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Two Simple Design Problems, Which Illustrate the Multi-Disciplinary Concept. Part II

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Abstract. Th. de Jong, A. Beukers and M. J. L. van Tooren [1, 2] considered two simple design problems, which illustrate the multi-disciplinary design concept.

Key words: multi-disciplinary design, composite, aluminium, tension, buckling.

Example 1: A Tension Loaded Plate with Stress Concentrations

In this application, the behaviour of a composite plate containing a circular hole subjected to an uni-axial tensile loading is considered. For reference purposes, an aluminium plate, having the same geometrical dimensions, will be used. Only its thickness t_a may differ from the thickness of the composite plate t_c. Stress calculations, in combination with an appropriate strength criterion, should yield the most suitable laminate lay-up for the considered geometry and loading condition. Since these calculations are out of the scope of this example, we will simply choose a laminate lay-up, which is known for its (relatively good) ability to tolerate stress concentrations. It is the $[(0_2/+45/-45)_n]_s$ lay-up, where n refers to the number of the basic $(0_2/+45/-45)$ group of plies in the laminates and 's' refers to a symmetrical stacking order. This lay-up yields the mechanical properties shown in Table I; only the properties relevant to the present calculations are given there.

The laminate properties calculated from the properties of the layers with the classical lamination theory (see, for instance, Jones [3]). The strength value has been calculated with Puck's criterion [4]. The ratio of plate width w and hole diameter D is assumed to be larger than 3. This implies that for the calculation of the stresses around the hole Letnitskii's expressions [5] for infinite anisotropic plates can be used. The calculations show, in combination with Puck's criterion, that the stress concentration in the minimum net section at the hole edge is critical for static failure,

$$\frac{\sigma}{\sigma_{\mathrm{gr}_c}} = 3.465 \tag{1}$$

Table I. Mechanical properties of some carbon fibre composite laminates. Values typical for 50% fibres by volume. The ultimate normal stress values are tension values.

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	σ_1 (MPa)	σ_2 (MPa)	τ_{12} (MPa)
UD	120	11.8	10.5	0.31	1400	51	91
$[(0_2/\pm 45)_n]_s$	76.6	26.4	21.0	0.49	659		
isotropic	52.5	52.5	21.0	0.25			

yielding a static failure gross stress

$$\bar{\sigma}_{\mathrm{gr_c}} = 190\ \mathrm{MPa}. \tag{2}$$

So, the relative static strength in the strongest direction of the laminate is only 29%. Aluminium alloys, at least those commonly used in aircraft structures, have a reasonable ductile behaviour. Consequently, we assume a complete minimum net section yielding in the aluminium plate at higher loads, resulting in a uniformly distributed stress at failure. The static, gross failure stress of this plate is then simply related to the material strength σ_u by

$$\bar{\sigma}_{\mathrm{gr_a}} = \sigma_u \frac{w - D}{w}. \tag{3}$$

Note that subscript 'a' refers to aluminium and 'c' to composite. If the composite plate and the aluminium plate are required to fail at the same load P_u,

$$P_u = \bar{\sigma}_{\mathrm{gr_c}} t_c w = \bar{\sigma}_{\mathrm{gr_a}} t_a w. \tag{4}$$

Their weight ratio is

$$\frac{W_c}{W_a} = \frac{t_c \gamma_c}{t_a \gamma_a} = \sigma_u \frac{(w - D)\gamma_c}{w \bar{\sigma}_{\mathrm{gr_c}} \gamma_a}, \tag{5}$$

where γ_c and γ_a are specific weights. In this calculations they are assumed 14.5 (typical for the composite considered here) and 27.3 N/dm^3, respectively. The ultimate tensile stress σ_u is chosen 405 Mpa, being the tensile strength of aluminium alclad 2024-T3.

In Figure 1, the weight ratio of the two plates is given as function of D/w. The graph shows that for relatively small holes the composite plate does not yield a weight reduction at all compared to the aluminium plate. This is due to the lack of plastic behaviour and related redistribution of stresses. The composite behaves brittle and, therefore, is sensitive to the presence of even a small hole, whereas the aluminium plate in that case behaves almost like a plate without a hole, a result of the net section yielding. For relatively larger holes this advantage is less dominant,

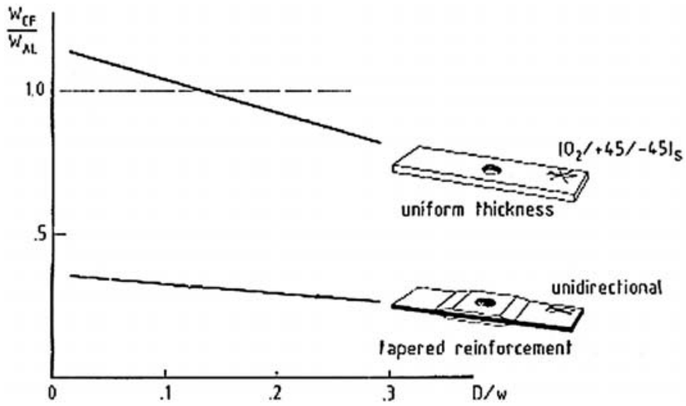

Figure 1. Design margin for a tension loaded plate with a hole.

and in the weight ratio it is overshadowed by the favourable specific weight of the composite.

