STABILITY OF A THREE-LAYERED ORTHOTROPIC CYLINDRICAL SHELL UNDER NONUNIFORM EXTERNAL PRESSURE

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§ 1. A three-layered shallow cylindrical shell consisting of two thin orthotropic layers of thickness $t_1 = t_2 = t$ and an orthotropic core of thickness t_3 is considered. The Kirchhoff–Love hypothesis is assumed to be valid for the outer layers, while transverse shear is taken into account in the core: the shear stresses are assumed to be uniformly distributed across the thickness. The transverse compressibility of the middle layer is neglected. The Poisson's ratios ν_1 and ν_2 of all three layers are the same.

The nonlinear equations of equilibrium of an element of the shell are written in the form [3]

$$\begin{gathered}
\delta_1 \frac{\partial^4 F}{\partial x^4} + 2\delta_3 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \delta_2 \frac{\partial^4 F}{\partial y^4} = -\frac{1}{R}\frac{\partial^2 W}{\partial x^2} + \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 - \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2};\\
I_1\left(C_1 \frac{\partial^2 f_1}{\partial x^2} + C_4 \frac{\partial^2 f_1}{\partial y^2} + C_3' \frac{\partial^2 f_2}{\partial x \partial y} + H_1 \frac{\partial^3 W}{\partial x^3} + H_3 \frac{\partial^3 W}{\partial x \partial y^2}\right) - f_1 = 0;\\
I_2\left(C_2 \frac{\partial^2 f_2}{\partial y^2} + C_4 \frac{\partial^2 f_2}{\partial x^2} + C_3' \frac{\partial^2 f_1}{\partial x \partial y} + H_2 \frac{\partial^3 W}{\partial y^3} + H_3 \frac{\partial^3 W}{\partial x^2 \partial y}\right) - f_2 = 0;\\
D_1 \frac{\partial^4 W}{\partial x^4} + 2D_3 \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 W}{\partial y^4} + H_1 \frac{\partial^3 f_1}{\partial x^3} + H_2 \frac{\partial^3 f_2}{\partial y^3}\\
+ H_3\left(\frac{\partial^3 f_1}{\partial x \partial y^2} + \frac{\partial^3 f_2}{\partial x^2 \partial y}\right) - \frac{\partial^2 F}{\partial x^2}\left(\frac{1}{R} + \frac{\partial^2 W}{\partial y^2}\right) - \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 W}{\partial x^2} + 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y} - q = 0,
\end{gathered} \tag{1.1}$$

where W(x, y) and F are functions of deflections and forces.

The functions f_1 and f_2 are introduced into the equations by the relationships

$$u_\beta = h\frac{\partial W}{\partial x} + f_1; \qquad v_\beta = h\frac{\partial W}{\partial y} + f_2. \tag{1.2}$$

They are zero, if the transverse shear in the core is not taken into account. At the same time $u_\beta = (u_1 - u_2)/2$; $u_\beta = (v_1 - v_2)/2$ (u_1, v_1, u_2, v_2 are the displacements of points of the middle surfaces of the outer layers along the x and y coordinate axes).

The position of the axes is chosen so that $y = R\vartheta$, $-L/2 \leq x \leq L/2$ (L is the length; R is the radius of the shell; ϑ is the angle measured along the directrix of the cylinder).

The coefficients δ_i, C_i, H_i, D_f, and I_f are the stiffness parameters of the three-layered construction. They are expressed in terms of the stiffnesses of constituent layers according to [1]

$$\delta_1 = \frac{1}{(2C_{22} + C_{22}^*)(1 - \nu_1\nu_2)}; \qquad \delta_2 = \frac{1}{(2C_{11} + C_{11}^*)(1 - \nu_1\nu_2)};$$

$$\delta_3 = \frac{1}{2(2C_{66} + C_{66}^*)} - \frac{\nu_2}{(2C_{22} + C_{22}^*)(1 - \nu_1\nu_2)};$$

