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**416.** Investigations of the Olefinic Acids. Part XVII. The Addition of Hydrogen Bromide to Heptenoic and Nonenoic Acids with Terminal Double Bonds.

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The addition of hydrogen bromide to  $\Delta^6$ -n-heptenoic and  $\Delta^8$ -n-nonenoic acids has been studied with reference to the effect of solvents and peroxides. The results are reviewed in the light of other work in this field.

The addition of hydrogen bromide to the heptenoic and nonenoic acids (preceding paper) was studied under various conditions. We also re-examined the behaviour of allylacetic acid in view of a discrepancy between our previous results (Linstead and Rydon, J., 1934,

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2001) and those of Kharasch and McNab (Chem. and Ind., 1935, 54, 989). Stated briefly, the American investigators (who used a different technique) had confirmed our observation that terminal additions of bromine occurred in the presence of peroxide, but obtained y-bromovaleric acid in the presence of anti-oxidants, whereas we obtained terminal addition in hexane even in the presence of hydrogen or anti-oxidants.\*

A repetition of our experiments in peroxide-free hexane with hydrogen bromide prepared catalytically from its elements has now confirmed our previous result that terminal addition occurred in the presence of both hydrogen and diphenylamine. This interesting anomaly is discussed later.

In the case of the higher acids, the direction of addition was determined, where possible, by direct comparison of the hydrobromide with material independently synthesised. For this purpose, 7-bromo-n-heptoic acid (IV) was prepared by the following method, the last stage of which follows the method of von Braun (Ber., 1906, **39**, 4362):

When such direct comparison was not possible, the bromo-acid was converted into the corresponding dibasic acid by esterification, condensation with ethyl sodiomalonate, and acid hydrolysis:

$$\begin{array}{c} \mathrm{CHRBr} \boldsymbol{\cdot} [\mathrm{CH}_2]_n \boldsymbol{\cdot} \mathrm{CO}_2 \mathrm{H} \longrightarrow \mathrm{CHRBr} \boldsymbol{\cdot} [\mathrm{CH}_2]_n \boldsymbol{\cdot} \mathrm{CO}_2 \mathrm{Et} \longrightarrow \\ (\mathrm{CO}_2 \mathrm{Et})_2 \mathrm{CH} \boldsymbol{\cdot} \mathrm{CHR} \boldsymbol{\cdot} [\mathrm{CH}_2]_n \boldsymbol{\cdot} \mathrm{CO}_2 \mathrm{Et} \longrightarrow \mathrm{CO}_2 \mathrm{H} \boldsymbol{\cdot} \mathrm{CH}_2 \boldsymbol{\cdot} \mathrm{CHR} \boldsymbol{\cdot} [\mathrm{CH}_2]_n \boldsymbol{\cdot} \mathrm{CO}_2 \mathrm{H} \end{array}$$

β-Methylsuberic acid (VIII), required for reference, was prepared synthetically from δ-acetovaleric ester (V) by the following series of reactions:

$$(V.) \ \text{Me•CO•[CH$_2]}_4 \cdot \text{CO$_2$Et} \xrightarrow{\text{CH$_2$Br•CO$_2$Et}, \ Zn} \text{CO$_2$Et•CH$_2$•CMe•[CH$_2]}_4 \cdot \text{CO} \ (VI.) \longrightarrow \\ (VII.) \ \text{CO$_2$Et•CH$:CMe•[CH$_2]}_4 \cdot \text{CO$_2$Et} \longrightarrow \text{CO$_2$H•CH$_2$•CHMe•[CH$_2]}_4 \cdot \text{CO$_2$H} \ (VIII.)$$

Our attempt to synthesise \u03b3-methylsebacic acid from 8-ketononoic acid by a similar process failed, but it confirmed some similar experiments by Barger, Robinson, and L. H. Smith (this vol., p. 718), an account of which appeared after the completion of our results. We obtained 8-ketononoic acid, identical with theirs, by the condensation of sodioacetoacetic ester with ethyl 6-bromohexoate. The bromo-ester was prepared, unlike that of Barger, Robinson, and L. H. Smith, by esterifying the product of the addition in petroleum solution of hydrogen bromide to  $\Delta^{5}$ -n-hexenoic acid (cf. Linstead and Rydon, J., 1934, 1995). This is another example of the formation of a terminal bromo-acid under these conditions.

