Z. Phys. A - Atomic Nuclei 324, 325–330 (1986)

Zeitschrift für Physik A
Atomic Nuclei

A Semi-Classical Model of Multi-Step Direct and Compound Nuclear Reactions

Wen Yuanqi, Shi Xiangjun*, Yan Shiwei, and Zhuo Yizhong
Institute of Atomic Energy, Beijing, China

Han Huiyi
Department of Physics, Normal College, Beijing, China

Received December 20, 1985

A semi-classical model of multi-step direct and compound nuclear reactions is proposed to describe the angular distributions of particles emitted from the inelastic scattering induced by a nucleon with an energy of several tens of MeV. The energy-angle correlation is exactly taken into account for the first few steps of the collision process (multi-step direct process) and the generalized master equation is employed for the following stages of collision process, using the energy-averaged kernel $G(\Omega \to \Omega')$ (multi-compound process). The calculations for $^{197}\mathrm{Au}(p, p')$, $^{120}\mathrm{Sn}(p, p')$ and $^{93}\mathrm{Nb}(n, n')$ show that the model can rather nicely reproduce the experimental data of double-differential cross sections.

PACS: 24.10; 24.60; 25.40

1. Introduction

In recent years pre-equilibrium nuclear reaction theories have been developed with great success for the description of the angular distributions of emitted particles based on the exciton model. This success is mainly due to the introduction of a "fast" particle concept into the generalized excition model [1], the proposal of using the nucleon-nucleon cross section in nuclear matter [2] (Fermi gas model) instead of free N-N scattering for the kernel $G(\Omega, \Omega')$, and taking into account the energy-angle correlation in the generalized master equations [3].

The consideration of the energy-angle correlation was restricted to the first collision by Costa et al. [3], and subsequently extended to higher steps by Iwamoto and Harada [4] in the never-come-back approximation. The importance of some other effects has also been investigated, such as finite size, refraction of the ingoing and outgoing waves [3], and the influence of the recoil nucleons in the target [4]. On the whole, the above-mentioned pre-equilibrium theories are quite good in reproducing the experimental data of the double-differential cross sections of nucleon-induced reactions at incident energies of several tens of MeV. However, it should be pointed out that the discrepancy between theories and experiments both in the forward- and in the backward-angle range has not yet been completely settled.

In this paper, a semi-classical model of multi-step direct and compound nuclear reactions is proposed to solve this problem. For the first few steps (one, two, three, ...) of the collision process the energy-angle correlation is exactly taken into account, following the "never-come-back" assumption ("multi-step direct process"); for the remaining steps the master equation with the energy-averaged kernel [2] ("multi-step compound process") is solved.

In this way we have calculated the double-differential cross sections for the $^{197}\mathrm{Au}(p, p')$, $^{120}\mathrm{Sn}(p, p')$ and $^{93}\mathrm{Nb}(n, n')$ reactions. We have obtained better calculated results as compared with [2] and [4].

Section 2 is devoted to the formulation of the proposed model. The calculated results are given in Sect. 3. In the last section we discuss the model and suggest some further improvements.

* Present address: Netherlands Energy Research Foundation (ECN), NL-1755 ZG Petten, The Netherlands

2. Formulation and Solution of the Semi-Classical Model

2.1. Multi-Step Direct Process: Energy-Angle Correlation

According to the dynamical mechanism of nuclear reactions with incident energies of several tens of MeV, the energy-angle correlation is very important for the first few collision steps. So, the extended generalized master equation with the angle-energy correlated kernel is employed only for the first few steps.

