Frequency of Flashovers of Clearances in Low-Voltage AC Power Circuits Due to Overvoltage Waves

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Abstract

The frequency distribution of the crest values of overvoltage waves in low voltage AC power circuits can be approximated by an exponential function. The flashover probability P of a clearance with an impulse withstand voltage $\hat{U}_{st}$ in an electrical equipment is given by $P = exp(-\alpha \hat{U}_{st})$. The annual number of flashovers of a single clearance in an unprotected house wiring system supplied by an unprotected overhead network depends upon the impulse withstand voltage, the latitude of the network, the annual number of thunderstorm days, the effective length and the height above ground of the overhead line. However, this does not mean, that every flashover triggers a dangerous power frequency follow current.

1 Purpose of the Investigation

Atmospheric discharges and switching actions produce overvoltage waves in supply networks. They do not endanger devices with a given impulse withstand voltage as long as the maximum amplitude of the voltage wave is less than the impulse withstand voltage of the equipment.

Greater values, leading to flashover of the clearance, can be tolerated as long as the wave energy is not sufficient to ionize the discharge channel to such an extent, that a power frequency follow current can build up.

Greater values can also be tolerated of which the maximum amplitudes lead in fact to flashover followed by power frequency follow currents, if the mean time between two consecutive voltage waves is approximately the expected lifetime of the equipment.

Analogous considerations apply to creepage distances, too.

The following is an estimate of the prevailing situation in an AC power circuit which is installed according to the rules of insulation co-ordination [1].

The expressions frequency distribution and partition function of overvoltage waves are used as abbreviations for frequency distribution and partition function of the absolute value of the maximum amplitude of overvoltage waves.

2 Experimental Data

Martzloff and *Gauper* [2] have shown, that a 1.2/50 μs voltage wave travelling along a $3 \cdot 3.3$ mm^2 installation cable propagates at a velocity of $2 \cdot 10^8$ m/s. Own measurements [3] of a PVC insulated NYM $3 \cdot 1.5$ mm^2 installation cable resulted in $1.7 \cdot 10^8$ m/s. The high velocity implies, that the leading edge of a 1.2/50 μs voltage wave is contained in a 200 m long section of the cable which, simplifying, one may consider to be a concentrated capacity.

In Italy [4] utilities have measured the frequency distribution of overvoltage waves in 17 representative unprotected low-voltage cable networks, which were not directly exposed to atmospheric influences ("non exposed networks"). The measuring period was one year.

In France [5] the "Electricité de France" has measured the frequency distribution of overvoltage waves between the outer phase conductor and ground in an unprotected low-voltage overhead network of unknown geometry at the supply entrance ("dérivation d'abonné") with the consumer load connected. The measuring period was three years.

In Austria [6] *Harich* and *Enders* investigated carefully the origin and shape of atmospheric overvoltage waves in several unprotected low-voltage overhead networks. They reported the frequency distribution of overvoltage waves between the outer phase conductor and ground in an overhead network of known geometry at the supply entrance with the consumer load connected. The measuring period was nine weeks from June until the beginning of September.

In Germany [7] the "Forschungsgemeinschaft Elektromagnetische Verträglichkeit" of the "Zentralverband Elektrotechnik- und Elektronikindustrie (ZVEI)" under the chairmanship of *Meissen* has investigated the occurrence of transient overvoltage waves in low-voltage AC power circuits of unknown geometry in industrial, commercial, residential and laboratory buildings between the phase conductor and ground and reported the frequency distribution of the observed overvoltage waves.

Biegelmeier [9] investigated the probability of power frequency follow currents as the function of the phase difference between a 1.2/50 μs test voltage wave and the line voltage.

3 Plausibility Considerations

The Italian [4], French [5] and German [7] measurements were not made at the supply entrance of a single consumer. However, the annual number of flashovers of a single clearance in an electrical equipment installed in a low-voltage AC power circuit, under consideration here, can be estimated, if plausible assumptions are made. These assumptions are:

- the distribution functions derived from the Italian, French and German measurements are also valid at the supply entrance of a single consumer;
- the low-voltage network is not protected by arresters;
- an overvoltage wave produced at any point of the low-voltage network propagates undamped within the network and the domestic installation;
- the low-voltage network is fed by a substation, the medium-voltage side of which is protected by an arrester;
- every cloud-to-cloud or cloud-to-ground lightning discharge in the vicinity of a low-voltage overhead line produces an overvoltage wave within the network. The crest voltage of the wave depends in particular upon the distance between the lightning and the line and upon the maximum amplitude of the lightning current.

4 Frequency Distribution and Partition Function of the Absolute Crest Values of Overvoltage Waves

Whenever a clearance with a given impulse withstand voltage is subjected to overvoltage waves then the annual number of flashovers can be calculated when the frequency distribution of the overvoltage waves is known. **Fig. 1** shows the frequency distribution of the absolute values of the maximum voltage amplitude of overvoltage waves observed in Italy, France, Austria and Germany during time periods of different lengths.

