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ORIGINAL ARTICLE

Clustering of non-point mass system of galaxies in the expanding Universe

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Abstract We derive the cosmic energy equation for the non-point mass system of galaxies (galaxies with halos) by using the adiabatic approximation for the growth of gravitational clustering of galaxies in the expanding Universe. The cosmic energy equation so derived represents the general form of conservation of energy for the expanding volume. Using the derived form of cosmic energy equation we try to study the evolution of correlation potential energy of the system. We also try to explore the condition under which the approximation of extensivity may be applied to the infinite gravitating non-point mass system of galaxies.

Keywords Cosmology · Clustering · Galaxy · Theory

1 Introduction

Galaxies cluster under the influence of their mutual gravitational force, in an expanding Universe. When mutual gravitational interactions of individual galaxies dominate, clustering can be described by quasi-equilibrium thermodynamics (Saslaw 2000; Saslaw and Hamilton 1984; Saslaw and Fang 1996) and statistical mechanics (Ahmad et al. 2002; Leong and Saslaw 2004). These theories are found to be in good agreement with observations (Sivakoff and Saslaw 2005). As long as the evolution is in quasi-equilibrium, we may be able to use thermodynamics. The main aspect in which thermodynamics helps us to move forward is that expansion of the system of galaxies is to a good approximation adiabatic (Saslaw 2000). The parameter on which the adiabatic approximation largely depends is the ratio of the linear gravitational clustering time scale τ_{grav}, to the expansion time scale τ_{expand} ($\alpha = \tau_{\text{grav}}/\tau_{\text{expand}}$). When $\alpha > 1$, the time scale for gravitational clustering to grow exceeds the global expansion time scale and evolution is approximately adiabatic. The adiabatic approximation becomes even better for the models in which α is greater. Gravitational clustering may be adiabatic but it is not necessary that it will be either isentropic or reversible (Saslaw 2000; Saslaw and Fang 1996).

For adiabatic evolution of gravitational clustering, the first law of thermodynamics leads us to the cosmic energy equation $\mathrm{d}(W + K)/\mathrm{d}t + \dot{R}(2K + W)/R = 0$ derived by Irvine (1961) and Layzer (1963) (Saslaw 2000; Saslaw and Fang 1996). Here W is the correlation potential energy, K the peculiar kinetic energy and R the scale factor. In an infinite and expanding region of the Universe, conservation of energy may be given by the cosmic energy equation. The cosmic energy equation describes the evolution of kinetic energy and the correlation potential energy with time in an expanding region of the Universe. In deriving the cosmic energy equation (Irvine 1961; Layzer 1963), it has been assumed that galaxies are point particles, which, in fact, is a crude approximation. Galaxies are extended structures, i.e., they have halos around them and thus may be called non-point masses. The extended nature of galaxies is being accounted for by using the soften-

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ing parameter ϵ which ranges as $0.01 \leq \epsilon \leq 0.05$ in units of constant cell size (Ahmad et al. 2002).

The cosmic energy equation for extended structures has been derived by Ahmad et al. (2009) from the concept of correlation functions and the Hamiltonian of the system. In the present paper we try to derive the same cosmic energy equation rather in a different way as compared to the derivation given by Ahmad et al. (2009). In Sect. 2 we derive the cosmic energy equation for extended structures from the adiabatic approximation. The equation so derived represents the conservation of energy and the manifestation of the first law of thermodynamics. The cosmic energy equation obtained in this section is the same as derived by Ahmad et al. (2009); however, the approaches are different. In Sect. 3 we study the evolution of the correlation potential with time. We compare the evolutions of the correlation potential energies for point and non-point masses. Section 4 describes the extensivity approximation for the non-point mass system of galaxies clustering under the influence of the gravitational force. Using the grand canonical ensemble, composed of cells, to describe the infinite system, it is worth to mention that the correlation within the cells is larger than the correlation between the cells. Therefore, extensivity is a good approximation for an infinite system. It is thus imperative to find the conditions under which extensivity approximation becomes good. Finally, we discuss the results in Sect. 5.

