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Permanence for nonautonomous predator–prey Kolmogorov systems with impulses and its applications

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ABSTRACT

In this paper, the general impulsive non-autonomous predator–prey Kolmogorov system is studied. Some new criteria on the permanence and ultimate boundedness are established. As applications of these results, some special models are studied, such as a class of impulsive non-autonomous Lotka–Volterra systems, impulsive Holling I-type functional response systems, impulsive Holling (m,n)-type functional response systems, impulsive Beddington–DeAngelis functional response systems, Leslie–Gower functional response systems and chemostat-type systems.

1. Introduction

In this paper, we consider the following two-species non-autonomous predator–prey Kolmogorov system with impulse

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t)f_1(t,x_1(t),x_2(t)), & \\ \frac{dx_2(t)}{dt} = x_2(t)f_2(t,x_1(t),x_2(t)), & t \neq t_k, \\ x_1(t_k^+) = h_{1k}x_1(t_k), & \\ x_2(t_k^+) = h_{2k}x_2(t_k), & k = 1,2,\ldots, \end{cases} \tag{1.1}$$

where we assume that $0 \leqslant t_1 < t_2 < \cdots < t_k < \cdots$ is impulsive time sequence and $\lim_{k\to\infty} t_k = \infty$, h_{ik} are positive constant for each $i = 1,2$ and $k = 1,2,\ldots$, functions $f_i(t,x_1,x_2)$ $(i = 1,2)$ are continuous for all $t \in R_{+0} = [0,\infty)$, $x_1 > 0$ and $x_2 \geq 0$. But, when $x_1 = 0$, $f_i(t,x_1,x_2)$ $(i = 1,2)$ may not have any definition for any $t \in R_{+0}$ and $x_2 \geq 0$.

System (1.1) include many well-known impulsive non-autonomous two-species predator–prey systems as its specific cases, for example

(1) Lotka–Volterra type system with impulse

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t)(b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)), & t \neq t_k, \\ \frac{dx_2(t)}{dt} = x_2(t)(-b_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)), & \\ x_i(t_k^+) = h_{ik}x_i(t_k), \quad i = 1,2, \ k = 1,2,\ldots. & \end{cases} \tag{1.2}$$

(2) Holling I-type functional response system with impulse

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t)(b_1(t) - a_{11}(t)x_1(t)) - x_2(t)\phi_1(t,x_1(t)), & t \neq t_k, \\ \frac{dx_2(t)}{dt} = x_2(t)(-b_2(t) + \phi_2(t,x_1(t)) - a_{22}(t)x_2(t)), & \\ x_i(t_k^+) = h_{ik}x_i(t_k), \quad i = 1,2, \ k = 1,2,\ldots, & \end{cases} \tag{1.3}$$

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where the function

$$\phi_i(t,x_1)=\begin{cases}\alpha_i(t)x_1, & 0\leqslant x_1\leqslant x_{10},\\ \alpha_i(t)x_{10}, & x_1>x_{10},\ i=1,2,\end{cases}$$

and $x_{10}>0$ is a constant.

(3) Holling (m,n)-type functional response system with impulse

$$\begin{cases}\frac{dx_1(t)}{dt}=x_1(t)(b_1(t)-a_{11}(t)x_1(t))-x_2\phi_1(t,x_1(t)), & t\neq t_k,\\ \frac{dx_2(t)}{dt}=x_2(t)(-b_2(t)+\phi_2(t,x_1(t))-a_{22}(t)x_2(t)), & \\ x_i(t_k^+)=h_{ik}x_i(t_k),\quad i=1,2,\ k=1,2,\ldots, &\end{cases}\tag{1.4}$$

where the function

$$\phi_i(t,x_1)=\frac{\alpha_i(t)x_1^m}{x_1^n+\beta_i(t)},\quad i=1,2.$$

(4) Beddington–DeAngelis functional response system with impulse

$$\begin{cases}\frac{dx_1(t)}{dt}=x_1(t)(b_1(t)-a_{11}(t)x_1(t))-x_2(t)\phi_1(t,x_1(t),x_2(t)), & t\neq t_k,\\ \frac{dx_2(t)}{dt}=x_2(t)(-b_2(t)+\phi_2(t,x_1(t),x_2(t))), & \\ x_i(t_k^+)=h_{ik}x_i(t_k),\quad i=1,2,\ k=1,2,\ldots, &\end{cases}\tag{1.5}$$

where the function

$$\phi_i(t,x_1,x_2)=\frac{\alpha_i(t)x_1^m}{1+\gamma_i(t)x_1^n+\omega_i(t)x_2},\quad i=1,2.$$

(5) Leslie–Gower system with functional response with impulse

$$\begin{cases}\frac{dx_1(t)}{dt}=x_1(t)(b_1(t)-a_{11}(t)x_1(t))-x_2(t)\phi_1(t,x_1(t)), & t\neq t_k,\\ \frac{dx_2(t)}{dt}=x_2(t)I(t,\frac{x_2(t)}{x_1(t)}), & \\ x_i(t_k^+)=h_{ik}x_i(t_k),\quad i=1,2,\ k=1,2,\ldots, &\end{cases}\tag{1.6}$$

where the function

$$\phi_1(t,x_1)=\frac{\alpha_1(t)x_1^2}{x_1^2+\gamma_1(t)x_1+\beta_1(t)}$$

and

$$I\left(t,\frac{x_2}{x_1}\right)=b_2(t)-\alpha_2(t)\left(\frac{x_2}{x_1}\right)^m.$$

(6) Impulsive Chemostat-type system

$$\begin{cases}\frac{dx_1(t)}{dt}=a_{11}(t)-b_1(t)x_1(t)-\phi_1(t,x_1(t))x_2(t), & t\neq t_k,\\ \frac{dx_2(t)}{dt}=x_2(t)(-b_2(t)+\phi_2(t,x_1(t))-a_{22}(t)x_2(t)), & \\ x_i(t_k^+)=h_{ik}x_i(t_k),\quad i=1,2,\ k=1,2,\ldots, &\end{cases}\tag{1.7}$$

where the function

$$\phi_i(t,x_1)=\frac{\alpha_i(t)x_1^m}{x_1^n+\beta_i(t)},\quad i=1,2.$$

As we well know, in the theory of mathematical, traditional predator–prey system are very important mathematical models which describe multi-species population dynamics in a non-autonomous environment. In order to make the model more accurate, there are many well-known two-species non-autonomous predator–prey systems, such as Lotka–Volterra type systems (see [1–3]), Holling I-type functional response system (see [1,3,4]), Holling (m,n)-type functional response system (see [1,3–5]), Beddington–DeAngelis functional response system (see [3,6–8]), Leslie–Gower system with functional response (see [1,3,9,10]), Chemostat-type system (see [3,11–14]). Many important and interesting results on the dynamical behaviors for such systems, such as the permanence, global asymptotic behavior and the existence and uniqueness of coexistence states (for example, positive periodic solution, positive almost periodic solution, etc.) can be found in above references and references there in.

However, biological species may undergo discrete changes of relatively short duration at a fixed time, such as fire, drought, flooding, crop-dusting, deforestation, hunting, harvesting, etc., the intrinsic discipline of biological species or ecological environment. For having a more accurate description of such system, we need to consider the impulsive differential equations.

In recent years, population models with impulsive perturbations have been intensively researched, such as the Lotka–Volterra model [15–18], Holling-type [19–24] and Beddington-type [25–28]. However, to our best knowledge, for general impulsive non-autonomous Kolmogorov predator–prey system (1.1), up until now, there is not any study work for the permanence of positive solutions. In addition, we also find that for the impulsive non-autonomous predator–prey Holling functional response systems, Beddington–DeAngelis functional response systems, Leslie–Gower functional response systems and impulsive non-autonomous chemostat-type systems, etc., there is also not any study work for the permanence of positive solutions.

In this paper, motivated by the above works, we study the permanence of positive solutions for general impulsive non-autonomous Kolmogorov predator–prey system (1.1) and establish a general criterion which is described by integrable form. The organization of this paper is as follows. In the next section, the impulsive non-autonomous single-species Kolmogorov system is considered and several useful lemmas are introduced. In Section 3, a general theorem for the permanence of system (1.1) is stated and proved. Finally in Section 4, as applications of above theorems, we will study the permanence of positive solutions for special cases.

2. Preliminaries

Let $R_{+0}=[0,\infty)$ and $R_+=(0,\infty)$. Let $\{t_k\}$ be a time sequence, satisfying $0\leqslant t_1<t_2<\cdots<t_k<\cdots$ and $t_k\to\infty$ as $k\to\infty$. In this section, as a preliminary we consider the following impulsive Komogorov system with a parameter

$$\begin{cases}\frac{du(t)}{dt}=u(t)g(t,u(t),\alpha), & t\neq t_k,\\ u(t_k^+)=h_ku(t_k), & k=1,2,\ldots,\end{cases}\tag{2.1}$$

where $g(t,u,\alpha)$ is a continuous function defined on $(t,u,\alpha)\in R_{+0}\times R_+\times[0,\alpha_0]$, $\alpha_0>0$ is a constant and h_k is positive constant for $k=1,2,\ldots$. We assume that for any $(t_0,u_0)\in R_{+0}\times R_+$ and $\alpha\in[0,\alpha_0]$ system (2.1) has a unique solution $u_\alpha(t)$ satisfying $u_\alpha(t_0)=u_0$. If $u_\alpha(t)>0$ on the interval of existence, the $u_\alpha(t)$ is said to be a positive solution. It is easy to see that $u_\alpha(t)$ is positive solution if the initial value $u_0>0$. For system (2.1) we introduce the following assumption:

(A1) For any $\sigma>1$, $g(t,u,\alpha)$ is bounded on $R_{+0}\times[\sigma^{-1},\sigma]\times[0,\alpha_0]$.

(A2) There are positive constants $k_1,k_2,\omega_1,\omega_2$ and $k_2>k_1$ such that

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_1}g(s,k_1,0)ds+\sum_{t\leqslant t_k<t+\omega_1}\ln h_k\right)>0,$$

$$\limsup_{t\to\infty}\left(\int_t^{t+\omega_2}g(s,k_2,0)ds+\sum_{t\leqslant t_k<t+\omega_2}\ln h_k\right)<0,$$

and function

$$h(t,v)=\sum_{t\leqslant t_k<t+v}\ln h_k$$

is bounded on $t\in R_+$ and $v\in[0,\omega_1)$.

(A3) Partial derivative $\partial g(t,u,\alpha)/\partial u$ exists for all $(t,u,\alpha)\in R_{+0}\times R_+\times[0,\alpha_0]$ and there is a nonnegative continuous function $q(t)$ and a constant $\omega>0$, satisfying

$$\liminf_{t\to\infty}\int_t^{t+\omega}q(s)ds>0,$$

and a continuous function $p(u)$, satisfying $p(u)>0$ for all $u\in R_+$, such that

$$\frac{\partial g(t,u,\alpha)}{\partial u}\leqslant -q(t)p(u)\quad\text{for all }(t,u,\alpha)\in R_{+0}\times R_+\times[0,\alpha_0].$$

(A4) Partial derivative $\partial g(t,u,\alpha)/\partial\alpha$ exist for all $(t,u,\alpha)\in R_{+0}\times R_+\times[0,\alpha_0]$, and for any constant $U>0$, $\partial g(t,u,\alpha)/\partial\alpha$ is also bounded on $(t,u,\alpha)\in R_{+0}\times(0,U]\times[0,\alpha_0]$.

In system (2.1), when parameter $\alpha=0$ we obtain the following system

$$\begin{cases}\frac{du(t)}{dt}=u(t)g(t,u(t),0), & t\neq t_k,\\ u(t_k^+)=h_ku(t_k), & k=1,2,\ldots.\end{cases}\tag{2.2}$$

Let $u_0^*(t)$ be a fixed positive solution of system (2.2) defined on R_{+0}.

Definition 2.1. System (2.2) is said to be *permanent*, if there are positive constants m and M such that

$$m \leqslant \liminf_{t\to\infty} u_0(t) \leqslant \limsup_{t\to\infty} u_0(t) \leqslant M,$$

for any positive solution $u_0(t)$ of system (2.2).

Definition 2.2. $u_0^*(t)$ is *globally uniformly attractive* on R_{+0}, if for any constants $\eta > 1$ and $\varepsilon > 0$ there is a constant $T(\eta,\varepsilon) > 0$ such that for any initial time $t_0 \in R_{+0}$ and any solution $u_0(t)$ of system (2.2) with $u_0(t_0) \in [\eta^{-1}, \eta]$, one has

$$|u_0(t) - u_0^*(t)| < \varepsilon \quad \text{for all } t \geqslant t_0 + T(\eta,\varepsilon).$$

We first have the following result by using a similar argument as Lemma 1 in [3].

Lemma 2.1. *Suppose that* (A1)–(A3) *hold, then*

(a) *System* (2.2) *is permanent.*

(b) *Each fixed positive solution* $u_0^*(t)$ *of system* (2.2) *is globally uniformly attractive on* R_{+0}.

