

Effect of Solvent Discreteness on Solvation

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Solvent discreteness or “graininess” is usually considered to affect solvation energetics by modifying the intermolecular structure of the solvent, which in turn modifies its dielectric constant and the solute–solvent configurations. In this work, we separate the effect of solvent discreteness from solvent structure and polarity, as well as the arrangement of solvent particles around the solute. Because it is rather difficult to do this separation with real solutions and their “realistic” models, we utilized translationally fixed dipole lattices, which allow such a separation. The polarity and the dielectric constant of a dipole lattice can be kept invariant as the number density of the dipoles is varied. The lattice spacing represents the degree of discreteness or coarseness in a lattice. Dipole lattices that polarize according to the continuum prediction, as well as truly interacting dipole lattices, lead to effective cavity sizes that are significantly smaller than the geometrically defined exclusion radius (the radius of a sphere around the center of a solute ion into which solvent particles cannot penetrate). The results are similar for lattices of opposite microscopic polarization tendencies and opposite ferroelectric divergence properties. Placing the solute in an interstitial or substitutional position does not cause a qualitative change in the results; increased solvent discreteness leads to smaller Born radii. To check whether this interesting result is peculiar to dipole lattice representations, we studied various forms of solute–solvent distribution function $g(r)$. We derived a formula that connects $g(r)$ to an effective cavity radius, with the approximation that the microscopic polarization follows the continuum prediction. The effective cavity radius is again found to be smaller than the exclusion radius. In agreement with the dipole-lattice analysis, the effective cavity radius decreases with increasing “graininess” of the solute–solvent pair distribution function; wavier $g(r)$ ’s lead to smaller effective cavity sizes. This result has important conceptual and practical implications in solvation modeling. Solvent discreteness becomes an important factor in solvation in its own right, distinct from its indirect effects felt through modified solvent properties and solute–solvent configurations. Also, in light of the present results, it should be possible to develop better parameterization schemes for simplified solvation models.

1. Introduction

Solvents are composed of discrete particles. However, especially in the context of solvation, it is not uncommon to regard this discreteness or “molecularity” as merely one of the determinants of the solvent polarity or dielectric constant. The degree of solvent discreteness has also been viewed as a secondary determinant of the “cavity size” in a continuum description of solvation because it partially determines the extent of the space around a solute where there are no solvent particles. A common approach involves a “solute envelope” determined from quantum mechanical¹ calculations or empirical potentials.² A constant length is then added¹ to this envelope or a solvent-sized probe is rolled over the molecular “surface” of the solute² in order to account for the nearest approach of solvent molecules. In other words, solvent discreteness has been implicitly viewed as a factor that influences solvation by either affecting other properties or simply defining a steric exclusion radius. The discrete structure of the solvent as a variable in its own right has largely been ignored. This is understandable, because the solvent discreteness is not a simple variable that is independent of other properties of the solvent, when real solvents or “realistic” models are considered. However, this difficulty does not logically mean that the discreteness itself is not an important variable. To address this issue, we utilize dipole lattices with no translational degrees of freedom. Such lattices allow a rigorous separation of polarity and structure. To ensure that

our analysis is physically relevant and generalizable, and does not reflect the idiosyncrasies of a particular lattice geometry, we employed simple cubic (sc) and face-centered cubic (fcc) lattices. This choice is proper, because sc and fcc dipole lattices have rather different interdipolar interactions, as indicated by their opposite dielectric divergence properties at high polarity and markedly different radial polarization profiles in response to an ionic solute.³ We use both Langevin dipole and Brownian dipole lattices, the latter having a higher degree of realism for representing polar solvents because of the presence of explicit fluctuations in interdipolar fields. On an equal footing with the analysis that uses dipole lattices, we present an analogous analysis using pair-distribution functions that one may find in more realistic solute–solvent models. Using simulations of the fluids themselves is impractical because it is impossible to vary the solvent discreteness cleanly without affecting other system properties such as polarity and dielectric constant. Nature made it difficult to detect the effect of solvent discreteness by entangling it with many other traits of a given solvent. Nevertheless, solvent molecularity itself, independent of its effects propagated through other system properties, affects solvation energetics significantly, as we show in subsequent sections.

Our analysis has two main components. The first involves using various dipole lattices as solvents, and is covered in

sections 2–5. This part of the analysis also contains dipole-lattice solvents that polarize according to the continuum prediction. The second main component of our analysis involves a mapping of continuum polarization onto the radial distribution function $g(r)$ of a solute–solvent system, and is covered in sections 6 and 7. In section 2 we briefly describe the dipole lattice models used in this study. In section 3, we present a brief summary of the continuum model of ionic solvation. In section 4 we derive the continuum dielectric prediction for the shielding factor by which the solvent dipoles are screened from the ionic field. We then proceed to use this continuum prediction for the ion-dipole shielding to cast the continuum dielectric model into a dipole lattice in section 5. The qualitative validity of the results from this transformation are verified by a direct comparison with fully-interacting “noncontinuum” dipole lattices of Langevin dipoles as well as Brownian dipoles. In section 6 we begin the second part of our analysis, and derive a formula that relates Born radius to the structure of the solvent around an ionic solute by casting continuum polarization onto the radial distribution function $g(r)$. The generality of the results from dipole lattices (section 5) is confirmed in section 7 by systematically varying the “graininess” of various forms of $g(r)$. Section 8 summarizes our conclusions.

2. Dipole Lattice Models

An important part of our analysis involves the Langevin Dipole Lattice (LDL).⁴ As its name implies, an LDL is composed of a lattice of point dipoles, the polarization of which in an electric field is represented by the well-known Langevin function

$$\mu = \left[\coth(y) - \frac{1}{y} \right] \mu_0 \mathbf{E}, \quad y = E \frac{\mu_0^2}{3k_B T} \quad (1)$$

where μ is the induced dipole, μ_0 is the permanent dipole moment, E is the magnitude of the electric field at the Langevin dipole, k_B is the Boltzmann constant, and T is the temperature. The Langevin function describes the net orientational polarization of a permanent dipole undergoing rotational Brownian motion due to the thermal energy of its surroundings. However, for small field strengths, a Langevin dipole is equivalent to a point polarizability with a magnitude of $\mu_0^2/3k_B T$. Thus an LDL can represent both polar and polarizable solvents.

Although a Langevin dipole is a rigorous replacement for a single dipole undergoing rotational diffusion in an electric field, an LDL lacks fluctuations in the interdipolar fields.^{3,4} A Brownian dipole lattice (BDL) is composed of point dipoles explicitly fluctuating in orientation, resulting in fluctuating interdipolar electric fields. Thus a BDL offers a higher degree of realism for representing the behavior of dipolar solvents. However, the presence of explicit orientational fluctuations makes “pure” BDLs (i.e., BDLs composed of permanent dipoles with no polarizability) unsuitable for representing nonpolar polarizable solvents. The equation of motion of a Brownian dipole and the algorithm for its propagation is given in ref 3.

The microscopic polarity

$$\eta = \rho \mu_0^2 / 3k_B T \quad (2)$$

determines the macroscopic dielectric properties of a given kind of dipole lattice, where ρ is the number density of dipoles. That is, once we know the lattice symmetry (e.g., sc or fcc) and type (e.g., LDL or BDL), the dielectric constant of a dipole lattice is only a function of η . By changing the lattice spacing, we

change ρ . However, η can be kept constant if μ_0^2 is adjusted accordingly. Because the dielectric constant is a unique function of η , the coarseness of a given type of lattice can be changed without changing its polarity or dielectric constant. However, it is important to realize that the macroscopic dielectric constants (ϵ) of LDLs and BDLs are very different³ functions of η .