2 Cosmic energy equation for extended structures (galaxies with halos)

When the growth of gravitational clustering in a region of the Universe takes place adiabatically, the first law of thermodynamics may be given as (Saslaw 2000)

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho R^3) + P\frac{\mathrm{d}}{\mathrm{d}t}(R^3) = 0. \tag{1}$$

Here ρ is the energy density of matter and P is the pressure. Obviously, (1) is the conservation of energy in the form of the first law of thermodynamics and explains that in any local volume, comoving with scale factor $R(t)$, the change in total energy is just the work done on its surroundings by the adiabatic change of that volume. Equation (1) is applicable to those scales on which the correlation function has the least contribution to the correlation energy or those scales over which the transfer of correlation energy is least. Besides, the peculiar velocities of galaxies are small compared to the velocity of light and the scale of clustering is small compared to the scale of the Universe; therefore, it is essentially a Newtonian problem.

We consider a spherical volume V such that there are N particles at temperature T. The total energy for such system is (Saslaw 2000; McQuarrie 2003)

$$U = \frac{3}{2}NT - \frac{N\bar{n}}{2}\int_V \phi(\mathbf{r})\xi(\mathbf{r})4\pi\mathbf{r}^2\,\mathrm{d}\mathbf{r}. \tag{2}$$

Here $\phi(\mathbf{r}) = -Gm^2/\mathbf{r}$ is the gravitational potential for point particles that all have mass m, average number density $\bar{n}$ and total number $N = \bar{n}V$. $\xi(\mathbf{r})$ is the two-point correlation function and we have taken energy units such that Boltzmann's constant $k = 1$. The first term on right hand side of (2) is the kinetic energy K and the second term is the correlation potential energy W. Therefore, (2) can be written in a compact form as

$$U = K + W. \tag{3}$$

Even on smaller scales, the adiabatic evolution is a good approximation, because the correlation energy or heat transfer over a boundary of a volume will generally be small compared to the correlation energy or heat within the volume.

The pressure for such system may be given as (Saslaw 2000; McQuarrie 2003)

$$P = \frac{NT}{V} - \frac{\bar{n}^2}{6}\int_V \mathbf{r}\frac{d\phi(\mathbf{r})}{d\mathbf{r}}\xi(\mathbf{r})4\pi\mathbf{r}^2\,\mathrm{d}\mathbf{r}. \tag{4}$$

Again, in (4) we have taken energy units in which Boltzmann's constant $k = 1$. It is worth to mention here that in (2), (3) and (4) it is assumed that the galaxies are point masses. However, as discussed earlier, this is a crude approximation. Observations suggest that the galaxies have extended structures (Sivakoff and Saslaw 2005). This extended structure of galaxies is taken into account by making use of a constant average softening parameter ϵ representing approximately all galaxies. The potential in case of extended structures varies as $(\mathbf{r}^2+\epsilon^2)^{-1/2}$ instead of $\mathbf{r}^{-1}$. The potential with softening parameter is known as a softened potential and such a potential is widely used in simulations of cosmological many body problem (Plumer 1911) in order to avoid the short distance singularity of the gravitational potential. The interaction potential between any two galaxies with extended structure may be given as (Ahmad et al. 2002, 2009)

$$\phi(\mathbf{r}) = -\frac{Gm^2}{(\mathbf{r}^2+\epsilon^2)^{1/2}}. \tag{5}$$

In (5) it is assumed that all galaxies are of the same mass. On substituting (5) in (2) we get the internal energy as

$$U = \frac{3}{2}NT - \frac{GN\bar{n}m^2}{2}\int_V \frac{\xi(\mathbf{r})}{(\mathbf{r}^2+\epsilon^2)^{1/2}}4\pi\mathbf{r}^2\,\mathrm{d}\mathbf{r}. \tag{6}$$

For gravitational galaxy clustering, the correlation function $\xi(\mathbf{r})$ usually taken is the two particle correlation function.

