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DETERMINATION OF THE STEADY STATE RESPONSE OF A VISCOELASTICALLY POINT-SUPPORTED RECTANGULAR PLATE

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The steady state response to a sinusoidally varying force is determined for a viscoelastically point-supported square or rectangular plate. For this purpose, the transverse deflection of the plate is written in a series of the product of the deflection functions of beams parallel to the edges, and the response equation is derived by the generalized Galerkin method. The natural boundary conditions of the plate which cannot be satisfied by the beam functions at the edges and the corners are appropriately compensated by suitable additions to the residual forces and moments. The method is applied to a square plate supported at four points symmetrically located at the corners or on the diagonals; the steady state response of the plate to a point force acting at the centre is calculated numerically, and the effects of the point supports on the vibration are studied.

1. INTRODUCTION

This paper presents an analysis of the steady state vibration of a viscoelastically point-supported square or rectangular plate in response to a sinusoidally varying force. Recently, point-supported plates have been extensively used in many industrial applications such as in shakers, machine foundations, thermostructures, etc. Therefore, the vibration problems of these plates have assumed practical importance and have been studied by many researchers. The vibration of square or rectangular plates simply supported at the corners was studied by Cox and Boxer [1], Kirk [2], and Reed [3], and plates supported at four points on the diagonals or the edges by Tso [4], Johns *et al.* [5, 6], Dowell [7], Mirza and Petyt [8, 9], Amba-Rao *et al.* [10–13] and Gorman [14, 15]. A multiple point-supported plate was studied by Kerstens [16]; a point-supported skew or rectangular orthotropic plate by Srinivasan and Munaswamy [17] and Narita [18], and an elastically point-supported plate by Leuner [19] and Laura and Gutiérrez [20]. However, these studies have all been confined to the free vibration of simply or elastically point-supported plates, and no papers have been presented giving results for the response of the viscoelastically point-supported plate reported here, except for the study of Das and Navaratna [21], who considered the free vibration of a viscoelastically point-supported plate.

For the purpose of this study, the transverse deflection of the point-supported plate is expressed in a series of the eigenfunctions of vibration of beams parallel to the edges, and the response equation is derived by use of the generalized Galerkin method proposed by Bassily and Dickinson [22]. In the method, the natural boundary conditions which the beam functions do not satisfy at the corners and the edges are compensated by suitable additions to the residual forces and moments acting there.

By the application of the method, the transverse deflection of the plate and the force transmissibility at the supports under the action of a point force have been calculated

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numerically for a square plate viscoelastically supported at four points symmetrically located at the corners or on the diagonals, and the results are presented in some illustrations.

2. ANALYSIS

Figure 1 shows a viscoelastically point-supported rectangular plate subjected to an external force. With the lengths of the two edges denoted by a and b, the Cartesian co-ordinates (x, y) are taken in the neutral surface of the plate as shown in the figure.

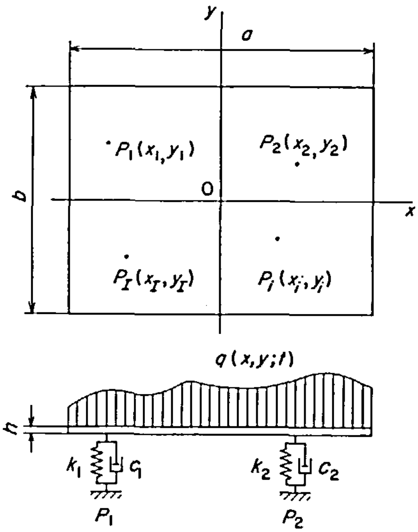

Figure 1. Viscoelastically point-supported rectangular plate subjected to an external force.

