Lüders Band Motion in Iron

K. PREWO, J. C. M. LI, AND M. GENSAMER

Lüders band motion during the nonuniform deformation of iron has been investigated and a model has been developed which relates band velocity and band front strain profile to dislocation dynamics. Band motion under both constant and varying applied stress has been experimentally determined and it has been shown that the effect of prestrain on band front velocity is associated with an increase in internal stress. Measured band front strain profiles have been compared with those predicted by the present formulation and good agreement has been achieved.

NONUNIFORM yielding is characteristic of a fairly wide variety of materials. Examples are polycrystalline Fe, Ta, Mo, and single crystals of Fe,[1] α Cu-Al,[2] α brass,[3,4] Cu-10 pct In,[3] austenitic stainless steel,[5] copper whiskers,[6] and even nylon fibers.[7] In each of these cases plastic deformation began in some localized region, due to either grip stresses or a notch. In the case of iron and steel these localized regions have been referred to as Lüders bands.[8,9] Studies of Lüders bands using etch pitting techniques[10-12] and electron microscopy[13,14] have shown that grains ahead of the band front exhibited microstrain through slip on some few systems in each grain. The passage of the Lüders front was then found to be associated with a large increase in dislocation density with the major portion of the strain increment taking place over a fairly short distance. This is in agreement with measured Lüders band strain profiles.[7,15-17] Hahn[18] and Gilman,[19] using a formulation established by Hart,[20] were able to derive relations which fit the features of these strain profiles rather well. Miklowitz[15] showed that the length of active front increased with increasing velocity of the Lüders band and Sylwestrowicz and Hall[21] showed that the stress concentration due to the geometrical shape of this active front was probably not an important factor in band motion.

The velocity of band motion was studied as a function of applied stress in several steels,[22-24,27] niobium,[25] Fe-3.14 pct Si[26] and nylon,[7] while it also has been suggested that the effective stress is the most important quantity in determining band velocity.[25,28-30]

It was the purpose of this investigation to further establish the relationship of Lüders band motion and strain profile to dislocation behavior. This was undertaken as part of a larger program concerning the brittle fracture of steel and is related to this subject by the dependence of fracture toughness on the extent and velocity of plastic zones at a crack tip.

EXPERIMENTAL METHOD

Two different compositions of iron were studied during this investigation. The first, Ferrovac iron, contained 50 ppm carbon and 13 ppm nitrogen while the second material contained 200 ppm carbon, 30 ppm nitrogen, 260 ppm chromium, and 200 ppm nickel as primary alloying constituents. Both of these materials were cold rolled into sheet having thicknesses of 10 to 20 mil. Tensile specimens were then machined and fully annealed in dry hydrogen. The tensile specimen configuration, shown in Fig. 1, consistently provided the nucleation of a single Lüders band at the narrow specimen end. This was accomplished without subjecting the specimen to a load corresponding to the material's upper yield point because of the effective stress concentration caused by the grip at the narrow end.

1) Lüders Band Motion

Lüders band velocity measurement was made by noting the time required for a given band front to cover the known distance between two parallel lines scribed lightly onto the specimen surface prior to final anneal-

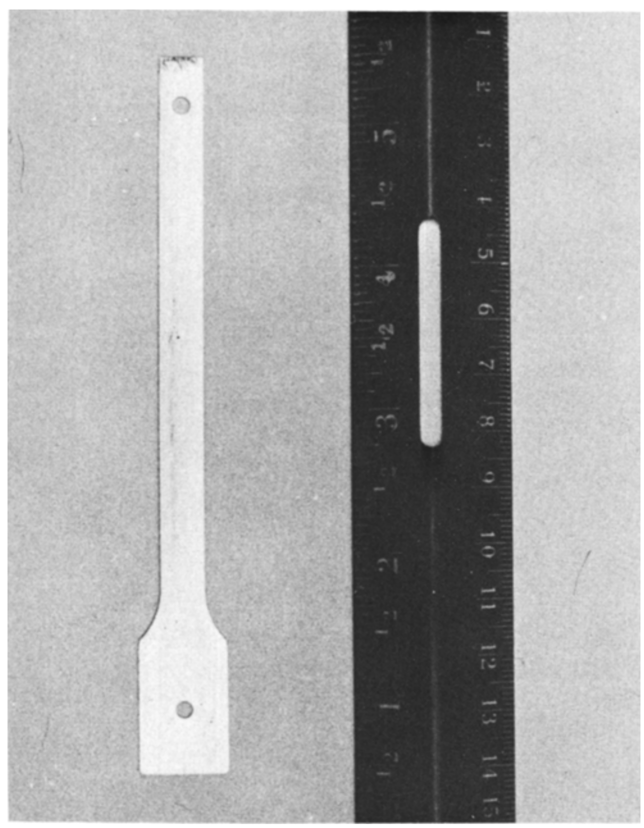

Fig. 1—Tensile specimen.

K. PREWO and J. C. M. LI, formerly Graduate Student and adjunct Professor, respectively, Henry Krumb School of Mines, Columbia University, are now Research Scientist, United Aircraft Research Laboratories, East Hartford, Conn. and Professor, Department of Mechanics and Aerospace Sciences, University of Rochester, Rochester, N. Y., respectively, M. GENSAMER is Professor Emeritus, Henry Krumb School of Mines. This paper is based upon a thesis submitted by K. PREWO in partial fulfillment of the requirements of the degree of Doctor of Engineering Science at Columbia University.

Manuscript submitted October 18, 1971.

ing. All mechanical testing was performed in tension using two different methods of load application. The first method subjected the test specimen to the constant crosshead velocity of an Instron TTC Universal Testing Machine. This velocity could be varied by 14 discrete intervals between 2×10^1 and 2×10^{-3} ipm. A constant applied load arrangement consisting of weights suspended from the lower specimen grip was also used to permit the application of loads smaller than that accompanying the lowest Instron crosshead velocity. All applied loads were measured using the load cells and recorder of the Instron machine to provide a continuous record of load vs time.

Lüders band velocities were determined as a function of stress in prestrained as well as fully annealed material. The prestrained specimens were of the higher carbon content iron and were initially deformed by propagating Lüders bands of a given strain completely along the length of each specimen. This resulted in a uniform strain along the entire length of each specimen. The subsequent aging at 200°F for 30 min in an inert atmosphere and tensile testing of these specimens resulted in the propagation of a second series of Lüders bands through prestrained material.

2) Stress Relaxation

Stress relaxation was performed on each specimen tested to determine the magnitude of the internal stress. This was accomplished by simply stopping the crosshead motion during Lüders band propagation. The subsequent variation of applied stress with time was continuously monitored on the Instron recorder. The form of the relaxation curves was independent of position of the Lüders band front along the specimen gage length. Contributions due to the inelastic response of the Instron testing machine were found to occur only in the first few seconds of testing and accounted for only 1 pct or less of the applied load.

