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Is Absorption Necessary in High-Energy Inelastic Processes?

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Summary. — We present two arguments against the presence of important absorptive effects in high-energy physics. The first comes from the phenomenological analysis of single-pion production in pp collisions and the second from the application of the random phase approximation to the unitarity condition for an inelastic process.

1. – Introduction.

Several high-energy inelastic processes present a marked accumulation of events in the low-momentum-transfer region. Many of these processes can be interpreted with the so-called one-pion exchange (OPE) model and we will fix our attention on such cases. Some of the most interesting examples of OPE reactions are

$$(1) \qquad p + p \to \mathcal{N}^*(p\pi^+) + n\,,$$

$$(2) \qquad \bar{p} + n \to \overline{\mathcal{N}^*}(\bar{p}\pi^-) + p\,,$$

$$(3) \qquad \bar{p} + p \to \overline{\mathcal{N}^*}(\bar{p}\pi^-) + \mathcal{N}^*(p\pi^+)\,,$$

$$(4) \qquad \pi^- + p \to \rho(\pi^-\pi^-) + n\,,$$

$$(5) \qquad \pi^+ + p \to \rho(\pi^+\pi^-) + \mathcal{N}^*(p\pi^+)\,,$$

$$(6) \qquad K^- + p \to K^*(K^-\pi^-) + \mathcal{N}^*(p\pi^+)\,,$$

$$(7) \qquad \gamma + p \to \omega(\pi^+\pi^-\pi^0) + p\,.$$

Two different formulations of the OPE model have been proposed:

i) The form factor formulation [1] (FFF), which introduces a cut-off function in the squared four-momentum Δ^2 of the virtual pion. This function is attributed to the structure of the strongly interacting particles and determined phenomenologically by fitting the Δ^2 distribution for reaction (1) at a given energy. Once this is done the theory is completely determined for reaction (1) at different energies and for reactions (2) and (3) at all energies. It is also completely determined for the remaining reactions if one assumes that there is a universal form factor for all the vertices of pionic origin. It turns out that the model can successfully explain the reactions (1) [1], (3) [2] (5) [3] and (7) [4]. It explains partially reaction (4) for which absolute normalization and $d\sigma/d\Delta^2$ are rather well reproduced, but some anisotropy in the Treiman-Yang angle [6] remains unexplained [5]. The FFF fails partially in the case of reactions (2) [7] and (6) [8] for which several distributions are well reproduced, but with normalizations about double and half of the experimental ones, respectively.

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[1] E. FERRARI and F. SELLERI: *Nuovo Cimento*, **27**, 1450 (1963).

[2] H. C. DEHNE, E. LOHRMANN, E. RAUBOLD, P. SÖDING, M. W. TEUCHER and G. WOLF: *Phys. Rev.*, **136**, B 843 (1964); T. FERBEL, A. FIRESTONE, J. SANDWEISS, D. TAFT, M. GAILLARD, T. W. MORRIS, W. J. WILLIS, A. H. BACHMAN, P. BAUMEL and R. M. LEA: *Phys. Rev.*, **138**, B 1528 (1965); K. BOECKMANN, *et al.*: to be published.

[3] M. ADERHOLZ, L. BONDÁR, W. BRAUNECK, M. DEUTSCHMANN, C. GROTE, H. KAUFMANN, U. KUNDT, K. LOANIUS, R. LEISTE, H. W. MEYER, R. POSE, D. C. COLLEY, W. P. DODD, B. MUSGRAVE, J. SIMMONS, K. BÖCKMANN, J. MOEKES, B. NELLEN, G. WINTER, V. BLOBEL, H. BUTENSCHÖN, J. DAMMAN, P. VON HANDEL, E. LOHRMANN, P. SCHILLING, M. W. TEUCHER, G. WOLF, J. M. BROWNLEE, I. BUTTERWORTH, F. I. CAMPAYNE, M. IBBOTSON, M. SAEED, N. N. BISWAS, K. GOTTSTEIN, D. LÜERS, P. SCHLAMP, N. SCHMITZ, J. WEIGL and W. WITTEK: *Phys. Rev.*, **138**, B 897 (1965).

