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Toeplitz Operators on Symmetric Siegel Domains

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Dedicated to Max Koecher on the occasion of his sixtieth birthday

0. Introduction

From the beginning Jordan algebras have been studied not only purely algebraically but also in view of applications to analysis, physics, and geometry. Examples are the Jordan algebraic formulation of quantum mechanics and the Jordan theoretic description of bounded symmetric domains in finite and infinite dimensions. Another more recent application concerns certain classes of Hilbert space operators, namely Wiener-Hopf operators on self-dual homogeneous cones [13] and Toeplitz operators on bounded symmetric domains [14–17]. The algebraic and spectral properties of these operators can be described in terms of the boundary or facial structure of the underlying "stratified" manifold [5] which in turn is closely related to the idempotents of the associated Jordan algebra or Jordan triple system.

In this paper we apply Jordan theoretic methods to study Toeplitz operators related to the nilpotent groups Σ of Heisenberg type which arise as distinguished boundaries of symmetric Siegel domains of the first or second kind. For domains of tube type, the C^*-algebra $\mathscr{T}_\Sigma$ generated by these operators is isomorphic to a Wiener-Hopf C^*-algebra via the Fourier transformation. On the other hand, $\mathscr{T}_\Sigma$ can always be realized as a closed ideal in the Toeplitz C^*-algebra of a bounded symmetric domain by means of a Cayley transformation. Therefore the results of [15, 16] can be applied to describe $\mathscr{T}_\Sigma$ in Jordan theoretic terms including an explicit realization of all its irreducible representations. As a corollary we obtain a new proof of the main result of [13] avoiding groupoid theoretic methods.

1. Boundary Components of Siegel Domains

In this paper we study operators associated with *symmetric Siegel domains*

$$D = D(Y, F) := \{(u, v) \in U \times V : 2\,\mathrm{Re}(u) - F(v, v) \in Y\}.$$

Here U and V are complex vector spaces of finite dimension, $\mathrm{Re}(u) := (u + u^*)/2$ denotes the real part of $u \in U$ with respect to an involution $u \mapsto u^*$ of U, and Y is an

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open non-void regular convex cone in the self-adjoint part $X := \{x \in U : x^* = x\}$ of U. Further, $F : V \times V \to U$ denotes a "Y-hermitian" mapping (conjugate-linear in the second variable) satisfying $F(b, v)^* = F(v, b)$ for all $b, v \in V$ and $F(v, v) \in \bar{Y} \setminus \{0\}$ whenever $v \neq 0$. In the special case $V = \{0\}$, $D(Y, F)$ reduces to a *tube domain*

$$D(Y) := \{u \in U : \mathrm{Re}(u) \in Y\}$$

over Y. Note that we consider "right half-planes" instead of the more familiar upper half-planes (cf. [12, Remark 10.10]). The boundary of $D = D(Y, F)$ is

$$\partial D = \{(u, v) \in U \times V : 2\,\mathrm{Re}(u) - F(v, v) \in \partial Y\},$$

where ∂Y denotes the boundary of Y in X. The subset

$$\Sigma := \{(u, v) \in U \times V : 2\,\mathrm{Re}(u) = F(v, v)\}$$

is called the *distinguished boundary* of D. Σ is a real analytic manifold isomorphic to $iX \times V$ via the mapping

$$iX \times V \ni (a, b) \mapsto (a + F(b, b)/2, b) \in \Sigma .$$

For any pair $(a, b) \in iX \times V$, the affine transformation

$$g_{a,b}(u, v) := (u + a + F(v, b) + F(b, b)/2, v + b)$$

leaves D invariant. For another such pair (a', b'), we have

$$g_{a,b} \circ g_{a',b'} = g_{(a,b)(a',b')},$$

where the product is defined by

$$(a, b)(a', b') := (a + a' + i\,\mathrm{Im}\,F(b', b), b + b') .$$

In this way, $\Sigma \approx iX \times V$ becomes a nilpotent Lie group. In the special case where $U = \mathbb{C}$, $V = \mathbb{C}^n$, and $F(v, b)$ is the scalar product on V, Σ is isomorphic to the Heisenberg group of dimension $2n + 1$.

By [6, Theorem 2.5], the tangent space $Z := U \times V$ of $D = D(Y, F)$ at a given point $e \in Y \subset D$ is a *Jordan triple system*. Let $\{xy^*z\} \in Z$ denote the Jordan triple product of $x, y, z \in Z$. An element c of Z is called a *tripotent* ("triple idempotent") if $\{cc^*c\} = c$. In this case there exists a *Peirce decomposition*

$$Z = Z_1(c) \oplus Z_{1/2}(c) \oplus Z_0(c),$$

where $Z_s(c) := \{z \in Z : \{cc^*z\} = sz\}$ [12, Theorem 3.13]. The base point $e \in Y$ is a tripotent of Z with $Z_0(e) = \{0\}$, and D has the following Jordan theoretic description [6, Theorem 2.5 and Proposition 4.6]: Z is a Jordan algebra under the product

$$z \circ w := \{ze^*w\}$$

and $U = Z_1(e)$ is a subalgebra with unit element e. The involution on U is given by $u^* = \{eu^*e\}$, and Y is the interior (with respect to X) of the closed convex cone $C := \{x^2 : x \in X\}$. The Y-hermitian mapping F on $V = Z_{1/2}(e)$ is given by $F(v, b) = 2\{vb^*e\}$.

According to [12, 6.1], the *holomorphic arc components* of a subset W of Z are the equivalence classes of W under the equivalence relation $z \sim w$ iff there exist finitely many holomorphic mappings $f_j : \Delta \to W$ $(0 \leqq j \leqq n)$ such that $z \in f_0(\Delta)$, $w \in f_n(\Delta)$, and $f_{j-1}(\Delta) \cap f_j(\Delta) \neq \emptyset$ for $1 \leqq j \leqq n$. Here Δ denotes the open unit disc in $\mathbb{C}$. Our next aim is to determine the holomorphic arc components of the closure $\bar{D}$ of a symmetric Siegel domain D.

