Some Remarks on the Goos-Hänchen and Imbert Effects.

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Summary. — An explanation of the Goos-Hänchen and Imbert effects from the viewpoint of geometrical optics is presented in this paper.

In a recent letter TROUP *et al.* (1) have expressed the opinion that the all the subtleties of the Goos-Hänchen effect (2,3) may be explained on purely classical grounds, in contrast to DE BROGLIE and VIGIER (4) who treat this effects as conflicting with classical predictions (the lack of an intermediale shift). We agree completly with arguments in (1) and we present below a simple explanation of both Goos-Hänchen's (2,3,5-10) (G.H.) and Imbert's (11-13) (I) effects in the spirit of geometrical optics. Finally the approach adopted in this paper (eq. (42)) confirms the lack of intermediate shift (1,4).

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(1) C. P. TROUP, L. A. FRANCEY, R. G. TURNER and A. TIRKEL: *Phys. Rev. Lett.*, **28**, 1540 (1972).
(2) F. GOOS and H. HÄNCHEN: *Ann. der Phys.*, **1**, 333 (1947).
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In fact, there exist many other classical explanations of both phenomena particularly the first one ([5-12,14]), but some questions remain open. It seems that (the sources of) the apparent discrepancies with classical predictions, recently much analysed, stem from the inadequate « language » of plane waves. It must be understood that plane waves obtained as a limiting case of spherical waves (when a source or set of sources is removed to infinity) can sometimes have very different total properties than the plane waves obtained as a direct solution of the Helmholtz equation, although local properties of both can be similar.

The present explanation confirms formulae published to date, particularly those which follow from the treatment of a diffraction problem ([5,6,8]). The method which we are using relates only to the general conclusions of optics, reducing the problem to the displacements of the image of the source ([15,16]).

As is known ([17]) the far-zone field $\boldsymbol{g}_1 \exp[ikr]/r$ determines uniquely the whole electromagnetic field (outside a sphere inside which the sources are contained), since

$$\boldsymbol{E}(r, \varphi, \theta) = \sum_{p=1}^{\infty} \boldsymbol{g}_p(\varphi, \theta) \frac{\exp[ikr]}{r^p}, \tag{1}$$

and $\boldsymbol{g}_{p+1}$ is calculated by differentation of $\boldsymbol{g}_p$ with respect to the arguments. From (1) it immediately follows that the radiation pattern $\boldsymbol{g}_1$ is

$$\boldsymbol{g}_1 = \lim_{r\to\infty} (r \exp[-ikr]\boldsymbol{E}). \tag{2}$$

Let us translate the co-ordinate system along the vector $\boldsymbol{a}$. In the new R, Φ, Θ system, the radiation pattern is

$$\boldsymbol{G}_1(\Phi, \Theta) = \lim_{R\to\infty} (R \exp[-ikR]\boldsymbol{E}) = \tag{3}$$

$$= \lim_{R\to\infty}\left[R \exp[ikR] \sum_{p=1}^{\infty} \boldsymbol{g}_p \frac{\exp[ik(R^2 - 2\boldsymbol{R}\cdot\boldsymbol{a} + a^2)^{\frac{1}{2}}]}{(R^2 - 2\boldsymbol{R}\cdot\boldsymbol{a} + a^2)^{p/2}}\right] = \exp[-ik\boldsymbol{a}\cdot\boldsymbol{e}_R]\boldsymbol{g}_1(\phi, \Theta),$$

where $e_R = \boldsymbol{R}/R$.

Thus, during the translation, the magnitude of the radiation pattern $|G_1|$ remains unchanged and only its argument ($\arg G_1$) changes.

This last observation (3) can be used to determine the position of a source, if the radiation pattern $\boldsymbol{G}_1$ in some co-ordinate system and the radiation pattern of the source $\boldsymbol{g}_1$ are known.

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([15]) J. RICARD: *Compt. Rend.*, **274**, 384 (1972).
([16]) O. COSTA DE BEAUREGARD and C. IMBERT: *Phys. Rev. Lett.*, **28**, 1211 (1972).
([17]) C. H. WILCOX: *Comm. Pure Appl. Math.*, **9**, 115 (1956).

But, if a source is complex, $\boldsymbol{g}_1$ can also be of the form

$$\boldsymbol{g}_1 = |g_\varphi| \exp[ih_\varphi]\boldsymbol{e}_\varphi + |g_\theta| \exp[ih_\theta]\boldsymbol{e}_\theta , \tag{4}$$

where h_i are real functions of the angles φ, θ. The question then arises: where is the source in reality? It will be on a normal to the constant-phase surfaces of the wave, in the centre of curvature, if this is determined uniquely ([18])—which, however, holds only for an elementary point source. If the source is extended, as a rule the principal curvatures of the constant-phase surface can be different, and we are able to determine only the normal to a constant-phase surface.

