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Ground movement analysis in deep iron mine using fuzzy probability theory

Wen-Xiu Li *, Sheng-Jie Liu, Ji-Fei Li, Zhan-Hua Ji, Qi Wang, Xia Yin

College of Machinery and Civil Engineering, Hebei University, Baoding 071002, China

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ABSTRACT

The analysis of the ground movements due to underground mining operation is one of the many important problems of rock mass mechanics. It is difficult to calculate the ground movement due to deep underground mining of iron ore accurately because of the complexity of the problems. In this paper, the application is described of the fuzzy probability measures to the analysis of ground movements. Based on the definition of the fuzzy probability measure, the theories for the two- and three-dimensional problems are developed and are applied to the analysis of ground movements duo to underground deep underground mining of iron ore.

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1. Introduction

Any underground engineering excavation (such as underground mining, railway, underground storehouses, etc.) will certainly result in rock mass displacements and ground movements of varying degrees [1]. The ground movements due to excavation operations especially mining of ore have often resulted in major disasters occurring throughout the world, frequently with considerable loss of life and damage to property.

In underground and surface mining, if a void is excavated in a rock continuum, load formerly carried on the rock in the opening will be transferred either to the rock surrounding the opening or to supports (pillars) within the opening, or both, and finally to the ground surface. This results in a macroscopically nonuniform deformation of the surface in the horizontal or vertical direction. If these uneven deformations or subsidence cannot effectively be controlled, then they will cause damage and even disaster, such as deformation or cracking of buildings.

To sum up, it is difficult to calculate the accurate displacement or subsidence of every point in a body of rock because of the complexity of the factors affecting mine subsidence. Instead, various approximate methods have been used for this calculation. In fact, the movement or subsidence of each point on a level of the overburden can be regarded as a fuzzy event [2,3]. In other words, this displacement or subsidence will take place at a fuzzy probability, and so the theory of fuzzy probability measures can be applied to describe the ground grovements due to deep mining of iron ore by pillarless sublevel caving method.

* Corresponding author. Tel.: +86 312 5079493; fax: +86 312 5079375.
E-mail address: Leewenxiu@yahoo.com.cn (W.-X. Li).

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2. Fuzzy probability measure models

2.1. Fuzzy probability measure

2.1.1. Fuzzy probability and probabilistic processes

In Ref. [4] (Abraham KANDEL, William J. BYATT. 1980), discussion centered on the algebra of fuzzy sets and on some of the properties of grades of membership in such fuzzy sets. In fact, in ordinary probability theory, probabilities are defined in a given sample space. The collection, in the discrete case, of points in the sample space, where an event *A* occurs, describes the event. Thus, an event is the same as an aggregate of sample points; that is, an event A consists of, or contains, certain points representing an experiment where *A* occurs. Then, by definition, for the discrete case, it follows that the probability of any event *A* is the sum of the probabilities of all sample points in it. We now turn to the question of grades of membership again. In the case wherein we deal with ordinary probability theory, grades of membership in a set can take on only the values unity or zero, corresponding to certain (unity) membership and no (zero) membership. Thus, for example, the grade of membership of an integer in the set of integers is unity, while for non-integer numbers, the grade of membership is zero.

There are, however, concepts for which the notion of either belonging, or not belonging, to a given set make little sense. We give a few examples. Heights are distributed probabilistically. Since, in principle, all heights are possible, we can assign, on the basis of exhaustive experiments, a probability density of heights $P(h)dh$ which gives the probability of having height in the interval $(h,h + dh)$. We want the distribution to be normalized so that $\int_0^\infty P(h)dh = 1$.

But if we now ask the question "what is the probability of the event 'tall' among people whose heights are distributed as above?", then the set of tall people must be defined. We choose to make this definition by incorporating a grade of membership $\mu_T(h) \in [0,1]$ in the set of tall people, with $\lim_{h\to 0}\mu_T(h) \to 0$ and $\lim_{h\to\infty}\mu_T(h) \to 1$. There are other questions which come to mind. We can ask 'what constitutes the set of crowded streets?' We ask "what constitutes a 'suitable' response of a circuit to an input signal?" To answer each of these questions, a suitable mathematics into which is built a notion of grade of membership in a set is needed.

Zadeh [5] has provided a framework such that inexact, or fuzzy, concepts can be discussed rigorously within the confines of an extension of probability theory.

2.1.2. Fuzzy probability theory

First several definitions of fuzzy probability measures are given in brief.

Definition 1. Suppose triplet (Ω,B,ρ) is a probability space, where Ω is a sample space and *B* is the fuzzy σ-field of Borel sets in Ω (or fuzzy σ-algebra), and ρ is the probability measure over Ω. A_1 and A_2 are two fuzzy sets in the Ω, and μ_{A_1}, μ_{A_2} (μ_{A_1}, μ_{A_2}: $\Omega \to [0,1]$) are two membership functions and μ_{A_1}, μ_{A_2} are Borel measurable.

In Definition 1, $A_1=\{x_1,x_2,\ldots,x_i\}$, $A_2 = \{y_1,y_2,\ldots,y_i\}$; where, x_i ($i = 1,2,\ldots,n$), y_i are points on the $x0y$ cross section.