Let p be a projection in X. Then $p_1 := p$ and $p_2 := e - p$ form a complete system of orthogonal tripotents of Z. By [12, Theorem 3.14], there exists a Peirce decomposition

$$Z = \sum_{0 \leqq i \leqq j \leqq 2}^{\oplus} Z_{ij},$$

where

$$Z_{ij} := \{z \in Z : 2\{p_k p_k^* z\} = (\delta_{ik} + \delta_{jk})z \text{ for } k = 1, 2\}.$$

Then $U = Z_{11} \oplus Z_{12} \oplus Z_{22}$, $V = Z_{01} \oplus Z_{02}$, and $Z_{00} = \{0\}$. Each summand of U is invariant under the involution. Put $X_p := X \cap Z_{22}$, $U_p := Z_{22} = X_p^{\mathbb{C}}$, and $X_p^{\perp} := X \cap (Z_{11} \oplus Z_{12})$. Then U_p is a Jordan algebra with unit element $e - p$ and, by [12, Theorem 3.14], $u^* = \{p_2 u^* p_2\}$ for all $u \in U_p$. Put $V_p := Z_{02}$ and $V_p^{\perp} := Z_{01}$. Then

$$Z_p := U_p \times V_p = Z_0(p)$$

is the Peirce 0-space of p, and $Y_p := \pi_p(Y)$ is the open cone of the formally real Jordan algebra X_p. Here $\pi_p : Z \to Z_p$ denotes the Peirce 0-projection. By [12, Theorem 3.14], $F(v, v) = 2\{vv^* p_2\} \in U_p$ for all $v \in V_p$, and the restriction $F_p := F | V_p \times V_p$ is a Y_p-hermitian mapping. It follows that

$$D_p := D(Y_p, F_p) = \{(u, v) \in Z_p : 2\,\mathrm{Re}(u) - F(v, v) \in Y_p\}$$

is a symmetric Siegel domain with base point $e - p$ and corresponding Jordan triple system Z_p.

1.1. Proposition. *For a symmetric Siegel domain $D = D(Y, F)$ of rank r, the partition into holomorphic arc components is given by*

$$\bar{D} = \bigcup_{0 \leqq k \leqq r} \ \bigcup_{(p, a, b) \in \Sigma_k} g_{a,b}(D_p), \tag{1.1}$$

where

$$\Sigma_k = \bigcup_{p \in \Pi_k} \{p\} \times iX_p^{\perp} \times V_p^{\perp}$$

and Π_k denotes the compact real-analytic manifold of all projections in X of rank k.

Proof. Every point $(u', v') \in g_{a,b}(D_p)$ satisfies

$$2\,\mathrm{Re}(u') - F(v', v') \in Y_p. \tag{1.2}$$

Therefore $g_{a,b}(D_p) \cap g_{a',b'}(D_{p'}) \neq \emptyset$ implies $Y_p \cap Y_{p'} \neq \emptyset$ and hence $p = p'$ by [2, Satz XI.7.4]. Moreover $b - b' \in V_p \cap V_p^{\perp} = \{0\}$ and $a - a' \in i(X_p \cap X_p^{\perp}) = \{0\}$. It follows that the sets $g_{a,b}(D_p)$ are pairwise disjoint. Now suppose $(u', v') \in \bar{D}$, i.e. $2\,\mathrm{Re}(u') - F(v', v') \in \bar{Y}$. By [2, Satz XI.7.4], there exists a projection $p \in X$ such that

(1.2) holds. Write $u' = u + c$, $v' = v + b$ with $u \in U_p$, $c \in U_p^\perp$, $v \in V_p$, and $b \in V_p^\perp$. Then

$$a := c - F(v, b) - F(b, b)/2 \in U_p^\perp$$

and (1.2) implies $(u, v) \in D_p$ and $\mathrm{Re}(a) = 0$. Hence (1.1) is a partition of $\bar{D}$. To conclude the proof, it suffices to show that for every holomorphic mapping $f: \Delta \to \bar{D}$, the condition $f(\Delta) \cap g_{a,b}(D_p) \neq \emptyset$ implies $f(\Delta) \subset g_{a,b}(D_p)$.

By [12, Theorem 10.8], $D = D(Y, F)$ is holomorphically equivalent to the open unit ball B of the Jordan triple system Z (for the spectral norm) via the *Cayley transformation* $g: D \to B$ defined by

$$g(u, v) := (u - e, 2^{1/2} v) \circ (e + u)^{-1}. \tag{1.3}$$

According to [12; Theorem 6.3], the partition into holomorphic arc components for the bounded symmetric domain B is given by

$$\bar{B} = \bigcup_{0 \leqq k \leqq r} \bigcup_{c \in S_k} c + B_c, \tag{1.4}$$

where S_k denotes the compact real-analytic manifold of all tripotents $c \in Z$ of rank k, and $B_c := B \cap Z_c$ is the open unit ball of the Peirce 0-space $Z_c := Z_0(c)$. By (1.3),

$$g(D_p) = -p + B_p$$

is a component of $\bar{B}$. Since $g_{a,b}$ is an automorphism of D, it follows that $g(g_{a,b}(D_p))$ is a component of $\bar{B}$. This implies the assertion. Q.E.D.

Note that the component for $k = 0$ is $D_0 = D$. The other components are called the *boundary components* of D. For $k = r$, we get the distinguished boundary $\Sigma = \Sigma_r$. Similarly, $B_0 = B$ whereas for $c \in S_r$, $c + B_c = \{c\}$ reduces to an extreme point of $\bar{B}$. By [12, Theorem 6.5], $S = S_r$ is the *Shilov boundary* of B. Let $\pi: Z \to U$ denote the Peirce 1-projection with respect to e. Then [12, Proposition 10.3] implies

$$g(\bar{D}) = \{z \in \bar{B} : \pi(z) - e \in U \text{ invertible}\}.$$

For $0 \leqq k \leqq r$, put $S_k^0 := S_k \cap g(\bar{D})$. Then $S^0 := S_r^0 = g(\Sigma)$. Define $S_k^\infty := S_k \backslash S_k^0$ and $S^\infty := S \backslash S^0 = S_r^\infty$. For each tripotent c, let S_c denote the Shilov boundary of the bounded symmetric domain B_c.

