Glow discharge in microfluidic chips for visible analog computing

Darwin R. Reyes,[a] Moustafa M. Ghanem,[b] George M. Whitesides[c] and Andreas Manz[a]

[a] *Department of Chemistry, Imperial College of Science, Technology and Medicine, London, UK SW7 2AY. E-mail: a.manz@ic.ac.uk; Fax: 44-207-594-5833; Tel: 44-207-594-5838*
[b] *Department of Computing, Imperial College of Science, Technology and Medicine, Huxley Building, 180 Queen's Gate, London, UK SW7 2BZ*
[c] *Department of Chemistry and Chemical Biology, Harvard University, 12 Oxford Street, Cambridge, MA 02138, USA*

Received 17th January 2002, Accepted 11th April 2002
First published as an Advance Article on the web 29th April 2002

Here we present a novel visible analog computing approach for solving a wide class of shortest path problems. Using a microfluidic chip for computation, based on the lighting up of a glow discharge, the solution to maze search problems, the solution of a network shortest path and k-shortest paths problems and the practical application of finding the shortest paths between several landmarks from a street map are presented. The solution and visible display (in real time) for these problems shows only a small difference in practical problem solving time among problems with varying differences in size.

Introduction

The time required for formulating and coding an efficient algorithm and the increased computational time needed to solve larger instances of a problem can pose limitations for digital computation. These limitations can be addressed, for many problems, by an analog approach simulating a physical system. Further advantages could be added to an analog approach if a visible representation of the results is available.[1] Over the last decade, novel computing approaches have been attempted. For example, DNA has been used to solve combinatorial problems, but the computation requires several PCR (polymerase chain reaction) and several agarose gel electrophoresis separations to obtain the solution.[2–7] Theoretically, the scalability of the approach builds on trading execution time for cheap DNA resources acting as processors. In practice, the redundancy of DNA is required and to date only the solutions to small problem sizes have been reported. Also recently, a computational non-deterministic polynomial (NP) complete problem, which lies within the area of combinatorial optimization problems, was solved using microfluidic devices.[8,9] Chiu *et al*. showed the feasibility of moving fluids in parallel to obtain the solution of the maximum clique problem in polynomial time. Their approach is based on coding an algorithm using the properties of microfluidic devices to search the space of possible solutions to the problem. Similar to the DNA computing approach, their approach also makes the traditional tradeoff between the execution time and the space resources required for solving the problem; solving larger such combinatorial problems in polynomial time requires exponentially increasing space. Here we present an analog computational approach to finding the solution to the shortest path problem in networks and planar graphs. Our system uses a microfluidic device for a computation based on the lighting up of a glow discharge to demonstrate a *natural* way of solving such problems. This work shows (i) the solution and visible display, in real time, for the shortest path between two points, (ii) the small difference in practical problem solving time among problems with large differences in size, and (iii) the easy extension of the solution to discover the k-shortest path between two points.

The approach presented here is suitable for a simple class of combinatorial optimization problems including maze searches, network flow problems and the computation of shortest paths both in networks and free space. The shortest path problem in networks and planar graphs is a classic computing problem. Admittedly efficient sequential algorithms exist for finding the shortest path in a network, for example, a simple implementation of Dijkstra's algorithm has complexity $O(n^2)$ where n is the number of vertices in the network.[10] With a little more programming effort more efficient sequential and parallel algorithms to the problem can be coded based on using complex data structures. There also exist efficient algorithms for finding the shortest path between two points in 2D Euclidean space with obstacles. However, finding the shortest path, with arbitrary precision, between two points in 3D Euclidean space with polyhedral objects is NP-hard and efficient algorithms are only known for finding an approximation of such a path within a given tolerance.[11,12] The solutions to these problems play an important role in many practical applications, such as geographical information systems, intelligent transportation systems, robot motion and navigation, among many others.

