6. V. T. Tomashevskii, "On the effect of transverse displacements and the stressed state on the stability of an anisotropic cylinder," Prikl. Mekh., 11, No. 4, 7-16 (1966).
7. P. F. Papkovich, The Structural Mechanics of a Vessel [in Russian], Vol. 1, Part 1, Moscow (1945).

# ONE METHOD FOR SOLVING THE PROBLEM OF THE DYNAMICAL STABILITY OF THIN-WALLED VISCOELASTIC STRUCTURES

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In the present work the dynamical problem of the stability of a viscoelastic orthotropic cylindrical shell in a geometrically nonlinear physically linear formulation has been investigated. As the criterion for dynamic "loss of stability," we take the attainment by the characteristic flexure of a value equal to the thickness of the shell. We shall take the relationship between the loading parameter P (under axial compression) and the time t as a linear law of growth in P, i.e., P = st, where s is the rate of loading. The derivation of the initial equations for the investigation of the stability of a viscoelastic orthotropic shell will be the same as for an elastic orthotropic shell [1]. We obtain the following system of equations [2] from the equilibrium equations and the compatibility equations for the deformation of the middle surface

$$\begin{gathered}\frac{h^3}{12}\nabla^4 w = L(w,\Phi)+\nabla_k\Phi+F-\rho_0 h\frac{\partial^2 w}{\partial t^2};\\ \nabla_\beta{}^4\Phi=-B_{11}\left(\frac{1}{2}L(w,w)+\nabla_k w\right)+B_{11}\int_0^t\Gamma_{11}(t-\tau)\left(\frac{1}{2}L^*(w,w)+\right.\\ \left.+\nabla^*{}_k w\right)d\tau,\end{gathered}\tag{1}$$

where $\Gamma_{11}(t)$ is the kernel of the rate of relaxation

$$L^*(w,w)=2\left[\alpha^*{}_2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\alpha^*{}_3\left(\frac{\partial^2 w}{\partial x\partial y}\right)\right];$$

$$\nabla^*{}_k\cdot=\alpha^*{}_2 k_1\frac{\partial^2\cdot}{\partial y^2}+\alpha^*{}_1 k_2\frac{\partial^2\cdot}{\partial x^2};$$

$$\nabla_\beta{}^4\Phi=\beta_1\frac{\partial^4\Phi}{\partial y^4}+\beta_2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\beta_3\frac{\partial^4\Phi}{\partial x^4};$$

$$F=\frac{h^3}{12}\int_0^t\left[B_{11}\frac{\partial^4 w}{\partial x^4}+2(\bar B_{12}+2\bar B)\frac{\partial^4 w}{\partial x^2\partial y^2}+\bar B_{22}\frac{\partial^4 w}{\partial y^4}\right]\times$$
$$\times\Gamma_{11}(t-\tau)\,d\tau.$$

The remaining differential operators will be the same as in the elastic case.

Let the shell be hinged at the ends by means of rigid ring frame supports, i.e., the following boundary conditions are satisfied: when x = 0, L, w(t, 0, y) = w(t, L, y) = 0; $M_x|_{x=0} = M_x|_{x=L} = 0$. We approximate the expression for the flexure as a function of the spatial coordinates and time by the expression [3]:

$$w=f_1(t)\sin\frac{m\pi x}{L}\sin\frac{ny}{R}+f_2(t)\sin^2\frac{m\pi x}{L}+\varphi f_1.\tag{2}$$

We assume that the shell is "nonideal" and has a certain initial deviation of the flexure $w_0$. We shall take this initial flexure in the same form as the total flexure.

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$$w_0=f_{1,0}\sin\frac{m\pi x}{4}\sin\frac{ny}{R}+f_{2,0}\sin^2\frac{m\pi x}{L}. \tag{3}$$

For this problem, the initial equations (1) will be

$$\begin{gathered}\frac{h^3}{12}\nabla^4(w-w_0)=L(w,\Phi)+\nabla_k\Phi+F-\rho_0 h\frac{\partial^2 w}{\partial t^2};\\ \nabla_\beta{}^4\Phi=-B_{11}\left[\frac{1}{2}L(w,w)-\frac{1}{2}L(w_0,w_0)+\nabla_k w-\nabla_k w_0\right]+\\ +B_{11}\int\limits_0^t\Gamma_{11}(t-\tau)\left[\frac{1}{2}L^*(w,w)-\frac{1}{2}L^*(w_0,w_0)+\nabla^*{}_k w-\nabla^*{}_k w_0\right]d\tau.\end{gathered} \tag{4}$$

