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Pressure drop and friction factor correlations of supercritical flow

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ABSTRACT

The determination of the in-tube friction pressure drop under supercritical conditions is important to the design, analysis and simulation of transcritical cycles of air conditioning and heat pump systems, nuclear reactor cooling systems and some other systems. A number of correlations for supercritical friction factors have been proposed. Their accuracy and applicability should be examined. This paper provides a comprehensive survey of experimental investigations into the pressure drop of supercritical flow in the past decade and a comparative study of supercritical friction factor correlations. Our analysis shows that none of the existing correlations is completely satisfactory, that there are contradictions between the existing experimental results and thus more elaborate experiments are needed, and that the tube roughness should be considered. A new friction factor correlation for supercritical tube flow is proposed based on 390 experimental data from the available literature, including 263 data of supercritical R410A cooling, 45 data of supercritical R404A cooling, 64 data of supercritical carbon dioxide (CO_2) cooling and 18 data of supercritical R22 heating. Compared with the best existing model, the new correlation increases the accuracy by more than 10%.

1. Introduction

Supercritical pipe flow has many applications. To cool nuclear reactors, supercritical water has been used for several decades (Pioro et al., 2004), and supercritical CO_2 also has some applications (Pioro et al., 2004; Duffey and Pioro, 2005). Since CO_2 became a promising alternative refrigerant due to the phaseout of conventional refrigerants that have ozone depleting effects and global-warming potential, the study of supercritical CO_2 cooling has attracted renewal interest (Fang et al., 2001; Cheng et al., 2008; Oh and Son, 2010; Dang and Hihara, 2004; Huai et al., 2005). There are other interests in supercritical cooling and heating. For example, supercritical CO_2 is used in solar collectors (Niu et al., 2011), hypersonic aircraft may use supercritical H_2 for active cooling (Dziedzic et al., 1993), and some promising alternative refrigerants like R410A and R404A may also operate with transcritical cycles (Garimella, 2008; Andresen, 2006; Mitra, 2005; Jiang, 2004).

Under supercritical pressures, the flow pattern is somewhat similar to the conventional single-phase flow, and no phase change takes place. However, the thermophysical properties of fluids change drastically during supercritical heating and cooling processes. In these circumstances, the pressure drop is greatly dependent on the local fluid temperature and the inner wall temperature, which makes the conventional single-phase friction factor correlations unsuitable.

The friction factor is not only used to calculate pressure drop, but also used to calculate heat transfer. For example, the widely used Gnielinski (1976) equation, which applies to single-phase turbulent isothermal tube flow, is of the form

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1.07+12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3}-1)} \tag{1a}$$

where the Darcy–Weisbach friction factor (friction factor for short) f is defined by Eq. (3), and the Nusselt number Nu and the Reynolds number Re are defined as the following, respectively:

$$\mathrm{Nu} = \frac{\alpha D}{\lambda} \tag{1b}$$

$$\mathrm{Re} = \frac{\rho V D}{\mu} \tag{1c}$$

The single-phase isothermal heat transfer equations are the base of supercritical heat transfer models (Fang et al., 2001; Cheng et al., 2008).

There are a number of experimental investigations related to the pressure drop under supercritical flow, especially in supercritical CO_2 cooling. Pioro et al. (2004) conducted a survey of hydraulic resistance of fluids flowing in channels at supercritical pressures. Duffey and Pioro (2005) conducted a survey of experimental heat transfer of supercritical CO_2 flowing inside channels. They listed no pressure drop experimental study after year 1986. There are

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Nomenclature

C_f	Fanning friction factor
c_p	specific heat at constant pressure (J/kg K)
D	inner diameter (m)
f	Darcy–Weisbach friction factor
f_{ac}	acceleration factor
g	acceleration due to gravity (m/s^2)
G	mass flux (kg/m^2 s)
h	specific enthalpy (J/kg)
L	tube length (m)
Nu	Nusselt number
p	pressure (Pa)
Pr	Prandtl number
q	heat flux from fluid to wall (W/m^2)
Re	Reynolds number
T	temperature (K)
V	velocity (m/s)

Greek symbols

α	heat transfer coefficient (W/m^2 K)
β	thermal expansion coefficient (1/K)
λ	thermal conductivity (W/m K)
Δp	pressure drop (Pa)
ε	average roughness height of tubes (m)
μ	dynamic viscosity (Pa s)
ρ	density (kg/m^3)

Subscripts

ac	acceleration
b	at fluid bulk temperature
f	at film temperature, $T_f=(T_b+T_w)/2$
fr	friction
g	gravity
h	hydraulic
in	inlet
iso	isothermal
out	outlet
pc	at pseudo-critical temperature
t	total
w	at inner wall temperature

some issues regarding the recent experimental study that should be addressed.

