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RESEARCH ARTICLE

Quantum corrections to the spectroscopy of a BTZ black hole via periodicity

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Abstract Quantum corrections to the area spectrum and the entropy spectrum of a BTZ black hole are calculated by equaling the motion period of an outgoing wave coming from the quantum corrections of the semiclassical action to the period of gravitational system with respect to the Euclidean time. We find that the area spectrum and the entropy spectrum are independent of the properties of particles. Furthermore, in the presence of higher-order quantum corrections, the area spectrum is found to be corrected by inverse area terms while the entropy spectrum is found to have a universal form, $\Delta S_{BH} = 2\pi$. Both results show that the entropy spectrum is independent of not only the BTZ black hole parameters but also the higher-order quantum corrections, which implies that the entropy spectrum is more natural than the area spectrum in quantum gravity theory.

Keywords Spectroscopy · Black hole · Periodicity

1 Introduction

Quantum effects in black holes have attracted much attention of theoretical physicists since Hawking's profound discovery of Hawking radiation [1,2]. Among them, the

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study of the quantization of black holes has been a very interesting topic. As the pioneer, Bekenstein [3–5] got the spacing of the area spectrum as $\Delta A = 8\pi l_p^2$, using the Hisenberg uncertainty principle, where l_p is the Planck length. But this result was challenged later by the important work of Hod in 1998, which [6] produced the quantized horizon area spectrum with the spacing $\Delta A = 4\ln 3 l_p^2$ coming from the real part of quasinormal frequencies of the black hole by employing Bohr's correspondence principle. However, Maggiore [7] doubted about the spacing value $4\ln 3 l_p^2$ made by Hod and suggested that in the large damping limit the imaginary part rather than the real part of the quasinormal frequencies of black holes should be responsible for the area spectrum, which could also lead to the Bekenstein's initial result. Following Maggiore's proposal, it was later showed that the area spectrum and entropy spectrum of Kerr black holes are also quantized evenly and these conclusions are consistent with the result of Bekenstein [8,9]. Until now Maggiore's proposal has been wildly used to study the area spectra for various black holes [10–16].

It is worth noting that some new interpretations can also give the area spectrum of black holes without using the quasinormal frequencies [17–21]. Among them, following the speculation of Maggiore [7] that the periodicity of a black hole may be the origin of the area quantization, Zeng et al. [22,23] showed that the area spectrum of the black holes can be produced by the periodicity of an outgoing wave. By equaling the motion period of a scalar particle to the period of gravity system in Kruskal coordinate with respect to the Euclidean time, they reproduced the area spectrum of the black holes, $\Delta A = 8\pi l_p^2$, which is just the result obtained by Bekenstein firstly. More importantly, for a rotating black hole, they obtained the general semiclassical area spectrum without using the small angular momentum assumption, which is necessary from the perspective of quasinormal modes [8,9]. Later on, the corrected area spectrum has been also investigated in [24,25] by equaling the motion period of an outgoing wave to the period of gravitational system in Kruskal coordinate with consideration of quantum corrections to the semiclassical action. Their results have shown that the formula for the area spectrum with quantum corrections is universal though it is not independent of the black hole parameters, which is a desirable result for the forthcoming quantum gravity theory of black holes.

Until now, some previous works have also investigated the area spectrum and the entropy spectrum of the BTZ black holes [23,26,27]. Using the real part of quasinormal frequencies, the area spectrum has been found to be not equally spaced in [26]. Following Maggiore's suggestion, using the imaginary part of quasinormal frequencies, one also got the spectrum of the horizon area with equally spaced value [27]. However, the entropy spectrum for a non-rotating (rotating) BTZ black hole was $4\pi(2\pi)$ respectively, which are not consistent with the Bekenstein's original results. Recently, using the periodicity of an outgoing wave, Ref. [23] got the area spectrum and the entropy spectrum of the BTZ black holes. Their results are same as the Bekenstein's original conclusions, which further confirms the Bekenstein's original hypothesis. In the present paper, the area spectrum and the entropy spectrum with quantum corrections of the rotating BTZ black holes are further calculated by the periodicity method. When calculating the entropy spectrum of the rotating BTZ black holes, the first law of thermodynamics plays a very important role. The results show that the area spectrum is corrected by inverse area terms, which indicates that the corrected area spectrum is not evenly spaced. However, the corrected entropy spectrum remains evenly spaced, which further confirms that the entropy spectrum should be universal independent of higher-order quantum corrections. Like the quantum tunneling method in [18], this periodicity method also treats the change of horizon area as the response of a particle running out from the horizon of a black hole. What's more, considering the special case that most of the previous works were mainly concentrated on scalar particles, we will be further investigate the case of Dirac particles in this paper. As a desirable result, we find that for both scalar particles and Dirac particles the area spectrum and entropy spectrum take the same form. Therefore, we can conclude that the spectroscopy of a black hole is irrelevant of the properties of particles in our case.