The equation of flexural vibration of the point-supported plate subjected to an external force $q(x, y; t)$ is expressed as

$$D\nabla^4 w+\sum_{i=1}^{I}\left(k_i w+c_i\frac{\partial w}{\partial t}\right)\delta(x-x_i)\,\delta(y-y_i)+\rho h\frac{\partial^2 w}{\partial t^2}=q(x,y;t), \tag{1}$$

in terms of the transverse deflection w, where ρ is the mass density, h is the plate thickness, and $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ denotes the Laplacian operator. The quantity D is the flexural rigidity of the plate expressed as $D=Eh^3/12(1-\nu^2)$, in terms of the Young's modulus E and Poisson's ratio ν. The constants k_i and c_i, respectively, are the spring constant and the damping coefficient of a point support $P_i(x_i, y_i)$, and $\delta(x-x_i)$ and $\delta(y-y_i)$ denote Dirac delta functions.

The steady state response (the transverse deflection) of the plate to a sinusoidally varying force $q=Q(x, y)\,\mathrm{e}^{\mathrm{j}\omega t}$ can be written as

$$w=W(x,y)\,\mathrm{e}^{\mathrm{j}\omega t}. \tag{2}$$

In this paper, $W(x, y)$ is a complex variable containing a phase angle. For simplicity of analysis, the following dimensionless variables are introduced:

$$\xi=x/a, \qquad \eta=y/b, \qquad \alpha=a/b\ \text{(side ratio)}, \qquad \bar{\nabla}^2=\partial^2/\partial\xi^2+\alpha^2\,\partial^2/\partial\eta^2,$$

$$F=Q(a^4/D),$$

$\kappa_i = k_i(a^3/bD)$ (stiffness parameter), $\gamma_i = c_i(a/b\sqrt{\rho hD})$ (damping parameter),

$$\lambda^2 = \rho h a^4 \omega^2/D \text{ (frequency parameter).} \tag{3}$$

Equation (1) can be rewritten as

$$L(W) = \bar{\nabla}^4 W(\xi, \eta) + \sum_{i=1}^{I} (\kappa_i + \mathrm{j}\gamma_i\lambda) W(\xi, \eta)\delta(\xi - \xi_i)\delta(\eta - \eta_i) - \lambda^2 W(\xi, \eta) - F(\xi, \eta) = 0. \tag{4}$$

The boundary conditions at the free edges are

$$M_x = -D(\partial^2 W/\partial x^2 + \nu\, \partial^2 W/\partial y^2) = 0,$$
$$V_x = -D\{\partial^3 W/\partial x^3 + (2-\nu)\, \partial^3 W/\partial x\, \partial y^2\} = 0 \qquad \text{at } x = \pm a/2,$$
$$M_y = -D(\partial^2 W/\partial y^2 + \nu\, \partial^2 W/\partial x^2) = 0,$$
$$V_y = -D\{\partial^3 W/\partial y^3 + (2-\nu)\, \partial^3 W/\partial x^2\, \partial y\} = 0 \qquad \text{at } y = \pm b/2. \tag{5}$$

When no point supports are located at the corners, the reaction forces must be also zero:

$$R = 2D(1-\nu)\, \partial^2 W/\partial x\, \partial y = 0. \tag{6}$$

Since the solution of equation (1) cannot be obtained analytically under the boundary conditions, the generalized Galerkin method is conveniently adopted here, with the eigenfunctions of unconstrained free beams parallel to the edges used as admissible functions. The transverse deflection of the plate is approximately expressed as

$$W(\xi, \eta) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} X_m(\xi) Y_n(\eta). \tag{7}$$

