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Joint ventures in patent contests with spillovers and the role of strategic budgeting☆

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A R T I C L E I N F O

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A B S T R A C T

We consider an R&D contest between n firms in the presence of external spillovers. Our analysis focuses on the effects of these spillovers on joint venture activities between firms. In particular, we are interested in how different budget responsibilities within the research joint venture (RJV) affect profits of firms taking part in the joint venture and profits of their non-cooperating rival firms. Three arrangements for RJVs are analyzed: First, cooperation, in which the firms participating in the joint venture completely share the research they create in the innovation process and each firm has a sovereign budget responsibility. Second, a collusive arrangement in which the participating firms not only share their research but have joint budget responsibilities in the sense that they make all strategic choices cooperatively and maximize joint profits. Third, a hierarchical form, in which the cooperating firms establish joint headquarters which have strategic budget responsibility in the sense that it can strategically subsidize R&D efforts of its member firms so as to maximize overall RJV profits. We show that the first two arrangements can be mimiced in the hierarchical structure and that a hierarchical structure is optimal if it completely subsidizes its members' R&D activities. In this case all rival firms are driven out of the contest.

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1. Introduction

Due to the increasing complexity of innovations and the simultaneous rise in development cost, a natural means for firms to keep up with the process of technological change is to combine research efforts and internalize technology spillovers. It is not surprising that research joint ventures (RJVs) as organizations jointly controlled by at least two participating firms whose primary purpose is to engage in cooperative research and development (R&D) have exploded during the last couple of decades.[1]

This phenomenon of RJVs has attracted a great deal of both theoretical and empirical work in the recent year. This literature is mainly focused on the performance evaluation of various forms of cooperative R&D relative to non-cooperating R&D for firms competing in a product market. A seminal contribution within the economic analysis of RJVs is that of d'Aspremont and Jacquemin (1988). Their analysis is conducted in terms of a two-stage model of oligopolistic competition with R&D spillovers. In the first stage, firms decide on their cost-reducing R&D either cooperatively or non-cooperatively. In the second stage they engage in quantity competition in the product market. Building on this framework, Kamien, Muller, and Zang (1992) analyze a model allowing a richer set of R&D cooperation scenarios. In particular, they compare two different types of RJVs: First, firms may coordinate their R&D efforts in order to maximize their joint product market profits while conducting R&D in separate labs. Second, firms may decide on their R&D efforts in a non-cooperative way but jointly agree to fully share their R&D activities among RJV partners. Kamien et al. (1992) refer to the former scenario as RJV cartelization, to the second one as RJV competition. They show that the cartelized RJV is superior to RJV competition as well as to two other scenarios in which firms either cooperatively or non-cooperatively decide on their R&D activities but do not share R&D efforts.

Concerning RJVs there are, of course, other possible modes of organizing the cooperation between RJV partners. Of particular interest and realized in several real world examples are RJVs in which firms coordinate their R&D activities in a hierarchical structure. Cooperating firms establish a new separate organizational entity that is jointly owned by all participating partners in the RJV and in charge of the coordination of all partners' R&D activities. In the following we call this type of cooperation RJV hierarchy: Cooperating firms set up an entity in the form of RJV headquarters that coordinate the R&D activities carried out in the firms' respective R&D units. The advantage of an RJV hierarchy comes from the headquarters' possibility to influence the R&D activities of their member firms strategically in the competition with non-cooperating firms. In particular, we assume that headquarters can subsidize the R&D costs of the participating firms in the RJV so as to maximize overall profits.[2]

Different to most of the existing literature on RJV following d'Aspremont and Jacquemin (1988), we do not analyze the organizational design of RJVs in the context of process innovation.[3] Instead, we focus on product innovations and model R&D as a patent contest, see Bagwell and Staiger (1997): Firms compete to enter a new market with an innovation. To be the first to serve the new market or to serve an existing market with a new product implies a quasi-monopoly in form of additional profits compared to other firms. To improve their innovation potential and to increase their chances of winning the R&D contest, firms compete by spending resources in innovation activities. These efforts cannot be recovered, whether the firm wins the patent or not. Examples for such R&D contests are high-technology industries in which the firm with the best idea wins an exclusive right for commercializing its idea, such as the contest sponsored by the Federal Communications Commission to develop the best technology for high-definition television, see Baye and Hoppe (2003), or the pharmaceutical industry

[1] See, for example, Hagedoorn and Schakenraad (1990, 1991, 1992) for the emergence of inter-firm cooperative agreements since the early 1980s.

[2] For other possibilities how the headquarter of the RJV could influence its members' incentives to invest in R&D, see Section 3.3.

[3] Some other contributions have extended d' Aspremont and Jacquemin' model in several ways: For example, Suzumura (1992) extends the model to n-firms, Poyago-Theotoky (1995) studies the endogenous number of participants in the RJV, Salant and Shaffer (1998) study optimal asymmetric strategies in RJV, Petit and Tolwinski (1999) introduce dynamics and asymmetries, and Molto, Georgantzis, and Orts (2005) consider cooperative R&D with endogeneous technology differentiation.

in which pharmaceutical firms spend huge amounts of money in the development of new drugs, see Jost and van der Velden (2006, 2007).[4]

Our analysis of different organization designs of RJVs is then conducted in a model with n firms competing for a patent in the presence of spillovers. In RJV competition we consider the case where two out of these n firms form an RJV and pool their R&D efforts. All firms in the industry, whether cooperating or not, decide on their own R&D efforts taking the R&D activities of others firms as given. In RJV cartelization we assume that the two cooperating firms not only share R&D efforts but also coordinate their R&D activities so as to maximize the sum of overall RJV profits.[5] Both scenarios are modelled in terms of a one-stage game. RJV hierarchy then is a two-stage model. In the first stage, the RJV headquarters choose how much of the R&D costs of their member firms will be subsidized by the RJV so as to maximize overall RJV profits. Given this choice, all firms and the two partners in the RJV choose R&D efforts in the second stage.

We derive the following results. First, RJV competition is superior relative to the fully non-cooperative case, that is, cooperating as well as non-cooperating firms benefit from the RJV relative to R&D competition. However, the profits of the cooperating firms within the RJV are higher than the ones of their competitors only if the number of contestants n is sufficiently high and the spillover rate is sufficiently low. These results can be intuitively explained in terms of the *cooperation dilemma effect*: Since each firm in the RJV benefits from the additional R&D efforts it gets from its partner – *the research sharing effect*– it reduces its own R&D efforts and free rides on the other firm's research. In turn, it behaves less aggressively in the contest than non-cooperating firms. Moreover, each non-cooperating rival firm will increase its efforts as a response to the lower R&D efforts of cooperating firms. Consequently, their winning probabilities increase. In sum, the cooperating firms basically benefit on the cost side, while rival firms basically gain on the benefit (probability) side. These benefits of non-cooperating firms are relatively higher compared to those of the cooperating firms if the number of competing firms in the industry is small. This is because a constant increase in R&D efforts increases the winning probability more the less firms are in the industry. An increasing level of competition thus diminishes these higher benefits of non-cooperative firms. In this case profits of the RJV partners are still lower than the ones of rival firms if the spillover rate is high. This is because the benefits of the research sharing effect get less important, that is, the additional fraction of research cooperating firms have available in the contest is small.

Second, and in line with the analysis of Kamien et al. (1992), we show that RJV cartelization is always more profitable for the cooperating firms than RJV competition. Moreover, non-cooperating firms have lower profits under RJV cartelization relative to the fully non-cooperative case. The intuition for these results follows immediately from the *joint maximization effect*: The externality caused by the*cooperation dilemma effect* and ignored by a firm under RJV competition is now internalized when firms choose how much to spend so as to maximize the RJV's combined profits. Hence, both the *joint maximization effect* and the *research sharing effect* jointly dominate the *cooperation dilemma effect*. It should be noted that an essential driver for this is the *research sharing effect*. To see this, suppose for a moment that there would be no research sharing in an RJV—this corresponds to the case of R&D cartelization in the terminology of Kamien et al. (1992). Then joint profit maximization of two firms in the industry is nothing else than a fusion of the previously independent firms. This is because the two separate firms within the cartelization act as if they were just one firm. In fact, their R&D efforts are half of the ones of the rival firms leading to profits for the cartelized firms that are less than the ones of the non-cooperating firms. Hence, without the research sharing effect firms in an RJV cartelization

[4] Competition that takes the form of a contest can be found in various markets. Typical examples for this form of competition are contests in markets with network externalitities where firms compete for a quasi-monopoly, see Besen and Farrell (1994), litigation contests for brand names, internet addresses or other exclusive assets that yield quasi-monopoly rents, see Huck, Konrad, and Müller (2000), promotion contests in the pharmaceutical industry where producers try to persuade physicians to prescribe their products instead of competing products, see Breyer and Zweifel (1999), or sales contests in the pre-1994 insurance market where insurance retailers try to attract potential customers by visiting or persuasive talking, see Rees and Kessner (1999).

[5] The reason for assuming that only two out of n firms can form a RJV is based on the main result of this paper: If an RJV with just two firms establishes an RJV hierarchy, strategic budgeting drives out all other firms from the innovation contest. See also our concluding remarks in Section 4.

are worse off than the non-cooperating firms. Thus, research sharing is the essential driver for the more aggressive behavior of the cooperating firms under joint profit maximization resulting in higher profits under RJV cartelization than those of the rival firms.

Third, and this is the main contribution of our analysis, we show that RJV hierarchy is the most desirable mode of organizational design for the RJV, at least in the eyes of cooperating firms. In particular, we show that it is optimal for the RJV headquarters to completely subsidize their members' R&D activities. In this case all rival firms are driven out of the contest. To see this note first that RJV competition and RJV cartelization can both be mimiced in a hierarchical structure. Indeed, if the headquarters do not subsidize their member firms at all, the cooperating RJV partners act independently as if they were in an RJV competition. And, if the headquarters subsidize R&D efforts of their member firms by 50%, the cooperating firms just behave as if they were in an RJV cartelization and jointly maximize profits. However, the headquarters can do better due to a *strategic budgeting effect*: The higher the amount of subsidization by the headquarters, the higher are their member firms' R&D efforts. Since R&D activities are strategic substitutes in R&D competition, more aggressive behavior of cooperating firms leads to less aggressive behavior of the non-cooperating firms.

This last result is related to findings in the literature on strategic delegation. Fersthman and Judd (1987) and Sklivas (1987) show that a firm may profit from giving his manager incentives not to maximize only profits but to put a positive weight on sales in order to induce a more aggressive market behavior. This result can also be confirmed in our model of a patent contest.[6] Indeed, if the headquarters delegate R&D activities to only one unit, it will be optimal to subsidize only part of the R&D costs of their unit. Our analysis then shows that strategic budgeting in the presence of two units is even more beneficial: If the headquarters delegate R&D activities to their two cooperating members, it will be optimal to fully subsidize the R&D costs of both units. The resulting aggressiveness of the cooperating firms then is so high that all rival firms are driven out of the R&D contest.[7]

More closely related to our modelling are the papers that extend the analysis of strategic delegation to multiproduct firms. For example, Vickers (1985) shows that inducing competition between divisions can be profitable because of its effect on the behavior of rival firms. Schwartz and Thompson (1986) show that divisionalization may lead incumbent firms to preempt entry into their industry.[8]

The rest of the paper is structured as follows. In Section 2, we introduce the basic contest model which we extend to the existence of spillovers. In the following third section, we apply the contest model to analyze the optimal organizational design of an RJV. In particular, we analyze the three different organizational scenarios discussed above. In the last section, we conclude our results and present options for future research.

2. R&D contests with spillovers

Consider an industry with n firms, $n \geq 3$. Suppose that these firms compete in introducing some patentable innovation and let V be the value of the patent, that is the monopoly rent a firm would capture if it innovated.

To win the patent, firms compete in innovative activities. Each firm i, $i = 1, \ldots, n$, simultaneously invests in R&D efforts. These investments are sunk and cannot be recovered, independent of whether the firm wins the patent or not.[9] Let x_i denote these R&D efforts of firm i. Due to spillovers, the probability that firm i is successful in the patent contest is a function of the "effective" R&D levels, X_1,

[6] See also Baik and Kim (1997), Kräkel (2002, 2005), Baik (2003) or Kräkel and Sliwka (2006) for the analysis of strategic delegation in contests. Different to our model, these articles of strategic delegation assume that the owner of a firm decides on an incentive scheme for his manager.

[7] See also Zhao (1999) who shows in a Stackelberg–Cournot context how joint ventures may be used to deter entry.

[8] Other work on the relationship between internal organizational design and competitive forces includes that of Gal-Or (1993), Sen (1993), Petit and Tolwinski (1999) or Bárcena-Ruiz and Espinosa (1999).

[9] These innovation efforts are typically chosen in a sequential structure of innovation, see e.g. Loury (1979) or Green and Scotchmer (1995) with two different types of uncertainty. First, R&D involves a technological uncertainty since innovation genuinely is a risky activity in the sense that it is uncertain whether and when own research effort leads to an innovation. Second, R&D involves a market uncertainty since other firms invest in the same patentable innovation and it is uncertain which firm innovates first. In the following we concentrate on market uncertainty.

$\ldots, X_i, \ldots, X_n$, rather than only of each firm's R&D efforts, $x_1, \ldots, x_i, \ldots, x_n$, see Kamien et al. (1992).[10] The magnitude of effective R&D efforts X_i is determined by the combined individual R&D efforts of all firms and an external spillover rate.[11] Let β denote this spillover rate with $\beta \in [0, 1]$. Then

$$X_i = x_i + \beta \sum_{i \neq j}^{n} x_j \text{ for } \beta \in [0, 1].$$

If $\beta = 1$, research generated by R&D activities spills over freely and thus behaves as a public good. If $\beta = 0$, then spillovers do not occur between both firms. That means that both firms can keep the results of R&D private.

