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General Covariance in General Relativity?

G. F. R. Ellis[1,2] and D. R. Matravers[1]

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The mathematical approach to General Relativity insists that all coordinate systems are equal. However physicists and astrophysicists in fact almost always use preferred coordinate systems not merely to simplify the calculations but also to help define quantities of physical interest. This suggests we should reconsider and perhaps refine the dogma of General Covariance.

1. INTRODUCTION

The standard doctrine of General Relativity, considered from a mathematical vantage point, is that all coordinate systems (and associated tensor bases) are equally as good as each other; hence our use of the tensor calculus, and methodological insistence on a equality of all coordinates. Mathematically considered, that is correct. However in reality this is rather misleading. The fact is that application of General Relativity to tackle specific physical problems is usually facilitated and much clarified by using coordinate or tetrad systems carefully adapted to the problem at hand, indeed one would be acting in a rather contrary manner if one were to use a generic, ill-adapted coordinate system in specific physical situations. We would almost always fail to fully solve the problem at hand if we did so.

Clearly one wishes to use fully covariant methods as far as one can, and physically we should always aim to ultimately refer to measured quan-

[1] Applied Mathematics Department, University of Cape Town, Rondebosch 7700, Cape Town, South Africa

[2] Department of Applied Mathematics, University of Portsmouth, Mercantile House, Hampshire Terrace, Portsmouth PO1 2EG, UK

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tities rather than coordinate dependent variables; nevertheless, complete analysis of specific physical problems almost always requires some coordinate choice, and making this choice appropriately is often a key to proper understanding of a problem.

The argument this far is presumably uncontroversial. The purpose of this paper is to consider an extension of this line of thought to more interesting and controversial areas. We will suggest that (even though this violates one of the most fondly held tenets of General Relativity theory) some significant physical issues can *only* be sensibly tackled using (explicitly or implicitly) particular, well-adapted coordinate systems; so that to adequately bridge the gap between mathematical theory and physical application, we may have to change from the stance that all coordinate systems are acceptable, to stating that some problems dictate use of coordinates chosen from an atlas that is specifically adapted to the particular problem at hand (which is a subset of the full atlas of all possible coordinate systems).

In what follows, we discuss this issue, pointing out specific examples of considerable interest (the Newtonian limit of General relativity, solar system studies, density perturbations in cosmology, study of the cosmological microwave background radiation anisotropies) where in fact this stance (the essential use of preferred coordinates) is usually adopted, even though it is not often advertised as such. One could in consequence, approaching these topics from a strict mathematical viewpoint, claim that the results attained are not physically significant, because they are not based on a fully covariant approach; or one can alternatively adopt a viewpoint such as that suggested here, claiming that such physical studies do indeed make sense, despite the restricted coordinate choices underlying them.

Specifically, the essential point is that while all coordinate systems are mathematically allowed, most of them are far too wiggly and unruly to be of any physical interest; for purposes of application, it makes sense, and indeed is desirable, to restrict coordinates to those that are suitably 'smooth' from a physical and geometric viewpoint (and that are adapted to any symmetries or near-symmetries there may be). This is still quite a wide class of coordinates, but is not nearly as large as the set of all differentiably smooth coordinates that is mathematically allowable.

This viewpoint is then a possible basis for tackling perhaps the most controversial area of this kind, namely the fact that physically we *always* refer to measured quantities that are coarse-grained (they are averaged over some space-time volume). Studies of the underlying averaging process are on very shaky ground, if we adopt the standard position of demanding full covariance of the procedure adopted; and this is the basis of controversy that has raged over various specific recent proposals aiming to examine the degree to which averaging the metric is non-commutative with calculating the field equations of a specific metric. The revised position advocated here, proposing a more restricted allowable coordinate choice in physical applications of General Relativity, may be required to enable some progress on this averaging issue; and that is required, if we are to seriously understand the relation of the mathematical models used in General Relativity theory to the reality they are supposed to represent.

2. PHYSICAL SITUATIONS AND ADAPTED COORDINATES

The essential point here is that in many cases one uses preferred coordinates not merely for calculational convenience, but also for understanding. They enable us to see what is there. Indeed in many cases it is use of specific coordinates that enables us to define the features we wish to investigate.

