Economic Modelling 46 (2015) 334–345

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness

Shilpi Pal [a], G.S. Mahapatra [b,*], G.P. Samanta [a]

[a] Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India
[b] Department of Mathematics, National Institute of Technology Puducherry, Karaikal 609 605, India

ARTICLE INFO

Article history:
Accepted 9 December 2014
Available online 28 January 2015

Keywords:
Ramp-type demand rate
Deterioration
Inflation
Shortage
Triangular fuzzy number
Inventory

ABSTRACT

Economic production quantity (EPQ) is the quantity of a product that should be manufactured in a single batch so as to minimize the total cost. In classical model EPQ only applies where the demand for a product and production rate is constant over the year. But in reality these parameters vary with time in different scenarios. In this paper we have considered a production inventory model for deteriorating items with ramp type demand rate under the effect of inflation and shortages under fuzziness. The deterioration rate is represented by a two-parameter Weibull distribution. As inflation erodes the value of money so we have also considered the effect of inflation when there is shortage in the stock under finite time horizon. Some parameters are vaguely or unclearly defined or whose values are imprecise or determined based on subjective beliefs of individuals. Therefore the inventory model is solved under fuzzy environment to evaluate the optimum solution of the model in different cases. We have optimized our solution by considering production time and production rate as decision variables in two separate cases. While incorporating symmetric triangular fuzzy number we use total λ-integral value to defuzzify the solution. Finally, utility of the model is presented by using some numerical examples and sensitivity analysis and the results are analyzed.

1. Introduction

Now a days, the facts like variation in the demand rate of an item, deterioration, shortages and inflation in the market, are in growing interest. Demand rates are of various types, sometimes it is linear or quadratic time dependent function, or it is a price and stock dependent type of function etc. Recently, many researchers have given considerable attention where the demand rate is a ramp type function of time. This type of demand rate is seen for a new brand item launching in the market. The demand pattern assumed here occurs not only for seasonal product, but also for fashion apparel, computer chips of advanced computer, spare parts, etc. The nature of demand for seasonal and fashionable products is increasing then steady then decreasing and finally becoming asymptotic. The demand of the item increases with time and then stabilizes after some time and ultimately becomes constant. Thus the demand rate is deterministic when any new brand product is launched in the markets, the demand rate linearly depends on time, and later it gets stabilized in the market.

It is noted that the effect of deterioration cannot be ignored in any inventory model. Deterioration means decay, wastage or damage in a way the item cannot be used further for its original purpose. Goods undergo deterioration over time. Food items (like fruits, vegetable, etc), radioactive substance, drugs, photographic films, etc. undergo spoilage during normal storage. Highly volatile liquids (like alcohol, turpentine, etc) undergo depletion over time by the process of evaporation. Thus decay is a natural phenomenon and it cannot be ignored. But to denote mathematically the leakage failure of the batteries and life expectancy of ethical drugs we use the Weibull distribution. We have considered the model with shortages while the unsatisfied demand is completely backlogged. Due to shortages, partially the inventory is backordered and partially there is lost in sales. Also we have taken into consideration a finite time horizon under inflation. We use inventory carrying cost to determine how much inventory we will keep on hand. Inflation affects the inventory model by increasing carrying costs (because the inflation pushes up the interest rate) which results in a small inventory level. This small quantity is misleading and results in an increase in inventory related costs. Hence, to calculate the optimum order size, cost to carry should be reduced by inflationary impact on interest cost.

In this model we have taken the ramp type function of time as demand rate and deterioration as Weibull's distribution. In reality, the rate and cost fluctuate with time due to various reasons in the market as well as in production unit. So we try to modify the model by fuzzifying the rates and costs to coincide with the realistic situation and check its effect in the model. In the model we have considered the inflationary effect under shortages and thus the lost in sales cannot be ignored. In Section 2 we have taken some assumptions and denoted the variables by some notations that we have used in this paper. In

* Corresponding author. Tel.: +91 9433135327; fax: +91 4368231665.
E-mail address: g_s_mahapatra@yahoo.com (G.S. Mahapatra).

http://dx.doi.org/10.1016/j.econmod.2014.12.031

Section 3 we have defined the inventory model with detailed analysis of the model along with the minimization in two different cases. In Section 4 we have observed the effect of fuzziness on the proposed inventory model due to imprecise parameters. It is followed by a numerical example and mathematical analysis has shown graphically the solution of the problem of this paper in Section 4. Lastly in Section 5 we have done the sensitivity analysis of the proposed inventory model.

1.1. Literature survey

In the classical inventory the demand rate was considered as a constant function of time. Covert and Philip (1973), and Philip (1974) considered an inventory model with a constant demand rate. Thereafter researchers started observing various types of demand function. Sometimes demand of the item depends on the on hand stock, price of the items, etc. This type of demand rate was discussed by Hou (2006) considering a stock-dependent demand rate for deteriorating items with shortages. Chung and Wee (2011) scheduled an inventory model for stock dependent selling rate and a replenishment plan for an integrated deteriorating item. Some notable researches were done by Yang et al. (2010) and Wu et al. (2006) in the direction of non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Sarkar and Sarkar (2013a) have also worked on the model with partial backlogging, time varying deterioration and stock dependent demand. The time varying demand rate actually reflects sales of the item in different phases of the time i.e., demand as well as the sales increases in the growth phase and decreases in the decline phase. This type of demand rate was discussed by many researchers like Hariga (1995) and Bose et al. (1995). Recently researchers are working on the demand pattern of new products launching in the market as a ramp type function of time. Ahmed et al. (2013) proposed a model for new EOQ policy considering ramp type demand rate for deteriorating items with partial backlogging. Skouri et al. (2011a,b) discussed ramp type demand rate with time dependent deterioration. Skouri et al. (2011a,b) also have formulated supply chain models for deteriorating item. Samanta and Bhowmick (2010) formulated a continuous order-level inventory model with ramp type demand rate and deterioration as the Weibull distribution. Agrawal and Banerjee (2011) developed an algorithm considering a two-warehouse inventory model with ramp type demand rate under shortages, where the constant fraction of shortages is backlogged. Wu (2001) worked on an inventory model with ramp type demand rate and Weibull distribution deterioration under shortage and partial backlogging, where it is a variable. An investigation on short life-cycle deteriorating product remanufacturing in a green supply chain inventory control system was developed by Chung and Wee (2011). An EOQ model for deteriorating items with planned backorder level was developed by Widyadana et al. (2011). A production inventory model with random machine breakdown and stochastic repair time was addressed by Widyadana and Wee (2011). Yang et al. (2002) considered the demand of the items as power demand pattern with shortages.

In the beginning researchers did not notice the effect of deterioration in the inventory model. But later they realized that deterioration is a natural phenomenon and so thereafter they started considering the effect of it, in their inventory models. Sarkar and Sarkar (2013b) have worked on variable deterioration and demand. Begum et al. (2012) proposed a replenishment policy with time proportional deterioration and no shortages. An inventory model with a two parameter Weibull distribution was developed by Covert and Philip (1973). It was further developed by Philip (1974). Both of the above papers by Covert and Philip did allow shortages. The effect of deterioration as the Weibull distribution was also dealt by Sharma and Chaudhury (2013) and Skouri et al. (2009). Wu and Ouyang (2000) formulated a replenishment policy for deteriorating items with ramp type demand rate.

Again the classical inventory models did not consider the effect of inflation and time value of the money. But in the last thirty–forty years the economic condition of most of the countries has changed and thus the effect of inflation and time value of money cannot be ignored. In recent past there is a heavy blow to the world economy and thus the time value of money is changing on a day-to-day basis. Buzacott (1975) was the first to consider the inflationary effect in the inventory model assuming constant inflation rate. Datta and Pal (1991); Jolai et al. (2006); Wee et al. (2008); Jaggi et al. (2011); and Neetu and Tomer (2012) have considered an inventory model for deteriorating items under the inflationary effect. Pal et al. (2014b) worked on price and stock dependent demand for deteriorating item under inflation and delay in payment. Yang (2012) worked on two-warehouse partial backlogging inventory models with a three parameter Weibull distribution deterioration under inflation. Uthavakumar and Rameswari (2012) worked on the economic order quantity model for deteriorating items with time discounting. Lin and Lin (2006) studied and proposed a purchasing model, which considers a case of time-varying deterioration, partial back-ordering that depends on the waiting time for backlogging, and time value of money over a finite horizon. Many other researchers have considered the EOQ model under shortage, inflation and finite time horizon. Yang (2011) worked on partial backlogging for a deteriorating item with time varying production rate and demand rate.