Proof. By assumption (A2), there are positive constants δ and T_0 such that for all $t \geqslant T_0$ we have

$$\int_t^{t+\omega_1} g(s,k_1,0)\mathrm{d}s + \sum_{t\leqslant t_k<t+\omega_1} \ln h_k > \delta \tag{2.3}$$

and

$$\int_t^{t+\omega_2} g(s,k_2,0)\mathrm{d}s + \sum_{t\leqslant t_k<t+\omega_2} \ln h_k < -\delta. \tag{2.4}$$

Since function $h(t,v) = \sum_{t\leqslant t_k<t+v} \ln h_k$ is bounded on $t \in R_+$ and $v \in [0,\omega_1)$, there is a positive constant H such that for any $t \in R_{+0}$ and $v \in [0, \max\{\omega_1,\omega_2\})$

$$|h(t,v)| = \left|\sum_{t\leqslant t_k<t+v} \ln h_k\right| < H. \tag{2.5}$$

Let $u(t)$ be any positive solution of Eq. (2.2). We first prove that there is a $\tau \geqslant T_0$ such that

$$k_1 \exp(-\alpha_1\omega_1 - H) \leqslant u(t) \leqslant k_2 \exp(\alpha_2\omega_2 + H) \quad \text{for all } t \geqslant \tau,$$

where $\alpha_i = \sup\{|g(t,k_i,0)| : t \in R_{+0}\}$. If $u(t) \geqslant k_2$, for all $t \geqslant T_0$, then from (A3) we have

$$\begin{aligned} u(t) &= u(T_0)\exp\left(\int_{T_0}^t g(s,u(s),0)\mathrm{d}s + \sum_{T_0\leqslant t_k<t} \ln h_k\right) \\ &\leqslant u(T_0)\exp\left(\int_{T_0}^t g(s,k_2,0)\mathrm{d}s + \sum_{T_0\leqslant t_k<t} \ln h_k\right). \end{aligned}$$

From (2.4), we easily obtain $u(t) \to 0$ as $t \to \infty$, which is a contradiction. Hence, $u(\tau_1) < k_2$ for some $\tau_1 \geqslant T_0$. Further, if there are $s_2 > s_1 \geqslant \tau_1$ such that $u(s_2) > k_2\exp(\alpha_2\omega_2 + H)$, $u(s_1) \leqslant k_2$, $u(s_1^+) \geqslant k_2$ and $u(t) \geqslant k_2$ for all $t \in (s_1,s_2)$, then we can choose an integer $p \geq 0$ such that $s_2 \in [s_1 + p\omega_2, s_1 + (p+1)\omega_2)$ and obtain

$$\begin{aligned} k_2\exp(\alpha_2\omega_2 + H) < u(s_2) &= u(s_1)\exp\left(\int_{s_1}^{s_2} g(t,u(t),0)\mathrm{d}t + \sum_{s_1\leqslant t_k<s_2} \ln h_k\right) \\ &\leqslant k_2\exp\left(\left(\int_{s_1}^{s_1+p\omega_2} + \int_{s_1+p\omega_2}^{s_2}\right) g(t,k_2,0)\mathrm{d}t + \left(\sum_{s_1\leqslant t_k<s_1+p\omega_2} + \sum_{s_1+p\omega_2\leqslant t_k<s_2}\right)\ln h_k\right) \\ &\leqslant k_2\exp\left(\int_{s_1+p\omega_2}^{s_2} g(t,k_2,0)\mathrm{d}t + \sum_{s_1+p\omega_2\leqslant t_k<s_2} \ln h_k\right) \leqslant k_2\exp(\alpha_2\omega_2 + H), \end{aligned}$$

which also is a contradiction. Therefore, we have

$$u(t) \leqslant k_2\exp(\alpha_2\omega_2 + H) \quad \text{for all } t \in [\tau_1,\infty).$$

Similarly, by (2.3), we can prove that there is a $\tau_2 \geqslant T_0$ such that

$$u(t) \geqslant k_1\exp(-\alpha_1\omega_1 - H) \quad \text{for all } t \in [\tau_2,\infty).$$

Choose $\tau = \max\{\tau_1, \tau_2\}$, $m = k_1 \exp(-\alpha_1\omega_1 - H)$ and $M = k_2 \exp(\alpha_2\omega_2 + H)$, then we obtain conclusion (a).

Here, we prove conclusion (b). For solution $u_0^*(t)$, by conclusion (a) there is a constant $M_1 > 1$ such that

$$M_1^{-1} \leqslant u_0^*(t) \leqslant M_1 \quad \text{for all } t \in R_{+0}. \tag{2.6}$$

For any constant $\eta > 1$ and $t_0 \in R_{+0}$, let $u_0(t)$ be the solution of system (2.1) with initial value $u_0(t_0) \in [\eta^{-1}, \eta]$. We can choose Lyapunov function as follows

$$V(t) = |\ln u_0(t) - \ln u_0^*(t)|.$$

For any $k = 1, 2, \ldots$, we have

$$V(t_k^+) = |\ln(h_k u_0(t_k)) - \ln(h_k u_0^*(t_k))| = V(t_k).$$

Calculating the Dini derivative, by (A3) we can obtain

$$\begin{aligned} D^+V(t) &= \operatorname{sgn}(u_0(t) - u_0^*(t))(g(t, u_0(t), 0) - g(t, u_0^*(t), 0)) = \frac{\partial g(t, \xi(t), 0)}{\partial u}|u_0(t) - u_0^*(t)| \\ &\leqslant -q(t)p(\xi(t))|u_0(t) - u_0^*(t)| \quad \text{for all } t \neq t_k,\ k = 1, 2, \ldots, \end{aligned} \tag{2.7}$$

where $\xi(t)$ is situated between $u_0(t)$ and $u_0^*(t)$. Hence, $V(t) \leqslant V(t_0)$ for all $t \geqslant t_0$. Consequently, by (2.6) we have

$$|\ln u_0(t)| \leqslant |\ln u_0^*(t)| + V(t_0) \leqslant \ln(\eta M_1^2) \quad \text{for all } t \geqslant t_0.$$

Hence, $\eta^{-1}M_1^{-2} \leqslant u_0(t) \leqslant \eta M_1^2$ for all $t \geqslant t_0$. Further by (2.6), we obtain

$$\eta^{-1}M_1^{-2}V(t) \leqslant |u_0(t) - u_0^*(t)| \leqslant \eta M_1^2 V(t) \quad \text{for all } t \geqslant t_0.$$

Consequently, by (2.7) it follows that

$$D^+V(t) \leqslant -q(t)M_0V(t) \quad \text{for all } t \neq t_k,\ k = 1, 2, \ldots, \tag{2.8}$$

where $M_0 = \eta^{-1}M_1^2 \min\{p(u) : \eta^{-1}M_1^{-2} \leqslant u \leqslant \eta M_1^2\}$. Since

$$\liminf_{t\to\infty} \int_t^{t+\omega} q(s)ds > 0,$$

we can choose positive constants δ and T_0 such that

$$\int_t^{t+\omega} q(s)ds \geqslant \delta \quad \text{for all } t \geqslant T_0.$$

Let $T_0' = t_0 + T_0$, for any $t \geqslant T_0'$, there is an integer $n_t \geqslant 0$ such that $t \in [T_0' + n_t\omega, T_0' + (n_t + 1)\omega)$. Integrating (2.8) from T_0' to t, we have

$$V(t) \leqslant V(T_0') \exp \int_{T_0'}^t (-M_0q(s))ds = V(T_0') \exp \left(\int_{T_0'}^{T_0'+\omega} + \cdots + \int_{T_0'+(n_t-1)\omega}^{T_0'+n_t\omega} + \int_{T_0'+n_t\omega}^t \right) (-M_0q(s))ds \leqslant V(T_0') \exp(-M_0\delta n_t).$$

Since $V(T_0') \leqslant V(t_0) \leqslant \ln(\eta M_1)$, we further have

$$V(t) \leqslant \ln(\eta M_1) \exp\left(-M_0\delta\omega^{-1}(t - T_0' - \omega)\right) = M_2(\eta) \exp\left(-M_0\delta\omega^{-1}(t - t_0)\right), \tag{2.9}$$

where $M_2(\eta) = \ln(\eta M_1) \exp(M_0\delta(1 + T_0/\omega))$. Hence, for any positive constant ε, from (2.9), there is a large enough $T(\eta, \varepsilon) \geqslant T_0$ such that

$$V(t) < \eta^{-1}M_1^{-2}\varepsilon \quad \text{for all } t \geqslant t_0 + T(\eta, \varepsilon).$$

Therefore, $|u_0(t) - u_0^*(t)| < \varepsilon$ for all $t \geqslant t_0 + T(\eta, \varepsilon)$. This shows that solution $u_0^*(t)$ is globally uniformly attractive on R_{+0}. This completes the proof. □

Lemma 2.2. *Suppose that* (A1)–(A4) *hold. Then* $u_\alpha(t)$ *converges to* $u_0(t)$ *uniformly for* $t \in [t_0, +\infty)$ *as* $\alpha \to 0$.

Proof. For any $(t, u, \alpha) \in R_{+0} \times R_+ \times [0, \alpha_0]$, since

$$g(t, u, \alpha) = g(t, u, 0) + \frac{\partial g(t, u, \xi)}{\partial \alpha}\alpha,$$

where $\xi \in (0, \alpha)$, by (A2) and (A4) there are constants $T_0 > 0, \gamma_0 \in (0, \alpha_0]$ and $\delta_0 > 0$ such that

$$\int_t^{t+\omega_1} g(s, k_1, \alpha)ds + \sum_{t \leqslant t_k < t+\omega_1} \ln h_k > \delta_0$$

and

$$\int_t^{t+\omega_2} g(s,k_2,\alpha)\mathrm{d}s + \sum_{t\leqslant t_k<t+\omega_2} \ln h_k < -\delta_0$$

for all $t \geqslant T_0$ and $\alpha \in [0,\gamma_0]$. Since function $h(t,v) = \sum_{t\leqslant t_k<t+v} \ln h_k$ is bounded on $t \in R_+$ and $v \in [0,\omega_1)$, there is a positive constant H such that for any $t \in R_{+0}$ and $v \in [0,\max\{\omega_1,\omega_2\})$

$$|h(t,v)| = \left|\sum_{t\leqslant t_k<t+v} \ln h_k\right| < H.$$

Let $u_\alpha(t)$ be any positive solution of Eq. (2.1). From this and by (A1) and (A3), using a similar argument as in Lemma 2.1, we can prove that there is a $\tau \geqslant T_0$ such that

$$k_1\exp(-\alpha_1\omega_1 - H) \leqslant u_\alpha(t) \leqslant k_2\exp(\alpha_2\omega_2 + H) \quad \text{for all } t \geqslant \tau,$$

where $\alpha_i = \sup\{|g(t,k_i,\alpha)| : t \in R_{+0}, \alpha \in [0,\alpha_0]\}$ for $i = 1,2$. Therefore, there is a constant $M > 1$, and M is independent of any α, such that

$$M^{-1} \leqslant u_\alpha(t) \leqslant M \quad \text{for all } t \geqslant t_0.$$

Let $V(t) = |\ln u_\alpha(t) - \ln u_0(t)|$. For any $k = 1,2,\ldots$, we have

$$V(t_k^+) = |\ln(h_k u_\alpha(t_k)) - \ln(h_k u_0(t_k))| = V(t_k).$$

Hence, $V(t)$ is continuous for all $t \in R_+$. Then similar argument as in Lemma 2 in [3], we can prove the conclusion of Lemma 2.2 is hold. □

A special case of system (2.2) is the following logistic system

$$\begin{cases} \frac{\mathrm{d}u(t)}{\mathrm{d}t} = u(t)(a(t) - b(t)u(t)), & t \neq t_k, \\ u(t_k^+) = h_k u(t_k), & k = 1,2,\ldots, \end{cases} \tag{2.10}$$

where $a(t)$ and $b(t)$ are bounded and continuous function defined on R_{+0}. From Lemma 2.1 we obtain following result.

Lemma 2.3. *Suppose $b(t) \geq 0$ and there are positive constants ω_1 and ω_2 such that*

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_1} a(s)\mathrm{d}s + \sum_{t\leqslant t_k<t+\omega_1} \ln h_k\right) > 0,$$

$$\liminf_{t\to\infty}\int_t^{t+\omega_2} b(s)\mathrm{d}s > 0$$

and function

$$h(t,v) = \sum_{t\leqslant t_k<t+v} \ln h_k$$

is bounded on $t \in R_{+0}$ and $v \in [0,\omega_1)$. Then the conclusions of Lemma 2.1 for system (2.10) hold.

Remark 2.1. System (2.10) has been studied by Hou and his collaborators in [18]. They obtained the same result with Lemma 2.3. Therefore, we improve and extent the results in Lemma 2.1 in [18], and our result is more general.

