Journal of Financial Services Research 4:93-110 (1990)
© 1990 Kluwer Academic Publishers

An Analysis of Market-Index Certificates of Deposit

ANDREW H. CHEN
Distinguished Professor of Finance, Edwin L. Cox School of Business, Southern Methodist University, Dallas, TX 75275 U.S.A.

JOHN W. KENSINGER
Assistant Professor of Finance, College and Graduate School of Business, University of Texas at Austin, Austin TX 78712-1179 U.S.A.

Abstract

We analyze the new market-index certificates of deposit (MICDs), which pay variable interest pegged to the performance of the Standard and Poor's (S&P) 500 stock market index. The pricing formulas as well as the equilibrium relationship between the participation percentages for both the call- and put-type MICDs are derived and discussed. From the implicit value of the option component we compute implied standard deviations (ISDs) for the market index, and find inconsistencies in the pricing policies of the recent issuers of MICDs. The terms under which "bear" versions are being offered are particularly unattractive, and offering better terms would create new opportunities for the issuers of MICDs. We also analyze the issuer's risk exposure, and discuss the natural as well as the dynamic hedging strategies.

In March 1987 Chase Manhattan Bank responded to competition for funds during a long-running bull market on Wall Street by offering market-index certificates of deposit (MICDs), a new product designed to provide investors a means for participating in the stock market while at the same time limiting their risk exposure. These MICDs are variable-rate certificates of deposit for which the interest rate is contingent upon the performance of a well-known stock market average such as the S&P 500 index—but with a guaranteed minimum interest rate. They are available in two versions: the call (or "bull") version offers an interest that rises in proportion to the rate of increase in the stock market index; the put (or "bear") version works in reverse, paying higher interest the more the stock market index declines.[1]

Fortune magazine chose the MICDs for inclusion in its 1987 list of "Products of the Year," on the grounds that their downside protection makes participation in the stock market attractive to a broader base of investors.[2] In addition, an investor who already has stock holdings in direct form, or through a mutual fund or pension fund, could use a put version of the MICD as a hedge. Since their introduction, MICDs have been offered by several other commercial banks and thrift institutions as well, including an innovative "capped" version with an upper bound on the interest rate.[3]

We thank George Kaufman, Richard Rendleman, Robert Eisenbeis, and two anonymous referees for valuable comments and suggestions, and gratefully acknowledge financial support for this work from a Summer Research Grant provided by the College of Business, University of Texas at Austin.

Although the growth of MICDs was retarded by the combined effects of the stock market crash in October 1987 as well as the thrift crisis, the demand is understandably large for investment vehicles that combine upside potential with guaranteed protection of principal—for example, guaranteed investment contracts from employee savings plans are forecast to rise 25 percent during 1989, to $40 billion. Because the capacity of the insurance industry to meet this demand will not keep pace with growth, however, banks are expected to pick up the slack.[4] With the recovery of the stock market from its 1987 woes, therefore, it is reasonable to expect significant growth potential for MICDs.

This article compliments the analysis presented by Chance and Broughton (1988) in several ways which should be useful not only to the issuers of MICDs but also to regulators. First, we generalize the pricing formulas to include both the minimum guaranteed interest rates and the maximum caps on the MICDs. In addition, we derive the equilibrium relationship between the call and put participation percentages. Finally, we examine two major decisions confronting the issuers of MICDs: pricing policy and risk management strategy.

The experience with MICDs, furthermore, holds some lessons that will be useful in coping with the newer contingent liabilities that are on the way in the immediate future. The dynamically evolving regulatory environment continues to open new opportunities, while at the same time creating new pitfalls for the unwary or reckless. In July 1989 the Commodity Futures Trading Commission (CFTC) officially opened the door for financial institutions to offer a variety of products such as CDs or bonds with interest contingent upon the price of gold, oil, or any other basic commodity,[5] and the activity in this area is expected to be substantial.[6] Ironically, this is another instance in which one body of regulators has addressed issues within its purview, only to create potential new problems for another group of regulators, reflecting the ongoing regulatory dialectic described in Kane (1977).

The article is organized as follows: in section 1 we analyze the MICDs and derive the theoretical terms for an equilibrium pricing policy. Then in section 2 we report empirical observations of actual pricing policy, and analyze the anomalies that have occurred. To some extent these anomalies reflect marketing considerations associated with the introduction of a new product; but in section 3 we analyze the alternative hedging strategies, and find that hedging behavior explains some of these anomalies as well. Finally, section 4 contains the concluding remarks.

1. Pricing the market-index CD contracts

The call version of the MICD pays interest equal to a stated percentage (referred to as the participation percentage) of the rate of increase in the stock market index, but pays the guaranteed minimum interest rate if the index falls. The put version works in reverse, paying a variable rate of interest that is linked to declines in the market index. The capped version places an upper bound on the interest rate the issuer is liable to pay, making it possible to offer a higher participation percentage than would otherwise be feasible.[7]

In order to express the terms of these arrangements formally, let us consider an MICD with principal of one dollar and maturity on day T. For easy reference, all the symbols used in the analysis are summarized in table 1. Let $\lambda(\kappa)$ be the guaranteed minimum (maximum) payment at maturity for an MICD with principal of one dollar.[8] Let γ_c (γ_p) be the participation percentage for the call (put) version of the MICD, stated as a decimal fraction. Since the uncertainty about the maturity payment arises from the volatility of the stock market, let us define a scaled index, S_t, which is the ratio of the raw S&P 500 index on day t divided by the raw index on day 0. Thus on the initial date of the contract, this scaled index is defined to be 1 (i.e., $S_0 \equiv 1$). For an MICD with a principal of one dollar, then, the payment at maturity is defined as follows:

$$\text{Maturity Payment (call version)} = \min\{\kappa, \max[\lambda, 1 + \gamma_c(S_T - 1)]\}.$$

For a call version of the MICD, that is, the rate of interest on the deposit is γ_c times the rate of increase in the stock market index from the initial date of the contract to the maturity date—subject to the boundaries established for the minimum and maximum interest rate.[9] The put version works in reverse, with the variable interest rate pegged to γ_p times the rate of decline in the index, so the maturity payment is defined as follows:

$$\text{Maturity Payment (put version)} = \min\{\kappa, \max[\lambda, 1 + \gamma_p(1 - S_T)]\}.$$

Table 1. List of symbols.