Generally, in the inventory model all the cost parameters are taken constant but in reality they are uncertain in nature. So researchers try to introduce the concept of fuzzy in their model. Fuzzy set was first introduced by Zadeh (1965). Then the researchers have used it in various fields. Giannoccaro et al. (2003) considered a fuzzy echelon form in an inventory management system. De and Sana (2013) worked on a fuzzy ordered inventory model with fuzzy shortage and fuzzy promotional index. Mahapatra and Roy (2006) worked on a reliability optimization model using fuzzy multi-objective mathematical programming. Chen and Liu (2007) presented the optimum profit model between the producers and the purchasers for the supply chain system with a pure procurement policy and this paper was further modified by Chen and Lu (2011) where they optimize the profit model by considering production, quantity and sale problem. Mahapatra et al. (2012) developed an imprecise space constraint EPQ model based on an intuitionistic fuzzy optimization technique. Pal et al. (2014a) developed an EPQ model for a ramp type demand with Weibull deterioration under inflation in finite time horizon in crisp and fuzzy environment.

2. Assumptions and notations

We have formulated our model using the following notations and assumptions.

2.1. Notations

D	Demand rate,
K	Production rate,
$\theta(t)$	Distribution of time for deterioration of the item where t denotes time of deterioration,
h	Holding cost per unit per unit time,

c_1 Purchase cost per unit item,
c_2 Setup cost for production per cycle,
c_3 Shortage cost,
c_4 Penalty cost of a lost sale including loss of profit,
B Fraction of stock demand backordered $(0 < B \le 1)$,
k Inflationary rate i.e., the difference between capital cost and cost after inflation,
T One replenishment cycle,
H Planning horizon,
m No. of replenishment during the planning horizon i.e., $m = \frac{H}{T}$,
T_j Time between start and end of jth replenishment cycle i.e., $T_0 = 0, T_1 = T, T_2 = 2T, \ldots, T_m = mT = H$.
Q_m Maximum inventory,
Q_s Minimum inventory i.e., inventory after shortage.

2.2. Assumption

(i) Demand rate $D = f(t)$ is assumed to be a ramp type function of time $f(t) = P[t - (t-\mu)H(t-\mu)], P > 0$ and $H(t)$ is a heaviside function $H(t-\mu) = \begin{cases} 1, & \text{if } t \ge \mu \\ 0, & \text{if } t < \mu \end{cases}$,
(ii) Deterioration of an item varies with time, here we consider it as a two parameter Weibull distribution of time, and the distribution function is $\theta(t) = \alpha\beta t^{\beta-1}, 0 < \alpha < 1, \beta \ge 1$ where t denotes time of deterioration,
(iii) Deterioration starts as soon as it comes in the inventory,
(iv) One item is considered in one replenishment cycle,
(v) Inflation is taken into consideration,
(vi) Shortages are allowed,
(vii) Delivery lead time is zero,
(viii) Model is considered under finite time horizon,
(ix) Production rate is greater than demand rate so $K = \gamma f(t)$ is the production rate where $\gamma > 1$,
(x) Demand during shortage is partially lost and partially backordered.

2.3. Formulation of the inventory model

According to the assumptions, we formulated mathematically the proposed inventory model. The graphical presentation of the proposed inventory model is shown in Fig. 1.

We analyze the deterministic inventory model for deteriorating items with shortages. Production of the item starts at time $t = 0$. From $t = 0$ to $t = t_1$ the model undergoes production as well as supplies for the demand in the market, and at $t = t_1$ the inventory level is maximum, Q_s. From $t = t_1$ to $t = t_2$ the inventory level decreases due to deterioration and demand. Also at time $t = t_2$, the inventory level reaches zero. Thereafter during $[t_2, t_3]$ the model undergoes shortage where the part of shortage is backlogged and part of it is lost in sales due to lack of inventory. Only the backlogged items are replaced by the next replenishments. During $[t_3, T_1]$ production resumes to make up for the part of shortage which backlogged the items. Thus the total number of backlogged items is replaced by the next replenishment.

The inventory level of the system at any time t over $[0, T]$ can be described by the following equations:

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = K - D = (\gamma - 1)Pt, \quad 0 \le t \le \mu \tag{1}$$

with the condition, $Q(0) = 0$,

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = (\gamma - 1)P\mu, \quad \mu \le t \le t_1 \tag{2}$$

with the condition, $Q(\mu) = \frac{1}{e^{\alpha\mu^\beta}} \int_0^\mu (\gamma - 1)Pxe^{\alpha x^\beta} dx$,

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -D = -P\mu, \quad t_1 \le t \le t_2 \tag{3}$$

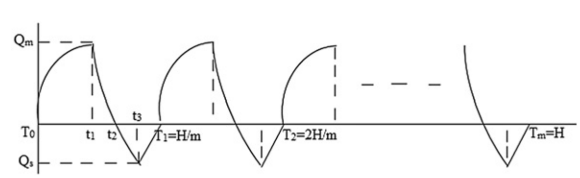

Fig. 1. Graphical representation of the inventory model.

with the condition, $Q(t_1) = Q_m$ and $Q(t_2) = 0$,

$$\frac{dQ(t)}{dt} = -BD = -BP\mu, \quad t_2 \le t \le t_3 \tag{4}$$

with the condition, $Q(t_2) = 0$ and $Q(t_3) = -Q_s$,

$$\frac{dQ(t)}{dt} = K-D = K-P\mu = (\gamma-1)P\mu, \quad t_3 \le t \le T_1 \tag{5}$$

with the condition, $Q(t_3) = -Q_s$ and $Q(T_1) = 0$.

From the differential equation (Eq. (1)) we obtain the inventory level of the proposed inventory model during time span $0 \le t \le \mu$ as follows:

$$Q(t) = e^{-\alpha t^\beta}\int_0^t (\gamma-1)Pxe^{\alpha x^\beta}dx, \ \ 0 \le t \le \mu.$$

Again from Eq. (2) we get,

$$Q(t) = e^{-\alpha t^\beta}\int_\mu^t (\gamma-1)P\mu e^{\alpha x^\beta}dx + c.$$

Now using the initial condition we get,

$$Q(t) = e^{-\alpha t^\beta}\int_\mu^t (\gamma-1)P\mu e^{\alpha x^\beta}dx + e^{-\alpha \mu^\beta}\int_0^\mu (\gamma-1)Pxe^{\alpha x^\beta}dx, \ \ \mu \le t \le t_1. \tag{6}$$

Now solving Eq. (3) we get,

$$Q(t) = e^{-\alpha t^\beta}\int_{t_1}^t -P\mu e^{\alpha x^\beta}dx + Q_m, \ \ t_1 \le t \le t_2. \tag{7}$$

From Eq. (7) and using $Q(t_2) = 0$ we get,

$$Q_m = e^{-\alpha t_2^\beta}\int_{t_1}^{t_2} P\mu e^{\alpha x^\beta}dx. \tag{8}$$

Again from Eq. (4) we get,

$$Q(t) = BP\mu(t_2-t), \ \ t_2 \le t \le t_3. \tag{9}$$

Now using Eq. (9) and $Q(t_3) = -Q_s$ we get,

$$Q_s = BP\mu(t_3-t_2). \tag{10}$$

Again from Eq. (5) we obtain,

$$Q(t) = (\gamma-1)P\mu(t-t_3)-Q_s, \ \ t_3 \le t \le T_1. \tag{11}$$

Now using Eq. (11) and $Q(T_1) = 0$, we get,

$$Q_s = (\gamma-1)P\mu(T_1-t_3). \tag{12}$$

2.3.1. Present value of holding cost (HC)

As we have to keep the item in stock during $[0, t_2]$, we calculate the cost of holding in this period. During $[t_2, t_3]$, the item undergoes shortage so there is no item to store. And during $[t_3, T_1]$ the items which are produced are used to overcome the backorder so no items are required to be held or stored during this period. Hence,

$$\begin{aligned} HC &= h\left\{\int_0^\mu Q(t)e^{-kt}dt + \int_\mu^{t_1} Q(t)e^{-kt}dt + \int_{t_1}^{t_2} Q(t)e^{-kt}dt\right\} \\ &= h\Bigg[\int_0^\mu\left(\int_0^t (\gamma-1)Pxe^{\alpha x^\beta}dx\right)e^{-kt-\alpha t^\beta}dt + \int_\mu^{t_1}[\left(\int_0^\mu (\gamma-1)Pxe^{\alpha x^\beta}dx\right)e^{-kt-\alpha\mu^\beta} \\ &\quad + \left(\int_\mu^t (\gamma-1)P\mu e^{\alpha x^\beta}dx\right)e^{-kt-\alpha t^\beta}]dt + \int_{t_1}^{t_2}\left[\left(\int_{t_1}^t -P\mu e^{\alpha x^\beta}dx)e^{-kt-\alpha t^\beta} + Q_m e^{-kt}\right]dt\right]. \end{aligned}$$