The normalized eigenfunctions $X_m(\xi)$ of flexural vibration of a free beam are given by

$$X_1(\xi) = 1, \qquad X_2(\xi) = 2\sqrt{3}\xi,$$
$$X_m(\xi) = \sqrt{2/\{\cosh^2(\beta_m/2) + \cos^2(\beta_m/2)\}}\, \{\cosh(\beta_m/2)\cos(\beta_m\xi)$$
$$+ \cos(\beta_m/2)\cosh(\beta_m\xi)\}, \qquad m = 3, 5, 7, \ldots,$$
$$X_m(\xi) = \sqrt{2/\{\sinh^2(\beta_m/2) - \sin^2(\beta_m/2)\}}\, \{\sinh(\beta_m/2)\sin(\beta_m\xi)$$
$$+ \sin(\beta_m/2)\sinh(\beta_m\xi)\}, \qquad m = 4, 6, 8, \ldots, \tag{8}$$

where the parameters β_m are obtained by calculating numerically the positive roots of the equation

$$\tan(\beta/2) \pm \tanh(\beta/2) = 0, \qquad m = \begin{Bmatrix} 3, 5, 7, \ldots \\ 4, 6, 8, \ldots \end{Bmatrix}. \tag{9}$$

The eigenfunctions $Y_n(\eta)$ are given by expressions of the same forms as equations (8). However, these assumed functions of a free beam satisfy only the edge conditions

$$\partial^2 W/\partial x^2 = \partial^3 W/\partial x^3 = 0 \qquad \text{at } x = \pm a/2,$$
$$\partial^2 W/\partial y^2 = \partial^3 W/\partial y^3 = 0 \qquad \text{at } y = \pm b/2, \tag{10}$$

and other edge and corner conditions are completely ignored. Therefore, the following residual forces and moments must be added to the plate equation:

$$\begin{aligned}(M_x)_r &= D\nu\, \partial^2 W/\partial y^2, \qquad (V_x)_r = D(2-\nu)\, \partial^3 W/\partial x\, \partial y^2 \qquad \text{at } x=\pm a/2,\\ (M_y)_r &= D\nu\, \partial^2 W/\partial x^2, \qquad (V_y)_r = D(2-\nu)\, \partial^3 W/\partial x^2\, \partial y \qquad \text{at } y=\pm b/2,\\ (R)_r &= 2D(1-\nu)\, \partial^2 W/\partial x\, \partial y \qquad \text{at } x=\pm a/2 \text{ and } y=\pm b/2. \end{aligned} \tag{11}$$

As the results, the loading terms to be compensated are given by

$$\begin{aligned} L_r(W) = \alpha^2 [\![& \nu(\partial/\partial\xi)[(\partial^2 W/\partial\eta^2)\{\delta(\xi+\tfrac{1}{2})-\delta(\xi-\tfrac{1}{2})\}]\\ &+(2-\nu)(\partial^3 W/\partial\xi\,\partial\eta^2)\{\delta(\xi+\tfrac{1}{2})-\delta(\xi-\tfrac{1}{2})\}\\ &+\nu(\partial/\partial\eta)[(\partial^2 W/\partial\xi^2)\{\delta(\eta+\tfrac{1}{2})-\delta(\eta-\tfrac{1}{2})\}]\\ &+(2-\nu)(\partial^3 W/\partial\xi^2\,\partial\eta)\{\delta(\eta+\tfrac{1}{2})-\delta(\eta-\tfrac{1}{2})\}\\ &+2(1-\nu)(\partial^2 W/\partial\xi\,\partial\eta)\{\delta(\xi+\tfrac{1}{2})-\delta(\xi-\tfrac{1}{2})\}\{\delta(\eta+\tfrac{1}{2})-\delta(\eta-\tfrac{1}{2})\}]\!], \end{aligned} \tag{12}$$

and the equation of vibration of the plate becomes

$$L(W)+L_r(W)=0. \tag{13}$$

In the Galerkin orthogonalization process, the following equations are derived:

$$\begin{aligned} &\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} L\left\{\sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}X_m(\xi)Y_n(\eta)\right\}X_k(\xi)Y_l(\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta\\ &\quad+\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} L_r\left\{\sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}X_m(\xi)Y_n(\eta)\right\}X_k(\xi)Y_l(\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta=0,\\ &\qquad\qquad k=1,2,\ldots,M;\ l=1,2,\ldots,N. \end{aligned} \tag{14}$$