In this diagram the natural logarithm of the number of overvoltage waves $\tilde{n}$ with crest values $\hat{U}$ falling within a given interval is plotted against the mid-value of the respective interval, the investigated voltage range being equally divided. It turns out that the measured values can be approximated by an exponential function, namely

$$\tilde{n} = \tilde{n}_0 \, e^{-\alpha \hat{U}} . \tag{1a}$$

Hence the differential frequency distribution of the crest values is

$$n^* = n_0 \, e^{-\alpha \hat{U}} , \tag{1b}$$

where n_0 denotes the initial value of the differential frequency distribution per measuring period at the location of the measurements and

$$n = \alpha \, e^{-\alpha \hat{U}} \tag{2}$$

the partition function. The approximation applies also in a generalized form to the German measurements of overvoltage waves within the installation of buildings

$$\tilde{n} = \tilde{n}_{01} \, e^{-\alpha_1 \hat{U}} + \tilde{n}_{02} \, e^{-\alpha_2 \hat{U}} . \tag{3}$$

In this case, however, the voltage range under investigation was not precisely equally divided. In eq. (1a) each functional value represents the number of all crest values within the voltage interval ΔU. Therefore each value is equal to the definite integral of the differential

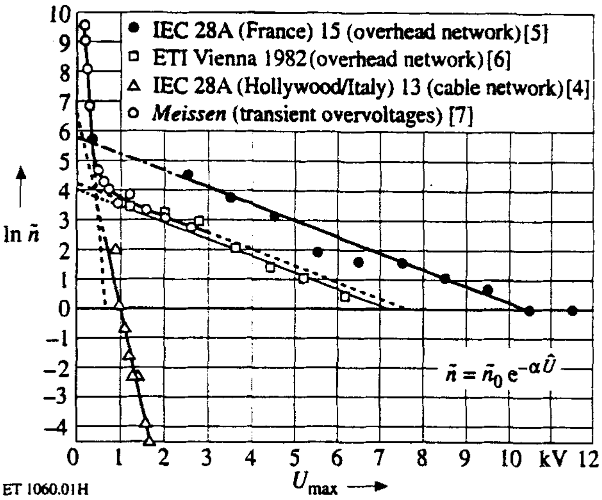

Fig. 1. Frequency distribution of the maximum amplitude of overvoltage waves travelling in low-voltage overhead and cable networks

frequency distribution between the limits of the voltage interval. Hence it follows because of eq. (1b) that

$$\tilde{n} = \tilde{n}_0 \, e^{-\alpha \hat{U}} = n_0 \int_{\hat{U}-\Delta\hat{U}/2}^{\hat{U}+\Delta\hat{U}/2} e^{-\alpha \hat{U}} \, d\hat{U} . \tag{4}$$

If $\tilde{n}_0$ and α are known then it follows for the initial value n_0 of the differential frequency distribution that

$$n_0 = \frac{\tilde{n}_0 \alpha \, e^{-\alpha \hat{U}}}{e^{-\alpha(\hat{U}-\Delta\hat{U}/2)} - e^{-\alpha(\hat{U}+\Delta\hat{U}/2)}} = \frac{\tilde{n}_0 \alpha}{e^{(\alpha/2)\Delta\hat{U}} - e^{-(\alpha/2)\Delta\hat{U}}} = \frac{\tilde{n}_0(\alpha/2)}{\sinh[(\alpha/2)\Delta\hat{U}]} , \tag{5}$$

and for the differential frequency distribution itself that

$$n^*(\hat{U}) = \frac{\tilde{n}_0(\alpha/2)}{\sinh[(\alpha/2)\Delta\hat{U}]} e^{-\alpha \hat{U}} = n_0 \, e^{-\alpha \hat{U}} . \tag{6}$$

The quantities $\tilde{n}_0$ and α can be inferred from Fig. 1. **Tab. 1** is a compilation of the experimental data taken

Measuring- Site	Period	$\tilde{n}_0$ [3] kV^{-1}	α kV^{-1}	n_0 [3]
IT [4]	1 a	572.5	6.55	5624
FR [5] Massif Central	3 a	314.2	0.552	310
AUT [6] Hartmannsdorf	63 d [1]	54.6	0.552	68
DE [7]	3428 h [2]	$\tilde{n}_{01}$: $1.33 \cdot 10^5$ $\tilde{n}_{02}$: 66.7	α_1: 17.88 α_2: 0.538	n_{01}: $3.26 \cdot 10^6$

1) Measuring period: June until the beginning of September

2) Season unknown. Measuring time spread among:
Industrial buildings: 1317 h
Commercial buildings: 1202 h
Residential buildings: 447 h
Laboratories: 462 h

3) Related to the measuring time

Tab. 1. Measuring data

from the original publications or inferred from Fig. 1. The table also includes the initial values of the differential frequency distributions calculated from eq. (5).

Tab. 1 and Fig. 1 show, that the decay constant α in the exponent of the frequency distribution of the unprotected French and Austrian network has the same value. The reason for concluding that the observed overvoltage waves in these networks were of atmospheric origin follows from the Austrian measurements, during which a thunderclap was recorded simultaneously with each overvoltage wave. This indicates, that the decay constant $\alpha = 0.552\ \mathrm{kV}^{-1}$ is characteristic of the frequency distribution of atmospheric overvoltage waves in unprotected low-voltage overhead networks.