Pioro et al.'s (2004) survey listed several friction factor correlations for supercritical flow. This is the only paper found which offered a systematic review in this regard. However, they only listed some correlations published before 1990 and gave no detailed analysis, evaluation or comparison. Therefore, it is necessary to carry out a comprehensive review and comparative study of existing supercritical friction factor correlations.

Among the available literature for the pressure drop of supercritical flow, only Pettersen et al. (2000) explicitly considered tube wall roughness. They estimated the roughness of the 0.787-mm multi-port extruded circular tubes at $\varepsilon = 1$ μm. However, they did not propose any correlation of supercritical friction factors. The effect of the tube roughness on supercritical friction factors should be considered, especially for flow in minichannels at large Reynolds number.

Moody (1944) gave a roughness of $\varepsilon = 1.5$ μm for drawing tubing of macro-size. Matkovic et al. (2009) tested a commercial copper tube with an inner diameter of 0.96 mm. They measured the friction factor during adiabatic flow of subcooled liquid and superheated vapour and found the internal surface average roughness $\varepsilon = 1.3$ μm. Sobierska et al. (2006) conducted experiments of flow boiling of water in a vertical microchannel with dimension of 0.86×2.0 mm^2 (width × depth, hydraulic diameter $D_h = 1.2$ mm). The surface roughness of the channel was determined to be $\varepsilon < 1$ μm. Steiner and Taborek (1992) proposed a correlation for flow boiling. For using the correlation, they gave channel surface roughness $\varepsilon = 0.1$–18 μm and standard value of $\varepsilon = 1$ μm if the surface roughness is unknown (Fernando et al., 2008). Thome and Hajal (2004) studied flow boiling heat transfer model for horizontal in-tube evaporation specifically for CO_2 and proposed a correlation for nucleate pool boiling with an assumption of $\varepsilon = 1$ μm. Cavallini et al. (2009) emphasized roughness issue in two-phase pressure drop calculations.

This paper conducts a critical review of the published literature in the experimental study of supercritical flow in the past decade and supercritical friction factor correlations. A new supercritical friction factor correlation, which considers the tube roughness, is to be proposed based on 390 experimental data from the available literature, among which 263 data are from supercritical R410A cooling (Garimella, 2008; Andresen, 2006; Mitra, 2005), 45 data from supercritical R404A cooling (Garimella, 2008; Jiang, 2004), 64 data from supercritical CO_2 cooling (Dang and Hihara, 2004) and 18 data from supercritical R22 heating (Yamshita et al., 2003). The detailed analysis of the existing correlations and the new correlation is performed thereafter using the same data bank.

2. Pressure drop equation

Attention should be made regarding the difference between the friction pressure drop and the acceleration pressure drop. The majority of recent experimental investigations into supercritical CO_2 did not distinguish them without any reasoning (Dang and Hihara, 2004; Huai et al., 2005; Yoon et al., 2003; Son and Park, 2006; Dang et al., 2007, 2010; Yun et al., 2007).

The total pressure drop of a straight pipe with a constant section area can be calculated with

$$\Delta p_t = \Delta p_{fr} + \Delta p_{ac} + \Delta p_g \tag{2}$$

where Δp_{fr} is the friction pressure drop, Δp_{ac} is the acceleration pressure drop, and Δp_g is the pressure drop due to the gravity.

$$\Delta p_{fr} = \frac{G^2}{2\rho} \cdot \frac{L}{D} f \tag{3}$$

$$\Delta p_{ac} = \frac{G^2}{2\rho} \cdot \frac{L}{D} f_{ac} \tag{4}$$

$$\Delta p_g = \pm g \left(\frac{\rho_{out} + \rho_{in}}{2} \right) L \sin\theta \tag{5}$$

where f_{ac} is the acceleration factor, θ is the pipe inclination angle to the horizontal plane, the sign "+" is for the upward flow, and the sign "−" is for the downward flow. The arithmetic average value of densities can be used only for short pipes in the case of strongly nonlinear dependency of the density versus temperature.