These can be also written as

$$\begin{aligned} &\sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}\Big\{(\beta_m^4+\alpha^4\beta_n^4)\delta_{mk}\delta_{nl}+2(1-\nu)\alpha^2[I_{11}]_m^{(k)}[J_{11}]_n^{(l)}\\ &\quad+\nu\alpha^2([I_{20}]_m^{k}[J_{02}]_n^{(l)}+[I_{02}]_m^{(k)}[J_{20}]_n^{(l)})\\ &\quad+\sum_{i=1}^{I}(\kappa_i+\mathrm{j}\gamma_i\lambda)X_m(\xi_i)X_k(\xi_i)Y_n(\eta_i)Y_l(\eta_i)-\lambda^4\delta_{mk}\delta_{nl}\Big\}\\ &=\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} F(\xi,\eta)X_k(\xi)Y_l(\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta, \qquad k=1,2,\ldots,M;\ l=1,2,\ldots,N, \end{aligned} \tag{15}$$

where δ_{mk} and δ_{nl} denote Kronecker's delta, and $[I_{pq}]_m^{(k)}$ and $[J_{pq}]_n^{(l)}$, respectively, are the defintie integrals

$$[I_{pq}]_m^{(k)}=\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{\mathrm{d}^pX_m}{\mathrm{d}\xi^p}\frac{\mathrm{d}^qX_k}{\mathrm{d}\xi^q}\mathrm{d}\xi, \qquad [J_{pq}]_n^{(l)}=\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{\mathrm{d}^pY_n}{\mathrm{d}\eta^p}\frac{\mathrm{d}^qY_l}{\mathrm{d}\eta^q}\mathrm{d}\eta. \tag{16}$$

Equation (15) is a set of linear equations in the unknown coefficients A_{mn}. The transverse deflection of the plate is determined from equation (7) by calculating the unknown coefficients. The magnitude of the reaction force $F_T\,\mathrm{e}^{\mathrm{j}\omega t}$ at point supports is given by

$$F_T=\sum_{i=1}^{I}(k_i+\mathrm{j}c_i\omega)W(x_i,y_i), \tag{17}$$

and therefore the force transmissibility at the supports is determined by

$$T_R = \frac{1}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} F(\xi, \eta)\, d\xi\, d\eta} \sum_{m=1}^{M} \sum_{n=1}^{n} A_{mn} \sum_{i=1}^{I} (\kappa_i + j\gamma_i\lambda) X_m(\xi_i) Y_n(\eta_i). \quad (18)$$

3. NUMERICAL RESULTS AND DISCUSSION

The present method can be applied to square or rectangular plates viscoelastically supported at many points under the action of any distributed forces. In this section, the steady state response to a point force $F(\xi, \eta) = F_0\, \delta(\xi)\, \delta(\eta)$ acting at the centre is calculatd numerically for a square plate viscoelastically supported at four points symmetrically located at the corners or on the two diagonals, where the parameters κ_i and γ_i are taken to have the same respective values at all the supports denoted by $\kappa_i = \kappa_s$ and $\gamma_i = \gamma_s$.

A brief explanation of the free vibration of an elastically point-supported plate is necessary in order for the response presented to be easily understood. The natural frequencies (the frequency parameters) of the plate are determined by calculating the eigenvalues λ of the frequency equation obtained by taking the damping parameter of the supports as $\gamma_s = 0$ and the forcing function as $F(\xi, \eta) = 0$ in equation (15), and the mode shapes of vibration are determined from equation (7) by calculating the eigenvectors corresponding to the eigenvalues.

