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TIKHONOV-REGULARIZED BISPECTRAL VARIATIONAL METHOD FOR OPTICAL SIGNAL RECONSTRUCTION

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A Tikhonov-regularized bispectral variational method is proposed for image restoration in the presence of strong phase distortions. This method combines a number of advantages of the bispectral approach, such as preservation and restoration of phase information, invariance to random shifts of the original signal, and no requirement of high-accuracy prior information about statistical properties of observed signals. In combination with the Tikhonov-regularized variational method, which is adapted to stable processing of large images, we obtain a fairly efficient image restoration method. Test results in the presence of atmospheric and underwater phase distortions reported in this article establish the advantages of the proposed method relative to the traditional recursive bispectral method.

Keywords: bispectrum, triple correlation, reconstruction of phase distortions, variational method, Tikhonov regularization, regularized gradient method

1. Introduction

Image processing applications often involve considerable difficulties due to distortions, errors, and noise of various origins and magnitudes. The statistical noise characteristics are generally a priori unknown or are known only partially and with large errors [1]. This situation is typical for signal detection and recognition in radar and sonar applications, optical and radio astronomy, and also in remote sounding systems and in medical diagnosis signal-processing systems. Implementation of various filtering methods [2] and noisy image recovery methods is attracting ever increasing interest among a wide circle of experts in many areas.

Traditional approaches that partially suppress various distortions include Wiener and inverse filtering methods [3, 4], linear and nonlinear filtering with two-dimensional and one-dimensional moving windows [5, 6], adaptive image restoration algorithms [7]. The efficiency of the first group of methods largely depends on the completeness of prior information about signal and noise statistics, which in practice is seldom available. The approaches using linear filtering theory in some cases lead to implementation of minimum mean square error image processing algorithms, for instance, in the presence of normally distributed additive noise. However, if the noise distribution is non-normal (e.g., mixed noise that includes an additive Gaussian component and impulse noise), better results are achieved with nonlinear filtering methods and algorithms. Methods of this group, despite the presence of specific dynamic distortions, ensure satisfactory suppression of mixed (non-Gaussian) noise under complete or partial prior uncertainty regarding noise properties. This is accomplished by using one of the many smoothing filters available at this time. Yet the efficiency of this group of methods is reduced for "impulse" signal processing, i.e., processing of signals that constitute a mixture of impulses whose length is

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comparable to the length of noise spikes, and also in cases when some of the observed signal realizations experience random shifts of the information component, e.g., due to turbulence in the signal propagation channel. We should also note that the limits of applicability of traditional filtering and signal recovery methods are highly sensitive to the signal/noise ratio on the system input. In most of the methods enumerated above the parameter estimation accuracy required in practice is attainable only for signal/noise ratios substantially greater than 1.

For many important signal and image processing applications, e.g., detection of weak radar and sonar targets or recovery of images of low-intensity objects in astronomy, the signal/noise ratio does not exceed 1 or is only slightly greater than 1. One of the promising approaches that ensure good results for image recovery problems under such conditions is based on the use of triple correlations and bispectra [8, 9].

Recall that triple correlation of the function $o(x, y)$ is defined as the function

$$o^{(3)}(x_1, y_1, x_2, y_2) = \int o^*(x, y) o(x + x_1, y + y_1) o(x + x_2, y + y_2)\, dx dy\,. \tag{1}$$

The bispectrum of the function $o(x, y)$ is the Fourier transform of its triple correlation:

$$O^{(3)} = F(o^{(3)})\,. \tag{2}$$

Although triple correlations and bispectra are relatively difficult to compute, their use is justified by the following fundamental advantages:

1. Triple correlation makes it possible to preserves the complex Fourier spectrum of the signal and thus restore the information about the phase characteristics of the original image.
2. Triple correlation of Gaussian noise goes to zero, which in some applications ensures robustness of algorithms that recover non-Gaussian signals from background Gaussian noise.
3. The bispectrum is invariant to displacement of the original signal.

