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Multiobjective optimization under uncertainty in tunneling: application to the design of tunnel support/reinforcement with case histories

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Abstract

In this paper, two questions common in tunnel design are addressed: (i) how to choose an optimum solution when more than one conflicting objective must be achieved; (ii) how to deal with data affected both by imprecision and randomness. Fuzzy Set Theory and Random Set Theory are used to develop a general interactive multiobjective procedure, which is then applied to the design of tunnel support/reinforcement. A case history illustrates how the procedure was successfully used in the preliminary design of a total of 40 km of tunnels in Central Italy. It is shown that the procedure allows the designer to become a knowledgeable decision maker because his interaction is required at the key points of the process, and because the trade-offs among the objective functions can be easily assessed. The designer's personal input is valued and clearly defined in its impact on the solution. The case history demonstrates that, without an optimization procedure, it is extremely likely (probability of 99%) that a solution is chosen, which either increases the costs without increasing safety, or decreases the safety without decreasing the costs. Finally, it is shown that both imprecision and randomness can be easily taken into account in tunnel design.

Keywords: Tunnel design; Support/reinforcement; Uncertainty

1. Introduction

The design of underground excavations entails daily decision-making processes under uncertainty. In this paper, two classes of problems will be investigated among those experienced by tunnel engineers.

The first problem originates in the fact that the design objectives are generally conflicting (e.g. safety and cost), to the extent that to attain one of them is not possible without missing at least one of the others. A compromise must, therefore, be achieved, and this is often performed at an empirical level by resorting to the engineer's or somebody else's past experience or intuition. However, the high costs of underground facilities and their strategic importance for and impact on the future of a country (and often mankind at large) are easily recognized. Moreover, new and unprecedented underground facilities are being built or will be designed in the near future, for which no past-experience exists. Therefore, it is felt that a more rational approach to the design of underground excavations is needed (Bieniawski, 1993; Tonon, 1995; Pelizza, 1997).

Multiobjective optimization, which dates back to the late 1970s in its structural engineering applications (e.g. Baier, 1977; Gerasimov and Repko, 1978), allows the designer to formulate each objective separately, and the set of 'good' alternatives, called Pareto optimal, to be determined (the reader is referred to Koski (1994) for a review of this topic, and a recent example of application to tunneling can be found in Kalamaras et al., 2000). The problem of choosing just one design among the set of Pareto optimal solutions is central but not closed in this context, because it involves a good deal of subjectivity. The application of Fuzzy Set Theory to multiobjective optimization is therefore unsurprising (the reader is referred to Klir and Yuan, 1995 for an

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introduction to Fuzzy Set Theory, and to Appendix A for some basic definitions). In fact, since the early papers by Rao (1987a,b,c), a vast amount of literature has been devoted to this subject. Because interaction with the decision maker is deemed necessary, the authors developed an interactive approach to the problem in hand (Tonon, 1995; Tonon and Bernardini, 1999) as an extension to that described by Sakawa (1993). In this first class of problems, uncertainty arises from the lack of a precise definition for the concept labeled as ‘good compromise solution’. This is a semantic source of uncertainty (vagueness) as opposed to ambiguity, i.e. the uncertainty caused by lack of information on a precisely defined physical property (e.g. friction angle for a soil layer) (Klir, 1994).

This latter kind of uncertainty represents the second class of problems faced by the engineer, and was tackled by the authors in Tonon et al. (2000a,b) by means of Random Set Theory (Robbins, 1944, 1945; Kendall, 1974; Matheron, 1975; Dubois and Prade, 1989, 1991). Random Set Theory deals with information affected by both randomness (disagreement resulting from the attempt to classify an object into two or more disjoint subsets) and imprecision (variety of alternatives that are left unspecified in a given situation). On the contrary, Probability Theory deals only with randomness (Klir, 1989); for example, Kalamaras et al. (2000) apply probabilistic techniques to the convergence-confinement method. Appendix A gives some basic definitions of Random Set Theory.

After emphasizing the importance of imprecision in rock engineering, Tonon et al. (2000a,b) showed how imprecise (i.e. set-valued) data can be used to construct membership functions. These membership functions possess a clear probabilistic meaning, and bounds on the probability of an event can be easily determined. Consequently, in an optimization setting, bounds on the probability that the solution respects the constraints can be effectively calculated (Tonon and Bernardini, 1998, 1999). This approach is the natural extension to reliability-based optimization procedures (see, for example, Frangopol and Moses, 1994 for a review) when basic data on parameters are so scarce or imprecise that a probabilistic approach is not durable. It also encompasses the convex model of uncertainty (Ben-Haim and Elishakoff, 1990; Elishakoff, 1995) adopted by Elishakoff et al. (1994).

At the outset, this paper briefly recalls the interactive model for multiobjective optimization previously propounded by the authors (Tonon and Bernardini, 1999).

Subsequently, this model is applied to the design of tunnel reinforcement, support and lining by setting three objectives: the minimization of the cost incurred by the Contractor, of the cost incurred by the Client, and of the probability of failure.

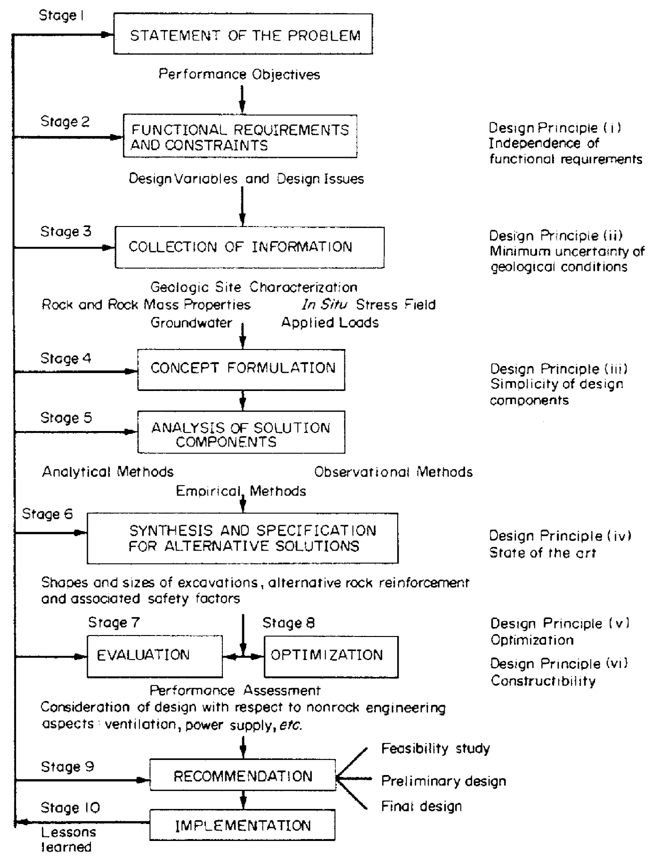


Fig. 1. Bieniawski’s design methodology and design principles. After Bieniawski (1993).

Finally, a case study is presented in which the authors’ interactive procedure was used in the preliminary design of four different route alternatives for State Road 77 Muccia-Colfiorito, Italy, involving approximately 40 km of tunnels.

2. Multiobjective optimization

2.1. The general context

Let us consider a typical design process for an underground excavation. In order to illustrate, let us refer to Bieniawski’s design methodology (1993) summarized in Fig. 1. In the present paper, it is assumed that the design objectives (Stage 1 in Fig. 1) as well as the design variables (Stage 2 in Fig. 1) can be mathematically quantified. It is believed the very process of defining objectives and design variables is very beneficial to the rationalization of the design process. A set of design variables defines a design solution (Stage 6 in Fig. 1), which has to satisfy some design constraints in order to be feasible. At this point, an evaluation and optimization process can be carried out (Stages 7 and 8).

For clarity, a multiobjective optimization procedure is firstly introduced for those design situations in which parameters are not affected by uncertainty. Next, the procedure is extended to encompass parameter uncertainty.

2.2. Multiobjective optimization with crisp parameters

2.2.1. Problem statement

Let us assume that the design problem can be mathematically formulated as follows:

$$\begin{aligned}&\text{minimize } f(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\right)\\ &\text{such that } g_i(\mathbf{x}) \leq 0 \quad i=1,\ldots,n\end{aligned} \tag{1}$$

where

- $\mathbf{x} \in R^s$ = vector of s design variables defining a design solution (e.g. points defining alignment and profile in space, steel set spacing, lining thickness, etc.).
- R = set of real numbers.
- $f_j(\mathbf{x})$:$R^s \to R$ = j-th objective function (e.g. construction cost for the Client, probability of failure, etc.).
- k = number of objective functions;
- $g_i(\mathbf{x})$:$R^s \to R$ = function defining the i-th constraint (e.g. pre-existing structures or geomorphological features along an alignment, maximum and minimum shotcrete thickness, etc.).
- n = number of constraints.

Let us define $X = \left\{\mathbf{x} | g_i(\mathbf{x}) \leq 0 \ \ i=1,\ldots,n\right\}$, $X \subset R^s$ the feasible set (Koski, 1994). The objective functions induce a space transformation $\mathbf{H}$: $R^s \to R^k$; let us define as *attainable criterion set* the image I of X through $\mathbf{H}$: $I = \left\{\mathbf{f}(\mathbf{x}) | \mathbf{x} \in X\right\}$, and as *criterion space* the space R^k in which I is embedded. As $f_j(\mathbf{x})$ varies on X, the set $Y_j = \left\{ y_j = f_j(\mathbf{x}) | \mathbf{x} \in X\right\}$ is described; Y_j is a compact set (a real interval) if X is compact and if $f_j(\mathbf{x})$ is continuous on X. The objective functions are often *globally conflicting* (Koski, 1994), i.e. they attain their minima at different points one from the other. Therefore, the optimality concept must be the Pareto one (Koski, 1994).

Definition 1. A vector $\mathbf{x}^* \in X$ is Pareto optimal for problem (1) if and only if there exists no $\mathbf{x} \in X$ such that $f_j(\mathbf{x}) \leq f_j(\mathbf{x}^*)$for $j=1,2,\ldots,k$ with $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one j.

It stems from Definition 1 that the set of Pareto optima is an (s-1)-dimensional sub-space of the design space R^s. Moreover, Definition 1 induces the following partial order relationship (Koski and Silvennoinen, 1987) on the attainable criterion set I:

$$\begin{aligned}&\forall \ \ P_1 = (f_{11}, f_{21}) \in I, P_2 = (f_{12}, f_{22}) \ \ \in I;\\ &P_1 \geq P_2 \ \text{ iff } \ f_{11} \leq f_{12} \ \text{ and } f_{21} \leq f_{22}\end{aligned} \tag{2}$$

Since I is only partially ordered, it is not possible to compare any attainable vector $\mathbf{f} \in I$ with all others (i.e. it is not possible to compare the performances of any solution with all others). Therefore, a further ordering criterion must be introduced if just one solution must be singled out, and this can be done solely on a subjective basis. The several methods proposed in the literature to solve multiobjective optimization problems (see Sakawa, 1993 and Koski, 1994 for a review) can be seen as attempts to obtain a further ordering on set I.

2.2.2. The interactive procedure

Given the imprecise nature inherent in human judgment in the ambit of multiobjective problems, Sakawa (1993) stresses that the decision maker may have a fuzzy goal expressed as '$f_j(\mathbf{x})$ should be substantially less than some value f_j^0'. This type of statement can be quantified by eliciting a corresponding membership function $\mu_j(f_j(\mathbf{x}))$, once the interval $Y_j = [f_j^{\min}, f_j^{\max}]$ has been determined by solving the following two optimization problems:

$$\underset{\mathbf{x} \in X}{\text{minimize}} \ f_j(\mathbf{x}) \tag{i}$$

$$\underset{\mathbf{x} \in X}{\text{maximize}} \ f_j(\mathbf{x}). \tag{ii}$$

The fuzzy set $M_j = \left\{\left(f_j, \mu_j(f_j)\right) | f_j \in Y_j\right\}$ is a mathematical model representing the degree of satisfaction of the attainable values of the j-th objective function (see Fig. 2a,b). It can be severe as in Fig. 2a, or permissive as in Fig. 2b; $\mu_j\left(f_j(\mathbf{x})\right) = 1$ expresses total satisfaction relative to $f_j(\mathbf{x})$, whereas $\mu_j\left(f_j(\mathbf{x})\right) = 0$ expresses total dissatisfaction relative to $f_j(\mathbf{x})$, and a continuous gradation can be modeled between these two extremes.

