CALCULATION OF FLEXIBLE NONHOLLOW SHELLS OF VARIOUS TYPES BY THE FINITE-ELEMENT METHOD

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UDC 539.384.4

1. Basic Relations. Flexible thin nonhollow shells of variable thickness $h(\alpha, \beta)$, variable curvature $k_\alpha(\alpha, \beta)$, $k_\beta(\alpha, \beta)$, $k_{\alpha\beta}(\alpha, \beta)$, with an initial flexure $w^0(\alpha, \beta)$ and coefficients of primary quadratic form $A(\alpha, \beta)$, $B(\alpha, \beta)$, are considered. The Kirchhoff–Love hypothesis is assumed to apply to all the shells. It is assumed that α, β, z form a curvilinear orthogonal coordinate system; u, v, w are the displacements of the median surface of the shell in the direction of the α, β, z axes, respectively. The deformation at a point on the median surface ε_α, ε_β, ε_z is expressed in terms of the displacements of this point u, v, w as follows

$$\begin{gathered}\varepsilon_\alpha^0 = u_{,\alpha}A^{-1} + A_{,\beta}(AB)^{-1}v - k_\alpha w + w^0_{,\alpha}w_{,\alpha}A^{-2}; \qquad \varepsilon^0_{\alpha\beta} = AB^{-1}(uA^{-1})_{,\beta} + \\ + BA^{-1}(vB^{-1})_{,\alpha} + 2k_{\alpha\beta}w + (w^0_{,\alpha}w_{,\beta} + w^0_{,\beta}w_{,\alpha})(AB)^{-1}; \\ \varepsilon_\alpha = \varepsilon_\alpha^0 + 0{,}5\Theta_\alpha^2; \qquad \varepsilon_{\alpha\beta} = \varepsilon^0_{\alpha\beta} + \Theta_\alpha\Theta_\beta; \qquad \Theta_\alpha = -A^{-1}w_{,\alpha} + k_\alpha u \\ (\alpha \rightleftarrows \beta), \quad (A \rightleftarrows B).\end{gathered} \tag{1.1}$$

Here and below, symbols analogous to $(\alpha \rightleftarrows \beta)$ indicate the existence of dependences obtained from those given by the replacement of the notation (α, β) by (β, α).

The increments in the curvature $\varkappa_\alpha$, $\varkappa_\beta$, $\varkappa_{\alpha\beta}$ are written in the form of well-known formulas; see Eq. (7.1.6) of [2]. The relations between the forces and moments, on the one hand, and the components of the median-surface deformation, on the other, take the form [1]

$$\begin{gathered}N_\alpha = C_\alpha(\varepsilon_\alpha + \nu_{\alpha\beta}\varepsilon_\beta); \qquad M_\alpha = D_\alpha(\varkappa_\alpha + \nu_{\alpha\beta}\varkappa_\beta); \qquad N_{\alpha\beta} = C_{\alpha\beta}\varepsilon_{\alpha\beta}; \\ M_{\alpha\beta} = D_{\alpha\beta}\varkappa_{\alpha\beta}; \qquad I = h/(1 - \nu_{\beta\alpha}\nu_{\alpha\beta}); \qquad C_\alpha = E_\alpha I; \qquad C_{\alpha\beta} = G_{\alpha\beta}h; \\ J = h^2/12; \qquad D_\alpha = C_\alpha J; \qquad D_{\alpha\beta} = C_{\alpha\beta}J; \; E_\alpha\nu_{\beta\alpha} = E_\beta\nu_{\alpha\beta},\end{gathered} \tag{1.2}$$

where E_α, E_β, $G_{\alpha\beta}$, $\nu_{\alpha\beta}$, $\nu_{\beta\alpha}$ are the elastic constants of the orthotropic material.

The rigidity matrix of a finite element is obtained using the Lagrange variational principle

$$\delta A - \delta W = 0, \tag{1.3}$$

where δA is the virtual work of the internal forces and moments; δW is the work of external forces.

The displacement field of points of the element in terms of the values of the displacement components and the angles of rotation at node points is written in the form of a sum of products of the form function Φ_i and the generalized coordinates q_n [3, 6-8]; q_n are the node unknown displacements

$$q = \{q_1, q_2, q_3, \ldots, q_n\}; \qquad q = \Phi_1 u^1 + \Phi_2 u^1_{,\alpha} + \Phi_3 u^1_{,\beta} + \Phi_4 u^1_{,\alpha\beta} + \ldots + \Phi_{16} w^4_{,\alpha\beta}. \tag{1.4}$$

Kazan' Physicotechnical Institute, Kazan' Branch, Academy of Sciences of the USSR. Translated from Prikladnaya Mekhanika, Vol. 20, No. 1, pp. 72-78, January, 1984. Original article submitted January 25, 1982.

