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LETTERS TO THE EDITOR

COMMENTS ON "FREE VIBRATIONS OF CIRCULAR CYLINDRICAL SHELLS"

In a recent paper, Ramamurti and Pattabiraman used axisymmetric finite elements to determine natural frequencies and mode shapes of cylindrical shells with various boundary conditions, including flanged ends for which experimental data were also obtained [1]. In earlier work, the writer determined natural frequencies of cylindrical shells with clamped ends, using the Rayleigh–Ritz method, obtained natural frequencies experimentally for shells with flanged ends and discussed the relationships between flange and shell dimensions for which the shell could be considered to be clamped [2]. Obviously the use of the finite element method to model the flanged end is an improvement on the writer's early empirical relationship based on experimental results.

The writer would like to comment on some of the numerical results given by Ramamurti and Pattabiraman. In Tables 1 and 2 they compare natural frequencies of clamped shells obtained from their finite element formulation and from the writer's earlier Rayleigh–Ritz approach. In Table 9 they compare the predictions of the finite element method with experimental results for a particular shell with flanged ends. In one of these predictions the shell is assumed to be clamped at the ends with an effective length $l = 44$ cm. If the flanges are sufficiently rigid to ensure complete constraint of the shell, the effective length of the shell should be 40 cm, which is the free length between the flanges (see Figure 9 of reference [1]). In Table 1 the writer has estimated the natural frequencies for the first three modes for a clamped shell of this length from the finite element and Rayleigh–Ritz methods; this has been done simply by interpolation between the results of Tables 1 and 2 of reference [1] for $l = 36$ and 44 cm respectively. For the moderate changes in frequency caused by this variation in length the assumption of a linear frequency-length relation should give acceptable estimates. The experimental results from reference [1] are also included in Table 1. The natural frequencies from the Rayleigh–Ritz method given in Table 1 are known to exceed exact solutions from thin shell theory by small amounts [3]. From the results of reference [3] and other results in his possession, the writer has estimated the small percentage differences by which the Rayleigh–Ritz solution exceeds the exact solution from thin shell theory, and thus has given a column of corrected frequencies in Table 1. These compare favourably with the experimental values and it would seem reasonable to assume that the dimensions of the flanges are sufficiently large to ensure that the ends of the shell are clamped with an effective length of $l = 40$ cm. It is noted that the natural frequencies from the finite element method, like those from the Rayleigh–Ritz method, are slightly high.

TABLE 1

Natural frequencies of clamped steel cylindrical shell (Hz); *length* = 40 cm; *thickness* = 0·362 cm; *radius* = 17·5 cm

m	n	Finite element method [1]	Rayleigh–Ritz method [2]	Exact solution from thin shell theory [3]	Experiment (shell with flanged ends) [1]
1	3	1280	1186	1110	1079
1	4	1082	950	900	847
1	5	1075	964	930	1008

Ramamurti and Pattabiraman mentioned that as their finite element formulation is based on three dimensional elasticity theory, their solutions are not restricted to thin shells. Considering cylindrical shells with simply supported ends, some results for a thickness to mean radius ratio of 0·2 are included in their Table 4. It is surprising that the authors have compared their finite element results with those labelled B in their Table 4, as the latter are based on Flügge's thin shell theory, and that they have not compared their results directly with those labelled A, which are based on Armenakas' solution from three dimensional elasticity theory. Considering the formulation of the finite element in reference [1], it would be hoped that natural frequencies obtained by this method for moderately thick shells would converge to Armenakas' values, rather than to any thin shell values when there is a significant difference between these two sets of values. Also the quoted values in Table 4 were given by the writer [4], together with frequencies from the Mirsky-Herrmann shell theory, which includes the effects of transverse shear deformation and rotatory inertia, in order to illustrate for moderately thick shells the close agreement between frequencies from three-dimensional elasticity theory and from a higher order shell theory and the divergence of frequencies based on a thin shell theory.

The effect on natural frequencies of neglecting in-plane inertia is illustrated in Table 5 of reference [1]. As these results relate to the shell previously considered in Table 2 of reference [1], its thickness to radius ratio is 0·0207 and thus rotatory inertia can be neglected. In such circumstances the effect of neglecting in-plane inertia is to increase the frequency by the approximate factor $(1 + 1/n^2)^{1/2}$ for $n > 0$. This approximate expression is based on the assumption that in the kinetic energy expression the ratio of the amplitude of the circumferential component of displacement to that of the radial component is $1/n$ and the amplitude of the axial component of displacement is small compared to the radial component. This is only a rough guide to the effect of neglecting in-plane inertia. In general, its accuracy increases as n increases and decreases as m increases. (It is also dependent on the shell geometry.) Frequency increases of 3 and 2% are predicted for $n = 4$ and 5 respectively; comparable values obtained from Table 5 of reference [1] are from 1·7 to 2·7 percent for $n = 4$ and from 1·9 to 2·1 percent for $n = 5$.

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AUTHORS' REPLY

We are most grateful to Professor Warburton for the nice comments made on our paper.

While we appreciate the ideas contained in his letter, we would like to make the following comments.

1. The "rigidity" of the flanges is a matter of engineering judgment and depending on the experimental results (amplitude measured on flanges and on shells) the equivalent