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The Ratio of the Momentum and Material Vertical Eddy Transfer Coefficients in Buoyancy Driven Turbulence

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With 2 Figures

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Summary

The eddy coefficients for vertical transfer are calculated for a hypothetical pattern of thermal convection possessing certain similarity properties and their ratio is found to be given by

$$\frac{K_M}{K_H} = \frac{\sigma}{1-\sigma} \frac{C^2}{3n}$$

where σ is the fraction of the area occupied by thermals, C is a constant giving the velocity of a thermal in terms of its buoyancy and size, and n represents its angle of spread. Typical values of this ratio are thought to be in the range 10^{-1} to 10^{-3}. A value correspondingly larger than unity can be expected in a stably stratified fluid.

The phenomenon is explained in terms of the elastic properties of the tangled vortex tubes which compose the turbulence: vertically oriented ones are well placed to produce a Reynolds stress, but transfer no material; horizontal ones produce no stress but transfer material effectively. Only the latter are generated by the buoyancy forces directly in thermal convection.

Zusammenfassung

Das Verhältnis der turbulenten Austauschkoeffizienten für Impuls und für Beimengungen in konvektionsbeeinflußter Turbulenz

Die turbulenten Austauschkoeffizienten für Vertikaltransport werden für eine hypothetische Art thermischer Konvektion berechnet, die gewisse Ähnlichkeits-

bedingungen erfüllt. Es zeigt sich, daß das Verhältnis der Koeffizienten gegeben ist durch

$$\frac{K_M}{K_H} = \frac{\sigma}{1-\sigma} \frac{C^2}{3n}.$$

σ ist der von Thermik bedeckte Anteil der betrachteten Fläche. C ist eine Konstante, die die Geschwindigkeit einer Thermikblase als Funktion ihres Auftriebs und ihrer Größe angibt. n stellt den Winkel dar, unter dem sich die Blase vergrößert. Typische Werte dieser Verhältniszahl liegen in der Größenordnung 10^{-1} bis 10^{-3}. Für eine stabil geschichtete Strömung können Werte dieser Zahl größer als 1 erwartet werden.

Das Verhalten dieser Zahl wird mittels der elastischen Eigenschaften von Wirbelröhren, welche die Turbulenz darstellen, zu erklären versucht: Vertikal angeordnete Röhren verursachen zwar eine Reynoldssche Schubspannung, transportieren jedoch keine Beimengungen. Horizontale Röhren dagegen erzeugen keine Schubspannung, sind jedoch im Transport von Beimengungen sehr ergiebig. Die Auftriebskräfte in thermischer Konvektion erzeugen lediglich horizontale Röhren.

1. Choice of a Case for the Estimation of K_M/K_H

Actual convention in the atmosphere is complicated by variations in time, and from place to place, of the heat flux near the surface, by the presence of a condensation level and a stable stratification of the environment of the convection clouds above it and by considerable fluxes of heat except near the surface through larger scale convergence than thermals. The purpose of this paper is to examine a very special and simple kind of thermal convection to see how its effectiveness for the transfer of heat and momentum compare with one another.

Because of the great variety of factors with physical dimensions affecting the motion, each with a region of the atmosphere in which it is more dominant, we cannot expect similarity. The argument we shall use depends very much on the motion being similar over a fair height range but the conclusions are not really restricted to that case so long as the convection is composed of thermals; they merely need rather careful interpretation.

In order to obtain similarity we envisage the motion of a body of buoyant fluid to be determined solely by its size and buoyancy (total weight deficiency). The motion as a whole is so turbulent that molecular transfer is negligible. The smaller eddies are generated by, and are proportional to the larger buoyancy produced motions namely the thermals or whatever they may be, and we study the motion at a sufficient distance from the boundary for radiative transfer and the details of the surface features to be of little importance. The eddies are essentially buoyancy driven, which means that we must be above the layer in which the eddies are predominantly generated by flow over a rough boundary.

The convection must not have a top near at hand, otherwise its presence will exert a complicating influence on the thermal size and there will be no similarity. Thermals must therefore grow indefinitely in size with height, and must therefore combine. Since the height, z, is the only length parameter imposed, the thermals must have a size, r, proportional to it: thus

$$z = n r \tag{1}$$

where n is a number determined by the geometry of the motion of thermals. The geometry must be independent of z, and the amalgamation of thermals must reduce the number with height so that the same fraction of a horizontal area is occupied by thermals at all heights.

