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Computational Materials Science 37 (2006) 323–327
COMPUTATIONAL MATERIALS SCIENCE
www.elsevier.com/locate/commatsci

Application of similarity theory in the laser forming process

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Received 23 June 2005; received in revised form 5 September 2005; accepted 6 September 2005

Abstract

Laser forming is a flexible forming technology. The deformation in the laser forming process primarily depends on process parameters, workpiece geometry and material properties. To acquire a comprehensive understanding of the deformation, numerous experiments are needed. However, sometimes the plate is huge, which leads to a high experimental expense. To reduce the number and the expense of the experiments, similarity method is adopted and the similarity conditions are derived using the dimensions theory. Numerical simulations of the real and similar plates are carried out. The comparison between the real and similar plate shows a good similarity in the temperature and deformation field.

Keywords: Laser forming; Similarity theory; Metal plate

1. Introduction

Laser forming is a promising technology in manufacturing, such as in shipbuilding, automobile, microelectronics and aerospace. It forms sheet metal by means of laser induced non-uniform thermal stress. In comparison with conventional metal forming methods, laser forming does not require a hard forming tool, and can shape complex curved surface and very small parts. What's more, many hard and brittle materials, such as titanium alloy, nickel alloy, ceramics, etc., can be processed [1–3].

Laser forming is a complex thermal–mechanical process. Understanding various aspects of laser forming is a challenging problem. To achieve the control strategy of the deformation, a large number of experiments are necessary. However, in practical applications, the plate is on occasion huge, such as some ship plates with dimensions 12 000–21 000 × 2000–4000 × 12–22 mm, thus, experimental expenses are high. In such a situation, the similarity method is considered to reduce the number and expense of experiments. Similarity theory has been widely applied in aeromechanics, hydrodynamics, problems of explosion and astrophysical problems [4]. In addition, Cai Zhipeng derived the similarity principle on welding temperature, strain and stress field using the methods of dimensional analysis and equation analysis [5,6]. The applications in laser forming, however, have not been studied in detail. In this work, the similarity conditions are derived for the laser forming process based on the analysis of the laser forming process, and the validity of similarity conditions is verified by the numerical results.

2. Similarity analysis

The following assumptions are made for the similarity analysis. We only consider the heat input by the laser beam. Heat generated by the plastic and phase transition is ignored. The material is elastic, perfectly plastic, homogeneous, and isotropic and obeys the Von Mises yield criterion. The metal plate is initially stress-free. Initial temperature and ambient temperature of different metal plates are the same.

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doi:10.1016/j.commatsci.2005.09.002

2.1. Temperature field

The temperature field in laser forming is mainly related with the heat flux I, the scanning speed v, the dimension of the plate l, the specific heat c, the density ρ and the heat conductivity λ. We choose the mass M, length L, time t and temperature T as the fundamental units. Consequently, the dimensions of the parameters will be

$$\begin{cases} [I] = [M^1, L^0, t^{-3}, T^0], \\ [v] = [M^0, L^1, t^{-1}, T^0], \\ [l] = [M^0, L^1, t^0, T^0], \\ [c] = [M^0, L^2, t^{-2}, T^{-1}], \\ [\rho] = [M^1, L^{-3}, t^0, T^0], \\ [\lambda] = [M^1, L^1, t^{-3}, T^{-1}], \\ [T] = [M^0, L^0, t^0, T^1]. \end{cases} \tag{1}$$

The dimensional relation can be written as [4]

$$\pi = (Mt^{-3})^a (Lt^{-1})^b L^c (L^2 t^{-2} T^{-1})^d (ML^{-3})^e (MLt^{-3} T^{-1})^f T^g. \tag{2}$$

Since the dimension of π is zero and the dimensions of both sides of Eq. (2) must be equal, the following relations are derived

$$\begin{cases} a + e + f = 0, \\ b + c + 2d - 3e + f = 0, \\ -3a - b - 2d - 3f = 0, \\ -d - f + g = 0. \end{cases} \tag{3}$$

