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IDENTITY AND MODALITY

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§1. *Modal theory and dyadicity.* Under their intended interpretation the axioms of the identity sign, '=', seem not especially well-served by their customary, model theoretic, semantic construal. On the face of it this is a dark and ominous saying, for it would appear that under their customary semantic construal, the identity axioms are anyhow *true.* One can but wonder what greater service is demanded of semantics and how, if that service is found wanting, a reasonable prescription for its improvement might go. All the same, on at least two counts identity is hard to make out. For one, there is the question of how the identity relation can be both dyadic and reflexive. But there is also a worry as to how statements of identity can agree in truth-value, yet differ in modal value in the special case where these statements differ from each other in their syntactic composition only at co-referential singular terms (in name only, as we shall say for short). Even if it proves to be genuine, the trouble does not obviously reveal its source, for if it can safely be presumed that something is amiss with the identity axioms on their intended interpretation, it may not be obvious whether we are to indict the axioms for failing our intuitions, or to fault the intuitions as unworthy of serious axiomatization.

Identity is axiomatized by four conditions, three of which proclaim it an equivalence relation, that is, a relation that is reflexive, symmetrical and transitive; and a fourth, by which things identical with one another have all their properties in common. Since identity is, or is taken to be, an equivalence relation (-in-extension), it exhaustively partitions its field, F, into the disjoint classes K_1, K_2,.... The K_i, which are sometimes called equivalence classes with respect to the given equivalence relation, are, in the case of identity, unit classes or singletons, classes with but one member. It is this that is striking, and unique, about identity, for there are consistent conditions under which any other equivalence relation would partition its field into equivalence classes of a cardinality of two or higher. Identity is that equivalence relation alone whose class of equivalence classes can contain naught but singletons.

Some equivalence relations are dyadic, and this, we deem it reasonable to suppose, means that such relations may obtain between objects taken pairwise from their fields. Thus, the material biconditional is dyadic, and it holds, when it does, between pairs of statements. Being of the same height is likewise two-place and obtains only between couples of the equitall. Identity is fashioned in like manner; it is made out to be dyadic, if only because it obtains between Cicero and Tully, Elizabeth II and Elizabeth Windsor, Venus and Hesperus, and so on and on. It is not hard to appreciate that a judgement of dyadicity in this sense is a syntactic verdict that is not easily upheld in the higher semantic courts. For, the identity relation does *not* obtain pairwise between elements from its field; identity's equivalence classes are singletons, never couples. Nor does this appear to be but an isolated quirk of identity, to be dealt with perhaps by special pleading. Every dyadic equivalence relation is reflexive, and so obtains between every element of its field and itself. But classes are not enlarged by repetitions, and the class of a given thing together with itself is never a pair, never other than a singleton. Thus, there is a paradox involved in our understanding of the identity relation.

Philosophers have learned that a paradox need not be an antimony, and so have learned a certain poise in the face of newly unearthed departures from the expected or the desirable. In the present case, it is a fairly straight-forward matter to muster support for the claim that our paradox is only apparent, and that it rests upon too shallow an understanding of dyadicity; and for the accompanying judgement, too, that where paradox proceeds from superficiality, there were but confusion and half-baked intuitions. Witness Peirce[1] :

> The Dyad is a mental Diagram consisting of two images of two objects, one existentially connected with one member of the pair, the other with the other, the one having attached to it, as representing it, a Symbol whose meaning is "First", and the other a Symbol whose meaning is "Second".

Of relations, it is our current understanding, that it to say, a myth that presently recommends itself over the alternatives, that they are classes of sequences, classes of those classes containing in some definite order those items between or among which the relation is taken intuitively to obtain. On this model, dyadic relations, for example, are construed as classes of *ordered couples*. This is not yet to advance beyond the paradox of the paragraph above, for if it is wrong to say that identity holds between such "pairs" of objects as Cicero and himself, it is at this stage of the enquiry not obviously less wrong to countenance the *ordered* "pair" Cicero and himself in that, or any, sequence. The one is no less counterfeit a couple than the other.

Ordered pairs require a construal that preserves both the dyadicity and the reflexivity of identity, as well as providing some semblance of the intuitive notions of order and coupleness. A third desideratum would be that ordered pairs be made out to be a breed of object that is reasonably well understood, as well understood, say, as classes are. Following Kuratowski we might say that the ordered pair $\langle x, y\rangle$ just is the ordinary couple, K, $\{\{x\}, \{x,y\}\}$. Now K is plainly a couple, and it is easy to associate with K an effective recipe for ordering.[2] What is more, Kuratowski classes satisfy the prime postulate on ordered pairs:

(1) $\langle x, y\rangle = \langle z, w\rangle$ only if $x = z$ and $y = w$.

It must be said, however, that Kuratowski-classes do not contain as members the objects that, intuitively speaking, go to make up the sequences with which they are identified. The ordered pair ⟨Cicero, Cataline⟩ contains neither Cicero nor Cataline, but only classes of these. This is not (yet) cause to repudiate Kuratowski's treatment of ordered pairs; indeed, for all we really know of sequences, this is a desirable or anyhow a mathematically tolerable consequence. However, in the case of couples that are members of the reflexive wing of a dyadic relation, we have ordered pairs $\langle x, x\rangle$, whose K-equivalent is $\{\{x\}, \{x,x\}\}$: but this just is the class $\{\{x\}\}$, or in some set theories (x), or even x itself,[3] and withal our second desideratum, conservation of some intuitive semblance of twoness and order, is failed on each count. And if in fact we, and model theory, are right to think that every equivalence class K_1, K_2,... with respect to the identity relation is a singleton, we have the ironical-seeming result that in the very respect in which the K-classes of Kuratowski overturn our intuitions of dyadicity,[4] the equivalence classes, K_i, hold fast to that primordial attribute of identity, namely, that when it obtains it does not relate couples. In some set theories[5] the equivalence class K_i associated with a truth of identity just is its associated K-class as well.

§2. *Wiener-classes.* If, in light of this last point, construal as K-classes were to be thought inadequate to our understanding of ordered pairs, we might choose an earlier, non-equivalent, strategem, due to Wiener, by which the ordered couple $\langle x,y\rangle$ is identified with the ordinary class $\{\{x\}, \{y, \Lambda\}\}$ where Λ is the empty class. This at once restores some appearance of naturalness to repeating sequences; the couple $\langle x,y\rangle$ emerges here as the genuine pair $\{\{x,\{x,\Lambda\}\}$ of manifest order, by the algorithm of four footnotes above. So we might do better to opt for Wiener's W-classes over K-classes. But no; for recall that model theory reconstructs the identity relation so as to make singletons out of its equivalence classes. Now, an equivalence class with respect to identify is precisely that class whose element has that relation borne to something. And if the relation with respect to that equivalence class is the class of W-classes "of" its relata, we now have the awkward consequence that the equivalence class K_i cannot be the appropriate W-class, even though, intuitively, they would seem to be one and the same, being, each of them, the class of that thing to which the identity relation is borne.[6] On the contrary, the former turns out to be a member of the latter. So now the difficulty seems to be that W-classes well enough serve our intuitions concerning the dyadicity of equivalence relations in general, only to fail us when the equivalence relation happens to be identity.

If we had solid and clear beliefs concerning dyadcitiy, identity, order and the like, if we could but confidently, and in general, distinguish the *sine qua non* from what Quine[7] has called "don't cares", it might be possible for us to abandon the cautious subjunctive seemings of these past few pages for rather more authoritative assurances of real trouble with identity theory. But since these antecedents are so clearly unrealized, it is left to us to philosophize. It is necessary once more to scrutinize our intuitions and to seek for their regimentation via new or anyhow different perspectives.

We face something of a dilemma. By model theory and Kuratowski, $\ulcorner\alpha = \beta\urcorner$ can be interpreted consistently and aright, but identity cannot be dyadic. By model theory and Wiener, identity is dyadic, but $\ulcorner\alpha = \beta\urcorner$ can be interpreted only incongruently, as involving a class that, by model theory is a singleton and, by Wiener, a pair. Either way, there is a question that cannot easily be slipped: is there for identity a uniform semantics by which truths of identity differing (syntactically) in name only can differ (semantically) as to modality?

It is commonly agreed that the Wiener- and the Kuratowski- treatments of sequences are, though non-equivalent, equally good, somewhat as Zermelo's and von Neumann's treatments of the natural numbers, though non-equivalent, are equally good. And since an understanding of our question does not seem to be substantially affected by whether we favour W-classes over K-classes, or vice-versa, we shall, when speaking of sequences, follow the more common practice of regarding them as K-classes.

To ask our question concerning identity may on first sight suggest the possibility that the symbol '=' is ambiguous; for to maintain concerning truths of identity that on their face differ in name only, the possibility of modal differences, amounts to rejecting any theory which accords to their proper parts the same semantic assignments, names to the same bearer, predicates to the same relations, quantified variables to the same values. That in turn argues the doubtfulness of the claim that these are sentences differing in name only, i.e. that are syntactically alike save for different (but co-referential) names. Thus, if "Cicero = Tully" and "Cicero = Cicero" are both truths of identity that differ in name only, a first occurrence of 'Tully' for a second of 'Cicero', and whose respective components differ, semantically speaking, not at all; and if each sentence has as its associated equivalence class the class whose sole member is that self-same Roman plagiarist of old, and if each has associated with it a sequence that is identified with a K-class whose sole member in each case is, again, that very Roman, then it is not easy to see why the one sentence should be counted necessary, and the other not. Any modal policy that cheerfully acquiesced in such a circumstance would run the risk of leaving the introduction of modal distinctions semantically unmotivated. It would also risk having to make it reasonable to believe that the modality of a sentence is not determined by the semantic assignments made to its components, which in turn cannot but prompt the question: upon what supracomponential basis shall we make semantic assignments to two sentences differing in name only, whose semantic profiles, componentially computed, are indistinguisable? One possible solution might be to put it that in every possible world, or, to take special note or our own puzzle-cases, in every possible world in which 'Cicero' and 'Tully' refer, repeated occurrences of a name in a given sentence refer to one and the same thing, whereas those of different names, do not. Thus is essayed a distinction between contingent and necessary co-referentiality over the class of names. But that would be to place an enormous burden on the most generous conception of possible worldhood. It is not even slightly less difficult to conjure up a world such that for every sentence of English in which a given name occurs twice or more, there are exactly that many distinct objects named, than it is to suppose a possible world with respect to which a sentence's every different name refers to a different object. It is incomparably more difficult (assuming indices to be names) to imagine a world in which every sentence containing different names refers to just one thing. In such a world, for example, proclamations of love are never less narcississtic than true.