This is because the gravitational interaction is pair wise. The two particle correlation function describes the probability of finding a galaxy in the neighbourhood of the other galaxies. For the statistically homogeneous system, correlations depend only on relative positions of volume elements and not on their absolute positions. Thus, the extended nature of galaxies (galaxies with halos) would hardly influence the two particle correlation function. Moreover, from simulations of the gravitational many body problem it is evident that the softening parameter softens the potential only and not the correlation function (Sheth and Saslaw 1996). Equation (6) may be put in a compact form as

$$U_\epsilon = K + W_\epsilon. \tag{7}$$

Here

$$W_\epsilon = -\frac{GN\bar{n}m^2}{2}\int_V \frac{\xi(\mathbf{r})}{(\mathbf{r}^2+\epsilon^2)^{1/2}} 4\pi \mathbf{r}^2\,\mathrm{d}\mathbf{r} \tag{8}$$

is the correlation potential energy for extended structures (or non-point masses). The correlations outside the volume have the least contribution to the correlation potential energy for non-point masses on the scales over which adiabatic evolution becomes a good approximation. It is evident that for $\epsilon = 0$ (i.e. point masses) (8) gives the correlation potential energy W for point masses.

The pressure P for non-point mass system of galaxies may be obtained on substituting (5) in (4) as

$$P_\epsilon = \frac{NT}{V} - \frac{G\bar{n}^2m^2}{6}\int_V \mathbf{r}^2 \frac{\xi(\mathbf{r})}{(\mathbf{r}^2+\epsilon^2)^{3/2}} 4\pi \mathbf{r}^2\,\mathrm{d}\mathbf{r}. \tag{9}$$

Now in order to solve (9) we may proceed as follows:

$$P_\epsilon = \frac{NT}{V} - \frac{G\bar{n}^2m^2}{6}\int_V \frac{(\mathbf{r}^2+\epsilon^2-\epsilon^2)\xi(\mathbf{r})}{(\mathbf{r}^2+\epsilon^2)^{3/2}} 4\pi \mathbf{r}^2\,\mathrm{d}\mathbf{r}. \tag{10}$$

The final form of the pressure equation for the non-point mass system of galaxies is

$$P_\epsilon = \frac{(3NT + W_\epsilon + \epsilon^2 W'_\epsilon)}{3V}. \tag{11}$$

The pressure equation obtained has a contribution from momentum transport produced by the kinetic energy density and also has a contribution from the correlated force of the gravitational interaction.

Here

$$W'_\epsilon == -\frac{GN\bar{n}m^2}{2}\int_V \frac{\xi(\mathbf{r})}{(\mathbf{r}^2+\epsilon^2)^{3/2}} 4\pi \mathbf{r}^2\,\mathrm{d}\mathbf{r} \tag{12}$$

is an additional term for the system of extended structures. It is obvious that (12) comes into existence only because of the softened potential and it measures the impact of the halotic structure of galaxies on their clustering. The presence of the term W'_ϵ in (11) shows that the softening parameter has a non-negligible effect on the thermodynamics of the non-point mass system of galaxies.

The energy density of matter in volume V is taken as ρ. Therefore, the total energy in volume V will be given by

$$\rho V = U_\epsilon = K + W_\epsilon. \tag{13}$$

As an approximation, the size of a cluster may be taken as spherical with definite radius (Saslaw 2000). Thus volume V of a spherical cluster with radius R will be $(4/3)\pi R^3$. Then the total energy in volume V will be $(4/3)\pi\rho R^3$. Therefore, the total energy may be written as

$$\begin{aligned} U_\epsilon &= K + W_\epsilon \\ &= \frac{4}{3}\pi\rho R^3. \end{aligned} \tag{14}$$

Also, we may write

$$\frac{\mathrm{d}}{\mathrm{d}t}(R^3) = \frac{\dot{R}}{R}(3R^3). \tag{15}$$

Taking the spherical size of a cluster into consideration and with the help of (11), (15) ultimately leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}(R^3) = \frac{\dot{R}}{R}\left[\frac{3}{4\pi}\frac{(2K + W_\epsilon + \epsilon^2 W'_\epsilon)}{P_\epsilon}\right]. \tag{16}$$

Using (14) and (16) in (1), the law of conservation of energy for the galaxy cluster takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}(K + W_\epsilon) + \frac{\dot{R}}{R}\big(2K + W_\epsilon + \epsilon^2 W'_\epsilon\big) = 0. \tag{17}$$

