Information Sciences 180 (2010) 4207–4222

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Determining objective weights with intuitionistic fuzzy entropy measures: A comparative analysis

Ting-Yu Chen [a,*], Chia-Hang Li [b,1]

[a] Department of Industrial and Business Management, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan
[b] Graduate Institute of Business Administration, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan

ARTICLE INFO

Article history:
Received 9 January 2009
Received in revised form 10 October 2009
Accepted 17 July 2010

Keywords:
Intuitionistic fuzzy entropy
Multiple-attribute decision-making
Intuitionistic fuzzy sets
Objective weight
Computational experiment

ABSTRACT

In this paper, we propose a new objective weighting method that employs intuitionistic fuzzy (IF) entropy measures to solve multiple-attribute decision-making problems in the context of intuitionistic fuzzy sets. Instead of traditional fuzzy entropy, which uses the probabilistic discrimination of attributes to obtain attribute weights, we utilize the IF entropy to assess objective weights based on the credibility of the input data. We examine various measures for IF entropy with respect to hesitation degree, probability, non-probability, and geometry to calculate the attribute weights. A comparative analysis of different measures to generate attribute rankings is illustrated with both computational experiments as well as analyses of Pearson correlations, Spearman rank correlations, contradiction rates, inversion rates, and consistency rates. The experimental results indicate that ranking the outcomes of attributes not only depends on the type of IF entropy measures but is also affected by the number of attributes and the number of alternatives.

1. Introduction

In multiple-attribute decision-making (MADM) analysis, a decision maker (DM) must evaluate alternatives with respect to each attribute, address attribute weights, and select the best result from the generated set of alternatives. The MADM approach provides an effective way to select among non-commeasurable and conflicting attributes. Some representative methods, such as the simple additive weighting method (SAW), the analytic hierarchy process (AHP), and the technique for order preference by similarity to ideal solution (TOPSIS), have been developed to solve MADM problems [11,18,28]. Numerical data cannot be used to accurately model real-life circumstances because human judgments and preferences include fuzziness and vagueness. For this reason, research has extended the use of fuzzy set theory proposed by Zadeh [39] to MADM methods [7,9,20,37]. Using the notion of a conventional fuzzy set, Atanassov [2] introduced the concept of an intuitionistic fuzzy set (IFS), which is a generalization of the fuzzy set, to provide additional information about indeterminacy degrees. Since IFSs can adequately measure the decision-making process of human beings and cope with incomplete information, the merits of IFSs have been applied in various fields, including logical reasoning [19], pattern recognition [16], and decision-making [36]. In particular, Li [22] and Lin et al. [24] used a linear programming model to assess the optimal attribute weights in MADM. Liu and Wang [25] adopted the max–min, max-center, and max–max methods, which were introduced by Chen and Tan [8] and Hong and Choi [14], to yield weights within an intuitionistic fuzzy context. Xu [35] utilized similarity measures to solve MADM problems. Yu et al. [38] presented a fuzzy optimization method to manage MCDM problems in terms of the inclusion

* Corresponding author. Tel.: +886 3 2118800x5678; fax: +886 3 2118700.
E-mail addresses: tychen@mail.cgu.edu.tw (T.-Y. Chen), m9540314@stmail.cgu.edu.tw (C.-H. Li).
[1] Tel.: +886 3 2118800x5860; fax: +886 3 2118700.

0020-0255/$ - see front matter
doi:10.1016/j.ins.2010.07.009

degree of IFSs. Li et al. [23] proposed a fractional programming method based on the TOPSIS to handle multiple-attribute group decision-making problems using IFSs.

The proper assessment of attribute weights plays an essential role in the MADM process because the variation of weight values may result in different final rankings of alternatives [18]. In general, the weights in MADM can be classified as subjective weights and objective weights depending on the method of information acquisition [26]. Subjective weights are obtained from preference information on attributes given by the DM, who provides subjective intuition or judgments on specific attributes. Objective weights are derived from the information in a decision matrix through mathematical models. The well-known approaches for generating subjective weights include AHP [28] and the Delphi method [17]. In terms of determining objective weights, one of the most-representative approaches is the entropy method, which expresses the relative intensities of attribute importance to signify the average intrinsic information transmitted to the DM [28,41]. Most research pertaining to MADM analysis in an IF environment has been executed using subjective weights [1,6,10]. However, there is little research on the extension of objective weighting approaches in MADM to IFSs.

In this study, we propose a new objective weighting method that combines intuitionistic fuzzy (IF) entropy measures to solve MADM problems. The entropy of a fuzzy set is a measure of the fuzziness of a fuzzy set. Although this is called entropy due to the concept's intrinsic similarity to Shannon entropy, the two functions measure fundamentally different types of uncertainty [31]. In the field of information theory, Shannon entropy is a measure of uncertainty associated with a random variable and is derived from the probability theory. However, the IF entropy measures used in our objective weighting method are non-probabilistic. De Luca and Termini [12] introduced a non-probabilistic entropy for fuzzy sets and formulated the axiomatic requirements with which an entropy measure should comply. Fuzziness is a feature of imperfect information and results from the lack of a clear distinction between the elements that do and do not belong to a set. Kaufmann [21] measured the degree of fuzziness of a fuzzy set using the metric distance between its membership function and the membership function of its nearest set. Szmidt and Kacprzyk [31] extended De Luca and Termini's axioms and proposed an entropy measure for IFSs called the IF entropy measure. Zeng and Li [42] expressed the axioms of Szmidt and Kacprzyk using the notation of interval-valued fuzzy sets. In summary, the concept of IF entropy that we apply in this paper differs from Shannon entropy; rather than drawing on information theory as is the case with Shannon entropy, our analysis is supported and evidenced by previous work on IF entropy [5,15,31,33,42].

The traditional entropy method focuses on the discrimination among data to determine attribute weights. If an attribute can discriminate the data more effectively, it is given a higher weight. In contrast, this study emphasizes the credibility of data in establishing attribute weights through IF entropy measures. Although it differs from the traditional entropy method, our method is nevertheless related to concepts associated with the traditional entropy method. Several IF entropy measures, including those introduced by Burillo and Bustince [5], Szmidt and Kacprzyk [31], Zeng and Li [42], Hung and Yang [15], and Vlachos and Sergiadis [32] as distinct theorems, were employed and compared using our objective weighting method. A computational experiment with simulation data was also designed to observe the ranking outcomes of attributes by applying different IF entropy measures.

The rest of this paper is organized as follows. Section 2 presents a collection of IF entropy measures and explains the determination of the specific measures used in the formal experiment. Section 3 illustrates an algorithm for the proposed method through a numerical example. Section 4 uses a computational experiment to compare some of the IF entropy measures implemented in the objective weighting method. The paper concludes in Section 5.

2. Entropy measures for intuitionistic fuzzy sets

2.1. Intuitionistic fuzzy sets

Atanassov [2] introduced the notion of intuitionistic fuzzy sets. An IFS A in X is defined as $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$. The values of $\mu_A(x)$ and $\nu_A(x)$ are the degrees of membership and non-membership of $x \in X$ in A, respectively. In addition, for each $x \in X$, $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$.

For each IFS A in X, we call $\pi_A = 1 - \mu_A(x) - \nu_A(x)$ the intuitionistic index of x in A. This is the hesitancy degree of x to A. For every A, $B \in$ IFSs(X), the following operations are defined:

1. $A \leqslant B$ if and only if $\mu_A(x) \leqslant \mu_B(x)$ and $\nu_A(x) \geqslant \nu_B(x)$ for all x in X.
2. $A = B$ if and only if $A \leqslant B$ and $B \leqslant A$.
3. $A \cap B = \{x, \min(\mu_A(x), \mu_B(x)), \max(\mu_A(x), \mu_B(x)) | x \in X\}$.
4. $A \cup B = \{x, \max(\mu_A(x), \mu_B(x)), \min(\mu_A(x), \mu_B(x)) | x \in X\}$.

2.2. Interval-valued fuzzy sets

An interval-valued fuzzy set (IVFS) in X is an expression A given by $A = \{(x, M_A(x)) | x \in X\}$, where $M_A : X \to [0,1]$ defines the interval degree of membership of an element x in the set $A \in X$ [29]. The interval $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ has a lower bound $M_{AL}(x)$ and an upper bound $M_{AU}(x)$ in the set A. Burillo and Bustince [5] and Sambuc [29] defined the following expressions for all A, $B \in$ IVFSs(X):

1. $A \leqslant B$ if and only if $M_{AL}(x) \leqslant M_{BL}(x)$ and $M_{AU}(x) \leqslant M_{BU}(x)$ for all $x \in X$.
2. $A \preceq B$ if and only if $M_{AL}(x) \leqslant M_{BL}(x)$ and $M_{AU}(x) \geqslant M_{BU}(x)$ for all $x \in X$.
3. $A = B$ if and only if $M_{AL}(x) = M_{BL}(x)$ and $M_{AU}(x) = M_{BU}(x)$ for all $x \in X$.
4. $A^c = \{x, \overline{M}_A(x) | x \in X\}$, where A^c is the complement of A.

In fact, IFS and IVFS can be viewed as equivalent thanks to various studies that have researched their relationship. Atanassov and Gargov [4] used the map f assigned to every IVFS(X) A and IFS(X). $B = f(A)$ is given by $\mu_B(x) = M_{AL}(x)$ and $\nu_B(x) = 1 - M_{AU}(x)$. The map g is assigned to every IFS(X)B and IVFS(X). $A = g(B)$ is given by $M_A(x) = [\mu_B(x), 1 - \nu_B(x)]$. Therefore, an IVFS can be transformed into an IFS form through this method.