This paper is organized as follows: In the first section, we give a briefly review of the rotating BTZ black hole. We find that the Euclidean-Kruskal section of the rotating BTZ black hole is a cyclic or single period system with period $\frac{2\pi}{\kappa_+}$, where κ_+ is the surface gravity of the event horizon. The area spectrum and the entropy spectrum of the rotating BTZ black holes stirred by both scalar particles and Dirac particles are calculated in the second section. Finally, some conclusions are given in the last section.

2 A brief review of the rotating BTZ black hole

The rotating Bañados-Teitelboim-Zanelli (BTZ) black hole is derived from the (2+1)-dimensional Einstein gravity with a negative cosmological constant $\Lambda = -1/l^2$ [28]. The corresponding line element in the usual Schwarzschild coordinates is

$$ds^2 = -N(r)^2 dt^2 + N(r)^{-2} dr^2 + r^2 (d\phi + N^{\phi}(r) dt)^2, \tag{1}$$

with the metric functions

$$N(r)^2 = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \; N^{\phi} = -\frac{J}{2r^2} \tag{2}$$

where M is the Arnowitt–Deser–Misner (ADM) mass, J is the angular momentum (spin) of the BTZ black hole and $-\infty < t < +\infty, \; 0 < r < +\infty, \; 0 \leq \phi \leq 2\pi$.

The BTZ spacetime is asymptotically anti-de Sitter, and has an event and a Cauchy horizon at

$$r_{\pm}^2 = \frac{1}{2} M l^2 (1 \pm \triangle), \triangle = \left[1 - \left(\frac{J}{Ml} \right)^2 \right]^{\frac{1}{2}} \tag{3}$$

with $r_+, \; r_-$ the outer (event) and inner (cauchy) horizon, respectively. The existence of an event horizon dose impose a bound on the angular momentum J, such that $|J| \leq Ml$.

In addition to the Hawking temperature T_H, horizon area A_H and angular velocity Ω_H at the event horizon of the BTZ black hole can be written as:

$$T_H = \frac{r_+^2 - r_-^2}{2\pi r_+ l^2} = \frac{M\triangle}{2\pi r_+}, \quad A_H = 2\pi r_+, \Omega_H = \frac{J}{2r_+^2}. \tag{4}$$

These physical quantities can also be found in many papers about investigating Hawking effect of the BTZ black hole, such as [29–36].

To avoid the dragging effect in a rotating black hole, one often perform the so-called dragging coordinate transformation as

$$d\phi = -N^\phi dt = \frac{J}{2r^2} dt = \Omega dt, \tag{5}$$

where Ω is the dragged angular velocity of the rotating BTZ black hole. Subsequently substituting Eq. (5) into Eq. (1), the line element takes the 2-dimensional form as

$$ds^2 = -N(r)^2 dt^2 + N(r)^{-2} dr^2, \tag{6}$$

Thus the two-dimensional Euclidean metric can be given as

$$ds_E^2 = N(r)^2 d\tau^2 + N(r)^{-2} dr^2, \tag{7}$$

where we have used the Euclidean time $\tau = -it$.