The probability p_i of firm i of winning the patent depends on the R&D efforts of all firms $p_i = p_i(x_1, \ldots, x_i, \ldots, x_n)$. With spillovers we assume that the probability function p_i is given by an extension of the framework of Tullock (1980):[12]

$$\begin{aligned} p_i(x_1, \ldots, x_i, \ldots x_n) &= \frac{X_i}{\sum_{j=1}^{n} X_j} \\ &= \frac{x_i + \beta \sum_{j \neq i} x_j}{X + (n-1)\beta X}. \end{aligned}$$

where X denotes total R&D efforts in the industry, $X = \sum_j x_j$. If $\beta = 1$, p_i is independent of the actual R&D efforts and identical to $1/n$. If $\beta = 0$, p_i is the originally so-called "contest success function", see Dixit (1987) or "production function for lotteries", see Szidarowszky and Okuguchi (1997).

Firms are risk neutral. Let Π_i denote firm i's expected profit in this R&D contest. Then each firm chooses its R&D efforts x_i to maximize

$$\Pi_i(n, \beta) = p_i(x_1, \ldots, x_i, \ldots, x_n)V - x_i. \tag{1}$$

Proposition 1. *The symmetric Nash-equilibrium with n firms in the innovation contest with spillovers is characterized by the following properties: The optimal R&D efforts are*

$$x_i^*(n, \beta) = x^*(n, \beta) = V(1-\beta)\frac{n-1}{(\beta(n-1)+1)n^2},$$

effective R&D efforts are

$$X_i^*(n, \beta) = X^*(n, \beta) = V(1-\beta)\frac{n-1}{n^2},$$

winning probabilities are

$$p_i^*(n, \beta) = p^*(n, \beta) = \frac{1}{n},$$

and profits in equilibrium are given by

$$\Pi_i^*(n, \beta) = \Pi^*(n, \beta) = V\frac{\beta(n^2-1)+1}{(\beta(n-1)+1)n^2} \text{ for } i = 1, \ldots, n.$$

[10] In his widely noticed review on spillovers and innovative activities, De Bondt (1996, p. 5) traces back the notation of the effective research effort X_i in the simple form $X_i = x_i + \beta x_j$ to at least Ruff (1969).

[11] See De Bondt (1996) for an overview over some of the relevant economic literature of spillovers in innovation.

[12] This form of market uncertainty in the R&D contest also arises from a more elaborate probability structure, see Fullerton and McAfee (1999). Moreover, Baye and Hoppe (2003) show that R&D tournaments in which the process of innovation follows a stochastic process are equivalent to our simple R&D contest function in which the probability to win equals the share in aggregated expenditure. In addition, for some stochastic processes the contest prize is constant with respect to aggregate efforts.

Proof. See Appendix. □

Proposition 1 shows that equilibrium R&D efforts are positive as long as spillovers are incomplete, i.e. $x^* > 0$ only if $\beta < 1$. Moreover, equilibrium R&D efforts are lower, the higher the degree of spillovers:

$$\frac{\partial x^*}{\partial \beta} = V \frac{1-n}{n(\beta(n-1)+1)^2} < 0$$

If there are complete spillovers in the industry, i.e. $\beta = 1$, then no firm has an incentive to invest in R&D. This is because all research created due to R&D activities would be beneficial to the same extent for all competitors and no firm could enhance its chances to win the R&D contest.

Although R&D efforts in equilibrium depend on the spillover rate, the probability p^* of each firm to win the R&D contest does not and is $1/n$ in equilibrium since all firms act symmetrically. Thus, the individual probability of winning negatively depends on the number of competitors in equilibrium, but it does not depend on spillovers. The reason relies on the fact that symmetric firms will spend all the same efforts and exhibit all the same amount of spillovers, while no technological risk may harm the probability. Competition, of course, also affects firms' R&D efforts in equilibrium:

$$\frac{\partial x^*}{\partial n} = V(\beta-1)\frac{\left(2n^2-4n+2\right)\beta+n-2}{\left(\beta(n-1)+1\right)^2 n^3} < 0 \text{ for } n \geq 2\,.$$

That is, a higher degree of competition leads to lower R&D efforts for each firm. This is consistent with the well-known proposition in the patent race model by Loury (1979).

3. Joint ventures in R&D contests

Suppose that two out of the n firms in the R&D contest cooperate in an RJV. We consider three forms of such a joint venture varying the degree of cooperation between the two partners: First, in scenario K called RJV competition, we assume that both partners decide independently on their own R&D efforts so as to maximize their individual profits, but pool their R&D efforts so as to fully share their research. That is, the internal spillover rate is One for the two participating firms.[13] Second, in scenario C called RJV cartelization, we not only assume complete research sharing between the partners, but also that partners coordinate their R&D activities so as to maximize joint profits. Third, in scenario H called RJV hierarchy, we consider an arrangement similar to scenario C, but assume that joint headquarters are established that can subsidize the R&D efforts of the two partners in order to maximize the sum of overall profits.

3.1. Scenario K: RVJ competition

Suppose that two out of n firms form an RJV and share all the research they create. However, each firm decides on its R&D efforts unilaterally, so as to maximize its individual profits. Denote with i, $i = 1, 2$ the two firms participating in the joint venture.

Consider first the effective R&D levels, $X_1, X_2, X_3, \ldots X_n$, if each firm's research efforts are $x_1, x_2, x_3, \ldots x_n$. The difference between the RJV partners and other competing firms in the contest are the additional spillovers the former get from the other partners. While before the cooperation they got βx_i from each other, now they additionally can make use of $(1-\beta)x_i$ of the other's research. Hence,

$$X_i = \begin{cases} x_1 + x_2 + \beta \sum\limits_{j \neq 1,2} x_j & \text{for} i = 1, 2 \\ x_i + \beta \sum\limits_{j \neq i} x_j & \text{for} i = 3, \ldots, n \end{cases}$$

[13] This assumption is consistent with the equilibrium that emerges when internal spillovers are endogenized, see e.g. Beath, Poyago-Theotoky, and Ulph (1998) or Amir, Evstigneev, and Wooders (2003).

and total effective R&D efforts are

$$\sum X_j = X + (n-1)\beta X + (1-\beta)(x_1 + x_2)$$

with $X = \sum_j x_j$. Given these effective R&D efforts, firms seek to maximize profits

$$\Pi_i = \begin{cases} \left((1-\beta)(x_1+x_2) + \beta X\right) \dfrac{V}{\sum X_j} - x_i & \text{for} i = 1, 2 \\ \left((1-\beta)x_i + \beta X\right) \dfrac{V}{\sum X_j} - x_i & \text{for} i = 3, \ldots, n \end{cases}.$$

Because of the symmetry of firms $i = 1, 2$ and of firms $i = 3, \ldots, n$, we are interested in a quasi-symmetric equilibrium in which both firms participating in the RJV choose the same R&D efforts x^*_{kj} and all rival firms choose x^*_{kr}. Let Π^*_{kj} denote the equilibrium profits of a firm within the RJV and let Π^*_{kr} be the equilibrium profits of a rival firm.

Proposition 2. *The quasi-symmetric Nash-equilibrium in a patent contest, in which two firms form an RJV and completely share all their research during the contest, has the following properties: The efforts, effective efforts and winning probabilities of the firms within the RJV are lower than the ones of rival firms,*

$$x^*_{kj} < x^*_{kr},\ X^*_{kj} < X^*_{kr} \text{and}\ p^*_{kj} < p^*_{kr}.$$

The profits of each firm within the RJV are greater than rival firms' profits if spillovers in the industry are lower than some critical value $\tilde{\beta}(n) < 1$,

$$\Pi^*_{kj} > \Pi^*_{kr}\ \text{for}\ \beta < \tilde{\beta}(n).$$

Proof. See Appendix. □

The proposition shows that forming an RJV is not always more profitable for the participating firms than for rival firms. As mentioned in Section 1, this result reflects the trade-off between the *cooperation dilemma effect* —because a cooperating firm in the RJV has full access to the research of its partner firm, it has a higher incentive to free ride than non-cooperating firms have – and the*research sharing effect*– cooperating firms in the RJV have an additional fraction $(1-\beta)$ of R&D efforts of their partners available in the contest: The cooperation dilemma effect implies that because of its research sharing cooperating firms invest less in R&D than their non-cooperating rival firms. This, however, implies less effective R&D effort for a non-cooperating firm and, as a response, it increases its own R&D efforts. In equilibrium then, its effective efforts, and, hence, its winning probability, are higher than the ones of a firm within the RJV. Whether this leads to higher profits for a non-cooperating firm than for a cooperating firm depends on the trade-off between the cooperation dilemma effect and the research sharing effect: If, for example, only three or four firms are in the industry, the cooperation dilemma effect always dominates the research sharing effect and research sharing is never beneficial, independent of the degree of spillovers. This is because if the number of firms n in the industry is small, the cost savings due to less R&D efforts do not compensate the reduced benefits of a cooperating firm from the patent contest due to its lower winning probability. If, however, the degree of competition becomes higher and more firms are in the industry, the expected benefits of cooperating firms becomes greater relative to the ones of its rival firms, whereas the opposite is true for the cost savings,

$$\frac{\partial}{\partial n}\frac{p^*_{kj}}{p^*_{kr}} > 0,\ \frac{\partial}{\partial n}\frac{x^*_{kj}}{x^*_{kr}} < 0.$$

Since these effects are lower the higher the rate of spillovers,

$$\frac{\partial}{\partial \beta}\left(\frac{\partial}{\partial n}\frac{p^*_{kj}}{p^*_{kr}}\right) > 0,\ \frac{\partial}{\partial \beta}\left(\frac{\partial}{\partial n}\frac{p^*_{kj}}{p^*_{kr}}\right) < 0,$$

there exists a critical value $\tilde{\beta}(n)$ which is an increasing function in n such that the profits of the RJV partners are higher than the ones of rival firms, if the spillover rate is lower than this value. Intuitively, if the spillover rate is high, the benefits of the research sharing effect become valuable only if the number of firms in the industry is sufficiently high.

In the next proposition we ask how R&D efforts, winning probabilities and profits of all firms change after the RJV is formed compared to the situation before cooperation. Thus, we compare the equilibrium results of R&D competition with the equilibrium results in an RJV competition.

Proposition 3. *An RJV in a patent contest, in which the cooperating firms completely share their R&D research, is always profitable for the participating firms, $\Pi^*_{kj} > \Pi^*$. However, it also increases the profits of the non-cooperating rival firms, $\Pi^*_{kr} > \Pi^*$. Moreover, cooperating firms spend less in* R&D, *$x^*_{kj} < x^*$ and have a lower winning probability $p^*_{kj} < p^*$, whereas the opposite is true for rival firms, that is $x^*_{kr} > x^*$* and *$p^*_{kr} > p^*$.*

Proof. See Appendix. □

Interestingly, both the cooperating and the non-cooperating firms benefit from the joint venture. Following proposition 2, however, the increase in profits for the participating firms in the RJV is lower than the one for the non-cooperating firms, $\Pi^*_{kr} > \Pi^*_{kj} > \Pi^*$, if the degree of competition is low or the spillover rate high. In such a situation, forming a RJV creates a public good and one would expect, that if firms' decisions to cooperate would be endogenized, no RJV would result. This intuition, however, proves to be wrong, see Jost (2010): In a coalition formation game in which firms have the opportunity to cooperate in a RJV and to form a coalition prior to their R&D activities, it turns out that the endogenous formation of RJVs is limited but that firms always have an incentive to cooperate. This result on the optimal size of competing RJVs differs from most of the findings in the literature on endogenous formation of R&D coalitions.[14] In Greenlee (2005), for example, larger research sharing RJVs earn higher profits, and benefit from the dispersion of rival firms across many small ventures. When joint ventures set jointly optimal research efforts, Yi and Shin (2000) establish that without research sharing firms benefit from having few partners and rival firms are organized in large RJVs.

Although an in-depth discussion of these differences is beyond the scope of this paper, three important features of the modelling in the present paper and in Jost (2010) are worth noting: First, as mentioned in Section 1, the mode of innovation differs—in contrast to all papers in the literature, we analyze product innovations instead of cost-reducing process innovations. Second, the mode of competition differs—whereas under process innovation firms after their R&D investments compete in a Cournot product market, we concentrate on a patent race. This guarantees that all strategic interactions occur and are exhausted at the R&D networking stage and implies that competition is of an all-or-nothing nature—either a firm wins the patent or does not. Hence, our patent contest is a mode of competition which is more aggressive than the moderate Cournot competition.[15] And third, while, in general, a firm's optimal R&D effort depends on the research efforts and affiliations of all rival firms, a comprehensive analysis of competing RJVs in the context of process innovations is too complicated whereas in Jost (2010) a complete characterization of firms' equilibrium R&D efforts and profits for arbitrary industry partitions is possible.[16]

[14] See, for example, Greenlee and Cassiman (1999), Yi and Shin (2000), Morasch (2000) or Greenlee (2005).

[15] The nature of market competition therefore has an important effect on the endogenous formation of coalitions which is in line with the result in Goyal and Joshi (2003) who show that, if competition is in prices, the empty network in which no firm cooperates will be the unique stable network.

[16] For research sharing joint ventures, Greenlee and Cassiman (1999) and Greenlee (2005) consider arbitrary industry partitions but assumes Cournot interaction, linear demand, and quadratic R&D costs to keep the analysis tractable. The analysis is less complicated for the case of joint profit maximization where firms' optimal R&D efforts depend only on aggregate R&D effort in the industry and not on the composition of other firms' investments. See Yi and Shin (2000) who consider joint profit maximizing RJVs for arbitrary industry partitions under a general demand structure or Morasch (2000) who assumes a linear Cournot model.

3.2. Scenario C: RJV cartelization

Suppose now that two out of the n firms forming an RJV not only share all the research they create in the R&D process but choose their R&D efforts cooperatively to maximize joint profits.

