# Coexistence of Superconductivity with Charge- or Spin- Density Waves

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# COEXISTENCE OF SUPERCONDUCTIVITY WITH CHARGE- OR SPIN- DENSITY WAVES

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## 1. Introduction

Once the Bardeen-Cooper-Schrieffer (BCS) seminal theory had been developed and accepted by the superconducting community, it was successfully applied to metals and alloys. However, the validity of the BCS scheme was confirmed fully only for substances (but not all!) with low critical temperatures, $T_c$'s. It is hardly surprising because the weak-coupling theory can be expressed through the reduced quantities in much the same way as the Van der Waals equation of state for real gases can be traced to the equation of corresponding states. Thus, all background physics contained in $T_c$, energy gap $\Delta$, electronic specific heat, ultrasound attenuation coefficient, etc. is formally hidden in equations to be compared with experiment. The "only" (but extremely important) exception is the electrodynamics, where the microscopic quantities cannot be eliminated neither at the very beginning of the calculations nor from the final results. That is why the "BCS" relationships for reduced quantities do not exist here.

In reality, the so-called strong-coupling deviations from the BCS scheme become conspicuous even for simple metals such as Pb, so that the details of the electron-phonon interaction, originally included into the BCS theory, or any other possible attraction leading to the Cooper pairing (which may be incorporated into the original scheme on equal footing [1]) manifest themselves explicitly. The situation becomes even more entangled for compounds with magnetic ions [2], superconductors with low densities of current carriers [3, 4], layered and quasi-one-dimensional compounds [1] and so on. Such compounds were called "exotic" some time ago [5], and their list expanded enormously in recent years [6], now including the vast majority of superconductors. This expansion seems quite natural since, strictly speaking, all superconductors are "exotic". However, low $T_c$'s in some of them result in a



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strong reduction of the departure from the BCS simplicity [7]. Nevertheless, the BCS theory and the Cooper pairing concept remain the cornerstone of almost all current sophisticated theories of superconductivity [6, 7, 8, 9].

Among the exotic superconductors there is a very interesting large group involving substances with charge- or spin-density waves (CDW's and SDW's) [1, 4, 5, 9, 10, 11]. Here two kinds of pairings, namely, Cooper pairing and dielectric one, coexisting in the same microscopic cells of the sample, compete for the same Fermi surface (FS) and lead to the emergence of two interplaying order parameters $\Delta$ and $\Sigma$, respectively [4, 12]. The occurrence of two energy gaps $\Delta$ and $|\Sigma|$ makes the methods of tunnel (TS) and point-contact spectroscopies (PCS) or scanning tunnel microscopy (STM) the most suitable and powerful tools to investigate such a class of superconductors. Bearing in mind the Kramers-Kronig relationship between the quasiparticle current $J$ and the nonstationary Josephson current $I$ [13], one may expect the dielectric gap to reveal itself also in the $I(V)$ measurements, where $V$ is voltage. From this point of view it is especially interesting and important to consider the most promising high-$T_c$ oxides where the available tunnel data are somewhat confusing [14, 15, 16, 17, 18, 19]. At the same time, other various experimental methods clearly show that SDW's and CDW's manifest themselves for non-superconducting compositions [6, 8, 9, 10] and often exist as pseudogap phantoms above $T_c$ in superconducting samples [9].

Below we present the theory of tunnel currents in junctions involving superconductors with CDW's and SDW's, founded on the BCS idea applied both to Cooper and to dielectric pairings, with the partial dielectrization (partial gapping) model of Bilbro and McMillan [20] used as the basis of the investigation. However, the original simplicity inherent to the TS of BCS superconductors or Peierls (excitonic) insulators is lost here due to the peculiar interplay between the underlying phenomena. Since the theories for SDW and CDW superconductors have much in common, we introduce also the notation DW to consider them simultaneously. The results describe well the data for high-$T_c$ oxides, $NbSe_3$, heavy-fermion compound $URu_2Si_2$ and other compounds.