The friction factor f is not to be confused with the friction coefficient, sometimes called the Fanning friction factor. Denoted the Fanning friction factor by C_f, it follows that

$$f = 4C_f \tag{6}$$

For long pipes at high heat fluxes and in critical and pseudo-critical regions, the integral value of densities should be used. Ornatskiy et al. (1980) proposed to calculate Δp_g at supercritical pressures as follows:

$$\Delta p_g = \pm g \left(\frac{h_{out}\rho_{out} + h_{in}\rho_{in}}{h_{out} + h_{in}} \right) L \sin\theta \tag{7}$$

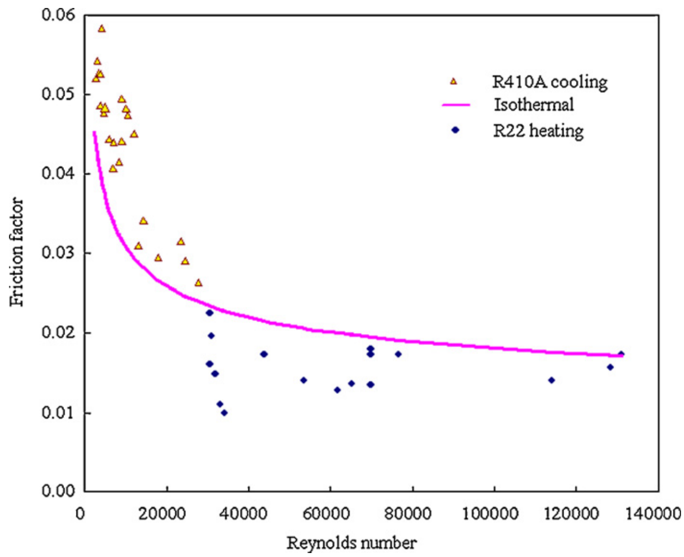

Fig. 1. Comparison of friction factors: supercritical vs. isothermal.

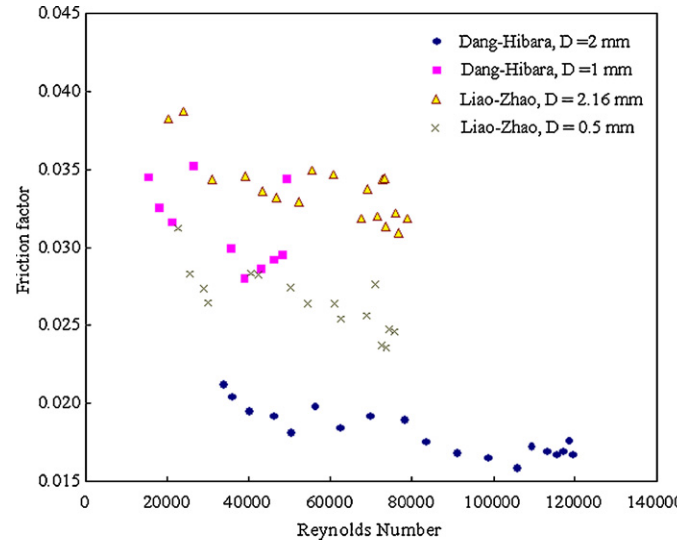

Fig. 2. Effect of tube sizes on supercritical friction factors.

For cooling processes, f_{ac} can be approximated with the one-dimensional approximation model (Petrov and Popov, 1988; Polyakov, 1991) as follows:

$$f_{ac} \approx 2D\rho_b \frac{d}{dL}\left(\frac{1}{\rho_b}\right) = \frac{-8q}{G}\left(\frac{\beta}{c_p}\right)_b \tag{8}$$

where d is the differential symbol. Substituting Eq. (8) into Eq. (4), it follows that

$$\Delta p_{ac} = G^2\left(\frac{1}{\rho_{b,out}} - \frac{1}{\rho_{b,in}}\right) \tag{9}$$

The Δp_{ac} equation above is also used for heating (Pioro et al., 2004), from which it can be seen that $f_{ac} < 0$ for cooling and $f_{ac} > 0$ for heating.

3. Recent experimental study of supercritical pressure drop

Our extensive literature survey found that most of experimental investigations into the pressure drop of supercritical flow in the past decade are supercritical CO_2 cooling, as shown in Tables 1 and 2. Table 1 shows the experiments of pure CO_2, while Table 2 lists those of CO_2 and oil mixture.

The experimental investigations into in-tube supercritical pressure drop other than supercritical CO_2 cooling in the past decade are presented in Table 3. Only Yamshita et al.'s (2003) supercritical R22 heating and Garimella's group's (Garimella, 2008; Andresen, 2006; Mitra, 2005; Jiang, 2004) supercritical R410A and R404A cooling are found.

From the information provided by the literature listed in Tables 1–3, the following can be obtained:

(1) The majority of the experimental study of the pressure drop of supercritical CO_2 cooling (Dang and Hihara, 2004; Huai et al., 2005; Yoon et al., 2003; Son and Park, 2006; Dang et al., 2007, 2010; Yun et al., 2007) in the past decade used an isothermal single-phase friction factor correlation to calculate the combining pressure drop of the friction and acceleration. The rationality of this practice lacks reasoning.

