

Some Studies in Molecular Orbital Theory III. Substitution in Aromatic and Heteroaromatic Systems

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Some Studies in Molecular Orbital Theory

III. Substitution in Aromatic and Heteroaromatic Systems*

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It is shown that LCAO molecular orbital theory, when combined with first-order perturbation theory, provides a very simple procedure for estimating how far electronic effects will modify the transition state energies for substitution reactions in alternant molecules. This procedure is shown to lead to the same qualitative predictions as resonance theory, but has the advantage that it gives at the same time quantitative information as to the relative effects of substitution at different points in a complex system. The theoretical results agree well with experiment as far as can be ascertained at present.

The approximations involved, and their relation to those of resonance theory, are discussed in detail.

1. INTRODUCTION

THE theory of resonance between classical valence-bond structures has proved triumphantly successful in explaining the stabilities and reactivities of organic molecules in terms of their electronic structures. However, up to the time of writing, the qualitative successes of the theory have not been extensively followed up with quantitative calculations.‡ This is primarily because in discussing molecules containing atoms other than carbon and hydrogen it is necessary to take into account ionic structures, and the contributions of such structures to the wave function and energy of a molecule are not easy to estimate by the valence-bond method. Almost all the quantitative calculations made in the past on polyatomic heteromolecules have been carried out by the method of molecular orbitals; and although the results generally agree with those of resonance theory, few attempts have been made to unite the ideas which lie behind the two approaches, or to discover how far agreement may be expected to extend beyond the special cases worked out.

In this paper it will be shown that the most significant general predictions of resonance theory, as to the reactivity of conjugated systems, do indeed follow from a combination of molecular orbital theory and perturbation theory; and that with the latter combination of theoretical tools it is possible to calculate quite simply the relative stabilities of isomeric systems containing heteroatoms. Such systems include both individual molecules and reaction complexes, so one can discuss both energies and rates of reaction in this way.

2. SUBSTITUTION IN AROMATIC SYSTEMS

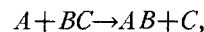
The attack of electrophilic (cationoid) and nucleophilic (anionoid) reagents on conjugated systems has

been discussed quantitatively from two points of view. On the one hand, calculations have been made¹ on the unsaturation energy of the transition complexes involved in substitution reactions; and it has been found that for a given reagent R and a series of different positions of attack on a given substrate S , the rate of reaction is higher the lower the energy of the π -electrons in the transition state RS . Such calculations justify the view that differences in substitution rate in conjugated systems are indeed determined principally by the changes in energy of the π -electrons; and this assumption will be made throughout the present paper. On the other hand,^{2,3} it has been demonstrated that if two positions in a conjugated molecule differ considerably in " π -electron density" q , then the position with the higher value of q is more susceptible to attack by electrophilic reagents, whereas nucleophilic reagents substitute more easily at the position of lower electron density.

Calculations on the transition state are of course more reliable than calculations of electron density for predicting the relative rates of analogous reactions, and in the following pages the former approach will be adopted as far as possible; but for certain types of substitution electron density calculations are much easier to make, and provide a useful supplement to the more rigorous approach.

3. SCOPE OF THIS PAPER

For the most part we shall be concerned with heterolytic substitution reactions, that is, bimolecular reactions of the type



where A is an electrophilic or nucleophilic reagent and BC is a conjugated molecule, not necessarily a hydrocarbon. Reactions of the above general type in which A is electrophilic have been called S_E2 reactions by

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‡ The procedure of counting resonance structures in order to estimate resonance energy has no basis in quantum theory, and should be relegated to the limbo of what Guggenheim has called "sesqui-empirical methods."

¹ G. W. Wheland, J. Am. Chem. Soc. **64**, 900 (1942).

² G. W. Wheland and L. Pauling, J. Am. Chem. Soc. **57**, 2091 (1935).

³ H. C. Longuet-Higgins and C. A. Coulson, Trans. Faraday Soc. **43**, 87 (1947).

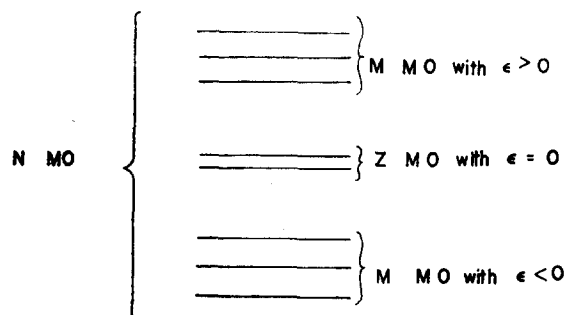
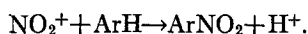
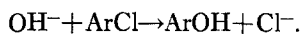


FIG. 1.

