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Field integrals error of undulator

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Abstract

The errors of undulator magnet field integrals are analyzed. The formulas of first and second field integral error are given as they are induced by field error σ_B and by positioning error σ_z, respectively. The requirement for the measurement points per period also is analyzed.

1. Introduction

The undulator is a significant device in the third generation synchrotron radiation sources and in free-electron lasers. The undulator magnetic field distribution must be optimized to minimize the effect of the device on the electron beam and to obtain as ideal a radiation spectrum as possible [1]. The influences of undulator imperfections on radiation spectrum can be characterized by the phase error that has been discussed in many papers [2–5]. To minimize the disturbing effect on the electron beam, the negligible values of the field integrals are required. First integral of the magnetic field determines the angular deflection of the electron beam after passing the undulator and induces the closed orbit distortion on the storage ring, while the second integral determines the orbital displacement. Therefore, sufficiently accurate determination of the field integral values are needed for the calculation of electron trajectory and radiation spectrum.

2. Field integrals error formula

The most common method of undulator magnet field measurement is the point-by-point measurement performed by Hall probe scanning. For sufficient accuracy a high density of points is used and to reduce the measurement time data are taken on-the-fly. From the measured data of undulator magnet field, sufficient precise field integrals can be given by trapezoid formula

$$I = \sum_{i=1}^{N_z} B_i \Delta z_i = \Delta z \sum_{i=1}^{N_z} B_i \tag{1}$$

where N_z is the number of data points, Δz is the spacing of the data (uniformly spaced data are considered in this paper). The length of the measured undulator is $N_z \Delta z = L = N\lambda_u$, λu is the period of the undulator, and N the number of periods.

Firstly, we consider the first field integral error introduced by statistical uncertainty in the Hall

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plate reading due to resolution and noise (therefore, the field error, can be regarded as independent of position). A random field error with rms.σ_B gives an rms. field integral error

$$\sigma_{I,B}^2 = \sum_{i=1}^{N_z} \left(\frac{\partial I}{\partial B_i}\right)^2 \sigma_{Bi}^2 = \sum_{i=1}^{N_z} \Delta z_i^2 \sigma_{Bi}^2 = N_z \Delta z^2 \sigma_B^2 \quad (2)$$

so

$$\sigma_{I,B} = \sqrt{L\Delta z}\sigma_B . \quad (3)$$

For the integral error due to positioning accuracy of the probe along the z-axis, we have

$$\sigma_{I,z}^2 = \sum_{i=1}^{N_z} \left(\frac{\partial I}{\partial z_i}\right)^2 \sigma_{zi}^2 \approx \sum_{i=1}^{N_z} \left(\frac{\partial B_i}{\partial z_i}\right)^2 \Delta z^2 \sigma_z^2 \quad (4)$$

where σ_z is the average of σ_{zi}. Taking undulator magnet field $B = B_0 \sin(k_u z)$, $k_u = 2\pi/\lambda_u$, for a large number of data points per period it has

$$\sum_{i=1}^{N_z} \left(\frac{\partial B_i}{\partial z_i}\right)^2 = k_u^2 B_0^2 \sum_{i=1}^{N_z} \cos^2 k_u z_i \approx k_u^2 B_0^2 \frac{N_z}{2}, \quad (5)$$

Eq. (4) becomes

$$\sigma_{I,z} = \pi \sqrt{\frac{2N}{n_z}} B_0 \sigma_z \quad (6)$$

where $n_z = \lambda_u/\Delta z = N_z/N$ is the number of data points per period. The formulas (3) and (6) are the result presented in Ref. [6].