5 Annual Number of Flashovers of Clearances in Special Networks as a Function of the Impulse Withstand Voltage

When a clearance is subjected to voltage waves it flashes over when the surge crest voltage exceeds its impulse withstand voltage and when the duration of the wave is long enough for a spark to develop. A clearance with an impulse withstand voltage $\hat{U}_{st}$ withstands according to the rules of insulation co-ordination [1] a 1.2/50 µs testing wave with a maximum voltage amplitude $\hat{U}$ as long as

$$\hat{U} \le \hat{U}_{st}, \tag{7}$$

otherwise it flashes over. When the differential frequency distribution of the overvoltage waves $n^*(\hat{U})$ is known and when the requirement for the wave duration is met, then the number of flashovers $N(\hat{U}_{st})$ of a clearance with the withstand voltage $\hat{U}_{st}$ per measuring period results because of eq. (6) from:

$$N(\hat{U}_{st}) = \int_{\hat{U}_{st}}^{\infty} n^*(\hat{U})\,\mathrm{d}\hat{U} = \frac{n_0}{\alpha}\,\mathrm{e}^{-\alpha\hat{U}_{st}}. \tag{8}$$

5.1 Flashovers in Cable Networks

The number of overvoltage waves in the Italian networks refers to one year. The annual number of flashovers of a clearance in an installation supplied from a cable network, $N_{a,cab}(\hat{U}_{st})$, follows from eq. (8) in conjunction with the data in Tab. 1:

$$N_{a,cab}(\hat{U}_{st}) = 859 \cdot \mathrm{e}^{-6.55\,\hat{U}_{st}}. \tag{9}$$

According to eq. (9) a clearance with a 4-kV-impulse withstand voltage would flash over with a frequency of $4 \cdot 10^{-9}$ per year. As it is plausible, that the Italian measurements are representative of properly earthed low-voltage cable networks, cable networks are not considered further.

5.2 Flashovers in the Installations of Buildings Due to Switching Overvoltage Waves

According to eq. (3) the German measurements in the installations of buildings can be approximated by the superposition of two exponential functions. Tab. 1 and Fig. 1 show, that within the accuracy of readings the decay constant α_2 of the second exponential function has the same value as in the case of overhead networks. This suggests, that the second term in eq. (3) is due to atmospheric overvoltage waves and that only the first term represents switching overvoltage waves. If one therefore assumes, that only the first term is representative of switching overvoltages, which unlike atmospheric waves are uniformly distributed over the measuring period t_m, then it follows from eqs. (6) and (8) for the annual number of flashovers of a clearance within the installation of a building due to switching overvoltage waves, $N_{a,sw}(\hat{U}_{st})$, that

$$N_{a,sw}(\hat{U}_{st}) = \frac{n_{01}\,t_a}{\alpha_1\,t_m}\,\mathrm{e}^{-\alpha_1\hat{U}_{st}}, \tag{10}$$

where t_a denotes the duration of one year (8 760 h). With the data in Tab. 1 it follows for $N_{a,cab}(\hat{U}_{st})$:

$$N_{a,sw}(\hat{U}_{st}) = 4.66 \cdot \mathrm{e}^{-17.88\,\hat{U}_{st}}. \tag{11}$$

A clearance with a 4-kV impulse withstand voltage would flash over with a frequency of $4 \cdot 10^{-26}$ per year. That means, that switching overvoltage waves can be neglected, when the clearances are designed to meet the requirements for atmospheric overvoltage waves in overhead networks.

5.3 Flashovers in Overhead Networks Due to Atmospheric Overvoltage Waves

The French measurements were made in the Massif Central, an area with 30 thunderstorm days per year, during three years and the Austrian measurements in the province of Steiermark, an area with 38 thunderstorm days per year, during nine weeks from June until the beginning of September. Therefore both measurements are not immediately comparable. In order to achieve comparability, the French and the Austrian results must be converted to the respective annual number of thunderstorm days, that means to 30 days for the Massif Central and to 38 days for the province of Steiermark.

If the initial value n_0 in eq. (6) does not relate to a measuring period of one year but to a deviating period with $T_0 = x\,T_d$ thunderstorm days, then the crest values of the annual overvoltage waves are distributed according to

$$n_a^* = \frac{n_0}{x}\,\mathrm{e}^{-\alpha\hat{U}_{st}}. \tag{12}$$

By substitution of eq. (12) into eq. (8) it follows for the annual number $N_a(\hat{U}_{st})$ of flashovers of a clearance with the impulse withstand voltage $\hat{U}_{st}$:

$$N_a(\hat{U}_{st}) = \frac{n_0}{x\,\alpha}\,\mathrm{e}^{-\alpha\hat{U}_{st}}. \tag{13}$$

5.3.1 Austrian Overhead Network

The distribution of the monthly thunderstorm days over the year and the annual number can be estimated from the world distribution of the monthly thunderstorm days [10]. The annual number amounts, thus estimated, to 38 thunderstorm days in agreement with the local observations. In the course of the measuring period from June until the first week of September, i. e. 92 days, readiness for measurement existed for 63 days.

