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NUMERICAL ANALYSIS OF THE DIFFRACTION OF A PLANE WAVE ON A PERIODIC GRATING OF LOADED STRIP ELEMENTS

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Diffraction on a plane periodic grating is one of the main problems in the study of electrodynamic properties of large antenna arrays. The use of strip elements in large antenna arrays is a highly promising technique, insofar as it relies on modern integrated printed circuit technology. The purpose of this study is to develop an algorithm for numerical solution of the diffraction problem and also to analyze the diffracted field as a function of grating parameters and the effect of impedance loads of the strip elements.

The diffraction problem is solved by a direct numerical method, which is applicable for a periodic grating of homogeneous elements in the form of narrow constant-curvature strip conductors embedded in a stratified homogeneous medium [1]. This class includes the traditional elements of antenna engineering, such as equiangular spirals, linear oscillators, and ring loops. The relevant class of elements can be extended if their strip structure is represented by a collection of constant-curvature fragments.

The method is based on derivation of a loaded integral equation for the current induced by the field of a plane wave inn an impedance-loaded strip element located in a spatial cell of the periodic grating (Floquet channel) with a transverse stratified-homogeneous core. The base element is a spiral. Other elements follow directly from the base spiral. The loaded integral equation is solved numerically by self-regularization [2] isolating the kernel singularities, local interpolation of the sought solution, and reduction of the equation to a stable system of linear algebraic equations on a selected set of collocation points. Numerical stability is ensured by domination of the diagonal elements of the system matrix.

We consider an unbounded periodic grating formed from plane spiral ribbons. A spiral consists of narrow strip conductors S_{con} (Fig. 1) of width $2d$ and length $2L$. We assume that $k_0d << 1$, where $k_0 = 2\pi/\lambda$, λ is the working wavelength. The spiral is placed at the boundary of the stratified-homogeneous medium with the parameters $\varepsilon(z)$, $\sigma(z)$, μ_0. The spiral elements coincide with the nodes of a rectangular lattice. The case of a skewed lattice is considered similarly.

The grating is excited by the field ($\mathbf{E}^0$, $\mathbf{H}^0$) of a plane linearly polarized wave incident at an angle (θ, φ) on the grating in the direction of the wavevector $\mathbf{k}$ (see Fig. 1). In the coordinate system (ξ, η, ζ), the wave field has the form

$$\mathbf{E}^0=\boldsymbol{\xi}_0Ae^{-jk\zeta},\quad \mathbf{H}^0=\boldsymbol{\eta}_0A/We^{-jk\zeta},$$

where A is the complex amplitude, $W = (\mu_0/\varepsilon)^{1/2}$. The orientation of this system of coordinates relative to the coordinate system (x, y, z) for a plane grating is completely determined by three angles θ, φ, α, where α is the polarization angle (see Fig. 1). Then for the wavevector $\mathbf{k}$ we have

$$\mathbf{k}=k(\mathbf{x}_0\sin\theta\cos\varphi+\mathbf{y}_0\sin\theta\sin\varphi-\mathbf{z}_0\cos\theta). \tag{1}$$

For the components of the field vector $\mathbf{E}^0$, we obtain

$$\begin{aligned} E_x&=Ae^{-jk\zeta}(\cos\alpha\sin\varphi+\sin\alpha\cos\theta\cos\varphi),\\ E_y&=Ae^{-jk\zeta}(-\cos\alpha\cos\varphi+\sin\alpha\cos\theta\sin\varphi),\\ E_z&=Ae^{-jk\zeta}\sin\alpha\sin\theta. \end{aligned} \tag{2}$$

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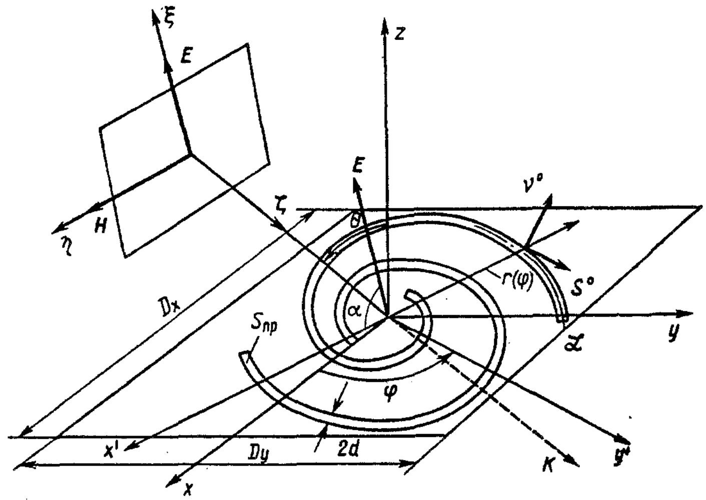

