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Finite Element Dynamic Analysis of Laminated Viscoelastic Structures

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Abstract This work is concerned with the dynamic behavior of laminated beam, plate and shell structures consisting of a viscoelastic damping layer constrained between two structural layers. Finite element models for modal, harmonic and transient analyses are developed. The dynamic interlaminar shear stresses are determined and presented under harmonic and transient loads. The effect of the damping ratio of the viscoelastic material is investigated. It is found that the viscoelastic material damping reduces the interlaminar stresses. The results also show the dependency of the viscoelastic material on frequency, hence, the effect of the viscoelastic material appears significantly under harmonic loading. In transient analysis, the importance of the viscoelastic material is observed in absorbing the impact and returning the structure to its original configuration.

Keywords Finite element · Viscoelastic · Laminated beam · Plate and shell · Dynamic interlaminar stresses

1 Introduction

Viscoelasticity is the material response that exhibits characteristics of both a viscous fluid and an elastic solid. Viscoelastic materials have been used to enhance damping in structures in three different ways: free-layer damping, constrained-layer damping and tuned viscoelastic damping. The properties of viscoelastic materials depend strongly on vibration frequency, dynamic loading, temperature and humidity. Viscoelastic materials are suitable for applications where weight is an important factor, mainly automotive and aerospace industries, to reduce vibration and noise [1].

In order to predict the viscoelastic material response under different loading conditions, some well-known models, such as the Maxwell model, the Kelvin-Voigt model, and the standard linear solid model, are usually used (Fig. 1). In these models, viscoelastic behavior

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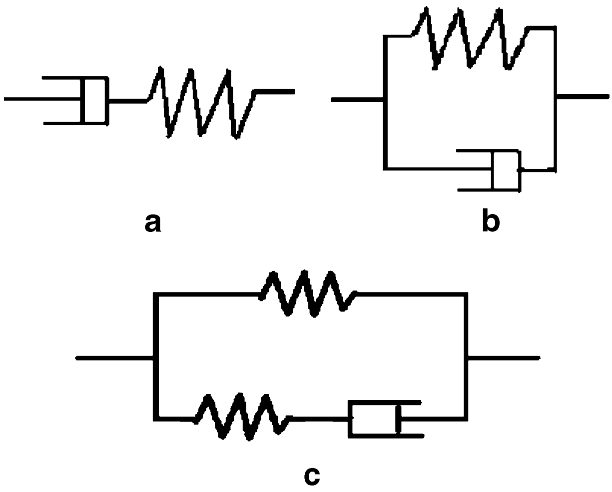

Fig. 1 Viscoelastic material models: **a** Maxwell model, **b** Kelvin-Voigt model, and **c** the standard linear solid model

is comprised of elastic and viscous components modeled as linear combinations of springs and dashpots, respectively. Each model differs in the arrangement of these elements.

Studying the structures that contain a viscoelastic damping layer has attracted many researchers. Suzuki [2] studied the dynamic load factors of viscoelastic beams, frames and rings. Dokmerci and AlpD [3] presented a continuum theory for viscoelastic composite beams. Many other researchers studied the dynamic response of viscoelastic beams [4–10], plates [11–16] and shells [17–23]. The previous invetigations have mainly concentrated on the damping characteristics of the viscoelastic material that resist noise and vibration in the structure. However, they did not concentrate on the interlaminar (interfacial) stresses developed between the viscoelastic layer and the basic structure on one side and the constraining layer on the other side. It is known that interlaminar stresses play a major role in the behavior of laminated structures. One of the possible results of interlaminar stresses is the delamination failure.

This work is concerned with the dynamic interlaminar (interfacial) stresses developed between the viscoelastic layer and the surrounding layers in laminated beam, plate and shell structures consisting of a viscoelastic damping layer constrained between two structural layers. Finite element models are developed and analyzed for modal, harmonic and transient analyses. The first natural frequencies are determined. The effect of the damping ratio of the viscoelastic material is investigated.

2 Material Properties

Each of the structures under consideration consists of three layers: a viscoelastic layer (host layer) sandwiched between two isotropic layers (constraining layers). The viscoelastic layer is made of HEREX C70.130 foam while the constraining layers are made of a structural material (soda-lima silica float glass).

