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Conditions for string stability

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Abstract

The stability properties of a system, comprising an infinite string of identical coupled linear subsystems, are investigated by using discrete Fourier transforms defined on the set of component state vectors. Two theorems are presented, one of which gives sufficient conditions on the eigenvalues of the state evolution matrix for the transformed system, to guarantee string stability in a defined sense, while the other establishes an equivalence between state-space and frequency-domain conditions.

Keywords: Frequency domain; Linear systems; State space; String stability; Vehicle convoys

1. Introduction

The concept of string stability [9] is by now well established in the literature of systems and control theory, particularly with regard to the dynamics of vehicle convoy systems [6], although it also has other potential applications. Loosely speaking, the idea is that in a system of coupled components, such as a platoon of vehicles [5], disturbances should always be attenuated as they propagate through the system, so that it asymptotically settles to a steady state. In order to formulate this concept mathematically, the system is considered as an infinite string, since otherwise the definition would simply reduce to that of stability in the ordinary sense. A real system, however, is always finite, and this consideration raises the question of how the formal definition relates to its application in practice. This issue may not present any significant difficulty, provided that the propagation is always in only one direction, because a real finite string can then be regarded as a truncation of an idealized infinite one, and the implication of string stability is that its length can be arbitrarily extended without causing any amplification of disturbance effects. On the other hand, if propagation in both forward and backward directions can take place, the situation is less clear, in that an extension to the length may affect the behaviour throughout the string, not just in the extended part. It is thus arguable that a reconsideration of the concept may be appropriate, with the aim of clarifying both the definition and its relation to practical application, which is what will be attempted in this paper.

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Specifically, the intention is to relate frequency-domain conditions, derived in a previous investigation [4] from the consideration of finite strings, to spectral conditions obtained by Fourier transformation of the state-space for an infinite string.

The structure of the paper is as follows. Section 2 sets up the mathematical description of the class of systems considered, and Section 3 briefly presents the characterization of string stability from the state-space and frequency-domain viewpoints. In Section 4, the analysis of string stability for linear systems is addressed, and the main results presented. Some applications are made in Section 5, and the conclusions discussed in Section 6.

2. System description

The system under consideration is assumed to be formed by the interconnection of identical components from an infinite set, labelled by an index which can range over all integer values. Because the components are identical and the interconnection structure is taken to be uniform, the system belongs to the class considered in [1] and can be analysed by similar methods, although the issues concerned here are different. For the ith component considered separately, there is a finite-dimensional state-vector $y_i(t)$ and an input $u_i(t)$, so that the components can be interconnected by making the input of each one depend on the states of others. If there were only a finite number of components, the structure would be similar to that considered in [3], but here the index i ranges over all integer values. As a result, the state of the whole system becomes infinite-dimensional, with $y_i(t)$ satisfying an equation of the form

$$\dot{y}_i = \varphi(y_{i-n}, \ldots, y_{i+n'})$$

for some nonnegative integers (n, n') and some, generally nonlinear, vector function $\varphi(\cdot)$. In the event that the system is linear, the corresponding equation is

$$\dot{y}_i = \sum_{r=-n'}^{n} A_r y_{i-r}, \tag{1}$$

where the A_r are constant matrices. Such a structure can arise from the interconnection of components having inputs u_i, with

$$\dot{y}_i = A_0 y_i + B u_i,$$

where

$$u_i = \sum_{r \neq 0} C_r y_{i-r}$$

so that

$$A_r = B C_r \tag{2}$$

for $r \neq 0$. Then, defining matrix functions

$$G_r(s) = C_r(sI - A_0)^{-1} B$$

and assuming zero initial conditions, the system description can be reformulated as

$$U_i(s) = \sum_{r \neq 0} G_r(s) U_{i-r}(s), \tag{3}$$

where $U_i(s)$ denotes the Laplace transform of $u_i(t)$.

Up to this point, it has not been necessary to specify the dimension of $u_i(t)$, but subsequent attention will be concentrated mainly on the scalar case, where $[B, C_r]$ are, respectively, column and row vectors $[b, c_r]$ so that

$$G_r(s) = c_r(sI - A_0)^{-1} b \tag{4}$$

is a scalar function as in [4]. Nevertheless, some of the results appear to be capable of generalization to the matrix case, as will be indicated later.