Fig. 1

An incident wave of general polarization can be decomposed into two waves. The wave with the vector E^0 lying in the plane of incidence is characterized by E-polarization. If the vector E^0 of the wave is perpendicular to the plane of incidence and parallel to the boundary of the media, the wave is characterized by H-polarization. The phase shift of the field at neighboring corners of the rectangular lattice is related to the incidence angle of the wave by the equalities

$$\psi_x = kD_x \sin\theta\cos\varphi, \quad \psi_y = kD_y \sin\theta\sin\varphi,$$

where D_x and D_y are the grating periods. The surface current j_s induced in the spiral strips creates a secondary field (E, H) which satisfies the almost-periodicity condition (Floquet condition). It suffices to consider this secondary field within one spatial cell of the grating.

Examining the geometry of the spiral strip in the curvilinear orthogonal coordinate system (s, ν) with elements of length $dl = h_1 ds$, $dt = h_2 dv$, where h_1, h_2 are Lamé constants, we use the condition $k_0 d << 1$. Then for the surface current j_s in the spiral we need only consider its longitudinal component j_s using the representation $j_s = I(l)/(d^2 - t^2)^{1/2}$, where $I(l)$ is the total spiral current [1]. This representation allows for the singularity of the current j_s on the edge of the spiral strip conductor.

The electrodynamic problem is formulated for the total field of the grating and involves finding the field (E, H) by solving the system of Maxwell's equations with continuity condition for the tangential field components on the discontinuity boundaries of the parameters in the medium, condition on the spiral strip conductor S_{con}, almost-periodicity condition in the coordinates x and y, and radiation condition at infinity ($z \to \infty$).

We solve the problem using the integral representation of the vector potential A for the spiral current j_s. Similarly to the fundamental solution, the vector potential of the current is representable in the form

$$A(M) = \frac{\mu_0}{4\pi} \iint\limits_{S_{np}} j_s(M_0)\,\hat{G}(M, M_0)\, d\sigma_{M_0}, \tag{3}$$

where $\hat{G}(M, M_0)$ is the tensor Green's function [3] which allows for the properties of the stratified medium. Then the field vectors (E, H) are determined from the relationships

$$\mathbf{H} = 1/\mu_0 \,\mathrm{rot}\,\mathbf{A}, \quad \mathbf{E} = -j\omega\,[\mathbf{A} + 1/(\omega^2\mu_0)\,\mathrm{grad}\,(1/\dot{\varepsilon}(z)\,\mathrm{div}\,\mathbf{A})], \tag{4}$$

where $\dot{\varepsilon}(z) = \varepsilon(z) - j\sigma(z)\omega$. We assume time dependence of the form $\exp(j\omega t)$ for the field.

For a spatial waveguide (Floquet channel) with a transverse stratified-homogeneous core the elements of the tensor Green's function in (3) are determined in [1]. Then relationships (3), (4) make it possible to obtain an integral representation of the solution which satisfies all the conditions, except the condition on the spiral strip conductor S_{con}. Assume that concentrated impedances Z_m, $m = 0, 1, \ldots, M$, are connected to the spiral arms. Allowing for these impedances, we impose the boundary condition in a narrow conductor S_{con} in the form

$$(\mathbf{E} + \mathbf{E}^0, \mathbf{s}^0)_{\mathscr{L}} = I(l) \sum_{m=0}^{M} Z_m \delta(l_k - l_m), \tag{5}$$

where $\mathfrak{L}$ is the coordinate line (see Fig. 1), $s0$ is the unit vector tangent to $\mathfrak{L}$, $\delta(l - l_m)$ is the delta function, l_m are the coordinates of the impedances along the spiral.

Substituting (3), (4) in the boundary condition (5), we obtain an integrodifferential equation for the spiral current. From this equation, similarly to [1], we can obtain a loaded integral equation of the form

$$\int_{-L}^{L} I(l_0)\, K(l, l_0)\, dl_0 + j2\pi/w \sum_{m=0}^{M} Z_m I(l_m) \sin|l - l_m| =$$
$$= -j2\pi/w \int_{-L}^{L} (E^0, S_u^0) \sin|l-u|\, du + c_1 \sin l + c_2 \cos l. \tag{6}$$

