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The physics of rainclouds

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## 1. INTRODUCTION

The intensive modern study of the physics of precipitation owes much to a classical paper of Bergeron (1933). He emphasized that heavy cloud, without precipitation, is frequently very stable and long-lived, and that the nature of the process leading to the release of precipitation, involving the transfer of a large mass of water from a state of dispersion in very numerous particles to one of concentration in a much smaller number of much larger particles, remained to be explained. After discussing various possibilities, he concluded that the only process of widespread significance, which would lead to the formation of 'real' rain and snow, involved the co-existence of two phases, the presence side-by-side of particles of ice and supercooled water, the latter being much the more numerous.

At temperatures below  $0^{\circ}\text{C}$ , the vapour pressure in equilibrium with water exceeds that in equilibrium with ice at the same temperature; the magnitude of this difference vanishes at  $0^{\circ}\text{C}$  and reaches a maximum of about 0.26 mb near  $-12^{\circ}\text{C}$ . A cloud containing both ice and water at the same temperature is thus unstable; the water drops will evaporate and the ice crystals will grow by direct crystallization from the vapour. Bergeron estimated that, in the neighbourhood of  $-12^{\circ}\text{C}$ , significant growth of the ice particles would occur in a few minutes and that this was sufficiently rapid to explain the initial stage of growth to raindrop size. Once the ice particles have become large enough to fall through the cloud droplets at an appreciable rate, other processes, such as direct capture, may become significant and, indeed, in the later stages of growth, predominant. The speed of the initial process depends not only on the difference between the equilibrium vapour pressures and on the diffusive properties of water vapour, but also on the speed with which the growing ice crystal can lose the latent heat released at its surface, and the water droplet gain from its surroundings the heat lost by evaporation. A difference of temperature of  $1^{\circ}\text{C}$  between ice and water would suffice to stop the transfer completely. Detailed calculations of the rate of growth of ice crystals under such conditions have been made by Houghton (1950).

Bergeron summed up his theory in the statement 'almost every real raindrop and all snowflakes originated around an ice crystal.' The crucial step in the release of precipitation was thus claimed to be the appearance in the cloud of ice crystals; this was deemed to require that the top of the cloud should attain a sufficiently low temperature, considerably below  $0^{\circ}\text{C}$ , the ice being presumed to originate mainly by the freezing of cloud droplets although the possibility of direct crystallization from the vapour, on suitable nuclei, was also envisaged.

Findeisen (1938) developed the picture in more detail, maintaining that ice-particles could only form directly from the vapour phase on suitable 'sublimation' nuclei; he too concluded that 'all appreciable formation of precipitation is initiated by sublimation.'

This view of the essential importance of the presence of ice in a raincloud became the working hypothesis of most meteorologists and there is indeed no doubt that most rainclouds in temperate latitudes do extend above the  $0^{\circ}\text{C}$  isotherm. There was never any justification for statements as sweeping as those in which these theories were summed up and the need for caution was expressed, for example, by Simpson (1941). Of recent years evidence has accumulated which shows conclusively that heavy rain can fall, under certain conditions, from clouds whose temperatures are nowhere below the freezing point.

Bowen (1950) and Ludlam (1951a), developing earlier ideas of Findeisen (1939) and Houghton (1938), have suggested that, in suitable circumstances, coalescence of droplets can provide the initial step in the growth of raindrops. This possibility was rejected by Bergeron (1933) who was contemplating a cloud of particles of uniform size. The new

picture starts from the assumption of the presence, near the cloud base, of a few particles appreciably bigger than the main population of cloud droplets and the occurrence of a significant updraught through a considerable thickness of cloud. The larger droplets are considered by Bowen to originate in the occasional coalescence of two cloud droplets of slightly different sizes. Ludlam suggests that, in maritime air masses, they may grow from the rare, very large, sea-salt nuclei. A larger droplet, whatever its source, will be carried up in the ascending air, although falling relative to the ordinary cloud droplets, some of which will be captured. The drop thus grows and, ultimately, if the updraught extends through a sufficient height, the rate of fall will exceed the speed of the updraught and the drop will return to the base of the cloud, still growing as it descends. Fig. 1 (after Bowen) shows the result of a computation of the trajectory of a drop growing in this way as it traverses a cloud with an updraught of 1 m/sec, average droplet diameter  $20\ \mu$  and cloud water content of  $1\ \text{g/m}^3$ . The initial volume of the drop is taken to be twice that of the cloud particles. It is clear that this process could produce raindrops, given a supply of suitable initial droplets, a sufficient updraught and a cloud of considerable thickness. The last proviso will usually mean in practice, in temperate latitudes, that the top of the cloud will extend above the  $0^\circ\text{C}$  isotherm and may contain ice. It may thus be a difficult matter to decide which of the two possible processes caused the initial growth; both may be active.

This review will be largely concerned with a survey of the problems which have arisen in the detailed development of the ideas which have just been outlined and with the comparison of the observed properties of rainclouds with theoretical predictions. In order to avoid ambiguity it is convenient, here, to define certain terms which will be used frequently. *Condensation* will signify the deposition of liquid water from the vapour, *sublimation* the deposition of crystalline ice from the vapour, *coalescence* the union of two drops, *accretion* the growth of an ice particle by collision with water drops and *aggregation* the collision and adherence of two or more pre-existing ice crystals.

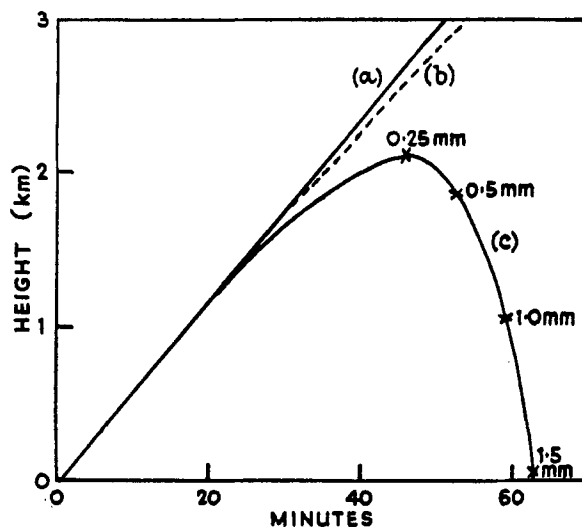


Figure 1. The trajectory of a drop growing by coalescence in a cloud with an updraught of 1 m/sec, average droplet diameter  $20\ \mu$  and cloud water content  $1\ \text{g/m}^3$  (after Bowen). Curve (a) represents the path of the air, (b) an average cloud droplet, (c) a droplet starting with twice the average volume. The positions of the growing drop when it attains diameters of 0.25, 0.5, 1.0 and 1.5 mm respectively are indicated on curve (c).

In the past few years great interest has arisen in the possibility of modifying the physical state of a cloud by artificial means and, in particular, of influencing the amount of precipitation or the time or place at which it occurs. Attempts have been made to accelerate ice formation by introducing fragments of solid carbon dioxide ('dry ice'), which causes intense localized cooling, or by introducing suitable freezing nuclei, silver iodide in particular. Alternatively, attempts have been made to promote the coalescence process by introducing hygroscopic nuclei (salt, for example), or even a spray of fine water drops, into a suitable part of the cloud. Numerous attempts at 'seeding' clouds have been made from aircraft, balloons and even from the ground. The proper design, control and assessment of such experiments is not easy nor is our understanding of all the processes involved sufficiently detailed and precise to be of much assistance in arriving at a just verdict. Spectacular effects have occurred on some occasions but, despite the claims which have been made in some quarters, the general consensus of opinion is that the possibility of influencing rainfall by such methods on a large scale, sufficient, for example, to be of economic importance, remains unproven. Valuable recent summaries of the state of this subject have been published by the World Meteorological Organization (1954) and by Absalom (1954).

## 2. OBSERVATIONAL EVIDENCE

### 2.1 *The growth of precipitation particles from ice crystals*

Bergeron (1933) and Findeisen (1939), although differing considerably in their detailed picture of the processes involved in the appearance of ice crystals in a cloud, were in

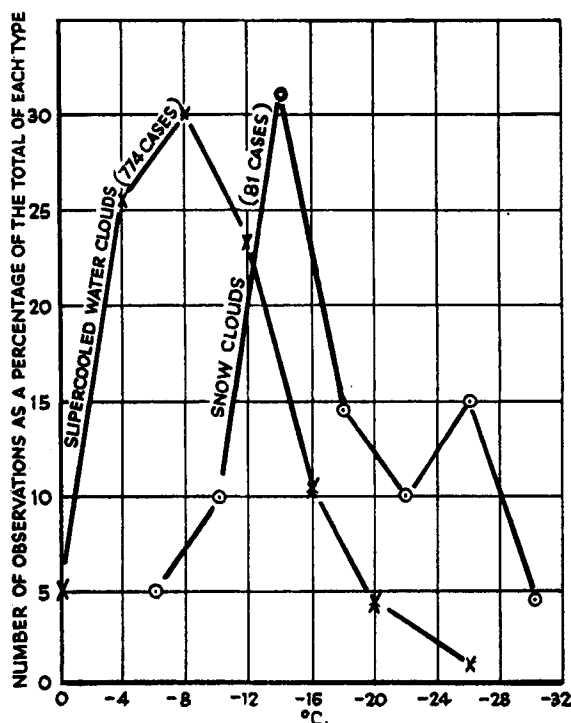


Figure 2. Graph of the results of Pepler's analysis of the cloud observations reported by German weather flights 1931-1935. The abscissa represents the temperatures of the cloud summits.

- 'Snow clouds': in which the observer reported that individual snow flakes could be seen and where very little or no ice accretion occurred on the aircraft.
- × 'Supercooled water clouds': in which the observer reported that no ice crystals could be seen and where ice accretion occurred on the aircraft.

agreement in claiming that 'almost every real raindrop... originates around an ice crystal' and that clouds formed entirely of water particles can only produce the small drops of drizzle. The picture of the essential process in the change from cloud droplets to raindrops, as the rapid growth of ice crystals, in an atmosphere nearly saturated with respect to water, came to be very generally accepted. While direct evidence for the dominance of the process in precipitation clouds was meagre, it was accepted *faute de mieux*; other possible processes seemed likely to be much too slow.

### 2.11 *The temperatures of cloud summits*

Several series of routine measurements of the air temperature at the tops of clouds have been analysed by Peppler (1940), Stickley (1940) and by Mason and Howorth (1952). These seem to show that, in temperate latitudes, most heavy rain falls from clouds which extend several thousand feet above the freezing level. Some of Peppler's results are plotted in Fig. 2; although the heights attained by the aircraft in many flights were not very great the analysis extended over four years and was very thorough. Mason and Howorth analysed soundings by aircraft from Aldergrove, Northern Ireland. In 99 cases of rain for which cloud-top temperatures were known, these were below  $-5^{\circ}\text{C}$  on 87 occasions. On the other hand, in 45 examples of drizzle, the cloud-top temperature was above  $-5^{\circ}\text{C}$  in 39 cases. Stickley reported that, of 312 cases of precipitating cloud in the southern U.S.A., the temperature of the cloud top was below  $0^{\circ}\text{C}$  on 292 occasions, while on 10 occasions when the cloud-top temperature was above  $0^{\circ}\text{C}$ , only drizzle was observed.

These analyses refer only to temperate latitudes and the lack of observations of tropical clouds may account for the initial easy acceptance of the Bergeron theory. Such results do not of course prove that the process occurs but establish the existence of one condition necessary for it to be possible.

### 2.12 *Radar observations and cloud seeding experiments*

In the last ten years the two new techniques of radar and of cloud seeding have provided strong evidence that the Bergeron process is frequently operative. Radar examinations of rainclouds have often revealed a marked intensification of the echo from a level near the  $0^{\circ}\text{C}$  isotherm. Hooper and Kippax (1950a) interpreted this 'melting band' as a clear indication that snowflakes were melting as they fell through this level. This interpretation has been confirmed by combined aircraft and radar observations (Bowen 1951) and the technique could be used to provide statistics of the frequency of rainfall associated with the Bergeron process. The melting band provides particularly striking evidence in favour of the Bergeron theory as its presence implies the absence of the strong updraughts (exceeding 1 m/sec) necessary to the Bowen-Ludlam coalescence process (cf. Section 5).

The effects of seeding a supercooled cloud with many ice crystals, produced by the low temperature of solid carbon dioxide, were observed by Kraus and Squires (1947), and the experiment has been repeated on many subsequent occasions, other nucleating agents such as silver iodide being sometimes used. Where appreciable amounts of rainfall, or a substantial modification of the cloud, are produced, the experiments give clear evidence of the operation of the Bergeron process. Such cases seem to be a small percentage of the attempted experiments and most of the work in this field has been characterized by a lack of careful analysis.

## 2.2 *The growth of precipitation particles by coalescence*

Well-documented observations of 'non-freezing rain' (rain of considerable intensity from clouds throughout which the temperature exceeded  $0^{\circ}\text{C}$ ) have been reported with increasing frequency since the war. Kotsch (1947) and Virgo (1950), for example, describe typical examples of non-freezing showers, observed from aircraft, while Mordy and Eber (1954) report many examples of moderate and heavy rain from warm clouds (cloud temperature everywhere exceeding  $+7^{\circ}\text{C}$ ) on an island in the Hawaiian group, together with details of rainfall rate, drop-size distribution, cloud depth and orographic effects. E. J. Smith (1951a, b) discusses several simultaneous aircraft and radar observations of non-freezing rain over New South Wales, in great detail. These papers have established, beyond doubt, the occurrence of rain from warm clouds. The examples they discuss are all, however, in warm or tropical maritime air and the question arises as to the frequency of occurrence of this type of rain deep inland and in cooler climates. The position here is unsatisfactory though various observations suggest that, in England, non-freezing rain is rare. Jones (1951) reports a single case of slight rain from a line of warm cumulus near London. Indirect evidence in favour of the occurrence of non-freezing rain in England has been given by L. G. Smith (1951) from a consideration of the association of heavy rain with strong electric fields. Usually clouds which produce rainfall rates exceeding about 15 mm/hr, produce intense electric fields (greater than 1,000 v/m) at the ground. These fields are generally thought to be associated with the presence of ice in the cloud. From an analysis of some observations taken over two years by Simpson at Kew, Smith finds that on 7 occasions, intense rain was associated with only weak electric fields. On 6 of these occasions, the temperature at the level of the cloud top, estimated from a tephigram, was above  $-3^{\circ}\text{C}$ , whereas on the numerous occasions on which intense rain was associated with strong electric fields the estimated cloud-top temperature was always below  $-10^{\circ}\text{C}$ . This evidence is extremely suggestive.

There are no published observations of non-freezing rain in continental air masses. Byers (1949) has pointed out that this may be due to the absence of the giant sea-salt nuclei which serve to form the large droplets necessary to initiate the Bowen-Ludlam process.

The occurrence of rain from warm convective clouds is strong but not conclusive evidence for the importance of the Bowen-Ludlam process. That this process is mainly responsible for non-freezing rain is indicated by some other lines of evidence. E. J. Smith, in his observations of clouds giving non-freezing rain, found that the rain-water content increased sharply towards the top of the cloud, a fact supported by some radar observations discussed by Bowen (1950). This level of high rain-water content may be tentatively identified with that at which the terminal velocities of the growing drops are just equal to the velocity of the updraught which has carried them aloft, and from which they subsequently fall as rain, in accordance with the Bowen-Ludlam theory (cf. Fig. 1).

Bowen (1952) has claimed successful stimulation of artificial rain on 4 out of 10 attempts by the injection of water droplets into the base of growing cloud. Rain, or rain and hail, fell within 40 min. The occurrence of hail on two of the successful attempts confuses the results, in that it might be considered evidence in favour of the Bergeron process!

It will be difficult to evaluate the importance of the Bowen-Ludlam process in temperate climates where the freezing level is usually below 3 km, because clouds deep enough to form rain by this process will usually extend above the freezing level. In such cases, it is impossible to distinguish between the Bergeron and Bowen-Ludlam processes by temperature measurements (cf. Section 4).

## 3. PRECIPITATION ELEMENTS

## 3.1 Physical properties of precipitation elements

## 3.11 Terminal velocities

In order to study the processes occurring in rainclouds it is important to know the velocities of fall of raindrops, hailstones, snowflakes and ice crystals.

