Essential Work of Fracture and J-Integral Measurements for Ductile Polymers

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SYNOPSIS

In the ductile tearing of polymers that neck before failure it is shown that the specific essential fracture work (w_e), consisting of the energies dissipated in forming and tearing the neck, is a material property for a given sheet thickness and is independent of specimen geometry. Work of fracture experiments using both double deep-edge notched (DENT) and deep-center notched tension (DCNT) geometries with different ligament lengths yielded almost identical w_e values for a grade of high-density polyethylene. These measurements for w_e are in fairly good agreement with the theoretical values based on the J integral evaluated along a contour surrounding the neck region near the crack tip. Under J-controlled crack growth conditions, it is shown that J_c obtained by extrapolation of the J_R curve to zero crack growth and the slope dJ/da are identical, respectively, to w_e and $4\beta w_p$ obtained from the straight line relationship between the specific total work of fracture (w_f) and ligament length (l).

INTRODUCTION

It is now well established that in characterizing the failure of ductile and toughened polymers the linear elastic fracture mechanics (LEFM) parameters, i.e., the critical potential energy release rate (G_c) and the critical stress intensity factor (K_c), are inappropriate. This is due to the large plastic zone which exists at the tip of a notch or defect which thereby invalidate the basic assumptions of LEFM. Much of the plastic flow work dissipated in this zone is not directly associated with the fracture process. Only that work which goes into the fracture process zone (FPZ) is a material constant and hereafter is called the specific essential work of fracture (w_e). In thin sheets of polymers the FPZ may be identifed with the neck region formed at the notch tip. The essential fracture work is therefore that required to form the neck and subsequently to initiate tearing of the neck. Experimentally, the essential fracture work (W_e) can be separated from the nonessential fracture work (W_p) (i.e., which is dissipated in the plastic zone outside of the FPZ) by conducting a series of total work of fracture (W_f) tests on deep double-edge notched tension (DENT) specimens with different ligament lengths (l) (see Fig. 1a). For this method to be successful the ligament should have been yielded prior to tear initiation at the notch tip and it should also be at least three times the specimen thickness (t) to avoid the plane strain–plane stress transition region. Because W_e is proportional to l and W_p to l^2, a straight line relationship exists between the specific total fracture work w_f ($= W_f/lt$) and ligament length (l) with the intercept at $l = 0$ giving the specific essential fracture work (w_e), i.e.,

$$w_f = w_e + \beta \omega_p l \qquad (1)$$

where β is a shape factor of the outer plastic zone which depends on the specimen geometry.

Equation (1) was first used by Cotterell and Reddell[1] to measure the specific essential fracture work for ductile tearing of thin sheet metals. The same technique was subsequently extended by us[2,3] to measure w_e for a range of ductile polymers in plane stress conditions using the DENT geometry. In order to show that w_e is a material property for a given

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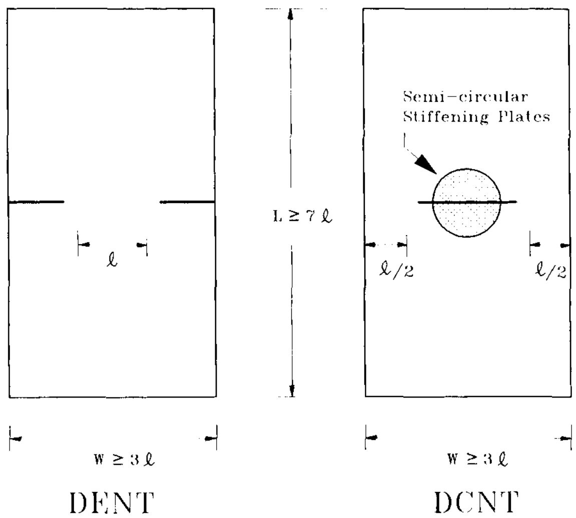

Figure 1. (a) Deep double-edge notched tension (DENT) specimen. (b) Deep center notched tension (DCNT) specimen.

sheet thickness and is independent of specimen geometry the deep single-edge notched tension (DSEN) geometry has been used in addition to the DENT specimens because eq. (1) is still valid,[2,3] though βw_p is not the same owing to the different shapes of the outer plastic zone surrounding the inner FPZ in the ligament region. There are, however, some problems with the DSEN geometry. Rather unlike the DENT geometry in which crack growth is symmetrical on either side of the ligament and the load-deflection curves are geometrically similar for different ligament lengths, in the DSEN geometry, as the fracture progresses toward the end of the ligament or for short initial ligament lengths, geometrical similarity to the load-deflection curves cannot be maintained due to the large rotations of the two halves of the specimen. This difficulty has limited the range of valid data that can be used for fracture analysis.[2,4] Another problem is that the state of stress in the ligament of the DENT and DSEN geometries is different, being biaxial for the former and uniaxial for the latter. It is not clear if w_e depends on the stress state although previous work[3] on several polyethylenes indicates that if the failure mode and the deformation of the ligament at tearing remain the same then w_e is identical for both the DSEN and DENT geometries.