In contrast to the relatively poor static behaviour of the composite plate, its ability to withstand cyclic loading is superior. The fatigue threshold is typically 60% of the relative static or residual strength, i.e., about 120 MPa in the present case. An identical stress level in the aluminium plate (30% of the relative static strength, $w/D = 20$ and $R = 0.05$) yields between 10^5 and 1,5 times 10^5 cycles to failure, Roebroeks [6]. Because the fatigue threshold of composites is such a high percentage of their static strength, composite structures are usually not fatigue critical. In the present example, it is apparently the static strength, which is the critical design consideration. The composite does not yield a considerable weight saving for that reason; its advantage obviously is an important improvement of lifetime.

Of course there is a possibility to reduce the weight of the composite plate by carefully tapering it. In the present case of tensile loading, this could be done by gradually leaving the ±45 – layers take care of a smooth stress transfer around the hole. Assuming that the weight of the reinforcement around the hole is 20% of the total weight of the composite plate. Further assuming that the UD material can be loaded to a 0.6% strain level at ultimate load, equivalent to a stress of 720 MPa. The thickness ratio of the UD part of the composite plate and the aluminium plate and the weight ratio of the two plates then are, respectively:

$$\frac{t_{\mathrm{UD}}}{t_{\mathrm{a}}} = \frac{\bar{\sigma}_{\mathrm{gr_a}}}{720} = \frac{405(w-D)}{720w} = 0.563\left(1 - \frac{D}{w}\right), \tag{6}$$

$$\frac{W_{\mathrm{c}}}{W_{\mathrm{a}}} = 1.2\frac{t_{\mathrm{UD}}\gamma_{\mathrm{c}}}{t_{\mathrm{a}}\gamma_{\mathrm{a}}} = 0.359\left(1 - \frac{D}{w}\right). \tag{7}$$

This weight ratio as function of D/w is also given in Figure 1. The reduction of the weight of the UD plate with tapered reinforcement, compared to the weight of the aluminium, is quite impressive, as Figure 1 shows. The disadvantages of this type of composite structure, however, are quite impressive as well. To mention the most important ones:

- It is difficult to fabricate and, therefore, expensive compared to the $[(0_2/+45/-45)_n]_s$ laminate with its uniform thickness. The tapering asks for a quite complicated procedure.
- The reinforcement is rather thick compared to the UD area. The thickness of the reinforced area is 3.8 times (!) the thickness of the UD material. So there is no smooth surface.
- The plate is fatigue sensitive in the tapered area.
  The UD area of the plate is relatively thin, which makes it sensitive to handling impact damage.

The two weight lines for the weight ratio in Figure 1 indicate the margin for a designer in which he can optimize between weight reduction, on one hand and reduction of fabrication costs and improved lifetimes on the other hand. It is obvious that the advantages of the two presented solutions must be carefully examined for each specific case. For outer skins, for example, the soft $[(0_2/+45/-45)_n]_s$ laminate might be considered, especially in damage-prone areas. In inner structures stiffer laminates, which are more weight-efficient but less damage tolerant, might be used effectively. More generally, however, we argue for a careful approach. In the presently discussed case it implies that we choose less weight reduction, cheaper fabrication and more reliability. This is even more important for compression loaded structures, since the tolerance to damage of composites in compression is often the critical design consideration.

If the $[(0_2/+45/-45)_n]_s$ laminate is applied, the global structure apparently is based on the required strength of the laminate with a stress raiser, in this case the circular hole. There is nothing of a local reinforcement, the laminate having a uniform thickness everywhere. This of course is the reason for the modest (or even no) weight reduction compared to the aluminum plate. It is noted, however, that the aluminium plate usually must be reinforced around the hole, due to its rather poor fatigue properties. So, in the case of the composite plate there might be some weight saving: not as a primary goal, but as an added benefit.