Definition 2. If A_1 and A_2 are two fuzzy events in the Ω, then we can define the fuzzy probability measures of A_1 and A_2 as follows:

$$\rho(A_1) = \int_{D_1} \mu_{A_1}(x)d\rho, \quad x \in (0,+\infty), \tag{1}$$

$$\rho(A_2) = \int_{D_2} \mu_{A_2}(y)d\rho, \quad y \in (0,+\infty), \tag{2}$$

Here Eqs. (1) and (2) are Lebesgue–Stieltjes intergrals.

In Definition 2, D_1 is integration area in the *x*-direction, such as $D_1 \in [0,200]$; and D_2 is integration area in the *y*-direction, such as $D_2 \in [0,300]$. Where, D_1 (D_1 = 200 m) is the mining range in the *x*-direction), D_2 (D_2 = 300 m) is the mining range in the *y*-direction).

Because μ_{A_1}, μ_{A_2} are Borel measurable, the Lebesgue–Stieltjes integrals exist.

Definition 3. Let A_1 and A_2 be two fuzzy events in the probability space (Ω,B,ρ). A_1 and A_2 are said to be independent if:

$$\rho(A_1A_2) = \rho(A_1)\rho(A_2), \tag{3}$$

where, A_1 A_2 is a product.

Because μ_{A_1}, μ_{A_2} are Borel measurable, the Lebesgue–Stieltjes integrals exist, and because *B* is a Borel field in the set Ω, it can readily verify that a fuzzy probability measure possesses the following properties:

Property 1. If $A \in B$, then:

$$0 \le \rho(A) \le 1. \tag{4}$$

Property 2. For any set Ω, we have:

$$\rho(\Omega) = 1. \tag{5}$$

Property 3. If $A_1, A_2 \in B$, and $A_1 \subset A_2$, then we have:

$$\rho(A_1) \leq \rho(A_2). \tag{6}$$

Property 4. $A_n \in B$, $A_n \in B$ for $1 \leqslant n < \infty$ (n is an integer), and A_i, A_j are mutually exclusive, i.e. a product $A_iA_j = \Phi(i \neq j)$, then we have:

$$\rho(\bigcup_{n=1}^{\infty}) = \sum_{n=1}^{\infty} \rho(A_n). \tag{7}$$

In fact, in ordinary probability theory, given a random variable X in one dimension, we define

$$\rho\{X = x\} = f(x), \tag{8}$$

where $f(x)$ is the probability density of the random variable X. Then we may define the cumulative distribution function $\rho(x)$, which gives $\rho\{X \leqslant x\}$, as

$$\rho(A_1) = \int_{-\infty}^{x} f(x')dx', \tag{9}$$

for $x' \in [-\infty, +\infty]$. Then with $d\rho(x) \equiv f(x)dx$, Eq. (9) can be written as

$$\rho(A_1) = \int_{-\infty}^{x} d\rho(x'). \tag{10}$$

It is held that $\lim_{x \to \infty} \rho(x) \equiv \int_{-\infty}^{+\infty} dp(x) = 1$; i.e., the distribution function is normalizable. For a fuzzy random variable X_A, by analogy with Eq. (8), we write

$$\rho\{X_A = x\} \equiv \mu_A(x)f(x), \tag{11}$$

thereby associating with each x a grade of membership in the set A. Then we define a quantity $\rho(A;x)$ as

$$\rho(A;x) \equiv \rho\{X_A \leqslant x\} = \int_{-\infty}^{x} \mu_A(x')d\rho(x'). \tag{12}$$

Corresponding to the normalization condition of ordinary probability theory, we write that $\lim_{x \to \infty} \rho(A, x) \equiv \rho(A)$, where

$$\rho(A) = \int_{-\infty}^{\infty} \mu_A(x)d\rho(x). \tag{13}$$

Eq. (13) is Zadeh's definition of a fuzzy event A in a one-dimensional space. We now seek to formalize the above argument to multi-dimensional spaces by referring directly to Zadeh's [16] paper.

A probability space is assumed to be a triplet (Ω, B, ρ), where B is the σ-field of Borel sets in Ω and ρ is a probability measure over Ω. A point in Ω is denoted by x.

Let a set $A \in B$. Then on defining a characteristic function of or grade of membership in, the set A by $\mu_A(x)$: $\Omega \to [0,1]$, we define the probability of A as

$$\rho(A) = \int_{\Omega} \mu_A(x)d\rho(x), \tag{14}$$

where

$$\int_{\Omega} dp(x = 1. \tag{15}$$

An alternative way of writing Eq. (14) is

$$\rho(A) = \langle \mu A \rangle = E\mu_A, \tag{16}$$

where we use the notation ‹› to denote the expected value of a function, and where $E\{\mu_A\}$ is the expected value of μ_A. What is meant by Eq. (14)? In one dimension, a fuzzy random variable X_A is, as defined in Eq. (12),

$$\rho(X_A \leq x) \equiv \rho\{A;x\} = \int_{-\infty}^{x} \mu_A(x')d\rho(x'), \tag{17}$$

if $x' \in (-\infty,\infty)$. In a multi-dimensional case, we have

$$\rho\{(X_A^{(1)} \leqslant x_1) \cap (X_A^{(2)} \leqslant x_2) \cap \cdots \cap (X_A^{(n)} \leqslant x_n)\} = \int_{-\infty}^{x_1} \cdots \int_{-\infty}^{x_n} \mu_A(x) d\rho(x)$$

for $x_i \in (-\infty,\infty)$, $\forall i$ in Ω, and where $\mu_A(X) = \mu_A(x_1, x_2, x_3, \ldots x_n)$ and: $d\rho(X) = f(x_1,x_2,\ldots,x_n)dx_1dx_2\ldots dx_n$. Then, as $x_i \to \infty$ ($i = 1,2,\ldots,n$), $\rho(A)$ is given by Eq. (14).

