

# EFFECT OF TANK SIZE ON $k_La$ AND MIXING TIME IN AERATED STIRRED REACTORS WITH NON-NEWTONIAN FLUIDS

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The most important scale-up parameters of aerated bioreactors are investigated in 42 and 340 L vessels, with water and various xanthan gum and carboxymethyl cellulose solutions. The study focuses mainly on mass transfer ( $k_La$ ) measurements under various operating conditions. The relevance of existing correlations is discussed. The traditional viscosity-contribution approach appears unable to predict the changes in  $k_La$  during scale-up and an alternative formulation is proposed. The effect of rheology on power consumption and mixing time is in fair agreement with works published on this topic.

**Keywords:** non-Newtonian rheology,  $k_La$ , xanthan gum, CMC, aerated stirred vessels

## INTRODUCTION

In various fermentation systems, such as mycelial aerobic cultures, sparged stirred reactors are usually chosen as they provide good mixing and high mass transfer levels (Leib et al., 2001). Depending on the organism and the culture conditions, broth rheology ranges from Newtonian to highly viscous non-Newtonian behaviour during growth and may show significant time variations (Allen and Robinson, 1990). The non-Newtonian behaviour is associated with high biomass concentration and is caused by interactions among the suspended filaments (Metz et al., 1979; Oolman and Blanch, 1986). The viscous nature of the fermentation broths can strongly affect transport properties. Poor mixing can result in problems associated with nutrient limitation, pH gradients, and insufficient oxygen transfer rates (Bylund et al., 1998; George et al., 1998; Gibbs et al., 2000). This is especially the case at large scale when mixing times are of the order of minutes. In the case of ethanol production from lignocellulosic biomass, large amounts of enzymes have to be produced by the filamentous fungi *Trichoderma reesei*. Therefore, the scale-up of aerated reactors in presence of non-Newtonian media which is addressed in this work is a major issue in the field of biofuels.

In many studies, model fluids are preferred to biological suspensions for the study of gas-liquid transfer properties in bioreactors.

They are cheaper and easy to use. As the rheological properties evolve during fermentation, the choice of a single model fluid is not relevant and several have to be used to represent the mycelial broth (Pedersen et al., 1993). Homogeneous polymer solutions such as carboxymethyl cellulose (CMC) (Yagi and Yoshida, 1975; Ranade and Ulbrecht, 1978; Nishikawa et al., 1981; Kawase and Moo-Young, 1988), and xanthan gum (XG) (Henzler and Obenroter, 1991; Pedersen, 1997; Garcia-Ochoa and Gomez, 1998) are often used to determine correlations for the mass transfer coefficient,  $k_La$ , in non-Newtonian systems. Fibre suspensions have also been used for their filamentous structure (Cooke et al., 1988; Dronawati et al., 1997; Benchapattarapong et al., 2005), despite possible difficulties in mimicking the real rheology of highly shear-thinning filamentous fungi. Model fluids are also used for

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the study of mixing time and power consumption in viscous media (Luong and Volesky, 1979; Nienow and Elson, 1988; Henzler and Obernosterer, 1991; Pedersen et al., 1993, 1994; Nienow et al., 1996; Vlaev et al., 2002; Albaek et al., 2008). A detailed discussion on rheological measurements of mycelial broth and model media is proposed by various authors (Metz and Kossen, 1977; Metz et al., 1979; Kim et al., 1983; Allen and Robinson, 1990; Olsvik and Kristiansen, 1994; Svihla et al., 1997).

In this study, un-gassed and gassed power consumption per unit volume ( $P_0$  and  $P_g$ ), mixing time ( $\theta$ ) and gas–liquid mass transfer coefficient ( $k_L a$ ) are investigated as these parameters are strongly linked and potentially influence mycelial fermentation yields. The effect of rheology is quantified by means of model fluids.

Power draw is a key parameter as it contributes significantly to the overall operating costs of industrial plants and influences heat and mass transfer phenomena, mixing and circulation times. Recently, Ascanio et al. (2004) reviewed the methods for measuring power consumption in stirred vessels. They point out that discrepancies are often found in the literature as friction losses are difficult to discount, even if a mechanical torque meter is used. The viscous dissipation is then difficult to measure accurately in small reactors (below  $\sim 30$  L). A large number of studies concern the relative power draw ( $P_g/P_0$ ), defined as the reduction in power consumption due to gas injection inside stirred reactors (Michel and Miller, 1962; Harnby et al., 1997; Hudcova et al., 1989; Tattersson, 1991; Paul et al., 2004). These results are established in Newtonian solutions but very few studies have focused on  $P_g/P_0$  when non-Newtonian liquids are involved (Luong and Volesky, 1979; Hudcova et al., 1989; Albaek et al., 2008). For this reason, particular attention has to be paid to this measurement here as power draw is used as a criterion for process scale-up and  $k_L a$  calculation.

Mixing is also a key variable in the study of bioreactors as it can intensify mass and heat transfer and can affect biological reactions. There are many experimental studies related to mixing time in water systems and air–water systems but few are dedicated to non-Newtonian media matching mycelial cultures. Most of the work reported was done under ungassed conditions (Visuri et al., 2007; Broniarz-Press and Woziwodzki, 2008; Pakzad et al., 2008). In gassed conditions, the mixing time may be different, especially at low stirring velocity, where aeration contributes to the mixing (Henzler and Obernosterer, 1991). Measurement and prediction of mixing times are also necessary to drive scale-down experiments with fermentation broths, in order to quantify the sensitivity to concentration gradients.

Gas–liquid mass transfer has been widely studied since the 1970's. Among the available methods, dynamic measurement based on dissolved oxygen (DO) probes is often preferred. This technique has been reviewed by several authors (Dunn and Heinsele, 1975; Dang et al., 1977; van't Riet, 1979; Linek et al., 1987). They point out that for accurate determination of  $k_L a$ , complete system dynamics need to be considered and consequently electrode characteristic time and gas/liquid hydrodynamics should be included. It has been found that very large errors can occur with oxygen probes applied to large reactors. When the residence time of the gas phase is not negligible versus the characteristic time scale of mass transfer,  $1/k_L a$ , the assumption of an instantaneous step on  $O_2$  partial pressure is not valid (van't Riet, 1979; Linek et al., 1987). The time response,  $\tau_p$ , of probes is also a potential problem when  $\tau_p \geq 1/k_L a$ . Another problem regarding dynamic methods applied to yield stress fluids or shear-thinning fluids is the presence of small stagnant bubbles in the liquid. This issue has been investigated recently by Pinelli (2005, 2007), and

is discussed later in Oxygen Mass Transfer Section. Steady-state chemical methods are also used. These methods are not recommended as they can lead to large errors (Gogate and Pandit, 1999), and their use inside polymer solutions needs fastidious validation studies. Independently of the choice of the method, the size of the reactors used to develop correlations is also a key point. In small reactors, most of the mass transfer takes place at the free surface at the top of the reactors, and this proportion decreases during scale-up (Humphrey, 1998). Many authors have measured  $k_L a$  in CMC solutions inside small reactors ( $V < 10$  L) (Yagi and Yoshida, 1975; Nishikawa et al., 1981; Arjunwadkar et al., 1998; Puthli et al., 2005) and the problems listed above concerning accuracy of  $k_L a$  and power measurements explain the disparity of results. Cabaret et al. (2008) measured mass transfer in CMC and glucose solutions inside unbaffled dual-impeller mixers of 7–14 L. Nishikawa et al. (1981) reported  $k_L a$  measurements in CMC solutions in reactors of various sizes (3–170 L). Cooke et al. (1988) used 0.4% CMC solutions to quantify  $k_L a$  inside 4 m<sup>3</sup> bioreactors, and proposed a correlation to predict  $k_L a$ , including a surprisingly strong effect of the viscosity:  $k_L a \propto \mu^{-1}$ . Several studies have also been done with XG solutions (Pedersen et al., 1994; Garcia-Ochoa and Gomez, 1998) inside small reactors of less than 25 L. Herbst et al. (1992) studied XG production inside 0.2–3 m<sup>3</sup> reactors by oxygen mass balance and validated the correlation of Kawase and Moo-Young (1988) at different scales. This is a very interesting study reports a strong effect of the choice of impellers but it covers a specific range of rheology and operating conditions, as it focuses on very high XG concentrations—up to 2%—and only one gas flow rate. Nearly all the authors agree on the negative impact of the apparent viscosity, with a power law contribution:  $k_L a \propto \mu^\lambda$ , with  $\lambda$  in the range of  $-0.41$  to  $-1$  (Cooke et al., 1988; Herbst et al., 1992; Garcia-Ochoa and Gomez, 2009). In conclusion, very few articles report  $k_L a$  measurements at different scales in presence of a shear-thinning fluid.

