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Waves in a Partially Ionized Paramagnetic Gas[1]

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The nature of electromagnetic wave propagation in a partially ionized paramagnetic gas in a static magnetic field is explored. It is necessary to characterize such a medium by second-order tensor expressions for both the permittivity and permeability, the former being that characteristic of an ordinary magnetoplasma, and the latter that for a magnetized paramagnetic medium. Employing linearized forms of these and Maxwell's equations leads to a general dispersion relation. For lossless conditions it is shown that the resonances and cutoffs associated with the free electrons and those due to the paramagnetic constituents are independent. This leads to some interesting, hypothetical propagation conditions that may exist in planetary or stellar atmospheres.

Application of this theory is made to the ionosphere, in which atomic oxygen is the major paramagnetic constituent. Accounting for collisional losses, it is shown that the upper limit of the absorption at the peak of the atomic oxygen electron spin resonance line in the ionosphere is only 5×10^{-4} times the background absorption due to the free electrons; the effect of Doppler broadening would be to decrease this number. The minute nature of this absorption line is supported by the apparently complete lack of experimental evidence of electron spin resonance effects on radio wave propagation in the terrestrial ionosphere. It is also shown that the internal radiation (noise) in the ionosphere must exhibit a similarly weak absorption line at the electron spin resonance, due to the nonequilibrium nature of the energy distribution of the free electrons.

1. Introduction

The theory of dispersion of electromagnetic waves in a partially ionized gas in a static magnetic field, based on the Appleton-Hartree approximation, appears to give an adequate explanation of experimental observations of radio wave propagation in the ionosphere. However, the presence of paramagnetic constituents, such as atomic oxygen in the $^3P_2$ ground state, in a planetary or stellar atmosphere, must produce a macroscopic effect on the dispersion. The purpose of this paper is to develop the theory of dispersion of electromagnetic waves in a partially ionized paramagnetic gas.

The macroscopic effect of a paramagnetic gas in a static magnetic field may be determined by consideration of the motion of the magnetization of the gas due to time varying fields. This approach was used by Hogan (1952) to derive the permeability tensor of a magnetized ferromagnetic material. Laboratory experiments on gas paramagnetism, such as electron spin resonance absorption, usually employ a quantum mechanical interpretation of the dispersion for a particular field in deducing properties or concentrations of the gas in question. The field used in these experiments is an eigenvector of the macroscopic permeability tensor of Hogan, when a gas is considered.

Laboratory electron spin resonance experiments in atomic oxygen by Rawson and Beringer (1952) and the theoretical analysis of these results by Abragam and Van Vleck (1953) indicate that the nonlinearity of the Zeeman splitting of the $^3P_2$ state should be the order of one part in 10^7 in the terrestrial field (of about 5×10^{-5} Weber meter^{-2}). The corresponding separation of Zeeman lines is about .1 Hz, which is negligible as compared to expected line widths. Thus, atomic oxygen should be an ideal paramagnetic constituent to consider.

Other laboratory experiments on atomic oxygen by Krongelb and Strandberg (1959) and by Marshall (1962) have employed the electron spin resonance absorption technique to determine diffusion and recombination rates. While emission lines have not been observed in laboratory experiments of this type, it has been suggested by Herman and Gibbons (1965) that a paramagnetic emission line should be detectable in the ionosphere, and that this would be a useful method for the determination of the atomic oxygen distribution. Subsequently, it was pointed out by Hodges and Colegrove (1966) that this prediction of an emission line was the result of incorrect mathematical manipulation. As line emission would be unlikely without a mechanism to continuously overpopulate the upper Zeeman levels, a more realistic appraisal of the problem indicates that a weak absorption line might be expected in the atmospheric noise within the ionosphere.

In the present paper the dispersion of plane waves in a partially ionized paramagnetic gas is derived. Neglecting losses, the effects of free electrons and paramagnetism on the dispersive nature of such a medium are elucidated through the use of a three-dimensional form of the Clemmow-Mullaly-Allis diagram. In relating this theory to waves in the ionosphere, losses are introduced and an upper limit on the ratio of the electron spin resonance absorption to that due to the free electrons is found. Finally the influence of nonequilibrium free electrons on internal radiation (or noise) is shown to result in an absorption line at the paramagnetic resonance.

