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A geodesic constant method for computing high-frequency mutual coupling between antennas on general quadric cylinders

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Contents: A Geodesic Constant Method (GCM) is outlined which provides a common approach to ray tracing on quadric cylinders in general, and yields all the surface ray-geometric parameters required in the UTD mutual coupling analysis of conformal antenna arrays in the closed form. The approach permits the incorporation of a shaping parameter which permits the modeling of quadric cylindrical surfaces of desired sharpness/flatness with a common set of equations. The mutual admittance between the slots on a general parabolic cylinder is obtained as an illustration of the applicability of the GCM.

Eine geodätische Konstanten-Methode zur Berechnung hochfrequenter wechselseitiger Kopplung zwischen Antennen auf allgemeinen Flächen zweiter Ordnung

Übersicht: Dargelegt wird eine geodätische Konstanten-Methode (GKM), die einen gemeinsamen Zugang zur Strahlverfolgung auf allgemeinen Zylindern zweiter Ordnung bietet. Sie liefert sämtliche strahlengeometrischen oberflächenbezogenen Parameter, die in der Uniformen Geometrischen Beugungstheorie (UTD) für die wechselseitige Kopplungsanalyse konformer Antennenarrays benötigt werden, in geschlossener Form. Der Zugang gestattet die Einbeziehung eines Formparameters, der die Modellierung von Zylinderflächen zweiter Ordnung mit gewünschter Spitzigkeit/Flachheit durch ein und dasselbe Gleichungssystem zuläßt. Die erhaltene wechselseitige Admittanz zwischen Schlitzen auf einem allgemeinen parabolischen Zylinder dient zur Veranschaulichung der Anwendbarkeit der GKM.

1 Introduction

Quadric cylinders are second degree surfaces where one of the principal curvatures is zero, leading to the developability of these surfaces. The family of quadric cylinders consists of the right circular, the elliptic, the general parabolic and the hyperbolic cylinders. These canonical surfaces are very useful in modeling many major components of aerospace bodies, such as fuselage [1] and wings [2].

It is often possible and desirable to locate conformal antenna arrays on aircraft wings and fuselage. Such distributed conformal arrays have a wide range of applications including those for enroute-flight navigation purposes, e.g., using the Global Positioning Systems (GPS) [3], airborne electronically scanning radars, electronic service module (ESM) systems, etc. The operating frequency of many of these systems is such that aircraft wings and bodies act as electrically large structures, i.e., structures with each linear dimension greater than a wavelength. This permits the antennas on aircraft bodies and wings to be treated in the high-frequency domain.

The analysis of such conformal antennas in the high-frequency domain can be readily efficiently carried out using the well-known Uniform Theory of Diffraction (UTD) [4]. The UTD in essence is a ray-theoretic formulation, where the electromagnetic surface-ray fields are expressed in terms of the surface ray-geometric parameters pertaining to a given surface. The UTD assumes that the surface is a perfectly conducting, convex, smooth and electrically large scatterer. Expressions for the ray-geometric parameters necessary for field evaluation through the UTD formulation are, in general, difficult to obtain for all but the simplest convex surfaces. In this paper, we outline the derivation of the surface-ray geometric parameters over quadric cylinders in general, in the closed form, to enable the use of the UTD formulations, for example, in the determination of the radiation characteristics of the arrays [5]. As an illustration, specific results on mutual coupling between element pairs on a general parabolic cylinder are provided in the paper.

2 Formulation

A general quadric cylinder is a regular surface [6] which may be expressed in a two-parameter form as

$$x = f(u); \quad y = g(u); \quad z = v; \tag{1}$$

where (u, v) are the geodesic coordinates and $f(u)$ and $g(u)$ are functions of u. In the particular case of the general parabolic cylinder, which is an important member of this class, the parametric equations reduce to

$$x = au; \quad y = u^2; \quad z = v. \tag{2}$$

In (2), a is a constant which acts as a "shaping parameter" determining the "sharpness" of the cylinder. The effect of the varying the value of a is illustrated in Fig. 1. Similar to the general parabolic cylinder, all the quadric cylinders

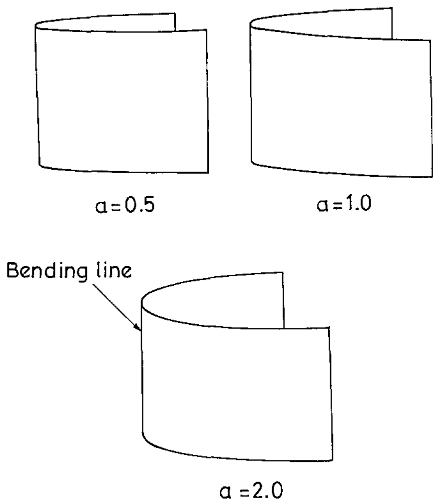

Fig. 1. The effect of the variation of the shaping parameter a on the shape of a general parabolic cylinder

can be generalized to incorporate the shaping parameter when their parametric equations are written in the Geodesic Coordinate System [7].

