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# On lot streaming in multistage production systems

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**Abstract**

Recently a model on processing a product in a multistage serial production system by overlapping production between stages is presented in the literature. For minimum in-process inventory the production flow is synchronized by shifting the lot, from a stage to the next in equal shipment sizes. An optimal solution procedure for obtaining the minimum cost of set up, transportation and inventory is also developed taking into account the non-negative set up and transportation times. In their paper, the authors ignored the recently published literature on the topic, where a model is developed to minimize the total cost by synchronizing the production flow in the system by shifting the lot from a stage to the next in equal and unequal shipment sizes, but without considering set up and transportation times. In this note, this model is extended to include the set up and transportation times. Two numerical examples are solved to illustrate the extended model.


*Keywords:* Single product; Serial production system; Overlapping production; Set up; Transportation time

## 1. Introduction

Deterministic constant lot size models for the production of a single product in a multistage serial production system are well known in the literature. In this system, a product is produced in a sequence of manufacturing operations separated by inventories of work-in-process (WIP). After completing an operation at a stage, the processed items are transported to the work-in-process inventory for the subsequent production stages. The customer demand is met up from the final production stage. It is a special case of an assembly network in which an item requires processing in strictly ordered stages. There is a tendency in multistage production systems for the work-in-process to increase at all stages unless the production flow is synchronized.

For the control of WIP inventory, Diponegoro and Sarkar (2002) have developed a model by transferring the lot to customers in batches (sublots). They considered a manufacturing system which procured raw materials and converted them to finished products of varying demand. The authors have proposed a decision rule to determine the production start time, lot and batch sizes with minimum cost of ordering of raw materials, manufacturing set up, raw material and finished

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product holding. Ramasesh et al. (2000) synchronized the production flow in a serial production system by transferring a lot from a stage to the next with equal-sized batches. The batches are transported from a stage to the next for processing without waiting for the entire production lot to be processed at the earlier stage before being moved to the next stage. Assuming a fixed set up cost for all stages and a fixed cost of transporting a batch through all stages, they developed an economic production lot size model taking into account the set up and transportation times. Though a brief review of the literature is given, they ignored the relevant papers of Goyal and Szendrovits (1986), and Hoque and Kingsman (1995). Goyal and Szendrovits (1986) developed a model of processing a single product in any number of serial production stages. This allowed the combination of equal and unequal batch sizes at a particular stage so that the equal batch size is the largest and does not exceed the transport equipment capacity. A heuristic procedure based on the concept of differentiation of the cost function was developed to determine the economic lot size and batch sizes for each stage. But this heuristic provided no way of estimating how close it was to the actual minimum cost solution. It merely gave comparisons with earlier published heuristics. Furthermore, the procedure did not naturally deal with certain specific cases that could easily occur, for example zero transportation times for batch transfer between stages and equal production rates at successive stages. Considering these specific cases Hoque and Kingsman (1995) had developed a modification to the model of Goyal and Szendrovits (1986), which enabled a number of properties that the optimal solution must satisfy to be determined. An algorithm giving the optimal solution was then derived based on these properties. Thus the model of Ramasesh et al. (2000) is a particular case of the modified model of Hoque and Kingsman (1995) except in the consideration of set up and transportation times and avoidance of the capacity constraint on the transport equipment. To cope with the model of Ramasesh et al. (2000), this paper extends the model of Hoque and Kingsman (1995) to include set up and transportation times. An algorithm leading to the optimal solution is derived. Superior behavior of this extended model as compared to the model of Ramasesh et al. (2000) is shown with the help of two numerical examples.

## 2. Assumptions and Notation

The assumptions and notation underpinning the model are as follows:

### 2.1. Assumptions

(1) Units of product are infinitely divisible and their production requires a fixed sequence of operations with finite and constant production rates at each stage.
(2) The production capacity is unrestricted and no back-logging is permitted in the system.
(3) The demand rate for the product is deterministic and constant over an infinite time horizon.
(4) The lot size is uniform and constant across all stages and only a single set-up cost is allowed for uninterrupted production at each stage.
(5) The lot can be shipped in equal or unequal sized batches from one stage to the next, but the shipment of each batch incurs a transportation cost. It is not necessary to wait until the whole lot is completed before a part of it can be transported and processed in the next stage.
(6) The total number of equal and unequal sized batches can be different at various stages.
(7) The unit holding-cost for each stage represents the cost of carrying one unit of physical inventory of a product after processing on the stage.
(8) There are $n$ production stages, $i = 1, \ldots, n$; $i = n$ represents the final stage that satisfies the demand.