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$$C_i = 2\left(C_{ii} + \frac{1}{6}C^*_{ii}\right); \qquad D_i = 2D_{ii} + D^*_{ii} + 2h^2C_{ii};$$

$$H_i = C_{ii}t + t_3\left(C_{ii} + \frac{1}{6}C^*_{ii}\right); \qquad I_i = \frac{t_3}{4G_{i3}} \quad (i = 1, 2); \tag{1.3}$$

$$C'_3 = 2\left(C_{66} + \frac{1}{6}C^*_{66}\right) + 2\nu_2\left(C_{11} + \frac{1}{6}C^*_{11}\right); \qquad C_4 = 2\left(C_{66} + \frac{1}{6}C^*_{66}\right);$$

$$D_3 = \frac{1}{2}\nu_2(2D_{11} + D^*_{11} + 2h^2C_{11}) + (2D_{66} + D^*_{66} + 2h^2C_{66});$$

$$H_3 = 2\left[C_{66}t + t_3\left(C_{66} + \frac{1}{6}C^*_{66}\right)\right] + \nu_2\left[C_{11}t + t_3\left(C_{11} + \frac{1}{6}C^*_{11}\right)\right].$$

Here

$$h = \frac{t + t_3}{2}; \qquad C_{ii} = \frac{E_i t_i}{1 - \nu_1\nu_2}; \qquad D_{ii} = \frac{E_i t_i^3}{12(1 - \nu_1\nu_2)} \quad (i = 1,2);$$

$$C_{66} = Gt; \qquad D_{66} = \frac{1}{12}Gt^3.$$

with the index * denoting the stiffness of the middle layer.

We consider the case when the edges of the shell are fixed to frames which are absolutely rigid in their plane but allow them to be freely rotated. Here the strain and relative motion of the layers along the tangent to the boundary are zero. We assume that boundary loads are absent. The boundary conditions are as follows:

$$W = 0; \qquad \frac{\partial^2 W}{\partial x^2} = 0; \qquad \frac{\partial^2 F}{\partial y^2} = 0; \qquad \frac{\partial^2 F}{\partial x^2} = 0; \qquad \frac{\partial f_1}{\partial x} = 0; \qquad f_2 = 0. \tag{1.4}$$

Using the nonlinear system of equations (1.1) we obtain the equations of stability "in the small" of the three-layered orthotropic shell. Considering the possibility of the existence, together with the non-perturbed mode of equilibrium, of other modes of equilibrium which are adjacent to the former, we write these equations in the dimensionless form

$$\frac{1}{\beta_1}\frac{\partial^4 F_1}{\partial \xi^4} + \frac{2}{\beta_3}\frac{\partial^4 F_1}{\partial \xi^2 \partial \vartheta^2} + \frac{1}{\beta_2}\frac{\partial^4 F_1}{\partial \vartheta^4} = -\frac{\partial^2 W_1}{\partial \xi^2} + 2\frac{\partial^2 W_0}{\partial \xi \partial \vartheta}\frac{\partial^2 W_1}{\partial \xi \partial \vartheta} - \frac{\partial^2 W_0}{\partial \xi^2}\frac{\partial^2 W_1}{\partial \vartheta^2} - \frac{\partial^2 W_0}{\partial \vartheta^2}\frac{\partial^2 W_1}{\partial \xi^2};$$

$$r_1\frac{\partial^2 \bar{f}_1}{\partial \xi^2} + r_4\frac{\partial^2 \bar{f}_1}{\partial \vartheta^2} + r'_3\frac{\partial^2 \bar{f}_2}{\partial \xi \partial \vartheta} + r_1 s_1\frac{\partial^3 W_1}{\partial \xi^3} + r_3 s_3\frac{\partial^3 W_1}{\partial \xi \partial \vartheta^2} - \bar{f}_1 = 0;$$

$$r_2\lambda\frac{\partial^2 \bar{f}_2}{\partial \vartheta^2} + r_4\lambda\frac{\partial^2 \bar{f}_2}{\partial \vartheta^2} + r'_3\lambda\frac{\partial^2 \bar{f}_1}{\partial \xi \partial \vartheta} + r_2 s_2\lambda\frac{\partial^3 W_1}{\partial \vartheta^3} + r_3 s_3\lambda\frac{\partial^3 W_1}{\partial \xi^2 \partial \vartheta} - \bar{f}_2 = 0;$$

$$\gamma_1\frac{\partial^4 W_1}{\partial \xi^4} + 2\gamma_3\frac{\partial^4 W_1}{\partial \xi^2 \partial \vartheta^2} + \gamma_2\frac{\partial^4 W}{\partial \vartheta^4} + \alpha_1\frac{\partial^3 \bar{f}_1}{\partial \xi^3} + \alpha_2\frac{\partial^3 \bar{f}_2}{\partial \vartheta^3} \tag{1.5}$$