According to **Fig. 2**, 24 thunderstorm days fall within the measuring period, the fraction

$T_0 = (63/92)\,24 = 16.4\ \mathrm{d}$

corresponding to the period of readiness for measuring. The annual number of flashovers of a clearance with the impulse withstand voltage $\hat{U}_{st}$ at the measuring site, $N_{a,AUT}(\hat{U}_{st})$, follows from eq. (13) and the data in Tab. 1:

$$N_{a,AUT}(\hat{U}_{st}) = 285 \cdot e^{-0.552\hat{U}_{st}}. \quad (14)$$

5.3.2 French Overhead Networks

In the Massif Central 30 thunderstorm days per year are observed. The annual number of flashovers of a clearance with the impulse withstand voltage $\hat{U}_{st}$ at the measuring site, $N_{a,FR}(\hat{U}_{st})$, follows from eq. (13) and the data in Tab. 1 in consideration of the measuring period of three years:

$$N_{a,FR}(\hat{U}_{st}) = 187 \cdot e^{-0.552\hat{U}_{st}}. \quad (15)$$

6 Annual Number of Flashovers in General Low-Voltage Overhead Networks Due to Atmospheric Overvoltage Waves as a Function of the Impulse Withstand Voltage, Latitude and Number of Annual Thunderstorm Days

The annual number of flashovers of a clearance, as deduced from the distribution of the absolute values of the surge crest voltages, is only valid at the measuring site. The measuring site is characterised by the latitude λ and the annual number of thunderstorm days T_d. The annual number of flashovers as a function of the impulse withstand voltage, the latitude and the thunderstorm days, $N_a(\hat{U}_{st}, \lambda, T_d)$, is the product of

- the flashover probability $P(\hat{U}_{st})$ of the clearance with the impulse withstand voltage $\hat{U}_{st}$ and
- the annual number $S(\lambda, T_d)$ of the atmospheric overvoltage waves occurring at the clearance as a function of the latitude and the local annual thunderstorm days:

$$N_a(\hat{U}_{st}, \lambda, T_d) = P(\hat{U}_{st})\, S(\lambda, T_d). \quad (16)$$

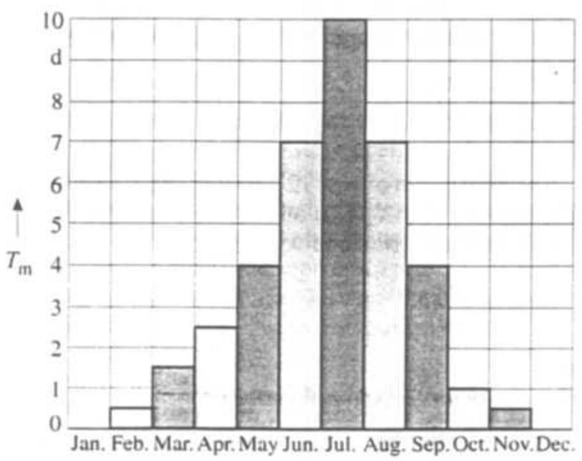

Fig. 2. Distribution of the monthly thunderstorm days T_m in Austria, estimated from "Distribution of thunderstorm days Part I – WMO 1953" [10]

6.1 Flashover Probability

The flashover probability of a clearance subjected to atmospheric overvoltage waves follows from eq. (6) and eq. (8)

$$P(\hat{U}_{st}) = \int_{\hat{U}_{st}}^{\infty} e^{-\alpha\hat{U}} d\hat{U} \Big/ \int_{0}^{\infty} e^{-\alpha\hat{U}} d\hat{U} = e^{-\alpha\hat{U}_{st}}. \quad (17)$$

6.2 Annual Number of Atmospheric Overvoltage Waves as a Function of Latitude and Local Annual Thunderstorm Days

If atmospheric overvoltages were only generated by cloud-to-ground lightning flashes, then the annual number of overvoltage waves arriving at the clearance, S^*, would result from the overvoltage-relevant ground-flash striking area A and the annual number of ground flashes per unit area N_g:

$$S^* = A\, N_g. \quad (18)$$

According to *Anderson* and *Erikson* [11], N_g depends on the annual number of thunderstorm days:

$$N_g = \beta\, T_d^{1.3}, \quad (19)$$

where

$\beta = 0.023\ \mathrm{km}^{-2}\,\mathrm{a}^{0.3}$.

