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REGULAR ARTICLE

Imitation and selective matching in reputational games

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Abstract This paper investigates *imitation* and *selective* matching in *reputational* games with an outside option. We identify two classes of such games, *ultimatum* and *trust* games. By selective matching we mean that short-run players have the possibility of selecting the long-run player they play against. We find that selective matching (unlike random matching) favors the equilibrium associated to reputation in the ultimatum game, but not in the trust game.

Keywords Reputation · Long-run equilibria · Selective matching · Games with an outside option

JEL Classifications C72 · C73 · L1

1 Introduction

Reputation effects are crucial elements in many interesting economic relationships. For instance, reputation may support foreign investments in developing countries, contribute to the maintenance of quality in 'experience good' markets, or deter potential competitors in monopolistic local markets. However, it has proven difficult to model such phenomena in finite and infinite horizon models.[1] In infinitely repeated games, an implication of the Folk theorem is that there exist multiple rational equilibria supporting almost all types of behavior, cooperation as well as defection, toughness and fairness. As a result,

[1]See Selten (1978), Kreps and Wilson (1982), Milgrom and Roberts (1982), Fudenberg and Levine (1989, 1992).

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the predictive power of the theory may be sharply reduced. In particular, we cannot identify situations in which reputation represents a significant factor in determining the outcome of the game.

This paper investigates the role of imitation and selective matching in supporting the reputation-building dynamics in infinitely repeated games with an outside option (GOO). In these games, which are traditional models of reputation, a sequence of short-run players (buyers or investors) first select whether to play a game with a long-run player (a firm or a country). An exit option is therefore available to short-run players. Due to this outside option, competition between long-run players seems a natural setting for most of GOO: consumers can use their outside option to switch firms freely, or countries may compete for investors. However, most studies on reputation formation have considered GOO in isolation, building models where a single long-run player faces a sequence of short-run players.[2] This accounts for two of the main ingredients of the whole story: (1) *social learning*, that is, the opportunity to learn from others (especially for long-run players), and (2) *selective matching*, that is, short-run players' possibility to select the long-run player they play against, when several long-run players are simultaneously considered.

In this paper, I use an evolutionary game model integrating both social learning (by way of imitation) and selective matching. Here, a finite population of long-run players repeatedly face a population of short-run players. Both types of players are boundedly rational agents who learn by imitating successful others. This follows a growing and recent literature on evolutionary games, in which players are assumed to imitate successful behaviors.[3] Some empirical studies have shown that imitation is a good approximation of economic behaviors in a variety of contexts.[4] Alternatively, some papers such as Kandori and Rob (1995) have assumed best response dynamics. Imitation dynamics requires less information on the structure of the game, and thus is more appropriate in a very complex environment where agents seek to minimize decision costs. The matching process between short-run and long-run players is specified following two polar scenarios: the *random* matching scenario, where short-run and long-run players are randomly matched, and the *selective* matching scenario, where a short-run player selects the long-run player against whom he plays. I use the random (pairwise) matching scenario as a benchmark as it is one of the two usual matching processes in evolutionary models.[5] The selective matching scenario gives short-run players some control over those against whom they play. This selection is endogenously incorporated in the

[2]Notable exceptions are Jackson and Kalai (1999) and Hörner (2002).

[3]See Robson and Vega-Redondo (1996), Vega-Redondo (1997), Tanaka (1999) or Alós-Ferrer (2004) for instance.

[4]See Offerman and Sonnemans (1998) and Pingle and Day (1996).

[5]The other matching process is the so-called playing-the-field. See for instance Vega-Redondo (1997) for a description of the differences between both matching processes.

model by assuming that short-run players strategy consists of choosing an action (enter the subgame or not) and of selecting a long-run player.

1.1 Motivations

Evolutionary game theory is an appropriate tool for capturing social learning and selective matching. First, evolutionary models consider populations of boundedly rational agents who are repeatedly matched in pairs to play a defined stage game. Thus, there are several repeated games played simultaneously which allows players to learn not only from their own experience but also from the experience of others' choices. As in Jackson and Kalai (1999), we examine reputational games when such games are not played in isolation, but instead allow a player to learn from other players by observing how they are playing the game.

Evolutionary game theory is also an ideal setup for analyzing matching processes. Due to the simultaneity of games, evolutionary models have to make some assumptions as to how players meet in each game stage. Literature offers various specifications regarding the matching mechanism and, as Oechssler (1997) and Robson and Vega-Redondo (1996) have shown, evolutionary models are quite sensitive to the specification of the matching process. In a general way, one may define two models of matching: the *fully global selection* model and the *group selection* model. In the former, interaction takes place within the entire population, where individuals are randomly matched to play a bilateral game.[6] In contrast, the second model of matching assumes that interaction takes place within relatively small subpopulations, where there is infrequent migration between subpopulations.[7] Both models share the assumption that the matching mechanism relies on a perfectly random process, removing *selective* considerations such as group or partner choice. Recently however, some evolutionary models have included the possibility of selective considerations in the matching process. For instance, Oechssler (1997) studies the coordination problem in a population partitioned into groups, where players can not only choose which action to take in the game, but also which group they want to join. Similarly, Bergstrom (2003) explores the possibility of partner choice in a multiplayer prisoners' dilemma game with voluntary matching. Larson (2004) also integrates selective matching in an evolutionary model, giving agents some control over those against whom they play. Our paper follows this line of research.

Using the bounded rationality set-up offered by evolutionary game theory, we follow the Selten (1978) intuition which underlies the necessity of a limited rationality approach in order to capture reputation effects. Furthermore, as noted by Abreu and Sethi (2003), traditional literature on reputation in games considers the possibility of one or more boundedly rational players leaving

[6] For more details on this class of matching mechanisms, see Fudenberg and Levine (1998).

