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OPTICAL PROPERTIES OF METALS WITH INHOMOGENEOUSLY HEATED ELECTRONS*

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Abstract

We propose a new approach for describing the optical properties of metals with inhomogeneously heated electrons in the skin layer. The absorption coefficient and phase shift of the reflected wave, which are not described by the usual Fresnel formulas, were found.

Keywords: Fresnel formula, skin layer, inhomogeneous temperature.

1. Introduction

Usually optical properties of massive specimens are studied using the classical Fresnel formulas (see, e.g., [1, 2]). If a transition layer with inhomogeneous permittivity is located on the specimen surface, one needs to refine the Fresnel formulas; this was done, for example, in [3–7] where, taking into account the small ratio of the transition-layer thickness to the radiation wavelength, corrections to the complex reflection coefficient were obtained. The optical properties of metals within a wide range of frequencies are determined by the response of electrons to the action of the field localized in the skin layer. At radiation frequencies ω much smaller than the plasma frequency of electrons ω_p, the skin-layer depth is small in comparison with the wavelength. Under such conditions, the approach of [3–7] can be used to consider the optical properties of metals. However, if the frequency ω approaches the plasma frequency ω_p, the use of this approach assumes taking into account a comparatively large number of correction terms to the classical Fresnel formulas. At the same time, with an increase in ω, the ratio of the effective collision frequency of electrons ν to ω decreases. The electron path during the field period proves to be small in comparison with the skin-layer depth; i.e., the conditions of a high-frequency skin effect are realized at frequencies ω not too small as compared with ω_p. At $\nu \ll \omega$, the metal permittivity can be presented as a series in terms of powers of $\nu/\omega \ll 1$, and an approximate solution of the field equations in the metal can be constructed using the perturbation theory for ν/ω. Such approach to describe the optical properties of metals is proposed in the present paper.

It is assumed that the frequency ν can change significantly along the thickness of the skin layer. The reason for this change is inhomogeneous heating of electrons when the laser radiation is absorbed in the skin layer (for details, see [8, 9]). Within an accuracy of the corrections quadratic with respect to a small parameter ν/ω, below we obtain the complex reflection coefficient, the absorption coefficient, and the

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phase shift of reflected wave. In the case where electrons in the skin layer are efficiently cooled due to the heat transfer into metal, the frequency ν varies slowly on the scale of the skin layer, and the general Fresnel formulas can be used with the corrections due to the dependence of permittivity on the electron temperature on the metal surface. In contrast, if the heat transfer is not efficient and the change in ν with respect to the skin layer should be taken into account, the absorption coefficient A and phase shift ϕ of the reflected wave are smaller than the values obtained using the usual Fresnel formulas. The stronger the heating of electrons in the skin layer and the higher the inhomogeneity of the corrections to the permittivity determined by the small ratio ν/ω, the greater the difference between the new values of A and ϕ and the known ones.

2. Basic Relations

Consider the interaction of a linearly polarized electromagnetic wave of the form

$$\frac{1}{2}\overrightarrow{E}_L \exp(-i\omega t + i\overrightarrow{k}\,\overrightarrow{r}) + \text{c.c.}, \qquad z \leqslant 0 \tag{1}$$

with the metal occupying the half-space $z \geqslant 0$. In formula (1), $\overrightarrow{E}_L = (0, E_L, 0)$, $\omega = kc$, ω is the frequency, k is the wavenumber, c is the speed of light, $\overrightarrow{k} = k(\sin\theta, 0, \cos\theta)$, and θ is the angle between the vector $\overrightarrow{k}$ and the direction of the axis Oz.

The magnetic field

$$\overrightarrow{B}_L = \frac{\overrightarrow{k}\,\overrightarrow{E}_L}{k} = E_L(-\cos\theta, 0, \sin\theta)$$

has the components along the axes Ox and Oz.