λ	= the guaranteed minimum payment at maturity for an MICD with principal of one dollar
κ	= the guaranteed maximum payment at maturity for an MICD with principal of one dollar
γ_c	= the participation percentage for the call version of the MICD, stated as a decimal fraction
γ_p	= the participation percentage for the put version of the MICD, stated as a decimal fraction
S_t	= the ratio of the raw S&P 500 index on day t divided by the raw index on day 0 (where day 0 is the issue date of the CD)
X_c	$= \frac{\lambda - 1}{\gamma_c} + 1$
χ_c	$= \frac{\kappa - 1}{\gamma_c} + 1$
X_p	$= \frac{1 - \lambda}{\gamma_p} + 1$
X_p	$= \frac{1 - \kappa}{\gamma_p} + 1$
$c(1,\tau,X,r,\sigma_2)$	$= N(d_1) - e^{-r\tau}XN(d_2)$
$N(\ldots)$	= Normal probability density function
d_1	$= \frac{\ln\left(\frac{1}{X}\right) + r\tau}{\sigma\sqrt{\tau}} + \frac{\sigma}{2}\sqrt{\tau}$
d_2	$= \frac{\ln\left(\frac{1}{X}\right) + r\tau}{\sigma\sqrt{\tau}} - \frac{\sigma}{2}\sqrt{\tau}$
τ	= the time to maturity of the CD, measured as a fraction of a year
r	= Treasury bill rate
σ^2	= instantaneous variance of the S&P 500 stock market index
$p(1,\tau,X,r,\sigma^2)$	$= e^{-r\tau}X(1 - N(d_2)) - (1 - N(d_1))$

The price of an MICD is not revised continuously on the floor of an exchange, but instead is manifested implicitly through the terms offered by the issuer at the beginning of the MICD's life (i.e., the participation percentage, as well as the minimum and maximum maturity payment). Since these parameters remain fixed over the life of the specific contract, and there is typically a substantial early withdrawal penalty that binds the deposit until maturity,[10] the appropriate focus of attention is on the fairness of the terms at the time the contract is initiated. Let us therefore begin by breaking down the MICDs into an equivalent package of Treasury securities and index options.

1.1. Replicating the market-index CDs

A call version of the MICD with a principal of one dollar can be replicated on the issue date by a portfolio containing: (1) a risk-free pure discount bond that matures on day T with maturity value λ; (2) γ_c call options on the scaled market index that expire on day T with exercise price X_c; and (3) a short position in γ_c call options on the scaled market index that expire on day T with exercise price χ_c, where X_c and χ_c are defined as follows:

$$X_c = \frac{\lambda - 1}{\gamma_c} + 1$$

$$\chi_c = \frac{\kappa - 1}{\gamma_c} + 1.$$

Suppose, for example, that λ were 1.04, κ were 1.15, and γ_c were .80. Then the scaled index would have to rise above 1.05 before the maturity payment on the MICD would exceed the guaranteed minimum. The combination of the bond and the first option in the replicating portfolio duplicates not only the guaranteed minimum but also the additional return which would occur if the scaled index exceeded 1.05 at the maturity date. If the scaled index exceeded 1.1875 at the maturity date, however, the cap would take effect, and the third component of the replicating portfolio duplicates this aspect of the MICD.

Likewise, the put version of the MICD with a principal of one dollar can be replicated at time 0 by a portfolio containing: (1) a risk-free pure discount bond that matures on day T with maturity value λ; (2) γ_p put options on the scaled market index that expire on day T with exercise price X_p; and (3) a short position in γ_p put options on the scaled market index that expire on day T with exercise price χ_p, where X_p and χ_p are defined as follows:

$$X_p = \frac{1 - \lambda}{\gamma_p} + 1$$

$$\chi_p = \frac{1 - \kappa}{\gamma_p} + 1.$$

1.2. Setting the participation percentages for market-index CDs

Since MICDs cannot be redeemed early under favorable terms, there is no viable early exercise opportunity. Thus the options involved in these contracts are European-type, so we can use the Black-Scholes option pricing model in order to determine fair pricing policies. Let $c(1,\tau,X,r,\sigma^2)$ represent the value of a call option on the scaled index with time to maturity of τ and exercise price X, when r is the risk-free interest rate and σ^2 is the instantaneous variance of the raw index.[11] To be in equilibrium at the time of issue (assuming the CD has principal of one dollar) the participation percentage must be set so that:

$$\lambda e^{-r\tau} + \gamma_c c(1,\tau,X_c,r,\sigma^2) - \gamma_c c(1,\tau,\chi_c,r,\sigma^2) = 1. \tag{1}$$

To derive the equilibrium condition for the put version of the MICD, let $p(1,\tau,X,r,\sigma^2)$ represent the value of a put option on the scaled index with time until maturity of τ and exercise price X, when the risk-free interest rate is r and the variance rate of the index is σ^2. In order to be in equilibrium at the time of issue, the "bear" participation percentage must be set so that:

$$\lambda e^{-r\tau} + \gamma_p p(1,\tau,X_p,r,\sigma^2) - \gamma_p p(1,\tau,\chi_p,r,\sigma^2) = 1. \tag{2}$$

To gain insight into the nature of the equilibrium participation percentages, let us consider the special case of the basic MICD which offers a guaranteed repayment of principal with no cap (λ equals 1 and κ is undefined). For the call version in this special case, the equilibrium participation percentage is the following:

$$\gamma_c = \frac{1-e^{-r\tau}}{c(1,\tau,1,r,\sigma^2)} \tag{3}$$

Because the guaranteed minimum maturity payment equals 1 in this case, the option is written at the money on day 0. Since the numerator in equation (3) represents the lower bound for the value of this option, it must be less than the denominator prior to maturity. (See Merton, 1973, for the proof.) Therefore, the equilibrium participation percentage is always less than 100 percent for the basic call MICD with a guaranteed return of principal and no cap. Furthermore, the equilibrium participation percentage increases with longer time to maturity, but decreases with increasing volatility of the market index.[12]