If a particle is released from rest in still air, it will accelerate to a terminal velocity which is reached effectively, by a raindrop, after falling about ten metres. The terminal velocity is determined by the mass, size and shape of the particle and by the density and viscosity of the air, and it thus varies with height. Best (1950a) has reviewed various measurements and gives an empirical formula for the velocity of a waterdrop at any height; the terminal velocities, at heights of 3 km and 6 km, are found to exceed those near the ground by about 10 per cent and 30 per cent respectively. The most recent and consistent measurements are those of Gunn and Kinzer (1949). It should be noted that Stokes's Law is applicable only for diameters less than about  $50\ \mu$ , the drag due to the wake becoming important for larger drops. This drag is considerably enhanced by the distortion of drops larger than 3 mm in diameter.

Few data are available on the terminal velocities of hailstones. Weickmann (1953) maintains that they have poor aerodynamic properties and a drag coefficient considerably greater than that of ideal spheres; he suggests an approximate formula for the velocity,  $v$  cm/sec,

$$v = 200 D^{\frac{1}{2}}, \text{ where } D \text{ is the diameter in mm.}$$

This implies that some earlier workers (e.g., Bilham and Relf 1937) have considerably overestimated the rates of fall of large hailstones.

There have been a number of attempts to measure the terminal velocities of snowflakes, a problem beset with practical difficulties. The recent results of Langleben (1954) are systematic and remarkably consistent, showing in a useful manner how the terminal velocity depends on the form and size of the flake. As a rough rule, it can be stated that ice crystals with diameters from  $300\ \mu$  to  $1,000\ \mu$  fall with velocities between 0.4 and 1.0 m/sec, while large dry snowflakes fall at about 1.5 to 2.0 m/sec.

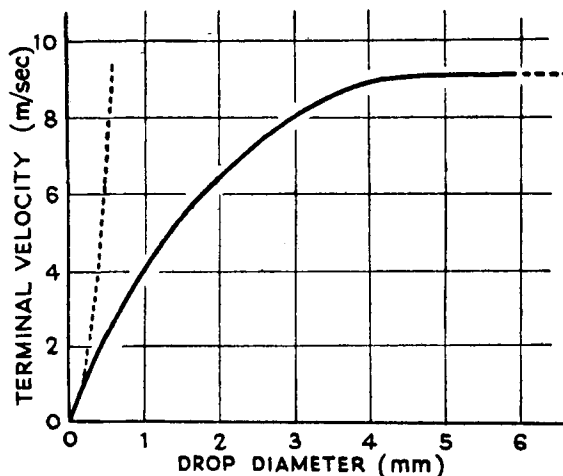


Figure 3. The terminal velocities of freely falling raindrops in air at N.T.P. (after Best 1950a). The pecked line shows the terminal velocities that would be attained if Stokes's Law were obeyed for all sizes of drop.

## 3.12 Collection efficiencies

The rate at which a drop, of mass  $m$  and diameter  $D$ , grows in falling a distance  $z$  relative to the air in a homogeneous cloud of droplets of diameter  $\delta$  and of liquid-water content  $w$ , is given by the equation :

$$\frac{dm}{dz} = E(D, \delta) \pi D^2 w/4$$

The factor  $E(D, \delta)$  is called the collection efficiency; it lies usually between 0 and 1 and represents the fact that some of the droplets lying in the path of the falling drop are swept around it and escape collision. The equation assumes that all collisions lead to coalescence, an assumption to be examined in the next section. The collection efficiency varies in a complex way with the diameters of the drop and droplets, as represented in Fig. 4 after Langmuir (1948). A droplet tends to follow the airflow around a falling drop, the deflecting force depending on the viscosity of the air. The form of the airflow and the velocity near the surface depend on the diameter of the drop; the ratio, of the inertia of a droplet to the viscous force acting on it, increases with droplet size and thus, for a given  $D$ ,  $E$  increases with  $\delta$ .

The main difficulty in computing values of collection efficiency concerns the exact form of the airflow around a falling particle. Langmuir (1948) assumed viscous flow for Reynolds numbers less than 60 and potential flow for greater values of Reynolds number. He further assumed that collision would occur for all droplets whose centres passed within a distance  $\frac{1}{2}D$  of the centre of the drop. The problem is very difficult when  $D$  and  $\delta$  become comparable. Ludlam (1951a) has suggested that, in this case, a better approximation to the truth is obtained by increasing the collection efficiencies given by Langmuir by a geometrical factor  $\{1 + \delta/(DE^{\frac{1}{2}})\}^2$ .

There has been no satisfactory experimental check on Langmuir's values of collection efficiency for drops, although values obtained by a similar method for cylinders have been verified. Measurements by Gunn and Hitschfeld (1951) of the growth, by coalescence, of drops (diameter 3.2 mm) falling through an artificial cloud, containing droplets with diameters from 5 to 50  $\mu$ , are consistent with Langmuir's values. Observations on natural clouds by Adderley (1953), on the other hand, suggest, if his results are taken at their face value, that the relative values of Langmuir's collection efficiencies, for drops of 0.5 and 1.0 mm diameter respectively, are wrong by a considerable factor. There is great uncertainty in the values of the collection efficiencies of small drops such as would be important in the initial formation of rain by a coalescence process.

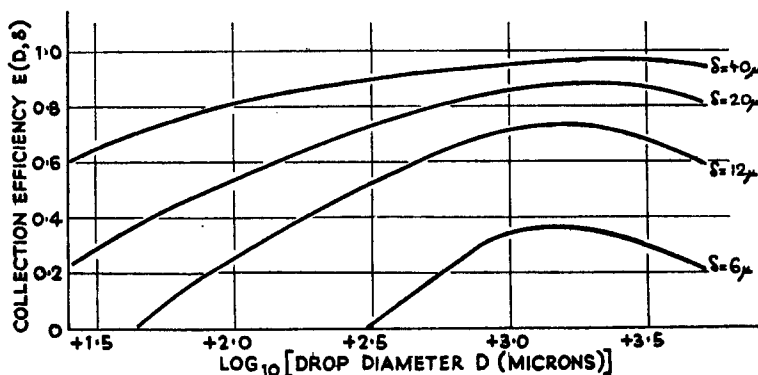


Figure 4. The collection efficiency  $E(D, \delta)$  for a drop of diameter  $D$  (microns) falling at its terminal velocity in air at N.T.P. through a homogeneous cloud of water droplets of diameter  $\delta$  (microns) (after Langmuir 1948).



Still less is known of the collection efficiencies of plate and needle-like ice crystals for droplets, and no computations are available. It is not even certain whether viscous or potential flow should be assumed for small Reynolds number (e.g., see Browne, Wexler 1953), yet calculations assuming viscous flow lead to values of collection efficiency roughly one-fifth of those obtained by the assumption of potential flow. In view of the importance of accretion processes, this is a most unsatisfactory state of affairs. Further knowledge will not come easily and all calculations must certainly be checked carefully by experiments.

### 3.13 *Coalescence of drops*

The total surface energy of two drops is greater than that of the composite drop formed by coalescence, so that energy is released by the process. Coalescence may conceivably occur as soon as two freely-falling drops touch, or it may occur only after the surfaces in contact have been flattened slightly. If the latter is the case, a certain amount of energy must be provided for the initial deformation with its consequent increase of surface area. In the former case, it is not obvious whether or not such a 'potential barrier' must be surmounted before coalescence can occur – there will be no potential barrier only if the drops coalesce in such a way that their total surface area decreases steadily throughout. Although no theoretical work on the problem has appeared, there has been considerable experiment to determine the conditions under which coalescence can occur.

Rayleigh (1879, 1882) observed that drops of diameter 1-2 mm, formed from unstable jets of water, coalesced upon collision only in the presence of a weak electric field. His conclusion that coalescence will not normally occur is apparently supported by the observations of Dady (1947) that no coalescence occurred in several thousand collisions between oil droplets  $4\ \mu$  in diameter. The use of an atomiser to produce the droplets in these experiments may, however, have given to them electric charges of like sign, leading to mutual electrostatic repulsion.

Diametrically opposite conclusions were reached by Dessens (1949) and Gabilly (1949). In Dessens's experiments, droplets of diameter about  $20\ \mu$ , attached to fine spider threads, were brought gradually together, and coalescence occurred always at contact. Gabilly allowed droplets suspended in an airstream to strike others attached to fine threads. Both these experiments can be criticized on the grounds that a droplet fixed to a support is distorted, so that cloud conditions are not simulated.

Indirect evidence that coalescence always occurs on collision between large drops and small droplets in a cloud was obtained by Gunn and Hirschfeld (1951) in the experiments discussed in the previous section.

It can be seen that the experimental results are apparently conflicting. The effects of surface impurities and electric fields may be important and more carefully controlled experiments would be welcome. Some indication of the results that might be expected is given by the following 'order-of-magnitude' reasoning. The surface energy  $S$  of a drop of radius  $r$  and surface tension  $\gamma$  dynes/cm is given by

$$S = 4\pi r^2 \gamma$$

The potential barrier to coalescence, if it exists at all, can be expected to depend on  $S$  and we may write

$$\Delta S \geq C 4\pi r^2 \gamma$$

where  $r$  is taken as the radius of the smaller of the two colliding drops and  $C$  is a dimensionless constant. The energy available for surmounting the potential barrier  $\Delta S$  will be the

difference between the total kinetic energies before and after coalescence. If the radius of the large drop is much greater than that of the small drop, then the kinetic energy available for overcoming the potential barrier is given by  $\frac{1}{2} m U^2$ , since the collision is inelastic, where  $m$  is the mass of the smaller drop and  $U$  the velocity of the larger drop. Since  $m = \frac{4}{3} \pi \rho r^3$ , the condition that coalescence should occur reduces to

$$r U^2 > 6 C \gamma / \rho$$

Putting  $\gamma = 80$  dynes/cm,  $\rho = 1$  gm/cm<sup>3</sup> and  $C = 10^{-2}$  (Browne 1952a), this reduces to  $r U^2 > 4.8$  cm<sup>3</sup> sec<sup>-2</sup>. Thus, in considering collisions between raindrops and cloud droplets, as in the experiments of Gunn and Hitschfeld, we put  $U = 300$  cm/sec, and find that coalescence will occur if the radius  $r$  of the cloud droplet is greater than  $0.5 \mu$ . On the other hand, the argument indicates that a cloud droplet of radius  $4 \mu$  will coalesce only with drops of radius greater than  $150 \mu$ , a result consistent with Dady's results. The kinetic energy available for coalescence may be increased in turbulent air and the minimum droplet radius required for coalescence reduced. In conclusion, we must regard the problem of coalescence as still undecided, even though the tacit assumption that coalescence always occurs on collision has so far led to no inconsistencies, and has enabled plausible theories of rain formation to be put forward.

### 3.14 The fragmentation of drops and splintering of ice crystals

Lenard (1904) observed that freely falling drops larger than 5.5 mm in diameter were unstable, and disintegrated. This fact is important as it provides a mechanism for 'chain reaction' rain formation (see Section 5).

Blanchard (1948) has made a recent study of the instability of drops and finds that in calm air, drops can reach a diameter of 7 mm before becoming unstable, but the slightest turbulence reduces the diameter of the greatest stable drop to 5 mm. Blanchard observed that an unstable drop broke up into two to ten large fragments together with numerous droplets. The results were little affected when the surface tension of water was reduced by contamination from 80 to 56 dyne/cm.

Although he observed that the shock of collision with a cloud droplet would sometimes disrupt a drop, Blanchard did not study the effects of shock produced by sudden

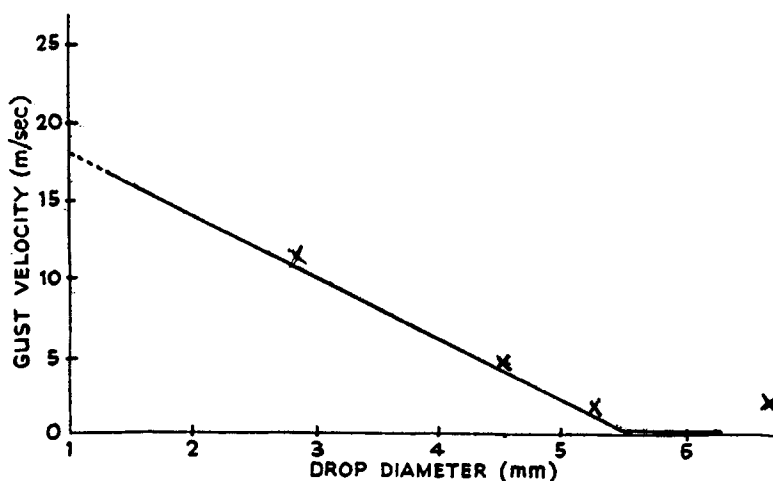


Figure 5. Graph showing the velocity of a sudden gust sufficient to disrupt a water drop falling at its terminal velocity through air at N.T.P. (after Hochschwender 1919).

violent gusts of air. Information on this matter is provided by some experiments of Hochschwender (1919), who gives a table showing, for various drop sizes, the minimum velocity of a drop, relative to the air, needed for disruption. By subtracting from this the terminal velocity of the drop, the graph above (Fig. 5) is obtained, giving the minimum sharp-edged gust velocity needed for disruption.

The splintering of ice crystals growing by accretion or sublimation has been observed by Findeisen (1943) and by Palmer (1950) who suggested that the process may provide ice-crystal nuclei greatly outnumbering the naturally occurring freezing nuclei (§ 3.22). Splintering seems to occur most readily from dendritic crystals, and for this reason it is difficult to assess its importance, because such crystals are rare in natural clouds at temperatures below  $-5^{\circ}\text{C}$ .

### 3.2 Growth processes

#### 3.21 Condensation nuclei

It is generally agreed that all natural cloud droplets formed from the vapour commence their growth around 'condensation nuclei.' Since the nature and concentration (number per unit volume) of the nuclei control the *initial* size distribution of cloud droplets, their determination is important.

As the relative humidity in cloud-free air is increased by cooling, condensation will occur first on the larger nuclei, other things being equal. If the cooling is sufficiently gradual, the initial droplets will be able to take up all the available water vapour, but if there is rapid cooling, the relative humidity may continue to rise, and condensation will commence on successively smaller nuclei. When different kinds of nuclei are present, condensation will favour the more hygroscopic ones. Wright (1939) gives the size of nucleus required for droplet formation at various relative humidities, on the assumption that the nuclei are minute droplets of common-salt solution of varying strengths.

Many estimates have been made of the concentration of condensation nuclei in the atmosphere, and a brief summary of the results is given below, taken from Simpson (1941) and Wigand (1919) :

	Mean values of nucleus concentration (nuclei/cm <sup>3</sup> )
Upper air (10 km height)	2
Sea air	950
Inland air away from towns	9,500
City air	150,000

Since these nucleus counts were made with Aitken counters they may be misleading. For example, sea air may contain relatively few hygroscopic nuclei while inland air may contain numerous small non-hygroscopic nuclei. Copious cloud could form in the sea air, with a slow rate of cooling, at lower relative humidity than over land, although the sea air would give a lower Aitken count !

Results of more interest are obtained from correlating weather conditions with nucleus counts. Moore (1952) and Woodcock (1950) have made nucleus counts at sea, determining total number with Aitken counters and measuring the size distribution of nuclei more massive than  $10^{-11}$  g by impactor techniques. From an analysis of such results, and from observations of the reduction in visibility caused by the larger hygroscopic nuclei (Wright 1939), Moore concludes that the number of large nuclei increases markedly with wave height and wind speed, and is able to explain why previous investigators had failed to detect this correlation. These results suggest that the larger nuclei in maritime air at sea level are almost entirely due to evaporated sea-spray droplets, a result supported by Moore's observations that the nucleus concentration at sea level falls when there is

vertical convective mixing in the air. Even inland, a large proportion of the larger (and therefore more important nuclei) appear to consist of sodium chloride (Dessens 1949). Dessens estimates that, inland in France, the average concentration of salt nuclei of radii greater than  $0.1 \mu$  is about  $100 \text{ cm}^{-3}$  and with radii greater than  $1 \mu$  of the order of  $1 \text{ cm}^{-3}$ . Even the latter figure is great enough to be important in the initiation of rain by the Bowen-Ludlam process.

