

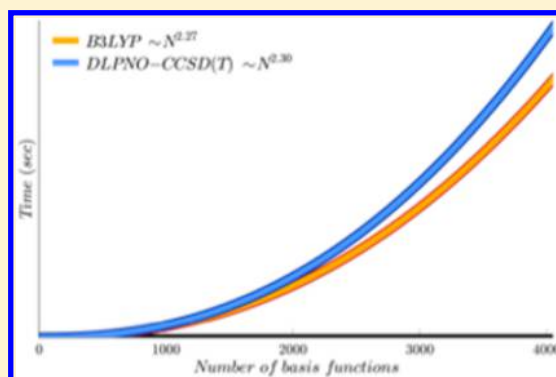
Is It Possible To Obtain Coupled Cluster Quality Energies at near Density Functional Theory Cost? Domain-Based Local Pair Natural Orbital Coupled Cluster vs Modern Density Functional Theory

Dimitrios G. Liakos and Frank Neese*

Max Planck Institute for Chemical Energy Conversion, Stiftstr. 32-34, D-45470 Mülheim an der Ruhr, Germany

S Supporting Information

ABSTRACT: The recently developed domain-based local pair natural orbital coupled cluster theory with single, double, and perturbative triple excitations (DLPNO-CCSD(T)) delivers results that are closely approaching those of the parent canonical coupled cluster method at a small fraction of the computational cost. A recent extended benchmark study established that, depending on the three main truncation thresholds, it is possible to approach the canonical CCSD(T) results within 1 kJ (default setting, TightPNO), 1 kcal/mol (default setting, NormalPNO), and 2–3 kcal (default setting, LoosePNO). Although thresholds for calculations with TightPNO are 2–4 times slower than those based on NormalPNO thresholds, they are still many orders of magnitude faster than canonical CCSD(T) calculations, even for small and medium sized molecules where there is little locality. The computational effort for the coupled cluster step scales nearly linearly with system size. Since, in many instances, the coupled cluster step in DLPNO-CCSD(T) is cheaper or at least not much more expensive than the preceding Hartree–Fock calculation, it is useful to compare the method against modern density functional theory (DFT), which requires an effort comparable to that of Hartree–Fock theory (at least if Hartree–Fock exchange is part of the functional definition). Double hybrid density functionals (DHDF's) even require a MP2-like step. The purpose of this article is to evaluate the cost vs accuracy ratio of DLPNO-CCSD(T) against modern DFT (including the PBE, B3LYP, M06-2X, B2PLYP, and B2GP-PLYP functionals and, where applicable, their van der Waals corrected counterparts). To eliminate any possible bias in favor of DLPNO-CCSD(T), we have chosen established benchmark sets that were specifically proposed for evaluating DFT functionals. It is demonstrated that DLPNO-CCSD(T) with any of the three default thresholds is more accurate than any of the DFT functionals. Furthermore, using the aug-cc-pVTZ basis set and the LoosePNO default settings, DLPNO-CCSD(T) is only about 1.2 times slower than B3LYP. With NormalPNO thresholds, DLPNO-CCSD(T) is about a factor of 2 slower than B3LYP and shows a mean absolute deviation of less than 1 kcal/mol to the reference values for the four different data sets used. Our conclusion is that coupled cluster energies can indeed be obtained at near DFT cost.



1. INTRODUCTION

During the past decade, major improvements in the field of local correlation methods were achieved, resulting, for example, in the calculation of the crambin molecule with 644 atoms and more than 8800 basis functions at the CCSD(T) level.¹ Along the way, different groups followed various paths in order to exploit every possible hint that could permit accurate and rigorous *ab initio* wave function methods to become more efficient. Strong evidence of the achieved progress is that researchers² started investigating the accuracy of the gold standard method, CCSD(T).^{3–7}

The hint for the construction of efficient and accurate wave function-based methods rests in the fact that dynamical electron correlation is, for the most part, a short-range effect. The long recognized problem at hand boils down to determining the best way to exploit the use of the inherent locality in the correlation energy in order to arrive at computationally attractive schemes. It should be stressed that it is extremely

important not to sacrifice accuracy in the process of designing local correlation methods. For practical applications, the errors should not exceed ~0.1% of the basis set correlation energy. While this appears to be a fairly stiff requirement, it must be realized that obtaining an inaccurate result fast is not useful for any chemical application. Hence, if one is aiming for an accuracy of about 1 kcal/mol, then the necessity of recovering about 99.9% of the correlation energy is an insurmountable consequence. Many local correlation methods fall short of this requirement or become computationally more expensive than the parent canonical methods at the time the desired accuracy is reached. These problems of balancing accuracy and efficiency may be at the heart of the fact that local correlation methods, so far, have not seen the large-scale use in computational chemistry that might have been anticipated given the fact that

Received: April 16, 2015

Published: July 24, 2015

the parent canonical CCSD(T) has nearly as many of the desirable features of a successful chemical theory. Thus, the dynamical electron correlation, while definitely being a local quantity, is not quite as local as one would initially hope, and, consequently, the design of efficient computational methods that optimally exploit the sparsity in the dynamical electron correlation is a formidable challenge.

In terms of local correlation approaches, two broad categories of methods exist. In the first category, instead of one unified calculation on the whole molecule, one fragments the system according to some automated or manual scheme and subsequently performs many individual calculations on small fragments of the molecule and then combines the results to obtain an overall correlation energy estimate. The following methods belong to this category: incremental correlation family of methods,^{8–16} cluster in molecule (CIM),^{17–23} natural linear-scaled coupled cluster,^{24,25} divide–expand–consolidate (DEC),^{26–28} and others.²⁹ These methods invariably involve many redundant steps, like integral transformations and amplitude optimizations, that arise from performing calculations on overlapping fragments. Their strong suit is their excellent parallelizability, which is impressively demonstrated by some recent DEC calculations.³⁰

In the second category, the calculations are performed for the whole molecule, but components of the problem (either in the form of electron pairs or in the form of molecular orbitals) that are not considered to be important are either treated in a more approximate fashion or completely ignored. The most popular of these methods originates from ideas that Pulay brought forward in the early 1980s and that were subsequently greatly extended and refined in the work by Werner, Schütz, and co-workers, who produced highly efficient local versions for the whole spectrum of post-HF methods. Thus, they were able to implement local MP2,^{31,32} CCSD,^{33–35} CCSD(T),³⁶ and iterative CCSDT-1b³⁷ methods.

In our group, we have followed a similar pathway toward direct local correlation methods. By insisting on high accuracy, we realized that the required domains become very large (ca. 30 atoms) and that domain-based local correlation methods become computationally very expensive. It appeared natural to investigate methods to compress the information content of the local external spaces that are contained in the correlation domains. This led to the revival of the pair natural orbital (PNO) approaches. The latter were pioneered in the early 1970s by Meyer,^{38–41} Ahlrichs and co-workers,^{42,43} Kutzelnigg, Staemmler, and co-workers,^{42,44} Taylor,⁴⁵ and others. By realizing that the PNO approach forms an ideal basis for local correlation treatment, the domain based local pair natural orbital coupled cluster method with single, double, and perturbative triple excitations (DLPNO-CCSD(T)) was developed.^{1,46–51}

The latter method, in our opinion, holds great promise for computational chemistry. It has been shown that the DLPNO-CCSD(T) method is efficient, accurate, and robust. Chemical applications for the method by our group^{1,52} and others^{53,54} have begun to appear in the literature. While, for the time being, the method is restricted to closed shell energies, the ultimate goal of the development is to make the method available to computational chemistry in the same way that density functional theory (DFT) already is.

Following a different direction, the DFT community has made significant progress toward solving known problems with the existing DFT methodology and removing some outstanding weaknesses of modern day functionals.^{55–65} Here, the proposed

solutions fall into three broad categories: (a) new functionals with different parametrizations, (b) functionals that step up in Jacob's ladder, and (c) *a posteriori* corrections for specific, recognized problems. Due to these efforts, the accuracy of DFT methods is now higher and the trust in these types of methods, even in fields that were inaccessible in the past, has significantly increased.

One might argue that local correlation approaches have advantages over DFT. Specifically, unlike realizations of DFT, correlated *ab initio* methods converge toward the exact solutions of the many-particle Schrödinger equation as the one- and many-particle bases approach completeness. While the exact solution remains intractable for large systems, the CCSD(T) method seems to sit well in the sweet spot between practicability and accuracy. Hence, its local correlation counterpart, provided one does not loose too much of the underlying rigor of CCSD(T), should be a method of choice for large-scale applications involving medium to large sized molecules.

