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On Bursts Produced by Muons and Electrons.

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Summary. — The problem of the size fluctuation of bursts produced by muons is considered in the light of the method adopted by us for the evaluation of the mean square deviation of the number of electrons produced in an electron-photon cascade. An explicit simple analytical expression for the mean square deviation of the number of electrons is derived. It is also shown how other problems of experimental interest can be dealt with by the use of multipoint density functions. In particular the sequent correlation functions developed in the earlier paper are used for the bursts produced by electrons.

Introduction.

The fluctuations in the size of showers produced by μ-mesons has received considerable importance in the interpretation of cosmic-ray events. Particularly in view of the occurrence of highly energetic muons and photons, it has become necessary to study the average size of the soft showers as well as their fluctuations about the mean size. Theoretical studies on the number fluctuation have been attempted as early as 1950 and have, however, not resulted in convenient expressions for the mean square deviation to be used in conjunction with experimental data (see for example RAMAKRISHNAN (1)). In view of this situation there have been successful attempts by Monte-Carlo calculations particularly by MESSEL and his group (2) who have obtained some results on the number distribution in certain energy ranges. Thus from a practical point

(1) ALLADI RAMAKRISHNAN: *Theory of Elementary Particles and Cosmic Rays* (London, 1962).
(2) J. C. BUTCHER and H. MESSEL: *Nucl. Phys.*, **20**, 15 (1960).

of view, the problem of estimating the fluctuation can be considered as a closed one particularly in view of the fact that the program of Monte-Carlo calculations (prepared by the Sydney group) can be fed into a computer by choosing the parametric values in the range corresponding to the experimental data. However, it is worth-while to examine whether simple analytical expressions can be obtained by a re-examination of the problem. The simple analytical expressions have a decided advantage of being used by any experimenter on the globe. Such an attempt in the opinion of the authors is really worth-while particularly in the light of some results obtained by us (3) in connection with the new approach to cascade theory put forward by RAMAKRISHNAN and SRINIVASAN (4) for the interpretation of events in nuclear emulsions (see also reference (5)). Section 1 of this paper deals with the number fluctuations of electrons produced in the entire shower due to a muon while in Sect. 2 we attempt the application of new densities in studying the properties of showers produced by μ-mesons and electrons.

1. – Let us consider the shower excited by a muon of energy E_0 incident at the top of the atmosphere corresponding to $t=0$. We assume that the shower develops due to the following fundamental processes:

i) radiation of a photon by the primary muon;

ii) radiation of a photon by a secondary electron and

iii) production of an electron-positron pair by a secondary photon.

When screening is complete, the cross-sections for these processes are given by

$$R_\mu(E'|E)\,\mathrm{d}E' = \frac{1}{(210)^2}\,R_1(E'|E)\,\mathrm{d}E'\,, \tag{1.1}$$

$$R_1(E'|E)\,\mathrm{d}E' = \left[\frac{E-E'}{E} - \left(\frac{4}{3}+\alpha\right)\left(1-\frac{E}{E-E'}\right)\right]\frac{\mathrm{d}E'}{E}\,, \tag{1.2}$$

$$R_2(E'|E)\,\mathrm{d}E' = \left[1 - \left(\frac{4}{3}+\alpha\right)\left(\frac{E'}{E}-\frac{E'^2}{E^2}\right)\right]\frac{\mathrm{d}E'}{E}\,, \tag{1.3}$$

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(3) S. K. SRINIVASAN, K. S. S. IYER and N. V. KOTESWARA RAO: *Zeit. Phys.*, **177**, 164 (1964).

(4) ALLADI RAMAKRISHNAN and S. K. SRINIVASAN: *Proc. Ind. Acad. Sci.*, **44**, 263 (1956).

