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Rubinowicz transform of the MTPO surface integrals

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ABSTRACT

The surface integral of the modified theory of physical optics is reduced to a line integral by using the Rubinowicz transform for the incident scattered fields by an arbitrary aperture in a black surface. The integral theorem of Kirchhoff is applied to the scattering geometry and the diffracted fields are expressed in terms of a line integral along the contour of the diffracting edge.

1. Introduction

The phenomenon of diffraction is the bending of waves by a discontinuity on their path of propagation. Since this process occurs in many of the actual geometries, it was compulsorily observed by the scientists who were trying to understand the nature of light by observations. The phenomenon was first recorded by Grimaldi in his experiment with a small aperture [1].[1] Newton also observed the diffraction of light, but he interpreted the process as the distortion of the traveling paths of the light particles by the edge [2]. The first wave based interpretation of the diffraction phenomenon was put forward by Young [3,4]. He proposed that the scattered field by an edge was the interference of two fields. One of these fields is the geometrical optics (GO) wave that passes through the unobstructed aperture without affected by the discontinuity. The second field is the boundary diffracted wave, which is reflected by the edge in a special manner. Although reasonable, the ideas of Young were dominated by the formulation of Fresnel since he only proposed a qualitative explanation. The quantitative support to the interpretation of Young was put forward by Maggi and Rubinowicz, independently [5,6]. They applied the diffraction theorem of Kirchhoff [7] to an arbitrary aperture for spherical wave incidence and obtained a line integral along the diffracting contour of the edge.

The resultant line integral is widely used in the literature in order to investigate the diffraction of inhomogeneous waves by apertures [8–11]. However this method of the boundary diffraction wave (BDW) leads to non-uniform field expressions which approach to infinity at the transition regions of the diffraction geometry [12,13]. For this reason we obtained novel line integral representations for the theory of BDW by using the surface integral of the modified theory of physical optics (MTPO) [14–17]. The process of the line integral reduction is performed by using the method of asymptotic reduction. It is the aim of this paper to obtain the line integrals of edge diffraction by applying the method of Rubinowicz to the MTPO integrals. The integral theorem of Kirchhoff will be applied to the scattering integral of MTPO for the diffraction of spherical waves by an arbitrary aperture in a black surface. The obtained line integral of edge diffraction will be compared with the previous results in Refs. [16,17]. An interesting feature of the BDW theory is the non-uniform nature of the line integral [6] although the line integral reduction of Rubinowicz is rigorous. The obtained solutions by the method of BDW, as mentioned before, are non-uniform thus approach to infinity at the transition regions. The surface integral of Kirchhoff is uniform in nature. For this reason the diffracted waves that are evaluated by the line integral reduction of the scattering integrals must also be finite everywhere in space. In order to investigate this inconsistency, the uniform version of the BDW theory based on MTPO will also be searched in this study.

A time factor of $\exp(jwt)$ will be considered and suppressed throughout the paper. It is important to note that the term of "diffracted" field is taken into account for a wave that is created by the edge contour of the scatterer. Furthermore the term of the "scattered" waves are used for the sum of the diffracted and GO fields.

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[1] Refs. [1], [2], [4] and [5] are secondary sources of information.

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2. Theory

The scalar expression of the scattered electromagnetic or optical field by an aperture in a black surface can be written as

$$u_s(P) = -\frac{1}{4\pi}\int\int_S \left(u\frac{\partial G}{\partial n_1} - G\frac{\partial u}{\partial n_1}\right)\mathrm{d}S \tag{1}$$

according to MTPO. S is the surface of the aperture. $\vec{n}_1$ is the modified unit vector of the surface [14]. G is the Green's function, which is the fundamental solution of the Helmholtz equation of

$$\nabla^2 G + k^2 G = 0. \tag{2}$$

u is the total field on the surface. The geometry of the problem is given in Fig. 1. In this study we will consider the incident field as a spherical wave, which has the expression of

$$u_i = \frac{e^{-jkr}}{r} \tag{3}$$

The Green's function is also a spherical wave and can be given by

$$G = \frac{e^{-jkR}}{R} \tag{4}$$

In Fig. 1, Q is the integration or scattering point. $\vec{n}_1$ divides the angle between the incident and scattered rays into two equal angles. R is the function of the observation and scattering points. The integral, in Eq. (2), can be rewritten as