1.2. Corollary. *Every component of $\bar{D}$ has the form*

$$g_{a,b}(D_p) = g^{-1}(c + B_c)$$

for a unique tripotent $c \in Z$. The mapping $\beta: S_k^0 \to \Sigma_k$, defined by $\beta(c) := (p, a, b)$, is bijective. Further $c \in S_k^\infty$ implies $c + S_c \subset S^\infty$.

Proof. The first assertion follows from the last part of the proof of Proposition 1.1. Hence $g(\bar{D})$ and $\bar{B} \backslash g(\bar{D})$ are unions of components of $\bar{B}$. The components in $g(\bar{D})$ of rank $r - k$ are precisely the sets $c + B_c$ for $c \in S_k^0$. Since the manifolds $g_{a,b}(D_p)$ for $p \in \Pi_k$ have rank $r - k$, Proposition 1.1 implies that β is bijective. For $c \in S_k^\infty$, $c + B_c$ is contained in the closed set $\bar{B} \backslash g(\bar{D})$. Hence

$$c + S_c \subset S \cap (\bar{B} \backslash g(\bar{D})) = S \backslash g(\Sigma) = S^\infty. \quad \text{Q.E.D.}$$

Let $\Sigma_p \approx iX_p \times V_p$ denote the distinguished boundary of the symmetric Siegel domain D_p.

1.3. Lemma. *For different components $g_{a,b}(D_p)$ and $g_{a',b'}(D_{p'})$ of $\bar{D}$ we have*

$$g_{a,b}(\Sigma_p) \neq g_{a',b'}(\Sigma_{p'}).$$

In case p and p′ have the same rank, we have

$$g_{a,b}(\Sigma_p) \not\subset g_{a',b'}(\Sigma_{p'}).$$

Proof. Since $g_{a,b}(\Sigma_p) = \Sigma \cap g_{a,b}(\bar{D}_p)$ and $g_{a,b}$ is an automorphism of D leaving Σ invariant, we may assume $(a', b') = 0$. Suppose $g_{a,b}(\Sigma_p) = \Sigma_q$ for projections $p, q \in X$. Then $(a, b) = 0$ since $0 \in \Sigma_q$ and $a \in iX_p^\perp$, $b \in V_p^\perp$. Hence

$$iX_p = \Sigma_p \cap iX = \Sigma_q \cap iX = iX_q$$

and therefore $e - p = e - q$. The second assertion is proved in a similar way. Q.E.D.

Specializing Proposition 1.1 to the case of tube domains, we get

1.4. Corollary. *For a symmetric tube domain $D = D(Y)$ of rank r, the partition into holomorphic arc components is given by*

$$\bar{D} = \bigcup_{0 \leq k \leq r} \bigcup_{p \in \Pi_k} \bigcup_{a \in iX_p^\perp} a + D(Y_p).$$

2. Toeplitz Operators

Toeplitz operators on Hardy spaces of holomorphic functions, a classical subject in the 1-dimensional case, are also of interest for functions of several complex variables. Classically, Toeplitz operators are studied either on the unit disc or on a suitable half-plane. The generalization of half-planes to the higher dimensional case are (symmetric) tube domains and Siegel domains.

By Sect. 1, the distinguished boundary Σ of a symmetric Siegel domain $D = D(Y, F)$ of rank r is a simply connected nilpotent Lie group. Let $L^2(\Sigma)$ denote the Lebesgue space over Σ with respect to a Haar measure on Σ. Let $H^2(\Sigma)$ denote the set of all holomorphic functions $h: D \to \mathbb{C}$ such that the functions $h_y: \Sigma \to \mathbb{C}$ for $y \in Y$, defined by $h_y(w) := h(y + w)$, form a bounded subset of $L^2(\Sigma)$. By [9, p. 334], $H^2(\Sigma)$ can be realized as a closed subspace of $L^2(\Sigma)$ by taking the limit function of (h_y) for $y \to 0$. It follows that $H^2(\Sigma)$ is a Hilbert space called the *Hardy space* over Σ. The orthogonal projection $\pi_\Sigma: L^2(\Sigma) \to H^2(\Sigma)$ is called the *Szegö projection* being induced by the Szegö kernel [9, (2.1)]. Let $\mathscr{C}_0(\Sigma)$ be the C^*-algebra of all continuous functions on Σ vanishing at infinity. For $\phi \in \mathscr{C}_0(\Sigma)$, the multiplication operator $M_\phi h := \phi h$ is a bounded operator on $L^2(\Sigma)$, and the *Toeplitz operator* T_ϕ on $H^2(\Sigma)$ is defined by the commutative diagram

$$\begin{array}{ccc} L^2(\Sigma) & \xrightarrow{M_\phi} & L^2(\Sigma) \\ \downarrow{\scriptstyle \pi_\Sigma} & & \downarrow{\scriptstyle \pi_\Sigma} \\ H^2(\Sigma) & \xrightarrow[T_\phi]{} & H^2(\Sigma). \end{array}$$

The mapping $\phi \mapsto T_\phi$ from $\mathscr{C}_0(\Sigma)$ to the C^*-algebra $\mathscr{L}(H^2(\Sigma))$ of all bounded operators on $H^2(\Sigma)$ is linear and positive, but not a homomorphism. In particular, Toeplitz operators are in general not normal. In order to study algebraic and spectral properties of these operators, it is therefore appropriate to consider the (non-unital) *Toeplitz C^*-algebra*

$$\mathscr{T}_\Sigma := C^*(T_\phi : \phi \in \mathscr{C}_0(\Sigma))$$

generated by all Toeplitz operators with "symbol function" $\phi \in \mathscr{C}_0(\Sigma)$. It will be shown that $\mathscr{T}_\Sigma$ can be realized as a closed ideal in the Toeplitz C^*-algebra associated with the bounded symmetric domain $B = g(D)$. Therefore the results of [15, 16] can be applied to determine the structure of $\mathscr{T}_\Sigma$ including an explicit description of all its irreducible representations.