[4] This can be seen by comparing the model with the data presented by U. MAOR at the 1965 *Conference on High-Energy Physics held in Athens, Ohio.*

[5] S. for instance, J. ALITTI, J. B. BATON, A. BERTHELOT, A. DAUDIN, B. DELER, O. GOUSSU, M. A. JABIOL, C. KOCHOWSKI, C. LEWIN, M. NEVEU-RENÉ, A. ROGOZINSKI F. SHIVELY, J. LABERRIGUE-FROLOW, NGUYEN HUU KHANH, C. OUANNÈS, M. SENÉ, L. VIGNERON, N. ARMENISE, S. MONGELLI, L. NITTI, A. ROMANO, V. ALLES-BORELLI, G. BENEDETTI, A. FORINO, G. GIACOMELLI, J. LITVAK, G. PUPPI, P. WALOSCHEK and W. WHITEHEAD: *Nuovo Cimento*, **29**, 515 (1963); L. BONDÁR, K. BONGARTZ, M. DEUTSCHMANN, E. KEPPEL, G. KRAUS, H. WEBER, D. C. COLLEY, W. P. DODD, J. SIMMONS, B. TALLINI, A. M. FREIRE-ENDLER,B. NELLER, G. WINTER, D. CORDS, CH. DEHNE E. LOHRMANN, P. SODING, M. TEUCHER, G. WOLF, J. M. BROWNLEE, J. BUTTERWORTH, F. J. CAMPAYNE, M. IBBOTSON, N. N. BISWAS, J. DERADO, K. GOTTSTEIN, D. LÜERS G. LÜTJENS and N. SCHMITZ: *Nuovo Cimento*, **31**, 729 (1964).

[6] S. B. TREIMAN and C. N. YANG: *Phys. Rev. Lett.*, **8**, 140 (1962).

[7] T. C. BACON, *et al.*: *Phys. Rev.*, to be published.

[8] A good agreement was obtained by G. GOLDHABER, W. CHINOWSKY, S. GOLDHABER, W. LEE and T. O'HALLORAN: *Phys. Lett.*, **6**, 62 (1963) who, however, used a quantitatively inaccurate description of the K* and $\mathcal{N}$* vertices.

ii) The formulation with absorption (9) (AF) in which a cut-off in Δ^2 is provided by the absorption of incoming and outgoing waves. The quantitative reduction is deduced, with some secondary assumptions, from elastic scattering angular distributions. The AF is very successful in the case of reaction (4) for which not only normalization and shape, but also the anisotropies in the Treiman-Yang angle are quantitatively reproduced (10). It fails, however, in every other case for which predictions have been worked out, at least for the OPE processes. For instance the normalization for reaction (3) is predicted five times larger than the measured one (11). Reaction (7) could be explained at the price of introducing a total absorption of the low partial waves, but this is very hard to accept in view of the fact that no initial-state absorption exists for photon-generated processes. The over-all phenomenological situation favors the FFF, although even the best explained reactions show deviations from the predictions of the FFF. These deviations present themselves as anisotropies in the population of the Treiman-Yang angle, or as correlations between the decay planes of simultaneously produced resonances (12).

In the following we will present two fundamental reasons *against* the presence of absorptive effects.

2. – The phenomenological argument.

The first argument is of phenomenological nature. Reaction (1) was studied experimentally at several energies. The original determination of the form factor (1) was carried out at 970 MeV where a total cross-section of 40 mb is divided about equally between elastic scattering and single-pion production. Only a fraction of a mb is left for reactions of double-pion production and of deuteron formation. It is therefore clear that the basic idea that there is absorption because of the competition between different anelastic channels does not apply here. Therefore one can assume that the form factor obtained at this energy is the true form factor associated with the presence of a virtual pion. The belief in the correctness of the above conclusion comes from the fact that once the form factor is obtained by fitting the laboratory kinetic energy spectrum of the neutrons from $pp \to pn\pi^+$ all the other distributions of this reaction

(9) For a review of the main ideas and a complete list of references see: J. D. JACKSON, J. T. DONOHUE, K. GOTTFRIED, R. KEYSER and B. E. Y. SVENSSON: *Phys. Rev.*, **139**, B 428 (1965).