[7] Biologists call this class of models *Haystack models*. See Bergstrom (2002).

unexplained the particular form of irrationality assumed. Modelling reputation formation with evolutionary game theory may provide the theory with formal descriptions of boundedly rational behaviors.

1.2 Results and predictions

Selective matching and imitation may play a key role in the analysis of reputation in GOO. We find that they support the equilibrium associated with reputation as the *long-run* equilibrium in the ultimatum game. However, in the trust game, the long-run outcome remains the subgame perfect equilibrium, under both random and selective matchings. This is because the selective short-run player reaction is too low in punishing defectors and then, as players are concerned about relative payoffs in imitation dynamics, players continue imitating those who reject cooperation. Even if our model cannot produce a clear-cut selection between equilibria based on the sole matching and learning considerations, it shows that, unlike the random matching benchmark, selective matching does not always favor subgame perfection.

Our result is very close of that obtained by Gale et al. (1995), who are motivated by similar concerns. They study a simple learning model (coming from the replicator dynamics) to which is added a source of noise: players may "mistakenly learn to play a strategy that is adapted to the wrong game". Under some asymmetric conditions, the model also shows that, applied to the ultimatum game, such a learning process leads to outcomes that are Nash equilibria but not subgame-perfect.

1.3 Possible applications

The model developed in this paper is applicable in several economic situations, as many GOO implicitly contain the idea of selective matching. For instance, the *quality* game represents a situation in which the short-run player begins by choosing whether or not to purchase a good from the long-run player, who in turn can produce high quality or low (see Klein and Leffler 1981; Shapiro 1983; Holmstrom 1999).[8] Considering a competitive setting, that is several long-run players, provides short-run players with the possibility of selective matching. Another interesting economic situation is the sovereign problem of foreign direct investment in less developed countries (see Eaton and Gersovitz 1983; Raff 1992). Such investments are prone to sovereign risk because they are sunk, and then the host country may expropriate without compensation or unilaterally change its tax policy. This situation may be described by the *investment* or *trust* game which is a reputational game with an outside option. Here, the outside option (to invest abroad) gives rise to the possibility of selective matching. Finally, even the chain-store game may be considered

[8]The same game is used in price dispersion models. See Salop and Stiglitz (1977).

with selective matching: as incumbent firms are viewed as local monopolists, potential competitors may choose the local market into which they will move.

1.4 Organisation of the paper

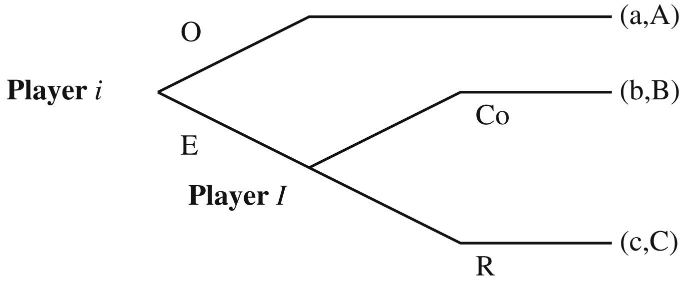
The rest of the paper is organized as follows: Section 2 presents the model. Section 3 derives results under random and selective matching. Section 4 discusses the results and suggests some extensions. Section 5 concludes.

2 The model

We consider two large but finite sets of long-run and short-run players,[9] respectively denoted $\mathbf{L} = \{1, ..., I, ..., N\}$ and $\mathbf{S} = \{1, ..., i, ..., n\}$. Let $N = n$.[10] Players I and i are repeatedly matched to play a stage game and adjust their behavior over time. The model consists of four elements: (1) an underlying game that describes the basic strategic environment; (2) a matching process specifying how players are paired to play the underlying game; (3) a learning mechanism that describes how agents learn about different strategies and sometimes switch to them; (4) a mutation mechanism that integrates some perturbations in the learning mechanism. These elements are described in turn in this section.

2.1 The underlying game

We focus on two-player games with an outside option. These games describe situations which may be summarized as follows



where (a, b, c) and (A, B, C) respectively denote players i and I payoffs. Player i may either choose to enter the subgame (E) or take his outside option (O). If player i decides to enter, the long-run player has the option to cooperate (Co)

[9] We keep here the denomination “short-run” and “long-run” players only for convenience, as all players repeatedly play the game.

[10] As will be easy to see, allowing population sizes to be different does not alter the results of the model, but highly complicates their exposition.

or to reject cooperation (R). For our purposes, there are two types of generic games with an outside option. I will first consider pure strategy equilibria.

(*i*) The *trust* game,[11] in which $b > a > c$ and $C > B > A$. This game has the same strategic structure as the quality (or price dispersion) game and the investment game. In the trust game, (O, R) constitutes the only Nash equilibrium of the game in pure strategy, which is also a subgame perfect equilibrium. Notice that outcome (E, Co), which is not a Nash equilibrium, Pareto-dominates (O, R), meaning that self-interested behavior makes everyone worse off. In a context of perfect rationality, when the game is repeated, the expected outcome to be sustained by reputation effects is (E, Co).

