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Thin Solid Films 270 (1995) 279–282

thin solid films

Comparison between analytical and FEM calculations for the contact problem of spherical indenters on layered materials

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Abstract

An analytical solution and a finite element calculation have been utilised to investigate the contact stresses due to elastic spherical indentation into coating/substrate systems. Such calculated stress distributions may be used for the optimal design of layer systems as well as, together with experimental penetration–force curves, to determine quantitative measures for toughness and adhesion of the film. Both approaches were compared using various systems composed of a higher modulus surface coating on a relatively low modulus substrate.

It was found that both methods agree very well and yield adequate stress distributions for various film thicknesses. Here we show the radial stress component $Yy = \sigma_r$ for six different systems.

Keywords: Contacts; Stress; Adhesion

1. Introduction

Despite the fact that more than one century has passed since the basic work of Hertz [1] the contact problem of spherical indentation is still of great interest [2,3]. Especially the spherical indentation into layered materials as well as the extension of the theory of Hertz for larger penetration (non-Hertzian indentation) which have not yet been adequately solved.

The aim of this work is to compare a finite element method (FEM) with an analytical model to describe the stress distribution in layered materials under spherical non-Hertzian load. Here, non-Hertzian means that the radius of the contact zone is in the order of the magnitude of the indenter tip radius, i.e. one cannot neglect the modification of the surface curvature during indention and the radial force component caused therefrom. The analytical model is described elsewhere [4]. To improve the agreement with the finite element model we always added to any coating/substrate compound a rigid half space with a fixed Youngs modulus of 100.000 GPa at a depth of 330 μm to simulate the restricted size of the finite element model at least in the vertical direction. But we were not able to simulate the free boundary conditions at the horizontal edge of the FEM model. To describe layered materials we have used Heaviside functions in order to simulate the co-ordinate dependence of the elastic parameters, Youngs modulus and Poissons ratio.

The finite element analysis was conducted using the ANSYS finite element program [5]. The method and model used are fully described in Ref. [3] and only the most relevant features are described here. The whole finite.element mesh is an arrangement of 330 quadrilateral, eight-node, isoparametric elements and the total number of nodes is 1088. To simulate contact conditions, three node interface elements are used. The horizontal and vertical dimensions of the space are 264 μm (horizontal) and 334–348 μm (vertical) in dependence on the investigated film thickness. Because of the loading symmetry, only half of the model is considered and the model is restrained at its base and along the axisymmetric axis. The loading conditions are specified by vertical prescribed displacements at the top of the indenter. The dimensions of the smallest elements are 1 μm in the vertical and 2 μm in the horizontal direction. The computations are performed on a 486 PC and the number of increments, iterations in each increment and other parameters are chosen in such a way as to obtain a converged solution in a reasonable period of time.

In all cases the penetration of a spherical indenter with a radius of 32 μm and a Youngs modulus of 600 GPa was modelled. The final penetration depth into the compound systems was 6 μm. We compare systems with a substrate

0040-6090/95/$09.50
SSDI 0040-6090(95)06922-4

Table 1

Compound	Youngs modulus (GPa)	Film thickness (μm)
T1E1	300	18
T1E3	500	18
T2E1	300	10
T2E3	500	10
T3E1	300	4
T3E3	500	4

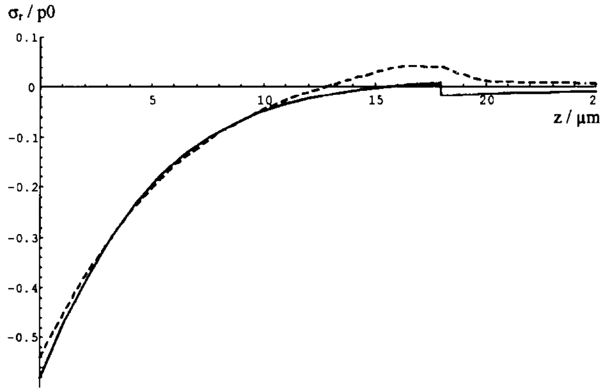

Fig. 1. σ_r for system T1E1 along the indenter axis.

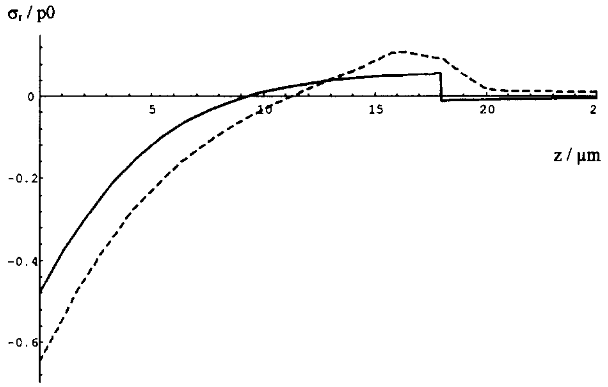

Fig. 2. σ_r for system T1E3 along the indenter axis.

