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ORIGINAL PAPER

Estimation of strength parameters of rock using artificial neural networks

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Abstract The accurate determination of geomechanical properties such as uniaxial compressive strength and shear strength requires considerable time in collecting appropriate samples, their preparation and laboratory testing. To minimize the time and cost, a number of empirical relations have been reported which are widely used for the estimation of complex rock properties from more easily acquired data. This paper reports the use of an artificial neural network to predict the deformation properties of Coal Measure rocks using dynamic wave velocity, point load index, slake durability index and density. The results confirm the applicability of this method.

Keywords Artificial neural networks · Rock properties · Strength parameters

Résumé La détermination précise des propriétés géomécaniques telles que la résistance à la compression simple et la résistance au cisaillement demande beaucoup de temps pour le choix des échantillons, leur préparation et la réalisation des essais de laboratoire. Afin de minimiser le temps et le coût, plusieurs relations empiriques ont été présentées, largement utilisées pour l'estimation des propriétés des roches à partir de données plus facilement obtenues. L'article présente l'utilisation d'un réseau de neurones artificiel destiné à prévoir les propriétés de déformation de roches d'une série houillère à partir de mesures de vitesses des ondes, de l'indice de compression entre pointes, l'indice de durabilité et la densité. Les résultats confirment l'applicabilité de cette méthode.

Mots clés Réseaux de neurones artificiels · Propriétés des roches · Paramètres de résistance

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Introduction

The geo-engineering characteristics of rock are complex and ill-defined due to the varied physical processes associated with the formation of these materials in time and space (Jaksa 1995; Singh et al. 2005). Some of the important design parameters which are difficult to establish can be indirectly determined using the relationship between the static and dynamic properties of rock (Sarkar et al. 2009). Inoue and Ohomi (1981) suggested a relationship between the uniaxial compressive strength (UCS), elastic wave velocity and density of weak rocks while Singh and Dubey 2000 and Singh et al. 2004) suggested empirical relationships between UCS and P-wave velocity, mainly for Coal Measure strata. As these empirical equations have limitations, notably being site specific, in this paper an attempt has been made to predict the strength parameters using artificial neural networks (ANN). A back-propagation feed-forward neural network has been used, with four input and two output parameters and a hidden layer. To check the sensitivity of the results, *t* tests were undertaken.

ANN is a widely used tool, well suited to complex problems where the relationship between the model variables is not well established (Hubick 1992; Millar and Clarici 1994). In such situations, expert judgment plays an important role. The ANN technique makes use of heuristic knowledge or pattern-matching techniques, as opposed to solving a set of mathematical equations (Lee and Sterling 1992; Singh et al. 2001). This approach has been used by various researchers to solve some complex problems with a good degree of accuracy (Millar and Hudson 1994; Lessard and Hadjigeorgiou 1999; Sirat and Talbot 2001).

Geological conditions at the site

The study area is in the Luhri region, Himachal Pradesh (Fig. 1) and is in the Shimla block which has been designated as part of Seismic Zone V (Narula et al. 2000).

The valley of the River Satluj is characterized by highly deformed, metamorphosed sedimentary and igneous rocks. Much of the deformation is in the form of major folds belonging to several different phases of tectonism. Tight recumbent as well as later more open isoclinal and anticlinal structures are present.

Laboratory investigations

A total of 120 samples of quartz–mica–schist, quartzite, slate and limestone were collected from different locations in the Luhri region and UCS, P-wave velocity, point load index, slake durability index and density were determined following ISRM (1981). The results in Table 1 are the average of three tests.

Artificial neural network (ANN)

ANN is an information processing system which simulates both structure and functions. It consists of numerous simple processing elements (neurons) capable of performing complex data processing and knowledge representation (Kosko 1994). The neural network is normally trained by processing a large number of input and output patterns to achieve matching and prediction. It is basically mapping the input and output values; hence, it has excellent interpolation capabilities, especially when the input data are noisy. Neural networks may be used as a substitute for auto-correlation, multivariable regression, linear regression, trigonometric and other statistical analysis techniques.

