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Analysis of steady-state heat transfer through mid-crustal vertical cracks with upward throughflow in hydrothermal systems

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SUMMARY

We conduct a theoretical analysis of steady-state heat transfer problems through mid-crustal vertical cracks with upward throughflow in hydrothermal systems. In particular, we derive analytical solutions for both the far field and near field of the system. In order to investigate the contribution of the forced advection to the total temperature of the system, two concepts, namely the critical Peclet number and the critical permeability of the system, have been presented and discussed in this paper. The analytical solution for the far field of the system indicates that if the pore-fluid pressure gradient in the crust is lithostatic, the critical permeability of the system can be used to determine whether or not the contribution of the forced advection to the total temperature of the system is negligible. Otherwise, the critical Peclet number should be used. For a crust of moderate thickness, the critical permeability is of the order of magnitude of 10^{-20} m^2, under which heat conduction is the overwhelming mechanism to transfer heat energy, even though the pore-fluid pressure gradient in the crust is lithostatic. Furthermore, the lower bound analytical solution for the near field of the system demonstrates that the permeable vertical cracks in the middle crust can efficiently transfer heat energy from the lower crust to the upper crust of the Earth.

KEY WORDS: theoretical solutions; heat transfer; vertical crack; upward throughflow; hydrothermal systems

1. INTRODUCTION

The study of heat transfer mechanisms in hydrothermal systems is important for the understanding of basic physics behind the orebody formation and mineralization in the upper crust of the Earth [1–6]. Generally, heat energy may be transferred within the crust of the Earth in the following forms: conduction, forced advection, buoyancy driven convection (i.e. free convection, natural convection) and a combination of them. Since the advective flow is usually generated by pore-fluid pressure gradient, heat transfer due to the advective flow is largely

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dependent on the pore-fluid pressure gradient distribution in hydrothermal systems. A typical example of this kind of the forced advective flow is the upward throughflow caused by the pore-fluid pressure gradient of lithostatic pressure value within the lower crust of the Earth. Extensive studies [7–13] have shown that the lithostatic pressure can be built up by the metamorphic fluids due to devolatilization and dehydration reactions, provided that the permeability is low enough to control the fluid flow in the lower crust of the Earth. It is noted that the forced advection mentioned in this study refers to the contribution of the metamorphic fluid flow to heat transfer within the crust of the Earth. Although heat transfer due to both conduction and buoyancy driven convective flow is caused by the vertical temperature gradient, there are some significant differences between these two heat transfer mechanisms. First, heat transfer due to pure conduction is caused by a subcritical temperature gradient, while heat transfer due to the buoyancy driven convective flow is caused by a critical or supercritical temperature gradient. For a given hydrothermal system, the critical vertical temperature gradient can be evaluated by the critical Rayleigh number, which is directly determinable from the classical theory in the field of convective flow in porous media [14–17]. Second, in the case of heat transfer due to conduction, both the rock mass and pore-fluid in a porous medium play important roles in transferring heat energy. However, in the case of heat transfer due to buoyancy driven convective flow, heat energy may be predominantly transferred by the pore-fluid in the porous medium (e.g. in the case of large Rayleigh numbers).

According to different heat transfer mechanisms, we have developed two different theoretical models to represent the process of orebody formation and mineralization in the upper crust of the Earth [18–21]. In the first model, heat is transferred by a combination of conduction and buoyancy driven convection and therefore, this model is called the temperature gradient driven orebody formation and mineralization model [19]. In this model, an orebody can be localized by the convective flow so that large ore deposits may be formed within the upper crust of the Earth. The restriction of this model is that, in order to generate the requisite convective flow, the permeability of the porous medium under consideration should be around or greater than the order of magnitude of 10^{-14} m^2, for a physically realistic upper crust. In the second model, heat is transferred by a combination of conduction, forced advection and buoyancy driven convection [20,21]. The basic characteristic of this model is that the forced advective flow is driven by the pore-fluid pressure gradient of lithostatic pressure gradient value and is controlled by the least permeable layer in the crust of the Earth [22]. For this model, if the boundary condition of thermal flux is applied at the bottom of the crust, the minimum permeability of the porous medium to generate the buoyancy driven convective flow can be reduced to the order of magnitude of 10^{-16} m^2, for a physically realistic thick crust [20,21]. This means that the buoyancy driven convective flow can be triggered in the deeper crust of the Earth, if this model is used. Since both pore-fluid pressure gradient and temperature gradient play important roles in the process of heat transfer, this model is called the mixed pore-fluid pressure and temperature gradients driven orebody formation and mineralization model. If the permeability of the crust becomes much smaller, even though the pore-fluid pressure gradient in the crust is maintained at the value of lithostatic pressure gradient, both the forced advection and buoyancy driven convection may become unimportant, from the heat transfer point of view. In this case, conduction is the main mechanism to transfer heat energy within the crust. Since the capability of transferring heat by conduction is limited, the upper crust may not be hot enough to enable orebody formation and mineralization to take place. To solve this problem, Connolly [6] presented a middle crust crack model to transfer heat energy efficiently through focused advective flow in the middle crust cracks. Although some numerical results have been produced from this model [6], the following two fundamental questions remain unanswered: (1) What is the critical value of permeability, below which the forced advective flow caused by the pore-fluid pressure gradient (of lithostatic pressure gradient value) becomes unimportant, from the heat transfer point of view? (2) Is it possible to obtain analytical solutions for the middle crust crack model, because it is difficult to use numerical solutions to draw general conclusions? It needs to be pointed out that the reason of posing the first question here is that permeability is an intrinsic and therefore unique property of the porous material in a given system. Although the fluid mass flux rate was widely used to describe the metamorphic fluid flow in the crust of the Earth, it is a derived and therefore non-unique quantity to describe the consequence of a resulting fluid flow. As we mentioned above, the fluid mass flux rate can be the consequence of a topography induced fluid flow [23,24], a buoyancy driven fluid flow (i.e. free convection flow) [14–17], a metamorphic fluid flow [7–11,13] and so forth. On the other hand, unlike other properties such as the fluid density, viscosity and specific heat, permeability is independent of both temperature and pressure in the given system. This means that the intrinsic permeability of a porous material is a better quantity to identify the possible heat transfer regime in the system, as we discussed in the beginning of this paragraph. Although Brady [3] presented an analytical solution for transient heat transfer problems in parallel channels, the solution is limited to the systems of periodic boundary conditions. The parallel channels may enhance the thermal flow in each individual channel, especially for steady-state heat transfer problems. Therefore, it is desirable to derive analytical solution for steady-state heat transfer in single channel systems.

