MECCANICA
23 (1988), 81-87

A SIMPLIFIED FLOOD ROUTING MODEL: VARIABLE PARAMETER MUSKINGUM (VPM)*

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SOMMARIO. I metodi di «flood routing» studiano la propagazione di un'onda di piena lungo un tratto di un corso d'acqua, assegnato l'andamento temporale della portata nella sezione di monte e le caratteristiche dell'alveo, e usualmente nell'ipotesi di assenza di perturbazioni provenienti da valle («condizione di valle passiva»). Viene qui proposto un procedimento di «flood routing», formalmente simile ad un «Muskingum» ma con i parametri variabili e calcolati per via idraulica; idoneo a stimare anche i livelli idrici; valido anche se i termini cinetici non sono del tutto trascurabili; che sfrutta l'irrilevanza della condizione di valle procedendo a cascata da monte a valle; che sfrutta, a vantaggio della semplicità, il fatto che per le normali onde di piena dei corsi d'acqua il cappio di portata è di dimensioni modeste. I risultati ottenuti sono molto migliori di quelli ottenibili con metodi a parametri costanti e, almeno per i casi in cui il cappio relativo è inferiore al 10%, paragonabili a quelli ottenuti con metodi molto più complessi ed onerosi.

SUMMARY. Flood routing methods are numerical methods for estimating the movement of a flood wave along a channel reach, on the basis of the knowledge of the discharge hydrograph at the upstream end and of the hydraulic characteristics of the reach and, usually, in the hypothesis that no perturbation is coming from downstream («free boundary condition»). The flood routing method wich is proposed is similar to the «Muskingum» one, but with variable and «hydraulic» parameters; it is able to estimate water levels too; is effective even if kinetic terms are not completely negligible; take advantage of the insignificance of the downstream condition and make it possible to obtain results starting upstream and proceeding downstream; for simplicity's sake, take advantage of the fact that the discharge loop of normal flood waves is quite small. Obtained results are much better that those obtainable from constant parameters methods and indeed, if the flood loop is less that 10%, very similar to those obtainable from more complex and time consuming models.

LIST OF SYMBOLS

x, t	channel distance, starting upstream; time
z	water surface height above datum
Q	volumetric rate of discharge
$P(x, z)$	steady rating curve
$q = Q - P$	flood loop
g	acceleration of gravity
A, B	cross section wetted area and free surface width
I, S	water surface slope and friction slope
c	kinematic wave velocity
F	Froude number
L, L_0, L_1, L_2, L_3	characteristic lengths of the channel
$T = L/c$	characteristic time of the channel
D	diffusion
p, l	time and space steps
K, X	Muskingum parameters
C_1, C_2, C_3, C_4	Muskingum coefficients

$f_x = \delta f/\delta x$, $f_t = \delta f/\delta t$ etc. for the partial derivatives.

1. INTRODUCTION

The term «FLOOD ROUTING» is used to describe methods for estimating the hydrograph of discharge (and sometimes of water level) at one or several sections on a channel reach, based on the knowledge of the discharge hydrograph at the upstream end of the reach as well as of the hydraulic characteristics of the intermediate channel.

For clarity's sake, it should be stated that usually:

– initial conditions are assumed to be steady flow;

– it is implicitly assumed that the downstream discharge can be predicted even though the state of the channel further downstream is not known; in a first approximation this means to set the steady rating curve, and in a second approximation, a «normal» deviation (called «loop») from this curve, since the channel flows on beyond the reach of interest.

The best known flood routing methods can be classified in three groups, namely:

– the hydrological methods, the parameters of which are not clearly related to the hydraulic characteristics of the channel; examples of these are the classical Muskingum method and the reservoirs cascade method;

– the simplified hydraulic methods; for example, the different versions of the kinematic and diffusion models with constant parameters or with variable parameters (see Price's VPD [1] or Todini's PAB [2]);

– the complete hydraulic methods which numerically solve the full Saint Venant equations.