The purpose of this article is to evaluate the relative merits of the DLPNO-CCSD(T) method compared to those of modern DFT approaches.

Given the great proliferation of theoretical approaches, some choices had to be made concerning the tested methods. For the wave function-based, only the DLPNO-CCSD(T) method is used. In a recent publication,⁶⁶ we proposed three different accuracy levels (different cutoff parameters defined in order of increasing accuracy as LoosePNO, NormalPNO, and TightPNO), and we are going to use all three of them. The main parameters that change between the three accuracy levels are T_{CutPairs} , T_{CutPNO} , and T_{CutMKN} and their values are given in Table 1.

Table 1. Values for the Cutoff Parameters for the Three Default Setups of the DLPNO-CCSD(T) Method

	LoosePNO	NormalPNO	TightPNO
T_{CutPairs}	10^{-3}	10^{-4}	10^{-5}
T_{CutPNO}	10^{-6}	3.33×10^{-7}	10^{-7}
T_{CutMKN}	10^{-3}	10^{-3}	10^{-4}

T_{CutPairs} mainly defines which electron pairs are going to be treated through CCSD(T) and which ones will be treated through MP2 method. Only pairs with pair correlation energies higher than T_{CutPairs} are included in the coupled cluster treatment. T_{CutPNO} defines the size of the virtual space for each electron pair. Only orbitals with occupation number higher than T_{CutPNO} are kept. The final truncation parameter, T_{CutMKN} , determines the size of the initial domain in which the PNOs are expanded. Atoms that have Mulliken population larger than T_{CutMKN} for a given localized MO i are included in the primary domain of orbital i .

For DFT, the following representative functionals are chosen: PBE,⁶⁷ PBE-D3,^{67–69} B3-LYP,^{70–72} B3LYP-D3,^{68–72} B3LYP-SCNL,^{70–74} M06-2X,⁷⁵ B2PLYP,⁷⁶ B2PLYP-D3,^{68,69,76} B2GP-PLYP,⁷⁷ and B2GP-PLYP-D3.^{77,78}

Given that excellent, large databases^{79,80} exist in the literature and that there are a large number of articles using these test sets,^{77,79,81–94} the purpose of this effort is to focus on the difficult cases that the DFT community uses as a testing ground to improve the robustness of the functionals. Thus, four different data sets were used, namely, ISOL6/11,^{88,95} HC7/11,^{96–99} π TC13,^{100–102} and NCCE31/05.^{82,99}

The article is constructed in the following way: in the Section 2, the details of the methods concerning programs, functionals,

and data sets will be discussed. In the Section 3, we will analyze the accuracy and timings of the various methods. Finally, in the Section 4, we will summarize our conclusions.

2. THEORY AND TECHNICAL DETAILS

2.1. Technical Details. The ORCA quantum chemistry package was used for all calculations.¹⁰³ Dunning's cc-pVnZ^{104,105} (pVnZ) and aug-cc-pVnZ (apVnZ) families of basis sets were used throughout. For cases where resolution of the identity (RI or DF) was used, the corresponding basis sets were employed.¹⁰⁶ All calculations were run in parallel using four processors for double- ζ basis sets and eight processors for triple- ζ basis sets.

The choice of reference values will be discussed separately for each data set since the reference values found in the literature were not uniform for all data sets. Thus, we have recalculated reference values using the procedures discussed individually below.

2.2. Functionals. As there have been hundreds of different density functionals proposed in the literature over the years, some selection had to be made for the present study. Our choice of density functionals in this study aims at including functionals that are representatives of the major classes that are presently in large-scale use in the computational chemistry community. No claim is made that the choice of functionals is unique. Our selection is guided by Jacob's ladder, as proposed in 2001 by Perdew.¹⁰⁷ In the first rung of the ladder, one finds functionals depending only on the local density (LDA). We have chosen to skip this rung in an effort to keep the results as compact as possible together with the well-known inaccuracy of LDA.^{90–92} Furthermore, LDA functionals do not present any significant improvement efficiencywise with respect to that of the next rung in the ladder, namely, GGA functionals.

Stepping up the ladder adds explicit dependence on gradients of density, resulting in what is called generalized-gradient approximations GGAs. Some of the most well-known GGA functionals are the HCTH,¹⁰⁸ PW91,^{109–111} BP86,^{70,112} BLYP,^{70,71} and PBE.⁶⁷ Of these functionals, we have chosen PBE because of its widespread use in the literature and its good performance, which is among the best for this type of functional, for the kind of problems treated here.^{88,90,92,98}

In the next rung on the ladder, the functionals depend also on the kinetic energy density (meta-GGA). We also skip this type of functionals since there are no timing advantages over GGA's and their performance is usually only marginally better.¹¹³

Adding a fraction of the exact Hartree–Fock exchange leads to hybrid functionals. This category contains some of the most successful and widely used functionals. Here, we will use two of them. The B3LYP^{70–72} functional is an obvious choice due to its popularity throughout the scientific community and its overall excellent performance. The second hybrid functional used is the M06-2X functional developed by Truhlar and co-workers.⁷⁵ M06-2X is a hybrid meta-GGA functional with 54% percent of EEX. It is reported to perform well for noncovalent interactions.

The next rung on the ladder includes functionals where one explicitly includes dependence on unoccupied orbitals. The original and still most successful functional in this group is Grimme's B2PLYP approach.⁷⁶ In this functional, a part of the correlation energy is calculated by an MP2-like expression. Using a similar form but different parametrizations, Martin's group proposed the B2GP-PLYP functional⁷⁷ that seems to be

one of the most accurate and robust functionals. Hence, it was also included.

We finally mention RPA methods^{59,114–118} that have recently gained much popularity in the DFT community and that hold great promise for the future. Klimeš,⁵⁸ in his recent perspective on van der Waals dispersion forces and DFT, stated that "...approaches based on the ACFDT such as RPA look promising. However, given their computational cost and complex setup it is likely that for the foreseeable future such methods will mainly be useful for *tour de force* reference style calculations rather than for routine studies". Notwithstanding algorithmic improvements,^{59,65,114–121} we await the widespread applicability of RPA approaches and have skipped them for the present study as well.

The problem at hand for this study concerns the category of chemical problems that are broadly referred to as non-covalent interactions. Noncovalent interactions pose a very serious challenge for density functionals, and no functional has yet been reported to produce robust and accurate results for this category of problems. The problem derives, mainly, from the fact that dispersion forces are a correlation problem that cannot be handled by local functionals. In parallel with the search for a functional that could solve the problem itself, two additional pathways gained significant popularity.

The first approach is to add a correction for dispersion forces after the SCF procedure has converged. This idea is simple, clear, and straightforward to implement. Various attempts⁸⁷ were made for a global, robust way to add the correction.^{122–126}

By far, the most popular approaches in the literature are due to Grimme.^{68,69} Thus, we are going to use this correction in combination with PBE, B3LYP, B2PLYP, and B2GP-PLYP.

The second way involves the use of nonlocal functionals. Here, the correction is based on the electron density and can be used in a self-consistent way, as demonstrated by Vydrov and Van Voorhis.⁷³ The most successful functional of this kind is VV10 (or in a non-self-consistent way, similar to the $-D$ approach as applied to many functionals by Grimme⁷⁴). In this study, we chose the B3LYP-NL functional, in a self-consistent way, since this functional was proposed by Grimme to be the most accurate one for this group.⁷⁴

2.3. Data Sets. The next question to address is which data sets to use. Given that the focus of this study is the problem of weak interactions we chose four data sets that contain nontrivial problems with weak interactions. All geometries for all data sets were taken from the Web site of Truhlar's group.^{80,92}

2.3.1. Molecules. The first data set is HC7–11.^{96–99,113,127} The geometries for this set were obtained at the MP2/6-311+G(d,p) level of theory. The set consists of seven difficult cases involving alkanes. The first three cases are isomerizations, and the rest are isodesmic stabilization reactions. The first two cases involve isomerization of D_{3d} -octahedrane ($(CH_2)_{12}$) that has been shown to present a significant challenge to DFT (especially B3LYP).⁹⁶ The basis set size of the larger molecule, using apVTZ basis set, is 828 basis functions, which makes the use of canonical CCSD(T) challenging but still possible.

