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REPLY

Multiscale Integration in Scale Relativity Theory

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Abstract We give a "direction for use" of the scale relativity theory and apply it to an example of spontaneous multiscale integration including four embedded levels of organization (intracellular, cell, tissue and organism-like levels). We conclude by an update of our analysis of the arctic sea ice melting.

Keywords Relativity · Scale · Macroscopic quantum mechanics

In their commentary to my (2010) paper, Auffray and Noble (2010), raise the important question of multiscale integration in biology and of the ability of the scale relativity theory to contribute by new insights and methods to a future possible solution of this problem (and of other questions in life and other sciences).

In order to give elements of answer, let us recall how the scale relativity theory can be used for practical applications.

The construction of the theory of scale relativity proceeds by extension and generalization with respect to currently existing theories. Its founding principles are the same as those on which these theories are founded (principles of relativity and covariance, of optimization—least action and geodesic principles), but applied also to scale transformations of the reference system. As a consequence, its equations are themselves extensions of the standard fundamental equations (Euler-Lagrange equations for particles and fields, energy equation). Moreover several equivalent representations of these equations have been established (geodesic form, Schrödinger quantum-mechanical form and fluid dynamical form with quantum potential), which connect various domains and methods often considered as totally or partially disconnected (quantum and classical mechanics, diffusion, hydrodynamics).

This allows one to suggest a fast way to apply it to various systems (where, for example, the current methods have failed): it consists in starting from the standard description and in

This paper is a response to Auffray and Noble (2010) commentary on my (2010) paper.

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looking for the possible existence of the additional terms introduced by the scale relativity theory.

More generally, let us give some "directions for use" of the scale reativity theory:

(I) Laws of Scale Transformation. The main new ingredient of the theory is the explicit introduction in the description (physical quantities and their equations) of explicit scale variables ("resolutions") achieving a "scale space". The theory does not deal (only) with the scaling properties of the standard variables, but also and mainly of these new variables. The scale laws of the coordinates which depend on them, then of the physical functions of these coordinates are obtained as consequences. The scale relativity approach writes these laws of scale transformation in terms of differential equations acting in the scale space. One recovers in this way the standard fractal laws with constant fractal dimension as the simplest possible laws, but one also generalizes them in many ways (including the possibility of quantum-like laws in scale space).

For a given system, one can therefore

(1) (i) attempt to analyse in a differential way the scale behavior of the system, then
(ii) write the corresponding differential equation,
(iii) solve them and
(iv) compare these solutions to the observational/experimental data, or, in a more empirical approach

(2) (i) look for:
- transitions from scale-dependence to scale-independence, and/or between different fractal dimensions;
- variations of the fractal dimensions, including linear variation, log-periodic fluctuations, divergence, etc…; then

(ii) study the cause for this deviation from pure self-similarity (scale force, geometric distorsion in scale space…).

(II) Laws of Motion. On the basis of the internal laws of scale which have been obtained for a given system in step (I), one now constructs the laws of motion (by defining and using a covariant derivative which includes the effects of the fractal geometry). The various representations of the laws of motion in scale relativity theory include the following forms and their generalizations:

- geodesic equation/fundamental equation of dynamics,
- Schrödinger equation,
- diffusion equations,
- hydrodynamics equations including a quantum potential.

The application of the scale relativity approach to a given system may therefore involve the following possibilities, depending on the standard description of the system:

(i) check for the existence in the studied system of the additional terms in the covariant total derivative and in the corresponding equation of dynamics;
(ii) look for signatures of a quantum-type system (probability density which is the square of the modulus of a wave function, existence of a phase involving interferences);
(iii) complete the diffusion Fokker-Planck type equation by a backward Fokker-Planck equation (as a consequence of microscopic time scale irreversibility);
(iv) check for the existence of an additional quantum-type potential in the hydrodynamic form of the equations.

We can now apply this method to the specific question of multiscale integration. Let us give a hint of what would be the successive steps of such an application (a fully developped description lies outside the scope of this short answer and will be the subject of future publications).

One starts from a "point", which represents the smallest scale considered (for example, intracellular "organelles"), then one writes a motion equation which can be integrated in terms of a macroscopic Schrödinger-type equation. Actually, the solutions of this Schrödinger equation are naturally multiscaled. It yields the density of probability of the initial "points", which describes a structure at a larger scale (the "cell" level). Now, while the "vacuum" (lowest energy) state usually describes one object (a single "cell"), excited states describe multi objects ("tissue-like" level), each of which being often separated by zones of null densities (therefore corresponding to infinite quantum potentials) which may represent "walls" (looking, e.g., like an Abrikosov lattice). Note that the resulting structure is not only qualitative, but also quantitative, since the relative sizes of these three levels can be obtained from the theoretical description. Finally, such a "tissue" of individual "cells" can be inserted in a growth equation which takes itself a Schrödinger form. Its solutions yield a new, larger level of organization, such as the "flower" of Fig. 12 of the paper. Finally, the matching conditions between the small scale and large scale solutions allow to connect the constants of these two equations, and therefore the quantitative scales of their solutions.

Let us conclude this answer by a short update of one of the questions also raised by Auffray and Noble's commentary, namely, that of the fast decrease of the arctic sea ice extent. We have given in Fig. 11 of the paper a fit of the US National Snow and Ice Data Center data up to 2008 by a critical law yielding a very close critical date of 2012. The 2009 minimum is now known: its value of 5.1 millions of square km is the third lowest value registered, of the order of the 2007 and 2008 values and it therefore confirms (within fluctuations) the acceleration. A simple model of fractal fracture of the sea ice (see Chmel et al. 2005) leads naturally to an exponential increase of the enlightened surface, and then of the melting. A fit of the data up to 2009 by such a model (which is close to the critical one) still yields a very close date of full ice melting in 2014–2015.

References

Auffray, C., & Noble, D. (2010). Scale relativity: An extended paradigm for physics and biology? *Foundations of Science*. doi:10.1007/s10699-010-9204-9. arXiv:0912.5508v2

Chmel, A., Smirnov, V. N., & Astakhov, M. P. (2005). The Arctic sea-ice cover: Fractal space-time domain. *Physica A: Statistical Mechanics and its Applications 357*(3), 556–564. doi:10.1016/j.physa.2005.04.009.

Nottale, L. (2010). Scale relativity and fractal space-time: Theory and applications. *Foundations of Science, 15*, 2(3), 101–152. doi:10.1007/s10699-010-9170-2. arXiv:0912.5508v2