A fuzzy event A in Ω is, according to Zadeh [5], a fuzzy set A whose membership function μ_A (x) is Borel measurable.

Definition 4. Assume that

$$\mu_A(X) \to \mu_A(X(t), t) \tag{18}$$

and is associated with a preassigned probability density $d\rho(X(t))$, then, in the n-dimensional case, we define $\rho_A(t)$ as:

$$\rho_A(t) = \int \mu_A(X(t), t) d\rho(X(t)). \tag{19}$$

Definition 5. In the one-dimensional case, let $d\rho = \rho dx$ to be such that

$$\rho = \rho(x, t), \tag{20}$$

while $\mu_A = \mu_A(x)$. Then we have:

$$\rho_A(t) = \int_D \mu_A(x)\rho(x, t)dx, \tag{21}$$

with the above definitions and theories as a basis, the engineering problems in practical cases, can now be described.

2.2. Applications of fuzzy probability in engineering

The term "fuzzy probability" have been applied to different concepts and formalized in various ways in past 30 years.

2.2.1. Applications of fuzzy probability in rockmass displacement assessment

The prediction of displacement of rock mass and their surface effects is an important problem of the rock mass mechanics in the excavation activities especially the coal and metal mining. Li [1] presented the fuzzy probability models and applied to the analysis of rockmass displacement due to deep mining of metal ore. The agreement of the theoretical results with the field measurements shows that our model is satisfactory and the formulae obtained are valid and thus can be effectively used for predicting the displacements and deformations and the safety evaluation of the buildings on the ground.

Rockmass displacement assessment is a geotechnical problem characterized by many sources of uncertainty. Some of them, e.g., are connected to the variability of rock and soil parameters involved in the analysis. In fact, the predictions of displacements and deformations of rock slopes are important in rock and soil mechanics and engineering [6,7]. Various approximate methods have been used for such calculations. The displacement or deformation of a rock mass can be regarded as a fuzzy event that takes place at a fuzzy probability. Therefore, the theory of fuzzy measures can be applied to describe the displacements and deformations of rockmass. Based on the results of the statistical analysis of a large amount of measured data in slope engineering, Li et al. [7] established the fundamental fuzzy model of displacements and deformations of rock slope by using the theory of fuzzy measures.

2.2.2. Risk assessment system of natural hazards

Karimi et al. [8] presented a system for assessing the risk of natural disasters, particularly under highly uncertain conditions, i.e. where neither the statistical data nor the physical knowledge required for a purely probabilistic risk analysis are sufficient. The theoretical foundation of this study is based on employing fuzzy set theory to complement the probability theory with an additional dimension of uncertainty. This would allow for expressing the likelihood of natural hazards by fuzzy probability. The fuzzy probability is characterized in terms of possibility-probability distributions, for which a new approach has been developed. The new approach has been compared with an alternative approach [7,8]. Moreover, uncertainties about the correlation of the parameters of hazard intensity, damage and loss, i.e. vulnerability relations, have been considered by means of fuzzy relations. The composition of fuzzy probability of hazard and fuzzy vulnerability relation yields the fuzzy probability of damage (or loss). The system has been applied for assessing the earthquake risk in Istanbul metropolitan area.

2.2.3. Reliability assessment method for pressure piping containing circumferential defects based on fuzzy probability

Randomness of the assessment parameters and fuzzy failure areas both exist in the reliability assessment of pressure piping containing circumferential defects. The use of fuzzy sets is considered in this context. It is pointed out that the failure probability of pressure piping containing circumferential defects is in fact a fuzzy failure probability, and a method to compute the fuzzy failure probability of pressure piping is proposed by Zhou [9]. Numerical examples that consider the fuzzy failure areas of pressure piping can have an effect on reliability estimation of pressure piping containing defects, and different assessment results can sometimes be obtained. Compared with the conventional reliability estimation method, which neglects the existence of fuzzy failure areas, the method proposed by Zhou [9] provides a more complete assessment of pressure piping containing defects.

2.2.4. Modeling attitude to risk in human decision processes

Divakaran and Terence [10] reported two studies based on a model of strategic telecommunication investment decisions from a research work involving a survey of executives. The first study involves building fuzzy probability models corresponding to each individual decision maker with the results grouped based on the decision makers' propensity to risk as determined by their degrees of disjunction. The Shapley indices and the interaction effects are determined for each pooled dataset corresponding to each group. To contrast this approach with those of conventional nomothetic comparisons of decision policies, the decision makers are grouped based on a clustering analysis of the individual linear regression models. The data for each cluster are pooled and the fuzzy probability measures learned from the dataset are analyzed for comparison purposes. The results not only serve as a demonstration of fuzzy probability measure analysis as a viable approach to studying qualitative decision making but also provide useful methodological insights into applying fuzzy probability measures to strategic investment decisions under risk.

