

# Optimum operating conditions of irreversible solar driven heat engines

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## Abstract

An optimal performance analysis of an internally and externally irreversible solar driven heat engine has been carried out. A Carnot-type heat engine model for radiative and convective boundary conditions was used to consider the effects of the finite-rate heat transfer and internal irreversibilities. The power and power density functions have been derived and maximization of these functions has been carried out for various design parameters. The optimum design parameters have been derived and the obtained results for maximum power (MP) and maximum power density (MPD) conditions have been compared. The effects of the technical parameters on the performance have been investigated.

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**Keywords:** Internal irreversibilities; Solar driven; Thermodynamics optimization; Power density; Finite-time thermodynamics

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## 1. Introduction

Solar driven heat engine systems, which consists of a solar collector and a heat engine have a large potential for saving fossil fuel and decreasing environmental pollution. Basic concepts of solar driven systems are detailed in [1–4].

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**Nomenclature**

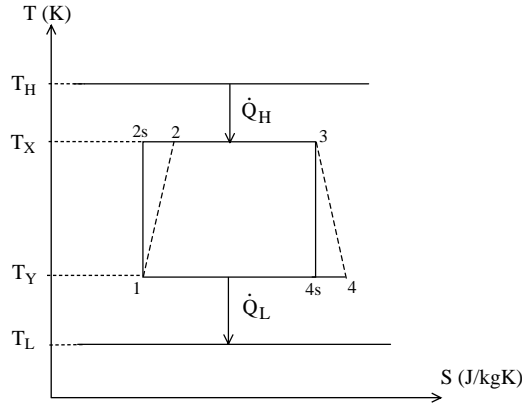
$A$	heat transfer area ( $\text{m}^2$ )
$I$	irreversibility parameter
$m$	mass of the working fluid (kg)
$\dot{Q}$	rate of heat transfer (W)
$R$	ideal gas constant (kJ/kg K)
$S$	entropy (J/kg K)
$T$	temperature (K)
$U$	overall heat transfer coefficient ( $\text{W}/\text{m}^2 \text{ K}$ )
$\dot{W}$	power output (W)
Greek symbols	
$\beta$	heat conductance parameter
$\eta$	thermal efficiency
$\tau$	extreme temperature ratio
Subscripts	
d	density
H	heat source
L	heat sink
max	maximum
mp	maximum power conditions
mpd	maximum power density conditions
X	warm working fluid
Y	cold working fluid
Superscripts	
—	dimensionless

Since the power optimization studies of heat engines using finite-time thermodynamics were started by Chambadal [5] and Novikov [6] much works has been undertaken on the performance analysis and optimization. In recent years, the performance of a solar driven heat engine using the technique of finite time thermodynamic analysis has been investigated. In these studies, the objective functions chosen for optimization are usually power output [7–14] and power density output [15,16].

The objective of the present study is to investigate the optimal performance of an irreversible solar driven Carnot-type heat engine. Maximum power and the maximum power density performances of the cycle are analyzed and the effect of the external and internal irreversibilities on the performance is also studied.

## 2. The theoretical model

An irreversible solar driven heat engine operating between two heat reservoirs temperatures ( $T_H$  and  $T_L$ ) is shown in Fig. 1. The temperatures of the fluid exchanging heat

Fig. 1.  $T$ - $S$  diagram of a Carnot-type solar-powered heat engine.

with the reservoirs at  $T_H$  and  $T_L$  are  $T_X$  and  $T_Y$ , respectively. Heat transfer from the hot reservoir is assumed to be radiation dominated and the heat flow rate  $\dot{Q}_H$  from the hot reservoir to the heat engine can be written

$$\dot{Q}_H = U_H A_H (T_H^4 - T_X^4) \quad (1)$$

where  $U_H$  is the hot side heat transfer coefficient and  $A_H$  is the heat transfer area of the hot side heat exchanger. On the other hand, convection heat transfer is assumed to be the main mode of heat transfer to the low temperature reservoir, and therefore, the heat flow rate  $\dot{Q}_L$  from the heat engine to the cold reservoir can be written as

$$\dot{Q}_L = U_L A_L (T_Y - T_L) \quad (2)$$

where  $U_L$  is the cold side heat transfer coefficient and  $A_L$  is the heat transfer area of the cold side heat exchanger. From the first law of the thermodynamics the net power output of the solar driven heat engine is

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = \left[ \beta \frac{(T_H^4 - T_X^4)}{T_H^3} - (T_Y - T_L) \right] U_L A_L. \quad (3)$$