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2. Dispersion in a General Linear Medium

The relationships of the fundamental parameters of an electromagnetic field in a source free region are given by Maxwell's curl equations:

$$\nabla_\wedge \mathbf{E} = -\frac{d}{dt}\mathbf{B} \tag{1}$$

$$\nabla_\wedge \mathbf{H} = \frac{d}{dt}\mathbf{D} \tag{2}$$

where **E** and **H** are respectively the electric and magnetic field intensitives, **D** is the electric displacement density, and **B** is the magnetic induction. Furthermore **D** and **B** may be considered as derived quantities due to the relations

$$\mathbf{D} = \epsilon_0 \mathbf{K}\mathbf{E} \tag{3}$$

$$\mathbf{B} = \mu_0 \mathbf{L}\mathbf{H} \tag{4}$$

where ϵ_0 and μ_0 are respectively the permittivity and permeability constants for free space, and **K** and **L** are tensors of second order in a general anisotropic medium.

Using these definitions, Maxwell's curl equations may be expressed:

$$\nabla_\wedge \mathbf{E} = -\mu_0 \mathbf{L}\frac{d}{dt}\mathbf{H}, \tag{5}$$

$$\nabla_\wedge \mathbf{H} = \epsilon_0 \mathbf{K}\frac{d}{dt}\mathbf{E}. \tag{6}$$

By performing the operation $\frac{1}{\epsilon_0}\nabla_\wedge \mathbf{K}^{-1}$ on both sides of (6) and substituting this result into the time derivative of (5), the resulting electromagnetic wave equation is

$$\left\{\nabla_\wedge \mathbf{K}^{-1}\nabla_\wedge + \epsilon_0\mu_0 \mathbf{L}\frac{d^2}{dt^2}\right\}\mathbf{H} = 0. \tag{7}$$

Alternatively, a wave equation for **E** may be obtained; this has the form

$$\left\{\nabla_\wedge \mathbf{L}^{-1}\nabla_\wedge + \epsilon_0\mu_0 \mathbf{K}\frac{d^2}{dt^2}\right\}\mathbf{E} = 0. \tag{8}$$

While (7) and (8) appear to differ notably, each is derivable from the other, and hence both characterize the same field.

In the remainder of this discussion it is useful to consider time harmonic fields which propagate as plane waves, such that **E** and **H** vary as

$$e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}$$

where ω represents angular frequency, **k** is the vector wave number, and **r** is a space position vector. (This is also analogous to consideration of one frequency and wave number of the Fourier transform of a more general field in a uniform medium.) Making this identification in (7) results in

$$\{\mathbf{k}_\wedge \mathbf{K}^{-1}\mathbf{k}_\wedge + k_0^2\mathbf{L}\}\mathbf{H} = 0 \tag{9}$$

where k_0 is the free space wave number (i.e., $k_0^2 = \omega^2\mu_0\epsilon_0$). For this expression to be equal to zero, it is necessary that

$$\text{Det}\,\{\mathbf{k}_\wedge \mathbf{K}^{-1}k_\wedge + \mathbf{k}_0^2\mathbf{L}\} = 0. \tag{10}$$

This is the dispersion relation for electromagnetic waves in a general linear anisotropic medium; a similar relation may be obtained from (8), which is

$$\text{Det}\,\{\mathbf{k}_\wedge \mathbf{L}^{-1}\mathbf{k}_\wedge + k_0^2\mathbf{K}\} = 0. \tag{11}$$

However, this is redundant in that it gives the same result as (10).

3. Application to a Partially Ionized Paramagnetic Gas

In the presence of a static magnetic field the collective, linearized motion of free electrons in a cold, collisionless plasma is governed by the Langevin equation

$$m\frac{d\mathbf{v}}{dt} = -e(\mathbf{E} + \mathbf{v}_\wedge \mathbf{B}_0) \tag{12}$$

where **v** is the mean electron velocity, $\mathbf{B}_0$ is the static magnetic induction, e is electronic charge, and m the electron mass. From this equation a relation between the free electron current density and the electric field may be found. Identifying this current as a component of electric displacement density leads to the proper form of **K** in (3), which in matrix notation is

$$\mathbf{K} = \begin{vmatrix} K' & -iK'' & 0 \\ iK'' & K' & 0 \\ 0 & 0 & K_0 \end{vmatrix} \tag{13}$$

where the static magnetic field is assumed to be directed along the z axis. For fields with time dependence as exp $(i\omega t)$, the entries of this matrix are

$$K' = 1 - \frac{X}{1-Y^2} \tag{14}$$

$$K'' = \frac{XY}{1-Y^2} \tag{15}$$

$$K_0 = 1 - X \tag{16}$$

where
$$X = \frac{N_e e^2}{\epsilon_0 m \omega^2} = \left(\frac{\omega_p}{\omega}\right)^2 \tag{17}$$

$$Y = \frac{B_0 e}{m\omega} = \frac{\omega_b}{\omega} \tag{18}$$

and N_e = electron concentration

ω_p = plasma frequency

ω_b = free electron gyrofrequency

and the use of rationalized MKS units is implied.