Assuming that all physical quantities of the similar model have the subscript m, the relations between the similar model and the real model are expressed as: $I_{\mathrm{m}}/I = C_I$, $v_{\mathrm{m}}/v = C_v$, $l_{\mathrm{m}}/l = C_l$, $c_{\mathrm{m}}/c = C_c$, $\rho_{\mathrm{m}}/\rho = C_\rho$, $\lambda_{\mathrm{m}}/\lambda = C_\lambda$, $T_{\mathrm{m}}/T = C_T$. Let $e = 1$, $f = -2$, $g = -1$, we can solve the above equations

$$a = 1, \quad b = 1, \quad c = 2, \quad d = 1. \tag{4}$$

Then

$$I_{\mathrm{m}} v_{\mathrm{m}} l_{\mathrm{m}}^2 c_{\mathrm{m}} \rho_{\mathrm{m}} \lambda_{\mathrm{m}}^{-2} T_{\mathrm{m}}^{-1} = I v l^2 c \rho \lambda^{-2} T^{-1}. \tag{5}$$

From the above equation, the relation between the similar model and the real model can be expressed as

$$C_I C_v C_l^2 C_c C_\rho C_\lambda^{-2} C_T^{-1} = 1. \tag{6}$$

Let $e = 0$, $f = -1$, $g = -1$, solving the above equations, we can get

$$a = 1, \quad b = 0, \quad c = 1, \quad d = 0. \tag{7}$$

Then

$$I_{\mathrm{m}} l_{\mathrm{m}} \lambda_{\mathrm{m}}^{-1} T_{\mathrm{m}}^{-1} = I l \lambda^{-1} T^{-1}. \tag{8}$$

From the above equation, we can obtain

$$C_I C_l C_\lambda^{-1} C_T^{-1} = 1. \tag{9}$$

In the laser forming process, the temperature of similar and real plate at the corresponding point should be the same, and material properties only depend on the temperature. When material of the two plates are the same, $C_c = C_\rho = C_\lambda = C_T = 1$, simplifying Eqs. (6) and (9), we get

$$\begin{cases} C_I = 1/C_l, \\ C_v = 1/C_l. \end{cases} \tag{10}$$

Therefore, the similarity conditions can be given as follows:

$$\begin{cases} I_{\mathrm{m}}/I = l_{\mathrm{m}}/l = 1/\kappa, \\ v_{\mathrm{m}}/v = 1/\kappa, \end{cases} \tag{11}$$

where κ is the rate of the physical dimension of the similar and the real plate.

2.2. Heat source

The power distribution of a continuous wave laser beam can be roughly expressed by a Gaussian function

$$I = \frac{2AP}{\pi R^2} e^{-\frac{2r^2}{R^2}}, \tag{12}$$

where P is the laser power, R the effective laser beam radius defined as the radius at which the power density decreases

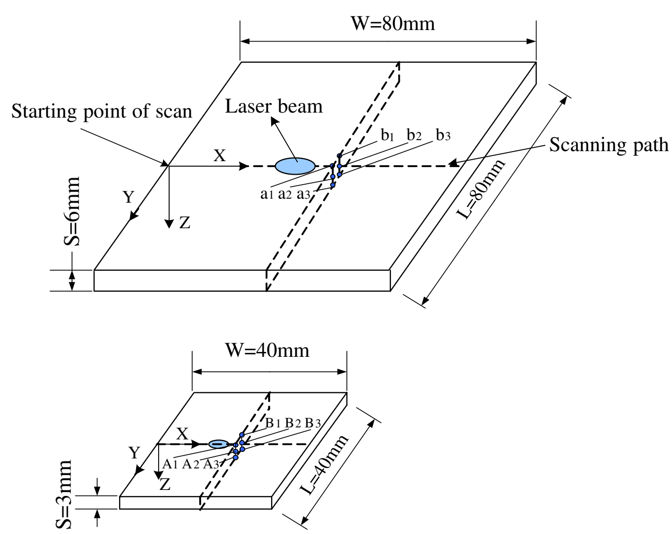

Fig. 1. The real plate and the similar plate.

Table 1
The parameters of the similar plate and the real plate

	Material	Laser power P (W)	Scanning speed v (mm/s)	Spot diameter d (mm)	Size l (mm)
Real plate	D36	1600	10	10	$80 \times 80 \times 6$
Similar plate	D36	800	20	5	$40 \times 40 \times 3$

to $1/e^2$, r the distance from the center of the heat source, A the laser absorption coefficient.