Taking the above into account, we derive analytical solutions for steady-state heat transfer through mid-crustal vertical cracks with upward throughflow in hydrothermal systems. The forthcoming contents of this paper are arranged as follows. After the problem to be considered is described, the related assumptions, on which the theoretical analysis is based, are given in Section 2. In Section 3, analytical solutions are derived for the far field, in which the effect of the crack on the solution is negligible, of the system. This far field solution is then used to answer the first question raised in the above paragraph. Similarly, the near field of the system is referred to as the crack and its near surroundings, in which the effect of the crack to the solution must be considered. The lower bound analytical solution for the near field of the system is derived in Section 4. This near field solution is used to answer the second question raised above. Finally, some conclusions drawn from this study are given in Section 5.

2. STATEMENT OF THE PROBLEM

The problem to be considered in this study is a crust of horizontal flat bottom and top. The crust is comprised of fluid-saturated porous materials and is heated uniformly from the bottom of the crust. There is a vertical planar crack in the middle of the crust. Figure 1 shows the geometry of the problem. Compared with the length scale of the crust, the thickness of the crack is very small. This means that the crack only affects the solution in its near surroundings [25]. For this reason, the whole domain of the problem can be divided into two sub-domains. The first sub-domain is the crack and its near surroundings and is defined as the near field of the system. The second sub-domain is the rest of the crust, in which the effect of the crack on the solution is negligible. This sub-domain is called the far field of the system. In the far field, the pore-fluid pressure gradient is assumed to be lithostatic. This implies that the Darcy's velocity in the

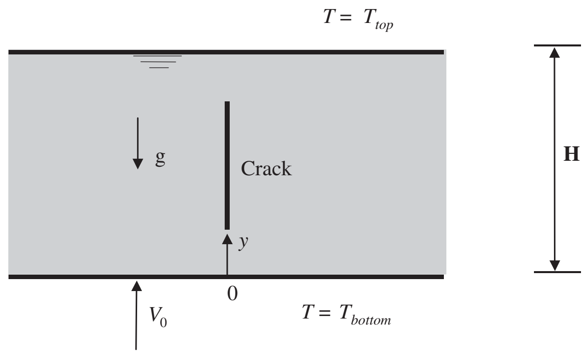


Figure 1. Geometry of the problem.

vertical direction due to the upward throughflow is constant throughout the far field of the system [22].

In order to derive analytical solutions for scientific and engineering problems, it is common practice to make assumptions [26,27]. According to Phillips [17], the assumptions must, however, be firmly based and clearly specified. The theoretical analysis to be carried out in this paper is based on the following assumptions.

(1) Heat transfer in the crust has reached a steady state. This means that the warm-up process of the whole crust has been completed in the forthcoming analysis. Therefore, the analytical results from this study can only provide a lower bound solution for the problem considered in this paper.
(2) The Rayleigh number of the hydrothermal system considered is subcritical, so that the buoyancy driven convective flow of the pore-fluid cannot take place in the system.
(3) Compared with the length scale of the crust, the thickness of the crack is very small. Therefore, the effect of the crack on the temperature solution is only of local significance.
(4) The pore-fluid pressure gradient in the overall crust is lithostatic. This implies that there is an upward throughflow and the resulting vertical Darcy's velocity is constant throughout the far field of the system [22]. Since the main focus is on heat transfer due to the forced advective flow, radioactive heat sources are ignored in the analysis.
(5) The permeability of the crack is much greater than that of the crust, so that the pore-fluid flow is highly focused within the crack. The focusing phenomenon of the pore-fluid can be considered using the analytical model presented by Connolly [6]. Therefore, the near field of the system can be divided into three regions: a collection region, a stabilized region and a dispersion region.
(6) The vertical Darcy's velocity within the crack is much greater than that in the far field of the system. Thus, the effect of the vertical Darcy's velocity in the far field is negligible for the analysis of heat transfer in the near field of the system.
(7) In the near field of the system, heat transfer is highly dominated in the vertical direction so that the heat transfer due to the forced advection in the horizontal direction can be neglected in the analysis, because either the horizontal velocity component or the temperature gradient in the horizontal direction is relatively very small. Since the length of the crack is much greater than the thickness of the crack, the one-dimensional crack

flow model [6] can be used to represent the main heat transfer phenomenon along the vertical axis of the crack.

