AIP | The Journal of Chemical Physics

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Citation: The Journal of Chemical Physics **1**, 133 (1933); doi: 10.1063/1.1749264
View online: http://dx.doi.org/10.1063/1.1749264
View Table of Contents: http://scitation.aip.org/content/aip/journal/jcp/1/2?ver=pdfcov
Published by the AIP Publishing

FEBRUARY, 1933 JOURNAL OF CHEMICAL PHYSICS VOLUME 1

Bond Energies and Mass Defects in Atomic Nuclei

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(Received November 25, 1932)

The problem of the interpretation of the mass defect curve is discussed in reference to the coupling of protons and electrons proposed by Latimer. Gamow's equation is modified by making the attractive energies the sum of the bond energies. For certain symmetrical nuclei approximate values for the mass defect may be calculated which are in good agreement with the experimental results.

GAMOW[1] has suggested that the energy of a nucleus built up of alpha-particles may be represented by an equation of the form:

$$E = CN_\alpha - f(Z^2/R), \tag{1}$$

where N_α is the number of alpha-particles, Z the charge on the nucleus, and R the radius. The first term postulates that the energy of attraction is proportional to the number of alpha-particles and the second term that the energy of repulsion is a function of the energy of the field produced by the charged nucleus.

It is obvious that such an expression is in qualitative agreement with the facts: with increasing values for N_α and Z, the mass defect curve at first increased but with higher atomic numbers the repulsive term becomes so large that the curve passes through a maximum and then decreases to such an extent that radioactive decomposition occurs. However, if one attempts to calculate from Eq. (1) a quantitative mass defect curve for the $4n$ proton series (i.e., the nuclei which may be considered as built up of alpha-particles and "cementing electrons"), it appears to be impossible to obtain agreement employing any reasonable assumptions regarding the energies of the extra electrons. Moreover, an inspection of the experimental values for the lighter elements of the series, shows that the increase in the mass defect is not regular. It is doubtless small for beryllium with 2 alpha-particles, larger for carbon with 3 particles, very much larger for oxygen and small again for the addition of the fifth alpha-particles to form neon.

[1] Gamow, *Der Bau des Atomkerns und die Radioaktiv.*, p. 21. Hirzel, Leipzig, 1932.

We wish to discuss these problems in terms of the nuclear structure proposed by Latimer.[2] The essential feature of this theory is the close packing of alpha-particles in a tetrahedral pattern. Each alpha-particle is pictured as a small tetrahedron with a proton at each corner and the coupling pattern is such that the tetrahedral angle between protons is maintained throughout the structure. It may be of significance that this is the angle required for the addition of half unit spin vectors by the quantum mechanics. The extra or "cementing" electron pairs enter whenever four protons come together at a point. The structure thus gives a basis for the entrance of the first electron pair at argon, 40, the second pair at zinc, 68, and in general gives the correct number of alpha-particles and extra electrons. Moreover, it offers an explanation for the maxima in the abundance curve for the lighter elements and for the termination of the radioactive decomposition series at lead.

If we adopt the viewpoint of this theory we should modify Eq. (1) by assuming that the attractive energy is proportional, not to the number of alpha-particles, but to the number and kinds of *bonds* between the alpha-particles.

$$E = \Sigma \text{ bond energies} - f(Z^2/R). \tag{2}$$

In the proposed structure for nuclei of the $4n$ proton type, there are but three kinds of bonds; those between 2 protons, between 3 protons and between 4 protons and an electron pair. Hence a knowledge of these three bond energies would permit a rough evaluation of the first term of this equation.

[2] Latimer, J. Am. Chem. Soc. **53**, 981 (1931).

The classical relativistic expression for the electromagnetic or field mass, $\frac{2}{3}Z^2/R$, assumes a uniform distribution of charge over a spherical shell. Now one of the interesting features of the proposed nuclear structure, is that the entrance of the extra electrons makes the interior of the lattice electrically neutral and the whole of the positive charge occurs on the surface of the lattice. Moreover, the structures for O^{16} (see Fig. 1), A^{40}, Xe^{124}] (see Fig. 2), and Po^{208}, are not far from spherical symmetry. So in these special cases the classical expression may give at least an approximate value for the field energy.