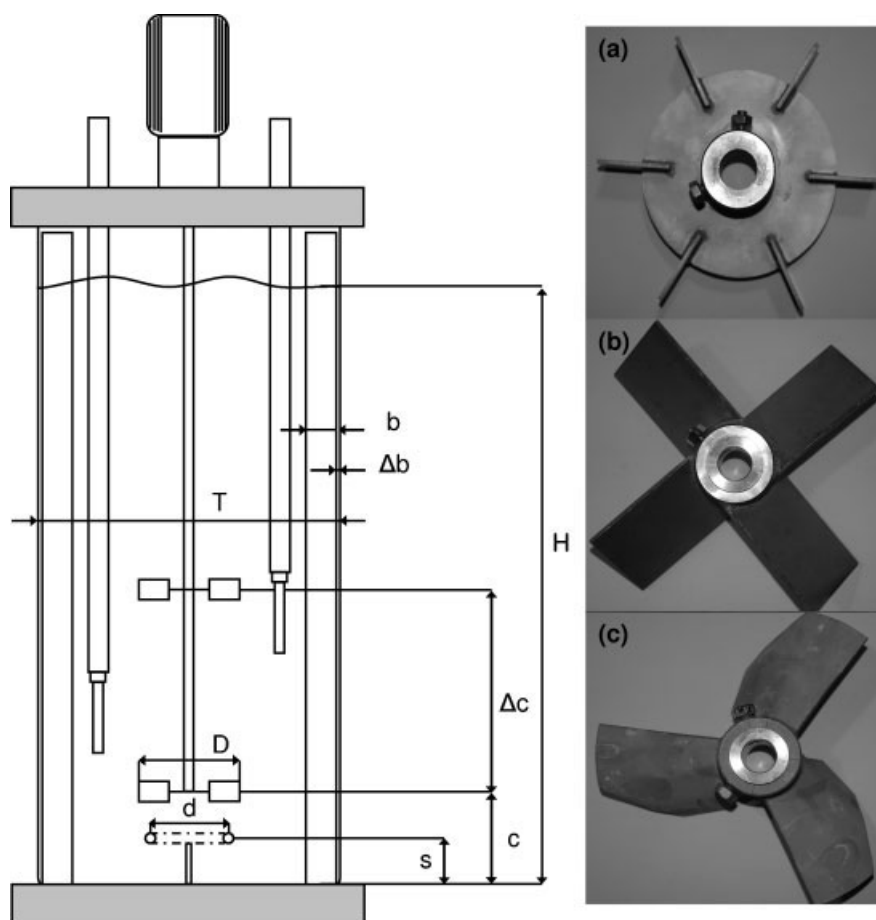
In this context, the present study proposes to complete the available data usable for the scale-up and design of aerated reactors involving shear-thinning fluids, by performing experiments at two different scales: 42 and 340 L. In order to mimic fermentation broths of interest, water, CMC and XG solutions at 0.25% and 0.5% (w/w) were used in a wide range of flow conditions.

## EXPERIMENTAL SETUP

### Stirred Vessels

Experiments were carried out in two geometrically similar multi-staged baffle tanks ( $H/T = 1$  or  $2$ ). The first tank ( $T_1$ ) was a glass vessel with a diameter  $T = 0.3$  m and a volume of liquid of 21 or 42 L. The second tank ( $T_2$ ) was a polymethyl methacrylate (PMMA) vessel with a diameter of 0.6 m and a volume of 170 or 340 L. The vessel equipment and geometrical definitions are shown in Figure 1. The characteristic dimensions of the tanks are listed in Table 1. Compressed air was introduced by two ring spargers with 1 mm holes (25 holes for  $T_1$  and 120 holes for  $T_2$ ). Gas flow rates ranged from  $1.38 \times 10^{-4}$  m<sup>3</sup>/s to  $5.5 \times 10^{-3}$  m<sup>3</sup>/s ( $V_s = 3.9 \times 10^{-3}$  or  $7.8 \times 10^{-3}$  m/s).

Various simple or double impeller combinations were tested with three types of impeller: a Rushton disc turbine (R), a down pumping four-blade 45° pitched turbine (PB) and an axial flow impeller, Mixel TT (Mixel S.A., Dardilly, France) axial flow impeller (TT). Stirrers are shown in Figure 1. The different configurations and impeller sizes are presented in Table 2. Mechanical stirring was supplied by two motors (Leroy Somer LS112M and



**Figure 1.** Geometry of the stirred vessels. The two tanks are homothetics. Data are reported in Table 1. On the right, the three types of stirrer used in this study: (a) six blade Rushton turbine (R); (b) four pitched blade turbine (PB) down flow and (c) Mixel TT (TT).

Sew Eurodrive DV100L4). The maximum rotational speeds were 16.6 (998 rpm) and  $8.85 \text{ s}^{-1}$  (531 rpm) for  $T_1$  and  $T_2$  respectively.

## Model Fluids

For the present work, two of the most common polymers (CMC and XG) were used. XG (G1253 Sigma) was supplied by Sigma-Aldrich Co, St. Louis, while CMC was supplied by two different manufacturers (C5013 Sigma 700 000 g/mol, Sigma-Aldrich and Fischer Scientific C/3100/53). Moderately high concentrations of polymer were chosen (0.25% and 0.5%) in order to simulate a fermentation broth (Pedersen et al., 1993). Sodium azide (0.02%) was used as a biocide for XG runs to avoid

contamination of the solution. According to Escudier et al. (2001), the influence of tap water and biocide on the viscosity is negligible. The polymer in powder form was added to 1 kg of tap water with 0.02% of sodium azide (for XG). Once the polymer had been added to the water, the solution was stirred until complete dissolution was achieved. The solution was then left for 24 h without any mixing to completely hydrate the polymer molecules.

## Rheological Measurements

Rheological measurements were carried out using an AR 2000 controlled stress rheometer (from TA Instruments, New Castle, DE). A 6-cm diameter,  $2^\circ$  aluminium cone geometry was used. The shear rate could be varied from 1 to  $1000 \text{ s}^{-1}$ . Measurements were carried out first with increasing shear rate for 120 s, and then

**Table 1.** Characteristic dimensions of the tanks  $T_1$  and  $T_2$

Tank	$T_1$ (21–42 L)	$T_2$ (170–340 L)
$T$ (m)	0.3	0.6
$D$ (m)	0.1	0.2
$d$ (m)	0.05	0.15
$s$ (m)	0.05	0.1
$H$ (m)	0.3–0.6	0.6–1.2
$c$ (m)	0.1	0.2
$\Delta c$ (m)	0.23	0.53
$b$ (m)	0.02	0.04
$\Delta b$ (m)	0.005	0.01

**Table 2.** Configuration and size of stirrers used in this work

Configuration	Lower impeller	Upper impeller	$D/T$ ( $T_1$ )	$D/T$ ( $T_2$ )
R	R	—	0.33	0.33
PB	PB	—	0.33–0.66	0.33–0.53
TT	TT	—	0.33–0.66	0.33–0.53
RPB	R	PB	0.33	0.33
RTT	R	TT	0.33	0.33
TTTT	TT	TT	0.33	0.33
RR	R	R	0.33	0.33

with decreasing shear rate for same length of time. Temperature was kept constant at 20°C. For the 0.25% CMC (Fischer Scientific) medium, because of the low viscosity, a Contraves Low Shear 30 viscometer with concentric cylinder geometry was used.

## Power Measurements

In this work, the torque was determined by an optical-electronic torque meter method (Dataflex 22/20 N/m and 22/100 N/m, KTR, Rheine, Germany, for  $T_1$  and  $T_2$ ). The accuracy of the measurement was  $\pm 0.5\%$  of the maximum torque ( $\pm 0.1$  and  $\pm 0.5$  N/m for  $T_1$  and  $T_2$ ). As suggested by several authors (Ascanio et al., 2004; Albaek et al., 2008), power losses due to friction were subtracted. All the torque measurements were made as an average value over a 30 min period at constant rotational speed and flow rate. Then, at the end of the run, the torque was subtracted from the measured torque at minimum stirred speed ( $1.9$  and  $1.3 \text{ s}^{-1}$  for  $T_1$  and  $T_2$ ). At this speed, the value of the torque was in the range of uncertainty of the measurement. Un-aerated and aerated power numbers were calculated at the maximum stirring speed to minimise uncertainties on the measurements. For the case of the  $R$  configuration, the un-aerated power number was measured at the maximum allowable speed avoiding air entrainment from the free surface. Most of the measurements were carried out in fully dispersed regime.

## Mixing Time Method

Mixing times were measured inside the two reactors filled with different media and with or without gas injection. Only the RPB configuration was studied. Two experimental methods were used depending on the media, in order to record mixing times corresponding to 95% of homogeneity ( $\theta_{95}$ ).

### Colorimetric method

A colorimetric method was used to measure mixing times in polymer solutions. Dye volumes of 0.025% of the total volume were injected at the level of the higher impeller. The injection device was specially designed to disperse the dye radially at the level of the impeller. Injection time was less than 2 s. A camera was used to record the dye injection and its mixing inside the reactor, and in-house image processing software was developed to calculate mixing times. Pictures were divided into 6 windows. The mean grey level inside each window was computed and normalised between 0 and 1. The variance method described by Brown et al. (2004) was used to calculate  $\theta_{95}$ :

$$\text{Log}(\sigma_{\text{RMS}}^2) = \text{Log} \left\{ \frac{1}{np} \sum_{i=1}^{np} (C_i - 1)^2 \right\} \quad (1)$$

where  $np$  is the number of probes and  $C_i$  the normalised signal of the probe  $i$ . Mixing was considered complete when  $\text{Log}(\sigma_{\text{RMS}}^2) = -2.6$ .

### Conductivity method

A conductivity method was used to measure mixing times in water. Injection volumes were 0.3% of the total volume. 250 g/L NaCl solutions were used. Several conductivity probes placed at different locations in the reactors were tested and the conductivity probe finally used to measure  $\theta_{95}$  was placed below the lower impeller. In these conditions, a comparison between the two methods showed that equal mixing times were measured with an accuracy of  $\pm 3$  s for mixing times in a range of 10–50 s.