2.1. Newtonian limit and PPN calculations

The paradigmatic, and fundamentally important, example is the way we usually derive the Newtonian limit of General Relativity. If a space-time is a perturbation of flat space-time, as envisaged in the Newtonian limit, this fact can only be demonstrated by (explicit or implicit) very particular choice of a time coordinate. The standard calculations (see e.g. Ref. 31, p.74–78) almost universally rely on use of inertial coordinates and then additionally use of a specific gauge condition analogous to the Lorentz gauge in electromagnetism. It is use of these coordinate conditions that enables us to demonstrate and validate the Newtonian limit, and the higher order approximations to this limit which are amongst the most important applications of General Relativity theory.

Indeed systematic tests of the validity of General Relativity are based in the weak gravitational field equations (Ref. 4, p.48–58) and in particular the PPN approximation, founded in a specific restricted coordinate choice (see Chapter 4 of Ref. 32, e.g. p.91: 'to discuss the Post-Newtonian limit properly, we must specify the coordinate system...'). These coordinates underlie our accurate understanding of the geometry and dynamics of the Solar system (verified to a very high accuracy through radar ranging experiments). While it might in principle be possible to write down in a more coordinate independent way, a set of conditions equivalent to choosing such coordinates, in fact this would be so cumbersome as to not be useful. Specification of a restricted set of coordinates is the best way to set up the post-Newtonian analysis that underlies solar system tests of General Relativity.

Similarly, restricted coordinate choices underlie the basic theory by which we understand emission of gravitational radiation by astrophysical systems (which starts with the linearised theory and then employs yet more gauge transformations, further restricting the allowed coordinate choice, see e.g. Ref. 31, p.78–83).

Now one can, and indeed should, insist on eventually calculating observable quantities in terms of the chosen coordinates, whatever those coordinates are. Then one could use the argument that any coordinates at all could have been chosen in order to carry out the calculation, so the role of these coordinates is inessential: they are merely a convenient scaffolding to use to arrive at the final answer, and once they have been dismantled, leave no visible mark on that result.

However the suggestion here is that this is incorrect: it is difficult even to write down the usual results that are the basis of astrophysical calculations of gravitational radiation and planetary motion, e.g. equation (4.4.49) in [31] and equation (4.4.12) in [4], without using special coordinates. While an elaborate mechanism of bi-tensors could in principle solve this problem and attain the necessary equations (cf. Refs. 7,8), this formalism — in principle a beautiful way to tackle such problems — in fact is so cumbersome it is not used for practical calculations. If one turns to the idea of chronometric invariants (Ref. 4, p.61–64), this often turns out to still be coordinate based, e.g. the tensors D^{ik} and A^{ik} in Blumberg's work (Ref. 4, p.63) refer to deformation and rotation of the chosen reference system: thus their real role is to facilitate better understanding of the coordinate system in use.

Tetrad systems can be used to give a basis for direct reference to physical quantities. They in essence give a more physically based extension of the tensor basis associated with coordinates, but also in principle can be arbitrarily chosen (just as coordinates are). To attain physically meaningful results, one will again in general restrict the tetrad choice to one adapted to the geometry and physics of the situation at hand. One can work out equations relative to such a suitably chosen tetrad (e.g. equation (2.3.72) in Ref. 4) and these do indeed refer to measurable quantities (in this case, the precession of spin relative to a comoving tetrad). The answers make physical sense if this tetrad is chosen appropriately; but then they are not expressed in explicitly covariant form.

2.2. Spherical and asymptotically flat states

Understanding spherically symmetric systems is in principle possible in any coordinate system (or associated tetrad basis), but in practice is only made transparent by use of coordinates adapted to the spherical symmetry. These can then be used to examine strong field phenomena such as spherical collapse to a black hole state, and accretion by the black hole. It is particularly in examining observational relations that the adapted coordinates play a crucial role.

Asymptotically flat states were initially examined using carefully crafted asymptotically flat coordinates and associated tetrads [3,25]; later these were more elegantly expressed in terms of Penrose's concept of conformal infinity, and the rate of fall off of quantities at infinity relative to particular coordinates. Again in principle one could use any coordinates to examine these situations, and the requisite definitions can be given in a coordinate free manner (Ref. 23, Ch.9; Ref. 26, p.29), but in practice one uses adapted coordinates and tetrads to illuminate what is going on (e.g. Ref. 23, p.292–294,332–334,347–350; Ref. 26, p.12–20).

