Dispersion and Absorption in Dielectrics II. Direct Current Characteristics

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Dispersion and Absorption in Dielectrics

II. Direct Current Characteristics*

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In the first paper of this series [J. Chem. Phys. 9, 341 (1941)], it was shown that the complex dielectric constant, ϵ^*, of many liquid and solid dielectrics is given by a single very general formula

$$\epsilon^* = \epsilon_\infty + (\epsilon_0 - \epsilon_\infty)/[1 + (i\omega\tau_0)^{1-\alpha}].$$

In this equation ϵ_0 and ϵ_∞ are the "static" and "infinite frequency" dielectric constants, $\omega = 2\pi$ times the frequency, τ_0 is a generalized relaxation time and α is a constant, $0 < \alpha < 1$. The transient current as a function of the time, t, after application of a unit constant potential difference has been calculated from this expression in series form. For times much less than τ_0, the time dependence is of the form $(t/\tau_0)^{-\alpha}$, and for times much greater than τ_0, it is of the form $(t/\tau_0)^{-(2-\alpha)}$. The transition between these extremes occurs for the range in which t is comparable with τ_0. The total absorption charge, which is the integral of the exact expression, is always finite. Although many transient data for dielectrics are of the predicted form, none have been taken over a sufficiently wide range of times adequately to test the result, nor is it yet possible to determine either the relaxation time or the static dielectric constant from available data.

I. INTRODUCTION

THE Debye theory of dispersion and absorption in dielectrics[1] leads to the formula

$$\epsilon^* = \epsilon' - i\epsilon'' = \epsilon_\infty + (\epsilon_0 - \epsilon_\infty)/(1 + i\omega\tau_0) \qquad (1)$$

for the complex dielectric constant ϵ^* as a function of frequency $\nu(=\omega/2\pi)$, where ϵ' and ϵ'' are the real and imaginary parts of ϵ^*, ϵ_0 and ϵ_∞ are the limiting (real) values of ϵ^* at very low and high frequencies, and τ_0 is the relaxation time. A result of this same form is predicted by other simple dispersion theories, as was discussed in the preceding paper under the same title[2] (hereafter referred to as I).

It has been shown that the transient current $I(t)$ following application of unit potential difference across the dielectric at time $t=0$ is given by

$$I(t) = (1/\tau_0)(\epsilon_0 - \epsilon_\infty)e^{-t/\tau_0}, \qquad (2)$$

if Eq. (1) is a correct description of the alternating current characteristics.[3] The result (2) is apparent by inspection of the equivalent circuit of the dielectric (cf. Fig. 2 of I) and may be derived by the Fourier integral theorem.[4]

It has long been known that Eq. (2) is not a satisfactory expression for the transient absorption current in dielectrics. In a considerable number of cases the experimental results are well fitted by the formula

$$I(t) = At^{-n}, \qquad (3)$$

where A is a constant and the exponent n usually but not always has a value between 0 and 1. The result (1) predicted by the Debye and other theories cannot, therefore, represent the alternating-current characteristics of these dielectrics. It is well known that Eq. (1) is in fact a poor

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

[1] P. Debye, *Polar Molecules* (Chemical Catalog Company, New York, 1929).

[2] K. S. Cole and R. H. Cole, J. Chem. Phys. 9, 341 (1941).

[3] See reference 1, p. 88. The current involved in setting up the high frequency polarization represented by ϵ_∞ has been omitted in Eq. (2). This is justified as this current flows for an extremely short time and is not ordinarily measured.

[4] J. R. Carson, *Electric Circuit Theory and Operational Calculus* (McGraw-Hill Book Company, New York, 1926).

approximation for many dielectrics. As the writers have shown in I, the complex dielectric constant is in many cases given by the formula

$$\epsilon^* = \epsilon_\infty + (\epsilon_0 - \epsilon_\infty)/[1+(i\omega\tau_0)^{1-\alpha}], \qquad (4)$$

the parameter α having values between 0 and 1.

