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FREQUENCY-DOMAIN CRITERIA OF ROBUST STABILITY: DISCRETE-TIME AND CONTINUOUS-TIME SYSTEMS, A UNIFIED APPROACH

Y. Z. TSYPKIN* AND K. FURUTA†

Tokyo Institute of Technology, Department of Control Engineering, 2-12-1, Oh-Okayama Meguro-ku, Tokyo 152, Japan.

SUMMARY

This paper is concerned with the stability and robust stability of linear discrete-time and continuous-time systems. The characteristic polynomials of linear systems are transformed into the general polynomials of discrete-time systems, and similarly to the hodogram of the discrete-time system, the stability and the robust stability for the general characteristic polynomials with parameter uncertainty are analysed by using the zero exclusion method, where the order of the system is not used specifically a condition for a stability.

KEY WORDS linear systems; robust stability; frequency methods

1. INTRODUCTION

The stability of the continuous system has been studied extensively since the pioneering work of Routh and Hurwitz.[1] Mikhailov[2] and Leonhard[3] used the hodogram of the characteristic equation along the imaginary axis for the analysis of stability. The stability of the discrete-time system has practically analysed the transformed characteristic equation from the discrete-time system by the bilinear transformation,[4] which has been used also for the pole assignment in a specified region.[5,6] When there are uncertainties for the parameters of characteristic polynomials, the robust stability should be studied. Kharitonov[7] gave the robust stability criterion for a characteristic polynomial under interval uncertainties of independent parameters. The robust stability condition given was the stability of four Kharitonov polynomials, which is proved easily by using the Mikhailov and Leonhard stability theorem. This last theorem states that the order of the system should be equal to the number of quadrants passed, when the angular velocity ω moves from 0 to ∞, for stability of a continuous-time system. But there exists a stability condition in that no encirclement of the origin by the hodogram of the characteristic polynomial assures the stability of a discrete-time system irrespective of the order of the system, i.e., there are no roots of the polynomial inside the unit disk.[8]

In this paper we present a method of the stability and the robust stability analysis for both

* Chair of Intelligent Control endowed by Nippon Steel, Tokyo Institute of Technology, on leave from the Institute of Control Sciences, Moscow.
† Author to whom all correspondence should be addressed.

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discrete-time and continuous-time linear systems by extending the previous method which does not use its order for the astability analysis. We consider transforming the continuous-time system into the discrete-time system and give a unified method of analysing robust stability of the continuous-time system. The method is applicable to the robust stability analysis of the delta operator and is also used to check $\mathscr{D}$-stability which places all zeros of the characteristic polynomial in the region $\mathscr{D}$ for the continuous case and outside of the region for the discrete case.

2. PROBLEM FORMULATION

This paper considers the analysis of the stability of the continuous-time and other represented systems by transforming into a generalized form of the discrete-time system. The characteristic polynomial of this system is represented by

$$G_D(q) = \sum_{k=0}^{n} a_k f_k(q) \tag{1}$$

where $f_k(q)$ is a polynomial with the order less than or equal to n

$$\deg f_k(q) \leqslant n$$

and

$$q = e^{-sT}$$

T is the sampling interval and can be chosen equal to 1 without loss of generality. The problem considered in this paper is whether all roots of (1) are outside of the disk or not. If all zeros of $G_D(q)$ are outside of the unit disk then the given system is stable. In the usual definition of z, the stability is given by the condition that all the roots of the characteristic polynomial are inside of the unit disk. The stability condition that all the zeros are placed outside of the unit disk gives interesting results in that the Nyquist-like loci of $G_D(e^{-j\omega})$ does not encircle the origin. This stability condition does not require the order of the polynomials which is required in the original Mikhailov loci. By transforming the characteristic polynomial of the continuous-time system to $G_D(q)$, the stability of the continuous-time system can be treated. This problem, therefore, concerns the stability analysis of both continuous-time and discrete-time systems in the unified fashion.

The function $f_k(q)$ will be generally taken as follows:

- For the characteristic polynomial of the discrete-time system, the $f_k(q)$ is written as

$$f_k(q) = q^k \tag{2}$$

- For the delta operator defined as[9]

$$\delta = \frac{1-q}{q} \tag{3}$$

  The stability of the polynomial

$$G_\delta(\delta) = \sum_{k=0}^{n} a_k \, \delta^k$$

  can be analysed by the polynomial obtained by substituting (3) into the above relation and multiplying q^n

$$f_k(q) = q^{n-k}(1-q)^k \tag{4}$$

- Similarly for the continuous system, the stability of the characteristic polynomial

$$G_s(s)=\sum_{k=0}^{n} a_k s^k \tag{5}$$

is analysed by the linear fractional transformation of

$$s=\frac{1-q}{1+q} \tag{6}$$

which transforms the outside of the disk to the left half complex plane.

