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# Chain length dependence of static longitudinal polarizabilities and hyperpolarizabilities in linear polyynes

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Ab initio calculations of the static longitudinal dipole polarizability  $\alpha_L$  and second dipole hyperpolarizability  $\gamma_L$  are reported for the linear  $C_{2n}H_2$  polyynes up to  $C_{44}H_2$ . Basis set requirements diminish with increasing chain length. The intermediate neglect of differential overlap (INDO) method is able to mimic the *ab initio* results if the Ohno-Klopman parameterization is used. The values per acetylenic linkage,  $\alpha_L/n$  and  $\gamma_L/n$ , converge very slowly with chain length; they are extrapolated to the infinite chain limit.

## I. INTRODUCTION

Materials with relatively large linear and nonlinear response properties are of widespread interest particularly in connection with telecommunications and device technology. Organic molecules with delocalized  $\pi$  electrons, such as conjugated chains, almost invariably have large dipole polarizabilities and hyperpolarizabilities. The dependence of these electric properties upon the chain length needs to be better understood to aid the design of more promising optoelectronic materials. High accuracy *ab initio* calculations, using large one-particle basis sets and including the effects of electron correlation, could lead to such an understanding. Unfortunately such calculations are not feasible currently for long-chain molecules, and hence simpler models are used for the prediction of their polarizabilities and susceptibilities.

Linear  $C_{2n}H_2$  polyynes provide a particularly simple example of chains with conjugated  $\pi$  systems. Some *ab initio* studies of the electric properties of very short linear polyynes have already been published.<sup>4–9</sup> Two heartening features have emerged from these and other studies. Electron correlation plays only a minor role in the dipole polarizabilities and hyperpolarizabilities of acetylene<sup>7,8</sup> and butadiyne,<sup>7,9</sup> and presumably the higher members of the  $C_{2n}H_2$  series as well. Moreover as the chain length increases, intramolecular polarization becomes relatively more important and the basis set requirements for meaningful calculations of the longitudinal, but not transverse, polarizabilities decrease.<sup>6,9–12</sup>

These two observations entice one to believe that coupled Hartree–Fock (CHF) calculations in a relatively small basis set may be adequate to study the chain length dependence of the longitudinal polarizabilities of the linear polyynes. The purpose of this paper is to report such a study of the linear polyynes from  $C_4H_2$  to  $C_{44}H_2$ . We begin with a very brief review of polarizabilities and the finite-field method in Sec. II. The selection of a relatively small basis set that is adequate for the longitudinal polarizabilities of the  $C_{2n}H_2$  polyynes is made and justified in Sec. III. The results and a discussion of the growth of the electric properties with chain length are given in Sec. IV. Hartree atomic units are used throughout.

## II. METHOD

The energy  $E_p$  of a neutral centrosymmetric molecule in a weak, homogeneous electric field may be written<sup>13</sup> as

$$E_{p} = E_{0} - \frac{1}{2}\alpha_{\alpha\beta}F_{\alpha}F_{\beta} - \frac{1}{24}\gamma_{\alpha\beta\gamma\delta}F_{\alpha}F_{\beta}F_{\gamma}F_{\delta} + \cdots$$
 (1)

in which  $F_{\alpha}$  is the field strength, and  $E_0$ ,  $\alpha$ , and  $\gamma$ , respectively are the energy, dipole polarizability, and second dipole hyperpolarizability tensors of the free molecule. The Greek subscripts denote Cartesian components (x, y or z) and a repeated subscript implies summation over the latter. We choose the z axis to coincide with the molecular axis and so the origin-independent  $\alpha$  and  $\gamma$  tensors have two and three independent components, respectively. For simplicity of notation, the longitudinal components  $\alpha_{zz}$  and  $\gamma_{zzzz}$  will be referred to as  $\alpha_L$  and  $\gamma_L$ , respectively. Similarly,  $\alpha_T$  and  $\gamma_T$  will denote the transverse components.