The behaviour of the thermals, although they are continually amalgamating, is assumed to be described by the equations for an isolated thermal. The chief grounds for this simplification are that a thermal is a very dissipative configuration of motion and achieves its ultimate motion pattern very quickly, probably in the course of rising about one diameter. The motion which turns the thermal inside out in such a way that entrained fluid is rapidly dispersed throughout the whole thermal, even if entrained over a small part of the surface, ensures that axial symmetry is rapidly re-established after amalgamation. We therefore write

$$w = C\,(g\,B\,r)^{1/2} \tag{2}$$

$$B = \Delta\varrho/\varrho \tag{3}$$

where w is the vertical velocity. C is a constant measured by experiment. B, w, and ϱ may be mean values of some particular values such as the maximum buoyancy anomaly, the rate of rise of the top and density.

In order to maintain similarity the buoyancy flux must be independent of height; otherwise a length would be introduced which would interfere. The environment of the thermals must be subsiding without local change of potential temperature, which must therefore be uniform. The convection is thus not warming the air but transporting buoyancy through it. An air mass is warmed by this type of thermal convection only when the environment is stably stratified, and in that case similarity would disappear unless the stratification varied in a unique way with height. Thus we could image a variety of thermal strengths, some of the feebler ones reaching an equilibrium level and becoming part of the environment, others being absorbed into more buyant ones as already envisaged.

The theoretical aspects of the argument to follow do not depend on the precise form which thermal convection takes. If enough were known about plumes from infinitesimal sources in a statically unstable medium calculations could be made on the assumption that convection was composed of them. But only thermals have been sufficiently studied for a numerical result to be obtained. One reason for this is that thermals bear a closer resemblance to what is observed in the atmosphere by glider pilots and by other means, and plumes seem to be rather a feature of laboratory experiments in which the mechanisms for heating the surface are different from that for ground heating.

Thermals have another advantage over plumes or any other configuration in this analysis because we need to know the buoyancy flux produced by them as well as the momentum flux. In a plume a detailed knowledge of the correlations between vertical velocity and horizontal velocity and buoyancy would have to be known. A thermal on the other hand transfers momentum and buoyancy as if the mean values pervaded the whole thermal and it had a closed boundary which contains the transferred material whose motion is known in terms of the mean values.

Fig. 1 is a visual impression of the thermal convection we are discussing.

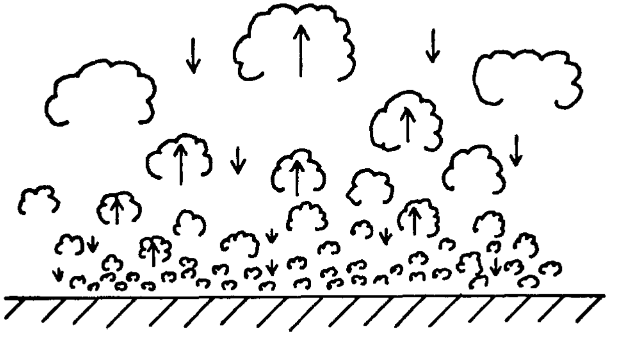

Fig. 1. When similarity prevails thermals must (a) have a size proportional to height (b) occupy the same fraction of a horizontal area at all heights. Consequently they must amalgamate during ascent so as to decrease in number. As they rise they also grow by entrainment of air of the environment, and so the rate of descent of the environment must decrease downwards, and such environment as reaches the lower levels has the same potential temperature as that higher up. The figure gives an impression of the pattern of thermals

2. The Details of the Similarity Regime of Convection by Thermals

The mean vertical velocity is assumed to be zero. This is equivalent to neglecting density variations in an isobaric surface in the equation of continuity of mass. Thus if σ is the fraction of the whole area occupied by thermals

$$\sigma w + (1-\sigma) w_e = 0 \tag{4}$$

where w and w_e are the upward velocities of the thermals and the environment.

