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Design Data for Efficient Axial Gradient Coils: Application to NMR Imaging

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A new design for an axial magnetic field gradient is described. Implemented in a four-coil configuration, it requires far less power than the conventional Maxwell pair, while maintaining the same field linearity. A practical design tool with a set of curves giving coil dimensions is proposed. Two realizations dedicated to NMR imaging are described and compared with the equivalent Maxwell pair. Substantial power reductions are achieved; in these cases, dc power is reduced by a factor of 5 and switching power by a factor of 15.

INTRODUCTION

Linear magnetic field gradients are required in many physics applications: shimming in high-resolution nuclear magnetic resonance spectroscopy (*1*), diffusion and flow measurements by NMR (*2–5*), inhomogeneous field measurements (*6*), and, more recently, spatial encoding in NMR imaging systems (*7*).

The purpose of this paper is to give a practical design tool for axial magnetic field gradients. The topic of transverse gradients will be treated in a subsequent paper. We consider here systems producing a field variation along the field direction. Two kinds of coil systems are often used: one pair of circular (*2, 3*) or polygonal (*8*) coils and sets of quadrupolar coils (*4–6, 9*).

When a field gradient featuring a high linearity over a large volume is required (as in whole-body NMR imaging) these designs do not have enough efficiency: the linearity-to-power ratio is usually very low. We present here a more efficient design using two coil pairs. Savings on switching power can reach a factor of 15 over the equivalent-linearity Maxwell pair.

This paper describes the mathematical basis of the design and provides a fast and simple graphical method for computing coil dimensions. Some implementation guidelines are also given. Finally, two original configurations dedicated to NMR imaging are presented. The latter offers a particularly easy and efficient implementation because the two sets of coils can be wound on a single cylinder, a feature which also optimizes access to the useful volume.

THEORETICAL ANALYSIS OF A z-GRADIENT SYSTEM

The magnetic induction B produced by a current can be computed from the Biot and Savart formula. In an NMR experiment only the field component B_z along the

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static field (z axis) is significant. When considering z-gradient systems, the most efficient configuration is a set of coils with cylindrical symmetry. In this case, the spherical coordinate development of B_z reduces to (*10, 11*)

$$B_z(M) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n B_z(0)}{dz^n} \rho^n P_n(\cos\theta) \qquad [1]$$

(where the coordinates correspond to Fig. 1 and the P_n are the Legendre polynomials).

The first z derivatives $B_n = d^n B_z(0)/dz^n$ are listed in the Appendix.

The simplest z-gradient device is built with a pair of circular loops (Maxwell pair). Opposite currents in the two loops ensure cancellation of all even-order z derivatives. Such a system is fully described by three parameters: R, I, and D (Fig. 1). One parameter, say R, fixes the size access to the central volume. The intensity I determines the gradient strength G. The last parameter is used to cancel B_3 (with $X = D/R = \sqrt{3}/2$). The first term contributing the nonlinearity is B_5.

When a higher linearity of the z gradient is required, higher-order derivatives must be canceled by using a greater number of loops. With n pairs, one has $3n$ independent parameters. Two parameters which we call "scaling parameters" fix the gradient strength G (one current) and the system size (one dimension). The remaining parameters are used to maximize B_1 and cancel odd terms up to B_{6n-3} included. In practice some parameters are often dedicated to geometrical requirements such as access or compactness, and optimization is deliberately limited to a lower

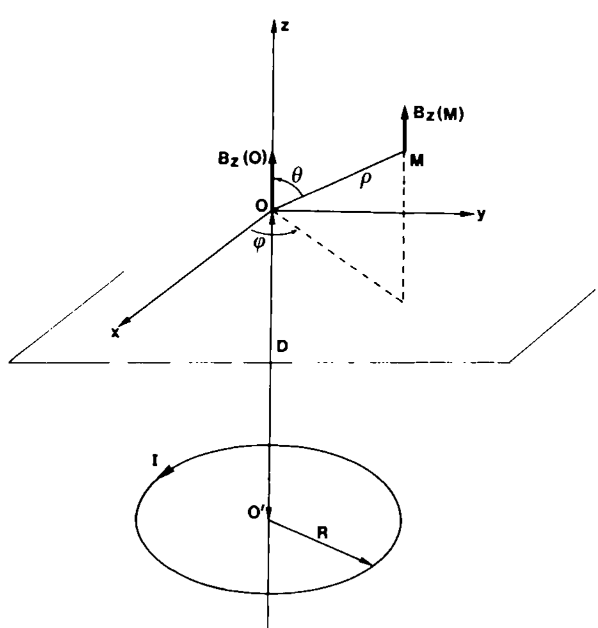

FIG. 1. Description of the coordinate system for computing the B_z component of the induction created at point M by a circular current loop.

order, often B_{6n-5}. In the case of a two-pair system, this means that odd terms up to B_7 included are effectively canceled.