(10) See, for example, I. DERADO, V. P. KENNEY, J. A. POIRIER and W. D. SHEPHARD: *Phys. Rev. Lett.*, **14**, 872 (1965).

(11) B. E. Y. SVENSSON: *Nuovo Cimento*, **39**, 667 (1965).

(12) G. GOLDHABER, J. L. BROWN, J. BUTTERWORTH, S. GOLDHABER, A. A. HIRATA, J. A. KADYK, B. C. SHEN and G. H. TRILLING: *Phys. Lett.*, **18**, 76 (1965).

and all the distributions of $pp \to pp\pi^0$ at the same energy can be quantitatively reproduced. Now comes the important point. When the so-obtained form factor is used to explain reaction (1) at much higher energies one obtains beatiful fits even though other inelastic channels take now a considerable share of the total cross-section. Were absorption present one would expect to find the empirical cross-sections for reaction (1) smaller than those predicted with the use of the form factor obtained at 970 MeV. This is not so and one concludes therefore on empirical grounds that absorption cannot play an important role at least for the intermediate energy region (1 to 5 GeV/c in the laboratory).

3. – The unitarity-analyticity argument.

We will next discuss in detail whether absorption is in any way implied by unitarity and analyticity of the S-matrix.

To fix the ideas let us consider the processes $\pi\mathcal{N} \to \rho\mathcal{N}$ and let us call $\frac{1}{2}g_l$ the partial-wave amplitudes describing it (here and in the following spin and isospin labels will be neglect for simplicity). Similarly let f_{1l} and f_{2l} be the partial-wave amplitudes for the elastic processes $\pi\mathcal{N} \to \pi\mathcal{N}$ and $\rho\mathcal{N} \to \rho\mathcal{N}$ respectively. Finally let $h_{(1,n)l}$ and $h_{(2,n)l}$ be the partial-wave amplitudes for the other final states n, which can be reached from $\pi\mathcal{N}$ and $\rho\mathcal{N}$ initial states, respectively. As is well known there are different forms of unitarity, one for every possible process. We will consider here the conditions:

$$\text{Im}\, f_{1l} - |f_{1l}|^2 = \tfrac{1}{4}|g_l|^2 + \sum_n |h_{(1,n)l}|^2 , \tag{8}$$

$$\text{Im}\, f_{2l} - |f_{2l}|^2 = \tfrac{1}{4}|g_l|^2 + \sum_n |h_{(2,n)l}|^2 , \tag{9}$$

$$\text{Im}\, g_l = f_{1l}^* g_l + g_l^* f_{2l} + 2\sum_n [h_{(1,n)l}^* h_{(2,n)l}] . \tag{10}$$

Consider first (8) and (9). The elastic partial-wave amplitudes $f_{jl} (j = 1, 2)$ can be written in terms of a complex phase shift $\delta_{jl} = \alpha_{jl} + i\beta_{jl}$ (α_{jl} real and β_{jl} real and positive) in the following way:

$$f_{jl} = \frac{1}{2i}[\exp[2i\delta_{jl}] - 1] . \tag{11}$$

It is easy to show that a necessary condition for (8) and (9) to be satisfied is [13]

$$|g_l|^2 \leqslant 1 . \tag{12}$$

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[13] U. AMALDI jr. and F. SELLERI: *Nuovo Cimento*, **31**, 360 (1964).

Let g_l^0 be the value of g_l predicted by the FFF. The calculation of $|g_l^0|^2$ for the process $\pi\mathcal{N} \to \rho\mathcal{N}$ was performed two years ago by AMALDI and the author ([13]) who obtained a slight violation of unitarity only for the $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ waves, if the form factor obtained from reaction (1) was used. Since then an improved calculation of the off-shell amplitude for $\pi\mathcal{N}$-scattering ([14]) has led to a new form factor ([15]) practically coincident with the old one for $\Delta^2 \leqslant 10\mu^2$, but considerably smaller at higher Δ^2. Therefore we have recalculated $|g_l^0|^2$ using the formulae of ref. ([13]) and the new form factor. The result is shown in Fig. 1. As one can see all the waves are well below the unitarity limit. Furthermore AMALDI, BIANCASTELLI and FRANCAVIGLIA have calculated $|g_l^0|^2$ for reaction (1) and found again no violations of unitarity ([16]). Other OPE processes are not likely to exceed the unitarity limit since they have considerably smaller total cross-sections than (1) and (4).