The motion of magnetization in a lossless paramagnetic medium with a degenerate ground state, in which Russell-Saunders coupling predominates, may be obtained from the equation

$$\frac{d\mathbf{M}}{dt} = \frac{ge}{2m}\,\mathbf{M}_\wedge \mathbf{H}_T \tag{19}$$

where $\mathbf{M}$ is the magnetization, g is the Landé g factor for the bound electron ground state, and $\mathbf{H}_T$ is the total magnetic field (both static and time varying components). This has been applied by Hogan (1952) to give an approximate description of the behavior of a ferromagnetic material. That result is also applicable to a paramagnetic gas, such as atomic oxygen in the $^3P_2$ ground state, in a weak magnetic field. The resulting tensor, $\mathbf{L}$, which characterizes the anisotropic nature of the permeability, has the form

$$\mathbf{L} = \begin{vmatrix} L' & iL'' & 0 \\ -iL'' & L' & 0 \\ 0 & 0 & L_0 \end{vmatrix}. \tag{20}$$

Neglecting losses the entries in this matrix may be expressed

$$L' = 1 - \frac{SG^2Y^2}{1-G^2Y^2} \tag{21}$$

$$L'' = \frac{SGY}{1-G^2Y^2} \tag{22}$$

$$L_0 = 1 \tag{23}$$

where S is the static magnetic susceptibility of the medium, Y is the parameter defined in (18), and G is one-half the Landé g factor. From these it is evident that the electron spin resonance condition is one for which $G^2Y^2 = 1$. The magnetic susceptibility of a gas is temperature dependent, and is given for an atomic species (in MKS units) by

$$S = \frac{\mu_0 N_a J(J+1) g^2 \mu_B^2}{3k_B T} \tag{24}$$

where

N_a = atomic concentration
J = electron total angular momentum quantum number
μ_B = Bohr magneton
k_B = Boltzmann's constant
T = gas temperature.

Obviously the magnetic susceptibility of a gas is necessarily a small quantity.

By substituting (13) and (20) into either (10) or (11) and performing some algebraic manipulation, a dispersion relation is found which has the form

$$An^4 + Bn^2 + C = 0 \tag{25}$$

where n is the refractive index of the medium defined by

$$n^2 = \frac{k^2}{k_0^2} \tag{26}$$

and the coefficients are

$$A = (K' \sin^2\theta + K_0 \cos^2\theta)(L' \sin^2\theta + L_0\cos^2\theta) \tag{27}$$

$$B = -\{(K_0K'L_+L_- + K_+K_-L_0L')\sin^2\theta + (K_0K_+L_0L_+ + K_0K_-L_0L_-)\cos^2\theta\} \tag{28}$$

$$C = K_0K_+K_-L_0L_+L_-. \tag{29}$$

The angle θ is that between the wave number $\mathbf{k}$ and the static magnetic field $\mathbf{B}_0$. Quantities denoted $K_\pm$ and $L_\pm$ are eigenvalues of the tensors $\mathbf{K}$ and $\mathbf{L}$, respectively, and are given by

$$K_\pm = K' \pm K'' \tag{30}$$

$$L_\pm = L' \pm L''. \tag{31}$$

It may be noted that for zero magnetic susceptibility, the tensor $\mathbf{L}$ becomes unity, and the solution to (25) is the Appleton-Hartree relation. Similarly, for no free electrons, the resulting dispersion relation is identical to that for an ideal ferromagnetic material.