$$+ \alpha_3\left(\frac{\partial^3 \bar{f}_1}{\partial \xi \partial \vartheta^2} + \frac{\partial^3 \bar{f}_2}{\partial \xi^2 \partial \vartheta}\right) - \frac{\partial^2 F_1}{\partial \xi^2} - \frac{\partial^2 F_0}{\partial \vartheta^2}\frac{\partial^2 W_1}{\partial \xi^2} - \frac{\partial^2 W_0}{\partial \xi^2}\frac{\partial^2 F_1}{\partial \vartheta^2}$$

$$- \frac{\partial^2 F_0}{\partial \xi^2}\frac{\partial^2 W_1}{\partial \vartheta^2} - \frac{\partial^2 W_0}{\partial \vartheta^2}\frac{\partial^2 F_1}{\partial \xi^2} + 2\frac{\partial^2 F_0}{\partial \xi \partial \vartheta}\frac{\partial^2 W_1}{\partial \xi \partial \vartheta} + 2\frac{\partial^2 W_0}{\partial \xi \partial \vartheta}\frac{\partial^2 F_1}{\partial \xi \partial \vartheta} = 0.$$

Here the index zero is used to denote quantities referring to the precritical stress–strain state. The same quantities without index constitute additional values of the functions which arise when modes of equilibrium branch out.

The dimensionless quantities are introduced by the relationships

$$F = KF_1; \qquad W = RW_1; \qquad f_1 = R\bar{f}_1; \qquad f_2 = R\bar{f}_2;$$

$$x = R\xi; \; K = R\sqrt{\frac{H_1H_2}{\omega}}; \; \omega = \sqrt{\delta_1\delta_2C_1C_2};$$

$$r_1 = \frac{C_1 I_1}{R^2};\ r_2 = \frac{C_2 I_2}{R^2};\ r_3' = \frac{C_3' I_1}{R^2};\ r_4 = \frac{C_4 I_1}{R^2};$$

$$\lambda = \frac{I_2}{I_1};\ r_3 = r_3' + r_4;\ s_i = \frac{H_i}{RC_i};\ \frac{1}{\beta_i} = \frac{\delta_i K}{R^2};\ \gamma_i = \frac{D_i}{K}. \tag{1.6}$$

The boundary conditions (1.4) are written in the form

$$W_1 = 0;\ \frac{\partial^2 W}{\partial \xi^2} = 0;\ \frac{\partial^2 F_1}{\partial \vartheta^2} = 0;\ \frac{\partial^2 F}{\partial \xi^2} = 0;\ \frac{\partial \bar{f}_1}{\partial \xi} = 0;\ \bar{f}_2 = 0. \tag{1.7}$$

§ 2. Let the shell be loaded by an external pressure which is constant along the generator and has the intensity

$$q = q_0 (1 + 2\varepsilon \cos N\vartheta). \tag{2.1}$$

We consider the case of a self-equilibrated load ($N \geq 2$). Solving the problem in the first approximation, in the role of the precritical state we take the nondeformed state of the shell without the boundary conditions taken into account. Then the initial stress state is determined by the compressive force

$$N_u = \frac{\partial^2 F_0}{\partial x^2} = -q_0 R (1 + 2\varepsilon \cos N\vartheta)$$

or in the dimensionless form

$$\frac{\partial^2 F_0}{\partial \xi^2} = -m_q^0 (1 + 2\varepsilon \cos N\vartheta), \tag{2.2}$$

where $m_q = qR^3/K$; ε is the parameter of nonuniformity of loading.

The solution of Eqs. (1.5), satisfying the boundary conditions (1.7), will be represented in the form of the series

$$\begin{gathered} W_1 = \sum_{m,n} A_{m,n} \cos l_m \xi \cos n\vartheta;\ F_1 = \sum_{m,n} B_{m,n} \cos l_m \xi \cos n\vartheta; \\ f_1 = \sum_{m,n} C_{m,n} \sin l_m \xi \cos n\vartheta;\ f_2 = \sum_{m,n} D_{m,n} \cos l_m \xi \sin n\vartheta. \end{gathered} \tag{2.3}$$

Here $l_m = m\pi R/L$ ($m = 1, 3, 5 \ldots$).