The second data set is ISOL6/11.^{88,95} It consists of isomerization energies of organic molecules. The molecules contain 24–35 atoms and were chosen by Luo et al.⁸⁸ from a larger set proposed by Huenerbein et al.⁹⁵ in order to perform more accurate reference calculations.

The third data set is the NCCE31/05 data set.^{82,98,99} This is a set composed of five smaller sets, HB6/04,⁸² CT7/04,⁸² DI6/04,⁸² WI7/05,⁸³ and PP55/05.⁸³ All smaller sets consist of

binding energies: HB6/04, six hydrogen-bonding dimers; CT7/04, seven charge-transfer complexes; DI6/04, six dipole interaction complexes; WI7/05, weak interaction complexes; and PP5/05, 5 π - π stacking complexes

Finally, the fourth data set is π TC13.^{100–102} This data set is composed of three smaller data sets containing allene–propyne isomerization energies (π IE3/06^{102,128}) and proton affinities of conjugated polyenes (PA-CP5/06¹⁰²) as well as conjugated Schiff bases (PA-SB5/06¹⁰²).

2.3.2. Reference Values. A point of obvious importance in judging the accuracy of a family of methods is the accuracy of the reference values themselves. While accurate gas-phase experimental data would be the ultimate reference data, many studies choose high-level calculations as reference in order to avoid possible complications from vibrational or other effects. The advantage of using theoretical reference data is that one can focus on the electronic energy alone. The established approach is to aim for the CCSD(T) complete basis set limit. While, for all test sets used in this study, reference data are available, we made a dedicated effort to check and possibly improve on this data in order to put our conclusions on as-solid-as-possible ground.

For the HC7/11 data set, the largest basis we were able to use, for all molecules, in conjunction with the CCSD(T) method, is apVTZ. Hence, the reference values were calculated by direct extrapolation of canonical CCSD(T) energies in conjunction with the apVDZ and apVTZ basis sets. We refer to this direct extrapolation scheme as EP1; hence, these values are denoted EP1(CCSD(T); apVDZ, apVTZ).^{129–132} In the EP1 scheme, the total energy is obtained using the following formula

$$E_{\text{CBS}}^{\text{EP1}}(M; X, Y) = \underbrace{\frac{E_{\text{HF}}(Y)e^{-\alpha\sqrt{X}} - E_{\text{HF}}(X)e^{-\alpha\sqrt{Y}}}{e^{-\alpha\sqrt{X}} - e^{-\alpha\sqrt{Y}}}}_{\text{Hartree-Fock}} + \underbrace{\frac{X^\beta E_{\text{corr}}(X) - Y^\beta E_{\text{corr}}(Y)}{X^\beta - Y^\beta}}_{\text{Correlation}} \quad (1)$$

where M is the method used, $E_{\text{HF}}(X)$ is the Hartree–Fock energy obtained with the basis set with cardinal number X (2 for apVDZ and 3 for apVTZ), $E_{\text{corr}}(X)$ similarly is the corresponding correlation energy, and α and β are constants depending on the basis sets. We prefer the values obtained this way over the ones reported by Peverati,⁹⁹ partially because it was not entirely clear to us by which method their values were obtained. In the [Supporting Information](#), we report results based on the original reference data. No qualitative differences to the conclusions drawn here arise.

For the ISOL6/11 data set, Luo proposed CCSD(T)-F12a/aug-cc-pVDZ and CCSD(T)-F12/aug-cc-pVDZ reference values. Unfortunately, the two sets of reference values show significant differences. For this reason, we have chosen to calculate a new set of reference values. Since it was not possible to perform canonical CCSD(T) calculations in conjunction with apVTZ basis set for all molecules, we used the EP2 extrapolation scheme.¹³² In this scheme, the total energy is approximated as

$$E_{\text{CBS}}^{\text{EP2}}(M; X, Y, X') = \underbrace{E_{\text{CBS}}^{\text{EP1}}(M; X, Y)}_{\text{method's } M \text{ CBS}} + \underbrace{(E_{\text{CCSD(T)}}(X') - E_M(X'))}_{\text{CCSD(T) adjustment}} \quad (2)$$

In the traditional EP2 method, basis sets X and X' are the same; however, this is not a theoretical requirement, and it can indeed be beneficial to use a different X . In this study, we used the EP2 extrapolation scheme, EP2(DLPNO-CCSD(T); apVDZ, apVTZ, apVDZ) using the DLPNO-CCSD(T) with TightPNO set up and full local MP2 guess. Our new reference

values are typically between the previous two sets of values. Again, in the [Supporting Information](#), we include all reference values and repeat the statistical analysis for all of them. Once more, no qualitatively significant changes are observed.

For the NCCE31/05 data set, it was possible to calculate single-point energies with the CCSD(T) method in conjunction with apVTZ basis set; thus, our reference values are the ones approximated through the EP1(CCSD(T); apVDZ, apVTZ) scheme. In the [Supporting Information](#), the reference values from Truhlar's web site are used, and no qualitative changes are observed compared to the results reported here. Finally, for the π TC13 data set, we calculated reference values in the same way as we did for ISOL6/11. In the [Supporting Information](#), we also include the reference values of Truhlar's web site. Beside a small general shift, no qualitative difference is observed.

For the π TC13 and ISOL6/11 sets, the reference values were not calculated through direct EP1(CCSD(T); apVDZ, apVTZ) extrapolation but through the EP2(DLPNO-CCSD(T); apVDZ, apVTZ, apVDZ) scheme that involves the DLPNO-CCSD(T) method. Hence, the question may be raised regarding whether the reference values are biased toward DLPNO-CCSD(T) values. In [Figure 1](#), we compare, for the

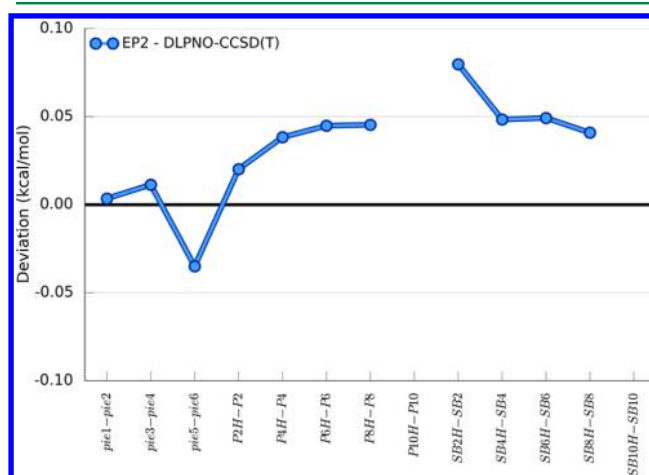


Figure 1. Accuracy of EP2-DLPNO-CCSD(T) reference reaction energies with respect to EP1(CCSD(T); apVDZ, apVTZ).

π TC13 data set, the reaction energies we used as reference values (using the EP2 scheme) with the ones calculated with direct EP1(CCSD(T); apVDZ, apVTZ) extrapolation.

One can see in [Figure 1](#) that the deviation exceeds 0.05 kcal/mol in only one case and is always smaller than 0.08 kcal/mol. Thus, we can safely conclude that for all four data sets the reference values were of direct EP1(CCSD(T); apVDZ, apVTZ) quality and are not biased in favor of DLPNO-CCSD(T).

The next question is if the reaction energies calculated using direct extrapolation of CCSD(T), in conjunction with augmented double and triple- ζ basis correlation consistent basis sets, are converged enough. In order to at least partially address this valid question, we used CCSD(T) extrapolations in conjunction with the cc-pVQZ and aug-cc-pVQZ basis sets and CCSD(T)-F12 calculations using the cc-pVDZ-F12 and cc-pVTZ-F12 basis sets, again for as many molecules as possible from the π TC13 data set. Finally, the EP2 scheme was used, but this time with MP2 method EP2(MP2; pVQZ, pVSZ, apVDZ). This amounts to the very popular focal point method.^{133,134}

In Figure S1, we plot the differences between these methods. From Figure S1, one can see that all different methods, except the one involving MP2 in the extrapolation, lie within 0.5 kcal/mol. Nevertheless, even though this difference is not negligible, no qualitative difference occurs from the use of different reference values, and it is evident that the conclusions drawn based on the EP1(CCSD(T); apVDZ, apVTZ) values should hold even if more accurate reference values than the present ones become available for the entire range of test sets. The focal point values involving the MP2 method appear to be of somewhat lower quality.

