Original Paper

phys. stat. sol. (b) **242**, No. 9, 1817–1819 (2005) / **DOI** 10.1002/pssb.200461735

On the phase diagram of a two-dimensional electron gas near integer fillings and fractions such as 1/5 and 1/7

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Received 11 October 2004, revised 5 May 2005, accepted 5 May 2005
Published online 29 June 2005

PACS 73.43.Cd

The phase diagram of the 2DEG at high magnetic fields is discussed in the light of recent experiments showing the presence of crystalline phases around integer as well as certain fractional fillings. We suggest that a Hall crystal is a proper competing phase compatible with the odd denominator rule that characterizes the fractional quantum Hall effect, and that near integer filling a pinned crystal coexists with an incompressible fluid with no longitudinal resistance.

The boundaries between different phases of the 2DEG at high magnetic fields has been the subject of much interest recently. Although originally it was believed that a Wigner Crystal (WC) would only form in the lower Landau level and below filling 1/7 [1], microwave resonance experiments showed that this state appeared around higher fractions such as 1/5 [2], and integer fillings like 1 and 2 [3]. Numerical diagonalization of the hamiltonian over a unit cell containing few particles with periodic boundary conditions also showed the presence of rich phases at a range of filling factors [4]. A schematic diagram of the melting curve characterizing the equilibrium between a Wigner solid and a Laughlin liquid for the thermodynamic limit was proposed more than a decade ago by Buhmann et al. [5, 6]. This was followed by a discussion based on a first order phase transition by Lea, March and Sung [7]. These authors subsequently applied microscopic theory, for both anyon and composite Fermion models, to demonstrate that the main features of the Buhmann et al. diagram could be explained by invoking the magnetization, of the de Haas–van Alphen character, of the Laughlin/CF liquid [8–10].

In this work we refine and extend somewhat the phase diagram of Buhmann et al. In the course of such refinement, we shall want to consider one further phase besides the Wigner solid: a 'Hall' crystal (HC) [11]. This state was found in early work by one of us (F.C.) [12, 13], using the Hartree–Fock (HF) approximation. It is a charge density wave with a fraction of an electron per unit cell that is conveniently taken to be 1/2. As ν is varied, the energy per particle presents cusps at all fillings with odd denominator, revealing a gap, while it forms a Fermi liquid at all even denominators in accordance with experiment. The cusps are hierarchically deployed, in such a way that fractions of smaller denominators are associated with larger gaps. Thus, the HC state at 1/5 presents a large gap while that at 2/11 has a smaller gap, and 3/19 is smaller still. Cabo and Claro later argued that the HC shows strong cohesive energy determined by cooperative ring exchange effects, making the HC a suitable candidate for the ground state

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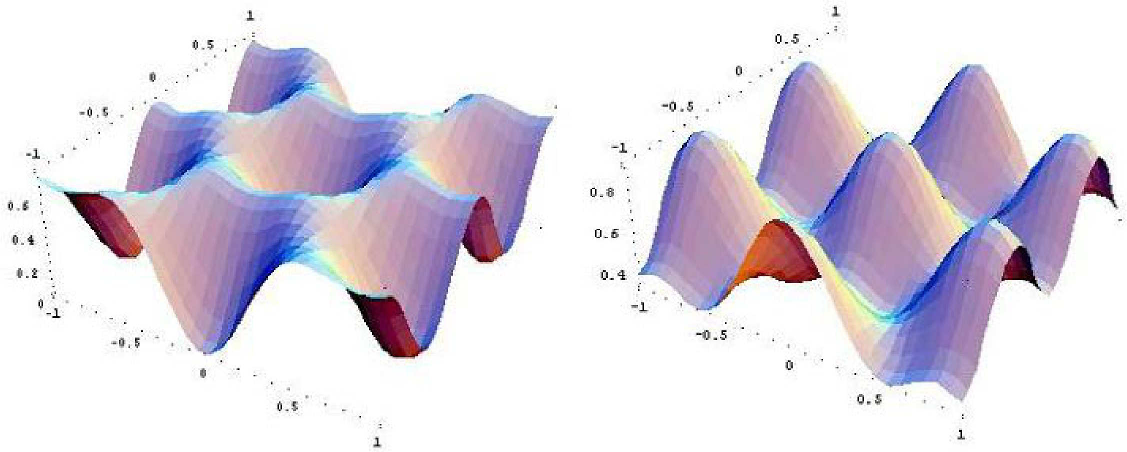

Fig. 1 (online colour at: www.pss-b.com) Charge density wave profile of the Hartree–Fock approximation for the electron distribution in the Hall crystal (left). For comparison, the Wigner crystal density is shown in the right.