Before showing the expressions, we recall the same set of symbols as used in [4] to denote the physical quantities:

$q(n\Omega\varepsilon t)$ the occupation probability of the composite system at time t in exciton state n with energy ε and direction Ω of the particle under consideration;

$W_{m\to n}(\Omega'\varepsilon'\to\Omega\varepsilon)$ the transition probability of the system per unit time from state $(m, \Omega', \varepsilon')$ to state (n, Ω, ε);

W_n the total emission rate of the system in state n;

$\lambda_{m\to n}$ intranuclear transition rate from state m to state n;

$G(\Omega'\varepsilon'\to\Omega\varepsilon)$ the energy-angle probability distribution of the nucleon-nucleon collision from state $(\Omega'\varepsilon')$ to state $(\Omega\varepsilon)$ in the nuclear matter.

Thus, the extended generalized master equation is written as follows [4]:

$$\frac{\mathrm{d}}{\mathrm{d}t}q(n\Omega\varepsilon t)=\sum_m\int\mathrm{d}\Omega'\int\mathrm{d}\varepsilon'\,q(m\Omega'\varepsilon' t)\,W_{m\to n}(\Omega'\varepsilon'\to\Omega\varepsilon)$$
$$-\{\sum_m\int\mathrm{d}\Omega'\int\mathrm{d}\varepsilon'\,W_{n\to m}(\Omega\varepsilon\to\Omega'\varepsilon')+W_n\}\,q(n\Omega\varepsilon t). \quad (1)$$

Denoting

$$W_{m\to n}(\varepsilon'\,\Omega'\to\varepsilon\Omega)=\lambda_{m\to n}G(\Omega'\varepsilon'\to\Omega\varepsilon) \quad (2)$$

and

$$G(\Omega\varepsilon\to\Omega'\varepsilon')=G(\Omega'\varepsilon\to\Omega\varepsilon')=\frac{1}{\bar\sigma}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon'\,\mathrm{d}\Omega'} \quad (3)$$

with

$$\bar\sigma=\int\int\mathrm{d}\varepsilon'\,\mathrm{d}\Omega'\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon'\,\mathrm{d}\Omega'}, \quad (4)$$

it follows that all the dynamical information of angular distribution and angle-energy correlation is contained in $G(\Omega\varepsilon\to\Omega'\varepsilon')$.

The solution to master equation (1) can be obtained in the form of a partial-wave expansion [5]:

Let

$$q(n\Omega\varepsilon t)=\sum_l\eta_l(n\varepsilon t)P_l(\cos\Theta), \quad (5)$$

$$\int G(\Omega'\varepsilon'\to\Omega\varepsilon)P_l(\cos\Theta')\,\mathrm{d}\Omega'=\mu_l(\varepsilon,\varepsilon')P_l(\cos\Theta), \quad (6)$$

$$\zeta_l(n\varepsilon)=\int_0^\infty\mathrm{d}t\,\eta_l(n\varepsilon t), \quad (7)$$

and

$$\tau(n\Omega\varepsilon)=\int_0^\infty\mathrm{d}t\,q(n\Omega\varepsilon t)=\sum_l\zeta_l(n\varepsilon)P_l(\cos\Theta). \quad (8)$$

Under the "never-come-back" assumption the time-integrated equation can be solved in a closed form and the solution is given by

$$\zeta_l(n\varepsilon)=\tau(n)\frac{2l+1}{4\pi}\int\mathrm{d}\varepsilon_2\int\mathrm{d}\varepsilon_3\ldots\int\mathrm{d}\varepsilon_{(n+2-n_0)/2}$$
$$\cdot\,\mu_l(E,\varepsilon_2)\,\mu_l(\varepsilon_2,\varepsilon_3)\ldots\mu_l(\varepsilon_{(n+2-n_0)/2},\varepsilon). \quad (9)$$