Hughes and Ingold⁴ and their school:[¶] a typical S_E2 reaction is the nitration of an aromatic compound, represented by the equation



If the substrate ArH contains $2m$ π -electrons distributed over n atomic orbitals, then the transition complex NO_2ArH^+ will have $2m-2$ π -electrons distributed over $n-1$ atomic orbitals. (The π -electrons of the nitro group are not counted, since they are not conjugated with the aromatic π -electrons in the transition state.) This statement applies to all S_E2 reactions in which A attacks an unsaturated carbon atom of BC . On the other hand, for S_N2 reactions, in which A is nucleophilic, the electronic mechanism is essentially different. A typical S_N2 reaction is the hydrolysis of aryl chlorides to phenols, according to the equation



In this type of reaction the transition complex HOArCl^- contains the same number, $2m$, of π -electrons as the substrate, these electrons being again distributed over $n-1$ atomic orbitals.

The first problem that will be considered is:

What can be said about the difference in activation energy between two S_E2 reactions or two S_N2 reactions having precisely similar electronic mechanisms? To make the question clearer, it is necessary to introduce the idea of *isoaromatic* systems. If two conjugated systems have the same number of π -electrons, moving in geometrically similar sets of atomic p orbitals, the two systems are said to be isoaromatic. Thus naphthalene, quinoline, and isoquinoline are all isoaromatic, and nitrobenzene is isoaromatic with the benzoate ion. Two

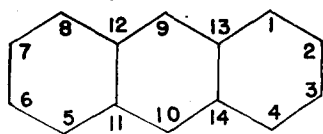


FIG. 2.

⁴ E. D. Hughes and C. K. Ingold, J. Chem. Soc. 244 (1935).

[¶] The symbols S_N2 and S_E2 have been most used in discussing substitution at a *saturated* carbon atom; but there seems to be no objection to removing this restriction.

reactions are said to be isoaromatic if and only if both the substrates *and* the transition complexes are isoaromatic. Thus the nitration of benzene is isoaromatic with the nitration of toluene in any position, and the 1-chlorination of naphthalene is isoaromatic with the 5-chlorination of isoquinoline; but the 2-nitration of naphthalene would not be isoaromatic with the 1-nitration of the same molecule because the transition complexes have different geometry.

The above question may now be put in the form: What will be the difference in activation energy between two isoaromatic S_E2 or S_N2 reactions? This question will be discussed in Sections 5 and 6, and it will be shown that the relative stabilities of isoaromatic systems can be understood very simply in terms of LCAO molecular orbital theory.

Another problem of importance is to predict theoretically the relative rates of two reactions which are not strictly isoaromatic. This problem is discussed in Section 8, and, as would be expected, the theory is not quite so straightforward as for isoaromatic reactions. Nevertheless, if one assumes that the activation energy of a reaction is correlated with the change in unsaturation energy in the early stages of reaction, it is possible to predict differences in reactivity on the basis of electron density values; and in many important cases these electron density values may be calculated to a first approximation without having to solve any secular equations.

It is obviously desirable to understand as far as possible how the results obtained by these quantitative methods are related to those of qualitative resonance theory; so an attempt has been made to interpret the latter theory in terms of the former, where this is possible, and to discover to what extent the two theories rely on the same physical principles.

4. ALTERNANT HETEROMOLECULES

In what follows, we shall be mainly concerned with heteromolecules satisfying the conditions: (a) The number of π -electrons is even, all these electrons being paired. (b) No odd-membered rings of conjugated atoms are present. (c) In the isoaromatic hydrocarbon all the MO of negative energy are filled, and all the MO of positive energy are empty.

Of these conditions the first rules out free radicals, and the second defines the class of alternant molecules,

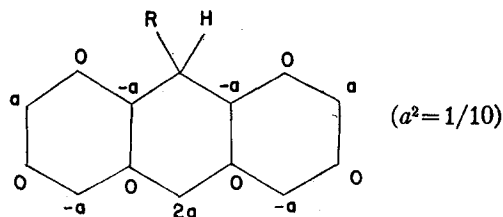


FIG. 3.

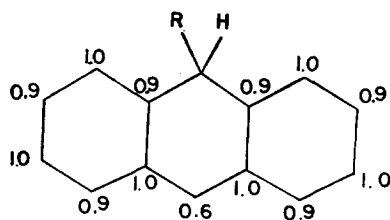


FIG. 4.

in the sense of Papers I* and II.† In order to see whether condition (c) holds for a given heteromolecule, it is necessary to find the arrangement of the MO energies in the isoaromatic hydrocarbon. Let N be the total number of MO (equal to the number of conjugated atomic orbitals), and let Z be the number of zero-energy MO. By the theorem quoted in Paper I, the numbers of MO of positive and negative energy must be equal, to M say; so $N = 2M + Z$, and the arrangement of MO energies is as shown in Fig. 1. For the parent hydrocarbon not to contain unpaired electrons, the levels Z must either all be filled or all be empty. So if P is the number of π -electrons, the following three cases may be distinguished:

(1) $Z = 0$, $P = 2M$. Hence, according to the Coulson-Rushbrooke theorem,⁵ $q_r = 1$ at every position.