Temperature fluctuations were of great importance during stress relaxation. Disturbances in the atmosphere surrounding the specimen were sufficient to alter the shape of the relaxation curve. To minimize such effects, care was taken that the air in the specimen vicinity was kept still and in most cases the specimen was surrounded by a glass dewar to insure thermal stability during testing.

Band front motion continued during the stress relaxation procedure. The extent of band motion during relaxation was measured by stopping the imposed crosshead motion at the instant the band front crossed a preselected scribed line on the specimen surface. The position of the band front, relative to this original scribed line at the conclusion of stress relaxation, determined the extent of motion.

3) The Lüders Band Front Strain Profile

The distribution of axial strain in a Lüders band was determined using grids of scribed lines on specimen surfaces. These grids were produced by a Leitz Miniload Hardness Tester equipped with a diamond pyramid indentor. Each grid consisted of a single 0.40 in. long line scribed axially down the center of the specimen and a set of short scribes perpendicular to and intersecting the axial one every 0.01 in.

A Lüders band front was allowed to propagate along the specimen until it reached a position in the grid network at which point the applied load was reduced. This reduction was accomplished in two different ways to produce contrasting results. In the first method the load was permitted to relax gradually as described in the previous section. In the second method the load was removed very suddenly by quickly removing a connecting pin from the lower extension rod of the mechanical test arrangement. Using this latter method the load dropped to zero in less than 1 sec.

The strain profile of the band front was then determined by comparing the spacings of the cross grid lines measured before and after deformation using a toolmaker's microscope. This profile consisted of the average strain developed in each of a series of 0.01 in. gage lengths along the specimen.

RESULTS AND DISCUSSION

1) Lüders Band Motion

The analysis of band motion has been carried out in a manner similar to that used to describe the dependence of dislocation velocity on effective stress.[31-33] At the head of an advancing band front the Lüders band velocity, V_L, is directly proportional to the product of the mobile dislocation density and average dislocation velocity and inversely proportional to the strain gradient.[18,20,34] Thus, it is proposed that the front velocity be written in a manner similar to that used to describe dislocation motion.

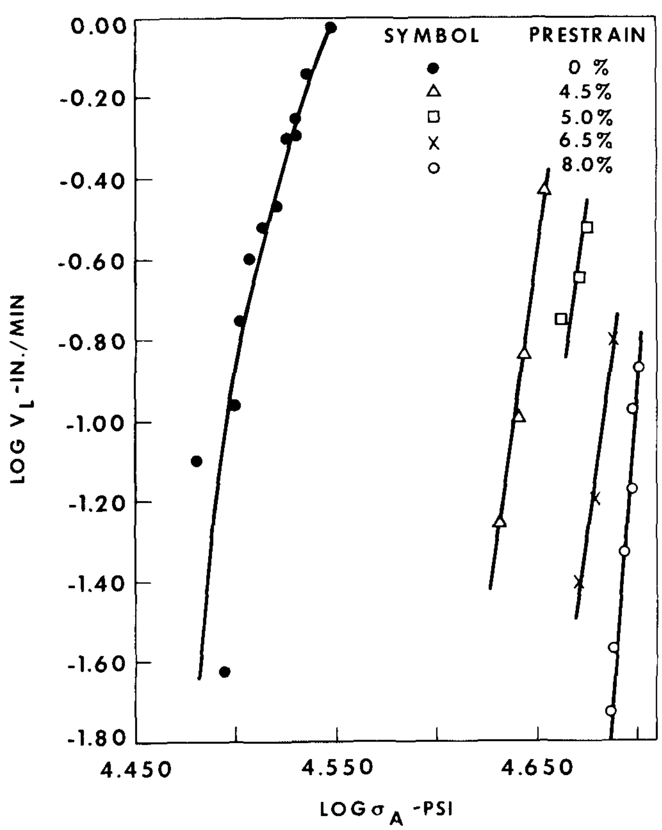

Fig. 2—Lüders band velocity as a function of applied stress at room temperature for prestrained specimens of iron containing 200 ppm carbon.

$$V_L = A(\sigma_A - \sigma_0)^m \quad [1]$$

In this equation σ_A is the applied stress, σ_0 the value of applied stress at which the band velocity goes to zero and A and m are constants independent of stress. The value of σ_0 corresponds to the internal stress opposing dislocation motion ahead of the advancing Lüders band. As will be shown, the value of σ_0 depends on the amount of specimen prestrain prior to band propagation. It has been shown previously[29,30] that σ_0 is also dependent on material composition and grain size. The quantity $(\sigma_A - \sigma_0)$ is the average effective stress for band motion, both stresses being determined on the basis of the unstrained specimen cross sectional area and applied load. To determine the magnitude of σ_0, a stress relaxation procedure has been formulated based on the work of Li and Gupta.[35] During a tensile test the sum of the rate of elastic deformation of specimen and machine plus the rate of plastic deformation of the specimen must equal the rate of crosshead motion. For a specimen with a single propagating band front under constant applied stress, the crosshead velocity is equal to the product $V_L \epsilon_L$ of the Lüders band velocity and the Lüders strain. When the crosshead velocity is stopped the stress relaxation that follows can be written in terms of band motion still occurring in the specimen. It is assumed that during stress relaxation; the plastic deformation of the specimen is still confined to the band front region, that V_L is described by Eq. [1], and that during stress relaxation the Lüders strain associated with a band front can be written as $\epsilon_L = B(\sigma_A - \sigma_0)^n$. In accord with this last assumption, the strain associated with a band front decreases with decreasing applied stress during stress relaxation. As will be shown, this causes an extended strain profile to characterize the specimen region along which relaxation occurred. The equation to describe ϵ_L during relaxation is proposed for ease of manipulation and is not a bad approximation over the range of strains en-

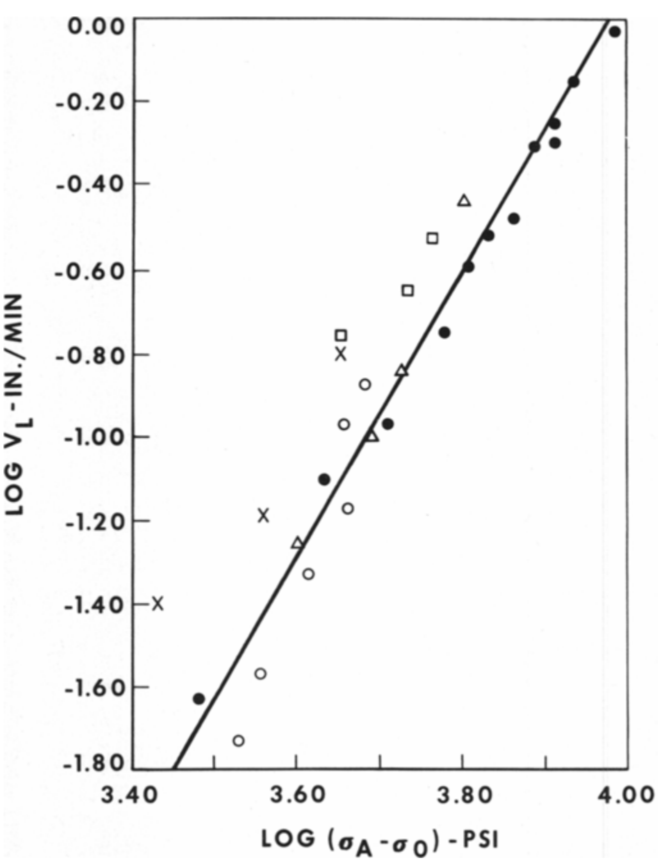

Fig. 3—Lüders band velocity data of Fig. 2 as a function of effective stress.

