COLLECTIVE ADMIXTURES IN THE ODD-PARITY STATES OF O^{16}

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Recent experiments [1] have shown that many of the low-lying excited states of O^{16} can be fitted into rotational bands. This is a strong indication that nearly all of the low-lying states have some appreciable deformed component in their wave functions. Here this effect is reproduced by mixing a deformed odd parity state with those obtained earlier [2] as 1 particle - 1 hole (1p - 1h) excitations in a spherical basis. In this manner an interpretation is given to the lowest-lying $T = 0$ states in O^{16}.

From the Nilsson diagram [3] we see that, for a nucleus prolate in the z-direction, a low-lying odd-parity state is expected when three nucleons are excited from the x-y direction in the number 4 orbital to the z-direction in the number 6 orbital. We use here, however, Elliott's [4] SU(3) formalism. The analogous SU(3) state has a wave function ψ(3p-3h) proportional to

$$A\left(\frac{z_1}{c}\right)A\left(\frac{z_2}{c}\right)A\left(\frac{z_3}{c}\right)B\left(\frac{x_1}{b}\right)B\left(\frac{x_2}{b}\right)B\left(\frac{x_3}{b}\right) \qquad (1)$$

where $A(\frac{z}{c}) = \left(\frac{z^2}{c^2} - \frac{1}{2}\right) \exp\left[-\left(\frac{x^2}{b^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)\right]$

and $B(\frac{x}{b}) = \frac{x}{b} \exp\left[-\left(\frac{x^2}{b^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)\right]$.

This reduces, for $b = c$, to Elliott's state of maximum weight of $\{6, 3\}$. Here we start from the empirical values for the two lowest-lying energy levels of a given angular momentum and see what mixing of deformed and spherical components this implies. Therefore in each of the three cases $J^{\pi} = 1^-$, 2^-, 3^- we are dealing with a 2×2 matrix, the off-diagonal element of which is

$$M(J, k) = \langle\phi^J(\text{1p-1h})|V|\psi^J(\text{3p-3h})\rangle \qquad (2)$$

where ψ^J is the component of ψ with angular momentum J and $c = ka$, $b^2 = a^2/k$. The problem is now reduced to evaluating M.

In the notation of Elliott and Harvey [5]

$$\psi\{\lambda, \mu\} = \sum_J a_J(\lambda, \mu)\psi^J\{\lambda, \mu\} \qquad (3)$$

and so

$$M(J,k) = \frac{1}{a_J(6, 3)} \langle\phi^J(\text{1p-1h})|V|\psi(\text{3p-3h})\rangle \qquad (4)$$

However, since V is a two-body operator, one of the three particles excited must be only a bystander, and so we can write

$$\begin{aligned}\psi(\text{3p-3h}) &= \psi(\text{1p-1h})\,\psi(\text{2p-2h}) \\ &= \psi\,\{2, 1\}\ \ \psi\,\{4, 2\}\end{aligned}$$

in the $\{\lambda, \mu\}$ notation. Therefore, M reduces to

$$\frac{a_J\{2, 1\}}{a_J\{6, 3\}} \langle\text{spherical } O^{16} \text{ core } |V| \text{ deformed 2p-2h}\rangle$$

which becomes a two-body matrix element, the overlap of the 14 inert particles in the spherical and deformed states being calculated separately. In addition there is a spin and isotopic spin factor of 3, in the case of the $[4^4]$ symmetry considered here. Since earlier work [6] has indicated that the effective internucleon interaction in O^{16} has a Serber character, there are no exchange contributions and the Wigner and Majorana components behave in the same manner. The interaction is, in fact, taken to be $\frac{1}{2} V_0 \exp(-r^2/a^2) \times \times (1 + P^M)$ where $V_0 = -41$ MeV, $a = 1.68$ fm, and $\alpha a = 1$, α being the oscillator parameter equivalent to $\hbar\omega = 14.7$ MeV. Also the $a_J\{\lambda, \mu\}$

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Table 1

k	β	J = 1	2	3
1.2	0.3	-1.98	-1.61	-1.38
1.3	0.45	-1.54	-1.25	-1.07
1.4	0.6	-1.12	-0.91	-0.78

are evaluated using an expression derived for the case of spherical symmetry. However, calculations by Yitzhak Sharon (private communication) show that the correction to this ratio of a_J's would be less than 5 per cent for the deformations considered here. The above constants lead to table 1 for $M(J,k)$ in MeV.

If the above mechanism is to explain the two observed 1^- states at 7.12 and 9.59 MeV, $M(J=1)$ can not exceed 1.235 MeV in magnitude. In fact $M(J=1) = -1.235$ MeV implies that the unperturbed positions of the 1p-1h and 3p-3h levels are degenerate at 8.355 MeV and that the observed levels are equal mixtures of 1p-1h and 3p-3h components.

Recent (p, p') experiments [7] have shown that the 2^- state observed at 8.88 MeV contains little single particle strength compared with the 12.52 MeV level. From table 1 we find that the amplitude of the 1p-1h component in the 8.88 MeV level for k = 1.2, 1.3, 1.4 is 0.52, 0.37, 0.26 respectively and the unperturbed position of the 3p-3h component 9.85, 9.38, 9.13 MeV.

Similarly in the 3^- state observed at 6.14 MeV, the 3p-3h component for k = 1.2, 1.3, 1.4 has an amplitude of 0.26, 0.20, 0.14 respectively and unperturbed positions 6.50, 6.35 and 6.25 MeV.

From the expression [8]

$$B(\mathrm{E2},\ 1^- \rightarrow 3^-,\ K=1) = \frac{e^2}{8\pi} Q_0^2 \tag{5}$$

for the rotational band corresponding to a deformed state with quadrupole moment Q_0, we obtain 92.5, 158, 244 e^2f^4 for k = 1.2, 1.3, 1.4. Therefore the B(E2) between the 1^- and 3^- levels observed at 7.12 and 6.14 MeV is 11.5, 11.7 and 10.7 e^2f^4, where we have considered the 1^- state to have maximum mixing, and B(E2, 1p-1h) [9] to be 5.7 e^2f^4. This is to be compared with the observed B(E2) [10] of ~ 65 e^2f^4.

In conclusion, we see that by introducing a 3p-3h level that mixes with the low-lying 1p-1h odd-parity states we are able to get several improvements over the usual 1p-1h theory. Firstly, in addition to accounting for the number of T = 0, 1^- and 2^- levels observed below 13 MeV, it introduces a mechanism for depressing, by well over an MeV, the lowest 1^- state which always came out too high in the 1p-1h calculations. Secondly, the E2 transition rate between the lowest 1^- and 3^- levels is increased by a factor of two over the value given by the 1p-1h theory. However, this is still a factor of 5 or 6 short of the experimental value. This is somewhat similar to the situation with respect to the $4^+ \rightarrow 2^+$ transition in O^{16}, where the recently reported value [11] is 49 single-particle units, a factor of ~ 3 greater than the SU(3) predictions inclusive of effective charge. Also we see that the unperturbed positions of the deformed states are in reasonable agreement with a $J(J+1)$ type of spectrum.

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