BASIS

Any set of linearly independent vectors $\mathbf{v}_1$, $\mathbf{v}_2, \ldots, \mathbf{v}_n$ such that every vector in a vector space can be written as a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is a *basis* of the space (provided that $\upsilon_1, \ldots, \upsilon_n$ belong to the space).

In general, there are many different possible bases for a given vector space. The minimum number of members needed for a basis is the *dimension* of the vector space.

Example. The vectors $b_1 = \binom{1}{1}$ and $b_2 = \binom{1}{-1}$ form a basis in the two-dimensional Euclidean space R^2. We can express the vector $\binom{2}{1}$ as

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{3}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

A basis consisting of *unit* vectors δ_i is called a *standard* or *canonical* basis. A canonical basis of R^n consists of vectors

$$\delta_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \delta_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \cdots, \delta_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

See also ORTHOGONAL TRANSFORMATION.