Optimisation of the Damping Resistance of Synchronous Machines Using a Quadratic Performance Criterion

W. Haager

Abstract

Starting from a simplified model of the salientpole engine in the form of the analytically computed transfer function the damping resistance is optimized. The applied performance criterion provides a formula for that damping resistance, at which rotor hunting, occurring for example at changes of the load, is damped down in the best possible way.

1 Introduction

At the operation of synchronous machines the damper cage has several duties:

- damping of harmonics of the primary mmf curve,
- damping of the inverse rotary field at asymmetric load (mainly important in generators),
- generation of an asynchronous torque for self-starting (mainly important for motors),
- damping of rotor hunting (of particular interest for motoring as well as for generating).

According to a particular weighting of these objectives various designs of the damper cage can be considered. For lack of an appropriate mathematical model the damper cage is usually designed in practice based on empirical data or according to thermal considerations for the asynchronous self-starting [1].

From the seven system equations of the synchronous machine in the rotor-fixed coordinate system in scaled representation [2] the calculation of the transfer function succeeds under neglection of a finite stator resistance analytically. The mechanical driving torque is assumed to be the input and the rotor angle to be the output. Hereby the dependence of the dynamic behaviour on the operating point and all the machine parameters, especially the damping resistance, is clearly recognizable [3].

From that an optimal design of the parameters of the damper cage is determined with the aid of a performance criterion, which is current in control engineering, aiming at damping down rotor hunting, as it occurs at step changes of the driving torque, in the best possible way.

At the mathematical discussion of the synchronous machine according to the two-axis theory the damper cage is split up into an equivalent d-axis and q-axis winding. The reactances of these equivalent windings are determined approximately by the geometrical layout of the damper bars; the resistance of the equivalent windings remains as the freely chooseable parameter, which can be adapted easily by appropriate choice of the conductor cross section or the material.

At very high values of the damping resistance (which equals the absence of a damper winding) as well as at very low values (owing to a large time constant the induced currents decay very slowly) the damping behaviour of rotor hunting is poor. The result of the optimisation, which succeeds at several simplifying assumptions analytically, yields a formula for that damping resistance, at which an optimal damping of rotor hunting can be expected.

2 Model

The transfer function of the salientpole engine with the deviation of the mechanical driving torque $\Delta m_A(s)$ from the stationary value as the input and the resulting deviation of the rotor angle $\Delta\vartheta(s)$ as the output is calculated from the system equations by linearisation and Laplace transform in [3]. Hereby the quiescent operating point is determined definitely by generator terminal voltage u_s, air-gap emf u_p, rotor angle ϑ and synchronous angular speed ω_s.

Owing to the large time constant τ_f the influence of the field winding on the dynamical behaviour at step changes of the driving torque is primarily manifested by a slow aperiodic movement, superposed to the damped oscillations, towards the new steady state. With the assumption of a vanishing field resistance ($\tau_f \to \infty$) the induced currents do not decay, which means a flux linkage of the field winding kept constant for all times. Under that assumption the aperiodic part of the transient motion is omitted, however, the oscillatory part, which is decisive for an optimisation, is yet remaining. For reason of an easy computability thus a finite resistance of the field winding is neglected in the following investigations. In that case the transfer function has fourth order:

$$F(s)=\frac{\Delta\vartheta(s)}{\Delta m_{\mathrm{A}}(s)}=\frac{b_2 s^2+b_1 s+b_0}{a_4 s^4+a_3 s^3+a_2 s^2+a_1 s+a_0} \tag{1}$$

with the coefficients

$$b_0 = x_{\mathrm{d}}^2 x_{\mathrm{q}} \sigma_{\mathrm{df}} \omega_{\mathrm{s}}^2,$$

$$b_1 = x_{\mathrm{d}} x_{\mathrm{q}} \omega_{\mathrm{s}}^2 (\tau_Q \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} x_{\mathrm{d}} + \tau_{\mathrm{D}} \sigma_{\mathrm{Df}} x_{\mathrm{d}}''),$$