For the put version of the MICD in this special case, equation (2) can be solved to find the following equilibrium value for the participation percentage on the issue date:

$$\gamma_p = \frac{1-e^{-r\tau}}{p(1,\tau,1,r,\sigma^2)}. \tag{4}$$

By Stoll's (1969) put-call parity we know that $p(1,\tau,1,r,\sigma^2) = c(1,\tau,1,r,\sigma^2) - 1 + e^{-r\tau}$. Substituting this into equation (4) makes it possible to show the relationship between γ_p and γ_c in this special case. Through put-call parity, equation (4) transforms to the following:

$$\gamma_p = \frac{\gamma_c}{1-\gamma_c}. \tag{5}$$

Perhaps most noteworthy is that $\gamma_p > 1$ for all values of $\gamma_c > 0.5$. A put version of the MICD could thus provide investors with a valuable hedging tool in a personal portfolio which contains direct or indirect investments in the stock market—a thousand-dollar holding of equilibrium-priced put MICDs could hedge a significantly larger holding of stocks. A potential market for very long-term put versions can be visualized, since they would be a useful tool for an investor with a substantial commitment in stocks and a goal of, say, retiring in ten years. For such an investor, an equilibrium-priced long-term put version of the MICD could offer a much more attractive hedging instrument than would otherwise be available from a series of short-term positions.

With positive guaranteed minimum interest rates or a cap, however, equations (1) and (2) cannot be solved algebraically for the equilibrium participation percentage on the issue date because the exercise prices of the options are functions of the participation percentage. Thus the simple special-case relationships expressed in equations (3) through (5) no longer hold—but they do continue to provide a useful base case for reference. When there is a positive guaranteed minimum interest rate the risk of investing in the CD is diminished, so its equilibrium participation percentage is reduced below the base case. The provision of a cap, however, reduces the value of the CD to the investor, requiring an upward adjustment of the participation percentage in order to restore equilibrium. Exact values for the equilibrium participation percentages in the general case can be found by numerical solution of equations (1) and (2).

2. Analysis of pricing policies

To begin our examination of pricing policy, let us consider the makeup of the replicating portfolio. Although the stock-market-related aspects may tend to dominate one's first impression of MICDs, in reality the Treasury-security component makes up most of the replicating portfolio when the terms of the CDs are set at equilibrium. From equations (1) and (2) one can see that when the conditions for equilibrium are met, the fraction of the replicating portfolio represented by Treasury securities is $\lambda e^{-r\tau}$—which is generally close to 1.[13] That is, the cost of buying a complete hedge for an MICD series on the issue date is a small fraction of the principal.[14]

In effect the bulk of the principal an issuer raises through an equilibrium-priced MICD series represents proceeds borrowed at the same rate as the U.S. Treasury, and the issuer can completely hedge its exposure to stock market risk by using the remainder of the principal to purchase offsetting index options. For example, with a maturity of one year, an interest rate of 7 percent on U.S. Treasury securities, and a guaranteed minimum maturity payment of \$1.04 for every dollar of principal, the Treasury securities represent 97 percent of the replicating portfolio. (Thus from every dollar of principal the issuer could set aside 3 cents for hedging, leaving 97 cents in proceeds.) The issuer could break even on this MICD series simply by investing the proceeds at the Treasury security rate. With equilibrium pricing, then, the MICD would offer substantial funding at a lower cost than traditional CDs—which typically break even at a premium above Treasury security interest rates. Equilibrium pricing, therefore, seems to be a desirable goal not only from the investors' point of view but also from the issuers'.[15]

2.1. *Observed pricing policies*

Nevertheless, issuers do not appear to be concerned about maintaining an equilibrium pricing policy—although the "price" of an MICD is established through the participation percentage, the issuers have chosen to adjust this parameter infrequently. To gain more insight into the actual pricing policies of issuers, we calculated the implied standard deviation (ISD) for the stock market index from participation percentages quoted in January 1988 by a large issuer (Chase Manhattan Bank) and a small issuer (Murray Savings Association of Dallas.)[16] The results are shown in table 2, ranked from lowest to highest ISD. The range of values is wide, reflecting significant inconsistencies. For comparison we calculated the ISD from the S&P 500 call options quoted by the Chicago Board Options Exchange on Friday, January 8, 1988, maturing March 1988, and found an

Table 2. Implied standard deviations in market-index certificates of deposit, January 1988.*

Issuer	ISD	Type	Minimum rate	Participation percentage	Maturity
Murray Savings	.19	Bull	0%	45%	6 mos.
	.19	Bull	0%	60%	1 year
	.24	Bear[a]	0%	100%	6 mos.
	.31	Bear	0%	45%	6 mos.
	.33	Bear[a]	0%	100%	1 year
	.37	Bear	0%	60%	1 year
Chase Manhattan	.38	Bull	0%	37%	1 year
	.39	Bull	4%	24%	1 year
	.40	Bull	4%	10%	3 mos.
	.45	Bull	0%	15%	3 mos.
	.45	Bear	4%	11%	3 mos.
	.47	Bear	0%	45%	1 year
	.47	Bear	4%	32%	1 year
	.50	Bear	0%	16%	3 mos.
	.50	Bull	4%	13%	6 mos
	.52	Bull	0%	20%	6 mos.
	.56	Bear	0%	23%	6 mos.
	.59	Bear	4%	15%	6 mos.

*For comparison, from prices quoted January 8, 1988, for the S&P 500 call options maturing in March 1988, we calculated an ISD of .40 for the market index.

[a]Capped at 20 percent maximum rate of interest.

Note: We used Treasury bill rates of 6.04 percent, 6.71 percent, and 7.12 percent for three-month, six-month, and 12-month maturities, respectively. (*Source: Wall Street Journal*, January 8, 1988.)