Another special case of system (2.2) is the following linear impulsive system

$$\begin{cases} \frac{\mathrm{d}u(t)}{\mathrm{d}t} = a(t) - b(t)u, & t \neq t_k, \\ u(t_k^+) = h_k u(t_k), & k = 1,2,\ldots, \end{cases} \tag{2.11}$$

where $a(t)$ and $b(t)$ are bounded and continuous functions defined on R_{+0}. From Lemma 2.1 we obtain Lemma 2.4.

Lemma 2.4. *Assume that $a(t) \geq 0$ and there are positive constants ω_1 and ω_2 such that*

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_1} b(s)\mathrm{d}s - \sum_{t\leqslant t_k<t+\omega_1} \ln h_k\right) > 0,$$

$$\liminf_{t\to\infty}\int_t^{t+\omega_2} a(s)\mathrm{d}s > 0$$

and function

$$h(t,v)=\sum_{t\leqslant t_k<t+v}\ln h_k$$

is bounded on $t\in R_+$ and $v\in[0,\omega_1)$. Then the conclusions of Lemma 2.1 for system (2.11) hold.

3. Main result

Let $R_+^2=\{(x_1,x_2):x_1>0,x_2>0\}$. For any point $(t_0,x_0)\in R_{+0}\times R_+^2$, let $x(t,x_0)=(x_1(t,x_0),x_2(t,x_0))$ be the solution of system (1.1) with initial condition $x(t_0,x_0)=x_0$. We easily prove that $x_i(t,x_0)>0\ (i=1,2)$ on the interval of existence if the initial value $x_0\in R_+^2$. Further, let $\alpha_0>0$ be a constant. For system (1.1), we introduce the following assumptions.

(B1) Function $f_1(t,x_1,x_2)$ satisfies the following conditions.

(1) Partial derivative $\partial f_1(t,x_1,x_2)/\partial x_2$ exists and is non-positive for all $(t,x_1,x_2)\in R_{+0}\times R_+^2$.

(2) Partial derivative $\partial f_1(t,x_1,x_2)/\partial x_1$ exist on $(t,x_1,x_2)\in R_{+0}\times R_+\times[0,\alpha_0]$ and there is a nonnegative continuous function $q(t)$ and a constant $\omega>0$ satisfying $\liminf_{t\to\infty}\int_t^{t+\omega}q(s)ds>0$, and a continuous function $p(x_1)$, satisfying $p(x_1)>0$ for all $x_1>0$, such that

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_1}\leqslant -q(t)p(x_1)\quad\text{for all }(t,x_1,x_2)\in R_{+0}\times R_+\times[0,\alpha_0].$$

(3) For any constant $\sigma>1$, $f_1(t,x_1,x_2)$ is bounded on $R_{+0}\times[\sigma^{-1},\sigma]\times[0,\alpha_0]$ and there is a constant $G_1=G_1(\sigma)>0$ such that

$$|f_1(t,x_1,x_2)|\leqslant G_1\quad\text{for all }t\in R_{+0}\text{ and }0<x_i\leqslant\sigma\ (i=1,2).$$

(4) There are positive constants ω_1,ω_2,k_1 and neighborhood U of origin $(0,0)$ such that

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_1}f_1(u,x_1,x_2)du+\sum_{t\leqslant t_k<t+\omega_1}\ln h_{1k}\right)>0$$

for any point $(x_1,x_2)\in U$ with $x_i>0\ (i=1,2)$ and

$$\limsup_{t\to\infty}\left(\int_t^{t+\omega_2}f_1(u,k_1,0)du+\sum_{t\leqslant t_k<t+\omega_2}\ln h_{1k}\right)<0$$

and function

$$h_1(t,v)=\sum_{t\leqslant t_k<t+v}\ln h_{1k}$$

is bounded on $t\in R_+$ and $v\in[0,\omega_1)$.

(B2) Function $f_2(t,x_1,x_2)$ satisfying the following conditions.

(1) Partial derivative $\partial f_2(t,x_1,x_2)/\partial x_2$ exists and is non-positive for all $(t,x_1,x_2)\in R_{+0}\times R_+^2$ and for any constant $K>0,\partial f_2(t,x_1,x_2)/\partial x_2$ is bounded on $(t,x_1,x_2)\in R_{+0}\times(0,K]\times[0,\alpha_0]$.

(2) There is a constant $\beta_0>0$ such that partial derivative $\partial f_2(t,x_1,x_2)/\partial x_1$ exists and is nonnegative for all $(t,x_1,x_2)\in R_{+0}\times(0,\beta_0]\times R_+$ and for any constant $K>\beta_0$,

$$\sup\{|f_2(t,x_1,x_2)|:t\in R_{+0},\ \beta_0\leqslant x_1\leqslant K,\ 0\leqslant x_2\leqslant K\}<\infty.$$

(3) There is a constant $\omega_3>0$ and neighborhood U of origin $(0,0)$ such that

$$\limsup_{t\to\infty}\left(\int_t^{t+\omega_3}f_2(u,x_1,x_2)du+\sum_{t\leqslant t_k<t+\omega_3}\ln h_{2k}\right)<0$$

for any point $(x_1,x_2)\in U$ with $x_i>0\ (i=1,2)$, and function $h_2(t,v)=\sum_{t\leqslant t_k<t+\omega_3}\ln h_{2k}$ is bounded on $t\in R_+$ and $v\in[0,\omega_3)$.

(B3) There is a large constant $k_2>0$ such that $\limsup_{t\to\infty}x_2(t)<k_2$ for any positive solution $(x_1(t),x_2(t))$ of system (1.1).

We consider the following impulsive single-species non-autonomous Kolmogorov system

$$\begin{cases}\frac{dx_1(t)}{dt}=x_1(t)f_1(t,x_1(t),0), & t\neq t_k,\\ x_1(t_k^+)=h_{1k}x_1(t_k), & k=1,2,\ldots.\end{cases}\tag{3.1}$$

By conditions (2), (3) and (4) of (B1), we see that system (3.1) satisfies all conditions of Lemma 2.1. Hence, by Lemma 2.1, each positive solution of system (3.1) is globally asymptotically stable. Let $x_{10}(t)$ be some fixed positive solution of system (3.1). On the permanence of system (1.1) we have the following result.

Theorem 3.1. *Suppose* (B1)–(B3) *hold. If there is a constant* $\omega_0 > 0$ *such that*

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_0} f_2(u,x_{10}(u),0)\mathrm{d}u + \sum_{t\leqslant t_k<t+\omega_0}\ln h_{2k}\right) > 0,$$

then system (1.1) *is permanent.*

Proof. Let $(x_1(t),x_2(t))$ be any positive solution of system (1.1). From condition (1) of (B1) we have

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} \leqslant x_1(t)f_1(t,x_1(t),0) \quad \text{for all } t \geq 0 \text{ and } t \neq t_k.$$

By the comparison theorem of impulsive system and since $x_{10}(t)$ is the globally asymptotically stable positive solution of system (3.1), we obtain that for any constant $\varepsilon > 0$ there is a constant $T = T(\varepsilon) > 0$ such that

$$x_1(t) \leqslant x_{10}(t) + \varepsilon \quad \text{for all } t \geqslant T. \tag{3.2}$$

From this and by (B3), it follows that all positive solutions of system (1.1) are defined on R_+ and ultimately bounded with boundary $L = \max\{L_1, k_2, \alpha_0, \beta_0\}$, where constant $L_1 > \max_{t\in R} x_{10}(t)$.

Let $\omega = \max\{\omega_0,\omega_1,\omega_2,\omega_3\}$. By the boundedness of function $h_i(t,v)$ $(i = 1,2)$ on $(t,v) \in R_+ \times [0,\omega)$, we have there is a positive constant H such that

$$|h_i(t,v)| = \left|\sum_{t\leqslant t_k<t+v}\ln h_{ik}\right| \leqslant H \quad \text{for all } t \in R_{+0},\ v \in [0,\omega). \tag{3.3}$$

For any s_1, s_2 and $s_2 \geqslant s_1 \geq 0$, integrating directly system (1.1) we have

$$x_1(s_2) = x_1(s_1)\exp\left(\int_{s_1}^{s_2} f_1(t,x_1(t),x_2(t))\mathrm{d}t + \sum_{s_1\leqslant t_k<s_2}\ln h_{1k}\right) \tag{3.4}$$

and

$$x_2(s_2) = x_2(s_1)\exp\left(\int_{s_1}^{s_2} f_2(t,x_1(t),x_2(t))\mathrm{d}t + \sum_{s_1\leqslant t_k<s_2}\ln h_{2k}\right). \tag{3.5}$$

In the following, we will use four claims to complete the proof of Theorem 3.1.

Claim 1.

There is a constant $\eta > 0$ *such that* $\limsup_{t\to\infty} x_1(t) > \eta$ *for any positive solution* $(x_1(t),x_2(t))$ *of system* (1.1).

In fact, from conditions (4) of (B1) and (2), (3) of (B2), there are positive constants $T_0, \varepsilon_0, \varepsilon_1$ and μ, satisfying $\varepsilon_0\exp(\alpha_1\omega_3 + H) < \alpha_0$ and $\varepsilon_1 < \beta_0$, where $\alpha_1 = \max_{t\in R_{+0}}|f_2(t,\beta_0,0)| < \infty$, such that for all $t \geqslant T_0$

$$\int_t^{t+\omega_1} f_1(u,\varepsilon_1,\varepsilon_0\exp(\alpha_1\omega_3 + H))\mathrm{d}u + \sum_{t\leqslant t_k<t+\omega_1}\ln h_{1k} > \mu \tag{3.6}$$

and

$$\int_t^{t+\omega_3} f_2(u,\varepsilon_1,\varepsilon_0)\mathrm{d}u + \sum_{t\leqslant t_k<t+\omega_3}\ln h_{2k} < -\mu. \tag{3.7}$$

If Claim 1 is not true, then there is a positive solution $(x_1(t),x_2(t))$ of system (1.1) such that $\limsup_{t\to\infty} x_1(t) < \varepsilon_1$. Hence, there is a $T_1 \geqslant T_0$ such that $x_1(t) < \varepsilon_1$ for all $t \geqslant T_1$. Firstly, we prove that there exist a $s_1 \geqslant T_1$ such that

$$x_2(t) \leqslant \varepsilon_0\exp(\alpha_1\omega_3 + H) \quad \text{for all } t \geqslant s_1. \tag{3.8}$$

In fact, we only need to consider the following three cases about $x_2(t)$.

Case I: there is a $s_1 \geqslant T_1$ such that $x_2(t) \geqslant \varepsilon_0$ for all $t \geqslant s_1$.

Case II: there is a $s_1 \geqslant T_1$ such that $x_2(t) \leqslant \varepsilon_0$ for all $t \geqslant s_1$.

Case III: $x_2(t)$ is oscillatory about ε_0 for all $t \geqslant T_1$.

We first consider Case I. Since $x_2(t) \geqslant \varepsilon_0$ for all $t \geqslant s_1$, then by conditions (1) and (2) of (B2) and (3.5) we have

$$x_2(t) \leqslant x_2(s_1)\exp\left(\int_{s_1}^{t} f_2(u,\varepsilon_1,\varepsilon_0)\mathrm{d}u + \sum_{s_1\leqslant t_k<t}\ln h_{2k}\right) \quad \text{for all } t \geqslant s_1.$$

From this and by (3.7) it follows $\lim_{t\to\infty} x_2(t) = 0$ which leads to a contradiction.

Next, we consider Case III. From the oscillation of $x_2(t)$ about ε_0, we can choose two sequences $\{\rho_n\}$ and $\{\rho_n^*\}$ satisfying

$$T_1 < \rho_1 < \rho_1^* < \cdots < \rho_n < \rho_n^* < \cdots$$

and

$$\lim_{n\to\infty}\rho_n=\lim_{n\to\infty}\rho_n^*=\infty$$

such that

$$x_2(\rho_n)\leqslant\varepsilon_0,\quad x_2(\rho_n^+)\geqslant\varepsilon_0,\quad x_2(\rho_n^*)\geqslant\varepsilon_0,\quad x_2(\rho_n^{*+})\leqslant\varepsilon_0,$$

$$x_2(t)\geqslant\varepsilon_0\quad\text{for all } t\in(\rho_n,\rho_n^*)\qquad\text{and}\qquad x_2(t)\leqslant\varepsilon_0\quad\text{for all } t\in(\rho_n^*,\rho_{n+1}).$$

For any $t\geqslant T_1$, if $t\in(\rho_n,\rho_n^*]$ for some integer n, then we can choose integer $l\geq 0$ and constant $0\leqslant\mu_1<\omega_3$ such that $t=\rho_n+l\omega_3+\mu_1$.