(5) S. K. SRINIVASAN, J. C. BUTCHER, B. A. CHARTRES and H. MESSEL: *Nuovo Cimento*, **9**, 77 (1958).

where $R_\mu(E'|E)\,\mathrm{d}E'$ denotes the probability per unit thickness that a μ-meson (an electron) of energy E radiates a quantum and drops to an energy between E' and $E'+\mathrm{d}E'$ while $R_2(E'|E)\,\mathrm{d}E'$ denotes the probability per unit thickness that a photon of energy E creates a pair one particle of which has an energy lying between E' and $E'+\mathrm{d}E'$. Let $\pi_i(n, E, E_0, t)$ be the probability of finding n electrons between 0 and t in a shower excited by a primary of i-th type, ($i=1, 2, 3$ denote respectively a muon, a photon and an electron) each of the electrons having an energy greater than E at the time of its production. In the first approximation we shall neglect the loss in energy of muons and electrons due to ionization. Then it is easy to see that $\pi_i(n, E, E_0, t)$ is a function only of $\varepsilon = E_0/E$ due to the homogeneous nature of the assumed cross-sections. To obtain the equations satisfied by $\pi_i(n, \varepsilon, t)$, we use the technique of invariant imbedding as demonstrated in the previous paper [6]:

$$\frac{\partial\pi_1(n,\varepsilon,t)}{\partial t} = -\pi_1(n,\varepsilon,t)\int_0^1 R_\mu(\varepsilon')\,\mathrm{d}\varepsilon' + \sum_{n_1+n_2=n}\int_0^1 R_\mu(\varepsilon')\pi_1\left(n_1,\frac{\varepsilon}{\varepsilon'},t\right)\pi_2\left(n_2,\frac{\varepsilon}{1-\varepsilon'},t\right)\mathrm{d}\varepsilon', \tag{1.4}$$

$$\begin{aligned}\frac{\partial\pi_2(n,\varepsilon,t)}{\partial t} = &-\pi_2(n,\varepsilon,t)\int_0^1 R_2(\varepsilon')\,\mathrm{d}\varepsilon' + \int_0^\varepsilon R_2(\varepsilon')\pi_2\left(n-1,\frac{\varepsilon}{1-\varepsilon'},t\right)\mathrm{d}\varepsilon' + \\ &+\sum_{m=0}^{\infty}\int_\varepsilon^{1-\varepsilon} R_2(\varepsilon')\pi_3\left(m,\frac{\varepsilon}{\varepsilon'},t\right)\pi_2\left(n-m-2,\frac{\varepsilon}{1-\varepsilon'},t\right)\mathrm{d}\varepsilon' + \\ &+\int_{1-\varepsilon}^1 R_2(\varepsilon')\pi_2\left(n-1,\frac{\varepsilon}{\varepsilon'},t\right)\mathrm{d}\varepsilon',\end{aligned} \tag{1.5}$$

$$\frac{\partial\pi_3(n,\varepsilon,t)}{\partial t} = -\pi_3(n,\varepsilon,t)\int_0^1 R_2(\varepsilon')\,\mathrm{d}\varepsilon' + \sum_{n_1+n_2=n}\int_0^1 R_2(\varepsilon')\pi_3\left(n_1,\frac{\varepsilon}{\varepsilon'},t\right)\pi_2\left(n_2,\frac{\varepsilon}{1-\varepsilon'},t\right)\mathrm{d}\varepsilon'. \tag{1.6}$$

We observe that (1.4) and (1.6) are valid for the entire range of ε while

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[6] A. N. MITRA: *Nucl. Phys.*, **3**, 262 (1957).

(1.5) is valid only for $0 \leqslant \varepsilon \leqslant \frac{1}{2}$. E for $\varepsilon > \frac{1}{2}\,\pi_2$ satisfies the equation

$$(1.5a)\qquad \frac{\partial \pi_2(n, \varepsilon, t)}{\partial t} = -\pi_2(n, \varepsilon, t)\int_0^1 R_2(\varepsilon')\, d\varepsilon' + \int_0^{1-\varepsilon} R_2(\varepsilon')\pi_3\left(n-1, \frac{\varepsilon}{1-\varepsilon'}, t\right) d\varepsilon' + \int_\varepsilon^1 R_2(\varepsilon')\pi_3\left(n-1, \frac{\varepsilon}{\varepsilon'}, t\right) d\varepsilon'\,.$$

It is interesting to note that in the case of a shower generated by an electron or a photon, (1.5) and (1.6) alone need be considered.

