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Curvaton reheating mechanism in inflation on warped Dvali–Gabadadze–Porrati brane

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Abstract

An impressed feature of inflation on warped Dvali–Gabadadze–Porrati (DGP) brane is that the inflationary phase exits spontaneously for a scalar inflaton field with exponential potential, which presents a graceful exit mechanism for the inflation. But its reheating mechanism leaves open. We investigate the curvaton reheating in inflation on warped DGP brane model. The reheating may occur in effectively 5-dimensional or 4-dimensional stage. We study the permitted parameter space of the curvaton field in detail. We demonstrate how the inflation model of the warped DGP brane is improved by the curvaton mechanism.

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1. Introduction

The inflation is a work schema, rather than a model. There are several inflation models since Guth's seminal work [1], for a review, see, for example [2]. Though the inflationary scenario successfully solved several problems in the standard big bang model, how to get an inflaton field from fundamental field theory keeps unsolved. In view of achievements and shortcomings of inflationary scenario, it is necessary to further our understanding of the inflationary scenario from a theoretical perspective.

The brane world scenario, in which the standard model particles are confined on the 3-brane while the gravitation can propagate in the whole space, is an important progress in high energy physics. Among various brane universe models, the one proposed in [3], called DGP model, is very interesting. In the DGP model, gravity appears 4-dimensional at short distances but is altered at distance large compared to some freely ad-

justable crossover scale r_0 through the slow evaporation of the graviton off our 4-dimensional brane world universe into an unseen, yet large, fifth dimension. The original DGP model was soon generalized to warped DPG model [4]. In some sense, it is a hybrid model sharing some features of Randall–Sundrum model [5] and some features of DGP model.

The inflation on warped DGP brane has been discussed in [6,7]. It is found that there may exist three stages in the inflation period: at the ultra high energy limit, the spacetime is effectively 4-dimensional; at middle energy region, the spacetime is effectively 5-dimensional; and then the spacetime undergoes the second 4-dimensional stage before nucleosynthesis (note that in the ultra low energy limit, such as the present day, the model can enter another 5-dimensional stage again). In a concrete numerical example of warped DGP inflation model, the universe starts to inflate at the 4-dimensional stage and ends at 5-dimensional stage [7]. In inflation scenarios, the exponential potential is an important example which can be solved exactly in the standard model. In addition, we know that such exponential potentials of scalar fields occur naturally in some fundamental theories such as string/M-theories. It is worth noting that in this inflation model based on the warped DGP brane

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world scenario, for an exponential potential, the inflationary phase can exit naturally. In this inflationary scenario, when the energy density decreases, the scalar field makes a transition into a kinetic energy dominated regime, which means the slow roll parameters becomes larger than 1, bringing inflation to an end. Since the inflaton survives this process without decay, an alternative reheating mechanism is required.

It is suggested in [8] that reheating took place via gravitational particle production. This particle production mechanism roots in the different vacuums of the inflationary phase and the kinetic phase. The different vacuums are related by Bogoliubov transformations. The vacuum state in the inflationary phase is no longer vacuum state measured in the kinetic phase: there are particles generated. However this mechanism is very inefficiency. So there is a rather long kinetic energy dominated regime, which leads to short-wavelength gravitational waves to reach excessive amplitudes [9]. The gravitational particle production mechanism is also plagued by its prediction on perturbation spectrum: it gives an spectrum far from scale invariant, while the observation data prefer a nearly scale invariant spectrum.

In this Letter we shall propose a curvaton reheating mechanism for the warped DGP inflation model. The curvaton mechanism is firstly suggested as an alternative mechanism to generate the primordial scalar perturbation which is responsible for the structure formation. In the curvaton scenario the primordial density perturbation originates from the vacuum fluctuation of some "curvaton" field σ, different from the inflaton field [10]. The curvaton reheating mechanism was firstly suggested in context of the quintessential inflation model in [11]. It is shown in [12] that the eta-problem of quintessential inflation can be also ameliorated under the curvaton hypothesis. And recently the curvaton reheating mechanism is applied to the different inflation models [13].

The organization of this Letter is as follows. For a thorough discussion on the curvaton reheating process, in the next section we briefly review the warped DGP inflation, stressing on the different stages of different effective dimensions. In this section we also reanalyze the example discussed in [7], pointing out the critical points of the dimension transition, which is very important for the studies on the permitted parameter space of the curvaton field. In Section 3 we traverse the constraint on the curvaton field. We investigate all the possible 3 cases: (1) The curvaton oscillates and decays in 5-dimensional stage; (2) the curvaton oscillates in 5-dimensional stage but decays in 4-dimensional stage; and (3) the curvaton oscillates and decays in 4-dimensional stage. In the last section we present our conclusions and discussions.

2. A short review on the warped DGP inflation model

For further discussions on curvaton reheating after inflationary phase, we first give a brief review on the warped DGP inflation model. For a thorough discussion, see [7].

We start from the Friedmann equation on the warped DGP brane. Assuming a Friedmann–Robertson–Walker (FRW) met-

ric on the brane, one derives the Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \big[\rho + \rho_0 \big(1 + \epsilon \mathcal{A}(\rho, a) \big) \big], \tag{1}$$

where as usual H is the Hubble parameter, a denotes the scale factor, ρ denotes the matter field density, k is the constant curvature of the maximal symmetric space of the FRW metric, μ is a parameter with dimension of mass, which denotes the strength of the induced gravity term on the brane, and ϵ denotes either $+1$ or -1, represents the two branches of this model. $\mathcal{A}$ is defined by

$$\mathcal{A} = \left[\mathcal{A}_0^2 + \frac{2\eta}{\rho_0} \left(\rho - \mu^2 \frac{\mathcal{E}_0}{a^4} \right) \right]^{1/2}, \tag{2}$$

where

$$\mathcal{A}_0 = \sqrt{1 - 2\eta \frac{\mu^2 \Lambda}{\rho_0}}, \quad \eta = \frac{6m_5^6}{\rho_0 \mu^2} \ (0 < \eta \leqslant 1), \tag{3}$$

