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Nucleus-Nucleus Potentials (*).

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Summary. — The real part of the heavy-ion optical potential is calculated by the folding technique, by dropping the contribution of the inner nucleons instead of using density-dependent potentials. An expression is derived for the potential when one of the nuclei is considered as two fragments.

1. – Introduction.

Nuclear potentials have been studied by several people, starting from an effective two-body interaction. The simplest approach is to fold the effective nucleon-nucleon interaction into the densities of both nuclei. SATCHLER (1) gives a review of the simple folding, together with the double folding. The interaction potential between the nuclei is calculated by folding the nuclear-density distribution function of a nucleus with the real part of the single-nucleon optical potential of the other nucleus. This model for the interaction potential has been used in the studies of α-nucleus scattering (2). The real part of nucleus-nucleus interaction for magic nuclei has been derived (3) from

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(1) G. R. SATCHLER: *Proceedings of the International Conference on Reactions between Complex Nuclei* (Nashville, Tenn., 1974).

(2) D. F. JACKSON: *Nucl. Phys. A*, **123**, 273 (1969); G. R. SATCHLER: *Nucl. Phys.*, **77**, 481 (1966).

(3) D. M. BRINK and Fl. STANCU: *Nucl. Phys. A*, **243**, 175 (1975); Fl. STANCU and D. M. BRINK: *Nucl. Phys. A*, **270**, 236 (1976).

the Skyrme interaction density function by taking into account the exchange effects. This real part of the potential is tested against the elastic-scattering data by adopting an imaginary part with the same geometry and variable strength and good agreement is found [4]. GOLDFARB [5] gives analytic forms for the real part of the heavy-ion optical potential through the folding technique, considering Yukawa terms for the effective potential between the nucleons.

The elastic scattering is fairly sensitive to the nucleon distortion. CRAMER *et al.* [6] obtained an excellent fit for the elastic data of $^{28}Si+^{16}O$ adopting the potential labelled E18 in ref. [6]. SATCHLER [7] used a combination of Gaussian potentials for the effective nucleon-nucleon interaction to calculate the $^{28}Si+^{16}O$ potential by the double-folding technique and he had to multiply his results by a factor of about 0.5 to get the potential of Cramer *et al.* [6] in the scattering region $(6\div10)$ fm. However, SINHA [8] shows that density-dependent potentials are appropriate for the effective interaction in cases in which the densities of the colliding nuclei overlap significantly, so that the saturation property of this interaction prevents the total density from increasing beyond a certain value and he shows that at the very tail of the potential the saturation term is negligible. But for heavier target-projectile systems the saturation term is not negligible, even for a separation larger than the touching radius. In general, he finds that the potential is weaker than the potential obtained by simple folding calculations, without density-dependent terms [9]. Furthermore, the exchange terms were found to give a small contribution [8,9].

In the present work, the nucleus-nucleus potential is calculated by adopting the double-folding technique and accounting for the saturation property by dropping the contribution of the inner nucleons. Moreover, an expression for the nucleus-nucleus potential is derived by considering one nucleus as two fragments. This expression is useful for the calculations of heavy-ion transfer reactions and of recoil effects.

2. – Model for nucleus-nucleus potential.

The double-folding form for the real part of the optical potential between two nuclei separated by a distance R is [1,5,9]

$$V(R)=\int\varphi_1(\boldsymbol{x})\varphi_2(\boldsymbol{y})\,V(|\boldsymbol{R}+\boldsymbol{x}-\boldsymbol{y}|)\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{y}\,, \tag{1}$$

(4) D. VAUTHERIN and D. M. BRINK: *Phys. Rev. C*, **5**, 626 (1972).
(5) L. J. B. GOLDFARB: *Nucl. Phys. A*, **301**, 497 (1978).
(6) J. G. CRAMER, R. M. DEVRIES, D. A. GOLDBERG, M. S. ZISMAN and C. F. MAGUIRE: *Phys. Rev. C*, **14**, 2158 (1976).
(7) G. R. SATCHLER: *Nucl. Phys. A*, **279**, 493 (1977).
(8) B. C. SINHA: *Phys. Rev. Lett.*, **33**, 600 (1974); *Phys. Rev. C*, **11**, 1546 (1975).
(9) D. M. BRINK and N. ROWLEY: *Nucl. Phys. A*, **219**, 79 (1974).

