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WKB-LIKE WAVE-FUNCTIONAL FOR A STRING STATE AND PATH-DEPENDENT PHASE FACTOR

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The variational calculation of the path-ordered phase factor is streamlined. We propose that the wave functional for a string state is approximately of the form

$$\Psi[x(\sigma)]=\prod_{\sigma'} K(x(\sigma'))\mathrm{P}\exp\left(ie\int A\,\mathrm{d}x\right).$$

The real factor K is shown to be proportional to the square root of the dielectric permeability in the Kogut–Susskind–'t Hooft model of colour flux collimation.

1. Introduction

Recently the variation of the path-dependent phase factor with respect to the path has been studied in various contexts [1–3], notably in the context of the string picture of hadrons. In spite of their tantalizing resemblance to the constraint equations in the dual string model, variational equations for the phase factor so far deduced have not contained very much physical substance. The underlying dynamics of colour flux squeezing has not been fully reflected in the phase-factor approach.

One of the purposes of this paper is to streamline the computation of the variation mentioned above. Kinematical calculations should be made easier before undertaking dynamical studies. We demonstrate that the previously obtained path-integral formula [4] for the phase factor is quite useful in carrying out the variation systematically. In particular, we show the interchangeability of the order of differentiation, pointing out an error in the literature [3].

We then propose a WKB approximate wave functional for the string state in the form

$$\Psi[x(\sigma)]=\prod_{\sigma'} K(x(\sigma'))\mathrm{P}\exp\left(ie\int A\,\mathrm{d}x\right). \tag{1.1}$$

The real factor in front of the path-ordered phase factor is a functional of $x[\sigma]$ peaked around a classical string configuration. Our approach is the following. With the lack of a fully fledged colour confinement theory, we adopt the Kogut–Susskind [5]–'t Hooft [6] model of colour squeezing as a phenomenological field theory. Namely, we are going to work in a system described by the lagrangian

$$L=-\tfrac{1}{4}\epsilon(\phi)F_{\mu\nu}^2+\tfrac{1}{2}(\partial_\mu\phi)^2-V(\phi)\,, \tag{1.2}$$

with a constraint that the colour flux from a quark to an antiquark has a given value e. We show that

$$K=\text{const}\,\sqrt{\epsilon} \tag{1.3}$$

in the WKB-like approximation. In the thin-tube limit, Ψ in (1.1) is shown to satisfy the same constraint equation appearing in the dual string model.

In sect. 2 we develop an algorithm for the phase factor on the basis of the result in ref. [4]. The WKB-like approximation to the string state is proposed in sect. 3 after a brief review of the Kogut–Susskind–'t Hooft model. Sect. 4 is devoted to discussions.

2. An algorithm for the phase factor

2.1. Preliminaries

As a starting point we restate the path-integral formula for the path-dependent phase factor which was established in ref. [4]. It reads

$$\begin{aligned}&\left(\mathrm{P}\exp\left(ie\int \tfrac{1}{2}\lambda^a A_\mu^a x'^\mu\,\mathrm{d}\sigma\right)\right)_{\alpha\beta}\\ &\qquad=\int[\mathrm{d}\xi\,\mathrm{d}\xi^\dagger]\tilde{\phi}_\alpha\phi_\beta\exp\left(i\int\mathscr{L}(\xi,\xi^\dagger)\,\mathrm{d}\sigma\right),\\ &\mathscr{L}=i\xi^\dagger\frac{\partial\xi}{\partial\sigma}+e\xi^\dagger\tfrac{1}{2}\lambda^a\xi A_\mu^a x'^\mu\,,\end{aligned} \tag{2.1}$$

where ξ is the three dimensional Grassmann spinor with λ^a being the Gell-Mann matrix and $x^\mu(\sigma)$ represents the path under consideration*.