$$u_s(P) = -\frac{1}{4\pi}\int\int_S \left(u\frac{\partial G}{\partial R}\frac{\partial R}{\partial n_1} - G\frac{\partial u}{\partial r_1}\frac{\partial r_1}{\partial n_1}\right)\mathrm{d}S \tag{5}$$

for the relation of

$$\frac{\partial r_1}{\partial n_1} = -\frac{\partial R}{\partial n_1} \tag{6}$$

is valid according to Fig. 1. The scattered field takes the form of

$$u_s(P) = -\frac{1}{4\pi}\int\int_S \left(u\frac{\partial G}{\partial R} + G\frac{\partial u}{\partial r_1}\right)\frac{\partial R}{\partial n_1}\mathrm{d}S \tag{7}$$

when Eq. (6) is used in Eq. (5). It is important to note that the screen divides the space into two parts with the aperture. We will apply the integral theorem of Kirchhoff two the half space where the observation points exists. The geometry, in Fig. 2, is taken into account.

A volume of V_2 is constructed in the closed surface of $A + B + C_\infty$. A is the surface of the aperture. B is the lateral surface of the truncated cone, whose generator pass through the edge contour of the aperture. C_∞ is the partial surface of a sphere whose radius has an infinite length. D is the surface of the screen. The integral theorem of Kirchhoff can be given by the equation of

$$-\frac{1}{4\pi}\int\int_{A+B+C_\infty} \left(u\frac{\partial G}{\partial R} + G\frac{\partial u}{\partial r_1}\right)\frac{\partial R}{\partial n_1}\mathrm{d}S = u_{\mathrm{GO}}(P) \tag{8}$$

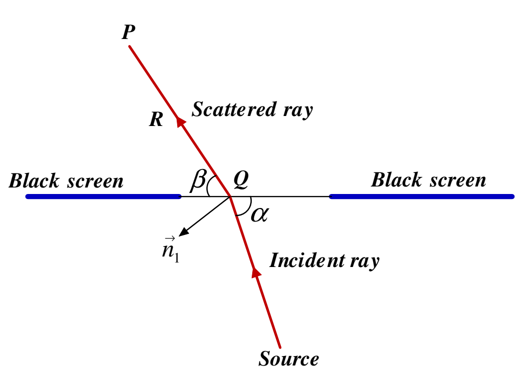

Fig. 1. Scattering of a wave by an aperture.

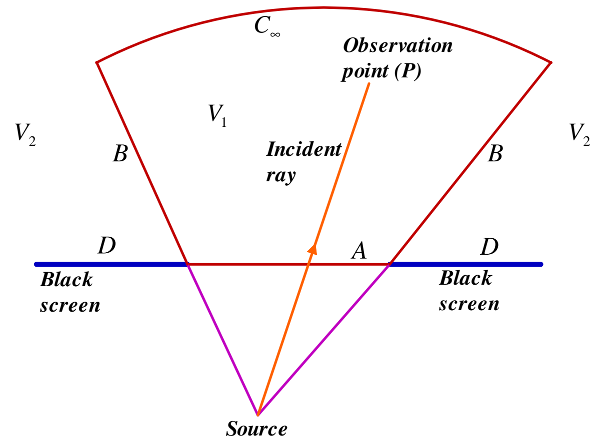

Fig. 2. Geometry for the Kirchhoff's theorem.

where $u_{\mathrm{GO}}(P)$ is the GO field, which can be defined as

$$u_{\mathrm{GO}}(P) = u_i(P)U(P) \tag{9}$$

for $U(P)$ is the unit step function that is equal to one for $P \in V_1$ and zero for $P \in V_2$. Since the surface is at infinity, C_∞ does not contribute to the integral, in Eq. (9), physically [18]. Eq. (8) reduces to

$$-\frac{1}{4\pi}\int\int_A \left(u\frac{\partial G}{\partial R} + G\frac{\partial u}{\partial r_1}\right)\frac{\partial R}{\partial n_1}\mathrm{d}S = u_{\mathrm{GO}}(P) + u_{\mathrm{d}}(P) \tag{10}$$

where $u_{\mathrm{d}}(P)$ is the diffracted wave, which is equal to

$$u_d(P) = \frac{1}{4\pi}\int\int_B \left(u\frac{\partial G}{\partial R} + G\frac{\partial u}{\partial r_1}\right)\frac{\partial R}{\partial n_1}\mathrm{d}S \tag{11}$$

