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ANALYSIS OF PLANAR MICROWAVE CIRCUITS WITH LUMPED-ELEMENTS BY CN-FDTD

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ABSTRACT: A three-dimensional implementation of the lumped-element by Crank-Nicolson finite-difference time-domain (CN-FDTD) algorithm has been presented in this article. Several examples of planar microwave circuits with lumped resistor, capacitor, and/or inductor are simulated and compared with traditional finite-difference time-domain method and measurements. The accuracy of CN-FDTD implementation for lumped elements in this article has been verified. © 2008 Wiley Periodicals, Inc. *Microwave Opt Technol Lett* 51: 113–116, 2009; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.23969

Key words: CN-FDTD, microstrip, lumped-element

1. INTRODUCTION

The traditional finite-difference time-domain (FDTD) method [1] is a very popular and robust approach. As the traditional FDTD method is based on an explicit time-stepping technique, there is one challenge for its implementation because of the time-step size imposed by the Courant-Friedrich-Levy (CFL) stability condition. This means that the time step Δt has a specific bound relative to the spatial discretizations. To relax the stability constraint, various time-domain techniques have been developed [2–4].

Recently, an unconditionally stable Crank-Nicolson finite-difference time-domain (CN-FDTD) method in the 2D case was developed and found that this scheme's accuracy is superior to that of the Alternating-Direction-Implicit-FDTD [5]. The CN algorithm [6] solved the discretized Maxwell's equations by a full time step size with one marching procedure, and averaged the right-hand-sides of the discretized Maxwell's equations at $n + 1$ time step and n time step. Later in 2006, Yang et al. [7] developed CN-FDTD in 3D case for the analysis of planar microwave circuits with only distributed elements.

In practical planar microwave circuits, however, there are not only distributed elements, such as transmission lines, but also lumped elements (e.g., resistor, capacitor, inductor, diode, and transistor). Therefore, to deal with the whole planar microwave

circuit, lumped elements must be considered carefully. Several researchers have studied the microstrip circuits including lumped-elements by traditional FDTD method, such as [8].

In this article, the CN-FDTD is extended to 3D microstrip circuits including lumped elements of resistor, capacitor, and inductor. And the validity of CN-FDTD formulation for lumped elements has been demonstrated. The efficiency of CN-FDTD scheme with lumped-elements is not included in this article and will be studied in the future.

2. MODELING OF LUMPED ELEMENTS BY 3D CN-FDTD

2.1. Formulation of 3D CN-FDTD

For simplicity, an isotropic, linear, nondispersive, and lossless medium with permittivity ϵ is considered in this article. The numerical formulation of the 3D CN-FDTD method can be found in [7]. The electromagnetic-field components are arranged on the cells in the same way as the conventional FDTD method. By inserting formulas of [7], a set of equations with coupled unknown E-field components can be obtained. This equation is a block tridiagonal, and the values of the E-field components can be obtained by solving this simultaneous equation. Thereafter, the H-field components are solved explicitly.

2.2. Modeling of Lumped Elements in CN-FDTD

By setting the coordinates as shown in Figure 1 and assuming the lumped elements in x direction, the current-voltage expressions of the lumped elements are, respectively,

$$I = U/R \text{ (for lumped resistor),} \quad (1)$$

$$I = CdU/dt \text{ (for lumped capacitor),} \quad (2)$$

$$I = \frac{1}{L} \cdot \int_0^t U dt \text{ (for lumped inductor),} \quad (3)$$

R , L , and C are, respectively, the resistance, inductance, and capacitance of the lumped elements.

Assume that the lumped element is smaller than one Yee's cell. According to the method of Sui et al. in 1992 [9], the relation between the current and the voltage of the lumped elements can be changed to the relation of electric and magnetic fields. Then it can be added in the traditional FDTD formulation [8].