$$f_k(q)=(1+q)^{n-k}(1-q)^k \tag{7}$$

In the above discussions the stability region is transformed into the outside of the unit disk. But there is the case that a specified region is transformed into the the unit disk to treat the so-called $\mathscr{D}$-stable problem. Let the region $\mathscr{D}$ be defined by the disk with the centre $\alpha + \mathrm{j}0$ and the radius r. If all roots of $G_D(q)=0$ are outside of the region $\mathscr{D}$, it is said to be $\mathscr{D}$-stable in the discrete system, since the transformation

$$\tilde{q}=rq+\alpha$$

maps the unit disk into $\mathscr{D}$ which is written also as

$$q=\frac{\tilde{q}-\alpha}{r} \tag{8}$$

By substituting $\tilde{q}$ into q of $G_D(q)$ and if the derived $G_D(q)$ has all roots outside of the unit disk, the original characteristic polynomial is $\mathscr{D}$-stable. Similarly for the continuous-time system $\tilde{q}$ defined as

$$\tilde{q}=\frac{r}{s-\alpha} \tag{9}$$

maps the region $\mathscr{D}$ into the outside of the unit disk, so

$$s=\frac{r+\alpha\tilde{q}}{\tilde{q}}$$

maps the unit disk into the outside of $\mathscr{D}$. If the derived general polynomial is stable, the original polynominal is $\mathscr{D}$-stable.

3. STABILITY OF CHARACTERISTIC POLYNOMIAL

If the characteristic polynomial with respect to q has no roots inside the unit disk, the corresponding system is stable.

To check the stability of the $G_D(s)$, the stability criterion is given by Reference 8 as follows. The polynomial $G_D(q)$ has all zeros outside of the unit disk if and only if

$$\Delta_0^{2\pi} \arg G_D(\mathrm{e}^{-\mathrm{j}\omega})=0 \tag{10}$$

where $\Delta_0^{2\pi} \arg G_D(\mathrm{e}^{-\mathrm{j}\omega})$ denotes the net change in the angle $\arg G_D(\mathrm{e}^{-\mathrm{j}\omega})$ as ω moves from 0 to 2π, which is given by the principle of the argument.

This is easily proved as follows. Assuming that the order of the polynomial is n and there exist k zeros inside of the unit disk, then the $\arg G_D(\mathrm{e}^{-\mathrm{j}\omega})$ is equal to $2k\pi$ as ω moves from 0 to 2π. If the given polynomial $G_D(q)$ has all zeros outside of the unit disk, then the original characteristic polynomial is stable and the above equation is proved.

Criterion: $G_D(q)$ is stable if and only if the characteristic locus of $G_D(\mathrm{e}^{-\mathrm{j}\omega})$ does not encircle the origin when ω moves from 0 to 2π

4. ROBUST STABILITY

In the previous section, the stability of the linear characteristic polynomial was discussed. In this section, the robust stability will be considered. The given characteristic polynomial $G_D(q)$ is represented by the sum of the nominal part $G_D^o(q)$ and the perturbation part $\gamma\ \delta G_D(q)$. Letting

$$G_D(q) \stackrel{\mathrm{d}}{=} G_D^o(q) + \gamma\ \delta G_D(q)$$

where γ is a positive constant and the set of the admissible perturbation is assumed given by $Q(\omega)$ which is defined by the set Γ

$$\delta G_D(q) \in \Gamma$$

as

$$Q(\omega) \stackrel{\mathrm{d}}{=} \mathrm{set}\{\gamma\ \delta G_D(\mathrm{e}^{-\mathrm{j}\omega}) \mid 0 \leqslant \omega < 2\pi\}$$

where

$$0 \in Q(\omega)$$

Then G_D is robust stable if and only if

- $G_D^o(q)$ is stable.
- $0 \bar{\in} G_D^o(\mathrm{e}^{-\mathrm{j}\omega}) + Q(\omega)$

The dual formulation of the zero exclusion principle[10-12] will be considered by writing

$$| G_D^o(\mathrm{e}^{-\mathrm{j}\omega}) + \gamma\ \delta G_D(\mathrm{e}^{-\mathrm{j}\omega}) | > 0$$

which is equivalent to the existence of $\varphi \in [0, 2\pi)$ such that

$$\mathrm{Re}\{(G_D^o(\mathrm{e}^{-\mathrm{j}\omega}) + \gamma\ \delta G_D(\mathrm{e}^{-\mathrm{j}\omega}))\mathrm{e}^{-\mathrm{j}\varphi}\} > 0$$

This relation is rewritten as

$$\mathrm{Re}\{G_D^o(\mathrm{e}^{-\mathrm{j}\omega})\mathrm{e}^{-\mathrm{j}\varphi}\} > \gamma \max_{\delta G_D \in \Gamma} \mathrm{Re}\{\delta G_D(\mathrm{e}^{-\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}(\pi-\varphi)}\} \tag{11}$$

For $0 \leqslant \varphi < 2\pi$ and $0 \leqslant \omega < 2\pi$, we define the following functions.

$$\eta(\omega, \varphi) = \mathrm{Re}\{G_D^o(\mathrm{e}^{-\mathrm{j}\omega})\mathrm{e}^{-\mathrm{j}\varphi}\} \tag{12}$$

$$\mu(\omega, \varphi) = \max_{\delta G_D \in \Gamma} \mathrm{Re}\{\delta G_D(\mathrm{e}^{-\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}(\pi-\varphi)}\} \tag{13}$$

