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ANISOTROPY AND TRANSVERSE ISOTROPY*

S. CRAMPIN**

ABSTRACT

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A number of authors in the exploration literature have written about anisotropy, but have restricted their discussions to wave propagation through rock having transverse isotropy with a vertical symmetry axis. This note shows that there are fundamental differences between transverse isotropy when the symmetry axis is vertical (normal to the free surface) and more general anisotropy with an azimuthal variation of properties. These differences are important now that effective azimuthal shear-wave anisotropy resulting from aligned cracks and pores is becoming recognized as a significant property of crustal rocks.

INTRODUCTION

The significance of anisotropy in seismic exploration has been discussed formally (e.g., Helbig 1981, 1983, Helbig and Mesdag 1982, Hake, Helbig and Mesdag 1984, Levin 1978, 1979, 1980, Radovich and Levin 1982). However, the discussion has been limited to wave propagation through rock with transverse isotropy oriented so that the symmetry axis is vertical and normal to the free surface. We shall say that such rock has *vertical transverse isotropy*. These discussions of vertical transverse isotropy are all theoretical. To the author's knowledge, the only published observations that reliably indicate the existence of vertical transverse isotropy, and not some more general anisotropic symmetry, are a much quoted paper by Jolly (1956) and recently a paper by Robertson and Corrigan (1983). Both papers refer to observations of shear-waves splitting into SV- and SH-polarizations in the top two or three hundred meter of near-surface shale. This is caused by the intrinsic vertical transverse isotropy of the shale.

Transverse isotropy (hexagonal symmetry) is one of the eight classes of aniso-

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** British Geological Survey, Murchison House, West Mains Road, Edinburgh, Scotland EH9 3LA, UK.

tropic (crystalline) symmetry, and all (uniform) elastic materials must belong to one of these eight classes. The classes range from, at one extreme, isotropy specified by two elastic constants and having spherical symmetry, to the other extreme of triclinic symmetry specified by up to 21 elastic constants and having no symmetry apart from inverse symmetry (so that it takes as long for a wave to travel from A to B as from B to A). Transverse isotropy is specified by five independent elastic constants and has an axis of rotational symmetry so that the properties are identical (isotropic) for all directions at right angles to the axis, and when the axis is vertical there is vertical transverse isotropy. This note suggests that there are more fundamental differences between structures possessing azimuthal anisotropy and vertical transverse isotropy than there are between those possessing vertical transverse isotropy and isotropy.

The reason for this note is that anisotropy-induced shear-wave splitting (shear-wave bi-refringence) is now almost routinely observed in three-component vertical seismic profiles (VSPs). Gal'perin (personal communication) recognizes shear-wave splitting in figs 41, 42, 43 and 45, and 66 and 67 of Gal'perin (1977). Shear waves at near vertical incidence through vertical transverse isotropy propagate in directions very close to the symmetry axis with nearly identical velocities and will display minimal shear-wave splitting. Thus, the shear-wave splitting observed in near vertical arrivals in VSPs, or in steep angled reflections, is diagnostic of azimuthal anisotropy, *not* vertical transverse isotropy. Since distributions of cracks and pores with preferred orientations are one of the major causes of effective anisotropy, there are considerable advantages to be gained from the correct interpretation of such azimuthal anisotropy (Crampin 1984).