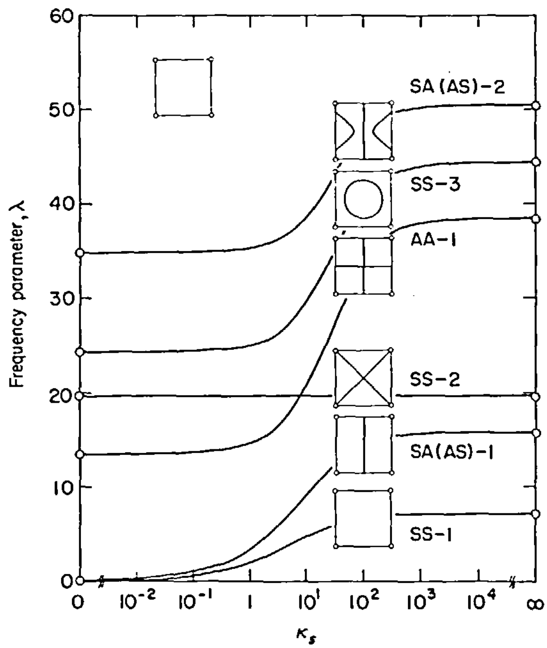

Figure 2. Frequency parameters of square plates elastically supported at the corners. $\nu = 0{\cdot}3$.

Figure 2 shows the frequency parameters λ versus the stiffness parameters κ_s of the supports for a square plate elastically supported at the corners. In a plate with structural symmetry, only symmetrical and antisymmetrical vibrations arise with respect to the two centre lines parallel to the edges. In this case, the natural frequencies and the dynamic responses can be calculated by taking only the odd or even terms in the series of equation

(7) corresponding to symmetrical or antisymmetrical vibration, respectively. The symbols SS and AA, respectively, represent symmetrical and antisymmetrical vibrations with respect to the centrelines. The symbol SA or AS represents symmetrical vibration with respect to one centreline and antisymmetrical vibration with respect to the other. For a square plate with structural symmetry, the SA- and AS-type vibrations have the same frequency parameters (degeneration of frequencies). The circled values on the ordinates at $\kappa_s = 0$ and ∞, respectively, represent the frequency parameters of an unconstrained free plate and a simply point-supported plate. With an increase of the parameters κ_s, the frequency parameters monotonically increase and ultimately become the values of a simply point-supported plate except for only the SS-2 vibration. Nodal lines arising in the SS-2 vibration coincide with the diagonals passing through the supports as seen in the mode shapes (the nodal patterns) of the figure, and therefore the frequency parameter remains constant without being affected by the variation of the stiffness parameters.

Figures 3 and 4, respectively, show the transverse deflections of the centre and the force transmissibility at the supports for a square plate viscoelastically supported at the corners and driven at the centre. In this case, only the SS-type vibrations arise in the plate because of the symmetry of the external force and the location of the supports. Within the frequency range of the figures, two resonant peaks appear at the natural frequencies of the SS-1 and SS-3 vibrations, and also antiresonant peaks or lowest values appear between adjacent resonant frequencies. The solid lines ($\gamma_s = 0$) represent the response curve of a plate with undamped elastic point supports, and the broken lines ($\gamma_s = \infty$) a plate with simple point supports. The two points of intersection P and Q of these two lines are fixed points, through which all the response curves pass, regardless of the damping parameters. The values λ corresponding to the points P and Q in the two figures are a little different from each other as seen in equation (17). Within a certain range of the frequencies, the force transmissibilities are less than unity, which indicates