 0038-5298/84/2001-0068$08.50

Here

$$\Phi_1 = f_1(\xi) f_1(\eta); \quad \Phi_2 = a g_1(\xi) f_1(\eta); \quad \Phi_3 = b f_1(\xi) g_1(\eta); \quad \Phi_4 = ab g_1(\xi) g_1(\eta);$$
$$\Phi_5 = f_2(\xi) f_1(\eta); \quad \Phi_6 = a g_2(\xi) f_1(\eta); \quad \Phi_7 = b f_2(\xi) f_1(\eta); \quad \Phi_8 = ab g_2(\xi) g_1(\eta);$$
$$\Phi_9 = f_2(\xi) f_2(\eta); \quad \Phi_{10} = a g_2(\xi) g_1(\eta); \quad \Phi_{11} = b f_2(\xi) g_2(\eta); \quad \Phi_{12} = ab g_1(\xi) f_2(\eta);$$
$$\Phi_{13} = f_1(\xi) f_2(\eta); \quad \Phi_{14} = a g_1(\xi) f_2(\eta); \quad \Phi_{15} = b f_1(\xi) g_2(\eta); \quad \Phi_{16} = ab g_1(\xi) g_2(\eta)$$
$$f_1(s) = 1 - 3s^2 + 2s^3; \qquad f_2(s) = 3s^2 - 2s^3; \qquad g_1(s) = s - 2s^2 + s^3;$$
$$g_2(s) = s^3 - s^2; \qquad \xi = \alpha/a; \qquad \eta = \beta/b;$$

$f_1(s)$, $f_2(s)$, $g_1(s)$, $g_2(s)$ are cubic Hermite polynomials; a, b are the linear dimensions of the element.

Expressing the virtual work of the internal and external forces in terms of the internal forces, loads, and variation in the deformation gives

$$-\iint_s (M_\alpha \delta\varkappa_\alpha + M_\beta \delta\varkappa_\beta + M_{\alpha\beta}\delta\varkappa_{\alpha\beta} + N_\alpha \delta\varepsilon_\alpha + N_\beta \delta\varepsilon_\beta + N_{\alpha\beta}\delta\varepsilon_{\alpha\beta}) AB d\alpha d\beta + \\ + \int_c t_n \delta v dc + \int_c [m_n, J^*]\delta J^* dc + \iint_s (L\delta v + [F, J]\delta J) AB d\alpha d\beta = 0, \tag{1.5}$$

where $v_1 = u$; $v_2 = v$; s is the area of the element; c is the contour bounding the surface s; L, F are the principal vector and principal moment of the external surfaces and volume forces; t_n, m_n are the principal vector and principal moment of the external forces acting on the boundary cross section of the shell; N_α, N_β, $N_{\alpha\beta}$ are the normal tangential and shear forces; M_α, M_β, $M_{\alpha\beta}$ are the flexural moments and torque.

Assuming that the forces acting on the contour are equal to zero, expanding the variations $\delta\varepsilon_\alpha$, $\delta\varepsilon_\beta$, $\delta\varepsilon_{\alpha\beta}$, $\delta\varkappa_\alpha$, $\delta\varkappa_\beta$, $\delta\varkappa_{\alpha\beta}$ in terms of the variation δq_i of the node unknowns, and regarding δq_i as arbitrary, it is found that

$$\iint_s [(a_1^i + a_{20}^i) N_\alpha + a_2^i N_\beta + (a_3^i + a_{19}^i) N_{\alpha\beta} - a_{14}^i M_\alpha - a_{15}^i M_\beta - \\ - a_{16}^i M_{\alpha\beta} - P_\alpha(\alpha, \beta)\Phi_i] AB d\alpha d\beta = 0 \;\; (\overrightarrow{\underleftarrow{A, B}}), \; (\overrightarrow{\underleftarrow{\alpha, \beta}}), \; (\overrightarrow{\underleftarrow{i, j}}); \tag{1.6}$$

$$\iint_s [-M_\alpha a_4^k - M_\beta a_5^k - M_{\alpha\beta} a_6^k + N_\alpha a_7^k + N_\beta a_8^k + N_{\alpha\beta} a_9^k - \Phi_k P_z(\alpha, \beta)] AB d\alpha d\beta = 0, \tag{1.7}$$

