STEADY STATE SCHEDULING OF A FLEXIBLE MANUFACTURING SYSTEM WITH PERIODIC RELEASING AND FLOW TIME CONSTRAINTS

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Abstract

This paper deals with the dynamic behaviour of an FMS when parts are released periodically. The minimal release period is induced by the most critical machine or fixture pallet type. Moreover, with a fixed period, the limited number of available pallets induces a maximal flow time for every part type. Thus, the objective of this paper is to determine a part release strategy and an activity schedule on every machine which allows to control every part flow time for steady state. An analysis method for this objective is presented in this paper. Its purpose is to obtain releasing and scheduling conditions ensuring a workshop steady state compatible with the considered constraints, especially part flow times. This method is based on the resolution of conflicts between activities processed on a common machine. This resolution uses limit times associated with each activity and it can modify these limit times. These modifications in turn can induce modifications for other activity limit times due to part routing, steady-state periodicity, and part flow time constraints. Thus, an iterative procedure has been defined. It refines steady-state feasibility conditions through limit times and sequencing conditions of activities processed on a common machine. The method is illustrated with two examples.

Keywords and phrases

FMS scheduling, periodic releasing strategies, FMS steady state.

1. Introduction

A flexible manufacturing system is generally dedicated to the production of various part types in small or medium quantities. It is characterized by a high level of automation which allows to successively process various part types with negligible set-up times or set-up costs. Thus, batch production is no longer necessary and various part types can be processed simultaneously in an FMS. Such a mixing allows to take account of the part routing heterogeneity more easily to increase machine utilization rate and to decrease the level of in-process parts.

In this paper, we consider the case where the FMS production objectives are expressed as the number of every part type to be performed over a certain horizon (for instance, 10 parts of type A, 20 parts of type B, 30 parts of type C per week). In such a case, the FMS's organization could aim at performing regularly the desired parts with the proposed ratios. Hitz [8,9] proposed a solution to this problem by releasing minimal part sets (MPS) periodically. An MPS is the minimal part set satisfying production ratios ($1A$, $2B$, $3C$ in the preceding example). It is this general release strategy that we use in this paper. With such a strategy, two types of questions occur:

- what is the FMS's behaviour when such a strategy is used?
- how to choose a release schedule and scheduling disciplines at the stations to obtain suitable performance for the FMS?

To the first question, answers can be found in [8,9] about the existence of a steady state for flow shops. In [7] and [10], a detailed analysis of the dynamic behaviour of an FMS with periodic releasing is developed. Upper bounds for the transient state duration and steady-state characteristics are determined. All these results use FIFO disciplines at the stations.

Solutions to the second question are more complicated. In [8,9], for the flow shop case, a branch and bound procedure is proposed to find a release sequence that is compatible with given in-process inventory levels while minimizing transient state duration. For both flow shop and job shop cases, a heuristic procedure can be found in [7] and [10] which attempts to reduce the work-in-process inventory level. FIFO disciplines are also used in these studies.

It must be noticed that if the priority rules are chosen a priori for the stations, it is not very easy to control the steady-state characteristics by means of the release sequence only [10]. If we intend to control the steady state, we need to define the release strategy and the scheduling rules for every machine.

But all the preceding results do not take into account one of the major characteristics found in an FMS: fixture pallets are very often needed to process parts. These pallets are available in a limited number. Thus, with the considered production objectives previously mentioned, pallet utilization induces a maximum flow time for the parts. Furthermore, it must be noticed that the work-in-process inventory level is also strongly connected with the part flow times.

This is the reason why, in this paper, we have chosen to take into account the pallet utilization problem and work-in-process level by defining a limited flow time for each part type. With these part flow times as data, the purpose of this paper is to search for periodic release strategies and scheduling conditions which ensure that a periodic steady state can be obtained. Only the job shop case is considered.

First of all, we give some feasibility conditions to choose these data, according to job shop and part characteristics. Thus, we assume in the following that the releasing period and the limited part flow times are given and have to be considered as constraints. With this as a basis, we propose a procedure to search for some characteristics of the part releasing and the activity scheduling which guarantee that there exists a steady state compatible with these constraints. These characteristics denote the influence of global constraints (machine utilization rate, part flow time) upon local decisions for releasing and scheduling (starting times, processing orders). Thus they can be used to search for releasing and scheduling strategies compatible with the given global constraints. The proposed procedure is based on results obtained for constraint-based analysis applied to job shop scheduling [4,5,6], which are summarized in sect. 3.

It should be noticed that the proposed approach does not try to define a unique solution as in the conventional schedule generation procedure [1,3], but gives information about characteristics of a set of releasing and scheduling strategies compatible with the considered constraints. Thus, such an approach could be very useful as a basis for real-time control strategy or adaptive scheduling, which is the final objective of our research.

2. Problem statement

2.1. PROBLEM DATA

A part I is characterized by its part routing, which is an ordered set of g_I activities. The jth activity of part I is denoted by (I, j) and it is characterized by:

- its processing time $p(I, j)$
- the machine $m(I, j)$ where it is processed.

Remark: It must be noticed that we restrict ourselves to the classical job shop in this paper. In fact, in actual FMSs, two sources of flexibility could exist:

alternate routing of parts

We assume that precise part routing has been chosen at another decision level (hierarchical decision structure) by taking account of the resource utilization problem [11]. At the considered decision level, an alternate routing would only be used as a possible solution when machine breakdown occurs. Furthermore, in practical situations, it seems that very often one of the alternate routes is considered to be more suitable.

machine groups (identical machines)

We intend to extend our approach to this case. Some of the following results, as critical period definition, are obvious for this general case. The principles for extension of constraint-based analysis to this general case have been defined [5], but their application to the particular problem considered in this paper has not yet been made.