If the principle of superposition[5] holds, it is possible to calculate the transient current corresponding to Eq. (4). The present paper describes the results of these calculations and the interpretation of transient current data in terms of them.

II. MATHEMATICAL DETAILS

The complex admittance $Y(i\omega)$ of a dielectric described by Eq. (4) is given by

$$Y(i\omega) = i\omega\epsilon^* = i\omega\{\epsilon_\infty + (\epsilon_0 - \epsilon_\infty)/[1+(i\omega\tau_0)^{1-\alpha}]\}, \qquad (5)$$

if a condenser of unit geometrical capacity is assumed. The transient current $I(t)$ following application of unit potential difference at time $t=0$ may be written as

$$I(t) = \frac{1}{\pi}\int_0^\infty \epsilon^*(i\omega)e^{i\omega t}d\omega. \qquad (6)$$

The direct-current transient may also be written as either a Fourier sine or cosine integral involving ϵ'' or ϵ', respectively, the formulas being[6]

$$I(t) = \frac{2}{\pi}\int_0^\infty \epsilon''(\omega)\sin\omega t d\omega, \qquad I(t) = \frac{2}{\pi}\int_0^\infty \epsilon'(\omega)\cos\omega t d\omega, \qquad (7)$$

the initial current involved in charging ϵ_∞ being omitted as before.

The direct-current transient is therefore determined by the frequency dependence of either ϵ' or ϵ''. Thus the behavior of the dielectric is completely determined if any one of the three functions $I(t)$, $\epsilon'(\omega)$, $\epsilon''(\omega)$ is known over the *complete* range of time or frequency. As a corollary, it is possible to calculate any one of these functions from any other. The expressions for $\epsilon'(\omega)$ and $\epsilon''(\omega)$ in terms of $I(t)$ are well known[7] and follow directly from Eqs. (7) by use of Fourier sine or cosine transforms. The mutual integral relations between ϵ' and ϵ'' are less familiar. They were originally given by Kramers[8] for the case of optical dispersion (i.e., for the frequency variation of ϵ_∞ in terms of the optical absorption and vice versa). The Kramers' relations are readily derived from Eqs. (7) by use of Fourier transforms as has been pointed out by Gross[9] and independently by one of the writers.[10]

The solution of Eq. (6) is easily obtained in series form by the Heaviside operational calculus.[4] Rewriting Eq. (6) as a series in descending powers of the Heaviside operator p and using the operational formula $p^{-n}\rightarrow t^n/\Gamma(1+n)$, one obtains

$$I(t) = \frac{\epsilon_0-\epsilon_\infty}{\tau_0}(1-\alpha)\left(\frac{t}{\tau_0}\right)^{-\alpha} \times\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{\Gamma[1+n(1-\alpha)]}\left(\frac{t}{\tau_0}\right)^{(n-1)(1-\alpha)}. \qquad (8)$$

In the case $\alpha=0$, Eq. (8) reduces, as it should, to the exponential form (2). For $t/\tau_0 \ll 1$, the current is given with sufficient accuracy by the leading term of the series:

$$I(t) = \frac{\epsilon_0-\epsilon_\infty}{\tau_0}\cdot\frac{1}{\Gamma(1-\alpha)}\left(\frac{t}{\tau_0}\right)^{-\alpha}, \quad t\ll\tau_0. \qquad (9)$$

It is also possible to obtain an asymptotic expansion in negative powers of t/τ_0:

$$I(t) = \frac{\epsilon_0-\epsilon_\infty}{\tau_0}(1-\alpha)\left(\frac{t}{\tau_0}\right)^{-(2-\alpha)} \times\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{\Gamma[1-n(1-\alpha)]}\left(\frac{t}{\tau_0}\right)^{-(n-1)(1-\alpha)}. \qquad (10)$$

For sufficiently large values of t/τ_0 the current is given by

$$I(t) = \frac{\epsilon_0-\epsilon_\infty}{\tau_0}\cdot\frac{(1-\alpha)}{\Gamma(\alpha)}\left(\frac{t}{\tau_0}\right)^{-(2-\alpha)}, \quad t\gg\tau_0. \qquad (11)$$