Because of these advantages, triple correlations and bispectra are successfully used for signal and image processing in sonar, astronomy [9–12], optics [13], biomedical signal processing [14], object recognition in images [15], remote sounding systems, etc. Despite the large number of publications on this subject, some important issues, such as enhancement of signals restored from estimated bispectra and analysis of bispectrum stability under non-Gaussian (e.g., mixed) noise, remain insufficiently studied.

Bispectral image processing methods are divided into direct methods and variational methods. Among direct methods, the so-called recursive method [9, 12, 16] is particularly popular: it solves the linear algebraic system for the bispectrum phase function under certain additional assumptions about its structure. This method has been successfully applied in work with relatively small images, especially with the addition of a smoothing filter.

Bispectral variational methods involve minimization of the discrepancy functional with respect to the bispectrum phase function [12, 17, 18]. This ensures more accurate image recovery and permits processing large volumes of data at lower cost. To suppress undesirable distortions and achieve higher stability, this method can be combined with regularization [17, 18].

2. Statement of the Problem

In this article we consider optical imaging systems that are typical of microscopy, when phase distortions arise near the plane of the observed object (Fig. 1). For the isoplanar case, the injection of distortions $p(x, y)$

in the phase of the original function can be formally allowed for in the same way as for changes introduced by the aperture $M(x, y)$ in the near-focal plane. At the optical system output, the modified spatial-frequency spectrum is Fourier transformed and we obtain a distribution of light amplitudes described by the equation

$$\mathrm{i} = F(M \cdot \exp\{ip\} \cdot F(o)) = o * h. \tag{3}$$

Thus, the distribution of complex amplitudes at the system output is a convolution of the input signal $o(x, y)$ with the impulse characteristic of a system of spatial transformations $h = F(M \cdot \exp\{ip\})$. Note that a similar optical scheme has been considered in [13] for the case of Gaussian diffusion blur.

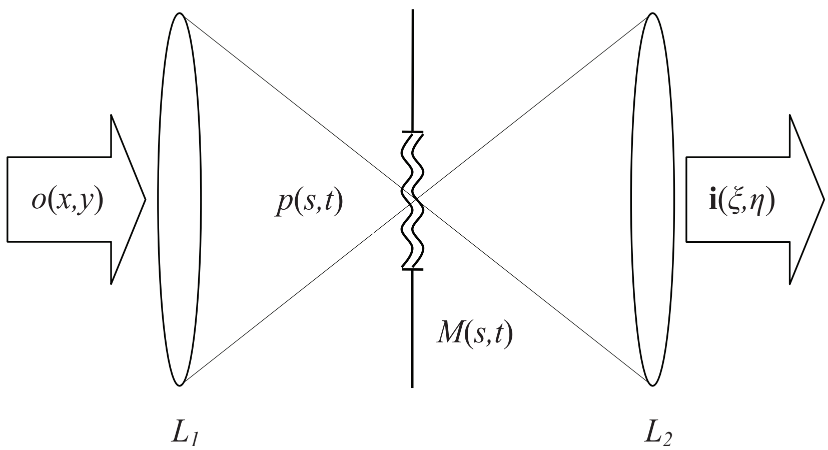


Fig. 1. Schematic diagram of the device for spatial transformation of images.

The aberrations introduced by the function $p(x, y)$ in the system may prove to be fairly strong, and one image is generally not sufficient for the recovery of the original object. In this situation we have to deal with a series of distorted images: they are processed sequentially by the algorithm, which then averages the result.

3. General Description of the Bispectral Approach

Following [9], we recall the main stages of the bispectral approach. After Fourier-transforming relationship (3), we obtain

$$I(u, v) = O(u, v) \cdot H(u, v), \tag{4}$$

where $I = F(i)$, $H = F(h)$, $O = F(o)$. To restore the function $O(u, v)$, we use its representation in terms of the absolute value $|O(u, v)|$ and phase $\varphi(u, v)$:

$$O(u, v) = |O(u, v)| \cdot \exp\{i\varphi(u, v)\}, \tag{5}$$

which permits restoring the absolute value and the phase of the sought function simultaneously. The absolute value is obtained from relationship (4):

$$|O(u,v)|^2 = \frac{|I(u,v)|^2}{|H(u,v)|^2 + \varepsilon},$$

where ε is a small parameter chosen with the purpose of avoiding division by zero. Note that here and in what follows, the implementation of the algorithm for a series of images uses the averages of $|O(u,v)|^2$ and $|H(u,v)|^2$ over all the images in the series.