To provide further evidence that the specific essential fracture work is a material constant for ductile polymers we need to use a specimen geometry that may eliminate some of these problems associated with the DSEN geometry. Thus we have chosen the center-notched tension (DCNT) geometry (Fig. 1b) in which the ligaments are made small enough to ensure complete plastic flow before tear initiation and to confine yielding to the ligament. Geometrical dissimilarity of crack growth and hence of load-deflection curves for different ligament sizes is easily avoided in this case and the state of stress in the ligament is predominantly biaxial for large ligaments and uniaxial for small ligaments. To overcome compressive buckling of the specimen, semicircular stiffening plates are attached in the central region above and below the notch (Fig. 1b). The stiffening plates are scaled linearly with the ligament length. Equation (1) is also valid for the DCNT geometry so the specific essential fracture work can be obtained in the same manner as the DENT specimens.

We present some fracture results for a grade of high-density polyethylene from both the DENT and DCNT geometries to show that w_e is truly a material property. $J_R-\Delta a$ crack resistance curves are also obtained for these two specimen geometries from the load-deflection diagrams and compared with the w_f-l curves. Difficulties with using the $J_R-\Delta a$ curves according to the ASTM E813-81 and E813-87 procedures to determine the crack initiation J_c value and its relevance to plane stress ductile fracture are discussed.

EXPERIMENTAL

The high-density polyethylene in 3 mm thick sheets was supplied by Cadillac Plastics (Australia) Pty Ltd for testing. Both DENT and DCNT specimens were cut from the sheets with dimensions shown in Figure 1. The ligament lengths (l) varied from 3 to 30 mm for the DENT geometry and from 10 to 30 mm for the DCNT geometry. Semicircular stiffening plates of 3 mm thickness made from brass were attached to both sides of the specimen, and this successfully avoided compressive buckling of the central region near the notch. All fracture experiments were conducted in an Instron 1195 testing machine with a crosshead speed of 0.50 mm/min and an ambient temperature of 20 ± 2°C. Since fracture propagation was stable it was possible to blip the load-deflection curves as the crack tip successively traversed the position grids printed on the ligament. This procedure was required to obtain the $J_R-\Delta a$ curve using the method based on the energetic formulation of the J integral,[5] i.e.,

$$J = -\frac{dU}{da} \tag{2}$$

where dU is the potential energy released per unit thickness during crack growth da.

True stress ($\bar{\sigma}$)–true strain ($\bar{\epsilon}$) relation was also obtained for the polyethylene using dumbbell tensile specimens with gauge length 25 mm. This gave the following equation: $\bar{\sigma} = 120\,\bar{\epsilon}^{0.438}$ (MPa) and a maximum load tensile strength (σ_o) of about 50 MPa and a 2% offset yield strength (σ_y) of 27 MPa.

RESULTS

Figure 2 gives a set of load-deflection curves for a range of ligament lengths of the DENT specimens and it is readily seen that these curves are geometrically similar. The blips on the curves represent crack growth of about 0.8 mm on either side of the notches. The work areas under the curves can be plotted against the initial ligament length according to eq. (1) and the results are given in Figure 3. As expected, a straight line relationship between w_f and l is obtained, and at zero ligament length the specific essential fracture work (w_e) for fracture propagation is about 36 kJ/m^2 and the slope βw_p is equal to 14 MJ/m^3. Similar results were also obtained for the DCNT specimens and Figure 4 shows a plot of w_f against l. Again, the straight line relationship obtained is in accordance with the prediction of eq. (1) and this gives w_e = 36.5 kJ/m^2 and βw_p $\doteq$ 10.4 MJ/m^3.

Figure 5 represents the load-point elongation (δ_f) at complete fracture results plotted against ligament length (l) for both the DENT and DCNT geometries. The critical crack tip opening displacement for fracture propagation (δ_c) can be obtained by extrapolating the straight lines to zero ligament length.[1,3] Thus, $\delta_c \doteq 1$ mm for both specimen geometries.

