of (58) and multiply the resulting expression by L_r to yield

$$L_r\dot{\eta}_\psi = L_r\dot{\psi}_d - L_r\dot{\hat{\psi}}_a\hat{\psi}_a - L_r\dot{\hat{\psi}}_b\hat{\psi}_b. \tag{59}$$

From the form of (59), we can see that the rotor flux tracking error dynamics requires substitution for $L_r\dot{\hat{\psi}}_a$ and $L_r\dot{\hat{\psi}}_b$ from the rotor flux observers defined in (17) and (18) respectively. To facilitate the formation of the dynamics for η_ψ, we first construct a more complete description for the rotor flux observers. Specifically, we substitute (19) and (56) into (17) and (18) for u_{O3}, u_{O4}, u_{c3} and u_{c4} which then calls for the substitution of (16) and (55) for u_{O1}, u_{O2}, u_{c1} and

u_{c2}. After simplifying the resulting expression for the rotor flux observers by isolating all of the explicit occurrences of stator current, we have

$$L_r\dot{\hat{\psi}}_a = \Omega_{a1} + \Omega_{a2}\tilde{I}_a + \Omega_{a3}\tilde{I}_b + \Omega_c\tilde{I}_b + \hat{R}_rM\tilde{I}_a + \Omega_3\eta_\tau + \bar{u}_{c3}, \tag{60}$$

$$L_r\dot{\hat{\psi}}_b = \Omega_{b1} + \Omega_{b2}\tilde{I}_a + \Omega_{b3}\tilde{I}_b - \Omega_c\hat{I}_a + \hat{R}_rM\hat{I}_b + \Omega_4\eta_\tau + \bar{u}_{c4}, \tag{61}$$

where Ω_{ai}, Ω_{bi} and Ω_c are auxiliary measurable functions that do not depend on stator current and are defined explicitly in Appendix B. The above form for the rotor flux observers allows us to design a voltage level input that forces the rotor flux tracking error to zero while minimizing any possible singularities. Specifically, if the *fictitious* controller (Kokotovic, 1992) that will be subsequently designed to force η_ψ to zero is dependent on stator current then its time derivative will contain multiple occurrences of the input stator voltage, which can ultimately lead to a large control singularity region.

After substituting the right-hand sides of (60) and (61) for $L_r\dot{\hat{\psi}}_a$ and $L_r\dot{\hat{\psi}}_b$ respectively into (59), we have

$$\begin{aligned} L_r\dot{\eta}_\psi = {} & L_r\dot{\psi}_d - \hat{\psi}_a\Omega_{a1} - \hat{\psi}_b\Omega_{b1} \\ & + \tilde{I}_a(-\hat{\psi}_a\Omega_{a2} - \hat{\psi}_b\Omega_{b2}) \\ & + \tilde{I}_b(-\hat{\psi}_a\Omega_{a3} - \hat{\psi}_b\Omega_{b3}) \\ & - \Omega_c(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a) + (-\Omega_3\hat{\psi}_a - \Omega_4\hat{\psi}_b)\eta_\tau \\ & - \hat{\psi}_a\bar{u}_{c3} - \hat{\psi}_b\bar{u}_{c4} - \hat{R}_rM(\hat{\psi}_a\hat{I}_a + \hat{\psi}_b\hat{I}_b). \end{aligned} \tag{62}$$

Since the dynamics of (62) lack a control input, we add and subtract a fictitious controller, denoted by u_I, to the right-hand side of (62) to obtain

$$\begin{aligned} L_r\dot{\eta}_\psi = {} & \Omega_5 + \tilde{I}_a(-\hat{\psi}_a\Omega_{a2} - \hat{\psi}_b\Omega_{b2}) \\ & + \tilde{I}_b(-\hat{\psi}_a\Omega_{a3} - \hat{\psi}_b\Omega_{b3}) + (\Omega_c - \Omega_3\hat{\psi}_a \\ & - \Omega_4\hat{\psi}_b)\eta_\tau - \hat{\psi}_a\bar{u}_{c3} - \hat{\psi}_b\bar{u}_{c4} \\ & - \hat{R}_rMu_I + \hat{R}_rM\eta_I, \end{aligned} \tag{63}$$

where (40) has been used to write the first term in the fourth line of (62) in terms of η_τ, Ω_5 is an auxiliary measurable function defined as

$$\Omega_5 = L_r\dot{\psi}_d - \hat{\psi}_a\Omega_{a1} - \hat{\psi}_b\Omega_{b1} - \Omega_c\tau_d,$$

and η_I is an auxiliary, measurable tracking error variable defined as

$$\eta_I = u_I - (\hat{\psi}_a\hat{I}_a + \hat{\psi}_b\hat{I}_b). \tag{64}$$

Based on the structure of (63), we design the fictitious control input u_I as follows:

$$\begin{aligned} u_I = \frac{1}{\hat{R}_rM} [& \Omega_5 + K_n\hat{\psi}_a^2(\Omega_{a2}^2 + \Omega_{a3}^2)\eta_\psi \\ & + K_n\hat{\psi}_b^2(\Omega_{b2}^2 + \Omega_{b3}^2)\eta_\psi + K_3\eta_\psi], \end{aligned} \tag{65}$$

where K_3 and K_n are positive control gains (it is important to note that u_I does not depend explicitly on stator current measurements; furthermore, we have designed the update law for $\hat{R}_r$ to prevent control singularities). After substituting u_I of (65) into (63), we obtain the following closed-loop error system for η_ψ:

$$\begin{aligned} L_r\dot{\eta}_\psi = {} & -K_3\eta_\psi + (\Omega_c - \Omega_3\hat{\psi}_a - \Omega_4\hat{\psi}_b)\eta_\tau \\ & + [\hat{\psi}_a(-\Omega_{a2}\tilde{I}_a - \Omega_{a3}\tilde{I}_b) \\ & - K_n\hat{\psi}_a^2(\Omega_{a2}^2 + \Omega_{a3}^2)\eta_\psi] \\ & + [\hat{\psi}_b(\Omega_{b2}\tilde{I}_a - \Omega_{b3}\tilde{I}_b) - K_n\hat{\psi}_b^2(\Omega_{b2}^2 \\ & + \Omega_{b3}^2)\eta_\psi] - \hat{\psi}_a\bar{u}_{c3} - \hat{\psi}_b\bar{u}_{c4} + \hat{R}_rM\eta_I, \end{aligned} \tag{66}$$

where the square-bracketed terms are nonlinear damping pairs (Kokotovic, 1992).

