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SCIENTIFIC AND TECHNICAL SECTION

EFFECT OF VOLUMETRIC STRESSED STATE ON YIELDING OF IRON AND CARBON STEELS. COMMUNICATION 1. PHYSICAL MODEL OF POLYCRYSTAL YIELDING WITH A COMPLEX STRESSED STATE

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Within the scope of a statistical model for polycrystal yielding a connection is demonstrated between the features of slip in a crystal lattice and the form of the yielding criterion for a metal under conditions of a complex stressed state. A physical interpretation is given for transformation of the ultimate yielding surface of metals and alloys with a BCC-lattice with a complex stressed state and low temperatures. It is shown that deviation from the Mises criterion is specified by the dependence of critical shear stress for a crystal τ_c *on the magnitude of normal stresses in the slip plane which is observed in BCC-crystals at low temperatures.*

Recently abundant experimental material has been accumulated on the effect of the form of stressed state on yielding of metals and alloys. The main features of plastic deformation under these conditions have been established. Applied to steels these features appear as the fact that at room temperature the yield strength is described with sufficient accuracy by the Mises criterion. However, at low temperature the yield strength curve is transformed so that with a change-over from uniaxial to biaxial tension there is a reduction in yield strength, and with compressive stresses there is a reverse effect (see Fig. 1). Under conditions of a linear stressed state the yield stress in compression exceeds that in tension (χ-effect [1]).

Comprehensive analysis of the features of low-temperature yielding of steels is given in [1]. There it is shown that the effect in question may be described with sufficient accuracy by a generalized yielding criterion. At the same time, it is not clear how the physical phenomena at the microlevel give rise to anomalies in the low-temperature plastic deformation of metals with a BCC-lattice with a complex stressed state (CSS). In addition, there is considerable interest in evaluating the magnitude of the effect in question in the case of triaxial tension since available experimental data were obtained mainly with a plane (biaxial) stressed state.

In view of this in the present communication we consider the physical nature of low-temperature yielding of polycrystals with a BCC-lattice with a CSS and it is devoted to an experimental study of the plastic deformation of steels with triaxial tension under low-temperature conditions.

Statistical Model for Yielding of a Polycrystal with a Complex Stressed State. Development of ideas of the physical nature of plastic deformation of metals under CSS conditions requires construction of a yielding model for a polycrystal proceeding from initial principles. In this case by initial principles we understand the assumption of crystallographic slip as the main mechanism of plastic deformation for a polycrystalline aggregate. In this arrangement the problem of modeling yielding of a polycrystal was formulated and resolved by Taylor [2]. Further development of this approach was carried out in [3-5]. However, use of classical methods for solving this problem is connected with a number of difficulties caused primarily by the complexity of mathematical provisions which do not make it possible to obtain fundamental relationships in analytical form. For these reasons the basis of the suggested approach is the idea of statistical description of plastic deformation of a polycrystal [6-8]. In this arrangement a polycrystalline aggregate is considered as a micro-inhomogeneous

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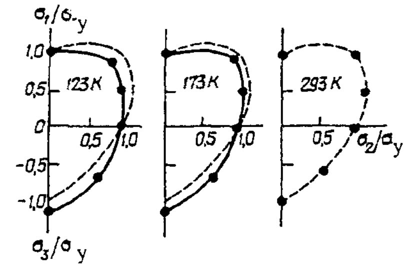

Fig. 1. Yield strength diagrams for steel 35 with a plane stressed state [14]. (Solid lines with points are experimental data, and broken lines are the Mises criterion.)

material in which the function for the distribution density of microstresses is described by a normal rule. Here the start of macroyielding is assumed to be connected with reaching a critical proportion of slip activation systems P_c:

$$P(\tau_{nv} \geq \tau_c) = P_c; \tag{1}$$

$$P(\tau_{nv} \geq \tau_c) = 1 - \int_{-\tau_c}^{+\tau_c} f(\tau_{nv}) d(\tau_{nv}), \tag{2}$$

where $P(\tau_{nv} \geq \tau_c)$ is the probability of reaching shear stress τ_c in slip systems by operating tangential microstresses τ_{nv}: $f(\tau_{nv})$ is the function of tangential microstress distribution density in slip systems.

