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Concavity Assumptions in Regulatory Models and the Capital Waste Controversy[1]

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Abstract
Sherman (1992, 197) concludes that "the wasteful use of capital [by a rate-of-return constrained monopolist] is motivated to avoid an inelastic region of demand." Previous analyses of capital waste by regulated firms often employ models with concavity restrictions on the profit and production functions. Here we demonstrate that these conventional assumptions in Averch-Johnson type models require demand to be everywhere elastic, ruling out the "avoidance" motive emphasized by Sherman. Although these highly restrictive assumptions are suitable for studying inefficient input mix, they are inappropriate when considering investment in unproductive capital.

1. Introduction

Sherman (1989; 1992) has rekindled the concern that rate-of-return constrained firms may have a profit incentive to invest in unproductive capital (pure capital waste). Similar conclusions were clearly established decades ago, yet this extreme form of capital bias by the regulated firm has yet to become widely appreciated. Westfield (1965) first demonstrated the incentive for a rate-of-return constrained monopoly to waste capital through the over-payment for productive assets. Bailey (1973) and Zajac (1970) later concluded that the firm will not waste capital provided the input has a positive marginal product. Sherman (1989; 1992) emphasizes the possibility that waste may result from the regulated monopolist's desire to avoid operation in the inelastic demand region.[2] Despite Sherman's efforts to clarify this issue, Train's recent discussion illustrates the controversy that persists on this important subject: "...the regulated firm will not purchase capital that does not serve a productive purpose..." (1991, 54).

Here we emphasize a mathematical oversight made by authors on both sides of the capital waste debate. We do not challenge the capital waste results established by Westfield,

1 This note is based on the appendix to my doctoral dissertation completed at The University of Tennessee, Knoxville, and supervised by John W. Mayo. Useful comments were also made by Ross Eriksson of The University of Tennessee, David Kaserman of Auburn University, David Mandy of the University of Missouri, and an anonymous referee. All conclusions and opinions expressed herein are mine and do not necessarily reflect the views or policies of the National Regulatory Research Institute (NRRI) or any organization associated with NRRI.

2 Also see Kennedy (1977).

Kennedy, and Sherman but question the suitability of the conventional Averch and Johnson (1962) model assumptions for studying the capital waste choice problem facing the regulated monopolist. As Sherman (1992) has clearly argued, understanding the capital waste issue requires one to first recognize that a rate-of-return constraint does not eliminate the profit maximizing monopolist's desire or ability to avoid inelastic demand. Previous authors apparently fail to recognize, however, that the customary assumptions of the Averch-Johnson framework, adopted by Bailey, Zajac, Train, and others, do not allow for an inelastic region of demand. Therefore, analytical investigation of the capital waste issue is moot when based on the Averch-Johnson model since, as observed by Sherman, "...waste of capital arises whenever the firm uses it as a means of avoiding inelastic demand" (1992, 197).

Below, we demonstrate that conventional concavity restrictions on both the production function and the profit or revenue function necessarily imply that marginal revenue is everywhere positive. This important implication has heretofore gone unnoticed by researchers using the Averch-Johnson framework. The highly restrictive form of these models may be suitable for identifying inefficient choice of input mix (i.e., the Averch-Johnson effect) but is inappropriate for considering pure capital waste choice by the regulated firm.

2. Concavity Restrictions and the Implications for Demand

Let $x = (K,L)$, where K is the level of capital input, L is the level of labor or noncapital input, and $x \in \Re^2_+$. Assuming market-clearing,[3] profit can be written in terms of vector x as

$$\pi(x) = f(x) \cdot p(f(x)) - wL - iK, \tag{1}$$

where $f(x)$ is a continuous production function, $p(f(x))$ is an inverse demand function, and w and i are fixed factor prices per unit of L and K, respectively. The revenue function can be written as

$$R(x) = f(x) \cdot p(f(x)). \tag{2}$$

We will consider two assumptions that characterize Averch-Johnson type models. First, most models of rate-of-return regulation restrict either the profit or the revenue function to be concave in input space (K,L).[4] Note that the profit function (1) is concave if and only if the revenue function (2) is concave (proof is provided in the Appendix).[5] The second conventional assumption of the Averch-Johnson model requires the production function, $f(x)$, to be quasi-concave (an axiom in neoclassical theory of the firm).[6] Though not explicitly assumed by all previous authors, convex technology is implied by their model descriptions.[7]

