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The Stark Effect for a Rigid Asymmetric Rotor*

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The Stark effect, arising from the interaction of a uniform electric field with a permanent electric dipole that is arbitrarily oriented within a rigid asymmetric rotor, and with a dipole induced in the rotor by the field, has been evaluated by perturbation methods. Tables are given for the perturbation of the energy levels so that, for $J \leq 2$, the rotational energies of an asymmetric molecule in an electric field may be readily approximated to terms quadratic in the electric field.

The effect of accidental degeneracy upon the Stark effect and line intensities has been considered. A qualitative discussion of certain features of Stark patterns is given that may be useful in the identification of rotational spectral lines.

INTRODUCTION

THE recent progress in microwave spectroscopy has made it possible to observe the Stark effect in the pure rotational spectra of many molecules.[1] The formulas for the influence of an homogeneous electric field upon the rotational energy levels of linear[2] and symmetric-rotor[3] molecules are well known. The case of the rigid asymmetric rotor has been treated by Penney.[4] The purpose of the present paper is to extend this treatment, to give certain general rules that may facilitate the identification of spectral lines and to provide tables from which the Stark effect may be approximated for rotational energy levels up to and including $J=2$. The results are accurate to and including terms quadratic in the electric field strength.

I. THE PERTURBATION OPERATOR

When an homogeneous electric field is applied to a molecule having a permanent electric dipole moment μ and a polarizability tensor $\mathcal{P}$, the Hamiltonian operator will contain the following terms in addition to those involving rotation-vibration.

$$\begin{aligned}\mathbf{H}' &= -E\sum_g \Phi_{Zg}\mu_g - \tfrac{1}{2}E^2\sum_{g,h}\Phi_{Zg}\Phi_{Zh}P_{gh}\\ &= E\mathbf{H}^{(1)}+E^2\mathbf{H}^{(2)},\end{aligned} \tag{1}$$

where E is the electric field, assumed to be along the space-fixed Z axis; g, h refer to the molecule-fixed principal axes of inertia x, y, z; μ_g are the components of the permanent dipole moment along the principal axes; P_{gh} are the components of the polarizability tensor referred to the principal axes of inertia; Φ_{Zg} are the direction cosines between the space fixed Z axis and the rotating molecule-fixed principal axes.

The first term in this expression is the orientational energy of the permanent dipole in the field, while the second is that of the induced dipole.

II. PERTURBATION THEORY NON-DEGENERATE CASE

Even when the asymmetric-rotor energy levels are otherwise distinct, they are nevertheless $(2J+1)$-fold degenerate in the quantum number M. Here J is the total angular-momentum quantum number and M is the magnetic quantum number. However this M degeneracy offers no difficulty to the application of the conventional perturbation theory.

To apply the conventional perturbation theory the matrix elements of Eq. (1) must be evaluated in terms of a basis of asymmetric-rotor wave

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[1] Various molecules are: OCS, T. W. Dakin, W. E. Good, D. K. Coles, Phys. Rev. **70**, 560 (1946); NH_3 D. K. Coles, W. E. Good, Phys. Rev. **70**, 979 (1946); SO_2, B. P. Dailey, S. Golden, E. B. Wilson, Jr., Phys. Rev. **72**, 871 (1947); H_2O, S. Golden, T. Wentink, M. W. P. Strandberg, Phys. Rev. **73**, 92 (1948).

[2] See, for example, J. H. Van Vleck, *Electric and Magnetic Susceptibilities* (Oxford Press, 1932), p. 152.

[3] See, for example, C. Mannebeck, Physik. Zeits. **28**, 72 (1927).

[4] W. G. Penney, Phil. Mag. **11**, 602 (1931).

TABLE I. The structure of the perturbation energy matrix, $\mathbf{H}^{(1)}$, due to a permanent dipole.

Species	A	B_z	B_y	B_x
A	$\cdots$	$\mu_z\Phi_{Zz}$	$\mu_y\Phi_{Zy}$	$\mu_x\Phi_{Zx}$
B_z	$\mu_z\Phi_{Zz}$	$\cdots$	$\mu_x\Phi_{Zx}$	$\mu_y\Phi_{Zy}$
B_y	$\mu_y\Phi_{Zy}$	$\mu_x\Phi_{Zx}$	$\cdots$	$\mu_z\Phi_{Zz}$
B_x	$\mu_x\Phi_{Zx}$	$\mu_y\Phi_{Zy}$	$\mu_z\Phi_{Zz}$	$\cdots$

TABLE II. The structure of the perturbation energy matrix, $\mathbf{H}^{(2)}$, due to an induced dipole.

Species	A	B_z	B_y	B_x
A	$\Sigma_g P_{gg}(\Phi_{Zg})^2$	$P_{xy}(\Phi_{Zx})(\Phi_{Zy})$	$P_{xz}(\Phi_{Zx})(\Phi_{Zz})$	$P_{yz}(\Phi_{Zy})(\Phi_{Zz})$
B_z	$P_{xy}(\Phi_{Zx})(\Phi_{Zy})$	$\Sigma_g P_{gg}(\Phi_{Zg})^2$	$P_{yz}(\Phi_{Zy})(\Phi_{Zz})$	$P_{xz}(\Phi_{Zx})(\Phi_{Zz})$
B_y	$P_{xz}(\Phi_{Zx})(\Phi_{Zz})$	$P_{yz}(\Phi_{Zy})(\Phi_{Zz})$	$\Sigma_g P_{gg}(\Phi_{Zg})^2$	$P_{xy}(\Phi_{Zx})(\Phi_{Zy})$
B_x	$P_{yz}(\Phi_{Zy})(P_{Zz})$	$P_{xz}(\Phi_{Zx})(\Phi_{Zz})$	$P_{xy}(\Phi_{Zx})(\Phi_{Zy})$	$\Sigma_g P_{gg}(\Phi_{Zg})^2$

functions. The latter may be expressed as linear combinations of symmetric-rotor wave functions,[5] all having the same values of J and M. The matrix elements of the direction cosines Φ_{Zg} evaluated in terms of a symmetric-rotor wave function basis are diagonal in M.[6] Hence it follows that in terms of the asymmetric-rotor functions the direction-cosine matrix elements are also diagonal in M, and the problem can be treated for each value of M separately.

Now, the asymmetric rotor wave functions belong to the representations A, B_x, B_y, B_z of the Four-Group.[7] The direction cosines Φ_{Zg} belong to the representations B_x, B_y, and B_z for $g=x$, y and z, respectively. Since non-vanishing matrix elements can be obtained only if the product of the direction cosine and the wave functions of the connected states belongs to representation A, the results of Table I are obtained for the non-vanishing matrix elements of $\mathbf{H}^{(1)}$.

Similarly, and by use of the fact that $(\Phi_{Zg})^2$ belongs to representation A, $\Phi_{Zx}\Phi_{Zy}$ to B_z, $\Phi_{Zz}\Phi_{Zx}$ to B_y, $\Phi_{Zy}\Phi_{Zz}$ to B_x, the results of Table II are obtained for the non-vanishing elements of $\mathbf{H}^{(2)}$.

From Table I it is seen that there are no diagonal elements of $\mathbf{H}^{(1)}$. Hence if the energy levels of the unperturbed asymmetric rotor are distinct and widely separated compared to the magnitude of the coupling perturbation, there are no contributions from terms linear in the field. Any first-order effect must arise from degeneracy between two or more levels of the unperturbed rotor. In the non-degenerate second-order perturbation treatment $\mathbf{H}^{(2)}$ will contribute only diagonal elements. A proof is given in Appendix A which shows that terms cubic in the field vanish in the non-degenerate case.

[5] S. C. Wang, Phys. Rev. **34**, 243 (1929).
[6] See, for example, P. C. Cross, R. M. Hainer, G. W. King, J. Chem. Phys. **12**, 210 (1944).
[7] See, for example, G. W. King, R. M. Hainer, P. C. Cross, J. Chem. Phys. **11**, 27 (1943).

The formula for the energy levels correct to the second order is, in the absence of accidental degeneracy, the usual perturbation result and is identical with the expression in Eq. (11) below.

III. PERTURBATION THEORY-DEGENERATE CASE

In order to provide a useful tabulation of the Stark effect that will be applicable to most asymmetric molecules, it is necessary to examine the cases of accidental degeneracy that may occur. The energy of the J_τ'th level of the asymmetric rotor is[7]

$$W^0{}_{J\tau}=\frac{(a-c)}{2}E_\tau{}^J(\kappa)+\frac{(a+c)}{2}J(J+1), \qquad (2)$$

where $a=\hbar^2/2I_a$, $b=\hbar^2/2I_b$, $c=\hbar^2/2I_c$; $I_a\leq I_b\leq I_c$ are the principal moments of inertia;

$$\kappa=(2b-a-c)/(a-c),$$

the asymmetry parameter; $E_\tau{}^J(\kappa)$ is a characteristic value of the asymmetric-rotor problem. It is convenient to write Eq. (2) as

$$W^0{}_{J\tau}=\frac{(a+c)}{2}[\alpha E_\tau{}^J(\kappa)+J(J+1)], \qquad (3)$$

where

$$\alpha=\frac{a-c}{a+c} \quad \text{and} \quad 0\leq\alpha\leq+1.$$

The lower limit corresponds to the spherically symmetric case while the upper limit corresponds to the linear case. It is readily observed that the terms in brackets in Eq. (3) are dimensionless; they may be thought of as a reduced energy.

Now for degeneracy between a number of unperturbed levels

$$W^0{}_{J\tau}=W^0{}_{J'\tau'}=W^0{}_{J''\tau''}=\cdots$$

or

$$\alpha E_\tau{}^J(\kappa)+J(J+1)=\alpha E_{\tau'}{}^{J'}(\kappa)+J'(J'+1)=\cdots,$$

for $J\neq J'$, $J\neq J''$, etc.,

$$\alpha=\frac{J'(J'+1)-J(J+1)}{E_\tau{}^J(\kappa)-E_{\tau'}{}^{J'}(\kappa)}=\frac{J''(J''+1)-J(J+1)}{E_\tau{}^J(\kappa)-E_{\tau''}{}^{J''}(\kappa)}=\cdots. \tag{4}$$

In particular for $J=J'$

$$\alpha[E_\tau{}^J(\kappa)-E_{\tau'}{}^{J'}(\kappa)]=0. \tag{5}$$

In this case ($J=J'$) the degeneracy is never exact except in the symmetric rotor limits but it can be very nearly exact for many pairs of levels even for molecules with the maximum asymmetry.

Equation (4) has been applied to the energy levels for $J\leq 3$ to determine which *pairs* of levels may become accidentally degenerate in the asymmetric rotor. Table III indicates which component of the permanent dipole becomes important in the accidental degeneracy.

From Table III it is clear that as both κ and α are varied, the frequent appearance of accidental degeneracy, even at low values of J,[8] makes it impractical to apply conventional perturbation theory. By a modification of the conventional perturbation theory, such as that employed by Van Vleck[9] and Jordahl,[10] an entirely satisfactory manner of handling these cases is obtained.

