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ON SIMULTANEOUS EXPLANATION OF LONG AND MEDIUM-TERM EMPLOYMENT CYCLES

BY

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1 INTRODUCTION

Taking for granted that longer periods of prosperity and depression alternate – a phenomenon indicated as 'long waves' – there is a range of macroeconomic phenomena which remain unexplained in business cycle theory. Theories about the causes of high versus low growth rates or high versus low employment rates in the structural or long-term sense either invoke purely exogenous factors such as changes in technological development or make use of static analysis of temporary equilibria or 'regimes.' As Goodwin (1986) remarks, 'long waves appear to be facts in search of a model.'

The present paper is an endeavour to show that endogenous behavioral mechanisms together with lagged responses may account for the existence of long-period (40–60 years or more) movements in output, employment and prices ('Kondratieff' cycles) alongside the well-known and well-analyzed cycles due to capital stock adjustment or inventory adjustment ('Juglar' or 'Kitchin' cycles). I would like to make clear from the outset that, though long waves are mentioned here, the model to be presented stresses the long-term changes in income shares and the technological determinants of employment much more than changing trends in output growth; first of all it analyses the long-run dynamics of employment and technological unemployment. The latter is a relevant aspect of long-run dynamics, whether one believes that long output fluctuations can be found empirically or not.[1] So the emphasis is on simultaneous modeling of long and medium employment cycles, due to 'classical' and 'Keynesian' mechanisms.

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1 *Cf.* Van Ewijk (1982) for a skeptical view on long waves in output, and Fitoussi/Georgescu-Roegen (1980) on the significance of chronic unemployment in long-term dynamics.

The method adopted is to construct a model based on rather straightforward macroeconomic relationships. The nonlinear accelerator model by Goodwin (1951) which gives rise to regular relaxation cycles (limit cycles) is extended and applied to two components of investment separately: investment in replacement and investment in expansion of production capacity, as suggested *e.g.* in Malinvaud (1982).[2] Thus two interacting limit cycles are generated which give rise to long and medium-term movements in a simultaneous model. Such a model cannot immediately be applied to describe the course of a real economy since one cannot expect macroeconomic coefficients to be constant for very long periods of time. But it may help to classify which dynamic response mechanisms may account for what kinds of macroeconomic movements.

It is not my contention that long waves are to be considered as a strictly periodic movement, but only that a stylized model of long waves is a means to study trend as an endogenous or partly endogenous phenomenon. Trend itself may thus be considered as subject to cyclical influences. In particular, I intend to show how the concise model adopted here may describe a wide range of combined long waves and business cycles and how the characteristics of both kinds of cycles may be interpreted in terms of behavioral parameters of the model.

Section 2 contains an exposition of the model; section 3 presents an analytical approach to solutions of the equation system, and section 4 explains the characteristics of the model by numerical simulation.

2 A NONLINEAR MODEL OF LONG WAVES AND BUSINESS CYCLES

The body of theories that explain business cycles as a disequilibrium phenomenon arising from volatile investment and lagged responses, includes essentially two methods of explanation. One is the multiplier-accelerator approach in linear or nonlinear form; *e.g.* Samuelson (1939), Hicks (1950), and Goodwin (1951). It establishes a dynamic interaction between capacity effects and demand effects of investment. So the *rate of capacity utilization* appears as the central variable here. The product market is in disequilibrium and the theory is clearly Keynesian in spirit.

The second approach relies on the distribution between wages and profits as a determinant of the level of investment and activity via profits; the level of activity again affects employment and thereby wages. This approach is represented by the predator-prey growth cycle model of Goodwin (1967) and other conflicting claims models in which the Phillips-curve mechanism plays an important role; see Desai (1973), Rose (1967), and Van der Ploeg (1983). The *income distribution between labor and capital* or profitability appears as the central variable in this class of models. Disequilibrium arises from labor mar-

2 It is interesting to note that the relevance of relaxation cycles for the study of business cycles was pointed out in *De Economist* – maybe for the first time – as early as 1930 by L. Hamburger (1930).

ket conditions, rather than from the product market. Profits, saving and investment are equal or move in consonance. Unemployment is not of the Keynesian but rather of the classical or Marxian type (*cf.* Malinvaud (1980) with Kaldor's comments).

The two approaches are thus entirely different if not mutually exclusive. Both lines of reasoning seem to have a counterpart in reality, but unfortunately they represent competing explanations of the same type of growth cycles. This apparent contradiction between a classical approach relying on full utilization of capacity and a Keynesian or demand side approach is observed in Goodwin (1983) and in Van der Ploeg (1983, p. 252). Therefore, in order to avoid the dilemma, I try to combine both lines of approach into a single model in which the capacity principle accounts for the medium-term business cycle, whereas the capital-labor antagonism accounts for long-term fluctuations.

The analysis is based upon the distinction of investment in capital deepening and widening respectively. Malinvaud (1982) introduces this approach and derives the change in capital stock from $K = kY^c$ (or capital coefficient times capacity output),

$$\mathrm{D}K = k\,\mathrm{D}Y^c + Y^c\,\mathrm{D}k$$

in which D indicates the derivative with respect to time.

In this way net investment is distinguished into two components: net expansion $k \cdot \mathrm{D}Y^c$ and net replacement $Y^c\,\mathrm{D}k$ which will be indicated as I^c and I^r respectively. So the net addition to capital stock is composed of firstly: the net effect on capacity (the capital coefficient remaining constant), or the *capital widening* effect of investment; and secondly: the excess of replacement over depreciation (capacity remaining constant), which is to be identified as the *capital deepening* effect of net investment.

If net investment in capacity expansion is defined as I^c, and net investment in replacement as I^r it follows that

$$I^c = k\,\mathrm{D}Y^c \quad \text{or} \quad \mathrm{D}\mathit{ln}\,Y^c = I^c/K$$

$$I^r = Y^c\,\mathrm{D}k \quad \text{or} \quad \mathrm{D}\mathit{ln}\,k = I^r/K$$

so that the net expansion-to-capital ratio equals the growth rate of production capacity, whereas it is also made clear that net replacement investment amounts to a change of the capital-output ratio k.[3]

The split of investment introduces a twofold dynamic mechanism. A medium-term cyclical mechanism is based on the capacity investment or widen-

3 The literature on investment behavior shows evidence that replacement investment on the one hand, and expansion investment on the other hand have distinctive determinants and mechanisms; see Eisner (1972) and Feldstein and Foot (1971).

ing process, which is guided by product market disequilibrium. The utilization rate operates as indicator of disequilibrium here. A long-term cyclical mechanism centers around the capital deepening or capital-labor substitution process; it is guided by labor market disequilibrium or discrepancy between desired and actual capital coefficients. The indicator for disequilibrium in the labor market is the discrepancy between actual and optimal distributive shares (from the employers' point of view). Comparing with the terminology of Sneessens/Drèze (1986), one may identify the rate of excess demand or the *demand gap* (with reverse sign) as central to the product market disequilibrium, and the *distributive gap* as driving force in the labor market. Thus, in the model presented below, the demand gap is controlling capacity investment, while the distributive gap affects replacement investment or capital deepening/shallowing.[4]

For the sake of clarity I keep the model and the number of behavioral parameters as small as possible, which somewhat limits the scope and form of cyclical patterns that may be simulated. The model consists of equations (2.1)–(2.17) below. First of all some definitions are presented.

$$k = K/Y^c \tag{2.1}$$

$$a = L^p/Y^c \tag{2.2}$$

$$\mathrm{D}K = I \tag{2.3}$$

$$I = I^c + I^r \tag{2.4}$$

$$q = Y/Y^c - 1 \qquad [\approx ln\,(Y/Y^c)] \tag{2.5}$$

$$u = (L^s - L^d)/L^s \quad [\approx ln\,(L^s/L^d)] \tag{2.6}$$

$$Z = C + I \tag{2.7}$$

$$V = a(p_l/p_y) \tag{2.8}$$

$$\upsilon = ln\,(V/V^d) \tag{2.9}$$

The capital-output ratio (k) is defined as the ratio of capital stock K and production capacity Y^c. The labor-output ratio (a) is defined as the ratio of demand for labor at full capacity L^p and capacity output Y^c. I represents net

4 A connection can be made with the Austrian idea of capital deepening as a factor in cyclical development; *cf.* Hayek (1939), Hicks (1973) and Zamagni (1984). It is transformed, however, by connecting it with the concept of replacement cycles and thus including it into an explanation of long employment cycles.

investment; I^c stands for net investment in expansion, whereas I^r represents net investment in replacement (or capital deepening).