If $\bar{\tau}$ is the mean buoyancy (reciprocal of absolute potential temperature) at any level and τ' and τ_e' are the mean anomalies of the thermals and the environment,

$$\bar{\tau} = \sigma(\bar{\tau} - \tau') + (1-\sigma)(\bar{\tau} - \tau'_e) \tag{5}$$

so that τ' is a positive quantity and τ_e' is negative, and

$$\sigma\tau' + (1-\sigma)\tau'_e = 0. \tag{6}$$

For a liquid we would replace τ by ϱ.
The buoyancy flux is

$$F = \sigma w \tau' + (1-\sigma) w_e \tau_e' = \frac{\sigma}{1-\sigma} w \tau' \tag{7}$$

by means of (4) and (6). We can express this as a function of height z above the apparent level of origin of the thermals as follows:

$$B = \tau'/\bar{\tau} \tag{8}$$

so that

$$w\tau' = C\left(g\frac{\tau'}{\tau} r\right)^{1/2} \tau' = C\,(g\tau'^3 z/n\bar{\tau})^{1/2}. \tag{9}$$

Now

$$\bar{\tau} = \tau_e - \tau'_e = \tau_e + \frac{\sigma}{1-\sigma}\tau' \tag{10}$$

so that because τ' tends to zero as z increases, $\bar{\tau}$ tends to τ_e which is the buoyancy of the subsiding environment. Since we are ignoring the region near the surface in which τ' is comparable with $\bar{\tau}$ and in which the thermals are infinitesimally small, we may put $\bar{\tau}$ equal to τ_e in (9) and obtain from (7)

$$F = \frac{\sigma C}{1-\sigma}\left(\frac{g}{n\tau_e}\tau'^3 z\right)^{1/2}. \tag{11}$$

For a flux independent of z it is therefore required that

$$\tau' \propto z^{-1/3}. \tag{12}$$

By differentiation of (10) we obtain

$$\frac{\partial \bar{\tau}}{\partial z} = \frac{\sigma}{1-\sigma} \frac{\partial \tau'}{\partial z} \tag{13}$$

and so

$$-\frac{\sigma \tau'}{1-\sigma} \bigg/ \frac{\partial \bar{\tau}}{\partial z} = -\tau' \bigg/ \frac{\partial \tau'}{\partial z} = 3z \tag{14}$$

which is a result required later.

We may also remark that the well known result that

$$\frac{\partial \bar{\tau}}{\partial z} \propto z^{-4/3} \tag{15}$$

is also obtained, and that this depends only on (a) the assumption that geometrical similarity exists, (b) the formula (2) which follows directly from that without any reference to a particular geometrical model, and (c) the assumption of buoyancy flux independent of height. It is therefore a purely mechanical result and is obtained without reference to heat. It is clear from the argument used here that it is not a valid formula close to the surface for two reasons: first, the physical mechanisms of radiation and molecular transfer which are dominant there are ignored, and secondly the Boussinesq approximation, made in obtaining (4) and again in writing τ_e for $\bar{\tau}$ in deriving (11) is not valid there.

In view of (12) we have immediately from (7) that

$$w_e \propto z^{1/3} \tag{16}$$

for F and σ independent of z. The picture of convention thus implies that the subsidence velocity of the environment decreases downwards because the subsiding air occupies the same area at all heights while the thermals are entraining some of it at every level. The upward velocity of the thermals is proportional to the same power of z, according to (4) so that they accelerate upwards. This is much more in accord with observation than the behaviour of an isolated thermal having a constant total buoyancy anomaly, for then

$$B \propto r^{-3} \tag{17}$$

and so with (2) and (1) we have

$$w \propto z^{-1} \tag{18}$$

The amalgamation of thermals thus has a most important effect. The acceleration must mean that the value of C which will be used, which is for isolated thermals, is not precisely correct for this convection, but the error is not significant in this context. The mechanisms by which a thermal approaches its limiting velocity are not like those of a normal system approaching the limit exponentially. The motion is established by the orderly operation of buoyancy forces and the region occupied by eddy motions increases if the thermal is decelerated. The configuration of the motion changes, and it is not a simple balance between an accelerating force and a brake proportional to the velocity.