$$b_2 = x_{\mathrm{d}} x_{\mathrm{d}}'' x_{\mathrm{q}} \sigma_{\mathrm{qQ}} \sigma_{\mathrm{Df}} \tau_{\mathrm{D}} \tau_Q \omega_{\mathrm{s}}^2,$$

$$\begin{aligned} a_0 = {} & u_{\mathrm{p}} u_{\mathrm{s}} \cos\vartheta\, x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \\ & + u_{\mathrm{s}}^2 \sin^2\vartheta\, x_{\mathrm{d}} (x_{\mathrm{q}} \sigma_{\mathrm{df}} + x_{\mathrm{q}} - 2 x_{\mathrm{d}} \sigma_{\mathrm{df}}) + u_{\mathrm{s}}^2 x_{\mathrm{d}} \sigma_{\mathrm{df}} (x_{\mathrm{d}} - x_{\mathrm{q}}), \end{aligned}$$

$$\begin{aligned} a_1 = {} & u_{\mathrm{p}} u_{\mathrm{s}} \cos\vartheta\, x_{\mathrm{q}} (\tau_Q \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} x_{\mathrm{d}} + \tau_{\mathrm{D}} \sigma_{\mathrm{Df}} x_{\mathrm{d}}'') \\ & + u_{\mathrm{s}}^2 \sin^2\vartheta [\tau_{\mathrm{D}} \sigma_{\mathrm{Df}} (x_{\mathrm{d}} x_{\mathrm{q}} - 2 x_{\mathrm{d}} x_{\mathrm{d}}'' + x_{\mathrm{q}} x_{\mathrm{d}}'') \\ & + \tau_Q x_{\mathrm{d}} (x_{\mathrm{q}} \sigma_{\mathrm{qQ}} (\sigma_{\mathrm{df}} + 1) - x_{\mathrm{d}} \sigma_{\mathrm{df}} (\sigma_{\mathrm{qQ}} + 1))] \\ & + u_{\mathrm{s}}^2 \tau_{\mathrm{D}} \sigma_{\mathrm{Df}} x_{\mathrm{d}}'' (x_{\mathrm{d}} - x_{\mathrm{q}}) + u_{\mathrm{s}}^2 \tau_{\mathrm{Q}} x_{\mathrm{d}} \sigma_{\mathrm{df}} (x_{\mathrm{d}} - x_{\mathrm{q}} \sigma_{\mathrm{qQ}}), \end{aligned}$$

$$\begin{aligned} a_2 = {} & u_{\mathrm{p}} u_{\mathrm{s}} \cos\vartheta\, x_{\mathrm{d}}'' x_{\mathrm{q}} \sigma_{\mathrm{qQ}} \sigma_{\mathrm{Df}} \tau_{\mathrm{D}} \tau_Q \\ & + u_{\mathrm{s}}^2 \sin^2\vartheta\, \tau_{\mathrm{D}} \tau_Q \sigma_{\mathrm{Df}} (x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{qQ}} - x_{\mathrm{d}} \sigma_{\mathrm{qQ}} x_{\mathrm{d}}'' - x_{\mathrm{d}} x_{\mathrm{d}}'' + x_{\mathrm{q}} \sigma_{\mathrm{qQ}} x_{\mathrm{d}}'') \\ & + u_{\mathrm{s}}^2 \tau_{\mathrm{D}} \tau_Q \sigma_{\mathrm{Df}} x_{\mathrm{d}}'' (x_{\mathrm{d}} - x_{\mathrm{q}} \sigma_{\mathrm{qQ}}) + x_{\mathrm{d}}^2 x_{\mathrm{q}} \sigma_{\mathrm{df}} \tau_{\mathrm{m}} \omega_{\mathrm{s}}^2, \end{aligned}$$

$$a_3 = x_{\mathrm{d}} x_{\mathrm{q}} \tau_{\mathrm{m}} \omega_{\mathrm{s}}^2 (\tau_{\mathrm{Q}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} x_{\mathrm{d}} + \tau_{\mathrm{D}} \sigma_{\mathrm{Df}} x_{\mathrm{d}}'').$$