Experimental

Steps to solve the problems

We posed several problems: a straight line, a maze (Fig. 1A), finding the shortest path in a network (Fig. 1B) and finding the shortest path in a street map (Fig. 1C). To obtain their solutions a series of steps were implemented. However, none of these steps required coding an algorithm or knowledge of computing science concepts. First, a graphical representation of the problem is required. Second, the starting and ending points are chosen. Third, the computation is performed when a breakdown voltage is applied between the two selected points in the graph. The result is displayed visually by a glow discharge that lights up between two electrodes in a low-pressure helium gas environment.[13]

Chips fabrication

To perform the computational process with this analog device the graphical representations of the problems were fabricated on soda-lime glass. The standard micro-fabrication procedures of wet etching and fusion bonding have been described elsewhere.[14] Briefly, the graphical representations were drawn in AutoCAD and transferred to a glass wafer (1.5 mm thick), coated with a positive photoresist and chromium (Nanofilm, Westlake Village, CA), using a direct write laser lithography system (HIMT GmbH, Germany). The exposed photoresist was developed and then the chrome layer was etched. The channels (in the glass) were etched with a HF solution, and had dimensions of *ca.* 250 µm in width and *ca.* 100 µm in depth. A glass cover (1.5 mm thick) was fusion bonded on the top of the glass chip. To introduce the gas flow, the channels were accessed by holes (*ca.* 400 µm) drilled into the cover plate and fused silica capillaries (Composite Metal Services Ltd., UK) of 250 µm id along with graphite ferrules (SWE, Milton Keynes, UK) were glued to the cover. Tungsten wires of 25 µm (AlfaAesar, Karlsruhe, Germany) were used as electrodes and were introduced through the side of the chip *via* the channels close to the chosen landmarks. The chips were sealed on the sides to avoid the entry of external gases. Fig. 2 shows the setup used to generate the glow discharge in the chip. Helium was introduced by one of the capillaries attached to the chip. The outlet capillary was connected to an external pressure sensor.

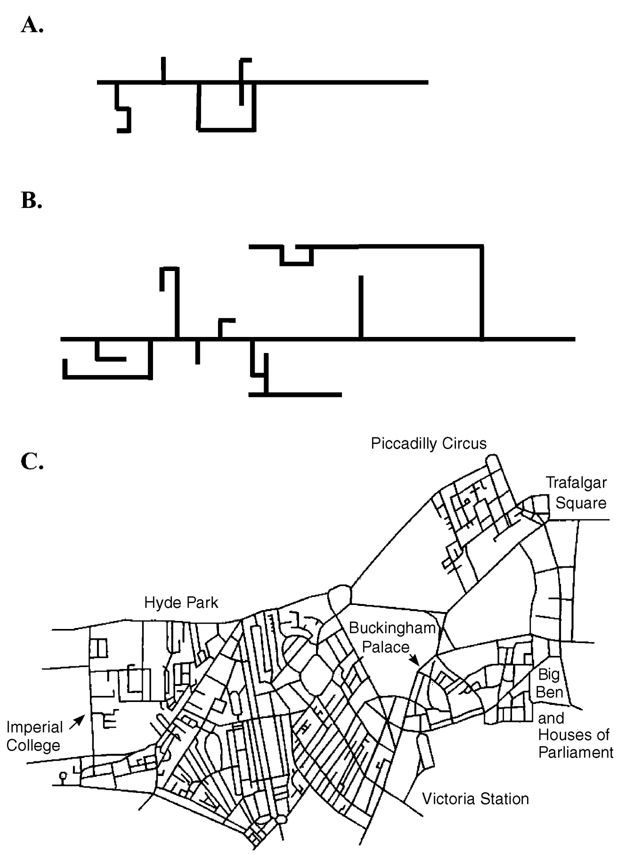

Fig. 1 Graphical representation of: (A) a maze problem with two possible paths and 5 decision points or nodes represented as a simple line; (B) a maze problem with one possible path and 11 nodes represented as a simple line; and (C) a network problem represented as a map of part of Central London.