Isolating the singularity, we rewrite the kernel $K(l, l_0)$ in (6) in the form

$$K(l, l_0) = \frac{4\varepsilon_0}{\pi(\varepsilon_1 + \varepsilon_0)} F(\pi/2, \alpha)/\sqrt{\rho^2 + d^2} +$$
$$+ \frac{1}{D_x D_y} \sum\sum_{i(m,n)} \left\{ e^{j\lambda_m x(l)} e^{j\lambda_n y(l)} (\mathrm{S}^0, \mathrm{S}_0^0) \times \right.$$
$$\times \left[\left(G_{0i} + \frac{\partial g_i}{\partial z}\Big|_{z=0} - \frac{\varepsilon_0}{(\varepsilon_1 + \varepsilon_0)\lambda_i} \right) e^{-j[\lambda_m x_0(l_0) + \lambda_n y_0(l_0)]} + \right.$$
$$\left. \left. + \frac{4\varepsilon_0}{(\varepsilon_1 + \varepsilon_0)\lambda_i} P_i(\lambda_i, d) \right] - \frac{1}{2} \frac{\partial g_i}{\partial z}\Big|_{z=0} \int_{-L}^{L} (S_u^0, S_0^0)\, S_m(l, u)\, du \right\},$$

where $F(\pi/2, \alpha)$ is a complete elliptic integral of the first kind, ρ is the distance between the points $M, M_0 \in \mathfrak{L}$, $x(l)$, $y(l)$, $x_0(l_0)$, $y_0(l_0)$ are the coordinates of these points as functions of the arc length used as a parameter, (s^0, s_0^0) is the scalar product of unit vectors, $\alpha = d/(\rho^2 + d^2)^{1/2}$,

$$S_m(l, u) = \sin|l-u|\, e^{j[\lambda_m x(u) + \lambda_n y(u)]},$$
$$\lambda_m = k_x + 2\pi m/D_x,\ \lambda_n = k_y + 2\pi n/D_y,\ m, n = 0, \pm 1, \pm 2, \ldots,$$

$k_x = \psi_x/D_x$, $k_y = \psi_y/D_y$, $\lambda_i = (\lambda_m^2 + \lambda_n^2)^{1/2}$, G_{0i} and $\partial g_i/\partial z$ are the elements of the tensor Green's function for parameter values λ_i [1], P_i is a complicated function obtained by isolating the singularity in the kernel. Using the representations (2) for the right-hand side of Eq. (6), we in turn obtain the relationships

$$(\mathbf{E}^0(u), \mathrm{s}_u^0) = E_x^0(u)(h_1 \cos\varphi_u + h_2 \sin\varphi_u) - E_y^0(h_1 \sin\varphi_u - h_2 \cos\varphi_u),$$
$$E_x^0(u) = A(\cos\alpha \sin\varphi + \sin\alpha \cos\theta \sin\varphi)\, e^{-jk[x(u)\sin\theta\cos\varphi + y(u)\sin\theta\sin\varphi]},$$
$$E_y^0(u) = A(-\cos\alpha\cos\varphi + \sin\alpha\cos\theta\sin\varphi)\, e^{-jk[x(u)\sin\theta\cos\varphi + y(u)\sin\theta\sin\varphi]}.$$

In (6) we assume that the spiral is located on the boundary of two media with the parameters ε_0, ε_1 and the linear dimensions are normalized by k_0. The coefficients C_1, C_2 are determined from the supplementary condition which stipulates that the current is zero at the spiral ends.

Note that in the absence of impedance loads on the spiral, we obtain from (6), by weak singularity of the kernel $K(l, l_0)$, a Fredholm integral equation of the first kind for the current $I(l)$. Numerical solution of this equation is best obtained by the self-regularization method [2]. This was the basis of the algorithm developed in [4], which reduces the solution of the integral equation on a discretization step $h = 2L/N$, where N is the number of collocation points, to the solution of a system of linear algebraic equations. The matrix of the system has diagonal domination, which ensures stable solution of the system.

The elements of a matrix with diagonal domination satisfy the conditions

$$|a_{ii}| - \sum_{i \neq j} |a_{ij}| = r_i > 0,\ i, j = 1, 2, \ldots, N. \tag{7}$$

As the measure of conditioning of the system we may take, say, $\max_i |a_{ii}| (\min_i r_i)^{-1}$, $i = 1, 2, \ldots, N$. If this quantity is not large, then the system is well conditioned and its numerical solution is stable.

