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Low complexity LMS-Type adaptive algorithm with selective coefficient update for stereophonic acoustic echo cancellation

Khaled Mayyas

Department of Electrical Engineering, Jordan University of Science and Technology, P.O. Box 3030, Irbid 221 10, Jordan

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ABSTRACT

Stereophonic acoustic echo cancellation (SAEC) has brought up recently much attention and found a viable place in a number of hands-free applications. In this paper, we propose an LMS-type algorithm for SAEC based on decomposing the long adaptive filter of each channel of the SAEC system into smaller subfilters. We further reduce the complexity of the algorithm by employing the selective coefficient update (SCU) method in each subfilter. This leads to a significant improvement in the convergence rate of the algorithm with low computational overhead. However, the algorithm has a high final mean-square error (MSE) at steady-state that increases as number of subfilters increases. A combined-error algorithm is presented that achieves fast convergence without compromising the steady state error level. Simulations demonstrate the convergence speed advantages of the combined-error algorithm.

1. Introduction

Beside the common problem of large filter orders in single channel acoustic echo cancellation, acoustic echo cancellation in stereophonic systems has brought with it extra difficulties in implementation. The two-channel input signals are highly correlated resulting in a very ill-conditioned covariance matrix. Additionally, stereophonic acoustic echo cancellation (SAEC) needs four adaptive filters, two per channel, where each can be of an order of thousands. This gives rise to essential challenges in the implementation of the SAE canceller due to the high computational complexity problem, and the very slow convergence of existing low complexity algorithms [1–3].

Recursive least squares (RLS) algorithms can provide excellent stereo acoustic echo cancellation. However, computational requirements for RLS-type algorithms are rather large and cannot be implemented directly in real time with current DSP processor resources [1,4,5]. Affine projection (AP) algorithms have been derived for SAEC [6,7]. Those algorithms are still computationally expensive for SAEC, whereas the fast versions suffer numerical instability problems. The transform-domain algorithms in [8–10] have low computational overhead, but similar to the RLS-type and AP-type algorithms, they are memory intensive and need long software codes to implement.

In [11], a stereophonic LMS algorithm based on the decomposition of the adaptive filter was introduced that leads to considerable improvements in convergence rate over the stereo LMS (SLMS) and with a complexity that is almost the same as that of the SLMS. A popular approach to lowering the computational complexity of the adaptive algorithm is to update only a subset of the adaptive filter coefficients per iteration. The resulting algorithm needs fewer arithmetic operations compared to its full-update counterpart. In [12], the decomposition technique was applied to the transform-domain (TD) algorithm [13] and algorithm complexity was reduced by using the selective coefficient update (SCU) approach [14]. In this paper, we reduce the computational complexity of the algorithm in [11] by using the SCU approach. The SCU algorithm was shown

E-mail address: mayyas@just.edu.jo

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to have the closest performance to its full-update (FU) counterpart while reducing the complexity of the FU algorithm. The proposed algorithm has the fast converging characteristic of the decomposition technique and the benefits of the SCU of low complexity with minimal performance losses, which make it amenable for SAEC. Analysis of the MSE convergence and steady-state performance of the algorithm for zero-mean white Gaussian signals are provided for the case when one coefficient of each subfilter is updated per iteration. Analysis will establish step-size bounds for MSE convergence and provide an expression for the algorithm steady-state MSE.

It will be noted that, due to the decomposition technique [11], the proposed algorithm has a higher misadjustment than the SLMS. This leads to the development of a variable-α combined-error algorithm, which is a hybrid of the proposed algorithm and the stereo NLMS (SNLMS). The combined-error algorithm achieves very fast convergence without compromising the steady state error level.

We should emphasize here two points. Firstly, we assume the real and practical situation that the length of the adaptive filter L is less than the length of the impulse responses in the transmission room, which ensures that the well-known non-uniqueness problem in SAEC does not occur [3]. A complete characterization of the nonuniqueness problem can be found in [3]. Secondly, the objective of the proposed algorithms is not to present a solution to the well-known misalignment problem in SAEC [3] (or reduce misalignment) that is due to the high crosscorrelation between the two inputs, but rather to introduce an algorithm for SAEC, where the condition number is high, that accelerates convergence and reduces the computational complexity of the SLMS algorithm.

2. Full-update stereophonic LMS algorithm based on the decomposition of the adaptive filter

In SAEC, the two-channel input signal vectors and filter coefficient vectors can be combined as $\mathbf{X}(n) = [x_1(n)x_1(n-1)\ldots x_1(n-L+1)x_2(n)x_2(n-1)\ldots x_2(n-L+1)]^T$, $\mathbf{W}(n) = [w_{11}(n)w_{12}(n)\ldots w_{1L}(n)w_{21}(n)w_{22}(n)\ldots w_{2L}(n)]^T$, respectively, and L is the adaptive filter length. In [11], the input vector $\mathbf{X}(n)$ and the weight vector $\mathbf{W}(n)$ are partitioned into M sub-vectors such that

$$\mathbf{X}(n) = [\mathbf{X}_1^T(n)\mathbf{X}_2^T(n)\ldots\mathbf{X}_M^T(n)]^T$$
$$\mathbf{W}(n) = [\mathbf{W}_1^T(n)\mathbf{W}_2^T(n)\ldots\mathbf{W}_M^T(n)]^T$$

where $\mathbf{X}_i(n)$ and $\mathbf{W}_i(n)$ have each P_i elements, and $2L = \sum_{i=1}^{M} P_i$. P_i, $i = 1,2\ldots,M$ need not to be equal for implementation purposes. However, we will assume throughout this paper that all subfilters have equal lengths, i.e., $P = P_1 = P_2 = \ldots = P_M$. We introduce M error signals as [11]

$$e_i(n) = d(n) - \sum_{j=1}^{i}\mathbf{W}_j^T(n)\mathbf{X}_j(n),\ i = 1,2,\ldots,M \tag{1}$$

and let each subfilter $\mathbf{W}_i(n)$ seek the minimum of the corresponding cost function $J_i(e_i(n)) = E\{e_i^2(n)\}$. Here, $d(n)$ is the output of the unknown system. Minimization of the instantaneous estimate of this cost function with respect to $\mathbf{W}_i(n)$ results in the LMS equation for each subfilter as[11],

$$\mathbf{W}_i(n+1) = \mathbf{W}_i(n) + \mu_i\mathbf{X}_i(n)e_i(n) \tag{2}$$

where μ_i is the adaptation step size, and $e_i(n)$ has the order update equation

$$e_i(n) = e_{i-1}(n) - \mathbf{W}_i^T(n)\mathbf{X}_i(n),\ i = 2,3,\ldots,M \tag{3}$$

with $e_1(n) = d(n) - \mathbf{W}_1^T(n)\mathbf{X}_1(n)$, and the system output error is given by

$$e_M(n) = d(n) - \sum_{j=1}^{M}\mathbf{W}_j^T(n)\mathbf{X}_j(n) = d(n) - \mathbf{W}^T(n)\mathbf{X}(n) \tag{4}$$