From the similarity conditions of the temperature field, we can obtain

$$\frac{2r_{\mathrm{m}}^2}{R_{\mathrm{m}}^2} = \frac{2r^2}{R^2}, \tag{13}$$

$$\frac{P_{\mathrm{m}}}{R_{\mathrm{m}}^2} / \frac{P}{R^2} = 1/\kappa. \tag{14}$$

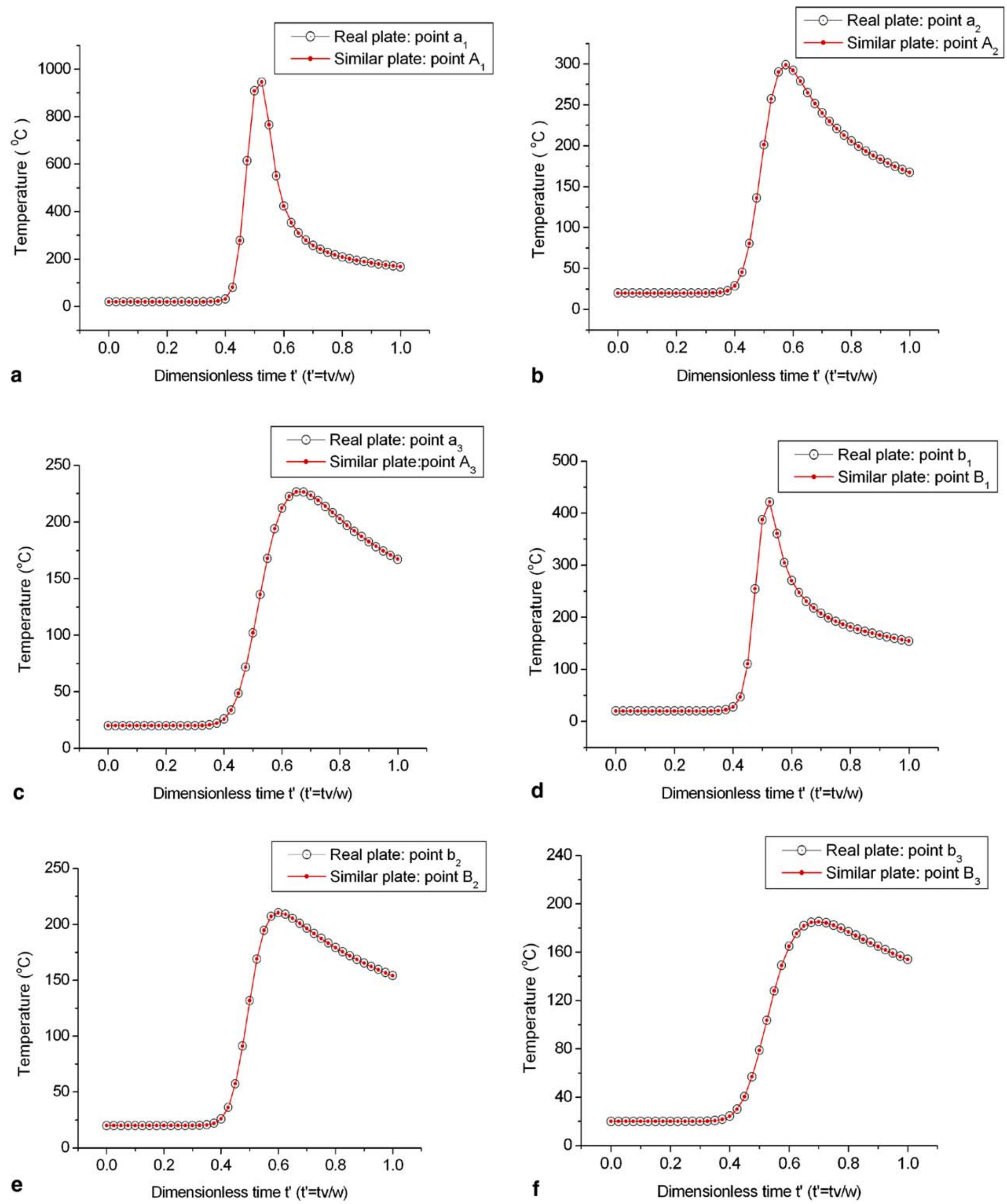

Fig. 2. The comparison of the temperature: (a) temperature history at point a_1 and point A_1; (b) temperature history at point a_2 and point A_2; (c) temperature history at point a_3 and point A_3; (d) temperature history at point b_1 and point B_1; (e) temperature history at point b_2 and point B_2 and (f) temperature history at point b_3 and point B_3.