The parametric equations of a general quadric cylinder may be differentiated with respect to the geodesic coordinates u and v to obtain the First Fundamental Coefficients [6] and the unit surface normal vector. The dot product of these First Fundamental Coefficients and the unit surface normal vector in turn lead to the Second Fundamental Coefficients [6]. The First and Second Fundamental Coefficients essentially are intrinsic to the surface, and are extensively used in differential geometric analysis to derive other surface parameters like the principal curvatures of the surface.

The ray-theoretic approaches [4, 8] require the determination of the geodesics between the source and observation points on the surface. A geodesic is a path along which the surface-ray field traverses. It is possible to have geodesics between any given pair of points on quadric cylinders, both in the anti-clockwise (right geodesic) as well as the clockwise (left geodesics) sense [4]. Also, the quadric cylinders will in general admit multiply-encircling geodesics [4] which we have designated as the higher-order geodesics [5, 7]. The standard equation of the geodesic in terms of the Christoeffel symbols (tensor notation) [6] is a second order differential equation, requiring two integrations which are usually difficult to perform. To obviate this difficulty, we have used the Geodesic Coordinate System, in which all quadric cylinders are coordinate surfaces. The use of the special properties the Geodesic Coordinate System permits us to express the geodesics on quadric cylinders through a first-order differential equation. For the right m^{th}-order geodesic (i.e., the geodesic that completely encircles the cylinder $(m-1)$ times in the anti-clockwise sense), the equation is as follows

$$\frac{\mathrm{d}v(u)}{\mathrm{d}u} = \frac{\pm h_{rm}(f_u^2 + g_u^2)^{1/2}}{(1 - h_{rm}^2)^{1/2}}. \tag{3}$$

The subscripts u and v refer to the partial derivatives with respect to u and v respectively and h_{rm} denotes the First Geodesic Constant [6, 7], of the right m^{th}-order geodesic.

The surface-ray parameters required in the high frequency mutual coupling analysis can be conveniently classified as [7]:

1. *Surface-dependent ray geometric parameters* which depend on the geometric nature of the surface alone and are expressed as a function of the geodesic coordinates (u, v) of the point at which they are evaluated. The unit surface normal vector and the principal curvatures are examples of these.

2. *Geodesic-dependent ray geometric parameters* depend on the nature of the individual space curves traced on surfaces, in addition to the geometric nature of the surface itself, for example the Frenet-frame field vectors $(\hat{\mathbf{t}}, \hat{\mathbf{n}}, \hat{\mathbf{b}})$ [6] and the radius of curvature along the geodesic.

3. *Interaction-dependent ray geometric parameters* which can be described only if the geodesic coordinates of the source and observation points are given. The arc length, the generalized torsion factor [4] and Fock parameter [4], and the blending functions [4] are examples of interaction-dependent ray geometric parameters.

It is possible to derive all these surface-ray geometric parameters in the closed form using the definition given in [4]. For example, the unit surface normal vector, which is a surface-dependent ray geometric parameter, can be expressed for a general quadric cylinder as

$$\hat{\mathbf{N}} = \frac{g_u\hat{\mathbf{i}} - f_u\hat{\mathbf{j}} + 0\hat{\mathbf{k}}}{(f_u^2 + g_u^2)^{1/2}}, \tag{4}$$

where $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are the orthonormal basis along the three rectangular coordinate directions.

The Geodesic Coordinate System permits the characterization of the surfaces by only two orthogonal parameters along u and v. The curvatures along these directions are called the principal curvatures (k_u and k_v). We have obtained these curvatures as

$$k_u = \frac{-f_u g_{uu} + g_u f_{uu}}{(f_u^2 + g_u^2)^{3/2}}, \tag{5}$$

$$k_v = 0. \tag{6}$$

The principal curvatures k_u and k_v in turn are used to obtain the radius of curvature ϱ_g (along the geodesic), by Euler's Theorem. Furthermore, the same k_u and k_v will also yield the two blending functions which were heuristically defined by Pathak et al. [4] to overcome certain anomalies in the UTD computation on convex surfaces.