2.2.5. Application of fuzzy probabilistic method in the general evaluation of regional atmosphere environment, water environment

The fuzzy probabilistic method is a general evaluation method of regional environmental quality. It is popular in the general evaluation of regional atmosphere environment, water environment, and so on. As the fuzzy probabilistic method requires a great deal of data on the studied area, it is inconvenient for environmental evaluation. To enhance the practicability of this method, Wang et al. [11] improved the fuzzy probability method by integrating sampling points of space and time. Result from a case study shows that the improvement is feasible with concise theory and less calculation. With good reliability, this method can save a lot of time and money in the course of sampling and can increase the efficiency of environmental evaluation.

2.2.6. A probability of fuzzy events approach to validating expert systems in a multiple agent environment

Daniel and O'Leary [12] developed a model that employs a probability of fuzzy events model of the process to determine how much validation should be done in a multiple agent setting. An example is given to illustrate the use of the model.

3. The fuzzy probability model for practical engineering problems

3.1. Two-dimensional problem

When the subsidence of overburden takes place due to the underground mining, some points on the corresponding surface will deviate from their equilibrium positions, and so displacements will occur (Fig. 1). Because ground subsidence is controlled by many factors, such as geologic conditions, the presence of groundwater, the properties of rock masses, mining conditions, etc., it is very difficult to predict ground subsidence (the magnitude and range of subsidence) accurately. Because the theory of fuzzy mathematics is generally taken to embrace the whole field of imprecisely described systems, the above theory of fuzzy probability can be used for this prediction.

Consider an $x0z$ cross section of the overburden; for the points x_i ($i = 1,2,\ldots,n$) on that plane, the expression for the fuzzy probability of subsidence can be established. The set (denoted by A_1) is called an "associated set", whose elements are the points of x-axis involved in the subsidence. Clearly, A_1 is a fuzzy subset in the set Ω.

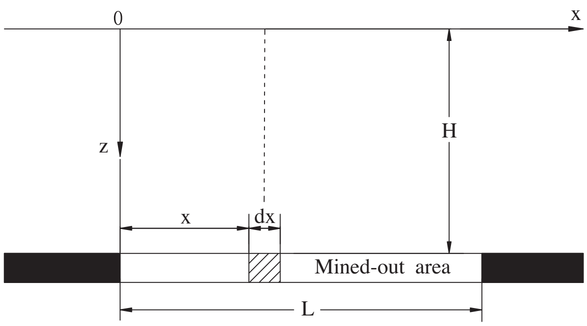

Fig. 1. A rectangular coordinate system.

From [1–3,5]we can see that the theory of fuzzy mathematics deals with the inexact concepts encountered in the real physical world, i.e. objects of a fuzzy type. By "inexact concept" we mean the set of those concepts which are uncertain in extension, and so the classification into which its elements are put does not have sharply defined boundaries (fuzziness).

In many cases, for the problem dealt with in this paper, the degree of membership can be assigned by the fuzzy statistical method, so as to study the indeterminacy by means of the deterministic system. The membership function for engineering problems can be assigned by the membership function of the points on the subsidence surface to the fuzzy subset A_1 (as mentioned above, A_1 is the set of the points on the subsidence surface).

As shown in Fig. 1, we choose a rectangular coordinate system such that the x-axis is parallel to the mined-out area (we only consider the points on the top of the mined-out area), and assign unity to the maximum degree of membership of x_i to A_1. We assume that the following expression hold:

$$\mu_{A_1}(x) = f(x_i), \tag{22}$$

where $f(x_i)$ is the relation function determined by using regression analysis according to the data measured in the field.

The statistical analysis is made by using a large amount of measured data, and the results obtained show that $f(x_i)$ is a special function, where x_i are the measured points on the subsidence surface. From Eq. (22) we have the following expression of the membership function (the regression method is used for this application):

$$\mu_{A_1}(x_i) = \xi_1(x_i/z)^{\xi_2} \cdot \exp(-\xi_3(x_i/z)^{\xi_4}), x_i \in (0, \infty). \tag{23}$$

The above density function (i.e. the density function for the subsidence) can be established using the statistical theory of probability. The type of the membership function (23) is the Gamma distribution function. In the case of the mining of ore, the subsidence of the ground surface depends on many factors. We shall consider the theory of the plane problems an particular cases of rock mass movements. In the $x0z$ plane of the rectangular coordinates system, let $d\rho(x_i)$ be the density function of the ground subsidence at point x_i in the $x0z$ cross-section, from [5,11–14] we have (the type of the density distribution is Gamma distribution):

$$d\rho(x_i) = (x/z)^{a_1} \cdot \exp(-a_2(x/z)^{a_3}), x_i \in (0, \infty), \tag{24}$$

where a_i ($i = 1,2,3$) are "fuzzy parameters".