The phase is restored by exploiting the relationship between the bispectra of the original image $O^{(3)}$, the observed image $I^{(3)}$, and the apparatus function $H^{(3)}$. On the one hand, since the bispectrum is computed as the Fourier transform of triple correlation, we have by (1) and (2)

$$O^{(3)}(u,u_1,v,v_1) = O(u,u_1)O(v,v_1)O^*(u+u_1,v+v_1)\,. \tag{6}$$

On the other hand, from (4) we obtain

$$I^{(3)}(u,u_1,v,v_1) = O^{(3)}(u,u_1,v,v_1)\cdot H^{(3)}(u,u_1,v,v_1)\,.$$

Using (5), (6) and the representation

$$I^{(3)}(u,u_1,v,v_1) = \left|I^{(3)}(u,u_1,v,v_1)\right|\cdot \exp\{i\beta(u,u_1,v,v_1)\}$$

we recall that the bispectrum of the transfer function $H^{(3)}(u,u_1,v,v_1)$ is real [9] and thus the phase multipliers are the same for the bispectra of the observed image and the original image:

$$\exp\{i\beta(u,u_1,v,v_1)\} = \exp\{i(\varphi(u,u_1)+\varphi(v,v_1)-\varphi(u+v,u_1+v_1))\}\,. \tag{7}$$

We will now review and compare two methods that can be used to find the sought phase from Eq. (7) – a recursive method and a variational method.

4. Recursive Bispectral Method

The recursive phase recovery method [9, 12] relies on a particular case of Eq. (7) corresponding to equality of phases of the complex exponentials:

$$\beta(u,u_1,v,v_1) = \varphi(u,u_1)+\varphi(v,v_1)-\varphi(u+v,u_1+v_1)\,. \tag{8}$$

After discretization on a grid, Eq. (8) is reduced to a system of linear algebraic equations. The operator of this system is non-invertible. However, under certain additional restrictions, the solution can be obtained from explicit formulas. Let us briefly review the implementation of this method.

For discretization we introduce grids with increments Δ and Δ_1 in the corresponding variables:

$$u_p = p\Delta, \quad u_{1p_1} = p_1\Delta_1, \quad v_q = q\Delta, \quad v_{1q_1} = q_1\Delta_1, \quad p, q, p_1, q_1 = -N \dots N,$$

and also the grid functions

$$O_{p,p_1} = O(u_p, u_{1p_1}), \quad O^{(3)}_{p,p_1,q,q_1} = O^{(3)}(u_p, u_{1p_1}, v_q, v_{1q_1}),$$

$$\beta_{p,p_1,q,q_1} = \beta(u_p, u_{1p_1}, v_q, v_{1q_1}), \quad \varphi_{p,q} = \varphi(u_p, v_q).$$

For these grid functions, noting that $r = p + q$, $r_1 = p_1 + q_1$, we obtain the phase relationship (8) for $q = 1$ in the form

$$\varphi_{r,r_1} = \varphi_{1,1} + \varphi_{r-1,r_1-1} - \beta_{r-1,1,r_1-1,1}.$$

Since $O(u,v)$ is an Hermite function and $\varphi_{r,r_1} = \varphi_{-r,-r_1}$, we have $\varphi_{0,0} = 0$. Now, assuming that the location of the restored signal is immaterial, we obtain $\varphi_{1,1} = 0$. The relationship between the functions β and φ on the given grid thus takes the form

$$\varphi_{2,1} = \varphi_{1,1} - \beta_{1,1,1,1},$$

$$\varphi_{3,1} = \varphi_{1,1} + \varphi_{2,1} - \beta_{2,1,1,1} = 3\varphi_{1,1} - \beta_{2,1,1,1} - \beta_{1,1,1,1},$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\varphi_{r,1} = r\varphi_{1,1} - \beta_{1,1,1,1} - \beta_{2,1,1,1} - \dots - \beta_{r-1,1,1,1}.$$

Hence we obtain the phase $\varphi(u,v)$, then $O(u,v)$ from (5), and finally the sought function $o(u,v)$ by inverse Fourier transformation: $o = F^{-1}(O)$.