Indeed, if the x-axis is in the direction of the main beam, in its vicinity we can write for h_φ or h_θ

$$h(\varphi, \theta) = \sum_{i,j=0}^{\infty} h_{ij}\varphi^i \left(\theta - \frac{\pi}{2}\right)^j . \tag{5}$$

Now, translating the co-ordinate system along the vector $\boldsymbol{a} = (a_x, h_{1,0}/k, -h_{0,1}/k)$, by (3), in the new frame x', y', z', we obtain the phase of radiation pattern

$$h' = -k(\boldsymbol{a}\cdot\boldsymbol{e}_{r'}) + h = ka_x + \sum_{i,j=0}^{\infty} h'_{ij}\varphi^{i+2}\left(\theta - \frac{\pi}{2}\right)^{j+2} , \tag{6}$$

where a_x is arbitrary.

If $h'_{1,0} = h'_{0,1}$ (which means that both centers of curvature of the constant-phase surface reduce to the one) then by the next translation along the x'-axis we can at last reduce the phase to the form

$$h'' = h''_0 + \sum_{i,j=0}^{\infty} h''_{ij}\varphi^{i+3}\left(\theta - \frac{\pi}{2}\right)^{j+3} , \tag{7}$$

but this is a rather exceptional case. It may also be impossible to reduce both h_φ and h_θ to the form (7) simultaneously.

Thus, observing the radiation pattern phase without linear terms in the direction of the main beam, one can be sure that the source or apparent source lies in this direction, although its position cannot be determined uniquely. Nevertheless an observer placed in the far zone will register the waves as incident from this direction.

We will define the co-ordinate system for which the phase of radiation pattern h is of form (6) as natural, and its distinguished direction as the phase centre line. The explanation of G.H. and I effects leads to the determination of a natural frame for the reflected beam.

([18]) M. Born and E. Wolf: *Principles of Optics* (Oxford, 1965).

At first we will examine the two-dimensional case. In this situation, since the centre of curvature of the flat curve is determined uniquely, it is also possible to find exactly the position of the apparent source.

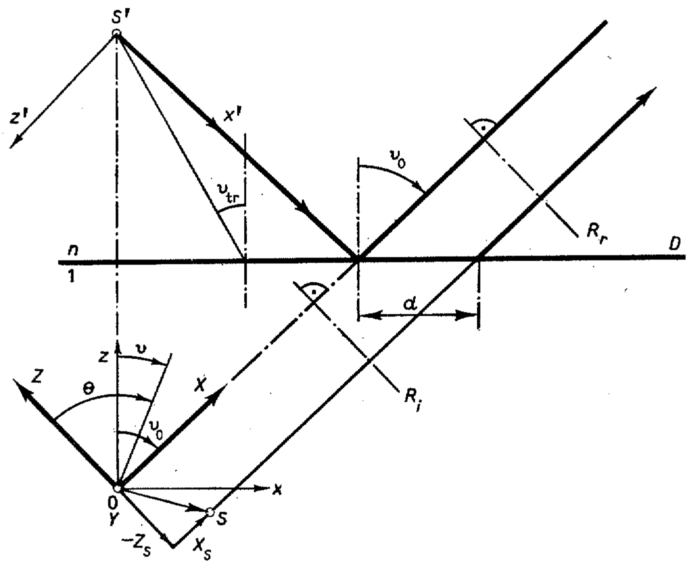

Fig. 1. – Propagation of the reflected beam.

Let suppose that the source is at point S' (Fig. 1) and it radiates an electromagnetic wave such that the angle between the central ray and the normal to plane D (which separates the two media) is θ_0. Moreover all the rays of the whole beam fulfil the conditions for total reflection $\left(\theta > \theta_{\text{tot refl}} = \arcsin(1/n)\right)$. If the source at S' has a natural radiation pattern g, and we assume that g is a slowly varying function in the vicinity of θ_0, all incident waves can be locally treated as plane waves. Then the reflection radiation will have a pattern (in the vicinity of θ_0)

$$G(\theta) = \Gamma(\theta)\, g(\theta)\,, \tag{8}$$

where Γ is a reflection coefficient. In the case of total reflection [5,13]

$$\Gamma_{\perp,\parallel} = \exp\left[-i\Xi_{\perp,\parallel}\right], \tag{9}$$

where

$$\left\{\begin{aligned} \Xi_\perp &= \Psi_\perp = 2\,\mathrm{arc\,tg}\,\frac{(\sin^2\theta - 1/n^2)^{\frac{1}{2}}}{\cos\theta}\,,\\ \Xi_\parallel &= \Psi_\parallel + \pi = 2\,\mathrm{arc\,tg}\,\frac{n^2(\sin^2\theta - 1/n^2)^{\frac{1}{2}}}{\cos\theta} + \pi\,. \end{aligned}\right. \tag{10}$$