As a next step, we will reduce the surface integral of Eq. (11) into a line integral along the contour of the diffracting edge. Two separate methods will be used with this aim. First of all the transform, introduced by Rubinowicz, will be taken into account. It is well known from the literature that this transform leads to non-uniform field expression [19]. As a second method, one part of the integral will be transformed into a Fresnel function in order to obtain a uniform expression for the line integral of the diffracted waves.

3. Rubinowicz transform

The surface integral, given by Eq. (11), will be reduced to a line integral along the edge contour of the aperture. The geometry, in Fig. 3, is taken into account. Q and Q_e are the scattering (integration) and edge points, respectively.

The cosine of the angle, between the modified unit normal vector of the surface and R is equal to

$$\cos\left(\vec{n}_1, \vec{R}\right) = \frac{\partial R}{\partial n_1} = \cos\frac{\pi - \beta}{2} \tag{12}$$

$\mathrm{d}S$ can be defined by the expression of

$$\mathrm{d}S = r_1\mathrm{d}r_1\mathrm{d}\theta \tag{13}$$

for a surface element on the truncated cone. It will be convenient to represent $\mathrm{d}\theta$ in terms of the length element of the edge contour. With this aim, the geometry, in Fig. 4, is taken into account. The line element of $\mathrm{d}l'$ is equal to $r_0\mathrm{d}\theta$. $\mathrm{d}l$ is the length element of the edge contour of the aperture. The relation of

$$\cos\left(\mathrm{d}\vec{l}, \mathrm{d}\vec{l}'\right) = \sin\left(\vec{r}_0, \mathrm{d}\vec{l}\right) \tag{14}$$

can be written according to Fig. 4.

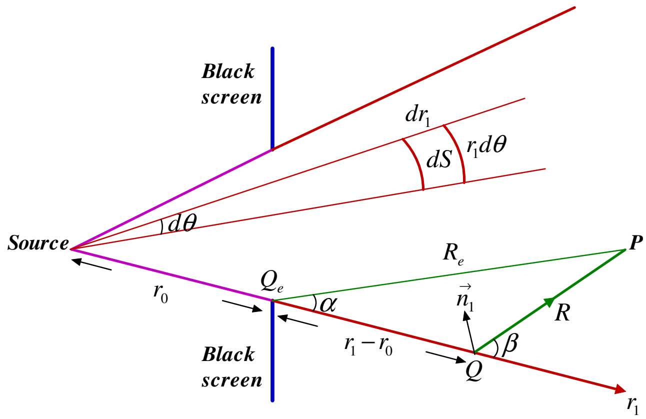

Fig. 3. Geometry of the Rubinowicz transform.

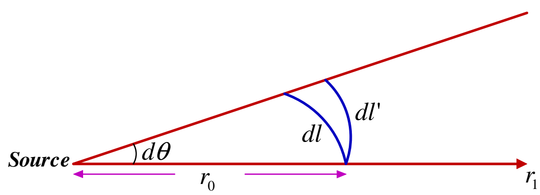

Fig. 4. Length elements of the cone and the edge.

One obtains the expression of

$$\mathrm{d}\theta = \frac{\sin\left(\vec{r}_0, \mathrm{d}\vec{l}\right)}{r_0}\mathrm{d}l \tag{15}$$

when the relation of

$$\mathrm{d}l' = \cos\left(\mathrm{d}\vec{l}, \mathrm{d}\vec{l}'\right)\mathrm{d}l \tag{16}$$

and Eq. (14) is taken into account. As a result the surface element can be written as

$$\mathrm{d}S = \frac{r_1}{r_0}\sin\left(\vec{r}_0, \mathrm{d}\vec{l}\right)\mathrm{d}r_1\mathrm{d}l \tag{17}$$

