# Molecular Realism in Default Models for Information Theories of Hydrophobic Effects

**M. A. Gomez,\* L. R. Pratt, G. Hummer, and S. Garde**

*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

*Received: January 28, 1999; In Final Form: March 16, 1999*

This letter considers several physical arguments about contributions to hydrophobic hydration of inert gases, constructs default models to test them within information theories, and gives information theory predictions using those default models with moment information drawn from simulation of liquid water. Tested physical features include packing or steric effects, the role of attractive forces that lower the solvent pressure, and the roughly tetrahedral coordination of water molecules in liquid water. Packing effects (hard-sphere default model) and packing effects plus attractive forces (Lennard-Jones default model) are ineffective in improving the prediction of hydrophobic hydration free energies of inert gases over the previously used flat default model. However, a conceptually simple cluster Poisson model that incorporates tetrahedral coordination structure in the default model is effective for these predictions. These results provide a partial rationalization of the remarkable performance of the flat default model with two moments in previous applications. The cluster Poisson default model thus will be the subject of further refinement.

## I. Introduction

The idea of constructing an information theory description of cavity formation in water[1] has reinvigorated the molecular theory of hydrophobic effects.[2−7] One advantage of this approach is that simple physical hypotheses can be expressed in default models. Given a fixed amount of specific information, the quality of the predictions gives an assessment of the physical ideas that are embodied in the underlying default model. Relevant physical ideas include: whether a direct description of dense fluid packings significantly improves the predictions; or whether incorporation of dewetting of hydrophobic surfaces is required; or whether specific expression of the roughly tetrahedral coordination of water molecules in liquid water is the most helpful next step for these theories. It is remarkable that the previous successes of the information models for the primitive hydrophobic effects have not required specific consideration of these physical points.

This letter considers these physical arguments, constructs default models to test them, and gives the results of information theory predictions using those default models with specific moment information drawn from simulation of liquid water. Occupancy moments are used as information. Complete moment information produces results that are independent of the default model. However, the goal is to judge the default models and the physical ideas that they express. Therefore, our judgments will center on the accuracy of the predictions with limited moment information. More specifically, we take the view that the quality of the prediction with two moments is critical because information for the first two occupancy moments, the mean and variance, is available from experiment.

Much of the technical work required to construct the default models considered involves molecular simulation calculations for fiducial systems. That technical work will be reported at a later time.

The application of the information theory approach more broadly than to liquid water immediately turns up cases where it works less well. Thus, a broader suite of default models will be a key to increasing the utility of this approach.

## II. Testing Physical Ideas of Hydrophobic Effects

The information theory approach studied here grew out of earlier studies of formation of atomic-sized cavities in molecular liquids.[8−11] It has led to new and simple views of entropy convergence in hydrophobic hydration[12] and of pressure denaturation of proteins.[13] A review of these developments has been given;[14] broader discussions are also available.[15,16]

The objective of the information theory prediction is the interaction part of the chemical potential of a hard core solute $\beta \Delta \mu = -\ln p_0$, where $p_0$ is the probability that the hard solute could be inserted into the system without overlap of van der Waals volume of the solvent; $1/\beta = k_B T$. This procedure depends on a default model $\hat{p}_n$ of the distribution $p_n$ of which $p_0$ is the $n = 0$ member. Two default models have been considered in previous work: (a) the "Gibbs default model" $\hat{p}_n \propto 1/n!$ that predicts a Poisson distribution when the moment $\langle n \rangle_0$ is the only information available and (b) the "flat default model" $\hat{p}_n = $ constant greater than 0, $n = 0$, 1, ..., $n_{max}$, and zero otherwise. The predictions obtained using these default models for the hydration free energy of inert gases in water are similar. Convergence to the correct result is nonmonotonic with increasing numbers of binomial moments $B_j = \left\langle \binom{n}{j} \right\rangle_0$ used.[14,16] Because of this nonmonotonic convergence, the most accurate prediction obtained with a small number of moments utilizes only the two moments, $B_1$ and $B_2$. Furthermore, the flat default model produces a distinctly more accurate prediction of this hydration free energy when only two moments are used than does the Gibbs default model.