## Mass Transfer

### Dynamic oxygen electrode method

The method used for the calculation of the volumetric mass transfer coefficient was the dynamic method based on DO probes. In this method, aeration is started in deoxygenated liquid and the DO concentration is measured.

Reasonable estimates can be obtained when a simple model of perfect mixing for the liquid and plug flow for the gas is assumed (MP model). The transport equations of oxygen concentration in each phase can be written as:

$$\frac{dC_l}{dt} = \frac{k_L a}{1 - \varepsilon_G} \left( \frac{C_g}{m} - C_l \right) \quad (2)$$

$$\frac{\partial C_g}{\partial t} = - \frac{Q_g}{S \times \varepsilon_G} \times \frac{\partial C_g}{\partial x} - \frac{k_L a}{\varepsilon_G} \left( \frac{C_g}{m} - C_l \right) \quad (3)$$

The oxygen concentration is measured at the middle of the vessel height (Nocentini, 1990; Nocentini et al., 1993). In our study, the electrode response could be neglected since a 'Clark-type' fast oxygen micro sensor (Ox 100 fast; Unisense, Aarhus, Denmark), with a response time shorter than 2 s, was used.

### Start-up method

A variant of the dynamic oxygen electrode method called 'start-up method' was used in this work. Agitation and nitrogen aeration were stopped until all bubbles had escaped from the vessel. Then, agitation was re-started, followed by air flow. According to Gogate and Pandit (1999), with this method, the effect of non-ideal mixing in the gas phase is largely suppressed.

## RESULTS

### Flow Curves

Flow curves of the polymer media are presented in Figure 2 for shear rates between 10 and  $300 \text{ s}^{-1}$ , which is the range expected

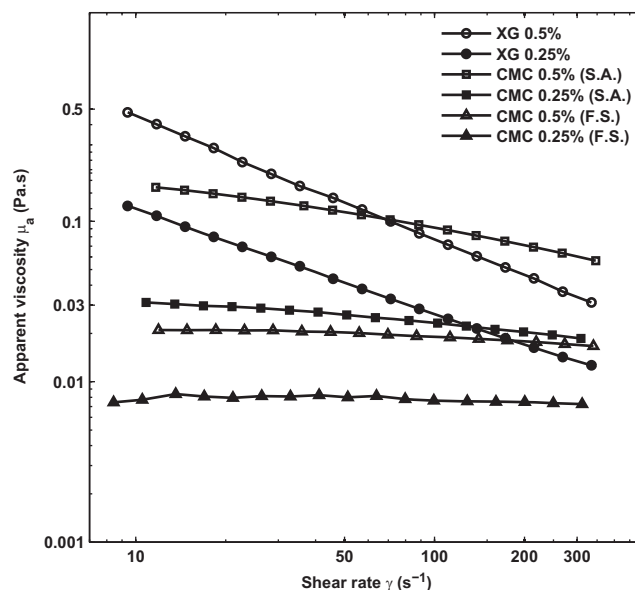


Figure 2. Apparent viscosity of model fluids versus shear rate.



**Table 3.** Rheological properties of model media fitted to the power law model (Equation 4) in the range of 10–300 s<sup>-1</sup>

Medium	n	K (Pa s <sup>n</sup> )
XG 0.5%	0.26	3.34
XG 0.25%	0.37	0.48
CMC 0.5% (Sigma-Aldrich)	0.65	0.44
CMC 0.25% (Sigma-Aldrich)	0.82	0.05
CMC 0.5% (Fischer Scientific)	0.90	0.03
CMC 0.25% (Fischer Scientific)	0.95	0.01

in the bulk of our stirred vessels. Apart from the CMC 0.25% (Fischer Scientific), all media could be considered as non-Newtonian shear-thinning liquids. The rheological data were fitted to the Ostwald de Waele model (power law) in this range (Equation 4), and had high regression coefficients. Results of data fitting are presented in Table 3.

$$\mu_a = K \times |\dot{\gamma}|^{n-1} \quad (4)$$

## Power Draw

### Ungassed Power number

For a stirrer system and for a Newtonian fluid, the Reynolds number (*Re*) is defined as:

$$Re = \frac{\rho_l N D^2}{\mu} \quad (5)$$

Following the assumption of Metzner and Otto (1957), the modified Reynolds number for non-Newtonian fluids becomes:

$$Re_m = \frac{\rho_l N^{2-n} D^2}{K (k_s^{n-1})} \quad (6)$$

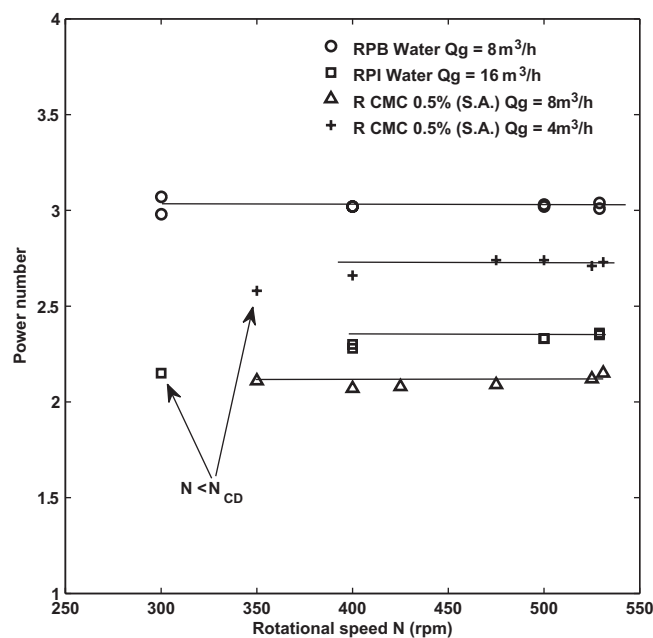
The power number is defined as follows:

$$Np_0 = \frac{P_0}{(\rho_l N^3 D^5)} = \frac{2\pi \times N \times M}{(\rho_l N^3 D^5)} \quad (7)$$

For all the media used in this work, power measurements were conducted at the maximum rotational speed so as to minimise the error measurements (500 rpm in *T*<sub>1</sub> and 300 rpm in *T*<sub>2</sub>). For the *R* configuration, the velocity was lowered in order to avoid air entrainment at the interface. The data reported in Table 4 are consistent with the work of Nienow et al. (1996).

**Table 4.** Ungassed Power number measured at the maximum stirred speed without air entrainment

Configuration/ <i>Np</i> <sub>0</sub>	<i>T</i> <sub>1</sub>	<i>T</i> <sub>2</sub>
R	3.93	3.97
PB	1.63	1.44
TT	0.64	0.56
RPB	5.22	5.93
RTT	4.55	5.28
TTTT	1.03	1.13
RR	6.80	—



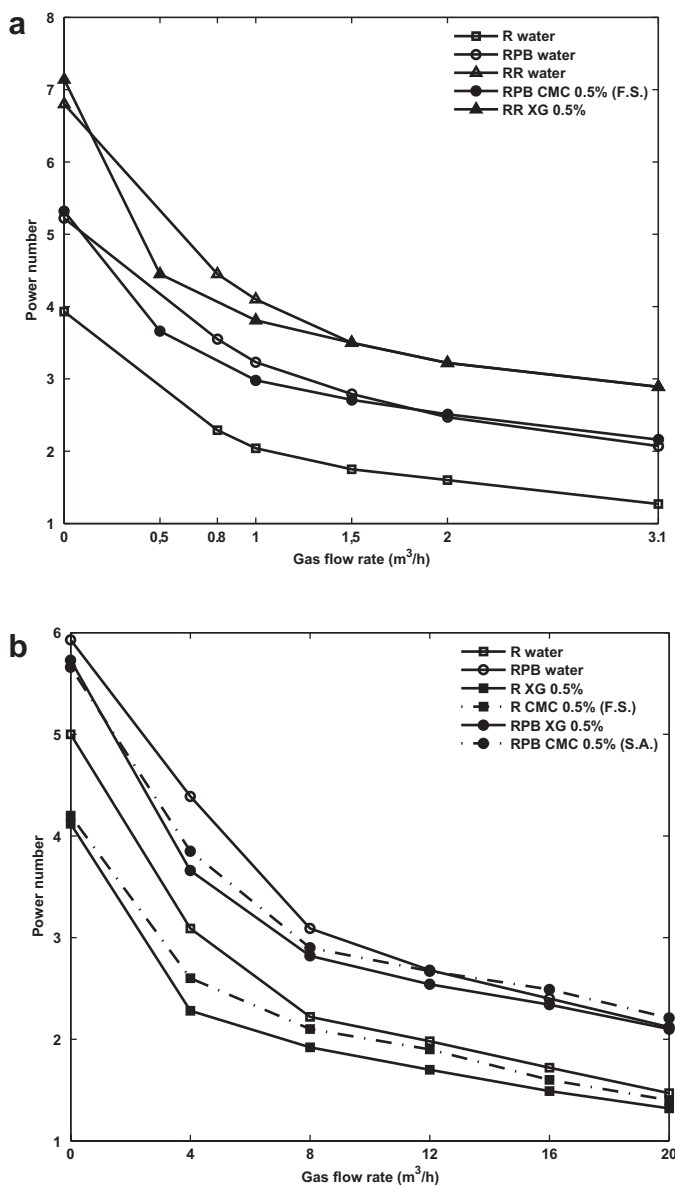
**Figure 3.** Power numbers at fixed value of gas flow rate measured in *T*<sub>2</sub> versus rotational speed.

