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The Spin (3/2, 1) Multiplet and Superspace Geometry[1]

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Abstract. We give a completely geometrical derivation of the de Wit-van Holten formulation of a supermultiplet with physical helicity states one and three-halves. We identify all of the components of the multiplet, derive the transformation laws, and construct an invariant action. Like supergravity the action is not expressible only in terms of field strengths.

Within the context of rigid $N=1$ supersymmetry it is well known that multiplets consist of physical components with helicities s and $s-1/2$. For the choice $s=3/2$, a spin (3/2, 1) or matter gravitino multiplet is thus obtained. Historically this multiplet was first discovered [1] in the construction of a consistent free-field theory for a spinor superfield. The theory must be free, the presence of consistent interactions for the spin 3/2 component field requires supergravity, i.e., local supersymmetry. The $N=1$ matter gravitino multiplet was later rediscovered [2] in an attempt to clarify the geometric structure of supersymmetric gauge theories. The matter gravitino multiplet has also played a role in extended supergravity theories. It was soon after the discovery of the matter gravitino multiplet as a rigid $N=1$ supermultiplet that the on-shell formulation of $N=2$ supergravity was found [3]. Also, the first off-shell form of $N=2$ supergravity required a reformulation [4] of the $N=1$ gravitino multiplet. This implied that, like supergravity, the matter gravitino multiplet possesses a number of inequivalent off-shell formulations. This can be understood as the ambiguity in which irreducible representations of a prepotential, Ψ_α, appear in the construction of an action. In Ref. 5 all of the independent supertensors which describe the Wit-van Holten formulation were constructed in terms of Ψ_α. A set of differential equations which the super-tensors satisfy were derived.

At this stage the understanding of the (3/2, 1) multiplet was very reminiscent of the earlier development of super Yang-Mills theories [6]. The prepotential and supertensors were known but there appeared to be no relation to the geometrical structure of gauge theories in x-space in terms of fiber bundles, connection, etc. This was unsatisfactory for two reasons. First, this was indicative of a lack of understanding of the superspace geometry of these theories. Second and more practical, to develop a background superfield method [7], a completely geometrical formulation of a theory must first be developed [8]. The fact that this condition was satisfied for the Yang-Mills theory permitted the construction. The geometrical quantities (connections, field strengths) essentially become background superfields and the unconstrained prepotentials become the quantum superfields. This is a general feature of supersymmetric gauge theories.

So a primary motivation for searching for a geometrical formulation of the matter gravitino multiplet is to develop a similar framework. This is particularly important in light of the observation that $N=1$ supergraph techniques appear to offer the

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most reasonable method for carrying out investigations of the quantum behavior of extended supergravity theories. The first step toward the development of a geometric formulation of the matter gravitino multiplet appeared [9] in considerations of reducing $N=2$ superspace to $N=1$ superspace. It was noted that such a reduction naturally leads to the introduction of $N=1$ superfields Ψ_A^ε and $\bar{\Psi}_A^{\dot\varepsilon}$. In other words, the theory must contain quantities that are similar to Yang-Mills superconnections but with an additional spinor index. It is the purpose of this work to show that by starting from Ψ_A^ε, its field strength and a set of suitable constraints, all of the results of [5] can be reconstructed from solely geometric means.

In the framework of superspace geometry⋆ the helicity (3/2, 1) multiplet is described in terms of spinorial superpotentials $\Psi_{A\dot\varepsilon}$ and $\bar{\Psi}_{A\dot\varepsilon}$ subject to gauge transformations of the form

$$\delta \Psi_A{}^\varepsilon = D_A \xi^\varepsilon, \qquad \delta \Psi_{A\dot\varepsilon} = D_A \bar{\xi}_{\dot\varepsilon}. \tag{1}$$

The super index $A \sim (a, \alpha, \dot\alpha)$ describes vectorial as well as spinorial indices. The covariant field strengths are given by

$$R_{BA}{}^x = D_B \Psi_A{}^x - (-)^{AB} D_A \Psi_B{}^x + T_{BA}{}^C \Psi_C{}^x \tag{2}$$

where the x index stands for ε and $\dot\varepsilon$ only. The super derivative $D_A \sim (\partial_a, D_\alpha, \bar{D}^{\dot\alpha})$ contains the usual space-time derivative ∂_a and the covariant derivative of rigid superspace. They satisfy the graded commutation relations

$$[D_B, D_A\} = D_B D_A - (-)^{AB} D_A D_B = -T_{BA}{}^c D_c \tag{3}$$

where $(-)^{AB}$ is equal to -1 if and only if A and B both denote spinor indices. The only nonvanishing component of the torsion $T_{BA}{}^C$ in rigid superspace is⋆⋆

$$T_\beta{}^{\dot\alpha c} = -i2(\sigma^c \varepsilon)_\beta{}^{\dot\alpha}. \tag{4}$$