$$\rho_0 = m_\lambda^4 + 6\frac{m_5^6}{\mu^2}. \tag{4}$$

Λ is defined as

$$\Lambda = \frac{1}{2} \left({}^{(5)}\Lambda + \frac{1}{6}\kappa_5^4 \lambda^2 \right), \tag{5}$$

where ${}^{(5)}\Lambda$ is the 5-dimensional cosmological constant in the bulk, κ_5 is the 5-dimensional Newton constant, and λ is the brane tension. Note that here there are three mass scales, μ, $m_\lambda = \lambda^{1/4}$ and $m_5 = \kappa_5^{-2/3}$. $\mathcal{E}_0$ is a constant related to Weyl radiation. Since we are interested in the inflation dynamics of the model, as usual, we neglect the curvature term and dark radiation term in what follows. Also in this Letter we restrict ourselves in the Randall–Sundrum critical case, that is $\Lambda = \frac{1}{2}({}^{(5)}\Lambda + \frac{1}{6}\kappa_5^4 \lambda^2) = 0$. Then the Friedmann equation (1) can be rewritten as

$$H^2 = \frac{1}{3\mu^2} \left[\rho + \rho_0 + \epsilon \rho_0 \left(1 + \frac{2\eta \rho}{\rho_0} \right)^{1/2} \right]. \tag{6}$$

Because only in the branch $\epsilon = -1$ the inflation exits spontaneously, we only consider this branch from now on. In the ultra high energy limit where $\rho \gg \rho_0 \gg m_\lambda^4$, the Friedmann equation (6) is

$$H^2 = \frac{1}{3\mu^2} (\rho + \epsilon \sqrt{2\rho\rho_0}). \tag{7}$$

This describes a four-dimensional gravity on the brane. In the intermediate energy region where $\rho \ll \rho_0$ but $\rho \gg m_\lambda^4$, for the branch with $\epsilon = -1$, the Friedmann equation changes to

$$H^2 = \frac{m_\lambda^4}{18m_5^6} \left(\rho + \frac{\rho^2}{2m_\lambda^4} - \frac{\mu^2 m_\lambda^4}{6m_5^6} \rho - \frac{\mu^2}{4m_5^6} \rho^2 \right). \tag{8}$$

And in low energy limit $\rho \ll m_\lambda^4 \ll \rho_0$, Friedmann equation (6) becomes

$$H^2 = \frac{1}{3m_p^2} \left[\rho + \mathcal{O} \left(\frac{\rho}{\rho_0} \right)^2 \right], \tag{9}$$

where $m_p^2 = \mu^2/(1 - \eta)$, m_p is 4-dimensional Planck mass. Carefully observe the conditions for (7), (9), and (8), one finds only when

$$\lambda \ll \frac{6m_5^6}{\mu^2}, \tag{10}$$

the universe really undergoes a 5-dimensional phase. Otherwise the universe will be always in a 4-dimensional evolution, which is without interest. Considering (10), and only leaving the most important term, (7)–(9) are further simplified to

$$H^2 = \frac{1}{3\mu^2}\rho, \tag{11}$$

$$H^2 = \frac{\rho^2}{36m_5^6}, \tag{12}$$

and

$$H^2 = \frac{\lambda}{18m_5^6}\rho, \tag{13}$$

respectively. Clearly, the above 3 equations describe effectively 4-dimensional gravity, 5-dimensional gravity, and 4-dimensional gravity, respectively. But the corresponding gravitational constants are different. The dimension transition from ultra high energy region to intermediate energy region occurs at

$$\rho'_{4.5} = \frac{12m_5^6}{\mu^2}, \tag{14}$$

$$H'_{4.5} = \frac{2m_5^3}{\mu^2}, \tag{15}$$

where have used (11) and (12). Similarly by using (12) and (13), the transition from intermediate energy region to low energy region occurs at

$$\rho_{4.5} = 2\lambda, \tag{16}$$

$$H_{4.5} = \frac{\lambda}{3m_5^3}. \tag{17}$$

Consider an inflation scalar field ϕ on the brane with exponential potential

$$V = \tilde{V}e^{-\sqrt{2/p}\frac{\phi}{\mu}}, \tag{18}$$

where $\tilde{V}$ and p are two constants. Introduce

$$u = \frac{V}{\rho_0}. \tag{19}$$

Recall the numerical example given in [7], $\eta = 0.99$, $\mu^2 = 0.01m_p^2 \sim (10^{17}\ \text{GeV})^2$, e-folds of the inflation $N = 60$, $p = 50$, the value of u when the inflation ends $u_{\text{end}} = 0.05$, the value of u when the cosmic scale observed today crosses the Hubble horizon during inflation $u_i = 36$, $\frac{\lambda}{\mu^4} = 6.7 \times 10^{-12}$, $\frac{m_5}{\mu} = 0.02$, $\frac{{}^{(5)}A}{\mu^2} = 6.7 \times 10^{-14}$.

Now we check the inflationary process of this example. First,

$$\frac{\lambda\mu^2}{6m_5^6} = 0.017 \ll 1, \tag{20}$$

hence the evolution of the universe really undergoes 4-dimensional stage, 5-dimensional stage, and then the other 4-dimensional stage. Second,

$$u_i = 36 \gg 1, \tag{21}$$

therefore the universe inflates in a 4-dimensional stage when the cosmic scale observed today crosses the Hubble horizon during inflation. Finally

$$u_{\text{end}} = 0.05 \ll 1, \qquad \frac{\rho}{m_\lambda^4} = 58.3 \gg 1, \tag{22}$$

hence the inflation phase exits in a 5-dimensional stage.

Then, in the model without curvaton field, the universe enters a kinetic period, in which the energy density decreases very fast, which soon restores the universe to be a 4-dimensional again before nucleosynthesis. But, as we have mentioned, such a scenario brings several serious problems because the gravitational particle production is far from efficient.