The time dependence of the absorption current $I(t)$ is most conveniently represented on a double logarithmic scale. From Eqs. (9), (11) the log I, log t characteristic is a curve with limiting slopes $-\alpha$ for $t\ll\tau_0$ and $-(2-\alpha)$ for $t\gg\tau_0$. The Fourier mate[11] $\mathfrak{M}(1+p^{1-\alpha})^{-1}$, to which the transient

[5] For a discussion of the superposition principle, see M. F. Manning and M. E. Bell, Rev. Mod. Phys. **12**, 215 (1940).

[6] See reference 4, p. 180.

[7] E. Von Schweidler, Ann. d. Physik **24**, 711 (1907). See also reference 5.

[8] H. A. Kramers, Atti Congr. dei Fisici Como, 545 (1927). See also reference 2.

[9] B. Gross, Phys. Rev. **59**, 748 (1941).

[10] R. H. Cole, Phys. Rev. **60**, 172A (1941); see also reference 2.

[11] G. A. Campbell and R. M. Foster, *Fourier Integrals for Practical Applications*, Bell Telephone Monograph B-584 (1931). The writers are very much indebted to Dr. Foster of the Bell Telephone Laboratories for making his calculations of the Fourier mate available to them.

current is proportional, has been calculated from the appropriate expansions over the range 10^{-6} to 10^{5} for t/τ_0 and for values 0.1, 0.3, 0.5, 0.7, 0.9 of α. Wilbur[12] has calculated the integral of the series (8) in the range $10^{-3}<t/\tau_0<10^{2}$ for $\alpha=0.1$, 0.2, 0.3, 0.4, 0.5. These values thus give the absorption charge as a function of time after application of unit potential difference. The absorption currents have been obtained from these results by graphical differentiation and

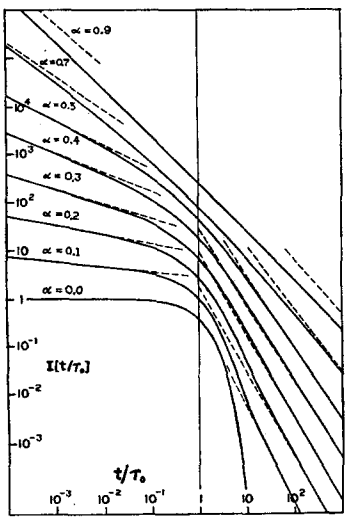

FIG. 1. Calculated transient currents as a function of time. The curves for increasing values of the parameter α have been successively displaced vertically upward by half-decade intervals for greater clarity.

these independent values agree within their limit of accuracy with the directly calculated values.

The transient currents for several values of α are shown graphically in Fig. 1. The limiting straight lines as predicted by Eqs. (8), (10) are also drawn in as dashed lines. Some of the numerical data from which Fig. 1 was drawn are given in Table I in which values of the current $I(t/\tau_0)$ are given for values of t/τ_0 at approximately equal logarithmic intervals. It has been found very useful in drawing curves such as those of Fig. 1 to draw in the limiting straight lines of slope $-\alpha$ and $-(2-\alpha)$. The intercepts of these lines on the axis $t/\tau_0=1$ as calculated from Eqs. (8), (10) are listed in Table II, together with the value of the intercept: $I(t/\tau_0=1)$. These data are also useful in matching experimental results to the predicted curves.

[12] The writers are indebted to Dr. D. T. Wilbur of Cornell University for use of his results.