3. DERIVATION OF ANALYTICAL SOLUTIONS FOR THE FAR FIELD OF THE SYSTEM

If the permeability of the crust is small enough, the Rayleigh number of the system can reach a subcritical value, because the Rayleigh number is directly proportional to the permeability of the crustal material. In this case, the buoyancy driven convection of pore-fluid cannot take place in the crust and therefore, conduction and the forced advection are two possible mechanisms to transfer heat energy in the hydrothermal system. However, when the permeability of the crust becomes even much smaller, the heat transfer due to the forced advection may become negligible and consequently, conduction becomes the major mechanism to transfer heat energy in the crust of the Earth. In order to find the critical value of the permeability, under which the contribution of the forced advection to the heat transfer in the crust is negligible, it is necessary to use the analytical solution for the temperature distribution in the far field of the system [28–30]. By subtracting the temperature due to conduction only from the total temperature due to both conduction and advection, we can evaluate the contribution of the forced advection to the total temperature. As a result, we can determine the condition, under which the contribution of the forced advection to the total temperature is negligible in the hydrothermal system considered. This condition can be further used to determine the critical value of the permeability of the crust.

As mentioned above, the Rayleigh number of the hydrothermal system considered here is subcritical so that we only need to derive the trivial solution for the temperature distribution in the far field of the system. Since the solution for the far field is identical to that for the whole crust without the crack, we can directly use this solution for the whole crust model, as shown in Figure 2. For this particular crust model, if the buoyancy driven convection is impossible (as we consider here), the problem can be treated as a one-dimensional problem in the theoretical analysis [28–30]. In this case, the dimensionless governing equation for describing the steady-

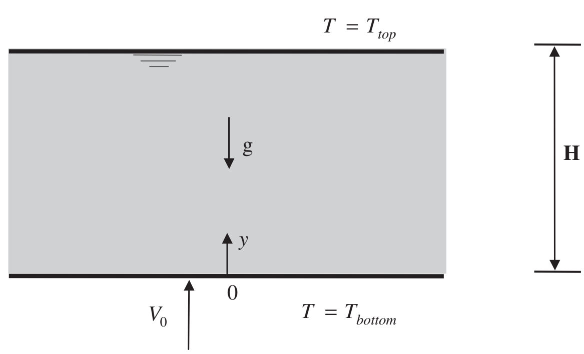


Figure 2. Geometry of the far field of the problem.

state heat transfer can be expressed as follows [30,17,16]:

$$\frac{\mathrm{d}^2 T^*}{\mathrm{d}y^{*2}} - Pe\frac{\mathrm{d}T^*}{\mathrm{d}y^*} = 0 \tag{1}$$

where T^* is the dimensionless temperature; Pe is the Peclet number of the hydrothermal system; y^* is the dimensionless co-ordinate in the vertical direction. These dimensionless quantities can be expressed as

$$y^* = \frac{y}{H}, \quad T^* = \frac{T - T_{\mathrm{top}}}{T_{\mathrm{bottom}} - T_{\mathrm{top}}} \tag{2}$$

$$Pe = \frac{H\rho_{\mathrm{f}} c_p}{\lambda_{\mathrm{e}}} V_0 \tag{3}$$

where T is temperature; T_{top} and T_{bottom} are the temperature at the top and bottom, respectively; y is the physical co-ordinate in the vertical direction; H is the thickness of the crust; ρ_{f} and c_p are the density and specific heat of the pore-fluid; λ_{e} is the thermal conductivity of the porous medium; and V_0 is the vertical Darcy's velocity due to the upward throughflow in the crust.

If the pore-fluid pressure gradient is lithostatic, the vertical Darcy's velocity in the crust can be evaluated using the following formula [31,7,22]

$$V_0 = \frac{K}{\mu}(\rho_{\mathrm{s}} - \rho_{\mathrm{f}})g \tag{4}$$

where ρ_{s} is the density of the solid matrix; μ is the dynamic viscosity of the pore-fluid; K is the permeability of the porous medium; g is the acceleration due to gravity.

Substituting Equation (4) into Equation (3) yields the following equation:

$$Pe = \frac{H\rho_{\mathrm{f}} c_p K}{\lambda_{\mathrm{e}}\mu}(\rho_{\mathrm{s}} - \rho_{\mathrm{f}})g \tag{5}$$

Equation (5) indicates that the Peclet number of the hydrothermal system is directly proportional to the permeability of the crustal material. This means that if the critical Peclet number, under which the contribution of the forced advection to the heat transfer in the crust is negligible, is determined, then the corresponding critical permeability can be straightforwardly evaluated from Equation (5), for a given hydrothermal system with upward throughflow.