Then following τ is defined.

$$\tau(\omega) = \max_{\varphi} \frac{\eta(\omega, \varphi)}{\mu(\omega, \varphi)} \tag{14}$$

where

$$\mu(\omega, \varphi) \neq 0$$

Theorem 1

The family of the general characteristic polynomial $G_D(q)$ is robust stable if and only if the nominal characteristic polynomial $G_D^o(q)$ is stable and

$$\tau(\omega) > \gamma, \qquad \omega \in [0, 2\pi] \tag{15}$$

Defining

$$\tilde{G}_D(e^{-j\omega}) = \frac{G_D^o(e^{-j\omega})}{|G_D^o(e^{-j\omega})|}\tau(\omega) \tag{16}$$

Then the robust stability is assured if and only if $\tilde{G}_D(e^{-j\omega})$ neither encircles the origin nor intersects the circle $C(\gamma, 0)$ with radius γ and centre in the origin when ω moves from 0 to 2π. In other words, if $\tilde{G}_D(e^{-j\omega})$ encircles the origin but intersects the circle $C(\gamma, 0)$, then the robust instability exists.

The proof is obvious from the previous discussion.

The theorem will be rewritten for the cases depending the properties of uncertainties.

Disc constraint.[13,14,18] In the first, the parameters $\{a_k\}$ are assumed uncertain complex values in the disk with the nominal value $\{a_k^o\}$ as

$$a_k \in C(\alpha_k, a_k^o) \tag{17}$$

where

$$C(\alpha_k, a_k^o) = \{a_k \in \mathbf{C}\colon |a_k - a_k^o| \leqslant \gamma\alpha_k\} \tag{18}$$

The nominal polynomial $G_D^o(q)$ is defined as

$$G_D^o(q) = \sum_{k=0}^{n} a_k^o f_k(q) \tag{19}$$

The robust stability is the stability of the systems with a class of parameters defined by (18) given by

$$\sum_{k=0}^{n} C(\alpha_k, a_k^o) f_k(q) = C\left(\gamma \sum_{k=0}^{n} \alpha_k |f_k(q)|, \sum_{k=0}^{n} a_k^o f_k(q)\right)$$

By substituting

$$\varphi(\omega) = \varphi^*(\omega) = \arg\{G_D^o(e^{-j\omega})\}$$

in (12) and (13), we obtain

$$\eta(\omega, \varphi^*) = |G_D^o(e^{-j\omega})|$$

$$\mu(\omega, \varphi) = \max_{\delta G_D \in \Gamma} \operatorname{Re}\{\delta G_D(e^{-j\omega}) e^{j(\pi - \varphi)}\} = \sum_{k=0}^{n} \alpha_k |f_k(e^{-j\omega})|$$

Then $\tau(\omega)$ of (14) yields

$$\tau(\omega) = \frac{|G_D^o(e^{-j\omega})|}{\sum_{k=0}^{n} \alpha_k |f_k(e^{-j\omega})|} \tag{20}$$

which gives

$$\tilde{G}_D(e^{-j\omega}) = \frac{G_D^o(e^{-j\omega})}{\sum_{k=0}^{n} \alpha_k |f_k(e^{-j\omega})|} \tag{21}$$

From Theorem 1, robust stability is assured if and only if $\tilde{G}_D(e^{-j\omega})$ does not encircle and not intersect the circle $C(\gamma, 0)$ when ω moves from 0 to 2π.

The uncertainty of the parameters can be more generalized by defining[16]

$$\left(\sum_{k=1}^{n} \left| \frac{a_k - a_k^0}{\alpha_k} \right|^p \right)^{1/P} < \gamma \tag{22}$$

In this case $\tilde{G}_D(e^{-j\omega})$ is given by

$$\tilde{G}_D(e^{-j\omega}) = \frac{G_D^0(e^{-j\omega})}{\left[\sum_{k=0}^{n} (\alpha_k |f_k(e^{-j\omega})|)^q\right]^{1/q}}, \qquad \text{where } \frac{1}{p} + \frac{1}{q} = 1 \tag{23}$$

Interval constraint.[10] This part concerns that the constraint is given by

$$|a_k - a_k^0| \leqslant \gamma \alpha_k \tag{24}$$

The perturbation of the characteristic polynomial is represented by

$$\delta G_D(q) = \sum_{k=0}^{n} \alpha_k \nu_k f_k(q), \ |\nu_k| \leqslant 1 \tag{25}$$

Letting the phase of $f_k(e^{j\omega})$ be $\theta_k(\omega)$, then the following relation is derived.