Capacity utilization in (2.5) is expressed as its deviation from a 'normal' level 1. So in effect, q is expressed as the rate of overutilization, or as rate of excess demand in terms of Sneessens/Drèze (1986); if $q > 0$ capacity utilization is 'above normal.' In (2.5) and (2.6), the approximation of $(x-1)$ by the natural log of x is applicable if x is close to one; thus the model is more easily solved.

Unemployment u as a fraction of 1 is the difference between labor demand L^d and supply L^s related to L^s (assumed = 1 throughout). In the present model unemployment is allowed to be negative as well as positive, because we want to analyze employment cycles with a given labor supply; though this may appear peculiar, it is identical to the approach in growth cycle models like Goodwin (1967) where employment cycles are described in terms of the employment ratio. The exogenously given L^s is a 'normal' supply; when demand exceeds supply the economy enters a situation of overemployment, not of rationing in the absolute sense. Real aggregate expenditure Z is defined as consumption C plus net investment I. V in (2.8) is the share of wages that would exist at full capacity:

$$V = (L^p/Y^c)(p_l/p_y) = a(p_l/p_y) = aw$$

in which p_l is the nominal wage index, p_y the nominal price index and w the real wage rate. So according to (2.9), υ is the logarithmic approximation of $(V - V^d)/V^d$, the relative deviation of V from V^d. As will appear below,

$$V^d = 1/(1+\beta)$$

which coincides with the production elasticity of labor and with the constant wage share corresponding with profit maximization by employers.

So the sign of $\upsilon = ln\,(V/V^d)$ indicates whether the wage share is above or below the level at which employers are in equilibrium. Although not strictly equivalent, υ is close to the concept of distributive gap as developed by Sneessens/Drèze (1986). The behavioral relationships of the model read as follows:

$$c = c_a + c \cdot Y \qquad (2.10)$$

$$\mathrm{D}Y = (Z - Y)/\tau \qquad (2.11)$$

$$a = (k/k_0)^{-\beta} \qquad (2.12)$$

$$L^d = (1+q) \cdot L^p \qquad (2.13)$$

$$I^c = k\,\mathrm{D}Y^c \qquad \text{[and so } I^r = Y^c\,\mathrm{D}k\text{]} \qquad (2.14)$$

$$I^c/Y^c = \pi \cdot sgn\, q \qquad 0 \leq \pi < 1 \qquad (2.15)$$

$$I^r/Y^c = \sigma \cdot sgn\, \upsilon \qquad 0 \leq \sigma < 1 \qquad (2.16)$$

in which the function sgn means:

$$sgn\, q = 1 \quad \text{if } q>0$$
$$sgn\, q = 0 \quad \text{if } q=0$$
$$sgn\, q = -1 \quad \text{if } q<0$$

$$\hat{p}_y = \mathrm{D}ln\, p_y = \Theta_1 \cdot v + \Theta_2 \cdot q \qquad \Theta_1, \Theta_2 \geq 0 \tag{2.17}$$

$$\hat{p}_l = \mathrm{D}ln\, p_l = -\varepsilon_1 \cdot v - \varepsilon_2 \cdot u \qquad \varepsilon_1,\ \varepsilon_2 \geq 0 \tag{2.18}$$

The consumption function is linear with autonomous consumption c_a and marginal propensity to consume c. To facilitate derivations, consumption units will be chosen such that $c_a = 1 - c$. For when net investment is zero, this implies that the stationary or 'cycle-free' level of income $c_a/(1-c)$ equals 1. The marginal rate of saving $1-c$ will be indicated by s below.

Equation (2.11) introduces the dynamic multiplier mechanism by a simple exponential lag of output on expenditure. It is written with a variable time lag τ. Evidently, τ has a substantial influence on the time distribution of the response of Y on Z and thus on the shape of the short cycle.

Equation (2.12) is equivalent to a Cobb–Douglas production function with constant returns to scale:

$$Y^c = [K/k_0]^{\beta/(1+\beta)} L^{p^{1/(1+\beta)}}.$$

So the production elasticity of labor $1/(1+\beta)$ is the equilibrium wage share in neo-classical production theory.[5] The optimal or desired capital-output ratio k^d is to be derived from profit maximization and the production function (see also Rose (1967)).

Clearly, for a given real wage rate $w = (p_l/p_y)$, equal to the marginal product of labor, the desired labor coefficient a^d amounts to

$$a^d = [w(1+\beta)]^{-1}$$

and the wage share as desired by employers will be

$$V^d = wa^d = 1/(1+\beta).$$

Substituting k^d for a^d gives the desired capital-output ratio k^d as

5 In fact the scale constant k_0 in (2.12) is such that both the labor coefficient (a) and (k/k_0) are defined as an index relative to 1. Thus k moves around k_0 and k_0 is related to the unit of time. When capital is measured in units of capacity y^c, the average capital-output ratio is 3 on a yearly basis and k_0 is taken to be 2, the unit of time is 1.5 years. So one has to choose k_0 equal to the average yearly capital-output ratio in capacity units in order to obtain the year as unit of time.

$$k^d = k_0[w(1+\beta)]^{1/\beta} \quad \text{and from (2.12)}$$
$$k^d/k = [aw(1+\beta)]^{1/\beta} = [V/V^d]^{1/\beta} = exp(\upsilon/\beta)$$

following from the definition of the distributive gap υ in (2.9). So the ratio of desired and actual capital-output ratio is a monotonous function of υ. Especially, $k^d > k$ is equivalent with $\upsilon > 0$ (and $V > V^d$). Therefore, υ is used to indicate a disequilibrium of employers with respect to their desired capital-output ratio and to determine the direction of change of k (see the explanation below). Whenever the wage share exceeds employers' claims, employers will engage in capital deepening. They start to increase k in order to increase labor productivity and reduce unit wage costs, and vice versa. The production function with substitution elasticity -1 facilitates analysis, because the desired wage share is a constant. Different assumptions are however possible.

The demand for labor is equal to the potential demand for labor multiplied with the rate of capacity utilization $(1+q)$. Combined with definition (2.2) it follows that $L^d = aY$.[6] This implies that the full capacity wage share $V = aw$ is at the same time equal to the actual wage share $w(L^d/Y)$.

The shape of the investment functions is of essential importance for the dynamic behavior of the model. The behavior of the two components of net investment is described in an extremely simplified manner according to (2.15) and (2.16). According to (2.15) net investment in additional capacity I^c is a positive fraction π of production capacity if $q > 0$ (or $Y > Y^c$), and a negative fraction $-\pi$ if $q < 0$. Expansion investment is zero when $q = 0$ (output equals capacity). This is nearly identical to Goodwin's (1951) formulation of the nonlinear accelerator which generates a stable limit cycle in output and capital around an unstable center where $DY = 0$ and $DK = 0$.[7,8]

6 Instead of (2.13) one might take

$$L^d = L^p(1+q)^{\Phi}$$

with $\Phi \neq 1$ which allows for labor hoarding. It would change the short-term dynamics but is not essential to an understanding of the model. So $\Phi = 1$ throughout.