The placement of the solute in the solvent lattice is an important factor in determining how the solvent structure appears to the solute, which in turn affects the solute–solvent interactions. Solute placement also affects solvent–solvent interactions by determining which solvent dipoles are displaced by the solute. We use two kinds of solute placements. In the first kind, the solute is at a lattice site that otherwise would be occupied by a solvent dipole (in the absence of a cavity). We call this a “substitutional” solute. In the second kind of placement, a solute is introduced into the center of a unit cell, without displacing a solvent dipole (which would be there in the absence of the cavity). We call this an “interstitial” solute.

3. Electric Potential in the Solvent and at the Solute

The electric potential generated by a continuum dielectric in response to a point charge q in a spherical cavity (the so-called reaction potential) is given by

$$V_0 = -q \left(1 - \frac{1}{\epsilon} \right) \left(\frac{1}{r'_c} - \frac{1}{R'} \right) \quad (3)$$

and the solute–solvent interaction energy W_{qu} is

$$W_{qu} = qV_0 \quad (4)$$

where r'_c is the effective cavity radius that gives the correct solvation energy for the given macroscopic dielectric constant ϵ . r'_c is also called the Born radius. V_0 is the electric potential at the solute point charge. R' is the effective radius of the spherical shell of continuum dielectric surrounding the cavity, and is given by

$$R' = \left[\frac{\left(\frac{N}{\rho} \right)}{\frac{4\pi}{3}} \right]^{1/3} \quad (5)$$

where N is the number of dipoles contained in a spherically truncated lattice without a cavity. In a dielectric continuum representation of the solvent, the reaction potential away from the solute and the cavity is independent of the cavity size and is determined entirely by the magnitude of the solute charge and the dielectric constant of the solvent. Because potential at r in a given dielectric is determined only by the charge contained in the inner sphere of radius r , it follows from eq 3 that the reaction potential inside a spherical sample of a continuum dielectric at a distance r from a charge q is given by

$$V(r) = -q \left(1 - \frac{1}{\epsilon} \right) \left(\frac{1}{r} - \frac{1}{R'} \right) \quad (6)$$

neglecting the “edge effects” near the dielectric discontinuity at the outer surface of the sphere. As Figure 1 illustrates, eq 6 provides a good prediction for $V(r)$ in dipole lattices, which have an archetypal molecularity. However, the reaction potential V_0 shows a strong dependence on the “degree of discreteness” (represented by the grid spacing a) of the lattice for a given ϵ . As a decreases, the potential at the center, V_0 , approaches the continuum prediction given by eq 3 with $r'_c =$

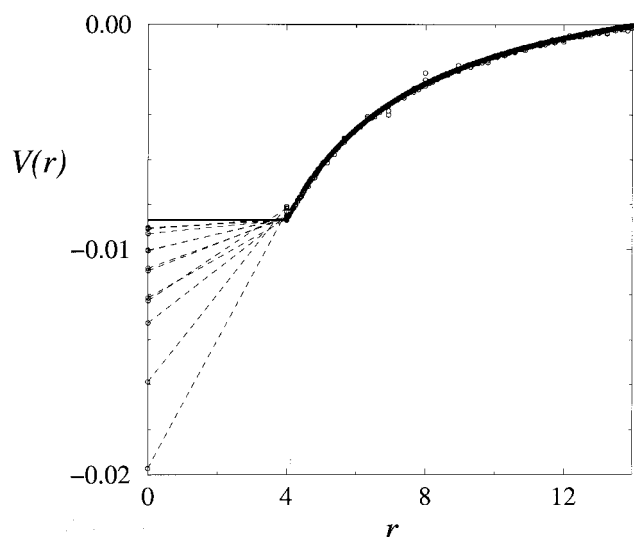


Figure 1. Electric potential $V(r)$ (in atomic units) within the solvent (i.e., $r \geq 4$ Å) for a given solvent polarity (e.g., sc LDLs with $\eta = 0.19$, corresponding to $\epsilon \approx 13$) follows the continuum prediction closely, and varies very little with solvent discreteness, represented by the lattice parameter a . By contrast, the electric potential V_0 at the ionic solute ($r = 0$) is strongly dependent on solvent discreteness, and to some extent on the lattice geometry. The systems shown are sc LDLs with a values of 0.58, 0.71, 1.00, 2.00, 2.31, 2.83, and 4.00 Å and fcc LDLs with a values of 1.02, 1.94, 2.67, 3.58, 4.62, and 8.00 Å which have V_0 (electric potential at the solute) values of -0.0090 , -0.0091 , -0.0093 , -0.0101 , -0.0110 , -0.0133 , -0.0159 , -0.0091 , -0.0100 , -0.0108 , -0.0121 , -0.0123 , and -0.0197 atomic units, respectively, for a charged solute with $q = 0.1$ atomic units. The solid line is the continuum prediction for $V(r)$ for a point charge of $q = 0.1$ at the center of a spherical cavity of radius $r_c = 4$ Å, with a solvent dielectric constant of 12.7.

r_c . Another prediction of the continuum theory is that the reaction potential should be constant across the cavity, i.e., $V(r \leq r'_c) = V_0$.

4. The Shielding Factor

In a continuum description, the shielding factor for the interaction between two ions in a solvent is equal to the dielectric constant of the solvent. Although it may be tempting to think that a solvent dipole is shielded from a solute charge also by a factor equal to ϵ , that is not the case even in a continuum description. The shielding factor for the interaction between a solvated charge and a solvent dipole is not simply equal to the dielectric constant. Scientific literature about this important aspect of electrostatic theory goes at least as far back as 1967⁵ for solvated charges and dipoles in polarizable media. Warshel and co-workers considered the solute–solvent shielding problem by microscopic simulations.^{4,6} Between 1977 and 1981 the nature of this shielding factor was clarified,^{7–11} but was still formally limited to polarizable media, and some ambiguity remained in what was exactly meant by “continuum”. In a recent article, Hyun et al.¹² utilized a formula that amounts to a “shielding” of the field of an ion for ion–solvent interactions in a Kirkwood–Fröhlich dielectric, but no proof was presented. We present here a general formula that applies to any isotropic dielectric regardless of the source of the dielectric response. The derivation, which follows the framework laid out earlier,^{7–11} is done with a continuum dielectric representation of the solvent in mind, although that is not made explicit until the usual macroscopic dielectric constant is utilized in the final expression. We start by considering the fundamental equation for the

relationship between electric fields and polarization in isotropic dielectrics, given by

$$4\pi\mathbf{P} = \mathbf{D} - \mathbf{E} \quad (7)$$

where \mathbf{D} is the so-called “dielectric displacement” and is equal to the field that would exist in the absence of the dielectric material. Using the relationship $\mathbf{D} = \epsilon\mathbf{E}$ for isotropic materials, and the fact that \mathbf{D} is the vacuum field due to the solute charge, we find

$$P = |\mathbf{P}| = \frac{q}{4\pi r^2} \left(1 - \frac{1}{\epsilon}\right) \quad (8)$$

where \mathbf{P} is along the distance vector connecting the solute charge with the solvent dipole (we drop the vector notation at this point because we are dealing with isotropic materials and thus all fields and polarizations are along the charge–dipole distance vector). Now if we express \mathbf{P} by microscopic descriptors (induced dipole magnitude, μ_ϵ and dipole density ρ) we obtain

$$\mu_\epsilon = \frac{q}{4\pi r^2 \rho} \left(1 - \frac{1}{\epsilon}\right) \quad (9)$$

for a given distance r from the charge q . The subscript ϵ indicates that μ_ϵ represents the induced polarization in a continuum dielectric. Equation 9 implies a continuum polarizability given by

$$\alpha_\epsilon = \frac{1}{4\pi\rho} \left(1 - \frac{1}{\epsilon}\right) \quad (10)$$

For a material consisting solely of permanent dipoles, $\mu = \mu_0 \langle \cos\theta \rangle$ where θ is the instantaneous angle between the charge–dipole vector and the dipole orientation, and $\langle \rangle$ refers to a time-average. The orientational polarizability of a solvent molecule with a permanent dipole is given by

$$\alpha_{\text{or}} = \mu_0^2 / 3k_B T \quad (11)$$

where μ_0 is the permanent dipole moment of the solvent molecule, k_B is the Boltzmann constant, and T is absolute temperature.