One of the main concerns in dealing with a viscoelastic material is the material properties, mainly the viscoelastic modulus. For such materials, the viscoelastic modulus is frequency-dependent. In addition, it is modeled as a complex number with the general form:

$$\widehat{E} = E(1 + i\eta) \tag{1}$$

Where E is the elastic modulus, i is the imaginary number and η is the viscosity. Both E and η are frequency-dependent. For the viscoelastic material under consideration (HEREX C70.130 foam), Meunier and Shenoi [13] presented the following expressions for the properties:

$$E = 4.9796 \ln f + 32.331, \ \eta = -0.013 \ln f + 0.2104 \tag{2}$$

where f is the frequency in Hz.

The density of HEREX C70.130 is 130 kg/m^3. The structural material (soda silica float glass) has a modulus of elasticity of 72 GPa and a mass density of 2550 kg/m^3.

3 Finite Element Models

The finite element models of the viscoelastic structures under consideration, namely the beam, the square plate and the cylindrical shell, are shown in Fig. 2. Modeling and analysis are carried on using the student version of the finite element software package ANSYS. The structural material is meshed with solid186 element, while the viscoelastic layer is meshed with visco89 element. Each of the two elements has 20 nodes, The layers are fully bonded by merging the nodes on the common boarders. Modal, harmonic and transient finite element analyses of the structures are performed. The finite element meshes were first refined till stable solutions of the natural frequencies are obtained. We set our own criterion for a stable solution where subsequent values of natural frequencies differ by less than 0.5%. Example of mesh refinement is shown in Table 1 for the beam case.

The viscoelastic beam under consideration is simply-supported on both ends. The plate is simply supported along all edges. The cylindrical shell is in a fixed-free condition.

4 Model Verification

In order to verify the present finite element models, we searched for published similar works. One similar work was found which is an experimental investigation of a viscoelastic plate done by Duser et al [11]. Therefore, our finite element model of the viscoelastic square plate is verified by comparing the predicted transverse deflections with the experimental results obtained by Duser et al [11]. The load on the plate is in the form of a uniform pressure applied on its upper surface. The pressure varies from 1000 Pa to 7000 Pa with a step of 1000 Pa. Figure 3 shows the results of the comparison in the form of the variations of the transverse deflections with the applied pressure, where a good agreement is found.

5 Results and Discussion

Finite element modal analysis of the simply-supported beam is first carried out to obtain the natural frequencies. Within a frequency range from 0 to 1000 rad/s, four natural frequencies