The following notation is adopted for later convenience. We write

$$W_{\alpha\beta}\equiv\int[\mathrm{d}\xi\,\mathrm{d}\xi^\dagger]\tilde{\phi}_\alpha\phi_\beta\,\mathrm{e}^{i\int L\,\mathrm{d}\sigma}\,, \tag{2.2}$$

$$\langle A(\sigma)\rangle\equiv\int[\mathrm{d}\xi\,\mathrm{d}\xi^\dagger]\tilde{\phi}_\alpha\phi_\beta A(\xi(\sigma),\xi^\dagger(\sigma))\,\mathrm{e}^{\mathrm{i}\int L\,\mathrm{d}\sigma}, \tag{2.3}$$

$$I^a\equiv\xi^\dagger\tfrac{1}{2}\lambda^a\xi\,.$$

* The notation here is slightly different from that of ref. [4].

It is easy to see that

$$\left\langle i\frac{\partial\xi}{\partial\sigma}+e\tfrac{1}{2}\lambda^a\xi A^a_\mu x'^\mu\right\rangle=0\,, \tag{2.4}$$

$$\langle I'^a\rangle=-ef^{abc}A^b_\mu x'^\mu\langle I^c\rangle\,, \tag{2.5}$$

$$\langle I'^a(\sigma_1)I'^b(\sigma_2)\rangle=if^{abc}\langle I^c(\sigma_1)\rangle\delta(\sigma_1-\sigma_2)+ef^{amn}A^m_\mu x'^\mu\langle I^n(\sigma_1)I^b(\sigma_2)\rangle\,, \tag{2.6}$$

etc., by repeated functional partial integrations. We will use the current algebraic formulae (2.5) and (2.6) later. Going back to the path-ordered form is quite easy, e.g.,

$$\langle I^a(\sigma)\rangle=\mathrm{P}\exp\left(ie\int_\sigma^{\sigma_\mathrm{f}}Ax'\,\mathrm{d}\sigma\right)\tfrac{1}{2}\lambda^a\,\mathrm{P}\exp\left(ie\int_{\sigma_\mathrm{i}}^{\sigma}Ax'\,\mathrm{d}\sigma\right),$$

$$\langle I^a(\sigma_1)I^b(\sigma_2)\rangle=\mathrm{P}\exp\left(ie\int_{\sigma_1}^{\sigma_\mathrm{f}}Ax'\,\mathrm{d}\sigma\right)\tfrac{1}{2}\lambda^a\,\mathrm{P}\exp\left(ie\int_{\sigma_2}^{\sigma_1}Ax'\,\mathrm{d}\sigma\right)$$
$$\times\tfrac{1}{2}\lambda^b\,\mathrm{P}\exp\left(ie\int_{\sigma_\mathrm{i}}^{\sigma_2}Ax'\,\mathrm{d}\sigma\right),\qquad(\sigma_1>\sigma_2)\,.$$

2.2. *The first variational derivative*

Looking at formula (2.2), we take the variational derivative of W,

$$-i\frac{\delta}{\delta x^\mu(\sigma)}W=\left\langle\frac{\delta}{\delta x^\mu(\sigma)}\left(e\int\mathrm{d}\sigma'\,I^a(\sigma')A^a_\nu(x(\sigma'))x'^\nu(\sigma')\right)\right\rangle$$
$$=e\int\mathrm{d}\sigma'\,\langle I^a(\sigma')(\partial_\mu A^a_\nu x'^\nu\delta(\sigma-\sigma')+A^a_\nu\delta^\nu_\mu\partial_{\sigma'}\delta(\sigma-\sigma'))\rangle\,. \tag{2.7}$$

Partial integration gives

$$e\langle I^a(\sigma)(\partial_\mu A^a_\nu-\partial_\nu A^a_\mu)x'^\nu\rangle-e\langle I'^a(\sigma)A^a_\mu(\sigma)\rangle$$

for the right-hand side of eq. (2.7). Using eq. (2.5) for the last term, we obtain

$$-i\frac{\delta}{\delta x^\mu(\sigma)}W=e\langle I^a(\sigma)F^a_{\mu\nu}(x(\sigma))\rangle x'^\nu\,. \tag{2.8}$$