The authors are carrying out a research into flood routing methods that:

– are of the «simplified hydraulic» type;

– can be used to provide reliable estimates not only of

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* Paper presented at the First Italian Meeting of Computational Mechanics held in Milan, June 24-26, 1986.

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discharge but also of water level;

– are accurate even when the kinetic terms are not completely negligible;

– take advantage of the insignificance of the downstream boundary condition and make it possible to obtain the results for the intermediate reaches starting upstream and proceeding downstream;

– for simplicity's sake take advantage of the small size of the discharge loop of usual flood waves.

The earliest results of this research, which were discussed at an international workshop held in Bologna in April 1985, are reported in [7] and are here summarized.

2. EQUATIONS OF MOTION AND CONTINUITY

Equations of motion and continuity in variables Q and z for a one-dimensional open channel flow are usually expressed by:

$$Q_t + (Q^2/A)_x + g\,A\,z_x + g\,A\,S = 0 \tag{1}$$

$$B\,z_t + Q_x = 0, \tag{2}$$

where A, B, $s = S/Q^2$ are the channel parameters depending on x and z.

We preferred to replace the water surface level z with steady flow discharge P which, according to the so-called local rating curve, corresponds to this level. This substitution leads to a formal reduction of the equations, as will be seen later.

To do this, it was necessary to transform the derivatives of A and z into derivatives of discharge P:

$$A_x = A_{x/p} + A_{p/x}\,P_x = H + (1/c)\,P_x \tag{3}$$

$$z_x = z_{x/p} + z_{p/x}\,P_x = -I + (1/cB)\,P_x$$

$$z_t = z_{p/x}\,P_t = (1/cB)\,P_t$$

where the symbol $A_{x/p}$ means $\delta A/\delta x$ with constant P, whereas A_x means $\delta A/\delta x$ with constant time, and so on.

If we substitute in (1) we get:

$$Q_t + (2Q/A)\,Q_x - (gA/cB - Q^2/cA^2)\,P_x = \tag{4}$$

$$= g\,A\,I + Q^2H/A^2 - g\,A\,s\,Q^2$$

Since the first member of eq. (4) is zero in steady flow, the second member is also zero, that is:

$$g\,A\,I + P^2H/A^2 - g\,A\,s\,P^2 = 0;$$

if we substitute in (4), we get:

$$Q_t + (2Q/A)\,Q_x - (gA/cB - Q^2/cA^2)\,P_x = g\,A\,I(1 - (Q/P)^2) \tag{5}$$

or even, if we let

$$L_0 = P^2/((P+Q)\,I\,B\,c) \tag{6}$$

$F = B\,Q^2/g\,A^3$, the Froude number (7)

$w = cA/Q$, the ratio of kinematic wave velocity to mean velocity (8)

$$L_1 = L_0\,(1 - F) \tag{9}$$

$$L_2 = L_0\,2\,w\,F \tag{10}$$

$$L_3 = L_0\,w^2\,F, \tag{11}$$

the equation of motion becomes:

$$Q - P + L_1\,P_x + L_2\,Q_x + (L_3/c)\,Q_t = 0$$

The equation of continuity, with the substitution outlined in (3), becomes:

$$P_t + c\,Q_x = 0 \tag{13}$$

The system of eqs.(12-13) is therefore equivalent to the Saint Venant equations (1-3), with a change in the state variables (Q, P instead of Q, z); the only parameter of the continuity equation is the kinematic velocity c which depends on x and P, while in the equation of motion there are three characteristic channel lengths (L_1, L_2, L_3) which depend on x, P, Q. The first of them is the main one, at least for low Froude numbers: it is the damping distance of the disturbances of P in steady flow.

If the variability of the parameters in the system of eqs. (12-13) is disregarded, two second-order equations in P and Q only can be easily derived:

$$Q_t + c\,Q_x - c\,L_1\,Q_{xx} + L_2\,Q_{xt} + (L_3/c)\,Q_{tt} = 0 \tag{14}$$

$$P_t + c\,P_x - c\,L_1\,P_{xx} + L_2\,P_{xt} + (L_3/c)\,P_{tt} = 0 \tag{15}$$

Eq. (14) is also known as Deymie's equation [3], even though with regard to a rectangular channel only.