Atmospheric overvoltage waves, however, are not only generated by cloud-to-ground, but also by cloud-to-cloud lightning flashes. The number of cloud-to-cloud flashes N_c depends on the annual number of cloud-to-ground flashes per unit area:

$$N_c = \gamma N_g. \quad (20)$$

Hence the total number of overvoltage wave generating lightning flashes N_{tot} amounts to

$$N_{tot} = N_c + N_g = (1+\gamma) N_g = \beta(1+\gamma)\, T_d^{1.3}, \quad (21)$$

and the total number of annual overvoltage waves S

$$S = A\, N_{tot} = A\, \beta\, (1+\gamma)\, T_d^{1.3}. \quad (22)$$

According to *Prentice* and *Mackerras* [8], γ depends on the latitude λ and the annual number of thunderstorm days T_d at the observation point:

$$\gamma = (\delta + \varepsilon \cos(3\lambda))\left(\xi + \frac{\eta T_d}{\vartheta - i\lambda}\right), \quad (23)$$

where the constants are

$\delta = 4.16, \quad \eta = 0.4$ a
$\varepsilon = 2.16, \quad \vartheta = 72$
$\xi = 0.6, \quad i = 0.98 \text{ deg}^{-1},$

and the region of validity

$T_d \leq 84 \text{ a}^{-1} \qquad \lambda \leq 60°.$

Hence the number of annual overvoltage waves amounts to

$$S(\lambda, T_d) = A\beta\left[1 + (\delta + \varepsilon\cos(3\lambda))\left(\xi + \frac{\eta T_d}{\vartheta - i\lambda}\right)\right]T_d^{1.3}. \tag{24}$$

6.2.1 Overvoltage-Relevant Ground-Lightning Flash-Striking Area: Calculation from Experimental Data

Setting $\hat{U}_{st}$ to zero in eq. (13) yields for the annual number of overvoltage waves at the observation point

$$S = n_0/(x\alpha). \tag{25}$$

From eqs. (24) and (25) it follows for the overvoltage-relevant ground-flash striking area:

$$A = \frac{n_0}{x\alpha\beta\left[1 + (\delta + \varepsilon\cos(3\lambda))\left(\xi + \frac{\eta T_d}{\vartheta - i\lambda}\right)\right]T_d^{1.3}}. \tag{26}$$

The data concerning the Austrian (AUT) and French (FR) overhead network are listed in **Tab. 2**.

From eq. (26) and the data in Tab. 2 it follows for the overvoltage-relevant ground-flash striking area in Austria $A_{AUT} = 27.65$ km^2 and in France $A_{FR} = 26.49$ km^2.

The dimensional co-incidence is remarkable but not obvious. The ground-flash striking area is a function of the extent of the network, because the annual number of overvoltage waves in an overhead network tends to zero as the extent of the network decreases if no overvoltages are transmitted from the protected medium voltage network to the properly earthed low-voltage side.

6.2.2 Overvoltage-Relevant Ground-Lightning Flash-Striking Area: Calculation from the Geometrical Data of the Overhead Network

A calculation of the overvoltage-relevant ground-lighting flash-striking area is possible if one assumes that

- the partition function of the crest values of atmospheric overvoltage waves along overhead lines and
- the partition function of the flash peak current amplitudes in the vicinity of the overhead line

are identical for any overhead network.

If by way of simplification the network is assumed to be linearly elongated with an effective length L and when a weighted mean distance $\bar{r}$ between the lightning stroke and the overhead line can be given, then the overvoltage relevant ground lightning flash striking area A is

$$A = 2\,\bar{r}\,L. \tag{27}$$

	AUT	FR
α	0.552 kV^{-1}	0.552 kV^{-1}
x	0.433	3
T_d	38 a^{-1}	30 a^{-1}
n_0	68 kV^{-1}	310 kV^{-1}
λ	46.5°	45.5°

Tab. 2. Network data

Rusck [12] gave a relationship between the crest value of overvoltage waves along overhead lines and the peak current amplitude $\hat{I}$ of the voltage generating lightning stroke

$$\hat{U} = \frac{\text{const}\, f h}{r}\hat{I} = \kappa\frac{h}{r}\hat{I}, \quad r \neq 0, \tag{28}$$

where h denotes the height above the ground of the overhead line, r the distance of the lightning stroke from the overhead line and f a factor between 1.07 Ω and 1.38 Ω, which depends on the propagation velocity of the flash leader.

Let the partition functions of the crest values of overvoltage waves and of the wave generating flash currents be

$$n_1 = n_1(\hat{U}), \qquad \hat{U}_1 \leq \hat{U} \leq \hat{U}_2, \tag{29}$$

$$n_2 = n_2(\hat{I}), \qquad \hat{I}_1 \leq \hat{I} \leq \hat{I}_2. \tag{30}$$

Multiplying of eq. (28) by $n_1(\hat{U})\, n_2(\hat{I})$ and omitting the accent for simplification's sake yields

$$U n_1(U)\, n_2(I) = \kappa(h/r)\, I\, n_1(U)\, n_2(I). \tag{31}$$

Integration between the limits U_1 and U_2 and I_1 and I_2 gives:

$$\left[\int_{U_1}^{U_2} U n_1(U)\mathrm{d}U\right]\int_{I_1}^{I_2} n_2(I)\mathrm{d}I = \kappa\frac{h}{\bar{r}}\left[\int_{I_1}^{I_2} I n_2(I)\mathrm{d}I\right]\int_{U_1}^{U_2} n_1(U)\mathrm{d}U \tag{32}$$

or

$$\bar{r} = \kappa h \frac{\left[\int_{I_1}^{I_2} I n_2(I)\mathrm{d}I\right] \Big/ \int_{I_1}^{I_2} n_2(I)\mathrm{d}I}{\left[\int_{U_1}^{U_2} U n_1(U)\mathrm{d}U\right] \Big/ \int_{U_1}^{U_2} n_1(U)\mathrm{d}U} = \kappa h \frac{\bar{I}}{\bar{U}}. \tag{33}$$