Then by condition (1) and (2) of (B2), (3.5) and (3.7) it follows that

$$\begin{aligned}
x_2(t)&=x_2(\rho_n)\exp\left(\int_{\rho_n}^t f_2(s,x_1,x_2)\mathrm{d}s+\sum_{\rho_n\leqslant t_k<t}\ln h_{2k}\right)\leqslant\varepsilon_0\exp\left(\int_{\rho_n}^t f_2(t,\varepsilon_1,\varepsilon_0)\mathrm{d}t+\sum_{\rho_n\leqslant t_k<t}\ln h_{2k}\right)\\
&=\varepsilon_0\exp\left(\left(\int_{\rho_n}^{\rho_n+l\omega_3}+\int_{\rho_n+l\omega_3}^t\right)f_2(t,\varepsilon_1,\varepsilon_0)\mathrm{d}t+\left(\sum_{\rho_n\leqslant t_k<\rho_n+l\omega_3}+\sum_{\rho_n+l\omega_3\leqslant t_k<t}\right)\ln h_{2k}\right)\\
&\leqslant\varepsilon_0\exp\left(\int_{\rho_n+l\omega_3}^t f_2(t,\varepsilon_1,\varepsilon_0)\mathrm{d}t+\sum_{\rho_n+l\omega_3\leqslant t_k<t}\ln h_{2k}\right)\leqslant\varepsilon_0\exp\left(\int_{\rho_n+l\omega_3}^t f_2(t,\beta_0,0)\mathrm{d}t+\sum_{\rho_n+l\omega_3\leqslant t_k<t}\ln h_{2k}\right)\\
&\leqslant\varepsilon_0\exp(\alpha_1\omega_3+H).
\end{aligned}$$

If there is an integer n such that $t\in(\rho_n^*,\rho_{n+1}]$, then we obviously have

$$x_2(t)\leqslant\varepsilon_0<\varepsilon_0\exp(\alpha_1\omega_3+H).$$

Therefore, let $s_1=\rho_1$, for Case III we always have

$$x_2(t)\leqslant\varepsilon_0\exp(\alpha_1\omega_3+H)\quad\text{for all } t\geqslant s_1.$$

Lastly, if Case II holds, then we directly have

$$x_2(t)\leqslant\varepsilon_0\exp(\alpha_1\omega_3+H)\quad\text{for all } t\geqslant s_1.$$

Therefore, (3.8) is true.

Finally, by conditions (1) and (2) of (B1), (3.4) and (3.8) we have

$$x_1(t)\geqslant x_1(s_1)\exp\left(\int_{s_1}^t f_1(u,\varepsilon_1,\varepsilon_0\exp(\alpha_1\omega_3+H))\mathrm{d}u+\sum_{s_1\leqslant t_k<t}\ln h_{1k}\right)$$

for all $t\geqslant s_1$. From this and by (3.6) it follows $\lim_{t\to\infty}x_1(t)=\infty$ which leads to a contradiction. Therefore, Claim 1 is true.

Claim 2.

There is a constant $\gamma>0$ such that $\liminf_{t\to\infty}x_1(t)>\gamma$ for any positive solution $(x_1(t),x_2(t))$ of system (1.1).

In fact, from (3.6) and (3.7) there is a constant $P>0$ such that

$$\int_t^{t+a}f_1(u,\varepsilon_1,\varepsilon_0\exp(\alpha_1\omega_3+H))\mathrm{d}u+\sum_{t\leqslant t_k\leqslant t+a}\ln h_{1k}>\varepsilon_1 \tag{3.9}$$

and

$$L\exp\left(\int_t^{t+a}f_2(u,\varepsilon_1,\varepsilon_0)\mathrm{d}u+\sum_{t\leqslant t_k<t+a}\ln h_{2k}\right)<\varepsilon_0 \tag{3.10}$$

for all $t\geqslant T_0$ and $a\geqslant P$, where constant L is given in the above. If Claim 2 is not true, then there is a sequence of initial value $\{x_n\}\subset R_+^2$ such that for the solution $(x_1(t,x_n),x_2(t,x_n))$ of system (1.1)

$$\liminf_{t\to\infty}x_1(t,x_n)<\frac{\eta}{n^2},\quad n=1,2,\ldots,$$

where constant η is given in Claim 1. From (3.3) we have that

$$e^{-H}\leqslant h_{1k}\leqslant e^H\quad\text{for all } k=1,2,\ldots.$$

Hence, we can choose an integer $K>e^H$, for any positive solution $x(t)$ of system (1.1), if

$$x_1(t_k) \geqslant \frac{\eta}{n} \quad \text{for some } k = 1, 2, \ldots,$$

then we have

$$x_1(t_k^+) = h_{1k} x_1(t_k) \geqslant e^{-H} \frac{\eta}{n} \geqslant \frac{\eta}{n^2} \quad \text{for all } n \geqslant K,$$

and if

$$x_1(t_k) \leqslant \frac{\eta}{n^2} \quad \text{for some } k = 1, 2, \ldots,$$

then we have

$$x_1(t_k^+) = h_{1k} x_1(t_k) \leqslant e^{H} \frac{\eta}{n^2} \leqslant \frac{\eta}{n} \quad \text{for all } n \geqslant K.$$

By Claim 1 and above inequality, we obtain that there exist two time sequence $\{s_q^{(n)}\}$ and $\{t_q^{(n)}\}$ such that for each $n = K, K+1, \ldots$,

$$0 < s_1^{(n)} < t_1^{(n)} < s_2^{(n)} < t_2^{(n)} < \cdots < s_q^{(n)} < t_q^{(n)} < \cdots,$$

$$s_q^{(n)} \to \infty, \; t_q^{(n)} \to \infty \quad \text{as } q \to \infty, \tag{3.11}$$

$$x_1(s_q^{(n)}, x_n) \geqslant \frac{\eta}{n}, \quad \frac{\eta}{n^2} < x_1(s_q^{(n)+}, x_n) \leqslant \frac{\eta}{n}, \tag{3.12}$$

$$\frac{\eta}{n^2} \leqslant x_1(t_q^{(n)}, x_n) < \frac{\eta}{n}, \quad x_1(t_q^{(n)+}, x_n) \leqslant \frac{\eta}{n^2}, \tag{3.13}$$

$$\frac{\eta}{n^2} \leqslant x_1(t, x_n) \leqslant \frac{\eta}{n} \quad \text{for all } t \in (s_q^{(n)}, t_q^{(n)}). \tag{3.14}$$

From the ultimate boundedness of system (1.1), there is a $T^{(n)} \geqslant T_0$ such that $x_i(t, x_n) \leqslant L$ $(i = 1, 2)$ for all $t \geqslant T^{(n)}$. Further, there is an integer $N_1^{(n)} > 0$ such that $s_q^{(n)} > T^{(n)}$ for all $q \geqslant N_1^{(n)}$. From condition (3) of (B1), there is a constant $G_1 = G_1(L) > 0$ such that

$$|f_1(t, x_1, x_2)| \leqslant G_1$$

for all $t \in R_{+0}$, $0 < x_i \leqslant L$ $(i = 1, 2)$. Hence, from (3.4) we have

$$\begin{aligned} x_1(t_q^{(n)+}, x_n) &= x_1(s_q^{(n)}, x_n) \exp\left(\int_{s_q^{(n)}}^{t_q^{(n)}} f_1(t, x_1(t, x_n), x_2(t, x_n)) \mathrm{d}t + \sum_{s_q^{(n)} \leqslant t_k \leqslant t_q^{(n)}} \ln h_{1k} \right) = x_1(s_q^{(n)}, x_n) \\ &\quad \times \exp\left(\int_{s_q^{(n)}}^{t_q^{(n)}} f_1(t, x_1(t, x_n), x_2(t, x_n)) \mathrm{d}t - \int_{s_q^{(n)}}^{t_q^{(n)}} f_1(t, \varepsilon_1, \varepsilon_0 \exp(\alpha_1 \omega_3 + H)) \mathrm{d}t + \int_{s_q^{(n)}}^{t_q^{(n)}} f_1(t, \varepsilon_1, \varepsilon_0 \exp(\alpha_1 \omega_3 + H)) \mathrm{d}t + \sum_{s_q^{(n)} \leqslant t_k \leqslant t_q^{(n)}} \ln h_{1k} \right). \end{aligned}$$

We can choose a positive integer $l_q^{(n)}$ such that $t_q^{(n)} = s_q^{(n)} + l_q^{(n)} \omega_1 + v_q^{(n)}$, where $v_q^{(n)} \in [0, \omega_1)$. Then from (3.6) and above equality, we can obtain

$$\begin{aligned} \frac{\eta}{n^2} &\geqslant x_1(t_q^{(n)+}, x_n) \geqslant x_1(s_q^{(n)}, x_n) \exp\left(-2G_1(t_q^{(n)} - s_q^{(n)}) + \int_{s_q^{(n)} + l_q^{(n)} \omega_1}^{t_q^{(n)}} f_1(t, \varepsilon_1, \varepsilon_0 \exp(\alpha_1 \omega_3 + H)) \mathrm{d}t + \sum_{s_q^{(n)} + l_q^{(n)} \omega_1 \leqslant t_k \leqslant t_q^{(n)}} \ln h_{1k} \right) \\ &\geqslant \frac{\eta}{n} \exp(-2G_1(t_q^{(n)} - s_q^{(n)}) - G_1 \omega_1 - 2H). \end{aligned}$$

Consequently, we have

$$t_q^{(n)} - s_q^{(n)} \geqslant \frac{\ln n - G_1 \omega_1 - 2H}{2G_1} \quad \text{for all } q \geqslant N_1^{(n)}, \; n = K, K+1, \ldots.$$

Thus, there is an integer $N_0 > K$ such that $\eta / N_0 < \varepsilon_1$ and

$$t_q^{(n)} - s_q^{(n)} > 2P \quad \text{for all } n \geqslant N_0, \; q \geqslant N_1^{(n)}.$$

For any $n > N_0$ and $q \geqslant N_1^{(n)}$, if $x_2(t, x_n) \geqslant \varepsilon_0$ for all $t \in [s_q^{(n)}, s_q^{(n)} + P]$, then by condition (1) and (2) of (B2), (3.5), (3.10) and (3.14) we obtain

$$\begin{aligned}\varepsilon_0 \leqslant x_2(s_q^{(n)}+P,x_n) &= x_2(s_q^{(n)},x_n)\exp\left(\int_{s_q^{(n)}}^{s_q^{(n)}+P} f_2(t,x_1(t,x_n),x_2(t,x_n))\mathrm{d}t + \sum_{s_q^{(n)}\leqslant t_k<s_q^{(n)}+P}\ln h_{2k}\right)\\ &\leqslant L\exp\left(\int_{s_q^{(n)}}^{s_q^{(n)}+P} f_2(t,\varepsilon_1,\varepsilon_0)\mathrm{d}t + \sum_{s_q^{(n)}\leqslant t_k<s_q^{(n)}+P}\ln h_{2k}\right) < \varepsilon_0.\end{aligned}$$

This leads to a contradiction. Hence, there is a $s_1 \in [s_q^{(n)}, s_q^{(n)}+P]$ such that $x_2(s_1,x_n) < \varepsilon_0$. We now prove that

$$x_2(t,x_n) \leqslant \varepsilon_0\exp(\alpha_1\omega_3 + H) \quad \text{for all } t \in [s_1, t_q^{(n)}]. \tag{3.15}$$

In fact, if there is a $s_2 \in [s_1, t_q^{(n)}]$ such that $x_2(s_2,x_n) > \varepsilon_0\exp(\alpha_1\omega_3+H)$, then there is a $s_3 \in [s_1,s_2)$ such that

$$x_2(s_3,x_n) \leqslant \varepsilon_0, \quad x_2(s_3^+,x_n) \geqslant \varepsilon_0 \quad \text{and} \quad x_2(t,x_n) \geqslant \varepsilon_0 \quad \text{for all } t \in (s_3,s_2].$$

Choose an integer $p \geq 0$ such that $s_2 \in [s_3+p\omega_3, s_3+(p+1)\omega_3)$, then from conditions (1) and (2) of (B2), (3.5), (3.7) and (3.14) we have

$$\begin{aligned}x_2(s_2,x_n) &= x_2(s_3,x_n)\exp\left(\int_{s_3}^{s_2} f_2(t,x_1(t,x_n),x_2(t,x_n))\mathrm{d}t + \sum_{s_3\leqslant t_k<s_2}\ln h_{2k}\right)\\ &\leqslant \varepsilon_0\exp\left(\left(\int_{s_3}^{s_3+p\omega_3} + \int_{s_3+p\omega_3}^{s_2}\right) f_2(t,\varepsilon_1,\varepsilon_0)\mathrm{d}t + \left(\sum_{s_3\leqslant t_k<s_3+p\omega_3} + \sum_{s_3+p\omega_3\leqslant t_k<s_2}\right)\ln h_{2k}\right)\\ &\leqslant \varepsilon_0\exp\left(\int_{s_3+p\omega_3}^{s_2} f_2(t,\varepsilon_1,\varepsilon_0)\mathrm{d}t + \sum_{s_3+p\omega_3\leqslant t_k<s_2}\ln h_{2k}\right) \leqslant \varepsilon_0\exp\left(\int_{s_3+p\omega_3}^{s_2} f_2(t,\beta_0,0)\mathrm{d}t + H\right) \leqslant \varepsilon_0\exp(\alpha_1\omega_3+H).\end{aligned}$$

This leads to a contradiction. Therefore, (3.15) is true.