After using the tortoise coordinate

$$\frac{dr^*}{dr} = N(r)^{-2} \tag{8}$$

or

$$r^* = r + \frac{1}{2\kappa_+} \ln \frac{r - r_+}{r_+} - \frac{1}{2\kappa_-} \ln \frac{r - r_-}{r_-}, \tag{9}$$

where $\kappa_\pm = \frac{M\triangle}{r_\pm}$ is the surface gravity on the outer (inner) horizon, we can obtain the Euclidean Kruskal section of the rotating BTZ black hole. Similar to the Schwarzschild black hole, this section is a cyclic or single periodic system, whose metric reads

$$ds_E^2 = N(r)^2 e^{-2k_+ r^*} (dX^2 + dY^2), (r > r_+) \tag{10}$$

where

$$iX = e^{\kappa_+ r^*} \sin \kappa_+ \tau, \tag{11}$$

$$Y = e^{\kappa_+ r^*} \cos \kappa_+ \tau, \tag{12}$$

Obviously, both X and Y in the Eqs. (11) and (12) are the periodic function of τ with the period

$$T_e = \frac{2\pi}{\kappa_+} = \frac{\hbar}{T_{BH}} \tag{13}$$

This period is of key importance for studying Hawking temperature with the temperature green function [37].

3 Quantum corrections to the spectroscopy via periodicity

In this section, we are interested in computing the corrected area spectrum and the corrected entropy spectrum of the rotating BTZ black holes via periodicity. We want to discuss the quantum corrected features of the rotating BTZ black holes and explore whether the corrected area spectrum and the corrected entropy spectrum for scalar particles take the same from as that of Dirac particles.

3.1 Spectroscopy arising from the radiation of scalar particles

To study the spectroscopy of the black holes with the periodicity method, one should first find the motion period of outgoing particles. It is well known that the period is related to the wave function, thus we will employ the following Klein–Gordon equation to investigate the wave function of the outgoing scalar particles

$$-\frac{\hbar^2}{\sqrt{-g}}\partial_\mu[g^{\mu\nu}\sqrt{-g}\partial_\nu]\phi = 0. \tag{14}$$

For the 2-dimensional line element in Eq. (6), the wave function ϕ is usually written as

$$\phi(r,t) = \exp\Big[-\frac{i}{\hbar}S(r,t)\Big], \tag{15}$$

where S is the action of the outgoing wave. Thus Eq. (14) can be written explicitly as

$$\begin{aligned}&\frac{i}{N(r)^2}\left(\frac{\partial S}{\partial t}\right)^2 - iN(r)^2\left(\frac{\partial S}{\partial r}\right)^2 - \frac{\hbar}{N(r)^2}\frac{\partial^2 S}{\partial t^2}\\&+\hbar N(r)^2\frac{\partial^2 S}{\partial r^2} + \hbar\partial_r(N(r)^2)\frac{\partial S}{\partial r} = 0,\end{aligned} \tag{16}$$

In order to exactly solve the Eq. (16), we choose the standard ansatz for $S(r,t)$, which is [38,39]

$$S(r,t) = S_0(r,t) + \sum_{i=1}^{\infty}\hbar^i S_i(r,t). \tag{17}$$

The Eq. (17) contains higher-order quantum corrections and $S_0(r,t)$ is the semiclassical action. Then substituting Eq. (17) into Eq. (16) and equating the different powers of $\hbar$ on both sides, we find each partial differential equation about $S_i(r,t)$ shares the same form as

$$\frac{\partial S_i}{\partial t} = \pm N(r)^2\frac{\partial S_i}{\partial r}, \tag{18}$$

where $i = 0, 1, 2, 3, \ldots$. Thus the solutions of $S_i(r, t)$ are not independent, and S_i are proportional to S_0 with a proportionality factor.

It is well known that according to (2 + 1) dimensions spacetime, in units $8G_3 = 1$ and $c = k_B = 1$, the Planck constant ($\hbar$) is of the order of Planck length (l_P) [40]. So after a dimensional analysis, the proportionality factor can be written as $\beta_i (r_+)^{-i}$ [39] and the action in Eq. (17) can be expanded as

$$S(r,t) = \left(1 + \sum_i \beta_i \frac{\hbar^i}{r_+^i}\right) S_0(r,t), \tag{19}$$

where β_i are dimensionless constant parameters . Our next step is to find the semi-classical action. In the dragging coordinate frame, the semiclassical action S_0 can be decomposed as

