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Simulation inferences for an availability system with general repair distribution and imperfect fault coverage

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ABSTRACT

We study the statistical inferences of an availability system with imperfect coverage. The system consists of two active components and one warm standby. The time-to-failure and time-to-repair of the components are assumed to follow an exponential and a general distribution respectively. The coverage factors for an active-component failure and for a standby-component failure are assumed to be the same. We construct a consistent and asymptotically normal estimator of availability for such repairable system. Based on this estimator, interval estimation and testing hypothesis are performed. To implement the simulation inference for the system availability, we adopt two repair-time distributions, namely, lognormal and Weibull and three types of Weibull distributions characterized by their shape parameters are considered. Finally, all simulation results are displayed in appropriate tables and curves for highlighting the performance of the statistical inference procedures.

1. Introduction

To gain and maintain competitive advantage, managers/engineers require a higher availability/reliability system for maintaining production/service machinery operations. Availability/reliability systems often utilize redundancy or using recovery/reconfiguration mechanisms to meet the required levels of fault tolerance. The normal operation of such systems depends on the system's ability to detect, isolate, and correctly accommodate failures of the redundant components. The probability of correctly accomplishing these tasks is termed coverage. For highly reliable systems, coverage has a significant effect on system's availability. However, some failures can remain undetected or uncovered, which can lead to system failure. Examples of the effect of uncovered faults can be found in computing systems, electrical power distribution networks, pipelines carrying dangerous materials, etc. (see Amari et al. [1] and Levitin and Amari [2]).

Numerous researches have worked on systems with imperfect fault coverage (abbreviated IFC). If the system is subject to IFC, the system reliability can decrease with increase in redundancy over some particular limit. The readers can refer to Levitin and Amari [2], Amari et al. [3–5], Chang et al. [6], Levitin [7], and Myers and Rauzy [8]. In the literature cited above, the entire system fails in the case of any uncovered failure.

Several studies have focused on assuming the time-to-repair follows an exponential distribution (see Yadavalli et al. [9], Chien et al. [10], and Ke et al. [11]). The two-unit system with lognormal repair time was investigated by Masters et al. [12] and Sridharan and Mohanavadivu [13]. Chandrasekhar et al. [14] considered a two-unit cold standby system in which the repair time distribution is a two-stage Erlangian. Recently, Wang et al. [15] investigated the cost benefit analysis of series

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systems with general repair times. In this article, we consider the statistical inference of a redundant repairable three-component system in which the time-to-repair of the component is assumed to follow a general distribution.

Statistical inferences can be used to estimate the system parameters/characteristics by using experimental data or simulation. However, the issues regarding statistical inferences for the availability/reliability systems with general repair time and imperfect coverage of failed component are rarely addressed in the literature. Asymptotic confidence limits for availability were investigated by Yadavalli et al. [9], Masters et al. [12], Sridharan and Mohanavadivu [13] and Chandrasekhar et al. [14]. Chien et al. [10] examined asymptotic confidence limits for performance measures of a repairable system with multiple unreliable service stations in which the repair time of failed components and service stations are assumed to be exponential. Ke et al. [11] studied Bayesian inferences of system characteristics for a redundant repairable system with exponential repair time and compared the reliability of the highest posterior density intervals and the asymptotic confidence limits.

The interested reader can refer to the published research works which mainly focused on *interval estimations* of system characteristics for *specified* repair distributions. The bottleneck of research works is probably due to the fact that system characteristics in terms of Laplace–Stieltjes transforms (LST) is not a convenient tool for statistical inferences and simulation analyses. By far, we have not found any published works on extending the statistical inferences of system characteristics for a repairable system to general repair distributions. This motivates us to explore statistical inferences for the steady-state availability of a repairable system with distribution-free repair times. To tackle the simulation difficulty of LST, we successfully derive the computable LST form in this paper.

This article is organized as follows. In Section 2, we describe the problem statements which include assumptions and definitions for the repairable system. In Section 3, we develop an estimator of system availability from a statistical standpoint. Using the estimator and its estimated variance, the asymptotic confidence interval of the availability is obtained. Moreover, the rule for testing (statistical hypothesis) of the system availability is developed and its power function is derived. In Section 4, to explore the performance of the estimator for system availability, we conduct a simulation study. In the simulation study, we first evaluate the accuracy of the confidence interval of system availability by computing their coverage percentage. Secondly, the power functions of the test under different sample sizes are generated and compared. For concreteness, we consider two different repair distributions, namely, lognormal and Weibull distribution with three different shape parameters. Finally, we draw some conclusions based on the study.