The integral of the edge diffracted waves can be represented as

$$u_d(P) = \frac{1}{4\pi}\int_C\int_{r_1=r_0}^{\infty}\left(u\frac{\partial G}{\partial R} + G\frac{\partial u}{\partial r_1}\right)\sin\frac{\beta}{2}\sin\left(\vec{r}_0, \mathrm{d}\vec{l}\right)\frac{r_1}{r_0}\mathrm{d}r_1\mathrm{d}l \tag{18}$$

when Eqs. (12) and (17) are used in Eq. (11). The lateral surface of B, in Eq. (11), consists of two line integrals. One of these integrals is the edge contour of the diffracting aperture, named by C and the second integral is along the r_1 axis. Eq. (18) can be arranged as

$$u_d(P) = -\frac{1}{4\pi}\int_C\frac{\sin\left(\vec{r}_0, \mathrm{d}\vec{l}\right)}{r_0} \times \int_{r_1=r_0}^{\infty}\left(2\mathrm{j}k + \frac{1}{r_1} + \frac{1}{R}\right)\frac{\exp\left[-\mathrm{j}k(r_1+R)\right]}{R}\sin\frac{\beta}{2}\mathrm{d}r_1\mathrm{d}l \tag{19}$$

We will evaluate the integral of

$$I = \int_{r_1=r_0}^{\infty}\left(2\mathrm{j}k + \frac{1}{r_1} + \frac{1}{R}\right)\frac{\exp\left[-\mathrm{j}k(r_1+R)\right]}{R}\sin\frac{\beta}{2}\mathrm{d}r_1 \tag{20}$$

in order to obtain the line integral representation of the diffracted waves. We will define a function of

$$g(r_1) = f(r_1)\frac{\exp\left[-\mathrm{j}k(r_1+R)\right]}{R} \tag{21}$$

the derivative of which is equal to

$$\frac{\mathrm{d}g(r_1)}{\mathrm{d}r_1} = \left[\frac{\mathrm{d}f(r_1)}{\mathrm{d}r_1} + \frac{\cos\beta - \mathrm{j}k(1-\cos\beta)R}{R}f(r_1)\right]\frac{\exp\left[-\mathrm{j}k(r_1+R)\right]}{R} \tag{22}$$

The derivative of R according to r_1 gives

$$\frac{\partial R}{\partial r_1} = -\cos\beta \tag{23}$$

for R is equal to $\sqrt{R_e^2 + (r_1 - r_0)^2 - 2R_e(r_1 - r_0)\cos\alpha}$ according to Fig. 3. $f(r_1)$ is found to be

$$f(r_1) = -\frac{1}{\sin(\beta/2)} \tag{24}$$

when Eq. (22) is equated to the integrand of Eq. (20). As a result the integral, in Eq. (20), can be evaluated as

$$I_1 = -\frac{1}{\sin(\beta/2)}\frac{\exp\left[-\mathrm{j}k(r_1+R)\right]}{R}\Bigg|_{r_1=r_0}^{\infty} \tag{25}$$

which yields the equation of

$$I_1 = \frac{1}{\sin(\alpha/2)}\frac{\exp\left[-\mathrm{j}k(r_0+R_e)\right]}{R_e} \tag{26}$$

since β and R are equal to α and R_e at Q_e. The line integral of the edge diffracted waves can be written as

$$u_d(P) = -\frac{1}{4\pi}\int_C\frac{\exp\left[-\mathrm{j}k(r_0+R_e)\right]}{R_e r_0}\frac{1}{\sin(\alpha/2)}\sin\left(\vec{r}_0, \mathrm{d}\vec{l}\right)\mathrm{d}l. \tag{27}$$

The integral has singularity at $\alpha = 0$, which represents the shadow boundary. For this reason Eq. (27) is not uniform.

4. Uniform representation of the line integral

We aim to obtain a uniform line integral representation for the edge diffracted fields. The integral of I can be rewritten as

$$I \approx 2\mathrm{j}k\int_{r_1=r_0}^{\infty}\frac{\exp\left[-\mathrm{j}k(r_1+R)\right]}{R}\sin\frac{\beta}{2}\mathrm{d}r_1 \tag{28}$$

for $k > > 1$. We will evaluate the integral, in Eq. (28), by using the method, which was given in Ref. [20]. The stationary phase of the integral can be found as $\beta_s = 0$ by equating the first derivative of the phase function to zero. The stationary phase value of the amplitude function will also be equal to zero for all of it's derivatives since a multiplier of $\mathrm{d}\beta/\mathrm{d}r_1$, which is equal to, $\sin\beta/R$ will come. Thus only the edge point, at and $\beta_e = \alpha$, will have a contribution on the integral. The method, in Ref. [20], can be defined as