$$G_D^0(e^{-j\omega})e^{-j\varphi} = \sum_{k=0}^{n} a_k^0 |f_k(e^{-j\omega})| e^{j(\theta_k(\omega)+\varphi)} \tag{26}$$

$$\mathrm{Re}\{G_D^0(e^{-j\omega})e^{-j\varphi}\} = \sum_{k=0}^{n} a_k^0 |f_k(e^{-j\omega})| \cos(\theta_k(\omega)+\varphi) \tag{27}$$

$$\max_{\delta G \in \Gamma} \mathrm{Re}\{\delta G(e^{-j\omega})e^{-j(\pi-\varphi)}\} = \sum_{k=0}^{n} \alpha_k |f_k(e^{-j\omega})| |\cos(\theta_k(\omega)+\varphi)| \tag{28}$$

Using above relations, the τ can be defined as

$$\tau(\omega) = \max_{\varphi \in [0 \cdot 2\pi)} \frac{\sum_{k=0}^{n} a_k^0 |f_k(e^{-j\omega})| \cos(\theta_k(\omega)+\varphi)}{\sum_{k=0}^{n} \alpha_k |f_k(e^{-j\omega})| |\cos(\theta_k(\omega)+\varphi)|} \tag{29}$$

The maximization of the τ is calculated at the point

$$\varphi_i = \pm \frac{\pi}{2} - \theta_i(\omega), \qquad i = 0, 1, 2, \ldots, n$$

Then the calculation of τ is done by

$$\tau(\omega) = \max_{0 \leqslant i \leqslant n} \frac{\left|\sum_{k=0}^{n} a_k^0 |f_k(e^{-j\omega})| \sin(\theta_i(\omega) - \theta_k(\omega))\right|}{\sum_{k=0}^{n} \alpha_k |f_k(e^{-j\omega})| |\sin(\theta_i(\omega) - \theta_k(\omega)|} \tag{30}$$

where

$$\tau(0) = \frac{\left|\sum_{k=0}^{n} \alpha_k^0 |f_k(1)\right|}{\sum_{k=0}^{n} a_k^0 |f_k(1)|} \tag{31}$$

$$\tau(\pi) = \frac{\left| \sum_{k=0}^{n} a_k^0 \, |f_k(-1)| \right|}{\sum_{k=0}^{n} \alpha_k \, |f_k(-1)|} \tag{32}$$

The robust stability can be analysed by testing that

$$\tilde{G}_D(\mathrm{e}^{-\mathrm{j}\omega}) = \frac{G_D^0(\mathrm{e}^{-\mathrm{j}\omega})}{|G_D^0(\mathrm{e}^{-\mathrm{j}\omega})|} \, \tau(\omega)$$

does not encircle and not intersect the disk $C(\gamma, 0)$ when when ω moves from 0 to 2π.

5. CLOSED-LOOP SYSTEM

For the plant

$$P(s) = \frac{B(s)}{A(s)} \quad \text{and} \quad P_D(q) = \frac{B\left(\frac{1-q}{1+q}\right)}{A\left(\frac{1-q}{1+q}\right)} = \frac{B_D(q)}{A_D(q)} \tag{33}$$

the controller is set

$$R(s) = \frac{C(s)}{D(s)} \quad \text{and} \quad R_D(q) = \frac{C\left(\frac{1-q}{1+q}\right)}{D\left(\frac{1-q}{1+q}\right)} = \frac{C_D(q)}{D_D(q)} \tag{34}$$

then the characteristic polynomial of the closed loop is represented by

$$G_D(q) = A_D(q) C_D(q) + B_D(q) D_D(q) \tag{35}$$

This section considers the case that the plant contains the uncertainty which is expressed by the perturbation of

$$A_D(q) = A_D^0(q) + \gamma \, \delta A(q)$$
$$B_D(q) = B_D^0(q) + \gamma \, \delta B(q)$$

where $A_D^0(q)$, $B_D(q)$ are fixed nominal polynomials. Let the characteristic polynomial be

$$G_D(q) = G_D^0(q) + \delta G(q) \tag{36}$$

where

$$G_D^0(q) = A_D^0(q) C_D(q) + B_D^0(q) D_D(q)$$
$$\delta G(q) = \delta A_D(q) C_D(q) + \delta B_D(q) D_D(q)$$

For the disk uncertainty. The maximization of $\tau(\omega)$ is given by choosing

$$\varphi(\omega) = \varphi^*(\omega) = \arg\{G_D^0(\mathrm{e}^{-\mathrm{j}\omega})\} \tag{37}$$

the $\tau(\omega)$ is

$$\tau(\omega) = \frac{|G_D^0(\mathrm{e}^{-\mathrm{j}\omega})|}{\left|C_D(\mathrm{e}^{-\mathrm{j}\omega})\right| \sum_{k=0}^{n} \alpha_k \left|f_k(\mathrm{e}^{-\mathrm{j}\omega})\right| + \left|D_D(\mathrm{e}^{-\mathrm{j}\omega})\right| \sum_{k=0}^{n} \beta_k \left|f_k(\mathrm{e}^{-\mathrm{j}\omega})\right|} \tag{38}$$

and $\tilde{G}_D(e^{-j\omega})$ is

$$\tilde{G}_D(e^{-j\omega}) = \frac{|G_D^o(e^{-j\omega})|}{\left|C_D(e^{-j\omega})\right| \sum_{k=0}^{n} \alpha_k \left|f_k(e^{-j\omega})\right| + \left|D_D(e^{-j\omega})\right| \sum_{k=0}^{n} \beta_k \left|f_k(e^{-j\omega})\right|} \tag{39}$$

The condition of the robust stability is given similarly to the previous chapter by

- $G_D^o(q)$ is stable.
- $|\tilde{G}_D(e^{-j\omega})| > \gamma$

or $\tilde{G}_D(e^{-j\omega})$ does not encircle and does not intersect $C(\gamma, 0)$.