To analyze RJV cartelization, consider the objective function of a firm i, $i=1, 2$, participating in the RJV. They choose their R&D efforts x_1 and x_2 to maximize joint profits

$$\max_{x_1,x_2} \quad 2\frac{X_{12}}{\sum X_j}V-(x_1+x_2), \text{where}$$
$$X_{12}=x_1+x_2+\beta\sum_{j=3,\ldots,n}x_j \text{ and}$$
$$\sum X_j = X+(n-1)\beta X+(1-\beta)(x_1+x_2) \text{ with}$$
$$X=x_1+x_2+\sum_{j=3,\ldots,n}x_j$$

All other firms $i=3, \ldots, n$, seek to maximize profits as usual by

$$\max_{x_i} \quad \frac{X_i}{\sum X_j}V-x_i, \text{where}$$
$$X_i=x_i+\beta\left(x_1+x_2+\sum_{j=3,\ldots,n,\, j\neq i}x_j\right).$$

Since both partners are symmetric, they will choose identical R&D efforts in equilibrium, $x_1^*=x_2^*=x_{cj}^*$. As before, we are interested in a quasi-symmetric equilibrium in which all rival firms choose identical R&D efforts in equilibrium, $x_3^*=\ldots=x_n^*=x_{cr}^*$. Let Π_{cj}^* denote the equilibrium profits of a firm participating in the RJV and Π_{cr}^* the equilibrium profits of the rival firms.

Proposition 4. *The quasi-symmetric Nash-equilibrium in a patent contest, in which firms form an RJV with research sharing and joint profit maximization, has the following properties: In general, the R&D efforts of firms within the RJV are greater than the ones of rival firms,*

$$\begin{aligned} x_{cj}^* &< x_{cr}^* \text{ for } n=3,4 \\ x_{cj}^* &> x_{cr}^* \text{ for } n\geq 5. \end{aligned}$$

The effective efforts and winning probabilities of the firms within the RJV are always greater than the ones of rival firms,

$$X_{cj}^*>X_{cr}^* \text{ and } p_{cj}^*>p_{cr}^*.$$

Moreover, the equilibrium profits of firms within the RJV are always strictly greater than those of their rivals:

$$\Pi_{cj}^*>\Pi_{cr}^*$$

Proof. See Appendix. □

Thus, if there are only one or two firms next to the RJV, then each firm participating in the RJV will spend less in the R&D contest than each rival firm. Otherwise, the firms within the RJV spend more efforts than their rivals in order to win the contest. Concerning profits, the proposition shows that in equilibrium the profits of a cooperating firm exceed its rival firms' profits. Only if there are complete spillovers in the industry, i.e. $\beta=1$, then cooperating and non-cooperating firms have identical profits.

These results are an immediate consequence of the *joint maximization effect* as discussed in Section 1: If both cooperating firms jointly maximize overall RJV profits, the *cooperation dilemma effect* ignored under RJV competition is internalized. Note, however, that the *research sharing effect* is essential for this result. In fact, as we prove in Appendix B, a cartelization without research sharing leads to R&D efforts for the cooperating firms that are half of the ones of the rival firms. This implies less profits of the cooperating firms relative to their rival firms. As a result, it is the research sharing effect that leads to more aggressive behavior of the cooperating firms under RJV cartelization resulting in higher profits than those of the rival firms.

Finally, we will answer the question whether the RJV cartelization is actually profitable. The answer is positive if a firm's equilibrium profits in R&D competition (Π^*) are lower than its profits Π^*_{cj} in an RJV cartelization.

Proposition 5. *In a contest model, in which two firms form an RJV cartelization, collusive behavior between the joining firms leads to profits that are always higher for each of the firm participating in RJV than before the cooperation. Although the RJV is profitable for the participating firms, it is disadvantageous for rival firms. That is*

$$\Pi^*_{cj} > \Pi^* > \Pi^*_{cr} \text{ for all } \beta \in [0, 1).$$

Moreover, if the size of spillovers is sufficiently large, R&D efforts as well as effective R&D efforts of the firms participating in RJV are greater than before cooperation but those of rivals firms are not.[17] That is, for all $\beta > \bar{\beta}(n)$

$$x^*_{cj} > x^* > x^*_{cr} \text{ and } X^*_{cj} > X^* > X^*_{cr}.$$

And finally, cooperation always increases the probability of winning the contest for each of the firm participating in RJV, that is

$$p^*_{cj} > p^* > p^*_{cr}.$$

Proof. See Appendix. □

This is a remarkable result and an immediate consequence of the discussion above. It predicts that firms always have an incentive to cooperate if they share their research efforts and cooperatively maximize their joint profits. On the other hand, profits for non-cooperating firms decrease due to the RJV. Only in case of complete spillovers, $\beta = 1$, all firms in the industry have identical profits, whether cooperating or not.

Note that under RJV cartelization profits Π^*_{cj} of firms participating in the RJV are always higher than under RJV competition, for all $n \geq 3$ and $0 \leq \beta < 1$. This follows from the three facts: First, the R&D effort levels of cooperating firms under RJV competition can always be implemented by the partners under joint profit maximization in case of RJV cartelization. Second, as argued above and in contrast to RJV competition, RJV cartelization allows the cooperating firms to internalize the cooperation dilemma effect which, in turn, leads to higher R&D efforts. And third, as a response of the higher R&D efforts, rival firms reduce their own R&D efforts. The last two effects together imply that the effective R&D efforts, the winning probabilities as well as the profits of cooperating firms are higher under RJV cartelization than under RJV competition.

3.3. Scenario H: RJV hierarchy

One may ask whether the cooperating firms can do better than in the RJV cartelization case. To answer this question, we consider scenario *H* in which the cooperating firms establish headquarters that play an active role in the contest. In particular, we assume that headquarters can subsidize the R&D costs of the participating firms in the RJV so as to maximize overall profits.[18]

[17] In fact, $X^*_{cr} > X^*$ for all $\beta \in [0, 1)$, see the proof of Proposition 5.

[18] There are two other possibilities how the headquarter of the RJV could influence its members' incentives: First, by setting an internal contest prize *W* if one of the joining firms wins the contest. However, since the incentives to invest in R&D are always

As in scenario *C*, suppose that two out of n firms cooperate in an RJV cartelization. Different to scenario *C* the newly established headquarters now use the following incentive mechanism to influence the success of their member firms. Suppose that headquarters subsidize the R&D expenditure of their member firm $i = 1, 2$ by a fixed subsidy F_i where F_i is set equal to $(1 - c)x_i$ after firm i has chosen its R&D effort x_i. The factor $c \in [0, 1]$ is credibly announced by the headquarters before their member firms engage in R&D. This implies that each member firm behaves as if the marginal costs of research is c rather than One. Then each participating firm maximizes the following profit function:

$$\Pi_i = p_i V - x_i + F_i \text{ for } i = 1, 2$$
$$\text{with } p_i = \frac{(1-\beta)(x_1 + x_2) + \beta X}{\sum X_j}.$$

Given their members' profits the headquarters then choose their subsidization to maximize overall profits of the RJV, that is

$$\max_c \quad \pi_{hj} = \Pi_1 + \Pi_2 - (F_1 + F_2).$$

We then model the competition between the RJV and the $n - 2$ non-cooperating firms as a two stage game. In the first stage the RJV's headquarters choose their subsidization parameter c. Given this choice, all firms and the two partners in the RJV spend efforts on R&D in the second stage.

As discussed in Section 1, subsidization then has two effects on the firms' R&D efforts in the contest. First, since each cooperating firm has to bear only a fraction of its actual R&D efforts, its incentives to invest in R&D are increased. The higher the subsidization, the higher these incentives. Second, as a response to the more aggressive behavior of the cooperating firms, rival firms will reduce their R&D efforts. Both effects, of course, imply that the probability for the cooperating firms to win the contest prize increases in subsidization whereas the winning probability of each rival firm decreases. Whether and to which extent subsidization then actually pays for the RJV depends on the marginal increase in getting the prize and on the marginal increase in overall R&D costs.

To solve for a subgame perfect equilibrium consider first the second stage. For a given $c \in [0, 1]$ chosen by the RJV's headquarters, the first order conditions for the non-merging firms $i = 3, \ldots, n$ are given by

$$\frac{\sum X_j - X_i(1 + (n-1)\beta)}{\left(\sum X_j\right)^2} V = 1,$$

and for the cooperating firms in the RJV, $i = 1, 2$

$$\frac{\sum X_j - X_i(2 + (n-2)\beta)}{\left(\sum X_j\right)^2} V = c.$$

Proposition 6. *In a contest model, in which two firms form an RJV hierarchy and the headquarters choose to subsidize their R&D efforts so that the implied marginal research costs for their RJV members are c, leads to the following equilibrium behavior: The efforts of the participating firms in the RJV are greater than the ones of rival firms,* $x_{hj}^* > x_{hr}^*$, *iff*

$$\frac{(n-2)(1 + n\beta - 2\beta)}{(n + n^2\beta - 2n\beta + 1 + \beta)} > c.$$

increasing in the contest prize, the optimal internal prize always equals the contest prize V. Second, by restricting knowledge sharing between the members to a rate $\eta \in [\beta, 1]$. Note, that the trade-off between the cooperation dilemma effect and the research sharing effect suggests that the optimal internal spillover rate will not always be maximal and equal to One, unless the headquarter internalizes the free-riding problem by joint profit maximization. Hence, our results for RJV hierarchy require complete research sharing.

Moreover, R&D *efforts of the cooperating firms as well as their profits are increasing in the amount of subsidization*, *whereas rivals' R&D efforts and profits are decreasing*:

$$\frac{\partial}{\partial c} x^*_{hj}(c) < 0,\ \frac{\partial}{\partial c} \Pi^*_{hj}(c) < 0$$
$$\frac{\partial}{\partial c} x^*_{hr}(c) > 0,\ \frac{\partial}{\partial c} \Pi^*_{hr}(c) > 0$$

Proof. See Appendix. □

Three remarks are worth noting: First, if headquarters subsidize R&D efforts by 50%, that is, $c = 1/2$, their member firms will just behave as if they were in an RJV cartelization and jointly maximize profits, that is $x^*_{hj}(\frac{1}{7}2) = x^*_{cj}$. Second, if there is no subsidization, $c = 1$, the two cooperating firms will act independently as if they were in an RJV competition, that is $x^*_{hj}(1) = x^*_{kj}$. Third, it turns out that the left hand side of the condition above is strictly between [1/4, 1). Hence, if subsidization is higher than this critical value, that is $c < 1/4$, the cooperating firms will always spend more than their rival firms. If, on the other hand, subsidization is lower than this critical value, that is $c \in [1/4, 1)$, this property depends on the competition in the industry and the size of spillovers. In particular, the left hand side is increasing in the number n of firms and in the spillover rate β.

To answer the question, whether headquarters can do better than letting their member firms act as in an RJV cartelization, consider Proposition 6. It shows that the higher the subsidization of R&D costs, the higher its member firms' profits. This is not surprising since the cooperating firms have to bear only part of the actual costs they invest in R&D. If this cost reduction then pays for the headquarters, depends on the marginal increase in the members' profits due to the subsidization.

Proposition 7. *In a contest model, in which two firms form an RJV hierarchy, the headquarters optimally fully subsidize the R&D expenditure of their members*, $c^* = 0$. *This leads to the following equilibrium behavior*: *Rival firms do not invest in R&D whereas the R&D efforts of the cooperating firms are decreasing in the degree of competition n and in the size of spillovers* β:

$$x^*_{hj}(0) = \frac{V}{2}(1-\beta)\frac{(n\beta - \beta + 2)}{(n\beta - 2\beta + 2)^2}$$
$$x^*_{hr}(0) = 0$$

Effective R&D efforts and winning probabilities of the cooperating firms are always greater than those of the rival firms,

$$X^*_{hj}(0) > X^*_{hr}(0) \,\text{and}\, p^*_{hj}(0) > p^*_{hr}(0).$$

Equilibrium profits of each firm in the RJV hierarchy and those of rival firms are

$$\Pi^*_{hj}(0) = V\frac{n\beta - \beta - \beta^2 + n\beta^2 + 2}{\left(n\beta - 2\beta + 2\right)^2}$$
$$\Pi^*_{hr}(0) = V\frac{\beta}{\left(n\beta - 2\beta + 2\right)}$$

and decreasing in competition. *Moreover*, *the cooperation firms' profits are almost always decreasing in the rate of spillovers whereas the ones of its rival firms are increasing in* β. *Compared to R&D competition rival firms loose and the cooperating firms win*:

$$\Pi^* > \Pi^*_{hr}(0) \,\text{and}\, \Pi^*_{hj}(0) > \Pi^*$$

Proof. See Appendix. □

The proposition shows the importance of the *strategic budgeting effect* discussed in Section 1 when forming a RJV. Note that for $\beta = 0$ rivals' profits are zero for they cannot benefit from spillovers from the cooperating firms which invest $x^*_{hj}(0) = V/4$ into R&D to win the contest prize with certainty, leading to overall profits of $\pi^*_{hj}(0) = V/2$ for the RJV hierarchy. On the other extreme, if $\beta = 1$, R&D efforts are public goods and the cooperating firms will not invest at all, $x^*_{hj}(0) = 0$, yielding identical profits of V/n for all firms in the industry.

4. The optimal design of a RJV and its stability

In Section 3 we considered three different forms of RJVs with different degrees of cooperation. In the following we will use these results and compare how R&D efforts, effective R&D levels, winning probabilities and profits change when various RJVs form. As we will see, the RJV hierarchy is superior to all the other forms of cooperations. This result then leads to two further questions answered in this section: First, is a two-firm RJV hierarchy actually stable in the sense that other non-cooperating firms have no incentive to form a competing RJV? Second, is it actually necessary to have two firms organized as RJV hierarchy or can a single firm already accomplish this?