Low pressure was obtained using a vacuum pump. A high voltage (HV) power supply provided the breakdown voltages used to perform the desired computation.

Problem solving time measurements

To measure the problem solving time a photomultiplier tube and the monitoring voltage outlet from the power supply were connected to an oscilloscope. An external trigger connection, for the whole system, was made to allow the potential to be applied while at the same time the voltage and the light emitted by the system were recorded as a function of time. From the graph obtained, the problem solving time is measured from the triggering time (t = 0) to the time when the glow discharge appears.

Chemicals

The Microposit 351 Developer (developing solution) was supplied by Chestech Ltd (UK). Perchloric acid–cerium ammonium nitrate solution used for chrome etching was obtained from Microchem Systems Ltd. (UK). HF was obtained from BDH (Merck, Dorset, UK).

Results and discussion

Figs. 3A, B and C show the channels of a straight line (2.0 mm long), a maze with a shortest path of 16.2 mm and 5 vertices (points where a decision has to be made either to turn left or right) and a more complicated maze with a path of 26.0 mm long and 11 vertices. Figs. 3D, E and F show the glow discharge when a voltage above the breakdown voltage was applied. In the case of the straight line (Fig. 3D), the glow discharge had only one straight path to follow while in the maze (Fig. 3E) there are two possible paths from one end to the other. The glow discharge lit up through the shortest path. Fig. 3F shows the glow discharge on a more complicated and longer maze.

A more complicated graphical representation of a network, which conforms a street map of central London, was studied next. Fig. 4 shows the graphical representation of a map of a part of London. The problem posed was to find the shortest path between Imperial College (IC) and several landmarks in London. Figs. 4A, B and C show the solutions found by the computation. Subsequent measuring proved that the plasma indeed provided the shortest path from IC to Victoria Station, Buckingham Palace and Trafalgar Square, respectively.

We observed another important characteristic on this device, which can be used to solve k-shortest paths problems. Just

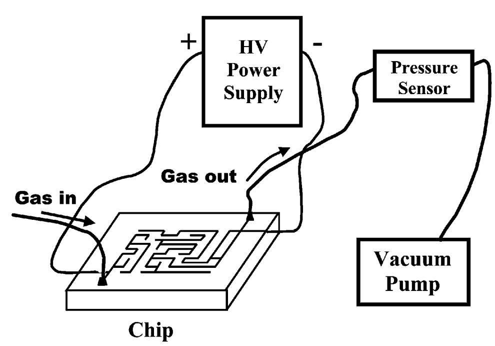

Fig. 2 Schematic layout of the experimental setup used to produce the glow discharge in the chips. The tungsten wires, used as electrodes (not shown), are introduced through the channels adjacent to the starting and ending points. The helium gas is introduced through the inlet capillary connected to the chip and is evacuated through the outlet capillary connected to the pressure sensor and vacuum pump.

above the breakdown voltage, the solution of the problem stayed on the shortest path, but as the voltage was increased the other paths started to appear in increasing order of length. Thus, Fig. 4D is the superimposition of multiple solutions for the same problem (IC to Victoria Station). This is an advantage of this analog computing device, since there is no need of additional steps, other than increasing the applied voltage, to obtain the solution for k-shortest paths problems.

The practical problem solving time for the network graphical representations was studied. The London map (Fig. 4) is a network of 456 vertices. If this were to be naively searched there exist 2^{456} ($\sim 10^{137}$) possible pathways. The 'algorithm' or solution method can be thought of as simply based on multiple breadth-first searches, each starting at one of the electrodes. The electric field explores all the possible pathways in parallel since its lines of force cover the whole graphical representation, but it is the higher field strength along the imaginary straight line between the two electrodes which defines the discharge path through the shortest path. This eliminates unsuitable solutions (*i.e.* the longer paths). The approach used by this device performed the computation of all the problems posed in less than 500 ms. However, the time needed to fabricate the microchip devices was not included in the computational time. In this case, the chip fabrication can take from two to three days, depending on the complexity of the structure (that is, if the drawing of the design is also included as the solving time). This does not pose a limit if a large number of solutions (*e.g.* all-shortest paths) are sought from the same chip.