The gradient of $R(x)$ is given by $\nabla R = (p + \partial p/\partial f)\, \nabla f$. Let $\varphi = (p + \partial p/\partial f)$, which is the marginal revenue function. It is obvious that any contour (isoquant) of $f(x)$ is also a contour (isorevenue curve) of $R(x)$. This can be shown by comparing the differentials of each function. The vector ∇f is orthogonal to every direction, dx, along the contour of $f(x)$. Thus,

3 Throughout, the level of quantity demanded always equals the level of output produced.

4 See, for example, Averch and Johnson (1962, 1055); Takayama (1969, 256); Zajac (1970, 118); Zajac (1972, 312); Bailey (1973, 74); and, more recently, Train (1991, 27-28).

5 This fact is also noted by Takayama (1969) and Bailey (1973).

6 See Das (1980, 456) and Varian (1992, 8-9).

7 See, for example, Train (1991, Chapter 1).

if $df = \nabla f \cdot dx = 0$, then $dR = \nabla R \cdot dx = \varphi \, \nabla f \cdot dx = 0$. Therefore, an isoquant necessarily coincides with some isorevenue curve.

Proposition: If $R(x)$ is concave and $f(x)$ is quasi-concave, then marginal revenue, φ, is positive for all x (i.e., demand is everywhere elastic).

Proof: Define sets V_f and V_R as

$$V_f = \{x \mid f(x) \geq f(x^*)\}$$

$$V_R = \{x \mid R(x) \geq R(x^*)\},$$

where x^* is some arbitrary input vector. The concavity restrictions placed on both $f(x)$ and $R(x)$ imply that V_f and V_R are convex sets. Since the contours of each function are equivalent and both sets are convex, it follows that $V_f = V_R$. At any point, x^*, we can select an arbitrarily small vector, h, such that $(x^* + h)$ is an interior element of $V_f = V_R$ and

$$f(x^* + h) - f(x^*) > 0. \tag{3}$$

This implies that

$$R(x^* + h) - R(x^*) > 0. \tag{4}$$

Applying the Mean Value Theorem, the difference in (3) is given by

$$df = \nabla f(x^* + \alpha h) \cdot h > 0,$$

where α is some element of the open interval $(0,1)$.[8] Similarly, the difference in (4) is given by

$$dR = \nabla R(x^* + \alpha h) \cdot h = \varphi \, \nabla f(x^* + \alpha h) \cdot h > 0.$$

It follows that $\varphi > 0$ for all x. QED

Some intuition for this result can be developed geometrically within the two-dimensional input space, (K,L). Figure 1 depicts an arbitrary pair of coinciding contours: an isoquant given by $Q^* = \{x \mid f(x) = f(x^*)\}$, and an isorevenue curve given by $R^* = \{x \mid R(x) = R(x^*)\}$. The diagonally shaded region to the "northeast" of the contour depicts the equivalent convex sets V_f and V_R, defined in the proof above. Consider an arbitrary input pair on the contour such as that given by x^* and an interior point such as $(x^* + h)$. Movement from x^* to $(x^* + h)$ necessarily results in higher output and higher revenue (by the definitions of V_f and V_R). Since higher output results in higher revenue, it must be the case that marginal revenue is positive at x^*. As demonstrated in the proof, the concavity restrictions on the production and revenue functions (implying convexity of V_f and V_R) are sufficient to guarantee that this will be true at all input combinations. Note that concavity assumptions on both functions are sufficient for positive marginal revenue at all x but not necessary. Relaxing one or both

8 This premise was improved by suggestions made by Ross Eriksson.

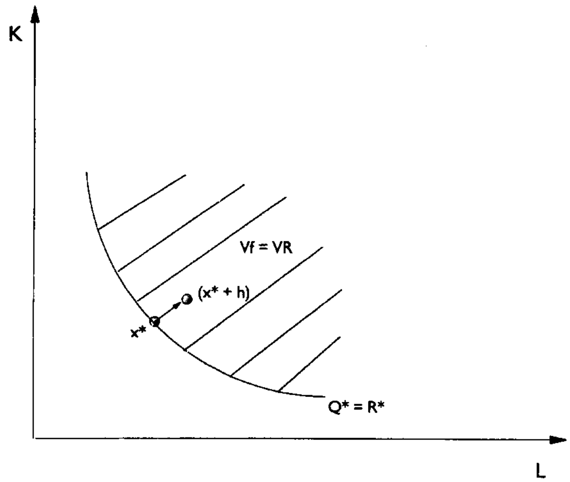

Figure 1. Coinciding Isoquant and Isorevenue Curves

assumptions eliminates the sufficient state.