After the k membership functions $\mu_j\left(f_j(\mathbf{x})\right)$ have been determined, another space transformation $\mathbf{F}$: $\left(I \subset R^k\right) \to R^k$ is available, from the attainable criterion set I to the space of the membership functions. Let us define as *attainable membership set* the set $I' = \left\{\left(\mu_1(f_1), \mu_2(f_2), \ldots, \mu_k(f_k)\right) \ | \mathbf{f} \in I\right\}$, which is the image of I through $\mathbf{F}$, and define as *membership space* the space R^k in which I' is embedded. Fig. 3 summarizes these transformations.

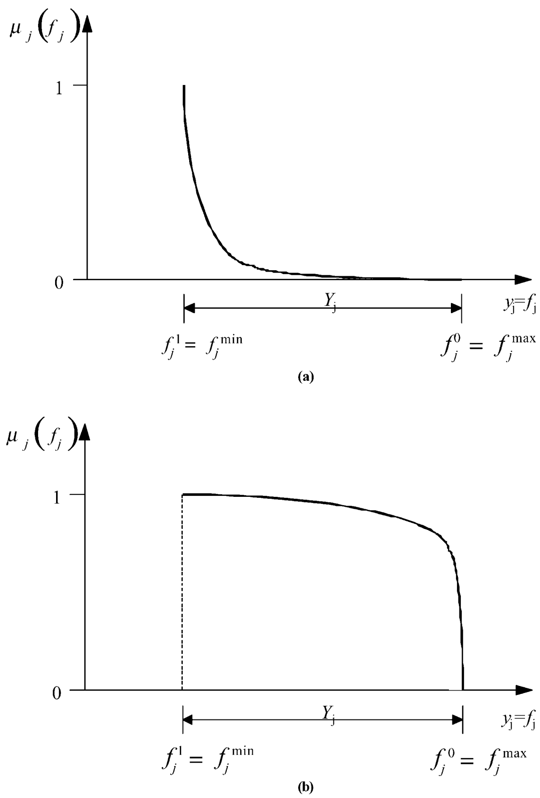


Fig. 2. Fuzzy sets modeling human judgement on the attainable values of the objective functions to be minimized: $\mu=1$ refer to total satisfaction; $\mu=0$ refers to total dissatisfaction; (a) severe judgment; (b) permissive judgment.

The optimality concept in the membership space comes from Sakawa (1993).

Definition 2. A vector $\mathbf{x}^* \in X$ is said to be an M-Pareto optimal solution to problem (1) if and only if there exists no $\mathbf{x} \in X$ such that $\mu_j(f_j(\mathbf{x})) \geq \mu_j(f_j(\mathbf{x}^*))$ for $j=1,2,...,k$ with $\mu_j(f_j(\mathbf{x})) > \mu_j(f_j(\mathbf{x}^*))$ for at least one j.

The relationships between M-Pareto solutions and Pareto solutions are discussed in Tonon and Bernardini (1999).

Transformation **F** 'stretches' set I in accordance with the decision maker's judgment and, at the same time, normalizes set I so that the final decision is independent both of the units used for quantifying the objective functions and of possible incommensurability between objective functions (e.g. cost and probability of failure). This indifference is not guaranteed by those methods that operate directly on the criterion space (e.g. Wierzbicki's method, 1979).

Moreover, every normalization formula proposed in the literature can be seen as a membership function $\mu_j(f_j(\mathbf{x}))$, and it is of the utmost importance to clearly point out the subjectivity of this step along with its consequences on the final decision (Tonon and Bernardini, 1999). Finally, it is evident that the membership function $\mu_j(f_j(\mathbf{x}))$ has no probabilistic meaning because it is a model of vagueness.

Now, it turns out quite natural to proceed as follows:

i. The feasible solution nearest (according to some metric) to $\bar{\mu}=(1,1,...,1)$ is searched for. If l_p metric (also called Minkowski distance, Kolmogorov and Fomin, 1961) is chosen, problem (1) becomes:

$$\underset{\mathbf{x}\in X}{\text{minimize}} \left(\sum \left| \mu_j(f_j(\mathbf{x})) - \bar{\mu}_j \right|^p \right)^{1/p} \text{ with } p \geq 1 \qquad (3)$$

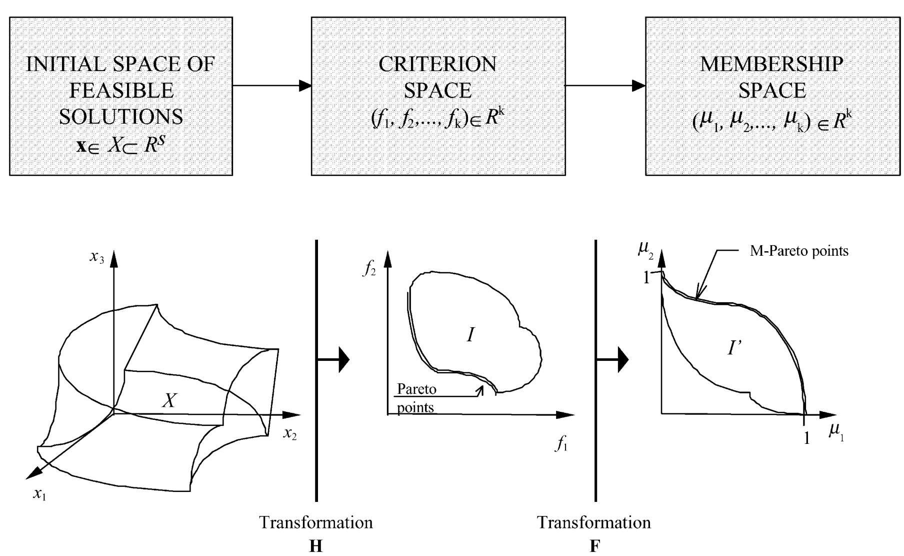


Fig. 3. Feasible space X mapped into the criterion space through the vector function **f** (transformation **H**) and into the membership space through the vector function μ (transformation **F**).

Obviously, if $\bar{\mu} \in I'$, the solution is $\bar{\mu}$ itself; in the following it is supposed that $\bar{\mu} \notin I'$. The conditions under which the nearest solution is an M-Pareto solution or a Pareto solution are discussed in Tonon and Bernardini (1999).

ii. If the solution to problem (3) is not satisfactory, aspiration levels of achievement $\bar{\mu} = (\bar{\mu}_1, \bar{\mu}_2, \ldots, \bar{\mu}_k)$ are specified for all the membership functions (Sakawa, 1993). This point $\bar{\mu}$ can be viewed as the natural extension to the reference point introduced by Wierzbicki (1979) in the criterion space.

2.3. Multiobjective optimization with fuzzy parameters

2.3.1. The need for incorporating imprecision

As discussed more extensively in Tonon et al. (2000a), imprecise probabilities are the natural consequence of set-valued measurements, which arise frequently:

1. Directly in real-world observations (geological or geomechanical surveys).
2. When statistical data are analyzed by means of histograms, even if measurements are point-valued (Bernardini, 1999).
3. When lack of direct experimental data forces one to resort to an expert's (or experts') opinion.

The latter is a very common occurrence in rock engineering (e.g. p. 143 in Einstein, 1996, and p. 58 in Einstein and Baecher, 1983), and, if Probability Theory is used (see Roberds, 1990 for the problems involved in this process), it leads to subjective probabilities (Savage, 1972). However, we know, through inspection, that our beliefs about many matters are indeterminate (p. 4 in Walley (1991)). When a group of experts is available (as often happens in major engineering projects), disagreement between individuals automatically generates imprecision, even if each expert had precise beliefs. The criticism against the additive axiom in subjective probabilities is a well-known issue (Fine, 1973) with historical roots (Shafer, 1978).

2.3.2. General formulation

As discussed in Section 1, it is assumed that:

1. Parameters are affected by both imprecision and randomness.
2. Fuzzy Sets are constructed as detailed in Tonon et al. (2000a) in order to take into account both imprecision and randomness.
3. Operations on parameters restricted by Fuzzy Sets are carried out as detailed in Tonon et al. (2000b) in order to preserve the probabilistic meaning of the basic information.

Let $\mathbf{v}_j$ denote a vector of parameters affecting the j-th objective function:

$$\mathbf{v}_j = \{v_{j,1}, \ldots, v_{j,d_j}\} \tag{4}$$

In Eq. (4), the allowed values of a parameter $v_{j,r}$ $r = 1, \ldots, d_j$ are restricted by a fuzzy number or a fuzzy relation $E_{j,r}$. Similarly, the parameters affecting the i-th constraint are ordered in a vector $\mathbf{u}_i$:

$$\mathbf{u}_i = \{u_{i,1}, \ldots, u_{i,k_i}\} \tag{5}$$

where the allowed values of $u_{i,r}$ $r = 1, \ldots, k_i$ are restricted by a fuzzy number or a fuzzy relation $F_{i,r}$.

The multiobjective optimization problem can be stated as follows:

$$\begin{aligned} &\underset{\mathbf{x}}{\text{minimize}}\ f_1(\mathbf{x}, \mathbf{v}_1), \ldots, f_k(\mathbf{x}, \mathbf{v}_k) \\ &\text{such that}\ \ g_i(\mathbf{x}, \mathbf{u}_i) \le 0 \quad i = 1, \ldots, n \end{aligned} \tag{6}$$

Following the proposal developed in Tonon and Bernardini (1998, 1999), an α-level, β_j, is chosen for each objective function, and an α-level, α_i, is chosen for each constraint. Then, each vector $\mathbf{v}_j$ is allowed to vary in a set Ξ_j, and every vector $\mathbf{u}_i$ is allowed to vary in a set Ψ_i, obtained as Cartesian products of the relevant α-cuts:

$$\Xi_j = \{E_{j,1}(\beta_j) \mathrm{x}, \ldots, \mathrm{x} E_{j,d_j}(\beta_j)\} \tag{7}$$

$$\Psi_i = \{F_{i,1}(\alpha_i) \mathrm{x}, \ldots, \mathrm{x} F_{i,k_i}(\alpha_i)\} \tag{8}$$

where $E_{j,r}(\beta_j)$ denotes the α-cut of level β_j of fuzzy set $E_{j,r}$ and $F_{i,r}(\alpha_i)$ denotes the α-cut of level α_i of fuzzy set $F_{i,r}$. In general, Ξ_j and Ψ_i are not convex sets.

The multiobjective optimization problem (6) can now be restated as:

$$\begin{aligned} &\underset{\mathbf{x},\mathbf{y}}{\text{minimize}}\ \ \mathbf{y} = (y_1, \ldots, y_k) \\ &\text{such that} \begin{cases} g_i(\mathbf{x}, \mathbf{u}_i) \le 0 \quad \forall \mathbf{u}_i \in \Psi_i; \quad i = 1, \ldots, n \\ -y_j + f_j(\mathbf{x}, \mathbf{v}_j) \le 0 \quad \forall \mathbf{v}_j \in \Xi_j; \quad j = 1, \ldots, k \end{cases} \end{aligned} \tag{9}$$

In formulation (9), the concept of anti-optimization conceived by Elishakoff (1995) has been adopted because the worst values of the parameters are considered for both the objective functions and the constraints. For every design $\mathbf{x}^*$, when $\mathbf{v}_j$ varies in the set Ξ_j, the following interval is defined:

$$\Gamma_j(\mathbf{x}^*) = [y_j^L(\mathbf{x}^*), y_j^R(\mathbf{x}^*)] = \left[\min_{\mathbf{v}_j \in \Xi} f_j(\mathbf{x}^*, \mathbf{v}_j),\ \max_{\mathbf{v}_j \in \Xi} f_j(\mathbf{x}^*, \mathbf{v}_j)\right] \tag{10}$$

It stems from Eq. (9) that the right extreme, $y_j^R(\mathbf{x}^*)$, must be minimized.