$$\frac{\exp(\mathrm{j}\pi/4)}{\sqrt{\pi}}\int_{\alpha_e}^{\infty}f(\alpha)\exp\left[\mathrm{j}kg(\alpha)\right]\mathrm{d}\alpha = \frac{f(\alpha_e)2t_e}{kg'(\alpha_e)}\exp\left[\mathrm{j}kg(\alpha_s)\right]\mathrm{sign}(t_e)F\left[|t_e|\right] \tag{29}$$

for $\mathrm{sign}(x)$ is the signum function, which is equal to 1 for $x > 0$ and -1, otherwise. $F[x]$ is the Fresnel function and can be defined by the equation of

$$F[x] = \frac{\exp(\mathrm{j}\pi/4)}{\sqrt{\pi}}\int_x^{\infty}\exp\left(-\mathrm{j}t^2\right)\mathrm{d}t. \tag{30}$$

α_s and α_e are the stationary phase and edge point values of α. $f(\alpha)$ and $g(\alpha)$ are the amplitude and phase functions of the integral, respectively. can be given by the expression of

$$t_e = -\sqrt{k\left[g(\alpha_s) - g(\alpha_e)\right]} \tag{31}$$

The phase function of the integral, in Eq. (28), is equal to

$$g(r_1) = -(r_1 + R) \tag{32}$$

which can also be expressed as

$$g(r_1) = -\frac{R_e \cos\alpha + r_0 - r_1(1-\cos\beta)}{\cos\beta} \tag{33}$$

according to the equation of $R\cos\beta = R_e\cos\alpha - (r_1 - r_0)$. The stationary phase value of the phase function can be found as

$$g(r_{1s}) = -(R_e\cos\alpha + r_0) \tag{34}$$

for $\beta_s = 0$. The phase function is equal to

$$g(r_{1e}) = -(r_0 + R_e) \tag{35}$$

at the edge point of the integral. Thus t_e reads

$$t_e = -\sqrt{2kR_e}\sin\frac{\alpha}{2} \tag{36}$$

according to Eq. (31). $g\prime(r_{1e})$ can be written as

$$g'(r_{1e}) = -2\sin^2\frac{\alpha}{2} \tag{37}$$

As a result, the uniform evaluation of Eq. (28) gives

$$I \approx 2\sqrt{2\pi}\exp(j\pi/4)k\frac{\exp[-jk(R_e\cos\alpha + r_0)]}{\sqrt{kR_e}}sign(t_e)F[|t_9 m|] \tag{38}$$

The line integral of the edge diffracted waves is found to be

$$u_d(P) \approx \frac{\exp(j\pi/4)k}{\sqrt{2\pi}}\int_C \frac{\exp[-jk(R_e\cos\alpha + r_0)]}{r_0\sqrt{kR_e}} \times \sin\left(\vec{r}_0, d\vec{l}\right)sign(t_e)F[|t_e|]dl \tag{39}$$

which is the uniform version of Eq. (27). Eq. (39) can be generally evaluated by the method of the stationary phase, but the phase function of the Fresnel integral can also be considered. For this reason the asymptotic expansion of the Fresnel function can be written as

$$\mathrm{sign}(x)F[|x|] = \exp(-jx^2)M(x) \tag{40}$$

where $M(x)$ can be defined by the expression of

$$M(x) = \frac{\exp(-j\pi/4)}{2\pi x}\sum_{n=1}^{\infty}\frac{\Gamma\left(n+\frac{1}{2}\right)}{\left(-jx^2\right)^n} \tag{41}$$

for $\Gamma(x)$ is the gamma function. Eq. (39) can be rewritten as

$$u_d(P) \approx \frac{\exp(j\pi/4)k}{\sqrt{2\pi}}\int_C \frac{\exp[-jk(R_e + r_0)]}{r_0\sqrt{kR_e}}\sin\left(\vec{r}_0, d\vec{l}\right)M(t_e)dl \tag{42}$$

when Eq. (40) is taken into account. After the evaluation of the line integral, the Fresnel function can be reconstructed.