Briefly, the method consists in applying to the matrix to be diagonalized (*viz.* $\mathbf{W}^0+\mathbf{H}'$, where $\mathbf{W}^0$ is the diagonal energy matrix of the asymmetric rotor and $\mathbf{H}'$ is the matrix of Eq. (1) evaluated with asymmetric-rotor wave functions) the following matrix.

$$\mathbf{S}=\mathbf{I}+\lambda\mathbf{S}^{(1)}+\lambda^2\mathbf{S}^{(2)}, \tag{6}$$

where λ is a parameter of smallness (to be associated with the electric field in the present case); $\mathbf{S}$ is unitary to second order in λ. This requires

$$\mathbf{S}^{(1)\dagger}=-\mathbf{S}^{(1)},\quad \mathbf{S}^{(2)}=\tfrac{1}{2}[\mathbf{S}^{(1)}]^2. \tag{7}$$

With no loss of generality, $\mathbf{S}^{(2)}$ may be taken to be hermitian. Now define $\mathbf{S}^{(1)}$ as follows.

$$S_{ij}{}^{(1)}=H_{ij}{}^{(1)}/W_i{}^0-W_j{}^0 \tag{8a}$$

for those values of i and j for which $W_i{}^0$ is not near $W_j{}^0$; for all other values k, l (i.e. the degenerate levels)

$$S_{kl}{}^{(1)}=0. \tag{8b}$$

With this definition, the application of $\mathbf{S}\cdots\mathbf{S}^\dagger$ to $(\mathbf{W}^0+\mathbf{H}')$ gives, for results correct to second order, the following matrix to be diagonalized:

$$\mathcal{H}=(\mathbf{W}^0+E\mathbf{H}_0{}^{(1)}+E^2\mathbf{H}^{(2)}+E^2\mathbf{H}^{(3)}). \tag{9}$$

Here $\mathbf{H}_0{}^{(1)}$ consists of those elements of $\mathbf{H}^{(1)}$ coupling degenerate levels, and

$$H_{jk}{}^{(3)}=\tfrac{1}{2}\sum_m{}'\left(\frac{H_{jm}{}^{(1)}H_{mk}{}^{(1)}}{W_j{}^0-W_m{}^0}+\frac{H_{jm}{}^{(1)}H_{mk}{}^{(1)}}{W_k{}^0-W_m{}^0}\right). \tag{10}$$

TABLE III. Appearances of accidental degeneracy in the asymmetric rotor. a, b, c, denote dipole-moment components along principal axes of least, intermediate, and greatest moment of inertia, respectively. The asterisk denotes no coupling term. Blanks indicate no accidental degeneracy possible.

J_τ	0_0	1_{-1}	1_0	1_{+1}	2_{-2}	2_{-1}	2_0	2_{+1}	2_{+2}
0_0									
1_{-1}									
1_0		c							
1_{+1}			a						
2_{-2}			b	c					
2_{-1}					c				
2_0						a			
2_{+1}							c		
2_{+2}								a	
3_{-3}						c	b	*	a
3_{-2}							*	c	b
3_{-1}								b	c
3_0									
3_{+1}									
3_{+2}									
3_{+3}									

[8] The formula given by Penney for $J=1$, must be restricted to case of non-degeneracy.

[9] J. H. Van Vleck, Phys. Rev. **33**, 467 (1929). See also E. C. Kemble, *Fundamental Principles of Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1937), p. 394.

[10] O. M. Jordahl, Phys. Rev. **45**, 87 (1934).

the prime indicating that the summation extends only over levels W_m^0 not degenerate with W_j^0 or W_k^0.

In diagonalizing $\mathcal{H}$, off-diagonal elements connecting non-degenerate levels will be ignored, since the transformation $\mathbf{S}$ has reduced them to second order and they therefore contribute no terms below fourth order to the energy. Consequently, $\mathcal{H}$ factors into small matrices, each associated with one group of degenerate levels. The diagonalization of these factors involves the solution of secular equations, usually no larger than second degree.

A. Diagonal Elements of $\mathcal{H}$

The diagonal elements of $\mathcal{H}$ are given by[11]

$$\mathcal{H}(J\tau M)=W^0{}_{J\tau}+E^2\sum_{J',\tau',g}{}' \frac{\mu_g{}^2[(\Phi_{Zg})_{J,\tau,M;J',\tau',M}]^2}{W^0{}_{J\tau}-W^0{}_{J'\tau'}} -E^2\sum_{J'\tau'g}\tfrac{1}{2}P_{gg}[(\Phi_{Zg})_{J,\tau,M;J'\tau',M}]^2, \quad (11)$$

where the prime over the summation indicates that the summation is to extend only over those values of $J'\tau'$ for which $W^0{}_{J'\tau'}$ is not near $W^0{}_{J\tau}$.

No cross products of the dipole moments occur because of symmetry restrictions (see Tables I and II). Consequently, the perturbation energy may be separated according to the components of the permanent and induced dipoles. The second-order perturbation arising from the gth component of the permanent dipole (to the degree of approximation indicated in Eq. (11)) is

$$[W_g{}^{(2)}]_{J\tau M}=(\mu_g)^2E^2\sum_{J',\tau'}{}' \frac{[(\Phi_{Zg})_{J,\tau,M;J',\tau'M}]^2}{W^0{}_{J\tau}-W^0{}_{J'\tau'}} \quad (12)$$

and that arising from the ggth component of the induced dipole is

$$[W_{gg}{}^{(2)}]_{J\tau M} =\tfrac{1}{2}P_{gg}E^2\sum_{J',\tau'}[(\Phi_{Zg})_{J,\tau,M;J',\tau',M}]^2, \quad (13)$$

so that Eq. (11) may be written as

$$\mathcal{H}(J\tau M)=W_{J\tau}{}^0+\textstyle\sum_g[W_g{}^{(2)}-W_{gg}{}^{(2)}]_{J\tau M}.$$

Making use of Table I of reference (6)

$$[W_g{}^{(2)}]_{J\tau M}=\mu_g{}^2E^2\left[\frac{(J^2-M^2)}{4J^2(4J^2-1)}\sum_{\tau'}{}'\left\{\frac{[(\Phi_{Zg})_{J,\tau;J-1,\tau'}]^2}{W^0{}_{J\tau}-W^0{}_{J-1,\tau'}}\right\}+\frac{M^2}{4J^2(J+1)^2}\sum_{\tau'\neq\tau}{}'\left\{\frac{[(\Phi_{Zg})_{J,\tau;J,\tau'}]^2}{W^0{}_{J\tau}-W^0{}_{J\tau'}}\right\} +\frac{(J+1)^2-M^2}{4(J+1)^2(2J+1)(2J+3)}\sum_{\tau'}{}'\left\{\frac{[(\Phi_{Zg})_{J,\tau;J+1,\tau'}]^2}{W^0{}_{J\tau}-W^0{}_{J+1,\tau'}}\right\}\right]. \quad (14)$$

Similarly

$$[W_{gg}{}^{(2)}]_{J\tau M}=\tfrac{1}{2}P_{gg}E^2\left[\frac{J^2-M^2}{4J^2(4J^2-1)}\sum_{\tau'}[(\Phi_{Zg})_{J,\tau;J-1,\tau'}]^2 +\frac{M^2}{4J^2(J+1)^2}\sum_{\tau'}[(\Phi_{Zg})_{J,\tau;J,\tau'}]^2+\frac{(J+1)^2-M^2}{4(J+1)^2(2J+1)(2J+3)}\sum_{\tau'}[(\Phi_{Zg})_{J,\tau;J+1,\tau'}]^2\right]. \quad (15)$$

For computational purposes it is convenient to separate out the coefficient of M^2 and remove the $\frac{1}{2}(a+c)$ from the W^0's (see Eq. (3)). One then obtains for the contribution of the second-order perturbation to the diagonal elements of $\mathcal{H}$

$$[W_g{}^{(2)}]_{J\tau M}=\frac{2\mu_g{}^2E^2}{a+c}[A_{J\tau}(\kappa,\alpha)+M^2B_{J\tau}(\kappa,\alpha)] \quad (16)$$

and

$$[W_{gg}{}^{(2)}]_{J\tau M}=\tfrac{1}{2}P_{gg}E^2[C_{J\tau}(\kappa)+M^2D_{J\tau}(\kappa)]. \quad (17)$$

In Appendix D the quantities A and B are tabulated for various values of the parameters κ and α. The quantities C and D have not been tabulated since, as shown in Appendix B, the terms arising from induced polarization are extremely small compared to those arising from

[11] The notation of reference 6 will be used.

the permanent dipole, and may be safely neglected.

B. Energy Calculations

By means of these tabulated quantities, the $\mathcal{H}(J\tau M)$'s may be obtained. In order to determine energy levels when degeneracy occurs a secular equation must be solved. When, as will probably most frequently be the case, the degeneracy occurs simply between a pair of levels, simple second-degree secular equations are obtained. These have the solutions

$$2W=\mathcal{H}(J\tau M)+\mathcal{H}(J'\tau'M)$$
$$\pm\{[\mathcal{H}(J\tau M)-\mathcal{H}(J'\tau'M)]^2+4|\xi^2|E^2\}^{\frac{1}{2}}. \quad (18)$$

The value of the off-diagonal element ξ is determined by the two states under consideration and will be discussed below.

Except for the fact that the $\mathcal{H}$'s are functions of E, Eq. (18) is virtually identical with that given by Penney for cases of slight symmetry.

A discussion of the ξ's requires the consideration of four separate cases. (Induced polarization terms are neglected.)

(1) $|\Delta J|=0$

Only one type of degeneracy need be considered under this case and that is the limiting symmetric-rotor degeneracy. It must be emphasized, however, that the rotor may be quite asymmetric and still have this near-degeneracy occur. Since these states are adjacent to one another[12] in the energy scale they have different symmetries. Therefore there will be a non-vanishing coupling term from $\mathbf{H}_0^{(1)}$ so that

$$E\xi=\mathcal{H}_{kl}=EH_{kl}^{(1)}+\frac{E^2}{2}\sum_m{}' H_{km}^{(1)}H_{ml}^{(1)}$$
$$\times\left(\frac{1}{W_k^0-W_m^0}+\frac{1}{W_l^0-W_m^0}\right), \quad (19)$$

where k, l, m refer to the unperturbed levels. From Table I, all terms vanish for which m has the same species representation as either k or l. Consequently, the non-vanishing terms of Eq. (19) contains $H_{kl}^{(1)}$, $H_{km}^{(1)}$, $H_{ml}^{(1)}$ corresponding to the three different direction cosines or dipole components. By Table I, reference 6, one of these only is a pure imaginary while the other two are real. Therefore, no first-order terms appear in $|\xi|^2$ and it reduces to

$$|\xi|^2=(H_{kl}^{(1)})^2=\frac{M^2}{4J^2(J+1)^2}\mu_g^2|(\Phi_{Zg})_{J\tau;J\tau'}|^2, \quad (20)$$

if second-order terms are neglected. g is determined by the symmetry of the two states. It is seen that there is no "mixing" of the dipole components for this case. Two expansions of Eq. (18) are possible. When $|\mathcal{H}_{(J\tau M)}-\mathcal{H}_{(J\tau'M)}|\gg 2|\xi E|$ the conventional second-order result is obtained. When the converse is true a term linear in E appears which is proportional to $|M|$.

(2) $|\Delta J|=1$*; unperturbed states have different symmetries*

The results are similar to the previous case except that now

$$|\xi|^2=|H_{kl}^{(1)}|^2=\frac{(J^{*2}-M^2)\mu_g^2|(\Phi_{Zg})_{J\tau;J'\tau'}|^2}{4J^{*2}(4J^{*2}-1)}, \quad (21)$$

where J^* is the larger of the two J's involved, g is determined by the symmetry of the two states.

Here, also, there is no "mixing" of the dipole-moment components. The usual second-order result is obtained when $|\mathcal{H}_{(J\tau M)}-\mathcal{H}_{(J'\tau'M)}|\gg 2|\xi E|$. When the converse is true a term linear in E appears which is proportional to $(J^{*2}-M^2)^{\frac{1}{2}}$.

(3) $|\Delta J|=1$*; unperturbed states have the same symmetry*

In this case $H_{kl}^{(1)}$, as well as terms for which m has the same species representation as k or l, will vanish. The sum in Eq. (19) will consist of a real linear combination of the squares of the dipole-moment components. Moreover since the direction-cosine matrix elements vanish for $|\Delta J|>1$, the following form for ξ is obtained

$$\xi=EM[J^{*2}-M^2]^{\frac{1}{2}}\sum_g \mu_g^2R_g(J,J',\tau,\tau'), \quad (22)$$

where J^* is the larger of J, J', g refers to the principal axes of inertia, and R_g depends upon the

[12] See, for example, Appendix I of reference 7. Also, G. H. Dicke and G. B. Kistiakowsky, Phys. Rev. **45**, 4 (1934).

quantities indicated (for a given asymmetric rotor).