Motivated by the form of (66), we now design a voltage control input relationship that forces η_I to zero. After taking the time derivative of (64), and multiplying the resulting expression by the constant $L_rL_OM_m$, we have

$$\begin{aligned} L_rL_OM_m\dot{\eta}_I = {} & L_OL_rM_m\dot{u}_I - L_rL_OM_m \\ & \times(\dot{\hat{\psi}}_a\hat{I}_a + \hat{\psi}_a\dot{\hat{I}}_a + \dot{\hat{\psi}}_b\hat{I}_b + \hat{\psi}_b\dot{\hat{I}}_b). \end{aligned} \tag{67}$$

To complete the open-loop description for η_I, we note that u_I of (65) is a function of $\hat{R}_r$, $\hat{\psi}_a$, $\hat{\psi}_b$, η_ψ, q, $\dot{q}$, ζ_a, ζ_b, $\hat{\theta}_m$, $\hat{M}_m$ and t; therefore the time derivative for u_I can be expressed as

$$\begin{aligned} \dot{u}_I = {} & \frac{\partial u_I}{\partial\hat{R}_r}\dot{\hat{R}}_r + \frac{\partial u_I}{\partial\hat{\psi}_a}\dot{\hat{\psi}}_a + \frac{\partial u_I}{\partial\hat{\psi}_b}\dot{\hat{\psi}}_b + \frac{\partial u_I}{\partial\eta_\psi}\dot{\eta}_\psi + \frac{\partial u_I}{\partial q}\dot{q} \\ & + \frac{\partial u_I}{\partial\dot{q}}\ddot{q} + \frac{\partial u_I}{\partial\zeta_a}\dot{\zeta}_a + \frac{\partial u_I}{\partial\zeta_b}\dot{\zeta}_b + \frac{\partial u_I}{\partial\dot{q}}\ddot{q} \\ & + \frac{\partial u_I}{\partial\hat{\theta}_m}\dot{\hat{\theta}}_m + \frac{\partial u_I}{\partial\hat{M}_m}\dot{\hat{M}}_m + \frac{\partial u_I}{\partial t}, \end{aligned} \tag{68}$$

where the above partial derivative terms are all calculated explicitly in Appendix C. After substituting (46) for $\ddot{q}$ into (68) and then substituting the resulting expression for $\dot{u}_I$, and the right-hand sides of (14) and (15) for $L_O\dot{\hat{I}}_a$ and $L_O\dot{\hat{I}}_b$ respectively into (67), and then rearranging the resulting expression in a manner to highlight the unmeasurable quantities and the unknown parameters, we have

$$\begin{aligned} L_OL_rM_m\dot{\eta}_I = {} & M_m\Omega_6 + \Omega_7 + \Omega_8\tilde{Z}_a + \Omega_9\tilde{Z}_b + Y_2\theta_m \\ & - \hat{R}_rM\eta_\psi - L_r\beta_3M_m(\hat{\psi}_aV_a + \hat{\psi}_bV_b), \end{aligned} \tag{69}$$

where the auxiliary measurable terms Ω_6, Ω_7, Ω_8, Ω_9, and $Y_2 \in \mathbb{R}^{1\times p}$ are given explicitly below:

$$\Omega_6 = L_O L_r\left(\frac{\partial u_I}{\partial \hat{R}_r}\dot{\hat{R}}_r + \frac{\partial u_I}{\partial \hat{\psi}_a}\dot{\hat{\psi}}_a + \frac{\partial u_I}{\partial \hat{\psi}_b}\dot{\hat{\psi}}_b + \frac{\partial u_I}{\partial \eta_\psi}\dot{\eta}_\psi + \frac{\partial u_I}{\partial q}\dot{q}\right) + L_O L_r\left(\frac{\partial u_I}{\partial \zeta_a}\dot{\zeta}_a + \frac{\partial u_I}{\partial \zeta_b}\dot{\zeta}_b + \frac{\partial u_I}{\partial \hat{\theta}_m}\dot{\hat{\theta}}_m + \frac{\partial u_I}{\partial \hat{M}_m}\dot{\hat{M}}_m + \frac{\partial u_I}{\partial t}\right), \tag{70}$$

$$\Omega_7 = L_O L_r\frac{\partial u_I}{\partial \dot{q}}\left[\alpha_1(\hat{\psi}_a\hat{I}_b - \hat{\psi}_b\hat{I}_a) + \alpha\left(\hat{\psi}_a\tilde{I}_b - \frac{L_O}{L_r}\tilde{I}_a I_b\right)\right] - L_O L_r\frac{\partial u_I}{\partial \dot{q}}\alpha_1 \times\left(\hat{\psi}_b\tilde{I}_a - \frac{L_O}{L_r}\tilde{I}_b I_a\right) + \hat{R}_r M\eta_\psi, \tag{71}$$

$$\Omega_8 = L_O\frac{\partial u_I}{\partial \dot{q}}\alpha_1 I_b,\quad \Omega_9 = -L_O\frac{\partial u_I}{\partial \dot{q}}\alpha_1 I_a,\quad Y_2 = -L_O L_r\frac{\partial u_I}{\partial \dot{q}}W_m. \tag{72}$$

Note the term $-\hat{R}_r M\eta_\psi$ has been added and subtracted to the right-hand side of (69) to cancel the $\hat{R}_r M\eta_I$ in (66) during the ensuing stability analysis; furthermore, the right-hand sides of (36), (17), (18), (66) and (20) can be substituted into (70) for $\dot{\hat{R}}_r$, $\dot{\hat{\psi}}_a$, $\dot{\hat{\psi}}_b$, $\dot{\eta}_\psi$, $\dot{\zeta}_a$ and $\dot{\zeta}_b$, respectively, while the subsequent adaptation laws will allow for substitution for $\dot{\hat{M}}_m$, and $\dot{\hat{\theta}}_m$.

Based on the structure of (69) and the subsequent stability analysis, we now design the voltage input control relationship to force η_I to zero as follows:

$$\hat{\psi}_a V_a + \hat{\psi}_b V_b = \frac{1}{L_r\beta_3}(K_4\eta_I + \Omega_6) + \frac{1}{L_r\beta_3\hat{M}_m}\times(Y_2\hat{\theta}_m + \Omega_7 + u_{c6}), \tag{73}$$

where K_4 is a positive control gain, and u_{c6} is an auxiliary control input that will be designed to facilitate the subsequent composite observer-controller stability analysis. To eliminate over-parameterization, the update laws for $\hat{M}_m$ and $\hat{\theta}_m$ will be defined explicitly later; furthermore, the update law for $\hat{M}_m$ will be designed to ensure $\hat{M}_m > 0$. After substituting (73) into (69), we can write the closed-loop error system for η_I in the following form:

$$L_O L_r M_m\dot{\eta}_I = -K_4 M_m\eta_I + \Omega_8\tilde{Z}_a + \Omega_9\tilde{Z}_b + Y_2\tilde{\theta}_m - \tilde{M}_m\hat{M}_m^{-1}(Y_2\hat{\theta}_m + \Omega_7 + u_{c6}) - \hat{R}_r M\eta_\psi - u_{c6}, \tag{74}$$

where $\tilde{\theta}_m$ and $\tilde{M}_m$ were defined in (10).