The recursive method is fairly simple to implement and has been successfully used with one-dimensional signals. In the two-dimensional case its application has been limited to small images, because the efficiency of the recursive method drops sharply due to its slowness on large images. The accuracy achieved by the recursive method is generally lower than the accuracy of the variational method. The recursive and variational methods are compared in the next section.

5. Variational Method

The recursive bispectral method uses Eq. (8), which is written in terms of phase functions and requires evaluation of the phase $\beta(u, u_1, v, v_1)$ of the observed signal bispectrum. Since, first, the linear system operator after discretization is non-invertible and, second, the input data are highly distorted, the problem is ill-posed [19], which in practice involves additional computational instabilities.

To reduce the negative effect of these instabilities and at the same time to be able to process large images or frames with numerous aberrations, we will move from working with phases to working with phase multipliers in (7). Furthermore, instead of solving Eq. (7) for phase multipliers of the corresponding bispectra, we formulate a variational problem that requires minimizing the discrepancy functional that describes the mean square deviation of the complex exponentials over the grid points [12]:

$$E(\varphi) = \sum_{p,q,k,l} \left((\mathrm{Re}\,\delta_{p,q,k,l})^2 + (\mathrm{Im}\,\delta_{p,q,k,l})^2 \right) \to \min ,$$

where $\delta_{p,q,k,l} = \exp\{i\beta_{p,q,k,l}\} - \exp\{i(\varphi_{p,q}+\varphi_{k,l}-\varphi_{p+k,q+l})\}$.

When choosing an appropriate minimization method for the given problem size, we see that the gradient method is preferred to the quasi-Newton method: although it has a lower convergence rate, the gradient method does not require evaluating and saving a large matrix of second derivatives.

The gradient method constructs a minimizing functional of the sequence $\varphi^{(m)} = \{\varphi^{(m)}_{p,q}\}$ by the following rule [20]:

$$\varphi^{(m+1)} = \varphi^{(m)} - b_m \nabla E\left(\varphi^{(m)}\right), \qquad b_m > 0, \qquad m = 0,1,\ldots .$$

The conjugate gradient method [13] is also used in applications; various stopping rules for the gradient method are given in [20]. Numerical analysis has shown that the algorithm is unstable under input aberrations.

Methods for unstable minimization problems are constructed by regularizing the standard minimization methods [20]. In regularized methods, the minimization of the objective functional is replaced with minimization of the Tikhonov functional

$$T(\varphi) = E(\varphi) + \alpha \|\varphi\|^2 \to \min, \qquad \alpha > 0 ,$$

where $\|\circ\|$ is the ℓ_2-norm. As a result, we obtain the regularized gradient method

$$\varphi^{(m+1)} = \varphi^{(m)} - b_m \left[\nabla E\left(\varphi^{(m)}\right) + 2\alpha_m \varphi^{(m)} \right], \qquad m = 0,1,\ldots, \tag{9}$$

where the regularization parameter α_m should be chosen consistently with the gradient method increment b_m. To ensure consistency of the parameters entering the Tikhonov function, we apply the iterative regularization principle with prior parameter choice [20]. As an example of the sequences α_m, b_m satisfying the required conditions, we can take the sequences