Now, we consider the second field integral:

$$II = \sum_{j=1}^{N_z} I_j \Delta z_j = \sum_{j=1}^{N_z} \left(\sum_{i=1}^{j} B_i \Delta z_i\right) \Delta z_j. \quad (7)$$

The rms. error introduced by field error σ_B is

$$\sigma_{II,B}^2 = \sum_{j=1}^{N_z} \left(\frac{\partial II}{\partial Bj}\right)^2 \sigma_{Bj}^2 \quad (8)$$

where

$$\frac{\partial II}{\partial B_j} = \Delta z_j \sum_{i=j}^{N_z} \Delta z_i = (N_z - j + 1)\Delta z^2. \quad (9)$$

Eq. (8) becomes

$$\sigma_{II,B}^2 = \sum_{j=1}^{N_z} (N_z - j + 1)^2 \Delta z^4 \sigma_B^2 = \Delta z^4 \sigma_B^2 \sum_{l=1}^{N_z} l^2 \approx \Delta z^4 \sigma_B^2 \frac{N_z^3}{3}. \quad (10)$$

Therefore, the second field integral error induced by σ_B is

$$\sigma_{II,B} = \frac{L^2}{\sqrt{3N_z}} \sigma_B. \quad (11)$$

For the second integral error induced by positioning error σ_z,

$$\sigma_{II,z}^2 = \sum_{j=1}^{N_z} \left(\frac{\partial II}{\partial z_j}\right)^2 \sigma_{zj}^2 \quad (12)$$

where

$$\frac{\partial II}{\partial z_j} = \frac{\partial II}{\partial B_j}\frac{\partial B_j}{\partial z_j} = \frac{\partial B_j}{\partial z_j} \Delta z_j \sum_{i=j}^{N_z} \Delta z_i = \frac{\partial B_j}{\partial z_j} \Delta z_j (N_z - j + 1)\Delta z. \quad (13)$$

Then Eq. (12) is written as

$$\sigma_{II,z}^2 = \Delta z^2 \sum_{j=1}^{N_z} \left(\frac{\partial B_j}{\partial z_j}\right)^2 \Delta z_j^2 (N_z - j + 1)^2 \sigma_{zj}^2 \approx \frac{N_z^3}{6} k_u^2 B_0^2 \Delta z^4 \sigma_z^2. \quad (14)$$

Therefore, we have

$$\sigma_{II,z} = \pi B_0 N \Delta z \sqrt{\frac{2N_z}{3}} \sigma_z. \quad (15)$$

The error of the second field integral has relation with the error of the first field integral

$$\sigma_{II} = \frac{L}{\sqrt{3}} \sigma_I \quad (16)$$

introduced by both field error σ_B and positioning error σ_z.

Besides, the uncertainty in the zero-field offset due to the resolution of the system and/or drift due to temperature fluctuations can introduce a field integral error

$$\Delta I = L\Delta B \quad (17a)$$

$$\Delta II = L^2 \Delta B/2. \quad (17b)$$

3. Discussion

Using dimensionless quantities, the rms error of undulator field integrals, Eqs. (3) and (6), can be re-written as

$$\sigma_{I,B} = \frac{K}{0.934}\sqrt{\frac{N}{n_z}}\frac{\sigma_B}{B_0}, \quad (\mathrm{T^*cm}) \tag{18}$$

$$\sigma_{I,z} = \frac{K\pi}{0.934 n_z}\sqrt{\frac{2N}{n_z}}\frac{\sigma_z}{\Delta z}, \quad (\mathrm{T^*cm}). \tag{19}$$

where $K = eBmck_u$ is a parameter of the undulator. The formulas of the second field integral error (Eqs. (11) and (15)) can also be written in a similar way. If both the field error σ_B and positioning error σ_z are considered, the error of the first field integral and the error of the second field integral are given by

$$\sigma_I = \frac{K}{0.934}\sqrt{\frac{2N}{n_z}}\sqrt{\frac{1}{2}\left(\frac{\sigma_B}{B_0}\right)^2 + \left(\frac{\pi\sigma_z}{n_z\Delta z}\right)^2}, \quad (\mathrm{T^*cm}) \tag{20}$$

$$\sigma_{II} = \frac{KL}{0.934}\sqrt{\frac{2N}{3n_z}}\sqrt{\frac{1}{2}\left(\frac{\sigma_B}{B_0}\right)^2 + \left(\frac{\pi\sigma_z}{n_z\Delta z}\right)^2}, \quad (\mathrm{T^*cm^2}). \tag{21}$$

