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Delay-dependent robust control for singular discrete-time Markovian jump systems with time-varying delay

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SUMMARY

The problem of delay-dependent robust stabilization for uncertain singular discrete-time systems with Markovian jumping parameters and time-varying delay is investigated. In terms of free-weighting-matrix approach and linear matrix inequalities, a delay-dependent condition is presented to ensure a singular discrete-time system to be regular, causal and stochastically stable based on which the stability analysis and robust stabilization problem are studied. An explicit expression for the desired state-feedback controller is also given. Some numerical examples are provided to demonstrate the effectiveness of the proposed approach. Copyright © 2009 John Wiley & Sons, Ltd.

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1. INTRODUCTION

During the last decade, there has been some remarkable theoretical and practical progress in stability, stabilization, and robust control of linear continuous-time systems [1–4]. For discrete-time systems with a time-delay, the existing results can be classified into two types: delay-independent and delay-dependent systems. Delay-dependent criteria take into account the maximum length of time-delay that the system can tolerate and thus be less conservative than delay-independent ones. A great number of results on this topic have been reported in the literatures [5–8].

Singular systems have received much attention since singular model can preserve the structure of practical systems and describe a large class of physical systems better than regular ones. Singular systems are also referred to as generalized systems, descriptor system, implicit systems, differential-algebraic systems or semi-state systems [9]. It should be pointed out that when the robust stability problem for uncertain singular time-delay systems is studied, it is required to consider not only stability robustness problem but robustness of regularity, absence of impulses (for continuous systems) and causality (for discrete systems) [10–12], while the latter two problems

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need not to be considered for state-space time-delay systems.

It is well known that stochastic modeling has come to play an important role in many branches of science and industry. An area of particular interest is Markovian jump systems, which can model stochastic systems with abrupt structural variation resulting from the occurrence of some inner discrete events in the system such as failures and repairs of machine in manufacturing systems, modifications of the operating point of a linearized model of a nonlinear system. A great number of fundamental results on Markovian jump systems in both the discrete and continuous contexts have been reported in the literature, see, e.g. [13–15] and references therein. Xu and Lam have investigated the problem of robust H_∞ control of singular discrete-time Markovian jump systems, where necessary and sufficient conditions for stochastic admissibility are obtained in terms of linear matrix inequalities (LMIs) [16, 17]. Zou and Wang *et al.*, have considered the reconstructability and detectability of the jump modes of discrete singular systems [18]. Ma and Liu *et al.* have discussed robust stochastic stability and stabilization of time-delay discrete Markovian jump singular systems with parameter uncertainties [19]. However, little effort has been devoted to studying the robust stabilization for the uncertain singular discrete-time systems with Markovian jumping parameters and time-varying delay.

In this paper, we investigate the delay-dependent robust stabilization problem for uncertain singular discrete-time Markovian jump system with time-varying delay. First, in terms of free-weighting-matrix approach, we present a delay-dependent criterion that provides a sufficient condition for an unforced nominal singular discrete-time system to be regular, causal and stochastically stable. Based on the result, the robust stabilization problem is studied and a state-feedback controller is constructed such that the resultant closed-loop system is regular, causal and stochastically stable. All the results presented in this paper do not require the decomposition of system matrices. The desired state-feedback controller can be obtained by solving the feasibility problem of strict LMIs, which is very simple by using interior-point algorithm.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geqslant Y$ (respectively, $X > Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. N is the set of natural numbers. The superscript 'T' represents the transpose. $\mathscr{E}\{.\}$ denotes the expectation operator with respect to some probability measure. $\mathscr{P}|\bullet|$ is the Euclidean vector norm while $\|\bullet\|$ stands for the spectral norm of a matrix. For a symmetric matrix, * denotes the matrix entries implied by symmetry.

2. PROBLEM FORMULATION

Consider a class of singular discrete-time Markovian jump systems with state delay described by

$$\begin{aligned} Ex(k+1) &= (A(r_k)+\Delta A(k,r_k))x(k)+(A_d(r_k)\\ &\quad +\Delta A_d(k,r_k))x(k-d(k))\\ &\quad +(B(r_k)+\Delta B(k,r_k))u(k)\\ x(k) &= \varphi(k), \quad -d_2\leqslant k\leqslant 0, \end{aligned} \tag{1}$$

where $x(k)\in R^n$, $u(k)\in R^m$, are the system state, control input, respectively. $d(k)$ is a time-varying delay satisfying

$$d_1\leqslant d(k)\leqslant d_2 \tag{2}$$

where d_1 and d_2 are known positive integers. $\varphi(k)$ is a compatible vector-valued initial function. The matrix $E\in R^{n\times n}$ may be singular, and we shall assume that $\operatorname{rank}(E)=r\leqslant n$. The parameter r_k represents a discrete-time, discrete-state Markov chain that takes values in a finite set $S=\{1,2,\ldots,s\}$ with transition probabilities

$$\Pr\{r_{k+1}=j\,|\,r_k=i\}=\pi_{ij} \tag{3}$$

where $\pi_{ij}\geqslant 0$, and for any $i\in S$,

$$\sum_{j=1}^{s}\pi_{ij}=1 \tag{4}$$

$A(r_k), A_d(r_k)$ and $B(r_k)$ are known real constant matrices with appropriate dimensions. For notational simplicity, in the sequel, for each possible $r_k=i, i\in S$, the matrix $A(r_k)$ will be denoted by A_i. It is assumed that the uncertainties are norm bounded and, for each

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$i \in S$, can be described as

$$[\Delta A(k, r(k)) \;\; \Delta A(k, r(k)) \;\; \Delta B(k, r(k))] \\ = M_i F_i(k)[N_{Ai} \;\; N_{Adi} \;\; N_{Bi}] \tag{5}$$

where M_i, N_{Ai}, N_{Adi} are known constant matrices with compatible dimensions for each $i \in S$, and $F_i(k)$ are unknown matrix functions satisfying

$$F_i^{\mathrm{T}}(k) F_i(k) \leqslant I \quad \forall i \in S \tag{6}$$

The nominal singular discrete-time Markovian jump system and time-varying delay system of system (1) are as follows:

$$\begin{aligned} Ex(k+1) &= A(r_k)x(k) + A_d(r_k)x(k-d(k)) \\ x(k) &= \varphi(k), \quad -d_2 \leqslant k \leqslant 0 \end{aligned} \tag{7}$$

where the variables follow the same definitions as those in (1).

For the system (7), we introduce the following definitions and lemmas, which will be used in the proof of our main results.

Definition 1

(i) For given scalars d_1, d_2 with $d_2 \geqslant d_1 > 0$, the system (7) is said to be regular if, for each $i \in S$, $\det(zE - A_i)$ and $\det(zE - (A_i + A_{di}))$ are not identically zero.
(ii) For given scalars d_1, d_2 with $d_2 \geqslant d_1 > 0$, the system (7) is said to be causal if, for each $i \in S$, $\deg(\det(zE - A_i)) = \mathrm{rank}(E)$ and $\deg(\det(zE - (A_i + A_{di}))) = \mathrm{rank}(E)$.
(iii) For given scalars d_1, d_2 with $d_2 \geqslant d_1 > 0$, the system (7) is stochastically stable if for any $x_0 \in R^n$ and $r_0 \in S$, there exists a scalar $\widetilde{M}(x_0, r_0) > 0$ such that

$$\lim_{v \to \infty} \mathscr{E}\left\{ \sum_{k=0}^{v} |x(k)|^2 | x_0, r_0 \right\} \leqslant \widetilde{M}(x_0, r_0) \tag{8}$$

where $x(k, x_0, r_0)$ denotes the solution to system (7) at time k under the initial conditions x_0 and r_0.
(iv) The system (7) is said to be stochastically admissible if it is regular, causal and stochastically stable.

We are interested in designing a state-feedback controller,

$$u(k) = K(r_k)x(k), \quad K(r_k) \in R^{m \times n} \tag{9}$$

such that the closed-loop system of (1) is regular, causal and stochastically stable.

The following lemmas are essential for the proofs in the subsequent sections.