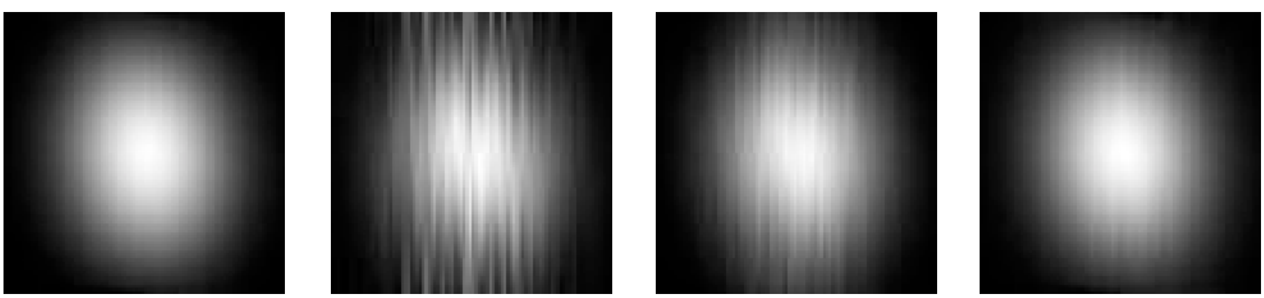
$$b_m = (1+m)^{-1/2}, \qquad \alpha_m = (1+m)^{-s}, \qquad 0 < s < 0.5 . \tag{10}$$

It is important to note that previous constructions of the variational method [12, 17, 18] used the squared-norm discrepancy functional for the linear system (8). This functional is convex. Therefore the corresponding Tikhonov functional in the iterative regularization method [17] with a fairly complex dependence of the regularization parameter on the current phase value can be made strongly convex, which simplifies the implementation of the minimization method.

The iterative regularization based on the discrepancy $E(\varphi)$ between the phase multipliers is better suited for applications, where we actually observe the phase multiplier $\exp\{i\beta(u,u_1,v,v_1)\}$, and not the phase $\beta(u,u_1,v,v_1)$ itself. Thus, in our problem, the Tikhonov functional $T(\varphi)$, generally speaking, is not strongly convex. However, allowing for the previously noted features of observed images (which, in the final analysis, provide invaluable prior information for the solution of the ill-posed problem) combined with prior choice of the regularization parameters, we manage to construct a regularized bispectral method for efficient image recovery. Other regularized phase-recovery methods that do not use information about the bispectrum can be found in [21].

6. Simulation of Image Recovery by Bispectral Variational Method

As noted previously, the recursive recovery method is adequate for restoring small images and signals, but the gradient method restores distorted images and signals with higher accuracy. The recursive image recovery method satisfactorily eliminates impulse noise on small images, although it takes somewhat longer than the bispectral variational method. Typical comparative results for the two methods run on a prototype example are shown in Fig. 2. In what follows we report the results obtained with the proposed variational method for various aberrations.



(i) **(ii)** **(iii)** **(iv)**

Fig. 2. Comparison of recursive and variational method: (i) original image without aberrations; (ii) image with aberrations; (iii) result produced by the recursive method; (iv) result produced by the variational method

6.1. Image Recovery in the Presence of Atmospheric Aberrations. One of the most common applications of the proposed method is in astrophysics, and therefore the first test was carried out on images with atmospheric aberrations shown in Fig. 3 [22]. As the original image we use an image with two bright spots – a common photograph in astrophysics (Fig. 4-iv). A series of aberrations was superimposed on the original image, creating a sequence of noisy images (Fig. 4-i) that were delivered to the input of the variational method. The images with aberrations in this example are essentially distinguished by their qualitative features from the original images: they show several false maxima of the light amplitude. Thus, one of the main subjective performance criteria for the recovery method is its ability to correctly restore the sought maxima while at the same time suppressing false maxima.

As we see from Fig. 4-ii, simple averaging does not produce the desired result: it provides only partial solution to our problem. On the other hand, the result produced by the variational method (9) with iterative regularization parameters chosen by (10) with $s = 0.25$ (Fig. 4-iii) constitutes an essential improvement: it ensures high-quality restoration of the sought amplitude maxima and of the original image as a whole.

Fig. 3. A series of atmospheric aberrations.