The results of the additions are summarised below:

Addition of hydrogen bromide to CH2:CH·[CH2]n·CO2H.

n.	Solvent for acid.	Diluent for HBr.	Catalyst.	Direction of addition.
4	None	Hydrogen	None	Mainly proximate
,,	Acetic acid Hexane	,,	"	Terminal
,,	nexane	,,	Diphenylamine	Mainly terminal
,,	None	Oxygen	Benzoyl peroxide	,, ,,
6	None	Hydrogen	None	Mainly proximate
,,	Acetic acid Hexane	,,	,,	Mixed Mainly terminal
,,		,,	Diphenylamine	•
,,	None	Oxygen	Benzoyl peroxide	Terminal "

Additions were fast in the presence of benzoyl peroxide, slow in the presence of diphenylamine, and intermediate in the other experiments.

\* J. C. Smith (Chem. and Ind., 1937, 56, 833, footnote) reports that addition in hydrocarbon solvents yielded the δ-bromo-acid in the presence of air, but in hydrogen gave an oil which readily lost hydrogen bromide. The y-isomeride is certainly the less stable of the bromo-acids, but as it can easily be obtained as a crystalline solid from suitable additions (Boorman, Linstead, and Rydon, J., 1933, 575), it seems probable that this product was a mixture.

These results are in general agreement with those obtained in our study of allylacetic acid, but are less clear-cut, possibly because the reactions were slower. They also agree in many respects with Ashton and Smith's observations on undecenoic acid (J., 1934, 435; Smith, loc. cit.). The persistent anomaly is the formation of the terminal bromoacids in hexane or petroleum solution in the presence of hydrogen and anti-oxidant, which we have now observed in the case of five acids of the general formula  $CH_2:CH\cdot[CH_2]_n\cdot CO_2H$  (where n=1, 2, 3, 4 and 6), but which is not shown by undecenoic acid (n=8), or by allylacetic acid in the experiments of Kharasch and McNab. This discrepancy may arise from some difference in technique or in the purity of materials, or perhaps in the varying capacity of acids (or of different samples of the same acid) to form a peroxide or similar catalyst.

In spite of the obvious importance of the effect of peroxide or oxygen in determining the orientation of additions of hydrogen bromide generally, we do not consider that the evidence at present available justifies the contention that solvents only influence orientation in so far as they prevent (or enhance) this effect. To take two examples: (i) Sherrill, Mayer, and Walker's experiments on additions to olefins show a profound influence of solvents in the absence of peroxide or oxygen (J. Amer. Chem. Soc., 1934, 56, 926, 1645). (ii) Vinylacetic acid yields  $\beta$ -bromobutyric acid in the absence of solvent or when dissolved in ether or acetic acid; in the presence of peroxide the  $\beta$ -bromo-acid is still produced, but the  $\gamma$ -bromo-acid is readily and completely formed in hexane even in the presence of hydrogen or anti-oxidants. In view of these facts the statement of J. C. Smith (loc. cit.) that the usual orientation of addition is reversed in presence of peroxide is unacceptable, as there is no direct evidence that peroxide has any influence at all in this case. It would be preferable to say that the usual orientation is reversed in hexane solution.

In spite of the uncertainty still attaching to the theoretical position, for practical purposes sufficient information has now been obtained to enable additions to be carried out at will in one or other of the two directions. Thus a terminal bromo-acid will be obtained when hydrogen bromide is passed into a solution of a terminally unsaturated acid in a hydrocarbon in the presence of oxygen or a peroxide. Non-terminal addition will predominate when the acid is undiluted and the reaction is carried out in hydrogen or in the presence of an anti-oxidant.

## EXPERIMENTAL.

Synthesis of 7-Bromo-n-heptoic Acid.—A solution of 10 g. of sodium hydroxide in 30 c.c. of water was added dropwise during an hour, with stirring, to a boiling mixture of 75 g. of pentamethylene dibromide, 24.6 g. of phenol, and 130 c.c. of water. Stirring and refluxing were continued for 5 hours. After cooling, the lower layer was run off, dried, and distilled, yielding 23 g. (29%) of 5-phenoxy-n-amyl bromide (I), b. p.  $160-165^{\circ}/11$  mm. (Found: Br, 32.4.  $C_{11}H_{15}OBr$  requires Br, 32.9%). Much pentamethylene dibromide was recovered and used in further experiments.