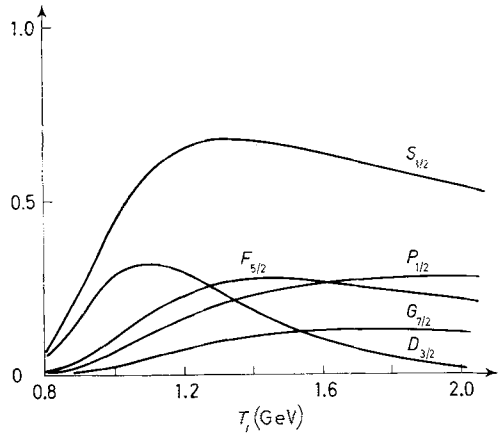

Fig. 1. – The squared moduli of the amplitudes g_l^0 as calculated from the OPE model for ρ production in $T = \frac{1}{2}$ $\pi\mathcal{N}$-collisions. The $T = \frac{3}{2}$ amplitudes are one half of the $T = \frac{1}{2}$ ones. The curves are plotted against T_l, the laboratory kinetic energy of the incoming π, and are calculated for a ρ mass of 750 MeV and a ρ width of 125 MeV. The squared moduli of the waves not explicitly shown never exceed 0.1.

Consider next the relation (10). It is impossible to obtain rigorous conditions from it because of the appearance of the unknown amplitudes $h_{(j,n)l}$ in the last term on the r.h.s. Fortunately, however, it is reasonable to put this term equal to zero since it results from the sum of a large number of complex quantities with uncorrelated phases, each small in modulus. This assumption, which is often referred to as random phase approximation, has been made (explicitly or not) in all the papers on the AF ([17]) and is of basic importance for the validity

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([14]) F. SELLERI: *Nuovo Cimento*, **40** A, 236 (1965).

([15]) The new « form factor » is well represented by the expression $8\mu^2/(\Delta^2 + 9\mu^2)$. That this and the old form factors have a variation with Δ^2 fully compatible with the analiticity properties of the vertex and propagator functions has been shown by the author in a paper presented at the 1964 *Boulder Conference on High-Energy Physics*.

([16]) Private communication from U. AMALDI.

([17]) For clear statements of this hypothesis see: A. DAR and W. TOBOCMAN: *Phys. Rev. Lett.*, **12**, 511 (1964) and A. BIAŁAS and L. VAN HOVE: *Nuovo Cimento*, **38**, 1385 (1965).

of the model in either formulation. By its virtue one can obtain from (10)

$$\text{(13)}\qquad \mathrm{Re}\, g_l \{\exp[-2\beta_{1l}]\sin 2\alpha_{1l} + \exp[-2\beta_{2l}]\sin 2\alpha_{2l}\} - \\ - \mathrm{Im}\, g_l \{\exp[-2\beta_{1l}]\cos 2\alpha_{1l} + \exp[-2\beta_{2l}]\cos 2\alpha_{2l}\} = 0$$

and

$$\text{(14)}\quad \mathrm{Re}\, g_l \{\exp[-2\beta_{1l}]\cos 2\alpha_{1l} - \exp[-2\beta_{2l}]\cos 2\alpha_{2l}\} + \\ + \mathrm{Im}\, g_l \{\exp[-2\beta_{1l}]\sin 2\alpha_{1l} - \exp[-2\beta_{2l}]\sin 2\alpha_{2l}\} = 0 .$$

The eqs. (13) and (14) have the general solution ([18]) $\beta_{1l} = \beta_{2l}$ and

$$\text{(15)}\qquad \frac{\mathrm{Im}\, g_l}{\mathrm{Re}\, g_l} = \mathrm{tg}\,(\alpha_{1l} + \alpha_{2l}) .$$