If the effects of Pauli principle and Fermi motion are taken into account, the expression for $\mu_l(\varepsilon,\varepsilon')$ is obtained as follows [2]

$$\mu_l(\varepsilon,\varepsilon')=\frac{1}{\bar\sigma}\int\mathrm{d}\Omega'\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon'\,\mathrm{d}\Omega'}P_l(\cos\Theta')$$
$$\begin{cases}=\dfrac{2\pi}{\bar\sigma}\left\{\displaystyle\int_{\alpha_2}^{\alpha_1}B\cdot P_l(\cos\Theta')\,\mathrm{d}\cos\Theta'\right.\\ \quad+\displaystyle\int_{\alpha_1}^{\beta_1}A\cdot P_l(\cos\Theta')\,\mathrm{d}\cos\Theta'\\ \quad\left.+\displaystyle\int_{\beta_1}^{\beta_2}B\cdot P_l(\cos\Theta')\,\mathrm{d}\cos\Theta'\right\}\ (\text{for }\varepsilon'>\varepsilon-E_f)\\ =\dfrac{2\pi}{\bar\sigma}\displaystyle\int_{\alpha_2}^{\beta_2}B\cdot P_l(\cos\Theta')\,\mathrm{d}\cos\Theta'\ (\text{for }\varepsilon'\leqq\varepsilon-E_f),\end{cases} \quad (10)$$

with

$$\bar\sigma=\sigma\left[1-\frac{7}{5}\frac{E_f}{\varepsilon}+\frac{2}{5}\frac{E_f}{\varepsilon}\left(2-\frac{\varepsilon}{E_f}\right)^{5/2}H(2E_f-\varepsilon)\right]. \quad (11)$$

In (3) and (11) $\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon'\,\mathrm{d}\Omega'}$ is the double-differential cross section following a single nucleon-nucleon collision in nuclear matter given by:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon'\,\mathrm{d}\Omega'}=$$
$$\begin{cases}A=\dfrac{3\sigma}{8\pi E_f}\sqrt{\dfrac{\varepsilon'}{E_f}}\dfrac{(\varepsilon-\varepsilon')}{Q\varepsilon} & (A\text{-area}),\\ B=\dfrac{3\sigma}{8\pi E_f}\sqrt{\dfrac{\varepsilon'}{E_f}}\dfrac{1}{Q\sqrt{Q\varepsilon}}\\ \quad\cdot[\varepsilon\varepsilon'\sin^2\Theta-(\varepsilon'-E_f)Q] & (B\text{-area}),\end{cases} \quad (12)$$

with

$$Q=\varepsilon+\varepsilon'-2\sqrt{\varepsilon\varepsilon'}\cos\Theta. \tag{13}$$

The A- and B-areas are those parts in the angle-energy plane of the emitted nucleon after collision in nuclear matter where the collisions are allowed to take place (see Fig. 1). In (11) and (12) σ is the total cross section of the free nucleon-nucleon scattering, which appears in both the denominator and the numerator of $\mu_l(\varepsilon,\varepsilon')$ and hence μ_l is independent of σ.

Finally, the contribution of the first few steps to the double-differential cross sections is given by

$$\frac{\mathrm{d}^2\sigma(a,b)}{\mathrm{d}\varepsilon\,\mathrm{d}\Omega}=\sigma_a\sum_{\substack{n=n_0\\ \Delta n=2}}^{N}\tau(n)\,W_n^b(\varepsilon)\,\tau(n\Omega\varepsilon)/\int\mathrm{d}\Omega\tau(n\Omega\varepsilon). \tag{14}$$

The summation runs over the first few exciton states ($n\leqq N$) for which the angle-energy correlation is exactly taken into account. The quantity $W_n^b(\varepsilon)$ is the emission rate of the composite system for exit channel b and the mean lifetime is given by

$$\tau(n)=\prod_{\substack{i=n_0\\ \Delta i=2}}^{n-2}\frac{\lambda_i^+}{\lambda_i^++W_i}\Big/(\lambda_n^++W_n). \tag{15}$$

Here, λ_i^+ is the exciton transition rate from the i-th exciton state to the $(i+2)$-th exciton state, and W_i is the total emission rate of the i-th exciton state. Please note that for $n=n_0$ one may replace $\tau(n\Omega\varepsilon)$ in (14) by (12).