For a nonpolar, polarizable solvent (such as noble gases) total polarizability is equal to the electronic polarizability of the solvent particles. Electronic polarizability involves the distortion of the electron distribution in a solvent particle. Another source of distortional polarizability is the flexibility of molecules. The nuclei in a molecule may change their relative positions in response to an external electric field, giving rise to a nuclear distortional polarization. For the purposes of the present work, electronic and nuclear polarizabilities can be grouped together into a distortional polarizability α_{dist} . As in Debye’s extension of Clausius–Mossotti equation,¹³ the generalized polarizability can be defined as

$$\alpha = \alpha_{\text{dist}} + \alpha_{\text{or}} = \alpha_{\text{dist}} + \frac{\mu_0^2}{3k_B T} \quad (12)$$

Defining a physically consistent generalized polarizability allows us to have a generalized polarity defined as

$$\eta = \rho\alpha \quad (13)$$

Equation 13 states that, as far as the static polarization is concerned, orientational and distortional polarizabilities are

indistinguishable. Therefore, a nonpolar but polarizable solvent can, in principle, have a high generalized polarity.

The induced dipole at a solvent molecule in the absence of the rest of the solvent is simply

$$\mu_1 = \alpha \frac{q}{r^2} = \frac{\eta}{\rho} \frac{q}{r^2} \quad (14)$$

where α is the generalized polarizability of solvent molecules. Because μ_ϵ given by eq 9 implicitly includes the effect of other solvent dipoles and the shielding due to them, we can define the shielding factor by comparing μ_1 and μ_ϵ . Thus,

$$\sigma(\epsilon, \eta) = \frac{\mu_1}{\mu_\epsilon} = 4\pi\eta \left(1 - \frac{1}{\epsilon}\right)^{-1} \quad (15)$$

where σ is the shielding factor. Because a relationship (such as the Clausius–Mossotti or Kirkwood–Fröhlich equations) between the microscopic quantities (ρ and μ_0) provides a connection between η and ϵ , eq 15 is easily transformable to a form that is a function only of ϵ within a given dielectric model. Equation 15 provides a completely general way to obtain the *continuum* prediction for local polarization within the solvent. Apart from the isotropic continuum formalism, the only “assumption” involved in deriving eq 15 is the representation of the polarization of the solvent molecule by that of a dipole. This assumption is implicitly made in continuum electrostatics through the use of dipole density \mathbf{P} . Within a dielectric framework, a purely dipolar representation is well-justified beyond the first solvation shell for neutral solvent molecules even if the permanent dipole moment of the solvent molecules is zero. At distances of practical interest the solvation contribution from higher solvent multipoles becomes negligible, and the solvation response comes from induced dipoles of orientational and distortional origin. For example, a recent *quadrupolar* continuum study¹⁴ revealed that the purely quadrupolar contribution is important only at solute–solvent distances of less than ~ 0.5 Å.¹⁵

By substituting the relationship between polarity η and dielectric constant ϵ into eq 15, we can obtain the local shielding as a function of ϵ (e.g., in a family of similar solvents or in the same solvent at different densities and temperatures). For a Clausius–Mossotti dielectric we have

$$\frac{\epsilon - 1}{\epsilon + 2} = \frac{4\pi}{3}\eta \quad (16)$$

and

$$\sigma_{CM}(\epsilon) = \frac{3\epsilon}{\epsilon + 2} \quad (17)$$

and for a Kirkwood–Fröhlich dielectric we have

$$\frac{(2\epsilon + 1)(\epsilon - 1)}{9\epsilon} = \frac{4\pi}{3}\eta g_K \quad (18)$$

and

$$\sigma_{KF}(\epsilon) = \frac{2\epsilon + 1}{3g_K} \quad (19)$$

Interestingly, the local shielding of the solute charge is qualitatively different for solvents that can be described by Clausius–Mossotti and Kirkwood–Fröhlich equations. Although the shielding of solute charge from solvent dipoles

saturates to a limit of 3 in the former, it increases approximately linearly (if we regard g_K as a slowly varying function of ϵ) with ϵ in the latter.

5. Casting Continuum into a Dipole Lattice

Having the continuum result for the local shielding of solvent dipoles from the solute charge, we can now derive the electric potential exerted by these “continuum” dipoles at the solute. We use dipole lattices (briefly described in section 2) as an archetype of a molecular solvent. The electric potential V_0 at the solute charge is a sum of the individual dipole contributions over the lattice, given by

$$V_0 = - \sum_{\text{lattice sites}} \frac{\xi \alpha}{r^2} \quad (20)$$

where ξ is the shielded field from the solute point charge, given by

$$\xi = \frac{q}{\sigma r^2} \quad (21)$$

Combining eqs 13, 15, 20, and 21 we obtain

$$V_0 = - \frac{q}{4\pi\rho} \left(1 - \frac{1}{\epsilon}\right) \sum_{\text{lattice sites}} \frac{1}{r^4} \quad (22)$$

From eqs 3 and 22 we can obtain the effective cavity radius:

$$r'_c = \left[\frac{1}{4\pi\rho} \sum_{\text{lattice sites}} \frac{1}{r^4} \right]^{-1} = \frac{1}{R'} \quad (23)$$

The lattice sum in eq 23 is different for sc and fcc lattices and also depends on the position of the solute within a unit cell. It is given by

$$\left(\sum_{\text{lattice sites}} \frac{1}{r^4} \right)_{\text{sc}} = \frac{C}{a^4} \sum_{i,j,k} \frac{1}{[(i - i_0)^2 + (j - j_0)^2 + (k - k_0)^2]^2} \quad (24a)$$

$$\left(\sum_{\text{lattice sites}} \frac{1}{r^4} \right)_{\text{fcc}} = \frac{C}{a^4} \left[\sum_{i,j,k} \frac{1}{[(i - i_0)^2 + (j - j_0)^2 + (k - k_0)^2]^2} + \sum_{i,j,k} \frac{1}{[(i - i_0)^2 + (j - j_0 - 1/2)^2 + (k - k_0 - 1/2)^2]^2} + \sum_{i,j,k} \frac{1}{[(i - i_0 - 1/2)^2 + (j - j_0)^2 + (k - k_0 - 1/2)^2]^2} + \sum_{i,j,k} \frac{1}{[(i - i_0 - 1/2)^2 + (j - j_0 - 1/2)^2 + (k - k_0)^2]^2} \right] \quad (24b)$$

for a solute charge at (i_0, j_0, k_0) where $C = \rho a^3$ ($C = 1$ and $C = 1/4$ for sc and fcc lattices, respectively). When (i_0, j_0, k_0) is $(0, 0, 0)$, the solute displaces a solvent dipole that would be there in the absence of the cavity, and thus is “substitutional”. When (i_0, j_0, k_0) is $(1/2, 1/2, 1/2)$, the solute is at the middle of a unit cell, and is “interstitial”.