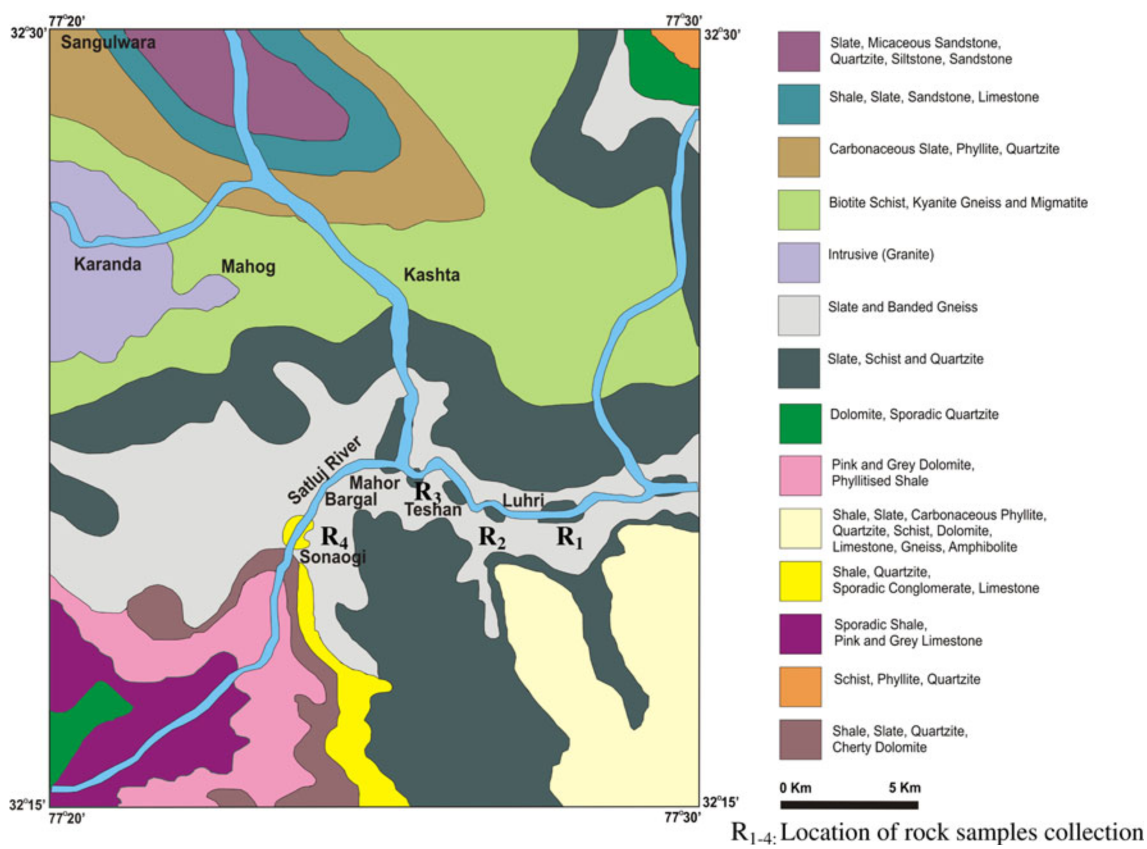

Fig. 1 Geological map of the study area (GSI 1999)