We put $N = 2$. Substituting (2.3) into Eqs. (1.5), with (2.2) taken into account, we obtain an infinite system of homogeneous algebraic equations for the determination of the coefficients of expansion $A_{m,n}$, $B_{m,n}$, $C_{m,n}$, $D_{m,n}$. This system decomposes with respect to n into two independent systems with even and odd indices. The coefficients $A_{m,2n}$ after elimination of $B_{m,2n}$, $D_{m,2n}$ are determined by the solution of the infinite system of equations

$$\begin{gathered} A_0 \alpha - 2^2 A_2 \varepsilon m_q = 0; \\ A_{2n} = (\Gamma_{2n} - m_q)(2n)^2 - \varepsilon m_q [A_{2n-2}(2n-2)^2 + A_{2n+2}(2n+2)^2] = 0, \end{gathered} \tag{2.4}$$

where $n = 1, 2, 3 \ldots$; the index m is omitted.

The coefficients A_{2n+1} are found from the system

$$\begin{gathered} A_1(\Gamma_1 - m_q) - \varepsilon m_q (A_1 + 3^2 A_3) = 0; \\ A_{2n+1}(\Gamma_{2n+1} - m_q) - \varepsilon m_q [A_{2n+3}(2n+3)^2 + A_{2n-1}(2n-1)^2] = 0. \end{gathered} \tag{2.5}$$

Here

$$\Gamma_n = \frac{1}{n^2}\left[\gamma_1 l_m^4 + 2\gamma_3 l_m^2 n^2 + \gamma_2 n^4 + \frac{l_m^4}{\frac{1}{\beta_1} l_m^4 + \frac{2}{\beta_3} l_m^2 n^2 + \frac{1}{\beta_2} n^4} - \Phi(r)\right];$$

$$\Phi(r) = l_m^2 (\alpha_1 l_m^2 + \alpha_3 n^2) \Phi_1(r) + n^2 (\alpha_2 n^2 + \alpha_3 l_m^2) \Phi_2(r);$$

$$\Phi_1(r) = \frac{(1 + r_2 \lambda n^2 + r_4 \lambda l_m^2)(r_1 s_1 l_m^2 + r_3 s_3 n^2) - r_3' n^2 (r_2 s_2 n^2 + r_3 s_3 l_m^2)}{(1 + r_1 l_m^2 + r_4 n^2)(1 + r_2 \lambda n^2 + r_4 \lambda l_m^2) - \lambda (r_3')^2 l_m^2 n^2};$$

$$\Phi_2(r) = \frac{(1 + r_1 l_m^2 + r_4 n^2)(r_2 s_2 n^2 + r_3 s_3 l_m^2)\lambda - r_3' l_m^2 (r_1 s_1 l_m^2 + r_3 s_3 n^2)\lambda}{(1 + r_1 l_m^2 + r_4 n^2)(1 + r_2 \lambda n^2 + r_4 \lambda l_m^2) - \lambda (r_3')^2 l_m^2 n^2};$$

$$\alpha = \gamma_1 l_m^4 + \beta_1 - r_1 s_1 \alpha_1 \frac{l_m^6}{1 + r_1 l_m^2}. \tag{2.6}$$

The least value of the parameter of critical load m_q^0 for a uniform pressure ($\varepsilon = 0$) is determined by varying, with respect to n, the expression $m_q^0 = \Gamma_n$. The critical value of the load, for uniform pressure, corresponds to $m = 1$.

The expression (2.6) in the case of isotropic layers coincides with the corresponding expression of [2]. The critical value of m_q for $\varepsilon \neq 0$ is established from the determinant of the system (2.4) or (2.5) equated to zero. These determinants, for the sake of calculating the least value of the parameter of critical load can be represented in the form of a sum of finite and infinite continued fractions. The determinant of the system (2.4) is transformed into

$$\Gamma_{2n} - m_q = \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{2n-2} - m_q) - \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{2n-4} - m_q) - \ldots - \cfrac{\varepsilon^2 m_q^2}{\Gamma_2 - m_q}}} + \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{2n+2} - m_q) - \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{2n+4} - m_q) - \ldots}} \tag{2.7}$$

The value of 2n is found from the minimum condition of Γ_{2n} for $\varepsilon = 0$. Therefore not a single one of the denominators of these fractions becomes zero. For $n \to \infty$ the numerator of the common term of the fractions remains constant, while the denominator increases without bounds. Consequently, the infinite continued fraction will be convergent.

The determinant of the system (2.5) also can be represented in the form of a sum of a finite and an infinite convergent continuous fraction

$$\Gamma_{2n-1} - m_q = \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{2n-3} - m_q) - \ldots - \cfrac{\varepsilon^2 m_q^2}{(\Gamma_1 - m_q) - \varepsilon m_q}} + \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{2n+1} - m_q) - \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{2n+3} + m_q) - \ldots}}. \tag{2.8}$$

To calculate the critical value of m_q for small ε, we can confine ourselves in these fractions to the first terms, and solve the relationship thus obtained by the method of successive approximations.

