

Numerical simulation of flow around square cylinder using different low-Reynolds number turbulence models

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Abstract: ABE-KONDOH-NAGANO, ABID, YANG-SHIH and LAUNDER-SHARMA low-Reynolds number turbulence models were applied to simulating unsteady turbulence flow around a square cylinder in different phases flow field and time-averaged unsteady flow field. Meanwhile, drag and lift coefficients of the four different low-Reynolds number turbulence models were analyzed. The simulated results of YANG-SHIH model are close to the large eddy simulation results and experimental results, and they are significantly better than those of ABE-KONDOH-NAGANO, ABID and LAUNDER-SHARMA models. The modification of the generation of turbulence kinetic energy is the key factor to a successful simulation for YANG-SHIH model, while the correction of the turbulence near the wall has minor influence on the simulation results. For ABE-KONDOH-NAGANO, ABID and LAUNDER-SHARMA models satisfactory simulation results cannot be obtained due to lack of the modification of the generation of turbulence kinetic energy. With the joint force of wall function and the turbulence models with the adoption of corrected swirl stream, flow around a square cylinder can be fully simulated with less grids by the near-wall.

Key words: low-Reynolds number turbulence model; flow around square cylinder; numerical simulation

1 Introduction

Flow around a square cylinder has extensive significance in engineering, such as large building, cooling tower, trestle of sea roof, tube of heat exchanger and so on^[1–3]. As the alternate producing and shedding of the back sides' separation vortex will influence the load imposed on the object, resonance may happen if the shedding frequency of separation vortex is close to that of the object's self-oscillation, which may damage the object. LES(large eddy simulation) has been utilized mostly in numerical simulation of flow around a square cylinder in recent years^[4–6], but Reynolds-average method also has advantages in engineering application because LES costs far more than Reynolds-average method. BOSCH and RODI et al^[7], KIMURA and HOSODA^[8], IACCARINO^[9], RODI^[10] and LÜBCKE^[11] investigated flow around a square cylinder with RANS (Reynolds-averaged Navier-Stokes). Many of them believed that the modification to the turbulence by the

near-wall had a significant influence on simulation results, and paid much attention to the modification of the near-wall turbulence model. But generally speaking, for those improved models, denser grids that work against engineering are needed by the near-wall. In this work, four different low-Reynolds number turbulence models were used to simulate the flow around a square cylinder and the key factors for satisfactory simulation were investigated.

2 Numerical methods

2.1 Turbulence models

Four different low-Reynolds number turbulence models, including ABE-KONDOH-NAGANO(AKN) model^[12–13], ABID(AB) model^[14], YANG-SHIH(YS) model^[15] and LAUNDER-SHARMA(LS) model^[16–17] were utilized to simulate the flow around a square cylinder.

2.2 Geometric model and grid partition

The region of geometric model was $14h \times 20h$, and

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the distance to entrance was $5h$, where h was the length of the square cylinder. Grids were asymmetrically partitioned. For the first row of the near-wall, dimensionless wall distance $y^+ < 1$, and inside the region of $y^+ < 5$, at least five nodes were set. Totally 12 000 nodes were partitioned in the entire region.

2.3 Boundary conditions

The velocity of inlet flow was fixed, with Reynolds number $Re = 2200$ and a 2% turbulence degree; periodical boundary conditions were applied for both sides of the square cylinder; local single-directional method was adopted to treat the outlet of the square cylinder.

2.4 Numerical simulation method

Equations were discretized with finite volume method. The differential format of convection was treated by QUICK format and the unsteady item was treated by second-order implicit format. The pressure-implicit with splitting of operators (PISO) arithmetic was utilized to couple pressure–velocity of each time-layer with dimensionless time step of 0.02.

3 Results and discussion

The simulation results show that the periodically unsteady flow around a square cylinder can be simulated with different low-Reynolds number turbulence models. Because the flow around a square cylinder is periodical and the unsteady low-Reynolds kappa-epsilon ($k-\epsilon$)

model is Reynolds-average, the simulation results can still be regarded as time-averaged results, but they vary periodically with time. Therefore, in this work, they were called time-averaged unsteady results. Time-averaged results with steady characteristic were obtained by averaging the time-averaged unsteady results of many periods.

