Band structure of superdeformed bands in odd-A Hg nuclei*

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Abstract Through particle-rotor model, band structure of superdeformed bands in odd-A Hg nuclei is analysed. An overall and excellent agreement between the calculated and observed kinematic and dynamic moments of inertia is obtained. The electromagnetic transition properties of SD bands can be used to identify the configuration with certainty.

Keywords: nuclear structure, superdeformed bands, particle-rotor model, electromagnetic transition.

In the study of superdeformed bands (SD bands) the cranked shell model with pairing appears to be greatly successful in the prediction and analysis of the band structure. Up to now, most of the data can be explained with the framework of the cranked shell model and the specific configurations of the SD bands have been proposed based on the comparison between the experimental and calculated dynamic moments of inertia $J^{(2)}$[1,2]. The cranked model emphasizes the central role of the rotational frequency. However, the rotational frequency is neither a directly measured quantity in physics nor an exact constant of motion in nuclei. It is a direct consequence of this kind of calculation that, after the quasiparticle alignments take place, $J^{(2)}$ will exhibit a downturn with increasing $\hbar\omega$. It is noted recently that a dramatic discrepancy appears between the experiments and the predictions of the cranked shell model. For example, the SD band of ^{192}Hg has been extended to high rotational frequencies ($\hbar\omega \approx 0.44$ MeV)[3] and the dynamic moment of inertia $J^{(2)}$ is found to continue to increase with rotational frequency. On the other hand, the calculation of the cranked shell model predicts that $J^{(2)}$-values start to decrease at about $\hbar\omega \approx 0.35$ MeV. It is pointed out[3,4] that the sensible variations of parameters used (such as monopole pairing strength and deformation parameters) cannot make radical improvement in the agreement between the experimental and calculated dynamic moments of inertia $J^{(2)}$ and the discrepancy still remains. Consequently, an important question is whether the assigned configurations of the SD bands on the basis of the cranked shell model are reliable. It is well known that the particle-rotor model as another powerful method is also used in the study of high spin states in nuclei. The total angular momentum I is treated as a good quantum number in the particle-rotor model, which is very important for the quantitative estimation of the electromagnetic transition probabilities. Using particle-rotor model many features of the rotational system have been illustrated. In ref. [5], the triaxial-particle-rotor model has been used to analyze SD bands in odd-A Tl nuclei. For the SD bands in odd-A Hg nuclei, the occupied orbitals of the odd neutron have

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been proposed based on the cranked shell model calculations. The occupation of [761 3/2] orbital favours ^{189}Hg; however, a [640 1/2] assignment cannot be ruled out[4]; [761 3/2] for ^{191}Hg (b1)[1] and [642 3/2] for ^{191}Hg(b2, b3)[6]; [512 5/2] or [624 9/2] for ^{193}Hg (b2, b3)[7] (^{193}Hg(b1) and ^{193}Hg(b4), in which the band crossing appears, are not considered in this paper). Since there are deficiencies in the cranked shell model and the uncertainties of these assigned configurations (for convenience, an occupied orbital of the odd neutron is called a configuration), there is need to reexamine the configurations of the SD bands. Generally speaking, mixture of different j-shells should be taken into account in the particle-rotor model calculations. In view of the neutron Woods-Saxon routhian diagram[1], it is found that: (i) there are two single particle gaps at $N = 112$ and $N = 116$, separated by two orbitals [512 5/2] and [624 9/2] with different parity. Therefore, the mixture of the two subshells is assumed to be negligible; (ii) similarly, [761 3/2] and [640 1/2] or [642 3/2] levels have opposite parity, and the mixing of different j-shells is also assumed to be negligible; (iii) the admixture of different j-shells may be caused by [640 1/2] and [642 3/2] orbitals. However, the experimental data show that ^{191}Hg(b2) and ^{191}Hg(b3) are signature partners. Consequently, the component of [640 1/2] level is considered to be small. In order to emphasize the physical idea the single j-shell particle-rotor model is used in the following study. Band structure of the superdeformed bands in odd-A Hg nuclei has been investigated. According to a comparison between the observed and the calculated values the occupied orbitals of the odd neutron are proposed. It is shown that in order to confirm the configuration assignment the measurements of the electromagnetic transition probabilities are required.