Table 1 Physico-mechanical properties of different rock types

Rock types	P-wave velocity, V_p (m/s)	Point load index, I_s (MPa)	Density, ρ (gm/cm^3)	Slake durability index (SDI %)	Uniaxial compressive strength, σ_C (MPa)	Shear strength, τ (MPa)
Limestone	3,845.35	3.91	2.70	97.98	84.50	16.56
Limestone	3,656.20	3.78	2.68	97.97	82.46	15.48
Limestone	3,548.13	3.62	2.67	97.58	80.25	14.91
Limestone	3,124.58	3.51	2.63	97.35	77.68	14.35
Limestone	3,106.42	3.47	2.60	97.21	77.20	13.89
Limestone	3,089.78	3.25	2.58	97.14	72.98	13.56
Limestone	3,047.13	3.19	2.57	97.01	70.05	13.01
Limestone	2,845.98	3.12	2.54	96.98	69.15	12.96
Limestone	2,679.14	2.87	2.51	96.72	68.84	12.54
Limestone	2,549.63	2.81	2.50	96.65	68.43	12.49
Slate	4,260.12	2.28	2.81	99.84	49.25	10.78
Slate	4,199.09	2.11	2.78	99.71	46.13	9.66
Slate	4,104.56	2.10	2.77	99.70	44.38	8.78
Slate	4,026.97	1.98	2.75	99.65	43.18	8.53
Slate	3,832.05	1.86	2.72	99.45	40.35	8.17
Slate	3,502.15	1.74	2.68	99.36	37.19	7.98
Slate	3,435.69	1.50	2.68	99.25	32.10	7.43
Slate	3,202.15	1.34	2.65	99.11	30.05	6.78
Slate	3,172.76	1.27	2.64	98.98	27.54	6.50
Slate	3,050.38	1.18	2.64	98.86	24.46	5.64
Quartzite	4,225.14	5.27	2.70	99.98	112.25	21.57
Quartzite	4,119.56	5.18	2.69	99.97	108.01	21.46
Quartzite	4,102.37	5.05	2.69	99.97	105.69	21.13
Quartzite	3,910.22	4.95	2.68	99.95	98.43	19.86
Quartzite	3,841.06	4.78	2.65	99.89	98.02	19.12
Quartzite	3,820.25	4.50	2.65	99.87	95.51	18.99
Quartzite	3,695.79	4.37	2.64	99.65	94.96	18.25
Quartzite	3,650.12	4.21	2.64	99.48	94.54	18.06
Quartzite	3,617.35	3.98	2.63	99.29	93.30	17.55
Quartzite	3,521.13	3.75	2.63	99.14	93.21	17.42
Quartz mica schist	2,489.25	1.78	2.69	97.18	28.45	6.78
Quartz mica schist	2,450.64	1.65	2.68	97.05	27.10	5.72
Quartz mica schist	2,302.68	1.42	2.68	96.60	27.02	5.38
Quartz mica schist	2,300.23	1.26	2.66	96.58	24.78	4.89
Quartz mica schist	2,278.45	1.19	2.66	96.57	24.36	4.75
Quartz mica schist	2,265.13	1.14	2.65	96.55	23.18	4.28
Quartz mica schist	2,200.07	1.10	2.64	96.50	22.80	4.17
Quartz mica schist	2,178.60	1.08	2.63	96.47	22.10	4.06
Quartz mica schist	2,145.56	1.05	2.62	96.38	20.58	3.95
Quartz mica schist	2,142.39	1.03	2.62	96.33	20.32	3.80

When data are analyzed using a neural network, it is possible to detect important predictive patterns that were not previously apparent to a non-expert. A particular network can be defined using three fundamental components—transfer function, network architecture and learning law (Simpson 1990)—and will depend on the nature of the problem to be solved.

Network training

A network first needs to be trained before new information can be interpreted. Several different algorithms are available for the training but the back-propagation algorithm is the most versatile and popular. The feed-forward back-propagation neural network (BPNN) consists primarily of