Certainly the presence of (azimuthal) anisotropy complicates the *conventional* analysis of shear-wave record sections: both conventional CDP gathers and conventional deconvolution techniques may seriously degrade the signal if anisotropy is not taken into account and correctly treated. However, it is suggested (Crampin 1985) that comparatively simple measurements of anisotropic parameters in displays of the three-dimensional particle displacements (*polarization diagrams*) can yield estimates of the orientations of liquid-filled cracks and pores. Such liquid-filled cracks and pores have only second-order effects on P-wave propagation and thus are nearly invisible to P-waves (Crampin 1978), which is the reason why this common phenomenon of crack-induced anisotropy has not been recognized earlier. In contrast, aligned liquid-filled cracks and pores split shear waves into components with different polarizations and different velocities, so that even quite weak effective anisotropy may write characteristic signatures into the waveforms (see fig. 1 of Crampin 1985) that can be easily recognized in polarization diagrams of the shear-wave displacements. This means that the isotropic P-wave velocity models of the shallow crust, which have been so successful in exploration seismics, can co-exist with models of crack-induced shear-wave anisotropy (estimated from shear-wave splitting) with little conflict or disagreement. Since the preferred orientations of cracks and pores are likely to be associated with preferred directions of flow in reservoir rocks, there are many, possibly very important, applications for such investigations of effective crack-induced anisotropy (Crampin 1984, 1985).

Body Waves in Rocks with Azimuthal Anisotropy

The velocities of body waves in an arbitrary direction through an anisotropic rock which does not possess vertical transverse isotropy cannot be written explicitly, but are roots of an equation which must be solved numerically. There are several ways of writing the equations; for example, they can be written as the roots of a sixth-order polynomial expression, or as the solution of three simultaneous equations in V^2. However, the most powerful and informative technique is to rotate the coordinate axis so that the direction of propagation is transformed into the x_1-direction. The velocities of the body waves can then be written as the eigen-values of a real linear 3×3 eigenvalue problem in V^2 (Crampin 1981):

$$(\mathbf{T} - \rho V^2 \mathbf{I})\mathbf{a} = 0\,, \tag{1}$$

where $\mathbf{T}$ is the 3×3 matrix with elements c_{i1j1} for $i, j = 1, 2, 3$, c_{ijkm} are elements of the fourth-order elastic tensor, ρ is the density, V is the phase velocity, $\mathbf{I}$ is the 3×3 identity matrix, and $\mathbf{a}$ is the displacement vector for the appropriate body-wave. Equation (1) must be solved numerically. However, eigenvalue problems have been used extensively in many branches of physics and engineering and a variety of solutions are readily available on most computers. Note that the sixth-order polynomial equivalent to (1) is the expansion of the determinant of $(\mathbf{T} - \rho V^2 \mathbf{I})$ equated to zero. Note also that there are several equivalent ways of writing the elastic constants: as elements c_{ijkm} of a fourth-order tensor for $i, j, k, m = 1, 2, 3$ as in (1); as elements c_{ij} of a second-order matrix for $i, j = 1, 2, \ldots 6$; or as the Love constants A, B, C, etc. (the equivalent relationships are given in Crampin and Radovich 1982, and elsewhere).

Since (1) is a third order eigenvalue problem, there are three roots in V^2 (the square means that the velocities are the same in opposite directions). There are three body-wave roots: a quasi longitudinal wave qP, having particle displacements which are usually close but not exactly parallel to the normal to the surface of constant phase; and two quasi shear-waves, qS1 and qS2, with different velocities. The three displacement directions corresponding to the three wave types (qP, qS1, qS2) belonging to one direction of phase propagation are analytically orthogonal. Thus, in any direction of phase propagation in anisotropic rocks two shear-waves may propagate, but—because of possible irregularities and cusps in the group-velocity surface (the *wave surface*)—more than two split shear waves may propagate along any ray direction. This means that an incident shear wave necessarily splits into several phases which are not polarized in and at right angles to the plane of propagation, except, possibly, in particular symmetry directions. The body waves are called *quasi* longitudinal and *quasi* transverse, because there is three-dimensional coupling of the body-wave displacements so that the displacements are not purely longitudinal and purely transverse. The dispersion of velocities with direction results in the energy in the seismic ray deviating from the normal to the surface of constant phase, so that, in general, the ray path is not normal to the surface of constant phase. All these phenomena, in general, can only be evaluated numerically and mean that computer techniques must be used to obtain almost every characteristic of wave propagation in anisotropic rocks.

Crampin (1985) shows, however, that for crack alignments which are probably appropriate for crustal rocks, comparatively simple measurements in polarization diagrams can be directly interpreted in terms of crack orientations.