(2) $Z > 0$, $P = 2M$. The electron densities are then given by,

$$q_r = 2 \sum_m (c_r^m)^2, \quad (1)$$

where m refers to a MO of negative energy. But, as shown in Paper I,

$$1 = 2 \sum_m (c_r^m)^2 + \sum_z (c_r^z)^2, \quad (2)$$

where z refers to a MO of zero energy. Hence,

$$q_r = 1 - \sum_z (c_r^z)^2. \quad (3)$$

(3) $Z > 0$, $P = 2M + 2Z$. In this case

$$q_r = 2 \sum_m (c_r^m)^2 + 2 \sum_z (c_r^z)^2. \quad (4)$$

Combining (2) with (4) we get

$$q_r = 1 + \sum_z (c_r^z)^2. \quad (5)$$

Now, as shown in Paper I, the number Z of zero-energy MO in an alternant hydrocarbon and their AO coefficients c_r^z , may be obtained without evaluating the roots of the secular determinant. So, if conditions (a), (b), and (c) are satisfied, the electron densities q_r may be found very easily.

* H. C. Longuet-Higgins, J. Chem. Phys. 18, 265 (1950), hereafter referred to as Paper I.

† H. C. Longuet-Higgins, J. Chem. Phys. 18, 275 (1950), hereafter referred to as Paper II.

⁵ C. A. Coulson and S. Rushbrooke, Proc. Camb. Phil. Soc. 36, 193 (1940).

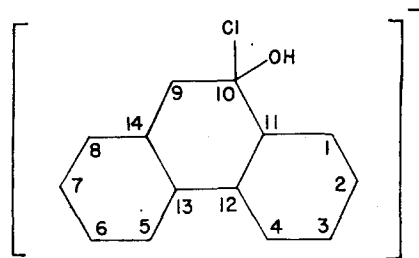


FIG. 5.

5. ISOAROMATIC REACTIONS

It will now be shown how first-order perturbation theory can be used to calculate approximately the difference in unsaturation energy between a heteromolecule and the isoaromatic hydrocarbon.

Denote by ϵ_0^i the energy of a π -electron in the MO ψ^i of the hydrocarbon. Then replacement of one or more carbon atoms by heteroatoms will change the value of ϵ_0^i to ϵ^i , say, where, by first-order perturbation theory,⁶

$$\epsilon^i = \epsilon_0^i + \int \psi^i H' \psi^i d\tau, \quad (6)$$

and H' denotes the difference between the effective 1-electron Hamiltonian for the hydrocarbon and the heteromolecule. Now in LCAO approximation,

$$\psi^i = \sum_r c_r^i \phi_r;$$

so the above equation becomes

$$\epsilon^i = \epsilon_0^i + \sum_r \sum_s c_r^i c_s^i \int \phi_r H' \phi_s d\tau.$$

The most important of the integrals under the double summation will be the Coulomb terms of the various heteroatoms, the Coulomb term of atom r being defined as $\int \phi_r H' \phi_r d\tau = \alpha_r$, say. Hence, to a fair approximation,

$$\epsilon^i = \epsilon_0^i + \sum_r (c_r^i)^2 \alpha_r.$$

Summing over the π -electrons we get

$$\mathcal{E} = \mathcal{E}_0 + \sum_r q_r \alpha_r, \quad (7)$$

where \mathcal{E} is the unsaturation energy of the heteromolecule, \mathcal{E}_0 that of the parent hydrocarbon, and q_r is the π -electron density at position r in the latter. Equation (7) was used with success in Paper II for interpreting the basic strengths of heterocycles. It will now be used to calculate the relative rates of isoaromatic reactions.

Equation (7) implies that alteration of the Coulomb integral of atom r in a substrate changes the unsatura-

⁶ L. Pauling and E. B. Wilson, *Introduction to Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1935).

tion energy by approximately $q_r\alpha_r$, q_r being the π -electron density at atom r in the isoaromatic hydrocarbon. For a given reaction of this substrate the perturbation α_r will therefore lower the activation energy by an amount

$$\alpha_r(q_r - q_r'), \quad (8)$$

where q_r' is the electron density at position r in the transition complex, or rather, in the hydrocarbon isoaromatic with it. Thus, if the perturbation increases the electron affinity of atom r ($\alpha_r < 0$), the reaction will be favored by a high value of q_r' , but if $\alpha_r > 0$, the reverse is true. The electron densities q_r and q_r' in Eq. (8) are calculated from the equations of Section 4.

6. EXAMPLES

(1) A typical problem in orientation theory is the following: To what extent will the rate of 9-nitration of anthracene be affected by methyl or aza-substitution at other positions in the molecule? For example, one might wish to estimate the relative rates at which 1-methyl, 2-methyl, 3-methyl, 4-methyl, and 10-methyl anthracenes nitrate in the 9-position, the numbering of the positions being as shown in Fig. 2. The effect of methyl substitution will be taken into account by supposing that it alters the Coulomb integral of the substituted carbon atom by an amount $\alpha > 0$. Now the electron densities on the carbon atoms in anthracene are all unity, by the Coulson-Rushbrooke theorem, so any differences in nitration rate between the various methyl anthracenes will be due to differences in the electron densities q_r' in the transition complex. In the transition complex, position 9 is no longer conjugated with the rest of the molecule, and two π -electrons have been removed from the aromatic system to form a σ -bond with the nitro group; so 12 π -electrons are left, distributed over 13 carbon atoms. Using the relations between resonance structures and MO established in Paper I, we deduce that in the transition state there is one MO of zero energy, and that its AO coefficients are finite only at positions 2, 4, 5, 7, 10, 12, and 13, where the plus charge is located in the principal resonance structures. It is easily verified that these coefficients c_r^z are as shown in Fig. 3. For the transition complex $Z=1$ and $N=13$, so $2M=12=P$. So Eq. (3) applies, and the electron densities q_r' in the transition complex are as shown in Fig. 4.