Substituting Eqs. (2) and (3) into the right-hand term of the second of the Eqs. (4) and integrating, we determine the function of the stresses in the middle surface

$$\Phi=B_{11}\left(K_1\cos\frac{2m\pi x}{4}+K_2\cos\frac{2ny}{R}+K_3\sin\frac{m\pi x}{L}\sin\frac{ny}{R}+K_4\sin\frac{3m\pi x}{L}\sin\frac{ny}{R}\right)-\frac{Py^2}{2}, \tag{5}$$

where P is the intensity due to the dynamic compressive forces; $K_i = K_i(f_1, f_0, m, n, L, R, h, \alpha^*_1, \alpha^*_2, \alpha^*_3, \Gamma_{11}(t)$ $(i=\overline{1,4})$. We apply the Bubnov–Galerkin methods to the first equation of the system (4):

$$\int\limits_0^L\int\limits_0^{2\pi R}X\sin\frac{m\pi x}{L}\sin\frac{ny}{R}\,dxdy=0; \tag{6}$$

$$\int\limits_0^L\int\limits_0^{2\pi R}X\sin^2\frac{m\pi x}{L}\,dxdy=0, \tag{7}$$

where

$$X=\frac{h^3}{12}\nabla^4(w-w_0)-L(w,\Phi)-\nabla_k\Phi-F+\rho_0 h\frac{\partial^2 w}{\partial t^2}.$$

Integrating Eqs. (6) and (7) and taking account of Eqs. (2), (3), and (5), we obtain the following nonlinear integrodifferential equations of the second order:

$$\begin{gathered}\frac{d^2\xi_1}{d\tau^2}+c^*{}_1\left(1-\frac{P}{T_1}\right)[(1+a_1)\xi_1+a_2\xi_1\xi_2+a_3\xi_1\xi_2{}^2+a_4\xi_2+\\ +a_5\xi_1{}^2+a_6\xi_1{}^3+a_8\xi_1{}^2\xi_2+a_7]=(\bar a_1+\bar a_6\xi_2)\int\limits_0^\tau\Gamma_{11}(\tau-s)\xi_1(s)\,ds+\\ +(\bar a_2+\bar a_{11}\xi_2)\int\limits_0^\tau\Gamma_{11}(\tau-s)\xi_1(s)\xi_2(s)\,ds+(\bar a_3+\bar a_7\xi_1)\int\limits_0^\tau\Gamma_{11}(\tau-s)\xi_1{}^2(s)\,ds+\\ +(\bar a_4+\bar a_8\xi_1)\int\limits_0^\tau\Gamma_{11}(\tau-s)\xi_2(s)\,ds+(\bar a_5+\bar a_9\xi_1+\bar a_{10}\xi_2)\int\limits_0^\tau\Gamma_{11}(s)\,ds+\bar a_{13}\xi_1\int\limits_0^\tau\Gamma_{11}(\tau-s)\xi_2(s)\,ds;\end{gathered}$$

$$\begin{gathered}\frac{d^2\xi_2}{d\tau^2}+c^*{}_2\left(1-\frac{P}{T_2}\right)[\xi_2+b_1\xi_1{}^2\xi_2+b_2\xi_1{}^2+b_3\xi_1+b_4\xi_1\xi_2+\\ +b_5]=\bar b_1\int\limits_0^\tau\Gamma_{11}(\tau-s)\xi_2(s)\,ds+(\bar b_2\xi_1+\bar b_3)\int\limits_0^\tau\Gamma_{11}(\tau-s)\xi_1(s)\,ds+\end{gathered} \tag{8}$$

$$+\bar{b}_4\int_0^\tau \Gamma_{11}(\tau-s)\xi_1(s)ds+(\bar{b}_5\xi_1+\bar{b}_6)\int_0^\tau \Gamma_{11}(\tau-s)\xi_1(s)\xi_2(s)ds+(\bar{b}_7\xi_1+\bar{b}_8)\int_0^\tau \Gamma_{11}(s)ds.$$