Using the same derivation in [7], the 3D CN-FDTD formulation in the mesh including lumped-elements in the CN-FDTD iteratives can be derived as follows,

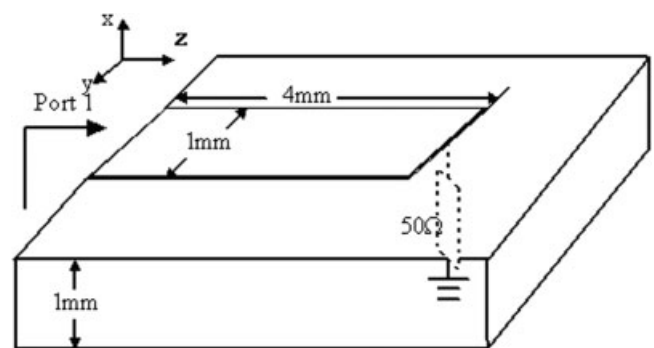


Figure 1 A microstrip line terminated by a resistor in x direction

$$\begin{aligned}
& \left[1 + \frac{2A_i}{dy^2} + \frac{2A_i}{dz^2} \right] E_x^{n+1}(i,j,k) - \frac{A_i}{dz^2} [E_x^{n+1}(i,j,k-1) + E_x^{n+1}(i,j,k+1)] - \frac{A_i}{dy^2} [E_x^{n+1}(i,j-1,k) + E_x^{n+1}(i,j+1,k)] \\
& + \frac{A_i}{dxdy} \left[E_y^{n+1} \left(i \left[1 + \frac{2A_i}{dy^2} + \frac{2A_i}{dz^2} \right] E_x^{n+1}(i,j,k) - \frac{A_i}{dz^2} [E_x^{n+1}(i,j,k-1) + E_x^{n+1}(i,j,k+1)] - \frac{A_i}{dy^2} [E_x^{n+1}(i,j-1,k) + E_x^{n+1}(i,j+1,k)] \right. \right. \\
& \left. \left. + \frac{A_i}{dxdy} [E_y^{n+1}(i-1,j,k) - E_y^{n+1}(i,j,k) - E_y^{n+1}(i-1,j+1,k) + E_y^{n+1}(i,j+1,k)] + \frac{A_i}{dxdz} [E_z^{n+1}(i-1,j,k) - E_z^{n+1}(i,j,k) - E_z^{n+1}(i \right. \right. \\
& \left. \left. - 1,j,k+1) + E_z^{n+1}(i,j,k+1)] \right] = \left[C_i - \frac{2A_i}{dy^2} - \frac{2A_i}{dz^2} \right] E_x^n(i,j,k) + \frac{A_i}{dy^2} [E_x^n(i,j+1,k) + E_x^n(i,j-1,k)] + \frac{A_i}{dz^2} [E_x^n(i,j,k+1) + E_x^n(i,j,k \\
& - 1)] + \frac{B_i}{dy} [H_z^n(i,j+1,k) - H_z^n(i,j,k) - \frac{B_i}{dz} [H_y^n(i,j,k+1) - H_y^n(i,j,k)] - \frac{A_i}{dxdy} [E_y^n(i,j+1,k) - E_y^n(i-1,j+1,k) - E_y^n(i,j,k) \\
& + E_y^n(i-1,j,k)] - \frac{A_i}{dxdz} [E_z^n(i,j,k+1) - E_z^n(i-1,j,k+1) - E_z^n(i,j,k) + E_z^n(i-1,j,k)] - \xi_i [dd_i \sum_{m=1}^n E_x^m(i,j,k)], \quad (4)
\end{aligned}$$

where

$$A_i = \begin{cases} \frac{dt^2}{(1+dd)4\epsilon\mu}, & i = R, C \\ \frac{dt^2}{4\epsilon\mu}, & i = L, \end{cases} \quad (5)$$

$$B_i = \begin{cases} \frac{dt}{(1+dd)\epsilon}, & i = R, C \\ \frac{dt}{\epsilon}, & i = L, \end{cases} \quad (6)$$

$$dd_i = \begin{cases} \frac{dtdx}{2R\epsilon dydz}, & i = R \\ \frac{Cdx}{\epsilon dydz}, & i = C \\ \frac{dxdt^2}{\epsilon Ldydz}, & i = L, \end{cases} \quad (7)$$

$$\xi_i = \begin{cases} 0, & i = R, C \\ 1, & i = L, \end{cases} \quad (8)$$

$$C_i = \begin{cases} \frac{1-dd_i}{1+dd_i}, & i = R \\ 1, & i = C, L. \end{cases} \quad (9)$$

If the lumped elements are located in y or z direction, the above Eqs. (4)–(7) can easily be changed accordingly.

3. SIMULATION RESULTS

Applying the CN scheme derived in the last section to Maxwell's equation results in a large sparse matrix equation for the E -field components. In this article, the symmetric successive over relaxation (SSOR) [7] preconditioned biconjugate-gradient algorithm (SSOR-BCG) is proposed to solve the large sparse matrix equation. The E -value at the last time step is used as the initial value for the iterative algorithm.

