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RESEARCH PAPER

A bi-objective time-dependent vehicle routing and scheduling problem for hazardous materials distribution

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Abstract Planning hazardous materials distribution routes for servicing a given set of orders within specified time windows is a problem frequently surfacing in a city logistics environment which is characterized by dynamic travel times. The hazardous materials distribution problem involves the determination of the sequence of deliveries and the corresponding paths assigned to each truck. This paper presents the formulation of the hazardous materials distribution problem as a bi-objective time-dependent vehicle routing problem with time windows. The paper presents the mathematical formulation of the problem as an integer network flow model with multiple objectives. The weighted-sum method is applied decomposing the bi-objective vehicle routing and scheduling problem to a series of single-objective instances of the problem, where the objective function is expressed by the weighted sum of the criteria under consideration. A route-building heuristic algorithm is presented for addressing each of the constituent single-objective problems, where stops are inserted iteratively in the front part of the unfinished route. A label-setting algorithm is integrated in the heuristic algorithm for solving the time-dependent shortest path problem with multiple intermediate stops arising after the insertion of any stop in the route. The proposed solution approach has been applied to a set of solvable test problems to assess the accuracy of the heuristic solutions. The results indicate a tolerable deviation of the heuristic solutions from the actual non-dominated solutions. In addition, the proposed algorithm was applied to a set of test problems resembling real-life problem cases. The average computational time needed for solving this type of test problems is not prohibitive.

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Introduction

Hazardous materials transportation constitutes a major economic activity. Large quantities of hazardous materials are transported and distributed annually throughout the world. In 2007, the quantity of hazardous materials transported in US was approximately 2.23 billion tons accounting for 17.8 % of the total quantity of goods shipped in US (US DOT 2010a). A key feature in hazardous materials transportation relates to the potential consequences of hazardous materials accidents which may involve fatalities, loss of property value, high recovery costs for the shippers/carriers, evacuation, or contamination of the area around the accident (Zografos and Davis 1989; Abkowitz et al. 2001; Saccomanno and Shortreed 1993). In the period between 2000 and 2009, there were 146,012 of hazardous materials road incidents in US, leading to 114 fatalities, 1,476 injuries, and approximately 462 M$ of damages (US DOT 2010b). Due to the severe and unpredicted consequences of a hazardous materials accident, the mitigation of the associated transportation risk constitutes a major objective for all stakeholders involved in hazardous materials transportation. Planning safe truck routes is considered as a major proactive measure contributing substantially to risk mitigation. Hazardous materials routing decisions can be grouped into two major categories: (1) single origin to single destination routing of full truckload shipments, and (2) hazardous materials routing for less-than truckload distribution. Although substantial research effort has been focused on the former type of routing decision (List et al. 1991; Erkut et al. 2007), limited work has been found on the latter. Hazardous materials distribution routing involves the determination of the routes for a set of trucks servicing a set of customers of known demand within given service time constraints. This type of routing decisions are encountered in planning hazardous materials distribution routes for the distribution of products such as LPG, heating oil, gasoline, and diesel. The relevant distribution routes are planned on a daily basis by determining the sequence of stops serviced by each truck (i.e., truck routes) and the path followed (route path) from the origin to the destination traversing the corresponding intermediate stops. The objective of this problem is to minimize the expected consequences of an accident and the transportation cost, expressed as linear function of the travel time or distance.

The potential customers are usually located in urban or suburban areas where substantial variation of traffic flow intensity is encountered throughout a day. In this setting, the travel time on a link of the underlying roadway network is time dependent and stochastic. A simplified version of this model results when the mean value of the travel time is used thus leading to deterministic travel times expressed as a function of the departure time from the upstream node (Miller-Hooks and Mahmassani 2000). The consequences of a hazardous materials accident may affect: (1) the roadway users traversing the link in which the accident occurs, and (2) the population residing in the impact area of the accident. Traffic flow on the links of the network may vary significantly throughout the day. Thus, the expected number of the roadway users traversing a link in which a hazardous materials accident could potentially occur, varies accordingly depending on the time of the day. Similarly the expected population of the residents aside each transportation link depends on the time of the day and the corresponding land use. In addition, the radius of the potential impacts of a hazardous materials accident depends among others on the mass of the load of the truck (TNO 2005). However, in case of hazardous materials distribution the load of the vehicles is reduced at each stop visited by the quantity delivered to the corresponding customer. In this context, the transportation risk on any link of the roadway network depends on the departure time from the upstream stop and the load of the truck while traversing the link.

This paper deals with the hazardous materials routing and scheduling problem assuming time-dependent travel times, and risk values varying with time and truck load. It is worth noting that considering the load of the truck in transportation risk calculation implies that the decision on the sequence of visiting a set of stops affects the path-finding problem between any pair of stops. In other words, the risk optimum path for servicing any two consecutive stops depends on their position in the route. The objective of this paper is to present a mathematical formulation and a solution algorithm for the emerging bi-objective time-dependent vehicle routing problem with time windows.

The remainder of this paper consists of seven sections. The following section presents previous related work and highlights the contribution of the work presented in this paper. Next, the proposed travel time model is presented, while the following section provides the definition of the risk measure considered in this study. The mathematical formulation of the vehicle routing and scheduling problem under study is discussed, while the next section provides the proposed solution approach. The following section presents the computational performance of the proposed solution algorithm on a set of routing problems developed by the authors, and the final section elaborates on the work presented in this paper and discusses directions for future research.

Previous related work

In general, the multi-objective vehicle routing problem with static cost attributes (Josefowiez et al. 2008) and the single-objective time-dependent vehicle routing and scheduling problem (Fleischmann et al. 2004) have been widely studied. However, no studies regarding the multi-objective time-dependent vehicle routing and scheduling problem have been found in the literature. This section aims to outline the major achievements on the above two neighboring research areas and highlight the novel issues of the work presented in this paper.

Given the increased interest of the carriers/shippers for the environmental impacts of their transport activities, recent studies on hazardous materials distribution have included transportation risk in the associated route planning problem (Zografos and Androutsopoulos 2004; Androutsopoulos and Zografos 2010; Pradhananga et al. 2010). Zografos and Androutsopoulos (2004) formulate the problem of less-than truckload hazardous materials transportation as a bi-objective vehicle routing problem with time windows assuming static travel times and risk values. The solution approach of the emerging bi-objective optimization problem is based on applying the weighted-sum method leading to a series of single-objective vehicle routing problems with time windows, each one solved through a route-building heuristic algorithm. In this formulation, it is implicitly assumed that each pair of customers is linked with an a priori specified shortest path. Pradhananga et al. (2010) enhance the work in Zografos and Androutsopoulos (2004) by taking into account explicitly the intermediate bi-objective path-finding problems between any pair of customers. The bi-objective vehicle routing problem with time windows is solved through a Ant-Colony System algorithm where each solution is constructed by adding a new customer at the end of the route by selecting a non-dominated path from the last stop of the route to any other stop that can be feasibly visited. It is worth noting, however, that the studies mentioned above on hazardous material distribution assume static travel times or risk attributes. In Androutsopoulos and Zografos (2010) the hazardous materials distribution problem for a pre-specified fixed sequence of delivery stops is modeled as a bi-criterion time-dependent path-finding problem with mandatory intermediate stops. The objective of this problem is to determine the non-dominated paths that traverse a given sequence of intermediate stops within specified service time windows. A solution approach based on the *k*-shortest path method is proposed for approximating the set of non-dominated solutions.

The study of the single-objective time-dependent vehicle routing and scheduling problem has been focused on incorporating different link travel time models into existing algorithms for the time-invariant version of the problem. In Malandraki and Daskin (1992), the travel time on any link (i,j) is expressed as a step function of the departure time from the stop i. However, the discontinuities of this travel time function may lead to significant underestimation or overestimation of travel time thus possibly leading to violation of time windows or suboptimal solutions, respectively. Moreover, this travel time model enables the possibility of overtaking, i.e., any vehicle traversing an arc (i,j) may arrive later than another vehicle departing later. Malandraki and Daskin (1992) clarified that any discontinuities in the travel time step function involving a decrease in the travel time (where the travel time for period m is higher than the travel time in period $m + 1$), may be fixed by allowing waiting time at the nodes of the network. However, the travel time function retains discontinuities if the travel time of period m is lower than the corresponding travel time in period $m + 1$. Moreover, the assumption of waiting time is not realistic in practice especially for hazardous materials distribution applications. Malandraki and Daskin (1992) adapt the sequential and the parallel route construction heuristic techniques in the time-dependent case to solve the emerging time-dependent vehicle routing problem. They argue that the computational burden is substantially increased since the travel times are no more symmetric while the computation of the additional cost (in terms of time) of inserting a new stop in a route is further complicated.

Ahn and Shin (1991) adopt a piecewise linear travel time function model which by assumption satisfies the first in first out (FIFO) property. They developed conditions for efficiently checking the time feasibility of a route after inserting a new stop, joining two partial routes, or performing an Or-Opt arc exchange for time-dependent travel times. They provide guidelines on incorporating the above feasibility tests in the insertion, savings, and tour improvement heuristics of the time-dependent vehicle routing problem with time windows.