3. The curvaton reheating

In this section we shall explore the permitted parameter region of the curvaton field which is responsible for particle production and for the structure formation.

3.1. The dynamics of the curvaton field

Curvaton is a new mechanism for the primordial curvature perturbation generation suggested in literatures in recent years. In contrast with the usual inflaton reheating process, the inflaton need not roll slowly. The inflation and reheating are charged by different fields, such that many hopeful inflation models survive. We here assume that the inflaton has no interactions with inflaton except the gravitational coupling. Hence, similar to the inflaton, the equation of motion of curvaton field σ can be written as

$$\ddot{\sigma} + 3H\dot{\sigma} + U'(\sigma) = 0. \tag{23}$$

For simplicity, we assume $U(\sigma) = \frac{1}{2}m^2\sigma^2$. We start to give a brief description of the dynamical evolution of the curvaton field. First, the curvaton coexists with the inflaton field throughout the inflationary phase, during which the inflaton energy density dominates the energy density of curvaton. Because the curvaton is effectively massless before oscillation, σ keeps at its initial value σ_i, where the subscript i denotes the value when the cosmic scale observed today crosses the Hubble horizon during inflation [10]. The next stage begins when the curvaton field becomes to oscillate, and this should happen at the kinetic epoch, otherwise its fluctuation can be suppressed by the fluctuation of inflaton.

For the sake of preventing a stage of curvaton-driven inflation, the universe must be still dominated by inflaton till this time. This condition imposes a constraint on the initial values the curvaton field. In this period the curvaton evolves as pressureless dust because its mean kinetic energy equals the mean potential energy. From (23) we see the curvaton becomes to oscillate when $H \simeq m$, while the universe can be effectively

5-dimensional or 4-dimensional. According to our assumption the universe is still dominated by inflaton, which behaves as stiff matter in kinetic epoch. Therefore we obtain, for the case of oscillating in 5-dimensional stage

$$m/H_{\text{kin}} = a_{\text{kin}}^6/a_{\text{osc}}^6, \tag{24}$$

where kin stands for the value when the universe exits from the inflationary phase and enters the kinetic energy dominated epoch. And for the case of oscillating in 4-dimensional stage

$$m/H_{\text{kin}} = \frac{a_{\text{kin}}^6}{a_{4.5}^6} \frac{a_{4.5}^3}{a_{\text{osc}}^3}, \tag{25}$$

where 4.5 stands for the transition point from a 5-dimensional stage to a 4-dimensional stage. In the oscillating period the energy density of the curvaton field evolves as dust,

$$\rho_\sigma = \rho_{\sigma i} a_{\text{osc}}^3/a^3, \tag{26}$$

where $\rho_{\sigma i} = m^2 \sigma_i^2/2$. The third stage is that of curvaton decay into radiation, which happens when the decay parameter $\Gamma = H$. Here we adopt a sudden decay approximation, which is a quite good approximation in curvaton models [14]. We see there are three free parameters of the curvaton field: the initial value of the field σ_i; the mass m; and the decay energy scale Γ.

If the curvaton oscillates in 5-dimensional stage, it may decay in 5-dimensional stage or 4-dimensional stage. If the curvaton oscillates in 4-dimensional stage, it must also decay in 4-dimensional stage. Hence there are three cases: (1) oscillation in 5-dimensional stage, decay in 5-dimensional stage; (2) oscillation in 5-dimensional stage, decay in 4-dimensional stage; and (3) oscillation in 4-dimensional stage, decay in 4-dimensional stage. Each of the three cases includes two subcases to be considered, depending whether the curvaton field decays before or after it becomes the dominant component of the universe. In the following subsections we shall discuss these cases one-by-one.

3.2. Case 1: oscillation and decay in 5-dimensional stage

In this subsection we present the constraints on the parameter space of the curvaton in the case of the curvaton oscillates and decays in 5-dimensional stage. First, we should ensure that the inflaton dominates the evolution of the universe when the curvaton starts to oscillate, which implies

$$\left.\frac{\rho_\sigma}{\rho_\phi}\right|_{\text{osc}} < 1, \tag{27}$$

where

$$\rho_\sigma = \rho_{\sigma i} = m^2 \sigma_i^2/2, \tag{28}$$

since we have assumed the σ is effectively a constant before oscillation, and by using (12),

$$\rho_\phi = 6m_5^3 m. \tag{29}$$

So we obtain

$$m\sigma_i^2 < 12m_5^3. \tag{30}$$

There are two subcases in this case depending the curvaton dominates the universe before or after decay. First, we study the curvaton begins to dominate the universe before decay. The events sequence is as follow: the curvaton starts to oscillate, the energy density of curvaton σ equals the energy density of inflaton ϕ, the curvaton decays, the universe transits from 5-dimensional phase 4-dimensional phase, the nucleosynthesis happens. One can translate this sequence into equation

$$m > H_{\text{eq1}} > \Gamma > H_{4.5} > H_{\text{nuc}}, \tag{31}$$

where H_{eq1} denotes the value of the Hubble parameter when the density of curvaton equals the density of inflaton in 5-dimensional stage, $H_{\text{nuc}} = 10^{-41} m_{\text{Pl}}$ denotes the value of the Hubble parameter when nucleosynthesis happens. The energy density of inflaton evolves as

$$\rho_\phi = \rho_{\text{kin}} \frac{a_{\text{kin}}^6}{a^6} = 6m_5^3 H_{\text{kin}} \frac{a_{\text{kin}}^6}{a^6}. \tag{32}$$

So we obtain

$$\left.\frac{\rho_\sigma}{\rho_\phi}\right|_{a=a_{\text{eq1}}} = \frac{m\sigma_i^2}{2} \frac{a_{\text{osc}}^3}{a_{\text{eq}}^3} \frac{1}{6m_5^3} \frac{a_{\text{eq1}}^6}{a_{\text{kin}}^6} = 1, \tag{33}$$

which yields

$$H_{\text{eq1}} = \frac{\sigma_i^4}{4} \frac{m^3}{36m_5^6}, \tag{34}$$

where we have used

$$\frac{m}{H_{\text{kin}}} \frac{a_{\text{osc}}^6}{a_{\text{kin}}^6} = 1, \tag{35}$$

and

$$H_{\text{eq1}} = H_{\text{kin}} \frac{a_{\text{kin}}^6}{a_{\text{eq1}}^6}. \tag{36}$$