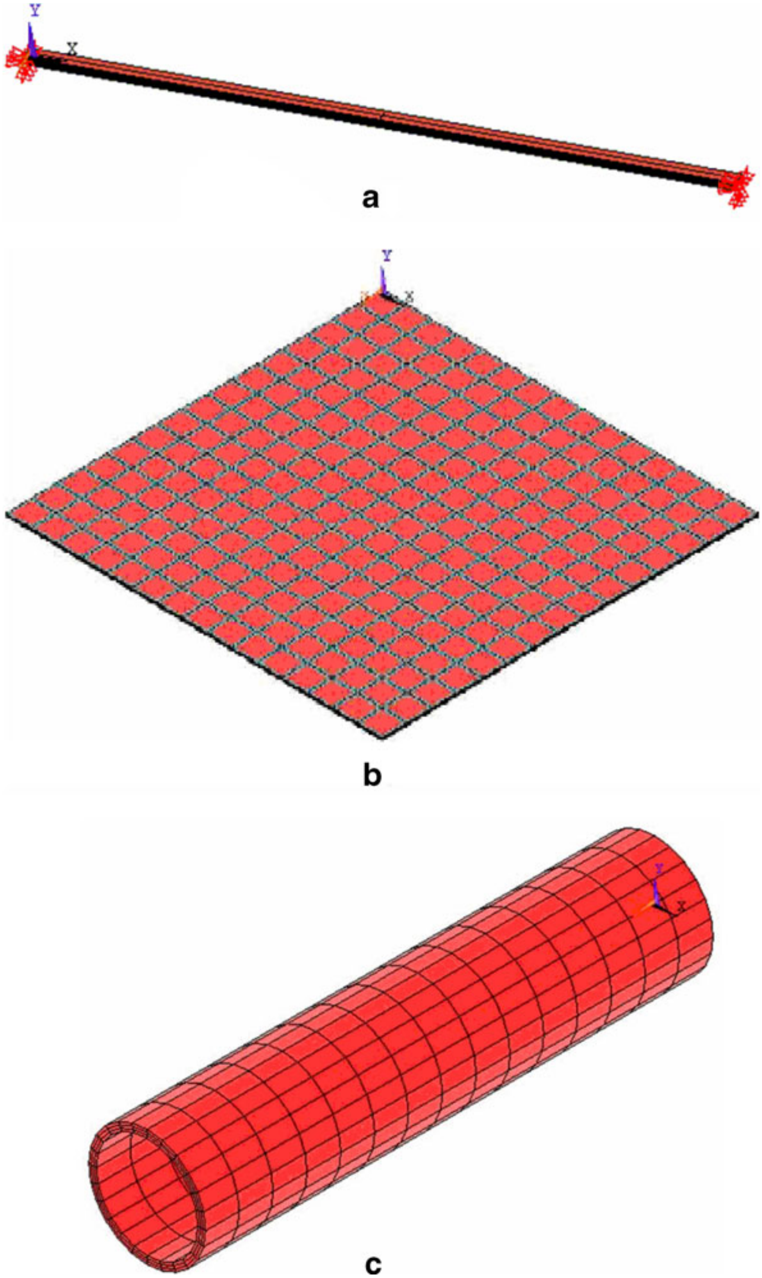

Fig. 2 Finite element models of the laminated viscoelastic structures **a** beam, **b** plate, and **c** cylindrical shell

are found with the values: 91.51 rad/s, 267.82 rad/s, 400.15 rad/s, and 654.33 rad/s. Then, a harmonic force is applied to the beam. The force, with a magnitude of 1000 N and a frequency range from 0 to 1000 rad/s, is applied at $x = L/3$, where x is the axial distance measured from the left support and L is the length of the beam. The deflections and the

Table 1 Mesh refinement results for the viscoelastic beam

Number of Elements	First Natural Frequency (Hz)	Second Natural Frequency (Hz)	Third Natural Frequency (Hz)	Fourth Natural Frequency (Hz)
100	17.354	42.457	73.329	79.945
150	16.794	42.459	71.846	87.756
200	15.873	42.561	67.337	94.654
250	14.738	42.578	64.328	99.846
300	14.696	42.590	63.967	103.518
350	14.605	42.605	63.694	103.853
400	14.565	42.625	63.686	104.140

interlaminar shear stresses are computed for different damping ratios of the viscoelastic material. Figure 4 compares the interlaminar harmonic deflection response of the finite element model of the viscoelastic sandwich beam for different damping ratios of the viscoelastic core. Since the viscoelastic material is frequency-dependent, variations are observed in the behavior of deflections as the forcing frequency is changed. As seen from the figure, more damping produces less deflections. The effect of the damping ratio is seen clearly at low frequencies where two natural frequencies exist: 91.51 rad/s and 267.82 rad/s. Figure 5 shows the interlaminar shear stress response for the viscoelastic sandwich beam model. The shear stress behavior is similar to the deflection, as can be seen that the higher the damping, the less the shear stress. This indicates the need to have a viscoelastic material with high damping in order to reduce the interlaminar shear stresses within the beam.

The response of the beam under transient loading is also investigated. The deflections and the interlaminar shear stresses are evaluated under the effect of a transient force of 1000 N applied to the beam at $x = L/3$ for 0.001 s. Figure 6 presents the results of the transient transverse deflections of the beam for different damping ratios of the viscoelastic material. Large deflections and vibration are observed in the early stages of time, and as time proceeds, the deflections die out. The damping effect is also clear here in reducing the