2.3. The entropy measure for intuitionistic fuzzy sets

The entropy of a fuzzy set describes the fuzziness degree of a fuzzy set, as first mentioned by Zadeh [39]. Traditional fuzzy entropy is derived from the concept of probability and measures the discrimination of attributes when applied in MADM. Nevertheless, the nature of IF entropy is distinct from traditional entropy because IF entropy explains data credibility. Since Atanassov [2] pioneered the basic concept of IFSs, researchers have proposed IF entropy measures from various perspectives, including hesitation degree, geometry, probability, and non-probability frameworks. We classify these measures into four groups based on the perspective from which they emerge.

2.3.1. Group 1: hesitation degree

Burillo and Bustince [5] defined the distance measure between IFSs by providing the following IF entropy measures:

$$E_{BB}^1(A) = \sum_{i=1}^{n}(1 - (\mu_A(x_i) + \nu_A(x_i))) = \sum_{i=1}^{n} \pi_A(x_i), \tag{1}$$

$$E_{BB}^2(A) = \sum_{i=1}^{n}(1 - (\mu_A(x_i) + \nu_A(x_i))^k), \quad k = 2, 3, \ldots, \infty, \tag{2}$$

$$E_{BB}^3(A) = \sum_{i=1}^{n} \left(1 - (\mu_A(x_i) + \nu_A(x_i)) \cdot e^{1-(\mu_A(x_i)+\nu_A(x_i))}\right), \tag{3}$$

$$E_{BB}^4(A) = \sum_{i=1}^{n} \left(1 - (\mu_A(x_i) + \nu_A(x_i)) \cdot \sin\left((\pi/2)(\mu_A(x_i) + \nu_A(x_i))\right)\right). \tag{4}$$

2.3.2. Group 2: geometry

Szmidt and Kacprzyk [31] proposed a new entropy method for IFSs. That provides a geometric interpretation of IFSs. The IF entropy is a ratio of distances between (F, F_{near}) and (F, F_{far}). We express it as follows:

$$E_{SK}(F) = \frac{(F, F_{near})}{(F, F_{far})}. \tag{5}$$

Note that (F, F_{near}) is the distance from F to the nearest point F_{near}, and (F, F_{far}) is the distance from F to the farthest point F_{far}. To calculate the entropy value using this formula, we must measure the distance between (F, F_{near}) and (F, F_{far}) in advance. Atanassov [3] suggested a direct generalization of distances for IFSs using classical set theory by replacing the characteristic functions of two sets with their membership functions. Nevertheless, Szmidt and Kacprzyk [30] indicated that Atanassov's distances are merely the orthogonal projections of the real distances and that they have limited geometrical interpretation. From a geometrical perspective, Szmidt and Kacprzyk accounted for the three-parameter characterization of IFSs in their development of new definitions of distances. Their approach ensures that the distances for fuzzy sets and IFSs can be easily compared because it reflects distances in a three-dimensional space. Two different distance measures introduced by Szmidt and Kacprzyk [30] are as follows. Let $X = \{x_1, x_2, \ldots, x_n\}$. The Hamming distance between the IFSs A and B belonging to IFS(X) is defined by

$$d_{SK}^1(A, B) = \sum_{i=1}^{n} \left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\right). \tag{6}$$

Szmidt and Kacprzyk also defined another Euclidean distance between the IFSs A and B belonging to IFS(X) as follows:

$$d_{SK}^2(A, B) = \sqrt{\sum_{i=1}^{n} \left[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2\right]}. \tag{7}$$

Wang and Xia [34] proposed two additional distance measures between the IFSs A and B belonging to IFS(X) as follows:

$$d^1_{WX}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|}{4}+\frac{\max(|\mu_A(x_i)-\mu_B(x_i)|,|\nu_A(x_i)-\nu_B(x_i)|)}{2}\right], \tag{8}$$

$$d^2_{WX}(A,B) = \frac{1}{\sqrt[l]{n}}\sqrt[l]{\sum_{i=1}^{n}\left(\frac{|\mu_A(x_i)-\mu_B(x_i)|}{2}+\frac{|\nu_A(x_i)-\nu_B(x_i)|}{2}\right)^l}. \tag{9}$$

We combine (6)–(9) with (5) and obtain the IF entropy measures (10)–(13), respectively, as follows:

$$E^1_{SK}(A) = \frac{\sum_{i=1}^{n}(|\mu_{near}(x_i)-\mu_A(x_i)|+|\nu_{near}(x_i)-\nu_A(x_i)|+|\pi_{near}(x_i)-\pi_A(x_i)|)}{\sum_{i=1}^{n}\left(|\mu_{far}(x_i)-\mu_A(x_i)|+|\nu_{far}(x_i)-\nu_A(x_i)|+|\pi_{far}(x_i)-\pi_A(x_i)|\right)}, \tag{10}$$

$$E^2_{SK}(A) = \frac{\sqrt{\sum_{i=1}^{n}\left[(\mu_{near}(x_i)-\mu_A(x_i))^2+(\nu_{near}(x_i)-\nu_A(x_i))^2+(\pi_{near}(x_i)-\pi_A(x_i))^2\right]}}{\sqrt{\sum_{i=1}^{n}\left[\left(\mu_{far}(x_i)-\mu_A(x_i)\right)^2+\left(\nu_{far}(x_i)-\nu_A(x_i)\right)^2+\left(\pi_{far}(x_i)-\pi_A(x_i)\right)^2\right]}}, \tag{11}$$

$$E^3_{SK}(A) = \frac{\sum_{i=1}^{n}\left[\frac{|\mu_{near}(x_i)-\mu_A(x_i)|+|\nu_{near}(x_i)-\nu_A(x_i)|}{4}+\frac{\max(|\mu_{near}(x_i)-\mu_A(x_i)|,|\nu_{near}(x_i)-\nu_A(x_i)|)}{2}\right]}{\sum_{i=1}^{n}\left[\frac{|\mu_{far}(x_i)-\mu_A(x_i)|+|\nu_{far}(x_i)-\nu_A(x_i)|}{4}+\frac{\max(|\mu_{far}(x_i)-\mu_A(x_i)|,|\nu_{far}(x_i)-\nu_A(x_i)|)}{2}\right]}, \tag{12}$$

$$E^4_{SK}(A) = \frac{\frac{1}{\sqrt[l]{n}}\sqrt[l]{\left(\frac{|\mu_{near}(x_i)-\mu_A(x_i)|}{2}+\frac{|\nu_{near}(x_i)-\nu_A(x_i)|}{2}\right)^l}}{\frac{1}{\sqrt[l]{n}}\sqrt[l]{\left(\frac{|\mu_{far}(x_i)-\mu_A(x_i)|}{2}+\frac{|\nu_{far}(x_i)-\nu_A(x_i)|}{2}\right)^l}}. \tag{13}$$

2.3.3. Group 3: non-probability

Zeng and Li [42] discussed the relationship between the similarity measure and entropy in IVFSs. They proved that the entropy of IVFS and the similarity measure can be reciprocally transformed into one another. To maintain consistency, the entropy of IVFS is transformed into an IF entropy format and is presented as follows:

$$E^1_{ZL}(A) = 1-\frac{1}{b-a}\int_a^b|\mu_A(x_i)-\nu_A(x_i)|dx, \tag{14}$$

$$E^2_{ZL}(A) = \frac{\int_a^b(\mu_A(x_i)\wedge\nu_A(x_i))dx}{\int_a^b(\mu_A(x_i)\vee\nu_A(x_i))dx}, \tag{15}$$

$$E^3_{ZL}(A) = 1-|\mu_A(x_i)-\nu_A(x_i)|, \tag{16}$$

$$E^4_{ZL}(A) = 1-\sqrt{(\mu_A(x_i)-\nu_A(x_i))^2}. \tag{17}$$

Vlachos and Sergiadis [33] presented a unified framework for subsethood, entropy, and cardinality for IVFSs based on the axiomatic skeleton; they also further stated interval-valued fuzzy versions of the entropy and entropy-subsethood theorems. We transform the interval-valued fuzzy entropy into an intuitionistic fuzzy version on account of concordance and present the equation as follows:

$$E^1_{VS}(A) = \frac{\sum_{i=1}^{n}(\min\{\mu_A(x_i),\nu_A(x_i)\}+\min\{1-\nu_A(x_i),1-\mu_A(x_i)\})}{\sum_{i=1}^{n}(\max\{\mu_A(x_i),\nu_A(x_i)\}+\max\{1-\nu_A(x_i),1-\mu_A(x_i)\})}. \tag{18}$$

In addition, Vlachos and Sergiadis [32] also presented an approach to discrimination measures for IFSs based on information theory; they derived an IF entropy measure from a generalized non-probabilistic entropy version of IFSs. The proposed entropy measure for IFSs is as follows:

$$E^2_{Vs}(A) = -(2n\ln 2)^{-1}D_{\mathrm{IFS}}(A,A^c)+1. \tag{19}$$

Note that $D_{\mathrm{IFS}}(A,B) = I_{\mathrm{IFS}}(A,B)+I_{\mathrm{IFS}}(B,A)$, and

$$I_{\mathrm{IFS}}(A,B) = \sum_{i=1}^{n}\left[\mu_A(x_i)\ln\frac{\mu_A(x_i)}{\frac{1}{2}(\mu_A(x_i)+\mu_B(x_i))}+\nu_A(x_i)\ln\frac{\nu_A(x_i)}{\frac{1}{2}(\nu_A(x_i)+\nu_B(x_i))}\right].$$

