FLUCTUATIONS OF THE COHERENCE FUNCTION OF METER RADIO WAVES SCATTERED IN A RANDOM COSMIC MEDIUM

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A qualitative analysis of the fluctuations of a wave field in both weak and strong scattering regimes is presented. It is shown that for meter radiowaves the scattering in the interstellar space leads to the appearance of stochastic multipaths and to a speckle structure of the source image. A speckle-interferometric method is applied to stochastic multipaths in the interstellar medium. An analytic solution for the cross-frequency field correlation function is investigated, and the widths of the coherence bands as functions of frequency are estimated.

In the case of meter and decameter band radiowaves, one of the main factors restricting the sensitivity and resolution power of instruments is the scattering in the propagation medium (ionosphere, interplanetary medium (IPM), or interstellar medium (ISM)). The effect of scattering in the propagation medium on radioastronomic measurements has been investigated in a large number of studies (see, e.g., [1, 2], and the references quoted therein). However, in most of these studies, it is actually considered the approximation of weak scattering, which is contingent on the radiowave band. The existing synthetic aperture systems and the radiointerferometers with independent registration work in the centimeter band, for which the scattering in the ionosphere or in the IPM is quite weak. As far as the ISM is concerned, the characteristic scales of wave front distortion in this band are not smaller, or even larger in order of magnitude than the maximum linear dimensions of the apertures synthesized by terrestrial instruments [3]. In the meter or decameter band radiowaves, the scattering intensity in IPM significantly depends on elongation. In order to reduce the effect in IPM, the source observations in these bands are performed at elongations exceeding 90-120 degrees. The scattering in the ionosphere is subject to considerable variations depending on the state of the ionosphere at the time of measurement. At moderate latitudes and a calm ionosphere, the typical regime is, for meter waveband, one of unsaturated or partly unsaturated fluctuations. In this case, the wave propagation is accompanied by a focusing effect [4], which leads to significant phase and amplitude fluctuations. Further qualitative analysis of the scattering in ISM shows that, in the case of observations of sources in the galactic plane, the scattering regime corresponds to saturated fluctuations.

In the present article, we shall theoretically analyze the effect of scattering upon the interferometer characteristics.

1. Let us assume that two spatially separated antennas receive waves identically polarized, and let us consider the scalar wave fields $E(\boldsymbol{r}_1, t_1)$ and $E(\boldsymbol{r}_2, t_2$, where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors of the antenna centers and t_1 and t_2 are the arrival times. The z-axis is taken from the source center towards the middle of the base vector $\boldsymbol{b}=\boldsymbol{r}_1-\boldsymbol{r}_2$; we shall also assume that both antennas are in a z = const plane. Introducing coordinates in this plane, $\boldsymbol{r}_{1,2}=\{z, \boldsymbol{\rho}_{1,2}\}$, the base vector $\boldsymbol{b} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$. From The Fourier transform of the field $\tilde{E}(\boldsymbol{r}, \omega)$ we separate a rapidly varying factor $\exp(ikz)$, where $k = \omega/c$ and c is the velocity of light. We shall assume now that the wave is propagating in a cold, quasineutral, cosmic plasma with a dielectric permeability $\varepsilon = \langle\varepsilon(\omega, \boldsymbol{r})\rangle + \delta\varepsilon(\omega, \boldsymbol{r})$ where the angular brackets indicate averaging over the ensemble of realizations. Here, $\langle\varepsilon\rangle = 1 - \omega_p^2/\omega^2$, where $\omega_p^2 = 4\pi e^2\langle N_e\rangle/m$ is the plasma frequency, $\langle N_e\rangle$ is the average electron density, e and m are the charge and mass of the electron, respectively, and $\delta\varepsilon = 4\pi e^2\delta N_e(r)/m\omega^2$ where δN_e is the fluctuating component of the electron density: $\langle\delta N_e\rangle = 0$. The relatively slowly variable complex field amplitude $U(\omega, \boldsymbol{r}) = E(\omega, \boldsymbol{r})\exp(-ikz)$ satisfies the parabolic equation

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$$2ik\frac{\partial U}{\partial z}+\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+[k^2(\langle\varepsilon\rangle-1)+k^2\delta\varepsilon]U=0. \tag{1}$$