Defining the generating function $g_i(u, \varepsilon, t)$ as

$$(1.7)\qquad g_i(u, \varepsilon, t) = \sum_{n=0}^{\infty} u^n \pi_i(n, \varepsilon, t)$$

and observing that the cumulant moments are given by

$$n_i^m(\varepsilon, t) = \frac{\partial^m}{\partial u^m} g_i(u, \varepsilon, t)\Big|_{u=1}\,,$$

we find

$$(1.8)\qquad \frac{\partial n_1^m(\varepsilon, t)}{\partial t} = -n_1^m(\varepsilon, t)\int_0^1 R_\mu(\varepsilon')\, d\varepsilon' + \sum_{i=0}^{m}\int_0^1 R_\mu(\varepsilon') n_1^i\left(\frac{\varepsilon}{\varepsilon'}, t\right) n_2^{m-i}\left(\frac{\varepsilon}{1-\varepsilon'}, t\right) d\varepsilon'\,,$$

We do not propose to go into the similar equations satisfied by $n_2^m(\varepsilon, t)$ and $n_3^m(\varepsilon, t)$ since they have been dealt with in great detail ([7]). (1.8) can be rewritten as

$$(1.9)\qquad \frac{\partial n_1^m(\varepsilon, t)}{\partial t} = -\int_0^1 R^\mu(\varepsilon')\left[n_1^m(\varepsilon, t) - n_1^m\left(\frac{\varepsilon}{\varepsilon'}, t\right)\right] d\varepsilon' + \int_0^1 R_\mu(\varepsilon')\, n_2^m\left(\frac{\varepsilon}{1-\varepsilon'}, t\right) d\varepsilon' + \sum_{i=1}^{m-1}\int_0^1 R_\mu(\varepsilon') n_1^i\left(\frac{\varepsilon}{\varepsilon'}, t\right) n_2^{m-i}\left(\frac{1-\varepsilon'}{\varepsilon}, t\right) d\varepsilon'\,.$$

(1.9) can be solved recursively if we know $n_1^i(\varepsilon, t)$. These are just the cumulant moments of the number distribution corresponding to a photon primary.

([7]) S. K. SRINIVASAN and K. S. S. IYER: *Nuovo Cimento*, 33, 273 (1964).

The Mellin transform solution of $n_1^1(\varepsilon, t)$ can be obtained without any difficulty (*). Using the well-known result for $n_2^1(\varepsilon, t)$ (see, for example, reference [5]), we obtain, after straightforward calculations,

$$(1.10)\qquad n_1^1(s,t) = \frac{B(s+1)\,C^\mu(s+1)}{s}\left[\frac{\mu(s+1)-D}{\lambda(s+1)}\left\{\frac{1-\exp[-A^\mu(s+1)t]}{A^\mu(s+1)} - \frac{\exp[-\lambda(s+1)t]-\exp[-A^\mu(s+1)t]}{A^\mu(s+1)-\lambda(s+1)}\right\} + \frac{D-\lambda(s+1)}{\mu(s+1)}\cdot\left\{\frac{1-\exp[-A^\mu(s+1)t]}{A^\mu(s+1)} - \frac{\exp[-\mu(s+1)t]-\exp[-A^\mu(s+1)]}{A^\mu(s+1)-\lambda(s+1)}\right\}\right],$$

where the functions, A, B, C, D are the same as those defined in the previous paper and the functions $A^\mu(s+1)$ and $C^\mu(s+1)$ are the corresponding functions associated with the muon cross-sections.

1·1. *Showers corresponding to large thickness.* – We next consider an interesting case corresponding to large t. Proceeding to the limit as t tends to infinity, we find

$$(1.11)\qquad n_1^1(s) = \frac{B(s+1)\,C^\mu(s+1)\,A(s+1)}{A^\mu(s+1)\,s[A(s+1)D-B(s+1)\,C(s+1)]}\,.$$

If we observe that $A^\mu(s+1)$ and $C^\mu(s+1)$ differ from $A(s+1)$ and $B(s+1)$ only by a constant factor arising from the mass, (1.11) can be identified to be the solution corresponding to an electron primary. This should be expected since we have made t infinite. In fact the result is true for all the moments of number distribution. Thus the muon characteristics of a shower are completely lost by making t tend to infinity. However we can introduce the decay constant of the μ-meson and this has a tendency to cut the size of the shower. The decay constant can be introduced into the equations.