In the study of Hill and Benton (1992), the travel time between stops (i,j) for a given departure time is estimated by the ratio of the distance between the stops with the average of the two local travel speed estimates around upstream stop (i) and downstream stop (j). They propose various methods (forecasting, sum of squares, subjective measurements) to estimate the speed value at a time t around any node of the network. The proposed solution approach implicitly assumes that the travel time function is calculated for an a priori selected path between any pair of customers. In practice, the fastest path between any pair of customers may change throughout the day which implies that the proposed method may lead to suboptimal solutions. Their solution approach is based on the adaptation of the savings and the Or-opt algorithms for time-dependent travel times.

Ichoua et al. (2003) propose a piecewise linear travel time function emerging from modeling the average travel speed on any arc as a step function of the time of the day (not the departure time from the upstream stop). This definition implies that the calculation of the travel time on any arc for a given departure time takes into account the changes of the speed encountered by a vehicle traversing the arc until it reaches the downstream stop. The resulting travel time function satisfies the FIFO property. However, it implicitly assumes that an a priori fixed path is specified for any pair of customers (thus ignoring the intermediate path-finding problems), leading to sub-optimal solutions. Ichoua et al. (2003) adapt a parallel Tabu Search algorithm (initially designed for the time-invariant problem), and apply it to various benchmark problems. The computational results indicate that the solutions from the time-dependent problem provided better solutions than the corresponding solutions produced from the time-invariant case. Donati et al. (2008) propose a multi ant-colony system for solving the time-dependent vehicle routing problem with time windows. In support of the proposed ant-colony system, they calculate in advance the table of time-dependent shortest travel times for every pair of stops and every possible departure time. Van Woensel et al. (2008) enhance the travel time model proposed by Ichoua et al. (2003) by estimating the time-dependent travel speeds using traffic flow theory formulas based on queuing models. The authors integrate the emerging travel time function in a Tabu Search algorithm for solving a set of benchmark vehicle routing problems.

A limitation of the travel time models proposed in the above studies for the time-dependent vehicle routing problem is that the travel time between any two customers is calculated for an a priori fixed path. Fleischmann et al. (2004) aim to address this issue by proposing a travel time function emerging from the shortest travel times paths between any pair of stops for every possible departure time. The proposed travel time function is piecewise linear derived from smoothing the step function expressing the shortest travel times between any pair of stops. The proposed smoothing process guarantees that the emerging travel times satisfy the FIFO property. However, for departure times where the travel time function slope is non-zero (i.e., it moves upwards or downwards) the corresponding travel times may not necessarily correspond to an actual path. Fleischmann et al. (2004) propose efficient procedures for checking time feasibility in iterations of savings and insertion heuristics when waiting time in any route is bounded from above. The computational results from applying the above heuristic algorithms for solving time-dependent and time-invariant vehicle routing problems imply that ignoring the fluctuations of travel time may lead to underestimation of the total travel time of the solutions and violation of the time window constraints for a substantial percentage of orders.

According to the knowledge of the authors, there exists no study addressing the bi-objective time-dependent vehicle routing problem with time windows. In any study on the single-objective time-dependent vehicle routing problem, the travel time between any pair of stops is expressed as a function of the departure time, emerging from addressing the corresponding time-dependent shortest path problems (Fleischmann et al. 2004). The travel time function is specified at a pre-processing stage, and thus considered available for solving the time-dependent vehicle routing problem. However, this convention regarding the path-finding problems between any pair of stops is not valid for the bi-objective time-dependent vehicle routing problem, since it no longer involves optimal paths. In the bi-objective case, the solution of the relevant path-finding problem refers to a set of non-dominated paths for each alternative departure time. Assuming a single path between any pair of stops may lead to dominated solutions. Moreover, even if all non-dominated paths were identified between any pair of consecutive stops within a given route, as was done by Pradhananga et al. (2010) for the static bi-objective problem, the entire set of combinations of intermediate paths must be searched to exclude any dominated route paths, i.e., not all combinations of intermediate non-dominated paths lead to non-dominated route paths. Clearly this task involves an exponential number of iterations.

The formulation presented in this paper extends the work in Androutsopoulos and Zografos (2010) where the routes and the sequence for visiting the customers within each route were pre-specified as opposed to the present work where identifying the truck routes is also part of the problem. Thus, the proposed formulation deals with the problem of identifying the truck routes while it takes into account explicitly the bi-objective path-finding problems between any pair of stops. The service time windows of the problem under study imply that if a truck arrives later than the latest allowable time, the service of the stop is canceled. The contingency of missing the time window of a customer involves additional cost and risk for the shipper/carrier. Thus, accurate estimation of the travel time is a major issue in modeling the problem under study.

Travel time model

A major issue in addressing the time-dependent vehicle routing problem relates to considering a discrete travel time function between any pair of stops which approximates as close as possible the actual continuous travel time function emerging from historical traffic data records (Fleischmann et al. 2004). The travel time models used in the existing studies imply that the changes in travel speed occur instantly. On the other hand, accuracy in estimating the expected travel times is a critical issue for assessing the time windows feasibility of routes. In this paper, it is assumed that the travel speed on any arc (i,j) is a continuous piece-wise linear function of the time, denoted by $v_{ij}(\tau)$. This assumption implies that the acceleration rate along any arc of the network is expressed as a step function of the time of the day. The travel time model adopted in this paper was proposed in Horn (2000). This paper enhances the work in Horn (2000) by presenting a new efficient procedure for implementing the above travel time model. For any particular day, it is assumed that any roadway link (i,j) is associated with a time series of average travel speeds $v_{ij}(\tau_k),\ k = 1, \ldots, q$. This type of data may be obtained from processing historical traffic data provided by a traffic management center. Figure 1 depicts a hypothetical example of the expected travel speeds for a given day over 15 min time intervals. A continuous travel speed function emerges from connecting any two consecutive travel speed data points with straight line segments (as illustrated in Fig. 1). Algebraically, this speed function is expressed by formula (1) defined over the time intervals $\tau \in [\tau_k, \tau_{k+1})$.

$$v_{ij}(\tau) := v_{ij}(\tau_k) + g_{ij}(\tau_k)[\tau - \tau_k], \quad \tau \in [\tau_k, \tau_{k+1}) \tag{1}$$

where $v_{ij}(\tau_k)$ and $g_{ij}(\tau_k)$ (denoted also v_{ij}^k and g_{ij}^k for simplicity) are the travel speed and acceleration/deceleration rate at time τ_k. The acceleration rate in an arc (i,j) is defined as a step function of the time of the day and it is estimated by formula (2) as the slope of the line segments connecting any two consecutive travel speed data points.

$$g_{ij}(\tau_k) := \frac{v_{ij}(\tau_{k+1}) - v_{ij}(\tau_k)}{\tau_{k+1} - \tau_k}. \tag{2}$$

The proposed travel speed model implies that any vehicle that traverses arc (i,j) may encounter more than one level of acceleration rate until it reaches node j. Thus, the travel time on any arc for a given departure time depends on the different acceleration rate values realized throughout the traversal of arc (i,j). In Horn (2000) the computation of the travel time on any arc (i,j) is performed by counting time from the departure time up to the point in time that a distance equal to the length of the arc is traversed based on the speed model defined in (1). In this paper, a new approach is proposed for calculating the travel time on any (i,j) given a departure time, based on the computation of the arrival time at node j. The new approach is computationally more efficient than the one proposed in Horn (2000), as it provides a closed-form formula for the travel time calculation. Although the proposed travel time model involves more computations than any of the other models used in vehicle routing and scheduling problems, it involves a more realistic estimation of the actual travel time variation since it includes smooth travel speed changes over a day while it satisfies the FIFO condition (Horn 2000).

Given that the arrival time τ_α at node j after traversing arc (i,j) is available (e.g., by applying the procedure proposed in Horn (2000) for the first τ_d), then a closed-form expression has been developed calculating the arrival time for any other departure time. In general, if a vehicle departing from node i at time $\tau_d \in [\tau_k, \tau_{k+1})$