Furthermore, it should be noticed that, due to tool availability and the limited size of tool magazines, machines are in fact very often dedicated to certain types of parts [2,11].

Let E be an MPS of N parts with Q types. E is made of N_A parts of each type A, in order to satisfy production ratios. The workshop is made of M machines. A fixture pallet type is associated with every part type; let b_A be the number of pallets of type A.

Sets E are periodically released with period T. E^{υ} is the υth released set and $(I,j)^{\upsilon}$ the jth activity of part I in the υth released set.

Let $c(I,j)^{\upsilon}$ be the starting time of activity $(I,j)^{\upsilon}$. It is assumed that the first activity of every part is performed on machine 1, named 'releasing machine': $m(I, 1) = 1 \;\forall I \in E$.

A periodic releasing with period T is characterized by:

$$\begin{cases} c(I, 1)^{\upsilon+1} = c(I, 1)^{\upsilon} + T & \forall I \in E \\ c(I, 1)^{\upsilon} < c(J, 1)^{\upsilon+1} & \forall I, J \in E . \end{cases}$$

To simplify presentation, transportation times are not taken into account, but they could be included without difficulty in the following results.

2.2. CONSTRAINTS ON THE RELEASING PERIOD

In order to obtain a steady state in the workshop, it is necessary that the amount of work associated with the periodic releasing can be performed by machines and pallets, which are limited resources. Non-blocking conditions can be expressed for machines and pallets:

(i) *non-blocking conditions for machines*

Let

$$P_k = \sum_{\substack{I \in E \\ m(I,j) = k}} p(I,j) \qquad k = 1, \ldots, M.$$

P_k is the work load on machine k induced by releasing a set E.

Thus we must have:

$$T \geqslant \max_{k = 1, \ldots, M} P_k = P_{m_c} .$$

So the releasing period has a lower bound issued from the machine with the greatest work load, named 'critical machine'.

(ii) *non-blocking conditions for pallets*

Let

$$P_A = \sum_{j=1}^{g_A} p(A, j) .$$

P_A is the total processing time of a part of type A. It represents the minimal flow time of this part in the workshop, which is the minimal utilization time of the pallet associated with type A.

Thus we must have:

$$b_A \cdot T \geqslant N_A \cdot P_A \qquad A = 1, \ldots, Q,$$

which implies:

$$T \geqslant \max_{A=1,\ldots,Q} \frac{N_A P_A}{b_A} = \frac{N_{A_c} P_{A_c}}{b_{A_c}} .$$

Then the releasing period has a lower bound issued from part type A_c, which is the more critical for pallet utilization.

By adding these two conditions, we conclude that the releasing period can be limited even by a machine or a part type. Finally we have:

$$T \geqslant \max \left[P_{m_c}, \frac{N_{A_c} P_{A_c}}{b_{A_c}} \right] = T_c .$$

T_c is the critical period. Every releasing such that $T < T_c$ leads to an unbounded increase of the number of parts waiting for a machine or a pallet. In the following, only periodic releasing such that $T \geqslant T_c$ is considered.

2.3. CONSTRAINTS ON THE PART FLOW TIMES

Let T be the releasing period chosen according to constraints presented in subsect. 2.2. Let D_I be the flow time of part I in the workshop. We get:

$$D_I = P_I + R_I ,$$

where R_I is the total waiting time of part I.

If we want to control part flow times in the workshop, we can choose a maximal flow time D_A for every part type A. Then the limited number of pallets imposes the following condition:

$$b_A T \geqslant N_A D_A \quad \text{or} \quad D_A \leqslant \frac{b_A T}{N_A} .$$

Thus, the number of available pallets imposes an upper bound on the part flow times. In particular, if period T is critical and induced by part type A_c, no waiting time is allowed for this type and $D_{A_c} = P_{A_c}$.

2.4. PROPOSED APPROACH

In the considered problem, there exists a strong connection between the releasing and scheduling policies and the steady-state characteristics (such as maximal number and mean number of parts waiting to be processed, part flow times). Thus, given a releasing period T chosen according to subsect. 2.2, it would be interesting to analyze the influence of maximal flow time constraints on releasing and scheduling policies. Such an analysis would be very complicated for the transient state, but it can be made easier if we restrict ourselves to the periodic steady state of period T such that:

$$\forall \upsilon,\ c(I,j)^{\upsilon+1} = c(I,j)^{\upsilon} + T, \qquad I = 1,\dots,N; \qquad j = 1,\dots,g_I .$$

Such an analysis is proposed in sect. 4. It is based on a 'constraint-based analysis', described in the following section. By using this analysis method, characteristics of activity schedules compatible with part flow time constraints are progressively defined. These characteristics are expressed as activity limit times and sequence conditions on machines.

The following notation is used:

$c^-(I,i)^\upsilon$ = earliest starting time of $(I,i)^\upsilon$;

$c^+(I,i)^\upsilon$ = latest starting time of $(I,i)^\upsilon$;

$f^-(I,i)^\upsilon$ = earliest finishing time of $(I,i)^\upsilon$;

$f^+(I,i)^\upsilon$ = latest finishing time of $(I,i)^\upsilon$;

with

$$f^-(I,i)^\upsilon = c^-(I,i)^\upsilon + p(I,i)$$
$$f^+(I,i)^\upsilon = c^+(I,i)^\upsilon + p(I,i)\,.$$

3. Principles for constraint-based analysis – [5]

This method is dedicated to scheduling problems where an earliest starting time and a latest finishing time can be associated with every activity, these activities being conflicting for resource utilization. In our case, resources are machines (see sect. 4).