To demonstrate the validity of the CN-FDTD including lumped elements in this article, several microstrip circuits with lumped

elements are simulated and compared with traditional FDTD method or measurement.

The first example is a microstrip line terminated by a lumped resistor of 50Ω , as shown in Figure 1. The resistor is in x direction, and it covers four cells and goes from the top to the ground. The substrate is filled with the dielectric material of $\epsilon_r = 9.6$ and the substrate thickness is 1 mm . The width of the microstrip line is also 1 mm so that the characteristic impedance of the microstrip line is 50Ω . The total framework has the dimension of $50 dx \times 25 dy \times 20 dz$ with the Yee cell size: $dx = 0.25 \text{ mm}$, $dy = 0.25 \text{ mm}$, and $dz = 0.25 \text{ mm}$. Figure 2 shows the reflection coefficient at Port 1. As can be seen, the results by CN-FDTD and traditional FDTD agree very well.

Next, two more examples are given for the analysis of complicated microstrip circuits with lumped elements.

First, a $\lambda/4$ composite right/left-handed (CRLH) open-circuit stub [10] is analyzed. And the structure is shown in Figure 3. The permittivity of the dielectric is 10.2 with the substrate thickness of

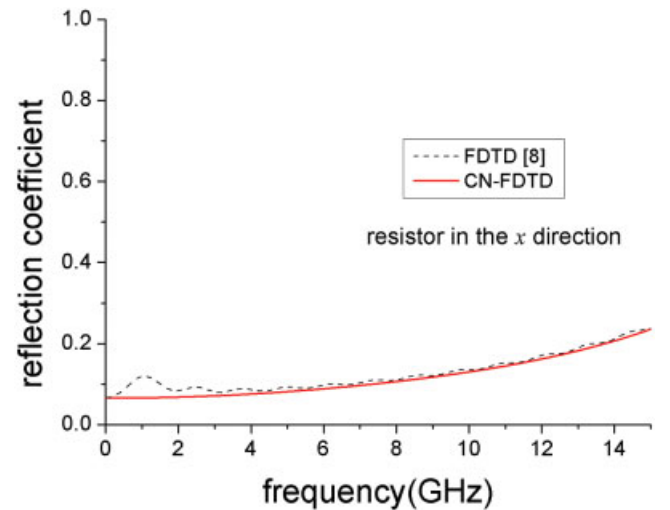


Figure 2 Comparison of reflection coefficient at port 1 of Figure 1 by CN-FDTD and traditional FDTD. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

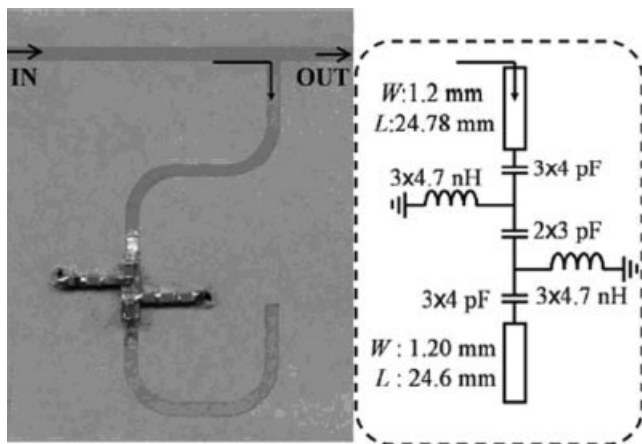


Figure 3 Schematic of the $\lambda/4$ CRLH open-circuit stub

1.27 mm. In CN-FDTD scheme, the total framework has the dimension of $16 dx \times 145 dy \times 40 dz$ with the Yee cell size: $dx = 0.3175$ mm, $dy = 0.5$ mm, and $dz = 0.3$ mm. Each capacitor covers one Yee cell in y direction, and each inductor covers 4 Yee cell in x direction. Figure 4 shows and compares the magnitude of S_{11} by CN-FDTD and measurements [10]. One can see that the results agree very well, which means the formulation of CN-FDTD in this article is correct and effective.