For a charge q in a “spherical” cavity (approximated by deleting dipoles at distances smaller than the cavity radius r_c) at the center of a concentric solvent sphere of radius R

TABLE 1: Lattice Sums for the sc Lattice

| substitutionally centered | | interstitially centered | |
|---------------------------|--------------|-------------------------|--------------|
| R_c/a | $\sum 1/r^4$ | R_c/a | $\sum 1/r^4$ |
| 1.00000 | 6.000 | 0.8660 | 14.222 |
| 1.41421 | 9.000 | 1.6583 | 17.396 |
| 1.73205 | 9.889 | 2.1794 | 18.459 |
| 2.00000 | 10.264 | 2.5980 | 19.162 |
| 2.23607 | 11.224 | 2.9580 | 19.789 |
| 2.44949 | 11.891 | 3.2787 | 19.996 |
| 2.82843 | 12.078 | 3.5707 | 20.292 |
| 3.00000 | 12.448 | 3.8405 | 20.623 |
| 3.16228 | 12.688 | 4.0926 | 20.708 |
| 3.31662 | 12.887 | 4.3301 | 20.867 |
| 3.46410 | 12.942 | 4.5552 | 21.035 |
| 3.60555 | 13.084 | 4.7697 | 21.127 |
| 3.74166 | 13.329 | 4.9749 | 21.245 |
| 4.00000 | 13.353 | 5.1720 | 21.346 |
| 4.12311 | 13.519 | 1000 | 23.702 |
| 4.24264 | 13.630 | | |
| 4.35890 | 13.696 | | |
| 4.47214 | 13.756 | | |
| 4.58258 | 13.865 | | |
| 4.69042 | 13.915 | | |
| 4.89898 | 13.956 | | |
| 5.00000 | 14.004 | | |
| 1000 | 16.520 | | |

(approximated by deleting dipole at distances larger than R), we have

$$V_0 = -\frac{q}{4\pi\epsilon_0} \left(1 - \frac{1}{\epsilon}\right) \left[\sum_{r \leq R} \frac{1}{r^4} - \sum_{r < r_c} \frac{1}{r^4} \right]^{-1} \quad (25)$$

Tables 1 and 2 list the lattice sums (to be used in eq 25) for spheres of various discreteness, for sc and fcc lattices, respectively, with interstitial and substitutional solutes. The magnitude of the solvation in what may be called a “continuum lattice” (calculated by eq 25) is significantly larger than the continuum prediction given by the Born expression (eq 3) when r'_c is taken to be equal to the “radius of exclusion” r_c .¹⁶

Figure 2 illustrates that, regardless of whether the lattice is sc or fcc, and regardless of whether the solute is placed interstitially or substitutionally, we have:

$$r'_c < r_c \quad (26)$$

Note that this inequality is obtained by casting the continuum description (which is the zero molecularity, or $\rho \rightarrow \infty$ limit of a dipolar model) onto a dipole lattice with finite molecularity ($\rho < \infty$). That is, the polarization of solvent dipoles follow precisely the continuum prediction, and lack the microscopic interplay of neighboring dipoles which would give rise to a somewhat irregular polarization profile that oscillates around the continuum prediction. Therefore the inequality in eq 26 is the result of the mere discreteness of the solvent, and excludes the secondary effects the discreteness would have on the polarization behavior of the materials.^{3,17} However, it is interesting to observe that the above result is in semiquantitative agreement with the solvation observed in “true” dipole lattices where the solvent dipoles are not forced to follow the continuum polarization and interact with each other. The dependence of the Born radius on lattice discreteness in a Langevin dipole lattice (Figure 3) is very similar to that in a “continuum lattice” (Figure 2). Brownian dipole lattices, which have explicit thermal fluctuations and exhibit a completely different relationship between the macroscopic dielectric constant and microscopic polarity, show much the same behavior (Table 3) as the continuum lattice or the Langevin dipole lattice. This “over-

TABLE 2: Lattice Sums for the fcc Lattice

| substitutionally centered | | interstitially centered | |
|---------------------------|--------------|-------------------------|--------------|
| R_c/a | $\sum 1/r^4$ | R_c/a | $\sum 1/r^4$ |
| 0.70711 | 48.000 | 0.50000 | 96.000 |
| 1.00000 | 54.000 | 0.86603 | 110.222 |
| 1.22474 | 64.667 | 1.11803 | 125.582 |
| 1.41421 | 67.667 | 1.50000 | 131.508 |
| 1.58114 | 71.507 | 1.65831 | 134.682 |
| 1.73205 | 72.396 | 1.80278 | 136.954 |
| 1.87083 | 76.314 | 2.06155 | 139.611 |
| 2.00000 | 76.689 | 2.17945 | 140.675 |
| 2.12132 | 78.467 | 2.29129 | 142.417 |
| 2.23607 | 79.427 | 2.50000 | 143.185 |
| 2.34521 | 80.220 | 2.59808 | 143.887 |
| 2.44949 | 80.887 | 2.69258 | 145.257 |
| 2.54951 | 82.591 | 2.87228 | 145.962 |
| 2.73861 | 83.444 | 2.95804 | 146.589 |
| 2.82843 | 83.632 | 3.04138 | 146.869 |
| 2.91548 | 84.296 | 3.20156 | 147.783 |
| 3.00000 | 84.666 | 3.27872 | 147.991 |
| 3.08221 | 85.464 | 3.35410 | 148.560 |
| 3.16228 | 85.704 | 3.50000 | 148.919 |
| 3.24037 | 86.140 | 3.57071 | 149.215 |
| 3.31662 | 86.338 | 3.64005 | 149.625 |
| 3.39116 | 86.701 | 3.77492 | 149.861 |
| 3.46410 | 86.756 | 3.84057 | 150.192 |
| 3.53553 | 87.294 | 3.90512 | 150.502 |
| 3.60555 | 87.436 | 4.03113 | 150.865 |
| 3.67423 | 87.963 | 4.09268 | 150.951 |
| 3.74166 | 88.208 | 4.15331 | 151.274 |
| 3.80789 | 88.322 | 4.27200 | 151.418 |
| 3.93700 | 88.721 | 4.33013 | 151.577 |
| 4.00000 | 88.745 | 4.38748 | 151.836 |
| 4.06202 | 89.098 | 4.50000 | 152.085 |
| 4.12311 | 89.264 | 4.55522 | 152.252 |
| 4.18330 | 89.420 | 4.60977 | 152.358 |
| 4.24264 | 89.531 | 4.71699 | 152.649 |
| 4.30116 | 89.882 | 4.76970 | 152.742 |
| 4.35890 | 89.949 | 4.82183 | 152.831 |
| 4.41588 | 90.075 | 4.92443 | 152.912 |
| 4.47214 | 90.135 | 4.97494 | 153.030 |
| 4.52769 | 90.249 | 5.02494 | 153.293 |
| 4.58258 | 90.358 | 5.12348 | 153.433 |
| 4.63681 | 90.617 | 5.17204 | 153.533 |
| 4.69042 | 90.667 | 5.22015 | 153.630 |
| 4.74342 | 90.904 | 5.31507 | 153.751 |
| 4.84768 | 91.078 | 5.36190 | 153.809 |
| 4.89898 | 91.120 | 5.40833 | 153.949 |
| 4.94975 | 91.300 | 5.50000 | 154.034 |
| 5.00000 | 91.348 | 5.54527 | 154.085 |
| 1000 | 101.303 | 1000 | 163.100 |

solvation” of the charge is in sharp contrast to the electric potential (generated in response to a solute charge) *within the solvent*. As implied by eq 6, and demonstrated in Figure 1, electric potential V within the solvent is fairly insensitive to the details of the cavity around the solute and the coarseness of the lattice. Because eq 22 contains no explicit, microscopic intermolecular interactions, it is clear that the extra electric potential at the solute is not due to a complicated interplay of dipolar interactions within a lattice geometry. Rather, it is a direct result of the mere discreteness of the solvent. It is possible to understand this result by noticing that the dipole density is infinite at the continuum limit, with vanishing magnitudes assigned to individual dipoles. At finite dipole densities of ordinary solvents, the collective electric field from dipoles of finite magnitude happens to decay at a length scale comparable to molecular dimensions, rather than vanishing abruptly where there is no solvent (as would be the case with dipoles of infinitesimal magnitude at the continuum limit of $\rho \rightarrow 0$). The collective solvent electric field thus penetrates into the geometrically defined cavity, and causes the associated electric