The power dissipated by two impellers was found to be close to the sum of the power dissipated individually (average error 8%). These results are in good agreement with the work of Hudcova et al. (1989) for unaerated dual impeller disc turbines, as the ratio of the spacing of the two impellers over the diameter ( $\Delta c/D$ ) is higher than 2.

### Gassed Power number

In the case of aerated systems and for most agitators, the power consumption is lower than in unaerated systems due to the presence of cavities behind the blades. In the present study, the dispersion regime ( $N > N_{CD}$ ) (Nienow, 1990) was investigated. In this regime, the gassed power number was found to be independent of the stirrer speed, *N*, as illustrated in Figure 3. *N*<sub>CD</sub> can be estimated following Nienow (1990): flooding of impellers is avoided when  $Fl_G < 30 Fr (D/T)^{3.5}$ . This condition was always respected during the experiments reported. In this figure, the power number was measured for a fixed gas flow rate and for different stirrer speeds. The gassed power number remained constant except when the stirrer velocity was lower than the minimum velocity for the total dispersion. The same results can be found in the work of Nocentini et al. (1988); and Abrardi et al. (1990). Below the minimum stirrer speed, *N*<sub>CD</sub>, a decrease in power number is observed. The decrease in power number is induced by the increase of the gas flooding number ( $Fl_G = Q_g/ND^3$ ). In consequence, for each configuration, including tank size and impeller type, and at given gas flow rate, a constant power number was assumed in the complete dispersion regime.

The effect of rheology and gas flow rate on power number is reported in Figure 4 for various configurations, media and stirred vessels. At lower gas flow rates, there is a slight influence of the rheology. In non-Newtonian media, the power loss due to the presence of gas is higher than in water. At higher gas flow rate, power numbers are the same as in water or within the measurement errors. The small effect of rheology on *P*<sub>g</sub> has already been mentioned in the literature (Luong and Volesky, 1979; Machon et al., 1980). Therefore, to establish *k*<sub>L</sub>*a* correlations, power



**Figure 4.** (a) Power numbers versus gas flow rate for  $T_1$  and for various configurations and media. (b) Power numbers versus gas flow rate for  $T_2$  and for various configurations and media.

number values measured in water were preferred because a large amount of data was available. Whatever the reactor size and the dual impeller configuration, power curves follow the same trends as those obtained with the same lower impeller in a single stage system ( $H/T = 1$ ). These data are in good agreement with the literature indicating that, in a multi-impeller system, aerated power draw follows the same trend as the lower impeller alone (Hudcova et al., 1989; Abrardi et al., 1990; Gogate et al., 2000). This is due to the fact that most of the gas from the sparger goes through the lower impeller. The gas cavities are mainly formed around the lower impeller and hence reduce the power level. Only a small amount of gas goes through other impellers.

Many correlations have been published since the 1960's to predict the relative power draw,  $P_g/P_0$ . First, mono-impeller systems fitted with Rushton turbines were investigated (Calderbank and Moo-Young, 1959; Michel and Miller, 1962; Nagata, 1975; Loiseau et al., 1977; Luong and Volesky, 1979). Then, multi-impeller sys-

tems were explored (Hudcova et al., 1989; Abrardi et al., 1990; Bouaifi et al., 2001; see Paul et al., 2004 for more details). Despite this large amount of data, it seems difficult to find a single correlation that is valid for any device. Hence it seems that  $P_g/P_0$  depends on: the geometry of the tank, the scale of the tank, the gas sparger design and location, the clearance, the  $D/T$  ratio, and the type of stirrer. Cited correlations were tested in our case but led to large errors ( $>40\%$ ). Moreover, most of them were not suitable for a null flow rate. Thus, it was decided to find another way to correlate  $P_g/P_0$  for both scales and all the configurations. The data for both vessels can be fitted by a correlation in terms of relative power draws,  $P_g/P_0$ . This correlation depends strongly on the flow rate and the tank diameter. Little influence of the stirrer to tank diameter ratio was found. The power number of the lower stirrer was added according to its dominant role as discussed above:

$$\ln\left(\frac{P_g}{P_0}\right) = -15.36 \times Q_g^{0.62} \times T^{-1.7} \left(\frac{D}{T}\right)^{0.51} \times Np_{0,ls}^{0.16} \quad (8)$$

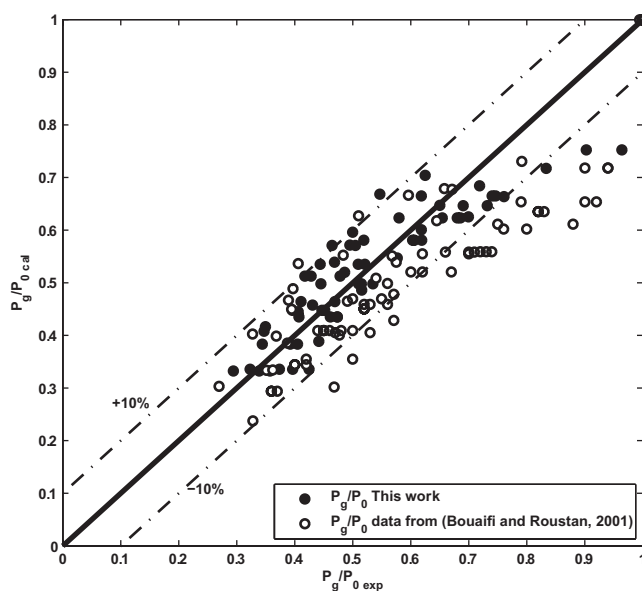
The accuracy of the correlation was 10% on average. As shown in Figure 5, the highest values of  $P_g/P_0$  were difficult to predict with this type of correlation. However, at industrial scale, high flow rates are recommended. Data were compared with the data of Bouaifi and Roustan (2001), who studied a number of multi-impeller configurations and were in acceptable agreement. As mentioned above, discrepancies could be attributed to differences of sparger and clearance.

## Mixing Times

### Ungassed mixing times

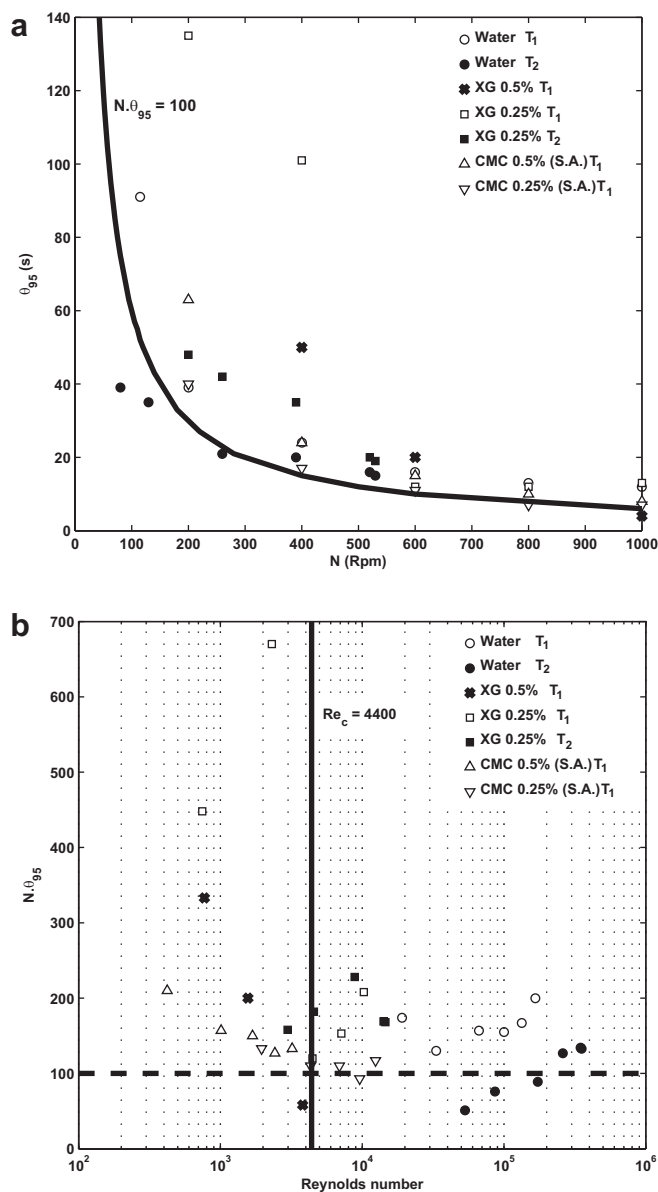
In single phase flows, mixing times are classically predicted by the following expression:

$$N \times \theta_{95} = A \times Re^B \quad (9)$$



**Figure 5.** Comparison between values of  $P_g/P_0$  calculated from Equation (8) and experimental measurements from Bouaifi and Roustan (2001) and this study.

where  $B$  is generally equal to zero in turbulent flows and  $A$  depends on the geometry of the reactor. Concerning the results presented, in the case of water,  $B$  is null and  $A$  is observed to be close to 100. This rather high value is explained by the use of a double-impeller system and the relatively low position of the higher impeller, at only  $H/T$ . Results are reported in Figure 6 following two classical formalisms. First,  $\theta_{95}$  is reported versus  $N$  (Figure 6a). In water,  $\theta_{95}$  roughly follows Equation 9 with  $A = 100$  and  $B = 0$ , in both reactors. But when non-Newtonian media are used, large differences between media and reactors are observed. Concerning XG solutions, at  $N > 500$  rpm, rheology have no effect on mixing time, but below this value mixing times are significantly higher in  $T_1$ . Mixing times are shorter in the bigger reactor. When CMC solutions were used, mixing times were increased compared to those for water, but less than in XG solutions. In Figure 6b, the results are reported according to the Reynolds number.