No non-frivolous solution can, of course, just *that* quickly be dismissed; and so it cannot automatically be assumed that the sentences $\ulcorner\alpha = \alpha\urcorner$ and $\ulcorner\alpha = \beta\urcorner$ must receive identical componential semantic assignments. Indeed, as we said, '=' *may* occur ambiguously from $\ulcorner\alpha = \alpha\urcorner$ to $\ulcorner\alpha = \beta\urcorner$, and that, if true, could well be the beginnings of an entirely satisfactory account of their modal difference. It does not particularly matter, for now, that the structure of the suppossed ambiguity does not leap out at one, for it may lie concealed at fairly deep levels.[8] What is more, there is no special reason to think that this would be an ambiguity announced by clear or obvious symptoms. On the contrary, our policy is to search it out, if it exists, not just to assume that it must exist. In fact, it may emerge that we decide to say that the contexts ⌜Necessarily...⌝ and ⌜Contingently...⌝ serve to disambiguate identity statements; but we are being previous.[9]

Of course, we should be careful not to pick our targets too finely. True, there are puzzling differences as between $\ulcorner\alpha = \alpha\urcorner$ and $\ulcorner\alpha = \beta\urcorner$ to which the supposition of ambiquity may be relevant. But just what *bears* this ambiguity might not be so easily isolated. It may well be the '=' of "$\ulcorner\alpha = \alpha\urcorner$" and "$\ulcorner\alpha = \beta\urcorner$"; but it might also be the expression "of identity" as in "Both

$\ulcorner \alpha = \alpha \urcorner$ amd $\ulcorner \alpha = \beta \urcorner$ are statements *of identity*". Indeed it might turn out that '=' does not even occur in $\ulcorner \alpha = \alpha \urcorner$ but rather that only α and $\ulcorner = \alpha \urcorner$ do, the latter intimating, not identity, but a slyly different notion of *self*-identity. So we would do well for the time being to speak less specifically of simply "an ambiguity concerning $\ulcorner \alpha = \alpha \urcorner$ and $\ulcorner \alpha = \beta \urcorner$".

§3. *A simple regimented language.* It is necessary to determine whether there is an ambiguity concerning $\ulcorner \alpha = \alpha \urcorner$ and $\ulcorner \alpha = \beta \urcorner$. To this end, we assume a large sublanguage of English, L, regimented by the basic logical apparatus of first order logic with individual constants (or names), but (for now) no definite descriptions, and with an identity-sign '=', the axioms for which are as yet unspecified. In particular, we suppose that:

I. $\ulcorner \textcircled{1} = \textcircled{2} \urcorner$ is the *identity-frame*, and individual variables upon entering the identity-frame in either place admit as substituends individual constants or individual variables and only these. Such formulas are also called *instances* of the identity-frame.

II. Only closed instances of the identity-frame and truth functional and quantificational constructions on them shall be reckoned to be identity-sentences.

III. All variables entering the identity-frame are to be thought of as having values, and all individual constants bearers or denotations; *unum nomen, unum nominatum.*

One final preliminary word. Ours is a relatively modest regimented language within which to conduct the examination of identity. It is perhaps less modest than impoverished for it is blind to such putative identities as,

(1) The number of planets is 9;

for either (1), like

(2) There are just 9 planets,

it is not an identity-statement, or (1) is an identity sentence containing a definite description.[10] The restriction against definite description frees *L* from such vexing difficulties as that from the truth of

(3) Identity is universally substitutive

and the necessity of

(4) $9 < 11$,

it is inescapable, and undesirable, that we have, from (1), the necessity of

(5) The number of planets <11.

Of course it can always be protested, in an apt phrase of Russell's concerning other matters, that our policy of not recognizing definite descriptions has none but the virtues of theft over honest toil. Indeed, to banish a problem is seldom the better way of dealing with it, and, although what in general to do with definite descriptions is still under advisement, (the most popular option being to banish them), it will suffice to note that the kind of problem illustrated by our reluctant passage to (5) can be generated in a description-free language similar to *L* in all other respects and so owes nothing very special to the availability or descriptions. For present purposes, descriptions need not trouble us.

An identical policy governs the remaining restriction. None of the problems peculiar to the identity relation *obviously* waits upon solutions to such puzzles as how best to deal with empty singular terms, and so we shall not worry about them either.[11]

§4. *Paradoxes of Identity.* The identity-relation is not a stranger to philosophical trouble. Frege could hardly have understated it more when he remarked that identity "gives rise to challenging questions which are not altogether easy to answer."[12] Frege, for one, wished to know[13] how the two identities

(6) $\alpha = \alpha$

and

(7) $\alpha = \beta$

should refer to exactly the same object, x, and to ascribe to x the very same thing (whatever that might be), and yet be such that only (7) is "informative". Not the least puzzling part of Frege's complaint derives from the unanalyzed state in which we meet the concept of *information*. But it is clear enough that an important aspect of Frege's concern deals with the (disputable) epistemological thesis that, whereas (6) is knowable apriori, (7) is knowable only aposteriori. Seen in this light, Frege's problem with identity is but a special case of the Paradox of Analysis and may accordingly be dubbed "an epistemic paradox". Even so, at least to the extent that Frege championed classical empiricistic causes to the effect, first, that all and only those truths knowable apriori are necessary, and, second, that all and only those truths that are necessary are analytic (i.e. logical truths of quantification theory) it is easy to associate, if not identify with, the epistemic paradox a modal counterpart, thus: If (6) is apriori, then by the one empiricist dogma, it is necessary, and so, by the other, analytic. There is, moreover, a standard conception of information, due to Shannon and Weaver[14] and others, by which analytic truths contain zero bits of information, which is certainly the kind of way in which Frege saw (6). This, despite the anachronism, somewhat strengthens the case for saying that Frege's reaction to the epistemic paradox was partly (probably very largely) a response to what I shall call the *modal paradox*[15] of identity, a famous formulation of which can be found in Carnap.[15] Carnap, among a host of others, supposed it natural (and true) to say of any two names α and β that

(8) If α and β name the same object then any and all truths remain true upon replacement in any and all contexts of α by β and of β by α.

Condition (8), which Carnap called "the Principle of Interchangeability" reflects a deep intuition to the effect that the identity-relation conforms to the principle of the indiscernibility of identicals; or, as it sometimes (though wrongly) is put, that identity is universally substitutive.

Now given

(9) Necessarily, $\alpha = \alpha$,

it is immediate from (7) and (8), by replacement of the second occurrence of 'α by 'β' that

(10) Necessarily, $\alpha = \beta$.[16]

It is useless, or anyhow questionable strategy at this stage, either to inveigh against (10) as manifestly preposterous, or to seek to champion it as logically misunderstood. It is true, of course, that sentences in the form of (10) are consequences – desirable consequences, in fact – of most systems of quantified modal logic (*qml*) with identity. In fact, in any such system containing *qml*wffs in the form of (10), and in which $\upsilon = \upsilon$ is a theorem and in which it is a rule that ϕ is a theorem only if ⌜Necessarily, ϕ⌝ is a theorem, such *qml*wffs are themselves there provable.[17] On the other hand, that very feature of *qml* is prominently cited in the charge that *qml* is a fundamentally untenable pursuit. This is not the time to beg one another's questions, and it will suffice to admit that there are interpretations of ⌜$\alpha = \beta$⌝ under which there is no room for an attribution of necessity. But, as *quid pro quo*, we claim leave to wonder out loud whether, under those very interpretations, it is desirable or defensible to regard ⌜$\alpha = \alpha$⌝ as necessary either. Considerations, below, will suggest an answer in the negative.

We would do well to attend to the modal paradox in what might be called "Carnap's form". Specifically there are two points to notice. For one, (8) the condition of Interchange rests heavily upon the formal syntactical concept of *context*. Standard treatments provide that a context[18] for a name is its immediate linguistic environment, or, as Prior has said,[19] whatever one "wraps around" a name to make a (closed) sentence.

Secondly, the purport of that uncontroversial half, L^*, of Leibniz's Law, the principle of the indiscernibility of identicals, according to which identical things share all their attributes, is frequently presumed to find an accurate and complete expression in (8). In particular, the idea that L^* holds across the board and without exception is taken to be nearly enough equivalent to the claim that identity is substitutive for all contexts. Whence, if a context is found not to tolerate, *salva veritate*, interchange of co-referential names, it is but a half-step to the severe conclusion, in defiance of L^*, that identical things do *not* always share their properties. Worse still, it is a very easy matter to construct contexts that plainly topple (8), and therewith, after that half-step, L^* as well. In the classical example, let " '①' has six letters" serve as the context for the name 'Cicero' in the true sentence,

(9″) 'Cicero' has six letters.

Application of (8) to the context of (9″) in the light of

(7′) Cicero = Tully

gives the falsehood

(10‴′) 'Tully' has six letters.

Yet no one believes, that on this account, either that Cicero is not identical with Tully, or that they are identical though the "one" enjoys attributes not available to the "other". Evidently it is necessary to deal cautiously with such formally syntacticized versions of L^* as (8).

A similar point awaits the context ⌜Necessarily ① = ①⌝. Here too is a context that will not suffer a name α to appear on the left flank together with even a co-referential name, β, on the right. But this alone should not persuade us that the "one" object has properties not had by the "other"; neither should we think that $\alpha = \beta$ is false. On the contrary, such interchange failures as these could be expected to prompt a robust skepticism concerning the belief that the idea of substitutivity of co-referential names in all contexts is an adequate linguistic proxy for the idea of identical objects sharing all their properties.[20]

True, neither the quotation-puzzle, (10''''), not the necessity-puzzle, (10), carries its method of repair on its sleeve. Concerning (10'''') there seems to be room for a number of adjustments. One such, often associated with Quine[21] involves saying that " '①' has six letters" is *not* a context, and that such names of names as

'Cicero'

and

'Tully'

are primitive in the sense that they no more contain the names

Tully

and

Cicero

than the word

rather

contains the word

rat.

Much can be said of this strategy, not all of it by any means good.[22] However, our current problems require no more pungent a reaction than these: (i) The syntactically pure notion of context is obscured by the strategy, not only to the point where the virtues of (8), the principle of interchange, over L^*, the indiscernibility of identicals, are hard to make out, but also to the point where there seems to be insufficient control of whether we might solve the necessity-puzzle with a similar, abrupt, stroke: viz, by deciding to say that ⌜Necessarily, ① = ②⌝ is not a context either. (ii) If a good, or good enough, solution to the quotation-puzzle turns upon detecting an ambiguity as between the context ⌜ϕ (①)⌝ and the (non-) context ⌜ϕ ('①')⌝, so very well might a good (enough) solution to the necessity-puzzle, (10), turn upon finding an ambiguity as between the context ⌜Necessarily, ① = ①⌝ and the (non-) context ⌜Necessarily, ① = ②⌝.

If our skepticism of late over the purported equivalence of (8) and L^* is half as worthy as it is vigorous, it behoves us to discover whether the modal paradox of identity is constructable independently of (8).[23] It is. For, unless we mistake the tone of Frege's remarks, it seems that he at least would be prepared to say:

I. The relational sign '=' occurs univocally in the truths $\ulcorner\alpha = \beta\urcorner$ and in $\ulcorner\alpha = \alpha\urcorner$.

II. In both $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$, the names α and β function in such a way that just one *individual* x is being "talked about".

III. Since '=' is univocal, then, whatever is asserted of x by $\ulcorner\alpha = \beta\urcorner$, that very thing is asserted of x by $\ulcorner\alpha = \alpha\urcorner$.

IV. Since $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$ are about one and the same thing and say the same thing about it, then $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$ are strongly enough equivalent to provide for the commonality of their modal values. If true they are both contingent or both necessary.