$$S_0(t,r) = (E - m\Omega_h)t + W(r), \tag{20}$$

where E is the energy of the emitted particles measured by the observer at the infinity, m denotes the angular quantum number about ϕ. Substituting Eq. (20) into Eq. (18), we can get the concrete formula of the radial component of semiclassical action. So the outgoing wave can be written as

$$\phi_{\text{out}} = \exp\left[-\frac{i}{\hbar}\left(1+\sum_i \beta_i \frac{\hbar^i}{r_+^i}\right)\left((E - m\Omega_h)t - (E - m\Omega_h)\int_0^r \frac{dr}{N(r)^2}\right)\right], \tag{21}$$

where we have incorporated Eq. (20) into Eq. (15). Thus the motion period of the outgoing wave can be given as

$$T_o = \frac{2\pi}{(\omega - \frac{m\Omega_h}{\hbar})\left(1 + \sum_i \beta_i \frac{\hbar^i}{r_+^i}\right)}. \tag{22}$$

It is obvious that the motion period of the outgoing wave in this space-time depends not only on the frequency but also on the dragging angular velocity at the event horizon. In addition to, different from the result in [23], the motion period in Eq. (22) is corrected with additional terms. In the previous work [23], the classical formula of area spectrum was obtained by equaling the motion period of an outgoing wave to the period of gravitational system, such as $T_o = T_e = \frac{2\pi}{\kappa_+} = \frac{\hbar}{T_{BH}}$. Here we will also resort to this skill to compute the quantum corrected area spectrum in the rotating BTZ black holes. According to the rotating BTZ black hole, the change in the event horizon area of a BTZ black hole can be obtained as

$$\Delta A = \frac{4\hbar}{T_{BH}}\left(\omega - \frac{m\Omega_h}{\hbar}\right), \tag{23}$$

where ω is the vibrational frequency of the scalar perturbed BTZ black hole. Taking into account Eq. (13) and Eqs. (22), (23) can be simplified as

$$\Delta A = 8\pi \tilde{l}_p^2 \frac{1}{\left(1+\sum_i \beta_i \frac{\hbar^i}{r_+^i}\right)}, \tag{24}$$

where $\tilde{l}_p^2 = \frac{\hbar G_3}{c^3}$ is re-scaled Plank length. Expanding Eq. (24) and taking into consideration Eq. (4), we get

$$\begin{aligned}\Delta A = 8\pi \tilde{l}_p^2 \Bigg[1 - \beta_1 \frac{8\pi\hbar}{A} - (\beta_2 - \beta_1^2)\left(\frac{8\pi\hbar}{A}\right)^2 \\ -(\beta_3 + \beta_1^3 - 2\beta_1\beta_2)\left(\frac{8\pi\hbar}{A}\right)^3 + \ldots \Bigg].\end{aligned} \tag{25}$$

It is obvious that the area spectrum of the BTZ black hole obtained via periodicity is corrected and the spacing is not evenly spaced.

Assuming that the first law of thermodynamics is still valid for the perturbed BTZ black hole,

$$T_{eff} dS_{bh} = dM - \Omega dJ \tag{26}$$

we can get the effective entropy S_{bh} as [39]

$$\begin{aligned} S_{bh} &= \frac{4\pi r_+}{\hbar} + 4\pi\beta_1 \log r_+ - 4\pi\beta_2\hbar\left(\frac{1}{r_+}\right) + o(\hbar^3) + const \ldots \\ &= S_{BH} + 4\pi\beta_1 \log S_{BH} - 16\pi^2\beta_2\left(\frac{1}{S_{BH}}\right) + o(\hbar^3) + const \ldots, \end{aligned} \tag{27}$$

where we have also used the unit $8G_3 = 1$. The first term in the equation (27) is the usual semiclassical Bekenstein-Hawking entropy $S_{BH} = \frac{A}{4\hbar G_3}$ while the other terms are corrections due to quantum effects. Comparing with the results of quantum corrected entropy for the BTZ black holes in previous works [41,42], which is $S_{bh} = S_{BH} - \frac{3}{2}\log S_{BH} + \ldots$, so the parameter β_1 can be fixed as $\beta_1 = -\frac{3}{8\pi}$. However, the other parameters $\beta_i (i > 1)$ can not be fixed in the present approach.