DISCUSSION

Specific Essential Work of Fracture as a Material Property

If the specific essential fracture work dissipated within the FPZ is a material property in thin sheets of ductile polymers, then this physical quantity has to be independent of specimen geometry. In earlier work on a range of ductile polymers[2,3] including several grades of polyethylenes we have shown that both the DENT and DSEN geometries give the same value of w_e provided the failure modes remain similar. This conclusion is reconfirmed here that for

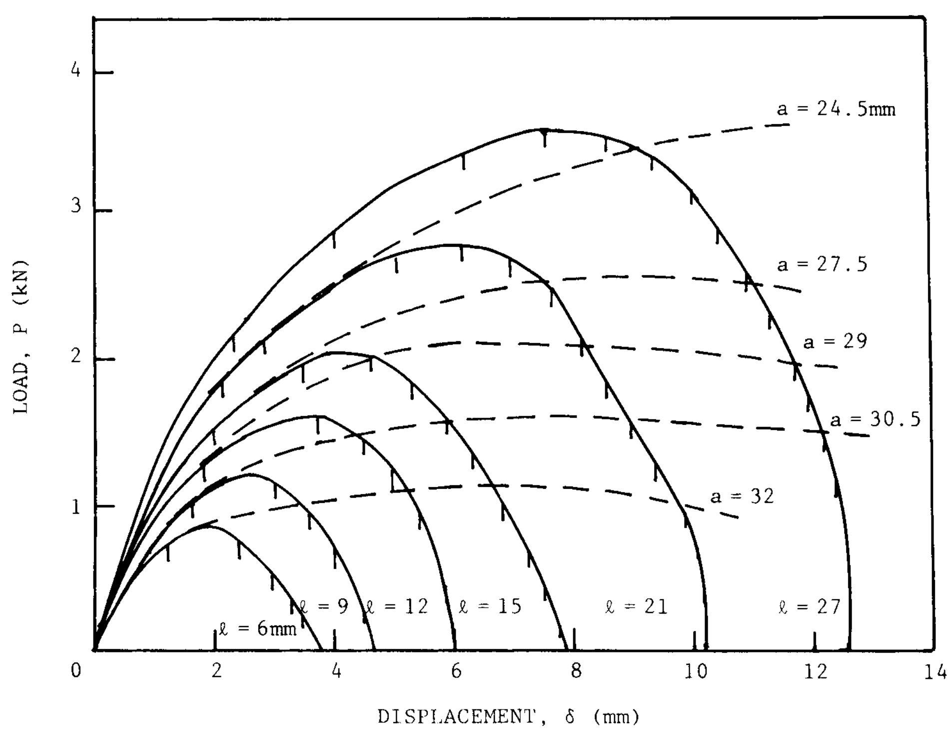

Figure 2. Load-deflection ($P - u$) curves for fracture in DENT polyethylene specimens. —— experimental curves; ----- interpolated constant crack length curves.

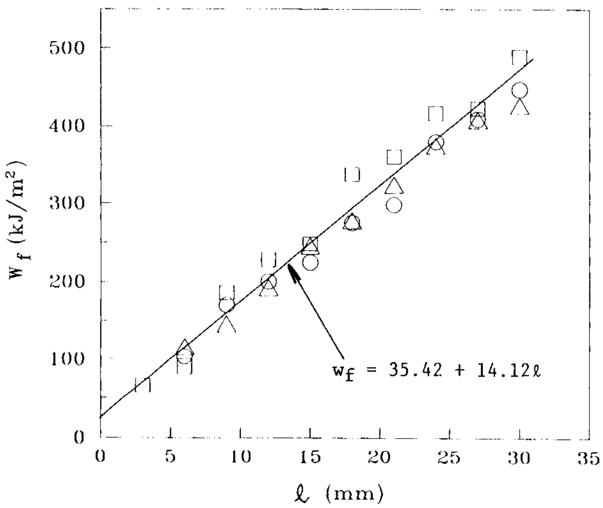

Figure 3. Plot of total specific work of fracture (w_f) against ligament length (l) for DENT specimens.

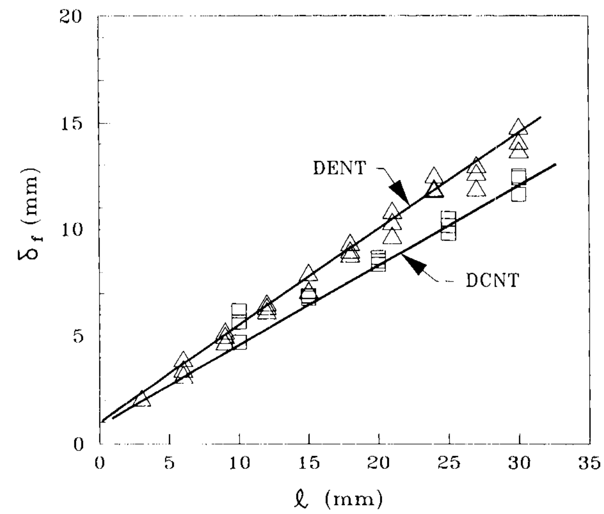

Figure 5. Plot of the elongation to fracture (δ_f) against ligament length (l) for DENT and DCNT geometries.