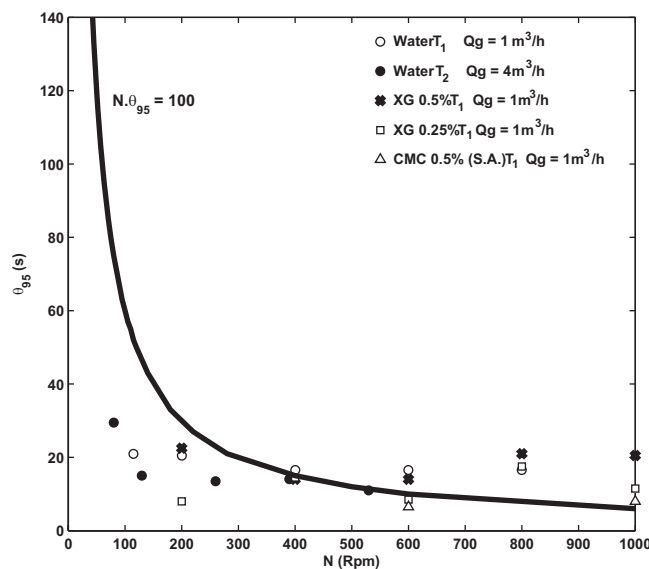


**Figure 6.** (a) Ungassed mixing time at 95% of homogeneity plotted versus the stirrer speed. Continuous line refers to  $N \times \theta_{95} = 100$ . (b)  $N \times \theta_{95}$  plotted versus the Reynolds number calculated from Equation (6). RPB configuration in water and in various model media in  $T_1$  and  $T_2$ .

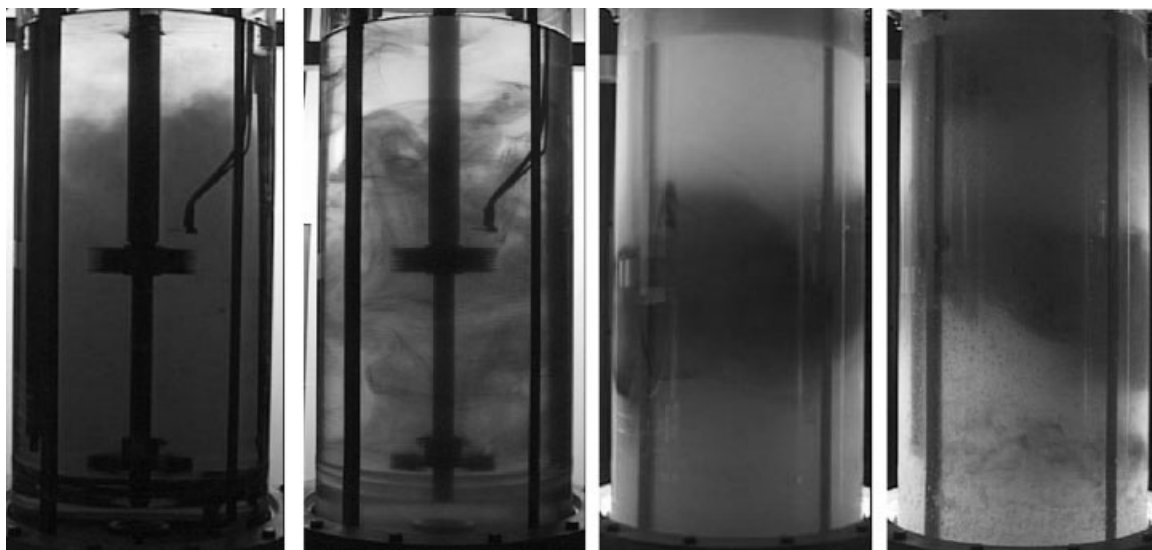
They show that above the critical  $Re$  number of 4400 reported by Cooke et al. (1988), mixing numbers  $N \cdot \theta_{95}$  can be considered as constant. Short mixing times induce high noise on mixing numbers in the range of the  $Re$  number. When  $Re_c < 4400$ , mixing numbers are no longer constant, as the fully turbulent regime is no longer achieved. The relation between mixing and Reynolds number depends on the medium: CMC and xanthan solutions show very different trends. For example, at  $Re \approx 1000$ ,  $N \times \theta_{95}$  varies approximately from 150 to 400 between CMC and XG solutions. Low Reynolds numbers were only studied in the smallest reactor, and measurements performed at the same stirring velocity in the bigger reactor corresponded to much higher  $Re$  numbers, that is,  $Re > 4400$ . Differences in  $Re$  numbers explain the decrease of  $\theta_{95}$  with scale-up at given  $N$ . But scale-up of industrial processes is more usually performed at constant  $P/V$ . Thus, for a given geometry,  $P/V \propto N^3 D^2$ , and, at given  $P/V$ ,  $N \propto D^{-2/3}$ .  $Re \propto ND^2 \propto D^{4/3}$  so  $Re$  increases during scale-up. In consequence, measurements made at  $Re < 4400$  in the small reactor are not relevant for scale-up, as fully turbulent flow is not reached. Measurements made in the bigger reactor are much more representative of industrial flows.

### Mixing times under gassed conditions

Similar measurements were performed under gassed conditions and are reported in Figure 7. Only one superficial velocity was studied ( $V_s = 7.8 \times 10^{-3}$  m/s). Comparison between the different media in the small reactor did not bring out any effect of rheology or stirring velocity, as all the mixing times were close to 15 s. The effect of the reactor size was also negligible in this range of operating conditions. At the lower stirring velocities, visual observations showed that mixing was induced by the gas flow, which generated recirculation loops in the reactors. At higher  $N$ , impellers dispersed the gas in the reactor and drove the mixing phenomena, but mixing times were higher than without gas flow. The increase in mixing time induced by the gas is often explained by the loss of power consumption induced by the gas as was discussed earlier (Cooke et al., 1988; Gao et al., 2000; Zhao et al., 2001). Some authors have thus suggested replacing  $N$  in equations similar to Equation (9) by  $(P_g/V)^{1/3}$ .  $P_g/P_0$  losses of  $\sim 40\%$  observed at



**Figure 7.** Gassed mixing time at 95% of homogeneity plotted versus the stirrer speed. Continuous line refers to  $N \times \theta_{95} = 100$ .



**Figure 8.** Tracer mixing after 4 s of injection for  $T_1$  and RPB at 400 rpm. From left to right: Water, CMC 0.5% (Sigma-Aldrich), XG 0.5%, XG 0.5% at  $V_s = 7.8 \times 10^{-3}$  m/s.

$V_s = 7.8 \times 10^{-3}$  m/s are consistent with deviations observed under gassed and ungassed mixing, when  $N > 400$  rpm.

Snapshots of dye mixing inside  $T_1$  are shown in Figure 8 after 4 s in water, in 0.5% CMC and XG solutions without gas and in XG solution with gas flow. Stirring velocity was 400 rpm. The photographs show that mixing in water and CMC are comparable, although turbulence is lower in CMC, but not complete as a part of the liquid at the top is not coloured yet. Mixing in ungassed XG is slower and a kind of cavern is observed in the middle of the reactor. This shows that axial mixing is poor in this medium. At the same time, mixing is better in presence of gas flow because bubbles induce axial mixing in the reactor. Dye starts to be observable at the bottom and in the higher part of the reactor. To conclude on mixing times, a marked effect of the rheology has been observed at low  $Re$  number without any gas flow. But the impact of rheology disappears under gassed conditions or without gas when  $Re$  is above  $\sim 4000$ .

## Oxygen Mass Transfer

When aerated viscous media were used, a small number of small bubbles were present in the tank even if aeration was stopped. These bubbles had a diameter of the order of 1 mm and a high residence time. According to Heijnen et al. (1980), these small bubbles are in equilibrium in the liquid phase and are considered as perfectly mixed. In our experiments, the maximum residual hold up was estimated to be 0.4% in XG and negligible in CMC solutions. Considering this measurement, the maximum error in  $k_L a$  was 15%. However, in this model, the system is assumed to be non-coalescent, which should not be the case in XG and CMC solutions as surface tension has been measured at 75 and  $65 \times 10^{-3}$  N/m for XG 0.5% (Sigma-Aldrich) and CMC 0.5% (Sigma-Aldrich) respectively. Moreover, a measurement with the same operating conditions ( $N$  and  $Q_g$ ) was repeated, varying the time between successive measurements and allowing the small bubbles to escape. The results showed no variations in  $k_L a$  value.

Another way to take the presence of small bubbles into account is to assume that they contribute to oxygen transfer (Muller and Davidson, 1992; Shetty et al., 1992; Kantak et al., 1995; Khare and Niranjana, 1995). With this approach, better information on bubble size, coalescence, and precise gas hold up are needed to

distinguish the contribution of small and large bubbles to  $k_L a$ . Consequently, it was decided to neglect the role of small bubbles on mass transfer.

## Resolution of the model

For each experiment,  $k_L a$  and 'start time' were initialised as parameters of the regression. The system was solved in a Matlab 7.9 code (The MathWorks Inc., Natick) following a least square method. Experimental and calculated absorption curves were compared and were in good agreement with the MP model even for the 'start time', which refers to the first increase of oxygen concentration in the liquid. For the liquid phase, various approaches were also tested, such as a cascade of ideal mixing with back mixing, corresponding to different recirculation and mixing times, but no significant changes were observed in the  $k_L a$  value when mixing time was short enough.