As a consequence of (2) and (3) the field strength satisfies a set of Bianchi identifies,

$$\oint_{CBA} (D_C R_{BA}{}^x + T_{CB}{}^D R_{DA}{}^x) = 0. \tag{5}$$

Initially we will constrain the geometric structure by imposing a set of covariant conditions on some of the components of $R_{BA}{}^x$. These are

$$R_{\beta\alpha}{}^x = 0, \qquad R^{\dot\beta\dot\alpha x} = 0, \tag{6}$$

$$R_\beta{}^{\dot\alpha\varepsilon} = \delta_\beta^\varepsilon \bar{\lambda}^{\dot\alpha}, \qquad \bar{R}^{\dot\beta}{}_{\alpha\dot\varepsilon} = \delta_{\dot\varepsilon}^{\dot\beta} \lambda_\alpha, \tag{7}$$

$$R_{\beta a}{}^\alpha = \delta_\beta^\alpha \bar{Y}_a, \qquad \bar{R}^{\dot\beta}{}_{a\dot\alpha} = \delta_{\dot\alpha}^{\dot\beta} Y_a, \tag{8}$$

$$(\bar\sigma^a)^{\dot\varepsilon\beta} \bar{R}_{\beta a\dot\varepsilon} - (\sigma^a)_{\varepsilon\dot\beta} R^{\dot\beta}{}_a{}^\varepsilon = 0. \tag{9}$$

We will now demonstrate how the de Wit-van Holten formulation of the multiplet containing physical helicities 3/2 and 1 as well as auxiliary fields emerges from this geometrical structure by solving the constraints. A solution to (6) is provided by introducing prepotentials Ψ^x and Σ^x in terms of which Ψ_α^x and $\bar{\Psi}_{\dot\alpha}^x$ can be expressed as

$$\begin{aligned} \Psi_\alpha{}^\varepsilon &= \tfrac{1}{2} D_\alpha (\Sigma^\varepsilon - \Psi^\varepsilon), & \bar{\Psi}^{\dot\alpha\varepsilon} &= \tfrac{1}{2} \bar{D}^{\dot\alpha} (\Sigma^\varepsilon + \Psi^\varepsilon), \\ \bar{\Psi}^{\dot\alpha}{}_{\dot\varepsilon} &= \tfrac{1}{2} \bar{D}^{\dot\alpha} (\bar\Sigma_{\dot\varepsilon} - \bar\Psi_{\dot\varepsilon}), & \Psi^\alpha{}_{\dot\varepsilon} &= \tfrac{1}{2} D^\alpha (\bar\Sigma_{\dot\varepsilon} + \bar\Psi_{\dot\varepsilon}). \end{aligned} \tag{10}$$

The superfields Σ^x and Ψ^x transform as

$$\begin{aligned} \delta \Sigma^x &= \sigma^x + \tau^x + 2\xi^x, \\ \delta \Psi^x &= \sigma^x - \tau^x, \end{aligned} \tag{11}$$

under combined gauge (parameter ξ^x) and pregauge (parameters σ^x and τ^x) transformations. The pregauge transformation leaves the potentials $\Psi_A{}^x$ inert if the conditions

$$\begin{aligned} D_\alpha \sigma_{\dot\varepsilon} &= 0, & \bar{D}^{\dot\alpha} \sigma_\varepsilon &= 0, \\ D_\alpha \tau^\varepsilon &= 0, & \bar{D}^{\dot\alpha} \bar\tau_{\dot\varepsilon} &= 0. \end{aligned} \tag{12}$$

Having used (7) to express Ψ_a^ε and $\bar\Psi_{a\dot\varepsilon}$ in terms of other superfields⋆

$$\begin{aligned} \Psi_\alpha{}^{\dot\alpha\varepsilon} &= i\tfrac{1}{4}[D_\alpha, \bar{D}^{\dot\alpha}] \Psi^\varepsilon \\ &\quad + \tfrac{1}{2}\partial_\alpha{}^{\dot\alpha} \Sigma^\varepsilon - i\tfrac{1}{2}\delta_\alpha{}^\varepsilon \bar\lambda^{\dot\alpha}, \\ \bar\Psi_\alpha{}^{\dot\alpha}{}_{\dot\varepsilon} &= -i\tfrac{1}{4}[D_\alpha, \bar{D}^{\dot\alpha}] \bar\Psi_{\dot\varepsilon} \\ &\quad + \tfrac{1}{2}\partial_\alpha{}^{\dot\alpha} \bar\Sigma_{\dot\varepsilon} - i\tfrac{1}{2}\delta_{\dot\varepsilon}{}^{\dot\alpha} \lambda_\alpha, \end{aligned} \tag{13}$$