The Shilov boundary S of B is homogeneous under the compact group K of all linear automorphisms of B. Let $L^2(S)$ denote the Lebesgue space for the normalized K-invariant measure on S. The *Hardy space* $H^2(S)$ over S is defined in a similar way as $H^2(\Sigma)$ (cf. [9, p. 342]). Let $\pi_S : L^2(S) \to H^2(S)$ denote the orthogonal (Szegö) projection. For any $f \in \mathscr{C}(S)$, let M_f denote the associated multiplication operator on $L^2(S)$, and define the Toeplitz operator T_f on $H^2(S)$ by the commutative diagram

$$\begin{array}{ccc} L^2(S) & \xrightarrow{M_f} & L^2(S) \\ \downarrow{\scriptstyle \pi_S} & & \downarrow{\scriptstyle \pi_S} \\ H^2(S) & \xrightarrow[T_f]{} & H^2(S) \end{array} .$$

Define the (unital) Toeplitz C^*-algebra over S,

$$\mathscr{T}_S := C^*(T_f : f \in \mathscr{C}(S))$$

generated by all Toeplitz operators with continuous symbol function. In [15, Theorem 3.8] it was shown that every tripotent $c \in Z$ induces an irreducible representation ("c-symbol")

$$\sigma_c : \mathscr{T}_S \to \mathscr{L}(H^2(S_c))$$

on the Hardy space $H^2(S_c)$ associated with the Shilov boundary S_c of the bounded symmetric domain $B_c \subset Z_c$ [cf. (1.4)]. This representation is uniquely determined by the property

$$\sigma_c(T_f) = T_{f_c}$$

for all $f \in \mathscr{C}(S)$, where $f_c \in \mathscr{C}(S_c)$ is defined by $f_c(z) = f(c+z)$. Let S_k denote the compact manifold of all tripotents of rank k. For $c \in S = S_r$, σ_c is the character of $\mathscr{T}_S$ determined by $\sigma_c(T_f) = f(c)$, whereas for $k = 0$, σ_0 is the faithful irreducible representation on $H^2(S)$ (cf. [15, Lemma 3.1]). The representations σ_c are pairwise inequivalent and constitute all irreducible representations of $\mathscr{T}_S$ up to unitary equivalence. This is a consequence of the following theorem, which is the main result of [16]:

2.1. Theorem. *The Toeplitz C^*-algebra $\mathcal{T}_S$ is solvable of length r and has the spectral components S_k for $0 \leqq k \leqq r$. More precisely, the null-spaces $\mathcal{I}_k := \operatorname{Ker}(\sigma_k)$ of the "k-symbol homomorphisms"*

$$\sigma_k := (\sigma_c)_{c \in S_k}$$

form a composition sequence

$$\{0\} = \mathcal{I}_0 \subset \mathcal{I}_1 \subset \mathcal{I}_2 \subset \dots \subset \mathcal{I}_{r-1} \subset \mathcal{I}_r \subset \mathcal{T}_S$$

of $\mathcal{T}_S =: \mathcal{I}_{r+1}$, with C^*-algebra isomorphisms

$$\mathcal{I}_{k+1}/\mathcal{I}_k \approx \mathscr{C}(S_k) \otimes \mathcal{K}(H_k),$$

where $\mathcal{K}(H_k)$ denotes the C^*-algebra of all compact operators on a Hilbert space H_k which is separable (infinite-dimensional) for $0 \leqq k < r$ and 1-dimensional for $k = r$.

It follows that $\mathcal{I}_r = \mathcal{T}_S'$ is the closed commutator ideal of $\mathcal{T}_S$ and $\mathcal{T}_S/\mathcal{T}_S' \approx \mathscr{C}(S)$. $\mathcal{I}_1$ consists of all compact operators on $H^2(S)$. The Hilbert spaces H_k have natural interpretations as suitable Hardy spaces. The Jacobson topology on the spectrum

$$\operatorname{Spec}(\mathcal{T}_S) = \bigcup_{0 \leqq k \leqq r} S_k$$

of $\mathcal{T}_S$ is non-hausdorff, and the closure of the topological subspace S_k of $\operatorname{Spec}(\mathcal{T}_S)$ coincides with the union of all S_j for $j \geqq k$ [4].

2.2. Lemma. *For every biholomorphic mapping $\gamma : D \to B$, there exists a (surjective) isometry*

$$\gamma_* : H^2(\Sigma) \to H^2(S)$$

such that, for every $\varphi \in \mathscr{C}_0(\Sigma)$, the diagram

$$\begin{array}{ccc} H^2(\Sigma) & \xrightarrow{T_\varphi} & H^2(\Sigma) \\ {\scriptstyle \gamma_*}\downarrow & & \downarrow{\scriptstyle \gamma_*} \\ H^2(S) & \xrightarrow[T_f]{} & H^2(S) \end{array} \tag{2.1}$$

commutes, where $f := \varphi \circ \gamma^{-1}$ belongs to $\mathscr{C}_0(S^0)$, identified with the closed ideal $\{f \in \mathscr{C}(S) : f | S^\infty = 0\}$ of $\mathscr{C}(S)$.

Proof. We can write $\gamma = g \circ \sigma$, where g is the Cayley transformation and σ is a holomorphic automorphism of D. From [9, Lemma 3.2, Definition 4.2, and Proposition 4.10] it follows that there exists a surjective isometry $\gamma_* : H^2(\Sigma) \to H^2(S)$ such that

$$(\gamma_* h)(z) = h(\gamma^{-1} z) \cdot j_\gamma(\gamma^{-1} z)$$

for all $h \in H^2(\Sigma)$ and $z \in B$, where $j_\gamma : D \to \mathbb{C}$ is holomorphic. Consider the subalgebra

$$\mathcal{A} := H^\infty(D) \cap \mathscr{C}_0(\Sigma)$$

of $\mathscr{C}_0(\Sigma)$. Then (2.1) commutes for all $\varphi \in \mathscr{A}$, since $T_\varphi = M_\varphi | H^2(\Sigma)$. Since $T_{\varphi^* \psi} = T_\varphi^* T_\psi$ for all $\varphi, \psi \in \mathscr{A}$, the Stone-Weierstrass theorem implies that (2.1) commutes for all $\varphi \in \mathscr{C}_0(\Sigma)$. Q.E.D.