The initial condition at z = 0 is

$$U(\omega, 0, \boldsymbol{\rho}) = U_0(\omega, \boldsymbol{\rho}), \tag{2}$$

where $U_0(\omega, \boldsymbol{\rho})$ is the Fourier transform of the source radiation field. The solution of equation (1) can be written in integral form, using Feynman's path integrals [5]:

$$U(\omega, z, \boldsymbol{\rho})=\int_{-\infty}^{\infty} d^2\rho_0\, U(\omega, \boldsymbol{\rho}_0)\, G(\boldsymbol{\rho}, z; \boldsymbol{\rho}_0, 0), \tag{3}$$

$$G(\boldsymbol{\rho}, z; \boldsymbol{\rho}_0, 0)=\int D\eta(z)\exp\left\{i\frac{k}{2}\int_0^z dz'[(d\eta/dz')^2+V(\omega, z)+\delta\varepsilon(\omega, \eta(z'), z')]\right\}. \tag{4}$$

Here, $V(\omega, z) = \langle\varepsilon\rangle - 1$, and $D\eta(z)$ is the differential in the space of continuous paths, $\eta(z)$. The integration in (4) is taken over all possible paths $\eta(z')$, with the end conditions $\eta(0) = \boldsymbol{\rho}_0$, $\eta(z) = \boldsymbol{\rho}$. We shall introduce now a few assumptions, which are usually made in the analysis of radiointerferometric measurements.

The radiation field of the cosmic source will be assumed spatially homogeneous, and the following inequalities will be assumed to hold:

$$T \gg \Delta f^{-1} \gg f_0^{-1}; \tag{5a}$$

$$k_0 d\gamma_i \ll 1; \tag{5b}$$

$$\Delta f \ll c/(b\gamma_i), \tag{5c}$$

in which T is the coherent build-up time, Δf is the bandwidth of registered frequencies, f_0 is the median filter frequency of the receiving system, d is the antenna size, γ_i is the angular source size, and $b = |\boldsymbol{b}|$. These assumptions imply that the average over temporal fluctuations of the received signal is equivalent with the ensemble average over the realizations of the random source field (inequality (5a)); that the width of the antenna directional span is considerably larger than the angular source size (5b); that the receiving antennas may be considered pointlike; and that within the frequency bandwidth the directional antenna span may be considered constant(5c) (the lags due to differences in wave paths in the angular range are within the limits of the coherence interval, $\tau_c \sim 1/\Delta f$.)

The nature of the wave scattering depends [6, 7] on the relation between the field coherence radius R_C and the magnitude of the Fresnel zone, $R_F = \sqrt{\lambda z}$. Correspondingly, the inequality $R_C \gg R_F$ defines the domain of weak, or unsaturated fluctuations. In this case, the path of the physical ray in (4) differs only slightly from the unperturbed ray, in the absence of fluctuations $\delta\varepsilon$. The integration in (4) is then taken along the unperturbed paths, and the coherence function (CF) of the field, $\Gamma = \langle U(\boldsymbol{\rho}_1, z)U^*(\boldsymbol{\rho}_2, z)\rangle$, has the expression $\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) \sim \exp(-D_S(\boldsymbol{b}, z))$, where $D_S(\boldsymbol{b}, z)$ is the structure function of the phase fluctuations at the base $\boldsymbol{b}$.

In the other limiting case, when $R_C \ll R_F$, the field emerging from the scattering layer consists of N rays, with $N \sim (R_F/R_C)^4 \gg 1$. The characteristic size of the region of effective scattering is $R_C \sim R_F^2/R_C \gg R_F$. Since the expression of the field CVF is the same in both weak and strong regimes, we can use the well known solution [5]. Using the Kolmogorov model for the spectrum of turbulence fluctuations (r)

$$R_c=\begin{cases}1{,}78\sigma_\varepsilon^{-6/5}K^{-6/5}L_s^{-3/5}L_0^{2/5}, & R_c \gg l_0\\ 2{,}15\sigma_\varepsilon^{-1}K^{-1}l_0^{1/6}L_s^{-1/3}L_0^{1/3}, & R_c \ll l_0\end{cases}, \tag{6}$$

