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# Noise Removal for Degraded Images with Poisson Noise Using M-Transformation and BayesShrink Method

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## SUMMARY

Median filters and other nonlinear filters have been investigated for restoration of degraded images with Poisson noise. Recently, subband image restoration using the wavelet transform has been attracting much attention. This method is effective for small-amplitude noise, but in the case of Poisson noise, large-amplitude noise exceeds the preset threshold and is not removed. In this study, we propose a new method of noise removal from degraded images with Poisson noise by using a combination of the M-transformation [5] and the wavelet BayesShrink method. © 2007 Wiley Periodicals, Inc. Electron Comm Jpn Pt 3, 90(11): 11–20, 2007; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/ecjc.20357

**Key words:** Poisson noise; M-transformation; wavelet; BayesShrink method.

## 1. Introduction

Numerous images of various kinds are being digitally processed in different fields. Such processing involves the restoration of degraded images.

Studies of image restoration with noise removal have been performed for many years, and a large variety of smoothing filters have already been proposed [1, 2]. Median filters are helpful for both edge preservation and impulse noise removal, which was impossible with conventional linear filters. However, these filters are not free of problems. In particular, images are processed without separation of edge and nonedge portions, and hence new blurring occurs. In addition, Wiener filters achieve much better preservation of edges and high-frequency portions than conventional linear filters, and are effective for white noise such as Gaussian noise.

Noise removal techniques based on wavelet transform have been proposed recently, and image restoration in the wavelet domain is attracting much attention [3]. Noise removal methods based on the wavelet transform involve threshold processing in every wavelet domain, thus providing signal preservation where the original signal components are dominant, and noise removal where noise is dominant [4]. An image is subjected to the wavelet transform, and the scaling factor and wavelet coefficients are obtained. It is known that noise concentrates around the wavelet coefficients. Noise can be removed by using the wavelet BayesShrink method proposed in Ref. 4. However, this method assumes white Gaussian noise, and its effectiveness for images degraded by Poisson noise is not clear.

In this study, we propose a method for removing Poisson noise by combining the M-transformation with the wavelet BayesShrink method. In the proposed method, the M-transformation, a new signal processing technique presented in Ref. 5, is used to transform Poisson noise, depending on the pixel value, into small-width irregular noise. Then the wavelet BayesShrink method, which is effective for small-width noise, is applied to the M-transformed image for noise removal. The proposed method has been validated by computer simulations.

## 2. Properties of Poisson Noise

Poisson noise is a signal-dependent noise that can be seen on photon images, and is also called quantum noise [6]. The photon image is an image formed by observation of photons, and is a special image dealt with in such fields as medicine and astronomy. Photons are generated discretely according to the Poisson distribution. Due to its signal dependence, Poisson noise is more difficult to remove than Gaussian noise, impulse noise, and other kinds of signal-independent noise.

### 2.1. Poisson distribution

The Poisson distribution is defined as follows in terms of the mean parameter $k$:

$$p(x;k) = \frac{e^{-k}k^x}{x!} \tag{1}$$

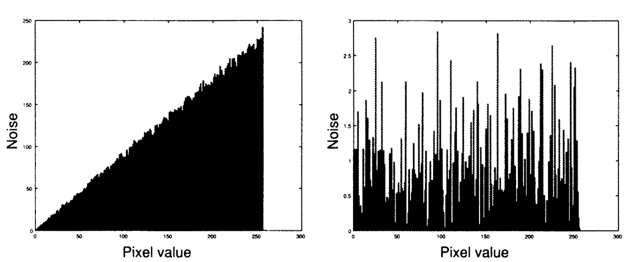
In the Poisson distribution, the mean and the variance are equal. That is, the Poisson distribution is governed by the single parameter $k$. As $k$ increases, the Poisson distribution approaches the Gaussian distribution.

### 2.2. Model of degradation by Poisson noise

The model of degradation due to Poisson noise is expressed as follows:

$$g(i,j) = \frac{1}{\lambda} Poisson(\lambda f(i,j)) \tag{2}$$

Here $f(i, j)$ is the original image and $g(i, j)$ is the degraded image. In the function *Poisson*(), the argument is the mean of the Poisson distribution, and the return value is a Poisson random generation function that returns Poisson random numbers. In addition, $\lambda$ is a constant that expresses the degree of image degradation. The smaller $\lambda$ is, the stronger the degradation. Since the degree of degradation depends on the number of detected photons, the fewer photons are detected, the smaller $\lambda$ is.



(a) Poisson noise (b) Gaussian noise

Fig. 1. Relationship between pixel value and degree of degradation.