ISD of .40.[17] Although the terms of two of the Chase Manhattan call MICDs were consistent with this calculation, Chase's participation percentages were smaller than the equilibrium values for the most part, while Murray Savings MICDs were closer to being priced in harmony with historical norms.[18]

To see if the anomalies persisted as MICDs became a seasoned product, we also calculated ISDs from the terms offered by Chase Manhattan and Murray Savings in January 1989 (see table 3).[19] With the increase in Treasury Bill rates and the lack of significant change in the terms of the MICDs from the previous year, the ISDs were even higher in 1989 than they were in 1988. This is surprising, since the volatility of the index had by then returned to the low end of the range of historic norms.[20]

Table 3. Implied standard deviations in market-index certificates of deposit, January 1989.*

Issuer	ISD	Type	Minimum rate	Participation percentage	Maturity
Murray Savings	.21	Bull	0%	65%	1 year
	.25	Bull	0%	45%	6 mos.
	.40	Bear	0%	45%	6 mos.
	.43	Bear	0%	65%	1 year
	.45	Bear[a]	0%	100%	6 mos.
	.50	Bear[a]	0%	100%	1 year
Chase Manhattan	.43	Bull	0%	40%	1 year
	.46	Bull	0%	33%	9 mos.
	.48	Bull	2%	32%	1 year
	.50	Bull	0%	18%	3 mos.
	.50	Bull	2%	27%	9 mos.
	.52	Bull	0%	25%	6 mos.
	.52	Bull	2%	15%	3 mos.
	.52	Bull	4%	25%	1 year
	.57	Bull	2%	20%	6 mos.
	.58	Bull	4%	20%	9 mos.
	.59	Bear	0%	45%	1 year
	.65	Bear	4%	32%	1 year
	.65	Bull	4%	10%	3 mos.
	.67	Bear	0%	16%	3 mos.
	.73	Bear	0%	23%	6 mos.
	.73	Bear	4%	11%	3 mos.
	.75	Bull	4%	13%	6 mos.
	.88	Bear	4%	15%	6 mos.

*For comparison, from prices quoted January 30, 1989, for the S&P 500 call options maturing in March 1989, we calculated an ISD of .16 for the market index.

[a]Capped at 20 percent maximum rate of interest.

Note: We used Treasury bill rates of 8.29 percent, 8.82 percent, 8.92 percent, and 8.97 percent for three-month, six-month, nine-month, and 12-month maturities, respectively. (*Source: Wall Street Journal*, January 31, 1989.)

One potential explanation is that a big issuer like Chase Manhattan is engaging in arbitrage, taking advantage of market imperfections that prevent customers from creating better homemade alternatives. If imperfections such as minimum contract size and high transactions costs for individuals make it possible for the issuer to receive $1.00 for a package that theoretically is worth, say, 99 cents, the issuer is enabled to borrow the proceeds for *less* than the Treasury security rate. Although this may provide part of the explanation, however, it does not fully explain why the best terms tend to be associated with longer term bull versions of the MICD. Pricing policies aimed at encouraging customers to choose longer maturities suggest that the issuer is more concerned with obtaining a committed funding source than with earning arbitrage profits.

The implied standard deviation for a capped call version of the MICD cannot be interpreted in the same way as the others, and so it is not reported in tables 2 or 3.[21] It is nevertheless possible to determine that the call versions of the Murray Savings Association's capped MICDs present another kind of anomaly. Both the six-month and one-year maturities offered guaranteed return of principal, a participation percentage of 100 percent and a cap of 20 percent. Solving equation (1) reveals that if the standard deviation of the index were .20, the 1988 equilibrium participation percentage would be 72 percent for the one-year CDs—and if the standard deviation were .40, it would be 65 percent. The participation percentage of 100 percent that was actually offered, therefore, renders the call version of this CD more favorable to the depositor than any homemade replica that could be constructed, distinguishing it among the various MICDs offered to the general public.[22] In order to explain this, an official of Murray Savings indicated that an effort was made to offer an attractive call version of this capped CD in order to build market share. (Chase Manhattan, moreover, offered attractive "loss-leader" terms on some of its MICDs when they were first introduced, and later lowered its participation percentages.[23])

Of course, regulators must be vigilant for a less laudable motive; a troubled institution may offer loss-leader terms on its MICDs as an alternative to offering high interest rates on conventional CDs. Since the price paid for funds is implicit rather than explicit in the case of MICDs, such an institution may issue them in an attempt to disguise the truth of its situation.

2.2. *The bias is greatest for the put version*

The distortions are most intense for the put versions of the MICDs. Murray Savings consistently offered the same participation percentage for *both* the call and put versions of its MICDs. Chase offered slightly higher participation percentages for the put versions of its MICDs, but in no case did the observed pricing policies for the basic MICDs (i.e., those with a guaranteed minimum interest rate of zero and no cap) approach the equilibrium relationship expressed in equation (5).

This is puzzling because a fairly priced put version of the MICD could be an attractive hedging tool for an investor with sizeable holdings in stocks, mutual funds, or stock-based pension funds, accompanied by frictions that place constraints on the mobility of the funds (such as tax considerations or pension rules). Thus, it appears to be worthwhile for banks to offer competitive long-term put versions of their MICDs. The depositors would gain valuable protection with no loss of principal, and in return the issuers would benefit from a new long-term source of funding. Observed pricing policies for the put versions of the MICDs, however, prevent them from achieving their potential.

Not only were the put versions of the stock MICDs out of equilibrium, however, they were introduced later than the call variety and were not as actively marketed—the emphasis was clearly placed upon the call version of the MICD.[24] In search of an explanation, we asked an executive of one of the issuers why the put-type CDs were treated this way. He readily acknowledged that higher participation percentages were possible in the case of the put MICDs, but observed that there were no competitive pressures to offer better terms—and, moreover, there were concerns among top management that such an offer might be interpreted negatively by regulators and potential customers. Because the interest rates on put-type MICDs would be higher in the wake of a stock market downturn, top management feared that high participation percentages would be interpreted as an indication of increased vulnerability in an economic slump. Rightly or wrongly, management worried that the general public would not understand the hedging used to resolve such risks.

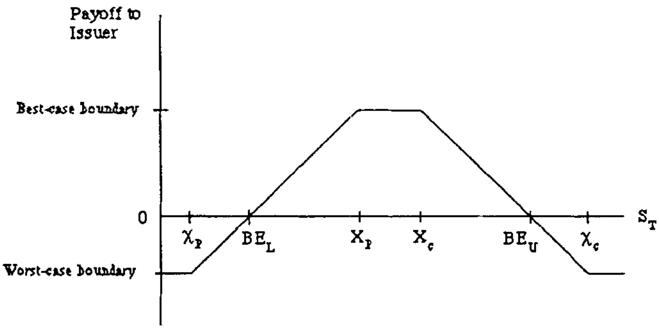
Given our observations of the anomalies in pricing MICDs during the introductory period of a new issuer or a new version, as well as the anomalous pricing of the put version, it appears that issuers have been concerned more about marketing-related issues (e.g., market share and an innovative image) than about achieving equilibrium pricing. We offer some rational financial explanations, however, in the next section.