(2) In the same Re, the friction factor of supercritical cooling is higher than that of the isothermal condition, and the friction factor of supercritical heating is lower than that of the isothermal counterpart, as shown in Fig. 1, where the R410A data are those of $D = 3.05$ mm from Garimella (2008) and Andresen (2006), the R22 data are those from Yamshita et al. (2003), and the isothermal single-phase friction factor for smooth tubes is calculated with the equation newly proposed by Fang et al. (2011)

$$f = 0.25\left[\log\left(\frac{150.39}{Re^{0.98865}} - \frac{152.66}{Re}\right)\right]^{-2} \tag{10}$$

which has the mean absolute relative deviation (MARD) of 0.02% and the maximum relative deviation (RD) of −0.05% in the range of $Re = 3000–10^8$ compared with the following Nikuradse (1933) equation

$$\frac{1}{\sqrt{f}} = 2\ \log(Re\sqrt{f}) - 0.8 \tag{11}$$

$$\mathrm{RD} = \frac{y(i)_{pred} - y(i)_{\exp}}{y(i)_{\exp}} \tag{12}$$

$$\mathrm{MARD} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y(i)_{pred} - y(i)_{\exp}}{y(i)_{\exp}}\right| \tag{13}$$

where y_{pred} is the predicted value, y_{exp} is the measured value, and N is the total data points.

(3) Effects of tube sizes on supercritical friction factors were contradictorily reported. From Liao and Zhao (2002) data, the friction factor reduces by around 15% as the tube diameter decreases from 2.16 mm to 0.5 mm, while the Dang and Hihara (2004) data show that the 1-mm tube has bigger fiction factors than those of the 2-mm tube, as shown in Fig. 2. Mitra (2005) studied 6.22- and 9.4-mm tubes and proposed a correlation with size correction as follows

$$f = af_{iso,b}\left(\frac{\rho_w}{\rho_b}\right)^b\left(\frac{D_{actual}}{D_{baseline}}\right)^c \tag{14}$$

where $f_{iso,b}$ is the isothermal single-phase friction factor calculated with Churchill (1977) equation at the fluid bulk temperature, D_{actual} is the actual channel inner diameter, $D_{baseline}$ equals to 9.4 mm, the maximum diameter in the experiments, and the exponent c is negative, which suggests that the smaller diameter has bigger friction factors. After tested 0.76-, 1.52- and 3.05-mm tubes, it turned out to be that the D correction does not hold. Therefore, Garimella (2008) and Andresen (2006) abandoned it in their final reports.

(4) Pettersen et al. (2000) addressed the tube roughness issue. Their test section is multi-port extruded circular tubes with the inner diameter of 0.787 mm, which is commonly used in mobile air conditioning. They compared their measured data with the predictions of single-phase friction factor equations for smooth tubes

Table 1
Experiments of pressure drop of supercritical CO_2 cooling.

Reference	Flow parameter: T_{in} (°C)/p_{in} (MPa)/G (kg/m^2 s)/q (kW/m^2)	Flow geometry: D (mm)/L (mm)/orientation/geometry	Results
Pettersen et al. (2000)	15–70/8.1–10.1/600–1200/10–20	0.787/540/horizontal/multi-port extruded circular tubes	Assuming ε = 1 μm, the Colebrook correlation predicted well.
Liao and Zhao (2002)	20–110/7.4–12/236–1179/not mentioned	0.50, 0.70, 1.10, 1.40, 1.55, 2.16/110/horizontal/single circular tube	The friction factor is seen to drop as the tube size reduced from 2.16 mm to 0.5 mm.
Yoon et al. (2003)	50–80/7.5–8.8/225–450/not mentioned	7.73/500/horizontal/single circular tube	Blasius' correlation has under-prediction. Deviations increase as Re increasing.
Dang and Hihara (2004)	30–70/8–10/200–1200/6–33	1, 2/500/horizontal/single circular tube	The 1-mm tube has bigger fiction factors than the 2-mm tube.
Huai et al. (2005)	22–53/7.4–8.5/113.7–418.6/0.8–9	1.31/500/horizontal/multi-port extruded circular tubes	Predictions of Blasius' equation are generally within 25%.
Son and Park (2006)	90–100/7.5–10/200–400/not mentioned	7.75/500/horizontal/single circular tube	Blasius' correlation has under-prediction. Deviations increase as Re increasing.

and for rough tubes, respectively. For the rough conditions, they assumed the roughness to be 1 μm. The mean relative deviation (MRD) is −6% for the smooth assumption and 1% for the rough assumption, and the MARD is 6% for the smooth assumption and 2% for the rough assumption. Pettersen et al. did not mention how they estimated the roughness. It can be reasoned that they overestimated the roughness because they used the single-phase isothermal correlations to calculate the pressure drop of supercritical flow.

$$\mathrm{MRD} = \frac{1}{N}\sum_{i=1}^{N}\frac{y(i)_{pred} - y(i)_{\exp}}{y(i)_{\exp}} \tag{15}$$

(5) Containing oil increases the pressure drop (Dang et al., 2007, 2010; Yun et al., 2007; Kuang et al., 2003). The pressure drop increase sharply with an increase from 1% in the oil concentration (Dang et al., 2007; Yun et al., 2007).