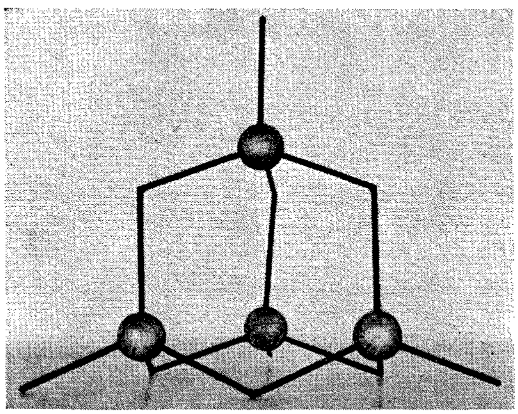

FIG. 1. Oxygen 16, four alpha-particles. Each alpha-particle is represented by a ball for the electron pair and four bars for the protons.

For these nuclei, then, we may set up expressions for the mass defect, i.e., the energy of formation from alpha-particles and electrons, in the following manner. We will write for the energy of the alpha-particle,

$$E_\alpha = E_{\alpha'} - \tfrac{2}{3}(Z_\alpha^2/R_\alpha), \quad (3)$$

where $E_{\alpha'}$ is the interaction energies of the four protons and two electrons. In oxygen, 16, we have 4 alpha-particles and 6 two-proton bonds, hence

$$E_0 = 4E_{\alpha'} + 6E_{2PB} - \tfrac{2}{3}(Z_0^2/R_0). \quad (4)$$

Then from (3) and (4) we may express the mass defect of oxygen in terms of the energy of the bond and the charges and radii of the alpha-particle and oxygen nucleus.

$$MD_0 = 6E_{2PB} - \tfrac{2}{3}(Z_0^2/R_0) + 4x\tfrac{2}{3}(Z_\alpha^2/R_\alpha). \quad (5)$$

For the heavier elements we will have similar expressions but these will contain also the energy of the three-proton bonds and the bond formed by 4 protons and a cementing electron pair. This latter has been represented in the models by a dark ball and we shall designate the energy of this bond as $E_{\alpha''}$.

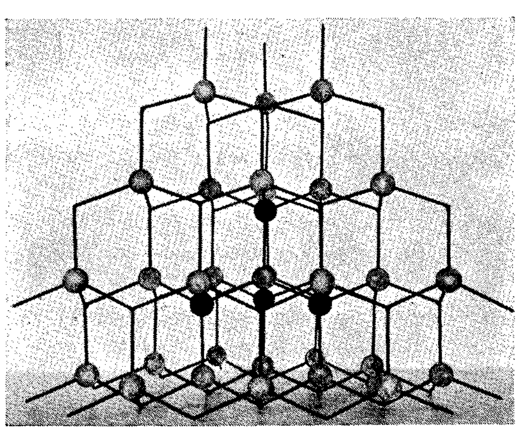

FIG. 2. Xenon 124, thirty-one alpha-particles and four extra electron pairs, which are represented by the black balls.

In general then we have four parameters which must be determined, three-bond energies and the lattice constant which determines the radius. These parameters, however, are not all independent and our choice of values is confined to narrow limits by experimental facts. Thus Gamow[1] uses 2.2×10^{-13} cm as the radius of the alpha-particle. The effective value in the nuclear lattice may be slightly larger than for the free alpha-particle, but should give a calculated value for the radius of the lead nucleus (or Po^{208}), probably not greater than 8×10^{-13}. We have used 2.4×10^{-13} as the lattice value and calculate the radii given in Fig. 3. In these calculations we have taken the average distance of the surface protons from the center of the lattice. This of course is only an approximation but should give comparable values which are sufficient in view of the somewhat arbitrary choice of this parameter.

Aston's experimental value for the mass defect of the alpha-particle is 0.029 mass (atomic weight) units. It would seem that the value for $E_{\alpha''}$ should be somewhat less in view of the sharing of protons between two groups. The energy of the two-proton bond should be about

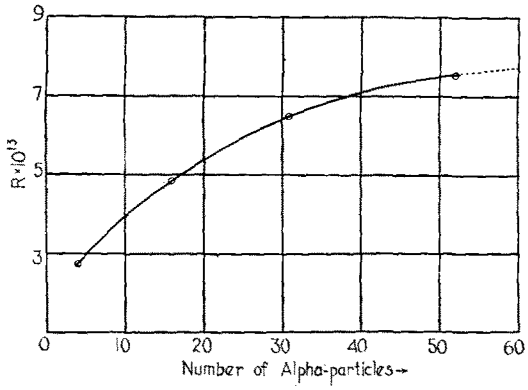

FIG. 3. Nuclear radius as a function of the number of alpha-particles.

one-third that of the three-proton bond and several considerations indicate that this in turn is less than $E_{\alpha''}$. The values which we have arbitrarily chosen to give the best agreement are $E_{2PB}=0.005$, $E_{3PB}=0.017$ and $E_{\alpha''}=0.020$, all expressed in mass units. The calculations are summarized in Table I.

