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THE RURAL–URBAN POPULATION BALANCE AGAIN

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This paper explores further the economics of the rural–urban population balance, using the method of comparative statics. It confirms the results obtained by Artle and his colleagues, using the method of excess demand analysis, thus demonstrating that the two methods of analysis are equivalent in this context. In analysing the effects of non-neutral technological change upon urbanization, it is essential to distinguish between the total elasticity of demand for manufactured products, and the income and (compensated) price elasticities, respectively. The empirical plausibility is questioned of assuming that total demand for the aggregate of manufactured goods is elastic.

1. Introduction

The economic determinants of the rural–urban population balance are of considerable interest both for the economic history of the industrialized part of today's world, and for the urbanization processes that are now going on in the developing nations. Some 35 years ago (1947), I published a paper on these topics, and quite recently (1977, 1979), Artle, Humes and Varaiya have returned to the same questions, proposing a new method of analysis and extending my earlier results.

It is my purpose here to examine further the relation between my original analysis and that set forth in the first paper by Artle et al., both with respect to the methods of analysis employed in these two papers, and with respect to one of their substantive findings. In particular:

(1) Artle et al. (1977, p. 186) state that in the case where there is an improvement in productivity in only one sector (say, manufacturing) and not in the other (say, agriculture), 'no definite conclusion about the change in the division of labor can be made for this case, within the framework of Simon's paper, unless a different method of analysis is used.'

(2) These authors (1977, p. 187) also state that the method of analysis they use 'provides sufficient information to draw fairly general conclusions about the direction of the flow of labor from one sector to the other. In particular, it will be shown that Simon's proposition about the effect of

equal rates of productivity increase and Baumol's case of "unbalanced growth" follow as special cases of a more general result.'

In the second and third sections of the present paper, I shall undertake to show that the method of analysis I used in my 1947 paper (comparative statics) and the method of analysis used by Artle, Humes and Varaiya (1977) (excess demand analysis) are in fact equivalent and lead to precisely the same conclusions. In particular, I will derive their result in the case where there is a productivity improvement in only one sector using my method of comparative statics.

In the fourth section of the paper I will consider the economic meaning of the Theorem 4.2 of Artle et al. (1977, p. 191) which states that: 'In the case of unbalanced growth, labor migrates towards the progressive (non-progressive) sector if the demand for its output is elastic (inelastic) to its own price.'

The analysis of these last two sections will remind us of some classical relations, discussed many years ago by Henry Schultz, among the different price and income elasticities that can be defined for models of the sort considered here.

2. Comparative statics and neutral technological change

The two papers under discussion postulate identical models of a two-sector economy, with equilibrium determined by constrained maximization of consumer utilities and producer profits. My analysis (comparative statics) was carried out by introducing changes in the productivities of one or both sectors, and then comparing the new equilibrium with the original one, in order to ascertain the changes that would take place in prices, quantity of output of each sector, and employment in each sector. Artle et al. compute the shifts in the prices that will prevent a shift of labor between the two sectors (i.e., prices that will equilibrate the new marginal values of product with the labor costs), and use these prices to compute the resulting excess demands for goods.

In order to show that the two methods obtain the same results, it is necessary for me to summarize my model briefly. Both methods of analysis require about the same amount of algebra to reach their conclusions, so I will employ my original procedures, and retain most of my original notation.

Consider commodities A and M of which the quantities q_a and q_m are produced with quantities x_a and x_m, respectively, of labor:

$$q_a = \xi(x_a), \qquad q_m = \eta(x_m). \tag{1}$$

The total supply of labor is fixed:

$$x_a + x_m = X = \text{constant}. \tag{2}$$

The utility of consumers is given by an index, $\phi = \phi(q_a, q_m)$. The profits of producers of A and M, respectively, are

$$\pi_a = q_a p_a - x_a, \qquad \pi_m = q_m p_m - x_m. \tag{3}$$

where p_a and p_m are commodity prices, while the wage rate is taken as unity. Then total money product is

$$I = q_a p_a + q_m p_m. \tag{4}$$

The conditions for equilibrium with utility and profit maximization are

$$\phi_a p_m = \phi_m p_a \quad \text{where} \quad \phi_Z = (\partial\phi/\partial q_Z), \tag{5}$$

$$p_a \xi_x = 1, \qquad p_m \eta_x = 1 \quad \text{where} \quad \xi_x = (\partial\xi/\partial x_a), \quad \eta_x = (\partial\eta/\partial x_m). \tag{6}$$

Sufficient conditions for maximization are given in equations (7) and (10) of my 1947 paper. They are

$$\phi_{aa} - 2(p_a/p_m)\phi_{am} + (p_a^2/p_m^2)\phi_{mm} < 0, \quad \text{and} \tag{7}$$

$$p_a(\partial\xi_x/\partial x_a) < 0, \qquad p_m(\partial\eta_x/\partial x_m) < 0, \tag{8}$$

where the ϕ_{xy} are second partial derivatives: $\partial^2\phi/\partial x\,\partial y$.