The solution to the dispersion relation, (25), may be expressed

$$n_\pm = \left\{\frac{1}{2A}\left[-B \pm \sqrt{B^2 - 4AC}\right]\right\}^{1/2} \tag{32}$$

where the subscript $\pm$ on n denotes the sign preceding the square root. Real values of n imply wave propagation; imaginary values imply evanescence. Because of the dependence of $n_{\pm}$ on θ, these roots form two surfaces of revolution about the direction of $\mathbf{B}_0$, each with equatorial symmetry; i.e., $n_{\pm}(\theta)=n_{\pm}(\pi-\theta)$. Among the interesting characteristics of the refractive index are the resonances, $|n|=\infty$, and cutoffs, $n=0$, which depend on the parameters S, X, Y, and θ. The condition necessary for one root of (32) to become infinite is

$$0=\frac{A}{-B\pm\sqrt{B^2-4AC}} \tag{33}$$

which requires either $A=0$ or $C=\infty$. (The condition $B=\infty$ is redundant inasmuch as the infinities of B are exactly those of C.) Thus one index surface will have a resonance when

$$\frac{A}{C}=0. \tag{34}$$

By means of a similar argument, it can be shown that one index surface becomes zero, or cutoff, when

$$\frac{C}{A}=0. \tag{35}$$

The ratio $\frac{C}{A}$ is the product of two terms:

$$\frac{C}{A}=\left(\frac{K_+K_-K_0}{K'\sin^2\theta+K_0\cos^2\theta}\right)\cdot\left(\frac{L_+L_-L_0}{L'\sin^2\theta+L_0\cos^2\theta}\right). \tag{36}$$

The first term is independent of magnetic susceptibility, the second is independent of the free electron concentration, but both involve the magnetic parameter Y. It may also be noted that the infinities and zeros of the first term are the resonances and cutoffs of a cold magnetoplasma; these are plotted, as in the Clemmow-Mullaly-Allis diagram, in the $X-Y^2$ plane of figure 1. Similarly, the paramagnetic resonances and cutoffs are due to the second term, and are plotted in the $S-Y^2$ plane. The ramification of the multiplicative nature of $\frac{C}{A}$ is that the magnetoplasma resonances and cutoffs are cylindrical surfaces defined by the curves in the $X-Y^2$ plane and generated parallel to the S axis, while the paramagnetic resonances and cutoffs are cylindrical surfaces generated parallel to the X axis. As a result of the intersections of these surfaces, there are created volumes in S, X, Y^2 space, in which the refractive index may be real even though it would be imaginary for either $S=0$ or $X=0$. While there are a number of interesting possibilities here, the practical limitations on magnetic susceptibility of a gas limit useful discussion to cases for $S<<1$.

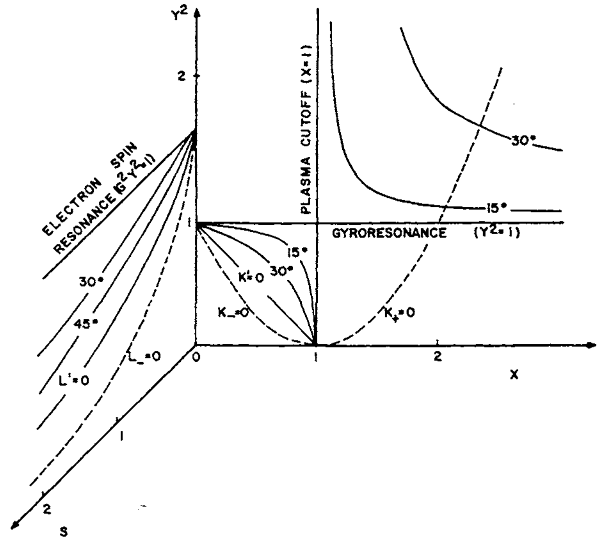

FIGURE 1. *Intersections of resonance and cutoff surfaces (solid and dashed lines respectively) in the* $X-Y^2$ *and* $S-Y^2$ *planes.*

To elucidate the various free electron and paramagnetic gas effects on the dispersive properties of the medium, it is convenient to use meridional plane graphs of the refractive index surfaces. In figure 2, various pairs of index surfaces for $S=0$, i.e., for an ordinary magnetoplasma, are plotted with the origin of each pair located at the appropriate $X-Y^2$ coordinate of the chart corresponding to the parameters which define the surfaces. Real values of refractive index are shown as solid lines and imaginary values as dashed lines. The z direction for the index surfaces is parallel to the Y^2 axis. This chart is of the type which has been used by Deschamps (1963), and with index surfaces replaced by those for phase velocity (these are essentially inverses of each other), it is similar to charts used by Allis, Buchsbaum and Bers (1963).