Equation (17) gives the cosmic energy equation for an extended system of galaxies which cluster under the influence of the gravitational force in the expanding Universe. The same cosmic energy equation for extended structures has been derived by Ahmad et al. (2009); however, in the present work we have taken a different approach to derive it. This basic dynamical result is applicable to both linear and nonlinear evolution. For the case where there is no expansion i.e. $\dot{R} = 0$, we may have

$$2K + W_\epsilon + \epsilon^2 W'_\epsilon = 0, \tag{18}$$

which is the virial theorem for extended structures. Obviously, here we can see that for $\epsilon = 0$ (18) will give us the virial theorem for point masses. Besides this, (17) may be put in its equivalent form, thus:

$$\frac{\mathrm{d}}{\mathrm{d}t}[R(K + W_\epsilon)] = -\dot{R}\big(K + \epsilon^2 W'_\epsilon\big). \tag{19}$$

The right hand side of (19) is always negative for an expanding Universe. This means that the total energy decreases as the Universe expands. This is a consequence of the adiabatic evolution of clustering of non-point masses (extended structures or galaxies with halos).

The cosmic energy equation given by (17) is a general one of which the particular cosmic energy equation for point masses is (Irvine 1961; Layzer 1963), in the special case when $\epsilon = 0$,

$$\frac{\mathrm{d}}{\mathrm{d}t}(K+W)+\frac{\dot{R}}{R}(2K+W)=0. \tag{20}$$

Therefore, we may here say that the introduction of the softening parameter as a measure of the extended nature of galaxies modifies the evolution of clustering and modification depends upon the value of ϵ taken. From (17) and (20) the difference between the evolution of point and non-point mass systems is clear. When we deal with non-point masses, the evolution of kinetic and potential energies with the expansion of volume is given by (17). However, when we observe a point mass system, the evolution of the kinetic and potential energies is given by (20).

3 Evolution of correlation potential energy for non-point masses (galaxies with halos)

To solve the cosmic energy equation for extended structures given by (17), we may suppose that the correction term $\epsilon^2 W'_\epsilon$ is small and we approximate $(W_\epsilon + \epsilon^2 W'_\epsilon)$ by $W_\epsilon(1+\eta)$, with η a positive constant but less than unity and independent of time (Ahmad et al. 2009). In this approximation, the cosmic energy equation becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}(K+W_\epsilon)+\frac{\dot{R}}{R}\big[2K+W_\epsilon(1+\eta)\big]=0. \tag{21}$$

The cosmic correlation potential energy is related to the peculiar kinetic energy by (Saslaw 1980; Ahmad et al. 2009)

$$K=-\beta_\epsilon(t)W_\epsilon. \tag{22}$$

Here $\beta_\epsilon(t)$ is a complicated function of time. We assume that the cluster is of a spherical shape and that the cosmic expansion has the form (Saslaw 1980)

$$R \sim t^{\alpha}. \tag{23}$$

Here the value of α depends upon the model of the Universe. For the Dirac Universe $\alpha = 1/2$, Einstein–de Sitter $\alpha = 2/3$ and Milne $\alpha = 1$. In order to get an idea of the expansion we suppose α to be a constant with a certain fixed value. With (22) and (23), (21) will become

$$\frac{\mathrm{d}}{\mathrm{d}t}(1-\beta_\epsilon)W_\epsilon+\frac{\alpha}{t}(\mathbf{\Delta}-2\beta_\epsilon)W_\epsilon=0. \tag{24}$$

Here $\mathbf{\Delta} = 1+\eta$ which is obviously greater than unity. The solution of (24) may be given as

$$W_\epsilon = W_\epsilon(0)(1-\beta_\epsilon)^{-1}\exp\left(\int\frac{\alpha(2\beta_\epsilon-\mathbf{\Delta})}{(1-\beta_\epsilon)t}\,\mathrm{d}t\right). \tag{25}$$

From (25) it is evident that the evolution of $W_\epsilon(t)$ depends upon the evolution of the function $\beta_\epsilon(t)$. Therefore, one is required to study the time evolution of $\beta_\epsilon(t)$ for extended structures. In (24) we may use $(1/W_\epsilon)\mathrm{d}W_\epsilon/\mathrm{d}t = q/t$ to find

$$(1-\beta_\epsilon)\frac{q}{t}-\frac{\mathrm{d}\beta_\epsilon}{\mathrm{d}t}+\frac{\alpha}{t}(\mathbf{\Delta}-2\beta_\epsilon)=0. \tag{26}$$