The closed form expression for the unit tangent vector along the forward direction of the geodesic, presented here as an example of a Geodesic-dependent ray geometric parameter, is expressed as

$$\hat{\mathbf{t}} = t_x\hat{\mathbf{i}} + t_y\hat{\mathbf{j}} + t_z\hat{\mathbf{k}}, \tag{7}$$

where

$$t_x = \frac{f_u(1-h_{rm}^2)^{1/2}}{(f_u^{\,2}+g_u^{\,2})^{1/2}}, \tag{7a}$$

$$t_y = \frac{g_u(1-h_{rm}^2)^{1/2}}{(f_u^{\,2}+g_u^{\,2})^{1/2}}, \tag{7b}$$

$$t_z = h_{rm}. \tag{7c}$$

Similarly, the derivative of the arc length $s(u)$ and the Fock parameter $\xi(u)$ with respect to u on a general quadric cylinder are expressed as

$$\frac{\mathrm{d}s}{\mathrm{d}u} = \frac{(f_u^{\,2}+g_u^{\,2})^{1/2}}{(1-h_{rm}^2)^{1/2}}, \tag{8}$$

$$\frac{\mathrm{d}\xi(u)}{\mathrm{d}u} = \frac{\pi^{1/3}(1-h_{rm}^2)^{1/6}\,(-f_u g_{uu}+g_u f_{uu})^{2/3}}{(f_u^{\,2}+g_u^{\,2})^{1/2}}. \tag{9}$$

The expression (9) follows from the definition of the generalized Fock parameter [4] and is related to the radius of curvature ϱ_g (along the geodesic). Equations (8) and (9) are examples of interaction-dependent parameters.

One of the highlights of the method outlined in this paper is that all these expressions are now a function of the First Geodesic Constant h_{rm} and their accuracy depends on the accuracy of h_{rm} alone. For this reason, we call this method the Geodesic Constant Method (GCM).

It may however be noted that serious difficulties exist in deriving many of the interaction-dependent parameters on a convex surface in general; these pertain to the integrations such as those encountered in Eqs. (3), (8) and (9). A major contribution of the work reported here is the success in obtaining these parameters analytically; for the specific case of the general parabolic cylinder, shown in Fig. 2, these are expressed in the closed form as

$$s(u) = \frac{1}{4(1-h^2)^{1/2}} \times \left[2u(a^2+4u^2)^{1/2} + a^2 \ln\left[2u+(a^2+4u^2)^{1/2}\right]\right]_{u=u_s}^{u=u_f}, \tag{10}$$

$$\xi(u) = \frac{(4\pi a^2)^{1/3}\,(1-h^2)^{1/6}}{2}\left[\ln\left[2u+(a^2+4u^2)^{1/2}\right]\right]_{u=u_s}^{u=u_f}. \tag{11}$$

The parameters $s(u)$ and $\xi(u)$ from (10) and (11) can now be utilized as in Appendix A to compute the generalized Fock integrals which is crucial to entire UTD ray-field computation.

It may be noted that the subscript "rm" of h_{rm} has been dropped in (10) and (11), since in the case of the open cylinders, such as the general parabolic and hyperbolic cylinders, there are no higher-order geodesics (or multiple-encirclements) of the cylinder.

To be able to use the parameters $s(u)$ and $\xi(u)$ defined by expressions (10) and (11) in the actual numerical computations of field strengths and/or mutual coupling coefficients, we need to use these parameters in an appropriate equation. Several options are available, starting with the basic GTD [1] or one of its derivatives such as the UTD [4]. We have used the surface magnetic field in terms of the surface rays given by Pathak et al. [4]. Furthermore, this equation requires the knowledge of the blending functions which can be readily derived using the definitions in Ref. [4].

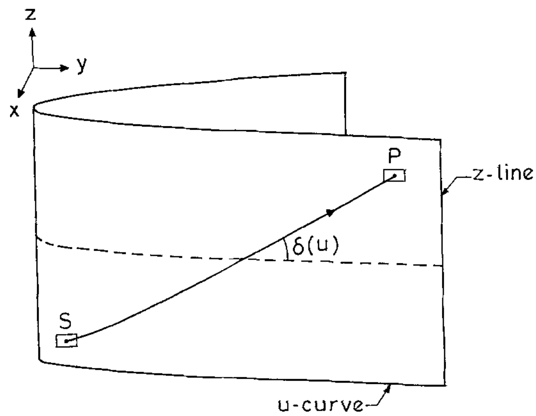

Fig. 2. Surface ray-path from the source point $S(u_s, v_s)$ to the observation point $P(u_f, v_f)$. The dashed line is the generator of the cylinder, and $\delta(u)$ is the angle that the geodesic makes with it