From Definition 2 we obtain the expression of the fuzzy probability for ground subsidence in the $x0z$ cross section:

$$\rho(A_1) = \int_D \xi_1 \left(\frac{x}{z}\right)^{\xi_2} \left(\frac{x}{z}\right)^{a_1} \cdot \exp\left\{-\left[\xi_3\left(\frac{x}{z}\right)^{\xi_4} + a_2\left(\frac{x}{z}\right)^{a_3}\right]\right\} dx, \tag{25}$$

where D is the mining range, $D \in (0, x_n)$.

The fuzzy probability of slope or tilt, $\rho_T(A_1)$, is the first derivative of the subsidence fuzzy probability, i.e.:

$$\rho_T(A_1) = \frac{\partial}{\partial x}\rho(A_1). \tag{26}$$

The fuzzy probability of curvature, $\rho_c(A_1)$, is the second derivative of subsidence fuzzy probability, i.e.:

$$\rho_c(A_1) = \frac{\partial^2}{\partial x^2}\rho(A_1). \tag{27}$$

To determine the horizontal displacement, it is necessary to calculate the fuzzy probability of horizontal displacement, $\rho_H(A_1)$. According to Li [13–16] that is proportional to the first derivative of the fuzzy probability of subsidence, i.e.:

$$\rho_H(A_1) = u\frac{\partial}{\partial x}\rho(A_1), \quad (0 < u \leqslant 1.0). \tag{28}$$

The fuzzy probability of horizontal strains, $\rho_s(A_1)$ is:

$$\rho_S(A_1) = \frac{\partial}{\partial x}\rho_H(A_1). \tag{29}$$

The value calculated from Eq. (25) is the fuzzy probability for ground subsidence. However, the mining thickness and subsidence factor must be taken into account for calculating the practical surface subsidence S.

$$S = \rho(A_1)K_i, \tag{30}$$

where S_ρ is the practical ground subsidence; K_i ($i = 1,2,3$) represents the parameters which depend on the mining method and the rock properties, and can be determined from the measured data of mining district. For example, given the fuzzy probability of surface subsidence in a mining district $\rho(A_1) = 0.7$, $K_1 = 25$ m, $K_2 = 0.2$, then $S = 3.5$ m.

3.2. Three-dimensional fuzzy models

When subsidence takes place in an underground rock mass, there must exist one subsidence point $C_i(x_i, y_i, z_i)$ on the corresponding ground surface which will allow the fuzzy probability $\rho(B)$ of the subsidence to be obtained in the three-dimensional case.

In the space rectangular coordinates, let $\rho(A_1)$ be a subsidence fuzzy probability of the fuzzy event A_1 on the $x0z$ plane, and $\rho(A_2)$ be a subsidence fuzzy probability of the fuzzy event A_2 on the $y0z$ plane. From Definition 3, we can find the subsidence fuzzy probability $\rho(A_1A_2)$ at the point $C_i(x_i,y_i,z_i)$ to be $\rho(A_1A_2) = \rho(B)$, and:

$$\rho(A_1A_2) = \rho(A_1) \cdot \rho(A_2). \tag{31}$$

In the $x0z$ plane, we have:

$$\mu_{A_1}(x_i) = \xi_1\left(\frac{x}{z}\right)^{\xi_2} \cdot \exp\left[-\xi_3\left(\frac{x}{z}\right)^{\xi_4}\right], \tag{32}$$

$$d\rho(x_i) = \left(\frac{x}{z}\right)^{a_1} \cdot \exp\left[-a_2\left(\frac{x}{z}\right)^{a_3}\right]. \tag{33}$$

Then we have:

$$\begin{aligned} \rho(A_1) = \textstyle\int_{D_1} \xi_1\left(\frac{x}{z}\right)^{\xi_2+a_1} \times \exp\{-[\xi_3(\frac{x}{z})^{\xi_4} + a_2(\frac{x}{z})^{a_3}]\}dx, \\ D_1 \in (0, x_n), n \to \infty. \end{aligned} \tag{34}$$

In the $y0z$ plane, we have:

$$\mu_{A_2}(y_i) = \eta_1\left(\frac{x}{z}\right)^{\eta_2} \cdot \exp\left[-\eta_3\left(\frac{x}{z}\right)^{\eta_4}\right], \tag{35}$$

$$d\rho(y_i) = \left(\frac{x}{z}\right)^{b_1} \cdot \exp\left[-b_2\left(\frac{x}{z}\right)^{b_3}\right]. \tag{36}$$

The fuzzy probability $\rho(A_2)$ can be written as:

$$\begin{aligned} \rho(A_2) = \textstyle\int_{D_2} \eta_1(\frac{y}{z})^{\eta_2+b_1} \times \exp\{-[\eta_3(\frac{y}{z})^{\eta_4} + b_2(\frac{y}{z})^{b_3}]\}dx, \\ D_1 \in (0, y_n), n \to \infty. \end{aligned} \tag{37}$$

From Eq. (31), $\rho(A_1A_2)$ can be determined by following:

$$\rho(B) = \rho(A_1) \cdot \rho(A_2) = \int_{D_1} \mu_{A_1}(x_i)d\rho(x_i) \cdot \int_{D_2} \mu_{A_2}(y_i)d\rho(y_i). \tag{38}$$

The fuzzy probability of tilt is:

In the x-direction:

$$\rho_{Tx}(B) = \frac{\partial}{\partial x}\rho(B). \tag{39}$$

In the y-direction:

$$\rho_{Ty}(B) = \frac{\partial}{\partial y}\rho(B). \tag{40}$$

The fuzzy probability of direction slope is:

$$\rho_{T\xi}(B) = \rho_{Tx}(B) \cdot \cos\zeta + \rho_{Ty}(B) \cdot \sin\zeta, \tag{41}$$

where ζ = direction angle.