Wave (1) is reflected from the surface $z = 0$ and penetrates into the metal. The field of the reflected wave reads

$$\frac{1}{2}\overrightarrow{E}_r \exp(-i\omega t + ikx\sin\theta - ikz\cos\theta) + \text{c.c.}, \qquad z \leqslant 0, \tag{2}$$

where $\overrightarrow{E}_r = R\overrightarrow{E}_L$ and R is the complex reflection coefficient. The magnetic field of the reflected wave is described by the relation

$$\overrightarrow{B}_r = RE_L(\cos\theta, 0, \sin\theta)$$

and the field in the metal is found as follows:

$$\overrightarrow{E}(\overrightarrow{r}, t) = \frac{1}{2}\overrightarrow{E}(z)\exp(-i\omega t + ikx\sin\theta) + \text{c.c.}, \qquad z \geqslant 0, \tag{3}$$

$$\overrightarrow{B}(\overrightarrow{r}, t) = -\frac{i}{k}\,\text{rot}\,\overrightarrow{E}(\overrightarrow{r}, t) = \frac{1}{2}\overrightarrow{B}(z)\exp(-i\omega t + ikx\sin\theta) + \text{c.c.}, \qquad z \geqslant 0, \tag{4}$$

where

$$\overrightarrow{E}(z) = (0, E(z), 0), \qquad \overrightarrow{B}(z) = \left(iE'(z)/k, 0, E(z)\sin\theta\right).$$

The electric and magnetic fields are continuous on the surface $z = 0$. Taking into account relations (1)–(4), we write the conditions for the field continuity as follows:

$$E_L(1 + R) = E(z)|_{z=0} \equiv E_y(z)|_{z=0}, \tag{5}$$

$$E_L\cos\theta(R - 1) = \frac{i}{k}E'(z)|_{z=0} \equiv B_x(z)|_{z=0}. \tag{6}$$

Introducing the surface impedance of the metal according to the definition

$$Z = -\left.[E_y(z)/B_x(z)]\right|_{z=0}, \tag{7}$$

we obtain the relation for the reflection coefficient R

$$R = \frac{Z\cos\theta - 1}{Z\cos\theta + 1}. \tag{8}$$

In contrast, excluding R from the system of equations (5) and (6), we obtain the boundary condition for determining the field $E(z)$ in the metal as follows:

$$\left.\left[\frac{1}{ik}E'(z) + E(z)\cos\theta\right]\right|_{z=0} = 2E_L\cos\theta. \tag{9}$$

3. Solution of the Field Equation in Metals

The field distribution in metals depends substantially on the equations connecting the current with the electric field. In the case of a high-frequency skin effect, the electron path during the period of the field change is small in comparison with the skin-layer depth. This fact forces one not to take into account the spatial dispersion in the field equation. In addition, under the high-frequency skin effect, the characteristic frequency of the electron–electron collisions ν is small with respect to the field frequency ω, which enables one to use the small parameter $\nu/\omega \ll 1$. At the same time, since the frequency ω is smaller than the frequency corresponding to the forbidden-band gap, the effect of interband transitions can be neglected. Under these conditions, we make use of the well-known expression for permittivity $\epsilon(z)$ to describe the metal response

$$\epsilon(z) = \epsilon_0 - \frac{\omega_p^2}{\omega[\omega + i\nu(z)]} \simeq \epsilon_1 + i\epsilon_2(z) + \delta\epsilon_1(z), \tag{10}$$

where ϵ_0 is the contribution of bound electrons and ions of the lattice to the metal permittivity, ω_p is the plasma frequency, and we use the following notation:

$$\epsilon_1 = \epsilon_0 - \frac{\omega_p^2}{\omega^2}, \qquad \epsilon_2(z) = \frac{\nu(z)}{\omega^3}\omega_p^2, \qquad \delta\epsilon_1(z) = \frac{\nu^2(z)}{\omega^4}\omega_p^2. \tag{11}$$

Since usually $|\epsilon_1| \gg 1$ and $\nu(z)/\omega \ll 1$, one has $|\epsilon_1| \gg \epsilon_2(z) \gg \delta\epsilon_1(z)$.