## 2. Theory

The model Hamiltonian of the DW superconductor has the form [14, 20, 21, 22]

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\mathrm{BCS}} + \mathcal{H}_{\mathrm{DW}}, \tag{1}$$

where $\mathcal{H}_0$ is the free-electron Hamiltonian, $\mathcal{H}_{\rm BCS}$ is the original BCS Hamiltonian, and

$$\mathcal{H}_{\rm DW} = -\Sigma \sum_{i=1}^{2} \sum_{\mathbf{p}\alpha} \left[1 + (2\alpha - 1)\Psi\right] a^{\dagger}_{i\mathbf{p}\alpha}\, a_{i,\mathbf{p}+\mathbf{Q},\alpha} + \text{H.c.} \tag{2}$$

is the DW Hamiltonian describing the electron-hole pairing. The operator $a^{\dagger}_{i\mathbf{p}\alpha}$ $(a_{i\mathbf{p}\alpha})$ is the creation (annihilation) operator of a quasiparticle with a quasimomentum $\mathbf{p}$ and spin projection $\alpha = \pm\frac{1}{2}$ from the $i$th FS section. Namely, $i = 1$ and $2$ for the nested sections where the electron spectrum is degenerate $\xi_1(\mathbf{p}) = -\xi_2(\mathbf{p} + \mathbf{Q})$, $\mathbf{Q}$ being the DW vector, while $i = 3$ for the rest of the FS where the dispersion relation for elementary excitations is described by the different function $\xi_3(\mathbf{p})$. Parameter $\Psi = 0$ (1) for CDW's (SDW's). The dielectric order parameter $\Sigma$ emerges on the nested FS sections, so the summation in Eq. (2) is carried out over them only. On the other hand, the *single* superconducting order parameter $\Delta$ appears on the whole FS. The ratio

$$\nu = N_{nd}(0)/N_d(0), \tag{3}$$

where $N_{d(nd)}(0)$ is the density of states for dielectrized (nondielectrized) part of the FS, characterizes the gapping degree of the metal.

The quantity $\Sigma$ can be taken as a $\Delta$ independent phenomenological function of the temperature $T$ [14, 23]. The order parameter $\Sigma$ is real and can be of either sign [22, 24]. We performed most calculations using the BCS curve for $\Sigma(T)$ because the specific choice is not important from the conceptual point of view. The $\Delta(T)$ dependence for DW superconductors can be easily found from our theory [25, 26, 27, 28], using the function $\Sigma(T)$ and appropriate values of $\nu$.

The normal $\mathsf{G}^{\alpha\beta}_{ij}(\mathbf{p};\omega_n)$ and anomalous $\mathsf{F}^{\alpha\beta}_{ij}(\mathbf{p};\omega_n)$ Matsubara Green's functions (GF) corresponding to the Hamiltonian (1) can be found from the Dyson-Gor'kov equations [11, 24, 26, 27, 28]. To calculate the tunnel currents we need temporal GF $F(\omega)$ and $G(\omega)$ rather than the temperature ones. The equivalence of the dielectrized FS sections 1 and 2, e.g., in the SDW superconductor reduces the number of GF for each electrode. Namely,

$$\begin{gathered}
F_{11}(\omega) = F_{22}(\omega) \equiv F_d(\omega), G_{11}(\omega) = G_{22}(\omega) \equiv G_d(\omega), \\
F_{12}(\omega) = F_{21}(\omega) \equiv F_{is}(\omega), G_{12}(\omega) = G_{21}(\omega) \equiv G_{is}(\omega), \\
F_{33}(\omega) \equiv F_{nd}(\omega), G_{33}(\omega) \equiv G_{nd}(\omega),
\end{gathered} \tag{4}$$

