MARKOV PROCESS MODEL FOR THE ACOUSTIC FIELD IN A WAVEGUIDE WITH A STATISTICALLY UNEVEN WALL

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A system of stochastic equations is derived for the amplitudes of the normal waves of an acoustic field in a waveguide with a statistically uneven wall. Equations for the second and fourth moments of the amplitude are derived in the Markov approximation. Mode intensity fluctuations in a waveguide with cylindrical irregularities are discussed.

In this paper we propose to derive a closed system of stochastic equations for the complex amplitudes of the normal waves of a scalar field in a waveguide with a statistically uneven wall. Equations for the second and fourth moments of the amplitude will be found from this system in a Markov process approximation. The approach taken in this paper is similar to the method originated for the solution of the problem of propagation of a wave beam in a medium with large-scale fluctuations of the dielectric constant and detailed, e.g., in [1].

Consider an acoustic waveguide formed from a perfectly rigid surface $z = D = \mathrm{const}$ and a completely yielding statistically uneven surface $z = \zeta(x, y)$, where x, y, z form a Cartesian coordinate system, and $\zeta(x, y)$ is a zero-mean ($\langle\zeta(x, y)\rangle = 0$), random, weakly inhomogeneous function. Assume that the speed of sound $c = c(z)$ depends in general on the coordinate z. The complex amplitude of the monochromatic field $\Phi(x, y, z)\exp(i\omega t)$ in the region $x \geqslant 0$, where there are no sources, satisfies the Helmholtz equation

$$\Delta\Phi(x, y, z) + (\omega^2/c^2(z))\,\Phi(x, y, z) = 0 \tag{1}$$

and boundary conditions at the waveguide walls:

$$\Phi(x, y, z)|_{z=\zeta(x, y)} = 0, \quad \partial\Phi(x, y, z)/\partial z|_{z=D} = 0.$$

Assuming that the unevenness is sufficiently flat and small compared with the wavelength of the sound, we will utilize, instead of the exact boundary condition at the uneven surface $z = \zeta(x, y)$, an effective boundary condition at the mean plane $z = 0$ [1, 2]:

$$[\Phi(x, y, z) + \zeta(x, y)(\partial/\partial z)\,\Phi(x, y, z)]_{z=0} = 0. \tag{2}$$

We will represent the solution of the wave equation (1) as a sum of the unattenuated modes of a regular waveguide [2, 3]:

$$\Phi(x, y, z) = \sum_{n=1}^{N} \varphi_n(z)\, S_n(x, y)/\sqrt{h_n}, \tag{3}$$

where $\varphi_n(z)$ is an orthonormal system of eigenfunctions $\left(\int_0^D \varphi_n(z)\varphi_m(z)\,dz = \delta_{nm}\right)$ of the operator $d^2/dz^2 + \omega^2/c^2(z)$, with a spectrum of eigenvalues h_n^2 (h_n^2 decreases monotonically with increasing n: $h_n^2 \to -\infty$ as $n \to \infty$), and satisfying the boundary conditions $\varphi_n(z)|_{z=0} = 0$, $d\varphi_n(z)/dz|_{z=D} = 0$, and N is the number of normal modes which are propagating (to which the values h_n correspond). In what follows, a waveguide possessing N unattenuated eigenmodes will be called an N-mode waveguide.

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In the region $x \geqslant 0$, Eq. (1) with boundary condition (2) is equivalent to the integral equation

$$\Phi(x, y, z) = \Phi^{(0)}(x, y, z) + \int_0^{\infty} dx' \int_{-\infty}^{\infty} dy' \zeta(x', y') \left[(\partial/\partial z')\Phi(x', y', z')(\partial/\partial z')G(x, x'; y, y'; z, z')\right]_{z'=0}, \quad (4)$$

where $G(x, x'; y, y'; z, z')$ and $\Phi^{(0)}(x, y, z)$ are a Green's function and the solution of the problem when $\zeta(x, y) \equiv 0$ (the initial field $\Phi^{(0)}(x, y, z)$ is a wave incident on the region $x \geqslant 0$). The solution $\Phi(x, y, z)$ is represented as a sum of fields scattered both in the forward (positive x) and backwards direction. We limit our considerations to only those waves which have experienced no backscattering. This means that in the integral equation (4), we ignore the range of integration $x < x' < +\infty$ [1, 4].