(*ii*) The *ultimatum* (mini)game, with $b > a > c$ and $A > B > C$. In this game, we can see player i as making either a high offer, O, or a low offer, E. The high offer is assumed to be always accepted (as $A > B > C$). After E, player I can either cooperate (Co) or reject the offer (R). This game is also similar to the chain-store game. It has two Nash equilibria (O, R) and (E, Co), with the latter subgame perfect. Thus, unlike the trust game, (E, Co) is a Nash equilibrium but here (O, R) is the expected outcome with reputation effects when the game is repeated. It turns out that this outcome is also a focal point in laboratory experiments.[12]

When mixed strategies are added to the analysis, both game types have a component of Nash equilibria. Let Ω_i and Ω_I stand for strategy spaces. Denote by $\Sigma^i = \Delta(\Omega_i)$ and $\Sigma^I = \Delta(\Omega_I)$ the spaces of mixed strategy of each type of player, with generic elements $\sigma^i \in \Sigma^i$ and $\sigma^I \in \Sigma^I$. Without loss of generality, let $c = 0$. Both type games have a continuum of Nash equilibria, containing equilibrium (O, R), given by

$$\Theta = \{(\sigma^i, \sigma^I) \in \Sigma^i \times \Sigma^I : \sigma^i_E = 0, \sigma^I_{Co} \leq \frac{a}{b}\}$$

where player i takes its outside option, and player I chooses strategy Co with probability at most a/b.

2.2 The matching process

Time is measured discretely and indexed by $t = 1, 2, 3....$ At period t, player $i \in \mathbf{S}$ is paired with player $I \in \mathbf{L}$ to play the generic game described above. I consider two polar scenarios describing the matching process between players i and I:

- The *random* matching scenario, in which each player i is randomly matched with exactly one player I to play the generic game once in each period. As $N = n$, each player I plays the game once in each period.

[11] See Kreps (1990).
[12] See Binmore et al. (1995).

- The *selective* matching scenario, where player i selects the long-run player I he plays against. The way player i operates the selection is described in the next section (it is part of the learning mechanism). Due to the selection component here, player I may be confronted with several players $i \in \mathbf{S}$ (all having chosen I), meaning that player I may play the game several times in one period. All players i play once in each period.

The random matching scenario provides the analysis with a benchmark case, as random matching is the usual assumption in evolutionary models. Notice that, in selective matching, long-run players act as in the random matching scenario. From their point of view, the matching follows a random process, even if short-run players are selective. The main reason supporting this assumption is that long-run players have no outside option in GOO, meaning that they play no active role in the matching process. Note also that both scenarios suppose different degrees of sophistication for short-run players, since selective matching requires more information about long-run players. We will discuss this in Section 5.

2.3 The learning mechanism

The learning mechanism specifies how players choose their strategies. Following Vega-Redondo (1997), I will consider an imitation dynamics in which players simply mimic the strategy of the most successful players.[13] Thus, they are assumed to be myopic[14] and adaptive. Agents do not form expectations about the future course of play, but take into account the decisions made in the past to determine their strategies. This means that changing from one strategy to another is dictated by such considerations as: How well do I perform compared to the other players? What is the strategy used by the most successful players?

We are interested in the description of the behavior adopted by the players in the long-run. Let $z_t = (z_t^s, z_t^l) \in Z \equiv \{z = (z^s, z^l) : 0 \le z^s \le n,\ 0 \le z^l \le N\}$ be the state at t of the evolutionary dynamics, where z_t^s and z_t^l represent respectively the number of players using O in population $\mathbf{S}$ and using R in population $\mathbf{L}$. For convenience, states $z_1 = (n, N)$, $z_2 = (0, 0)$, $z_3 = (0, N)$ and $z_4 = (n, 0)$ will be directly written $z_1 = (O, R)$, $z_2 = (E, Co)$, $z_3 = (E, R)$ and $z_4 = (O, Co)$.

Let $X_t = (X_{1t}, ..., X_{It}, ..., X_{Nt})$ represent the strategy-profile of long-run players at period t, with $X_{It} \in \{Co, R\}$. In the same way, define $x_t = (x_{1t}, ..., x_{it}, ..., x_{nt})$ as the strategy-profile of short-run players at t, where $x_{it} \in \{O, E\}$ in the random matching scenario and $x_{it} = (a_{it}, I)$ with $a_{it} \in \{O, E\}$ and $I \in \mathbf{L}$ in the selective matching scenario. This is because x_{it} consists of choosing

[13] Alternatively, evolutionary games consider best response dynamics which, relative to imitation dynamics, required much more information (as players need to know the whole structure of the game).

[14] On the justification of the myopic assumption, see Section 5.

an action $a_{it} \in \{O, E\}$ and selecting a long-run player $I \in \mathbf{L}$ when the matching is selective.[15] Consequently, player i's strategy is a couple $x_{it} = (a_{it}, I)$ in the selective matching scenario. Define also the payoff-profiles associated respectively to x_t and X_t as $\pi_t = (\pi_{1t}, ..., \pi_{it}, ..., \pi_{nt})$ and $\Pi_t = (\Pi_{1t}, ..., \Pi_{It}, ..., \Pi_{Nt})$.

At the beginning of each period, all players i and I choose a pure strategy and stick to it for the duration of the period. In the random matching model, players play exactly once in each period, so that both π_{it} and Π_{It} depend on the strategies of the two randomly matched partners.