the particular grade of polyethylene studied in this work using 3 mm DENT and DCNT specimens, the specific essential fracture work is indeed a material constant and equal to 36 kJ/m^2. (see Figs. 3 and 4). That the critical crack-opening displacement for fracture propagation is also identical for both DENT and DCNT geometries (i.e., $\delta_c \doteq 1$ mm from Fig. 5) lends further support to the conclusion that the specific essential fracture work is a material property (see eq. (4) later).

The physical meaning of the specific essential fracture work has been given earlier by us[2,3] in terms of a critical J integral evaluated along a contour **S** bordering the fracture process or necked-down zone of width d at the notch tip. Thus,

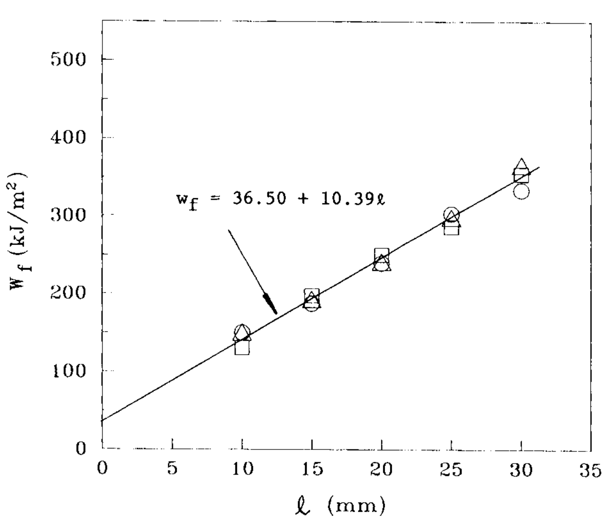

Figure 4. Plot of total specific work of fracture (w_f) against ligament length (l) for DCNT specimens.

$$J_c = \int_{\mathbf{S}} \left[W^* \, dy - \mathbf{T} \frac{\partial \mathbf{u}}{\partial x} \, ds \right] \tag{3}$$

where W^* is the strain energy density which can be expressed as $\int \bar{\sigma} \, d\bar{\epsilon}$ and the traction $\mathbf{T}$ is the stress $\sigma(\delta)$ acting across the FPZ and a function of the crack tip-opening displacement δ. Eq. (3) can be simplified to

$$w_e = J_c = \int_o^{\bar{\epsilon}_n} \bar{\sigma} \, d\bar{\epsilon} \int_o^d dy + \int_o^{\delta_c} \sigma(\delta) \, d\delta \tag{4a}$$

$$= d \int_o^{\bar{\epsilon}_n} \bar{\sigma} \, d\bar{\epsilon} + \int_{d\epsilon_n}^{\delta_c} \sigma(\delta) \, d\delta \tag{4b}$$

where $\bar{\epsilon}_n$ and ϵ_n are the true and engineering strains at necking. The first term of eq. (4b) represents the plastic flow work to form a neck and the second term is the additional work required to tear the neck to initiate fracture propagation. In general, $d = \alpha t$ where $\alpha \leq 1$ and $\sigma(\delta)$ is taken as a constant equal to the maximum load tensile strength σ_o. The critical crack-opening displacement δ_c is expected to vary directly with d if a critical strain at fracture (ϵ_c) criterion is applied (i.e., ϵ_c and δ_c/d). Equation (4b) can be simplified using these not unreasonable assumptions to:

$$w_e = J_c = \frac{\alpha t k \bar{\epsilon}_n^{\,n+1}}{(n+1)} + \sigma_o \alpha t (\epsilon_c - \epsilon_n). \tag{5}$$

where the true stress–true strain relationship is given by $\bar{\sigma} = k\bar{\epsilon}^n$.

In ductile plane stress fracture the crack tip-opening displacement at tear *initiation* δ_i usually is lower than that for *propagation* δ_c. This means that J_c for tear propagation is larger than initiation as can be seen from eqs. (4) and (5) by replacing δ_c with δ_i.

It is interesting to note that in *plane stress* conditions the specific essential fracture work w_e (and hence J_c) for fracture propagation is linearly dependent on specimen thickness t. Experimental results in support of this prediction have been obtained by us for several aluminum alloys in the thickness range 0.5–3.2 mm. However, we do not have any results for ductile polymers. It must also be realized that eq. (5) will not hold when the failure mode departs from plane stress to plane strain.

