IL NUOVO CIMENTO VOL. LIV B, N. 2 11 Aprile 1968

On the Proton Pairing Energies.

D. BANERJEE
Palit Laboratory of Physics - Calcutta

(ricevuto il 20 Febbraio 1968)

From an analysis of alpha- and beta-disintegration energies of the heavy nuclides, GLUECKAUF (1) has shown that the pairing energy of protons exceeds that of neutrons in heavy nuclei. KOWARSKI (2) has postulated that such a situation exists in regions of open-shell neutrons, in order to explain the occurence of beta-labile elements $_{43}$Tc and $_{61}$Pm beyond closed neutron shells. KOHMAN (3) has suggested theoretical justification for this and has mentioned that this situation is probably fairly general except for light elements.

In a previous communication (4), we have studied in detail the variation of neutron pairing energies with total nucleon number and also with the total angular momentum of the orbit in which the pair is present. Here, we have examined the dependence of proton pairing energies for even-even nuclei on the nucleon number, the kind of particles which compose the pair and the state occupied by the pair and have also examined Mayer's (5,6) form critically.

The pairing energy P_p of the protons is the difference in the nucleon binding energies between the even proton and the preceding odd proton.

Thus, the pairing energy for the Z-th and $(Z-1)$-th protons in a nucleus with N neutrons is

$$P_p(Z, N) = B_p(Z, N) - B_p(Z-1, N)\,, \tag{1}$$

where Z is an even number and $B_p(Z, N)$ is the energy of the binding of the last Z-th proton to the nucleus with N neutrons.

Expressing the relation (1) in terms of the total binding energy $E(Z, N)$ we get

$$\begin{aligned} P_p(Z, N) &= [E(Z, N) - E(Z-1, N)] - [E(Z-1, N) - E(Z-2, N)] = \\ &= E(Z, N) + E(Z-2, N) - 2E(Z-1, N)\,. \end{aligned} \tag{2}$$

(1) E. GLUECKAUF: *Proc. Phys. Soc.*, **61**, 25 (1948).
(2) L. KOWARSKI: *Phys. Rev.*, **78**, 477 (1950).
(3) T. P. KOHMAN: *Phys. Rev.*, **85**, 530 (1952).
(4) D. BANERJEE: *Nuovo Cimento*, **52** B, 233 (1967).
(5) M. G. MAYER and J. H. D. JENSEN: *Elementary Theory of Nuclear Shell Structure* (New York, 1955).
(6) M. G. MAYER: *Phys. Rev.*, **78**, 16 (1950).

Several authors (7-10) have failed to find the correlation between the local values of the pairing energies and the total angular momentum J of the orbit.

To investigate the variation of P_p, we have plotted the graph as shown in Fig. 1. The relation (2) has been used to calculate the proton pairing energies from the mass