We now suppose that a technological advance occurs in both sectors. We designate by $\bar{q}_a, \bar{q}_m$, etc., the new values of the variables, and we suppose the new production function to be of the form:

$$\bar{a}_a = (1+\rho)\xi(\bar{x}_a), \qquad \bar{q}_m = (1+\sigma)\eta(\bar{x}_m), \quad \rho, \sigma > 0, \tag{9}$$

The other equations are unchanged in form.

Next, we calculate the changes in the equilibrium values of the variables, defining

$$\delta q_a = \bar{q}_a - q_a, \qquad \delta q_m = \bar{q}_m - q_m \quad \text{etc.} \tag{10}$$

Now by straightforward, if tedious, algebra we can express δx_a as a function of ρ and σ [equivalent to equation (22) of Simon (1947)]:

$$\delta x_a = [p_a p_m/(\gamma + p_a p_m \varepsilon)][-\rho(q_a\beta + \phi_m p_a) + \sigma(q_m\alpha + \phi_a p_m)], \tag{11}$$

where

$$\alpha=(p_a\phi_{mm}-p_m\phi_{am})<0,\ \beta=(p_m\phi_{aa}-p_a\phi_{ma})<0,\quad \text{and}$$

$$\gamma=(p_m\beta+p_a\alpha)<0;\ \ \varepsilon=(\phi_m p_a^2\xi_{xx}+\phi_a p_m^2\eta_{xx})<0,$$

where the inequalities follow from the conditions for stability of equilibrium.

The denominator of (11) is negative, but in the numerator there are both positive terms ($-\rho q_a\beta, \sigma\phi_a p_m$) and negative terms ($-\rho\phi_m p_a, \sigma q_m\alpha$), so that the sign of δq_a is indeterminate.

In Simon (1947, eqs. (24)–(30)), it was shown that the ratio of income elasticities of demand for M and A, hence the ratio of changes in quantities demanded when relative prices are fixed is: $\beta q_a/\alpha q_m$. If the income elasticity of demand for manufactured goods is more than the income elasticity of demand for agricultural goods, then this ratio will be greater than unity.

Now consider the numerator of (11) in the special case where $\rho=\sigma$ (neutral technological change). Up to a positive multiplier, it becomes

$$-q_a\beta+q_m\alpha-\phi_m p_a+\phi_a p_m. \tag{12}$$

The third and fourth terms cancel, by (5), while the first dominates the second, under the assumption just made that their ratio exceeds unity. Hence, under the assumptions of neutral technological change and greater income elasticity of demand for manufactured goods than for agricultural goods, δx_a will be negative — agricultural employment will decline with rising productivity. This is the result derived in equation (34) of Simon (1947).

3. Non-neutral technological change

Artle et al. (1977) show that, under certain assumptions about price elasticities, agricultural employment will decline even if there is technological advance in manufacturing alone. They further claim that the method of excess demand analysis is essential for the derivation of this result. I shall derive the same result by the method of comparative statics, and then will examine the meanings of the price elasticities that are postulated.

From (4), and remembering that I is fixed, since the wage rate is the numeraire, we have

$$0=p_a\delta q_a+q_a\delta p_a+p_m\delta q_m+q_m\delta p_m \tag{13}$$

and from (5),

$$\phi_a\delta p_m+\phi_{aa}p_m\delta q_a+\phi_{am}p_m\delta q_m=\phi_{ma}p_a\delta q_a+\phi_{mm}p_a\delta q_m+\phi_m\delta p_a. \tag{14}$$

Solving (13) for δq_m, substituting this value in (14), and simplifying — remembering our previous definitions for α, β, and γ, we obtain

$$\gamma\,\delta q_a=(p_m\phi_m-\alpha q_a)\delta p_a-(p_m\phi_a+\alpha q_m)\delta p_m. \tag{15}$$

Taking p_a and p_m as our independent variables, we can read off the price elasticities of demand for agricultural goods:

$$(p_a/q_a)\partial q_a/\partial p_a=[p_a(p_m\phi_m-\alpha q_a)]/[q_a\gamma], \quad \text{and} \tag{16}$$

$$(p_m/q_a)\partial q_a/\partial p_m=-[p_m(p_m\phi_a+\alpha q_m)]/[q_a\gamma]. \tag{17}$$

By symmetry, we have also

$$(p_m/q_m)\partial q_m/\partial p_m=[p_m(p_a\phi_a-\beta q_m)]/[q_m\gamma]. \tag{18}$$

Now suppose the absolute value of the price elasticity of demand for M is greater than unity, while the corresponding elasticity for A is less than unity:

$$(p_a/q_a)(\partial q_a/\partial p_a)>-1, \qquad (p_m/q_m)(\partial q_m/\partial p_m)<-1, \tag{19}$$

The first of these relations is, by (16), equivalent to

$$p_ap_m\phi_m-p_a\alpha q_a<-q_ap_m\beta-q_ap_a\alpha, \quad \text{or} \tag{20}$$

$$-p_a\phi_m-q_a\beta>0. \tag{21}$$

In the same way, from (18) and (19), we get

$$p_m\phi_a+q_m\alpha>0. \tag{22}$$

If we now return to (11), we see that (21) and (22) guarantee that the numerator will be positive, hence that ∂x_a will be negative. Moreover, if $\rho=0$, we require only the second inequality in (19), while if $\sigma=0$, we require only the first inequality. But these results encompass Theorems 4.2 and 4.3 of Artle et al., showing that the theorems can be proved by the method of comparative statics — and by essentially the same path as with the method of excess demand analysis.