3. Definitions of stability

The formulation of definitions and criteria for string stability can be addressed in terms of either time or frequency-domain concepts, and the purpose of this investigation is to relate these two approaches.

3.1. Time domain

For interconnected systems in the state-space form (1), typical characterizations of string stability are as follows [9]. The system is said to be *string stable* if, given any $\varepsilon > 0$, there is a $\delta > 0$ such that

$$\sup_i \|y_i(0)\| \leqslant \delta \Rightarrow \sup_i \|y_i(t)\| \leqslant \varepsilon \quad \forall t > 0$$

and *asymptotically string stable* if it is string stable and also $\exists \Delta > 0$ such that

$$\sup_i \|y_i(0)\| \leqslant \Delta \Rightarrow \sup_i \|y_i(t)\| \to 0$$

as $t \to \infty$. Similarly, for any $p \geqslant 1$, it is said to be ℓ_p-*string stable* if, given any $\varepsilon > 0$, there is a $\delta > 0$ such that

$$\left[\sum_i \|y_i(0)\|^p\right]^{1/p} \leqslant \delta \Rightarrow \left[\sum_i \|y_i(t)\|^p\right]^{1/p} \leqslant \varepsilon \quad \forall t > 0$$

which reduces to the definition of string stability when $p \to \infty$. Here, however, it will be convenient to use slightly modified definitions, as follows.

Definition 1. The system is said to be *practically string stable* if, given any $\varepsilon > 0$, there is a $\delta > 0$ such that

$$\sum_i \|y_i(0)\| \leqslant \delta \Rightarrow \sup_i \|y_i(t)\| \leqslant \varepsilon \quad \forall t > 0.$$

Definition 2. The system is said to be *practically asymptotically string stable* if it is practically string stable and also $\exists \Delta > 0$ such that

$$\sum_i \|y_i(0)\| \leqslant \Delta \Rightarrow \sup_i \|y_i(t)\| \to 0$$

as $t \to \infty$.

Remark 1. It is clear from the definitions that (*asymptotic*) *string stability* implies *practical* (*asymptotic*) *string stability*, though the converse may not necessarily be true.

Remark 2. For the application of Definitions 1 and 2 to linear systems, it is sufficient that the initial state satisfies $y_i(0) \neq 0$ for only a finite set of $\{i\}$, because this ensures that

$$\sum_i \|y_i(0)\| \leqslant \delta$$

for some $\delta > 0$, and the scaling is irrelevant on account of the linearity.

3.2. Frequency domain

On the other hand, at least for linear systems, a more convenient characterization may be obtained from a frequency-domain viewpoint, by considering how disturbances propagate along the string. In the vehicle-following context, the input $u_i(t)$ is normally a scalar, representing the driving force or motor torque applied to the ith vehicle. Then, if the motion of each vehicle is influenced only by the one immediately ahead of it, so that the Laplace-transformed system description (3) becomes

$$U_i(s) = G(s)U_{i-1}(s)$$

for some scalar transfer function $G(s)$, it has been shown [8] that an appropriate condition for string stability is $|G(\jmath\omega)| < 1$ for all real $\omega \neq 0$, which ensures that disturbances are attenuated as they propagate. Furthermore, it has been argued [4] that a similar analysis can be made more generally, and specifically for the cases where the control of each vehicle takes account either of those an arbitrary number of places ahead, or of those immediately ahead and astern. For the first of these cases, corresponding to $n' = 0$ in (1), the condition found there is

$$|z_k(\jmath\omega)| < 1$$

for all k with $1 \leqslant k \leqslant n$ and all $\omega \neq 0$, where $z_k(s)$ are the roots of the equation

$$\sum_{r=1}^{n} G_r(s) z^{-r}(s) = 1$$

while for the second case, obtained by taking $n = n' = 1$, the condition is

$$|z_1(\jmath\omega)| < 1 < |z_{-1}(\jmath\omega)|$$

for all $\omega \neq 0$, where $z_1(s)$ and $z_{-1}(s)$ are the roots of

$$G_{-1}(s)z(s) + G_1(s)z^{-1}(s) = 1$$

in the notation of this paper. It does not, however, seem to be clear how the conditions obtained in this way relate to the state-space definitions, and the treatment given here is intended to establish a more secure connection between these approaches to the issue, as well as indicating how the results might be generalized.