2.2. *Multi-Step Compound Process by Using the Energy-Averaged Kernel*

Since it is expected that the memory of the angle-energy correlation is gradually lost at more complicated stages of the reaction process, the energy-averaged kernel $G(\Omega\to\Omega')$ can be used for these equilibration stages in the generalized master equation [1-3]:

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}q(n\Omega t)\\&=\sum_m\int\mathrm{d}\Omega'\,q(m\Omega' t)\,W_{m\to n}(\Omega'\to\Omega)\\&\quad-\{\sum_m\int\mathrm{d}\Omega'\,W_{n\to m}(\Omega\to\Omega')+W_n\}\,q(n\Omega t),\end{aligned} \tag{16}$$

where

$$W_{m\to n}(\Omega'\to\Omega)=\lambda_{n\to m}\,G(\Omega'\to\Omega). \tag{17}$$

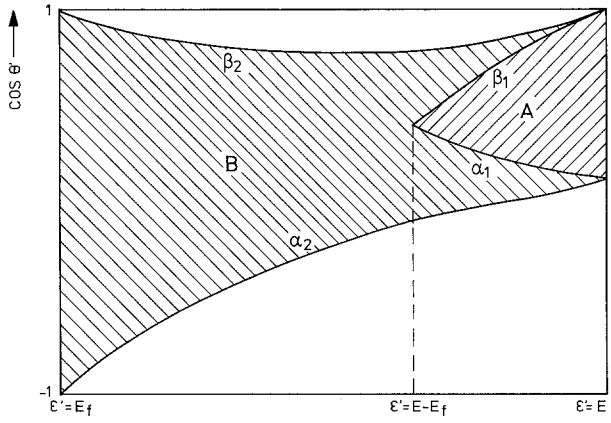

Fig. 1. A sketch of the integration region for energy ε' and angle Θ' of the nucleons after a collision when the effects of Pauli princile and Fermi motion are taken into account. The regions denoted by A and B indicate where the collisions are allowed to take place

For these later stages the "never-come-back" approximation would cause a serious error. Therefore the full master equation is solved, including both forward and backward transitions.

Let

$$q(n\Omega t)=\sum_l\eta_l(nt)\,P_l(\cos\Theta), \tag{18}$$

$$G(\Omega'\to\Omega)=G(\Omega\to\Omega')=\frac{1}{\bar\sigma}\int\mathrm{d}\varepsilon'\,\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon'\,\mathrm{d}\Omega'}, \tag{19}$$

$$\int G(\Omega'\to\Omega)\,P_l(\cos\Theta')\,\mathrm{d}\Omega'=\mu_l P_l(\cos\Theta), \tag{20}$$

and

$$\zeta_l(n)=\int_0^\infty\eta_l(nt)\,\mathrm{d}t, \tag{21}$$

then an exact closed-form solution of the time-integrated master equation for partial waves l is obtained [2] in the form

$$\zeta_l(n)=\frac{2l+1}{4\pi}\tau_l(n)\,\mu_l F_n\prod_{\substack{k=n_0\\ \Delta k=2}}^{n-2}\mu_l\lambda_k^+\,\tau_l(k)\,F_k \tag{22}$$

and

$$t(n,\Theta)=\int_0^\infty\mathrm{d}t\,q(n\Omega t)=\Sigma_l\zeta_l(n)\,P_l(\cos\Theta). \tag{23}$$

From (19-20) we have

$$\mu_l=\frac{1}{\bar\sigma}\int\mathrm{d}\varepsilon'\int\mathrm{d}\Omega'\,\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon'\,\mathrm{d}\Omega'}\,P_l(\cos\Theta). \tag{24}$$