Lemma 1 (*Xie and Souza* [*20*])
Given matrices Ω, Γ and Ξ with appropriate dimensions and with Ω symmetrical, then

$$\Omega + \Gamma F \Xi + \Xi^{\mathrm{T}} F^{\mathrm{T}} \Gamma^{\mathrm{T}} < 0$$

for any F satisfying $F^{\mathrm{T}} F \leqslant I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Omega + \varepsilon \Gamma \Gamma^{\mathrm{T}} + \varepsilon^{-1} \Xi^{\mathrm{T}} \Xi < 0$$

Lemma 2 (*Xie and Souza* [*20*])
For symmetric positive-definite matrix Q and matrices P, R with appropriate dimensions, the following inequality holds:

$$PR^{\mathrm{T}} + RP^{\mathrm{T}} \leqslant RQR^{\mathrm{T}} + PQ^{-1}P^{\mathrm{T}} \tag{10}$$

3. MAIN RESULTS

First, in terms of free-weighting-matrix approach and LMI, we provide a sufficient condition under which the nominal system (7) is regular, causal and stochastically stable, which will play a key role in solving the below mentioned problems.

Theorem 1
For given scalars d_1, d_2 with $d_2 \geqslant d_1 > 0$, the nominal system (7) is stochastically admissible if there exist positive-definite symmetric matrices Q, Z, and matrices

$$X_i = \begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \geqslant 0, \quad N_{1i}, N_{2i}$$

$$P_i = P_i^{\mathrm{T}}, \quad i = 1, 2, \ldots, s$$

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such that

$$E^{\mathrm{T}} P_i E \geqslant 0 \quad \text{(11a)}$$

$$\Xi_i = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & d_2(A_i - E)^{\mathrm{T}} Z \\ * & \Xi_{22i} & d_2 A_{di}^{\mathrm{T}} Z \\ * & * & -d_2 Z \end{bmatrix} < 0 \quad \text{(11b)}$$

$$\Psi_i = \begin{bmatrix} X_{11i} & X_{12i} & N_{1i} \\ * & X_{22i} & N_{2i} \\ * & * & Z \end{bmatrix} \geqslant 0 \quad \text{(11c)}$$

where

$$\Xi_{11i} = (d_2 - d_1 + 1) Q + A_i^{\mathrm{T}} \widetilde{P}_i A_i - E^{\mathrm{T}} P_i E + N_{1i} E + E^{\mathrm{T}} N_{1i}^{\mathrm{T}} + d_2 X_{11i}$$

$$\Xi_{12i} = A_i^{\mathrm{T}} \widetilde{P}_i A_{di} + E^{\mathrm{T}} N_{2i}^{\mathrm{T}} - N_{1i} E + d_2 X_{12i}$$

$$\Xi_{22i} = A_{di}^{\mathrm{T}} \widetilde{P}_i A_{di} - Q - N_{2i} E - E^{\mathrm{T}} N_{2i}^{\mathrm{T}} + d_2 X_{22i}$$

$$\widetilde{P}_i = \sum_{j=1}^{s} \pi_{ij} P_j$$

Proof

Under the condition of the theorem, we first prove the stochastic stability of the nominal system (7). To this end, for each $r_k = i, i \in S$, we choose a stochastic Lyapunov function candidate to be

$$\begin{aligned} V(x(k), r_k) &= V_1(x(k), r_k) + V_2(x(k), r_k) + V_3(x(k), r_k) + V_4(x(k), r_k) \\ V_1(x(k), r_k) &= x^{\mathrm{T}}(k) E^{\mathrm{T}} P_i E x(k) \\ V_2(x(k), r_k) &= \sum_{\theta=-d_2+1}^{0} \sum_{l=k-1+\theta}^{k-1} y^{\mathrm{T}}(l) E^{\mathrm{T}} Z E y(l) \\ V_3(x(k), r_k) &= \sum_{l=k-d(k)}^{k-1} x^{\mathrm{T}}(l) Q x(l) \\ V_4(x(k), r_k) &= \sum_{\theta=-d_2+2}^{d_1+1} \sum_{l=k-1+\theta}^{k-1} x^{\mathrm{T}}(l) Q x(l) \end{aligned} \quad (12)$$

where $y(k) = x(k+1) - x(k)$. Now, define

$$\Delta V(x(k), r_k) = \mathscr{E}\{V(x(k+1), r_{k+1} | x(k), r_k = i\} - V(x(k), r_k = i)$$

yields

$$\begin{aligned} \Delta V_1(x(k), r_k) &= x^{\mathrm{T}}(k+1) E^{\mathrm{T}} \left(\sum_{j=1}^{s} \pi_{ij} P_j \right) E x(k+1) - x^{\mathrm{T}}(k) E^{\mathrm{T}} P_i E x(k) \\ &= (A_i x(k) + A_{di} x(k - d(k)))^{\mathrm{T}} \times \left(\sum_{j=1}^{s} \pi_{ij} P_j \right) (A_i x(k) x + A_{di}(k - d(k))) \\ &\quad - x^{\mathrm{T}}(k) E^{\mathrm{T}} P_i E x(k) \end{aligned} \quad (13)$$

$$\begin{aligned} \Delta V_2(x(k), r_k) &= d_2 y^{\mathrm{T}} E^{\mathrm{T}} Z E y^{\mathrm{T}} - \sum_{l=k-d_2}^{k-1} y^{\mathrm{T}}(l) E^{\mathrm{T}} Z E y(l) \\ &\leqslant d_2 y^{\mathrm{T}} E^{\mathrm{T}} Z E y - \sum_{l=k-d(k)}^{k-1} y^{\mathrm{T}}(l) E^{\mathrm{T}} Z E y(l) \\ &= d_2 (E x(k+1) - E x(k))^{\mathrm{T}} \times Z (E x(k+1) - E(x)) - \sum_{l=k-d(k)}^{k-1} y^{\mathrm{T}}(l) E^{\mathrm{T}} Z E y(l) \\ &= d_2 ((A_i - E) x(k) + A_{di} x(k - d(k)))^{\mathrm{T}} \times Z ((A_i - E) x(k) + A_{di} x(k - d(k))) \\ &\quad - \sum_{l=k-d(k)}^{k-1} y^{\mathrm{T}}(l) E^{\mathrm{T}} Z E y(l) \end{aligned} \quad (14)$$

$$\Delta V_3(x(k), r_k) = \sum_{l=k+1-d(k+1)}^{k} x^{\mathrm{T}}(l) Q x(l) - \sum_{l=k-d(k)}^{k-1} x^{\mathrm{T}}(l) Q x(l)$$

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$$
\begin{aligned}
&= x^{\mathrm{T}}(k)Qx(k) + \sum_{l=k+1-d(k+1)}^{k-1} \\
&\quad \times x^{\mathrm{T}}(l)Qx(l) - x^{\mathrm{T}}(k-d(k))Qx(k-d(k)) \\
&\quad - \sum_{l=k+1-d(k)}^{k-1} x^{\mathrm{T}}(l)Qx(l) \\
&= x^{\mathrm{T}}(k)Qx(k) + \sum_{l=k+1-d(k)}^{k-1} x^{\mathrm{T}}(l)Qx(l) \\
&\quad - x^{\mathrm{T}}(k-d(k))Qx(k-d(k)) \\
&\quad - \left(\sum_{l=k+1-d_1}^{k-1} x^{\mathrm{T}}(l)Qx(l) \right. \\
&\quad \left. + \sum_{l=k+1-d(k+1)}^{k-d_1} x^{\mathrm{T}}(l)Qx(l) \right) \qquad (15)
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\sum_{l=k+1-d_1}^{k-1} x^{\mathrm{T}}(l)Qx(l) + \sum_{l=k+1-d(k+1)}^{k-d_1} x^{\mathrm{T}}(l)Qx(l) \\
&\leqslant \sum_{l=k+1-d(k)}^{k-1} x^{\mathrm{T}}(l)Qx(l) + \sum_{l=k+1-d_2}^{k-d_1} x^{\mathrm{T}}(l)Qx(l)
\end{aligned}
$$

we have

$$
\begin{aligned}
\Delta V_3(x(k), r_k) &\leqslant x^{\mathrm{T}}(k)Qx(k) - x^{\mathrm{T}}(k-d(k))Qx(k-d(k)) \\
&\quad + \sum_{l=k+1-d_2}^{k-d_1} x^{\mathrm{T}}(l)Qx(l)) \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\Delta V_4(x(k), r_k) &= (d_2 - d_1)x^{\mathrm{T}}(k)Qx(k) \\
&\quad - \sum_{l=k+1-d_2}^{k-d_1} x^{\mathrm{T}}(l)Qx(l)) \qquad (17)
\end{aligned}
$$