For the polyethylene studied here we have $t = 3$ mm, $\alpha \doteq 0.3$, $n = 0.438$, $k = 120$ MPa, $\bar{\epsilon}_n = 0.438$, $\epsilon_n = 0.55$, $\sigma_o = 50$ MPa, and $\delta_c = 1$ mm. Thus from eq. (5), the estimated value of the specified essential fracture work is 47 kJ/m^2 which is in fairly good agreement with the experimental value of 36 kJ/m^2 measured from the DENT and DCNT specimens.

Comparisons of the J Integral and Essential Fracture Work Methods for Characterizing Ductile Fracture

Under strictly J-controlled crack growth conditions[6]

$$\frac{l\sigma_o}{J_R} \gg 1 \tag{6}$$

and

$$\frac{l}{J_R}\frac{dJ}{da} \gg 1 \tag{7}$$

the crack resistance J_R curve is approximately a linear function of crack growth Δa and is given by:[7]

$$J_R = J_c + \frac{dJ}{da}\Delta a \tag{8}$$

for small growth. J_c is normally identified with crack initiation in plane strain but in plane stress it should be interpreted as the crack propagation value. J_R can also be obtained from the energetic definition already given in eq. (2). Thus integrating J_R to complete fracture for both the DENT and DCNT geometries we have

$$U = \frac{W_f}{2t} = J_c\frac{l}{2} + \frac{dJ}{da}\frac{l^2}{8} \tag{9a}$$

or

$$w_f = J_c + \frac{l}{4}\frac{dJ}{da}. \tag{9b}$$

Comparisons of eqs. (1) and (9b) shows that $w_e \equiv J_c$ and $\beta w_p \equiv \frac{1}{4}\frac{dJ}{da}$. To validate these equalities we have to construct the J_R–Δa crack resistance curves in accordance with the procedures outlined by Begley and Landes.[5] This requires the construction of load-deflection (P–u) curves for different constant crack lengths using blunt notches to discourage crack propagation. Straight lines for U against a for varying load-point deflections and subsequently the calibration curve of J against u are obtained. J_R–Δa can now be plotted using the experimental P–u curves from the fracture experiments noting the values of u and the corresponding crack lengths during stable crack propagation. Hodgkinson and Williams[8] have used similar procedures to obtain J_R–Δa curves for highly deformable polymers. However, they suggested that constant crack length curves could be constructed by interpolation between crack length positions marked on the P–u curves such as shown in Figure 2. This assumes that the deformation fields are identical for a given ligament length, whether that ligament size is achieved by crack growth from a larger ligament or not. Hence they did not use blunt notches to obtain the J–u calibration curve. We have followed both the Begley and Landes[5] and the Hodgkinson and Williams[8] methods using J–u calibration curves based on "blunt" and "sharp" notches, respectively, to derive the J_R–Δa curves for the DENT specimens as shown in Figures 6 and 7. Only those data which satisfy both eqs. (6) and (7) are included in these two figures. Here we take both $l\sigma_o/J_R$ and $(l/J_R)(dJ/da)$ to be bigger than 4. The large scatter of the experimental data points are caused by the usually unequal crack growths on either side of the notches. Nevertheless, we can fit a straight line through the data according to eq. (8). Thus, both Figures 6 and 7 give the same J_c of 34 kJ/m^2 at zero crack growth but dJ/da have different values of 43 and 89 MJ/m^3 depending on whether the J–u calibration curve is based on "blunt" notches or "sharp" notches. The J-controlled crack growth data for the DCNT specimens are plotted in Figure 8 and there is much less scatter in this case. A straight line relationship between J_R and Δa is again

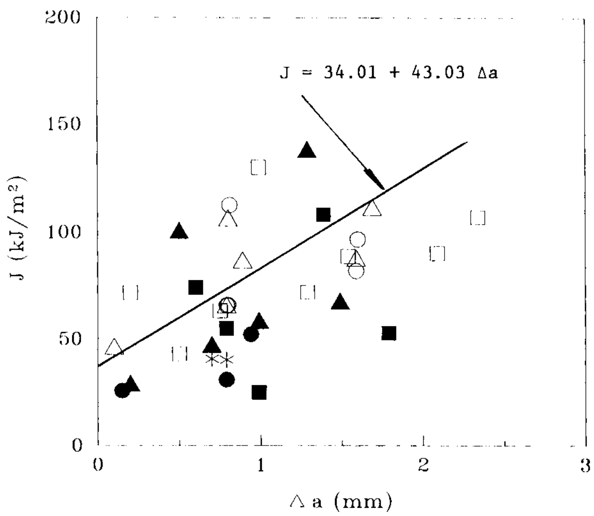

Figure 6. J_R versus Δa plot for DENT specimens based on the method of Begley and Landes.[5] Δ l = 30 mm; □ l = 27 mm; ○ l = 24 mm; ▲ l = 21 mm; ■ l = 18 mm; ● l = 15 mm; * l = 12 mm.