Where  $\beta$  is the ratio of the heat transfer areas and the heat conductance parameter and defined as

$$\beta = \frac{U_H A_H}{U_L A_L} T_H^3 \quad (4)$$

Assuming an ideal gas, the maximum volume in the cycle  $V_4$  can be written as

$$V_{\max} = V_4 = mRT_4/P_4 = mRT_Y/P_{\min} \quad (5)$$

where  $m$  is the mass of the working fluid and  $R$  is the ideal gas constant. In the analysis, the minimum pressure in the cycle ( $P_4$ ) is taken to be constant. The power density defined as the ratio of power to the maximum volume in the cycle [11,12] then takes the form

$$\dot{W}_d = \dot{W}/V_{\max} = \left[ \beta \left( \frac{T_H^4 - T_X^4}{T_H^3} \right) - (T_Y - T_L) \right] \frac{U_L A_L P_{\min}}{m R T_Y} \quad (6)$$

From the second law of thermodynamics for an irreversible cycle, the change in the entropies of the working fluid for heat addition and heat removing processes yields,

$$\oint \frac{\delta \dot{Q}}{T} = \frac{\dot{Q}_H}{T_X} - \frac{\dot{Q}_L}{T_Y} < 0. \quad (7)$$

One can rewrite the inequality in Eq. (7) as

$$\frac{\dot{Q}_H}{T_X} = I \frac{\dot{Q}_L}{T_Y} \quad 0 < I < 1. \quad (8)$$

With the above definition  $I$  becomes

$$I = \frac{\dot{Q}_H T_Y}{\dot{Q}_L T_X} = \frac{T_X (s_3 - s_2) T_Y}{T_Y (s_4 - s_1) T_X} = \frac{s_3 - s_2}{s_4 - s_1}. \quad (9)$$

From Eqs. (1), (2) and (6)

$$T_Y = \frac{T_L}{1 - \frac{\beta}{I} \frac{(T_H^4 - T_X^4)}{T_X T_H^3}}. \quad (10)$$

The thermal efficiency of the irreversible heat engine is

$$\eta = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{T_Y}{I T_X} \quad (11)$$

By using Eqs. (3), (6), (10) and (11) one can obtain the dimensionless power output  $[\bar{W} = \dot{W}/(U_L A_L T_L)]$ , dimensionless power density  $[\bar{W}_d = \dot{W}_d/(P_{\min} U_L A_L/T_Y)]$  and thermal efficiency ( $\eta$ ), respectively as,

$$\bar{W} = \beta \left( \frac{T_H^4 - T_X^4}{T_L T_H^3} \right) - \left( \frac{T_Y - T_L}{T_L} \right) \quad (12)$$

$$\bar{W}_d = \beta \left( \frac{T_H^4 - T_X^4}{T_Y T_H^3} \right) - \left( \frac{T_Y - T_L}{T_Y} \right) \quad (13)$$

$$\eta = 1 - \frac{T_L T_H^3}{I T_X T_H^3 - \beta (T_H^4 - T_X^4)}. \quad (14)$$

Since the Eqs. (12) and (13) are functions of the working fluid temperatures ( $T_X$ ,  $T_Y$ ), this functions can be maximized with respect to  $T_X$  and  $T_Y$ . This optimization

procedure has been carried out numerically and the results are presented in next section.

### 3. Results and discussion

The variations of the dimensionless power output ( $\bar{W}$ ) and power density output ( $\bar{W}_d$ ) with respect to thermal efficiency ( $\eta$ ) for various extreme temperature ratios ( $\tau = T_H/T_L$ ) are shown in Fig. 2(a) and (b), respectively. From the observation of these figures one can