3. Hedging strategies for issuers

3.1. Using index options and natural partial hedges

In the analysis above, we have assumed that the issuer hedges the stock market risk completely by purchasing the options in the replicating portfolios for each series of MICDs it issues. Other degrees of hedging, of course, are possible. To begin assessing such alternatives let us consider the extreme of no offsetting option hedges at all. An issuer with both put and call MICDs outstanding has a natural position that straddles the index, so there is already a built-in cushion against stock market fluctuations. To analyze the risks involved in an unhedged position, let n_c (n_p) represent the number of call (put) CDs with principal of one dollar in a given issue, for a combined total number of call and put MICDs equal to n_c plus n_p. Let $\tilde{\rho}$ represent the uncertain rate of return the issuer earns from investing the deposited funds. At maturity the issuer will have loan repayments equal to $e^{\rho\tau}(n_c + n_p)$ available to make the required payment to depositors. If the value of the index falls between X_p and X_c at maturity, neither the call MICDs nor the put MICDs will earn more than the guaranteed minimum interest rate, so the issuer will keep $(n_c + n_p)(e^{\rho\tau} - \lambda)$. The payoff function in figure 1 is therefore flat in the region between X_p and X_c.

Figure 1. Illustration of issuer's exposure.



Note: The best-case boundary is equal to $(n_c + n_p)(e^{\rho\tau} - \lambda)$. If the index rises above X_c, the worst-case boundary is $(n_c + n_p)e^{\rho\tau} - \lambda n_p - \kappa n_c$; and if the index falls below X_p, the worst-case boundary is $(n_c + n_p)e^{\rho\tau} - \lambda n_c - \kappa n_p$.

Every point by which the value of the index at maturity exceeds X_c will cost the issuer $\gamma_c n_c$ dollars, and likewise every point by which the value of the index at maturity falls below X_p will cost the issuer $\gamma_p n_p$ dollars.[25] The issuer earns a profit, however, so long as the value of the index at maturity falls within the bounds of the upper and lower break-even points represented by BE_U and BE_L in figure 1, which are defined as follows:

$$\text{Upper break-even point} = BE_U = X_c + (e^{\rho\tau} - \lambda)\,\frac{(n_c+n_p)}{\gamma_c n_c} \tag{6}$$

$$\text{Lower break-even point} = BE_L = X_p - (e^{\rho\tau} - \lambda)\,\frac{(n_c+n_p)}{\gamma_p n_p}. \tag{7}$$

The more the issuer is able to earn from its investments, the wider the gap between the lower and upper break-even points. The issuer's risk exposure depends not only upon the variability of the rate of change in the stock market index but also upon its correlation with the rate of return on the loan portfolio. The more positive the correlation (i.e., the more situations that dictate paying high interest on the call MICDs tend to be associated with better-than-expected loan portfolio performance), the less the issuer's risk exposure.[26] With a high ratio of call- to put-type CDs, furthermore, the lower break-even is nonexistent, making a hedge against a falling market unnecessary. Thus a high correlation between the stock market index and the performance of the loan portfolio motivates the issuer to seek a high call/put ratio, which may help explain the biases observed in the pricing and marketing of the put-type CDs.

When the MICD specifies a cap, the issuer has a modified butterfly spread on the index, allowing the potential losses to be contained within the coverage of the issuer's capital reserves.[27] Since the MICD product was designed to provide a means for the issuer to compete with stock market investments for deposits, it can be argued that containing the issuer's exposure to stock market fluctuations within the bounds of capital coverage is a reasonable alternative to eliminating such risk altogether.

In the absence of a cap, however, a partial hedge can be used to limit the issuer's exposure to stock-market risk. The issuer could form a partial hedge by purchasing $n_c\gamma_c$ call options with exercise price X_U and $n_p\gamma_p$ put options with exercise price X_L. To define X_U and X_L, let R be the dollar reserve against losses on this issue of MICDs, and H be the cost of the options purchased for hedging.[28] Then for any given level of reserves, X_U and X_L can be found by simultaneously solving the following expressions:

$$X_U = BE_U + \frac{R-H}{\gamma_c n_c}, \tag{8}$$

$$X_L = BE_L - \frac{R-H}{\gamma_p n_p}. \tag{9}$$

Although the cost of a partial hedge depends on the exercise prices chosen, the solutions can be found by numerical techniques.[29]

3.2. *Regulatory problems associated with partial hedging and dynamic hedging*

MICDs would not create any new stock-market worries for regulators if all issuers formed an exact hedge at the initiation of each issue, by purchasing option contracts on the S&P 500 index in the quantities specified in equations (1) and (2).[30] An issuer may argue that its newly created exposure to stock market risk is hedged by the correlation between returns on the stock market and the performance of its loan portfolio, however. With both call and put MICDs outstanding, furthermore, the issuer has a straddle on the stock market index, and it is possible for a partial hedge to contain potential losses within bounds the issuer may contend are covered by its capital reserves. In the case of capped CDs, moreover, such a partial hedge is already in place. By increasing the probability of insolvency, issuers which seek permission for no hedging or partial hedging create new regulatory oversight burdens.

Adding to the regulators' cause for concern about MICDs, some issuers (including Chase Manhattan) have announced the intention to manage their exposure to stock market risk by means of "dynamic hedging"—applying portfolio insurance techniques using futures contracts on the S&P 500 index. The essence of dynamic hedging is to create synthetic options for substitution into the hedges described above, by means of a frequently adjusted position in index futures contracts[31]—so the distinction between dynamic hedging and hedging with options contracts, therefore, reflects a different method rather than a different objective. Such dynamic hedging strategies offer valuable advantages for issuers but create costly problems for regulators. For a large issuer, dynamic hedging offers savings in transactions costs. Another obvious advantage of dynamic hedging is that synthetic options can be created with maturities or exercise prices that are not available from listed options—hedging with option contracts, which offer a finite set of exercise prices and maturities, is less flexible than dynamic hedging, which theoretically can be accomplished for any exercise price or maturity.