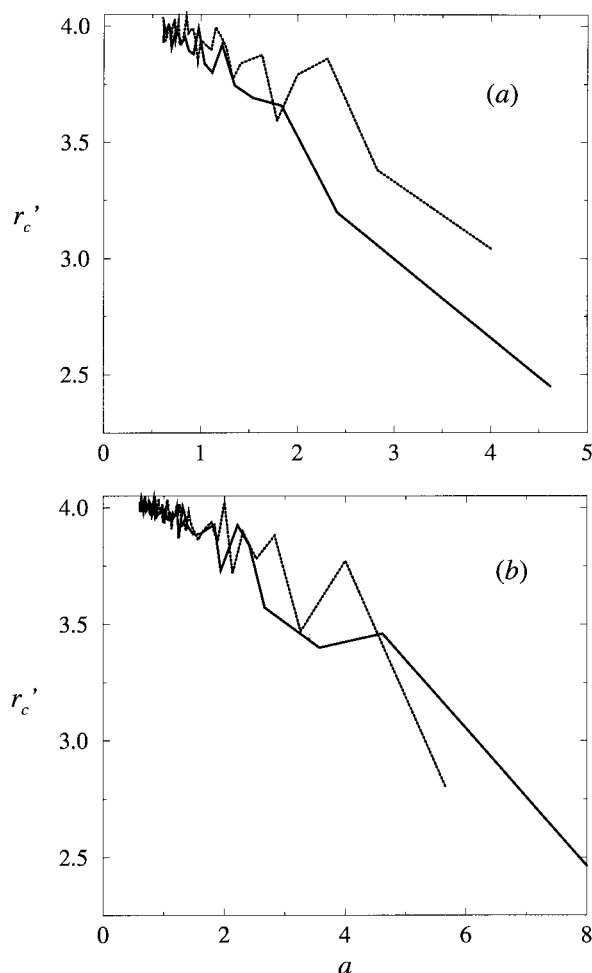


Figure 2. Effective cavity radius r'_c (given by eq 23) for a charged solute in a “continuum lattice” (see section 5), as a function of lattice parameter a which represents the degree of solvent discreteness. Results for sc and fcc lattices are shown in panels (a) and (b), respectively. Solid lines are for interstitially placed solutes, and the dotted lines are for substitutionally placed solutes (see section 2 for definitions). Both r'_c and a are in angstroms.

potential to continue increasing in magnitude. As Figure 4 shows, the magnitude of $V(r)$ continues to increase for $r < r_c$ before leveling off to V_0 , while continuum theory would predict that $V(r) = V_0$ for $0 \leq r \leq r_c$. It appears that the point at which $V(r)$ starts leveling off to its final value of V_0 corresponds roughly to the effective cavity radius r'_c . However, we did not find a simple and general relationship to capture this correspondence quantitatively for all degrees of solvent discreteness, geometry, and polarity. Although Figure 3 showed that the effective cavity radius is dependent on solvent polarity, implying that a purely geometric definition is incomplete, we nevertheless also explored the behavior of a “density-consistent” effective radius r_c^ρ given by

$$r_c^\rho = \left[\frac{\left(\frac{N_d}{\rho} \right)}{\frac{4\pi}{3}} \right]^{1/3} \quad (27)$$

where r_c^ρ is the radius of a sphere that would hold the N_d solvent particles displaced by the constructed cavity. Although there are no van der Waals forces in the dipole lattices determining how large the solvent particles are, r_c^ρ indirectly produces such a quantity. Figure 5 shows the remarkable fact

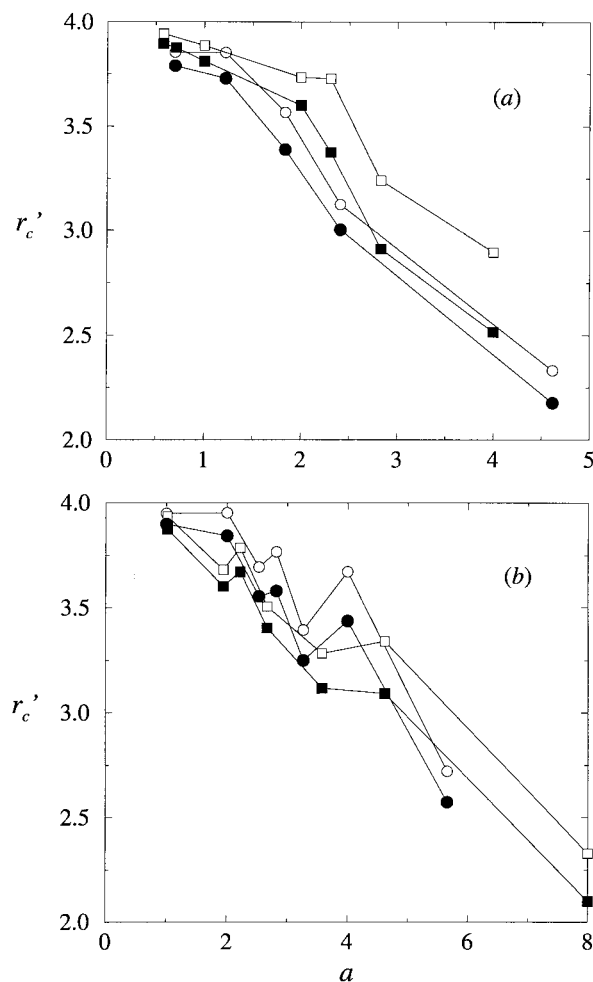


Figure 3. Effective cavity radius r'_c for a charged solute ($q = 0.1$ atomic units) in LDLs (see section 2 for system definition), as a function of lattice parameter a which represents the degree of solvent discreteness. Results for sc and fcc LDLs are shown in panels (a) and (b), respectively. Open and filled circles are for solutes placed interstitially in LDLs with $\eta = 0.05$ ($\epsilon = 1.795$) and $\eta = 0.19$ ($\epsilon \approx 13$), respectively. Open and filled squares are for solutes placed substitutionally in LDLs with $\eta = 0.05$ ($\epsilon = 1.795$) and $\eta = 0.19$ ($\epsilon \approx 13$), respectively. Both r'_c and a are in angstroms.

TABLE 3: Effective Cavity Radii for an Ionic Solute ($r_c = 4$ Å) in a Lattice of Brownian Dipoles

| system | r'_c/r_c |
|---|------------|
| sc lattice, substitutional solute | |
| $\eta = 1/3$ ($\epsilon = 10.7$), $a = 1$ Å, | 0.977 |
| $\eta = 1/3$ ($\epsilon = 10.7$), $a = 2$ Å, | 0.915 |
| $\eta = 1/3$ ($\epsilon = 10.7$), $a = 4$ Å, | 0.630 |
| sc lattice, interstitial solute | |
| $\eta = 1/3$ ($\epsilon = 10.7$), $a = 2.4121$ Å, | 0.780 |
| $\eta = 1/3$ ($\epsilon = 10.7$), $a = 4.6188$ Å, | 0.588 |
| fcc lattice, interstitial solute | |
| $\eta = 1/3$ ($\epsilon = 21.5$), $a = 4$ Å, | 0.858 |
| $\eta = 1/3$ ($\epsilon = 21.5$), $a = 8$ Å, | 0.532 |

that the effective continuum cavity radius r'_c for ion solvation closely tracks r_c^ρ , a quantity that is derived from the average empty volume around solvent particles. r'_c nearly quantitatively captures r'_c for $\eta = 0.05$ ($\epsilon = 1.8$), but the agreement for $\eta = 0.19$ is semiquantitative. For the largest possible value of a that is consistent with $r_c = 4$ Å with an interstitial solute, r'_c vanishes because no solvent particles are displaced by the “cavity” and a sensible comparison with r'_c cannot be made. For a substitutional solute, r_c^ρ does not vanish for the largest