Finally, since

$$x_2(t,x_n) \leqslant \varepsilon_0\exp(\alpha_1\omega_3+H) \quad \text{for all } t \in [s_q^{(n)}+P, t_q^{(n)}],$$

by conditions (1) and (2) of (B1), (3.4), (3.9) and (3.14) we obtain

$$\begin{aligned}\frac{\eta}{n^2} &\geqslant x_1(t_q^{(n)+},x_n) = x_1(s_q^{(n)}+P,x_n)\exp\left(\int_{s_q^{(n)}+P}^{t_q^{(n)}} f_1(t,x_1(t,x_n),x_2(t,x_n))\mathrm{d}t + \sum_{s_q^{(n)}+P\leqslant t_k\leqslant t_q^{(n)}}\ln h_{1k}\right)\\ &\geqslant \frac{\eta}{n^2}\exp\left(\int_{s_q^{(n)}+P}^{t_q^{(n)}} f_1(t,\varepsilon_1,\varepsilon_0\exp(\alpha_1\omega_3+H))\mathrm{d}t + \sum_{s_q^{(n)}+P\leqslant t_k\leqslant t_q^{(n)}}\ln h_{1k}\right) > \frac{\eta}{n^2}.\end{aligned}$$

This leads to a contradiction. Therefore, Claim 2 is true.

Claim 3.

There is a constant $\alpha > 0$ such that $\limsup_{t\to\infty} x_2(t) > \alpha$ for any positive solution $(x_1(t),x_2(t))$ of system (1.1).

In fact, by

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_0} f_2(s,x_{10}(s),0)\mathrm{d}s + \sum_{t\leqslant t_k<t+\omega_0}\ln h_{2k}\right) > 0,$$

there are positive constants T_0, $\varepsilon_0 < \alpha_0$ and δ_0 such that for any continuous function $u(t)$ defined on R_{+0}, satisfying

$$|u(t) - x_{10}(t)| < \varepsilon_0 \quad \text{for all } t \geqslant T_0,$$

one has

$$\int_t^{t+\omega_0} f_2(s,u(s),\varepsilon_0)\mathrm{d}s + \sum_{t\leqslant t_k<t+\omega_0}\ln h_{2k} \geqslant \delta_0 \quad \text{for all } t \geqslant T_0. \tag{3.16}$$

Consider the following system

$$\begin{cases}\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = x_1(t)f_1(t,x_1(t),\alpha), & t \neq t_k,\\ x_1(t_k^+) = h_{1k}x_1(t_k), & k = 1,2,\ldots,\end{cases} \tag{3.17}$$

where $\alpha \in (0, \alpha_0]$ is a parameter. Let $x_{1\alpha}(t)$ be the solution of system (3.17) with initial value $x_{1\alpha}(0) = x_{10}(0)$. Since $f_1(t, x_1, \alpha)$ satisfies (A1)–(A4), by Lemma 2.1, $x_{1\alpha}(t)$ is globally asymptotically stable. Further, by Lemma 2.2, we obtain that $x_{1\alpha}(t)$ uniformly for $t \in R_+$ converges to $x_{10}(t)$ as $\alpha \to 0$. Hence, there is a $\alpha > 0$ and $\alpha < \varepsilon_0$ such that

$$x_{1\alpha}(t) > x_{10}(t) - \frac{\varepsilon_0}{2} \quad \text{for all } t \in R_{+0}. \tag{3.18}$$

If Claim 3 is not true, then there is a positive solution $(x_1(t), x_2(t))$ of system (1.1) such that $\limsup_{t\to\infty} x_2(t) < \alpha$. Hence, there is a $T_1 > T_0$ such that $x_2(t) < \alpha$ for all $t \geqslant T_1$. Since

$$\frac{dx_1(t)}{dt} \geqslant x_1(t) f_1(t, x_1(t), \alpha) \quad \text{for all } t \geqslant T_1 \text{ and } t \neq t_k,$$

by the comparison theorem of impulsive differential system and global asymptotic stability of solution $x_{1\alpha}(t)$, we obtain that there is a $T_2 \geqslant T_1$ such that

$$x_1(t) \geqslant x_{1\alpha}(t) - \frac{\varepsilon_0}{2} \quad \text{for all } t \geqslant T_2. \tag{3.19}$$

On the other hand, by (3.2) there is a $T_3 \geqslant T_2$ such that

$$x_1(t) \leqslant x_{10}(t) + \varepsilon_0 \quad \text{for all } t \geqslant T_3.$$

Hence, from (3.18) and (3.19) it follows that

$$|x_1(t) - x_{10}(t)| < \varepsilon_0 \quad \text{for all } t \geqslant T_3. \tag{3.20}$$

By (3.5) and condition (1) of (B2) we obtain

$$x_2(t) \geqslant x_2(T_3) \exp\left(\int_{T_3}^{t} f_2(s, x_1(s), \varepsilon_0) ds + \sum_{T_3 \leqslant t_k < t} \ln h_{2k} \right)$$

Thus, from (3.16) and (3.20) we finally obtain $\lim_{t\to\infty} x_2(t) = \infty$ which leads to a contradiction. Therefore, Claim 3 is true.

Claim 4.

There is a constant $\beta > 0$ such that $\liminf_{t\to\infty} x_2(t) > \beta$ for any positive solution $(x_1(t), x_2(t))$ of system (1.1).

In fact, if Claim 4 is not true, then there is a sequence of initial value $\{x_n\} \subset R_+^2$ such that, for the solution $(x_1(t, x_n), x_2(t, x_n))$ of system (1.1),

$$\liminf_{t\to\infty} x_2(t, x_n) < \frac{\alpha}{n}, \quad n = 1, 2, \ldots,$$

where constant α is given in Claim 3. From (3.3) we have that

$$e^{-H} \leqslant h_{2k} \leqslant e^{H} \quad \text{for all } k = 1, 2, \ldots.$$

Hence, we can choose an integer $K > e^H$, for any positive solution $x(t)$ of system (1.1), if

$$x_2(t_k) \geqslant \alpha \quad \text{for some } k = 1, 2, \ldots,$$

then we have

$$x_2(t_k^+) = h_{2k} x_2(t_k) \geqslant e^{-H}\alpha > \frac{\alpha}{n} \quad \text{for all } n \geqslant K,$$

and if

$$x_2(t_k) \leqslant \frac{\alpha}{n} \quad \text{for some } k = 1, 2, \ldots,$$

then we have

$$x_2(t_k^+) = h_{2k} x_2(t_k) \leqslant e^{H} \frac{\alpha}{n} < \alpha \quad \text{for all } n \geqslant K.$$

By Claim 3, for every $n = K, K+1, \ldots$ there are two time sequence $\{s_q^{(n)}\}$ and $\{t_q^{(n)}\}$, satisfying

$$0 < s_1^{(n)} < t_1^{(n)} < s_2^{(n)} < t_2^{(n)} < \cdots < s_q^{(n)} < t_q^{(n)} < \cdots$$

and $\lim_{q\to\infty} s_q^{(n)} = \infty$, such that

$$x_2(s_q^{(n)}, x_n) \geqslant \alpha, \quad \frac{\alpha}{n} < x_2(s_q^{(n)+}, x_n) \leqslant \alpha, \tag{3.21}$$

$$\frac{\alpha}{n} \leqslant x_2(t_q^{(n)}, x_n) < \alpha, \quad x_2(t_q^{(n)+}, x_n) \leqslant \frac{\alpha}{n}, \tag{3.22}$$

$$\frac{\alpha}{n} \leqslant x_2(t, x_n) \leqslant \alpha \quad \text{for all } t \in (s_q^{(n)}, t_q^{(n)}). \tag{3.23}$$

From (3.2), Claim 2 and the ultimate boundedness of system (1.1), we obtain that for every n there is a $T^{(n)} > T_0$ such that

$$\gamma \leqslant x_1(t, x_n) \leqslant x_{10}(t) + \varepsilon_0, \quad \gamma \leqslant x_{10}(t) \tag{3.24}$$

and $x_i(t, x_n) \leqslant L\ (i = 1, 2)$ for all $t \geqslant T^{(n)}$, where constants ε_0 and T_0 is given in (3.16) and constant γ is given in Claim 2. Further, for every n there is an integer $N_1^{(n)} > 0$ such that $s_q^{(n)} > T^{(n)}$ for all $q \geqslant N_1^{(n)}$. By condition (2) of (B2) there is a constant $G_2 > 0$ such that

$$|f_2(t, x_1, x_2)| \leqslant G_2$$

for all $t \in R_{+0}$, $\gamma \leqslant x_1 \leqslant L$ and $0 \leqslant x_2 \leqslant L$. Hence, from (3.5), (3.16) and (3.24) we obtain

$$x_2(t_q^{(n)+}, x_n) = x_2(s_q^{(n)}, x_n) \exp\left(\int_{s_q^{(n)}}^{t_q^{(n)}} f_2(t, x_1(t, x_n), x_2(t, x_n))\mathrm{d}t + \sum_{s_q^{(n)} \leqslant t_k \leqslant t_q^{(n)}} \ln h_{2k}\right) = x_2(s_q^{(n)}, x_n)$$
$$\times \exp\left(\int_{s_q^{(n)}}^{t_q^{(n)}} (f_2(t, x_1(t, x_n), x_2(t, x_n)) - f_2(t, x_{10}(t), \varepsilon_0))\mathrm{d}t + \int_{s_q^{(n)}}^{t_q^{(n)}} f_2(t, x_{10}(t), \varepsilon_0)\mathrm{d}t + \sum_{s_q^{(n)} \leqslant t_k \leqslant t_q^{(n)}} \ln h_{2k}\right).$$

We can choose an integer $l_q^{(n)}$ such that $t_q^{(n)} = s_q^{(n)} + l_q^{(n)}\omega_0 + v_q^{(n)}$, where $v_q^{(n)} \in [0, \omega_0)$. Then from (3.16) and above equality, we can obtain

$$\frac{\alpha}{n} \geqslant x_2(t_q^{(n)+}, x_n) \geqslant x_2(s_q^{(n)}, x_n) \exp\left(-2G_2(t_q^{(n)} - s_q^{(n)}) + \int_{s_q^{(n)} + l_q^{(n)}\omega_0}^{t_q^{(n)}} f_2(t, x_{10}(t), \varepsilon_0)\mathrm{d}t + \sum_{s_q^{(n)} + l_q^{(n)}\omega_0 \leqslant t_k \leqslant t_q^{(n)}} \ln h_{2k}\right)$$
$$\geqslant \alpha \exp(-2G_2(t_q^{(n)} - s_q^{(n)}) - G_2\omega_0 - 2H).$$

Consequently, we have

$$t_q^{(n)} - s_q^{(n)} \geqslant \frac{\ln n - G_2\omega_0 - 2H}{2G_2} \quad \text{for all } q \geqslant N_1^{(n)},\ n = K, K+1, \ldots.$$

By (3.16) there are positive constants P and r such that

$$\int_t^{t+a} f_2(s, u(s), \varepsilon_0)\mathrm{d}s + \sum_{t \leqslant t_k \leqslant t+a} \ln h_{2k} \geqslant r, \tag{3.25}$$

for all $t \in R_{+0}$ and $a \geqslant P$.