In addition, a dynamic hedging strategy allows the flexibility for dealing with the risk exposure created by aging issues through the pricing policies associated with new issues. A rising market, for example, creates the need to add to the issuer's long position in index futures as a hedge against aging call MICDs that are outstanding. As an alternative, however, the outstanding call MICDs could be offset in the issuer's aggregate position by establishing a pricing policy that encourages a preponderance of put MICDs to be sold. The newly issued put versions of the MICD will then generate a profit for the issuer if the stock market index continues to rise, which will partially offset the losses generated by seasoned call-type CDs.[32] Thus, apparent anomalies in pricing policy may simply reflect hedging activity.

Issuers who use dynamic hedging will, therefore, have a competitive advantage relative to issuers who hedge with options contracts. A major stock market fluctuation, however, could hurt weak banks that move heavily into MICD products, engage in aggressive pricing policies, and hedge inadequately. Conservative regulatory policies, on the other hand, result in less attractive terms for consumers (i.e., lower guaranteed minimum interest rates, lower participation percentages, or lower caps) and reduce the issuers' ability to compete against mutual funds.

4. Concluding remarks

We have derived the general pricing formulas for the call and put versions of MICDs, and examined the pricing policies and hedging strategies for their issuers. From observed quotations we have also derived implied standard deviations, compared them with implied standard deviations calculated from the prices of exchange-traded index options, and found inconsistencies in the terms being offered—not only between issuers but among MICDs of different maturities and types offered by the same issuer. In particular, the terms of the put MICDs are out of equilibrium. Moreover, this situation represents a potentially significant untapped opportunity for issuers of MICDs, because a long-term put version of the MICD offers a useful hedging instrument for an individual with investments in a mutual fund or stock-based pension fund. Such a hedging vehicle would be particularly attractive to an investor whose tax situation makes it undesirable to liquidate the stock position, and who would therefore be willing to pay a premium.

MICDs have the potential to expand significantly the investment opportunities available to the general public, while offering financial institutions a means to compete with stock market investments for deposits. MICDs can be replicated by portfolios that consist primarily of Treasury securities—only a small portion of the replicating portfolios consist of options on the stock market index. Complete hedging against stock market fluctuations can therefore be accomplished by purchasing offsetting index options, which is not difficult for regulators to monitor. Because the issuer's exposure to stock market risk can be hedged completely by spending a small fraction of the proceeds (less than 5 percent in many cases), there is the appearance that MICDs could proliferate without necessarily creating any new problems concerning the safety of the banking system.

Appendix: Some illustrations of hedging with options

Suppose an issue of MICDs includes $2 million of call MICDs and $1 million of put MICDs with participation percentages of 60 percent for both call MICDs and put MICDs, no cap, and guaranteed return of principal. The MICDs mature in one year, and the initial level of the index is 200. Then a one point gain (loss) in the index translates into a rise (fall) of 0.5 percent. With a participation percentage of 60 percent, this translates into a 0.3 percent gain for the depositors who were fortunate enough to pick the direction of the index correctly. For each point the market index rises above the initial level, this translates to $6,000 for the call MICDs. Each point the index falls below the initial level translates into $3,000 for the put MICDs. To hold its losses at a given level, then, the issuer would need to hold 6,000 call options on the raw index and 3,000 put options in its hedge portfolio.

For the issue just described, let us further assume that the standard deviation of the market index is .2, the Treasury security rate is 6 percent compounded annually, and the issuer wishes to contain potential losses under $25,000. The issuer then buys 6,000 call options with exercise price of 235 and 3,000 put options with exercise price of 130, and invests the remainder of the funds at 8 percent compounded annually. Each call has a fair value of approximately $8.12, each put has a fair value of approximately $0.07, and the total cost of hedging is $48,920. Upon settlement of the MICDs the issuer's proceeds would range from a maximum profit of $187,170 (which occurs when the index is at 200 on the maturity date) to a maximum loss of $22,830 (which occurs if the index is above 235 or below 130 at maturity). As a percentage of the total principal of the issue, the issuer's proceeds range from a maximum of 6.24 percent to a minimum of -0.76 percent.

By reducing the exercise price of the call options and increasing the exercise price of the put options, the hedge can be adjusted in order to bring the range of potential outcomes into an acceptable span (i.e., within the coverage of the issuer's capital reserves). The hedge can be modified to keep the profit margin above zero, for instance, by choosing a call exercise price of 225 and a put exercise price of 145. The total cost of this hedge is $67,134, and the maximum possible profit for the issuer is 5.58 percent of the total principal. If the index rises above 225 the issuer's profit margin will be reduced to 0.6 percent, and if the index falls below 145 the profit margin will be just under 0.1 percent. Thus there is no chance of loss but a possibility for very attractive profits.

In the extreme, the issuer or its regulators may desire to eliminate stock market risk altogether by hedging with at-the-money put and call options. To protect the above issue of MICDs, the call options would cost approximately $21.79 each, the put options approximately $10.47 each, and the total cost of hedging would be $162,153. With the remaining $2,837,847 invested at 8 percent compounded annually, the issuer would expect to collect $3,064,875 at the end of the year. The interest on the CDs would be covered by the proceeds from unwinding the hedge, leaving the issuer a profit of $64,875 regardless of the outcome for the stock market index (which translates to a margin of 2.16 percent of the principal).