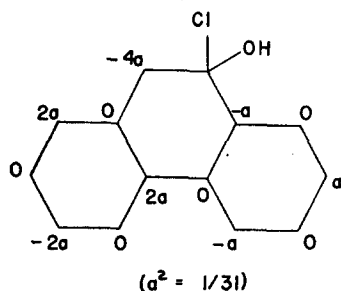


FIG. 6.

By Eq. (8), therefore, methyl substitution at the various positions lowers the activation energy by the following amounts:

1-methyl	+0.0 α
2-methyl	+0.1 α
3-methyl	+0.0 α
4-methyl	+0.1 α
10-methyl	+0.4 α .

Therefore, 10-methyl substitution lowers the activation energy for 9-nitration four times as much as 2-methyl or 4-methyl substitution, whereas 1- or 3-methyl substitution has no effect. Now in anthracene itself the 1 and 4 positions are obviously equivalent, and so are the 2- and 3-positions. From the experimental point of view, therefore, the significant predictions are that a 1-methyl group activates the 10-position but not the 9-position, a 2-methyl group activates the 9-position but not the 10-position, and a 10-methyl group activates the 9-position, the three energy shifts being in the ratio 1:1:4.

(2) A precisely analogous discussion is possible of how aza-substitution affects the anionoid activity of a given position in an aromatic system. Suppose we are interested in the relative effects of aza-substitution in different places on the hydrolysis of the chlorine atom in 10-chloro-phenanthrene. It is probably a good approximation to neglect the conjugation of the Cl atom with the aromatic system, in which case the π -electron densities q_r will be unity at all positions in the substrate. So here again, any differences in rate of hydrolysis between the various aza-derivatives will be ascribed to differences in q_r' , and hence in energy, in the corresponding transition complexes. The transition complexes probably have the general structure shown in Fig. 5 with 14 π -electrons associated with 13 carbon atoms. The rules given in Paper I demand that there shall be one zero-energy MO, with AO coefficients vanishing at all positions except 2, 4, 6, 8, 9, 11, and 13; and it is easily checked that the coefficients c_r^z are in fact as shown in Fig. 6. So $Z=1$, $M=\frac{1}{2}(N-Z)=6$, $P=14=2M+2Z$, and Eq. (5) may be applied. The electron densities q_r' in the transition complex are therefore as given in Fig. 7. Now if α is the Coulomb term for aromatic nitrogen, the activating effect of aza-substitution at position r is, by Eq. (8),

$$\alpha(1 - q_r').$$

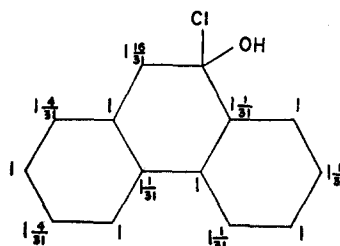


FIG. 7.

We conclude that aza-substitution in positions 1, 2, 3, 4, 5, 6, 7, 8, and 9 lowers the activation energy for the hydrolysis of the chlorine atom by amounts in the ratios 0:1:0:1:0:4:0:4:16. Whether these figures are quantitatively correct or not, they certainly agree well with the observation that 10-chloro-phenanthridine is much more easily hydrolyzed than other aza-10-chloro-phenanthrenes.

(3) The ionization of "reactive" methyl groups was discussed briefly in Paper II. This is not an aromatic substitution, of course, but nevertheless the present ideas may be applied. We may compare the energies of ionization of the methyl group in toluene and the methyl groups in the three aza-toluenes (picolines). Ionization of the methyl group gives a system isoaromatic with the benzyl anion, which has eight π -electrons and seven MO, one of which has zero energy; so the π -electron densities in the ion are given by Eq. (5). They are as shown in Fig. 8 (see Paper II). Hence aza-substitution at the meta-position does not affect the activity of the methyl group, whereas aza-substitution at the ortho or para positions activates the methyl group by an amount

$(1/7)\alpha$. A further point is that the group NR^+ , which

has a higher electron affinity than N , will also have a larger value of α , and this is why quaternation of α - and γ -picolines greatly enhances the reactivity of their methyl groups.

(4) A more complicated example is the problem of where S_E2 substitution should occur in the molecule 2,4-diamino-pyridine. The presence of three heteroatoms in this molecule probably makes it less accurate to apply Eq. (7) than in the above three examples; but all the same it is interesting to see what the theory predicts. The numbering of the atoms is as shown in Fig. 9: the attacking reagent may enter in any one of positions 3, 4, or 6. The transition complexes formed by attack of an electrophilic reagent in positions 3, 4, and 6 are all isoaromatic, differing only in the position of the nuclear nitrogen atom. (This is, of course, why the example has been chosen.) The parent hydrocarbon of the three transition complexes would have seven MO (this being the number of conjugated AO), one having zero energy, with coefficients as shown in Fig. 10(a). There are eight π -electrons, so the zero-energy MO contains two electrons, and the π -electron densities are as shown in Fig. 10(b).