where

$$a_1^i = \Phi_{i,\alpha} A^{-1}; \quad a_2^i = B_{,\alpha}(AB)^{-1}\Phi_i; \quad a_3^i = \Phi_{i,\beta} B^{-1} + AB^{-1}(A^{-1})_{,\beta}\Phi_i;$$
$$a_4^k = \Phi_{k,\alpha\alpha} A^{-2} + A^{-1} A_{,\alpha}^{-1} \Phi_{k,\alpha} + A^{-1} B^{-2} \Phi_{k,\beta} A_{,\beta};$$
$$a_6^k = 2(AB)^{-1}(\Phi_{k,\alpha\beta} - A^{-1} A_{,\beta} \Phi_{k,\alpha} - B^{-1} B_{,\alpha} \Phi_{k,\beta}); \quad a_7^k = A^{-2}(w_{,\alpha}^0 + w_{,\alpha})\Phi_{k,\alpha} - k_\alpha \Phi_k;$$
$$a_9^k = (AB)^{-1}[(w_{,\alpha}^0 + w_{,\alpha})\Phi_{k,\beta} + (w_{,\beta}^0 + w_{,\beta})\Phi_{k,\alpha}] - 2k_{\alpha\beta}\Phi_k;$$

a_5^i, a_8^k, a_{10}^i, a_{11}^i, a_{12}^i are obtained from a_4^k, a_7^k, a_3^i, a_2^i, a_1^i when $(\overrightarrow{\underleftarrow{\alpha, \beta}})$, $(\overrightarrow{\underleftarrow{A, B}})$; Φ_i Φ_j, Φ_k are the basic functions corresponding to the displacements u, v, w.

Expressing the internal forces N_α, N_β, $N_{\alpha\beta}$, M_α, M_β, $M_{\alpha\beta}$ in terms of the desired node-point displacements q_n by means of Eqs. (1.1) and (1.2), Eqs. (1.6) and (1.7) are obtained in vectorial form

$$Bq_n + R_q = R_p, \tag{1.8}$$

where q_n is the node-unknown vector; B is the rigidity matrix; R_q is a vector which is a function of the desired q, i.e., of nonlinear terms; R_p is the external-load vector for a single element.

The components of the rigidity matrix take the form

$$b_{ip} = \iint_s (Q^{ip}_{\alpha\beta} + \tilde{Q}^{ip}_{\alpha\beta})\, ABd\alpha d\beta \;\; (\overleftrightarrow{i, j}),\; (\overleftrightarrow{A, B});\quad b_{ij} = b_{ji} = \iint_s S^{ij}_{\alpha\beta} ABd\alpha d\beta; \tag{1.9}$$

$$b_{ik} = b_{ki} = \iint_s [T^{ik}_{\alpha\beta} + F^{ik}_{\alpha\beta} + \tilde{F}^{ik}_{\alpha\beta}]\, ABd\alpha d\beta;\quad b_{kp} = b^1_{kp} + b^2_{kp},$$

where

$$b^1_{kp} = \iint [H^{kp}_{\alpha\beta} + \lambda^{kp}_{\alpha\beta} + (w^0_{,\alpha})^2 L^{kp}_{\alpha\beta} + w^0_{,\alpha} w^0_{,\beta} S^{kp}_{\alpha\beta} + w^0_{,\alpha} (F^{kp}_{\alpha\beta} + R^{kp}_{\alpha\beta})]\, ABd\alpha d\beta;$$

$$Q^{ip}_{\alpha\beta} = c_\alpha a^i_1 (a^p_1 + \nu_{\alpha\beta} a^p_2) + c_{\alpha\beta} a^i_3 a^p_3 + c_\beta a^i_2 (a^p_2 + \nu_{\beta\alpha} a^p_1) \text{ from } b^1_{kp} \Rightarrow b^2_{kp} \text{ for } (\overleftrightarrow{\alpha, \beta});$$

$$S^{ij}_{\alpha\beta} = c_\alpha a^i_1 (a^j_{11} + \nu_{\alpha\beta} a^j_{12}) + c_{\alpha\beta} a^j_{10} a^i_3 + c_\beta (a^j_{12} + \nu_{\alpha\beta} a^j_{11})\, a^i_2;\; T^{ik}_{\alpha\beta} = w^0_{,\alpha} Q^{ik}_{\alpha\beta} + w^0_{,\beta} S^{ik}_{\alpha\beta}$$