Notes

1. Chase Manhattan was the first bank to offer a market index product in the form of an insured certificate of deposit, but these MICDs were preceded by several varieties of bonds with interest contingent upon a stock market index. Swedish Export Credit (SEK) pioneered the so-called "bull and bear" bonds in mid-1986. The redemption price of these bonds (which mature in July 1991) is linked to the Nikkei average of 225 stocks on the Tokyo exchange. (A detailed description is given in Walmsley, 1988, p. 299.) In August 1986 the Salomon Brothers investment banking firm offered a product called S&P 500 Index Subordinated Notes (SPINs) maturing in 1990 with a 2 percent annual coupon and a maturity payment linked to the increase in the stock market index. A bear version, Reverse SPINs, was later introduced in September 1987. In October 1987, Merrill Lynch offered an index version of its Liquid Yield Option Notes (LYONS) with zero coupon, four-year maturity, and maturity payment linked to the New York Stock Exchange Composite Index. Then Portfolio Income Notes (PINs) debuted in December 1987, designed to appreciate in direct correlation with declines in the New York Stock Exchange (NYSE) Index, while providing guaranteed return of principal.
2. *Fortune*, December 7, 1987, pp. 120–125.
3. By the close of 1987 at least ten institutions were offering MICDs. After Chase Manhattan introduced the "bull" version in March, Fleet National Bank of Providence, RI, introduced the first "bear" version in September 1987. Next, Murray Savings Association of Dallas and Citicorp Select Investments of Sioux Falls, SD, introduced a capped version of the MICD. Other issuers include Northern Trust Bank and First National Bank in Chicago, AmeriFirst Bank in Miami, American Pioneer Savings Bank in Orlando, First City Bancorporation in Houston, and Shearson Lehman Brothers, Inc. (which began offering MICDs through its affiliate, Boston Safe Deposit & Trust Co.).
4. Reported by *The Wall Street Journal*, October 3, 1989, p. A1.
5. Reported by *The Wall Street Journal*, July 18, 1989, p. C6. At the same time, the Commission also issued a policy statement clearing the way for swap transactions involving commodities, opening a "safe harbor" for most swap transactions to escape regulation by the CFTC.
6. *The Wall Street Journal*, September 26, 1989, p. C1, reports that "commodity swaps have progressed faster and further in the two months since the CFTC policy statement than did interest-rate swaps in two years when they began." This rapid growth is attributed at least in part to greater sophistication among customers and hence more ready acceptance of new products. To the extent that the same can be said for retail customers, commodity-contingent CDs should have substantial growth potential as well.
7. Murray Savings Association of Dallas, for example, offered MICDs in both capped and uncapped versions. In January 1989, the participation percentages on its uncapped Wall Street CDs were 45 percent for the six-month maturity and 65 percent for the one-year maturity, for both the call and put versions, with guaranteed return of principal. In exchange for a cap of 20 percent on the interest rate, however, depositors could get participation percentages of 100 percent in both the call and put versions.
8. If a CD with a one-year maturity has a guaranteed minimum interest rate of 4 percent, for example, with an upper limit of 15 percent, then λ would be 1.04 and κ would be 1.15.
9. Suppose, for example, a bullish depositor with an investment of $1,000 could obtain an MICD with a one-year maturity which promised a variable interest pegged at 70 percent of the rate of increase in the S&P 500 index, but with a guaranteed return of principle. (These terms were actually quoted by Chase Manhattan in July 1987, and revised terms are regularly announced for new CDs.) If an investor deposited $1,000 into one of these call MICDs when the S&P 500 index was at 200 and the index climbed to 240 a year later, the investor would earn interest of 14 percent and receive a maturity payment of $1,140. If the index dropped to 190, however, the investor would get back all of the principal, and would have nothing to lament except the interest that could have been earned by making a safe investment in Treasury Bills (or fixed-rate CDs).
10. There is a substantial penalty for early withdrawal that limits the liquidity of MICDs. At Chase Manhattan, no interest whatsoever is paid on the account in the event of early withdrawal. In addition, Chase charges an early withdrawal penalty equal to 0.5 percent of the depositor's principal for each remaining full four-week period in the term of the account. Murray Savings charges an early withdrawal penalty of 10 percent of principal, regardless of the length of time remaining until maturity. Thus even if the CD is "out of the money," it is generally more valuable alive than dead. Chance and Broughton (1988) analyze the

optimal early redemption strategy and find that if the penalty (as a percentage of the principal) exceeds the risk-free interest rate for the time remaining to maturity, early redemption is not desirable. Stiff early withdrawal penalties, therefore, make these CDs a reasonably reliable source of funding to the issuer for the full time to maturity; and given the purpose behind their creation, it is reasonable to expect that such penalties will continue to be a common feature of MICDs.

11. The variance used for calculating the value of options on the scaled index is the same as that for the raw index, since the function defining the value of an option is homogeneous of degree 1 with respect to the exercise price and the price of the underlying asset. Merton (1973) proves that for any scalar k, the value of $c(1,\tau,X,r,\sigma^2) = \frac{1}{k}\, c(k,\tau,kX,r,\sigma^2)$. Thus it is straightforward for a hedger to adjust the scale of the replicating portfolio in order to determine how many options on the raw index to buy for hedging purposes.
12. With increasing time to maturity, the numerator in equation (3) increases more rapidly than the denominator (see Black and Scholes, 1973), so the equilibrium participation percentage is higher for longer maturities. With increasing volatility of the underlying index, ceteris paribus, the numerator in equation (3) remains constant while the denominator increases.
13. Since the value of the Treasury security component is λe^{-rt}, and the total package has an equilibrium value of \$1.00 by definition, the fraction represented by Treasury securities, therefore, is also λe^{-rt}.
14. Equation (1) rearranges to the following: $\gamma_c[c(1,\tau,X_c,r,\sigma^2) - c(1,\tau,\chi_c,r,\sigma^2)] = 1 - \lambda e^{-r\tau}$. That is, the condition for equilibrium is that the cap, floor, and participation percentage be set so the cost of the hedge portfolio of options is just equal to the amount left over after setting aside the present value of the guaranteed minimum maturity payment. Thus the fraction of the replicating portfolio represented by the option component is $1 - \lambda e^{-r\tau}$ (a reasonable approximation is simply $\tau(1 + r - \lambda)$). The same conclusion follows for the put version, from equation (2).
15. When transactions costs for buying the hedging options are taken into account, of course, the issuer's net cost of funds would be modestly higher than the Treasury bill rate. Transactions costs, however, are a very small portion of the principal, will generally be lower for the bank than for its retail customers, and can readily be taken into account in determining the equilibrium participation percentage.
16. We solved equations (1) and (2) for the implied standard deviation for each issue of MICDs, using the Black-Scholes Option Pricing Model applied via the technique described in Whaley (1982). We used Treasury security rates quoted for January 8, 1988.
17. This implied volatility of .40 is higher than the historic norm (which ranges from .16 to .22), reflecting the heightened uncertainty in the wake of the recent market "crash" of October 1987.
18. In those cases in which maturities and guaranteed minimum interest rates of the Chase MICDs and the Murray MICDs are all identical, the participation percentages can be compared directly. In the case of the one-year, zero minimum return CDs, for instance, Murray offered 60 percent participation versus 37 percent for Chase. For comparisons involving different guaranteed minimums at a given maturity, however, the ISDs facilitate comparison. When the theoretical values of the CDs listed in tables 2 and 3 are calculated (using equations 1 and 2) the rankings are identical to those given by the ISDs shown in the tables.
19. We obtained quotations on January 31, 1989. The quoted terms had remained constant for several months, and had not changed significantly over the past year.
20. From prices quoted January 30, 1989, for the S&P 500 call options maturing in March 1989, we calculated an ISD of .16 for the market index.
21. For the Murray Savings one-year call MICD with guaranteed repayment of principal, participation percentage of 100 percent, a cap of 20 percent, and a T-bill rate of 7.12 percent, equation (1) balances at two different levels of volatility: .02 and .67. The low solution results because the short option is virtually worthless, while the value of the long option approaches its lower bound (which equals the amount available for the purchasing the call option position). The high solution results from the necessity for the short option to become sufficiently valuable for equation (1) to balance, and is very high relative to historical norms. Adding to the difficulty of interpretation, this solution is very sensitive to changes in the T-bill rate. Substituting a rate of 6 percent, for example, changes the solution to about 200 percent; while a T-bill rate of 9 percent changes the solution to about 20 percent.
22. Since the six-month maturity offered the same terms, it was clearly also superior to the homemade replica.