5. Application: diffraction of a Gaussian beam by a half-plane

In this section the diffraction process of a Gaussian beam by a black half-plane, which does not reflect or transmit the incident waves, will be investigated by using the uniform line integral of the edge diffracted waves, given in Eq. (42). The half-plane is laying at $S = \{x \in (0,\infty), y = 0, z \in (-\infty,\infty)\}$. We will consider the Gaussian beam as a line source with complex source coordinates [21–24]. This method provides simplicity in the operations. First of all the integral is evaluated for an actual line source and then the resultant fields are transformed to complex coordinates in order to obtain the diffraction of a Gaussian beam by considering the principle of analytic continuation.

The geometry of the problem is given in Fig. 5. The line source is placed at (x_0, y_0). The incident field can be defined as

$$u_i = u_0\frac{\exp(-jkR_i)}{\sqrt{kR_i}} \tag{43}$$

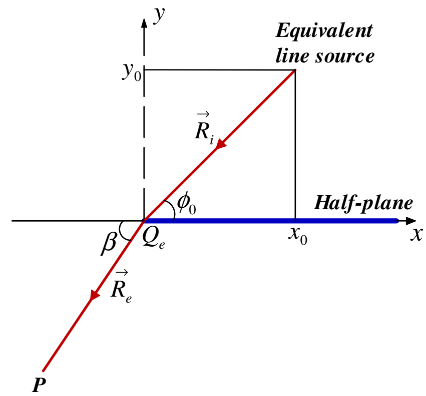

Fig. 5. Edge diffraction by a half-plane.

for R_i is equal to $\sqrt{(x-x_0)^2 + (y-y_0)^2}$. Its complex form which represents a Gaussian beam has the expression of

$$R_i = \sqrt{(x-x_0)^2 + (y-y_0-jb)^2} \tag{44}$$

according to which the angle of incidence gives

$$\phi_0 = tg^{-1}\frac{y_0 + jb}{x_0} \tag{45}$$

b is a real constant that represents the beam waist of a cylindrical Gaussian beam [22,23]. R_e is equal to $\sqrt{\rho^2 + (z-z')^2}$. At the edge point of Q_e, the incident field can be expressed as

$$u_i = u_0\frac{\exp(-jk\rho_0)}{\sqrt{k\rho_0}} \tag{46}$$

for ρ_0 is $\sqrt{x_0^2 + y_0^2}$ for a real field. The comparison of the real field and a Gaussian beam is given in Fig. 6. The distance of observation (ρ) is equal to 6λ for λ is the wave-length. x_0 and y_0 are zero.

The length element of $d\vec{l}$ is equal to $dz'\vec{e}_z$. Thus the term of $\sin\left(\vec{r}_0, d\vec{l}\right)$ is one. α gives $\beta + \phi_0$. The line integral of the edge diffracted waves can be written as

$$u_d(P) \approx \frac{\exp(j\pi/4)ku_0}{\sqrt{2\pi}}\frac{\exp(-jk\rho_0)}{\sqrt{k\rho_0}}\int_{z'=-\infty}^{\infty}\frac{\exp(-jkR_e)}{\sqrt{kR_e}}M(t_e)dz' \tag{47}$$

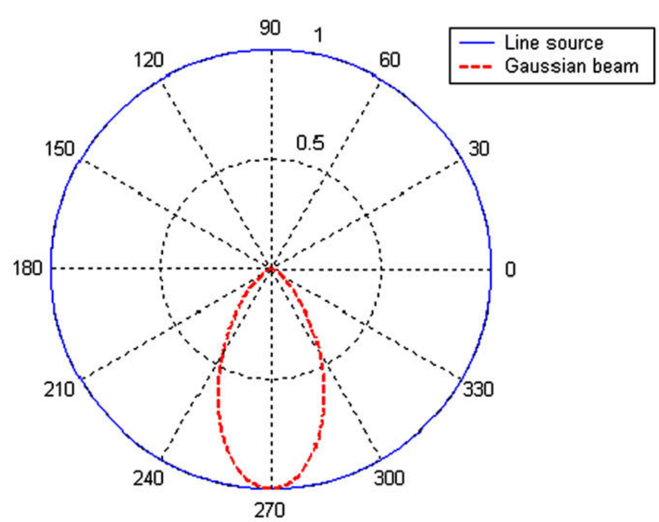

Fig. 6. Comparison of a line source with the Gaussian beam.

