Appendix 1

Co-ordinate transformation

The introduction of the variable $\zeta = y/h$ modifies the energy and continuity equations. For the sake of generality the film thickness will be assumed to be a function of x, z and t and the transverse co-ordinate y will be measured from a fixed origin in order to allow for the possibility of motion of the bearing. The change of variable has a subtle effect on partial derivatives which is best explained by introducing a completely fresh set of co-ordinates:

$$x' = x$$

$$\zeta = (y - y_0(x,z,t))/h(x,z,t)$$

$$z' = z$$

$$t' = t$$

Partial derivatives transform according to the chain rule as follows:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'} - \frac{1}{h}\left(\frac{\partial y_0}{\partial x'} + \zeta\frac{\partial h}{\partial x'}\right)\frac{\partial}{\partial \zeta}$$

$$\frac{\partial}{\partial y} = \frac{1}{h}\frac{\partial}{\partial \zeta}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z'} - \frac{1}{h}\left(\frac{\partial y_0}{\partial z'} + \zeta\frac{\partial h}{\partial z'}\right)\frac{\partial}{\partial \zeta}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \frac{1}{h}\left(\frac{\partial y_0}{\partial t'} + \zeta\frac{\partial h}{\partial t'}\right)\frac{\partial}{\partial \zeta}$$

The term $\mathrm{d}T/\mathrm{d}t$ on the right-hand-side of the energy equation transforms as follows:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

$$= \frac{\partial T}{\partial t'} + u\frac{\partial T}{\partial x'} + \frac{v^*}{h}\frac{\partial T}{\partial \zeta} + w\frac{\partial T}{\partial z'}$$

The effective transverse velocity v^* is defined by:

$$v^* = v - \left(\frac{\partial y^*}{\partial t'} + u\frac{\partial y^*}{\partial x'} + w\frac{\partial y^*}{\partial z'}\right),\quad y^* = y_0 + \zeta h$$

At the bearing surface:

$$v_0^* = v_0 - \left(\frac{\partial y_0}{\partial t} + u_0\frac{\partial y_0}{\partial x} + w_0\frac{\partial y_0}{\partial z}\right) = 0$$

and at the journal surface:

$$v_1^* = v_1 - \left[\frac{\partial}{\partial t}(y_0 + h) + u_1\frac{\partial}{\partial x}(y_0 + h) + w_1\frac{\partial}{\partial z}(y_0 + h)\right] = 0$$

So that v^* is zero at both surfaces.

The continuity equation for fluid flow is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

After substituting the above expressions for the partial derivatives and rearranging, the equation becomes:

$$\frac{\partial}{\partial t'}(\rho h) + \frac{\partial}{\partial x'}(\rho h u) + \frac{\partial}{\partial \zeta}(\rho v^*) + \frac{\partial}{\partial z'}(\rho h w) = 0$$

Integrating with respect to ζ from 0 to 1 leads to:

$$\frac{\partial}{\partial t'}\left(h\int_0^1 \rho\,\mathrm{d}\zeta\right) + \frac{\partial}{\partial x'}\left(h\int_0^1 \rho u\,\mathrm{d}\zeta\right) + \frac{\partial}{\partial z'}\left(h\int_0^1 \rho w\,\mathrm{d}\zeta\right) = 0$$

The generalised Reynolds' equation is based on this integrated continuity equation.

Letters

I wish to comment on the article entitled "Rubber – Ice Friction and Vehicle Handling" by A.D. Roberts in the February 1981 issue, vol. 14, no. 1.

It seems to me that the experiment differs from tyre conditions in several ways and these should be discussed in order to avoid unjustified conclusions.

1. A tyre is warmed during use by the hysteresis involved in flexing the periphery from round to flat when in contact with the road. The heating depends on the amount of hysteresis in the compound and on vehicle parameters, eg the running deflection decided by the designer. The rubber hemisphere is free from such heating, instead of which it has friction heating which may be totally different in temperature, distribution and dependence on speed, being nil at the start of slip, whereas the tyre is warm before slip.
2. The pressure distribution under a tyre is not Hertzian as under an elastic sphere but is governed by inflation pressure, stiffness and hysteresis in flexing and unflexing. The form is likely to encourage the trapping of a film of liquid, as in elasto-hydrodynamic lubrication of rollers, but on a larger scale and including a large part of the contact zone.
3. We are not told the contact pressures in the experiment; these affect the melting point of ice.
4. The effect of grit-loaded compounds (like ink-erasers) is likely to be much more striking under trapped-film conditions than under spherical contact. A coarse grit is likely to allow some escape of liquid and also may make contact with the solid ice through the liquid film, giving grip. At some temperatures, the high pressure under a grit particle may allow a dimple to form by pressure melting. This may enhance the grip further during rolling, thus postponing the skid.

Perhaps the experiments could be continued with a fluid-filled pad, kept at a representative temperature by infra-red rays, additional to the lighting needed for observing the contact zone (Fig 1).

My interest in the subject dates from a period when I published a number of short articles on automotive design topics.

P. Polak,
Department of Mechanical Engineering,
The University of Sheffield,
Mappin Street, Sheffield, UK, S1 3JD

(continued)