We use the finite-field method<sup>14</sup> to calculate the response properties from energies of the molecule perturbed by various fields. Since the energies are obtained from self-consistent-field (SCF) solutions of the Roothaan-Hartree-Fock equations,<sup>15</sup> using the GAUSSIAN 86 program,<sup>16</sup> this method is equivalent to the coupled Hartree-Fock procedure.<sup>17</sup> The accuracy of the finite-field calculations was controlled by choosing fields carefully and keeping enough terms in Eq. (1) to ensure that the calculated properties agreed with those obtained by using one less term in Eq. (1) and with values obtained from induced dipole moments.

The experimental  $r_0$  geometry of butadiyne, <sup>18</sup> and calculated geometries for hexatriyne <sup>19</sup> and octatetrayne <sup>6</sup> have been used as in our previous work. <sup>9</sup> For the higher members of the  $C_{2n}H_2$  sequence, we use an idealized geometry obtained from the calculated geometry of octatetrayne. <sup>6</sup> This idealized geometry is defined by R(C-H) = 107 pm, R(C-C) = 140 pm, and R(C = C) = 118 pm.

## **III. SELECTION OF BASIS SETS**

The requirements for small basis sets that can provide adequate longitudinal response properties of chains have been studied previously.  $^{6,11,12}$  In order to validate our results, we decided to conduct an extensive basis set study on linear butadiyne  $C_4H_2$  for which accurate polarizabilities at the CHF level were calculated by Maroulis and Thakkar.  $^9$ 

TABLE I. Basis sets.

STO-3G from Refs. 15 and 16 3-21G from Refs. 15 and 16 4-31G from Refs. 15 and 16 6-31G from Refs. 15 and 16 SV = (9s5p/4s) contracted to [3s2p/2s], Ref. 20. H scale DZ = (9s5p/4s) contracted to [4s2p/2s], Ref. 21. H scale factor = 1  $SV + H_S = SV + H_{:S}(0.0483)$ SV + Hp = SV + H:p(0.208) $SV + C_S = SV + C:s(0.05)$ SV + Cp = SV + C:p(0.0365) $SV + C_pHs = SV + C:p(0.0365) + H:s(0.0483)$ SVp = SV + C:p(0.0365) + H:p(0.208)SVpd = SVp + C:d(0.13)B1 = [4s3p2d/3s2p1d] from Ref. 9 B3 = [5s3p2d/3s2p1d] from Ref. 9 B2 = [4s3p2d1f/3s2p1d] from Ref. 9

Table I lists 16 different basis sets of Gaussian-type functions (GTF), and Table II lists the longitudinal and transverse polarizabilities and hyperpolarizabilities obtained with these basis sets.

First consider the longitudinal properties. The minimal STO-3G basis <sup>15</sup> recovers only 61% and 13%, respectively of the accurate CHF  $\alpha_L$  and  $\gamma_L$  computed <sup>9</sup> in the [4s3p2d1f/3s2p1d] basis B2. The split-valence 3-21G basis <sup>15</sup> is already sufficient to obtain 88% of  $\alpha_L$  but only 27% of  $\gamma_L$ . The 4-31G basis set <sup>15</sup> does not provide any significant improvement over 3-21G. The 6-31G basis set <sup>15</sup> is a bit better and gets 91% and 31%, respectively, of  $\alpha_L$  and  $\gamma_L$ . The Dunning-Hay<sup>20</sup> split-valence (SV) basis set, which does not have the sp-shell constraint of the Pople split-valence sets, <sup>15</sup> does noticeably better obtaining 94% and 47% of  $\alpha_L$  and  $\gamma_L$ , respectively. Since the response properties are essentially valence properties, the Dunning

TABLE II. Tests of various basis sets on response properties of butadiyne C<sub>4</sub>H<sub>2</sub>. Conversion factors to SI units are  $e^2a_0^2E_H^{-1}=1.648\ 778\times 10^{-41}$  C<sup>2</sup> m<sup>2</sup> J<sup>-1</sup> and  $e^4a_0^4E_H^{-3}=6.235\ 378\times 10^{-65}$  C<sup>4</sup> m<sup>4</sup> J<sup>-3</sup> for  $\alpha$  and  $\gamma$ , respectively.