The accuracy of the prediction utilizing the flat default model is remarkable. On the other hand, the Gibbs default model is conceptually more natural in this framework because the Poisson distribution is the usual uncorrelated result here. So, the effectiveness of the flat default model relative to the Gibbs model is additionally puzzling. The work that follows addresses these issues.

---

\* Corresponding author. E-mail: gomez@lanl.gov.

Letters

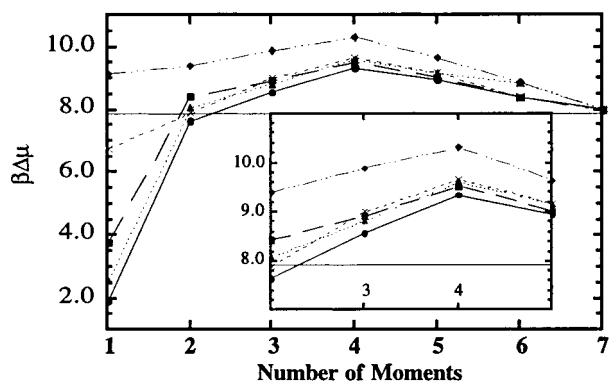*J. Phys. Chem. B, Vol. 103, No. 18, 1999* **3521**



**Figure 1.** Convergence with number of binomial moments of $\beta\Delta\mu$ predicted using several default models for a spherical solute with distance of closest approach $\lambda = 3.0$ Å for water oxygen atoms. Identifications are: diamonds (dash−dot lines), hard-sphere default; crosses (short dash line), Lennard-Jones default; squares (long dash line), Gibbs default; triangles (dotted line), cluster Poisson default; circles (gray line), flat default. For this value of $\lambda$, binomial moments $B_j$ are nonzero through $j = 9$. The horizontal line is the prediction with all nine moments included. With only two moments, the Lennard-Jones default model makes the best prediction. However, the differences are slight with the exception of the hard-sphere model. The results for the hard-sphere default model were obtained from NPT Monte Carlo calculations at $\beta p^* = 2.989$. The average density was $\rho d^3 = 0.633 \pm 0.002$ with $d = 2.67$ Å. The Lennard-Jones model was obtained from NPT Monte Carlo calculations at $p = 0.0$ and $T^* = 1.103$. $\sigma = 2.67$ Å, and the value $\epsilon/k_B = 272$ K obtained from a fit of zero-pressure data. The mean density was $\rho^* = 0.624 \pm 0.001$. A system size of 256 particles was sufficient for both simulations.

It deserves emphasis that the overall distribution $p_n$ is well described by the information theory with the first two moments, $B_1$ and $B_2$. It is the prediction of the extreme member $p_0$ that makes the differences in these default models significant.

**a. Packing.** A first idea is that the default model should contain a direct description of dense fluid packings that are central to the theory of liquids.[17] Accordingly, we computed $p_n$ for the fluid of hard spheres of diameter $d = 2.67$ Å at a density $\rho d^3 = 0.633$. Those computations used specialized importance sampling and will be reported later. Typical predictions for the hydrophobic hydration free energies of atomic size solutes obtained using those results as a default model are plotted in Figure 1, which shows the nonmonotonic convergence with increasing number of occupancy moments obtained from the flat and the Gibbs default models. The predictions obtained using the hard-sphere results as a default model are different but not improved in the essential aspects. Direct convergence is only seen if four or more moments are included. Though the convergence is more nearly monotonic from the beginning, the prediction obtained from a two-moment model is worse than for the flat and the Gibbs default cases.

