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A gauge theory of the Hamiltonian reduction for the rational Calogero–Moser system ☆

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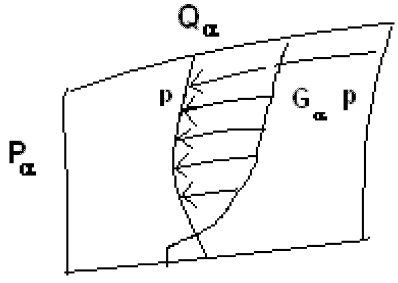
Abstract

A gauge theory equivalent to the Hamiltonian reduction scheme for rational Calogero–Moser model is presented.

1. Introduction

Hamiltonian reduction [1,2] is one of the most powerful methods of constructing integrable models. In particular, the celebrated Calogero–Moser system can be described within this framework, both in degenerate [3,4] as well as in general elliptic case [5].

The general reduction scheme can be described as follows. There is a symplectic manifold (M, ω) on which a Lie group G acts in a symplectic way. It is assumed that this action is strongly Hamiltonian. One selects a G-invariant function H on M as a Hamiltonian of the dynamical system to be reduced. Clearly, H has vanishing Poisson brackets with all Hamiltonians generating the action of G. The reduced phase space is defined as follows. Let g be the Lie algebra of G; for any $\xi \in g$ let h_ξ denote the corresponding Hamiltonian. Obviously, h_ξ is a linear function of ξ, so that $h_\xi(p)$, $p \in M$, defines an element of g^*; the mapping $\mu : M \to g^*$, $\langle \mu(p), \xi \rangle = h_\xi(p)$ is called the momentum map. In order to define the reduced phase space one selects an element $\alpha \in g^*$ and consider the subset $P_\alpha \subset M$ corresponding to $\mu(p) = \alpha$. Under suitable assumptions P_α is a submanifold of M. However, P_α is in general not symplectic: $\omega|_{P_\alpha}$ is degenerate. Fortunately, this degeneracy can be easily characterized: let $G_\alpha \subset G$ be a stability subgroup of α under coadjoint action. It appears then that $Q_\alpha = P_\alpha / G_\alpha$ (which, again, under some suitable assumptions is a manifold) is symplectic, i.e., $\omega|_{Q_\alpha}$ is nondegenerate. Now, with H being G-invariant the momentum map is a constant of motion so that P_α is invariant under dynamics generated by H. The essence of the method is that the trajectories on P_α when projected on Q_α are Hamiltonian, the Hamiltonian function being $H|_{Q_\alpha}$.



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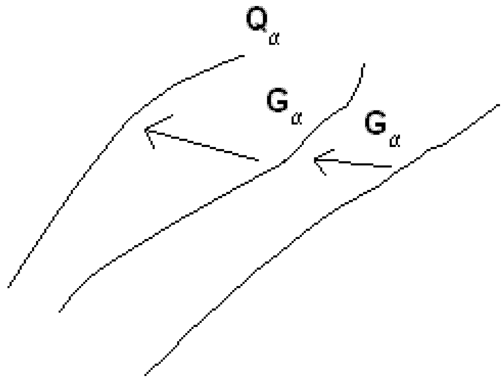
One can now pose the question what is the Lagrangian theory behind this scheme. To answer this question let us note that we are eventually looking for is the dynamics in terms of canonical variables on Q_α. It is obvious that any function of these variables can be regarded as G_α-invariant function on P_α (any G_α invariant function on P_α is constant on G_α orbits so it is a function on Q_α). Therefore, a given reduced trajectory on Q_α corresponds to an infinite set of curves on P_α, all of them being related to each other by the time-dependent action of G_α.



This suggests strongly that the corresponding Lagrangian dynamics possess gauge symmetry related to G_α.

It seems that the above reasoning is fairly general, i.e., the Hamiltonian reduction can be described within the framework of Lagrangian gauge theory. The details of this general scheme will be published elsewhere. Here we consider a particular example of (degenerate) Calogero–Moser systems [6,7]. An explicit construction of relevant gauge theory has been given by Polychronakos [8]. In order to obtain a Lagrangian theory he made first the trick consisting essentially in taking $G_\alpha = G$. This is achieved by viewing the value α of momentum map as a dynamical variable transforming under G according to coadjoint representation. The advantage is that the constraint concerning the value of momentum map becomes now G-invariant, $G_\alpha = G$. On the other hand, the new dynamical variables do not appear in Hamiltonian, so they are frozen on any particular trajectory. The reduced theory appears then to be the gauge theory with the gauge group G while the reduced dynamics corresponds to the temporal gauge.