3. Rise through Wind Shear

This problem was discussed at length by HALL [2], whose paper includes the main result of this one but without a full discussion of the implications of its derivation and its consequences. An isolated thermal experiences a vertical acceleration relative to its

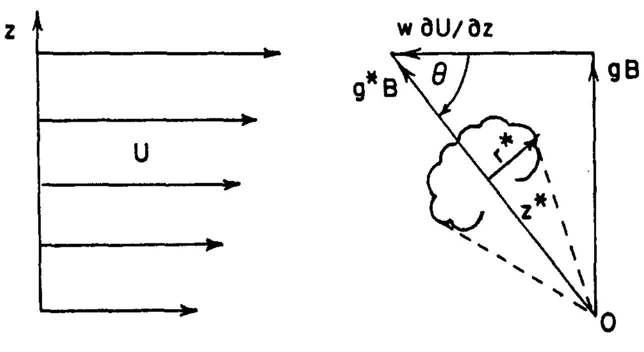

Fig. 2. Coordinates and notation for a thermal rising through wind shear. It moves as if subjected to a buoyancy $g^* B$ which is the vector addition of the actual buoyancy $g B$ and the relative acceleration of the environment $w \partial U / \partial z$

environment equal to $g B$. When it rises through wind shear, $\partial U / \partial z$, its environment is accelerated relative to it with a horizontal acceleration $w \partial U / \partial z$. The total acceleration is therefore the vector sum of these, whose magnitude we can represent by $g^* B$. If Θ is the inclination of the axis of symmetry of the thermal to the horizontal (see Fig. 2)

$$g^* B = g B / \sin \Theta = w \frac{\partial U}{\partial z} / \cos \Theta \tag{19}$$

$\Theta = 90^0$ if there is no shear.

The width of the inclined thermal, r^*, and, z^*, the distance from its instantaneously apparent origin in space, are related to its total inclined speed $w/\sin\Theta$ and relative horizontal velocity u by

$$\frac{w}{\sin\Theta} = \frac{u}{\cos\Theta} = C\,(g^* B\, r^*)^{1/2} = C\left(\frac{g}{\sin\Theta}\, B\, \frac{r}{\sin\Theta}\right)^{1/2} \tag{20}$$

and so

$$w = C\,(g B r)^{1/2} \tag{21}$$

and

$$\frac{u}{\partial U/\partial z} = \frac{w \cot\Theta}{w^{-1} g B \cot\Theta} = C^2 r \tag{22}$$

which is required later.

This part of the argument could not be sustained in such a simple form for plume convection because a bent over plume is certainly not like a vertical plume tilted over: each section of it is much more like a section of a two-dimensional thermal. Even this argument for thermals is restricted by the assumption that the environment has a single value for its horizontal velocity as far as the thermal is concerned. This means that the typical velocities due to the relative motion must be large compared with the differences in velocity due to the wind shear in the thermal's neighbourhood. Thus

$$r\,\frac{\partial U}{\partial z} \ll w \tag{23}$$

is a requirement.

The assumption in HALL's paper [2] that the two components of the relative acceleration can be compounded has been given some additional justification by some experiments by RICHARDS [5] on puffs. A body of fluid ejected into an otherwise stationary environment of the same density assumes a configuration of velocity which is for practical purposes identical with that of a thermal. This was a rather unexpected result because it was thought that the region of mixing in a thermal arose from buoyancy forces. A puff can be made without a mixing region in the form of a ring vortex, but if care is taken to make it initially turbulent it retains the thermal-like velocity pattern. Thus the buoyant effects represented in the vertical accelerations can be compounded with inertial effects due to the horizontal accelerations.

4. The Ratio K_M/K_H

By equating the buoyancy flux implied by the definition of K_H to the formula for the flux in (7) we obtain

$$-K_H \frac{\partial \bar{\tau}}{\partial z} = \frac{\sigma}{1-\sigma} w \tau' \tag{24}$$

and so

$$K_H = -\frac{\sigma}{1-\sigma} w \tau' \Big/ \frac{\partial \bar{\tau}}{\partial z} = 3 w z \tag{25}$$

by (14).