Using Maclaurin's approximation, we get,

$$
\begin{aligned}
&= h(\gamma-1)P\left\{\int_0^{\mu}\left[\int_0^{t}x\left(\sum_{n=0}^{\infty}\frac{\left(\alpha x^{\beta}\right)^n}{n!}\right)dx\right]\sum_{n=0}^{\infty}\frac{\left(-kt-\alpha t^{\beta}\right)^n}{n!}dt+\int_{\mu}^{t_1}\left\{\left[\int_0^{\mu}x\left(\sum_{n=0}^{\infty}\frac{\left(\alpha x^{\beta}\right)^n}{n!}\right)dx\right]\sum_{n=0}^{\infty}\frac{\left(-kt-\alpha\mu^{\beta}\right)^n}{n!}\right.\right.\\
&\quad\left.\left.+\left[\mu\int_{\mu}^{t}\left(\sum_{n=0}^{\infty}\frac{\left(\alpha x^{\beta}\right)^n}{n!}\right)dx\right]\left[\sum_{n=0}^{\infty}\frac{\left(-kt-\alpha t^{\beta}\right)^n}{n!}\right]dt\right\}+hP\mu\right.\\
&\quad\times\left\{\int_{t_1}^{t_2}\left\{\left[-\int_{t_1}^{t}\left(\sum_{n=0}^{\infty}\frac{\left(\alpha x^{\beta}\right)^n}{n!}\right)dx\right]\sum_{n=0}^{\infty}\frac{\left(-kt-\alpha t^{\beta}\right)^n}{n!}+\left[\int_{t_1}^{t_2}\left(\sum_{n=0}^{\infty}\frac{\left(\alpha x^{\beta}\right)^n}{n!}\right)dx\right]\sum_{n=0}^{\infty}\frac{\left(-kt-\alpha t_2^{\beta}\right)^n}{n!}\right\}dt\right\} \qquad (13)\\
&= hP\left[(\gamma-1)\left(\frac{\mu^3}{6}\left(1-\frac{k\mu}{4}\right)+\frac{\alpha\beta\mu^{\beta+3}\left(\beta^2+5\beta+8\right)}{2(\beta+1)(\beta+2)(\beta+3)}+\frac{\mu t_1(t_1-\mu)}{2}+k\mu t_1^2\left(\frac{\mu}{4}-\frac{t_1}{3}\right)-\frac{\alpha\mu t_1\left[\mu^{\beta+1}\left(\beta^2+3\beta+4\right)-\beta t_1^{\beta+1}\right]}{2(\beta+1)(\beta+2)}+\frac{\alpha\mu^2 t_1^{\beta+1}}{\beta+1}\right)\right.\\
&\quad\left.+\mu\left\{\frac{(t_2-t_1)^2}{2}-\frac{\alpha\beta t_2^{\beta+2}}{\beta+2}+\frac{2\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1 t_2\left(2\beta t_2^{\beta}-t_1^{\beta}\right)}{\beta+1}-\alpha t_1^2 t_2^{\beta}-\frac{k}{6}\left(t_2^3-3t_2t_1^2+2t_1^3\right)\right\}\right].
\end{aligned}
$$

2.3.2. Present value of purchase cost (PC)

To produce an item the supplier has to purchase the raw material. Since the items are produced during $[0, T_1]$ and $[t_3, T_1]$ (production during this period is to overcome the shortage), we calculate purchase cost during the above period as

$$PC=\gamma c_1\left(\int_0^{\mu}Ptdt+\int_{\mu}^{t_1}P\mu dt+\int_{t_3}^{T_1}P\mu e^{-kt_3}dt\right)=\gamma c_1P\mu\left(t_1-\frac{\mu}{2}+(1-kt_3)(T_1-t_3)\right). \qquad (14)$$

2.3.3. Present value of deterioration cost (DC)

There is no deterioration of items during the period $[t_2, T_1]$. So the total no. of deteriorated items in $[0, T_1]$ is same as deterioration in $[0, t_2]$.

$$
\begin{aligned}
&\text{Total no. of deteriorated items in } [0,t_2]=d_1\\
&\quad=\text{Production in } [0,\mu]+\text{ Production in } [\mu,t_1]-\text{ Demand in } [0,\mu]-\text{Demand in } [\mu,t_2]\\
&\quad=\gamma\int_0^{\mu}Ptdt+\gamma\int_{\mu}^{t_1}P\mu dt-\int_0^{\mu}Ptdt-\int_{\mu}^{t_2}P\mu dt=\frac{1}{2}P\gamma(2t_1-\mu)-\frac{1}{2}P\mu(2t_2-\mu).
\end{aligned}
$$

Therefore the deterioration cost $DC = gd_1$

$$=\frac{1}{2}Pg\{\gamma(2t_1-\mu)-\mu(2t_2-\mu)\}. \qquad (15)$$

2.3.4. Present value of shortage cost (SC)

In our inventory model shortages hold during $[t_2, t_3]$ and to overcome the backorders due to shortages items are produced during $[t_2, T_1]$. Thus the total time period for shortages is $[t_2, T_1]$.

$$
\begin{aligned}
SC&=c_3\int_{t_2}^{t_3}-Q(t)e^{-k(t_2+t)}dt+c_3\int_{t_3}^{T_1}-Q(t)e^{-k(t_3+t)}dt\\
&=c_3\int_{t_2}^{t_3}BP\mu(t-t_2)(1-k(t_2+t))dt+c_3\int_{t_3}^{T_1}-[(\gamma-1)P\mu(t-t_3)-(\gamma-1)P\mu(T_1-t_3)](1-k(t_3+t))dt \qquad (16)\\
&=c_3P\mu\left[B(t_3-t_2)\left\{\frac{(t_3-t_2)}{2}-\frac{k}{3}\left(t_3^2+t_2t_3-2t_2^2\right)\right\}+(\gamma-1)\frac{(T_1-t_3)^2}{6}\{3-k(T_1+5t_3)\}\right]
\end{aligned}
$$

2.3.5. Present value of lost cost (LC)

During the shortage period $[t_2, t_3]$, there is an opportunity for loss in sale. So the present worth lost cost for one replenishment interval is

$$LC=c_4(1-B)P(t_3-t_2)e^{-kt_3}\approx c_4(1-B)P(t_3-t_2)(1-kt_3). \qquad (17)$$

2.3.6. Present value of setup cost (STC)

The 1st cycle has two setup costs. The first one occurs at the starting of the cycle and the second one occurs at $t = t_3$. The production set-up cost of subsequent cycle at $t = iT = T_i$ where $i = 2, 3, \ldots, m - 1$

$$SC=\begin{cases}c_2+c_2e^{-kt_3}=c_2+S, & \text{for the 1st cycle}\\ c_2e^{-kt_3}=S, & \text{for the other cycle}\end{cases}. \qquad (18)$$