The critical proportion of activated slip systems clearly governs the amount of plastic deformation of a polycrystal and therefore this formulation for the start of macroyielding corresponds to the currently adopted method for experimental determination of the yielding stress with a prescribed allowance for the amount of residual strain.

Within the scope of this approach it is possible to obtain an expression for macroyielding stress with a CSS proceeding from the conditions for slip at the microlevel. If slip in a crystalline lattice obeys the Schmidt rule (τ_c = const), then in the Voigt approximation the macroyielding condition has the form [9]

$$\sigma_i = \frac{3(2 + 3\gamma)}{Z_c \sqrt{5(2 + \gamma^2)}} \tau_c, \tag{3}$$

where σ_i is macrostress intensity; γ is lattice elastic anisotropy parameter; Z_c is normalized integration limit in (2).

Relationship (3) coincides with the known phenomenological Mises criterion (σ_i = const) which as established by experiment describes with sufficient accuracy yielding of polycrystalline metals at room temperature.

A typical feature of slip in crystals of BCC-metals in the low-temperature region ($T \leq 293$ K) is the dependence of shear strength τ_c on the magnitude of normal stresses σ_{nv} operating in a slip plane [10, 11]. To a first approximation this relationship may be described by a linear function

$$\tau_c = \tau_c^0(1 - \alpha\, \sigma_{nv}); \tag{4}$$

where τ_c^0 is the value of τ_c with $\sigma_{nv} = 0$; α is a coefficient characterizing the sensitivity of shear strength to the magnitude of normal stresses.

The condition for the start of macroyielding of a polycrystal in this case has the form

$$P\left\{\tau_{nv} \geq \tau_c \middle| \tau_c = \tau_c^0(1 - \alpha\, \sigma_{nv})\right\} \geq P_c; \tag{5}$$

$$P\left\{\tau_{nv} \geq \tau_c \middle| \tau_c = \tau_c^0(1 - \alpha\, \sigma_{nv}\right\} = 1 - \int_{-\infty}^{+\infty} d(\sigma_{nv}) \int_{-\tau_c}^{+\tau_c} \varphi(\tau_{nv}, \sigma_{nv}) d(\tau_{nv}), \tag{6}$$

where $\varphi(\tau_{nv}, \sigma_{nv})$ is a density function for the combined distribution of tangential τ_{nv} and normal σ_{nv} microstresses in slip systems.

TABLE 1. Calculated and Experimental Values of Parameters K_m and χ Characterizing the Level Deviation from the Mises Criterion with Biaxial ($\sigma_1 = \sigma_2$) Tension and Uniaxial Compression

T, K	K_m^e, [14]	K_m^t, (8)	δ_{K_m}, %	χ^e, [1]	χ^t, (8)	δ_χ, %
293	1,00	0,998	− 1,2	1,00	0,996	− 0,4
223	0,99	0,976	− 1,4	0,90	0,95	+ 5,6
173	0,98	0,961	− 1,94	0,85	0,918	+ 8,0
123	0,86	0,947	+ 10,1	0,83	0,887	+ 6,9

Note. δ_{K_m} and δ_χ are relative error for theoretical values $K_m{}^T$ and χ^T

In explicit form the yielding criterion taking account of features of low-temperature slip in a BCC-lattice is conveniently presented as follows [12]:

$$\sigma_i = K_m \sigma_y. \tag{7}$$

Here σ_y is yield strength with uniaxial tension;

$$K_m = \frac{K_1(1 - P_c)\sqrt{2\pi\,(1 - r_t^2)} + \alpha K_2 \tau_c^0}{K_1(1 - P_c)\sqrt{2\pi\,(1 - r_m^2)} + \alpha K_2 \tau_c^0 J}, \tag{8}$$

where J is a parameter characterizing stressed state stiffness [1],

$$J = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sigma_i}; \tag{9}$$

σ_1, σ_2, σ_3 are principal stresses; r_t and r_m are correlation coefficients for tangential and normal microstresses in slip systems with a uniaxial and multiaxial stressed state respectively; K_1 and K_2 are coefficients depending on lattice elastic constants.