The system of eqs.(12-13), as well as eqs. (14) and (15), are still hyperbolic and have the same characteristic velocities as the Saint Venant equations'.

In the case of quasi-kinematic waves, it is possible to simplify the expression of the parameters (taking advantage of the assumption that $Q/P \approx 1$) and even of the equations themselves, by combining the space-time derivatives of the discharges P and Q, as well be discussed later.

3. SIMPLIFIED EQUATIONS FOR «QUASI-KINEMATIC» WAVES

For quasi-kinematic flood waves, it is possible to compound the first derivatives of P and Q:

$$Q_t \approx P_t = -c\,Q_x \approx -c\,P_x \tag{16}$$

Consequently, the equation of motion (12) approximates with:

$$Q - P + L\,P_x = 0 \tag{17}$$

or

$$Q - P - T\,Q_t = 0 \tag{18}$$

where a single channel characteristic length L is introduced instead of L_1, L_2, L_3 and consequently a characteristic time T is defined by:

$$L = L_1 + L_2 - L_3 = L_0\,(1 - F\,(1 - w)^2) \tag{19}$$

$$T = L/c \tag{20}$$

As was mentioned earlier on when discussing steady flow, L and T can be attributed the meaning of damping length and time for a quasi kinematic wave. Note that L and T are usually positive, but they become negative when the so-called

Vedernikov number ($F(1 - w)^2$) is greater than 1: in this case a steady flow becomes unstable.

It should be noted that when F tends to vanish, the length L_1 becomes much greater than the other two, and therefore equation (17) tends to be exact: consequently, equations (14) and (15) can be simplified as follows:

$$Q_t + c\,Q_x - c\,L\,Q_{xx} = 0 \tag{21}$$

$$P_t + c\,P_x - c\,L\,P_{xx} = 0 \tag{22}$$

which are known as equations of convection-diffusion: note that the parameter ($c\ L$) is more exact but not extremely different from the diffusion D defined by $D = Q\,/\,(2\,I\,B)$.

On the contrary, for F approaching 1, L_1 is much smaller than the other two and therefore equation (18) is more accurate than eq. (17); consequently equations (14) and (15) can be simplified as follows

$$Q_t + c\,Q_x + L\,Q_{x\,t} = 0. \tag{23}$$

$$P_t + c\,P_x + L\,P_{xt} = 0. \tag{24}$$

These equations can be defined as being of the «Muskingum type», as will be seen in the next paragraph.

4. SIMPLIFIED, «MUSKINGUM» TYPE EQUATIONS

The Muskingum-Cunge model can also be considered to be a simplified version of the system (12-13) (Fig. 1). In effect, this model is usually expressed in a finite difference form by the following linear relation between the discharges at the extremes of a space-time interval:

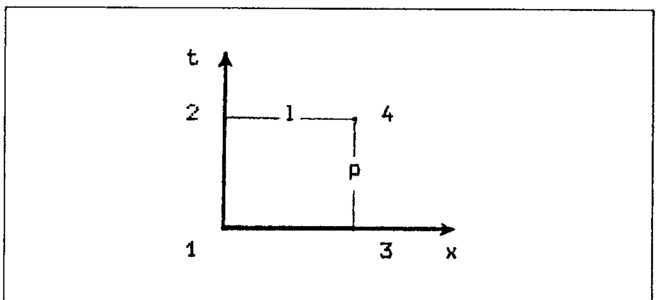


Fig. 1. The space-time scheme of Muskingum model.

$$C_1\,Q_1 + C_2\,Q_2 + C_3\,Q_3 + C_4 Q_4 = 0 \tag{25}$$

where the coefficients C are:

$$C_1 = + K - K\,(1 - 2X) + p \tag{26}$$

$$C_2 = - K \ + K\,(1 - 2X) + p$$

$$C_3 = + K + K\,(1 - 2X) - p$$

$$C_4 = - K - K\,(1 - 2X) - p$$

Eq. (25) can be rewritten in the form:

$$K(Q_1 - Q_2 + Q_3 - Q_4) + K(1 - 2X)(- Q_1 + Q_2 + Q_3 - Q_4) + + p(Q_1 + Q_2 - Q_3 - Q_4) = 0. \tag{27}$$