where $\sigma_\varepsilon^2 = 1.77r_\varepsilon^2\lambda^4\langle\delta N_e^2\rangle$, r_e is the electron classical radius, and L_S the thickness of the scattering layer. Following [8, 9], we have also introduced fluctuation parameters of the electronic plasma density: the dispersion $\langle\delta N_e^2\rangle$, the effective thickness of the inhomogeneity layer, L_S, and the external, L_0, and internal, l_0, fluctuation scales. For ISM in the galactic plane, $\langle\delta N_e^2\rangle \simeq 10^{-4}$ cm^{-6}, $L_0 \simeq 10^{20}$ cm, $L_S \simeq 10^{21}$ cm, $l_0 = 10^{10}$ cm. For IPM, according to estimates made in the vicinity of the terrestrial orbit, $\langle\delta N_e^2\rangle \simeq 10^{-2}$ cm^{-6}, $L_0 \simeq 10^{13}$ cm, $l_0 \simeq 10^6$ cm, $L_S \simeq 10^{14}$ cm. These estimates describe approximately

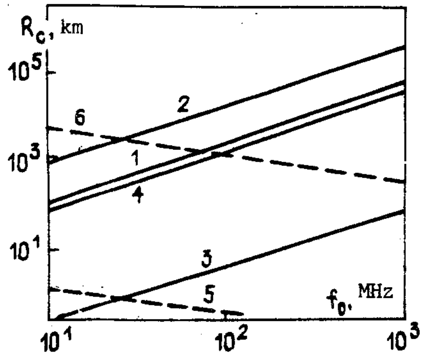

Fig. 1

the scattering conditions in IPM for observations of sources with an elongation $\varphi > 90°$. For ionosphere, we assume that the main contribution to scattering originates in the F layer, with $\langle \delta N_e{}^2 \rangle \simeq 10^5\ \mathrm{cm}^{-6}$, $L_0 \simeq 10^6$ cm, $L_S \simeq 10^7$ cm. In Fig. 1 we give the coherence radius R_c as a function of frequency, f_c, for ISM, IPM, and ionosphere, as well as the Fresnel zone $R_F(f_0)$ for typical positions in the scattering layer. The curves have been labelled as follows: 1) R_c in ISM in the galactic disk; 2) R_c in the circumpolar region of ISM; 3) R_c in the calm ionosphere; 4) R_c in IPM at $\varphi > 90°$; 5, 6) the magnitude R_F of the Fresnel zone in ionosphere and IMP, respectively. The inhomogeneity transport velocity was taken to be v_S = 50 km/sec in ISM, and v_i = 100 m/sec in ionosphere.

The coherence build-up time in a correlation receiver is restricted by the displacement time of the distorted wave front over a coherence radius. From Fig. 1 it appears clearly that the main contribution to the coherence build-up time comes from ionosphere and IPM. In the meter bandwidth, the scattering regime in ionosphere corresponds to weak, unsaturated fluctuations. By contrast, the scattering in ISM occurs in the regime of strong, saturated fluctuations, and is accompanied by stochastic multipaths.

When the averaging processes in the stochastic multipaths and unsaturated fluctuations regimes are compared, one must note the following. If the CF is measured instantaneously in the weak fluctuation regime, the field is at each point of observation a physical ray with random incidence of the wave front direction, which results in random phase differences on the base $b = \rho_1 - \rho_2$. Time averaging, equivalent to ensemble averaging over the realizations of leads to a combination of waves with different angles of incidence, resulting in the well known expression of CF for long exposures.

In the case of stochastic multipaths, even when CF is measured only once (averaged over a time $T \ll T_c$) a large number of rays with different angles of incidence are compounded. Thus, the phase structure of the field is statistically similar to the phase in the weak fluctuation regime, although the amplitude is, of course, different since the field energy is distributed over a considerably larger region. The transversal size R_S of this region is of the order of magnitude $R_S \sim z\langle \vartheta_S{}^2 \rangle^{1/2} \simeq R_F{}^2/R_c$, where $\langle \vartheta_S{}^2 \rangle^{1/2}$ is the effective width of the angular spectrum of scattered waves. By analogy with the optical speckle-interferometry [10], one may say that in the ISM stochastic multipath regime, one finds a speckle structure of the CF in one realization. Consequently, quantities like the speckle size R_c, the size R_S of the region filled with speckles, and the number N of speckles vary more slowly from one realization to another than in the case of refractive interstellar scintillation.