The main basis of the method is to use an initial set of limit times to solve some conflicts and, in turn, to use solutions of these conflicts to update limit times. Thus, iterative procedures can be defined which progressively refine the characteristics of feasible schedules (limit times, sequence conditions). The main basis of the method is now described for the single machine scheduling problem.

N parts have to be scheduled on a single machine. The activity to perform part I is characterized by

- its processing time $p(I)$,
- its earliest starting time $c^-(I)$,
- its latest finishing time $f^+(I)$,

where $c^+(I) = f^+(I) - p(I)$; $f^-(I) = c^-(I) + p(I)$. Characterization of feasible schedules is obtained by considering conflicting activities for machine utilization.

3.1. CONFLICTING ACTIVITIES

Two activities I and J are absolutely ordered if:

$$f^+(I) \leqslant c^-(J) \quad \text{or} \quad f^+(J) \leqslant c^-(I)\,.$$

Two activities I and J are relatively ordered if there exists between them a sequence condition (part routing constraint, similar activities located in two MPS successively released....).

Two activities are conflicting if they are neither absolutely nor relatively ordered. Thus the analysis can be made by considering only conflicting activities, characterized by:

$$f^+(I) > c^-(J) \quad \text{and} \quad c^-(I) < f^+(J)\,.$$

3.2. FEASIBILITY CONDITIONS OF A SEQUENCE

The sequencing relationship "I precedes J" is denoted by $I \prec J$. Let I, J be an ordered pair of activities associated with a subset of activities $\mathcal{H}$. If

$$c^+(J) - f^-(I) < \sum_{K \in \mathcal{H}} p(K)$$

the sequences such that $I \prec \mathcal{H} \prec J$ are not feasible. The sequential condition prohibiting such sequences is denoted by $\mathfrak{D}(I, \mathcal{H}, J)$. Let $\{\mathfrak{D}\}$ be the set of conditions such as $\mathfrak{D}(I, \mathcal{H}, J)$. The following theorem can be proved [6] :

THEOREM

An N-activity sequence is feasible iff it satisfies the set of conditions $\{\mathfrak{D}\}$.

Thus, starting from an initial set of limit times, it is possible to characterize directly the feasible sequences; but this leads to a very awkward enumerative procedure, and gives no information about the feasible limit times. Nevertheless, it should be noticed that some sets of conditions $\mathfrak{D}$ can be used to update limit times and thus it is possible to define an attractive iterative procedure.

3.3. ITERATIVE PROCEDURE – EXAMPLE

Let I be an activity associated with a subset of activities $\mathcal{H}$. If

$$\max_{K \in \mathcal{H}} f^+(K) - \sum_{K \in \mathcal{H}} p(K) < f^-(I)$$

the sequences such that $I \prec \mathcal{H}$ are not feasible, and there exists a sequencing relationship between activity I and at least one activity of $\mathcal{H}$. So the limit time $c^-(I)$ may be updated. Such a condition is denoted by $\mathfrak{D}^\star(I, \mathcal{H})$ and it is such that:

$$\mathfrak{D}^\star(I, \mathcal{H}) = \bigcup_{K \in \mathcal{H}} \mathfrak{D}(I, \mathcal{H} - K, K).$$

A similar condition $\mathfrak{D}^\star(\mathcal{H}, I)$ can be obtained if:

$$c^+(I) < \min_{K \in \mathcal{H}} c^-(K) + \sum_{K \in \mathcal{H}} p(K),$$

so the limit time $f^+(I)$ can be updated. An interesting special case arises for a pair of activities when $c^+(J) < f^-(I)$, which leads to conditions such that

$$\mathfrak{D}^\star(I, J) = \mathfrak{D}(I, \emptyset, J) \Leftrightarrow J \prec I.$$

Then there exists a sequencing relationship between the two activities I and J. As a result, the limit times $c^-(I)$ and $f^+(J)$ can be updated.

The use of these updating conditions in an iterative procedure is illustrated in the following elementary example of four activities to be performed on a single machine. Data of the problem and the two steps of the iterative procedure are presented in table 1. At the first step, three conditions $\mathfrak{D}^\star$ with $|\mathcal{H}| = 1$ are found by

Table 1

Illustrative example for a single machine problem analysis

		Initial limit times							
I	p(I)	$c^-(I)$	$c^+(I)$	$f^-(I)$	$f^+(I)$	$c^-(I)$	$c^+(I)$	$f^-(I)$	$f^+(I)$
1	3	7	8	10	11	7	8	10	11
2	2	3	11	5	13	3	11	5	13
3	3	0	12	3	15	0	$\underline{10}$	3	$\underline{13}$
4	6	8	13	14	19	$\underline{10}$	13	$\underline{16}$	19
Sequencing relationships		$1 \prec 4$ $2 \prec 4$ $3 \prec 4$				$\mathfrak{D}^\star(1,\{2,3\})$ $\Leftrightarrow$ 2 or 3 $\prec 1$			

using initial limit times. Condition ($1 \prec 4$) allows to update limit time $c^-(4)$ and condition ($3 \prec 4$) allows to update limit time $f^+(3)$. The new value of $f^+(3)$ can be used at the second step to find a new condition $\mathfrak{D}^\star(1, \{2, 3\})$.

It is not possible to again update the limit times and so find other conditions.