Here eq1 labels the time when the density of curvaton equals the density of inflaton in 5-dimensional stage. And then (31) becomes

$$m > \frac{\sigma_i^4}{4} \frac{m^3}{36m_5^6} > \Gamma > \frac{\lambda}{3m_5^3} > H_{\text{nuc}}. \tag{37}$$

Because of transitivity (37) is a fairly strong constraint on m, Γ, and σ_i, which includes

$$m > \frac{\sigma_i^4}{4} \frac{m^3}{36m_5^6}, \tag{38}$$

$$m > \Gamma, \tag{39}$$

$$m > \frac{\lambda}{3m_5^3}, \tag{40}$$

$$m > H_{\text{nuc}}, \tag{41}$$

$$\frac{\sigma_i^4}{4} \frac{m^3}{36m_5^6} > \Gamma, \tag{42}$$

$$\frac{\sigma_i^4}{4} \frac{m^3}{36m_5^6} > \frac{\lambda}{3m_5^3}, \tag{43}$$

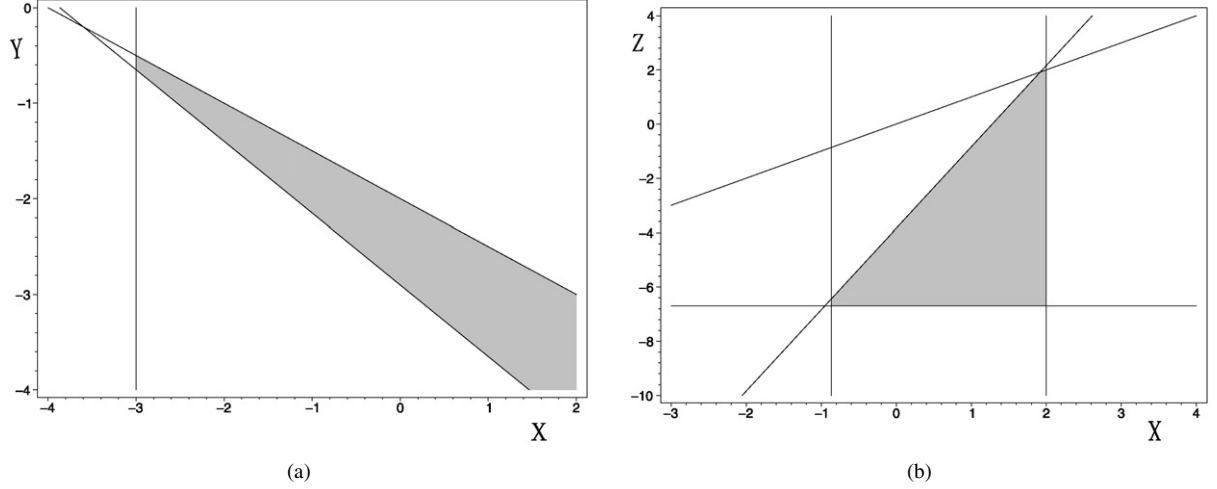(a)   (b)

Fig. 1. The subcase decay after domination of case 1. In this and all the following figures we adopt the numerical example of the parameters in warped DGP model given in [7], where $\eta = 0.99$, $\mu^2 = 0.01 m_p^2 \sim (10^{17} \text{ GeV})^2$, $\frac{\lambda}{\mu^4} = 6.7 \times 10^{-12}$, $\frac{m_5}{\mu} = 0.02$, $\frac{^{(5)}A}{\mu^2} = 6.7 \times 10^{-14}$, and the shadowed region represents the permitted region. (a) y versus x, in which we take $z = -3$. (b) z versus x, in which we take $y = -3$.

$$\frac{\sigma_i^4}{4} \frac{m^3}{36 m_5^6} > H_{\text{nuc}}, \tag{44}$$

$$\Gamma > \frac{\lambda}{3 m_5^3}, \tag{45}$$

$$\Gamma > H_{\text{nuc}}, \tag{46}$$

$$\frac{\lambda}{3 m_5^3} > H_{\text{nuc}}. \tag{47}$$

Note that (30) is just (38). They represent the same relation between ρ_ϕ and ρ_σ to be compared at different times.

We plot a figure to display the parameter space permitted clearly, in which we will use the numerical example in [7]. In this example

$$\frac{\lambda}{3 m_5^3 H_{\text{nuc}}} = 3 \times 10^{34} > 1, \tag{48}$$

hence Eqs. (38)–(47) reduce to

$$m > \frac{\sigma_i^4}{4} \frac{m^3}{36 m_5^6}, \tag{49}$$

$$m > \Gamma, \tag{50}$$

$$m > \frac{\lambda}{3 m_5^3}, \tag{51}$$

$$\frac{\sigma_i^4}{4} \frac{m^3}{36 m_5^6} > \Gamma, \tag{52}$$

$$\frac{\sigma_i^4}{4} \frac{m^3}{36 m_5^6} > \frac{\lambda}{3 m_5^3}, \tag{53}$$

$$\Gamma > \frac{\lambda}{3 m_5^3}. \tag{54}$$

Define

$$x = \log \frac{m}{\mu}, \tag{55}$$

$$y = \log \frac{\sigma_i}{\mu}, \tag{56}$$

$$z = \log \frac{\Gamma}{\mu}. \tag{57}$$

Substitute the values of the parameters in warped DGP model in this example, (49)–(54) become

$$x + 2y < -4, \tag{58}$$

$$x > -6.55, \tag{59}$$

$$x > z, \tag{60}$$

$$4y + 3x > z - 8.16, \tag{61}$$

$$4y + 3x > -14.6, \tag{62}$$

$$z > -6.55, \tag{63}$$

respectively. We show the permitted parameter region in Fig. 1.