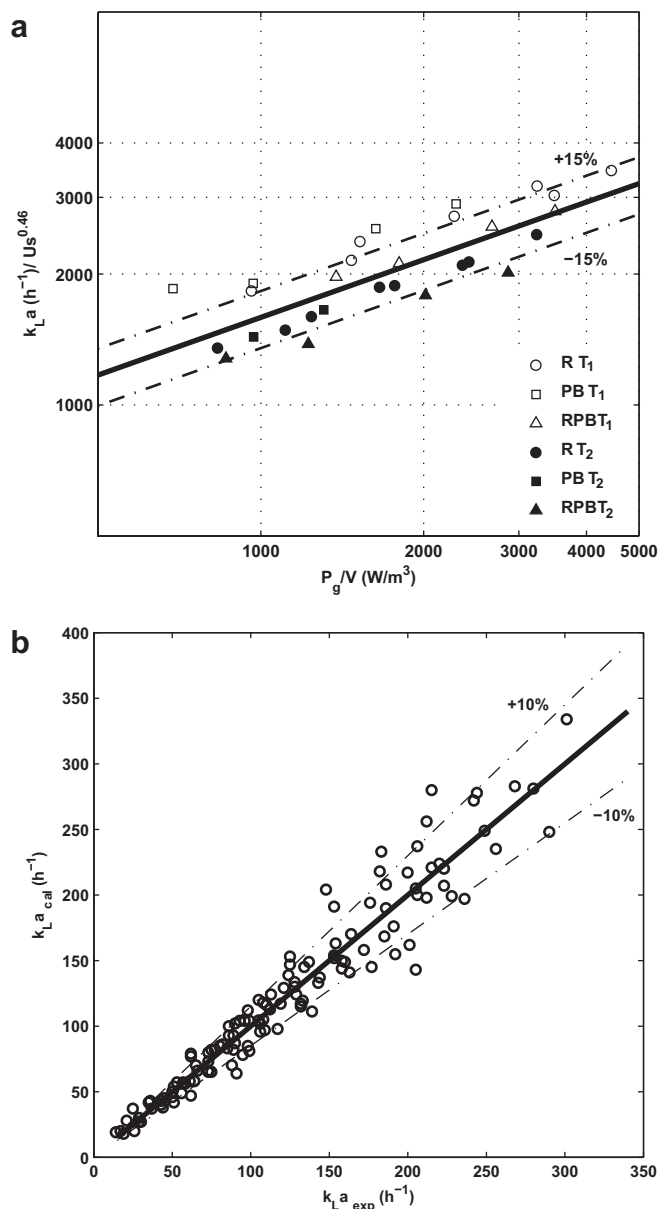
## Mass transfer in water

Different correlations are available in the literature concerning the calculation of the volumetric mass transfer coefficients.  $k_L a$  values are usually given by a correlation of the form:

$$k_L a = C \left( \frac{P_g}{V} \right)^\alpha (V_s)^\beta \quad (10)$$

Available correlations are usually established for one type of stirred configuration and/or one size of vessel. In this study, mass transfer measurements were performed for both tanks,  $T_1$  and  $T_2$ , and for various types of stirrer and configuration (single or double). At the same power input,  $P_g/V$ , and gas flow rate,  $Q_g$ , it was found that the mass transfer coefficient,  $k_L a$ , was approximately independent of the type of stirrer (data not shown). This result is consistent with the work of Bouaifi and Roustan (1998) who studied a number of configurations of a double impeller stirrer system. Then, although there was a small difference between the  $k_L a$  values obtained for the two reactors (Figure 9a), it was decided to establish one correlation for all types of stirrer, configuration, and tank size. Experimental measurements were correlated in the form of Equation (10) with an average error of 11% (Figure 9b), with  $C = 0.02$ ,  $\alpha = 0.44$  and  $\beta = 0.448$ . Table 5 summarises





**Figure 9.** (a) Effect of the tank size on mass transfer in water.  $k_L a$  measured in various configurations are divided by  $U_s^{0.46}$  and plotted versus  $P_g/V$ . (b) Parity diagram of Equation (10). Water measurements.

the other correlations that were found in the literature. Data obtained with other correlations were compared to our experimental values and showed an average error of between 20% and 31%.

These different correlations were compared (data not shown) for both superficial gas velocities used in this study. A moderate divergence in  $k_L a$  prediction was seen but the values obtained were within the range of published works. The small differences can be attributed to experimental conditions. For example, a multi-impeller reactor ( $H/T \geq 3$ ) usually gives a higher value of gas hold up and volumetric mass transfer is improved. Then, as outlined above, the measurement method can lead to other values of  $k_L a$ . For instance, Moucha et al. (1995) used the dynamic pressure method. The size of the tank is also important and data for a relatively large tank ( $>100$  L) are scarce.

### Mass transfer in non-Newtonian media

Mass transfer experiments performed with viscous media were analysed in order to point out the effect of rheology on  $k_L a$ , and the evolution of  $k_L a$  during scale-up. In Figure 10a and b, the  $k_L a$  values for  $T_1$  and for the model media are presented. No distinction was made between the type of stirrer and configuration. As discussed above, for a given medium and various impeller configurations, equal volumetric power draws and flow rate gave similar values of  $k_L a$ , considering the precision of measurements. The negative influence of the rheology was observed. Values of  $k_L a$  in water were two or three times higher than the values in model media. Moreover, the values obtained in XG 0.25% were higher than the value of XG 0.5%. For XG 0.5% and CMC 0.5%, the values of  $k_L a$  were very similar for  $T_1$ . In Figure 11, the impact of the size of the vessels is shown. A loss of  $k_L a$  were observed with the tank size, but the effect clearly depends on the rheology of the media, as it is small concerning water ( $\sim 15\%$ ) and much stronger concerning most viscous media, and more pronounced for XG 0.5%. In Figures 10 and 11, lines present the value of  $k_L a$  obtained with the model discussed later in the study.

### Average shear rate

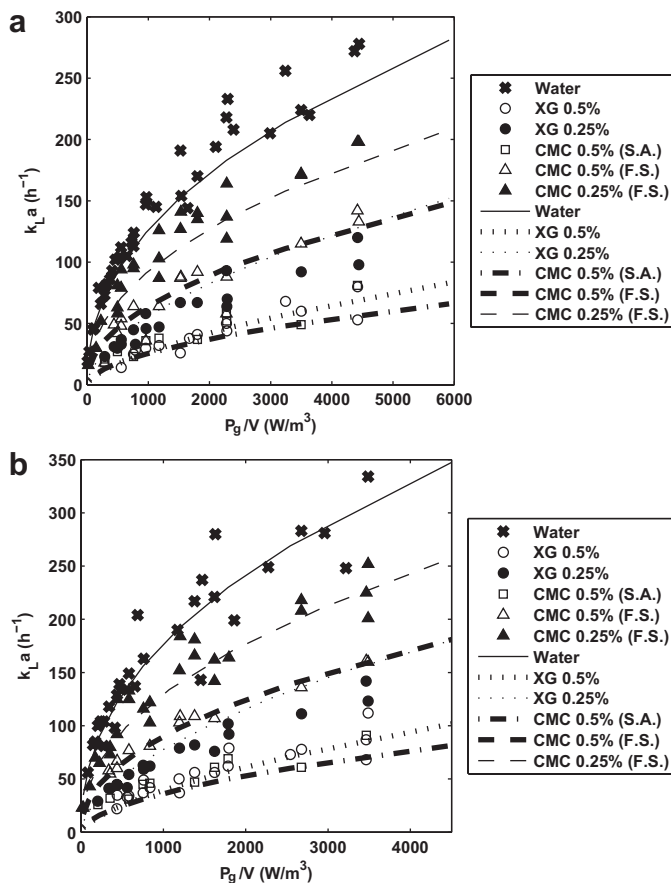
In the case of non-Newtonian fluids, apparent viscosity depends on the shear rate (Equation 4). Therefore, a definition of the appropriate shear rate is required. Shear rate is not uniform in the tanks (Sanchez Perez et al., 2006), and it decreases with the distance from the impeller. It may fall to zero in stagnant zones. The most commonly used correlation is that proposed by Metzner and Otto (1957). These authors assume that the fluid motion in the vicinity of the impeller can be described by a volume averaged shear rate,  $\dot{\gamma}_{av}$ , which is linearly related to the rotational speed of the impeller:

$$\dot{\gamma}_{av} = k_s \times N \quad (11)$$

The constant,  $k_s$ , depends on the impeller and tank configuration. Usually, a value between 10 and 13.5 is taken (Nienow and