From (14)–(18), we can get

$$
\begin{aligned}
&\Delta V(x(k), r_k) \\
&\quad = (A_i x(k) + A_{di} x(k-d(k)))^{\mathrm{T}} \\
&\quad \times \left(\sum_{j=1}^{s} \pi_{ij} P_j \right) (A_i x(k) + A_{di} x(k-d(k))) \\
&\quad - x^{\mathrm{T}}(k) E^{\mathrm{T}} P_i E x(k) \\
&\quad + d_2((A_i - E)x(k) + A_{di} x(k-d(k)))^{\mathrm{T}} \\
&\quad \times Z((A_i - E)x(k) + A_{di} x(k-d(k))) \\
&\quad - \sum_{l=k-d(k)}^{k-1} y^{\mathrm{T}}(l) E^{\mathrm{T}} Z E y(l) - x^{\mathrm{T}}(k-d(k)) \\
&\quad \times Qx(k-d(k)) + (d_2 - d_1 + 1)x^{\mathrm{T}}(k)Qx(k) \qquad (18)
\end{aligned}
$$

For each $i \in S$ and any appropriately dimensioned matrices N_{1i}, N_{2i},

$$
X_i = \begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \geqslant 0
$$

the following equations are true:

$$
2x^{\mathrm{T}}(k)N_{1i} \times \left[Ex(k) - Ex(k-d(k)) - \sum_{l=k-d(k)}^{k-1} Ey(l) \right] = 0 \qquad (19)
$$

$$
2x^{\mathrm{T}}(k-d(k))N_{2i} \times \left[Ex(k) - Ex(k-d(k)) - \sum_{l=k-d(k)}^{k-1} Ey(l) \right] = 0 \qquad (20)
$$

$$
\begin{aligned}
&\sum_{l=k-d_2}^{k-1} \xi_1^{\mathrm{T}}(k) X_i \xi(k) - \sum_{l=k-d(k)}^{k-1} \xi_1^{\mathrm{T}}(k) X_i \xi(k) \\
&\quad = d_2 \xi_1^{\mathrm{T}}(k) X_i \xi(k) - \sum_{l=k-d(k)}^{k-1} \xi_1^{\mathrm{T}}(k) X_i \xi_1(k) \geqslant 0 \qquad (21)
\end{aligned}
$$

where $\xi_1(k) = [x^{\mathrm{T}}(k) \ \ x^{\mathrm{T}}(k-d(k))]^{\mathrm{T}}$. Adding the terms on the right sides of Equations (19)–(21) to (18), we have

$$
\begin{aligned}
\Delta V(x(k), r_k) &\leqslant \xi_1(k) \Phi_i \xi_1(k) \\
&\quad - \sum_{l=k-d(k)}^{k-1} \xi_2^{\mathrm{T}}(k,l) \Psi_i \xi_2(k,l) \qquad (22)
\end{aligned}
$$

where Ψ_i is defined in (11c),

$$\xi_2(k,l)=[x^{\mathrm{T}}(k)\ \ x^{\mathrm{T}}(k-d(k))\ \ y^{\mathrm{T}}(k)E^{\mathrm{T}}]^{\mathrm{T}}$$

$$\Phi_i=\begin{bmatrix}\Phi_{11i} & \Phi_{12i}\\ * & \Phi_{22i}\end{bmatrix}$$

with

$$\Phi_{11i}=(d_2-d_1+1)Q+A_i^{\mathrm{T}}\widetilde{P}_iA_i-E^{\mathrm{T}}P_iE+d_2(A_i-E)^{\mathrm{T}}\times Z(A_i-E)+N_{1i}E+E^{\mathrm{T}}N_{1i}^{\mathrm{T}}+d_2X_{11i}$$

$$\Phi_{12i}=A_i^{\mathrm{T}}\widetilde{P}_iA_{di}+d_2A_i^{\mathrm{T}}ZA_{di}+d_2(A_i-E)^{\mathrm{T}}ZA_{di}+E^{\mathrm{T}}N_{2i}^{\mathrm{T}}-N_{1i}E+d_2X_{12i}$$

$$\Phi_{22i}=-Q-N_{2i}E-E^{\mathrm{T}}N_{2i}^{\mathrm{T}}+d_2X_{22i}+A_{di}^{\mathrm{T}}\widetilde{P}_iA_{di}+A_{di}^{\mathrm{T}}ZA_{di}$$

By the Schur complement, we can show that (11b) guarantees

$$\Phi_i<0 \tag{23}$$

By (23) and (11c) we have

$$\Delta V(x(k),r_k)<0 \tag{24}$$

which implies that there exists a scalar $\alpha>0$ such that

$$\mathscr{E}\{V(x(k+1),r_{k+1}|x(k),r_k=i\}-V(x(k),r_k=i)<-\alpha|x(k)|^2 \tag{25}$$

Then, following the same lines as in the proof of Theorem 2.1 in [21], we can deduce that there exists a scalar $\delta>0$ such that

$$\lim_{v\to\infty}\mathscr{E}\left\{\sum_{k=0}^{v}|x(k)|^2|x_0,r_0\right\}\leqslant\delta V(x_0,r_0)$$

Then, by Definition 1, we deduce that the system (7) is stochastically stable.

Next, we will show the system (7) is regular and causal.

From (23), we have

$$\Phi_i=\begin{bmatrix}\Phi_{11i} & \Phi_{12i}\\ * & \Phi_{22i}\end{bmatrix}<0 \tag{26}$$

By the Schur complement and Lemma 2, it is easy to show that

$$\begin{aligned}0>\Phi_{11i}&=(d_2-d_1+1)Q+A_i^{\mathrm{T}}\widetilde{P}_iA_i-E^{\mathrm{T}}P_iE\\&\quad+d_2(A_i-E)^{\mathrm{T}}Z(A_i-E)+N_{1i}E\\&\quad+E^{\mathrm{T}}N_{1i}^{\mathrm{T}}+d_2X_{11i}\\&\geqslant A_i^{\mathrm{T}}\widetilde{P}_iA_i-E^{\mathrm{T}}P_iE+N_{1i}E+E^{\mathrm{T}}N_{1i}^{\mathrm{T}}\end{aligned} \tag{27}$$

$$\begin{aligned}0&>\Phi_{11i}+\Phi_{12i}(-\Phi_{22i})^{-1}\Phi_{12i}^{\mathrm{T}}\\&\geqslant\Phi_{11i}+\Phi_{12i}+\Phi_{12i}^{\mathrm{T}}+\Phi_{22i}\\&\geqslant(A_i+A_{di})^{\mathrm{T}}\widetilde{P}_i(A_i+A_{di})-E^{\mathrm{T}}P_iE\end{aligned} \tag{28}$$

We choose two nonsingular matrices $\hat{G}$ and $\hat{H}$ such that

$$\hat{G}E\hat{H}=\begin{bmatrix}I & 0\\ 0 & 0\end{bmatrix} \tag{29}$$

and write

$$\hat{G}A_i\hat{H}=\begin{bmatrix}\hat{A}_{1i} & \hat{A}_{2i}\\ \hat{A}_{3i} & \hat{A}_{4i}\end{bmatrix},\quad \hat{G}A_{di}\hat{H}=\begin{bmatrix}\hat{A}_{d1i} & \hat{A}_{d2i}\\ \hat{A}_{d3i} & \hat{A}_{d4i}\end{bmatrix}$$

$$\hat{G}^{-\mathrm{T}}P_i\hat{G}^{-1}=\begin{bmatrix}\hat{P}_{1ii} & \hat{P}_{2i}\\ \hat{P}_{3i} & \hat{P}_{4i}\end{bmatrix}$$

$$\begin{aligned}\hat{H}^{\mathrm{T}}N_{1i}\hat{G}^{-1}&=\begin{bmatrix}\hat{N}_{11i} & \hat{N}_{12i}\\ \hat{N}_{13i} & \hat{N}_{14i}\end{bmatrix}\\ \hat{H}^{\mathrm{T}}N_{2i}\hat{G}^{-1}&=\begin{bmatrix}\hat{N}_{21i} & \hat{N}_{22i}\\ \hat{N}_{23i} & \hat{N}_{24i}\end{bmatrix}\end{aligned} \tag{30}$$

for $i=1,2,\ldots,s$, where the partitions of $\hat{G}A_i\hat{H}$, $\hat{G}A_{di}\hat{H}$, $\hat{G}^{-T}P_i\hat{G}^{-1}$, $\hat{H}^{\mathrm{T}}N_{1i}\hat{G}^{-1}$, $\hat{H}^{\mathrm{T}}N_{2i}\hat{G}^{-1}$ are compatible with that of $\hat{G}E\hat{H}$ in (29).