The corresponding boundary conditions for the problem are:

$$T^* = 0 \quad (\text{at } y^* = 1)$$

$$T^* = 1 \quad (\text{at } y^* = 0) \tag{6}$$

The solution for Equation (1) with the boundary conditions in Equation (6) can be derived and expressed as follows:

$$T^*(y^*) = \frac{1}{1 - \mathrm{e}^{-Pe}} + \frac{1}{1 - \mathrm{e}^{Pe}} \mathrm{e}^{Pe\, y^*} \tag{7}$$

Similarly, if the Peclet number is equal to zero, the temperature due to pure conduction can be derived and expressed as

$$T^*_{\mathrm{conduction}}(y^*) = 1 - y^* \tag{8}$$

Clearly, the contribution of the forced advection to the temperature distribution is the difference between Equations (7) and (8). This difference can be expressed as

$$\Delta T^*(y^*) = T^*_{\text{advection}}(y^*) = y^* - \frac{1 - e^{Pe\, y^*}}{1 - e^{Pe}} \tag{9}$$

Differentiating Equation (9) with respect to y^* leads to the following equation:

$$\frac{d[\Delta T^*(y^*)]}{dy^*} = 1 + \frac{Pe}{1 - e^{Pe}} e^{Pe\, y^*} \tag{10}$$

Allowing the above differentiation to be equal to zero yields the following condition:

$$y^* = \frac{1}{Pe} \ln\left(\frac{e^{Pe} - 1}{Pe}\right) \tag{11}$$

Note that under this condition, $\Delta T^*(y^*)$ has reached the maximum value for the problem considered. Therefore, by substituting Equation (11) into Equation (9), we can obtain the maximum contribution of the forced advection to the temperature in the crust.

$$T^*_{\text{advection}}(y^*)|_{\max} = \frac{1}{e^{Pe} - 1} + \frac{1}{Pe}\left[\ln\left(\frac{e^{Pe} - 1}{Pe}\right) - 1\right] \tag{12}$$

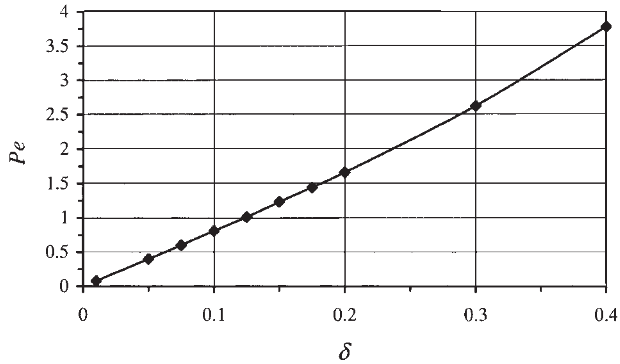
Since the maximum value of the dimensionless temperature in the system is one, the ratio of the maximum temperature contribution due to the forced advection to the maximum total temperature can be readily expressed as

$$\delta = \frac{1}{e^{Pe} - 1} + \frac{1}{Pe}\left[\ln\left(\frac{e^{Pe} - 1}{Pe}\right) - 1\right] \tag{13}$$

Therefore, for a given value of δ, we can determine the value of the corresponding Peclet number using Equation (13). This means that if we set a standard value of δ, under which the maximum contribution of the forced advection to the temperature is negligible, then we can find the corresponding critical value of the Peclet number from Equation (13).

Figure 3 shows the variation of the Peclet number with the maximum contribution of the forced advection to the maximum total temperature. It is observed that when the Peclet number is less than 6, the Peclet number varies linearly with the maximum contribution of the forced advection to the maximum total temperature. However, when the Peclet number is greater than 6, the relationship between the Peclet number and the maximum contribution of the forced advection to the maximum total temperature is highly non-linear. This indicates that when the upward throughflow becomes stronger and stronger, the contribution of the forced advection to the overall heat transfer becomes greater and greater.

Now let us show how to use the $Pe - \delta$ curve in Figure 3 to determine the critical value of the Peclet number. Supposing the standard value of δ is set to be 0.01. This means that if the maximum contribution of the forced advection to the maximum total temperature is below 1%, the contribution of the forced advection is negligible in the whole heat transfer process. Using



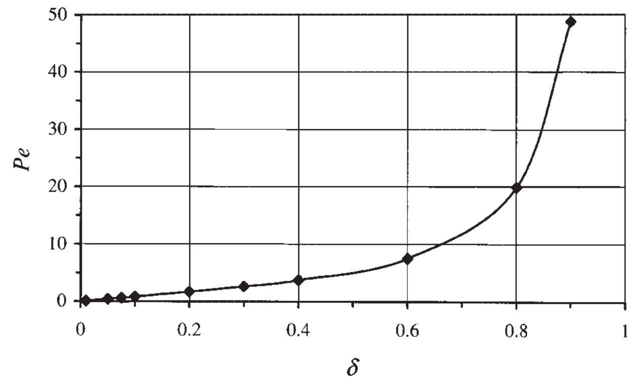


Figure 3. Variation of Peclet number with the maximum contribution of advection to temperature.

the $Pe - \delta$ curve in Figure 3, we can find that the corresponding critical value of the Peclet number is equal to 0.08. As mentioned above, we can use Equation (5) to calculate the corresponding critical value of the permeability of the system.

$$K = \frac{\lambda_e \mu Pe}{H \rho_f c_p (\rho_s - \rho_f) g} \tag{14}$$

For the purpose of illustrating how to use Equation (14) to determine the critical permeability of a system, the numerical values for the related properties, which were used by Connolly [6], are used here. Connolly [6] justified the appropriateness of using those values in his paper. With a crust of 30 km thick taken as an example, the following parameters [32,6] are used to determine the critical value of the permeability of the system: The dynamic viscosity of the pore-fluid is 10^{-4} N s/m^2; The density of the pore-fluid is 800 kg/m^3; The specific heat of the pore-fluid is 4000 J/(kg °C); The thermal conductivity coefficient of the porous medium is 2.25 W/(m °C); The density of the solid matrix is 2800 kg/m^3. Substituting these parameters into Equation (14), we find that the corresponding critical value of the permeability of the system is equal to 0.95×10^{-20} m^2. This means that for the crust of moderate thickness, if the permeability of the system is below the order of magnitude of 10^{-20} m^2, conduction becomes the overwhelming mechanism to transfer heat energy, even through the pore-fluid pressure gradient is maintained at the value of lithostatic pressure gradient. Thus, the first fundamental question raised in Section 1 has been successfully answered in this section.