III. SIGNIFICANCE OF THE RESULTS

It is apparent from Fig. 1 and Eqs. (9), (11) that experimental data requiring a time dependence of the form t^{-n} are consistent with the alternating-current characteristic expressed by Eq. (4) and discussed in I. It is further evident that data for which a slope of value less than unity is found over the entire time interval of measurement have not been taken at sufficiently long times to reveal the departures shown in Fig. 1. This situation has an exact counterpart in the complex plane locus (ϵ'' *versus* ϵ') of the dielectric constant. This may be seen by calculating the steady state relations for $\epsilon'(\omega)$, $\epsilon''(\omega)$ required by Eq. (9) if the superposition principle is valid. It is well known that in this limiting case one obtains[13]

$$\epsilon'(\omega)=(\epsilon_0-\epsilon_\infty)(\omega\tau_0)^{-(1-\alpha)}\sin\tfrac{1}{2}\alpha\pi,$$
$$\epsilon''(\omega)=(\epsilon_0-\epsilon_\infty)(\omega\tau_0)^{-(1-\alpha)}\cos\tfrac{1}{2}\alpha\pi. \qquad (12)$$

The complex plane locus defined by Eq. (12) is a straight line of slope $\tan(1-\alpha)\pi/2$. This is simply the limiting slope of the circular arc locus for $\omega\tau_0\gg1$, as discussed in I. As pointed out there, one has from this limiting behavior no estimate of either the relaxation time or the static dielectric constant for the reason that the measurements do not extend to sufficiently low frequencies. In the present direct-current case one has no information about either of these quantities for the reason that the measurements do not extend to sufficiently long times.

A similar argument applies to data for which values of α greater than unity are found, the time intervals of measurement being in this case considerably greater than the relaxation time τ_0. In these cases then, the measurements do not extend to sufficiently short times to determine $\epsilon_0-\epsilon_\infty$ or τ_0 and the situation corresponds to the limiting slope $-\tan(1-\alpha)\pi/2$ of the circular arc locus at low frequencies ($\omega\tau_0\ll1$).

For times comparable with the relaxation time a transition between the limiting behaviors outlined above is to be expected; that is, the locus is a curve of increasing negative slope as shown in Fig. 1 and corresponds to the major part of the circular arc locus for the alternating-current

[13] See reference 5 for details of the calculation.

TABLE I. Transient currents for various values of the parameter α.

t/τ_0 \ α	0.0	0.1	0.3	0.5	0.7	0.9
1×10^{-4}	0.9999	2.350	12.18	55.43	184.4	209.0
2	0.9998	2.192	9.880	38.92	111.6	107.4
5	0.9995	1.999	7.480	24.26	56.13	44.41
1×10^{-3}	0.9990	1.867	6.049	16.88	33.07	22.71
2	0.9980	1.742	4.878	11.66	19.31	11.59
5	0.9950	1.574	3.643	7.054	9.328	4.747
1×10^{-2}	0.9900	1.456	2.894	4.745	5.300	2.410
2	0.9802	1.338	2.268	3.131	2.966	1.220
5	0.9512	1.169	1.585	1.733	1.341	
1×10^{-1}	0.9048	1.020	1.156	1.061	0.719	
2	0.8187	0.8410	0.791	0.618		
5	0.6065	0.5478	0.411	0.275		
1	0.3679	0.2996	0.211	0.137	0.073	0.025
2	0.1353			0.0627		
5	0.00674			0.0200		
10	0.000045	0.00212	0.00605	0.00794		
20		0.000413	0.00172	0.00294	0.0029	
50		0.000062	0.000335	0.000775	0.00101	0.00048
1×10^{2}		0.000016	0.000099	0.000278	0.000430	0.000235
2			0.000030	0.000099	0.000185	0.000115
5				0.000025	0.000059	0.000045

characteristic. This correlation of the direct-current and alternating-current characteristics is shown in Fig. 2 in which the log I, log t curve and the complex plane locus are shown, one above the other with corresponding quantities indicated. The figure is drawn for the case $\alpha=0.5$, but a similar situation holds for other values in the range $0<\alpha<1$.

Two objections have often been raised to the empirical expression (3) or its equivalent (9) for the absorption current. The first of these is that Eq. (2) predicts an infinite current for $t=0$. It should, however, be realized that the same thing is true of a perfect capacity on applying a discontinuous change in potential and that the constant phase angle impedance element in the equivalent circuit (see Fig. 2 of I) has the characteristics of both resistance and capacity. The significant quantity is the total charge stored in the dielectric. This total charge is finite for any finite time, being the time integral of Eq. (9).