The electron density in the transition complex is therefore highest on one of the NH_2 groups and on the unsubstituted atom next to the position of attack. Aza-substitution of the latter atom will therefore lend the transition complex most stability, so 2,4-diamino-pyridine should be most sensitive to electrophilic reagents in the 6-position. What resonance theory would predict in this case is not at all clear, but it might be

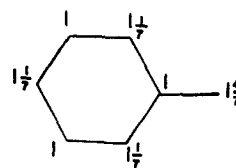


FIG. 8.

thought that the 4- and 6-positions would be deactivated by the nuclear heteroatom, and that S_E2 substitution would therefore occur at the 3-position. This is an interesting point which should be checked experimentally.

(5) The problem of orientation by substituents in the benzene ring has been solved qualitatively many times over, and semiquantitatively by Wheland and Pauling.² However, it is instructive to see how the results of the latter authors may be understood from the present viewpoint. The substituent X in $\text{C}_6\text{H}_5\text{X}$ affects the aromatic ring in either or both of two ways: (1) it may alter the Coulomb integral of the adjacent carbon atom† (inductive effect), and (2) the atom or residue X may itself possess π -electrons which can conjugate with the ring π -electrons (electromeric effect). Insofar as the inductive effect is the more important, as it probably is in toluene and the anilinium ion, the ortho, meta, and para S_E2 or S_N2 reactions of $\text{C}_6\text{H}_5\text{X}$ are isoaromatic, and may be discussed as follows:

The transition states for substitution at the ortho, meta, or para positions are all isoaromatic with a 5-carbon chain having four or six π -electrons according as S_E2 or S_N2 substitution is being considered. This 5-carbon chain has one zero-energy MO, with coefficients as shown in Fig. 11. Therefore, in S_E2 reactions the positive charge on the transition complex is located equally at the three positions ortho and para to the position being attacked, and the same applies to the negative charge in S_N2 reactions. The electron densities q_i' at the various positions in the three possible types of transition complex are therefore as follows:

	$q_1' = q_5'$	$q_2' = q_4'$	q_3'
S_E2	$\frac{2}{3}$	1	$\frac{2}{3}$
S_N2	$1\frac{1}{3}$	1	$1\frac{1}{3}$
S_R2	1	1	1.

(Here S_R2 means bimolecular substitution by free radicals. For such reactions the transition complex contains five mobile electrons, so by the Coulson-Rushbrooke theorem the electron density is unity at each of the five carbon atoms.) If we denote by α the difference

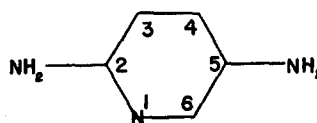
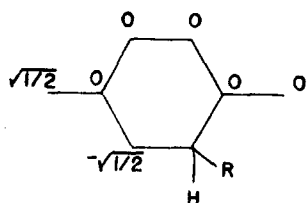
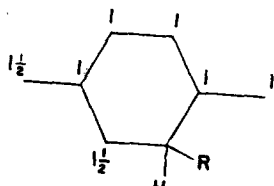


FIG. 9.

† Strictly speaking, the inductive effect also affects the other ring atoms; but this can be ignored in first approximation.

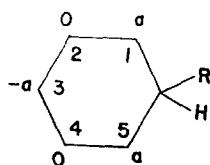


(a)



(b)

FIG. 10.



(a = 1/√3)

FIG. 11.

in Coulomb integral between the substituted carbon atom and the others, then the activating effects of the substituent X on the nine types of substitution are as follows:

	Ortho	Meta	Para
S_E2	$\frac{1}{3}\alpha$	0	$\frac{1}{3}\alpha$
S_N2	$-\frac{1}{3}\alpha$	0	$-\frac{1}{3}\alpha$
S_R2	0	0	0.

This shows that in first approximation S_R2 reactions are unaffected by inductive substituents, as are S_E2 and S_N2 reactions in the meta position. Substituents such as NR_3^+ , for which $\alpha < 0$ ($+I$ substituents) deactivate the ortho and para positions equally toward electrophilic reagents; whereas substituents such as CH_3 with $\alpha > 0$ ($-I$ substituents) have precisely the opposite effect. The above argument follows similar lines to one given by Dewar⁷ in his recent book, and perhaps makes clearer why the inductive effect discriminates between the meta positions and the others.

When the electromeric effect of X is more important than the inductive effect, the complication arises that the transition complexes corresponding to substitution in the ortho, meta, and para positions are not isoaromatic. In order to study the relative reactivities of these three positions, it is necessary either to solve the secular equations completely for the transition states, as was done by Wheland,¹ or to fall back on calculations of

electron density at the different positions in the substrate, as done by Wheland and Pauling.² If the latter procedure is accepted as reliable, the electron densities can often be estimated without solving the secular equations, by the method outlined in Section 2, and this means a considerable saving of labor. This point will be taken up in Section 8.