2.3. *Higher derivatives*

It is now a matter of exercises to calculate the functional derivatives of W for higher orders. We simply write down the second and the third derivatives which may be enough for an illustration how things are going on:

$$-i\frac{\delta^2 W}{\delta x^\mu(\sigma_1)\delta x^\nu(\sigma_2)}=e\langle D_\nu F_{\mu\rho}\rangle x'^\rho\delta(\sigma_1-\sigma_2)$$
$$+e\langle F_{\mu\nu}(\sigma_1)\rangle\partial_{\sigma_1}\delta(\sigma_1-\sigma_2)+ie^2\langle F_{\mu\rho}(\sigma_1)F_{\nu\sigma}(\sigma_2)\rangle x'^\rho(\sigma_1)x'^\sigma(\sigma_2)\,, \tag{2.9}$$

$$-i\frac{\delta^3 W}{\delta x^\lambda(\sigma_1)\delta x^\mu(\sigma_2)\delta x^\nu(\sigma_3)} = e\langle D_\lambda D_\nu F_{\mu\rho}(\sigma_2)\rangle x'^\rho(\sigma_2)\delta(\sigma_2-\sigma_3)\delta(\sigma_1-\sigma_2)$$
$$+e\langle D_\lambda F_{\mu\nu}(\sigma_2)\rangle\partial_{\sigma_2}\delta(\sigma_2-\sigma_3)\delta(\sigma_1-\sigma_2)$$
$$+e\langle D_\nu F_{\mu\lambda}(\sigma_2)\rangle\partial_{\sigma_2}\delta(\sigma_2-\sigma_1)\delta(\sigma_2-\sigma_3)$$
$$+ie^2\{\langle D_\nu F_{\mu\rho}(\sigma_2)F_{\lambda\sigma}(\sigma_1)\rangle x'^\rho(\sigma_2)x'^\sigma(\sigma_1)\delta(\sigma_2-\sigma_3)$$
$$+\langle D_\lambda F_{\mu\sigma}(\sigma_2)F_{\nu\rho}(\sigma_3)\rangle x'^\rho(\sigma_2)x'^\sigma(\sigma_3)\delta(\sigma_1-\sigma_2)$$
$$+\langle D_\lambda F_{\nu\sigma}(\sigma_3)F_{\mu\rho}(\sigma_2)\rangle x'^\rho(\sigma_2)x'^\sigma(\sigma_3)\delta(\sigma_1-\sigma_3)\}$$
$$+ie^2\{\langle F_{\mu\nu}(\sigma_2)F_{\lambda\sigma}(\sigma_1)\rangle x'^\sigma(\sigma_1)\partial_{\sigma_2}\delta(\sigma_2-\sigma_3)$$
$$+\langle F_{\mu\lambda}(\sigma_2)F_{\nu\sigma}(\sigma_3)\rangle x'^\sigma(\sigma_3)\partial_{\sigma_2}\delta(\sigma_2-\sigma_1)$$
$$+\langle F_{\mu\sigma}(\sigma_2)F_{\nu\lambda}(\sigma_3)\rangle x'^\sigma(\sigma_2)\partial_{\sigma_3}\delta(\sigma_3-\sigma_1)\}$$
$$+ie^4\langle F_{\lambda\kappa}(\sigma_1)F_{\mu\rho}(\sigma_2)F_{\nu\sigma}(\sigma_3)\rangle x'^\kappa(\sigma_1)x'^\rho(\sigma_2)x'^\sigma(\sigma_3)\,. \tag{2.10}$$

Here an abbreviated notation

$$\langle I^a(\sigma)F^a_{\mu\nu}(x(\sigma))\rangle \equiv \langle F_{\mu\nu}(\sigma)\rangle$$

is used.