we proceed with (8). First of all, from the explicit definitions (2) together with (10) and (13) we find:

$$\begin{aligned} R_{\beta\alpha}{}^{\dot\alpha}{}_\varepsilon &= i\tfrac{1}{4}\varepsilon_{\beta\alpha} D^2 \bar{D}^{\dot\alpha} \Psi_\varepsilon - i\tfrac{1}{2}\varepsilon_{\varepsilon\alpha} D_\beta \bar\lambda^{\dot\alpha}, \\ R_{\dot\beta\alpha}{}^{\dot\alpha}{}_{\dot\varepsilon} &= i\tfrac{1}{4}\delta_{\dot\beta}{}^{\dot\alpha} \bar{D}^2 D_\alpha \bar\Psi_{\dot\varepsilon} - i\tfrac{1}{2}\delta_{\dot\varepsilon}{}^{\dot\alpha} \bar{D}_{\dot\beta} \lambda_\alpha. \end{aligned} \tag{14}$$

Now from the constraint (8) in the form

$$\begin{aligned} R_{\beta a\varepsilon} + R_{\varepsilon a\beta} &= 0, \\ R_{\dot\beta a\dot\varepsilon} + R_{\dot\varepsilon a\dot\beta} &= 0, \end{aligned} \tag{15}$$

we learn after some rearrangement of indices, that

$$\begin{aligned} D_\varepsilon(\bar\lambda^{\dot\alpha} + D^\alpha \bar{D}^{\dot\alpha} \Psi_\alpha) &= 0, \\ \bar{D}_{\dot\varepsilon}(\lambda_\alpha + \bar{D}_{\dot\alpha} D_\alpha \bar\Psi^{\dot\alpha}) &= 0, \end{aligned} \tag{16}$$

⋆ Our conventions and notation are the same as in [10]

⋆⋆ For more details on superspace geometry see e.g. [8], [13], [15] and [14]

⋆ In spinor notation [16], vector indices are replaced by spinor indices, $V_a \to V_{\alpha\dot\alpha} = \sigma^a{}_{\alpha\dot\alpha} V_a$ and hence $\{D_\alpha, \bar{D}^{\dot\alpha}\} = -i2\partial_\alpha{}^{\dot\alpha}$

and, of course,

$$R_{\beta\alpha\dot{\alpha}}{}^{\varepsilon} = -i\tfrac{1}{2}\delta^{\varepsilon}_{\beta} D_{\alpha}\bar{\lambda}_{\dot{\alpha}},$$
$$R_{\beta\alpha\dot{\alpha}}{}^{\dot{\varepsilon}} = -i\tfrac{1}{2}\delta^{\dot{\varepsilon}}_{\beta} \bar{D}_{\dot{\alpha}}\lambda_{\alpha}. \tag{17}$$

This concludes our discussion of (8) and we turn to the final restriction (9). Once again using earlier definition and results, we find

$$R_{\beta\alpha\dot{\alpha}\dot{\varepsilon}} = -i\tfrac{1}{2}\varepsilon_{\dot{\alpha}\dot{\varepsilon}} D_{\beta}\lambda_{\alpha} - i\tfrac{1}{4}\varepsilon_{\beta\alpha} D^2 \bar{D}_{\dot{\alpha}}\bar{\Psi}_{\dot{\varepsilon}},$$
$$R_{\beta\alpha\dot{\alpha}\varepsilon} = i\tfrac{1}{2}\varepsilon_{\alpha\varepsilon}\bar{D}_{\beta}\bar{\lambda}_{\dot{\alpha}} + i\tfrac{1}{4}\varepsilon_{\beta\dot{\alpha}}\bar{D}^2 D_{\alpha}\Psi_{\varepsilon}. \tag{18}$$

Utilizing spinor indices (9) takes the form

$$\varepsilon^{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}}(R_{\beta\alpha\dot{\alpha}\dot{\beta}} - R_{\dot{\beta}\alpha\dot{\alpha}\beta}) = 0, \tag{19}$$

which by means of (18) tells us that

$$D^{\alpha}\lambda_{\alpha} - \bar{D}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} - \tfrac{1}{2}D^2\bar{D}_{\dot{\alpha}}\bar{\Psi}^{\dot{\alpha}} + \tfrac{1}{2}\bar{D}^2 D^{\alpha}\Psi_{\alpha} = 0. \tag{20}$$