7. DISCUSSION

The conclusions arrived at so far may be summarized as follows: (1) For a given reaction, if the substrate has electron density q_r at position r , and the electron density is q_r' at the same position in the transition complex, then alteration of the Coulomb integral of atom r by an amount α will lower the activation energy for the reaction by an amount $\alpha(q_r - q_r')$. (2) For many types of reaction, such as those discussed in the preceding section, the electron densities q_r and q_r' depend only on the AO coefficients c_r belonging to zero-energy MO, so the relative activating effects of $\pm I$ substituents at different positions may be quantitatively calculated without solving the secular equations.

There is for many types of reaction a correspondence between the above results and the predictions of qualitative resonance theory. For instance, in example (2) of the previous section the π -electron densities are unity at all positions in 10-chloro-phenanthrene, but the electron densities q_r' in the transition complex with the OH ion exceed unity at those atoms which bear the negative charge in one or more principal resonance structures. Therefore the transition complex is stabilized by an increased positive field at one of the charged positions; and this is precisely what is implied by the statement

that resonance structures containing N^- are more stable than those containing CH^- . The fundamental connect-

ing link between the resonance and MO predictions is the fact, proved in Paper I, that in alternant hydrocarbon ions the ionic charge is distributed entirely over the atoms that bear the charge in the principal resonance structures. This result may be regarded as a kind of correspondence principle between resonance theory and LCAO MO theory, and is frequently illuminating when one wishes to interpret the former theory in terms of the latter.

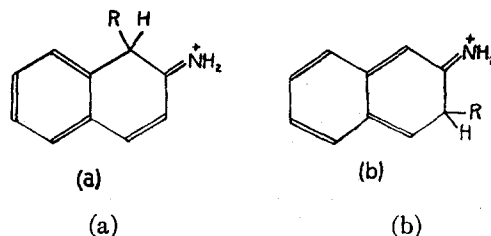


FIG. 12.

⁷ M. J. S. Dewar, *Electronic Theory of Organic Reactions* (Oxford University Press, London, 1949).

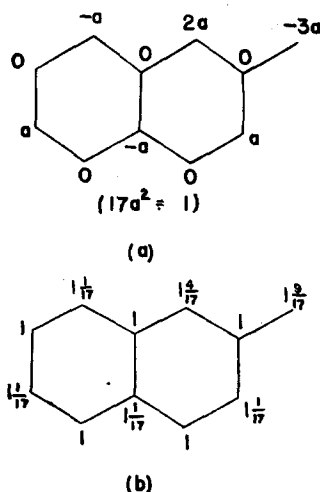


FIG. 13.

8. REACTIONS WHICH ARE NOT ISOAROMATIC

In the preceding sections we have seen how it is possible to estimate differences in activation energy for two or more reactions in which both the substrates and the transition complexes are isoaromatic. It is clearly desirable to see if an equally simple treatment can be given for reactions in which the transition complexes differ in their geometry. For example, it should be possible to understand in simple terms why 2-naphthylamine directs electrophilic reagents into position 1 rather than position 3, say. The most rigorous kind of calculation would be to solve the secular equations for the complexes (a) and (b) in Fig. 12, and thus obtain the difference in their unsaturation energies. If this is done, it does indeed turn out that (b) is less stable than (a); but this sort of calculation becomes prohibitively laborious when the aromatic system contains more than two rings. On the other hand, the electron distribution in a heteromolecule is often a good index of its behavior in S_E2 and S_N2 substitutions; so we shall now apply the methods outlined in Section 4 to the calculation of electron densities in heteromolecules.

To return to the example of 2-naphthylamine, two different extreme approximations can be made in calculating the π -electron densities in this molecule. The first is to assign the Coulomb integral of the nitrogen atom an infinite negative value; in this case two π -electrons would be localized on the NH_2 group, the remaining ten having the same distribution as in naphthalene, so that the π -electron density would be unity at every carbon atom. The other approximation is to assume that the nitrogen atom has the same Coulomb integral as aromatic carbon, so that we have in effect a carbanion with 11 atomic orbitals and 12 π -electrons. The principal resonance structures for this carbanion all have just one carbon atom bearing an unshared pair of π -electrons, so there is one zero-energy MO whose AO coefficients are finite only at these "active" atoms. According to the

definitions of Section 5, therefore, $Z=1$, $2M=10$, $P=12=2M+2Z$, so the molecule is of Type (3), and the electron densities may be calculated from Eq. (5). To find the coefficients c_r^z we first observe that in any principal resonance structure the negative charge must be located either at the external position or at a position separated from it by an even number of bonds; c_r^z therefore vanishes at positions 2, 4, 5, 7, and 9. It is then obvious that the remaining AO coefficients have the values shown in Fig. 13(a) and that the π -electron densities are as given in Fig. 13(b).