Figure 3 is a similar chart, with index surfaces plotted for the same set of X and Y^2 parameters, but with $S=.1$, an unreasonably large value for a gas, chosen to exaggerate the effect of a paramagnetic constituent. The value $G=\frac{3}{4}$, appropriate for atomic oxygen, is used. On this chart, the magnetoplasma resonances and cutoffs are indicated as in previous figures and, in addition, those for the paramagnetism are shown. The latter are straight horizontal lines corresponding to constant values of Y^2, because the plane of the chart corresponds to a fixed value of S.

The effect of the paramagnetic constituent is most apparent in the region of figure 3 bounded by the paramagnetic cutoff ($L_-=0$) and resonance ($G^2Y^2=1$) lines. Elsewhere, the index surfaces are distorted, but maintain their essential characteristics as determined by the free electrons alone. Surfaces plotted for $X=0$ show the influence of only the paramagnetism.

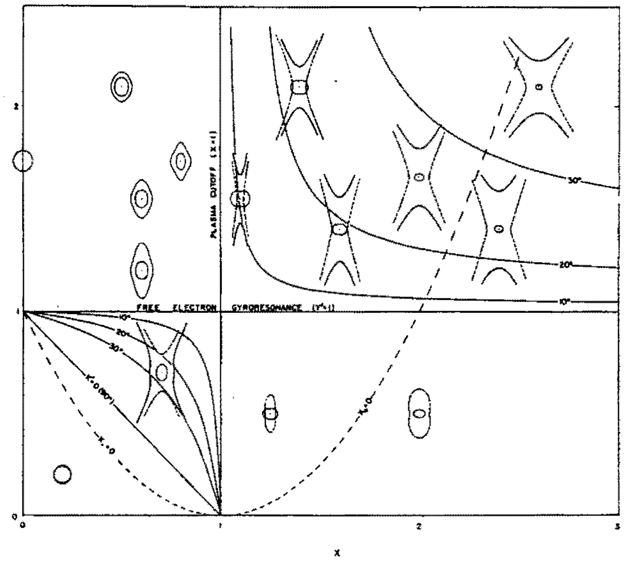

FIGURE 2. *Refractive index surfaces for various conditions in a magnetoplasma with no paramagnetic constituent.*

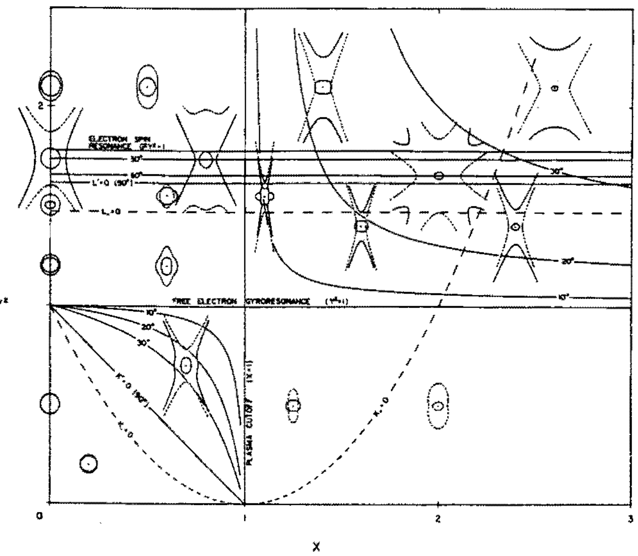

FIGURE 3. *Refractive index surfaces for various conditions in a partially ionized paramagnetic gas with* $S=0.1$ *and* $G=\frac{3}{4}$.

In the region between the lines $L_-=0$ and $L'=0$, and for $X<1$, the introduction of paramagnetism causes one of the index surfaces to be imaginary, or cutoff. This is similar to the effect in a magnetoplasma between the $K_-=0$ and $K'=0$ lines. The $L'=0$ line corresponds to resonance conditions at $\theta=90°$; $G^2Y^2=1$ corresponds to resonance at $\theta=0$. Between these the angle of the resonance is determined by the infinity of (36). The index surfaces for $\theta=30°$, which are representative of those for $X<1$, show that the angle of the resonance is a boundary between evanescence and propagation for the same surface which was completely cutoff before. This effect is similar to that produced by the magnetoplasma in the triangular region bounded by the lines $X=1$, $Y^2=1$, and $K'=0$.