Let $\bar{x}_{1\alpha}(t)$ be the solution of system (3.17) with the initial condition $\bar{x}_{1\alpha}(s_q^{(n)}) = x_1(s_q^{(n)}, x_n)$. Since for any n, q and $t \in [s_q^{(n)}, t_q^{(n)}]$ we have $x_2(t, x_n) \leqslant \alpha$ and

$$\frac{\mathrm{d}x_1(t, x_n)}{\mathrm{d}t} \geqslant x_1(t, x_n) f_1(t, x_1(t, x_n), \alpha),$$

by the comparison theorem of impulsive system it follows that

$$x_1(t, x_n) \geqslant \bar{x}_{1\alpha}(t) \quad \text{for all } t \in [s_q^{(n)}, t_q^{(n)}]. \tag{3.26}$$

By Lemma 2.1, the solution $x_{1\alpha}(t)$ is globally uniformly asymptotically stable. From (3.24) we obtain

$$\gamma \leqslant x_1(s_q^{(n)}, x_n) \leqslant L \quad \text{for all } q \geqslant N_1^{(n)}.$$

Hence, there is a constant $T_2 \geqslant P$ and T_2 is independent of any n and $q \geqslant N_1^{(n)}$, such that

$$\bar{x}_{1\alpha}(t) > x_{1\alpha}(t) - \frac{\varepsilon_0}{2} \quad \text{for all } t \geqslant s_q^{(n)} + T_2. \tag{3.27}$$

Choose an integer $K_0 \geqslant K$ such that $n \geqslant K_0$ and $q \geqslant N_1^{(n)}$,

$$t_q^{(n)} - s_q^{(n)} > 2T_2.$$

Further, from (3.18), (3.24), (3.26) and (3.27) we obtain

$$|x_1(t, x_n) - x_{10}(t)| < \varepsilon_0 \tag{3.28}$$

for all $t \in [s_q^{(n)} + T_2, t_q^{(n)}]$. Hence, when $n \geqslant K_0$ and $q \geqslant N_1^{(n)}$, by (3.5), (3.23) and condition (1) of (B2) it follows

$$x_2(t_q^{(n)+}, x_n) \geqslant x_2(s_q^{(n)} + T_2, x_n) \exp\left(\int_{s_q^{(n)}+T_2}^{t_q^{(n)}} f_2(t, x_1(t, x_n), \varepsilon_0)dt + \sum_{s_q^{(n)}+T_2 \leqslant t_k \leqslant t_q^{(n)}} \ln h_{2k}\right).$$

Finally, from (3.21), (3.23), (3.25) and (3.28) we have

$$\frac{\alpha}{n} \geqslant \frac{\alpha}{n} \exp\left(\int_{s_q^{(n)}+T_2}^{t_q^{(n)}} f_2(t, x_1(t, x_n), \varepsilon_0)dt + \sum_{s_q^{(n)}+T_2 \leqslant t_k \leqslant t_q^{(n)}} \ln h_{2k}\right) > \frac{\alpha}{n},$$

which leads to contradiction. Therefore, Claim 4 is true.

Finally, from Claims 1–4 we see that Theorem 3.1 is proved and this completes the proof of Theorem 3.1. □

Remark 3.1. If system (1.1) without the impulsive effective, that is $h_{ik} = 1$ for all $i = 1, 2$ and $k = 1, 2, \ldots$, then Theorem 3.1 is the same with the Theorem 1 in [3]. Therefore, our result extents the corresponding results on the permanence for the general non-autonomous predator–prey Kolmogorov systems in [3].

In Theorem 3.1, we assume that component $x_2(t)$ of all positive solutions $(x_1(t), x_2(t))$ of system (1.1) is ultimately bounded, i.e. (B3). In the following we will establish a criterion which assures the ultimate boundedness of positive solutions of system (1.1). We first introduce the following assumptions:

(B4) There is a constant $\omega_4 > 0$ such that for any positive constant $M > \beta_0$ there is a large $s = s(M) > 0$ such that

$$\limsup_{t\to\infty}\left(\int_t^{t+\omega_4} \max_{\beta_0 \leqslant x_1 \leqslant M} f_2(u, x_1, s)du + \sum_{t \leqslant t_k < t+\omega_4} \ln h_{2k}\right) < 0.$$

(B5) The function $f_1(t, x_1, x_2)$ is continuous for all $(t, x_1, x_2) \in R_{+0} \times R_{+0}^2$ and there is a constant $\omega_5 > 0$ such that for any positive constant M, ε and $M > \varepsilon$ there is a large $s = s(M, \varepsilon) > 0$ such that

$$\limsup_{t\to\infty}\left(\int_t^{t+\omega_5} \max_{\varepsilon \leqslant x_1 \leqslant M} f_1(u, x_1, s)du + \sum_{t \leqslant t_k < t+\omega_5} \ln h_{1k}\right) < 0.$$

Furthermore, for any constant $G > 0$

$$\sup\{|f_2(t, x_1, 0)| : t \in R_+, 0 \leqslant x_1 \leqslant G\} < \infty.$$

On the ultimate boundedness of system (1.1) we have the following result.

Theorem 3.2. *Suppose that* (B1) *and* (B2) *hold. If* (B4) *or* (B5) *holds, then system* (1.1) *is ultimately bounded.*

Proof. Choose a constant L_1 such that $L_1 > \max\{\beta_0, \max_{t\in R} x_{10}(t)\}$, where $x_{10}(t)$ is some fixed positive solution of system (3.1) and constant β_0 is given in condition (2) of (B2). For any positive solution $(x_1(t), x_2(t))$ of system (1.1), by (3.2) there is a large $T_0 > 0$ such that

$$x_1(t) \leqslant L_1 \quad \text{for all } t \geqslant T_0. \tag{3.29}$$

Let $\omega = \max\{\omega_i : i = 0, \ldots, 5\}$, by the boundedness of function $h_i(t, v)$ $(i = 1, 2)$ on $(t, v) \in R_+ \times [0, \omega)$, we have there is positive constant H such that (3.3) hold.

Now, we consider component $x_2(t)$. Firstly, we let (B4) hold. From (B4), for constant L_1, there are $s = s(L_1) > 0$, $T_1 \geqslant T_0$ and $\delta > 0$ such that

$$\int_t^{t+\omega_4} \max_{\beta_0 \leqslant x_1 \leqslant L_1} f_2(u, x_1, s)du + \sum_{t \leqslant t_k < t+\omega_4} \ln h_{2k} < -\delta \quad \text{for all } t \geqslant T_1. \tag{3.30}$$

Firstly, we prove that there exist a $s_1 \geqslant T_1$ such that

$$x_2(t) \leqslant s\exp(\alpha_2\omega_4 + H) \quad \text{for all } t \geqslant s_1, \tag{3.31}$$

where $\alpha_2 = \max\{f_2(t, x_1, s) : t \in R_{+0},\ \beta_0 \leqslant x_1 \leqslant L_1\}$. In fact, we only need to consider the following three cases for $x_2(t)$,

Case I: there is a $s_1 \geqslant T_1$ such that $x_2(t) \geqslant s$ for all $t \geqslant s_1$.

Case II: there is a $s_1 \geqslant T_1$ such that $x_2(t) \leqslant s$ for all $t \geqslant s_1$.

Case III: $x_2(t)$ is oscillatory about s for all $t \geqslant T_1$.

We first consider Case I. Since $x_2(t) \geqslant s$ for all $t \geqslant s_1$, then by condition (1) and (2) of (B2) and (3.5) we can obtain

$$x_2(t) \leqslant x_2(s_1)\exp\left(\int_{s_1}^t f_2(u, x_1(u), s)du + \sum_{s_1 \leqslant t_k < t} \ln h_{2k}\right) \leqslant x_2(s_1)\exp\left(\int_{s_1}^t \max_{\beta_0 \leqslant x_1 \leqslant L_1} f_2(u, x_1, s)du + \sum_{s_1 \leqslant t_k < t} \ln h_{2k}\right)$$

for all $t \geqslant s_1$. From this and (3.30) it follows that $\lim_{t\to\infty} x_2(t) = 0$ which leads a contradiction.

Next, we consider Case III. From the oscillation of $x_2(t)$ about s, we can choose two sequences $\{\rho_n\}$ and $\{\rho_n^*\}$ satisfying $T_1 < \rho_1 < \rho_1^* < \cdots < \rho_n < \rho_n^* < \cdots$ and $\lim_{n\to\infty}\rho_n = \lim_{n\to\infty}\rho_n^* = \infty$ such that

$$x_2(\rho_n) \leqslant s,\ x_2(\rho_n^+) \geqslant s, \quad x_2(\rho_n^*) \geqslant s, \quad x_2(\rho_n^{*+}) \leqslant s,$$

$$x_2(t) \geqslant s \quad \text{for all } t \in (\rho_n, \rho_n^*) \qquad \text{and} \qquad x_2(t) \leqslant s \quad \text{for all } t \in (\rho_n^*, \rho_{n+1}).$$

For any $t \geqslant T_1$, if $t \in (\rho_n, \rho_n^*]$ for some integer n, then we can choose integer $l \geq 0$ and constant $0 \leqslant \mu_1 < \omega_4$ such that $t = \rho_n + l\omega_4 + \mu_1$. Then by condition (1) and (2) of (B2), (3.5) and (3.30) we have

$$\begin{aligned}
x_2(t) &\leqslant s\ \exp\left(\left(\left(\int_{\rho_n}^{\rho_n+\omega_4} + \cdots + \int_{\rho_n+(l-1)\omega_4}^{\rho_n+l\omega_4} + \int_{\rho_n+l\omega_4}^{t}\right) \max_{\beta_0\leqslant x_1\leqslant L_1} f_2(t,x_1,s)\mathrm{d}t\right.\right. \\
&\quad \left. + \left(\sum_{\rho_n\leqslant t_k<\rho_n+\omega_4} + \cdots + \sum_{\rho_n+(l-1)\omega_4\leqslant t_k<\rho_n+l\omega_4} + \sum_{\rho_n+l\omega_4\leqslant t_k<t}\right)\ln h_{2k}\right) \\
&\leqslant s\ \exp\left(\int_{\rho_n+l\omega_4}^{t} \max_{\beta_0\leqslant x_1\leqslant L_1} f_2(t,x_1,s)\mathrm{d}t + \sum_{\rho_n+l\omega_4\leqslant t_k<t} \ln h_{2k}\right) \leqslant s\exp(\alpha_2\omega_4 + H).
\end{aligned}$$

If there is an integer n such that $t \in (\rho_n^*, \rho_{n+1}]$, then we obviously have

$$x_2(t) \leqslant s < s\exp(\alpha_2\omega_4 + H).$$

Therefore, let $s_1 = \rho_1$, for Case III we always have

$$x_2(t) \leqslant s\exp(\alpha_2\omega_4 + H) \quad \text{for all } t \geqslant s_1.$$

Lastly, if Case II holds, then we directly have

$$x_2(t) \leqslant s\exp(\alpha_2\omega_4 + H) \quad \text{for all } t \geqslant s_1.$$

Therefore, (3.31) is true.

Next, we let (B5) hold. Let $\alpha_3 = \max\{|f_1(t,x_1,0)| : t \in R_{+0}, 0 \leqslant x_1 \leqslant L_1\}$. By condition (3) of (B2) there are positive constant $T_1, \delta_0, \varepsilon_0, \varepsilon_1$, $\varepsilon_0\exp(\alpha_3\omega_5 + H) < \beta_0$ and $\varepsilon_1 < L_1$ such that

$$\int_t^{t+\omega_3} f_2(u, \varepsilon_0\exp(\alpha_3\omega_5 + H), \varepsilon_1)\mathrm{d}u + \sum_{t\leqslant t_k<t+\omega_3} \ln h_{2k} < -\delta_0 \quad \text{for all } t \geqslant T_1. \tag{3.32}$$

Further, by (B5) there are constant $s > L_1$, $T_2 \geqslant T_1$ and δ_1 such that

$$\int_t^{t+\omega_5} \max_{\varepsilon_0\leqslant x_1\leqslant L_1} f_1(u,x_1,s)\mathrm{d}u + \sum_{t\leqslant t_k<t+\omega_5} \ln h_{1k} < -\delta_1 \quad \text{for all } t \geqslant T_2. \tag{3.33}$$

We first prove $\liminf_{t\to\infty} x_2(t) \leqslant s$ for any positive solution $(x_1(t), x_2(t))$ of system (1.1). In fact, if $\liminf_{t\to\infty} x_2(t) > s$, then there is a $T_3 \geqslant \max\{T_0, T_2\}$ such that $x_2(t) \geqslant s$ for all $t \geqslant T_3$. If $x_1(t) \geqslant \varepsilon_0$ for all $t \geqslant T_3$, then by condition (1) of (B1), (3.4) and (3.29) we have

$$x_1(t) \leqslant x_1(T_3)\exp\left(\int_{T_3}^t f_1(u,x_1(u),s)\mathrm{d}u + \sum_{T_3\leqslant t_k<t} \ln h_{1k}\right) \leqslant x_1(T_3)\exp\left(\int_{T_3}^t \max_{\varepsilon_0\leqslant x_1\leqslant L_1} f_1(u,x_1,s)\mathrm{d}u + \sum_{T_3\leqslant t_k<t} \ln h_{1k}\right)$$

for all $t \geqslant T_3$. Consequently, by (3.33) it follows $\lim_{t\to\infty} x_1(t) = 0$, which leads to a contradiction. Hence, there is a $s_1 \geqslant T_3$ such that $x_1(s_1) < \varepsilon_0$.