4. DERIVATION OF ANALYTICAL SOLUTIONS FOR THE NEAR FIELD OF THE SYSTEM

Based on the assumptions made in Section 2, the near field of the system is divided into three regions: a collection region, a stabilized region and a dispersion region [6], as shown in Figure 4, where L_{C}, L_{S} and L_{D} are the length for the collection, stabilized and dispersion regions, respectively. In the collection region, the pore-fluid is focused into the crack, while in the dispersion region, the pore-fluid is dispersed out of the crack. However, the total amount of the pore-fluid flux is constant in these three regions, from the mass conservation point of view. For this reason, the total thickness of the system (i.e. the summation of the lengths of these three regions) can be used approximately to consider the energy conservation along the vertical

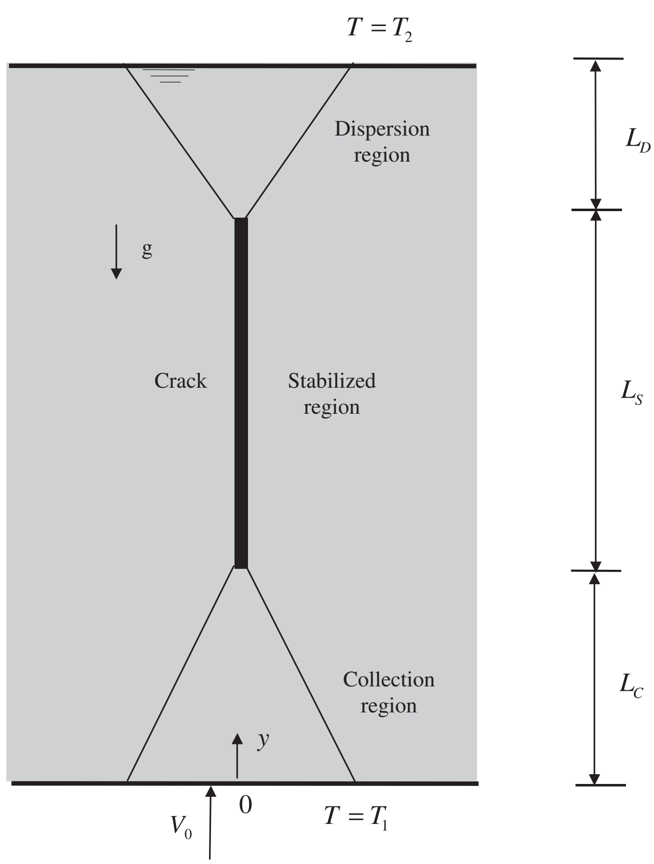


Figure 4. Geometry of the near field of the problem.

direction of the system. Since either the horizontal velocity component or the temperature gradient in the horizontal direction is relatively very small, heat transfer is highly dominated in the vertical direction so that heat transfer due to the forced advection in the horizontal direction can be neglected in the analysis. Under this assumption, Connolly [6] gave the energy conservation equation for the planar crack model in a transient state as follows:

$$(\rho c_p)_{\mathrm{e}} \frac{\mathrm{d}T}{\mathrm{d}t} = \lambda_{\mathrm{e}} \left(\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} + \frac{\mathrm{d}^2 T}{\mathrm{d}y^2} \right) - \rho_{\mathrm{f}} c_p^{\mathrm{f}} V \frac{\mathrm{d}T}{\mathrm{d}y} \tag{15}$$

where

$$(\rho c_p)_{\mathrm{e}} = \phi \rho_{\mathrm{f}} c_p^{\mathrm{f}} + (1-\phi)\rho_{\mathrm{s}} c_p^{\mathrm{s}}, \quad \lambda_{\mathrm{e}} = \phi \lambda_{\mathrm{f}} + (1-\phi)\lambda_{\mathrm{s}} \tag{16}$$

where ρ_{f} and ρ_{s} are the densities of the pore-fluid and rock mass; λ_{f} and c_p^{s} are the thermal conductivity of the pore-fluid and rock mass; c_p^{f} and c_p^{s} are the specific heat of the pore-fluid and rock mass, respectively; ϕ is the porosity of the porous medium, V is the vertical component of the fluid velocity.

It is noted that Equation (15) is exactly the same as the governing equation used by Connolly [6] in his numerical study, except for the replacement of the vertical fluid mass flux with the vertical component of the fluid velocity.

From the mathematics point of view, there are two basic ways to obtain a steady-state solution for the above-mentioned governing equation. The first way is to derive an analytical solution for the transient state governing equation and find the limit of the transient state solution as time variable approaches infinity. If the limit of the transient state solution exists, then this limiting solution is the steady-state solution. The second way is to simply eliminate the term with time variable in the left-hand side of Equation (15) and directly derive the steady-state solution. The existence of the steady-state solution means that the system considered can reach a steady state. Therefore, it may be considered as a useful method to examine whether or not a particular system can reach a steady state.