We refer to this algorithm as the full-update stereo decomposition LMS (FU-SDLMS), and the stereophonic acoustic echo cancellation setup with filter decomposition is shown in Fig. 1.

The arithmetic complexity of the FU-SDLMS algorithm, in terms of number of multiplications and additions, is almost the same as that of the traditional stereo LMS algorithm. The algorithm adds only M multiplications for the calculation of $\mu_i e_i(n), i = 1,2,\ldots,M$, which is a negligible amount.

3. Selective coefficient update stereophonic DLMS (SCU-SDLMS)

A widely accepted approach to reducing the computational cost of the adaptive algorithm is to update a portion of the adaptive filter coefficients at each iteration. The complexity overhead of the FU-SDLMS can be lowered by incorporating this approach in its update mechanism. We will show that the selective coefficient update (SCU) method, which belongs to the family of partial coefficient update algorithms, leads to the closest possible performance to the full-update algorithm.

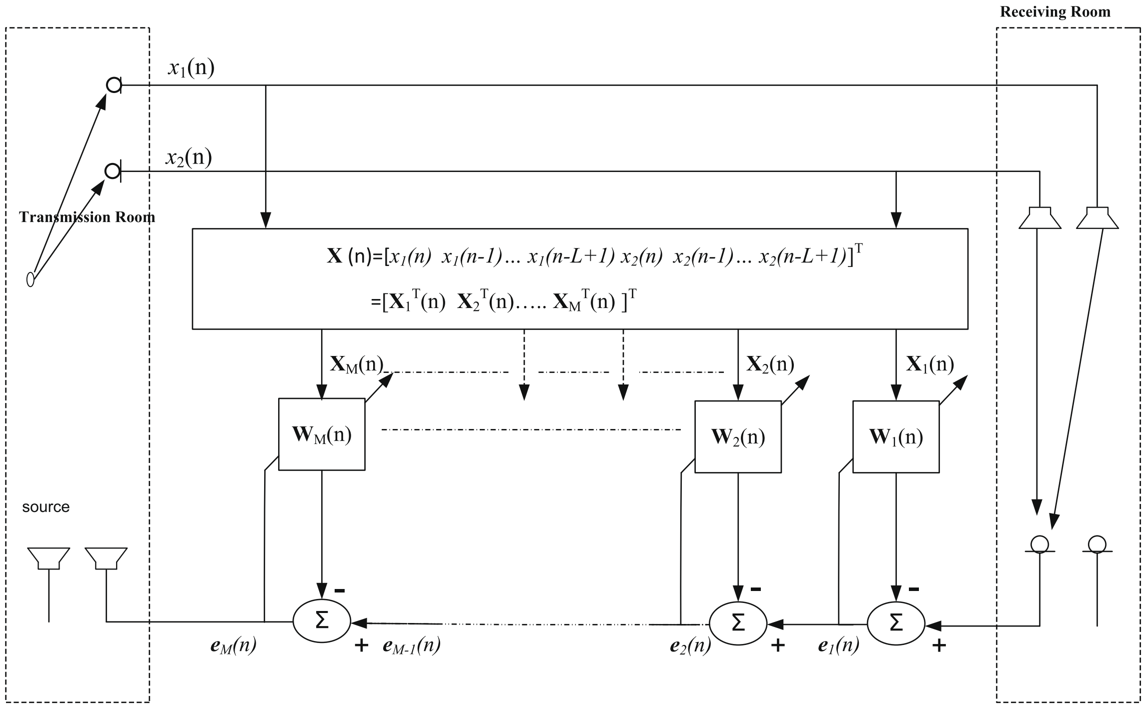

Fig. 1. The stereophonic acoustic echo cancellation setup with filter decomposition.

We propose here that for each subfilter, N coefficients out of the subfilter P coefficients are updated per iteration. Define the diagonal $P \times P$ matrix $\mathbf{A}_i(n)$ that has N ones at entries indicated by $r_j, j = 1, 2 \ldots, N$, and zeros elsewhere. Then, the partial update version of the algorithm in Eq. (2) is given by

$$\mathbf{W}_i(n+1) = \mathbf{W}_i(n) + \mu \mathbf{A}_i(n)\mathbf{X}_i(n)e_i(n) \tag{5}$$

where $i = 1, 2, \ldots, M$. We notice from Eq. (5) that only N coefficients of the i-th subfilter specified by the nonzero entries in $\mathbf{A}_i(n)$ are updated, whereas the remaining P-N coefficients stays unchanged. The selection of those N coefficients will be based on the following minimization criterion [14]: choose the N coefficients of $\mathbf{W}_i(n)$ to be updated at each iteration that result in the minimum of the respective instantaneous square *a posteriori* error $e_{pi}(n)$ defined as

$$e_{pi}(n) = e_{i-1}(n) - \mathbf{X}_i^T(n)\mathbf{W}_i(n+1) \tag{6}$$

where $i = 1, 2, \ldots, M$, and $e_0(n) = d(n)$. It is easy to see that $e_{pi}(n)$ in Eq. (6) can be expressed as

$$e_{pi}(n) = e_i(n) - \mathbf{X}_i^T(n)\Delta\mathbf{W}_i(n) \tag{7}$$

where $\Delta\mathbf{W}_i(n) = \mathbf{W}_i(n+1) - \mathbf{W}_i(n)$. Substituting the recursion in Eq. (5) into Eq. (7), and squaring the results, we get

$$e_{pi}^2(n) = e_i^2(n)(1 - \mu||\mathbf{A}_i(n)\mathbf{X}_i(n)||^2)^2 \tag{8}$$