3. RESULTS AND DISCUSSION

3.1. Accuracy. In Figure 2, we plot the mean absolute deviation (MAD) for the chosen methods in conjunction with

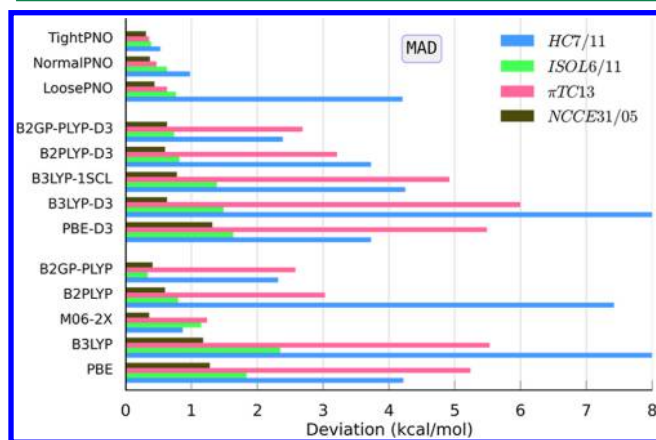


Figure 2. Mean absolute deviation (MAD) for various methods and data sets.

the aug-cc-pVTZ basis set. In the Supporting Information, we report all calculated values and additional statistical analysis.

The first comment one can make about Figure 2 is that the expected result for DLPNO-CCSD(T) with the TightPNO setup is extremely accurate even for these difficult cases. The largest absolute error observed with DLPNO-CCSD(T)/TightPNO is 0.54 kcal/mol. Given the fact that some of the error must be attributed to the deficiencies of the apVTZ basis set, the great power of the method becomes evident. Evidently, much of this basis set error could be easily removed through extrapolation. However, this is not done here in order to document the accuracy of the method in conjunction with one reasonable basis set. In Section 3.4, the study will be extended to cover additional basis sets.

The results based on the NormalPNO settings of DLPNO-CCSD(T) closely follow those of the TightPNO setup in accuracy; these results are more robust and accurate than all of the functionals tested. The MAD for the four sets is 0.98, 0.63, 0.37, and 0.47 kcal/mol for HC7/11, ISOL6, NCCE 31/05, and π TC13, respectively, well inside the 1 kcal/mol limit that defines chemical accuracy. The absolute maximum deviations are 1.82, 1.37, 1.75, and 1.44 kcal/mol, respectively, confirming the robustness of the setup even in these extreme cases.

Next, we turn our attention to the LoosePNO setup. The thresholds for LoosePNO were designed with efficiency in mind. Nevertheless, from Figure 2, one can conclude that LoosePNO still demonstrates overall better behavior than any of the

functionals used. There are cases where different functionals perform better than LoosePNO, but none of them is consistently better. Thus, if accuracy is the only criterion, based on the reported tests, then we believe that LoosePNO DLPNO-CCSD(T) is still superior to the tested functionals.

The worst case for LoosePNO happens in the HC7/11 data set. The worst performance comes in reaction 6 of the set that describes the following reaction: adamantane \rightarrow 3 ethylene + 2 ethyne. Grimme calculated the heats of formation of adamantane, ethylene, and ethyne using various methods.¹³⁵ From his calculations, one can see that the error of MP2 for adamantane is of the opposite sign compared to the errors of ethylene and ethyne, meaning that when one calculates this reaction energy the error would increase. In addition, the error for adamantane is quite large (9.1 kcal/mol). In 2006, Grimme published a detailed analysis concerning the stereoelectronic effects in alkanes and the errors of DFT.¹²⁷ He concluded that the main cause of error in the description of branched alkanes is the interpair correlation. Given that in LoosePNO the threshold for pairs that are described with MP2 is looser, we believe that this MP2 effect is the cause for the larger deviation. The error with MP2 aug-cc-pVTZ for this reaction is 14.79 kcal/mol, and with LoosePNO, -8.00 kcal/mol. As expected when one uses the NormalPNO setup, the error decreases significantly (-1.29 kcal/mol) while with TightPNO one obtains an accurate result (error = -0.74 kcal/mol).

Concerning the functionals, the expected Jacob's ladder behavior is obtained here, too. Thus, the double hybrid functionals produce robust and accurate energies, as already reported in the literature.^{77,84,87,136,137} The behavior of M06-2X is very good, being, in many cases, the most accurate among the various DFT functionals. However, this is not the observed behavior in the literature, where it is usually better than hybrid functionals but somewhat inferior to double hybrid functionals.^{77,87,90,136} One should keep in mind, of course, that most of the data sets used here were also used for the parametrization of M06-2X. Between PBE and B3LYP, in their dispersion-corrected and non-corrected versions, B3LYP performs better in most cases, but the cases where PBE is superior are not negligible.

Finally, as has been shown in the past,^{87,88,91,137} dispersion corrections improve the results in most cases. Comparing the nonlocal correction with the $-D3$ correction, it seems that the nonlocal dispersion behaves slightly better than the $-D3$ correction, at least for the case of B3LYP tested here. This finding is in agreement with previous observations.⁷⁴

3.2. Timings. Next, we turn our attention to computational efficiency. In Figure 3, we plot, for the same methods as before, the total wall clock time required for the calculations. The results will vary somewhat if computational packages other than ORCA are used. However, in our experience, the ORCA package is sufficiently efficient in order to justify the reported timings. All calculations were performed on a computer where other jobs were also running. Thus, small inaccuracies are to be expected, but the general conclusions should remain. In order to obtain a better graphical representation, we plotted the time needed for each method divided by the time required by B3LYP rather than plotting the actual execution times. Obviously, this choice is arbitrary, but it has no effect concerning the conclusions and should make the results easier to visualize. In the Supporting Information, all absolute timings are reported.

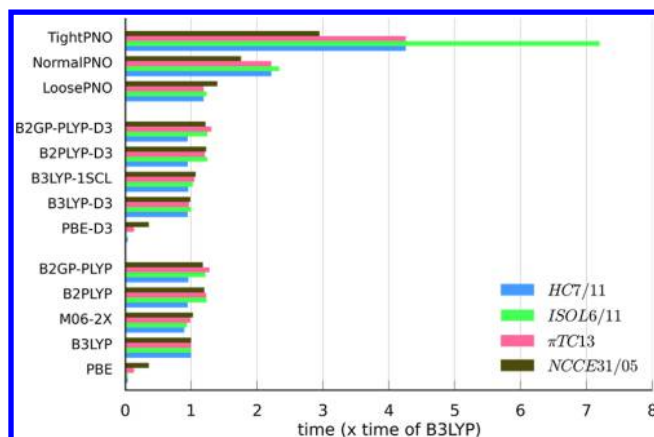


Figure 3. Average timings for various methods and data sets with respect to B3LYP timings.

In terms of efficiency, one can distinguish three broad categories. The first one contains the GGA functionals PBE and PBE-D3. These functionals exploit the efficiency of the resolution of the identity (density fitting) approximation^{138–145} and the lack of exact exchange. Hence, they are extremely efficient.

The next group contains the remaining functionals and also the DLPNO-CCSD(T) method with the LoosePNO threshold settings. All of these different methods require essentially the same amount of time as that for B3LYP. Specifically, the computational cost of LoosePNO DLPNO-CCSD(T) is 1.19, 1.24, 1.19, and 1.40 times that of B3LYP for HC7/11, ISOL6/11, π TC13, and NCCE31/05, respectively.

In the third group, one finds the NormalPNO and TightPNO setups of DLPNO-CCSD(T). For the NormalPNO setup, the average timings are 2.22, 2.34, 2.22, and 1.76 times those for HC7/11, ISOL6/11, π TC13, and NCCE31/05, respectively. Thus, NormalPNO DLPNO-CCSD(T) requires approximately twice the amount of time of a B3LYP calculation. Given the superiority in accuracy reported above, this appears to be a small price to pay for the increased accuracy.