Example 2: Buckling of a Composite Plate

The second example of optimization between reduction of production costs, improved lifetime and a modest weight saving is a compression loaded composite plate. Just as in the previous example, an aluminium plate will be used as reference. The case of buckling for which a simple analytical solution for the buckling load is available will be considered, namely the in-plane loaded, locally buckling rectangular plate with four simply supported edges. The plate consists of a sufficient number of layers, and its stacking order is supposed to result in a (nearby) isotropic behaviour, both in-plane and out-of-plane. Moreover, the geometrical dimensions l and w and the wave pattern are supposed to be critical, yielding the minimum value of the buckling load. The well-known expression for this minimum is

$$N_{\mathrm{cr}} = \frac{1}{3}\frac{\pi^2 E t^3}{w^2(1-\nu^2)}. \tag{8}$$

Because of its unfavourable post-buckling behaviour, we do not allow the composite plate to buckle in the whole load range of an airplane. So the critical buckling load in this case is defined as the ultimate load. The aluminium plate, on the other hand, is allowed to buckle at the operational limit load, limit load and ultimate load being related with the classical factor 1.5. This results in the requirement for the respective buckling loads.

$$N_{\mathrm{cr_c}} = 1.5 N_{\mathrm{cr_a}}. \tag{9}$$

With the plates having the same dimensions l and w this requirement, after substitution of (8) directly yields the thickness ratio (t_c/t_a), whereas their weight ratio is

$$\frac{W_c}{W_a} = \frac{t_c \gamma_c}{t_a \gamma_a}. \tag{10}$$

Values of E and ν of the isotropic laminate are given in Table I. With E and ν of the aluminium plate taken as 72 GPa and 0.33, the numerical values of the thickness ratio and the weight ratio become 1.29 and 0.687, respectively. So the composite plate gives a weight reduction of more than 30% compared to the aluminium plate, although its critical buckling load is required to be 50% higher! It may be clear that, in terms of weight, the composite plate has a high structural efficiency. This is mainly due to the low specific weight of the composite (14.5 compared to 27.3 N/mm^3 of the aluminium).

The low specific weight can also be ‘translated’ onto other advantages, not only pure weight reduction. To show the margin in which a designer can optimize between various requirements, the composite plate is supposed to have a thickness yielding the same weight per unit width as the aluminium plate. This is equivalent to an increase of the thickness with a factor 1.456 (= 1/ 0.687) compared to the original thickness. Expression (8) then shows that, for an identical buckling load (note that this is a force per unit width), the width w of the plate can be increased as well, in the present example by almost 76%. The margin for the plate width is shown in Figure 2. If the considered plate is part of the skin of a stiffened panel, the increase of w is equivalent to the increase of the spacing of the stiffeners and, therefore, to a proportional decrease of their number.

Now the advantages of the increased thickness become apparent:

– Fabrication is easier compared to the panel with the original skin thickness, due to a reduced number of parts and fewer (difficult) joints.
– Inspection is easier during fabrication.

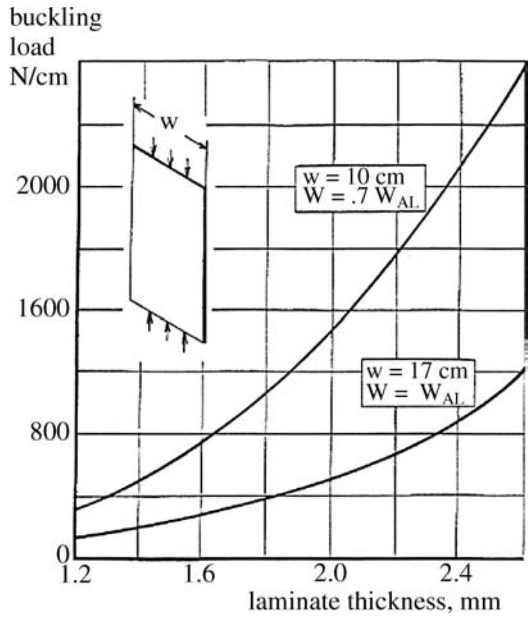

Figure 2. The design margin for compression loaded plate.

- At identical load levels, the stress is almost 32% lower in the most impact sensitive area, the skin. The panel will be more damage tolerant and less (compression) fatigue sensitive; therefore, it will have an improved lifetime. Moreover, if damage occurs, it will be more local and easier to inspect and repair.
- Thermal insulation is better.

Of course the higher price of the thicker skin material is a disadvantage. It will, however, easily be cancelled out by the lower price of the reduced number of stiffeners.

Finally, there is the added benefit of a modest weight reduction. Although the panel skin was supposed to have the same weight as the aluminium counter part, the total weight of a panel will be lower due to the reduced number of the stiffeners. Thus, even when weight reduction is not made a primary goal, clever application of composite design principles can yield a wide range of structural and manufacturing advantages including a reduced airframe weight.

Conclusions

From these two examples can be concluded that weight saving should not be the main goal to improve a design. If the weight saving gained by exchanging aluminium with composites is used to improve other aspects of a design like less stress concentrations, better fatigue life, better thermal and acoustic properties and reduction of parts, the production and maintenance cost can be reduced. Therefore a multidisciplinary concept might be more competitive than the lightest possible design.

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