Likewise, for a given design $\mathbf{x}^*$, the interval:

$$[h_i^L(\mathbf{x}^*), h_i^R(\mathbf{x}^*)] = \left[\min_{\mathbf{u}_i \in \Psi_i} g_i(\mathbf{x}^*, \mathbf{u}_i), \max_{\mathbf{u}_i \in \Psi_i} g_i(\mathbf{x}^*, \mathbf{u}_i)\right] \tag{11}$$

is described when $\mathbf{u}_i$ varies in the set Ψ_i. According to the formulation in Eq. (9), the solution $\mathbf{x}^*$ is feasible if and only if:

$$h_i^R(\mathbf{x}^*)\le 0 \tag{12}$$

and, consequently, the feasible set is defined as:

$$X=\left\{\mathbf{x}\ :h_i^R(\mathbf{x}^*)\le 0,\ i=1,\ldots,n\right\} \tag{13}$$

With this notation in hand, Problem (9) can then be rewritten as follows:

$$\begin{aligned}&\underset{\mathbf{x}}{\text{minimize}}\ \left(y_1^R(\mathbf{x}),\ldots,y_k^R(\mathbf{x})\right)\\ &\text{such that}\ h_i^R(\mathbf{x})\le 0\quad i=1,\ldots,n\end{aligned} \tag{14}$$

Problem (14) is identical to Problem (1) with the substitutions:

$$f_j\to y_j^R;$$

$$g_i\to h_i^R$$

When these substitutions are made, Definitions 1 and 2 still hold, and Problem (14) can be solved by means of the interactive procedure described in Section 2.2 (the reader is referred to Tonon and Bernardini, 1999, for the implementation details).

2.3.3. Probability that a solution satisfies the constraints

Let $\mathbf{x}^*$ be a given design. As shown in Tonon and Bernardini (1998, 1999), the probability that the actual value of $f_j(\mathbf{x}^*,\mathbf{v}_j)$ belongs to $\Gamma_j(\mathbf{x}^*)$, is bounded by the following inequalities:

$$\begin{aligned}\text{Nec}=1-\beta_j\le\text{Pro}\Big(&y_j^L(\mathbf{x}^*)\\ &\le f_j(\mathbf{x}^*,\mathbf{v}_j)\le y_j^R(\mathbf{x}^*)\Big)\le 1=\text{Pos}\quad j=1,\ldots,k\end{aligned} \tag{15}$$

where Nec is called Necessity and Pos is called Possibility (see Appendix A for basic definitions). Similarly, the probability that the actual value of $g_i(\mathbf{x}^*,\mathbf{u}_i)$ is smaller than $h_i^R(\mathbf{x}^*)$, is bounded by the following inequality:

$$\begin{aligned}&\text{Nec}=1-\alpha_i\le\text{Pro}\Big(g_j(\mathbf{x}^*,\mathbf{u}_i)\le h_i^R(\mathbf{x}^*)\Big)\le 1=\text{Pos}\\ &\quad i=1,\ldots,n\end{aligned} \tag{16}$$

Since $h_i^R(\mathbf{x}^*)\le 0$ [see Eq. (14)], the bounds in Eq. (16) are also the bounds on the probability that the i-th constraint is actually respected.

Inequalities (15) and (16), which can be demonstrated by means of Random Set Theory and Possibility Theory, stress the probabilistic meaning of the fuzzy sets which restrict the values of the uncertain parameters. Therefore, it is evident that this meaning of fuzzy sets is radically different from that of fuzzy sets M_j used in Section 2.2, where the judgment on an ill-defined concept was to be quantified. Here, one simply wants to extend the imprecise and dissonant information on the parameters to the solution of the optimization problem. The presence of upper and lower bounds on the probability of an event in Eqs. (15) and (16) is a direct consequence of the imprecision affecting the data on the parameters.

Inequalities (15) and (16) hold that, by choosing the α-levels β_j and α_i, the lower bound on the probability (the Necessity) is chosen, whereas the upper bound (the Possibility) is always equal to 1. Therefore, by setting the α-levels β_j and α_i, the designer can establish the level of the lower bound of the probability that a constraint is respected or that an objective function is minimized. This is similar to reliability based optimization techniques (Frangopol and Moses, 1994), in which one chooses the probability that a constraint is respected or that an objective function is minimized. However, the effectiveness of the numerical procedures and their robustness are dramatically greater in the present approach (Tonon and Bernardini, 1998, 1999).

2.3.4. Reliability as an objective function

In general, every design $\mathbf{x}\in R^s$ satisfies the i-th constraint up to a threshold value of Necessity:

$$N_i^{th}(\mathbf{x})=1-\alpha_i^{th}(\mathbf{x}) \tag{17}$$

The designer can decide to maximize this lower bound, i.e. to minimize $\alpha_i^{th}(\mathbf{x})$, which becomes a new objective function. This is also similar to reliability optimization techniques in which the reliability of a constraint is maximized (Frangopol and Moses, 1994).

Let us suppose that the designer wants to maximize the reliability relative to the first m constraints ($m\le n$); then the optimization problem reads:

$$\begin{aligned}&\underset{\mathbf{x}}{\text{minimize}}\left(y_1^R(\mathbf{x}),\ldots,y_k^R(\mathbf{x}),\alpha_1^{th}(\mathbf{x}),\ldots,\alpha_m^{th}(\mathbf{x})\right)\\ &\text{such that}\ h_i^R(\mathbf{x})\le 0\quad i=m+1,\ldots,n\end{aligned} \tag{18}$$

If these first m constraints are all considered equally important (e.g. all of them refer to ultimate limit states), then one can define:

$$\alpha^{th}(\mathbf{x})=\max\left\{\alpha_1^{th}(\mathbf{x}),\ldots,\alpha_m^{th}(\mathbf{x})\right\} \tag{19}$$

and restate problem (18) as:

$$\begin{aligned}&\underset{\mathbf{x}}{\text{minimize}}\left(y_1^R(\mathbf{x}),\ldots,y_k^R(\mathbf{x}),\alpha^{th}(\mathbf{x})\right)\\ &\text{such that}\ h_i^R(\mathbf{x})\le 0\quad i=m+1,\ldots,n\end{aligned} \tag{20}$$

Problem (20) is similar to Problem (14), and thus can be solved by means of the interactive procedure described in Section 2.2. The reader is referred to Tonon and Bernardini (1999) for the details on the solution strategy for the optimization problem in Eq. (20).

3. Application to tunnel support/lining design

The procedures described in Section 2 are applied to the design of the support/reinforcement and lining of tunnels in rock (Tonon, 1995; Tonon et al., 1999; Mammino and Tonon, 1997). With reference to Bieniawski's design methodology in Fig. 1, at Stage 4 the design concept is formulated. To illustrate, in this paper, three concepts are used:

1. Rockbolts and shotcrete + final lining.
2. Steel sets and shotcrete + final lining.
3. Rockbolts, steel sets and shotcrete + final lining.

The method of analysis chosen (Stage 5 in Fig. 1) is the convergence-confinement method (Hoek and Brown, 1980; Panet, 1995; Mammino and Tonon, 1997), which implies that the rock–structure interaction problem is axisymmetric. It is, thus, assumed that an equivalent continuum model is appropriate for the rock mass, which does not exhibit swelling or squeezing phenomena, and for which the long-term strength concept can be applied (Ladanyi, 1974; Gill and Ladanyi, 1986). The convergence curve formulation is that propounded in Brown et al. (1983) for elasto-brittle-plastic behavior of the rock mass. The initial convergence prior to installation of the primary support is determined as proposed by Corbetta et al. (1991) and Nguyen-Minh and Guo (1996). The support/lining confinement curve and design resistance are evaluated as explained in Mammino and Tonon (1997) and summarized in Appendix B.

It must be emphasized that any ground–structure interaction model can be used within the proposed procedure, and that the convergence-confinement method was chosen in order to illustrate in simple terms the application to tunnel support/lining.

The uncertain parameters (Stage 3 in Fig. 1) are the initial Geological Strength Index (GSI) (Hoek and Brown, 1997), the lithostatic pressure p_0, and a parameter K which multiplies the initial value of GSI to obtain a long-term value for GSI. Initial and long-term rock mass strength and deformability parameters are derived from the initial and long-term GSI values, respectively, by means of correlation (Hoek and Brown, 1997).

The objective functions chosen are three: the cost incurred by the Contractor, the cost incurred by the Client, and the probability of failure.

3.1. Design variables

Four structural elements are considered: steel sets, shotcrete, rockbolts, and final cast-in-place concrete lining. These elements are defined by six design variables, x_1 through x_6:

a. Steel sets: x_1 = section type; x_2 = cross-sectional depth; x_3 = spacing along the tunnel axis.
b. Shotcrete: x_4 = thickness.
c. Ungrouted rockbolts: x_5 = rockbolt spacing (a square grid is chosen in order to reduce the number of design variables).
d. Final cast-in-place concrete lining: x_6 = thickness.

Components a, b, c form the primary support. All components are arranged in three different combinations:

- Combination 1 is composed of components a + b + d. This combination is defined by four design variables: x_1, x_2, x_3, x_6.
- Combination 2 is composed of components b + c + d. This combination is defined by three design variables: x_4, x_5, x_6.
- Combination 3 is composed of components a + b + c + d. This combination is defined by five design variables: x_1, x_2, x_3, x_5, x_6.

In combinations 1 and 3, the shotcrete thickness is not included. This is because the shotcrete thickness is assumed to be equal to the steel set cross-sectional depth so as to allow an easy installation of the geofabric and of the waterproofing membrane before casting the final lining.

3.2. Constraints

Two groups of constraints have been considered: physical-geometrical constraints and safety constraints.

3.2.1. Physical-geometrical constraints

Rockbolt spacing cannot be less than 0.5 m because a smaller spacing is both impossible from a constructional viewpoint and would eventually increase the rock fracturing. Hoek and Brown (1980) recommend that the maximum rockbolt spacing must be equal to the minimum of the following three quantities:

- Half the rockbolt length.
- 1.5 times the average joint spacing.
- 2 m (this is also necessary in order to support the welded wire fabric, if present)

However, it must be realized that the second requirement is not valid when the joint spacing is in the order of 0.2–0.3 m or less because the rockbolt spacing would be less than 0.5 m. Therefore, the second requirement is used only when the joint spacing is larger than 0.5 m.

Finally, from a practical viewpoint, rockbolt spacing can only take discrete values with a 0.1-m step. Summarizing, the constraints relative to rockbolts read as follows (in the following, lengths are given in mm):

$$\begin{cases} g_1: x_5 \geq 500 \text{ mm} \\ g_2: x_5 \leq i_{\max} \\ g_3: i_{\max} = \min\left\{\frac{l}{2}, (1.5 \cdot \text{joint spacing, if joint spacing} \geq 500 \text{ mm}), 2000 \text{ mm}\right\} \\ g_4: x_5 = 500 + 100 \cdot n, \quad n = 0,1,2... \end{cases} \tag{21}$$

The steel set spacing is constrained as follows:

$$\begin{cases} g_5: x_3 \geq 500 \text{ mm} \\ g_6: x_3 \leq 1500 \text{ mm} \\ g_7: x_3 = 500 + 250 \cdot n, \quad n = 0,1,2,... \end{cases} \tag{22}$$

The steel set type is to be chosen among the Italian I-beam sections IPE, HEA, HEB:

$$g_8: \; x_1 \in \{\text{IPE,HEA,HEB}\} \tag{23}$$

The steel set cross-sectional depth is constrained within the limits:

$$\begin{cases} g_9: x_2 \geq 100 \text{ mm} \\ g_{10}: x_2 \leq 200 \text{ mm} \\ g_{11}: x_2 = 100 + 20 \cdot n, \quad n = 0,1,2,... \end{cases} \tag{24}$$

The shotcrete thickness must be comprised between the two limits:

$$\begin{cases} g_{12}: \; x_4 \geq 50 \text{ mm} \\ g_{13}: \; x_4 \leq 200 \text{ mm} \end{cases} \tag{25}$$

If steel sets are used, the following constraint g_{14} must be respected:

$$g_{14}: \; x_4 = x_2 \tag{26}$$

Else, the following constraint g_{15} must be respected:

$$g_{15}: x_4 \in \{50, \; 100, \; 150, \; 200\} \tag{27}$$

The minimum thickness for the cast-in-place lining cannot be less than 300 mm in order to allow for a proper concrete pouring. Therefore, the cast-in-place lining thickness is constrained as follows:

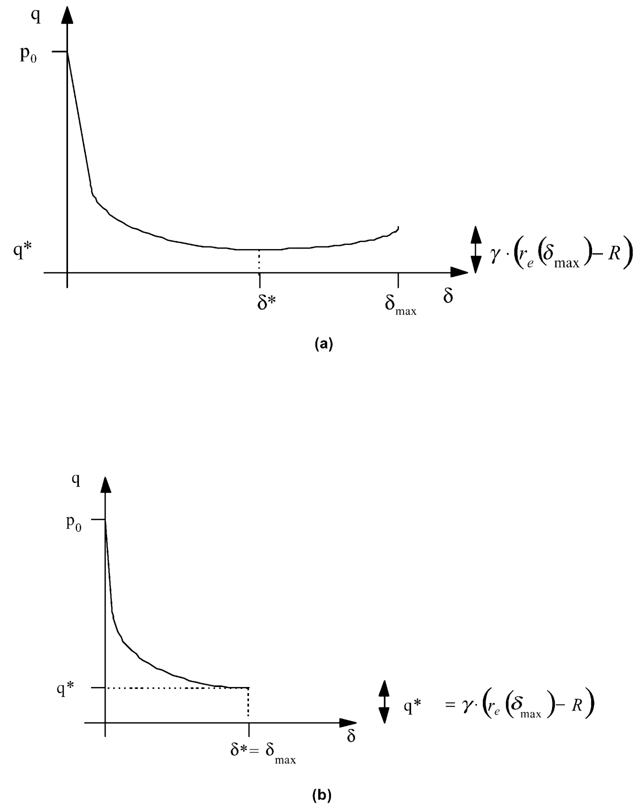


Fig. 4. (a) Convergence curve with an internal minimum. (b) Convergence curve without an internal minimum. q = pressure exerted by the rock mass onto the lining; δ = tunnel wall displacement; p_0 = in situ lithostatic stress; γ = broken rock mass unit weight; r_e = plastic radius; R = tunnel radius.

$$\begin{cases} g_{16}: x_6 \geq 300 \text{ mm} \\ g_{17}: x_6 \leq 1000 \text{ mm} \\ g_{18}: x_6 = 300 + 50 \cdot n, \quad n = 0,1,2,... \end{cases} \tag{28}$$

3.2.2. Safety constraints

Two kinds of safety constraints have been deemed appropriate for the design of tunnels:

1. Short-term constraints relative to the primary support, which is loaded because of the removal of the three-dimensional arching effect of the tunnel face. This is a construction phase.
2. Long-term constraints relative to both primary support and final lining, which are loaded by the relaxation of the rock mass around the tunnel. This is a service phase.

3.2.2.1. Short-term safety constraints. The first short-term safety constraint makes sure that the primary support/lining is not too deformable, and dangerous loosening of the rock mass does not occur (Brox and Hagedorn, 1999). Modeling of this phenomenon is very difficult, and its present understanding very poor.

In the present formulation, it is simplistically assumed that the weight of the rock mass in the plastic annulus is supported by the primary support/lining (Hoek and Brown, 1980). As a consequence, the convergence curve may (Fig. 4a) or may not (Fig. 4b) display an internal minimum. In either case, the intersection with the confinement curve must fall to the left of the minimum, and this is ensured by the following constraint (Mammino and Tonon, 1997):

$$g_{19}:\ \delta_0\left(d,p_{i_I}\right)+\frac{q^{*}\cdot R}{k_{t,i}}\leq\delta^{*}\ t,i=1,i\ ;2,i\ ;3,i \tag{29}$$

where:

- q^* = minimum pressure exerted on the lining calculated using the short-term values of the rock mass strength and deformability parameters;
- δ^* = tunnel wall convergence associated with the minimum pressure q^*;
- δ_0 = tunnel wall convergence before installation of the primary support/lining stiffness [see Eq. (B.3)];
- R = tunnel radius;
- k = primary support/lining stiffness calculated as described in Appendix B;
- subscript t refers to combination 1, 2 or 3; and
- subscript i refers to primary support/lining.

The second short-term safety constraint ensures that equilibrium is reached between the rock mass around the tunnel and the primary support/lining:

$$g_{20}:\ \delta_{tun}\left(p_{i_I}\right)=\delta_0\left(d,p_{i_I}\right)+\frac{p_{i_I}+\gamma\cdot\left(r_e\left(p_{i_I}\right)-R\right)}{k_{t,i}}\cdot R \tag{30}$$

where:

- p_{i_I} = pressure exerted by the primary support/lining on the rock mass excluding the broken rock contribution;
- γ = yielded rock unit weight;
- $r_e\left(p_{i_I}\right)$ = short-term plastic radius [see Eq. (B.2)]; and
- $\delta_{tun}\left(p_{i_I}\right)$ = short-term convergence curve [(see Eq. (B.1)].

The third short-term safety constraint ensures that the pressure exerted on the lining is not greater than the design capacity of the primary support/lining, $q_{t,i}$:

$$g_{21}:\ p_{i_I}+\gamma\cdot\left(r_e\left(p_{i_I}\right)\ -R\right)\leq q_{t,i} \tag{31}$$

In accordance with EuroCodes and the Italian National Code, the design capacity of every structural element contributing to $q_{t,i}$ is quantified as follows (see Appendix B):

$$\text{design capacity}=\frac{\text{characteristic resistance (time-dependent for elements } b \text{ and } c)}{\text{partial safety factor (phase-dependent)}} \tag{32}$$

3.2.2.2. Long-term safety constraints. The constraints for long-term safety are similar in concept to their short-term counterparts. The deformability constraint reads:

$$g_{22}:\ \delta_I+\frac{\left(q^{*}_{\infty}-q_I\right)\cdot R}{k_{t,\infty}}\leq\delta^{*}_{\infty} \tag{33}$$

where:

- q^*_∞ = minimum pressure exerted on the lining calculated using the long-term values of the rock mass strength and deformability parameters;
- δ^*_∞ = tunnel wall convergence associated with q^*_∞;
- $\delta_I=\delta_{tun}\left(p_{i_I}\right)$ = short-term tunnel wall convergence;
- $q_I=p_{i_I}+\gamma\cdot\left(r_e\left(p_{i_I}\right)\ -R\right)$: short-term pressure exerted on the primary support/lining; and
- $k_{t,\infty}$ = long-term stiffness of support/lining combination t.

The long-term equilibrium constraint is as follows:

$$g_{23}:\ \delta_{tun\infty}\left(p_{i_{II}}\right)=\delta_I+\frac{\left[p_{i_{II}}+\gamma\cdot\left(r_{e\infty}\left(p_{i_{II}}\right)\ -R\right)\right]\ -q_I}{k_{t,\infty}}\cdot R \tag{34}$$

- $p_{i_{II}}$ = long-term pressure exerted by the lining on the rock mass excluding the broken rock contribution
- $r_{e\infty}\left(p_{i_{II}}\right)$ = long-term plastic radius [see Eq. (B.2)]
- $\delta_{tun\infty}\left(p_{i_{II}}\right)$ = long-term convergence curve [see Eq. (B.1)]

The long-term capacity constraint reads:

$$g_{24}:\ p_{i_{II}}+\gamma\cdot\left(r_e\left(p_{i_{II}}\right)\ -R\right)\leq q_{t,\infty} \tag{35}$$

where $q_{t,\infty}$ is the long-term capacity of the support/lining, whose element contribution is calculated by means of Eq. (32).

3.3. Objective functions

The economic objective towards the cost minimization is two-fold, as on the one side, the cost incurred by the Client (first objective function) must be minimized, and on the other, the cost met by the Contractor (second objective function) is of concern. Both of these objective functions were fitted on a market research basis as described below.

Since all safety constraints are affected by fuzzy parameters and they all refer to ultimate limit states, the lower bound of the probability of failure is the threshold value $\alpha^{\text{th}}(\mathbf{x})$ defined in Eq. (19), which summarizes the safety of the support/lining. This lower bound is the third objective function to be minimized.

3.3.1. Cost incurred by the Client

The costs of the structural elements from *a* to *d*, which are defined by the design variables as explained in Section 3.1, are evaluated as follows on the basis of available price lists.

3.3.1.1. Steel sets. The unit price, *p1cen*, comprises the steel beam, bending operations, steel plates, nuts and bolts, braces, and installation. The cost per tunnel meter length is:

$$L1cen = p1cen \cdot A \cdot 10^{-6} \cdot \rho \cdot \left(\frac{6.28 \cdot R}{x_3}\right) \tag{36}$$

where:

- A = steel set cross-sectional area in mm^2 (which is a function of design variables x_1 and x_2);
- $\rho = 7950$ kgf/m^3 = steel unit weight;
- R is the tunnel radius in mm; and
- x_3 = steel set spacing in mm

The market unit price (updated June 1995) is *p1cen* = 2000 £/kgf.

3.3.1.2. Shotcrete. The unit shotcrete price is relative to a square meter, and is available for 100-mm and 200-mm thickness. Unit price includes shotcrete production, steel fibers, admixtures, and installation. Let:

- p_2 = price per m^2 of a 100-mm thick shotcrete; and
- p_3 = price per m^2 of a 200-mm thick shotcrete.

The interpolated cost of shotcrete is thus:

$$c1s' = p_2 + \frac{p_3 - p_2}{100}(x_4 - 100) \tag{37}$$

where x_4 is the shotcrete thickness in mm.

However, shotcrete rebound and the need for filling voids and irregularities of the rock surface ('lost shotcrete') account for an average 100% increase in the shotcrete volume actually used. Therefore, the actual cost per square meter of tunnel wall is:

$$c1s = 2 \cdot \left[p_2 + \frac{p_3 - p_2}{100}(x_4 - 100)\right] \tag{38}$$

Finally, the cost per tunnel meter is:

$$L1s = 2 \cdot \left[p_2 + \frac{p_3 - p_2}{100}(x_4 - 100)\right] \cdot \left(6.28 \cdot \frac{R}{1000}\right) \tag{39}$$

The market unit prices (updated June 1995) are:

- p_2 = 38 000 £/m^2
- p_3 = 63 000 £/m^2

It is to be noted that the price per cubic meter increases as the thickness decreases (380 000 £/m^3 for a 100-mm thickness, 315 000 £/m^3 for a 200-mm thickness). This is due to the greater impact of the 'lost' shotcrete on the smaller thickness.

3.3.1.3. Rockbolts. The available price list gives rockbolt prices (including fabrication, delivery and installation) for three rockbolt lengths, i.e. 3 m, 4.5 m and 6 m. By interpolating these values, the following cost per rockbolt is obtained:

$$\begin{aligned} c1b = {} & \frac{p2b - 2 \cdot p3b + p4b}{4.5} \cdot \left(\frac{L}{1000}\right)^2 \\ & + \frac{-3.5 \cdot p2b + 6 \cdot p3b - 2.5 \cdot p4b}{1.5} \cdot \frac{L}{1000} + 6 \\ & \cdot p2b - 8 \cdot p3b + 3 \cdot p4b \end{aligned} \tag{40}$$

where:

- L = rockbolt length in mm;
- $p2b$ = cost of a 3-m-long rockbolt;
- $p3b$ = cost of a 4.5-m-long rockbolt; and
- $p4b$ = cost of a 6-m-long rockbolt.

The cost per meter of tunnel is:

$$L1b = \frac{c1b}{x_5^2} \cdot 6.28 \cdot R \cdot 1000 \tag{41}$$

where the rockbolt grid spacing, x_5, and the tunnel radius, R, are expressed in mm.

The market unit prices (updated June 1995) are:

- $p2b$ = 105 000 £;
- $p3b$ = 125 000 £; and
- $p4b$ = 178 000 £.

It is to be noted that the price per unit length of a rockbolt is minimum for a length of 4.5 m. This can be easily justified by realizing that, for smaller lengths, there is a greater impact of the cost for equipment movement, whereas for greater lengths installation and grouting costs prevail. However, this trend was not observed in the analysis of the Contractor's costs.

3.3.1.4. Cast-in-place lining. The price list available reports the following two unit prices:

- $p3c$ = concrete cost per cubic meter (production, delivery, and pumping into formwork); and
- $p4c$ = mechanized formwork cost per m^2 of tunnel wall (production, delivery, assembly, and movement)

The cost per meter of tunnel is:

$$L1c = \left(p3c \cdot \frac{x_6}{1000} + p4c\right) \cdot \left(6.28 \cdot \frac{R}{1000}\right) \tag{42}$$

where x_6 is the lining thickness in mm.

The market unit prices (updated June 1995) are:

- $p3c$ = 130 000 £/m^3; and
- $p4c$ = 30 000 £/m^2.

3.3.2. Cost incurred by the Contractor

The cost estimates described below were obtained by means of research with specialty contractor TREVI s.p.a. (Cesena, Italy).

3.3.2.1. Steel sets. The cost of 1 kgf of finished steel set is composed of the two following components:

1. Cost N1 for 1 kgf of steel sets delivered to the construction yard (comprising bending, welding, nuts and bolts, steel plates, accessories, etc.).
2. Cost N2 for delivering 1 kgf of steel sets from the construction yard to the tunnel face, installing the steel sets, and backfilling.

