

Energy Formula and Potential Distribution of Diatomic Molecules

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Taking as general expression of a diatomic molecular potential V(r) the following one,

$$V = -D + D\xi^{2}, \quad 1 - \xi = \frac{(1+a)(1+\epsilon)}{1+b} \frac{x+b}{(x+a)(x+\epsilon)},$$

$$x = e^{(1+k)\rho}, \quad \frac{1}{1+k} = \frac{1}{1+a} + \frac{1}{1+\epsilon} - \frac{1}{1+b},$$

$$\rho = \left[h\nu_{e}/2(B_{e}D)^{\frac{1}{2}}\right](r-r_{e})/r_{e},$$
(1)

 r_e being the equilibrium distance of the atoms and D, B_e and $h\nu_e$ well-known spectroscopic constants, the energy formula may be expressed as follows,

$$1 - \left(-\frac{E}{D}\right)^{\frac{1}{2}} = \frac{h\nu_{\epsilon}}{2D}(n + \frac{1}{2}) + \frac{A(h\nu_{\epsilon}/2D)^{2}(n + \frac{1}{2})^{2}}{1 - \kappa(h\nu_{\epsilon}/2D)(n + \frac{1}{2})},$$

$$A = \frac{1}{2}(1 - a_{2}), \quad \kappa = 1 - a_{2} + \frac{a_{2} - a_{4}}{1 - a_{2}},$$

$$(2)$$

 a_2 and a_4 being coefficients in the expansion of $d\rho/d\xi$ in power series of ξ,

$$d\rho/d\xi = 1 + a_1\xi + a_2\xi^2 + a_3\xi^3 + a_4\xi^4 + \cdots$$
 (3)

The rotational constant B depends upon the vibrational quantum number n in the following way,

$$B = B_{\epsilon} - \alpha (n + \frac{1}{2}) - \alpha_{2} (n + \frac{1}{2})^{2} - \cdots$$

$$\alpha / B_{\epsilon} = \frac{h\nu_{\epsilon}}{2D} \left[3a_{1} \left(\frac{2(B_{\epsilon}D)^{\dagger}}{h\nu_{\epsilon}} \right) - 3 \left(\frac{2(B_{\epsilon}D)^{\dagger}}{h\nu_{\epsilon}} \right)^{2} \right],$$

$$\alpha_{2} / B_{\epsilon} = \left(\frac{h\nu_{\epsilon}}{2D} \right)^{2} \left[\left(\frac{15}{4} a_{3} - 2a_{1}a_{2} \right) \left(\frac{2(B_{\epsilon}D)^{\dagger}}{h\nu_{\epsilon}} \right) - \left(3a_{2} + \frac{45}{8} a_{1}^{2} \right) \left(\frac{2(B_{\epsilon}D)^{\dagger}}{h\nu_{\epsilon}} \right)^{2} + 15a_{1} \left(\frac{2(B_{\epsilon}D)^{\dagger}}{h\nu_{\epsilon}} \right)^{3} - \frac{15}{2} \left(\frac{2(B_{\epsilon}D)^{\dagger}}{h\nu_{\epsilon}} \right)^{4} \right].$$

$$(4)$$

The coefficients a_1 , a_2 , a_3 , a_4 , may be derived from the expression

$$\rho = \frac{1}{1+k} \log \frac{1}{1-\xi} \left\{ 1 + k\xi - \frac{2k_1k_2(1+k)\xi^2}{1 + (k_1 + k_2 + 2k_1k_2)\xi + \sqrt{[1+(k_1 + k_2 + 2k_1k_2)\xi]^2 - 4k_1k_2(1+k_1)(1+k_2)\xi^2}} \right\},$$

$$k_1 = \left(\frac{1}{1+b} - \frac{1}{1+a} \right) (1+k), \qquad k_2 = \left(\frac{1}{1+b} - \frac{1}{1+c} \right) (1+k).$$
(5)

If now this energy formula can be adjusted to the observed vibrational energy levels of a molecule, the coefficients a_2 and a_4 are already given. The coefficient a_1 may easily be found from the first order dependance of B upon the quantum number n. Thus the constants of the potential curve, k, k_1 and k_2 , or, a, b and c are defined and may be calculated from a_1 , a_2 and a^4 and so we have an analytical

In Fig. 1 some potential curves are given, the solid at the minimum point the dotted curves are narrower in

expression of the potential distribution of the molecule. curve being the well-known potential curve given by Morse.1 With the same curvature and the same asymmetry

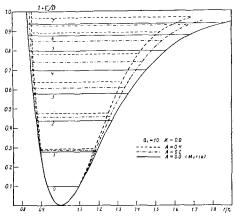


Fig. 1. Dependance of the upper levels of the narrowness of the potential

the upper part, thus raising progressively the upper energy levels

In Fig. 2 quite another variation possibility of the curves is illustrated, the energy formula being unaltered and only the symmetry of the curves being changed.

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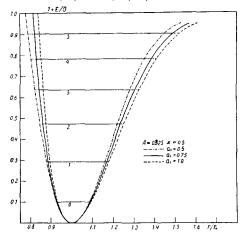


Fig. 2. Potential curves with different asymmetry but with identical