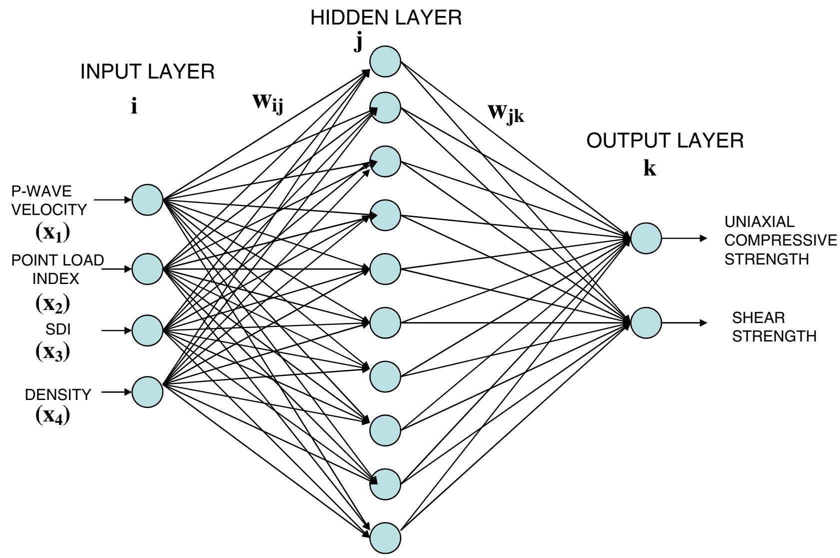

Fig. 2 Three layer feed-forward back-propagation neural net

at least three layers: input, hidden and output. Each layer consists of a number of neurons and each neuron is interconnected to the next layer through weights, i.e., neurons in the input layer send output as input to neurons in the hidden layer, which similarly sends data as input to the output layer. The number of input and output neurons is the same as the number of input and output variables; the number of hidden layers and number of neurons in the hidden layer/s varies according to the complexity of problem.

To differentiate between the different processing units, values called biases are introduced in the transfer functions. These biases are referred to as the temperature of a neuron. Except for the input layer, all neurons in the back-propagation network are associated with a bias neuron and a transfer function. The bias is much like a weight, except that it has a constant input of 1, while the transfer function filters the summed signals received from this neuron. These transfer functions are designed to map the net output of a neuron or layers to the actual output; they are simple step functions—either linear or nonlinear. The application of these transfer functions depends on the purpose of the neural network. The output layer produces the computed output vectors corresponding to the solution.

During training of the network, data are processed through the input layer to the hidden layer/s and then the output layer (forward pass). In this layer, the outputs are compared with the measured values (the true output). The difference between them (or error) is processed back through the network (backward pass), updating the individual weights of the connections and the biases of the individual neurons. The input and output data are generally represented as vectors called training pairs (Khandelwal and Singh 2007). This process is repeated for all the training pairs in the dataset, until the error reaches a threshold minimum defined by a corresponding cost function; usually the root mean squared error (RMSE) or summed squared error (SSE).

In Fig. 2 the jth neuron is connected with a number of inputs

$$X_i = (X_1, X_2, X_3, \ldots X_n).$$

The net input values in the hidden layer will be:

$$\mathrm{Net}_j = \sum_{i=1}^{n} x_i w_{ij} + \theta_j,$$

where x_i is the input unit, w_{ij} the weight on the connection of ith input and jth neuron, θ_j the bias neuron (optional), and n the number of input units. The net output from the hidden layer is calculated using the algorithmic sigmoid function

$$O_j = f(\mathrm{Net}_j) = {}^{1}\!/_{1} + \mathrm{e}^{-(\mathrm{Net}_j + \theta_j)}.$$

The total input to the kth unit is

$$\mathrm{Net}_k = \sum_{j=1}^{n} w_{jk} o_j + \theta_k$$

where, θ_k is the bias neuron, w_{jk} the weight between jth neuron and kth output.Thus the total output from kth unit will be

$$O_k = f(\mathrm{Net}_k).$$

In the learning process, the network is presented with a pair of patterns; an input pattern and a corresponding desired output pattern. The network computes its own output pattern using its (probably incorrect) weights and thresholds and the actual output is compared with the desired output. The error at any output in layer k is

Table 2 Comparison between measured and predicted values by ANN of the output variables