3.1 Time-averaged unsteady results

The flow lines of different phases predicted by YS, AB and LES models^[11] are shown in Fig.1. A period T is divided into 20 phases with the same time interval, and the results of four phases, $1T/20$, $5T/20$, $8T/20$ and $11T/20$, are also shown in Fig.1. The results of AKN, LS and AB models are similar because the space is limited. So parts of simulated results are listed here.

Fig.1 shows that the simulated results of YS model are distinct from those of AB model and approach those of LES model. But the results of AB model cannot accurately reflect the development and dropping of the eddies behind the square cylinder. Generally, these vortices fall off the square cylinder until they become very strong. Also according to the simulated results of AB model, the eddies of both sides seldom vary over the periods, which is essentially different from that of LES.

3.2 Time-averaged steady results

The time-averaged results of experiment^[18], LES results^[10] and the simulated results of this work are listed in Table 1. The Strouhal numbers (St) obtained by the four different models in this work conform well to those

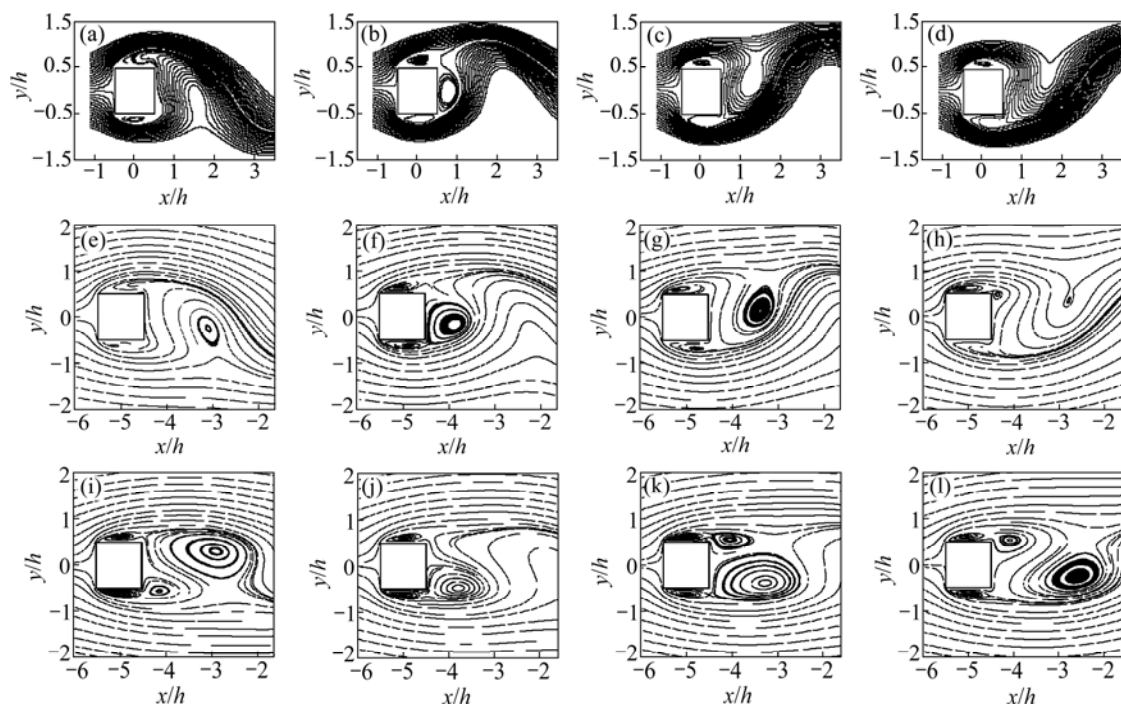


Fig.1 Comparison of RANS and LES in different phases: (a) LES model, $t=1T/20$; (b) LES model, $t=5T/20$; (c) LES model, $t=8T/20$; (d) LES model, $t=11T/20$; (e) YS model, $t=1T/20$; (f) YS model, $t=5T/20$; (g) YS model, $t=8T/20$; (h) YS model, $t=11T/20$; (i) AB model, $t=1T/20$; (j) AB model, $t=5T/20$; (k) AB model, $t=8T/20$; (l) AB model, $t=11T/20$