Thus, similarity condition of heat source can be expressed as follows:

$$\begin{cases} R_m/R = \kappa, \\ P_m/P = \kappa. \end{cases} \tag{15}$$

2.3. Stress and strain field

For the real plate and similar plate, the temperature field and the geometries of the plastic zone are similar, and all of the stresses reach yield limit. Suppose that the plastic zones

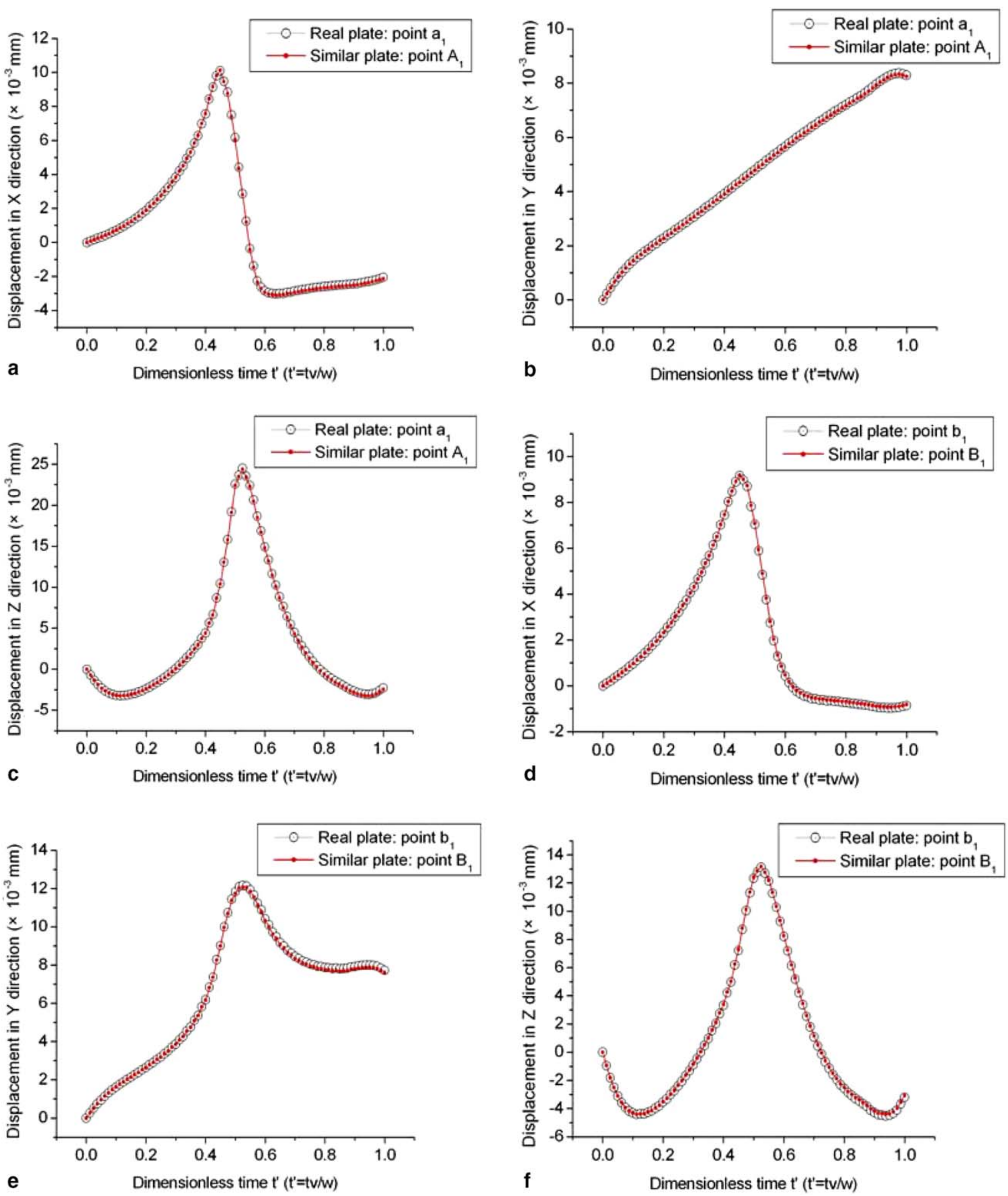

Fig. 3. The comparison of the displacement: (a) displacement history in X direction at point a_1 and point A_1; (b) displacement history in Y direction at point a_1 and point A_1; (c) displacement history in Z direction at point a_1 and point A_1; (d) displacement history in X direction at point b_1 and point B_1; (e) displacement history in Y direction at point b_1 and point B_1; (f) displacement history in Z direction at point b_1 and point B_1.