4. Linear systems

If the system equations are linear, their solution can be accomplished through the use of Laplace and Fourier transforms, as in [2] for a linearized model. For this purpose, it is convenient to introduce a discrete Fourier transform of the infinite-dimensional state-vector by defining

$$\tilde{y}(\theta, t) = \sum_i y_i(t) \exp(-\jmath \mathrm{i}\theta) \tag{5}$$

with the Laplace transform

$$Y(\theta, s) = \int_0^\infty \tilde{y}(\theta, t) \exp(-st)\, \mathrm{d}t$$

so that, from Eq. (1),

$$\dot{\tilde{y}}(\theta, t) = F(\theta)\tilde{y}(\theta, t) \tag{6}$$

where

$$F(\theta) = \sum_{r=-n'}^{n} A_r \exp(-\jmath r\theta) \tag{7}$$

giving

$$Y(\theta, s) = [sI - F(\theta)]^{-1}\tilde{y}(\theta, 0)$$

or equivalently

$$\tilde{y}(\theta, t) = \exp[F(\theta)t]\tilde{y}(\theta, 0) \tag{8}$$

as the solution, from which

$$y_i(t) = \sum_k \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[F(\theta)t + \jmath(\mathrm{i} - k)\theta I]\, \mathrm{d}\theta\, y_k(0)$$

by inverting the Fourier transformation. More compactly, by defining

$$\Gamma_i(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[F(\theta)t + \jmath \mathrm{i}\theta I]\, \mathrm{d}\theta \tag{9}$$

the solution can be written as

$$y_i(t) = \sum_k \Gamma_{i-k}(t) y_k(0) \tag{10}$$

displaying the dependence of the state-vector for each component subsystem on the initial conditions of them all. By expanding the matrix exponential function in the expression for $\Gamma_i(t)$, it can be seen that, if $n' = 0$ as in the *look-ahead* case of convoy control, then $\Gamma_i(t) = 0$ for $i < 0$, so that the state of each subsystem is affected only by those earlier in the sequence, as would be expected. Similarly, if $n = 0$, then $\Gamma_i(t) = 0$ for $i > 0$, so that the propagation is in the opposite direction.

4.1. Stability analysis

A natural approach to the analysis of stability for linear systems in state-space form is to consider the spectrum of the state evolution matrix. Here, because of Eq. (6), the evolution matrix for the Fourier-transformed state-vector (5) is $F(\theta)$, and the matrix exponential $\exp[F(\theta)t]$ appears as the state transition matrix in solution (8). In order to elucidate its properties, the following lemmas will be useful.

Lemma 1. *For any real or complex square matrix A, there exists a unitary matrix U such that*

$$U^{-1}AU = D + Q,$$

where D is a diagonal matrix of eigenvalues of A, and Q is a matrix whose only nonzero entries are on its superdiagonal, with Euclidean spectral norm satisfying

$$\|Q\| \leqslant \|A\|.$$

Proof. The proof is an extension of that given in [7], and proceeds by induction on the order N of the matrices, noting that the result obviously holds for $N = 1$. Suppose it holds for order $N - 1$, and A has order N. Construct a unitary matrix U with columns $u_1, u_2, \ldots, u_N$ as follows: u_1 is an eigenvector of A, corresponding to some eigenvalue α; u_2 is a linear combination of u_1 and A^*u_1 where * denotes Hermitian conjugate, unless these are linearly dependent, in which case u_2 is an arbitrary vector orthogonal to u_1; the remainder are arbitrarily chosen so as to complete an orthonormal set, and are consequently orthogonal to A^*u_1. Hence,

$$U^{-1}AU = \begin{pmatrix} \alpha & w \\ 0 & P \end{pmatrix},$$

where w is a row vector of length $N - 1$ with all elements zero except the first, and P is a square matrix of order $N - 1$. Then, since by hypothesis there is a unitary transformation which converts P into the required form, it follows that the same is true for A. This establishes the form of Q, whence $\|Q\|$ is equal to the maximum modulus of its elements, and so

$$\|Q\| \leqslant \|U^{-1}AU\| = \|A\|$$

since a unitary transformation preserves the norm. □

Lemma 2. *If every eigenvalue of A has nonpositive real part, then*

$$\|\exp(At)\| \leqslant \sum_{k=0}^{N-1} \frac{\|A\|^k t^k}{k!} \quad \forall t > 0.$$