We are now going to check the interchangeability of the differentiations in eq. (2.9) which has been alluded to in the introduction. Let us examine the difference

$$-i\left(\frac{\delta^2}{\delta x^\mu(\sigma_1)\delta x^\nu(\sigma_2)}-\frac{\delta^2}{\delta x^\nu(\sigma_2)\delta x^\mu(\sigma_1)}\right)W = e\langle D_\nu F_{\mu\rho}-D_\mu F_{\nu\rho}\rangle x'^\rho\delta(\sigma_1-\sigma_2)$$
$$+e\{\langle F_{\mu\nu}(\sigma_1)\rangle\partial_{\sigma_1}\delta(\sigma_1-\sigma_2)-\langle F_{\nu\mu}(\sigma_1)\rangle\partial_{\sigma_2}\delta(\sigma_1-\sigma_2)\}\,. \tag{2.11}$$

The first term can be rewritten as

$$\langle D_\rho F_{\mu\nu}(\sigma_1)\rangle x'^\rho\delta(\sigma_1-\sigma_2) = e\,\partial_{\sigma_1}\langle F_{\mu\nu}(\sigma_1)\rangle\delta(\sigma_1-\sigma_2)$$

with the help of the Bianchi identity and eq. (2.5). Careful manipulations of the second term in the right-hand side of eq. (2.11) give

$$e\{\langle F_{\mu\nu}(\sigma_1)\rangle\partial_{\sigma_1}\delta(\sigma_1-\sigma_2)-\langle F_{\mu\nu}(\sigma_2)\rangle\partial_{\sigma_1}\delta(\sigma_1-\sigma_2)\}$$
$$= e\,\partial_{\sigma_1}\{\langle F_{\mu\nu}(\sigma_1)\rangle\delta(\sigma_1-\sigma_2)-\langle F_{\mu\nu}(\sigma_2)\rangle\delta(\sigma_1-\sigma_2)\}$$
$$-e\,\partial_{\sigma_1}\langle F_{\mu\nu}(\sigma_1)\rangle\delta(\sigma_1-\sigma_2)$$
$$= -e\,\partial_{\sigma_1}\langle F_{\mu\nu}(\sigma_1)\rangle\delta(\sigma_1-\sigma_2)\,.$$

Hence the first and the second terms in the right-hand side of eq. (2.11) cancel each other. We have confirmed that

$$\left(\frac{\delta^2}{\delta x^\mu(\sigma_1)\delta x^\nu(\sigma_2)}-\frac{\delta^2}{\delta x^\nu(\sigma_2)\delta x^\mu(\sigma_1)}\right)W = 0\,. \tag{2.12}$$

Taking a contraction of μ and ν in eq. (2.9) and letting $\sigma_1 = \sigma_2 = \sigma$, we reproduce an "equation of motion":

$$-\frac{\delta^2 W}{\delta x^\mu(\sigma)\delta x_\mu(\sigma)} = -ie\langle D^\mu F_{\mu\rho}\rangle x'^\rho \delta(0) + e^2\langle F_{\mu\rho}(\sigma) F^\mu{}_\sigma(\sigma)\rangle x'^\rho(\sigma) x'^\sigma(\sigma)\,. \tag{2.13}$$

This formula will be used in sect. 3. Its integrability is guaranteed by eq. (2.12).

3. WKB-like approximation to the string state

As stated in sect. 1, eq. (2.13) in sect. 2, which has been derived by many people, has little physical content unless the colour collimation is taken into account in some way. In this section we take the Kogut–Susskind–'t Hooft model as the simplest field theoretical model of collimated coloured electric flux. The model is more or less phenomenological in the sense that it has not yet been "derived" from canonical QCD and does not enjoy renormalizability. Our investigation of the string state is also phenomenological, exactly to the same extent as the Kogut–Susskind–'t Hooft model.