Now we turn to subcase 2: curvaton dominates the universe after decay. The event sequence is slightly different from the above subcase: the curvaton starts to oscillate, the curvaton decays, the energy density of curvaton σ equals the energy density of inflaton ϕ, the universe transits from 5-dimensional phase 4-dimensional phase, the nucleosynthesis happens, or by equation

$$m > \Gamma > H_{\text{eq1}} > H_{4.5} > H_{\text{nuc}}. \tag{64}$$

Just mimicking the discussion of the last subcase, we plot the permitted parameter region in Fig. 2.

3.3. Case 2: oscillation in 5-dimensional stage, decay in 4-dimensional stage

In this subsection we deduce the constraints on the parameter space of the curvaton in the case of the curvaton oscillates in 5-dimensional stage, but decays in 4-dimensional stage. The condition that the inflaton dominates the evolution of the universe when the curvaton starts to oscillate is the same as in last case, which is just (30).

This case also include two subcases depending the curvaton dominates the universe before or after decay. First, we study the curvaton begins to dominate the universe before decay. The
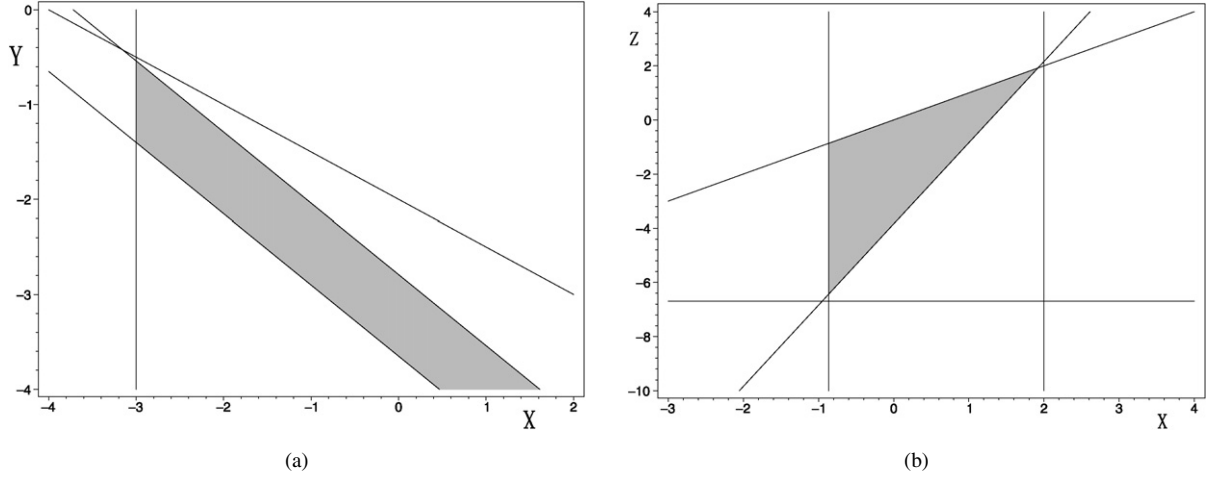
Fig. 2. The subcase decay after domination of case 1. (a) y versus x, in which we take $z = -3$. (b) z versus x, in which we take $y = -3$.
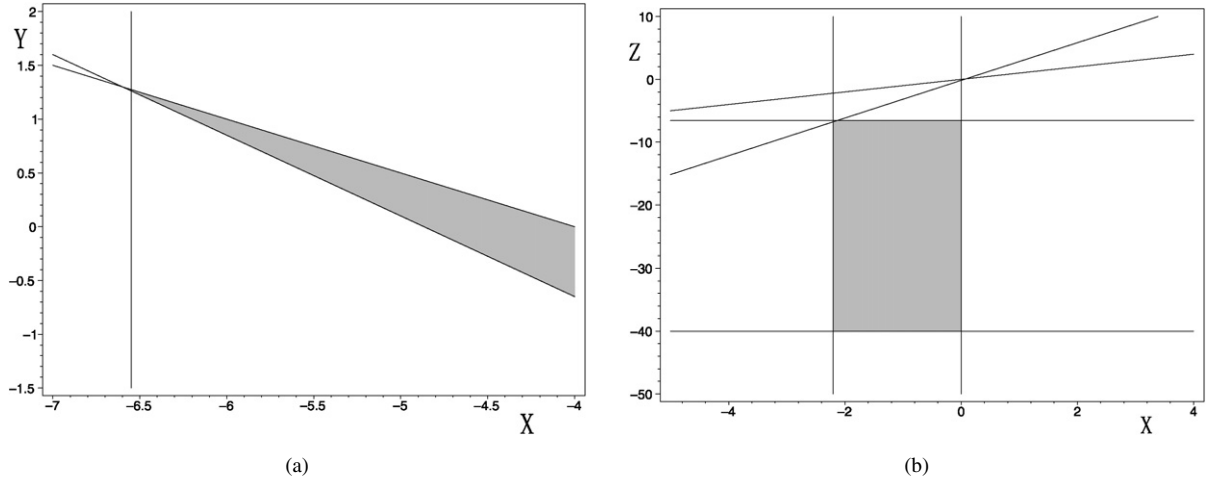

Fig. 3. The subcase decay after domination of case 2, where the energy density of curvaton equals the energy density of inflaton happening in 5-dimensional stage. (a) y versus x, in which we set $z = -10$. (b) z versus x, in which we set $y = -2$.

events sequence is as follow: the curvaton starts to oscillate, the universe transits from 5-dimensional phase 4-dimensional phase, the energy density of curvaton σ equals the energy density of inflaton ϕ, the curvaton decays, the nucleosynthesis happens; or the curvaton starts to oscillate, the energy density of curvaton σ equals the energy density of inflaton ϕ, the universe transits from 5-dimensional phase 4-dimensional phase, the curvaton decays, the nucleosynthesis happens. One can translate the sequences into equation

$$m > H_{eq1} > H_{4.5} > \Gamma > H_{nuc}, \tag{65}$$

or

$$m > H_{4.5} > H_{eq2} > \Gamma > H_{nuc}, \tag{66}$$

where eq2 labels the value of a variable when the density of curvaton equals the density of inflaton in latter 4-dimensional stage, while the curvaton begins to oscillate in 5-dimensional stage. We demonstrate the numerical result of (65) in Fig. 3, with the same parameters of the warped DGP model in the above case.