$$(\overleftrightarrow{i, j}),\; (\overleftrightarrow{\alpha, \beta}),\; (\overleftrightarrow{A, B});\quad F^{ik}_{\alpha\beta} = (c_\alpha a^i_1 a_{13} - c_{\alpha\beta} a^i_3 2k_{\alpha\beta} + c_\beta a^i_2 a_{21})\, \Phi_k;$$

$$H^{kp}_{\beta\alpha} + H^{pk}_{\alpha\beta} = D_\alpha a^k_4 (a^p_4 + \nu_{\alpha\beta} a^p_5) + D_\beta a^k_5 (a^p_5 + \nu_{\beta\alpha} a^p_4) + D_{\alpha\beta} a^k_6 a^p_6;$$

$$\lambda^{kp}_{\alpha\beta} = (c_\alpha a_{13} k^2_\alpha + 2c_{\alpha\beta} k^2_{\alpha\beta})\, \Phi_k \Phi_p \;\; (\overleftrightarrow{\alpha, \beta}),\; (\overleftrightarrow{k, p});\quad R^{kp}_{\alpha\beta} = c_\alpha a_{13} \Phi_k A a^p_1 -$$

$$- 2c_{\alpha\beta} (AB)^{-1} k_{\alpha\beta} \Phi_k \Phi_{p,\beta};\quad \tilde{Q}^{ip}_{\alpha\beta} = D_\alpha a^i_{14} (a^p_{14} + \nu_{\alpha\beta} a^p_{15}) +$$

$$+ D_\beta a^i_{15} (a^p_{15} + \nu_{\beta\alpha} a^p_{14}) + D_{\alpha\beta} a^i_{16} a^p_{16};\quad \tilde{F}^{ik}_{\alpha\beta} = D_\alpha a^i_{14} (a^k_4 + \nu_{\alpha\beta} a^k_5) +$$

$$+ D_\beta a^i_{15} (a^k_5 + \nu_{\beta\alpha} a^k_4) + D_{\alpha\beta} a^i_{16} a^k_6.$$

The terms that are nonlinear with respect to the desired node displacement are written in the form

$$r_{qi} = \iint_s (\Psi^i_{\alpha\beta} + \tilde{\Psi}^i_{\alpha\beta})\, ABd\alpha d\beta;\qquad r_{qk} = \iint_s [Q^k_{\alpha\beta} + Q^k_{\beta\alpha} + \tilde{Q}^k_{\alpha\beta} + \tilde{Q}^k_{\beta\alpha}]\, ABd\alpha d\beta. \tag{1.10}$$

Here

$$\Psi^i_{\alpha\beta} = 0.5c_\alpha (a^i_1 + a^i_{20})\, (A^{-2} w^2_{,\alpha} + \nu_{\alpha\beta} B^{-2} w^2_{,\beta}) + c_{\alpha\beta} (a^i_3 + a^i_{19})\, (AB)^{-1} w_{,\alpha} w_{,\beta} +$$

$$+ 0.5c_\beta a^i_2 (B^{-2} w^2_{,\beta} + \nu_{\beta\alpha} A^{-2} w^2_{,\alpha}) \quad (\overleftrightarrow{i, j}),\; (\overleftrightarrow{\alpha, \beta}),\; (\overleftrightarrow{A, B});\quad Q^k_{\alpha\beta} + Q^k_{\beta\alpha} =$$

$$= c_\alpha (\varepsilon^0_\alpha + \nu_{\alpha\beta} \varepsilon^0_\beta)\, w_{,\alpha} a^k_1 + c_\beta 0.5\, (A^{-2} w^2_{,\alpha} + \nu_{\alpha\beta} B^{-2} w^2_{,\beta})\, a^k_7 + c_\beta a^k_8 0.5 \times$$

$$\times (A^{-2} \nu_{\beta\alpha} w^2_{,\alpha} + w^2_{,\beta} B^{-2}) + c_\beta (\varepsilon^0_\alpha \nu_{\beta\alpha} + \varepsilon^0_\beta)\, B^{-2} w_{,\beta} \Phi_{k,\beta} + (AB)^{-1} (w_{,\alpha} \Phi_{k,\beta} +$$

$$+ w_{,\beta} \Phi_{k,\alpha})\, c_{k,\alpha} \varepsilon^0_{\alpha\beta} + c_{\alpha\beta} a^k_9 A^{-1} B^{-1} w_{,\alpha} w_{,\beta} - P_z (\alpha, \beta)\, \Phi_k;\quad \tilde{\Psi}^i_{\alpha\beta} = 0.5c_\alpha (a^i_1 + a^i_{20}) \times$$