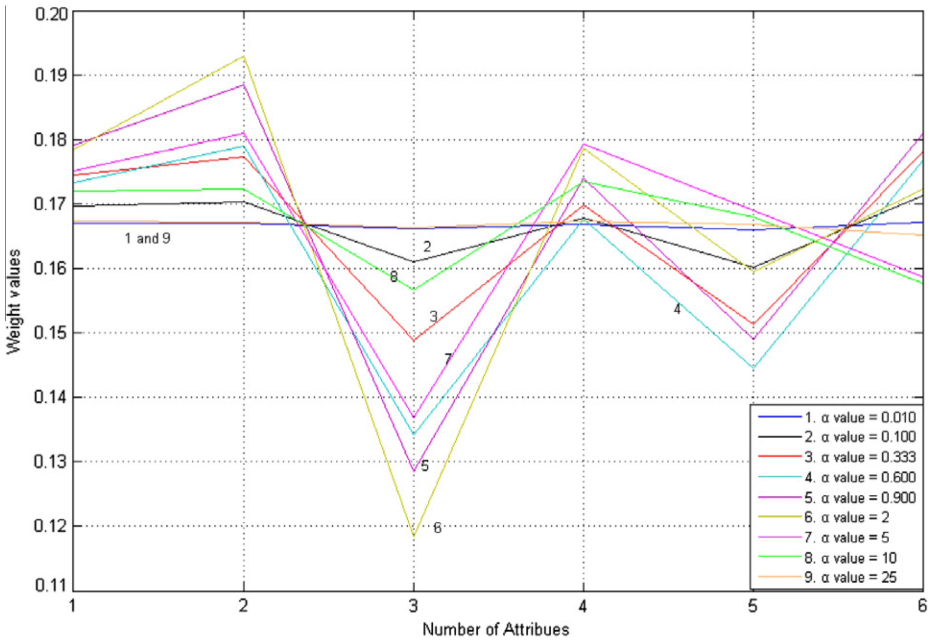


Fig. 1. Test of α values in G4.

2.3.4. Group 4: probability

Hung and Yang [16] exploited the concept of probability that originated from Havrda and Charavát's concept of entropy $H^{\alpha}_{hc}(q)$ [13] and Rényi's notion of entropy $H^{\beta}_{r}(q)$ [27] to define IF entropy. They proposed two families of entropy measures for IFSs as follows:

$$E^{\alpha}_{HC}(A) = \begin{cases} \frac{1}{\alpha-1}\left[1-\left(\mu^{\alpha}_{A}+v^{\alpha}_{A}+\pi^{\alpha}_{A}\right)\right], & \alpha \neq 1 (\alpha > 0); \\ \mu_A \log \mu_A - v_A \log v_A - \pi_A \log \pi_A, & \alpha = 1, \end{cases} \tag{20}$$

and

$$E^{\beta}_{R}(A) = \frac{1}{1-\beta} \log\left(\mu^{\beta}_{A}+v^{\beta}_{A}+\pi^{\beta}_{A}\right), \quad 0 < \beta < 1. \tag{21}$$

Although there are four different groups of IF entropy measures, measures in the same group originate from a common concept or background. Therefore, we select a representative measure in each group and conduct a computational experiment to compare the difference between these measures, which is then applied in the proposed objective weighting method to calculate attribute weights. We choose (4) for group 1, (12) for group 2, (16) for group 3, and (20) for group 4 and rename them G1, G2, G3, and G4, respectively. Note that G4 has the parameter α, which requires a fixed value. We use several values of α, including $\alpha = 0.010$, $\alpha = 0.100$, $\alpha = 0.333$, $\alpha = 0.600$, $\alpha = 0.900$, $\alpha = 2$, $\alpha = 5$, $\alpha = 10$ and $\alpha = 25$, to test the effect of α on attribute weights; the result is shown in Fig. 1.

The abscissa represents "number of attributes," and the ordinate represents the "weight values." There are nine lines in Fig. 1, but line 1 and line 9 are nearly overlapping. From line 2 to line 6, this figure shows divergence as the value of αincreases. When the value of α exceeds 2, it starts to converge from line 7 to line 9. Thus, when the value of α is very small or large, the lines are nearly horizontal, and so we remove these situations from the experiment. For G4, we set $\alpha = 0.1$ because line 2 shows little fluctuation.

3. Research method

3.1. Algorithm

To demonstrate the process of determining the objective weights, an algorithm is developed here; this approach is similar that employed by Zeleny [40]. The difference is that Zeleny adopted traditional entropy measures to calculate entropy values, whereas we employed IF entropy measures. Assume that there are m alternative measures $P_i(i=1,2,\ldots,m)$ to be performed over n attributes $x_j(j=1,2,\ldots,n)$. The intuitionistic fuzzy decision matrix D is expressed as follows:

$$D=\begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{array}\begin{array}{c} \begin{array}{ccccccccc} x_1 & & x_2 & & \ldots & x_j & & \ldots & x_n \end{array} \\ \begin{bmatrix} (\mu_{11},\nu_{11},\pi_{11}) & (\mu_{12},\nu_{12},\pi_{12}) & \cdots & (\mu_{1j},\nu_{1j},\pi_{1j}) & \cdots & (\mu_{1n},\nu_{1n},\pi_{1n}) \\ (\mu_{21},\nu_{21},\pi_{21}) & (\mu_{11},\nu_{11},\pi_{11}) & \cdots & (\mu_{2j},\nu_{2j},\pi_{2j}) & \cdots & (\mu_{2n},\nu_{2n},\pi_{2n}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (\mu_{i1},\nu_{i1},\pi_{i1}) & & & (\mu_{ij},\nu_{ij},\pi_{ij}) & \cdots & (\mu_{in},\nu_{in},\pi_{in}) \\ \vdots & & \vdots & \vdots & & \vdots \\ (\mu_{m1},\nu_{m1},\pi_{m1}) & (\mu_{m2},\nu_{m2},\pi_{m2}) & \cdots & (\mu_{mj},\nu_{mj},\pi_{mj}) & \cdots & (\mu_{mn},\nu_{mn},\pi_{mn}) \end{bmatrix} \end{array}.$$

Step 1 : Calculate the entropy values of each IFS by using the IF entropy measures introduced above. Each performance value in the decision matrix D is turned into an entropy value E_{ij} of the following form:

$$D=\begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{array}\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & \ldots & x_j & \ldots & x_n \end{array} \\ \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1j} & \cdots & E_{1n} \\ E_{21} & E_{22} & & \vdots & & E_{2n} \\ \vdots & \vdots & \vdots & & & \vdots \\ E_{i1} & & & E_{ij} & \cdots & E_{in} \\ \vdots & & & \vdots & & \vdots \\ E_{m1} & E_{m2} & \cdots & E_{mj} & \cdots & E_{mn} \end{bmatrix} \end{array}.$$

Step 2 : Normalize the IF entropy values in the decision matrix using the following equation:

$$h_{ij}=\frac{E_{i1}}{\max(E_{i1})},\frac{E_{i2}}{\max(E_{i2})},\ldots,\frac{E_{ij}}{\max(E_{ij})},\quad i=1,2,\ldots,\quad m;\ j=1,2,\ldots,n. \tag{22}$$

We use h_{ij} to label the normalized value. The normalized decision matrix is thus shown as follows:

$$D=\begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{array}\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & \ldots & x_j & \ldots & x_n \end{array} \\ \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1j} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2j} & \cdots & h_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \\ h_{i1} & & & h_{ij} & \cdots & h_{in} \\ \vdots & & & \vdots & & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mj} & \cdots & h_{mn} \end{bmatrix} \end{array}.$$

Step 3 : Calculate the objective attribute weights by applying the following transformer:

$$w_j=\frac{1}{n-T}\times(1-a_j). \tag{23}$$

Note that $a_j=\sum_{i=1}^{m}h_{ij}$ and $T=\sum_{j=1}^{n}a_j$. The w_j stands for the objective weight value of attribute x_j. a_j represents the summation of the normalized entropy values corresponding to attribute x_j. T is the summation of a_j. If the summation of IF entropy values to one specific attribute is lower, we assign it a higher weight, and vice versa.

3.2. Numerical example and discussion

In this section, we use a numerical example to illustrate the proposed method for determining objective weights in a MADM problem. Suppose that the MADM problem refers to four alternatives on six attributes. The intuitionistic fuzzy decision matrix D is given as follows:

$$
\begin{array}{cccc}
 & x_1 & x_2 & x_3 \\
\begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix} &
\multicolumn{3}{c}{\begin{bmatrix} (0.39,0.33,0.28) & (0.31,0.46,0.23) & (0.11,0.35,0.54) \\ (0.13,0.58,0.29) & (0.27,0.49,0.24) & (0.31,0.68,0.01) \\ (0.39,0.49,0.12) & (0.70,0.10,0.20) & (0.47,0.02,0.51) \\ (0.54,0.34,0.12) & (0.95,0.00,0.05) & (0.08,0.02,0.90) \end{bmatrix}} \\
 & x_4 & x_5 & x_6 \\
\begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix} &
\multicolumn{3}{c}{\begin{bmatrix} (0.24,0.69,0.07) & (0.36,0.16,0.48) & (0.01,0.53,0.46) \\ (0.91,0.08,0.01) & (0.01,0.14,0.85) & (0.04,0.72,0.24) \\ (0.21,0.62,0.17) & (0.01,0.04,0.95) & (0.26,0.11,0.63) \\ (0.22,0.27,0.51) & (0.19,0.02,0.79) & (0.08,0.46,0.46) \end{bmatrix}}
\end{array}.
$$

Note that P_i represents the alternative, and x_j represents the attribute. In the decision matrix, each element is expressed in the IFS format. For example, the performance value of the alternative P_1 with respect to the attribute x_1 is (0.39, 0.33, 0.28).

