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A functionally graded structural design of mirrors for reducing their thermal deformations in high-power laser systems by finite element method ☆

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Abstract

In this paper, a new method is proposed to effectively lower the high temperature, smooth the thermal stress and reduce the thermal deformation of mirrors in high-power laser systems utilizing functionally graded materials (FGMs). Two kinds of laser source are discussed at the same time. One is the doughnut-shaped laser, and the other is the Gaussian-distributed laser. Numerical results from finite element analysis show that the thermal deformation of silicon and copper mirrors can be reduced by one order of magnitude, and the temperature rise is also lowered obviously by this means. The effects of slope regulation of the functions for representation of thermophysical and mechanical properties on thermal deformation of mirrors are discussed to meet the requirement of optimum design and manufacture of FGM mirrors for the future.

Keywords: Thermal deformation; Mirrors; Functionally graded structure

1. Introduction

In designing high-power laser systems, a fundamental problem need to be solved is that the thermal deformation of heated mirrors must be reduced dramatically to generate highly directional laser beams and make their transmission without perturbations, otherwise the thermal lenses effect will be produced and cause aberration and fail of laser systems under high heat fluxes. To eliminate this effect or reduce it to be acceptable, many theoretical and experimental works have been conducted in related research fields. In 1974, Apollonov et al. [1] derived the analytical expression of thermal deformation of mirrors under Gaussian-type laser heating, and compared it with their experiments for metal and alloy mirrors under 1 kW of power density that was consistent with the analytical one. Evans [2] studied the thermal lenses effects for spherical, parabolic and Gaussian surfaces under given thermal deformations. Beside the mechanism research of deformation, many efforts have been devoted to reducing the thermal deformation of heated mirrors through experiments. Two approaches, including selection and cooling of mirror materials, are usually concerned. The comparison of different mirror materials were firstly done by Anthony et al. [3], whose experimental results show that silicon and copper are the most competitive materials due to their relatively high thermal conductivities and low thermal expansion coefficients. With these mirrors, variant cooling techniques were designed. Although, instead of direct liquid cooling of rear and circumferential surfaces of mirrors, alternatives that change the structure of mirror were usually adopted, because the liquid pressure in direct cooling will generate additional deformation. These changes of structures, including cavity, annular channel, honeycomb, etc., present significant figure of merit in reducing thermal deformation [4–7]. Meanwhile, the

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numerical analysis of Herrit et al. [8] shows that the thermal deformation of silicon mirror is less than that of copper under the same laser irradiation and liquid cooling fluxes. This has made silicon mirror a popular selection for high-power laser systems with power density up to 10 kW. A most recent design is to make silicon mirror into a composite mirror that composes a copper layer between it and its cooling substrate filled with phase-change material (PCM), by which it has combined the advantages of high thermal conductivity of copper and low thermal expansion coefficient of silicon [9]. The complex mirror that fills with PCMs, e.g., paraffin/carbon powder, decreases the thermal distortion of silicon mirror to 0.25 μm under irradiation of laser power of 2 kW. However, few experimental results and theoretical analyses can be attained from literatures for laser power higher than 10 kW.

The motivation for using functionally graded materials (FGMs) is their advantages of superior stress relaxation and capabilities of withstanding high temperatures and large temperature gradients. FGMs that consisting typically of thermal-resisting ceramic and fracture-resisting metal are composites that the volume fractions of the constituents vary gradually, resulting in continuously transition in physical properties, e.g., thermal conductivity, specific heat capacity, density, thermal expansion coefficient, Young's modulus, and shear modulus. In the last two decades, research of FGMs focused on heat and stress analysis, performance estimation and material design. The readers are referred to related literatures by, e.g., Miyamoto et al. [10] for more detailed comprehension on FGMs. However, the most realistic method for representing the continuous gradation of the material properties of FGMs is the volume fractions and rules of mixture, which complicate the analytical solution of FGM problems at the same time. The use of finite element method in such problems is one of the most effective tools to overcome such difficulties.

In this work, finite element method is used for both heat equation and equation of motion to analyze the thermal deformation of mirrors under high-power density heating of Gaussian-type and doughnut-shaped lasers (and can be also some combinations of the two), assuming that the irradiation power is not yet sufficient to melt the mirror surface. The backward difference method and implicit Newmark algorithm are adopted for the heat equation and equation of motion, respectively. The thermoelastic deformation at the mirror surface under spatial-dependent laser irradiation is calculated using standard elements of finite element program generator (FEPG) and is figurized through FEPG's user element interface, GiD®. Comparisons of results under different heating power densities and characteristic beam radii are given in section three. The influence of physical properties of FGMs is then discussed, from which we expect to find a new guideline for mirror design in high laser power systems through regulation of the functions for representation of the material properties.