2. Let us examine now the applicability of the speckle-interferometry method in the saturated fluctuations regime. In this case, one makes a series of short "exposures" (build-up over a relatively short time $T < T_c$) of the CF $\Gamma(\rho_1, \rho_2)$ at a fixed projection of the base vector on the image plane. In each measurement, one determines the modulus squared $|\Gamma(\rho_1, \rho_2)|^2$ of the CF (the squares of the sine and cosine components of the interferometer output are added up), and this is averaged over the ensemble of realizations obtained during a time $T_{tot} \gg T_c$. Usually, the total averaging time T_{tot} is restricted by either the capacity of the magnetic support or the possible measurement of the base vector projection, i.e., by the field of view.

Let us examine the ensemble average of the mutual CF squared modulus:

$$K(\rho_1, \rho_2) = \langle |\Gamma(\rho_1, \rho_2)|^2 \rangle = c^2 S_A \int dq_1 \int dq_2\, A(k_0 + q_1)\, A^*(k_0 + q_2) \times$$

$$\times \int d^2\vartheta \int d^2 n B(\vartheta + n, 2) B(\vartheta - n, 2) \int d^2\eta_1 \dots \int d^2\eta_4 \times$$
$$\times \exp[i(pn + q\vartheta)[\eta_1 - \eta_2 - (\eta_3 - \eta_4)]] \times$$
$$\times \langle G(k_1, \rho_1, \eta_1, L_s) G^*(k_1, \rho_2, \eta_2, L_s) G^*(k_2, \rho_1, \eta_3, L_s) G^*(k_2, \rho_2, \eta_4, L_s)\rangle \quad (7)$$

at the exit from the ISM layer of thickness L_S. Here

$$B(n) = \int d^2\rho_0 \langle U_0(\rho' + \rho_0) U_0^*(\rho')\rangle \exp(-ikn\rho_0) \quad (8)$$

is the angular distribution of the source radiobrightness and A(k) is the squared modulus of the transfer function of the receiving device. The integration over the frequency bandwidth $\Omega_{1,2}$ can be replaced by an integration over the wavenumber $q_{1,2} = \Omega_{1,2}/c$, $p = (k_1 + k_2)/2$, $k_{1,2} = k_0 + q_{1,2}$, $q = q_1 - q_2$. If $R_C \ll R_F$, the field of meter radiowaves at the exit from the ISM layer has gaussian statistics. Consequently, the fourth moment of the field in (7) can be asymptotically approximated by a product of two correlators. The main contribution is given by only two terms:

$$\begin{aligned} &\langle G(k_1, \rho_1, \eta_1, L_s) G^*(k_1, \rho_2, \eta_2, L_s) G^*(k_2, \rho_1, \eta_3, L_s) G(k_2, \rho_2, \eta_4, L_s)\rangle \simeq \\ &\simeq \langle G(k_1, \rho_1, \eta_1, L_s) G^*(k_1, \rho_2, \eta_2, L_s)\rangle \langle G^*(k_2, \rho_1, \eta_3, L_s) \times \\ &\times G(k_2, \rho_2, \eta_4, L_s)\rangle + \langle G(k_1, \rho_1, \eta_1, L_s) G^*(k_2, \rho_1, \eta_3, L_s)\rangle \times \\ &\times \langle G^*(k_1, \rho_2, \eta_2, L_s) G(k_2, \rho_2, \eta_4, L_s)\rangle . \end{aligned} \quad (9)$$

The neglected terms are of the order $\exp(-2\langle(\delta S)^2\rangle)$, where $\langle(\delta S)^2\rangle$ is the dispersion of the scattered field phase fluctuations. In the saturated fluctuations regime, $\langle(\delta S)^2\rangle \gg 1$. Substituting the first term of the sum (9) in (7) yields the ensemble averaged squared modulus of the CF. Consider the moment

$$\Gamma_s = \langle G(k_1, \rho_1, \eta_1, L_s) G^*(k_2, \rho_1, \eta_3, L_s)\rangle . \quad (10)$$