To find $r_j, j = 1, 2, \ldots, N$ indicating the set of N coefficients to be updated for the i-th subfilter, we notice that for a given value of $e_i^2(n)$ and μ, $e_{pi}^2(n)$ achieves its minimum when $||\mathbf{A}_i(n)\mathbf{X}_i(n)||^2$ is maximized. This implies that the N coefficients of the i-th subfilter to be updated at each iteration are those that correspond to the N largest $X_{ir_j}^2(n)$ (or $|X_{ir_j}|$), $j = 1, 2, \ldots, P$, where $X_{ir_j}(n)$ is the r_jth element of $\mathbf{X}_i(n)$, thus resulting in the closest possible performance to the full-update algorithm. The selection of those N values can be performed using the Heapsort algorithm [16], which requires $O(P\log_2(N+1))$ comparisons. We denote the algorithm as the Selective Coefficient Update Stereophonic DLMS (SCU-SDLMS).

3.1. Mean-square analysis of the SCU-SDLMS algorithm

The mean-square convergence properties of the SCU-SDLMS algorithm will be investigated in this section. To make the analysis tractable, we consider the case when $N = 1$, i.e., one coefficient of each i-th subfilter is updated per iteration, and assume that the input signals $x_1(n)$ and $x_2(n)$ are zero mean white Gaussian. Moreover, we assume that the desired signal can be described by any of the following equations

$$d(n) = \sum_{j=1}^{i} \mathbf{X}_j^T(n)\mathbf{W}_j^* + z_i(n), \; i = 1, 2, \ldots, M \tag{9}$$

where $z_i(n)$ accounts for the modeling error and noises corrupting $d(n)$, and $\mathbf{W}_i^*$ is the steady-state mean weight solution of the i-th subfilter, given that $E\{\mathbf{W}_j(n)\}\ \forall j < i$ had already converged to $\mathbf{W}_j^*$. $z_i(n)$ is assumed zero-mean stationary and independent of $\mathbf{X}_i(n)$, with a variance

$$\epsilon_{min}^i = E\{z_i^2(n)\} \tag{10}$$

Moreover, $z_i(n)$ is independent of $z_j(n)$ for $i \neq j$. The commonly used assumption in adaptive filtering that $\mathbf{X}_i(n)$ is statistically independent of $\mathbf{W}_i(n)$ will also be employed.

Define the coefficient error vector

$$\mathbf{V}_i(n) = \mathbf{W}_i(n) - \mathbf{W}_i^* \tag{11}$$

then Eq. (5) becomes

$$\mathbf{V}_i(n+1) = \mathbf{V}_i(n) + \mu\mathbf{A}_i(n)\mathbf{X}_i(n)e_i(n) \tag{12}$$

Recall that the mean-square error of the SCU-SDLMS algorithm is given by

$$\epsilon(n) = E\{e_M^2(n)\} \tag{13}$$

where $e_M(n)$ can be found from Eq. (4). Substituting Eqs. (11) in (9) and (4), then using them in Eq. (13) along with our assumptions result in,

$$\epsilon(n) = \epsilon_{min}^M(n) + \sum_{i=1}^{M}\sum_{j=1}^{P} E\{X_{ij}^2(n)\}E\{V_{ij}^2(n)\} \tag{14}$$

Let *max* be the index of the coefficient of the i-th subfilter to be updated at time instant n i.e., $V_{imax}(n)$. Using Eq. (9) in (12) yield

$$V_{imax}(n+1) = (1-\mu X_{imax}^2(n))V_{imax}(n) - \mu\sum_{\substack{l=1\\ l\neq max}}^{P} X_{imax}(n)X_{il}(n)V_{il}(n) - \mu X_{imax}(n)\sum_{j=1}^{i-1}\mathbf{X}_j^T(n)\mathbf{V}_j(n) + \mu z_i(n)X_{imax}(n) \tag{15}$$

Taking the expected value of the square of Eq. (15), and after some manipulations we get,

$$E\{V_{imax}^2(n+1)\} = (1-2\mu\sigma_{xi}^2 + 3\mu^2\sigma_{xi}^4)E\{V_{imax}^2(n)\} + \mu^2\sigma_{xi}^4\sum_{\substack{l=1\\ l\neq max}}^{P} E\{V_{il}^2(n)\} + \mu^2\sigma_{xi}^2\sum_{j=1}^{i-1}\sum_{l=1}^{P}\sigma_{xj}^2 E\{V_{jl}^2(n)\} + \mu^2\sigma_{xi}^2\epsilon_{min}^i \tag{16}$$

where we have used $E\{X_{imax}^2(n)\} = E\{X_{ij}^2(n)\} = \sigma_{xi}^2, \forall j = 1,2,\ldots,P$. This indicates that $x_1(n)$ and $x_2(n)$ can have a different variance but partitioning is performed such that $E\{X_{ij}^2(n)\} = \sigma_{xi}^2, \forall j = 1,2,\ldots,P$. Similar to [14], we assume that $E\{V_{imax}^2(n)\} = E\{V_{ij}^2(n)\} = C_i(n), \forall j = 1,2,\ldots,P$, and that subfilter coefficients have an equal probability of being updated at each iteration, i.e., the probability of updating any coefficient of the i-th subfilter at each sample time is $\frac{1}{P}$. Therefore, for $j \neq max$

$$C_i(n+1) = \left(1-\frac{1}{P}\right)E\{V_{ij}^2(n+1)\} + \frac{1}{P}E\{V_{imax}^2(n+1)\} \tag{17}$$