Finally, if one needs to proceed to the TightPNO setup, the corresponding times relative to B3LYP become 4.26, 7.20, 4.26, and 2.95. The worst case happens in the ISOL6 data set, which requires, on average, 7.20 times that of B3LYP. We should nevertheless mention that, in terms of execution time, TightPNO DLPNO-CCSD(T) is still a huge improvement compared to the canonical CCSD(T) method, where, for example, we could not perform the calculations for ISOL6/11 with the apVTZ basis set. For the HC7/11 data set with the apVTZ basis set, the DLPNO-CCSD(T) calculation with the TightPNO setup is up to 35 times faster than the canonical one.

3.3. Effect of System Size. It is valid to ask how these relative times will change for molecules that are larger than the molecules included in the test set (maximum size, ~ 30 atoms). In particular, the double hybrid DF's make use of a MP2-like step that scales as $O(N^5)$. For the relatively small molecules here, this step is insignificant due to the use of the RI-approximation in the MP2 step. However, for large molecules, it will eventually become dominant. By contrast, the coupled cluster part of DLPNO-CCSD(T) is near linear in its scaling. Hence, its execution time relative to the quadratic scaling SCF procedure will, rather, become more favorable as the system size increases.

In order to demonstrate this behavior, we performed calculations in a series of alkanes with up to 70 carbon atoms. Obviously, linear alkane chains do not represent realistic real-life systems. However, they are commonly employed in order to demonstrate the asymptotic scaling of a given method. The same behavior is expected for real three-dimensional molecules. However, in the latter case, the asymptotic scaling will be reached much later, i.e., for much larger molecules.

In Figure 4, we plot the scaling of DLPNO-CCSD(T) with NormalPNO settings as well as, for comparison, the

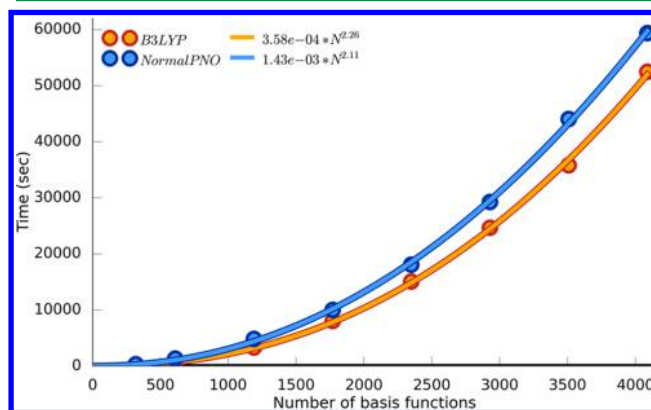


Figure 4. Scaling of the NormalPNO DLPNO-CCSD(T) and B3LYP methods in conjunction with cc-pVTZ for linear alkanes C_nH_{2n+2} for n up to 70.

corresponding B3LYP timings. It is obvious that the asymptotic scaling of both calculations is nearly identical. This is, in fact, the anticipated result, since in both cases the self-consistent field process has asymptotic quadratic scaling. It is also obvious from Figure 4 that the extra cost of DLPNO-CCSD(T) over B3LYP is very limited. It should also be mentioned that in both cases no additional accelerations, such as RIJCOSX,^{146,147} have been performed. They would mainly reduce the prefactor of the computation and apply in the same way to both methods.

3.4. Effect of Basis Set. As a workhorse for this study, we used the apVTZ basis set. Nevertheless, as one of the reviewers noted, this basis set may still be too small for a wave function-based correlation calculation, whereas it is too large for a DFT study. Hence, it is interesting to investigate how the results of the present study change in response to changes in the one-particle basis set. All results are documented in detail in the Supporting Information. Here, we will only allude to the most important conclusions.

The first important test is to determine the effective scaling of each method with respect to basis set extension. This is best done using a family of basis sets of increasing size, such as the correlation consistent basis sets. Hence, we performed calculations with the cc-pVnZ family of basis sets for $n = D, T, Q$ (Figure 5).

From the results, a well-known fact is obtained, namely, that the canonical CCSD(T) method has an explosive cost with respect to basis set extension: going from pVDZ to pVQZ, an increase of two cardinal numbers, leads to an increased cost factor of 136 for π TC13. This is fundamentally different in DLPNO-CCSD(T) since the number of PNOs per pair is approaching a constant as the basis set is approaching completion.⁵² Since the integral transformation to obtain the integrals over these PNOs is a major factor in the DLPNO-CCSD(T) calculation, the cost increase is still significant but

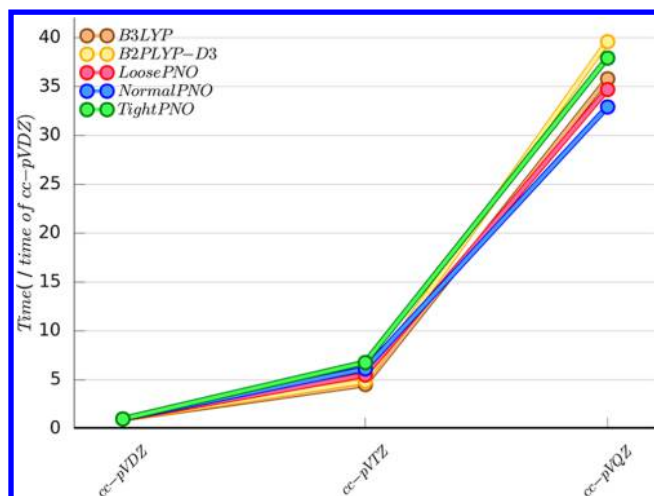


Figure 5. Effective scaling of various methods with respect to the basis set for the π TC13 data set.

not nearly as steep as that for canonical CCSD(T). In fact, when going from pVDZ to pVTZ for the NormalPNO settings, the computational wall time increases by roughly a factor of 6, whereas for B3LYP, the corresponding factor is 4.52. By contrast, the corresponding factor for canonical CCSD(T) is 12.5. In our experience, these effects are even more pronounced for larger basis sets.

In the next step, the affect of the one-particle basis set on the accuracy of each method was studied (Figure 6).

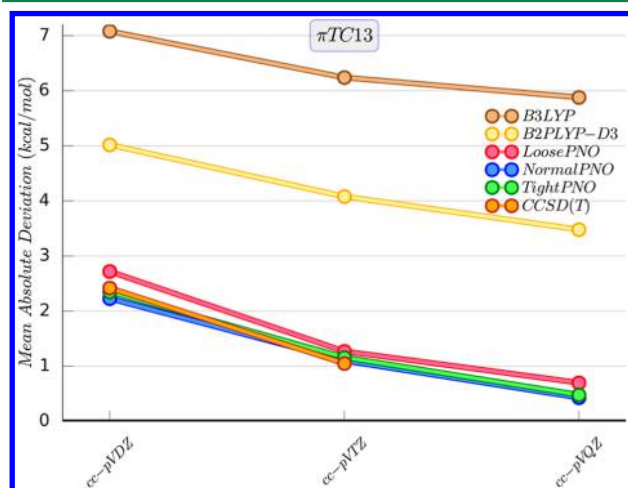


Figure 6. Convergence of methods with the one-particle basis set with respect to the EP2(DLPNO-CCSD(T); apVDZ, apVTZ, apVQZ) reference.

Again, these findings reinforce the well-known behavior that the convergence of the electronic energies with respect to the size of the one-particle basis set is slower for wave function-based correlation methods than it is for DFT. However, the effects are not drastic. Either method increases significantly in accuracy when going from a double- to a triple- ζ basis set. For the wave function methods, this is probably mandatory in order to realize their full potential. However, a triple- ζ basis set brings either method close to the limiting accuracy for chemical applications.

In order to put this analysis on an even more sound basis, we repeated all calculations with def2-SVP,¹⁴⁸ def2-TZVPP,^{148,149}

and def2-QZVPP¹⁵⁰ that are frequently used for DFT calculations. All results are summarized in the [Supporting Information](#). It emerges that all conclusions that were drawn from the calculations with the apVTZ basis set hold, in general, for practically all basis sets tested here. Obviously, DFT in a double- ζ basis set is much more efficient than DLPNO-CCSD(T) in a triple- ζ basis set (e.g., B3LYP/pVDZ is 11 times faster than NormalPNO DLPNO-CCSD(T)/pVTZ). However, the accuracy one reaches in such calculations is far inferior. In fact, DLPNO-CCSD(T) with the NormalPNO settings using a double- ζ basis, at least for the test sets studied here, is still significantly more accurate than any of the studied DFT functionals with an extended basis set.