Using the Green's function representation (4) and taking into account the statistical homogeneity of the fluctuations $\delta\varepsilon$, we are led to a simple path integral:

$$\begin{aligned} \Gamma_s = \frac{iq}{2\pi L_s} \exp\left[i\frac{q}{2L_s}\left(\rho_1 - R_0 + \frac{p}{q}\rho_0\right)^2\right] \times \\ \times \int Dv(z) \exp\left\{ -i\varkappa \int_0^{L_s} dz' \left[\frac{1}{2}(dv/dz')^2 - i\frac{p^2}{\varkappa} H_\varepsilon(v(z'), z')\right]\right\}, \end{aligned} \quad (11)$$

where $\kappa = k_1 k_2/q$, $R_0 = (\eta_1 + \eta_2)/2$, $\rho_0 = \eta_1 - \eta_3$ and the path $v(z')$ satisfies the end conditions $v(0) = \rho_0$, $v(L_S) = 0$:

$$H_\varepsilon(v(z)) = 2\int d^2\varkappa_\perp \Phi_\varepsilon(0, \varkappa_\perp)[1 - \cos(\varkappa_\perp v(z))] . \quad (12)$$

The main contribution to the integral (11) comes from paths which are a distance from each other not exceeding the field coherence radius R_C. For turbulent isotropic fluctuations of the cosmic plasma, in the meter bandwidth, $l_0 \simeq 10^{10}$ cm and $R_C \ll l_0$. In this case, the structure function of the dielectric permeability fluctuations $\delta\varepsilon$ can be taken as

$$H_\varepsilon = \alpha v^2(z) ,$$

where $\alpha = H_\varepsilon''(0)/2$, $v = |v|$. Hence,

$$\Gamma_s = B\exp\left[i\frac{q}{2L_s}\left(\rho_1 - R_0 + \frac{p}{q}\rho_0\right)^2 - i\frac{\varkappa\nu}{2}\operatorname{ctg}(\nu z)\rho_0^2\right], \quad (13)$$

where

$$B = -\frac{k_1 k_2}{4\pi^2 L_s^2}\left(\frac{\nu z}{\sin(\nu z)}\right)\exp(-q^2\gamma_0/16) , \quad (14)$$

with $\nu^2 = -2i\alpha q$, $\gamma_0 = \pi L_S \int d^2\varkappa_\perp \Phi_\varepsilon(0, \varkappa_\perp) = \langle(\delta s)^2\rangle/k_0{}^2$, and the asterisk indicating complex conjugation.

Substituting the obtained expression in the correlator (7) we find

$$K = c^2 S_A^2 \int dp \int dq\, F(p, q)\,[B_1(p, q, b) + B_2(p, q, b)]\,, \tag{15}$$

where

$$B_1(p, q, b) = \int d^2\theta \int d^2 n B(\theta + n/2) B(\theta - n/2) \times \exp[ib(pn + q\vartheta) - (p^2 + q^2/4) L_s H_\varepsilon(b)]\,; \tag{16}$$

$$B_2(p, q, b) = |\cos(\nu z)|^{-2} \exp(-q^2 \gamma_0/8) \times \int d^2\vartheta \int d^2 n B(\theta + n/2) B(\theta - n/2) \exp\Big\{i(pn + q\theta)b - 2\,\mathrm{Im}\Big[\frac{\lambda^2 n^2 L_s}{2\varkappa}(\mathrm{tg}(\nu z)/\nu z)\Big]\Big\}, \quad \lambda = (p^2/q)(1 - q^2/4p^2), \quad F(p, q) = A(p + q/2) A(p - q/2). \tag{17}$$

As can be seen from (16), $B_1(p, q, b)$ is the squared modulus of the averaged CF. This term introduces no new, other than (5c), restrictions upon the frequency bandwidth. In the expression (17) there are two parameters which determine ΔF, the coherence bandwidth in the frequency domain or, equivalently, the difference between wavenumbers, $\Delta q = 2\pi\Delta F/c$, describing a significant reduction of the cross-frequency correlation function, $B_2(p, q, L_S)$. A first characteristic scale for q appears in the exponent of $\exp(-q^2\gamma_0/8)$. For a mean square deviation of the optical path fluctuations equal to $\sqrt{\gamma_0}/2$, the difference between phase fluctuations at the frequencies ω_1 and ω_2 is $q\sqrt{\gamma_0}/2$. The quantity $q^2\gamma_0/8 = q^2\langle(\delta S)^2\rangle/k_0^2$ determines the dispersion of the differential phase difference for a frequency shift $\Delta\omega = \omega_1 - \omega_2$. Hence, the coherence bandwidth is $\Delta F_1 = f_0\langle(\delta S)^2\rangle^{-1/2}$. This expression assumes averaging over the finite time T_{tot}. Unlike self-averaging, which is due to the superposition of a large number of microrays, the averaging over the fluctuations of the optical path, γ_0, pertains only to the median ray defined by the sum of incidence directions of the set of rays corresponding to one realization. The finite value of the averaging time T_{tot}, actually restricts the maximum fluctuation scales of the dielectric permeability of the cosmic plasma to a value $L_{tot} = |v_\perp| T_{tot}$.