By the definition of U we have that if u and u_e are the horizontal velocity anomalies of the thermals and the environment,

$$U = \sigma (U-u) + (1-\sigma)(U-u_e) \tag{26}$$

and so

$$\sigma u + (1-\sigma) u_e = 0. \tag{27}$$

By the definition of K_M

$$-K_M \frac{\partial U}{\partial z} = -\sigma u w - (1-\sigma) u_e w_e = -\frac{\sigma}{1-\sigma} u w \tag{28}$$

and so by means of (1) and (22) we obtain

$$K_M = \frac{\sigma}{1-\sigma} \frac{u w}{\partial U/\partial z} = \frac{\sigma}{1-\sigma} C^2 w r = \frac{\sigma}{1-\sigma} \frac{C^2}{n} w z. \tag{29}$$

Therefore

$$\frac{K_M}{K_H} = \frac{\sigma C^2}{(1-\sigma) 3 n}. \tag{30}$$

5. The Numerical Value of K_M/K_H

The argument above does not really require that there should be a very deep layer in which the similarity exists. K_M and K_H could be expressed in terms of the local value of r, and all we assume really is that the mechanics of the individual thermal, whatever sort of environment it is in, are rather like those of the similarity regime. The only factor in (30) which would be significantly altered by changing the regime is the number 3 which arises because of the law represented in (12). Convection with a buoyancy flux decreasing with height could not have a very different value, and much more uncertainty arises in estimating σ.

The factors C^2 and n are those which are most characteristic of the particular convection that occurs, and these have to be obtained from experiment because there is no theoretical means of obtaining them. C represents the relationship of w to B and r and gives the rate of rise of a buoyant mass; n represents the rate of widening or mixing of the buoyant fluid as it rises: neither of these can be in error by more than a fraction of an order of magnitude.

The most complete set of experiments on thermals has been made by RICHARDS [4] and in the place of (2) he found it more consistent to write

$$w = {}^1/_2\, C_1\, n^{3/2} (m B g z)^{1/2} \tag{31}$$

where the volume of the thermal is $m r^3$. Thus our C is given by

$$C = {}^1/_2\, C_1\, n^{1/2} m^{1/2}. \tag{32}$$

He found that although C and n varied considerably from thermal to thermal while remaining constant for the life of each thermal, C_1 was very much more nearly the same for all thermals with a value of 0.73. C, on the other hand, ranged from about 1.7 to 7. Also m is about 3. Thus

$$\frac{C^2}{n} = {}^1/_4\, C_1{}^2 m = 0.4 \text{ roughly.} \tag{33}$$

Values obtained in the earlier experiments were somewhat lower than this but SAUNDERS [6] obtained values of $C = 1.5$, $n = 5$ from observations of cumulus clouds in Sweden. His larger value of n probably represents the effect of erosion of the thermals in the stably stratified environment with evaporation.

The value of σ varies quite a lot. Glider pilots find that up-currents are much stronger than the intervening down-currents, which can scarcely be detected in fair weather cumulus conditions in the environment of thermals below cloud. HARDY and OTTERSTEN [3] and others have obtained radar pictures of thermals giving $0.1 < \sigma < 0.6$. In a cloud layer σ could range from as much as 0.5 to as little as 0.01 or less in oceanic areas. Where the visible cloud occupies as much as half the area the up-currents occupy a much smaller fraction of the area. Thus we can probably say that

$$\frac{1}{3} > \sigma \tag{34}$$

and in some circumstances very small values could be found.

The quantity σ enters into the formula for K_M/K_H because if σ is very small, for a given mean buoyancy gradient the buoyancy anomaly is very large: consequently a reduction to a very few intense thermals makes them each carry a lot of buoyancy but they do not carry a corresponding increase in momentum.

If we put $\sigma = 0.1$ we obtain

$$\frac{K_M}{K_H} = \frac{\sigma}{1-\sigma} \frac{C^2}{3n} = 0.013 \tag{35}$$

and we could expect values to range from about 0.1 to perhaps 10^{-3}. It appears therefore that thermals are much more efficient at transferring buoyancy (or heat) than momentum.

6. Convection by Cumulus in a Stratified Environment

The configurations of motion and buoyancy anomaly could be very similar around a cloud thermal to what we have supposed for dry ones, but because the environment is stably stratified our calculation of the heat transfer has to be altered. The estimate we have just made would be more correct if K_H were taken to represent the water vapour transfer coefficient because humidity is not far from uniform in the environment of the cumulus and the nonuniformities have a negligible affect on the motion anyway.

Near the top of a layer containing a steady amount of cumulus the continual evaporation of the clouds produces a very significant amount of downdraughts which might cause a mean upward vertical velocity in the environment as explained by FRASER [1]. This leads to an overall cooling of the layer. However the result still represents the fact that the thermals will be a very good agent for the transfer of water vapour but poor for momentum.