7 Goodwin uses, in fact, two different approaches in his (1951) article; one is a capital stock adjustment mechanism following from the response of investment to desired capital stock, the other is a truncated and therefore nonlinear accelerator function for investment. There is some theoretical difference between the two. Capital stock adjustment forces capacity towards output, while the nonlinear accelerator allows output and capacity to move away from each other because capacity does not appear as an argument in the investment function. In the context of this paper where long-term movements are taken into account, the capital stock adjustment approach is preferable; see also Thio (1984). For a solution of the Goodwin (1951) model in terms of the utilization rate, *cf.* Allen (1959, Chapter 8.2).

8 A difference with Goodwin (1951) is that (2.15) (and also 2.16) is symmetric. Investment and disinvestment are equally fast. This assumption is only made to achieve the utmost notational and analytical simplicity; this symmetry could be avoided by writing a linear function of *sgn q* instead of *sgn q* itself. In that case a difference in the relative length of upswing and downswing would emerge. I neglect this aspect here for it does not add much to an understanding of the working of the model.

Equation (2.16) together with (2.15) accounts for the possibility to simulate long-wave movements alongside the medium-term cycle. The split of investment in two components both regulated by a nonlinear accelerator mechanism is decisive as to the dynamic behavior of the model. Equation (2.16) is analogous to (2.15). I^r/Y^c is a positive or negative proportion σ according to the sign of υ, which is equivalent to the sign of $k^d - k$.[9]

The implication of (2.16) is that as long as employers are satisfied with the actual profit share (being in accordance with the production function and profit maximization) they maintain the existing capital and labor coefficients. This situation is possible but unstable, just as the stationary state in the Goodwin (1951) model. Whenever the actual wage share is below the level that employers view as the 'right' level ($\upsilon < 0$) they start to increase the labor coefficient by decreasing the capital-output ratio; they substitute labor for capital, and increase the demand for labor. But if $\upsilon > 0$ (positive distributive gap) they increase the capital-output ratio in order to increase labor productivity and diminish the demand for labor.

Equation (2.16) implies that *the rate of change of the capital-output ratio is limited to σ in both directions*:

$$\mathrm{D}k = I^r/Y^c = \pm\sigma.$$

The adaptation of production technique, the process of capital deepening and shallowing, in response to changes in relative prices does not take place instantly; it is supposed to take time. This delay in the adaptation of actual to desired capital-output ratio may account for periodic movements of long duration. In the present model the movements of technical coefficients, unemployment and the wage share exhibit a limit cycle.[10]

The formulation of (2.16) is in accordance with theories of capital accumula-

9 So (2.16) is a simplified statement of

$$I^r/Y^c = \min\,[\sigma, \delta(k^d - k)] \quad \text{if } k^d > k$$
$$I^r/Y^c = \max[\sigma, \delta(k^d - k)] \quad \text{if } k^d < k$$

in which σ and δ are non-negative constants; in the same way, (2.15) is a simplification of an accelerator mechanism with upper and lower bounds. Such a formulation yields an S-shaped investment function with upper and lower bounds as in nonlinear or flexible accelerator types of investment relations. Also, one could make the floor and ceiling in (2.16) asymmetric for upswing ($\upsilon > 0$) and downswing ($\upsilon < 0$), which would affect the relative length of both phases of the cycle.

10 Instead of (2.15) and (2.16) one might use more sophisticated investment relations, for instance by assuming a decision lag as in Goodwin (1951). Thus the crude saw tooth patterns of production capacity and capital output ratio that emerge (see also the pictures in section 4) could be smoothed, and the discontinuities in the first derivatives $\mathrm{D}k$ and $\mathrm{D}Y^c$ evaded. The resulting differential equations would become second order and far more complex; a sketch of an analytical approach to coupled second order equations in Goodwin-type models is found in Evans and Fradellos (1982).

tion that allow for limited substitution between labor and capital; *e.g.* because change in technical coefficients takes place at the margin as is the case in vintage growth models. Sneessens/Drèze (1986; p. S99) use a similar argument.

Some general justification of the rather special investment functions may be gained from Nickell (1978, Chapters 6 and 11). He shows that especially uncertainty and investment lags account for a spread in time of the adjustment of the firm to desired capital stock when demand shocks occur. The same goes for adjustment to the desired capital-output ratio in putty-clay models where substitution takes time. The sluggish dynamic behavior of investment is represented here by a speed limit to the adjustment to demand shocks and relative price shocks. If adaptation to the desired technical coefficients could take place instantly and without uncertainty and cost, one could of course return to an equilibrium description of the economy and the stationary solution would then be applicable.

The exposition of the model concludes with two equations (2.17) and (2.18) for wage and price formation that are necessary for the endogenous determination of the real wage rate $w = p_l/p_y$ and the wage share. These imply a negotiation model for wage and price formation in an economy that is unionized or characterized by monopolistic competition on both sides of the market (as developed by Turnovsky and Pitchford (1978)). It is more elaborate than the usual real wage equation in predator-prey models such as Goodwin (1967), and thus more flexible as to the time shape of the adjustment of nominal wage and price levels and income shares to incentives from labor and product market.

The employers' mark-up is derived from their optimal profit share and therefore equal to the reciprocal of the wage share as desired by employers: $1+\beta$. Defining the target price level as

$$p_y^* = \text{unit labor cost } x \text{ markup} = ap_l\,(1+\beta)$$

it follows that

$$ln\,(p_y^*/p_y) = ln\,[aw(1+\beta)] = ln\,(V/V^d) = \upsilon$$

Thus, remembering that

$$ln\,(p_y^*/p_y) \approx (p_y^* - p_y)/p_y$$

it is clear that an explanation of inflation in (2.17) from υ amounts to a partial adjustment process of the price level towards its target, with adjustment speed Θ_1. The influence of the market power of employers on the rate of inflation is expressed by $\Theta_2 q$. This is analogous to Turnovsky and Pitchford (1978).

In the same vein (2.18) gives nominal wage increase as a partial adjustment of the actual wage level to a target wage level. The target wage level p_l^* is derived from a target wage share V^d:

$$p_l^* = V^d(p_y/a)$$

and again

$$(p_l^* - p_l)/p_l \approx ln\,(p_l^*/p_l) = ln(V^d/V) = -\upsilon$$

So nominal wage increase in (2.18) is explained as the speed of adjustment (ε_1) of the wage level p_l to the target p_l^*. Wage demands are also affected by the market power of workers as expressed by unemployment u. The desired wage share V^d is assumed to be the reciprocal of the mark-up claimed by employers, so that income claims are 'consistent' (add up to one). This assumption may be interpreted as a dominating position of the employers, because they force workers to accept their own equilibrium income share. Again this is done for the sake of simplicity only. If wage and price claims exceed 1, one would get an additional constant in (2.18) and persistent (though steady) inflation.

In conclusion of this section a few remarks on the nature of the present model are in order. Our investment functions may be conceived of as a simple dynamic version of an investment model in which both capacity utilization and profitability are considered as factors influencing investment and employment. Especially Malinvaud (1982), also (1989), has been working in this direction. The model may, in this respect, be understood as a dynamic counterpart to rationing models.