where $\varphi_1(\boldsymbol{x})$, $\varphi_2(\boldsymbol{y})$ are the nucleon density distribution functions of nuclei 1 and 2 measured from their own centres and v is the nucleon-nucleon potential. It can be seen from eq. (1) that each nucleon of the projectile nucleus is treated essentially as free and, therefore, the saturation property of the two-body interaction, which prevents the nuclear density from increasing beyond a certain limit, is ignored. Thus, the potential calculated in this way is found to be over-estimated [7,8]. An approach was suggested by SINHA [8], to get the correct order of magnitude of the potential, by using a density-dependent two-body interaction to account for the saturation property of the two-body interaction. Now the density function $\varphi_2(y)$ of nucleus 2 can be written in terms of the single-particle wave function $\psi_{n_i l_i j_i m_i}$ of a nucleon in the state $(n_i l_i j_i)$ as

$$\varphi_2 = \sum_{i=1}^{N_2} \sum_{m_i} |\psi_{n_i l_i j_i m_i}(\boldsymbol{y})|^2 = \sum_{i=1}^{N_2} \frac{2l_i + 1}{4\pi} |\psi_{n_i l_i}(y)|^2, \tag{2}$$

where N_2 is the number of the nucleons in nucleus 2. But, as the wave function of the nucleus is an antisymmetric one, then each nucleon contributes the same to $\varphi_2(y)$, so one can drop the summation in eq. (2) and multiply the single-particle density by the number of nucleons in the state $(n_i l_i j_i)$, thus

$$\varphi_2(y) = \sum_{l,j} \frac{2l+1}{4\pi} N_{lj} |\psi_{nl}(y)|^2 \tag{3}$$

with $\sum_{l,j} N_{lj} = N_2$.

Substituting eq. (3) in eq. (1) and for the region of light nuclei, one can use the oscillator wave function for $\psi_{nl}(y)$ and one gets for $V(R)$ the following form:

$$V(R) = \sum_{nl} V_n^0 N_{lj} |A_l|^2 \frac{2l+1}{4\pi} \cdot \int \varphi_1(x) j_0(2i\beta_n y r) \exp[-(\beta_n + 2\alpha) y^2] y^{2l+2} \exp[-\beta_n r^2] \, \mathrm{d}\boldsymbol{r} \, \mathrm{d}y \, . \tag{4}$$

Here $\psi_{nl} = A_l r^l \exp[-\alpha r^2]$, and for $v(|\boldsymbol{r} - \boldsymbol{y}|)$ a combination of Gaussian potentials is used [7], *i.e.*

$$v(|\boldsymbol{r} - \boldsymbol{y}|) = -\sum_n v_n^0 \exp[-\beta_n |\boldsymbol{r} - \boldsymbol{y}|^2] \, .$$

Equation (4) is evaluated for $1p$-nuclei and adopting a Gaussian form for $\varphi(x)$ (nucleus 1 is also in the region of light nuclei, although one can adopt a Fermi shape for $\varphi_1(x)$ and apply the method to heavier target nuclei) and finally the following expression is obtained for the nucleus-nucleus potential:

$$V(R) = 2\pi^2 \varphi_0 \sum \exp\left[-\left(\sigma_n - \frac{\sigma_n^2}{\sigma_n^1}\right) R^2\right] (A_n + B_n R^2) \tag{5}$$

with

$$A_n = \frac{1}{(2\sigma_n^1)^{\frac{3}{2}}}\left[\frac{V_n^0 N_0|A_0|^2}{[2(2\alpha+\beta_n)]^{\frac{3}{2}}} + \frac{9V_n^0 N_{1j}|A_1|^2}{[2(2\alpha+\beta_n)]^{\frac{5}{2}}}\right] + \frac{9\beta_n^2 N_{1j} V_n^0|A_1|^2}{2^{\frac{3}{2}}(2\sigma_n^1)^{\frac{5}{2}}(2\alpha+\beta_n)^{\frac{7}{2}}},$$

$$B_n = \frac{3\gamma^2\beta_n^2 N_{1j}|A_1|^2 V_n^0}{8[\sigma_n^1(2\alpha+\beta_n)]^{\frac{7}{2}}}, \qquad \sigma_n = \beta_n - \frac{\beta_n^2}{2\alpha+\beta_n}, \qquad \sigma_n^1 = \sigma_n + \gamma$$

and $\varrho_1 = \varrho_0 \exp[-\gamma x^2]$.