The aim of the present note is to construct the Lagrangian dynamics for Calogero–Moser model without enlarging the set of dynamical variables. It is not surprising that the resulting Lagrangian theory is a gauge theory with the gauge group G broken down explicitly to G_α.

2. Calogero model as a gauge theory

2.1. Hamiltonian reduction

We start with a brief description of general reduction method mentioned above as applied to rational Calogero–Moser model [3].

As an unreduced phase space T one takes the set of pairs of traceless Hermitian $n \times n$ matrices A and B:

$$T = \left\{(A, B);\ A = A^+,\ B = B^+,\ \operatorname{Tr} A = 0 = \operatorname{Tr} B\right\}. \tag{1}$$

The symplectic form on T is given by

$$\omega = \operatorname{Tr}(dB \wedge dA) \tag{2}$$

and implies the following Poisson brackets:

$$\{A_{ij}, B_{mn}\} = \delta_{in}\delta_{jm} - \frac{1}{N}\delta_{ij}\delta_{mn}. \tag{3}$$

There is a natural symplectic action of $SU(N)$ group on T,

$$SU(N) \ni U : (A, B) \to \left(UAU^+, UBU^+\right), \tag{4}$$

which generates a Hamiltonian vector fields ζ on T,

$$\zeta(A, B) = \left(\{A, H_{\hat\zeta}\}, \{B, H_{\hat\zeta}\}\right), \tag{5}$$

with Hamiltonians

$$H_{\hat\zeta}(A, B) = \operatorname{Tr}\left(i[A, B]\hat\zeta\right), \quad \hat\zeta \in sU(N). \tag{6}$$

Hence, the momentum map reads

$$\Phi(A, B) = i[A, B]. \tag{7}$$

Dynamics on our unreduced phase space is defined by $SU(N)$-invariant Hamiltonian

$$H(A, B) = \frac{1}{2}\operatorname{Tr} B^2. \tag{8}$$

The element α of $sU(N)$ algebra and the reduced phase space Q_α constructed along lines sketched in the introduction read

$$\begin{aligned}
&\alpha = ig\big(v^+ \otimes v - I\big), \quad v = (1, 1, \ldots, 1),\\
&Q_\alpha = P_\alpha / G_\alpha,\\
&P_\alpha = \big\{(A, B) \in T,\ [A, B] = \alpha\big\},\\
&G_\alpha = \big\{C \in SU(N),\ C\alpha C^+ = \alpha\big\}.
\end{aligned} \tag{9}$$

In other words, Q_α consists of pairs of matrices (Q, L) which solve momentum map equation $\Phi(A, B) = \alpha$ and cannot be related by G_α action.

One can show that these matrices can be parametrized in the following nice way:

$$\begin{aligned}
&Q(q) = \operatorname{diag}(q_1 \ldots q_N),\\
&\big(L(q, p)\big)_{ij} = p_j \delta_{ij} + ig(1 - \delta_{ij}) \frac{1}{q_i - q_j};
\end{aligned} \tag{10}$$

here $L(p, q)$ is nothing but the Lax matrix of rational Calogero–Moser system.

In such a way the unreduced dynamics given by $H(A, B) = 1/2 \operatorname{Tr} B^2$ with constraint $\Phi(A, B) = \alpha$ when solved "modulo" the transformations from stability group G_α provides the rational C–M model.

2.2. Lagrangian gauge theory

Now, to discuss the equivalence of rational C–M model to some gauge theory let us consider the $SU(N)$ gauge theory (we assume that we are in the center-of-mass system) broken explicitly by linear term. The "matter" transforms according to the adjoint representation of $SU(N)$. The simplest Lagrangian reads

$$L(q, \dot q) = \frac{1}{2}\big(\dot q_\alpha + f_{\alpha\beta\gamma} q_\beta A_\gamma\big)^2 - v_\alpha A_\alpha, \tag{11}$$

where $f_{\alpha\beta\gamma}$ are $SU(N)$ structure constants, v_α is a fixed vector in adjoint representation of $SU(N)$ and A_α is a gauge field. L is invariant (up to a total derivative) under the stability subgroup $G_v \subset SU(N)$ of v_α:

$$\begin{aligned}
&q_\alpha \to q'_\alpha = D_{\alpha\beta}(t)_{q_\beta},\\
&A_\alpha \to A'_\alpha = D_{\alpha\beta}(t) A_\beta - \varrho_\alpha(t),\\
&f_{\alpha\beta\gamma} \varrho_\gamma = \dot D_{\alpha\varrho}(t) D^{-1}_{\varrho\beta}(t),\\
&D_{\alpha\beta}(t) v_\beta = v_\alpha.
\end{aligned} \tag{12}$$