In Step 1, we illustrate the G1 measure to calculate the IF entropy of each IFS. The IF entropy of alternative P_1 over the attribute x_1 is 0.25. The remaining performance values in the decision matrix are also calculated using G1, and the IF entropy values are provided in the following matrix.

$$
\begin{array}{cc}
 & \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix} \\
\begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix} & \begin{bmatrix} 0.25 & 0.22 & 0.36 & 0.07 & 0.35 & 0.34 \\ 0.26 & 0.22 & 0.01 & 0.01 & 0.20 & 0.22 \\ 0.12 & 0.19 & 0.35 & 0.16 & 0.07 & 0.35 \\ 0.12 & 0.05 & 0.14 & 0.35 & 0.26 & 0.34 \end{bmatrix}
\end{array}.
$$

In Step 2, we normalize the matrix by applying (22). The normalized outcomes are as follows:

$$
\begin{array}{cc}
 & \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix} \\
\begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{matrix} & \begin{bmatrix} 0.96 & 1.00 & 1.00 & 0.20 & 1.00 & 0.97 \\ 1.00 & 1.00 & 0.03 & 0.03 & 0.57 & 0.63 \\ 0.46 & 0.86 & 0.97 & 0.46 & 0.20 & 1.00 \\ 0.46 & 0.23 & 0.39 & 1.00 & 0.74 & 0.97 \end{bmatrix}
\end{array}.
$$

In Step 3, we use a transformer to generate the objective attribute weights. We use the attribute x_1 as an example in calculating the final objective weight. In this numerical example, $a_1 = 2.88$, $a_2 = 3.09$, $a_3 = 2.39$, $a_4 = 1.69$, $a_5 = 2.51$, $a_6 = 3.57$, and $T = 16.13$, and so $w_1 = 0.19$. By employing the above successive steps, we obtain the weight of each attribute by using the other IF entropy measures (i.e., G2, G3 and G4). The detailed results are shown in Table 1.

Inspecting the fixed attributes shown in Table 1, we see that the attribute weights vary by the IF entropy measure used. For a clearer means of illustrating the difference between measures, descending rankings are presented in Table 1 according to the weight values. For instance, x_1 performs the best according to G4; nevertheless, it is in second place according to G2 and G3, and it is in third place under G1. Table 1 shows that ranking consequences vary as we employ different IF entropy measures. For further examination, the indices of the Pearson correlation coefficients and Spearman rank correlation coefficients were utilized to observe the relationship between the two rankings of attributes that are generated by each pair of two different IF entropy measures.

In Table 2, both Pearson correlation coefficients and Spearman rank correlation coefficients indicate that G2 has a highly positive relationship with G3. The two rankings of attribute weights generated by G2 and G3 are identical. However, with the exception of G2 and G3, the other pairs of measures show only a slight correlation. According to this numerical example, it seems that the use of IF entropy measures has an influence on the ranking of attribute weights.

Table 1
The results for weight values and ranks.

Measures	Attributes					
	x_1	x_2	x_3	x_4	x_5	x_6
G1	0.19 (3)[a]	0.20 (2)	0.14 (5)	0.07 (6)	0.15 (4)	0.25 (1)
G2	0.21 (2)	0.13 (5)	0.17 (3)	0.10 (6)	0.24 (1)	0.15 (4)
G3	0.21 (2)	0.13 (5)	0.18 (3)	0.12 (6)	0.22 (1)	0.14 (4)
G4	0.181 (1)	0.148 (6)	0.166 (4)	0.170 (3)	0.158 (5)	0.177 (2)

[a] Ranking orders are in parentheses.

Table 2
The results for correlations.

	Comparisons					
	G1 vs. G2	G1 vs. G3	G1 vs. G4	G2 vs. G3	G2 vs. G4	G3 vs. G4
Pearson correlation coefficient	0.195	0.035	0.105	0.973	0.088	0.147
Spearman rank correlation coefficient	0.086	0.086	0.086	1.000	0.086	0.086

4. Experiment analysis

The ranking of attribute weights plays a crucial role in decision-making because the first and second places of attributes in the ranking are essential in choosing among alternatives. In general, relatively important attributes are often endowed with higher weights. As a result, if the variation among weight rankings is significant, especially with respect to the first two attributes in the ranking, we may be led to select a different alternative. Because the above numerical example demonstrates that using different IF entropy measures in objective weighting may lead to different attribute rankings, we use a great amount of simulation data in our experiment to gain insights into the proposed weighting method, which implements comparisons of several various IF entropy measures.

The random data are generated to form MADM problems with all possible combinations of $4,6,8,\ldots,22$ alternatives and $4,6,8,\ldots,22$ attributes. Hence, 100 (=10×10) different instances are examined in this study. For each instance, 1000 different intuitionistic fuzzy decision matrices D are randomly produced under the preliminary condition of IFSs. Therefore, a total of 100,000 (=100×1000) sets of experimental cases are generated. In the following sections, the differences between the IF entropy measures are discussed in terms of some comparative indices. The Pearson correlation coefficients, the Spearman rank correlation coefficients, and the consistency rates belonging to consistent indices are utilized to examine the concordance of rankings yielded by different IF entropy measures. The inconsistent indices, which include the contradiction rates and the inversion rates, are employed to observe the discrepancies between consequent rankings.

4.1. Analysis of average r-values

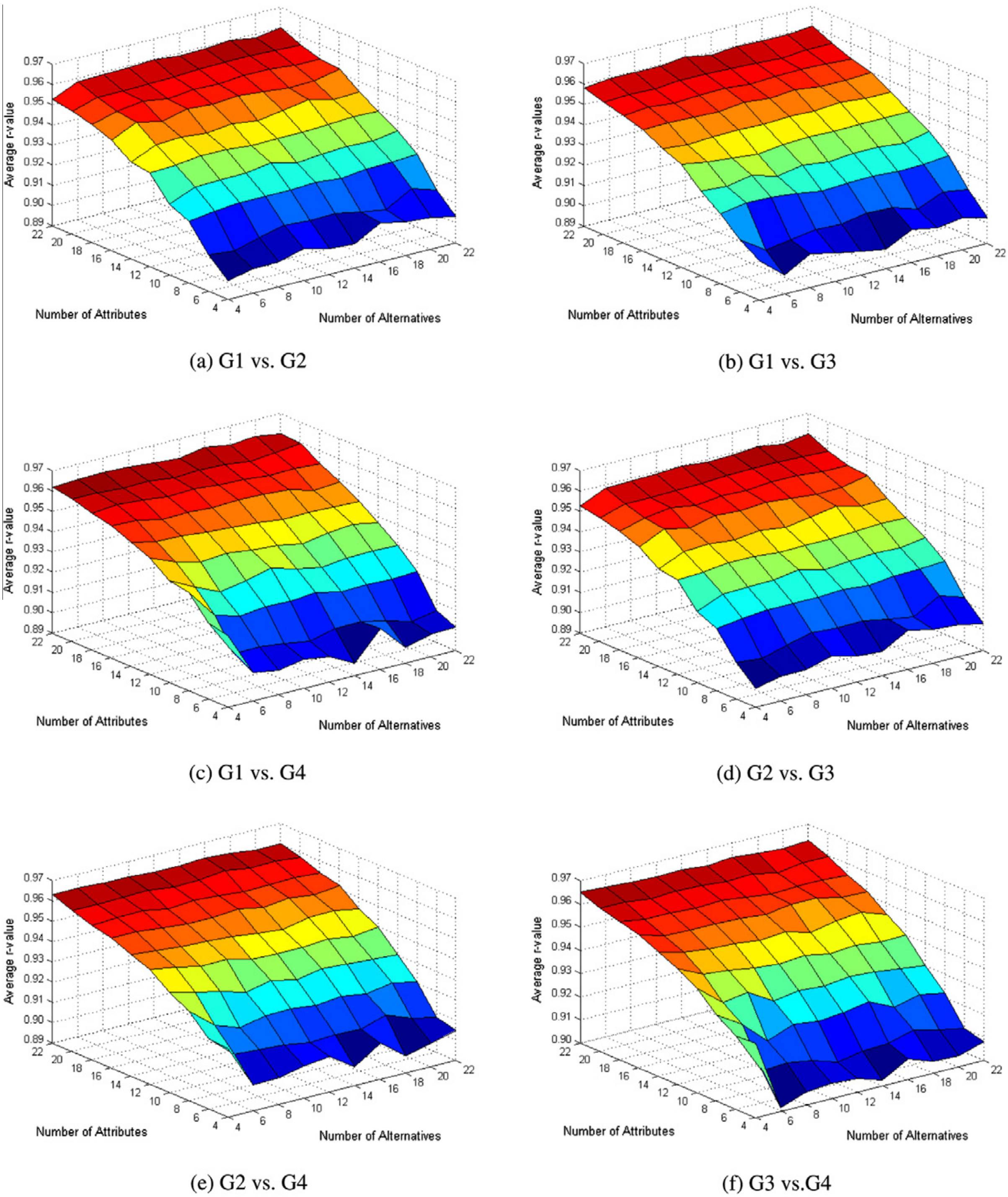
There are six subordinate figures in Fig. 2, corresponding to C_2^4 comparisons of IF entropy measures in our objective weighting method. The numbers of both the attributes and alternatives increase from 4 to 22. The six figures not only have a similar shape but also have high average r-values. We now consider figure (a) in further detail. In (a), the average r-values are located between 0.90 and 0.96, which suggests that applying G1 and G2 may result in highly similar rankings of attributes. The average r-values rise with the increase in the number of attributes; however, whether the number of alternatives is large or small, the average r-value maintains an equivalent level. Hence, the effect of the number of attributes on the average r-values is very strong, whereas the effect of the number of alternatives does not have such significance. The high average r-values result from the postulate on weights, which stipulates that all weights should be between 0 and 1 and that the summation of weights should equal 1. Thus, we use the Spearman rank correlation to further understand the relationship between each pair of IF entropy measures.