3.1. The Kogut–Susskind–'t Hooft model

The model is described by the action:

$$S = -\tfrac{1}{4}\int d^4x\, \epsilon(\phi)(F^a_{\mu\nu})^2 + \int d^4x\, [\tfrac{1}{2}(\partial_\mu\phi)^2 - V(\phi)]\,, \tag{3.1}$$

where the dielectric permeability ϵ is assumed to be a function of the colour singlet scalar field ϕ. We do not specify the functions ϵ and V except that they have a common zero $\phi = \phi_0$ [7]. According to ref. [5], the variation of the action (3.1) should be taken with the constraint that a given coloured electric flux goes between the fixed sources (quark and antiquark. The authors of ref. [5] showed that an axially symmetric tube-like solution exists in the model (3.1). Here we presume the existence of a more general shape of the tubes than just a straight rod. In particular, in the string limit of a thin tube, we may have

$$\epsilon(x) = k\iint \sqrt{-\det g}\,\delta^4(x - X(\sigma\tau))\, d\sigma\, d\tau\,, \tag{3.2}$$

where $X^\mu(\sigma\tau)$ is the space-time position of the string with an arbitrary parametrization (σ, τ). The induced metric $g_{\alpha\beta}(\alpha, \beta = \tau, \sigma)$ is defined by

$$(g) = \begin{pmatrix} \dfrac{\partial X^\mu}{\partial\tau}\dfrac{\partial X_\mu}{\partial\tau} & \dfrac{\partial X^\mu}{\partial\sigma}\dfrac{\partial X_\mu}{\partial\tau} \\[2ex] \dfrac{\partial X^\mu}{\partial\tau}\dfrac{\partial X_\mu}{\partial\sigma} & \dfrac{\partial X^\mu}{\partial\sigma}\dfrac{\partial X_\mu}{\partial\sigma} \end{pmatrix}. \tag{3.3}$$

k is a dimensionful constant, which can in principle be determined from the original action of our model. With a permeability like (3.2) which is localized on the string position, the coloured electric flux running along the string is given by

$$k\dot{X}^{\mu}X'^{\nu}F^{a}_{\mu\nu}.$$

Then the constraint stated below eq. (3.1) can be restated as

$$k\dot{X}^{\mu}X'^{\nu}F^{a}_{\mu\nu}=-eI^{a}\sqrt{-\det g}\,, \tag{3.4}$$

where I^a is the colour spin of the quark (antiquark) under consideration. Following Kogut and Susskind, we incorporate the constraint (3.4) into the action in the form of Lagrange multiplier:

$$S^{\text{eff}}=\iint \mathrm{d}\sigma\,\mathrm{d}\tau\,\lambda^{a}(k\dot{X}^{\mu}X'^{\nu}F^{a}_{\mu\nu}+eI^{a}\sqrt{-\det g})$$
$$-\tfrac{1}{4}k\iint \mathrm{d}\sigma\,\mathrm{d}\tau\,\sqrt{-\det g}\,(F^{a}_{\mu\nu})^{2}+\phi\text{-related terms}. \tag{3.5}$$

The action (3.5) is effective in the sense that the solution for ϕ is substituted in the action. The variation with respect to $F^{a}_{\mu\nu}$ gives

$$F^{a,\mu\nu}=\lambda^{a}\frac{\dot{X}^{\mu}X'^{\nu}-\dot{X}^{\nu}X'^{\mu}}{\sqrt{-\det g}}. \tag{3.6}$$

Combining eqs. (3.4) and (3.6) we have

$$\lambda^{a}=\frac{e}{k}I^{a}. \tag{3.7}$$

The colour field strength (3.6) with λ^a given by (3.7) reduces the effective action to

$$S^{\text{eff}}\to-\frac{e^{2}(I^{a})^{2}}{2k}\iint \mathrm{d}\sigma\,\mathrm{d}\tau\,\sqrt{-\det g}+\phi\text{-related terms}. \tag{3.8}$$

We can not go further in the same naivete to evaluate the ϕ-related terms. Some arguments will be given in the discussion. We do not claim that the derivation of the action (3.8) of Nambu–Goto type [8] is new, which may be another (non-rigorous) way round what people have (non-rigorously) stated. From this rather lengthy digression we return to the problem of the string state with the background field theory (3.1) in mind.