### 2.2. Notation

#### 2.2.1. Variables

$Q$ = uniform lot size ($Q$ is infinitely divisible) through all stages,

$m_i$ = total number of batches at stage $i$ ($m_i$ is a positive integer),
$e_i$ = number of unequal sized batches at stage $i$ ($e_i$ is a positive integer),
$z_i$ = the smallest batch size at stage $i$.

### 2.2.2. Parameters

$D$ = constant demand rate,
$P_i$ = constant production rate at stage $i$,
$F_i$ = setup cost per lot at stage $i$,
$T_i$ = transportation cost per batch at stage $i$,
$s_i$ = setup time of a machine at stage $i$,
$t1_i$ = transfer time of a batch from a stage $i$ to the next,
$t2_i$ = returning time of a transport vehicle after shifting a batch from stage $i$ to the next,
$g_i$ = capacity of the transport equipment at a stage $i$,
$c_i$ = unit inventory holding cost per unit per unit time at stage $i$,
$k_i$ = ratio of production rates at a stage $i$, $\mathrm{Max}(P_i/P_{i+1}, P_{i+1}/P_i)$,
$P_{n+1} = D$, the demand for the final product.

It may be pointed out that production rate of each stage must exceed the demand rate $D$, i.e. $P_i \geqslant D$, for all $i = 1$ to $n$.

## 3. The original model

Hoque and Kingsman (1995) developed a heuristic solution procedure to minimize the total cost of set up, inventory holding and transportation per unit time for processing a single product in a multistage serial production system. The lot is transferred with equal and/or unequal sized batch shipments. The model presented by them can be expressed as

$$\text{minimize } AQ + B/Q + \sum H(Q, m_i, e_i)$$

subject to

$$Q \leqslant \frac{g_i}{k_i^{e_i-1}} f(m_i, e_i) \quad \text{for } i = 1, 2, \ldots, n, \tag{1}$$

where

$$A = D \sum_{i=1}^{n} \left( \frac{c_i}{2} \left| \frac{1}{P_i} - \frac{1}{P_{i+1}} \right| \right),$$

$$B = D \sum_{r=0}^{e_i-1} F_i,$$

$$H(Q, m_i, e_i) = \frac{m_i b_i}{Q} + \frac{Q a_i}{f(m_i, e_i)},$$

$$b_i = DT_i,$$

$$a_i = \frac{Dc_i}{\mathrm{Max}[P_i, P_{i+1}]},$$

$$f(m_i, e_i) = (m_i - e_i) k_i^{e_i-1} + \sum_{r=0}^{e_i-1} k_i^r$$

and $Q > 0$, $m_i \geqslant e_i$, and $e_i \geqslant 1$ all $i$.

The smallest batch size at the $i$th stage is $z_i$, given by

$$z_i = \frac{Q}{f(m_i, e_i)}. \tag{2}$$

The different transfer batch sizes are $z_i, k_i z_i, k_i^2 z_i, \ldots, k_i^{e_i-1} z_i$. Constraint (1) is derived by ensuring that the largest batch size does not exceed the capacity of the transport equipment. For given $Q$, $m_i$, $H(Q, m_i, e_i)$ is shown as a non-decreasing function of $e_i$ and for given $Q$, $m_i$, $e_i$, $H(Q, m_i + n, e_i + n)$ is proved to be a convex function in $n$ ($n = 1, 2, 3, \ldots$).