Fig. 5 shows a graph of computational time for all the network graphical representations as a function of applied voltage. Light emission and voltage in the system were measured at the same time by triggering off the power supply voltage. We found that the computation time slightly decreased when the applied voltage was increased, solving the problem in slightly shorter time. The difference in computation time with increasing size/difficulty of the problems was found to be insignificant.

Conclusions

The advantage of our approach in solving the above examples is that it operates directly to the representation of the problem itself rather than operating on the space of all possible solutions. Problems with more vertices could be easily fitted into this device either by shrinking the dimensions of the graphical representation, using a bigger glass wafer, or using the total area of the glass wafer. Clearly inaccuracies in fabrication must be accounted for and the precision of the desired solution will affect how a physical problem (*e.g.* the map of London) can be mapped to a microfluidic network representation. Using the total area of the wafer in our current experimental set up would allow us to fit approximately 1000 vertices in the chip. Clearly, the scalability of the approach will be limited by the handling of higher voltages needed to obtain the glow discharge in longer paths. This would be expected to set the limit.

Our analog computing device provides a solution within practical solution time, if the time for fabrication is not taken into account. In addition, it was found that when the size of problems is increased the difference in computational time is insignificant. Another important feature of this device is the possibility of obtaining alternate answers (k-short paths) using the same representation.