DESIGN PROCEDURE FOR A TWO-PAIR GRADIENT SYSTEM

A two coil-pair design (Fig. 2) is fully described with six parameters. In addition to the scaling parameters, a third arbitrary parameter (often the minimum bore size or coil spacing) is usually needed for access to the central volume. Three parameters remain available for cancellation of the derivatives ΣB_3, ΣB_5, ΣB_7:

$$\begin{aligned}\sum B_3 &= I_1 \cdot B_3(X_1)/R_1^2 + I_2 \cdot B_3(X_2)/R_2^2 = 0\\ \sum B_5 &= I_1 \cdot B_5(X_1)/R_1^4 + I_2 \cdot B_5(X_2)/R_2^4 = 0\\ \sum B_7 &= I_1 \cdot B_7(X_1)/R_1^6 + I_2 \cdot B_7(X_2)/R_2^6 = 0\end{aligned} \qquad [2]$$

with $X_1 = D_1/R_1$ and $X_2 = D_2/R_2$.

To solve this system, we fix I_1, R_1, and X_1. We eliminate the unknowns I_1/I_2 and R_1/R_2 and obtain X_2 as a solution of an eigth-order polynomial equation:

$$F(X_1)(3 - 4X_2^2)(-64X_2^6 + 336X_2^4 - 280X_2^2 + 35) = -8X_2^4 + 20X_2^2 - 5 \qquad [3]$$

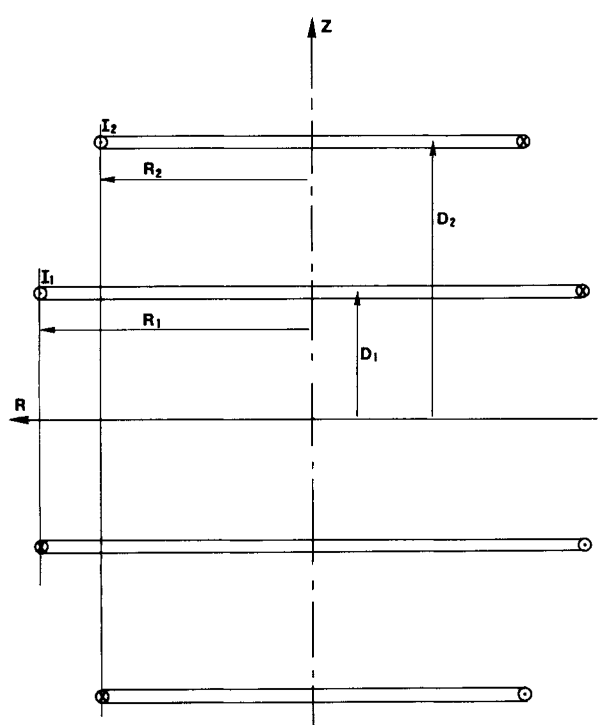

FIG. 2. Parameters of a two-coil pair system. In the case of GRAD500 design $D_1 = 0.16$ m, $R_1 = 0.33$ m, $N_1 = 40$ turns, $D_2 = 0.344$ m, $R_2 = 0.255$ m, $N_2 = 99$ turns. Nominal gradient: $G = 50$ G/m at 5 A.

with

$$F(X_1) = B_5^2(X_1)/[B_3(X_1) \cdot B_7(X_1)] \quad [4]$$

I_1/I_2 and R_1/R_2 are then obtained from Eq. [2].

The four sets of solutions are not equally sound. To achieve the best efficiency, $B_1(X_1)$ and $B_1(X_2)$ must have the same sign: I_1 and I_2 currents flow in same direction. Furthermore, linearity is improved if contributions of the two pairs of coils to nonvanishing high-order derivatives have opposite signs. With these constraints only one X_2 solution is obtained for each X_1 value.