1 Theoretical model

Suppose that one odd quasi-neutron moving in a triaxially-deformed potential is coupled to a rotating core. The corresponding Hamiltonian can be written as a sum of the rotor part and intrinsic part[8]:

$$H = H_{\mathrm{rot}} + H_{\mathrm{intr}}. \tag{1}$$

The rotor Hamiltonian is given by

$$H_{\mathrm{rot}} = \sum_{k=1}^{3} \frac{\hbar^2}{2J_k}(I - j)_k^2, \tag{2}$$

where J_k is the inertia moments associated with rotation about the intrinsic kth axis. In the numerical calculations, we take the moments of inertia of hydrodynamical type:

$$J_k = \frac{4}{3}J_0(I)\sin^2\left(\gamma + k\frac{2\pi}{3}\right), k = 1,2,3, \tag{3}$$

where $J_0(I)$ is a smooth function of the total angular momentum I of the SD band. Since the transition energies of the SD bands can be accurately reproduced[9–11] by a two-parameters ab formula[12,13], we take

$$J_0(I) = J_{00}\frac{1 + \sqrt{1 + bI(I+1)}}{2}, \tag{4}$$

where parameters J_{00} and b are estimated by fitting γ-transition energies of the core. The intrinsic Hamiltonian is expressed by

$$H_{\mathrm{intr}} = \sum_{\nu}(\varepsilon_\nu - \lambda)a_\nu^+a_\nu + \frac{\Delta}{2}\sum_{\mu\nu}\delta(\tilde{\mu}, \nu)\,(a_\mu^+a_\nu^+ + a_\nu a_\mu), \tag{5}$$

where λ is Fermi level estimated from the Nilsson level scheme for a particular SD band. Δ is energy gap, which is not precisely known for superdeformed nuclei. However, the value of the energy gap in SD bands is much less than that in the normally-deformed nuclei[1,5]. The single-particle energies ε_ν are calculated by using the triaxially-deformed quadrupole potential:

$$V(\gamma) = \frac{\kappa}{j(j+1)}\left\{[3j_2^2 - j(j+1)]\cos\gamma + \sqrt{3}(j_2^2 - j_1^2)\sin\gamma\right\}, \tag{6}$$

where κ is used as an energy unit in the single j-shell model, which is determined depending on the value of deformation parameter β and estimated from the splitting of the single j-shell around $\beta\approx 0.5$ in Nilsson level scheme. For example, the typical value of κ is $\kappa\approx 8$ Mev for $j_{15/2}$-shell, $\kappa\approx 6$ MeV for $i_{13/2}$, $i_{11/2}$ and $g_{9/2}$-shells and $\kappa\approx 5$ MeV for $h_{9/2}$-shell. The diagonalization of the total Hamiltonian eq. (1) is performed by using BCS one-quasiparticle states for the particle wave functions. The kinematic and dynamic moments of inertia are extracted from the calculated γ-transition energies $E_\gamma(I) \equiv E_\gamma(I \to I-2)$. Using the resulting wave functions the calculation of $B(\mathrm{E2})$ and $B(\mathrm{M1})$ values is straightforward. In order to compare them with the experimental data we assume that the level spins of the SD bands in odd-A Hg nuclei have been assigned[10,11,14]. The calculated γ-transition energies of the SD band are normalized on the basis of transition energy $E_\gamma(\mathrm{cal.}, I) = E_\gamma(\mathrm{exp.}, I)$ for a given I to determine energy unit κ.

2 Results and discussion

By using single j-shell particle-rotor model, the calculations of the SD bands in odd-A Hg nuclei have been carried out. The energy gap is taken to be $\Delta = 0.045\kappa$[5] for all odd-A Hg nuclei and the other parameters used have been adjusted to reproduce the observed transition energies as well as possible. The model parameters and the root-mean-square deviation of transition energies for different configurations are given in table 1.