2.3. Corollary. *The adjoint mapping*

$$\mathrm{Ad}(g_*)T := g_* T g_*^{-1}$$

*associated with the Cayley transformation $g: D \to B$ is a *-isomorphism from $\mathscr{T}_\Sigma$ onto a closed *-subalgebra of $\mathscr{T}_S$. Every $\varphi \in \mathscr{C}_0(\Sigma)$ satisfies*

$$\|T_\varphi\| = \sup |\varphi(\Sigma)| . \tag{2.2}$$

Proof. Since Theorem 2.1 implies $\|T_f\| = \sup |f(S)|$ for all $f \in \mathscr{C}(S)$, we have

$$\|T_\varphi\| = \|\mathrm{Ad}(g_*) T_\varphi\| = \|T_{\varphi \circ g^{-1}}\| = \sup |\varphi(\Sigma)| . \quad \text{Q.E.D.}$$

For the Cayley transformation g, [9, Definition 4.2] implies

$$j_g(w) = \mathscr{S}_e(w)^{-1} := \mathscr{S}(w, e)^{-1}$$

for all $w \in D$, where $\mathscr{S}: (D \cup \Sigma) \times D \to \mathbb{C}$ denotes the Szegö kernel of D [holomorphic in the first variable, cf. [9, (2.1)]] and the Haar measure on Σ has been normalized such that $\mathscr{S}(e, e) = 1$.

2.4. Proposition. *The Toeplitz C^*-algebra $\mathscr{T}_\Sigma$ acts irreducibly on $H^2(\Sigma)$.*

Proof. Let $\pi: Z \to U$ denote the Peirce 1-projection onto U and let $N: U \to \mathbb{C}$ be the norm function of the Jordan algebra U [2]. Then N is a polynomial of degree $r = \mathrm{rank}(U)$ satisfying $N(e) = 1$. By [12, Proposition 10.3], we have

$$S^\infty = \{z \in S : N(e - \pi z) = 0\} .$$

Let X^r_{sym} denote the symmetric r^{th} power of a set X. By [18, Appendix V, Theorem 4A], the mapping

$$Z \ni z \mapsto \langle \lambda_1^z, \ldots, \lambda_r^z \rangle \in \mathbb{C}^r_{\mathrm{sym}} ,$$

defined by $N(\lambda e - \pi z) = (\lambda - \lambda_1^z) \ldots (\lambda - \lambda_r^z)$, is continuous. For $z \in \bar{B}$, we have $|\lambda_j^z| \leq 1$ since $e - u \in U$ is invertible for all $u \in B \cap U$. Let Δ denote the open unit disc and consider the continuous function $f: \bar{\Delta} \to \mathbb{C}$ satisfying $f(\lambda)^r = 1 - \lambda$ and $f(0) = 1$. By [18, Appendix V, Lemma 3H], $F\langle \lambda_1, \ldots, \lambda_r \rangle := f(\lambda_1) \ldots f(\lambda_r)$ defines a continuous function $F: \bar{\Delta}^r_{\mathrm{sym}} \to \mathbb{C}$ satisfying $\mathrm{Re}(F) \geqq 0$ since $\arg f(\lambda) \leqq \pi/2r$ for all $\lambda \in \bar{\Delta}$. Further, $\mathrm{Re}(F) > 0$ on Δ^r_{sym}. The continuous mapping $H: \bar{B} \to \mathbb{C}$, defined by $H(z) := F\langle \lambda_1^z, \ldots, \lambda_r^z \rangle$, satisfies $H(z)^r = N(e - \pi z)$, $H(0) = 1$, and $\mathrm{Re}(H) \geqq 0$. Further, H is holomorphic on B and $\mathrm{Re}(H) > 0$ on $S \backslash S^\infty$. Hence $h(z) := \exp(-H(z))$ satisfies $h | S^\infty = 1$ and $|h| < 1$ on $S \backslash S^\infty$. Since $S^\infty \subset S$ has measure 0, the dominated convergence theorem implies $(1 - h^n) f \to f$ in $H^2(S)$ whenever $f \in H^2(S)$. Hence the algebra

$$\mathscr{A}_S = \{f \in \mathscr{C}(\bar{B}) : f | B \text{ holomorphic, } f | S^\infty = 0\}$$

is dense in $H^2(S)$. By Lemma 2.2, it follows that $\mathscr{S}_e \cdot \mathscr{A}$ is a dense subspace of $H^2(\Sigma)$. Now suppose π is a projection on $H^2(\Sigma)$ commuting with $\mathscr{T}_\Sigma$. Put

$f := \pi(\mathcal{S}_e) \in H^2(\Sigma)$. Then

$$\pi(\mathcal{S}_e \cdot \varphi) = \pi(T_\varphi \mathcal{S}_e) = T_\varphi(\pi \mathcal{S}_e) = \varphi f \tag{2.3}$$

for all $\varphi \in \mathcal{A}$. For all $w \in D$, [9, (2.4)] implies

$$\begin{aligned}(\mathcal{S}_e \cdot \varphi | \pi(\mathcal{S}_w)) &= (\pi(\mathcal{S}_e \cdot \varphi) | \mathcal{S}_w) = (\varphi f | \mathcal{S}_w) \\ &= \overline{\varphi(w) f(w)} = \overline{f(w)/\mathcal{S}_e(w)}\,(\mathcal{S}_e \cdot \varphi | \mathcal{S}_w).\end{aligned}$$

Since $\mathcal{S}_e \cdot \mathcal{A}$ is dense and $\mathcal{S}_w \neq 0$, $f(w)/\mathcal{S}_e(w) \in \{0, 1\}$ is an eigenvalue of π. By continuity, $f/\mathcal{S}_e$ is constant, and (2.3) implies $\pi = 0$ or $\pi = \mathrm{id}$. Q.E.D.

2.5. Theorem. *Let $D = D(Y, F)$ be a symmetric Siegel domain of rank r. For every holomorphic arc component $g_{a,b}(D_p)$ of $\bar{D}$, there exists an irreducible representation*

$$\varrho_p^{a,b} : \mathcal{T}_\Sigma \to \mathcal{L}(H^2(\Sigma_p))$$

of $\mathcal{T}_\Sigma$ on the Hardy space $H^2(\Sigma_p)$ associated with the symmetric Siegel domain $D_p = D(Y_p, F_p)$ in Z_p and its distinguished boundary $\Sigma_p \approx iX_p \times V_p$. This representation is uniquely determined by the property

$$\varrho_p^{a,b}(T_\phi) = T_{\phi_{a,b}} \tag{2.4}$$

for all $\phi \in \mathcal{C}_0(\Sigma)$, where $\phi_{a,b}(z) := \phi(g_{a,b}(z))$ for all $z \in \Sigma_p$. The representations $\varrho_p^{a,b}$ are pairwise inequivalent.