Consider the field $\Phi(x, y, z)$, which in the plane z = const takes the form of a beam of plane waves propagating within a narrow angle along the positive-going x axis. The possibility of such an approximate description of $\Phi(x, y, z)$ is prescribed by the conditions [5]

$$\sqrt{\langle\zeta^2\rangle} \ll \lambda_n, \Lambda_n; \quad (5a)$$

$$\lambda_n \ll l_\perp, l_\parallel; \quad (5b)$$

$$\lambda_n \ll \rho_\perp, \quad (5c)$$

where $\lambda_n = 2\pi/h_n$, Λ_n is the characteristic scale on which the eigenfunction $\varphi_n(z)$ varies, $l_\perp$ and $l_\parallel$ are the transverse (y coordinate) and longitudinal (x coordinate) correlation lengths of irregularities, and $\rho_\perp$ is the transverse coherence length of the wave field. Satisfying the inequalities (5a) and (5b) makes it possible to utilize the approximate boundary condition (2) and to represent the field in the form (3). Condition (5b) also permits one to neglect backscattering, and relation (5c) is a narrowness criterion for the wave beam.

Expand the Green's function in unattenuated normal waves [3]:

$$G(x, x'; y, y'; z, z') = \frac{i}{4}\sum_{n=1}^{N} \varphi_n(z)\varphi_n(z') H_0^{(2)}\left(h_n\sqrt{(x-x')^2+(y-y')^2}\right), \quad (6)$$

where $H_0^{(2)}$ is the Hankel function of the second kind. Substitute into Eq. (4) the sum of normal waves (3), Eq. (6), and the expansion of the initial field $\Phi^{(0)}(x, y, z)$ in modes, whence, taking into account the orthogonality of the eigenfunctions $\varphi_n(z)$, we derive a system of integral equations for $S_n(x, y)$:

$$S_n(x, y) = S_n^{(0)}(x, y) + \frac{ih_n}{2}\sum_{m=1}^{N} a_n^m \int_0^x dx' \int_{-\infty}^{\infty} dy' \zeta(x', y') S_m(x', y') H_0^{(2)}\left(h_n\sqrt{(x-x')^2+(y-y')^2}\right) =$$

$$S_n^{(0)}(x, y) + i\sum_{m=1}^{N} a_n^m \int_0^x dx' \int_{-\infty}^{\infty} dy' \zeta(x', y') S_m(x', y') \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \frac{h_n}{\sqrt{h_n^2-k^2}} \exp\left[-i\sqrt{h_n^2-k^2}(x-x') - ik(y-y')\right], \quad (4a)$$

$$n = 1, 2, \ldots, N,$$

where

$$a_n^m = (1/2)\varphi_n'(0)\varphi_m'(0)/\sqrt{h_n h_m}.$$

It is well known [1, 2] that for each scattering event, the incident wave changes propagation direction by a characteristic angle with a magnitude of order $\lambda_n/l_\perp$ or $\lambda_n/l_\parallel$. Therefore, the main contribution in formation of the wave beam comes from plane waves in the integral representation (4a) with $|k/\sqrt{h_n^2-k^2}|\lesssim \lambda_n/l_\perp,\ \lambda_n/l_\parallel \ll 1$ (i.e., $|k|\lesssim k_{\max}\sim 1/l_\perp,\ 1/l_\parallel$). On this basis, we keep only the first terms of the Taylor expansion $\sqrt{h_n^2-k^2}\approx h_n - k^2/2h_n + \ldots$ and change the function $h_n(h_n^2-k^2)^{-1/2}\exp[-i\sqrt{h_n^2-k^2}(x-x')]$ in (4a) to $\exp[-i(h_n - k^2/2h_n)\times(x-x')$ (which is equivalent to recasting the Green's function in the Fresnel approximation):

$$S_n(x,y) = S_n^{(0)}(x,y) + i\sum_{m=1}^{N} a_n^m \int_0^x dx' \int_{-\infty}^{\infty} dy' \zeta(x',y') \times$$

$$S_m(x',y')\frac{1}{2\pi}\int_{-\infty}^{\infty} dk \exp[-i(h_n - k^2/2h_n)(x-x') - ik(y-y')]. \tag{4b}$$