### 3.22 Ice-forming nuclei

It was long assumed that atmospheric ice crystals were formed directly on nuclei in a manner analogous to the condensation of water drops. Wegener (1910, 1920) suggested that such 'sublimation nuclei' would be effective as soon as the air became saturated with respect to ice. The matter was not investigated further until Krastanow (1941) showed that the formation of ice crystals in pure water vapour is an inherently improbable process except at temperatures below  $-70^\circ\text{C}$ . At higher temperatures ice crystals are not normally formed until water saturation is reached, and then probably by the freezing of liquid water. The work in this field has been reviewed by Mason and Ludlam (1950).

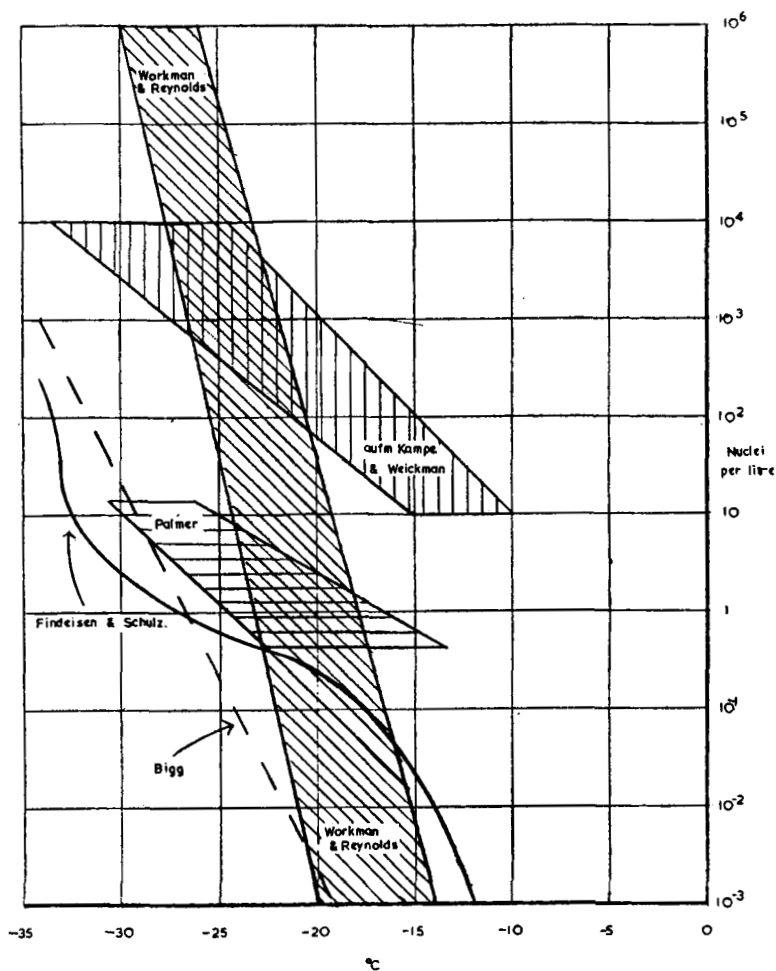


Figure 6. The spectra of freezing nuclei observed by different workers (after Aufm Kampe and Weickmann 1951). Bigg calculated the spectrum shown from his observations of the supercooling of distilled water (Bigg 1953a, b). He assumed a rate of ascent of 20 m/sec.

Findeisen and Schulz (1944) reported some careful measurements of the concentration of ice-forming nuclei, using a large chamber of volume  $2 \text{ m}^3$  in which clouds could be produced by adiabatic expansion. The large size of the chamber allowed the use of very slow rates of expansion, corresponding to updraughts in natural clouds of  $5 \text{ m/sec}$ . They assumed that each ice crystal seen had been formed on a freezing nucleus, and published the well-known 'freezing-nucleus spectrum' reproduced in Fig. 6. There have been several investigations of atmospheric air by an expansion technique (e.g., Cwilog 1947, Palmer 1949); the conditions, in these experiments, become similar to those in natural clouds as the size of the cloud chamber is increased and, on this account, the experiments of Findeisen and Schulz are probably the most satisfactory.

In a second group of experiments clouds have been produced by mixing a little warm moist air with a large volume of cold saturated air, frequently by the expedient of breathing into a cold tank. Large cold rooms have been used,  $10 \text{ m}^3$  by Workman and Reynolds (1949a),  $30 \text{ m}^3$  by Aufm Kampe and Weickmann (1951), for some of these experiments and the large volume has sometimes been thought to provide a closer approach to atmospheric conditions. It is clear, however, that the individual cloud droplets, and freezing nuclei if present, are cooled much more rapidly in the formation of such a mixing cloud than in any natural cloud, and the results of such experiments are relevant to atmospheric processes only if the rate of cooling does not influence the temperature at which ice crystals appear. Findeisen used different rates of adiabatic cooling, all much slower than the rate at which the temperature of the nuclei must fall in a mixing cloud, and for this slow cooling he found more ice crystals for lower rates of expansion. This trend was reversed for rates of cooling corresponding to updraughts exceeding  $50 \text{ m/sec}$ . Bigg (1953a) has also found that the freezing temperatures of water drops fall slightly at higher rates of cooling (*vide infra*). Simultaneous measurements, using the two techniques, of the nucleus spectrum at one place should elucidate the importance of this effect.

The interest of the freezing-nucleus concept led to the neglect of an alternative source of ice crystals, the freezing of fully-grown cloud droplets. As it is impossible to follow the history of a freely-falling cloud drop, we have no direct knowledge of the temperatures at which they freeze, but there have been many observations of the freezing of drops supported on cooled surfaces. The earlier work, which has been reviewed by Mason and Ludlam, did not greatly help our understanding of ice formation in the atmosphere. Recent work by Heverley (1949), Dorsch and Hacker (1950) and Brewer and Palmer (1951) suggests that, in general, smaller drops freeze at lower temperatures and this effect has been more fully investigated by Bigg (1953a, b). In order to reduce the effects of surface contamination he observed the freezing of drops supported between two immiscible liquids. He concluded that the probability  $P$  of freezing depended only on  $T_s$ , the degree of supercooling below  $0^\circ\text{C}$ , the volume  $V$  of the drop and the time  $t$  for which it had been kept at the lower temperature: his results could be represented by the equation

$$VtK \exp(aT_s - 1) = -\ln(1 - P) \simeq +P \text{ when } P \ll 1$$

where  $a \simeq 1$  and  $K$  is of order  $10^{-8}$ . Bigg established that these results could be reproduced with distilled water from various sources and with different supporting liquids; suggesting that the equation would also hold for cloud drops, he has re-interpreted Findeisen's results. With assumptions concerning the humidity of air in the chamber and the concentrations of condensation nuclei, he was able to calculate the size and concentration of the cloud droplets and hence the concentration of ice crystals which should be observed under differing conditions. At temperatures below about  $-20^\circ\text{C}$  the values deduced agreed well with Findeisen's measurements, but at higher temperatures Findeisen's crystals were the more numerous by a factor of about 10.

Bigg's results are of considerable interest, although, as he emphasized, they do not tell us anything about the physical process of freezing. The results of cloud-chamber experiments on a single day could be explained either in terms of the properties of freezing nuclei or by the freezing of drops. If, however, the nucleus spectra for several days are compared, there are frequently discrepancies of 5-10°C between the temperatures at which some particular concentration of ice crystals was observed. These might be explained by variations in the concentration of *condensation* nuclei, which would cause similar expansions to produce droplets of different sizes. It would thus be a valuable experiment to repeat cloud-chamber measurements, the samples having first been diluted with cleaned air to standard condensation-nucleus concentration. If the variation of the spectrum were still observed, the constants in Bigg's equations would require daily adjustment and this would seem to suggest that the freezing of the cloud drops was brought about by variable, presumably foreign, particles and one would be very close to a freezing-nucleus theory.

### 3.23 *The nature of freezing nuclei*

A certain amount of work has been carried out to determine the nature of freezing nuclei, with the primary object of detecting geographical influences. Schaefer (1951) finds that dust varies in its efficacy with its source, but the variation is not strikingly large, while Vonnegut (private discussion) has suggested that atmospheric pollution may inactivate many nuclei formed near the ground. Most experiments to detect ice-forming nuclei have been performed at the earth's surface, usually in populous industrial districts. Even so, the results shown in Fig. 6 do not agree well, and there has been no careful investigation of the variation of the nucleus spectrum with geographical position, with origin of the local air mass, or with height in the atmosphere. E. J. Smith and Heffernan (1954) have carried in an aircraft a chamber of 76-litre capacity in which clouds were formed by mixing. They found that, in the range of height 1-3.5 km, the concentration of ice crystals in the cloud chamber tended to be greater above an inversion but otherwise showed little correlation with height.

Considerable interest has been aroused by a recent suggestion (Bowen 1953) that meteoric dust particles may act as extremely efficient freezing nuclei. Bowen finds that the rainfall, averaged over fifty years at several stations in the southern hemisphere, shows peaks on certain dates about 29 days after some of the major annual meteor showers. He considers that these peaks are real, and not caused by statistical fluctuations. If this is accepted, and the point will of course demand a very searching scrutiny, it would be hard to escape the conclusion that there is a 'cosmic' influence acting on the rainfall. It is believed on experimental and theoretical grounds that, while meteor particles more than a few microns in radius (mass greater than  $10^{-11}$  g) evaporate completely in their passage through the upper atmosphere to give heat, light and ionization, smaller particles are unscathed and will fall to the tropopause in about one month or longer. Bowen suggests that it is these 'micrometeorites' which can act as freezing nuclei. The theory is attractive, but certain objections can be raised on astronomical grounds. It is believed (Kaiser 1953) from the evidence of radio echo and visual observations, and from certain theoretical considerations, that for meteors fainter than 5th magnitude, the diurnal flux of sporadic meteors incident on the earth greatly exceeds the maximum diurnal flux during any of the major annual showers (with the possible exception of the Arietid daytime shower). If this law holds down to the micrometeorites, which there is no reason to doubt, then the additional flux of micrometeorites produced during a meteor shower would not be noticeable. To overcome this difficulty, it could be argued that freezing nuclei are produced by the flaking of the brighter larger visual meteors, which

outnumber the comparable sporadic meteors considerably during meteor showers. There is little evidence for such flaking, except possibly for meteors brighter than third magnitude (Ceplecha 1953) but even if we assume that such meteors are completely pulverised into fragments of mass  $10^{-11}$  g spread along the path of the meteor (10 km or more), it can be shown that the concentration of the particles near the earth's surface will be less than  $0.1\text{ m}^{-3}$ . This seems insufficient to affect markedly the rate of freezing of supercooled droplets in a cloud. However, it is clear that Bowen's suggestion needs more than a superficial examination before a conclusion is reached, and his observations, if they are valid, will be hard to explain by any other theory.

3.24 Condensation, evaporation and sublimation

Jeffreys (1918) showed that when the growth of a particle is governed by the diffusion towards it of vapour the rate of growth is given by the equation

$$\frac{dm}{dt} = 4\pi\Delta CM (p - p_s)/RT$$

where  $m$  is the mass of the particle,  $C$  is the electrostatic capacity of a conductor of the same shape and size,  $\Delta$  is the diffusion coefficient for water vapour in air,  $p_s$  and  $p$  are the water-vapour pressures at the surface of the particle and at a remote point in the vapour respectively;  $M$ ,  $R$ ,  $T$  are molecular weight of water vapour, gas constant per mole and absolute temperature respectively. In the case of spherical drops  $C$  is equal to the radius. Frössling (1938) pointed out that the motion of the drop through the air would increase the supply of vapour and, to allow for his effect, multiplied the right-hand side of the equation by a wind factor  $f$ . His experimental results indicated that  $f$  varied with the Reynolds number  $Re$  of the flow round the drop ( $f = 1 + b (Re)^{\frac{1}{2}}$ ;  $b$  is approximately 0.3). These results were confirmed for the evaporation of freely-falling drops by Kinzer and Gunn (1951).

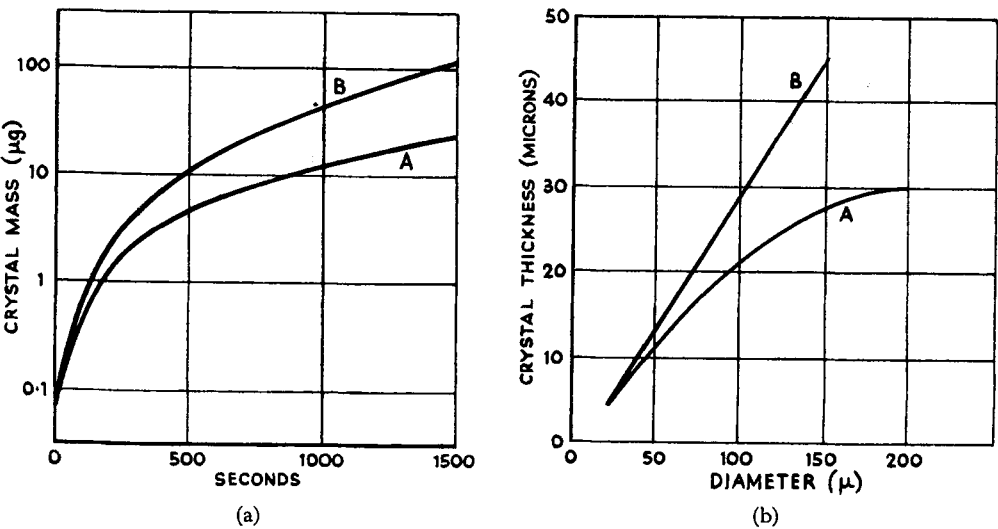


Figure 7. (a) The growth of ice crystals in a water-saturated atmosphere at  $-10^{\circ}\text{C}$  (the wind factor was assumed constant  $= 2.5$ ). Curve A shows the growth if the crystal thickness is equal to one-fifth of the radius, and curve B shows the growth if the thickness is constant  $= 10\text{ }\mu$ . The crystals are initially identical with a mass of  $6.25 \times 10^{-8}\text{ }\mu\text{g}$  and circular shape. (b) Reynolds's measurements of the thickness of ice crystals grown at about  $-18^{\circ}\text{C}$ . Curve A, plane dendritic crystals; curve B, hexagonal plate crystals.

The value of  $p_s$  may be adjusted to allow for the effects of the curvature of the drop and for the heating or cooling produced by the latent heat of vaporization; the drop will usually be at the ventilated wet-bulb temperature (Kinzer and Gunn 1951). If the drops contain hygroscopic substances the change in  $p_s$  may, with caution, be deduced from Raoult's law; the numerous pitfalls in such a calculation have been emphasized by Macdonald (1953).

The growth of ice crystals by sublimation has been discussed by Houghton (1950) and Mason (1953); they replace  $C$  in the equation above by the capacity of a conductor of an appropriate simple geometrical shape. The equation does not predict how the crystal will grow but only the instantaneous rate of growth for a particular size and shape. Fig. 7a shows the effect of different assumptions about the thickness of plate crystals. Mason (1953) and Reynolds (1952) have measured the rate of growth of small ice crystals in supercooled water clouds but their crystals were too small to show in Fig. 7a. Reynolds also observed the relation between the radius and the thickness of plate and dendrite crystals and his results are shown in Fig. 7b. Weickmann (1945) has reported observations of the forms of atmospheric ice crystals and Nakaya (1951) has reproduced many of these but, unfortunately, the conditions at the surface of the artificially grown crystals were not very well defined.

The growth of a dendritic crystal cannot be calculated from the equation above, since the value of  $C$  is unknown, although it might be found from model experiments. Mason has used dimensional arguments to estimate that hexagonal plates should commence dendritic growth when the diameter exceeds about  $100\ \mu$ . Browne (1953) and Murgatroyd (private communication) report however that dendrites are rare in nimbostratus and hexagonal plates as large as  $400\ \mu$  are common.

#### 4. PHYSICAL PROPERTIES OF RAINCLOUDS

##### 4.1 *Layer clouds*

The physical properties of rain-forming layer clouds will be discussed in an effort to present a general description of their large- and small-scale features. The discussion will be restricted to pure layer clouds, thus excluding systems of cumulonimbus embedded in nimbostratus. In England, nimbostratus usually gives a rainfall rate of from 1 to 3 mm/hr.