Discussion of Results. It follows from (7) that the dependence of shear strength on the magnitude of normal stresses in a slip plane at the microlevel departs from the Mises criterion. The degree of this deviation increases with an increase in spherical macrostress tensor component.

Thus it is possible to formulate a theorem for the connection of flow conditions at the micro- and macrolevels. Whereas at the microlevel the maximum tangential stress criterion is fulfilled, at the macrolevel it corresponds to the Mises criterion. The dependence of shear strength on the magnitude of normal stresses in a slip system develops at the microlevel as the effect of hydrostatic stress component on yield strength.

These assumptions are in good agreement with available experimental data. It is well known that slip in single-crystal metals with a FCC-lattice obeys the Schmidt rule (the criterion of maximum tangential stresses) over a wide temperature range. Testing of polycrystals with this type of lattice (Al, Cu) indicates that the Mises criterion for these metals is fulfilled both at room ($T = 293$ K) and at low ($T = 78$ K) temperatures [13]. There is no deviation from the Mises criterion also with low-temperature tests on steels with an FCC-lattice [14].

According to current ideas the deviation from the Schmidt rule observed in BCC-crystals is connected with the effect of normal stresses on the height of Peierls–Nabbarro screw dislocation barriers which is governed by features of the structure of the nucleus for these dislocations in the BCC-lattice [15]. This makes it possible to obtain an expression for coefficient α in (8) [16]:

$$\alpha = \alpha' \frac{\tau_p^0}{\tau_c^0}, \tag{10}$$

where $\tau_p{}^0$ is Peierls–Nabbarro stress with $\sigma_{nv} = 0$; α' is a coefficient characterizing the sensitivity of the Peierls–Nabbarro barrier value to the level of normal stresses.

Taking account of (10) the product $\alpha\tau_c{}^0$ in expression (8) may be substituted by $\alpha'\tau_p{}^0$. This means that the increase in the level of deviation from the Mises criterion observed in BCC-metals at low temperature and also the reduction in parameter χ is connected with an increase in the magnitude of Peierls–Nabbarro stresses.

Given in Table 1 are calculated and experimental values of K_m and χ for steel 35. Calculations were performed for α – Fe: $K_1 = 0.226$, $K_2 = 0.346$, $r_t \approx r_m = -0.3507$ [12]; $P_c = 0.208$ [9]. According to experimental data obtained for silica iron single crystals with T = 195 K, $\tau_c{}^0$ = 223 MPa, α = $1.9 \cdot 10^{-4}$ MPa^{-1} [10]; $\tau_p{}^0$ = 96 MPa [17] with equation (10) α' = $4.41 \cdot 10^{-4}$ MPa^{-1}. The temperature dependence for Peierls–Nabbarro stress with T = 98-293 K [17] was approximated by the relationship

$$\tau_p^0 = a(1 - \beta T), \tag{11}$$

where a = 283.1 MPa; β = 0.00335 K^{-1}.

Thus, it is possible to conclude that to a first approximation the connection between slip conditions in a lattice and macroyielding of a polycrystal is as follows. If at a microlevel the criterion of maximum tangential stresses (Schmidt rule) is fulfilled, then at the macrolevel yielding is described by the Mises criterion. The dependence of shear strength on the magnitude of normal stresses operating in a slip plane appears at the macrolevel as the effect of hydrostatic macrostress tensor components on the yield strength.

Deviation from the Schmidt rule, which occurs with low-temperature slip in a BCC-lattice, is the main reason for the effect of transforming the yield strength curve for iron and steels with a CSS and the χ-effect with a uniaxial stressed state.

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