V. $\ulcorner\alpha = \beta\urcorner$ is true but not necessary.

VI. $\ulcorner\alpha = \alpha\urcorner$ is true but not contingent.

VII. Contradiction.

The first part of the argument, to step III, simply recapitulates what we have come to regard as the received opinion on identity. In any event, it reflects the model theoretic treatment of '=', of §1, according to which equivalence classes with respect to the identity relation are singletons; and it also reflects the ordinary pre-axiomatic prefiguration, by which there is just one relation up for axiomatization.

Step IV may strike one as a gross *non sequitur*. Gross, since having associated our proof with Frege, we have not even taken note of Frege's express renunciation of sameness-of-reference-together-with-sameness-of-what-is-ascribed as a criterion of sameness of propositions, or thoughts. It was Frege, after all, who urged upon us the propositional difference between $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$ as evidence of there being some further ingredient (the sense of *Eigennamme*) in a satisfactory recipe for propositional sameness.[24] However, the appearance deceives. Step IV makes no mention of propositional sameness; it speaks only of a strong equivalence between $\alpha = \beta$ and $\alpha = \alpha$, a circumstance consistent with their being no propositions at all. The equivalence claimed is one by which $\ulcorner \alpha = \alpha \urcorner$ and $\ulcorner \alpha = \beta \urcorner$ cannot have different modal values; and nothing in Frege's doctrine of sense and reference obviously defeats the equivalence.[25] It suffices therefore to embed the modal claim in the equivalence, and to leave propositions to those who know how to manage them.

The immediate lessons of the reformulated paradox are entirely clear, provided we tolerate the picturesque idiom of possible worlds. Let ϕ be any necessary truth and ψ any contingent truth, and suppose that ϕ is semantically equivalent to ψ; that is, suppose that either $\ulcorner \phi \equiv \psi \urcorner$ is a valid sentence of first order logic or that ϕ and ψ mutually entail one another in the sense in which "being red" entails "being coloured". Then

[a] Since ϕ and ψ are semantically equivalent, there is no possible world, W, in which ϕ is true and ψ is false, or ϕ is false and ψ true.

[b] Since ϕ is necessary, there is no possible world in which ϕ is false.

[c] Yet, since ψ is contingent, there is at least possible world in which it is untrue.[26]

Evidently the premiss-set {[a], [b], [c]} is inconsistent, and at least one premiss must go. If [b] is dropped, then, since ϕ is true, it must be replaced by $\ulcorner \phi$ is contingent$\urcorner$ (together with appropriately selected consequences of that fact) and we are left with one half of our puzzle, namely, that there are no necessary identities. Yet we have the other half of it if [c] is banished, since in that event no identities will be contingent.[27] And if [a] is abandoned, we must adjust to saying either that sentences that say the same about the same are not semantically equivalent, or that they are and ϕ and ψ never meet that condition. Of these two last alternatives, it is hard to suppress the thought that, orthographic and superficial morphophonemic similarities apart, $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$ do not have an identical semantic structure. Of course, it can be argued, as Frege did, that the semantic differential is borne by the names in $\ulcorner\alpha = \alpha\urcorner$ and $\ulcorner\alpha = \beta\urcorner$, or it can be ventured that it is borne by the identity sign '='; or, as we have earlier suggested, we may be speaking of a syntactic ambiguity, a feature of entire sentences, not their parts. Either way, the identity-sentences differ in meaning.

§5. *Trouble for empiricism.* The modal paradox poses a dilemma. Given the necessity of $\ulcorner\alpha = \alpha\urcorner$, every identity is necessary if true; given the contingency of $\ulcorner\alpha = \beta\urcorner$, no identity, if true, is necessary. Thus, if it is contingent whether The Morning Star is identical with The Evening Star[28], then it is not true, and if true then necessary. Whence the necessity or the falsehood of every identity statement.[29] Yet, if false they are negations of necessary statements, and so necessarily false. Hence, given that any identity $\ulcorner\alpha = \alpha\urcorner$ is necessarily true we have

(11) No identity-statement, true or false, is contingent.

Defeated dogmas rarely fall alone. So it is with our current difficulty. It is, for example, a commonplace of empiricistic textbooks that, roughtly speaking, necessary truths are true just by meanings and contingent truths are true by facts as well. But consider: is the statement that The Morning Star = The Evening Star true by meaning alone? If not, then, by the one horn of our identity-dilemma and by the empiricistic dogma under review, it is contingent and untrue.

There is also a kindred epistemological thesis that must be called into doubt. It holds that because necessary truths are true just by meanings they are insusceptible of empirical confirmation or disconfirmation. How could it be otherwise, the rhetoric sometimes goes, since such sentences are "factually vacuous."[30] Contingent truths, on the other hand, true by facts as well, have factual content. True, this is so rough a sketch as barely to fall short of caricature, but it well enough, and without prejudice, serves the point at hand. For we need but ask: what can a philosopher, of positivistic fancy, make of identity-statements drawn, say, from the natural sciences? If contingent, they are open to empirical scrutiny, and so are false. If true, they are necessarily true, but empirically unscrutinizable and barren of content. Whether sensations be identical with neutral C-states, whether The Morning Star be identical with The Evening Star, if to be decided by neurophysiology or astronomy are to be decided in the negative; and if to be judged in the affirmative are not to be decided by neurophysiology or astronomy. But, then, by what? Or are we reduced so quickly to pleading Deliverances of the Intellect?

§6. *Contingent identities.* It take it to be a condition of the adequacy of what follows that identity receive a construal, C, whereby $\ulcorner\alpha = \beta\urcorner$ can comfortably be claimed to be contingent, and a construal, K, whereby $\ulcorner\alpha = \alpha\urcorner$ can reasonably be said to be necessary. We do not, however, make it an absolute condition that the C be the same construal as K, for we wish to preserve the option of saying that the most desirable manner in which to solve the modal paradox is by discerning an ambiguity concerning $\ulcorner\alpha = \alpha\urcorner$ and $\ulcorner\alpha = \beta\urcorner$.

How, then, shall be understand $\ulcorner\alpha = \beta\urcorner$? It is clear enough that we cannot, with Frege of the *Begriffsschrift*-period,[31] say with respect to a contingent identity, "$a = b$", for example

(12) $a = b$ iff the referent of 'a' = the referent of 'b'.

(12) cannot be accepted since its right side is a statement in the very form whose general structure is under question. Now this is not to say that sentences such as (12) have no proper role to play concerning identity. For example (12) might occur in a semantical theory for a formal language in which $\ulcorner\alpha = \beta\urcorner$ occurs. If it were our present task simply to recite in some semantic metalanguage truth conditions of $\ulcorner\alpha = \beta\urcorner$ in some object language, (12) might well do the trick. However, our concern is not merely to give an interpretation of $\ulcorner\alpha = \beta\urcorner$; it is primarily to

understand *identity*. It is a larger, philosophical undertaking in the light of which (12) can be seen to be inadequate.

Neither can the difficulty be removed by re-application of the self-same definitional procedure; for the contingent identity that is the right hand of (12) would be re-written as

(13) The referent of 'the referent of '*a*' ' = the referent of 'the referent of '*b*' '

which is no less an identity-statement and no less in need of an accounting.[32]

The wiser counsel is that we try what we all seem sooner or later to say anyway, namely:

(14) $a = b$ iff '*a*' and '*b*' refer to the same thing.

However, (13) was true and clear, but unhelpful, (14) is true (probably) but not clear.[33] I think it can be said to be obvious that 'the same thing' is not a definite description.[34] There is, it seems, no definite description

(15) (ιx) (x is the same thing),

unless, perhaps, it is just

(15′) (ιx) (x is a thing).

which, although tolerably a definite description, affords a counter-reading of (14), thus:

(14′) $a = b$ iff '*a*' and '*b*' refer to (ιx)(x is a thing),

whereby "$a = b$" is true only in case there is but one thing in the universe.

There is, of course, another way of saying what we wanted to say at (14). It is better said this new way, if only because more structure is apparent.

(14″) $a = b$ iff there is something that '*a*' and '*b*' both name.

Structurally, (14″) is relatively easy to deal with. In place of the inscrutable 'the same thing' of (14) we have the relatively translucent 'something or other' of ordinary quantification theory:

(16) $a = b$ iff $\exists x$ ('a' names x & 'b' names x)

What is more, accepting (16) need not force us into a decision on referential ambiguity in L, since (16) is adequately clear independently of such a policy. So far, then, there is room via (16) for even the most "fanatical mono-denotationalist".[35]

Now, it needs to be appreciated that (16) invites a powerful-seeming objection. It is that while "$a = b$" does not assert, or entail,[36] the existence of any names, its paraphrase under (16) does; hence, (16) is false. However, there is another argument that one might have made: Since "$(\exists x)$('a' names x & 'b' names x)" asserts or entails that there are names, and since (16) is true, it follows that "$a = b$" asserts or entails, not just virtually implies, the existence of names.

Obviously the set {" "$a = b$" neither asserts nor entails the existence of names." "$(\exists x)$('a' names x & 'b' names x)" asserts or entails the existence of names", (16)} is an inconsistent set; and, if looks mean anything, "$a = b$" does not look as though it asserts or entails the existence of names. But since the moral of preceding sections has been that sentences $\ulcorner\alpha = \beta\urcorner$ *disguise their structure*, one cannot repose any real confidence in the look of "$a = b$". Given the problems that bedevil such sentences, how they look will represent at least some departure from how they are. On the other hand, we saw, from (14) on, that every even half-intuitive paraphrase of "$a = b$" is one in which the existence of names is asserted or entailed. Since (14) and (14″) are very plainly true, it is surely a condition on (16), rather than a deficiency of it, that it preserve that feature, not extirpate it. This does not yet preclude the possibility contrary to what we all think, and apace with the opinion (which I share) that, in general, there is little philosophic gain in semantic ascension[37], that (14) and (14″) sentences are not true after all. I do not think that they are not true, though I have not been able to construct a proof of this. Thus, it is necessary to admit that the analysis of $\ulcorner\alpha = \beta\urcorner$ is relativized to the truth of sentences in the form of (14) and (14″).

Another fundamental feature of the current analysis is that, in effect, it professes itself an exception to that skepticism concerning semantic ascents in philosophy. But perhaps the exception is more apparent than real. No one would seriously proclaim

(17) 'cat' is a common noun

the result of a semantic ascent from

(17′) cat is a common noun;

for to talk of words is not always to change the subject. It is never to change the subject when the subject is those words. In such cases, there are procedures to be disenjoined or discouraged, but they are not procedures of ascent. Rather they are manoeuvres of semantic *descent*, as from (17) to (17′). It is not exactly, though somewhat, so with contingent identities, or so I believe. There, too, one does not semantically ascend from "$a = b$" to "$(\exists x)$('a' names x & 'b' names x)"; instead one declines to make the journey downwards.