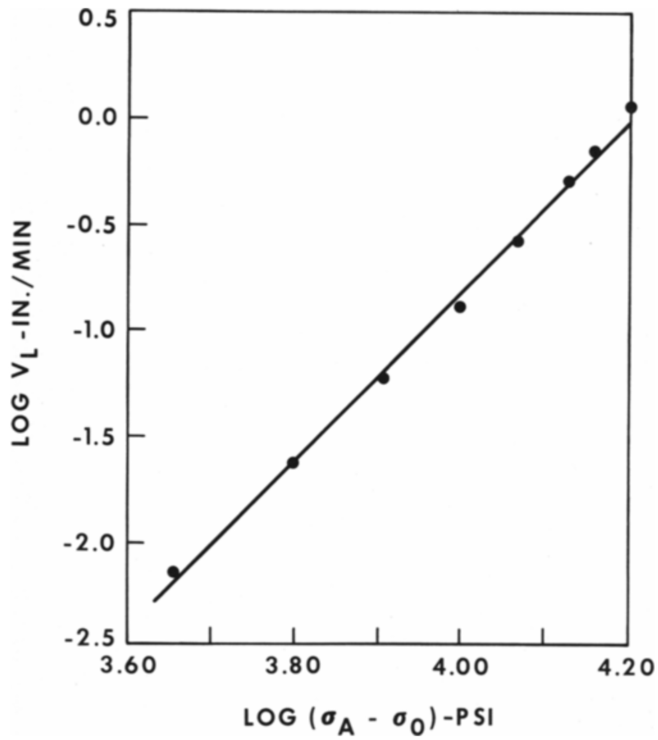

Fig. 4—The dependence of band velocity on effective stress for Ferrovac iron.

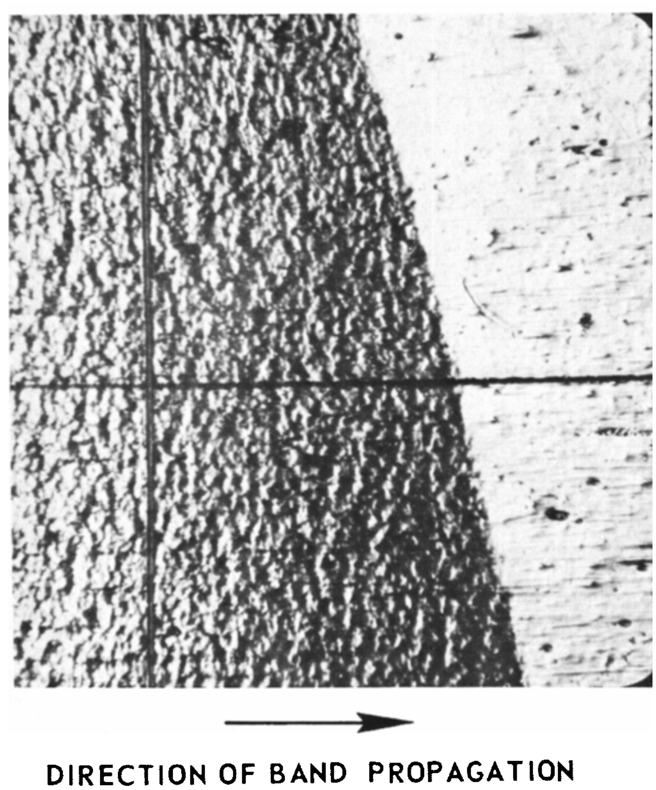

Fig. 5—Extent of Lüders band motion during stress relaxation.

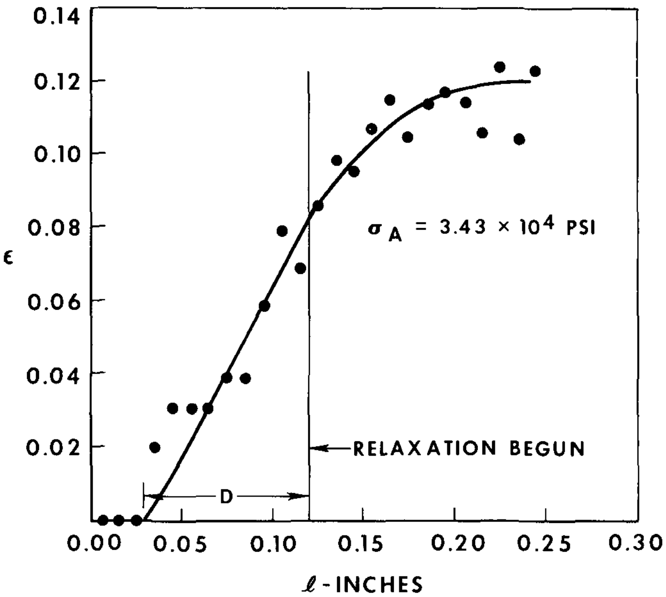

Fig. 6—The measured strain profile of a relaxed Lüders band.

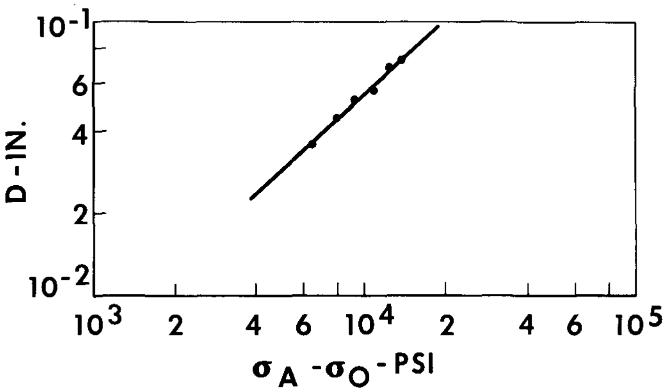

Fig. 7—The extent of band motion D during stress relaxation as a function of the maximum effective stress.