If steps (p, l) are made to tend to zero, the terms in Q can be expressed by means of the derivatives:

$$K(-2\,p\,Q_t) + K(1 - 2X)(-p\,l\,Q_{xt}) + p(-2\,l\,Q_x) = 0 \tag{28}$$

or even:

$$Q_t + (1/K)\,Q_x + l(0.5 - X)\,Q_{xt} = 0. \tag{29}$$

The latter equation is identical with the equation (23), with the following links between the parameters:

$$K = 1/c, \quad X = 0.5 - L/l \tag{30}$$

Cunge [4] and Dooge [5] found a similar relation between the Muskingum parameters and the hydraulic characteristics of the channel, but except that there D/c appeared instead of the parameter L, and this limited the range of its effectiveness to cases of very slow flow.

Just to mention a special case, note that if a space step twice as long as L (the characteristics length of the channel), is used, the parameter X is zero, which amounts to assuming that there is a univocal relation between the volume of water in the channel reach and the outflowing discharge. In this case the channel is schematically represented by a series of reservoirs in cascade.

5. THE «STANDARD» FLOOD LOOP EQUATION

If we assume that there are no disturbances from downstream (which is inherent in the concept of flood routing), the behaviour of water level $P(t)$, at a channel section, given hydrograph $Q(t)$ at the same section, depends only on the hydraulic characteristics of the channel downstream, and this dependence weakens as the ratio of the distance from the section itself to the characteristic length L of the channel increases. Here the corresponding behaviour of $q(t) = Q(t) - P(t)$ is called «standard loop».

An analytical expression of this loop can therefore be deduced from eqs. (12-13). If we rewrite them in terms of q and Q, we get:

$$q = L_1\,q_x - \{(L_1 + L_2)/c\}\,q_t + T\,Q_t \tag{31}$$

To obtain the standard loop expression, q_x must be removed.

In a first approximation, as was already discussed in paragraph 3, for quasi-kinematic waves the first two second side terms in equation (31) can be disregarded (cf. equation (18)).

A better approximation for the expression of the loop q can be obtained from eq. (31) by expressing q_x in kinematic terms:

$$q_t + c\,q_x \approx 0 \tag{32}$$

Hence we get:

$$q \approx - T_1\,q_t + T\,Q_t, \text{ where } T_1 = (2\,L_1 + L_2)/c. \tag{33}$$

In the numerical model used later on, a formula is needed in order to derive $P(t + p)$ based on $Q(t + p)$ and previous values of Q and P.

A simple but fairly accurate formula can be obtained by disregarding the variability of parameters T, T_1 in eq. (33):

$$q(p) = q(0)\exp(-p/T_1) + \qquad (34)$$

$$+ (T/T_1) \int_0^p \exp(-(p - t)/T_1)\, Q_t(t)\, dt$$

If $Q(t)$ approximates with a polynomial of the second order, we get:

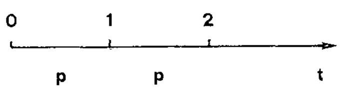
$$q_2 = q_1 \exp(-p/T_1) + T\,p\,b + \qquad (35)$$

$$+ T(1 - \exp(-p/T_1))\ (a - b\,T_1)$$

where:

$$a = (Q_2 - Q_0)/(2\,p)$$

$$b = (Q_2 - 2\,Q_1 + Q_0)/p^2$$



6. THE SIMPLIFIED MODEL: VARIABLE PARAMETER MUSKINGUM (VPM)

As a first application, the developments of the previous chapter make it possible to formulate a Muskingum type flood routing model for quasi-kinematic waves, which operates not only with regard to discharge Q but also to water level P, thus allowing an effective updating of parameters.