The influence of the d-axis damper winding is attenuated considerably by the field winding. The performance function, defined for the optimisation (in section 3), is – unless the q-axis damping resistance r_{Q} is far apart from its optimal value – nearly independent of the d-axis damping resistance r_{D} in wide ranges, which justifies the further simplification $r_{\mathrm{D}} \to \infty$ (or $\tau_{\mathrm{D}} = 0$). Thus eq. (1) leads to the transfer function

$$F(s)=\frac{\Delta\vartheta(s)}{\Delta m_{\mathrm{A}}(s)}=\frac{b_1 s+b_0}{a_3 s^3+a_2 s^2+a_1 s+a_0} \tag{2}$$

with

$$b_0 = \omega_{\mathrm{s}}^2 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}},$$

$$b_1 = \omega_{\mathrm{s}}^2 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \tau_{\mathrm{Q}} \sigma_{\mathrm{qQ}},$$

$$\begin{aligned} a_0 = {} & u_{\mathrm{s}}^2 \sin^2\vartheta (x_{\mathrm{q}} \sigma_{\mathrm{df}} + x_{\mathrm{q}} - 2 x_{\mathrm{d}} \sigma_{\mathrm{df}}) \\ & + u_{\mathrm{s}} \sigma_{\mathrm{df}} (u_{\mathrm{p}} x_{\mathrm{q}} \cos\vartheta + u_{\mathrm{s}} x_{\mathrm{d}} - u_{\mathrm{s}} x_{\mathrm{q}}), \end{aligned}$$

$$\begin{aligned} a_1 = {} & u_{\mathrm{s}}^2 \tau_{\mathrm{Q}} \sin^2\vartheta (x_{\mathrm{q}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} + x_{\mathrm{q}} \sigma_{\mathrm{qQ}} - x_{\mathrm{d}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} - x_{\mathrm{d}} \sigma_{\mathrm{df}}) \\ & + u_{\mathrm{s}} \sigma_{\mathrm{df}} \tau_{\mathrm{Q}} (u_{\mathrm{p}} x_{\mathrm{q}} \sigma_{\mathrm{qQ}} \cos\vartheta + u_{\mathrm{s}} x_{\mathrm{d}} - u_{\mathrm{s}} x_{\mathrm{q}} \sigma_{\mathrm{qQ}}), \end{aligned}$$

$$a_2 = \omega_{\mathrm{s}}^2 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \tau_{\mathrm{m}},$$

$$a_3 = \omega_{\mathrm{s}}^2 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \tau_{\mathrm{Q}} \sigma_{\mathrm{qQ}} \tau_{\mathrm{m}}.$$

Fig. 1 shows the comparison of the frequency responses $F(\mathrm{j}\omega)$ of the synchronous machine at full-load rating (machine parameters taken from [3]), in general (curve *1*), under neglection of the resistance of the field winding (curve *2*) and with the additional assumption of a damper winding only in the quadrature axis (curve *3*). At other operating points the differences are even smaller and vanish entirely at no-load operation ($\vartheta = 0$).