Taking into account all assumptions listed above, one has the equation for the field in the metal

$$E''(z) - \frac{1}{d^2}E(z) + k^2[i\epsilon_2(z) + \delta\epsilon_1(z)]E(z) = 0, \qquad z \geqslant 0, \tag{12}$$

where $d = \left(k\sqrt{|\epsilon_1| + \sin^2\theta}\right)^{-1}$ is the characteristic depth of the skin layer. The field $E(z)$ described by Eq. (12) satisfies the boundary condition (9) and the requirement that the field does not deeply penetrate inside the metal

$$E(z \to \infty) = 0, \qquad E'(z \to \infty) = 0. \tag{13}$$

The solution of Eq. (12) is found in the form

$$E(z) = E_a(z)\exp[i\psi(z)], \tag{14}$$

where $E_a(z)$ and $\psi(z)$ are the real amplitude and phase of the field. From (12), we have the following system of equations for $E_a(z)$ and $\psi(z)$:

$$E_a''(z) - E_a(z)[\psi'(z)]^2 + \left[k^2\delta\epsilon_1(z) - \frac{1}{d^2}\right] E_a(z) = 0, \tag{15}$$

$$2E_a'(z)\psi'(z) + E_a(z)\psi''(z) + k^2\epsilon_2(z)E_a(z) = 0. \tag{16}$$

In view of Eqs. (9) and (13), the functions $E_a(z)$ and $\psi(z)$ satisfy the boundary conditions

$$[E_a'(z) - 2kE_L\cos\theta\sin\psi(z)]\big|_{z=0} = 0, \tag{17}$$

$$[E_a(z)\psi'(z) + kE_a(z)\cos\theta - 2kE_L\cos\theta\cos\psi(z)]|_{z=0} = 0, \tag{18}$$

$$E_a(z\to\infty) = 0, \qquad E_a'(z\to\infty) = 0, \qquad E_a(z\to\infty)\psi'(z\to\infty) = 0. \tag{19}$$

Since our aim is to construct an approximate solution of Eqs. (15) and (16), we make use of the small parameter $\nu(z)/\omega \ll 1$. The functions $E_a(z)$ and $\psi(z)$ are found as a series in terms of powers of $[\nu(z)/\omega]^n \ll 1$, $n = 1, 2, \ldots$

$$E_a(z) = \sum_{n=0}^{\infty} E_n(z), \qquad \psi(z) = \sum_{n=0}^{\infty}\psi_n(z), \tag{20}$$

where $E_n = E_n(z)$ and $\psi_n = \psi_n(z)$ are of the order of $[\nu(z)/\omega]^n$.

In the zeroth order with respect to the small parameter, taking into account the correlations $\epsilon_2(z) \sim |\epsilon_1|\nu(z)/\omega$ and $\delta\epsilon_1(z) \sim \epsilon_2(z)\nu(z)/\omega$, from (15)–(20) we obtain

$$E_0'' - \frac{1}{d^2}E_0 - E_0(\psi_0')^2 = 0, \tag{21}$$

$$2E_0'\psi_0' + E_0\psi_0'' = 0, \tag{22}$$

and the following boundary conditions:

$$[E_0' - 2kE_L\cos\theta\sin\psi_0]\big|_{z=0} = 0, \tag{23}$$

$$[E_0\psi_0' + kE_0\cos\theta - 2kE_L\cos\theta\cos\psi_0]|_{z=0} = 0, \tag{24}$$

$$E_0(z\to\infty) = 0, \qquad E_0'(z\to\infty) = 0, \qquad E_0(z\to\infty)\psi_0'(z\to\infty) = 0. \tag{25}$$

Multiplying Eq. (22) by $E_0 \neq 0$, we obtain $E_0^2\psi_0' = \text{const}$. In accordance with the boundary conditions (25), at $z\to\infty$ one obtains $E_0^2\psi_0' = 0$ and, since $E_0 \neq 0$, we obtain

$$\psi_0' = 0 \tag{26}$$

for all $z \geqslant 0$.