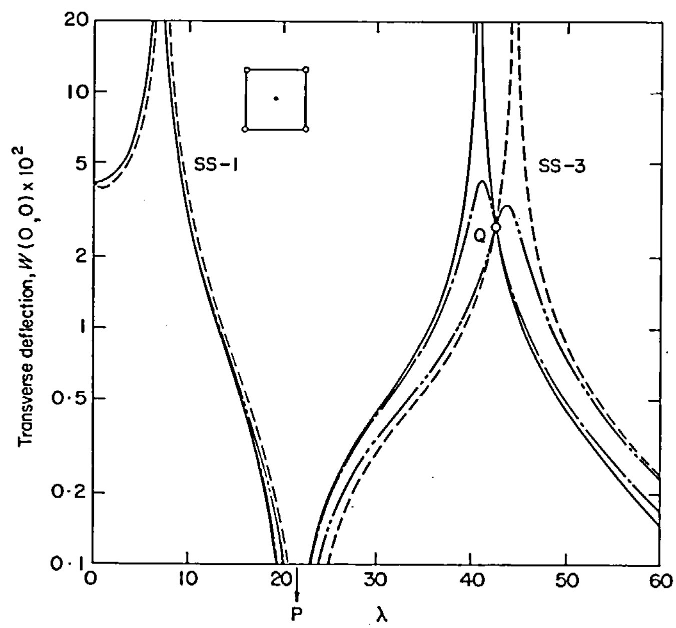

Figure 3. Transverse deflections of the centre of square plates viscoelastically supported at the corners under the action of a centre-force. $\nu = 0{\cdot}3$; $\kappa_s = 100$. γ_s: ——, 0; — - —, 1; — - - —, 5; - - - - -, ∞.

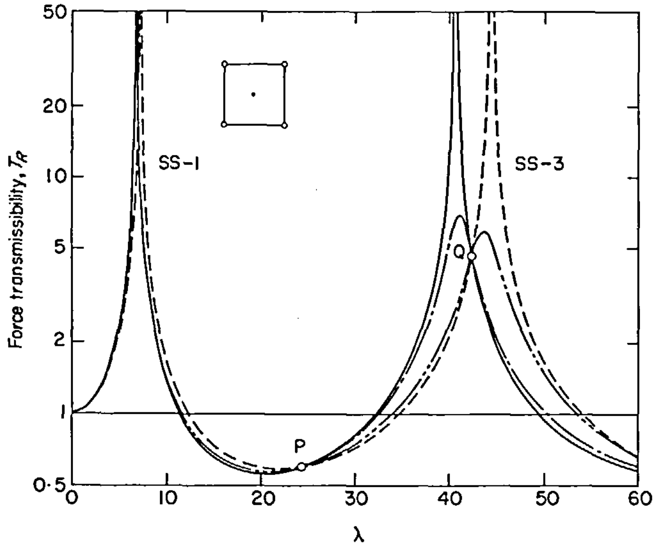

Figure 4. Force transmissibilities of square plates viscoelastically supported at the corners under the action of a centre-force. $\nu = 0{\cdot}3$; $\kappa_s = 100$. γ_s: ——, 0; —·—, 1; —--—, 5; ------, ∞.

the possibility of vibration isolation. The SS-2 vibration with the nodal lines passing through the supports does not arise in the plate.

Figure 5 shows the frequency parameters λ versus the location of the point supports for a square plate with undamped supports symmetrically located on the diagonals, where (ξ_s, η_s) denote the absolute values of the co-ordinates of all the supports, and the circled

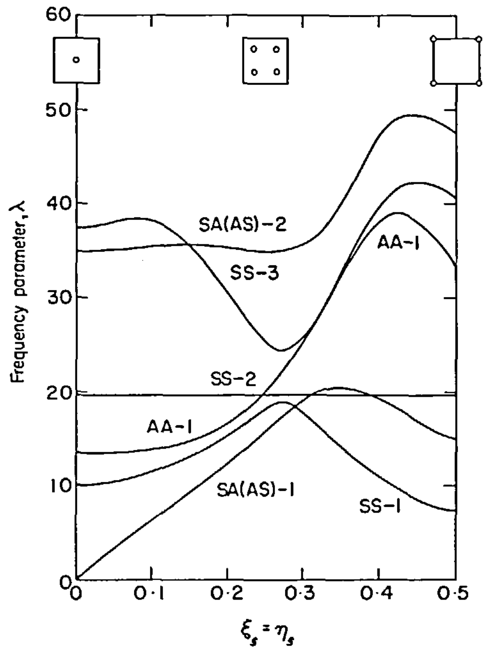

Figure 5. Frequency parameters of square plates elastically supported at four points symmetrically located on the diagonals. $\nu = 0{\cdot}3$; $\kappa_s = 100$.

points inside each square at the top of the figure indicate the location of the supports. With the variation of the location of the supports, the frequency parameters change in a wave-like manner, except for the SS-2 vibration. If the plate were clamped at any points without being simply supported, the frequency parameter of the SA(AS)-1 vibration would not approach zero as ξ_s and η_s approached zero, but would approach the value of the plate with a fully clamped, central-point support.