$$\times (a^i_{17} + \nu_{\alpha\beta} a^i_{18}) + 0.5 a^i_2 c_\beta (a^i_{18} + \nu_{\beta\alpha} a^i_{17}) + (a^i_3 + a^i_{19})\, c_{\alpha\beta} a^i_{19};$$

$$a_{13} = k_\alpha + \nu_{\alpha\beta} k_\beta;\quad a_{21} = k_\beta + \nu_{\beta\alpha} k_\alpha;\quad a^i_{14} = A^{-1} (k_\alpha \Phi_{i,\alpha} + k_{\alpha,\alpha} \Phi_i);$$

$$a^p_{15} = (AB)^{-1} k_\alpha \Phi_p B_{,\alpha};\quad a^i_{16} = (AB)^{-1} [A \Phi_{i,\beta} k_\alpha - k^2_\alpha \Phi_i (k_\alpha A^{-1})_{,\beta}];$$

$$a^k_{17} = B^{-2} \Phi_{k,\beta} w^0_{,\beta} - k_\beta \Phi_k;\quad a^k_{18} = (w^0_{,\alpha} w_{,\beta} + w_{,\beta} w^0_{,\alpha} + 2k_{\alpha\beta} w)\, (AB)^{-1};$$

$$a^k_{19} = k_\alpha k_\beta \Phi_l \Phi_m - B^{-1} \Phi_{k,\beta} k_\alpha \Phi_i;\qquad a^i_{20} = k^2_\alpha \Phi_{l,\alpha} \Phi_l - A^{-1} w_{,\alpha} \Phi_l k_\alpha.$$

Using the method of general iteration [4], the solution of the system of nonlinear relations in Eq. (1.8) is determined. If Eq. (1.8) is written in the form

$$q = B^{-1} R (q_j) = f (q_j),$$

iteration is performed according to the scheme

$$q^{j+1} = q^j + \beta_j (q^*_j - q_j);\quad q^*_j = f (q_j). \tag{1.11}$$

Iteration continues until the condition $\|q_j{}^* - q_j\| \leqslant \varepsilon$ is satisfied, where ε is the specified accuracy of solution of Eq. (1.8).

If the calculations must be performed for a sequence of loads or flexures, the initial approximation at an arbitrary stage of loading q_j^{0i} is found by extrapolating the solutions obtained from the previous stages of loading by means of the formula

$$q_j^{0i} = q_j^{i-1} + \frac{P_i - P_{i-1}}{P_{i-1} - P_{i-2}}(q_j^{i-1} + q_j^{i-2}). \tag{1.12}$$

For transition from one form of shell to another, it is sufficient to change the coefficients of the first quadratic form $A(\alpha, \beta)$, $B(\alpha, \beta)$ and their derivatives. The coefficients $A(\alpha, \beta)$, $B(\alpha, \beta)$ may vary from element to element.

2. Results of Calculations. Using a specially written program, calculations were performed for shells of rectangular plan with values of the coefficients of the first quadratic form $A(\alpha, \beta) = B(\alpha, \beta) = 1$ and with $A(\alpha, \beta) = 1$, $B(\alpha, \beta) \neq 1$. The program was tested on problems which may be solved by the finite-difference method [5] and the finite-element method [6].

The following notation is expedient: a, b are the dimensions of the shell in plan; $P(\alpha, \beta)$ is the transverse load; $\zeta = w/h_0$ is the dimensionless shell flexure; $\xi = \alpha/a$, $\eta = \beta/b$ are dimensionless coordinates; $\sigma_\xi = \sigma_\alpha b^2/Eh_0^2$, $\sigma_\eta = \sigma_\beta b^2/Eh_0^2$ are dimensionless total-stress parameters; σ_ξ^M, σ_η^M are the membrane stresses and σ_ξ^u, σ_η^u the flexural stresses in the direction of the coordinates ξ, η; $k_\xi = k_\alpha a^2/h_0$, $k_\eta = k_\beta b^2/h_0$ are curvature parameters; $p^* = pb^4/Eh_0^4$; $\lambda = b/a$; $\delta = h(\alpha, \beta)/h_0$ is the dimensionless thickness; $E_\alpha = E_\beta = E$, $\nu_{\alpha\beta} = \nu_{\beta\alpha} = \nu$.

In the calculations, it was assumed that

$$\lambda = 1;\ k_\xi = 0;\ k_\eta = 40;\ \delta = 1;\ \nu = 0.3.$$

The dependences $P^*(\zeta)$, $P^*(\sigma)$ for uniformly loaded hinged and rigidly fixed plates and cylindrical panels of rectangular plan were compared with the data of [5, 6]. In the linear case, the membrane stresses at the median line of half a truncated conical shell were compared [8].