The corresponding equations of motion read

$$f_{\alpha\beta\gamma} q_\beta \big(\dot q_\alpha + f_{\alpha\zeta\sigma} q_\zeta A_\sigma\big) - v_\gamma = 0, \tag{13}$$

$$\frac{d}{dt}\big(\dot q_\alpha + f_{\alpha\beta\gamma} q_\beta A_\gamma\big) - f_{\alpha\beta\gamma} A_\beta \big(\dot q_\gamma + f_{\gamma\zeta\sigma} q_\zeta A_\sigma\big) = 0. \tag{14}$$

Let us pass to the Hamiltonian formalism. The canonical momenta are

$$\begin{aligned}
&p_\alpha \equiv \frac{\partial L}{\partial q_\alpha} = \dot q_\alpha + f_{\alpha\beta\gamma} q_\beta A_\gamma,\\
&\pi_\alpha \equiv \frac{\partial L}{\partial \dot A_\alpha} = 0.
\end{aligned} \tag{15}$$

The theory is constrained, $\pi_\alpha \approx 0$ being the primary constraints. The Hamiltonian is constructed according to the standard rules [9],

$$H = p_\alpha q_\alpha - L = \frac{1}{2} p_\alpha^2 - f_{\alpha\beta\gamma} p_\alpha q_\beta A_\gamma + A_\alpha v_\alpha, \tag{16}$$

and yields the following canonical equations:

$$\dot q_\alpha = p_\alpha - f_{\alpha\beta\gamma} q_\beta A_\gamma, \tag{17}$$

$$\dot A_\alpha = u_\alpha, \tag{18}$$

$$\dot p_\alpha = f_{\alpha\beta\gamma} A_\beta p_\gamma, \tag{19}$$

$$\dot \pi_\alpha = f_{\alpha\beta\gamma} p_\beta q_\gamma - v_\alpha, \tag{20}$$

resulting from $\tilde H \equiv H + u_\alpha \pi_\alpha$, u_α being the Lagrange multipliers.

Following the standard procedure [9] we obtain the secondary constraints. First, $\dot\pi_\alpha \approx 0$ gives, together with (20),

$$\chi_\alpha \approx f_{\alpha\beta\gamma} p_\beta q_\gamma - v_\alpha = 0, \tag{21}$$

which is nothing but the momentum map condition. Taking again a time derivative of (21) and using Jacobi identity we arrive at

$$\tilde\chi_\alpha \equiv f_{\alpha\beta\gamma} v_\beta A_\gamma \approx 0. \tag{22}$$

This completes the list of constraints since $\dot\chi_\alpha \approx 0$ yields

$$f_{\alpha\beta\gamma} v_\beta u_\gamma \approx 0, \tag{23}$$

which is a constraint for Lagrange multipliers.

The full set of constraints reads

$$\pi_\alpha \approx 0 \text{ — the primary constraints,} \tag{24}$$

$$\left.\begin{aligned} &\chi_\alpha \equiv f_{\alpha\beta\gamma} p_\beta q_\gamma - v_\alpha = 0 \\ &\tilde{\chi}_\alpha \equiv f_{\alpha\beta\gamma} u_\beta A_\gamma = 0 \end{aligned}\right\} \text{ — the secondary constraints.} \tag{25}$$

The nontrivial Poisson brackets are

$$\begin{aligned} &\{\pi_\alpha, \tilde{\chi}_\beta\} = -f_{\alpha\beta\gamma} v_\gamma, \\ &\{\chi_\alpha, \chi_\beta\} = -f_{\alpha\beta\gamma}(\chi_\alpha + v_\gamma). \end{aligned} \tag{26}$$