We start with a comparison of the different forms of cooperation in the patent contest. Using the notation and the characterizations of the various RJV forms, we can establish the following result:

Proposition 8. *In general, the RJV hierarchy is the form of cooperation that leads to highest profits for the cooperating firms,*[19]

$$\Pi^*_{hj}(0) > \Pi^*_{cj} > \Pi^*_{kj} > \Pi^*_{kr} > \Pi^* > \Pi^*_{cr} > \Pi^*_{hr}(0).$$

In particular, in the RJV hierarchy R&D efforts are highest [20]

$$x^*_{hj}(0) > x^*_{kr} > x^*_{cj} > x^* > x^*_{cr} > x^*_{kj} > x^*_{hr}(0)$$

as well as the winning probabilities of the cooperating firms:[21]

$$p^*_{hj}(0) > p^*_{cj} > p^*_{kr} > p^* > p^*_{cr} > p^*_{kj} > p^*_{hr}(0)$$

Concerning effective R&D levels we have [22]

$$X^*_{kr} > X^*_{hj}(0) > X^*_{cj} \gtrless X^*_{hr}(0) \gtrless X^* \gtrless X^*_{cr} > X^*_{kj}.$$

Proof. See Appendix. □

We nicely see how the intensity of cooperation between the two firms in the RJV increases their profitability: Starting from the equilibrium under non-cooperation, RJV competition where firms only benefit from research sharing leads to lower R&D efforts and winning probabilities for the cooperating firms due to the cooperation dilemma effect. As a response, rival firms' R&D efforts and winning probabilities increase relative to a situation without cooperation. If, however, the cooperating partner mutually choose their R&D efforts, behavior switches in the sense that now their R&D efforts are higher than in the pure competition scenario so that their rival firms' R&D efforts are lower. This improves the winning probability of both firms under RJV cartelization but reduces the ones of their rivals. Under RJV hierarchy then both firms can even due better. The fact, that their effective R&D levels are not the highest in this scenario then results from the non-active behavior of their rivals.

Given this superiority of a RJV hierarchy, we will now consider the stability of this two-firm RJV. While endogenizing the coalition formation process is beyond the scope the present paper, we concentrate on two stability criteria: First, whether two non-cooperating firms ever have an incentive to

[19] $\Pi^*_{hj}(0) > \Pi^*_{cj}$ holds whenever $\beta < 0.170\,82$ for $n = 3$, $\beta < 0.736\,44$ for $n = 4$ and for all $n \geq 5$ with $\beta \in [0, 1]$. $\Pi^*_{cj} > \Pi^*_{kj}$ holds whenever $n \geq 4$ for all $\beta \in [0, 1]$. And $\Pi^*_{kj} > \Pi^*_{kr}$ holds for $\beta < \tilde{\beta}(n)$, see Proposition 2.

[20] $x^*_{cj} > x^*$ is true for $n = 5$ and all $\beta > 0.268$, for $n = 6$ and all $\beta > 0.065$, and all $n \geq 7$ independent of $\beta \in [0, 1]$. And $x^* > x^*_{cr}$ is true for $n = 4$ and all $\beta < 0.2$, for $n = 5$ and all $\beta < 0.565$, and for all $n \geq 6$ independent of $\beta \in [0, 1]$.

[21] $p^*_{cj} > p^*_{kr}$ is true whenever $n \geq 5$ for all $\beta \in [0, 1]$.

[22] $X^*_{cj} \gtrless X^*_{hr}(0)$ is true whenever $\beta \lesseqgtr \beta(n)$. $X^*_{hr}(0) \gtrless X^*$ depends on whether $\beta \gtrless (1/-8n + 4n^2 + 4)(n^2 + 4 - 6n + n\sqrt{4n + n^2 - 4})$. And $X^* \gtrless X^*_{cr}$ is equilivalent to $\beta \lesseqgtr (1/16n - 14n^2 + 4n^3 - 6)(14n - 8n^2 + n^3 - 6 + n\sqrt{20n - 12n^2 + n^4 - 8})$.

form a competing RJV hierarchy given the existence of the initial RJV? Second, whether a single firm by strategically delegating its R&D activities to its R&D department can imitate a RJV hierarchy? The following proposition answers these questions:

Proposition 9. *In a contest model,*

1. *in which two firms form an RJV hierarchy, rival firms have no incentive to form a competing RJV.*
2. *strategic delegation within a single firm never excludes its rival firms from the contest and leads to less profits compared to the RJV hierarchy.*

Proof. See Appendix. □

Both results are remarkable: The first finding shows that the first-mover advantage of a RJV hierarchy leads to a strong competitive advantage: It not only excludes rival firms from the patent race but also excludes the formation of a second RJV. In this sense, forming a RJV hierarchy implies a sustainable competitive advantage in the industry. Moreover, the second finding strengthens the usefulness of strategic delegation in competition. In the standard literature on strategic delegation, see e.g. Fersthman and Judd (1987) and Sklivas (1987), it is assumed that the owner of a firm delegates market activities to *one* manager. To induce a more aggressive market behavior of the manager it is then optimal to compensate him not only according to profits but to put some positive weight, for example, on sales. In the context discussed here this implies that, if a firm delegates its R&D activities to one R&D department, the headquarters of this firm would not always subsidize their entire R&D costs. In particular, strategic budgeting in this context does not lead to a preemptive behavior so that two firms are necessary to accomplish this. The result also shows that, depending on the industry structure, two firms are necessary to achieve the strategic delegation effect. This implies a strong motive for mergers in those industries.

5. Conclusion and discussion

In this paper we apply a contest approach to analyze firms which compete in discovering and commercializing an innovation. The winner of the contest gets a valuable patent as a reward. We consider research spillovers during the innovation process from each innovative firm to each of its rivals.

The main conclusion of our analysis is that in addition to research sharing and joint profit maximization it is favorable for an RJV to establish joint headquarters that subsidize R&D costs of RJV member firms. In the optimum the headquarters should fully subsidize their members, leading to an extreme aggressive behavior in the contest. In fact, even if only two firms participate in such an RJV hierarchy, rival firms are driven out of the market.

This result, of course, relies crucially on our mode of competition between firms as well as on the innovation we studied. As mentioned before, modelling innovation as a patent contest introduces a strong competitive force into firms' interactions: either a firm wins the patent leading to a monopoly rent or it looses the contest without recovering its R&D investments. In an environment, in which the mode of competition is less aggressive – for example, under process innovation in a Cournot market – our result that a two firm RJV hierarchy drives all competitors out of the contest might not hold: rival firms still gain some benefits from their R&D investments since competition is not of an all-or-nothing nature.

But even in the context of a patent contest our result depends crucially on the innovation process studied. We assumed in this paper that (1) the probability of winning the contest is given by an extension of the standard "lottery" contest function, and (2) the research function is linear in R&D efforts. Using a more general contest success function of the form

$$p_i(x_1, \ldots, x_i, \ldots x_n) = \frac{X_i^r}{\sum_{j=1}^{n} X_j^r}$$

with a parameter $r > 0$ might lead to different results. Note, that the parameter r is important for the marginal impact of an increase in a contestant's effort. If, for example, r is close to zero, the probability of winning the contest is almost independent of firms' R&D efforts. Hence, subsidizing research by the headquarters of an RJV hierarchy is less beneficial, and, in turn, rival firms might still invest in R&D.

The same is true, if we modify the research cost function. If, for example, research costs are of the form

$$c(x_i) = x_i^{\alpha}$$

with $\alpha > 0$ sufficiently high, costs are flat near zero but extremely high for medium or high levels of research. In this case, the RJV hierarchy might not find it profitable to subsidize R&D to a point where all rival firms are driven out of the contest. On the other hand, its competitors might still invest in some R&D although the probability of winning is small.

Of course, it would be also of interest to analyze the advantage of an RJV hierarchy in the context of endogenous RJV formation where more than one RJV can be formed with an arbitrary number of members.[23] Although such an analysis is beyond the scope of this paper, one might expect that in response to an RJV hierarchy other firms also intend to cooperate to avoid preemption. Moreover, given multiple RJV hierarchies were formed in equilibrium, it would also be interesting to analyze how headquarters would then subsidize their R&D departments. One would expect that, if rivals did not stop doing research, headquarters would not fully subsidize R&D to reduce the intensity of competition.

Our result sheds some light on two initiatives by the German Research Foundation (DFG) and the German Ministry of Education and Research to promote "cutting edge" research at German universities. The Excellence Initiative, for example, tries to create outstanding conditions for young scientists by funding research schools. In the field of economics, the "Bonn Graduate School of Economics" and the school for "Empirical and Quantitative Methods in the Economics and Social Science" at the University of Mannheim are founded by this program. Of course, both schools are competing for young scientists. Several other economics departments are not founded by this program. Applying our model of RJV hierarchy to this example – the state as the headquarters subsidizes two competing economic departments by covering research activities – then leads to the prediction that all other universities in Germany have an incentive to reduce their efforts for attracting and promoting young scientists in the field of economics.[24] Of course, these two schools are well prepared for international competition. In fact, one aim of the Excellence Initiative was to strengthen international competitiveness of German universities. Similarly, the Transregio Program tries to foster excellent research within a network of more than two universities in Germany. In economics, such a Transregio collaboration research center is the Transregio "Governance and the Efficiency of Economic Systems" with members from the universities in Berlin, Bonn, Mannheim and Munich. In this case, the DFG as the headquarters subsidizes research of several scientists at these member departments who compete in the international market for excellent publications. Knowledge transfer between scientists in this program is provided by internal conferences that gives them a forum to present their work. As with the Excellence Initiative, the aim of funding is to strengthen competitiveness of German scientists in the international community, it might, however, discourage other scientists.

Our findings also shed new light on the literature discussing firms' incentives to merge. The classical literature on mergers, for example, Salant, Switzer, and Reynolds (1983) states that a merger of less than two-third of all firms in the industry would be unprofitable for the merging firms. This is also true for mergers in innovation contests although the results by Jost and van der Velden (2006) show that small synergies between the merging firms tremendously lower this upper bound. Different to our argumentation, however, the literature on mergers assumes that a merger is not more than just a change in the number of firms. For example, a merger of two firms in an industry consisting of n identical firms simply reduces the number of firms to $(n-1)$. But if a merger of two firms results in

[23] For the endogeneous formation of RJVs see, for example, Greenlee (2005), Greenlee and Cassiman (1999), Morasch (2000), or Yi and Shin (2000) in the context of process innovation and Jost (2010) for a patent contest.

[24] Of course, this prediction implicitly requires universities to be profit maximizers.

$(n-2)$ old firms and one new firm that differs somehow from the old ones, our analysis leads to the conclusion that profits and R&D efforts of all firms in the market strongly depend on the way how the merged firm is organized. In particular, if the newly merged firm keeps the "old" R&D departments as fully functional entities and the firm's headquarters strategically delegate R&D budgets to these subunits, a merger is extremely favorable for the merged firm: A complete subsidization of the R&D costs of its departments drives all rival firms out of the contest.

Appendix A. Proofs

Proof of Proposition 1: Consider the optimization problem of firm i with respect to its R&D efforts x_i

$$\max_{x_i} \quad \frac{x_i + \beta \sum_{j \neq i} x_j}{\sum_{j=1}^{n} x_j + (n-1)\beta \sum_{j=1}^{n} x_j} V - x_i.$$

Let $(x_1^*, \ldots, x_i^*, \ldots, x_n^*)$ be a Nash-equilibriuim. Then the optimal x_i^* satisfies the following first-order condition

$$\frac{\sum_{j=1}^{n} \overset{*}{x}_j + (n-1)\beta \sum_{j=1}^{n} \overset{*}{x}_j - \left((1-\beta)\overset{*}{x}_i + \beta \sum_{j=1}^{n} \overset{*}{x}_j \right) (1 + (n-1)\beta)}{\left(\sum_{j=1}^{n} \overset{*}{x}_j + (n-1)\beta \sum_{j=1}^{n} \overset{*}{x}_j \right)^2} V = 1$$

Consider a symmetric Nash-equilibrium with $x^* = x_1^* = \ldots = x_n^*$. Then x^* satisfies the condition

$$\frac{nx^* + (n-1)\beta nx^* - x^*(1 + \beta(n-1))(1 + (n-1)\beta)}{(nx^* + (n-1)\beta nx^*)^2} V = 1$$

and we obtain the following equilibrium condition for R&D efforts:

$$x^* = V(1-\beta)\frac{n-1}{\left(1+\beta(n-1)\right) n^2}.$$

Due to the symmetry, $p_1^* = \ldots = p_n^* = 1/n$ and effective R&D efforts are

$$X_i^* = X^* = x^* + \beta(n-1)x^* = V(1-\beta)\frac{n-1}{n^2}.$$

By substituting the equilibrium R&D efforts in the profit function, profits in equilibrium are given by

$$\begin{aligned} \Pi^* &= \frac{1}{n}V - x^* \\ &= \frac{1}{n}V - V(1-\beta)\frac{n-1}{(\beta n - \beta + 1)n^2} \\ &= V\frac{\beta(n^2-1)+1}{(\beta(n-1)+1)n^2}. \end{aligned}$$

Proof of Proposition 2:

Consider the first-order condition for profit maximization of a firm participating in the RJV. For $i = 1, 2$ these are given by

$$\frac{\sum X_j - X_i(2+(n-2)\beta)}{\left(\sum X_j\right)^2}V = 1,\text{hence,}$$

$$\frac{\left(\sum X_j\right)^2}{V} - \sum X_j = -X_i(2+(n-2)\beta)$$

The first order conditions for the rival firms $i = 3, \ldots, n$ are given by

$$\frac{\sum X_j - X_i(1+(n-1)\beta)}{\left(\sum X_j\right)^2}V \quad = 1,\text{hence,}$$

$$\frac{\left(\sum X_j\right)^2}{V} - \sum X_j \qquad = -X_i(1+(n-1)\beta)$$

Because of the symmetry of firms $i = 1, 2$ and of firms $i = 3, \ldots, n$, we look for a quasi-symmetric equilibrium, that is an equilibrium in which $x_1^* = x_2^* = x_{kj}^*$ and $x_3^* = \ldots = x_n^* = x_{kr}^*$. Then the effective R&D efforts are

$$X_{kj}^* = 2x_{kj}^* + (n-2)\beta x_{kr}^* \text{and} X_{kr}^* = (1+(n-3)\beta)x_{kr}^* + 2\beta x_{kj}^*$$

and the two first order conditions lead to

$$\begin{aligned}
\left((1+(n-3)\beta)x_{kr}^* + 2\beta x_{kj}^*\right) \quad & (1+(n-1)\beta) \\
& = \left(2x_{kj}^* + (n-2)\beta x_{kr}^*\right)(2+(n-2)\beta), \text{ i.e.} \\
x_{kj}^* \quad & = \frac{(1+\beta)}{2((n-1)\beta+2)}x_{kr}^*.
\end{aligned}$$