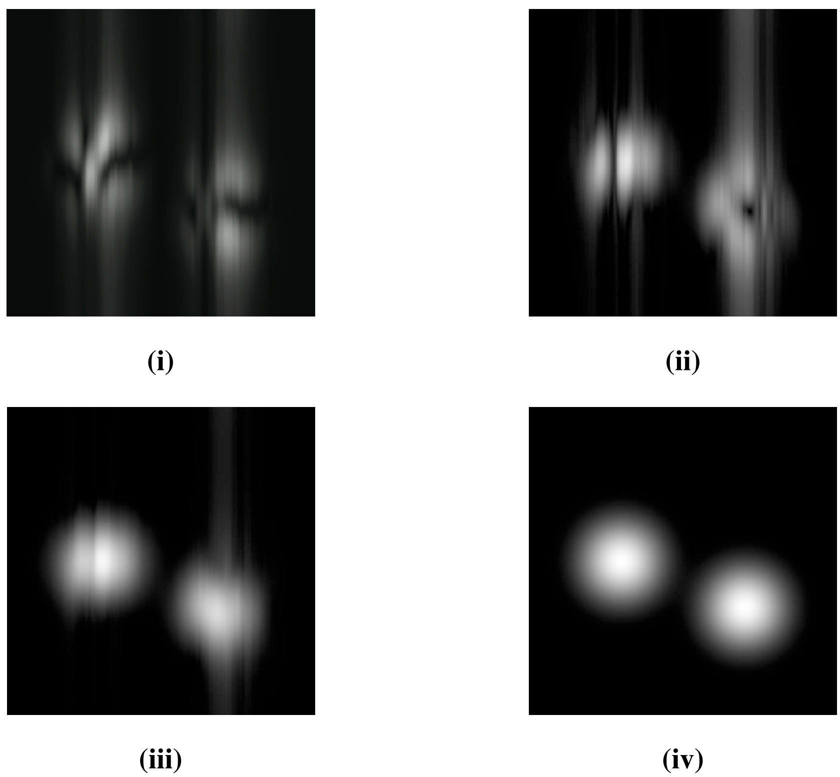


Fig. 4. Recovery of image in the presence of atmospheric aberrations. (i) one image from a series of images with aberrations; (ii) simple averaging of the series of images; (iii) outcome of the variational method; (iv) original image without aberrations.

6.2. Image Recovery in the Presence of Underwater Aberrations. Applications often involve recovery of images with various wave aberrations, e.g., aberrations produced by moving water waves. Aberrations of this kind can be observed in sonar problems. The distorted image suffers from blurring and partial loss of color. The input data consist of a series of distorted images, and the aberrations themselves have periodic structure. For numerical analysis of the efficiency of our variational method and its comparison with known results, we apply the model of wave aberrations from [16]. In modeling such aberrations, the displacement of each point that recedes from its original position is described by the following rules:

$$d_x = h \cdot p\left(1 - \frac{1}{n}\right) + \mathrm{N}, \qquad d_y = h \cdot q\left(1 - \frac{1}{n}\right) + \mathrm{N},$$

where h is the distance between the water surface and the bottom, n is the water reflection coefficient, N is white noise, p and q are the assumed displacements of water waves with different velocities and lengths:

$$p = -\sum a_j u_j \cos\{u_j x + v_j y - \omega_j t\}, \qquad q = -\sum a_j v_j \cos\{u_j x + v_j y - \omega_j t\}.$$

As a result, we obtain different types of waves that differ by depth and blurriness. This model contains certain assumptions. The first assumption is that a part of the water surface is stationary and planar. The second assumption is that the mean aberration at every point of the water surface is zero, given a sufficiently long observation time. Both assumptions are consistent with the real-life problem. Real data, however, contain also other aberrations, e.g., aberrations produced by wind or distortion of water waves by objects that fall into the water.

Let us consider the application of the regularized variational method using the example of two images with wave aberrations – monochrome text (Fig. 5) and a color image (Fig. 6). Figure 5-i is a typical text image with noise in the form of wave aberrations. Alongside geometrical text distortions, we also observe irregular noise. Such a situation arises, for instance, when we attempt to restore text written on a surface submersed under a layer of water in windy weather. Another cause of such aberrations is the convective motion of heated air layers (e.g., above a road surface), which produces essential optical aberrations.