Popolansky [13] gave for $n_2(I)$ a log-normal partition function

$$n_2(I) = \frac{1}{\sqrt{2\pi}\,\sigma}\frac{1}{I}\mathrm{e}^{-\frac{1}{2}\left[\frac{\ln(I/I_0)}{\sigma}\right]^2}. \tag{34}$$

Hence it follows for $\bar{I}$ that

$$\bar{I} = \int_{I_1}^{I_2} \mathrm{e}^{-\frac{1}{2}\left[\frac{\ln(I/I_0)}{\sigma}\right]^2}\mathrm{d}I \Big/ \int_{I_1}^{I_2}\frac{1}{I}\mathrm{e}^{-\frac{1}{2}\left[\frac{\ln(I/I_0)}{\sigma}\right]^2}\mathrm{d}I. \tag{35}$$

Setting

$$I/I_0 = \mathrm{e}^{t\sigma}$$

results in:

$dI = I_0 \sigma e^{t\sigma} dt,$

$t = [\ln(I/I_0)]/\sigma.$

Hence it follows for the denominator in eq. (35) that

$$\int_{I_1}^{I_2} \frac{1}{I} e^{-\frac{1}{2}\left[\frac{\ln(I/I_0)}{\sigma}\right]^2} dI = \sigma \int_{t_1}^{t_2} e^{-\frac{1}{2}t^2} dt, \tag{36}$$

and for the nominator that

$$\int_{I_1}^{I_2} e^{-\frac{1}{2}\left[\frac{\ln(I/I_0)}{\sigma}\right]^2} dI = I_0 \sigma \int_{t_1}^{t_2} e^{t\sigma - \frac{1}{2}t^2} dt$$

$$= I_0 \sigma e^{\frac{\sigma^2}{2}} \int_{t_1}^{t_2} e^{-\frac{1}{2}(t-\sigma)^2} dt. \tag{37}$$

Substituting t in eq. (37) by

$t = x + \sigma$

yields definitively for the nominator in eq. (35):

$$\int_{I_1}^{I_2} e^{-\frac{1}{2}\left[\frac{\ln(I/I_0)}{\sigma}\right]^2} dI = I_0 \sigma e^{\frac{\sigma^2}{2}} \int_{x_1}^{x_2} e^{-\frac{1}{2}x^2} dx. \tag{38}$$

From eqs. (36) and (38) it follows:

$$\bar{I} = I_0 e^{\frac{\sigma^2}{2}} \int_{x_1}^{x_2} e^{-\frac{1}{2}x^2} dx \Bigg/ \int_{t_1}^{t_2} e^{\frac{1}{2}t^2} dt. \tag{39}$$

Substituting the tabulated error function $\emptyset(x')$ [14]

$$\emptyset(x') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x'} e^{-\frac{1}{2}x^2} dx$$

into eq. (39) yields:

$$\bar{I} = I_0 e^{\frac{\sigma^2}{2}} \frac{\emptyset\left[\frac{\ln(I_2/I_0)}{\sigma} - \sigma\right] - \emptyset\left[\frac{\ln(I_1/I_0)}{\sigma} - \sigma\right]}{\emptyset\left[\frac{\ln(I_2/I_0)}{\sigma}\right] - \emptyset\left[\frac{\ln(I_1/I_0)}{\sigma}\right]}. \tag{40}$$

For $I_1 = 0$ and $I_2 = \infty$ the fraction in eq. (40) is equal to one and the weighted mean peak current $\bar{I}$ finally becomes:

$$\bar{I} = I_0 e^{\sigma^2/2}. \tag{41}$$

The partition function of the overvoltage peak values is:

$$n_1(\hat{U}) = \alpha e^{-\alpha \hat{U}}. \tag{2}$$

Hence it follows for the weighted mean crest value $\bar{U}$ of atmospheric overvoltage waves:

$$\bar{U} = \int_{U_1}^{U_2} U e^{-\alpha U} dU \Bigg/ \int_{U_1}^{U_2} e^{-\alpha U} dU. \tag{42}$$

Partial integration of the nominator yields:

$$\bar{U} = \frac{e^{-\alpha U_1}(U_1 + 1/\alpha) - e^{-\alpha U_2}(U_2 + 1/\alpha)}{e^{-\alpha U_1} - e^{-\alpha U_2}}. \tag{43}$$

With $U_1 = 0$ and $U_2 = \infty$ it follows from eq. (43):

$$\bar{U} = 1/\alpha. \tag{44}$$

From eq. (33) together with eqs. (41) and (44) it follows that the weighted mean distance $\bar{r}$ is:

$$\bar{r} = \alpha \kappa h I_0 e^{\sigma^2/2}. \tag{45}$$

From this and eq. (27) it follows that the overvoltage-relevant ground-lightning flash-striking area A under investigation is:

$$A = 2 L h \alpha \kappa I_0 e^{\sigma^2/2}. \tag{46}$$

According to *Popolansky* and *Erikson* [15] the log-normal partition function of the flash peak current amplitudes is characterised by the parameters $\sigma_{\ln I} = 0.74$ and $I_0 = 30$ kA.