Comparison of the data obtained in the present work with those of [5, 6, 8] shows agreement of the solutions at the center of the shell with a difference (of up to 5-8%) in the stresses at the contour [5, 6].

Consider a uniformly loaded shell of rectangular plan with a curvature varying according to the law $k_\eta = [1 - (\eta - 1)^n]^k$, $k_\xi = 0$, $n = 2$, $\eta = 2\beta b^{-1}$, $k_\eta = 40$; the shell thickness decreases linearly according to the law $\delta(\eta) = 2\delta_0(1-\eta)$, $0 \leqslant \eta \leqslant 0.5$, $\delta_0 = 1$ (Fig. 1). The influence of variability of the curvature, variability of the thickness, the presence of a hole, and a combination of conditions of rigid fixing and free edges on the stress–strain state of the shell is investigated.

The results of the investigation are shown in Figs. 1-3, where ζ_0, $\sigma_{\xi,0}$ are the flexure and stress at the center of the shell. In Fig. 1, curve 1 corresponds to the dependence $P(\sigma_{\xi,c})$ (load–stress at point C), curve 2 to $P(\sigma_{\xi,0})$ (load–stress at the center), and curve 3 to $P(\sigma_{\xi,b})$ (load–stress at point B). The coordinates of points C and B are ($\xi = 1/6$, $\eta = 0$) and ($\xi = 1/2$, $\eta = 0$), respectively. Comparison of the results for shells of constant curvature with the parameters $k_\xi = 0$, $k_\eta = 40$ (curve 1) and of variable curvature shows that the curve of $P^*(\zeta)$ for the second case lies considerably below the corresponding curve of $P^*(\zeta)$ for a cylindrical panel. The continuous curves in Figs. 1 and 2 correspond to solutions for shells with a hole and the dashed curves to those for shells without a hole.

It is evident from Fig. 2 that the presence of a hole has little influence on the shell flexure at a distance of 1/6 from the hole at point C. At point B, the curve of $P^*(\sigma_{\xi,b})$ for shells with and without a hole is almost the same. The largest total stress $\sigma_{\eta,0}$ is