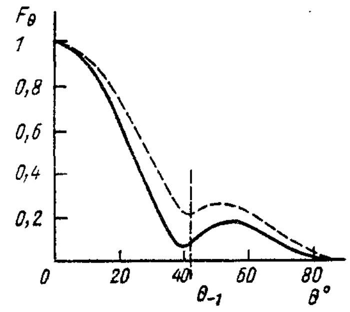

Fig. 2

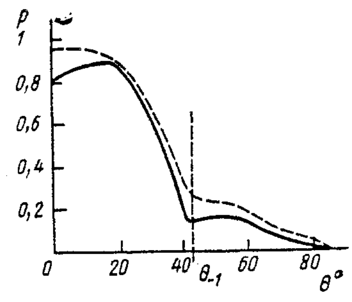

Fig. 3

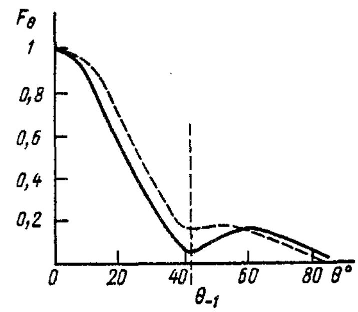

Fig. 4

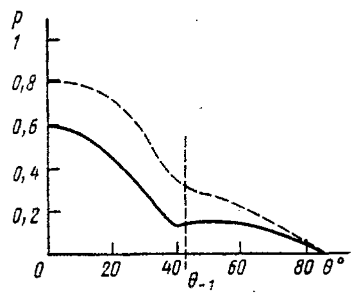

Fig. 5

The algorithm of [4] is applicable also to the solution of the loaded integral equation (6) when conditions (7) are satisfied. The presence of impedance loads in the spiral in general has an adverse effect on system conditioning. These factors restrict the value of impedance loads depending on their location along the spiral strips. The impedance loads therefore can be selected only in compliance with inequalities (7) or by checking the conditions of numerical instability of the system solution.

Having determined the current $I(l)$ of the spiral grating element from the solution of the integral equation (6), we can find the characteristics of the diffracted field. This field is represented by a collection of plane waves propagating in the direction of the angles $\theta_{\nu\mu}$, $\varphi_{\nu\mu}$ and determined by solving the system of equations

$$\sin\theta\cos\varphi + \sin\theta_{\nu\mu}\cos\varphi_{\nu\mu} = \lambda/D_x\nu, \quad \nu = 0, \pm 1, \pm 2, \ldots,$$

$$\sin\theta\sin\varphi + \sin\theta_{\nu\mu}\sin\varphi_{\nu\mu} = \lambda/D_y\mu, \quad \mu = 0, \pm 1, \pm 2, \ldots.$$

For the indices $\nu = 0$, $\mu = 0$ we have a wave reflected from the surface of the grating in the direction corresponding to the reflection of the incident wave. Similarly to the directional diagram of an element of a phased antenna array, we define the directional diagram of a spiral element of a reflector antenna array (a partial diagram) for the field components E_θ, E_φ in the form

$$E_{\theta,\varphi}(\theta,\varphi) = \int_{-L}^{L} I(l_0) f_{\theta,\varphi} e^{-jh_1 l_0 (S^0, S_0^0)\sin\theta} dl_0,$$

$$f_\theta = \cos\theta\{[h_1(S^0, S_0^0) - h_2(\nu^0, S_0^0)] G_{0i}(\lambda_i)\cos\theta + j(S^0, S_0^0) g_i(\lambda_i)\sin\theta\},$$

$$f_\varphi = -\cos\theta[h_1(\nu^0, S_0^0) + h_2(S^0, S_0^0)].$$

The values of G_{0i} and g_i are taken for $i = 0$. These expressions make it possible to determine the polarization coefficient $p = F_\theta/F_\varphi$.

The algorithm to solve the problem was implemented in FORTRAN under Dubna OS for the BÉSM-6 computer. As an example, we present the numerical results obtained for partial diagrams of a loaded equiangular two-turn spiral $r(\varphi) = r_0\exp(a\varphi)$ (see Fig. 1) located in a periodic grating cell with dimensions $D_x = D_y = 0.6\lambda$. The dimensions of the spiral are $L = 2.5\lambda$, $a = 0.01$, $r_0 = 0.05\lambda$, $d = 0.01\lambda$ and it is placed on a dielectric layer of thickness $D = 0.01\lambda$ with $\varepsilon_1 = 9$ at a distance 0.25λ from the shield. The number of terms in the representation of the kernel of the integral equation (6) is 81 with discretization step $h = 0.1\lambda$, which ensures a relative error of not more than 5% in the calculation of the spiral current. The time to compute the partial diagrams for one incidence angle is around 60 min.

Figure 2 plots the normalized values of the partial diagrams F_θ for E-polarization for an unloaded spiral (solid curve) and a spiral loaded by the impedance $Z = j \cdot 900\ \Omega$ at the end point (dashed curve). Figure 3 plots the polarization coefficient for this case. Note that the loading of the spiral results in a more efficient excitation and accordingly increases the diffracted field. Figures 4 and 5 show the E-polarization partial diagrams for spirals loaded with input impedances $Z = j \cdot 300\ \Omega$ (solid curve) and $Z = -j \cdot 300\ \Omega$ (dashed curve) and the corresponding polarization coefficients p. The diffracted field acquires an additional phase shift, whose change is 45° for an angle normal to the grating plane. In these figures, θ_{-1} is the angle corresponding to the first diffraction maximum for the given grating.

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