In the case of selective matching, as indicated above, player I may be confronted with several players i, having chosen $I \in \mathbf{L}$ in their strategy. Thus, Π_{It} depends not only on X_{It} and x_{it} with $x_{it} = (E, I)$, but also on the number of players i having chosen I. Let $\mathbf{S}_{I,t} \equiv \{i \in S : x_{it} = (E, I)\} \subseteq \mathbf{S}$ be the subset of players i having chosen I at t, we can define player I's payoff as follows

$$\Pi_{It}(X_{It}, \mathbf{S}_{I,t}) = \begin{cases} 0 & \text{if } \mathbf{S}_{I,t} = \emptyset \ \forall X_{It}, \\ |\mathbf{S}_{I,t}| \ B & \text{if } X_{It} = Co \text{ and } \mathbf{S}_{I,t} \neq \emptyset, \\ |\mathbf{S}_{I,t}| \ C & \text{if } X_{It} = R \text{ and } \mathbf{S}_{I,t} \neq \emptyset. \end{cases}$$

in which A is normalized to zero. This allows us to eliminate the payoff increase resulting from attracting more competitors in the ultimatum game, which is not appropriate in that case. Fixing $A = 0$ eliminates this effect.

At period $t + 1$, player $i \in \mathbf{S}$ observes (x_t, π_t), that is all previous payoffs in $\mathbf{S}$ with the corresponding strategies. Similarly, player $I \in \mathbf{L}$ observes (X_t, Π_t). Player $i \in \mathbf{S}$ (respectively player $I \in \mathbf{L}$) is assumed to find the maximal payoff in π_t (respectively Π_t) and then to imitate the corresponding strategy. Formally, player $i \in \mathbf{S}$ chooses $x_{kt} \in x_t$ such that

$$k \in argmax_{j \in \mathbf{S}} \{\pi_{jt}\}. \tag{1}$$

In the same way, player $I \in \mathbf{L}$ chooses $X_{Kt} \in X_t$ such that

$$K \in argmax_{J \in \mathbf{L}} \{\Pi_{Jt}\}. \tag{2}$$

Let $B_{\mathbf{S}}(t-1) = \{x_{kt-1} \in x_t : \pi_{kt-1} \geq \pi_{jt-1}, \forall j \in \mathbf{S}\}$ be the set of strategies achieving the highest payoff in $\mathbf{S}$ at the previous period. When $B_{\mathbf{S}}(t-1)$ is *not* a singleton, that is, when several strategies give the maximal payoff, one of them is chosen at random according to a probability distribution with the full support on $B_{\mathbf{S}}(t-1)$. The same assumption applies to population $\mathbf{L}$.

Games with an outside option have a particular strategic structure in which a long-run player facing no effective entry (i.e., only facing O-users) does not reveal his strategy. This is the case in most extensive-form games where some of the information sets may not have been reached. In such a situation, the very last behavior of a long-run player cannot be observed by other long-run

[15] Following Oechssler (1997), E and O are called *actions* since in our model of selective matching short-run players strategy indicates both a long-run player and an action choice.

players and thus cannot generate imitation.[16] For the sake of simplicity here, I assume that when the strategy of a long-run player is not revealed at a given period, it is considered as unmodified relative to the last time it could have been observed. Formally,

Assumption 1 *When $X_{It-1} \in X_{t-1}$ cannot be observed at the beginning of t, then $X_{It-1} = X_{I\tau}$ with $\tau < t$ the last period in which player $I \in \mathbf{L}$ faced an E-user.*

Thus, in such a case, player J, looking at the performance of player I at the beginning of t, considers the couple $(X_{I\tau}, \Pi_{t-1})$, that is, the last *observable* strategy with the payoff just realized. When τ does not exist (for instance, at the very beginning of the game), one may assume that players select their strategies at random in order to first obtain information about them and then to follow the imitation dynamics.

Inertia is an important aspect of evolutionary models. They consider that not all agents react instantaneously to their environment, but rather gradually adjust their strategy following the learning mechanism. Formally, each player independently with some fixed probability $\phi \in (0, 1]$ has the opportunity to update his strategy in each given period. As we will see in Section 5, assuming different levels of inertia in $\mathbf{S}$ and $\mathbf{L}$ may have an effect on the results of the evolutionary dynamics.

2.4 Mutations

Besides the learning mechanism, *mutation* is the other force acting on agents' strategies. It refers to a situation where an individual randomly switches to a new strategy. After the completion of the learning adjustment, each agent independently changes his strategy with a small probability ε. The learning process is then perturbed.[17] In economic contexts, the mutation phenomenon may be interpreted as *experimentation* of non-optimal strategies, in the sense of Eqs. 1–2, or the entry of a new player who knows nothing about the game.[18]

Learning dynamics (1)–(2) combined with the mutation mechanism generate a Markov chain over the finite state space Z. The existence of a small probability $\varepsilon > 0$ ensures that the process has a *unique* stationary distribution summarizing the long-run behavior of the system, regardless of initial

[16]Most evolutionary game models postulate that agents simply look at the *immediate* past and use it as a one-point predictor of what will happen next. This is called *static expectation*. Young (1993) was the first to introduce a process of expectation formation into stochastic evolutionary models. As extensive-form games may present unreached information sets, evolutionary models investigating these games have proposed an extension of the static-expectations approach. See, for instance, Nöldeke and Samuelson (1993), who developed the analysis of evolutionary stability in extensive form games.

[17]As mentioned by Samuelson (1997), mutation is a residual capturing whatever has been excluded when modelling selection.

[18]On this point, see Canning (1989).

conditions.[19] The latter characteristic of the model is particularly interesting when the learning mechanism presents several stationary states (which is the case here, as we will see in the next section), since it may permit a selection to be made between them. Our goal is to find the *stochastically stable* state or the *long-run* equilibrium (*LRE*) of the game, assuming that $\varepsilon \to 0$. We have to compute the number of mutations required in the transitions between absorbing states of the learning mechanism. The *LRE* is simply the one requiring the fewest mutations.[20]

Notice that mutation effects may be limited in extensive-form games. As mentioned in Section 2.3, in such games, information sets may not have been reached. Consequently, if a mutation changes an agent's strategy at a decision node that is not currently reached, then this has no effect on the agent's payoff and is not observed by others (and thus cannot generate imitation). This has some consequences on the result of the model.