2. Problem formulation

As shown in Fig. 1a mirror with thickness L and radius R are coordinated in cylindrical system. From the generalized thermoelastic theories, we can deduce the following constitutive equations [11], of which the linear energy equation is

$$\rho c_{\mathrm{E}} \dot{T} = k T_{,ii} \tag{1}$$

and the elastodynamic (Cauchy–Navier) displacement equation of motion for linearly elastic material behavior is

$$\rho \ddot{u}_i = (\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - (3\lambda + 2\mu)\alpha T_{,i}, \tag{2}$$

where ρ is the density, c_{E} the specific heat capacity at constant strain, T the absolute temperature, k the thermal conductivity, u_i the displacement tensor, λ and μ the Lame coefficients, and α the linear thermal expansion coefficient. In the cases concerned, the high-power laser irradiation time is very short, so that one can consider the heat conduction process to be within an axisymmetric and two-dimensional semi-infinite medium, with isotropic physical properties and without energy generation.

The mirror is kept at a uniform initial temperature T_0 and is irradiated at the surface $z = 0$ for times $\tau > 0$ by a heat flux with specified spatial profiles that can be descried as follows [12]:

$$q = q_0 \left[A + (1 - A) \frac{r^2}{d^2} \right] \mathrm{e}^{-r^2/\mathrm{d}^2}, \tag{3}$$

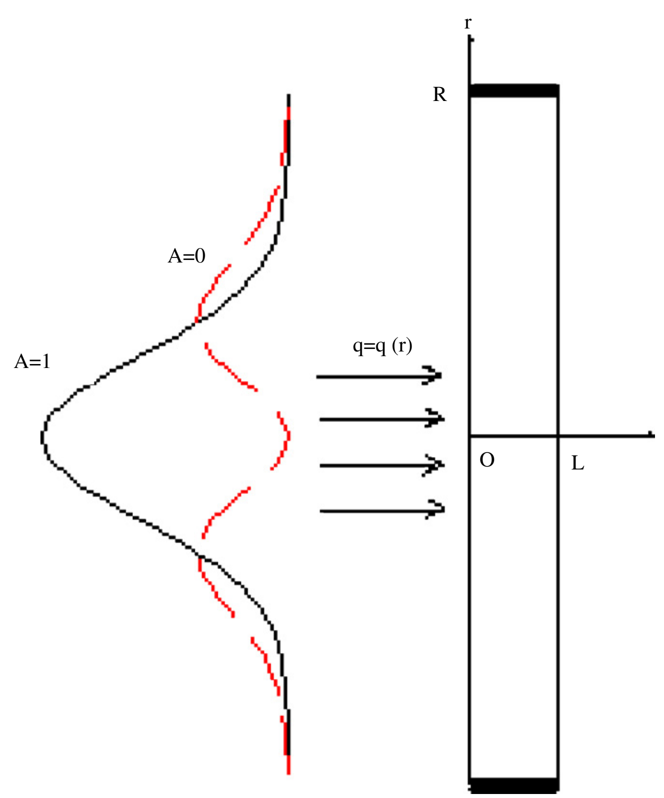

Fig. 1. A mirror heated by Gaussian-type ($A = 1$)/doughnut-shaped ($A = 0$) laser in cylindrical coordination.

where q_0 is the maximum heat flux for a Gaussian laser source and contains important information about surface properties, e.g., reflectivity. The parameter d is the characteristic beam radius that represents the circular boundary within the Gaussian source that 63% of the total beam power incident to the surface. The parameter A is the fraction of the total heat flux that contains the Gaussian mode and therefore falls in the range $0 \leqslant A \leqslant 1$. For simplicity, we discuss in this work with $A = 0$ and 1 and the heat is fully absorbed by the medium. The choice of these profiles is based upon the assumption that they approximate the beam outputs from some common laser sources. The circumferential surface of mirror is clamped and thermal insulated.

For mirror surface, the following condition must be concerned to include the effect of thermal radiation:

$$-k\nabla T = \sigma\varepsilon\left(T^4 - T_f^4\right), \tag{4}$$

where σ is the Stefan–Boltzmann constant, ε thermal emissivity, T_f ambient temperature. During the laser heating process, the temperature of mirror will be much higher than the ambient temperature ($T \gg T_f$), thus one can make this approximation that

$$T^4 \approx T_f^4 + \left.\frac{\mathrm{d}T^4}{\mathrm{d}T}\right|_{T=T_f}(T - T_f) = T_f^4 + 4T_f^3(T - T_f) \tag{5}$$

and the nonlinear boundary condition can be simplified as

$$-k\nabla T = 4\sigma\varepsilon T_f^3 T. \tag{6}$$

3. Analyses

3.1. Thermal deformation of pure material

In FEPG system, the partial differential equations are formulated into virtual work equations. The backward difference method is used for time discretization of the heat equation. And the implicit Newmark algorithm is adopted for time discretization of the equation of motion, which is in wave form that containing the first and second time derivatives of its governing variable. The equation of motion is solved at each time step iteratively with the solution of heat equation at the same time step, and vice versa. For mirror thickness $L = 0.5\,\mathrm{cm}$ and radius $R = 2\,\mathrm{cm}$, 100×25 finite elements are discretized with time discretization of 0.6 ms. The total irradiation time is set to be 300 ms. The general process of finite element implementation is referred to our previous work [13]. And the related material properties used in calculation are shown in Table 1 [14,15].