## 4. The extended model

### 4.1. The cost function

Following Goyal and Szendrovits (1986), it can easily be shown that consideration of transportation times adds a constant term

$$D \sum_{i=1}^{n} c_i t1_i$$

to the inventory cost per unit time over all stages. Thus the cost function transforms to

$$AQ + B/Q + C + \sum H(Q, m_i, e_i),$$

$$\text{where } C = D \sum_{i=1}^{n} c_i t1_i. \tag{3}$$

### 4.2. Additional constraints for set up and transfer times

#### 4.2.1. Set up time constraint:

The length of each production cycle, the time between starting successive production runs at any stage, is $Q/D$. The actual processing time at each stage is $Q/P_i$. There must be sufficient time in each production cycle to set up the machine at stage $i$ for the next production run,$s_i$, plus some idle non-production time $I_i$. Hence

$$\text{Production cycle time} = \frac{Q}{P_i} + I_i + s_i = \frac{Q}{D}.$$

The idle time must be non-negative for a feasible solution to exist. So it follows that the lot size $Q$ must satisfy

$$Q \geqslant s_i \bigg/ \left(\frac{1}{D} - \frac{1}{P_i}\right). \tag{4}$$

#### 4.2.2. Transfer time constraint

*Case 1*: When $P_i \geqslant P_{i+1}$; the smallest transfer batch size is first. Having transported the first batch to the stage $i+1$ and then returned to $i$, transport equipment will be ready to transport the second batch at time $z/P_i + t1_i + t2_i$. So

$$\frac{z_i}{P_i} + t1_i + t2_i \leqslant \frac{z_i}{P_i} + \frac{k_i z_i}{P_i}$$

which implies $\dfrac{k_i z_i}{P_i} \geqslant t1_i + t2_i$.

To maintain continuous production flow by avoiding extra inventory, the second batch finished at stage $i$, must be ready for processing at stage $i+1$ at time $z_i/P_i + t1_i + z_i/P_{i+1}$. So $z_i/P_i + t1_i + z_i/P_{i+1} = z_i/P_i + t1_i + k_i z_i/P_i$ which implies $z_i \geqslant P_{i+1}(t1_i + t2_i)$.

*Case 2*: When $P_i < P_{i+1}$; the smallest transfer batch size is last. To allow time for transfer, the last transfer batch must have been completed at stage $i$ $z_i/P_{i+1} + t1_i$ time units before the end of cycle. The last but one batch $(m_i - 1)$ must start being processed on stage $i+1$, a time $k_i z_i/P_{i+1} + z_i/P_{i+1}$ before end of cycle. Thus the transport equipment will be available at stage $i$ a time $k_i z_i/P_{i+1} + z_i/P_{i+1} - t2_i$ before the end of cycle. But this must occur on or before the last transfer batch is completed at stage $i$, so must have

$$k_i z_i/P_{i+1} + z_i/P_{i+1} - t2_i \geqslant z_i/P_{i+1} + t1_i.$$

Substituting for $k_i$ it reduces to $z_i \geqslant P_i(t1_i + t2_i)$.

Similarly it can be shown that these constraints are the constraints for transferring every other batch from a stage to the next in due time. Therefore, the constraint is expressed generally as $z_i \geqslant M_i t_i$ where $M_i = \mathrm{Min}(P_i, P_{i+1})$ and $t_i = t1_i + t2_i$. Substituting for $z_i$ from (2) and simplifying it transforms to

$$Q \geqslant M_i t_i \left[(m_i - e_i)k_i^{e_i - 1} + \sum_{r=0}^{e_i - 1} k_i^r\right]. \tag{5}$$

Thus objective function (3) along with constraints (1), (4) and (5) represents the extended model.

## 5. The solution of the extended model

In the original model, for a given value of $Q$, the objective function was considered as the sum of separable functions of $m_i$ and $e_i$ only, the values for the $i$th stage and independent of the values for other stages. So the optimal values were those satisfying constraint (1) which minimized the partial cost functions $H(Q, m_i, e_i)$ for all $i$. Thus the first step in the procedure was to determine the minimum values of the partial cost functions for a given value of $Q$, taking into account the integer nature of $m_i$ and $e_i$. Total cost function was shown to be non-convex in nature in $Q$. Hence a directed search procedure over values of $Q$ was used. See for details Hoque and Kingsman (1995).

In the extended model inequalities (1) and (5) implies

$$\left[\frac{g_i}{k_i^{e_i - 1}} - M_i t_i\right]\left[(m_i - e_i)k_i^{e_i - 1} + \sum_{r=0}^{e_i - 1} k_i^{-r}\right] \geqslant 0.$$

The second part of the product in the left-hand side of this inequality is always positive, so $g_i/k_i^{e_i - 1} \geqslant M_i t_i$. The largest value of the left-hand side of this inequality is $g_i$ when $e_i = 1$ and hence $g_i/M_i \geqslant t_i$ for all $i$ is the necessary condition for feasibility of the model.

