



Pattern formation due to non-linear vortex diffusion

Rinke J. Wijngaarden^a, R. Surdeanu^a, J.M. Huijbregtse^a, J.H. Rector^a, B. Dam^a, J. Einfeld^b,
R. Wördenweber^b and R. Griessen^a

^aDivision of Physics and Astronomy, Faculty of Sciences, Vrije Universiteit, De Boelelaan 1081,
NL-1081 HV Amsterdam, The Netherlands

^bInstitut für Schicht- und Ionentechnik (ISI), Forschungszentrum Jülich, D-52425 Jülich, Germany

Penetration of magnetic flux in $\text{YBa}_2\text{Cu}_3\text{O}_7$ superconducting thin films in an external magnetic field is visualized using a magneto-optic technique. A variety of flux patterns due to non-linear vortex diffusion is observed: (1) Roughening of the flux front with scaling exponents identical to those observed in burning paper including two distinct regimes where respectively spatial disorder and temporal disorder dominate. In the latter regime Kardar-Parisi-Zhang behavior is found. (2) Fractal penetration of flux with Hausdorff dimension depending on the critical current anisotropy. (3) Penetration as 'flux-rivers'. (4) The occurrence of commensurate and incommensurate channels in films with anti-dots as predicted in numerical simulations by Reichhardt, Olson and Nori. It is shown that most of the observed behavior is related to the non-linear diffusion of vortices by comparison with simulations of the non-linear diffusion equation appropriate for vortices.

1. Introduction

It has gradually become clear that vortices penetrating thin films of high- T_c superconductors exhibit a whole range of interesting phenomena, which are very similar to those observed in other systems displaying non-linear diffusion. Although it was known in principle that flux penetration is described by non-linear diffusion of vortices, it was not realized until very recently, however, that small inhomogeneities in the sample or small thermal fluctuations may lead to large spatial deviations in the vortex penetration process. Consequently, a whole range of interesting patterns can be observed in vortex matter. Here we compare fractal penetration, roughening of flux fronts and profiles and river-like vortex penetration in thin films.

2. Experimental

The local magnetic field in the superconducting thin films is measured using a magneto-optical technique[1]. Placed directly on top of the sample is a Bi-doped YIG film with in plane anisotropy, which has a very large Faraday effect. The sample assembly is placed on a specially designed holder, which is inserted in a commercial 1 T magnet sys-

tem with cryostat. The set-up is optically equivalent to a cryogenic polarisation microscope, and the Faraday rotation due to the local magnetic field in the superconductor is transformed into an intensity image. From this image the local magnetic field can be derived[2].

3. Fractal Flux penetration

Fractal flux penetration was first discussed by Surdeanu et al. [3] in $\text{Ti}_2\text{Ba}_2\text{CuO}_{6+x}$ thin films. A typical example of such behaviour is shown in fig. 1.

It is found that the fractal exponent (or Hausdorff dimension) is about 1.2 for films where the in-plane critical currents are isotropic, while this value decreases towards 1.0 with increasing in-plane anisotropy. The anisotropy can be varied in these films by changing the miscut angle of the substrate. In $\text{YBa}_2\text{Cu}_3\text{O}_7$ thin films on NdGaO_3 a similar behavior is observed, while the anisotropy is changed by the annealing procedure.

4. Kinetic roughening of flux fronts

In other $\text{YBa}_2\text{Cu}_3\text{O}_7$ thin films on NdGaO_3 kinetic roughening of flux fronts was found. The flux front is defined as the line which separates the

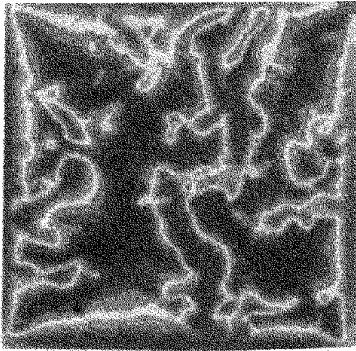


Figure 1. Grey-tone magneto-optical image of flux penetrating into a square $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+x}$ thin film with isotropic in-plane critical currents [3]. For a color version see ref. [4].

vortex free (Meissner) region in the sample from the region which contains vortices. Figure 2 is a three-dimensional representation of the vortex density. At the edge of the sample the intensity is highest due to the thin film geometry.

Experimentally, the flux front f is defined as the intersection of this intensity landscape L with a threshold plane H at a level t equal to three times the standard deviation of the intensity in the Meissner region. The resulting front is indicated in white in figure 2 and is also shown in figure 3a.