are cut away at the same time the same forces with the opposite direction are applied. Thus, the similarity of thermal deformations may translate into the similarity of elastic body. According to the equilibrium equation, constitutive equations, geometric equation, boundary conditions of force and displacement of elastic body, the similarity conditions of stress and strain field using the method of equation analysis can be obtained [5], stress similarity: $C_\sigma = \sigma_m/\sigma = 1$, strain similarity: $C_\varepsilon = \varepsilon_m/\varepsilon = 1$, displacement similarity: $C_u = u_m/u = k$.

3. Verification

To verify the similarity conditions, a finite element numerical analysis of the real and similar plates (shown in Fig. 1) was made using the finite element code ANSYS. The plates are heated along the centerline by a moving laser beam from one end to the other. The simulation of the laser forming process is realized by thermal-structure analysis. In the simulation, we assume that the materials are isotropic and continuous, and heat generated by plastic deformation is negligible because it is small compared with heat input by the laser beam. In the model of the finite element analysis, non-linear analysis is used according to the characteristics of the forming process. The Von Mises criterion is used as the yield criterion in the simulation. The material of sheet metal is shipbuilding steel, D36. The laser is CW laser, and the thermal load is given in the form of the thermal flux density which obeys a Gaussian distribution. The surfaces are cooled through free convection with atmosphere, and the plate is clamped at one side. The real plate is twice as large as the similar plate. Table 1 shows the parameters and used for the similar plate and the real plate based on similarity conditions.

3.1. Temperature field

The time histories of the temperatures at points a_1 $(40,40,0)$, a_2 $(40,40,3)$, a_3 $(40,40,6)$, b_1 $(40,44,0)$, b_2 $(40,44,3)$, b_3 $(40,44,6)$, A_1 $(20,20,0)$, A_2 $(20,20,1.5)$, A_3 $(20,20,3)$, B_1 $(20,22,0)$, B_2 $(20,22,1.5)$ and B_3 $(20,22,3)$ are shown in Fig. 2. It is difficult to compare the temperature distribution of the real plate and the similar plate because the heating time is different. For the convenience, we adopt the dimensionless time t' as the abscissa, $t' = tv/w$, where t is the heating time, w the width of the plate. When t' is equal to 0.5, the laser beam is passing the points a_1, a_2, a_3, b_1, b_2, b_3, A_1, A_2, A_3, B_1, B_2 and B_3. As seen from the figure, the maximal temperature difference at points with the same dimensionless time is less than 0.5 °C. The temperature distribution of the similar plate is very similar to the temperature distribution of the real plate. Thus, if two plates meet the similarity conditions, they have a similar temperature field.

3.2. Deformation field

Fig. 3 shows the relations between the displacement in X, Y and Z directions at point a_1, A_1, b_1 and B_1 and the dimensionless time. For comparison purpose, the value of displacement at point a_1 and b_1 is equal to half of the real numerical value according to the similarity conditions. It can be seen from this figure that the deformations are the similar when the plate geometry and the process parameters meet the similarity conditions.

4. Conclusions

To achieve accurate laser forming, it is necessary to design the process parameters for different plates. However, the plate in practical applications is huge. To reduce the number and the expense of the experiments, a similarity method has been adopted, and the following conclusions are drawn:

1. The similarity conditions for the laser forming process are derived using the dimensions theory.
2. The temperature and deformation fields of the real and similar plates have been shown to be similar through numerical simulations. Thus, the similarity conditions proposed are considered correct.

This investigation shows that by using similarity theory it is possible to predict the temperature field and deformation behavior of much larger plates through the analysis of much smaller ones providing the similarity conditions are met. Therefore, it is of significance in reducing the process expense and time for the practical forming of much larger plates.

Acknowledgement

It is gratefully acknowledged that the work presented in this paper has been supported by the Science and Technology department of Shanghai (No. 04dz11002). Thanks also goes to Professor Xinmin Lai and Professor Jun Ni for their helpful comments.

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