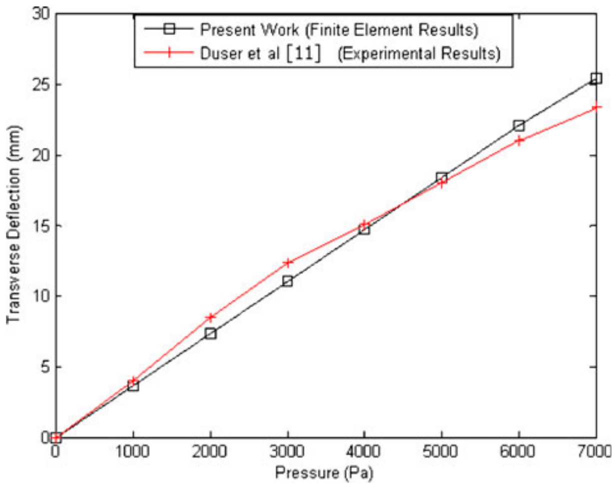

Fig. 3 Comparison between the present finite element results and the experimental results obtained by Duser et al. [11]

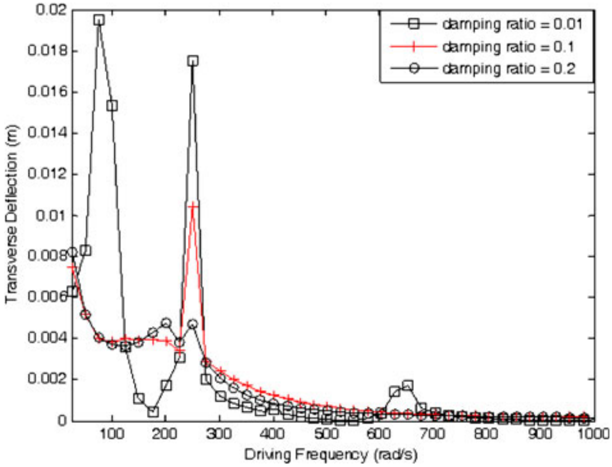

Fig. 4 Variations of the transverse deflections of the viscoelastic beam with the frequency of a harmonic force applied at $x = L/3$ for different damping ratios

deflections and hence the vibrations. The induced transient interlaminar shear stresses within the beam are shown in Fig. 7. In the early stages of time, high stresses are observed, which is due to the applied force. Due to damping, these stresses vanish with time.

The steps followed in the modeling of the beam are repeated for the case of the viscoelastic simply-supported plate under consideration. Modal analysis is carried out to obtain the natural frequencies. Within a frequency range from 0 to 1000 rad/s, four natural frequencies are also found with the values: 178.91 rad/s, 325.01 rad/s, 657.35 rad/s, and 895.90 rad/s. Then, a harmonic transverse load of 1000 N is applied to the plate at its mid-point with a frequency varying from 0 to 1000 rad/s. Finally, a transient force of 1000 N is applied at mid-point of the plate for 0.001 s. Figure 8 presents the variations of the

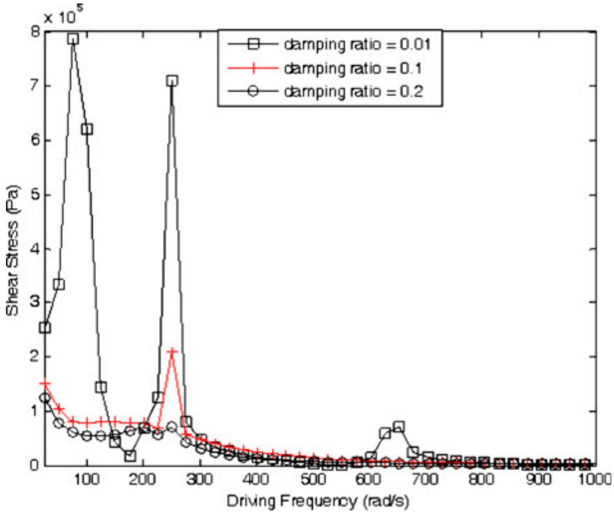

Fig. 5 Variations of the interlaminar shear stresses in the viscoelastic beam with the frequency of a harmonic force applied at $x = L/3$ for different damping ratios