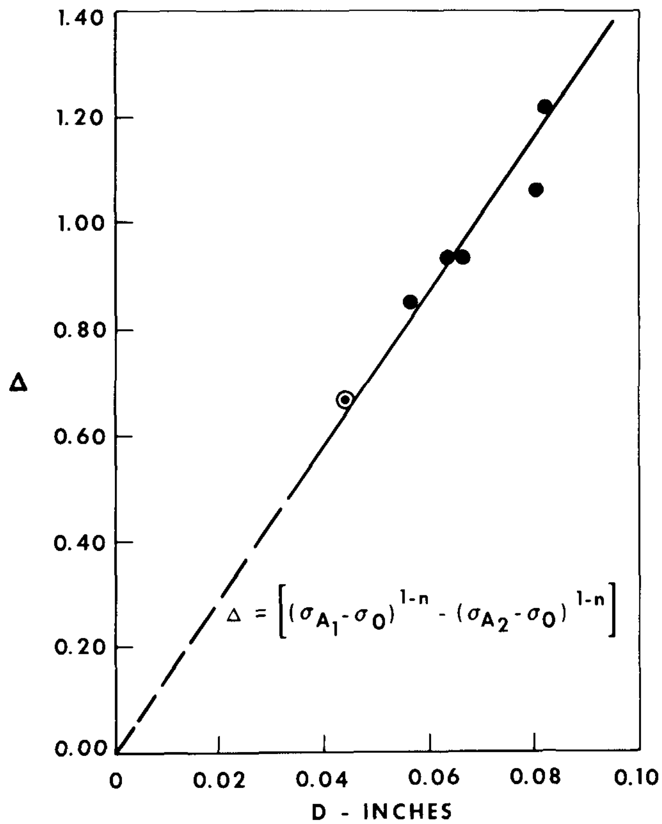

Fig. 8—The extent of band motion D during stress relaxation for the stress function Δ.

countered during the stress relaxations studied. The rate of stress relaxation can then be written as

$$-\frac{d\sigma_A}{dt} = \theta V_L \epsilon_L = AB\theta(\sigma_A - \sigma_0)^{m+n} = SK(t + a)^{S-1} \quad [2]$$

where t represents time measured from the start of stress relaxation, θ is a constant dependent on the elastic constants of the testing machine and specimen, K and a are constants and $S = 1/(1 - m - n)$. A more detailed derivation of Eq. [2] can be found elsewhere.[29] Eq. [2] suggests that if the log of $(d\sigma_A/dt)$ is plotted as a function of the log of $(t + a)$, a straight line having slope $(S - 1)$ will result. Or, if log $(d\sigma_A/dt)$ is plotted vs log t, at long times the slope should equal $(S - 1)$. Thus, the value of $(m + n)$ can be determined and then the value of σ_0 using

$$\sigma_0 = \frac{\sigma_{A_1} - C\sigma_{A_2}}{1 - C} \quad [3]$$

where the subscripts 1 and 2 refer to times t_1 and t_2 on the same stress relaxation curve and $C = [(t_1 + 2)/(t_2 + a)]^S$. The Lüders band velocity can then be plotted as a function of effective stress, $(\sigma_A - \sigma_0)$.

The dependence of Lüders band velocity on applied stress, load divided by original specimen cross sectional area, for fully annealed and prestrained plus aged iron containing 200 ppm carbon is presented in Fig. 2. Band velocity, at a given applied stress, decreases significantly with increasing degree of prestrain. A separate stress relaxation was performed on each specimen to determine the magnitude of the internal stress ahead of the band front σ_0, by use of Eqs. [2] and [3]. The internal stress was found to increase with increasing degree of prestrain and when plotted on the basis of effective stress, the band velocities for all specimens exhibited almost a single straight line dependence, Fig. 3. The value of m obtained from the line drawn in Fig. 3 is 3.5. Deviations from the line are smaller for values of prestrain of 0, 4.5, and 8 pct while at 5 and 6.5 pct the deviations are larger. This lack of total agreement is probably due to the fact that the values of $(\sigma_A - \sigma_0)$ are only about 10 pct of those of σ_A so that any lack of precision in determining σ_0 is amplified in the value of effective stress. However, the observed value of $m = 3.5$ is in good agreement with the value of m obtained previously[30] for similar material tested at room temperature. The value of m, obtained by a similar procedure to that used here, was 4.0 ± 0.5 and was independent of material grain size and carbon content.[30] This m value is also found to be in agreement with the values of m^*, the dislocation velocity effective stress exponent, obtained for iron by a variety of direct and indirect testing techniques.[36-40]

2) Extent of Band Front Motion During Stress Relaxation

The dependence of Lüders band velocity on effective stress for Ferrovac iron specimens containing 50 ppm carbon is presented in Fig. 4. Once again stress relaxation procedures based on Eqs. [2] and [3] were used to obtain the value of σ_0. The value of m is equal to 4.0. Fig. 5 is a photograph of the surface of a Ferrovac iron tensile specimen used to obtain the data in Fig. 4. The roughened region in the figure is the material behind the band front. The scribed line normal to the direction of band propagation marks the position at which the crosshead of the Instron machine was stopped and stress relaxation began. The band front continued to move under decreasing stress until it finally halted at the position noted. Fig. 6 illustrates this same point by indicating the variation of strain behind another band front at the conclusion of a stress relaxation procedure. In this figure the band front moved approximately 0.10 in.

The band propagates during the decrease of stress with time in stress relaxation. This stress decrease is described by the above analysis of the stress relaxation phenomenon. Thus if the known stress dependence of band velocity is combined with the relaxation formulation, it should be possible to describe the extent of band motion during relaxation. The following provides such an analysis.

The distance the band can move during a relaxation is given by the integral of the velocity over the period of time of relaxation.

$$D = \int_{t=0}^{t=t} V_L dt = \int_{\sigma_A @ t=0}^{\sigma_A @ t=t} V_L (dt/d\sigma_A) d\sigma_A \quad [4]$$

Eq. [2] provides the required expression for the rate of change in applied stress and by substituting Eq. [2] into Eq. [4], and recalling the expression for Lüders band velocity, the final form for D is achieved.

$$D = \frac{1}{\theta B(1-n)} (\sigma_A - \sigma_0)^{1-n} \Big|_{\sigma_A @ t=t}^{\sigma_A @ t=0} \quad [5]$$

The limits of integration are the stress at the onset of relaxation, *i.e.* the maximum stress the Lüders band propagates under for the particular relaxation, and the stress at the end of the stress drop, *i.e.* at time $t = t$. If the stress were allowed to relax until all band motion stopped at the stress level σ_0, Eq. [5] would become

$$D = \frac{1}{\theta B(1-n)} (\sigma_{A_1} - \sigma_0)^{1-n} \quad [6]$$

where σ_{A_1} is the applied stress at the onset of relaxation. For this case, Eq. [6] predicts that a plot of log D, the distance traveled in a relaxation, vs log $(\sigma_A - \sigma_0)$ should have a slope of $(1 - n)$.