Note that $X_{kj}^* < X_{kr}^*$ iff $2x_{kj}^* < x_{kr}^*$ and

$$2x_{kj}^* < x_{kr}^* \text{iff} \frac{(1+\beta)}{((n-1)\beta+2)} < 1 \text{ iff} \, 0 < (n-2)\beta + 1$$

which is always true. Hence, $x_{kj}^* < x_{kr}^*$ and $X_{kj}^* < X_{kr}^*$. The last property also implies that $p_{kj}^* < p_{kr}^*$. Substituting x_{kj}^* into total effective R&D efforts gives

$$\begin{aligned}
\sum_j \overset{*}{X} & = X^* + (n-1)\beta X^* + (1-\beta)2x_{kj}^* \\
& = x_{kj}^*(4+2\beta(n-2)) + x_{kr}^*(n-2)(1+(n-1)\beta). \\
& = x_{kj}^* \frac{2((n-2)(n^2-2n+2)\beta^2 + \beta(3n^2+6-8n) + 2n-2)}{1+\beta}.
\end{aligned}$$

Let

$$A := \frac{2((n-2)(n^2-2n+2)\beta^2 + \beta(3n^2+6-8n) + 2n-2)}{1+\beta}.$$

Then the first order condition for $i = 3, \ldots, n$ gives

$$\frac{A^2(x_{kj}^*)^2}{V} - Ax_{kj}^* = Bx_{kj}^*$$

with

$$B := -\frac{2(1+(n-1)\beta)}{(1+\beta)}(1+(n-2)\beta)(2+(n-2)\beta),$$

hence,

$$\begin{aligned} x^*_{kj} &= \frac{(A+B)}{A^2}V \\ &= \frac{(n-2)(1-\beta)(2+\beta n-\beta)(1+\beta)}{2(\beta n+1-2\beta)(\beta n^2+2n-2\beta n-2+2\beta)^2}V. \end{aligned}$$

Given the equilibrium R&D efforts x^*_{kj} of the two firms participating in the RJV, rival firms' R&D efforts are determined by

$$\begin{aligned} x^*_{kr} &= \frac{2((n-1)\beta+2)}{\left(1+\beta\right)}x^*_{kj} \\ &= \frac{(2+\beta n-\beta)^2(n-2)\left(1-\beta\right)}{\left(\beta n+1-2\beta\right)\left(n^2\beta+2n-2\beta n-2+2\beta\right)^2}V. \end{aligned}$$

Effective R&D efforts then read as

$$\begin{aligned} X^*_{kj} &= Cx^*_{kj} \text{ with } C = \frac{2}{1+\beta}\left((1+\beta)+\beta(n-2)\left((n-1)\beta+2\right)\right) \\ &= \frac{(n-2)(1-\beta)\left(\beta n+2-\beta\right)\left((1+\beta)+\beta(n-2)((n-1)\beta+2)\right)}{\left(\beta n+1-2\beta\right)\left(\beta n^2+2n-2\beta n-2+2\beta\right)^2}V, \\ X^*_{kr} &= Dx^*_{kr} \text{ with } D = \left((n-2)\beta+1\right)\frac{\left((n-2)\beta+2\right)}{\left((n-1)\beta+2\right)} \\ &= \frac{(n-2)(1-\beta)\left((n-1)\beta+2\right)\left((n-2)\beta+2\right)}{\left(\beta n^2+2n-2\beta n-2+2\beta\right)^2}V. \end{aligned}$$

In equilibrium, winning probabilities are given by

$$\begin{aligned} p^*_{kj} &= \frac{C}{A} = \frac{n\beta-\beta+1}{2n+2\beta-2n\beta+n^2\beta-2} \\ p^*_{kr} &= \frac{D}{A}\frac{2\left(2+n\beta-\beta\right)}{1+\beta} = \frac{n\beta-2\beta+2}{2n+2\beta-2n\beta+n^2\beta-2} \end{aligned}$$

and equilibrium profits read as

$$\begin{aligned} \Pi^*_{kj} &= p^*_{kj}V - x^*_{kj} = \frac{V}{A^2}\left(AC-A-B\right), \\ \Pi^*_{kr} &= p^*_{kr}V - x^*_{kr} = \frac{2((n-1)\beta+2)}{(1+\beta)}\frac{V}{A^2}\left(AD-A-B\right). \end{aligned}$$

Substituting A, B, C and D we find

$$\frac{1}{V}(\Pi^*_{kj}-\Pi^*_{kr}) = \frac{1}{2}(1-\beta)\frac{((n^2-7n+6)\beta-(n^2-n-2)\beta^2-2(4-n))}{(\beta n^2-2\beta n+2n+2\beta-2)^2(\beta n+1-2\beta)}$$

Of course, if $\beta = 1$ and there are complete spillovers in the industry, all firms will earn the same profits in the R&D contest. In this case there is no difference between firms inside or outside the RJV. For $\beta < 1$ we have

$$((n^2 - n - 2)\beta^2 - (n^2 - 7n + 6)\beta + 2(4 - n)) < 0 \,\text{iff}$$
$$\tilde{\beta}(n) := \frac{n^2 - 7n + 6 + \sqrt{(n^4 - 6n^3 + 21n^2 - 68n + 100)}}{2(n^2 - n - 2)} > \beta$$

Since $\tilde{\beta}(n)$ is strictly increasing in n with $\tilde{\beta}(n) \to 1$ for $n \to \infty$ the proposition follows. Moreover, $\tilde{\beta}(n) \leq 0$ for $n = 3, 4$. **Proof of Proposition 3:**

Using the equilibrium R&D efforts in Proposition 1 and 2, we have

$$x_{kr}^* > x^* \text{ iff } \frac{2((n-1)\beta + 2)}{(1+\beta)} \frac{(A+B)}{A^2} > \frac{(1-\beta)(n-1)}{(1+\beta(n-1))n^2}$$

which is equivalent to $(2 + \beta n - \beta)^2 (n-2)(1 + \beta(n-1))n^2 - (n-1)(\beta n + 1 - 2\beta)(n^2\beta + 2n - 2\beta n - 2 + 2\beta)^2 > 0$ and satisfied for all $n \geq 3$ and $\beta \in [0, 1)$. Moreover,

$$x_{kj}^* < x^* \text{ iff } \frac{(A+B)}{A^2} < \frac{(1-\beta)(n-1)}{(1+\beta(n-1))n^2}$$

which is equivalent to $(2 + \beta n - \beta)(n-2)(1+\beta)(1 + \beta(n-1))n^2 - 2(n-1)(\beta n + 1 - 2\beta)(n^2\beta + 2n - 2\beta n - 2 + 2\beta)^2 < 0$ and satisfied for all $n \geq 3$ and $\beta \in [0, 1)$.

Concerning winning probabilities, Proposition 1 and 2 yield

$$p_{kr}^* = \frac{n\beta - 2\beta + 2}{2n + 2\beta - 2n\beta + n^2\beta - 2} > \frac{1}{n}$$

iff $2 > 2\beta$ and

$$p_{kj}^* = \frac{n\beta - \beta + 1}{2n + 2\beta - 2n\beta + n^2\beta - 2} < \frac{1}{n}$$

iff $(\beta - 1)(n - 2) < 0$ which are both always satisfied.

Finally, if we compare the profits of a firm participating in an RJV before and after the cooperation, the results of Proposition 1 and 2 gives

$$\Pi_{kj}^* > \Pi^* \text{iff} \frac{V}{A^2}(CA - A - B) > V \frac{\beta n^2 + 1 - \beta}{(\beta(n-1)+1)n^2}$$
$$\text{iff} \frac{2(1-\beta)(\beta n - 2\beta + 1)}{(1+\beta)^2} E < 0$$

with $E = n^5\beta^2(\beta - 1) - n^4\beta(8\beta - 3)(\beta - 1) + n^3(-2 + 25\beta^3 - 48\beta^2 + 21\beta) - 2n^2(\beta - 1)(21\beta^2 - 27\beta + 4) + 8n(5\beta - 2)(\beta - 1)^2 - 8(2\beta - 1)(\beta - 1)^2$. Calculation then shows that $E < 0$ for all $n \geq 3$ and $\beta \in [0, 1)$. Moreover,

$$\Pi_{kr}^* > \Pi^* \text{iff} \frac{V}{A^2} \frac{2(2 + n\beta - \beta)}{1+\beta}(DA - A - B) > V \frac{\beta n^2 + 1 - \beta}{(\beta(n-1)+1)n^2}$$
$$\text{iff} (1-\beta)F < 0$$

with $F = -3n^4\beta^3 + n^4\beta^2 + 13n^3\beta^3 - 17n^3\beta^2 + 2n^3\beta - 22n^2\beta^3 + 46n^2\beta^2 - 24n^2\beta + 20n\beta^3 - 48n\beta^2 + 36n\beta - 8n - 8\beta^3 + 20\beta^2 - 16\beta + 4$. Calculation then shows that $F < 0$ for all $n > 3$ and $\beta \in [0, 1)$. **Proof of Proposition 4:**

bye Consider the first-order conditions for profit maximization in the RJV with respect to x_1, respectively x_2,:

$$\frac{\sum X_j - X_{12}(2+(n-2)\beta)}{\left(\sum X_j\right)^2} 2V = 1,$$

Both first-order conditions are identical, hence $x_1^* = x_2^* = x_{cj}^*$. Consider second the first order condition for a rival firm

$$\frac{\sum X_j - X_i(1+(n-1)\beta)}{\left(\sum X_j\right)^2} V = 1.$$

Since all rival firms are identical, a quasi-symmetric equilibrium is given by $x_3^* = \ldots = x_n^* = x_{cj}^*$. Then total R&D efforts are

$$X = 2x_{cj}^* + (n-2)x_{cr}^*$$

and total research associated with R&D efforts can be written as

$$\sum X_j = (2x_{cj}^* + (n-2)x_{cr}^*)(1+(n-1)\beta) + 2(1-\beta)x_{cj}^*.$$

Substituting these expressions into the first-order conditions and rearranging these then leads to the following results:

$$x_{cj}^* = \frac{1}{4}\frac{(n-2)(1-\beta)(2-\beta+n\beta)(n^2\beta+n-4n\beta-1+5\beta)}{(1-2\beta+n\beta)(n^2\beta+2n-3n\beta-3+3\beta)^2}V$$

$$x_{cr}^* = \frac{1}{2}\frac{(n-2)(1-\beta)(2-\beta+\beta n)^2}{(1-2\beta+\beta n)(\beta n^2+2n-3\beta n-3+3\beta)^2}V$$

Then

$$\begin{array}{ll} x_{cj}^* > x_{cr}^* \text{ iff} & (n^2\beta+n-4n\beta-1+5\beta) > 2(2-\beta+\beta n) \\ \text{iff} & n((n-6)\beta+1)-5+7\beta > 0 \text{ iff } n \geq 5. \end{array}$$

Effective R&D efforts in equilibrium then are

$$\begin{aligned} X_{cj}^* &= 2x_{cj}^* + \beta(n-2)x_{cr}^* \\ &= \frac{1}{2}\frac{(1-\beta^2)(n\beta-\beta+2)(n^2-3n+2)}{(2n+3\beta-3n\beta+n^2\beta-3)^2}V \\ X_{cr}^* &= x_{cr}^* + 2\beta x_{cj}^* + \beta(n-3)x_{cr}^* \\ &= \frac{(1-\beta)(n-2)(n^2\beta^2-3n\beta^2+3n\beta+2\beta^2-5\beta+2)}{(2n+3\beta-3n\beta+n^2\beta-3)^2}V \end{aligned}$$

and $X_{cj}^* > X_{cr}^*$ iff $(1-\beta)(n\beta-\beta+2)(n^2-5n+6) > 0$ which is always true. This, of course, implies that $p_{cj}^* > p_{cr}^*$. To compare the equilibrium profits of the RJV and rival firms in this situation, the difference can be obtained by

$$\begin{aligned} \Pi_{cj}^* - \Pi_{cr}^* &= \frac{X_{12} - X_i}{\sum X_j}V - (x_{cj}^* - x_{cr}^*) \\ &= \frac{(1-\beta)}{4}\frac{E}{(1-2\beta+\beta n)(\beta n^2+2n-3\beta n-3+3\beta)^2}V. \end{aligned}$$

with $E = n\beta^2(21n-33) + \beta^2 n^3(n-7) + \beta n^2(3n-14) + \beta(23n-16) + 2n(n-2) + 22\beta^2 - 2$. Calculation then shows that $E > 0$ for all $n \geq 3$ and $\beta \in [0, 1]$. **Proof of Proposition 5:**

