LITERATURE CITED

1. I. M. Gryaznov et al., Abstracts of 5th All-Union Conf. "Problems of Metrological Support for Measurement-Data Processing Systems" [in Russian], VNIIFTRI, Moscow (1984), p. 351.
2. All-Union State Standard (GOST) 26.003-80: ESSP: Interface System for Measuring Devices with Byte-Serial and Bit-Parallel Data Exchange: Compatibility Requirements [in Russian].
3. N. I. Gorelikov et al., Interface for Programmable Instruments in Experiment-Automation Systems [in Russian], Nauka, Moscow (1981).
4. I. M. Gryaznov et al., Abstracts of 5th All-Union Conf. "Problems of Metrological Support for Measurement-Data Processing Systems" [in Russian], VNIIFTRI, Moscow (1984), p. 355.

ESTIMATION OF ERRORS OF MASS DETERMINATION IN ACCOUNTING OF PETROLEUM AND PETROLEUM PRODUCTS

E. V. Zolotov

UDC 531.7.089.6:665.6(083.96)

Economy of fuel reserves is directly related to the provision of universal and effective stock taking. The latter includes various operations: receipt and distribution of petroleum and petroleum products (below, products), inventory, calculation of balances of distributed and received products, etc. The accounting operations are based on measurements of product mass. Mass, volume-mass dynamic and static, and hydrostatic methods are used for such measurements.

Often, the errors of single measurements are identified with the errors of determination of product mass when accounting operations are performed. However, this can only take place in a product-batch receipt-distribution operation and when the entire mass is measured completely, i.e., one measurement is made. But in most cases of accounting operations, the desired product masses are the sum of the results of many measurements. For example, in distribution or receipt of products at transfer storage and distribution centers, the product mass can be determined by summation of the results of measurements of each tank car that arrives or in several graduated tanks with which these measurements were made. In calculating reserves for an enterprise, the mass of products is determined by summation of the masses received and transferred in a given time interval from various suppliers and to various consumers.

The problem of finding methods of estimating the errors of determination of product mass with allowance for the characteristics of the measurement methods and means used and the possibilities of obtaining the initial information is examined below.

First of all, it is necessary to find an equation for the error of determination of product mass when this mass M is the sum of the results of single measurements of masses m_i, i.e.,

$M=\sum_{i=1}^{n} m_i$, where n is the number of single mass measurements.

In this case, it is obvious that the absolute error of determining the total mass ΔM is equal to the algebraic sum of the absolute measurement errors of the masses Δm_i:

$$\Delta M=\sum_{i=1}^{n} \Delta m_i.$$

Since single measurements are, as a rule, made by various measurement means with different working- and surrounding-medium parameters, the measurements can be considered random uncorrelated values. Therefore, the errors of a function ΔM are estimated using the variances of the arguments Δm_i. In accordance with [1], the variance of a sum of random values is equal to the sum of their variances $D(\Delta M)=\sum_{i=1}^{n} D(\Delta m_i)$. This variance can be expressed in terms of the standard deviation (SD):

Translated from Izmeritel'naya Tekhnika, No. 6, pp. 63-65, June, 1985.

0543-1972/85/2806-0568$09.50

$$\sigma(\Delta M)^2 = \sum_{i=1}^{n} \sigma(\Delta m_i)^2. \tag{1}$$

We transform (1), using the relative SDs of the error of the sum $\sigma(\delta M)$ and the error of single measurements $\sigma(\delta m)$ expressed in percent. In this case, we assume that the measurement errors belong to one general population, which can occur when the same measurement methods are used. Then the SD $\sigma(\delta m)$ is the error characteristic of all single measurements. With allowance for the above, we obtain

$$\sigma(\delta M)^2 = \frac{\sigma(\Delta M)^2}{M^2} 100^2 = \sum_{i=1}^{n} \frac{\sigma(\delta m)^2 m_i^2 100^2}{100^2 M^2} = \sigma(\delta m)^2 \sum_{i=1}^{n} \frac{m_i^2}{M^2}. \tag{2}$$

We represent m_i as $m_i = \bar{m} + \varepsilon m_i$, where $\bar{m} = \left(\sum_{i=1}^{n} m_i\right)/n$ is the mean value of the measured masses and εm_i are the absolute deviations of single measurements from the mean value.