3.2. String state

We have seen that the field equations

$$D^{\mu}(\epsilon(\phi)F^{a}_{\mu\nu})=0\,, \tag{3.9}$$

$$\partial^{2}\phi+V'(\phi)+\tfrac{1}{4}\epsilon'(\phi)(F^{a}_{\mu\nu})^{2}=0\,, \tag{3.10}$$

have classical tube-like solutions. It is by now well-known that classical solutions of field equations can be interpreted in the context of the WKB approximation in quantum field theory. The coordinates $X^{\mu}(\sigma)$ of the tube core are then so-called collective coordinates and turn into dynamical variables.

In this subsection, we propose that the WKB approximated state corresponding to the tube-like solution has a form*

$$\Psi[x(\sigma)]=\prod_{\sigma'} K(x(\sigma'))\mathrm{P}\exp\left(ie\int A\,\mathrm{d}x\right), \tag{3.11}$$

where the real factor K will be determined in due course. To what equation is our Ψ an approximated solution? Having seen that the Kogut–Susskind–'t Hooft model reduces to the Nambu–Goto string action in the limiting case, we expect the Virasoro constraint equation [8] to be the expression which Ψ [eq. (3.11)] approximately satisfies. Let us study the second functional derivative of Ψ:

$$-\frac{\delta^2}{\delta x^{\mu}(\sigma)\delta x_{\mu}(\sigma)}\Psi[x(\sigma)]=\prod_{\sigma'} K(x(\sigma'))\left(-\frac{\delta^2}{\delta x^{\mu}(\sigma)\delta x_{\mu}(\sigma)}W\right)$$
$$-2\prod_{\sigma'} K(x(\sigma'))\delta(0)\frac{\partial \log K(x(\sigma))}{\partial x_{\mu}(\sigma)}\frac{\delta W}{\delta x^{\mu}(\sigma)}-\left\{\frac{\delta^2}{\delta x^{\mu}(\sigma)\delta x_{\mu}(\sigma)}\prod_{\sigma'} K(x(\sigma'))\right\}W,$$

W being the path-ordered phase factor. The first and the second terms can be deformed by eqs. (2.13) and (2.8). The third term is ignored according to the usual WKB recipe. Hence, we have

$$-\frac{\delta^2}{\delta x^{\mu}(\sigma)\delta x_{\mu}(\sigma)}\Psi[x(\sigma)]\approx\prod_{\sigma'} K(x(\sigma'))\{-ie\langle D^{\mu}F_{\mu\rho}\rangle x'^{\rho}\delta(0)+e^2\langle F_{\mu\rho}(\sigma)F^{\mu}{}_{\sigma}(\sigma)\rangle$$
$$\times x'^{\rho}(\sigma)x'^{\sigma}(\sigma)-2ie\frac{\partial \log K(x(\sigma))}{\partial x_{\mu}(\sigma)}\langle F_{\mu\rho}\rangle x'^{\rho}\delta(0)\}\,. \tag{3.12}$$

Using the field equation (3.9), we rewrite eq. (3.12) as

$$-\frac{\delta^2}{\delta x^{\mu}(\sigma)\delta x_{\mu}(\sigma)}\Psi[x(\sigma)]\approx\prod_{\sigma'} K(x(\sigma'))\left\{ie\frac{\partial \log \epsilon(x(\sigma))}{\partial x_{\mu}(\sigma)}\langle F_{\mu\rho}\rangle x'^{\rho}\delta(0)\right.$$
$$\left.-2ie\frac{\partial \log K(x(\sigma))}{\partial x_{\mu}(\sigma)}\langle F_{\mu\rho}\rangle x'\delta(0)+e^2\langle F_{\mu\rho}(\sigma)F^{\mu}{}_{\sigma}(\sigma)\rangle x'^{\rho}x'^{\sigma}\right\}. \tag{3.13}$$