In comparison with (10), μ_l can easily be obtained:

$$\mu_l=\int_{E_f}^{E}\mu_l(E,\varepsilon')\,\mathrm{d}\varepsilon'. \tag{25}$$

Thus, the contribution of exciton state n to the double-differential cross section is

$$\frac{1}{\sigma_a}\left(\frac{d^2\sigma_b}{d\varepsilon d\Omega}\right)_n = t(n, \Theta)\, W_n^b(\varepsilon). \tag{26}$$

2.3. Double-Differential Cross Sections

In this paper it is proposed to incorporate the above-mentioned two approaches in one model code. The double-differential cross section is then given by the following final expression

$$\frac{d^2\sigma_b}{d\varepsilon d\Omega} = \sigma_a \left\{ \sum_{\substack{n=n_0 \\ \Delta n=2}}^{N} \tau(n)\, W_n^b(\varepsilon) \frac{\tau(n\Omega\varepsilon)}{\int d\Omega \tau(n\Omega\varepsilon)} + \sum_{\substack{n=N+2 \\ \Delta n=2}}^{\bar{n}} t(n, \Theta)\, W_n^b(\varepsilon) \right\}. \tag{27}$$

It is noticed that the present formulation is inspired by the quantum-mechanical FKK theory [6] and could be viewed as a semi-classical approximation to this theory. The present model is much more simple than the FKK-theory, which is quite complicated both in the formulation and in the modelling.

3. Calculated Results

Based on the present approach the cross sections for inelastic scattering of the reactions ^{197}Au(p, p'), $^{120}Sn(p, p')$ and ^{93}Nb(n, n') have been calculated and have been compared with the experimental data. For ^{197}Au(p, p') and ^{120}Sn(p, p) the incident energy is 62 MeV and the outgoing energies are 50, 40 and 30 MeV. For ^{93}Nb(n, n') the incident energy is 14.6 MeV and the outgoing energies are 8 and 4 MeV respectively.

As a rough estimate for the finite nuclear size the number of partial waves is cut off at l_{max} on the basis of the classical approximation of $hk = Rl_{max}$. However, an analytical expression (12) is employed for the first step $(n = n_0)$ instead of a truncated Legendre expansion in order to avoid the appearance of negative calculated values of double-differential cross sections at some backward angles. No refraction effects have been included in the calculations. In order to test the importance of the energy-angle correlation separate calculations of one-, two- and three-steps of direct processes, followed by the multi-step compound process were performed. The results are shown in Figs. 2 to 4. As a comparison the results from the model of [2] are also plotted there.