When $M=0$, ξ vanishes, so that Eq. (18) gives the conventional second-order result. When the unperturbed energy levels are distinct (i.e. $|W_{J_\tau}{}^0-W_{J'\tau'}{}^0|\gg 2|\xi E|$) the conventional second-order result is also obtained. However when there is degeneracy, no general conclusions may be drawn regarding the expansion of Eq. (18), This is seen from the fact that $[\mathcal{H}_{(J\tau M)}-\mathcal{H}_{(J'\tau' M)}]$ reduces to terms in E^2, so that all terms under the radical are of the same order, E^4. Then only terms quadratic in the electric field are introduced from the radical, so that this case of degeneracy does not give rise to a first-order Stark effect.

(4) $|\Delta J|=2$

This case will not be considered in any great detail except to note that for its dependence upon M and E,

$$|\xi|^2\propto E^2[(J+1)^2-M^2][(J+2)^2-M^2] \quad (23)$$

where J is the lower of the two J's involved.

Degeneracies for which $|\Delta J|\geq 3$ result in vanishing ξ.

In actual practice the effect of degeneracy may not be as complicated as might be supposed from the foregoing. For example, when the dipole moment lies along one of the principal axes it is easy to show that the sum in Eq. (19) vanishes for states having different symmetries. Another simplification occurs when only certain states are allowed by virtue of nuclear-symmetry restrictions. Then certain degeneracies cannot occur.

IV. INTENSITIES

The intensities of the various component lines in the presence of an electric field involve the elements of the dipole-moment matrix, evaluated in terms of the basis of functions which diagonalize the energy, when the field is applied. For transitions involving non-degenerate levels these matrix elements are simply obtained from $\mathbf{SH}^{(1)}\mathbf{S}^\dagger$. They will contain small terms linear and quadratic in E which are added to $\mathbf{H}^{(1)}$. Only quite small changes in intensity are to be expected due to the field. It is interesting to observe, however, that non-vanishing elements may be expected, for finite fields, between states having the same symmetry; they may likewise occur as terms linear in E when $|\Delta J|\leq 3$. The selection rule on M is, of course, unchanged.

An important modification of the selection rules (with the exception of those on M) becomes possible when an accidental degeneracy occurs. Consider a pair of degenerate levels that have a non-vanishing element $-EH_{kl}{}^{(1)}$ connecting them. Then the proper linear combinations of asymmetric-rotor wave functions $A_k{}^0$ and $A_l{}^0$ are

$$\begin{cases} A_1=\dfrac{1}{\sqrt{2}}\left(A_k{}^0-\dfrac{H_{kl}{}^{(1)}}{|H_{kl}{}^{(1)}|}A_l{}^0\right), \\ A_2=\dfrac{1}{\sqrt{2}}\left(\dfrac{H_{kl}{}^{(1)}}{|H_{kl}{}^{(1)}|}A_k{}^0+A_l{}^0\right). \end{cases} \quad (24)$$

The elements of the dipole-moment matrix coupling either of these two states to some other state with wave function $A_m{}^0$ are

$$\begin{cases} H_{1m}{}^{(1)}=\dfrac{1}{\sqrt{2}}H_{km}{}^{(1)}-\dfrac{1}{\sqrt{2}}\dfrac{H_{kl}{}^{(1)}}{|H_{kl}{}^{(1)}|}H_{lm}{}^{(1)}, \\ H_{2m}{}^{(1)}=\dfrac{1}{\sqrt{2}}\dfrac{H_{kl}{}^{(1)}}{|H_{kl}{}^{(1)}|}H_{km}{}^{(1)}+\dfrac{1}{\sqrt{2}}H_{lm}{}^{(1)}. \end{cases} \quad (25)$$

To make the case non-trivial, it may be assumed that neither $H_{km}{}^{(1)}$ nor $H_{lm}{}^{(1)}$ vanishes, which corresponds to a permitted transition between the unperturbed state m and the unperturbed states k or l. Since $H_{kl}{}^{(1)}\neq 0$, by hypothesis, k and l correspond to states having different species representations. Only when m corresponds to a state having a species representation that differs from both the states corresponding to k and l will all of the terms in Eqs. (25) be non-vanishing. Then, application of the line of argument immediately following Eq. (19), indicates that the three dipole-moment components must be involved in Eqs. (25). Since only one of these components is a pure imaginary while the other two are real, Eqs. (25) can differ only in phase. If either $H_{km}{}^{(1)}$ or $H_{lm}{}^{(1)}$ vanish, again Eqs. (25) differ only in phase.

Since the absolute values of the dipole-moment matrix elements are involved in the expression for the line intensity, either of Eqs. (25) will give the same value. This means, if transitions are normally permitted between, say, state m and state l but not between state m and state k, that transitions between the latter become possible if states k and l are degenerate. Furthermore, since degeneracy may occur between states for which $|\Delta J|=1$, it is clear that transitions having non-vanishing field-independent probabilities are now possible for $|\Delta J|=2$.

Other cases of degeneracy require the examination of the secular equation involving the degenerate levels, but will not be carried out here. It is important to observe that the change in selection rules which may be brought about by degeneracy results in a doubling (or, more generally, splitting) of the Stark components of a spectral line. When a first order term is introduced by the degeneracy a symmetrical Stark pattern results. When the degeneracy introduces only second-order terms, a splitting of the Stark components may be expected at higher fields.

V. APPLICATION TO ANALYSIS

Even though quantitative theoretical calculations for $J>2$ may not be available, it is nevertheless possible to utilize observed Stark effects to advantage in analyzing rotational spectra.

In what follows the effects due to induced polarization will be disregarded. Also, the discussion will be restricted to representative cases of non-degeneracy and of degeneracy[13] with first-order Stark effect (i.e. cases (1) and (2) above). Second-order terms will be neglected when first-order terms arise.

A. Formulas for Transition Frequencies, $\Delta M=0$

For those transitions in which the electric vector of the radiation is parallel to the electric field the following formulas may be obtained for the frequency displacement of each component, measured relative to the position of the unsplit line.

(a) *No Degeneracy*

$$\Delta\nu_M=(A'+B'M^2)E^2; \tag{26a}$$

(b) *Degeneracy,*[13] $|\Delta J|=0$

$$\Delta\nu_M=\pm F'|M|E; \tag{26b}$$

(c) *Degeneracy,*[13] $|\Delta J|=1$

$$\Delta\nu_M=\pm\{G'[J^{*2}-M^2]^{\frac{1}{2}}\}E; \tag{26c}$$

where A', B', F', G' are coefficients that are independent of M; J^* is the larger of the two J's involved in the degeneracy. A spectral line will, under the influence of an electric field, exhibit significant differences in appearance according to whether case (a), (b), or (c) applies.

Since $|M|\leq J$, a completely resolved spectral line will have $(J+1)$ components for case (a), where J is the smaller of the two J's involved in the transition. When degeneracy occurs, there will be $(2J+1)$ components.[14] If complete resolution is not attained it is nevertheless possible to determine the $|M|$ value of certain of the resolved components. From this information a good estimate of $|M|_{\max}$ may usually be determined, with the result that the smaller of the two J's involved is determined. How this may be accomplished depends upon which of Eq. (26a)–(26c) is applicable. The necessary formulas are given below.

(a) Observe that successive components, when resolved, must differ by unity in their value of M. For two such components, therefore

$$\begin{aligned}\Delta\nu_M-\Delta\nu_{M\pm1}&=B'E^2[M^2-(M\pm1)^2],\\&=\mp B'E^2(2M\pm1).\end{aligned} \tag{27a}$$

If there are at least three components then, ordering the components so that M corresponds to one extreme, $M\pm1$ to the intermediate and $M\pm2$ to the other extreme, one obtains

$$r=\frac{\Delta\nu_M-\Delta\nu_{M\pm1}}{\Delta\nu_{M\pm1}-\Delta\nu_{M\pm2}}=\frac{2M\pm1}{2M\pm3}. \tag{27b}$$

Solving,

$$M=\pm\left(\frac{3r-1}{2-2r}\right). \tag{27c}$$

Consistency of the solution may be determined by evaluating $B'E^2$ and then determining the frequency displacement of the component cor-

[13] Specifically, discussion of degeneracy will be restricted to transitions which either involve one non-degenerate level or two degenerate levels whose energy dependence on M is precisely the same.

[14] Not all components may be observable; for example when $\Delta J=0$, the $M=0$ transition will be forbidden.

responding to some other value of M and comparing with observation.

In general, experimental errors may be such that Eq. (27c) may not lead to an integral value for M, particularly if M is very large. In such a case, however, certain limits may be imposed upon the smaller J involved in the transition.

(b) For this case one may observe that

$$\Delta\nu_M - \Delta\nu_{M\pm 1} = \pm F'E, \tag{28a}$$

so that

$$M = \pm \frac{\Delta\nu_M}{\Delta\nu_M - \Delta\nu_{M\pm 1}}. \tag{28b}$$

As under case (a) consistency should be established by comparing the frequency displacement computed for another value of M with the observed spectrum.

(c) For this case one readily obtains

$$(\Delta\nu_M)^2 - (\Delta\nu_{M\pm 1})^2 = \pm G'^2E^2(2M\pm 1). \tag{29a}$$

Proceeding as in case (a),

$$s = \frac{(\Delta\nu_M)^2 - (\Delta\nu_{M\pm 1})^2}{(\Delta\nu_{M\pm 1})^2 - (\Delta\nu_{M\pm 2})^2} = \frac{2M\pm 1}{2M\pm 3}. \tag{29b}$$

Solving,

$$M = \pm \frac{3s-1}{2-2s}. \tag{29c}$$

B. Formulas for Transition Intensities, $\Delta M = 0$

To facilitate the analysis, the dependence of the intensity upon the quantum number M may be employed. For the selection rule $|\Delta M| = 0$, the following relationships are obtained (neglecting changes due to frequency).[15]

(*I*) $|\Delta J| = 0$ *transitions.*

For these transitions it is known that the line strength is

$$I_M = PM^2; \tag{30}$$

(*II*) $|\Delta J| = 1$ *transitions.*

For these transitions it is known that the line strength is

$$I_M = Q(J^{*2} - M^2), \tag{31}$$

where P, Q are parameters depending upon the quantum numbers (other than M) and the asymmetry of the molecule; J^* is the larger of the two quantum numbers of the levels involved in the transition.

A factor of $\frac{1}{2}$ must be applied to those transitions for which $M = 0$.

The results previously considered must be modified for case (*I*). From Eq. (30) it is seen that the component for which M equals zero has a vanishing line strength. Hence, under conditions of complete resolution only J components will be observed for case (a), and $2J$ components for cases (b) and (c), above.

The following relations may be obtained.

(*I*) $|\Delta J| = 0$ *transitions*

$$M = \pm \frac{(I_M/I_{M\pm 1})^{\frac{1}{2}}}{1-(I_M/I_{M\pm 1})^{\frac{1}{2}}}. \tag{32}$$

(*II*) $|\Delta J| = 1$ *transitions*

$$M = \pm\left(\frac{3t-1}{2-2t}\right), \tag{33}$$

where

$$t = \left(\frac{I_M - I_{M\pm 1}}{I_{M\pm 1} - I_{M\pm 2}}\right).$$

These relationships together with those given above for the frequency displacements permit the quantum number M to be evaluated by two independent methods. In practice, the formulas involving frequency displacements may be expected to yield somewhat better results since the frequencies are generally capable of being measured more accurately. Equations (32) and (33) may then serve as a convenient means of checking the results so obtained.

The formulas presented above have been arrayed to give a value of the quantum number M corresponding to a particular component of a partially or completely solved line. It should be emphasized, however, that to determine M one must know which of the above cases is actually applicable to a given line. In general, this may not be known beforehand, so it will be necessary to apply each of the formulas until a pair of equations (*viz.* for frequency displacements and relative intensities) is found that describe the experimental data adequately.