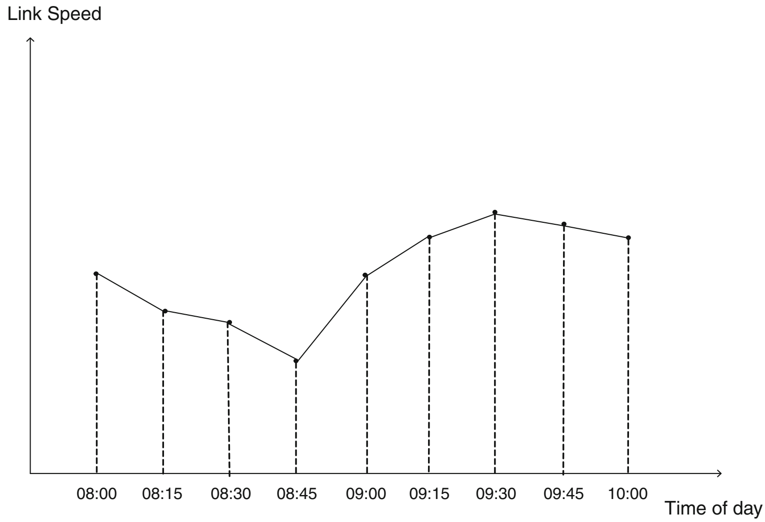

Fig. 1 Graphical display of the travel speed on any arc of the transportation network as a piecewise linear function of the time

arrives at node j at some other time interval, i.e., $\tau_a \in [\tau_{k+m}, \tau_{k+m+1}),\ m > 0$, the distance traveled may be expressed by (3):

$$l_{ij} = \int_{\tau_d}^{\tau_{k+1}} \{v_{ij}^k + g_{ij}^k[\tau - \tau_k]\}\,\mathrm{d}\tau + \sum_{h=k+1}^{k+m-1} \int_{\tau_h}^{\tau_{h+1}} \{v_{ij}^h + g_{ij}^h[\tau - \tau_h]\}\,\mathrm{d}\tau \\ + \int_{\tau_{k+m}}^{\tau a} \{v_{ij}^{k+m} + g_{ij}^{k+m}[\tau - \tau_{k+m}]\}\,\mathrm{d}\tau \tag{3}$$

Assume a marginal delay Δd of the departure time for which $(\tau_d + \Delta d) \in [\tau_k, \tau_{k+1})$ and the corresponding arrival delay Δa is such that $(\tau_a + \Delta a) \in [\tau_{k+m}, \tau_{k+m+1})$. The same distance traversed for departure time $(\tau_d + \Delta d)$ is expressed through formula (4):

$$l_{ij} = \int_{\tau_d+\Delta d}^{\tau_{k+1}} \{v_{ij}^k + g_{ij}^k[\tau - \tau_k]\}\,\mathrm{d}\tau + \sum_{h=k+1}^{k+m-1} \int_{\tau_h}^{\tau_{h+1}} \{v_{ij}^h + g_{ij}^h(\tau - \tau_h)\}\,\mathrm{d}\tau \\ + \int_{\tau_{k+m}}^{\tau_a+\Delta a} \{v_{ij}^{k+m} + g_{ij}^{k+m}[\tau - \tau_{k+m}]\}\,\mathrm{d}\tau \tag{4}$$

The existence of Δa such that $(\tau_a + \Delta a) \in [\tau_{k+m}, \tau_{k+m+1})$ is based on the fact that the first integral in (3) is a continuous function of $\tau_d + \Delta d$ around τ_d while the last integral in (3) is a continuous function of $(\tau_a + \Delta a)$ around τ_a. Combining formulae (3) and (4) leads to the quadratic equation (5).

$$\frac{1}{2}g_{ij}^{k+m}\Delta a^2 + \{g_{ij}^{k+m}[\tau_a - \tau_{k+m}] + v_{ij}^{k+m}\}\Delta a - \frac{1}{2}g_k\Delta d^2 - \{g_{ij}^k[\tau_d - \tau_k] + v_{ij}^k\}\Delta d = 0 \tag{5}$$

Solving (5) with respect to Δa leads to solution expressions (6) and (7) depending on whether $g_{ij}^{k+m} \neq 0$, or not.

$$\begin{aligned}\Delta a(\tau_d + \Delta d) = \quad & \left(\frac{1}{g_{ij}^{k+m}}\right)\Big\{-\Big[g_{ij}^{k+m}[\tau_a - \tau_{k+m}] + v_{ij}^{k+m}\Big] \\ & + \left\{\left\{g_{ij}^{k+m}[\tau_a - \tau_{k+m}] + v_{ij}^{k+m}\right\}^2\right. \\ & \left. +2g_{ij}^{k+m}\left\{\frac{1}{2}g_{ij}^k\Delta d^2 + \left\{g_{ij}^k[\tau_d - \tau_k] + v_{ij}^k\right\}\Delta d\right\}\right\}^{1/2}\Bigg\} \quad \text{if } g_{ij}^{k+m} \neq 0\end{aligned} \tag{6}$$

$$\Delta a(\tau_d + \Delta d) = \frac{1}{v_{ij}(\tau_{k+m})}\left\{\frac{1}{2}g_{ij}^k\Delta d^2 + \left\{g_{ij}^k[\tau_d - \tau_k] + v_{ij}^k\right\}\Delta d\right\} \quad \text{if } g_{ij}^{k+m} = 0. \tag{7}$$

For the special case that $\tau_d \in [\tau_k, \tau_{k+1})$ and $\tau_a \in [\tau_k, \tau_{k+1})$ (i.e., a vehicle departs from i and arrives at j within the same time interval), then τ_a can be calculated by solving the following equation with respect to τ_a:

$$l_{ij} = \int_{\tau_d}^{\tau_\alpha} \{v_{ij}^k + g_{ij}^k[\tau - \tau_k]\}\, \mathrm{d}\tau \tag{8}$$

where l_{ij} is the length of arc (i,j). Solving Eq. (8) leads to expressions (9) or (10) of τ_a, depending on whether the acceleration rate in interval $[\tau_k, \tau_{k+1})$ is zero or not.

$$\tau_\alpha := \tau_d + \frac{l_{ij}}{v_{ij}^k}, \text{ if } g_{ij}^k = 0 \tag{9}$$

$$\tau_a := \frac{1}{g_{ij}^k}\left\{-[v_{ij}^k - g_{ij}^k\tau_k] + \sqrt{[v_{ij}^k - g_{ij}^k\tau_k]^2 - 2g_{ij}^k\left[\tau_k\tau_d g_{ij}^k - \frac{1}{2}g_{ij}^k\tau_d^2 - \tau_d v_{ij}^k - l_{ij}\right]}\right\} \quad \text{if } g_{ij}^k \neq 0. \tag{10}$$

In conclusion, delaying the departure time for traversing arc (i,j) by Δd, the corresponding arrival time at node j is delayed by the quantity given in (6) or (7), provided that $\tau' := (\tau_d + \Delta d) \in [\tau_k, \tau_{k+1})$ and $(\tau_a + \Delta a) \in [\tau_{k+m}, \tau_{k+m+1})$. The function $\Delta a(\tau_d)$ (and therefore the corresponding travel time) is defined on the basis of time intervals $[\tau_k^h, \tau_k^{h+1}), h = 1, \ldots, n_{q_i}$ of possible departure times in which the parameters of (6) or (7) remain unchanged. The time intervals $[\tau_k^h, \tau_k^{h+1})$ constitute partitions of the time intervals $[\tau_k, \tau_{k+1})$. For any arc (i,j), the procedure proposed in Horn (2000) is run only once for departure time τ_0 to calculate the corresponding arrival time at node j. The arrival time for any other departure time is

Table 1 Input data for the hypothetical example for applying the proposed travel time model

Time period (t_i, t_{i+1})	Speed observation (at t_i)	Acceleration rate
0–10	20	0.6
10–15	23	−0.2
15–20	22	0.8

calculated by identifying the time interval $[\tau_k^h, \tau_k^{h+1})$ in which the departure time lies and apply either formulae (6) or (7). A numerical example is given to illustrate the process for identifying the time periods $[\tau_k^h, \tau_k^{h+1})$ and clarify their usability in calculating travel times. Assume a link of 2 km length where the average travel speeds from historical data are presented in Table 1. If departure from the upstream node occurs at time 0 min, then the arrival time at the downstream node of the link is 5.86 min, which lies in the second time period between 5 and 10 min. However, for a departure time of 5 min the corresponding arrival time is at 10.23 min, lying outside the second time period. The latest departure time within the first time period so that the arrival time lies in the second period [5,10) is 4.75 min. Leaving the upstream node later than that time will lead to arrival time lying within the third time period. Thus, the first time interval in which formula (6) could be applied is [0,4.75), by setting g_{ij}^{k+m} equal to -0.2 km/h^2, τ_a to 5.86 min, τ_{k+m} to 10 min (or 0.166 h), v_{ij}^{k+m} to 23 km/h, g_{ij}^{k} to 0.6 km/h^2, τ_k to 0, and v_{ij}^{k} to 20 km/h.

Risk estimation

In this study, the risk associated with any arc of the network is expressed by the expected consequences of a potential hazardous materials accident. In this type of risk analysis, any arc of the network is associated with two attributes: the probability of a hazardous materials accident (π_{ij}) and the expected population exposure (Pop_{ij}) in case a hazardous materials accident occurs anywhere within the specific arc. The probability of a hazardous materials accident on any arc is given by the following formula (Erkut et al. 2007):

$$\pi_{ij} := P[A_{ij}]P[R_m|A_{ij}]P[I_m|R_m] \tag{11}$$

where $P[A_{ij}]$ denotes the probability of a roadway accident on arc (i,j), $P[R_m|A_{ij}]$ is the probability for a release of hazardous material type m given a roadway accident on arc (i,j) involving the relevant truck, and $P\ [I_m|R_m]$ the probability of an incident (depending on the type of hazardous materials transported) given a release from the tank of the truck. The population exposure is expressed by the total population within the expected impact area of a hazardous materials accident within arc (i,j) and the expected population of roadway users occupying the corresponding arc. In this study, the impact area is modeled by a danger circle (Erkut et al. 2007) with radius equal to λ. Thus, assuming that the population density anywhere alongside arc (i,j) is uniform, the population exposure is given by the product of the population density of the area surrounding arc (i,j) with the acreage of the impact area. It should be clarified that the assumption of uniform population density is referred to the area around each link of the network. In practice, the mathematical network representation is actually designed so as the above assumption becomes valid.