The change made in (4a) is applicable to (4b) if the difference of the phase excursions of a plane wave with $|k| = k_{\max}$ in (4a) and (4b) (i.e., the quantity $(x-x')|\sqrt{h_n^2-k^2}-h_n+k^2/2h_n| \sim (x-x')k_{\max}^4/8h_n^3 \sim (x-x')\lambda_n^3/l^4$) is small compared with unity. This requirement leads to a limitation on the path length

$$x \ll l_\perp^4/\lambda_n^3,\quad l_\parallel^4/\lambda_n^3. \tag{5d}$$

Applying the differential operator $\partial/\partial x + ih_n + (i/2h_n)\partial^2/\partial y^2$ to both the left-hand and right-hand sides of each of Eqs. (4b), it is not difficult to show that system (4b) is transformed into a system of parabolic equations for the normal wave amplitude, with initial conditions $S_n^{(0)}(0, y) = S_n(0, y)$:

$$\left(\frac{\partial}{\partial x} + ih_n + \frac{i}{2h_n}\frac{\partial^2}{\partial y^2}\right) S_n(x,y) = i\zeta(x,y)\, a_n^m S_m(x,y), \tag{7}$$

where we sum over the index m from 1 to N.

The energy flux density $T_n(x, y)$ of an individual mode may be expressed in terms of the function $S_n(x, y)$ in the following fashion:

$$T_n(x,y) = |S_n|^2 e_1 + \left(\frac{i}{2h_n}\right)\left(S_n\frac{\partial S_n^*}{\partial y} - S_n^*\frac{\partial S_n}{\partial y}\right) e_2, \tag{8}$$

where e_1 and e_2 are unit vectors in the x and y directions. Multiply the parabolic equation for the amplitude $S_n(x, y)$ by the field $S_n^*(x, y)$, add to this the equation for the complex conjugate amplitude $S_n^*(x, y)$ multiplied by the field $S_n(x, y)$, and sum the resulting expression over all n from 1 to N. Then, taking the symmetry $a_n^m = a_m^n$ of the coefficients into account and utilizing Eq. (8), we obtain the following conservation law for the energy flux of the total field $T(x,y) = \sum_{n=1}^{N} T_n(x,y)$ (i.e., for the horizontal component of the energy flux density of the field $\Phi(x, y, z)$ integrated over z):

$$\operatorname{div} T(x,y) = 0,\qquad \sum_{n=1}^{N}\int_{-\infty}^{\infty} dy\, |S_n(x,y)|^2 = \text{const.} \tag{9}$$

It is apparent from Eq. (9) that the total energy flux carried by the acoustic field does not vary along the path, and is statistically independent of the random height variations of the surface irregularities $\zeta(x, y)$.

The solution of the system (7) of parabolic equations obeys the principle of dynamic causality: The field $S_n(x, y)$ is determined only by values of $\zeta(x', y')$ for $x' < x$, and is functionally independent of $\zeta(x', y')$ for $x' > x$. If we take the random function $\zeta(x, y)$ to be δ-correlated in x ($l_\parallel = 0$), then $S_n(x, y)$ is statistically independent of

$\zeta(x', y')$ for $x' > x$ (when $l_{||} \neq 0$, the amplitudes $S_n(x, y)$ are partially correlated with irregularities in the layer $x < x' < x + l_{||}$). We therefore solve below the equations for the moments of the normal wave amplitudes in the Markov process approximation.

Taking the random field $\zeta(x, y)$ to be normal and formally changing the correlation function of the irregularities $\langle\zeta(x_1, y_1)\zeta(x_2, y_2)\rangle$ into an effective correlation function

$$\delta(x_1 - x_2) \int_{-\infty}^{\infty} dx' \, \langle \zeta(x_2 + x', y_1) \zeta(x_2, y_2) \rangle,$$

we may apply to the system (7) a standard argument for the Markov approximation ([1], Ch. 7). For the second and fourth moments of the amplitudes $S_n(x, y)$, we then obtain the equations

$$\left[\frac{\partial}{\partial x} + i(h_n - h_\nu) + \frac{i}{2h_n}\frac{\partial^2}{\partial y_1^2} - \frac{i}{2h_\nu}\frac{\partial^2}{\partial y_2^2}\right] D_n^\nu(x, y_1, y_2) =$$
$$= -N_n^l(x, y_1) D_l^\nu(x, y_1, y_2) - N_l^\nu(x, y_2) D_n^l(x, y_1, y_2) + M_{nk}^{\nu l}(x, y_1, y_2) D_l^k(x, y_1 y_2); \tag{10}$$