Measured values		Predicted values		Percentage errors	
Uniaxial compressive strength, σ_C (MPa)	Shear strength, τ (MPa)	Uniaxial compressive strength, σ_C (MPa)	Shear strength, τ (MPa)	Uniaxial compressive strength (%)	Shear strength (%)
84.5000	16.5600	85.3110	16.5865	0.9507	0.1597
82.4600	15.4800	82.8870	15.8944	0.5151	2.6075
80.2500	14.9100	80.4107	14.9815	0.1998	0.4774
77.6800	14.3500	77.9082	14.1552	0.2929	1.3764
77.2000	13.8900	77.0143	14.0218	0.2412	0.9397
72.9800	13.5600	72.7144	13.2110	0.3652	2.6414
70.0500	13.0100	70.2402	12.8392	0.2707	1.3301
69.1500	12.9600	69.5840	12.9689	0.6237	0.0685
68.8400	12.5400	68.7385	12.5518	0.1476	0.0938
68.4300	12.4900	67.8404	12.4326	0.8690	0.4618
49.2500	10.7800	51.7282	11.2636	4.7908	4.3422
43.1800	8.5300	43.9148	8.8759	1.6733	3.8970
37.1900	7.9800	36.9826	8.1029	0.5608	1.5168
32.1000	7.4300	32.2499	7.3751	0.4647	0.7438
27.5400	6.5000	27.4499	6.5266	0.3282	0.4073
112.2500	21.5700	111.2001	21.8976	0.9442	1.4960
105.6900	21.1300	106.2914	21.1814	0.5658	0.2425
94.9600	18.2500	95.1510	18.3136	0.2007	0.3475
20.5800	3.9500	21.4475	3.7967	4.0448	4.0374
108.0100	21.4600	108.1099	21.5201	0.0925	0.2791

$$et = t_k - O_k$$

where, t_k is the desired output, and O_k the actual output.The total error function is given by

$$E = 0.5 \sum_{k=1}^{n} (t_k - O_k)^2.$$

Training of the network is basically a process of arriving at optimum weightings in the network. The descent-down error surface is made using the following rule:

$$\nabla W_{jk} = -\eta(\delta E/\delta W_{jk}),$$

where, η is the learning rate parameter, and E the error function. The update of weights for the $(n+1)$th pattern is given as

$$W_{jk}(n+1) = W_{jk}(n) + \nabla W_{jk}(n).$$

Similar logic applies to the connections between the hidden and output layers. This procedure is repeated with each pattern pair assigned for training the network. Each pass through the training cycle is called an epoch. The process is then repeated for as many epochs as needed until the error within the user-specific goal is reached.

A feed-forward BPNN was developed for the prediction of the strength properties of the sample using dynamic wave, point load index, slake durability index and density and comparing the predicted values with the measured ones. The neural network contains four input neurons, two output neurons and one hidden layer with ten hidden neurons. Figure 2 indicates the network used in the study.

Results and discussion

A hundred sets of data were used for the training and testing of the network and 20 randomly selected data which had not been used for training purpose were used for the validation and prediction of the output variables. Table 2 gives the measured and predicted values and the percentage error. It shows that the predicted values are very near the measured ones and, except for one test; the percentage errors are also small.

Figure 3 depicts the performance graph obtained during the training of the network. The percentage error varies between 0.09 and 4.79 in the case of uniaxial compressive strength, whereas for shear strength the error percentage varies between 0.06 and 4.34 (Figs. 4, 5). The graph plotting predicted and determined values indicates a very good correlation ($R = 0.99$) and confirms the applicability of ANN (Figs. 6, 7). This is also indicated in the bar charts