The investment process gives rise to *four states of the economy*: positive or negative expansion investment may go along with either positive or negative replacement investment, according as $q > 0$ or < 0 and $\upsilon > 0$ or < 0 respectively. One could characterize these states as *absolute* or *relative scarcity* or *abundance* of capital in the following sense:

υ	q	implying	I^r	I^c	
+	+		+	+	absolute scarcity of capital
+	−		+	−	relative scarcity of capital
−	+		−	+	relative abundance of capital
−	−		−	−	absolute abundance of capital

In a state of absolute scarcity, capital is in short supply relative to labor (because $\upsilon > 0$) as well as relative to output (because $q > 0$); therefore it is expanding in both directions. Relative scarcity is a state in which capital is scarce with respect to labor but not with respect to output; capital is then deepening but not widening. An analogous description goes for capital abundance; in case of absolute abundance with both υ and q negative, capital will be contracting in both senses.

The different states of the economy correspond to phases of cyclical development. A positive sign of q corresponds to the upswing of the capacity adjustment cycle, and a positive sign of v to the upswing of what may be called the capital deepening or substitution cycle. A state of capital scarcity will coincide with an expansionary climate, and general wage and price increases. It will be the 'prosperity' phase of the long periodic movement; the trend is in upward direction. Conversely, abundance of capital coincides with general contraction and price deflation. It constitutes the 'depression' phase of the long wave with the trend in its cyclical downturn.

These different states of the economy correspond with a particular analysis of unemployment. Conceptually, unemployment is to be decomposed as

$$\begin{aligned} u &= (L^s - L^d)/L^s \\ &= (L^s - L^p)/L^s + (L^p - L^d)/L^s. \end{aligned}$$

Borrowing the terminology of Sneessens/Drèze (1986), unemployment $(L^s - L^p)$ is due to lack of jobs or a *capital gap*, and unemployment $(L^p - L^d)$ is due to a *demand gap*. Quantitatively, with $L^s = 1$, this amounts to

$$u = -ln\, aY = -ln\, a - ln\, Y^c - ln\, (Y/Y^c)$$

so

$$u = \beta \cdot ln\, (k/k_0) - ln\, Y^c - q \quad [= u_{tec} + u_{cap} + u_{dem}].$$

Thus unemployment can be decomposed into a utilization rate effect (if q is subnormal); a capacity effect (if $ln\, Y^c$ is subnormal) and a substitution or capital deepening effect leading to technological unemployment (if the capital-output ratio index $ln\, (k/k_0)$ is above normal). I define the three effects as *technological, demand deficient*, and *capacity deficient unemployment* (or u_{tec}, u_{dem}, and u_{cap} respectively). Malinvaud (1980) plus comments contains a discussion of different types of unemployment; see also Driehuis (1988), Kuipers (1988), Sneessens/Drèze (1986), and Muysken (1989).

Clearly, capital gap related unemployment is to be distinguished in two parts, u_{tec} and u_{cap}. So I propose a distinction of the capital gap concept into a *quantitative* capital gap $-y^*$, which refers to job shortage on account of capacity shortage (deficient expansion of capital), and a *qualitative* capital gap βk^*, which refers to technological unemployment due to excessive capital deepening. In line with Malinvaud one could indicate demand deficient unemployment as Keynesian, and capacity deficient and technological unemployment as classical and Marxian respectively.[11] One should keep in mind that all

11 Regarding the usual distinction of structural and cyclical unemployment, the demand deficient type of unemployment would be called cyclical, and the capacity deficient and technological types would be called structural. However, in the present model the different types of unemployment are all cyclical, so that this distinction is not very useful in the present context.

three types may occur in positive or negative direction, and in all different combinations; such as positive technological unemployment with overutilization of capacity and so on.

In section 4 these concepts will be used to analyze employment cycles in terms of a succession of different states or unemployment regimes.

3 REMARKS ON THE ANALYTICAL SOLUTION OF THE MODEL

By substitution one may reduce the system (2.1)–(2.18) to a system of differential equations in four variables and from there get a fairly accurate idea of how it works. As is clear from the nonlinear investment function the crucial variables are q and υ, as indices of demand gap and of distributive gap. These regulate the dynamic movement of the system. The variables that are immediately affected by q and υ *via* the investment function are production capacity Y^c and capital-output ratio k respectively. The latter two variables are transformed as follows:

$y^* = ln\, Y^c$ index of the level of capacity

$k^* = ln\,(k/k_0)$ index of the capital-output ratio

so that

$\mathrm{D}y^* = \mathrm{D}ln\, Y^c = I^c/K$ the relative rate of change of capacity

$\mathrm{D}k^* = \mathrm{D}ln\, k = I^r/K$ the relative rate of change of k.

Then, the model of section 2 can be reduced to a system of four differential equations in q, υ, y^* and k^*.[12]

$$\mathrm{D}q = -(s/\tau)\cdot q - (s/\tau)\cdot y^* + (\pi/\tau - \pi/k_0)\cdot sgn\, q + (\sigma/\tau)\cdot sgn\, \upsilon \tag{3.1}$$

$$\mathrm{D}\upsilon = -(\Theta_2 - \varepsilon_2)\cdot q + \varepsilon_2\cdot y^* - (\Theta_1 + \varepsilon_1)\cdot \upsilon - \varepsilon_2 \beta\cdot k^* - \beta(\sigma/k_0)\cdot sgn\, \upsilon \tag{3.2}$$

$$\mathrm{D}y^* = (\pi/k_0)\cdot sgn\, q \tag{3.3}$$

$$\mathrm{D}k^* = (\sigma/k_0)\cdot sgn\, \upsilon \tag{3.4}$$

The system is nonlinear because of the terms with $sgn\, q$ and $sgn\, \upsilon$ on the right hand side. In particular the derivatives show discontinuities where q and υ change sign.

The four admitted investment regimes: positive and negative expansion and

12 An appendix with derivation will be made available by the author on request.

replacement investment, will correspond to four branches for the solution of the equations. One obtains an upward and a downward branch for the capacity investment cycle and an upward and downward branch for the replacement investment cycle. The clearest way to study the phase relations is to take the derivative of equation (3.1), denoting the second derivative as D^2q. This yields

$$\begin{aligned} D^2q &= -(s/\tau)\cdot Dq-(s/\tau)\cdot Dy^* \\ &= -(s/\tau)\cdot Dq-(s/\tau)(\pi/k_0)\cdot sgn\, q \end{aligned}$$

since the derivatives of *sgn q* and *sgn υ* are zero *except* at switch points of *sgn q* and *sgn υ*, where Dq is discontinuous. Defining $x = Dq$ renders

$$Dx = -(s/\tau)\cdot x-(s/\tau)(\pi/k_0)\cdot sgn\, q \tag{3.5}$$

or

$$Dx = -Ax - B\, sgn\, q$$

So, because the system (3.1)–(3.4) is non-autonomous, the phase relationship (3.5) cannot be expressed in x only. The phase diagram is linear in x and has two branches, when q is negative (AB) or positive (CD): see Figure (3.1a).

Within the first quadrant where $q<0$ and $x = Dq$ is positive and *increasing*, q must reach zero and the economy switches to the lower branch where $q>0$. At the switch point L where $q=0$, Dq or x is subject to an investment shock of twice $(\pi/\tau-\pi/k_0)$, due to the term $(\pi/\tau-\pi/k_0)\cdot sgn\, q$ in equation (3.1). Therefore point M is to the right of point L. The model yields a *limit cycle KLMN*

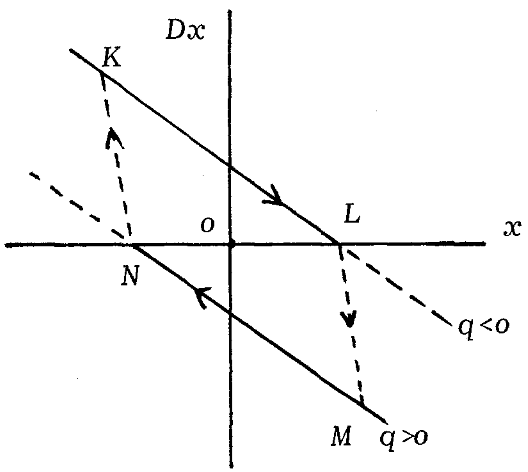

Figure 3.1a – Phase diagram for x; equation (3.5)

in x around a center $(0,0)$. The phase diagram of q, the rate of overutilization itself, is sketched in Figure (3.1b).