Note that for $\forall j \neq max$, $E\{V_{ij}^2(n+1)\} = E\{V_{ij}^2(n)\} = C_i(n)$. Substituting Eq. (16) in Eq. (17) results in

$$C_i(n+1) = \left(1-2\frac{\mu}{P}\sigma_{xi}^2 + \frac{\mu^2}{P}\sigma_{ix}^4(P+2)\right)C_i(n) + \mu^2\sigma_{xi}^2\sum_{j=1}^{i-1}\sigma_{xj}^2 C_j(n) + \frac{\mu^2}{P}\sigma_{ix}^2\epsilon_{min}^i \tag{18}$$

To ensure the convergence of Eq. (18), the step-size μ should be bounded by $0 < \mu < \frac{2}{(P+2)\sigma_{xi}^2}$, and the second term $\mu^2\sigma_{xi}^2\sum_{j=1}^{i-1}\sigma_{xj}^2 C_j(n)$ on the right hand side of Eq. (18) should be bounded. It is easy to see that the boundedness of this term is guaranteed by

$$0 < \mu < \frac{2}{(P+2)\sigma_{xj}^2},\ \forall j = 1,2,\ldots,i-1 \tag{19}$$

Thus, MSE convergence and, hence, stability of the algorithm are ensured by

$$0 < \mu < \frac{2}{(P+2)\sigma_{xi}^2},\ \forall i = 1,2,\ldots,M \tag{20}$$

This implies that μ should be chosen as

$$0 < \mu < \frac{2}{(P+2)\begin{pmatrix} max(\sigma_{xi}^2) \\ i = 1,2,\ldots,M \end{pmatrix}} \tag{21}$$

Assuming that the algorithm step-size is chosen according to Eq. (21), and using Eq. (18) in Eq. (14), the steady-state MSE $\epsilon(\infty)$ of the SCU-SDLMS has the form

$$\epsilon(\infty) = \epsilon_{min}^M + \sum_{i=1}^{M}\left(\frac{\mu P\sigma_{xi}^2\epsilon_{min}^i + \mu P^2\sigma_{xi}^2\sum_{j=1}^{i-1}\sigma_{xj}^2 C_j(\infty)}{2-\mu(P+2)\sigma_{xi}^2}\right) \tag{22}$$

Eq. (22) shows that the excess MSE of the SCU-SDLMS [15], represented by the second term on the right hand side of Eq. (22), is due to two sources. The first is a result of fluctuations of the subfilters coefficients, and the second is due to the partial decoupling imposed by the decomposition procedure where each i-th subfilter is influenced by all j-th subfilters for $j < i$.

In general, it is difficult to analytically show that $\epsilon(\infty)$ $(M > 1)$ of the proposed algorithm is larger than that of the SLMS algorithm. Nevertheless, this can be shown for the special case when $\mu = \mu_{\text{SLMS}}$, and all elements of $\mathbf{X}(n)$ are statistically independent with $E\{(X^{(r)}(n))^2\} = \sigma^2$, where $X^{(r)}(n)$ is the rth element of $\mathbf{X}(n)$. Also, we assume that there exists $\mathbf{W}_*$ such that

$$d(n) = \mathbf{X}^T(n)\mathbf{W}_* + \xi(n) \tag{23}$$

where $\xi(n)$ is a zero-mean white noise, independent of $\mathbf{X}(n)$, and of variance $\epsilon_{\min}$. For simplicity we choose $M = 2$ such that $P = L$. Recall that for the SLMS algorithm and under the above assumptions [17],

$$\epsilon(\infty)_{\text{SLMS}} = \epsilon_{\min} + \frac{\sum_{j=1}^{2L}\frac{\mu_{\text{SLMS}}\lambda_j\epsilon_{\min}}{2-2\mu_{\text{SLMS}}\lambda_j}}{1-\sum_{j=1}^{2L}\frac{\mu_{\text{SLMS}}\lambda_j}{2-2\mu_{\text{SLMS}}\lambda_j}} \tag{24}$$

where $\lambda_j,\ j = 1,2,\ldots,2L$ are the eigenvalues of $\mathbf{R} = E\{\mathbf{X}(n)\mathbf{X}^T(n)\}$. In this case, $\lambda_j = \sigma^2\ \forall j$. By examining (9) and (10) we find that

$$\epsilon_{\min}^1 = \epsilon_{\min} + \varrho \tag{25}$$

and

$$\epsilon_{\min}^2 = \epsilon_{\min} \tag{26}$$

where $\varrho = \sigma^2\sum_{r=L+1}^{2L}(W_*^{(r)})^2 > 0$, and $W_*^{(r)}$ is the rth element of $\mathbf{W}_*$. Substituting (25) and (26) into (22), and comparing the result with that from (24), one can easily see that

$$\epsilon(\infty) > \epsilon_{\text{SLMS}}(\infty) \tag{27}$$

where we have assumed a small μ such that $\mu P\sigma^2 \ll 1$ and $2\mu L\sigma^2 \ll 1$. Eq. (27) illustrates the misadjustment problem of the algorithm in (2), which makes its direct implementation in SAEC undesirable. Next, we present an algorithm that provides a solution to this problem.

4. Combined-error selective coefficient update algorithm

We propose here an algorithm that trades between convergence speed and misadjustment using only one parameter. The algorithm is a combination of the SCU decomposition algorithm and the SNLMS algorithm where its equation is given by

$$\mathbf{W}_i(n+1) = \mathbf{W}_i(n) + \mathbf{A}_i(n)\mathbf{X}_i(n)\left(\frac{\mu\alpha e_i(n)}{\mathbf{X}_i^T(n)\mathbf{X}_i(n)+\delta} + \frac{\mu(1-\alpha)e_M(n)}{\mathbf{X}^T(n)\mathbf{X}(n)+\delta}\right),\ i = 1,2,\ldots,M \tag{28}$$

where $0 < \delta \ll 1$, and $1 \geqslant \alpha \geqslant 0$. The step size is normalized by the input signal power to eliminate the dependency of its value on the input signal statistics and to simplify its choice. We refer to the algorithm as the Combined-Error SCU Stereophonic Decomposition NLMS (CE-SCU-SDNLMS). The complexity of the CE-SCU-SDNLMS is $2L + M(N+2) + 6$ multiplications and $2L + M(N+2) + 3$ additions while that of the SNLMS is $4L + 3$ multiplications and $4L + 2$ additions[1]. This implies that as compared to the SNLMS, the CE-SCU-SDNLMS requires $M(P-N-2)-3$ fewer multiplications, and $M(P-N-2)-1$ less additions.[2] The Combined-Error FU-SDNLMS (CE-FU-SDNLMS) algorithm [11] can be obtained similarly at the expense of an additional 5 multiplications and 5 additions over the FU-SDNLMS. When $\alpha = 1$, we get the SCU-SDNLMS

[1] We assumed that $r(n) = \mathbf{X}^{\mathbf{T}}(n)\mathbf{X}(n)$ is calculated using $r(n) = r(n-1) + x_1^2(n) + x_2^2(n) - x_1^2(n-L) - x_2^2(n-L)$, which requires 2 multiplication, 4 additions, and 3 memory locations.