4.2. Analysis of average ρ-value

In this section, Fig. 3 presents the results of the average ρ-values between each pair of measures. The six subordinate figures in Fig. 3 appear as inconsistent shapes; in addition, some figures fail to show a clear trend. For a better explanation of these figures, we provide illustrations of trends in different directions. In (a), (d), and (e), the number of attributes decreases from 22 to 4, and the number of alternatives increases from 4 to 22. In (b), both the number of attributes and the number of alternatives reduce from 22 to 4. In (c) and (f), the number of attributes increases from 4 to 22, and the number of alternatives decreases from 22 to 4.

In general, (a) and (e) have common shapes and average ρ-values, which are between 0 and 0.2. The effect of the number of attributes is evident in that the average ρ-values increase as the number of attributes goes down, but the effect of the number of alternatives is not conspicuous. (c) and (f) also have similar shapes, but we find higher average ρ-values in (f) (ρ = 0.10–0.50) than in (c) (ρ = –0.05–0.35). The effects of the number of attributes and alternatives are very significant in both (c) and (f). The average ρ-values increase with an increase in the number of attributes, but they decrease with an increase in the number of alternatives.

In (b), the effects of neither the number of attributes nor the number of alternatives are evident. The average ρ-values decrease quickly to a specific level as the number of attributes or the number of alternatives slightly increases. In (d), the average ρ-values located between 0 and 0.7 are, on average, higher than in the other figures in Fig. 3. Thus, we could infer that using G2 and G3 can produce more similar rankings of attributes than the others. The effect of the number of attributes is obvious in (d) in which the average ρ-values increase significantly as the number of attributes decreases from 22 to 4. The effect of the number of alternatives is not as obvious as that of the attributes. The average ρ-values increase dramatically as



(a) G1 vs. G2 (b) G1 vs. G3

(c) G1 vs. G4 (d) G2 vs. G3

(e) G2 vs. G4 (f) G3 vs.G4

Fig. 2. The results for average r-values.

the number of alternatives increases from 4 to 6, but the average ρ-values merely increase slightly, while the number of alternatives increases from 6 to 22.

In Fig. 2, each subordinate figure has a similar shape in which the coefficients become significantly higher when the number of attributes increases. In contrast, the shape in Fig. 3 presents irregular conditions, and the coefficients are much lower. The attribute weights given by ordinal or ratio data indeed have an influence on the extent of correlation between the IF entropy measures. The r-values and the ρ-values must complement one other due to the deficiency in both of them. The Pearson correlation analysis often provides high values due to the postulate on weight. The Spearman correlation analysis

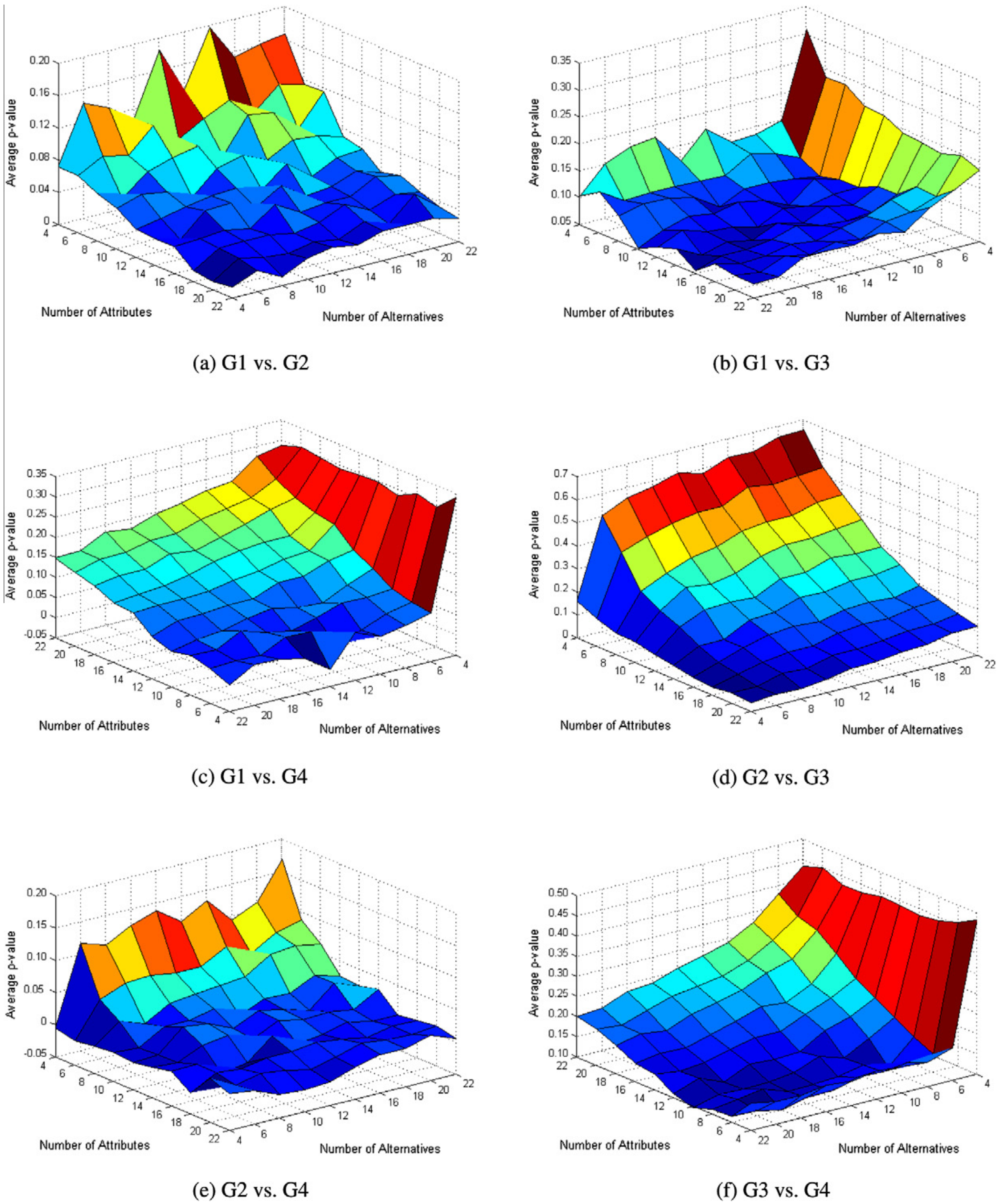


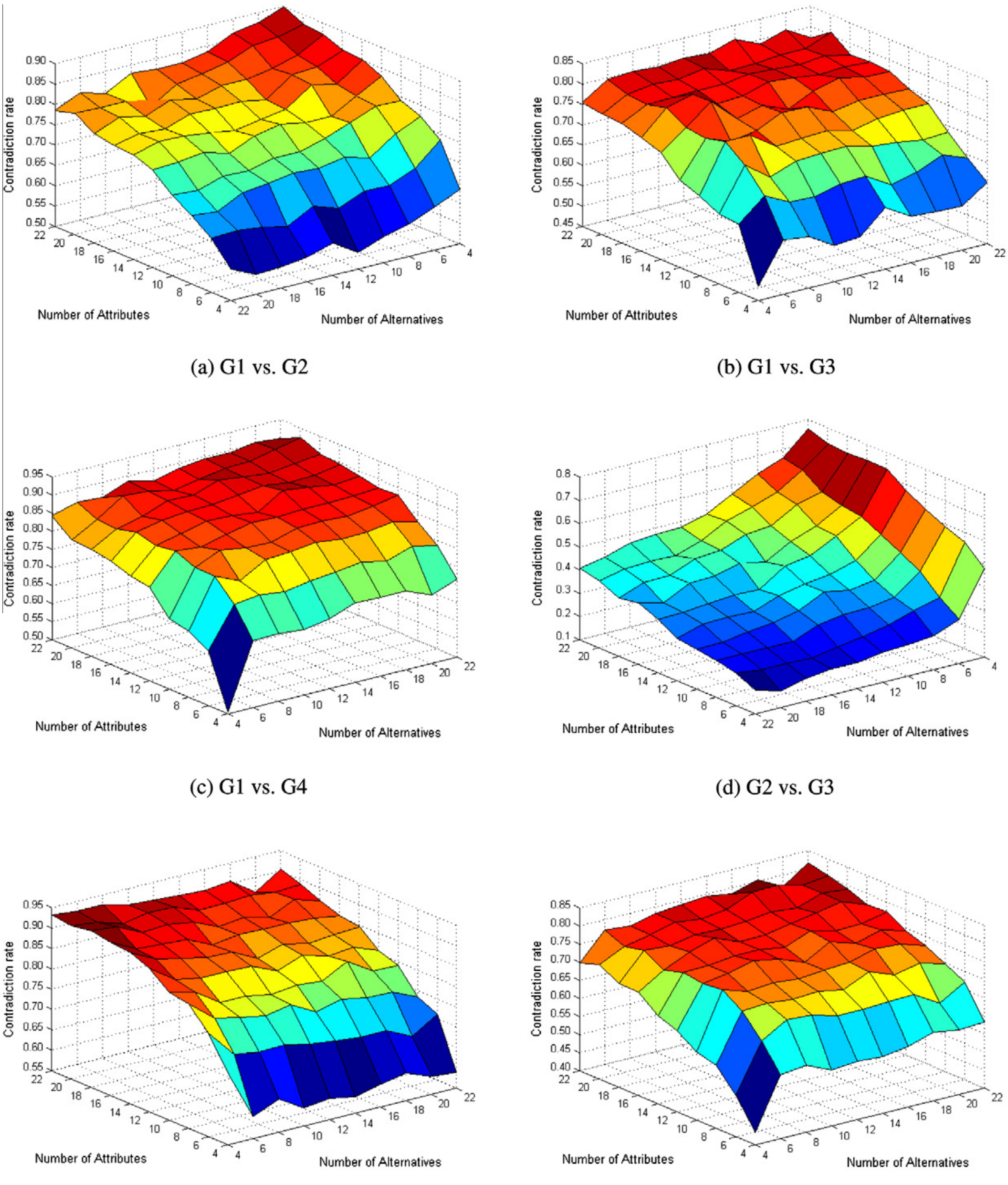
Fig. 3. The results for average ρ-values.