2. Model description

We consider a redundant repairable three-component system in which two components are active and the other is a standby. When an active component fails, it is immediately replaced by a standby if it is available. It is assumed that the switchover time is instantaneous. The switch from standby to active is perfect. By perfect, we mean the standby is always available and there is no time lag between switches. It is also assumed that the coverage factor is the same for an active-component failure as that for a warm standby-component failure and is denoted by c. When a fault/failure is present in an active component (or warm standby component), it may be immediately detected, located, and recovered with a probability c. After recovering, the failed component is ready for repair. Quantity c is called the coverage factor or coverage probability. (see Trivedi [16])

Active components and warm standby components are considered repairable. It is assumed that each of the active components fails independently of the others and follows an exponential time-to-failure distribution with parameter λ. If one of the active components fails, the warm standby component will replace it immediately as long as one is available. Moreover, we assume that the standby component fails independently of all the others and has an exponential time-to-failure distribution with parameter α ($\lambda > \alpha > 0$). Assume that the active component or standby component failure in the unsafe failure state is cleared by a reboot, and the delay for an active unit (or standby unit) is at rate β and follows an exponential distribution. Let the time-to-repair of the components be independent and identically distributed random variables following a general distribution $B(u)$ ($u > 0$), with probability density function $b(u)$ ($u > 0$), a LST $B^*(s)$ and mean repair time b_1. Furthermore, we define that the unsafe failure state of the system is that when any one of the breakdowns is not recovered. We continue with the assumption that active-component failure (or standby-component failure) in the unsafe failure (UF) state is cleared by a reboot. A component fails when the standby is emptied for which we define as the state of safe failure (SF), i.e., system failure. The steady-state transition for the availability system is given in Fig. 1.

As the state transition diagram given in Fig. 1 shows, initially two active components and one warm standby are normal (in State 3, which the numeric 3 represents the number of normal components is three). The fault in state 3 may be detected with probability c or not detected with probability $1-c$. In the former case, the system is still functioning in state 2, while in the later case, an unsafe failure has occurred (State UF). When an UF occurs, the cause of the fault is found by a testing which may indicates that a reboot action is required. If a component failure occurs when the system is in state 2 (standby unavailable), the system fails and enters state SF. After pinpointing the cause of the fault that occurs in the system in state 2, a reboot action is not required because that system failure is repaired inherently. If a failed component is repaired when the system is in state 2, the system is recovered and enters state 3. The repair is completed when system is in SF state, the system enters state 2. When a standby component switches over successfully, its failure characteristics become those of the active component (i.e., failure rate changes from α to λ). If an active or a standby component fails, it is immediately repaired. The repair is independent of the failure of components. Once a component is repaired, it instantly resumes standby status unless the system has failed.

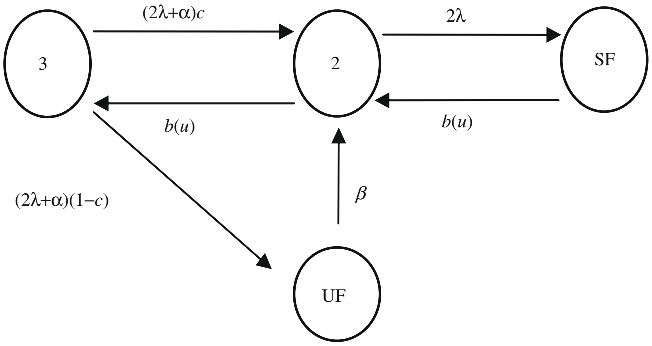

Fig. 1. The steady-state transition for the availability system.

2.1. Practical justification of the model

A number of practical problems arise that may be formulated as one in which the availability of system is often improved by redundancy or using recovery/reconfiguration mechanisms. One particular problem where this model can be applied is in the study of a power system. Consider a power plant with a capacity of 30 MW which is provided by a serial system of two 15 MW active generators. Besides the two generators, there is one standby generator so that when a generator breaks down, a standby generator is immediately substituted and thus the availability of the system is improved. Generator in operation or standby state is subject to breakdowns. When an active generator (or standby generator) fails, it may be successfully detected, located and recovered with an overall coverage (probability) c. When one of the breakdowns is not recovered, the system enters the UF state. Active-generator failure (or standby-generator failure) in UF state is possibly cleared by a reboot action, that is, some type of breakdowns for the generators may be recovered by restart (reset). An example of the aforementioned recovery is circuit breaker tripped can sometimes be tested and then recovered by reset (before repairing). When the standby is emptied, the system will enter SF state if a generator failure (i.e., system failure). At this time, the system doesn't produce the 30 MW power.

3. Consistent and asymptotically normal estimator of system availability

As before, the system is a serial of two active 15 MW components and one standby 15 MW component. If the active component fails, the warm standby component will replace the broken component immediately. From Wang and Chiu [17], the system availability (denoted by A_v) is as follows

$$A_v = \frac{\beta[2\lambda + \alpha - \alpha B^*(2\lambda)]}{2\lambda\{\beta B^*(2\lambda) + (2\lambda + \alpha)[\beta b_1 + (1-c)B^*(2\lambda)]\}}.$$