3. COSMOLOGY

The situation in cosmology is similar. There is a preferred rest frame and time coordinate in standard cosmology, and using any other coordinates simply obscures what is happening. The Cosmic Microwave Background Radiation determines the preferred rest frame (and associated time coordinate) to high accuracy.

The dynamics of the standard model can be dealt with largely in a coordinate-free manner, but observational relations cannot. There are a small family of preferred spatial coordinates that focus either on spatial isotropy or homogeneity, and make observational analysis easy. The subject is completely opaque if other, ill-adapted coordinates are used.

3.1. Inhomogeneities and the gauge issue

In studying inhomogeneities in cosmology, in principle we can use any coordinates; so the standard variable $\delta\rho/\rho$ is in fact arbitrary — it has no physical meaning, because its value depends completely on the time coordinate chosen in the real, lumpy universe (the gauge choice we make); for example we can always choose the zero-density perturbation gauge, where the surfaces of constant time are taken to be the surfaces of constant density, so that $\delta\rho/\rho = 0$ [12].

This is the reason for the need for gauge invariant methods [1,12]. But the intriguing point is that despite this, many astrophysicists continue to use particular gauges expressed in terms of particular coordinate choices; the philosophy is that if observational quantities are calculated, and one keeps track of the gauge freedom involved, then all will be fine (see e.g. Ref. 9). Covariant and gauge invariant methods are not that widely used, despite being available. Analyses essentially based in particular coordinate systems are often preferred.

3.1.1. Perturbations within the horizon

What makes the issue particularly apparent is the frequent claim that gauge problems do not occur if one considers the situation 'within the horizon' (a loose phrase meaning 'when we consider scales smaller than the instantaneous Hubble scale'). Now this is simply not true, if we adopt the usual approach that any differentiable coordinates can be chosen, for then the gauge problem remains completely unaffected by whether the inhomogeneity considered is within the horizon or not. Even on the most local scale one can still choose the zero-density perturbation gauge. Why then the claim that there is no gauge problem within the horizon?

What is apparently in mind here is the approach advocated in this paper: that one is envisaging not allowing arbitrary coordinates, but rather coordinates that are smooth in a differential geometry sense. For example if one considers spatial coordinates based on geodesics orthogonal to a chosen worldline, corresponding to a local Newtonian limit for an observer on that worldline [10], then no longer can the surfaces of constant time follow arbitrary wiggles in the spatial density. The zero-density perturbation gauge is then ruled out, and the apparent density perturbations with respect to this (restricted) set of allowed smooth coordinates does indeed have a physical meaning locally: an apparent inhomogeneity occurs at the central word line if and only if there is a real inhomogeneity (as measured by the covariant and gauge invariant variables) there.[3]

Thus the astrophysically based picture makes sense, if we allow only privileged sets of smooth coordinates. This illustrates the main point of this paper: linking physical or astrophysical reality to the theory of General Relativity strongly suggests a restriction of allowable coordinates to those that are smooth in a physical sense. This bridges the gap between astrophysical practice and relativity ideology. In fact analyses of the growth of structure in the expanding universe, and the associated velocity flows, are mostly done in particular coordinates associated with taking a quasi-Newtonian approach to local astrophysics (see e.g. Refs. 22,2). That makes sense if we adopt the viewpoint of this paper, but is on shaky ground otherwise.

3.1.2. Scalar, vector, and tensor perturbations

A particular associated issue is that often a separation is made of perturbation modes into scalar, vector, and tensor parts [1]. However that separation is non-locally defined [28] and is in effect dependent on the coordinate choice made (for example, the Bardeen formalism is gauge-

[3] The situation away from the central world line is not so simple (Ref. 13); a more complex, but unique, relation will hold between apparent and covariant density variables.

invariant only if one restricts coordinate changes to the vector kinds when vector modes are investigated, and to the scalar kind when scalar modes are the theme of interest; see Ref. 29). This means that, like the splitting of redshift into Doppler and gravitational parts (see Ref. 25), this splitting is of heuristic value only; it is not invariant under all coordinate transformations.

This gauge dependence may be taken either as implying such a splitting has no real physical meaning; or that interesting physical questions are usefully examined in terms of restrictions on the allowed family of coordinate systems (and associated restricted gauge choices). It is the latter option we are investigating in this paper.