We know that in a muon shower the decay probability per unit thickness is given by

$$k = \frac{mc}{\tau E},$$

where m is the rest energy, τ the lifetime of the muon. In this expression E is the only varying quantity. For large values of E_0, it is usually assumed (see, for example, reference [6]) that the change in energy of the muon is small

(*) We use the same functional symbol n_1, to denote the moment as well as its Mellin transform, the distinction being apparent from the context.

compared to the initial muon energy. Thus in the expression for k, E is replaced by E_0. However this is not a consistent procedure since muons may lose considerable part of the energy while traversing a very large thickness of matter. We shall estimate the correction on this assumption and compare it with the results obtained by taking the exact expression for k. Thus the effect of the decay constant can be incorporated by redefining $A^{\mu}(s+1)$ as

$$A^{\mu}(s+1) = K + \frac{1}{(210)^2} A(s+1) , \tag{1.12}$$

where

$$K = \frac{mc}{\tau E_0} . \tag{1.13}$$

$C^{\mu}(s+1)$ remains unaltered and is equal to $(210)^{-2}C(s+1)$. Since K is very small, $n_1^1(s)$ can be estimated easily. Retaining only the first power of K, we obtain

$$n_1^1(s) = B(s+1)\,C(s+1)\left[1 - \frac{(210)^2 K}{A(s+1)}\right] \cdot \frac{1}{s[A(s+1)D - B(s+1)\,C(s+1)]} . \tag{1.14}$$

Inverting the above expression, we find

$$n_1^1(\varepsilon) = 0.443\, e^{y} - (210)^2 K (0.4377\, e^{y} - 0.335\, y) . \tag{1.15}$$

For the primary muon energy of the order of 1000 GeV, the correction is of the order $10^{-4}\, e^{y}$.

We next use the exact expression for K. In this case, we obtain a difference equation for the mean number:

$$A(s+1)\,n_1^1(s) = C(s+1)\,n_2^1(s) - (210)^2 K\, n_1^1(s-1) = \\ = C(s+1)\,n_2^1(s) - (210)^2 K\, \delta n_1^1(s) , \tag{1.16}$$

where δ is a difference operator. Thus

$$\big(A(s+1) - (210)^2 K\delta\big)\, n_1^1(s) = C(s+1)\,n_2^1(s) . \tag{1.17}$$

Since $(210)^2K$ is small for fairly large muon primary energies, we can solve the difference equation to first order in $(210)^2K$:

$$n_1^1(s) = n_3^1(s) - \frac{(210)^2\, K\, B(s)\, C(s)\, A(s)}{A(s+1)\;(s-1)\,[A(s)\,D - B(s)\,C(s)]} . \tag{1.18}$$

Inverting the above expression, we find

$$n_1^1(\varepsilon) = 0.443\, e^y - (210)^2 K (0.2897\, e^{2y})\,. \tag{1.19}$$

Again for muon primary energy of the order of 1000 GeV, the correction is of the order $10^{-4} e^{2y}$. Thus (1.19) and (1.18) differ by a factor e^y and except for small values of y, the difference between the two expressions is fairly large. Table I shows the mean numbers as well as the correction due to decay.

TABLE I. – *Mean number of electrons produced in a muon burst.*

y	Mean number	Correction due to decay
2	3.27	$0.145 \cdot 10^{-2}$
3	8.90	$1.08 \cdot 10^{-2}$
4	24.2	$7.98 \cdot 10^{-2}$
5	65.7	$0.589 \cdot 10^{-1}$
6	179.0	4.36
7	486.0	32.20
8	1320.0	238.0

Corrections to the higher moments of the number distribution can easily be obtained. Using the same methods as were employed in reference [3], we can calculate the correction to the mean square number. The mean square number of electrons produced in the entire shower is given by

$$\varepsilon\{[n(\varepsilon)]^2\} = 0.2102\, e^{2y} + 0.656\, e^y + 1.2354 - \\ - (210)^2 K\{0.017\, e^{3y} + 0.41\, e^{2y} + 0.3\, e^y + 0.225\, y\}\,.$$

Table II gives the values of mean square number, mean square deviation as well as the correction due to decay. The corrections are fairly small for fairly small values of y.