Let X and Y represent the time-to-failure of the active component and standby component respectively, Z and W represent the repair time and reboot delay time of failed components respectively. Let μ_x, μ_y, μ_z and μ_w be the means of X, Y, Z and W, respectively. Assume that $\mathbf{X} = (X_1, \ldots, X_n)$, $\mathbf{Y} = (Y_1, \ldots, Y_n)$, $\mathbf{Z} = (Z_1, \ldots, Z_n)$ and $\mathbf{W} = (W_1, \ldots, W_n)$ are random samples of size n, respectively, for X, Y, Z and W. Let $\overline{X}, \overline{Y}, \overline{Z}$ and $\overline{W}$ denote the respective sample means of $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ and $\mathbf{W}$. According to the Law of Large Numbers, $\overline{X}, \overline{Y}, \overline{Z}$ and $\overline{W}$ are the respective consistent estimators of μ_x, μ_y, μ_z and μ_w, where $\mu_x = 1/\lambda$, $\mu_y = 1/\alpha$, $\mu_z = b_1$ and $\mu_w = 1/\beta$. Thus we have $\hat{\lambda} = \overline{X}^{-1}, \hat{\alpha} = \overline{Y}^{-1}, \hat{b}_1 = \overline{Z}$ and $\hat{\beta} = \overline{W}^{-1}$. An estimator of A_v is given by

$$\widehat{A}_v = \frac{\hat{\beta}[2\hat{\lambda} + \hat{\alpha} - \hat{\alpha} B^*(2\hat{\lambda})]}{2\hat{\lambda}\{\hat{\beta} B^*(2\hat{\lambda}) + (2\hat{\lambda} + \hat{\alpha})[\hat{\beta}\hat{b}_1 + (1-c)B^*(2\hat{\lambda})]\}}. \tag{1}$$

In general, the distributions of X, Y, Z and W are unknown and the exact distribution of $\widehat{A}_v$ is complicated. Under the assumption that $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ and $\mathbf{W}$ are mutually independent, we derive the asymptotic distribution of $\widehat{A}_v$ as follows.

By the Central Limit Theorem (CLT, see Casella and Berger [18]), we have

$$\overline{X} \xrightarrow{D} N\left(\frac{1}{\lambda}, \frac{1}{n\lambda^2}\right), \quad \overline{Y} \xrightarrow{D} N\left(\frac{1}{\alpha}, \frac{1}{n\alpha^2}\right),$$
$$\overline{Z} \xrightarrow{D} N\left(b_1, \frac{\sigma_z^2}{n}\right), \quad \overline{W} \xrightarrow{D} N\left(\frac{1}{\beta}, \frac{1}{n\beta^2}\right),$$

where σ_z^2 is the variance of Z, and $\xrightarrow{D}$ denotes convergence in distribution. Using the Delta method (see Casella and Berger [18]), we get

$$\hat{\lambda} = \overline{X}^{-1} \xrightarrow{D} N\left(\lambda, \frac{\lambda^2}{n}\right), \tag{2}$$

$$\hat{\alpha} = \overline{Y}^{-1} \xrightarrow{D} N\left(\alpha, \frac{\alpha^2}{n}\right), \tag{3}$$

$$\hat{b}_1 = \overline{Z} \xrightarrow{D} N\left(b_1, \frac{\sigma_z^2}{n}\right), \tag{4}$$

$$\hat{\beta} = \overline{W}^{-1} \xrightarrow{D} N\left(\beta, \frac{\beta^2}{n}\right). \tag{5}$$

Let

$$U(\lambda, \alpha, \beta, b_1) = \beta[2\lambda + \alpha - \alpha B^*(2\lambda)], \tag{6}$$
$$V(\lambda, \alpha, \beta, b_1) = 2\lambda\{\beta B^*(2\lambda) + (2\lambda + \alpha)[\beta b_1 + (1-c)B^*(2\lambda)]\}. \tag{7}$$

By the invariance property, $U(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1)$ and $V(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1)$ are the consistent estimators of Eqs. (6) and (7), respectively.

The asymptotic variance of $U(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1)$ (the detailed derivation is given in Appendix A) is evaluated as

$$Var[U(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1)] = Var(2\hat{\beta}\hat{\lambda}) + Var[\hat{\beta}\hat{\alpha}(1 - B^*(2\hat{\lambda}))] + 2Cov[2\hat{\beta}\hat{\lambda}, \hat{\beta}\hat{\alpha}(1 - B^*(2\hat{\lambda}))] = A + B + 2C, \tag{8}$$

where

$$A = \frac{4(2n+1)}{n^2}\lambda^2\beta^2,$$
$$B = \frac{4(n+1)^2}{n^3}\alpha^2\beta^2\lambda^2[B^{*(1)}(2\lambda)]^2 + \frac{2n+1}{n^2}\alpha^2\beta^2[1 - B^*(2\lambda)]^2,$$
$$C = \frac{2\alpha\beta^2\lambda}{n}(1 - B^*(2\lambda)),$$

and $B^*(\lambda)$ is the LST of Z by setting $s = \lambda$, and $B^{*(1)}(\lambda)$ is the first derivative of $B^*(s)$ with given $s = \lambda$. Using the Delta method again, we have