It should be borne in mind that the frequency

[15] Here field-free intensities are employed. See, for example, Table I, reference 6.

displacements and relative intensity measurements give two entirely different kinds of information about the transition giving rise to the spectral line under observation. The frequency displacement measurements give information dealing with the degeneracies among the asymmetric-rotor levels and information leading to a determination of a quantum number J, which is the smaller of the two involved in the transition. The relative intensity measurements give more pertinent information pertaining to the transition itself (i.e. whether $|\Delta J|=0$ or $|\Delta J|=1$).

C. Formulas for Transition Frequencies, $|\Delta M|=1$

For transitions in which the electric vector of the radiation is perpendicular to the electric field, the Stark pattern is moderately more complex than in the cases previously considered. A certain simplification is obtained when it is realized that transitions corresponding to

$$(-|M|\to-|M|-1) \text{ and } (|M|\to|M|+1)$$

give rise to the same frequency. Similarly for

$$(-|M|\to-|M|+1) \text{ and } (|M|\to|M|-1).$$

Accordingly, the frequency displacements may be classified into two groups, as indicated. The following formulas result.

(*a′*) *No degeneracy*

$$\begin{aligned}\Delta\nu_M{}^{(+)}&=\{A'+B'M^2+B''|M|\}E^2,\\ \Delta\nu_M{}^{(-)}&=\{A'+B'M^2-B''|M|\}E^2;\end{aligned}\tag{34a}$$

(*b′*) *Degeneracy*,[13] $|\Delta J|=0$

$$\begin{aligned}\Delta\nu_M{}^{(+)}&=\pm\{F'|M|+F''\}E,\\ \Delta\nu_M{}^{(-)}&=\pm\{F'|M|-F''\}E;\end{aligned}\tag{34b}$$

(*c′*) *Degeneracy*,[13] $|\Delta J|=1$

$$\begin{aligned}\Delta\nu_M{}^{(+)}&=\pm\{G'(J^{*2}-M^2)^{\frac{1}{2}}\\ &\qquad+G''(J^{*2}-(|M|+1)^2)^{\frac{1}{2}}\}E\\ \Delta\nu_M{}^{(-)}&=\pm\{G'(J^{*2}-M^2)^{\frac{1}{2}}\\ &\qquad+G''(J^{*2}-(|M|-1)^2)^{\frac{1}{2}}\}E\end{aligned}\tag{34c}$$

where $\Delta\nu_M{}^{(+)}$ corresponds to $|M|\to|M|+1$; $\Delta\nu_M{}^{(-)}$ correspond to $|M|\to|M|-1$.

Since $|M|\leq J$, a completely resolved spectral line will have $(2J+1)$ component for case (a′) where J is the smaller of the two J's involved in the transition. For case (b′) there will be $(4J+2)$ components if $F''\neq0$ and $(2J+1)$ components otherwise. A similar statement applies to case (c′), depending upon whether or not G'' vanishes. Presumably, the more frequently occurring cases of (c′) will have $G''=0$, corresponding to a transition that involves at least one nondegenerate level.

D. Formulas for Transition Intensities, $|\Delta M|=1$

The explicit formulas permitting a solution for $|M|$ can be obtained readily if the $\Delta\nu^{(+)}$ and $\Delta\nu^{(-)}$ groups of components can be distinguished from one another. Fortunately, this can be done on the basis of the relative strengths of the lines. The following relations hold.

(*I′*) $|\Delta J|=0$ *transitions*

$$I_M{}^{(+)}=P\{J^2-J-M^2+|M|\},\tag{35a}$$

$$I_M{}^{(-)}=P\{J^2-J-M^2-|M|\}.\tag{35b}$$

From these equations it is readily seen that the $\Delta\nu^{(+)}$ group of components have intensities that are greater than that of the $\Delta\nu^{(-)}$ group (except for $M=0$, when they are the same).

(*II′*) $\Delta J=-1$ *transitions*

$$I_M{}^{(+)}=Q(J-|M|)(J-|M|-1),\tag{36a}$$

$$I_M{}^{(-)}=Q(J+|M|)(J+|M|-1);\tag{36b}$$

(*II″*) $\Delta J=+1$ *transitions*

$$I_M{}^{(+)}=Q(J+|M|)(J+|M|+1),\tag{37a}$$

$$I_M{}^{(-)}=Q(J-|M|)(J-|M|+1).\tag{37b}$$

Here J is the larger of the two J's involved in the transition. Note that in each case the $\Delta\nu^{(+)}$ and $\Delta\nu^{(-)}$ groups may be distinguished from each other on the basis of intensities. However, in case (II′) the $\Delta\nu^{(-)}$ group is the more intense group, while in case (II″) it is the $\Delta\nu^{(+)}$ group. The $M=0$ components of both groups will have the same intensity and frequency displacement.

No detailed formulas will be given from which the M's corresponding to the various components may be determined. However, it may be noted that the mean frequency displacement ($\Delta\nu_M{}^{(+)}$

$+\Delta\nu_M^{(-)}$) reduces (for cases (a′) and (b′) above) to formulas already given for $\Delta M=0$ transitions. When it is possible to effect this combination for a number of different M values, those formulas may be used. In addition the frequency displacement differences $(\Delta\nu_M^{(+)}-\Delta\nu_M^{(-)})$ may serve to check a determination of M. In the particular case where F'' and G'' are zero, the formulas pertaining to $\Delta M=0$ are applicable insofar as frequency displacements are concerned.

APPENDIX A

Proof That the Third-Order Perturbation Terms Vanish in the Non-Degenerate Case

It may be shown that for the non-degenerate case the coefficient of the term cubic in the field strength is

$$W_{ii}^{(3)}=-\sum_{j\neq i}\frac{H_{ii}H_{ij}H_{ji}}{(W_i^0-W_j^0)^2}+\sum_{j\neq i}\sum_{k\neq i}\frac{H_{ij}H_{jk}H_{ki}}{(W_i^0-W_j^0)(W_i^0-W_k^0)},$$

where i, j, k refers to the pairs of quantum numbers J, τ, and H is the matrix element of the dipole moment matrix (see Eq. (1)). For the present case $H_{ii}=0$, so

$$W_{ii}^{(3)}=\sum_{j\neq i}\sum_{k\neq i}\frac{H_{ij}H_{jk}H_{ki}}{(W_i^0-W_j^0)(W_i^0-W_k^0)}.$$

Since H is hermitian, this becomes

$$W_{ii}^{(3)}=\sum_{j>k}\sum_{k}\frac{(H_{ij}H_{jk}H_{ki})+(H_{ij}H_{jk}H_{ki})^*}{(W_i^0-W_j^0)(W_j^0-W_k^0)},$$

since those terms for which $j=k$ must vanish (i.e. $H_{jj}=0$).

Now those terms for which any two of i, j, or k correspond to the same species of wave function must also vanish. Consequently, only these terms remain for which i, j, and k correspond to different species of functions. Consequently, H_{ij}, H_{jk}, H_{ki}, must correspond to the three different direction cosines (or dipole-moment components), as indicated in Table I.

Since one of these only[6] (depending upon the representation) is pure imaginary, while the other two are real, it follows that

$$W_{ii}^{(3)}=0.$$

APPENDIX B

Estimate of the Order of Magnitude of Polarization Terms

Any contribution attributable to polarization will depend upon the departure of the polarizability ellipsoid from a sphere. To make an estimate of the effect under the most favorable conditions, imagine the polarizability ellipsoid degenerating into a line. Then the contribution to the energy is

$$W_p<\tfrac{1}{2}\alpha\langle\cos^2\theta\rangle_{\text{Av}}E^2,$$

where α is the polarizability, θ is the angle between the line of polarization and the direction of the applied electric field E.

To contribute to the Stark effect we require

$$\Delta W_p<\tfrac{1}{2}\alpha E^2\Delta\langle\cos^2\theta\rangle_{\text{Av}},$$

where Δ refers to the difference in the values in the upper and lower states. Clearly, since $\cos^2\theta<1$

$$\Delta W_p\ll\tfrac{1}{2}\alpha E^2.$$

Since $\alpha=10^{-24}$ e.s.u. in order of magnitude, the frequency displacement for $E=10^4$ volts/cm.

$$\Delta\nu_p\ll 0.08 \text{ mc/sec.},$$

which can be safely neglected.

APPENDIX C

Use of Tabulated Line Strengths to Evaluate the Direction-Cosine Matrix Elements

When extension to larger values of J of the present tables of Stark coefficients is desired, use may be made of the tables of line strengths of reference 6. From Eq. (14), above, it is seen that the only quantities needed are $(\Phi_{Zg})^2{}_{J\tau;J'\tau'}$. From Eq. (19) of reference 6, the line strength is defined as

$$\lambda=\sum_{FMM'}|\Phi_{Fg}|^2{}_{J\tau M;J'\tau'M'}=3|\Phi_{Zg}|^2{}_{J;J'}\times|\Phi_{Zg}|^2{}_{J\tau;J'\tau'}\times\sum_{M,M'}|\Phi_{Zg}|^2{}_{J,M;J'M'}.$$

Two cases need be considered.

(*a*) $|J-J'|=1$. In this case, solving for $|\Phi_{Zg}|^2{}_{J\tau;J'\tau'}$ and substituting for $|\Phi_{Zg}|^2{}_{J;J'}$ and $\sum_{M,M'}|\Phi_{Zg}|^2{}_{JM;J'M}$

making use of Table I of reference 6,

$$|\Phi_{Zg}|^2{}_{J\tau;J'\tau'}=4J^*\lambda,$$

where J^* is the larger of the pair (J, J').
(*b*) $|J-J'|=0$. In this case

$$|\Phi_{Zg}|^2{}_{J\tau;J\tau'}=\frac{4J(J+1)}{2J+1}\lambda.$$

Substitution into Eq. (14), above with the frequencies determined from term values, or computed, gives immediately $W^{(2)}$.

APPENDIX D

Tables of Reduced Stark Coefficients

In preparing the tables of $A(\kappa, \alpha)$ and $B(\kappa, \alpha)$ the energy matrix of the asymmetric rotor was evaluated in terms of the type I^r representation of symmetric-rotor basis functions, making use of the results given in reference 7. The Wang transformation **X** was then applied. Since the energy levels of the asymmetric rotor are known as explicit functions of the parameter $\kappa(J\leq 3)$, it was possible to solve for the elements of the orthogonal transformation matrix **T** as explicit functions of κ. These elements were then evaluated for $\kappa=-1(0.1)+1$. The calculations were carried out independently for each element to eight significant figures. The magnitude of the characteristic vectors of T differed from unity by no more than four in the eighth place. The values of the elements were rounded off to the seventh decimal place.

Next, the direction-cosine matrices were evaluated by applying $\mathbf{T}\cdots\mathbf{T}'$ to the matrices $\|(\Phi_{Zg})_{JK;J'K'}\|$.[11] The rules of spectroscopic stability[6] were applied to check the transformation. The deviations from the sum rule were no more than three in the seventh figure. An additional check was made through the symmetry of the transformed direction-cosine matrices: these matrices for negative κ values have the same numerical values as for positive values, but have the labelling of the rows and columns inverted.[6] The deviations between several computed values and those obtained by symmetry did not exceed two in the seventh place.

The reduced energy levels (see Eq. (3)) of the asymmetric rotor were calculated from the explicit formulas for $\kappa=-1(0.1)+1$. To obtain the energy values for positive κ, use was made of the relation

$$E_\tau{}^J(\kappa)=-E_{-\tau}{}^J(-\kappa).$$

The calculations were carried out to eight significant figures and were rounded off to seven. Differences were formed between the various energy levels.

The reduced second-order Stark coefficients were then evaluated with the aid of Eq. (14) for $\kappa=-1(0.2)+1$ and $\alpha=0.1(0.2)0.9$. A separate computation was carried out for the accidentally degenerate levels so that Eq. (18) may be applied when degeneracy does occur. The final tabulation was rounded off to six decimal places, and fifth differences taken and smoothed.