Owing in particular to its simplicity, this model (which we shall call «Variable Parameter Muskingum» or VPM for short) in our view offers an effective alternative to the similar VPD model (Variable Parameter Diffusion) proposed by Price and the PAB model (Parabolic and Backwater) suggested by Todini.

The VPM model uses the simplified differential eqs. (23) and (24). These equations in their finite difference version in a Muskingum-like four-point diagram become:

$$C_1\,Q_1 + C_2\,Q_2 + C_3\,Q_3 + C_4\,Q_4 = 0 \qquad (36)$$

$$C_1\,P_1 + C_2\,P_2 + C_3\,P_3 + C_4\,P_4 = 0$$

where coefficients C are related to the characteristics of the channel (c, L) and to the steps (l, p) by eqs. (26) and (30).

It must be pointed out that using also equation (36) for the «levels» P and the complete expression for the parameter L (cf. eq. 19) instead of D/c, the C coefficients are better evaluated than in similar models, such as Muskingum-Cunge [5], or Ponce and Yevjevich [8].

Since an interest reach of a channel made up of n subreaches is to be examined, eqs. (36) are 2 times n in number, that is one less than the unknowns (the upstream discharge being known). The missing condition is usually a free boundary condition, set at the downstream end of the reach when the flow is subcritical or at the upstream one when it is supercritical. This condition can be expressed by the standard loop equation (35).

In order to compute both discharges and levels starting upstream and proceeding downstream by one subreach at a time in a way similar to the classical Muskingum procedure, the boundary equation (35) can be set upstream even for a subcritical flow, as long as the channel is not too irregular.

This is possible in the VPM model on account of the intrinsic stability of eqs. (36): in fact, discharges Q or P at point 4 are directly derived as weighted averages of the known values at the other three points and it is apparent that all of these weight factors are always smaller than one in absolute value, which ensures good numerical stability. Moreover these factors are also all positive, which prevents numerical oscillations even if dampened, if the computing steps (p, l) are carefully selected in the relation to the channel parameters (L, c), i.e. if l and $(p\,c)$ are not too small and not too different, more precisely if

$$|(p\,c - l)| < (2\,L) \text{ and } (p\,c + l) > (2\,L).$$

Note that if steps $l = (p\ c)$ and $l = (2\ L)$ are selected, the weight factors are all equal to each other.

7. NUMERICAL EXAMPLES

7.1. Comparison with an «exact» model

The VPM procedure described in the previous chapter was applied to the study of flood routing in a few cases of practical interest. We restricted our examples to prismatic channels with constant slope and friction, i.e. with parameters not depending on distance x.

A 20 km long reach of three prismatic channels having a double trapezoidal section, as is shown in Fig. 2, was considered. The Gauckler-Strickler friction coefficient has been assumed to be 30 for the main channel and 20 for the floodplains.

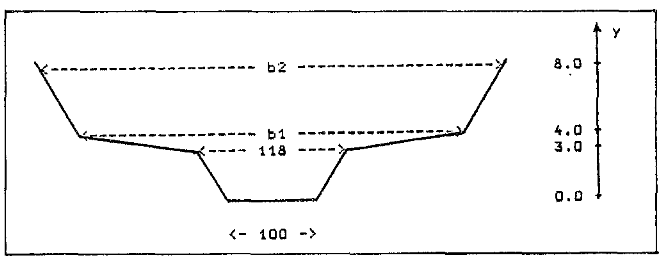


Fig. 2. Cross section of the channels: dimensions (m).

The other main geometric and hydraulic parameters of the channels are:

Channel type		*Slow*	*Reference*	*Fast*
b1: floodplain width (m)		324	174.	174.
b2: floodplain width (m)		348	198.	198.
Channel slope (m/km)		0.1	1.	4.
F (**)		0.01	0.16	0.51
c (**)	(m/s)	1.07	2.86	5.81
L (**)	(m)	16286.	1170.	183.
T (**)	(s)	15187.	409.	31.

(**) Reported values are for a discharge P = 1200 mc/s.