	$lpha_L$	$10^{-4}\gamma_L$	$\alpha_T$	$10^{-2}\gamma_T$
STO-3G	50.674	0.256	3.469	-0.1
3-21G	73.003	0.534	9.110	0.1
4-31G	73.960	0.584	10.429	0.3
6-31G	75.016	0.620	11.336	0.5
sv	77.942	0.944	15.249	5.5
DZ	77.947	0.945	15.308	5.5
SV+Hs	78.126	0.961	16.171	6.9
SV + Hp	78.516	1.061	16.455	6.2
SV+Cs	78.241	0.980	17.167	8.9
SV + Cp	79.806	1.888	16.589	29.3
SV + CpHs	79.961	1.905	17.613	41.4
$SV_p$	80.456	2.049	17.870	30.9
$SV_{pd}$	80.879	2.227	28.676	40.3
В1 <sup>а</sup>	82.624	2.083	30,253	57.3
B3ª	82.570	2.058	30,284	55.0
B2ª	82,646	2.014	30.451	74.0

<sup>\*</sup>Reference 9.

double-zeta<sup>21</sup> basis leads to virtually the same results as the SV set

Previous studies<sup>6,11,12</sup> on unsaturated hydrocarbons have noted that diffuse s- and p-GTF on the carbon atoms are required to obtain response properties more accurate than those predicted by split-valence basis sets. However, in many of these studies the contributions of the diffuse sand p-GTF on the carbons have not been separated, and diffuse s- and p-GTF on the hydrogens have not been considered. To clarify these issues, the SV basis was enlarged in four different ways by the addition of even-tempered diffuse s- or p-GTF shells on the carbon or hydrogen atoms. The  $\gamma_L$  values in Table II confirm that the most important of these diffuse functions is a p-GTF shell on the carbons just as it is for the  $C_{2n}H_{2n+2}$  polyenes;<sup>11</sup> the SV +Cp basis set recovers 97% and 94% of the accurate  $\alpha_L$ and  $\gamma_L$ , respectively, and is the smallest basis set that leads to a reasonable  $\gamma_L$ . However, the next most important diffuse function is a p-GTF shell on the hydrogens; this has not been noted in earlier studies<sup>6,11,12</sup> which did not explore diffuse p-GTF on the hydrogens. Addition of both these p-GTF shells leads to our favoured SVp basis set that has a [3s3p/2s1p] composition. The addition of a d-GTF shell on the carbons, with an exponent taken from Ref. 9, leads to the SVpd set; this set may be unbalanced since it predicts a  $\gamma_L$  which seems a bit too large relative to the most accurate CHF value.9

Next consider the transverse properties. Table II confirms clearly that small basis sets which do reasonably well for the longitudinal properties are simply not flexible enough to handle the transverse properties. The SVp basis recovers only 59% and 42% of  $\alpha_T$  and  $\gamma_T$  although it is quite good for the longitudinal properties. In fact, it is well known that f-GTF are needed to obtain accurate  $\gamma_T$  values for linear molecules. 5,8,9 Thus, a comparison of the B2 and B3 sets shows that  $\sim$ 23% of  $\gamma_T$  for butadiyne comes from the addition of an f-shell on carbon. We do not calculate the transverse properties for the higher polyynes because we are unable to use large enough basis sets.

## IV. RESULTS AND DISCUSSION

We calculated  $\alpha_L$  and  $\gamma_L$  for the higher polyynes with six basis sets, the SVp and SVpd sets recommended above and the smaller STO-3G, 3-21G, 6-31G, and SV sets. This allows us to check the basis set dependence of the response properties of the linear  $C_{2n}H_2$  polyynes as a function of chain length. In a similar study Hurst, Dupuis, and Clementi<sup>11</sup> found that basis set requirements diminish with increasing chain length in the nonlinear  $C_{2n}H_{2n+2}$  polyenes. We demonstrate that this is the case in the  $C_{2n}H_2$  chains as well.