Subsequently using Eqs. (22) and (26), we can also obtain

$$\Delta S_{bh} = \frac{\hbar}{T_{eff}}\left(\omega - \frac{m\Omega_h}{\hbar}\right) = 2\pi, \tag{28}$$

where $T_{eff} = \frac{\hbar}{T_o} = T_{BH}(1 + \sum_i \beta_i \frac{\hbar^i}{r_+^i})^{-1}$ has been used. This result shows that the entropy spectrum is evenly spaced. It is interesting that the result (28) is obtained in the presence of higher-order quantum corrections. Also, the same result was reproduced in [23] without consideration of higher-order quantum corrections. Thus we can get the desirable conclusion that the entropy quantum of the rotating BTZ black holes is independent of the inclusion of higher-order quantum corrections, which is consistent with the results in the previous works [18–23,43].

3.2 Spectroscopy arising from the radiation of a Dirac particle

In this section, we intend to study the corrected area spectrum and the corrected entropy spectrum of the BTZ black holes due to the radiation of Dirac particles. For simplify in this paper we only study the case of massless Dirac particles, whose motion equation can be written as

$$i\gamma^{\mu}\nabla_{\mu}\psi = 0, \tag{29}$$

where the covariant derivative is given by,

$$\begin{aligned} &\nabla_{\mu} = \partial_{\mu} + \frac{i}{2}\Gamma^{\alpha}{}_{\mu}{}^{\beta}\Sigma_{\alpha\beta}; \quad \Gamma^{\alpha}{}_{\mu}{}^{\beta} = g^{\beta\nu}\Gamma^{\alpha}_{\mu\nu}; \\ &\Sigma_{\alpha\beta} = \frac{i}{4}\Big[\gamma_{\alpha}, \gamma_{\beta}\Big]\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu} \end{aligned} \tag{30}$$

For the metric in Eq. (6), the γ matrices are defined as

$$\gamma^{t} = \frac{1}{N(r)}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}; \quad \gamma^{r} = N(r)\begin{pmatrix} 0 & \sigma^{3} \\ \sigma^{3} & 0 \end{pmatrix},$$

where σ^3 is

$$\sigma^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{31}$$

Thus the Eq. (29) can be written as

$$i\gamma^{\mu}\partial_{\mu}\psi - \frac{1}{2}\Big(g^{tt}\gamma^{\mu}\Gamma^{r}_{\mu t} - g^{rr}\gamma^{\mu}\Gamma^{t}_{\mu r}\Big)\Sigma_{rt}\psi = 0. \tag{32}$$

To exactly solve the Eq. (32), we employ the following wave function ansatz for the spin up and spin down ψ as

$$\psi_{\uparrow}(t,r) = \begin{pmatrix} A(t,r) \\ 0 \\ B(t,r) \\ 0 \end{pmatrix}\exp\left[\frac{i}{\hbar}S_{\uparrow}(t,r)\right] \tag{33}$$

$$\psi_{\downarrow}(t,r) = \begin{pmatrix} 0 \\ C(t,r) \\ 0 \\ D(t,r) \end{pmatrix}\exp\left[\frac{i}{\hbar}S_{\downarrow}(t,r)\right], \tag{34}$$

where $S(r, t)$ is the one particle action which will be expanded in powers of $\hbar$. Here we will only solve the spin up case explicitly since the spin down case is fully analogous. As stated in [44], since the second terms within the first bracket of Eq. (32) do not involve the single particle action, we can omit its contributions to the wave function. Substituting Eq. (33) into Eq. (32), we can obtain the following two equations:

$$\left(\frac{iA}{N(r)}\partial_t S_\uparrow + BN(r)\partial_r S_\uparrow\right) = 0 \tag{35}$$

$$\left(-\frac{iB}{N(r)}\partial_t S_\uparrow + AN(r)\partial_r S_\uparrow\right) = 0. \tag{36}$$