TABLE II. – *Mean square number of electrons produced in a muon burst.*

y	Mean square	Correction due to decay
3	17.3	$2.98 \cdot 10^{-3}$
3	91.3	$2.87 \cdot 10^{-2}$
4	641.0	$3.71 \cdot 10^{-1}$
5	4660.3	5.98
6	$343 \cdot 10^2$	109.0
7	$2.58 \cdot 10^5$	810.0

2. – In this Section we shall deal with another class of problems that can be studied by the use of sequent correlation densities introduced in the previous paper (7). Suppose a primary is incident at $t=0$ and observations pertaining to secondary electrons are made at $t>0$. It is worth-while to seek the number spectrum of electrons produced earlier in thickness prior to t say at t_1 $(<t)$. Thus we can ask for $N(E, E_0, t_1, t)$ mean number of electrons that are produced between 0 and t_1 and have an energy greater than E at $t>t_1$ (E_0 being the energy of a primary electron).

Thus $N(E, E_0, t_1, t)$ is given by

$$N(E, E_0, t_1, t) = \frac{1}{2\pi i} \int\limits_{\sigma-i\infty}^{\sigma+i\infty} \frac{P_1^1(1, t_1, s_2, t)}{(s-1)} \left(\frac{E_0}{E}\right)^{s-1} \mathrm{d}s\,, \tag{2.1}$$

where P_1^1 is given by (4.14a) of the previous paper.

It is rather difficult to invert the above expression analytically. However it can be calculated by the method of steepest descents. Table III gives the values of $N(E/E_0, t_1, t)$ for $t_1=1$ and various values of t and $y=\log E_0/E$.

TABLE III. – $\varepsilon\{n(y, t_1, t_2)\}$ $t_1=1$.

y	t_2 = 3	4	5	6	7
3	0.1408	0.0637	0.0108	0.0023	0.0004
4	0.3763	0.1295	0.0453	0.0130	0.0082
5	0.5340	0.3386	0.0857	0.0445	0.0139
6	0.4253	0.5239	0.2927	0.0954	0.0133
7	0.2678	0.2112	0.5182	0.2575	0.0462

Table III shows that most of the electrons that are produced between 0 and 1 drop down considerably in energy by the time they traverse 3 or 4 cascade units. Such a table can be used in conjunction with the study of « young » and « old » electrons that are observed at any particular thickness.

Next we deal with the $\mathscr{F}\pi$ functions that were introduced in the earlier paper. Such a function is useful in interpreting portions of showers accompanied or not by primary. We shall develop the theory for a primary electron; extensions to muon problems can be achieved by such modifications as were introduced in Sect. **1**.

$\mathscr{F}\pi_1(E_1, t_1; E_2, E_3, t/E_0)\,\mathrm{d}E_1\mathrm{d}E_2\mathrm{d}E_3$ denotes the probability that an electron of primitive energy between E_1 and $E_1+\mathrm{d}E_1$ is created between t_1 and $t_1+\mathrm{d}t_1$ in the shower excited by an electron of energy E_0, the primary and the secondary dropping down to an energy in the intervals $(E_2, E_2+\mathrm{d}E_2)$ and $(E_3, E_3+\mathrm{d}E_3)$, respectively. To obtain the differential equation satisfied by this function, we consider the first infinitesimal interval $(0, \Delta)$ of thickness t. In $(0, \Delta)$ the primary electron may radiate a photon and drop to a lower energy in which case we have two independent primaries (electron and photon). If the primary electron does not radiate, the contribution to the density function is the density itself except that the thicknesses are reduced by Δ. Thus by simple probability arguments we obtain

$$(2.2)\qquad \left(\frac{\partial}{\partial t_1}+\frac{\partial}{\partial t}\right)\mathscr{F}\pi_1(E_1, t_1; E_2, E_3, t\,|\,E_0) =$$

$$= -\int_0^{E_0} R_1(E'\,|\,E_0)\,\mathrm{d}E'\,\mathscr{F}\pi_1)\,(E_1, t_1, E_2, E_3, t\,|\,E_0) +$$

$$+\int_0^{E^0} R'(E'\,|\,E_0)\,\mathscr{F}\pi_1(E_2, t_1, E_2, E_3, t\,|\,E')\,\mathrm{d}E' +$$

$$+\int_0^{E^0} R_1((E'\,|\,E_0)\,\mathscr{F}_1^2(E_1, t_1; E_2, E_3, t_1\,|\,E_0-E')\,\pi(E_3, t\,|\,E')\,\mathrm{d}E'\,.$$