The next step is to identify the first- and second-class constraints. Let $a_{\underline{\alpha}}$ be the generators of G_v, $(a_{\underline{\alpha}})_{\varrho\sigma} \equiv a_{\underline{\alpha}\beta} f_{\beta\varrho\sigma}$, while $b_{\underline{\underline{\alpha}}}$ $((b_{\underline{\underline{\alpha}}})_{\varrho\sigma} \equiv b_{\underline{\underline{\alpha}}\beta} f_{\beta\varrho\sigma})$ the remaining ones. Obviously

$$a_{\underline{\alpha}\beta} f_{\beta\varrho\sigma} v_\sigma = 0. \tag{27}$$

Let us define

$$\begin{aligned} &\pi_{\underline{\alpha}} \equiv a_{\underline{\alpha}\beta}\pi_\beta, &&\pi_{\underline{\underline{\alpha}}} \equiv b_{\underline{\underline{\alpha}}\beta}\pi_\beta, \\ &\chi_{\underline{\alpha}} \equiv a_{\underline{\alpha}\beta}\chi_\beta, &&\chi_{\underline{\underline{\alpha}}} \equiv b_{\underline{\underline{\alpha}}\beta}\chi_\beta, \\ &\tilde{\chi}_{\underline{\alpha}} \equiv a_{\underline{\alpha}\beta}\tilde{\chi}_\beta, &&\tilde{\chi}_{\underline{\underline{\alpha}}} \equiv b_{\underline{\underline{\alpha}}\beta}\tilde{\chi}_\beta. \end{aligned} \tag{28}$$

It follows then from (27) that $\tilde{\chi}_{\underline{\alpha}} = 0$ as an identity, while

$$\{\pi_{\underline{\alpha}}, \cdot\} \approx 0, \qquad \{\chi_{\underline{\alpha}}, \cdot\} \approx 0, \tag{29}$$

which means that $\pi_{\underline{\alpha}}$ and $\chi_{\underline{\alpha}}$ are first class constraints. In order to deal with the remaining ones let us denote by $(a_{\beta\underline{\gamma}}, \tilde{b}_{\beta\underline{\gamma}})$ the matrix inverse to $(a_{\underline{\alpha}\beta}, b_{\underline{\underline{\alpha}}\beta})^{\mathrm{T}}$,

$$\begin{aligned} &a_{\underline{\alpha}\beta}\tilde{a}_{\beta\underline{\gamma}} = \delta_{\underline{\alpha}\underline{\gamma}}, \qquad a_{\underline{\alpha}\beta}\tilde{b}_{\beta\underline{\underline{\gamma}}} = 0, \\ &b_{\underline{\underline{\alpha}}\beta}\tilde{a}_{\beta\underline{\gamma}} = 0, \qquad b_{\underline{\underline{\alpha}}\beta}\tilde{b}_{\beta\underline{\underline{\gamma}}} = \delta_{\underline{\underline{\alpha}}\underline{\underline{\gamma}}}, \\ &\tilde{a}_{\alpha\underline{\gamma}} a_{\underline{\gamma}\beta} + \tilde{b}_{\beta\underline{\underline{\gamma}}} b_{\underline{\underline{\gamma}}\beta} = \delta_{\alpha\beta}, \end{aligned} \tag{30}$$

and define new variables which extend $\pi_\alpha \to (\pi_{\underline{\alpha}}, \pi_{\underline{\underline{\alpha}}})$ to the full canonical transformation

$$A_{\underline{\alpha}} \equiv \tilde{a}_{\beta\underline{\alpha}} A_\beta, \qquad A_{\underline{\underline{\alpha}}} = \tilde{b}_{\beta\underline{\underline{\alpha}}} A_\beta; \tag{31}$$

the inverse transformations read

$$A_\alpha = a_{\underline{\beta}\alpha} A_{\underline{\beta}} + b_{\underline{\underline{\beta}}\alpha} A_{\underline{\underline{\beta}}}, \tag{32}$$

$$\pi_\alpha = \tilde{a}_{\alpha\underline{\beta}}\pi_{\underline{\beta}} + \tilde{b}_{\alpha\underline{\underline{\beta}}}\pi_{\underline{\underline{\beta}}}. \tag{33}$$

We supply (32) with the analogous transformation for u_α,

$$u_\alpha = a_{\underline{\beta}\alpha} u_{\underline{\beta}} + b_{\underline{\underline{\beta}}\alpha} u_{\underline{\underline{\beta}}}. \tag{34}$$