To establish the validity of the above analysis, tests were run to see if Eq. [6] could be used to describe the stress dependence of band position at the conclusion of relaxation. Specimens of Ferrovac iron were tested according to the method described in the experimental procedure. The minimum load observed at the conclusion of relaxation was found to be just 10 pct above that calculated for σ_0 using the stress relaxation analysis so that a plot of log D vs log $(\sigma_A - \sigma_0)$ should approximately yield a straight line. Fig. 7 shows that this is the case. The value of $(1 - n)$ determined from this straight line was 0.9. The velocity data, shown in Fig. 4, provides the value of $m = 4.0$, while from stress relaxation the value of $(m + n)$ was found to equal 4.2. Thus, a predicted value of $(1 - n) = 0.8$ is in good agreement with that of 0.9 obtained above from the stress dependence of relaxation distance.

Further verification of the above formulation is provided by band relaxation distances measured for specimens of iron containing 200 ppm carbon. In this case the value of lowest relaxed stress was considerably higher than the value of σ_0 determined by stress relaxation so that the complete form of Eq. [5] must be used. The data are presented in Fig. 8. The expected dependence of extent of band motion, D, on stress function is observed.

3) The Lüders Band Strain Profile

The Lüders band front is a region of transition from localized slip and microstrain occurring ahead of the band and the steady-state condition left in its wake that is characterized by a level of fairly constant strain, *i.e.* the Lüders strain. In this region of transition the applied and internal stresses vary with position due to the rather abrupt change in strain. The following is a model intended to describe the Lüders band strain profile and the dynamic behavior of Lüders bands in terms of the stress dependence of dislocation velocity.

When the yielding of a material occurs through the propagation of a Lüders band, it is usually observed that the applied load remains constant. It often is presumed then that the material deforms from the onset of yielding to the Lüders strain under constant applied true stress, *i.e.* this is a region of zero work hardening. This presumption is not correct. Despite the fact that the load P does remain constant, the applied stress varies with position in the band because of the diminution in cross sectional area a associated with the increase in strain in the band front. The applied stress, σ_a, can be written in terms of the longitudinal strain, ϵ, at any position in the band.

$$\sigma_a = (P/a) = \sigma_A e^{\epsilon} \quad [7]$$

In Eq. [7], as in the previous equations, σ_A refers to the stress applied to the unstrained cross sectional area of the specimen. The strain rate at any position l in the band front can be described in terms of the band front velocity and the strain gradient at that position.

$$\dot{\epsilon} = V_L d\epsilon/dl \quad [8]$$

The strain rate can also be written in terms of dislocation velocity and the dependence of dislocation velocity on stress.[31,33,41]

$$\dot{\epsilon} = \phi \rho b V = \phi \rho b c (\sigma_a - \sigma_i)^{m^*} \quad [9]$$

In this equation ρ is the density of mobile dislocations, ϕ is an orientation factor resolving the dislocation produced displacements into the direction of observed strain, V is the average mobile dislocation velocity, c is the dislocation velocity at unit effective stress, σ_i is the internal stress at the particular position in the band front of interest and m^* is the dislocation velocity stress exponent. Combining the above expressions for plastic strain rate and integrating the resultant expression with respect to l, where l is chosen equal to zero at the band front tip at which the strain ϵ is equal to zero, an expression describing the variation of strain

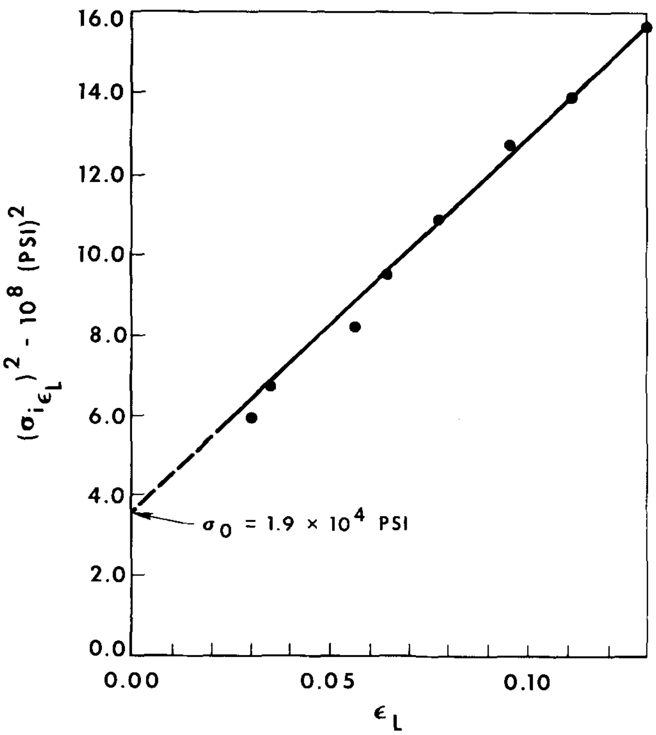

Fig. 9—The square of the internal stress at the Lüders strain as a function of the Lüders strain.

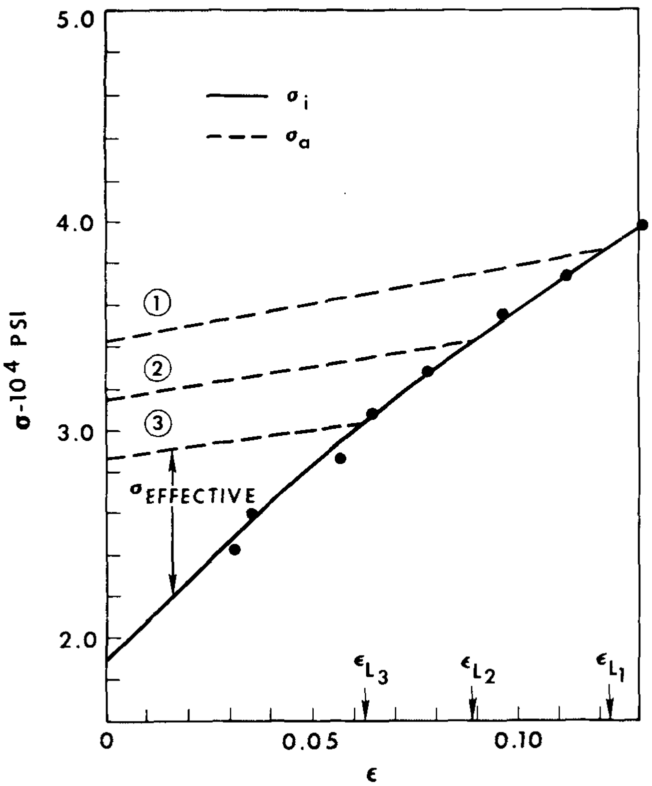

Fig. 10—The strain dependence of the applied stress and internal stress in a Lüders band front.

as a function of position in the Lüders band is obtained.

$$\frac{l}{V_L} = \int_{\epsilon=0}^{\epsilon=\epsilon_l} \frac{d\epsilon}{\phi\rho b c(\sigma_a - \sigma_i)^{m^*}} \qquad [10]$$

The dependence of both σ_i and ρ on plastic strain must be ascertained prior to numerical evaluation of Eq. [10].