5.1. *Composite observer–controller analysis*

To analyze the composite observer–controller tracking error system while also providing for the design of the adaptation laws and the remaining auxiliary observer and control inputs, we define the following non-negative function:

$$V_f = V_p + \tfrac{1}{2}L_r\eta_\psi^2 + \tfrac{1}{2}L_O L_r M_m\eta_I^2 + \tfrac{1}{2}\Gamma_1^{-1}\tilde{M}_m^2 + \tfrac{1}{2}\tilde{\theta}_m^{\mathrm{T}}\Gamma_2^{-1}\tilde{\theta}_m, \tag{75}$$

where V_p was defined in (53), Γ_1 is a positive scalar control gain, $\Gamma_2 \in \mathbb{R}^{p\times p}$ is a positive-definite, diagonal, gain matrix. After taking the time derivative of (75), substituting (66) and (74) into the resulting expressions for $L_r\dot{\eta}_\psi$ and $L_O L_r M_m\dot{\eta}_I$ respectively, and then cancelling the common terms, we have

$$\begin{aligned}\dot{V}_f = {} & \dot{V}_p - K_3\eta_\psi^2 - K_4 M_m\eta_I^2 + \eta_I\Omega_8\tilde{Z}_a \\ & + \eta_I\Omega_9\tilde{Z}_b + (\Omega_c - \Omega_3\hat{\psi}_a - \Omega_4\hat{\psi}_b)\eta_\psi\eta_\tau \\ & + [-\hat{\psi}_a\Omega_{a2}\tilde{I}_a\eta_\psi - K_n\hat{\psi}_a^2\Omega_{a2}^2\eta_\psi^2] \\ & + [-\hat{\psi}_b\Omega_{b2}\tilde{I}_a\eta_\psi - K_n\hat{\psi}_b^2\Omega_{b2}^2\eta_\psi^2] - \eta_\psi\hat{\psi}_a\bar{u}_{c3} \\ & + [-\hat{\psi}_a\Omega_{a3}\tilde{I}_b\eta_\psi - K_n\hat{\psi}_a^2\Omega_{a3}^2\eta_\psi^2] \\ & + [-\hat{\psi}_b\Omega_{b3}\tilde{I}_b\eta_\psi - K_n\hat{\psi}_b^2\Omega_{b3}^2\eta_\psi^2] - \eta_\psi\hat{\psi}_b\bar{u}_{c4} \\ & + \tilde{\theta}_m^{\mathrm{T}}[Y_2^{\mathrm{T}}\eta_I - \Gamma_2^{-1}\dot{\hat{\theta}}_m] \\ & + \tilde{M}_m[-\hat{M}_m^{-1}(Y_2\hat{\theta}_m + \Omega_7 + u_{c6})\eta_I - \Gamma_1^{-1}\dot{\hat{M}}_m] \\ & - \eta_I u_{c6}.\end{aligned} \tag{76}$$

After applying the nonlinear damping argument (Kokotovic, 1992) to the four square-bracketed terms on the third to the sixth lines of (76) and then substituting for $\dot{V}_p$ from (57), we can collect common terms to form the following upper bound for $\dot{V}_f$:

$$\begin{aligned}\dot{V}_f \le {} & -\left(K_O - \frac{2}{K_n}\right)\tilde{I}_a^2 - \left(K_O - \frac{2}{K_n}\right)\tilde{I}_b^2 - K_1 r^2 \\ & - K_2 M_m\eta_\tau^2 - K_3\eta_\psi^2 - K_4 M_m\eta_I^2 - \eta_I u_{c6} \\ & + \eta_\tau[(\Omega_c - \Omega_3\hat{\psi}_a - \Omega_4\hat{\psi}_b)\eta_\psi - u_{c5}] \\ & + \tilde{Z}_a(\eta_I\Omega_8 - \bar{u}_{c3}) + \tilde{Z}_b(\eta_I\Omega_9 - \bar{u}_{c4}) \\ & + \tilde{\theta}_m^{\mathrm{T}}(W_m^{\mathrm{T}}r + Y_1^{\mathrm{T}}\eta_\tau + Y_2^{\mathrm{T}}\eta_I - \Gamma_2^{-1}\dot{\hat{\theta}}_m) \\ & + \tilde{M}_m(\Omega_m - \Gamma_1^{-1}\dot{\hat{M}}_m) - \eta_\psi\hat{\psi}_a\bar{u}_{c3} - \eta_\psi\hat{\psi}_b\bar{u}_{c4},\end{aligned} \tag{77}$$

where the auxiliary function Ω_m is given by

$$\Omega_m = (\ddot{q}_d + \alpha\dot{e})r - \hat{M}_m^{-1}(Y_1\hat{\theta}_m + \Omega_1 + u_{c5})\eta_\tau - \hat{M}_m^{-1}(Y_2\hat{\theta}_m + \Omega_7 + u_{c6})\eta_I. \tag{78}$$

Now, based on the fifth and sixth lines of (77), we design the parameter update laws as follows:

$$\dot{\hat{\theta}}_m = \Gamma_2(W_m^{\mathrm{T}} r + Y_1^{\mathrm{T}} \eta_\tau + Y_2^{\mathrm{T}} \eta_I), \tag{79}$$

$$\dot{\hat{M}}_m = \begin{cases} \Gamma_1 \Omega_m & \text{if } \hat{M}_m > \underline{M}_m, \\ \Gamma_1 \Omega_m & \text{if } \hat{M}_m = \underline{M}_m \text{ and } \Omega_m \geq 0, \\ 0 & \text{if } \hat{M}_m = \underline{M}_m \text{ and } \Omega_m < 0, \end{cases} \tag{80}$$

where $\hat{M}_m(0) = \underline{M}_m$ (note that the above update law for $\hat{M}_m$ ensures that $\hat{M}_m > 0$). See Bridges *et al.* (1995) and Lozano *et al.* (1992) for details. Now, based on the third and fourth lines of (77), we design the auxiliary observer inputs $\bar{u}_{c3}$ and $\bar{u}_{c4}$, which were injected at (56), and the auxiliary control input u_{c5}, which was injected at (51), as follows:

$$\begin{aligned} \bar{u}_{c3} &= \eta_I \Omega_8, \quad \bar{u}_{c4} = \eta_I \Omega_9, \\ u_{c5} &= (\Omega_c - \Omega_3 \hat{\psi}_a - \Omega_4 \hat{\psi}_b)\eta_\psi. \end{aligned} \tag{81}$$