To investigate (66) we need to find H_{eq2} in advance. Under this situation the curvaton red shifts as

$$\rho_\sigma = \rho_{\sigma i} \frac{a_{osc}^3}{a_{4.5}^3} \frac{a_{4.5}^3}{a^3}, \tag{67}$$

and inflaton red shifts as

$$\rho_\sigma = 6m_5^3 H_{kin} \frac{a_{kin}^6}{a_{4.5}^6} \frac{a_{4.5}^6}{a^6}. \tag{68}$$

The Hubble parameter evolves as

$$H = H_{kin} \frac{a_{kin}^6}{a_{4.5}^6} \frac{a_{4.5}^6}{a^6}. \tag{69}$$

The energy density of curvaton equals the energy density of inflation when $a = a_{eq2}$, which means

$$\left. \frac{\rho_\sigma}{\rho_\phi} \right|_{a=a_{eq2}} = 1, \tag{70}$$

hereby we derive

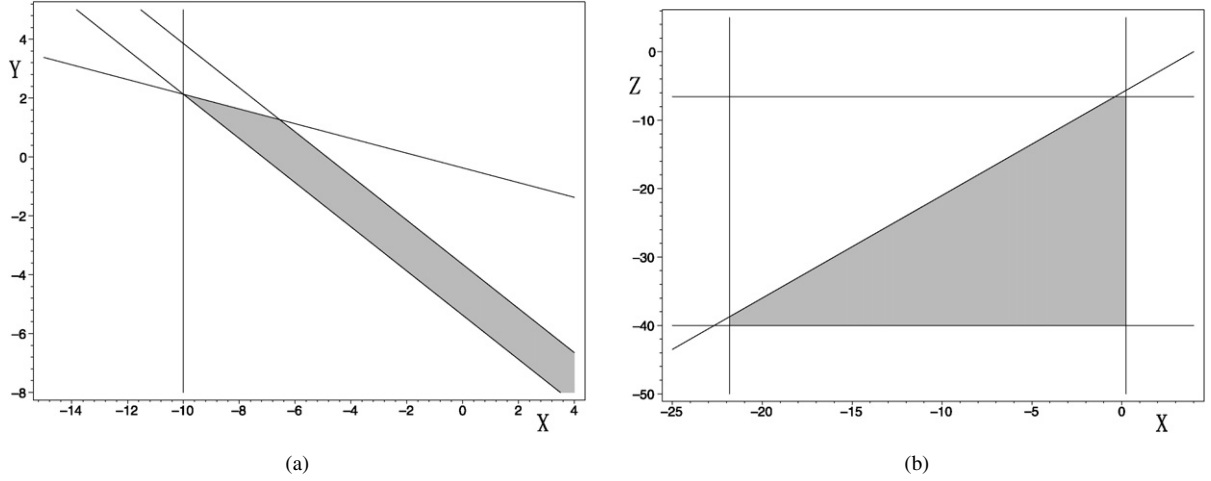$$H_{eq2} = \frac{m\sigma_i^2}{12m_5^3} \sqrt{\frac{\lambda m}{3m_5^3}}, \tag{71}$$

Fig. 4. The subcase decay after domination of case 2, where the energy density of curvaton equals the energy density of inflaton happening in 4-dimensional stage. (a) y versus x, in which we set $z = -10$. (b) z versus x, in which we set $y = -4$.
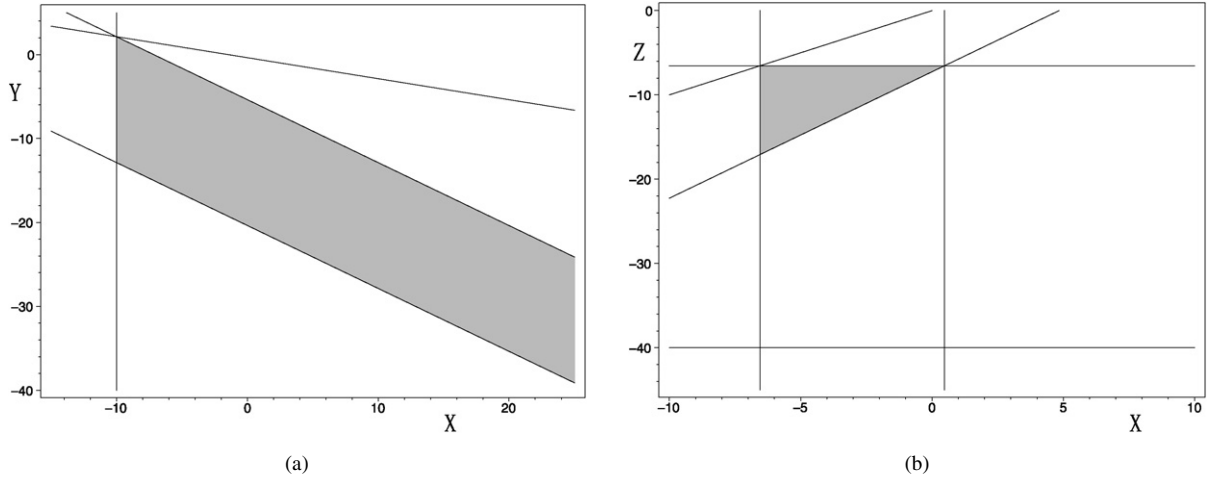

Fig. 5. The subcase decay after domination of case 2. (a) y versus x, in which we set $z = -10$. (b) z versus x, in which we set $y = -4$.