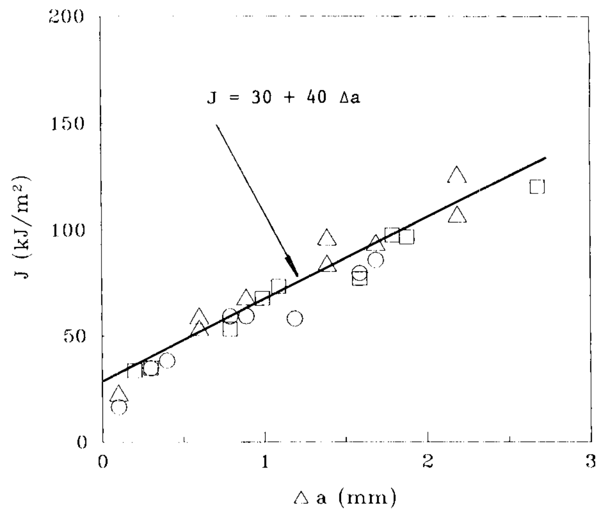

Figure 8. J_R versus Δa plot for DCNT specimens. Δ l = 30 mm; □ l = 25 mm; ○ l = 20 mm.

obtained and we have J_c = 30 kJ/m² and dJ/da = 40 MJ/m³. Table I compares w_e to J_c and $4\beta w_p$ to dJ/da for both the DENT and DCNT geometries. Indeed, in support of the equivalence of eqs. (1) and (9b), there is very good agreement between w_e and J_c as well as between $4\beta w_p$ and dJ/da (if the J–u calibration curve based on blunt notches is used for the construction of the J_R–Δa crack resistance curve). The value of dJ/da (= 89 MJ/m³) based on the method of Hodgkinson and Williams using interpolated constant crack lengths is considerably larger than $4\beta w_p$ (= 56 MJ/m³). These tabulated results have therefore supported our earlier suspicion[2] that in comparing the w_f–l plot to the J_R–Δa plot the latter curve cannot be accurately obtained from the method suggested by Hodgkinson and Williams[8] because of the unavoidable inaccuracies in constructing the J-u calibration curve using interpolated constant crack length curves.

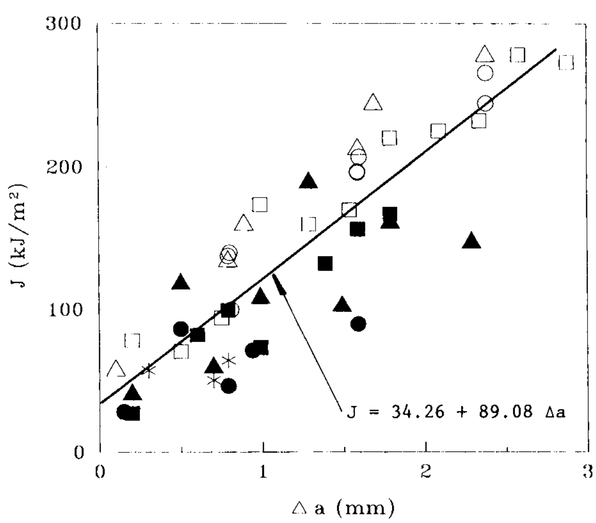

Figure 7. J_R versus Δa plot for DENT specimens based on the method of Hodgkinson and Williams.[8] Symbols are the same as in Figure 6.

Equation (9) has also been obtained by Vu-Khanh[9] for impact fracture testing of ductile polymers. He suggested that both J_c and dJ/da $\left(\text{which he rewrote as the tearing modulus } T_a = (E/\sigma_y^2)\,\dfrac{dJ}{da}\right)$ are material properties. Quite clearly, since dJ/da is related to βw_p, which is dependent on specimen geometry, it cannot be a material property. His experimental results do not sup-

Table I. Comparisons of J_c (kJ/m²) and dJ/da (MJ/m³) Obtained from the Work of Fracture for DENT and DCNT Specimens and from the Method of Begley and Landes

DENT	DCNT	J_c	DENT	DCNT	dJ/da
w_e			$4\beta w_p$		
	36	30		42	40
35		34	56		43
35[a]		34[a]	56[a]		89[a]

[a] Based on the method of Hodgkinson and Williams (8).

port this claim either. Only w_e (or J_c) is a material property.