It is as if we were proposing, just now, that a philosophically competent understanding of contingent identities of the form $\ulcorner\alpha = \beta\urcorner$ requires that we do this: If we were to imagine $\ulcorner\alpha = \beta\urcorner$ to occur in some standard formal object language, and $\ulcorner\exists\nu$('α' names ν &'β' names ν)$\urcorner$ to give its truth conditions in a semantic metalanguage, then identities in English should be regimented, so as explicitly to display the structure of the latter, rather than the former. But why this insistence on metalinguistic treatment of identity sentences in English? Surely, there are no intuitive motivations for it? But there are, I think. It is agreed that "Tully was an orator" is true independently of their being such a name as 'Tully'. Thus although "$\exists x$('Tully' names x & x was an orator)" is a truth condition of "Tully was an orator" it is not a condition of that Tully was an orator. In a language lacking 'Tully' other sentences are in principle available which are true by exactly the same semantic assignments by which "Tully was an orator" is true. But this is not so with "Tully = Cicero". If English lacked 'Tully' and 'Cicero' it could not express the identity of Tully and Cicero. Even supposing the availability of a different pair of names each naming that selfsame Roman statesman, say, "Publius" and "Philosophicus", English could convey that Publius is identical with Philosophicus, but not that Tully is Cicero.

How is this peculiar dependence of facts of contingent identity upon names to be represented? If meaning-the-same were a clear enough notion to bear the intended theoretical burdens, I would hazard it that it is an adequacy condition on theories of meaning for L that "Tully = Cicero" and "Something is named both by 'Tully' and by 'Cicero' " both mean-the-same; but that "Tully was an orator", and "$\exists x$('Tully' names x & 'was an orator' is true of x)" do not.

§7. *Languages without names.* There are, however, other difficulties with the analysis of $\ulcorner\alpha = \beta\urcorner$ that are on the face of it rather serious. One such problem is that first order theories with identity need not have singular terms, even where they can contain axioms for identity; hence they can formulate identity sentences, and presumably prove some of them, even without the resources of names. And since the axioms for identity, in any well-made theory, implicitly define, or provide an analysis of, the identity-relation, our own (as we shall say "referential") analysis seems wrong.

No doubt some such difficulty exists for the referential analysis. But it would not do to think that this is a perfect argument. Even waiving doubts about the eliminability of singular terms,[38] the argument causes trouble only if the axioms of identity provide, in a nameless first order theory, for contingent identity-sentences of the sort we have been discussing. Yet it is doubtful that the proviso is met. For, if such an identity theory is grafted onto a base sentential modal logic, whereby $\ulcorner\alpha = \alpha\urcorner$ emerges as necessary, then if $\ulcorner\alpha = \beta\urcorner$ is a theorem of that theory, it too will be necessary[39] and the axioms of the theory will implicitly define or analyze identity in such a way that no contingent identites are begotten. In fact, in the light of this, it cannot yet even be assumed that L will contain the standard identity axioms; for our earlier sections were designed to cast them in some doubt under their usual model theoretic semantics. Still, the point of the objection can be salvaged: *However* we regiment L it ought surely to be the case that some sentences, such as

(18) $(\exists x)(y = x)$,

will be true and contingent,[40] and will remain true and contingent even under the "objectual" interpretation of the quantifiers,[41] and hence so remain, even if contains no names, or, at least, no policy for allowing inferences such as that to

(19) $y = a$

where 'a' is a name.

§8. *Opacity.* A further, and I think somewhat over-rated, problem concerning a referentialist account of contingent identities involves the question of opacity. In particular, it is the problem of whether we should allow

(20) The Babylonians discovered that The Morning Star = The Evening Star

to be mapped onto

(21) $(\exists x)$ (The Babylonians discovered that 'The Morning Star' names x & 'The Evening Star' names x),

which is aid to force us into attributions of English, several centuries prematurely, to people who could not, in any case, be presumed to have such knowledge even had English suffered a much earlier start.

So we must deal with nameless identity theories and opacity. We turn first to opacity.

Let us suppose that 'Dā' and 'Nīt' are English transliterations of the Babylonian expressions for 'The Morning Star' and 'The Evening Star'. It is natural to think that (20) is an ellipsis, ambiguous as between

(20′) The Babylonians discovered that what they called 'Dā' is identical with what they call 'Nīt'

and

(20″) The Babylonians discovered that what we call 'The Morning Star' is identical with what we call 'The Evening Star'.

It may seem that (20) is tugged more in the direction of (20″) than (20′), if only because (20) actually contains 'The Morning Star' and 'The Evening Star' and does not obviously contain 'Dā' and 'Nīt'. And since, in fact, I can know (20) to be true without having the faintest idea (as evidently I do not) of what expressions the Babylonians used in place of 'The Morning Star' and 'The Evening Star', yet cannot not know this if I know (20′) and *can* not-know this is I know (20″) the ambiguity is resolved in favour of (20″).

Imagine now that our language L has been enriched not only by safe policies on quotation but also by procedures for mixing in epistemic modal operators. There is reason to expect that our identity theory will be able to locate in L a structurally illuminating paraphrase of (20).

It ought to be possible to say in L what the Babylonians discovered, without our having to know Babylonian or they English. As rough first try, let (20) be construed in L in some such fashion as:

(21) $(\exists x)$ $(\exists y)$ $(\exists z)$ (x names z & 'The Morning Star' names z & y names z. & 'The Evening Star' names z & the Babylonians discovered that (x names z & y names z)).

If however, we let x and y be the same Babylonian name say 'Dā' where 'Dā' names Venus, then (21) is true and (20) might be false; so, although it is on the right track, (21) requires some repairs. The usual method of adjustment of (21) would be by addition of a distinctness clause '$x \neq y$; but, here, that is a device which is not available to us. Instead, we shall distinguish x from y by means of a suitable range of linguistic predicates, P_1, P_2,..., such that for every two expressions of L one will have a given P_i and the other not. Thus, since the language of L is at most countably infinite, we may alphabetically well-order all the expressions of L, such that for every i, j ($i < j$ or $j > i$) and every expression ϕ of L, $\ulcorner P_i\ (\phi)\urcorner$ is true only if $\ulcorner P_j\ (\phi)\urcorner$ is not. Then, using the predicate dummies 'P_1,' 'P_2,..., 'P_j', it is possible to formulate (21) as

(21') $(\exists x)\ (\exists y)\ (\exists z)$ (x has P_i & y has P_j) & (x names z & 'The Morning Star' names z. &. y names z & 'The Evening Star' names z. & the Babylonians discovered that (x names z & y names z)).

The advantages of (21') are fairly evident. (21') says that there are two names that name what 'The Morning Star' and 'The Evening Star', respectively, name ("what we call 'The Morning Star' ", etc.), and that the Babylonians discovered those names (theirs, not ours) to have a common bearer z.

Actually, there is no good reason to take (21') as a truth *de re*, for although the Babylonians were, they need not have been, familiar with the common bearer, z, of the Babylonian equivalents of 'The Morning Star' and 'The Evening Star'. It would be advisable to reflect this circumstance in our paraphrase, say by re-writing the last clause of (21') as follows:

(22) & (The Babylonians discovered that $(\exists w)$(x names w & y names w))

Gloss: "...and the Babylonians discovered that there was something or other that x and y both name."
Even so, (21') rewritten in the spirit of (22), may still seem problematic. Suppose that z = Venus and w = a royal poodle named by a neighbouring Pharoah x and y. Then, provided x and y also name Venus, (21) could be true without the Babylonians having any idea that x and y are Venus.

The problem however is misconceived. Nothing about (20) forbids[42] our adding, "But the Babylonians had no inkling of what Venus is, even that it is a heavenly body. All that they could manage, from the remains of an Ionian scroll, was that Dā and Nīt are one and the same". And, certainly, I might know nothing of Cicero and Tully, save that they are in fact some same individual. In general, to know the truth of an identity-statement $\ulcorner\alpha = \beta\urcorner$ need not involve the truth of any statement $\ulcorner(\exists\nu)K_0(v = \alpha)\urcorner$ or $\ulcorner(\exists v)(K)(v = \beta)\urcorner$ where 'K' is the epistemic operator for knowledge, and the subscript '0' names the person making the identity claim. So the question is, even though we admit the possibility that one could know that $\alpha = \beta$ without knowing who "they" are, *i.e.* without having any beliefs at all as to the identity of the common bearer, shall we also countenance one's having *false* beliefs as to the identity of the common bearer, even where the intended astronomical candidate is not the only object co-designated by the names in question? The answer, no doubt, hinges upon whether we can agree that discoveries of contingent identities *must* include historical, or astronomical, or, in general, specific non-linguistic discoveries as well. For example, in the relatively less complicated climate of one's own everyday environment, we need to decide whether a contingent identity "$a = b$" is better conveyed by

(23) 'a' names b

than by

(16) $(\exists x)$('a' names x & 'b' names x).

It is not especially important that, in very many actual cases, to know (16) is also to know (23); nor should we accord too much point to the circumstance that, in standard cases, it is rather difficult to come to know (16) without coming to know some such statement as (23). But in view of the really quite evident consistency of "Yes, they knew that a and b were one and the same, yet they did not know what "they" were – in fact, they actually misidentified y as the common object denoted by 'a' and 'b' ", I do not see any way of keeping the problem unsolved.

Suppose we were to call (23) an *identification-statement* and (16) a *(pure) statement of contingent identity;* then our thesis is that the referential account is designed to accommodate, and does, identity-statements alone, and should not be required to meet further conditions appropriate to the analysis of identification-statements. We may also note that the referential analysis has the force of annulling a number of bothersome difficulties arising from substitution into opaque contexts. For example.

(24) Zachary explicitly denied that a = b

ought not generate, even in case "a = b" is true,

(25) Zachary explicitly denied that a = a.

On the referential analysis (24) is

(24′) Zachary explicitly denied that
($\exists x$) ('*a*' names x & '*b*' names x)

and (25) is

(25′) Zachary explicitly denied that
($\exists x$) ('*a*' names x and '*a*' names x).

Not only does (25′) not follow from (24′), its not following involves no breach of substitutivity principles (nor of indiscernibility requirements, either) since " '$\mathbb{O}$' names x" is not a substitutional contex with respect to $\mathbb{O}$.

Similarly (by the way),

(26) Barbarossa was so-called because of his red beard

does not, even in the face of "Barbarossa = Frederick I" entail

(27) Frederick I was so-called because of his red beard.

For (26) wants construal as

(26′) Barbarossa was called 'Barbarossa' because of his red beard.

and (27) as

(27′) Frederick I was called 'Frederick I' because of his red beard.

But (27′) does not follow from (26′) without violating ordinary substitution-procedures, since these do not disturb quotation-contexts. Needless to say, the result of substituting in (27) 'Frederick I' everywhere for

Barbarossa

yields the harmless

(27″) Frederick I was called 'Barbarossa' because of his red beard.

§9. *Namelessness.* We shall have to deal with the objection that the referential analysis of contingent identity-statements commits one to a language containing names. How shall the objection from nameless, though identity-capable, languages be dealt with?It is an easy matter to re-write our referential analysis, replacing it with its *denotational* counterpart, thus:

(16*) $a = b$ iff $(\exists x)$('a' Δ x & 'b' Δ x).[43]

As for quantified nameless identity formulas, these too can be handled denotationally. We put it, first, that L's former capacity for referring to its own names is augmented by the related capacity for *quantifying over its own variables* (and names, as well). We also specify the predicate of L '① Δ ②' by which to represent the denotation-relation. Our strategy, at this stage is to seek for denotational paraphrases of the standard identity axioms. In so doing, our object is as much to test the axioms as it is to test the denotational account of identity.