where the subscript $nd$ corresponds to the nondielectrized part 3 of the FS, $d$ to the nested (dielectrized) parts 1 and 2, and $is$ to the intersection excitonic (electron-hole) pairing between a quasiparticle from the part 1 and a **Q**-shifted quasiparticle from the part 2. All other GF vanish. Functions $F_{d,nd,is}(\omega)$ and $G_{d,nd,is}(\omega)$ are generated in the conventional manner [29]. First, the thermal GF are integrated over the momentum $\mathbf{p}$ and the analytical continuation is made to the real axis of the variable $i\omega_n$. For SDW superconductors the result is

$$\mathsf{G}^R_{nd}(\omega) = -\pi\omega s(\Delta), \tilde{\mathsf{F}}_{nd}(-i\omega) = \pi\Delta s(\Delta),$$

$$\mathsf{G}^R_{d}(\omega) = -\frac{\pi\omega}{2}\left[s(D_+) + s(D_-)\right], \tilde{\mathsf{F}}_{d}(-i\omega) = \frac{\pi}{2}\left[D_+ s(D_+) + D_- s(D_-)\right],$$

$$\mathsf{G}^R_{is}(\omega) = -\frac{\pi}{2}\left[D_+ s(D_+) - D_- s(D_-)\right], \tilde{\mathsf{F}}_{is}(-i\omega) = -\frac{\pi\omega}{2}\left[s(D_+) - s(D_-)\right]. \tag{5}$$

Here $D_\pm = \Delta \pm \Sigma$, $s(D) = 1/\sqrt{D^2 - (\omega + i0)^2}$, the superscript $R$ reflects the retarded character of the GF. Second, the functions (5) are connected to the relevant temporal GF $F(\omega)$ and $G(\omega)$ by the dispersion relations[29].

For CDW superconductor the GF are much more simple [14]. The function $\tilde{\mathsf{F}}_{is} \equiv 0$, and other functions involve a single $\Sigma$-dependent combined "gap" $D = \sqrt{\Delta^2 + \Sigma^2}$ rather than $D_\pm$.

To calculate the total tunnel current $I$ through the junction we use the conventional tunnel Hamiltonian approach [29], according to which the Hamiltonian has the form:

$$\mathcal{H}_{\rm tun} = \mathcal{H} + \mathcal{H}' + \mathcal{T}. \tag{6}$$

The left- and right-hand-side electrodes of the junction are described in Eq. (6) by the terms $\mathcal{H}$ and $\mathcal{H}'$, respectively, which coincide with the Hamiltonian (1) with an accuracy of notations. Hereafter primed entities including sub- and superscripts correspond to the right-hand-side of the junction. The tunnel term $\mathcal{T}$ is of the form

$$\mathcal{T} = \sum_{i,i'=1}^{3} \sum_{\mathbf{pq}'\alpha} \mathsf{T}^{ii'}_{\mathbf{pq}'} a^\dagger_{i\mathbf{p}\alpha} a_{i'\mathbf{q}'\alpha} + \text{H.c.}, \tag{7}$$

where $\mathsf{T}^{ii'}_{\mathbf{pq}'}$ are the tunnel matrix elements. The general expression for $I(T)$ obtained in the lowest order of the perturbation theory in $\mathcal{T}$ is a sum of functionals depending on temporal GF $F(\mathbf{p},\tau)$ and $G(\mathbf{p},\tau)$, where $\tau$ denotes time [29]. The GF $F(\mathbf{p},\tau)$ and $G(\mathbf{p},\tau)$, integrated over $\mathbf{p}$ variable, are connected to $F(\omega)$ and $G(\omega)$ by Fourier transformation. Making the assumptions [14] that (i) all matrix elements $\mathsf{T}^{ii'}_{\mathbf{pq}'}$ are equal and not influenced by the existence of $\Delta$ and $\Sigma$, in the spirit of the standard Ambegaokar-Baratoff approach [30], and (ii) the current $I$ is independent of the relative spatial orientation of the junction plane and the DW vector $\mathbf{Q}$, we introduce the universal tunnel resistance $R$ :

$$R^{-1} = 4\pi e^2 N(0) N'(0) \left\langle |\mathsf{T}|^2 \right\rangle_{\mathrm{FS}} . \tag{8}$$