In using the tables, the factor $(2/a+c)$ must be applied to take into account the fact that *reduced* energies have been employed (see Eq. (16)). The various A's and B's are given for each level and dipole-moment component. In addition, a separate tabulation is given for the accidentally degenerate levels which will permit of easier interpolation (*viz.* interpolation of reciprocal) in regions of degeneracy. To get the entire reduced Stark coefficient for no degeneracy it is necessary to add the contributions from the accidentally degenerate parts to that given for the individual level. The signs given are for the first indicated level of the pair. For the remaining one, the signs should be reversed. For degeneracy, Eq. (18) must be applied.

Reduced Stark coefficients for dipole-moment component along principal axis of smallest moment of inertia.

Level		$\alpha\diagdown\kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
		0.1	−.184162	−.182149	−.180180	−.178253	−.176367	−.174520	−.172712	−.170940	−.169205	−.167504
		0.3	−.233100	−.223714	−.215054	−.207039	−.199601	−.192678	−.186220	−.180180	−.174520	−.169205
0_0	A_a	0.5	−.317460	−.289855	−.266667	−.246914	−.229885	−.215054	−.202020	−.190476	−.180180	−.170940
		0.7	−.497512	−.411523	−.350877	−.305810	−.271003	−.243309	−.220751	−.202020	−.186220	−.172712
		0.9	−1.149425	−.709220	−.512820	−.401606	−.330033	−.280112	−.243309	−.215054	−.192678	−.174520

Reduced Stark coefficients for dipole-moment component along principal axis of smallest moment of inertia.—*Continued.*

Level		$\alpha \backslash \kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
1_{-1}	A_a	0.1	.110496	.109278	.108075	.106888	.105717	.104561	.103420	.102294	.101183	.100086
		0.3	.139843	.134088	.128671	.123564	.118742	.114182	.109860	.105758	.101858	.098142
		0.5	.190401	.173343	.158650	.145856	.134604	.124620	.115685	.107628	.100310	.093618
		0.7	.298186	.244913	.206427	.177239	.154294	.135714	.120298	.107243	.095985	.086120
		0.9	.687107	.416573	.294253	.224222	.178605	.146318	.122077	.103037	.087524	.074486
	B_a	0.1	−.165745	−.163931	−.162154	−.160412	−.158704	−.157030	−.155389	−.153779	−.152199	−.150650
		0.3	−.209786	−.201307	−.193458	−.186171	−.179386	−.173054	−.167130	−.161575	−.156354	−.151439
		0.5	−.285696	−.260727	−.239662	−.221649	−.206065	−.192445	−.180436	−.169764	−.160213	−.151610
		0.7	−.447681	−.369870	−.314762	−.273667	−.241826	−.216410	−.195638	−.178326	−.163661	−.151064
		0.9	−1.033846	−.636058	−.458179	−.357260	−.292176	−.246664	−.213001	−.187050	−.166390	−.149511
1_0	A_a	0.1	−.055402	−.055096	−.054795	−.054496	−.054201	−.053908	−.053619	−.053333	−.053050	−.052770
		0.3	−.070671	−.069204	−.067797	−.066445	−.065147	−.063898	−.062696	−.061539	−.060423	−.059347
		0.5	−.097561	−.093023	−.088889	−.085106	−.081633	−.078431	−.075472	−.072727	−.070175	−.067797
		0.7	−.157480	−.141844	−.129032	−.118343	−.109290	−.101523	−.094787	−.088889	−.083682	−.079051
		0.9	−.408163	−.298508	−.235294	−.194175	−.165289	−.143885	−.127389	−.114286	−.103627	−.094787
	B_a	0.1	.013850	.013774	.013699	.013624	.013550	.013477	.013405	.013333	.013263	.013193
		0.3	.017668	.017301	.016949	.016611	.016287	.015974	.015674	.015385	.015106	.014837
		0.5	.024390	.023256	.022222	.021277	.020408	.019608	.018868	.018182	.017544	.016949
		0.7	.039370	.035461	.032258	.029586	.027322	.025381	.023697	.022222	.020921	.019763
		0.9	.102041	.074627	.058824	.048544	.041322	.035971	.031847	.028571	.025907	.023697
1_1	A_a	0.1	−.055096	−.054201	−.053333	−.052493	−.051680	−.050891	−.050125	−.049383	−.048662	−.047962
		0.3	−.069204	−.065147	−.061539	−.058309	−.055402	−.052770	−.050378	−.048193	−.046189	−.044346
		0.5	−.093023	−.081633	−.072727	−.065574	−.059702	−.054795	−.050633	−.047059	−.043956	−.041237
		0.7	−.141844	−.109290	−.088889	−.074906	−.064725	−.056980	−.050891	−.045977	−.041929	−.038536
		0.9	−.298508	−.165289	−.114286	−.087336	−.070671	−.059347	−.051151	−.044944	−.040080	−.036166
	B_a	0.1	.013774	.013550	.013333	.013123	.012920	.012723	.012531	.012346	.012166	.011990
		0.3	.017301	.016287	.015385	.014577	.013850	.013193	.012595	.012048	.011547	.011087
		0.5	.023256	.020408	.018182	.016393	.014925	.013699	.012658	.011765	.010989	.010309
		0.7	.035461	.027322	.022222	.018727	.016181	.014245	.012723	.011494	.010482	.009634
		0.9	.074627	.041322	.028571	.021834	.017668	.014837	.012788	.011236	.010020	.009042
2_{-2}	A_a	0.1	.026284	.025775	.025012	.023958	.022605	.020984	.019157	.017210	.015234	.013308
		0.3	.033264	.031617	.029771	.027697	.025422	.023001	.020520	.018079	.015768	.013658
		0.5	.045274	.040759	.036457	.032298	.028296	.024498	.020982	.017831	.015111	.012856
		0.7	.070819	.057088	.046408	.037683	.030374	.024224	.019129	.015047	.011940	.009747
		0.9	.162326	.094060	.061305	.041255	.027418	.017391	.010156	.005209	.002218	.000895
	B_a	0.1	−.013218	−.013670	−.014960	−.017317	−.020893	−.025690	−.031531	−.038088	−.044954	−.051731
		0.3	−.016668	−.016168	−.015965	−.016122	−.016688	−.017666	−.019009	−.020619	−.022373	−.024149
		0.5	−.022689	−.020826	−.019421	−.018439	−.017861	−.017663	−.017799	−.018201	−.018788	−.019479
		0.7	−.035560	−.029561	−.025469	−.022595	−.020592	−.019259	−.018457	−.018072	−.018002	−.018156
		0.9	−.082252	−.051154	−.037352	−.029650	−.024831	−.021731	−.019755	−.018576	−.017982	−.017816
2_{-1}	A_a	0.1	.013191	.013115	.013036	.012956	.012873	.012788	.012701	.012612	.012521	.012429
		0.3	.016822	.016436	.016033	.015616	.015186	.014747	.014300	.013847	.013388	.012925
		0.5	.023211	.021981	.020739	.019498	.018270	.017060	.015872	.014708	.013568	.012453
		0.7	.037409	.033124	.029225	.025692	.022487	.019565	.016888	.014419	.012128	.009988
		0.9	.096327	.066911	.048725	.036288	.027204	.020186	.014548	.009868	.005871	.002375
	B_a	0.1	−.009160	−.009110	−.009059	−.009008	−.008958	−.008908	−.008858	−.008809	−.008759	−.008710
		0.3	−.011685	−.011438	−.011198	−.010964	−.010736	−.010513	−.010297	−.010086	−.009880	−.009679
		0.5	−.016129	−.015362	−.014650	−.013987	−.013368	−.012789	−.012246	−.011735	−.011254	−.010800
		0.7	−.026029	−.023381	−.021168	−.019291	−.017678	−.016274	−.015041	−.013948	−.012970	−.012089
		0.9	−.067393	−.048894	−.038094	−.031002	−.025980	−.022227	−.019309	−.016969	−.015045	−.013429
2_0	A_a	0.1	.013118	.012901	.012689	.012480	.012274	.012072	.011874	.011679	.011487	.011298
		0.3	.016473	.015474	.014558	.013714	.012933	.012208	.011531	.010897	.010300	.009736
		0.5	.022130	.019302	.017011	.015112	.013508	.012128	.010923	.009856	.008898	.008027
		0.7	.033699	.025617	.020395	.016724	.013985	.011846	.010113	.008668	.007429	.006340
		0.9	.070609	.038016	.025357	.018575	.014309	.011344	.009133	.007394	.005962	.004737
	B_a	0.1	−.009110	−.008961	−.008817	−.008677	−.008542	−.008410	−.008281	−.008156	−.008035	−.007917
		0.3	−.011442	−.010768	−.010165	−.009622	−.009132	−.008686	−.008278	−.007904	−.007560	−.007241
		0.5	−.015379	−.013483	−.011991	−.010787	−.009793	−.008958	−.008246	−.007631	−.007094	−.006619
		0.7	−.023445	−.018026	−.014612	−.012262	−.010543	−.009230	−.008192	−.007349	−.006649	−.006057
		0.9	−.049305	−.027181	−.018691	−.014194	−.011405	−.009503	−.008119	−.007064	−.006229	−.005550
2_1	A_a	0.1	−.026309	−.026021	−.025740	−.025465	−.025195	−.024931	−.024673	−.024420	−.024172	−.023929
		0.3	−.033300	−.031959	−.030722	−.029577	−.028514	−.027525	−.026603	−.025740	−.024931	−.024172
		0.5	−.045352	−.041408	−.038095	−.035273	−.032841	−.030722	−.028860	−.027211	−.025740	−.024420
		0.7	−.071073	−.058789	−.050125	−.043687	−.038715	−.034758	−.031536	−.028860	−.026603	−.024673
		0.9	−.164204	−.101317	−.073260	−.057372	−.047148	−.040016	−.034758	−.030722	−.027525	−.024931
	B_a	0.1	.002993	.003617	.005149	.007830	.011813	.017094	.023471	.030614	.038066	.045416
		0.3	.003723	.003793	.004177	.004953	.006173	.007833	.009868	.012160	.014564	.016938
		0.5	.005053	.004746	.004691	.004919	.005452	.006278	.007354	.008604	.009936	.011265
		0.7	.007907	.006636	.005897	.005569	.005589	.005908	.006466	.007192	.008010	.008850
		0.9	.018253	.011338	.008394	.006930	.006240	.006038	.006166	.006514	.006990	.007521

Reduced Stark coefficients for dipole-moment component along principal axis of smallest moment of inertia.—*Continued.*