Therefore the total cost in present value is (TC):

$$\begin{aligned} TC &= c_2 + (DC + PC + HC + LC + S + SC)\sum_{i=1}^{m} e^{-(i-1)kT} \\ &\approx c_2 + (DC + PC + HC + LC + S + SC)\frac{1-e^{-kmT}}{1-e^{-kT}} \end{aligned} \tag{19}$$

$$\begin{aligned} &= c_2 + \Bigg(\frac{1}{2}Pg\{\gamma(2t_1-\mu)-\mu(2T_1-\mu)\} + \gamma c_1 P\mu\left(t_1-\frac{\mu}{2}+(1-kt_3)(T_1-t_3)\right) + hP\Bigg[(\gamma-1)\Bigg\{\frac{\alpha\beta\mu^{\beta+3}\left(\beta^2+5\beta+8\right)\mu^3}{2(\beta+1)(\beta+2)(\beta+3)24}(4-k\mu) \\ &+\frac{\mu t_1(t_1-\mu)}{2} + k\mu t_1^2\left(\frac{\mu}{4}-\frac{t_1}{3}\right) + \frac{\alpha\mu^2 t_1^{\beta+1}}{\beta+1} - \frac{\alpha\mu t_1\left[\mu^{\beta+1}\left(\beta^2+3\beta+4\right)-\beta t_1^{\beta+1}\right]}{2(\beta+1)(\beta+2)}\Bigg\} + \frac{\mu\alpha t_1 t_2\left(2\beta t_2^{\beta}-t_1^{\beta}\right)}{\beta+1} \\ &+\mu\left\{\frac{(t_2-t_1)^2}{2} - \frac{\alpha\beta t_2^{\beta+2}}{\beta+2} + \frac{2\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \alpha t_1^2 t_2^{\beta} - \frac{k}{6}\left(t_2^3-3t_2t_1^2+2t_1^3\right)\right\}\Bigg] + c_4(1-B)P(t_3-t_2)(1-kt_3) + c_2e^{-kt_3} \\ &+c_3P\mu\left[B(t_3-t_2)\left\{\frac{(t_3-t_2)}{2}-\frac{k}{3}\left(t_3^2+t_2t_3-2t_2^2\right)\right\} + (\gamma-1)\frac{(T_1-t_3)^2}{6}\{3-k(T_1+5t_3)\}\right]\Bigg)\frac{1-e^{-mkT}}{1-e^{-kT}}. \end{aligned} \tag{20}$$

We observe that TC is a function of t_1, t_2, t_3, m and γ. But for the sake of simplicity, we simplified t_1 and t_3 in terms of t_2, m and γ. Considering Eqs. (6), (7) and (8) we get t_1 in terms of t_2, m and γ.

$$e^{-\alpha\mu^{\beta}}\int_0^{\mu}(\gamma-1)Pxe^{\alpha x^{\beta}}dx + e^{-\alpha t_1^{\beta}}\int_{\mu}^{t_1}(\gamma-1)P\mu e^{\alpha x^{\beta}}dx = e^{-\alpha t_2^{\beta}}\int_{t_1}^{t_2}P\mu e^{\alpha x^{\beta}}dx$$

Also considering Eqs. (10) and (12) we get t_3 in terms of t_2.

$$BP\mu(t_3-t_2) = (\gamma-1)P\mu(T_1-t_3)$$

Expanding the exponential and neglecting the second and higher order terms of α and after simplifying the above two equations we get,

$$t_1 = \frac{1}{\gamma}\left(t_2-\alpha t_2^{\beta}\left(\frac{\beta t_2}{\beta+1}+\frac{t_2}{\gamma}+\frac{(\gamma-1)\mu}{2\gamma}\right)+\xi_1\right) \tag{21}$$

$$t_3 = \frac{1}{B+\gamma-1}((\gamma-1)T_1+Bt_2) \tag{22}$$

where $\xi_1 = (\gamma-1)\left(\frac{\mu}{2}+\frac{\alpha\left(\beta^2+3\beta+4\right)\mu^{\beta+1}}{2(\beta+1)(\beta+2)}\right)$

Case 1. Let t_2 be the decision variable (for fixed production rate (γ)).

Thus the total cost TC is a function of t_2 and m. Now we can optimize i.e., minimize the total cost $TC(t_2, m)$ by obtaining the time taken t_2^* of our model, using the necessary condition for $\frac{dTC(t_2,m^*)}{dt_2} = 0$ for a given value (i.e., fixed value) of m (a positive integer), where the optimum value m^* is obtain from those values of m for which the total cost is minimum. To ensure that the objective function is convex, the derived value of $TC(t_2^*, m^*)$ must satisfy the sufficient condition:

$$\frac{dTC^2(t_2,m^*)}{dt_2^2} \geq 0. \tag{23}$$

Here we want to optimize the time i.e., at want time the inventory should finish so that we obtain minimum total cost.

Case 2. Let the production rate (γ) be the decision variable (for fixed t_2).

Thus in this case the total cost TC is a function of γ and m. So we can optimize i.e., minimize the total cost $TC(\gamma, m)$ by obtaining the optimum production rate γ^* for our model, using the necessary condition for $\frac{dTC(\gamma,m^*)}{d\gamma} = 0$ for a given value (i.e., fixed value) of m (a positive integer), where the optimum value m^* is obtain from those values of m for which the total cost is minimum. To ensure that the objective function is convex, the derived value of $TC(\gamma^*, m^*)$ must satisfy the sufficient condition:

$$\frac{dTC^2(\gamma,m^*)}{d\gamma^2} \geq 0. \tag{24}$$

Here we have optimized the production rate γ to know at what rate the producer should produce so that the total cost is minimized (though there is a shortage).

Since in both cases, TC is a very complicated function with high powers in the expression it is impossible to show the analytical validity of the above sufficient condition i.e., of Eqs. (23) and (24). Thus the inequality of Eqs. (23) and (24) is assessed and shown numerically.

3. Effect of fuzziness of parameter on proposed inventory model

Fuzzy set (see Appendix A) is providing a new mathematical tool that helps us to handle imprecise and ambiguous notion of any inventory model. The holding cost (h), purchasing cost (c_1) and inflation rate (k) of the proposed inventory model are considered as fuzzy, since in reality the above rates and costs fluctuate with time due to various reasons in the market as well as in production place. So we try to modify the model by fuzzifying the above rate and costs to coincide with the real situation and check the effect on the model. The holding cost h, purchasing cost c_1, and inflation rate k are replaced by $\widetilde{h}$, $\widetilde{c}_1$ and $\widetilde{k}$ which are respective triangular fuzzy number and it is represented by $\widetilde{h} = (h-\delta_1, h, h+\delta_1)$, $\widetilde{c}_1 = (c_1-\delta_2, c_1, c_1+\delta_2)$, and $\widetilde{k} = (k-\delta_3, k, k+\delta_3)$, where $0<\delta_1<1$, $0<\delta_2<1$ and $0<\delta_3<1$. This triangular fuzzy number $\widetilde{h}$, $\widetilde{c}_1$ and $\widetilde{k}$ can be used to fuzzify the total costs TC and this determines the fuzzy total expected cost per unit time

$$
\begin{aligned}
TC\left(\widetilde{h},\widetilde{c}_1,\widetilde{k}\right) &= c_2 + \Bigg(g\left(\frac{1}{2}P\gamma(2t_1-\mu)-\frac{1}{2}P\mu(2T_1-\mu)\right) + \gamma\widetilde{c}_1P\mu\left(t_1-\frac{\mu}{2}+\left(1-\widetilde{k}t_3\right)(T-t_3)\right) \\
&+\widetilde{h}P\Bigg[(\gamma-1)\Bigg(\frac{\mu^3}{6}\left(1-\frac{\widetilde{k}\mu}{4}\right)+\frac{\alpha\beta\mu^{\beta+3}\left(\beta^2+5\beta+8\right)}{2(\beta+1)(\beta+2)(\beta+3)}+\frac{\mu t_1(t_1-\mu)}{2}+\widetilde{k}\mu t_1^2\left(\frac{\mu}{4}-\frac{t_1}{3}\right)+\frac{\alpha\mu^2t_1^{\beta+1}}{\beta+1} \\
&-\frac{\alpha\mu t_1\left[\mu^{\beta+1}\left(\beta^2+3\beta+4\right)-\beta t_1^{\beta+1}\right]}{2(\beta+1)(\beta+2)}\Bigg)+\mu\Bigg\{\frac{(t_2-t_1)^2}{2}-\frac{\alpha\beta t_2^{\beta+2}}{\beta+2}+\frac{2\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1t_2\left(2\beta t_2^{\beta}-t_1^{\beta}\right)}{\beta+1} \\
&-\alpha t_1^2t_2^{\beta}-\frac{\widetilde{k}}{6}\left(t_2^3-3t_2t_1^2+2t_1^3\right)\Bigg\}\Bigg]+c_2e^{-\widetilde{k}t_3}+c_3P\mu\Bigg[(\gamma-1)\frac{(T_1-t_3)^2}{6}\left\{3-\widetilde{k}(T_1+5t_3)\right\} \\
&+B(t_3-t_2)\left\{\frac{(t_3-t_2)}{2}-\frac{\widetilde{k}}{3}\left(t_3^2+t_2t_3-2t_2^2\right)\right\}\Bigg]+c_4(1-B)P(t_3-t_2)\left(1-\widetilde{k}t_3\right)\Bigg)\left(\frac{1-e^{-m\widetilde{k}T}}{1-e^{-\widetilde{k}T}}\right).
\end{aligned} \tag{25}
$$

The total λ-integral value is a convex combination of the left and right integral values with the degree of optimism. The left integral value represents the pessimist point of view whereas, the right integral value represents the optimistic point of view for decision making purpose. Larger the value of λ, the higher the degree of optimism.