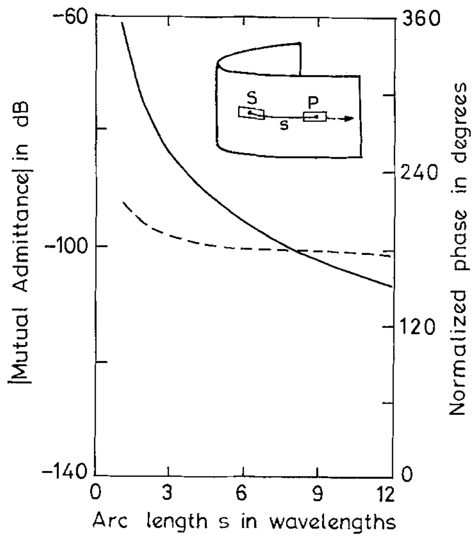

Fig. 3. Mutual admittance vs. arc length for separation along u-axis between two rectangular slots ($0.5\lambda \times 0.2\lambda$) on a general parabolic cylinder of shaping parameter $a = 0.25$. The z-axis separation between the slots is kept constant at 0.0λ. Magnitude ———, Phase - - - -

Mutual coupling between finite-dimensional ($0.5\lambda \times 0.2\lambda$) slots over general parabolic cylinders of varying shaping parameter has been computed and two specific results are presented here to illustrate the applicability of the mathematical model outlined in this paper. Figures 3 and 4 show the mutual admittance magnitude and phase plots for increasing separation between the two slots along the u-axis. The z-axis separation between the centroids of the slots is taken as zero and 0.5λ, in Figs. 3 and 4, respectively.

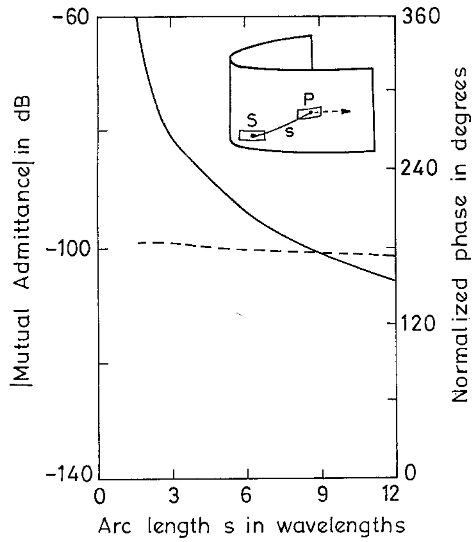

Fig. 4. Mutual admittance vs. arc length for separation along u-axis between two rectangular slots ($0.5\lambda \times 0.2\lambda$) on a general parabolic cylinder of shaping parameter $a = 0.25$. The z-axis separation between the slots is kept constant at 0.5λ. Magnitude ———, Phase - - - -

3 Discussion

The importance of a generalized analysis of quadric cylinders is emphasized by the fact that segments of such cylinders are extensively employed to describe many complex physical structures, such as those found on aerospace bodies. The fuselage [1, 9] and wings [2, 10] of aircraft are two such examples. The analysis presented here has been successfully employed by describing an aircraft wing as a hybrid quadric cylinder, i.e., a combination of a truncated general parabolic cylinder and a right (semi-) circular cylinder [2]. Wing cross-sections of various shapes, such as are useful in different flight regimes, may be approximated by manipulating the shaping parameters. It has been possible to compute the mutual coupling over such hybrid wings for the different flight regimes, using the analysis outlined in this paper [7, 10].

For the particular cases of the right circular cylinder and the general parabolic cylinder, the expressions obtained for the ray parameters are in the closed form, since the geodesic constant h is also derivable in the closed form. For the coordinate surfaces of the Elliptic-cylinder Coordinate System, such as the elliptic and general hyperbolic cylinders, h can be expressed explicitly in terms of the asymptotic series expansions of the incomplete elliptic integral functions, and hence the explicit nature of the ray parameters as a function of h as derived here is still valid. However, the determination of the value of h requires a simple univariate numerical procedure. Thus, the Geodesic Constant Method outlined in this paper is capable of analyzing the high-frequency mutual coupling between the antennas on the quadric cylinders in general.