##### 4.11 *Cloud depth and extent*

Clouds of this type often cover an area of thousands of square kilometres and are commonly associated with frontal systems. The depth of layer clouds and the height of the cloud base have been investigated by several workers (see § 2.11). Mason and Howorth (1952) give a rough rule relating the thickness of rain-forming stratus to the height of cloud base: clouds whose thickness exceeds the height of the cloud base by more than 1 km will precipitate. The data on which this statement is based were all obtained in one region, viz. Northern Ireland, and some caution should be used in generalizing from them. The fact that a cloud of given depth will precipitate only if its base is below a certain level presumably depends on the fact that a low cloud base tends to imply air of high humidity mixing ratio and a cloud of large liquid water content.

##### 4.12 *Updraught and turbulence*

No direct measurements of updraught in layer cloud are available. This is not surprising; only a very small updraught can be expected. Bannon (1948) has estimated



updraughts from observations of the mean rainfall rate below layer cloud by the assumptions that the cloud is in a steady state and that the water falling from a vertical column comes entirely from the release of water in the vertical upcurrent. His calculations indicate that the updraught speed will vary from 5 cm/sec up to 20 cm/sec for rainfall rates in the range 1-3 mm/hr. Updraughts may also be estimated crudely from the convergence along trough lines in depressions (e.g., Hewson and Longley 1944); the values estimated in this way show fair agreement with those obtained by Bannon.

The turbulence in layer clouds is probably also slight. It presumably arises mainly from the vertical shear of the horizontal winds, and this is generally only of the order of  $2 \times 10^{-3} \text{ sec}^{-1}$ . Few turbulence measurements have been reported. Browne (1952a) has estimated, from radar observations, the intensity of turbulence (ratio of mean square fluctuating velocity to mean wind speed) in warm frontal layer cloud as less than 0.1 per cent at a height of 3 km, but this value seems exceptionally small.

#### 4.13 Humidity

It has generally been assumed that in nimbostratus the air is saturated with respect to water. This is doubtless justified below the freezing level, but may not be at greater heights where the presence of ice crystals introduces a complication. No direct measurements are available, because of their difficulty. The matter will be discussed at length in Section 5.

#### 4.14 Liquid-water content and droplet-size distributions

There is a surprising lack of published systematic information on the liquid-water contents and droplet-size distributions in layer cloud, yet many measurements must have been made. Kline and Walker (1951) give liquid-water contents above the freezing level ranging from 0.05-1.3 g/m<sup>3</sup>. Cunningham (1951) measured liquid-water contents, with a rate-of-icing meter, from 0.05-0.2 g/m<sup>3</sup> in clouds giving rainfall rates between 1 and 2 mm/hr, and from 0.1-0.2 g/m<sup>3</sup> in a layer cloud giving a rainfall rate of 7 mm/hr. Other indirect estimates were made by Browne (1952a) from absolute measurements of the radar-echo intensity below the freezing level in precipitating layer cloud. He finds liquid-water contents of less than 0.05 g/m<sup>3</sup> for three clouds giving rainfall rates from 0.9-1.1 mm/hr, and values of 0.40-0.55 g/m<sup>3</sup> for a cloud giving rainfall rates from 2.5-3.5 mm/hr.

Measurements of cloud-droplet size have been reported for non-precipitating layer cloud, in regions below the freezing level, by Frith (1951) who gives values of droplet-size distribution showing diameters up to 30  $\mu$ , with 50 per cent of the total water content contained in droplets of diameters between 12  $\mu$  and 18  $\mu$ . Kline and Walker (1951) find that the bulk of the water in supercooled layer clouds is contained in droplets with mean diameter 12  $\mu$ , the largest droplets having diameters of about 40  $\mu$ .

#### 4.2 Convective rainclouds

Rain from convective cloud is characterized by its variability and short duration as compared with layer-cloud rain. In a moderate April shower in England, rain may fall for perhaps half-an-hour, and its maximum intensity may reach 10-15 mm/hr. In a heavy thunderstorm, rain may fall from a single 'cell' (see below) for as long as one hour, and its peak intensity, in this country, will usually be about 50-80 mm/hr, though higher values are not uncommon.

#### 4.21 Cumulonimbus cells

A great advance in the study of convective rain was made when it was realized that well-developed cumulonimbus clouds contained one or more 'cells,' each with a fairly definite life-cycle (*The Thunderstorm* 1949). The existence of these cells is clearly seen in numerous radar range-height photographs (Fig. 8). Initially, in the cumulus stage, a cell is growing and characterized by an updraught which lasts on the average for some 20 min. In the mature stage, precipitation is released, and the drag of the falling drops (which is equal to their weight) initiates a downdraught which spreads downwind through the cell, shutting off the further supply of moist air. In the final dissipating stage, the cell subsides and evaporates slowly, often to be absorbed by another cell. A large thundercloud may contain, at any instant, cells in all stages. Each cell converts into precipitation about  $10^7$  to  $10^8$  kg of water (Braham 1952). It has been observed that the release of precipitation frequently triggers off the growth of a new cell downwind. This may be brought about by a layer of cold air, about 1 km in depth, spreading out from the downdraught at ground level and causing lifting of unstable air downwind.

Cumulonimbus cells have an area of the order of a few square kilometres, often only a fraction of the total base area of the cloud. It must not be assumed that conditions are uniform over the whole horizontal extent of a cell which may include several regions of particularly strong updraughts (*Thunderstorm* 1949, Kuettner 1950). Cloud base is usually at a height of 1 to 3 km, while the height of the cloud top varies widely. In a typical moderate shower, the depth of the cloud might be 3-5 km, but in a violent thunderstorm, the cloud may be 15 km deep and may reach the tropopause (*Thunderstorm* 1949). Bowen (1950) and Ludlam (1951a, 1952) suggest that convective clouds cannot rain unless their depth exceeds about 2 km and this is well supported by observation.

#### 4.22 Updraughts and turbulence

The mean updraught in a cumulonimbus cell varies from 1 m/sec (small cloud) to perhaps 10 m/sec (thunderstorm). As might be expected from the equation of continuity, the updraughts increase upwards, and the greatest are found at about the 5-km level (*Thunderstorm* 1949). Updraughts have been measured by aircraft and by observing the rate of rise of the radar-echoing volume in a cloud, the two values being in excellent agreement. Higher values of updraught in cumulonimbus have been occasionally reported, Gillmer and Nietsch (1944) quote a report by a glider pilot of 100 m/sec, while Bilham and Relf (1937) have deduced velocities of 130 m/sec, on the assumption that the updraught must be sufficient to support the hailstones which sometimes fall from these clouds (see § 3.11). However, certain calculations (Gaviola and Fuertes 1947, Ludlam 1951a) suggest that hailstones can grow to the observed size without being supported by updraughts. Sil (1950) has estimated updraughts of up to 6 m/sec from the rainfall rate, on the hazardous assumptions that the cloud is in a steady state and that there is no entrainment (see below). Downdraughts in cumulonimbus cells occur, over regions about 1 km in diameter, and with modal value of 5 m/sec. The greatest speeds are found near the ground (*Thunderstorm* 1949).

Considerable effort has been put into measuring the vertical components of turbulent gusts in cumulonimbus (Jones 1949, 1954; *Thunderstorm* 1949). As might be expected, the greatest gusts are found in the regions of large vertical shear on the edges of cells. The value of the gust speed in terms of the equivalent sharp-edged gust (see e.g., Eyraud 1949) is obtained from vertical accelerations experienced by aircraft, and is commonly in the range 0 to 10 m/sec. Little is known of the spectrum of turbulence in convective cloud.

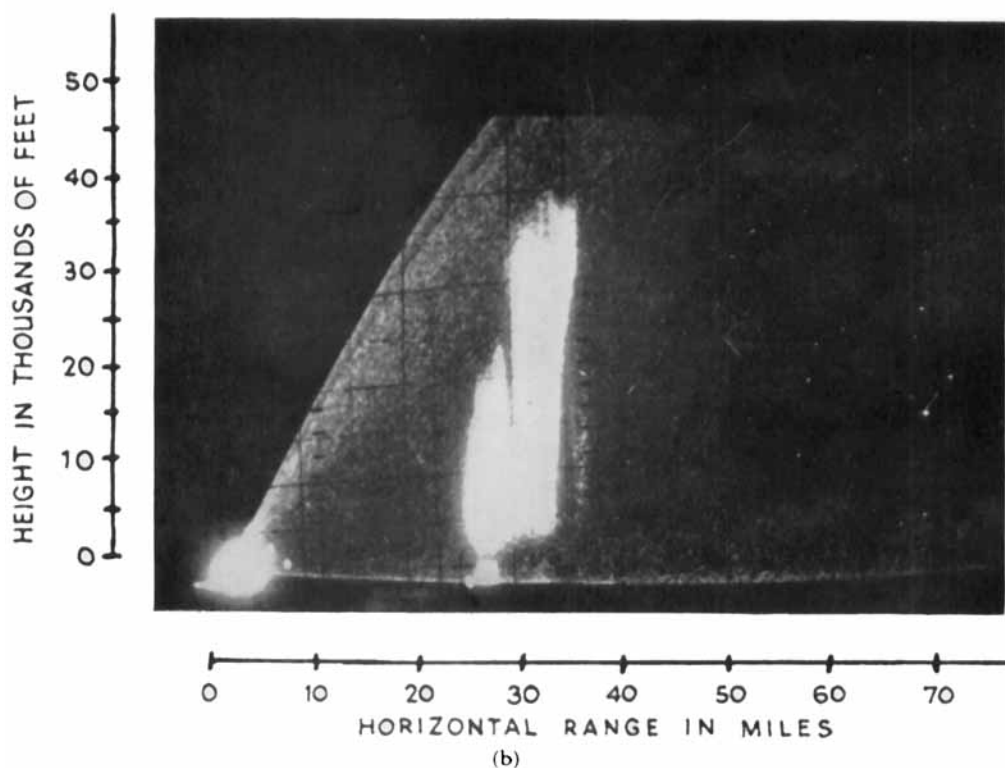
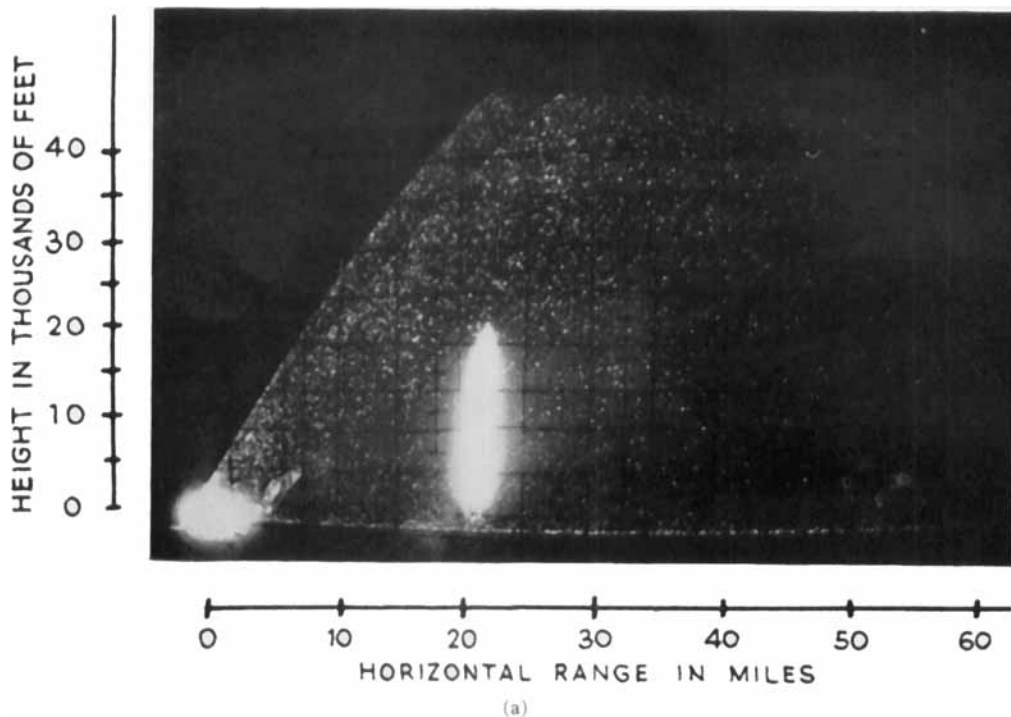


Figure 8. Radar echoes from cumulonimbus clouds showing cell structure. Fig. 8a is a fine example of an echo from a single isolated cell. Fig. 8b shows a pair of cells, of which that on the left has just reached the mature stage and is developing vigorously at the edge of an older cell. The tapering appearance of the latter may be due to attenuation of the radio waves by liquid water, which might be expected to show an increased concentration towards the base of the cells (cf. § 6.1). It should be noted that the scales of the horizontal and vertical axes are unequal, with a ratio of about 5 : 1, so that, in fact, the echoing regions have almost equal height and breadth.

The radar operated at a wavelength of 10 cm with a pulse-length of  $2 \mu\text{sec}$ , and used an aerial scanning in a vertical plane and giving a beam-width measured between half-power points of  $15^\circ$  in azimuth and  $3^\circ$  in elevation. The photographs were obtained by R. F. Jones at East Hill, Dunstable, and are reproduced by permission of the Director of the Meteorological Office.

#### 4.23 *Composition of convective clouds*

All types of precipitation elements may be found in convective cloud – rain, hail, graupel, snow and ice crystals. There has been little systematic observation of their occurrence in different regions of cloud at various stages in its development. The Thunderstorm Project (1949) found snow above 5 km in over 70 per cent of the observations in Florida and Ohio; its frequency increased with height. Hail seems to be rare and was encountered in less than 5 per cent of the observations, being found most often between 3 and 5 km above sea level; it is probably even more rare in convective shower clouds. It is suggested (*Thunderstorm* 1949) that hail is found, if at all, mostly during the mature stage; the evidence for this is very slender. Observers at the ground, on the other hand, report that hail, especially in the form of graupel (soft hail or snow pellets), is very common in precipitation from cumulonimbus clouds. Prohaska (1900) analyzed observations by an extensive network of observers in Austria, and showed that hail is often distributed by a storm over a track 1-2 km in width, and many kilometres long. His findings are supported by some valuable observations made at a mountain observatory on the Zugspitze (10,000 ft) by Kuettner (1950), who concludes that graupel occurs in 75 per cent of storms. The apparent discrepancy between reports from observers on the ground and from those in aircraft might be explained by the break-up of pellets of graupel which strike the windscreen of an aircraft, so giving the appearance of snow rather than of hail. For instance, one of the present authors noticed during a shower that pellets of graupel hitting the windscreen of a car travelling at a speed of 25 mi/hr (40 km/hr) remained intact, but at greater speeds they broke on impact.

The presence of liquid water, indicated by aircraft icing, was reported in roughly one-third of the observations at 9 km, the greatest height for which reports are given.

Ice crystals become a noticeable feature in the dissipating stage, when they predominate above 8 km (*Thunderstorm* 1949) forming the familiar anvil,\* but their presence in earlier stages of the cell's life is a matter for conjecture. Thus little can be deduced from these observations concerning the importance of ice in initiating the release of rain.

#### 4.24 *Liquid-water content and drop-size distribution*

Liquid-water contents in large cumulonimbus have usually been measured by 'rate-of-icing' meters and such methods can give serious underestimates if the water content is so high that the rapid release of latent heat of fusion prevents some of the deposited water from freezing (Ludlam 1951b). Warner and Newnham (1952) have made determinations in small cumulus by measuring the change in electrical conductivity of absorbent paper exposed to the cloud, while others have used impactor techniques (Diem 1948, Squires and Gillespie 1952, Weickmann and Aufm Kampe 1953). The results may be briefly summarized as follows: in large thunderstorms, the liquid-water content reaches a maximum at a level about 2 km above cloud base (a result not predicted by simple theories) with a value of about 4-6 g/m<sup>3</sup>. In small shower clouds, the water content is 0.2-0.5 g/m<sup>3</sup>, and is usually less than would be expected from calculations on the basis of adiabatic ascent through the cloud base. This is almost certainly due to what has been called 'entrainment,' the mixing of the environment with ascending cloud air (Stommel 1947). There is no doubt that entrainment occurs, though there is disagreement on its mechanism (Malkus; also Scorer and Ludlam 1953). Even in large cumulonimbus, it can produce as much as 100 per cent dilution of the cloud air for every 400 mb of ascent

\* It is not certain that the fibrous appearance of the anvil is an entirely reliable sign of glaciation. One of the authors recalls an occasion when the moon, visible through anvil cloud blowing off from distant cumulonimbus, was surrounded by a water diffraction corona.