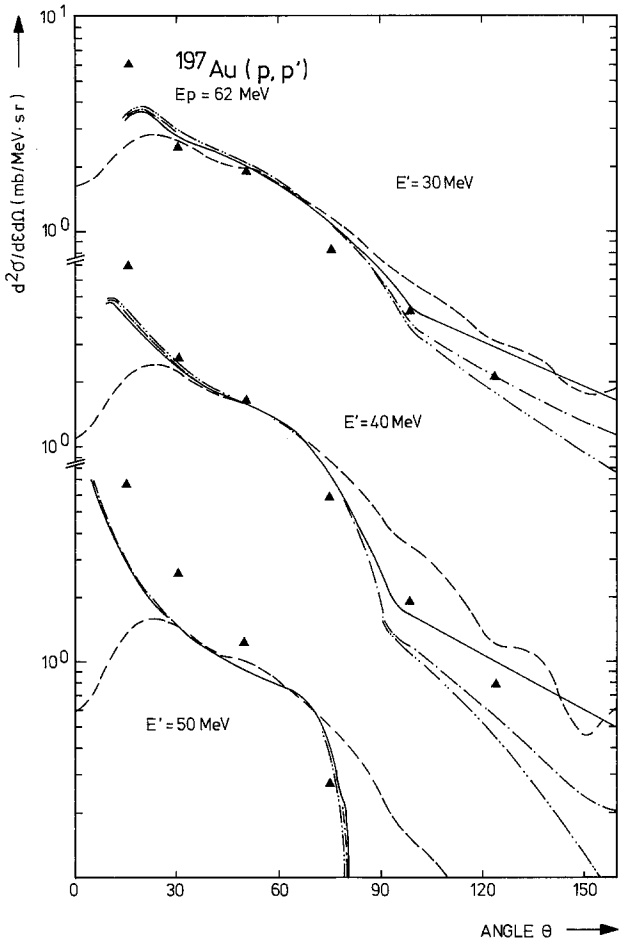

Fig. 2. The double-differential cross sections as functions of the angle of outgoing nucleons for the reaction ^{197}Au(p, p'), calculated with the present model. The full curves represent the calculation taking into account the energy-angle correlation for only the first step, the single-dotted-dashed curves for the first two steps, and the double-dotted-dashed curves for the first three steps. The dashed curves are obtained from the model of [2] as a comparison. The triangles represent experimental data, taken from [7]

From Figs. 2 and 3, it is seen that at 50-MeV outgoing energy the restriction of the energy-angle correlation to the first collision only is quite sufficient for both the Au(p, p') and Sn(p, p') reactions at $E_p = 62$ MeV (see the full curves). However, at 30-MeV outgoing energy it would be better to take into account the correlation for the first two steps (see the single-dotted-dashed lines). Even the first three steps of angle-energy correlation are needed for the case of 8-MeV outgoing energy in the reaction of ^{93}Nb(n, n') with $E_n = 14.6$ MeV (see Fig. 4).

Another question is: does it really matter if one takes the multi-step compound part to be isotropic, instead of using the complete multi-step compound calculation? To answer this, a calculation has been

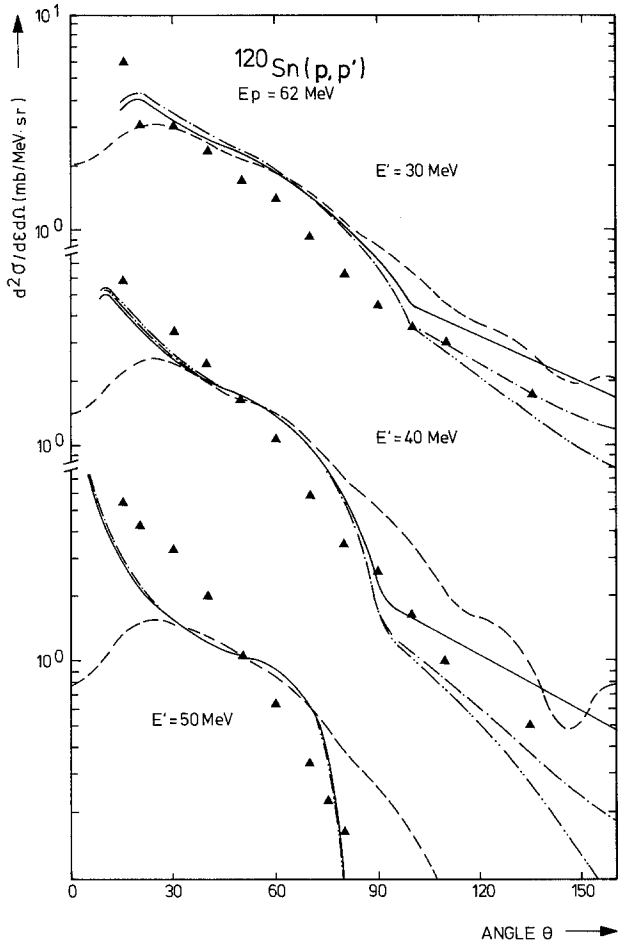

Fig. 3. The double-differential cross sections for the reaction $^{120}Sn(p, p')$. The curves are the same as in Fig. 2