Table 1 Comparison of results of experiments, LES and RANS in this work

Method	x_R/h	\bar{C}_d	\tilde{C}_d	\tilde{C}_l	Sr
Experiment ^[18]	1.38	2.10			0.132
LES ^[10]	1.32	2.20	0.140	1.01	0.130
RANS	AKN model	3.39	1.68	0.020	0.130
	AB model	3.26	1.70	0.001	0.130
	LS model	2.68	1.81	0.030	0.133
	YS model	1.54	2.10	0.060	0.147

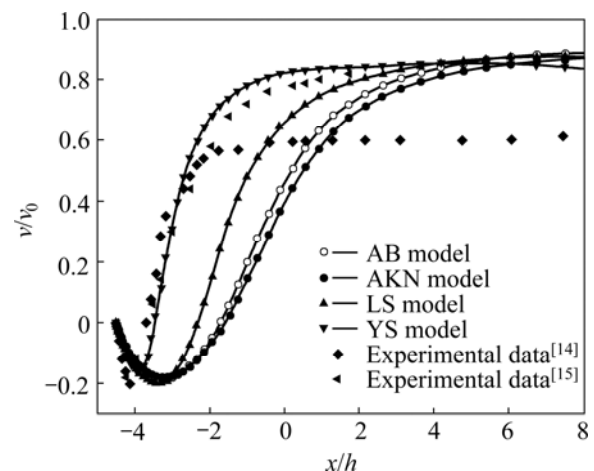
Note: x_R/h is re-attached distance; \bar{C}_d is average of drag coefficients; \tilde{C}_d is root-mean-square of drag coefficients; \tilde{C}_l is root-mean-square of lift coefficients; Sr is Strouhal number.

of the experimental results^[18] and LES results^[10], which approves that the characteristics of unsteady fluid flows around a square cylinder can be reflected by RANS in this work. Compared with LYN experimental results in Ref.[18], the re-attached distances x_R/h , simulated by AB, AKN and LS models respectively, are several times longer, but the maximum difference between YS result and experimental result is no more than 16%.

For the average of drag coefficient \bar{C}_d , the simulated result of YS model agrees well with experimental result, but the results of other models differ greatly from experimental result. As to the root-mean-square values of drag coefficients \tilde{C}_d , the simulated results of AB, AKN and LS models are almost equal to zero, and results of YS model are also different from experimental results. According to the comparison of flow field in different phases, the dropping positions of eddies simulated by YS model and AB model are 20%–35% and 30%–55% respectively, farther away from the square cylinder than that of LES model. For the root-mean-square of lift coefficient \tilde{C}_l , the results of AB, AKN and LS models greatly differ from experimental results, while that of YS model agrees well with them. This can be also explained by the comparison of flow lines of different phases in Fig.1. According to the simulated results of YS model, there are small eddies on both sides of the square cylinder, and their positions and states vary continuously over the whole period, corresponding to the movement of flow in the wake of the cylinder, whereas results of AB model show that eddies on both sides remain almost constant. Therefore, as to periodical characteristics, results of YS model are closer to those of LES.

Fig.2 shows the comparison of velocity distribution along the stream with experimental results of LYN^[18] and DURÃO^[19]. It can be seen that the results of AB, AKN and LS models all differ greatly from those of experiments, especially near the wake of the cylinder. But comparatively, the results of LS model agree better with experimental results than those of the other two models. It can be also seen from Fig.2 that, there are remarkable differences between DURÃO's and LYN's experimental results. The difference is not very obvious

near the wake of the cylinder, but it becomes serious far away from the wake of the cylinder. As to the simulation of wake flow far away from the cylinder, results of YS model agree very well with the experimental results of DURÃO, but differ somewhat from those of LYN. And interestingly, the Reynolds number, utilized in this work, is very close to that of LYN experiment ($Re=21\ 400$), while differ greatly from that of DURÃO's experiment ($Re=14\ 000$). In the wake flow far away from the cylinder, the results of LES^[4–6, 7, 10–11, 20] are generally close to the experiment results of DURÃO, but differ from those of LYN.