Table 1 Model parameters and root-mean-square deviation

SD bands	Occupied orbitals	Model parameters						σ/keV
		γ	Δ/κ	λ/κ	$J_{00}\kappa$	$b\times10^4$	κ/MeV	
^{193}Hg(b2, b3)	[624 9/2]	3°	0.045	0.55	480	1.9	5.435 2	1.1
	[512 5/2]	0°	0.045	0.12	500	1.8	5.553 5	1.2
^{191}Hg(b1)	[761 3/2]	0°	0.045	−0.72	620	3.5	8.616 7	0.8
^{191}Hg(b2, b3)	[642 3/2]	5°	0.045	−0.35	500	1.1	5.356 5	2.3
	[642 3/2]	4.5°	0.045	−0.35	500	1.1	5.362 6	2.6
^{189}Hg	[761 3/2]	3°	0.045	−0.75	650	4	9.497 6	1.0
	[640 1/2]	0°	0.045	−0.82	700	4	9.617 3	1.2
	[642 3/2]	3°	0.045	−0.5	500	2.1	5.907 5	0.5
	[651 1/2]	3°	0.045	−0.85	500	1.6	5.786 4	0.2

2.1 ^{193}Hg(b2) and ^{193}Hg(b3)

In fig. 1, the calculated kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ are compared with the observed ones for ^{193}Hg(b2) and ^{193}Hg(b3). The exit spin I_0 is taken to be 21/2 and 19/2[10,11,14], corresponding to transition energy $E_\gamma(I_0+2\to I_0) = 254.3$ and 233.7 keV, for bands 2 and 3, respectively. On the left-hand side of fig. 1, ^{193}Hg(b2) and ^{193}Hg(b3) are assumed to be signature partners built upon [624 9/2] orbital; thus, the $i_{13/2}$-shell particle-rotor model is used in the calculation. On the right-hand side of the figure, the $h_{9/2}$-shell is used since ^{193}Hg(b2) and ^{193}Hg(b3) are supposed to be signature partners built upon [512 5/2] orbital. Parameters used in the calculation are shown in table 1. From this figure it can be seen that the calculated kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ are in good agreement with the experimental data for both [624 9/2] and [512 5/2] orbitals of the odd neutron, which implies that the observed transition energies of bands 2 and 3 can be reproduced accurately by using particle-rotor model. In fact, the root-mean-square deviation of the transition energies σ is 1.1 keV for [624 9/2] configuration and 1.2 keV for [512 5/2] configuration. The root-mean-square deviation σ is defined as

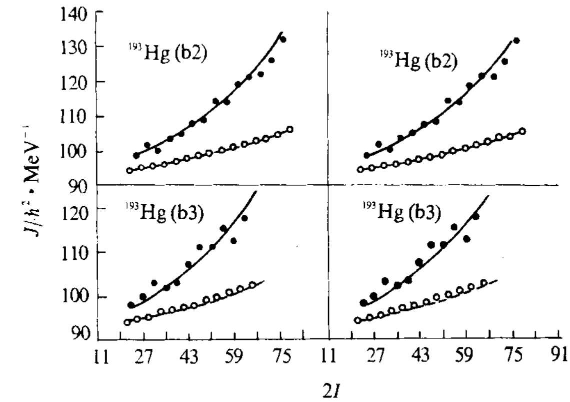

Fig. 1. Calculated kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ are compared with the observed ones for ^{193}Hg(b2) and ^{193}Hg(b3). The solid and dashed lines denote $J^{(2)}$ and $J^{(1)}$ respectively and the full circles and empty circles are observed values of $J^{(2)}$ and $J^{(1)}$, respectively[7].

$$\sigma = \sqrt{\frac{1}{n}\sum_{I} | E_{\gamma}(\text{cal.}, I) - E_{\gamma}(\text{exp.}, I) |^2}, \tag{7}$$

where n is the number of the transitions involved in the SD bands. For example, n is 25 for bands 2 and 3 of ^{193}Hg. Using the same parameters, 25 measured transition energies can be reproduced to an accuracy of 1 keV, which shows that: (i) an odd-A superdeformed nucleus can be regarded as one odd neucleon moving in a triaxially-deformed potential coupled to a rotating core; (ii) the exit spin assignments for ^{193}Hg(b2) and ^{193}Hg(b3) may be correct; (iii) the occupied orbital of the odd neutron cannot be assigned uniquely based solely on γ-transition energies.