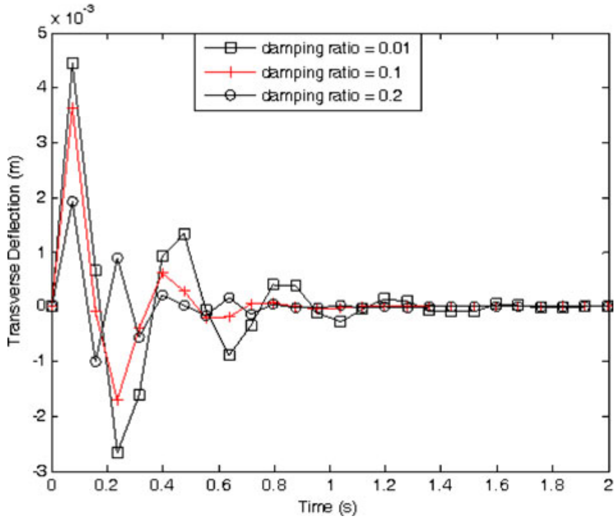

Fig. 6 Variations of the transverse deflections of the viscoelastic beam with time under the effect of a transient force applied at $x = L/3$ for different damping ratios

interlaminar shear stresses within the plate with the frequency of the harmonic load for different values of the damping ratio of the viscoelastic layer. The effect of damping in reducing the shear stresses is clear from the figure. For the light damping case, the peaks in the graph indicate the locations of the natural frequencies of the plate. Figure 9 shows the variations of the interlaminar shear stresses with time under the action of a transient force applied to the simply-supported plate at its mid-point. It is observed that the stresses initially rise rapidly to their maximum values close to the time of application of the force

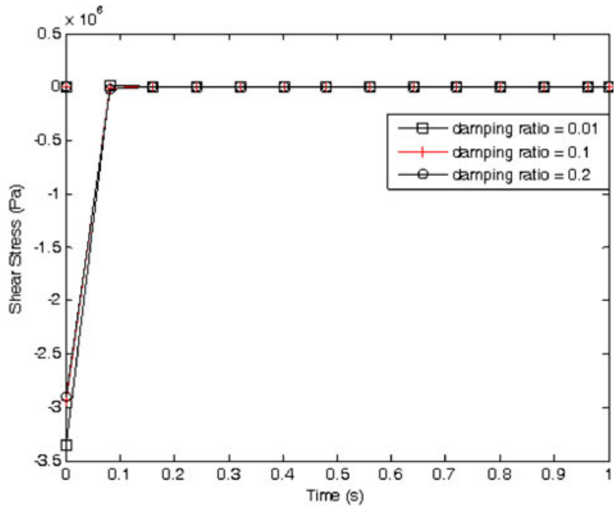

Fig. 7 Variations of the interlaminar shear stresses in the viscoelastic beam with time under the effect of a transient force applied at $x = L/3$ for different damping ratios

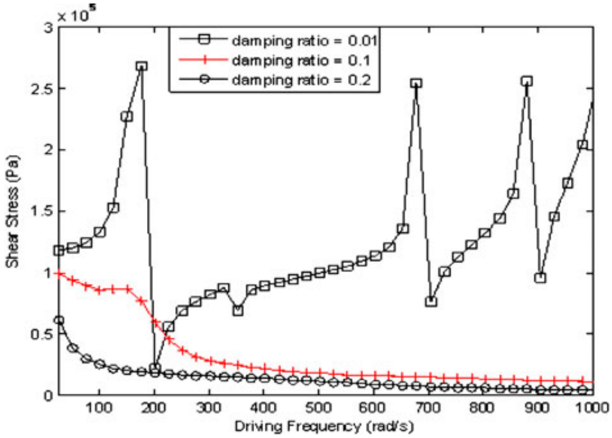

Fig. 8 Variations of the interlaminar shear stresses in the plate with the frequency of a harmonic force applied at its mid-point for different damping ratios

and then vanish as time proceeds due to the presence of the viscoelastic layer. The effect of damping is also clear in reducing the stresses, i.e, stresses are less under heavier damping.

A procedure similar to the beam and plate is repeated for the fixed-free cylindrical shell under consideration. Modal analysis produced only one natural frequency within a frequency range from 0 to 1000 rad/s, with a value of 788 rad/s. A harmonic radial force of 1000 N is applied to the shell at its free end with a frequency range from 0 to 1000 rad/s. Finally, a transient force of 1000 N is applied to the shell at its free end for 0.001 s. Figures 10 and 11 show the results of the computations of the finite element model in the form of the variations of the interlaminar shear stresses for different values of damping ratios of the viscoelastic layer. The results are similar to the beam and plate models.