is so sensitive that the values fluctuate easily. Hence, it is necessary to explain the difference between the IF entropy measures simultaneously using the r-values and the ρ-values.

4.3. Analysis of contradiction rates

The contradiction rate can be observed for the two top rankings by using different IF entropy measures. If the first-place attributes of two rankings are different, then we count again. For example, if the ranking of a set of six attributes is equal to

(2, 6, 3, 1, 4, 5) (i.e., $x_2 > x_6 > x_3 > x_1 > x_4 > x_5$) based on G1 while another method using G2 yields (6, 2, 3, 1, 4, 5), then a ranking contradiction of the best alternative has occurred. The results of the contradiction rates between both entropy measures are shown in Fig. 4. Except for (a) and (d), all figures present trends in the same direction. In (a) and (d), the number of attributes increases from 4 to 22, while the number of alternatives decreases from 22 to 4. In the other figures, both the number of attributes and the number of alternatives increase from 4 to 22. Basically, the values of contradiction rates in (a), (b), (c),



(a) G1 vs. G2

(b) G1 vs. G3

(c) G1 vs. G4

(d) G2 vs. G3

(e) G2 vs. G4

(f) G3 vs. G4

Fig. 4. The results for contradiction rates.

(e), and (f) are similar and lie between 0.45 and 0.90. In (a), the effects of both the number of attributes and the number of alternatives are significant. The contradiction rates significantly increase as the number of attributes increases; the contradiction rates also increase as the number of alternatives decreases.

In (d), contradiction rates lie between 0.2 and 0.8 and are much lower than for other pairs of measures. Hence, we can infer that the rankings yielded by G2 and G3 are alike. The shape of (d) is similar to that of (a), and the effects of both the number of attributes and the number of alternatives are evident. The contradiction rates increase when the number

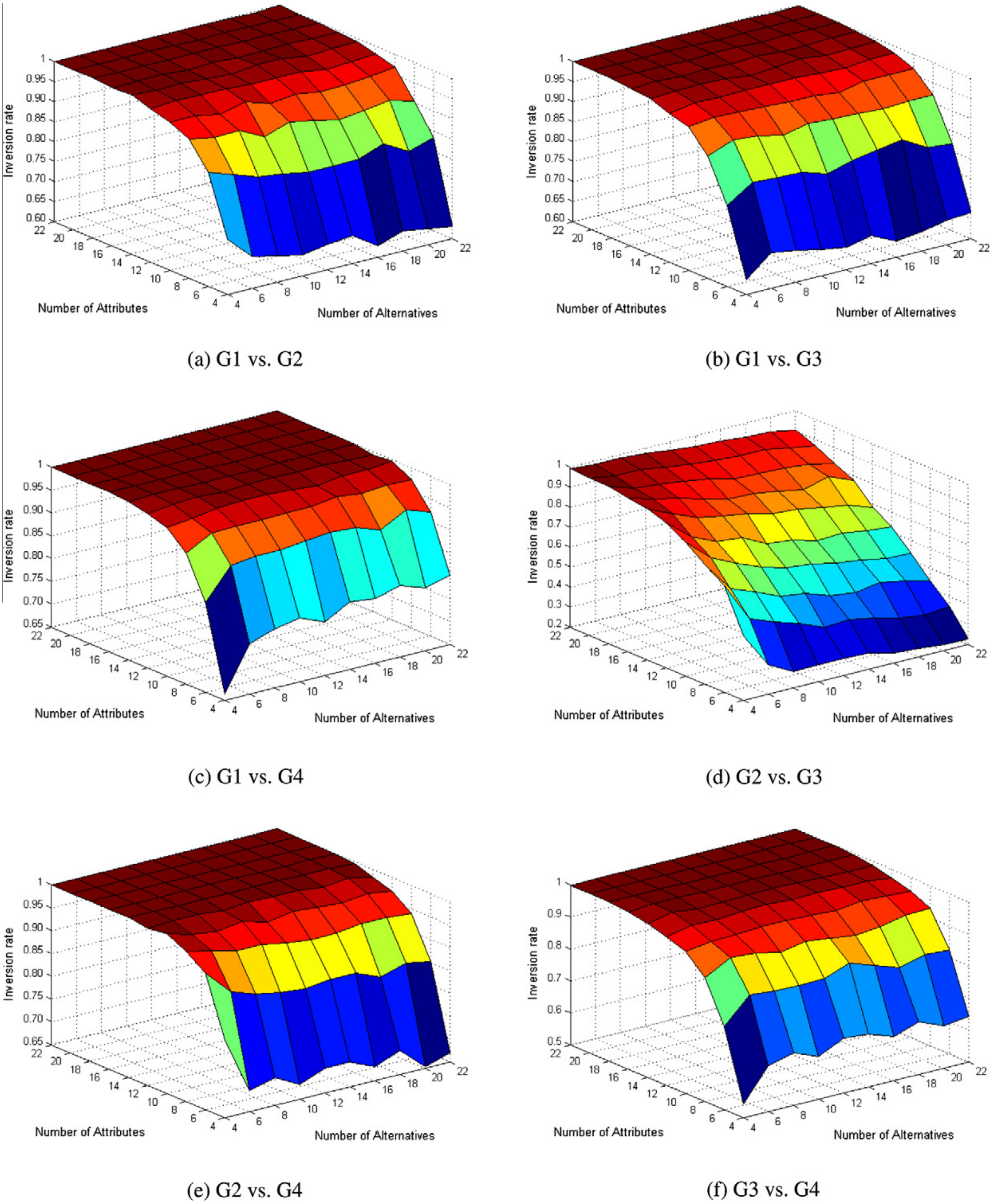


Fig. 5. The results for inversion rates.

of attributes increases from 4 to 22, but conversely, the contradiction rates increases when the number of alternatives decreases from 22 to 4.

In (e), the effect of the number of attributes is distinct, but the effect of the number of alternatives is not evident. The contradiction rates increase significantly when the number of attributes increases; however, the contradiction rates decrease slightly when the number of alternatives increases. Figures (b), (c), and (f) are all similar based on their shapes and contradiction rates. Moreover, the contradiction rates increase with an increase in the number of attributes or alternatives.