$$U(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1) \xrightarrow{D} N(\mu_u, \sigma_u^2), \tag{9}$$

where $\mu_u = U(\lambda, \alpha, \beta, b_1)$ and $\sigma_u^2 = Var[U(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1)]$. On the other hand,

$$V(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1) \xrightarrow{P} V(\lambda, \alpha, \beta, b_1), \tag{10}$$

where $\xrightarrow{P}$ denotes convergence in probability.Using the Slutsky's theorem (see Casella and Berger [18]) on Eqs. (9) and (10), we get

$$\widehat{A}_v = \frac{U(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1)}{V(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1)} \xrightarrow{D} N\left(A_v, \frac{\sigma_u^2}{V^2(\lambda, \alpha, \beta, b_1)}\right). \tag{11}$$

Let $\sigma_{A_v} = \sigma_u / V(\lambda, \alpha, \beta, b_1)$ and

$$\hat{\sigma}_{Av} = \frac{\sigma_u}{V(\hat{\lambda}, \hat{\alpha}, \hat{\beta}, \hat{b}_1)}. \tag{12}$$

From Eq. (10), $\hat{\sigma}_{A_v}^2$ is a consistent estimator of $\sigma_{A_v}^2$. By the Slutsky's theorem, we deduce that

$$\frac{\widehat{A}_v - A_v}{\hat{\sigma}_{Av}} \xrightarrow{D} N(0, 1), \tag{13}$$

that is, $\widehat{A}_v$ is a consistent and asymptotically normal (CAN) estimator with approximate variance $\sigma_{A_v}^2$.

3.1. Confidence interval for A_v

Using the statistic given in Eq. (13), we can construct a confidence interval for A_v for the availability system with a repair distribution-free system. Assume that z_α be the upper αth quantile of the standard normal distribution. Thus the approximate $100(1-\alpha)\%$ confidence interval of A_v is as follows

$$\left(\widehat{A}_v - z_{\alpha/2}\hat{\sigma}_{A_v}, \widehat{A}_v + z_{\alpha/2}\hat{\sigma}_{A_v}\right). \tag{14}$$

3.2. Hypothesis test for A_v

Besides point and interval estimation of A_v, hypothesis testing concerning A_v is another inference problem of concern to system engineers. We want to test the hypothesis

$$H_0: \ A_v \le A_v^* \text{vs.} \ H_1: \ A_v > A_v^*, \tag{15}$$

where A_v^* is a constant level of A_v. The rejection region can be constructed by choosing c such that

$$\sup_{A_v \in \Theta_0} P\left(Z > c + \frac{A_v^* - A_v}{\hat{\sigma}_{A_v}}\right) = \alpha \tag{16}$$

where $\Theta_0 = \{A_v | A_v \le A_v^*\}$ and α is the pre-specified significance level of the test. From Eq. (16) we obtain $c = z_\alpha$, thus the rejection region is

$$\frac{\widehat{A}_v - A_v^*}{\hat{\sigma}_{A_v}} > z_\alpha. \tag{17}$$

3.3. Power function for A_v

A most powerful level α test of Eq. (15) has power function

$$\beta_n(A_v) = P\left(Z > z_\alpha + \frac{A_v^* - A_v}{\hat{\sigma}_{A_v}}\right). \tag{18}$$

The power function $\beta_n(A_v)$ depend on the unknown parameter A_v and the sample size n. The probability of the Type I error can be obtained from $\beta_n(A_v)$, $A_v \in \Theta_0$.

4. Simulation study

In order to examine the performance of $\widehat{A}_v$, we conduct a simulation study on the statistical behaviors of the availability for the repair system. Firstly, the assurance of the confidence interval of A_v of the availability system is evaluated by coverage percentage. Secondly, using the rejection rules, we compare the power function of the test under different values of n and A_v. Finally, the Type I error rates for the A_v can be simulated. We consider two commonly used repair time distributions, namely, lognormal and Weibull in our numerical simulations. These two distributions play important roles in reliability theory. Further, Weibull distribution with scale parameter a and shape parameter b is subdivided into three types according to the shape parameter b, in which the component has decreasing repair rate if $b < 1$ (DRR), the constant repair rate if $b = 1$ (CRR), and increasing repair rate if $b > 1$ (IRR). In the simulation experiment, we denote W_{DRR}, W_{CRR} and W_{IRR} by setting $b = 0.5$, 1.0 and 2.0, respectively for the Weibull distribution. Since the LSTs of lognormal and Weibull distributions are implicit, the expressions for $\widehat{A}_v$ cannot be solved explicitly. Thus the simulation experiments are difficult to implement. To tackle this problem, we derive the computable forms of the LST of lognormal and Weibull in Appendix B. For simplicity, all numerical simulations are performed on the following specific parameter values: $\lambda = 1$, $\alpha = 0.01$, $b_1 = 1/30$, and $\beta = 0.025$.