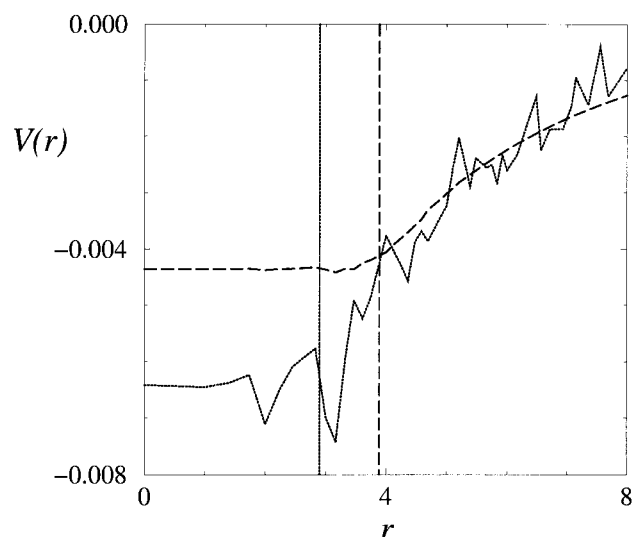


Figure 4. Electric potential V (in atomic units) as a function of distance r (in angstroms) from a charged solute ($q = 0.1$ atomic units) in a cavity with $r_c = 4$ Å. V is calculated on a three-dimensional simple cubic mesh with a lattice parameter of 1 Å. The dotted and dashed lines are for sc LDLs with $a = 4$ Å and 1 Å, respectively. Both lattices have a polarity of $\eta = 0.05$ ($\epsilon = 1.795$). The vertical lines serve to mark the values of the respective effective cavity radii r'_c calculated by fitting the LDL V_0 values to the Born equation (eq 3), and appear to correspond roughly to the radial distance where $V(r)$ starts (coming from outside the cavity) leveling off to its final value of V_0 .

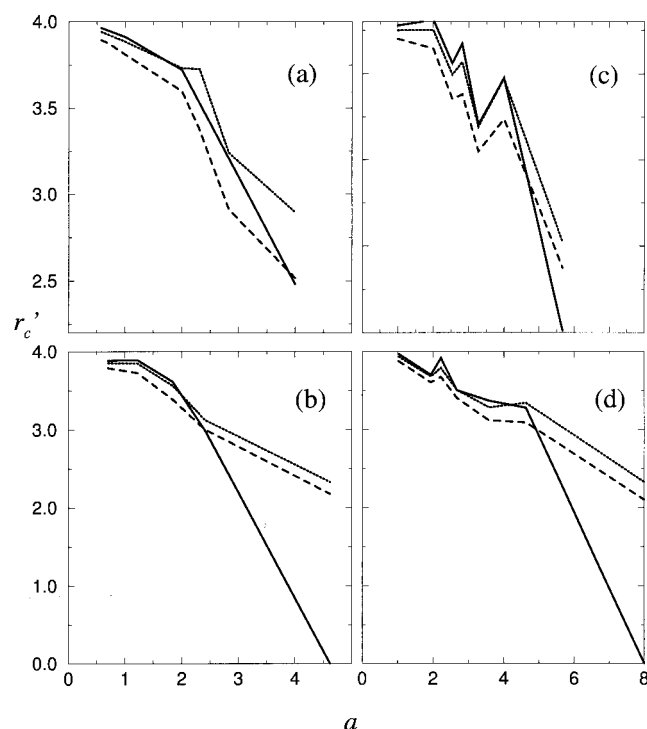


Figure 5. Effective cavity radius r'_c and the “density-consistent” estimate r_c^ρ for a charged solute ($q = 0.1$ atomic units) in LDLs as a function of the lattice parameter a that represent solvent discreteness. Dotted and dashed lines are for LDLs with $\eta = 0.05$ ($\epsilon = 1.795$) and $\eta = 0.19$ ($\epsilon \approx 13$) respectively. The solid lines show r'_c . Panels (a) and (b) show the results for sc LDLs with substitutionally and interstitially placed solute, respectively. Analogously, panels (c) and (d) show the results for fcc LDLs with substitutionally and interstitially placed solute, respectively.

possible value of a consistent with $r_c = 4$ Å; however it underestimates r'_c for the cases that we studied. The spatial range in which the collective electric fields (due to the solvent

dipoles, especially at the edge of the cavity) within the cavity decay to a value indistinguishable from a continuum solvent is roughly equal to the “size” of the solvent particle. Even though this coincidence might seem intuitive at first, there is no fundamental reason for a continuum cavity “wall” to follow the steric shape (or the “electron cloud”) of the solvent molecules. When we remove this potentially confusing issue by employing solvent “molecules” that have no such electron clouds, we nevertheless obtain effective cavity sizes that would be predicted from such steric considerations. There appears to be an intriguing “balance” in the magnitudes of seemingly unrelated microscopic properties.

Because over-solvation and the consequent inequality in eq 26 is present in both sc and fcc lattices¹⁸ with substitutional or interstitial solute, it is safe to assume that the effect of solvent discreteness is also present in more “realistic” solvents. Nevertheless, in the next section we will further check the validity of the conclusions drawn from the dipole lattices using more realistic descriptions.

6. Casting Continuum into the Solute–Solvent Radial Distribution Function $g(r)$

The equations derived for lattices have analogues in more realistic descriptions of solvents. Berne and co-workers¹⁹ utilized a formula that connects the solute–solvent radial distribution function to the effective cavity radius, given by

$$r'_c = [\int_0^\infty g(r)r^2 dr]^{-1} \quad (28)$$

Concepts related to eq 28 have appeared in the literature.²⁰ However, eq 28 itself was used in the literature¹⁹ without proof and can be derived in a fashion completely analogous to eq 23. The continuum prediction for V_0 is the integral of the electric potentials exerted by the solvent-induced dipoles created by the electric field of the charge shielded by the continuum estimate of the screening factor σ (eq 15). Thus,

$$V_0 = \int_0^\infty \frac{q}{\sigma r^2} \frac{\alpha}{r} \rho g(r) 4\pi r^2 dr \quad (29)$$

where $\rho g(r)$ gives the local density of dipoles, and $4\pi r^2 dr$ is the volume element. Because the product $\rho\alpha$ is equal to the dimensionless polarity parameter η , $\rho \rightarrow \infty$ and $\alpha \rightarrow 0$ limits necessary²¹ for the continuum description can be realized. We then have

$$V_0 = 4\pi \frac{q\eta}{\sigma} \int_0^\infty \frac{g(r)}{r^2} dr \quad (30)$$

Combining eqs 15 and 30 yields

$$V_0 = q \left(1 - \frac{1}{\epsilon} \right) \int_0^\infty \frac{g(r)}{r^2} dr \quad (31)$$

By equating the Born formula (eq 3 with $R \rightarrow \infty$) and eq 31, we obtain eq 28.

7. Systematic Analysis of Discreteness in the Radial Distribution Function

To make the transition from lattice models to solutions with arbitrary solute–solvent pair distribution functions, we first

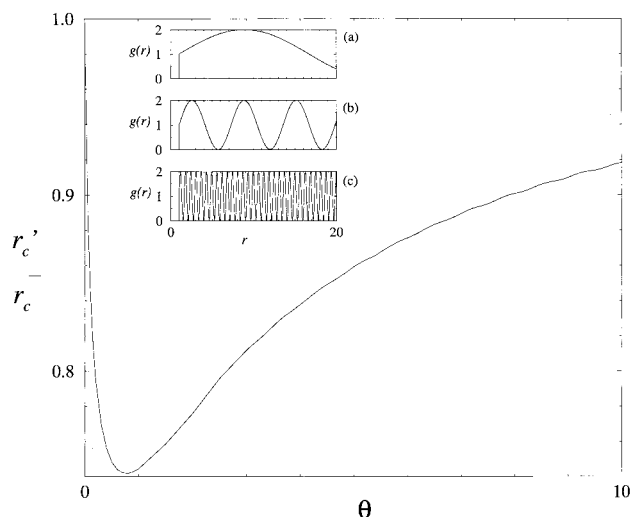


Figure 6. Effective cavity radius r'_c calculated from eq 28 with $g(r)$ given by eq 32, plotted as a function of the parameter θ . Insets (a), (b), and (c) show eq 32 with $\theta = 0.1$, $\theta = 0.8$, and $\theta = 10$, respectively, all with $b = 1$. Very small and very large values of θ lead to less discreteness, albeit in different ways (see text). This nonmonotonic variation of the “perceived” discreteness is tracked by r'_c .

examine the behavior of infinitely periodic $g(r)$'s of the form

$$g(r) = 0, \quad r < r_c$$

$$g(r) = 1 + b \sin\left[\frac{(r - r_c)}{r_c}\theta\right], \quad r \geq r_c \quad (32)$$

shown in the insets of Figure 6. The parameter b gives the depth of the oscillations in $g(r)$, and is a simple and direct measure of the “discreteness” of a system described by eq 32, whereas θ changes the effective molecularity in a complicated way. At $\theta \rightarrow 0$, $g(r)$ is unity for all r , corresponding to the continuum limit. We can also recover the continuum limit by making the “structure” of $g(r)$ infinitely fine, i.e., using $\theta \rightarrow \infty$. Minimum r'_c is obtained at $\theta = 0.8$ (see Figure 6). By contrast, r'_c decreases straightforwardly with increasing b , as shown in Figure 7. The graininess of $g(r)$ leads to r'_c values as low as $\sim 0.7r_c$; a significant reduction from the continuum limit.