Here $e$ is the elementary charge, the total density of states $N(0) = N_d(0) + N_{nd}(0)$, angular brackets $< \ldots >_{\mathrm{FS}}$ imply averaging over the FS. Then, in the adiabatic approximation $V^{-1} \frac{dV}{d\tau} \ll T_c$ for the ac bias voltage $V(\tau) \equiv V_{\mathrm{right}}(\tau) - V_{\mathrm{left}}(\tau)$ across the Josephson junction, we obtain the expression for the total current $I$ through the junction made up of the DW superconductors(cf. with Ref. [29]):

$$I[V(\tau)] = \sum_{i=1}^{9} [I_i^1(V) \sin 2\phi + I_i^2(V) \cos 2\phi + J_i(V)], \tag{9}$$

where $\phi = \int^\tau eV(\tau) d\tau$, $I^1 = \sum_{i=1}^9 I_i^1$ is the Josephson current, $I^2 = \sum_{i=1}^9 I_i^2$ is the interference pair-quasiparticle current, and $J = \sum_{i=1}^9 J_i$ is the quasiparticle current. The explicit cumbersome expressions for $I_i^{1,2}$ and $J_i$ are given in Ref. [14].

When obtaining Eq. (9), we assume the strong DW pinning. The phases of the superconducting order parameters are, as usual [30], considered free, with their difference obeying the given above Josephson relationship connecting it to the bias voltage.

## 3. Results

Below we shall confine ourselves to symmetrical junctions, i.e., when both electrodes are identical DW superconductors. At the same time, the junctions made up of *thermodynamically identical* superconductors ought to be further classified as genuinely symmetrical ($s$) or formally symmetrical with broken symmetry ($bs$). It is convenient to group nine components of each current amplitude in the following manner:

$$\begin{gathered}
I^{1,2}_{(b)s1} = I^{1,2}_1, I^{1,2}_{(b)s2} = I^{1,2}_4, I^{1,2}_{(b)s3} = I^{1,2}_5 + I^{1,2}_7, \\
I^{1,2}_{(b)s4} = I^{1,2}_9, I^{1,2}_{(b)s5} = I^{1,2}_2 + I^{1,2}_3, I^{1,2}_{(b)s6} = I^{1,2}_6 + I^{1,2}_8, \\
J_{(b)s1} = J_1, J_{(b)s2} = J_4, J_{(b)s3} = J_5 + J_7, \\
J_{(b)s4} = J_9, J_{(b)s5} = J_2 + J_3, J_{(b)s6} = J_6 + J_8.
\end{gathered} \tag{10}$$

For $s$-junctions $\nu = \nu', \Sigma = \Sigma', \Delta = \Delta'$, and (the notations $F'_{ij} \equiv F_{i'j'}$ and $G'_{ij} \equiv G_{i'j'}$ are used)

$$F_{11} = F'_{11} = F_d, G_{11} = G'_{11} = G_d, F_{33} = F'_{33} = F_{nd}, G_{33} = G'_{33} = G_{nd}, \tag{11}$$

$$F_{12} = F'_{12} = F_{is}, G_{12} = G'_{12} = G_{is}. \tag{12}$$

Then the initial current amplitude components $i = 2,3$ and $i = 6,8$ exactly compensate each other in pairs, and the total current (9) through the junction may be written as

$$\begin{aligned} I_s &= \sum_{i=1}^{4}[I^1_{si}(V)\sin 2\phi + I^2_{si}(V)\cos 2\phi + J_{si}(V)] \ for\ SDW \\ &= \sum_{i=1,3,4}[I^1_{si}(V)\sin 2\phi + I^2_{si}(V)\cos 2\phi] + \sum_{i=1}^{4} J_{si}(V) \ for\ CDW. \end{aligned} \tag{13}$$

For the current amplitudes $I^{1,2}_{si}$ and $J_{si}$ the usual symmetry relations hold:

$$I^1_{si}(-V) = I^1_{si}(V), I^2_{si}(-V) = -I^2_{si}(V), J_{si}(-V) = -J_{si}(V), i = 1\ldots 4. \tag{14}$$

Hence, these relations are also valid for total amplitudes. For $s$-junctions current-voltage characteristics (CVC's) of all three currents do not depend on the sign of $\Sigma$.