Let us use the second way to derive the steady-state solution for Equation (15). For this purpose, the following equation is considered in this analysis:

$$\lambda_{\mathrm{e}} \left(\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} + \frac{\mathrm{d}^2 T}{\mathrm{d}y^2} \right) - \rho_{\mathrm{f}} c_p^{\mathrm{f}} V \frac{\mathrm{d}T}{\mathrm{d}y} = 0 \tag{17}$$

Note that the solution of the total temperature can be divided into two parts: the unperturbed and perturbed ones. The unperturbed solution is the base solution in the far field of the system, while the perturbed solution is the solution due to the existence of the planar crack. In this regard, the following equations exist:

$$T = T_{\mathrm{base}} + T_{\mathrm{change}} \tag{18}$$

where T_{change} is the perturbed temperature due to the planar crack; T_{base} is the unperturbed temperature in the far field of the system. For the system considered in this study, the unperturbed temperature can be expressed as

$$T_{\mathrm{base}} = T_1 - k_{\mathrm{T}} y \tag{19}$$

where T_1 is the temperature at the bottom of the system; k_{T} is the vertical temperature gradient in the system.

Substituting Equations (18) and (17) into Equation (17) yields the following equation:

$$\lambda_e\left(\frac{d^2T_{change}}{dx^2}+\frac{d^2T_{change}}{dy^2}\right)-\rho_f c_p^f V\left(\frac{dT_{change}}{dy}-k_T\right)=0 \tag{20}$$

Note that the term, $\lambda_e(d^2T_{change}/dx^2)$, represents the heat lose rate in the lateral direction of the crack due to lateral heat conduction from the crack. This term can be approximately determined as follows. As shown in Figure 5, the half-width of the crack is W_c and the influence length, in which the temperature changes from T_{change} (at $l=0$) to zero (at $l=L_{inf}$), is L_{inf}. For a slice of the crack and surrounding rock, the thickness of the slice is assumed to be Δy. The net heat gain of the surrounding rock is expressed as

$$H_{gain}=\lambda_e\frac{d}{dl}\left(\frac{dT_{change}}{dl}\right)\Delta y L_{inf} \tag{21}$$

Clearly, dT_{change}/dl can be approximately expressed as

$$\frac{dT_{change}}{dl}\approx\frac{0-T_{change}}{L_{inf}}=-\frac{T_{change}}{L_{inf}} \tag{22}$$

Substituting Equation (22) into Equation (21) yields the following equation:

$$H_{gain}=\lambda_e\frac{T_{change}}{L_{inf}}\Delta y \tag{23}$$

Similarly, the net heat lose from the crack within the slice is

$$H_{lose}=-\lambda_e\frac{d^2T_{change}}{dx^2}\Delta y W_c \tag{24}$$

The conservation of thermal energy within the slice requires that the net heat gain in the surrounding rock be equal to the net heat lose from the crack. This leads to the following equation:

$$\frac{d^2T_{change}}{dx^2}=-\frac{T_{change}}{L_{inf}W_c} \tag{25}$$

Therefore, Equation (20) can be rewritten as

$$\frac{d^2T_{change}}{dy^2}-\alpha_1\frac{dT_{change}}{dy}-\alpha_2 T_{change}+\alpha_1 k_T=0 \tag{26}$$

where

$$\alpha_1=\frac{\rho_f c_p^f V}{\lambda_e},\quad \alpha_2=\frac{1}{L_{inf}W_c} \tag{27}$$

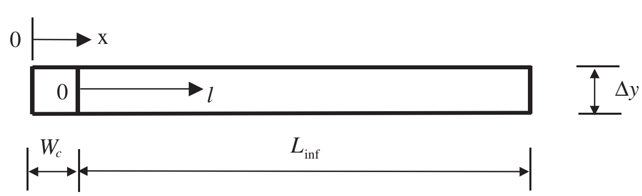


Figure 5. Consideration of lateral heat lose from the crack.

It is obvious that the particular solution to Equation (26) is as follows:

$$\hat{T}_{\text{particular}} = \frac{\alpha_1 k_{\mathrm{T}}}{\alpha_2} \tag{28}$$

Letting $\alpha_1 k_{\mathrm{T}}$ term be zero in Equation (26) results in the following homogeneous equation:

$$\frac{\mathrm{d}^2 T_{\text{change}}}{\mathrm{d}y^2} - \alpha_1 \frac{\mathrm{d}T_{\text{change}}}{\mathrm{d}y} - \alpha_2 T_{\text{change}} = 0 \tag{29}$$

The solution to Equation (29) can be derived and expressed as

$$T_{\text{change}}(y) = C_1 \mathrm{e}^{\beta_1 y} + C_2 \mathrm{e}^{\beta_2 y} \tag{30}$$

Therefore, the general solution to Equation (29) can be expressed as

$$T_{\text{change}}(y) = C_1 \mathrm{e}^{\beta_1 y} + C_2 \mathrm{e}^{\beta_2 y} + \frac{\alpha_1 k_{\mathrm{T}}}{\alpha_2} \tag{31}$$

where

$$\beta_1 = \frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_2}}{2}, \quad \beta_2 = \frac{\alpha_1 - \sqrt{\alpha_1^2 + 4\alpha_2}}{2} \tag{32}$$