TABLE II. Values of $I(t/\tau_0)$ for $t/\tau_0=1$; intercepts with axis $t/\tau_0=1$ of asymptotic functions valid for $t/\tau_0\ll1$ and for $t/\tau_0\gg1$.

α	$I(t/\tau_0)$	$I(t/\tau_0\ll1)$	$I(t/\tau_0\gg1)$
0.0	0.3679	1.000	0.000
0.1	0.2996	0.936	0.0946
0.2	0.254	0.859	0.174
0.3	0.211	0.770	0.234
0.4	0.172	0.672	0.271
0.5	0.137	0.564	0.282
0.7	0.073	0.334	0.231
0.9	0.025	0.105	0.0936

The second objection is that the total charge $Q(t)$ is not finite for an infinite time of charging, as this is given by

$$Q(t)=A\int_0^t t^{-\alpha}dt,$$

which does not converge for $\alpha<1$ as $t\to\infty$. This cannot be the case for the true reversible absorption current, as it would require that the dielectric be capable of storing an infinite amount of charge, all of which could be recovered on discharge. The answer is of course that Eq. (9) is valid only for short times ($t\ll\tau_0$) and does not account for the approach to equilibrium or "saturation." The integral of Eq. (6), which is the general expression for a dielectric, does converge to the proper limiting value ($\epsilon_0-\epsilon_\infty$), as can be seen intuitively from the equivalent circuit of Fig. 2b in I.

IV. EXPERIMENTAL EVIDENCE

It must be realized that the discussion above is concerned only with the reversible absorption current. If any comparison is to be made with experimental data, it is necessary that the effect of any irreversible "Joule" conductivity be taken into account. The charging current in a dielectric after application of a constant potential difference must ultimately reach a steady value if such a conductivity exists. This steady value may well

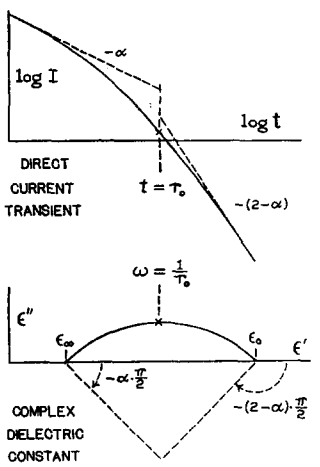

FIG. 2. Comparison of transient direct-current and steady-state alternating-current characteristics of dielectrics exhibiting dispersion.

be sufficiently large to mask completely the absorption current at large times and so make difficult or impossible any comparison with the predicted curves of Fig. 1 for large values of time.

This difficulty in determining the true absorption current can be avoided by measuring the discharge current when the dielectric is short-circuited. In this case, the Joule conductivity is negligible and the absorption current, because of its reversibility, is given by Eqs. (8), (10) except for sign. This is, however, strictly true only if the dielectric has been kept at a constant potential for an infinite time before short-circuiting to measure the discharge current. In practice, this means that the time of charging must be long in comparison with the relaxation time τ_0. If τ_0 is of the order of days or weeks, as the discussion below indicates may often be the case, this condition has been not at all satisfied in many experiments.

If the charging time is comparable with the relaxation time, one has to consider the response of the dielectric to a potential difference in the form of a rectangular pulse rather than a unit function. Such a pulse can be regarded as the sum of two equal and opposite unit functions with the discontinuities separated by the charging interval. Because of the superposition principle, the resulting absorption current is then the algebraic sum of the currents due to these two unit functions applied separately.

It should perhaps be mentioned that it is formally possible to calculate the variation of potential difference across a dielectric with time if the transient current expression is known and the superposition principle is valid. Gross[14] has considered the problem in detail for various conditions of charging and the reader is referred to his papers for a more detailed discussion.