Equation (5) is adopted to calculate the real part of the ^{16}O-^{28}Si potential, and the results are shown in fig. 1. In the numerical calculation, the standard

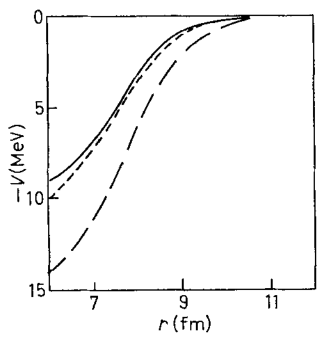

Fig. 1. – The ^{16}O-^{28}Si optical potential: the solid line is Cramer *et al.* (6) potential, the broken line gives the present results, taking all nucleons in consideration, while the dashed line gives those taking only the nucleons of the last state in consideration.

values of α, β, γ and ϱ_0 are used, while the form used by SATCHLER (7) and others (10) for the nucleon-nucleon effective potential, *viz.*

$$v(r) = -5.44 \exp[-0.292r^2] - 12.548 \exp[-0.415r^2],$$

is adopted.

As said before, the value of the potential calculated by this method is found to be over-estimated, indeed, it is found that one has to multiply the resulting values of the potential when all of the nucleons are considered (broken line in fig. 1) by a normalization factor of the order of 0.5 to get the values of the Cramer *et al.* potential (solid line) in the scattering region $(6\div10)$ fm. This factor is nearly the same as that found by SATCHLER (7).

In the work of Sinha (8) the nuclear density is taken into account by using a two-body density-dependent effective interaction. Thus, the internal nucleons in both nuclei are shielded by the outer nucleons and, for the internal ones to participate in the interaction process, the two nuclei must get close to each

(10) I. REICHSTEIN and Y. C. TANG: *Nucl. Phys. A*, **139**, 144 (1969).

other in such a way that a significant overlap occurs and the nuclear density increases up to the violation of the saturation property. So, if the contribution from these inner nucleons of the projectile nucleus is dropped and they are treated as an inert core (these inner nucleons were treated similarly in the theories of direct nuclear reactions) and only the contribution from the outer nucleons is taken into account, the nuclear density for all the considered nucleons does not exceed the proper value and keeps the saturation property unviolated. In fig. 1 the dashed line gives the potential when only the $1p_{\frac{3}{2}}$ nucleons are considered. No appreciable difference between the Cramer *et al.* potential and the present results is observed. This result shows that it is possible by simple calculations rather than other approaches [3,8] to obtain the right order of magnitude of the real part of the optical potential.

3. – Nucleus-nucleus potential considering one nucleus as two fragments.

It was found that the optical potential of the deuterons and of the helions can be calculated in terms of the potentials of their constituents at the appropriate energies [11]. In this section, the single-folding form is adopted to write the real part of the nucleus-nucleus potential, by considering one nucleus as two fragments, in terms of the potentials between each fragment and the other nucleus. The single-folding form for the nucleus-nucleus potential is [1]

$$V_{12}(\boldsymbol{r}) = \int \varphi_1(\boldsymbol{r}_1)\, V_2(\boldsymbol{r}_1 - \boldsymbol{r})\, \mathrm{d}\boldsymbol{r}_1\,, \tag{6}$$

here $V_2(\boldsymbol{r}_1 - \boldsymbol{r})$ is the potential between the single nucleon of nucleus 1 and nucleus 2. If one considers nucleus 2 as two fragments A and B, then the potential $V_{12}(\boldsymbol{r})$ may be written in the form [11] (if one neglects spin complications)

$$V_{12}(\boldsymbol{r}) = \int |\Phi(\boldsymbol{x})|^2 \big(\theta_{\mathrm{A}}(\boldsymbol{r}_{\mathrm{A}}) + \theta_{\mathrm{B}}(\boldsymbol{r}_{\mathrm{B}})\big)\, \mathrm{d}\boldsymbol{x} = V_{\mathrm{A}}(\boldsymbol{r}) + V_{\mathrm{B}}(\boldsymbol{r})\,, \tag{7}$$

where $\boldsymbol{r}_{\mathrm{A}}$ and $\boldsymbol{r}_{\mathrm{B}}$ are the separation distances of the centres of nuclei A and B, respectively, measured from the centre of nucleus 1; and $\Phi(\boldsymbol{x})$ is the relative wave function describing the motion of nuclei A and B in nucleus 1. From eq. (6) one writes for the potential $V_{\mathrm{A}}(\boldsymbol{r})$

$$V_{\mathrm{A}}(\boldsymbol{r}) = \int |\Phi(\boldsymbol{x})|^2 \varphi_1(\boldsymbol{r}_1)\, \theta_{\mathrm{A}}\left(\boldsymbol{r}_1 - \boldsymbol{r} - \frac{m_{\mathrm{A}}}{m_2}\, \boldsymbol{x}\right) \mathrm{d}\boldsymbol{x}\, \mathrm{d}\boldsymbol{r}_1\,. \tag{8}$$