The time path of the 'short' cycle *i.e.* the cycle of the utilization rate index q may be derived as[13]

$$q(t) = 2(\pi/s)(sgn\ q)[1 - exp\{-(s/\tau)\cdot t\}] - (sgn\ q)(\pi/k_0)\cdot t$$

The period of the short cycle amounts to

$$TS = 4k_0/s.$$

A phase diagram for υ is to be derived in roughly the same manner as for q. Because of interactions between the long and the medium-term cycle, however, it is necessary to simplify by smoothing the effects of the q-cycle. We may do so by neglecting the effects of capacity utilization. We assume:

$$q = 0 \quad \text{and} \quad y^* = (\sigma/s)\cdot sgn\ \upsilon.$$

Thus, in the long run, capacity y^* adapts to the level as determined by the multiplier effect of investment in capital deepening (σ/s). When drawing a phase diagram, we consider only the long-cyclical change of capacity and neglect the medium term adjustments of capacity and output. Actually, a pic-

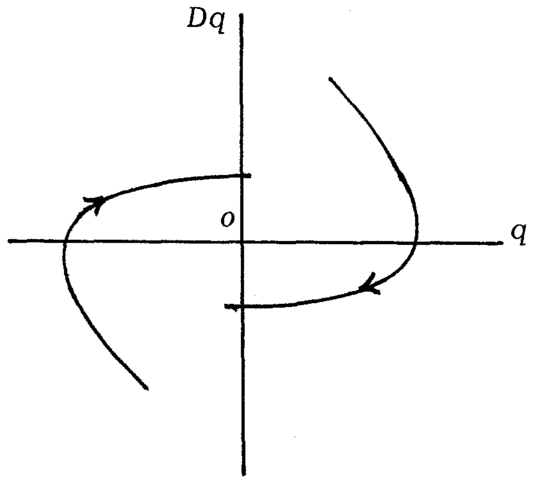

Figure 3.1b – Phase diagram for q

13 An appendix with a solution of the time path of q, as well as an approximation of the time path of υ will be made available by the author on request.

ture of the *interaction* of long and medium-term movements may only be obtained by numerical simulation and has to wait for section 4 below. The relationship for $\mathrm{D}\upsilon$, smoothed in this sense, is:

$$\mathrm{D}\upsilon = -(\Theta_1 + \varepsilon_1) \cdot \upsilon - \beta\varepsilon_2 \cdot k^* - (\beta\sigma/k_0) \cdot sgn\ \upsilon + \varepsilon_2(\sigma/s) \cdot sgn\ \upsilon$$

$\mathrm{D}\upsilon$ is differentiable except at $\upsilon = 0$, where $\mathrm{D}\upsilon$ is subject to a shock equal to twice $[-(\beta\sigma/k_0) + \varepsilon_2(\sigma/s)] \cdot sgn\ \upsilon$. For all other υ, and defining $y = \mathrm{D}\upsilon$, the derivative renders

$$\mathrm{D}y = -(\Theta_1 + \varepsilon_1) \cdot y - \beta\varepsilon_2(\sigma/k_0) \cdot sgn\ \upsilon \tag{3.6}$$

or

$$\mathrm{D}y = -Fy - G\, sgn\ \upsilon.$$

Again, the phase relationship is linear in y with two branches according to the sign of υ – see the phase diagram in Figure (3.2a). Because of smoothing, the diagram shows the phase relationship of a 'pure' long cycle for υ, undisturbed by the movement of q.

On the branch with $\upsilon < 0$, the variable $y = \mathrm{D}\upsilon$ becomes positive and increasing, so υ will approach zero at point L before G/F at the y-axis. Then the economy jumps to M on the other branch with $\upsilon < 0$. At the switch, as said before, the value of y shifts by $-2(\varepsilon_2/s - \beta/k_0) \cdot \sigma$. Thus a limit cycle along $KLMN$ is the resulting movement.

In Figure (3.2b) the phase diagram of υ is presented. It has basically the same

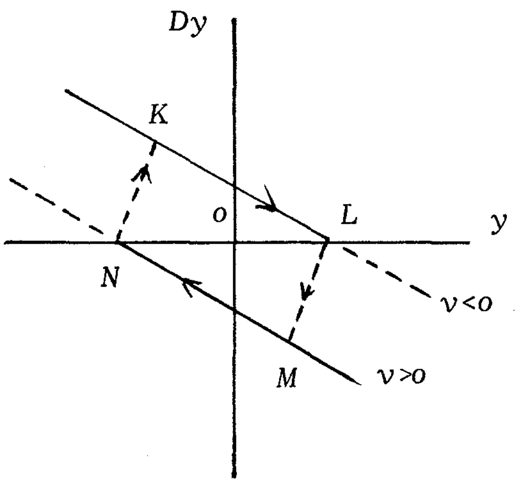

Figure 3.2a – Phase diagram for y; equation (3.6)

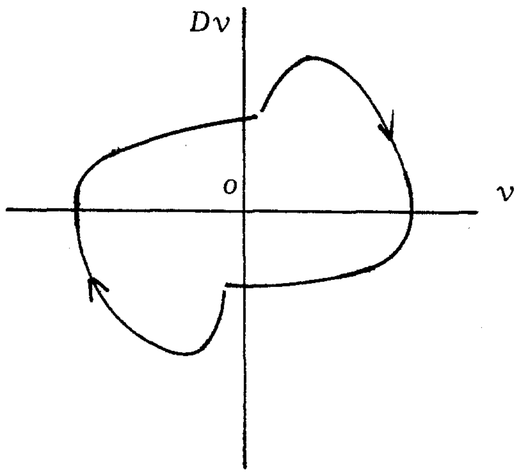

Figure 3.2b – Phase diagram for υ

form as that of q. Again the discontinuity in $D\upsilon$ occurs, which is typical of relaxation cycles.

The time path of the (smoothed) long wave or υ-cycle requires the same kind of approximations. It is given by

$$\upsilon(t) \approx -(G/F)(sgn\ \upsilon) \cdot t + (G/F)(sgn\ \upsilon) \cdot (TL/2) \cdot [1 - exp(-Ft)]$$

and the full period TL of the long wave equals

$$TL \approx 4[1/(\Theta_1 + \varepsilon_1) - 1/\varepsilon_2 + k_0/(\beta s)]$$

which will generally be larger than TS because β is substantially smaller than 1. Apparently, the periodicities for υ and q are *independent* of the arbitrarily chosen investment boundaries σ and π. This is an important feature because it shows that the existence of two connected cycles is basically determined by the *existence* of speed limits to adjustment, and not by their quantitative values. The latter affect the amplitudes only.

Concluding this section it may be said that the characteristics of the mutually interdependent cycles in income distribution and capacity utilization can be represented analytically by some surprisingly simple relationships which are open to direct interpretation in terms of the behavioral coefficients of the underlying model.

4 SOME NUMERICAL ILLUSTRATIONS

In order to get a clear picture of the behavior in time of the main variables, the model (2.1)–(2.18) can be solved numerically for given values of the parameters. As will be seen, the analytical characteristics that have been derived in section 3 by approximation are vindicated by the results of numerical solution of the original model of section 2. Three examples follow: the q and the υ-cycle isolated, and the mixed case. Comments concentrate on the change in unemployment regimes.