The analysis of dielectric phenomena in terms of potential rather than current variations does not appear to be very useful experimentally as the mathematical expressions involved are difficult to handle in numerical form. Gross has considered the case in which the transient current is given by a formula of the form (9). He obtains an approximate expression for the variation of potential across an open-circuited dielectric as a series of functions involving Mittag-Leffler functions.[15] The complications involved in any attempt to evaluate explicitly the potential variations required by a more general formula such as Eq. (8) for the transient current can readily be appreciated.

The additive property of absorption currents is not particularly useful in practice for the reason that one does not know the value τ_0 in advance. If, however, the charging time is small compared with τ_0, the current is essentially that for a unit impulse and is given by the derivative of Eqs. (8), (10) with respect to t/τ_0. In this case one should expect limiting slopes $-(1+\alpha)$ and $-(3-\alpha)$ on the log I *vs.* log t plot for t small and large, respectively. One cannot expect to evade the difficulties in determining the long time behavior by any such simple expedient as this. The reason is simply that the long time response of the dielectric does not come into play for a very short charging time. As a consequence, the current for large values of time will be extremely small and correspondingly difficult to measure.

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[14] B. Gross, Zeits. f. Physik **107**, 217 (1937); *ibid.* **108**, 598 (1938).

[15] The Mittag-Leffler function encountered is similar to, but not to be confused with, Eq. (8). The series (8) can be written in terms of the derivative of this function of order $1-\alpha$. For a discussion of the Mittag-Leffler function, see H. T. Davis, *The Theory of Linear Operators* (Principia Press, Bloomington, Indiana, 1936).

Because of the complications discussed above, it is necessary to be circumspect in analyzing experimental results. With these difficulties in mind, representative examples of transient current data in the literature will now be considered.

Experiments showing the existence of "anomalous" discharge in dielectrics were made by R. Kohlrausch in 1854.[16] This observer measured the decay of charge on the plates of glass condensers after removal of the charging potential. His values for the loss of charge $Q_0-Q(t)$, where $Q(t)$ is the charge at time t and Q_0 is the initial charge, are shown in Fig. 3 plotted against time. The data on a double logarithmic plot are well fitted by a straight line of slope 0.3. As the loss of charge is the integral of the discharge current, the expression for the discharge current is of the form $I(t)=At^{-0.7}$. The point already emphasized, that a formula of this kind can be true only for times small compared to the relaxation time of the dielectric, is brought out in another way in Fig. 3, as the curve for $Q_0-Q(t)$ must approach the value Q_0 indicated by the dashed line for very large values of time. It is seen that, while there is some evidence of this flattening out, a time of $5\cdot10^3$ seconds is still small compared with the relaxation time.

It should be pointed out that these data are not strictly comparable to the results of Section II, as Kohlrausch measured the charge on open circuit. The discharge current was thus maintained by leakage and the potential difference across the dielectric was not constant as assumed in the derivation above.[17]

Most of the available results on absorption currents *versus* time are for practically important insulators such as quartz, amber, hard rubber. It is unfortunate that there are few direct-current

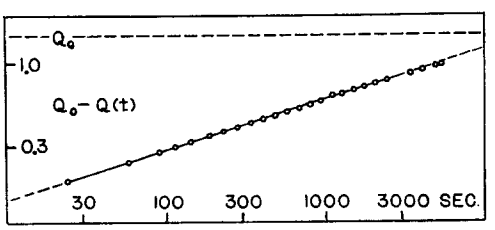

FIG. 3. Loss of absorption charge from glass as a function of time (from data of R. Kohlrausch).

[16] R. Kohlrausch, Pogg. Ann. **91**, 56 (1854).
[17] For a detailed discussion of such cases, see reference 14.

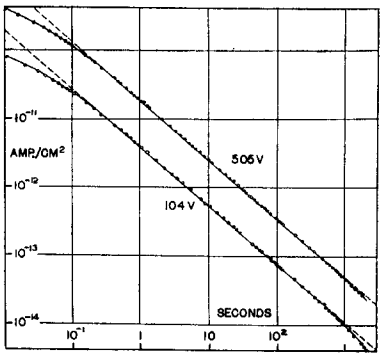

FIG. 4. Discharge currents in mica for two values of original potential difference across the dielectric (from data of Voglis).

data for dielectrics similar to the ones considered in I for which there are alternating-current data available as well.