The plot of solutions X_2, I_1/I_2, R_1/R_2 versus X_1 is given in Fig. 3. This set of curves allows a quick design of an axial gradient system: the choice of one (and only one) arbitrary parameter among X_1, X_2, R_1/R_2 and I_1/I_2 followed by a reading of the set of curves determines all the other reduced parameters. The choice of this arbitrary parameter allows the design to be customized to the particular problem considered (e.g., access optimization). When a more accurate solution is needed, a simple polynomial resolution implemented on a pocket calculator is used to solve Eq. [3]. Then with a proper choice of two scaling parameters (one current and one dimension), the real scale system is fully described. This thin-coil solution is either implemented as such or further refined including implementation constraints such as a wire thickness.

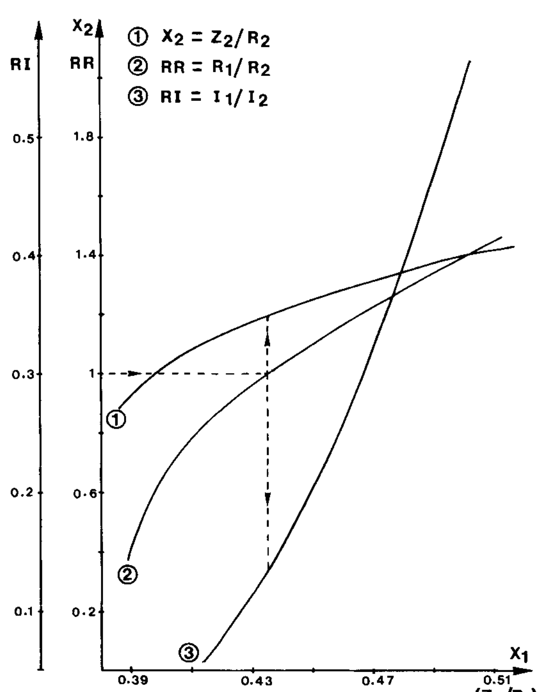

FIG. 3. Multiple-entry graph for a two-coil pair design. Choosing one arbitrary parameter among $X_1 = D_1/R_1 = Z_1/R_1$, $X_2 = D_2/R_2 = Z_2/R_2$, RR and RI provides a unique solution (e.g., GRAD600 design with $RR = 1$).

For the latter case, we have written a full optimization program (*12*) similar to the nonlinear program previously described (*13*). Running on a microcomputer, it takes into account finite-section coils and solves the system of Eq. [2].

In the former case, the solution remains valid only if the realization does not depart too much from the thin-coil approximation: the ratio r of the thickness of windings to the system size should remain small. Note that (i) if the symmetry of the system is maintained upon implementation, a finite r introduces no even impurities into the gradient; (ii) a finite r does not suppress the existence of a unique solution to system [2]. It only slightly changes the position of the optimum solution.

Mechanical or electrical tuning of the system (fine adjustments of spacings or current ratios) is possible and will partially compensate this error allowing better cancellation of odd impurities.

To minimize errors due to winding thickness or to properly take it into account, care should be taken that the winding matches closely the optimization situation: current leads and returns should be matched as best as possible; and windings should be realized circular with steps, rather than helicoidal (see Fig. 4), thus maintaining full cylindrical symmetry.

TWO GRADIENT SYSTEMS FOR NMR IMAGING

Our first system (GRAD500) was designed to be fitted into a 1000-G Walker Scientific air-core magnet (*14*). We specified a 0.5-m bore access to the patient, therefore we fixed $R_2 = 0.255$ m. With a cylinder length of 0.7 m, $X_2 = D_2/R_2 = 1.35$. The reading of Fig. 3 gives $X_1 = 0.483$, $R_1/R_2 = 1.29$ $I_1/I_2 = 0.404$ which leads to the parameters of Fig. 2. The linearity is about 0.6% over a 0.4 m diameter sphere in the central region, which is substantially better than the 5% obtained with a single pair of same size.

The system dissipates about 80 W at $G = 50$ G/m. This set of coils is currently used in our head NMR imaging system producing steady state free precession (SSFP) and 2D FT images (*15*).