Example 1: The pure medium-term employment cycle [Figure 4.1]
When the effects of income distribution are suppressed, the model exhibits a limit cycle in capacity utilization q only. This is attained by putting $\sigma = 0$ so that net replacement investment is zero. The example is close to the Goodwin (1951) model. The plot in Figure 4.1 is based on the following values for the coefficients:

$$s = 0.5; \quad k_0 = 1; \quad \pi = 0.02; \quad \tau = 0.5; \quad h = 0.2$$

and others zero or irrelevant.[14] The period of the cycle settles to $4k_0/s = 8$ time periods or 'years' as should be expected from the results of section 3; and with $h = 0.2$ this implies 40 time units in simulation (see Figure 4.1a).

Employment (Figure 4.1b) is the mirror image of output. It equals $-q - y^*$ or $u_{dem} + u_{cap}$; there is no technological unemployment.

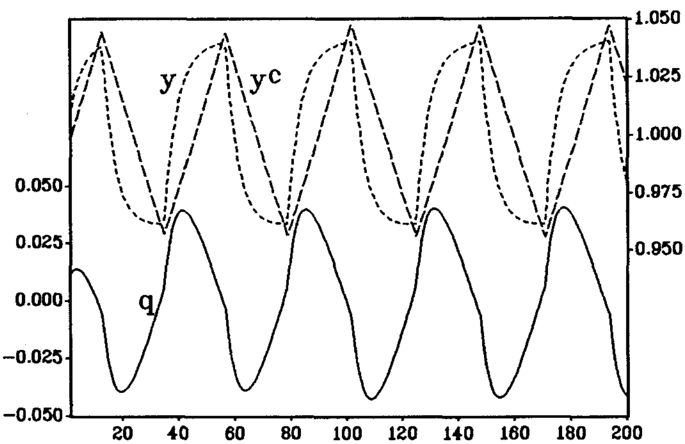

Figure 4.1a – q left scale; y, y^c right scale

14 The parameter h indicates the fraction of one time period which is used as the unit of time in the approximation of continuous time in all simulations below, so that $1/h$ represents the number of steps in a time unit.

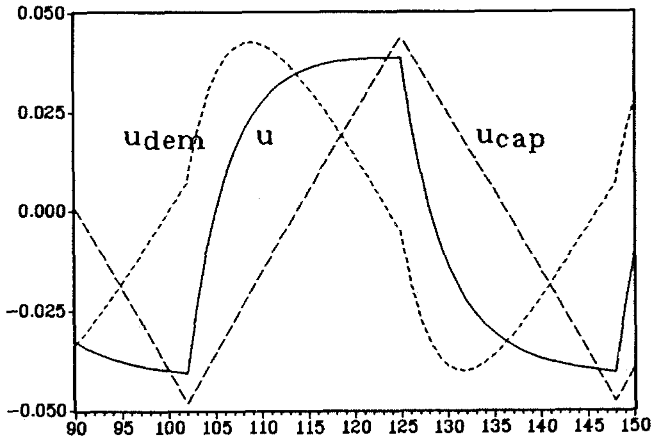

Figure 4.1b

Capacity Y^c shows a *two-phase* cycle with upswing and downswing determined by the sign of q. Utilization q shows a *four-phase* cycle pattern. The upswing (with q positive) is divided into the *recovery* phase, characterized by acceleration of overutilization q caused by the positive investment demand impulse, and the *boom* during which q decreases as capacity catches up. The downswing embraces the *crisis*, a breakdown of q because of a negative investment impulse, and the *depression* with q increasing to zero again as capacity is scrapped. Clearly, capital is scarce relative to output during recovery and boom ($q>0$) and abundant during crisis and depression.

In scheme, one obtains the following cyclical changes in the composition of unemployment. Signs indicate the sign of the variable, arrows its direction of change.

Decomposition of unemployment for stylized q-cycle:

	u	u_{dem}	u_{cap}	gaps
recovery	$+ \downarrow L^d<L^s$	$- \downarrow L^d>L^p$	$+ \downarrow L^p<L^s$	$L^p<L^d<L^s$
boom	$- \downarrow L^d>L^s$	$- \uparrow L^d>L^p$	$- \downarrow L^p>L^s$	$L^d>L^p>L^s$
crisis	$- \uparrow L^d>L^s$	$+ \uparrow L^d<L^p$	$- \uparrow L^p>L^s$	$L^p>L^d>L^s$
depression	$+ \uparrow L^d<L^s$	$+ \downarrow L^d<L^p$	$+ \uparrow L^p<L^s$	$L^d<L^p<L^s$

During the pure capacity adjustment cycle, the 'capital gap' between L^s and L^p is purely quantitative (in the sense mentioned in section 2): it is fully coincident with capacity related unemployment, since technological unemployment is absent.

Example 2: The pure long employment cycle [Figure 4.2]
If, contrary to example 1, π and thus investment in expansion of capacity is put zero, the capacity adjustment cycle is extinguished and a pure image of the long wave emerges in the variable υ and the accompanying movements in technical coefficients and unemployment, without 'disturbance' by medium-term movements. The chosen coefficients are:

$$\pi = 0; \quad \sigma = 0.015; \quad k_0 = 2; \quad s = 1/3; \quad \tau = 1;$$

$$\Theta_1 = 0.05; \quad \Theta_2 = 0; \quad \varepsilon_1 = 0.05; \quad \varepsilon_2 = 0.25; \quad \beta = 1/3; \quad h = 1.$$

The results are given in Figure 4.2 for a time span of 400 periods (equal to 'real' time periods since $h = 1$). The periodicity of the cycle should be 96 time units (see section 3). The plot of the capital-output ratio k clearly shows this (Figure 4.2a). The turning points of k coincide with $\upsilon = 0$ or wage share in 'equilibrium.'

The capacity cycle Y^c degenerates into a constant, whereas utilization q reflects a pure demand effect of capital deepening investment (Figure 4.2b).

A *two-phase* cycle appears in the capital output ratio k with an upswing for $\upsilon > 0$. The movement of the distributive gap υ can be considered as a *four-phase* pattern. Recovery starts with υ positive and increasing, accelerated by the fall of u_{dem}. Boom follows with υ decreasing to zero as the effect of technological unemployment starts to prevail. During crisis υ decreases below zero, accelerated by increasing u_{dem}; and during depression it increases again to zero because the decrease of u_{tec} is prevailing. The upswing is the phase with inflationary pressure.