After substituting the auxiliary control inputs given by (81) and the update laws given by (80) and (79) into (77), we can collect the common terms to form the new upper bound for $\dot{V}_f$:

$$\begin{aligned} \dot{V}_f \leq & -\left(K_O - \frac{2}{K_n}\right)\tilde{I}_a^2 - \left(K_O - \frac{2}{K_n}\right)\tilde{I}_b^2 - K_1 r^2 \\ & - K_2 M_m \eta_\tau^2 - K_3 \eta_\psi^2 - K_4 M_m \eta_I^2 \\ & + \eta_I(-\eta_\psi \hat{\psi}_a \Omega_8 - \eta_\psi \hat{\psi}_b \Omega_9 - u_{c6}). \end{aligned} \tag{82}$$

Now based on the third line of (82), we design the auxiliary control input u_{c6}, which was injected at (73), as follows:

$$u_{c6} = (-\Omega_8 \hat{\psi}_a - \Omega_9 \hat{\psi}_b)\eta_\psi; \tag{83}$$

hence $\dot{V}_f$ of (82) can be upper-bounded in the following concise form:

$$\dot{V}_f \leq -\lambda_3 \|x\|^2, \tag{84}$$

where

$$\lambda_3 = \min\left\{K_1, K_2 M_m, K_3, K_4 M_m, K_O - \frac{2}{K_n}\right\}, \tag{85}$$

$$x = [\tilde{I}_a \quad \tilde{I}_b \quad r \quad \eta_\tau \quad \eta_\psi \quad \eta_I]^{\mathrm{T}} \in \mathbb{R}^6. \tag{86}$$

5.2. *Voltage control input calculation*

Given the voltage control input relationships of (51) and (73), we can calculate the transformed voltage control inputs V_a and V_b as follows:

$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = C^{-1} \begin{bmatrix} \text{right-hand side of (51)} \\ \text{right-hand side of (73)} \end{bmatrix}, \tag{87}$$

where

$$C = \begin{bmatrix} \hat{\psi}_b & -\hat{\psi}_a \\ \hat{\psi}_a & \hat{\psi}_b \end{bmatrix} \in \mathbb{R}^{2\times 2}. \tag{88}$$

Given the definition of C in (88), it is easy to see that C is not invertible if $\hat{\psi}_a^2(t) + \hat{\psi}_b^2(t) = 0$; hence, the proposed controller exhibits a control singularity at $\hat{\psi}_a^2(t) + \hat{\psi}_b^2(t) = 0$.

6. MAIN RESULT

Theorem. If the sufficient control gain condition

$$K_O > \frac{2}{K_n} \tag{89}$$

is satisfied and

$$\hat{\psi}_a^2(t) + \hat{\psi}_b^2(t) \neq 0 \tag{90}$$

then the proposed observer–controller ensures asymptotic position tracking as follows:

$$\lim_{t\to\infty} e(t) = 0. \tag{91}$$

Proof. First, note that V_f of (75) can be bounded as

$$\lambda_1 \|z\| \leq V_f \leq \lambda_2 \|z\|, \tag{92}$$

where

$$\begin{aligned} \lambda_1 = \tfrac{1}{2} \min\{ & L_O, 1, R_r, M_m, L_O L_r M_m, L_r, \\ & \Gamma_r, \Gamma_1^{-1}, \lambda_{\min}\{\Gamma_2^{-1}\}\}, \end{aligned} \tag{93}$$

$$\begin{aligned} \lambda_2 = \tfrac{1}{2} \max\{ & L_O, 1, R_r, M_m, L_O L_r M_m, L_r, \\ & \Gamma_r, \Gamma_1^{-1}, \lambda_{\max}\{\Gamma_2^{-1}\}\}, \end{aligned} \tag{94}$$

$$\begin{aligned} z = [& x^{\mathrm{T}} \quad \tilde{Z}_a \quad \tilde{Z}_b \quad \eta_a \quad \eta_b \quad \tilde{R}_r \quad \tilde{M}_m \quad \tilde{\theta}_m^{\mathrm{T}}]^{\mathrm{T}} \\ & \in \mathbb{R}^{12+p}, \end{aligned} \tag{95}$$

where x was defined in (86). If the control gain condition given by (89) is satisfied, we know that λ_3 in (85) will be positive; therefore, since V_f is positive-definite and $\dot{V}_f$ is negative-semidefinite, we know that V_f is upper-bounded by $V_f(0)$ and lower-bounded by zero, and hence $z \in L_\infty^{12+p}$ (Sastry and Bodson, 1989) and $x \in L_\infty^6$. Also, it is easy to show from (84) that $x \in L_2^6$ (Sastry and Bodson, 1989). Since $r \in L_\infty$, it is easy to show that $e \in L_\infty$, $\dot{e} \in L_\infty$ (hence $q \in L_\infty$ and $\dot{q} \in L_\infty$). Given the assumption that θ_m, M_m and R_r are unknown constants, we know that $\hat{\theta}_m \in L_\infty$, $\hat{M}_m \in L_\infty$ and $\hat{R}_r \in L_\infty$, since $\tilde{\theta}_m \in L_\infty$, $\tilde{M}_m \in L_\infty$ and $\tilde{R}_r \in L_\infty$. Given the definition of η_ψ in (58), we know that $\hat{\psi}_a \in L_\infty$ and $\hat{\psi}_b \in L_\infty$, since $\eta_\psi \in L_\infty$. Also, since $\tilde{Z}_a \in L_\infty$, $\tilde{Z}_b \in L_\infty$ and $\tilde{I}_a \in L_\infty$, $\tilde{I}_b \in L_\infty$, we know that $\tilde{\psi}_a \in L_\infty$ and $\tilde{\psi}_b \in L_\infty$ (hence $\psi_a \in L_\infty$ and $\psi_b \in L_\infty$). In addition, since $\eta_a \in L_\infty$, $\eta_b \in L_\infty$, $\tilde{Z}_a \in L_\infty$ and $\tilde{Z}_b \in L_\infty$, we can use (13) to show that $\zeta_a \in L_\infty$ and $\zeta_b \in L_\infty$. Utilizing the above information, we are now able to show that τ_d of (41) and u_I of (65) are both bounded, and hence (40) and (64) can be used to show that $\hat{\psi}_a \hat{I}_b - \hat{\psi}_b \hat{I}_a \in L_\infty$, $\hat{\psi}_a \hat{I}_a + \hat{\psi}_b \hat{I}_b \in L_\infty$. Therefore we can multiply $(\hat{\psi}_a \hat{\psi}_b - \hat{\psi}_b \hat{I}_a$ by $\hat{\psi}_a$, $\hat{\psi}_a \hat{I}_a + \hat{\psi}_b \hat{I}_b$ by $\hat{\psi}_b$, and add the results together to obtain the bounded quantity $(\hat{\psi}_a^2 + \hat{\psi}_b^2)\hat{I}_b$, which shows, as a result of