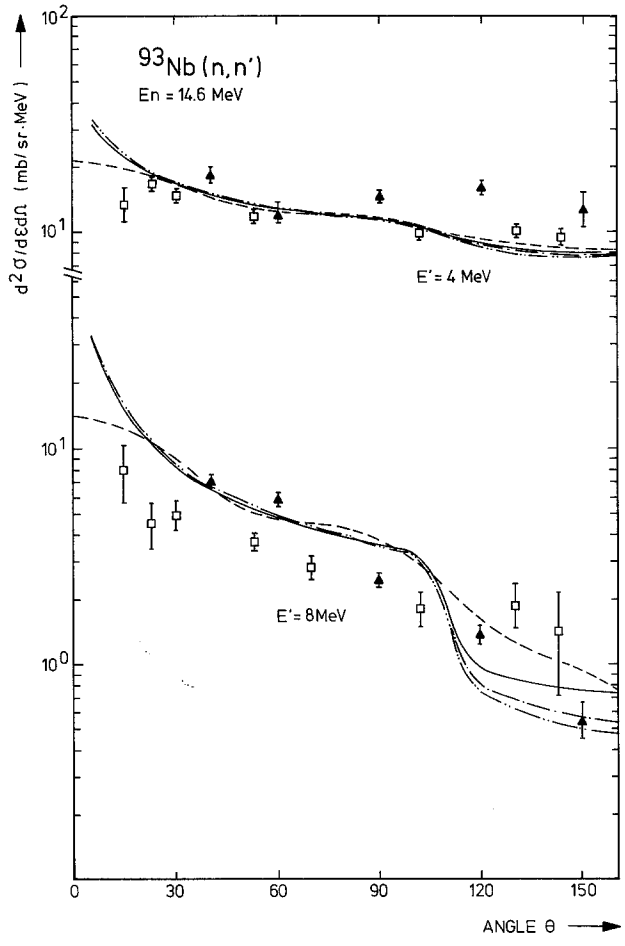

Fig. 4. The double-differential cross sections for the reaction $^{93}Nb(n, n')$. The curves are the same as in Fig. 2. The experimental data are adopted from [8] (triangles) and [9] (squares)

performed with the model of Iwamoto and Harada [4] for the first two or three collisions combined with the evaporation model for the following steps. Figure 5 shows the results for the $^{120}Sn(p, p')$ reaction and reveals that the addition of the complete multi-step compound calculation gives better results than that of the evaporation model in the backward-angle region. For the $^{197}Au(p, p')$ reaction, this is quite the same.

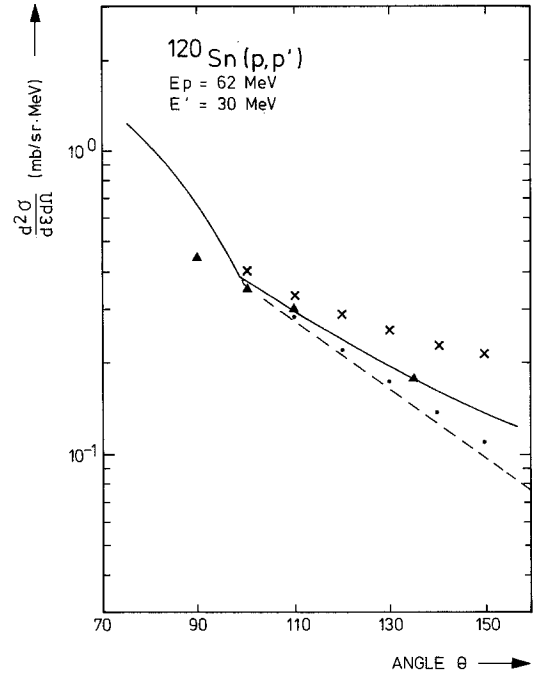

Fig. 5. Comparison of two calculations. The full curve shows the results of the two-step energy-angle correlation model followed by the complete multi-step-compound model. The points indicated by asterisks show the isotropic approximation instead of the complete multi-step-compound model. The dashed line represents the results of the three-step energy-angle correlation model followed by the complete multi-step-compound model. The points indicated by dots show the corresponding isotropic approximation. The triangles represent experimental data same as in Fig. 3

4. Conclusion

1) It seems that the present model rather nicely reproduces the experimental data of double-differential cross sections in a broad range of incident energies from about 10 MeV to several tens of MeV. It is also shown that the calculated results give better agreement with the experimental data than those of [2] in the forward-angle region and those of [4] in the backward region.