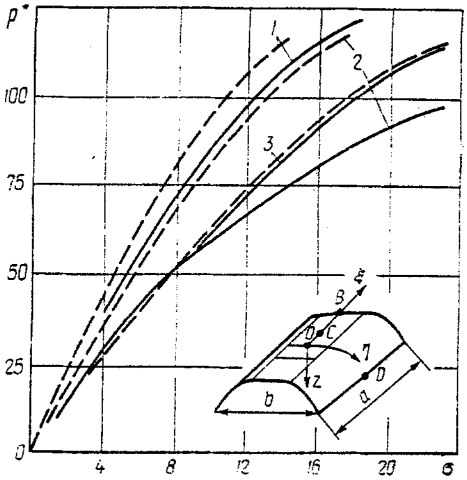

Fig. 1

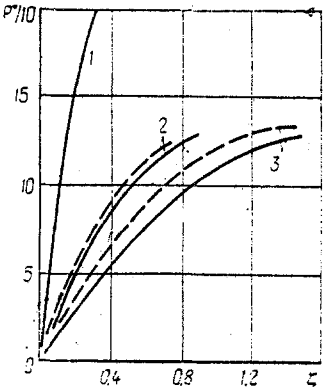

Fig. 2

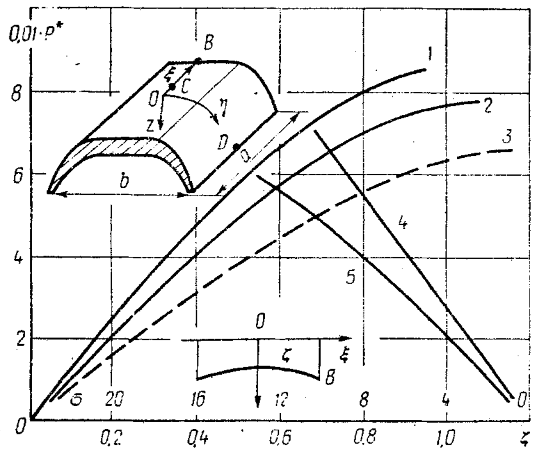

Fig. 3

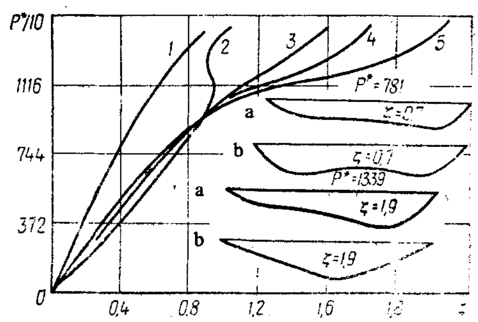

Fig. 4

obtained at the hole. Curve 2 in Fig. 2 corresponds to the dependence $P(\zeta_c)$ at point C (ξ = 1/6, η = 0), curve 3 to $P(\zeta_0)$ (load—flexure at the center), and curve 1 to $P(\zeta_0)$ for a continuous shell of constant curvature $k_\xi = 0$; $k_\eta = 40$. For a shell of variable thickness and curvature rigidly fixed along the generatrix and with free curvilinear edges, the maximum flexure is obtained at the free edge of the shell (at point B; Fig. 3). The normal stress $\sigma_{\xi,B}$ at the free edge is small in comparison with the other $\sigma_{\eta,B}$. Curve 1 in Fig. 3 corresponds to $P(\zeta_0)$, curve 2 to $P(\zeta_B)$, curve 3 to $P(\zeta^B_{c\xi})$, curve 4 to $P(\sigma^u_{\eta,D})$, and curve 5 to $P(\sigma^u_{\xi,B})$. The coordinates of points C, D, and B are (ξ = 1/6, η = 0), (ξ = 0, η = b/2), and

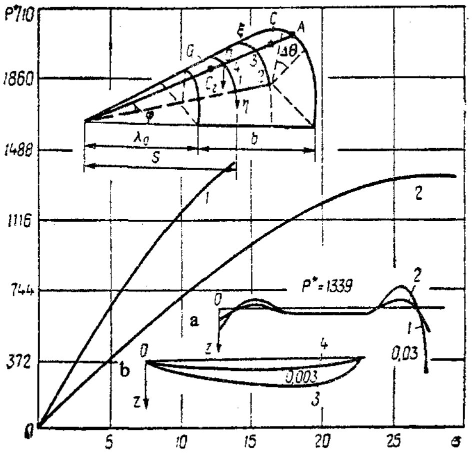

Fig. 5

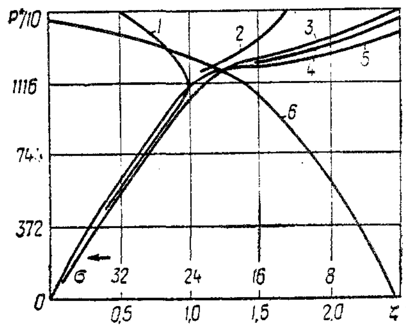

Fig. 6

(ξ = $a/2$, η = 0), respectively. The dashed curve in Fig. 3 shows the rotation $\zeta^{B}_{,\xi}$ at point B.

The dependence of the stress–strain state of truncated nonhollow conical shells on the presence of a hole may also be investigated. Boundary conditions of rigid fixing are considered, and the load is assumed to be uniform. The coordinate system and shell geometry shown in Fig. 5 are assumed. The finite element is determined by the dimensions $d\xi$, $d\eta$, i.e., Δb, $Bd\theta$, where $B(\xi, \eta) = \Delta b(\xi + \bar{S})\sin\Phi$, $\xi = \alpha/\Delta b$, $\bar{S} = S/\Delta b$, Δb or $\Delta\xi_H$ is the dimension of the element along the coordinate ξ; b is the length of the shell generatrix $\nu_{\alpha\beta} = \nu_{\beta\alpha} = \nu$; $A(\xi, \eta) = 1$; $E_\alpha = E_\beta = E$; $\xi_H = \lambda_0/B$; λ_0 is the distance from the vertex of the conical shell along the generator to the upper base; $h = h_0/b$. In the calculations, the following values are assumed: $\nu = 0.3$; $\xi_N = 2$; $\Phi = 0.205$; $\theta = \pi$; $\bar{\sigma}^u = \sigma^u(1 - \nu_{\beta\alpha}\nu_{\alpha\beta})/Eh_0$; $\sigma^M = \sigma^M(1 - \nu_{\alpha\beta}\nu_{\beta\alpha})/E$.