(90), that $\hat{I}_b \in L_\infty$ (and, by a similar argument, $\hat{I}_a \in L_\infty$). Therefore $I_a \in L_\infty$ and $I_b \in L_\infty$, since $\tilde{I}_a \in L_\infty$ and $\tilde{I}_b \in L_\infty$. From the right-hand sides of (51) and (73), we are now able to show that the voltage control inputs are bounded. Furthermore, the closed-loop error systems can now be used to show that $\dot{x} \in L_\infty^6$. Finally, since $x \in L_\infty^6$, $x \in L_2^6$ and $\dot{x} \in L_\infty^6$, Barbalat's lemma (Sastry and Bodson, 1989) can be used to show that

$$\lim_{t\to\infty} x(t) = 0 \Rightarrow \lim_{t\to\infty} \tilde{I}_a, \tilde{I}_b, r, \eta_\tau, \eta_\psi, \eta_I = 0.$$

Given the definition of the filtered tracking error given in (9), we can use standard linear arguments to illustrate (91) (Slotine and Li, 1991). □

Remark 6.1. The control singularity represented by (90) illustrates a drawback of the proposed approach; however, we note that the singularity is avoided asymptotically, since $\lim_{t\to\infty} \eta_\psi = 0$ (i.e. the estimated flux is moved away from the singularity in the limit). In addition, we note that since the singularity occurs for the *estimated* flux, it is easy to construct some *ad hoc* means of avoided the singularity during actual implementation. In comparison with the singularity-free control designs proposed in Hu *et al.* (1993), Espinosa-Perez and Ortega (1994), Dawson *et al.* (1994b) and Ortega *et al.* (1995) we note that all of these approaches required exact knowledge of the rotor resistance.

Remark 6.2. One important practical reason for performing rotor flux tracking is the need to prevent the rotor flux from entering its nonlinear saturation region. However, as illustrated by the main result of the paper, the rotor flux tracking objective is actually not fulfilled, since the observer–controller strategy does not ensure that the observed rotor flux converges to the actual rotor flux. Unfortunately, this pitfall could limit the practical usefulness of the proposed controller; however, we note that it is very difficult to precisely observe an unknown state (i.e. rotor flux) whose dynamics are nonlinear and contain an unknown parameter (i.e. rotor resistance).

7. SIMULATION RESULTS

To verify the performance of the proposed adaptive observer–controller scheme, we simulated a simple mechanical load driven by an induction motor. Since we are interested in utilizing the induction motor for relatively low-speed, high-performance position tracking applications such as robotics, we selected the mechanical load as a single-link robot. Therefore $W_m(q, \dot{q})\theta_m$ in (1) can be reasonably modelled as

$$W_m(q, \dot{q})\theta_m = B\dot{q} + N \sin q, \tag{96}$$

where B is the viscous friction coefficient. The terms M_m in (1) and N in (96) are given by

$$M_m = J_m + \tfrac{1}{3}mL_0^2, \quad N = \tfrac{1}{2}mgL_0,$$

where J_m is the rotor inertia, g is the gravity coefficient, m is the link mass and L_0 is the link length. For simulation purposes, we selected the Baldor model M3541, two-pole induction motor, which has a rated speed of 3450 rev min^{-1} rated current of 2.7 A, and rated voltage of 230 V. The values of the electromechanical system parameters were selected to be

$$J_m = 1.87 \times 10^{-4}\,\mathrm{kg\,m^2}, \quad m = 0.401\,\mathrm{kg},$$
$$L_0 = 0.305\,\mathrm{m}, \quad g = 9.81\,\mathrm{kg\,m\,s^{-2}},$$
$$L_r = 0.306\,\mathrm{H}, \quad L_s = 0.243\,\mathrm{H},$$
$$M = 0.225\,\mathrm{H}, \quad R_r = 2.12\,\Omega, \quad R_s = 3.05\,\Omega,$$
$$n_p = 1, \quad B = 0.003\,\mathrm{N\,m\,s\,rad^{-1}}.$$

The desired position trajectory was selected as the following smooth-start sinusoid:

$$q_d(t) = \tfrac{1}{2}\pi \sin(4t)(1 - e^{-0.1t^3}) \quad \mathrm{rad},$$

where $q_d(0) = \dot{q}_d(0) = \ddot{q}_d(0) = \dddot{q}_d(0) = 0$. The desired flux trajectory ψ_d of (58) was selected as

$$\psi_d = 0.3\frac{2e^{-0.25t}}{1 + e^{0.5t}} + 1.7\,\mathrm{Wb}^2.$$

The initial rotor flux estimates were set to

$$\hat{\psi}_a(0) = \hat{\psi}_b(0) = \sqrt{2}\,\mathrm{Wb}, \tag{97}$$

such that $\eta_\psi(0) = 0$. To better mimic actual operation, we selected the rotor resistance to be the following slowly-time-varying function (i.e. we assume $\dot{R}_r \approx 0$):

$$R_r = 2.12 + 4.24(1 - e^{-0.1t})\,\Omega.$$

The initial value for the estimate of R_r was selected as $\hat{R}_r(0) = 1.06\,\Omega$, which is 50% of its lowest value, and $\underline{R}_r$ was set equal to $0.5\,\Omega$. The initial value for the estimate of M_m was also selected to be 50% of its actual value as shown: $\hat{M}_m(0) = \underline{M}_m = 0.0065\,\mathrm{kg\,m^2}$. The initial rotor position was set equal -0.3 rad. The other initial values for the parameter update laws and state variables were all set equal to zero. The best tracking performance was established using the following control gain values:

$$\alpha = 2, \quad K_1 = 0.1, \quad K_2 = 0.3, \quad K_3 = 4, \quad K_4 = 1,$$
$$K_O = 5, \quad K_n = 0.01, \quad \Gamma_r = \Gamma_1 = \Gamma_{21} = \Gamma_{22} = 0.01.$$

The resulting rotor position tracking error is

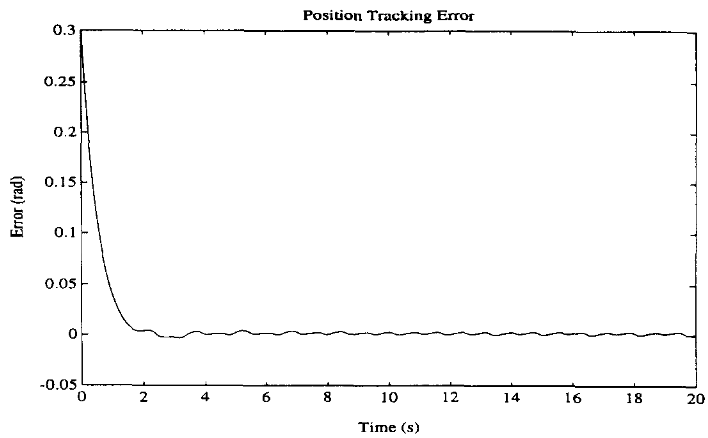

Fig. 1. Position tracking error.