where we have used (35), which is still valid because the curvaton oscillate in 5-dimensional stage. From now on we follow the same program in case 1. (66) converts to ten equations:

$$m > \frac{\lambda}{3m_5^3}, \tag{72}$$

$$m > \frac{m\sigma_i^2}{12m_5^3}\sqrt{\frac{\lambda m}{3m_5^3}}, \tag{73}$$

$$m > \Gamma, \tag{74}$$

$$m > H_{\text{nuc}}, \tag{75}$$

$$\frac{\lambda}{3m_5^3} > \frac{m\sigma_i^2}{12m_5^3}\sqrt{\frac{\lambda m}{3m_5^3}}, \tag{76}$$

$$\frac{\lambda}{3m_5^3} > H_{\text{nuc}}, \tag{77}$$

$$\frac{\lambda}{3m_5^3} > \Gamma, \tag{78}$$

$$\frac{m\sigma_i^2}{12m_5^3}\sqrt{\frac{\lambda m}{3m_5^3}} > \Gamma, \tag{79}$$

$$\frac{m\sigma_i^2}{12m_5^3}\sqrt{\frac{\lambda m}{3m_5^3}} > H_{\text{nuc}}, \tag{80}$$

$$\Gamma > H_{\text{nuc}}. \tag{81}$$

We show the permitted parameter region of the curvaton field in this subcase in Fig. 4.

The other subcase is that the curvaton starts to dominate the universe after decay, which means,

$$m > H_{4.5} > \Gamma > H_{\text{eq2}} > H_{\text{nuc}}. \tag{82}$$

Completely following the same procedures of the above subcase, we draw Fig. 5 to show the permitted region of this subcase.

3.4. Case 3: oscillation and decay in 4-dimensional stage

Surely, the curvaton can oscillate and decay after the universe arriving at the 4-dimensional phase. First we study the condition that the universe is dominated by inflaton when the curvaton starts to oscillate. In this latter 4-dimensional stage,

the energy density of inflation red shifts according to (13),

$$\rho_\phi = \frac{18m_5^6}{\lambda}H^2. \tag{83}$$

Hence

$$\left.\frac{\rho_\sigma}{\rho_\phi}\right|_{H=m} < 1, \tag{84}$$

from which we derive

$$\sigma_i^2 < \frac{36m_5}{\lambda}, \tag{85}$$

where we have used $\rho_\sigma = m^2\sigma_i^2/2$. Therefore we can directly get the constraint on the initial value of the curvaton if it oscillates and decays in a 4-dimensional stage.

Also, there are two subcases: one is that the curvaton dominates the universe before decay; the other is that the curvaton dominates the universe after decay. The events sequence of the former subcase: the curvaton starts to oscillate, the universe transits from 5-dimensional phase 4-dimensional phase, the energy density of curvaton σ equals the energy density of inflaton ϕ, the curvaton decays, the nucleosynthesis happens. The sequence of Hubble parameters is

$$H_{4.5} > m > H_{eq3} > \Gamma > H_{nuc}, \tag{86}$$

where eq3 stands for the value of a variable when the density of curvaton equals the density of inflaton in latter 4-dimensional stage, at the same time the curvaton also begins to oscillate in 4-dimensional stage. The energy density of the curvaton field decreases as (26). And the energy density of the inflaton field decreases as

$$\rho_\phi = \rho_{4.5}\frac{a_{4.5}^6}{a^6}. \tag{87}$$

At the equality point,

$$\left.\frac{\rho_\sigma}{\rho_\phi}\right|_{a=a_{eq3}} = \frac{m^2\sigma_i^2}{2\rho_{4.5}}\frac{a_{osc}^3}{a_{4.5}^3}\frac{a_{eq3}^3}{a_{4.5}^3} = 1. \tag{88}$$

Substitute

$$\frac{a_{osc}^3}{a_{4.5}^3} = \frac{H_{4.5}}{m}, \tag{89}$$

$$\frac{a_{eq3}^3}{a_{4.5}^3} = \frac{H_{4.5}}{H_{eq3}}, \tag{90}$$

in (88), we obtain

$$H_{eq3} = \frac{m\sigma_i^2}{2\rho_{4.5}}H_{4.5}^2 = \frac{m\sigma_i^2}{36}\frac{\lambda}{m_5^6}, \tag{91}$$

where we have used (16) and (17). So (86) yields

$$\frac{\lambda}{3m_5^3} > m, \tag{92}$$

$$\frac{\lambda}{3m_5^3} > \frac{m\sigma_i^2}{36}\frac{\lambda}{m_5^6}, \tag{93}$$

$$\frac{\lambda}{3m_5^3} > \Gamma, \tag{94}$$

$$\frac{\lambda}{3m_5^3} > H_{nuc}, \tag{95}$$

$$m > \frac{m\sigma_i^2}{36}\frac{\lambda}{m_5^6}, \tag{96}$$

$$m > \Gamma, \tag{97}$$

$$m > H_{nuc}, \tag{98}$$

$$\frac{m\sigma_i^2}{36}\frac{\lambda}{m_5^6} > \Gamma, \tag{99}$$

$$\frac{m\sigma_i^2}{36}\frac{\lambda}{m_5^6} > H_{nuc}, \tag{100}$$

$$\Gamma > H_{nuc}. \tag{101}$$

Note that (85) is also just (96). They represent the same relation between ρ_ϕ and ρ_σ to be compared at different times. With the same discussions used before and by the same group of parameters of warped DGP model, we obtain Fig. 6 to show the permitted parameter region of the curvaton in this subcase.