**Table 5.** Correlations for volumetric mass transfer prediction in the form of Equation (10)

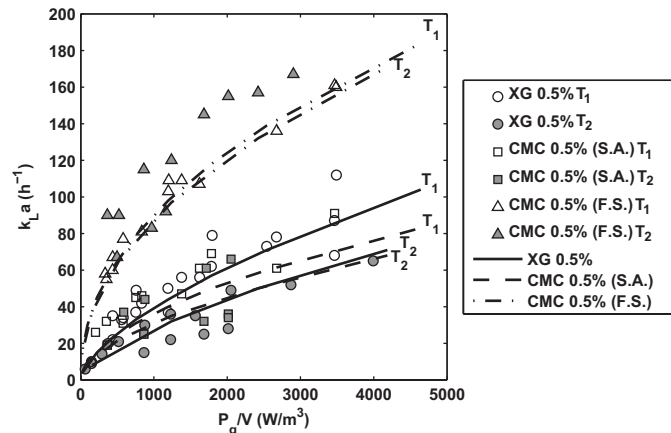
Refs.	Correlation	$T$ (m)	$V_s$ (mm/s)	Average error (%)	$H/T$	Stirrer
This study	$k_L a = 0.020 (P_g/V)^{0.44} (V_s)^{0.448}$	0.3–0.6	3.9–7.8	11	1 and 2	Various
Nocentini et al. (1993)	$k_L a = 0.015 (P_g/V)^{0.59} (V_s)^{0.55}$	0.232	1.5–10	24	4	4R
Moucha et al. (1995)	$k_L a = 0.018 (P_g/V)^{0.58} (V_s)^{0.588}$	0.19	2.12–8.4	20	4	4R
Linek et al. (1996)	$k_L a = 0.009 (P_g/V)^{0.637} (V_s)^{0.54}$	0.19	2.12–8.4	23	4	4R
Linek et al. (1987)	$k_L a = 0.005 (P_g/V)^{0.593} (V_s)^{0.4}$	0.29	2.21–4.2	22	1	R
Vasconcelos et al. (2000)	$k_L a = 0.006 (P_g/V)^{0.66} (V_s)^{0.51}$	0.392	0.8–6.5	25	2	Various
Bouaifi and Roustan (1998)	$k_L a = 0.022 (P_g/V)^{0.5} (V_s)^{0.6}$	0.43	5.4–18	31	2	Various
van't Riet (1979)	$k_L a = 0.026 (P_g/V)^{0.4} (V_s)^{0.5}$	0.14–1.25	3–15	28	—	—



**Figure 10.** (a)  $k_La$  measurements, performed in  $T_1$  and for  $V_s = 3.9 \times 10^{-3}$  m/s and for various model media, plotted versus the aerated volumetric power draw. Continuous lines refer to Equation (16). (b)  $k_La$  measurements, performed in  $T_1$  and for  $V_s = 7.8 \times 10^{-3}$  m/s and for various model media, plotted versus the aerated volumetric power draw. Continuous lines refer to Equation (16).

Elson, 1988). Calderbank and Moo-Young (1959) proposed a modification of the relationship with a function of the flow behaviour index:

$$\dot{\gamma}_{av} = k_s \times N \left( \frac{4n}{3n+1} \right)^{(n/(1-n))} \quad (12)$$



**Figure 11.** Comparison between  $k_La$  obtained in  $T_1$  and  $T_2$  for  $V_s = 7.8 \times 10^{-3}$  m/s and for various model media. Continuous lines refer to Equation (16).

These two approaches are based on laminar measurements and are pointed out by Herbst et al. (1992) as providing very similar results for XG solutions. In cases of turbulent flows, these correlations have to be used with caution. Based on dimensional considerations, Henzler and Kauling (1985) proposed a correlation that should be valid in fully turbulent flows:

$$\dot{\gamma}_{av} \propto \left( \frac{P}{V} \times \frac{1}{\mu_a} \right)^{0.5} \propto \left( \frac{P}{V} \times \frac{1}{K} \right)^{(1/(1+n))} \quad (13)$$

However, these authors could not reasonably determine the proportionality constant and assumed a value of 1.

In this work, these various approaches were tested to correlate our experimental results.

### Validity of correlations

Correlations for the prediction of  $k_La$  in non-Newtonian systems can be classified into two types: the dimensional form and the non-dimensional form. The first derives from Equation (10) and is in the form:

$$k_La = C \left( \frac{P_g}{V} \right)^\alpha (V_s)^\beta (\mu_a)^\lambda \quad (14)$$

with a value of  $\lambda$  in the range of  $-0.41$  to  $-1$ .

**Table 6.** Comparison of the fitted correlations with our experimental results

Data used for the fitting	Equations	Shear rate	Exponent on			Average error (%)	
			$P_g/V$	$V_s$	$\mu_a$	Data used for correlation	Prediction of $T_2$ model media
Water $T_1$ /Water $T_2$ /MM $T_1$	14	Metzner and Otto (1957)	0.45	0.50	-0.32	17	50
Water $T_1$ /Water $T_2$ /MM $T_1$	14	Henzler and Kauling (1985)	0.42	0.48	-0.32	17	54
Water $T_1$ /Water $T_2$ /MM $T_1$	14	Calderbank and Moo-Young (1959)	0.45	0.50	-0.31	16	50
No fitting	15	Calderbank and Moo-Young (1959)	$(9+n)/(10(1+n))$	0.5	$(1/((2(1+n))-0.25))$	33*	41*

\* With a correcting factor of 1.15.

Herbst et al. (1992) listed the non-dimensional correlations for the prediction of  $k_L a$  in non-Newtonian systems. These authors performed  $k_L a$  measurement during Xanthan production in tanks with volumes ranging from 0.072 to 3 m<sup>3</sup>. They point out the difficulties of using this type of correlation without any modification of the constants. The best fit on their measurements after modification is given by the correlation of Kawase and Moo-Young (1988):

$$k_L a = C \times \sqrt{D_{O_2}} \times \rho_1^{3/5} \times \frac{(P_g/\rho_1 V)^{9+4n/10(1+n)}}{(K/\rho_1)^{1/2(1+n)} \sigma^{3/5}} \left( \frac{V_s}{V_t} \right)^{0.5} \left( \frac{\mu_a}{\mu_w} \right)^{-0.25} \quad (15)$$

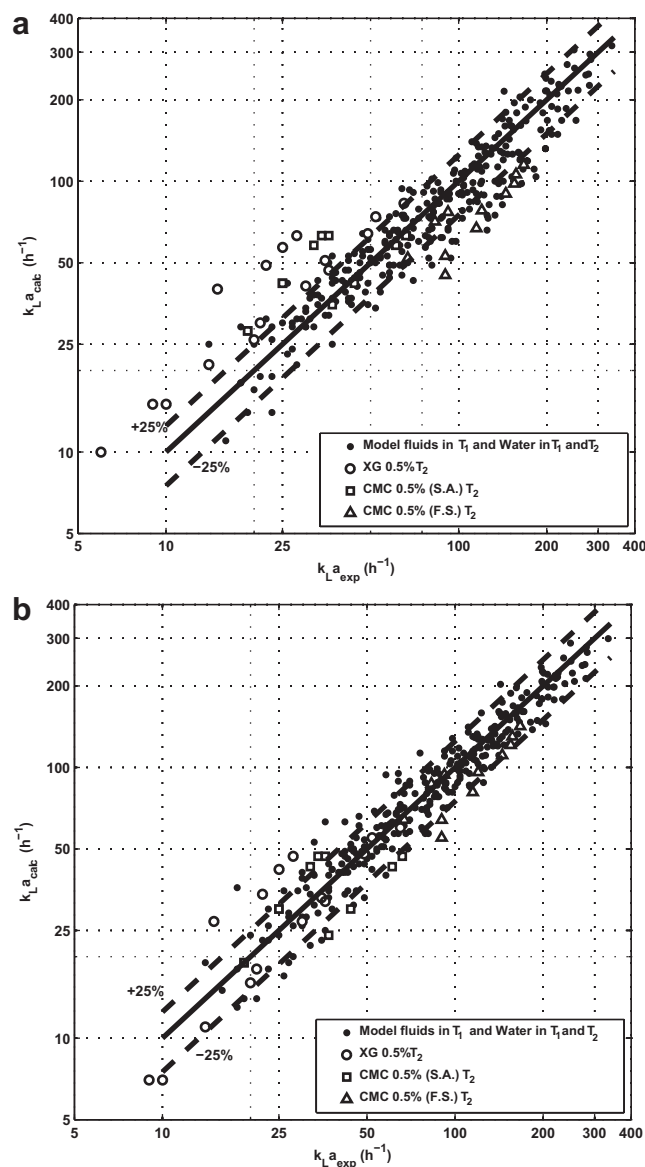
with  $C=0.675$ .

This correlation allows more importance to the flow curves parameters  $K$  and  $n$ . Concerning media of interest in this study, the exponent on the power draw ranges from 0.65 for water to 0.80 for the 0.5% XG (Sigma-Aldrich).

The relevancy of correlations concerning scale-up was quantified by fitting parameters only with small-scale experiments ( $T_1$ ). Experiments performed inside the larger tank  $T_2$  were then used for the validation of the correlation. The three ways of calculating the shear rate were also tested. Optimal values of fitted parameters are presented in Table 6. Different expressions for shear rate furnish very similar parameters of Equation (14) ( $\alpha=0.42$  or 0.45,  $\beta=0.48$  or 0.5 and  $\lambda=-0.31$  or  $-0.32$ ), and also very similar average errors on  $T_1$  and  $T_2$  data. The exponent on  $\lambda$  is slightly lower than generally found in the literature. For example, Nishikawa et al. (1981) found a value of  $-0.5$ , Yagi and Yoshida (1975) predicted a value of  $-0.4$ , Ranade and Ulbrecht (1978) found a value of  $-0.41$  and, more recently, Cabaret et al. (2008) found values between  $-0.25$  and  $-0.5$  in dual impeller mixers, depending on the positions of the shafts. The exponent on the power dissipated remains the same as in water using Equations (11 and 13). Concerning the superficial velocity, the exponent marginally decreases from 0.448 to 0.42 using Equation (12), compared to  $k_L a$  values obtained in water. A global comparison between experimental data and data given by Equation (14), using Equation (11) for shear rate calculation, is reported in Figure 12a. The agreement between predicted and experimental  $k_L a$  is good for all the data series (error  $\sim 17\%$ ), except concerning the most viscous solutions in tank  $T_2$ , for which the correlation overestimates  $k_L a$  by approximately 45–55%.