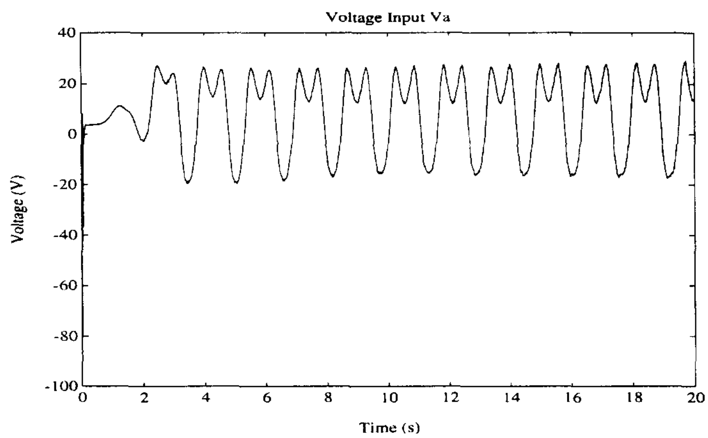

Fig. 2. Voltage control input V_a.

shown in Fig. 1, the voltage control inputs V_a and V_b are shown in Figs 2 and 3 respectively. From Fig. 1, we can see that good tracking performance has been achieved under the proposed adaptive observer-based controller. The current estimation errors $\tilde{I}_a$ and $\tilde{I}_b$ are shown in Figs 4 and 5 respectively. The flux estimation errors $\tilde{\psi}_a$ and $\tilde{\psi}_b$ are shown in Figs 6 and 7 respectively. To illustrate the parameter update law trajectory, we plotted $\hat{R}_r$ and $\hat{B}$ (i.e. the estimate of the friction coefficient B) in Figs 8 and 9 respectively.

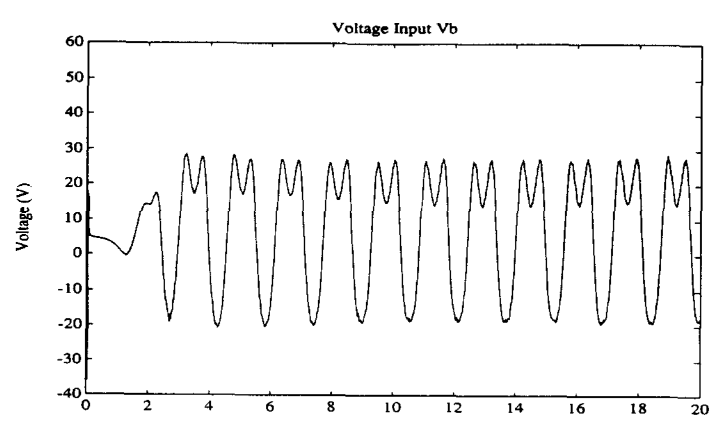

Fig. 3. Voltage control input V_b.

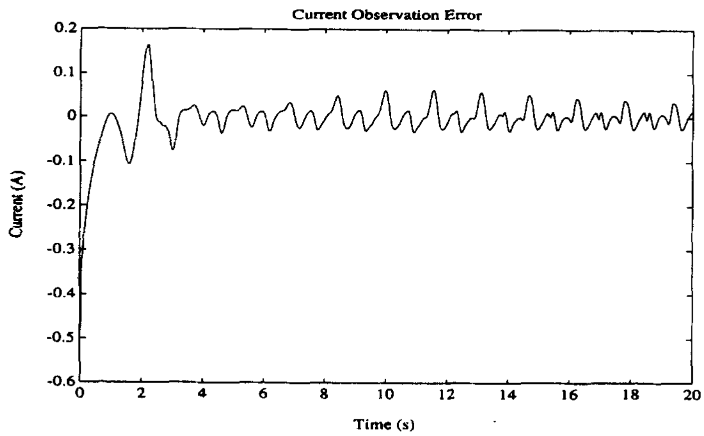

Fig. 4. Current observation error $\tilde{I}a$.

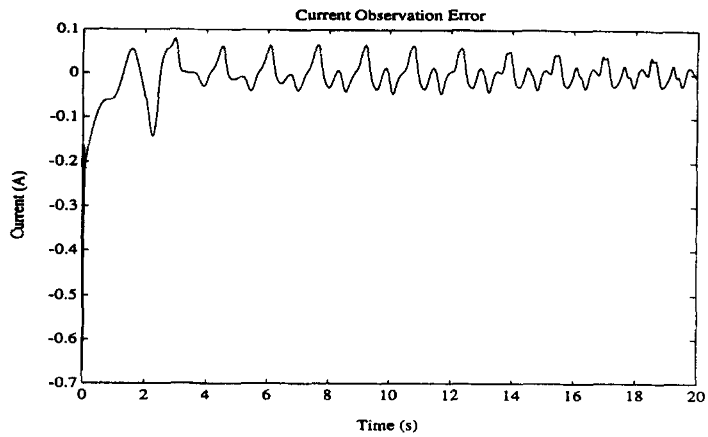

Fig. 5. Current observation error $\tilde{I}_b$.

Remark 7.1. The selection of the control parameters in the simulation does not satisfy the conditions given in (89); however, this control gain requirement is only a sufficient, conservative condition generated by the Lyapunov-like stability argument. It also should be noted that the control singularity of (90) never occurred during the simulation given the initial rotor flux estimates of (97). In Fig. 1, which depicts the position tracking error, there exist some small oscillations as the tracking error becomes smaller. Because of the complexity of the proposed controller, we believe that these oscillations are generated by computer simulation error.