Solving for $\beta_\epsilon(t)$, (26) may give us

$$\beta_\epsilon(t)=\frac{q+\alpha\mathbf{\Delta}}{q+2\alpha}+\left[\frac{\beta_{\epsilon 0}(2\alpha+q)-(q+\alpha\mathbf{\Delta})}{q+2\alpha}\right]\left(\frac{t_0}{t}\right)^{2\alpha+q}. \tag{27}$$

Here $\beta_{\epsilon 0}$ is the value of $\beta_\epsilon(t)$ at time $t=t_0$. For the point mass approximation, i.e. as $\mathbf{\Delta}\to 1$, (27) is

$$\beta(t)=\frac{q+\alpha}{q+2\alpha}+\left[\frac{\beta_0(2\alpha+q)-(q+\alpha)}{q+2\alpha}\right]\left(\frac{t_0}{t}\right)^{2\alpha+q}, \tag{28}$$

which is the same as that obtained by Saslaw (2000). Here $\beta_{\epsilon o}=\beta_o$ as the softening parameter $\epsilon = 0$. For long times, the memory of the initial value is lost, suggesting that an equilibrium or stationary state is reached. Obviously, this can be seen to be the case in any expanding Universe, because $\int t^{-1}\alpha\,\mathrm{d}t$ increases with time. Therefore,

$$\lim_{t\to\infty}\beta_\epsilon(t)=\frac{q+\alpha\mathbf{\Delta}}{q+2\alpha}. \tag{29}$$

For point masses, (29) will be given by

$$\lim_{t\to\infty}\beta(t)=\frac{q+\alpha}{q+2\alpha}. \tag{30}$$

From (29) and (30) it is obvious that for long times, we may suppose that $\beta_\epsilon(t)$ changes slowly and one may replace it by its average value $\bar{\beta}_\epsilon$. Therefore, (24) can be written as

$$\frac{\mathrm{d}W_\epsilon}{\mathrm{d}t}=\frac{\alpha}{t}\frac{(2\bar{\beta}_\epsilon-\mathbf{\Delta})}{(1-\bar{\beta}_\epsilon)}W_\epsilon. \tag{31}$$

After solving (24) we may get the evolution of the correlation potential energy as

$$W_\epsilon(t)=W_\epsilon(o)\left(\frac{t}{t_o}\right)^{\psi}. \tag{32}$$

Here $W_\epsilon(o)$ is the value of $W_\epsilon(t)$ at $t=t_o$. Besides we have

$$\psi=\frac{(2\bar{\beta}_\epsilon-\mathbf{\Delta})}{(1-\bar{\beta}_\epsilon)}\alpha. \tag{33}$$