The *symmetry axiom*

(28) $(\forall x)(\forall y)(x = y \cdot \equiv \cdot y = x)$

nicely goes over into

(28′) $(\forall x)(\forall y)(\forall z)(x \,\Delta\, z \;\&\; y \,\Delta\, z \cdot\equiv y \,\Delta\, z \;\&\; x \,\Delta\, z)$,

wherewith the symmetry of '=' is matched by the commutativity of '&'.

The *transitivity axiom*

$$(29)\ (\forall x)(\forall y)(\forall z)\,(x = y\ \&\ y = z \cdot\!\rightarrow x = z)$$

is taken into

$$(29')\ (\forall x)(\forall y)(\forall z)(\forall w)((x \Delta w\ \&\ y \Delta w \cdot \&\ y \Delta w\ \&\ z \Delta w) \cdot\!\rightarrow\!\cdot\ x \Delta w\ \&\ z \Delta w).$$

The *special identity-axiom* (the indiscernibility of identicals),

$$(30)\ (\forall x)(\forall y)(x = y \cdot\!\rightarrow \mathrm{F}x \equiv \mathrm{F}y),$$

can be construed as

$$(31^{*})\ (\forall x)(\forall y)(\forall z)\,(x \Delta z \rightarrow (\mathrm{F}x \equiv y \Delta z))$$

or as

$$(31^{**})\ (\forall x)(\forall y)(\forall z)(x \Delta y\ \&\ y \Delta z \rightarrow\ \vdash \mathrm{F}x \text{ iff } \vdash \mathrm{F}y)$$

provided, of course that we have in L a reasonably controllable notion of theoremhood.

The *reflexivity axiom* is not at all difficult to paraphrase denotationally. But it should be remarked that if sentences of the form $\ulcorner\alpha = \alpha\urcorner$ are necessary, then, since our current concern is to capture contingent identities only, it is undesirable that we employ a denotational paraphrase of the reflexivity axiom that makes reflexive identities other than contingent, even if by the standard reflexivity axiom they are necessary. The standard axiom

$$(32)\ (\forall x)(x = x)$$

can be rewritten as

$$(32^{*})\ (\forall x)(\exists y)(y \Delta x\ \&\ y \Delta x)$$

which simplifies to

$$(32^{**})\ (\forall x)(\exists y)(y \Delta x)$$

According to (32**) everything whatever is denoted – is either named or is a value of a bound variable, and although true and only self-defeatingly denied, (32**) is not a necessary truth *de dicto*.[44]

If it can be agreed that our denotational theory gives a map μ from the standard identity axioms into its denotational counterparts such that μ preserves truth, then a theory L under our additions is complete in its contingent-identity theory relative to the standard axioms. Thus it should not happen that any everyday statement of contingent identity serves to overturn the mapping. It seems indeed that none does. For example, μ replaces,

(33) $(\forall x)(\exists y)(x = y)$

which is true, with

(33*) $(\forall x)(\exists x)(y \Delta x)$,

which is also true. And μ takes

(34) $(\exists x)(\forall y)(y = x)$

which is false, in a universe of more than one object, to

(34*) $(\exists x)(\forall y)(y \Delta x)$

which likewise is false in a universe of more than one denoting phrase.

Opaque sentences such as

(35) Zachary believes that Cicero = Cicero, yet disbelieves that Cicero = Tully,

which is, let us say, a truth of English, is mapped onto

(35*) Zachary believes that $(\exists x)$('Cicero' Δ x yet he disbelieves that 'Tully' Δ x),

which would also be true, provided Zachary's language is the language of (35). And

(36) Since Sally does not believe that Tom = T.J., then, although she wants to marry Tom, she doesn't want to marry T.J.

becomes

(36*) Since Sally does not believe that $(\exists x)$('Tom' Δ x & 'T.J.' Δ x, then, although Sally wants to marry

Tom she doesn't want-to-be-true "Sally marries T.J.")[45]

§11. *Necessary identities.* The first part of the main project is concluded. We have, in the denotational account, a theory of identity-statements of the form $\ulcorner\alpha = \beta\urcorner$ that both preserves their contingency and also complies with the entrenched inclinations of us all to understand such sentences co-designationally. But the denotational theory also provides *one* solution to the modal paradox. It will be recalled that, on the denotational view, there is a map μ that takes statements such as "$a = a$" into statements such as "$(\exists x)(`a' \,\Delta\, x)$" which are contingent on their face. Thus, under μ-paraphrase, every identity-statement, $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$, is contingent, and the modal paradox is resolved.

True, a paradox is resolved, but only at the cost of a dilemma reinforced. The dilemma, that truths of identity cannot differ in modal value, is a trivial consequence of our denotational solution of the paradox: every identity is contingent; hence identities cannot be distinguished from one another modally. It is hard to resist the thought that we are landed in a disaster. It is clear to most of us that there is at least one philosophically tolerable conception of necessity – it needn't be *positivistic analyticity* and it needn't be rationalistic synthetic apriority; it could be a pragmatic notion of near-total immunity from revision, or entrenchment – under which at least *two* classes of identity statements collect nothing but sentences that are necessary. The one class comprises statements $\ulcorner\alpha = \alpha\urcorner$ of self-identity; the other, the equations of mathematics.

Philosophers have held widely diverging opinions of self-identity. Bishop Butler was moved to declare that a thing is what it is and not another thing; and Wittgenstein was troubled to know what statements of self-identity asserted, and concluded that they assert nothing. Certainly self-identity is not an ordinary, garden-variety property. It is not, for example, a universal; no self-identity property is instantiable by more than one individual. True, every individual is self-identical, but self-identity is not a property such that every individual has *it*. There is something Bergmannian about self-identity. Bergmann's "characters" "depend_1" upon the entities possessing them.[46] So the class of self-identities bears a one-to-one correspondence to the class of individuals, and this correspondence is invariant under change in membership. Every entity has its self-identity, and not that of any other, as a Butler might say; every case of self-identity is unique to just one individual. Bergmann's concept of dependency_1 is, of course, adumbrated in the Scholastic doctrine of essence.[47]

Now it should be mentioned that despite its odd-seeming ontological peculiarities, self-identity can be expressed in the idiom of ordinary first order logic with equality,[48] thus:

Def. a is self-identical iff $\exists x \forall y (y = z \,.\equiv.\, y = \mathrm{a})$

But the entire presumption of our current task is that equality (ordinary numerical identity) is not well enough understood, and so we can hardly here be encourged by Def. How, then, *shall* we deal with self-identity, and what are the obstacles which lie before us?

On the face of it, the denotationalist account of identity won't do for self-identity; in particular, on the denotational account, the ordinary reflexivity axiom "a = a" gives way to "$\exists$x('a' Δ x)" which hardly seems to convey what "a is self-identical" conveys. What is more, it is plain that "$\exists$x('a' Δ x)" is not a necessary truth, whereas by the conventional wisdom "*a* is self-identical" *is*. Indeed it is arguable that "$\exists x$('a' Δ x)" does not say what "*a* is self-identical" says; but it is also arguable, and I think true, that "*a* is self-identical" is *not* a necessary truth. Surely *a*'s self-identity is not a circumstance that prevails in every possible world, but rather only those in which the entity named by '*a*' is resident. So, although the denotationalist treatment of "*a*=*a*" does not convey the idea of *a*'s self-identity, it can hardly be faulted for mapping "*a*=*a*" onto a contingent truth. To the contrary, to the extent that it does so, it is a *good* mapping for the reflexive identities $\ulcorner \alpha = \alpha \urcorner$.[49]

Having now met with the idea that sentences such as

(37) a is self-identical

are expressible by means of ordinary identity in non-reflexive form, and with the idea that the structure of

(38) $a=a$

is less than manifest, and that (37) be construed as

(39) $(\exists x)(\forall y)(y = x \equiv a = y)$,

it is useful to remind ourselves of our current thesis that statements in the form of (39) are contingent identities for which there ought to exist a denotational paraphrase. If that thesis is sound, and if sentences such as "$a=a$" are taken as conveying a's self-identity, then we are led to a denotational paraphrase of "$a = a$", and therewith reinforcement is found for the view that to regard "$a=a$" as a necessary truth *de dicto* was a mistake from the outset. Whereupon, a horn over which our dilemma is escaped.

More soberly put, it seems we may say that neither "$a = a$" nor "a is self-identical" is necessary *de dicto*. However, since one's self-identity is an *essential* attribute, it may also be said that these are, though contingent truths *de dicto*, necessary truths *de re*. We shall shortly return to this point, but for now I shall briefly venture some thoughts upon that Tractarian skepticism, according to which identity statements are inscrutable, according to which, that is, identity-statements say nothing.

§12. *Inscrutability*. Supposing that self-identity statements admit of a tolerable denotational paraphrase, it would be interesting to determine whether there remains any room at all for the Tractarian conviction that self-identity is deeply inscrutable. In particular we might ask whether such a paraphrase relies, at all importantly, on what it *shows* as distinct from what it says. The paraphrase proceeds from

(39) $(\exists x)(\forall y)(y = x \cdot \equiv \cdot a = y)$

to

(40) $(\exists x)(\exists z)(\forall y)(x \,\Delta\, z \;\&\cdot y \,\Delta\, z \equiv \text{‘}a\text{’} \,\Delta\, z)$

which reads,

(40′) There is an object z such that no denoting phrase denotes it only unless ‘a’ denotes it.

In like fashion, the statement

(41) $(\forall z)(\exists x)(\forall y)(y = x \cdot \equiv \cdot\, y = z)$,

to the effect that everything is self-identical – is what it is and not another thing, would go over into

(42) $(\forall z)(\exists x)(\exists y)\,((x \,\Delta\, y) \rightarrow (\forall w(w \,\Delta\, y \cdot \equiv \cdot\, w \,\Delta\, z))$,

which is glossed as,

(42′) For all objects z if there is something y that is denoted by x then, no denoting phrase denotes y unless it denotes z.