(6) Some contradictions exist between different experiments. Therefore, more elaborate experiments are needed.

4. Review of supercritical friction factor correlations

Andresen (2006) and Garimella (2008) obtained the following supercritical friction factor correlation from experiments of supercritical R410A and R404A cooling

$$f = af_{iso,b}\left(\frac{\mu_w}{\mu_b}\right)^b \tag{16}$$

Table 2
Experiments of pressure drop of supercritical cooling of CO_2 with oil.

Reference	Flow parameter: T_{in} (°C)/p_{in} (MPa)/G (kg/m^2 s)/q (kW/m^2)/oil conc.	Flow geometry: D (mm)/L (mm)/orientation/geometry	Results
Dang et al. (2007)	20–70/8–10/200–1200/12–24/oil up to 5%	1, 2/500/horizontal/single circular tube, copper	Δp do not increase considerably until the oil concentration reaches 1%, but increases sharply with an increase from 1% in the oil concentration
Yun et al. (2007)	40–80/8.4–10.4/200–400/20–25/oil up to 4%	1/600/horizontal/multi-port extruded circular tubes	When oil concentration increased from 0% to 4%, Δp increased by 4.8 times
Dang et al. (2010)	30–70/8–10/400–1200/12–24/oil up to 5%	2/500/horizontal/single grooved tube with 6.3° helix angle	Δp for the grooved tube areproportional to those for a smooth tube
Kuang et al. (2003)	30–50/9/890/not clear/oil up to 5%	D = 0.79 mm microchannels	Immiscible oil has more negative influence on the pressure drops than the miscible oil

Table 3
Experiments of in-tube pressure drop of supercritical flow other than supercritical CO_2.

Reference	Flow parameter: T_{in} (°C)/p_{in}/G (kg/m^2 s)/q (kW/m^2)	Flow geometry: D (mm)/L (mm)/orientation/geometry	Substance
Yamshita et al. (2003)	72–80/5.5 Mpa/700/0–60	4.4/600/vertical/single circular tube	R22
Garimella (2008), Andresen (2006), Mitra (2005), Jiang (2004)	30–110/1.0, 1.1, 1.2 × p_{crit}/200–800/not available.	0.76, 1.52/304.8–508/horizontal/multi-port extruded circular tubes. 3.05, 6.22, 9.4/323.8, 292/horizontal/single circular tube.	R410A (D = 0.76–9.4 mm). R404A (D = 6.22, 9.4 mm).

where the constant a and the exponent b are equal to 1.16 and 0.91 for the liquid-like region, 1.31 and 0.25 for the pseudo-critical transition region, and 1.19 and 0.17 for the gas-like region, respectively, and $f_{iso,b}$ is the friction factor calculated with the following Churchill (1977) equation at the fluid bulk temperature without considering roughness ε:

$$f_{iso} = 8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12} + A^{-3/2}\right]^{1/12} \tag{17a}$$

$$A = \left(\frac{37,530}{\mathrm{Re}}\right)^{16} + \left\{-2.457\ln\left[\left(\frac{7}{\mathrm{Re}}\right)^{0.9} + 0.27\frac{\varepsilon}{D}\right]\right\}^{16} \tag{17b}$$

Yamshita et al. (2003) investigated experimentally the heat transfer and pressure drop of R22 flowing in a uniformly heated vertical tube of a diameter 4.4 mm under supercritical pressure. They found that the measured friction factors were lower than those predicted with the isothermal friction factor equation and obtained a correlation with the deviation of ±15% as follows

$$f = f_{iso,b}\left(\frac{\mu_w}{\mu_b}\right)^{0.72} \tag{18}$$

where f_{iso} is calculated with

$$f_{iso} = \frac{0.314}{0.7 - 1.65\ \log \mathrm{Re} + (\log \mathrm{Re})^2} \tag{19}$$