Using a computer, the quantity m_q was calculated from Eqs. (2.7) and (2.8) for any values of ε. In the fractions we retained a number of terms such that the error of calculation of m_q did not exceed 10^{-4}.

As an example we consider a shell with the geometrical parameters $t_3/t = 7.5$, $t/R = 0.0028$ for the three cases of elastic parameters

$$\frac{E_2}{E_1} = 1;\ \frac{E_2^*}{E_2} = 0.1;\ \frac{E_1^*}{E_1} = 0.1;\ \frac{G}{E_1} = \frac{1}{2(1 - \nu_1 \nu_2)};$$
$$\nu_1 = \nu_2 = \nu = 0.15;\ \frac{E_1}{G_{13}} = 426.25;\ \frac{G_{13}}{G_{23}} = 1; \tag{2.9}$$

$$\frac{E_2}{E_1} = \frac{1}{3};\ \frac{G}{E_1} = 0.4;\ \nu_1 = 0.15;\ \nu_2 = 0.05; \tag{2.10}$$

$$\frac{E_2}{E_1} = 3;\ \nu_1 = 0.15;\ \nu_2 = 0.45. \tag{2.11}$$

The results of the calculations are presented in Figs. 1, 2, 3, and 4. The graphs in Fig. 1 reflect the variation of the parameter of critical load m_q^0 for $\varepsilon = 0$, as a function of quantity L/R. Curve 1 corresponds to the first (isotropic) case; curve 2 corresponds to the case when $E_2/E_1 = 1/3$; curve 3 corresponds to the case when $E_2/E_1 = 3$.

In Fig. 2 we have shown the dependence of the ratio m_q/m_q^0 on the value of the parameter ε which characterizes nonuniformity of loading, for the isotropic shell. Curves 1 and 2 have been plotted for the ratio $R/L = 0.2$; curves 3 and 4 for $R/L = 0.4$; curve 5 for $R/L = 0.8$. These graphs have been obtained, respectively, from Eqs. (2.7) and (2.8). For small values of ε the difference between curves of the type (3, 4) and (1, 2) is considerable. However, as ε increases, the curves asymptotically merge.