With $X>1$, it is possible for the discriminant of (25), i.e., B^2-4AC, to become negative. This leads to two roots which are complex conjugates, suggesting one wave which grows in amplitude and one which decays. However, it can be shown that under these conditions, the polarizations of the two characteristic waves are not orthogonal, and must therefore be coupled so as to yield a continuous transfer of energy between the two modes. The result is apparently representable by a single real surface, which represents the real part of the index over the directions for which this condition exists, such as is the case in the index surfaces for $X=1.1$ and Y^2 between the lines corresponding to $L_-=0$ and $L'=0$ of figure 3. At still larger values of X, the discriminant of (25) does not become negative for any θ, and the effect of the combination of resonances due to the free electrons and the paramagnetism is to cause one surface to be imaginary both along and transverse to the magnetic field. Biconical regions of propagation exist, bounded by the resonance cones for each constituent.

This is of course an exaggerated picture of the dispersive nature of a partially ionized paramagnetic gas. In an actual gas, the value of S will be considerably smaller than that chosen for this discussion.

4. The Effect of Finite Losses

In a magnetoplasma, the collisions of free electrons with ions, atoms, or molecules provide a mechanism for the removal of ordered momentum of electrons. This is usually accounted for by the inclusion of a loss term in the Langevin equation, i.e.,

$$m\frac{d\mathbf{v}}{dt}=-e(\mathbf{E}+\mathbf{v}_\wedge\mathbf{B}_0)-m\mathbf{v}\nu_e \tag{37}$$

where ν_e is the effective electron collision frequency for momentum transfer. The terms of the permittivity tensor $\mathbf{K}$, obtained from this equation, are

$$K'=1-\frac{X(1-iZ)}{(1-iZ)^2-Y^2} \tag{38}$$

$$K''=\frac{XY}{(1-iZ)^2-Y^2} \tag{39}$$

$$K_0=1-\frac{X}{1-iZ} \tag{40}$$

where

$$Z=\frac{\nu_e}{\omega}. \tag{41}$$

Losses in the paramagnetic gas may be included by introducing a term which accounts for the damping force on the precessing dipole moment in the equation of motion of the magnetization. As given by Hogan (1952) the form of (20) becomes

$$\frac{d\mathbf{M}}{dt}=\frac{ge}{2m}\left[\mathbf{M}_\wedge\mathbf{H}_T-\frac{\alpha}{M}(\mathbf{M}_\wedge\mathbf{M}_\wedge\mathbf{H}_T)\right] \tag{42}$$

where α is a damping parameter. This results in

$$L'=1-\frac{SGY[GY(1+\alpha^2)+i\alpha]}{(1-iGY\alpha)^2-G^2Y^2} \tag{43}$$

$$L''=\frac{SGY}{(1-iGY\alpha)^2-G^2Y^2} \tag{44}$$

$$L_0=1 \tag{45}$$

for the terms of the tensor $\mathbf{L}$.

Of particular importance to the present discussion is the effect of the inclusion of these loss terms on the practicability of detecting the effect of a paramagnetic gas in a magnetoplasma. To aid in this problem it is helpful to note that solutions to the dispersion relation (32), have particularly simple forms for propagation in the principle directions $\theta=0$ and $\theta=\frac{\pi}{2}$. These are

$$n_+(0)=\sqrt{K_+L_+},\qquad n_-(0)=\sqrt{K_-L_-} \tag{46}$$

and

$$n_+\left(\frac{\pi}{2}\right)=\sqrt{\frac{K_0L_+L_-}{L'}},\qquad n_-\left(\frac{\pi}{2}\right)=\sqrt{\frac{K_+K_-L_0}{K'}}. \tag{47}$$

The most significant effect of the paramagnetic resonance is its influence on $n_-(0)$; this is also the index which governs propagation along the magnetic field in the whistler mode and the electron spin resonance absorption in laboratory experiments as well.