Figure 1 shows the relations between the original image signal and the degree of image degradation for Poisson noise with $\lambda = 0.1$, and Gaussian noise with $\sigma = 20$. In the diagrams, the pixel value is plotted on the horizontal axis, and the difference in pixel value between the original image and the degraded image is plotted on the vertical axis. As can be seen from Fig. 1(a), Poisson noise grows in amplitude directly with the pixel value of the original image. On the other hand, Fig. 1(b) indicates that the amplitude of Gaussian noise is random, being independent of the pixel value of the original image. That is, Poisson noise is signal-dependent in contrast to Gaussian noise.

## 3. Conventional Wavelet BayesShrink Method [4]

Wavelet shrinkage is a method of removing noise from images. In wavelet shrinkage, an image is subjected to the wavelet transform, the wavelet coefficients are found, the components with coefficients below a threshold are replaced with zeros, and the image is then reconstructed. In particular, the BayesShrink method has been attracting attention recently as an algorithm for setting different thresholds for every subband. Here subbands are frequency bands that differ from each other in level and direction. The BayesShrink method is effective for images including Gaussian noise. The observation model is expressed as follows:

$$Y = X + V \tag{3}$$

where $Y$ denotes the wavelet transform of the degraded image, $X$ denotes the wavelet transform of the original image, and $V$ denotes the wavelet transform of the noise component with a Gaussian distribution $N(0, \sigma^2)$. Since $X$ and $V$ are mutually independent, the following is true for the respective variances:

$$\sigma_Y^2 = \sigma_X^2 + \sigma^2 \tag{4}$$

Below we explain threshold setting by the BayesShrink method.

A. Estimation of standard deviation of noise [6]

$$\tilde{\sigma} = \frac{Median(|X_{i,j}|)}{0.6745} \tag{5}$$

Here $X_{i,j}$ denotes the coefficients in the diagonal subband obtained by 1-level wavelet decomposition.

B. Estimation of subband variances

$$\tilde{\sigma}_Y^2 = \frac{1}{n^2}\sum_{i,j=1}^{n} Y_{i,j}^2 \tag{6}$$

Here $n^2$ is the total number of coefficients in the subband of interest, and $Y_{i,j}$ denotes the coefficients in the subband of interest.

C. Calculation of threshold $T_s$

The threshold $T_s$ can be calculated from the results of the above steps A and B as follows:

$$T_s = \beta \frac{\tilde{\sigma}^2}{\tilde{\sigma}_X} \tag{7}$$

Here $\tilde{\sigma}_X$ is found as follows:

$$\tilde{\sigma}_X = \begin{cases} \sqrt{\tilde{\sigma}_Y^2 - \tilde{\sigma}^2}, & \tilde{\sigma}_Y^2 \geq \tilde{\sigma}^2 \\ 0, & \tilde{\sigma}_Y^2 < \tilde{\sigma}^2 \end{cases} \tag{8}$$

Thus, wavelet shrinkage is implemented by the BayesShrink method using different thresholds $T_s$ for all frequency components.

This method is effective against small-amplitude noise included in images. However, in the case of Poisson noise, large-amplitude noise components exceed the threshold, and such components cannot be removed by threshold processing.

## 4. Proposed System

To solve the problems of the BayesShrink method, we here propose noise removal for images degraded by Poisson noise by combining the M-transformation, which converts Poisson noise into small-amplitude irregular noise, with the wavelet BayesShrink method. The processing flow of the proposed system is shown in Fig. 2.

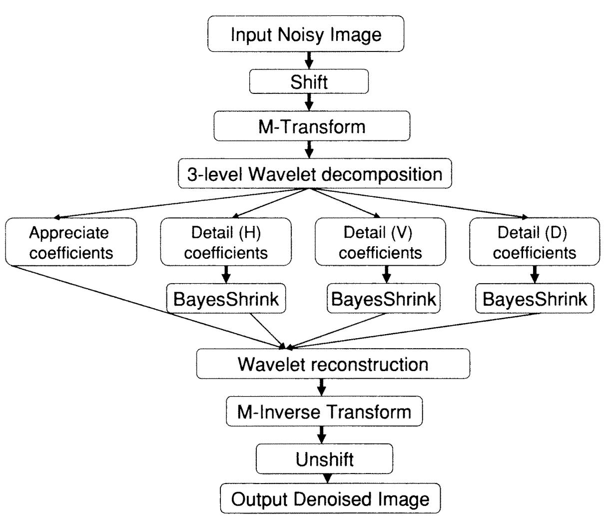



Fig. 2. Procedures of proposed denoising system.

Below we explain the first translation-invariant method (average-over-shifts), and then the combination of the M-transformation with wavelet BayesShrink.