We first compare R&D efforts in both scenarios. Using the results from Proposition 1 and 4 we have $x_{cj}^* > x^*$ whenever $(n-2)(2-\beta+n\beta)(n^2\beta+n-4n\beta-1+5\beta)(1+\beta(n-1))\ \ n^2 - 4(1-2\beta+n\beta)(n^2\beta+2n-3n\beta-3+3\beta)^2(n-1)$ is positive. This is true for $n \geq 6$ and all $\beta \in [0, 1)$. The term is negative for $n \leq 5$ and β sufficiently small. Moreover, $x_{cr}^* < x^*$ whenever $2(n-1)(1-2\beta+\beta n)\ \ (\beta n^2+2n-3\beta n-3+3\beta)^2 - (1+\beta(n-1))n^2(n-2)(2-\beta+\beta n)^2 < 0$. This is true if β is greater than critical value $\bar{\beta}(n)$. With respect to effective R&D effort we find $X_{cj}^* > X^*$ since $-n^3\beta^2+n^3\beta+6n^2\beta^2-6n^2\beta+2n^2-10n\beta^2+16n\beta-6n+6\beta^2-12\beta+6>0$ for $n \geq 3$ and all $\beta \in [0, 1)$. And, $X_{cr}^* > X^*$, iff $2n^3\beta^2-n^3\beta-7n^2\beta^2+8n^2\beta-2n^2+8n\beta^2-14n\beta+6n-3\beta^2+6\beta-3>0$ which is true if β is greater than critical value $\bar{\bar{\beta}}(n)$. For the winning probabilities, calculation shows

$$nX_{cr}^* < \sum_j \overset{*}{X} = \frac{(1-\beta)(n-2)(n\beta-\beta+2)}{2n+3\beta-3n\beta+n^2\beta-3}$$

iff $(\beta-1)(n-3)(n\beta-\beta+2)<0$, which is always true. Hence $p_{cr}^* < p^*$. And,

$$nX_{cj}^* > \sum_j \overset{*}{X}$$

iff $(1-\beta)(n-2)^2(n-3)>0$, which is always true, hence $p_{cj}^* > p^*$. Finally, using the notation

$$\begin{aligned}
F &:= (n-2)(1-\beta)(2-\beta+n\beta)(n^2\beta+n-4n\beta-1+5\beta),\\
G &:= (1-2\beta+n\beta)(n^2\beta+2n-3n\beta-3+3\beta)^2,\\
H &:= (2-\beta+\beta n)^2(1-\beta)(n-2).
\end{aligned}$$

so that equilibrium R&D efforts can be written as

$$x_{cj}^* = \frac{1}{4}\frac{F}{G}V \text{ and } x_{cr}^* = \frac{1}{2}\frac{H}{G}V,$$

equilibrium profits can be calculated as

$$\begin{aligned}
\Pi_{cj}^* &= \left(\frac{F+(n-2)\beta H}{(F+H(n-2))((n-1)\beta+1)+(1-\beta)F} - \frac{1}{4}\frac{F}{G}\right)V \text{ and}\\
\Pi_{cr}^* &= \left(\frac{H(1+(n-3)\beta)+\beta F}{(F+H(n-2))\big((n-1)\beta+1\big)+(1-\beta)F} - \frac{1}{2}\frac{H}{G}\right)V.
\end{aligned}$$

Comparing these profits with the equilibrium profits in R&D competition, see Proposition 1, we find

$$\begin{aligned}
\frac{\Pi_{cj}^* - \Pi^*}{V} &= \left(\frac{F+(n-2)\beta H}{(F+H(n-2))\big((n-1)\beta+1\big)+(1-\beta)F} - \frac{1}{4}\frac{F}{G}\right)\\
&\quad - \frac{\beta n^2+1-\beta}{\big(\beta n-\beta+1\big)\,n^2}\\
&= \frac{1}{4}\frac{(1-\beta)I}{\big(1-2\beta+n\beta\big)\big(n^2\beta+2n-3n\beta-3+3\beta\big)^2\big(1-\beta+n\beta\big)\,n^2}
\end{aligned}$$

with $I = -36 + 130n^2\beta - 252n\beta + 48n + 144\beta - 14n^2 + 70n^4\beta^2 - 60n^4\beta^3 + 142n^2\beta^3 - 180n\beta^3 + 72\beta^3 - 3n^3\beta^3 + 384n\beta^2 - 254n^2\beta^2 - 180\beta^2 + 9n^3\beta + 2n^3\beta^2 - 24n^4\beta - 10n^6\beta^3 + 4n^6\beta^2 - 4n^3 + 2n^4 + 38n^5\beta^3 + n^7\beta^3 + 5n^5\beta - 30n^5\beta^2$. Calculation then shows that $I > 0$ for all $n \geq 3$ and $\beta \in [0, 1)$. And for rival firms,

$$\begin{aligned}\frac{\Pi^*_{cr} - \Pi^*}{V} &= \left(\frac{H(1+(n-3)\beta)+\beta F}{(F+H(n-2))\left((n-1)\beta+1\right)+(1-\beta)F} - \frac{1}{2}\frac{H}{G}\right) \\ &\quad - \frac{\beta n^2 + 1 - \beta}{\left(\beta n - \beta + 1\right)n^2} \\ &= -\frac{1}{2}\frac{(1-\beta)J}{\left(1-2\beta+n\beta\right)\left(n^2\beta+2n-3n\beta-3+3\beta\right)^2\left(1-\beta+n\beta\right)n^2}\end{aligned}$$

with $J = 90n\beta^3 + n^6\beta^3 - 36\beta^3 - 82n^2\beta^3 - 5n^5\beta^3 + 3n^4\beta^3 + 29n^3\beta^3 - 6n^4\beta^2 + 146n^2\beta^2 + 90\beta^2 + 3n^5\beta^2 - 192n\beta^2 - 37n^3\beta^2 - 72\beta + 126n\beta + 2n^4\beta - 72n^2\beta + 8n^3\beta + 6n^2 - 24n + 18$. Calculation shows that $J > 0$ for all $\beta \in [0, 1]$. **Proof of Proposition 6:**

Rewrite the first order conditions for profit maximization of a non-cooperating firm $i = 3, \ldots, n$ as

$$\sum X_j - X_i(1+(n-1)\beta) = \frac{\left(\sum X_j\right)^2}{V}$$

and of a cooperating firm $i = 1, 2$ as

$$\left(\sum X_j - X_i(2+(n-2)\beta)\right)\frac{1}{c} = \frac{\left(\sum X_j\right)^2}{V}.$$

Since all rival firms as well as the two firms in the RJV are identical, a quasi-symmetric equilibrium is given by $x_3^* = \ldots = x_n^* = x_{hr}^*$ and $x_1^* = x_2^* = x_{hj}^*$. Then effective R&D efforts are

$$\begin{aligned}&X_{hj}^* = \beta(n-2)x_{hr}^* + 2x_{hj}^* \text{ and} \\ &X_{hr}^* = \left(1+\beta(n-3)\right)x_{hr}^* + 2\beta x_{hj}^*\end{aligned}$$

and total effective R&D efforts are

$$\begin{aligned}\sum \overset{*}{\underset{j}{X}} &= (1+(n-1)\beta)X^* + (1-\beta)2x_{hj}^* \\ &= (1+(n-1)\beta)(n-2)x_{hr}^* + 2(2+(n-2)\beta)x_{hj}^*.\end{aligned}$$

Using these expressions, the left hand sides of the first order conditions can be calculated as

$$\begin{aligned}&\sum \overset{*}{\underset{j}{X}} - X_{hr}^*(1+(n-1)\beta) = ax_{hj}^* + bx_{hr}^* \\ &\left(\sum \overset{*}{\underset{j}{X}} - X_{hj}^*(2+(n-2)\beta)\right)\frac{1}{c} = \frac{d}{c}x_{hr}^*\end{aligned}$$

with

$$\begin{aligned}a &= 2\left(2+(n-1)\beta\right)(1-\beta) \\ b &= (n-3)(1-\beta)\left(1+\beta(n-1)\right) \\ d &= \left(1+\beta(n-2)\right)(1-\beta)(n-2).\end{aligned}$$

Hence,

$$ax_{hj}^* + bx_{hr}^* = \frac{d}{c}x_{hr}^*\text{, i.e.,}$$
$$x_{hj}^* = x_{hr}^* \frac{d-bc}{ac}.$$

Note that

$$x_{hj}^* < x_{hr}^* \text{ iff}$$
$$K < c \,\text{aaaaaa with}\, K := \frac{(n-2)\left(1+n\beta-2\beta\right)}{\left(n+n^2\beta-2n\beta+1+\beta\right)}.$$

Then $(\partial/\partial n)K > 0$, $(\partial/\partial\beta)K > 0$ and $K = 1/4$ for $n = 3$ but $K < 1$ for all n and $\beta \in [0, 1)$. Inserting x_{hj}^* into total effective R&D efforts gives

$$\begin{aligned}\sum_j \overset{*}{X} &= \left((1+(n-1)\beta)(n-2)x_{hr}^* + 2(2+(n-2)\beta)x_{hj}^*\right) \\ &= \left(2(2+(n-2)\beta)\frac{d-bc}{ac} + (1+(n-1)\beta)(n-2)\right)x_{hr}^*.\end{aligned}$$

Then the first order condition for $i = 3, \ldots, n$ gives

$$\frac{d}{c}x_{hr}^* = \frac{1}{V}\left(2\left(2+(n-2)\beta\right)\frac{d-bc}{ac} + (1+(n-1)\beta)(n-2)\right)^2\left(x_{hr}^*\right)^2$$

hence,

$$x_{hr}^*(c) = Vc\frac{\left(2-\beta+n\beta\right)M}{L^2\left(1-2\beta+n\beta\right)}\text{, and}$$
$$x_{hj}^*(c) = \frac{V}{2}\frac{\left((n-2)\left(1-2\beta+n\beta\right) - c(n-3)\left(1+n\beta-\beta\right)\right)M}{L^2\left(1-2\beta+n\beta\right)}.$$

with

$$L := \left(n^2\beta + 2cn\beta - 4n\beta - 2c\beta + 4\beta + 2n - 4 + 2c\right)$$
$$M := \left(2-\beta+n\beta\right)(1-\beta)(n-2)$$

Calculation then shows that

$$\begin{aligned}\frac{\partial}{\partial c}x_{hr}^* &= \frac{\left(2-\beta+n\beta\right)M\left((n-2)\left(2+n\beta-2\beta\right) - 2c\left(1+n\beta-\beta\right)\right)}{L^3\left(1-2\beta+n\beta\right)} \\ &> 0 \\ \frac{\partial}{\partial c}x_{hj}^* &= -\frac{M\left(n\beta-\beta+1\right)\left((n-2)\left(n^2\beta-n\beta+2n-2-2\beta\right) - 2c(n-3)\left(n\beta-\beta+1\right)\right)}{2L^3\left(n\beta-2\beta+1\right)} \\ &< 0\end{aligned}$$

Profits can then be written as

$$\begin{aligned}\Pi^*_{hj}(c) &= \frac{X^*_{hj}}{\sum_j \overset{*}{X}} V - cx^*_{hj} \\ &= V\frac{c\left(n\beta-\beta-n+3\right)+(n-2)}{L} \\ &- \frac{Vc}{2}\frac{\left((n-2)\left(1-2\beta+n\beta\right)-c(n-3)\left(1+n\beta-\beta\right)\right)M}{L^2\left(1-2\beta+n\beta\right)} \\ \Pi^*_{hr}(c) &= \frac{X^*_{hr}}{\sum_j \overset{*}{X}} V - x^*_{hr} \\ &= V\frac{n\beta-2\beta+2c}{L} - Vc\frac{\left(2-\beta+n\beta\right)M}{L^2\left(1-2\beta+n\beta\right)}\end{aligned}$$

Then

$$\frac{\partial}{\partial c}\Pi^*_{hj} = \frac{V(n-2)M}{2L^3\left(1-2\beta+n\beta\right)}N < 0$$

since $N := 2n^3\beta^2c - 3n^3\beta^2 - 14c\beta^2n^2 + 18n^2\beta^2 + 6cn^2\beta - 9n^2\beta - 36n\beta^2 + 28c\beta^2n + 36n\beta - 30cn\beta + 4cn - 6n + 24\beta^2 - 16c\beta^2 - 36\beta + 30c\beta + 12 - 14c$ is increasing in c and $N = -n^3\beta^2 + 4n^2\beta^2 - 3n^2\beta - 8n\beta^2 + 6n\beta - 2n + 8\beta^2 - 6\beta - 2 < 0$ for all $n \geq 3$, $\beta \in [0, 1]$ and $c = 1$ and

$$\frac{\partial}{\partial c}\Pi^*_{hr} = \frac{VM}{L^3\left(1-2\beta+n\beta\right)}O > 0$$

since $O := n^3\beta^2 - 7\beta^2n^2 + 6c\beta^2n^2 + 2n^2\beta - 16c\beta^2n + 16\beta^2n + 14cn\beta - 10n\beta + 10c\beta^2 - 12\beta^2 + 12\beta - 18c\beta + 8c$ is increasing in c and $O = n^3\beta^2 - 7\beta^2n^2 + 2n^2\beta + 16\beta^2n - 10n\beta - 12\beta^2 + 12\beta > 0$ for all $n \geq 3$, $\beta \in [0, 1]$ and $c = 0$. Note that for $c = 1/2$ equilibrium R&D efforts simplify to

$$\begin{aligned} x^*_{hr}\left(\frac{1}{2}\right) &= \frac{1}{2}V\frac{\left(2-\beta+n\beta\right)^2(1-\beta)(n-2)}{\left(1-2\beta+n\beta\right)\left(n^2\beta+2n-3n\beta-3+3\beta\right)^2} = x^*_{cr}, \text{ and} \\ x^*_{hj}\left(\frac{1}{2}\right) &= \frac{1}{4}V\frac{\left(n^2\beta+n-4n\beta-1+5\beta\right)\left(2-\beta+n\beta\right)(1-\beta)(n-2)}{\left(1-2\beta+n\beta\right)\left(n^2\beta+2n-3n\beta-3+3\beta\right)^2} = x^*_{cj}\end{aligned}$$

and that for $c = 1$ equilibrium R&D efforts are

$$\begin{aligned} x^*_{hr}(1) &= \frac{\left(2+\beta n-\beta\right)^2(n-2)(1-\beta)}{\left(\beta n+1-2\beta\right)\left(n^2\beta+2n-2\beta n-2+2\beta\right)^2}V = x^*_{kr}, \text{ and} \\ x^*_{hj}(1) &= \frac{V}{2}\frac{(1+\beta)\left(2-\beta+n\beta\right)(1-\beta)(n-2)}{\left(n^2\beta-2(1-\beta)(n-1)\right)^2\left(1-2\beta+n\beta\right)} = x^*_{kj}.\end{aligned}$$