Most of the studies on modeling hazardous materials transportation risk assume that the accident probability and the population exposure are time invariant (Erkut et al. 2007). However, in practice both attributes may be considered time dependent (Miller-Hooks and Mahmassani 1998, 2000). In particular, the probability of a hazardous materials accident depends on various dynamic travel conditions (Karkazis and Boffey 1995; Verma and Verter 2007; Zografos and Androutsopoulos 2008) including variations of the traffic flow intensity and the prevailing meteorological conditions (e.g., fog, rain, snow). In addition, the population density of the areas exposed to transportation risk may also vary during different parts of the day due to the daily mobility of the residents. Moreover, the number of vehicles potentially affected by a hazardous materials accident depends of the traffic intensity of the corresponding arc. The impact area of a hazardous material accident depends on: (1) the load of the truck, and (2) dynamic characteristics of the prevailing meteorological conditions, especially in the case where the accident consequences relate to the dispersion of pollutants. In this type of accidents, the wind speed and direction play key role in the dispersion of the consequences. To incorporate the above time-dependent variations of the risk attributes in the routing model proposed in this paper, the risk on any arc of the network is modeled as a step function of the departure time from the upstream node and the load of the truck (q), given by (12).

$$R_{ij}^{\tau}(q) = \pi_{ij}^{\tau} Pop_{ij}^{\tau}(q) \quad \tau \in \mathrm{T}, q \in [m_k, m_{k+1}] \tag{12}$$

where T is the discrete time horizon used for the travel time function.

The risk measure defined in (12) captures the variations in risk, which are not taken into account in the classical model of risk for routing applications (Erkut et al. 2007). An issue that arises in incorporating (12) in the routing problem under study, is that time-dependent risk measure does not necessarily satisfy the FIFO conditions, and thus the solution of any risk minimization routing problem may involve loops at the stops (waiting time is not allowed by assumption).

The computation of risk through the use of (12) requires for every link of the network the estimation of: (1) the variations of accident probabilities throughout the day, (2) the impact area of a potential hazardous materials accident for different values of the load of the truck, type of hazardous materials carried, and weather conditions, and (3) the population density on the area under study throughout the day. Time-dependent accident probabilities on any link of the network may be produced by models that estimate accident probabilities based on the major causes of an accident and the physical and operational characteristics of the link including those varying in time, e.g., weather conditions and traffic intensity (Zografos and Androutsopoulos 2008). The estimation of the expected radius of the consequences of a hazardous materials accidents depends among others on the load and type of the material carried, the environmental conditions (e.g., temperature, wind direction and speed), the physical dimensions of the tank, and the diameter of the damage of the tank due to the accident (TNO 2005). The specification of the load interval $[m_k, m_{k+1}]$ needed to define the risk function may be specified through a pre-processing stage where various alternative accident scenarios should be designed and run through the use of an appropriate software (Zografos and Androutsopoulos 2008). The static population density may be obtained from the census data. The variations of the population density of an area throughout the day may be estimated by making appropriate assumptions based on the land use of the area and other relevant socioeconomic characteristics which may be found in the census data, e.g., US tract or block data (US Census Bureau 2012).

Definitely, all of the above issues require further analysis and modeling which, however, are not addressed in this paper. This paper aims only to highlight the above issues arising in computing risk according to (12) and indicate relevant directions for future research.

Problem definition and mathematical formulation

Assume that graph $G(N, A)$ denotes the mathematical representation of a roadway network, where N is the set of nodes, i.e., major roadway junctions, origin, destination, and intermediate stops (if any), and A the corresponding set of arcs each one representing the roadway segment connecting any two neighboring nodes of the roadway network. It is further assumed that the origin of the problem is at node $s_0 \in N$, the n intermediate stops (customers) are located at the nodes in $S := \{s_1, \ldots, s_n\} \subseteq N$, while the destination is at node s_{n+1} (not necessarily different from the geographical location of s_0). The numbering of the nodes in set S is arbitrary and does not imply any visiting order. Each intermediate stop s_k is associated with a given service time window denoted as $[a^e_{s_k}, a^l_{s_k}]$, where $a^e_{s_k}$ denotes the earliest service start time at stop s_k and $a^l_{s_k}$ denotes the corresponding latest service start time. If a vehicle arrives at stop s_k before $a^e_{s_k}$ then the service of the customer at that stop will be delayed until time $a^e_{s_k}$ is reached. On the other hand if the vehicle arrives at s_k later than $a^l_{s_k}$ the service of the corresponding stop is canceled. The operation of the depot is constrained within an earliest departure time from the origin denoted by $a^e_{s_o}$ and a latest arrival time at the destination denoted by $a^l_{s_{n+1}}$. Note also that upon arrival at any intermediate stop s_k, a mandatory fixed service time $t^s(s_k)$ is required. Each arc (i, j) of the graph is associated with travel time and risk, denoted by $(c^1_{(i,j)}(\tau), c^2_{(i,j)}(\mathrm{q}, \tau))$, respectively, where $c^2_{(i,j)}(\mathrm{q}, \tau)$ represents the risk on arc (i, j) as a function to departure time from node i and the load q of the truck.

Given a set of trucks V with capacity $K_v (v \in V)$, the proposed bi-objective time-dependent vehicle routing and scheduling problem aims to determine the routes (sequence of stops) and the associated paths traversing each route that optimize the criteria under consideration. A major constraint of the problem under study is that each customer is serviced by exactly one vehicle which visits the customer only once. In practice, however, there may exist solutions which may involve passing through a stop more than once, i.e., in the same solution, there may exist a path that passes through a given stop to service it and several other paths which pass through the stop without providing any service. However, under the above constraint any solution of this type is excluded from the solutions space, thus limiting unnecessarily the solutions space of the problem. A similar issue arises for the origin and destination which must be visited by each truck exactly once. This issue may be resolved by slightly modifying the underlying network. In particular, a copy i' of the origin, the destination, and each node i hosting a customer is created and connected with the original node with two new dummy arcs (i, i') and (i', i). The arc (i, i') is associated with a static travel time equal to the service time of the corresponding customer, and zero risk value. The arc (i', i) is associated with zero travel time and risk. Figure 2 illustrates the above transformation of the network. In this transformed network, any path may pass through any non-dummy node of the network with no restriction. However, passing through the dummy nodes is permitted only once.

Although no waiting is assumed in the problem under study, the formulation of the problem presented is generic including the case that unlimited waiting is allowed on the nodes of the network. The mathematical formulation of the proposed problem is given by (13)–(27).

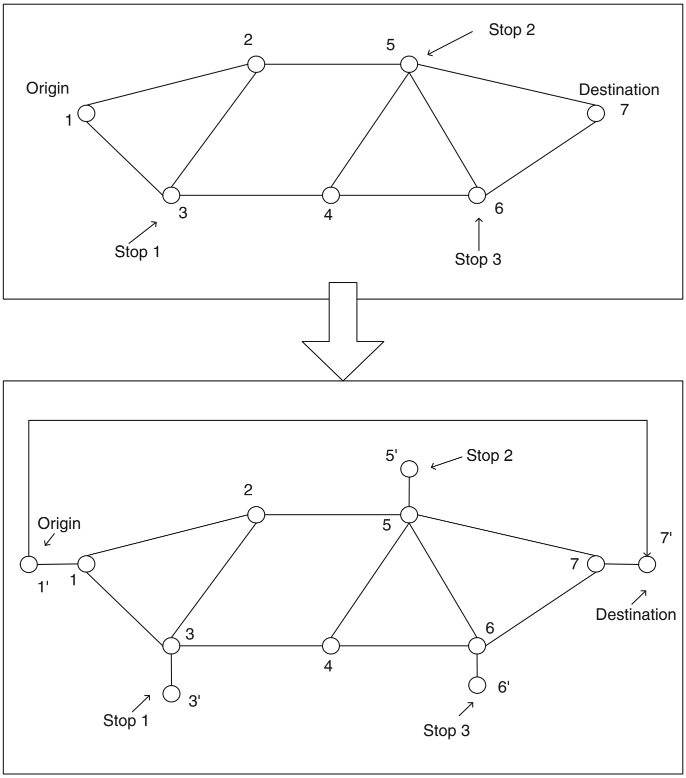

Fig. 2 Network transformation

$$\mathrm{Min}(Z_1, Z_2) \tag{13}$$

Subject to:

$$\sum_{\tau \in T} \sum_{i \in N} \sum_{v \in V} x^{\tau}_{isv} = 1 \quad s \in S \tag{14}$$

$$\sum_{\tau \in T} \sum_{j \in N} x^{\tau}_{ijv} - \sum_{\tau \in T} \sum_{j \in N} x^{\tau}_{jiv} = 0 \quad v \in V,\ i \in N \backslash \{s_0, s_{n+1}\} \tag{15}$$