### 4.1. Average-over-shifts

When noise is removed by using wavelets, the translation-invariant method (average-over-shifts) [7] is employed to suppress the Gibbs phenomenon that occurs in the inverse wavelet transform. This method consists of:

(a) Acquisition of cyclic shift data for the pixel values of the image of interest
(b) M-transformation of shift data
(c) 2D discrete wavelet transform
(d) Threshold processing of wavelet coefficients
(e) 2D discrete inverse wavelet transform
(f) Reset of shifted values to original position (shift-back)
(g) Averaging of all shifts

### 4.2. Combining M-transformation with wavelet BayesShrink method

#### 4.2.1. M-transformation [5]

The binary series $m_i$ ($0 \leq i \leq N-1$) is defined as follows using the M-series $a_i$ (= 0 or 1) with a period of $N$ (= $2^n - 1$) generated from an $n$-th order primitive polynomial defined on the Galois field $GF(2)$:

$$m_i = (-1)^{a_i} \tag{9}$$

Using this $m_i$, an $N \times N$ matrix $M_i$ is formed as follows:

$$M_i = \begin{bmatrix} m_i & m_{i-1} & \cdots & m_{i-N+1} \\ m_{i+1} & m_i & \cdots & m_{i-N+2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{i+N-1} & m_{i+N-2} & \cdots & m_i \end{bmatrix} \tag{10}$$

By the nature of the M-series, the following is true for the transposed matrix $M_i^T$:

$$M_i^T M_i = \begin{bmatrix} N & -1 & \cdots & -1 \\ -1 & N & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & N \end{bmatrix} \tag{11}$$

The inverse matrix is

$$
(M_i^T M_i)^{-1} = \frac{1}{N+1} \begin{bmatrix} 2 & 1 & \cdot & \cdot & \cdot & 1 \\ 1 & 2 & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 1 & 1 & \cdot & \cdot & \cdot & 2 \end{bmatrix} \tag{12}
$$

Let $x(i\Delta t)$, or simply $x(i)$, denote the signal obtained from $x(t)$ by sampling at an interval $\Delta t$. The M-transformation $T_M$ of $x(i)$ is defined as

$$
X_i = M_i T_M \tag{13}
$$

Here

$$
X_i = \{x(i), x(i+1), \cdots, x(i+N-1)\}^T
$$
$$
T_M = (\alpha_0, \alpha_1, \cdots, \alpha_{N-1})^T \tag{14}
$$

The M-transformation $T_M$ is expressed as follows:

$$
T_M = (M_i^T M_i)^{-1} M_i^T X_i \tag{15}
$$

In addition, the following can be derived from Eq. (13):

$$
x(i) = \sum_{j=0}^{N-1} \alpha_j m_{i-j} \qquad (0 \le i \le N-1) \tag{16}
$$

As is evident from the definition of the M-transformation $T_M$, a signal $X_i$ can be thought of as a sum of weights of the M-series with respect to $T_M$. This is equivalent to considering $x_i$ as $m_i$ transmitted through a filter with an impulse response series $\alpha_i$. This, again, is similar to a signal transmitted through a filter with white noise.

#### 4.2.2. Combination of M-transformation and wavelet BayesShrink method

Let $IA$ denote the M-transformation of image $X$ degraded with Poisson noise:

$$
X = \{x(i,j) \quad 0 \le i \le m-1, 0 \le j \le m-1\}
$$
$$
IA = M_i^{-1} X \tag{17}
$$

Here $m \times m$ is the number of pixels in the image. The following equation can be derived from Eqs. (12) and (15):

$$
\begin{aligned}
IA &= M_i^{-1} X \\
&= (M_i^T M_i)^{-1} M_i^T X \\
&= \frac{1}{N+1} \begin{bmatrix} 2 & 1 & \cdot & \cdot & \cdot & 1 \\ 1 & 2 & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 1 & 1 & \cdot & \cdot & \cdot & 2 \end{bmatrix} M_i^T X
\end{aligned} \tag{18}
$$

Poisson noise is signal-dependent noise, and when the conventional BayesShrink method is used for noise removal, the noise included in areas with high pixel values is not removed. However, Poisson signals with high values can be approximated by multiple signals. If signal $X_i$ is impulse noise $IM_j$ composed of impulses with magnitudes $im_j$ at positions $j$, it can be expressed as follows:

$$
IM_j = (0, 0, \cdots, im_j, 0, \cdots, 0)^T \tag{19}
$$

In addition, element $\alpha_i$ of its M-transformation $A_i m$ is as follows:

$$
\alpha_i = \frac{1}{N+1}(m_{i+1} - 1) im_j \tag{20}
$$

As suggested by Eq. (20), applying the M-transformation to impulse noise produces small-amplitude M-series noise. Therefore, when a Poisson signal is subjected to the M-transformation, it becomes a small-amplitude M-series signal. Usually, data gathered using Poisson-noise degraded images follow the Poisson distribution. Equation (20) indicates that when Poisson noise exists in a degraded image, the M-transformation of such an image is a sum of small-amplitude M-series signals, thus exhibiting Gaussian irregularity.