The following dimensionless parameters are introduced here: $\xi_1=\frac{f_1}{h}$; $\xi_2=\frac{f_2}{h}$; $\xi_{1,0}=\frac{f_{1,0}}{h}$; $\xi_{2,0}=\frac{f_{2,0}}{h}$; $\tau=\frac{ct}{R}$; $c^*_1=k_1k_2^2m^2\pi^2\hat{P}_0$; $c^*_2=\left(\frac{4}{3}\,m^4\pi^4k_2^2-\frac{1}{\beta_3k_3^2}\right)$; $T_1=k_1\hat{P}_0B_{11}$; $T_2=k_1B_{11}\left(\frac{1}{3}\,m^2\pi^2k_2k_3^2+\frac{3}{4\beta_3m^2\pi^2k_2k_3}\right)$; $k_1=\frac{h}{R}$; $k_2=\frac{R}{L}$; $k_3=\frac{h}{L}$; $a_i$, $\bar{a}_i$, $b_i$, $\bar{b}_i$, certain constants which depend on the rigidity parameters and the dimensions of the shell; $f_1$, deflection of the shell; c, velocity of propagation of sound in the material of the shell; h, thickness; R, radius; L, length; $\hat{P}_0$, static critical loading; $B_{11}$, principal modulus of elasticity of the shell; $f_2$, ratio of the "nonsymmetric" component of the flexure to the "symmetric" component; m, number of wave along the generatrix; n, number of waves along the circumference; and $\Gamma_{11}(t)$, kernel of the relaxation rate which takes the form $\Gamma_{11}(t) = A_0 l^{-\beta_0 t} t^{\alpha_0-1}$.

In solving the system of equations (8) and other dynamical problems of viscoelasticity, we normally encounter certain difficulties, one of which is the computation of the integral operators appearing in the system of equations (8):

$$\int_0^\tau \Gamma(\tau-s)\xi_1(s)ds;\quad \int_0^\tau \Gamma(\tau-s)\xi_1^2(s)ds;\quad \int_0^\tau \Gamma(\tau-s)\xi_1(s)\xi_2(s)ds. \tag{9}$$

If the material has a small viscous characteristic, the integrodifferential equations of motion are solved by the method of averaging [3]. When this is done, it becomes impossible to reveal the effect of the nonlinear and viscous properties of the system on the vibrational process.

In the given case, allowing for what has been said above concerning the computation of integral operators of the form (9), we shall proceed in the following manner. Let the following initial conditions be satisfied when $\tau = 0$: $\xi_1(0) = \xi_{1,0}$; $\xi_2(0) = \xi_{2,0}$; $\left.\frac{d\xi_1}{d\tau}\right|_{\tau=0}=\left.\frac{d\xi_2}{d\tau}\right|_{\tau=0}=0.$

In the case of carefully prepared shells, the amplitude of the initial flexure is taken in the calculation to be approximately equal to 0.001, i.e., 0.0001 of the thickness h. Hence, here we shall take

$$\xi_1(0)=\xi_2(0)=0.0001.$$

Further, we shall represent the functions $\xi_1(\tau)$ and $\xi_2(\tau)$ which occur in the integral operators (9) in the form of a power series

$$\xi_1(\tau)=\sum_{k=0}^{n} p_k\tau^k;\quad \xi_2(\tau)=\sum_{k=0}^{n} q_k\tau^k. \tag{10}$$

Let $\xi_1(\tau)$ and $\xi_2(\tau)$ be real principal functions which are differentiable with respect to $\tau$ in the interval $0 \le \tau \le \infty$.

Then the functions $\xi_1(\tau)$ and $\xi_2(\tau)$ can be expanded in a Taylor's series at the point $\tau_0 = 0$:

$$\begin{aligned}\xi_1(\tau)&=\xi_1(0)+\xi'_1(0)\tau+\frac{1}{2!}\xi''_1(0)\tau^2+\ldots+\frac{1}{n!}\xi_1^{(n)}\tau^n;\\ \xi_2(\tau)&=\xi_2(0)+\xi'_2(0)\tau+\frac{1}{2!}\xi''_2(0)\tau^2+\ldots+\frac{1}{n!}\xi_2^{(n)}\tau^n.\end{aligned} \tag{11}$$

Comparing the series (10) and (11), respectively, we find the unknown coefficients

$$p_k=\frac{1}{k!}\xi_1^{(k)}(0);\quad q_k=\frac{1}{k!}\xi_2^{(k)}(0)\quad (k=\overline{1,n}). \tag{12}$$