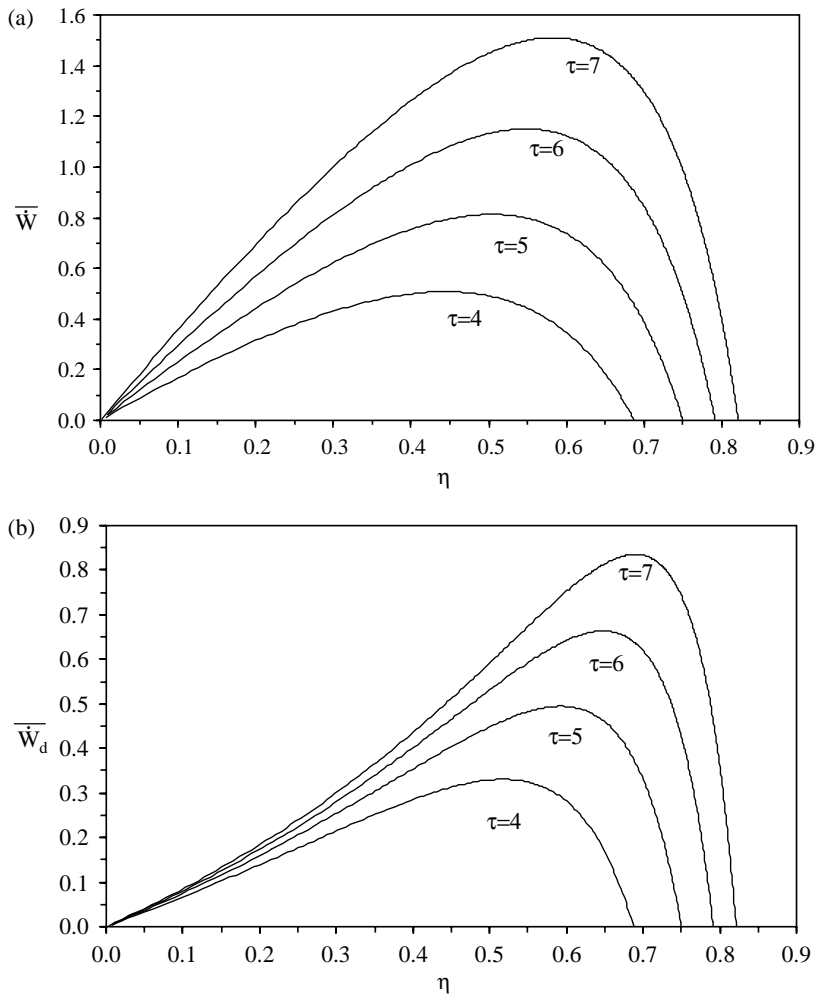


Fig. 2. Variations of the dimensionless power (a) and dimensionless power density (b) with respect to thermal efficiency for various  $\tau$  values ( $\beta=1$ ,  $I=0.8$ ).

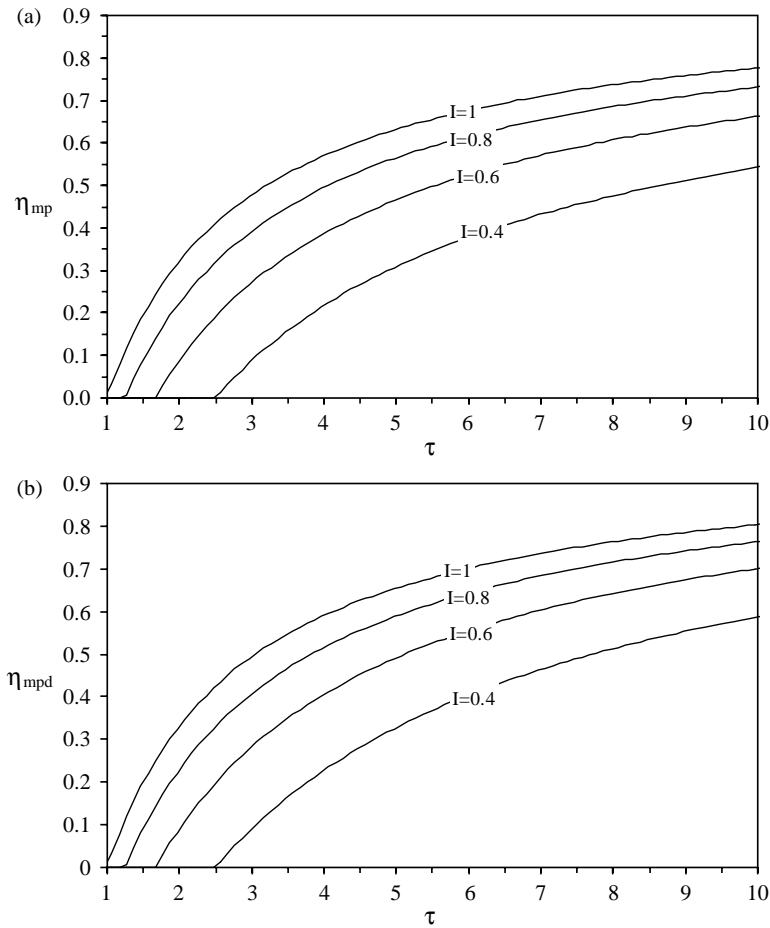


Fig. 3. Variations of the thermal efficiency at maximum power (a) and maximum power density (b) conditions with respect to  $\tau$  values for various  $I$  values ( $\beta=1$ ).