In view of relation (26), from Eq. (21) and the boundary conditions (23) and (24) at $z = 0$, we derive

$$E_0(z) = E_0(0)e^{-z/d} \equiv \frac{2E_L kd\cos\theta}{\sqrt{1+(kd\cos\theta)^2}}\exp\left(-\frac{z}{d}\right), \tag{27}$$

$$\psi_0 = -\arctan\left(\frac{1}{kd\cos\theta}\right). \tag{28}$$

Using the linear approximation in $\nu(z)/\omega$, from Eqs. (15)–(20) we obtain the equations for E_1 and ψ_1 as follows:

$$E_1'' - \frac{1}{d^2}E_1 = 0, \tag{29}$$

$$\psi_1'' - \frac{2}{d}\psi_1' = -k^2\epsilon_2(z), \tag{30}$$

along with the following boundary conditions:

$$(E_1' - \psi_1 k E_0 \cos\theta)\big|_{z=0} = 0, \tag{31}$$

$$(E_0\psi_1' + E_0'\psi_1 + kE_1\cos\theta)|_{z=0} = 0, \tag{32}$$

$$E_1(z\to\infty) = 0, \qquad E_1'(z\to\infty) = 0, \qquad E_0(z\to\infty)\psi_1'(z\to\infty) = 0. \tag{33}$$

Taking account of the last of conditions (33), which follows from the fact that at $z\to\infty$ the magnetic field is equal to zero, from Eq. (30) we obtain

$$\psi_1' = k^2 \exp\left(\frac{2z}{d}\right)\int_z^\infty dz'\epsilon_2(z')\exp\left(-\frac{2z'}{d}\right). \tag{34}$$

From Eqs. (29) and (34), taking into account the boundary conditions (31)–(33), we obtain E_1 and ψ_1 as follows:

$$\begin{aligned} E_1(z) &= E_1(0)e^{-z/d} \equiv -E_0(0)\psi_1(0)kd\cos\theta\exp\left(-\frac{z}{d}\right) \\ &\equiv -2E_L\exp\left(-\frac{z}{d}\right)\frac{k^4d^3\cos^2\theta}{[1+(kd\cos\theta)^2]^{3/2}}\int_0^\infty dz'\epsilon_2(z')\exp\left(-\frac{2z'}{d}\right), \end{aligned} \tag{35}$$

$$\psi_1(z) = \frac{d}{2}k^2\left[e^{2z/d}\int_z^\infty dz'\epsilon_2(z')e^{-2z'/d} + \int_0^z dz'\epsilon_2(z') + \frac{1-(kd\cos\theta)^2}{1+(kd\cos\theta)^2}\int_0^\infty dz'\epsilon_2(z')e^{-2z'/d}\right]. \tag{36}$$

Corrections to the amplitude E_2 and phase ψ_2 quadratic in $\nu(z)/\omega$ satisfy the equations

$$E_2'' - \frac{1}{d^2}E_2 = E_0[(\psi_1')^2 - k^2\delta\epsilon(z)], \tag{37}$$

$$\psi_2'' - \frac{2}{d}\psi_2' = 0 \tag{38}$$

and the boundary conditions

$$\left(E_2' - \psi_2 kE_0\cos\theta + \frac{1}{2}E_0'\psi_1^2\right)\bigg|_{z=0} = 0, \tag{39}$$

$$\left(E_1\psi_1' + E_0\psi_2' + E_0'\psi_2 + kE_2\cos\theta + \frac{1}{2}E_0\psi_1^2 kcos\theta\right)\bigg|_{z=0} = 0, \tag{40}$$

$$E_2(z\to\infty) = 0, \qquad E_2'(z\to\infty) = 0, \qquad E_0(z\to\infty)\psi_2'(z\to\infty) = 0. \tag{41}$$

In view of Eq. (38) and the last of the boundary conditions (41), we have $\psi_2' = 0$, i.e., $\psi_2 = \psi_2(0)$. Taking into account the first of the boundary conditions (41), from Eq. (37) we obtain

$$\begin{aligned} E_2(z) &= E_2(0)e^{-z/d} - d\sinh\left(\frac{z}{d}\right)\int\limits_z^\infty dz' e^{-z'/d}E_0(z')\Big\{[\psi_1'(z')]^2 - k^2\delta\epsilon_1(z')\Big\} \\ &\quad - de^{-z/d}\int\limits_0^z dz' \sinh\left(\frac{z'}{d}\right)E_0(z')\Big\{[\psi_1'(z')]^2 - k^2\delta\epsilon_1(z')\Big\}. \end{aligned} \tag{42}$$