The strain dependence of σ_i can be quantified by recalling Eq. [8]. When the strain rate in a moving band front becomes zero the position in the front has been reached where the strain gradient has also become zero. This point of constant strain, characterized by the Lüders strain, must also be a position at which the applied and internal stresses are equal, *i.e.* the effective stress is zero. Thus, if the applied stress is calculated from Eq. [7] for a particular Lüders strain, its value can be considered to equal the internal stress σ_i for that particular strain

$$\sigma_{i\epsilon_L} = \sigma_A e^{\epsilon_L} \qquad [11]$$

The Lüders strain was determined as a function of applied stress σ_A and the values of internal stress were calculated at each strain using Eq. [11]. The values of $\sigma_{i\epsilon_L}$ and ϵ_L are plotted in Fig. 9. The observed dependence of internal stress on plastic strain is consistent with previous observations.[42,43] It has been shown experimentally[42] and predicted theoretically[43] that the internal stress depends on the square root of the total dislocation density ρ_t. It was also found that, for polycrystalline iron, the total dislocation density can be described as a linear function of strain over a fairly large range of strain. This suggests that the values of internal stress, $\sigma_{i\epsilon_L}$, should be squared and plotted as a function of the measured Lüders strain ϵ_L. The data of Fig. 9 exhibit this dependence. The internal stress at zero strain is equal to the value obtained for σ_0 using the earlier described stress relaxation procedure. This is expected since σ_0 corresponds to the level of internal stress ahead of the Lüders band front and is thus associated with the unstrained specimen condition.

It now is possible to describe the effective stress for all strains in the band front. Fig. 10 shows graphically how this effective stress varies with strain. The applied stress curve varies, depending upon the magnitude of the applied load, while the internal stress curve remains unaffected. The point where the internal and applied stress curves intersect, *i.e.* where the effective stress goes to zero, occurs at the Lüders strain. Thus, in Fig. 10 for the applied stress curves 1, 2, and 3 the resultant Lüders strains are ϵ_{L_1}, ϵ_{L_2}, and ϵ_{L_3}, respectively. From this figure it can be seen how the Lüders strain can be increased to very large values by simply applying larger and larger values of applied load. In fact, for sufficiently large loads the entire stress strain history of the material being tested can be confined to the Lüders band region because the intersection of the applied and internal stress curves can then occur at strains in excess of the fracture strain. In this case the fracture strain is defined as that level of strain at which a specimen would fail if it exhibited deformation uniformly throughout its gage length. This same phenomenon was found to occur by Low and Feustel[44] in low carbon steel specimens tested at low temperatures as well as by other investigators.[45,46]

The dependence of mobile dislocation density on strain in the band front, required for the evaluation of Eq. [10], is more difficult to obtain. This quantity, unfortunately, has not been directly measured; however, in polycrystalline iron indirect experimental evidence indicates that, despite huge increases in the total dis-

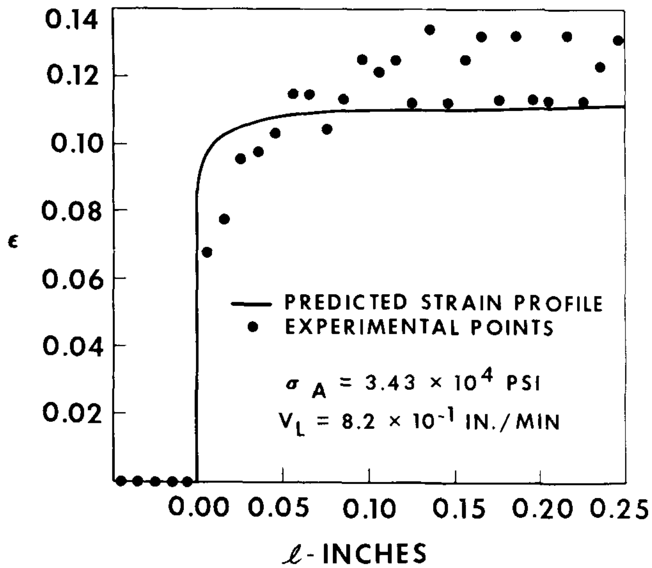

Fig. 11—Lüders band strain profile.

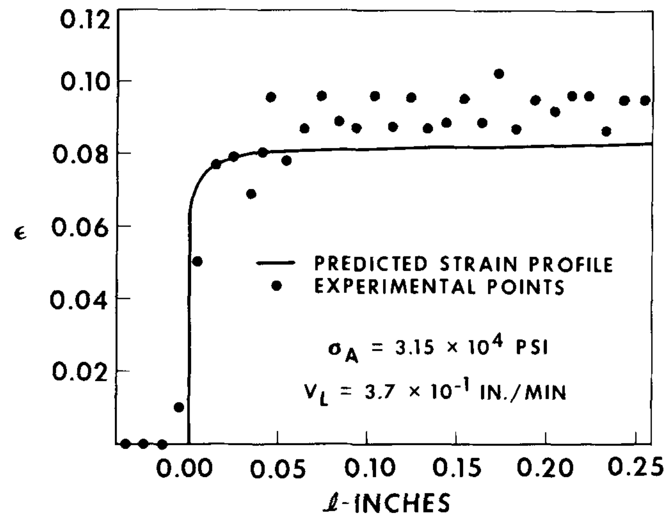

Fig. 12—Lüders band strain profile.

location density, the mobile dislocation density remains very nearly independent of strain.[36,47,48] This constancy will be assumed to apply in the present band front model. It also will be assumed that the value of m^* is equal to m for Lüders band velocities. This has been documented in the past for iron[29] and silicon iron.[30]

Using these assumptions and the variation of effective stress with strain, it is possible to rewrite Eq. [10]

$$\frac{l}{V_L} = \int_{\epsilon=0}^{\epsilon=\epsilon} \frac{d\epsilon}{k\left[\sigma_A e^{\epsilon} - \alpha\sqrt{\rho_0 + k\epsilon}\right]^m} \qquad [12]$$

where all the quantities except K ($K = \phi\rho b c$) are now known. The expression in the denominator can be simplified further if $\sigma_A e^{\epsilon}$ is approximated by $\sigma_A(1 + \epsilon)$. Using this simplification, Appendix I shows the further manipulation of Eq. [12] so that it can be integrated to yield the strain as a function of distance from the tip of a Lüders band front. The value of m is taken equal to 4.0.