For the interval uncertainties. Let the polynomials of the controller be described by

$$C_D^o(e^{-j\omega}) = |C_D^o(e^{-j\omega})|\, e^{j\phi_C(\omega)}$$
$$D_D^o(e^{-j\omega}) = |D_D^o(e^{-j\omega})|\, e^{j\phi_D(\omega)}$$
$$f_k(e^{-j\omega}) = |f_k(e^{-j\omega})|\, e^{-j\theta_k(\omega)}$$

and uncertainties of the denominator and the numerator be represented by

$$\delta A(q) = \sum_{k=0}^{n} \alpha_k \nu_k f_k(q) \tag{40}$$

$$\delta B(q) = \sum_{k=0}^{n} \beta_k \mu_k f_k(q) \tag{41}$$

where

$$|\nu_k| \leqslant 1, \; |\mu_k| \leqslant 1$$

then the τ is given by

$$\tau(\omega) = \max_{\varphi} \frac{\left|C_D(e^{-j\omega})\right| \sum_{k=0}^{n} a_k^o \left|f_k(e^{-j\omega})\right| \cos(\theta_k(\omega) + \varphi - \phi_C(\omega)) + \left|D_D(e^{-j\omega})\right| \sum_{k=0}^{n} b_k^o \left|f_k(e^{-j\omega})\right| \cos(\theta_k(\omega) + \varphi - \phi_D(\omega))}{\left|C_D(e^{-j\omega})\right| \sum_{k=0}^{n} \alpha_k \left|f_k(e^{-j\omega})\right| \left|\cos(\theta_k(\omega) + \varphi - \phi_C(\omega))\right| + \left|D_D(e^{-j\omega})\right| \sum_{k=0}^{n} \beta_k \left|f_k(e^{-j\omega})\right| \left|\cos(\theta_k(\omega) + \varphi - \phi_D(\omega))\right|} \tag{42}$$

The maximization can be achieved just by taking over n points as

$$\tau(\omega) = \max_{0 \leqslant i \leqslant n} \frac{\left|\left|C_D(e^{-j\omega})\right| \sum_{k=0}^{n} a_k^o \left|f_k(e^{-j\omega})\right| \sin(\theta_i(\omega) - \theta_k(\omega) + \phi_C(\omega)) + \left|D_D(e^{-j\omega})\right| \sum_{k=0}^{n} b_k^o \left|f_k(e^{-j\omega})\right| \sin(\theta_i(\omega) - \theta_k(\omega) - \phi_D(\omega))\right|}{\left|C_D(e^{-j\omega})\right| \sum_{k=0}^{n} \alpha_k \left|f_k(e^{-j\omega})\right| \left|\sin(\theta_i(\omega) + \theta_k(\omega) + \phi_C(\omega))\right| + \left|D_D(e^{-j\omega})\right| \sum_{k=0}^{n} \beta_k \left|f_k(e^{-j\omega})\right| \left|\sin(\theta_i(\omega) - \theta_k(\omega) + \phi_D(\omega))\right|} \tag{43}$$

The robust stability condition is also given by

- $G_D^o(q)$ is stable.
- $|\tilde{G}_D(e^{-j\omega})| > \gamma$

or $\tilde{G}_D(e^{-j\omega})$ does not encircle and does not intersect $C(\gamma, 0)$.

6. EXAMPLES

In this section, the robust stability of the polynomial given by Argoun[17]

$$G(s) = \sum_{k=0}^{6} a_k s^k \tag{44}$$

is analysed by the method presented by the paper, where

$$a^o = (433\cdot5, 667\cdot25, 502\cdot72, 251\cdot25, 80\cdot25, 14\cdot0, 1)$$
$$\alpha = (43\cdot35, 33\cdot36, 25\cdot137, 15\cdot075, 5\cdot6175, 1\cdot4, 0\cdot1)$$

The case that the parameters perturbation is defined by the disk constraint $G_D^o(e^{-j\omega})$ is shown in Figure 1, where the lower figure shows the behaviour near the origin, and the corresponding $\tau(\omega)$ is depicted in Figure 2. The minimum value of the gamma is given by 0·87 for the disk constraint. The hodogram of $\tilde{G}_D(e^{-j\omega})$ is shown in Figure 3, which does not intersect the disk with the diameter of 0·87. The interval constraint case is also analysed by $\tau(\omega)$ in Figure 4. γ is found 1·24 and the given case is found robust stable for the interval constraint. The hodogram of $\tilde{G}_D(e^{-j\omega})$ is depicted in Figure 5. The analysis is done without using the order of the polynomial explicitly.