Figures 3(a) and 3(b) show an image degraded by Poisson noise with $\lambda = 0.1$ and its M-transformation on a 256-grade scale, and Fig. 3(c) shows the restored image after the inverse M-transformation. The pixel values of the degraded image and its M-transformation are given in Figs. 3(d) and 3(e), respectively. Figures 4(a) and 4(b) show an original image free of noise and its M-transformation on a

(a) (b) (c)

(Cameraman$\lambda = 0.1$)

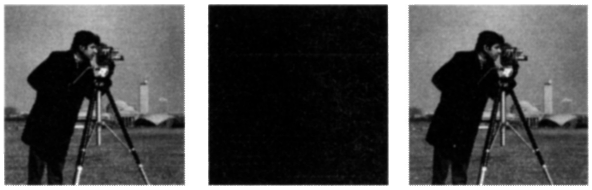
| | | | | |
|---|---|---|---|---|
| 109 | 240 | 180 | 190 | 240 |
| 130 | 150 | 180 | 170 | 240 |
| 190 | 150 | 150 | 190 | 200 |
| 200 | 170 | 150 | 200 | 210 |
| 170 | 180 | 170 | 140 | 160 |
| 120 | 210 | 120 | 210 | 109 |
| 210 | 130 | 170 | 250 | 170 |

(d)

| | | | | |
|---|---|---|---|---|
| 0.665 | 20.66 | 29.39 | 31.16 | 39.80 |
| 37.15 | 54.84 | 32.15 | 59.88 | 22.15 |
| 105.0 | 107.5 | 156.1 | 132.5 | 166.5 |
| 105.7 | 76.74 | 35.74 | 61.72 | 10.74 |
| 4.456 | 49.54 | 34.41 | 25.45 | 10.54 |
| 53.12 | 9.374 | 28.12 | 83.80 | 77.62 |
| 70.37 | 123.6 | 90.33 | 73.68 | 85.33 |

(e)

Fig. 3. Degraded image and M-transformation. (a) Degraded image; (b) Image after M-transformation; (c) Image after inverse M-transformation; (d) Pixel values of degraded image; (e) Pixel values of M-transformed image



(a) (b) (c)

(Cameraman$\lambda = 0.1$)

Fig. 4. Original image and M-transformation.
(a) Original image; (b) Image after M-transformation;
(c) Image after inverse M-transformation.

256-grade scale, and Fig. 4(c) shows the restored image after the inverse M-transformation. Figures 3 and 4 indicate that the pixel values of an image become small irregular signals after M-transformation. Therefore, by applying the conventional BayesShrink method to the M-transformation *IA* of Poisson-degraded image *X* it becomes possible to remove Poisson noise, in particular large-amplitude noise.

The method of Poisson noise removal proposed in this study can be summarized as follows. First, M-transformation is performed on the degraded image, and *IA* is found:

$$IA = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,n-1} \\ \alpha_{2,1} & \alpha_{2,2} & \cdots & \alpha_{2,n-1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \alpha_{m-1,1} & \alpha_{m-1,2} & \cdots & \alpha_{m-1,n-1} \end{bmatrix} \tag{21}$$

Then the M-transformation *IA* is subjected to the regular orthogonal wavelet transform, and the wavelet coefficients are calculated. In the regular orthogonal wavelet transform, a power of 2 of the signal length is required. Since the M-transformation signal is $(2^n - 1) \times (2^n - 1)$, $\alpha_{i,n}$ and $\alpha_{m,j}$ as shown below are added:

$$\begin{aligned} \alpha_{i,n} &= \alpha_{i,n-1} \\ \alpha_{m,j} &= \alpha_{m-1,j} \end{aligned} \tag{22}$$

The BayesShrink algorithm is applied to the wavelet coefficients thus obtained, using thresholds calculated by Eq. (7), and the noise components are removed. The M-transformation *IB* after noise removal is subjected to the inverse transformation to restore the original image without noise.

## 5. Simulations

We applied the proposed Poisson noise removal system to the restoration of actual degraded images. Here we explain the parameter settings used in simulations.

### 5.1. Number of shifts

In the present study, the average-over-shifts algorithm is employed to suppress the Gibbs phenomenon in wavelet reconstruction. When using average-over-shifts, the PSNR characteristics are examined versus the number of shifts to find an optimal number $n$. A computing time of $O(n^2)$ is taken, and thus increases with the square of the number of shifts. Therefore, if the number of shifts is set too large, the computing time increases dramatically.

In this study, considering PSNR convergence and computing time, we set the number of shifts to 4 in both the horizontal and vertical directions, that is, the total number was $4 \times 4$ shifts.

### 5.2. Test images

We used three images, Cameraman, Building, and Airplane. All the images are monochrome with a size of 256 $\times$ 256 pixels, and 8 bits per pixel.

The images were degraded by adding noise, and were then subjected to restoration processing. The noise was Poisson noise, with the parameter $\lambda$, expressing the degree of degradation, set to 0.01, 0.1, 0.5, 1.0, and 1.5.