Next, we examine a more general and realistic functional form for $g(r)$:

$$g(r) = 0, \quad r < r_c$$

$$g(r) = \left[1 + e^{-\frac{(r-r_c)}{r_c\tau}}\right] b \sin\left[\frac{(r - r_c)}{r_c}\theta\right], \quad r \geq r_c \quad (33)$$

The exponential factor ensures that the oscillations in $g(r)$ dampen at large r , as in any realistic fluid mixture. For $r \rightarrow 0$, the exponential relaxes immediately and destroys any discreteness in $g(r)$, and the continuum limit is recovered. For $\tau \rightarrow \infty$, the oscillations persist indefinitely, maximizing the molecularity that the solute senses (see the inset in Figure 8). Figure 8 shows that r'_c/r_c decreases with increasing τ (or increasing discreteness). With a well-defined exclusion radius, solvent molecularity causes a sizable contraction in the effective cavity radius r'_c with respect to r_c .

We now examine the $g(r)$ of a hard sphere fluid which has a venerable history in studying the behavior of liquids. In a recent article,²² Boublik presented analytical formulas that

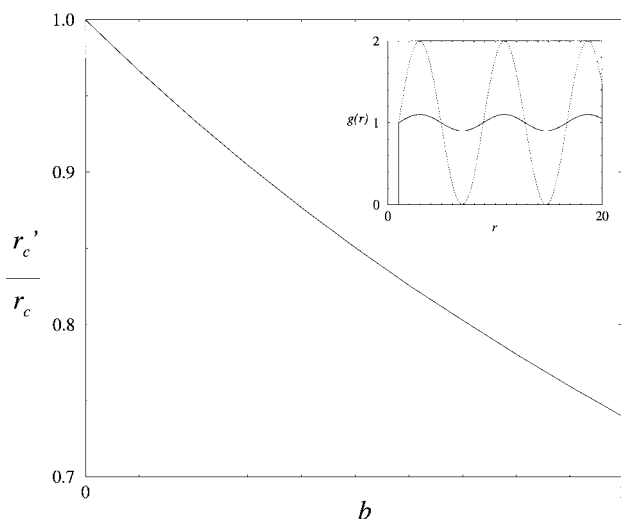


Figure 7. Effective cavity radius r'_c calculated from eq 28 with $g(r)$ given by eq 32, plotted as a function of the parameter b . The inset shows eq 32 with $b = 0.1$ (solid curve) and $b = 1.0$ (dotted curve), both with $\theta = 0.8$. Solvent discreteness quite clearly increases with the magnitude of the oscillations in $g(r)$, given by b . As the figure clearly shows, r'_c decreases steadily with increasing discreteness of $g(r)$.

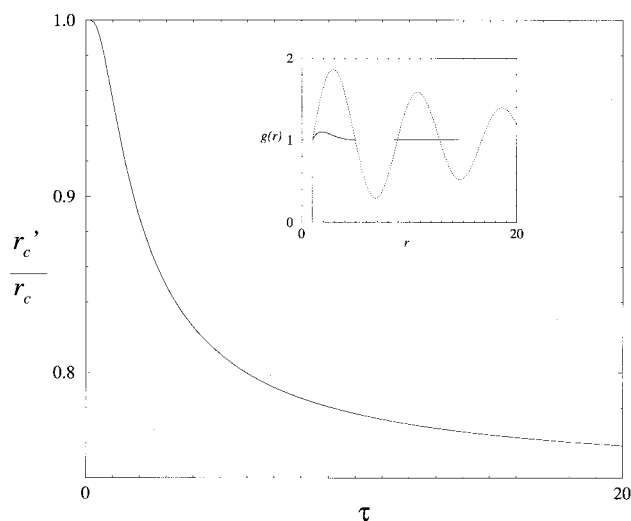


Figure 8. Effective cavity radius r'_c calculated from eq 28 with $g(r)$ given by eq 33, plotted as a function of the parameter b . The inset shows eq 33 with $\tau = 1$ (solid curve) and $\tau = 20$ (dotted curve), both with $b = 1$ and $\theta = 0.8$. The magnitude and persistence of oscillations in $g(r)$ increase with τ . r'_c tracks this relationship between solvent discreteness and τ .

accurately represent the radial distribution functions of hard sphere mixtures. For example, for a pure hard sphere fluid (i.e., when the solute is identical to the solvent particles) we have

$$\ln g(r) = -\ln(1 - y) + \frac{y}{1 - y} \left(7 - 6r + \frac{1}{2}r^3\right) +$$

$$\frac{y^2}{2(1 - y)^2} \left(15 - 18r - \frac{3}{4}r^2 + 3r^3\right) +$$

$$\frac{y^3(2 - y/3)}{(1 - y)^3} \left(1 - \frac{3}{2}r + \frac{1}{2}r^3\right), \quad r \leq r_m$$

$$g(x) = 1 + Ae^{-B(r-r_m)} \cos\left[\frac{\pi}{c}(r - r_m)\right], \quad r > r_m \quad (34)$$

where r_m is the radial distance where the first minimum of $g(r)$

occurs, and can be obtained from the equation

$$\frac{3}{2}[1 + 3w + w^2(2 - y/3)]r_m^2 - \frac{3}{4}wr_m - \left[6 + 9w + \frac{3}{2}w^2(2 - y/3)\right] = 0 \quad (35)$$

where $w = y/(1 - y)$. $c = 2(r_f - 1)$ where r_f denotes the distance for which $g(r) = 1$, and can be determined from the cubic equation

$$\begin{aligned} &\frac{w}{2}[1 + 3w + w^2(2 - y/3)]r_f^3 - \frac{3}{8}w^2r_f^2 - \\ &w\left[6 + 9w + \frac{3}{2}w^2(2 - y/3)\right]r_f + \\ &\left[7w + \frac{15}{2}w^2 + w^3(2 - y/3) - \ln(1 - y)\right] = 0 \quad (36) \end{aligned}$$

A and B are equal to $g(r_m) - 1$ and $1.2(1 - y)$ respectively. The level of the discreteness in $g(r)$ increases as the value of the packing ratio y is increased (Figure 9, the inset). As Figure 9 shows, r'_c/r_c decreases from 1 to 0.74 as the packing ratio y is varied between 0 and 0.6, confirming the inverse relationship between solvent discreteness and effective cavity radius for ionic solvation. It also demonstrates again that the effective cavity radius is smaller than the exclusion radius when the solvent is discrete.