(The additional term π in (10) follows from the continuity of Ξ at the point $\theta = \arcsin(1/n)$, when $\Gamma_\perp = 1$, whereas $\Gamma_\parallel = -1$.)

Now we can ask at what point the apparent source with initial phase $-\delta$ and radiation pattern $\boldsymbol{g}$ must be placed in order to give the radiation pattern $\boldsymbol{G}$ (with respect to frame xz). From (3) we want to have

$$\delta + k_1(\boldsymbol{a}\cdot\boldsymbol{e}_R) \equiv \Xi\,, \tag{11}$$

where in frame XYZ (Fig. 1)

$$\left|\begin{aligned} &\boldsymbol{e}_R = \boldsymbol{e}_X \sin\Theta + \boldsymbol{e}_Z \cos\Theta\,,\\ &\boldsymbol{a} = \boldsymbol{e}_X X_s + \boldsymbol{e}_Z Z_s\,,\\ &\Theta = \pi/2 + (\theta - \theta_0)\,,\\ &k_1 = 2\pi/\lambda_1 = 2\pi n/\lambda_0\,. \end{aligned}\right. \tag{12}$$

The identity (11) may be satisfied only approximately. Expanding in a Taylor series on both sides in (11) we get

$$\begin{aligned}[\delta + k_1 x_s - \Xi(\theta_0)] + [-k_1 Z_s - \Psi'(\theta_0)][\theta - \theta_0] &+ \\ + [-k_1 x_s - \Psi''(\theta_0)](\theta - \theta_0)^2/2 &\equiv 0\,,\end{aligned} \tag{13}$$

and the co-ordinates of our apparent source S are

$$X_s = -\frac{1}{k_1}\Psi''(\theta_0)\,, \qquad Z_s = -\frac{1}{k_1}\Psi'(\theta_0) \tag{14}$$

for both $\perp$ and $\parallel$ polarization. The initial phase shift is

$$\delta = -[\Xi(\theta_0) + \Psi''(\theta_0)]\,. \tag{15}$$

Employing (9) and (10) we immediately get in the vicinity of $\theta_0 \simeq \theta_{\text{tot refl}}$

$$Z_s = \frac{-\lambda_1[(1 - 1/n^2)\sin\theta_0 N^2]}{\pi[\cos^2\theta_0 + N^4(\sin^2\theta_0 - 1/n^2)][\sin^2\theta_0 - 1/n^2]^{\frac{1}{2}}} \simeq \frac{-\lambda_1}{\pi n}\,\frac{N^2}{(\sin^2\theta_0 - 1/n^2)^{\frac{1}{2}}}\,, \tag{16}$$

where $N = n$ for polarization $\parallel$,

$= 1$ for polarization $\perp$.

Also

$$X_s = \frac{\lambda_1}{\pi}\,\frac{(1 - 1/n^2)^{\frac{1}{2}}}{n}\,\frac{N^2}{(\sin^2\theta_0 - 1/n^2)^{\frac{3}{2}}}\,. \tag{17}$$

The shifts $d_\perp$, $d_\parallel$ along the surface separating the two media are then

$$d = \frac{-Z_s}{\cos\theta_0} \simeq \frac{\lambda_1}{\pi n}(1-1/n^2)^{-\frac{1}{2}} N^2(\sin^2\theta_0 - 1/n^2)^{-\frac{1}{2}}\,, \tag{18}$$

which is in good agreement with previous work [5-12,14]. It is to be noted that X_s, Z_s, d for fixed θ_0 do not depend on R, thus preserving their values as $R\to\infty$. It is worth-while to emphasize that the whole argument is correct only if $g_1 \exp[ikR]/R$ approximates the total field correctly, *i.e.* if R is sufficiently great and the amplitude distribution of the beam varies slowly, which restricts the angle of incidence to the interval $\pi/2 \gg \theta > \theta_{\text{tot refl}} + \varepsilon$, where ε is half of the beam width. Outside this region the amplitude distribution of the reflected beam in a separating plane must be taken into account [5-12,14].