3 Analysis

In this section, we characterize the long-run behavior of the evolutionary dynamics considering both random and selective matching processes. We find that selective matching has important implications on the *LRE* of the dynamics.

In the framework of the imitation rule considered here, extinct strategies are required to remain extinct without mutations.[21] This means that in *monomorphic* states, i.e., states in which all players use the same strategy, the learning mechanism cannot bring (alone) new strategies in the population. In such states players cannot observe the gain of an *unplayed* strategy so they cannot imitate it.

Let $T_0(z, z')$ be the transition matrix of dynamics (1)–(2) between states $z, z' \in Z$; the corresponding m-step transition matrix is denoted by $T_0^m(z, z')$. As customary, we define a *limit set* $A \subset Z$ of the imitation dynamics, that is, a set containing *absorbing* states, as a closed set under finite chains of positive-probability transitions. Formally, a set $A \subset Z$ is a *limit set* if

(1) $\forall z \in A, \forall z' \notin A, T_0(z, z') = 0,$
(2) $\forall z, z' \in A, \exists m \in \mathbb{N}$ such that $T_0^m(z, z') > 0.$

Denote by $\mathcal{A}$ the collection of limit sets of the imitation dynamics. When A is a singleton, for instance $A = \{z^*\}$, learning mechanism (1)–(2) cannot escape

[19] See Kandori et al. (1993) and Young (1993).

[20] For more details on this result, see Kandori et al. (1993), Samuelson (1997), and Vega-Redondo (1996).

[21] This follows the Robson and Vega-Redondo (1996) model as well as the biologists formulation of evolutionary dynamics. However, it constitutes a point of departure with Kandori et al. (1993). On this point, see Robson and Vega-Redondo (1996).

from $\{z^*\}$ without experimentation or mutation, so it remains in $\{z^*\}$ forever. We can state the following result.

Proposition 1 *Under random matching,* $\mathcal{A} = \{\{z_q\} : q = 1, 2, 3, 4\}$ *in any type of game with an outside option. Under selective matching, we find that* $\mathcal{A} = \{\{z_q\} : q = 1, 2, 3, 4\}$ *in the ultimatum game, and* $\mathcal{A} = \{\{z_q\} : q = 1, 2, 3, 4,\ Z' \equiv \{(z'^s, z'^l) : z'^s = 0,\ \ 0 < z'^l < N\}\}$ *in the trust game, for all* $|\mathbf{S}_I| = (0, 1, ..., n)$ *and* $I \in \mathbf{L}$.

Proof Observe first that z_1, z_2, z_3 and z_4 are the all monomorphic states of dynamics (1)–(2). By A3, no shift in strategy can take place from these states, as players cannot imitate unplayed strategies. Thus, z_1, z_2, z_3 and z_4 are all absorbing states under both random and selective matching.

On the other hand, a polymorphic state cannot be absorbing under random matching. Let $z' \in Z$ be a polymorphic state. z' is a candidate to stationarity only if players of the same population earn identical payoffs in using different strategies. Suppose that this is the case. By assumption, there is always positive probability that all players $I \in \mathbf{L}$ (respectively $i \in \mathbf{S}$) choose the same strategy in $B_{\mathbf{L}}(t-1)$ ($B_{\mathbf{S}}(t-1)$), forcing the imitation dynamics to come back towards a monomorphic state.

Consider now states $z' \in Z' \equiv \{(z'^s, z'^l) : z'^s = 0,\ 0 < z'^l < N\}$. As $0 < z_t'^l < N$, player i's strategy is $x_{it} = (E, J)$ with the restriction $J \in \mathbf{L}_{Co,t} \equiv \{I \in L : X_{It} = Co\} \subsetneq \mathbf{L}$. This means that all R-users denoted $K \notin \mathbf{L}_{Co,t}$ are avoided by selective short-run players at t. In the trust game, we then have $\Pi_{Jt} = |\mathbf{S}_{J,t}|B > \Pi_{Kt} = 0$ as $|\mathbf{S}_{J,t}| > 0$. Following imitation rule, players $K \notin \mathbf{L}_{Co,t}$ should imitate strategy Co in subsequent periods $\tau > t$. However, by construction, all players $i \in \mathbf{S}$ will continue to imitate $x_{it} = (E, J)$ with $J \in \mathbf{L}_{Co,t} \equiv \{I \in L : X_{It} = Co\}$, so that imitation of Co by $K \notin \mathbf{L}_{Co,t}$ cannot be revealed. Then, by A2, $X_{K\tau} = X_{Kt} = R$ for some $\tau > t$, so that $z_\tau = z'_t$, meaning that states $z'_t \in Z'$ are absorbing in the trust game.

In the ultimatum game, by contrast, one observes $\Pi_{Jt} = |\mathbf{S}_{J,t}|B < \Pi_{Kt} = 0$ as $B < 0$ in states $z' \in Z'$. Then, players $J \in \mathbf{L}_{Co,t}$ shift to R during periods $\tau > t$ and this is revealed here, since these players were selected at t by short-run players (and they continue to imitate this strategy). Thus, states $z' \in Z'$ are not absorbing states in the ultimatum game. □

The previous result indicates that, under random matching, an absorbing state of the dynamics has to be monomorphic, and that all monomorphic states are absorbing states. This comes from the specification of the imitation dynamics, in particular Assumption 3. No mutation is needed to move from any polymorphic state to an absorbing state, so that a *LRE* must be monomorphic when players meet at random. By contrast, under selective matching, some polymorphic states can be absorbing in the trust game, as mentioned above. The intuition behind this result is that a R-user at $t-1$ who tries to revise his choice at t cannot be revealed due to the structure of the game as discussed above. This will be no longer true with perturbations introduced by mutations.