Using Eq. [A-3], the value of K determined in the Appendix, and the values of I determined by the computer method, it is possible to calculate the distance behind the band front at which each strain occurs. The resultant strain profiles are plotted as continuous curves in Figs. 11 and 12 for each of the applied stresses shown. It should be remembered that the measured points shown in these figures represent average values of strain over 0.01 in. long sections of specimens. Although the agreement between experiment and calculation is not exact, the general features of the measured profiles are predicted.

The most notable characteristic of both the measured and calculated strain profiles is the very large local strain rates produced in the band front as a result of steep strain gradients. The maximum strain rates are those at the very tips of the bands. For Figs. 11 and 12 they are 67 sec^{-1} and 30 sec^{-1}, respectively. These strain rates are several orders of magnitude larger than those calculated conventionally by taking the applied crosshead velocity and dividing by the specimen gage length. For example, the overall strain rate for the specimen represented in Fig. 11 would be calculated to be 5×10^{-4} sec^{-1}.

The fact that the predicted profiles are indeed similar to those experimentally determined lends support to the observation that $m = m^*$. At the very tip of the Lüders band it can be shown by combining Eqs. [8] and [9] that,

$$V_L = \frac{\phi\rho b c(\sigma_A - \sigma_0)^{m^*}}{(d\epsilon/dl)} = A(\sigma_A - \sigma_0)^m \qquad [13]$$

In Eq. [13] the value of internal stress, σ_i, is set equal to σ_0, the internal stress characteristic of the material into which the band front tip is advancing. If all the quantities in Eq. [13], except σ_A, remain constant for a particular set of velocity measurements, it is clear that the stress dependence of Lüders band motion is controlled by the stress dependence of dislocation motion in the band front and that $m = m^*$. This equality then makes the observation that m is independent of carbon content for iron[30] consistent with the work of other investigators who have been able to show that this is true for m^*.[47,49] In a like manner, the fact that m is not affected by prestrain is consistent with results of several studies.[36,47]

Other investigators have also observed rather steep strain profiles associated with Lüders band fronts. Glenn and Morrison,[14] through the use of transmission electron microscopy, found that the band front was associated with a rapid increase in dislocation density. This increase occurred over a distance equivalent to only 3 or 4 grain diameters. Krafft[24] also measured the strain profiles of impact-loaded steel specimens and found extremely steep strain gradients. On the other hand, Miklowitz[15] reported more gradual increases in strain extending over distances of about 10 mm. These distances increased with increasing lower yield stress and may possibly be attributable to band motion taking place during load removal prior to strain profile measurement. As shown in Fig. 6, if great care is not exercised to halt the mechanical test abruptly, the resultant strain profile will indeed spread out over a large distance. This distance increases with increasing applied stress. The profile thus measured does not correspond to the strain profile of the band while it is in motion.

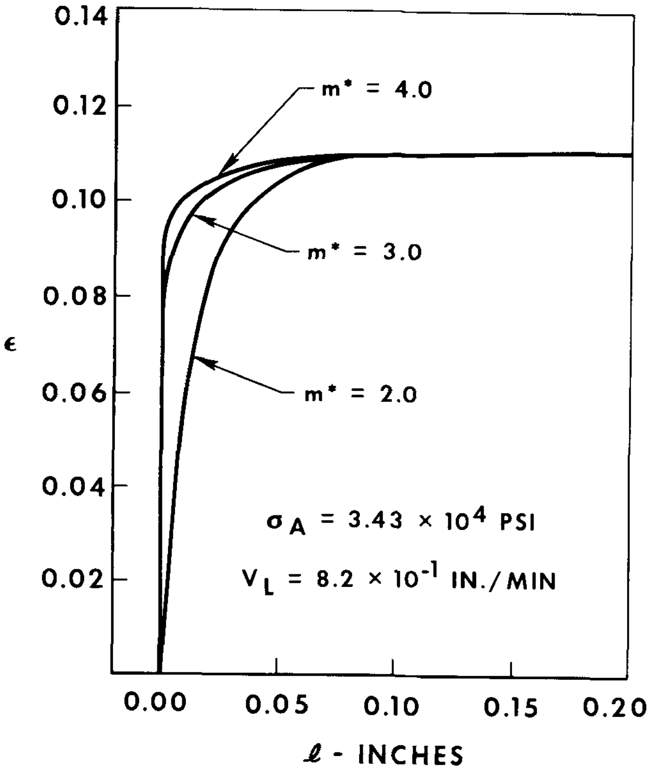

Fig. 13—The predicted variation of Lüders band strain profile with m^*.

It is suggested that the methods used in the above analysis can provide the basis for the description of the plastic zone size existing in the stress fields at the tips of static and moving cracks. In the case of a moving crack the stress on a material element will vary with both position relative to the crack tip and time. Both of these factors, however, can readily be accounted for and it is expected that this procedure will aid in explaining the effects of grain size, prestrain, temperature and composition on fracture.

Calculations of strain profiles were also performed for values of m^* equal to 3.0 and 2.0. In each case all quantities used were equal to those established in the previous analysis except for the value of m^*. The resultant strain profiles are shown in Fig. 13 for $\sigma_A = 3.43 \times 10^4$ psi. As m^* decreases the severity of the strain gradient also decreases.

SUMMARY

It has been shown that the stress dependence of Lüders band propagation and the strain profile of the Lüders band front are described well by a formulation based on dislocation dynamics. The concept of effective stress for band motion has been used and it has been shown that the internal stress accounts for the increased applied stress required to propagate Lüders bands in the prestrained material. Band front motion under constant applied stress and also during stress relaxation have both been successfully analyzed.

The strain gradient associated with the Lüders band front has been related to dislocation motion and strain hardening behavior. Local strain rates in the band front are considerably higher than those calculable on the basis of total specimen extension rate and gage length. These high levels of strain rate are associated with steep strain gradients at the band tip which are characteristic of the band front during motion.

APPENDIX

Eq. [12] can be written in the form

$$\frac{l}{V_L} = \int_{\epsilon=0}^{\epsilon=\epsilon} \frac{d\epsilon}{K[(\sigma_A(1+\epsilon) - \alpha(\rho_0 + k\epsilon)^{1/2}]^4} \qquad \text{[A-1]}$$

Letting $X^2 = (\epsilon + \rho_0/k)$

$$\frac{l}{V_L} = \frac{2}{K} = \int_{x=0}^{x=\sqrt{\epsilon+\rho_0/K}} \frac{x\,dx}{[a + bx + cx^2]^4} = \frac{2}{K} I \qquad \text{[A-2]}$$

The value of the integral I was obtained through the use of an IBM 360 system for the values of σ_A shown in Fig. 10.