It is suggestive to introduce the abbreviations

$$W_{\alpha} = \lambda_{\alpha} + \bar{D}_{\dot{\alpha}} D_{\alpha}\bar{\Psi}^{\dot{\alpha}} - \tfrac{1}{2}\bar{D}^2\Psi_{\alpha},$$
$$\bar{W}^{\dot{\alpha}} = \bar{\lambda}^{\dot{\alpha}} + D^{\alpha}\bar{D}^{\dot{\alpha}}\Psi_{\alpha} - \tfrac{1}{2}D^2\bar{\Psi}^{\dot{\alpha}}, \tag{21}$$

which together with the properties,

$$\{D^2, \bar{D}^{\dot{\alpha}}\} = -2D^{\alpha}\bar{D}^{\dot{\alpha}}D_{\alpha},$$
$$\{\bar{D}^2, D_{\alpha}\} = -2\bar{D}_{\dot{\alpha}}D_{\alpha}\bar{D}^{\dot{\alpha}}, \tag{22}$$

of the spinorial derivative reduces the system of Eqs. (16) and (20) to

$$\bar{D}^{\dot{\alpha}} W_{\alpha} = D_{\alpha}\bar{W}^{\dot{\alpha}} = 0, \tag{23}$$

$$D^{\alpha}W_{\alpha} - \bar{D}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} = 0. \tag{24}$$

It is well known [8] that a solution to this system of equations is*

$$W = \bar{D}^2 D_{\alpha} V, \qquad \bar{W}^{\dot{\alpha}} = D^2\bar{D}^{\dot{\alpha}} V, \tag{25}$$

with V a real unconstrained superfield, subject to pre-gauge transformations $\delta V = \frac{1}{2}(\Omega + \bar{\Omega})$. What do we know about Ω? First, due to (21) and (25) we are finally able to express λ_{α} and $\bar{\lambda}^{\dot{\alpha}}$ in terms of the prepotentials,

$$\lambda_{\alpha} = \bar{D}^2 D_{\alpha} V - \bar{D}_{\dot{\alpha}} D_{\alpha}\bar{\Psi}^{\dot{\alpha}} + \tfrac{1}{2}\bar{D}^2\Psi_{\alpha},$$
$$\bar{\lambda}_{\dot{\alpha}} = D^2\bar{D}_{\dot{\alpha}} V - D^{\alpha}\bar{D}_{\dot{\alpha}}\Psi_{\alpha} + \tfrac{1}{2}D^2\bar{\Psi}_{\dot{\alpha}}. \tag{26}$$

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* In our notation

$D^2 = D^{\alpha}D_{\alpha}, \quad \bar{D}^2 = \bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}},$

$[\bar{D}^2, D_{\alpha}] = -i4\partial_{\alpha}{}^{\dot{\alpha}}\bar{D}_{\dot{\alpha}}, \quad [D^2, \bar{D}^{\dot{\alpha}}] = -i4\partial_{\alpha}{}^{\dot{\alpha}}D^{\alpha},$

$D^{\alpha}\bar{D}^2 D_{\alpha} - \bar{D}_{\dot{\alpha}}D^2\bar{D}^{\dot{\alpha}} = 0, \quad [D^2, \bar{D}^2] = i4\partial_{\alpha}{}^{\dot{\alpha}}[D^{\alpha}, \bar{D}_{\dot{\alpha}}]$

Furthermore, by construction λ_{α} and $\bar{\lambda}^{\dot{\alpha}}$ are left invariant by all gauge transformations and therefore we learn

$$\tfrac{1}{2}\bar{D}^2 D_{\alpha}(\Omega + \bar{\Omega}) - \bar{D}_{\dot{\alpha}} D_{\alpha}\delta\bar{\Psi}^{\dot{\alpha}} + \tfrac{1}{2}\bar{D}^2\delta\Psi_{\alpha} = 0,$$
$$\tfrac{1}{2}D^2\bar{D}^{\dot{\alpha}}(\Omega + \bar{\Omega}) - D^{\alpha}\bar{D}^{\dot{\alpha}}\delta\Psi_{\alpha} + \tfrac{1}{2}D^2\delta\bar{\Psi}^{\dot{\alpha}} = 0. \tag{27}$$

This is solved if $\delta\Psi_{\alpha} = -D_{\alpha}\Omega$ and $D^2\Omega = 0$. Therefore, under a pregauge transformation of V, the spinorial prepotential Ψ^{α} also transforms. Summarizing we observe that we solved the constraints (6)-(9) in terms of unconstrained prepotentials (V, Ψ^{α}, and $\bar{\Psi}_{\dot{\alpha}}$) and a trivial compensating superfield Σ_{α}. The totality of pregauge transformations is given by

$$\delta\Psi^{\alpha} = \sigma^{\alpha} - D^{\alpha}\Omega,$$
$$\delta\bar{\Psi}^{\dot{\alpha}} = \bar{\sigma}^{\dot{\alpha}} - \bar{D}^{\dot{\alpha}}\bar{\Omega},$$
$$\delta V = \tfrac{1}{2}(\Omega + \bar{\Omega}), \tag{28}$$

where the parameters of the transformation are subject to the constraints

$$\bar{D}_{\dot{\alpha}}\sigma_{\alpha} = D_{\alpha}\bar{\sigma}_{\dot{\alpha}} = 0,$$
$$D^2\Omega = \bar{D}^2\bar{\Omega} = 0. \tag{29}$$