Since we deal with the perturbed solution for temperature, the corresponding boundary conditions of the problem are as follows:

$$T_{\text{change}}(y) = 0 \quad (\text{at } y = 0) \tag{33}$$

$$T_{\text{change}}(y) = 0 \quad (\text{at } y = L_{\mathrm{C}} + L_{\mathrm{S}} + L_{\mathrm{D}}) \tag{34}$$

Using these boundary conditions, two constants involved in Equation (29) can be determined.

$$\begin{aligned} C_1 &= -\frac{\alpha_1 k_{\mathrm{T}}(1 - \mathrm{e}^{\beta_2 H})}{\alpha_2(\mathrm{e}^{\beta_1 H - \mathrm{e}^{\beta_2 H}})} \\ C_2 &= \frac{\alpha_1 k_T(1 - \mathrm{e}^{\beta_1 H})}{\mathrm{e}^{\beta_1 H} - \mathrm{e}^{\beta_2 \mathrm{H}}} \end{aligned} \tag{35}$$

where

$$H = L_{\mathrm{C}} + L_{\mathrm{S}} + L_{\mathrm{D}} \tag{36}$$

Adding the perturbed and unperturbed temperature together yields the total temperature along the axis of the planar crack.

$$T_{\text{total}}(y) = C_1 \mathrm{e}^{\beta_1 y} + C_2 \mathrm{e}^{\beta_2 y} + \frac{\alpha_1 k_{\mathrm{T}}}{\alpha_2} + T_1 - k_Y y \tag{37}$$

Up to now, we have derived the analytical solutions for the near field of the system, so that the second question raised in Section 1 has been successfully answered.

As an application example of the analytical solutions, we calculate the temperature distribution along the vertical axis of the crack in a crust of moderate thickness. The following parameters [6] are used in the calculation: The density of the pore-fluid is 800 kg/m^3; The specific heat of the pore-fluid is 4000 J/(kg °C); The thermal conductivity coefficient of the crack is 2.25 W/(m °C); The total length of the system 30 km. The temperature at the top and bottom is 20 and 620°C, respectively. The half width of the crack is 500 m and the width of the influence

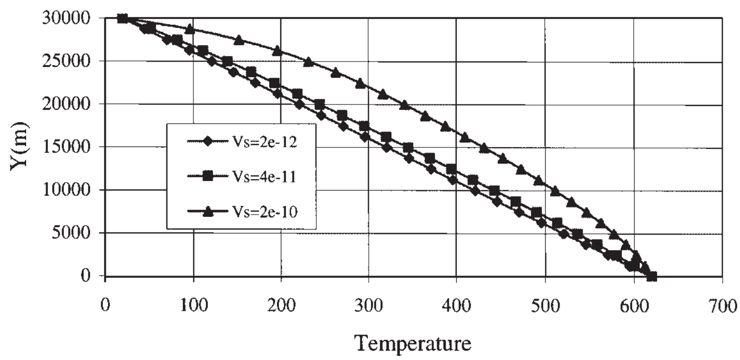


Figure 6. Effect of the vertical velocity on the temperature distribution.

region is 50 km. Since the vertical Darcy's velocity in the stabilized region is directly proportional to the permeability of the crack, it can be used to examine the effect of the crack permeability on the temperature distribution in the near field of the system. For this reason, three different values of the Darcy's velocity, namely 2×10^{-12}, 4×10^{-11} and 2×10^{-10} m/s, have been used in the calculation.

Figure 6 shows the effect of the vertical Darcy's velocity on the temperature distribution along the vertical axis of the crack. It is observed that when the vertical Darcy's velocity is equal to 2×10^{-12} m/s, the corresponding permeability is 10^{-20} m^2 under the condition that the pore-fluid pressure gradient is lithostatic. Since the critical permeability of the system is 10^{-20} m^2, the temperature distribution is mainly due to heat conduction. As mentioned before, in this case, the contribution of the forced advection to the temperature distribution is negligible. However, with the increase in the vertical Darcy's velocity, the contribution of advection to the temperature distribution becomes more and more important. This demonstrates that the more permeable vertical cracks can efficiently transfer heat energy from the lower crust to the upper crust of the Earth.

5. CONCLUSIONS

In this paper, we have derived analytical solutions to answer the following two fundamental questions: (1) What is the critical permeability in the Earth's crust, below which the advective flow caused by the pore-fluid pressure gradient (of lithostatic pressure gradient value) becomes unimportant, from the heat transfer point of view? (2) Can we obtain analytical solutions for the middle crust crack model?

The analytical solution for the far field of the system indicates that if the pore-fluid pressure gradient in the crust is lithostatic, the critical permeability of the system can be used to determine whether or not the contribution of the forced advection to the total temperature of the system is negligible. Otherwise, the critical Peclet number should be used. For a crust of moderate thickness, the critical permeability is of the order of magnitude of 10^{-20} m^2, under which heat conduction is the overwhelming mechanism to transfer heat energy, even though the pore-fluid pressure gradient in the crust is lithostatic.

The lower bound analytical solution for the near field of the system demonstrates that the permeable vertical cracks in the middle crust can efficiently transfer heat energy from the lower crust to the upper crust of the Earth.