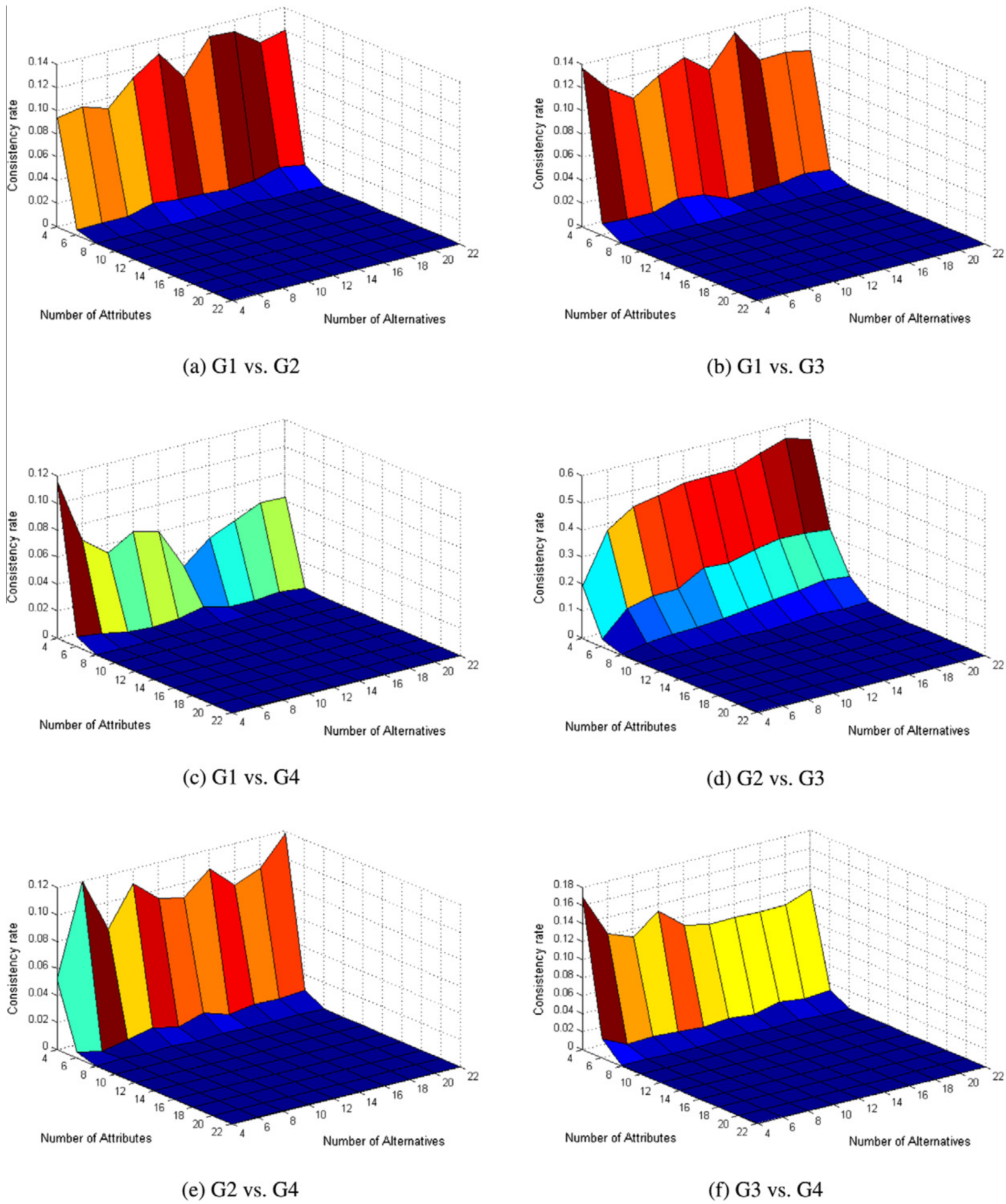


Fig. 6. The results for consistency rates.

4.4. Analysis of inversion rates

For the inversion rate, we separate the ranking into two parts. The first part includes the first-place up to the middle of the ranking, and the second part includes the middle to the last place in the ranking. If there is any attribute that appears in the first part of ranking 1 and also appears in the second part of ranking 2, we count ranking 1 one time, and vice versa. For example, if a ranking based on G1 of a set of six attributes is equal to ($\underline{2, 5, 3}$, $\underline{1, 4, 6}$) but G2 yielded ($\underline{4, 5, 3}$, $\underline{1, 2, 6}$), then a case of a ranking inversion between better worse attributes has occurred. Six figures in Fig. 5 have similar shapes, except for (d). Both the number of attributes and the number of alternatives increase from 4 to 22. In (a), the inversion rates lie between 0.6 and 1.0. The effect of the number of attributes is obvious, but the effect of the number of alternatives is not. The inversion rates soar drastically as the number of attributes grows larger than 4. When the number of attributes exceeds 10, the inversion rates approach an apex and maintain that value. Figures (b), (c), (e), and (f) all resemble (a) in terms of shape as well as inversion rate trend.

In (d), the inversion rates are much lower than in the other figures. The inversion rates do not increase in a dramatic fashion but rather increase gradually as the number of attributes increases. It is worth mentioning that the number of attributes has a regular and slight effect on the inversion rates. Overall, the pair (G2, G3) yields relatively small inversion rates compared to the others. Even if the number of attributes is larger than 10, the rate is still less than 0.8.

The contradiction rates and inversion rates help us examine the difference between the IF entropy measures from the perspective of inconsistency. They have something in common; that is, the rates expand with an increase in the number of attributes. However, the inverse condition performs more intensively than the contradictive condition insofar as the inversion rates are higher than the contradiction rates.

4.5. Analysis of consistency rates

The consistency rate is used to measure the level of concordance between two complete rankings generated by different IF entropy measures. When the two rankings are identical, we add 1 to our count. The detailed results are shown in Fig. 6. The number of attributes decreases from 22 to 4, and the number of alternatives increases from 4 to 22. The shapes and values of all six subordinate figures are similar, except for (d). In (a), the consistency rates are between 0 and 0.14, and the effect of the number of attributes shows a slight trend. The consistency rates roughly approach 0.12 when the number of attributes is 4 but abruptly arrive at a low-point of zero as the number of attributes increases. In contrast to the number of attributes, the effect of the number of alternatives does not show a prominent trend.

In (d), the consistency rates that lie between 0 and 0.6 are higher than the values in other subordinate figures. Hence, the pair (G2, G3) has a higher consistency than other pairs of measures. The effect of the number of attributes and alternatives are both slight in (d). The consistency rates decrease drastically as the number of attributes increases from 4 to 10. However, the consistency rates increase gradually as the number of alternatives increases from 4 to 22, but this effect only occurs when the number of attributes is very small.

Although the consistency and inversion rates aim to show concordance and discordance among rankings, respectively, the shapes of the figures present opposite results. That is, both rates display a shape of platform that represents low consistency and high inversion. The figures imply that it is difficult to obtain fully equivalent rankings by different IF entropy measures. The attribute in the first part of a given ranking 1 may easily appear in the second part of another ranking 2. As a consequence, inversion happens in accordance with these inconsistencies.

4.6. Regression analysis

A second-order regression model was used to capture the relationship of the numbers of alternatives, numbers of attributes, and different IF entropy measures with Spearman correlation coefficients. A dependent variable ρ represents the average Spearman correlation coefficients of the pairs of measures. The independent variables include m, m^2, n, n^2, mn, $L_{(G1,G3)}$, $L_{(G1,G3)}m$, $L_{(G1,G3)}n$, $L_{(G1,G4)}$, $L_{(G1,G4)}m$, $L_{(G1,G4)}n$, $L_{(G2,G3)}$, $L_{(G2,G3)}m$, $L_{(G2,G3)}n$, $L_{(G2,G4)}$, $L_{(G2,G4)}m$, $L_{(G2,G4)}n$, $L_{(G3,G4)}$, $L_{(G3,G4)}m$, and $L_{(G3,G4)}n$. Let $L_{(G1,G3)}$ be a dummy variable that is equal to one if the correlation coefficient is obtained from the pair of (G1, G3); otherwise, it is 0. m and n are variables of the numbers of alternative and attribute, respectively. Let ε be the effect caused by some other factors not considered here; that is, it is an error term. The regression model is demonstrated as follows:

$$\begin{aligned}\rho = {} & \lambda_0 + \lambda_1 m + \lambda_2 m^2 + \lambda_3 n + \lambda_4 n^2 + \lambda_5 mn + \lambda_6 L_{(G1,G3)} + \lambda_7 L_{(G1,G3)} m + \lambda_8 L_{(G1,G3)} n + \lambda_9 L_{(G1,G4)} + \lambda_{10} L_{(G1,G4)} m \\ & + \lambda_{11} L_{(G1,G4)} n + \lambda_{12} L_{(G2,G3)} + \lambda_{13} L_{(G2,G3)} m + \lambda_{14} L_{(G2,G3)} n + \lambda_{15} L_{(G2,G4)} + \lambda_{16} L_{(G2,G4)} m + \lambda_{17} L_{(G2,G4)} n + \lambda_{18} L_{(G3,G4)} \\ & + \lambda_{19} L_{(G3,G4)} m + \lambda_{20} L_{(G3,G4)} n + \varepsilon. \end{aligned} \tag{24}$$

The total sample size is 600 (i.e., 10 different numbers of alternatives $\times$ 10 different numbers of attributes $\times$ 6 different pairs of IF entropy measures). The regression results are generated by using the entry procedure in SPSS and are presented in Table 3. The coefficients and p-values indicate that some independent variables are not significant, including $L_{(G1,G4)}$, $L_{(G2,G4)}m$, and $L_{(G2,G4)}n$. The regression model is significant in terms of the p-value; $R^2$and adjusted-$R^2$are 0.889 and 0.885, respectively.

Table 3
Regression results.

Variable	Coefficients	p-value	Variable	Coefficients	p-value
Constant	0.208	0.000	$L_{(G1,G4)}n$	0.014	0.000
m	−0.004	0.020	$L_{(G2,G3)}$	0.318	0.000
m^2	0.000	0.000	$L_{(G2,G3)}m$	0.009	0.000
n	−0.019	0.000	$L_{(G2,G3)}n$	−0.020	0.000
n^2	0.001	0.000	$L_{(G2,G4)}$	−0.051	0.008
mn	0.000	0.000	$L_{(G2,G4)}m$	0.000	0.883
$L_{(G1,G3)}$	0.073	0.000	$L_{(G2,G4)}n$	0.001	0.335
$L_{(G1,G3)}m$	−0.006	0.000	$L_{(G3,G4)}$	0.158	0.000
$L_{(G1,G3)}n$	0.004	0.000	$L_{(G3,G4)}m$	−0.013	0.000
$L_{(G1,G4)}$	0.024	0.206	$L_{(G3,G4)}n$	0.012	0.000
$L_{(G1,G4)}m$	−0.010	0.000			

R^2 = 0.889; adj-R^2 = 0.885; F = 232.428; F(p-value) = 0.000.