To a solution of sodium ethoxide prepared from 3·23 g. of sodium in 40 c.c. of absolute alcohol, 22·5 g. of ethyl malonate were added, followed by 34 g. of the phenoxyamyl bromide. The mixture was refluxed on the steam-bath overnight, and the product worked up in the usual manner, yielding 24 g. (53%) of ethyl 5-phenoxy-n-amylmalonate (II), b. p. 200—204°/4 mm. 22 G. of this ester were refluxed for an hour with 15 g. of potassium hydroxide, 20 c.c. of water, and 5 c.c. of ethyl alcohol. The alcohol was removed by means of the pump, and the cooled residue acidified. Filtration, followed by drying on a porous tile, gave 15 g. (83%) of 5-phenoxy-n-amylmalonic acid, m. p. 116—119°, which on decarboxylation at 150° for 3 hours and crystallisation of the product from light petroleum yielded 9 g. (72%) of 7-phenoxy-n-heptoic acid (III), m. p. 55° (Found: C, 69·7; H, 8·0. Calc.: C, 70·3; H, 8·1%) (von Braun, loc. cit., gives m. p. 56—57°).

6 G. of the phenoxyheptoic acid were refluxed for 20 hours with 30 c.c. of hydrobromic acid ( $d \cdot 1.48$ ); the product was slowly distilled through a long column until the temperature at the head rose to  $120^{\circ}$ . The residue was diluted with water and extracted with ether. Distillation of the dried extract yielded 1.5 g. (30%) of 7-bromo-n-heptoic acid (IV), b. p.  $146-148^{\circ}/4$  mm., which solidified on cooling; m. p.  $29^{\circ}$  (Found: Br, 37.7. Calc.: Br, 38.3%) (von Braun, loc. cit., gives m. p.  $30-31^{\circ}$ ).

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Synthesis of  $\beta$ -Methylsuberic Acid.—55.6 G. of  $\gamma$ -bromobutyric acid (m. p. 29°; prepared by adding hydrogen bromide to vinylacetic acid in hexane solution; Boorman, Linstead, and Rydon, *loc. cit.*) were treated with an ice-cold mixture of 30.6 c.c. of absolute alcohol and 8·1 c.c. of concentrated sulphuric acid. After standing at 0° for 4 hours and at room temperature for 2 days, the product was poured on ice, and the ester isolated in the usual way; yield 50 g. (77%), b. p. 98—100°/20 mm.

17.3 G. of ethyl acetoacetate were added to sodium ethoxide prepared from 3 g. of sodium and 36 c.c. of absolute alcohol. The mixture was treated with 26 g. of ethyl  $\gamma$ -bromobutyrate and refluxed on the steam-bath for 24 hours. On being worked up in the usual way, 21 g. (65%) of ethyl  $\alpha$ -acetyladipate, b. p. 165—180°/15—20 mm., were obtained. This was refluxed for 20 hours with 5 vols. of constant-boiling hydrochloric acid; distillation of the product yielded 10 g. (81%) of  $\delta$ -acetyl-n-valeric acid, b. p. 158°/4 mm., m. p. 31—32°. Attempts to reduce this to 6-heptolactone were unsuccessful.