Second, a Wilkinson power divider operating at 1 GHz and 1.8 GHz [11] is presented. The structure of this power divider is illustrated in Figure 5. Each branch consists of two sections of microstrip line with 33.8 mm (length) and 0.7887 mm (width), and 33.2 mm (length) and 1.3 mm (width), respectively. The values of resistor R , inductor L , and capacitor C are 100 Ω , 9 nH, and 1.5 pF, respectively. The permittivity of the dielectric is 3.38 and the substrate thickness is 0.81 mm. In CN-FDTD scheme, the total framework has the dimension of $16dx \times 50 dy \times 100 dz$ with the Yee cell size: $dx = 0.2025$ mm, $dy = 0.4$ mm, and $dz = 1$ mm. In this example, the resistor, capacitor, and inductor all cover 3 Yee cell in y direction. Simulation results of S_{11} , S_{21} , and S_{23} agree well with those by measurements, as shown in Figures 6–8. As can be seen, S_{11} is less than -25 dB in the frequency range of 0.84 GHz

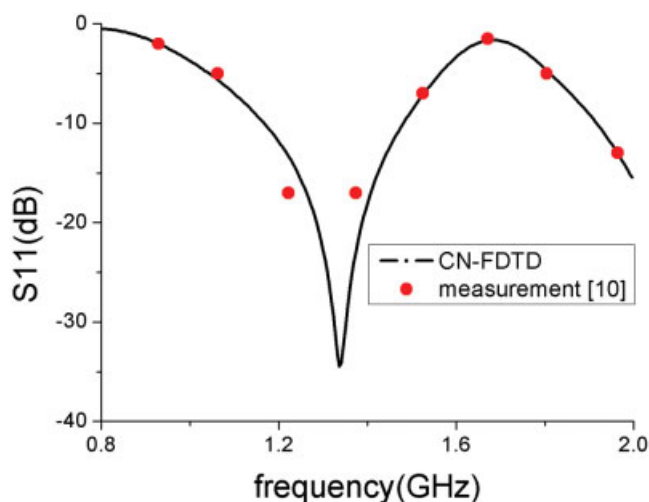


Figure 4 Comparison of S_{11} by CN-FDTD and measurement for Figure 3. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

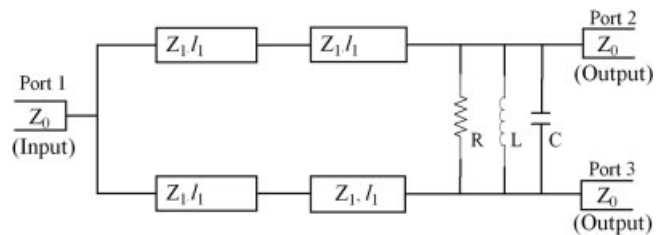


Figure 5 A dual-frequency Wilkinson power divider

to 1.15 GHz and 1.6 GHz to 1.95 GHz. S_{21} varies from -3.19 dB to -3 dB in the frequency range of 0.6 GHz to 1.99 GHz and from -3 dB to -3.06 dB in the frequency range of 1.63 GHz to 1.82 GHz. And the isolation between Port 2 and Port 3 is less than -20 dB in frequency range of 0.92 GHz to 1.01 GHz and 1.72 GHz to 1.82 GHz.

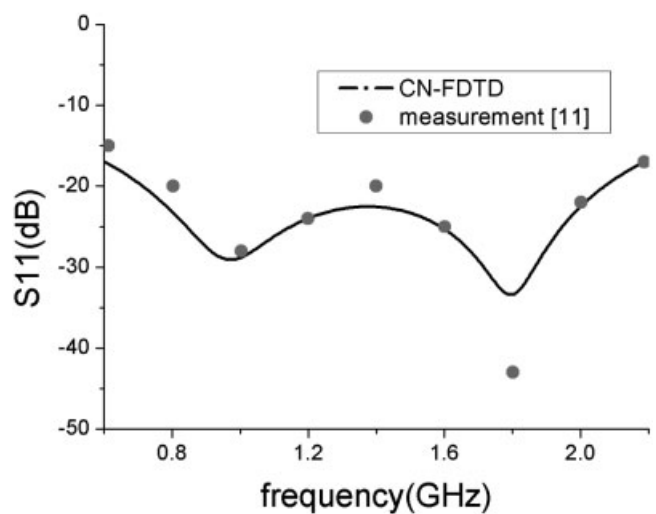


Figure 6 Comparison of S_{11} by CN-FDTD and measurement for Figure 5. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