The constants $\psi_2(0)$ and $E_2(0)$ calculated from the boundary conditions (39) and (40) at $z = 0$ have the form

$$\begin{aligned} E_2(0) &= E_L\frac{2k^3d^2\cos\theta}{[1+(kd\cos\theta)^2]^{3/2}}\int\limits_0^\infty dz e^{-2z/d}\left\{\delta\epsilon_1(z) - k^2\left[e^{2z/d}\int\limits_z^\infty dz'\epsilon_2(z')e^{-2z'/d}\right]^2\right\} \\ &\quad + E_L\frac{kd\cos\theta[2(kd\cos\theta)^2-1]}{[1+(kd\cos\theta)^2]^{5/2}}\left[k^2 d\int\limits_0^\infty dz\epsilon_2(z)e^{-2z/d}\right]^2, \end{aligned} \tag{43}$$

$$\begin{aligned} \psi_2(0) &= \frac{k^3d^2\cos\theta}{1+(kd\cos\theta)^2}\int\limits_0^\infty dz e^{-2z/d}\left\{\delta\epsilon_1(z) - k^2\left[e^{2z/d}\int\limits_z^\infty dz'\epsilon_2(z')e^{-2z'/d}\right]^2\right\} \\ &\quad - \frac{kd\cos\theta}{[1+(kd\cos\theta)^2]^2}\left[k^2 d\int\limits_0^\infty dz\epsilon_2(z)e^{-2z/d}\right]^2. \end{aligned} \tag{44}$$

4. Absorption Coefficient and Phase Shift

The obtained above formulas for the field enable one to calculate the complex reflection coefficient (8) within an accuracy of the corrections quadratic with respect to a small parameter ν/ω. According to (5)–(8) and (14), R is expressed through the functions $\psi'(z)$ and $[\ln E_a(z)]'$ at $z = 0$

$$R = \left\{\frac{k\cos\theta - \psi'(z) + i[\ln E_a(z)]'}{k\cos\theta + \psi'(z) - i[\ln E_a(z)]'}\right\}\Bigg|_{z=0} \equiv re^{i\phi}, \tag{45}$$

where r is the absolute value of the reflection coefficient and ϕ is the phase shift of the reflected wave. In view of the definition (45), the phase shift is described by the relation

$$\phi = \pi + \arctan\left\{\frac{2k\cos\theta E_a'(z)/E_a(z)}{k^2\cos^2\theta - [\psi'(z)]^2 - [E_a'(z)/E_a(z)]^2}\right\}\Bigg|_{z=0} \tag{46}$$

and r is related to the absorption coefficient as follows:

$$A = 1 - r^2 = \frac{4k\cos\theta\psi'(z)}{[k\cos\theta + \psi'(z)]^2 + [E_a'(z)/E_a(z)]^2}\Bigg|_{z=0}. \tag{47}$$

Taking into account the approximate relation

$$\frac{d}{dz}\ln E_a(z) \simeq -\frac{1}{d} + \frac{d}{dz}\left[\frac{E_2(z)}{E_0(z)}\right] \tag{48}$$

and formulas (26), (27), (34), and (42), from Eqs. (46) and (47) we obtain

$$A = \frac{4k^3d^2\cos\theta}{1+(kd\cos\theta)^2}\int\limits_0^\infty dz\epsilon_2(z)e^{-2z/d}, \tag{49}$$

$$\begin{aligned}\phi = \pi + \arctan\Bigg\{&\frac{2kd\cos\theta}{1-(kd\cos\theta)^2}\left[1 - \frac{k^4d^2}{1-(kd\cos\theta)^2}\left[\int\limits_0^\infty dz\epsilon_2(z)e^{-2z/d}\right]^2\right.\\ &\left.+\frac{1+(kd\cos\theta)^2}{1-(kd\cos\theta)^2}k^2d\int\limits_0^\infty dze^{-2z/d}\left(\delta\epsilon_1(z) - \left[ke^{2z/d}\int\limits_z^\infty dz'\epsilon_2(z')e^{-2z'/d}\right]^2\right)\right]\Bigg\}.\end{aligned} \tag{50}$$