After some algebraic manipulation, the expression for $n_-(0)$ may be written

$$n_-(0)=\left\{\left[1-\frac{X_1}{\psi\left(\psi-\frac{1}{G}-iZ_1\right)}\right]\left[1-\frac{S(1+i\alpha)}{\psi-1-i\alpha}\right]\right\}^{1/2} \tag{48}$$

where ψ is the ratio of applied frequency to that of the electron spin resonance, or

$$\psi=\frac{1}{GY} \tag{49}$$

and X_1 and Z_1 represent the parameters X and Z evaluated for $\psi=1$. Near $\psi=1$, the second term of this expression exhibits a resonance, the imaginary part of which has the Bloch - Lorentz line-shape characteristic of collisional broadening. This is in agreement with the line shapes for the atomic oxygen electron spin resonance measurements of Marshall (1962). Hence, the proper value for α is roughly the ratio of the atomic oxygen collision frequency, ν_a, to the angular electron spin resonance frequency. The condition necessary for Doppler broadening to have negligible influence on the line shape is

$$\frac{m_ac^2}{2k_BT}\left(\frac{\nu_a}{G\omega_b}\right)^2\gg 1 \tag{50}$$

where m_a is the atomic mass, and c is the velocity of light. Collisional losses must be considered only as a lower limit when condition (50) is not satisfied.

5. Paramagnetism in the Ionosphere

The magnetic susceptibility for atomic oxygen in the ${}^3P_2$ ground state, from (24), is

$$S=3.5\times10^{-29}\,\frac{N_a}{T} \tag{51}$$

where the concentration N_a is in units of meter^{-3}. For typical ionospheric conditions, as given by Johnson (1965), the order of magnitude of S is less than 10^{-15}. Therefore, for $G=\frac{3}{4}$ (appropriate for atomic oxygen), $\psi=1$, and Z_1 assumed small, (48) may be expressed approximately by

$$n_-(0)\cong\sqrt{1+3X_1}\left\{1-\frac{i}{2}\left[\frac{9X_1Z_1}{1+3X_1}+\frac{\alpha S\psi}{(\psi-1)^2+\alpha^2}\right]\right\}. \tag{52}$$

The electron spin resonance appears in the imaginary part of $n_-(0)$, a quantity sometimes referred to as the absorptive index. Detection of this resonance requires the ability to discern the paramagnetic absorption in the presence of the free electron absorption. The ratio of these quantities is

$$R=\frac{S(1+3X_1)}{9X_1Z_1\alpha}. \tag{53}$$

Based on values of atomic oxygen concentration and temperature given by Johnson (1965), electron concentration and electron collision frequency given by Hanson (1965), atomic oxygen collision frequency (assumed to be roughly that for neutral molecules) from the U.S. Standard Atmosphere (1962), and assuming $B_0=5\times10^{-5}$ Weber meter^{-2}, the maximum expected value of R in the ionosphere occurs at nighttime under quiet sun conditions at about 180 km, where (50) is satisfied. However, this upper limit for R is only 5×10^{-4}; thus the maximum paramagnetic absorption is much less than that for free electrons in the ionosphere.

The detection of a resonance which perturbs the ambient absorption index by less than one part in a thousand may be possible with a carefully executed experiment. It is not surprising, in view of this upper limit on R for the ionosphere, that there is a lack of experimental observation of propagation phenomena associated with the electron spin resonance of atomic oxygen.

6. Radiation

In this portion of the discussion it is assumed that the free electrons and paramagnetic atoms exist in a neutral buffer gas. The latter acts essentially as an infinite heat sink to which is eventually transferred any excess energy absorbed from electromagnetic waves by the free electrons or paramagnetic atoms. Thus the energy transferred between the absorbing species is negligible. It is reasonable that the neutral and the paramagnetic constituents should be nearly in thermal equilibrium, but in the presence of external radiation the electron energy distribution would be non-Maxwellian. Conditions in the ionosphere are similar to these.

The Poynting vector for radiation with wave number between $\mathbf{k}$ and $\mathbf{k}+d\mathbf{k}$ traveling in the range of directions $\mathbf{\Omega}$ to $\mathbf{\Omega}+d\mathbf{\Omega}$ will be denoted $\mathbf{I}(\mathbf{k},\mathbf{\Omega})d\mathbf{k}d\mathbf{\Omega}$ where the direction of energy flow, $\mathbf{\Omega}$, is the normal to the refractive index surface at $n=\frac{k}{k_0}$. The rate of change of the Poynting vector in the $\mathbf{k}$ direction may be expressed

$$\frac{\partial}{\partial r_k}\mathbf{I}(\mathbf{k},\mathbf{\Omega})d\mathbf{k}d\mathbf{\Omega}=-\frac{2\omega}{c}\chi(\mathbf{k})\mathbf{I}(\mathbf{k},\mathbf{\Omega})d\mathbf{k}d\mathbf{\Omega}+\mathbf{Q}(\mathbf{k},\mathbf{\Omega})d\mathbf{k}d\mathbf{\Omega} \tag{54}$$

where $\chi(\mathbf{k})$ is the absorptive index associated with $\mathbf{k}$ and $\mathbf{Q}(\mathbf{k},\mathbf{\Omega})d\mathbf{k}d\mathbf{\Omega}$ is the net volume emission rate of the medium for this mode of propagation.