4. CONCLUSIONS

For many years, DFT has served the scientific community well, and there is no question in our minds that this situation will persist for the foreseeable future. However, we do believe that the strong emphasis on DFT will diminish to some extent given the drastic improvements in efficiency that the wave function-based approaches have seen in recent years. The latter still require very significant development before their linear scaling realizations are as universally applicable as DFT. However, we believe that we have demonstrated that it is feasible to obtain accurate wave function energies at a computational cost that is comparable to that of rank 4 DFT. Hence, one might argue that higher ranked functionals should be compared against DLPNO-CCSD(T) since they do not offer computational advantages over the latter. Whether their accuracy can be pushed to the same level is an open question. The benchmarks assembled here indicate that double hybrid functionals, while being accurate, are still inferior to DLPNO-CCSD(T). Using the three defined levels of accuracy, LoosePNO, NormalPNO, and TightPNO,⁶⁶ it is possible for the user to strike the desired balance between accuracy and efficiency.

We have shown above that with NormalPNO thresholds, DLPNO-CCSD(T) is about a factor of 2 slower than B3LYP and shows a mean absolute deviation of less than 1 kcal/mol from the reference data for the four different, difficult data sets used. The robustness and accuracy of the method for all data sets tested were consistently better than those with even the most advanced functionals used. We believe that this investment in computer time is justified in view of the quality of the results obtained.

Finally, the TightPNO setup is more expensive than B3LYP by about 3 to 4.5 times. Together with the apVTZ basis set, it produces results close to the CCSD(T)/CBS level while being orders of magnitude more efficient.

With respect to extensions of system size, we have shown that the effective scaling of DFT and DLPNO-CCSD(T) is effectively dominated by the scaling of the self-consistent field equations, which is typically quadratic to cubic unless additional approximations are introduced. Full linear scaling is achievable for either DFT or DLPNO-CCSD(T).

With respect to extensions of the one-particle basis sets, we have shown that all conclusions hold if one compares DFT and DLPNO-CCSD(T) for the same basis set. However, DLPNO-CCSD(T), being a wave function-based correlation method converges more slowly to the basis set limit than does DFT. However, at least for the test cases considered here, for practical applications, a triple- ζ basis sets brings both DFT and DLPNO-CCSD(T) sufficiently close to their basis set limit. In either case, a double- ζ basis set leads to significantly inferior

results. However, DLPNO-CCSD(T) in a double- ζ basis is, at least for the test cases studied here, still significantly more accurate than any of the DFT functionals using an extended basis set.

Thus, the main conclusion of this work is that coupled cluster energies can indeed be obtained at near DFT cost. To the best of our knowledge, this has been demonstrated for the first time here in a systematic fashion. Thus, for a given computational chemistry study, computational efficiency no longer dictates that DFT has to be chosen in favor of coupled cluster theory. We believe that exciting opportunities for computational chemistry are emerging from this methodology.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: [10.1021/acs.jctc.5b00359](https://doi.org/10.1021/acs.jctc.5b00359).

All values for the cc-pVTZ basis set and all methods used (XLSX).

Figures with the behavior of chosen methods for all combinations of data set and basis set (PDF).

■ AUTHOR INFORMATION

Corresponding Author

*E-mail: Frank.Neese@cec.mpg.de.

Notes

The authors declare no competing financial interest.

■ REFERENCES

- (1) Riplinger, C.; Sandhoefer, B.; Hansen, A.; Neese, F. *J. Chem. Phys.* **2013**, *139*, 134101.
- (2) Řezáč, J.; Hobza, P. *J. Chem. Theory Comput.* **2013**, *9*, 2151–2155.
- (3) Bartlett, R. J.; Purvis, G. D. *Int. J. Quantum Chem.* **1978**, *14*, 561–581.
- (4) Purvis, G. D.; Bartlett, R. J. *J. Chem. Phys.* **1982**, *76*, 1910–1918.
- (5) Pople, J. A.; Head-Gordon, M.; Raghavachari, K. *J. Chem. Phys.* **1987**, *87*, 5968–5975.
- (6) Raghavachari, K.; Trucks, G. W.; Pople, J. A.; Head-Gordon, M. *Chem. Phys. Lett.* **1989**, *157*, 479–483.
- (7) Lee, T. J.; Rendell, A. P.; Taylor, P. R. *J. Phys. Chem.* **1990**, *94*, 5463–5468.
- (8) Stoll, H. *J. Chem. Phys.* **1992**, *97*, 8449–8454.
- (9) Stoll, H. *Chem. Phys. Lett.* **1992**, *191*, 548–552.
- (10) Stoll, H. *Phys. Rev. B: Condens. Matter Mater. Phys.* **1992**, *46*, 6700–6704.
- (11) Friedrich, J.; Hanrath, M.; Dolg, M. *J. Chem. Phys.* **2007**, *126*, 154110.
- (12) Friedrich, J.; Dolg, M. *J. Chem. Phys.* **2008**, *129*, 244105.
- (13) Friedrich, J.; Dolg, M. *J. Chem. Theory Comput.* **2009**, *5*, 287–294.
- (14) Friedrich, J. *J. Chem. Theory Comput.* **2012**, *8*, 1597–1607.
- (15) Friedrich, J.; Walczak, K. *J. Chem. Theory Comput.* **2013**, *9*, 408–417.
- (16) Friedrich, J.; Hänchen, J. *J. Chem. Theory Comput.* **2013**, *9*, 5381–5394.
- (17) Li, S.; Ma, J.; Jiang, Y. *J. Comput. Chem.* **2002**, *23*, 237–244.
- (18) Li, S.; Shen, J.; Li, W.; Jiang, Y. *J. Chem. Phys.* **2006**, *125*, 074109.
- (19) Li, W.; Piecuch, P.; Gour, J. R.; Li, S. *J. Chem. Phys.* **2009**, *131*, 114109.
- (20) Li, W.; Piecuch, P. *J. Phys. Chem. A* **2010**, *114*, 8644–8657.
- (21) Li, W.; Piecuch, P. *J. Phys. Chem. A* **2010**, *114*, 6721–6727.
- (22) Guo, Y.; Li, W.; Li, S. *J. Phys. Chem. A* **2014**, *118*, 8996–9004.
- (23) Li, W.; Li, S. *Sci. China: Chem.* **2014**, *57*, 78–86.
- (24) Flocke, N.; Bartlett, R. J. *J. Chem. Phys.* **2004**, *121*, 10935–10944.
- (25) Hughes, T. F.; Flocke, N.; Bartlett, R. J. *J. Phys. Chem. A* **2008**, *112*, 5994–6003.
- (26) Ziolkowski, M.; Jansik, B.; Kjaergaard, T.; Jorgensen, P. *J. Chem. Phys.* **2010**, *133*, 014107.
- (27) Kristensen, K.; Ziolkowski, M.; Jansik, B.; Kjaergaard, T.; Jorgensen, P. *J. Chem. Theory Comput.* **2011**, *7*, 1677–1694.
- (28) Hoyvik, I. M.; Kristensen, K.; Jansik, B.; Jorgensen, P. *J. Chem. Phys.* **2012**, *136*, 014105.
- (29) Rolik, Z.; Kallay, M. *J. Chem. Phys.* **2011**, *135*, 104111.
- (30) Kristensen, K.; Kjaergaard, T.; Høyvik, I.-M.; Ettenhuber, P.; Jorgensen, P.; Jansik, B.; Reine, S.; Jakowski, J. *Mol. Phys.* **2013**, *111*, 1196–1210.
- (31) Schütz, M.; Hetzer, G.; Werner, H.-J. *J. Chem. Phys.* **1999**, *111*, 5691–5705.
- (32) Rauhut, G.; Pulay, P.; Werner, H.-J. *J. Comput. Chem.* **1998**, *19*, 1241–1254.
- (33) Hampel, C.; Werner, H. J. *J. Chem. Phys.* **1996**, *104*, 6286–6297.
- (34) Schutz, M. *Phys. Chem. Chem. Phys.* **2002**, *4*, 3941–3947.
- (35) Schutz, M.; Manby, F. R. *Phys. Chem. Chem. Phys.* **2003**, *5*, 3349–3358.
- (36) Schütz, M.; Werner, H.-J. *Chem. Phys. Lett.* **2000**, *318*, 370–378.
- (37) Schütz, M. *J. Chem. Phys.* **2002**, *116*, 8772–8785.
- (38) Meyer, W. *Methods of Electronic Structure Theory*; Schaefer, H. F., III, Ed.; Plenum Press: New York, 1977.
- (39) Meyer, W. *Theor. Chem. Acc.* **1974**, *35*, 277–292.
- (40) Meyer, W. *J. Chem. Phys.* **1973**, *58*, 1017–1035.
- (41) Meyer, W. *Int. J. Quantum Chem.* **1971**, *S5*, 341–348.
- (42) Ahlrichs, R.; Driessler, F.; Lischka, H.; Staemmler, V.; Kutzelnigg, W. *J. Chem. Phys.* **1975**, *62*, 1235–1247.
- (43) Ahlrichs, R.; Driessler, F. *Theor. Chem. Acc.* **1975**, *36*, 275–287.
- (44) Staemmler, V.; Jaquet, R. *Theor. Chem. Acc.* **1981**, *59*, 487–500.
- (45) Taylor, P. R. *J. Chem. Phys.* **1981**, *74*, 1256–1270.
- (46) Neese, F.; Hansen, A.; Liakos, D. G. *J. Chem. Phys.* **2009**, *131*, 064103–15.
- (47) Neese, F.; Hansen, A.; Wennmohs, F.; Grimme, S. *Acc. Chem. Res.* **2009**, *42*, 641–648.
- (48) Neese, F.; Wennmohs, F.; Hansen, A. *J. Chem. Phys.* **2009**, *130*, 114108–18.
- (49) Liakos, D. G.; Hansen, A.; Neese, F. *J. Chem. Theory Comput.* **2011**, *7*, 76–87.
- (50) Hansen, A.; Liakos, D. G.; Neese, F. *J. Chem. Phys.* **2011**, *135*, 214102–20.
- (51) Riplinger, C.; Neese, F. *J. Chem. Phys.* **2013**, *138*, 034106.
- (52) Liakos, D. G.; Neese, F. *J. Chem. Theory Comput.* **2015**, *11*, 2137–2143.
- (53) Hansen, A.; Bannwarth, C.; Grimme, S.; Petrović, P.; Werlé, C.; Djukic, J.-P. *ChemistryOpen* **2014**, *3*, 177–189.
- (54) Calbo, J.; Ortí, E.; Sancho-García, J. C.; Aragón, J. *J. Chem. Theory Comput.* **2015**, *11*, 932–939.
- (55) Dwyer, A. D.; Tozer, D. J. *J. Chem. Phys.* **2011**, *135*, 164110.
- (56) Peach, M. J. G.; Tozer, D. J.; Handy, N. C. *Int. J. Quantum Chem.* **2011**, *111*, 563–569.
- (57) Burke, K. *J. Chem. Phys.* **2012**, *136*, 150901.
- (58) Klimeš, J.; Michaelides, A. *J. Chem. Phys.* **2012**, *137*, 120901.
- (59) Eshuis, H.; Bates, J.; Furche, F. *Theor. Chem. Acc.* **2012**, *131*, 1084.
- (60) Sun, J.; Xiao, B.; Fang, Y.; Haunschild, R.; Hao, P.; Ruzsinszky, A.; Csonka, G. I.; Scuseria, G. E.; Perdew, J. P. *Phys. Rev. Lett.* **2013**, *111*, 106401.
- (61) Gledhill, J. D.; Peach, M. J. G.; Tozer, D. J. *J. Chem. Theory Comput.* **2013**, *9*, 4414–4420.
- (62) Hao, P.; Sun, J.; Xiao, B.; Ruzsinszky, A.; Csonka, G. I.; Tao, J.; Glindmeyer, S.; Perdew, J. P. *J. Chem. Theory Comput.* **2013**, *9*, 355–363.
- (63) Becke, A. D. *J. Chem. Phys.* **2014**, *140*, 18A301.
- (64) Borgoo, A.; Green, J. A.; Tozer, D. J. *J. Chem. Theory Comput.* **2014**, *10*, 5338–5345.