Pre- and post-multiplying (12a), (27) and (28) by $\hat{H}^{\mathrm{T}}$ and $\hat{H}$, respectively, we have

$$\hat{P}_{1i}\geqslant 0 \tag{31}$$

$$\begin{bmatrix}\times & \times\\ \times & W_{1i}\end{bmatrix}<0 \tag{32}$$

$$\begin{bmatrix} \times & \times \\ \times & W_{2i} \end{bmatrix} < 0 \tag{33}$$

where $\times$ represents a matrix which will not be used in the following discussion, and

$$W_{1i} = A_{2i}^{\mathrm{T}}\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{1j}\right)A_{2i}+A_{4i}^{\mathrm{T}}\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{3j}\right)A_{2i}$$

$$+A_{2i}^{\mathrm{T}}\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{2j}\right)A_{4i}+A_{4i}^{\mathrm{T}}\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{4j}\right)A_{4i} \tag{34}$$

$$W_{2i} = (A_{2i}^{\mathrm{T}}+A_{d2i}^{\mathrm{T}})\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{1j}\right)(A_{2i}+A_{d2i})$$

$$+(A_{4i}^{\mathrm{T}}+A_{d4i}^{\mathrm{T}})\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{3j}\right)(A_{2i}+A_{d2i})$$

$$+(A_{2i}^{\mathrm{T}}+A_{d2i}^{\mathrm{T}})\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{2j}\right)(A_{4i}+A_{d4i})$$

$$+(A_{4i}^{\mathrm{T}}+A_{d4i}^{\mathrm{T}})\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{4j}\right)(A_{4i}+A_{d2i}) \tag{35}$$

From (32) and (33), it is easy to see

$$W_{1i} < 0 \tag{36}$$

$$W_{2i} < 0 \tag{37}$$

Obviously, (36) together with (37) implies that A_{4i} and $(A_{4i}+A_{d4i})$ are nonsingular for each $i \in s$. Suppose, by contradiction that A_{4i} and $(A_{4i}+A_{d4i})$ are singular for $i=1,2,\ldots,s$. Then, there exist two vectors ξ_{1i} and ξ_{2i} such that $A_{4i}\xi_{1i}=0$ and $(A_{4i}+A_{d4i})\xi_{2i}=0$. Hence, for $i=1,2,\ldots,s$, we have

$$\xi_i^{\mathrm{T}}A_{2i}^{\mathrm{T}}\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{1j}\right)A_{2i}\xi_i<0 \tag{38}$$

and

$$\xi_{2i}^{\mathrm{T}}(A_{2i}^{\mathrm{T}}+A_{d2i}^{\mathrm{T}})\left(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{1j}\right)(A_{2i}+A_{d2i})\xi_{2i}<0 \tag{39}$$

On the other hand, for $i=1,2,\ldots,s$, from (31) we have $(\sum_{j=1}^{s}\pi_{ij}\hat{P}_{1j})\geqslant 0$, which is a contradiction. Therefore, by Definition 1, it is easy to show that the system (7) is regular and causal.

This conclusion together with the conclusion that the system (7) is stochastically stable implies that the system (7) is stochastically admissible. This completes the proof. □

Remark 1

It is obvious that the free matrices N_{1i} and N_{2i} in (19) and (20) express the relationship among $Ex(k)$, $Ex(k-d(k))$ and $\sum_{l=k-d(k)}^{k-1}Ey(l)$. These free matrices can be easily determined by solving LMI (11).

Remark 2

Theorem 1 provides a delay-dependent condition for the system (7) to be stochastically admissible. The condition is an LMI, which can be easily and numerically solved by using interior-point algorithm. If the matrices N_{1i}, N_{2i} and X_i in (11b) and (11c) are set to zero, and $Z=\varepsilon$ (ε is a sufficient small positive scalar), Theorem 1 is identical to the delay-independent criterion.

Now, we consider the nominal singular discrete-time system of the system (1) described by

$$\begin{aligned} Ex(k+1) &= A(r_k)x(k)+A_d(r_k)x(k-d(k)) \\ &\quad +B(r_k)u(k) \\ x(k) &= \varphi(k), \quad -d_2\leqslant k\leqslant 0 \end{aligned} \tag{40}$$

where the variables follow the same definitions as those in (1). In the following theorem, we extend Theorem 1 to design a controller $u(k)=K_ix(k)$ for the above system such that the resultant closed-loop system is stochastically admissible.

Theorem 2

For given scalars d_1, d_2 with $d_2\geqslant d_1>0$, the nominal system (40) is stochastically admissible if there exist positive-definite symmetric matrices Q, Z, and matrices

$$X_i=\begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix}\geqslant 0, \quad N_{1i}, N_{2i}, P_i=P_i^{\mathrm{T}}$$

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and scalars ε_i, $i=1,2,\ldots,s$ such that (11a), (11c) and

$$B_i^{\mathrm{T}}\widetilde{P}_i B_i+\varepsilon_i I>0 \quad (41a)$$

$$\Theta_i=\begin{bmatrix}\Xi_{11i} & \Xi_{12i} & d_2(A_i-E)^{\mathrm{T}}Z & 0\\ * & \Xi_{22i} & d_2 A_{di}^{\mathrm{T}}Z & A_{di}^{\mathrm{T}}\widetilde{P}_i B_i\\ * & * & -d_2 Z & d_2 Z B_i\\ * & * & * & -\varepsilon_i I\end{bmatrix}<0 \quad (41b)$$

where Ξ_{11i}, Ξ_{12i}, Ξ_{22i} and $\widetilde{P}_i$ are defined in (11b). Furthermore, a suitable stabilizing feedback control law is given by

$$u(k)=-2(B_i^{\mathrm{T}}\widetilde{P}_i B_i+\varepsilon_i I)^{-1}B_i^{\mathrm{T}}\widetilde{P}_i A_i x(k)$$

Proof
Applying the controller $u(k)=K_i x(k)$ to the system (40) yields

$$\begin{aligned}Ex(k+1)&=(A(r_k)+B(r_k)K(r_k))x(k)\\ &\quad +A_d(r_k)x(k-d(k))\end{aligned} \quad (42)$$

From Theorem 1, we know that (42) is stochastically admissible if there exist matrices Q, Z, V_i, X_i, N_{1i}, N_{2i}, $P_i=P_i^{\mathrm{T}}$, $i=1,2,\ldots,s$, defined in (41) such that (11a), (11c) and the following matrix inequalities hold

$$\Gamma_i=\begin{bmatrix}\Gamma_{11i} & \Gamma_{12i} & d_2((A_i+B_iK_i)-E)^{\mathrm{T}}Z\\ * & \Gamma_{22i} & d_2A_{di}^{\mathrm{T}}Z\\ * & * & -d_2 Z\end{bmatrix}<0 \quad (43)$$

where

$$\begin{aligned}\Gamma_{11i}&=(d_2-d_1+1)Q+(A_i+B_iK_i)^{\mathrm{T}}\widetilde{P}_i(A_i+B_iK_i)\\ &\quad -E^{\mathrm{T}}P_iE+N_{1i}E+E^{\mathrm{T}}N_{1i}^{\mathrm{T}}+d_2X_{11i}\\ \Gamma_{12i}&=(A_i+B_iK_i)^{\mathrm{T}}\widetilde{P}_iA_{di}+E^{\mathrm{T}}N_{2i}^{\mathrm{T}}-N_{1i}E+d_2X_{12i}\\ \Gamma_{22i}&=A_{di}^{\mathrm{T}}\widetilde{P}_iA_{di}-Q-N_{2i}E-E^{\mathrm{T}}N_{2i}^{\mathrm{T}}+d_2X_{22i}\\ \widetilde{P}_i&=\sum_{j=1}^{s}\pi_{ij}P_j\end{aligned}$$