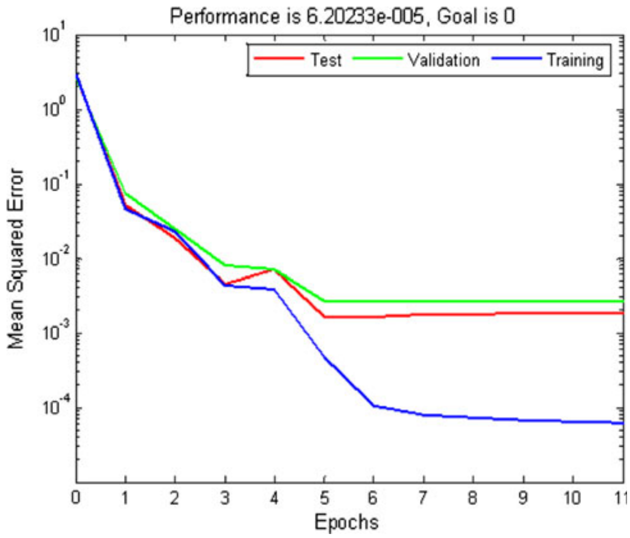

Fig. 3 Graph obtained during training of network

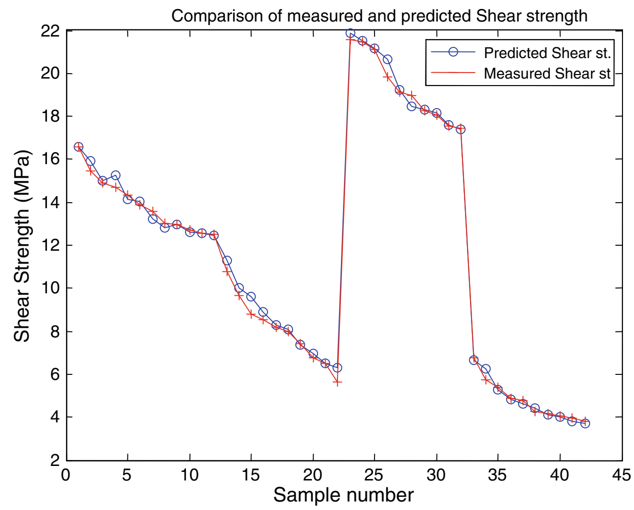

Fig. 5 Comparison of measured and predicted values of shear strength (ANN model)

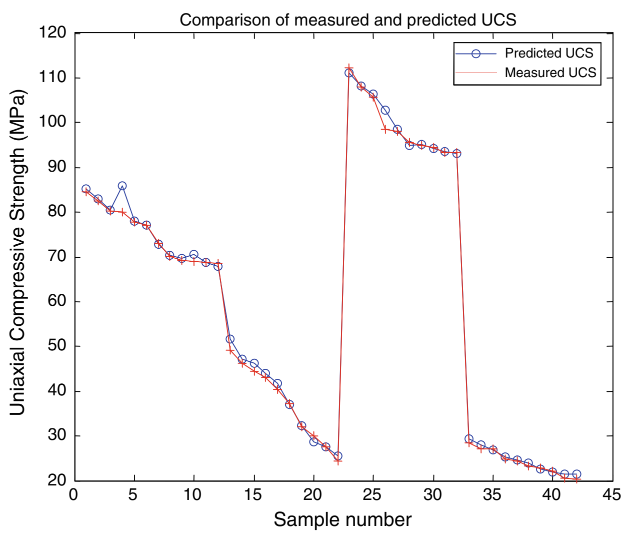

Fig. 4 Comparison of measured and predicted values of UCS (ANN model)

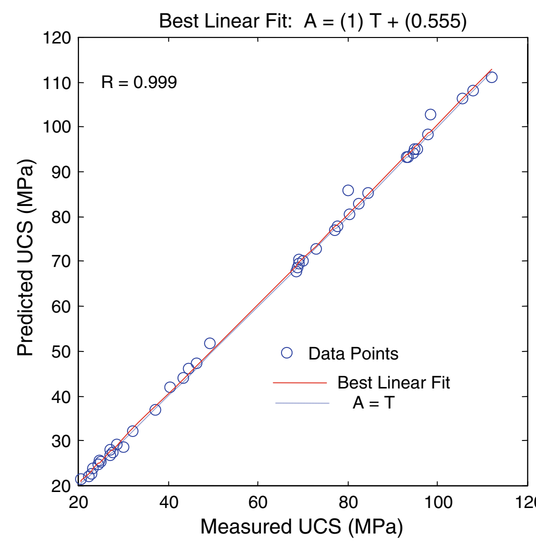

Fig. 6 Correlation between measured and predicted values of UCS (ANN model)

for uniaxial compressive strength and shear strength for different data set shown in Figs. 8, 9.