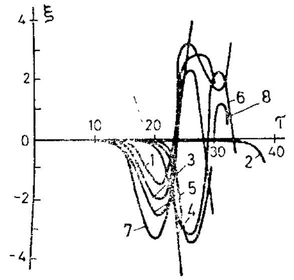


Fig. 1

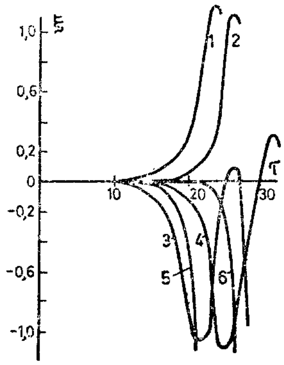


Fig. 2

Fig. 1. M = 1; N = 5; $S = 2\cdot10^6$ kgf/cm²·sec; $A_0$ = 0 (1, 2), 0.0044 (3, 4), 0.01 (5, 6), 0.02 (7, 8); 1,3,5,7 refer to $\xi_1$ and 2,4,6,8 refer to $\xi_2$.

Fig. 2. M = 1, N = 3, $\alpha$ = 0.025, $\beta$ = 0.05, A = 0.0044. $S = 0.8\cdot10^6$ (1, 2), $1.2\cdot10^6$ (3, 4), and $3\cdot10^6$ kgf/cm²·sec (5, 6).

The derivatives $\xi_1^{(k)}(0)$ and $\xi_2^{(k)}(0)$ $(k = \overline{1, n})$ are determined from the system of equations (8) at $\tau = 0$. Then Eqs. (10), taking account of Eqs. (12), assume the form

$$\xi_1(\tau)=\sum_{k=0}^{n}\frac{1}{k!}\xi_1^{(k)}(0)\tau^k;\quad \xi_2(\tau)=\sum_{k=0}^{n}\frac{1}{k!}\xi_2^{(k)}(0)\tau^k. \tag{13}$$

It can be proved using d'Almbert's criterion that the series (13) are absolutely convergent. The radius of convergence is equal to infinity. Substituting, respectively, Eqs. (13) into Eq. (9), we obtain, after several transformations, the expressions for the integral convolutions

$$\int_0^\tau \frac{e^{-\beta(\tau-s)}}{(\tau-s)^{1-\alpha}}\sum_{k=0}^{n}p_k s^k ds=\sum_{k=1}^{n}p_{1,n-k}(\tau)\left[p_{2,k}\frac{e^{-\beta\tau}}{\tau^{1-\alpha}}+\gamma_k\int_0^\tau\frac{e^{-\beta s}}{s^{1-\alpha}}ds\right]-\sum_{k=0}^{n}p_k\tau^k\int_0^\tau\frac{e^{-\beta s}}{s^{1-\alpha}}ds, \tag{14}$$

where

$$p_{1,n-k}=\sum_{i=0}^{n-k}b_i\tau^i;\quad p_{2,k}=\sum_{j=0}^{k}c_j\tau^j;\quad p_{2,0}(\tau)=0.$$

The expressions for the other integral convolutions, which are subsequently substituted into Eqs. (8), are obtained in a similar way. The system of equations (8), with allowance for Eq. (14), is solved using the Runge–Kutta method.

The results of the dynamical loading of a shell at a constant rate of increase in the load according to a linear law p = st are shown in Figs. 1 and 2. The rate of loading is taken as being equal to $s = 2\cdot10^6$ kgf/cm²·sec. Curves 1-8 in Fig. 1 respectively characterize the change in $\xi_1(\tau)$ and $\xi_2(\tau)$ for various different viscosity parameters for the shell material. As the viscosity of the material increases, the absolute values of $\xi_1$ and $\xi_2$ increase correspondingly during the first half period of the vibrations. With the passage of time the nonlinear vibrational process acquires a complex nature, i.e., there is a dynamic loss of stability with a sharp increase in the flexures. The various different series of curves shown in Fig. 2 refer to loads with rates of $s = 0.8\cdot10^6$, $1.2\cdot10^6$, and $3.0\cdot10^6$ kgf/cm²·sec, for the case of viscoelastic shells. Here, intense increases in the flexures $\xi_1(\tau)$ and $\xi_2(\tau)$ are observed as the rate of loading is increased. It can be seen from the curves which have been presented that the dynamical loss in stability in viscoelastic shells takes place more rapidly than in elastic shells. The following initial data was used in the calculations: $B_{11} = 2.32\cdot10^5$ kgf/cm², $B_{11}/\rho_0 c^2 = 1$, $B_{22}/B_{11} = n_1 = 0.78$, $B_{12}/B_{11} = n_2 = 0.004$, $2B/B_{11} = n_3 = 0.6$, $k_1 = 0.01$, $k_2 = 0.1$, $k_3 = 0.001$, $\Gamma_{22}(\tau)/\Gamma_{11}(\tau) = N_2 = 0.38$, $\Gamma_{12}(\tau)/\Gamma_{11}(\tau) = N_1 = 0.06$, $\Gamma(\tau)/\Gamma_{11}(\tau) = N_3 = 0.107$, $\alpha_2^* = N_1$;