$$\sum_{\tau \in T} \sum_{j \in N} x^{\tau}_{s_0 jv} = 1 \quad v \in V \tag{16}$$

$$\sum_{\tau \in T} \sum_{j \in N} x^{\tau}_{s_0 jv} - \sum_{\tau \in T} \sum_{j \in N} x^{\tau}_{js_{n+1}v} = 0 \quad v \in V \tag{17}$$

$$\sum_{i \in \Gamma^{-1}_j} \sum_{\tau' \in \{l : l + c^1_{(i,j)}(l) = \tau - t^s_j\}} x^{\tau'}_{ijv} - \sum_{i \in \Gamma^{+1}_j} x^{\tau}_{ijv} + x^{\tau-1}_{jjv} = x^{\tau}_{jjv} \quad \tau \in \mathrm{T},\ j \in N \tag{18}$$

$$\sum_{\tau \in D_i(s)} \sum_{v \in V} \sum_{i \in \Gamma^{-1}_s} x^{\tau}_{isv} = 1, \quad s \in S \tag{19}$$

$$\sum_{\tau \in A_j(s)} \sum_{v \in V} \sum_{j \in \Gamma^{-1}_s} x^{\tau}_{sjv} = 1, \quad s \in S \tag{20}$$

$$\omega^{\tau}_{iv} - \omega^{\tau'}_{jv} + (1 - x^{\tau}_{ijv})\mathrm{M} \geq d_j \quad (i,j) \in A,\ i \neq s_n,\ v \in V,\ \tau' = \tau + c^1_{(i,j)}(\tau) + t^s_j \tag{21}$$

$$\omega^{\tau}_{s_0 v} \leq K_v \quad v \in V,\ \tau \in \mathrm{T} \tag{22}$$

$$\omega^{\tau}_{s_n v} = 0 \tag{23}$$

$$\varphi^{\tau}_{ijv} + (1 - x^{\tau}_{ijv})M \geq \omega^{\tau}_{iv} \tag{24}$$

$$x^{\tau}_{ijv} \in \{0, 1\} \tag{25}$$

$$\omega^{\tau}_{iv} \geq 0, \tag{26}$$

$$\varphi^{\tau}_{ijv} \geq 0 \tag{27}$$

where $\Gamma^{-1}_{s_k} := \{i \in N : (i, s_k) \in A\}$, $D_i(s_k) := \{\tau : \alpha^e_{s_k} \leq \tau + c^1_{(i,s_k)}(\tau) \leq \alpha^l_{s_k}\}$ is the set of discrete departure times from any node i to an adjacent to stop s_k so that the arrival time at s_k is feasible, and $A_j(s_k) := \{\tau : \tau - t^s_{s_k} \leq \alpha^l_{s_k}\}$ the set of possible departure times from stop s_k. Variables (x^{τ}_{ijv}) takes value 1 if truck v uses link (i,j) departing from node i at time τ, and value 0 otherwise. Variables $(\omega^{\tau}_{iv} \geq 0)$ denote the load of the truck (v) right after leaving node i at time (τ), variables $(\omega^{\tau}_{s_0 v} \geq 0)$ denote the load of the truck v right after leaving depot at time (τ), while variables φ^{τ}_{ijv} denote the load of vehicle v traversing arc (i,j) at time τ. Constraint (14) implies that any intermediate stop s is visited only once by exactly one vehicle. Constraint (15) states that if a truck v enters a node i other than the origin and the destination, then the same truck also leaves node i. Thus, constraints (14) and (15) impose that each of the customers is serviced by exactly one vehicle. Constraint (16) imposes to each truck v to leave the origin s_0 while constraint (17) states that if a truck leaves the origin then at some point in time it should arrive at the destination s_{n+1}. Note that in Fig. 1, an additional arc (s_0, s_{n+1}) is included connecting directly the origin with the destination. Any solution for which $x^{\tau}_{s_0 s_{n+1} v}$ is equal to 1, implies that the corresponding truck v is not actually used. Constraints (16) and (17) in combination with the existence of arc (s_0, s_{n+1}) imply that the number of vehicles used is implicitly specified from the solution of the problem so as to optimize the objective functions. Constraint (18) implies that if a truck v has already arrived at a node j at time τ, it either waits until at least $\tau+1$ or it immediately leaves node j. If no waiting is allowed then (18) becomes:

$$\sum_{i\in\Gamma_j^{-1}} \sum_{\tau'\in\{l:l+c^1_{(i,j)}(l)=\tau-t^s_j\}} x^{\tau'}_{ijv} - \sum_{k\in\Gamma_j^{+1}} x^{\tau}_{kjv} = 0 \quad \tau \in \mathrm{T},\ j \in N \tag{18'}$$

where $\Gamma_j^{+} := \{i \in N : (j,i) \in A\}$.

Constraint (19) implies that any stop s is visited by a truck no later than the corresponding latest arrival time a^l_s. Constraint (20) implies that the service of any stop s is not allowed to start earlier that the corresponding earliest service start time a^e_s. Constraint (21) implies that if a truck uses link (i,j) then the change of the load of the vehicle when leaving node i from the load when leaving node j is d_j (demand in node j) at least. Note that for technical reasons any node of the network is associated with a demand value d_j. However, for nodes not hosting any customer the demand is set equal to 0. Constraint (22) implies that the load of any truck v when leaving the depot is at most equal to capacity of the truck (K_v). Finally constraint (23) implies that every truck must arrive empty at the destination while constraint (24) defines the variables φ^{τ}_{ijv} on the basis of variables ω^{τ}_{iv}. The objective functions expressed in (28) and (29) of the mathematical model express the total travel time (Z_1) and risk (Z_2) calculated by the sum of the travel time and risk of the arcs in any solution to the problem.

$$\mathrm{Z}_1 := \sum_{\tau\in\mathrm{T}} \sum_{i\in\Gamma^{-1}(s_{n+1})} \sum_{v\in V} (\tau x^{\tau}_{i s_{n+1} v}) - \sum_{\tau\in\mathrm{T}} \sum_{j\in\Gamma^{+1}(s_0)} \sum_{v\in V} (\tau x^{\tau}_{s_0 j v}) \tag{28}$$

$$Z_2 := \sum_{\tau\in\mathrm{T}} \sum_{(i,j)\in A} \sum_{v\in V} (R^{\tau}_{ij}(\varphi^{\tau}_{ijv})). \tag{29}$$

Solution algorithm

The proposed problem involves the determination of the set of non-dominated solutions based on the following definition: a solution $R_1 := \{rp_1^{R_1}, \ldots, rp_{m_1}^{R_1}\}$ is non-dominated if and only if there does not exist $R_2 := \{rp_1^{R_2}, \ldots, rp_{m_2}^{R_2}\}$ such that $R_1 \,/\!= R_2$ with $c_j(R_1) \geq c_j(R_2)$ for $j := 1,2$ and $c_{j_0}(R_1) > c_{j_0}(R_2)$ for at least one $j_0 \in \{1,2\}$. It is evident that if a solution $R_1 := \{rp_1^{R_1}, \ldots, rp_{m_1}^{R_1}\}$ is non-dominated then any of its route paths $(rp_i^{R_1})$ is non-dominated for the corresponding sequence of stops (route) $r_i^{R_1}$, i.e., there does not exist a path that passes through the sequence of stops of this route and outperforms route path $(rp_i^{R_1})$ in both risk and travel time criteria. Based on this finding the intuition behind the proposed solution approach is to build solutions based on non-dominated route paths.

The algorithm for solving this problem is based on the application of the weighted-sum method (Ehrgott 2005). The proposed bi-objective vehicle routing and scheduling problem is decomposed to a series of single-objective instances of the problem, where the objective function is expressed by the weighted sum of the criteria under consideration:

$$C(R;\overline{w}) = \sum_{j=1}^{2} w_j c_j(R) \tag{30}$$

where $w_j \in [0,1]$ and $\sum_{j=1}^{2} w_j = 1$. The proposed solution approach is heuristic. Given that the solution space (or decision space) of the problem defined by (13)–(27) is not convex (any feasible solution includes a 0-1 vector defining variables x_{ijv}^{τ}), solving to optimality the emerging single-objective problems aims to determine only a subset of the non-dominated solutions of the underlying combinatorial problem (Ehrgott 2005). In particular, applying the weighting method for solving the bi-objective problem at hand and provided that the emerging single-objective problems are solved to optimality, may result to the identification of those solutions corresponding to the extreme points of the convex hull of the decision space (Ehrgott 2005). Thus, non-dominated solutions lying in the interior of the convex hull of the extreme decision points (e.g., see Fig. 3) cannot be specified through the proposed approach. Therefore, the application of the weighted-sum method may achieve the determination of a subset of the non-dominated solutions of the underlying problem.