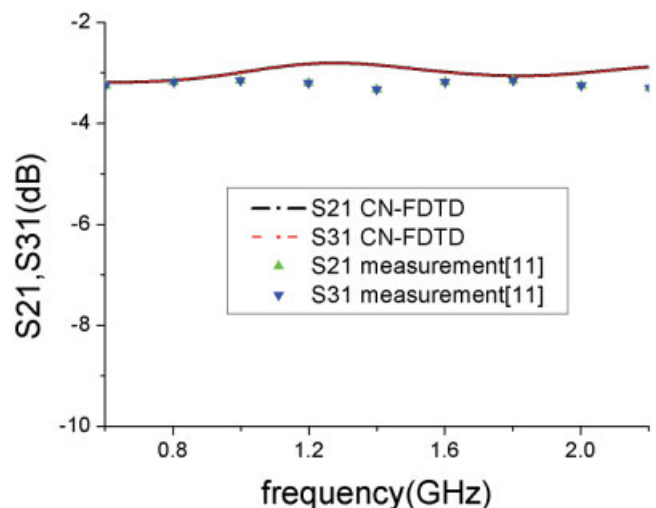


Figure 7 Comparison of S_{21} and S_{31} by CN-FDTD and measurement for Figure 5. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

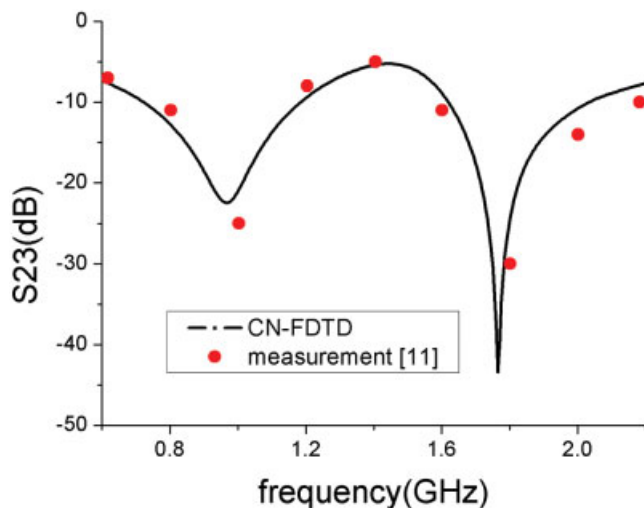


Figure 8 Comparison of S_{23} by CN-FDTD and measurement for Figure 5. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

4. CONCLUSION

In this article, the implicit three-dimensional unconditionally stable 3D CN-FDTD is presented for planar microwave circuits with lumped elements. The accuracy of 3D CN-FDTD in this article is verified by three examples. Good agreements are obtained with traditional FDTD method or measurement. The efficiency of CN-FDTD scheme with lumped-elements will be studied in the future and given in next article.

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ELECTROMAGNETIC SCATTERING FROM INHOMOGENEOUS IMPEDANCE PERIODIC ROUGH SURFACES BY TRANSFORM TECHNIQUE

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ABSTRACT: In this study, electromagnetic scattering from inhomogeneous impedance periodic surfaces have been solved by means of transformation of problem into equivalent problem, that is, scattering from plane represented by transformed boundary condition. Transformed boundary condition is determined by functions of the shape and impedance of the surface. Then, transformed equivalent problem is solved by means of series expansion method using Floquet modes. This transformation makes the problem simple formulation and computational effectively without involving calculation of slowly converging periodic Green's function. Results and computational times obtained by transform method and those obtained by Method of Moment (MoM) technique are compared. Good agreements are observed in results. It is also observed that transform method needs much less computational time than MoM method. © 2008 Wiley Periodicals, Inc. *Microwave Opt Technol Lett* 51: 116–119, 2009; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.23954

Key words: electromagnetic scattering; periodic surfaces; impedance boundary condition

1. INTRODUCTION

Scattering of electromagnetic waves from rough surfaces is of importance from both theoretical and practical points of view. One of the main applications of such problems is the ground wave propagation modeling of the electromagnetic waves. Although several methods have been developed for the solution of such kind of problems it still needs to be treated with effective methods. Scattering from perfectly conducting surfaces with particular periodic roughness, such as sinusoidal or trapezoidal, are investigated in [1] and [2], respectively. For arbitrary roughness, one obtains integral equation whose kernel is periodic Green's function and, this integral equation is solved by any numerical methods such as MoM or T-Matrix. Scattering from perfectly conducting periodic rough surface has been investigated by MoM [3–5] and T-Matrix [5, 6]. Because of the fact that evaluation of periodic Green's function slowly converges, these methods need much computational time. Therefore, more computational effective methods for evaluation of periodic Green's function have been investigated [7].

In this study, new solution approach for solution of scattering from periodic rough surfaces is developed. To make the problem