These perturbations will allow us to see the somewhat fragile stability of polymorphic states $z' \in Z'$ in the trust game.

Notice that, under selective matching and without mutation, it could happen that not all long-run players are selected in absorbing states in which $z^s = 0$, i.e, in states in which all short-run players play E.[22] In such states, without mutation, players only imitate strategies used in the past (which means here an action and a long-run player) without experimenting. As a result, if a long-run player is not selected in the initial state (that is, not present in the strategy space of short-run players), he will not be in the subsequent states. That will disappear when introducing mutation.

Consider now the possibility of mutations. According to Kandori and Rob (1995), the *LRE* are those requiring the fewest mutations in the transitions between limit sets. Here, the limit sets are singletons $\{z_1\}$, $\{z_2\}$, $\{z_3\}$, $\{z_4\}$ and $\{z'\} \in Z'$ so that we are interested by transitions between these absorbing states. Define first an A-tree as a collection of directed branches (A^0, A^1), with A^1 the successor of A^0, in which

(1) except for A, each limit set has a unique successor,
(2) there are no closed loops.

Let H_A be the set of A-trees. The cost of transition between two limit sets $A, A' \in \mathcal{A}$ is denoted by $C(A, A')$. This cost represents the minimum number of mutations to achieve A' from A over time. The *LRE* are the states having minimum $C(A, A')$, that is, they are solutions to the following program[23]

$$\min_{A \in \mathcal{A}} \min_{h \in H_A} \sum_{(A', A'') \in h} C(A', A''). \tag{3}$$

In words, the *LRE* are states whose minimum-cost trees are themselves minimum across absorbing states.

3.1 The ultimatum game

In this section, we compute the long-run equilibria in the ultimatum game under both matching scenarios. Consider first the basic model of matching, that is the random matching process. Then, we can state

Proposition 2 *In the ultimatum game, $z_2 = (E, Co)$ is the LRE under random matching.*

Proof We have to find the minimum-cost trees that are themselves minimum across absorbing states. From Proposition 1, we know that $\mathcal{A} = \{\{z_q\} : q = 1, 2, 3, 4\}$. It will be shown that there exists an z_2-tree h_{z_2} such that $C(h_{z_2}) =$

[22] This is underlined in the statement of Proposition 1 by specifying for all possible cardinality of subset $\mathbf{S_I}$ and for all $\mathbf{S_I}$.

[23] See Kandori and Rob (1995), Proposition 4.

$|\mathcal{A}| = 4$ whereas for all $z_q \neq z_2$ with $z_q \in \mathcal{A}$, every z_q-tree h_{z_q} implies $C(h_{z_q}) > 4$, so that z_2 is the unique solution to program (3).

Consider $z_3 = (E, R)$ and one mutation $J \in \mathbf{L}$ playing Co. Then, $\Pi_J = B > \Pi_I = C\ \forall I \neq J$, so that mutant $J \in \mathbf{L}$ is imitated and the system moves to $z_2 = (E, Co)$. In the same way, consider $z_4 = (O, Co)$ and one mutant $j \in \mathbf{S}$ playing E. As $\pi_j = b > \pi_i = a\ \forall i \neq j$, mutant j generates imitation of E in population $\mathbf{S}$. Thus, one mutation is sufficient to escape from both states $z_3 = (E, R)$ and $z_4 = (O, Co)$.

Assume now that the system is in $z_1 = (O, R)$. Notice that, due to the structure of the game, one mutation $J \in \mathbf{L}$ playing Co cannot be revealed until it is matched with a mutant $j \in \mathbf{S}$ playing E. In this event, $\Pi_J = B < \Pi_I = A$ $\forall I \neq J$ but $\pi_j = b > \pi_i = a\ \forall i \neq j$. As $\phi \in (0, 1]$, there is a positive probability that all players $i \in \mathbf{S}$ adjust their strategy towards E during the subsequent period, inducing a transition from $z_1 = (O, R)$.

On the other hand, two mutations are not sufficient to move from $z_2 = (E, Co)$. Assume $n \geq 3$. One mutation $j \in \mathbf{S}$ playing O cannot alone generate imitation since $\pi_j = a < \pi_i = b\ \forall i \neq j$; in the same way, a mutant $J \in \mathbf{L}$ playing R cannot operate a transition as $\Pi_J = C < \Pi_I = B\ \forall I \neq J$. Further, two paired simultaneous mutations (O in $\mathbf{S}$ and R in $\mathbf{L}$) do not induce a transition, since mutant $J \in \mathbf{L}$ playing R cannot be revealed by mutant $j \in \mathbf{S}$ playing O.

As a result, $C(h_{z_2}) = 4$ as one mutation is required in each of both states z_3 and z_4 plus at least two mutations in z_1, whereas $C(h_{z_q}) > 4$ for all $z_q \neq z_2$ (one mutation in z_3 and z_4, plus more than two in z_2), which completes the proof. □

On the other hand, selective matching favors different behaviors in the long run, that is,

Proposition 3 *In the ultimatum game, $z_1 = (O, R)$ is the LRE under selective matching.*

Proof Unlike the random matching scenario, selective matching allows the system to escape from $z_2 = (E, Co)$ with only one mutation, but leaves unchanged the number of mutations required in other states. As a result, $C(h_{z_1}) = 3$,whereas, for all $z_q \neq z_1$ with $z_q \in \mathcal{A}$ every z_q-tree h_{z_q} implies $C(h_{z_q}) > 3$.