(Thunderstorm 1949). On occasions, however, and particularly in large cumulonimbus clouds, the liquid-water content is found to be considerably greater than that expected from a simple calculation. Braham (1952) and Malkus (1953) have suggested that this can occur when a cell grows in an environment made moister than the average by the dissipation of previous cells, so that one cell feeds on the remains of its ancestors.

The mean droplet concentration (i.e. number per unit volume) in small cumulus is greater than that in larger clouds (cumulus congestus, cumulonimbus) which show, on the other hand, a higher proportion of droplets exceeding  $50\ \mu$  in diameter (Weickmann and Aufm Kampe 1953). This may provide evidence for the growth of cloud droplets by coalescence, a point which will be discussed more fully in Section 5. However, droplets of diameter about  $300\ \mu$  have often been detected in appreciable concentrations even in cumulus humilis (Ludlam, Browne and Day 1954). Their presence is very surprising, in view of the short lifetime, about 20 min, of these clouds.

## 5. CLOUD BUDGETS

### 5.1 Layer clouds

Measurements of the radar echo from precipitating clouds may be used to supplement the information given by the rainfall rate and by measurements of the raindrop-size distribution at the ground. If all such information is used together, there is sufficient to justify numerical estimates of mass, concentration (number per unit volume) and rate of growth of the precipitating particles. The case of rain-forming layer cloud is attractive,

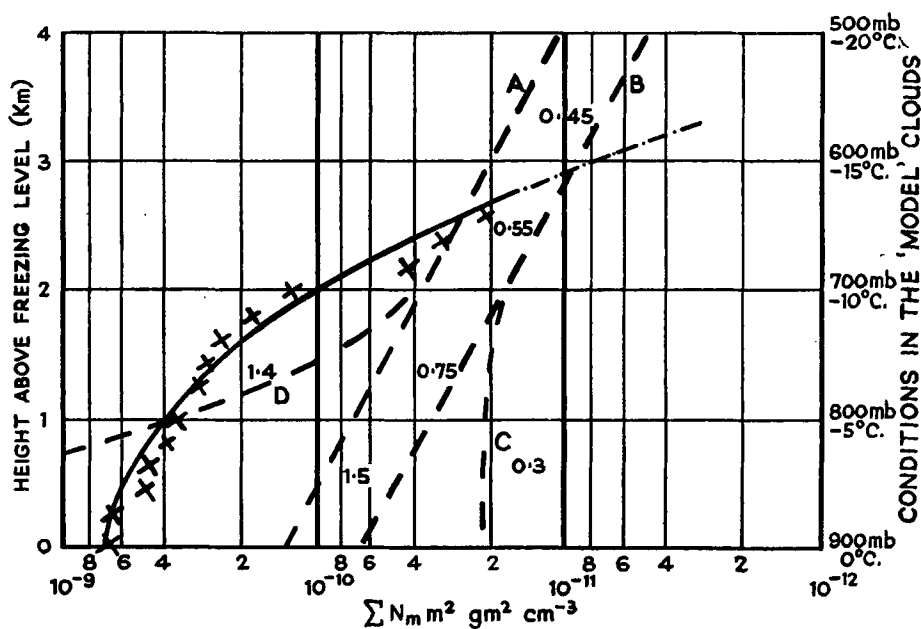


Figure 9. Graphs of  $\sum N_m m^2$  against height above freezing level. The points  $\times$  show the means of the observed values for three cases of widespread steady rain giving about 1 mm/hr at the ground. The small-chain line shows the trend of the echoes as they become too weak to observe. The broken lines show the values of  $\sum N_m m^2$  calculated for model clouds giving a rainfall of 1 mm/hr. A. Concentration of particles 2/litre; B. Concentration of particles 1/litre; C. Particles of mass greater than  $100\ \mu\text{g}$  release splinters of mass  $10\ \mu\text{g}$  at the rate of one for every 500 m fall; D. Particle concentration reduced by aggregation, by a factor of 2 for every 500 m fall below the  $-10^\circ\text{C}$  isotherm. The figures on the graph give the calculated relative humidity in an equilibrium cloud, expressed as a fraction of the difference in vapour pressure between water and ice at the prevailing temperature. For this calculation the crystals are assumed to be circular discs, with thickness equal to one-fifth of the radius.

because conditions are reasonably uniform over a wide area, and as a tentative preliminary we may assume, with Bannon (1948), that the cloud is in a steady state. In other words, it can be assumed that all the water condensed from the updraught at any level is removed at the same rate by the growth of the precipitation particles falling through that level. The water budget of the cloud is balanced.

### 5.11 The radar echo

The intensity of the radar echo from a cloud is proportional to the quantity  $\Sigma N_m m^2$ , where  $N_m$  is the concentration of particles of mass  $m$  each with a shape factor of unity (see Section 6). Measurements of the echo intensity from different heights in a cloud, together with absolute measurements of the power and sensitivity of the radar set, enabled Browne (1952a) to investigate the magnitude and variation with height in layer cloud of the quantity  $\Sigma N_m m^2$ . Fig. 9 shows the measurements of  $\Sigma N_m m^2$  in the cloud above the freezing level, for three cases of widespread warm-frontal rain, with precipitation rates of 0.9, 1.1 and 1.2 mm/hr, and freezing levels at 2,200 m, 1,180 m, and 960 m respectively.

In principle one could, by the following method, obtain information from these measurements about the way in which the precipitation particles in the cloud are growing. Starting from measurements of the rainfall rate and drop-size distribution  $N_m$  at the ground, a step-by-step calculation will yield values of  $N_m$  and  $m$  at any height in the cloud, for a given set of assumptions about the processes of growth and about the conditions in the cloud. The values so obtained could then be compared with the quantity  $\Sigma N_m m^2$  obtained from the radar measurements as a test of the validity of the assumptions. The necessary combination of measurements is not yet available for an individual cloud, but it is instructive to compare the values of  $\Sigma N_m m^2$  calculated for a mathematically simple model cloud with the values measured by Browne. Wexler (1952) and Browne (1952a) have both considered such simple models, and have attempted to relate them to Browne's echoes, although Wexler did not attempt to obtain quantitative agreement between the calculated and measured values of  $\Sigma N_m m^2$ . Their conclusions were very different, though neither made the assumption of a steady state.

### 5.12 A model cloud

The simplest model is a cloud in a steady state in which ice crystals are growing by sublimation. The rainfall rate is taken to be 1 mm/hr and the wet-bulb potential temperature of the air rising through the cloud base is taken as 5°C. Bannon's calculations (1948) show that under these conditions, the updraught in the cloud will be 5.5 cm/sec, and that water will be condensed from it at the rate  $I$  shown in the table :

Temperature (°C)	+ 2.5	− 2.5	− 7.5	− 12.5	− 17.5
Pressure (mb)	950	850	750	650	550
$I \times 10^{-11}$ (g cm <sup>-3</sup> sec <sup>-1</sup> )	9.5	7.5	5.8	4.3	3.0

This table will be used to calculate the increase in mass of particles as they fall through the cloud, given their concentration and terminal velocity, without any assumptions as to their manner of growth.

Best (1950b) has summarized the available information about raindrop-size distributions. In an earlier paper (1947) he gives an expression for the mean mass of raindrops  $\bar{m}$  ( $\mu\text{g}$ ), which can be re-written in the form :

$$\bar{m} = 180 p^{0.75}$$

where  $p$  is the rainfall rate in mm/hr. Thus for the model cloud under consideration,

the mean raindrop mass at the ground is  $180\ \mu\text{g}$ . Since the radar evidence suggests that the mass of the drops changes by less than 10 per cent as they fall from the freezing level to the ground, the ice crystals in the model will be assumed to be homogeneous with a mass, just above the freezing level, of  $180\ \mu\text{g}$ . Their terminal velocity will be taken as constant, with a value of  $70\ \text{cm/sec}$  (cf. § 3.11) so that the crystal concentration is 2/litre. The step-by-step calculation of the mass of the crystals at higher levels gives the values of  $\Sigma N_m m^2$  plotted in Fig. 9, curve A. Curve B shows the values of  $\Sigma N_m m^2$  for a similar cloud giving raindrops twice as massive and with half the concentration.

If the ice crystals are formed near the top, or coldest part of the cloud, their concentration may be increased as they fall by the release of ice splinters. This would retard the growth of the precipitating particles, and curve C shows the reduced values of  $\Sigma N_m m^2$  that would be expected if splinters of mass  $10\ \mu\text{g}$  were released at the rate of one per 500 m of fall by all crystals of mass greater than  $100\ \mu\text{g}$ . The reverse effect can occur where crystals aggregate to form snowflakes, and curve D shows the marked increase in  $\Sigma N_m m^2$  to be expected when aggregation reduces the concentration of particles by a factor of two for every 500 m fallen below the  $-10^\circ\text{C}$  isotherm. Variants of the cloud model which include processes such as aggregation and splintering will, of course, give a rainfall which does not agree with the relation given by Best.

A more realistic investigation would take account of the unequal sizes of the ice crystals. Most measurements of the size distribution in rain show that the mean square mass  $(\overline{m})^2$  of the particles exceeds the square of the mean mass  $(\overline{m})^2$  by a factor which can be as great as ten. The calculated values of  $\Sigma N_m m^2$  for a homogeneous cloud will therefore be less than those for a natural cloud giving the same precipitation rate. This might remove the discrepancy shown in Fig. 9 between the calculated and measured values of  $\Sigma N_m m^2$  at the freezing level, but seems unlikely to make the form of the calculated curve fit the observations.

### 5.13 *The humidity in ice clouds*

The assumption of a steady state has given the rate of growth of ice crystals. The relative humidities needed to achieve this rate can be calculated from the equations in § 3.24, by making suitable assumptions about the shape of the crystals. The figures on the curves in Fig. 9 give values of the supersaturation computed in this manner, and expressed as a fraction of the vapour-pressure difference between ice and water surfaces at the relevant temperatures. The crystals were assumed to be circular discs, of thickness equal to one-fifth of their radius. The calculated humidities are sensitive to this assumption about the crystal thickness: the thinner the crystals, the larger their radius, and the lower the supersaturation needed for a given rate of growth. The principal difference between the model clouds of Browne and Wexler lay precisely in the thickness assumed for their crystals. Browne was led to conclude that his crystals would not grow fast enough even at water saturation unless accretion was postulated, while Wexler found that the ice crystals must splinter in order to give a radar echo resembling that observed.

In the case of a pure ice cloud the supersaturation can change to maintain the equilibrium rate of growth of the ice crystals. For example, should the crystal concentration fall, equilibrium could be maintained by a rise in the relative humidity. But if liquid droplets are present in the cloud, the relative humidity is held close to water saturation, so that equilibrium can no longer be maintained by variations in humidity. It may thus be unrealistic to speak of a true steady state in a precipitating water cloud. Measurements are needed to establish whether or not warm-frontal nimbostratus contains supercooled water droplets above the freezing level.

#### 5.14 Discussion

The estimates of relative humidity indicate a difficulty. If atmospheric ice crystals are formed only when the relative humidity rises to water saturation, as is found in the laboratory, they could not be formed at the top of our equilibrium cloud, where the relative humidity only slightly exceeds ice saturation (Fig. 9). If the crystals become very scarce at the top of the cloud, the relative humidity may rise to water saturation. Freezing nuclei would then be enabled to form ice crystals there, but this would cause the relative humidity to fall again. Thus the cloud would no longer be in true equilibrium, though the size of the irregularities that would occur is uncertain, and might be small enough (with linear dimensions of the order of ten metres) to give an apparently uniform radar echo and precipitation pattern at the ground.

Marshall (1953) has reported radar observations of a rather different type of irregularity. He has observed bands of enhanced echo intensity from the upper parts of nimbostratus, well above the freezing level, and has interpreted them, in agreement with Browne (1952b), as the echoes from precipitation streaks. The streaks appear to retain their identity in some cases for hours, and it is suggested that they arise from 'generating elements' at the top of the cloud, which must in this case provide a steady supply of ice particles with similar falling speeds. The nature of the processes leading to the formation and subsequent maintenance of precipitation streaks is intriguing, and calls for further investigation.

The model cloud represented by curve A in Fig. 9 contained a concentration of ice crystals of 2/litre. If these crystals were formed by freezing nuclei carried in the air rising through the cloud base, the necessary concentration of freezing nuclei would be greater in the ratio of the terminal velocity of the ice crystals to the velocity of the up-draught. The resulting value of 30 freezing nuclei/litre was estimated by Palmer (1949), while Aufm Kampe and Weickmann (1951) report similar estimates by an independent method. The source of the ice crystals is uncertain, but if they are formed by the freezing nuclei detected in the measurements shown in Fig. 6, the required concentration will not appear until the temperature reaches  $-20^{\circ}\text{C}$  to  $-25^{\circ}\text{C}$ . However, rain, which has probably been formed by the Bergeron process, frequently falls from layer clouds with summit temperatures above  $-15^{\circ}\text{C}$ . Thus there appears to be a shortage of freezing nuclei, and further investigations seem called for.

Brewer and Palmer (1949) have suggested that the additional ice crystals may originate as small splinters of ice released from a growing crystal. A few freezing nuclei might thus provide a much larger number of ice crystals, and might even initiate a self-sustaining chain-reaction.

To conclude this section, it is suggested that widespread steady rain from a layer cloud in true equilibrium is rare, and that small-scale equilibrium is impossible if the cloud contains both liquid water and ice above the freezing level. The precipitating ice crystals would then grow by accretion as well as by sublimation, but there could be no automatic control of the equilibrium as in an ice cloud. Calculations are difficult for this case, partly because the collection efficiencies of ice crystals are unknown.

### 5.2 Convective rainclouds

#### 5.21 Macrophysical problems

The meteorologist would be satisfied with his knowledge of convective cloud if it enabled him to forecast the amount and rate of rainfall on a given day merely from detailed local air soundings and knowledge of the synoptic situation. There are wide gaps in his understanding which prevent him from achieving this ideal.



At present the heights of base and summit of clouds, less than about 3 km deep, can be predicted with an accuracy of perhaps  $\pm 200$  m from aerological soundings, provided that there is no large-scale vertical motion. There is a greater uncertainty in predictions of the heights of deeper clouds. Convergence increases convective activity and subsidence inhibits it, but their detailed effects have not been fully investigated.

The vigour with which a convective cloud builds up (that is, the updraught within the cloud) depends not only upon the instability of the air, but also upon the way in which hot air is fed in, often from sources on the ground (Scorer and Ludlam 1953), and the possibility of forecasting updraught velocities to better than  $\pm 50$  per cent seems remote. The reasons which determine whether the convection will manifest itself in the form of a few clouds of large base area, or in the form of many clouds of small base area, are imperfectly understood, but it is probably again determined largely by the distribution of heat sources. The problem is important, because the effects of entrainment upon the liquid-water content in the cloud are reduced as its base area is increased to provide the convective core with a protective shield of cloud. It has already been seen (§ 4.2) that aerological soundings are an unreliable guide to the liquid water content of cumuliform cloud.

In a paper based on the observations of the United States Thunderstorm Project, Braham (1952) discusses the history of the water vapour carried into a thunderstorm cell. He finds that, of some  $10^9$  kg of water vapour that may enter a single cell, only 50 per cent condenses: the remainder enriches the entrained air and passes through the top of the cloud. Of the condensed water, roughly two-fifths evaporate into the down-draught initiated by the release of precipitation, a further two-fifths are accounted for in the cloud remaining after the storm and by evaporation from the boundary of the cloud, and the rest (about  $1.2 \times 10^8$  kg) falls to the ground as rain. These estimates are very rough. As Braham points out, some thunderstorms in arid regions produce no precipitation at the ground; all the rain has been evaporated into the downdraught to maintain saturation during adiabatic descent. Braham estimates that the total energy release in a cell – the total of latent heats of evaporation and fusion – is of the order of  $10^{22}$  erg. Over 95 per cent of this is ultimately used in altering the energy state of the environment, that is to say, in ‘overturning’ the atmosphere, while most of the remaining energy is used in carrying liquid water aloft. Lightning discharges account for less than one per cent of the total energy.

It is clearly of importance to compile similar water and energy budgets for smaller convective showerclouds. Even if the resources of the Thunderstorm Project are not available, most of the necessary facts could be obtained by relatively simple experiments (see, e.g., Malkus 1953).