Relevance of the ASTM E813 Standard for J-Integral Measurement

The J-integral method[5] was originally proposed to measure the crack initiation value for *plane strain* fracture in ductile metals and has now been standardized by the ASTM with two versions E813-81 and E813-87 using multiple single-edge notched bend (SENB) specimens. Although this experimental technique has been extended to measure the plane strain fracture toughness of a range of ductile and toughened polymers and blends[10–14] there are some reservations about the use of the crack "blunting line" concept[15] to determine J_c for crack initiation and the interpretation of the $J_R-\Delta a$ curve[16] in general. In the ASTM E813-81 the "crack blunting line," $J = 2\sigma_y \Delta a$, is used to intersect the $J_R-\Delta a$ line obtained by linear regression of the crack growth data to give a measure of J_c for crack initiation. If the specimen thickness exceeds 25 (J_c/σ_y) then J_c is a valid plane strain fracture toughness value. Narisawa and Takemori[15] recently showed that for several impact-modified polymers the blunting line concept is irrelevant for these toughened materials, since J_c obtained at the intersection points are higher than the real values corresponding to crack growth directly observed on polished side surfaces. Due to the complex nature of the damage that occurs at the crack tip, crack growths measured on fracture surfaces as specified by E813-81 are often overestimated for small growth. However, inspection of the $J_R-\Delta a$ curves given by Narisawa and Takemori suggests that the true J_c can be obtained by extrapolating the straight line relationship for $J_R-\Delta a$ to zero crack growth. [This procedure was identical to what we did in Figures 6 and 8 to obtain J_c for plane stress fracture in accordance with eq. (8).]

Huang[16] obtained J-integral measurements of two rubber-toughened nylons using the SENB geometry and both E813-81 and E813-87 procedures by excluding experimental data outside of the two offset lines at Δa equal to 0.6% and 6% of the ligament length drawn parallel to the crack blunting line. (This is different from the exclusion lines at offsets of 0.15 and 1.5 mm recommended by E813-87). Instead of a linear regression line, E813-87 requires that a power law curve be fitted to the data, i.e., $J = \beta(\Delta a)^\gamma$, where β and γ are numerical constants. J_c is now defined by the intersecting point of this power law curve and an offset line drawn at 0.2 mm parallel to the blunting line. Physically, J_c is no longer the crack initiation value but it corresponds to a crack growth of 0.2 mm. When analyzing his data in accordance with E813-81, Huang[16] found that J_c increases linearly with specimen thickness whether or not the plain strain thickness requirement is satisfied. Such results are not totally unexpected because of the way data are selected to conform to the J-controlled crack growth conditions (i.e., limited to 6% ligament length) and the crack blunting line is used in the analysis. This difficulty is removed with E813-87 when all the data for different specimen sizes and thicknesses are combined together and a single $J_R-\Delta a$ curve is obtained. However, as remarked earlier the determination of J_c from this curve at $\Delta a = 0.2$ mm is not relevant for crack initiation. Huang[16] also emphasized and confirmed Narisawa and Takemori's remark that the crack-blunting line concept is inapplicable for analyzing the fracture data of ductile polymers.

Before proceeding further, it must be emphasized that there is no theoretical or physical basis for the modified procedures (J. Landes, personal communication) to determine J_c as given in E813-87. For many metals and alloy steels, the two methods of analysis (i.e., E813-81 and E813-87) do not give very different J_c results. Direct application of these J-integral measurement techniques (modified or not) to characterize fracture toughness for ductile polymers is problematic as already pointed out by Huang[16] and Narisawa and Takemori.[15] We agree completely with these investigators[14,15] that the crack-blunting line is inappropriate and for *plane stress* ductile fracture propagation the data exclusion lines given in E813-87 are also irrelevant. To limit data to J-controlled crack growth we need only to satisfy eqs. (6) and (7). (In this work we have chosen the left-hand side terms of these equations to be larger than 4.) Then J_c for crack propagation is determined by extrapolation of the linear $J_R-\Delta a$ crack resistance curve to $\Delta a = 0$ (see Figs. 6 and 8). Because of the equivalence of the J integral and the essential work of fracture methods discussed earlier, it is much easier to use the latter method to determine w_e (and hence J_c), which is a material property for ductile fracture propagation. We suggest therefore that the essential fracture work method be used for toughness characterization of ductile polymers in plane stress.