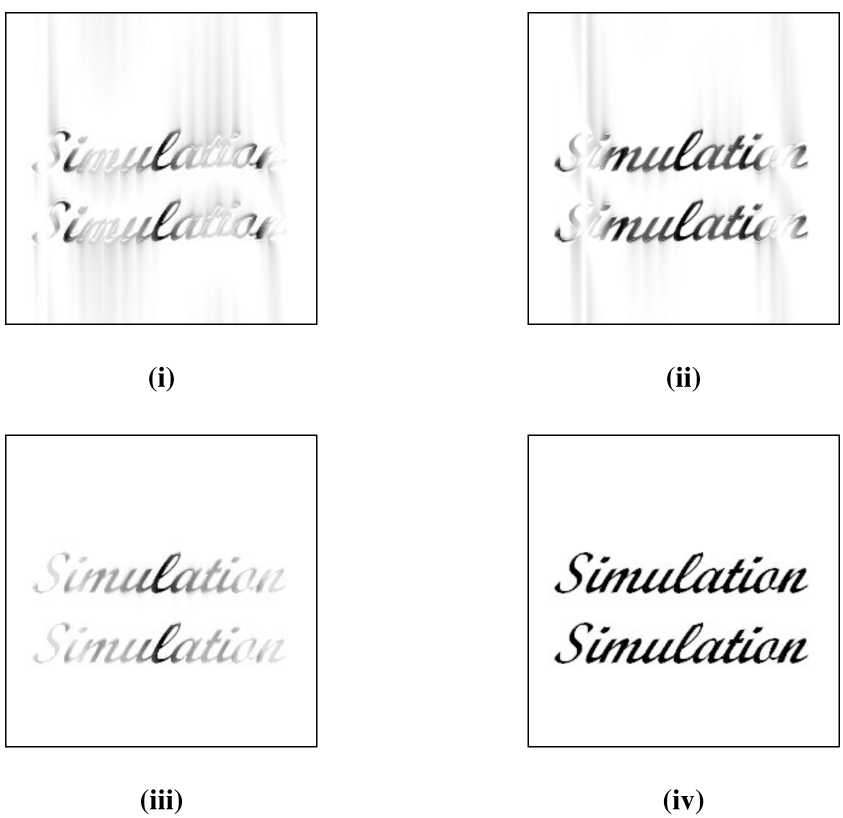


Fig. 5. Restoration of a text image with wave aberrations: (i) one image from a series of images with aberrations; (ii) result produced by recursive method; (iii) result produced by variational method; (iv) original aberration-free image

The series of images in Fig. 5 illustrates the removal of wave aberrations by the bispectral variational method. The result produced by the recursive method (Fig. 5-ii), although less noisy than the original image

(Fig. 5-iv), is clearly inferior to the image in Fig. 5-iii, restored by the bispectral variational method (9), (10) with $s = 1/3$.

Let us compare the quality of images with the same type of aberrations after restoration by the regularized variational method and the recursive method [16]. The recursive method generates an image which is more accurate than any image obtained by averaging, but less accurate than the result produced by the variational regularized method, even though the recursive method used a series of 100 images for restoration while the variational method used only 10 images.

Another example of correction of underwater aberrations (Fig. 6) illustrates the results produced by the regularized variational method for color images. The image restored by the regularized method (Fig. 6-iii) more accurately conveys the color of the original image than the image restored by averaging a series of images with aberrations (Fig. 6-ii). A specific feature of this example is that it clearly demonstrates contrast enhancement compared with the noisy image.