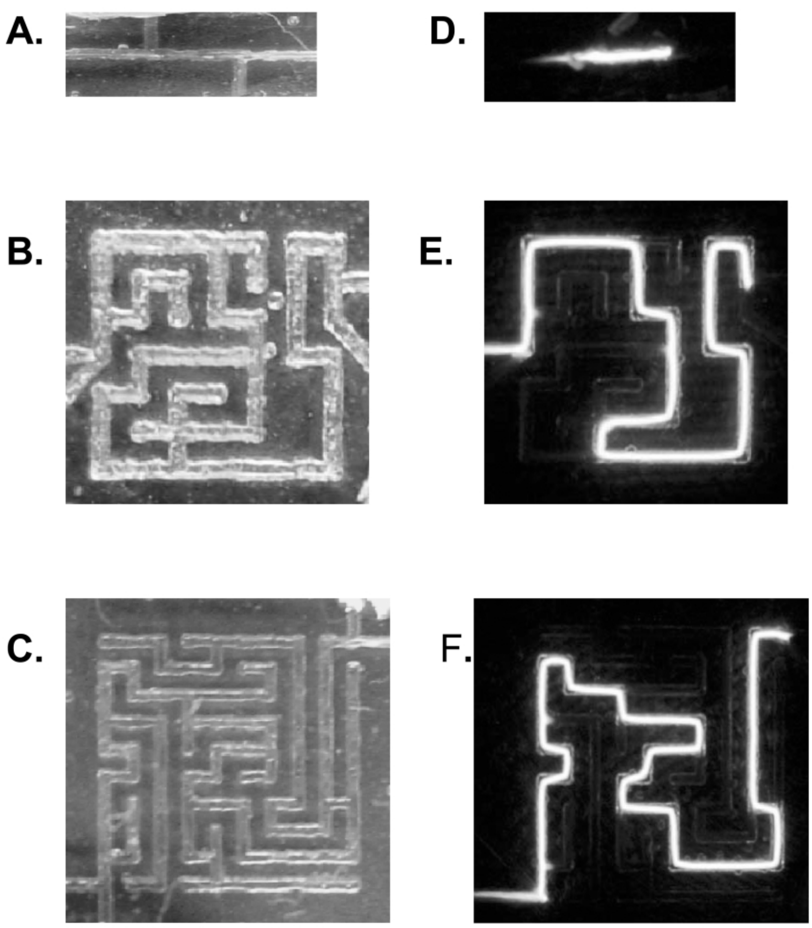

Fig. 3 Images of (A) a straight channel, (B) a maze and (C) a maze of higher complexity. (D) A glow discharge in (A) when 1.12 kV is applied at 11 torr. (E) Image of glow discharge showing the route for the shortest path in the maze (12 torr and 4.02 kV). (F) Image of the glow discharge showing the route from one end to the other of a more complex maze (5.77 kV and 161 torr).

Our microfluidic approach can be extended to solve problems other than the shortest paths problems. Clearly, solving more complex problems, *e.g*. Traveling Salesman Problem and Maximum Clique Problem, requires the ability to impose various constraints on the search conducted. One direct method could be based on first coding the possible solution space as a graph and then searching this graph. However, if we were to follow this approach, we would expect to face the same experimental increase in space requirement as in Chiu's approach.[8] Even though we have not yet tried our approach on NP problems, we believe our results could be a starting point for entirely novel (mathematical) applications of microfluidic channel systems.