The degraded and processed images Cameraman with $\lambda$ of 0.1 and 0.5 are shown in Figs. 5 and 6, respectively. In the diagrams, (a) is the original image, (b) is the noisy image, (c) is the image processed by MF (median filter), (d) is the image processed by WF (Wiener filter), (e) is the image processed by AS-BS (average-over-shifts + BayesShrink), and (f) is the image processed by the proposed system.

### 5.3. Simulation results

The following results were obtained by simulations.

- Conventional smoothing filters

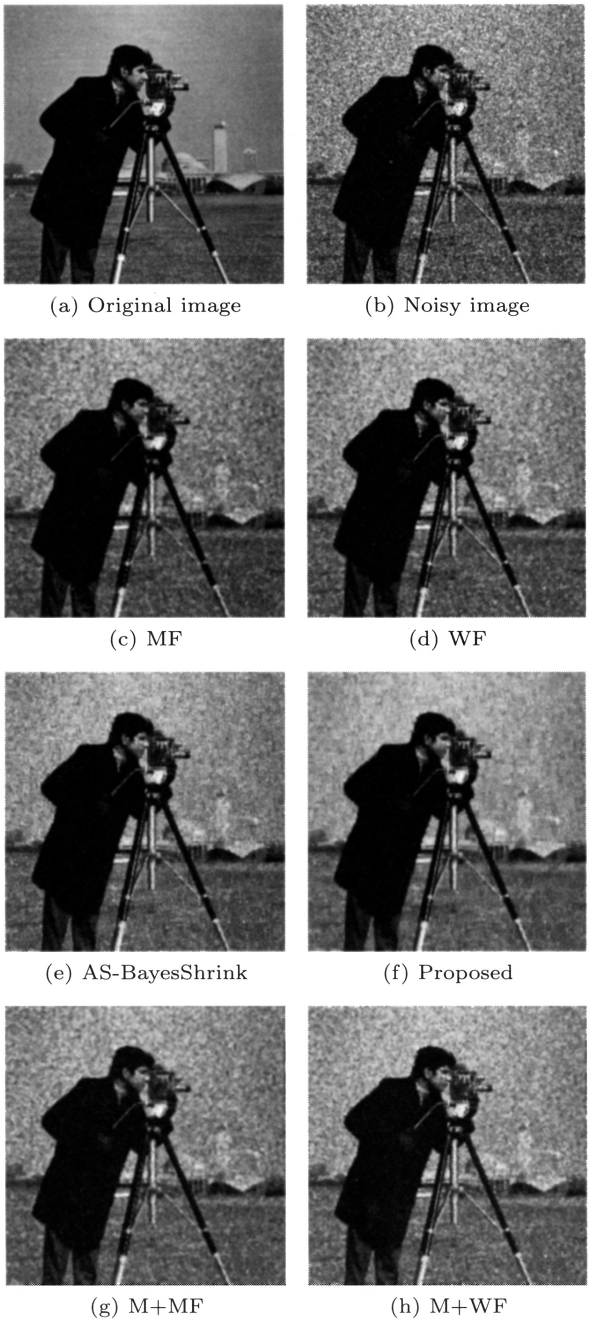
Results obtained by $3 \times 3$ median filter (MF)
Results obtained by $3 \times 3$ Wiener filter (WF)
Results obtained by M-transformation + WF
Results obtained by M-transformation + MF

- Conventional wavelet transform with threshold processing

Results obtained by combination of BayesShrink method [4] and average-over-shifts method (AS-BS)

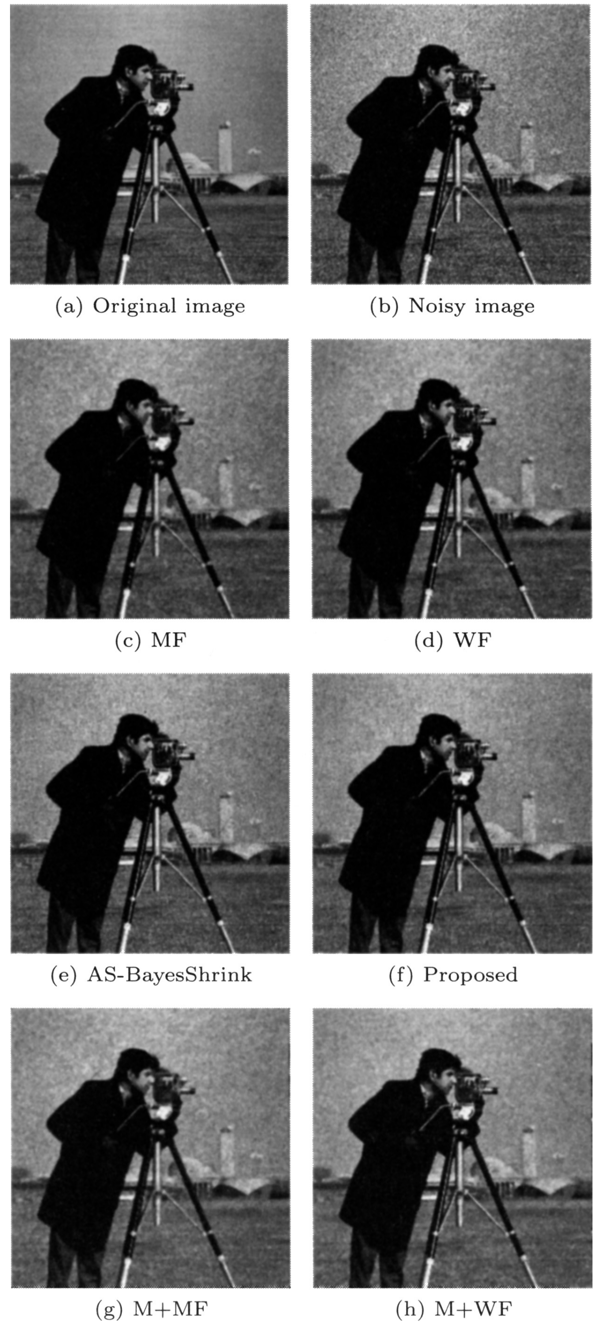
- Proposed system (combination of M-transformation with AS-BS processing)
- Estimation of every method