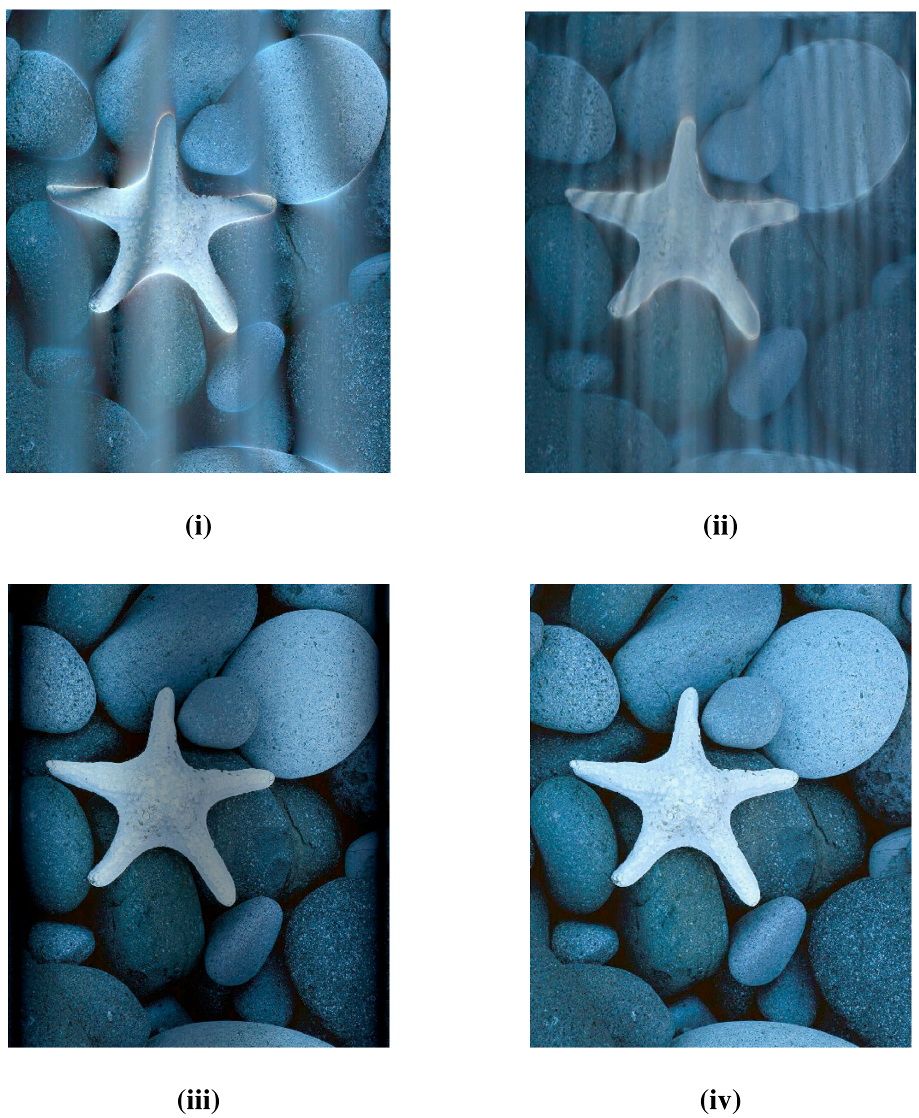


Fig. 6. Restoration of a color image with wave aberrations: (i) one image from a series of image with underwater aberrations; (ii) result produced by recursive method; (iii) result produced by variational method; (iv) original aberration-free image.

7. Conclusion

The traditional signal recovery method using estimated bispectra is a promising tool for image processing, because it allows preserving and restoring phase information, has low sensitivity to additive noise, and is invariant to random shifts of the original signal. Furthermore, the bispectral signal recovery method does not require quantitative prior information about the statistical properties of the observed processes. The efficiency of bispectral analysis in many applications is reduced, for instance, when the signal/noise ratio (for Gaussian noise) is less than 1 or when the additive noise does not follow the normal distribution. However, in these cases also the application of bispectral methods as well as the Tikhonov-regularized variational bispectral methods proposed in this article leads to significant enhancement of image restoration quality. The proposed image restoration method preserves all the previously noted strengths of bispectral analysis.

A distinctive feature of the iterative regularization method is its reliance on the discrepancy between phase multipliers, which is more appropriate in optical applications. Simulation results and comparisons of different methods suggest that the combined signal restoration methods ensure improved stability (compared with traditional bispectral processing). This approach makes it possible to implement an image restoration method that integrates the advantages of bispectral analysis with the advantages of regularized optimization methods. The proposed combined method can be applied to image and signal processing in systems exposed to high-intensity noises under conditions of prior uncertainty regarding the quantitative statistical characteristics of signals and noises. This applies to radar and sonar systems, optical systems for restoration of astronomical images, and systems for the detection of small objects.

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