The minimal STO-3G set contains "only" 10n+2 basis functions for  $C_{2n}H_2$  in contrast with the split-valence 3-21G, 6-31G, and SV sets that each contain 18n+4 basis functions, and the SVp and SVpd basis sets that contain 24n+10 and 34n+10 basis functions, respectively. Thus, we were able to extend the STO-3G calculations to  $C_{44}H_2$ , the 3-21G calculations to  $C_{26}H_2$ , the 6-31G calculations to  $C_{22}H_2$ , the SV and SVp calculations to  $C_{18}H_2$ , and the

TABLE III. Longitudinal polarizabilities  $\alpha_L$  for the  $C_{2n}H_2$  polyynes calculated in various ways. See caption to Table II for conversion factors.

n	INDO(MN)	INDO(OK)	STO-3G	3-21G	6-31G	sv	$SV_p$	SVpd
2	66.635	67.466	50.674	73.003	75.016	77.942	80.456	80.879
3	122.78	123.46	91.558	130.44	132.76	135.84	138.71	139.94
4	199.05	200.03	145.41	206.53	209.00	212.15	215.68	218.23
5	284.40	286.65	205.60	292.56	294.83	297.51	301.71	305.74
6	375.74	380.29	269.84	385.61	387.35	389.05	393.78	
7	471.50	479.22	336.95	483.87	484.84	485.20	490.46	
8	570.30	581.93	405.96	585.80	585.83	584.00	590.37	
9	671.24	687.36	476.25	690.31	689.29	686.23	691.09	
10	<i>7</i> 73.71	794.77	547.40	796.70	794.52			
11	877.30	903.62	619.16	904.44	901.04			
12	981.74	1013.5	691.34	1013.2				
13	1086.8	1124.3	763.84	1122.7				
14	1192.3	1235.6	836.56					
15	1298.3	1347.5	909.46					
16	1404.5	1459.7	982.49					
17	1511.0	1572.2	1055.6					
18	1617.6	1684.9	1128.8					
19	1724.4	1797.9	1202.1					
20	1831.4	1910.9	1275.5					
21	1938.5	2024.2	1348.8					
22	2045.6	2137.5	1422.2					
23	i	2250.9	e e		*			
24		2364.4						

SVpd calculations to  $C_{10}H_2$ . Convergence difficulties prevented us from carrying the 6-31G and SV computations out as far as the 3-21G calculations. Some of these computations were not small. The largest STO-3G calculation involved 222 basis functions constructed from 666 primitive GTF for  $C_{44}H_2$ , and the largest SVp calculation involved 226 basis functions constructed from 518 primitive GTF for  $C_{18}H_2$ . For the sake of comparison, we include finite-field SCF calculations based on the semiempirical intermediate neglect of differential overlap (INDO) method, <sup>22</sup> as implemented by Ridley and Zerner, <sup>23</sup> using both the Mataga–Nishimoto<sup>24</sup> (MN) and Ohno–Klopman <sup>25,26</sup> (OK) parametrizations.

#### A. Ab initio results

Table III lists our calculated  $\alpha_L$  values. Our 3-21G results for C<sub>6</sub>H<sub>2</sub> and C<sub>8</sub>H<sub>2</sub> are essentially the same as those reported earlier by Bodart et al.4 and Chopra et al.,6 with very minor differences arising from the slightly different geometries used. The SVpd results for C<sub>6</sub>H<sub>2</sub> and C<sub>8</sub>H<sub>2</sub> are within 2% of our previous values obtained in larger basis sets. The SVp results are similar and hence we use them as the reference for the higher polyynes. The STO-3G values are too low by amounts decreasing from 37% to 31% as n increases from 2 to 9. By contrast, the 3-21G split-valence set is quite good; its prediction of  $\alpha_L$  is too low by only 9% for C<sub>4</sub>H<sub>2</sub> and this discrepancy decreases to 1% for C<sub>14</sub>H<sub>2</sub>. In fact, for n > 5, there are only small differences of < 2%among the  $\alpha_L$  values obtained from the three split-valence sets and the SVp and SVpd sets. Noting that electron correlation<sup>7-9</sup> lowers  $\alpha_L$  of  $C_2H_2$  and  $C_4H_2$  by only 1% and 4% of the respective SCF values, it is not far fetched to conclude that, for n > 5, even the 3-21G SCF results are adequate for the purposes of a study such as this one.