Figure 9 shows a schematic of w_f versus l plot for different specimen thickness (t) for the work of fracture tests conducted on, for example, the DENT geometry. As t is increased, the straight lines in the plane stress region are moved upward to give in-

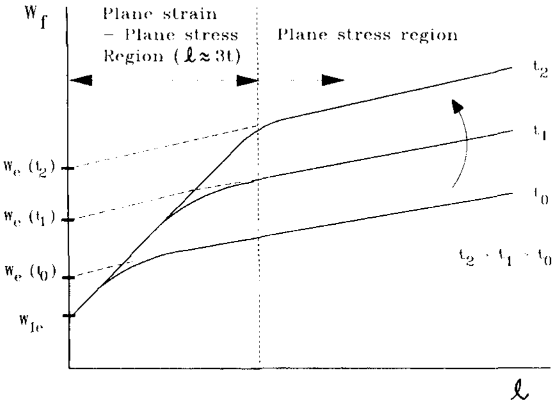

Figure 9. Schematic of the w_f-l relationship for varying specimen thickness t in the plane stress and the plane stress–plane strain transition regions.

creasing w_e values [as required by eq. (5)]. If the shape of the outer plastic zone surrounding the FPZ is invariant with t, βw_p is constant and these lines are parallel to one another. Otherwise, the slopes are expected to decrease with increasing t. When the ligament is less than about $3t$, the fracture data are in the plane strain–plane stress transition region[2] and the straight line relationship between w_f and l does not necessarily hold. However, extrapolation of these data to zero ligament length gives w_{Ie}, which is equal to the plane strain toughness J_{Ic} value if the thickness condition $25(w_{Ie}/\sigma_y)$ is satisfied.[2,17] Otherwise, w_{Ie} is only a "near" plane strain toughness value and is dependent on t. In the very small ligament size region toward pure plane strain, we expect the fracture data for all t under J-controlled crack growth to fall on approximately one single curve and hence extrapolate to a *single* w_{Ie} ($\equiv J_{Ic}$) value at zero ligament length.

Concerning the relationship between w_f and l in the plane stress–plane strain transition region Yap et al.[17] have shown that for a high-impact polystyrene (HIPS) 4.5 mm in thickness a straight line is obtained which on extrapolation to zero ligament length gives a valid plane strain fracture toughness. By assuming in this transition region that the specific essential fracture work w_e is linearly dependent on ligament size l and the plastic work βw_p to be independent of l, Saleemi and Nairn[18] recently showed that a linear relationship between w_f and l, i.e.,

$$\begin{aligned} w_f &\doteq w_{Ie} + \gamma(t)l + \beta(t)w_p(t)l \\ &\doteq w_{Ie} + [\gamma(t) + \beta(t)w_p(t)]l \end{aligned} \tag{10}$$

also holds for two toughened nylons and several polyethylenes. Because of the difficulties in deciding the range of data within the plane stress–plane strain transition region to fit eq. (10), they subtracted the plastic work $\beta w_p l$ from the specific work w_f to obtain w_e and plotted this against l. The intercept of this straight line relationship between w_e and l at zero ligament size gives w_{Ie}, which is shown to be independent of sheet thickness $t(1.6 \sim 3.2$ mm) and in agreement with independently determined J_{Ic} values. Admittedly, there is no theoretical basis for eq. (10), and its use to determine *plane strain* fracture toughness w_{Ie} needs to be more firmly established over a range of sheet thickness for any given ductile polymeric material. In Figure 9, it is postulated that, in the near plane strain very small ligament region, w_f is linearly dependent on l but independent of t. This seems to be at odds with eq. (10) which suggests different straight lines of w_f versus l for different t. Clarification on this aspect is also necessary in any future work.

CONCLUSIONS

The specific essential work of fracture (w_e) is proven to be a material property for the tearing of ductile polymers using a grade of high-density polyethylene as an example by conducting simple work of fracture experiments on two different specimen geometries: DENT and DCNT. Two work components physically make up this specific essential fracture work: that to form a neck at the notch tip and that to tear the neck. Crack resistance J_R–Δa curves are also obtained using the J-integral method outlined by Begley and Landes[5] for the same DENT and DCNT geometries. It is shown that $J_c \equiv w_e$ and $\dfrac{dJ}{da} \equiv 4\beta w_p$ for J-controlled crack growth. The ASTM E813-81 and E813-87 procedures cannot be directly applied to determine J_c for fracture propagation in plane stress. Also, the crack-blunting line concept is inapplicable for ductile polymers. It is concluded that in toughness characterization of ductile polymers in *plane stress* fracture it is more convenient to use the essential work of fracture method outlined here.

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