for a real line source. The integral, in Eq. (47), can be evaluated by using the method of the stationary phase [14]. The stationary phase point is found to be $z_s = z$ when the first derivative of the phase function is equated to zero. β is equal to $\phi - \pi$ at the edge point. As a result one obtains

$$u_d(P) \approx u_0 \frac{\exp(-jk\rho_0)}{\sqrt{k\rho_0}} \exp(-jk\rho) M(t_{es}) \tag{48}$$

for the edge diffracted fields. t_{es} is equal to $-\sqrt{2k\rho}\cos[(\phi - \phi_0)/2]$. The Fresnel function can be found by using Eq. (40). The resultant field expression can be written as

$$u_d(P) \approx u_0 \frac{\exp(-jk\rho_0)}{\sqrt{k\rho_0}} \exp[jk\rho\cos(\phi - \phi_0)] \mathrm{sign}(t_{es}) F[|t_{es}|] \tag{49}$$

The diffracted Gaussian beam can be obtained by using the uniform method, introduced in Ref. [25]. The complex value of ρ_0 is $\sqrt{x_0^2 + (y_0 + jb)^2}$. The detour parameter of t_{es} is also complex because of ϕ_0. The uniform diffracted Gaussian beam can be represented as

$$u_d(P) \approx u_0 \frac{\exp(-jk\rho_0)}{\sqrt{k\rho_0}} \times \exp[jk\rho\cos(\phi - \phi_0)] \mathrm{sign}(a - b) F[|a - b| + \mathrm{sign}(a - b)\eta] \tag{50}$$

for η is $b\sqrt{2}\exp(j\pi/4)$. a and b are the real and imaginary parts of t_{es}, respectively. Eq. (50) can be rewritten as

$$u_d(P) \approx u_0 \frac{\exp(-jkR_i)}{\sqrt{kR_i}} \mathrm{sign}(a - b) F[|a - b| + \mathrm{sign}(a - b)\eta] \tag{51}$$

when the expression of

$$\frac{\exp(-jkR_i)}{\sqrt{kR_i}} \approx \frac{\exp(-jk\rho_0)}{\sqrt{k\rho_0}} \exp[jk\rho\cos(\phi - \phi_0)] \tag{52}$$

is taken into account for $\rho_0 > > \rho$. The GO field can be given by the equation of

$$u_{GO}(P) \approx u_0 \frac{\exp(-jkR_i)}{\sqrt{kR_i}} U(b - a) \tag{53}$$

according to Ref. [25]. $U(x)$ is the unit step function, which is equal to one for $x > 0$ and zero, otherwise. The total field is the sum of the GO and diffracted waves.

6. Numerical results

In this section, the diffracted Gaussian beam will be examined for different places of the source coordinates according to the x axis. The beam axis is directed towards the direction of $\phi = 270°$ as shown in Fig. 6. Eqs. (51) and (53) will be used for the plot of the diffracted and GO waves. The equation of

$$u_t(P) \approx u_0 \frac{\exp(-jkR_i)}{\sqrt{kR_i}} F[t_{es}] \tag{54}$$

will be considered for the total field.

Fig. 7 shows the variation of the total, GO and diffracted fields versus the observation angle. x_0 and y_0 are equal to 7λ and 10λ, respectively. The observation distance is 6λ. The transition region (shadow boundary) lays nearly at $\phi = 235°$ and the discontinuity of the Go field is compensated by the diffracted wave at this boundary. It can be seen that the smooth construction of the GO wave is changed into a variable structure with the interference of the diffracted field. The total field is continuous everywhere.