In the second, the robust stability of the polynomial of the delta operator[18]

$$G(s) = \sum_{k=0}^{3} \delta^k \tag{45}$$

is analysed, where

$$a^o = (1, 3, 3, 1)$$
$$\alpha = (0\cdot5, 1, 1, 0\cdot5)$$

It is found that $G_D^o(e^{-j\omega}) = 1$ so it is a point at $1 + j0$. In this disk constraint case $\tau(\omega)$ shown in Figure 6 and the hodogram of $\tilde{G}_D(e^{-j\omega})$ is depicted in Figure 7. From the figures, γ is given by 0·10. For the interval constraint case is analysed by $\tau(\omega)$ in Figure 8 and the hodogram of $\tilde{G}_D(e^{-j\omega})$ is depicted in Figure 9. From the figures, γ is given by 0·23.

Similarly to the robust stability analysis of the general polynomials, the robust stability of the closed loop can be analysed, which is much easier than the previously proposed methods.[19]

The plant transfer function is represented by

$$P(s) = \frac{B(s)}{A(s)} = \frac{b_1 s}{1 - s + a_2 s^2 + s^2} \tag{46}$$

and the controller transfer function is

$$R(s) = \frac{D(s)}{C(s)} = \frac{3}{1 + s} \tag{47}$$

with the parameters

$$a^o = (1, -1, 4\cdot 2, 1)$$
$$\alpha = (0, 0, 0\cdot 8, 0)$$
$$b^o = (0, 1\cdot 5)$$
$$\beta = (0, 0\cdot 5)$$

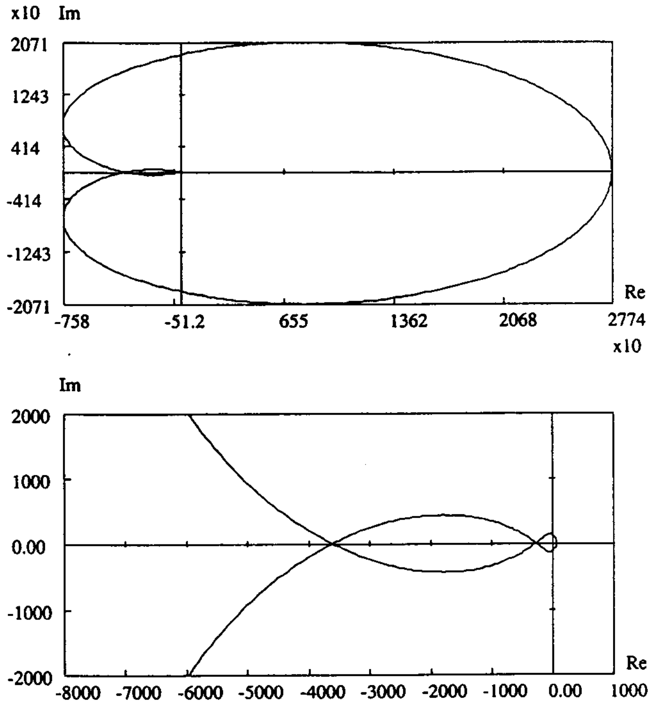

Figure 1. $G_b^o(e^{-j\omega})$ in the analysis of polynomial $G(s)$

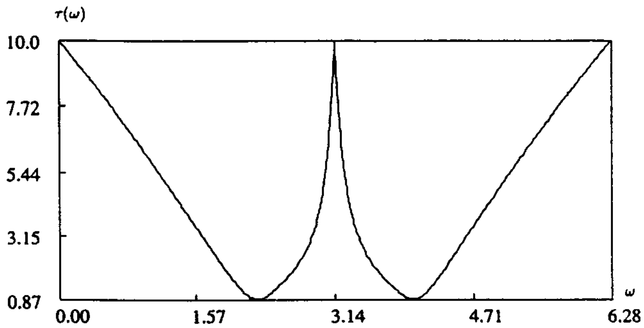

Figure 2. $\tau(\omega)$ for the disk constraint case in the analysis of polynomial $G(s)$