The fuzzy probability of curvature in the x-direction is:

$$\rho_{Cx}(B) = \frac{\partial}{\partial x}\rho_{Tx}(B). \tag{42}$$

In the y-direction is:

$$\rho_{Cy}(B) = \frac{\partial}{\partial y}\rho_{Ty}(B). \tag{43}$$

The fuzzy probability of directional curvature is:

$$\rho_{C\zeta}(B) = \rho_{Cx}(B) \cdot \cos^2\zeta + 2\rho_{Cxy}(B)\sin\zeta\cos\zeta + \rho_{Cy}(B) \cdot \sin^2\zeta, \tag{44}$$

where, ζ = direction angle; $\rho_{Cy}(B)$ = the second partial derivative of $\rho(B)$ with respect to x and y.

$$\rho_{Cxy}(B) = \frac{\partial^2}{\partial x \partial y}\rho(B).$$

The fuzzy probability of horizontal displacement in the x-direction is:

$$\rho_{Hx}(B) = \varepsilon_1 \frac{\partial}{\partial x}\rho_{Tx}(B). \tag{45}$$

In the y-direction is:

$$\rho_{Hy}(B) = \varepsilon_2 \frac{\partial}{\partial y}\rho_{Ty}(B). \tag{46}$$

The fuzzy probability of directional displacement is:

$$\rho_{H\zeta}(B) = \varepsilon_i \frac{\partial}{\partial \zeta}\rho_{T\zeta}(B), \tag{47}$$

where ε_1, ε_2 are engineering parameters, $0 < \varepsilon_1, \varepsilon_2 \leqslant 1$.

The fuzzy probability of horizontal strain in the x-direction is:

$$\rho_{Sx}(B) = \frac{\partial}{\partial x}\rho_{Hx}(B). \tag{48}$$

In the y-directions is:

$$\rho_{Sy}(B) = \frac{\partial}{\partial y}\rho_{Hy}(B). \tag{49}$$

The fuzzy probability of directional strain is:

$$\rho_{Sxy}(B) = u\frac{\partial^2}{\partial x \partial y}\rho(B). \tag{50}$$

The fuzzy probability of directional strain is:

$$\rho_{S\zeta}(B) = \rho_{Sx}(B)\cdot\cos^2\zeta + 2\rho_{Sxy}(B)\sin\zeta\cos\zeta + \rho_{Sy}(B)\cdot\sin^2\zeta. \tag{51}$$

3.3. Fuzzy model and its time-dependent processes

The dynamic problem (i.e. time-dependent process) of rock mass displacement is an important subject in rock mass mechanics.

In this section the equations satisfied by $\rho(B_t)$, when either $\mu(\cdot)$ and $dp(\cdot)$ vary with time, will be derive.

According to Li [1–3,13–16], the membership function $\mu_{A_1}(x_i,t)$ and $\mu_{A_1}(y_i,t)$ can be determined by the following formulas:

$$\mu_{A_1}(x_i,t) = \xi_1\left(\frac{x}{z}\right)^{\xi_2}\cdot\exp\left[-\xi_3\left(\frac{x}{z}\right)^{\xi_4}\right]\times[1-\exp(\tilde{a}_1 t)], \tag{52}$$

$$\mu_{A_1}(y_i,t) = \eta_1\left(\frac{y}{z}\right)^{\eta_2}\cdot\exp\left[-\eta_3\left(\frac{y}{z}\right)^{\eta_4}\right]\times[1-\exp(\tilde{b}_1 t)]. \tag{53}$$

From Definition 3, we have:

$$\rho(B_t) = \int_{\Omega_1}\mu_{A_1}(x_i,t)d\rho(x_i)\cdot\int_{\Omega_2}\mu_{A_2}(y_i,t)d\rho(y_i). \tag{54}$$

3.4. The case of multi-seam mining

For multi-seam mining or layer mining in a minable seam, according to the theory of fuzzy mathematics the fuzzy probability of the ground subsidence can be determined by the following formula:

$$\rho_m(A_{11}\cup A_{12}) = \rho_m(A_{11}) + \rho_m(A_{12}). \tag{55}$$

By using the above-mentioned results, we have:

$$\rho_m(A_{11} \cup A_{12}) = \int_{D_1} \mu_{A_{11}}(x_i)d\rho(x_i) \cdot \int_{D_2} \mu_{A_{12}}(x_i)d\rho(x_i). \tag{56}$$

A similar argument can be made for the three-dimensional problems of multi-seam mining.

4. Application of fuzzy models to engineering problems

In order to demonstrate the application of the formula for the fuzzy probability of ground surface subsidence some examples are given of the practical application of the above theoretical results.

4.1. Example 1. West mining area, Guanzhuang Iron Mine, Shandong Province, China

This mining area has a thick seam, mining thickness 60 m, dip 16°–25°, and a mining maximum depth of 1100 m. The seam was mined by the pillarless sublevel caving mining method.