Now the gloss of (40) makes it clear that (40) turns upon L's policy for names. It is easy to see that (40) actually exploits this policy, by which naming is likened to a function one-one and into from names to bearers; otherwise, interpreting ‘a’ in (4) as the name ‘Elizabeth’, we shall have it, e.g. that Elizabeth Windsor and St. Elizabeth of Hungary are self-identical-with-each-other. That policy on names – *unum nomen, unum nominatum* is fortunately the standard one for formal languages and for regimented languages such as L, and in embracing it, therefore, we can achieve a desirable consequence without calling down on ourselves a charge of palpable *ad hoc*ness. This, then, is what (40) relies upon. It depends, without saying so (indeed without having all the *resources* for saying so) upon the policy for names in L. There is a sense, therefore, in which (40) shows, without saying, that but one object, the individual a, satisfies its conditions. Formula (42) likewise relies on a similar device. The ordinary canons of quantification theory, together with what (42) actually says, provide, concerning the variables, ‘x’, ‘y’, that, with respect to the formula (42), the value of ‘x’ is a denoting phrase that denotes precisely the value of 'y'. But (42) also tells us that *any denoting phrase*, w, denoting z, does so iff it also denotes that self-same value of 'y'. Clearly then, any denoting phrase w is either a name or a bound variable. If it is a name, then (42) and our conditions on names in L imply that w names at all only if it names just that entity that 'y' here names. If w is an existentially bound variable, then w refers to z only if it refers to that self-same value of 'y', i.e. if, in the formula (42), x and w are co-designating. Finally, if w is a universally bound variable, then w refers to everything; hence it is trivial that w refers to y just in case w refers to z. However, since in ordinary quantification theory, L is not allowed to suffer empty domains, then where w is a universally quantified variable meeting the conditions of (42), it follows that there is an existentially quantified variable, y, that also meets them. But, by the previous argument, y and x are co-designating with respect to (42). Whence, in all three cases, nothing but y could be y : y is self-identical.

In the cases both of (40) and (42) there is something of an analogue of that inscrutability with which Wittgenstein's *Tractatus* charges the self-identity attribute. In the former case, it is that (40) relies upon a condition on naming that cannot, as L has been specified, be stated in L; in the latter, (42) relies upon a condition on quantified variables which L cannot state. For, (40) does *not* say (and could not, as L presently stands) that naming is a function one-one and into; and (42) does not say (and could not, as L presently stands) that in a given sentence occurrences of the same existentially bound variable denote the self-same entity at each such occurrence.

This is not to say, however, that these analogues of Tractarian inscrutability which seem to attend even the quite ordinary sentences of quantification theory, are doomed to stay inscrutable or are desirably left that way. It is entirely possible that L once again be extended so as to include enough of its own metalanguage to formulate the conditions by which its names are governed and under which its quantified variables operate. In either case, however, it is unlikely that the new formulations would work well without the resources of the ordinary identity-idiom, '='. Whereupon, a denotationalist identity theory for the newly enriched L would lapse.

§13. *Necessity de re.* Necessity *de dicto* is syntactically accommodated by an operator '$\Box$' taking sentences into sentences, thus: if ϕ is a sentence so too is $\ulcorner\Box\phi\urcorner$. In like fashion necessity *de re* may be syntactically represented by a predicate operator, '$\Box$', taking open sentences into open sentences; thus if $\ulcorner\phi\ \nu\urcorner$ is an open sentence then so too is $\ulcorner\phi^{\Box}\nu\urcorner$. $\ulcorner\phi^{\Box}\nu\urcorner$ may be read "ν necessarily ϕs" or "ν necessarily has ϕ". If we imagine for some object α that $\ulcorner\phi^{\Box}\alpha\urcorner$ is true, then we suppose that in every possible world, this one included, in which α occurs, α has ϕ. That is, having ϕ is for α a property such that were α to lose it, there would be nonesuch as α – nothing whatever would be α. It would be a substantial change, which α would be powerless to survive. Thus we might define $\ulcorner\phi^{\Box}\alpha\urcorner$ as follows:

$$(40^*)\quad \phi\,\alpha\ \&\ (\neg\phi\,\alpha \prec \neg(\exists\nu)(\alpha=\nu)).$$

That is, α necessarily has ϕ iff α has ϕ and α's not having ϕ strictly implies that nothing whatever is α. We now have a means of accommodating the non-*de dicto* necessity of $\ulcorner\alpha=\alpha\urcorner$. Rather than attaching to $\ulcorner\alpha=\alpha\urcorner$, the sentence operator 'necessarily' we re-write $\ulcorner\alpha=\alpha$ as α is necessarily identical with $\alpha\urcorner$:

$$(40^{**})\quad \alpha =^{\Box} \alpha\,,$$

and then paraphrase *denotationally* in recognition of '=' and *essentialistically* in recognition of the modal superscript '$\Box$', as follows:

$$(40^{***})\quad (\exists\nu)(\ulcorner\alpha\urcorner\Delta\ \nu\ \&\ (\neg(\ulcorner\alpha\urcorner\Delta\ \nu)\prec\neg(\exists\nu_0)(\nu_0\,\Delta\ \nu))).$$

Statements, $\ulcorner\alpha=^{\Box}\alpha\urcorner$, of necessary-self-identity, *de re*, are now read as asserting that something is denoted by $\ulcorner\alpha\urcorner$ and that were it otherwise, nothing would denote it. But, as the last clause is, in (40***), self-defeating, 'ν' there denoting what is said to be denoted by nothing, its antecedent is also self-defeating, and its negation, the first conjunct of (40***), is therefore *in a sense* necessary – necessary, even though a *de dicto* contingency.

True, the current paraphrase of (40**) may not strike one as deeply intuitive, for (40***) seems only to say that $\ulcorner\alpha\urcorner$ names something and that necessarily either $\ulcorner\alpha\urcorner$ names it or nothing whatever denotes it, which latter must be false since 'it' has already denoted it; whence $\ulcorner\alpha\urcorner$ is somehow *destined* to name what it already names. And that seems false of names.[50]

Repairs of a kind are ready to hand, however; for recall that the straight denotationist paraphrase of a sentence $\ulcorner\alpha = \alpha\urcorner$would give,

$$(40^{****})\quad (\exists\upsilon)(\ulcorner\alpha\urcorner\triangle\upsilon\ \&\ \ulcorner\alpha\urcorner\triangle\upsilon)$$

which was simplified by dropping "$\&\ \ulcorner\alpha\urcorner\triangle\upsilon$". If we were to leave the simplification undone, then (40***) would read

$$(40^{*****})\quad (\exists\upsilon)((\ulcorner\alpha\urcorner\triangle\upsilon\ \&\ \ulcorner\alpha\urcorner\triangle\upsilon)\ \&$$
$$\neg(\ulcorner\alpha\urcorner\triangle\upsilon\,\&\ \ulcorner\alpha\urcorner\,\Delta\,\nu \prec \neg(\exists\upsilon_0)(\upsilon = \upsilon_0))),$$

which now conveys that if υ, being named by $\ulcorner\alpha\urcorner$ in a sentence ϕ, is not named by each occurrence of $\ulcorner\alpha\urcorner$ in ϕ, then it is named by nothing, which at least has the merits of being a plausible condition on names. And since (40*****) also has it that α exists or that there is such a thing as α, then in virtue of the exact coincidence between the class of somethings and the class of self-identicals, (40*****) and (40**) are satisfied by just the same non-linguistic entities.

I certainly do not feel inclined to prosyletize in behalf of (40*****) nor to agitate over whether it successfully preserves truth and avoids a paradox. It seems to me much more important to determine whether if (40*****) is permitted to cash out attributions of necessary self-identity, than a like toleration be extended to $\ulcorner\alpha = \beta\urcorner$. There is, after all, *some* inclination to think that since Cicero just could not fail to be Tully (they being the same man), that there attributions of identity as well are somehow necessary.[51] If that intuition were thought worthy of attention, it would be a fairly straightforward matter to take e.g. "Cicero = Tully", first into "Cicero is necessarily Tully" or "Cicero = $\Box$ Tully", and thence into

$$(40'''''')\quad (\exists x)((\text{`Cicero'} \vartriangle x \ \&\ \text{`Tully'} \vartriangle x) \\ \&\ ((\neg(\text{`Cicero'} \vartriangle x \ \&\ \text{`Tully'} \vartriangle x) \prec \\ \neg(\exists y)(y \vartriangle x))),$$

for which a reading could be ventured whereby 'Cicero' and 'Tully' co-designate in a sentence ϕ, and if that were not so, say if 'Cicero' names x in ϕ and "Tully" there names something else, then nothing there would denote x. Again the moral for which we strain is that in specified contexts, co-designating terms remain co-designating or else.

The important point is that *if* the present treatment is extended from sentences $\ulcorner\alpha = \alpha\urcorner$ to sentences $\ulcorner\alpha = \beta\urcorner$ as well, our modal paradox is vanquished, for under this analysis every identity, $\ulcorner\alpha = \alpha\urcorner$, $\ulcorner\alpha = \beta\urcorner$, is co-modal, each being contingent *de dicto* and necessary *de re.* But even on the face of it, the present treatment of "Cicero = Tully" simply defies belief; (40'''''') is false, and that is all there is to it. And in so saying, limits upon the denotationist treatment of identity seem fairly easily and naturally set as follows: (A) There exists a well entrenched use of an identity idiom which calls for a denotationist analysis. That analysis best or most naturally serves modally unvarnished identity sentences of mixed names (e.g. "Cicero = Tully"); but it extends in a tolerable way to mononominative identity sentences such as "Cicero = Cicero". In fact, it takes the identity axioms into truth-preserving denotationist correspondences which reveal *all* identity sentences to be *de dicto* contingent (which is true, notwithstanding the semantic tradition) and *de re* contingent as well (a troublesome consequence). However, under the denotationist map, identities of the form "$a = a$", (statements of self-identity) though contingent both *de dicto* and *de re* emerge as propositions of which their denial is self-defeating. Hence there exists a *uniform* semantic treatment of all four identity axioms which preserves contingency where it should be preserved, which does not provide for even *de re* necessity where the tradition teaches us to look for it, but provides in its place the weaker necessity-notion of self-sustenance (i.e. the contradictory of self-defeatingness). (B) It is precisely at this point – the point at which the denotationist treatment is most open to traditional hostility, that a different notion of identity competes for centre stage. For if, for statements such as "$a = a$", we really are to insist upon a stronger notion of necessity than self-sustenance, what could be a readier remedy than to read these as statements of self-identity, as statements of a form which on their face make a good claim upon necessity *de re*. Well and good, we would have it then that "$(a = a)$ iff $a = {}^{\square} a$", and in turn that "$a = {}^{\square}a$ iff $a = a$ & $(a \neq a \prec \neg (\exists x)(a = x))$", which in *its* turn is taken by our denotationist map into "$(\exists x)(\text{'}a\text{'} \vartriangle x \,\&\, \neg (\text{'}a\text{'} \vartriangle x \prec \neg (\exists y)(y = x)))$". But here, too, the best we can say for the modal character of the denotationist paraphrase is that it is self-defeating to deny. Why, then, did we think it necessary or opportune to take "$a = a$ & $(a \neq a \prec \neg (\exists x)(a = x))$" into its denotationist paraphrase? Doubtless it was because it is rife with occurrences of the identity-sign for which, up till now in this enquiry, only the *denotationist* account of identity has been sanctioned as an acceptably intelligible treatment. Once said, it is immediate that here is an identity concept to which the denotationist account is not adequate. It falls to us therefore seriously to entertain the prospect that identity and self-identity are different and logically distinct properties; and that an equal success of the denotationist analysis for them both could not be supposed a triumph in the name of univocity, but a disaster in which importantly different notions are confused. And if it should be that, as Kripke supposed, there exists a decent rendering of "Cicero = Tully" by which it is necessary, the failure of the denotationist account to capture this necessity tells not against the account but rather against "Cicero = Tully" admitting of only those readings which attach to it a single notion of identity.