Using the total λ-integral value as done by Mahapatra and Roy (2006) (see Appendix A) for the fuzzy parameter of the proposed inventory model, we get,

$$
I_\lambda\left(\widetilde{h}\right) = h+\left(\lambda-\frac{1}{2}\right)\delta_1,\;\; I_\lambda(\widetilde{c}_1) = c_1+\left(\lambda-\frac{1}{2}\right)\delta_2,\;\; I_\lambda\left(\widetilde{k}\right) = k+\left(\lambda-\frac{1}{2}\right)\delta_3.
$$

Thus defuzzifying Eq. (25) we get,

$$
\begin{aligned}
TC' &= c_2+\Bigg(g\left(\frac{1}{2}P\gamma(2t_1-\mu)-\frac{1}{2}P\mu(2T_1-\mu)\right)+\gamma\left(c_1+\left(\lambda-\frac{1}{2}\right)\delta_2\right)P\mu\left(\left(1-\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)t_3\right)(T-t_3)\right. \\
&\left.+t_1-\frac{\mu}{2}\right)+\left(h+\left(\lambda-\frac{1}{2}\right)\delta_1\right)P\Bigg[(\gamma-1)\Bigg(\frac{\mu^3}{6}\left(1-\frac{\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)\mu}{4}\right)+\frac{\alpha\beta\mu^{\beta+3}\left(\beta^2+5\beta+8\right)}{2(\beta+1)(\beta+2)(\beta+3)}+\frac{\mu t_1(t_1-\mu)}{2} \\
&+\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)\mu t_1^2\left(\frac{\mu}{4}-\frac{t_1}{3}\right)-\frac{\alpha\mu t_1\left[\mu^{\beta+1}\left(\beta^2+3\beta+4\right)-\beta t_1^{\beta+1}\right]}{2(\beta+1)(\beta+2)}+\frac{\alpha\mu^2t_1^{\beta+1}}{\beta+1}\Bigg)+\mu\Bigg\{\frac{(t_2-t_1)^2}{2}-\frac{\alpha\beta t_2^{\beta+2}}{\beta+2} \\
&-\alpha t_1^2t_2^{\beta}+\frac{2\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1t_2\left(2\beta t_2^{\beta}-t_1^{\beta}\right)}{\beta+1}-\frac{\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)}{6}\left(t_2^3-3t_2t_1^2+2t_1^3\right)\Bigg\}\Bigg]+c_2e^{-\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)t_3} \\
&+c_4(1-B)P(t_3-t_2)\left(1-\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)t_3\right)+c_3P\mu\Bigg[B(t_3-t_2)\left\{\frac{(t_3-t_2)}{2}-\frac{\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)}{3}\left(t_3^2+t_2t_3-2t_2^2\right)\right\} \\
&+(\gamma-1)\frac{(T_1-t_3)^2}{6}\left\{3-\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)(T_1+5t_3)\right\}\Bigg]\Bigg)\left(\frac{1-e^{-m\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)T}}{1-e^{-\left(k+\left(\lambda-\frac{1}{2}\right)\delta_3\right)T}}\right).
\end{aligned} \tag{26}
$$

4. Numerical example

In this section we present a numerical example in support of our proposed inventory model under crisp and fuzzy sense. While launching a new brand item (like seasonal items or fashionable products) in the market the demand rate follows a ramp type function of time. Demand is considered as 100 unit per cycle. The item obviously undergoes deterioration (or gets expired) with time and it is in the form $\alpha\beta t^{\beta-1}(\alpha = 0.001$ and $\beta = 2)$ which costs 2$ per unit time. To produce the item the retailer has to purchase raw material at 2$ per unit item and has spent 4$ to set up for the production cycle and 6$ per unit to hold the item in the store. During shortages, which cost 3$, part of the customers leave if the item is not available and part of the customers wait for the item to come in the market and hence it is

Table 1
Optimal solution of the inventory model for different replenishments.

m	T in years	t_1^* in years	t_2^* in years	t_3^* in years	TC^*
2	5	1.088	1.431	3.81	1723.55
3	3.33	1.05	1.375	2.681	1731.92
4	2.5	1.024	1.336	2.112	1637.54
5*	2*	1.006*	1.309*	1.77*	1524.11*

Table 2
Optimal solution of the inventory model in fuzzy environment.

λ	m	T in years	t_1^* in years	t_2^* in years	t_3^* in years	TC^{**}
Optimistic	2	5	0.999	1.298	3.766	1622.74
$\lambda = 1$	3	3.33	0.974	1.26	2.642	1677.22
	4	2.5	0.954	1.231	2.077	1604.05
	5	2	0.94	1.21	1.737	1504.95
About optimistic	2	5	1.051	1.376	3.792	1683.7
$\lambda = 0.7$	3	3.33	1.018	1.327	2.665	1710.53
	4	2.5	0.995	1.292	2.097	1624.74
	5	2	0.979	1.268	1.756	1517.17
Moderate	2	5	1.088	1.431	3.81	1723.55
$\lambda = 0.5$	3	3.33	1.05	1.375	2.681	1731.92
	4	2.5	1.024	1336	2.112	1637.54
	5	2	1.006	1.309	1.77	1524.11
About pessimistic	2	5	1.146	1.519	3.84	1781.92
$\lambda = 0.2$	3	3.33	1.102	1.452	2.706	1762.52
	4	2.5	1.071	1.406	2.135	1654.94
	5	2	1.05	1.375	1.792	1532.36
Pessimistic	2	5	1.189	1.583	3.861	1819.75
$\lambda = 0$	3	3.33	1.138	1.506	2.724	1781.76
	4	2.5	1.104	1.456	2.152	1665.15
	5	2	1.082	1.422	1.807	1536.21

backordered. Let 0.25 fraction of stock demand get backordered. The penalty cost for the lost in sell is 6$. The inflation in the market is considered as 12% where the planning horizon is 10 years and is considered under various replenishment cycles i.e., $m = 2, 3, 4, 5$.

Data considered to illustrate the models are as follows: $c_1 = 2$, $c_2 = 4$, $c_3 = 3$, $c_4 = 6$, $g = 2$, $h = 6$, $P = 100$, and $k = 0.12$. The planning horizon is $H = 10$ years, $\mu = 0.8$, $\beta = 2$, $\alpha = 0.001$, $T = H/m$, and $B = 0.25$.

Case 1. Let us consider that to produce the item and to keep pace with the demand the retailer keeps the production rate 1.5 times the demand rate i.e., $\gamma = 1.5$. (t_2 be the decision variable).

Then for different values of $m = 1, 2, \ldots, H$, we get the optimum value of TC^* for corresponding t_2^* and hence obtain the values of t_1^*, t_3^*. The optimal results are given in Table 1.

We observe from Table 1 that as the value of m increases, i.e., as our production cycle becomes smaller, the optimal total cost first increases and then decreases and we have the minimum (or lowest value) optimal total cost at $m = 5$ and the highest value of total cost is obtained at $m = 3$. The total cost is minimum for $m = 5$ and it is 1524.11$ and the optimal replenishment time is $t_1^* = 1.006$, $t_2^* = 1.309$ and $t_3^* = 1.77$. Thus we observe from Table 1 as well as from Fig. 2 that if we increase the time of production cycle (i.e., for few no. of replenishment i.e., for less m) it takes more money to invest in the inventory i.e., it costs more for the retailer (in our case comparing with $m = 5$). Thus the retailer deals with short term of production cycle which costs him less as our main aim is to minimize the total cost. This is realistic because in long term cycle money gets wasted in holding the item and preventing the item from deterioration and loss due to deterioration. Now the optimal solution of the proposed inventory due to imperfections in the parameter of the inventory model is shown in Table 2 with the sense of degree of optimism. With the effect of different values of λ degree of optimism, we will see the effect of m on the inventory model. From Table 2 we observe that as the value of λ decreases the total cost increases for a given m.

Let us consider $\delta_1 = 1$, i.e., the holding cost can deviate by 1 unit, $\delta_2 = 0.2$, i.e., the purchase cost can deviate by 0.2 unit, and $\delta_3 = 0.02$ i.e., the inflation can deviate by 0.02 unit.