The helicity 3/2 field being built in from the outset in the geometric structure, we identify the potential of the helicity 1 field as the value at $\theta = \bar{\theta} = 0$ of the real superfield

$$B_{\alpha\dot{\alpha}} = \tfrac{1}{2}[D_{\alpha}, \bar{D}_{\dot{\alpha}}]V + \tfrac{1}{2}D_{\alpha}\bar{\Psi}_{\dot{\alpha}} - \tfrac{1}{2}\bar{D}_{\dot{\alpha}}\Psi_{\alpha}, \tag{30}$$

with the gauge transformation (which follows from (28) and (30))

$$\delta B_{a} = -i\tfrac{1}{2}\partial_{a}(\Omega - \bar{\Omega}). \tag{31}$$

The full set of component fields, containing auxiliary fields as well, might be obtained by an explicit superfield expansion of the prepotentials, taking into account the gauge transformations. We find it, however, more convenient to display to 20 bosonic and 20 fermionic degrees of freedom as components of the gauge invariant superfield $R_{AB}{}^{x}$. Its spinorial components $R_{\beta\alpha}{}^{x}$, $R^{\dot{\beta}\dot{\alpha}x}$ and $R_{\beta}{}^{\dot{\alpha}x}$ are fixed by the constraints (6) and (7). The other components of $R_{BA}{}^{x}$ can be expressed in terms of the three superfields $W_{\alpha\beta}$, λ_{α}, and P (and their conjugates).

$$R_{\beta\alpha\dot{\alpha}\varepsilon} = i\tfrac{1}{2}\varepsilon_{\beta\varepsilon}D_{\alpha}\bar{\lambda}_{\dot{\alpha}},$$
$$R_{\dot{\beta}\alpha\dot{\alpha}\dot{\varepsilon}} = -i\tfrac{1}{2}\varepsilon_{\dot{\beta}\dot{\varepsilon}}\bar{D}_{\dot{\alpha}}\lambda_{\alpha}, \tag{32}$$

$$R_{\beta\alpha\dot{\alpha}\dot{\varepsilon}} = \varepsilon_{\beta\alpha}\varepsilon_{\dot{\alpha}\dot{\varepsilon}}P + \varepsilon_{\beta\alpha}\bar{W}_{\underset{\smile}{\dot{\alpha}\dot{\varepsilon}}} - i\tfrac{1}{4}\varepsilon_{\dot{\alpha}\dot{\varepsilon}}\oint_{\alpha\beta} D_{\alpha}\lambda_{\beta},$$

$$R_{\dot{\beta}\alpha\dot{\alpha}\varepsilon} = \varepsilon_{\dot{\beta}\dot{\alpha}}\varepsilon_{\alpha\varepsilon}P + \varepsilon_{\dot{\beta}\dot{\alpha}}W_{\underset{\smile}{\alpha\varepsilon}} + i\tfrac{1}{4}\varepsilon_{\alpha\varepsilon}\oint_{\dot{\alpha}\dot{\beta}} D_{\dot{\alpha}}\lambda_{\beta}, \tag{33}$$

$$R_{ba\varepsilon} = -iD_\varepsilon \hat{F}_{ba},$$
$$R_{ba\dot{\varepsilon}} = i\bar{D}_{\dot{\varepsilon}} \hat{F}_{ba}, \tag{34}$$
$$\hat{F}_{ba} = \partial_b B_a - \partial_a B_b. \tag{35}$$

The vector field strength $\hat{F}_{ba}$ is identified as a certain linear combination of $R_{\beta a\dot{\varepsilon}}$ and $R^{\dot{\beta}}{}_a{}^{\varepsilon}$. In spinor indices* where

$$\hat{F}_{ba} \to \hat{F}_{\beta\dot{\beta}\alpha\dot{\alpha}} = -2\varepsilon_{\beta\alpha} F_{\underset{\smile}{\dot{\beta}\dot{\alpha}}} + 2\varepsilon_{\dot{\beta}\dot{\alpha}} F_{\underset{\smile}{\beta\alpha}}, \tag{36}$$

one finds:

$$F_{\underset{\smile}{\dot{\beta}\dot{\alpha}}} = -\tfrac{1}{4} W_{\underset{\smile}{\dot{\beta}\dot{\alpha}}} - i\tfrac{1}{16}(\bar{D}_{\dot{\beta}}\bar{\lambda}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}}\bar{\lambda}_{\dot{\beta}}),$$
$$F_{\underset{\smile}{\beta\alpha}} = -\tfrac{1}{4} W_{\underset{\smile}{\beta\alpha}} + i\tfrac{1}{16}(D_\beta\lambda_\alpha + D_\alpha\lambda_\beta). \tag{37}$$