Demanding that the imaginary part be vanishing in eq. (3.13), we obtain

$$K=\mathrm{const}\sqrt{\epsilon}\,. \tag{3.14}$$

When the colour flux is sufficiently collimated, as Nambu stated [1], we will have

* Here we write a factor related to the collective coordinates $X^{\mu}(\sigma)$. It is understood that for the full wave functional the field fluctuation part should be multiplied in the form of a gaussian or δ-function depending on the approximation scheme.

the Virasoro constraint equation. In the string limit of a thin tube, with eqs. (3.6) and (3.7), we have

$$-\frac{\delta^2}{\delta x^{\mu}(\sigma)\delta x_{\mu}(\sigma)}\Psi[x(\sigma)] \approx -\frac{e^4(I^a)^2(I^b)^2}{k^2}x'^2\Psi[x(\sigma)]\,. \tag{3.15}$$

If the ϕ-related terms in eq. (3.8) were equal to the first term we would have

$$S^{\text{eff}} \overset{?}{\to} -\frac{e^2(I^a)^2}{k}\int\!\!\int \mathrm{d}\tau\,\mathrm{d}\sigma\,\sqrt{-\det g}\,,$$

which gives the same constraint equation as eq. (3.15) up to the correct normalization. We will discuss the problem of the factor 2 in sect. 4.

Let us look at the obtained wave functional

$$\Psi[x(\sigma)] \approx \text{const}\sqrt{\prod_{\sigma'}\epsilon(x(\sigma'))}\,\mathrm{P}\exp\left(ie\int A_{\mu}\,\mathrm{d}x^{\mu}\right) \tag{3.16}$$

in more detail. First we observe that our $\Psi[x(\sigma)]$ is a localized functional of $x^{\mu}(\sigma)$ around the core $X^{\mu}(\sigma)$ of the classical tube solution, since $\epsilon(x)$ is. Perhaps this may be understood more clearly if we consider a straight rod located along the third axis with $x^0=\tau$, $x^3=\sigma$, $x^1=x^2=0$ in eq. (3.2) (the expression for the dielectric permeability ϵ). The probability for the string being of the shape $x^{\mu}(\sigma)$ is then given by

$$\begin{aligned}|\Psi[x(\sigma)]|^2 &= \text{const}\prod_{\sigma'}\epsilon(x(\sigma'))\\ &= \text{const}\prod_{\sigma'}\delta(x^1(\sigma'))\delta(x^2(\sigma'))\,.\end{aligned} \tag{3.17}$$

Eq. (3.17) certainly meets the intuitive picture of a string since only the transverse motion of the string is dynamical.

4. Summary and discussion

We have reformulated the variational differentiations of the path-ordered phase factor intending to streamline the calculus. Then it is proposed that the wave functional of the string state is of WKB type with the WKB phase factor replaced by the path-ordered phase factor in gauge theory. We adopt the Kogut–Susskind–'t Hooft model as the simplest colour collimating field theoretical model in which the phenomenological dielectric permeability plays an important role. The WKB-like amplitude is shown to be proportional to square root of the dielectric permeability ϵ and exhibits a localization around the classical string configuration. Our WKB wave functional approximately satisfy the Virasoro condition and is consistent with Nambu's argument [1].

The Kogut–Susskind–'t Hooft model is phenomenological in the sense that the field-dependent dielectric permeability ϵ, which is vitally important in the model, is yet to be derived from the highly respected QCD. We can, however, make some arguments about ϵ. If the phenomenological model is indeed derivable from QCD, the lagrangian is effective in the sense that

$$\int \mathrm{d}\phi \exp\left(i\int [\tfrac{1}{2}(\partial\phi)^2 - V(\phi) - \tfrac{1}{4}\epsilon(\phi)F^2]\,\mathrm{d}^4x]\right)$$
$$\sim \int \mathrm{d}q\,\mathrm{d}\bar{q} \exp\left(i\int [-\tfrac{1}{4}F^2 + \bar{q}(i\partial\!\!\!/ - eA\!\!\!/)q]\,\mathrm{d}^4x\right). \tag{4.1}$$