Also if we compare between Tables 2 and 3, we observe that the total cost is minimum if we consider a fuzzy parameter in optimistic sense as compare to a crisp parameter. We also observe that if we consider one fixed m then the total cost decreases as we move from a pessimistic to

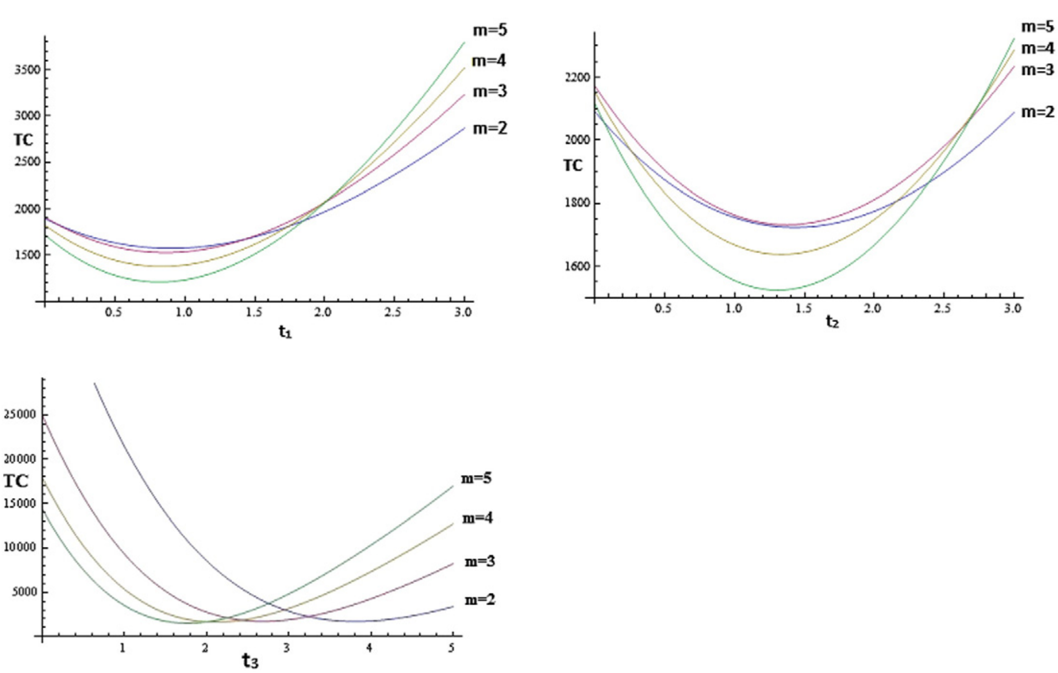

Fig. 2. Graphical presentation of total cost versus time for different m.

Table 3
Sensitivity analysis due to changes of cost parameters.

Parameters	% change	t_1^* in years	t_2^* in years	t_3^* in years	TC_2^*	% change in TC^*
	− 20	1.073	1.409	1.803	1397.2	− 8.33
c_1	− 10	1.039	1.358	1.786	1461.43	− 4.11
	10	0.974	1.26	1.753	1585.25	4.01
	20	0.942	1.212	1.737	1644.91	7.93
	− 20	0.838	1.056	1.685	1396.69	− 8.36
c_4	− 10	0.922	1.182	1.727	1465.52	− 3.84
	10	1.091	1.436	1.812	1572.54	3.18
	20	1.176	1.563	1.854	1610.93	5.7
	− 20	1.03	1.35	1.783	1499.7	− 1.6
g	− 10	1.02	1.33	1.777	1512.04	− 0.79
	10	0.992	1.288	1.763	1535.9	0.77
	20	0.978	1.267	1.756	1547.42	1.53
	− 20	0.993	1.289	1.763	1520.1	− 0.26
c_3	− 10	1.1	1.3	1.767	1522.13	− 0.13
	10	1.012	1.318	1.773	1526.02	0.13
	20	1.018	1.327	1.776	1527.88	0.25
	− 20	1.11	1.465	1.822	1474.21	− 3.27
h	− 10	1.054	1.381	1.794	1500.92	− 1.52
	10	0.965	1.247	1.749	1544.48	1.34
	20	0.929	1.193	1.731	1562.51	2.52

optimistic point of view. Now, since our task is to minimize total cost we propose to consider the parameters in fuzzy environment than in crisp environment. The model is realistic, since in reality the holding cost, purchase cost and inflation are varying and indeterministic in nature. Also if we observe the pessimistic point of view we see that the total cost increases which is not desirable in reality. Thus the model is better in fuzzy environment than in crisp environment.

From Fig. 3 we observe that for each value of λ, as the value of m increases, i.e., as we have a shorter production cycle, the total cost first increases then it decreases and its minimum is obtained for the largest value of m. Also as the value of λ increases the total cost decreases. It is also noticed that when the model is pessimistic or about to be pessimistic the optimal total cost decreases as the production cycle becomes shorter. Thus as the model in management viewpoint manifests that if there is any chance of or any possibility of increase in holding cost, purchase cost and inflation rate then the total cost decreases (since λ is a factor which represents fuzziness or possibilities, if λ increases, holding cost, purchase cost and inflation rate increase), which is good for the retailer. Thus we observe that for the possibilities of maximum value of holding cost, purchase cost and inflation rate and for the shortest replenishment cycle the total cost is minimum.

The shortages in the item generally affect the inventory model. Since during shortages there is a chance of loss in sales and partly it is backordered subsequently. In our model shortages do not affect the total cost a lot, which is good from a retailer's point of view. Here in our inventory model we noticed the effect of shortages for various replenishment cycles with the help of Fig. 4.

5. Sensitivity analysis

As per the crisp values of Case 1, the sensitivity analysis presented in this section, it is observed that the highest total cost is at $m = 3$, and the minimum is at $m = 5$. As our intension is to minimize the total cost we consider $m = 5$ to do our sensitivity analysis. Thus the optimal value for the total cost for $m = 5$ is $TC^* = 1524.11\$$ and the optimal time period for the replenishment cycle is $t_1^* = 1.006$, $t_2^* = 1.309$, and $t_3^* = 1.77$.

From above sensitivity analysis we can draw the following observations

1) The total cost TC is moderately sensitive to purchase cost (c_1), penalty cost (c_4), and holding cost (h) and less sensitive to the set-up cost (c_2), shortage cost (c_3) and deterioration cost (g).
2) It is observed that total cost TC increases with an increase in purchase cost of the raw material (c_1), holding cost of items (h) and penalty cost for loss in sale (c_4).
3) Again for an increase in less sensitive parameters like set-up cost (c_2), shortage cost (c_3) and deterioration cost (g) the total cost TC increases. An increase in cost for setting up a new production unit (set-up cost (c_2)), loss to retailers due to shortages (c_3) and deterioration due to damages and spoilages (g) invariably increases the total cost.

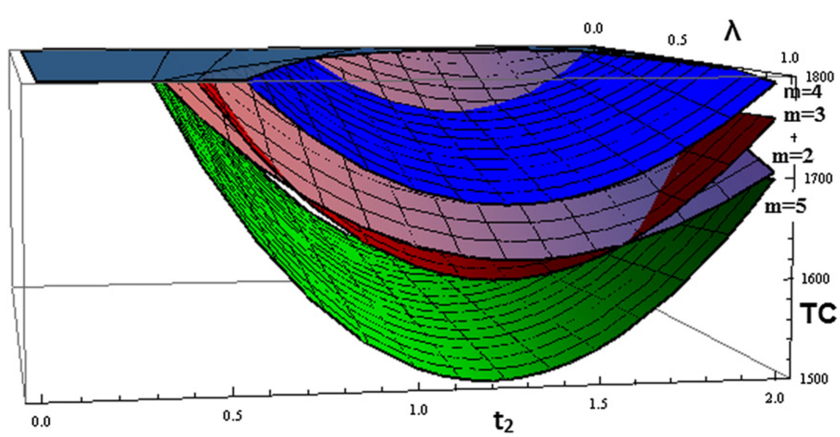

Fig. 3. Changes in total cost and time for different degrees of optimism.