The initial value of the lot size can be estimated as

$$Q_0 = \text{Max}\left[\frac{B + \sum_{i=1}^{n} b_i}{A + \sum_{i=1}^{n} a_i}, S1\right],$$

where the first term is obtained by equating the differentiated value of the objective function (for $m_i = 1$, $e_i = 1$ for all $i$) with respect to $Q$ to zero and $S1 = \max_i[s_i/(1/D - 1/P_i)]$ [from (4)].

### 5.1. Determination of the minimal partial cost

The Constraint (1) is always satisfied for $e_i = 1$ and constraint (5) becomes $Q \geqslant M_i m_i t_i$. So if $t_i > 0$, for given $Q$, $m_i$ cannot exceed $Q/M_i t_i$. Denote this largest value of $m_i$ by $ml_i$ so that $ml_i = \text{trunc}(Q/M_i t_i)$.

In the special case when $T_i = 0$, the smallest batch size is assumed to be 1 to limit the total number of batches by the lot size. In case of $T_i > 0$, $k_i > 1$, for given $Q$, with $m0_i = \text{int}[Q/g_i) + 1$ and the corresponding $e0_i$, the largest integer satisfying (1) and for

$$a_{\max} = \text{int}\left[\varepsilon_0 + \frac{1}{(k_i - 1)k_i^{e0_i - 1}}\right],$$

$$\varepsilon_0 = (m0_i - Q/g_i) - \left(e0_i - \sum_{r=0}^{e0_i - 1} k_i^{-r}\right).$$

Hoque and Kingsman (1995) found out the minimum of the partial cost functions at each of the following intervals (with the help of some formulae they developed):

$(m0_i + n, e0_i + n + a)$ for all $n$ such that $N_a \leqslant n < N_{a+1}$ and $0 \leqslant a < a_{\max}$ assuming $N_0 = 0$,

$(m0_i + n, e0_i + n + a_{\max})$ for all $n \geqslant N_{a_{\max}}$,

$(m0_i + n, e0_i + n)$ for all $n \geqslant 1$ when $m0_i = e0_i$ or $a_{\max} = 0$,

where

$$N_a \leqslant -\frac{\log\{1 - (a - \varepsilon_0)(k_i^{e0_i} - k_i^{e0_i - 1})\}}{\log k_i} \leqslant N_a + 1.$$

Note that the partial cost function is shown to be a convex function of $n$ in each of the intervals above.

Now for given $Q \geqslant Q_0$, if the values of $m_i$ and $e_i$ associated with the minimal partial cost at any of the intervals above thus obtained satisfy the transfer time constraint (5), give the minimum value of the same in the concerned interval for the extended model. If not, decrease that associated values of $m_i$ and $e_i$ successively by 1 at each step until they satisfy the transfer time constraint or the value of $m_i$ reaches the left end point of the interval. In the latter case, if the value of $m_i$ with its associated value of $e_i$ does not satisfy the transfer time constraint, decrease only that value of $e_i$ keeping $m_i$ the same until the transfer time constraint is satisfied. In both cases calculate the partial cost at the value of $m_i$ with its associated largest value of $e_i$ that satisfies the transfer time constraint. Identify this partial cost as the local minimal partial cost.

Observe that if the values of $m_i$, $e_i$ at the local minimal partial cost are both increased by 1, then these increased values do not satisfy the transfer time constraint. But, if only the value of $m_i$ (except $m_i$ as left-hand end point of an interval) is increased by 1, either with the present value of $e_i$ or reducing it by 1 at each step the transfer time constraint may be satisfied and may give lower partial cost. However, if $H(Q, m_i, e_i) < H(Q, m_i + 1, e_i + 1)$, then $H(Q, m_i, e_i) < H(Q, m_i + n, e_i + n)$ for $n = 1, 2, \ldots$, since for given $m_i$, $e_i$, $H(Q, m_i, e_i)$ is a convex function in $n$.

Thus $H(Q, m_i, e_i) < H(Q, m_i + n, en_i)$ for $en_i \leqslant e_i + n$, since for given $Q$ and $m_i$, $H(Q, m_i, e_i)$, is a non-increasing function of $e_i$, because of non-decreasing nature of $f(m_i, e_i)$.