If the absorption is sufficiently small it is possible to approximate $\chi(\mathbf{k})$ as the sum

$$\chi(\mathbf{k})\cong\chi_e(\mathbf{k})+\chi_m(\mathbf{k}) \tag{55}$$

where $\chi_e(\mathbf{k})$ is proportional to Z and $\chi_m(\mathbf{k})$ is proportional to α, as was done in the derivation of (52). As the rate of emission of each constituent is independent of the other, the net emission in this approximation, as given by Kirchoff's law, is

$$\mathbf{Q}(\mathbf{k},\mathbf{\Omega})=2\frac{\omega}{c}\{\chi_e(\mathbf{k})\mathbf{I}_e(\mathbf{k},\mathbf{\Omega})+\chi_m(\mathbf{k})\mathbf{I}_m(\mathbf{k},\mathbf{\Omega})\} \tag{56}$$

where $\mathbf{I}_e(\mathbf{k},\mathbf{\Omega})$ and $\mathbf{I}_m(\mathbf{k},\mathbf{\Omega})$ are respectively the radiation energy densities which would be present in a medium of similar dispersion if only the free electrons or the paramagnetic atoms, respectively, contributed to absorption.

Neglecting spatial gradients of the distribution of electron energy, only the external radiation has a derivative in the $\mathbf{k}$ direction. This may be identified as $-\frac{2\omega}{c}\chi(\mathbf{k})$ times the Poynting vector for the external radiation. Thus the internal radiation energy density may be identified approximately as

$$\mathbf{I}_{int}(\mathbf{k},\mathbf{\Omega})=\frac{\chi_e(\mathbf{k})\mathbf{I}_e(\mathbf{k},\mathbf{\Omega})+\chi_m(\mathbf{k})\mathbf{I}_m(\mathbf{k},\mathbf{\Omega})}{\chi_e(\mathbf{k})+\chi_m(\mathbf{k})}. \tag{57}$$

As it is physically necessary that χ_e and χ_m be real positive quantities, the internal radiation intensity is bounded by $|\mathbf{I}_e|$ and $|\mathbf{I}_m|$.

In the ionosphere the major paramagnetic constituent, atomic oxygen, is in approximate thermal equilibrium with the remainder of the neutral gas. Thus $|\mathbf{I}_m|$ may be considered to be thermal radiation. However, the mean electron energy may greatly exceed that of the neutral gas. Thus

$$|\mathbf{I}_e|>|\mathbf{I}_{int}|>|\mathbf{I}_m| \tag{58}$$

is a necessary condition in the ionosphere. Furthermore, the effect of a maximum of χ_m, such as occurs at the paramagnetic resonance frequency, is to cause $|\mathbf{I}_{int}|$ to decrease as an absorption line. Again the detection of such a resonance in the ionosphere would require the ability to discern a deviation in the spectral intensity of radiation of considerably less than one part in a thousand.

7. Conclusion

Addition of a neutral paramagnetic constituent to a magnetoplasma results in a medium for which both the permeability and permittivity are second-order tensors. In the lossless case the characteristic cutoffs and resonances for electromagnetic waves separate into two categories: those determined by the free electrons, as in an ordinary magnetoplasma; and those which depend on the static magnetic susceptibility of the paramagnetic constituent, as in an ideal ferromagnetic material.

The results of this study may eventually be applied to waves in planetary or stellar atmospheres if these contain large concentrations of paramagnetic gases. As applied to the earth's ionosphere, where atomic oxygen is the dominant paramagnetic constituent, it is shown that under the most optimistic circumstances the resonant paramagnetic absorption line due to atomic oxygen in the ionosphere corresponds to less than 10^{-3} times the free electron absorption. This explains the lack of evidence of paramagnetic resonance phenomena in ionsopheric experiments, and as well indicates the futility of attempting to determine the distribution of atomic oxygen by means of paramagnetic resonance experiments.

8. References

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