The dielectric permeability ϵ gets a contribution from the vacuum polarization in the background gauge field. Perhaps it may not be a wild conjecture that the permeability or the polarizability, which is non-vanishing only along the string, comes from the virtual quark and antiquark pairs in the shape of a macromolecule. In the same loose sense as (4.1) we may suggest that

$$\langle 0|\bar{q}\,\mathrm{P}\exp\left(ie\int A\,\mathrm{d}x\right)q|M\rangle \sim \sqrt{\epsilon}\,\mathrm{P}\exp\left(ie\int A\,\mathrm{d}x\right), \tag{4.2}$$

effectively, where $|M\rangle$ is the meson state under consideration.

We turn to the problem of under what circumstances the functional differential equation (3.15) coincides with the constrained equation derivable from the reduced effective action (3.8). It has already been pointed out that the agreement of the two equations occurs if the gauge field energy equals the scalar field energy in the expression (3.15). We would like to look at the virial property of our system in the simple case of a straight tube. In order to simplify the argument we take the abelian model. The non-abelian generalization is straightforward. Let us assume that $\epsilon(\phi)$ has a form

$$\epsilon(\phi) \sim k^2(\phi - \phi_0)^{2\alpha}. \tag{4.3}$$

Stability requires [7]

$$\alpha \geqslant 1. \tag{4.4}$$

The potential V will have the form

$$V(\phi) \sim \tfrac{1}{2}\mu(\phi - \phi_0)^2. \tag{4.5}$$

The gauge and scalar field energies per unit length [5] are given by

$$W_\mathrm{g} = \tfrac{1}{2}\int_0^\infty \mathrm{d}\rho\,\rho D^2(\rho)k^{-2}(\phi - \phi_0)^{-2\alpha}, \tag{4.6}$$

$$W_\mathrm{s} = \tfrac{1}{2}\int_0^\infty \mathrm{d}\rho\,\rho[(\partial_\rho\phi)^2 + \mu(\phi - \phi_0)^2], \tag{4.7}$$

respectively. The constraint of flux conservation and the field equation for ϕ are given by

$$D=\lambda k^2(\phi-\phi_0)^{2\alpha}, \tag{4.8}$$

$$-\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\rho\frac{\mathrm{d}\phi}{\mathrm{d}\rho}+\mu(\phi-\phi_0)-\alpha D^2k^{-2}(\phi-\phi_0)^{-2\alpha-1}=0, \tag{4.9}$$

where λ is the Lagrange multiplier [5]. With eqs. (4.6)–(4.9) it is easy to derive the virial relation

$$W_s=\alpha W_g. \tag{4.10}$$

Hence, we can conclude that if the marginally allowed value $\alpha=1$ [7] is realized, the functional equation (3.15) agrees with the constraint equation derivable from the action. Incidentally, the string limit of a thin tube will be attained if we take $\mu\to\infty$, at least in the straight tube case.

So far we have deliberately avoided any consideration of the end points (quark and antiquark). First, we should have been more careful about the terminals of the path in taking the functional differentiations in eq. (2.13). Second, in the Kogut–Susskind–'t Hooft model, the field configurations near the sources are something nasty to be tackled, though an intuitive spoon-like picture [7] may be drawn. This problem remains to be studied in detail.

The models of extended hadrons, which ours is among, is partially motivated by improvements of the dual string model [8]. Its modification seems particularly important in the lower energy states because of its notorious tachyon state and of the difficulty of the zero-point energy leading to the bizarre space-time dimension 26. We would like to remark that our present approach may also be useful for bag-like field configurations corresponding to lower energy states. In any case, the probability density $|\Psi|^2$ is proportional to ϵ which has a support inside the bag. Of course, our localized wave functional will satisfy an equation other than the Virasoro condition in this case.

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