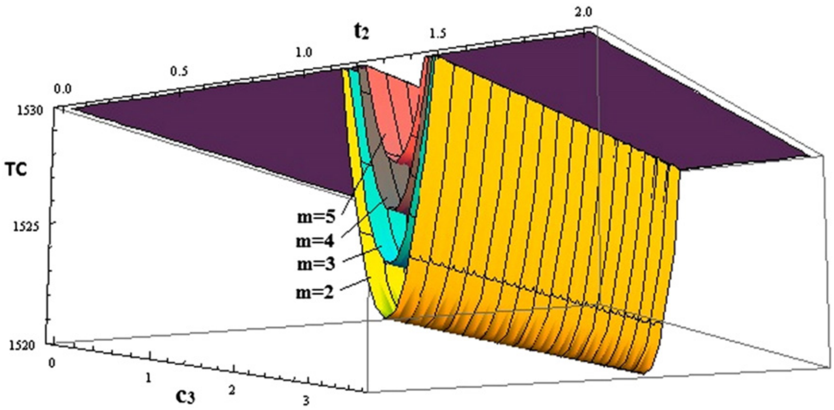

Fig. 4. Graphical representation of shortages for various replenishment cycles.

From Table 4 we observe that:

(i) The percentage change in the optimal cost is highly sensitive for the parameters like P, γ, and k, and is moderately sensitive to B, and less sensitive to μ.

(ii) The total cost TC increases with an increase in highly sensitive parameters P and γ and decreases with the inflation rate (k). With an increase in the demand rate (P), the manufacturing cost increases and hence the total cost increases. An increase in the production rate (γ) gives a chance for deterioration and holding inventory which can increase the total cost. Finally with an increase in inflation (k), time value of money increases which results in a decrease in total cost.

(iii) There is a decrease in TC with an increase in moderately sensitive parameter B and less sensitive parameter μ as increases in backorder lead to a decrease in inventory cost, resulting in a decreasing TC.

Case 2. Let us consider that the inventory ends at time $t_2 = 1.5$ (production rate (γ) be the decision variable).

Then for different values of $m = 1, 2, \ldots, H$, we get the optimum value of TC^* for the optimized production rate γ^* and hence corresponding to $t_2 = 1.5$ in years, we obtain the values of t_1, t_3. The optimal results are given in Table 5.

From Table 5 and Fig. 5 we observe that it would be better to consider a smaller replenishment cycle, to get the minimum total cost than for the large replenishment cycle. It is also observed that as the length of the replenishment cycle decreases (i.e., as the cycle becomes shorter) the production rate γ increases. This situation is obvious because, for a shorter cycle to fulfill the demand of the customer we have to produce at a much faster rate.

6. Conclusion

This paper deals with an EPQ model for deteriorating items with a new pattern of demand rate which is initially linear and then becomes constant. For ramp type demand rate of any new brand of consumer goods appearing in the market, management has to take vital decisions while maintaining inventory with least cost. Again a retailer in supermarket has to deal with such problem of highly perishable seasonal product where deterioration effect is considerable. The model is considered under the effect of inflation with deterioration as two parameters of Weibull distribution function of time. Shortages are allowed with partial backorder and partial loss in sale. The management may allow special cases as completely backlogged ($B = 1$) or not allowing shortages ($B = 0$). We have taken two cases where t_2 and γ are considered as decision variables. Sensitivity analysis in the first case reflects that the model is sensitive to the demand rate, production rate and inflation which signifies that these parameters can be considered as powerful tools by management for inventory control. In the second case the

Table 4
Sensitivity analysis due to changes of parameters.

Parameter	% change	t_1^* in years	t_2^* in years	t_3^* in years	TC_2^*	% change in TC_2^*
μ	− 20	0.98	1.31	1.77	1523.83	− 0.02
	− 10	0.991	1.306	1.769	1524.87	0.05
	10	1.024	1.316	1.772	1521.87	− 0.15
	20	1.044	1.327	1.776	1518.44	− 0.37
P	− 20	1.006	1.309	1.77	1222.2	− 19.81
	− 10	1.006	1.309	1.77	1373.16	− 9.9
	10	1.006	1.309	1.77	1675.06	− 9.9
	20	1.006	1.309	1.77	1826.01	19.81
k	− 20	1.046	1.369	1.79	1690.09	10.89
	− 10	1.026	1.339	1.78	1604.54	5.28
	10	0.986	1.278	1.759	1448.4	− 4.97
	20	0.965	1.247	1.749	1377.06	− 9.65
γ	− 20	1.191	1.349	1.638	1293.79	− 15.11
	− 10	1.106	1.352	1.73	1429.74	− 6.19
	10	0.924	1.264	1.756	1594.24	4.6
	20	0.859	1.225	1.815	1648.7	8.17
B	− 20	1.078	1.416	1.833	1570.21	3.02
	− 10	1.04	1.359	1.801	1547.3	1.52
	10	0.976	1.263	1.754	1500.87	− 1.52
	20	0.948	1.221	1.708	1477.75	− 3.04

Table 5
Optimal solution of the inventory model for different replenishments.

m	T in years	t_1 in years	t_3 in years	γ^*	TC^*
2	5	1.312	3.016	1.191	1630.09
3	3.33	1.305	2.312	1.199	1579.88
4	2.5	1.267	1.994	1.244	1501.99
5^*	2^*	1.229	1.77	1.294^*	1410.03^*

production rate affects the total cost of inventory significantly. Moreover, in both cases it is observed that a minimum value of total cost is obtained when we have a shorter production cycle and this can help managers to plan their ordering cycle accordingly. We have also considered two cases, one when the parameters are crisp variables and second when the parameters are fuzzy variables. On calculating we observed that the total cost in the crisp case is same as that of the fuzzy case when $\lambda = 0.5$, which reflects a moderately optimistic decision-maker's viewpoint. We have also done sensitivity analysis to illustrate our example in crisp variable.

Acknowledgments

The authors are heartily thankful to the editor and reviewers for their detailed, constructive and valuable comments that help us to improve the quality of the paper.

Appendix A

Fuzzy set (Zadeh, 1965) is providing a new mathematical tool which helps us to handle imprecise and ambiguous notion in daily life.

Fuzzy set

A fuzzy set $\tilde{A}$ in a universe of discourse X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x) : x \in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function of x in $\tilde{A}$ which maps $\mu_{\tilde{A}} : X \to [0, 1]$.

Now consider a symmetric triangular fuzzy number $\tilde{h} = (h-\delta, h, h+\delta)$ whose membership is as follows

$$\mu_{\tilde{h}}(x) = \begin{cases} 0 & \text{if } x \le h-\delta \\ \mu_{L\tilde{h}}(x) = \dfrac{x-h+\delta}{\delta} & \text{if } h-\delta \le x \le h \\ \mu_{R\tilde{h}}(x) = \dfrac{h+\delta-x}{\delta} & \text{if } h \le x \le h+\delta \\ 0 & \text{if } x \ge h+\delta \end{cases}.$$

Now to use the total λ-integral value we have to calculate:Let λ be the degree of optimism, and a pre-assigned parameter and $\lambda \in [0, 1]$.

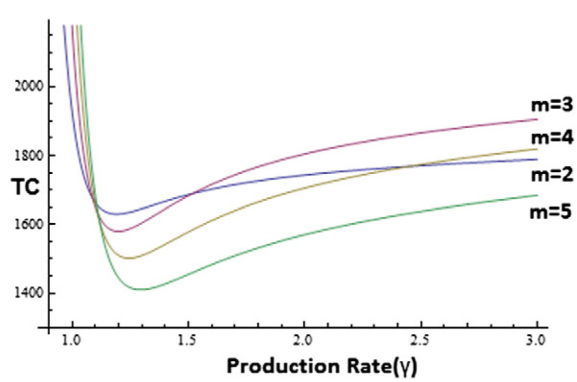

Fig. 5. Geometrical representation of total cost versus production rate for different m.

The graded mean value (or, the total λ-integral value) (Mahapatra and Roy, 2006) of $\tilde{h}$ is denoted as $I_\lambda(\tilde{h})$ and it is defined as:

$$I_\lambda(\tilde{h}) = \lambda I_R(\tilde{h}) + (1-\lambda) I_L(\tilde{h}) \tag{27}$$

where, $I_R(\tilde{h}) = \int_0^1 (\mu_{R\tilde{h}})^{-1} \alpha d\alpha$, and $I_L(\tilde{h}) = \int_0^1 (\mu_{L\tilde{h}})^{-1} \alpha d\alpha$ are the right and left interval values respectively of $\tilde{h}$.

Now, $(\mu_{R\tilde{h}})^{-1}\alpha = h+\delta-\alpha\delta$, and $(\mu_{L\tilde{h}})^{-1}\alpha = \delta\alpha + h - \delta$.

$\therefore I_R(\tilde{h}) = h + \delta/2$, and $I_L(\tilde{h}) = h - \delta/2$.