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(11) P. E. Hodgson: *Nuclear Reactions and Nuclear Structure* (Oxford, 1971); J. R. Rook: *Nucl. Phys.*, **61**, 219 (1965); A. Y. Abul-Magd and M. El-Nadi: *Prog. Theor. Phys.*, **35**, 798 (1966).

By expanding θ_A as

$$\exp\left[-\frac{m_A}{m_2}\,\boldsymbol{x}\cdot\boldsymbol{\nabla}(\boldsymbol{r}_1-\boldsymbol{r})\right]\theta_A(\boldsymbol{r}_1-\boldsymbol{r})\,,$$

where m_A and m_2 are the masses of A and 2, and using the Saxon-Woods form for

$$\theta_A(\boldsymbol{r}_1-\boldsymbol{r})=V_{A_0}\left(1+\exp\left[\frac{|\boldsymbol{r}_1-\boldsymbol{r}|-R_{A_0}}{\Delta}\right]\right)^{-1},$$

then

$$V_A(\boldsymbol{r}_A)=V_A\sum_{n=0}^{\infty}(-)^n\exp\left[-\frac{n}{\Delta}(R_{A_0}-|\boldsymbol{r}_1-\boldsymbol{r}|)\right]\exp\left[-\frac{n}{\Delta}\left(\frac{m_A}{m_2}\,\boldsymbol{x}\cdot(\boldsymbol{r}_1-\boldsymbol{r})\right)\right],\tag{9}$$

where V_{A_0}, R_{A_0} and Δ are the depth, the half-way radius and the diffuseness of the potential between the single nucleon of nucleus 1 and nucleus A (fragment of nucleus 2). In heavy-ion scattering the important partial waves have a large angular momentum compared to the relative angular momentum between the fragments of nucleus 2, hence one can use a Gaussian form for $\Phi(\boldsymbol{x})$, *i.e.* $\Phi(\boldsymbol{x})=(N/4\pi)\exp[-\alpha x^2]$, and the potential $V_A(\boldsymbol{r}_A)$ becomes

$$V_A(\boldsymbol{r}_A)=\frac{\sqrt{\pi}\,N^2}{(4\alpha)^{\frac{3}{2}}}V_{A_0}\sum_{n=0}^{\infty}(-)^n\exp\left[\frac{\gamma^2n^2}{\Delta^2}\right]\int\exp\left[-\frac{n}{\Delta}(R_{A_0}-|\boldsymbol{r}_1-\boldsymbol{r}|)\right]\varphi_1(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}_1\tag{10}$$

with $\gamma^2=m_A^2/8m_2^2\alpha$.

The factor $\exp[\gamma^2n^2/\Delta^2]$ can be, to a good approximation, put equal to $\cosh(\sqrt{2}\gamma n/\Delta)$, and substitution from eq. (9) gives

$$V_A(\boldsymbol{r}_A)=\frac{\sqrt{\pi}V_{A_0}N^2}{2(4\alpha)^{\frac{3}{2}}}\left\{\int\varphi_1(\boldsymbol{r}_1)\,\theta^-(\boldsymbol{r}_1-\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}_1+\int\varphi_1(\boldsymbol{r}_1)\,\theta^+(\boldsymbol{r}_1-\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}_1\right\},\tag{11}$$

here θ^- and θ^+ mean the half-way radius R_{A_0} of the single-nucleon potential replaced, respectively, by $R_{A_0}-\gamma$ and $R_{A_0}+\gamma$.