The topography in the district is complex, the maximum slope is 20°. The strata comprise purplish red slate and clay. From the observed data, the following parameters can be obtained: $\xi_1 = 0.3206$, $\xi_2 + a_1 = 3.2013$, $\xi_3 = 2.9231$, $\xi_4 = 3.8052$, $z = 520$.

From these data, a theoretical subsidence curve can be plotted using the formulas given above and is compared with the in situ field measurements (Fig. 2).

The results of calculation with the fuzzy probability method indicated that the predicted value was in good agreement with the actually measured data (Fig. 2).

4.2. Example 2. North mining area, Guanzhuang Iron Mine, Shandong Province, China

The overlying strata consist of black shale and dense dolomite in this mining area. The mining thickness of the seam is 40 m, dip 18°–22°, and the mining depth 546 m.The seam was mined by the pillarless sublevel caving mining method.

From the geologic feature and the data measured, the following parameters were determined: $\xi_1 = 0.2343$, $\xi_2 + a_1 = 4.3521$, $\xi_3 = a_2 = 5.2123$, $\xi_4 = a_3 = 5.3492$, $z_1 = 546$.$\eta_1 = 0.1891$, $\eta_2+a_1 = 4.3762$, $\eta_3 = a_2 = 5.0221$, $\eta_4 = a_3 = 6.4024$, $z_1 = 546$.

From these data, the theoretical values of fuzzy probability $\rho(A_1A_2)$ were obtained using the formulas given above. The practical calculations show that the theoretical results is in good agreement with the observed data (Fig. 3). The three-dimensional theoretical results are given in Fig. 4.

4.3. Example 3. Prediction of surface subsidence due to inclined coal-seam mining: Yangjinlan mining area, Changchun CB Mine, Jilin Province

The mining thickness of the seam is 1.80 m, strike NE40°, inclination SE130°, average dip 41°.

To protect the surface buildings from damage, two stations were set up to detect the displacement of rock mass. No. 202 station was established in September 1991. The No. 202 stope face lies in a stope district of drift. The overlying strata consist of argillaceous limestone and purple sandy shale intercalated with siltstone and fine- and medium-grained sandstone between them.

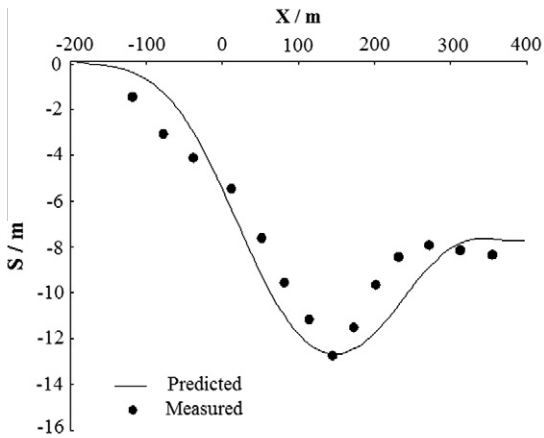

Fig. 2. The data points and the theoretical curve.

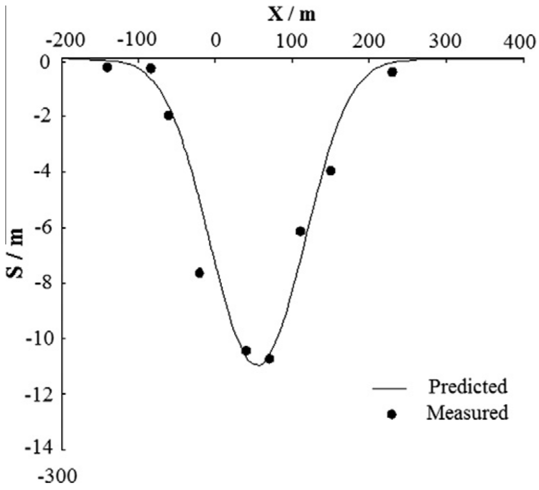

Fig. 3. The data points and the theoretical curve.