4. Interpretation of the price elasticities

How are the inequalities in (19), and especially the second one, to be interpreted? Using (4) and (5), first let us calculate the effect that a change in real income, I, would have upon the consumption of A, holding prices constant:

$$\delta I = p_a \delta q_a + p_m \delta q_m, \tag{23}$$

$$(\phi_{aa}p_m - \phi_{ma}p_a)\delta q_a = (\phi_{mm}p_a - \phi_{am}p_m)\delta q_m. \tag{24}$$

Substituting for δq_m in (24) and simplifying, we get

$$\beta\delta q_a = (\alpha/p_m)(\delta I - p_a q_a), \quad \text{or} \tag{25}$$

$$\partial q_a/\partial I = \alpha/\gamma \quad \text{and, since} \quad \partial I/\partial p_a = q_a, \tag{26}$$

$$\partial q_a/\partial p_a = \alpha q_a/\gamma. \tag{27}$$

But the right-hand side of (27) is essentially the second term in the numerator of (16). Hence this term measures the *income effect* of a change in price of A.

Again, using (4) and (5) let us calculate the effect of price changes in both A and M, of such magnitude as to hold I constant. In this case, from (13) and (14), we get

$$p_a \delta q_a = -p_m \delta q_m, \tag{28}$$

$$\beta\,\delta q_a + \phi_a\,\delta p_m = -\alpha(p_a/p_m)\delta q_a + \phi_m\,\delta p_a \quad \text{from which} \tag{29}$$

$$\partial q_a/\partial p_a = p_m\phi_m/\gamma. \tag{30}$$

But the right-hand side of (30) is the first term in the numerator of (16). Hence this term measures the effect of a *compensated change* in price of A (i.e., a change in the relative prices of A and M that does not alter real income).

Thus, by (27) and (30), we have decomposed the total elasticity of demand of A with respect to its price [eq. (16)] into the sum of a compensated relative price elasticity, (30), and an income elasticity, (27), respectively.

Now the assumption that the total elasticity of demand of one of the commodities, say M, be greater than unity in absolute value is seen to imply that the first term in (22), which is positive, dominates the second term, which is negative. We see that these terms are identical with those that appear in the numerator of (17). That is to say, they are the components of the cross-elasticity of demand for A with respect to a change in price of M. The first term, $p_m\phi_a$, represents the decrease in demand for A produced by a (compensated) decrease in price of M; while the second term, $q_m\alpha$, represents the increase in demand for A produced by an increase in income (the income effect of the reduced price of M). Thus, that the total demand elasticity for M be positive is equivalent to the cross-elasticity of demand being negative — a decrease in the cost of producing M causing a decrease in the consumption of A.

The question of whether these assumptions are likely to be satisfied in a real economy is, of course, an empirical one, which cannot be decided by theory or debate. If A and M were close substitutes, it would be a plausible assumption. However, in the case of the aggregates of manufactured and agricultural goods, respectively, it seems an excessively strong assumption to postulate that a reduction in price of manufactured goods will cause such a great substitution of manufactured for agricultural products that — in spite of the contrary income effect — the consumption of agricultural goods would actually decrease. This would seem especially unlikely to be the case in underdeveloped countries where foodstuffs, available at not much above the subsistence level, are the principal product of agriculture.

If we do not make this assumption, however, how do we account for the apparent fact that urbanization goes forward in some economies where agricultural efficiency appears not to change while manufacturing technology is advancing? Several alternative conditions can produce this scenario. One of these is disguised unemployment in agriculture, where the marginal productivity of labor in agriculture is essentially zero. Another, and probably more significant one, is that most of the economies we observe are not closed, but engage in international trade on a significant scale. Under these circumstances, a reduction in manufacturing costs could lead to increased export of manufactures in exchange for increased food imports. This kind of shift appears to have occurred at various points in history in the cases of Japan, India, and China, for example, and earlier, in the classical case of England.

5. Conclusion

In this paper I have explored further the economics of the rural–urban population balance. Using the method of comparative statics, I have confirmed the results obtained by Artle and his colleagues (1977) with excess demand analysis, thus demonstrating that the two methods of analysis are equivalent.

The formulas obtained by the method of comparative statics permit an interpretation of total elasticity of demand for a product as compounded of an income effect and a (compensated) price effect, respectively. This interpretation, in turn, allows some assessment of the plausibility of assuming, under various conditions, that the total demand for a commodity aggregate like manufactured goods will be elastic. It can be seen, in fact, that this is a very strong, and perhaps even unrealistic, assumption, for it requires that the substitution between the two aggregates be so great as to overcome the income effects on their consumption.

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