Table IV lists our calculated  $\gamma_L$  values. Our 3-21G results for C<sub>6</sub>H<sub>2</sub> and C<sub>8</sub>H<sub>2</sub> are in complete agreement with those reported by Chopra et al.6 The SVp and SVpd results for C<sub>6</sub>H<sub>2</sub> and C<sub>8</sub>H<sub>2</sub> are actually larger than our previous values<sup>9</sup> of  $10^{-5}\gamma_L = 0.634$  and 1.52 a.u., respectively obtained with larger basis sets. In the case of C<sub>6</sub>H<sub>2</sub> we attribute this to imbalance in the SVp and SVpd bases. However, our previous  $\gamma_L$  value<sup>9</sup> for  $C_8H_2$  is probably too low because of a decision, unwise in retrospect, to jettison the diffuse p-GTF on the carbons. The SVp values can be used as a reference for the higher polyynes because they are as good as the SVpd ones. The STO-3G values are far too low, by amounts decreasing from 87% in C<sub>4</sub>H<sub>2</sub> to 56% in  $C_{18}H_2$ . The 3-21G value is too low by 74% in  $C_4H_2$  but this discrepancy decreases rapidly to a tolerable 24% for  $C_{10}H_2$  and then to 6% for  $C_{18}H_2$ . The 6-31G and SV results are similar to the 3-21G values except for smaller nin which case they are noticeably better. Electron correlation<sup>8,9</sup> lowers  $\gamma_L$  of  $C_2H_2$  and  $C_4H_2$  by only 13% and 17% of the respective SCF values. Hence, our longitudinal hyperpolarizabilities, except the STO-3G ones, should be useful at least in a qualitative sense.

#### B. Semiempirical INDO results

Since semiempirical calculations entail considerably less computational effort than *ab initio* calculations, it is important to examine how well the INDO results can mimic the *ab initio* calculations in the 3-21G and larger basis sets.

First, consider the longitudinal polarizabilities  $\alpha_L$ . Both sets of INDO  $\alpha_L$  are consistently lower than the SVp results, although this discrepancy decreases as the chain length increases. Thus, the INDO(MN)  $\alpha_L$  value is 17%

TABLE IV. Longitudinal hyperpolarizabilities  $10^{-5}\gamma_I$  of the  $C_{2n}H_2$  polyynes computed in various ways. See caption to Table II for conversion factors.

n	INDO(MN)	INDO(OK)	STO-3G	3-21G	6-31 <b>G</b>	SV	$SV_p$	SVpd
2	0.200	0.083	0.0256	0.0534	0.0620	0.0944	0.205	0.223
3	1.10	0.480	0.171	0.336	0.367	0.464	0.675	0.677
4	3.40	1.55	0.604	1.16	1.23	1.43	1.76	1.77
5	7.76	3.57	1.46	2.84	2.95	3.27	3.72	3.73
6	14.4	6.68	2.78	5.54	5.66	6.09	6.64	
7	22.9	10.9	4.56	9.29	9.42	9.92	10.6	
8	33.6	16.1	6.73	14.1	14.1	14.7	15.3	
9	45.6	22.2	9.23	19.7	19.7	20.2	21.0	
10	59.3	29.1	12.0	26.1	26.0			
11	73.7	36.5	14.9	33.2	32.8			
12	88.6 ,	44.5	18.0	40.7				
13	105	52.8	21.2	48.6				
14	121	61.5	24.4					
15	138	70.4	27.8					
16	156	79.5	31.2					
17	173	88.9	34.6			•		
18	190	98.3	38.0	* *				
19	208	108	41.5			*		
20	225	118	45.0	•				
21	243	127	48.5					
22	261	137	52.0					
23		147						
24		157						

lower than the SVp value at  $C_4H_2$ , but this discrepancy is reduced to 3% for  $C_{18}H_2$ . Similarly, the INDO(OK)  $\alpha_L$  is 16% lower than the SVp value for  $C_4H_2$  but this discrepancy decreases to 1% for  $C_{18}H_2$ . A comparison for somewhat longer chains can be made with the 3-21G results. Interestingly, the INDO(OK)  $\alpha_L$  are lower than the 3-21G results for smaller chains but exceed them by increasing amounts in  $C_{24}H_2$  and  $C_{26}H_2$ . Previous studies  $^{12}(a)$ ,  $^{27}$  have found INDO  $\alpha_L$  to be consistently lower than ab initio results obtained in split-valence or larger basis sets; this may be because of the INDO parametrization used or, more probably, because the chains examined were too short for the crossover to be seen.