The question is: will the electron densities in 2-naphthylamine be nearer to those shown in Fig. 13(b) or to those in naphthalene? The following argument suggests that Fig. 13(b) probably gives not too inaccurate a picture of the electron distribution in the amine. In calculating the coefficients given in Fig. 13(a), it has been assumed that the Coulomb integral of the external atom is the same as that of a ring atom. This assumption leads to a considerably greater negative charge on the external atom than on any of the ring atoms, so that the electron affinity of the external atom would be reduced more than that of the others. So a negative value of α for the external atom would be required to make the field self-consistent, and it is even possible that the electron distribution given in Fig. 13(b) is a better approximation for the amine than for the isoaromatic carbanion. Be that as it may, the actual electron densities in the amine almost certainly lie between those in naphthalene and those given in Fig. 13(b), and the latter probably gives a more or less correct picture of the relative net charges at the carbon atoms. Certainly the electron densities given in Fig. 13(b) accord well with the high reactivity of the 1-position in 2-naphthylamine toward electrophilic reagents as compared with any of the other positions.

The elementary calculations just described for 2-naphthylamine may be extended to disubstituted alternant hydrocarbons, provided that the heterosubstituents are separated by even numbers of unsaturated bonds. As an example, let us work out the electron distribution in the diamine depicted in Fig. 14. If we carry out the starting process described in Paper I, starting with one of the NH_2 groups, we find that the other NH_2 group is also starred, and that there are altogether 11 starred atoms and nine unstarred atoms. Therefore there must be $11-9=2$ zero-energy MO in the isoaromatic hydrocarbon; and both of these are distributed entirely over

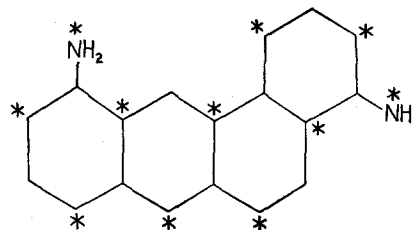


FIG. 14.

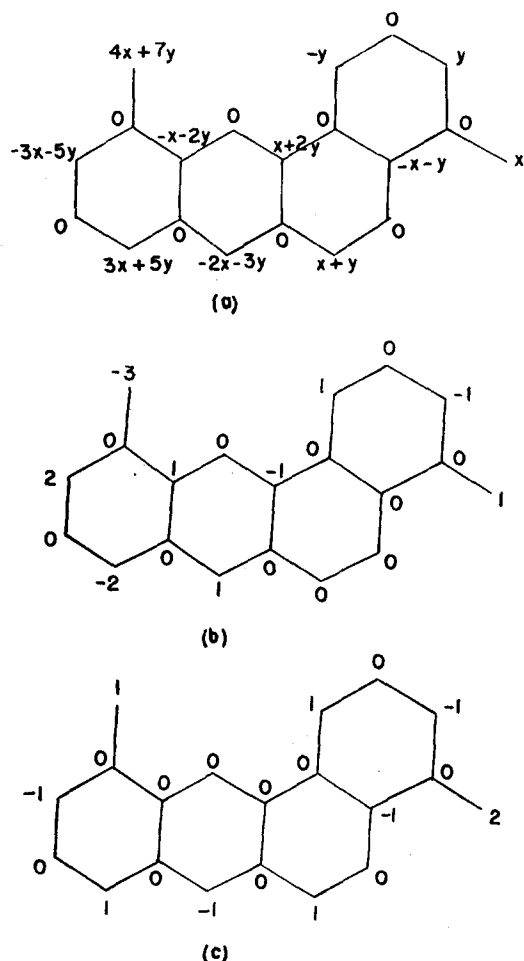


FIG. 15.

the starred atoms, since in every principal resonance structure there are nine double bonds, and every unstarred atom is therefore doubly bonded in all the structures. To find the net charges at the various atoms we require the AO coefficients for the two zero-energy MO only. These MO are not completely determined, —any orthonormal combination is equally valid— but the sum of the squares of their coefficients at a given atom is determinate, being invariant under an orthogonal transformation of the two MO. Denote by x and y the

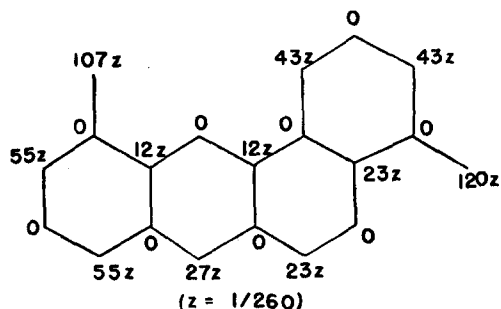


FIG. 16.

AO coefficients of a zero-energy MO at the left-hand NH_2 group and the ring position ortho to it. Then the other coefficients must be as shown in Fig. 15(a). Putting $x=1$, $y=-1$ and $x=2$, $y=-1$, we get the linearly independent (but not normalized or orthogonal) combinations of AO shown in Figs. 15(b) and 15(c), respectively. By well-known methods we can obtain from these combinations two orthonormal MO, and the sum of the squares of their AO coefficients at each position are given in Fig. 16.¶ In view of Eq. (7), the numbers in Fig. 16 also represent the net negative charges at the various positions, except at the nitrogen atoms, which have net positive charges equal to the differences of these numbers from unity. As in the case of 2-naphthylamine, the true π -electron densities are probably nearer to unity than the values in Fig. 16, but the order is probably more or less correct. The diamine should therefore be more reactive in the left-hand ring than in the right-hand one.