(C) It is true, of course, that in my earlier work on self-identity (Woods [22] and [5]) self-identity is a *defined* notion in a quantified S2 system with (ordinary) identity. That analysis, however, was not directed to an understanding of ordinary identity, but rather to an understanding of essential attributes, of which self-identity was the preferred paradigm. The object of that analysis was to persuade certain philosophers that essentialism could be made sense of provided they understood quantified S2 with identity (and in [5], it was pointed out that no known attack upon quantified modal logic found its mark in such a language). Here, however, my dialectical purposes are different. Here I venture to think that (ordinary) identity is not, as indicated by its customary semantics, well understood, and so I cannot here seriously propose my former identity-ridden treatment of self-identity. But neither need I continue to believe that self-identity stands in need of a *definition*. Indeed, it is tempting to paraphrase Scott:

> What is [self-identity]? A very good question. So good, in fact, that we should not even try to answer it. We could assume that being [self-identical] is a primitive concept – that is, harmless: any sufficiently clear concept can be made harmless.[52]

§14. *A predicate constant.* By these lights, then, another, briefer option would be to take very seriously the (definitional) inscrutability of self-identity, indeed to acquiesce in it totally. To that end, L could be extended by introduction of the primitive monadic predicate constant '$I(\textcircled{1})$', read as '$\textcircled{1}$ is self-identical', whose axiomatic treatment might be expected to parallel rather closely, say the Meyer-Lambert treatment of the existence-predicate '$E\,!\,\textcircled{1}$'.[53]

The important point is that on either of the two treatments, of $\ulcorner \alpha = \alpha \urcorner$, which now abbreviates either a denotational-sentence in the manner of (40) or a primitive self-identity sentence, in the manner of

(43) $I(\alpha)$,[54]

differs so strikingly from $\ulcorner\alpha = \beta\urcorner$ that it would be arbitrarily conservative to suppose that there exists a single relation, identity, whose uniform theory consists in providing a common theory of inference or a theory of truth for all sentences $\ulcorner\alpha = \alpha\urcorner$ and $\ulcorner\alpha = \beta\urcorner$. Here too a more natural verdict is that here we have *different* "relations". That is to say: identity is multivocal as between $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$.[55] This demolishes at once the Frege-toned argument of §4 in behalf of the modal paradox, an argument that relies, absolutely, upon the univocity of identity as between $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$. If identity is there multivocal, the substitutions needed to proceed, e.g., from $\ulcorner$Necessarily, $\alpha = \alpha\urcorner$[56] to $\ulcorner$Necessarily, $\alpha = \beta\urcorner$ are simply not available merely on the strength of $\ulcorner$Necessarily, α is self-identical$\urcorner$ together with $\ulcorner(\exists\nu)(\text{‘}\alpha\text{’} \vartriangle \nu \;\&\; \text{‘}\beta\text{’} \vartriangle \nu)\urcorner$. The most that substitutionally arises from *that* pair is the entirely welcome $\ulcorner$Necessarily, β is self-identical$\urcorner$.

One virtue certainly of the multivocal solution to Fregean paradoxes is that we have an effortless idiom in which to capture all of Kripke's hunch that seems to be true. If we wish to say that there is a sense in which "Cicero = Tully" is necessary, and if it seems a hopeful device for saying this, to say no matter how awkwardly "Cicero = Tully $\rightarrow \Box$ (Cicero = Tully)", it is open to us to say,

$$(\exists x)((\text{‘Cicero’} \vartriangle x \;\&\; \text{‘Tully’} \vartriangle x) \;\&\; I^{\Box}\, x)$$

in which the two quite distinct identity notions coalesce over Kripke's idea.

§15. *Mathematical identities.* How, finally, do the mathematically equalities fare – e.g. "7 + 5 = 12", which, even by Kant and the Intuitionists are scarcely to be thought of as merely contingent? It is not especially urgent, for our accommodation of the modal paradox of identity, that we settle on a treatment of the mathematical variety, since, whatever, the exact judgement on their modality might be, they are all in any case co-modals, and so cannot generate our puzzle. What follows, therefore, is brief and tentative.

By the crude measure of truth in all worlds, "7 + 5 = 12" seems destined to hold only of every possible world containing the integers. And metaphysical raging concerning necessary existence obscures, and counsels caution on, the point of whether every possible world should be reckoned to house the integers. Perhaps it will suffice to grant that "7 + 5 = 12" is a truth preserved in every possible world containing the integers (and the operation of addition); for that is to grant, or anyhow to insinuate, that "7 + 5 = 12" is at least a necessary truth *de re*, and hence necessary in some sense.

Conceivably the chief attraction of this view of the modality of arithmetical identities is that it leaves the door open for a denotationist map, the arguments of which are assigned contingent images. Its chief drawback is that it seems to want a map whose *de dicto* contingent images are nonetheless necessary in some sense. And it is not clear that the denotationist procedure can be varied in the desired way, unless that is, we are able to content ourselves with the measures in §13.

Clearly the *unvarnished* denotationist procedures of earlier sections seem to work here at least up to preservation of truth. Thus

(44) $7 + 5 = 12$

is taken to

(45) $(\exists x)(\text{`}7 + 5\text{'} \vartriangle x \,\&\, \text{`}12\text{'} \vartriangle x)$[57] ;

and, where 'x' and 'y' vary through the reals, the connexivity property conveyed by

(46) $(\forall x)(\forall y)(x < y \vee y < x \vee x = y)$.

is preserved by

(47) $(\forall z)(\forall x)(\forall y)(x < y \vee y < x \vee (z \vartriangle x \equiv z \vartriangle y))$.

The desideratum to provide an idiom in which to reflect the necessity (in some sense) of sentences (44), (46) and their like, might perhaps be thought to be honoured without abandoning the denotationist treatment of mathematical identities. For, once again, precisely in the sense of § 13, it might be possible to modalize, or essentialize, the identity predicate '=' thus: '$=^{\Box}$'. Instead of distinguishing two alethic expressions for identity-sentences, 'true' and 'necessary', we once more distinguish two identity relations, 'is identical with' and its essentialistic counterpart, 'is necessarily identical with'. As before, the rationale of the choice is to afford ourselves a reasonably natural means of discharging a necessity concept less powerful than the notion of *de dicto* necessity. And, as before, where the *de re* or essentialist necessity

(48) $\phi^{\Box}\alpha$

is taken to

(49) $\phi\alpha \,\&\, (\neg\phi\alpha \prec \neg(\exists\nu)(\alpha = \nu))$,

so here the *de re* necessary truth of mathematical identity e.g.

(50) $2 + 2 =^{\Box} 4$

is taken to

(51) $(\exists x)\,(('2 + 2' \vartriangle x \;\&\; '4' \vartriangle x) \;\&$
$(\neg('2 + 2' \vartriangle x \;\&\; '4' \vartriangle x) \prec$
$\neg(\exists y)(x = y))$.

The terminal clause, itself an identity-formula, becomes under paraphrase

(52) $\neg(\exists w)\,(w \vartriangle x)$

Thus the full paraphrase of "$2 + 2 = {}^{\Box}4$" is

(53) $(\exists x)(('2 + 2' \vartriangle x \;\&\; '4' \vartriangle x) \;\&$
$(\neg('2 + 2' \Delta\, x \;\&\; '4' \Delta\, x) \prec \neg\,(\exists y\,(y \Delta\, x))))$

The *de re* truth, "2 + 2 is necessarily identical with 4" is now understood to convey that the names '2 + 2' and '4' name a common number, and that if it were not so, say if '2 + 2' named

x but '4' did not, then no denoting phrase whatever would denote x. But, as this last clause is itself self-defeating,[58] 'x' therein denoting x the negation of its antecedent is revealed to be necessary in a sense, the sense namely in which to be necessary is to be the negation of a sentence that is self-defeating.

Obviously, however, as it stands, (53) won't do. It plunges the expressions '2 + 2' and '4' into a fanatically co-denotational role which easily might have gone unacknowledged by the actual unfolding of the history of the English language. Possibly the denotation relation, $\triangle$, could be constrained in such a way as to reflect only "English today"; but unless those constraints were exceedingly refined and selective, to say nothing of powerful, we would land ourselves in the obligation to say e.g. that it is essentialistically necessary that Paris is the capital of France.

What this begins to suggest is the possibility that in ordinary mathematical English the identity sign is prodigally employed. Plainly enough identity is needed for substitutional procedures, and when so used, a denotational paraphrase seems an entirely natural device. But is it necessary, to say nothing of perspicuous, to record the fact that the pair 2, 2 sums to 4 with

$$2 + 2 = 4,$$

or the fact that the pair 2, 2 products to 4 with

$$2 \times 2 = 4?$$

If one wished to convey the essentialistic necessity of "$2 + 2 = 4$", it is always open to say,

$$(2, 2) \text{ sums to } 4 \ \& \ (\neg((2, 2) \text{ sums to } 4) \prec \neg\iota(4))$$

in which the only identity idiom crops up somewhat honorifically in the terminal clause, to give full force to the *arithmetic* attributions. And even though one wishes to preserve the inference

$$\begin{array}{l} 2+2=4 \\ \underline{2 \times 2=4} \\ \therefore 2+2=2 \times 2 \end{array}$$

what is lost by saying that if (2, 2) sums to 4, and also products to 4, then (2, 2) both sum and product to the same thing?:

(2, 2) sums to 4

(2, 2) products to 4

$(\exists x)$((2, 2) sums to x & (2, 2) products to x)

Similarly, it is customary to write functional expressions thus:

$$f(y, z) = x\,.$$

But is it any more obvious, to say nothing of *mathematically* important, that it be done that way rather than, say, this way:

x is f-image of (y, z)?

And who will say that this is a disguised identity, who will not also say, unconvincingly, the same thing for:

x is the father of y

or

x is to the right of y?

True, the identity idioms seem to convey uniqueness better than the predication idiom. They seem largely a notational convenience, rather than a metaphysical or mathematical necessity. In fact, however, the uniqueness of the sum of 2 and 2 is not conveyed by the identity sign '=', but rather by the fact that "the sum of" (or in verb form, "sums to") is a functional expression. But even if it were not so, we could always avail ourselves of a different means of handling uniqueness:

((2, 2) sums to 4) & $(\forall z)$((2, 2) sums to $z \rightarrow$ '4' $\triangle$ z))[59]

But the role of the identities in mathematical English is a large subject, which we set aside for the time being. Perhaps it will here suffice to say that, on the face of it, there is nothing particularly *mathematically* germane about the identity relation; and, on the face of it, the metaphysical significance of self-identity attaches to mathematics in no especially mathematical way. True, *everything* is what it is and not another thing – everything is self-identical, whether individuals, numbers, or sets. And, yes, often it is necessary to say that this variable and that have a common value. The predicate 'I' serves the former need; the denotationist transcription, the latter. The *mathematically substantive* 'is's are not perspicuously made out to be identity-'is's; they are 'is's of predication or membership. And, finally, if mathematical truths are necessary, the source of *that* necessity is scarecely to be found in the identity idioms appropriate to mathematical discourse.