The events sequence of the latter subcase: the curvaton starts to oscillate, the universe transits from 5-dimensional phase 4-dimensional phase, the curvaton decays, the energy density
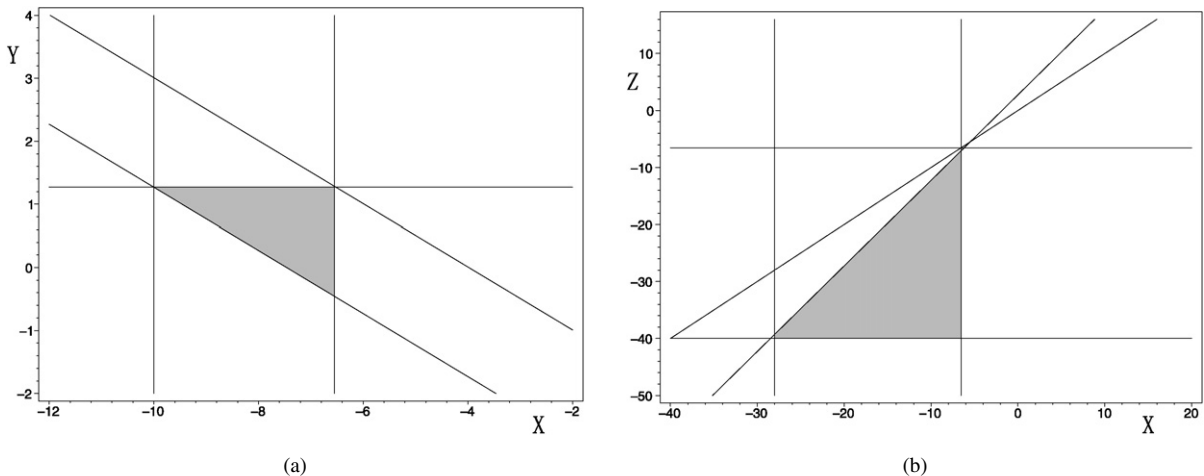

(a)   (b)

Fig. 6. The subcase decay after domination of case 3. (a) y versus x, where we fix $z = -10$. (b) z versus x, where we fix $y = 1$.

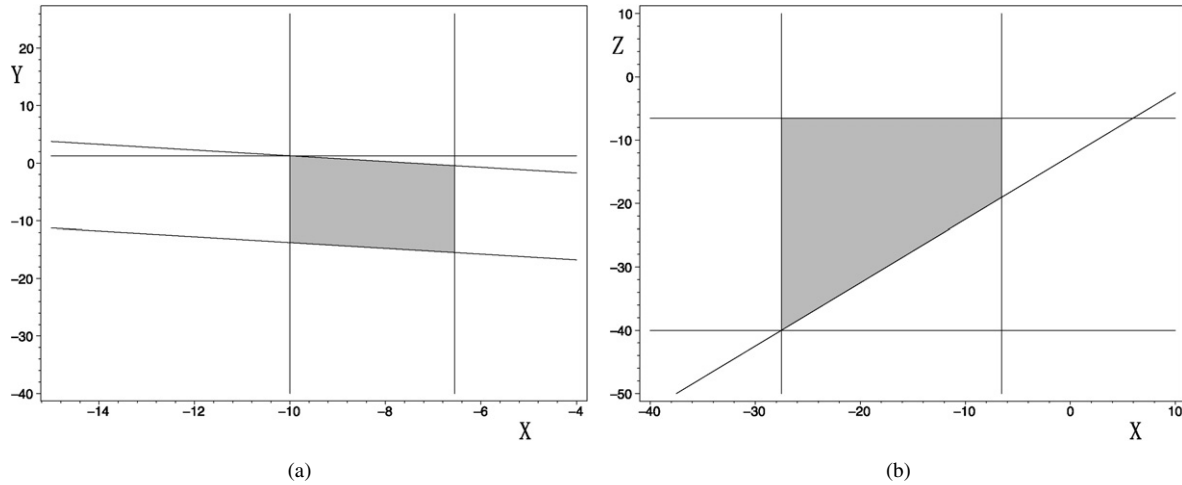(a)   (b)

Fig. 7. The subcase decay before domination of case 3. (a) y versus x, where we fix $z = -10$. (b) z versus x, where we fix $y = -5$.

of curvaton σ equals the energy density of inflaton ϕ, the nucleosynthesis happens. The sequence of Hubble parameters is

$$H_{4.5} > m > \Gamma > H_{eq3} > H_{nuc}. \tag{102}$$

We also show the permitted parameter region of curvaton in this subcase in Fig. 7. From Figs. 1–7 we see that generally speaking if the curvaton oscillates in a 5-dimensional stage the permitted region of m, σ_i is open for the value of x can be arbitrarily large if y is small enough, while if the curvaton oscillates in a 4-dimensional stage the permitted region of m, Γ is close. In contrast with the permitted region of m, σ, the permitted region of m, Γ always keeps close. So the permitted parameter region of the curvaton is much more ample when it oscillates in the 5-dimensional stage than in the 4-dimensional stage, that is, a reasonable curvaton model is much easier if the curvaton becomes to oscillate in the 5-dimensional stage.

In all of these cases there exists viable curvaton model satisfying the requirements such as a enough high reheating energy scale, sufficient particle generation mechanism for nucleosynthesis (comparing to "gravitational production mechanism").

4. Conclusion and discussion

The inflation model on DGP brane is of very interest and attraction because the universe can exit the inflationary phase spontaneously without any additional mechanism for an exponential potential, which is generally a serious problem for the ordinary inflation model with an exponential potential. However, this model suffer from the problem that the particles generated by gravitation is far from efficiency when nucleosynthesis happens.

In this Letter we investigate the curvaton mechanism in warped DGP model. We find it can hurdle the inefficient particle production problem with fairly ample parameter regions. Because the curvaton may oscillates and decay in a 5-dimensional stage or 4-dimensional stage, we discuss the 3 cases, say, oscillation in 5-dimensional stage, decay in 5-dimensional stage; os-

cillation in 5-dimensional stage, decay in 4-dimensional stage; and oscillation in 4-dimensional stage, decay in 4-dimensional stage, respectively. We plot figures for every case to show permitted parameter regions clearly.

Other constraints on the parameter of the curvaton field, such as the fluctuations for structure formation generated by the curvaton and the primordial gravitational wave etc., should be further studied in the future work.

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