The solution approach of the bi-objective time-dependent vehicle routing and scheduling problem involves the solution of the emerging single-objective problem for various values of the weights w_j. In general, the adaptation of route building and route improvement heuristics to the time-dependent vehicle routing problem has been a common solution strategy in the relevant literature (Malandraki and Daskin 1992; Hill and Benton 1992). However, any adaptation of heuristic algorithms to the time-dependent vehicle routing and scheduling problem presented in the existing studies assumes that the cost (e.g., travel time) function between any pair of stops is available in advance. In the case of the single-objective problems under study, where the objective function is the weighted sum of the total travel time and risk of the scheduled route paths (i.e., route paths enhanced with the departure time from each constituent node), the corresponding cost function depends on the weights (w_1, w_2), and thus it cannot be available in advance. Therefore, any solution algorithm for the emerging single-objective time-dependent vehicle routing and scheduling problem should deal simultaneously with two problems: (1) a sequencing problem aiming to specify the sequence of stops serviced by each truck, and (2) the path-finding problem between any two consecutive stops.

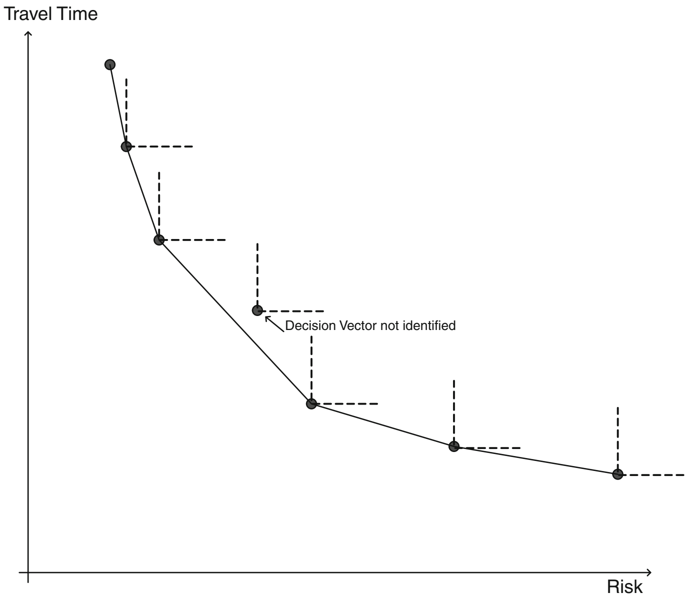

Fig. 3 An indicative representation of the decision points that correspond to the non-dominated solutions of the problem under study

A sequential route construction heuristic was developed to solve the emerging single-objective time-dependent vehicle routing and scheduling problems. The development of the proposed algorithm was based on the following features of the emerging single-objective problems: (1) the link cost function does not satisfy the FIFO consistency assumption (i.e., leaving later from the upstream node does not necessarily imply higher cost for traversing the link), (2) inserting a new customer in a route right after a customer i causes changes in the load of the vehicle before and after customer i and thus a recalculation of the total risk of the route is required. In particular, trying out the insertion of any unrouted stop s_k within a position (s_i, s_{i+1}) of a partially built route with m intermediate stops involves the recalculation of the shortest scheduled paths from the origin s_0 to the destination s_{n+1} through the intermediate mandatory stops $\{s_1, \ldots, s_i, s_q, s_{i+1}, \ldots, s_m\}$. However, given that the shortest scheduled paths from s_{i+1} to the destination s_{n+1} through the stops $\{s_{i+2}, \ldots, s_m\}$ are readily available (calculated in previous iterations), the path-finding problem reduces to searching for the scheduled paths from s_0 to s_{i+1} through $\{s_1, \ldots, s_i, s_q\}$ such that when joined with the corresponding shortest scheduled paths from s_{i+1} to the destination s_{n+1} they yield optimal scheduled route paths.

Based on the above discussion, limiting the candidate insertion position of an unrouted stop only on the first insertion position (nearest neighbor technique), i.e., between the origin and the first stop of the partial route or the destination (if the route is empty), is expected to simplify the emerging path-finding problem. In this case, only the scheduled paths from s_0 to s_1 through the unrouted stop s_k should be taken into account since the optimal paths for all possible departure times already found from s_1 to the destination s_{n+1} through the intermediate stops (customers) $\{s_2, \ldots, s_q\}$ are not affected by the insertion of a new customer in the first position of the partial route. Any other insertion algorithm would imply solving the above-mentioned path-finding problems for each candidate insertion position in the route taking into account the new risk values due to the changes in vehicle load caused by serving the new candidate customer.

Thus, the proposed heuristic algorithm is based on the adaptation of the sequential route-building technique for the time-invariant problem where each route is constructed by inserting one stop in the first candidate position of the route (between the origin and the first stop) at each iteration. Figure 4 presents the flowchart of the relevant heuristic for solving the emerging single-objective time-dependent vehicle routing and scheduling problems. The routes are constructed sequentially, and at each iteration one stop is selected for insertion at the start of the route. The selection of the stop for insertion is performed on the basis of the minimum insertion metric presented in (31) calculated for every unrouted stop (i) which can be feasibly inserted in the first position of the route:

$$\eta(s_0, s_i, s_1; \overline{w}) = \left(\frac{\sum_{\tau=\tau^e}^{\tau^l} \theta_r^{\tau}(s_i; w_1, w_2)}{\tau^l - \tau^e} \right) \cdot \left(\frac{a_{s_{n+1}}^l - a_{s_i}^l}{\tau^e - a_{s_1}^e} \right) \tag{31}$$

where τ^e, τ^l denote the earliest and latest departure times from the origin for which at least one feasible scheduled path through $\{s_i, s_1, \ldots, s_{n_r}\}$ exists, $\theta_r^{\tau}(s_i; w_1, w_2)$ denotes the cost value (in terms of c) of the optimal route-path departing at time τ for a given vector of weights w. The calculation of $\theta_r^{\tau}(s_i; w_1, w_2)$ for each candidate stop s_i is achieved by the application of the backward label-setting (Chabini 1998) algorithm for solving the arising path-finding problem with mandatory intermediate stops (Androutsopoulos and Zografos 2010) mentioned above. The ratio in the first parenthesis of (31) expresses the average cost value of the route (r) enhanced with stop s_i inserted at the start of the route over all possible departure times. Thus, this part of the selection metric favors any stop that outperforms the remaining candidate stops in terms of the average cost value of the corresponding route paths. The intuition behind considering the average cost value of the route paths and not the optimum value among all possible departure times is that the set of the actual possible departure times from the origin is expected to be progressively reduced as the process moves and new stops are inserted in front of the stop s_i. In particular, any new stops inserted in the front part of the route with early service time windows tend to reduce the latest departure times from the origin.

On the other hand, the ratio in the second parenthesis of (31) expresses a measure of lateness of the time windows of the stops. In other words, since the rationale of the proposed algorithm implies that new stops are only inserted at the start of the route, the stops for which the upper limit of their time window is closer to the depot closing time while their lower limit is farther from the depot opening time, should be inserted in the route as early as possible. The objective of including this measure in the insertion metric is to build routes with as many stops as possible, thus leading to minimum number or routes.

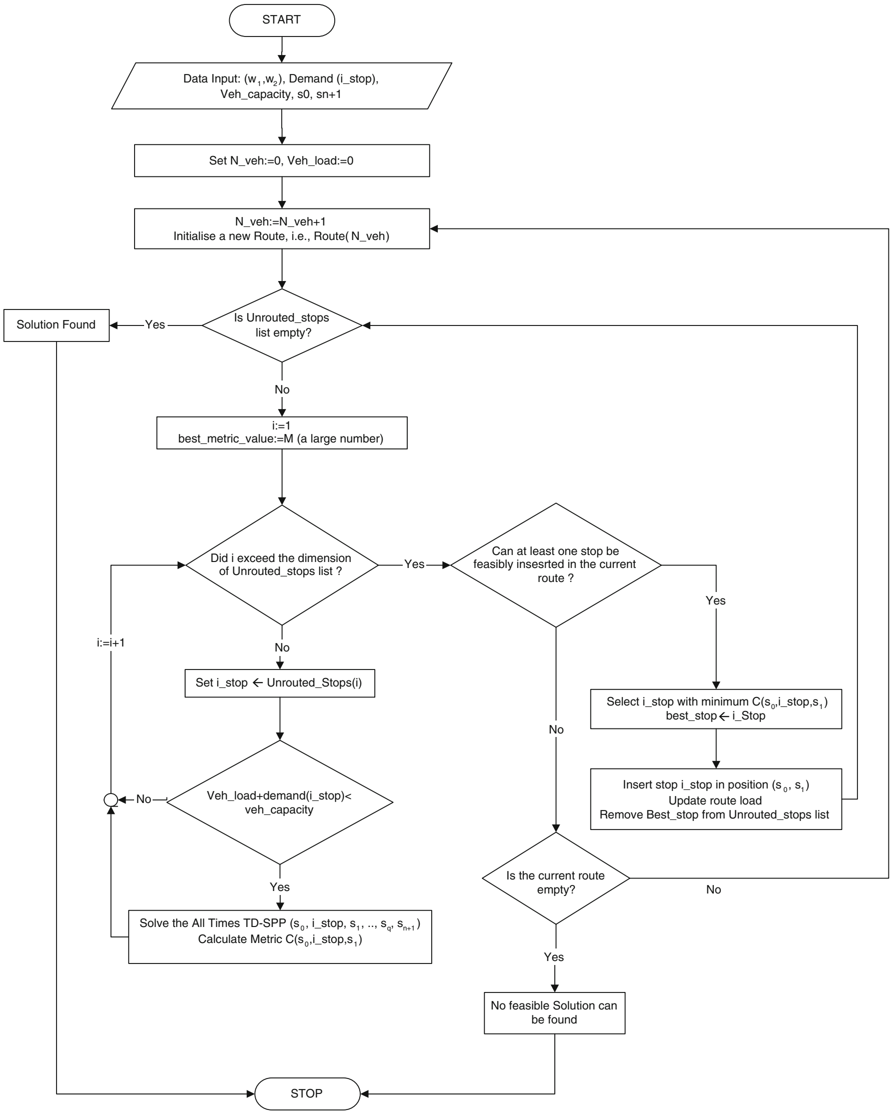

Fig. 4 Flowchart of the heuristic algorithm for solving the single-objective (weighted sum of travel time and risk) time-dependent vehicle routing and scheduling problem