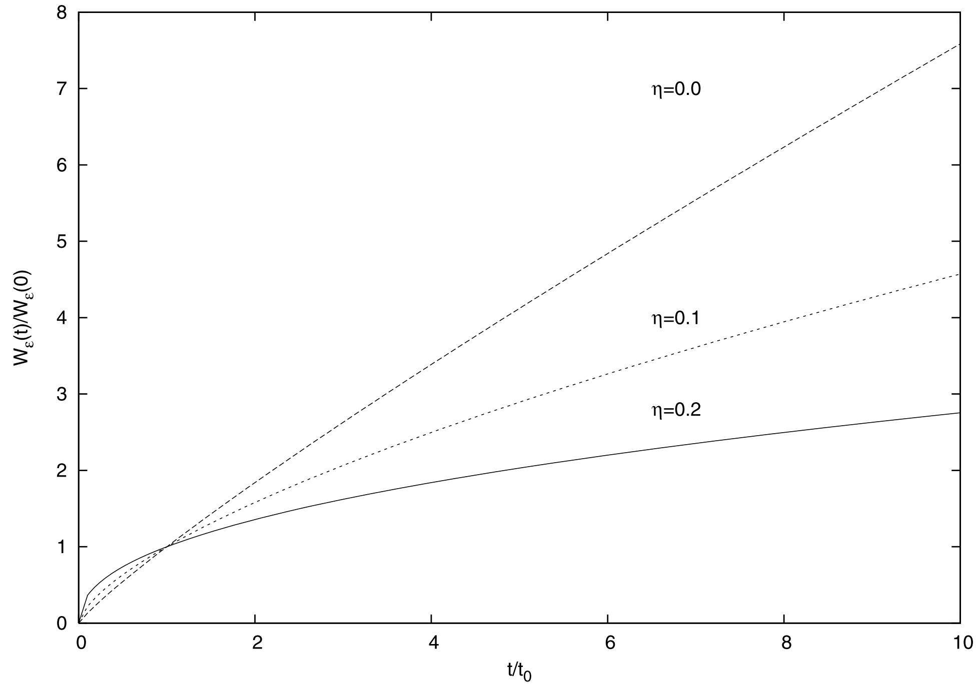


Fig. 1 Variation of $W_\epsilon(t)/W_\epsilon(0)$ against t/t_0 for different values of η

Figure 1 shows the variation of the ratio $W_\epsilon(t)/W_\epsilon(0)$ with respect to the ratio t/t_o for different values of ψ. The values of ψ have been computed for $\bar{\beta}_\epsilon = 0.7$, $\alpha = 3/2$ with $\eta = 0.0$, 0.1 and 0.2. It is observed that as the value of η increases, the evolution of the correlation potential energy decreases. In other words, we may say that as the extended nature of the system increases, the evolution of the correlation potential energy decreases. Obviously, for a point mass system of galaxies the value of $\mathbf{\Delta}$ is unity with $\eta = 0$. Thus, the value of ψ for point masses may emerge as

$$\psi' = \frac{(2\bar{\beta} - 1)}{(1 - \bar{\beta})}\alpha. \tag{34}$$

For the scales on which the parameter β_ϵ changes slowly, the evolution of the correlation potential energy for extended structures is given by (32). Obviously, the evolution of the correlation potential energy for point masses is different. The difference becomes more and more appreciable as the value of ϵ is increased. The quantity $\Delta\psi = \psi - \psi'$ will itself explain the evolution of non-point masses compared to the evolution of point masses. $\Delta\psi$ may be small, but it is non-zero for non-point masses. This means that the clustering of galaxies, with halotic structure, has dynamics different from point mass galaxies.

4 Extensivity approximation for non-point mass system of galaxies

For infinite systems, the extensivity is a good approximation. This is because, for infinite systems, we use the grand canonical ensemble of cells whose size is greater than the extent of the correlations. In this case, thermodynamic functions are essentially extensive. It is worth to note that for any cell size, the essential requirement for extensivity is that the correlation energy between the cells be much less than the correlation energy within an average cell (Saslaw 2000; Saslaw and Fang 1996). We may take a grand canonical ensemble containing all cells of size V with radius R_1. The extent of the correlation is defined as the correlation length. For the case where the ensemble cells are separated by much more than a correlation length, and if the size of the each cell is somewhat larger than the correlation lenght, the extensivity approximation is reasonable (Sheth and Saslaw 1996). If we consider a power law form for the correlation function, we have $\xi(\mathbf{r}) = \xi_o \mathbf{r}^{-\gamma}$ (Saslaw 1980, 2000). Then the correlation energy, for point masses, within a spherical volume $(4\pi R_1^3)/3$ is (Saslaw 2000)