Adopting a Karman model for the fluctuation spectrum in ISM, one can estimate the phase fluctuation dispersion, $\langle(\delta S)^2\rangle_{tot}$. Let us introduce the time for inhomogeneity transport over l_0: $T_0 = \ell_0/v_S$, $v_S = |\ _\perp|$. Then, for $T_{tot} \ll T_0$.

$$\langle(\delta S)^2\rangle_{tot} \simeq 8{,}5\cdot 10^{-22}\lambda^2 L_s \langle(\delta N_e)^2\rangle L_0^{-2,3} \varkappa_m^{-5/3}\, (T_{tot}/T_0)^{8/3} \exp[-(T_0/T_{tot})^2] \tag{18}$$

and for $T_{tot} \gg T_0$,

$$\langle(\delta S)^2\rangle_{tot} \simeq 2\cdot 10^{-22}\lambda^2 L_s \langle(\delta N_e)^2\rangle L_0^{-2/3} (v_s T_{tot})^{5/3}\,. \tag{19}$$

Here, all scales are measured in cm, and $[v_S]$ in cm/s. Correspondingly, the coherence bandwidth is

$$\Delta F_1 \simeq 0{,}363\cdot 10^{-8} f_0^2 (T_{tot}/T_0)^{1/3} \exp((T_0/T_{tot})^2/2)\,, \quad T_{tot} \ll T_0\,.$$

$$\Delta F_1 \simeq 0{,}5\cdot 10^{-9} f_0^2 (T_{tot}/T_0)^{-5/6}, \quad T_{tot} \gg T_0\,. \tag{20}$$

In Fig. 2 we show the limiting frequency bandwidth ΔF_1, for f_0 = 30, 100 and 300 MHz as function of the averaging time, assuming that v_S = 50 km/sec, $l_0 \sim 10^{10}$ cm and $T_0 \simeq 30$ min. As it can be seen, the fluctuations of the optical path do not impose any significiant limitations upon the bandwidth of registered frequencies.

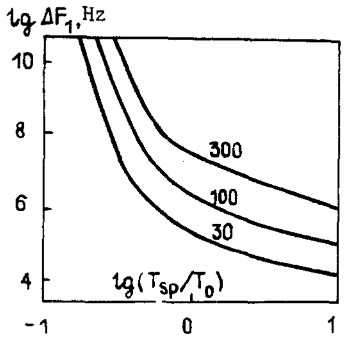

Fig. 2

A second characteristic scale for the frequency correlation in (17) is provided by the term

$$|\cos(\nu L_s)|^{-2} \sim \exp(-\sqrt{qL_s\langle\vartheta_s^2\rangle}), \tag{21}$$

in which $\langle\vartheta_s^2\rangle^{1/2} \sim (k_0R_c)^{-1}$ is the effective width of the angular spectrum of the scattered field. This term is practically independent of the averaging time T_{tot}, and is related to stochastic multipaths in ISM. It follows from (21) that the effective width in the space of wavenumbers is $q_{eff} \sim [L_s\langle\vartheta_s^2\rangle]^{-1}$, and the coherence bandwidth is

$$\Delta f \leqslant \Delta F_2 = cq_{eff}/2\pi \sim c/L_s\langle\vartheta_s^2\rangle. \tag{22}$$

This expression can be interpreted as showing that the maximum delay time τ_s within the limits of the scattering cone is given in terms of its width $\langle\vartheta_s^2\rangle$ by the relation: $\tau_s \sim L_s\langle\vartheta_s^2\rangle/2c$.

For a coherent superposition of the scattered waves it is necessary that the weight of the spatial wave packet be situated within the limits of the coherence train, i.e., that $\tau_s \lesssim \Delta F_2^{-1}$, which implies the inequality (22). The coherence width (22) determines the effective registration band within the limits of which the contribution of the expression (17) in modulus square has a significant value. In Fig. 3 we show the limiting frequency bandwidth F_2 as a function of the median filter frequency (line 1). It is obvious that even small registration bands impose restrictions upon the sensitivity of the receiving system.