The front is clearly not smooth and while it is penetrating further into the superconductor becomes increasingly rougher. Analysis of such fronts [5] in terms of the roughness exponent α shows that there are clearly two regimes. At the initial stage of flux penetration, we find $\alpha = 0.64$ while at later stages $\alpha = 0.46$ is observed. The first regime is characterized by a dominant influence of the static disorder in the sample. Indeed we find $\alpha = 0.64$ very close to the value $\alpha = 0.66$ for the Directed Percolation Depinning model, which is based on pure static disorder. At later stages during flux penetration the cumula-

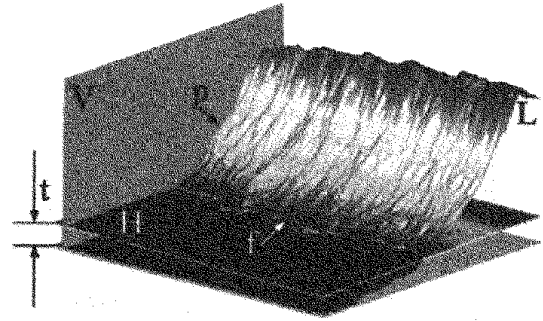


Figure 2. Three-dimensional representation of the vortex density in a $\text{YBa}_2\text{Cu}_3\text{O}_7$ film on NdGaO_3 [5]. The high ridge corresponds to the edge of the sample, the 'plain' in the foreground to the Meissner region. For a color version see ref. [4].

tive effect of random thermal fluctuations takes over and the system becomes more Kardar-Parisi-Zhang-like [6] where $\alpha = 0.50$ is expected.

It is very striking that the same two regimes with the same two exponents were also found for the slow combustion of paper [7], [8]. Indeed the edge of burned paper, fig. 3b is very similar to the edge of the Meissner region, fig 3a. Also it turns out that the growth exponents β for the two systems are approximately equal.

5. Theory

From Maxwell's relation $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ with in the flux creep regime $\vec{E} = \rho(j) \vec{j} = \rho_f \vec{j} e^{-\frac{U(j)}{kT}}$, where $\rho(j)$ is the current dependent resistivity, ρ_f is the flux-flow resistivity and $U(j)$ is the current dependent activation energy for vortex hopping, one can derive the following non-linear equation for one dimensional flux penetration in very thick superconductors:

$$\frac{\partial \rho(\vec{r}, t)}{\partial t} = \rho a(j) \frac{\partial^2 \rho(\vec{r}, t)}{\partial x^2} + b(j) \left[\frac{\partial \rho(\vec{r}, t)}{\partial x} \right]^2 \quad (1)$$

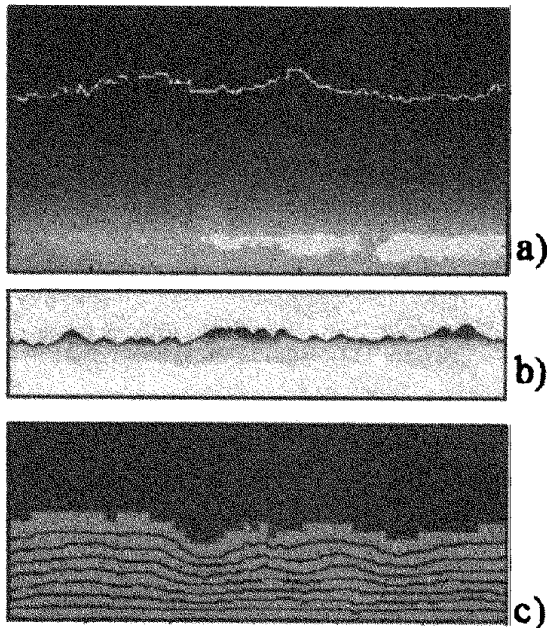


Figure 3. Images of rough edges developping during (a) the penetration of vortices in a superconducting thin film (the fluxfront is the thin white line), (b) the slow combustion of paper and (c) during a numerical simulation. For a color version see ref. [4].

The coefficients $a(j)$ and $b(j)$ depend only weakly on the form of $U = U(j)$ and are approximately constant[5]. Hence this equation is similar to the porous medium equation (for which $a(j) \equiv b(j)$), which is in turn similar to the KPZ equation:

$$\frac{\partial h(\vec{r}, t)}{\partial t} = \nu \nabla^2 h(\vec{r}, t) + \frac{\lambda}{2} [\nabla h(\vec{r}, t)]^2 + \eta(\vec{r}, t) \quad (2)$$

where $\eta(\vec{r}, t)$ is a noise term. In any case, the resistivity obeys a nonlinear diffusion equation. In the flux flow regime, it is found that the local magnetisation (or vortex density) obeys a porous medium equation, too.

Although in the case of thin films there are modifications necessary due to the geometry, the KPZ equation may still be a reasonable model, since it is the lowest order non-linear diffusion equation.

6. Simulations

To investigate the consequences of eq. 1, we started numerical simulations. The properties of the sample are taken into account by using $\rho = \rho_0 |j/j_0|^\sigma$. Here ρ_0 is position dependent and characterizes the inhomogeneities of the sample and $|j/j_0|^\sigma$ is a numerically easy way to include the highly non-linear nature of the current voltage characteristics. A typical front obtained in this way is shown in fig. 3c. The roughness exponents obtained from these preliminary simulations are with $\alpha \approx 0.4$ slightly smaller than those obtained in the real experiment.