The candidate stop with the minimum cost metric is selected and actually inserted in the route. If no stop can be feasibly inserted in the route, then that route is closed and the construction of a new route is commenced. Upon the closure of a route, one scheduled path has been determined for each possible departure time from the origin. Among the list of scheduled paths from the origin, the one with the minimum cost value is held, while the remaining are excluded from further consideration. The algorithm terminates when all stops have been inserted in a route.

The process described above is repeated for different vectors of the weights $(w, 1-w)$ such that $w \geq 0$. Assume that L is a list of ranges of values of w and Ξ is the list for storing the non-dominated solutions identified during the process. List Ξ is created by inserting the solution identified in each execution of the process described above. Any new solution is compared with the existing solutions in Ξ in terms of travel time and risk. If it is not dominated by any of the existing solutions in Ξ, then it is inserted in Ξ while any of the existing solutions dominated by the new solution are excluded from Ξ. Otherwise the new solution is excluded from further consideration. In the beginning of the process both lists L and Ξ are assumed to be empty. Initially, the problem is solved for w equal to 1 (i.e., risk criterion is disregarded) resulting to R_1, and w equal to 0 (i.e., travel time minimization criterion is disregarded) leading to solution R_0. Both solutions are inserted in Ξ. Then the problem is solved for w equal to 0.5 (the middle point of the range [0,1]) resulting to solution $R_{0.5}$ which is inserted in Ξ. If the solution emerging from this problem satisfies $c^1(R_0) > c^1(R_{0.5})$ then the ranges [0,0.5] are placed at the end of list L. Otherwise the range [0,0.5] is excluded from further consideration. Similarly, if solution $R_{0.5}$ emerging from this problem satisfies $c^1(R_{0.5}) > c^1(R_1)$ then the range [0.5,1] is placed at the end of list L. Otherwise the range [0.5,1] is excluded from further consideration. An iterative procedure follows in which at each iteration k, a range (w_{i_k}, w_{j_k}) is drawn from list L and the problem corresponding to the middle of this range is solved. After solving the problem for $w_k = \frac{w_{i_k}+w_{j_k}}{2}$ leads to solution R_{w_k} which is inserted in Ξ. If the solution emerging from this problem satisfies $c^1(R_{w_{i_k}}) > c^1(R_{w_k})$ and the length of range $[w_{i_k}, w_k]$ is above a threshold value, then this range is placed at the end of list L. Otherwise the range $[w_{i_k}, w_k]$ is excluded from further consideration. Moreover, if solution R_{w_k} satisfies $c^1(R_{w_{i_k}}) > c^1(R_{w_k})$ and the length of range $[w_k, w_{j_k}]$ is above a threshold value then this range is placed at the end of list L. Otherwise the range $[w_k, w_{j_k}]$ is excluded from further consideration. This process terminates when the list L becomes empty. The flowchart of the above process is provided in Fig. 5. The binary search performed for the weighted-sum method is illustrated in Fig. 6.

Computational performance

The proposed heuristic algorithmic approach for solving the bi-objective time-dependent vehicle routing and scheduling problem was applied for solving a set of test problems to assess its performance in terms of accuracy and computational time. The accuracy of the algorithm was assessed on small test problems which were designed so as to comply with the structure of real-life problems while still be solvable by a mixed integer programming (MIP) solver. To simplify the non-linear objective function of risk, the effect of the load on the risk values was not taken into account and thus each link was assumed to have time-dependent load-invariant risk values. This type of test problems were developed on a 49 nodes grid-like network involving five randomly located customers. The demand for each customer was randomly specified within range 2–4 tons while the capacity of each truck was

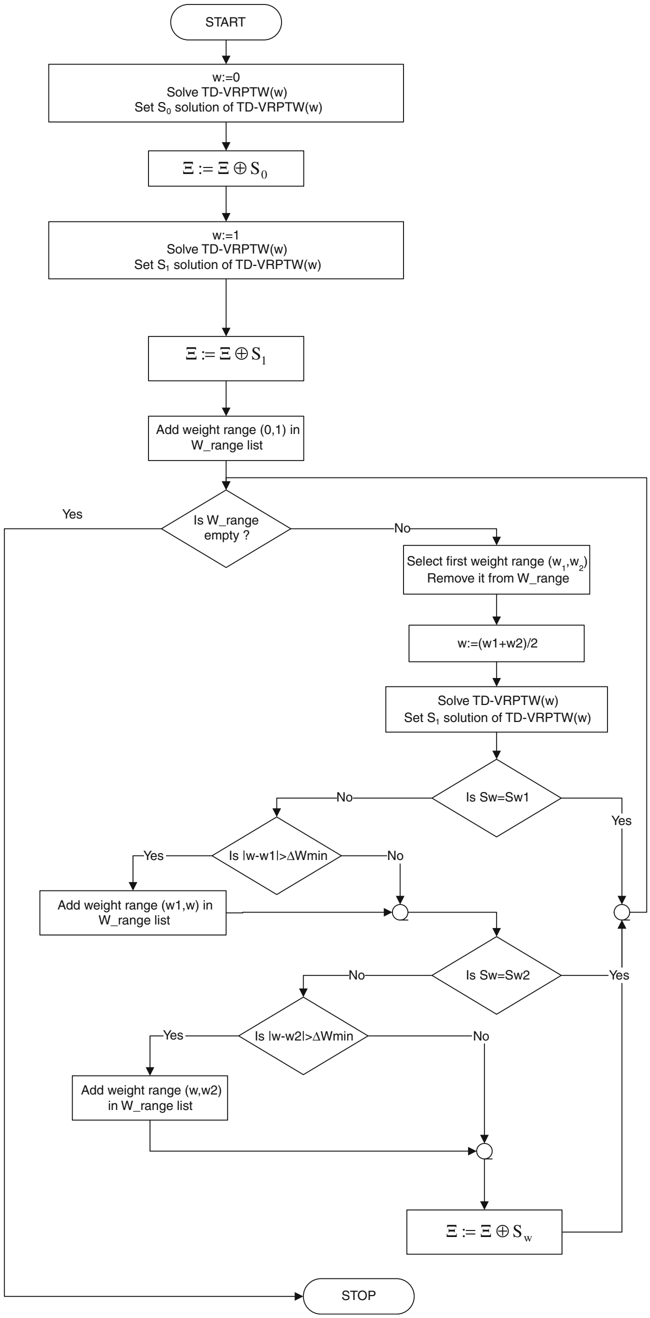

Fig. 5 Flowchart of the overall heuristic approach for solving the bi-objective time-dependent vehicle routing and scheduling problem

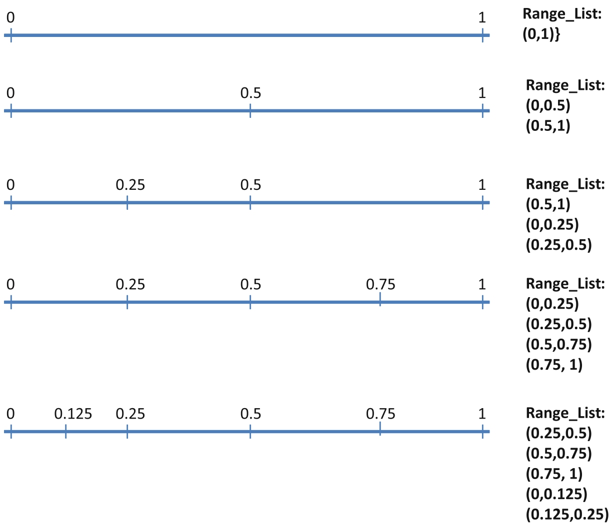

Fig. 6 Illustration of the way that the ranges of weights are scanned in the proposed application of the weighted-sum method

assumed equal to 10 tons. The origin of the truck was different from the destination while the time period between the earliest possible departure time from the origin until the latest arrival time at the destination was set equal to 60 min. In this setting the time window for each customer was 10 min. The width of service time windows and the operational time horizon of the depot were kept narrow to enable the exact solution of the emerging time-dependent vehicle routing and scheduling problems. Although the selected width for both time periods do not comply with the real-life case, their ratio resembles the actual relation of real-life service time windows and the depot operational daily time horizon. The underlying network included 168 links. The length of each link was randomly specified within the range of 600 and 900 m. Speed measurements were created for every link of the network. The speed measurements were randomly generated for each link separately on a 10-min basis, assuming the following average speed variation pattern: (1) 60–40 km/h in the first period, (2) 40–20 km/h for the second period, and (3) 60–50 km/h in the third period. The duration of each of these periods was 20 min.