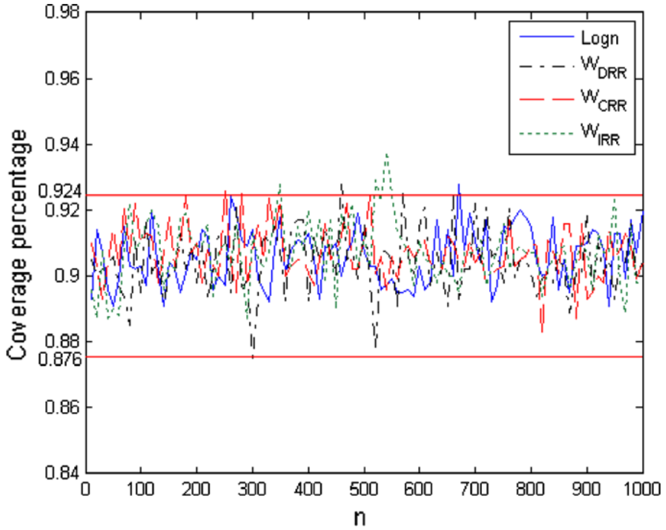

Fig. 2. Coverage percentages of the 90% CI for A_v.

Table 1
The mean length and the standard deviation of A_v.

Repair d	$n = 10$	$n = 20$	$n = 30$	$n = 40$	$n = 50$	$n = 60$	$n = 70$	$n = 80$	$n = 90$	$n = 100$
log n	0.0242	0.0161	0.0128	0.0110	0.0098	0.0088	0.0082	0.0076	0.0073	0.0068
	(0.0118)	(0.0051)	(0.0030)	(0.0022)	(0.0019)	(0.0015)	(0.0013)	(0.0012)	(0.0010)	(0.0009)
W_{DRR}	0.0234	0.016	0.0128	0.0110	0.0097	0.0089	0.0082	0.0076	0.0072	0.0068
	(0.0115)	(0.0053)	(0.0033)	(0.0024)	(0.0019)	(0.0015)	(0.0014)	(0.0012)	(0.0010)	(0.0009)
W_{CRR}	0.0234	0.0162	0.0129	0.0109	0.0098	0.0089	0.0082	0.0077	0.0072	0.0068
	(0.0118)	(0.0051)	(0.0034)	(0.0024)	(0.0019)	(0.0016)	(0.0014)	(0.0012)	(0.0010)	(0.0009)
W_{IRR}	0.0237	0.0160	0.0128	0.0110	0.0099	0.0088	0.0082	0.0077	0.0073	0.0069
	(0.0111)	(0.0049)	(0.0032)	(0.0023)	(0.0019)	(0.0016)	(0.0013)	(0.0012)	(0.0010)	(0.0009)

The value in the parenthesis is the standard deviation.

4.1. Confidence interval

The confidence interval $\left(\hat{A}_v - z_{\alpha/2}\hat{\sigma}_{A_v}, \widehat{A}_v + z_{\alpha/2}\hat{\sigma}_{A_v}\right)$ for the availability system is simulated as follows. Let the random samples of size n for active component time-to-failure $(X_1, \ldots, X_n)$, the standby component time-to-failure $(Y_1, \ldots, Y_n)$, the repair time $(Z_1, \ldots, Z_n)$ and reboot delay time $(W_1, \ldots, W_n)$ be drawn from X, Y, Z and W respectively. We can compute $\hat{\lambda}, \hat{\alpha}, \hat{b}_1$ and $\hat{\beta}$ from Eqs. (2)–(5). Then the estimate $\widehat{A}_v$ and its variance $\hat{\sigma}^2_{A_v}$ can be evaluated by substituting Eq. (1) and Eq. (12). The 90% confidence interval is given by

$$(\widehat{A}_v - 1.645\hat{\sigma}_{A_v}, \widehat{A}_v + 1.645\hat{\sigma}_{A_v}). \tag{19}$$

The simulation is replicated for $N = 1000$ times and then we compute the coverage probability. The coverage probability can be estimated by the proportion of the number of times the true values are contained in the constructed confidence intervals. Let I_j denote the indicator function of the event that which the true value is contained in the jth replication. We consider $n = 10, 20, \ldots, 1000$ in the N replications. The number of confidence intervals covering the true availability A_v obeys the binomial distribution with parameters $N = 1000$ and $p = 0.9$. According to the CLT

$$\frac{\sum_{j=1}^{1000} I_j - 900}{\sqrt{1000 \times 0.9 \times 0.1}} = \frac{\bar{I} - 0.9}{\sqrt{0.9 \times 0.1/1000}} \sim N(0,1).$$

Thus the 99% confidence interval for the coverage percentage itself is

$$0.9 \mp 2.576\sqrt{0.9 \times 0.1/1000} = (0.876, 0.924). \tag{20}$$

Fig. 2 gives the coverage percentages of 90% confidence intervals of A_v for various values of n and different repair distributions. Lots of coverage percentages points fall into the theoretical interval (0.876, 0.924) and fluctuate within. Moreover, the simulation results for the mean length and the standard deviation of the 90% confidence intervals for A_v are shown in Table 1. We observe that the mean lengths of confidence intervals for four types of repair distributions are very close, and the standard deviation decreases and then converges to 0 when n increases.