see that as  $\tau$  increases, the performance of the solar driven heat engine gets better for both the MP and MPD conditions. The thermal efficiencies at MP and MPD conditions also increase as  $\tau$  increases. This situation can be seen more clearly in Fig. 3 which shows the variations of the thermal efficiency at MP conditions ( $\eta_{mp}$ ) and the thermal efficiency at MPD conditions ( $\eta_{mpd}$ ) for various irreversibility parameter ( $I$ ). For the chosen values of  $I$  and  $\tau$ , the thermal efficiency of MPD is greater than the thermal efficiency at MP condition, i.e.  $\eta_{max} > \eta_{mpd} > \eta_{mp}$ .

The variations of the dimensionless power and power density with respect to thermal efficiency are shown for various internal irreversibility parameter in Fig. 4(a) and (b) and for various  $\beta$  values in Fig. 5(a) and (b). From these figures, one can observe

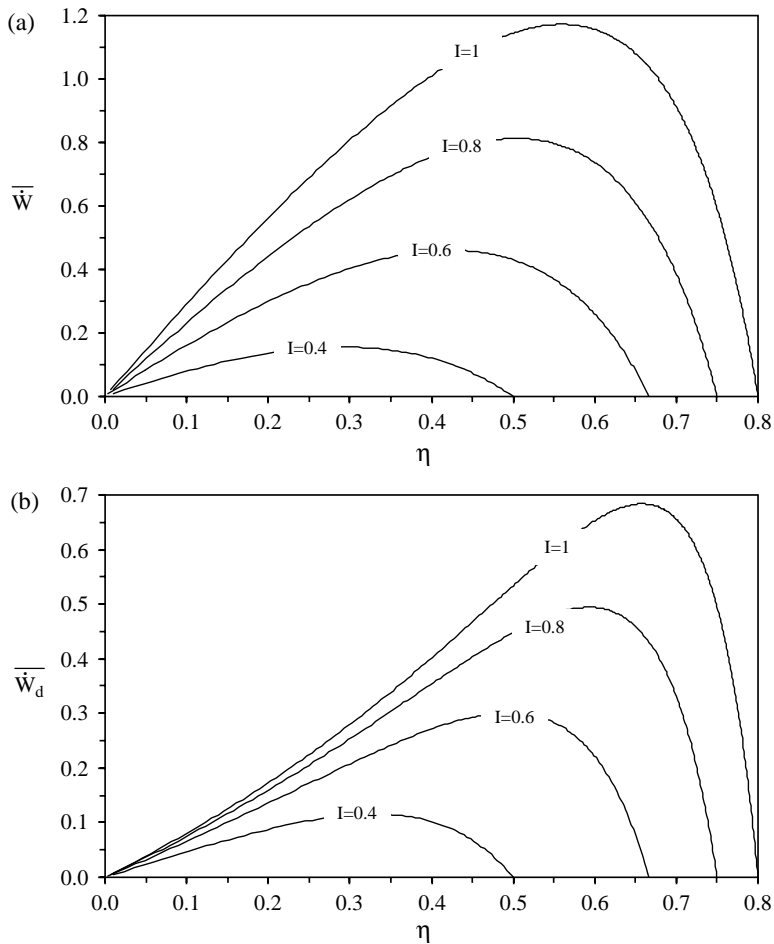


Fig. 4. Variations of the dimensionless power (a) and dimensionless power density (b) with respect to thermal efficiency for various  $I$  values ( $\beta=1$ ,  $\tau=5$ ).

the effects of the internal irreversibility and  $\beta$  on the global and optimal performances. As the internal irreversibility and  $\beta$  increase the global and the optimal performances gradually decrease. For all values of  $I$  and  $\beta$  the dimensionless power output is greater than the dimensionless power density output. The comparison of the dimensionless power and power density outputs can be seen in Fig. 6. Point (a) shows the maximum power situation and point (b) shows the situation of maximum power density. The design parameters according to the power performance analysis should be chosen close to point a for working situations where power is important. When we consider engine size together with thermal efficiency and power, the optimal operating range moves closer to point b. If the design parameters are selected at MPD conditions instead of