This issue arises in particular in examining the effect of these various perturbation modes on anisotropies of cosmic background radiation. There is considerable effort going on at present into distinguishing if anisotropies are due to scalar, vector, or tensor modes; but this is not a fully covariant and gauge-invariant activity, as we have just seen. Nevertheless it helps us understand the physical processes taking place, provided we drop the requirement that our analyses be fully covariant.

3.1.3. Non-linear Inhomogeneities

It is not surprising that essentially the same is true when non-linear inhomogeneities are investigated. One can set up a covariant description of some cases of interest, but proving consistency of the field equations — which can be done by covariant methods in the linear case — in the non-linear case demands use of a particular, well-adapted tetrad [17]. Detailed astrophysical applications of utility, such as use of the Zel'dovich approximation and its relativistic analogues [18,19], are based on specific coordinate systems.

Examining observational relations in non-linear cases — even the Tolman–Bondi spherically symmetric case — is rather difficult; use of observational coordinates however makes many observational relationships transparent, because the null geodesic are then already solved [21].

4. NUMERICAL APPROXIMATIONS AND AVERAGING

What is clear from the above is that analysis of inhomogeneities is where the issue raised here becomes important. This applies particularly to numerical investigations and multi-scale analysis.

4.1. Numerical relativity and the Regge calculus

A rapidly expanding area of work is numerical analysis of solutions and in particular of the growth of inhomogeneities. This work is by its very nature manifestly not covariant; a coordinate grid is set up that corresponds to use of particular adapted coordinates, and the way those coordinates should be chosen is the subject of considerable debate, for example one sees advocated maximal time slicing (Ref. 14, p.8) plus the quasi-isotropic gauge (Ref. 14, p.9) in examining axisymmetric gravitational collapse; study of planar numerical cosmologies is based on Cartesian coordinates (Ref. 6, p.124); and so on.

Equally use of the Regge calculus approximation [33] is based on a space-time splitting into flat regions, joined along faces, that is in essence equivalent to choosing a particular set of coordinates as the basis of the approximation. As in the previous case, those coordinates are always chosen in a geometrically smooth manner.

4.2. The averaging problem

As preferred coordinates are almost always used to understand inhomogeneity, it is not surprising that they could play a central role in studying the controversial issues arising in looking at averaging of geometries, and the effects of non-commutativity of averaging and calculating the field equations [11]. Just as in the case of studying growth of structure in cosmology, the point is that looking at local inhomogeneities on a variety of length scales is facilitated by using coordinates that are as smooth as possible, indeed that avoid varying on any except the largest length scales. This then enables us to look at the effects of various smaller scale inhomogeneities in a physically meaningful way.

Indeed as in other studies, particular smooth coordinate choices are either explicitly or implicitly used in the various studies of this averaging issue (e.g. Refs. 27,16,35,34,5). This has led to skepticism and controversy about their results. However the suggestion here is that this feature is probably inevitable if one is to understand this issue; and that furthermore if one does not understand it, then our understanding of the meaning and dynamics of our cosmological models is on rather shaky ground (we may easily be using the wrong effective field equations because we are not taking it into account).

5. GENERAL DYNAMICS

The issue arises again in any use of a canonical formalism, for example in quantum gravity studies, which must separate out time and so break the overall covariance of the theory. This form of the theory is non-covariant in principle; and relating it to the covariant form of the theory will go awry if one chooses a physically badly behaved time function. There is an implicit underlying assumption that the surfaces $\{t = \text{const}\}$ used to define the canonical theory will be chosen to be reasonably smooth, if physically meaningful results are to emerge.

One of the oldest aspects of the theme pursued here is the use of specific tetrads or special coordinates to define energy-momentum complexes of various kinds (pursued by Weyl, Einstein, and Levi-Civita in the 1930s, and by Møller, Plebański, and others later on). All these attempts have been rejected by the majority of the community of relativists because of their coordinate dependence, but there is still a continual striving from the physicist's side to search for a meaningful concept of gravitational energy (and entropy) both in general, and in specific situations (for example, in studies of stellar structure or of star clusters). This links back to the theme of the Newtonian limit of General Relativity, because we do know there is a good concept of gravitational energy in that case — so we should be able to derive that feature from the relativity equations, by use of suitably adapted coordinates (as are required to attain the Newtonian limit, which was our starting point). This becomes of practical importance in studies of stability of stellar structure, for example, when one goes beyond the Newtonian limit.