Body Waves in Rock with Vertical Transverse Isotropy

Wave propagation in transversely isotropic solids with an *arbitrary orientation* of the symmetry axis has an azimuthal variation of properties, and must be treated in the same way as general azimuthal anisotropy if the boundary conditions across plane horizontal interfaces are going to be easily accommodated. However, when the symmetry axis is *vertical* (specifically, normal to parallel plane interfaces), transverse isotropy has unique properties, in particular, the eigenvalue equation (1) factorizes so that the P- and SV-wave motion in the plane of incidence is independent of the (perpendicular) SH-wave particle motion. The *only other situation* where this separation of P- and SV-wave motion from SH-motion can occur is in an *isotropic* structure with horizontal interfaces.

The factorization of (1) is a fundamental simplification: the velocity of the SH-wave can be written explicitly, and the expressions for the velocities of the P- and SV-waves involve a binomial equation which can be solved with, at most, a square root. White (1965) gives convenient expressions for the body-wave velocities

$$V_{qP} = [(X + Y)/2\rho]^{1/2}, \tag{2}$$

$$V_{qSV} = [(X - Y)/2\rho]^{1/2} \tag{3}$$

and

$$V_{qSH} = [(Ns^2 + Lc^2)/\rho]^{1/2}, \tag{4}$$

where

$$X = (As^2 + Cc^2 + L),$$

$$Y = \{[(A - L)s^2 - (C - L)c^2]^2 + 4(F + L)^2 s^2 c^2\}^{1/2},$$

A, C, F, L, and N are the appropriate Love constants, $s = \sin\theta$, $c = \cos\theta$, and θ is the angle between the direction of (phase) propagation and the vertical symmetry axis. As with general anisotropy, there are three body waves: a qP-wave, and two quasi shear waves, qSV and qSH. The two quasi shear waves again have orthogonal polarizations but now are polarized strictly in a vertical plane and in a transverse horizontal direction.

The coupling of P- and SV-motion in vertical planes and the complete independence of the SH-motion is similar to the behavior in plane-layered isotropic solids. Correspondingly, the ability to write the three body-wave velocities explicitly, in terms of the elastic constants and the direction of propagation through the solid, means that many of the techniques and processes used for analysis of *isotropic* structures can be applied directly to propagation in *vertical transverse isotropy*, with many of the simplifications that that implies.

Conclusions

As Crampin, Chesnokov and Hipkin (1984) have stressed, vertical transverse isotropy is such an exceptional example of propagation in anisotropic solids that it is misleading to class vertical transverse isotropy with general anisotropy. Shear waves in vertical transverse isotropy separate (split) strictly into SV- and SH-displacements and do not possess the important and diagnostic information carried by the more general polarizations of shear waves in azimuthal anisotropy (Crampin 1984, 1985). The differences are so great and the implications so misleading that it would be useful if the term (vertical) *transverse isotropy* rather than *anisotropy* were cited in the titles of such publications. Levin and Helbig usually do this, but others are not so correct. This use of *anisotropy* for (vertical) *transverse isotropy* is also common in earthquake seismology, where it has similar misleading implications.

It is particularly important to stress the difference between (azimuthal) anisotropy and (vertical) transverse isotropy now that shear-wave splitting is almost routinely observed when shear vibrators are recorded in VSPs. As indicated in the Introduction, shear-wave splitting observed in VSPs, or in steep angled reflections, is diagnostic of azimuthal anisotropy, not transverse isotropy. Such splitting is likely to be caused by propagation through aligned cracks and pores (Crampin 1984, 1985). In such effective anisotropy, there are variations with azimuth and the shear-waves split into distinct polarizations which are not SV- and SH-polarizations except in specific symmetry directions. Attempts to interpret this splitting in terms of vertical transverse isotropy by conventional time-section analysis could be seriously misleading, as well as discarding the valuable information about crack orientations, preferred directions of flow, and stress orientations contained in shear-wave polarizations through aligned cracks and pores.

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