From relations (49) and (50), one can see that the phase shift of the reflected wave and the absorption coefficient depend on the explicit form of the effective frequency of the electron–electron collisions $\nu(z)$, which determines the corrections to permittivity $\epsilon_2(z)$ and $\delta\epsilon_1(z)$ [see (11)]. In metals, $\nu(z) = \nu_{\rm ep}+\nu_{\rm ee}(z)$, where $\nu_{\rm ep}$ is the frequency of the electron–phonon collisions and ν_{ee} is the frequency of the electron–electron collisions, which proceed with the umklapp of the quasimomentum. The frequency $\nu_{\rm ep}$ depending on the lattice temperature varies slowly on the scale of the skin layer. In contrast, the frequency $\nu_{ee} = a(\kappa T)^2/\hbar\epsilon_F$ [10], where $\hbar$ is Planck's constant, κ is the Boltzmann constant, ϵ_F is the Fermi energy, and a is a numerical coefficient, is proportional to the squared temperature of electrons $T(z)$; the last one may have a scale of inhomogeneity of the order of the skin-layer depth. In particular, under conditions of a high-frequency skin effect the evolution of the electron temperature in metals is described by the equation [8]

$$C\frac{\partial T}{\partial t} = \frac{4}{c}I(t)\nu(z)\exp\left(-\frac{2z}{d}\right) + \frac{\partial}{\partial z}q, \tag{51}$$

where $C = \pi^2N\kappa^2T/2\epsilon_F$ is the heat capacity of electrons at the electron density N, $I(t)$ is the intensity of radiation which heats electrons and slowly varies during time $2\pi/\omega$, and q is the density of the heat flux. As shown in [8], with efficient heat transfer from the skin layer, the solution to Eq. (51) is the function $T(z,t)$ slowly varying at a distance of the order of d as well as during the time $2\pi/\omega$. Herewith, with efficient heat transfer, the frequency $\nu(z)$ and the functions $\epsilon_2(z)$ and $\delta\epsilon_1(z)$ can be considered as approximately homogeneous in the skin layer and slowly varying during times much greater than $2\pi/\omega$.

Using the approximations $\epsilon_2(z) \simeq \epsilon_2(0)$ and $\delta\epsilon_1(z) \simeq \delta\epsilon_1(0)$, from Eqs. (45), (49), and (50) we obtain

$$R = \left.\frac{kd\cos\theta - i\left[1 - \frac{1}{2}(kd)^2(i\epsilon_2 + \delta\epsilon_1) + \frac{1}{8}(kd)^4\epsilon_2^2\right]}{kd\cos\theta + i\left[1 - \frac{1}{2}(kd)^2(i\epsilon_2 + \delta\epsilon_1) + \frac{1}{8}(kd)^4\epsilon_2^2\right]}\right|_{z=0}, \tag{52}$$

$$A = A_F = 2\frac{k^3d^3\cos\theta}{1 + (kd\cos\theta)^2}\epsilon_2(0), \tag{53}$$

$$\phi = \phi_F = \pi + \arctan\left\{\frac{2kd\cos\theta}{1 - (kd\cos\theta)^2}\left[1 - \frac{1}{8}\epsilon_2^2(0)\frac{k^4d^4}{1 - (kd\cos\theta)^2}\left(3 + k^2d^2\cos^2\theta\right) + \frac{1 + (kd\cos\theta)^2}{1 - (kd\cos\theta)^2}\frac{1}{2}(kd)^2\delta\epsilon_1(0)\right]\right\}. \tag{54}$$

Relations (52)–(54) are nothing else than the Fresnel formulas for metals under the condition of a high-frequency skin effect, when the ratio $\nu(z)/\omega$ and the spatial dispersion are small. According to (52)–(54), reflection and absorption are determined by the permittivity on the metal surface. Herewith, the corrections $\epsilon_2(0)$ and $\delta\epsilon_1(0)$ are slowly varying functions of time; they change with the evolution of the electron temperature in the skin layer.