Expression [A-2] was evaluated for the interval of strain ranging from zero to that strain at which the effective stress went to zero for each of these three cases. This was done in steps of 0.001 in strain. Thus the result was sets of values of I as a function of strain. For each value of strain it is possible to calculate the corresponding coordinate of position l using Eq. [A-2]. At the tip of the band both l and ϵ are equal to zero.

$$l = \frac{2IV_L}{K} \qquad \text{[A-3]}$$

Expression [A-3] can provide the general form of the strain profiles desired in units of position (Kl) vs ϵ. However, to determine the absolute magnitudes of the positions corresponding to each strain, K must be evaluated. This is accomplished by using one point on the measured strain profile in Fig. 11. The average strain at a position 0.08 in. behind the front of the band is 0.11. Using Eq. [A-3], the value of I determined by the computer for $\sigma_A = 3.43 \times 10^4$ psi, $\epsilon = 0.11$ and $V_L = 0.82$ ipm, is

$$K = \frac{2IV_L}{l} = 7.27 \times 10^{-14} \text{ (psi)}^{-4} \text{ (in.)}^{-1}$$

This value was then used in all further calculations.

ACKNOWLEDGMENT

The authors are pleased to acknowledge the financial support of the Army Office of Research under contract DA-31-124-ARO(D)-382.

REFERENCES

1. H. Paxton and I. J. Baer: *AIME Trans.*, 1955, vol. 203, p. 989.
2. T. J. Koppenal and M. E. Fine: *Trans. TMS-AIME*, 1961, vol. 221, p. 1178.
3. B. J. Brindley, J. H. Corderoy, and R. W. Honeycomb: *Acta Met.*, 1962, vol. 10, p. 1043.
4. G. R. Piercy, R. W. Cahn, and A. H. Cottrell: *Acta Met.*, 1955, vol. 3, p. 331.
5. G. Meyrick and H. W. Paxton: *Trans. TMS-AIME*, 1964, vol. 230, p. 1010.
6. K. Yoshida, *et al.*: *J. Phys. Soc. Japan*, 1968, vol. 24, p. 1099.
7. B. N. Dey: *J. Appl. Phys.*, 1967, vol. 38, p. 4144.
8. W. Lüders: *Dinglers Polytech. J.*, 1860, p. 18.
9. A. Poibert: *Mem. Artillerie Franc.*, 1842, vol. 5, p. 525.
10. J. C. Suits and B. Chalmers: *Acta Met.*, 1961, vol. 9, p. 854.
11. W. E. Carrington and D. McLean: *Acta Met.*, 1965, vol. 13, p. 493.
12. D. V. Wilson: *Mater. Sci. J.*, 1967, vol. 1, p. 40.
13. T. W. Barbee and R. A. Huggins: ONR Rept., 1965.
14. W. B. Morrison and R. C. Glenn: *J. Iron Steel Inst.*, 1968, vol. 206, p. 611.
15. J. Miklowitz: *J. Appl. Mech.*, 1947, vol. 14, p. 31.

16. D. B. C. Taylor: *J. Mech. Phys. Solids,* 1958, vol. 3, p. 38.
17. D. B. C. Taylor and L. E. Malvern: *Response of Metals to High Velocity Deformation,* Interscience, 1961.
18. G. T. Hahn: *Acta Met.,* 1962, vol. 12, p. 727.
19. P. Gillis and J. Gilman: *J. Appl. Phys.,* 1965, vol. 36, p. 3370.
20. E. W. Hart: *Acta Met.,* 1955, vol. 3, p. 146.
21. W. Sylwestrowicz and E. O. Hall: *Proc. Phys. Soc.,* 1951, vol. B64, p. 495.
22. J. C. Fisher and H. C. Rogers: *Acta Met.,* 1956, vol. 4, p. 180.
23. J. F. Butler: *Mech. Phys. Solids,* 1962, vol. 10, p. 313.
24. J. M. Krafft: *Acta Met.,* 1962, vol. 10, p. 85.
25. H. Conrad and G. Stone: *J. Mech. Phys. Solids,* 1964, vol. 12, p. 139.
26. D. W. Moon and T. Vreeland: *Scripta Met.,* 1968, vol. 2, p. 35.
27. J. F. Butler: *Acta Met.,* 1962, vol. 10, p. 257.
28. H. Conrad: *J. Mech. Phys. Solids,* 1963, vol. 11, p. 437.
29. K. Prewo, J. C. M. Li, and M. Gensamer: *Scripta Met.,* 1970, vol. 4, p. 777.
30. K. Prewo, J. C. M. Li, and M. Gensamer: *Proc. Second Int. Conf. On Strength of Metals and Alloys,* Asilomar, 1970.
31. W. G. Johnston and J. J. Gilman: *J. Appl. Phys.,* 1959, vol. 30, p. 129.
32. D. F. Stein and J. R. Low: *J. Appl. Phys.,* 1960, vol. 31, p. 362.
33. J. C. M. Li: *Can. J. Phys.,* 1967, vol. 45, p. 493.
34. J. C. M. Li and Y. T. Chou: *Met. Trans.,* 1970, vol. 1, p. 1145.
35. I. Gupta and J. C. M. Li: *Met. Trans.,* 1970, vol. 1, p. 2323.
36. I. Gupta: Dr. Eng. Sci. Thesis, Columbia University, 1968.
37. T. L. Altshuler and J. W. Chirstian: *Phil. Trans. Royal Soc.,* 1967, vol. 261A, p. 253.
38. H. Yada: *Acta Met.,* 1967, vol. 15, p. 1255.
39. I. M. Bernstein: *Acta Met.,* 1969, vol. 17, p. 249.
40. A. Turner and T. Vreeland: *Acta Met.,* 1970, vol. 18, p. 1225.
41. E. Orowan: *Proc. Phys. Soc. London,* 1940, vol. 82, p. 8.
42. A. S. Keh: *Direct Observation of Imperfections in Crystals,* J. B. Newkirk, ed., p. 213, Interscience, 1962.
43. J. C. M. Li: *Direct Observation of Imperfections in Crystal,* J. B. Newkirk, ed., p. 234, Interscience, 1962.
44. J. R. Low and R. G. Feustel: *Acta Met.,* 1953, vol. 1, p. 185.
45. Winlock and Leiter: *Trans. ASM,* 1937, vol. 25, p. 181.
46. Winlock and Lavergne: *Trans. ASM,* 1933, vol. 21, p. 109.
47. J. T. Michalak: *Acta Met.,* 1965, vol. 13, p. 213.
48. M. Nagumo and Matsuda: *Trans. Jap. Inst. Metals,* 1968, vol. 9, p. 885.
49. Y. Nakada and A. S. Keh: *Acta Met.,* 1968, vol. 16, p. 903.
50. J. J. Gilman: *J. Appl. Phys.,* 1965, vol. 36, p. 2722.