Next, consider the longitudinal hyperpolarizabilities  $\gamma_L$ . The INDO  $\gamma_L$  results are extremely sensitive to the parametrization scheme as noticed earlier by Papadopoulos, Waite, and Nicolaides<sup>28</sup> for  $C_2H_4$ . Using the SVp values as the reference, we see that the INDO(MN)  $\gamma_L$  are too large, except by chance in  $C_4H_2$ , by amounts ranging from 63% in  $C_6H_2$  to 117% in  $C_{18}H_2$ . On the other hand, the INDO(OK)  $\gamma_L$  are too low for the smaller chains but are larger than the SVp values for n>5; however, they remain within 6% of the SVp values for n>4. Moreover, the INDO(OK) values exceed the 3-21G results by amounts decreasing steadily from 55% in  $C_4H_2$  to only 9% in  $C_{26}H_2$ . These comparisons suggest that the INDO(OK) results can mimic the 3-21G and SVp results out to much longer chain lengths.

#### C. Extrapolation to infinite chain length

Finally, it is interesting to consider the growth of the response properties with chain length, and their extrapolation to infinite chain length.<sup>29</sup> Such a study based on static polarizabilities and hyperpolarizabilities may not tell us

anything important about experimental susceptibilities because the magnitude of the dispersion depends not only on the optical frequency used but also the specific optical process. Nevertheless, studying the static limit is a start at understanding how these properties vary with chain length. Virtually all previous *ab initio* studies on extrapolation to infinite chain length have been carried out at the static limit.<sup>29</sup>

Previous calculations<sup>6,9</sup> on  $C_6H_2$  and  $C_8H_2$  had been used to determine that  $\alpha_L$  and  $\gamma_L$  for  $C_{2n}H_2$  were growing as  $n^{1.5}$  and  $n^{3.0}$ , respectively. Similarly, the current SVpd results for n=4,5 suggest that  $\alpha_L$  and  $\gamma_L$ , respectively are growing as  $n^{1.51}$  and  $n^{3.3}$ . It is not unreasonable to suppose that the growth slows down as the chain length increases. The data in Tables III and IV show precisely that sort of behavior. For example, for n=8,9 the growth of the SVp  $\alpha_L$  and  $\gamma_L$ , respectively, has slowed to  $n^{1.34}$  and  $n^{2.7}$ , and for n=12,13 the 3-21G  $\alpha_L$  and  $\gamma_L$  are growing as "only"  $n^{1.28}$  and  $n^{2.2}$ , respectively.

For large enough n, one might expect both  $\alpha_L$  and  $\gamma_L$  to grow linearly with n like the scalar quadrupole moment listed in Table V. Unfortunately, this leveling off of the  $\alpha_L/n$  and  $\gamma_L/n$  curves is apparent only for the unreliable STO-3G and INDO(MN) data and the reasonably accurate INDO(OK) data which extend out to  $C_{44}H_2$ . Since the INDO(OK) data give us the best chance of extrapolation to the infinite chain limit, we extended the calculations to  $C_{46}H_2$  and  $C_{48}H_2$  (see Tables III and IV). The limiting values of the properties per acetylenic linkage can be estimated from least squares fits of these quantities,  $^{27,29,30}$  or their logarithms,  $^{11,26}$  to power series in 1/n. Thus,

$$A(n) = a_0 + a_1/n + a_2/n^2 + \dots + a_k/n^k$$
 (2)

TABLE V. Scalar quadrupole moments of the  $C_{2n}H_2$  polyynes computed with various bases. The conversion factor to SI units is  $ea_0^2 = 4.486\,554 \times 10^{-40}$  C m<sup>2</sup>.