The superfields in (7) and (33) satisfy the following set of identities,

$$D^2\bar{\lambda}_{\dot{\alpha}} = 0, \qquad \bar{D}^2\lambda_\alpha = 0, \tag{38}$$
$$D_\gamma \bar{W}_{\underset{\smile}{\dot{\beta}\dot{\alpha}}} = 0, \qquad \bar{D}_{\dot{\gamma}} W_{\underset{\smile}{\beta\alpha}} = 0, \tag{39}$$
$$D_\gamma P = i\tfrac{1}{8} D^2\lambda_\gamma, \qquad \bar{D}_{\dot{\gamma}} P = i\tfrac{1}{8}\bar{D}^2\bar{\lambda}_{\dot{\gamma}}, \tag{40}$$
$$D^\beta W_{\underset{\smile}{\beta\alpha}} - i\tfrac{1}{8}D^2\lambda_\alpha - i\tfrac{1}{4}D_\alpha\bar{D}^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}} + \partial_\alpha{}^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}} = 0,$$
$$\bar{D}^{\dot{\beta}} \bar{W}_{\underset{\smile}{\dot{\beta}\dot{\alpha}}} - i\tfrac{1}{8}\bar{D}^2\bar{\lambda}_{\dot{\alpha}} - i\tfrac{1}{4}\bar{D}_{\dot{\alpha}} D_\alpha\lambda^\alpha - \partial^\alpha{}_{\dot{\alpha}}\lambda_\alpha = 0, \tag{41}$$

which just imply the consistency of our constraints (6) with the Bianchi identities (5). The component fields may now be obtained by the technique first developed for flexible (curved) superspace [11]. We apply all combinations of spinorial derivatives to the basic superfields and take the resulting superfield at $\theta = \bar{\theta} = 0$ (this operation will be denoted by a vertical bar) with the results:

$$F_{ba}| = \partial_b B_a(x) - \partial_a B_b(x),$$
$$R_{ba}{}^{\varepsilon}| = \partial_b \psi_a{}^{\varepsilon}(x) - \partial_a \psi_b{}^{\varepsilon}(x),$$
$$R_{ba\dot{\varepsilon}}| = \partial_b \bar{\psi}_{a\dot{\varepsilon}}(x) - \partial_a \bar{\psi}_{b\dot{\varepsilon}}(x),$$
$$P| = P(x),$$
$$\lambda_\alpha| = \lambda_\alpha(x), \qquad \bar{\lambda}^{\dot{\alpha}}| = \bar{\lambda}^{\dot{\alpha}}(x),$$
$$D_\beta\lambda_\alpha| = 2\varepsilon_{\beta\alpha}[\bar{J}(x) + iP(x)] - i2t_{\underset{\smile}{\beta\alpha}}(x),$$
$$\bar{D}_{\dot{\beta}}\bar{\lambda}_{\dot{\alpha}}| = 2\varepsilon_{\dot{\beta}\dot{\alpha}}[J(x) - iP(x)] + i2\bar{t}_{\underset{\smile}{\dot{\beta}\dot{\alpha}}}(x),$$
$$\bar{D}_{\dot{\alpha}}\lambda_\alpha| = i2\,Y_{\alpha\dot{\alpha}}(x),$$
$$D_\alpha\bar{\lambda}_{\dot{\alpha}}| = i2\,\bar{Y}_{\alpha\dot{\alpha}}(x),$$
$$D^2\lambda_\alpha| = \chi_\alpha(x),$$
$$\bar{D}^2\bar{\lambda}_{\dot{\alpha}}| = \bar{\chi}_{\dot{\alpha}}(x), \tag{42}$$

where $B_{ba}(x)$, $\psi_{ba\varepsilon}(x)$, and $\psi_{ba\dot{\varepsilon}}(x)$ are the field strengths of the vector and Rarita-Schwinger fields. The remaining quantities $\lambda_\alpha(x)$, $P(x)$, $J(x)$, $\bar{J}(x)$, $Y_a(x)$, $\bar{Y}_a(x)$, $t_{\beta\alpha}(x)$, $\bar{t}_{\dot{\beta}\dot{\alpha}}(x)$, and $\chi^\alpha(x)$ are the auxiliary fields of the multiplet. All the other component fields in the θ expansion of $R_{BA}{}^{x}$ are expressed in terms of the 20 fermionic and bosonic fields or their space-time derivatives. The free Rarita-Schwinger and Maxwell equations for the physical fields are obtained from the superfield equations $\lambda_\alpha = \bar{\lambda}_{\dot{\alpha}} = 0$. We have thus achieved the completely geometrical derivation of the (3/2, 1) multiplet as anticipated in Ref. [5]. For the construction of its invariant action, we note that it can be written as

$$S = -\tfrac{1}{8}\int d^4x\,d^4\theta[(\Psi^\alpha \bar{R}_{\beta\alpha}{}^{\beta} + \bar{\Psi}_{\dot{\alpha}} R_\beta{}^{\dot{\alpha}\beta})$$
$$-i2V((\bar{\sigma}^a)^{\dot{\beta}\beta}\bar{R}_{\beta a\dot{\beta}} + (\sigma_a)_{\beta\dot{\beta}} R^{\dot{\beta}a\beta})]. \tag{43}$$