Next, we conduct the partial differential for the results between measures, and display them as follows:

$$\frac{\partial\hat{\rho}}{\partial\hat{m}} = -0.004 - 0.006 \cdot L_{(G1,G3)} - 0.001 \cdot L_{(G1,G4)} + 0.009 \cdot L_{(G2,G3)} - 0.013 \cdot L_{(G3,G4)}, \tag{25}$$

$$\frac{\partial\hat{\rho}}{\partial\hat{n}} = -0.019 + 0.002 \cdot n + 0.004 \cdot L_{(G1,G3)} + 0.014 \cdot L_{(G1,G4)} - 0.002 \cdot L_{(G2,G3)} + 0.001 \cdot L_{(G2,G4)} + 0.012 \cdot L_{(G3,G4)}. \tag{26}$$

From (25), we know that the effects of alternatives are negative. In contrast with the pair (G1, G2), the effects of $L_{(G1,G4)}$, $L_{(G1,G4)}$, and $L_{(G3,G4)}$ are also negative, while the effect of $L_{(G2,G3)}$ is positive. In (26), the effect of the attributes is negative, but the larger number of attributes reduces the negative effect. In contrast with the pair (G1, G2), the effects of $L_{(G1,G3)}$, $L_{(G1,G4)}$, $L_{(G2,G4)}$, and $L_{(G3,G4)}$ are all positive, while the effect of $L_{(G2,G3)}$ is negative.

5. Conclusions

By applying IFSs to MADM problems, we can obtain more accurate values from the incomplete and complex information often provided by decision makers in real-life. A great number of methods, including subjective methods that generate or calculate attribute weights, have been studied by researchers. In this study, we introduce a new weighting method based on objective weights that uses the credibility of data in order to compute attribute weights. Making use of the IF entropy measures, this new method utilizes a transformer to convert the IF entropy value into an objective weight. On the basis of the objective weights, we can further combine the subjective weights yielded by decision makers to receive compromise weights that may better represent the real weights when solving MADM problems.

Because the literature on IF entropy measures includes various perspectives, we classified these measures into four groups based on hesitation degree, geometry, probability, and non-probability frameworks. Furthermore, to investigate the differences among IF entropy measures, a computational experiment was employed in which several combinations of ten differently sized sets of attributes and alternatives were utilized to rank attribute weights. The Pearson correlation coefficients, Spearman rank correlation coefficients, contradiction rates, inversion rates, and consistency rates indicate that using different IF entropy measures may generate divergent attribute weights and ranking outcomes. In addition, the number of attributes and alternatives can also increase or decrease the variation among IF entropy measures. On the whole, the IF entropy measures from the perspectives of geometry and non-probability can generate relatively similar rankings of attribute weights; in particular, when the number of attributes is relatively small and the number of alternatives is relatively large, the two IF entropy measures have lower contradiction rates, lower inversion rates, and higher consistency rates.

However, the experimental results provide only limited information to identify which two IF entropy measures in the objective weighting method presented here generate similar rankings. That is, our simulation data cannot tell us which measure is best used in reality. By using evidence-based data, follow-up research is suggested to determine the best IF entropy measure, which may be more appropriate in real-life situations.

References

[1] B. Ashtiani, F. Haghighirad, A. Makui, G.A. Montazer, Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets, Applied Soft Computing 9 (2009) 457–461.
[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.
[3] K. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications, Springer, Heidelberg, 1999.
[4] K. Atanassov, G. Gargov, Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31 (1989) 343–349.
[5] P. Burillo, H. Bustince, Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets, Fuzzy Sets and Systems 78 (1996) 305–316.
[6] C.T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems 114 (2000) 1–9.
[7] S.J. Chen, C.L. Hwang, Fuzzy Multiple Attribute Decision Making Methods and Applications, Springer-Verlag, New York, 1992.
[8] S.M. Chen, J.M. Tan, Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 67 (1994) 163–172.

[9] T.H. Chen, T.C. Wang, Using the fuzzy multi-criteria decision making approach for measuring the possibility of successful knowledge management, Information Sciences 179 (2009) 355–370.
[10] S.Y. Chou, Y.H. Chang, C.Y. Shen, A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes, European Journal of Operational Research 189 (2008) 132–145.
[11] C.W. Churchman, R.L. Ackoff, An approximate measure of value, Journal of the Operations Research Society of America 2 (1954) 172–187.
[12] A. De Luca, S. Termini, A definition of nonprobabilistic entropy in the setting of fuzzy theory, Journal of General Systems 5 (1972) 301–312.
[13] M.E. Havrda, F. Charavát, Quantification method of classification processes: concept of structural a-entropy, Kybernetica 3 (1967) 30–35.
[14] D.H. Hong, C.H. Choi, Multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 114 (2000) 103–113.
[15] W.L. Hung, M.S. Yang, Fuzzy entropy on intuitionistic fuzzy sets, International Journal of Intelligent Systems 21 (2006) 443–451.
[16] W.L. Hung, M.S. Yang, On the *J*-divergence of intuitionistic fuzzy sets with its application to pattern recognition, Information Sciences 178 (2008) 1641–1650.
[17] C.L. Hwang, M.J. Lin, Group Decision Making under Multiple Criteria: Methods and Applications, Springer, Berlin, 1987.
[18] C.L. Hwang, K. Yoon, Multiple Attribute Decision Making-Methods and Applications, A State-of-the-Art Survey, Springer-Verlag, New York, 1981.
[19] Y.C. Jiang, Y. Tang, J. Wang, S. Tang, Reasoning within intuitionistic fuzzy rough description logics, Information Sciences 179 (2009) 2362–2378.
[20] G. Kahraman, G. Buyukozkan, N.Y. Ates, A two phase multi-attribute decision-making approach for new product introduction, Information Sciences 177 (2007) 1567–1582.
[21] A. Kaufmann, Introduction to the Theory of Fuzzy Subsets, Fundamental Theoretical Elements, vol. 1, Academic Press, New York, 1975.
[22] D.F. Li, Multiatribute decision making models and methods using intuitionistic fuzzy sets, Journal of Computer System Sciences 70 (2005) 73–85.
[23] D.F. Li, Y.C. Wang, S. Liu, F. Shan, Fractional programming methodology for multi-attribute group decision-making using IFS, Applied Soft Computing 9 (2009) 219–225.
[24] L. Lin, X.H. Yuan, Z.Q. Xia, Multicriteria fuzzy decision-making methods based on intuitionistic fuzzy sets, Journal of Computer and Systems Sciences 73 (2007) 84–88.
[25] H.W. Liu, G.J. Wang, Multi-criteria decision-making methods based on intuitionistic fuzzy sets, European Journal of Operational Research 179 (2007) 220–233.
[26] J. Ma, Z.P. Fan, L.H. Huang, A subjective and objective integrated approach to determine attribute weights, European Journal of Operational Research 112 (1999) 397–404.
[27] A. Rényi, On measures of entropy and information, in: Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability, 1961, pp. 547–561.
[28] T.L. Saaty, The Analytic Hierarchy Process, McGraw-Hill, New York, 1980.
[29] R. Sambuc, Application l'Aide au Diagnostic en Pathologie Thyroidienne, Ph.D. Thesis, University of Marseille, France, 1975.
[30] E. Szmidt, J. Kacprzyk, Distances between intuitionistic fuzzy sets, Fuzzy Sets and Systems 114 (2000) 505–518.
[31] E. Szmidt, J. Kacprzyk, Entropy for intuitionistic fuzzy sets, Fuzzy Sets and Systems 118 (2001) 467–477.
[32] I.K. Vlachos, G.D. Sergiadis, Intuitionistic fuzzy information-applications to pattern recognition, Pattern Recognition Letters 28 (2007) 197–206.
[33] I.K. Vlachos, G.D. Sergiadis, Subsethood, entropy, and cardinality for interval-valued fuzzy sets – An algebraic derivation, Fuzzy Sets and Systems 158 (2007) 1384–1396.
[34] W. Wang, X. Xia, Distance measure between intuitionistic fuzzy sets, Pattern Recognition Letters 26 (2005) 2603–2609.
[35] Z. Xu, Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making, Fuzzy Optimization and Decision Making 6 (2007) 109–121.
[36] Z.S. Xu, Intuitionistic preference relations and their application in group decision making, Information Sciences 177 (2007) 2363–2397.
[37] J.L. Yang, H.N. Chiu, G.H. Tzeng, R.H. Yeh, Vendor selection by integrated fuzzy MCDM techniques with independent and interdependent relationships, Information Sciences 178 (2008) 4166–4183.
[38] C. Yu, Y. Luo, S. Liu, A fuzzy optimization method for multi-criteria decision-making problem based on the inclusion degree of intuitionistic fuzzy sets, in: Proceedings of International Conference on Intelligent Computing, Lecture Notes in Computer Science, Shanghai, 2008, pp. 332–339.
[39] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[40] M. Zeleny, The attribute-dynamic attribute model, Management Science 23 (1976) 12–26.
[41] M. Zeleny, Multiple Criteria Decision Making, Springer, NY, 1982.
[42] W. Zeng, H. Li, Relationship between similarity measure and entropy of interval valued fuzzy sets, Fuzzy Sets and Systems 157 (2006) 1477–1484.