Meulenkamp and Alvarez (1999) used a neural network model to predict the strength using equotip hardness and reported a correlation coefficient of 0.967 between the UCS predicted using the ANN model and the measured UCS. Singh et al. (2001) also used ANN for the prediction of the strength properties of some schistose rocks based on their petrographic properties. The statistical analyses gave inconsistent correlations with a high degree of error compared with the results using the neural network model. Tiryaki (2008) predicted (UCS) and modulus of elasticity (E) of intact rocks for mechanical excavation. The study indicated that ANN models are more versatile than multiple nonlinear regression models. The R values obtained using ANN model were 0.63 for UCS and 0.71 for modulus of elasticity. Yilmaz and Yuksek (2008) used ANN to determine the UCS and E of gypsum and obtained correlations of $R^2 = 0.93$ for UCS and 0.91 for E. Sarkar et al. (2009) estimated the strength parameters using the dynamic wave for sandstone rock types and obtained a correlation coefficient 0.98 for UCS, again supporting the applicability of ANN for the estimation of strength parameters of rock.

Conclusions

The present study confirms that ANN is a useful tool for predicting rock strengths which are not clearly established

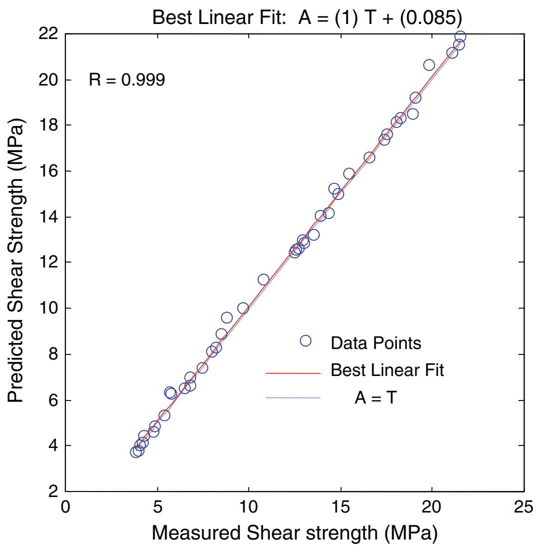

Fig. 7 Correlation between measured and predicted values of shear strength (ANN model)

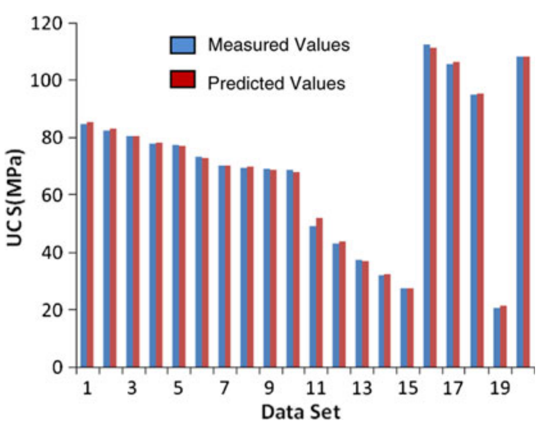

Fig. 8 *Bar chart* of UCS for data set number 1–20

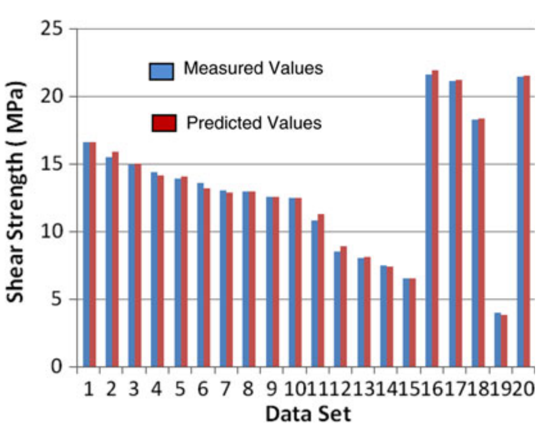

Fig. 9 *Bar chart* of shear strength for data set number 1–20

using empirical relationships, avoiding the subjectivity inherent in finding best fit equations. Using simple input parameters, strong correlations and limited percentage errors were found in the prediction of important parameters including uniaxial compressive strength and shear strength. The work demonstrates that this approach can save time and cost and it is hoped that it will encourage further development of this method for predicting rock strength parameters.

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