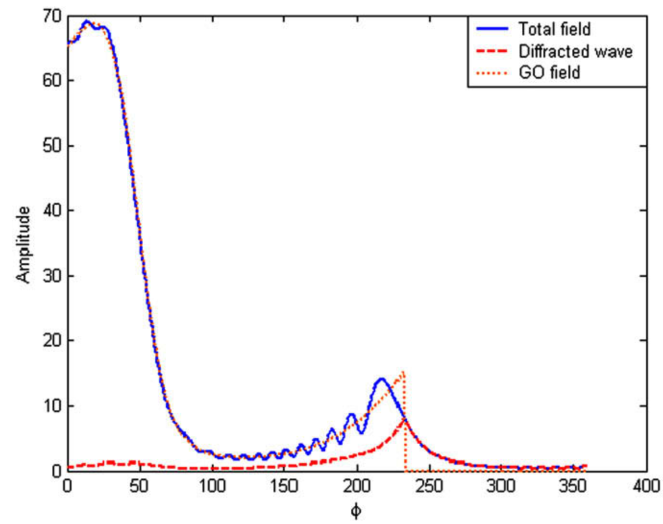

Fig. 7. Scattered fields for $x_0 = 7\lambda$.

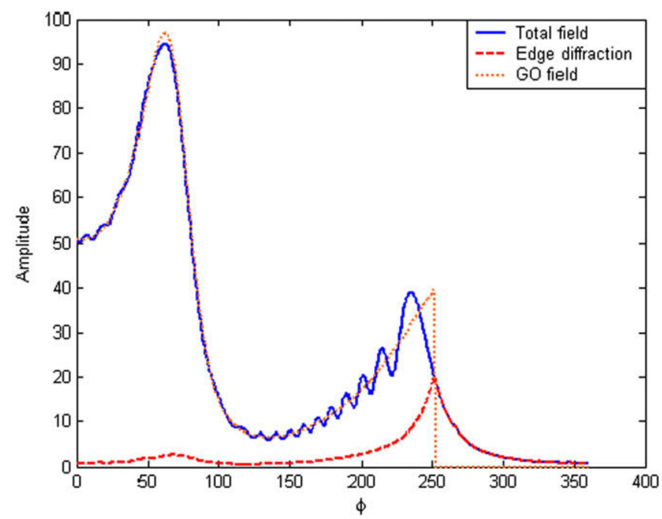

Fig. 8. Scattered fields for $x_0 = 3\lambda$.

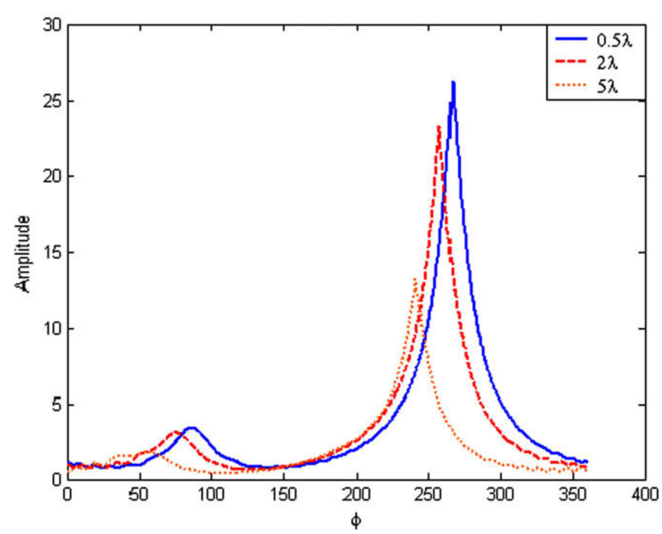

Fig. 9. Variation of the diffracted wave versus x_0.

Fig. 8 depicts the variation of the scattered waves with respect to the observation angle for x_0 is equal to 3λ. It is observed that the place of the shadow boundary is shifted to $\phi \approx 250^\circ$ since the source approaches to the edge of the half-plane. This is an expected result.

In Fig. 9, the variation of the diffracted wave with respect to the observation angle for various values of x_0 is plotted. It can be seen that the placement of the transition region shifts to the right as the source comes nearer the edge of the half-plane. Also the amplitude of the diffracted wave increases in this case.

7. Conclusion

In this study, the transform of Rubinowicz is applied to the surface integrals of MTPO. The resultant line integral is in harmony with the previous results [16]. Since the actual edge diffracted wave is uniform everywhere and finite at the transition regions, a uniform version of the line integral is derived by evaluating the r part of the integral with the method, introduced in Ref. [20]. The uniform integral is applied to the problem of diffraction of a Gaussian beam by a half-plane and the evaluated fields are examined numerically. The results obey the theoretical assumptions.

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