The cost per meter of tunnel is:

$$L2cen = p2cen \cdot A \cdot 10^{-6} \cdot \rho \cdot \left(\frac{6.28 \cdot R}{x_3} \right) \tag{43}$$

where $p2cen = \mathrm{N1} + \mathrm{N2}$.

The market values (updated June 1995) are: N1 = 1100 £/kgf, N2 = 350 £/kgf.

3.3.2.2. Shotcrete. The material costs for producing one cubic meter of shotcrete are itemized as follows:

1. cement: M1 = [quantity (500 kg/m^3)]·[unit price (105 £/m^3)] = 500·105 = 52 500 £/m^3;
2. sand (grain size less than 4 mm): M2 = [quantity (0.8 m^3/m^3)]·[unit price (30 000 £/m^3)] = 0.8·30 000 = 24 000 £/m^3;
3. pea gravel (grain size between 4 and 8 mm): M3 = [quantity (0.4 m^3/m^3)]·[unit cost (25 000 £/m^3)] = 0.4·25 000 = 10 000 £/m^3;
4. accelerating admixture: M4 = [quantity (60 l/m^3)]·[unit cost (500 £/l)] = 60·500 = 30 000 £/m^3; and
5. steel fibers: M5 = [quantity (50 kg/m^3)]·[unit cost (1900 £/kg)] = 50·1900 = 95 000 £/m^3.

Production costs are as follows:

1. Labor cost: M6 = [Number of workers per crew (4)]·[h/m^3 (1/7)]·[£/h (50 000 £/h)] = 4·(1/7) ·50 000 = 28 571 £/m^3; and
2. Equipment: M7 = 10 000 £/m^3.

The total cost for one cubic meter of shotcrete is:

$$p1s = \sum_{i=1}^{6} Mi = 250\,071 \quad £/\mathrm{m}^3 \approx 250\,000 \quad £/\mathrm{m}^3 \tag{44}$$

Based on the Contractor's experience, the percentage of lost shotcrete is equal to 125% for a 50-mm thickness, and 75% for a 200-mm thickness. By interpolating between these two values, the cost for one meter of tunnel is:

$$L2s = \left(p1s \cdot \frac{36.25 - 0.05 \cdot x_4}{15} \cdot \frac{x_4}{1000} \right) \cdot \left(6.28 \cdot \frac{R}{1000} \right) \tag{45}$$

3.3.2.3. Rockbolts. The cost for one meter of rockbolts is composed as follows:

1. Cost *k1* for 1 m drilling (2 in diameter).
2. Cost *k2* for 1 m of rebar (24 mm diameter), comprising cutting, threading, steel plate and nut.
3. Cost *k3* for bond length created using either resin or cement grout.

The total cost per one meter of tunnel is:

$$L2b = \frac{c2b \cdot L}{x_5^2} \cdot 6.28 \cdot R \cdot 1000 \tag{46}$$

where $c2b = k1 + k2 + k3$.

The market values (updated June 1995) are: *k1* = 20 000 £/m, *k2* = 2000 £/m, *k3* = 3000 £/m.

3.3.2.4. Cast-in-place lining. The cost of 1 m^3 of concrete is itemized as follows:

1. Cost J1 for 1 m^3 of concrete, cylindrical characteristic strength = 25 MPa: J1 = 90 000 £/m^3.
2. Cost J2 for pouring and vibrating 1 m^3 of concrete: J2 = 900 £/m^3.
3. Labor cost for concrete pouring and vibrating: J3 = [number of hours (0.2 h/m^3)]·[£/h (45 000 £/h)] = 0.2·45 000 = 9000 £/m^3.

The formwork cost is evaluated as follows:

1. Equipment cost, J4 = 10 000 £/m^2.
2. Labor cost for operating the formwork, J5 = [number of hours (0.2 h/m^2)]·[£/h (45 000 £/h)] = 0.2·45 000 = 9000 £/m^2.

The cost per one meter of tunnel is:

$$L2c = \left(p1c \cdot \frac{x_6}{1000} + p2c \right) \cdot \left(6.28 \cdot \frac{R}{1000} \right) \tag{47}$$

where:

- $p1c = \mathrm{J1} + \mathrm{J2} + \mathrm{J3}$
- $p2c = \mathrm{J4} + \mathrm{J5}$

3.3.3. Summary of objective functions

Let $f1$ be the cost incurred by the Client, and $f2$ be the cost incurred by the Contractor. The objective functions are quantified as follows for the three support/lining combinations:

Combination 1:

$$f1 = L1cen + L1s + L1c \tag{48}$$

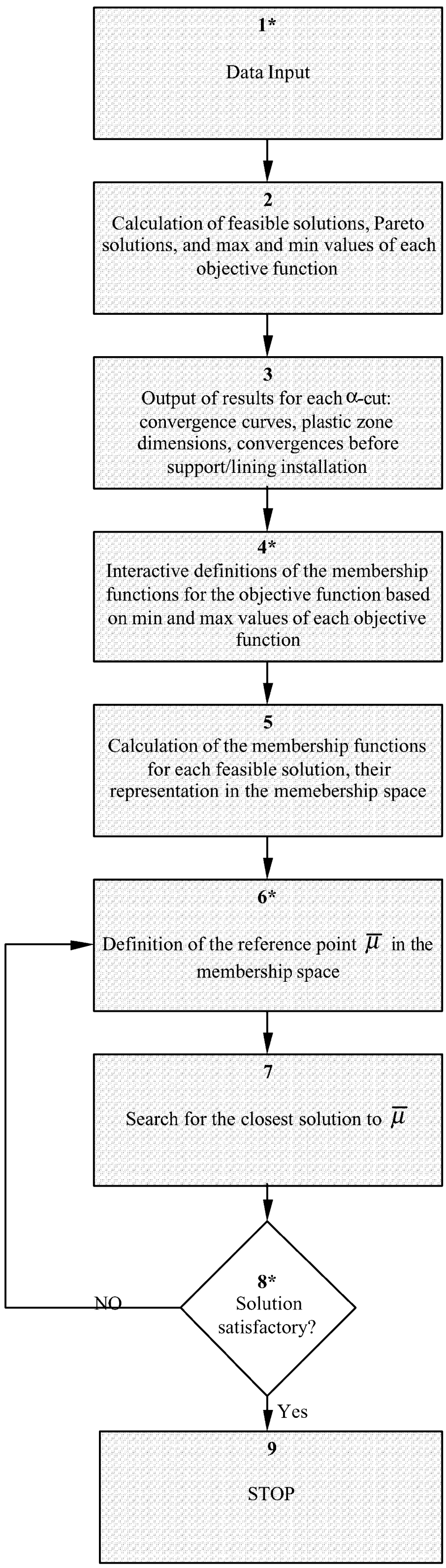


Fig. 5. Flow chart of the optimization procedure implementation in the interactive computer program TUNNEL.

$$f2 = L2cen + L2s + L2c \tag{49}$$

Combination 2:

$$f1 = L1b + L1s + L1c \tag{50}$$

$$f2 = L2b + L2s + L2c \tag{51}$$

Combination 3:

$$f1 = L1b + L1cen + L1s + L1c \tag{52}$$

$$f2 = L2b + L2cen + L2s + L2c \tag{53}$$

3.4. Implementation of the design procedure

The design procedure described above was implemented in the interactive computer program TUNNEL for Mathematica (Tonon, 1995; Mammino and Tonon, 1997). Fig. 5 depicts the flow chart of the program. Steps requiring interaction with the decision maker are marked with a '*' symbol.

At step 1, data are input. The information on the three parameters constrained by fuzzy sets is given by means of their α-cuts.

The determination of the feasible solutions is carried out at Step 2 by simple enumeration because the design variables are discrete, and their number is limited to 6 (Manohar Kamat and Mesquita, 1994). Pareto solutions are singled-out, and the range $[f_j^{\min}, f_j^{\max}]$ calculated for each objective function.

At Step 3, for each α-level of the fuzzy parameters, the worst-case short-term and long-term convergence curves are plotted along with the short-term and long-term plastic radii. Furthermore, the program gives a printout of the worst-case values for the fuzzy parameters, and the relevant short-term and long-term rock mass properties. Because the convergence before support/lining installation, δ_0, depends on the primary support/lining stiffness [see Eq. (B.3)], a range of calculated δ_0 values is furnished, together with a range of circumferential strains, useful for detecting possible squeezing problems.

Step 4 requires the interaction with the decision maker because a judgment on the attainable interval $[f_j^{\min}, f_j^{\max}]$ of each objective function must be elicited. This is accomplished in three steps:

i. The completely satisfying value f_j^1 of the j-th objective function is chosen, with the constraint:

$$f_j^1(\mathbf{x}) \leq f_j^{\min} \tag{54}$$

ii. The completely dissatisfying value f_j^0 of the j-th objective function is chosen, with the constraint:

$$f_j^0(\mathbf{x}) \geq f_j^{\max} \tag{55}$$

Constraints (54) and (55) are necessary in order for

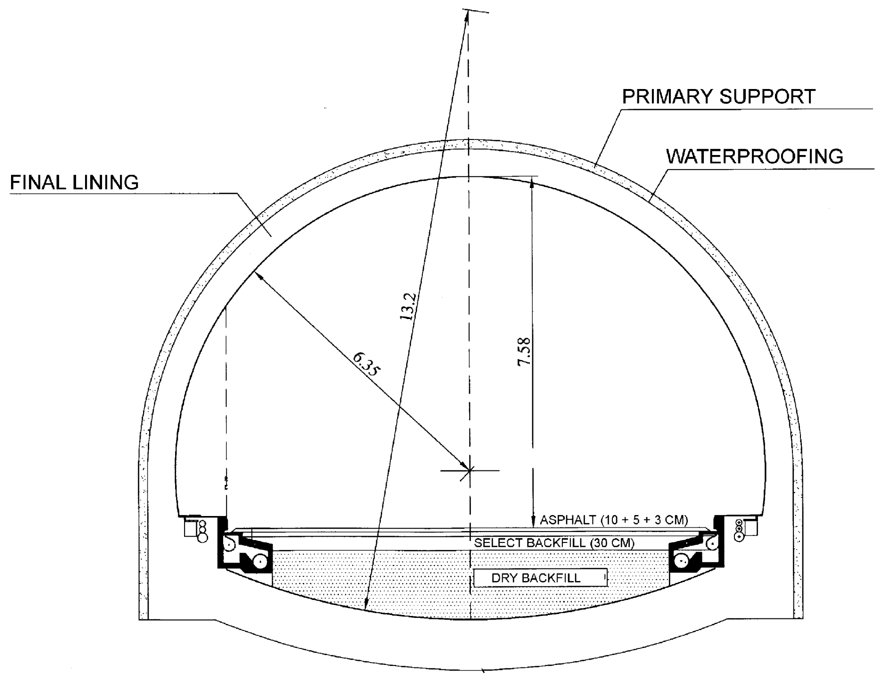


Fig. 6. Typical tunnel cross-section for the State Road 77 tunnels. Rockbolts are not shown for clarity. Dimensions are in meters, unless otherwise specified.

the transformation **F** to be regular (Tonon and Bernardini, 1999).

iii. The membership functions of fuzzy sets $M_j = \left\{\left(f_j, \mu_j(f_j)\right) \times | f_j \in Y_j\right\}$, $j = 1, 2, 3$, are of the form:

$$\mu_j(f_j) = \frac{1 - e^{a \cdot (f_j - f_j^0)/(f_j^1 - f_j^0)}}{1 - e^a} \tag{56}$$

where a is a parameter that controls the shape of the membership function. The judgment is severe if $a > 0$ (see Fig. 2a), and the larger the value of a, the more severe the judgment is. On the contrary, the judgment is permissive if $a < 0$ (see Fig. 2b), and the more negative the value of a, the more permissive the judgment is. A 'neutral' judgment (linear between f_j^1 and f_j^0) is obtained for a value of a close to zero; in the author's experience $a = 0.1$ provides a sufficiently straight membership function.

Pop-out windows allow the user to interactively input the values required in Steps 4.1 through 4.3. At the end of Step 4.3, the j-th membership function is plotted, and the user can decide to accept it, or to modify it.

The program proceeds to Step 5 with the calculation of the membership functions for each feasible solution. These values are then plotted in the (3-D) membership space.

Steps 6 again requires the decision maker's intervention because the reference point $\bar{\mu}$ in the membership space must be chosen. The point $\bar{\mu} = (1,1,1)$ is frequently chosen at the first iteration.