The importance of this result is that in parameterising the effects of cumulus in large scale numerical models it will be adequate to introduce a very large buoyancy or vapour transfer coefficient while transferring no momentum as the same time. This raises the question of how to represent the buoyancy transfer. If we made the transfer coefficient infinite we would simply put temperature gradient as soon as the cumulus appeared equal to that which cumulus convection ultimately gives rise to. If we wished to have a time lag we should express the transfer in terms of the departure of the gradient from this value, which would itself depend on whether the whole air mass became saturated or retained a constant cloud amount.

The main feature of cumulus convection which renders this treatment of heat transfer inapplicable is that most of the heat transfer is by subsidence or ascent of the environment. That the clouds transfer a negligible amount of heat was demonstrated by Fraser [1]. The lapse rate of the environment is not related to the buoyancy anomaly of the thermals, and so we cannot obtain results like those for the similarity regime.

7. The Mechanism of Selective Transfer

The effect of buoyancy forces in modifying the transfer by eddies has been discussed in a limited way by Scorer [7, 8]. It is appropriate to state the ideas anew here in the light of the other arguments presented.

In order to produce a Reynolds stress the eddy motion must have its energy increased at the expense of the mean shear. It is well known that vorticity is increased by the stretching of vortex lines, but if the vortex lines of a chaotic pattern of vorticity (i.e. turbulence) are stretched by the shearing motion of the fluid on a large scale the kinetic energy of the eddy motion is only increased thereby if the vorticity is in the form of tubes rather than sheets — "spaghetti rather than lasagne". Stretched sheets merely become thinner and the vorticity is increased merely by bringing the fluid of different velocities closer together. But stretched tubes acquire more energy of rotation. That spaghetti-like vorticity behaves like elastic strands can be demonstrated by calculating conditions under which waves can be propagated along vortex tubes. Waves cannot be propagated along a vortex sheet but disturbances remain in situ and become distorted by the redistributed velocity pattern.

We take cylindrical polar coordinates (r, Θ, z) with fluid velocity components $(0, r\Omega, 0)$ in the undisturbed state and add a small perturbation (u, v, w). To consider a tube of vorticity we assume that $\Omega = \Omega(r)$. The equation of motion are

$$\frac{Du}{Dt} - \frac{(v + r\Omega)^2}{r} = -\frac{1}{\varrho}\frac{\partial p'}{\partial r} = -\frac{1}{\varrho}\frac{\partial p_0}{\partial r} - \frac{\partial p}{\partial r} \tag{36}$$

$$\frac{D}{Dt}(v + r\Omega) + \frac{u(v + r\Omega)}{r} = 0$$

$$\frac{Dw}{Dt} = -\frac{1}{\varrho}\frac{\partial p'}{\partial z} = -\frac{\partial p}{\partial z}$$

where, with constant density ϱ and pressure equal to

$$p' = p_0 + \varrho p \tag{37}$$

we have represented the perturbation of pressure by ϱp. If there is a perturbation proportional to $\exp i\,(kz - \sigma t)$, corresponding to waves propagated along the rotation axis,

$$\begin{aligned} \frac{D}{Dt} &= \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z} \\ &= -i\sigma \text{ for the perturbation,} \\ &= u\frac{\partial}{\partial r} \text{ for the undisturbed motion.} \end{aligned} \tag{38}$$

The linear terms of the equations are

$$\left.\begin{aligned} i\sigma u + 2\Omega v &= \frac{\partial p}{\partial r} \\ -i\sigma v + u\frac{\partial}{\partial r}(r\Omega) + u\Omega &= 0 \\ i\sigma w &= \frac{\partial p}{\partial z} \end{aligned}\right\} \tag{39}$$

and the equation of continuity is

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0. \tag{40}$$

Eliminating p, v, and w, we get

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \left[\frac{1}{r^2} + k^2\left(1 - \frac{4\Omega^2 + 2r\Omega\Omega'}{\sigma^2}\right)\right]u = 0 \tag{41}$$

where

$$\Omega' = \partial\Omega/\partial r.$$

In order that waves shall be trapped in the tube it is necessary that u be zero on the axis, have an oscillatory form for small values of r and an exponential form for large values. We simplify the analysis by writing

$$\Phi = r^{1/2}u, \quad x = kr, \quad S = 2\Omega/\sigma \tag{42}$$

and then for a region in which Ω is constant we find

$$\frac{\partial^2\Phi}{\partial x^2} = \left(\frac{3}{4x^2} + 1 - S^2\right)\Phi \tag{43}$$

and for a region of zero vorticity in which

$$\Omega = \frac{K}{r^2}, \quad \Omega' = -\frac{2\,\Omega}{r} \tag{44}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = \left(\frac{3}{4\,x^3} + 1\right) \Phi \tag{45}$$