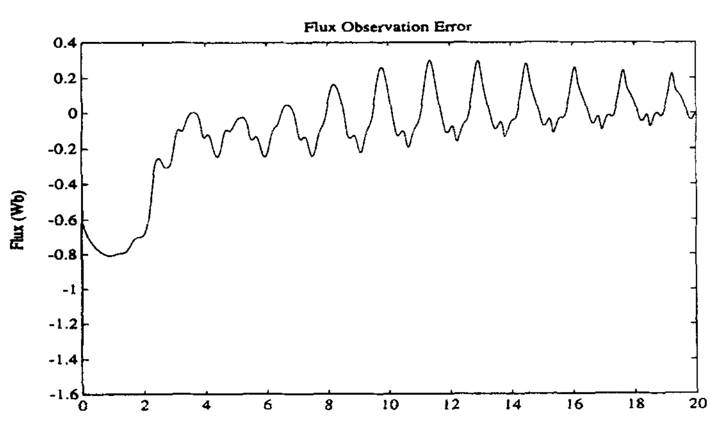

Fig. 6. Flux observation error $\tilde{\psi}_a$.

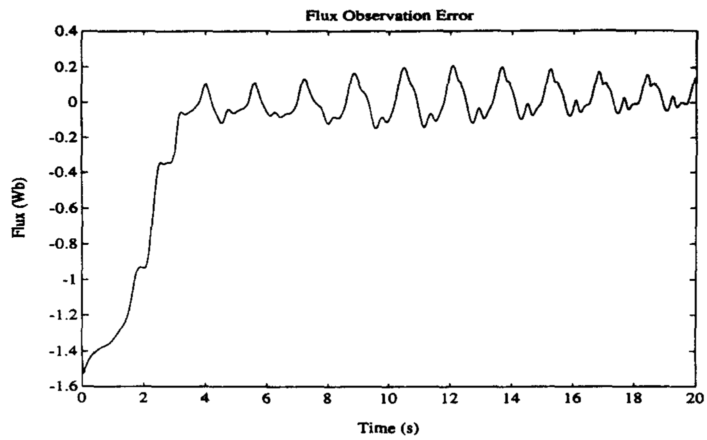

Fig. 7. Flux observation error $\tilde{\psi}_b$.

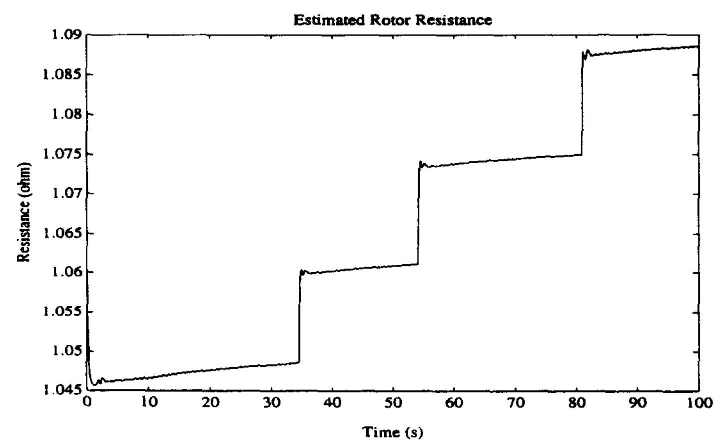

Fig. 8. Estimated rotor resistance $\hat{R}_r$.

Remark 7.2. It should be noted that the proposed control strategy is indeed complex when compared with some of the more traditional induction motor controllers (Krause, 1986); however, previous experimental results presented in Hu *et al.* (1995a, b) point out that good position tracking can be obtained with present microprocessor technology, even for a controller of similar complexity. Specifically, the present DSP technology allows a controller of this complexity to be calculated in approximately 1 ms; hence we believe that relatively low-cost implementation of such control strategies has become achievable and will become even more

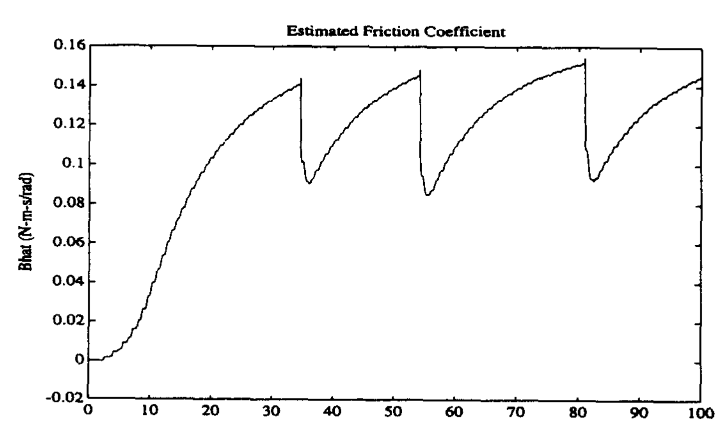

Fig. 9. Estimated friction coefficient $\hat{B}$.

practical in the future. In addition, we note that the need for exact knowledge of most of the electrical parameters represents a disadvantage of the proposed controller; however, the simultaneous estimation of all of the electromechanical parameters and the rotor flux has yet to be reported, as far as we know. Hence we believe future work will have to address this important issue.

8. CONCLUSIONS

We have presented an adaptive, partial-state feedback, position tracking controller for the full-order, nonlinear dynamic model representing an induction motor actuating a mechanical subsystem. The proposed controller provides asymptotic rotor position tracking while compensating for parametric uncertainty in the form of the rotor resistance and a general, *uncertain* mechanical subsystem. While the observer–controller does not require measurement of rotor flux or rotor current, it does exhibit a singularity when the magnitude of the estimated rotor flux is zero; however, the singularity is avoided asymptotically, since the estimated flux is moved away from the singularity in the limit. Simulation results have been provided to verify the effectiveness of the approach. Future research will involve eliminating the control singularity using similar methods to those outlined in Dawson *et al.* (1994b) and Ortega *et al.* (1995), and performing experimental validation as previously done by Dawson *et al.* (1994b) and Hu *et al.* (1995a).

Acknowledgements—This work is supported in part by the U.S. National Science Foundation Grants DDM-931133269 and DMI-9457967, DOE Contract DE-AC21-92MC29115, Office of Naval Research Grant URI-3139-YIP01, and the Union Camp Corporation.