[2] Typically, $\frac{P}{2} \geqslant N$. Therefore, $(M(P-N-2)-3) > 0$ and $(M(P-N-2)-1) > 0$.

algorithm. As α decreases to 0, the SCU-SDNLMS is de-emphasized and the SNLMS characteristics will appear more in the combined-error algorithm. As a result, less misadjustment is expected with a corresponding decrease in convergence rate. When $\alpha = 0$, the CE-SCU-SDNLMS algorithm in Eq. (28) becomes the traditional SNLMS algorithm with SCU. To illustrate this, we plot in Fig. 2 the MSE of a two-channel system identification example with $L = 32$ and $M = 4$ for different values of α. The two unknown systems are FIR filters, each of length 32. A zero mean white noise of variance 0.0001 is added to the desired signal. μ is chosen to be $\mu = 1$, and $N = 4$ such that 16 coefficients out of the two adaptive filters 64 coefficients are updated. The input signals are attained by passing a zero-mean white Gaussian signal through two FIR filters, each has 200 taps. Results are averaged over 100 independent runs. It is clear also from Fig. 2 that the convergence speed of the SCU-SDNLMS is very close to the FU-SDNLMS though only 25% of the two adaptive filters coefficients were updated at each iteration.

An attractive observation of the algorithm in Eq. (28) is that switching α to a smaller value during the adaptation process does not disturb the process. On the other hand, it brings about better performance in terms of deriving the algorithm to lower steady-state error ($M \geqslant 2$). This suggests that α can be made variable. Initially, α is set to 1 to achieve fast convergence, and gradually α is decreased until it reaches 0 or the algorithm attains the desired final steady-state error. An estimate of the MSE $E\{e_M^2(n)\}$ can be computed using the simple recursive equation $\hat{\epsilon}(n) = \beta\hat{\epsilon}(n-1) + (1-\beta)e_M^2(n)$ where β $(0 < \beta < 1)$ is the exponential weighting factor that determines the averaging time constant, i.e., the quality of estimation. The algorithm also keeps monitoring the steady-state error for abrupt changes, i.e., if $\hat{\epsilon}(n) >$ desired level. If detected, the algorithm enters the fast convergence mode setting $\alpha = 1$ and repeating the procedure.

In Fig. 2, we plot the MSE evolution of the CE-SCU-SDNLMS and CE-FU-SNLMS with variable α for the same example above. Here, α is set initially to 1, and when the MSE reached a prespecified level, α is decreased in steps of 0.1 every 100 samples. The prespecified value is close to the steady state MSE value of the algorithm attained when $\alpha = 1$ and was obtained experimentally. It is obvious that the variable-α algorithms possess significant performance improvements over the SNLMS while maintaining its robustness, and at a cost of extra $2M + 3$ multiplications for the CE-FU-SDNLMS, and with 49 less multiplications for the CE-SCU-SDNLMS. It should be pointed out that the mechanism of controlling α can be tailored to the characteristics and parameters of the application to achieve the best possible performance results.

An analytical development of the combined-error algorithm in Eq. (28) is likely to provide useful insight into the transient behavior of the algorithm. However, the high correlation between $e_i(n)$ and $e_M(n)$ leads to many complex terms that are difficult to simplify. This limits the usefulness of such an analysis since it would not clearly quantify the performance of