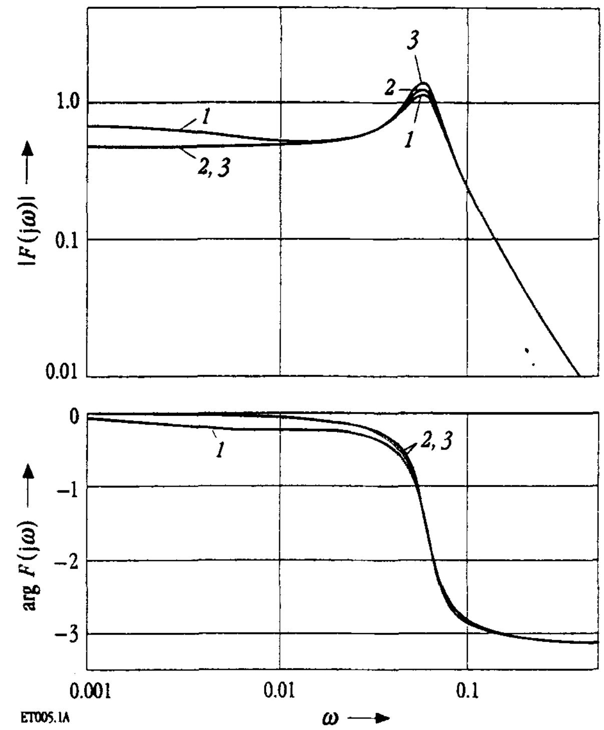

Fig. 1. Frequency responses of the synchronous machine at full-load rating (*1* in general; *2* under neglection of the field resistance; *3* at additional assumption of a damper winding only in q-direction)

3 Optimisation

For appraisal of the quickness of a transient motion the deviation $e(t)$ of the interesting variable (in the present case the deviation of the rotor angle $\Delta\vartheta$ due to a step change of the driving torque) from the new steady state ($\Delta\vartheta_{\mathrm{e}}$) can be assigned to a performance index; the transient motion is optimized by minimisation of that performance index.

The so-called ISE-criterion, a time domain integral ("Integral of Squared Error"), which weights the deviation from the new steady state quadratically, can be calculated directly in the frequency domain according to the Parseval Theorem [4]:

$$I_{\mathrm{ISE}} = \int_0^{\infty} e^2(t)\,\mathrm{d}t = \lim_{s_1 \to \mathrm{j}\infty} \frac{1}{2\pi\mathrm{j}} \int_{-s_1}^{s_1} E(s)E(-s)\,\mathrm{d}s. \tag{3}$$

For a third order function

$$E(s) = \mathcal{L}\{e(t)\} = \frac{c_2 s^2 + c_1 s + c_0}{d_3 s^3 + d_2 s^2 + d_1 s + d_0}$$

the following result is obtained:

$$I_{\mathrm{ISE}} = \frac{c_2^2 d_0 d_1 + (c_1^2 - 2c_0 c_2) d_0 d_3 + c_0^2 d_2 d_3}{2 d_0 d_3 (d_1 d_2 - d_0 d_3)} . \tag{4}$$

An example of a performance function for the machine model according to eq. (1) in dependence on the d-axis damping resistance r_{D} and the q-axis damping resistance r_{Q} at the operating point $p_{\mathrm{w}} = 1$ and $p_{\mathrm{b}} = 0$ is shown in **Fig. 2**. At least close to the optimal value of r_{Q} the performance index depends only very little on r_{D}, at other operating points this dependence is even smaller and vanishes entirely at no-load operation. Thus the optimisation can be confined to the q-axis damping resistance, the machine model without a d-axis damper winding ($\tau_{\mathrm{D}} = 0$) according to eq. (2) seems justified.

At a step change of the driving torque

$$\Delta\vartheta(\tau) = \sigma(\tau) \quad \text{or} \quad \Delta\vartheta(s) = 1/s$$

the steady-state deviation of the rotor angle $\Delta\vartheta_{\mathrm{e}}$ is obtained from eq. (2) by application of the final-value theorem of the Laplace transform:

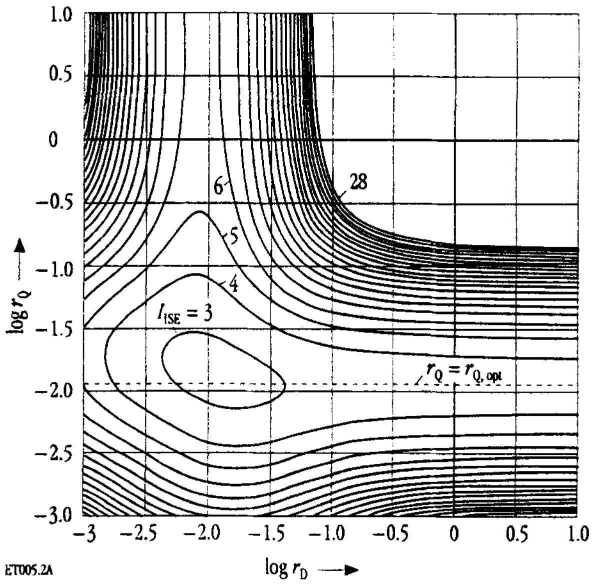

Fig. 2. Performance function according to eq. (3) in dependence of r_{D} and r_{Q}, contour lines at the values $I_{\mathrm{ISE}} = 3, 4, 5, 6, \ldots, 28$

$$\Delta\vartheta_{\mathrm{e}} = \lim_{\tau\to\infty} \Delta\vartheta(\tau) = \lim_{s\to 0} F(s)$$

$$= \frac{x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \omega_{\mathrm{s}}^2}{u_{\mathrm{p}} u_{\mathrm{s}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \cos\vartheta + u_{\mathrm{s}}^2 \sin^2\vartheta (x_{\mathrm{q}}\sigma_{\mathrm{df}} + x_{\mathrm{q}} - 2x_{\mathrm{d}}\sigma_{\mathrm{df}}) + u_{\mathrm{s}}^2 \sigma_{\mathrm{df}} (x_{\mathrm{d}} - x_{\mathrm{q}})} . \tag{5}$$

With

$$E(s) = (1/s)[\Delta\vartheta_{\mathrm{e}} - F(s)] \tag{6}$$

the performance index is calculated under eq. (4).

Deriving with respect to the changeable parameter τ_{Q} and setting the derivative to zero

$$\mathrm{d} I_{\mathrm{ISE}} / \mathrm{d}\tau_{\mathrm{Q}} = 0 \tag{7}$$

yields the square of the optimal time constant of the q-axis damper winding:

$$\begin{aligned}\tau_{\mathrm{Q,opt}}^2 = {} & [u_{\mathrm{s}}^2 \sin^2\vartheta\, \omega_{\mathrm{s}}^2 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \tau_{\mathrm{m}} (x_{\mathrm{q}}\sigma_{\mathrm{df}} + x_{\mathrm{q}} - 2x_{\mathrm{d}}\sigma_{\mathrm{df}}) \\ & + \omega_{\mathrm{s}}^2 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}}^2 \tau_{\mathrm{m}} (u_{\mathrm{s}}^2 x_{\mathrm{d}} - u_{\mathrm{s}}^2 x_{\mathrm{q}} + x_{\mathrm{q}} u_{\mathrm{p}} u_{\mathrm{s}} \cos\vartheta)] \\ & \cdot [(u_{\mathrm{s}} \sin\vartheta)^4 ((\sigma_{\mathrm{df}} + 1)^2 x_{\mathrm{q}}^2 \sigma_{\mathrm{qQ}}^2 - (\sigma_{\mathrm{df}} + 1) \\ & \cdot (5\sigma_{\mathrm{qQ}} - 1) x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} + (7\sigma_{\mathrm{qQ}}^2 - 4\sigma_{\mathrm{qQ}} + 1) x_{\mathrm{d}}^2 \sigma_{\mathrm{df}}^2 \\ & - u_{\mathrm{s}}^2 \sin^2\vartheta (u_{\mathrm{p}} u_{\mathrm{s}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} \cos\vartheta (5 x_{\mathrm{d}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} - x_{\mathrm{d}} \sigma_{\mathrm{df}} \\ & - 2 x_{\mathrm{q}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} - 2 x_{\mathrm{q}} \sigma_{\mathrm{qQ}}) + u_{\mathrm{s}}^2 \sigma_{\mathrm{df}} (9 x_{\mathrm{d}}^2 \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}}^2 \\ & - 7 x_{\mathrm{d}}^2 \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} + 2 x_{\mathrm{d}}^2 \sigma_{\mathrm{df}} - 8 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}}^2 + 2 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} \\ & - 3 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{qQ}}^2 + x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{qQ}} + 2 x_{\mathrm{q}}^2 \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}}^2 + 2 x_{\mathrm{q}}^2 \sigma_{\mathrm{qQ}}^2)) \\ & + (u_{\mathrm{p}} u_{\mathrm{s}} x_{\mathrm{q}} \sigma_{\mathrm{df}} \sigma_{\mathrm{qQ}} \cos\vartheta)^2 + u_{\mathrm{p}} u_{\mathrm{s}}^3 x_{\mathrm{q}} \sigma_{\mathrm{df}}^2 \sigma_{\mathrm{qQ}} \cos\vartheta \\ & \cdot (3 x_{\mathrm{d}} \sigma_{\mathrm{qQ}} - x_{\mathrm{d}} - 2 x_{\mathrm{q}} \sigma_{\mathrm{qQ}}) + u_{\mathrm{s}}^4 \sigma_{\mathrm{df}}^2 (3 x_{\mathrm{d}}^2 \sigma_{\mathrm{qQ}}^2 - 3 x_{\mathrm{d}}^2 \sigma_{\mathrm{qQ}} \\ & + x_{\mathrm{d}}^2 - 3 x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{qQ}}^2 + x_{\mathrm{d}} x_{\mathrm{q}} \sigma_{\mathrm{qQ}} + x_{\mathrm{q}}^2 \sigma_{\mathrm{qQ}}^2)]^{-1} .\end{aligned} \tag{8}$$

From that the optimal damping resistance follows by

$$r_{\mathrm{Q,opt}} = x_{\mathrm{Q}} / \tau_{\mathrm{Q,opt}} . \tag{9}$$

The optimal damping resistance depends on the operating point (u_{p}, u_{s}, ϑ), its shape in dependence on active power p_{w} and reactive power p_{b} (with $u_{\mathrm{s}} = 1$) is shown in **Fig. 3**. Active power and reactive power as determination of an operating point match practical conditions mostly better than air-gap emf and rotor angle; the relation to these quantities is given by

$$\begin{aligned} u_{\mathrm{p}} \cos\vartheta = {} & \{[(p_{\mathrm{w}}^2 + p_{\mathrm{b}}^2) x_{\mathrm{d}} x_{\mathrm{q}} \omega_{\mathrm{s}}^2 + p_{\mathrm{b}} u_{\mathrm{s}}^2 \omega_{\mathrm{s}} (x_{\mathrm{d}} + x_{\mathrm{q}}) + u_{\mathrm{s}}^4] \\ & \cdot (p_{\mathrm{b}} \omega_{\mathrm{s}} x_{\mathrm{q}} + u_{\mathrm{s}}^2)\} / \{u_{\mathrm{s}} [(p_{\mathrm{w}}^2 + p_{\mathrm{b}}^2) \omega_{\mathrm{s}}^2 x_{\mathrm{q}}^2 \\ & + 2 p_{\mathrm{b}} u_{\mathrm{s}}^2 \omega_{\mathrm{s}} x_{\mathrm{q}} + u_{\mathrm{s}}^4]\} ,\end{aligned} \tag{10}$$

$$\sin^2\vartheta = \frac{p_{\mathrm{w}}^2 x_{\mathrm{q}}^2 \omega_{\mathrm{s}}^2}{(p_{\mathrm{w}}^2 + p_{\mathrm{b}}^2) \omega_{\mathrm{s}}^2 x_{\mathrm{q}}^2 + 2 p_{\mathrm{b}} u_{\mathrm{s}}^2 \omega_{\mathrm{s}} x_{\mathrm{q}} + u_{\mathrm{s}}^4} , \tag{11}$$