Petrov and Popov (1988) calculated the pressure drop of the water, helium, and carbon dioxide at supercritical pressures in the boundary conditions of the wall temperature T_w = constant and the heat flux q = constant, based on which they obtained

$$\frac{f}{f_{iso,b}} = \left(\frac{\mu_w}{\mu_b}\right)^{1/4} + 0.17\left(\frac{\rho_w}{\rho_b}\right)^{1/3}\frac{|f_{ac}|}{f_{iso,b}} \tag{20}$$

where f_{iso} is calculated with the Filonenko equation

$$f_{iso} = (0.79\ \ln \mathrm{Re} - 1.64)^{-2} \tag{21}$$

Earlier in 1985, Petrov and Popov calculated the pressure drop of the turbulent pipe flow of supercritical CO_2 cooling in the range of $\mathrm{Re}_w = 1.4 \times 10^4$–$7.9 \times 10^5$ and $\mathrm{Re}_b = 3.1 \times 10^4$–$8 \times 10^5$, and obtained a friction factor correlation of the form

$$f = f_{iso,w}\frac{\rho_w}{\rho_b}\left(\frac{\mu_w}{\mu_b}\right)^s \tag{22a}$$

where f_{iso} is calculated with the Filonenko equation, and

$$s = 0.023\left(\frac{|q|}{G}\right)^{0.42} \tag{22b}$$

Tarasova and Leont'ev (1968) studied the friction factor in heated pipes at supercritical pressures and proposed that

$$f = f_{iso,b}\left(\frac{\mu_w}{\mu_b}\right)^{0.22} \tag{23}$$

They reported a deviation of ±5% between experimental points and their fitting.

Popov (1967) proposed to calculate the frictional factor of supercritical CO_2 with the correlation of the form

$$f = f_{iso,b}\left(\frac{\rho_f}{\rho_b}\right)^{0.74} \tag{24}$$

where f_{iso} is calculated with the Filonenko equation, and the subscript f denotes evaluated at the film temperature T_f. $T_f = (T_b + T_w)/2$. The author reported that the equation had uncertainty of ±5%.

Kutateladze (1962) proposed the equation of the form

$$f = f_{iso,b}\left(\frac{2}{\sqrt{T_w/T_b} + 1}\right)^2 \tag{25}$$

Mikheev (1956) introduced that the friction factor of non-isothermal flow of water and other fluids could be calculated with

$$f = f_{iso,b}\left(\frac{Pr_w}{Pr_b}\right)^{1/3} \tag{26}$$

where f_{iso} is calculated with the Filonenko equation.

5. A new supercritical friction factor correlation

A new supercritical friction factor correlation is proposed based on the regression analysis of 390 experimental data, including 263 data of R410A and 45 data of R404A under supercritical cooling (Garimella, 2008; Andresen, 2006), 64 data of supercritical CO_2 cooling (Dang and Hihara, 2004), and 18 data of supercritical R22 heating (Yamshita et al., 2003), all of which were given graphically. The commercial software GetData Graph Digitizer is used to translate the experimental data points on the figures into digital data, and the NIST REFPROP is used to determine the fluid thermophysical properties corresponding to the given experimental conditions of pressures and temperatures.

Extensive computer tests have been conducted to develop a new correlation based on the 390 experimental data, using the regression analysis methodology described by Fang et al. (2010) and Fang and Xu (2011a). The methodology is summarized as the following:

(1) To choose independent variables among those in Eqs. (16)–(26).
(2) To construct possible correlations based on the chosen independent variables.
(3) To conduct regression analysis using available software and data bank to find out the best model that has the highest corrected coefficient of determination R_c^2 and the lowest MARD.

The corrected coefficient of determination R_c^2 is defined as

$$R_c^2 = 1 - \frac{(N-1)(1-R^2)}{N-m} = 1 - \frac{(N-1)s^2}{SS_{Total}} \tag{27}$$

where m is the number of parameters estimated, N is the number of total samples, R^2 is the coefficient of determination, SS_{Total} is the total sum of squares, and s^2 is the residual mean square (Myers, 2005).

The software used for the regression is 1stOpt (First Optimization), a leading worldwide software platform for numerical optimization analysis, especially in the area of curve fitting, nonlinear regression and parameter estimation of nonlinear complex engineering models (7D-Soft High Technology Inc., 2010).