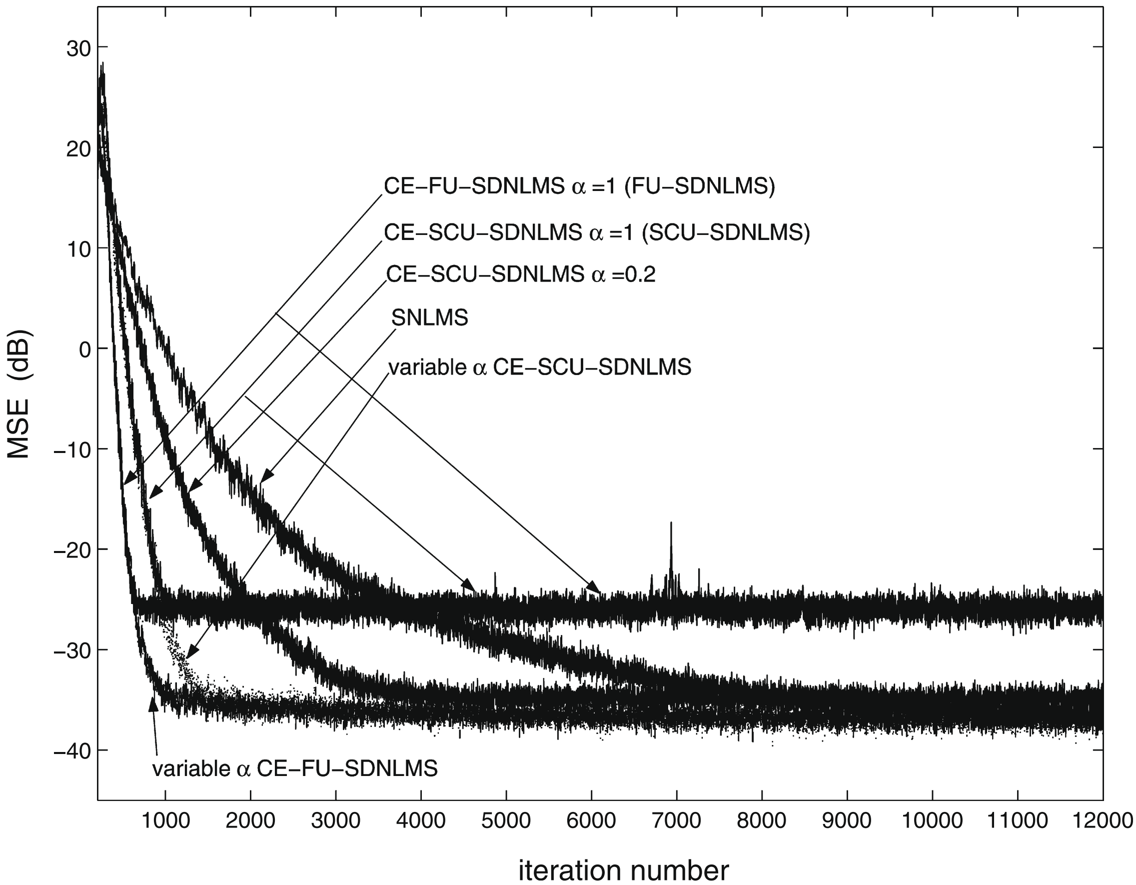

Fig. 2. MSE of the SNLMS, CE-FU-SDNLMS and CE-SCU-SDNLMS with fixed and variable α for $M = 4$ and $N = 4$.

the algorithm. Nevertheless, the initial behavior of the algorithm when $\alpha \approx 1$ can approximately be characterized by that of the subfilter algorithm, which is described and analyzed in the previous sections, while its steady-state properties are those of the well studied SLMS algorithm.

5. Simulations

The nonlinear method proposed in [3] has proven to be very effective in the solution of the misalignment problem in SAEC [2,3]. It has become an intrinsic part of the SAEC structure and has been incorporated in several recently proposed algorithms for SAEC [2]. The performance of the CE-SCU-SDNLMS and CE-FU-SDNLMS can be further improved by employing this method in both algorithms. Also, it is incorporated in all other algorithms used in simulations. We will use an improved version of this method to process the transmission room signals as [2]

$$\acute{x}_1(n) = x_1(n) + \alpha\frac{x_1(n) + |x_1(n)|}{2}$$
$$\acute{x}_2(n) = x_2(n) + \alpha\frac{x_2(n) - |x_2(n)|}{2}$$

where $\alpha = 0.5$.

We compare the performance of the variable-α CE-SCU-SDNLMS and CE-FU-SDNLMS with the SNLMS, and the stereo FRLS (SFRLS) [15]. The impulse responses of the receiving room are each of length 4096 points. They are obtained in an actual room and truncated to 512 points. The length of each adaptive filter is $L = 312$. The two channel signals $x_1(n)$ and $x_2(n)$ are generated by convolving the source signal $s(n)$ with the two impulse responses of the transmission room, each of length 4096 points and are measured in an actual room. The source signal is zero-mean white Gaussian with unity variance. The step sizes for the variable-α algorithms and the SNLMS are chosen to be $\mu = \mu_{SNLMS} = 0.8$. For the variable-α algorithms $M = 8$, and α is initially one and was decreased in steps of 0.1 every 1000 samples when the MSE reached a pre-specified level. The pre-specified value is close to the steady state MSE value of the variable-α algorithm attained when $\alpha = 1$ and was obtained experimentally. The CE-SCU-SDNLMS is used with $N = 39$, and $N = 19$, which correspond respectively to 50%,

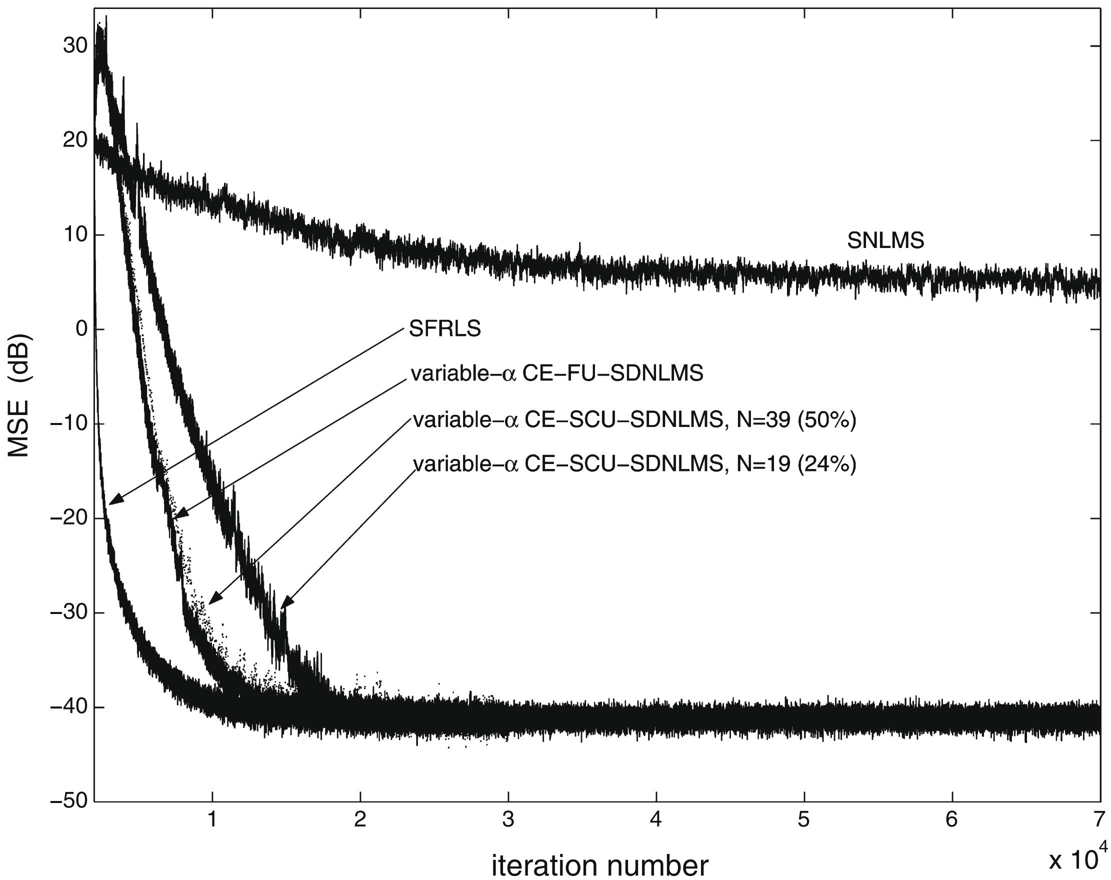

Fig. 3. Comparison of MSE of various adaptive algorithms for example one.