The only restriction imposed on g_l by (10) is thus on its phase, which has to equal the sum of the real parts of the phase shifts for $\pi\mathcal{N}$ and $\varrho\mathcal{N}$ elastic scattering. Obviously the OPE amplitude g_l^0 does not satisfy (15) in general, since $\mathrm{Im}\, g_l^0 = 0$ ([19]). It should be noted, however, that at high energy the α's are probably either near zero or, having resonated at lower energy, near π. In either case $\mathrm{Im}\, g_l$ is small compared to Re g_l and g_l^0 can be a resonable zeroth approximation. It is very important to stress that (10) does not give rise to reductions of the modulus of g_l^0. This fact seems to us in strong contradiction with the idea that there « must » be some absorption from the competition of other inelastic channels. The situation is completely different for the elastic an the inelastic amplitudes. For the former all the inelastic channels give rise to absorption since the squared moduli of the corresponding matrix elements appear in the r.h.s. of (8) and (9) and force the phase shifts to become complex. For the latter the inelastic processes give probably a vanishing contribution to the r.h.s. of (10) and the elastic processes do not generate absorption, but merely a change of the phase of the OPE amplitudes.

Let us now be more rigorous and solve the following problem. Construct a function g_l having certain singularities on the left-hand side which, alone, would give to g_l the value g_l^0 and having a cut on the right with given phase $\alpha_l = \alpha_{1l} + \alpha_{2l}$. A straightforward application of the Cauchy theorem gives

$$\text{(16)}\qquad g_l = g_l^0 + \frac{1}{\pi}\int_{\omega_0}^{\infty} \frac{\mathrm{Im}\, g_l' \mathrm{d}\omega'}{\omega' - \omega - i\varepsilon} ,$$

([18]) For simplicity we treat now the ϱ as a stable particle.

([19]) This solution can also be obtained as a particular case from eq. (3.8) of the quoted paper by A. Białas and L. Van Hove.

where

$$g_l^0 = -\frac{1}{2\pi i}\int_L \frac{\operatorname{disc}\{g_l'\}\,d\omega'}{\omega'-\omega}\,. \tag{17}$$

is the contribution from the left-hand cut L which we suppose known and could be given, for instance, by the OPE model with or without form factor and

$$\operatorname{Im} g_l' = \exp[-i\alpha_l']\sin\alpha_l' g_l'\,, \tag{18}$$

because the phase of g_l equals α_l on the right-hand cut.

The coupling of (16) with (18) gives rise to an Omnès problem which has the well-known solution

$$g_l(\omega) = \left[g_l^0(\omega)\cos\alpha_l + \Omega(\omega)\frac{P}{\pi}\int_{\omega_0}^{\infty}\frac{g_l^0(\omega')\sin\alpha_l'}{\Omega(\omega')(\omega'-\omega)}\,\mathrm{d}\omega'\right]e^{i\alpha_l} \tag{19}$$

with

$$\Omega(\omega) = \exp\left[\frac{P}{\pi}\int_{\omega_0}^{\infty}\frac{\alpha_l'\mathrm{d}\omega'}{\omega'-\omega}\right]. \tag{20}$$

At high energy it is reasonable to assume that α_l tends to zero. We can then divide the integral appearing in (19) into the sum of two integrals, one relative to the region from ω_0 to $\bar{\omega}$ where α_l is nonnegligible and one from $\bar{\omega}$ to ∞ which is practically zero. The first however tends to zero for ω increasing like ω^{-1} and therefore we get the solution

$$g_l(\omega) \simeq g_l^0(\omega)\,. \tag{21}$$

In conclusion we can state that sufficient conditions for the validity of a model which assumes only left-hand side singularities for a certain inelastic process are [20]:

1) The validity of the random-phase approximation which allows one to drop the last term in (10).

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[20] Notice that our eq. (16) is not in contradiction with the result obtained by BALL and FRAZER [J. S. BALL and W. R. FRAZER: *Phys. Rev. Lett.*, **14**, 746 (1965)]. These authors produced an inelastic matrix element showing explicitly all the correct analyticity properties on both cuts and could show that their result reduced to the absorptive formula only in the limit of vanishing absorption.

2) The condition that α_l becomes rather small at high energy. In practice α_l needs only to be smaller than 10° to insure that no radical changes in the modulus of g_l arise from (19).

4. – Conclusions.