We can see that the errors of field integrals are proportional to the undulator parameter K. Generally, $\sigma_B/B \sim 10^{-4}$, $\sigma_z \sim 10^{-3}$ cm, $\lambda_u \sim$ cm, therefore from the above formulas the effect of positioning errorσ_z is larger than the effect of the field error σ_B. For example, $K/0.934 = 2$, $N = 50$,$n_z = 100$, $\lambda_u = 4$ cm, then the field integral error induced by the field error and positioning error will be $\sigma_{I,B} \approx 1.4$ Gscm and $\sigma_{Iz} \approx 15.7$ Gscm, respectively. If an uncertainty of 0.1 Gs is present in the zero-field offset, by Eqs. (17a) and (17b) it will lead to an integral error of $\Delta I = 20$ Gscm, $\Delta II = 0.20$ Tcm^2 for example. The uncertainty in the zero-field offset has a large effect on the value of field integrals. Care must be taken to accurately calibrate the Hall plate and keep the environment of the measurement stable.

In this paper we neglected the effect of positioning error on spacing of the data. If the data is taken by step-by-step, but not by on-the-fly, Eq. (1) should be

$$I = \frac{1}{2}\sum_{i=1}^{N_z}(B_i + B_{i-1})(z_i - z_{i-1}) \tag{22}$$

and $\partial I/\partial z_i$ in Eq. (4) becomes

$$\begin{aligned}\frac{\partial I}{\partial z_i} &= \frac{1}{2}\frac{\partial B_i}{\partial z_i}(z_{i+1} - z_{i-1}) - \frac{1}{2}(B_{i+1} - B_{i-1}) \\ &= \frac{\partial B_i}{\partial z_i}\Delta z - \frac{1}{2}(B_{i+1} - B_{i-1}) \\ &\approx B_0 \cos(k_u z_i)[k_u\Delta z - \sin(k_u\Delta z)]\end{aligned} \tag{23}$$

where $k_u\Delta z = 2\pi/n_z \ll 1$. So one can see that the effects of the two terms in the above equation counteracted with each other. Therefore, with the effect of the positioning error on the spacing neglected, the previous results on the field integrals error due to positioning accuracy are fairly conservative.

Finally, we analyse the requirement of the measurement points per period. Still taking a sinusoidal distribution for undulator magnet field, a position variation of δz, gives rise to a variation of field strength

$$\frac{\delta B}{B} = \frac{2\pi}{\lambda_u} ctg(k_u z)\delta z. \tag{24}$$

At peak field point

$$\left|\frac{\delta B}{B_0}\right| = \left(\frac{2\pi}{\lambda_u}\delta z\right)^2. \tag{25}$$

If δz, the positioning deviation of the peak point here, is aroused by the spacing of the data (Δz), it has $\delta z \le \Delta z/2$, then for the number of data points per period n_z, we have

$$n_z = \frac{\lambda_u}{\Delta z} \sim \frac{\pi}{\sqrt{|\delta B/B_0|}}. \tag{26}$$

Assuming the physical demand for magnet field to be $\Delta B/B$, it should have

$$\left|\frac{\delta B}{B_0}\right| < \frac{\Delta B}{B}. \tag{27}$$

Otherwise it cannot be determined whether the demand is satisfied. If the field error of the measurement system is σ_B/B, then we can make

$$\left|\frac{\delta B}{B_0}\right| \ge \frac{\sigma_B}{B} \tag{28}$$

because a smaller is not of much meaning. From the above three equations, we give the following requirement for the number of data points per period:

$$\frac{\pi}{\sqrt{\Delta B/B}} \leq n_z \leq \frac{\pi}{\sqrt{\sigma_B/B}}. \tag{29}$$

Typically $\Delta B/B \sim 10^{-3}, \sigma_B/B \sim 10^{-4}$, then $100 \leq n_z \leq 300$, which is difficult for the undulator with short periods.

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