Now, we suppose that there exist scalars $\varepsilon_i>0, i=1,2,\ldots,s$ such that

$$B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI>0 \quad (44)$$

and set

$$K_i=-2(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}B_i^{\mathrm{T}}\widetilde{P}_iA_i \quad (45)$$

Then, from (43) we have

$$\begin{aligned}\Gamma_i&=\Xi_i+\begin{bmatrix}A_i^{\mathrm{T}}\widetilde{P}_iB_iK_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}\widetilde{P}_iA_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}\widetilde{P}_iB_iK_i & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\\ &\quad +\begin{bmatrix}K_i^{\mathrm{T}}\\ 0\\ 0\end{bmatrix}\begin{bmatrix}0\\ (B_i^{\mathrm{T}}\widetilde{P}_iA_{di})^{\mathrm{T}}\\ (d_2B_i^{\mathrm{T}}Z)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\\ &\quad +\begin{bmatrix}0\\ (B_i^{\mathrm{T}}\widetilde{P}_iA_{di})^{\mathrm{T}}\\ (d_2B_i^{\mathrm{T}}Z)^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}K_i^{\mathrm{T}}\\ 0\\ 0\end{bmatrix}^{\mathrm{T}}\end{aligned} \quad (46)$$

where Ξ_i is defined in (11b).

By Lemma 2, we can deduce that there exist scalars $\varepsilon_i>0$, $i=1,2,\ldots,s$, where ε_i is defined in (44) such that

$$\begin{aligned}\Gamma_i&\leqslant\Xi_i+\begin{bmatrix}A_i^{\mathrm{T}}\widetilde{P}_iB_iK_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}\widetilde{P}_iA_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}\widetilde{P}_iB_iK_i & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\\ &\quad +\varepsilon_i\begin{bmatrix}K_i^{\mathrm{T}}\\ 0\\ 0\end{bmatrix}\begin{bmatrix}K_i^{\mathrm{T}}\\ 0\\ 0\end{bmatrix}^{\mathrm{T}}\\ &\quad +\varepsilon_i^{-1}\begin{bmatrix}0\\ (B_i^{\mathrm{T}}\widetilde{P}_iA_{di})^{\mathrm{T}}\\ (d_2B_i^{\mathrm{T}}Z)^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}0\\ (B_i^{\mathrm{T}}\widetilde{P}_iA_{di})^{\mathrm{T}}\\ (d_2B_i^{\mathrm{T}}Z)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\end{aligned} \quad (47)$$

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From (44), (45), we have

$$
\begin{aligned}
&A_i^{\mathrm{T}}\widetilde{P}_iB_iK_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}\widetilde{P}_iA_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}\widetilde{P}_iB_iK_i+\varepsilon_iK_i^{\mathrm{T}}K_i\\
&\quad=-2A_i^{\mathrm{T}}\widetilde{P}_iB_i(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}B_i^{\mathrm{T}}\widetilde{P}_iA_i\\
&\qquad-2A_i^{\mathrm{T}}\widetilde{P}_iB_i(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}B_i^{\mathrm{T}}\widetilde{P}_iA_i\\
&\qquad+4A_i^{\mathrm{T}}\widetilde{P}_iB_i(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}B_i^{\mathrm{T}}\widetilde{P}_iB_i\\
&\qquad\times(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}B_i^{\mathrm{T}}\widetilde{P}_iA_i\\
&\qquad+\varepsilon_iA_i^{\mathrm{T}}\widetilde{P}_iB_i(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}\\
&\qquad\times(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}B_i^{\mathrm{T}}\widetilde{P}_iA_i\\
&\quad=-4A_i^{\mathrm{T}}\widetilde{P}_iB_i(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}B_i^{\mathrm{T}}\widetilde{P}_iA_i\\
&\qquad+4A_i^{\mathrm{T}}\widetilde{P}_iB_i(B_i^{\mathrm{T}}\widetilde{P}_iB_i+\varepsilon_iI)^{-1}B_i^{\mathrm{T}}\widetilde{P}_iA_i\\
&\quad=0
\end{aligned}
\tag{48}
$$

Using (47), (48) and employing the Schur complement, we obtain that (47) is equivalent to (41b). This completes the proof. □

Now, we generalize Theorem 2 to the uncertain case, and give the following sufficient condition on robust stochastic admissibility.

Theorem 3
For given scalars d_1, d_2 with $d_2 \geqslant d_1 > 0$, the closed-loop system associated with (9) and (1) is stochastically admissible if there exist positive-definite symmetric matrices Q, Z, and matrices

$$
X_i=\begin{bmatrix} X_{11i} & X_{12i}\\ * & X_{22i}\end{bmatrix}\geqslant 0,\quad N_{1i}, N_{2i}, P_i=P_i^{\mathrm{T}}
$$

and scalars ε_i, λ_{1i}, λ_{2i}, λ_{3i}, λ_{4i}, α_i, $i=1,2,\dots,s$, such that (11a), (11c), (41a) (49a) and (49b) hold

$$
V_i=M_i^{\mathrm{T}}\widetilde{P}_iM_i+\alpha_iI>0 \tag{49a}
$$

$$
\begin{bmatrix}
\Theta_{11i} & \Theta_{12i} & \Theta_{13i} & 0 & 0 & 0 & 0 & A_i^{\mathrm{T}}\widetilde{P}_iM_i & A_i^{\mathrm{T}}\widetilde{P}_iM_i & 0 & 0\\
* & \Theta_{22i} & \Theta_{23i} & A_{di}^{\mathrm{T}}\widetilde{P}_iB_i & 0 & 0 & 0 & A_{di}^{\mathrm{T}}\widetilde{P}_iM_i & A_{di}^{\mathrm{T}}\widetilde{P}_iM_i & 0 & 0\\
* & * & -d_2Z & d_2ZB_i & 0 & 0 & 0 & d_2ZM_i & d_2ZM_i & d_2ZM_i & 0\\
* & * & * & \Theta_{44i} & 0 & 0 & 0 & 0 & B_i^{\mathrm{T}}\widetilde{P}_iM_i & 0 & 0\\
* & * & * & * & \Theta_{55i} & 0 & 0 & I & I & 0 & 0\\
* & * & * & * & * & -I & 0 & 0 & I & 0 & 0\\
* & * & * & * & * & * & -I & 0 & 0 & 0 & V_i\\
* & * & * & * & * & * & * & -\lambda_{1i}I & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & -\lambda_{2i}I & 0 & 0\\
* & * & * & * & * & * & * & * & * & -\lambda_{3i}I & 0\\
* & * & * & * & * & * & * & * & * & * & -\lambda_{4i}I
\end{bmatrix}<0 \tag{49b}
$$

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where

$$\Theta_{11i} = (d_2-d_1+1)Q + A_i^{\mathrm{T}}\widetilde{P}_i A_i - E^{\mathrm{T}} P_i E$$
$$+N_{1i}E + E^{\mathrm{T}} N_{1i}^{\mathrm{T}} + d_2 X_{11i} + \lambda_{1i} N_{Ai}^{\mathrm{T}} N_{Ai}$$
$$\Theta_{12i} = A_i^{\mathrm{T}}\widetilde{P}_i A_{di} + E^{\mathrm{T}} N_{2i}^{\mathrm{T}}$$
$$-N_{1i}E + d_2 X_{12i}, \quad \Theta_{13i} = d_2(A_i - E)^{\mathrm{T}} Z$$
$$\Theta_{22i} = A_{di}^{\mathrm{T}}\widetilde{P}_i A_{di} - Q - N_{2i}E$$
$$-E^{\mathrm{T}} N_{2i}^{\mathrm{T}} + d_2 X_{22i} + \lambda_{2i} N_{Adi}^{\mathrm{T}} N_{Adi}$$
$$\Theta_{23i} = d_2 A_{di}^{\mathrm{T}} Z, \quad \Theta_{44i} = -\varepsilon_i I + \lambda_{3i} N_{Bi}^{\mathrm{T}} N_{Bi}$$
$$\Theta_{55i} = -V_i + \lambda_{4i} N_{Bi}^{\mathrm{T}} N_{Bi}$$
$$\widetilde{P}_i = \sum_{j=1}^{s} \pi_{ij} P_j$$