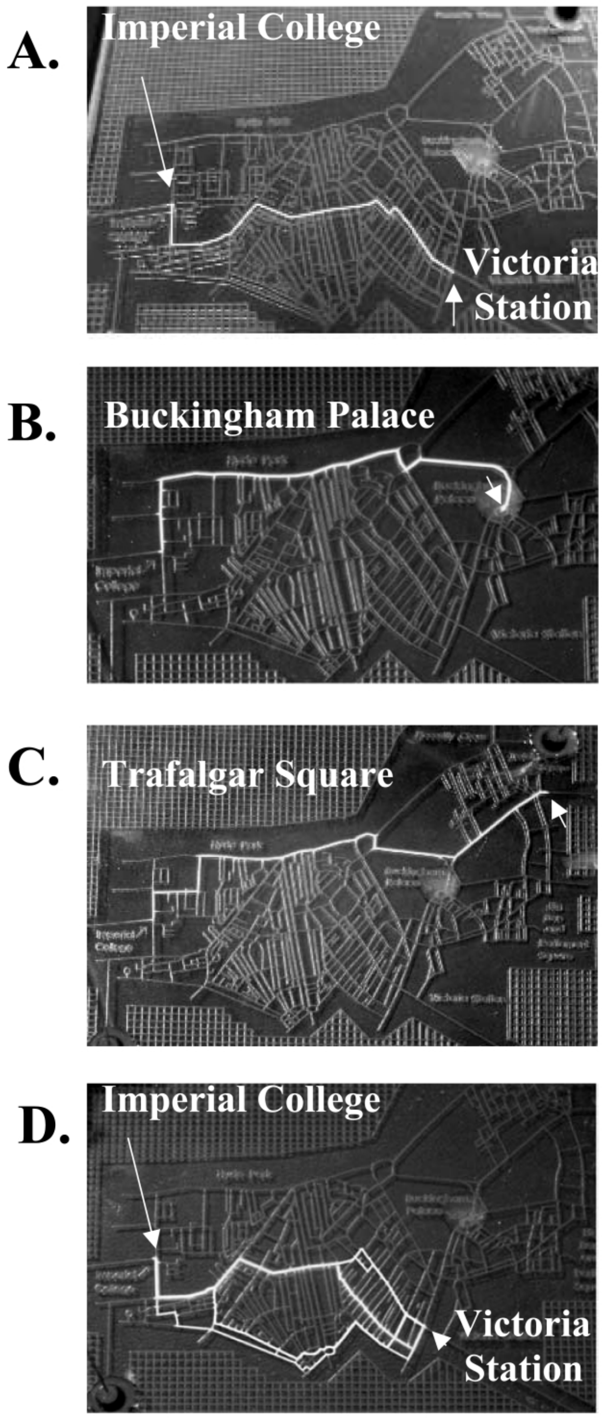

Fig. 4 Image of the London map showing the shortest path between (A) Imperial College (IC) and Victoria Station, (B) IC to Buckingham Palace and (C) IC to Trafalgar Square, by means of a visible glow discharge. A voltage was applied between the points that correspond to the two landmarks. Applied voltages were 28.0, 25.2 and 28.0 kV, respectively, and the pressure was 486 torr. (D) Superimposed image of alternate solutions between IC and Victoria Station. Applied potential was increased from 28 kV to *ca.* 30 kV. Pressure was 486 torr.

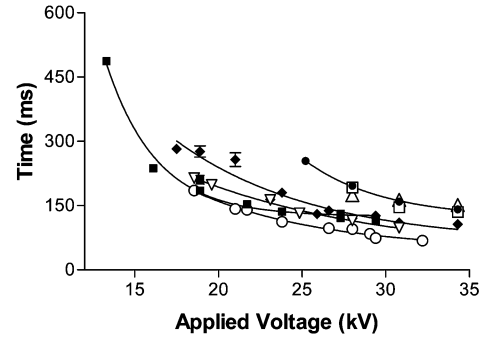

Fig. 5 Problem solving time measured as a function of the applied voltage for: a straight line 2 mm long (■) at 27 torr, a straight line 13.5 mm long (▽) at 24 torr, a simple maze (○) at 27 torr, a complex maze (◆) at 159 torr, route from Imperial College to Buckingham Palace (●), route from Imperial College to Victoria Station (□) and route from Imperial College to Big Ben (△) all at 486 torr.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. INT-0000462. We acknowledge BBSRC (Britain) for clean room facilities. D.R.R. also thanks Y. Kwok, R. Alicea-Maldonado and L. Maldonado-Baez for proof reading and comments on the manuscript and J. Eijkel, G. Jenkins, O. Naji for valuable discussion and technical support.

References

1 A. W. Phillips, *Economica*, 1950, **17**, 283.
2 L. M. Adleman, *Science*, 1994, **266**, 1021.
3 R. J. Lipton, *Science*, 1995, **268**, 542.
4 F. Guarnieri, M. Fliss and C. Bancroft, *Science*, 1996, **273**, 220.
5 Q. Ouyang, P. D. Kaplan, S. M. Liu and A. Libchaber, *Science*, 1997, **278**, 446.
6 K. Sakamoto, H. Gouzu, K. Komiya, D. Kiga, S. Yokoyama, T. Yokomori and M. Hagiya, *Science*, 2000, **288**, 1223.
7 Q. H. Liu, L. Wang, A. G. Frutos, A. E. Condon, R. M. Corn and L. M. Smith, *Nature*, 2000, **403**, 175.
8 D. T. Chiu, E. Pezzoli, H. K. Wu, A. D. Stroock and G. M. Whitesides, *Proc. Natl. Acad. Sci. USA*, 2001, **98**, 2961. Moving fluids in a 3D microfluidic device is used to search, in parallel, all the potential solutions for the Maximum Clique Problem (NP complete problem) in polynomial time. The strength of the system is its high parallelism and the weakness is that the physical size of the device increases exponentially with the number of vertices.
9 J. S. McCaskill, *Biosystems*, 2001, **59**, 125.
10 E. W. Dijkstra, *Numerische Mathematik*, 1959, **1**, 269.
11 J. H. Reif and J. A. Sorter, *J. Assoc. Comput. Machinery*, 1994, **41**, 1013.
12 D. Lee and F. Preparata, *Networks*, 1984, **14**, 393.
13 J. C. T. Eijkel, H. Stoeri and A. Manz, *Anal. Chem.*, 1999, **71**, 2600.
14 D. J. Harrison, A. Manz, Z. H. Fan, H. Ludi and H. M. Widmer, *Anal. Chem.*, 1992, **64**, 1926.