Let us assume in (15) an infinitely narrow registration band: $q = 0$, $p = k_0$. In the unit frequency band,

$$K = S_A^2 \int d^2\vartheta \int d^2 n B(\vartheta + n/2)B(\vartheta - n/2) \times \exp(ik_0 nb)[\exp[-D_s(L_s, b)] + \exp(-(4/3)n^2/n_s^2)], \tag{23}$$

where $n_s = (kL_s\langle\vartheta_s^2\rangle^{1/2})^{-1} = R_c/L_s$ is the angular dimension of the coherence radius. The first term in (23) is the product between the modulus squared CF of the source field $\Gamma_i(\upsilon)$ and the modulus squared of the transfer function of the medium, $\exp(-D_s(L_s, \upsilon))$. This term ensures the suppression of all spatial frequencies b/λ, which exceed the spatial coherence frequency, R_c/λ. The second term in (23), when $kn_sb \sim R_cb/(\lambda L_s) \gg 1$ and if the angular dimension of the source is $\gamma_i > n_s$, is of the order $n_sS_i^2/\gamma_i$, where S_i is the source flux density. Therefore, for a wide registration band, $\Delta f \gg \Delta F_2$, the average modulus squared of the CF at the exit from the ISM layer is equivalent with the ensemble average of the CF. The scattering in ISM imposes a fundamental limit upon the angular resolution of a synthetic aperture system (SAS), equal to the scattering angle $\langle\vartheta_s^2\rangle^{1/2}$ of the ISM. It follows hence that an increase in the interferometers bases inside a SAS, above the coherence radius R_c in ISM, is not advisable, as it does not improve the angular resolution.

We examine now the effect of stochastic multipaths on the scintillation of sources in ISM. From the viewpoint of the weak scattering theory, we would conclude that the angular resolution of the scintillation method is of the order of the angular size of the Fresnel zone, $\vartheta_F \sim 1/kR_F \sim \sqrt{\lambda/L_s}$, i.e., it deteriorates as the wavelength increases. However, this

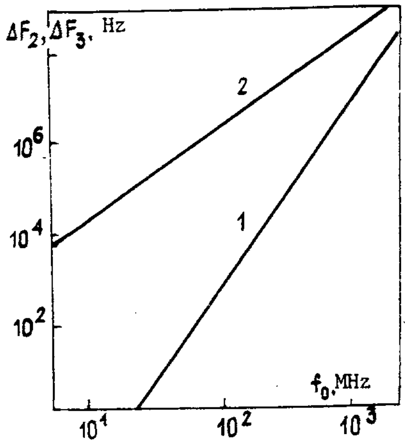

Fig. 3

conclusion is only true within the limits of weak scattering, when the characteristic transverse coherence scale is the Fresnel zone, R_F. For stochastic multipaths, in the saturated fluctuations regime in ISM, the coherence scale is R_c which actually decreases as the wavelength increases, like λ^{-1}. In this case, the scintillation method has an angular resolution of the order of the n_s-angular dimension of the coherence radius, and it may theoretically be of the order of 10^{-6} angular seconds. However, such a resolution is only possible if the registration bandwidth Δf does not exceed ΔF_2, and this drastically restricts the system sensitivity.

It can be shown that the scintillation index

$$m^2 = (K(b=0) - |\langle\Gamma(0)\rangle|^2)/|\langle\Gamma(0)\rangle|^2 \tag{24}$$

of a pointlike source is, in two limiting cases, respectively, equal to

$$m^2 \simeq \begin{cases} 1, & \Delta f \ll \Delta F_2 \\ (\Delta F_2/\Delta f) \ll 1, & \Delta f \gg \Delta F_2 \end{cases}. \tag{25}$$

Let us consider the expression for the fluctuation spectrum of the scintillation index $V_m(\nu)$ for a pointlike source (ν is the frequency).

If $\Delta f \ll \Delta F_2$ and $\nu^2 \ll \nu_s{}^2 f_0/\Delta f$,

$$V_m(\nu) \simeq \exp(-\nu^2/\nu_s^2), \tag{26}$$

where $\nu_s = T_s{}^{-1} = v_s/R_c$, and T_s is the characteristic scintillation period, corresponding to the mixing time for the diffraction pattern of the scattered field on a scale of order R_c. For f_0 = 300 MHz, T_s = 100 s.