Furthermore, a suitable stabilizing feedback control law is given by

$$u(k) = -2(B_i^{\mathrm{T}}\widetilde{P}_i B_i + \varepsilon_i I)^{-1} B_i^{\mathrm{T}}\widetilde{P}_i A_i x(k)$$

Proof

Replacing A_i by $A_i + M_i F_i N_{Ai}$, A_{di} by $A_{di} + M_i F_i N_{Adi}$ and B_i by $B_i + M_i F_i N_{Bi}$ in (41b), the following inequality is obtained:

$$\Omega_i = \begin{bmatrix} \Omega_{11i} & \Omega_{12i} & d_2(A_i + M_i F_i N_{Ai} - E)^{\mathrm{T}} Z & 0 \\ * & \Omega_{22i} & d_2(A_{di} + M_i F_i N_{Adi})^{\mathrm{T}} Z & \Omega_{24i} \\ * & * & -d_2 Z & d_2 Z(B_i + M_i F_i N_{Bi}) \\ * & * & * & -\varepsilon_i I \end{bmatrix} < 0 \quad (50)$$

where

$$\Omega_{11i} = (d_2-d_1+1)Q - E^{\mathrm{T}} P_i E + N_{1i}E + E^{\mathrm{T}} N_{1i}^{\mathrm{T}} + d_2 X_{11i}$$
$$+A_i^{\mathrm{T}}\widetilde{P}_i A_i + N_{Ai}^{\mathrm{T}} F_i^{\mathrm{T}} M_i^{\mathrm{T}}\widetilde{P}_i A_i + A_i^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Ai}$$
$$+N_{Ai}^{\mathrm{T}} F_i^{\mathrm{T}} M_i^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Ai}$$

$$\Omega_{12i} = E^{\mathrm{T}} N_{2i}^{\mathrm{T}} - N_{1i}E + d_2 X_{12i}$$
$$+A_i^{\mathrm{T}}\widetilde{P}_i A_{di} + N_{Ai}^{\mathrm{T}} F_i^{\mathrm{T}} M_i^{\mathrm{T}}\widetilde{P}_i A_{di} + A_i^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Adi}$$
$$+N_{Ai}^{\mathrm{T}} F_i^{\mathrm{T}} M_i^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Adi}$$
$$\Omega_{22i} = -Q - N_{2i}E - E^{\mathrm{T}} N_{2i}^{\mathrm{T}} + d_2 X_{22i}$$
$$+A_{di}^{\mathrm{T}}\widetilde{P}_i A_{di} + N_{Adi}^{\mathrm{T}} F_i^{\mathrm{T}} M_i^{\mathrm{T}}\widetilde{P}_i A_{di} + A_{di}^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Adi}$$
$$+N_{Adi}^{\mathrm{T}} F_i^{\mathrm{T}} M_i^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Adi}$$
$$\Omega_{24i} = A_{di}^{\mathrm{T}}\widetilde{P}_i B_i + N_{Adi}^{\mathrm{T}} F_i^{\mathrm{T}} M_i^{\mathrm{T}}\widetilde{P}_i B_i + A_{di}^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Bi}$$
$$+N_{Adi}^{\mathrm{T}} F_i^{\mathrm{T}} M_i^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Bi}$$
$$\widetilde{P}_i = \sum_{j=1}^{s} \pi_{ij} P_j$$

Now, we suppose that, there exist scalars $\alpha_i > 0$, $i = 1, 2, \ldots, s$, such that

$$V_i = M_i^{\mathrm{T}}\widetilde{P}_i M_i + \alpha_i I > 0 \quad (51)$$

Then, using (42) (43), and employing Lemma 2, we have

$$\Omega_i = \overline{\Omega}_i + \begin{bmatrix} N_{Ai}^{\mathrm{T}} F_i^{\mathrm{T}} \\ N_{Adi}^{\mathrm{T}} F_i^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} M_i^{\mathrm{T}}\widetilde{P}_i M_i \begin{bmatrix} N_{Ai}^{\mathrm{T}} F_i^{\mathrm{T}} \\ N_{Adi}^{\mathrm{T}} F_i^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

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$$+\begin{bmatrix} 0 \\ N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ (M_i^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Bi})^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ (M_i^{\mathrm{T}}\widetilde{P}_i M_i F_i N_{Bi})^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} 0 \\ N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

$$\leqslant \overline{\Omega}_i+\begin{bmatrix} N_{Ai}^{\mathrm{T}}F_i^{\mathrm{T}} \\ N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} V_i \begin{bmatrix} N_{Ai}^{\mathrm{T}}F_i^{\mathrm{T}} \\ N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

$$+\begin{bmatrix} 0 \\ N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ (V_i F_i N_{Bi})^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ (V_i F_i N_{Bi})^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \qquad (52)$$

where

$$\overline{\Omega}_i=\begin{bmatrix} \overline{\Omega}_{11i} & \overline{\Omega}_{12i} & d_2(A_i+M_iF_iN_{Ai}-E)^{\mathrm{T}}Z & 0 \\ * & \overline{\Omega}_{22i} & d_2(A_{di}+M_iF_iN_{Adi})^{\mathrm{T}}Z & \overline{\Omega}_{24i} \\ * & * & -d_2Z & d_2Z(B_i+M_iF_iN_{Bi}) \\ * & * & * & -\varepsilon_i I \end{bmatrix}<0 \qquad (53)$$

with

$$\overline{\Omega}_{11i}=(d_2-d_1+1)Q-E^{\mathrm{T}}P_iE+N_{1i}E+E^{\mathrm{T}}N_{1i}^{\mathrm{T}}+d_2X_{11i} +A_i^{\mathrm{T}}\widetilde{P}_iA_i+N_{Ai}^{\mathrm{T}}F_i^{\mathrm{T}}M_i^{\mathrm{T}}\widetilde{P}_iA_i+A_i^{\mathrm{T}}\widetilde{P}_iM_iF_iN_{Ai}$$

$$\overline{\Omega}_{12i}=E^{\mathrm{T}}N_{2i}^{\mathrm{T}}-N_{1i}E+d_2X_{12i}+A_i^{\mathrm{T}}\widetilde{P}_iA_{di} +N_{Ai}^{\mathrm{T}}F_i^{\mathrm{T}}M_i^{\mathrm{T}}\widetilde{P}_iA_{di}+A_i^{\mathrm{T}}\widetilde{P}_iM_iF_iN_{Adi}$$

$$\overline{\Omega}_{22i}=-Q-N_{2i}E-E^{\mathrm{T}}N_{2i}^{\mathrm{T}}+d_2X_{22i}+A_{di}^{\mathrm{T}}\widetilde{P}_iA_{di} +N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}}M_i^{\mathrm{T}}\widetilde{P}_iA_{di}+A_{di}^{\mathrm{T}}\widetilde{P}_iM_iF_iN_{Adi}$$

$$\overline{\Omega}_{24i}=A_{di}^{\mathrm{T}}\widetilde{P}_iB_i+N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}}M_i^{\mathrm{T}}\widetilde{P}_iB_i+A_{di}^{\mathrm{T}}\widetilde{P}_iM_iF_iN_{Bi}$$

$$\widetilde{P}_i=\sum_{j=1}^{s}\pi_{ij}P_j$$

Using the Schur complement, we obtain from (52) that

$$\Omega_i\leqslant\begin{bmatrix} \overline{\Omega}_{11i} & \overline{\Omega}_{12i} & d_2(A_i+M_iF_iN_{Ai}-E)^{\mathrm{T}}Z & 0 & N_{Ai}^{\mathrm{T}}F_i^{\mathrm{T}} & 0 & 0 \\ * & \overline{\Omega}_{22i} & d_2(A_{di}+M_iF_iN_{Adi})^{\mathrm{T}}Z & \Omega_{24i} & N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}} & N_{Adi}^{\mathrm{T}}F_i^{\mathrm{T}} & 0 \\ * & * & -d_2Z & d_2Z(B_i+M_iF_iN_{Bi}) & 0 & 0 & 0 \\ * & * & * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & * & * & -V_i & 0 & N_{Bi}^{\mathrm{T}}F_i^{\mathrm{T}}V_i \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix}<0 \qquad (54)$$