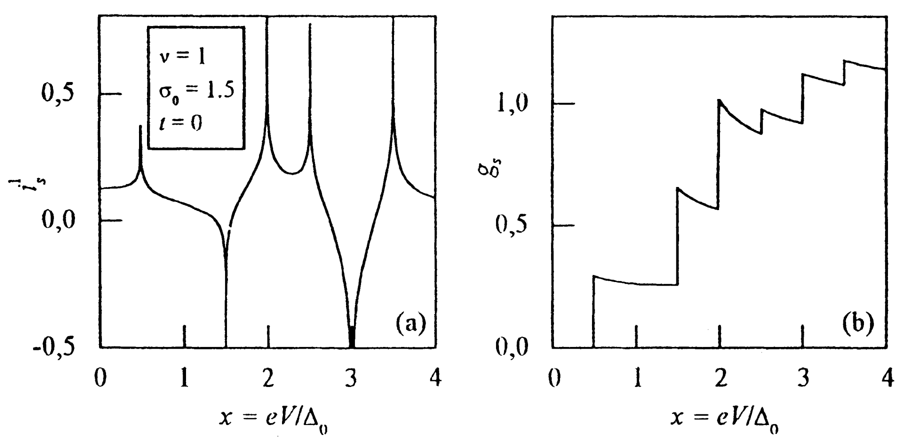


*Figure 1.* Dependences (a) of the dimensionless Josephson current amplitude $i^1_s$ and (b) of the differential quasiparticle conductivity $g_s$ on the dimensionless voltage $x$ for symmetrical tunnel junction between SDW superconductors.

The CVC for the dimensionless Josephson current amplitude $i^1_s(x) \equiv I^1_s eR/\Delta_0$, where $x \equiv eV/\Delta_0$ and $\Delta_0$ is the superconducting gap in the absence of the dielectrization, as well as the differential quasiparticle conductivity $g_s(x) = dj_s/dx$, where $j_s \equiv J_s eR/\Delta_0$ is the reduced quasiparticle current amplitude, are shown in Fig. 1 for the SDW case. The feature points are seen at voltages $eV = 2\Delta, 2|D_\pm|, |D_+| + |D_-|$, and $|D_\pm| + \Delta$. One can see that the structure of the curves is much more involved than for BCS superconductors. It is also remarkable that the *dielectric* gap influences strongly the CVC for coherent Josephson *supercurrent.*

The situation for junctions involving CDW superconductors is similar but less cumbersome. The respective $i_s^1(x)$ and $g_s(x)$ are depicted in Fig. 2. The feature points clearly seen in the figures correspond to the voltages $eV = 2\Delta, D + \Delta$, and $2D$.