In order to identify the configuration assignments for bands 2 and 3 of ^{193}Hg the electromagnetic transition probabilities, calculated by using particle-rotor model, are shown in fig.2, where the left part is for $B(M1; I \rightarrow I-1)$, and the right part for $B(E2; I \rightarrow I-1)$ and $B(E2; I \rightarrow I-2)$. The $B(E2)$ values in fig.2 are expressed in the unit of axially-symmetric values. Parameters used are $g_l = 0$, $g_s = -2.29$, and $g_R = 0.42$, $e_{\text{eff}}\langle j | r^2 | j\rangle / Q_0 = 0.1e$, while the others are the same as those used in fig.1. From this figure, it is seen that: (i) there is a considerable difference in $B(M1)$ values between two configurations, namely $B(M1; I \rightarrow I-1) \approx 0.8\ \mu_N^2$ for the [624 9/2] configuration and $0.04\ \mu_N^2$ for the [512 5/2] configuration; (ii) in low spin region, the $B(E2; \Delta I = 1)$ values in the [624 9/2] configuration are much less than those in the [512 5/2] configuration. For the former the $B(E2; \Delta I = 1)$ values decrease slowly with increasing spin, while for the latter the $B(E2; \Delta I = 1)$ values decrease much faster and show the signature dependence after $I > 55/2$; (iii) the $B(E2; \Delta I = 2)$ values in the [624 9/2] configuration are less than those in the [512 5/2] configuration for $I < 39/2$, while the opposite occurs for $I > 39/2$. In addition, the $B(E2; \Delta I = 2)$ values in the [624 9/2] configuration are close to unity in high spin region. Consequently, although the observed transition energies can be reproduced

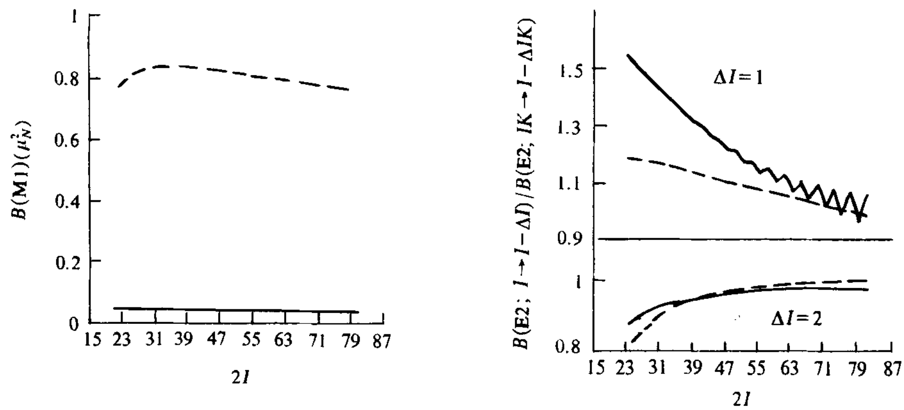

Fig. 2. Calculated electromagnetic transition probabilities are plotted against angular momentum I for ^{193}Hg(b2) and ^{193}Hg(b3). The left-hand side is for $B(M1; I\rightarrow I-1)$, while the right-hand side for $B(E2; I\rightarrow I-1)$ and $B(E2; I\rightarrow I-2)$. The solid lines denote the occupied [512 5/2] orbital of the odd neutron, the dashed lines the occupied [624 9/2].

for both [624 9/2] and [512 5/2] configurations, there is a remarkable difference in the properties of the electromagnetic transitions between them. Therefore, one can utilize this difference to identify the occupied orbital of the odd neutron.

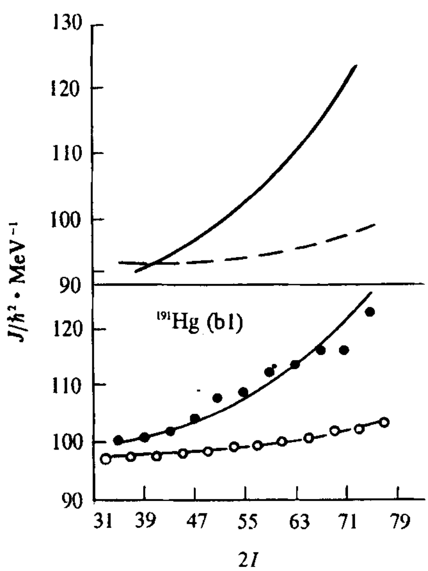

Fig. 3. Calculated kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ are compared with the observed data for ^{191}Hg(b1) (the lower portion). The calculated values of $J^{(1)}$ and $J^{(2)}$ for another possible SD band are also given in the upper part of the figure. The symbols are the same as those used in figure 1.