A. Mica

One example for which both kinds of data are available is mica. The results of Voglis[18] for the discharge current over a time range 10^{-2} to 10^3 seconds are shown in Fig. 4. Over most of this range the curve is a straight line of slope -0.87 and hence $\alpha=0.87$. MacLeod[19] has measured the resistance and capacity over the frequency range $500-10^6$ cycles and his results give a value $\alpha=0.88$.

The departure from the $t^{-\alpha}$ formula for large values of time is, as mentioned by Manning and Bell,[5] in the necessary direction that the total charge recovered be finite. Voglis suggested that the flattening of the $\log I-\log t$ characteristic for t small is evidence that the $t^{-\alpha}$ formula breaks down for sufficiently small times. It is interesting to note that this conclusion is contradicted by MacLeod's results which correspond to times as small as 10^{-6} second and require the value $\alpha=0.88$.

Hippauf and Stein[20] have also made measurements on mica in the time range $4\cdot10^{-3}-2\cdot10^{-1}$ second and they found a value $\alpha=0.24$, which is not at all in agreement with Voglis and MacLeod. Some unpublished measurements of R. F. Field are interesting in this connection. Field has made alternating-current measurements on stained

[18] G. M. Voglis, Zeits. f. Physik **109**, 52 (1938).
[19] H. J. MacLeod, Phys. Rev. **21**, 53 (1923).
[20] E. Hippauf and R. Stein, Physik. Zeits. **39**, 90 (1938).

samples of mica over the frequency range $25-10^4$ cycles. He finds in one case that the data for loss factor *versus* capacity plot into a circular arc for which $\alpha=0.46$, and the relaxation time $\tau_0=0.016$ sec. His measurements of the absorption current for the time range $1.5-100$ seconds give a straight line of slope -0.92 on the $\log I-\log t$ plot. The natural conclusion is that one is dealing here with two more or less distinct dispersion regions characterized by separate values of τ_0, α. It is quite possible that the higher frequency dispersion is not a true characteristic of the mica itself but arises from imperfections in the sample. Whatever the cause, it is evident that the question can be resolved only when data over a wide range of frequency and time are available.

The existence of more than one region of dispersion is a possibility which may very well have to be considered in other dielectrics as well. For example, the data of Yager[21] on polyvinyl chloride and chloroprene in the frequency range $10^3-3\cdot10^7$ cycles give evidence of the same type of thing.[22]

The various results for mica have been mentioned here chiefly to indicate the complications which may arise in the analysis of dielectric data. It is, however, significant that all the evidence for mica is consistent with the constant phase angle characteristic of dielectrics generally.

B. Glass

Voglis has also made measurements on glass, the data again falling on a straight line of slope -0.74 over the major portion of the time range 10^{-2} to 10^3 seconds. The slope increases slightly at the longer times but the departures are so small that it is impossible to estimate from them the value of the relaxation time. It can only be concluded that it is considerably greater than 10^3 seconds. A number of other observers have made measurements over various time intervals for different glasses and find the same sort of behavior. Benedict,[23] for example, finds values $\alpha=0.69-0.75$ for soda lime glass in the range 0.4 to $100\cdot10^{-3}$ second; Tank[24] finds a value 0.70 for times up to 0.2 second.

C. Amber

The absorption current has been measured by a number of observers, the results being in not too good agreement. Seidl[25] finds values $0.88-0.91$ for α in the time range 1–60 minutes, indicating a still larger value of the relaxation time. On the other hand, the data of Neumann[26] plotted in Fig. 5 appear to show a transition between slopes -0.7 and -1.5. These estimates are necessarily rough as the range of times is small and the consistency of the data not very good. If these data constitute an example of the behavior predicted by Eqs. (9), (11), they require the values $\alpha\sim0.7$, $\tau_0\sim30$ minutes. Gnann[27] finds a slope 1.11 on the $\log I-\log t$ plot for times greater than about 15 minutes, which corresponds to $\alpha=0.89$. His data do not extend to sufficiently short times to permit an accurate estimate of the other limiting slope, but it appears from his figure to be of the order 0.9 as it should according to Eqs. (9), (11).