Consider that the system is in $z_2 = (E, Co)$ at t and suppose that one mutation $J \in \mathbf{L}$ playing R occurs at the same time. In this event, one observes $\Pi_{Jt} = |\mathbf{S}_{J,t}|C < \Pi_{It} = |\mathbf{S}_{I,t}|B\ \forall I \neq J$, as $A > B > C$ and by A1 $|\mathbf{S}_{J,t}| = |\mathbf{S}_{I,t}|$. At the next period, mutant $J \in \mathbf{L}$ is not imitated. However, selective short-run players will avoid strategy (E, J) as $\pi_j < \pi_i$ with $x_{jt} = (E, J)\ \forall i \neq j$ (i.e, they avoid mutant $J \in \mathbf{L}$) so that, even if mutant $J \in \mathbf{L}$ changes its strategy (returns to Co) at t, this is not revealed. By A2, players $I \neq J$ will consider that $X_{J\tau} = R$ (the last observable strategy) with $\Pi_{J\tau} = A$ (the last realized payoff). Then they have to compare $\Pi_{J\tau} = A = 0$ to $\Pi_{I\tau} = |\mathbf{S}_{J,t}|B$ during periods $\tau > t$. As $B < 0$ in the ultimatum game, players $I \neq J$ start to imitate strategy R, which in turn generates imitation of O in $\mathbf{S}$.

On the other hand, selective matching does not change the number of mutations necessary to escape from $z_1 = (O, R)$. Afresh one mutant $J \in \mathbf{L}$ playing Co cannot be revealed until it is matched with a mutant $j \in \mathbf{S}$ playing E. When this occurs, there is a positive probability that all players $i \in \mathbf{S}$ imitate $x_{jt} = (E, J)$ as $\pi_j > \pi_i$ $\forall i \neq j$, which induces a transition from $z_1 = (O, R)$ towards state $z' \in Z'$.

Finally, observe that one mutation is sufficient to escape from both states $z_3 = (E, R)$ and $z_4 = (O, Co)$, even under selective matching, so that $C(h_{z_1}) = 3$, whereas for all $z_q \neq z_1$ with $z_q \in \mathcal{A}$ every z_q-tree h_{z_q} implies $C(h_{z_q}) > 3$. □

The stronger stability of state $z_1 = (O, R)$ under selective matching comes from the possibility for short-run players to (1) avoid long-run players using R and (2) select long-run players using Co. As a result, Co-users always confront entries whereas R-users are protected by selective matching which permits them to realize the highest gain in population $\mathbf{L}$. Selective short-run players then lead the imitation dynamics to favor strategy R at the expense of strategy Co. Unlike random matching, selective matching may support reputation formation in the ultimatum game by preventing the random entries in games where long-run players reject cooperation.

3.2 The trust game

We now turn to the trust game. Unlike the ultimatum game, the presence of selective short-run players cannot here support reputation effects. This is stated in the following proposition.

Proposition 4 *In the trust game, $z_1 = (O, R)$ is the LRE under both random and selective matching processes.*

Proof Notice first that one mutation is sufficient to escape from both states $z_3 = (E, R)$ and $z_4 = (O, Co)$. In $z_3 = (E, R)$, one mutation $j \in \mathbf{S}$ playing O realizes a better payoff as $\pi_j = a > \pi_i = c$ $\forall i \neq j$. Similarly, one mutation $j \in \mathbf{S}$ playing E from $z_4 = (O, Co)$ earns $\pi_j = b > \pi_i = a$ $\forall i \neq j$.

From Proposition 1, we know that under selective matching we have also to consider absorbing states $z' \in Z' \equiv \{(z'^s, z'^l) : z'^s = n,\ 0 < z'^l < N\}$. Here, one mutation $x_{jt} = (E, K)$ in population $\mathbf{S}$ with $K \in \mathbf{L}_R \equiv \{I \in L : X_I = R\}$ is sufficient to reveal that player K has changed his strategy and now play Co, so that the system escapes from $z' \in Z'$. (Recall that, in the trust game, Co earns a higher profit when confronted to selective short-run players).

Consider now state $z_1 = (O, R)$. From this state, one mutation $J \in \mathbf{L}$ playing Co cannot be revealed until it is matched with a mutant $j \in \mathbf{S}$ playing E. In this event, one observes $\Pi_J = B > \Pi_I = A$ $\forall I \neq J$ and $\pi_j = b > \pi_i = a$ $\forall i \neq j$, and the system will end in a state near to $z_1 = (O, R)$, in which all players in $\mathbf{S}$ will select mutant $J \in \mathbf{L}$. Even if other players in $\mathbf{L}$ imitate strategy Co, this shift in strategy will become effective only in presence of other mutations in $\mathbf{S}$, meaning that a complete transition from $z_1 = (O, R)$ requires even more than two paired mutations.

It remains to show that one mutation can induce a transition from $z_2 = (E, Co)$. Consider a mutation $J \in \mathbf{L}$ playing R. As $\Pi_J = C > \Pi_I = B \; \forall I \neq J$, mutation J generates imitation in population $\mathbf{L}$ during the subsequent periods. Consequently, $C(h_{z_1}) = 3 + |Z'|$ with $|Z'| = N - 2$, whereas $C(h_{z_q}) > N + 1$ for all $z_q \neq z_1$ with $z_q \in \mathcal{A}$, and $C(h_{z'}) > N + 1$ for all $z' \in Z'$, so that $z_1 = (O, R)$ is the LRE. □

The idea behind Proposition 4 is the following. Recall first that, in the trust game, $z_2 = (E, Co)$ is Pareto-dominant but does not constitute a Nash equilibrium. As a result, a long-run player using strategy R from state $z_2 = (E, Co)$ earns immediately a higher payoff than Co-users. This player may then initialize imitation of R in population $\mathbf{L}$.