Thus after a reflection, the constant-phase surfaces are distorted in such a manner that when observed in the far zone the waves appear to come from the point S instead of the point O, which is an image of the real source S' (compare the saddle-point method [5,19]).

The above-mentioned representation of the constant-phase surfaces by the spheres suggests yet another possibility—that of finding the co-ordinates of an apparent source as a centre of curvature of the constant-phase surfaces at the point θ_0.

The family of constant-phase surfaces of a reflected beam is given by

$$k_1 R - \Xi = C \qquad \text{or} \qquad R = (1/k_1)\,[\Xi(\theta) + C]\,, \tag{19}$$

where C is constant. Performing a calculation of the centre of curvature at the point θ_0, we get

$$X_C = \left.\frac{R(R'^2 - RR'')}{R^2 + 2R'^2 - RR''}\right|_{\theta_0} \xrightarrow[C\to\infty]{} -R''(\theta_0) = -\frac{1}{k_1}\Psi''(\theta_0)\,, \tag{20}$$

$$Z_C = \left.\frac{-R'(R^2 + R'^2)}{R^2 + 2R'^2 - RR''}\right|_{\theta_0} \xrightarrow[C\to\infty]{} -R'(\theta_0) = -\frac{1}{k'}\Psi'(\theta_0)\,, \tag{21}$$

where R', R'' are the first and second derivatives of (19) with respect to θ.

Thus, in the limit, when $R\to\infty$, *i.e.* for a very distant source S', we reach the same result as previously.

Since so far our explanation of the G.H. effect has been based on the two-dimensional approach, we have not found an explanation for the Imbert effect.

Now we will consider the three-dimensional case.

Let the incident waves in the plane R_i (Fig. 1) and the reflected wave in

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(19) J. PICHT: *Ann. der Phys.*, **3**, 433 (1929).

the plane R_r be $\boldsymbol{E}_i$ and $\boldsymbol{E}_r$, respectively, with polar components

$$\boldsymbol{E}_i = V_\Phi \boldsymbol{e}_\Phi + V_\Theta \boldsymbol{e}_\Theta\,, \qquad \boldsymbol{E}_r = W_\Phi \boldsymbol{e}_\Phi + W_\Theta \boldsymbol{e}_\Theta\,, \tag{22}$$

where $\boldsymbol{e}_R$, $\boldsymbol{e}_\Phi$, $\boldsymbol{e}_\Theta$ are unit vectors in polar co-ordinates connected with the frame XYZ. The unit vector $\boldsymbol{e}_\Phi$ remains tangential to the surface of separation only in the plane $Y = 0$. On the other hand, the vector $\boldsymbol{e}_\varphi$, connected with the frame xyz, is always tangent to D. Changing the co-ordinate system, multiplying by Γ and returning to the previous one, we have

$$W = SV\,, \qquad S = U^{-1}\Gamma U\,, \tag{23}$$

where

$$W = \begin{bmatrix} W_\Phi \\ W_\Theta \end{bmatrix}, \quad V = \begin{bmatrix} V_\Phi \\ V_\Theta \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \Gamma_\perp & 0 \\ 0 & \Gamma_\parallel \end{bmatrix}. \tag{24}$$

The matrix U is a matrix of the rotation of the frame along the Y-axis and its elements are the scalar products of the unit vectors

$$U = \begin{bmatrix} e_\varphi \cdot e_\Phi & e_\varphi \cdot e_\Theta \\ e_\theta \cdot e_\Phi & e_\theta \cdot e_\Theta \end{bmatrix}. \tag{25}$$

(In (23) we neglect the term $\exp[ikl]$, where l is the distance between R_i and R_r.)

As before, we assume that the waves are locally plane, which is equivalent to neglecting the slow variation of the radiation pattern. The matrix U is orthogonal ($U^{-1} = U^T$). In the case of total reflection the matrix Γ is unitary ($\Gamma^* = \Gamma^{-1}$). The scattering matrix S will then also be unitary, leading to the conservation of the energy current

$$|W_\Phi|^2 + |W_\Theta|^2 = |V_\Phi|^2 + |V_\Theta|^2\,. \tag{26}$$

Since U, as the matrix of the rotation of the frame, can be always written in the form