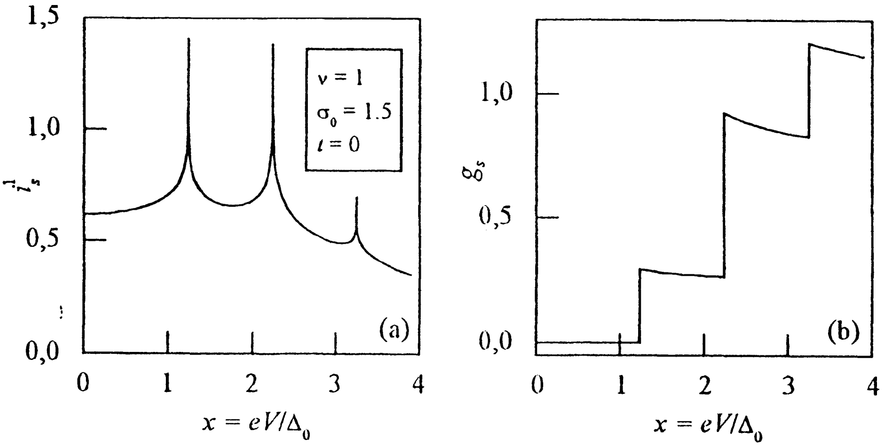


*Figure 2.* **The same as in Fig. 1 but for CDW superconductors.**

For formally symmetrical junctions involving identical SDW superconductors an alternative opportunity may be realized. Namely, the *symmetry breaking* can take place, i.e., the left-hand partially-gapped electrode possessing, say, a positive dielectric order parameter $\Sigma > 0$ and the right-hand one having a negative parameter $\Sigma' = -\Sigma < 0$, or vice versa. In both cases the junction is nonsymmetrical in reality, although $|\Sigma| = |\Sigma'|$ and all macroscopical properties of each separated electrode are *identical* due to the thermodynamical equivalence of SDW superconductors with equal $\Delta$'s and $|\Sigma|$'s [21, 26]. However, if the junction concerned is a part of the electric circuit, it will serve as a phase-sensitive indicator of the symmetry breaking between the electrodes [15, 31, 32]. Such a phenomenon comprises a new macroscopical manifestation of the symmetry breaking in many-body systems. The corresponding CVC's are substantially different from their genuinely symmetrical counterparts. Really, for this state of a junction $\nu = \nu'$, $\Delta = \Delta'$, $\Sigma' = -\Sigma$, Eqs. (11) hold but

$$F_{12} = -F'_{12} = F_{is}, G_{12} = -G'_{12} = G_{is}. \tag{15}$$

Then the current amplitude components $i = 2,3$ and $i = 6,8$ do not compensate but enhance each other. So,

$$
\begin{aligned}
I_{bs} &= \sum_{i=1}^{6}[I^1_{bsi}(V)\sin 2\phi + I^2_{bsi}(V)\cos 2\phi + J_{bsi}(V)] \ for\ SDW \\
&= \sum_{i=1,3,4}[I^1_{bsi}(V)\sin 2\phi + I^2_{bsi}(V)\cos 2\phi] + \sum_{i=1}^{6} J_{bsi}(V) \ for\ CDW.
\end{aligned}
\tag{16}
$$

The symmetry relations for the $bs$-current components indexed by $bs$ 1 to $bs4$ are the same as for the $s$-junction [see Eq. (14)]

$$
I^1_{bsi}(-V) = I^1_{bsi}(V), I^2_{bsi}(-V) = -I^2_{bsi}(V), J_{bsi}(-V) = -J_{bsi}(V), i = 1\ldots 4,
\tag{17}
$$

with the $bs2$ terms changing signs as compared to their $s2$ counterparts. However, now the $bs5$ and $bs6$ components do not vanish, having the unusual properties

$$
I^1_{bsi}(-V) = -I^1_{bsi}(V), I^2_{bsi}(-V) = I^2_{bsi}(V), J_{bsi}(-V) = J_{bsi}(V), i = 5,6.
\tag{18}
$$

It affects crucially the total current amplitudes $I^{1,2}_{bs}(V)$ and $J_{bs}(V)$, making them neither symmetrical nor antisymmetrical in $V$. Therefore, in the $bs$-junctions the CVC's of all total currents for SDW case and only of $J_{bs}$ for CDW case depend on the voltage polarity. They are also sensitive to the sign of $\Sigma = -\Sigma'$ because the current components have the following symmetry properties:

$$
\begin{aligned}
&I^{1,2}_{bs1,2,3,4}(-\Sigma) = I^{1,2}_{bs1,2,3,4}(\Sigma), I^{1,2}_{bs5,6}(-\Sigma) = -I^{1,2}_{bs5,6}(\Sigma), \\
&J_{bs1,2,3,4}(-\Sigma) = J_{bs1,2,3,4}(\Sigma), J_{bs5,6}(-\Sigma) = -J_{bs5,6}(\Sigma).
\end{aligned}
\tag{19}
$$

With changing $\Sigma$ sign, the different $V$-polarity branches are interchanged.