6. PHYSICALLY PREFERRED COORDINATE SYSTEMS

The argument, then, is that while in principle all coordinates are allowed, some are better than others in any realistic physical situation, to the extent that we may not properly understand many physical situations unless we use adapted coordinates. If this is so, then it should be recognized in the way we phrase our understanding of GR. That would enable a rapprochement between the more mathematical and the more physical relativists.

What is involved here is a type of symmetry breaking: the theory is covariant but the specific models we employ to understand various physical and astrophysical situations break that symmetry. The solutions of the equations that underlie the theory do not in general have all the symmetries of those equations. The family of physically useful solutions may have a smaller family of symmetries than the full set of solutions of the equations.

Given this understanding, the debate turns to considering the various options for choice of such 'smoothed-out' coordinate systems that can best help us understand the physics (see Ref. 1 for an instructive discussion of many of them in the cosmological context).

6.1. Newton-like coordinates

One is to use coordinates that locally give the best approach to Newtonian theory in a general curved space-time; either normal coordinates, or proper time along a worldline plus geodesic coordinates orthogonal to the worldline [30,10]. An alternative approach to obtaining Newton-like behaviour, in the cosmological context, is choice of timelike surfaces of minimal shear [1].[4]

6.2. Harmonic coordinates

A second overall option is to choose harmonic coordinates, which in any case underlie many approximation schemes and studies of gravitational radiation. These give smoothed-out coordinates not adapted to any particular world line or surfaces, and their virtues have been extolled by many, in particular [15].

6.3. Symmetry based coordinates

A third overall option is to note that in many cases the best coordinates are those adapted to symmetry groups or asymptotic symmetry groups, for example, the coordinates used to examine asymptotically flat space-times, where the asymptotic symmetries define coordinates (and associated tetrads) that are then the basis for examining energy, mass, and the 'news' functions.

Thus one could aim to choose coordinates associated with such symmetries or almost symmetries. The problem, however, is that while we understand well the asymptotic symmetries at infinity of asymptotically flat space-times, we have been less successful in defining local 'almost symmetries' — which could be used to do the same essential job, but locally rather than at infinity. They would be used to define local coordinates which as smoothly as possible represent local almost — isometries.

7. A MORE REALISTIC DOGMA

It is not clear which of these is best; to determine that will require experimentation, and may well lead to different answers in different cases, The overall point remains: some coordinate systems are better than others for understanding physical problems, and even sometimes for defining the essential variables that enable us to understand those problems. The key comment then is that it may be sometimes that in particular cases, they may be *essential* to the required understanding (at least in practical terms). The need is for some criterion of smooth coordinates in a geometrical rather than differential sense; a family of preferred coordinates that do not wave and vary in an arbitrary way, but rather are as smooth as possible in a general curved space-time, in a disciplined way.

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[4] Although they are sometimes advocated, choice of zero-shear coordinates is almost never possible, and when they are possible they are not necessarily unique [20].

Given such a choice of a restricted set of coordinates, we still need the tensor calculus to relate physical quantities in different coordinates, and to understand the nature of the coordinates in use. We will still use coordinate-free (covariant) methods when possible. However the suggestion is that we allow for and acknowledge the privileged nature of smoothed-out coordinates, which are in fact preferable over arbitrarily oscillating or varying coordinates in order to properly understand physically important situations. Thus we do not abandon covariance of the theory, but move from general covariance (all coordinate systems are allowed, no matter how unsuitable) to restricted or physical covariance (the coordinates we use for physical applications exclude those that are wildly oscillating or are in other ways exceptionally badly adapted to the system at hand).

An elaborate enough covariant description may eventually succeed in describing the essential features that can be easily described by using suitably adapted or smoothed coordinates, without explicitly using them; but it is much simpler and straightforward to simply use such coordinates, because that leads in the easiest and simplest way to the answer. Thus there are indeed preferred coordinates in almost every situation; their use is the best way to understand the problem at hand. In particular smoothed out coordinates are sometimes needed to understand physically important situations, such as averaging; acknowledging this preferred role of particular families of coordinates may be the best thing to do in making progress with complex physical problems.

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