Three possible upstream discharge waves were then considered, expressed by:

$$Q(t) = Q_{in} + (Q_{pe} - Q_{in}) \{\exp(1 - t/t_{pe})\, t/t_{pe}\}^n \qquad (37)$$

with the following values of the parameters:

Wave type	*Slow*	*Reference*	*Fast*
Q_{in}: initial discharge (mc/s)	200.	200.	200.
Q_{pe}: peak discharge (mc/s)	3200.	1200.	1200.
t_{pe}: time to peak (s)	60000.	20000.	5000.
n: shape parameter	4.	4.	4.

Channels and waves were chosen that exhibited a wide range of variation of Froude numbers and of «kinematicity» of the flood: actually it is clear that the ratio of the time to peak of the upstream hydrograph (t_{pe}) to the characteristic time of the channel (T) is related to the relative dimension of the loop (q/Q) and, consequently, to the amount of peak attenuation. As was shown earlier eq. (18) in a first approximation is:

$$q = T\,Q_t$$

and, where the waveform is the same, Q_t is proportional to $(Q_{pe} - Q_{in})/t_{pe}$. The (T/t_{pe}) ratio can therefore be assumed as being an approximate «kinematicity» index of the flood.

For every wave-channel combination a run with VPM method was done in order to compute the upstream P hydrograph (P_u) and the downstream P, Q hydrographs (P_d, Q_d). The computational time step p was small enough to describe the upstream hydrograph, that is about 1/10 of t_{pe}; a space step l was chosen which was not too different from (p c), in order to obtain a good space-time ratio as is seen in chapter 6.

The results thus obtained were compared with those obtained using the method of characteristics for the solution the full St. Venant equations: the computed point grid is irregular and trapezoidal approximation is used to estimate the parameters (see Cunge et al. [6]). The initial and upstream conditions are the same as those used for the VPM model. The «free boundary condition» was set at a channel section at an adeguate distance downstream from the last section of interest.

The results are fully given in [7] and summarized in the table at the foot of the page.

It is apparent that the accuracy of the hydrographs computed with the VPM model is heavily dependent on the «kinematicity» of the flood wave, which can be easily estimated from the ratio (T/t_{pe}):

– when this ratio is smaller than about 1%, results are virtually exact (that is why some results are not reported in the table)

– as long as this ratio is smaller than about 10%, the VPM values of peak times and peak attenuation differ from the actual values by less than 10%

– when t_{pe} of the upstream wave becomes similar to the channel's T, the upstream levels computed with the «standard loop equation (35)» are still acceptable, but downstream propagation is very badly approximated.

7.2. Comparison with other approximate models

For a comparison of VPM with other approximate models,

max q_u/Q_u : maximum relative upstream flood loop
max Q_u, P_u, Q_d, P_d : peak values of P, Q discharges up, downstream (mc/s)
tim Q_u, P_u, Q_d, P_d : peak times minus t_{pe} (s)
CAR, VPM : calculated using CAR or VPM model

channel		*slow channel*			*reference channel*			*fast channel*	
wave	*fast*	*ref.*	*slow*	*fast*	*ref.*	*slow*	*fast*	*ref.*	*slow*
max Q_u	1200	1200	3200	1200	1200	3200	1200	1200	3200
t_{pe}	5000	20000	60000	5000	20000	60000	5000	20000	60000
T/t_{pe}	3.037	.759	.295	.082	.020	.008	.006	.002	.001
max q_u/Q_u	.55	.42	.36	.14	.043	.024	.017		
max P_u CAR	664	888	2727	1165	1197	3199	1200		
max P_u VPM	748	875	2691	1162	1197	3199	1200		
tim P_u CAR	1460	4620	9800	380	430	500	30		
tim P_u VPM	270	2720	8800	410	440	600	30		
max Q_d CAR	389	778	2892	942	1170	3193	1182		
max Q_d VPM	340	588	2598	906	1169	3192	1182		
tim Q_d CAR	13930	15410	11200	7410	7350	4800	3520		
tim Q_d VPM	1520	13330	14600	7840	7440	4800	3520		
max P_d CAR	324	616	2464	929	1167	3192	1182		
max P_d VPM	280	515	2302	896	1167	3191	1182		
tim P_d CAR	19940	21840	21900	7790	7790	5300	3550		
tim P_d VPM	2350	21910	24200	8160	7860	5400	3350		