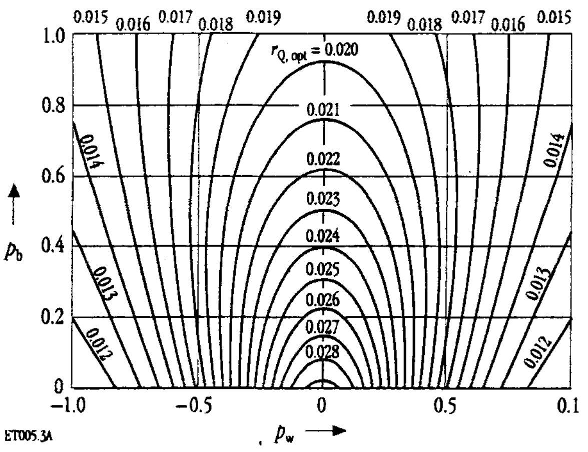

Fig. 3. Optimal damping resistance in dependence on p_{w} and p_{b}

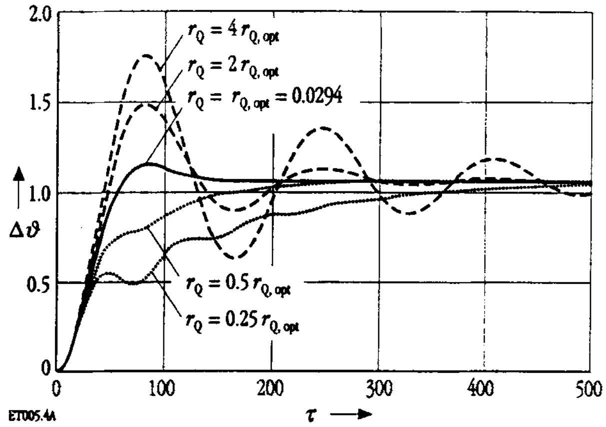

Fig. 4. Rotor hunting $\Delta\vartheta(\tau)$ due to a step change of the driving torque $\Delta m_A(\tau)$ at no-load operation at various values of the damping resistance

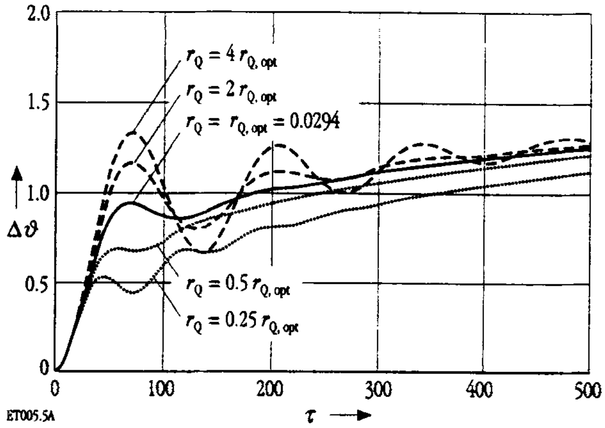

Fig. 5. Rotor hunting $\Delta\vartheta(\tau)$ due to a step change of the driving torque $\Delta m_A(\tau)$ at full-load rating at various values of the damping resistance

which follows from the system equations of the synchronous machine using the relations

$$p_w = u_s i_s \cos\varphi \;,\; p_b = u_s i_s \sin\varphi$$

by elementary transformations.