Proof. By Corollary 1.2, there exists a tripotent $c \in Z$ such that $g_{a,b}(D_p) = g^{-1}(c + B_c)$, where g is the Cayley transformation. Therefore $\gamma(z) + c = g(g_{a,b}(z))$ defines a biholomorphic mapping $\gamma : D_p \to B_c$ with $\gamma(\Sigma_p) \subset S_c$. By Lemma 2.2, the commutative diagram

$$\begin{tikzcd} \mathcal{T}_S \arrow[r, "\sigma_c"] & \mathcal{L}(H^2(S_c)) \\ \mathcal{T}_\Sigma \arrow[u, "\mathrm{Ad}(g_*)"] \arrow[r, "\varrho_p^{a,b}"'] & \mathcal{L}(H^2(\Sigma_p)) \arrow[u, "\mathrm{Ad}(\gamma_*)"'] \end{tikzcd} \tag{2.5}$$

defines a representation $\varrho_p^{a,b}$ of $\mathcal{T}_\Sigma$ satisfying (2.4). Since

$$\varrho_p^{a,b}(\mathcal{T}_\Sigma) = \mathcal{T}_{\Sigma_p}$$

by (2.4), Proposition 2.4 implies that $\varrho_p^{a,b}$ is irreducible. Now let $g_{a,b}(D_p)$ and $g_{a',b'}(D_{p'})$ be different components. By Lemma 1.3 we may assume that there exists $\varphi \in \mathcal{C}_0(\Sigma)$ vanishing on $g_{a',b'}(\Sigma_{p'})$ but not on $g_{a,b}(\Sigma_p)$. By (2.2) and (2.4) this implies

$$T_\varphi \in \mathrm{Ker}(\varrho_{p'}^{a',b'}) \setminus \mathrm{Ker}(\varrho_p^{a,b}). \tag{2.6}$$

Hence $\varrho_p^{a,b}$ and $\varrho_{p'}^{a',b'}$ are inequivalent. Q.E.D.

2.6. Corollary. *Via $\mathrm{Ad}(g_*)$, $\mathcal{T}_\Sigma$ can be realized as a closed ideal of $\mathcal{T}_S$.*

Proof. Since Corollary 1.2 shows $c + S_c \subset S^\infty$ for all $c \in S_k^\infty$, the closed ideal

$$\mathcal{J} := \bigcap_{k=1}^{r} \bigcap_{c \in S_k^\infty} \mathrm{Ker}(\sigma_c)$$

of $\mathcal{T}_S$ contains $\mathcal{T}_\Sigma$ (realized as a closed *-subalgebra of $\mathcal{T}_S$). Now let σ be an irreducible representation of $\mathcal{J}$. By [3, Proposition 2.11.2], we may assume that $\sigma=\sigma_c|\mathcal{J}$ for some $c\in S_k^0$. For $\beta(c)=(p,a,b)\in\Sigma_k$ (cf. Corollary 1.2), the commuting diagram (2.5) shows that $\sigma|\mathcal{T}_\Sigma$ is irreducible. If $\sigma'=\sigma_{c'}|\mathcal{J}$ is another irreducible representation of $\mathcal{J}$ not equivalent to σ, then $c'\neq c$ and (2.6) shows that $\sigma|\mathcal{T}_\Sigma$ and $\sigma'|\mathcal{T}_\Sigma$ are not equivalent. Since $\mathcal{T}_S$ is a postliminal C^*-algebra by Theorem 2.1, [3, Propositions 4.3.5 and 11.1.6] imply $\mathcal{T}_\Sigma=\mathcal{J}$. Q.E.D.

2.7. Corollary. *The representations $\varrho_p^{a,b}$ constitute all irreducible representations of $\mathcal{T}_\Sigma$, up to unitary equivalence.*

Proof. Apply Theorem 2.1, Corollary 2.6 and [3, Proposition 2.11.2]. Q.E.D.

The representations $\varrho_p^{a,b}$ will now be used to prove the main result of this paper, a structure theorem for $\mathcal{T}_\Sigma$ analogous to Theorem 2.1. Endow the vector bundle Σ_k over Π_k with the topology induced by the bijection $\beta: S_k^0\to\Sigma_k$.

2.8. Theorem. *Let $D=D(Y,F)$ be a symmetric Siegel domain of rank r with distinguished boundary Σ. Then the Toeplitz C^*-algebra $\mathcal{T}_\Sigma$ is solvable of length r, with spectral components given by the vector bundles Σ_k over Π_k for $0\leqq k\leqq r$. The null-spaces $\mathcal{J}_k:=\operatorname{Ker}(\varrho_k)$ of the k-symbol homomorphisms*

$$\varrho_k:=(\varrho_p^{a,b})_{(p,a,b)\in\Sigma_k}$$

form a composition sequence $(\mathcal{J}_k)_{0\leqq k\leqq r}$ of $\mathcal{T}_\Sigma=\mathcal{J}_{r+1}$, with C^*-algebra isomorphisms

$$\mathcal{J}_{k+1}/\mathcal{J}_k\approx\mathcal{C}_0(\Sigma_k)\otimes\mathcal{K}(H_k),$$

where H_k is separable for $0\leq k<r$ and 1-dimensional for $k=r$. In particular, $\mathcal{J}_1=\mathcal{K}(H^2(\Sigma))$, $\mathcal{J}_r$ is the closed commutator ideal of $\mathcal{T}_\Sigma$ and the mapping $T_\varphi\mapsto\varphi$ induces an isomorphism $\mathcal{T}_\Sigma/\mathcal{J}_r\approx\mathcal{C}_0(\Sigma)$.