Unlike the ultimatum game, selective short-run players play no role here. As $\phi \in (0, 1]$, there is always a positive probability that strategy R is imitated by all long-run players before the reaction of selective short-run players. Indeed, selective short-run players once informed could (1) avoid R-users and (2) select Co-users as a matter of priority indicating that strategy Co may realize the best payoff in population $\mathbf{L}$. This means that considering that selective short-run players always learn more quickly than long-run players, this may have an effect on the result in Proposition 4. However, this requires additional assumptions as regards the imitation rule as considering only cases in which short-run players adjust their strategy before or infinitely faster than the long-run players. It seems to us that such assumptions are too extreme to be considered as satisfactory.

4 Discussion and extensions

4.1 Random versus selective short-run players

The main result of our model suggests that long-run equilibria in GOO depend on the matching process specifications, in particular, whether it is selective or random for short-run players. Thus, one may ask what sustains the presence of *selective* short-run players in these games. More precisely, the important questions are: which context favors the existence of selective compared to random short-run players under evolutionary competition? Can both types of short-run players coexist in the population? In order to answer these questions, notice that the main difference between both types of short-run players depends on information players have to gather in the imitative process. Both player types have to consider the payoff earned and the associated strategy. However, as a strategy for a selective player is constituted by two elements (an action and a long-run player) we could suppose that gathering information for a selective player has an added cost relatively to a random type.

Models on evolutionary competition between player types (see Stahl 1993; Barnerjee and Weibull 1995; Heller 2004) postulate that higher informed players incur a cost in learning compared to others. This literature has shown that, depending on the nature of the game, informed types can either come to dominate the population, become extinct or coexist along with less-informed types. In these models, the question is addressed in *symmetric* normal form games. The reputation games we studied in this paper are *asymmetric* games. Thus, the analysis regarding evolutionary competition between selective and random short-run players constitutes an interesting extension of this paper.

4.2 Selective matching and the myopa assumption

Evolutionary models consider myopic and adaptive players, that is to say agents who do not form expectations about the future course of play but simply take into account the decisions made in the past to determine their strategies. In order to justify myopia, evolutionary game theory assumes large population of players randomly matched. As agents are randomly paired, the incentive to try to alter the future play of opponents is small enough to be negligible. But at the same time, due to the random matching of players, reputation effects are ruled out by evolutionary models. Thus, at first glance, using evolutionary game theory to investigate reputation effects may appear counter-intuitive.

As we have seen in Section 1, evolutionary models have evolved to incorporate selective consideration in matching, which in turn is part of many economic situations. When the random matching assumption is weakened, at least in part, *inertia* is the other assumption justifying myopia in evolutionary models. Evolutionary models assume that not all agents react instantaneously to their environment, but rather gradually adjust their strategy following the selection mechanism. This is a good justification of myopic behavior: as players know that only a small segment of agents change their actions, strategies that prove to be effective today are likely to remain effective in the near future.

4.3 The related literature

Our model may be related to the evolutionary literature on equilibrium selection[24] and, in particular, to Nöldeke and Samuelson (1993) and Binmore et al. (1995). They show that evolutionary models provide support to experimental outcomes, and thus can yield outcomes that significantly differ from subgame perfection.

Other evolutionary analyses have considered reputation effects. For instance, Burke and Prasad (2002) study the emergence of institutions that facilitate lending within the context of a reputational model of debt. In traditional models, lending can occur in equilibrium because players value the

[24] On the interplay between evolutionary game theory and the equilibrium selection problem, see Samuelson (1997).

credit relationship itself. Burke and Prasad generalize the repeated model of sovereign debt to a population game with several borrowers and lenders, a situation in which the value of a relationship with any single partner is diminished. Another study using evolutionary game theory in a reputational context is Abreu and Sethi (2003). They start by observing that the literature on reputation in games uses boundedly rational players, leaving unexplained the presence of such players in the analysis as well as the particular forms of irrationality assumed. Investigating the relative survival of various behavioral types, they show that the presence of nonrational or behavioral types are necessary to evolutionary stability.

Finally, the idea of selective matching in reputational games is also found in Hörner (2002). He considers a situation in which consumers can only assess the quality of a seller's product by purchasing and consuming it (i.e., *experience good* markets). Unlike the traditional analysis, he investigates such a situation in a competitive environment where many consumers and firms repeatedly trade. Consumers have an outside option represented by the possibility to freely switch firms at any time. In this setting, he shows how competition supports the existence of equilibria in which firms always exert high effort.

5 Conclusion

This paper investigated reputation in population games with an outside option considering both imitation of success and *selective* matching. Contrary to the random model, we find that selective matching, by allowing short-run players to select the long-run player against whom they play, sustains the equilibrium traditionally associated with reputation formation in the ultimatum game, but not in the trust game.

Games with an outside option are representative of many economic situations in which reputation effects play a crucial role. In these games, selective matching is a natural assumption, since in the first stage one player selects whether to play a game with a second player or not. A useful extension of the present paper would be to investigate the survival of the two types of short-run players under evolutionary competition, adding a cost to selective (and more rational) short-run players. This is left to future work.

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