The results are compared in terms of the PSNR (Peak Signal to Noise Ratio).



(a) Original image (b) Noisy image
(c) MF (d) WF
(e) AS-BayesShrink (f) Proposed
(g) M+MF (h) M+WF

Fig. 5. Noisy image and processed image (Cameraman, λ = 0.1).



(a) Original image (b) Noisy image
(c) MF (d) WF
(e) AS-BayesShrink (f) Proposed
(g) M+MF (h) M+WF

Fig. 6. Noisy image and processed image (Cameraman, λ = 0.5).

The PSNR is defined as follows, using the maximum brightness signal $S_{\max}$ and the mean square error (MSE) between the processed image and the reference image:

$$PSNR = 20\log_{10}\left(\frac{255}{MSE}\right) \tag{23}$$

$$MSE = \frac{1}{H \times L}\sum_{i=1}^{H}\sum_{j=1}^{L}\left(d(i,j) - o(i,j)\right)^2 \tag{24}$$

Here $d(i, j)$ is the pixel value of the processed image, and $o(i, j)$ is the pixel value of the reference image.

## 6. Discussion

As is evident from Figs. 5 and 6, edge blurring is more perceptible than noise removal in the case of MF. This is because signal preservation is not a strong point of MF, and

Table 1. Value of PSNR (Cameraman)

| | $\lambda = 0.01$ | $\lambda = 0.1$ | $\lambda = 0.5$ | $\lambda = 1.0$ | $\lambda = 1.5$ |
|---|---|---|---|---|---|
| Noisy | 9.77 | 17.63 | 24.43 | 27.40 | 29.14 |
| MF | 13.52 | 22.46 | 25.71 | 26.43 | 26.62 |
| WF | 15.86 | 23.56 | 29.00 | 30.34 | 30.90 |
| M+MF | 13.47 | 22.28 | 25.40 | 26.07 | 26.29 |
| M+WF | 15.96 | 23.65 | 29.33 | 30.49 | 31.00 |
| AS-BS | 18.69 | 23.89 | 28.41 | 30.33 | 31.75 |
| Proposed | 18.99 | 25.31 | 29.38 | 31.39 | 32.59 |

Table 3. Value of PSNR (Airplane)

| | $\lambda = 0.01$ | $\lambda = 0.1$ | $\lambda = 0.5$ | $\lambda = 1.0$ | $\lambda = 1.5$ |
|---|---|---|---|---|---|
| Noisy | 8.71 | 16.66 | 22.70 | 25.62 | 27.39 |
| MF | 12.76 | 21.88 | 26.02 | 27.03 | 27.49 |
| WF | 14.24 | 23.13 | 27.95 | 29.49 | 30.28 |
| M+MF | 12.21 | 21.61 | 25.71 | 26.43 | 26.62 |
| M+WF | 14.86 | 23.56 | 28.20 | 29.65 | 30.50 |
| AS-BS | 15.37 | 24.28 | 28.60 | 30.54 | 31.68 |
| Proposed | 15.92 | 24.50 | 28.70 | 30.64 | 31.89 |

edges cannot be preserved appropriately. WF shows better noise removal than MF. However, noise removal in both WF and MF is accompanied by edge smoothing. This is because filtering is applied uniformly to the entire image, including both flat portions and edges. AS-BS offers good results in terms of edge preservation. The reason is that the thresholds set independently in every subband of the wavelet domain function properly for both noise removal and edge preservation. However, large-amplitude noise exceeds the thresholds, and thus is not removed. The proposed system shows good results for both noise removal and edge preservation. This is because Poisson noise is converted into small-amplitude irregular noise by the M-transformation, so that even the noise components with originally large amplitude are removed successfully.