Proof. Identify $\mathcal{T}_\Sigma$ with a closed ideal of $\mathcal{T}_S$ via $\operatorname{Ad}(g_*)$. Then Theorem 2.5 implies

$$\mathcal{J}_k=\mathcal{I}_k\cap\mathcal{T}_\Sigma=\operatorname{Ker}(\sigma_k^0)\cap\mathcal{T}_\Sigma, \tag{2.7}$$

where σ_k^0 denotes the representation

$$\sigma_k^0=(\sigma_c)_{c\in S_k^0}$$

of $\mathcal{T}_S$. By [16, Lemma 3.3], the continuous field

$$(H^2(S_c))_{c\in S_k^0}$$

of Hilbert spaces over S_k^0 is trivial. It follows that the associated continuous field

$$\mathcal{K}_k^0:=(\mathcal{K}(H^2(S_c)))_{c\in S_k^0}$$

of elementary C^*-algebras over S_k^0 (with sections vanishing at infinity, cf. [3, 10.4]) is also trivial. Hence there exist C^*-algebra isomorphisms

$$\mathcal{K}_k^0\approx\mathcal{C}_0(S_k^0)\otimes\mathcal{K}(H_k)\approx\mathcal{C}_0(\Sigma_k)\otimes\mathcal{K}(H_k), \tag{2.8}$$

where the Hilbert space H_k is separable for $0 \leqq k < r$ and 1-dimensional for $k=r$. For every $A \in \mathscr{J}_{k+1}$, [16, Theorem 3.12] implies that

$$\sigma_k(A) = (\sigma_c(A))_{c \in S_k}$$

is a continuous field of compact operators, and $\sigma_c(A)=0$ for all $c \in S_k^\infty$. It follows that $\sigma_k^0(\mathscr{J}_{k+1}) \subset \mathscr{K}_k^0$.

By Theorem 2.1, $\sigma_c(\mathscr{J}_{k+1}) = \mathscr{K}_c := \mathscr{K}(H^2(S_c))$ for all $c \in S_k^0$. By (2.2), there exist $\varphi \in \mathscr{C}_0(S^0)$ and $A \in \mathscr{J}_{k+1}$ such that

$$\sigma_c(T_\varphi A) = T_{\varphi_c} \cdot \sigma_c(A) \neq 0 .$$

Since $\mathscr{K}_c$ is a simple C^*-algebra, we get $\sigma_c(\mathscr{J}_{k+1}) = \mathscr{K}_c$. Now suppose $k \geqq 1$ and let $c' \in S_k^0$ be different from c. By (2.6), we may assume that $T_\varphi \in \operatorname{Ker}(\sigma_{c'})$. It follows that

$$\sigma_c(\mathscr{J}_{k+1} \cap \operatorname{Ker}(\sigma_{c'})) = \mathscr{K}_c .$$

Applying [3, Lemma 10.5.3], we get $\sigma_k^0(\mathscr{J}_{k+1}) = \mathscr{K}_k^0$ for $0 \leqq k \leqq r$. Now apply (2.8). Q.E.D.

3. Wiener-Hopf Operators

The results of Sect. 2 will now be applied to study Wiener-Hopf operators on self-dual homogeneous cones. Since these operators are equivalent to Toeplitz operators on symmetric tube domains via the Fourier transformation, Theorem 2.8 yields a new proof of the main result of [13] independent of the theory of groupoid C^*-algebras. Wiener-Hopf operators play an important role in the theory of integral equations (cf. the bibliography of [13]).

Suppose C is a self-dual homogeneous cone in a real vector space X of finite dimension. Then

$$C = \{x \in X : (x|C) \geqq 0\}$$

for a suitable inner product $(\,|\,)$ on X, and the interior Y of C is non-void and homogeneous under linear transformations. It is well-known [8] that there exists a formally real Jordan algebra structure on X such that $C = \{x^2 : x \in X\}$. The unit element e of X is contained in Y. Let $U := X^{\mathbb{C}}$ denote the complexified Jordan algebra. Then the tube domain $D := D(Y)$ is homogeneous under affine transformations, and $u \to u^{-1}$ is the symmetry of D about e. Hence D is a symmetric tube domain with distinguished boundary iX.

The character group of the locally compact abelian group X (under addition) can be identified with iX via the pairing

$$iX \times X \ni (a, x) \to e^{(a|x)} \in U_1(\mathbb{C}) .$$

Endow X with a Haar measure dx whose dual measure on iX is the image measure under multiplication by i. By [1, Theorem 36D], the Fourier transformation

$$\mathscr{F} h(a) := \int_X e^{-(a|x)} h(x)\, dx$$

induces a (surjective) isometry $\mathscr{F}: L^2(X) \to L^2(iX)$. According to the Paley-Wiener Theorem, in the generalized form due to Bochner (cf. [10, Lemma 2.2]), $\mathscr{F}$ maps the closed subspace

$$L^2(C) := \{h \in L^2(X) : \operatorname{Supp}(h) \subset C\}$$

of $L^2(X)$ onto the Hardy space $H^2(iX)$. Let $\pi_C : L^2(X) \to L^2(C)$ denote the orthogonal projection. Then there exists a commutative diagram

$$\begin{array}{ccc} L^2(X) & \xrightarrow{\mathscr{F}} & L^2(iX) \\ {\scriptstyle \pi_C}\downarrow & & \downarrow{\scriptstyle \pi_{iX}} \\ L^2(C) & \xrightarrow[\mathscr{F}]{} & H^2(iX) \end{array} .$$

By [11, Theorem 31A], the (non-unital) Banach *-algebra $L^1(X)$ acts on $L^2(X)$ by assigning to each $\psi \in L^1(X)$ the convolution operator $F_\psi h := \psi * h$ on $L^2(X)$. Define the *Wiener-Hopf operator* W_ψ on $L^2(C)$ by the commutative diagram

$$\begin{array}{ccc} L^2(X) & \xrightarrow{F_\psi} & L^2(X) \\ {\scriptstyle \pi_C}\downarrow & & \downarrow{\scriptstyle \pi_C} \\ L^2(C) & \xrightarrow[W_\psi]{} & L^2(C) \end{array} ,$$

i.e.,

$$(W_\psi h)(y) = \int_C \psi(y-x) h(x) dx$$

for all $y \in C$ and $h \in L^2(C)$. The assignment $\psi \mapsto W_\psi$ is linear and positive, but not a homomorphism. Define the (non-unital) *Wiener-Hopf C^*-algebra* on $L^2(C)$,

$$\mathscr{W}_C := C^*(W_\psi : \psi \in L^1(X)) .$$

For any $\psi \in L^1(X)$, the Fourier transform $\phi := \hat{\psi} = \mathscr{F}\psi$ belongs to $\mathscr{C}_0(iX)$ by the Riemann-Lebesgue lemma, and there exists a commutative diagram

$$\begin{array}{ccc} L^2(X) & \xrightarrow{F_\psi} & L^2(X) \\ {\scriptstyle \mathscr{F}}\downarrow & & \downarrow{\scriptstyle \mathscr{F}} \\ L^2(iX) & \xrightarrow[M_\phi]{} & L^2(iX) \end{array} .$$