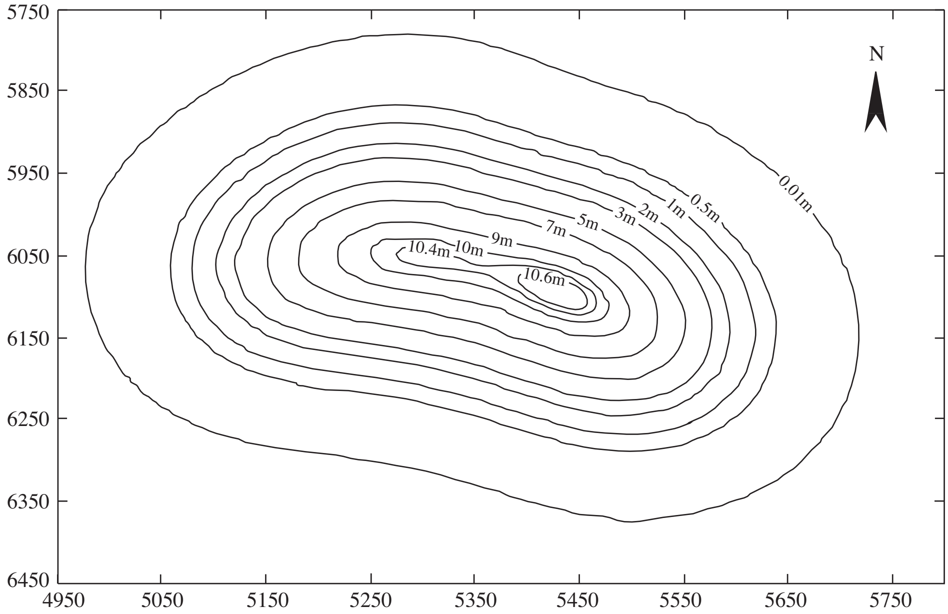

Fig. 4. Theoretical results calculated using fuzzy model (m).

The stope face has a strike length 330 m, inclination length 130 m, and a mining depth $z = 115$ m (mining depth at the boundary of dip entry, $z = 115$ m). The seam was mined by longwall retreat mining method.

From the geologic feature and the data measured, the parameters were determined. The engineering parameters can be determined by the fuzzy theory (Table 1).

In order to demonstrate the application of the formula for the fuzzy probability measures of rock mass displacements some examples are given of the practical application of the above theoretical results.

From these data, a theoretical subsidence curve was plotted using the above formulas and was compared with the observed data (Fig. 5).

Table 1
The parameters of rock mass displacements.

Parameters	K_1 (mm)	K_2	z (m)	ξ_1	$\xi_2 + a_1$	$\xi_3 = a_2$	$\xi_4 = a_3$
Values	1800	0.5831	115	10.1301	1.8525	1.5917	2.8525

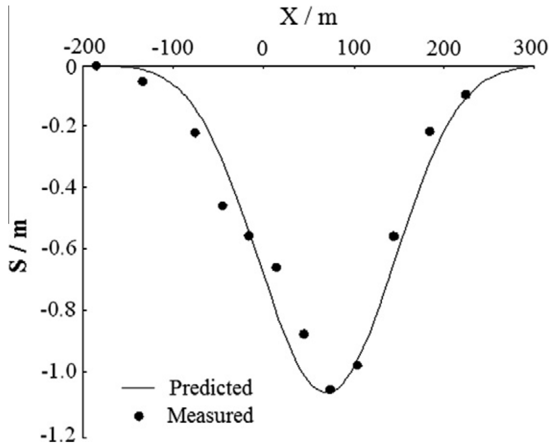

Fig. 5. The data points and the theoretical curve for ground subsidence due to inclined seam mining.

The results of the calculation with the fuzzy model indicate that the predicted value was in good agreement with the measured data (Fig. 5).

5. Conclusions

In this paper, the mathematical theory of fuzzy probability is used to solve the problems of rock mass mechanics due to excavation, especially the mining of deep iron ore. The use of the theoretical of fuzzy mathematics to the evaluation of movements and deformations and to describe the basic feature of the ground surface due to excavation, is a new concept, especially for the deep mining of metal ore.

In this paper, by applying the concepts of fuzzy probability measures to actual cases of excavation, mining, surface movement and subsidence have been analyzed and the corresponding membership function established. The approximate fuzzy probability has been calculated and compared with the recorded data obtained from monitoring stations. The comparison shows that the theoretical prediction is in agreement with the observations.

The novel contributions of the paper:

(1) Fuzzy models of ground subsidence, tilt, curvature, horizontal displacement and horizontal strain due to deep mining of thick metal ore are given.
(2) Applications of fuzzy probability models in deep underground mining of thick metal ore (depth > 500 m, seam thickness > 20 m) are given. Such as "*4.1 Example 1. West mining area, Guanzhuang Iron Mine, Shandong Province, China.*". This mining area has a thick seam, mining thickness 60 m, and a mining depth of 520 m. And "*4.2 Example 2. North mining area, Guanzhuang Iron Mine, Shandong Province, China*". The mining thickness of seam is 40 m, and the mining depth 546 m.
(3) Two- and three-dimensional models and theoretical results are given (see equations (31)–(56), Fig. 4 in the paper.
(4) The subsidence prediction methodology, developed and tested for flat seam mining, has been extended to inclined seam extraction. Applications of fuzzy probability models in underground mining of inclined coal seam are given. Such as "*4.3 Example 3. Yangjinlan mining area, Changchun CB Mine, Jilin Province, China.*". This mining area has a inclined coal seam (average dip 41°), mining thickness 1.80 m, and a mining depth at the boundary of dip entry, $z = 115$ m).
(5) The subsidence time-dependent processes is given (see equations (52)–(54)).
(6) The fuzzy model of multi-seam mining processes is given (see equations (55),(56)).
(7) The fuzzy analysis system–systematic study method with two- and three-dimensional theoretical models is established.
(8) The method presented in this paper can be applied to predicting surface displacement and/or deformation of rock, and so provide a theoretical basis for ensuring the safety of mines.

The formulas derived in this paper have been confirmed by a large amount of measured data. The theory of fuzzy probability, therefore, is valid for solving the engineering problems of excavation. It is to be expected that the wide application of the fuzzy mathematics theory will play an important role in the future.

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