It can be noted that the  $k_L a$  values calculated for tank  $T_2$  and for XG 0.5% (Sigma-Aldrich) and CMC 0.5% (Sigma-Aldrich) are over estimated by the correlation. In contrast, the correlation falls in the prediction of relatively low shear thinning model media CMC 0.5% (Fischer Scientific). When this type of correlation is used to predict  $k_L a$  values in tank  $T_2$ , observed errors in Figure 12a are of the order of 59% for the equation of Metzner and Otto (1957); and Calderbank and Moo-Young (1959) and 63% for the equation proposed by Henzler and Kauling (1985).

Using the correlation proposed by Kawase and Moo-Young (1988), the average error is almost constant between the two vessels, but rather high. This correlation seems to be a little more appropriate for predicting the scale-up of  $k_L a$  concerning highly viscous media, as pointed out by Herbst et al. (1992). However, the correlation falls in the prediction of low viscous media and water. Using a classical power law correlation for the prediction of our experiments leads to large errors during scale-up. Indeed, it is not possible to fit a single correlation in good agreement with all experimental data. For instance, a parameter regression based



**Figure 12.** (a) Parity diagram of Equation (14) with the average shear rate calculated from Equation (11). (b) Parity diagram of Equation (16) with the average shear rate calculated from Equation (11).

only on high shear thinning media would find a higher exponent on the apparent viscosity ( $-0.59$ ). This observation leads us to introduce a novel formulation that better reproduces the physical reality of viscosity in an aerated tank. The influence of the apparent viscosity in Equation (14) is replaced by the factor  $1/(1 + \mu_a/\mu_c)$  with  $\mu_c$  defined as a critical apparent viscosity in the tank. The correlation becomes:

$$k_L a = C \left( \frac{P_g}{V} \right)^\alpha (V_s)^\beta \left( 1 + \frac{\mu_a}{\mu_c} \right)^{-1} \quad (16)$$

Below this value, the impact of the rheology on mass transfer is moderate and above, the effect is increased. The value of this constant was determined by data fitting of the correlation on our experiments ( $C=0.023$ ,  $\alpha=0.44$ ,  $\beta=0.47$  and  $\mu_c=0.018$  Pa/s). The average shear rate was calculated by the Metzner and Otto (1957) correlation. The result of this fitting is reported in Figure 12b. An exponent on  $(1 + \mu_a/\mu_c)$  was tested but remains close

to  $-1$ . Scale-up prediction is improved as the average error on model media in  $T_1$  and water in  $T_1$  and  $T_2$  is 15% and 23% for the predication of  $k_L a$  for model media in  $T_2$ . This correlation was reported in Figures 10 and 11, and an acceptable agreement was observed with data. The effect of rheology and scale-up are well predicted by Equation (16). But, however useful this improved correlation may be, caution has to be exercised concerning the interpretation of its physical meaning. Very strong variations of local fluid properties exist in a stirred reactor, and correlations based only on macroscopic lengths are very risky if used outside their validation range.

## CONCLUSION

The first step of this study was to investigate power consumption in non-Newtonian media. It was found that, in the range of studied, the influence of rheology on power consumption was very similar to that found with water, under gassed or ungassed conditions. A correlation was proposed to account for the decreasing power efficiency due to aeration. This correlation is available for a wide choice of stirrer and tank configurations and a diameter ranging from 0.3 to 0.6 m.

Then, mixing times were studied. In an unaerated system, two regimes of mixing were identified. Below a critical Reynolds number ( $Re_C \approx 4000$ ), mixing is impacted by rheology. Mixing time of a viscous medium increases with its shear thinning behaviour, or with the fluid index  $n$ , of the solution. Dead zones are observed near baffles and far away from the stirrer. When the Reynolds number is higher than  $Re_C$ , flow seems to be totally controlled by turbulence since no differences are observed between media. In fully turbulent aerated systems, mixing times are slightly lower than in unaerated systems and are of the same order as in water. At low stirring rate, mixing time is controlled by the gas flow, which keeps it at low values ( $\theta < 20$  s).

Finally, gas-liquid mass transfer was characterised in various media and stirring conditions. For all experiments, it was observed that, at given power consumption, the type of stirrer had no influence on the mass transfer. The results obtained in water are in good agreement with the literature, and a small effect of the size of the tank is quantified on  $k_L a$ . According to literature, values of  $k_L a$  determined in viscous model media are significantly reduced compared to water.

With most viscous media, a strong effect of the tank size is demonstrated. Discrepancies found between the experiments presented and correlations available in the literature can be attributed to the fact that studies are usually made at one size of reactors and with one medium.

Two types of correlation were tested in this study. First, a classical power law was tested on the apparent viscosity using three methods for the calculation of average shear rate. It appeared that for all the methods, the results were unsatisfactory and failed in the prediction of  $k_L a$  in tank  $T_2$ . The correlation of Kawase and Moo-Young (1988) was also tested and gave better results for the model media in tank  $T_2$ . However, mass transfers in water and in low viscosity media were not predicted satisfactorily. Then, the effect of viscosity was modelled following a new expression, and the factor  $1/(1 + \mu_a/\mu_c)$  was introduced in  $k_L a$  correlation. Benefits of this formulation are highlighted by the good agreement with experimental data acquired with all model fluids and both size of tanks.

However, the calculation of the average shear remains uncertain, and the physical meaning of the regressed critical viscosity is not clear. In the future, local investigations will be necessary

to analyse interactions between distribution of shear rates, bubble size and mass transfer. Concerning the understanding of mass transfer phenomena inside fermentation broths, the effect of the heterogeneous texture of the real media should also be investigated via the use of fibre suspensions as model fluids.

## NOMENCLATURE

$a$	specific area ( $\text{m}^{-1}$ )
$A$	constant in Equation (9)
$B$	constant in Equation (9)
$b$	baffle width (m)
$\Delta b$	space between baffle and tank (m)
$c$	impeller clearance (m)
$\Delta c$	impeller spacing (m)
$C_i$	normalised signal of the probe $i$
$C$	constant in Equations (10, 14–16)
$C_g$	oxygen concentration in the gas phase ( $\text{mol}/\text{m}^3$ )
$C_l$	oxygen concentration in the liquid phase ( $\text{mol}/\text{m}^3$ )
$d$	sparger clearance (m)
$D$	stirrer diameter (m)
$D_{O_2}$	diffusivity ( $\text{m}^2/\text{s}$ )
$Fl_G$	Flooding number ( $Fl_G = Q_g/ND^3$ )
$Fr$	Froude number ( $Fr = N^2 D/g$ )
$H$	height of liquid in tank (m)
$k_s$	Metzner–Otto constant ( $k_s = 11.5$ )
$k_L$	liquid side mass transfer ( $\text{m}/\text{s}$ )
$k_L a$	volumetric mass transfer coefficient ( $\text{s}^{-1}$ )
$K$	fluid consistency factor ( $\text{Pa} \cdot \text{s}^n$ )
$m$	proportionality coefficient
$M$	torque ( $\text{N} \cdot \text{m}$ )
$n$	flow index in power law model
$N$	stirrer speed ( $\text{s}^{-1}$ )
$N_{CD}$	minimum stirrer speed for complete dispersion ( $\text{s}^{-1}$ )
$Np_0$	ungassed Power number
$Np_{ls}$	ungassed Power number of the lower stirrer
$np$	number of probes
$P_g$	aerated power (W)
$P_0$	un-aerated power (W)
$Q_g$	gas flow rate ( $\text{m}^3/\text{s}$ )
$Re$	Reynolds number
$Re_m$	modified Reynolds number
$T$	tank diameter (m)
$V$	volume ( $\text{m}^3$ )
$V_s$	superficial gas velocity ( $\text{m}/\text{s}$ )
$V_t$	terminal rise velocity ( $\text{m}/\text{s}$ ) in Equation (12), $V_t = 0.265 \text{ m/s}$

## Greek Symbols

$\alpha$	exponent on the volumetric power draw in the correlation for $k_L a$ prediction
$\beta$	exponent on superficial velocity in the correlation for $k_L a$ prediction
$\dot{\gamma}$	shear rate ( $\text{s}^{-1}$ )
$\dot{\gamma}_{av}$	average shear rate ( $\text{s}^{-1}$ )
$\varepsilon_G$	gas hold-up
$\theta$	mixing time (s)
$\theta_{95}$	mixing time corresponding to 95% homogeneity (s)
$\lambda$	exponent on apparent viscosity in the correlation for $k_L a$ prediction
$\mu_a$	apparent viscosity ( $\text{Pa} \cdot \text{s}$ )
$\mu_c$	critical apparent viscosity ( $\text{Pa} \cdot \text{s}$ )



$\mu_w$	viscosity of water (Pa · s)
$\mu$	viscosity (Pa · s)
$\rho_l$	density (kg/m <sup>3</sup> )
$\sigma_{RMS}$	variance of $C_i$
$\sigma$	surface tension (N/m)
$\tau$	shear stress (Pa)

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