$$\left[\frac{\partial}{\partial x} + i(h_n + h_m - h_\nu - h_\mu) + \frac{i}{2h_n}\frac{\partial^2}{\partial y_1^2} + \frac{i}{2h_m}\frac{\partial^2}{\partial y_2^2} - \right.$$
$$\left. - \frac{i}{2h_\nu}\frac{\partial^2}{\partial y_3^2} - \frac{i}{2h_\mu}\frac{\partial^2}{\partial y_4^2}\right] J_{nm}^{\nu\mu}(x, y_1, y_2, y_3, y_4) = -N_n^l(x, y_1) J_{lm}^{\nu\mu} -$$
$$- N_m^l(x, y_2) J_{nl}^{\nu\mu} - N_l^\nu(x, y_3) J_{nm}^{l\mu} - N_l^\mu(x, y_4) J_{nm}^{\nu l} + M_{nl}^{\nu k}(x, y_1, y_3) J_{km}^{l\mu} +$$
$$+ M_{nl}^{\mu k}(x, y_1, y_4) J_{km}^{\nu l} + M_{ml}^{\nu k}(x, y_2, y_3) J_{nk}^{l\mu} + M_{ml}^{\mu k}(x, y_2, y_4) J_{nk}^{\nu l} -$$
$$- M_{nm}^{kl}(x, y_1, y_2) J_{kl}^{\nu\mu} - M_{kl}^{\nu\mu}(x, y_3, y_4) J_{nm}^{kl}, \quad n, m, \nu, \mu = 1, 2, \ldots, N, \tag{11}$$

where

$$D_n^\nu(x, y_1, y_2) = \langle S_n(x, y_1) S_\nu^*(x, y_2) \rangle,$$
$$J_{nm}^{\nu\mu}(x, y_1, y_2, y_3, y_4) = \langle S_n(x, y_1) S_m(x, y_2) S_\nu^*(x, y_3) S_\mu^*(x, y_4) \rangle,$$
$$M_{nm}^{\nu\mu}(x, y_1, y_2) = a_n^\nu a_m^\mu \int_{-\infty}^{\infty} dx' \, \langle \zeta(x + x', y_1) \zeta(x, y_2) \rangle,$$
$$N_n^m(x, y) = (1/2) M_{nl}^{ml}(x, y, y),$$

and the indices l and k are summed from 1 to N.

Since the null approximation for the parameter $l_{\|}$ ($l_{\|} = 0$) satisfies the Markov random process approximation, one of the conditions for Eqs. (10) and (11) to be applicable is that the longitudinal correlation length $l_{\|}$ for irregularities must be small compared with the longitudinal scale of mode interference $l_{nm} = 2\pi/|h_n - h_m|$ and the scale L_0 at which processes associated with multiple scattering from irregularities becomes manifest:

$$l_{\|} \ll l_{nm}, L_0. \tag{12}$$

Note also that for the system (10) and (11), one must consider, instead of (5d), a weaker limit on the path length,

$$x \ll (l_\perp/\lambda_n)^4 \, l_{nm}, \ (l_{\|}/\lambda_n)^4 \, l_{nm}, \tag{5e}$$

which follows from the fact that the even moments are defined by the product

$$H_0^{(2)}\left(h_n \sqrt{(x - x')^2 + (y - y')^2}\right) H_0^{(2)}\left(h_m \sqrt{(x - x'')^2 + (y - y'')^2}\right)$$

(see Eq. (4a)), and therefore the quantity

$$|(x - x')\, k_{\max}^4/8h_n^3 - (x - x'')\, k_{\max}^4/8h_m^3| \sim x\lambda_n^4/l_{nm}\, t^4$$

must be small compared with unity, not the difference of plane wave phase shifts in (4a) and (4b). In particular, condition (5e) is absent for a single-mode waveguide (N = 1).

Some partial consequences of the solution of the differential equations (10) are presented in [5, 6]. Here we will study the intensity fluctuations of modes in a waveguide with irregularities which are cylindrically statistically uniform in x:

$$\int_{-\infty}^{\infty} dx' \langle \zeta(x + x', y_1)\, \zeta(x, y_2)\rangle = \sigma^2 = \mathrm{const}\,(N_n^l = \mathrm{const},\ M_{nm}^{\nu\mu} = \mathrm{const}).$$

Assume that as initial condition, we are given

$$D_n^\nu(0, y_1, y_2) = D_n^{\nu(0)} = \mathrm{const}$$

and

$$J_{nm}^{\nu\mu}(0, y_1, y_2, y_3, y_4) = J_{nm}^{\nu\mu(0)} = \mathrm{const}.$$