Although in the heteromolecules discussed, relative net charges may be calculated with little labor, this can by no means always be done, even for alternant molecules. If a molecule contains two heterosubstituents separated by odd numbers of unsaturated bonds, as in the hydroquinones, the isoaromatic hydrocarbon will not in general have any zero-energy MO, so Eq. (7) cannot be applied. But provided all the heteroatoms are in the same class when the starring process is carried out, the parent hydrocarbon will have enough zero-energy MO to accommodate all the extra electrons, and the approximate electron distribution may be calculated by the simple method just described.

9. CONCLUSIONS

Two quite different lines of thought have been pursued in this paper. On the one hand, it has been shown how differences in activation energy between isoaromatic reactions may be calculated by first-order perturbation theory, provided conditions (a), (b), and (c) of Section 4 are satisfied. On the other hand, it has been shown that the relative reactivities of different positions in a heteromolecule can be estimated from the calculated electron distribution in the parent hydrocarbon ion. It must be emphasized that calculations of the former kind are probably much more reliable than those of the latter sort. There are two main reasons for this. (1) As has already been said, calculations on the transition state provide a much firmer basis for predicting relative reaction rates than calculations of electron density in the substrate, although in a given substrate there often seems to be a close correspondence between the calculated electron distribution and the observed chemical

¶ Alternatively, we can use the result that if $\chi_1 = \sum_r a_r \phi_r$ and $\chi_2 = \sum_r b_r \phi_r$ are two linearly independent combinations of AO, and if $\sum_r c_r \phi_r$ and $\sum_r d_r \phi_r$ are two orthonormal combinations of χ_1 and χ_2 , then

$$c_i^2 + d_i^2 = \frac{a_i^2 \sum_r b_r^2 - 2a_i b_i \sum_r a_r b_r + b_i^2 \sum_r a_r^2}{\sum_r a_r^2 \sum_r b_r^2 - (\sum_r a_r b_r)^2}.$$

behavior. (2) It is certainly not correct to neglect the Coulomb terms of the heteroatoms in working out the electron distribution in a heteromolecule. It might be argued that the latter objection applies as much to calculations of transition state energies as to calculations of electron distribution; however, this seems not to be so, for the following reasons. In calculating the unsaturation energy of a heteromolecule or transition complex by Eq. (7), we are essentially using first-order perturbation theory; and it is well known that the first-order correction to the energy of a perturbed system is obtained by integrating the perturbation over the unperturbed wave function. In other words, even an inaccurate wave function gives the energy fairly accurately if the right Hamiltonian is used, and this is the basis of Eq. (7). On the other hand, the electron densities in a heteromolecule depend on the AO coefficients in the π -orbitals, and these coefficients differ in first order between a heteromolecule and the parent hydrocarbon. So the electron densities calculated in Section 8 should not be taken too literally, although their relative values are probably correct.

Before concluding, it seems worth while to compare briefly the present ideas with those of qualitative resonance theory. The first point to be made is that neither approach pretends to do more than account for the energy changes due to electronic conjugation in a molecule; other energy terms, and all entropy terms, have to be ignored altogether. Therefore both theories should be most successful when used to account for the relative stabilities of two species, or the relative rates of two reactions, in which these other energy and entropy terms are nearly equal. Second, the results of Paper I establish, and the examples of Sections 6 and 8 illustrate, the fact that the approximate quantitative theory outlined here is bound to give qualitatively the same π -electron distribution in an alternant molecule as would arise from resonance among the principal resonance structures *only*, the higher excited structures being absent altogether. Third, Eq. (7) shows that a

heteroatom confers most stability on a conjugated system if the isoaromatic hydrocarbon has a high electron density at the corresponding position; and this is physically equivalent to the statement that a heteroatom is relatively more stable with two unshared p electrons than a carbon atom. (A heteroatom here means an atom to the right of carbon in the Periodic Table.) So a large measure of agreement is to be expected in general between the results of MO theory and predictions based on the principal resonance structures only.

The methods of calculation outlined are not to be regarded as a substitute for the more precise method of Wheland¹ in cases for which (a) the required Coulomb integrals are all known with moderate certainty, and (b) solution of the secular equations is not prohibitively laborious; and, of course, there are many important molecules to which the present calculations cannot be applied, even in principle. However, for problems such as example 2 in Section 6, the labor involved in applying Eqs. (3) and (5) is infinitesimal compared with that required to set up and solve the secular equations for all the species involved; and the difference between the results of the two kinds of calculation is not likely to be as significant as the error due to uncertainty in the Coulomb term for aromatic nitrogen.⁸ As is well known, the labor of solving the secular equations increases rapidly with the size of the molecule, particularly if there is no symmetry; and to solve the secular equations for all the possible transition states that could be formed by the diamine in Fig. 14 would be extremely tedious, and not very illuminating. The present method, it is hoped, provides a moderately balanced compromise between accuracy and speed for calculations on complex systems; and perhaps provides a clearer picture of why the more exact calculations come out in the way that they do.

My thanks are due Dr. J. R. Platt for much stimulating discussion.

⁸ C. A. Coulson and H. C. Longuet-Higgins, Proc. Roy. Soc. A192, 16 (1947).