- (65) Mezei, P. D.; Csonka, G. I.; Ruzsinszky, A.; Sun, J. *J. Chem. Theory Comput.* **2015**, *11*, 360–371.
- (66) Liakos, D. G.; Sparta, M.; Kesharwani, M. K.; Martin, J. M. L.; Neese, F. *J. Chem. Theory Comput.* **2015**, *11*, 1525–1539.
- (67) Perdew, J. P.; Burke, K.; Ernzerhof, M. *Phys. Rev. Lett.* **1996**, *77*, 3865.
- (68) Grimme, S.; Ehrlich, S.; Goerigk, L. *J. Comput. Chem.* **2011**, *32*, 1456–1465.
- (69) Grimme, S.; Antony, J.; Ehrlich, S.; Krieg, H. *J. Chem. Phys.* **2010**, *132*, 154104–19.
- (70) Becke, A. D. *Phys. Rev. A: At, Mol, Opt. Phys.* **1988**, *38*, 3098–3100.
- (71) Lee, C. T.; Yang, W. T.; Parr, R. G. *Phys. Rev. B: Condens. Matter Mater. Phys.* **1988**, *37*, 785–789.
- (72) Becke, A. D. *J. Chem. Phys.* **1993**, *98*, 5648–5652.
- (73) Vydrov, O. A.; Van Voorhis, T. *J. Chem. Phys.* **2010**, *133*, 244103.
- (74) Hujo, W.; Grimme, S. *J. Chem. Theory Comput.* **2011**, *7*, 3866–3871.
- (75) Zhao, Y.; Truhlar, D. G. *J. Phys. Chem. C* **2008**, *112*, 4061–4067.
- (76) Grimme, S. *J. Chem. Phys.* **2006**, *124*, 034108–16.
- (77) Karton, A.; Tarnopolsky, A.; Lamère, J.-F.; Schatz, G. C.; Martin, J. M. L. *J. Phys. Chem. A* **2008**, *112*, 12868–12886.
- (78) Goerigk, L.; Grimme, S. *Phys. Chem. Chem. Phys.* **2011**, *13*, 6670–6688.
- (79) Goerigk, L.; Grimme, S. *J. Chem. Theory Comput.* **2011**, *7*, 291–309.
- (80) Truhlar, D. Minnesota Database. <http://comp.chem.umn.edu/db/index.html>.
- (81) Grimme, S. *J. Comput. Chem.* **2004**, *25*, 1463–1473.
- (82) Zhao, Y.; Truhlar, D. G. *J. Chem. Theory Comput.* **2005**, *1*, 415–432.
- (83) Zhao, Y.; Truhlar, D. G. *J. Phys. Chem. A* **2005**, *109*, 5656–5667.
- (84) Grimme, S.; Steinmetz, M.; Korth, M. *J. Org. Chem.* **2007**, *72*, 2118–2126.
- (85) Goerigk, L.; Grimme, S. *J. Chem. Theory Comput.* **2010**, *6*, 107–126.
- (86) Karton, A.; Gruzman, D.; Martin, J. M. L. *J. Phys. Chem. A* **2009**, *113*, 8434–8447.
- (87) Riley, K. E.; Pitoňák, M.; Jurečka, P.; Hobza, P. *Chem. Rev.* **2010**, *110*, 5023–5063.
- (88) Luo, S.; Zhao, Y.; Truhlar, D. G. *Phys. Chem. Chem. Phys.* **2011**, *13*, 13683–13689.
- (89) Hujo, W.; Grimme, S. *Phys. Chem. Chem. Phys.* **2011**, *13*, 13942–13950.
- (90) Grimme, S.; Steinmetz, M. *Phys. Chem. Chem. Phys.* **2013**, *15*, 16031–16042.
- (91) Risthaus, T.; Grimme, S. *J. Chem. Theory Comput.* **2013**, *9*, 1580–1591.
- (92) Peverati, R.; Truhlar, D. G. *Philos. Trans. R. Soc., A* **2014**, *372*, 20120476.
- (93) Steinmetz, M.; Hansen, A.; Ehrlich, S.; Risthaus, T.; Grimme, S. *Accurate Thermochemistry for Large Molecules with Modern Density Functionals*; Springer: Berlin, 2014; pp 1–23.
- (94) Goerigk, L. *J. Chem. Theory Comput.* **2014**, *10*, 968–980.
- (95) Huenerbein, R.; Schirmer, B.; Moellmann, J.; Grimme, S. *Phys. Chem. Chem. Phys.* **2010**, *12*, 6940–6948.
- (96) Schreiner, P. R.; Fokin, A. A.; Pascal, R. A.; de Meijere, A. *Org. Lett.* **2006**, *8*, 3635–3638.
- (97) Zhao, Y.; Truhlar, D. G. *Org. Lett.* **2006**, *8*, 5753–5755.
- (98) Zhao, Y.; Truhlar, D. *Theor. Chem. Acc.* **2008**, *120*, 215–241.
- (99) Peverati, R.; Zhao, Y.; Truhlar, D. G. *J. Phys. Chem. Lett.* **2011**, *2*, 1991–1997.
- (100) Zhao, Y.; Schultz, N. E.; Truhlar, D. G. *J. Chem. Phys.* **2005**, *123*, 161103.
- (101) Zhao, Y.; Truhlar, D. G. *J. Chem. Phys.* **2006**, *125*, 194101.
- (102) Zhao, Y.; Truhlar, D. G. *J. Phys. Chem. A* **2006**, *110*, 10478–10486.
- (103) Neese, F. *Wiley Interdisciplinary Reviews: Computational Molecular Science* **2012**, *2*, 73–78.
- (104) Dunning, J. T. H. *J. Chem. Phys.* **1989**, *90*, 1007–1023.
- (105) Kendall, R. A.; Dunning, T. H.; Harrison, R. J. *J. Chem. Phys.* **1992**, *96*, 6796–6806.
- (106) Weigend, F.; Köhn, A.; Hättig, C. *J. Chem. Phys.* **2002**, *116*, 3175–3183.
- (107) Perdew, J. P.; Schmidt, K. *AIP Conf. Proc.* **2001**, *577*, 1–20.
- (108) Boese, A. D.; Handy, N. C. *J. Chem. Phys.* **2001**, *114*, 5497–5503.
- (109) Perdew, J. P.; Chevary, J. A.; Vosko, S. H.; Jackson, K. A.; Pederson, M. R.; Singh, D. J.; Fiolhais, C. *Phys. Rev. B: Condens. Matter Mater. Phys.* **1992**, *46*, 6671–6687.
- (110) Perdew, J. P.; Chevary, J. A.; Vosko, S. H.; Jackson, K. A.; Pederson, M. R.; Singh, D. J.; Fiolhais, C. *Phys. Rev. B: Condens. Matter Mater. Phys.* **1993**, *48*, 4978–4978.
- (111) Perdew, J. P.; Burke, K.; Wang, Y. *Phys. Rev. B: Condens. Matter Mater. Phys.* **1996**, *54*, 16533–16539.
- (112) Perdew, J. P. *Phys. Rev. B: Condens. Matter Mater. Phys.* **1986**, *33*, 8822.
- (113) Wodrich, M. D.; Corminboeuf, C.; Schleyer, P. v. R. *Org. Lett.* **2006**, *8*, 3631–3634.
- (114) Furche, F. *Phys. Rev. B: Condens. Matter Mater. Phys.* **2001**, *64*, 195120.
- (115) Scuseria, G. E.; Henderson, T. M.; Sorensen, D. C. *J. Chem. Phys.* **2008**, *129*, 231101.
- (116) Ángyán, J. G.; Liu, R.-F.; Toulouse, J.; Jansen, G. *J. Chem. Theory Comput.* **2011**, *7*, 3116–3130.
- (117) Eshuis, H.; Furche, F. *J. Phys. Chem. Lett.* **2011**, *2*, 983–989.
- (118) Heßelmann, A.; Görling, A. *Mol. Phys.* **2011**, *109*, 2473–2500.
- (119) Bleiziffer, P.; Heßelmann, A.; Görling, A. *J. Chem. Phys.* **2013**, *139*, 084113.
- (120) Bates, J. E.; Furche, F. *J. Chem. Phys.* **2013**, *139*, 171103.
- (121) Burow, A. M.; Bates, J. E.; Furche, F.; Eshuis, H. *J. Chem. Theory Comput.* **2014**, *10*, 180–194.
- (122) Elstner, M.; Porezag, D.; Jungnickel, G.; Elsner, J.; Haugk, M.; Frauenheim, T.; Suhai, S.; Seifert, G. *Phys. Rev. B: Condens. Matter Mater. Phys.* **1998**, *58*, 7260–7268.
- (123) Elstner, M.; Hobza, P.; Frauenheim, T.; Suhai, S.; Kaxiras, E. *J. Chem. Phys.* **2001**, *114*, 5149–5155.
- (124) Wu, X.; Vargas, M. C.; Nayak, S.; Lotrich, V.; Scoles, G. *J. Chem. Phys.* **2001**, *115*, 8748–8757.
- (125) Wu, Q.; Yang, W. *J. Chem. Phys.* **2002**, *116*, 515–524.
- (126) Zimmerli, U.; Parrinello, M.; Koumoutsakos, P. *J. Chem. Phys.* **2004**, *120*, 2693–2699.
- (127) Grimme, S. *Angew. Chem., Int. Ed.* **2006**, *45*, 4460–4464.
- (128) Woodcock, H. L.; Schaefer, H. F.; Schreiner, P. R. *J. Phys. Chem. A* **2002**, *106*, 11923–11931.
- (129) Klopper, W.; Kutzelnigg, W. *J. Mol. Struct.: THEOCHEM* **1986**, *135*, 339–356.
- (130) Helgaker, T.; Klopper, W.; Koch, H.; Noga, J. *J. Chem. Phys.* **1997**, *106*, 9639–9646.
- (131) Halkier, A.; Helgaker, T.; Jørgensen, P.; Klopper, W.; Koch, H.; Olsen, J.; Wilson, A. K. *Chem. Phys. Lett.* **1998**, *286*, 243–252.
- (132) Liakos, D. G.; Neese, F. *J. Phys. Chem. A* **2012**, *116*, 4801–4816.
- (133) Allinger, N. L.; Fermann, J. T.; Allen, W. D.; Schaefer, H. F., III *J. Chem. Phys.* **1997**, *106*, 5143–5150.
- (134) Császár, A. G.; Allen, W. D.; Schaefer, H. F. *J. Chem. Phys.* **1998**, *108*, 9751–9764.
- (135) Grimme, S. *J. Phys. Chem. A* **2005**, *109*, 3067–3077.
- (136) Korth, M.; Thiel, W. *J. Chem. Theory Comput.* **2011**, *7*, 2929–2936.
- (137) Grimme, S. *Org. Lett.* **2010**, *12*, 4670–4673.
- (138) Baerends, E. J.; Ellis, D. E.; Ros, P. *Chem. Phys.* **1973**, *2*, 41–51.
- (139) Whitten, J. L. *J. Chem. Phys.* **1973**, *58*, 4496–4501.
- (140) Dunlap, B. I.; Connolly, J. W. D.; Sabin, J. R. *J. Chem. Phys.* **1979**, *71*, 3396–3402.