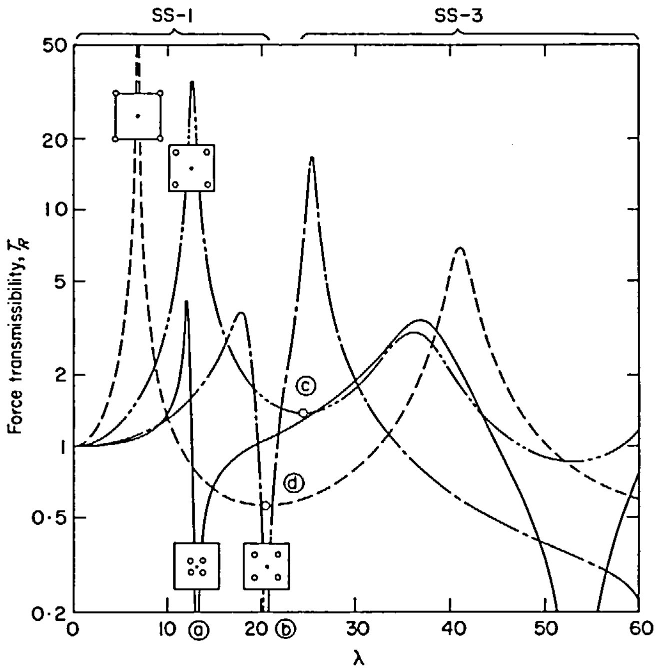

Figure 6. Force transmissibilities of square plates viscoelastically supported at four points symmetrically located on the diagonals under the action of a centre-force. $\nu = 0{\cdot}3$; $\kappa_s = 100$; $\gamma_s = 1$. $\xi_s = \eta_s$: ——, 1/8; — - —, 2/8; — - - —, 3/8; - - - - -, 4/8.

Figure 6 shows the force transmissibilities at the supports for viscoelastically point-supported plates driven at the centre. The curves are considerably affected by the locations of the supports. If the locations of the supports are appropriately chosen, certain resonant peaks disappear and wide frequency ranges where the transmissibilities become small, resulting in vibration isolation, can be obtained.

Figure 7 shows the transverse deflections of the plates at the resonant SS-1 and SS-3 peaks and the antiresonant ones ⓐ, ⓑ, ⓒ and ⓓ shown in Figure 6. The broken lines represent the contour lines of the transverse deflections where the maximum values are taken to be 100.

To examine the accuracy of the numerical results obtained here, the frequency parameters of a completely free and a corner point-supported square plate obtained by the method are compared in Table 1 with the values obtained by other authors [18, 23]. In the calculation of this paper, 6×6 terms of the series were used for each type of vibration. The values of the present authors are in good agreement with those of other authors.

The numerical computations presented here were carried out on a HITAC M-280H computer of the Hokkaido University Computing Center.