It is very important to stress that the two assumptions made in the previous Section have been made in all the « derivations » of the absorptive formula. Therefore we are faced with a definite contradiction between our results and those of the authors who first proposed the absorption model in high-energy physics. Our argument is so simple and straightforward that we cannot imagine anything wrong with it. We believe that the error made by the other authors arises from an undue generalization of the concept of complex potential.

To illustrate this point let us consider a conceptually simple example. Suppose that a negative pion hits a hypernucleus containing a Λ. We may try to calculate the probability of observing the reaction

$$\pi^- + \Lambda \to K^- + n \,. \tag{22}$$

Obviously such a probability will be much smaller than that of observing the same reaction on free lambdas because the nuclear matter surrounding the Λ will give rise to an attenuation of the incoming and outgoing waves. This situation can be discussed by introducing a complex potential the imaginary part of which gives rise to absorption in the initial and final states. If one assumes that also a proton is surrounded by a complex potential one derives absorptive formulae also for high-energy interactions, but this assumption is highly doubtful because one of the elements of the reasoning fails there. In the high-energy case there cannot be a discussion of a given reaction (*e.g.* $\pi\mathcal{N} \to \rho\mathcal{N}$) in terms of a simpler one because the whole treatment is based on considering the $\mathcal{N}$ as the basic entity for which one *needs* not to assume further structure. One *can* do so, for instance by considering the nucleon as a composite system of three quarks. If one does so and wants to describe the process $\pi\mathcal{N} \to \rho\mathcal{N}$ in terms of the more elementary one $\pi + \text{quark} \to \rho + \text{quark}$ one can consider the absorption due to the other two quarks. But as long as one ignores a possible substructure of the nucleon and uses left-hand side singularities for the over-all process $\pi + \mathcal{N} \to \rho + \mathcal{N}$ one must not introduce absorptive corrections. The parallel situation is found also at the level of nuclear physics. For instance the reaction

$$\gamma + {}^6\text{Li} \to {}^4\text{He} + p + n \tag{23}$$

has been studied by using Feynman graph techniques by WATAGHIN and col-

laborators ([21]) without considering absorptive effects in the final state. They consider a model in which the ^{6}Li emits a deuteron and an ^{4}He the latter behaving as a spectator. The incoming γ disintegrates the d in p and n. Of course they have to introduce a coupling constant and a form factor for the vertex ^{6}Li-d-^{4}He and a propagator for the virtual deuteron. Alternatively they could have used a description in which the deuteron preexists in the ^{6}Li nucleus and corrected the matrix element for *free* photodisintegration of the deuteron with absorptions of the final wave. Therefore there is some kind of equivalence between the absorptive corrections on one side and the product of vertex function times propagator on the other. Either formulation is correct for reaction (23). One cannot, however, use both of them simultaneously by applying absorptive corrections to the amplitude calculated with the Feynman rules. Similarly in high-energy physics one cannot introduce absorptive corrections to a graph like for instance the one-pion exchange graph of the process $\pi\mathcal{N} \to \rho\mathcal{N}$.

Finally we would like to comment briefly on the two assumptions mentioned at the end of the previous Section. The second one should be a reasonable first approximation to reality, while the first one is difficult to understand physically even though it is widely accepted. It is clear that when it fails important corrections to g_l^0 are introduced by unitarity. It would however be misleading to consider such corrections as absorptive ones in view of the previous discussion.

([21]) S. FERRONI *et al.*: University of Torino, preprint (1965).

Note added in proof.

After completion of this work the papers by E. J. SQUIRES [*Nuovo Cimento*, **34**, 1328 (1964); **39**, 300 (1965)] were brought to our attention. Some absorptive effects can be obtained with his treatment, but there is an additional assumption (eq. (6) of the second paper) which is avoided in our discussion and which can account for the different conclusions reached.

RIASSUNTO

Presentiamo due argomenti contro la presenza di importanti effetti assorbitivi nella fisica delle alte energie. Il primo deriva da un'analisi fenomenologica dei processi di produzione negli urti protone-protone ed il secondo dall'applicazione dell'approssimazione delle fasi non correlate alla condizione di unitarietà per un processo anelastico.