Proof of Proposition 7:

Given its members' profits $\Pi^*_{hj}(c)$ the headquarters choose c to maximize overall RJV's profits

$$\pi^*_{hj}(c) = 2\Pi^*_{hj}(c) - 2(1-c)x^*_{hj}(c).$$

Using the derivations above RJV's profit reads as

$$\pi^*_{hj}(c) = 2V\frac{c\left(n\beta - \beta - n + 3\right) + (n-2)}{L}$$
$$-V\frac{\left((n-2)\left(1-2\beta+n\beta\right) - c(n-3)\left(1+n\beta-\beta\right)\right)M}{L^2\left(1-2\beta+n\beta\right)}$$

with

$$L := (n^2\beta + 2cn\beta - 4n\beta - 2c\beta + 4\beta + 2n - 4 + 2c)$$
$$M := (2-\beta+n\beta)(1-\beta)(n-2)$$

Marginal RJV's profits with respect to subsidization c then are

$$\frac{\partial}{\partial c}\pi^*_{hj}(c) = -VM\frac{P}{L^3\left(1-2\beta+n\beta\right)} < 0$$

since $P := n^4\beta^2 + 6n^3\beta^2 c - 12n^3\beta^2 - 30n^2\beta^2 c + 45n^2\beta^2 + 46cn\beta^2 - 68n\beta^2 + 36\beta^2 - 22c\beta^2 + 3n^3\beta - 25n^2\beta + 12cn^2\beta + 62n\beta - 44cn\beta - 48\beta + 36c\beta + 2n^2 + 6cn - 10n + 12 - 14c$ is increasing in c and $P = (n-2)(n^3\beta^2 - 10n^2\beta^2 + 3n^2\beta + 2n + 25n\beta^2 - 19n\beta - 18\beta^2 - 6 + 24\beta)$ is positive for $c = 0$ and all $n \geq 3$, $\beta \in [0, 1]$. Hence, it is optimal to set $c^* = 0$. This implies R&D efforts

$$x^*_{hr}(0) = 0$$
$$x^*_{hj}(0) = \frac{V}{2}(1-\beta)\frac{(n\beta-\beta+2)}{(n\beta-2\beta+2)^2}$$

and profits

$$\pi^*_{hj}(0) = V\frac{n\beta - \beta - \beta^2 + n\beta^2 + 2}{\left(n\beta-2\beta+2\right)^2}$$

$$\Pi^*_{hr}(0) = V\frac{\beta}{(n\beta-2\beta+2)}$$

Simple calculation then shows that

$$\frac{\partial}{\partial n}x^*_{hj}(0) < 0 \text{ and } \frac{\partial}{\partial \beta}x^*_{hj}(0) < 0$$
$$\frac{\partial}{\partial n}\pi^*_{hj}(0) < 0 \text{ and } \frac{\partial}{\partial \beta}\pi^*_{hj}(0) < 0 \text{for } n > 4,\ \beta < \frac{1}{3} \text{ for } n = 4$$
$$\frac{\partial}{\partial n}\Pi^*_{hr}(0) < 0 \text{ and } \frac{\partial}{\partial \beta}\Pi^*_{hr}(0) > 0.$$

To see the last part of the proposition, compare profits after forming the RJV with firms' profits Π^* before. Then

$$\Pi^*_{hr}(0) - \Pi^* = -V(1-\beta)\frac{\beta n^2 + n\beta - 2\beta + 2}{\left(n\beta-2\beta+2\right)\left(1+n\beta-\beta\right)n^2} < 0 \text{ and}$$

$$\pi^*_{hj}(0) - 2\Pi^* = V(1-\beta)\frac{Q}{\left(n\beta-2\beta+2\right)^2\left(1+n\beta-\beta\right)n^2} > 0$$

and $Q = 5\beta^2 n^2 - 6\beta^2 n^3 + 8n\beta^2 + n^4\beta^2 - 8\beta^2 - 9\beta n^2 - 8n\beta + 16\beta + 3n^3\beta - 8 + 2n^2$ is positive for all $n > 3$ and $\beta \in [0, 1)$. **Proof of Proposition 8:**

We use our previous results from Proposition 1 to 7. Then

$$x^*_{hj}(0) > x^*_{kr} > x^*_{cj} > x^* > x^*_{cr} > x^*_{kj} > x^*_{hr}(0)$$

follows[25] from[26]

- $x^*_{hj}(0) > x^*_{kr}$ since $(\beta n + 1 - 2\beta)(n^2\beta + 2n - 2\beta n - 2 + 2\beta)^2 - 2(n\beta - 2\beta + 2)^2(2 + \beta n - \beta)(n-2) > 0$,
- $x^*_{kr} > x^*_{cj}$ since $(n^2\beta + n - 4n\beta - 1 + 5\beta)(n^2\beta + 2n - 2\beta n - 2 + 2\beta)^2 - 4(n^2\beta + 2n - 3n\beta - 3 + 3\beta)^2 (2 + \beta n - \beta) > 0$,
- $x^*_{cj} > x^*$ whenever $(n-2)(2-\beta+n\beta)(n^2\beta+n-4n\beta-1+5\beta) \quad (\beta(n-1)+1)n^2 - 4(1-2\beta+n\beta)(n^2\beta+2n-3n\beta-3+3\beta)^2 \quad (n-1)>0$ which is true for $n=5$ and all $\beta>0.268$, for $n=6$ and all $\beta>0.065$, and all $n \geq 7$ independent of $\beta \in [0, 1]$ (note that $x^*_{kr} > x^*$ since $(2+\beta n-\beta)^2(n-2)(1+\beta(n-1))n^2 - (\beta n + 1 - 2\beta)(n^2\beta + 2n - 2\beta n - 2 + 2\beta)^2(n-1) > 0$),
- $x^* > x^*_{cr}$ whenever $2(n-1)(1-2\beta+\beta n)(\beta n^2 + 2n - 3\beta n - 3 + 3\beta)^2 - n^2(\beta(n-1)+1)(n-2)(2-\beta+\beta n)^2 > 0$ which is true for $n=4$ and all $\beta<0.2$, for $n=5$ and all $\beta<0.565$, and for all $n \geq 6$ independent of $\beta \in [0, 1]$ (note that $x^* > x^*_{kj}$ since $(n-2)(2+\beta n-\beta)(1+\beta)(1+\beta n-\beta)n^2 - 2(\beta n + 1 - 2\beta)(\beta n^2 + 2n - 2\beta n - 2 + 2\beta)^2(n-1) < 0$),
- $x^*_{cr} > x^*_{kj}$ since $(1+\beta)(\beta n^2 + 2n - 3\beta n - 3 + 3\beta)^2 - (\beta n^2 + 2n - 2\beta n - 2 + 2\beta)^2 \left(2 - \beta + \beta n\right) < 0$,
- $x^*_{cr} > 0 = x^*_{hr}(0)$.

Concerning effective R&D efforts,

$$X^*_{kr} > X^*_{hj}(0) > X^*_{cj} \gtrless X^*_{hr}(0) \gtrless X^* \gtrless X^*_{cr} > X^*_{kj}$$

follows[27],[28] from[29]

- $X^*_{kr} > X^*_{hj}(0)$ since $40n + 228\beta - 260n^2\beta^2 + 304n^2\beta^3 + 48n^3\beta^2 - 112n^2\beta^4 - 112n^3\beta^3 - n^4\beta^2 + 64n^3\beta^4 + 15n^4\beta^3 - 18n^4\beta^4 + 2n^5\beta^4 - 248n\beta - 284\beta^2 + 156\beta^3 - 32\beta^4 + 472n\beta^2 + 72n^2\beta - 360n\beta^3 - 4n^3\beta + 96n\beta^4 - 4n^2 - 68 < 0$,
- $X^*_{hj}(0) > X^*_{cj}$ since $2(n\beta - \beta + 2)(2n + 3\beta - 3n\beta + n^2\beta - 3)^2 > (n\beta - 2\beta + 2)^2(1+\beta)(n\beta - \beta + 2)(n^2 - 3n + 2)$ is always true
- $X^*_{cj} \gtrless X^*_{hr}(0)$ whenever $12n + 26\beta - 34n^2\beta^2 + 12n^2\beta^3 + 11n^3\beta^2 - 5n^3\beta^3 - n^4\beta^2 + n^4\beta^3 - 44n\beta - 28\beta^2 + 10\beta^3 + 48n\beta^2 + 24n^2\beta - 16n\beta^3 - 4n^3\beta - 4n^2 - 8 \lesseqgtr 0$ which is true whenever $\beta \lesseqgtr \beta(n)$ (note that $X^*_{hj}(0) > X^*_{hr}(0)$ is always true),
- $X^*_{hr}(0) \gtrless X^*$ whenever $-4n^2\beta^2 + 2n^2\beta + 8n\beta^2 - 12n\beta + 4n - 4\beta^2 + 8\beta - 4 \lesseqgtr 0$ which is true whenever $\beta \gtrless \frac{1}{-8n+4n^2+4}(n^2 + 4 - 6n + n\sqrt{4n + n^2 - 4})$ (note that $X^*_{cj} > X^*$ since $6n + 12\beta - 6n^2\beta^2 + n^3\beta^2 - 16n\beta - 6\beta^2 + 10n\beta^2 + 6n^2\beta - n^3\beta - 2n^2 - 6 < 0$),
- $X^* \gtrless X^*_{cr}$ whenever $-2n^3\beta^2 + n^3\beta + 7n^2\beta^2 - 8n^2\beta + 2n^2 - 8n\beta^2 + 14n\beta - 6n + 3\beta^2 - 6\beta + 3 \gtrless 0$ which is true whenever $\beta \lesseqgtr \frac{1}{16n - 14n^2 + 4n^3 - 6}\left(14n - 8n^2 + n^3 - 6 + n\sqrt{20n - 12n^2 + n^4 - 8}\right)$ (note that $X^*_{cj} > X^*_{cr}$ is always true),
- $X^*_{cr} > X^*_{kj}$ since $4n + 11\beta - 15n^2\beta^2 + 9n^2\beta^3 + 4n^3\beta^2 - 5n^3\beta^3 + n^4\beta^3 - 15n\beta - 7\beta^2 + \beta^3 + 18n\beta^2 + 6n^2\beta - 7n\beta^3 - 5 > 0$.

[25] This is true for $n=5$ and all $\beta>0.268$, for $n=6$ and all $\beta>0.065$, and all $n \geq 7$ independent of $\beta \in [0, 1]$.
[26] This is true for $n=4$ and all $\beta<0.2$, for $n=5$ and all $\beta<0.565$, and for all $n \geq 6$ independent of $\beta \in [0, 1]$.
[27] This is true whenever $\beta \lesseqgtr \beta(n)$.
[28] This depends on whether $\beta \gtrless \frac{1}{-8n+4n^2+4}\left(n^2 + 4 - 6n + n\sqrt{4n + n^2 - 4}\right)$.
[29] This is equilivalent to $\beta \lesseqgtr \frac{1}{16n-14n^2+4n^3-6}(14n - 8n^2 + n^3 - 6 + n\sqrt{20n - 12n^2 + n^4 - 8})$.

Concerning winning probabilities,

$$p^*_{hj}(0) > p^*_{cj} > p^*_{kr} > p^* > p^*_{cr} > p^*_{kj} > p^*_{hr}(0)$$

follows[30] from

- $p^*_{hj}(0) > p^*_{cj}$ since $\left(1-\beta\right)(n-2)(n\beta-\beta+2)$ is always positive,
- $p^*_{cj} > p^*_{kr}$ whenever $n \geq 5$ for all $\beta \in [0, 1]$,
- $p^*_{kr} > p^*$ since $2(1-\beta)$ is always positive (note that $p^*_{cj} > p^*$ since $(1-\beta)\left(n^2-5n+6\right)$ is always positive),
- $p^* > p^*_{cr}$ since $(1-\beta)(n-3)$ is always positive,
- $p^*_{cr} > p^*_{kj}$ since $(1-\beta)\left(1+\beta\right)$ is always positive,
- $p^*_{kj} > p^*_{hr}(0)$ since $\left(1-\beta\right)\left(n\beta+2\right)$ is always positive.

Finally, with respect to profits,

$$\Pi^*_{hj}(0) > \Pi^*_{cj} > \Pi^*_{kj} > \Pi^*_{kr} > \Pi^* > \Pi^*_{cr} > \Pi^*_{hr}(0)$$

follows[31],[32] from[33]

- $\Pi^*_{hj}(0) > \Pi^*_{cj}$ whenever $\beta < 0.170\,82$ for $n = 3$, $\beta < 0.736\,44$ for $n = 4$ and for all $n \geq 5$ with $\beta \in [0, 1]$.
- $\Pi^*_{cj} > \Pi^*_{kj}$ whenever $n \geq 4$ and $\beta \in [0, 1]$ but not for $n = 3$ and $\beta \in [0, 1]$.
- $\Pi^*_{kj} > \Pi^*_{kr}$ for $\beta < \tilde{\beta}(n)$, see Proposition 2,
- $\Pi^*_{kr} > \Pi^*$, see Proposition 3,
- $\Pi^* > \Pi^*_{cr}$ since $n^6\beta^3 - 5n^5\beta^3 + 3n^5\beta^2 + 3n^4\beta^3 - 6n^4\beta^2 + 2n^4\beta + 29n^3\beta^3 - 37n^3\beta^2 + 8n^3\beta - 82n^2\beta^3 + 146n^2\beta^2 - 72n^2\beta + 6n^2 + 90n\beta^3 - 192n\beta^2 + 126n\beta - 24n - 36\beta^3 + 90\beta^2 - 72\beta + 18$ is always positive,
- $\Pi^*_{cr} > \Pi^*_{hr}(0)$ since $6\beta - 5n^2\beta^2 + n^3\beta^2 - 6n\beta - 8\beta^2 + 10n\beta^2 + 2n^2\beta + 2$ is always positive.