The simplest case in which waves can be trapped, that is transmitted along the tube, illustrates the situation well. It is one of solid rotation surrounded by irrotational motion. Trapping occurs if the coefficient of Φ takes the opposite sign in (43) for large r to that in (45) for small r. This is always the case if $\Omega \neq 0$ and r is large enough. Clearly the larger S, i. e. the larger Ω or the smaller σ, the smaller will be the radius within which the waves will be trapped, for Φ will decay exponentially when

$$S^2 > 1 + \frac{3}{4\,x^2}. \tag{46}$$

It is required at least that

$$S^2 > 1, \text{ i. e. } 2\,\Omega > \sigma \tag{47}$$

and so tubes cannot trap oscillations of frequency greater than their vorticity; but all frequencies much smaller are trapped at a very small radius.

The disturbances represented by the mean shear are of very low frequency, and so they are all trapped and the vortex tubes behave like elastic fibres and absorb energy when they are stretched. The motion is not reversible because the tubes are all the time distorting one another, and soon become randomly oriented again after the stretching. For shorter period large scale deformations however the eddies would introduce some visco-elastic properties into the fluid.

It is interesting that randomly chaotic vorticity, which is isotropic turbulence, initially produces no Reynolds stress, but as soon as a deformation makes it anisotropic, a continued deformation feeds "elastic" energy into the tubes.

For an incompressible inviscid fluid the vorticity equation is

$$\frac{D\,\omega}{D\,t} = (\omega \text{ grad})\, v - \frac{1}{\varrho} \text{ grad } \varrho \times \left(g - \frac{D\,v}{D\,t}\right) \tag{48}$$

and in motions produced by small buoyancy the acceleration is small compared with g, so that the last term can be ignored. The first term on the right represents the advection of the vortex lines with the fluid. Clearly the vorticity produced by gravity is perpendicular to g; and this represents the vortex ring motion characteristic of thermals. But in horizontal mean motion with a vertical gradient, horizontal vortex lines are not stretched, and so thermals cannot initially produce any Reynolds stress. After a vertical displacement has taken place the rings become inclined as indicated in Fig. 2: but in thermals the motion is being continually destroyed by smaller eddies and recreated by the buoyancy forces, so there is a tendency all the time to create horizontal vortex tubes, and no Reynolds stress.

On the other hand horizontal vortex rings are the best configuration of vorticity for transferring material, or heat or pollution or anything fixed to the material, while vertical vortex lines, which are in a good position to produce a Reynolds stress do not transfer material and are not created in a buoyancy driven eddy motion.

It is a reasonable conclusion that since in a stable stratification the horizontal vortex lines which are transferring material vertically are put into reverse by gravity, a stable stratification will inhibit material transfer. This is obvious on any picture of material transfer, but the vertically oriented vortex tubes will not be stopped by the gravity field, and so, except in so far as the general level of turbulence is lower in a stably stratified fluid anyway, the Reynolds stresses will not be affected. We can therefore expect that in this case

$$K_M/K_H > 1. \tag{49}$$

Unsteady two dimensional stable waves in a horizontally stratified fluid can transfer momentum vertically: for example the waves produced in the lee of a suddenly introduced obstacle transfer momentum from the obstacle and therefore through the fluid to a finite volume of it. These are motions which transfer momentum and no material property at all, and for which, therefore

$$K_M/K_H = \infty. \tag{50}$$

This does not justify the description of the wave motion as a form of turbulence but it does imply that momentum is of a different nature from other transferable properties in fluids. Reynolds analogy between the momentum and material transfer is not generally valid.

The propagation of momentum by waves is really of quite a different kind and cán be excluded from the discussion of eddy transfer mechanisms on the ground either that it is not a diffusive motion or that it is a motion with a clearly defined scale which does not decrease with time.

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