7. Rough profile

Evidently, the flux density landscape as shown in fig. 2 contains much more information than just the shape of the flux front. In particular it is interesting to compare its crosssection p with a vertical plane V with the shape that is observed for rice that is carefully poured between two glass plates[9]. For the superconductor we find a surface fluctuation roughness of ~ 0.25 in good agreement with experiments on rice.

8. Antidots

To investigate the possible effects of a regular pinning array on flux penetration and roughening, we studied a $\text{YBa}_2\text{Cu}_3\text{O}_7$ film with a triangular lattice of $2 \mu\text{m}$ diameter antidots spaced at $10 \mu\text{m}$. To enhance the contrast at the flux front, we show in fig. 4 the difference in measured intensity for 32 mT and 30 mT external field. The flux is penetrating from top to bottom along the anti-dots rows, the direction is indicated by the long black arrow. Interestingly, the flux penetration is not uniform for each row of anti-dots: there is a preferential penetration along particular rows (shown brighter; one of these is along the long black arrow), leading to an enhanced

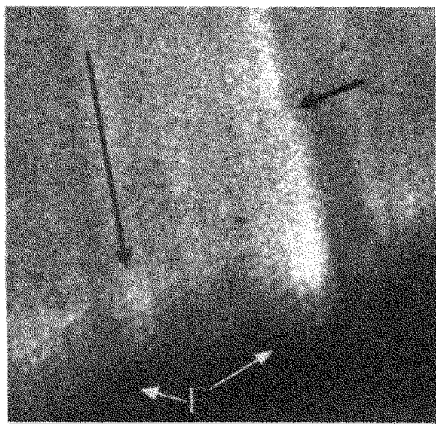


Figure 4. Flux penetration in a $\text{YBa}_2\text{Cu}_3\text{O}_7$ film with a square antidot lattice. The antidot spacing is $10\ \mu\text{m}$. For a color version see ref. [4].

progression of the flux front (at F) at the end of these rows hence enhancing roughening. Note also the kink at the short black arrow, where the stream of entering vortices hops over to an adjacent row. These phenomena are very similar to observations based on numerical simulations by Nori et al. [10] and will be discussed more extensively elsewhere [11].

9. Rivers

Occasionally we also observe flux penetration, which is very similar to a river network draining towards an outlet. This type of flux penetration seems to be correlated to the presence of weak links between the grains in the film.

10. Discussion

In conclusion, we have found a whole range of interesting patterns which develop during flux penetration in thin films with various microstructure. Remarkably, from X-Ray scattering we find that films with the best epitaxy show the clearest roughening and that with decreasing epitaxy the

flux penetration becomes smoother.

Furthermore we have found that the superconducting vortex system is an ideal system to study roughening phenomena, a.o. because the experiments are repeatable, unlike experiments on paper or on porous media.

This work is part of the research program of the Stichting Fundamenteel Onderzoek der Materie (FOM), which is financially supported by the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

REFERENCES

1. M.R. Koblishka and R.J. Wijngaarden, *Supercond. Sci. and Technol.* **8** (1995) 199
2. R.J. Wijngaarden, H.J.W. Spoelder, R. Surdeanu and R. Griessen, *Phys Rev B* **54** (1996) 6742
3. R. Surdeanu, R.J. Wijngaarden, B. Dam, J. Rector, R. Griessen, C. Rossel, Z.F. Ren and J.H. Wang, *Phys Rev B* **58** (1998) 12467
4. For color figures see: <http://www.nat.vu.nl/~vstof/colfigs/2000/rw-mms.htm>
5. R. Surdeanu, R.J. Wijngaarden, E. Visser, B. Dam, J. Rector and R. Griessen, *Phys. Rev. Lett.* **83** (1999) 2054
6. M. Kardar, G. Parisi, Y.-C. Zhang, *Phys. Rev. Lett.* **56** (1986) 889
7. J. Maunukela, M. Myllys, O.-P. Kähkönen, J. Timonen, N. Provatas, M.J. Alava, T. Ala-Nissila, *Phys. Rev. Lett.* **79** (1997) 1515
8. L.A.N. Amaral and H.A. Makse, *Phys. Rev. Lett.* **80** (1998) 5706
9. A. Mølthe-Sørensen, J. Feder, K. Christensen, V. Frette, T. Tøssang, *Phys. Rev. Lett.* **83** (1999) 764
10. C. Reichhardt, C.J. Olson and F. Nori, *Phys. Rev. B* **58** (1998) 6534
11. R. Surdeanu, R.J. Wijngaarden, J. Einfeld, R. Wördenweber and R. Griessen, submitted for publication.