At Step 7, the solution closest to the reference point is determined (this is done by enumeration, in order to avoid local minima). The default metric is the Euclidean metric, i.e. $p = 2$ in Eq. (3).

If the decision maker is satisfied with the solution, the program is terminated, otherwise trade-offs among the objective functions can be interactively assessed by going back to Step 6, and by moving the reference point in the membership space.

4. Case study: tunnels on State Road 77, Italy

A preliminary engineering study was carried out for the upgrading of State Road 77 between the towns of Muccia and Colfiorito located in the Marche Region, in Central Italy. Four different alignments were proposed, each involving from 9 to 13 tunnels with lengths from 180 m to 2 km each. The typical tunnel cross-section is shown in Fig. 6.

All alignments follow the Chienti Valley, which cuts at nearly 90° the 'Dorsale Umbro-Marchigiana', a series of ridges with North-East vergence. The tunnels cross two stratigraphic series:

- The older series (from Jurassic to Upper Eocene) is composed of mainly limestone formations (locally known as 'Corniola', 'Calcari Diasprini', 'Maiolica', 'Scaglia Bianca', and 'Scaglia Rossa'), whose stratigraphic continuity is interrupted by two clayey-marly layers (locally known as 'Marne a Fucoidi' and 'Rosso Ammonitico'). The resistance of the limestone formations to weathering allowed the rough morphology created by more recent tectonic activity to be maintained over geologic time.

- The more recent series (Upper Eocene to Medium Miocene) is composed of clayey-marly formations (locally known as 'Scaglia Variegata', 'Scaglia Cinerea', 'Bisciaro', and 'Schlier'). This series has been subjected to erosion and weathering processes, and is thus discontinuous and presents a smooth, rolling morphology dotted with low seated landslides.

4.1. Section design

The geology encountered along the tunnels was subdivided into nine categories, each characterized by different values of the overburden and of the rock mass quality. The optimization procedure presented in Section 3 was used to design the reinforcement/lining for each category. In order to illustrate, two examples are described here:

1. Example 1 is typical of a limestone formation of the Jurassic-Upper Eocene series.
2. Example 2 is typical of a clayey-marly formation of the Upper Eocene-Middle Miocene series.

The parameters common to both examples are listed in Table 1. The degradation parameter of the Geological Strength Index, K, is assumed to be affected by uncertainty, and its values are constrained by the fuzzy set depicted in Fig. 7a (Mammino and Tonon, 1997).

Following Tonon et al. (2000a) and Appendix A, the information on K given by the fuzzy set in Fig. 7a is to be interpreted as follows. It is surely possible that over time the Geological Strength Index be reduced by 80% [Pos(0.8) = 1]; however, it is not necessary that an 80%

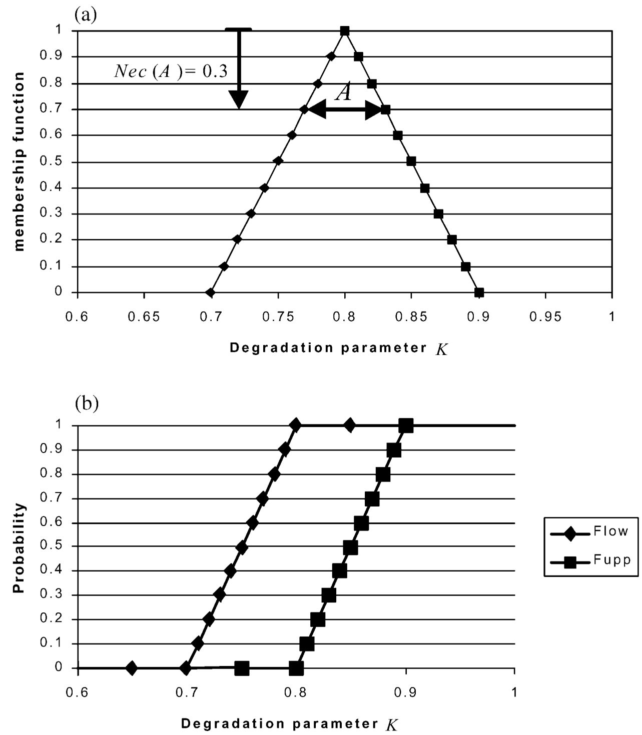
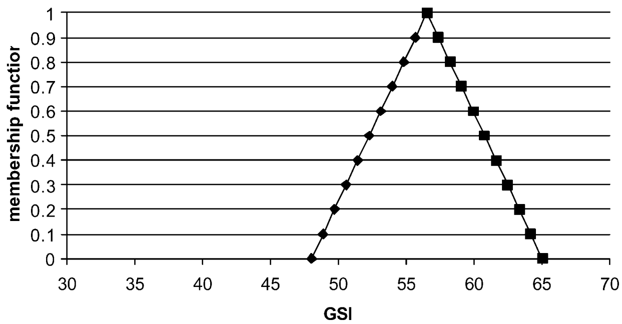


Fig. 7. (a) Fuzzy set of the degradation parameter of the Geological Strength Index, K. (b) Upper and lower cumulative distribution functions derived from the fuzzy set of the degradation parameter of the Geological Strength Index, K, shown in (a).

Table 1
Common parameters for Examples 1 and 2

Parameter	Value
Poisson's ratio of the rock mass	0.2
Hoek–Brown failure criterion m parameter	7
Unit weight of the broken rock in the plastic annulus	0.02 MN/m^3
Tunnel radius	6.5 m
Distance between primary lining installation and the tunnel face	2 m
Poisson's ratio of shotcrete and concrete, ν_c	0.2
Young's modulus of the shotcrete in short-term calculations, $E_{shot,i}$	7 GPa
Young's modulus of the shotcrete in long-term calculations, $E_{shot,\infty}$	7 GPa
Design resistance of the shotcrete for short-term loads, $\sigma_{shot,i}$	4.565 MPa
Design resistance of the shotcrete for long-term loads, $\sigma_{shot,\infty}$	10.6 MPa
Rockbolt diameter	24 mm
Steel Young's modulus, E_s	210 GPa
Constant which takes into account the deformability of the steel plate and of the anchor, Q	0.024 m/MN
Anchor length of rockbolts	1.5 m
Tensile resistance of a rockbolt for short-term applications, $T_{bolt,i}$	172 kN
Tensile resistance of a rockbolt for long-term applications, $T_{bolt,\infty}$	138 kN
Average value of the overbreak filled with shotcrete, t_b	5 cm
Yield stress of the steel, f_{yk}	240 MPa
Steel coefficient, γ_m	1
Young's modulus of the final lining concrete, E_{conc}	25 GPa
Design resistance of the concrete for long-term loads, σ_{conc}	12.75 MPa



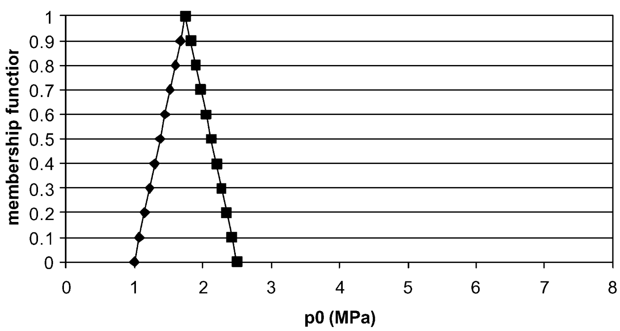


Fig. 8. Fuzzy sets for Example 1: (a) Geological Strength Index for short-term calculations; (b) in situ stress p_0.

reduction occurs [Nec(0.8)=0]. Upper and lower limits can be calculated on the probability that K belongs to a specified interval A; this is done by combining Eq. (A.6a) and Eq. (A.12) for the lower bound, and by using Eq. (A.12) for the upper bound. For example, if $A=[0.77, 0.83]$ as shown in Fig. 7a, the lower bound on the probability is:

$$\mathrm{Nec}(A)=\mathrm{Bel}(A)=1-\mathrm{Pos}(A^c)$$
$$=1-\sup_{K\in A^c}\left(\mu(k)\right)=1-0.7=0.3$$

and the upper bound on the probability is:

$$\mathrm{Pos}(A)=\sup_{k\in A}\left(\mu(k)\right)=1$$

Eqs. (A.7a) and (A.7b) allow the upper and lower cumulative distribution functions to be determined as shown in Fig. 7b. They are the envelope of all possible cumulative distribution functions that are compatible with the imprecise and random data. As explained above, the procedure considers the worst-case scenario among all possible scenarios created by the envelope of distribution functions.

Finally, according to Eqs. (A.14) and (A.15), and Eq. (A.8), the expected interval of K is [0.75, 0.85]; this underlines that, when randomness is 'filtered out' by means of the expectation operator, only imprecision remains (Dubois and Prade, 1989). A similar discussion applies to the fuzzy sets introduced hereafter.

As far as the optimization procedure is concerned, the following choices are made:

- Step 4: $f_j^1(\mathbf{x})=f_j^{\min}$; $f_j^0(\mathbf{x})=f_j^{\max}$. The parameter a (which controls the shape of the membership functions $\mu_j(f_j)$, [see Eq. (56)] is set as 0.1, i.e. a 'neutral' judgment is given on the objective functions.
- Step 6: at the first iteration, the reference point in the membership space is set as $\bar{\mu}=(1,1,1)$.
- Step 7: the Euclidean metric is chosen, i.e. $p=2$ in Eq. (3).

4.2. Example 1

The rock mass is composed of a good quality limestone, as can be seen from Fig. 8a, which shows the values of the Geological Strength Index constrained by a fuzzy set. The rock mass is assumed to be elasto-brittle-plastic. The uniaxial compressive strength of the intact rock is equal to 50 MPa. The overburden is not very high, and Fig. 8b illustrates the fuzzy set that quantifies the uncertainty on the in situ stress p_0.

The optimum solution comprises 15 cm of shotcrete, rockbolts at 2 m spacing, no steel sets, and a 50-cm thick cast-in-place final lining. The free-stressing length of the rockbolts is calculated as 5 m. The coordinates of the optimum solution in the membership space are $\mu=(0.996, 0.995, 1)$, i.e. the level of satisfaction relative to the first objective function (cost incurred by the Client) is equal to 0.996, the level of satisfaction relative

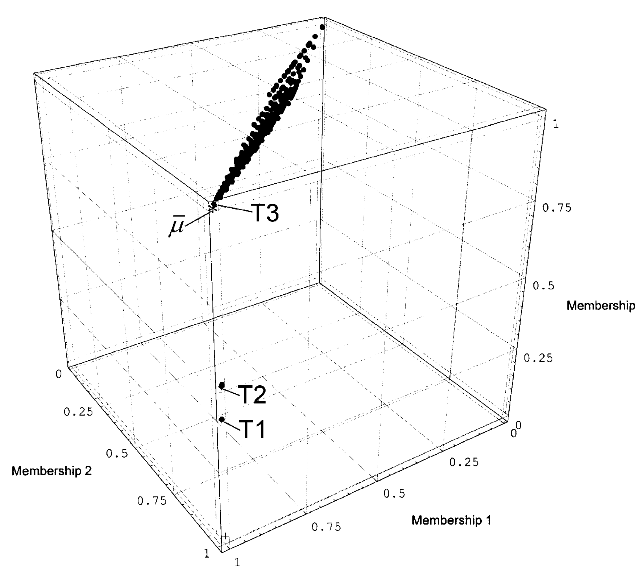


Fig. 9. Membership space for Example 1; T1, T2 and T3 denote the three M-Pareto solutions, $\bar{\mu}=(1,1,1)$ denotes the reference point.