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REFERENCES

1. Bickle MJ, McKenzie D. The transport of heat and matter by fluids during metamorphism. *Contributions to Mineralogy and Petrology* 1987; **95**:384–392.
2. Bjorlykke K, Mo A, Palm E. Modelling of thermal convection in sedimentary basins and its relevance to diagenetic reactions. *Marine and Petroleum Geology* 1998; **5**:338–351.
3. Brady J. The role of volatiles in the thermal history of metamorphic terrains. *Journal of Petrology* 1988; **29**: 1187–1213.
4. England PC, Thompson AB. Pressure–temperature–time paths of regional metamorphism: heat transfer during the evolution of regions of thickened crust. *Journal of Metamorphic Petrology* 1989; **83**:205–226.
5. Hoisch TD. The thermal effects of pervasive and channelized fluid flow in the deep crust. *Journal of Geology* 1991; **99**:69–80.
6. Connolly JAD. Mid-crustal focused fluid movements: thermal consequences and silica transport. In *Fluid Flow and Transport in Rocks: Mechanics and Effects*, Jamtveit B, Yardley BWD (eds). Chapman & Hall: London, 1997.
7. Connolly JAD, Ko SC. Development of excess fluid pressure during dehydration of the lower crust. *Terra Abstracts* 1995; **7**:312.
8. Etheridge MA, Wall VA, Vernon RH. The role of the fluid phase during regional metamorphism and deformation. *Journal of Metamorphic Petrology* 1983; **83**:205–226.
9. Fyfe WS, Price NJM, Thompson AB. *Fluids in the Earth's Crust*. Elsevier: Amsterdam, 1978.
10. Peacock SM. Numerical constraints on rates of metamorphism, fluid production and fluid flux during regional metamorphism. *Geological Society of America Bulletin* 1989; **101**:476–485.
11. Yardley BWD, Bottrell SH. Silica mobility and fluid movement during metamorphism of the Connemara schists, Ireland. *Journal of Metamorphic Geology* 1992; **10**:453–464.
12. Yardley BWD, Lloyd GE. Why metasomatic fronts are really sides. *Geology* 1995; **23**:53–56.
13. Norton D, Knapp R. Transport phenomena in hydrothermal systems: nature of porosity. *American Journal of Sciences* 1990; **277**:913–936.
14. Horton CW, Rogers FT. Convection currents in a porous medium. *Journal of Applied Physics* 1945; **16**:367–370.
15. Lapwood ER. Convection of a fluid in a porous medium. *Proceedings of the Cambridge Philosophical Society* 1948; **44**:508–521.
16. Nield DA, Bejan A. *Convection in Porous Media*. Springer-Verlag: New York, 1992.
17. Phillips OM. *Flow and Reactions in Permeable Rocks*. Cambridge University Press: Cambridge, 1991.
18. Zhao C, Mühlhaus HB, Hobbs BE. Finite element analysis of steady-state natural convection problems in fluid-saturated porous media heated from below. *International Journal for Numerical and Analytical Methods in Geomechanics* 1997; **21**:863–881.
19. Zhao C, Hobbs BE, Mühlhaus HB. Finite element modelling of temperature gradient driven rock alteration and mineralization in porous rock masses. *Computer Methods in Applied Mechanics and Engineering* 1998a; **165**:175–187.
20. Zhao C, Hobbs BE, Mühlhaus HB. Theoretical and numerical analyses of convective instability in porous media with upward throughflow. *International Journal for Numerical and Analytical Methods in Geomechanics* 1999a; **23**:629–646.
21. Zhao C, Hobbs BE, Mühlhaus HB. Finite element analysis of heat transfer and mineralization in layered hydrothermal systems with upward throughflow. *Computer Methods in Applied Mechanics and Engineering* 2000a; **186**:49–64.
22. Zhao C, Hobbs BE, Mühlhaus HB. Analysis of pore-fluid pressure gradient and effective vertical-stress gradient distribution in layered hydrodynamic systems. *Geophysical Journal International* 1998b; **134**:519–526.
23. Garven G, Freeze RA. Theoretical analysis of the role of groundwater flow in the genesis of stratabound ore deposits: mathematical and numerical model. *American Journal of Science* 1984; **284**:1085–1124.

24. Deming D, Nunn JA. Numerical simulations of brine migration by topographically driven recharge. *Journal of Geophysical Research* 1991; **96**:2485–2499.
25. Zhao C, Hobbs BE, Mÿhlhaus HB, Ord A. Finite element analysis of flow patterns near geological lenses in hydrodynamic and hydrothermal systems. *Geophysical Journal International* 1999b; **138**:146–158.
26. Zhao C, Steven GP. Analytical solutions for transient diffusion problems in infinite media. *Computer Methods in Applied Mechanics and Engineering* 1996; **129**:29–42.
27. Zhao C, Hebblewhite BK, Galvin JM. Analytical solutions for mining induced horizontal stress in floors of coal mining panels. *Computer Methods in Applied Mechanics and Engineering* 2000b; **184**:125–142.
28. Carslaw HS, Jaeger JC. *Conduction of Heat in Solids* Clarendon Press: Oxford, 1988.
29. Genuchten M van, Alvel WJ. Analytical solutions of the one-dimensional convection-dispersion solute transport equation. *Technical Bulletin*, U.S. Department of Agriculture, 1982.
30. Bear J, Bachmat Y. *Introduction to Modelling of Transport Phenomena in Porous Fractured Media*. Kluwer Academic Press: Dordrecht, MA, 1990.
31. Hanson RB. Effects of fluid production on fluid flow during regional and contact metamorphism. *Journal of Metamorphic Geology* 1992; **10**:87–97.
32. Walther JV, Orville PM. Volatile production and transport in regional metamorphism. *Contributions to Mineralogy and Petrology* 1982; **79**:252–257.