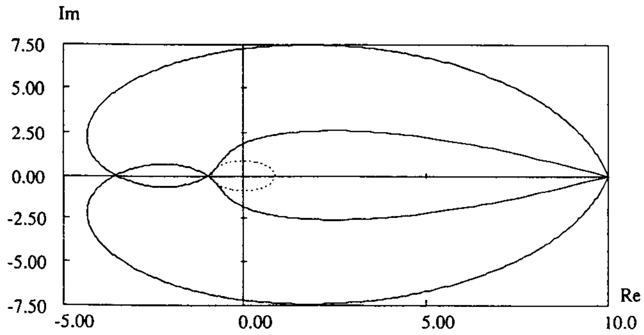

Figure 3. The hodogram of $\tilde{G}(e^{-j\omega})$ for the disk constraint case

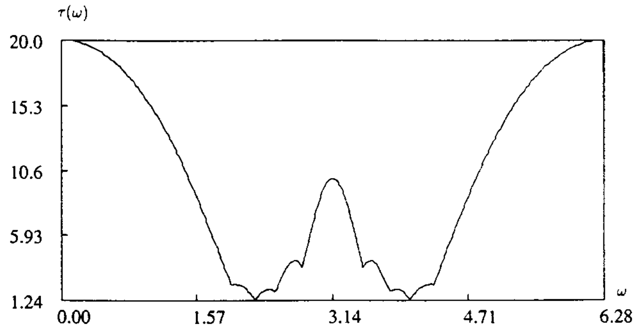

Figure 4. $\tau(\omega)$ for the interval constraint case

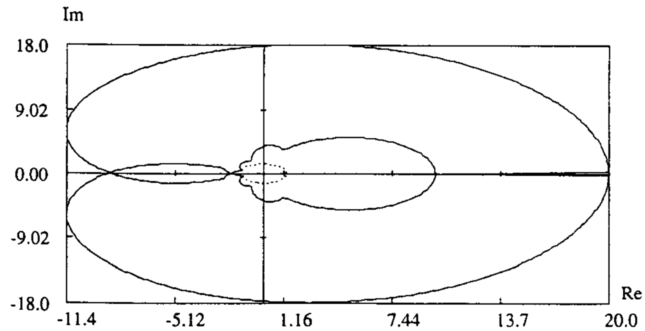

Figure 5. The hodogram of $\tilde{G}_D(e^{-j\omega})$ for the interval constraint case

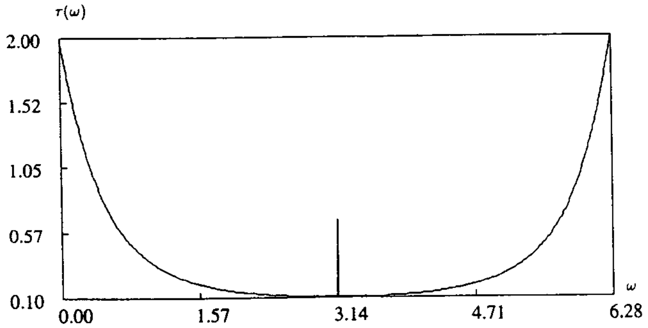

Figure 6. $\tau(\omega)$ for the disk constraint case in the analysis of robust stability of polynomial $G(\delta)$

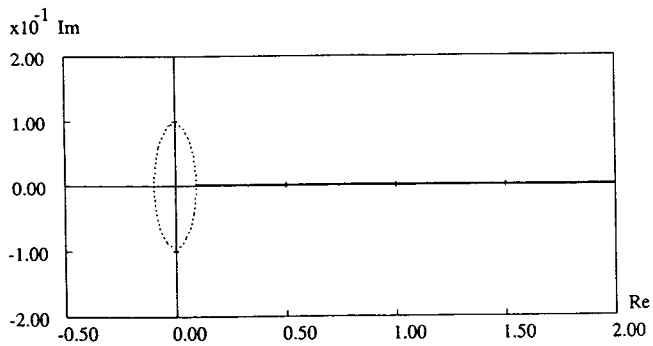

Figure 7. The hodogram of $\tilde{G}_D(e^{-j\omega})$ for the disk constraint case

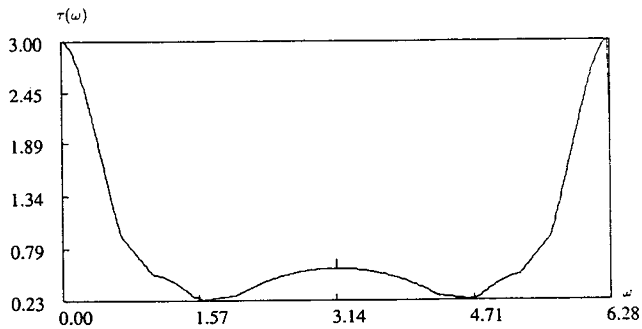

Figure 8. $\tau(\omega)$ for the interval constraint in the analysis of robust stability of polynomial $G(\delta)$

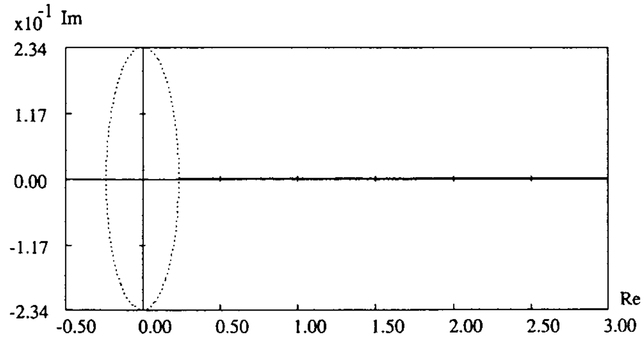

Figure 9. The hodogram of $\tilde{G}(e^{-j\omega})$ for the interval constraint case