**b. Attractive Interactions among Solvent Molecules.** A next idea is that attractive forces between solvent molecules might play a significant role for these properties because attractive forces lower the pressure of the solvent. Dehydration of hydrophobic surfaces becomes a principal consideration for solutes larger in size than the solvent molecules, but perhaps such effects are being felt already for atomic solutes. Accordingly, we computed $p_n$ for the Lennard-Jones liquid studied by Pratt and Pohorille[10] for which attractive interactions were adjusted so that the macroscopic pressure of the solvent would be approximately zero. This Lennard-Jones system thus gives a cavity formation free energy for atomic-sized cavities that is about the same as that of common liquid water simulation models. The results of Figure 1 confirm this latter point but
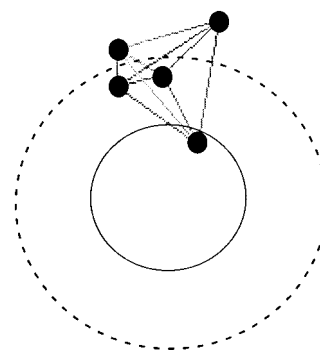


**Figure 2.** A tetrahedral cluster, the observation volume (sphere with solid line), and the augmented volume (sphere with dashed line). The cluster may contribute occupants of the observation volume only if the center is within the augmented volume.

also show that the convergence with number of moments is again nonmonotonic and not better than for the flat default model. Again, direct monotonic convergence is only seen after four occupancy moments are included.

**c. Tetrahedral Coordination of Solvent Molecules.** The final idea checked here is whether the predictions of cavity formation free energies are improved by incorporating a tetrahedral coordination structure for water molecules in liquid water. We use a cluster Poisson model to accomplish this.[18] The physical picture is one of tetrahedral clusters of water molecules with prescribed intracluster correlations but random positions and orientations.

A molecular cluster may contribute occupants to a specific observation volume only if the center of the cluster is an occupant of a larger augmented volume; see Figure 2. Definition of this augmented volume will depend on the structures of the clusters and the choice of cluster center. We then consider the generating function $\rho(z)$ for the probability $\rho_N$ that $N$ cluster centers are present in the augmented volume:

$$\rho(z) = \sum_{N=0} z^N \rho_N \tag{1}$$

We assume that $N$ is Poisson distributed $\rho(z) = e^{-\langle N\rangle(1-z)}$ with $\langle N\rangle$ the product of the density of clusters and the volume of the augmented region.

Next we consider the generating function $g(z)$ defined by the conditional probabilities, $g_n$, that a cluster with center in the augmented volume contributes $n$ oxygen atom occupants to the observation volume

$$g(z) = \sum_{n=0} z^n g_n \tag{2}$$

Defining the generating function for the probabilities of numbers of oxygen in the observation volume

$$p(z) \equiv \sum_{n=0} z^n p_n \tag{3}$$

we can express

$$p(z) = \rho(g(z)) \tag{4}$$

This is a standard result of probability theory.[19] $\ln p(z)$ is a polynomial function of $z$. Extraction of the series coefficients from eq 3 provides the desired default model. The numerical effort resides only in the computation of the $g_n$.

In this study, the clusters are assumed to be tetrahedra with the oxygen atom of a water molecule at the center and at each

vertex. Thus we take $\langle N \rangle = \rho v / 5$, with $v$ the volume of the augmented region and $\rho$ the molecular density of the solvent water. The O−O intracluster near-neighbor distance, the distance of a point of a tetrahedron from its center, is 2.67 Å, and the augmented volume is a sphere with radius $\lambda + 2.67$ Å. The coefficients of $g(z)$ are obtained from a Monte Carlo calculation that randomly positions a tetrahedron with center in the augmented volume and counts how many O-points of the cluster occupy the observation volume.

Figure 1 shows the predictions for cavity formation free energy obtained with the cluster (tetrahedron) Poisson default model. The nonmonotonic convergence is still evident. The prediction utilizing two moments is more accurate than that utilizing the Gibbs default model and similar to the predictions made by the flat default or the Lennard-Jones default in the best cases considered here for those models.