It has been seen that outstanding problems in the ‘macrophysics’ of convective clouds are the prediction of the base area of clouds, the amount of entrainment, the updraught velocity and the liquid-water content. We may by-pass these problems, only to meet more; given the heights of cloud base and top, area of cloud base, central updraught velocity and the liquid-water content at all levels, can a detailed history of the subsequent rainfall be predicted? These problems may be subdivided into those concerned with the initial formation of precipitation elements, and those concerned with the subsequent growth to raindrop size.

#### 5.22 *The initiation of rainfall*

The two main processes put forward to account for the initial formation of rain in convective clouds have already been described in Section 1. They can be regarded as competing to form the first precipitation element which we may regard as a particle

of mass greater than  $10\ \mu\text{g}$  (e.g., a drop of diameter  $0.27\ \text{mm}$ ). Once formed, a precipitation element will grow further by processes discussed in § 5.23 and, in so doing, will convert the growing into a decaying cloud.

The broad differences between the two initiation processes are that the Bergeron process requires the presence of a few ice particles among many supercooled water droplets, whereas the Bowen-Ludlam process requires a droplet population containing a few large members, together with a marked updraught and a sufficiently deep cloud. Most workers regard the giant sea-salt nuclei as the most likely source of these large droplets, accepting the implication that non-freezing rain is thus improbable in continental air masses. However, even in the absence of giant condensation nuclei, the initial droplet population formed at cloud base might conceivably alter in such a way that it contains many large droplets (of radius greater than  $30\ \mu$ ) by the time the air has risen a few hundred metres. Schumann (1940) has studied the effects of coalescence on droplet-size distributions, on the assumptions that collision frequency is independent of droplet size and that every collision results in coalescence. His calculations indicate that this process will create droplets sufficiently large to initiate the Bowen-Ludlam process only in a time considerably longer than 30 min, which may be taken as the lifetime of most droplets in cumuliform cloud. However, Schumann ignored the effects of turbulence in promoting collisions, effects which may be very important, as shown in a recent paper by East and Marshall (1954). For example, to increase the number of collisions between cloud droplets by factors from 5 to 10, the turbulent velocity spectrum must be such that the Fourier components of velocity in the frequency range 100-200 c/s have an amplitude of the order of 2 cm/sec. Although little is known of the spectrum of turbulence in clouds, these parameters do not seem unreasonable. It would thus seem possible that turbulence can play an important role in changing the initial droplet population in such a way that rain formation by the Bowen-Ludlam process becomes feasible, even in the absence of giant hygroscopic condensation nuclei.

It is often assumed that the presence of ice in showerclouds provides unambiguous evidence for the Bergeron process. However, the work of Bigg (1953b) has shown that the probability of a drop freezing under given conditions may be approximately proportional to its volume, so that a cloud containing large supercooled raindrops is very much more likely to contain ice than one which contains only droplets. This may account for Bowen's observations (discussed in § 2.2) that clouds seeded with large water droplets sometimes produced hail as well as rain. It is probably safe to assume that the Bowen-Ludlam process accounts for the initial release of any rain from convective clouds, whether or not they contain ice, whose tops do not extend above the  $-6^\circ\text{C}$  isotherm. This is the highest temperature at which ice crystals have been reported in cumuliform cloud (Coons, Jones and Gunn 1949; quoted by Bigg 1953a).

Several workers have studied the role played by ice in the initial release of precipitation, from measurements of the height at which the first radar echo from a thunderstorm appears. A radar echo from a cloud usually indicates the presence of elements larger than  $10\ \mu\text{g}$  in considerable numbers, so that its first appearance can be taken as a sign that the release of precipitation has begun. Workman and Reynolds (1949b) and the Thunderstorm Project (1949) found that the initial radar echo nearly always appears near the  $-10^\circ\text{C}$  isotherm; Battan (1953) on the other hand, concludes from an analysis of Thunderstorm Project data for Ohio that the initial echo frequently appears below the freezing level. Wexler (1953) has criticized Battan's work on the grounds that, under certain conditions, echoes from ice may be too weak to detect. Even if Battan's analysis is rejected, it is difficult to accept the conclusion that initial echoes near the  $-10^\circ\text{C}$  isotherm necessarily imply the presence of ice. For example, in a cloud of liquid-water

content  $1 \text{ g/m}^3$  and updraught  $3 \text{ m/sec}$ , droplets growing by coalescence would give an appreciable radar echo only after they had been carried some  $3 \text{ km}$  above cloud base, a level which might well lie near the  $-10^\circ\text{C}$  isotherm. It can be seen that observations of the initial radar echo alone cannot lead to unambiguous conclusions. They would be more valuable if correlated with simultaneous observations of the actual updraught (measured directly) and the liquid-water content.

### 5.23 The growth of precipitation elements

The general expression for the rate of growth of a precipitation element by both accretion and condensation is given by (cf. Section 3),

$$\frac{dm}{dt} = \underbrace{\pi r^2 v w E}_{\text{collisions}} + \underbrace{4\pi\Delta f C M p / (RT)}_{\text{condensation}}$$

where  $r$  = effective radius (cm) of element of mass  $m$  (g).

$v$  = terminal velocity in cm/sec.

$w$  = liquid-water content in  $\text{g/cm}^3$ .

$E$  = mean collection efficiency.

$\Delta$  = diffusion coefficient ( $\text{H}_2\text{O}$  in air) in  $\text{cm}^2 \text{ sec}^{-1}$ .

$f$  = Frössling's wind factor (cf. Section 3).

$p$  = vapour-pressure difference over the element ( $\text{dyne/cm}^2$ ).

$C$  = constant numerically equal to electrostatic capacity of the element (cf. Section 3) in cm.

Although this expression is generally difficult to integrate, a great simplification is usually possible in convective showerclouds, where the rate of growth by collision is far greater than that by condensation. For this condition to hold, it can easily be shown (putting  $\Delta = 0.2 \text{ cm}^2 \text{ sec}^{-1}$ ,  $M/RT = 8 \times 10^{-10} \text{ cm}^{-2} \text{ sec}^2$ ,  $p = 200 \text{ dyne/cm}^2$ ,  $E = 0.3$ ,  $v = 50 \text{ cm sec}^{-1}$ ) that the radius of the element must exceed a limit given approximately by

$$r > 10^{-8} w^{-1} \text{ cm}$$

Thus for a cloud of liquid-water content  $1 \text{ g/m}^3$ , the growth of a precipitation element of radius greater than  $100 \mu$  will be almost entirely due to accretion. We may thus reasonably assume that the element is nearly spherical when it is big enough to be detectable by radar. Under these conditions, the rate of increase in the radius of the element with respect to distance  $h$ , fallen relative to the ground, is given by

$$\frac{dr}{dh} = \frac{w E(r)}{4\rho (1 - W/v)}$$

where  $\rho$  is the density of the element assumed homogeneous,  $W$  = updraught in the cloud. In the case of a water drop ( $\rho = 1 \text{ g/cm}^3$ ) this expression can be integrated fairly easily, as has been done by Bowen (1950) and Ludlam (1951a) (cf. Section 1). When dealing with ice elements, there is uncertainty in the value of  $\rho$ , which may also vary from the centre to the outside of the element. For example, when the rate of accretion is small ( $w$  and  $r$  small) the accreted water freezes immediately to form a soft rime (graupel) with many air inclusions, so that the density of the element may fall as low as  $0.3 \text{ g/cm}^3$ . As

the rate of accretion rises, the element forms a pellet of opaque hail, with somewhat increased density. Ludlam (1952) has shown that for high rates of accretion (large  $w$ , large  $r$ ), the latent heat released by the freezing of the accreted water may be sufficient to raise the surface of the hailstone to  $0^{\circ}\text{C}$ . The water will now freeze only slowly, producing clear ice. Ludlam has suggested that the onion-like structure of some hailstones – alternate layers of clear and opaque ice – has been produced by their passage through regions of cloud with alternately high and low liquid-water content, clear ice being formed where the liquid-water content is so high that the accreted water does not freeze instantaneously, and opaque ice where the rate of accretion is low. Coste (1940) has pointed out, however, that opaque rime does not necessarily imply the presence of air inclusions. Ice may also be rendered opaque by dissolved gases coming out of solution on freezing (Leduc 1906).

Ludlam also considers that when the rate of accretion is so high that the hailstone acquires a coating of water, the water will blow off into the wake before freezing, and so provide further precipitation elements. The thickness and diameter of the water layer necessary for this to happen are not known (though they might be found out by wind-tunnel experiments), so that the suggestion must be regarded as speculative.

Langmuir (1948) pointed out that if a drop growing by coalescence reaches a diameter of 5 mm it will disrupt into a few large and many small fragments (§ 3.14). The large fragments would continue falling as rain, but the small fragments could be carried to the top of the cloud in an updraught and so recommence the process, which would thus become a self-propagating chain-reaction. The importance of this process has probably been overestimated; the conditions necessary for it to happen are so critical that they will be realized very seldom. Browne 1952a (see also Wormell 1953) has found at the core of large thunderclouds a typical radar echo with amplitude increasing exponentially with decreasing height, suggesting the occurrence of a similar type of chain-reaction which is, however, not self-propagating, but sustained by a supply of small raindrops from above. He has shown that, in order to fit both the observed rainfall rate and the absolute echo intensity, it is necessary to assume that the drops break when their diameter exceeds 3 mm. As shown in Section 3 this would require the presence of frequent gusts of the order of 10 m/sec in the cloud, a requirement which has only rarely been met (§ 4.22). However, this type of radar echo could also be explained by a straightforward coalescence process in the presence of an updraught of velocity greater than 1 m/sec.

## 6. USE OF RADAR IN CLOUD STUDIES

Although radar observations may be made to yield various types of information relevant to the physics of clouds, the applications of this technique have been hitherto largely qualitative. Radar has been used for ‘watching’ clouds rather than for probing them. Since the literature on the subject is contained largely in non-meteorological journals a brief review is timely. The pictorial uses of radar are well known and will not be discussed here (see, e.g., Jones 1950). In many ways radar has advantages over aircraft for the experimenter – the initial cost is less, running costs are negligible, the reliability is greater and, lastly, no disturbance is caused to a cloud when a radar beam is directed into it. It can easily be shown that the heating due to absorption of the incident energy is negligible. The disadvantages of radar are that it cannot, unless airborne, cover a large area in a search for clouds, and that it can be used to investigate only a few parameters of the cloud, namely, its motion and extent, the presence or absence of ice, the mean square mass of water or ice present per unit volume, the shape of the particles and their velocity spectrum. This last quantity can lead to a direct measure of turbulence and updraught.

6.1 *The physics of scattering by meteorological particles*

The theory of the scattering of radio waves by meteorological particles, of diameter less than one-tenth of the wavelength, predicts that the echo intensity (power received) is proportional to the quantity  $\Sigma N_m m^2$ , where  $N_m$  is the number of particles of mass  $m$  per unit volume of cloud (Ryde 1946). The full equation expressing this may be written

$$I = \frac{C' \Sigma [N_m m^2 s_m(\kappa) f_m(\kappa)] \exp \left( - \int_0^R \gamma(R) dR \right)}{\lambda^4 R^2}$$

where  $I$  = power received,

$C'$  = a constant involving several parameters of the radar set,

$\lambda$  = the radar wavelength,

and  $R$  = the range from which the echo is received,

$\kappa$  = is the dielectric constant of the scattering particles,

$s(\kappa)$  = a shape factor, equal to unity for spherical particles,

$f(\kappa)$  = a scattering factor, approximately equal to  $(\kappa - 1)^2/(\kappa + 2)^2$ ,

$m$  = is the mass of a particle,

$N_m$  = the number of particles, of mass  $m$ , per unit volume,

and  $\gamma(R)$  = the 'two-way' absorption coefficient.

This assumes that the cloud completely fills the radar beam. If it does not, as may happen at ranges greater than 10 km, the exponent of  $R$  is changed from 2 to 4.

For wavelengths between 1 and 10 cm, the values of  $f(\kappa)$  for water and ice are respectively 0.92 and 0.17, so that, other things things being equal, a water cloud returns an echo five times more intense than that returned by a similar ice cloud.

Marshall, Langille and Palmer (1947) were the first to investigate Ryde's equation experimentally and were able to confirm the proportionality between  $I$  and  $\Sigma N_m m^2$  for precipitation rates up to 10 mm/hr. Hooper and Kippax (1950b) verified the equation absolutely using radars operating simultaneously on wavelengths of 1.3, 3 and 9 cm, during steady rain, giving precipitation rates up to 2 mm/hr. Their work was extended by Browne (1952a) to showers giving precipitation rates up to 6 mm/hr and to snow. In all these measurements for rain and for snow, the predicted and measured values of echo intensity agreed to within the limits of experimental error, about  $\pm 30$  per cent. Recently, attention has been turned to scattering by non-spherical particles. Labrum (1952a) and independently Browne and Robinson (1952) have shown theoretically that non-spherical particles depolarise incident plane-polarised radiation, and that the shape factor  $s(\kappa)$  is approximately unity for all dry ice particles but may vary between 0.1 and 10 for melting ice particles or spheroids of water. These predictions were confirmed by Labrum (1952b) in a remarkable series of measurements on individual particles suspended in a waveguide, and by Browne and Robinson (1952) and Hunter (1954), who measured the cross-polarized component in the echo from clouds containing non-spherical particles. Scattering by inhomogeneous particles has been investigated by Aden and Kerker (1951) and by Labrum (1952a, b) who have shown that when only one-third of a sphere of ice has melted, the scattering cross-section has already risen to 0.91 of that obtaining when the sphere has wholly melted.

Attenuation of radio waves is produced by gases, water vapour, liquid water and ice and decreases rapidly with increasing wavelength. At wavelengths greater than 1.5 cm, only liquid water produces appreciable attenuation, but at shorter wavelengths water vapour and oxygen become important. Goldstein (1949) gives tables of the attenuation coefficient  $\gamma$  found under various conditions. Another form of attenuation, often more important than that due to the atmospheric constituents, arises in the thin film of rainwater or layer of snow which may be deposited on the radar scanner or perspex 'radome.' For example, with a rainfall rate of about 20 mm/hr, the film of water on a radome reduced the intensity of echo by 17 db (or 98 per cent) at a wavelength of 1.3 cm (Hooper 1949).

It can be seen that the theory of scattering and attenuation of radio waves, by cloud particles of any size, shape and composition, rests on a solid basis of experimental fact supported firmly by theory. This being so, it is remarkable that so little attempt has been made to exploit radar for a quantitative study of clouds. Some of the difficulties in the path of the experimenter are briefly surveyed in the remainder of this section.

It is impossible to use brightness-modulated displays for making accurate intensity measurements, for not only is the brightness of the area representing the echo a complicated function of echo intensity, but only a small range of intensities can be represented at any instant, the reason for this being that unless strong signals are 'limited,' they cause defocusing of the electron beam. Deflection-modulated displays permit accurate measurement of echo intensity at the cost of showing only a one-dimensional representation of the cloud at any instant. The ideal is to use both types of display simultaneously. Provided that great care is taken, in calibrating the radar, to determine the constant  $C'$  in Ryde's equation (see e.g., Hooper and Kippax 1950b), the quantity  $\Sigma N_m m^2$  may be determined to within 30 per cent. If this error seems excessive in comparison with that usually expected from physical measurements, it should be remembered that adjacent regions of a cloud may return echoes differing in intensity by factors of more than a million.

After measurement, the quantity  $\Sigma N_m m^2$  must be converted into some other more readily apprehended, such as drop mass  $m$  and concentration  $N_m$ . In an attempt to do this Browne (1952a) has measured the rainfall rate at the ground, this being equal to  $\Sigma N_m m v$ , where  $v$  is the terminal velocity of a drop of mass  $m$ , and from simultaneous radar measures of  $\Sigma N_m m^2$ , using a vertically directed beam, has found, by trial and error, a mean value of  $m$  at the cloud base (and hence a mean-cube value of drop diameter). From the variation of echo intensity with height, values of mean drop mass can then be deduced for all heights, given sufficiently steady precipitation. This method is open to criticism, in that the physical significance of mean particle mass at any height is obscure unless the particle-size distribution is also known. A surer approach would be to measure the drop-size distribution at the ground and then work upwards, but the size distribution may well vary considerably with height. In the next section a possible method of determining the drop-size distribution by radar will be discussed.