So, increase both the values of $m_i$ and $e_i$ obtained at the local minimal partial cost, by 1 and calculate the partial cost at these increased values. If this partial cost is greater than the local minimal partial cost, then the present local minimal partial cost is the minimal partial cost for the $i$th stage. Otherwise, keeping the increased value of $m_i$ the same, decrease only the value of $e_i$ successively by 1 at each step until the transfer time constraint is satisfied. Calculate the partial cost at the increased value of $m_i$ with its associated largest value of $e_i$ that satisfies the transfer time constraint. Record the smaller of this partial cost and the local minimal partial cost as the new local minimal partial cost. Again increase both the values of $m_i$ and $e_i$ (associated with the partial cost calculated last) by 1 and repeat the process of obtaining the local minimal partial cost until the partial cost corresponding to the increased values of $m_i$ and $e_i$ by 1 is greater than that of the one obtained without increasing them, or a increased value of $m_i$ reaches $ml_i$, the largest feasible value of $m_i$. In the former case, the local minimal partial cost is the minimal partial cost for the $i$th stage. In the later case, for the value of $m_i$, determine the largest value of $e_i$ that satisfy the transfer time constraint and calculate the partial cost at these values. Record the smaller of this partial cost and the local minimal partial cost as the minimal partial cost in the concerned interval for the $i$th stage. The minimum of the minimum partial costs thus found for all intervals gives the minimum partial cost at this stage. Sum of these minimum partial costs for all stages gives the total minimum partial cost for a fixed value of $Q$. Thus using a directed search over values of $Q$ (as mentioned earlier) the minimal total cost for the extended model can be obtained.

## 6. Numerical examples

Following the solution technique of the extended model two numerical examples are solved. One is the example illustrated in Ramasesh et al. (2000) and the other one is artificially generated with positive transportation times. Data for the former is given in Table 1.

Ramasesh et al. fixed set up cost for all stages and the fixed cost of moving a batch through all stages are given as \$220 and \$20, respectively. The values of $F_i$ and $T_i$ in the above table are obtained respectively by dividing \$220 and \$20 by 9. They found the optimal lot size as 5205 and it is transferred from a stage to the next in 5 equal batches of size 1041. So the transport capacity is taken as 1041. They assumed 250 working days per year and 480 minutes per working day but the processing time is given in minutes. Based on these given values the production rates and set up times are found out in years. The comparative results of this problem obtained by using the techniques of this extended model and that of Ramasesh et al. (2000) is given in Table 2.

Table 1
Data for the 9-items example

| $I$ | $T_i$ | $F_i$ | $C_i$ | $g_i$ | $t1_i$ | $s_i$ | $P_i$ | $D$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 2.22 | 24.44 | 0.24 | 1041 | 0 | 0.00063 | 171 429 | |
| 2 | 2.22 | 24.44 | 0.24 | 1041 | 0 | 0.00096 | 666 666 | |
| 3 | 2.22 | 24.44 | 0.24 | 1041 | 0 | 0.004 | 315 789 | |
| 4 | 2.22 | 24.44 | 0.24 | 1041 | 0 | 0.00021 | 80 000 | |
| 5 | 2.22 | 24.44 | 0.24 | 1041 | 0 | 0.0065 | 222 222 | 50 000 |
| 6 | 2.22 | 24.44 | 0.24 | 1041 | 0 | 0.0012 | 600 000 | |
| 7 | 2.22 | 24.44 | 0.24 | 1041 | 0 | 0.00033 | 85 715 | |
| 8 | 2.22 | 24.44 | 0.24 | 1041 | 0 | 0.0031 | 400 000 | |
| 9 | 2.22 | 24.44 | 0.40 | 1041 | 0 | 0.00021 | 250 000 | |

Table 2
Comparative results for an example considering zero transfer times for all stages

| Model | Lot size | (Total no. of batches, no. of unequal sized batches) in ascending order of stages | Total annual cost | Percentage reduction in total cost |
|---|---|---|---|---|
| Ramasesh et al. (2000) | 5205 | (5,1) | \$6168.25 | 8.85% |
| This extended model | 5353 | For all stages (6,2) | \$5622.27 | |