Rusck gave for κ the value

$\kappa = 30 f.$

Substituting the most unfavourable f-value yields:

$\kappa_{max} = 41.4\ \Omega.$

The decay constant in the exponent of the partition function amounts according to Tab. 1 to $\alpha = 0.552$ kV^{-1}. The conductors of medium- and low-voltage overhead networks are only at such a height above ground that they can definitely not be touched. The height h varies with the height of the supplied buildings. Assuming $h \approx 10$ m to be a safe conductor height above ground the weighted mean distance $\bar{r}$ between overhead line and lightning flash strike turns out to be $\bar{r} = 8.98$ km.

The network geometry is only known for the Austrian overhead network. **Fig. 3** shows the structure of this network. The approximate "linear" extension L is 1.45 km. Hence it follows from eq. (46) for the overvoltage-relevant ground-lightning flash striking area $A_{AUT} = 26.1$ km^2.

Considering the uncertainty of the assumptions and of the measuring data, this value is in satisfactory agreement with the area of 27.7 km^2 as calculated from

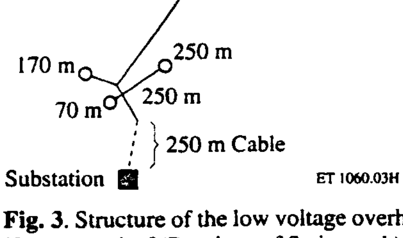

Fig. 3. Structure of the low voltage overhead network at Hartmannsdorf (Province of Steiermark) in Austria according to *Harich* and *Enders* [6]

eq. (27). The agreement shows, that eq. (46) describes satisfactorily the dependence of the overvoltage-relevant ground-lightning flash-striking area on the geometrical structure of an unprotected overhead network. This, in particular, is valid for the dependence on the linear extension.

6.2.3 *Annual Number of Flashovers of a Clearance: Calculation from Geometric Network Data*

The annual number of flashovers of a clearance, N_a, is a function of

- the "linear" extension and
- the height of the network conductors above ground as well as of
- the impulse withstand voltage of the clearance,
- the annual number of thunderstorm days and
- the latitude of the location of the network.

It follows from eqs. (16), (17), (24) and (46):

$$N_a = N_a(L, h, \hat{U}_{st}, T_d, \lambda) = 2\,L\,h\,\alpha\,\beta\,\kappa\,I_0\left[1 + (\delta + \varepsilon\cos(3\lambda))\left(\xi + \frac{\eta T_d}{\vartheta - i\lambda}\right)\right] \cdot T_d^{1.3}\,e^{(\sigma^2/2) - \alpha\hat{U}_{st}}. \quad (47)$$

If the overvoltages are limited to $\hat{U}_{max}$, then the term $e^{-\alpha\hat{U}_{st}}$ has to be replaced by the difference

$$\Delta = e^{-\alpha\hat{U}_{st}} - e^{-\alpha\hat{U}_{max}}. \quad (48)$$

The maximum length of a low-voltage overhead network cannot attain any value. It is technically restricted and limited by the admissible voltage drop and the connected consumer load. In Germany low-voltage overhead networks are predominantly found in rural areas. As far as possible, the line is fed in the middle of its total length. In this case the length of the network corresponds approximately to the maximum linear extension of the area served, if the admissible voltage drop does not set a limit.

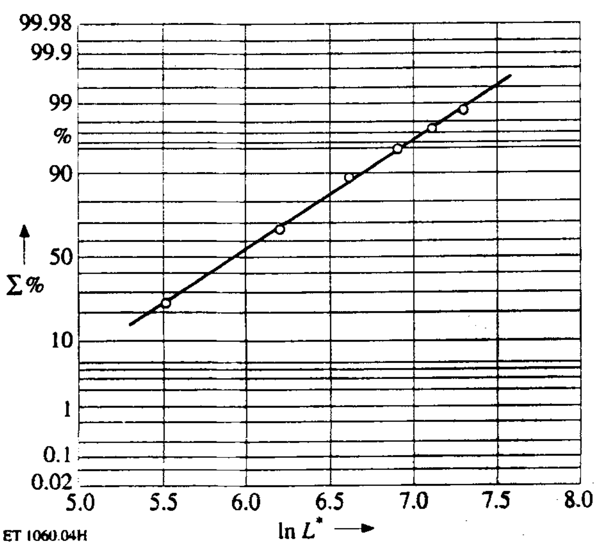

Fig. 4. Accumulated frequency distribution of the maximum linear extension of 277 rural areas in Southern Germany between 11° 5'... 11°30'30" E and 49°41'... 49°53'30" N (L maximum linear extension; L^* normalized maximum linear extension; L_0 = 372.4 m; $\sigma_{\ln L^*}$ = 0.61; $L_{90\%}$ = 812 m)

Fig. 4 shows the cumulative frequency distribution of the maximum "linear" extension of 277 areas of a rural region in Southern Germany with an area of 710 km^2. The distribution is a log-normal distribution with the parameters L_0 = 372 m and $\sigma_{\ln L^*}$ = 0.61. Fig. 4 shows, that 90 % of all lengths are less than or equal to 812 m.