The limit times and the sequencing relationships which have been found are necessary feasibility conditions. They define four sequences which can be feasible:

(2 3 1 4, 3 2 1 4, 2 1 3 4, 3 1 2 4).

In fact, in this example these four sequences are feasible, and each limit time is effective for at least one of these sequences.

4. Search for feasibility conditions of a periodic steady state

By applying the preceding results to periodic release strategies for FMSs, we will try to define necessary conditions for the feasibility of a periodic steady state. These conditions will be expressed as limit time values and sequencing relationships.

The successive updating of these limit times will increase the earliest starting times and decrease the latest finishing times, thus improving the definition of the time intervals allocated to the activities.

4.1. CONSIDERED CONSTRAINTS

4.1.1. Part flow times

As presented in the preceding sections, very often in an FMS, one of the objectives is that the flow time of a part I would be not greater than D_I. The proposed procedure will thus have to maintain the following relationships:

$$\forall I \in \{1, 2, \ldots, N\}$$

$$\forall \upsilon \geqslant 1$$

$$c^-(I, 1)^\upsilon \geqslant f^-(I, g_I)^\upsilon - D_I$$

$$c^+(I, 1)^\upsilon \geqslant f^+(I, g_I)^\upsilon - D_I .$$

Consequences of this statement are the following:

(1) Any increase of $f^-(I, g_I)^\upsilon$ may induce an increase of $c^-(I, 1)^\upsilon$:

$$c^-(I, 1)^\upsilon = \max \{c^-(I, 1)^\upsilon, f^-(I, g_I)^\upsilon - D_I\} .$$

(2) Any decrease of $c^+(I, 1)^\upsilon$ may induce a decrease of $f^+(I, g_I)^\upsilon$:

$$f^+(I, g_I)^\upsilon = \min \{f^+(I, g_I)^\upsilon, c^+(I, 1)^\upsilon + D_I\} .$$

4.1.2. Periodic steady state

We consider periodic steady states of period T only. Thus, any modification of an activity $(I, i)^{\upsilon_0}$ limit time must be systematically transmitted to all other activities $(I, i)^\upsilon$ by using the following relationships:

$$\forall \upsilon \in \mathbb{N}^\star,\quad c^-(I,i)^\upsilon = (\upsilon-\upsilon_0)T + c^-(I,i)^{\upsilon_0}$$

$$c^+(I,i)^\upsilon = (\upsilon-\upsilon_0)T + c^+(I,i)^{\upsilon_0}\ .$$

4.1.3. Part routing

It is obvious that an interval of at least $p(I, i)$ units of time must separate the starting times of the two activities $(I, i)^\upsilon$ and $(I, i+1)^\upsilon$. In the same way, the finishing time of activity $(I, i)^\upsilon$ is lower than the finishing time of activity $(I, i+1)^\upsilon$ minus $p(I, i+1)$. This induces the following statements:

(1) Any increase of $c^-(I, i)^\upsilon$ may induce an increase of $c^-(I, i+1)^\upsilon$:

$$c^-(I,i+1)^\upsilon = \max\{c^-(I,i+1)^\upsilon, c^-(I,i)^\upsilon + p(I,i)\}\ .$$

(2) Any decrease of $f^+(I, i+1)^\upsilon$ may induce a decrease of $f^+(I, i)^\upsilon$:

$$f^+(I,i)^\upsilon = \min\{f^+(I,i)^\upsilon, f^+(I,i+1)^\upsilon - p(I,i+1)\}\ .$$

4.2. PRINCIPLE OF THE PROCEDURE

4.2.1. Notion of 'pivot part'

The results we intend to obtain are interesting not for the absolute values of the limit times (which depend on the time origin), but for the amplitude and relative location of the time intervals associated with activities. This means that to study the periodic steady state, we can make an arbitrary choice for the time origin. Thus, this time origin can be the starting time of an arbitrary part I, which will be called the 'pivot part'.

In such a way, we will obtain a set $\mathcal{E}(I)$ of necessary conditions for the steady state compatible with the constraints of subsect. 4.1. These conditions will be sequence conditions and limit times $c^-(J, j)^\upsilon$ and $f^+(J, j)^\upsilon$ calculated by using a time origin such that $c^-(I, 1) = c^+(I, 1) = 0$. These limit times in fact represent amplitudes and locations of the time intervals associated with activities, when releasing time of part I is chosen as the time origin.

4.2.2. Limit time initialization

Part I is chosen as 'pivot part'; thus the release time of I is the time origin. Then part I, and the $(N-1)$ parts released after it constitutes an MPS; let us decide to call it MPS number one. Limit time initialization can be made according to the following:

(1) *For part I:*

We have $c^-(I, 1)^1 = c^+(I, 1)^1 = 0$.

Thus

$$\forall v \in \mathbb{N}^*, \quad \forall i \in \{1, 2, \ldots, g_I\}:$$

$$c^-(I, i)^v = (v-1)T + \sum_{k=1}^{i-1} p(I, k)$$

and

$$f^-(I, i)^v = (v-1)T + \sum_{k=1}^{i} p(I, k)\,.$$

Part I must have a flow time not greater than D_I. This implies

$$f^+(I, g_I)^1 = c^+(I, 1)^1 + D_I = D_I\,.$$

Thus

$$\forall v \in \mathbb{N}^*, \quad \forall i \in \{1, 2, \ldots, g_I\}$$

$$f^+(I, i)^v = (v-1)T + D_I - \sum_{k=i+1}^{g_I} p(I, k)$$

and

$$c^+(I, i)^v = (v-1)T + D_I - \sum_{k=i}^{g_I} p(I, k).$$

(2) *For any other part K:*

Any part K included in the $(N-1)$ parts following the pivot part I is released at an earliest time $p(I, 1)$ and at a latest time $T - p(K, 1)$.