n	STO-3G	3-21G	6-31G	SV	SVp	SVpd
2	6.5	10.3	9.6	9.7	9.7	9.9
3	10.6	16.2	15.0	14.9	14.9	15.1
4	15.3	23.1	21.1	20.8	20.8	21.0
5	20.4	30.5	27.7	27.2	27.1	27.4
6	25.7	38.3	34.6	33.9	33.7	
7	31.2	46.4	41.8	40.8	40.6	
8	36.8	54.8	49.2	47.9	47.7	
9	42.6	63.4	56.8	55.2	54.8	
10	48.4	72.2	64.5			
11	54.3	81.1	72.4			
12	60.3	90.1				
13	66.3	99.2				
14	72.3					
15	78.3	-				
16	84.4					
17	90.4					
18	96.5					
19	102.6					
20	108.7					
21	114.8					
22	120.9					

in which A(n) can be  $\alpha_L/n$  or  $\gamma_L/n$  or their logarithms.

We fit  $\alpha_L/n$  and  $\gamma_L/n$  using the INDO(OK), 3-21G, and SVp data. The INDO(MN) and STO-3G data are too poor to be worth fitting, the 6-31G and SV results are similar to the 3-21G ones and do not extend as far, and the SVpd results cannot be extrapolated because they extend only to n=5. The degree of each fit (k) was chosen to be the smallest that lowers the standard deviation to 85% of the estimated truncation error in our data (0.01 a.u. for  $\alpha_L$  and  $10^{-5}\gamma_L$ ), or to minimize the standard deviation in

cases where the former objective could not be met. A fit was also made with a polynomial containing one extra term. Both fits were repeated after discarding one or two short chains. The results are listed in Table VI. The scatter in the values can be used to estimate the uncertainty in the limits. Thus, it seems that in the infinite chain limit, the INDO(OK), 3-21G, and SVp values of  $\alpha_L/n$  saturate to  $114.5\pm0.6$ ,  $113.5\pm1.5$ , and  $109\pm3$  a.u., respectively. Similarly, in the infinite chain limit, the INDO(OK), 3-21G, and SVp values of  $\gamma_L/n$  seem to saturate to  $(1.09\pm0.01)$ 

TABLE VI. Limiting values of  $\alpha_L/n$  and  $10^{-5}\gamma_L/n$ . The range is the smallest and largest n included in the fit, k is the degree of the fitting polynomial, cf. Eq. (2),  $\nu$  is the number of degrees of freedom in the fit, and  $\sigma$  is the standard deviation. See caption to Table II for conversion factors.

Range	$\alpha_L/n$				$10^{-5}\gamma_L/n$				
	k	ν	σ	Limit	k	ν	σ	Limit	
INDO(OK)						•			
2,24	5	17	0.023	114.7	4	18	0.014	10.9	
2,24	6	16	0.002	113.9	5	17	0.013	11.0	
3,24	4	17	0.035	115.2	4	17	0.013	11.0	
3,24	5	16	0.003	114.1	5	16	0.007	10.7	
4,24	4	16	0.011	114.6	3	17	0.009	10.8	
4,24	5	15	0.002	114.0	4	16	0.009	10.9	
3-21G									
2,13	5	6	0.018	114.4	4	7	0.019	10.4	
2,13	6	5	0.002	112.2	5	6	0.004	11.5	
3,13	3	7	0.015	115.1	3	7	0.040	9.5	
3,13	4	6	0.016	115.0	4	6	0.004	11.1	
4,13	3	6	0.016	115.1	3	6	0.012	10.3	
4,13	4	5	0.002	113.8	4	5	0.004	11.2	
$SV_p$									
2,9	4	3	0.051	111.5	4	3	0.013	9.1	
2,9	5	2	0.046	105.8	5	2.	0.008	11.0	
3,9	3	3	0.044	110.0	3	3	0.021	8.0	
3,9	4	2	0.049	108.1	4	2	0.009	10.3	

 $\times 10^6$ ,  $(1.05\pm0.10)\times 10^6$ , and  $(0.95\pm0.15)\times 10^6$  a.u., respectively. Clearly, the SVp extrapolations suffer from our inability to compute SVp polarizabilities beyond n=9. Considering all the above estimates together, we suggest that the Hartree-Fock limit values of  $\alpha_L/n$  and  $\gamma_L/n$  saturate to  $110\pm10$  and  $(1.0\pm0.3)\times 10^6$  a.u., respectively.

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