We are now building a new gradient system (GRAD600). Three requirements specify this new design:

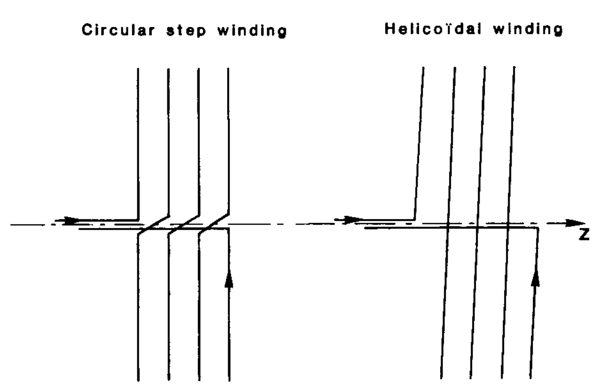

FIG. 4. Cylindrical symmetry and absence of return loop are achieved with circular step winding.

(i) To utilize the larger bore access (0.75 m) of our new 1000-G Drusch magnet (*16*), we want a minimum access diameter of 0.6 m.

(ii) To simplify assembly, the whole gradient system, including X and Y transverse gradients, is wound on a single cylinder, resulting in the condition $R_1 = R_2$.

(iii) The nonlinearity of the gradient must not spoil the main magnet homogeneity (50 mG over a 0.4-m diameter spherical volume). Therefore, with a gradient strength $G = 50$ G/m, the required linearity is about 0.5% over the volume.

When a two-coil-pair design is used, requirement (iii) allows the cylinder radius to be 0.32 m as per (i). Moreover requirement (ii) fixes $R_1/R_2 = 1.$ and the unscaled solution is obtained from Fig. 3: $X_1 = 0.433$, $X_2 = 1.19$, $I_1/I_2 = 0.133$. The GRAD600 actual parameters are given in Fig. 5.

The complete field simulation (*13*) performed at a computer center gives us the error map (real field–perfectly linear field) of Fig. 6. This map is drawn for a gradient strength $G = 50$ G/m and demonstrates an nonlinearity of about 20 mG (0.2%) in a 0.4-m-diameter sphere, which is twice less than the main magnet inhomogeneity.

With the same specifications, a single pair would have a 0.6-m radius (here requirement (iii) is critical), and the parameters would be $R = 0.6$ m, $D = 0.5196$ m, $N = 17$ turns, and $I = 130$ A for $G = 50$ G/m. The dc power would then be five times larger.

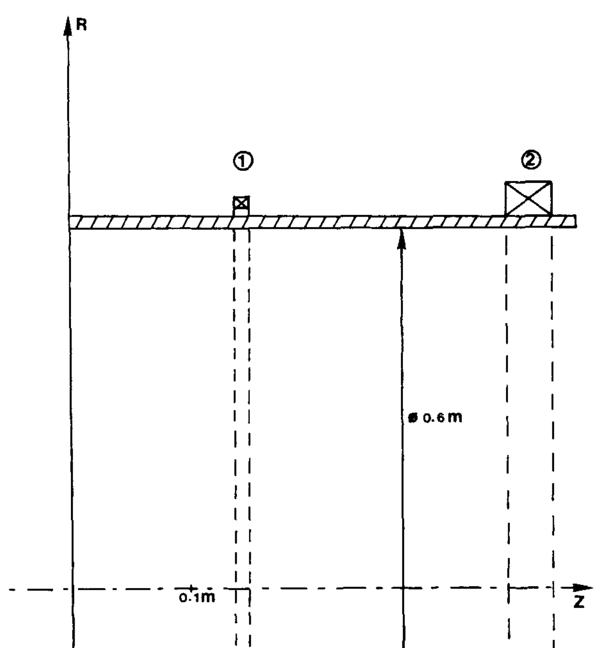

FIG. 5. GRAD600 design: $D_1 = 0.139$ m, $R_1 = 0.32$ m, $N_1 = 2$ turns, $D_2 = 0.38$ m, $R_2 = 0.32$ m, $N_2 = 15$ turns. Nominal gradient: $G = 50$ G/m at 53.4 A ($P \sim 350$ W).

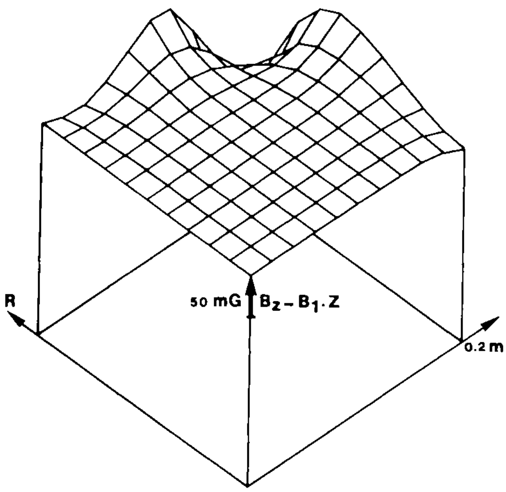

FIG. 6. Three-dimensional representation of the theoretical GRAD600 error map over a 200-mm radius sphere. Because of planar-cylindrical antisymmetry of the gradient device, only one-fourth of the map is represented. Gradient strength is $G = 50$ G/m, therefore a 0.5% nonlinearity corresponds to a 50-mG error.