All the simulations are performed by the mathematical program MATLAB7.1 and the computational time is acceptable. For example, for each curve (from $n = 10$ to 1000, step 10) of Fig. 1, it takes about 20 min on a PC (Intel® Pentium® D CPU 2.80 GHz, 2.79 GHz, 1.49 GB RAM, and running Microsoft Windows XP Professional Version 2002, Service Pack 3 set up).

4.2. Power function

In addition to constructing confidence interval of A_v, we compute the power function of the hypothesis test when $A_v^* = 0.9$ and n are given. From Section 4.1, we simulate 1000 times and the results are recorded as $(\widehat{A}_{v(1)}, \hat{\sigma}^2_{A_v(1)}), (\widehat{A}_{v(2)}, \hat{\sigma}^2_{A_v(2)}), \ldots, (\widehat{A}_{v(1000)}, \hat{\sigma}^2_{A_v(1000)})$. Let $I_j(A_v)$ denote the indicator function of

$$\left\{\frac{\widehat{A}_{v(j)} - A_v^*}{\hat{\sigma}_{A_v(j)}} > z_\alpha\right\} \quad \text{for } j = 1, \ldots, 1000. \tag{21}$$

We get $\hat{\beta}_n(A_v) = \sum_{j=1}^{1000} I_j(A_v)/1000$, which is a simulated value of the power $\beta_n(A_v)$. The simulation results for the power function of A_v for various values of n and different repair distributions are illustrated in Fig. 3. From the figure we observe that for any given A_v, the power $\hat{\beta}_{50}(A_v) > \hat{\beta}_{30}(A_v) > \hat{\beta}_{10}(A_v)$ for $A_v > A_v^*$, and all curves approach 1 when A_v approaches 1 or n is sufficiently large. Moreover, the numerical results depict that $\beta_{n_1} > \beta_{n_2}$ for $n_1 > n_2$, and β_n approaches to the maximum power 1 as A_v approaches 1 or n is large enough. For a fixed sample size n, the order of the powers of repair distribution is W_{DRR} > Lognormal $\approx W_{CRR} > W_{IRR}$.

The Type I error is simulated as follows. We explore the performance of the rejection rules given in Eq. (17) for the availability configuration. We consider the probability of Type I error when $A_v = A_v^*$, that is $\beta_n(A_v^*)$. We set the significance level of the test to be $\alpha = 0.05$. The simulation results for the Type I error rate of A_v are shown in Fig. 4. $\sum_{j=1}^{1000} I_j$ follows the binomial distribution with parameters $N = 1000$ and $p = 0.05$. According to the CLT,

$$\frac{\sum_{j=1}^{1000} I_j - 50}{\sqrt{1000 \times 0.05 \times 0.95}} = \frac{\bar{I} - 0.05}{\sqrt{0.05 \times 0.95/1000}} \sim N(0,1).$$

The 99% confidence interval for the coverage percentage itself is

$$0.05 \mp 2.576\sqrt{0.05 \times 0.95/1000} = (0.0322, 0.0678). \tag{22}$$

From Fig. 4, we find that the simulated Type I error of A_v performs well when $n > 200$ but becomes low (close to 0.0322) and fluctuates in the theoretical interval shown in Eq. (22).