Now while we do not attach any great importance to the quantitative agreement of the experimental and our calculated mass defects, we do believe it to be significant that the number and kinds of bonds in these structures are consistent with the reasonable values for the four parameters.

The assumption of the constancy of the $E_{\alpha'}$ value throughout all of the structures is doubtless not strictly true: especially is this so in the case of the one alpha-particle in xenon and the four alpha-particles in polonium which lie inside the lattice, and may therefore be considered as equivalent to the $E_{\alpha''}$. Subtracting the difference between 0.029 and 0.020 for these particles one obtains the mass defects given in brackets. It would seem that the value for E_{2PB} should be greater than one-third of E_{3PB}, but in view of the approximate nature of the calculations, the discrepancy is not important.

The large effect of the nuclear charge in the case of the heavier elements explains a number of anomalies in the abundance of these elements. Thus for the elements of even atomic number, the $4n-2$ proton type becomes more abundant than the $4n$ type which predominates with the lighter elements. This may be interpreted as meaning, that for large Z, the effect on the mass defect of reducing the charge by the removal of two protons, is much greater than the energy of

TABLE I. *Summary of calculations of mass defect.*

	$R\times10^{13}$	Bond energy	Δ Field energy	Mass defect Calculated	Experimental*
O_{16}	2.75	$6\times0.005=0.030$	$0.0239-0.0068=0.017$	0.013	0.009
A_{40}	3.90	$9\times0.005=0.045$			
		$3\times0.017=0.051$			
		$1\times0.020=0.020$			
		0.116	$0.085-0.017=0.068$	0.048	0.050
Zn_{64}	4.86	$6\times0.005=0.030$			
		$12\times0.017=0.204$			
		$1\times0.020=0.020$			
		0.254	$0.189-0.027=0.162$	0.092	0.095
Xe_{124}	6.49	$12\times0.005=0.060$			
		$24\times0.017=0.408$			
		$4\times0.020=0.080$			
		0.548	$0.460-0.053=0.407$	0.141 (0.132)	0.133
Po_{208}	7.56	$18\times0.005=0.090$			
		$40\times0.017=0.680$			
		$10\times0.020=0.200$			
		0.970	$0.958-0.089=0.869$	0.101 (0.065)	<0.095

* Aston's values as tabulated by Gamow.[1] The value for Po^{208} should be less than that for Pb^{208} which is 0.095, since the former is unstable with respect to the latter.

binding these two protons. Hence Po^{208} which one would expect to be abundant from the symmetry of the lattice model, is unstable with respect to the loss of two protons to give Pb^{206}, and also in this case to the addition of an extra electron pair to give Pb^{208}.

The effect of the charge explains also why Xe^{124}, adds an alpha-particle and electron pair to the face of its structure to form Xe^{128} rather than simply an alpha-particle to a corner to form Ba^{128}. The first process gives no change in field energy and an increase of 0.003 in bond energy ($E_{\alpha''}-E_{3PB}$). The second process gives an increase of $3E_{2PB}$ or 0.015 mass units in bond energy but the effect of the increase in charge is -0.035 mass units, or a net energy of 0.020 mass units against the reaction. Aston's value for the difference in mass defect for Xe^{124}, and Xe^{128}, is 0.004 units in agreement with the 0.003 which we calculate.

As another point of some interest, we may calculate the total energy evolved in the radioactive decomposition of uranium into lead. Uranium has eight alpha-particles and three electron pairs more than lead and we postulate that these form the most stable bonded group on the surface of the lead structure. Then using the extrapolated value for the radius of uranium, Fig. 3, and the same bond energies as before, we calculate 0.052 mass units for the total change, as compared with 0.048 units given by Rutherford and Chadwick,[3] as the best calorimetric value.

With the exception of oxygen, 16, the lighter elements are too unsymmetrical to attempt an evaluation of the field energy. However, the irregular increase in the mass defect is roughly proportional to the number of two-proton bonds which the theory indicates, and is added confirmation, we believe, of the point of view which we have presented.

[3] Rutherford, Chadwick and Ellis, *Radiations from Radioactive Substances*, p. 163, MacMillan, 1930.