$$\alpha^*_1=\frac{n_1N_2+n_2(N_1-1)-n_2^2N_1}{n_1-n_2^2};$$

$$\alpha^*_3=\frac{n_1(1+N_2-N_3)+n_2(1-N_1)+n_2^2(N_3-2N_2)+n_2(N_1-N_3)n_3}{n_1-n_2^2}.$$

## LITERATURE CITED

1. P. M. Ogibalov and M. A. Koltunov, "The stability of orthotropic viscoelastic shells," Prikl. Mekh., 3, No. 8, 1-10 (1967).
2. A. I. Karimov and T. Mavlyanov, "The investigation of the stability of cylindrical shells of the tunnel type," in: The Dynamics of Footings, Fundaments, and Subterranean Constructions [in Russian], Part 2, Tashkent (1977), pp. 132-135.
3. A. S. Vol'mir, The Nonlinear Dynamics of Disks and Shells [in Russian], Moscow (1973).
4. M. A. Koltunov, Creep and Relaxation [in Russian], Moscow (1976).

# STRESS STATE OF A GLASS-REINFORCED PLASTIC CYLINDRICAL SHELL WITH A BRANCH PIPE UNDER AXIAL COMPRESSION

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In shell constructions we often encounter holes with strengthening elements. One such construction is a cylindrical shell weakened by a hole into which there is fixed a branching shell. Such a joining of shells exerts a negative influence on the load carrying capacity of the unit of intersecting shells, while at the same time, it is brought about by design considerations and requires detailed theoretical and experimental study. Investigations into the stress—strain state of the unit of joining cylindrical shells have been conducted during the last 15-17 years and as yet are not completely finished. Usually these investigations have been devoted to the connections of isotropic materials, with the exception of [1].

1. We investigate the stress state of thin orthotropic cylindrical shells intersecting at right angles, the principal directions of elasticity of the shells coinciding with the lines of the principal curvature. The quantities referring to the basic shell are denoted by capital letters, while those referring to the branch are denoted by lower-case letters. The combined shell has two planes of symmetry: the longitudinal plane, containing the axes of the basic shell and the branch, and the transverse plane perpendicular to the longitudinal and including the axis of the branch pipe.

The point of intersection of the axes of the shells is taken as the origin of the coordinates $\xi = x/r$ and $\Xi = X/R$, where r and R are the radii of the middle surface of the branch shell and the basic shell, respectively. Measurement of the circumferential coordinates $\varphi = y/r$ and $\Phi = Y/R$ is taken from the generators $l$ and L at places of intersection of the middle surfaces of the shells with the longitudinal plane. The directions of the coordinates are chosen as is shown in Fig. 1. For $0 < \nu \leq 0.5$ ($\nu = r/R$) the maximum deviation of the curve along which the shells contact one another from a circle of radius r does not exceed 5% [2]. Therefore, for $0 < \nu \leq 0.5$ we approximately take the line of intersection of the shells to be a circle. On the middle surface of the basic shell we introduce a polar system of coordinates ($\rho = \rho_1/r$, $\varphi$), the origin of which is placed at the point where the axis of the branch shell passes through the middle surface of the basic shell.

The stress state of the construction of intersecting shells is assumed to consist of the basic momentless state and the perturbed state caused by the presence of the hole in the basic shell and the branch shell fixed in it. The perturbation of the stress state in the vicinity of the joint of the shells has a local character and propagates into a comparatively small zone [3]. Therefore, for its description we can use equations of the theory of shallow shells [4]. The components of the basic and the perturbed stress states must satisfy the boundary conditions [2] on the joining line, while the components of the perturbed state must also satisfy conditions at infinity [4].

Under axial compression of the basic shell by a load P, the components of the basic state for the branch are zero, while for the basic shell in the polar coordinate system the components are given by the expressions

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