From (54), we have

$$
\begin{aligned}
\Omega_i \leqslant{}& \Lambda_i \\
&+\begin{bmatrix} N_{Ai}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i^{\mathrm{T}} \begin{bmatrix} (M_i^{\mathrm{T}}\widetilde{P}_i A_i)^{\mathrm{T}} \\ (M_i^{\mathrm{T}}\widetilde{P}_i A_{di})^{\mathrm{T}} \\ (d_2 M_i^{\mathrm{T}} Z)^{\mathrm{T}} \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} \\
&+\begin{bmatrix} 0 \\ N_{Adi}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i^{\mathrm{T}} \begin{bmatrix} (M_i^{\mathrm{T}}\widetilde{P}_i A_i)^{\mathrm{T}} \\ (M_i^{\mathrm{T}}\widetilde{P}_i A_{di})^{\mathrm{T}} \\ (d_2 M_i^{\mathrm{T}} Z)^{\mathrm{T}} \\ (M_i^{\mathrm{T}}\widetilde{P}_i B_i)^{\mathrm{T}} \\ I \\ I \\ 0 \end{bmatrix}^{\mathrm{T}} \\
&+\begin{bmatrix} 0 \\ 0 \\ d_2 Z M_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i \begin{bmatrix} 0 \\ 0 \\ 0 \\ N_{Bi}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ N_{Bi}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} F_i^{\mathrm{T}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_i \end{bmatrix}^{\mathrm{T}} \\
&+\begin{bmatrix} (M_i^{\mathrm{T}}\widetilde{P}_i A_i)^{\mathrm{T}} \\ (M_i^{\mathrm{T}}\widetilde{P}_i A_{di})^{\mathrm{T}} \\ (d_2 M_i^{\mathrm{T}} Z)^{\mathrm{T}} \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix} F_i \begin{bmatrix} N_{Ai}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} \\
&+\begin{bmatrix} (M_i^{\mathrm{T}}\widetilde{P}_i A_i)^{\mathrm{T}} \\ (M_i^{\mathrm{T}}\widetilde{P}_i A_{di})^{\mathrm{T}} \\ (d_2 M_i^{\mathrm{T}} Z)^{\mathrm{T}} \\ (M_i^{\mathrm{T}}\widetilde{P}_i B_i)^{\mathrm{T}} \\ I \\ I \\ 0 \end{bmatrix} F_i \begin{bmatrix} 0 \\ N_{Adi}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} \\
&+\begin{bmatrix} 0 \\ 0 \\ 0 \\ N_{Bi}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i^{\mathrm{T}} \begin{bmatrix} 0 \\ 0 \\ d_2 Z M_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} \\
&+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_i \end{bmatrix} F_i \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ N_{Bi}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} < 0
\end{aligned}
\tag{55}
$$

where

$$
\Lambda_i = \begin{bmatrix}
\Lambda_{11i} & \Lambda_{12i} & d_2(A_i - E)^{\mathrm{T}} Z & 0 & 0 & 0 & 0 \\
* & \Lambda_{22i} & d_2 A_{di}^{\mathrm{T}} Z & A_{di}^{\mathrm{T}} \widetilde{P}_i B_i & 0 & 0 & 0 \\
* & * & -d_2 Z & d_2 Z B_i & 0 & 0 & 0 \\
* & * & * & -\varepsilon_i I & 0 & 0 & 0 \\
* & * & * & * & -V_i & 0 & 0 \\
* & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & -I
\end{bmatrix}
$$

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with

$$\Lambda_{11i} = (d_2-d_1+1)Q - E^{\mathrm{T}} P_i E + N_{1i} E + E^{\mathrm{T}} N_{1i}^{\mathrm{T}} + d_2 X_{11i} + A_i^{\mathrm{T}} \widetilde{P}_i A_i$$

$$\Lambda_{12i} = E^{\mathrm{T}} N_{2i}^{\mathrm{T}} - N_{1i} E + d_2 X_{12i} + A_i^{\mathrm{T}} \widetilde{P}_i A_{di}$$

$$\Lambda_{22i} = -Q - N_{2i} E - E^{\mathrm{T}} N_{2i}^{\mathrm{T}} + d_2 X_{22i} + A_{di}^{\mathrm{T}} \widetilde{P}_i A_{di}$$

By Lemma 1, it can be shown that (55) holds for any F_i satisfying (6), if and only if there exist scalars $\lambda_{ji}>0$ $(j=1,2,\ldots,4)$, $i\in S$ such that

$$\Lambda_i + \lambda_{1i} \begin{bmatrix} N_{Ai}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} N_{Ai}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \lambda_{1i}^{-1} \begin{bmatrix} (M_i^{\mathrm{T}} \widetilde{P}_i A_i)^{\mathrm{T}} \\ (M_i^{\mathrm{T}} \widetilde{P}_i A_{di})^{\mathrm{T}} \\ (d_2 M_i^{\mathrm{T}} Z)^{\mathrm{T}} \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (M_i^{\mathrm{T}} \widetilde{P}_i A_i)^{\mathrm{T}} \\ (M_i^{\mathrm{T}} \widetilde{P}_i A_{di})^{\mathrm{T}} \\ (d_2 M_i^{\mathrm{T}} Z)^{\mathrm{T}} \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \lambda_{2i} \begin{bmatrix} 0 \\ N_{Adi}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ N_{Adi}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

$$+ \lambda_{2i}^{-1} \begin{bmatrix} (M_i^{\mathrm{T}} \widetilde{P}_i A_i)^{\mathrm{T}} \\ (M_i^{\mathrm{T}} \widetilde{P}_i A_{di})^{\mathrm{T}} \\ (d_2 M_i^{\mathrm{T}} Z)^{\mathrm{T}} \\ (M_i^{\mathrm{T}} \widetilde{P}_i B_i)^{\mathrm{T}} \\ I \\ I \\ 0 \end{bmatrix} \begin{bmatrix} (M_i^{\mathrm{T}} \widetilde{P}_i A_i)^{\mathrm{T}} \\ (M_i^{\mathrm{T}} \widetilde{P}_i A_{di})^{\mathrm{T}} \\ (d_2 M_i^{\mathrm{T}} Z)^{\mathrm{T}} \\ (M_i^{\mathrm{T}} \widetilde{P}_i B_i)^{\mathrm{T}} \\ I \\ I \\ 0 \end{bmatrix}^{\mathrm{T}} + \lambda_{3i}^{-1} \begin{bmatrix} 0 \\ 0 \\ d_2 Z M_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d_2 Z M_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \lambda_{3i} \begin{bmatrix} 0 \\ 0 \\ 0 \\ N_{Bi}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ N_{Bi}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

$$+ \lambda_{4i} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ N_{Bi}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ N_{Bi}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}} + \lambda_{4i}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_i \end{bmatrix}^{\mathrm{T}} < 0 \tag{56}$$

which, by the Schur complement, is equivalent to (49b). This completes the proof. □

Remark 3

Theorem 3 provides a sufficient condition for robust stochastic stabilization for uncertain singular discrete-time system with Markovian jumping parameters and time-varying delay. Desired controller gain matrices for system (1) can be constructed through the solutions of LMIs, which can be solved efficiently by algorithms such as the interior-point method. It is worth noting that, if set $d_1=d_2=\text{const}$, Theorems 2 and 3 can be reduced to const time-delay conditions. Note that the methods in [16, 17] did not consider time delay.