Level		$\alpha\diagdown\kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
2_2	A_a	0.1	−.026272	−.025672	−.024761	−.023585	−.022193	−.020614	−.018879	−.017028	−.015122	−.013222
		0.3	−.033239	−.031433	−.029316	−.027045	−.024711	−.022362	−.020016	−.017702	−.015458	−.013326
		0.5	−.045220	−.040376	−.035671	−.031322	−.027420	−.023933	−.020788	−.017928	−.015329	−.012983
		0.7	−.070677	−.056316	−.045287	−.036893	−.030469	−.025429	−.021336	−.017910	−.014986	−.012472
		0.9	−.161622	−.092777	−.061682	−.044575	−.034022	−.026910	−.021739	−.017752	−.014545	−.011910
	B_a	0.1	.002915	.002812	.002628	.002364	.002021	.001608	.001139	.000633	.000113	−.000399
		0.3	.003690	.003457	.003154	.002789	.002374	.001918	.001435	.000940	.000453	−.000010
		0.5	.005021	.004456	.003874	.003294	.002728	.002175	.001639	.001125	.000644	.000203
		0.7	.007850	.006231	.004953	.003936	.003106	.002403	.001788	.001240	.000753	.000325
		0.9	.017956	.010285	.006783	.004807	.003531	.002615	.001900	.001312	.000816	.000396
1_1-1_0	B_a	0.1	25.000000	8.333333	5.000000	3.571428	2.777778	2.272727	1.923076	1.666667	1.470588	1.315789
		0.3	8.333333	2.777778	1.666667	1.190476	.925926	.757576	.641025	.555555	.490196	.438596
		0.5	5.000000	1.666667	1.000000	.714286	.555556	.454545	.384615	.333333	.294118	.263158
		0.7	3.571429	1.190476	.714286	.510204	.396825	.324675	.274725	.238095	.210084	.187970
		0.9	2.777778	.925926	.555556	.396825	.308642	.252525	.213675	.185185	.163399	.146199
2_0-2_{-1}	B_a	0.1	.925926	.308642	.185185	.132275	.102881	.084175	.071225	.061728	.054466	.048733
		0.3	.308642	.102881	.061728	.044092	.034294	.028058	.023742	.020576	.018155	.016244
		0.5	.185185	.061728	.037037	.026455	.020576	.016835	.014245	.012346	.010893	.009747
		0.7	.132275	.044092	.026455	.018896	.014697	.012025	.010175	.008818	.007781	.006962
		0.9	.102881	.034294	.020576	.014697	.011431	.009353	.007914	.006859	.006052	.005415
2_2-2_1	B_a	0.1	288.88888	30.453380	10.370598	4.993661	2.843947	1.789652	1.204169	.850946	.624897	.473659
		0.3	96.296296	10.151128	3.456866	1.664553	.947983	.596551	.401390	.283649	.208299	.157886
		0.5	57.777778	6.090677	2.074120	.998732	.568789	.357930	.240834	.170189	.124979	.094732
		0.7	41.269829	4.350483	1.481514	.713380	.406278	.255665	.172024	.121564	.089271	.067666
		0.9	32.098765	3.383710	1.152289	.554851	.315994	.198850	.133797	.094550	.069433	.052629
2_2-3_3	A_a	0.1	−.000012	−.000104	−.000247	−.000362	−.000395	−.000343	−.000242	−.000133	−.000049	−.000005
		0.3	−.000029	−.000226	−.000500	−.000702	−.000747	−.000642	−.000452	−.000251	−.000094	−.000011
		0.5	.000073	.001380	.022416	−.011169	−.006789	−.004939	−.003503	−.002263	−.001213	−.000357
		0.7	.000016	.000170	.000478	.000803	.000958	.000867	.000609	.000322	.000111	.000011
		0.9	.000009	.000091	.000242	.000388	.000447	.000399	.000280	.000150	.000053	.000006
2_2-3_{-3}	B_a	0.1	.000001	.000012	.000027	.000040	.000044	.000038	.000027	.000015	.000005	.000001
		0.3	.000003	.000025	.000056	.000078	.000083	.000071	.000050	.000028	.000010	.000001
		0.5	−.000008	−.000153	−.002491	.001241	.000754	.000549	.000389	.000252	.000135	.000040
		0.7	−.000002	−.000019	−.000053	−.000089	−.000106	−.000096	−.000068	−.000036	−.000012	−.000001
		0.9	−.000001	−.000010	−.000027	−.000043	−.000050	−.000044	−.000031	−.000017	−.000006	−.000001

Reduced Stark coefficients for dipole-moment component along principal axis of intermediate moment of inertia.

Level		$\alpha\diagdown\kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
0_0	A_b	0.1	−.166667									
		0.3	−.166667									
		0.5	−.166667									
		0.7	−.166667									
		0.9	−.166667									
1_{-1}	A_b	0.1	−.052632									
		0.3	−.058824									
		0.5	−.066667									
		0.7	−.076923									
		0.9	−.090909									
	B_b	0.1	.013158									
		0.3	.014706									
		0.5	.016667									
		0.7	.019231									
		0.9	.022727									
1_0	A_b	0.1	.120398	.123372	.126692	.130296	.134076	.137894	.141601	.145066	.148195	.150945
		0.3	.124601	.127676	.131049	.134647	.138355	.142034	.145543	.148763	.151620	.154085
		0.5	.128105	.131201	.134549	.138068	.141641	.145133	.148413	.151379	.153973	.156180
		0.7	.131070	.134141	.137423	.140828	.144243	.147539	.150597	.153329	.155690	.157677
		0.9	.133611	.136632	.139825	.143103	.146356	.149461	.152314	.154837	.156998	.158799
	B_b	0.1	−.155099	−.155843	−.156673	−.157574	−.158519	−.159474	−.160400	−.161266	−.162049	−.162736
		0.3	−.156150	−.156919	−.157762	−.158662	−.159589	−.160509	−.161386	−.162191	−.162905	−.163521
		0.5	−.157026	−.157800	−.158637	−.159517	−.160410	−.161283	−.162103	−.162845	−.163493	−.164045
		0.7	−.157768	−.158535	−.159356	−.160207	−.161061	−.161885	−.162649	−.163332	−.163923	−.164419
		0.9	−.158403	−.159158	−.159956	−.160776	−.161589	−.162365	−.163078	−.163709	−.164249	−.164700
1_{+1}	A_b	0.1	−.047619									
		0.3	−.043478									
		0.5	−.040000									
		0.7	−.037037									
		0.9	−.034483									
	B_b	0.1	.011905									
		0.3	.010870									
		0.5	.010000									
		0.7	.009259									
		0.9	.008621									

Reduced Stark coefficients for dipole-moment component along principal axis of intermediate moment of inertia.—*Continued.*

Levels		$\alpha\diagdown\kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
2_{-2}	A_b	0.1	−.031035	−.031957	−.032971	−.034054	−.035173	−.036284	−.037344	−.038317	−.039180	−.039923
		0.3	−.036285	−.037494	−.038771	−.040077	−.041359	−.042565	−.043644	−.044564	−.045314	−.045897
		0.5	−.043673	−.045367	−.047098	−.048796	−.050381	−.051773	−.052912	−.053770	−.054350	−.054678
		0.7	−.054842	−.057439	−.060023	−.062459	−.064601	−.066319	−.067530	−.068211	−.068403	−.068187
		0.9	−.073686	−.078280	−.082771	−.086852	−.090195	−.092527	−.093705	−.093749	−.092822	−.091161
	B_b	0.1	.003448	.003551	.003663	.003784	.003908	.004032	.004149	.004257	.004353	.004436
		0.3	.004032	.004166	.004308	.004453	.004596	.004729	.004849	.004952	.005035	.005100
		0.5	.004853	.005041	.005233	.005422	.005598	.005753	.005879	.005974	.006039	.006075
		0.7	.006094	.006382	.006669	.006940	.007178	.007369	.007503	.007579	.007600	.007576
		0.9	.008187	.008698	.009197	.009650	.010022	.010281	.010412	.010417	.010314	.010129
2_{-1}	A_b	0.1	.030784	.034885	.038807	.042148	.044760	.046699	.048109	.049129	.049874	.050426
		0.3	.037205	.041682	.045837	.049271	.051874	.053751	.055075	.056007	.056669	.057147
		0.5	.045272	.050091	.054422	.057892	.060466	.062236	.063466	.064311	.064899	.065315
		0.7	.055748	.060876	.065340	.068810	.071293	.072990	.074131	.074899	.075425	.075790
		0.9	.069949	.075359	.079920	.083365	.085767	.087373	.088433	.089135	.089609	.089935
	B_b	0.1	−.010730	−.011186	−.011622	−.011993	−.012283	−.012499	−.012655	−.012769	−.012852	−.012913
		0.3	−.012304	−.012801	−.013263	−.013644	−.013934	−.014142	−.014289	−.014393	−.014467	−.014520
		0.5	−.014290	−.014825	−.015306	−.015692	−.015976	−.016174	−.016311	−.016405	−.016470	−.016517
		0.7	−.016878	−.017448	−.017944	−.018329	−.018605	−.018794	−.018921	−.019006	−.019064	−.019105
		0.9	−.020398	−.021000	−.021506	−.021889	−.022156	−.022334	−.022452	−.022530	−.022583	−.022619
2_0	A_b	0.1	−.023810									
		0.3	−.023810									
		0.5	−.023810									
		0.7	−.023810									
		0.9	−.023810									
	B_b	0.1	.002646									
		0.3	.002646									
		0.5	.002646									
		0.7	.002646									
		0.9	.002646									
2_1	A_b	0.1	.014388	.015000	.015806	.016881	.018326	.020264	.022806	.025974	.029595	.033284
		0.3	.013980	.014614	.015436	.016513	.017935	.019808	.022218	.025163	.028462	.031754
		0.5	.013480	.014116	.014929	.015983	.017356	.019140	.021403	.024127	.027132	.030083
		0.7	.012950	.013575	.014368	.015386	.016701	.018391	.020510	.023031	.025779	.028444
		0.9	.012419	.013028	.013796	.014774	.016028	.017625	.019611	.021952	.024476	.026902
	B_b	0.1	−.008213	−.008281	−.008370	−.008489	−.008650	−.008865	−.009148	−.009500	−.009902	−.010312
		0.3	−.007592	−.007663	−.007754	−.007873	−.008032	−.008240	−.008507	−.008835	−.009201	−.009567
		0.5	−.007053	−.007124	−.007214	−.007331	−.007484	−.007682	−.007934	−.008236	−.008570	−.008898
		0.7	−.006583	−.006652	−.006741	−.006854	−.007000	−.007188	−.007423	−.007703	−.008008	−.008305
		0.9	−.006169	−.006237	−.006322	−.006431	−.006570	−.006748	−.006968	−.007228	−.007509	−.007778
2_2	A_b	0.1	.013028	.010569	.007820	.004848	.001760	−.001316	−.004253	−.006945	−.009331	−.011386
		0.3	.012555	.010004	.007187	.004183	.001103	−.001921	−.004766	−.007337	−.009580	−.011484
		0.5	.012029	.009450	.006638	.003676	.000678	−.002228	−.004926	−.007334	−.009409	−.011151
		0.7	.011496	.008928	.006160	.003277	.000392	−.002374	−.004914	−.007156	−.009070	−.010664
		0.9	.010978	.008446	.005742	.002955	.000194	−.002428	−.004814	−.006903	−.008673	−.010137
	B_b	0.1	−.007874	−.007188	−.006421	−.005590	−.004722	−.003850	−.003009	−.002229	−.001529	−.000921
		0.3	−.007237	−.006527	−.005746	−.004912	−.004055	−.003208	−.002404	−.001672	−.001025	−.000471
		0.5	−.006692	−.005976	−.005198	−.004381	−.003551	−.002743	−.001988	−.001308	−.000718	−.000218
		0.7	−.006221	−.005510	−.004746	−.003953	−.003158	−.002393	−.001686	−.001057	−.000517	−.000064
		0.9	−.005811	−.005110	−.004366	−.003601	−.002843	−.002120	−.001459	−.000876	−.000379	−.000034
1_0-2_{-2}	A_b	0.1	−.020951	−.023914	−.027223	−.030815	−.034585	−.038393	−.042090	−.045545	−.048666	−.051407
		0.3	−.031394	−.033904	−.036800	−.039988	−.043341	−.046710	−.049944	−.052920	−.055559	−.057827
		0.5	−.062588	−.058228	−.056771	−.056936	−.058034	−.059626	−.061400	−.063144	−.064726	−.066081
		0.7	−.830826	−.206071	−.124148	−.098817	−.087799	−.082414	−.079677	−.078265	−.077516	−.077083
		0.9	.063395	.133895	.664525	−.373715	−.180240	−.133399	−.113446	−.102908	−.096606	−.092479
	B_b	0.1	.005238	.005979	.006806	.007704	.008646	.009598	.010523	.011386	.012167	.012852
		0.3	.007848	.008476	.009200	.009997	.010835	.011677	.012486	.013230	.013890	.014457
		0.5	.015647	.014557	.014193	.014234	.014509	.014906	.015350	.015786	.016182	.016520
		0.7	2.457708	.051518	.031037	.024704	.021950	.020604	.019919	.019566	.019379	.019271
		0.9	−.015849	−.033474	−.166131	.093429	.045060	.033350	.028361	.025727	.024152	.023120
1_1-1_{-1}	B_b	0.1	1.250000									
		0.3	.416667									
		0.5	.250000									
		0.7	.178571									
		0.9	.138889									
$2_{-1}-3_{-3}$	A_b	0.1	−.018443	−.022539	−.026455	−.029792	−.032399	−.034334	−.035739	−.036755	−.037497	−.038045
		0.3	−.025850	−.030071	−.034014	−.037271	−.039724	−.041471	−.042682	−.043515	−.044090	−.044490
		0.5	−.043202	−.045165	−.047619	−.049762	−.051328	−.052353	−.052974	−.053322	−.053498	−.053565
		0.7	−.131405	−.090681	−.079365	−.074849	−.072509	−.070977	−.069805	−.068836	−.068008	−.067289
		0.9	.126148	11.634484	−.238095	−.150943	−.009168	−.110171	−.102314	−.097081	−.093318	−.090468
	B_b	0.1	.002049	.002504	.002939	.003310	.003600	.003815	.003971	.004084	.004166	.004227
		0.3	.002872	.003341	.003779	.004141	.004414	.004608	.004742	.004835	.004899	.004943
		0.5	.004800	.005018	.005291	.005529	.005703	.005817	.005886	.005925	.005944	.005952
		0.7	.014601	.010076	.008818	.008317	.008057	.007886	.007756	.007648	.007556	.007477
		0.9	−.014017	−1.292720	.026455	.016772	.001019	.012241	.011368	.010787	.010369	.010052