§16. *Ambiguity.* The sundry idioms of identity are rather more enigmatic, intractably so it sometimes seems, than we would want them to be. But there can be little *uncontroversial* doubt but what that very unstability automatically enriches our philosophic options. I said, earlier, that the paradoxes that seem to begger even our most innocent paradigms of identity force us into philosophical postures, and that that in turn commends us to review our intuitions on identity and to rifle through myriad frames of reference in search of a perspective that does some justice to the intuitions whilst aborting paradoxes. Needless to say, the baby is not to be discharged with the bathwater; when the balances have been weighed, the new perspective on identity should afford not less than a disequilibrium between intuitiveness and paradox that favours intuitiveness more than do the customary perspectives.

Speaking of perspectives, the modal paradox of identity prompts, more often than not, one or two responses, each of which is rooted in and reinforces that perspective by which '=' is unambiguous as between $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$, and by which the identity relation is correctly enough captured by the standard identity axioms under their customary semantic interpretation.

On the one response,[60] the axioms hold unexceptionably, and the modal paradox is thought to be dispatchable by some or other severely skeptical renunciation of the modals. On the other response,[61] the axioms holds nearly without qualification; the exception, which the modal paradox nicely spotlights, is the fourth axiom, indiscernibility of identicals, which is tolerated only in non-modal environments.[62]

My own point of departure has been, not to renounce the modals, and not to deny the validity of Indiscernibility; rather it has been to hazard that the modal paradox is a symptom of trouble elsewhere, with identity itself. And, although other approaches have here been essayed, our chief perspective has been that identity is ambiguous; that where ϕ is an identity-sentence, the modals 'Necessarily ⓪' and "Contingently ⓪' serve, to some extent, to disambiguate ϕ; that unless the modals are so seen, paradox results; and that identity-sentences ⌜Contingently, ϕ⌝ are, better made out to be expressly metalinguistic. Whence the denotationist map μ for contingent identities.

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NOTES

1 C.S. Peirce [1]. vol. 2, § 316, quoted in Quine [2] p.257.

2 As an example we give the recipe as it applies to the *four*-membered family of classes $\Gamma = \{x,y,z,w\}, \{x\}, \{x,y,z\}, \{y,z\}$. The first member of Γ is the least class with respect to Γ, that is that class that is a subset of every member of Γ. So $\{x\}$ is first in the ordering. The remaining members are generated by the condition: the *i*th class of the order is the least + (i-1) class with respect to Γ. So $\{x,y\}$

is second, $\{x,y,z\}$ is third and $\{x.y.z.w\}$ is fourth. The example is easily generalized.

[3] See Quine [4].

[4] The putative non-dyadicity of identity (save in a trivial syntactic sense) suggests non-reflexivity.

[5] For example, in the von Neumann-Bernays-Gödel theory, or in Quine's NF.

[6] It would be well to recall that the sequence $\langle x,y\rangle$ is *not* the identity relation, I, of which $\langle x,y\rangle$ is but one member. For, of course, we would not mind equivalence classes of the identity relation being members of I, but that is not what the Wiener theory compels, or allows, them to be.

[7] Quine [2] p.182.

[8] It may even be a syntactic ambiguity in the sense of Chomsky [3]. p.28.

[9] To be explicit, we are asking whether '=', in its use as a sign of strict numerical identity, is ambiguous. It is already assumed that the English expression 'is identical with' is ambiguous as between strict and qualitative identity.

[10] The first alternative is argued in Woods [5].

[11] Nor do we overlook the disputed (Hooker [7], pp.951-961) claim (Quine [2] pp.181 ff.) that constant singular terms need not appear in any canonical representation of languages adequate to the demands of science. But it is worth noting that procedures for their elimination require the resources of the identity-relation, which is the very object of our enquiry.

[12] Frege [8], p.56.

[13] Frege [8], p.56.

[14] Shannon and Weaver [9].

[15] Carnap [10], pp.98 and 133 ff.

[16] An array of like consequences is equally unavoidable and at least as undesirable. From (9) "Necessarily, α is just as tall as α" we have (10′), "Necessarily, α is just as tall as β"; and likewise, (10″), "Necessarily, α exactly resembles β", and (10‴), "Necessarily, α is congruent with β", and so on.

[17] See Føllesdal [11].

[18] In the sense of "context" relevant to talk of substitution *salva veritate.*

[19] Prior [12], p.195.

[20] Thus, some skeptical philosophers take the view that, however imperfect it might be, our understanding of the concept of *property* does not invite subscription to the view that for every context, c, there is a property that c "Expresses".

[21] But for a word of historical caution, see Kaplan [27], p.207.

[22] For example, there is the objection that no account of *words* and of *word*-containment has been proferred which would rationalize our saying that the word 'rat' does not occur in the word 'rather'. Certainly by such pre-reflective tests as orthography it does so occur. A less contentious-seeming claim is that what 'rather' denotes or means, or how it is used, is not a function of what 'rat' denotes or means. But even this is open to doubt; it may be that the meaning of "He

wants to go through Moose Jaw" is not in part a function of the meaning of 'to', but it would be counter-intuitive to conclude that 'to' has no occurrence there.

23 Otherwise, we continue to risk confusing a problem in the theory of formal systems (e.g. when is a string a context?) with a similar-seeming but different philosophical or ontological problem (e.g. how is identity to be understood?)

24 So Frege clearly takes the view that because $\ulcorner\alpha = \beta\urcorner$ and $\ulcorner\alpha = \alpha\urcorner$ do not mean the same, they need not modally coincide. Frege thought that they did not mean the same because α and β do not mean the same. But his main point would have remained untouched had he managed to show that they do not mean the same because '=' is ambiguous as between its two occurrences.

25 Apart from this, it might also be argued, in the manner of Carnap [10], p.136, that the doctrine of sense and reference was invented primarily to deal with modal problems, and that it clearly does not do this adequately. Thus it is not totally off-colour to think that a rejection of step IV is not convincing if based solely upon an unreflective acceptance of that doctrine.

26 We do not here depend upon adjudication of the NN-CN dispute, concerning the question, of statements imputing modal values, what *their* modality *shall* be. In particular, in this argument, $\ulcorner\phi \equiv \psi$ is necessary$\urcorner$ can itself be contingent, as can the sentences $\ulcorner\phi$ is necessary$\urcorner$ and $\ulcorner\psi$ is contingent$\urcorner$.

27 "Just so!", by the lights of the second half of "Two Dogmas of Empiricism". Well, not quite: for, let us replace 'necessity' with 'near-total immunity from revision' or 'entrenchedness' and 'contingency' with 'susceptibility'; the result now is either that every identity is entrenched or every identity susceptible, which is not what even the supporters of "Two Dogmas" would be expected to believe.

28 Of course $\mathcal{L}$ does not currently countenance definite descriptions; but since 'The Morning Star' and 'The Evening Star' have, in Strawson's metaphor, "grown capital letters", [13], p.186, they are here treated as names.

29 In support of the verdict of standard systems of qml, e.g. Marcus's QS4 of [14] p.282 and a system of Fitch [15], p.112 ff. Quine concludes that qml is incoherent, and Wiggins [16] that '① $= a$' is not a nice predicate. A discussion of Quine's agitations over qml was the main business of Woods [5].

30 Recall, they have zero information in Shannon's and Weaver's sense.

31 Frege [17], §8. There is some doubt that this is exactly Frege's view, partly because Frege does not yet clearly distinguish between the sense and reference of names (He speaks instead of their content (Inhalt)). Cf. Prior [12].

32 If, *pace* Frege, we choose to regard the expressions on either side of the identity-sign in (13) as definite descriptions, not names, (13) cannot even be expressed in $\mathcal{L}$, and so the *Begriffsschrift*-solution would fail in $\mathcal{L}$.

33 Notice also that (12) - (13) require us to have for $\mathcal{L}$ a suitable strategy for quotation. Let us for now assume that this is so, subject to some later remarks.

34 Of course, if it were, (14) could not be an expression of $\mathcal{L}$.

35 The *mot* is Kaplan's [27] p.212.

36 Of course, it is a self-defeating thing to say (Hintikka [18] pp.32-33) " "$a = b$" but there aren't any names." Following Hintikka I shall say that ϕ virtually implies ψ iff ϕ & $\neg\psi$ is merely a self-defeating thing to say; and that ϕ asserts or entails ψ iff ϕ & $\neg\psi$ is logically inconsistent. We are speaking, at the moment, of entailment or assertion, not of virtual implication.

37 See Quine [2], § 56.

38 See footnote 11.

39 See § 5, above.

40 That is, it may be desirable to conduct the enquiry into identity in a non-standard theory, such as the Meyer-Lambert free quantification theory [19].

41 Concerning which, see Quine [2], p.166, and Binkley [20].

42 *Pace* Geach [21].

43 'Δ' stands for the denotation relation which covers the reference-capacity of both names and of (bound) variables.

44 In Woods [22] it is argued that $\alpha = \alpha$ is not anyhow a necessary truth *de dicto*. We return to the point below.

45 Both (36) and (36**) contain two want-constructions, one *de re* and the other *de dicto* – "wants" and "wants-to-be-true". See Woods [23].

46 Bergmann [24], pp.244-5.

47 Aquinas [25], Chapter 3; Suarez [26], Section II, para. 8.

48 Woods [22], p.181.

49 Let us also remark that another option is opened to us via the earlier distinction between pure identity statements and identification statements.

50 Cf. a similar, though less bullying, view of (*vivid*) names, due to Kaplan [27] pp.220 ff.

51 Thus: "identity is a *philosophic* relation"; "identity is a relation of *reason*"; "identity is an *internal* relation", etc. etc. And see for example Saul Kripke [28], pp.144 ff. for an expression of the view that all identities are necessary.

52 Dana Scott, "Advice on Modal Logic", in K. Lambert (ed.), *Philosophical Problems in Logic* (Dordrecht: Reidel Publishing Co. 1970), p.144. Scott, of course, was speaking of *individuals*, not of self-identity.

53 K. Lambert and R.K. Meyer [19].

54 Of course $\ulcorner\mathfrak{I}\,\Box(\alpha)\urcorner$ could express the (*de re*) necessity of α's self-identity. $\ulcorner\mathfrak{I}\,\Box(\alpha)\urcorner$ could be construed thus: $\ulcorner(\exists v)(`\alpha' \Delta v \prec \mathfrak{I}(\alpha))\urcorner$.

55 It is *not*, however, to say that sentences $\ulcorner\alpha = \alpha\urcorner$ are ill-formed in the language of the denotationist theory of contingent identity-sentences, as we have seen.

56 Assuming, for the moment, that sentences $\ulcorner\alpha = \alpha\urcorner$ are indeed *de dicto* necessities.

57 Note, also that the *unum nomen – unum nominatum* convention provides that '7 + 5' and '12' have in (45) but one common bearer.

58 Alternatively, the terminal clause of (51) could be construed "$\neg\mathfrak{I}(x)$", which is necessarily false *de re*. Under this construal, "2 + 2 is necessarily identical with 4"

would be animated by two notions of identity, one amenable to the denotationist treatment, the other amenable to the predicate-constant treatment.

59 Provided we relaxed the *unum nomen – unum nominatum* condition on names.

60 E.g. Quine [29], [30] and [31].