Inserting (32)–(34) into the constraints $\pi_\alpha \approx 0$, $\chi_\alpha \approx 0$, $\tilde{\chi}_\alpha \approx 0$ we conclude that these constraints are equivalent to the following ones:

$$\left.\begin{aligned} &\pi_{\underline{\alpha}} = 0 \text{ — primary} \\ &\chi_{\underline{\alpha}} \equiv a_{\underline{\alpha}\beta}(f_{\beta\gamma\delta} p_\gamma q_\delta - v_\beta) \\ &\quad = 0 \text{ — secondary} \end{aligned}\right\} \text{ Ist class,} \tag{35}$$

$$\left.\begin{aligned} &\pi_{\underline{\underline{\alpha}}} = 0 \text{ — primary} \\ &\chi_{\underline{\underline{\alpha}}} \equiv b_{\underline{\underline{\alpha}}\beta}(f_{\beta\gamma\delta} p_\gamma q_\delta - v_\beta) \\ &\quad = 0 \text{ — secondary} \\ &\tilde{\chi}_{\underline{\underline{\alpha}}} \equiv A_{\underline{\underline{\alpha}}} = 0 \text{ — secondary} \end{aligned}\right\} \text{ IInd class.} \tag{36}$$

As always the Lagrange multipliers related to second-class constraints are fixed; here $u_{\underline{\underline{\alpha}}} = 0$. On the other hand, the arbitrary functions entering the general solution to Lagrange equations are $A_{\underline{\alpha}}(t)$; consequently, $\dot{u}_{\underline{\alpha}} \equiv A_{\underline{\alpha}}(t)$ are also arbitrary.

In order to put the theory in Dirac form let us consider first the second-class constraints. The matrix of Poisson brackets of these constraints takes the form

$$\begin{bmatrix} \{\pi_{\underline{\underline{\alpha}}}, A_{\underline{\underline{\beta}}}\} & 0 \\ 0 & \{\chi_{\underline{\underline{\alpha}}}, \chi_{\underline{\underline{\beta}}}\} \end{bmatrix}, \tag{37}$$

where, on the constrained submanifold,

$$\{\chi_{\underline{\underline{\alpha}}}, \chi_{\underline{\underline{\beta}}}\} = -b_{\underline{\underline{\alpha}}\gamma} b_{\underline{\underline{\beta}}\delta} f_{\gamma\delta\varrho} v_\varrho \equiv V_{\underline{\underline{\alpha}}\underline{\underline{\beta}}}. \tag{38}$$

It is not difficult to prove from the definition of $b_{\underline{\underline{\alpha}}\beta}$ that this matrix is nonsingular which confirms our classification (35), (36). One can now write out the basic Dirac brackets. First we note that due to the form of (37) the variables $A_{\underline{\underline{\alpha}}}$, $\pi_{\underline{\underline{\alpha}}}$ do not enter the Dirac bracket — they disappear from the theory altogether. For the remaining variables we find

$$\begin{aligned} &\{A_{\underline{\alpha}}, \pi_{\underline{\beta}}\}_D = \delta_{\underline{\alpha}\underline{\beta}}, \\ &\{q_\alpha, q_\beta\}_D = \Delta_{\alpha\beta\xi\chi} q_\xi q_\chi, \\ &\{p_\alpha, p_\beta\}_D = \Delta_{\alpha\beta\xi\chi} p_\xi p_\chi, \\ &\{q_\alpha, p_\beta\}_D = \delta_{\alpha\beta} + \Delta_{\alpha\beta\xi\chi} q_\xi p_\chi, \end{aligned} \tag{39}$$

where $\Delta_{\alpha\beta\xi\chi} \equiv v^{-1}_{\underline{\underline{\varrho}}\underline{\underline{\sigma}}} b_{\underline{\underline{\varrho}}\gamma} b_{\underline{\underline{\sigma}}\tau} f_{\gamma\alpha\xi} f_{\tau\beta\chi}$.