If there is a $s_2 > s_1$ such that $x_1(s_2) > \varepsilon_0\exp(\alpha_3\omega_5 + H)$, then there is a $s_3 \in (s_1, s_2)$ such that

$$x_1(s_3) \leqslant \varepsilon_0, \quad x_1(s_3^+) \geqslant \varepsilon_0 \quad \text{and} \quad x_1(t) \geqslant \varepsilon_0 \quad \text{for all } t \in (s_3, s_2].$$

Choose an integer $p \geq 0$ such that $s_2 \in [s_3 + p\omega_5, s_3 + (p+1)\omega_5)$, then by condition (1) of (B1), (3.5) and (3.33) we have

$$\begin{aligned}
x_1(s_2) &= x_1(s_3)\exp\left(\int_{s_3}^{s_2} f_1(t,x_1(t),x_2(t))\mathrm{d}t + \sum_{s_3\leqslant t_k<s_2} \ln h_{1k}\right) \\
&\leqslant \varepsilon_0\exp\left(\left(\left(\int_{s_3}^{s_3+\omega_5} + \cdots + \int_{s_3+(p-1)\omega_5}^{s_3+p\omega_5} + \int_{s_3+p\omega_5}^{s_2}\right) \max_{\varepsilon_0\leqslant x_1\leqslant L_1} f_1(t,x_1,s)\mathrm{d}t\right.\right. \\
&\quad \left. + \left(\sum_{s_3\leqslant t_k<s_3+\omega_5} + \cdots + \sum_{s_3+(p-1)\omega_5\leqslant t_k<s_3+p\omega_5} + \sum_{s_3+p\omega_5\leqslant t_k<s_2}\right)\ln h_{1k}\right) \leqslant \varepsilon_0\exp(\alpha_3\omega_5 + H),
\end{aligned}$$

which is contradictory. Hence, we have $x_1(t) \leqslant \varepsilon_0 \exp(\alpha_3\omega_5 + H)$ for all $t \geqslant s_1$. From this, by conditions (1) and (2) of (B2) and (3.5) we have

$$x_2(t) \leqslant x_2(s_1)\exp\left(\int_{s_1}^{t} f_2(u, \varepsilon_0 \exp(\alpha_3\omega_5 + H), s)\mathrm{d}u + \sum_{s_1 \leqslant t_k < t} \ln h_{2k}\right) \quad \text{for all } t \geqslant s_1.$$

Therefore, by (3.32) it follows $\lim_{t\to\infty} x_2(t) = 0$ which leads a contradiction.

Further, we prove that there is a constant $L_2 > 0$ such that for any positive solution $(x_1(t), x_2(t))$ of system (1.1),

$$\limsup_{t\to\infty} x_2(t) \leqslant L_2. \tag{3.34}$$

In fact, if (3.34) is not true, then there is a sequence of initial value $\{x_n\} \subset R_+^2$ such that for the solution $(x_1(t,x_n), x_2(t,x_n))$ of system (1.1)

$$\limsup_{t\to\infty} x_2(t, x_n) > (2s+1)n, \quad n = 1, 2, \ldots.$$

From (3.3) we have that

$$e^{-H} \leqslant h_{2k} \leqslant e^{H} \quad \text{for all } k = 1, 2, \ldots.$$

Hence, we can choose an integer $K > e^H$, for any positive solution $x(t)$ of system (1.1), if

$$x_2(t_k) \geqslant (2s+1)n \quad \text{for some } k = 1, 2, \ldots,$$

then we have

$$x_2(t_k^+) = h_{1k}x_1(t_k) \geqslant e^{-H}(2s+1)n > 2s \quad \text{for all } n \geqslant K,$$

and if

$$x_2(t_k) \leqslant 2s \quad \text{for some } k = 1, 2, \ldots,$$

then we have

$$x_2(t_k^+) = h_{2k}x_2(t_k) \leqslant e^H 2s < (2s+1)n \quad \text{for all } n \geqslant K.$$

Hence, for every n there are two time sequence $\{s_q^{(n)}\}$ and $\{t_q^{(n)}\}$, such that for each $n = K, K+1, \ldots$

$$0 < s_1^{(n)} < t_1^{(n)} < s_2^{(n)} < t_2^{(n)} < \cdots < s_q^{(n)} < t_q^{(n)} < \cdots,$$

$$s_q^{(n)} \to \infty, \quad t_q^{(n)} \to \infty \quad \text{as } q \to \infty,$$

$$x_2(s_q^{(n)}, x_n) \leqslant 2s, \quad (2s+1)n > x_2(s_q^{(n)+}, x_n) \geqslant 2s, \tag{3.35}$$

$$2s < x_2(t_q^{(n)}, x_n) \leqslant (2s+1)n, \quad x_2(t_q^{(n)+}, x_n) \geqslant (2s+1)n, \tag{3.36}$$

$$2s \leqslant x_2(t, x_n) \leqslant (2s+1)n \quad \text{for all } t \in (s_q^{(n)}, t_q^{(n)}). \tag{3.37}$$

For every n we can choose $T^{(n)} > T_0$ such that

$$x_1(t, x_n) \leqslant L_1 \quad \text{for all } t \geqslant T^{(n)}.$$

Further, there is an integer $N_1^{(n)} > 0$ such that $s_q^{(n)} > T^{(n)}$ for all $q \geqslant N_1^{(n)}$. From condition (2) of (B2), there is a constant $G_2 = G_2(L_1) > 0$ such that

$$|f_2(t, x_1, x_2)| \leqslant G_2 \quad \text{and} \quad |f_2(t, \varepsilon_0 \exp(\alpha_3\omega_3 + H), \varepsilon_1)| \leqslant G_2$$

for all $t \in R_{+0}$, $\beta_0 \leqslant x_1 \leqslant L_1$, $0 \leqslant x_2 \leqslant L_1$. By conditions (1) and (2) of (B2) and (3.5) we have

$$\begin{aligned} x_2(t_q^{(n)+}, x_n) &= x_2(s_q^{(n)}, x_n)\exp\left(\int_{s_q^{(n)}}^{t_q^{(n)}} f_2(t, x_1(t,x_n), x_2(t,x_n))\mathrm{d}t + \sum_{s_q^{(n)} \leqslant t_k \leqslant t_q^{(n)}} \ln h_{2k}\right) \\ &\leqslant x_2(s_q^{(n)}, x_n)\exp\left(\int_{s_q^{(n)}}^{t_q^{(n)}} (f_2(u, x_1(t,x_n), 0) - f_2(u, \varepsilon_0 \exp(\alpha_3\omega_5 + H), \varepsilon_1))\mathrm{d}u + \int_{s_q^{(n)}}^{t_q^{(n)}} f_2(u, \varepsilon_0 \exp(\alpha_3\omega_5 + H), \varepsilon_1)\mathrm{d}u + \sum_{s_q^{(n)} \leqslant t_k \leqslant t_q^{(n)}} \ln h_{2k}\right). \end{aligned}$$

We can choose an integer $l_q^{(n)}$ such that $t_q^{(n)} = s_q^{(n)} + l_q^{(n)}\omega_3 + v_q^{(n)}$, where $v_q^{(n)} \in [0, \omega_3)$. Then from (3.32), (3.35) and (3.36)

$$(2s+1)n \leqslant x_2(t_q^{(n)+},x_n) \leqslant 2s\exp\left(2G_2(t_q^{(n)}-s_q^{(n)}) + \int_{s_q^{(n)}+l_q^{(n)}\omega_3}^{t_q^{(n)}} f_2(t,\varepsilon_0\exp(\alpha_3\omega_3+H),\varepsilon_1)du + \sum_{s_q^{(n)}+l_q^{(n)}\omega_3\leqslant t_k\leqslant t_q^{(n)}} \ln h_{2k}\right)$$
$$\leqslant 2s\exp(2G_2(t_q^{(n)}-s_q^{(n)}) + G_2\omega_3 + 2H).$$

Consequently,

$$t_q^{(n)} - s_q^{(n)} \geqslant \frac{\ln n - G_2\omega_3 - 2H}{2G_2} \quad \text{for all } q \geqslant N_1^{(n)}.$$

By condition (1) of (B1), (3.32) and (3.33) there is a constant $P > 0$ such that

$$\int_t^{t+a} f_2(u,\varepsilon_0\exp(\alpha_3\omega_5+H),s)du + \sum_{t\leqslant t_k<t+a} \ln h_{2k} < -\ln H \tag{3.38}$$

and

$$\int_t^{t+a} \max_{\varepsilon_0\leqslant x_1\leqslant L_1} f_1(u,x_1,s)du + \sum_{t\leqslant t_k\leqslant t+a} \ln h_{1k} < \ln(\frac{\varepsilon_0}{2L_1}) \tag{3.39}$$

for all $t \in R_{+0}$ and $a \geqslant P$. Choose an integer $N_0 > K$ such that

$$t_q^{(n)} - s_q^{(n)} > 2P \quad \text{for all } n \geqslant N_0,\ q \geqslant N_1^{(n)}.$$

For any $n \geqslant N_0$ and $q \geqslant N_1^{(n)}$, if $x_1(t,x_n) \geqslant \varepsilon_0$ for all $t \in [s_q^{(n)}, s_q^{(n)}+P]$, then by condition (1) of (B1), (3.4), (3.37) and (3.39) we have

$$x_1(s_q^{(n)}+P,x_n) \leqslant L_1\exp\left(\int_{s_q^{(n)}}^{s_q^{(n)}+P} f_1(t,x_1(t,x_n),2s)dt + \sum_{s_q^{(n)}\leqslant t_k<s_q^{(n)}+P} \ln h_{1k}\right)$$
$$\leqslant L_1\exp\left(\int_{s_q^{(n)}}^{s_q^{(n)}+P} \max_{\varepsilon_0\leqslant x_1\leqslant L_1} f_1(t,x_1,s)dt + \sum_{s_q^{(n)}\leqslant t_k<s_q^{(n)}+P} \ln h_{1k}\right) < \varepsilon_0,$$

which is a contradiction. Hence, there is a $s_1 \in [s_q^{(n)}, s_q^{(n)}+P]$ such that $x_1(s_1,x_n) < \varepsilon_0$. Further, a similar argument as in the above we obtain that

$$x_1(t,x_n) \leqslant \varepsilon_0\exp(\alpha_3\omega_5+H) \quad \text{for all } t \in [s_1, t_q^{(n)}].$$

Therefore, from (3.5), (3.37), (3.38) and conditions (1) and (2) of (B2)

$$x_2(t_q^{(n)+},x_n) \leqslant (2s+1)n\exp\left(\int_{s_q^{(n)}+P}^{t_q^{(n)}} f_2(t,\varepsilon_0\exp(\alpha_3\omega_5+H),s)dt + \sum_{s_q^{(n)}+P\leqslant t_k\leqslant t_q^{(n)}} \ln h_{2k}\right) < (2s+1)n.$$

This leads to a contradiction with (3.36). Therefore, (3.34) is true. So system (1.1) is ultimately bounded. This completes the proof. □

Remark 3.2. In (B5), we require that the function $f_1(t,x_1,x_2)$ has definition and is continuous for $x_1 = 0$. For the case $f_1(t,x_1,x_2)$ is discontinuous, Teng has obtained a similar result with (B5) for the predator–prey Kolmogorov system without impulse in [3]. Therefore, there is an interesting open problem is whether system (1.1) has a similar result with (B5) for the case $f_1(t,x_1,x_2)$ is discontinuous.

4. Application

In this section, we will apply the above theorems to discuss the permanence of positive solutions for systems (1.2)–(1.7). We first assume in the systems (1.2)–(1.7) that $0 \leqslant t_1 < t_2 < \cdots < t_k < \cdots$ is impulsive time sequence and $\lim_{k\to\infty} t_k = \infty$ and h_{ik} for each $i = 1,2$ and $k = 1,2,\ldots$ are positive constants, $b_i(t)$, $a_{ij}(t)$, $\alpha_i(t)$, $\beta_i(t)$, $\gamma_i(t)$ and $\omega_i(t)$ $(i,j = 1,2)$ are bounded and continuous functions on R_{+0}, $a_{ij}(t) \geq 0$, $\alpha_i(t) \geq 0$, $\beta_i(t) > 0$, $\gamma_i(t) \geq 0$ and $\omega_i(t) \geq 0$ for all $t \in R_{+0}$, $i,j = 1,2$, there is a positive constant ω_2 such that

$$\liminf_{t\to\infty} \int_t^{t+\omega_2} a_{11}(s)ds > 0.$$

In addition, we assume that

(H_1) There is a positive constant ω_1 such that

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_1} b_1(s)\mathrm{d}s+\sum_{t\leqslant t_k<t+\omega_1}\ln h_{1k}\right)>0$$

and function

$$h_1(t,v)=\sum_{t\leqslant t_k<t+v}\ln h_{1k}$$

is bounded on $t\in R_{+0}$ and $v\in[0,\omega_1)$.

(H_2) There is a positive constant ω_1 such that

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_1} b_1(s)\mathrm{d}s-\sum_{t\leqslant t_k<t+\omega_1}\ln h_{1k}\right)>0$$

and function

$$h_1(t,v)=\sum_{t\leqslant t_k<t+v}\ln h_{1k}$$

is bounded on $t\in R_{+0}$ and $v\in[0,\omega_1)$.