Appendix A

1 Generalized Fock Parameter

The UTD requires the determination of a length-type parameter, called the generalized Fock parameter, defined as [4]

$$\xi(P, S) = \int_S^P \frac{m(s)}{\varrho_g(s)}\,ds = \int_{u_s}^{u_f} \frac{m}{\varrho_g}\left(\frac{ds}{du}\right) du, \tag{A.1}$$

where $S(u_s, v_s)$ and $P(u_f, v_f)$ are the source and observation points, and

$$m = \left[\frac{k\varrho_g}{2}\right]^{1/3}, \quad \text{in which} \quad k = 2\pi/\lambda. \tag{A.2}$$

$\xi(P, S)$ is used in defining the soft and hard Fock integrals, $U(\xi)$ and $V(\xi)$ respectively, which are complex contour integrals [4]

$$U(\xi) = \frac{\xi^{3/2}\, e^{j3\pi/4}}{\pi^{1/2}} \int_{\infty e^{-j2\pi/3}}^{\infty} \frac{W_2'(t)\, e^{-j\xi t}}{W_2(t)}\, dt, \tag{A.3}$$

$$V(\xi) = \frac{\xi^{1/2}\, e^{j\pi/4}}{2\pi^{1/2}} \int_{\infty e^{-j2\pi/3}}^{\infty} \frac{W_2(t)\, e^{-j\xi t}}{W_2'(t)}\, dt, \tag{A.4}$$

where $W_2(t)$ is the Fock-type Airy function defined as

$$W_2(t) = \frac{1}{\pi^{1/2}} \int_{\infty e^{-j2\pi/3}}^{\infty} \exp\left[tZ - \frac{1}{3} Z^3\right] dZ \tag{A.5}$$

and $W_2'(t)$ denotes its derivative with respect to t. $U(\xi)$ and $V(\xi)$, in turn are employed in obtaining the generalized Fock integrals $\tilde{U}(\xi)$ and $\tilde{V}(\xi)$ required in the UTD formulation [4]

$$\tilde{U}(\xi) = \left[\frac{ks}{2m(S)\, m(P)}\right]^{3/2} U(\xi), \tag{A.6}$$

$$\tilde{V}(\xi) = \left[\frac{ks}{2m(S)\, m(P)}\right]^{1/2} V(\xi). \tag{A.7}$$

2 UTD Mutual Coupling Formulation

For computing the mutual admittance, we obtain the surface magnetic field element $dH_m(P\,|\,S)$, at the observation point $P(u_f, v_f)$ due to the magnetic current moment $dp_m(S)$ because of the radiating element at the source point $S(u_s, v_s)$ as given in [4]

$$dH_m(P\,|\,S) = \frac{-jk}{4\pi}\, dp_m(S)$$

$$\times \left\{ 2Y_0 \left(\hat{\mathbf{b}}(S)\, \hat{\mathbf{b}}(P) \left[\left(1 - \frac{j}{ks}\right) \tilde{V}(\xi) \right.\right.\right.$$

$$+ D^2 \left(\frac{\mathrm{j}}{ks}\right)^2 (\Lambda_s \tilde{U}(\xi) + \Lambda_c \tilde{V}(\xi))$$

$$\left. + \tilde{T}_0{}^2 \frac{\mathrm{j}}{ks} (\tilde{U}(\xi) - \tilde{V}(\xi)) \right]$$

$$+ \hat{\mathbf{t}}(S)\, \hat{\mathbf{t}}(P) \left[D^2 \frac{\mathrm{j}}{ks} \tilde{V}(\xi) + \frac{\mathrm{j}}{ks} \tilde{U}(\xi) \right.$$

$$\left. - 2 \left(\frac{\mathrm{j}}{ks}\right)^2 (\Lambda_s \tilde{U}(\xi) + \Lambda_c \tilde{V}(\xi)) \right]$$

$$+ (\hat{\mathbf{t}}(S)\, \hat{\mathbf{b}}(P) + \hat{\mathbf{b}}(S)\, \hat{\mathbf{t}}(P))$$

$$\left. \times \left[\frac{\mathrm{j}}{ks} \tilde{T}_0 (\tilde{U}(\xi) - \tilde{V}(\xi)) \right] \right) \Bigg\} \cdot D \, \frac{\mathrm{e}^{-jks}}{s} \qquad \text{(A.8)}$$

where $\mathrm{j} = \sqrt{-1}$, and Y_0 is the characteristic admittance. The ray divergence factor D, the generalized torsion factor $\tilde{T}_0$ and the blending functions Λ_s, Λ_c [4] are examples of the Interaction-dependent ray geometric parameters described in Section 2. All the surface ray-geometric parameters appearing on the right hand side of Eq. (A.8) can be derived using the Geodesic Constant Method, described in this paper. The mutual admittance between two finite-dimensional slots is then obtained by performing a double-aperture area integration [4].

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