Capital is scarce relative to *labor* during recovery and boom (υ positive) and

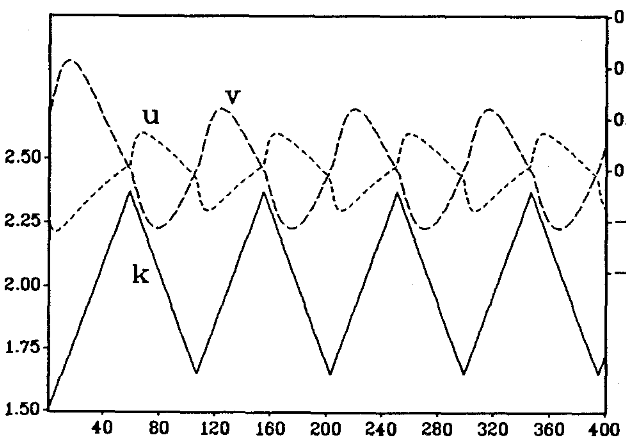

Figure 4.2a – k left scale; u, υ right scale

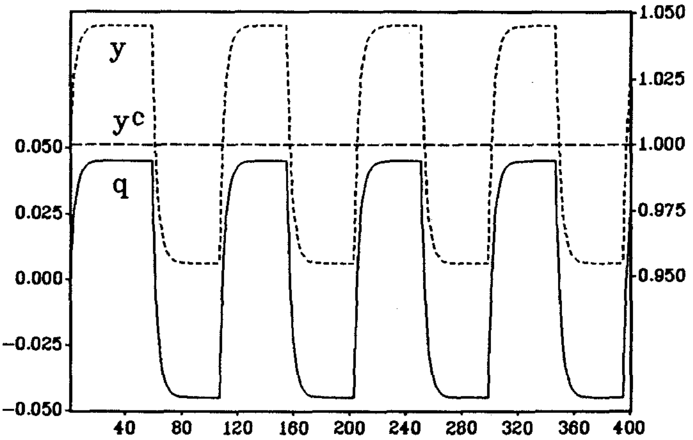

Figure 4.2b – q left scale; y, y^c right scale

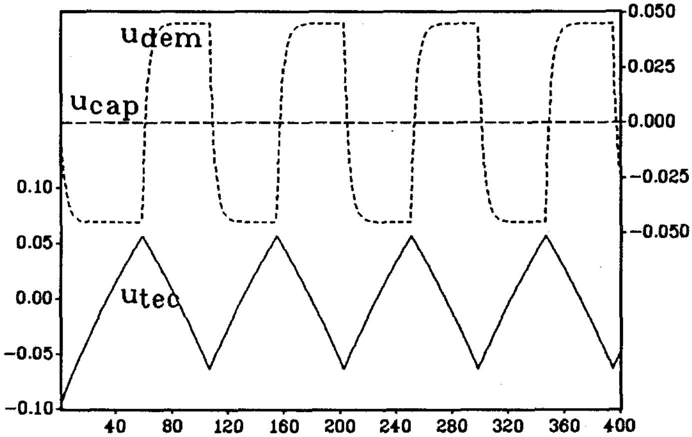

Figure 4.2c – u_{tec} left, u_{dem}, u_{cap} right scale

abundant during crisis and depression. Applying the distinction of section 2, either absolute scarcity or absolute abundance of capital is found, since the sign of q coincides with the sign of v in this case.

The dynamic pattern of unemployment during the stylized long wave may be developed in the same way as above, remembering that now $u_{cap}=0$. The capital gap is purely qualitative now: it is fully absorbed by technological unemployment.

Decomposition of unemployment for stylized υ-cycle

	u	u_{dem}	u_{tec}	gaps
recovery	$- \downarrow L^d>L^s$	$+ \downarrow L^d<L^p$	$- \uparrow L^p>L^s$	$L^p>L^d>L^s$
boom	$- \uparrow L^d>L^s$	$- \downarrow L^d>L^p$	$+ \uparrow L^p<L^s$	$L^p<L^s<L^d$
crisis	$+ \uparrow L^d<L^s$	$- \uparrow L^d>L^p$	$+ \downarrow L^p<L^s$	$L^p<L^d<L^s$
depression	$+ \downarrow L^d<L^s$	$+ \uparrow L^d<L^p$	$- \downarrow L^p>L^s$	$L^p>L^s>L^d$

The unemployment path and its decomposition for the υ-cycle are apparently different from that for the q-cycle.

Example 3: Long wave plus intermediate cycle [Figure 4.3]
The final example is of the mixed type with both replacement and expansion investment. The coefficients are:

$$\pi = 0.02; \quad \sigma = 0.02; \quad k_0 = 1; \quad s = 0.5; \quad \tau = 0.5;$$

$$\Theta_1 = 0.1; \quad \Theta_2 = 0.2; \quad \varepsilon_1 = 0.1; \quad \varepsilon_2 = 0.2; \quad \beta = 0.25; \quad h = 0.2.$$

One should find exactly 4 q-cycles of 8 'years' in one long wave of 32 'years' (or 160 simulation periods, for $h = 0.2$), which is vindicated by Figure 4.3a. Obviously, if the relevant capital output ratio k_0 is for instance 1.5 on a yearly basis in reality, a time period in the model corresponds to 1.5 years in historical time, so that the period length would be 48 years. Figure 4.3a shows the expected mixed long and intermediate range fluctuations for the main variables.

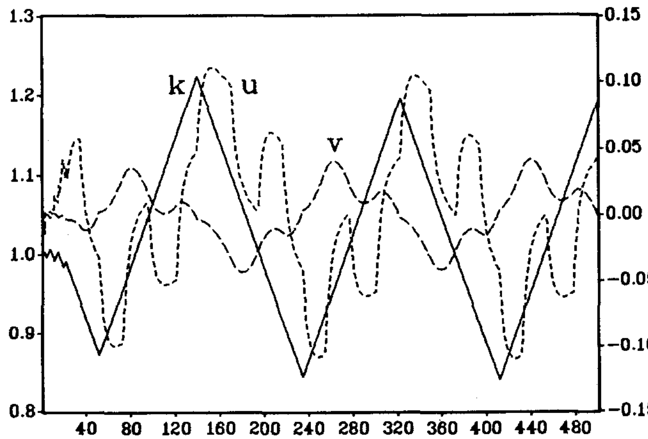

Figure 4.3a – k left scale; u, υ right scale

The repetitive time pattern of the variables, that is characteristic for the limit cycle solutions of the model, is apparent from this figure.

The other variables that result from this example are shown in Figures 4.3b–e. To keep the pictures clear, they are presented in a 'close-up' for one complete long wave from through to through (or approximately simulation periods 240–400, corresponding to 32 'years').

Output, capacity and utilization rate are given in Figure 4.3b. The turning points of the long wave are now accompanied by an extended utilization cycle,