## 6.2 Doppler shifts in radar cloud echoes

Radio waves reflected from objects moving with a line-of-sight velocity  $v$  suffer a change in frequency, or Doppler shift, given by :

$$\delta f = 2v/\lambda$$

where  $\lambda$  is the radio wavelength. In a cloud a large number of reflecting objects are moving relative to one another besides possessing a mean line-of-sight velocity. The

mean velocity produces a frequency shift in the radar echo, while the relative motions produce fluctuations in the intensity of the echo. The fluctuations are conveniently described either by their power spectrum or by their auto-correlation function (see, e.g., Siegert (1943); Booker, Ratcliffe and Shinn 1950). Consider a radar with a vertically-directed beam of finite angular width. The cloud particles in the beam will possess a mean line-of-sight velocity by virtue of their fall under gravity and of any vertical draughts acting upon them. They will be moving relative to one another because they possess different terminal velocities, because of turbulence and because any horizontal motion due to wind will possess a small line-of-sight component which varies across the finite width of the beam; these effects will all contribute towards causing the echo to fluctuate in intensity. Finally, spurious fluctuations and frequency shifts in the echo will arise in the radar equipment itself – the formidable difficulties inherent in minimising these have been discussed elsewhere (Browne 1952a). When several factors are contributing simultaneously to the fluctuation in an echo, their effects may be separated by a simple theorem (Browne, to be published) which states that the observed temporal autocorrelation function of the echo is equal to the product of the autocorrelation functions of the fluctuations which would be produced by each factor acting separately. Thus, if echo fluctuations are being produced by differences in terminal velocities, by turbulence and by horizontal wind, the turbulence may be deduced from measurements of the fluctuations if both the velocity of the wind and the drop-size distributions are known.

Measurements of the fluctuations in radar echoes from clouds have been made by Hilst (1949) and by Browne (1952a). Hilst used a nearly horizontal beam and recorded the power spectrum of the fluctuations directly. He interpreted his results in terms of the variation of wind with height and of certain simplified drop-size distributions. Hilst's apparatus had the disadvantage that changes in the power spectrum occurring over periods less than about two minutes were not detected. Browne, on the other hand, used a vertical beam and measured the autocorrelation function of the echo by a straightforward but rather cumbersome method. He was able to show, under steady warm-frontal conditions, that the snow particles above the freezing level possessed a root-mean-square velocity relative to one another of 20 cm/sec, while the raindrops below the freezing level had a relative R.M.S. velocity of 120 cm/sec. Both these authors measured only relative motions among cloud particles. The measurement of the mean velocity of the particles presents greater difficulties but Barratt and Browne (1953) have described a simple if somewhat restricted method of overcoming them, and have reported a measurement of downdraught in a showercloud. The simultaneous measurements of the vertical velocities of drops and the total echo intensity from them can yield a determination of the drop-size distribution (Bartnoff and Atlas 1951).

### 6.3 *The melting band*

During the war, Canadian radar operators noticed that a layer near the freezing level in rainclouds returned an enhanced radar echo, to which they gave the name 'bright band.' Lately, this term has been replaced by 'melting band,' a convenient suggestion due to Bowen. The melting band provides many valuable clues to the cloud physicist (see Section 2) and a brief discussion of its origin is given here. Ryde (1946) gave the following explanation: suppose that snowflakes of unit reflectivity are falling towards the freezing level. As they melt, their reflectivity increases five times, but when melting is complete and the wet flakes have collapsed to form drops, their terminal velocity also increases by about five times, with, consequently, a five-fold decrease in particle concentration  $N_m$ . The echo intensity thus returns to its original value. Hooper and Kippax

(1950a) after an extensive series of measurements, found that Ryde's explanation was substantially correct, while Bowen (1951) has confirmed that the region of the cloud which returns the melting-band echo contains crystals of melting ice. Austin and Bemis (1950) and Browne (1952a) have modified Ryde's theory to include factors such as the aggregation of snowflakes near the melting region, vertical draughts, and variation of the radar pulse-length. A major modification to Ryde's theory has been necessitated by Labrum's recent work (1952a, b) on the shape factor.

Quantitatively, the intensities of the echoes from just above, within, and just below the melting region ( $I_i$ ,  $I_m$  and  $I_w$  respectively) can be written approximately :

$$I_m/I_i = s(\kappa_w) [f(\kappa_w)/f(\kappa_i)] (m_w/m_i)^2$$

$$I_m/I_w = s(\kappa_w) [(v_w - W)/(v_i - W)]$$

provided that the radar-pulse length is smaller than twice the depth of the melting region. The symbols have the meaning given in § 6.1 with the addition  $W$  = updraught velocity, while the suffixes  $i$  and  $w$  refer to the states of ice and liquid water respectively. A schematic diagram to explain how the various terms in the equations arise is shown in Fig. 10.

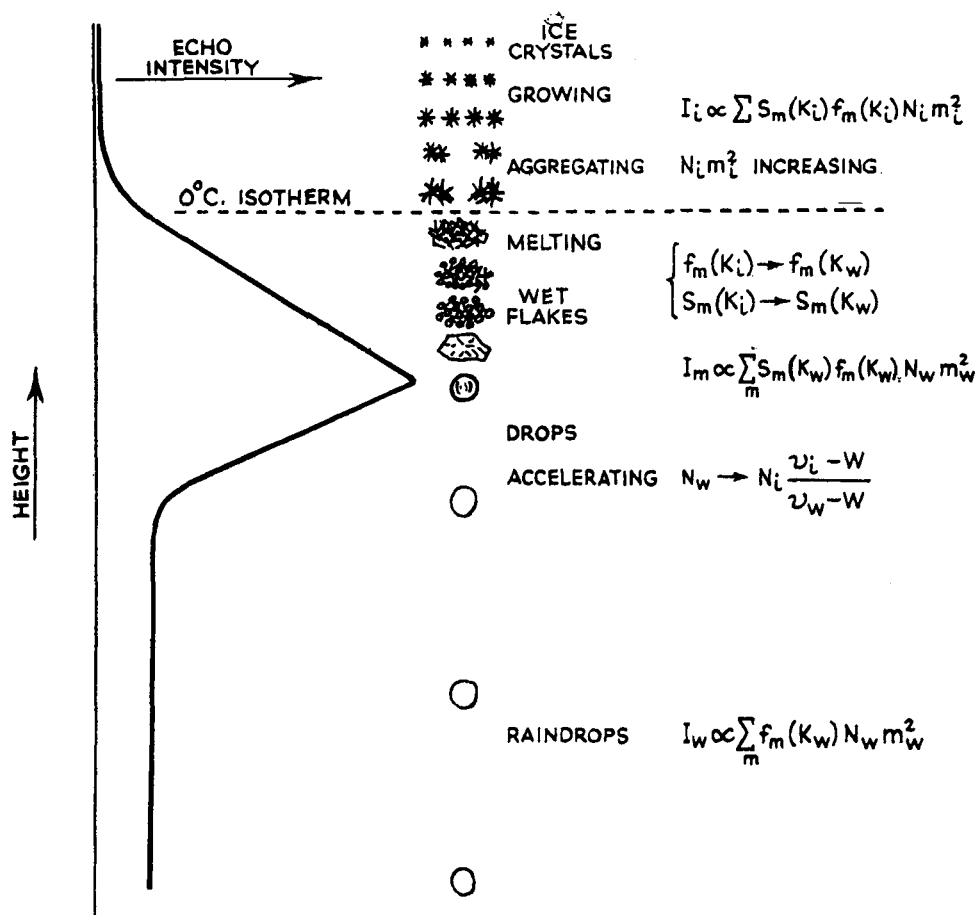


Figure 10. Schematic diagram to illustrate the formation of a 'melting band' in the radar echo from a raincloud. On the left, the radar-echo intensity after correction for inverse-square variation with range is plotted against height. In the centre are symbolised the particles giving rise to the radar echo at various heights, together with an indication of their behaviour and physical state. The formal relations on the right express the effect on the echo intensity of the various processes occurring in the cloud, and should be compared with the appropriate equations in the main text, where the symbols are also defined.



## 7. CONCLUSION

Cloud physics has reached a stage at which the various processes leading to rain are understood qualitatively. One of the principal gaps in our knowledge concerns the actual conditions to be found inside rain-clouds – liquid-water contents, drop, crystal and droplet-size distributions, updraughts, the spectrum of turbulence (cf. § 5.22) and, in layer clouds, relative humidities above the freezing level. Although satisfactory methods for measuring some of these quantities, particularly liquid-water contents and droplet-size distributions, have been developed, there has been a tendency to postpone carrying out systematic measurements until the apparatus has been brought to a state of unnecessary perfection. The many neglected potentialities of radar have already been discussed in this connection in the previous section.

The influence of condensation and freezing nuclei upon global and temporal variations of rainfall offers an interesting field of study. Much valuable information could come from observations such as those of Smith and Heffernan (§ 3.23), but a more rapid yield of knowledge might be gained by a study of the fate of the individual artificial nuclei used in cloud-seeding experiments. It should not prove difficult to seed suitable cumulus clouds at base level with finely ground common salt (NaCl) which had been converted into the radio-active form containing the isotope  $\text{Na}^{24}$  by irradiation in a nuclear pile. It would be necessary to carry out the irradiation a few hours before the seeding experiment because the highly beta-active sodium-24 has a half-life of only 15 hr. Preliminary calculations indicate that rain falling from clouds which had been seeded with salt prepared in this way would be sufficiently radio-active to give an easily measurable disintegration rate per gramme, a careful measurement of which would indicate whether the rain had actually been initiated by the seeding, or whether the salt particles had merely been collected by rain which had formed naturally. The possibilities of seeding with radio-active silver iodide do not appear to be favourable, as none of the many artificial isotopes of silver or iodine are sufficiently radio-active for the purpose.

In his review of cloud physics in this *Journal* in 1941, Sir George Simpson concluded by asking some stimulating questions, which subsequent research has gone far to answer. We try to follow his example with some further questions :

- (a) Is a steady-state process of rain formation possible, even in layer cloud ?
- (b) Do ice crystals in nimbostratus usually grow in a water-saturated environment ?
- (c) Do raindrops in the core of deep convective clouds grow as pellets of graupel, and are freezing nuclei concerned in the formation of the pellets ?
- (d) Are the giant droplets necessary to the Bowen-Ludlam process likely to form naturally in continental air masses far inland ?

## 8. ACKNOWLEDGMENTS

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## REFERENCES

- |  |      |   |
|--|------|---|
| Absalom, H. W. L.                      | 1954 | 'Artificial production of precipitation,' <i>Air Ministry, Met. Res. Cttee., M.R.P.</i> , No. 834.                  |
| Adderley, E. E.                        | 1953 | 'The growth of raindrops in cloud,' <i>Quart. J. R. Met. Soc.</i> , <b>79</b> , p. 380.                             |
| Aden, A. L. and Kerker, M.             | 1951 | 'The scattering of electromagnetic waves from two concentric spheres,' <i>J. Appl. Phys.</i> , <b>22</b> , p. 1242. |
| Aufm Kampe, H. J. and Weickmann, H. K. | 1951 | 'The effectiveness of natural and artificial aerosols as freezing nuclei,' <i>J. Met.</i> , <b>8</b> , p. 283.      |

- Austin, P. M. and Bemis, A. C. 1950 'A quantitative study of the bright band in radar precipitation echoes,' *J. Met.*, **7**, p. 145.
- Bannon, J. K. 1948 'The estimation of large-scale vertical currents from the rate of rainfall,' *Quart. J. R. Met. Soc.*, **74**, p. 57.
- Barratt, P. and Browne, I. C. 1953 'A new method for measuring vertical air currents,' *Quart. J. R. Met. Soc.*, **79**, p. 550.
- Bartnoff, S. and Atlas, D. 1951 'Microwave determination of particle-size distribution,' *J. Met.*, **8**, p. 130.
- Battan, L. J. 1953 'Formation and spread of precipitation in convective clouds,' *Ibid.*, **10**, p. 311.
- Bergeron, T. 1933 'On the physics of cloud and precipitation,' *Proc. Verb. Assoc. Mët. U.G.G.I.*, Lisbon, p. 156.
- Best, A. C. 1947 'The size distribution of raindrops,' *Air Ministry, Met. Res. Cttee., M.R.P.*, No. 352.
- 1950a 'Empirical formulae for the terminal velocity of water drops falling through the atmosphere,' *Quart. J. R. Met. Soc.*, **76**, p. 302.
- 1950b 'The size distribution of raindrops,' *Ibid.*, **76**, p. 16.
- Bigg, E. K. 1953a 'The formation of atmospheric ice crystals by the freezing of droplets,' *Ibid.*, **79**, p. 510.
- 1953b 'The supercooling of water,' *Proc. Phys. Soc. B*, **66**, p. 688.
- Bilham, E. G. and Relf, E. F. 1937 'The dynamics of large hailstones,' *Quart. J. R. Met. Soc.*, **63**, p. 149.
- Blanchard, D. C. 1948 'Observations on the behaviour of water drops at terminal velocity in air,' *Project Cirrus, Occ. Rep., G.E.C.*, No. 7, Schenectady.
- Booker, H. G., Ratcliffe, J. A. and Shinn, D. H. 1950 'Diffraction from an irregular screen with application to ionospheric problems,' *Phil. Trans. Roy. Soc.*, **242**, p. 579.
- Bowen, E. G. 1950 'The formation of rain by coalescence,' *Austr. J. Sci. Res.*, **A3**, p. 193.
- 1951 'Radar observations of rain and their relation to mechanisms of rain formation,' *J. Atmos. Terr. Phys.*, **1**, p. 125.
- 1952 'A new method of stimulating convective cloud to produce rain and hail,' *Quart. J. R. Met. Soc.*, **78**, p. 37.
- 1953 'The influence of meteoric dust on rainfall,' *Austr. J. Phys.*, **6**, p. 498.
- Braham, R. R. 1952 'The water and energy budgets of a thunderstorm and their relation to thunderstorm development,' *J. Met.*, **9**, p. 227.
- Brewer, A. W. and Palmer, H. P. 1949 'Condensation processes at low temperatures and the production of new sublimation nuclei by the splintering of ice,' *Nature*, **164**, p. 312.
- 1951 'Freezing of supercooled water,' *Proc. Phys. Soc. B*, **64**, p. 765.
- Browne, I. C. 1952a Dissertation, Cambridge University.
- 1952b 'Precipitation streaks as a cause of radar upper bands,' *Quart. J. R. Met. Soc.*, **78**, p. 590.
- 1953 Contribution to a discussion, *Ibid.*, **79**, p. 442.
- Browne, I. C. and Robinson, N. P. 1952 'Cross polarisation of the radar melting band,' *Nature*, **170**, p. 1078.
- Browne, I. C., also Wexler, R. (*reply*) 1953 'The collection efficiency of ice crystals,' *Quart. J. R. Met. Soc.*, **79**, p. 549.
- Byers, H. 1949 'Condensation nuclei and precipitation,' *J. Met.*, **6**, p. 363.
- Byers, H. and Braham, R. R. (Eds.) 1949 *The Thunderstorm*, U.S. Weather Bureau, Washington, D.C.
- Cepelcha, Z. 1953 'Meteor photographs. II. The photographed breakage of meteor 131a,' *Bull. Ast. Inst. Czech.*, **4**, p. 113.
- Coste, J. H. 1940 'The trapping of air in the formation of rime,' *Quart. J. R. Met. Soc.*, **66**, p. 366.
- Cunningham, R. M. 1951 'Some observations of natural precipitation processes,' *Bull. Amer. Met. Soc.*, **32**, p. 334.
- Cwiling, B. M. 1947 'Sublimation in a Wilson chamber,' *Proc. Roy. Soc. A*, **190**, p. 137.
- Dady, G. 1947 'Contribution à l'étude des précipitations,' *Comptes Rendus, Paris*, **225**, p. 1349.
- Dessens, H. 1949 'The use of spiders' threads in the study of condensation nuclei,' *Quart. J. R. Met. Soc.*, **75**, p. 23.
- Diem, M. 1948 'Messungen der Grosse von Wolkenelementen, II,' *Met. Rund.*, **1**, p. 261.