Then because of the cylindrical nature of the random field $\zeta(x, y)$, the second and fourth moments of the normal wave amplitudes are independent of the variable y:

$$D_n^\nu(x, y_1, y_2) = D_n^\nu(x), \quad J_{nm}^{\nu\mu}(x, y_1, y_2, y_3, y_4) = J_{nm}^{\nu\mu}(x).$$

(In the present instance, the y coordinate independence of the moments is equivalent to the statistical uniformity of the acoustic field in y.) The systems (10) and (11) then have the partial solutions

$$D_n^n(x) = \mathrm{const}, \quad D_n^m(x) \equiv 0, \quad J_{nn}^{nn}(x) = 2J_{nm}^{nm}(x) = \mathrm{const},\ n \neq m,$$

and the remaining functions $J_{nm}^{\mu\nu}(x) \equiv 0$. It is possible on the basis of physical considerations to demonstrate that these partial solutions are asymptotic solutions when $x \to +\infty$, which is the case independent of the initial values

$$D_n^{\nu(0)} \equiv S_n^{(0)} S_\nu^{(0)*}, \quad J_{nm}^{\nu\mu(0)} = S_n^{(0)} S_m^{(0)} S_\nu^{(0)*} S_\mu^{(0)*}.$$

The integrals

$$\sum_{n=1}^{N} D_n^n(x) = J_0, \qquad \sum_{n,m=1}^{N} J_{nm}^{nm}(x) = J_0^2, \tag{13}$$

follow directly from the energy conservation law (9) (or from Eqs. (10) and (11)); here J_0 = const is the total energy flux carried by the field. Making use of (13), we obtain for the asymptotic solutions

$$D_n^n(x) = J_0/N, \quad J_{nn}^{nn}(x) = 2J_0^2/N(N + 1). \tag{14}$$

The solutions given describe the stationary distribution of energy and of the mean square energy in the modes. Thus, as $x \to +\infty$, intensity fluctuations of an individual mode approach

$$f_n = \frac{J_{nn}^{nn}(x) - [D_n^n(x)]^2}{[D_n^n(x)]^2} = \frac{N-1}{N+1},$$

and the coherence coefficient of the stochastic components of the intensities of the different modes takes the value

$$k_{nm}(x) = \frac{J_{nm}^{nm}(x) - D_n^n(x)\, D_m^m(x)}{[J_{nn}^{nn} - (D_n^n)^2]^{1/2}\, [J_{mm}^{mm} - (D_m^m)^2]^{1/2}} = -\frac{1}{N-1}.$$

For a waveguide with a cylindrically irregular surface, the generatrix of which is parallel to the y axis, and with a field that is statistically uniform in y, only the plane wave with k = 0 enters into the integral expansions (4a) and (4b). Consequently, the transition from (4a) to (4b) is rigorous, and there are no conditions (5d) or (5e) limiting the path length.

Let us solve systems (10) and (11) in a two-mode waveguide (N = 2). To do so, we represent the second and fourth moments of the normal wave amplitudes in the form

$$D_n^{\nu}(x) = A_n^{\nu}(x)\exp\left[i(h_\nu - h_n)x\right]$$

and

$$J_{nm}^{\nu\mu}(x) = B_{nm}^{\nu\mu}(x)\exp\left[i(h_\nu + h_\mu - h_n - h_m)x\right].$$

The characteristic scale of variation of the functions $A_n^{\nu}(x)$ and $B_{nm}^{\nu\mu}(x)$ (as well as the functions $D_n^n(x) = A_n^n(x)$ and $J_{nm}^{nm}(x) = B_{nm}^{nm}(x)$) is of the same order of magnitude as the quantity $L_0 \sim (M_{12}^{12})^{-1} = \alpha^{-1}$, where $\alpha = \sigma^2(\varphi_1(0))^2(\varphi_2(0))^2/4h_1h_2$, and is large compared with the longitudinal scale of mode interference $l_{12} = 2\pi/|h_1 - h_2|$ (see [2]). We therefore average the equations for $A_n^{\nu}(x)$ and $B_{nm}^{\nu\mu}(x)$ over some spatial scale L, which is large compared with l_{12} but small compared with L_0. Then in the equations for the functions $A_n^{\nu}(x)$ and $B_{nm}^{\nu\mu}(x)$ terms containing the rapidly oscillating factor $\exp[i(h_1 - h_2)x]$ vanish, these terms being responsible for the partial correlation of the fluctuating components of the normal waves (see [5]). As a result, the low-contrast interference pattern of the field due to the partial correlation of the stochastic components of the modes is smoothed out, and the solution of Eqs. (10) and (11) with initial conditions $S_1(0) = 1$, $S_2(0) = 0$ (only one mode is excited at the wavelength input) takes the form