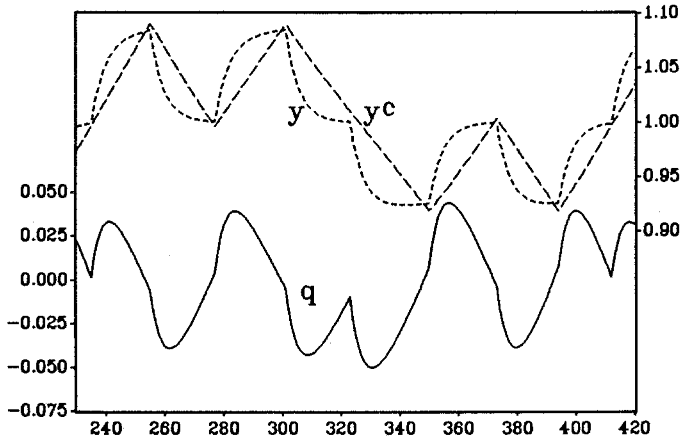

Figure 4.3b – q left scale; y, y^c right scale

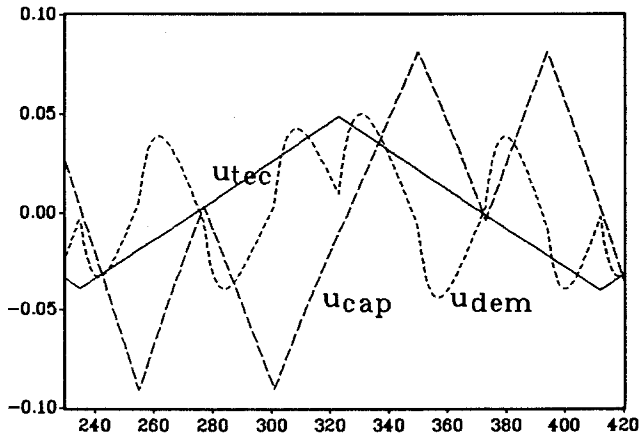

Figure 4.3c

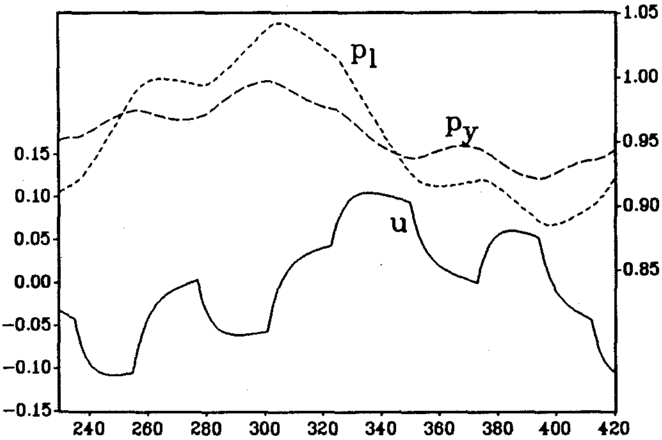

Figure 4.3d – u left scale; p_l, p_y right scale

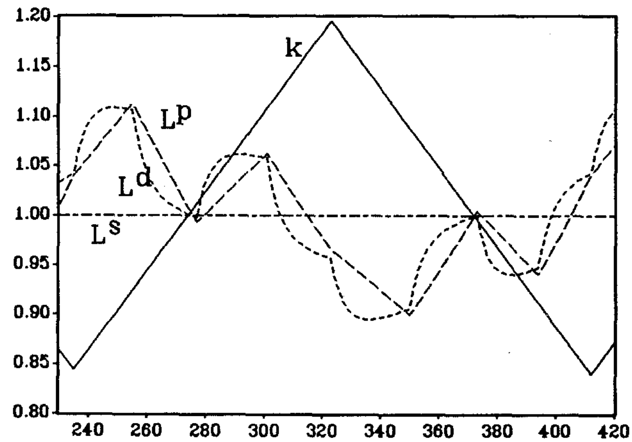

Figure 4.3e – Mixed long/medium employment cycle

which raises capacity to above-average levels during the upswing. So at the upper turning point of the long wave an extended recession or 'crisis' occurs as the onset of deflation, whereas the reverse takes place at the lower turning point. The analysis of unemployment shows that the upward and downward trends of unemployment are accounted for by the changes in technological and capacity related unemployment (Figure 4.3c). The peaks of long cyclical unemployment occur after the upper turning point of the long wave. Figure 4.3d shows how capital shortage coincides with increasing price and wage levels (or inflation exceeding its trend).

Figure 4.3e shows, again in a close-up of one long cycle, actual and potential labor demand, from which the demand and capital gaps appear. Clearly L^d is the mirror image of total unemployment. Figure 4.3e represents the typical picture of a long employment cycle as generated by the model.

The position of the demand gap is subject to cyclical change, which implies a *medium-term employment cycle.* The *long employment cycle* is basically determined by the movement of L^p with respect to L^s (the capital gap). Just as in the pure long wave example, the recovery is triggered by capital scarcity due to past capital shallowing. Excess demand for labor occurs as both components of the capital gap, k^* and y^*, point to the same direction. During the boom L^p decreases as k increases. So in the upswing of the long employment cycle, unemployment has an upward trend, starting from a position of overemployment. The downswing is triggered by capital abundance, and both components of the capital gap give rise to unemployment; technological unemployment is at a high *level* because of past capital deepening, and capacity related unemployment *increases* sharply because capital deepening stops: the extended recession of the intermediate employment cycle. The depression phase of the long employment cycle shows gradual decrease of unemployment from its high level as capital abundance diminishes.

So in conclusion, the typical movements during the *upswing* of a long employment cycle should be:

- output and capacity exceed the trend level; the recovery is marked by an extended period of overutilization (negative demand gap);
- the share of wages and the distributive gap increase during recovery and fall during the boom;
- technological unemployment soars steadily because capital deepening or 'rationalization' takes place; the capital gap in the qualitative sense (βk^*) starts negative but increases;
- the capital gap in the quantitative sense ($-y^*$) is negative since capacity y^* is positive;
- the development of labor productivity exceeds the trend;
- prices and wages increase; inflation exceeds the trend because unemployment starts at a low level.

For the downswing of the long employment cycle the reverse is to be expected.

If one would ask, just by way of a thought experiment, where we find ourselves at present, the answer should be: in the depression phase of a long employment cycle, of which the peak was placed in the early seventies. Unemployment is still high but decreasing; especially, technological unemployment has come down after a prolonged period of productivity slowdown due to reverse capital deepening (at least according to the model). The wage share is low in this phase, but already increasing.

So, looking at the coming decade, the present model would 'predict' that the recovery phase of the long employment cycle could set in after transition to renewed capital deepening investment or rationalization. This would at first generate high investment demand, low unemployment; inflationary tendencies and excess demand would prevail. The model suggests that available technological possibilities would then be used to increase labor productivity. Thus the groundwork would be laid for future technological unemployment, just as it happened during the fifties and sixties; or, for that matter, during the twenties, which could be interpreted as a period of rationalization during the boom of a long upswing preceding the Great Depression (*cf.* Tinbergen (1933)).

Although direct testing of a very stylized theoretical model like the present one looks hard, it may capture some typical features of long and medium employment cycles. Especially, it may help to provide a link between, on the one hand, unemployment analysis in terms of 'regimes' or disequilibrium constellations in labor and goods markets, and, on the other hand, the cyclical analysis of unemployment in the context of nonlinear dynamic models.

5 CONCLUSION

The model presented here sheds light on the interaction of long and intermediate-term employment cycles. The former are related with long-term capital-labor substitution (labor market dynamics), the latter with capacity adjustment (product market dynamics). The method adopted is to apply the nonlinear accelerator mechanism to investment in expansion on the one hand and to replacement investment on the other.

The category of phenomena in reality to which the model refers, is the succession of periods with low and high unemployment, inflation and deflation, capital scarcity and abundance *versus* labor scarcity and abundance.

The emphasis is on the analysis of employment cycles in the long and medium run, rather than output cycles. In the long run, restructuring of capital in the sense of capital deepening and shallowing is the predominant cyclical phenomenon affecting the labor market situation. Thus the apparent trend-like movements of employment and unemployment over longer stretches of time, which transcend the framework of the 'normal' business cycle, are endogenised in a nonlinear analysis of product and labor market disequilibria.

The model joins the disequilibrium tradition in unemployment theory and permits a dynamic analysis of different types of unemployment regimes in terms of demand gaps and distributive gaps, corresponding to unemployment due to deficient demand, deficient capacity, and technological unemployment.

The results in the form of time path and periodicity of cyclical movements have a clear interpretation in terms of the behavioral parameters of the model. Because of the specific approach to investment behavior, the model, though of small shape, is able to capture a wide range of cyclical patterns.

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Summary

ON SIMULTANEOUS EXPLANATION OF LONG AND MEDIUM-TERM EMPLOYMENT CYCLES

Long and medium-term employment cycles are simultaneously analyzed in a dynamic disequilibrium model, which applies the Goodwin (1951)-type nonlinear accelerator to capital widening and capital deepening investment separately. Capacity utilization and profitability are the variables that control this – purely endogenous – dynamic process. A link is provided between the disequilibrium analysis of unemployment by Malinvaud *et al.* and nonlinear cycle theories. Different unemployment regimes, *i.e.* deficient demand, deficient capacity, deficient labor intensity as causes of unemployment, emerge in a dynamic context. Time path and periodicity have a straight interpretation in terms of the behavioral parameters of the model.