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APPENDIX A—PARTIAL DERIVATIVE TERMS FOR t_d

The partial derivative terms defined in (44) are given explicitly as follows:

$$\frac{\partial \tau_d}{\partial t}=\frac{\alpha K_1}{\alpha_1}\dot{q}_d+\frac{\alpha \hat{M}_m+K_1}{\alpha_1}\ddot{q}_d+\frac{\hat{M}_m}{\alpha_1}\dddot{q}_d,$$

$$\frac{\partial \tau_d}{\partial q}=\frac{1}{\alpha_1}\left(\frac{\partial W_m}{\partial q}\hat{\theta}_m-\alpha K_1\right),$$

$$\frac{\partial \tau_d}{\partial \dot{q}}=\frac{1}{\alpha_1}\left(-\alpha\hat{M}_m+\frac{\partial W_m}{\partial \dot{q}}\hat{\theta}_m-K_1\right),$$

$$\frac{\partial \tau_d}{\partial \hat{M}_m}=\frac{\ddot{q}_d+\alpha\dot{e}}{\alpha_1},\quad \frac{\partial \tau_d}{\partial \hat{\theta}_m}=\frac{W_m}{\alpha_1}.$$

APPENDIX B—DEFINITIONS OF Ω_{ai}, Ω_{bi} AND Ω_c

The measurable terms Ω_{ai}, Ω_{bi} and Ω_c defined in the flux observers of (60) and (61), are given by

$$\Omega_{a1}=-\hat{R}_r\hat{\psi}_a-\beta_2\dot{q}\hat{\psi}_b-\frac{\hat{R}_r}{L_r}\zeta_a-\alpha_1\hat{\psi}_b r,$$

$$\Omega_{a2}=\Omega_{b3}=-K_O+\frac{\hat{R}_rL_o}{L_r}+\hat{R}_rM,$$

$$\Omega_c=-\frac{\alpha_1}{L_r}r(1+L_O),$$

$$\Omega_{a3}=\frac{\dot{q}}{L_r}\beta_2(L_O-1)-\frac{\alpha_1}{L_r}r(1+L_O)=-\Omega_{b2},$$

$$\Omega_{b1}=-\hat{R}_r\hat{\psi}_b+\beta_2\dot{q}\hat{\psi}_a-\frac{\hat{R}_r}{L_r}\zeta_b+\alpha_1\hat{\psi}_a r.$$

APPENDIX C—PARTIAL DERIVATIVE TERMS FOR $\dot{u}_I$

The partial derivative terms defined in (68) are given explicitly by

$$\frac{\partial u_I}{\partial t}=\frac{L_r}{\hat{R}_rM}\ddot{\psi}_d+\frac{1+L_O}{\hat{R}_rML_r}\alpha_1\ddot{q}_d\Big(\tau_d+r\frac{\partial\tau_d}{\partial\dot{q}_d}-2K_n\hat{\psi}_a^2\eta_\psi\Omega_{a3}+2K_n\hat{\psi}_b^2\eta_\psi\Omega_{b2}\Big)$$
$$+\frac{1+L_O}{\hat{R}_rML_r}\alpha\dot{q}_d(\alpha_1\tau_d+K_1r-2\alpha_1K_n\hat{\psi}_a^2\eta_\psi\Omega_{a3}+2\alpha_1K_n\hat{\psi}_b^2\eta_\psi\Omega_{b2})$$
$$+\frac{1+L_O}{\hat{R}_rML_r}r\hat{M}_m\dddot{q}_d,$$

$$\frac{\partial u_I}{\partial\zeta_a}=\frac{1}{ML_r}\hat{\psi}_a,\quad \frac{\partial u_I}{\partial\zeta_b}=\frac{1}{ML_r}\hat{\psi}_b,$$

$$\frac{\partial u_I}{\partial q}=\frac{1+L_O}{\hat{R}_rML_r}\alpha_1\Big(-\alpha\tau_d+r\frac{\partial\tau_d}{\partial q}+2\alpha K_n\hat{\psi}_a^2\eta_\psi\Omega_{a3}-2\alpha K_n\hat{\psi}_b^2\eta_\psi\Omega_{b2}\Big),$$

$$\frac{\partial u_I}{\partial\dot{q}}=\frac{1+L_O}{\hat{R}_rML_r}\alpha_1\times\left\{-\tau_d+r\frac{\partial\tau_d}{\partial\dot{q}}+2\alpha K_n\eta_\psi\Omega_{a3}\hat{\psi}_a^2\times\left[1+\frac{L_O-1}{\alpha_1(L_O+1)}\beta_2\right]\right\}$$
$$-\frac{1+L_O}{\hat{R}_rML_r}\alpha_1\left\{2\alpha K_n\hat{\psi}_b^2\eta_\psi\Omega_{b2}\times\left[1+\frac{L_O-1}{\alpha_1(L_O+1)}\beta_2\right]\right\},$$

$$\frac{\partial u_I}{\partial\hat{\psi}_a}=\frac{1}{\hat{R}_rM}[-\Omega_{a1}+\hat{R}_r\hat{\psi}_a-\hat{\psi}_b(\beta_2\dot{q}+\alpha_1r)+2K_n\Omega_{a2}^2\hat{\psi}_a\eta_\psi+2K_n\Omega_{a3}^2\hat{\psi}_a\eta_\psi],$$

$$\frac{\partial u_I}{\partial\hat{\psi}_b}=\frac{1}{\hat{R}_rM}[-\Omega_{b1}+\hat{R}_r\hat{\psi}_b+\hat{\psi}_a(\beta_2\dot{q}+\alpha_1r)+2K_n\Omega_{b2}^2\hat{\psi}_b\eta_\psi+2K_n\Omega_{b3}^2\hat{\psi}_b\eta_\psi],$$

$$\frac{\partial u_I}{\partial\eta_\psi}=\frac{1}{\hat{R}_rM}(K_n\Omega_{a2}^2\hat{\psi}_a^2+K_n\Omega_{a3}^2\hat{\psi}_a^2+K_n\Omega_{b2}^2\hat{\psi}_b^2+K_n\Omega_{b3}^2\hat{\psi}_b^2+K_3),$$

$$\frac{\partial u_I}{\partial\hat{M}_m}=\frac{1+L_O}{\hat{R}_rML_r}\alpha_1r\frac{\partial\tau_d}{\partial\hat{M}_m},$$

$$\frac{\partial u_I}{\partial\hat{\theta}_m}=\frac{1+L_O}{\hat{R}_rML_r}\alpha_1r\frac{\partial\tau_d}{\partial\hat{\theta}_m}$$

$$\frac{\partial u_I}{\partial\hat{R}_r}=-\frac{u_I}{\hat{R}_r}+\frac{1}{\hat{R}_rM}\left(\hat{\psi}_a^2+\hat{\psi}_b^2+\frac{\hat{\psi}_a}{L_r}\zeta_a+\frac{\hat{\psi}_b}{L_r}\zeta_b\right)+\frac{2}{\hat{R}_rM}K_n\eta_\psi(\Omega_{a2}\hat{\psi}_a^2+\Omega_{b3}\hat{\psi}_b^2)\left(\frac{L_O}{L_r}+M\right).$$