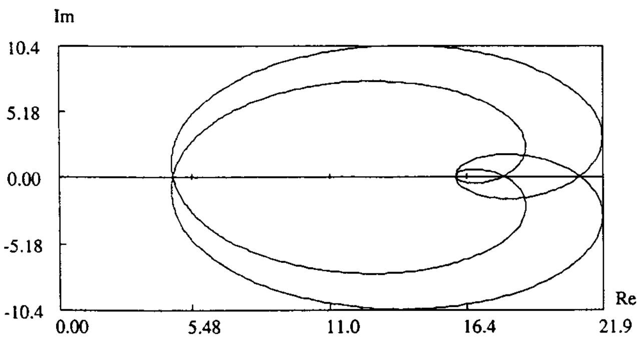

Figure 10. $\tilde{G}_D^o(e^{-j\omega})$ in the analysis of the continuous closed-loop system

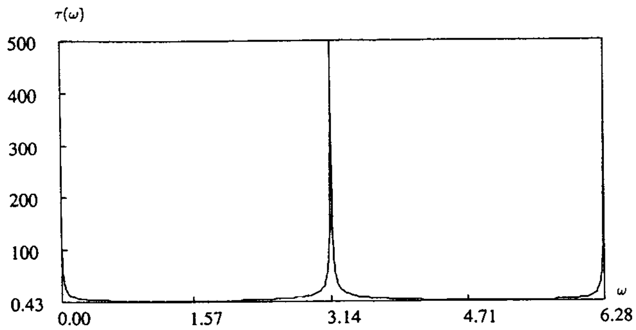

Figure 11. $\tau(\omega)$ for the disk constraint in the analysis of the continuous closed-loop system

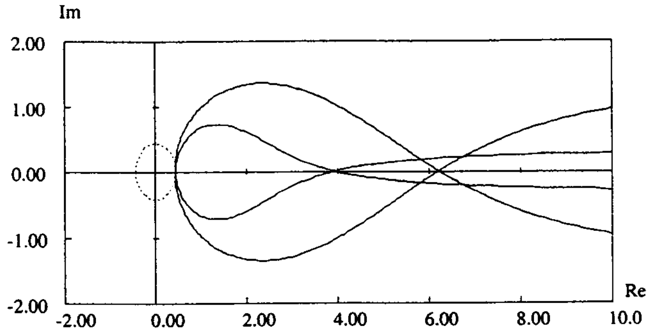

Figure 12. The hodogram of $\tilde{G}_D(e^{-j\omega})$ for the disk constraint case in the analysis of the continuous closed-loop system

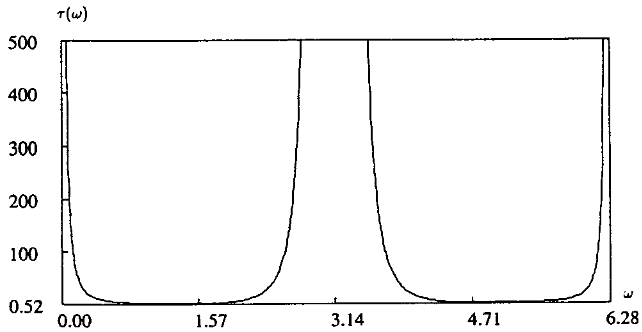

Figure 13. $\tau(\omega)$ for the interval constraint case in the analysis of the continuous closed-loop system

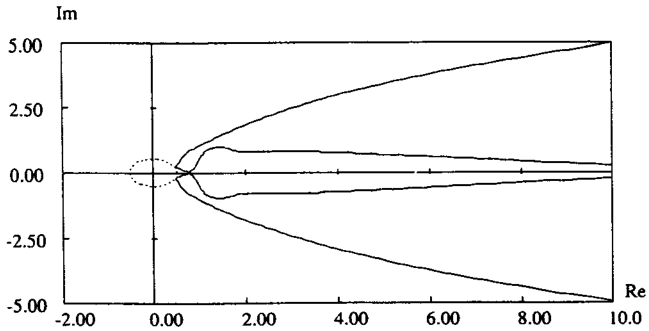

Figure 14. The hodogram of $\tilde{G}_D(e^{-j\omega})$ for interval constraint case in the analysis of the continuous closed-loop system

The disk constraint case is analysed by $G_D^o(e^{-j\omega})$ and $\tau(\omega)$ shown in Figures 10 and 11 respectively. The hodogram of $\tilde{G}_D(e^{-j\omega})$ is depicted in Figure 12. From the figures, γ is given by 0·43. For the interval constraint case is analysed by $\tau(\omega)$ in Figure 13 and the hodogram of $\tilde{G}_D(e^{-j\omega})$ is depicted in Figure 14. From the figures, γ is given by 0·52.

7. CONCLUSIONS

This paper presents a unified approach to analysing the robust stability of the discrete-time and continuous-time systems. The characteristic polynomials of both systems are transformed into the generalized polynomial form represented by the sum of polynomials with uncertain coefficients. We can use the approach in the robust analysis of the characteristic polynomial of the delta operator. The uncertainties are considered in detail for the cases of the disk and interval constraints. The approach gives the promising approach to the analysis of the characteristic polynomials represented by the sum of the complex polynomials with uncertain parameters.

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