and 24% of the total number of the two adaptive filter 624 coefficients. The SFRLS exhibits the fastest convergence speed with a complexity of $32L$ multiplications per sample period [2,15], which is computationally expensive for direct real time implementations. It is observed from Fig. 3 that the variable-α CE-FU-SDNLMS and CE-SCU-SDNLMS offer a dramatic improvement in performance over the SNLMS. There is no noticeable difference in performance between the CE-FU-SDNLMS and CE-SCU-SDNLMS with $N = 39$ though only 50% of the CE-SCU-SDNLMS filter coefficients are updated per iteration. As expected, the learning curve of the CE-SCU-SDNLMS with $N = 19$ exhibits slower convergence but still offers significant improvement in convergence rate over the SNLMS while requiring 793 multiplications and 795 additions. On the other hand, the SNLMS needs 1251 multiplications and 1250 additions.

The above experiment is repeated for the variable-α CE-FU-SDNLMS and CE-SCU-SDNLMS algorithms, and the SNLMS with a speech input signal of a male. The *echo-return loss enhancement*(ERLE), defined as

$$ERLE[dB] = 10\log_{10}\left(\frac{E\{d^2(n)\}}{E\{e^2(n)\}}\right) \tag{29}$$

is plotted in Fig. 4. The algorithms are used with the same parameters. Results are averaged over a sliding window of 256 samples. We can see that the CE-SCU-SDNLMS sustains its good performance properties in this practical example where it operates as well as the CE-FU-SDNLMS when $N = 39$, and outperforms the SNLMS algorithm for $N = 39$ and $N = 19$.

In the following example, we demonstrate the validity of the mean-square analytical results derived for the SCU-SDLMS in Eq. (5). We compare results for the steady-state MSE $\epsilon(\infty)$ of the SCU-SDLMS algorithm obtained from theoretical analysis via Eq. (22) with results of simulations for $M = 1$ and $M = 8$, and different values of μ and SNR. Results for the SLMS algorithm are also presented for comparison[17]. The values of μ and SNR are selected to cover several experimental conditions of low and high SNR, and small and large step sizes. The input signals $x_1(n)$ and $x_2(n)$ are zero mean white Gaussian with unity variance. The two receiving room impulse responses are each of 32 points, and $L = 32$. The SCU-SDLMS is used with $N = 1$. Simulation results were averaged over 200 independent runs. Table 1 shows that Eq. (22) predicts closely the actual $\epsilon(\infty)$ as obtained from simulations. Also, it shows that the SCU-SDLMS converges with a higher misadjustment than the SLMS for $M \geqslant 2$, and that its misadjustment increases as M increases. Notice that the theoretical results for the SCU-SDLMS with $M = 1$ match those of the SLMS algorithm. The SCU-SDLMS with $M = 1$ becomes the traditional SCU-SLMS, and results are

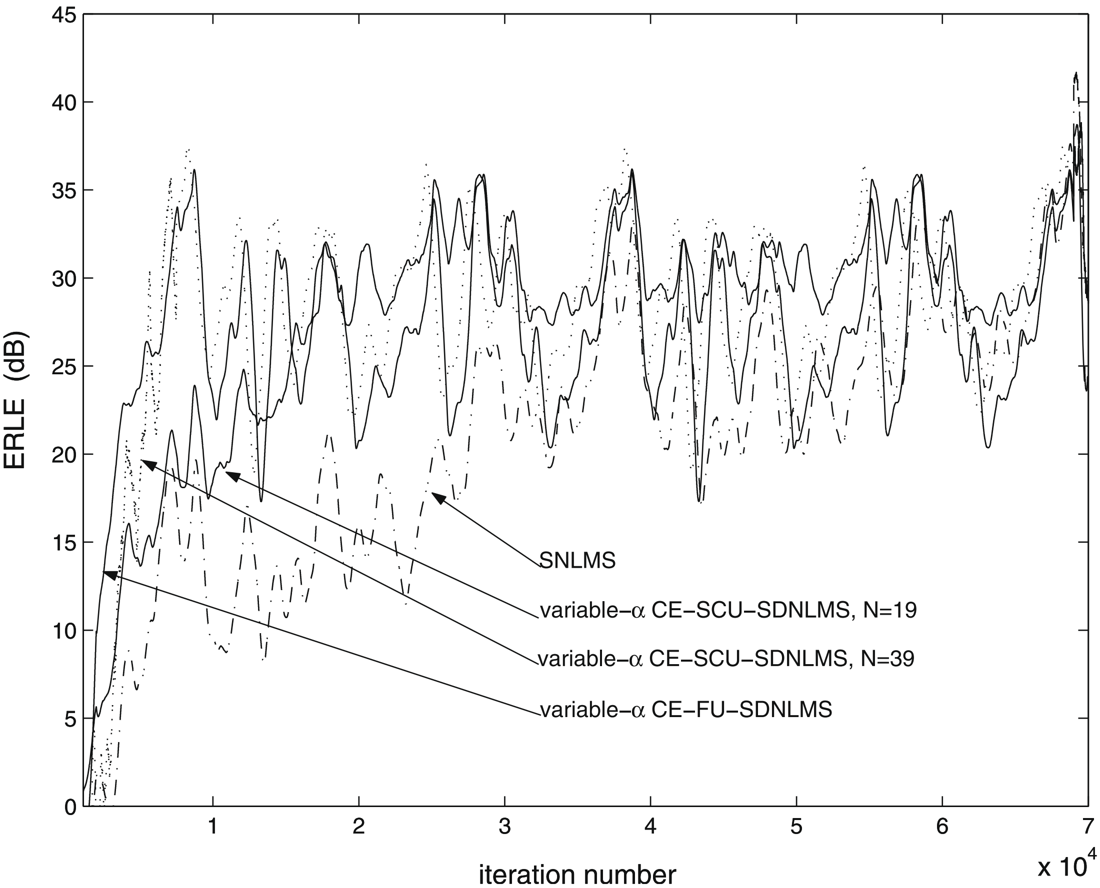

Fig. 4. Comparison of ERLE of the variable-α CE-FU-SDNLMS and CE-SCU-SDNLMS, and the SNLMS for a speech input.