The risk value on each arc and time interval was selected from the uniform distribution defined on the range from 1 to 9 (times 10^{-4}) (List et al. 1991; Chang et al. 2005). Although the ranges for both the risk and the travel time were arbitrarily selected, the objective function was scaled to incorporate the different magnitudes of the scales of travel time and risk.

The proposed mathematical model has been integrated in a mathematical programming development application (AIMMS v3.2). The objective function modeled in AIMMS was set equal to the weighted sum of two criteria, the travel time and risk. In the AIMMS application, each of the test problems was addressed by solving the emerging single-objective time-dependent vehicle routing and scheduling problems for twenty pairs of weights ranging from {0,1} up to {1,0} with a step of 0.05 (i.e., as w_1 increases by 0.05, w_2 decreases by the same quantity). The comparison of the exact solutions specified by the AIMMS application with those specified by the heuristic was performed on the basis of calculating the percentage difference of the travel time and risk between each heuristic solution from the exact solution with the minimum Euclidean distance in the decision space. Table 2 presents the travel time and risk values of the exact and heuristic solutions and the associated percentage deviation as described above. In particular, Table 2 presents the vector of travel time and risk values for each solution of the test problems identified with the heuristic algorithm. The vector of each heuristic solution is associated (placed in the same line) with the corresponding vector of the exact solution lying closer (in terms of Euclidean distance in the decisions vector space) to the heustic solution vector. Any other solution vector identified by the exact application which is not associated with any of the heuristic solutions is placed in the last column of the table. Based on these results the average difference between AIMMS and heuristic solutions in terms of travel time and risk was 10.1 and 15.4 %, respectively. Although for some problem instances the heuristic algorithm identified solutions with criteria vector lying very close (in terms of Euclidean distance in the decisions space) to the solutions vectors of the AIMMS application, in either cases the heuristic solutions deviated substantially from the corresponding exact solutions.

The worst heuristic solution found in terms of travel time involved 36.3 % travel time deviation from the travel time of the its closest non-dominated solution. The worst heuristic solution in terms of risk involves a deviation of 48.28 % of its risk value from the risk value of its closet non-dominated solution. It should be pointed out that these percentage deviations provide in Table 2 can be considered as upper bounds since the exact solution method does not specify the entire set of non-dominated solutions and thus leaving the possibility of the existence of an exact solution closer to the heuristic solution than any of the identified non-dominated solutions. In addition it is worth noting that the average computational time required by the heuristic algorithm for solving the test problem is substantially smaller than the corresponding time required by the AIMMS application (i.e. 15 vs. 5,000 s).

The performance of the proposed algorithmic approach in terms of computational time was assessed on a set of test problems generated on a 100 nodes network. The underlying network included 360 links while the length, travel speed function, and the risk values for any link were produced with the same procedure that was used for the 49 nodes network. Four categories of test problems were generated using two alternative numbers of customers (i.e., 10 vs. 20 customers) and two alternative depot time windows (i.e., 120 vs. 180 min). Ten test problems were generated for each of the above problem categories. In the problems that involved depot time window of 120 min, the length of the customers' time windows was 30 min while in the problems involving 180 min depot time window the corresponding time window length was 45 min. In all problem categories, the demand ranged randomly between 2 and 4 tons while the trucks capacity was 10 tons. The service time was also generated randomly within the range of 10–20 min. Table 3 presents the average computational time for solving the test problems under each of the above-mentioned problem categories. In addition, Table 3 presents the average number of the single-

Table 2 Results of the AIMMS application and the heuristic algorithm on the five customers test problems defined on the 50 node network

Problem test ID	Heuristic sol. (travel time and risk)	Exact sol. (travel time and risk)	% travel time deviation	% risk deviation	Other exact solutions
Test-1	(38, 50)	(31, 40)	22.6	25.0	(52, 32)
	(41, 41)	(35, 33)	17.1	24.2	
	(43, 39)	(35, 33)	22.9	−2.6	
Test-2	(45, 47)	(33, 48)	36.36	−2.13	(44, 28)
	(47, 43)	(43, 29)	9.30	48.28	
	(51, 41)	(43, 29)	15.91	46.43	
	(57, 40)	(46, 27)	23.91	48.15	
Test-3	(31, 37)	(31, 36)	0.0	2.8	–
	(32, 36)	(31, 36)	3.1	0.0	
	(38, 34)	(36, 33)	5.6	3.0	
	(42, 33)	(41, 32)	2.4	3.1	
Test-4	(38, 60)	(33, 54)	15.2	11.1	(35, 49)
	(39, 57)	(33, 54)	18.2	5.6	
	(43, 50)	(36, 47)	25.0	6.4	
	(51, 42)	(44, 38)	11.4	10.5	
	(53, 40)	(52, 36)	1.9	11.1	
Test-5	(23, 32)	(23, 29)	0	10.3	(25, 26)
Test-6	(27, 53)	(27, 45)	0.0	17.8	–
	(29, 41)	(28, 41)	3.6	0.0	
	(30, 38)	(30, 38)	0.0	0.0	
	(31, 37)	(31, 37)	0.0	0.0	
	(35, 36)	(41, 33)	12.9	−2.8	
	(53, 33)	(60, 32)	−13.2	3.1	
Test-7	(55, 45)	(48, 40)	14.6	12.5	(39, 43), (40, 42)
Test-8	(47, 68)	(40, 50)	17.5	36.0	(50, 37), (53, 36)
	(49, 60)	(40, 50)	22.5	39.5	
	(51, 50)	(44, 43)	15.9	16.3	
Test-9	(36, 35)	(39, 53)	0.0	0.0	(46, 35)
	(44, 34)	(40, 42)	0.0	0.0	
Test 10	(40, 53)	(33, 43)	21.2	23.3	(38, 36)
	(47, 46)	(46, 35)	2.2	31.4	
	(53, 45)	(46, 35)	15.2	28.6	
Average Comp. Time	15 s	5,000 s	Average: 10.1	Average: 15.4	

objective time-dependent vehicle routing and scheduling problems solved and the average number of the emerging alternative solutions. As it was expected this type of problems could not be solved by the AIMMS application (CPLEX 12 solver).

Table 3 Computational results of the proposed heuristic algorithm on the 10 and 20 customers test problems defined on a 100 nodes grid-like network

Test problem	Number of customers	Depot time window (min)	Average number of problems solved	Average number of solutions	Average computational time (s)	Average comp. time per problem solved (s)
1	10	120	17	7	104.5	5.9
2	10	180	18	7	233.8	12.8
3	20	120	20	5	281.4	13.9
4	20	180	20	8	512.75	25.2

The computational time for solving any of the bi-objective time-dependent vehicle routing and scheduling problems depends on the number of the emerging single-objective problems solved. The maximum solving time was encountered in the problems with 20 customers and a depot operational time horizon of 180 min, where each single-objective problem was solved in 25.2 s on average. Moreover, based on the results of Table 3, the computational time almost doubles when the depot operational time horizon increases by 50 %. This fact may be attributed to the computational burden for solving the intermediate time-dependent path-finding problems at each iteration of the heuristic algorithm which depend among other on the width of the time horizon. Thus, the computational performance of the proposed algorithmic approach could be further improved by integrating speed-up techniques to solve the intermediate path-finding problems between any pair of stops.

Concluding remarks

In this paper, hazardous materials distribution is modeled as a bi-objective time-dependent vehicle routing problem with time windows. This type of vehicle routing problem is defined on a transportation network (not a complete graph) and thus, its solution involves the identification of paths traversing the intermediate stops with feasible schedules. The travel time and risk attributes associated with each arc are assumed time dependent. Accuracy of travel time estimation is a critical issue in this problem since it leads to the identification routes with increased reliability in satisfying service time windows. The accuracy in estimating travel time is envisaged by smoothing the travel speed function on any arc of the network. A new risk model is proposed which takes into account time-dependent accident probabilities and load-dependent population exposure. Although the proposed algorithm for solving the hazardous materials distribution problem takes into account explicitly the effect of the load of the truck in the estimation of the transportation risk, the assessment of the computational performance of the algorithm was based on load-invariant risk values.

A number of issues have been identified that merit further research. More specifically, the computational results indicate that the proposed model for hazardous materials distribution may facilitate the associated decision making process by identifying alternative routing solutions capturing the travel time and risk trade-off. However, further research is required to improve the performance of the solution algorithm in terms of accuracy. In addition, further work is required to develop methodologies for estimating the time and load-dependent risk values on the links of the network. Furthermore, work presented in paper may be extended to cover other categories of routing and scheduling problems for freight distribution. Along this line, work under way by the authors extends the proposed model for the situation of heavy goods vehicles distribution, where the objective functions taken into consideration involve the emissions produced over a route and the travel time.

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