29 G. of acetylvaleric acid were refluxed on the steam-bath for 48 hours with a solution of 1.5 g. of anhydrous hydrogen chloride in 100 c.c. of absolute alcohol, and yielded 28 g. (81%) of ethyl δ-acetyl-n-valerate (V), b. p. 132-138°/16 mm. This was refluxed overnight with 27 g. of ethyl bromoacetate and 15 g. of zinc in 500 c.c. of dry benzene. The product was cooled and poured into dilute hydrochloric acid. Extraction with ether, followed by drying and distillation, gave 12 g. of a product, b. p. 120-125°/1 mm., which is considered to be mainly the lactonic ester (VI) (Found: C,  $62\cdot3$ ; H,  $8\cdot5$ .  $C_{11}H_{18}O_4$  requires C,  $61\cdot7$ ; H,  $8\cdot4\%$ ). It was dissolved in 15 c.c. of absolute alcohol and saturated with hydrogen bromide. After standing overnight, the alcohol was removed under reduced pressure, and the residue treated with water; extraction with ether, followed by drying and distillation, afforded 4 g. of partly unsaturated material, b. p. 149-152°/1 mm. This was mixed with 1.2 g. of pyridine and 5 c.c. of benzene and cooled in ice while 2.5 g. of thionyl chloride were added with stirring. After standing overnight, the product was poured into water and extracted with ether; distillation of the dried extract yielded 1.6 g. of an ester, mainly ethyl 2-methyl- $\Delta^1$ -n-hexene-1:6-dicarboxylate (VII), b. p. 160—162°/10 mm. (Found: C, 65.0; H, 9.7. C<sub>13</sub>H<sub>22</sub>O<sub>4</sub> requires C, 64.4; H, 9.1%).

Sulpnur compounds were removed by subjecting the unsaturated ester, dissolved in moist ether (100 c.c.), to the action of aluminium amalgam (from 1·5 g. of aluminium foil) for 2 days. The ester recovered from the ethereal filtrate was dissolved in 30 c.c. of glacial acetic acid and shaken in hydrogen with 0·1 g. of Adams's catalyst; the calculated amount of hydrogen for one double bond was taken up in 30 minutes, after which absorption ceased. Distillation of the filtered product gave 0·9 g. of ethyl β-methylsuberate, b. p. 120—125°/5 mm. This was refluxed for 20 hours with 10 c.c. of constant-boiling hydrochloric acid; crystals were deposited on cooling, and a further crop was obtained on concentration of the mother-liquor. Crystallisation from benzene-light petroleum (b. p. 60—80°) gave 0·5 g. of β-methylsuberic acid (VIII), m. p. 83° [Found: C, 57·4; H, 8·9; equiv., 92·8. C<sub>9</sub>H<sub>16</sub>O<sub>4</sub> (dibasic) requires C, 57·4; H, 8·5%; equiv., 94·0].

Attempted Synthesis of  $\beta$ -Methylsebacic Acid.—The compounds mentioned in this section have all recently been described by Barger, Robinson, and L. H. Smith (loc. cit.) and we therefore only give details of those experiments which differ from theirs.

Dry hydrogen bromide was passed into 53 g. of Δ<sup>5</sup>-n-hexenoic acid (Linstead and Rydon, J., 1934, 1995), dissolved in 300 c.c. of light petroleum (b. p. 60—80°), until the product was saturated. The bromo-acid (90 g.), obtained on removal of the solvent, was cooled in ice and treated with a cold mixture of 7·7 c.c. of concentrated sulphuric acid and 61 c.c. of absolute alcohol. After standing in ice for 4 hours and at room temperature for 2 days, the mixture was poured into water and worked up in the usual way. The product (85 g.), b. p. 126—130°/18 mm., was shown to be ethyl 6-bromo-n-hexoate by malonation: 2 G. were added to ethyl sodiomalonate, prepared from 0·2 g. of "molecular" sodium and 1·6 g. of ethyl malonate in benzene, and the mixture heated on the steam-bath for 2 days. The resulting ester, isolated in the usual manner, was hydrolysed without distillation by refluxing for 24 hours with 20 c.c. of constant-boiling hydrochloric acid. On cooling, 0·7 g. of a solid, m. p. 132—134°, crystallised; recrystallisation showed this to be suberic acid, m. p. and mixed m. p. 137°.

8-Ketononoic Acid.—The following conditions give a rather better yield than that of Barger, Robinson, and L. H. Smith. 85 G. of ethyl 6-bromohexoate were added to ethyl sodioacetoacetate prepared from sodium (7.5 g.), absolute alcohol (75 g.), and ethyl acetoacetate (45 g.). After refluxing overnight, the product was worked up in the usual way,

65 g. (63%) of ethyl  $\alpha$ -acetylsuberate, b. p.  $200^{\circ}/2-3$  mm., being obtained. This was refluxed for 20 hours with 50 c.c. of concentrated hydrochloric acid and 100 c.c. of water. After distillation, the fraction, b. p.  $165-170^{\circ}/1.5$  mm., was freed from neutral material by means of sodium bicarbonate solution; redistillation then gave 28 g. (68%) of 8-ketononoic acid, b. p.  $148^{\circ}/0.8$  mm., m. p.  $40^{\circ}$  (Found: C, 62.8; H, 9.3; equiv., 173.6. Calc.: C, 62.8; H, 9.3; equiv., 172). Barger, Robinson, and L. H. Smith give m. p.  $40.5-42^{\circ}$ . The ethyl ester boiled at  $145-147^{\circ}/15$  mm. (Found: C, 66.0; H, 10.0. Calc.: C, 66.0; H, 10.0%).