Reduced Stark coefficients for dipole-moment component along principal axis of intermediate moment of inertia.—*Continued.*

Levels		$\alpha \backslash \kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
2_0-2_{-2}	B_b	0.1	.385824	.326893	.272357	.223059	.179694	.142657	.111948	.087171	.067634	.052491
		0.3	.128608	.108964	.090786	.074353	.059898	.047552	.037316	.029057	.022545	.017497
		0.5	.077165	.065379	.054471	.044612	.035939	.028532	.022390	.017434	.013527	.010498
		0.7	.055118	.046699	.038908	.031866	.025671	.020380	.015993	.012453	.009662	.007499
		0.9	.042869	.036322	.030262	.024784	.019966	.015851	.012439	.009686	.007515	.005832
2_1-2_{-1}	B_b	0.1	.046296									
		0.3	.015432									
		0.5	.009259									→
		0.7	.006614									
		0.9	.005144									
2_1-3_{-1}	A_b	0.1	−.003224	−.003831	−.004632	−.005701	−.007143	−.009076	−.011614	−.014778	−.018396	−.022081
		0.3	−.005989	−.006272	−.006833	−.007709	−.008973	−.010716	−.013019	−.015873	−.019095	−.022320
		0.5	−.042052	−.017290	−.013024	−.011900	−.012065	−.013079	−.014810	−.017143	−.019849	−.022564
		0.7	.008375	.022854	−.242173	−.026073	−.018409	−.016780	−.017172	−.018634	−.020665	−.022813
		0.9	.003808	.006880	.016041	.136436	−.038821	−.023401	−.020431	−.020408	−.021551	−.023068
	B_b	0.1	.000358	.000426	.000515	.000634	.000794	.001009	.001291	.001642	.002044	.002454
		0.3	.000666	.000697	.000759	.000857	.000997	.001191	.001447	.001764	.002122	.002480
		0.5	.004672	.001921	.001447	.001322	.001341	.001453	.001646	.001905	.002205	.002507
		0.7	−.000931	−.002539	.026908	.002897	.002045	.001864	.001908	.002070	.002296	.002535
		0.9	−.000423	−.000764	−.001782	−.015160	.004314	.002600	.002270	.002268	.002395	.002563
2_2-2_0	B_b	0.1	.052491	.067634	.087171	.111948	.142657	.179694	.223059	.272357	.326893	.385824
		0.3	.017497	.022545	.029057	.037316	.047552	.059898	.074353	.090786	.108964	.128608
		0.5	.010498	.013527	.017434	.022390	.028532	.035939	.044612	.054471	.065379	.077165
		0.7	.007499	.009662	.012453	.015993	.020380	.025671	.031866	.038908	.046699	.055118
		0.9	.005832	.007515	.009686	.012439	.015851	.019966	.024784	.030262	.036322	.042869
2_2-3_{-2}	A_b	0.1	−.002792	−.002382	−.001935	−.001480	−.001050	−.000677	−.000384	−.000179	−.000058	−.000006
		0.3	−.005501	−.004578	−.003651	−.002757	−.001944	−.001253	−.000715	−.000337	−.000111	−.000011
		0.5	−.186936	−.058776	−.032184	−.020158	−.013102	−.008448	−.005226	−.002974	−.001420	−.000375
		0.7	.005845	.005423	.004722	.003795	.002763	.001782	.000984	.000436	.000131	.000012
		0.9	.002878	.002592	.002199	.001734	.001250	.000806	.000450	.000203	.000063	.000006
	B_b	0.1	.000310	.000265	.000215	.000164	.000117	.000075	.000043	.000020	.000006	.000001
		0.3	.000611	.000509	.000406	.000306	.000216	.000139	.000079	.000038	.000012	.000001
		0.5	.020771	.006531	.003576	.002240	.001456	.000939	.000581	.000330	.000158	.000042
		0.7	−.000650	−.000603	−.000525	−.000422	−.000307	−.000198	−.000109	−.000049	−.000015	−.000001
		0.9	−.000320	−.000288	−.000244	−.000193	−.000139	−.000090	−.000050	−.000023	−.000007	−.000001

Reduced Stark coefficients for dipole-moment component along principal axis of largest moment of inertia.

Level		$\alpha \backslash \kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
0_0	A_c	0.1	−.165838	−.164204	−.162602	−.161031	−.159490	−.157978	−.156494	−.155039	−.153610	−.152207
		0.3	−.164204	−.159490	−.155039	−.150830	−.146843	−.143062	−.139470	−.136054	−.132802	−.129702
		0.5	−.162602	−.155039	−.148148	−.141844	−.136054	−.130719	−.125786	−.121212	−.116959	−.112994
		0.7	−.161031	−.150830	−.141844	−.133869	−.126743	−.120337	−.114548	−.109290	−.104493	−.100100
		0.9	−.159490	−.146843	−.136054	−.126743	−.118623	−.111483	−.105152	−.099502	−.094429	−.089847
1_{-1}	A_c	0.1	−.052219	−.051414	−.050633	−.049875	−.049140	−.048426	−.047733	−.047059	−.046404	−.045767
		0.3	−.057307	−.054496	−.051948	−.049628	−.047506	−.045558	−.043764	−.042105	−.040568	−.039139
		0.5	−.063492	−.057971	−.053333	−.049383	−.045977	−.043011	−.040404	−.038095	−.036036	−.034188
		0.7	−.071174	−.061920	−.054795	−.049140	−.044543	−.040733	−.037523	−.034783	−.032415	−.030349
		0.9	−.080972	−.066445	−.056338	−.048900	−.043197	−.038685	−.035026	−.032000	−.029455	−.027285
	B_c	0.1	.013055	.012854	.012658	.012469	.012285	.012107	.011933	.011765	.011601	.011442
		0.3	.014327	.013624	.012987	.012407	.011877	.011390	.010941	.010526	.010142	.009785
		0.5	.015873	.014493	.013333	.012346	.011494	.010753	.010101	.009524	.009009	.008547
		0.7	.017794	.015480	.013699	.012285	.011136	.010183	.009381	.008696	.008104	.007587
		0.9	.020243	.016611	.014085	.012225	.010799	.009671	.008757	.008000	.007364	.006821
1_0	A_c	0.1	−.047506	−.047281	−.047059	−.046838	−.046620	−.046404	−.046189	−.045977	−.045767	−.045558
		0.3	−.043197	−.042644	−.042105	−.041580	−.041068	−.040568	−.040080	−.039604	−.039139	−.038685
		0.5	−.039604	−.038835	−.038095	−.037383	−.036697	−.036036	−.035398	−.034783	−.034188	−.033613
		0.7	−.036563	−.035651	−.034783	−.033956	−.033168	−.032415	−.031696	−.031008	−.030349	−.029718
		0.9	−.033956	−.032949	−.032000	−.031104	−.030257	−.029455	−.028694	−.027972	−.027285	−.026631
	B_c	0.1	.011877	.011820	.011765	.011710	.011655	.011601	.011547	.011494	.011442	.011390
		0.3	.010799	.010661	.010526	.010395	.010267	.010142	.010020	.009901	.009785	.009671
		0.5	.009901	.009709	.009524	.009346	.009174	.009009	.008850	.008696	.008547	.008403
		0.7	.009141	.008913	.008696	.008489	.008292	.008104	.007924	.007752	.007587	.007429
		0.9	.008489	.008237	.008000	.007776	.007564	.007364	.007174	.006993	.006821	.006658
1_1	A_c	0.1	.117019	.112960	.108978	.105191	.101720	.098674	.096122	.094081	.092516	.091360
		0.3	.119637	.112772	.106291	.100294	.094886	.090147	.086116	.082774	.080054	.077858
		0.5	.121606	.112112	.103463	.095693	.088847	.082950	.077988	.073891	.070550	.067834
		0.7	.123076	.111126	.100597	.091402	.083489	.076802	.071257	.066729	.063064	.060096
		0.9	.124156	.109911	.097754	.087416	.078713	.071492	.065593	.060832	.057014	.053943
	B_c	0.1	−.153633	−.151393	−.149196	−.147071	−.145047	−.143152	−.141401	−.139799	−.138336	−.136995
		0.3	−.153062	−.147810	−.142852	−.138196	−.133854	−.129833	−.126131	−.122734	−.119615	−.116741
		0.5	−.152353	−.144307	−.136977	−.130306	−.124253	−.118777	−.113837	−.109382	−.105357	−.101704
		0.7	−.151542	−.140904	−.131532	−.123252	−.115929	−.109453	−.103725	−.098650	−.094136	−.090099
		0.9	−.150656	−.137610	−.126479	−.116911	−.108646	−.101485	−.095263	−.089835	−.085075	−.080871

Reduced Stark coefficients for dipole-moment component along principal axis of largest moment of inertia.—*Continued.*