There is, however, one more feature of the geometry of this multiplet which we have not discussed. We note that there is a component spin gauge field B_a present. But we have not considered the supergeometry associated with this field. To this end we introduce a superconnection Γ_A such that $\Gamma_a| = B_a$. Denoting the field strength associated with Γ_A by $\hat{F}_{BA}$

$$\hat{F}_{BA} = D_B\Gamma_A - (-)^{AB} D_A\Gamma_B + T_{BA}{}^{c}\Gamma_C, \tag{44}$$

we can ask what constraints are satisfied by $\hat{F}_{BA}$. The answer is found to be

$$\hat{F}_{\beta\alpha} = -i(\Psi_{\alpha\beta} + \Psi_{\beta\alpha}),$$
$$\hat{\bar{F}}^{\dot{\beta}\dot{\alpha}} = i(\bar{\Psi}^{\dot{\alpha}\dot{\beta}} + \bar{\Psi}^{\dot{\beta}\dot{\alpha}}), \tag{45}$$
$$\hat{F}_\beta{}^{\dot{\alpha}} = -i(\Psi^{\dot{\alpha}}{}_\beta + \bar{\Psi}_\beta{}^{\dot{\alpha}}), \tag{46}$$
$$\hat{F}_{\beta a} = i\Psi_{a\beta}, \qquad \hat{\bar{F}}^{\dot{\beta}}{}_a = -i\bar{\Psi}_a{}^{\dot{\beta}}, \tag{47}$$

where $\hat{F}_{ba}$ is again given by (36) and (37). The Bianchi identifies for $\hat{F}_{BA}$ are found to simply reproduce some of the results implied by (6)–(9). In fact if Γ_A (and $\hat{F}_{BA}$) are introduced, a priori, along with $\Psi_A{}^{x}$ (and $R_{BA}{}^{x}$), then we may take as constraints Eqs. (6) and (7) together with (45), (46) and (47). Using the Bianchi identities for the $\hat{F}_{BA}$ one can show that the spin 3/2 part of Eq. (47) is redundant and further derive Eqs. (8) and (9).

From the point of view of fiber bundles, Eqs. (45)–(47) are very unusual. If we regard $\Psi_A{}^{\varepsilon}$, $\bar{\Psi}_{A\dot{\varepsilon}}$, and Γ_A as superconnections for a *supergroup*, then $R_{BA}{}^{\gamma}$, $R_{BA}{}^{\dot{\gamma}}$, and F_{BA} are curvatures for the bundle.

* In addition, in our conventions, one has

$$\hat{F}_{\underset{\smile}{\beta\alpha}} = \tfrac{1}{4}\oint_{\beta\alpha} \partial_\beta{}^{\dot{\alpha}} v_{\alpha\dot{\alpha}}, \qquad \hat{\bar{F}}_{\underset{\smile}{\dot{\beta}\dot{\alpha}}} = -\tfrac{1}{4}\oint_{\dot{\beta}\dot{\alpha}} \partial^\alpha{}_{\dot{\alpha}} v_{\alpha\dot{\beta}},$$

and the Bianchi identity for $\hat{F}_{ba}$ reads

$$\partial^\beta{}_{\dot{\alpha}}\hat{F}_{\underset{\smile}{\beta\alpha}} + \partial_\alpha{}^{\dot{\beta}}\hat{\bar{F}}_{\underset{\smile}{\dot{\beta}\dot{\alpha}}} = 0.$$

Equations (45)–(47) show that the curvature in some "directions" is proportional to the connections.