Addition of Hydrogen Bromide.—The general procedure followed that used in our previous work. The hydrogen bromide was, however, prepared by the method of Ruhoff, Burnett, and Reid ("Organic Syntheses," 1935, 15, 35). It was dried over phosphoric oxide and freed from bromine by means of a long column of copper turnings. The apparatus was so arranged that the current of hydrogen bromide could be replaced or augmented by either hydrogen or oxygen. The additions were carried out in U-tubes fitted with ground-glass joints and cooled at 0°. Before the reaction these were swept out with either hydrogen or oxygen. The passage of hydrogen bromide was continued until an excess had been taken up. The product was kept overnight, and the solvent and excess of reagent removed. When a solid bromo-acid was formed, it was, if possible, identified by direct comparison. More generally, the bromo-acid was esterified by the method of Wohlgemuth (Ann. Chim., 1914, 2, 298), and the distilled bromo-ester condensed with ethyl sodiomalonate as previously described (J., 1933, 573); it was generally desirable to remove low-boiling products (lactone, etc.) by distillation. The resulting tribasic ester was hydrolysed by refluxing with concentrated hydrochloric acid (5 vols.); the dibasic acid was identified by direct comparison.

- (1)  $\Delta^4$ -Pentenoic acid in hexane; hydrogen; no catalyst. Two experiments were carried out; in the first a little free bromine was allowed to pass over with the hydrogen bromide; in the second it was rigorously excluded. In both cases  $\delta$ -bromovaleric acid, m. p. and mixed m. p.  $40^{\circ}$ , was obtained.
- (2) Same acid in hexane; hydrogen; diphenylamine (1%). 2.6 G. of acid. Esterification of the oily bromo-acid gave 0.8 g. of bromo-ester, b. p.  $122-126^{\circ}/22$  mm. Malonation and hydrolysis gave 0.2 g. of dibasic acid; crystallisation from benzene-light petroleum (b. p.  $60-80^{\circ}$ ) gave pimelic acid, m. p. and mixed m. p.  $104^{\circ}$ .
- (3)  $\Delta^6$ -Heptenoic acid; no solvent; hydrogen; no catalyst. 2.8 G. of acid. The bromo-ester (3.6 g.) had b. p. 135°/17 mm.; malonation and hydrolysis gave 1.2 g. of dibasic acid, m. p. 63—66°. Recrystallisation from benzene-light petroleum gave an acid, m. p. 78°, mixed m. p. with  $\beta$ -methylsuberic acid 83°.
- (4) Same acid in hexane; hydrogen; no catalyst.  $2\cdot3$  G. of acid. The bromo-acid ( $3\cdot2$  g.), m. p. 17— $19^\circ$ , gave  $2\cdot1$  g. of bromo-ester, b. p. 144— $145^\circ/22$  mm. (Found: Br,  $32\cdot7$ . Calc. for  $C_9H_{17}O_2$ Br: Br,  $33\cdot8\%$ ). Malonation gave  $2\cdot0$  g. of tribasic ester, b. p. 138— $144^\circ/27$  mm., hydrolysis of which gave  $1\cdot7$  g. of dibasic acid. After one crystallisation from benzene-light petroleum this had m. p. 104— $105^\circ$ , mixed m. p. with azelaic acid  $105^\circ$ .
- (5) Same acid in acetic acid; hydrogen; no catalyst. 9.7 G. of acid. The bromo-acid (15.8 g.) gave, on esterification, 14.3 g. of bromo-ester, b. p.  $132^{\circ}/14$  mm. Malonation and hydrolysis gave 3 g. of dibasic acid, m. p.  $80^{\circ}$ ; one crystallisation from benzene-light petroleum gave  $\beta$ -methylsuberic acid, m. p.  $82^{\circ}$ , mixed m. p.  $83^{\circ}$ .
- (6) Same acid, no solvent; oxygen; benzoyl peroxide (1%). 2·2 G. of acid. The bromo-ester  $(2\cdot7$  g.), b. p.  $129-132^{\circ}/13$  mm., gave, on malonation and hydrolysis, 1·4 g., m. p.  $95-100^{\circ}$ . Crystallisation from benzene-light petroleum gave azelaic acid, m. p. and mixed m. p.  $105^{\circ}$ .
- (7) Same acid in hexane; hydrogen; diphenylamine (1%). 2·3 G. of acid. The bromo-ester (2·5 g.), b. p. 130°/10 mm., gave, on malonation and hydrolysis, 0·4 g. of acid, m. p. 100—102°. Crystallisation from benzene-light petroleum gave azelaic acid, m. p. and mixed m. p. 104°.
- (8)  $\Delta^8$ -Nonenoic acid; no solvent; hydrogen; no catalyst. 4·1 G. of acid gave 3·4 g. of bromo-acid. The bromo-ester (2·7 g.), b. p. 146—48°/8 mm., gave, on malonation, 1·0 g. of tribasic ester, b. p. 195—200°/4 mm. Hydrolysis gave 0·5 g. of dibasic acid, m. p. 65—75°; crystallisation, first from benzene-light petroleum and then from dilute acetic acid, gave  $\beta$ -methylsebacic acid, m. p. 75—76° (Found: C, 61·0; H, 9·4.  $C_{11}H_{20}O_4$  requires C, 61·1; H, 9·2%).
- (9) Same acid in acetic acid; hydrogen; no catalyst. 4.2 G. of acid gave 6.3 g. of bromoacid. The bromo-ester (5.9 g.), b. p.  $142-146^{\circ}/8$  mm., gave, on malonation, 1.5 g. of tribasic ester, b. p.  $160-165^{\circ}/0.6$  mm. Hydrolysis gave 0.7 g. of dibasic acid; crystallisation from