2.2 ^{191}Hg

2.2.1 ^{191}Hg(b1). A comparison between the calculated and observed kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ for ^{191}Hg(b1) is given in fig. 3[15]. Assuming that the configuration of ^{191}Hg(b1) is [761 3/2] and the exit spin I_0 of ^{191}Hg(b1) is 31/2, corresponding to $E_\gamma(I_0+2\rightarrow I_0)=350.6$ keV[10,11,14], it can be seen that an excellent agreement between the calculated and observed data is obtained. The root-mean-square deviation σ is 0.8 keV for ^{191}Hg(b1). Therefore, ^{191}Hg(b1) can be regarded as an SD band built upon the favoured [761 3/2] ($7_{3/2}$) orbital. The calculated kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ for another band built upon the unfavoured $7_{3/2}$ orbital, is also given in the upper portion of fig. 3. The calculation shows that the energy difference $E(u, I)-E(f, I)$ between two bands is about 100—700 keV.

2.2.2 ^{191}Hg(b2) and ^{191}Hg(b3). In fig. 4, a comparison between the calculated and observed kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ for two excited bands in ^{191}Hg is given. The exit spin I_0 of ^{191}Hg(b2) and ^{191}Hg(b3) is taken to be 25/2 and 27/2, corresponding to transition energies $E_\gamma(I_0+2\rightarrow I_0)=292.0$ keV and 311.8 keV, respectively[10,11,14]. From fig.4 and table 1, it can be seen that: (i) the dynamic moments of inertia $J^{(2)}$ are very sensitive to the changes of the triaxial deformation γ; (ii) ^{191}Hg(b2) and ^{191}Hg(b3) are signature partners built upon [642 3/2] orbital; (iii) the root-mean-square deviation of the transition energies $\sigma\approx 2.5$keV. The results cannot be sensibly improved by an adjustment of the model parameters in a reasonable range. We consider the admixture of [640 1/2] and [642 3/2] configurations probably important.

2.3 ^{189}Hg

The root-mean-square deviations σ for four different occupied orbitals in ^{189}Hg are given in table 1. Parameters used, except for energy gap, have been adjusted to reproduce the observed data as well as possible. The exit spin I_0 of the SD band in ^{189}Hg is taken to be 31/2, corresponding to transition energies $E_\gamma(I_0+2\rightarrow I_0)=366.0$ keV[14]. In fig.5, the calculated kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ for [651 1/2] configuration in ^{189}Hg are compared with the experimental data. The calculated values of $J^{(1)}$ and $J^{(2)}$ for another SD band in ^{189}Hg, which probably exists, are also given in the lower part of this figure. From table 1 and fig.5, it can be seen that: (i) when the odd neutron occupies [761 3/2] or [640 1/2] orbitals, the calculated results can hardly agree with the observed data by using physically reasonable model parameters, while the parameters given in table 1 are unreasonable. The reason is that the energy unit κ estimated from the splitting of the single j-shell around $\beta=0.5$ in the Nilsson level scheme is about 8 and 6 MeV for $j_{15/2}$ and $i_{11/2}$ shell, respectively, while from the normalization κ becomes 9.5

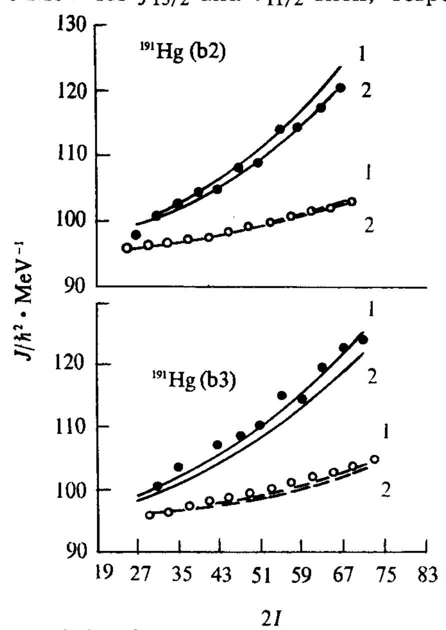

Fig.4. Calculated kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ are compared with the observed data for ^{191}Hg(b2) and ^{191}Hg(b3). Lines 1 and 2 denote $\gamma=5°$ and 4.5°, respectively. The symbols are the same as those used in figure 1.