We also examined a very early analytical radial distribution function suggested by Kirkwood,²³ and given by

$$g(r) = e^{-W(r)/kT} \quad (37)$$

$$W(r)/kT = U(r)/kT - \rho\omega_0 J(r)$$

$$\omega_0 = 4\pi b^3/3$$

$$J(r) = \frac{1}{\omega_0} \{ \omega_{bb} + 2(\omega_{ab} - \omega_{bb})(1 - e^{u_0/k_B T}) + (\omega_{aa} - 2\omega_{ab} + \omega_{bb})(1 - e^{u_0/k_B T})^2 \}$$

$$\frac{\omega_{bb}}{\omega_0} = 1 - 3r/4b + \frac{1}{16}(r/b)^3, \quad 0 < r \leq 2b$$

$$= 0, \quad r > 2b$$

$$\frac{\omega_{aa}}{\omega_0} = (a/b)^3 \left[1 - 3r/4a + \frac{1}{16}(a/b)^3 \right], \quad 0 < r \leq 2b$$

$$= 0, \quad r > 2b$$

$$\begin{aligned} \frac{\omega_{ab}}{\omega_0} = & \frac{1}{2} \left\{ \left[1 - \frac{3}{4} \left(1 - \frac{a^2 - b^2}{r^2} \right) \left(\frac{r}{b} \right) + \right. \right. \\ & \left. \frac{1}{16} \left(1 - \frac{a^2 - b^2}{r^2} \right)^3 \left(\frac{r}{b} \right)^3 \right] + \left(\frac{a}{b} \right)^3 \left[1 - \frac{3}{4} \left(1 + \frac{a^2 - b^2}{r^2} \right) \left(\frac{r}{a} \right) + \right. \\ & \left. \left. \frac{1}{16} \left(1 + \frac{a^2 - b^2}{r^2} \right)^3 \left(\frac{r}{a} \right)^3 \right] \right\}, \quad (a^2 - b^2)^{1/2} < r \leq a + b \end{aligned}$$

$$= 0, \quad r > a + b$$

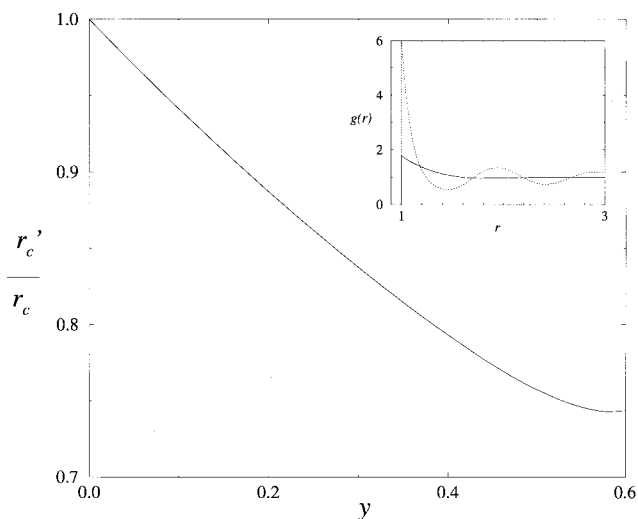


Figure 9. Effective cavity radius r'_c of a solute in a hard-sphere solvent, calculated from eq 28 with $g(r)$ given by eq 34, plotted as a function of the packing parameter y . The inset shows eq 34 with $y = 0.2$ (solid curve) and $y = 0.5$ (dotted curve). The magnitude and persistence of oscillations in $g(r)$ increase with y . r'_c tracks this relationship between solvent discreteness and y .

where $U(r)$ is the square-well potential between the solute and the solvent, given by

$$\begin{aligned} U(r) &= \infty, \quad 0 \leq r \leq b \\ &= -u_0, \quad b \leq r \leq a \\ &= 0, \quad a < r \end{aligned} \quad (38)$$

When we use the $\rho\omega_0 = 2$, $u_0/k_B T = 1$, and $a/b = 1.25$, as suggested by Kirkwood in ref 23, we obtain $r'_c/r_c = 0.807$. Again, the effective continuum cavity radius is significantly smaller than the exclusion radius.

To ensure that there is an unambiguous exclusion radius, all $g(r)$'s in our analysis have their first nonzero values greater than or equal to 1. The $g(r)$ functions that we have used in our analysis have either a simple physical basis such as a hard-sphere fluid or a fairly regular mathematical form. However, it is possible to construct a $g(r)$ that has a very narrow peak near the solute, followed by a large region where $g(r)$ has a small value. Because the narrow peak would technically define an "exclusion radius" and the "empty" (small $g(r)$) region would lead to reduced solvation, one could obtain a Born radius that is larger than the nominal "exclusion radius". However, such cases are either limited to unphysical $g(r)$'s or result from the intramolecular structure of the solvent molecules and simply blur the concept of exclusion radius itself.

8. Summary and Discussion

We studied the effect of solvent discreteness on solvation by studying the way it modifies the Born radius for ionic solvation. Asking a simple question such as "What is the effect of solvent molecularity on solvation, independent of all other effects?" is very difficult, if not impossible, with real liquids and their realistic models. One is not free to change one aspect of a laboratory solvent without affecting the other properties of the solvent or the solute-solvent system. For instance, a change in polarity inevitably modifies the intermolecular structure, because the molecules are free to move and respond to changes in intermolecular forces. To obtain a qualitative answer to the above question, we used two distinct approaches. In the first

approach, we used Langevin and Brownian dipole lattices as solvents. Because translationally fixed dipole lattices are legitimate dielectrics that allow a separation of polar and structural properties, we utilized them for investigating the direct effect of solvent discreteness by varying their lattice parameters. In the second approach, we investigated the effect of the graininess in $g(r)$ on the Born radius. The assumption of continuum polarization in the solvent results in an expression that is independent of polarity, and it enables us to study the direct effect of solvent discreteness within that approximation.

We found a universally inverse relationship between the degree of solvent discreteness and the amount of "oversolvation" of an ion in a cavity defined by an exclusion radius r_c . Dipole lattices that polarize according to the continuum prediction, as well as truly interacting dipole lattices, lead to effective cavity sizes that are significantly smaller than the geometrically defined exclusion radius. Cavity radii that are calculated from the number density of solvent dipoles provide fairly good estimates of the observed effective cavity radii, particularly at low solvent polarity. The effective cavity radius is found to correspond approximately to the radial distance where the electric potential due to solvent dipoles levels off to its final value at the center of the cavity. Thus, the spatial decay length for the collective solvent electric field within the cavity is roughly equal to the "size" of a solvent particle, reflecting an intriguing "balance" in the magnitudes of these seemingly unrelated microscopic quantities. The values of the effective cavity radii and the observed trends are similar for sc and fcc dipole lattices with different ways of placing the solute within the lattice geometry. Obtaining the same trend in every case indicates that our conclusions are quite general.

To ensure that the observed inequality $r'_c < r_c$ is not peculiar to dipole lattice representations, we studied various forms of solute-solvent distribution functions in the second part of our analysis. We derived a formula that connects $g(r)$ to an effective cavity radius, with the approximation that the microscopic polarization follows the continuum prediction. The effective cavity radius decreases with increasing graininess of the solute-solvent pair distribution function, wavier $g(r)$'s leading to smaller effective cavity sizes. The cavity radius is found to be consistently smaller than what would be predicted from the exclusion radius.

In the present work, we isolated the effect of solvent discreteness as a variable in its own right. The effect of solvent discreteness, distinct from the indirect effects felt through modified solvent properties and solute-solvent configurations, turns out to be an important factor. Our analysis indicates that the direct effect of discreteness on solvation energy (or Born radius) can easily exceed 20%. To put this result in perspective, we must remember that solvation free energy of a typical

monovalent ion is ~ 100 kcal/mol. In chemical processes, 20 kcal/mol or more is a very significant amount of energy. Alternatively, for a typical exclusion radius, the solvation free energy of an ion in a *discrete* solvent with $\epsilon = 5$ can be equal to that in a *continuum* solvent with $\epsilon = \infty$. We believe that this new insight could help in developing better parametrization schemes for simplified solvation models by clarifying the meaning of the cavity radius.

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- (16) In a more realistic description, the radius of exclusion would correspond to the sum of van der Waals radii of the solute and solvent particles.
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