**Fig.2** Comparison of streamwise velocity profiles in wake centerline of cylinder ($y/h=0$)

3.3 Discussion

As to simulation of flow around a square cylinder, not all diverse low-Reynolds number turbulence models can get good results, but YS model is an exception. KATO and LAUNDER^[21] found that simulated results of flow around a square cylinder with high-Reynolds number turbulence model, i.e. KATO-LANUDER(KL) model, also agreed well with experimental results. One characteristic, shared by YS and KL models, is that both models modify the generation of turbulence kinetic energy (P) in equation (the modification to S_{ij} in turbulence models corresponds to time-averaged rate of stress change of mainstream). The modification in YS

model is mainly about the modification to damping function f_1 in ε equation^[22].

$$f_1 = \frac{1}{1 + 1/\sqrt{Re_t}} \quad (1)$$

where Re_t is the turbulence Reynolds number. But in AB, AKN and LS models, $f_1=1$.

Instead of standard $k-\varepsilon$ model, KIMURA and HOSODA^[8] adopted non-linear $k-\varepsilon$ model to correct Boussinesq assumption, and further correct swirl stream. IACCARINO et al^[9] applied normal-velocity relaxation turbulence model (V2F model) to his research, and calculated additionally other variables \bar{v}'^2 and f , in which the former refers to the scale of turbulence velocity vertical to wall, and the latter is the re-distribution item of \bar{v}'^2 equation. The expression of f_1 is

$$f_1 = 1 + 0.045\sqrt{k/\bar{v}'^2} \quad (2)$$

Differing from general low-Reynolds number turbulence $k-\varepsilon$ model, f_1 contains modification to P although it does not work as damping function. YS, KL and V2F models modify P with different forms, but they all modify swirl stream in the flow field. Basically, f_1 modifies the deficiency of Boussinesq assumption like non-linear $k-\varepsilon$ models.

As to AB, AKN and LS models, f_1 approaches 1, when the flow is far away from the wall of cylinder and the turbulence Reynolds number Re_t is very large. In LS model, k equation and ε equation are corrected considering the diversity of kinetic energy dissipation of boundary layer. But generally speaking, these modifications will not influence the simulated results remarkably.

According to the above analysis, the modification of near-wall turbulence exerts little influence on the simulated results, and modification the generation of turbulence kinetic energy is the key to successful simulation. So by combining wall function with turbulence models and making use of corrected swirl stream, flow around a square cylinder can be solved very well. The advantage of this method in engineering application is that it can obviously reduce the number of grids.

4 Conclusions

1) By adopting AKN, AB, YS and LS models to simulate the unsteady flow around a square cylinder in different phases flow field, and comparing with LES results and experimental results, it is found that YS model can achieve better results than AKN, AB and LS models.

2) By comparing the time-averaged results calculated from the four adopted models, including the

re-attached distance x_R/h , the average of drag coefficients \bar{C}_d , the root-mean-square value of drag coefficients \tilde{c}_d , the root-mean-square value of lift coefficients \tilde{c}_l , and the Strouhal number St with the experimental results, LES results, and the RANS simulated results, it can be also concluded that, YS model's simulation results are closer than those of AKN, AB and LS models. Especially for root-mean-square values of drag coefficients \tilde{C}_d , the results of AB, AKN and LS models differ from experimental results by nearly 1 magnitude.

3) By comparing the results of velocity along the stream with experimental results of LYN and DURÃO and the RANS simulated results, results of AB, AKN and LS models greatly differ from those of experiments. But comparatively, the results of LS model agree better with experimental results than AB and AKN models. It can be also seen that the results of YS model are closer to the experiment results of DURÃO, but differ from those of LYN, just as the results of LES.

4) The conclusion can be safely drawn that the key to a successful simulation of YS model lies in the modification of the generation of turbulence kinetic energy (P) in equation, while AB, AKN and LS models get inferior simulated results because they only modify the turbulence near the wall, instead of modifying the generation of turbulence kinetic energy (P). So, instead of the modification of the turbulence near the wall, the modification of the generation of turbulence kinetic energy (P) is the key to a successful simulation. The joint force of wall function and the turbulence models with the adoption of corrected swirl stream can help a satisfactory simulation of flow around a square cylinder with less grids by the near-wall.

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