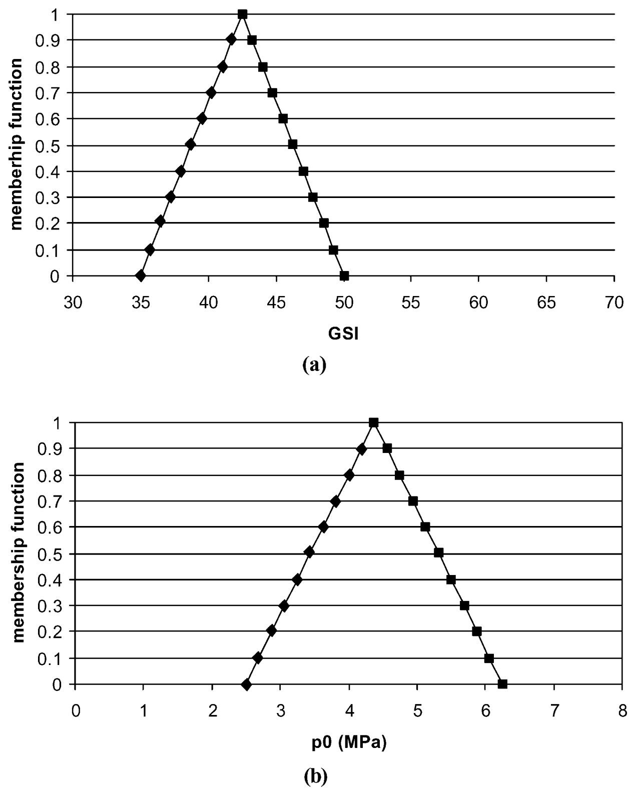


Fig. 10. Fuzzy sets for Example 2: (a) Geological Strength Index for short-term calculations; (b) in situ stress p_0.

to the second objective function (cost incurred by the Contractor) is equal to 0.995, and the level of satisfaction relative to the third objective function (probability of failure) is equal to 1. Because a high degree of satisfaction was reached for all three objective functions, it was decided that the optimum solution found at the first iteration was satisfactory, and was accepted for the preliminary design.

Fig. 9 depicts the three-dimensional membership space: each feasible solution is represented by a dot. The letter $\bar{\mu}$ denotes the reference point, and the optimal solution is denoted as T3. It can be noticed that nearly all feasible solutions guarantee total satisfaction of the third objective function (i.e. the minimization of the probability of failure). However, only three feasible solutions out of 1106 are M-Pareto solutions (they are denoted as T1, T2 and T3 in Fig. 9), i.e. there are only three 'good' feasible solutions out of 1106. The remaining 1103 feasible solutions either increase the costs without increasing safety, or decrease the safety without decreasing the costs.

4.3. Example 2

In Example 2, the tunnel must be excavated in a marly rock mass of fair quality, whose Geological Strength Index is quantified by the fuzzy set depicted in Fig. 10a. The rock mass is assumed to be elasto-perfectly-plastic. The uniaxial compressive strength of the intact rock is equal to 15 MPa. Fig. 10b illustrates the fuzzy set of the in situ stress p_0, which shows a large uncertainty.

Fig. 11 illustrates the three-dimensional membership space; 11 feasible solutions out of 1106 are M-Pareto solutions, and are denoted as T1 to T11 in Fig. 11.

At the first iteration, the optimum solution (closest to $\bar{\mu}=(1,1,1)$) features a 14-cm thick shotcrete, steel sets HEB 140 at 0.6 m spacing, no rockbolts, and a 70-cm thick cast-in-place final lining. The degree of satisfaction for the three objectives is quantified as (0.82, 0.83, 0.79). This solution is denoted as T9 in Fig. 11.

Due to the less favorable geomechanical properties of the rock mass and the higher in situ stress, the trade-offs between safety and cost had to be considered more closely in this example than in Example 1. Therefore, it was decided to move the reference point to $\bar{\mu}_2=(0.9,0.9,1)$, i.e. to slightly loosen the requirements for a cheap design. The optimum solution obtained at the second iteration was made of 20 cm of shotcrete, steel sets HEA 200 at 0.75 m spacing, rockbolts at 1.2 m grid spacing, and a 70-cm thick cast-in-place final lining. The coordinates of the optimum solution in the membership space are $\mu=(0.77, 0.78, 0.9)$. This solution is denoted as T10 in Fig. 11. It was decided that solution T10 was preferable to T9, because the increase in safety was deemed more important than the slight increase in cost.

5. Conclusions and future work

A procedure for multiobjective optimization under uncertainty has been applied to the design of tunnel

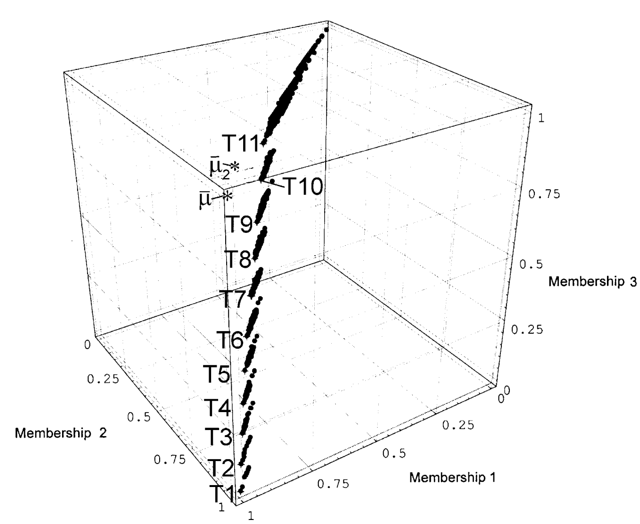


Fig. 11. Membership space for Example 2; the M-Pareto solutions are denoted as T1 to T11, $\bar{\mu}=(1,1,1)$ denotes the reference point of the first iteration, $\bar{\mu}_2=(0.9,0.9,1)$ denotes the reference point of the second iteration.

support/reinforcement. The three objective functions considered were the costs incurred by the Contractor and by the Client, and the probability of failure. The importance of the interaction with the decision maker has been stressed, so that the subjectivity inherent in each design process is not diminished by the use of information sciences. Rather, designers can quantitatively and knowledgeably make their decisions by intervening at the key points of the process. Both randomness and imprecision affecting the parameters that control the rock mass behavior were considered.

The procedure was completely implemented in an interactive computer program within Mathematica environment. This computer program was used in the preliminary design of the tunnels for State Road 77 in Central Italy, encompassing a total of approximately 40 km of tunnels. The following conclusions can be drawn from the case history presented:

- Only a very small percentage ($<1\%$) of the feasible solutions were 'good' solutions (here called M-Pareto solutions), all other feasible solutions either increased the costs without increasing safety, or decreased the safety without decreasing the costs. It is, thus, very probable (99%) that a traditional approach to tunnel design without optimization procedures would choose a solution that either increases the costs without increasing safety, or decreases the safety without decreasing the costs.
- Difficult geomechanical conditions make the choice of the optimum solution challenging and the trade-offs among the objective functions must be assessed by the decision-maker by means of an interactive procedure.
- The cost of the most expensive feasible solution is approximately 7 times as high as the cost of the cheapest feasible solution. However, the most expensive M-Pareto solution is only 3 times as expensive as the cheapest M-Pareto solution. This implies that the mere determination of the M-Pareto solutions can significantly help in cutting the costs. Additionally, without an optimization procedure, given the high probability of choosing a non M-Pareto solution, it is very likely that unnecessarily high cost increases are unknowingly introduced by the designer. This is a doubtful premise in rational tunnel design.
- The cost incurred by the Client turned out to be sensibly proportional to the cost incurred by the Contractor. As a consequence, the many feasible solutions that are not M-Pareto solutions just increase the Contractor's gain, without actually increasing the safety. This is also a doubtful premise in rational tunnel design.
- Both imprecision and randomness can be easily taken into account in tunnel design.

Mammino and Tonon (1997) applied the multiobjective optimization procedure described in this paper to the results produced by program DAT (Design Aids for Tunneling) (Einstein, 1996), when a solution does not exist that minimizes both construction time and cost. For brevity, this application was not described in the present paper. In DAT, the construction of a tunnel is simulated a number of times through the geology given by a probabilistic parameter profile; the output consists in cost and time distributions, and in consumed and produced resources. Thus, the decision-making problem is outside the scope of the DAT, as it is the occurrence of non-probabilistic information.

The future work will consist in applying the multiobjective optimization procedure described in this paper to other tunneling design problems such as the choice of the optimum alignment, the choice of the most appropriate excavation method, the design of a tunnel boring machine cutterhead, etc.

Finally, it is believed that information sciences will ever more contribute to a rational design of underground excavations, in which the designers' personal input is valued and clearly defined in its impact on the solution. Ample room for improvement exists in this area.

Acknowledgments

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Appendix A:

Some basic definitions of Random Set Theory and Fuzzy Set Theory are given in the following. For a fuller discussion on this subject, the reader is referred to Dubois and Prade (1988), Zimmermann (1991), Wang and Klir (1992), Klir and Yuan (1995), Tonon et al. (2000a,b).

Definition A1. Suppose M observations were made of a parameter $u \in U$, each of which resulted in an imprecise (non-specific) measurement given by a set A of values. Let c_i denote the number of occurrences of the set $A_i \subseteq U$, and $\mathscr{P}(U)$ the set of all the subsets of U (power set of U). A frequency function m can be defined, called *basic probability assignment*, such that:

$$m: \mathscr{P}(U) \to [0,1] \tag{A.1}$$

$$m(\varnothing)=0 \tag{A.2}$$

$$\sum_{A\in\mathscr{P}(U)} m(A)=1 \tag{A.3}$$

If $m(A_i)>0$ (i.e. if A_i has occurred at least once), A_i is called *focal element*. A *random set* is the pair $(\mathscr{F},m)$ where $\mathscr{F}$ is the family of all N focal elements; if U is the Cartesian product of p sets, i.e. $U=U_1\times....\times U_\mathrm{p}$, then one speaks of a *random relation*.

Definition A2. Because of the presence of imprecision, it is not possible to calculate the probability of a generic $u\in U$ or of a generic subset $E\subseteq U$, but only a lower and upper bound on this probability:

$$\mathrm{Bel}(E)\le \mathrm{Pro}(E)\le \mathrm{Pl}(E) \tag{A.5}$$

where, denoted E^c the complement of E:

$$\mathrm{Bel}(E)=\sum_{A_i:A_i\subseteq E} m(A_i)=1-\mathrm{Pl}(E^c) \tag{A.6a}$$

$$\mathrm{Pl}(E)=\sum_{A_i:A_i\cap E\neq\varnothing} m(A_i)=1-\mathrm{Bel}(E^c) \tag{A.6b}$$

$\mathrm{Bel}(E)$ is called *Belief Measure*, and $\mathrm{Pl}(E)$ is called *Plausibility Measure*; both measures are not additive.

Definition A3 When the set U is the real line, the two following limit cumulative probability distribution functions can be defined:

$$F_{\mathrm{upp}}(u)=\mathrm{Pl}\left(\{u'\in U:u'\le u\}\right)=\sum_{A_i:u\ge \inf(A_i)} m(A_i) \tag{A.7a}$$

$$F_{\mathrm{low}}(u)=\mathrm{Bel}\left(\{u'\in U:u'\le u\}\right) =\sum_{A_i:u\ge \sup(A_i)} m(A_i) \tag{A.7b}$$

They are an envelope of all the possible cumulative distribution functions compatible with the data. It stems from Eqs. (A.7a) and (A.7b) that the calculation of the expectation makes sense and is the interval:

$$\left[\sum_{i=1}^{N} m_i\cdot\inf(A_i),\quad \sum_{i=1}^{N} m_i\cdot\sup(A_i)\right] \tag{A.8}$$

Definition A4. If U is a collection of objects denoted generically by u, then a fuzzy set F in U is a set of ordered pairs

$$F=\left\{\left(u,\mu_F(u)\right)|u\in U\right\} \tag{A.9}$$

where $\mu_F(u): U\to[0,\infty)$ is called membership function of u in F. If $\sup_u \mu_F(u)=1$, the fuzzy set is called normal. In this paper, fuzzy sets are all normal. If $U=U_1\times...\times U_p$, the fuzzy set is called a fuzzy relation.

Definition A5. The crisp set of elements that belong to the fuzzy set F at least to the degree α is called the α-level set, or α-cut of F

$$F(\alpha)=\left\{u\in U|\mu_F(u)\ge\alpha\right\} \tag{A.10}$$

If $\alpha_1>\alpha_2$, then $F(\alpha_1)\subseteq F(\alpha_2)$.

Definition A6 When $\mathscr{F}$ can be ordered in such a way that $A_i\subset A_{i+1}$ $i=1,...,N$-1, then $(\mathscr{F},m)$ is said to be *consonant*. The focal elements can thus be seen as the α-cuts of the fuzzy set F whose membership function is:

$$\mu_F(u)=\mathrm{Pl}\left(\{u\}\right)=\sum_{A_i:u\in A_i} m(A_i) \tag{A.11}$$

In this case, Plausibility coincides with Possibility (Pos) and Belief with Necessity (Nec) in Possibility Theory. In this interpretation, a fuzzy set is a model of ambiguity and not of vagueness.

Definition A7. Every possibility measure Pos on a power set $\mathscr{P}(u)$ is uniquely determined by a possibility distribution function via the formula

$$\mathrm{Pos}(A)=\sup_{u\in A}\pi_F(u) \tag{A.12}$$

for each $A\in\mathscr{P}(u)$.