It is not difficult to show that this theory is equivalent to the Hamiltonian reduction for C–M model. Let $v = (1, \dots, 1)$ be the vector in defining representation of $SU(N)$ and

$$v_\alpha \equiv \nu \operatorname{Tr}\bigl(\lambda_\alpha \bigl(v \otimes v^+ - 1\bigr)\bigr), \tag{40}$$

where $\{\lambda_\alpha\}$ is a basis of $sU(N)$ such that $[\lambda_\alpha, \lambda_\beta] = i f_{\alpha\beta\gamma}\lambda_\gamma$, $\operatorname{Tr}(\lambda_\alpha\lambda_\beta) = \delta_{\alpha\beta}$. Therefore, $G_v \subset SU(N)$ consists of all matrices such that $Uv = e^{i\theta}v$, i.e., $G_v = S(U(N-1) \times U(1))$, $\dim G_v = (N-1)^2$. The initial set of dynamical variables (i.e., the "large" phase space to be reduced) consist of $4(N^2 - 1)$ variables $q_\alpha, p_\alpha, A_\alpha, \Pi_\alpha$. Now, the Ist and IInd class constraints are given by Eqs. (34) and (35), respectively. The primary constraints $\Pi_\alpha \approx 0$ eliminate Π_α altogether. The secondary constraints $\chi_{\underline{\alpha}} \approx 0$, $\chi_{\underline{\underline{\alpha}}} \approx 0$ determine the preimage of momentum map. The remaining IInd class secondary constraints $A_{\underline{\underline{\alpha}}} \approx 0$ reduce the number of gauge fields to that of the dimension of residual gauge symmetry G_v. It is important to note that we are considering only the G_v-invariant sector of our theory. This is because, on Hamiltonian picture level, the reduced phase space is *not* the preimage of the momentum map but rather the latter divided by the action of G_v group; therefore, all points on a given orbit are identified. Any function on phase space, when reduced to the submanifold of constraints, depends a priori on q_α, p_α and $A_{\underline{\alpha}}$. The invariance under time-dependent G_v transformations implies that it does not depend on $A_{\underline{\alpha}}$ and is a function of G_v-invariants built out of q_α and p_α. To get a number of degrees of freedom of find theory let us note that the number of first-class constraints (34) equals twice the dimension of G_v, $2(N-1)^2$. The number of second-class ones (35) is $3(\dim G - \dim G_v) = 6N - 6$. Therefore, there remains $4(N^2 - 1) - 2(N-1)^2 - (6N - 6) = 2N^2 - 2N$ degrees of freedom which is just the dimension of preimage of momentum map $N^2 - 1$, plus the number of $A_{\underline{\alpha}}$ variables, $(N-1)^2$. For G_v-invariant functions the number of independent variables is $2N^2 - 2N$ minus the number of $A_{\underline{\alpha}}$ variables, $(N-1)^2$, minus the dimension of G_v, again $(N-1)^2$, which gives $2N - 2$, as it should be. Summarizing, the second-class constraints reduce the system to the one based on G_v invariance with "matter fields" q_α, p_α being reduced to the preimage of momentum map.

In order to put the dynamics in the form which is explicitly equivalent to C–M one we can fix the temporal gauge $A_{\underline{\alpha}} = 0$. Then all primary constraints become second-class and the modified Dirac bracket does not contain A_α and π_α variables. The only constraints are now

$$\begin{aligned} &\chi_{\underline{\alpha}} \equiv a_{\underline{\alpha}\beta}(f_{\beta\gamma\delta}p_\gamma q_\delta - v_\beta) = 0 \text{ — Ist class,} \\ &\chi_{\underline{\underline{\alpha}}} \equiv b_{\underline{\underline{\alpha}}\beta}(f_{\beta\gamma\delta}p_\gamma q_\delta - v_\beta) = 0 \text{ — IInd class,} \end{aligned} \tag{41}$$

which is equivalent to

$$f_{\beta\gamma\delta}p_\gamma q_\delta - v_\beta = 0, \tag{42}$$

while Hamiltonian equations become

$$\dot{q}_\alpha = p_\alpha, \qquad \dot{p}_\alpha = 0. \tag{43}$$

Identifying $A \equiv q_\alpha\lambda_\alpha$, $B \equiv p_\alpha\lambda_\alpha$ we get

$$\omega = \operatorname{Tr}(dB \wedge dA), \qquad H = \frac{1}{2}\operatorname{Tr} B^2,$$

$$\left.\begin{aligned} \dot{A} &= B \\ \dot{B} &= 0 \end{aligned}\right\} \tag{44}$$

and the standard constraint

$$[A, B] = i\nu\bigl(v \otimes v^+ - I\bigr); \tag{45}$$

the invariant under time-independent G_v-transformations solutions to this constraint are provided by C–M model as it was described in the introduction.

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