Proof. Lemma 1 gives

$$U^{-1}(sI - A)U = sI - D - Q$$

and so

$$U^{-1}(sI - A)^{-1}U = \sum_{k=0}^{N-1} (sI - D)^{-1}[Q(sI - D)^{-1}]^k$$

from the form of Q. Then, taking inverse Laplace transforms, the result follows by induction on k. □

4.2. Main results

The following theorem gives sufficient conditions, on the eigenvalue locations of the state evolution matrix, for the forms of string stability characterized by Definitions 1 and 2.

Theorem 1. *Suppose all the eigenvalues of $F(\theta)$, for all real θ, lie strictly to the left of $-\sigma$ in the complex plane. Then the system is practically string stable if $\sigma \geqslant 0$, and practically asymptotically string stable if $\sigma > 0$.*

Proof. Because the eigenvalues of $F(\theta)$ are continuous functions of θ, Lemma 2 can be applied to $A = F(\theta) + \sigma' I$ for some $\sigma' > \sigma$, showing that $\exists M > 0$ with

$$\|\exp[F(\theta)t]\| \leqslant M \exp(-\sigma t)$$

for all $t \geqslant 0$ and all real θ. The result then follows by using Eqs. (9) and (10), together with Definitions 1 and 2. □

The next step is to relate the evolution matrix (7) to the transfer functions $G_r(s)$ defined in (4), using the expression

$$sI - F(\theta) = sI - A_0 - \sum_{r \neq 0} bc_r \exp(-\jmath r\theta)$$

derived from (2), together with the determinantal identity

$$\det[I - BC] \equiv \det[I - CB]$$

which holds for any matrix pair $[B, C]$ of compatible dimensions, to give

$$\det[sI - F(\theta)] = \left[1 - \sum_{r \neq 0} G_r(s) \exp(-\jmath r\theta)\right] \times \det(sI - A_0) \tag{11}$$

so that the spectrum of $F(\theta)$ comprises only those values of s for which

$$\sum_{r \neq 0} N_r(s) \exp(-\jmath r\theta) = \det(sI - A_0),$$

where

$$G_r(s) = \frac{N_r(s)}{\det(sI - A_0)}$$

with

$$N_r(s) = c_r \operatorname{adj}(sI - A_0)b$$

being the numerator polynomials of $G_r(s)$. In this connection, it will be convenient to define

$$\Phi(s, z, \lambda) = 1 - \lambda \sum_{r \neq 0} G_r(s) z^{-r}, \tag{12}$$

where λ is a scalar parameter, so that

$$\Phi(s, z, 0) = 1$$

and

$$\Phi(s, \exp(\jmath\theta), 1) = \frac{\det[sI - F(\theta)]}{\det(sI - A_0)} \tag{13}$$

from (11) and (12).

It will now be shown that the following conditions are related:

(I) All the nonzero eigenvalues of $F(\theta)$ are in the open left-half-plane for all real θ.

(II) All the eigenvalues of A_0 are in the open left-half-plane.
(III) The equation for z given by

$$\sum_{r\neq 0} G_r(\jmath\omega)z^{-r} = 1 \tag{14}$$

has n roots with $|z| < 1$ and n' roots with $|z| > 1$ for all real $\omega \neq 0$.

Theorem 2. (a) I *implies* III. (b) II *and* III *together imply* I.

Proof. (a) Considering the root loci of (14) for z as functions of ω, it follows that because

$$G_r(\jmath\omega) \to 0$$

in the limit $\omega \to \pm\infty$, there are then n roots approaching zero and n' roots tending to infinity. Consequently, if condition III were to be violated, then from (12) and (13), there would be some real θ and $\omega \neq 0$ such that

$$\det[\jmath\omega I - F(\theta)] = 0$$

contradicting condition I.