On the other hand, given a possibility measure Pos, a possibility distribution function is uniquely determined via the formula [see Eq. (A.11)]

$$\pi_F(u)=\mathrm{Pos}\left(\{u\}\right) \tag{A.13}$$

Because Pos is a plausibility measure and a plausibility measure uniquely determines a probability assignment, given a possibility distribution function $\pi_F(u)$, the underlying consonant random set is determined by Eq. (A.12). More explicitly, let $L(F)=\{\alpha_1>...>\alpha_n\}$ be the set of positive membership in F, then $\mu_F(u)$ is equivalent to the unique consonant random set $(\mathscr{F}, m)$ defined by

$$\mathscr{F}=\{{}^{\alpha_i}F|i=1,...,n\} \tag{A.14}$$

$$m({}^{\alpha_i}F)=\begin{cases}\alpha_i-\alpha_{i-1} & i=1,...,n-1\\ \alpha_n & i=n\\ 0 & \text{else}\end{cases} \tag{A.15}$$

Appendix B: Convergence curve

Let m and s be the Hoek–Brown peak parameters for the rock mass, and let a subscript r denote residual

Table 2
Formulas for stiffness and resistance to be used with the combinations defined in the main text

	Combination 1		Combination 2		Combination 3	
	Stiffness	Resistance	Stiffness	Resistance	Stiffness	Resistance
Primary Support/Lining	$k_{1,i}=k_a+k_{b,i}$	$q_{1,i}=q_a+q_{b,i}$	$k_{2,i}=k_{b,i}+k_c$	$q_{2,i}=q_{b,i}+q_{c,i}$	$k_{3,i}=k_a+k_{b,i}+k_c$	$q_{3,i}=q_a+q_{b,i}+q_{c,i}$
Complete Lining	$k_{1,\infty}=k_a+k_{b,\infty}+k_d$	$q_{1,\infty}=q_a+q_{b,\infty}+q_d$	$k_{2,\infty}=k_{b,\infty}+k_c+k_d$	$q_{2,\infty}=q_{b,\infty}+q_{c,\infty}+q_d$	$k_{3,\infty}=k_a+k_{b,\infty}+k_c+k_d$	$q_{3,\infty}=q_a+q_{b,\infty}+q_{c,\infty}+q_d$

values. Moreover, let σ_c denote the uniaxial compressive strength of the intact rock. The convergence curve is (Brown et al., 1983):

$$\delta_{tun}(p_i)=-\frac{M\cdot\sigma_c}{G\cdot(1+f)}\cdot\left[\frac{f-1}{2}+\left(\frac{r_e(p_i)}{R}\right)^{f+1}\right]\cdot R \qquad \text{(B.1)}$$

where:

$$M=-\frac{m}{8}+\frac{1}{2}\cdot\sqrt{p_0\cdot\frac{m}{\sigma_c}+\left(\frac{m}{4}\right)^2+s}$$

$$G=\frac{E}{2(1+\nu)}$$

where E = deformation modulus of the rock mass; ν = Poisson's ratio of the rock mass; and f = parameter controlling the rock mass dilatancy =

$$\begin{cases} 1+\dfrac{m}{2}\cdot\left(m\cdot\dfrac{\sigma_3}{\sigma_c}+s\right)^{-0.5} & \text{associated plasticity} \\ 1 & \text{no plastic volumetric strain} \end{cases}$$

$$r_e(p_i)=R\cdot\exp\left[N-\frac{2}{m_r\cdot\sigma_c}\cdot\sqrt{m_r\cdot\sigma_c\cdot p_i+s_r\cdot\sigma_c^2}\right]$$

= radius of the plastic annulus (B.2)

$$N=\frac{2}{m_r\cdot\sigma_c}\cdot\sqrt{m_r\cdot\sigma_c\cdot p_0+s_r\cdot\sigma_c^2-m_r\cdot M\cdot\sigma_c^2}$$

If the long-term rock mass properties are used, the expression in Eq. (B.1) is denoted as δ_{tun_∞}

Initial convergence

$$\delta_0(d,p_{i_I})=\left[0.55+0.45\cdot\frac{\delta_{tun}(p_{i_I})}{\delta_{tun}(0)}-0.42\cdot\left(1-\frac{\delta_{tun}(p_{i_I})}{\delta_{tun}(0)}\right)^3\right]\cdot\left\{\delta_\infty\cdot\left[0.29+0.71\cdot\left(1-e^{-1.5\cdot(D/\chi)^{0.7}}\right)\right]\right\} \qquad \text{(B.3)}$$

where:

$$D=\frac{d}{R}$$

d = distance between the tunnel face and the point where the primary support/lining is installed; $\delta_{tun}(p_i)$ = convergence curve given by Eq. (B.1)

$$\chi=\frac{\delta_{tun}(0)}{\delta_{e,\infty}}$$

$$\delta_{e,\infty}=\frac{1+\nu}{E}\cdot p_0\cdot R$$

p_{i_l} = pressure exerted on the rock mass by the primary support, excluding the yielded rock contribution.

Steel sets

It is assumed that a continuous layer of shotcrete is used to block the steel sets, rather than wooden blocks, which are unreliable and subject to deterioration over time. The stiffness of the steel sets is (Mammino and Tonon, 1997):

$$\frac{1}{k_a}=\frac{x_3\cdot R}{E_a\cdot A_a}+\frac{x_3\cdot t_b}{E_{\text{shot}}\cdot b\cdot R} \quad \text{(B.4)}$$

where:

- x_3 = steel sets spacing along the tunnel axis;
- E_a = Young's modulus of the steel sets;
- A_a = steel set cross-sectional area;
- E_{shot} = Young's modulus of the shotcrete;
- t_b = average value of the overbreak filled with shotcrete; and
- b = steel set flange width.

The design resistance of the steel sets is given by the formula (Mammino and Tonon, 1997):

$$q_a=\frac{f_{yk}\cdot A_a}{R\cdot x_3\cdot\gamma_m} \quad \text{(B.5)}$$

where:

- f_{yk} = yield stress of the steel; and
- γ_{m} = steel coefficient [1 per Italian Code (Ministero dei Lavori Pubblici, 1996, Part II, Section 4.0.3.1); 1.05 per Italian implementation of Eurocode 3 (Ministero dei Lavori Pubblici, 1996, Part II, Section 8); 1.1 per Eurocode 3 (CEN, 1993, Section 5.1.1)]

Shotcrete The stiffness of the shotcrete shell as a primary lining element and in the long term is, respectively:

$$k_{b,i}=\frac{E_{\text{shot},i}}{1-\nu_c^2}\cdot\frac{x_4}{R} \quad \text{(B.6)}$$

$$k_{b,\infty}=\frac{E_{\text{shot},\infty}}{1-\nu_c^2}\cdot\frac{x_4}{R} \quad \text{(B.7)}$$

where:

- $E_{\text{shot},i}$ = Young's modulus of the young shotcrete. Because of the rheological properties of the young shotcrete, an equivalent Young's modulus of 7 GPa is recommended (Pottler, 1990).
- $E_{\text{shot},\infty}$ = Young's modulus of the shotcrete for long-term calculations.
- ν_c = Poisson's ratio of shotcrete.
- x_4 = shotcrete thickness.

The resistance of the shotcrete shell as an element of the primary lining and in the long term is, respectively:

$$q_{b,i}=\sigma_{\text{shot},i}\cdot\frac{x_4}{R} \quad \text{(B.8)}$$

$$q_{b,\infty}=\sigma_{\text{shot},\infty}\cdot\frac{x_4}{R} \quad \text{(B.9)}$$

where :

- $\sigma_{\text{shot},i}$ = design resistance of the shotcrete for short-term loads [$0.5f_{ckj}$ per Italian Code and per Italian implementation of Eurocode 2 (Ministero dei Lavori Pubblici, 1996, Part I, Section 4.0.2 and 4.2.1.2), $0.533f_{ckj}$ per Eurocode 2 (CEN, 1992, Section 2.3.3.2)].
- f_{ckj} = cylindrical characteristic resistance of the shotcrete at the peak of its stress history, i.e. approximately 1 diameter from the face according to Pottler (1990).
- $\sigma_{\text{shot},\infty}$ = design resistance of the shotcrete for long-term loads ($0.425f_{ck}$ per Italian Code and per Italian implementation of Eurocode 2 [Ministero dei Lavori Pubblici, 1996, Part I, Sections 4.0.2 and 4.2.1.2), $0.453f_{ck}$ per Eurocode 2 (CEN, 1992, Section 2.3.3.2)].
- f_{ck} = cylindrical characteristic resistance of the shotcrete at 28 days.

If there are steel sets, the steel sets-shotcrete shell can be treated as a composite structure. In this case, $\sigma_{\text{shot},i}$ would increase by approximately 6%, and $\sigma_{\text{shot},\infty}$ by 25% (Mammino and Tonon, 1997).

Rockbolts

The stiffness of the rockbolt system is (Hoek and Brown, 1980):

$$k_c=\frac{R}{x_5^2}\cdot\left(\frac{4\cdot l}{\pi\cdot d_{\text{bolt}}{}^2\cdot E_s}+Q\right)^{-1} \quad \text{(B.10)}$$

where:

- x_5 = rockbolt spacing;
- l = free-stressing length of the rockbolt;
- d_{bolt} = rockbolt diameter;
- E_s = steel Young's modulus; and
- Q = constant which takes into account the deformability of the steel plate and of the bond length.

The resistance of the rockbolt system as a primary lining element in the short-term and in the long-term is, respectively:

$$q_{c,i}=\frac{T_{\text{bolt},i}}{x_5^2} \quad \text{(B.11)}$$

$$q_{c,\infty}=\frac{T_{\text{bolt},\infty}}{x_5^2} \quad \text{(B.12)}$$

where $T_{\text{bolt},i}$ (resp. $T_{\text{bolt},\infty}$) is the tensile resistance of a rockbolt for short-term (resp. tong-term) applications. For example, following AICAP (1991) recommendations and the Italian Code (Ministero dei Lavori Pubblici, 1996, Part I, Tables 2-I and 6-I), one obtains:

$$T_{\text{bolt},i}=0.783\frac{\pi\cdot d_{\text{bolt}}^2}{4}f_{yk} \tag{B.13}$$

$$T_{\text{bolt},\infty}=0.626\frac{\pi\cdot d_{\text{bolt}}^2}{4}f_{yk} \tag{B.14}$$

where f_{yk} is the characteristic yield stress of the rockbolt steel.

Final lining

The stiffness of the final lining ring is:

$$k_d=\frac{E_{\text{conc}}}{1+\nu_c}\cdot\frac{x_6}{R}\cdot\frac{2\cdot[1-x_6/(2\cdot R)]^2}{(1-2\cdot\nu_c)\cdot[1-x_6/(2\cdot R)]^2+(1-x_6/R)^2} \tag{B.15}$$

where:

- ν_c = Poisson's ratio of concrete.
- x_6 = thickness of the final lining.
- E_{conc} = Young's modulus of the concrete. Because of the long duration of the loads applied, the creep of the concrete must be taken into account. As shown in Mammino and Tonon (1997), this is accomplished by using the following formula:

$$E_{\text{conc}}=\frac{E_{c,28}}{1+\frac{\phi_\infty\cdot\xi}{2}} \tag{B.16}$$

- $E_{c,28}$ = Young's modulus of the concrete at 28 days for short-term loads.
- ϕ_∞ = creep coefficient, which depends on the nominal dimension of the structure ($2x_6$ for the final lining) and on the age of the concrete at the moment of loading, e.g. (CEN, 1992, Table 3.3).
- ξ = ratio between the deformation rate of the rock mass and the deformation rate of the concrete $\approx 1/4$

The resistance of the final lining is (Mammino and Tonon, 1997):

$$q_d=\sigma_{\text{conc}}\cdot\frac{R^2-(R-x_6)^2}{2\cdot R^2} \tag{B.17}$$

where σ_{conc} is the design resistance of the concrete for long-term loads [$0.425 f_{\text{ck}}$ per Italian Code and per Italian implementation of Eurocode 2 (Ministero dei Lavori Pubblici, 1996, Part I, Sections 4.0.2 and 4.2.1.2), $0.453 f_{\text{ck}}$ per Eurocode 2 (CEN, 1992, Section 2.3.3.2)].

Combinations 1, 2 and 3

Table 2 summarizes stiffness and resistance formulas to be used with the combinations defined in the main text.

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