When dynamic gradients are to be rapidly established (*17, 18*), the switching power P_s becomes the critical issue. This quantity is defined as the switching energy divided by the switching time T. A straightforward derivation shows that, within a given geometry

$$P_s = kR^2D^3G^2/T \qquad [5]$$

where k is a geometrical factor. Both this formula and the numerical simulation show that the two-coil-pair system needs 15 to 20 times less power than the equivalent Maxwell pair.

DISCUSSION

When a high-performance gradient system is required, simple Maxwell pairs are not convenient. A two-coil-pair design increases the overall efficiency: the same field linearity is obtained with smaller coils, hence with less electrical power. This is even more true when switching power, not dc power, is considered.

To build an optimum z-gradient device in regard with design requirements, one can follow the sequence: (1) one thin section pair, (2) several thin section pairs, (3) several pairs with thick winding optimization, and finally (4) optimization with distributed windings. This further reduces the switching power and voltage at a slight expense on dc efficiency.

All these steps are not always necessary to meet the required specifications. As one proceeds along this sequence, mathematical resolutions give way to numerical optimizations which involve more computing work and a more realistic simulation of the projected system.

APPENDIX

The z component of the field produced by a current loop (Fig. 1) at the point 0 is

$$B_z(0) = \frac{\mu_0 I}{R(1 + X^2)^{3/2}}$$

where $X = D/R$ and $\mu_0 = 4\pi\ 10^{-7}$ H/m

The first odd z derivatives are listed below:

$$B_1 = \frac{-3\mu_0 IX}{R^2(1 + X^2)^{5/2}}$$

$$B_3 = \frac{15\mu_0 IX(3 - 4X^2)}{R^4(1 + X^2)^{7/2}}$$

$$B_5 = \frac{315\mu_0 IX(-8X^4 + 20X^2 - 5)}{R^6(1 + X^2)^{9/2}}$$

$$B_7 = \frac{2835\mu_0 IX(-64X^6 + 336X^4 - 280X^2 + 35)}{R^8(1 + X^2)^{11/2}}.$$

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REFERENCES

1. W. A. Anderson, *Rev. Sci. Instrum.* **32,** 241 (1961).
2. H. Y. Carr and E. M. Purcell, *Phys. Rev.* **94,** 630 (1954).
3. J. E. Tanner, *Rev. Sci. Instrum.* **36,** 1086 (1965).
4. G. Ödberg and L. Ödberg, *J. Magn. Reson.* **16,** 342 (1974).
5. I. Zupancic and J. Pirs, *J. Phys. E* **9,** 79 (1976).
6. H. Breuer, *Rev. Sci. Instrum.* **36,** 1666 (1965).
7. P. C. Lauterbur, *Nature (London)* **242,** 190 (1973).
8. J. S. Blicharski and W. T. Sobol, *J. Magn. Reson.* **46,** 1 (1982).
9. R. S. Parker, I. Zupancic, and J. Pirs, *J. Phys. E* **6,** 899 (1973).
10. M. W. Garrett, *J. Appl. Phys.* **22,** 1091 (1951).
11. F. Romeo and D. I. Hoult, *Magn. Reson. Med.* **1,** 44 (1984).
12. Program "ZGRAD," Information available on request from H. Saint-Jalmes.
13. H. Saint-Jalmes, J. Taquin, and Y. Barjhoux, *Rev. Sci. Instrum.* **52,** 1501 (1981).
14. Walker Scientific Inc. USA, Plasmaflux system M4074, 1980.
15. H. Saint-Jalmes, J. Taquin, and Y. Barjhoux, Techniques for spatially localised NMR, University of Surrey, Guildford, U.K., 19–21 Sept. 1984.
16. Societe Drusch, France, Descriptif 821.014, 1982.
17. P. Mansfield and I. L. Pykett, *J. Magn. Reson.* **29,** 355 (1978).
18. L. F. Feiner and P. R. Locher, *Appl. Phys.* **22,** 257 (1980).