- (141) Feyereisen, M.; Fitzgerald, G.; Komornicki, A. *Chem. Phys. Lett.* **1993**, *208*, 359–363.
- (142) Vahtras, O.; Almlöf, J.; Feyereisen, M. W. *Chem. Phys. Lett.* **1993**, *213*, 514–518.
- (143) Kendall, R. A.; Früchtel, H. A. *Theor. Chem. Acc.* **1997**, *97*, 158–163.
- (144) Weigend, F.; Häser, M. *Theor. Chem. Acc.* **1997**, *97*, 331–340.
- (145) Weigend, F.; Häser, M.; Patzelt, H.; Ahlrichs, R. *Chem. Phys. Lett.* **1998**, *294*, 143–152.
- (146) Neese, F.; Wennmohs, F.; Hansen, A.; Becker, U. *Chem. Phys.* **2009**, *356*, 98–109.
- (147) Izsák, R.; Neese, F. *J. Chem. Phys.* **2011**, *135*, 144105.
- (148) Schafer, A.; Horn, H.; Ahlrichs, R. *J. Chem. Phys.* **1992**, *97*, 2571–2577.
- (149) Weigend, F.; Ahlrichs, R. *Phys. Chem. Chem. Phys.* **2005**, *7*, 3297–3305.
- (150) Weigend, F.; Furche, F.; Ahlrichs, R. *J. Chem. Phys.* **2003**, *119*, 12753–12762.