Thus we have:

$$\forall v \in \mathbb{N}^{\star}, \quad \forall i \in \{1, 2, \ldots, g_K\}$$

$$c^{-}(K, i)^{v} = p(I, 1) + (v-1)T + \sum_{k=1}^{i-1} p(K, k)$$

and

$$f^{-}(K, i)^{v} = p(I, 1) + (v-1)T + \sum_{k=1}^{i} p(K, k).$$

As the part K must have a flow time lower than D_K, we have:

$$f^{+}(K, g_K)^{1} = c^{+}(K, 1)^{1} + D_K = T - p(K, 1) + D_K .$$

Thus

$$\forall v \in \mathbb{N}^{\star}, \quad \forall i \in \{1, 2, \ldots, g_K\}$$

$$f^{+}(K, i)^{v} = vT - p(K, 1) + D_K - \sum_{k=i+1}^{g_K} p(K, k)$$

and

$$c^{+}(K, i)^{v} = vT - p(K, 1) + D_K - \sum_{k=i}^{g_K} p(K, k).$$

4.2.3. *Analysis of conflicting activities*

According to subsect. 3.1, two activities $(H, h)^{k}$ and $(J, j)^{v}$ to be performed by the same machine are conflicting if they are not relatively ordered and if their time intervals are overlapping, that is to say:

$$f^{+}(H, h)^{k} > c^{-}(J, j)^{v} \quad \text{and} \quad c^{-}(H, h)^{k} < f^{+}(J, j)^{v} .$$

By applying the results presented in subsect. 3.3 to conflicting activities, we obtain three types of sequential relationships and associated updating conditions:

(i) if $\max\limits_{(H,h)^k \in \mathcal{H}} f^+(H,h)^k + \sum\limits_{(H,h)^k \in \mathcal{H}} p(H,h)^k < f^-(J,v)^v$

then there exists the condition $\mathfrak{D}^\star[(J,j)^v, \mathcal{H}]$, which in turn may update $c^-(J,j)^v$:

$$c^-(J,j)^v = \max\left\{c^-(J,j)^v, \min_{(H,h)^k \in \mathcal{H}} f^-(H,h)^k\right\}.$$

(ii) if $c^+(J,j)^v < \min\limits_{(H,h)^k \in \mathcal{H}} c^-(H,h)^k + \sum\limits_{(H,h)^k \in \mathcal{H}} p(H,h)^k$

then there exists the condition $\mathfrak{D}^\star[\mathcal{H}, (J,j)^v]$, which in turn may update $f^+(J,j)^v$:

$$f^+(J,j)^v = \min\left\{f^+(J,j)^v, \max_{(H,h)^k \in \mathcal{H}} c^+(H,h)^k\right\}.$$

(iii) if $c^+(J,j)^v < f^-(H,h)^k$

then there exists the condition $\mathfrak{D}^\star[(I,j)^v, (H,h)^k]$, which in turn may update $c^-(H,h)^k$ and $f^+(J,j)^v$:

$$c^-(H,h)^k = \max\{c^-(H,h)^k, f^-(J,j)^v\}$$

$$f^+(J,j)^v = \min\{f^+(J,j)^v, c^+(H,h)^k\}.$$

Remarks

(1) Two activities $(H,h)^1$ and $(H,h)^{1+k}$ are not conflicting activities.

(2) As soon as two activities are such that their time intervals do not overlap, they are no longer conflicting.

Thus, it is useful to construct, for any activity $(H,h)^1$, a set $F[(H,h)^1]$ of activities conflicting with $(H,h)^1$. Such a set will be updated when limit times are updated. $F[(H,h)^1]$ is made of all the activities $(J,j)^v$ performed by the same machine as $(H,h)^1$, such that:

$$\begin{cases} (J,j)^{\upsilon} \neq (H,h)^1 \\ \text{time intervals of } (H,h)^1 \text{ and } (J,j)^{\upsilon} \text{ are overlapping.} \end{cases}$$

It should be noticed that υ is initially lower than or equal to the number of the first MPS released after the latest finishing time of all the parts belonging to the first MPS.

4.2.4. *Summary of the procedure*

Due to the periodic nature of the steady states, it will be sufficient to study only conflicts between activities of the first MPS and activities belonging to the associated sets F. The evolution of the procedure can be summarized in the following diagram (fig. 1).