The total λ-integral value of $\tilde{h}$ is

$$I_\lambda(\tilde{h}) = \lambda(h+\delta/2) + (1-\lambda)(h-\delta/2) = h + \lambda\delta - \delta/2 = h + \left(\lambda - \frac{1}{2}\right)\delta.$$

Similarly, $I_\lambda(\tilde{c}_1) = \lambda(c_1+\delta/2) + (1-\lambda)(c_1-\delta/2) = c_1 + \lambda\delta - \delta/2 = c_1 + (\lambda - \frac{1}{2})\delta$ and $I_\lambda(\tilde{k}) = \lambda(k+\delta/2) + (1-\lambda)(k-\delta/2) = k + \lambda\delta - \delta/2 = k + (\lambda - \frac{1}{2})\delta$.

References

Agrawal, S., Banerjee, S., 2011. Two-warehouse inventory model with ramp-type demand and partially backlogged shortages. Int. J. Syst. Sci. 42 (7), 1115–1126.
Ahmed, M.A., Al-Khamis, T.A., Benkherouf, L., 2013. Inventory models with ramp type demand rate, partial backlogging and general deterioration rate. Appl. Math. Comput. 219 (9), 4288–4370.
Begum, R., Sahoo, R.R., Sahu, S.K., 2012. A replenishment policy for items with price dependent demand, time-proportional deterioration and no shortages. Int. J. Syst. Sci. 43 (5), 903–910.
Bose, S., Goswami, A., Chaudhuri, K.S., 1995. An EOQ model for deteriorating items with linear time dependent demand rate and shortage under inflation and time discounting. J. Oper. Res. Soc. 46, 771–782.
Buzacott, J.A., 1975. Economic order quantity with inflation. Oper. Res. Q. 26, 553–558.
Chen, S.L., Liu, C.L., 2007. Procurement strategies in the presence of the spot market — an analytical framework. Prod. Plan. Control 18, 297–309.
Chen, C.H., Lu, C.L., 2011. Optimal profit model considering production, quality and sale problem. Int. J. Syst. Sci. 42 (12), 1917–1933.
Chung, C.J., Wee, H.M., 2011. Short life-cycle deteriorating product remanufacturing in a green supply chain inventory control system. Int. J. Prod. Econ. 129, 195–203.
Covert, R.P., Philip, G.C., 1973. An EOQ model for items with Weibull distribution deterioration. AIIE Trans. 5, 323–326.
Datta, T.K., Pal, A.K., 1991. Effect of inflation and time value of money on an inventory model with linear time dependent demand rate and shortages. Eur. J. Oper. Res. 52, 326–333.
De, S.K., Sana, S.S., 2013. Fuzzy order quantity inventory model with fuzzy shortage quantity and fuzzy promotional index. Econ. Model. 31, 351–358.
Giannoccaro, I., Pontrandolfo, P., Scozzi, B., 2003. A fuzzy echelon approach for inventory management in supply chains. Eur. J. Oper. Res. 149 (1), 185–196.
Hariga, M.A., 1995. An EOQ model for deteriorating items with shortages and time-varying demand. J. Oper. Res. Soc. 46, 398–404.
Hou, K.L., 2006. An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. Eur. J. Oper. Res. 168, 463–474.
Jaggi, C.K., Khanna, A., Verma, P., 2011. Two-warehouse partial backlogging inventory model for deteriorating items with linear trend in demand under inflationary conditions. Int. J. Syst. Sci. 42 (7), 1185–1196.
Jolai, F., Moghaddam, R.T., Rabbani, M., Sadoughian, M.R., 2006. An economic production lot size model with deteriorating items, stock dependent demand, inflation and partial backlogging. Appl. Math. Comput. 181, 380–389.
Lin, Y., Lin, C., 2006. Purchasing model for deteriorating items with time-varying demand under inflation and time discounting. Int. J. Adv. Manuf. Technol. 27 (7–8), 816–823.
Mahapatra, G.S., Roy, T.R., 2006. Fuzzy multi-objective mathematical programming on reliability optimization model. Appl. Math. Comput. 174, 643–659.
Mahapatra, G.S., Mandal, T.K., Samanta, G.P., 2012. An EPQ model with imprecise space constraint based on intuitionistic fuzzy optimization technique. J. Mult. Valued Log. Soft Comput. 19 (5–6), 409–423.
Neetu, Tomer, A.K., 2012. A deteriorating inventory model under variable inflation when supplier credits linked to order quantity. Procedia Eng. 38, 1241–1263.
Pal, S., Mahapatra, G.S., Samanta, G.P., 2014a. An EPQ model of ramp type demand with Weibull deterioration under inflation and finite horizon in crisp and fuzzy environment. Int. J. Prod. Econ. 156, 159–166.
Pal, S., Mahapatra, G.S., Samanta, G.P., 2014b. An inventory model of price and stock dependent demand rate with deterioration under inflation and delay in payment. Int. J. Syst. Assur. Eng. Manag. 5 (4), 591–601.
Philip, G.C., 1974. A generalized EOQ model for items with Weibull distribution deterioration. AIIE Trans. 6, 159–162.

Samanta, G.P., Bhowmick, J., 2010. A deterministic inventory system with Weibull distribution deterioration and Ramp type demand rate. Electron. J. Appl. Stat. Anal. 3 (2), 92–114.
Sarkar, B., Sarkar, S., 2013a. An improved inventory model with partial backlogging, time varying deterioration and stock dependent demand. Econ. Model. 30, 924–932.
Sarkar, B., Sarkar, S., 2013b. Variable deterioration and demand — an inventory model. Econ. Model. 31, 548–556.
Sharma, V., Chaudhury, R.R., 2013. An inventory model for deteriorating items with Weibull distribution with time dependent demand. Res. J. Manag. Sci. 2 (3), 28–30.
Skouri, K., Konstantaras, I., Papachristos, S., Ganas, I., 2009. Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. Eur. J. Oper. Res. 192 (1), 79–92.
Skouri, K., Konstantaras, I., Papachristos, S., Teng, J.T., 2011a. Supply chain models for deteriorating products with ramp type demand under permissible delay in payments. Expert Syst. Appl. 38, 14861–14869.
Skouri, K., Konstantaras, I., Manna, S.K., Chaudhuri, K.S., 2011b. Inventory models with ramp type demand rate, time dependent deterioration rate, unit production cost and shortage. Ann. Oper. Res. 191 (1), 73–95.
Uthavakumar, R., Rameswari, M., 2012. Economic order quantity for deteriorating items with time discounting. Int. J. Adv. Manuf. Technol. 58 (5–8), 817–840.
Wee, H.M., Lo, S.T., Yu, J., Chen, H.C., 2008. An inventory model for ameliorating and deteriorating items taking account of time value of money and finite planning horizon. Int. J. Syst. Sci. 39 (8), 801–807.
Widyadana, G.A., Wee, H.M., 2011. Optimal deteriorating items production inventory models with random machine breakdown and stochastic repair time. Appl. Math. Model. 35, 3495–3508.
Widyadana, G.A., Cardenas-Barron, L.E., Wee, H.M., 2011. Economic order quantity model for deteriorating items with planned backorder level. Math. Comput. Model. 54, 1569–1575.
Wu, K.S., 2001. An EOQ inventory model for items with Weibull distribution deterioration, ramp type demand rate and partial backlogging. Prod. Plan. Control Manag. Oper. 12 (8), 787–793.
Wu, K.S., Ouyang, L.Y., 2000. A replenishment policy for deteriorating items with ramp type demand rate. Proc. Natl. Sci. Counc. ROC(A) 24, 279–286.
Wu, K.S., Ouyang, L.Y., Yang, C.T., 2006. An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging. Int. J. Prod. Econ. 101, 369–384.
Yang, H.L., 2011. A partial backlogging production-inventory lot-size model for deteriorating items with time-varying production and demand rate over a finite time horizon. Int. J. Syst. Sci. 42 (8), 1397–1407.
Yang, H.L., 2012. Two-warehouse partial backlogging inventory models with three parameter Weibull distribution deterioration under inflation. Int. J. Prod. Econ. 138, 107–116.
Yang, H.L., Teng, J.T., Chern, M.S., 2002. A forward recursive algorithm for inventory lot-size models with power form demand and shortages. Eur. J. Oper. Res. 137, 394–400.
Yang, H.L., Teng, J.T., Chern, M.S., 2010. An inventory model under inflation for deteriorating items with stock dependent consumption rate and partial backlogging shortages. Int. J. Prod. Econ. 123, 8–19.
Zadeh, L.A., 1965. Fuzzy set. Inf. Control. 8, 338–353.