Table 3
Data for 12-items example

| $I$ | $T_i$ | $F_i$ | $c_i$ | $g_i$ | $t1_i$ | $s_i$ | $P_i$ | $D$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 20 | 0.24 | 2000 | 0.0003 | 0.00063 | 171 429 | |
| 2 | 2 | 20 | 0.25 | 2000 | 0.0008 | 0.00096 | 666 666 | |
| 3 | 2 | 20 | 0.26 | 2000 | 0.0062 | 0.004 | 315 789 | |
| 4 | 2 | 20 | 0.27 | 2000 | 0.0072 | 0.00021 | 80 000 | |
| 5 | 2 | 20 | 0.28 | 2000 | 0.0012 | 0.0065 | 222 222 | |
| 6 | 2 | 20 | 0.29 | 2000 | 0.0021 | 0.0012 | 600 000 | 45 000 |
| 7 | 2 | 20 | 0.30 | 2000 | 0.0015 | 0.00033 | 85 715 | |
| 8 | 2 | 20 | 0.31 | 2000 | 0.002 | 0.0031 | 400 000 | |
| 9 | 2 | 20 | 0.32 | 2000 | 0.006 | 0.00021 | 250 000 | |
| 10 | 2 | 20 | 0.33 | 2000 | 0.003 | 0.007 | 100 000 | |
| 11 | 2 | 20 | 0.34 | 2000 | 0.0023 | 0.00014 | 150 000 | |
| 12 | 2 | 20 | 0.35 | 2000 | 0.0050 | 0.00018 | 225 000 | |

Table 4
Comparative results for 12-items example considering non-zero transfer times for all stages

| Model | Lot size | (Total no. of batches, no. of unequal sized batches) in ascending order of stages | Total annual cost | Percentage reduction in total cost |
|---|---|---|---|---|
| Ramasesh et al. (2000) | 4500 | (3,1) | \$7288.45 | |
| The extended model | 4696 | (3,1),(3,2),(3,1),(4,1),(3,2),(3,1), (3,1),(3,2),(3,1),(4,3),(3,2),(4,1) | \$6602.53 | 9.41% |

Data for the second numerical example is given below in Table 3.

In solving the problem by the method of Ramasesh et al. (2000) fixed set up cost for all stages and the fixed cost of moving a batch through all stages are taken as \$240 and \$24, respectively. Average of the inventory costs in the first 11 stages is used in all these stages. The comparative results are given in Table 4.

## 7. A comparative study of the two techniques

Ramasesh et al. (2000) developed their model based on the assumption of transferring the lot in equal sized batches through all stages, while in our model we assumed that the lot is transferred from a stage to the next in equal or unequal or equal and unequal sized batches. Besides, we developed three constraints, one based on restricted capacities of the transport equipments and the others based on the set up time of stages and transfer times of batches from a stage to the next. In our model it is shown that the optimal lot size must satisfy the set up and transfer time constraints. They have considered set up and transfer times but without any such constraints. So, their model may lead to an infeasible solution. Ramasesh et al. (2000) developed their model considering same inventory costs at all stages for the WIP and higher inventory cost at the finished product stage. Generally for the WIP, inventory costs increase as semi-finished items are shifted to the next stage. Based on this idea we developed the model considering same or different inventory costs at different stages. Ramasesh et al. (2000) have not shown any way of calculating the optimal value of the number of sub-lot sizes whereas we have shown the ways of calculating the optimal numbers of total and unequal sized batches. Because of more generalized assumptions and restrictions in our model, there appear more mathematical complexities and hence more computational complexities in finding out its solution. However, development of a systematic solution algorithm and availability of powerful programming software now a days, have made the computational process so simple that one can obtain the solution of the model easily within expected time. Thus, our realistic assumptions and restrictions have led to the significant reduction in the total cost obtainable without any difficulty.

## 8. Conclusion

In this note the failure of the lot streaming technique developed by Ramasesh et al. (2000) in obtaining the minimal cost of set up, transportation and inventory in a multistage serial production system is highlighted. For a comparative study, the technique developed by Hoque and Kingsman (1995) in obtaining the same type of minimal cost in a multistage serial production system is extended to include non-negative set up and transportation times. Two numerical examples, one solved by Ramasesh et al. (2000) and the other artificially generated, are solved following the algorithm developed in this extended model and significant cost reductions are shown.

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