In Fig. 3, the dependences $i^1_{bs}(x) \equiv I^1_{bs}eR/\Delta_0$ and $g_{bs}(x) = dj_{bs}/dx$ are displayed for SDW superconductors. Here l.h.s. and r.h.s. branches differ substantially due to peculiar compensations and amplifications of logarithmic singularities and jumps originating from various current components.

The plot $j_{bs}(x)$ in the CDW case resembles (with corresponding simplifications) that for the $bs$-junctions between SDW superconductors and was published elsewhere[15, 31].

The frustrated junction between DW superconductors can be treated as a discrete analog, with respect to the relative phase difference, of the Josephson junction. Nevertheless, it is radically different from the phase-coherent weak link between two Peierls insulators with sliding CDW's considered by Artemenko and Volkov [33]. Unlike these authors, we assume the pinning of the $\Sigma$ and $\Sigma'$ phases, therefore ruling out coherent effects.

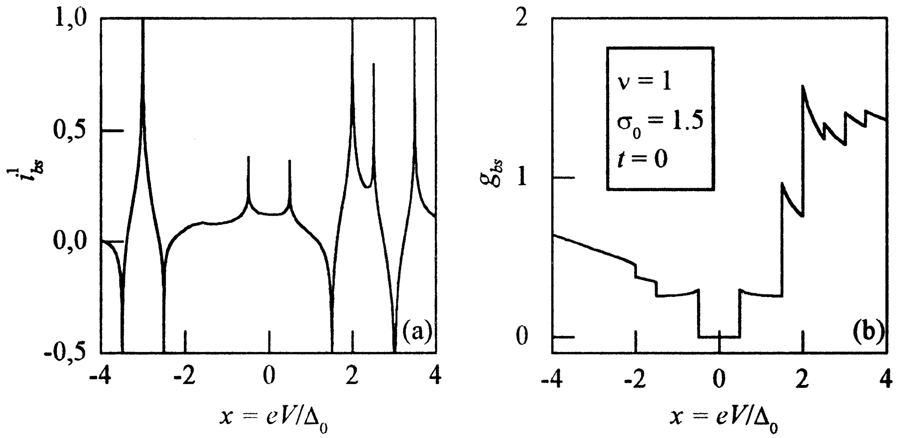


*Figure 3.* **Dependences (a) of the dimensionless Josephson current amplitude $i^1_{bs}$ and (b) of the differential quasiparticle conductivity $g_{bs}$ on the dimensionless voltage $x$ for tunnel junction with broken symmetry between formally identical SDW superconductors.**

Nevertheless, the junction concerned feels the difference or coincidence between the dielectric order parameter signs. Thus, the symmetry breaking in the symmetrical junction serves as a detector of the order-parameter phase multiplicity in electrodes. This is also common to nonsymmetrical junctions [23].

## 4. Discussion

Prior to comparing our results with experimental data, we should specify partially-dielectrized substances where Cooper and electron-hole pairings coexist. They are 2H-$NbSe_2$, $NbSe_3$, $Tl_2Mo_6Se_6$, $R_5Ir_4Si_{10}$ ($R$ labels various rare-earth elements), $Lu_5Rh_4Si_{10}$, $Li_{0.9}Mo_6O_{17}$, $BaPb_{1-x}Bi_xO_3$, $Rb_xWO_3$, $(PO_2)_4(WO_3)_{2m}$, Chevrel and Laves phases, A15 compounds for the CDW case (see references. in Refs. [4, 5, 6, 10, 11, 14]), $Cr_{1-x}Re_x$, $URu_2Si_2$, $(TMTSF)_2X$ ($X =$ $AsF_6$, $PF_6$, $ClO_4$), $RNi_2B_2C$ for the SDW case (see references in Refs. [2, 5, 10, 23]). As for high-$T_c$ oxides, we compiled numerous observations of structural anomalies above $T_c$ and indicated [10] as early as in 1992 that the partial dielectrization concept might be valid for them similar to their lower-$T_c$ relatives [4].