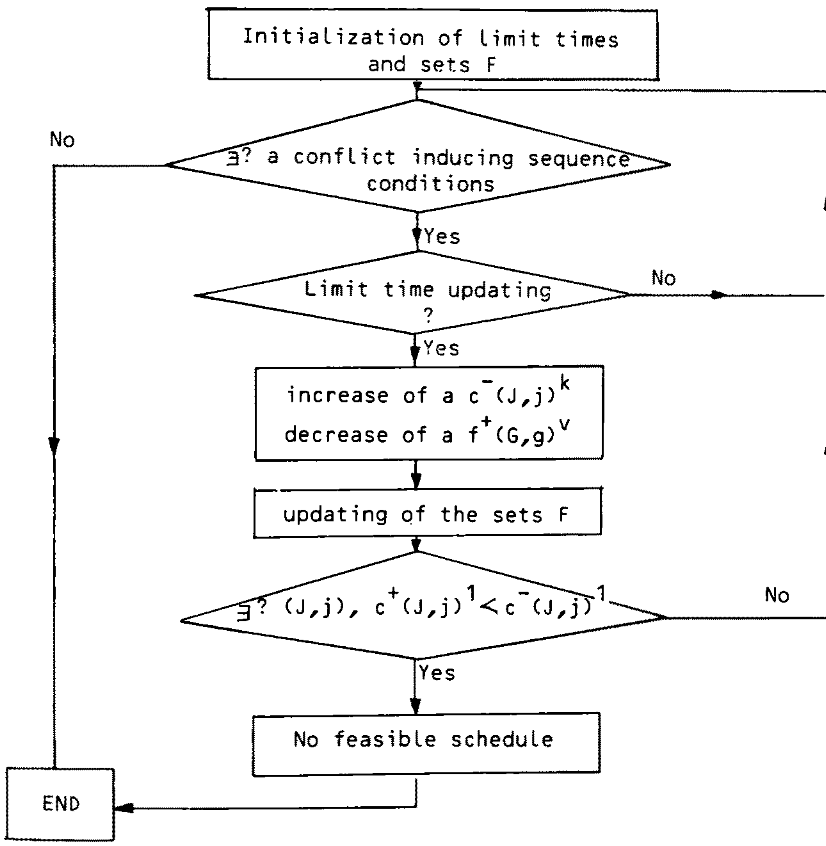

Fig. 1. Flow chart of the general procedure.

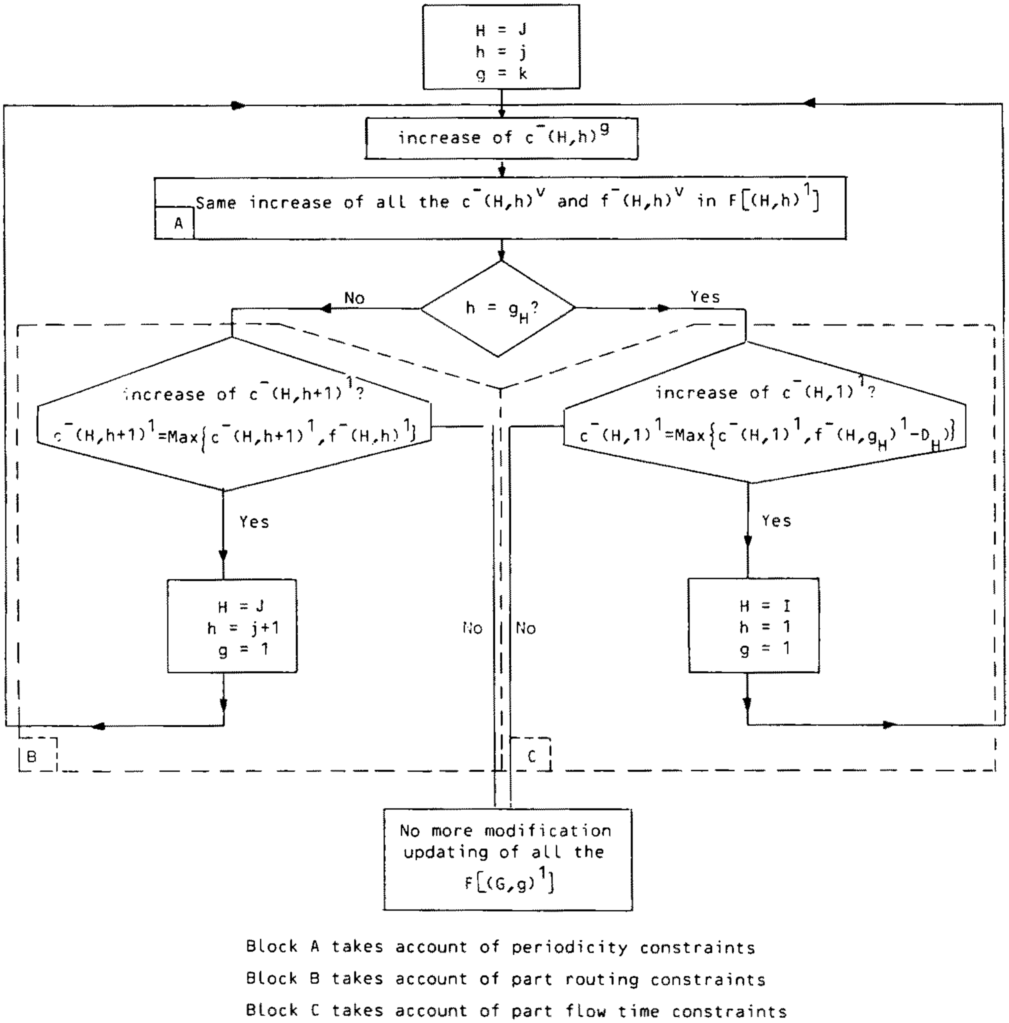

Fig. 2. Flow chart of the time updating procedure.

The procedure associated with an increase of a time $c^-(J,j)^k$ is described in the above diagram (fig. 2). A similar diagram can be defined for updating latest finishing times $f^+(J,j)^k$. In this case, the first test detects if the considered activity is the first one of J. In the case of a positive answer, connection is made to the flow time constraint in order to update, if necessary, $f^+(J, g_j)^1$; if not, connection is made to the part routing constraint in order to update, if possible, $f^+(J, j-1)^k$.

4.2.5. *Illustrative examples*

First example: Three parts have to be performed on three machines according to the following data:

$$[m(I,j)] = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \qquad [p(I,j)] = \begin{bmatrix} 2 & 4 & 2 \\ 2 & 3 & 4 \\ 4 & 3 & 1 \end{bmatrix}.$$

An MPS is made of one part of every type:

$$E = \{1, 2, 3\}.$$

Flow time of every part is minimal (no waiting time); this implies:

$$D_1 = 8, \quad D_2 = 9, \quad D_3 = 8.$$

The period is chosen critical (induced by machine 2): $T = 10$.