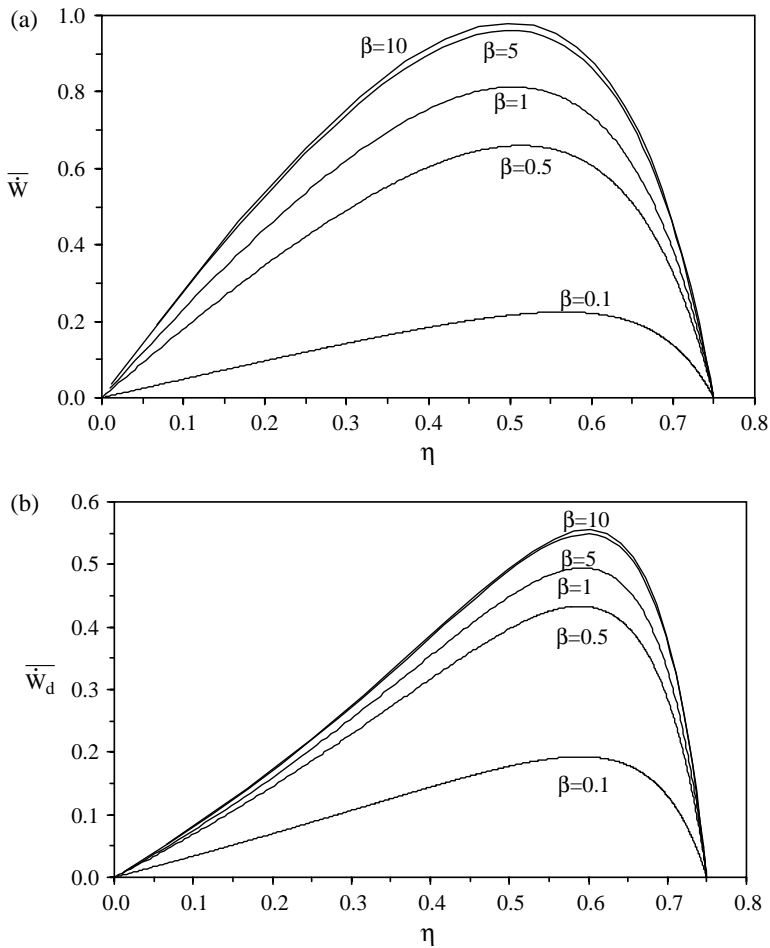


Fig. 5. Variations of the dimensionless power (a) and dimensionless power density (b) with respect to thermal efficiency for various  $\beta$  values ( $I=0.8$ ,  $\tau=5$ ).

MP condition the thermal efficiency increases as much as  $\Delta\eta = \eta_{\text{mpd}} - \eta_{\text{mp}}$  but the power reduces by  $\Delta\dot{W} = \dot{W}_{\text{mp}} - \dot{W}_{\text{mpd}}$ .

#### 4. Conclusions

A performance analysis using finite time thermodynamics has been carried out for an irreversible solar driven heat engine. An objective function has been defined as the ratio of power to the maximum volume in the cycle. The results were compared with the results of performance analysis based on maximum power. The analysis



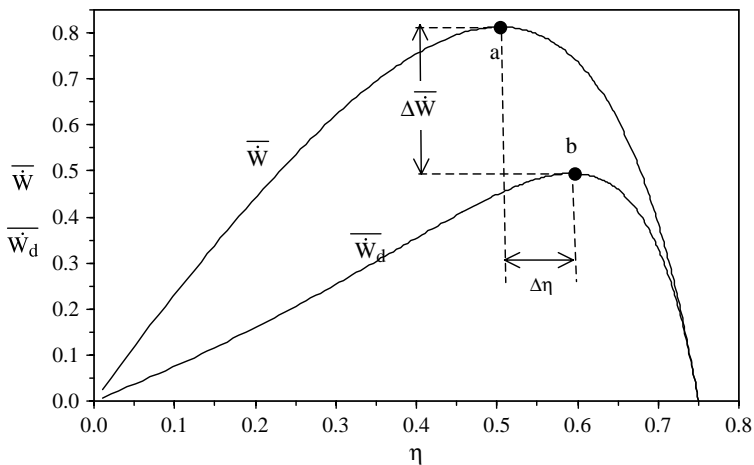


Fig. 6. Comparison of the dimensionless power and power density outputs with respect to thermal efficiency ( $\beta=1$ ,  $I=0.8$ ,  $\tau=5$ ).

showed that the design parameters at maximum power density lead to more efficient solar driven heat engines than those at maximum power output conditions. The effect of the irreversibilities on the global and optimal performances is also studied. Although the irreversibilities result in a reduction of the solar driven heat engine's power and efficiency, the maximum power density conditions still give a better performance in terms of thermal efficiency and size. The analysis may provide a basis for both determination of the optimal operating conditions and design parameters for real solar powered heat engines.

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