[14]. Its second order correlation correction turns out to be an order of magnitude larger than that for the Wigner solid [15]. This strong enhancement may be understood on the basis of the electron densities of the competing states, shown in Fig. 1. As can be observed in the figure, the HC presents marked percolating channels, suggesting that the Wannier like localized states of the problem have appreciable overlap. This is in contrast with the WC structure that shows isolated gaussian-shaped peaks with no connecting channels between them. To be fully operative, cooperative rings of exchange effects require long paths with large exchange probabilities along them and these are more easily found in the HC state than in the WC.

The first experimental evidence for a Wigner crystal at low fillings was the finding of a sharp increase in the longitudinal resistance at the edges of the Hall plateau around filling 1/5, understood to arise from a pinned state with space charge fluctuations [16]. In a recent experiment, Pan et al. found that upon increasing the temperature this wing resistance decreased, developing structure consistent with the formation of further fractional quantum Hall effect (FQHE) at fractions such as 2/11, 3/17, 2/13 and 3/19 [17]. One interpretation of this finding is that the pinned WC formed at low temperatures melts down into a Laughlin liquid as the temperature is increased. Another interpretation is that the low temperature phase is a pinned HC exhibiting the characteristic FQHE odd-denominator rule, which becomes depinned as the sample is heated. Further heating finally destroys the FQHE with a temperature boundary profile consistent with the gap hierarchy exhibited by the HC state.

From the above description the following picture emerges. Fractions of small denominator such as 1/1, 1/3, 1/5, 1/7 and others form a liquid state of null resistance down to zero temperature. If a Hall crystal were to form at such fractions, pinning and the ensuing rise in longitudinal resistance would occur at lower temperatures than presently reported, possibly below a few mK. As the filling departs from such selective fractions a pinned crystal is formed, that could be a WC or a HC. Both would give a microwave signal and either model is consistent with the data of Refs. [2] and [3]. Both are also consistent with the data of Pan et al. [17], although the physics is quite different: in the WC model there is a high temperature FQHE liquid phase, as originally suggested by Buhmann et al. [5], while in the HC model the crystal is itself a FQHE state that becomes depinned as the temperature rises.

In connection with $\nu = 1$ and 2 we would like to add some comments related to the recent detection of crystalline states by Chen et al. [3], around such integral fillings. They find that the magnetic field region in which the microwave absorption shows peaks is well inside the plateaus [18]. Then, the question about the compatibility of the crystalline nature of the ground state and the quantized value of the Hall conductivity over a range of filling fractions emerges. We believe that the occurrence of such an effect is consistent with the results of Cabo and Martinez [19]. It is argued there that the linear response of a

2DEG in the integral quantum Hall regime can be roughly described as a tendency of the impurities to act as effective charge reservoirs, receiving or releasing the exact amount of electrons to ensure the local integral filling condition in large sample areas. The mechanism is triggered by the presence of a fairly sharp cusp in the energy per particle, making preferable energy-wise for the extra electrons or holes to occupy other available states than to join the main integral Hall state, which is preserved across de plateau. The following picture then seems a plausible way to explain the findings of Ref. [3]. First, when the filling factor is near the values $\nu = 1, 2$ for a very clean sample, it can be expected that the charge reservoirs receiving the excess or defect charges are the small number of imperfections present. However, as the filling deviates more and more from the integer value moving towards fractions such as ν = 10/9 or 8/9, it could happen that the limited number of localization centers in the high quality sample are saturated to their capacity. Excess or defect charges would then cluster around impurities forming pinned crystalline regions which could be viewed as a system of dynamically generated localization centers. It is possible that such mechanism is also present near major fractions, such as 1/5, 1/7 and others.

Acknowlegdements N.H.M. and A.C. acknowledge the kind hospitality of the AS ICTP allowing this collaborative work and wish to thank Prof. V. E. Kravtsov for the stimulating atmosphere provided. F.C. thanks the support of FONDECYT through grants 1020829 and 7020829.

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