The integrals in eq. (11) are evaluated by BRINK and ROWELY [9] who, analytically adopting reasonable approximations and using the Saxon-Woods form for both φ_1 and $\theta(\boldsymbol{r}_1-\boldsymbol{r})$ with the same diffuseness, found the following result:

$$\int\varphi_1(\boldsymbol{r}_1)\,\theta^{(\pm)}(\boldsymbol{r}_1-\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}_1=\frac{\pi\varphi_0V_{A_0}}{4r}f^{\pm}(r)\tag{12}$$

with

$$f^{(\pm)}(r)=2\Delta(r+\Delta)^2[r+\Delta-(R_{A_0}^{(\pm)}+R_A)]\exp\left[-\frac{r-(R_{A_0}^{\pm}+R_A)}{\Delta}\right],$$

$$\varphi_1=\varphi_{01}\left(1+\exp\left[\frac{r_1-R_A}{\Delta}\right]\right)^{-1}.$$

Equations (7), (11) and (12) show that the nucleus-nucleus potential, when one of the nuclei is considered as two fragments, can be expressed as a sum of four terms, two for each fragment.

These results are applied to the cases of

$$^{56}\mathrm{Ni}\text{-}^{208}\mathrm{Pb} \quad \text{as} \quad (^{16}\mathrm{O}+^{40}\mathrm{Ca})\text{-}^{208}\mathrm{Pb} \qquad (\text{fig. } 2)$$

and

$$^{56}\mathrm{Ni}\text{-}^{90}\mathrm{Zr} \quad \text{as} \quad (^{16}\mathrm{O}+^{40}\mathrm{Ca})\text{-}^{90}\mathrm{Zr} \qquad (\text{fig. } 3).$$

In the numerical calculations the values given by BRINK and STANCU (3) for V_0, R_0 and Δ are adopted. As may be seen from the figures, the agreement

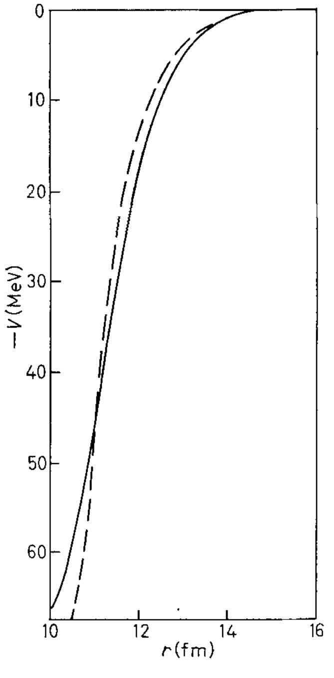

Fig. 2.

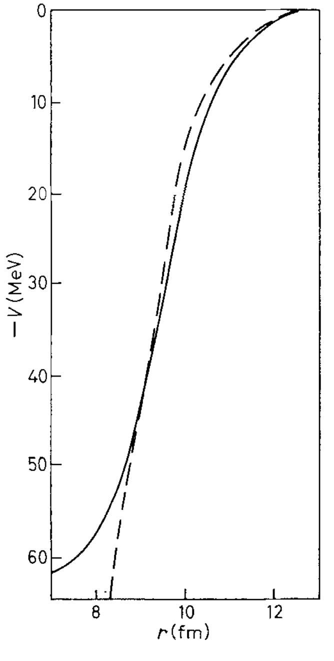

Fig. 3.

Fig. 2. – The solid line represents the ^{56}Ni-^{208}Pb potential, the dashed line gives the potential when the ^{56}Ni nucleus is considered as ^{40}Ca+^{16}O.

Fig. 3. – The solid line represents the ^{56}Ni-^{90}Zr potential, the dashed line gives the potential when the ^{56}Ni nucleus is considered as ^{40}Ca+^{16}O.

is excellent for the scattering region of the potential. The separation of the potential into the potentials of the constituents of one nucleus has a big advantage in the calculations of transfer reaction between heavy ions, specially in the study of recoil effects (12).

(12) M. A. SHARAF: *Phys. Lett.*, to be published.

● RIASSUNTO (*)

Si calcola la parte reale del potenziale ottico degli ioni pesanti per mezzo della tecnica a strati, trascurando il contributo dei nucleoni più interni invece di usare potenziali dipendenti dalla densità. Si deriva l'espressione per il potenziale quando uno dei nuclei è considerato come somma di due frammenti.

(*) *Traduzione a cura della Redazione.*

Потенциалы взаимодействия между ядрами.

Резюме (*). — С помощью метода свертки, отбрасывая вклад внутренних нуклонов вместо использования потенциалов, зависящих от плотности, вычисляется вещественная часть оптического потенциала для взаимодействия тяжелых ионов. Выводится выражение для потенциала, когда одно из ядер рассматривается как два фрагмента.

(*) *Переведено редакцией.*