Although our solution is completely correct at the classical level, there is a modification that can be made which simplifies the quantization of the theory. In [5] the parameter Ω was not subject to the constraint $D^2\Omega=0$. In general in supersymmetric gauge theories [12], the straightforward use of the Fadeev-Popov quantization procedure is valid only for unconstrained and chiral gauge parameters. Here Ω is linear*. But this condition can be avoided by introducing a compensator Φ and changing $\Psi_\alpha{}^\varepsilon$ and $\bar{\Psi}^{\dot\alpha}{}_{\dot\varepsilon}$ according to

$$\Psi_\alpha{}^\varepsilon \to \tfrac{1}{2} D_\alpha(\Sigma^\varepsilon - \Psi^\varepsilon) + \tfrac{1}{4}\delta_\alpha{}^\varepsilon \bar{\Phi},$$
$$\bar{\Psi}^{\dot\alpha}{}_{\dot\varepsilon} \to \tfrac{1}{2}\bar{D}^{\dot\alpha}(\bar{\Sigma}_{\dot\varepsilon} - \bar{\Psi}_{\dot\varepsilon}) + \tfrac{1}{4}\delta_{\dot\varepsilon}{}^{\dot\alpha}\Phi, \tag{48}$$

and still satisfy the constraint $R_{\alpha\beta}{}^x=0$ if $D_\alpha\bar{\Phi}=0$ and $\bar{D}^{\dot\alpha}\Phi=0$. The superfield Φ is another compensator. In the presence of Φ, the action in (43) must be modified by the additional terms,

$$-\tfrac{1}{32}\int d^4x\, d^2\theta\, \Phi[\bar{D}_{\dot\alpha} R_\alpha{}^{\dot\alpha\alpha} + i2(\sigma_a)_{\beta\dot\beta} R^{\dot\beta a\beta}] + \text{h.c.}, \tag{49}$$

where the scalar compensator transforms only by the Ω-parameter as $\delta\Phi=\bar{D}^2\bar{\Omega}$. The quantity multiplying Φ is proportional to J which satisfies $\bar{D}_{\dot\alpha}J=0$ due to the second result in (40). The modified action (the sum of (43) and (49) is now gauge invariant up to surface terms without implying $D^2\Omega=0$. We have thus given a completely geometrical description of the de Wit-van Holten formulation of the (3/2, 1) multiplet in rigid (flat) superspace**.

We have not discussed the geometry of the Ogievetsky-Sokatchev formulation of the (3/2, 1) multiplet. This is presently under study. At this point, it is clear that it has a more elaborate geometrical structure. The geometrical starting point necessitates the introduction of a spinorial superconnection $\Psi_A{}^x$, two superconnections Γ_A and A'_A, and a super two-form f_{AB}. This latter quantity indicates that a Cartan integrable system must play a rôle in this geometry. Results will be reported elsewhere.

* A linear superfield Ω satisfies $D^2\Omega=0$

** The generalisation to flexible superspace can be obtained using the methods discussed in [17]

References

1. V.I. Ogievetsky, E. Sokatchev: JETP Lett. **23**, 58 (1976); J. Phys. **A10**, 2021 (1977)
2. S.J. Gates, Jr.: Phys. Rev. **D16**, 1727 (1977)
3. S. Ferrara, P. van Nieuwenhuizen: Phys. Rev. Lett. **37**, 1669 (1976)
4. E.S. Fradkin, M.A. Vassiliev: Lett. Nuovo Cimento **25**, 79 (1979); B. de Wit, J.W. van Holten: Nucl. Phys. **B155**, 530 (1979)
5. S.J. Gates, Jr., W. Siegel: Nucl. Phys. **B164**, 484 (1980)
6. A. Salam, J. Strathdee: Phys. Lett. **51B**, 39 (1974); S. Ferrara, B. Zumino: Nucl. Phys. **B79**, 413 (1974)
7. M.T. Grisaru, W. Siegel, M. Rocek: Nucl. Phys. **B159**, 429 (1979)
8. J. Wess: Proceedings of the VIIIth GIFT International Seminars on Theoretical Physics (Salamanca, 1977); S.J. Gates, Jr.: M.I.T. Preprint #605, Feb. 1977 (unpublished)
9. S.J. Gates, Jr.: In: Superspace and supergravity. Hawking, S.W., Rocek, M. (eds.). Cambridge: Cambridge University Press 1981
10. R. Grimm: Proceedings of the Spring School on Supergravity, Miramere-Trieste 1981 (to be published)
11. J. Wess, B. Zumino: Phys. Lett. **79B**, 394 (1978)
12. W. Siegel: In: Superspace and supergravity. Hawking, S.W., Roček, M. (eds.). Cambridge: Cambridge University Press 1981
13. B. Zumino: Proceedings of the NATO Advanced Study Institute on Recent Developments in Gravitation, Cargèse, Corsica, 1978, NATO Advanced Study Institutes Series B. p. 405. New York, London: Plenum Press
14. J. Wess, J. Bagger: Supersymmetry and supergravity. Princeton Series in Physics. Princeton, New Jersey: Princeton University Press 1983
15. S.J. Gates, M.T. Grisaru, M. Roček, W. Siegel: Superspace. Reading: Benjamin Cummings Publishing Co. 1983
16. R. Penrose: Ann. Phys. **10**, 171 (1960)
17. S.J. Gates, Jr., A. Karlhede, U. Lindström, M. Roček: Stony Brook Preprint ITP-SB-83-54