### III. Discussion

Each of the default models newly considered here makes specific assumptions about $n$-body correlations. If the default model were the same as the experimental distribution, the limitation of the data to two moments would not be significant. The optimization would be unaffected by the number of experimental moments used.

Evidently, the specific assumptions for $n$-body correlations with the hard-sphere and Lennard-Jones default models do not improve the pattern of high-order correlations. It is helpful to consider using a prediction produced with a limited number of moments, say $n \leq 4$, as a default model for the predictions as shown in Figure 1. In such a case, the graph would be a horizontal line until new experimental information is incorporated, e.g., with $n > 4$. With this background, it is striking that the accuracy of the best $n = 2$ predictions is not improved until $n > 6$. A default model must refine seven-body correlations to improve upon the best current predictions in these circumstances.

A second point of discussion is that the biggest difference between the Lennard-Jones and the cluster Poisson model is in simplicity. Though the differences in the predictions seen here are not dramatic, the cluster Poisson model is simpler. This is particularly true for the dependence on thermodynamic state and the potential for further development. That the cluster Poisson model expressing tetrahedral coordination appears to be a helpful new direction is intuitive and encouraging. However, the fact that the predictions are not dramatically improved suggests that this sort of tetrahedral coordination is not the only or principal physical feature relevant for improved predictions of cavity formation.[20]

The Lennard-Jones default model incorporates some of the dewetting phenomena that are expected to become more pronounced as the solute size increases.[21] Figure 3 shows the variation of hydration free energy with solute size obtained with the different default models and two moments. At the smallest solute size shown, all the models give the same result. In the solute size range of 2.2−2.8 Å, the cluster Poisson model gives the best results overall. For larger solute sizes, the cluster Poisson model results overestimate the hydration free energy. At this point, results from the Lennard-Jones default model cross the simulation results and become slightly too small for the larger solute sizes shown.

### IV. Conclusion

We conclude that direct incorporation of dense fluid packing effects (hard-sphere default model) and packing effects plus attractive forces that lower the pressure of the solvent (Lennard-
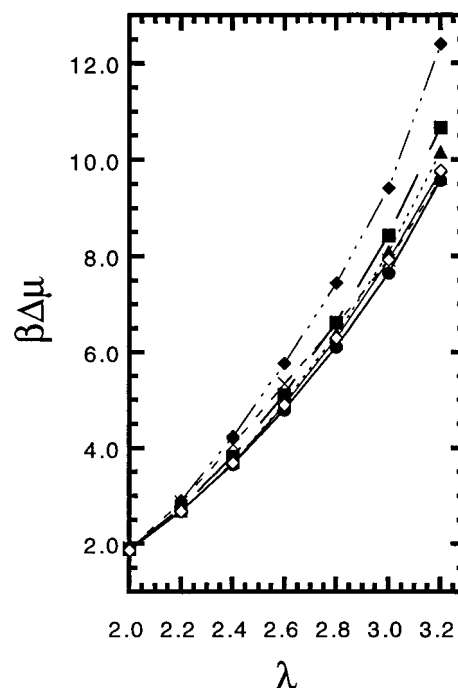


**Figure 3.** The variation of hydration free energy with solute size obtained with the different default models. The different models are identified as in Figure 1. In addition, simulation results are shown as open diamonds. At the smallest solute size shown, all the models give the same result. In the solute size range 2.2−2.8 Å, the cluster Poisson model gives the best results overall. For larger solute sizes, the cluster Poisson model results overestimate the hydration free energy. At this point, results from the Lennard-Jones default model cross the simulation results and become slightly too small for the larger solute sizes shown.