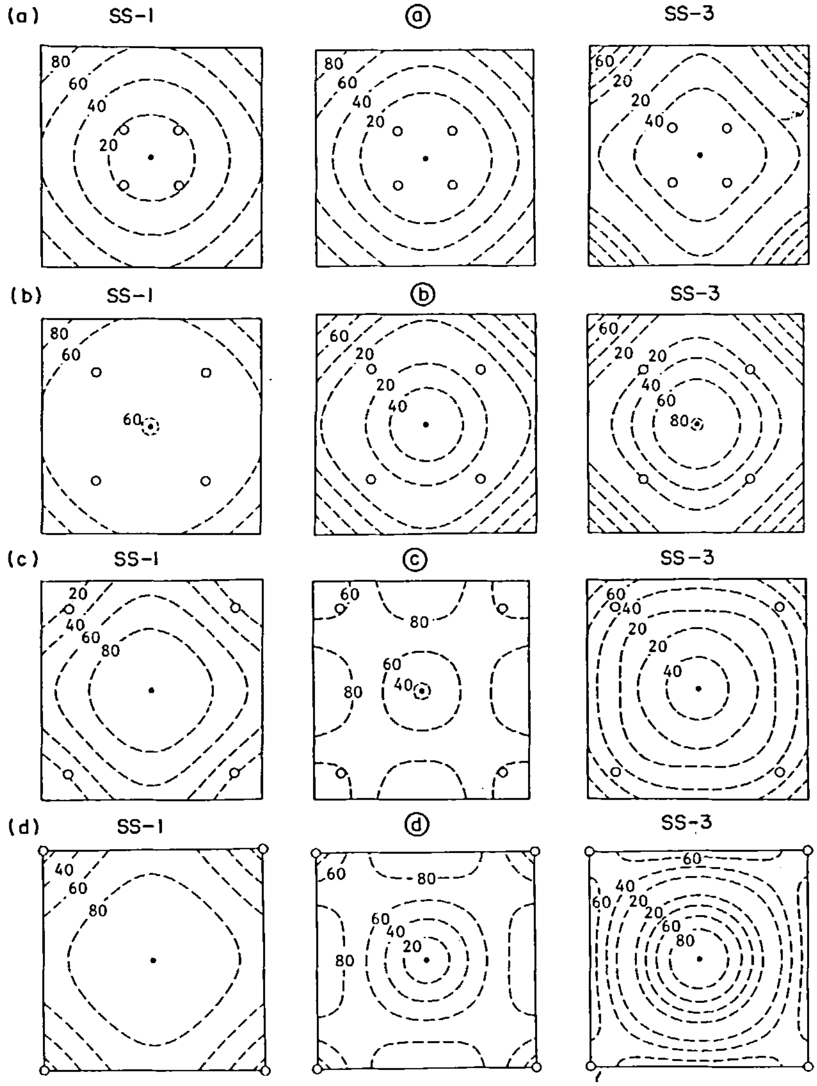

Figure 7. Transverse deflections of square plates viscoelastically supported at four points symmetrically located on the diagonals under the action of a centre-force. $\nu=0{\cdot}3$; $\kappa_s=100$; $\gamma_s=1$. (a) $\xi_s=\eta_s=1/8$; (b) $\xi_s=\eta_s=2/8$; (c) $\xi_s=\eta_s=3/8$; (d) $\xi_s=\eta_s=4/8$.

TABLE 1

Comparison of frequency parameters λ for a free and simply point-supported square plate; $\nu=0{\cdot}3$, $\gamma_s=0$

Vibration mode	$\kappa_s=0$ Present	$\kappa_s=0$ Gorman [23]	$\kappa_s=\infty$ Present	$\kappa_s=\infty$ Narita [18]
SS-1	0·0	0·0	7·137	7·111
SA(AS)-1	0·0	0·0	15·80	15·77
AA-1	13·48	13·47	38·58	38·43
SS-2	19·68	19·60	19·68	19·60
SS-3	24·35	24·27	44·38	44·37
SA(AS)-2	34·92	34·80	50·56	50·38
SA(AS)-3	61·36	61·08		

4. CONCLUSIONS

The steady state response to a sinusoidally varying force has been studied for a viscoelastically point-supported square or rectangular plate.

The response equation is derived from the equation of vibration of the point-supported plate by the generalized Galerkin method with the eigenfunctions of free beams used as admissible functions.

By the application of the method, the response curves to a sinusoidally varying point force acting at the centre have been calculated numerically for square plates viscoelastically supported at four points at the corners or on the diagonals, together with the natural frequencies of undamped point-supported plates.

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