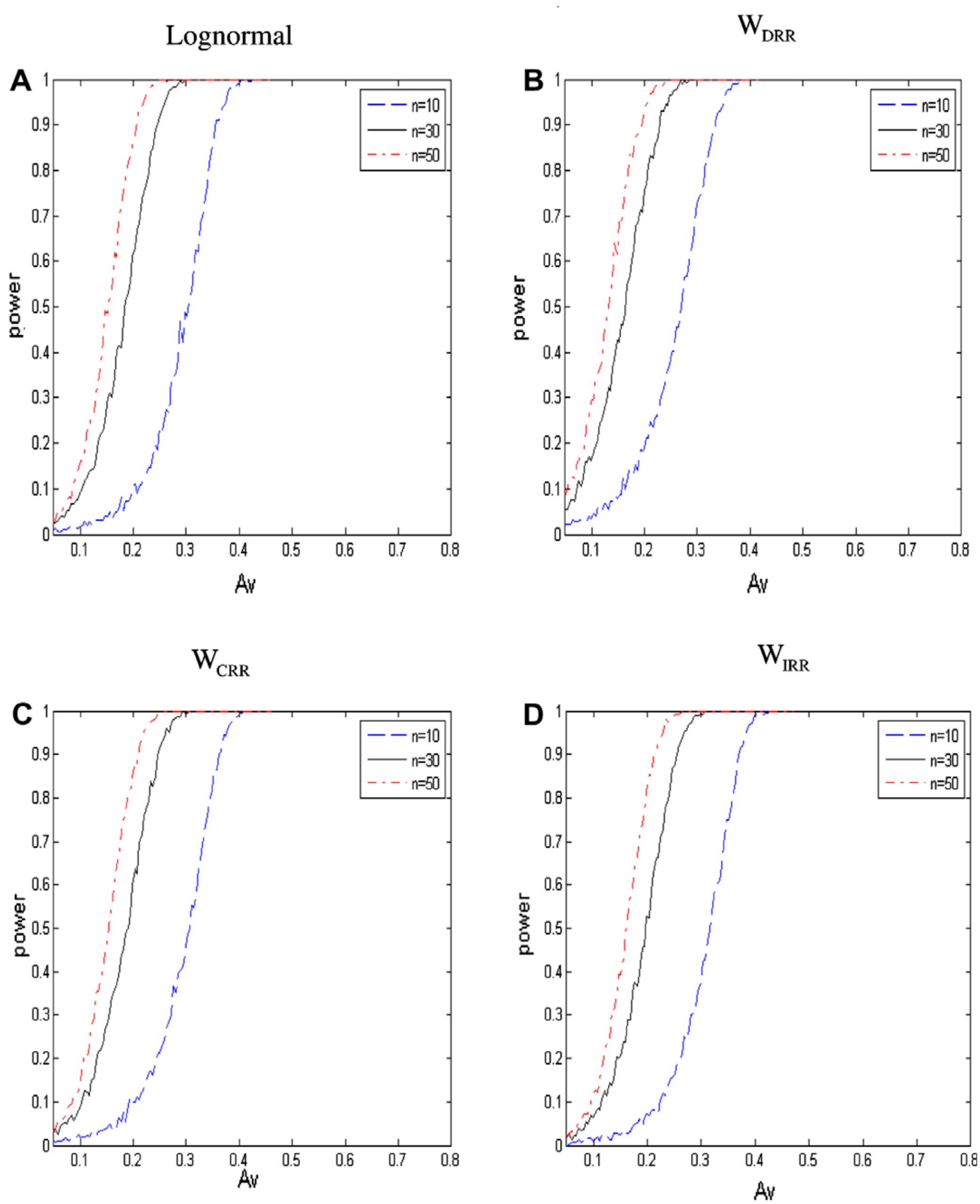

Fig. 3. The power function of $\widehat{A}_v$.

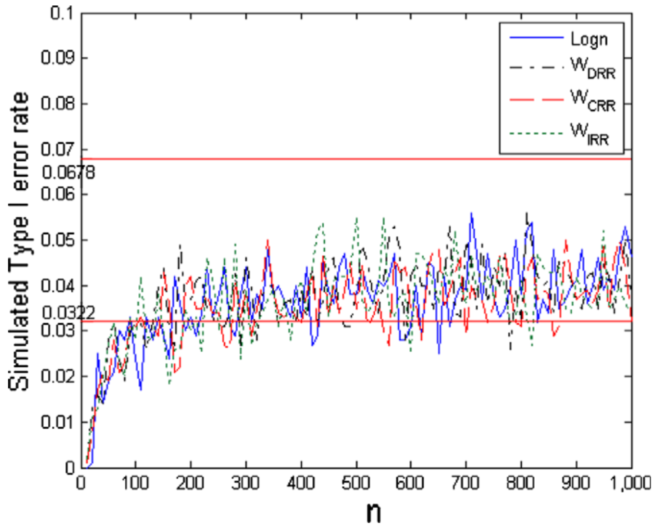

Fig. 4. The Type I errors of $\widehat{A}_v$.

5. Conclusions

We develop statistical inferential procedures for an availability system with imperfect coverage. A consistent and asymptotically normal estimator of the availability is derived. To evaluate the performance of this estimator, we specifically derive a simpler computation form for the lognormal and Weibull distribution. Simulation results clearly show that $\widehat{A}_v$ performs well in interval estimation and power performance. We highly recommend managers/engineers use the estimator to evaluate system availability. Future investigation can try to incorporate a more complex system (such as non-homogeneous) with different failure criteria. In addition, the effect of switching failures on a repairable system or more complex system may also be a problem of interest.

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Appendix A. The detailed derivation of variance of $\boldsymbol{U}(\hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{b}}_1)$

$$A = Var(2\hat{\beta}\hat{\lambda}) = 4E(\hat{\beta}^2\hat{\lambda}^2) - [2E(\hat{\beta}\hat{\lambda})]^2 = 4\left(\frac{\beta^2}{n} + \beta^2\right)\left(\frac{\lambda^2}{n} + \lambda^2\right) - 4\beta^2\lambda^2 = \frac{4(2n+1)}{n^2}\lambda^2\beta^2,$$