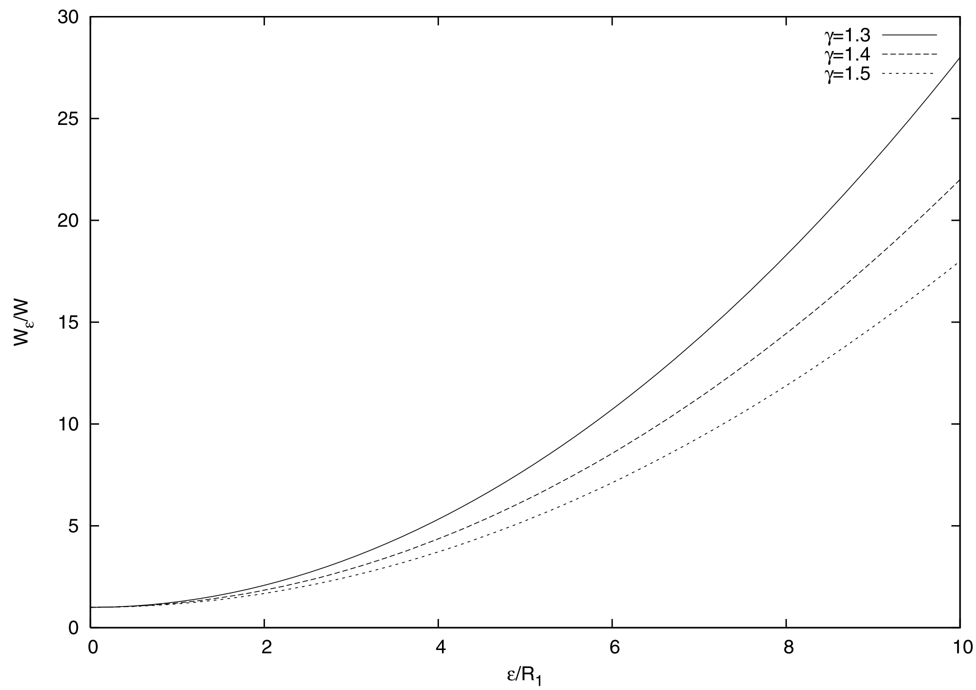
$$\begin{aligned} W_V &= -\bar{n}V \int_0^{R_1} \frac{Gm^2}{2\mathbf{r}} \xi(\mathbf{r}) 4\pi \mathbf{r}^2 \, d\mathbf{r} \\ &= -2\pi G m^2 \bar{n}^2 V \xi_o \frac{R_1^{2-\gamma}}{(2-\gamma)}. \end{aligned} \tag{35}$$

The correlation energy for point masses within the spherical volume $2V$ (i.e. of radius $2^{1/3}R_1$) is (Saslaw 2000)

$$W_{2V} = -2\pi G m^2 \bar{n}^2 2V \xi_o \frac{(2^{1/3}R_1)^{2-\gamma}}{(2-\gamma)}. \tag{36}$$

Obviously, for the approximation of extensivity we must have $|W_{2V}/2W_V| = 2^{(2-\gamma)/3} \approx 1$, which is the case when

Fig. 2 Variation of W_ϵ/W against ϵ/R_1 for different values of γ



$\gamma \gtrsim 1$. For the observed value, $\gamma \gtrsim 1.7$, it holds to a few percent (Saslaw 2000).

Since the introduction of the softening parameter ϵ in the potential $\phi(\mathbf{r})$ has modified the thermodynamics of extended structures (as we learned from preceding sections), it would be interesting to find any modification in the extensivity approximation for extended structures. For this, we may write (8) as

$$W_\epsilon = -\frac{Gm^2\bar{n}^2V}{2}\int_0^{R_1}\left(1+\frac{\epsilon^2}{\mathbf{r}^2}\right)^{-1/2}\xi(\mathbf{r})4\pi\mathbf{r}\,d\mathbf{r}. \tag{37}$$

Now using the power law form of the correlation function, the correlation energy within the spherical volume V will be

$$W_\epsilon = -2\pi Gm^2\bar{n}^2V\xi_o\int_0^{R_1}\mathbf{r}^{(1-\gamma)}\left(1+\frac{\epsilon^2}{\mathbf{r}^2}\right)^{-1/2}d\mathbf{r}. \tag{38}$$

For an average halo around the galaxy we may assume that $\epsilon/\mathbf{r}$ is a small quantity besides taking into consideration that each galaxy is having an individual halo (Sivakoff and Saslaw 2005). This means (38) may be put thus:

$$\begin{aligned} W_\epsilon(V) &\approx -2\pi Gm^2\bar{n}^2V\xi_o\int_0^{R_1}\mathbf{r}^{(1-\gamma)}\left(1-\frac{1}{2}\frac{\epsilon^2}{\mathbf{r}^2}\right)d\mathbf{r} \\ &= W_V\left[1+\lambda(\gamma)\frac{\epsilon^2}{R_1^2}\right], \end{aligned} \tag{39}$$

where $\lambda(\gamma) = (2-\gamma)/(2\gamma)$.