Levels		$\alpha\diagdown\kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
2_{-2}	A_c	0.1	−.029919	−.028576	−.027265	−.026030	−.024911	−.023943	−.023146	−.022525	−.022070	−.021757
		0.3	−.034085	−.031087	−.028419	−.026075	−.024060	−.022375	−.021011	−.019945	−.019138	−.018545
		0.5	−.039583	−.034089	−.029687	−.026135	−.023279	−.021011	−.019245	−.017899	−.016895	−.016160
		0.7	−.047195	−.037736	−.031082	−.026205	−.022555	−.019810	−.017756	−.016236	−.015124	−.014319
		0.9	−.058434	−.042261	−.032619	−.026279	−.021879	−.018742	−.016482	−.014856	−.013689	−.012854
	B_c	0.1	.003324	.003175	.003029	.002892	.002768	.002660	.002572	.002503	.002452	.002417
		0.3	.003787	.003454	.003158	.002897	.002673	.002486	.002335	.002216	.002126	.002061
		0.5	.004398	.003788	.003299	.002904	.002567	.002334	.002138	.001989	.001877	.001796
		0.7	.005244	.004193	.003454	.002912	.002506	.002201	.001973	.001804	.001680	.001591
		0.9	.006493	.004696	.003624	.002920	.002431	.002082	.001831	.001651	.001521	.001428
2_{-1}	A_c	0.1	−.023691	−.023458	−.023229	−.023004	−.022784	−.022568	−.022356	−.022148	−.021944	−.021744
		0.3	−.023458	−.022784	−.022148	−.021547	−.020978	−.020437	−.019924	−.019436	−.018971	−.018529
		0.5	−.023229	−.022148	−.021164	−.020263	−.019436	−.018674	−.017969	−.017316	−.016708	−.016152
		0.7	−.023004	−.021547	−.020263	−.019124	−.018106	−.017191	−.016364	−.015613	−.014928	−.014300
		0.9	−.022784	−.020978	−.019436	−.018106	−.016916	−.015926	−.015022	−.014215	−.013490	−.012835
	B_c	0.1	−.040125	−.032774	−.025319	−.018181	−.011792	−.006506	−.002517	.000172	.001712	.002346
		0.3	−.011646	−.009262	−.006839	−.004518	−.002444	−.000734	.000547	.001397	.001866	.002035
		0.5	−.005970	−.004615	−.003229	−.001896	−.000705	.000272	.000997	.001466	.001711	.001780
		0.7	−.003552	−.002660	−.001734	−.000838	−.000035	.000622	.001104	.001408	.001555	.001579
		0.9	−.002219	−.001600	−.000941	−.000293	.000291	.000768	.001114	.001325	.001418	.001418
2_0	A_c	0.1	.026619	.022384	.018756	.015989	.014062	.012792	.011977	.011457	.011123	.010904
		0.3	.031722	.025637	.020589	.016789	.014124	.012329	.011129	.010314	.009743	.009326
		0.5	.037924	.029281	.022476	.017526	.014136	.011884	.010389	.009377	.008668	.008148
		0.7	.045623	.033396	.024423	.018209	.014106	.011458	.009739	.008596	.007806	.007234
		0.9	.055436	.038087	.026438	.018843	.014045	.011053	.009164	.007935	.007100	.006504
	B_c	0.1	−.010211	−.009628	−.009117	−.008704	−.008388	−.008147	−.007961	−.007809	−.007681	−.007568
		0.3	−.011484	−.010418	−.009503	−.008758	−.008168	−.007698	−.007315	−.006994	−.006717	−.006472
		0.5	−.013032	−.011305	−.009905	−.008806	−.007957	−.007294	−.006766	−.006333	−.005968	−.005654
		0.7	−.014955	−.012311	−.010324	−.008848	−.007754	−.006931	−.006294	−.005786	−.005369	−.005019
		0.9	−.017406	−.013460	−.010762	−.008885	−.007560	−.006601	−.005883	−.005326	−.004880	−.004517
2_1	A_c	0.1	.013846	.013295	.012792	.012341	.011945	.011608	.011331	.011114	.010956	.010854
		0.3	.013317	.012568	.011889	.011283	.010753	.010299	.009921	.009618	.009385	.009218
		0.5	.012741	.011862	.011073	.010375	.009768	.009251	.008822	.008476	.008207	.008010
		0.7	.012164	.011200	.010343	.009592	.008944	.008395	.007941	.007576	.007293	.007083
		0.9	.011607	.010590	.009693	.008913	.008245	.007683	.007220	.006849	.006561	.006348
	B_c	0.1	−.008137	−.008044	−.007957	−.007877	−.007802	−.007735	−.007674	−.007621	−.007574	−.007534
		0.3	−.007479	−.007319	−.007169	−.007029	−.006899	−.006779	−.006669	−.006569	−.006479	−.006397
		0.5	−.006916	−.006712	−.006521	−.006345	−.006182	−.006033	−.005897	−.005773	−.005660	−.005559
		0.7	−.006430	−.006196	−.005980	−.005782	−.005601	−.005435	−.005285	−.005149	−.005026	−.004915
		0.9	−.006006	−.005753	−.005522	−.005310	−.005119	−.004945	−.004788	−.004646	−.004519	−.004404
2_2	A_c	0.1	.015148	.017160	.019063	.020717	.021971	.022703	.022880	.022616	.022158	.021769
		0.3	.014675	.016529	.018201	.019554	.020449	.020792	.020582	.019965	.019203	.018555
		0.5	.014120	.015834	.017431	.018437	.019072	.019147	.018690	.017866	.016942	.016168
		0.7	.013542	.015132	.016458	.017397	.017838	.017726	.017109	.016164	.015158	.014325
		0.9	.012971	.014451	.015647	.016441	.016736	.016491	.015770	.014757	.013713	.012860
	B_c	0.1	.034293	.026356	.018335	.010680	.003859	−.001745	−.005927	−.008690	−.010221	−.010800
		0.3	.006432	.003468	.000507	−.002279	−.004714	−.006655	−.008030	−.008855	−.009218	−.009239
		0.5	.001289	−.000645	−.002550	−.004311	−.005811	−.006959	−.007715	−.008100	−.008183	−.008055
		0.7	−.000668	−.002141	−.003572	−.004869	−.005943	−.006728	−.007199	−.007380	−.007335	−.006639
		0.9	−.001598	−.002804	−.003958	−.004985	−.005811	−.006385	−.006691	−.006756	−.006640	−.006408
1_0-1_{-1}	B_c	0.1	1.315789	1.470588	1.666667	1.923077	2.272727	2.777778	3.571429	5.000000	8.333333	25.000000
		0.3	.438596	.490196	.555556	.641026	.757576	.925926	1.190476	1.666667	2.777778	8.333333
		0.5	.263158	.294118	.333333	.384615	.454545	.555556	.714286	1.000000	1.666667	5.000000
		0.7	.187970	.210084	.238095	.274725	.324675	.396825	.510204	.714286	.190476	3.571428
		0.9	.146199	.163399	.185185	.213675	.252525	.308642	.396825	.555556	.925926	2.777778
1_1-2_{-2}	A_c	0.1	−.018016	−.014829	−.011716	−.008792	−.006181	−.003989	−.002286	−.001087	−.000360	−.000037
		0.3	−.027275	−.021635	−.016563	−.012106	−.008329	−.005285	−.002990	−.001410	−.000465	−.000047
		0.5	−.056106	−.039986	−.028249	−.019430	−.012768	−.007830	−.004323	−.002005	−.000654	−.000066
		0.7	.982747	−.263412	−.095945	−.049179	−.027335	−.015102	−.007797	−.003466	−.001103	−.000110
		0.9	.050357	.057418	.068712	.092589	.193987	−.211759	−.039729	−.012798	−.003527	−.000331
	B_c	0.1	.004504	.003707	.002929	.002198	.001545	.000997	.000572	.000272	.000090	.000009
		0.3	.006819	.005409	.004141	.003027	.002082	.001321	.000748	.000353	.000116	.000012
		0.5	.014027	.009997	.007062	.004857	.003192	.001958	.001081	.000501	.000163	.000016
		0.7	−.245687	.065853	.023986	.012295	.006834	.003776	.001949	.000867	.000276	.000027
		0.9	−.012589	−.014355	−.017178	−.023147	−.048497	.052940	.009932	.003199	.000882	.000083
$2_{-1}-2_{-2}$	B_c	0.1	.473659	.624897	.850946	1.204169	1.789653	2.843947	4.993661	10.372655	30.453386	288.888889
		0.3	.157886	.208299	.283649	.401390	.596551	.947983	1.664554	3.457552	10.151128	96.296286
		0.5	.094732	.124979	.170189	.240834	.357931	.568790	.998732	2.074531	6.090677	57.777778
		0.7	.067666	.089271	.121564	.172024	.255665	.406278	.713380	1.481808	4.350483	44.269839
		0.9	.052629	.069433	.094550	.133797	.198850	.315994	.554851	1.152517	3.383710	32.098765
2_0-3_{-3}	A_c	0.1	−.014358	−.010277	−.006803	−.004190	−.002414	−.001296	−.000633	−.000263	−.000078	−.000007
		0.3	−.020502	−.014457	−.009524	−.005891	−.003436	−.001879	−.000939	−.000402	−.000123	−.000012
		0.5	−.035839	−.024367	−.015873	−.009918	−.005956	−.003413	−.001820	−.000849	−.000291	−.000033
		0.7	−.142277	−.077483	−.047619	−.031360	−.022335	−.018577	−.029454	.007603	.000807	.000044
		0.9	.072227	.065674	.047619	.026993	.012761	.005394	.002076	.000694	.000169	.000013
	B_c	0.1	.001595	.001142	.000756	.000466	.000268	.000144	.000070	.000029	.000009	.000001
		0.3	.002278	.001606	.001058	.000655	.000382	.000209	.000104	.000045	.000014	.000001
		0.5	.003982	.002708	.001764	.001102	.000662	.000379	.000202	.000094	.000032	.000004
		0.7	.015809	.008609	.005291	.003484	.002482	.002064	.003273	−.000845	−.000090	−.000005
		0.9	−.008025	−.007297	−.005291	−.002999	−.001418	−.000599	−.000231	−.000077	−.000019	−.000002

Reduced Stark coefficients for dipole-moment component along principal axis of largest moment of inertia.—*Continued.*

Level		$\alpha\diagdown\kappa$	−0.9	−0.7	−0.5	−0.3	−0.1	+0.1	+0.3	+0.5	+0.7	+0.9
		0.1	.048733	.054466	.061728	.071225	.084175	.102881	.132275	.185185	.308642	.925926
		0.3	.016244	.018155	.020576	.023742	.028058	.034294	.044092	.061728	.102881	.308642
2_1-2_0	B_c	0.5	.009747	.010893	.012346	.014245	.016835	.020576	.026455	.037037	.061728	.185185
		0.7	.006962	.007781	.008818	.010175	.012025	.014697	.018896	.026455	.044092	.132275
		0.9	.005415	.006052	.006859	.007914	.009353	.011431	.014697	.020576	.034294	.102881
		0.1	−.002693	−.002163	−.001684	−.001260	−.000896	−.000593	−.000354	−.000177	−.000063	−.000007
		0.3	−.005305	−.004143	−.003145	−.002301	−.001604	−.001044	−.000613	−.000303	−.000106	−.000011
	A_c	0.5	−.175669	−.048888	−.023771	−.013249	−.007648	−.004334	−.002290	−.001044	−.000341	−.000035
		0.7	.005647	.004988	.004276	.003527	.002763	.002014	.001320	.000724	.000278	.000033
		0.9	.002779	.002373	.001962	.001556	.001170	.000817	.000512	.000269	.000099	.000011
2_1-3_{-2}												
		0.1	.000299	.000240	.000187	.000140	.000100	.000066	.000039	.000020	.000007	.000001
		0.3	.000590	.000460	.000349	.000256	.000178	.000116	.000068	.000034	.000012	.000001
	B_c	0.5	.019519	.005432	.002641	.001472	.000850	.000482	.000255	.000116	.000038	.000004
		0.7	−.000627	−.000554	−.000475	−.000392	−.000307	−.000224	−.000147	−.000081	−.000031	−.000004
		0.9	−.000309	−.000264	−.000218	−.000173	−.000130	−.000091	−.000057	−.000030	−.000011	−.000001
		0.1	−.003094	−.003301	−.003428	−.003415	−.003194	−.002718	−.002002	−.001169	−.000443	−.000048
		0.3	−.005750	−.005420	−.005093	−.004676	−.004091	−.003296	−.002324	−.001313	−.000478	−.000051
	A_c	0.5	−.040563	−.015134	−.009904	−.007413	−.005686	−.004186	−.002769	−.001496	−.000520	−.000056
		0.7	.008026	.018433	−.178707	−.017879	−.009319	−.005733	−.003426	−.001739	−.000570	−.000061
		0.9	.003652	.005855	.011139	.043408	−.025827	−.009095	−.004490	−.002076	−.000630	−.000067
2_2-3_{-1}												
		0.1	.000344	.000367	.000381	.000379	.000355	.000302	.000222	.000130	.000049	.000005
		03.	.000639	.000602	.000566	.000520	.000455	.000366	.000258	.000146	.000053	.000006
	B_c	0.5	.004507	.001681	.001100	.000824	.000632	.000465	.000308	.000166	.000058	.000006
		0.7	−.000892	−.002122	.019856	.001987	.001035	.000637	.000381	.000193	.000063	.000007
		0.9	−.000406	−.000651	−.001238	−.004823	.002870	.001011	.000499	.000231	.000070	.000008