Considering that $M = n\bar{m}$, we express (2) as

$$\sigma(\delta M)^2 = \sigma(\delta m)^2 \sum_{i=1}^{n} \frac{(\bar{m} + \varepsilon m_i)^2}{n^2 \bar{m}^2} = \sigma(\delta m)^2 \left(\frac{1}{n} + \frac{2 \sum_{i=1}^{n} \varepsilon m_i}{n^2 \bar{m}} + \frac{1}{n} \frac{\sum_{i=1}^{n} \varepsilon m_i^2}{n \bar{m}^2} \right). \tag{3}$$

However, in (3)

$$\frac{2 \sum_{i=1}^{n} \varepsilon m_i}{n^2 \bar{m}} = \frac{2 \sum_{i=1}^{n} (m_i - \bar{m})}{n^2 \bar{m}} = \frac{2}{n} \left(\frac{\sum_{i=1}^{n} m_i}{n\bar{m}} - \frac{n\bar{m}}{n\bar{m}} \right) = 0,$$

therefore,

$$\sigma(\delta M)^2 = \sigma(\delta m)^2 \left[\frac{1}{n} + \frac{1}{n} \sigma(\xi m)^2 \right], \tag{4}$$

where $\sigma(\xi m) = \left(\sum_{i} \varepsilon m_i^2 \right) / n\bar{m}^2$, i.e., is the variance of the relative values of individual mass measurements.

Thus, it follows from (4) that when the initial data are the results of single measurements performed by the same methods, the SD of the relative error of determining the total product mass is a function of the SD of the relative error of the single measurements, the number of such measurements, and the variance of the relative values of their results:

$$\sigma(\delta M) = \sigma(\delta m) \sqrt{1 + \sigma(\xi m)^2} / \sqrt{n}.$$

The relative error of determining the total mass in this case has the form

$$\delta M = \pm t \sigma(\delta m) \sqrt{1 + \sigma(\xi m)^2} / \sqrt{n}, \tag{5}$$

where t is the coefficient of the interval error estimate for a given confidence coefficient, which is a function of the error distribution.

In accounting practice, cases occur in which single measurements of mass are made by different methods and means. For example, at petroleum-product distribution centers, the received and transferred products can be accounted for simultaneously by accouting means, graduated tanks with level gauges, counters of various types, distributing columns, scales, etc. (mass and volume-mass dynamic and static measurement methods). It is natural that the measurement errors of different methods can belong to different universes and, therefore, have different SDs $\sigma(\delta m)$.

In this case, all results of single mass measurements should be grouped according to the methods or means used and the errors of determining the total mass of products should be calculated individually by (5) for each z-th group. Using (1), it is easy to show that the relative error of determining the entire total mass in this case has the form

$$\delta M = \sqrt{\sum_{z} \left[\frac{M_z}{M} t_z \sigma(\delta M)_z \right]^2}. \tag{6}$$

Thus, a theoretical solution of the problem of estimation of the error of product-mass determination with accounting by (5) and (6) is obtained. In a number of cases, however, it is difficult to find the arguments of function (5).

In general, to find the SD of the error and the variance of the relative values of the results of single measurements of mass, the distributions of these values must be known. The distribution of the relative values of the results of single mass measurements can be determined when a sample of these values of sufficient size over a particular time interval is available. Such a sample can be obtained by analyzing accounting data. As a rule, it is impossible to find the error distribution in this way, due to lack of knowledge of the actual error values for each measurement. Therefore, we shall consider another method.

From a practical point of view, to find the distribution and SD it is sufficient to determine the relative frequency (probability) of the appearance of errors in all possible and given j intervals of their values. The number of appearances of such errors N_j can be represented as the sum of the products of the number of measurements with given accuracy (appearances of error) l_{jg} made over a particular time by each instrument of a group of "g" measurement means with identical accuracies multiplied by their number n_g', i.e.,

$$N_j = \sum_{g=i}^{q} l_{jg} n_g', \quad j = 1, 2, 3, \ldots, q. \tag{7}$$

In this case, it should be borne in mind that l_{jg} and n_g' are random independent events for each group of instruments, while each term is a mutually exclusive event. Therefore, with allowance for (7), to find the desired probabilities of error appearance in given intervals of their values we can (according to [1]) use theorems of probability addition and multiplication and represent them as

$$P(\delta)_j = \frac{N_j}{n} = \sum_{g=i}^{q} P(\delta)_{jg} P(n')_g = \sum_{g=i}^{q} \frac{l_{jg}}{l} \frac{n_g'}{N'}, \tag{8}$$

where $P(\delta)_{jg}$ is the probability of appearance of a given error with measurements by each instrument, $P(n')_g$ is the probability of appearance of measurement means that provide a given error, l is the number of measurements by each instrument over the time interval, and N' is the number of instruments participating in the measurements.