The analysis procedure is presented in table 2, with part 1 as the pivot part. The times updated at every iteration are underlined.

Let us consider, for instance in the first iteration, the sequencing relationship $(3, 2)^1 \prec (1, 2)^2$. This relationship is induced by $c^+(3, 2)^1 < f^-(1, 2)^2$ and induces an updating of $f^+(3, 2)^1$. By taking account of part routing constraint or flow time constraint, $f^+(3, 3)$ is in turn updated. The procedure converges in four iterations and defines a unique steady state compatible with the constraints (fig. 3). This very simple example is strongly constrained and illustrates the various mechanisms used in the analysis procedure. It should be noticed that in this case only sequencing relationships such that $\mathfrak{D}^*(I, J)$ are used.

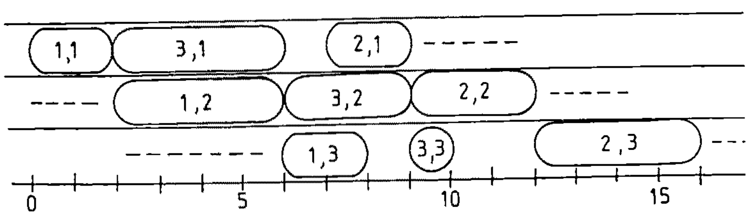

Fig. 3. Bar chart of the steady state.

	(J,j)	p(J,j)	$c^-(J,j)^1$	$f^+(J,j)^1$	$c^-(J,j)^2$	$f^+(J,j)^2$	$c^-(J,j)^1$	$f^+(J,j)^1$	$c^-(J,j)^2$	$f^+(J,j)^2$
m(J,j) = 1	(1,1)	2	0	2	10	12	0	2	10	12
	(2,1)	2	2	10	12	20	4	9	14	19
	(3,1)	4	2	10	12	20	2	9	12	19
m(J,j) = 2	(1,2)	4	2	6	12	16	2	6	12	16
	(2,2)	3	4	13	14	23	6	12	16	22
	(3,2)	3	6	13	16	23	6	12	16	22
m(J,j) = 3	(1,3)	2	6	8	16	18	6	2	16	18
	(2,3)	4	7	7	17	27	9	16	19	26
	(3,3)	1	9	14	19	24	9	13	19	23
sequencing relationships inducing time updating			$(1,2)^1 < (2,2)^1$ $(2,2)^1 < (1,2)^2$ $(3,2)^1 < (1,2)^2$				$(3,1)^1 < (2,1)^1$			
			Step 0				Step 1			

	(J,j)	$c^-(J,j)^1$	$f^+(J,j)^1$	$c^-(J,j)^2$	$f^+(J,j)^2$	$c^-(J,j)^1$	$f^+(J,j)^1$	$c^-(J,j)^2$	$f^+(J,j)^2$
m(J,j) = 1	(1,1)	0	2	10	12	0	2	10	12
	(2,1)	6	9	16	19	7	9	17	19
	(3,1)	2	7	12	17	2	6	12	16
m(J,j) = 2	(1,2)	2	6	12	16	2	6	12	16
	(2,2)	8	12	18	22	9	12	19	22
	(3,2)	6	10	16	20	6	9	16	19
m(J,j) = 3	(1,3)	6	8	16	18	6	8	16	18
	(2,3)	11	16	21	26	12	16	22	26
	(3,3)	9	11	19	21	9	10	19	20
sequencing relationships inducing time updating		$(3,2)^1 < (2,2)^1$							
		Step 2				Step 3			

Table 2. Illustrative example for a job shop periodic steady-state analysis.

Second example: Three parts have to be performed on four machines according to the following data:

$$[m(I,j)] = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & - \\ 1 & 3 & 2 & 4 \end{bmatrix} \qquad [p(I,j)] = \begin{bmatrix} 2 & 3 & 3 & 2 \\ 2 & 7 & 2 & - \\ 2 & 3 & 4 & 2 \end{bmatrix}$$

An MPS is made of one part of every type:

$$E = \{1, 2, 3\}.$$

Part flow times are the following:

$$D_1 = 12, \quad D_2 = 11, \quad D_3 = 14.$$

The period is chosen critical (induced by machine 2): $T = 13$.

The analysis procedure is presented in table 3, with part 2 as the pivot part. The times updated at every iteration are underlined.

Table 3

	$p(J,j)$	c^-	f^+	c^-	f^+	c^-	f^+	
$(1,1)^1$	2	2	13	2	$\underline{9}$	2	9	
$(2,1)^1$	2	0	2	0	2	0	2	$m = 1$
$(3,1)^1$	2	2	13	$\underline{4}$	$\underline{12}$	4	12	
$(1,2)^1$	3	4	18	4	$\underline{12}$	4	12	$m = 2$
$(3,3)^1$	4	7	23	$\underline{12}$	$\underline{22}$	12	$\underline{20}$	
$(1,3)^1$	3	7	21	$\underline{9}$	$\underline{15}$	9	15	
$(2,2)^1$	7	2	9	2	9	2	9	$m = 3$
$(3,2)^1$	3	4	19	$\underline{9}$	$\underline{15}$	9	15	
$(1,4)^1$	2	10	23	$\underline{12}$	$\underline{19}$	12	19	
$(2,3)^1$	2	9	11	9	11	9	11	$m = 4$
$(3,4)^1$	2	11	25	$\underline{16}$	$\underline{24}$	16	$\underline{22}$	
Sequencing relationships inducing time updating		$(2,2)^1 < (1,3)^1$ $(1,3)^1 < (2,2)^2$ $(2,2)^1 < (3,2)^1$ $(3,2)^1 < (2,2)^2$		$(3,4)^1 < (2,3)^2$				
		Step 0		Step 1		Step 2		