As in Sect. **1** we note that $\mathscr{F}\pi_1$ is a function only of $\varepsilon_1 = E_1/E_0$, $\varepsilon_2 = E_2/E_0$, $\varepsilon_3 = E_3/E_0$ and hence we write it as $\mathscr{F}\pi_1(\varepsilon_1, t_1; \varepsilon_2, \varepsilon_3, t)$. Defining the Mellin's transform of $\mathscr{F}\pi_1$ with respect to ε_1, ε_2 and ε_3 as

$$(2.3)\qquad \mathscr{F}\pi_1(s_1, t_1; s_2, s_3, t) = \iiint \mathscr{F}\pi_1(\varepsilon, t_1; \varepsilon_2, \varepsilon_3, t)\,\varepsilon_1^{s_1-1}\varepsilon_2^{s_2-1}\varepsilon_3^{s_3-1}\,\mathrm{d}\varepsilon_1\,\mathrm{d}\varepsilon_2\,\mathrm{d}\varepsilon_3\,;$$

we obtain

$$(2.4)\qquad \left(\frac{\partial}{\partial t_1}+\frac{\partial}{\partial t}\right)\mathscr{F}\pi_1(s_1, t_1; s_2, s_3, t) = -A(s_1+s_2+s_2-2)\,\mathscr{F}\pi_1(s_1, t_1; s_2, s_3, t) + \\ + \mathscr{F}_1^2(s_1, t_1; s_2, r)\,\pi(s_3, t)\,a_2(s_1+s_2-1, s_3)\,.$$

(2.4) can be solved by taking a double Laplace transformation with respect to t_1 and t and using the solutions for $\mathscr{F}_1^2(st, t_1; s_2, t)$ and $\pi(s_3, t)$. After some calculation we obtain

$$(2.5)\qquad \mathscr{F}\pi_1(s_1, t_1; s_2, s_3, t) = \frac{a_1(s_1+s_2-1, s_3)\,B(s_1+s_2-1)}{\mu(s_1+s_2-1)-\lambda(s_1+s^2-1)}\cdot \\ \cdot\left[\varphi(t_1)\exp[-A(s_2)-A(s_3)] + \varphi(t_1-t)\exp[-A(s_1+s_2+s_3-2)t]\right],$$

where

$$(2.6)\qquad \varphi(t_1) = \frac{\mu(s_1+s_2-1)-D}{A(s_1+s_2+s_3-1)-A(s_3)-\lambda(s_1+s_2-1)}\cdot$$

$$\cdot\left[\exp[-\{\lambda(s_1+s_2-1)-A(s_2)\}t_1]-\exp[-[A(s_1+s_2+s_3-2)-A(s_2)-A(s_2)]t_1\right]-$$

$$-\frac{D-\lambda(s_1+s_2-1)}{[A(s_1+s_2+s_3-2)-A(s_3)-\mu(s_1+s_2-1)}\cdot$$

$$\cdot\left[\exp[-\{\mu(s_1+s_2-1)-A(s_2)\}t_1]-\exp[-\{A(s_1+s_2+s_3-2)-A(s_2)-A(s_3)\}t_1]\right].$$

The mean number of electrons that are produced between 0 and t_1 and are followed by the primary and remain above an energy E is given by

$$(2.7)\qquad N(E, E_0, t_1) = \frac{1}{(2\pi i)^2}\int_{\sigma-i\infty}^{\sigma+i\infty}\int_{\sigma-i\infty}^{\sigma+i\infty}\mathscr{F}\pi_1(1, t_1, s_2, s_3)\exp[y(s_2+s_3-2)]\,\mathrm{d}s_2\,\mathrm{d}s_3\,,$$

where

$$y = \log\frac{E_0}{E}\,.$$

(2.7) can be numerically evaluated by the saddle point method.

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In conclusion, one of us (S.K.S.) wishes to thank Prof. R. VASUDEVAN for many interesting discussions.

RIASSUNTO (*)

Si studia il problema della fluttuazione delle dimensioni dei bursts prodotti dai muoni, alla luce del metodo da noi adottato per valutare la divergenza quadratica media del numero di elettroni prodotti in una cascata elettrone-fotone. Si deduce una semplice espressione analitica esplicita per la divergenza quadratica media del numero di elettroni. Si mostra anche come si possono trattare altri problemi di interesse sperimentale con l'uso delle funzioni di densità a molti punti. In particolare si usano le funzioni di correlazioni consecutive, sviluppate nell'articolo precedente, per i burst prodotti da elettroni.

(*) *Traduzione a cura della Redazione.*