$$\begin{aligned}
B &= Var[\hat{\beta}\hat{\alpha}(1 - B^*(2\hat{\lambda}))] = E[\hat{\beta}^2\hat{\alpha}^2(1 - B^*(2\hat{\lambda}))^2] - \{E[\hat{\beta}\hat{\alpha}(1 - B^*(2\hat{\lambda}))]\}^2 \\
&= \left(\frac{\alpha^2}{n} + \alpha^2\right)\left(\frac{\beta^2}{n} + \beta^2\right)\left\{\left[\frac{d}{d\lambda}(1 - B^*(2\lambda))\right]^2\frac{\lambda^2}{n} + (1 - B^*(2\hat{\lambda}))^2\right\} - [\beta\alpha(1 - B^*(2\lambda))]^2 \\
&= \frac{4(n+1)^2}{n^3}\alpha^2\beta^2\lambda^2[B^{*(1)}(2\lambda)]^2 + \frac{2n+1}{n^2}\alpha^2\beta^2[1 - B^*(2\hat{\lambda})]^2, \\
C &= Cov[2\hat{\beta}\hat{\lambda}, \hat{\beta}\hat{\alpha}(1 - B^*(2\hat{\lambda}))] = E\left[2\hat{\alpha}\hat{\beta}^2\hat{\lambda}(1 - B^*(2\hat{\lambda}))\right] - E(2\hat{\beta}\hat{\lambda}) \cdot E[\hat{\beta}\hat{\alpha}(1 - B^*(2\hat{\lambda}))] \\
&= 2\alpha\left(\frac{\beta^2}{n} + \beta^2\right)\lambda(1 - B^*(2\lambda)) - 2\beta\lambda\alpha\beta(1 - B^*(2\lambda)) \\
&= \frac{2\alpha\beta^2\lambda}{n}(1 - B^*(2\lambda)).
\end{aligned}$$

Appendix B. The computation of Laplace transform

Let $X \sim f_X(x)$. The LST of X is $B^*(s) = \int_0^\infty e^{-sx} \cdot f_X(x)\,dx$. In order to change the upper limit of the definite integral for LST from infinity to 1, we use the transformation $y = 1/(1 + x)$. The LST can be rewritten as

$$B^*(s) = \int_0^1 \exp\left(-s\left(\frac{1}{y}-1\right)\right)\cdot f_X\left(\frac{1}{y}-1\right)\frac{1}{y^2}\,dy. \tag{B1}$$

The differentiation of $B^*(s)$ with respect to s is

$$\frac{d}{ds}B^*(s) = -\int_0^1 \left(\frac{1}{y}-1\right)\exp\left(-s\left(\frac{1}{y}-1\right)\right)\cdot f_X\left(\frac{1}{y}-1\right)\frac{1}{y^2}\,dy. \tag{B2}$$

Using Simpson integration on Eqs. (B1) and (B2), we can get the value $B^*(s)|_{s=\lambda}$ equals the area under the curve

$$\exp\left(-s\left(\frac{1}{y}-1\right)\right)\cdot f_X\left(\frac{1}{y}-1\right)\frac{1}{y^2}$$

from 0 to 1, and the value $\frac{d}{ds}B^*(s)\big|_{s=\lambda}$ equals the area under the curve

$$\left(\frac{1}{y}-1\right)\exp\left(-s\left(\frac{1}{y}-1\right)\right)\cdot f_X\left(\frac{1}{y}-1\right)\frac{1}{y^2}$$

from 1 to 0, respectively.

(i) If $X \sim \log N(\mu, \sigma^2)$, the probability density function is

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma x}e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}}.$$

We have

$$B^*(s) = \int_0^1 e^{-s\left(\frac{1}{y}-1\right)}\cdot\frac{1}{\sqrt{2\pi}\sigma y(1-y)}e^{-\frac{(\ln(1-y)-\ln(y)-\mu)^2}{2\sigma^2}}\,dy$$

and

$$\frac{d}{ds}B^*(s) = -\int_0^1 e^{-s\left(\frac{1}{y}-1\right)}\cdot\frac{1}{\sqrt{2\pi}\sigma y^2}e^{-\frac{(\ln(1-y)-\ln(y)-\mu)^2}{2\sigma^2}}\,dy.$$

(ii) If $X \sim \text{Weibull}(a, b)$, the probability density function is

$$f_X(x) = abx^{b-1}e^{-ax^b}.$$

We have

$$B^*(s) = \int_0^1 ab\cdot\frac{(1-y)^{b-1}}{y^{b+1}}e^{-\left(\left(\frac{1-y}{y}\right)+a\left(\frac{1-y}{y}\right)^b\right)}\,dy$$

and

$$\frac{d}{ds}B^*(s) = -\int_0^1 ab\,\frac{(1-y)^b}{y^{b+2}}e^{-s\left(\frac{1}{y}-1\right)+a\left(\frac{1-y}{y}\right)^b}\,dy.$$

Remark: It should be noted that the computation of $\int_0^1 f(\cdot)\,dx$ is more simpler and more stable than $\int_0^\infty f(\cdot)\,dx$. Also, the computation time of $\int_0^1 f(\cdot)\,dx$ is much less than that of $\int_0^\infty f(\cdot)\,dx$.

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