The data source files do not give any roughness information. Assuming ε = 1 μm for the multi-port extruded circular tubes and ε = 0.5 μm for the single circular tubes and using the regression analysis methodology described above, a new correlation is obtained as follows

$$f = f_{iso,b}\left(\frac{\mu_w}{\mu_b}\right)^{0.49\left(\frac{\rho_f}{\rho_{pc}}\right)^{1.31}} \tag{28}$$

where f_{iso} is the isothermal single-phase friction factor for rough tubes, which is calculated with the equation newly proposed by Fang et al. (2011)

$$f_{iso} = 1.613\left[\ln\left(0.234\left(\frac{\varepsilon}{D}\right)^{1.1007} - \frac{60.525}{Re^{1.1105}} + \frac{56.291}{Re^{1.0712}}\right)\right]^{-2} \tag{29}$$

where the actual channel roughness should be used. If the actual channel roughness is not known, it is suggested to assume ε = 1 μm for multi-port extruded channels and ε = 0.5 μm for single channel tubes. For non-circular channels, the hydraulic diameter should be used.

Table 4
Evaluation of the available models based on the experimental data.

	Correlation	New correlation	Yamashitah	Petrov–Popov (1988)	Andresen–Garimella	Popov	Tarasova–Leont'ev	Kutateladze	Mikheev	Petrov–Popov (1985)	Isothermal smooth
All Data	MARD	**17.6**[d]	**19.5**	**20.1**	**18.3**	**21.6**	**22.3**	**25.0**	**25.5**	**37.0**	**25.9**
	MRD	−4.9	−7.1	−13.0	4.9	−15.5	−16.9	−19.3	−17.4	14.5	−20.1
	[a]	30.0	28.5	28.7	40.5	24.1	22.6	21.5	22.3	21.0	20.5
	[b]	64.9	59.0	52.1	63.9	48.7	48.5	40.5	42.1	49.2	38.2
	[c]	82.1	81.0	79.2	79.2	76.2	73.3	67.7	66.4	68.7	65.6
R410A	MARD	**17.5**[d]	**19.5**	**20.9**	**17.3**	**22.1**	**23.5**	**26.0**	**26.5**	**37.7**	**27.0**
	MRD	−5.1	−7.7	−15.9	1.6	−18.5	−20.7	−24.1	−21.8	18.1	−25.2
	[a]	36.1	32.7	28.5	38.8	26.6	22.4	19.4	21.3	24.0	18.6
	[b]	64.6	60.1	52.9	66.5	51.0	49.8	43.0	42.2	51.3	40.3
	[c]	79.5	79.1	75.3	82.9	71.1	67.3	63.9	62.0	65.8	63.8
R404A	MARD	**18.6**[d]	**23.2**	**19.8**	**6.9**	**23.6**	**23.9**	**28.4**	**25.1**	**58.1**	**29.4**
	MRD	−13.2	−7.2	−19.5	−5.9	−22.1	−23.9	−28.4	−20.5	32.4	−29.4
	[a]	13.3	11.1	22.2	80.0	2.2	0.0	0.0	4.4	8.9	0.0
	[b]	73.3	44.4	37.8	97.7	20.0	22.2	2.2	22.2	42.2	0.0
	[c]	84.4	80.0	91.1	100.0	88.9	84.4	68.9	75.6	73.3	53.3
CO_2	MARD	**19.0**[d]	**18.1**	**16.9**	**26.2**	**17.1**	**16.7**	**16.3**	**20.0**	**19.9**	**16.3**
	MRD	0.2	−3.7	−5.9	19.5	−5.5	−6.5	−8.0	−7.0	0.9	−8.3
	[a]	9.4	20.3	31.3	29.7	29.7	34.4	42.2	39.1	18.8	42.2
	[b]	56.3	59.4	59.4	42.2	59.4	59.4	56.3	54.7	50.0	54.7
	[c]	89.1	85.9	89.1	54.7	89.1	89.1	85.9	81.3	84.4	84.4
R22	MARD	**12.5**[d]	**14.7**	**21.0**	**34.1**	**26.1**	**20.9**	**33.4**	**37.1**	**35.8**	**36.4**
	MRD	0.7	−10.6	20.0	28.1	9.1	19.1	33.1	18.0	−33.7	36.2
	[a]	55.6	38.9	38.9	5.6	22.2	38.9	33.3	22.2	16.7	22.2
	[b]	77.8	77.8	50.0	16.7	50.0	55.6	44.4	44.4	33.3	44.4
	[c]	88.9	94.4	72.2	61.1	72.2	77.8	55.6	55.6	44.4	55.6

[a] Percentage of data points within the RD of ±10%.
[b] Percentage of data points within the RD of ±20%.
[c] Percentage of data points within the RD of ±30%.
[d] A bold line is the first line of the given data group.