Hence we get a commutative diagram

$$\begin{array}{ccc} L^2(C) & \xrightarrow{W_\psi} & L^2(C) \\ {\scriptstyle \mathscr{F}}\downarrow & & \downarrow{\scriptstyle \mathscr{F}} \\ H^2(iX) & \xrightarrow[T_\phi]{} & H^2(iX) \end{array} .$$

Since $\mathscr{F}(L^1(X))$ is a dense subalgebra of $\mathscr{C}_0(iX)$, we get

3.1. Lemma. *The Fourier transformation* $\mathscr{F}: L^2(C) \to H^2(iX)$ *induces a* C^**-algebra isomorphism*

$$\mathrm{Ad}(\mathscr{F}): \mathscr{W}_C \to \mathscr{T}_{iX}$$

onto the Toeplitz C^**-algebra over the distinguished boundary* iX *of* $D = D(Y)$.

3.2. Theorem. *For every projection* $p \in X$ *and every* $a \in iX_p^\perp$ *there exists an irreducible representation*

$$\tau_p^a: \mathscr{W}_C \to \mathscr{L}(L^2(C_p))$$

of $\mathscr{W}_C$ *on the* L^2*-space of the self-dual homogeneous cone* $C_p := \pi_p(C) \subset X_p$ *which is uniquely determined by the property*

$$\tau_p^a(W_{\psi_1 \psi_2}) = \hat{\psi}_1(a) \cdot W_{\psi_2}$$

for all continuous functions $\psi_1: X_p^\perp \to \mathbb{C}$ *and* $\psi_2: X_p \to \mathbb{C}$ *with compact support. The representations* τ_p^a *are pairwise inequivalent and constitute all irreducible representations of* $\mathscr{W}_C$*, up to unitary equivalence.*

Proof. By Lemma 3.1 and Theorem 2.5, the commutative diagram

$$\begin{tikzcd}
\mathscr{T}_{iX} \arrow[r, "\varrho_p^a"] & \mathscr{L}(H^2(iX_p)) \\
\mathscr{W}_C \arrow[u, "\mathrm{Ad}(\mathscr{F})"] \arrow[r, "\tau_p^a"'] & \mathscr{L}(L^2(C_p)) \arrow[u, "\mathrm{Ad}(\mathscr{F}_p)"']
\end{tikzcd}$$

defines an irreducible representation τ_p^a of $\mathscr{W}_C$ on $L^2(C_p)$, and the representations τ_p^a are pairwise inequivalent and constitute all irreducible representations of $\mathscr{W}_C$, up to unitary equivalence. Here $\mathscr{F}_p$ denotes the Fourier transformation associated with X_p. For $\psi \in L^1(X)$ put $\phi := \hat{\psi} \in \mathscr{C}_0(iX)$. Then

$$\varrho_p^a(\mathrm{Ad}(\mathscr{F}) W_\psi) = \varrho_p^a(T_\phi) = T_{\phi_a}.$$

For every $u \in iX_p$, we have

$$\phi_a(u) = \phi(a+u) = \int_X e^{-(a+u|x)} \psi(x) dx.$$

In case $\psi = \psi_1 \cdot \psi_2$, this implies

$$\begin{aligned}
\phi_a(u) &= \int_{X_p^\perp} e^{-(a|x')} \psi_1(x') dx' \cdot \int_{X_p} e^{-(u|x'')} \psi_2(x'') dx'' \\
&= \hat{\psi}_1(a) \cdot (\mathscr{F}_p \psi_2)(u) = \hat{\psi}_1(a) \cdot \phi_2(u).
\end{aligned}$$

Therefore

$$\mathrm{Ad}(\mathscr{F}_p)(\tau_p^a(W_{\psi_1 \psi_2})) = \hat{\psi}_1(a) T_{\phi_2} = \hat{\psi}_1(a) \cdot \mathrm{Ad}(\mathscr{F}_p)(W_{\psi_2}). \quad \text{Q.E.D.}$$

Note that for $p = 0$, $\tau_0 := \tau_0^0$ is the faithful irreducible representation of $\mathscr{W}_C$ on $L^2(C)$, whereas for $p = e$ and $a \in iX$, $\tau^a := \tau_e^a$ is the character of $\mathscr{W}_C$ determined by $\tau^a(W_\phi) = \hat{\phi}(a)$.

Applying Theorem 2.8 (for the special case of tube domains), we obtain a new proof of [13, Theorem 6.6]:

3.3. Theorem. *For every self-dual homogeneous cone C in X of rank r, the Wiener-Hopf C^*-algebra $\mathscr{W}_C$ is solvable of length r, with spectral components given by the vector bundles*

$$\Sigma_k = \bigcup_{p \in \Pi_k} \{p\} \times iX_p^{\perp}$$

over Π_k $(0 \leqq k \leqq r)$. The null-spaces $\mathscr{I}_k := \operatorname{Ker}(\tau_k)$ of the k-symbol homomorphisms

$$\tau_k := (\tau_p^a)_{(p,a) \in \Sigma_k}$$

form a composition sequence $(\mathscr{I}_k)_{0 \leqq k \leqq r}$ of $\mathscr{W}_C := \mathscr{I}_{r+1}$, with C^*-algebra isomorphisms

$$\mathscr{I}_{k+1}/\mathscr{I}_k \approx \mathscr{C}_0(\Sigma_k) \otimes \mathscr{K}(H_k),$$

where H_k is separable for $0 \leqq k < r$ and 1-dimensional for $k = r$.

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