A, B can be expended in powers of $\hbar$ as

$$A = A_0 + \sum_i \hbar^i A_i; \quad B = B_0 + \sum_i \hbar^i B_i, \tag{37}$$

where $i = 1, 2, 3, \ldots\ldots$. Now taking $S_\uparrow = S(r, t)$, substituting Eqs. (17) and (37) into Eqs. (35) and (36), and equating the different powers of $\hbar$ on both sides, we obtain two sets of equations. Following the similar skill in [44], we obtain a simplified relation for all the equations as

$$\partial_t S_i = \pm N(r)^2 \partial_r S_i, \tag{38}$$

where $i = 1, 2, 3, \ldots$. It is obvious that this result is similar to the case of the scalar particles. That is, S_i are proportional to S_0 with a proportionality factor. The action in this case hence also can be written as Eq. (19). Then in order to get the action, we will first solve the semiclassical action. Exacly solving Eq. (38) for the case $i = 0$, we obtain

$$S_0(r, t) = (E - m\Omega_h)t \mp (E - m\Omega_h) \int_0^r \frac{dr}{N(r)^2}, \tag{39}$$

where we have used Eqs. (19) and (20). Thus the complete solutions for the action are given as

$$S(r, t) = \left(1 + \sum_i \beta_i \frac{\hbar^i}{r_+^i}\right) \left[(E - m\Omega_h)t \pm (E - m\Omega_h) \int_0^r \frac{dr}{N(r)^2}\right]. \tag{40}$$

Further the outgoing waves of the Dirac equation in the BTZ black holes can be written as

$$\psi_{\text{out}} \sim \exp\left[-\frac{i}{\hbar}(1 + \sum_i \beta_i \frac{\hbar^i}{r_+^i}) \left((E - m\Omega_h)t - (E - m\Omega_h) \int_0^r \frac{dr}{N(r)^2}\right)\right], \tag{41}$$

where we have used Eq. (33). It is obvious that the wave functions of the spin up Dirac particles have a period with

$$T_o = \frac{2\pi}{(\omega - \frac{m\Omega_h}{\hbar})\left(1 + \sum_i \beta_i \frac{\hbar^i}{r_+^i}\right)}, \tag{42}$$

which is same as the one of the scalar particles. Subsequently employing the previous similar skill, we can also get the corrected area spectrum and the corrected entropy spectrum of the rotating BTZ black hole for the Dirac particles. The results are same as the case of scalar particles. Therefore we could conclude that the corrected area spectrum and the corrected entropy spectrum of the rotating BTZ black hole are independent of the properties of particles in our approach.

4 Conclusions

In the present paper, the area spectrum and entropy spectrum of the rotating BTZ black holes with quantum corrections have been calculated by a new simple semiclassical scheme. We begin with calculating the corrected area spectrum in the rotating BTZ black holes by equaling the motion period with quantum corrections of an outgoing wave to the period of gravity system in the Euclidean time. We successfully get the corrected motion period for both the outgoing scalar particles and the Dirac particles, when exactly solving the Klein–Gordon equation and Dirac equation respectively with the quantum corrections. Using this period from the outgoing waves, we obtain the quantum corrected area spectrum of the rotating BTZ black holes. Further we calculate the quantum corrected entropy and entropy spectrum of the rotating BTZ black holes based on the effective first law of thermodynamics. The results show that in the presence of quantum corrections the area spectrum acquires inverse area terms as additional terms and the entropy spectrum maintains a universal form as $\Delta S_{BH} = 2\pi$. Thus we conclude that the entropy spectrum is not only independent of the BTZ black hole parameters, but also independent of higher-order quantum corrections, which indicates that the entropy spectrum could be more natural than the area spectrum for the quantization of the black holes. This interesting result may be expected to shed some light on the future study of the quantum gravity theory.

It is worth noting that for the periodicity method in this paper, we only adopt the Hod's proposal to study the area spectrum. In fact, we also can employ the Kunstatter's proposal, namely base on the relation

$$I = \int \frac{dE}{\omega} = \int \frac{dH}{\omega} = n\,\hbar, \tag{43}$$

to study the area spectrum of the black hole. Based on Eq. (43), we also can obtain the result as in Eq. (24). Furthermore, because the quasinormal mode frequency is not used in the present paper, the limit of small angular momentum, which has been used in [27], is unnecessary and there is also no confusion on whether the real part or imaginary part is responsible for the area spectrum.

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