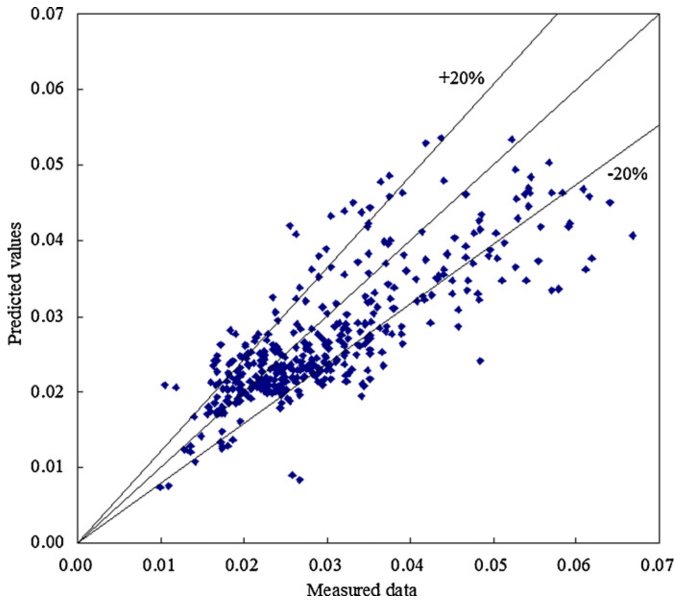

Fig. 3. Comparison of predicted values of the new equation with the 390 experimental data.

Eq. (29) has the MARD of 0.2% and the maximum RD of 0.6% in the range of Re = 3000–10^8 and ε/D = 0.0–0.05 compared with Colebrook (1938–39) equation

$$\frac{1}{\sqrt{f}} = -2\ \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \tag{30}$$

As indicated in Table 4, the proposed new equation has an MRAD of 17.6%, increasing the accuracy by more than 10% compared with the Yamshita et al.'s model, which is the best existing model as evaluated in the following section. The majority of the new model predictions have a RD of 20% (Fig. 3). In generating a heat transfer equation for in-tube supercritical CO_2 cooling, the new equation produced the best results (Fang and Xu, 2011b).

6. Evaluation of available supercritical friction factor correlations

The same bank of the 390 experimental data is used for the evaluation. The comparative results of the available models are shown in Table 4, where the MARD, MRD and the percentile of data points within the RD of ±10%, ±20% and ±30% are listed and the "isothermal smooth" means the single-phase friction factor correlation for isothermal flow in smooth tubes.

The order in Table 4 reflects the rank of the correlations based on their overall performances. The new correlation is the best, followed by Yamshita et al.'s (2003) and Petrov–Popov's (1988). For all data, the Andresen–Garimella correlation has the lower MARD than the Yamshita et al. correlation and the Petrov and Popov (1988) correlation have. However, the Andresen–Garimella correlation has much bigger MARDs for CO_2 and R22, which lowers its rank. Besides, 79% of the data used for the comparison are those for correlating the Andresen–Garimella model, which makes it overranked. The Petrov–Popov (1985) correlation has the biggest MARD because it behaves abnormally when the ρ_w/ρ_b is too big or too small. The correlations of Kutateladze, Mikheev and the single-phase friction factor correlation for isothermal flow in smooth tubes are very close. The predictions of the single-phase friction factor correlation for isothermal flow in smooth tubes are listed for reference.

7. Conclusions

Most of experimental investigations into the pressure drop of supercritical flow in the past decade are supercritical CO_2 cooling without or with oil, other than which only Yamshita et al.'s supercritical R22 heating and Garimella's group's supercritical R410A and R404A cooling are found.

In dealing with the supercritical CO_2 cooling, the majority of the study in the past decade did not differentiate the friction pressure drop and the acceleration pressure drop, using an isothermal single-phase friction factor correlation to calculate the combining pressure drop of the two types. This will cause more errors.

The experimental results of supercritical flow are not consistent well, such as the contradiction of the effect of the tube diameter on the friction factor (Fig. 2). More experiments are necessary to obtain the correlation with higher accuracy.

The effect of tube roughness on supercritical friction factors has not properly addressed. The Andresen–Garimella correlation uses the Churchill single-phase friction factor model, which would provide the opportunity to consider roughness. However, the Andresen–Garimella correlation was developed assuming smooth tubes and using the coefficient *a* during the curve fitting. The coefficient *a* already reflects the effect of roughness and thus prohibits the user from using roughness in calculation.

There are 8 existing correlations available to deal with supercritical friction factors. Based on the 390 experimental data of supercritical R410A, R404A and CO_2 cooling and supercritical R22 heating, the top two existing correlations are Yamshita et al.'s (2003) and Petrov–Popov's (1988).

The following new correlation for supercritical friction factors is proposed

$$f = f_{iso,b}\left(\frac{\mu_w}{\mu_b}\right)^{0.49\left(\frac{\rho_f}{\rho_{pc}}\right)^{1.31}}$$

It reduces the MARD by more than 10% compared with the best existing model.

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