2) For incident energies of several tens of MeV, the energy-angle correlation included in the first-step collision [3] is quite sufficient for the most energetic outgoing particle, because the contribution of the first step is dominant in this case. However, with the decrease of outgoing energies the energy-angle correlation of more steps (at least two steps) should be taken into account. This feature was also pointed out by Tamura et al. [10] who used a multistep-direct reaction (MSDR) method.

3) Strictly speaking, for the most energetic outgoing particles the present model still fails to provide very satisfactory results in both the forward and backward-angle regions, where the collisions are kinematically forbidden in the semi-classical N-N kernel. In this case a complete quantum-mechanical treatment might be necessary.

However, one should keep in mind that the experimental data at very forward and at very backward angles are not very accurate. On the other hand, in the theory isotropic N-N scattering in the C.M. system is assumed, but experimentally there is considerable forward-backward peaking over much of the energy range. Therefore, a more realistic N-N angular distribution might be useful in future work.

4) The calculations also show that the cut-off value of l might affect the calculated results considerably. The classical estimate of l_{max} seems not good enough to account for the finite size of the nucleus. A possible improvement is to perform the local-density approximation, that is, each l corresponds to a different density, so that the finite size and the diffuseness of the surface can be more properly taken into account.

The authors thank Dr. H. Gruppelaar and Dr. J.M. Akkermans for some constructive suggestions.

References

1. Mantzouranis, G., Weidenmüller, H.A., Agassi, D.: Z. Phys. A - Atoms and Nuclei **276**, 145 (1976)
2. Sun Ziyang, Wang Shunuan, Zhang Jingshang, Zhuo Yizhong: Z. Phys. A - Atoms and Nuclei **305**, 61 (1982)
3. Gosta, C., Gruppelaar, H., Akkermans, J.M.: Phys. Rev. C **28**, 587 (1983)
4. Iwamoto, A., Harada, K.: Nucl. Phys. A **419**, 472 (1984)
5. Akkermans, J.M.: Phys. Lett. **82** B, 20 (1979); Phys. Rev. C **22**, 73 (1980)
6. Feshbach, H., Kerman, A., Koonin, S.: Ann. Phys. (NY) **125**, 429 (1980)
7. Bertrand, F.E., Peele, R.W.: Phys. Rev. C **8**, 1045 (1973); ORNL-4460 (1969), ORNL-4471 (1970)
8. Takahashi, A., Yamamoto, J., Murakami, T., Oshima, K., Oda, H., Fujimoto, K., Ueda, M., Fukazawa, M., Yanagi, Y., Miyaguchi, J., Sumita, K.: Oktavian Report A-83-01 (1983), Osaka University
9. Hermsdorf, D., Meister, A., Sassonoff, S., Seeliger, D., Seidel, K., Shahin, F.: Report Zfk-277, Zentralinstitut für Kernforschung Rossendorf bei Dresden (1974)
10. Tamura, T., Udagawa, T., Lenske, H.: Phys. Rev. C **26**, 379 (1982)

Wen Yuanqi
Yan Shiwei
Zhuo Yizhong
Institute of Atomic Energy
Beijing
China

Shi Xiangjun
Netherlands Energy Research
Foundation - ECN
Postbus 1
NL-1755 ZG Petten
The Netherlands

Han Huiyi
Department of Physics
Normal College
Beijing
China