Equation (39) represents the correlation potential energy W_ϵ for extended structures in terms of the correlation potential energy W for point masses. For a particular value of γ, the modification of the correlation potential energy depends on the ratio ϵ/R_1. This ratio represents the extended nature of galaxies in terms of cell size. For the point mass approximation, $\epsilon/R_1 = 0$. As the value of ϵ/R_1 increases from zero, the problem becomes one of the non-point mass. For the non-point masses, the suitable values of ϵ/R_1 range between 0 and 1 (Ahmad et al. 2002). The impact of the extended nature of galaxies on their correlation energy is obvious from Fig. 2. Figure 2 shows the variation of the ratio W_ϵ/W with respect to the parameter ϵ/R_1 for different values of γ, i.e. γ =1.3, 1.4, and 1.5. As the value of ϵ/R_1 increases, the ratio W_ϵ/W becomes appreciably different from unity, which means that the correlation potential energy W_ϵ for extended structures is greater than the correlation potential energy W for point masses. Besides, as the value of γ is increased, the function $\lambda(\gamma)$ decreases and thereby the ratio W_ϵ/W decreases. This makes the impact of the extended nature of galaxies on the ratio W_ϵ/W small, for small values of ϵ/R_1. Though the impact is small, however, it is non-negligible. The correlation potential energy for volume $2V$ with radius $(2^{1/3}R_1)$ may be given as

$$W_\epsilon(2V) = W_{2V}\left[1+\lambda(\gamma)\frac{\epsilon^2}{2^{2/3}R_1^2}\right]. \tag{40}$$

Proceeding our mathematical argument we may write

$$\frac{W_\epsilon(2V)}{2W_\epsilon(V)} = \frac{W_{2V}}{2W_V}\left(\frac{1+\lambda(\gamma)\frac{\epsilon^2}{2^{2/3}R_1^2}}{1+\lambda(\gamma)\frac{\epsilon^2}{R_1^2}}\right). \tag{41}$$

On substituting the value of $W_{2V}/2W_V$ in (41) we have

$$\frac{W_\epsilon(2V)}{2W_\epsilon(V)} = 2^{(2-\gamma)/3}\alpha(\gamma, \epsilon/R_1). \tag{42}$$

Here

$$\alpha(\gamma, \epsilon/R_1) = \left(\frac{1+\lambda(\gamma)\frac{\epsilon^2}{2^{2/3}R_1^2}}{1+\lambda(\gamma)\frac{\epsilon^2}{R_1^2}}\right). \tag{43}$$

Like point mass systems, the extensivity approximation for extended structures may hold when the ratio $W_\epsilon(2V)/2W_\epsilon(V) \approx 1$. A careful observation shows that the ratio $W_\epsilon(2V)/2W_\epsilon(V)$ will be approximately equal to unity as $\gamma \gtrsim 1$. This may be put in other words: that the extensivity approximation for extended systems becomes effective as $\gamma \gtrsim 1$, which is also the case for point mass systems.

5 Summary and conclusion

We derive the cosmic energy equation for non-point mass structures by using the adiabatic approximation for gravitational growth of galaxy clustering in a volume which expands as the universe is expanding. The adiabatic approximation gives the first law of thermodynamics with $\delta Q = 0$ and this turns out to be the cosmic energy equation. The cosmic energy so obtained is the same as derived by Ahmad et al. (2009). The cosmic energy equation for non-point masses is a general one, and the cosmic energy equation for point masses is a special case of it. As the softening parameter $\epsilon \to 0$, the cosmic energy equation for non-point masses reduces to the cosmic energy equation for point masses.

The behavior of the clustering of an extended structure can be understood by analyzing the evolution of the correlation potential energy. The correlation potential energy for extended masses largely depends upon the form of the parameter $\beta_\epsilon(t)$. So it is necessary to study the form of $\beta_\epsilon(t)$. After initial or early relaxation, $\beta_\epsilon(t)$ changes very slowly and we may replace it by an average value $\bar{\beta}_\epsilon$. This approximation produces the W_ϵ as proportional to t^ψ, with ψ as constant.

In case of an infinite system, the extensivity is a good approximation. Since we take the grand canonical ensemble to represent an infinite system, we choose the cell size such that the correlation length is much smaller than the cell size. Besides, the correlation between cells is ignorable as compared to the strength of correlation within each cell. The power law of the correlation function is helpful in finding out the circumstances when the extensivity approximation becomes applicable. It is interesting to note that both point and non-point mass systems follow the extensivity approximation as $\gamma \gtrsim 1$.

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