23. For instance, Chase Manhattan Bank offers an institutional version of the MICD with a $1 million minimum investment. Maturities extend up to 36 months, and Chase Manhattan at one time offered a participation rate as high as 115 percent with guaranteed return of principal (in July 1987). This clearly defies home-made replication. Even when MICDs are priced as loss leaders, however, they can still provide a source of funds that is competitive with traditional CDs. For example, if the issuer chooses a pricing policy that results in receiving $1.00 for a package theoretically worth, say, $1.01, it could hedge away stock market risk and still receive the net proceeds at a cost about 1 percent above the Treasury security rate.
24. Although the bias in pricing policy is clear enough, the bias in promotion and marketing is more subtle. For example, a large issuer maintains a toll-free telephone quotation service for the call version, supported by its main office—but the operators are unaware that the put version even exists. After several inquiries about the put version, we finally tracked down the information from a subsidiary in another city—and had to pay for the calls.
25. When the value of the scaled index at the maturity date falls below X_p or above X_c, therefore, the slope of the payoff function in figure 1 changes of $\gamma_p n_p$ or $-\gamma_c n_c$, respectively.
26. To the extent that the performance of the stock market index reflects overall economic performance, it would be reasonable to anticipate a positive correlation with loan portfolio performance.
27. If the index exceeds χ_c, the worst case boundary is $(n_c + n_p)e^{\rho\tau} - \lambda n_p - \kappa n_c$; and if the index falls short of χ_p, the worst case boundary is $(n_c + n_p)e^{\rho\tau} - \lambda n_c - \kappa n_p$. So long as sufficient reserves are set aside to carry the maximum burden of potential loss, no further hedging against the index is required. When the dollar amount of sales of call MICDs is identical to that for put MICDs, the issuer has a classic butterfly spread on the index. (If the issue is not balanced, the spread becomes lopsided.) Even though figure 1 is drawn to show a negative worst-case payoff to the issuer, moreover, it could be positive if the cap were sufficiently small.
28. The cost of hedging is defined as follows: $H = n_c\gamma_c\, c(1,\tau,X_U,r,\sigma^2) + n_p\gamma_p\, p(1,\tau,X_L,r,\sigma^2)$.
29. The cost of hedging, of course, would reduce the amount available for investment in the issuer's loan portfolio, which in turn reduces the investment earnings and alters the old break-even points accordingly. The numerical solution technique readily adapts to this requirement, however. See the appendix for some illustrations of partial hedges formed in this fashion.
30. The shorter maturities can be hedged with listed index options. The one-year CDs can be hedged with options written over-the-counter by market makers such as Salomon Brothers.
31. Ideally, such a hedge should be adjusted continuously, but in practice it would be done at discrete intervals and so would not be theoretically perfect.
32. If call MICDs with one year until maturity were issued, for example, and then six months passed during which the index increased, the issuer could reduce its risk exposure by encouraging the sale of new put MICDs with six-month maturities.

References

Benston, G. and Kaufman, G. *Risk and Solvency Regulation of Depository Institutions: Past Policies and Current Options*. Monograph Series in Finance and Economics, Salomon Brothers Center for the Study of Financial Institutions, New York University Graduate School of Business Administration, Monograph 1988-1, 1988.

Black, F. and Scholes, M. "The Valuation of Option Contracts and a Test of Market Efficiency." *Journal of Finance* 27 (1972), 399–417.

Black, F. and Scholes, M. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (1973), 637–659.

Chance, D. and Broughton, J. "Market Index Depository Liabilities: Analysis, Interpretation, and Performance." *Journal of Financial Services Research* 1 (1988), 335–352.

Chiras, D. and Manaster, S. "The Informational Content of Option Prices and a Test of Market Efficiency." *Journal of Financial Economics* 6 (1978), 213–234.

Kane, E. "Good Intentions and Unintended Evil: The Case Against Selective Credit Allocation." *Journal of Money, Credit and Banking* (February 1977), 55–69.

King, S.R. and Remolona, E.M. "The Pricing and Hedging of Market Index Deposits." *Federal Reserve Bank of New York Quarterly Review* 12 (Summer 1987)), 9–20.

Latane, H. and Rendleman, R. "Standard Deviations of Stock Price Ratios Implied in Option Prices." *Journal of Finance* 31 (1976) 369–381.

Merton, R. "Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science* 4 (1973) 141–183.

Schmallensee, R. and Trippi, R. "Common Stock Volatility Estimates Implied by Option Premia." *Journal of Finance* 33 (1978) 129–147.

Walmsley, J. *The New Financial Instruments*. New York: John Wiley & Sons, 1988.

Whaley, R. "Valuation of American Options on Dividend-Paying Stocks." *Journal of Financial Economics* 10 (1982), 29–58.