The PSNR values for Cameraman, Building, and Airplane are given, respectively, in Tables 1, 2, and 3. As is evident from the tables, MF produces a worse PSNR than the other methods. The reason is that MF is an efficient filter for impulse noise removal, and large-amplitude noise is removed. Moreover, for $\lambda > 1.0$, the processed image has a poorer PSNR than the degraded image. This is because the degree of degradation is small, and the edge portions are removed excessively.

On the other hand, M + MF has a worse PSNR than MF. This is because the M-transformation makes the noise more Gaussian, and the MF effect disappears. WF obviously outperforms MF, and the performance improves further in M + WF. This is because WF itself is effective for Gaussian noise. AS-BS achieves basically good results

Table 2. Value of PSNR (Building)

| | $\lambda = 0.01$ | $\lambda = 0.1$ | $\lambda = 0.5$ | $\lambda = 1.0$ | $\lambda = 1.5$ |
|---|---|---|---|---|---|
| Noisy | 9.24 | 17.62 | 24.50 | 27.45 | 29.23 |
| MF | 13.11 | 23.21 | 27.26 | 28.25 | 28.65 |
| WF | 15.43 | 23.38 | 28.28 | 29.38 | 29.87 |
| M+MF | 13.15 | 23.16 | 27.16 | 28.14 | 28.53 |
| M+WF | 15.58 | 23.55 | 28.57 | 29.39 | 30.03 |
| AS-BS | 17.02 | 24.14 | 28.77 | 30.85 | 32.29 |
| Proposed | 17.73 | 24.68 | 28.96 | 30.93 | 32.32 |

compared to the other methods. This can be attributed to the setting of appropriate thresholds for each level and direction, which results in effective noise removal and edge preservation. The best results are achieved by the proposed method. This is because large-amplitude noise that could not be removed by AS-BS is removed by the BayesShrink method applied after the M-transformation.

Based on the above results, we may conclude that in images degraded by Poisson noise, a removal system combining the M-transformation with the wavelet BayesShrink method is more effective than conventional wavelet shrinkage.

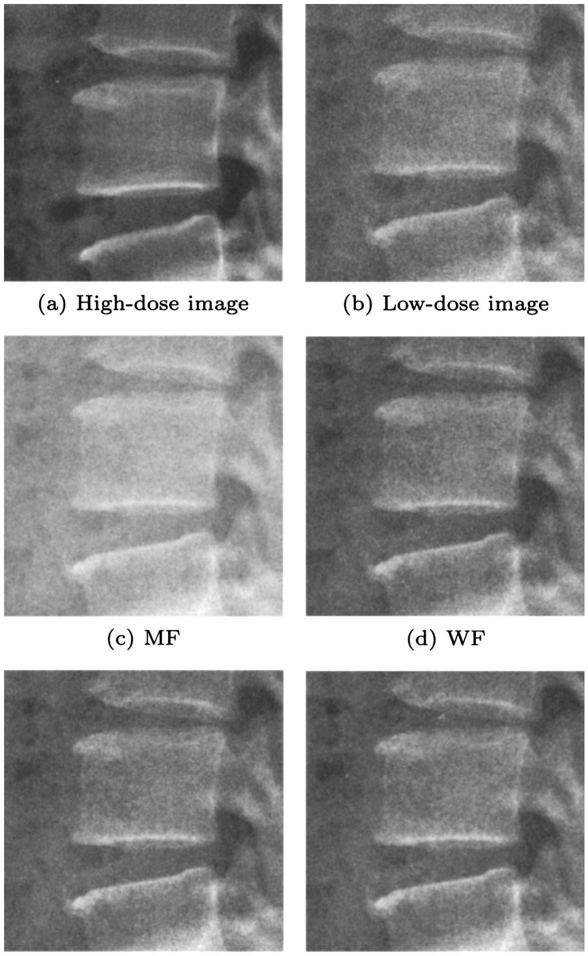
## 7. Application to Medical Images

In the radiographs currently used for medical image diagnosis, the granularity varies with exposure, quantum mottle, and other parameters. In particular, the granularity of X-ray images deteriorates with decreasing exposure. When the image is created, the emitted X-rays arrive at the receiver discretely, so that fluctuations occur in time and space. These fluctuations follow the Poisson distribution. Thus, the images become degraded with Poisson noise. In this section, we use the proposed method to remove noise from medical images.

### 7.1. Test images

We dealt with part of a spinal radiograph obtained at a comparatively high dose (*E.I.* = $2.99 \times 10^{-7}$ C/kg) as shown in Fig. 7(a). This image was degraded with Poisson noise, and then restored. The parameter $\lambda$ of the Poisson noise, expressing the degree of degradation, was set to 0.1 and 1.0. The image shown in Fig. 7(b) with a more realistic dose of *E.I.* = $3.37 \times 10^{-8}$ C/kg was also processed.