The results of the calculation are shown in Figs. 4-6, in the form of dependences $P^*(\zeta)$ and $P^*(\sigma^u)$ for various points of the shells. Curve 1 in Fig. 4 corresponds to $P(\zeta_N)$, curve 2 to $P(\zeta_E)$, curve 3 to $P(\zeta_C)$, curve 4 to $P(\zeta_L)$, and curve 5 to $P(\zeta_M)$; the coordinates of points N, E, C, L, M are (2, 0), $\left(\zeta_N + \frac{5}{6};\ 5\pi/6\right)$, $\left(\zeta_N + 1, \frac{\pi}{2}\right)$, $\left(\xi_N + \frac{5}{6};\ \frac{\pi}{2}\right)$, and $\left(\zeta_N + \frac{4}{6};\ \frac{\pi}{2}\right)$, respectively.

Curve 1 in Fig. 5 corresponds to $P(\sigma^u_{\xi,G})$ and curve 2 to $P(\sigma^u_{\xi,A})$. Curve 1 in Fig. 6 corresponds to $P(\zeta_E)$, curve 2 to $P(\zeta_C)$, curve 3 to $P(\zeta_L)$, curve 4 to $P(\zeta_F)$, curve 5 to $P(\zeta_M)$, and curve 6 to $P(\sigma^u_{\xi,A})$. In Fig. 4, curves of the flexure along the lines $\Theta = \pi/2$ (a) and $\xi = \xi_N + 5/6$ (b) at various stages of loading are shown. In Fig. 5a, b, curve 1 traces the flexural stress σ^u_ξ, while curve 2 traces σ^u_η, while curve 3 corresponds to the membrane stress σ^M_η and curve 4 to σ^M_ξ at the median line with $\Theta = \pi/2$. In the shell, there is an edge effect; the maximal flexural stress is at the contour.

Note that the largest flexure for a conical shell with a hole $\left(\xi_N + 1/3 \leqslant \xi \leqslant \xi_N + 1/2, \pi/3 \leqslant \Theta \leqslant \frac{2\pi}{3}\right)$ (Fig. 6) and without a hole (Fig. 4) is observed at point M $\left(\xi = \frac{4}{6} + \xi_N,\ \Theta = \pi/2\right)$, and then at point L ($\xi = \xi_N + 5/6$, $\Theta = \pi/2$).

In the course of loading, the flexure at points C, L, M increases but at point E ($\xi = \xi_N + 5/6$, $\Theta = 5\pi/6$) it begins to decrease at a certain time. The flexural and total stress at the contour in the shell with a hole is close to that in a shell with no hole. The calculations show that the largest stress σ^u_ξ, σ^u_η is obtained at the lower base of the truncated conical shell, while the greatest membrane stress is in the region of greatest flexure of the shell. The curves of $P^*(\zeta_C)$ at point C are close for shells with and without a hole.

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ANALYSIS OF SPHERICAL DOMES UNDER NONUNIFORM PRESSURE

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UDC 534.231

Investigation of the carrying capacity of spherical domes subjected to the pressure of snow is of practical interest.

Existing sources [1, 3] permit these loads to be given with a sufficient degree of approximation. Thus, for instance, a snow load of constant intensity can be assumed distributed uniformly over the horizontal projection of the dome only for very shallow surfaces. For domes with a greater rise, a symmetric snow load should be given in conformity with Fig. 1a: uniformly distributed and of constant intensity P_0 for $\theta \leqslant 25°$, and equal to zero for $\theta \geqslant 60°$. The load for surfaces with inclination from 25 to 60° is given by interpolation.

However, it is impossible to use formulas (105)-(108) in [3] in the analysis of domes under a nonsymmetric snow load. Indeed, if the normal component of the snow load equals $Z = 0.4P_0(1+\sin\theta\sin\varphi)$, then the maximal load turns out to be at $\theta = 0.5\pi$, i.e., on the vertical surface of the hemisphere where it has no horizontal components, while the minimal load is at the shell apex.

Using the indications of [3] as before, a nonsymmetric snow load can be obtained by transferring 40% of the snow from one side to the other of the dome as in Fig. 1b (inversely symmetric load) and adding to the axisymmetric as shown in Fig. 1c (one-sided load).

Taking all this into account, approximate formulas are proposed in [1] for the analysis of spherical domes under symmetric and nonsymmetric snow action, based on membrane theory [6]. Here the snow load is approximated by a continuous function which is not differentiable in the domain under consideration ($|\theta| \leqslant \pi/2$).

From the aspect of the general theory of shells [4], which is used in the method being proposed, such an approximation is not applicable. Hence, a snow load must be approximated not only by a continuous but also by a twice differentiable function which is selected as statically equivalent to the initial function [1].

Institute of Mechanics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Prikladnaya Mekhanika, Vol. 20, No. 1, pp. 78-83, January, 1983. Original article submitted October 13, 1982.

 0038-5298/84/2001-0074$08.50