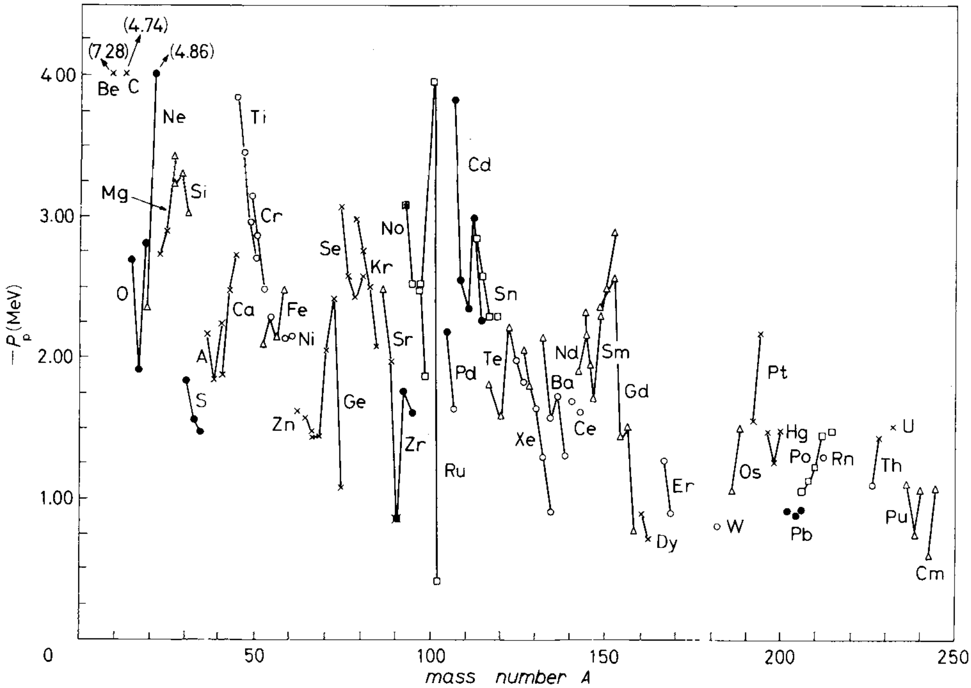

Fig. 1. – •, $J = \frac{1}{2}$; ×, $J = \frac{3}{2}$; △, $J = \frac{5}{2}$; ○, $J = \frac{7}{2}$; □, $J = \frac{9}{2}$.

table of KONIG *et al.* (11). We have omitted the nuclei with odd N because of the influence of residual n-p interaction. The points representing the pairing energies of the same elements are connected by a straight line. The values of J are taken from the table of nuclear constants (12).

We see that the proton pairing energies generally increase with increase of J. There are, however, exceptions for mass numbers 18, 58, 106, 112, 126, 132 and 142 where a decrease in P_p with increase of J is observed. We have seen that the neutron pairing energies depend little on the number of paired protons in the nucleus (4). The analogous rule for protons is less pronounced.

The position of the pair in the shell has a great influence on the proton pairing energies. For the same J as the shell becomes filled the proton pairing energies decrease. They increase as the proton number deviates from the magic numbers and

(7) K. S. QUISENBERRY, T. T. SCOLMAN and A. O. NIER: *Phys. Rev.*, **104**, 461 (1956).
(8) C. F. GIESE and J. L. BENSON: *Phys. Rev.*, **110**, 712 (1958).
(9) R. R. RIES, R. A. DAMEROW and W. H. JOHNSON jr.: *Phys. Rev.*, **132**, 1662 (1963).
(10) R. A. DAMEROW, R. R. RIES and W. H. JOHNSON jr.: *Phys. Rev.*, **132**, 1673 (1963).
(11) L. A. KONIG, J. H. MATTAUCH and A. H. WAPSTRA: *Nucl. Phys.*, **31**, 18 (1962).
(12) L. R. B. ELTON: *Introductory Nuclear Theory* (London, 1959).

reach the maximum value either at the middle of the shell or at the beginning. The same character has also been observed for neutron pairing energies (4). Thus the low values of P_p for Zn (90) and Gd (158) lead to the presence of subshells in the region $Z = 40$ and $Z = 64$. From the analysis of double nucleon separation energies ZELDES *et al.* (13) have predicted the presence of a subshell at $Z = 66$.

We have adopted the same method as for the neutron pairing energies (4), to find the dependence of proton pairing energies on the nucleon number. It has been found that P_p varies as $A^{-\frac{1}{3}}$, instead of A^{-1} as predicted by Mayer's theory. The mean deviation of the P_p from the expression given for the neutron pairing energies (4), namely,

$$P_p = -[20/A^{\frac{1}{3}} + 16J/A] \tag{3}$$

comes out of the order of ± 0.60 MeV. A better agreement is obtained by writing

$$P_p = -16[1/A^{\frac{1}{3}} + J/A]\,, \tag{4}$$

when the mean deviation of the proton pairing energies is of the order of ± 0.52 MeV. Thus, we get in general a lower value of the proton pairing energies than the neutron pairing energies, which contradicts the idea of KOHMAN (3). This is also clear from a comparative analysis of Fig. 1 in ref. (4) and Fig. 1 of this paper.

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Thanks are due to Prof. A. K. DUTTA for his interest in this work.

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(14) N. ZELDES, M. GRONAU and A. LEV: *Nucl. Phys.*, **63**, 1 (1965).