Youngs modulus of 200 GPa and the film parameters given in Table 1. The Poissons ratio in the substrate, indenter and all films is assumed to be 0.25.

2. Results

The best way to compare the two different approaches is to compare the stress distribution they provide. Therefore we present several figures for the systems mentioned above showing the radial normal stress $Yy(\mathrm{FEM}) = \sigma_r$ (analytical model) along the surface and interface (in radial direction r) and along the axis of indentation, which is z in our case. We chose this stress component because it is very important for the formation of cracks and is strongly influenced by the above-mentioned radial force component in the case of non-Hertzian indentation. All stress values are normalised by the maximum stress in the z direction p_0 (in the centre of the contact zone).

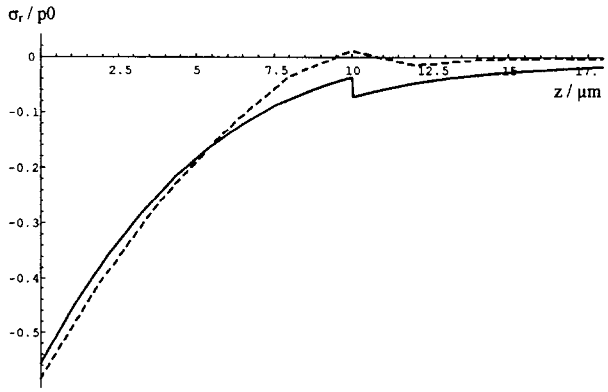

Fig. 3. σ_r for system T2E1 along the indenter axis.

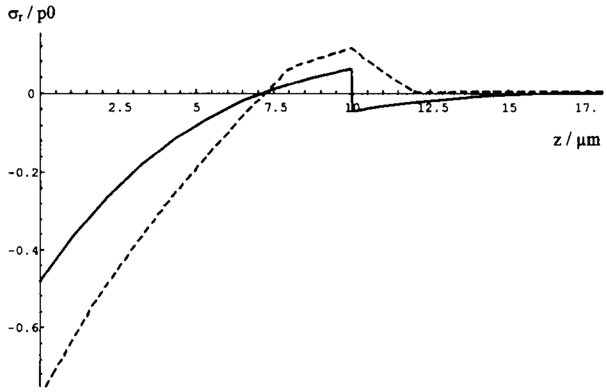

Fig. 4. σ_r for system T2E3 along the indenter axis.

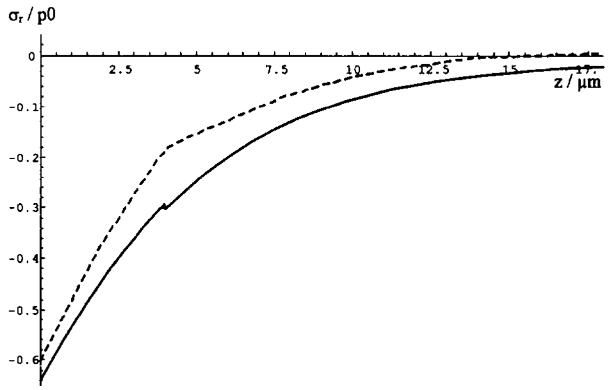

Fig. 5. σ_r for system T3E1 along the indenter axis.

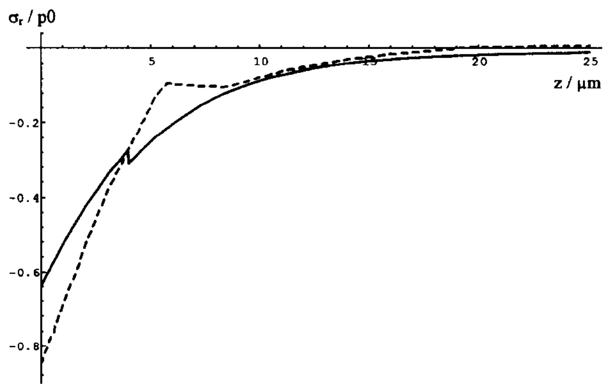

Fig. 6. σ_r for system T3E3 along the indenter axis.