$$U = \begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix}, \tag{27}$$

S becomes

$$S = \begin{bmatrix} \Gamma_\perp - (\Gamma_\perp - \Gamma_\parallel)\sin^2\Omega & \frac{1}{2}(\Gamma_\perp - \Gamma_\parallel)\sin 2\Omega \\ \frac{1}{2}(\Gamma_\perp - \Gamma_\parallel)\sin 2\Omega & \Gamma_\parallel + (\Gamma_\perp - \Gamma_\parallel)\sin^2\Omega \end{bmatrix}, \tag{28}$$

where Ω is an angle between φ and Φ at any point on the separation plane and

$$\cos\Omega = (\boldsymbol{e}_\varphi \cdot \boldsymbol{e}_\Phi) . \tag{29}$$

Let us find the location of the phase-centre line for the field which in the plane perpendicular to the X-axis has a distribution

$$E = \exp\left[-i[\Xi(\theta) + 2\alpha\Omega(\varphi, \theta)]\right] . \tag{30}$$

Ω is given by (29) and $\alpha = +1, -1$, or 0.

We will use relation (11), writing

$$k_1(\boldsymbol{a}\cdot\boldsymbol{e}_R) = \Xi + 2\alpha\Omega , \tag{31}$$

and compare the linear terms in a series expansion of both sides in (31). We get

$$(k_1 Y \sin\theta_0 - 2\alpha\Omega'_\varphi)|_{0,\theta_0} = 0 , \qquad (k_1 Z + 2\alpha\Omega'_\theta + \Psi'_\theta)|_{0,\theta_0} = 0 , \tag{32}$$

where $\boldsymbol{a} = (X, Y, Z)$.

Since $\Omega'_\varphi|_{0,\theta_0} = \cos\theta_0$, $\Omega'_\theta|_{0,\theta_0} = 0$, the co-ordinates of the phase-centre line are

$$Y = \frac{2\alpha}{k_1}(\operatorname{tg}\theta_0)^{-1} , \tag{33}$$

$$Z = -\frac{1}{k_1}\Psi'(\theta_0) , \tag{34}$$

giving the longitudinal and lateral (transverse) displacements. The remaining component X cannot be determined, since the phase pattern in (30) may have different principal centres of curvature and is not reducible to a single point source but only to a set of sources for which a phase-centre line was found above.

This means of course, that any co-ordinate system with an origin on the line (33)-(34) will be natural for the distribution (30).

Returning to the matrix S, it is seen that, if the incident wave has a perpendicular polarization $\perp$, $\boldsymbol{E}_i = \boldsymbol{e}_\Phi$, by (28), the reflected wave $\boldsymbol{E}_r$, in the reference plane $\boldsymbol{R}_r$, will have both components

$$\begin{aligned}\boldsymbol{E}_{r(\perp)} &= [\Gamma_\perp - (\Gamma_\perp - \Gamma_\parallel)\sin^2\Omega]\,\boldsymbol{e}_\Phi + [\tfrac{1}{2}(\Gamma_\perp - \Gamma_\parallel)\sin 2\Omega]\,\boldsymbol{e}_\Theta = \\ &= \tfrac{1}{2}\left(\exp[-i\Psi_\perp] + \exp[-i\Psi_\parallel]\right)\boldsymbol{e}_\Phi + \tfrac{1}{4}[\exp[-i(\Psi_\perp + 2\Omega)] + \\ &\quad + \exp[-i(\Psi_\parallel + 2\Omega)]]\,(\boldsymbol{e}_\Phi + i\boldsymbol{e}_\Theta) + \\ &\qquad + \tfrac{1}{4}[\exp[-i(\Psi_\perp - 2\Omega)] + \exp[-i(\Psi_\parallel - 2\Omega)]](\boldsymbol{e}_\Phi - i\boldsymbol{e}_\Theta) ,\end{aligned} \tag{35}$$

which, as shown in the previous considerations, is equivalent to six sources placed at different points in the reference plane $\boldsymbol{R}_r$, as presented in Fig. 2.

(In this picture we have adopted the designation that the vector shifted in phase $\pi/2$ is removed from the origin of its local co-ordinate systems.)