For $\Delta f \gg \Delta F_2$ and $\nu^2 \ll \nu_s{}^2 f_0/\Delta f$,

$$V_m(\nu) \simeq 2(\Delta F_2/\Delta f)\exp(-\nu^2/\nu_s^2). \tag{27}$$

As can be seen from (26) and (27), the fluctuation spectrum of the scintillation index in ISM has a width of the order of ν_s (in the plane of the galactic disk, $\nu_s \simeq 10^{-2}$ Hz, f_0 = 300 MHz.) If the registered frequencies band Δf does not exceed ΔF_2 within the scattering cone, the waves add up coherently and interfere with a characteristic period T_s. If not, the fluctuations diminish due to the fact that the waves with different frequencies are not correlated and differ in magnitude by a value larger than ΔF_2.

For the elimination of phase distortions in SAS, due to ionosphere and IPM at large elongations, where there exists an unsaturated fluctuation regime, the philosophy of speckle-interferometry is totally analogous to the one adopted in Optics [10].

Let us consider the ensemble average (during a time T_{tot}) of the coherence modulus squared. In the weak fluctuations approximation, when in the ionosphere $R_c \gg R_F$ and $R_c \gg \alpha$, where α is the linear dimension of the telescopic aperture, we can write for $K(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, L_s)$:

$$K(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, L_s) = K(\boldsymbol{b}, L_s) = \int d\boldsymbol{p} \int d\boldsymbol{q} F(\boldsymbol{p}, \boldsymbol{q}) \times$$
$$\times \int d^2\boldsymbol{\vartheta} \int d^2\boldsymbol{n} B(\boldsymbol{\theta} + \boldsymbol{n}/2) B(\boldsymbol{\theta} - \boldsymbol{n}/2) \exp[i\boldsymbol{b}(\boldsymbol{p}\boldsymbol{n} + \boldsymbol{q}\boldsymbol{\vartheta}) - 2(\boldsymbol{q}^2/\boldsymbol{p}^2) D_s(L_s, \boldsymbol{b})] \tag{28}$$

The subsequent consideration of scattering in ISM for leads to replacing of the initial radiobrightness distribution in (28) by the effective brightness distribution defined by the integrand terms in (16). Evidently, if

$$(q^2/p^2) D_s(L_s, \boldsymbol{b}) \ll 1 \tag{29}$$

one practically excludes from the average modulus squared of CF in (28) any phase distortions due to ionosphere, and all spatial frequencies up to the diffraction limit b/λ pass without attenuation.

The inequality (29) represents a restriction on the limiting registration band due to ionosphere. Using the relation between the phase structure function of CF, $D_s(\boldsymbol{b}, L_s)$, and the effective width of the angular spectrum of scattered waves $\langle\vartheta_s{}^2(\boldsymbol{b})\rangle \sim D_s(\boldsymbol{b}, L_s)/(kb)^2 \sim C_\varepsilon{}^2 L_s b^{-1/3}$, with $b \gg l_0$, one can write

$$\Delta f \leqslant \Delta F_3 = \frac{\lambda}{b} f_0 \langle \vartheta_s^2(b) \rangle^{-1/2} \sim f_0^2 \, b^{-5/6}. \tag{30}$$

In Fig. 3 we reproduce estimates of the limiting frequency band ΔF_3 (line 2), due to random refraction in the ionosphere, using (30). For smaller base lengths, the random gradients of the wave fronts in IPM do not, in general, imply restrictions upon the limiting frequency band. As the estimates given in Fig. 3 show, the restrictions on Δf due to ionosphere are not significant in the meter wavelength range.

To summarize, we can draw the following conclusions.

1) If the registration band of the receiving system does not exceed the coherence bandwidth, the scattering of the radioemission of cosmic sources in the meter bandwidth in ISM imposes a limit on the angular resolution of SAS, equal to the effective scattering angle in ISM.

2) Scattering in the ionosphere leads to a limitation of the coherence build-up time to about 100 s. There are no essential limitations of the registered frequencies bandwidth due to scattering in ionosphere and IPM.

3) To eliminate phase distortions due to ionosphere and IPM, one can use the methods of speckle-maximization and self-calibration. The resolution of SAS can then be brought up to the diffraction limit, provided it does not exceed $\langle \vartheta_S{}^2 \rangle^{1/2}$ in ISM.

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