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benzene-light petroleum gave two products. The more soluble was crystallised from dilute acetic acid and identified as  $\beta$ -methylsebacic acid, m. p. and mixed m. p.  $73-76^{\circ}$ . The less soluble was recrystallised from water and identified as nonane-1:9-dicarboxylic acid, m. p.  $107^{\circ}$ , mixed m. p.  $109^{\circ}$ . An authentic specimen of this compound was prepared by oxidation of  $\omega$ -hydroxyundecoic acid, for a sample of which we are much indebted to Professor A. C. Chibnall, F.R.S.

(10) Same acid in hexane; hydrogen; no catalyst. 1 G. of acid. Esterification of the bromo-acid gave 1·4 g. of bromo-ester, b. p. 150—152°/9 mm. Malonation and hydrolysis gave 0·4 g. of dibasic acid; crystallisation from benzene-light petroleum, and then from water, gave nonane-1: 9-dicarboxylic acid, m. p. and mixed m. p. 110°.

(11) Same acid in hexane; hydrogen; diphenylamine (1%). 4·3 G. of acid. Esterification of the bromo-acid gave 6 g. of bromo-ester, b. p. 148—152°/8 mm. Malonation gave 3·2 g. of tribasic ester, b. p. 165—175°/0·2—0·4 mm., which on hydrolysis gave a dibasic acid, m. p. 105°, mixed m. p. with nonane-1: 9-dicarboxylic acid 108—110°.

(12) Same acid; no solvent; oxygen; benzoyl peroxide (1%). 3 G. of acid gave 3.5 g. of solid 9-bromo-n-nonoic acid, m. p. 37—38° (Found: Br, 34.0. C<sub>9</sub>H<sub>17</sub>O<sub>2</sub>Br requires Br, 33.8%). Esterification gave 3.0 g. of bromo-ester, b. p. 150—154°/10 mm., which, on malonation, gave 2.9 g. of tribasic ester, b. p. 175—180°/1 mm. Hydrolysis and recrystallisation from benzene-light petroleum gave nonane-1: 9-dicarboxylic acid, m. p. 107°, mixed m. p. 107—108°.

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