- Dorsch, R. G. and Hacker, P. T. 1950 N.A.C.A. Tech. Notes, No. 2142.
- East, T. W. R. and Marshall, J. S. 1954 'Turbulence in clouds as a factor in precipitation,' *Quart. J. R. Met. Soc.*, **80**, p. 26.
- Eyraud, R. 1949 'Influence des courants verticaux sur les avions,' *J. Sci. Mété.*, **1**, p. 23.
- Findeisen, W. 1938 'Die kolloidmeteorologischen Vorgänge bei der Niederschlagsbildung,' *Met. Z.*, **55**, p. 121.
- 1939 'The problem of the formation of raindrops in clouds which are entirely composed of water,' *Ibid.*, **58**, p. 365.
- Findeisen, W. and Findeisen, E. 1943 'Untersuchungen über die Eissplitterbildung an Reifschichten,' *Ibid.*, **60**, p. 145.
- Findeisen, W. and Schulz, G. 1944 'Experimentelle Untersuchungen über die atmosphärische Eisteilchenbildung, I,' *Forsch. Erf. Reich. Wetterdienst.*, A, No. 27.
- Frith, R. 1951 'The size of cloud particles in stratocumulus cloud,' *Quart. J. R. Met. Soc.*, **77**, p. 441.
- Frössling, N. 1938 'Über die Verdunstung fallender Tropfen,' *Gerl. Beitr. Geophys.*, **52**, p. 170.
- Gabilly, A. 1949 'Sur le rôle que peuvent jouer les rafales dans la coalescence des gouttelettes d'un nuage,' *Ann. Géophys.*, **5**, p. 232.
- Gaviola, E. and Fuertes, F. A. 1947 'Hail formation, vertical currents and the icing of aircraft,' *J. Met.*, **4**, p. 117.
- Gillmer, T. and Nietsch, E. 1944 *Clouds, Weather and Flight*, New York (Van Nostrand), p. 117.
- Goldstein, H. 1949 'Absorption and scattering of microwaves by the atmosphere,' from *Radio Wave Propagation*, New York (Academic Press), p. 269.
- Gunn, K. L. S. and Hitschfeld, W. 1951 'A laboratory investigation of the coalescence between large and small water-drops,' *J. Met.*, **8**, p. 7.
- Gunn, R. and Kinzer, G. D. 1949 'The terminal velocity of raindrops,' *J. Met.*, **6**, p. 243.
- Heverley, J. R. 1949 'Supercooling and crystallisation,' *Trans. Amer. Geophys. Uni.*, **30**, p. 205.
- Hewson, E. and Longley, R. 1944 *Meteorology, Theoretical and Applied*, New York (Wylie), p. 358.
- Hilst, G. R. 1949 *Weather Radar Res. Tech. Rep.*, No. 9, A, M.I.T.
- Hochschwender, E. 1919 Dissertation, Heidelberg University, transl. by W. R. Lane, Min. of Supply, C.D.R.E., Porton, 1949.
- Hooper, J. E. N. 1949 'Measurements of radar echo intensities from precipitation,' *Min. Supply, Rep.*, No. T2116, T.R.E. Malvern.
- Hooper, J. E. N. and Kippax, A. A. 1950a 'The bright band - a phenomenon associated with radar echoes from falling rain,' *Quart. J. R. Met. Soc.*, **76**, p. 125.
- 1950b 'Radar echoes from meteorological precipitation,' *Proc. I.E.E.*, London, **97**, Pt. 1, p. 89.
- Houghton, H. G. 1938 'Problems connected with the condensation and precipitation processes in the atmosphere,' *Bull. Amer. Met. Soc.*, **19**, p. 152.
- 1950 'A preliminary quantitative analysis of precipitation mechanisms,' *J. Met.*, **7**, p. 363.
- Hunter, I. M. 1954 'Polarisation of radar echoes from meteorological precipitation,' *Nature*, **173**, p. 165.
- Jeffreys, H. 1918 'Some problems of evaporation,' *Phil. Mag.*, **35**, p. 270.
- Jones, R. F. 1949 Contribution to a discussion on 'Turbulence in clear air and in cloud,' *Met. Mag.*, **78**, p. 114.
- 1950 'Radar weather echoes,' *Ibid.*, **79**, pp. 109, 143, 170, 198.
- 1951 'Rain from non-freezing clouds,' *Ibid.*, **80**, p. 273.
- 1954 'Five flights through a thunderstorm belt,' *Quart. J. R. Met. Soc.*, **80**, p. 377.
- Kaiser, T. R. 1953 'Radio-echo studies of meteor ionisation,' *Adv. in Phys.*, London, **2**, p. 495.
- Kinzer, G. D. and Gunn, R. 1951 'The evaporation, temperature and thermal relaxation-time of freely falling water drops,' *J. Met.*, **8**, p. 71.
- Kline, D. B. and Walker, J. A. 1951 N.A.C.A. Tech. Notes, No. 2306.
- Kotsch, W. J. 1947 'An example of colloidal instability in clouds in tropical latitudes,' *Bull. Amer. Met. Soc.*, **28**, p. 87.
- Krastanow, L. 1941 'Beitrag zur Theorie der Tropfen- und Kristallbildung in der Atmosphäre,' *Met. Z.*, **58**, p. 37.

- Kraus, E. B. and Squires, P. 1947 'Experiments on the stimulation of clouds to produce rain,' *Nature*, **159**, p. 489.
- Kuettner, J. 1950 'Electrical and meteorological conditions inside thunderclouds,' *J. Met.*, **7**, p. 322.
- Labrum, N. R. 1952a 'The scattering of radio waves by meteorological particles,' *J. Appl. Phys.*, **23**, p. 1324.
- 1952b 'Some experiments on centimetre wavelength scattering by small obstacles,' *Ibid.*, **23**, p. 1320.
- Langleben, M. P. 1954 'The terminal velocity of snowflakes,' *Quart. J. R. Met. Soc.*, **80**, p. 174.
- Langmuir, I. 1948 'The production of rain by a chain-reaction in cumulus clouds at temperatures above freezing,' *J. Met.*, **5**, p. 175.
- Leduc, A. 1906 'Sur la densité de la glace,' *Comptes Rendus*, Paris, **142**, p. 149.
- Lenard, P. 1904 'Über Regen,' *Met. Z.*, **21**, p. 249.
- Ludlam, F. H. 1951a 'The production of showers by the coalescence of cloud droplets,' *Quart. J. R. Met. Soc.*, **77**, p. 402.
- 1951b 'The heat economy of a rimed cylinder,' *Ibid.*, **77**, p. 402.
- 1952 'The production of showers by the growth of ice particles,' *Ibid.*, **78**, p. 543.
- Ludlam, F. H.,  
Browne, I. C. and Day, G. 1954 'Observations of small shower clouds,' (in publication).
- Malkus, J. S. 1953 'Some results of a trade cumulus cloud investigation,' *Woods Hole Oceanogr. Inst. Tech. Rep.*, No. 23 (unpublished M.S.).
- Malkus, J. S., Scorer, R. S.  
and Ludlam F. H. (reply) 1953 'Bubble theory of penetrative convection,' *Quart. J. R. Met. Soc.*, **79**, p. 288.
- Marshall, J. S. 1953 'Precipitation trajectories and patterns,' *J. Met.*, **10**, p. 25.
- Marshall, J. S., Langille, R. C.  
and Palmer, W. McK. 1947 'Measurement of rainfall by radar,' *J. Met.*, **4**, p. 186.
- Mason, B. J. 1953 'The growth of ice crystals in a supercooled water cloud,' *Quart. J. R. Met. Soc.*, **79**, p. 104.
- Mason, B. J. and Howorth, B. P. 1952 'Some characteristics of stratiform clouds over Northern Ireland in relation to their precipitation,' *Ibid.*, **78**, p. 226.
- Mason, B. J. and Ludlam, F. H. 1950 'The microphysics of clouds,' *Rep. Progr. Phys., Phys. Soc.*, **14**, p. 147.
- McDonald, J. E. 1953 'Erroneous cloud physics application of Raoult's Law,' *J. Met.*, **10**, p. 68.
- Moore, D. J. 1952 'Measurements of condensation nuclei over the North Atlantic,' *Quart. J. R. Met. Soc.*, **78**, p. 596.
- Mordy, W. A. and Eber, L. E. 1954 'Observations of rainfall from warm clouds,' *Ibid.*, **80**, p. 48.
- Nakaya, U. 1951 'The formation of ice crystals,' *Compendium of Meteorology*, Amer. Met. Soc., p. 207.
- Palmer, H. P. 1949 'Natural ice particle nuclei,' *Quart. J. R. Met. Soc.*, **75**, p. 17.
- 1950 Contribution to a discussion, *Cent. Proc. R. Met. Soc.*, p. 63.
- Peppler, W. 1940 'Unterkühlte Wasserwolken und Eiswolken,' *Forsch. Erf. Reichs. Wetter.*, B, No. 1.
- Prohaska, K. 1900 'Die jährliche und tägliche Periode der Gewitter und Hagel-fälle in Steiermark und Kaernten,' *Met. Z.*, **17**, p. 327.
- Rayleigh, Lord 1879 'The influence of electricity on colliding water drops,' *Collected Papers*, **1**, p. 372.
- 1882 'Further observations upon liquid jets,' *Ibid.*, **2**, p. 103.
- Reynolds, S. E. 1952 'Ice crystal growth,' *J. Met.*, **9**, p. 36.
- Ryde, J. W. 1946 'The attenuation and radar echoes produced at centimetre wavelength by various meteorological phenomena,' *Meteorological Factors in Radio-Wave Propagation*, Phys. Soc. and R. Met. Soc., p. 169.
- Schaefer, V. J. 1951 'Snow and experimental meteorology,' *Compendium of Meteorology*, Amer. Met. Soc., p. 221.
- Schumann, T. E. W. 1940 'Theoretical aspects of the size distribution of fog particles,' *Quart. J. R. Met. Soc.*, **66**, p. 195.
- Scorer, R. S. and Ludlam, F. H. 1953 'Convection in the atmosphere,' *Quart. J. R. Met. Soc.*, **79**, p. 317.
- Siebert, A. J. F. 1943 'The fluctuations in signal from many independently moving scatterers,' *Rad. Lab. Rep.*, M.I.T., No. 465.
- Sil, J. M. 1950 'Relationship between vertical currents and intensity of precipitation,' *Indian J. Met., Geophys.*, **1**, p. 52.

- Simpson, G. C. 1941 'On the formation of cloud and rain,' *Quart. J. R. Met. Soc.*, **67**, p. 99.
- Smith, E. J. 1951a 'Observations of precipitation with an airborne radar,' *Austr. J. Sci. Res. A*, **3**, p. 214.
- 1951b 'Observations of rain from non-freezing clouds,' *Quart. J. R. Met. Soc.*, **77**, p. 33.
- Smith, E. J. and Heffernan, K. J. 1954 'Airborne measurements of the concentration of natural and artificial freezing nuclei,' *Ibid.*, **80**, p. 182.
- Smith, L. G. 1951 Contribution to discussion, *Ibid.*, **77**, p. 683.
- Squires, P. and Gillespie, C. A. 1952 'A cloud droplet sampler for use on aircraft,' *Ibid.*, **78**, p. 387.
- Stickley, A. R. 1940 'An evaluation of the Bergeron-Findeisen precipitation theory,' *Month. Weath. Rev.*, **68**, p. 272.
- Stommel, H. 1947 'Entrainment of air into a cumulus cloud,' *J. Met.*, **4**, p. 491.
- Thunderstorm, The 1949 See Byers and Braham 1949.
- Virgo, S. 1950 'Tropical rainfall from cloud which did not extend to the freezing level,' *Met. Mag.*, **79**, p. 237.
- Warner, J. and Newnham, T. D. 1952 'A new method of measurement of cloud water content,' *Quart. J. R. Met. Soc.*, **78**, p. 46.
- Wegener, A. 1910 'Über die Eisphase der Wasserdampfs in der Atmosphäre,' *Met. Z.*, **27**, p. 453.
- 1920 'Frost Übersättigung und Cirren,' *Ibid.*, **37**, p. 8.
- Weickmann, H. K. 1945 'Formen und Bildung der Atmosphärischen Eiskristallen,' *Beitr. Phys. frei. Atmos.*, **28**, p. 12.
- 1953 'Observational data on the formation of precipitation in cumulonimbus cloud,' *Thunderstorm Electricity*, Univ. of Chicago Press, p. 67.
- Weickmann, H. K. and 1953 'Physical properties of cumulus clouds,' *J. Met.*, **10**, p. 204.
- Aufm Kampe, H. J. 1952 'Precipitation growth in stratiform clouds,' *Quart. J. R. Met. Soc.*, **78**, p. 363.
- Wexler, R. 1953 'Radar echoes from a growing thunderstorm,' *J. Met.*, **10**, p. 285.
- Wigand, A. 1919 'Die vertikale Verteilung der Kondensationskerne in der freien Atmosphäre,' *Ann. Phys.*, **59**, p. 689.
- Woodcock, A. H. 1950 'Sea salt in a tropical storm,' *J. Met.*, **7**, p. 397.
- Workman, E. J. and Reynolds, S. E. 1949a *New Mexico Sch. Mines Prog. Rep.*, No. 6.
- 1949b 'Electrical activity related to thunderstorm cell growth,' *Bull. Amer. Met. Soc.*, **30**, p. 142.
- Wormell, T. W. 1953 'Atmospheric Electricity: Some recent trends and problems,' *Quart. J. R. Met. Soc.*, **79**, p. 3.
- World Meteorological Organization 1954 'Artificial inducement of precipitation with special reference to the arid and semi-arid regions of the world,' *Tech. Note*, No. 1.
- Wright, H. L. 1939 'Atmospheric opacity: a study of visibility observations in the British Isles,' *Quart. J. R. Met. Soc.*, **65**, p. 411.

## APPENDIX

The desire to carry out a quick order-of-magnitude calculation is often frustrated by a lack of readily-found constants. Some rough constants are appended here, in the hope that they may be of use to those who, like the authors, are not so familiar with cloud physics as to have all the data at their finger-tips. The sources of most of the values are given in this review; the rest are standard.

## 1. Terminal velocities (cm/sec)

Ice crystals (diameter $100\ \mu - 1,000\ \mu$ )	70
Small snowflakes	100
Large snowflakes	150

Raindrops and droplets of diameter  $D$ 

$D$	$10\ \mu$	$50\ \mu$	$100\ \mu$	$500\ \mu$	1 mm	3 mm	5 mm
$v$ (cm/sec)	0.3	6	30	200	400	800	900

2. Collection efficiencies  $E(D)$

			For homogeneous clouds containing droplets of diameter (a) 12 $\mu$ , (b) 20 $\mu$					
$D$			50 $\mu$	100 $\mu$	300 $\mu$	1 mm	3 mm	5 mm
(a)	12 $\mu$	$E(D)$	·05	·25	·5	·7	·7	·6
(b)	20 $\mu$	$E(D)$	·4	·6	·7	·9	·9	·8

3. Vapour pressure difference over ice in a water-saturated atmosphere

The heating of the ice by latent heat has been taken into account						
$T$ ( $^{\circ}\text{C}$ )	0	— 5	— 10	— 15	— 20	— 25
$p$ (dyne/cm <sup>2</sup> )	0	130	160	180	170	150

4. Various constants

- Density of air (N.T.P.) =  $1.3 \times 10^{-3}$  g cm<sup>-3</sup>
- Viscosity of air (N.T.P.) =  $1.8 \times 10^{-4}$  g cm<sup>-1</sup> sec<sup>-1</sup>
- Kinematic viscosity of air (N.T.P.) =  $0.14$  cm<sup>2</sup> sec<sup>-1</sup>
- Diffusion coefficient of water vapour in air (N.T.P.) =  $0.2$  cm<sup>2</sup> sec<sup>-1</sup>  $\propto$  ( $^{\circ}\text{K}$ )<sup>3</sup>
- $M/RT$  (water vapour) =  $8 \times 10^{-10}$  sec<sup>2</sup> cm<sup>-1</sup> (see Section 3)

5. Typical parameters for rainclouds

Height and temperature of cloud base assumed to be 1½ km and 5°C respectively				
	Height of cloud top (km)	Updraught velocity (m/sec)	Liquid-water content (g/m <sup>3</sup> )	Precipitation rate (mm/hr)
Layer cloud Ns	5	0.1	0.1	2
Small shower cloud (Cu congestus) (Cb)	4	2	0.5	10
Thundercloud Cb	8	10	2	80