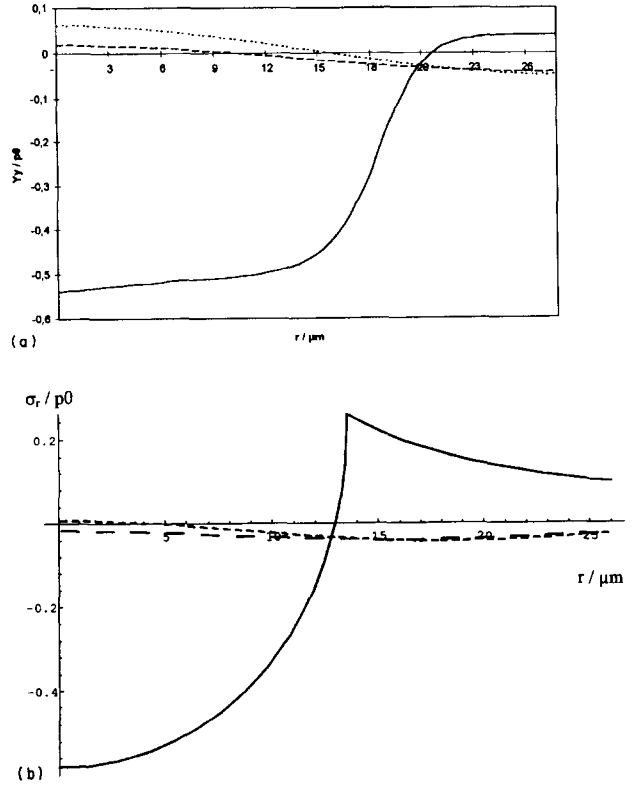

Fig. 7. (a) $Yy = \sigma_r$ for system T1E1 along surface and interface, FEM method; (b) σ_r for system T1E1 along surface and interface, analytically; (c) σ_r for system T1E1 along surface and interface, analytically with included tangential force (second approximation with respect to Ref. [4]).

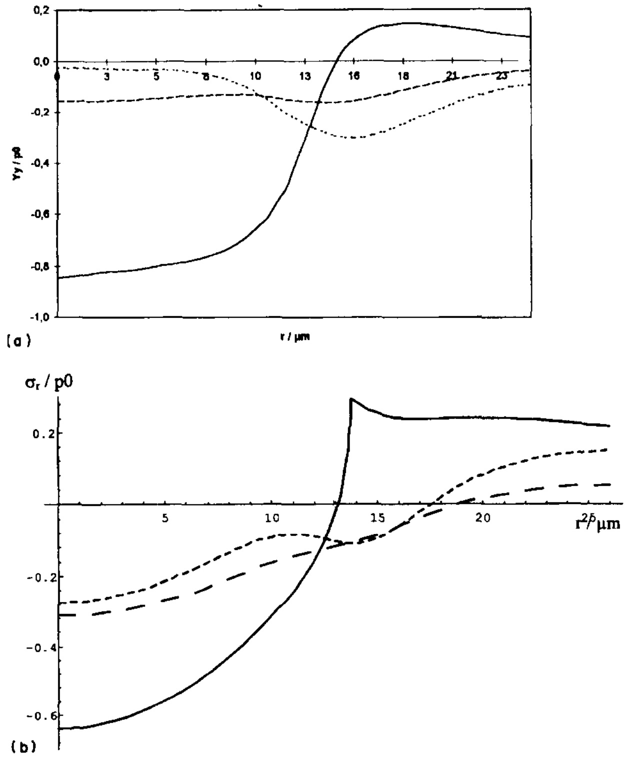

Fig. 8. (a) $Yy = \sigma_r$ for system T3E3 along surface and interface, FEM method; (b) σ_r for system T3E3 along surface and interface, analytically.

Figs. 1-6 show the normalised σ_r stress along the axis of indentation for all systems, where dashed lines denote the curves obtained by the FEM and solid lines those from the analytical method.

In addition, Fig. 7 and Fig. 8 show the stress distribution along the surface (solid line) and near the interface in the film (small dashes) and substrate (big dashes) for the two compounds T1E1 and T3E3.

3. Discussion

Practically all essential features of the radial stress components calculated by the two methods were in good agreement; especially the stress distributions along the axis of indentation for low differences of the Youngs moduli of the film and substrate are a good comparison. Somewhat bigger differences were obtained in the direction of r especially at the surface (Fig. 7 and Fig. 8). Because the parameters chosen for the system T1E1 are very close to the homogeneous case one would expect the typical sharp tensile stress maximum near the contact zone edge which could lead to the well-known Hertzian cracks (Fig. 7(b)). If one analytically includes the above-mentioned tangential force component caused by the curvature of the surface (Fig. 7(c)) this maximum becomes smoother but is again visible near the contact edge [4]. A possible explanation for the absence of this maximum in the FEM results as well as for the other deviations could be the restricted dimension of the whole finite element mesh and its free boundaries in the horizontal direction which are not fully adequate to a mathematical half space. This leads to a less or more stronger bending of the film in contrast to the stretching in the half space case especially for films with bigger Youngs moduli. Our analytical calculations however are based on this case. The bending effect increases with the differences of the Youngs moduli between the film and substrate. In addition, the deviations may be caused by the element size which is for the thinnest films under investigations in the order of magnitude of film thickness. Finally the methods differ in the calculation of stress limits at the surface and interface, where in the case of the FEM method one is forced to take average values of the neighbouring elements.

4. Conclusions

All in all both the analytical and FEM model can be used to calculate the principal stress distribution and the deformation state for the discussed contact problem and may therefore be used for the analysis of corresponding experimental results [6,7]. The influence of the tangential force component caused by the curvature of the surfaces for larger penetrations is important. The advantage of the FEM method is the possibility to involve plastic behaviour and arbitrary horizontal and vertical dimensions, while the analytical method calculates much faster and has practically no restriction in the number of layers or their parameters.

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