Firstly, we consider the following single-species non-autonomous logistic impulsive system and linear impulsive system

$$\begin{cases}\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}=x_1(t)(b_1(t)-a_{11}(t)x_1(t)), & t\neq t_k,\\ x_1(t_k^+)=h_{1k}x(t_k), & k=1,2,\ldots\end{cases}\tag{4.1}$$

and

$$\begin{cases}\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}=a_{11}(t)-b_1(t)x_1(t), & t\neq t_k,\\ x_1(t_k^+)=h_{1k}x(t_k), & k=1,2,\ldots.\end{cases}\tag{4.2}$$

Let $x_{10}(t)$ be some fixed positive solution of system (4.1) or (4.2). If the assumption (H_1) holds, then $x_{10}(t)$ is globally uniformly asymptotically stable for system (4.1). For system (4.2), If the assumption (H_2) holds, by Lemma 2.4, then we have $x_{10}(t)$ is globally uniformly asymptotically stable.

We first consider the two species non-autonomous predator–prey Lotka–Volterra system (1.2). Directly applying Theorems 3.1 and 3.2 we have the following result.

Theorem 4.1. *Assume that assumption* (H_1) *holds and there is a constant* $\omega_3>0$ *such that*

$$\limsup_{t\to\infty}\left(-\int_t^{t+\omega_3} b_2(s)\mathrm{d}s+\sum_{t\leqslant t_k<t+\omega_3}\ln h_{2k}\right)<0,$$

$$h_2(t,v)=\sum_{t\leqslant t_k<t+v}\ln h_{2k}$$

is bounded on $t\in R_+ \text{and} v\in[0,\omega_3)$ *and there is a constant* $\omega_4>0$ *such that*

$$\liminf_{t\to\infty}\int_t^{t+\omega_4} a_{22}(s)\mathrm{d}s>0\quad\text{or}\quad\liminf_{t\to\infty}\int_t^{t+\omega_4} a_{12}(s)\mathrm{d}s>0.$$

If there is a constant $\omega_0>0$ *such that*

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_0}(-b_2(s)+a_{21}(s)x_{10}(s))\mathrm{d}s+\sum_{t\leqslant t_k<t+\omega_0}\ln h_{2k}\right)>0,$$

then system (1.2) *is permanent.*

We next consider two species non-autonomous predator–prey Holling-type functional response systems (1.3) and (1.4). Applying Theorems 3.1 and 3.2 we have the following result.

Theorem 4.2. *Assume that assumption* (H_1) *holds,* $\inf_{t\geq 0}a_{11}(t)>0$ *and there is a constant* $\omega_3>0$ *such that*

$$\limsup_{t\to\infty}\left(-\int_t^{t+\omega_3} b_2(s)\mathrm{d}s+\sum_{t\leqslant t_k<t+\omega_3}\ln h_{2k}\right)<0,$$

$$h_2(t,v) = \sum_{t\leqslant t_k<t+v} \ln h_{2k}$$

is bounded on $t \in R_{+0}$ and $v \in [0,\omega_3)$, *and there is a* $\omega_4 > 0$ *such that*

$$\liminf_{t\to\infty} \int_t^{t+\omega_4} \alpha_1(s)ds > 0 \quad \text{or} \quad \liminf_{t\to\infty} \int_t^{t+\omega_4} a_{22}(s)ds > 0.$$

If there is a constant $\omega_0 > 0$ *such that*

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_0} (-b_2(s) + \phi_2(s, x_{10}(s)))ds + \sum_{t\leqslant t_k<t+\omega_0} \ln h_{2k}\right) > 0,$$

then system (1.3) and (1.4) *are permanent.*

Proof. In fact, for the two species non-autonomous predator–prey Holling I-type functional response impulsive system (1.3). Comparing with system (1.1), we have

$$f_1(t,x_1,x_2) = \begin{cases} b_1(t) - a_{11}(t)x_1 - \alpha_1(t)x_2, & 0 \leqslant x_1 \leqslant x_{10}, \\ b_1(t) - a_{11}(t)x_1 - \frac{\alpha_1(t)x_{10}}{x_1}x_2, & x_1 > x_{10}, \end{cases}$$

and

$$f_2(t,x_1,x_2) = \begin{cases} -b_2(t) + \alpha_2(t)x_1 - a_{22}(t)x_2, & 0 \leqslant x_1 \leqslant x_{10}, \\ -b_2(t) + \alpha_2(t)x_{10} - a_{22}(t)x_2, & x_1 > x_{10}. \end{cases}$$

Calculating the partial derivative of $f_i(t,x_1,x_2)$ $(i = 1,2)$ with respect to x_1 and x_2 we have

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_1} = \begin{cases} -a_{11}(t), & 0 \leqslant x_1 < x_{10}, \\ -a_{11}(t) + \frac{\alpha_1(t)x_{10}}{x_1^2}x_2, & x_1 > x_{10}, \end{cases} \tag{4.3}$$

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_2} = \begin{cases} -\alpha_1(t), & 0 \leqslant x_1 < x_{10}, \\ -\frac{\alpha_1(t)x_{10}}{x_1}, & x_1 > x_{10}, \end{cases}$$

$$\frac{\partial f_2(t,x_1,x_2)}{\partial x_1} = \begin{cases} \alpha_2(t), & 0 \leqslant x_1 < x_{10}, \\ 0, & x_1 > x_{10}, \end{cases} \tag{4.4}$$

and

$$\frac{\partial f_2(t,x_1,x_2)}{\partial x_2} = -a_{22}(t).$$

From (4.3) we see that, when $0 \leqslant x_1 < x_{10}$, obviously

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_1} \leqslant -a_{11}(t) \quad \text{for all } t \in R_{+0},$$

and when $x_1 > x_{10}$, since the function

$$g(t,x_1) = \frac{\alpha_1(t)x_{10}}{x_1}$$

is bounded for all $t \in R_{10}$ and $\inf_{t\geq 0} a_{11}(t) > 0$, there is constant $\alpha_0 > 0$ such that

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_1} \leqslant -\frac{1}{2}a_{11}(t)$$

for all $t \in R_{+0}$ and $0 \leqslant x_2 \leqslant \alpha_0$. Therefore, (B1), (B2) and (B4) or (B5) hold. Thus, as a consequence of Theorems 3.1 and 3.2, we obtain that system (1.3) is permanent.

Similar argument as in Theorem 4 in [3], we can obtain that system (1.4) is permanent. This complete the proof. □

Further, we consider the two species non-autonomous predator–prey Beddington–DeAngelis functional response impulsive system (1.5) and Leslie–Gower functional response system (1.6). Applying Theorems 3.1 and 3.2 we have the following results.

Theorem 4.3. *Assume that assumption* (H_1) *holds,* $\inf_{t\geq 0} a_{11}(t) > 0$ *and there is a constant* $\omega_3 > 0$ *such that*

$$\limsup_{t\to\infty}\left(-\int_t^{t+\omega_3} b_2(s)ds + \sum_{t\leqslant t_k<t+\omega_3} \ln h_{2k}\right) < 0$$

and

$$h_2(t,v)=\sum_{t\leqslant t_k<t+v}\ln h_{2k}$$

is bounded on $t\in R_{+0}$ and $v\in[0,\omega_3)$. *If there is a constant* $\omega_0>0$ *such that*

$$\liminf_{t\to\infty}\int_t^{t+\omega_0}(-b_2(s)+\phi_2(s,x_{10}(s)))\mathrm{d}s+\sum_{t\leqslant t_k<t+\omega_0}\ln h_{2k}>0,$$

then system (1.5) is permanent.

Proof. Comparing with system (1.1), we have

$$f_1(t,x_1,x_2)=b_1(t)-a_{11}(t)x_1-\frac{\alpha_1(t)x_1^{m-1}}{1+\gamma_1(t)x_1^n+\omega_1(t)x_2}x_2$$

and

$$f_2(t,x_1,x_2)=-b_2(t)+\frac{\alpha_2(t)x_1^m}{1+\gamma_2(t)x_1^n+\omega_2(t)x_2}.$$

Calculating the partial derivative of $f_i(t,x_1,x_2)$ $(i=1,2)$ with respect to x_1 and x_2 we have

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_1}=-a_{11}(t)-\frac{[(m-1)(1+\omega_1(t)x_2)+(m-1-n)\gamma_1(t)x_1^n]\alpha_1(t)x_1^{m-2}x_2}{[1+\gamma_1(t)x_1^n+\omega_1(t)x_2]^2}, \tag{4.5}$$

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_2}=-\frac{\alpha_1(t)x_1^{m-1}(1+\gamma_1(t)x_1^n)}{[1+\gamma_1(t)x_1^n+\omega_1(t)x_2]^2},$$

$$\frac{\partial f_2(t,x_1,x_2)}{\partial x_1}=\frac{\alpha_2(t)x_1^{m-1}(m[1+\omega_2(t)x_2]+(m-n)\gamma_2(t)x_1^n)}{[1+\gamma_2(t)x_1^n+\omega_2(t)x_2]^2} \tag{4.6}$$

and

$$\frac{\partial f_2(t,x_1,x_2)}{\partial x_2}=-\frac{\alpha_2(t)\omega_2(t)x_1^m}{[1+\gamma_2(t)x_1^n+\omega_2(t)x_2]^2}.$$

From (4.5) we see that, when $m\geqslant n+1$, obviously

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_1}\leqslant -a_{11}(t)$$

for all $t\in R_{+0}$, $x_1\in R_+$ and $x_2\in R_{+0}$, and when $m\leqslant n$, since the function

$$g(t,x_1,x_2)=\frac{[(m-1)(1+\omega_1(t)x_2)+(m-1-n)\gamma_1(t)x_1^n]\alpha_1(t)x_1^{m-2}}{[1+\gamma_1(t)x_1^n+\omega_1(t)x_2]^2}$$

is bounded for all $t\in R_{+0}$ and $x_1,x_2\in R_+$, there is a small enough constant $\alpha_0>0$ such that

$$\frac{\partial f_1(t,x_1,x_2)}{\partial x_1}\leqslant -\frac{1}{2}a_{11}(t)$$

for all $t\in R_{+0}$, $x_1\in R_+$ and $0\leqslant x_2\leqslant\alpha_0$. Further, from (4.6) it is obvious that there is a constant $\beta_0>0$ such that

$$\frac{f_2(t,x_1,x_2)}{\partial x_1}\geq 0$$

for all $t\in R_{+0}$, $0<x_1\leqslant\beta_0$ and $x_2\in R_{+0}$. Therefore, (B1), (B2) and (B4) hold. Thus, as a consequence of Theorems 3.1 and 3.2, we obtain that system (1.5) is permanent. □

Theorem 4.4. *Assume that* $\inf_{t\geq0}a_{11}(t)>0$ *and there is a constant* $\omega_3>0$ *such that* $\liminf_{t\to\infty}\int_t^{t+\omega_3}\alpha_2(s)\mathrm{d}s>0$. *If there is a constant* $\omega_0>0$ *such that*

$$\limsup_{t\to\infty}\left(\int_t^{t+\omega_0}b_2(s)\mathrm{d}s+\sum_{t\leqslant t_k<t+\omega_0}\ln h_{2k}\right)<0,$$

then system (1.6) is permanent.

We lastly consider the two-species non-autonomous predator chemostat type impulsive system (1.7).

Theorem 4.5. *Assume that assumption* (H_2) *holds and*

(a) $\inf_{t\geq 0} b_1(t) > 0$.

(b) *There is a constant* $\omega_3 > 0$ *such that*

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_3} b_2(s)\mathrm{d}s - \sum_{t\leqslant t_k<t+\omega_3} \ln h_{2k}\right) > 0$$

and

$$h_2(t,v) = \sum_{t\leqslant t_k<t+v} \ln h_{2k}$$

is bounded on $t \in R_{+0},\ v \in [0, \omega_3)$.

(c) *There is a constant* $\omega_4 > 0$ *such that*

$$\liminf_{t\to\infty}\int_t^{t+\omega_4} a_{22}(s)\mathrm{d}s - \sum_{t\leqslant t_k<t+\omega_4} \ln h_{2k} > 0.$$

If there is a constant $\omega_0 > 0$ *such that*

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega_0} (-b_2(s) + \phi_2(s, x_{10}(s)))\mathrm{d}s + \sum_{t\leqslant t_k<t+\omega_0} \ln h_{2k}\right) > 0,$$

then system (1.7) *is permanent.*

Remark 4.1. If system (1.2)–(1.7) without the impulsive effective, that is $h_{ik} = 1$ for all $i = 1, 2$ and $k = 1, 2, \ldots$, Theorems 4.1,4.2,4.3,4.4,4.5 are the same with Theorems 3–7 in [3]. Therefore, our result extents the corresponding results for the permanence for systems (1.2)–(1.7) without impulse in [3].

Remark 4.2. By our best knowledge, on the permanence for some special impulsive non-autonomous predator–prey such as systems (1.2)–(1.7) have not been studied. Therefore, our results in Theorems 4.1,4.2,4.3,4.4,4.5 is new.

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