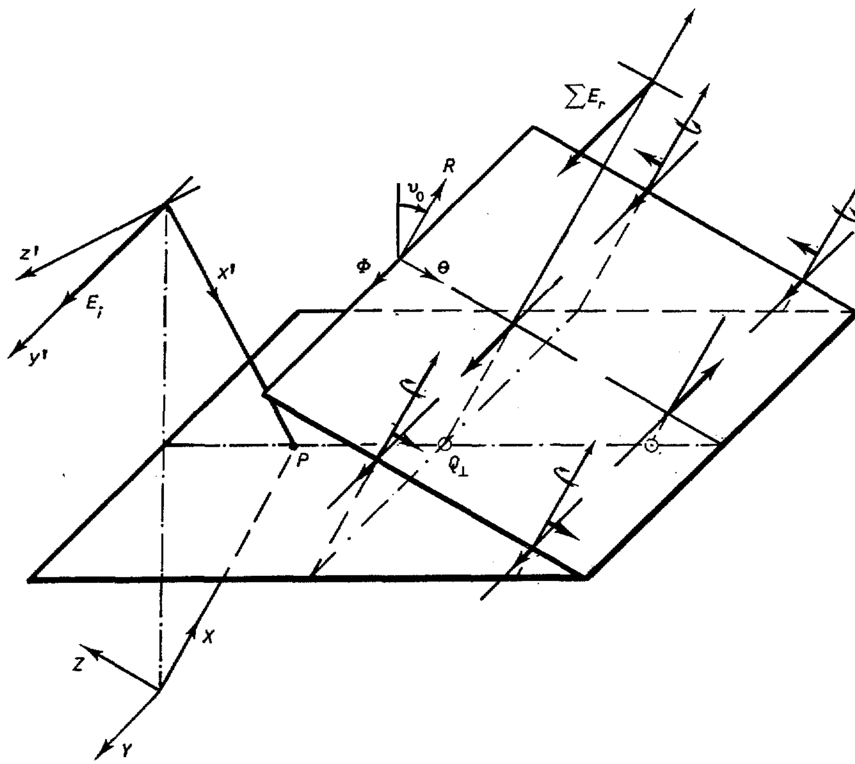

Fig. 2. – The structure of the reflected beam if the incident wave has polarization $E_\perp = 1\cdot e_\Phi$.

But, on the other hand, if the beam is sufficently narrow, $\Omega \max \ll 1$, by (28) we have

$$E_{r(\perp)} \simeq \Gamma_\perp e_\Phi\,, \tag{36}$$

as can also be observed in Fig. 2. Contributions from the different sources cancel each other and only the field denoted as $\sum E_r$ remains and will be seen as originating from the point $Q_\perp$.

Similary, if the incident wave is $E_i = |-i| e_\Theta$ (we multiply by $(-i)$ for the purposes of further derivation), the reflected wave is

$$\begin{aligned} E_{r(\parallel)} = \frac{i}{2}\left(\exp[-i\Psi_\parallel] - \exp[-i\Psi_\perp]\right) e_\Theta + \tfrac{1}{4}[\exp[-2(\Psi_\perp + 2\Omega)] + \\ + \exp[-i(\Psi_\parallel + 2\Omega)]](e_\Phi + ie_\Theta) + \\ + \tfrac{1}{4}[\exp[-i(\Psi_\perp - 2\Omega)] + \exp[-i(\Psi_\parallel - 2\Omega)]](-e_\Phi + ie_\Theta)\,. \end{aligned} \tag{37}$$

Also, by (28)

$$E_{r(\parallel)} = i\Gamma_{\parallel}\, e_{\Theta} \qquad \text{if} \qquad \Omega_{\max} \ll 1\,, \tag{38}$$

symbolically presented in Fig. 3.