The STM observations of CDW's in $YBa_2Cu_3O_{7-x}$ [34], and the pseudogap manifestations above $T_c$ in neutron scattering for $La_{2-x}Sr_xCuO_4$ [35] and $YBa_2Cu_3O_{6.6}$ [36], Raman scattering for $Bi_2Sr_2(Ca_{0.62}Y_{0.38})Cu_2O_{8+\delta}$ and $YBa_2Cu_3O_{7-x}$ [37], NMR measurements for $HgBa_2Ca_2Cu_3O_{8+\delta}$ [38] and $YBa_2Cu_4O_8$ [39] seem to be the direct proof of the CDW existence in high-$T_c$ superconductors. But it still remains unclarified whether their dielectric order parameters are of $s$- or $d$-symmetry, although most experimenters consider pseudogaps as $d$-ones [40, 41, 42]. It is worth noting that the extremely positive curvature of the upper critical field in $Bi_2Sr_2CaCu_2O_8$ [43] correlates well with our predictions for DW superconductors [11], despite that the authors explain their results in terms of preformed pairs (bipolarons), the latter concept accepted also in Refs. [44, 45]. The identification of pseudogaps with CDW or SDW gaps has been made in Refs. [9, 46, 47, 48] too.

TS and PCS measurements validated the partial dielectrization picture for two toy-compounds: $NbSe_3$ with CDW's [49] and $URu_2Si_2$ with SDW's [50, 51, 52]. In particular, CVC's are asymmetrical with complex structure of feature points for differential conductivity $G^{diff}(V)$. Moreover, homocontacts $URu_2Si_2-URu_2Si_2$ reveal the symmetry breaking predicted by us earlier for systems with DW's [32].

Again, the interpretation of TS and PCS results for high-$T_c$ oxides is hampered by the uncertainties of the superconducting order parameter symmetry [9, 16, 17, 19]. Namely, $s$-type gaps $\Delta$ are seen for $HgBa_2CuO_4$ [53, 54] and $Nd_{1.85}Ce_{0.15}CuO_{4-\delta}$ [53, 55], whereas V-like $G^{diff}(V)$ are appropriate to $YBa_2\,Cu_3O_{7-x}$ [16, 53], $Bi_2Sr_2CaCu_2O_{8+\delta}$ [16, 17, 53, 56] and $Tl_2Ba_2CuO_6$ [19, 53]. On the other hand, our theory has been developed for $s$-like superconductivity. Thus, it is possible now to make only preliminary comparison with different oxides, although our treatment could be easily modified for the $d$-case.

Conductance asymmetry in $S-I-N$ structures was observed for $Bi_2Sr_2CaCu_2O_{8+\delta}$ [57, 58]. In $S-I-S$ junctions the peaks of $G^{diff}(V)$ were found [57] for $|eV| \approx 2\Delta$ and dips for $|eV| \approx 3\Delta$. If one considers, in accordance with Refs. [40, 41], $|\Sigma|$ and $\Delta$ to be of the same order of magnitude far below $T_c$, then these features should be attributed to our $2\Delta$ and $\Delta + D$, with singularity at $|eV| \approx 2D$ being smeared. Two gap-like peculiarities were also seen for $HgBa_2Ca_2Cu_3O_{8-x}$ [59].

Applying simple BCS-based theories, to which ours positively belongs, to high-$T_c$ superconductors one should bear in mind the feasibility of more complex "superspin" structure of the order parameters [60]. The relatively simple interplay between DW-driven gap $|\Sigma|$ and superconducting gap $\Delta$ assumed here might be replaced then by a SO(n) picture including order parameter component mixing [60] and the emergence of current-density or spin-current-density waves [48]. So far, such a generalization lacks direct experimental evidence and lies beyond the scope of this article.

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