The situation is different if heating of electrons in the skin layer is relatively fast and their cooling due to the heat transfer deep inside the metal can be neglected. Then, taking into account the quadratic dependence of ν on the electron temperature, from (51) we have

$$\frac{\nu(z,t)}{\nu(t_0)} = \exp\left[\alpha\exp\left(-\frac{2z}{d}\right)\right], \tag{55}$$

where the parameter α is determined by the integral of the intensity of the radiation heating the electrons

$$\alpha = \alpha(t) = \frac{16a}{\pi^2 N\hbar c}\int\limits_{t_0}^{t} dt' I(t'), \tag{56}$$

t_0 is the time moment when the heating pulse is switched on, and we assume that the frequency $\nu(t_0)$ does not dependent on the coordinate.

Relation (55) makes it possible to present the corrections ϵ_2 and $\delta\epsilon_1$ (11) in the form

$$\epsilon_2(z) = \epsilon_2(0)\exp\left\{\alpha\left[\exp\left(-\frac{2z}{d}\right) - 1\right]\right\}, \tag{57}$$

$$\delta\epsilon_1(z) = \delta\epsilon_1(0)\exp\left\{2\alpha\left[\exp\left(-\frac{2z}{d}\right) - 1\right]\right\}, \tag{58}$$

where $\epsilon_2(0)$ and $\delta\epsilon_1(0)$ are slowly varying functions of time. For the absorption coefficient (49) and phase shift of reflected wave (50), in view of (57) and (58), we obtain

$$A = A_F \frac{1}{\alpha}(1 - e^{-\alpha}) < A_F, \tag{59}$$

$$\begin{aligned}\tan\phi - \tan\phi_F &= \frac{1 + (kd\cos\theta)^2}{[1 - (kd\cos\theta)^2]^2}(kd)^3\delta\epsilon_1(0)\cos\theta\left[\frac{1}{2\alpha}(1 - e^{-2\alpha}) - 1\right] \\ &- \frac{(kd)^5\cos\theta}{[1 - (kd\cos\theta)^2]^2}\frac{1}{2}[\epsilon_2(0)]^2\left[\frac{1}{\alpha^2}(1 - e^{-\alpha})^2 - 1\right] - \frac{1 + (kd\cos\theta)^2}{[1 - (kd\cos\theta)^2]^2} \\ &\times \frac{1}{4}[\epsilon_2(0)]^2(kd)^5\cos\theta\left[\frac{2}{\alpha}e^{-2\alpha}\int_0^\alpha \frac{d\xi}{\xi}e^{\xi}(e^{\xi} - 1) - \frac{1}{\alpha^2}(1 - e^{-\alpha})^2 - 1\right].\end{aligned} \tag{60}$$

From (59), one sees that inhomogeneous heating of electrons leads to a relative decrease in the absorption coefficient as compared with the value A_F calculated using the Fresnel formula. The stronger the temperature inhomogeneity, the stronger the relative decrease in A. Since the ratio of the skin-layer depth d to the inverse wavenumber $1/k = \lambda/2\pi$, where λ is the wavelength, is of the order of $kd \sim \omega/\omega_p$, then at $\omega \ll \omega_p$ the main term in Eq. (60) is the summand proportional to $\delta\epsilon_1(0)$. The difference of the phase shift ϕ from ϕ_F described by the Fresnel formula is small. Herewith, the value ϕ is smaller than ϕ_F. The larger the temperature inhomogeneity along the skin layer, the greater the decrease in the phase shift.

5. Conclusions

The developed theory is of interest for considering metals with hot electrons, when the dominant event is scattering due to the electron–electron collisions proceeding with the quasimomentum umklapp. Studies of hot-electron states are topical in view of the large number of experiments on the interaction of high-power femtosecond laser pulses with metals. Such experiments realize comparatively easily the conditions where the electron temperature on the metal surface considerably exceeds that of the lattice, and the frequency of the electron–electron collisions, though it increases with time, still remains small as compared with the radiation frequency. Under these conditions, the newly obtained expressions for the absorption coefficient and phase shift of the reflected wave should be used instead of the Fresnel formulas.

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