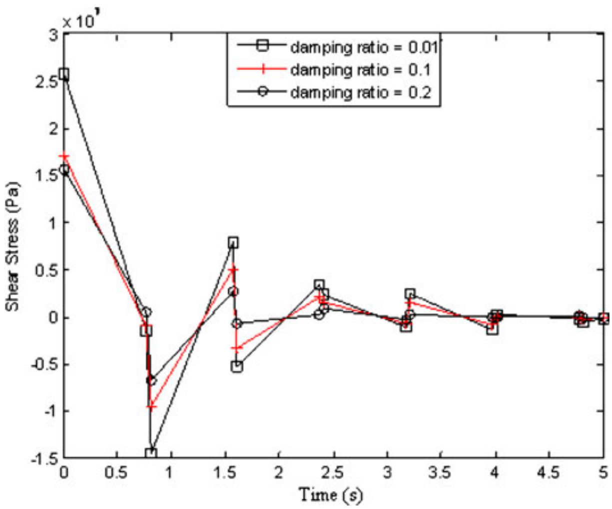

Fig. 9 Variations of the interlaminar shear stresses in the plate with time under the effect of a transient force applied at its mid-point for different damping ratios

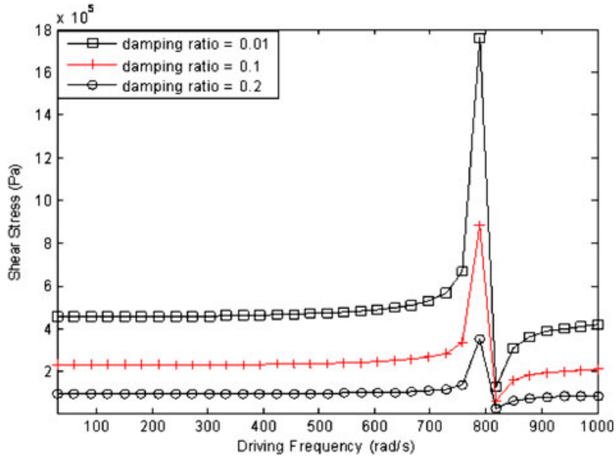

Fig. 10 Variations of the interlaminar shear stresses in the cylindrical shell with the frequency a harmonic force applied at its free end for different damping ratios

6 Concluding Remarks

The dynamic behavior of viscoelastic laminated beam, plate and shell structures is investigated using finite element analysis. Modal, harmonic and transient models are developed and solved for deflections and stresses. The effect of the damping ratio of the viscoelastic material is investigated. It is found that the viscoelastic material damping reduces the interlaminar stresses. The results also show the dependency of the viscoelastic material on frequency, hence, the effect of the viscoelastic material appears significantly under harmonic loading. In transient analysis, the importance of the viscoelastic material is observed in absorbing the impact and returning the structure to its original configuration.

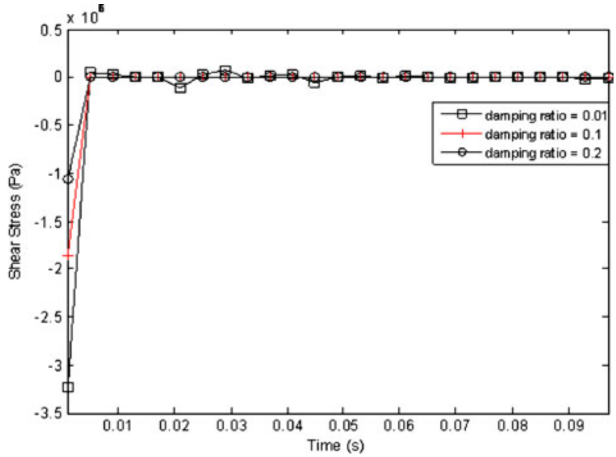

Fig. 11 Variations of the interlaminar shear stresses in the cylindrical shell with time under a transient force applied at its free end for different damping ratios

Finally it is recommended to verify the present findings, which are based on finite element results, by analytical and experimental investigations.

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