Rotor hunting is appearing unpleasantly most likely in the vicinity of no-load operation, moreover the damping behaviour is worst in that case. Thus an optimal design of the damper cage with regard to that operating point seems appropriate. The assumption $\vartheta = 0$ leads to a considerable simplification of the result; with the additional assumption $u_s = 1$ and $\omega_s = 1$, which can be regarded as valid for a machine operating at an infinite bus, the optimal damping resistance follows from eqs. (8) and (9) to

$$r_{Q,opt} = x_Q \sqrt{\frac{\sigma_{qQ}^2 [3x_d(u_p x_q - x_q + x_d) + x_q^2(u_p - 1)^2] + \sigma_{qQ} x_d (x_q - 3x_d - u_p x_q) + x_d^2}{x_d x_q \tau_m (u_p x_q - x_q + x_d)}} \tag{12}$$

or in dependence of the reactive power p_b to

$$r_{Q,opt} = x_Q \sqrt{\frac{\sigma_{qQ}^2 (x_q^2 p_b^2 + 3x_q p_b + 3) - \sigma_{qQ}(x_q p_b + 3) + 1}{x_q \tau_m (x_q p_b + 1)}}\,. \tag{13}$$

These formulas provide an easily calculable guidance value for that damping resistance, at which an optimal damping of rotor hunting can be expected, which is illustrated in **Fig. 4**. The oscillations of the rotor angle due to a step change of the driving torque at optimal design of the damping resistance (according to eq. (12)) are compared with those occurring at other damping resistances. Also at other operating conditions than no-load operation a good damping behaviour can be achieved with that optimal design, which is corroborated by **Fig. 5**, showing rotor hunting at rated load ($p_w = 0.8$, $p_b = 0.6$) and various values of the damping resistance.

At higher values of the stator resistance the synchronous machine shows a tendency to instability; even when the simplification $r_s = 0$ is not fulfilled approximately any more, it turns out that in many cases the stability behaviour can be improved by a proper design of the damping resistance with eq. (12) or (13) as a guide value.

4 Conclusion

A model of the synchronous machine in the form of the transfer function, derived from the system equations, allows under certain simplifications with the aid of a quadratic performance criterion an optimisation of the damping resistance analytically. The result is an easily calculable guide value for the design of the damper cage, at which an optimal damping of rotor hunting can be expected.

5 List of symbols

i_s	stator current
m_A	mechanical driving torque
p_b	reactive power
p_w	active power
r_D	resistance of d-axis damper winding
r_f	field resistance
r_Q	resistance of q-axix damper winding
u_p	air-gap emf (internal voltage of generator)
u_s	generator terminal voltage
x_D	reactance of d-axis damper winding

x_d	d-axis synchronous reactance
x_d''	d-axis substransient reactance
x_{ij}	mutual reactance of two windings (in general)
x_Q	reactance of q-axis damper winding
x_q	q-axis synchronous reactance
ϑ	rotor angle
σ_{ij}	mutual leakage coefficients between two windings (in general)
τ	per unit time
τ_D	time constant of d-axis damper winding
τ_f	time constant of field winding
τ_m	acceleration constant
τ_Q	time constant of q-axis damper winding
ω_s	synchronous angular speed
I_{ISE}	performance index according to the ISE-criterion

References

[1] *Vogt, K.:* Elektrische Maschinen; Berechnung rotierender elektrischer Maschinen. 4. Ed., Berlin: VEB Verl. Tech., 1988

[2] *Kleinrath, H.:* Stromrichtergespeiste Drehfeldmaschinen. 1. Ed., Wien: Springer, 1980

[3] *Haager, W.:* Investigation of damping behaviour of hunting of the salientpole engine by means of the analytically computed transfer function. etzArchiv 12 (1990) no. 8, pp. 245 – 250

[4] *Newton, G.; Gould, L.; Kaiser, J.:* Analytical design of linear feedback controls. 4. Ed., New York: Wiley, 1967

Manuscript received on August 3, 1990

The Author

Dipl.-Ing. Dr. Wilhelm Haager (31) is assistant at the Institute for Electrical Control Engineering, Technical University of Vienna/Austria. He studied Electrical Engineering (Power Engineering) at the Technical University of Vienna. His major interests are in modelling and optimisation of technical systems. (TU-Wien, Institut für Elektrische Regelungstechnik, Gußhausstraße 27 – 29, A-1040 Wien, T +43 1/5 88 01-39 08)