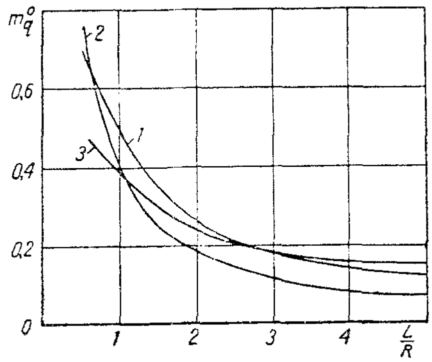

Fig. 1

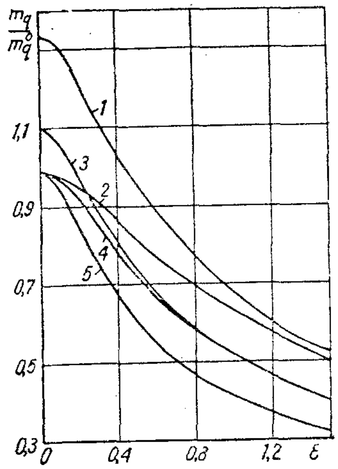

Fig. 2

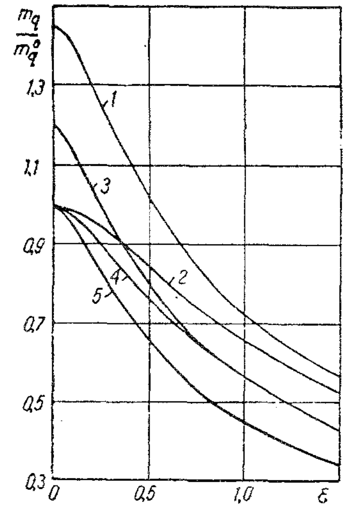

Fig. 3

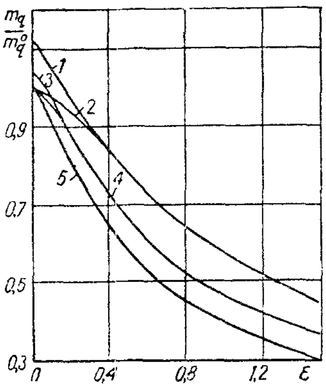

Fig. 4

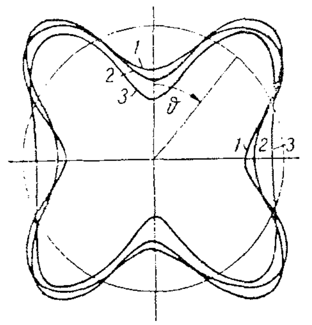

Fig. 5

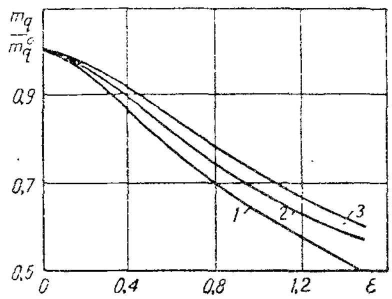

Fig. 6

As we see, the increase in the nonuniformity of loading manifests itself more strongly in the value of critical load for short shells than for long shells. In Figs. 3 and 4 the same curves are shown for shells with orthotropic layers. In Fig. 3 $E_2/E_1 = 1/3$, while Fig. 4 $E_2/E_1 = 3$. In spite of the presence in Eq. (2.8) of the quantity ε in an odd power, the graphs in all cases are symmetric about the ordinate axis. This is explained by the character of variation of the quantity Γ_n with n.

The graphs thus obtained allow us to conclude that the calculation of the critical loads for $\varepsilon \neq 0$ must be carried out according to Eq. (2.7) or (2.8), depending on the number of waves (n) with which the shell loses its stability for $\varepsilon = 0$. If n is even, then Eq. (2.7) is used; if n is odd, then Eq. (2.8) is used.

We consider the influence of the nonuniformity of loading ($N = 2$) on the mode of stability loss. At the section $x = 0$ the form of the deformed surface can be represented by the series

$$W = \sum_{n=0}^{\infty} A_{2n} \cos 2n\vartheta. \tag{2.12}$$

It is not difficult to show that this series converges. If for $\varepsilon = 0$ the shell loses stability with $2n_1$ waves and the amplitude of the wave is taken equal to unity, then for nonuniform pressure, as ε decreases, the coefficient A_{2n} must tend to unity. Using the norming condition

$$1 = 2A_0^2 + A_2^2 + A_4^2 + \dots , \tag{2.13}$$

we determine the expansion coefficients A_{2n}. The values of A_{2n} thus calculated allowed us to plot (Fig. 5) the form of the deformed surface at the section $x = 0$, dependent on the quantity ε for a shell with the parameters (2.9). This shell for $\varepsilon = 0$ loses stability with four waves. The curves 1, 2, and 3 are plotted respectively for $\varepsilon = 0$; $\varepsilon = 0.1$, and $\varepsilon = 1$.

§ 3. In the same way we can consider the cases N = 3, N = 4, and so forth. For N = 3 the system for determining the expansion coefficients A_n is decomposed into three independent systems. However, in each case the mode of stability loss will possess only one plane of symmetry. The determinant of the system can be represented in the form of a sum of finite and infinite continued fractions. If for $\varepsilon = 0$ the shell loses stability with n_1 waves, then the equations for determining the critical loads assume the form

$$m_q = \Gamma_{n_1} - \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{n_1-3} - m_q) - \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{n_1-6} - m_q) - \ldots - \cfrac{\varepsilon^2 m_q^2}{\Gamma_1 - m_q}}} - \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{n_1+3} - m_q) - \cfrac{\varepsilon^2 m_q^2}{(\Gamma_{n_1+6} - m_q) - \ldots}}. \tag{3.1}$$

For N = 4 the system decomposes into forms with one, $\vartheta = 0$, and two, $\vartheta = 0$ and $\vartheta = \pi/2$, planes of symmetry. The results of computations for these forms of the load, for R/L = 0.2, are indicated in Fig. 6. Comparing the values obtained (the curves 1, 2, and 3 for N = 2; N = 3; N = 4) with the first case, we can observe that for N > 2 the influence of nonuniformity of loading is less substantial than for N = 2. In the limiting case for $N \to \infty$ the critical load $m_q \to m_q^0$, since $\varepsilon^2 m_q^2/(\Gamma_{n_1+N} - m_q) \to 0$.

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