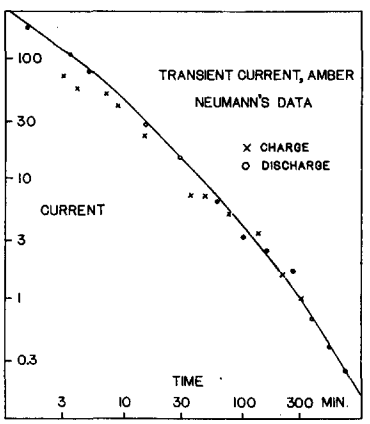

FIG. 5. Transient currents for charge and discharge of amber (from data of H. Neumann).

In view of the disagreements among the various data, they can hardly be said to confirm the results of the present paper. They do, however, give evidence of the transition region for $t\sim\tau_0$

[21] W. A. Yager, Trans. Electrochem. Soc. **74** (preprint), (1938); Bell Telephone Monograph B-1099.

[22] The writers are indebted to Mr. R. F. Field of the General Radio Company for pointing this out and for much helpful discussion of the general subject of this paper, as well as for permission to quote his results for mica.

[23] R. R. Benedict, Trans. A. I. E. E. **49**, 221 (1930).

[24] F. Tank, Ann. d. Physik **48**, 307 (1915).

[25] F. Seidl, Zeits. f. Physik **91**, 318 (1934).

[26] H. Neumann, Zeits. f. Physik **45**, 717 (1927).

[27] W. Gnann, Physik. Zeits. **36**, 222 (1935).

which must occur for any dielectric exhibiting dispersion.

D. Other Data

There are many other data which could be cited here as examples of the $t^{-\alpha}$ time dependence. The majority of these measurements are on such things as various melting point paraffins, impregnated paper for condenser insulation, various waxes and rosins, and other insulation of commercial importance. For the most part, values of α less than one have been observed with little evidence of an increase in negative slope on the $\log I - \log t$ plot. For such materials, it can only be concluded that the times involved were very much less than the relaxation time. An estimate of the static dielectric constant is even less possible, as was the case with the alternating-current data on several solid dielectrics discussed in I.

V. CONCLUSION

The examples discussed above are believed to be representative of experimental results in the literature. The list is by no means complete and it appears probable that the number of examples which could be found to illustrate the $t^{-\alpha}$ time dependence is limited chiefly by one's patience in searching the literature.

The data which have been considered do not give much evidence on the question of the necessary change in time dependence if the total absorption charge is to remain finite. As a consequence, the limiting expression (11) required by the results of alternating-current data cannot as yet be checked from transient current measurements. Such results as do exist are at least not inconsistent with the predictions of the present paper. It is therefore not unreasonable to suppose that the "constant phase angle" characteristic very generally required by alternating-current data will be further substantiated by more complete measurements of absorption currents.

The importance of extending transient current measurements to longer times should be emphasized. Unless this is done one has only a part of the complete picture. These times may extend to the order of hours or even days in some cases, but without such data it is impossible to do more than set lower limits for the values of the relaxation time and static dielectric constant. About all that can be said at present about many solid dielectrics is that both these quantities must be very much larger than is ordinarily supposed. To avoid large errors in correcting for steady-state conduction currents, discharge rather than charging currents should be measured. If these results are to be interpreted without ambiguity, the dielectric must previously have been charged for a time long in comparison with the relaxation time.

It is important to realize that measurements of transient absorption currents in dielectrics give identically the same information as do steady-state alternating-current measurements. Either type of measurement determines the electrical characteristics of a dielectric for which the superposition principle is valid. The choice between the two is then dictated by convenience in measurement and analysis, the transient current method being obviously more suitable if the characteristic relaxation time is of the order of seconds or longer.