Table 1
Comparison of theoretical and experimental SCU-SDLMS ($N = 1$) and SLMS steady state MSE.

μ	SNR (dB)	Predicted $\epsilon(\infty)$ (dB) Eq. (22)		Measured $\epsilon(\infty)$		Predicted $\epsilon_{SLMS}(\infty)$	Measured $\epsilon_{SLMS}(\infty)$
		M = 1	M = 8	M = 1	M = 8	(dB) [17]	(dB)
0.02	40	−37.02	−23.71	−35.33	−23.50	−37.02	−35.38
0.02	20	−17.28	−16.25	−15.46	−16.17	−17.28	−15.29
0.008	40	−39.40	−29.03	−38.66	−28.79	−39.40	−38.74
0.008	20	−19.40	−18.50	−18.72	−18.57	−19.40	−18.76
0.001	40	−39.92	−36.57	−39.85	−36.54	−39.92	−39.71
0.001	20	−19.93	−19.81	−19.94	−19.83	−19.93	−19.85

in agreement with findings in [14] where it has been shown that the SCU-SLMS algorithm with one coefficient update has the same steady-state error as the SLMS algorithm.

6. Conclusions

In this paper, we presented a reduced complexity LMS-type algorithm based on decomposing the long adaptive filter into smaller subfilters and employing the selective coefficient update (SCU) method in each subfilter. The MSE convergence behavior of algorithm has been analyzed for zero-mean white Gaussian input signals and $N = 1$. Step size bounds that guarantee mean-square convergence, and an expression for the final MSE were derived. Analysis was validated through simulations showing good agreement.

The proposed algorithm was shown to have high misadjustment. A combined-error algorithm was introduced that offers a superior performance with less computational complexity when compared to the standard stereo NLMS algorithm.

References

[1] Breining C, et al., Acoustic echo cancellation, An application of very-high-order adaptive filters. In: IEEE Signal Processing Magazine; July 1999. p. 42–69.
[2] Gay SL, Benesty J. Acoustic signal processing for telecommunication. Kluwer Academic Press; 2000.
[3] Benesty J, Morgan DR, Sondhi MM. A better understanding and an improved solution to the specific problems of stereophonic acoustic echo cancellation. IEEE Trans Speech Audio Process 1998;6(2):156–65.
[4] Eneroth P, Gay SL, Gaensler T, Benesty J. An implementation of a stereophonic acoustic echo canceler on a general purpose DSP. In: Proc ICSPAT; 1999.
[5] Benesty J, Amand F, Gilloire A, Grenier Y. Adaptive filtering algorithms for stereophonic acoustic echo cancellation. In: Proc. ICASSP; 1995. p. 3099–3102.
[6] Benesty J, Duhamel P, Grenier Y. A multi-channel affine projection algorithm with applications to multichannel acoustic echo cancellation. IEEE Signal Process Lett 1996;3(2):35–7.
[7] Amand F, Benesty J, Gilloire A, Grenier Y. A fast two-channel projection algorithm for stereophonic acoustic echo cancellation. In: Proc ICASSP; 1996. p. 949–952.
[8] Benesty J, Morgan DR. Frequency-domain adaptive filtering revisited, generalization to the multi-channel case, and application to acoustic echo cancellation. In: Proc ICASSP; 2000. p. 789–792.
[9] Mayyas K, Aboulnasr T. A stereophonic low complexity subband adaptive algorithm. In: Proc ISCAS; 2001. p. 725–728.
[10] Mayyas K. Stereophonic acoustic echo cancellation using lattice orthogonalization. IEEE Trans on Speech Audio Process 2002;10(7):517–25.
[11] Mayyas K. Stereophonic low complexity algorithms based on the decomposition of the adaptive filter. In: 1st IEEE international symposium on signal processing and information technology ISSPIT'2001, Cairo, Egypt, December 28–30; 2001. p. 196–199.
[12] Mayyas K, Aboulnasr T. Reduced complexity transform domain adaptive algorithm with selective coefficient update. IEEE Trans Circ Syst II: Express Briefs 2004;51(3):136–42.
[13] Narayan SS, Peterson AM, Narasimha MJ. Transform domain LMS algorithm. IEEE Trans Acoust Speech and Signal Process 1983;ASSP-31:609–15.
[14] Aboulnasr T, Mayyas K. Complexity reduction of the NLMS algorithm via selective coefficient update. IEEE Trans Signal Process 1999;47(5):1421–4.
[15] Haykin S. Adaptive filtering theory. Englewood Cliffs, NJ: Prentice Hall; 1996.
[16] Knuth DE. Sorting and searching, the art of computer programming, 2nd ed., vol. 3. Reading, MA: Addison-Wesley; 1973.
[17] Feuer A, Weinstein E. Convergence analysis of the LMS filters with uncorrelated gaussian data. IEEE Trans. Acoust Speech, Signal Process 1985;ASSP-33(1):222–30.

Khaled Mayyas received the B.S.E and M.Sc. degrees from the Jordan University of Science and Technology, Irbid, Jordan, in 1990 and 1992 and the Ph.D. degree from the University of Ottawa, Ottawa, Ont., Canada, in 1995, all in Electrical Engineering.
He is currently a Professor of Electrical Engineering and Dean of the Faculty of Engineering at the Jordan University of Science and Technology, Irbid, Jordan. He was a visiting Research Associate at the School of Information Technology and Engineering, University of Ottawa, in summers of 96, 97, 98, 99, and 2002. His research interests are in various aspects of adaptive digital signal processing and their applications, including the development of efficient algorithms.