(b) With condition II satisfied, it is sufficient, because of (13), to show that when $|z| = 1$, the graph of $\Phi(\jmath\omega, z, 1)$ does not encircle the origin as ω goes from $-\infty$ to ∞. Now, by considering the dependence of (12) on λ, one can write

$$\Phi(s, z, \lambda) = \Psi(s, \lambda) \prod_{r=1}^{n} \left[1 - \frac{z_r(s, \lambda)}{z}\right] \times \prod_{r'=1}^{n'} \left[1 - \frac{z}{z_{-r'}(s, \lambda)}\right]$$

with $\Psi(s, \lambda)$ independent of z and $\Psi(s, 0) = 1$, where if condition III is satisfied, then

$$|z_r(\jmath\omega, 1)| < 1 < |z_{-r'}(\jmath\omega, 1)|$$

for all $-n' \leqslant -r' < 0 < r \leqslant n$ and $\omega \neq 0$. Also, the graph of $\Psi(\jmath\omega, 1)$ cannot encircle the origin because if it did so, there would exist λ with $0 < \lambda < 1$ and $\omega \neq 0$ such that $\Psi(\jmath\omega, \lambda) = 0$, and then $\Phi(\jmath\omega, z, \lambda) = 0$ independently of z, which is impossible from (12). Consequently, the nonencirclement criterion is satisfied, and so condition I is established. $\square$

The significance of Theorem 2 is that, under the assumption of asymptotic stability for the separate subsystems (i.e., condition II), it establishes an equivalence between the state-space-based condition I, on the eigenvalue distribution of the evolution matrix $F(\theta)$, and the frequency-domain condition III, on the roots of Eq. (14). It thus provides a further justification and extension for the analysis previously given [4], on the basis of disturbance propagation, for the cases $n' = 0$ (or similarly $n = 0$) and $n = n' = 1$, where condition III was also obtained. Moreover, an inspection of the analysis suggests that similar results could be obtained if $G_r(s)$ were a matrix, by replacing Eq. (14) with

$$\det\left[I - \sum_{r\neq 0} G_r(\jmath\omega)z^{-r}\right] = 0$$

and appropriately modifying the required distribution of its roots. Condition I, however, is slightly weaker than the assumptions of Theorem 1, in that it allows $F(\theta)$ to have eigenvalues equal to zero for some θ, as is indeed necessary in a typical application to vehicle convoying [4], where Eq. (14) has a root at $z = 1$ for $\omega = 0$. Consequently, a more precise analysis is needed in order to determine whether or not condition I actually ensures the (practical) string stability properties in such a case. This may well be possible, e.g., by exploiting the integration over θ in Eq. (9), but will not be attempted here. Instead, some examples will be presented, which indicate that the implications of condition I (or, consequently, II and III) are stronger than what has so far been proved.

5. Examples

In order to verify the results of the preceding section and illustrate their application, the following examples will now be considered.

Example 1. Suppose that $y_i(t)$ is scalar and that $n = n' = 1$, giving the simplest case of bidirectional propagation. Then $F(\theta)$ is also scalar, with

$$F(\theta) = A_0 + (A_1 + A_{-1})\cos\theta - \jmath(A_1 - A_{-1})\sin\theta$$

so that condition I requires

$$|A_1 + A_{-1}| \leqslant -A_0$$

and either $A_1 = A_{-1}$ or $A_1 + A_{-1} \neq 0$, while condition II becomes simply $A_0 < 0$. Also,

$$\Phi(s, z, 1) = 1 - \left(\frac{A_1 z^{-1} + A_{-1} z}{s - A_0} \right)$$

so the equation $\Phi(\jmath\omega, z, 1) = 0$ becomes

$$A_1 z^{-1} + A_{-1} z = \jmath\omega - A_0$$

from which condition III is obtained, and it is then straightforward to verify compatibility with the results of Theorem 2.