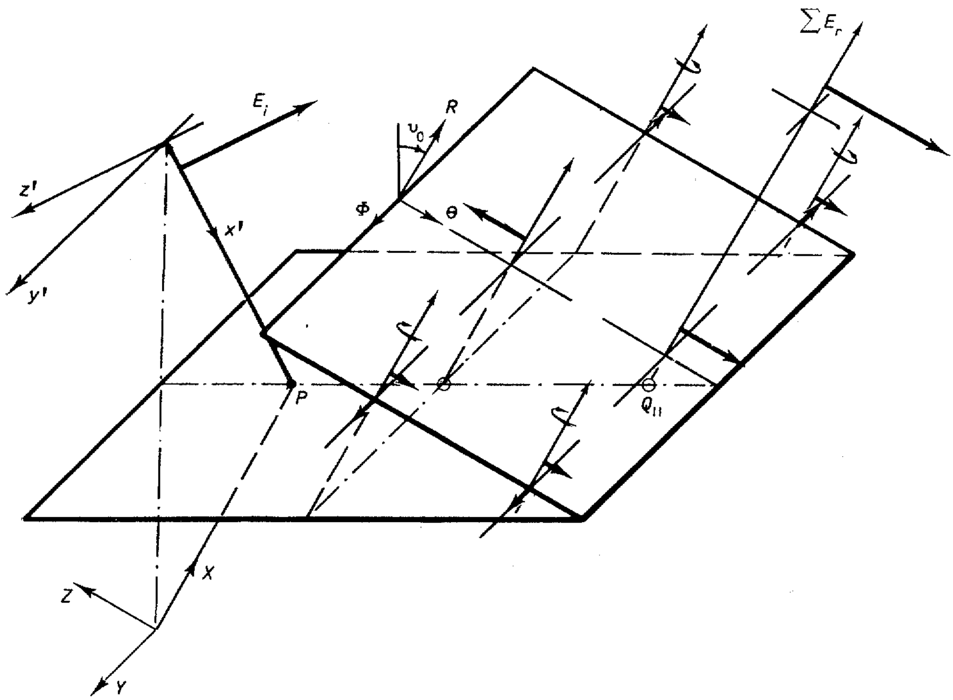

Fig. 3. – The structure of the reflected beam if the incident wave has polarization $E_{\parallel} = (-i)\, e_{\Theta}$.

Assuming an incident wave with circular polarization

$$E_i = e_{\Phi} - i e_{\Theta}\,,$$

we get

$$E_{r(c)} = \begin{bmatrix} \frac{1}{2}(\Gamma_{\perp} - \Gamma_{\parallel}) \exp[-2i\Omega] + \frac{1}{2}(\Gamma_{\perp} + \Gamma_{\parallel}) \\ \frac{i}{2}(\Gamma_{\perp} - \Gamma_{\parallel}) \exp[-2i\Omega] - \frac{i}{2}(\Gamma_{\perp} + \Gamma_{\parallel}) \end{bmatrix} = \tag{39}$$

$$= \begin{bmatrix} \frac{1}{2}(\exp[-i\Psi_{\perp}] + \exp[-i\Psi_{\parallel}]) \exp[-2i\Omega] + \frac{1}{2}(\exp[-i\Psi_{\perp}] - \exp[-i\Psi_{\parallel}]) \\ \frac{1}{2}(\exp[-i\Psi_{\perp}] + \exp[-i\Psi_{\parallel}]) \exp[-2i\Omega] - \frac{i}{2}(\exp[-i\Psi_{\perp}] - \exp[-i\Psi_{\parallel}]) \end{bmatrix}.$$

But near the critical angle $\Psi_\perp \to 0$ and $\Psi_\parallel \to 0$, so that the second terms in (39) are small. Then, for $\Psi_\perp$, $\Psi_\parallel \ll 1$

$$E_{r(c)} \simeq \exp\left[-i[\tfrac{1}{2}(\Psi_\perp + \Psi_\parallel) + 2\Omega]\right]\begin{bmatrix}1\\ i\end{bmatrix}. \tag{40}$$

This of course expresses a circularly polarized wave, but with an opposite helicity to that of the incident wave. By (30) and (34) it is shifted laterally as well as longitudinally. The last shift is naturally a meam value of $d_\parallel$ and $d_\perp$, as is depicted in Fig. 4. The result is identical to that obtained by « adding » Fig. 2 and 3.