Proof of Proposition 9:

(1) To show that rival firms never have an incentive to form a competing RJV if a two-firm RJV hierarchy already exists, suppose that two firms $i = 1, 2$ already formed a RJV hierarchy and that the headquarters announced to subsidize R&D expenditures with a fixed subsidy F_{1i} where F_{1i} is set equal to $(1-c_1)x_i$ for $i = 1, 2$ and $c_1 \in [0, 1]$. Suppose two rival firms $r = 3, 4$ would form a competing RJV hierarchy and its headquarters announce a fixed subsidy F_{2r} where F_{2r} is set equal to $(1-c_2)x_r$ for $r = 3, 4$ and $c_r \in [0, 1]$. For simplicity assume that all other rival firm left the patent contest. Then the two rival firm $r = 3, 4$ chooses x_r to maximize

$$\pi_r = \frac{(1-\beta)(x_3+x_4)+\beta X}{\sum X_j} V - c_2 x_r$$

whereas the two firm in the initial RJV $i = 1, 2$ chooses x_i to maximize

$$\pi_i = \frac{(1-\beta)(x_1+x_2)+\beta X}{\sum X_j} V - c_1 x_i,$$

with

$$\sum X_j = 2X(1+\beta), X = x_1 + x_2 + x_3 + x_4.$$

[30] This is true whenever $n \geq 5$ for all $\beta \in [0, 1]$.

[31] This holds whenever $\beta < 0.170\,82$ for $n = 3$, $\beta < 0.736\,44$ for $n = 4$ and for all $n \geq 5$ with $\beta \in [0, 1]$.

[32] This holds whenever $n \geq 4$ for all $\beta \in [0, 1]$.

[33] This holds for $\beta < \tilde{\beta}(n)$, see Proposition 2.

The first order conditions for profit maximization of the two rival firm $r = 3, 4$ are

$$\frac{\sum X_j - \left((1-\beta)(x_3+x_4)+\beta X\right)2(1+\beta)}{\left(\sum X_j\right)^2} = \frac{c_2}{V}$$

and of a initially cooperating firm $i = 1, 2$ as

$$\frac{\sum X_j - \left((1-\beta)(x_1+x_2)+\beta X\right)2(1+\beta)}{\left(\sum X_j\right)^2} = \frac{c_1}{V}.$$

Since the F.O.C.s for firms $i = 1, 2$, respectively $r = 3, 4$, in the RJV are identical, a quasi-symmetric equilibrium is given by $x_1^* = x_2^* = x_{h1}^*$ and $x_3^* = x_4^* = x_{h2}^*$. Then effective R&D efforts are

$$X_{h1}^* = 2\beta x_{h2}^* + 2x_{h1}^* \text{ and}$$
$$X_{h2}^* = 2x_{h2}^* + 2\beta x_{h1}^*$$

and total effective R&D efforts are

$$\sum_{hj} \overset{*}{X} = 2(1+\beta)(x_{h1}^* + x_{h2}^*).$$

Using these expressions, the first order conditions for $r = 3, 4$ gives

$$\begin{aligned}\frac{c_2}{V} &= \frac{2(1+\beta)(x_{h1}^*+x_{h2}^*) - ((1-\beta)2x_{h2}^* + \beta 2(x_{h1}^*+x_{h2}^*))2(1+\beta)}{(2(1+\beta)(x_{h1}^*+x_{h2}^*))^2} \\ &= \frac{(x_{h1}^*(1-2\beta) - x_{h2}^*)}{2(1+\beta)(x_{h1}^*+x_{h2}^*)^2}\end{aligned}$$

and similarly for $i = 1, 2$ as

$$\frac{c_2}{V} = \frac{\left(x_{h2}^*\left(1-2\beta\right) - x_{h1}^*\right)}{2(1+\beta)\left(x_{h1}^*+x_{h2}^*\right)^2}.$$

Hence,

$$\begin{aligned} c_1\left(x_{h1}^*\left(1-2\beta\right) - x_{h2}^*\right) &= c_2\left(x_{h2}^*\left(1-2\beta\right) - x_{h1}^*\right), \text{ that is} \\ x_{h1}^* &= \frac{c_2}{c_1}x_{h2}^* \end{aligned}$$

and the F.O.C. for $i = 1, 2$ reduces to

$$x_{h2}^* = \frac{c_1\left(1-2\beta\right) - c_2}{2\left(1+\beta\right)(c_1+c_2)^2}.$$

Hence, if the headquarters of the initial RJV choose $c_1 = 0$, R&D expenditures of the cooperating rival firms are Zero, $x_{h2}^* = 0$ independent of the cost subsidy c_2.

(2) To prove the second part of the proposition suppose that one out of n firm in an R&D contest is structured hierarchically and that the headquarters of this firm can subsidize the R&D costs of their R&D department. In particular, suppose that $n - 1$ firms, $i = 2, \ldots, n$, choose R&D efforts to maximize profits, that is, their maximization problem is given by

$$\max_{x_i} \frac{x_i + \beta \sum_{j \neq i} x_j}{\sum_{j=1}^{n} x_j + (n-1)\beta \sum_{j=1}^{n} x_j} V - x_i \text{ for } i = 2, \ldots, n.$$

and that firm $i=1$ is hierarchically structured in the sense that an R&D department undertakes R&D on behalf of the firm's headquarters. Suppose that the headquarters subsidize the R&D expenditure of their department by a fixed subsidy F_1 where F_1 is set equal to $(1-c)x_1$ after firm 1 has chosen its R&D effort x_1. As argued in Chapter 3.3 the R&D department then maximizes the following profit function:

$$\max_{x_1} \quad \frac{x_1+\beta\sum_{j=2}^{n}x_j}{\sum_{j=1}^{n}x_j+(n-1)\beta\sum_{j=1}^{n}x_j}V-cx_1.$$

For a given $c\in[0,1]$ let $(x_1^*(c),\ldots,x_i^*(c),\ldots,x_n^*(c))$ be a Nash-equilibrium. Then the optimal $x_i^*(c), i=2,\ldots,n$ satisfies the following first-order condition

$$\frac{(1-\beta)(1+(n-1)\beta)\left(x_1^*(c)+\sum_{j=2,j\neq i}^{n}\overset{*}{\underset{j}{x}}(c)\right)}{\left((1+(n-1)\beta)\left(x_1^*(c)+\sum_{j=2}^{n}\overset{*}{\underset{j}{x}}(c)\right)\right)^2}V=1$$

whereas $x_1^*(c)$ satisfies

$$\frac{(1-\beta)(1+(n-1)\beta)\left(\sum_{j=2}^{n}\overset{*}{\underset{j}{x}}(c)\right)}{\left((1+(n-1)\beta)\left(x_1^*(c)+\sum_{j=2}^{n}\overset{*}{\underset{j}{x}}(c)\right)\right)^2}V=c$$

Consider a quasi-symmetric Nash-equilibrium with $x_h^*(c)=x_1^*(c), x_r^*(c)=x_2^*(c)=\ldots=x_n^*(c)$. Then the first order conditions read as:

$$(1-\beta)(1+(n-1)\beta)\left(x_h^*(c)+(n-2)x_r^*(c)\right)=\frac{(1+(n-1)\beta)^2\left(x_h^*(c)+(n-1)x_r^*(c)\right)^2}{V}$$

$$\frac{(1-\beta)(1+(n-1)\beta)(n-1)}{c}x_r^*(c)=\frac{(1+(n-1)\beta)^2\left(x_h^*(c)+(n-1)x_r^*(c)\right)^2}{V}$$

Hence

$$x_h^*(c)=\left(\frac{n-1}{c}-(n-2)\right)x_r^*(c).$$

Inserting this into the first order condition gives

$$x_r^*(c)=V(1-\beta)\frac{c(n-1)}{(1+(n-1)\beta)(n-1+c)^2},$$

$$x_h^*(c)=V(1-\beta)\frac{(n-1-c(n-2))(n-1)}{(1+(n-1)\beta)(n-1+c)^2}.$$

The headquarters then choose c^* to maximize overall profits

$$\Pi_h^*(c) = V\frac{\left(1+\beta c\right)(n-1)-c(n-2)}{(1+(n-1)\beta)(n-1+c)} - V(1-\beta)\frac{(n-1-c(n-2))(n-1)}{\left(1+(n-1)\beta\right)(n-1+c)^2}.$$

The first order condition

$$\frac{\partial}{\partial c}\Pi_h^*(c) = \frac{V(1-\beta)(n-1)(3c+n-2cn-1)}{\left(n\beta-\beta+1\right)(c+n-1)^3} = 0$$

implies that the optimal c^* satisfies

$$c^* = \frac{n-1}{2n-3} < 1.$$

Then

$$\frac{\partial}{\partial n}c^* < 0$$

and

$$\Pi_h^*(c^*) = \frac{V}{4}\frac{\left(4n\beta-5\beta+1\right)}{(n-1)\left(n\beta-\beta+1\right)} < \frac{1}{2}\pi_{hj}^*(0) = V\frac{n\beta-\beta-\beta^2+n\beta^2+2}{2\left(n\beta-2\beta+2\right)^2}$$

for

$$-(1-\beta)(2n^3\beta^2-15n^2\beta^2+6n^2\beta+30n\beta^2-28n\beta+4n-18\beta^2+26\beta-8) < 0.$$

Moreover, rivals' profits in equilibrium are

$$\frac{V}{4}\frac{\left(4n\beta(n-2)+3\beta+1\right)}{(n-1)^2\left(n\beta-\beta+1\right)}$$

positive.

Appendix B. R&D Cartelization

We prove in the following that if two out of n firms coordinate their R&D activities so as to maximize the sum of overall profits but do not share R&D research, their profits will be lower than the ones of non-cooperating firms. Kamien et al. (1992) refer to this scenario as R&D Cartelization.

To analyze R&D cartelization, consider the objective function of a firm i, $i = 1, 2$, participating in the R&D cartel. It chooses its R&D efforts to maximize joint profits

$$2\left(x_c+\beta x_c+\beta\sum_{j\neq 1,2}x_j\right)\frac{V}{\sum X_j} - 2x_{nj}\text{,where}$$

$$\sum X_j = X+(n-1)\beta X \text{ with } X = 2x_{nj}+\sum_{j=3,\ldots,n}x_j$$

All other firms $i = 3, \ldots, n$, seek to maximize profits as usual by

$$\max_{x_i}\quad \left((1-\beta)x_i+\beta X\right)\frac{V}{\sum X_j} - x_i\text{,where}$$

$$X_i = x_i+\beta\left(2x_{nj}+\sum_{j=3,\ldots,n,\, j\neq i}x_j\right).$$

Since both partners in the RJV are symmetric, they will choose identical R&D efforts, $x_1^* = x_2^* = x_{nj}^*$. Consider the first-order condition for profit maximization in R&D cartelization:

$$\frac{(1+\beta)\sum X_i - \left((1-\beta)x_{nj} + \beta X\right) 2(1+(n-1)\beta)}{\left(\sum X_j\right)^2} V = 1, \text{ hence,}$$

$$\beta \sum X_j - 2\left((1-\beta)x_{nj} + \beta X\right)(1+(n-1)\beta) = \frac{\left(\sum X_j\right)^2}{V} - \sum X_j$$

The first order conditions for the rival firms $i = 3, \ldots, n$ are given by

$$\frac{\sum X_j - X_i(1+(n-1)\beta)}{\left(\sum X_j\right)^2} V = 1, \text{ hence,}$$

$$\frac{\left(\sum X_j\right)^2}{V} - \sum X_j = -X_i(1+(n-1)\beta)$$

Because of the symmetry of firms $i = 3, \ldots, n$, we look for a quasi-symmetric equilibrium, that is an equilibrium in which $x_3^* = \ldots = x_n^* = x_{nr}^*$. Then the effective R&D efforts are

$$X_{nj} = (1+\beta)x_{nj}^* + (n-2)\beta x_{nr}^* \text{ and } X_{nr} = (1+(n-3)\beta)x_{nr}^* + 2\beta x_{nj}^*$$

and the two first order conditions lead to

$$\beta \sum X_j - 2\left((1-\beta)x_{nj}^* + \beta X\right)(1+(n-1)\beta) = -X_i(1+(n-1)\beta),$$

$$\text{i.e. } x_{nj}^* = \frac{1}{2} x_{nr}^*.$$

Substituting x_{nj}^* into total effective R&D efforts gives

$$\sum X_j = (1+(n-1)\beta)(n-1)x_{nr}^*$$

and the first order condition becomes

$$\begin{aligned} \frac{(1+(n-1)\beta)(n-1)^2}{V} x_{nr}^* &= (n-2)(1-\beta) \\ x_{nr}^* &= V \frac{(n-2)(1-\beta)}{(1+(n-1)\beta)(n-1)^2}. \end{aligned}$$

Profits then are

$$\Pi_{nj}^* = \frac{1}{2} V \frac{1 - 4\beta n + \beta + 2\beta n^2}{\left(1+\beta n - \beta\right)(n-1)^2}$$

$$\Pi_{nr}^* = V \frac{1 - 2\beta n + \beta n^2}{\left(1+\beta n - \beta\right)(n-1)^2}$$

Hence,

$$\begin{aligned} \Pi_{nr}^* - \Pi_{nj}^* &= V \frac{1 - 2\beta n + \beta n^2}{\left(1+\beta n - \beta\right)(n-1)^2} - \frac{1}{2} V \frac{1 - 4\beta n + \beta + 2\beta n^2}{\left(1+\beta n - \beta\right)(n-1)^2} \\ &= \frac{1}{2} V \frac{1-\beta}{\left(1+\beta n - \beta\right)(n-1)^2} > 0 \end{aligned}$$

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