3. Modelling Implications

It has long been recognized that certain aspects of regulation can cause a firm to be motivated to operate in regions of its production frontier that no unregulated monopoly would find profitable. As a result, some conventional model assumptions become inappropriate when analyzing a regulated firm, for although they are nonbinding in traditional models, they rule out important behavioral possibilities for the regulated firm.[9] Here concavity assumptions are shown to fall in that category of seemingly harmless but actually restrictive assumptions that many researchers have overlooked.[10]

As should be clear from the proposition, profit or revenue concavity in the input space is no weak assumption.[11] When combined with a neoclassical production function, profit concavity restricts demand to be everywhere elastic. Contrasting this result with the discussions by Bailey (1973, 74-75) and Train (1991, Chapter 1), we find that these authors are incorrect in their presumption that demand in their models has an inelastic region. Furthermore, our proposition indicates that "proofs" demonstrating operation in the elastic region of demand within the Averch-Johnson framework are, at best, immaterial.

Perhaps the most important implication of the result is that for analysis on the capital waste problem. Models designed for pure capital waste inquiry should relax the profit (revenue) concavity restriction common to Averch-Johnson type models.[12] Conversely,

9 For example, use of a neoclassical cost function based on cost-minimization is inappropriate for study of a rate-of-return constrained firm.

10 The author is indebted to an anonymous referee for this characterization of the result.

11 Takayama (1969) also stresses the strength of the concave profit assumption. However, he apparently does not recognize the implication established here.

12 Similarly, researchers who analyze "used and useful" criteria imposed by regulators should realize that

research with a central focus on inefficient input substitution or some other aspect of regulation can invoke the concavity assumption on profit to avoid potential complications that would arise if the firm had an incentive to purchase unproductive capital.[13]

4. Conclusions

Sherman argues that pure capital waste becomes a possibility when the rate-of-return constrained firm uses this strategy to avoid operating in the inelastic region of demand. Since demand is everywhere elastic in the Averch-Johnson model, such a profit incentive never arises. Unlike earlier non-waste results, our finding does not rely on the sign of the marginal product of capital. Models intended for investigating the potential investment by regulated firms in unproductive capital must, therefore, relax the concave profit assumption traditionally used in most input choice analyses. The conventional Averch-Johnson assumptions are useful for identifying inefficient input mix but fail to provide a useful framework for analyzing the pure capital waste issue. Sherman (1992) avoids this problem by relaxing the concave revenue (profit) assumption.[14] Future consideration of the capital waste choice problem must follow Sherman's lead in this regard.

Appendix

Proof of the following statement:

The profit function, $\pi(x) = R(x) - wL - iK$, is concave iff the revenue function, $R(x) = f(x) \cdot p(f(x))$, is concave.

If $\pi(x)$ is concave, then for any two points $x^a = (K_a, L_a)$ and $x^b = (K_b, L_b) \in \Re^2_+$,

$$\pi[\lambda x^a + (1-\lambda)\, x^b] \geq \lambda\pi(x^a) + (1-\lambda)\, \pi(x^b) \quad 0 \leq \lambda \leq 1.$$

Substituting $R(x)$, this inequality becomes

$$R[\lambda x^a + (1-\lambda)\, x^b] - w \cdot [\lambda L_a + (1-\lambda)\, L_b] - i \cdot [\lambda K_a + (1-\lambda)\, K_b]$$

$$\geq \lambda R(x^a) + (1-\lambda)\, R(x^b) - w \cdot [\lambda L_a + (1-\lambda)\, L_b] - i \cdot [\lambda K_a + (1-\lambda)\, K_b].$$

Since the second and third terms on both sides of this inequality are identical:

$$R[\lambda x^a + (1-\lambda)\, x^b] \geq \lambda R(x^a) + (1-\lambda)\, R(x^b).$$

Therefore, $R(x)$ is concave. QED

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the regulated firm in their model must first have an incentive to purchase unproductive capital which may require that profit be nonconcave in input space.

13 For example, Blank (1995) analyzes the regulator's choice of constraints but ignores the pure capital waste issue and the possibility of "used and useful" provisions (topics not central to his investigation) because profit concavity is imposed.

14 Although Sherman (1992) does not explicitly relax the concave revenue assumption, non-concavity in the input space is implied by his figure 1, p. 199.

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