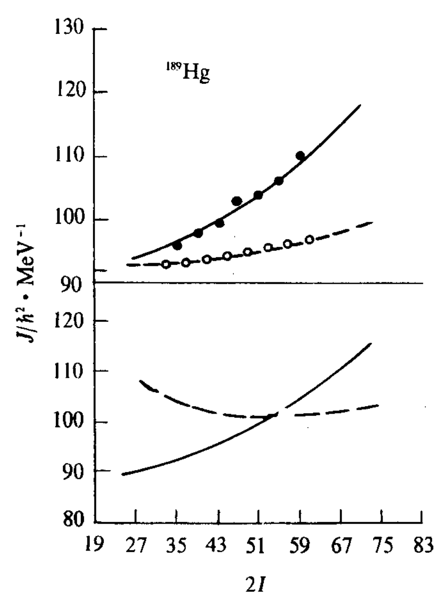

Fig.5. Calculated kinematic and dynamic moments of inertia $J^{(1)}$ and $J^{(2)}$ are compared with the observed values for ^{189}Hg. The symbols are the same as those used in figure 1.

MeV and 9.6 MeV, respectively. It is noted that κ is proportional to β. Consequently, κ is too large, which implies that the quadrupole deformation β used in calculations is unreasonable. Moreover, the inertia parameter J_{00} of the core in ^{189}Hg is 68.4 and 72.9 $\hbar^2$MeV^{-1} for [761 3/2] and [640 1/2] configurations, respectively, which are much less than 88.7 $\hbar^2$MeV^{-1} estimated for ^{190}Hg SD band in ref. [10]. (ii) In the case of [651 1/2] or [642 3/2] configuration, the calculated transition energies, with physically reasonable model parameters, coincide with the observed data quite well. For example, $\kappa \approx 5.8$ MeV, $J_{00} \approx 86.2$ $\hbar^2$ MeV^{-1} and $\sigma - 0.2$ keV for [651 1/2] configuration. (iii) However, the occupation of the odd-neutron in [651 1/2] or [642 3/2] orbitals, for the moment, cannot be assigned with certainty. (a) Since both levels belong to $g_{9/2}$-shell, if the exit spin $I_0 = 31/2$ the observed SD band in ^{189}Hg is an unfavoured band (u-band). Why can the favoured band (f-band) not be observed? After having assumed the exit spin $I_0 = 29/2$[14], the calculations for the four configurations mentioned above have been performed again. For the [761 3/2] and [640 1/2] configurations, the observed results cannot be reproduced by using particle-rotor model, while for the occupation of [651 1/2] or [642 3/2] orbitals the calculated values are in good agreement with the observed values and the root-mean-square deviation σ is 0.6 and 1.3 keV, respectively. Thus, the observed SD band in ^{189}Hg is a favoured band built upon $g_{9/2}$-shell. Furthermore, because of the large signature splitting of the [651 1/2] routhians, the energies of u-band are expected to lie 100—400 keV higher than those of f-band in the investigated region of the angular momentum. As a result, the u-band should be more difficult to observe. Therefore, at present, only a single SD band is measured. (b) Since [651 1/2] level is far away from the Fermi level, the neutron is hardly excited. It is noted that the parameters used in Nilsson potential or Woods-Saxon potential are obtained for the normally-deformed and the low spin states. It is not clear whether these parameters are still suitable for the superdeformed and very high spin states. Thus, it is possible to raise the position of the [651 1/2] level by an appropriate adjustment of the potential parameters. However, since the number of the measured transition energies is too small the calculated results are insensitive to the exit spin I_0. It is desirable to obtain more experimental data in order to certainly assign the configuration of SD band in ^{189}Hg (e.g. extend the SD band to higher angular momentum region, measure its signature partner, etc.).

3 Conclusion

Using the particle-rotor model, band structure of superdeformed bands in odd-A Hg nuclei has been investigated.

(i) ^{193}Hg(b2) and ^{193}Hg (b3) are a pair of signature partners. It is difficult to assign the occupied [624 9/2] or [512 5/2] orbital of the odd neutron based solely on γ-transition energies. It is suggested that the measurement of the electromagnetic transition probabilities can be used as a criterion to determine which orbital is occupied.

(ii) ^{191}Hg(b1) is an f-band built upon [761 3/2] orbital, of which signature partner is expected to lie 100—700 keV higher, and ^{191}Hg(b2) and ^{191}Hg(b3) are a pair of signature partners built upon [642 3/2] orbital.

(iii) According to our calculations, the possible occupation of [761 3/2] or [640 1/2] orbital of the odd neutron in ^{189}Hg has been ruled out. However, the observed data are not sufficient, for the moment, to assign the occupied [651 1/2] orbital with certainty.

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