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Table I. Upper bound on d_2 for various d_1 in Example 1.

d_1	0	1	2	3	4	5
Theorem 1	2	3	3	3	4	5

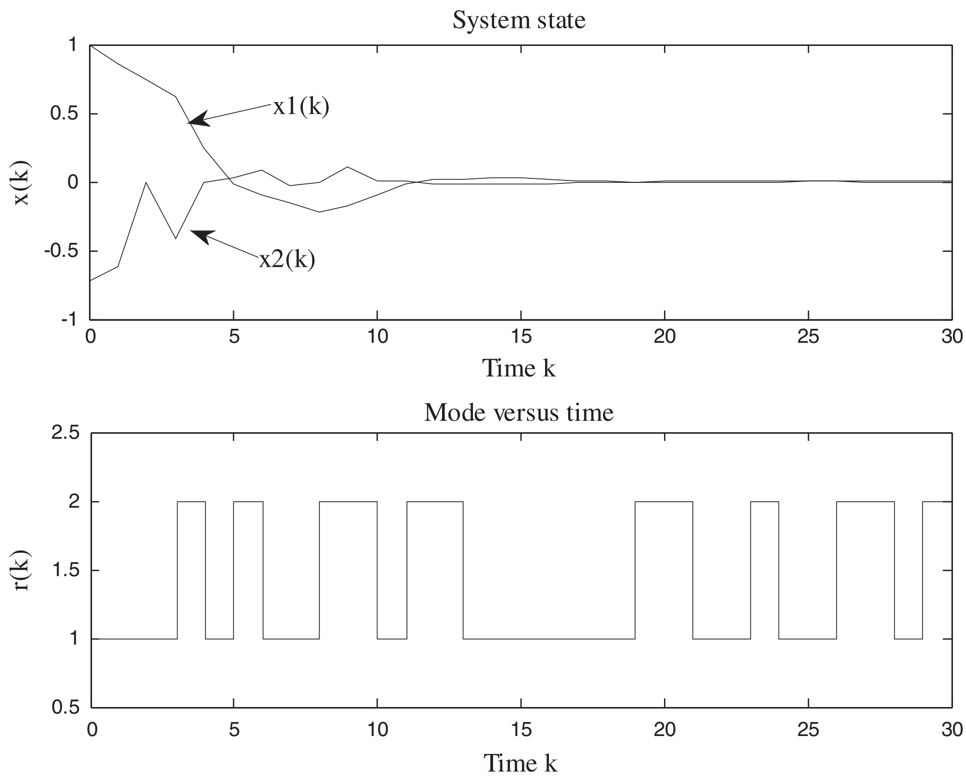

Figure 1. The response of the state vector $x(k)$ of the system (7) (Example 1).

4. NUMERICAL EXAMPLES

In this section, we provide two numerical examples to demonstrate the effectiveness and applicability of the proposed condition.

Example 1
Consider the system (7) with two-mode. Without loss of generality, we let

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

and the data of this system are as follows:

$$A_1 = \begin{bmatrix} 0.86 & 0.1 \\ 0.7 & 0.97 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.33 & 0 \\ -0.1 & -0.1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.83 & -0.6 \\ 0 & -1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.35 & 0 \\ -0.1 & -0.12 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}$$

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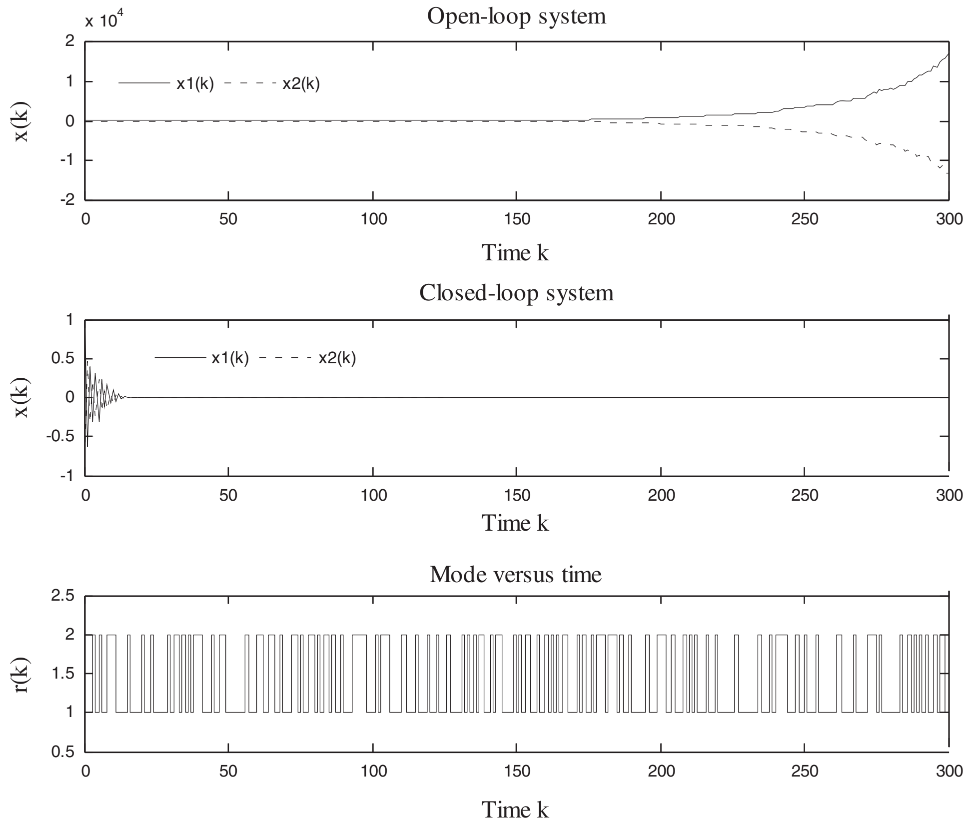

Figure 2. The response of the state vector $x(k)$ of the system (1) (Example 2).

We use the Matlab LMI Control toolbox to solve (11a), (11b) and (11c). First, we consider the case where $d_1 = d_2 = \text{const}$, that is, $d(k) = \text{const}$. The criterion in this example yields a value of 5, in [19] and can be obtained in the same time upper bound. It should be pointed out that the approach in [19] needs the original system matrices to be decomposed and then a new system to be constructed based on the decomposed matrices. Therefore, the new constructed system matrices are required to evaluate the approach, which increase extra computation. However, our results do not require the decomposition of system matrices. Furthermore, we consider the case where $d(k)$ is time-varying, i.e. $d_1 \neq d_2$, and Table I lists the upper bound on d_2 for various d_1 by this method. Note that methods in [16, 17, 19] did not consider the case.

The responses of the state vector $x(k)$ in system (7) with $d(k) = \text{round}(\sin(k)+2)$ (satisfying $1 \leqslant d(k) \leqslant 3$) for Example 1 are shown in Figure 1, which further illustrate the effectiveness of the proposed condition.

Example 2

Consider the system (1) with two-mode. The data of this system are as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.93 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.9 & 0.2 \\ 0.7 & 0.83 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.1 & -0.02 \\ -0.1 & -0.1 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} 0.12 & -0.1 \\ -0.1 & -0.1 \end{bmatrix}$$

$$B_1=\begin{bmatrix}-1 & 0.5\\ 0.2 & 5\end{bmatrix},\quad B_2=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Pi=\begin{bmatrix}0.6 & 0.4\\ 0.7 & 0.3\end{bmatrix}$$

$$F_1(k)=F_2(k)=\begin{bmatrix}\sin(3k) & 0\\ 0 & \sin(5k)\end{bmatrix}$$

$$\begin{aligned}M_1&=M_2=N_{A1}=N_{A2}=N_{Ad1}=N_{Ad2}\\ &=N_{B1}=N_{B2}=0.1I\end{aligned}$$

We use the Matlab LMI toolbox to solve the LMIs in Theorem 3. Given $d_1=1$, we obtain the upper bound on d_2 which is 4. By Theorem 3, the gain matrices of a robust controller can be obtained as

$$K_1=\begin{bmatrix}1.3613 & 0.0882\\ -0.3336 & -0.3758\end{bmatrix}$$

$$K_2=\begin{bmatrix}1.7088 & 0.6872\\ -0.3947 & -1.6767\end{bmatrix}$$

The open-loop system is unstable with $d(k)=\mathrm{round}(1.5\sin(k)+2.5)$ (satisfying $1\leqslant d(k)\leqslant 4$), and applying this controller makes the closed-loop system stochastically stable, see Figure 2.

5. CONCLUSION

In this paper, we have studied the stochastic stability and the robust control of singular discrete-time system with Markovian jumping parameters and time-varying delay. In terms of free-weighting-matrix approach and LMIs, a delay-dependent condition is presented to ensure a singular discrete-time system to be regular, causal and stochastically stable, and based on which, the stability analysis and robust stabilization problem are studied. All the obtained results are formulated in terms of strict LMIs involving no decomposition of the system matrices, which makes the design procedure relatively simple and reliable. A robustly stabilizing state-feedback controller can be constructed through the numerical solutions of LMIs.

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