Probability $P(n')_g$ can be found if we know the theoretical or empirical distribution of the maximum instrument errors, which is easily determined from the results of checking of the measurement means used when actual values are indicated in their certificates. To find probability $P(\delta)_{jg}$, it is sufficient to know the type of error distribution of each instrument, which can be determined by analyzing the structure of the metrological characteristics of the given measurement means. For a normal error distribution, the probability of errors in a given interval with measurements by each instrument can be determined by means of a Laplace function [1]. With a uniform error distribution, if the distribution is centered and symmetrical, expression (8) takes the form $P(\delta)_j = \sum_{g=i}^{q} \left(\frac{1}{2q}\right)_g \frac{n_g'}{N'}$, where q is the number of the error interval analyzed.

From the data obtained by the methods indicated above, using the well-known equation of [1], we can find the SD of the relative error of single mass measurements:

$$\sigma(\delta) = \sqrt{\sum_{j=1}^{q} P(\delta)_j \delta_j^2} \tag{9}$$

When finding the relative error of determining product mass using a volume-mass method of measurement, it should be borne in mind that the masses m_i are measured by indirect methods and defined as the product of the volume V_i multiplied by the average density ρ_i over a given time interval. Since V_i and ρ_i are independent random values, according to [2] the variance of the relative error of an indirect measurement of mass has the form

$$\sigma(\delta m)^2 = \sigma(\delta V)^2 + \sigma(\delta\rho)^2, \tag{10}$$

where $\sigma(\delta V)^2$ and $\sigma(\delta\rho)^2$ are the variances of the relative errors of volume and average-density measurements.

In this case, (9) is used to determine $\sigma(\delta V)^2$ and $\sigma(\delta\rho)^2$, and then (10) is used to find the desired relative error of single measurements of mass.

As was indicated above, to find the variances of the relative values of the results of single measurements, it is necessary to know the distributions of the corresponding values, which can be found by methods of mathematical statistics by processing the entire universe of measurements or a sampling of it. However, this solution of the problem is very time-consuming. At the same time, simpler methods can be used in some cases.

If at least the type of distribution of the measured value for each instrument for a particular time interval, which can be established from a limited sample, and the number of measurement means with definite upper and lower measurement-value ranges used are known, the empirical distribution of this quantity is found as is the error distribution of single measurements using (8) and (9). In this case, in (8), $P(\delta)_j$ is the probability of appearance of a measured value in a given range interval, $P(n')_g$ is the probability of appearance of instruments with a particular measurement range that covers the interval in question, l_{jg} is the number of measurements in a given interval j made by each instrument of a group "g" of instruments with identical ranges, and n_g' is the number of instruments in the given group.

This method can be successfully used to measure the volumes of graduated tanks, turbine counters, etc. When a volume-mass measurement method is used, when $m_i = V_i\rho_i$, the relative values of the measured masses can be represented as $m_i/\bar{m} = V_i\rho_i/\bar{V}\bar{\rho}$, where $\bar{V}$ and $\bar{\rho}$ are the average volume and density for the universe of measurements.

In view of the randomness and independence of V_i and ρ_i, the desired variance of the relative values of single measurements of mass, according to [1], will have the form

$$\sigma(\delta m)^2 = \sigma(V_i/\bar{V})^2\,\sigma(\rho_i/\bar{\rho})^2 + \sigma(\rho_i/\bar{\rho})^2 + \sigma(V_i/\bar{V})^2,$$

where $\sigma(V_i/\bar{V})^2$ is the variance of the relative values of the measured volumes and $\sigma(\rho_i/\bar{\rho})^2$ is the variance of the relative density values.

When variance is determined using (8), it should be borne in mind that $P(V)_j = P(V_j/\bar{V})$ and $P(\rho)_j = P(\rho_j/\bar{\rho})$.

Thus, the above analytic relations are necessary and sufficient for estimation of the error of determination of the mass of petroleum and petroleum products when these masses are the sum of the results of a set of measurements. The effectiveness of the method increases with an increase in the number of single measurements employed.

LITERATURE CITED

1. E. S. Ventsel', Probability Theory [in Russian], Nauka, Moscow (1964).
2. G. D. Burdun and B. N. Markov, Fundamentals of Metrology [in Russian], Standartov, Moscow (1975).