the example given by Price in [1] was selected: a rectangular channel, 50 m wide, 100 km long, 1 m/km steep, Manning friction coefficient $n = 0.035$, initial steady flow with discharge of 100 m^3/s, upstream hydrograph with a peak of 900 m^3/s and time to peak of 1 day, and waveform according to eq. (37) with exponent $n = 16$. The characteristic time T is about 6 minutes, therefore the wave is virtually kinematic.

Price reports the behaviour of the downstream discharges computed in accordance with the following four models (see Fig. 3):

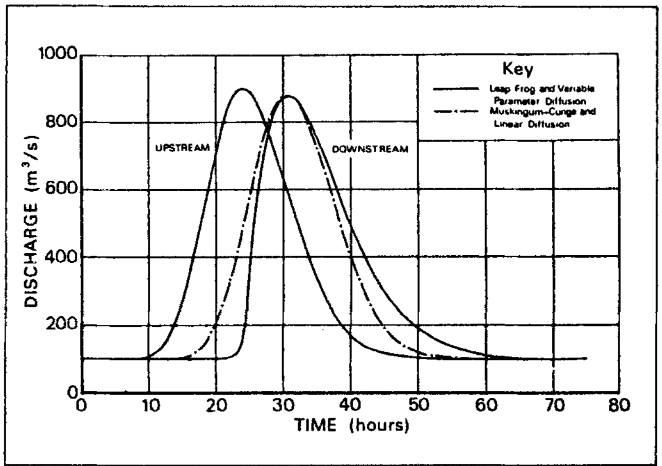


Fig. 3. The downstream hydrographs obtained by different models.

– Muskingum-Cunge with constant parameters and steps of 10 km and 1 h.

– Linear Diffusion, with constant parameters and steps of 5 km and 0.2 h.

– VPD with the same steps.

– Explicit Leapfrog Method, as the «exact» method of comparison.

Using the same data and steps, (5 km and 0.2 h) the VPM method gave results that were indistinguishable from those of the last two methods, while being much simpler and less time consuming. It should be pointed out that VPM and VPD results were far better than those obtained from the same models but with constant parameters.

8. CONCLUSIONS

For flood routing purposes, the characteristic dimensions of the channel (L and T) here introduced seem to be sufficient to summarize the geometrical and hydraulic features of a channel.

The wave can be considered quasi-kinematic if the flood loop is small ($q \ll Q$), which occurs (for normal hydrograph shapes) when growth time is high compared to the characteristic time of the channel.

The shape of the loop is well approximated by eq. (35) given herein, as long as the wave is «quasi-kinematic». It can therefore be used as a free boundary condition for any computational model.

The formulas used to calculate the characteristic length L, that take into account, though approximately, the kinetic terms, enable to improve the estimate of parameters for any model that uses the diffusion or Muskingum equations

A variation of the Muskingum-Cunge model is proposed as an initial application of the above. This variation (called VPM):

– uses the standard loop equation to estimate the «level» P upstream;

– starts upstream and proceeds downstream in a cascade fashion, like the Muskingum model, to calculate not only the discharges but also the levels P along the whole channel;

– updates the parameters on the basis of the levels calculated.

This VPM model was tried on prismatic channels, including also those having composite cross sections; «quasi-kinematic» waves gave excellent results, indeed identical with those that can be obtained from more complex and time consuming models such as the Leap-Frog or the VPD.

We also intend to test an other flood routing model which also uses the quasi-kinematic wave hypothesis and the simple Muskingum cascade procedure, while being more accurate and more suitable fro strongly non-prismatic channels.

Received: October 7, 1986; in revised form: November 26, 1987.

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