$$\overline{D_1^1(x)} = \frac{1 + e^{-2\alpha x}}{2}, \quad \overline{D_2^2(x)} = \frac{1 - e^{-2\alpha x}}{2}, \quad \overline{J_{12}^{12}(x)} = \frac{1 - e^{-6\alpha x}}{6},$$

$$\overline{J_{11}^{11}(x)} = \frac{1}{3} + \frac{e^{-2\alpha x}}{2} + \frac{e^{-6\alpha x}}{6}, \quad \overline{J_{22}^{22}(x)} = \frac{1}{3} - \frac{e^{-2\alpha x}}{2} + \frac{e^{-6\alpha x}}{6},$$

where the overbar signifies averaging: $\overline{D_n^n(x)} = \frac{1}{L}\int_x^{x+L} dx' D_n^n(x')$, $\overline{J_{nm}^{nm}(x)} = \frac{1}{L}\int_x^{x+L} dx' J_{nm}^{nm}(x')$. The qualitative behavior of intensity fluctuations of the modes f_1 and f_2 is plotted in Fig. 1. An analysis of the solution shows that on a scale of $1/2\alpha$, not only are the energy fluxes carried by the modes equal ($D_1^1(x) = D_2^2(x)$), but so are their fluctuations. Allowing for the interference structure of the field leads to the appearance of weak, attenuated oscillations of the functions $f_1(x)$ and $f_2(x)$ near the average curves depicted in Fig. 1, with a

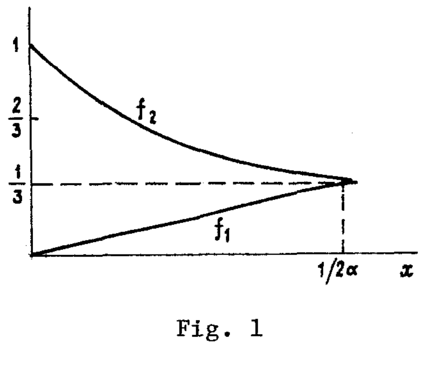

Fig. 1

period of oscillations close to l_{12}. If the energy flux carried by the first mode exceeds the mean value $D_1^1(x)$ for some realization, then the energy conservation law (9) dictates that the energy flux carried by the second mode will fall short of the average value $D_2^2(x)$ by just the same amount. The variance in the intensity of normal waves is therefore the same, and the coherence coefficient for the stochastic components of the intensity along the path is constant and equal to $k_{12}(x) = -1$.

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THE PROBLEM OF BACKSCATTERING IN THREE-DIMENSIONAL, RANDOMLY INHOMOGENEOUS MEDIA

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The statistical characteristics of backscattering in the diffusional approximation are studied on the basis of a nonlinear equation for the forward-scattered field for three-dimensional, randomly inhomogeneous media. Expressions are obtained for the moments of the reflection coefficient, and they are used to analyze the intensity moments of backscattering on a concrete example of a spectrum of the incident field.

In connection with problems of the probing of inhomogeneous media and radar, there is great interest in the problem of backscattering (BS) in three-dimensional, randomly inhomogeneous media. This problem has been considered by many authors within the framework of statistical theory using various methods (see, e.g., [1-3]). In [4, 5], BS was studied on the basis of a nonlinear equation for the forward-scattered field. The nonlinear equation, for which one sets up a problem with initial data, allows one to obtain a system of equations for the statistical characteristics of the BS field (moments) in the diffusional approximation. Solving this system in a first approximation with respect to fluctuation intensity allows one to construct a modified perturbation theory differing considerably from the Born approximation.

The role of the modified perturbation theory was studied on the concrete example of stratified, randomly inhomogeneous media [6], since there one can also obtain an exact solution of the system of equations for the statistical characteristics of the BS field, obtained in the diffusional approximation. In comparing the two solutions it was found that the modified perturbation theory, besides the fact that it is applicable only in the case of small BS, also does not allow for effects which accumulate with time, which results in false damping of the BS intensity moments. In this connection, and also in connection with questions of long-range probing of inhomogeneous media and radar, in three-dimensional

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