An example for the annual number of flashovers of a clearance in a low-voltage overhead network of 812 m linear extension and 12 m height above ground is given in **Fig. 5** as a function of the annual thunderstorm days. The 50.12° latitude is that of Frankfurt a. M. in Germany.

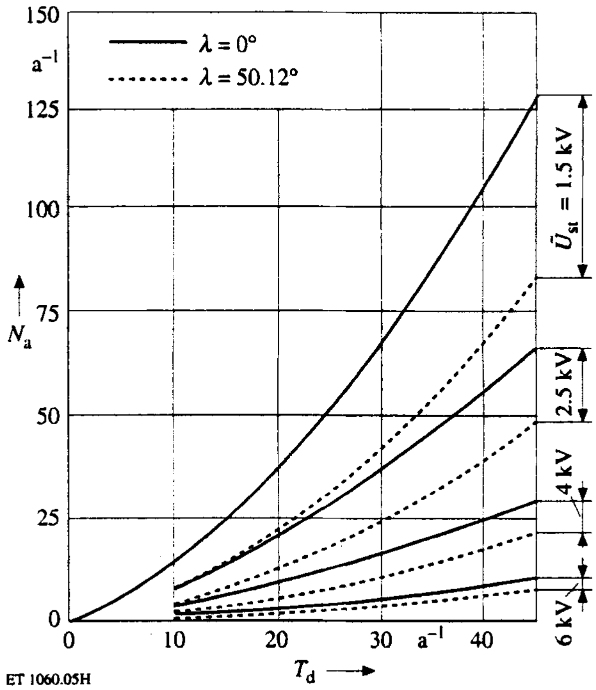

Fig. 5. Annual flashovers N_a of a clearance in an unprotected low-voltage overhead network of 812 m length and 10 m height above ground as a function of the annual thunderstorm days T_d (Parameters : impulse withstand voltage $\hat{U}_{st}$ and latitude λ)

7 Flashovers with Power-Frequency Follow Current

The curves in Fig. 5 represent the annual number of flashovers of a clearance due to atmospheric overvoltage waves. Flashovers of this kind must not endanger the clearance of an electrical device on any account. A flashover, however, is dangerous if it triggers a self-sustaining power-frequency follow current between the electrodes connected to the line voltage.

A discharge becomes self-sustaining, when the current density j in the discharge channel exceeds a critical value j_{cr}. Hence the condition for a self-sustaining discharge is $j > j_{cr} = (\Sigma E)_{cr}$, where Σ denotes the electrical conductivity and E the electrical field strength in the discharge channel. The conductivity is a function of the degree of ionisation which in turn depends on the voltage- wave energy, dissipated in the discharge plasma. According to *Harich* and *Enders* [6], the longest experimentally observed duration of an atmospheric overvoltage wave travelling along an overhead line is $1.3 \cdot 10^{-4}$ s. This means that the wave energy is nearly adiabatically converted within the discharge channel. *Harich* and *Enders* have measured 4 Ws as the maximum energy content of atmospheric overvoltage waves in low-voltage overhead networks.

The energy W of a voltage wave travelling along a line with the surge resistance Z is given by:

$$W = (1/Z)\int_0^\infty U(t)^2 \,dt. \tag{49}$$

This means, that the wave energy depends on the shape and duration of the wave and may even for a given maximum voltage amplitude attain arbitrary values between zero and a maximum of 4 Ws.

The electrical power-frequency field strength in the discharge channel oscillates periodically between zero and a maximum value. This means, that not only the initial conductivity in the discharge channel, caused by the flashover of the overvoltage wave, but also the electrical power-frequency field strength may vary between zero and a maximum value. Hence not every overvoltage wave, which leads to a flashover, can trigger a dangerous power-frequency follow current. This has been proved by *Biegelmeier* [9] by systematic variation of the phase angle between a 1.2/50 µs testing voltage wave and the power-frequency voltage. For these reasons, the curves in Fig. 5 represent limiting values.

8 Final Remarks

The annual number of flashovers which triggered power-frequency follow currents observed in installation practice is much less than the annual number calculated from eq. (47). The annual number of flashovers which trigger power-frequency follow currents could, however, be estimated if

- the partition function of the energy of atmospheric overvoltage waves in overhead networks is known with the crest voltage as parameter, and if furthermore
- the flashover behaviour of clearances at mains voltage, which are triggered by overvoltage waves is known with the wave energy, the crest voltage, the phase angle and the impulse withstand voltage as parameters.

This could be the program of future experimental work.

Accordingly, eq. (47) is valid for creeping distances and solid insulations of electrical equipment in domestic installations, which are supplied by overhead lines.

In the case of creepage distances eq. (47) gives the maximum number of annual flashovers, which, as in the case of clearances, could, but need not, initiate follow-up currents.

In the case of solid insulations, eq. (47) gives the annual number of subjections to overvoltage waves with a crest value greater then or equal to $\hat{U}_{st}$.

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