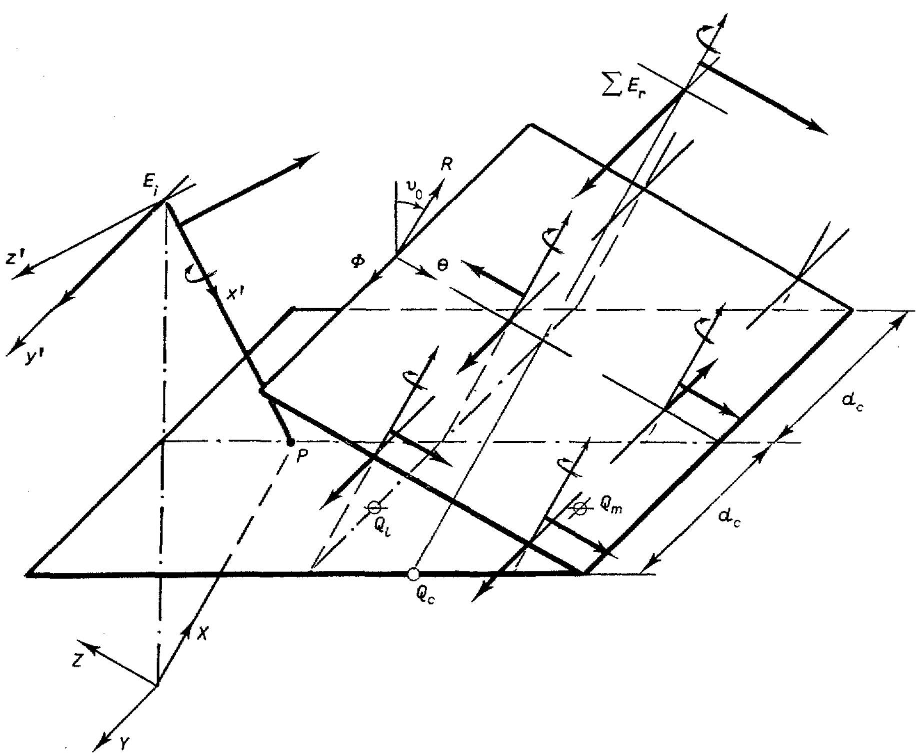

Fig. 4. – The structure of the reflected beam if the incident wave has circular polarization $E_C = e_\Phi - ie_\Theta$.

Let us consider the situation with an arbitrary, but linearly polarized wave. We have

$$V = \begin{bmatrix}\cos\alpha\\ \sin\alpha\end{bmatrix}, \tag{41}$$

and from (28) we get

$$W=\begin{bmatrix}\Gamma_\perp\cos\alpha+(\Gamma_\perp-\Gamma_\parallel)\sin\Omega\sin(\alpha-\Omega)\\ \Gamma_\parallel\sin\alpha+(\Gamma_\perp-\Gamma_\parallel)\sin\Omega\cos(\alpha-\Omega)\end{bmatrix}\simeq\begin{bmatrix}\exp[-i\Psi_\perp]\cos\alpha\\ -\exp[-i\Psi_\parallel]\sin\alpha\end{bmatrix},\tag{42}$$

if $\Omega_{\max}\ll 1$, which obviously denotes us two waves originating at points $Q_\perp$ and $Q_\parallel$ with intensities $\cos\alpha$ and $\sin\alpha$, respectively [1,4,12]. (One can check that a frame which would be natural for both waves simultaneously does not exist.)

Thus and the G.H. and I effects are the consequence of the displacement of the reflected beam's fictitious sources due to the distortions of the constant-phase surfaces. The results are in good agreement with previous work, but this method also explains the mechanism of creation of the reflected beam. Note that for spherical waves the matrix S is always of nondiagonal form; however for the infinite plane waves it is diagonal ($S\equiv\Gamma$). Consequently, a finite beam of a given linear polarization ($\parallel$ or $\perp$) will produce reflected waves of both polarizations although on the axis of the reflected beam, in the far zone, the field of « spurious » polarization vanishes.

We must stress that our explanation is based on the assumption of local planarity of the incident waves, which is typical for geometrical optics and leads to unitarity of the scattering matrix S. Consequently, it does not involve any normal transport of energy to the second medium, similarly as in [15]. However this transport exists [9,19] occuring only on the sides of a beam, since, if the radiation pattern changes rapidly is some region, the wave is no longer plane even locally. The propagation vector $\boldsymbol{k}$ in this region is different from that for a quasi-plane wave in a central part of the beam, or, in other words, we must take into account also the second term in the series (1). Nevertheless due to the conservation of energy this last circumstance leads to a useful method for the determination of both shifts [7,11,20]. If the incident angle approaches $\pi/2$, our conclusions would require some postulates about the amplitude, since both the phase and amplitude distribution in the separating plane determine the final shape of the reflected beam, and the amplitude ceases to be uniform. In the simplest case this leads than to the Kirchhoff formula, or similar ones [10].

Finally, as is seen from the first part of (39), the I effect ought to hold also for angles somewhat smaller than the angle of total reflection, since then $|\Gamma_\perp|\approx|\Gamma_\parallel|\approx 1$ and $E_{r(c)}\sim\exp[-2_i\Omega]$; however a more precise calculation must also include a change of amplitude.

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(20) M. Lenc: *Lett. Nuovo Cimento*, **5**, 236 (1972).

● RIASSUNTO (*)

In questo articolo si presenta una spiegazione degli effetti di Goos-Hänchen e Imbert dal punto di vista dell'ottica geometrica.

(*) *Traduzione a cura della Redazione.*

Некоторые замечания относительно эффектов Гуса-Хенхена и Имберта.

Резюме (*). — В этой статье предлагается объяснение эффектов Гуса-Хенхена и Имберта с точки зрения геометрической оптики.

(*) *Переведено редакцией.*