With regard to Theorem 1, it may be noted that although condition I is weaker than the requirements, it is actually a little stronger than is necessary, in this case, to ensure the conclusions. For instance, even if $A_0 = 0$ and $A_1 = -A_{-1} = a/2$, corresponding to the state-space equation

$$\dot{y}_i = \frac{a}{2}(y_{i-1} - y_{i+1})$$

then

$$F(\theta) = -\jmath a \sin \theta$$

and so from (9),

$$\Gamma_i(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(\mathrm{i}\theta - at \sin \theta)\, \mathrm{d}\theta$$

which is just the Bessel function $\Gamma_i(t) = J_i(at)$, with the property

$$\Gamma_i(t) \to 0$$

as $t \to \infty$. Consequently, the system is practically asymptotically string stable in the sense of Definition 2.

Example 2. Denoting the position of the *i*th vehicle in a convoy by x_i, and its velocity by v_i, a simple bidirectional control law is given by

$$\dot{v}_i = k_P(x_{i-1} - 2x_i + x_{i+1}) + k_V(v_{i-1} - 2v_i + v_{i+1})$$

with

$$\dot{x}_i = v_i,$$

where k_P and k_V are constant gains. In practice, of course, the convoy will be of finite length, but here the idealized case of an infinitely long platoon is considered. Defining the state-vector

$$y_i = \begin{pmatrix} x_i \\ v_i \end{pmatrix}$$

and setting up the corresponding state equations, the matrix $F(\theta)$ becomes

$$F(\theta) = A_0 + A_1 \exp(-\jmath\theta) + A_{-1} \exp(\jmath\theta)$$

with the coefficient matrices

$$A_0 = \begin{pmatrix} 0 & 1 \\ -2k_P & -2k_V \end{pmatrix}, \quad A_1 = A_{-1} = \begin{pmatrix} 0 & 0 \\ k_P & k_V \end{pmatrix}$$

giving

$$F(\theta) = \begin{pmatrix} 0 & 1 \\ -2k_P[1 - \cos\theta] & -2k_V[1 - \cos\theta] \end{pmatrix}$$

so that

$$\det[sI - F(\theta)] = s^2 + 2(1 - \cos\theta)(k_V s + k_P)$$

from which conditions I and II are both satisfied when $k_P > 0$ and $k_V > 0$, confirming the results of Theorem 2 by comparison with the use of condition III in [4].

It is of interest also to consider the case where $k_P > 0$ and $k_V = 0$, so that

$$F(\theta) = \begin{pmatrix} 0 & 1 \\ -4k_P \sin^2(\theta/2) & 0 \end{pmatrix}$$

which gives

$$\exp[F(\theta)t] = \begin{pmatrix} \cos[\rho(\theta)t] & \sin[\rho(\theta)t]/\rho(\theta) \\ -\rho(\theta)\sin[\rho(\theta)t] & \cos[\rho(\theta)t] \end{pmatrix}$$

where

$$\rho(\theta) = 2\sqrt{k_P} \sin\frac{\theta}{2}$$

with the result that $\Gamma_i(t)$ can again be expressed in terms of Bessel functions. One finds from (9) that

$$\Gamma_i(t) = \begin{pmatrix} J_{2i}(2t\sqrt{k_P}) & \int_0^t J_{2i}(2t'\sqrt{k_P})\,\mathrm{d}t' \\ \sqrt{k_P}\,[J_{2i-1}(2t\sqrt{k_P}) - J_{2i+1}(2t\sqrt{k_P})] & J_{2i}(2t\sqrt{k_P}) \end{pmatrix}$$

which remains bounded as $t \to \infty$. The system is therefore practically string stable in the sense of Definition 1, even though the conditions assumed are weaker than those in Theorem 1.

6. Conclusions

The main results of the paper are Theorems 1 and 2. Theorem 1 gives sufficient conditions on the spectrum of the evolution matrix for the Fourier-transformed state vector, to ensure string stability in the sense of Definitions 1 and 2. Theorem 2 establishes the connection between a condition on the evolution matrix, closely related to those assumed in Theorem 1, and a frequency-domain criterion previously developed. On the basis of these results, and some illustrative examples, it is argued that state-space and frequency-domain methods can be, in effect, applied interchangeably to the analysis of string stability for linear systems, according to whichever is computationally more convenient.

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