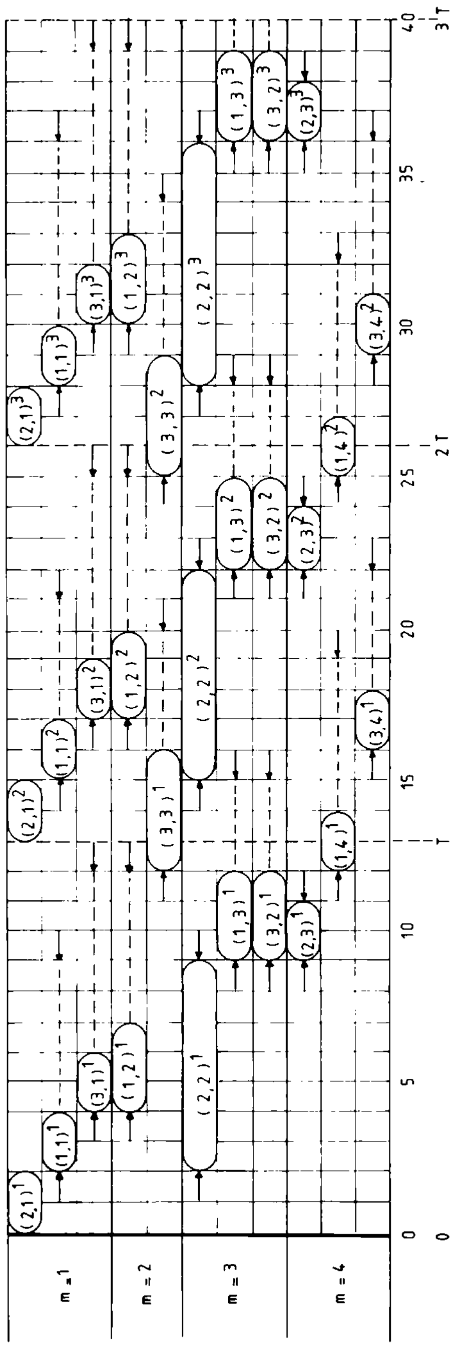

Fig. 4. Bar chart of the steady-state characteristics of activities.

The procedure converges in two iterations. In this case, parts are totally ordered on machine 2 ((1, 2) $\prec$ (3, 3)) and on machine 4 ((2, 3) $\prec$ (1, 4) $\prec$ (3, 4)). Possible permutation remains on machine 1, between (1, 1) and (3, 1) and on machine 3, between (1, 3) and (3, 2). However, it should be noticed that these permutations can not be chosen independently. Thus, only three sequences are feasible. Figure 4 gives for every activity the time interval on which it has to be located.

5. Conclusions

The steady state analysis proposed in this paper can be made by using any part as a pivot. Results obtained for a chosen pivot part are only necessary conditions for feasibility of activity schedules in steady state. Thus, different conditions can be obtained when the choice of the pivot part is modified.

Studies are now being developed in order to be able to make a synthesis of the different results issued from different choices of the pivot part. This synthesis will define conditions for the relative locations of activities, independent of time origin. These conditions are related to some characteristic parameters of the releasing sequence. Use of these conditions for the definition of a releasing sequence and scheduling strategies on the machines is now being studied and we intend to develop software on these bases.

References

[1] K.R. Baker, *Introduction to Sequencing and Scheduling* (Wiley, New York, 1974).

[2] M. Berrada and K. Stecke, A branch and bound algorithm for FMS machine loading, *Proc. 1st ORSA/TIMS Conf. on Flexible Manufacturing Systems*, Ann Arbor, USA (1984).

[3] R. Conway, M. Maxwell, L. Miller, *Theory of Scheduling* (Addison Wesley, 1967).

[4] J. Erschler, F. Roubellat and J.P. Vernhes, Finding some essential characteristics of the feasible solutions for a scheduling problem, Oper. Res. 24, 4(1976)774.

[5] J. Erschler, G. Fontan and F. Roubellat, Potentiels sur un graphe non conjonctif et analyse d'un problème d'ordonnancement à moyens limités, RAIRO, série verte, vol. 13, no. 4 (1979).

[6] J. Erschler, F. Roubellat and J.P. Vernhes, Characterizing the set of feasible sequences for *n* jobs to be carried out on a single machine, Euro. J. Oper. Res. 4, 3(1980) 189.

[7] J. Erschler, D. Levèque and F. Roubellat, Periodic loading of flexible manufacturing systems, *Advances in Production Management Systems* (Bordeaux, 1982).

[8] K.L. Hitz, Scheduling of flexible flow shops, Tech. Rep. 879, L.I.D.S. Massachusetts Institute of Technology (March, 1979).

[9] K.L. Hitz, Scheduling of flexible flow shops II, Tech. Rep. 1049, L.I.D.S. Massachusetts Institute of Technology (October 1980).

[10] D. Levèque, Lancement périodique de produits dans un atelier flexible, Thèse de Docteur-Ingénieur, Toulouse (1982).

[11] K. Stecke, Formulation and solution of nonlinear integer production planning problems for FMS, Management Science 29, 3(1983)273.