Jones default model) are ineffective in improving the prediction of hydrophobic hydration free energies of inert gases over the previously used Gibbs and flat default models. However, a cluster Poisson model that incorporates tetrahedral coordination structure in the default model is intuitive, simple to implement, and is effective for these predictions. These results provide a partial rationalization of the remarkable performance of the flat default model with two moments in previous applications. The specific cluster Poisson default model used here is primitive and will be the subject of further refinement.

**References and Notes**

(1) Hummer, G.; Garde, S.; Garcia, A. E.; Pohorille, A.; Pratt, L. R. *Proc. Natl. Acad. Sci. U.S.A.* **1996**, *93*, 8951.

(2) Berne, B. J. *Proc. Natl. Acad. Sci. U.S.A.* **1996**, *93*, 8880.

(3) Crooks, G. E.; Chandler, D. *Phys. Rev. E* **1997**, *56*, 4217.

(4) Mountain, R. D.; Thirumalai, D. *Proc. Natl. Acad. Sci. U.S.A.* **1998**, *95*, 8436−8440.

(5) Wallqvist, A.; Covell, D. G.; Thirumalai, D. *J. Am. Chem. Soc.* **1998**, *120*, 427−428.

(6) Arthur, J. W.; Haymet, A. D. J. *J. Chem. Phys.* **1998**, *109*, 7991.

(7) Lum, K.; Chandler, D.; Weeks, J. D. *J. Phys. Chem.*, in press.

(8) Pohorille, A.; Pratt, L. R. *J. Am. Chem. Soc.* **1990**, *112*, 5066.

(9) Pratt, L. R. *CLS Division 1991 Annual Review*; National Technical Information Service U.S. Department of Commerce, 5285 Port Royal Rd., Springfield, VA 22161, 1991, LA-UR-91-1783.

(10) Pratt, L. R.; Pohorille, A. *Proc. Natl. Acad. Sci. U.S.A.* **1992**, *89*, 2995.

(11) Pratt, L. R.; Pohorille, A. In *Proceedings of the EBSA 1992 International Workshop on Water-Biomolecule Interactions*; Palma, M. U., Palma-Vittorelli, M. B., Parak, F., Eds.; Societa Italiana di Fisica: Bologna, 1993.

(12) Garde, S.; Hummer, G.; García, A. E.; Paulaitis, M. E.; Pratt, L. R. *Phys. Rev. Lett.* **1996**, *77*, 4966.

(13) Hummer, G.; Garde, S.; García, A. E.; Paulaitis, M. E.; Pratt, L. R. *Proc. Natl. Acad. Sci. U.S.A.* **1998**, *95*, 1552.

(14) Hummer, G.; Garde, S.; García, A. E.; Paulaitis, M. E.; Pratt, L. R. *J. Phys. Chem. B* **1998**, *102*, 10469.

(15) Pratt, L. R. In *Encyclopedia of Computational Chemistry*; Schleyer, P. v. R., Allinger, N. L., Clark, T., Gasteiger, J., Kollman, P. A., Schaefer, H. F., III, Schreiner, P. R., Eds.; John Wiley & Sons: Chichester, 1998.

(16) Pratt, L. R.; Garde, S.; Hummer, G. In *New Approaches in Liquid State Theory. Inhomogeneities and Phase Separation in Simple, Complex and Quantum Fluids*. Caccamo, C., Hansen, J.-P., Stell, G., Eds.; NATO Advanced Study Institute; Kluwer: Dordrecht, 1999.

(17) Chandler, D.; Weeks, J. D.; Andersen, H. C. *Science* **1983**, *220*, 787.

(18) Neyman, J.; Scott, E. L. *Sci. Am.* **1956**, September, 187.

(19) Karlin, S.; Taylor, H. M. *A First Course in Stochastic Processes*; Academic Press, Inc.: New York, 1975.

(20) Arthur, J. W.; Haymet, A. D. J. *J. Chem. Phys.* **1999**, *110*, 5873.

(21) Hummer, G.; Garde, S. *Phys. Rev. Lett.* **1998**, *80*, 4193.