Figure 7 shows the processing results for the realistic low-dose image. In particular, (a) is the high-dose image, (b) is the noisy image, (c) is the image processed by MF, (d) is the image processed by WF, (e) is the image processed by AS-BS, and (f) is the image processed by the proposed method.



(a) High-dose image (b) Low-dose image

(c) MF (d) WF

(e) AS-BS (f) Proposed

Fig. 7. Noisy image and processed image (Spine, *E.I.* = $3.37 \times 10^{-8}$ C/kg).

### 7.2. Evaluation method

The quantitative evaluation of quality of the processed images was based on the PSNR and CNR (Contrast to Noise Ratio).

The PSNR and CNR for medical images are defined as follows [8, 9]:

$$PSNR = 20\log_{10}\left(\frac{1}{MSE}\right) \tag{25}$$

$$MSE = \frac{1}{H \times L}\sum_{x,y}^{H,L}\left(\frac{(d(x,y)}{max(D)} - \frac{o(x,y)}{max(O)}\right)^2$$
$$(d(x,y) \in D, o(x,y) \in O) \tag{26}$$

$$CNR = \frac{|\mu_d - \mu_u|}{\sqrt{0.5(\sigma_d^2 + \sigma_u^2)}} \tag{27}$$

Here $d(x, y)$ is the pixel value of the processed image, $o(x, y)$ is the pixel value of the reference image, $max(D)$ is the maximum pixel value of the processed image, and $max(O)$ is the maximum pixel value of the reference image. In addition, $\mu_d$ and $\sigma_d$ are the mean and standard deviation in the DROI (Desired Region Of Interest), and $\mu_d$ and $\sigma_d$ are the mean and standard deviation in the UROI (Undesired Region Of Interest). In this study, edge portions of bones are considered as DROI, and the image background is considered as UROI.

### 7.3. Discussion

As indicated by Figs. 7(c) and 7(d), MF and WF do not assure sufficient noise removal. This is because both MF and WF apply filtering to the entire image, and in the process of noise removal, areas free of noise are affected as well. In AS-BS, good edge preservation is observed. The reason is that the thresholds set independently in every subband of wavelet domain function properly for both noise removal and edge preservation. However, large-amplitude noise exceeds the thresholds, and thus is not removed. The proposed system shows good results in terms of noise removal and edge preservation. This is because Poisson noise is converted into small-amplitude irregular noise by means of the M-transformation, so that even noise with an originally large amplitude is removed successfully.

Now consider the PSNR and CNR data presented in Tables 4 and 5. First, WF proves worse than other methods in terms of both PSNR and CNR. This is because WF is effective for Gaussian noise removal, and functioned properly on low-amplitude noise in our case. MF has better performance than WF. This can be attributed to the fact that medical images usually have fewer edge portions than ordinary images. AB-BS showed better results than the other conventional methods. This is because appropriate thresholds were set for each level and direction, resulting in effective noise removal and edge preservation. The best results are shown by the proposed method. This is because large-amplitude noise that could not be removed by AS-BS is removed by the BayesShrink method after the M-transformation.

Table 4. Value of PSNR (spine)

| | $\lambda = 0.1$ | $\lambda = 1.0$ | $E.I = 3.37 \times 10^{-8}$ C/kg |
|---|---|---|---|
| Noisy | 7.14 | 9.75 | 20.06 |
| MF | 9.22 | 11.47 | 22.12 |
| WF | 7.82 | 10.61 | 21.87 |
| AS-BS | 11.02 | 11.75 | 22.05 |
| Proposed | 11.52 | 12.01 | 22.36 |

Table 5. Value of CNR (spine)

| | $\lambda = 0.1$ | $\lambda = 1.0$ | $E.I = 3.37 \times 10^{-8}$ C/kg |
|---|---|---|---|
| Noisy | 3.34 | 4.54 | 6.82 |
| MF | 4.48 | 4.89 | 7.18 |
| `WF | 4.43 | 4.88 | 7.16 |
| AS-BS | 4.89 | 4.95 | 7.20 |
| Proposed | 4.94 | 5.01 | 7.28 |

## 8. Conclusions

The objective of this study is Poisson noise removal from degraded images, which is difficult to achieve by the conventional wavelet BayesShrink method. We propose a new noise removal method that combines the M-transformation with the wavelet BayesShrink method. In the proposed method, the M-transformation is first applied in order to convert Poisson noise into small-amplitude irregular noise, and then the noise is removed by the wavelet BayesShrink method, resulting in better Poisson noise removal performance. Simulations show that the proposed method is better than the conventional wavelet transform and allows successful restoration of degraded images.

In addition, we validated the proposed method on actual medical radiographs degraded by Poisson noise.

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