Fig. 2 shows the temperature profiles at $\tau = 300\,\mathrm{ms}$ in pure silicon mirror for $d = 0.2\,\mathrm{cm}$ and $q_0 = 0.5 \times 10^7\,\mathrm{W/m^2}$. One can conclude from Fig. 2 that the high-temperature value will actually results in heat loss through radiation for the Gaussian laser source, since the temperature has already reached to an order of magnitude of 10^3K. Due to axisymmetricity, only half of the cross section of the mirror is drawn in Fig. 2. Similar results can be visualized for pure copper mirror.

To observe the thermomechanical behavior under different types of laser, we vary the values of d and q_0, respectively, to study their impact on thermal deformation at the heating mirror center, $r = 0$ and $z = 0$, as shown in Figs. 3 and 4. Fig. 3 shows the thermal deformation under different heat fluxes in pure silicon mirror for $d = 0.2\,\mathrm{cm}$. For the Gaussian laser source, the thermal deformation in the silicon mirror increases with the laser power density. Although, for the doughnut-shaped laser source, the thermal deformation decreases with the laser power density due to the spatial distribution of heat flux. Fig. 4 illustrates the influence of the characteristic beam radius on thermal deformation for $q_0 = 0.5 \times 10^7\,\mathrm{W/m^2}$. It can be observed clearly that, comparing to the Gaussian laser source, the mirror distorts more obviously when the characteristic beam radius decreases for doughnut-shaped laser heating. This tendency can also be explained by the fact that the amplitude of laser power density is sharply reduced by the factor d^{-2} in the function profile of the doughnut-shaped laser source. For the Gaussian laser source, this large variation is weakened and negligible. With the above-mentioned comparisons, we choose $d = 0.2\,\mathrm{cm}$ and $q_0 = 0.5 \times 10^7\,\mathrm{W/m^2}$ for the following discussions to explore the possibility of applying FGMs to the reduction of mirror distortion when designing the corresponding laser systems.

3.2. Thermal deformation of FGM

Based on the calculation of pure materials, we discuss in this part on the mirrors made of complex FGM composites. When describing the thermophysical and mechanical properties of FGMs, different functions for the continuous gradation can be considered. For example, Noda et al. [16] considered exponential functions for representation of continuous gradation of the material properties in analytical solution of FGM problems: $\alpha = \alpha_0 e^{\beta z}$, $E = E_0 e^{\gamma z}$, $v = v_0 e^{\gamma z}$, and $k = k_0 e^{\delta z}$, where E is

Table 1
Thermophysical and mechanical properties of pure silicon and copper [14,15]

Material	k ($\mathrm{W\,m^{-1}\,K^{-1}}$)	ρ ($\mathrm{kg\,m^{-3}}$)	c_E ($\mathrm{J\,kg^{-1}\,K^{-1}}$)	λ (Pa)	μ (Pa)	α ($\mathrm{K^{-1}}$)
Si	157	2329	695	6×10^{10}	8×10^{10}	2.33×10^{-6}
Cu	388	8968	385	1.0×10^{10}	4.8×10^{10}	1.7×10^{-5}

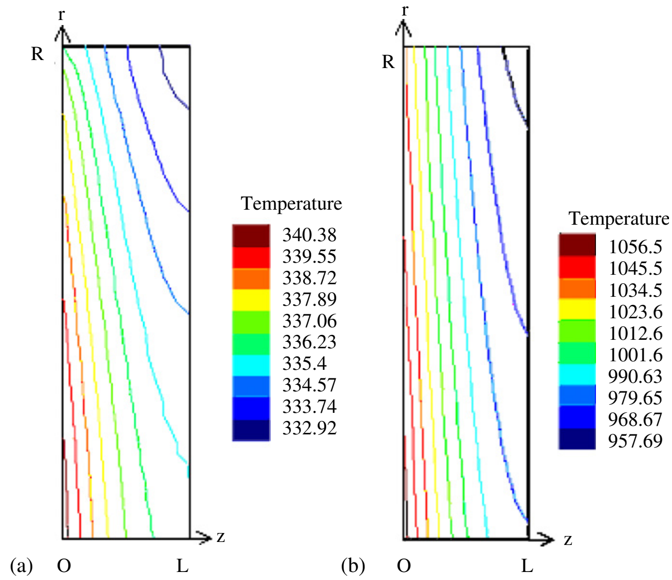

Fig. 2. Isothermal surfaces in the silicon mirror.

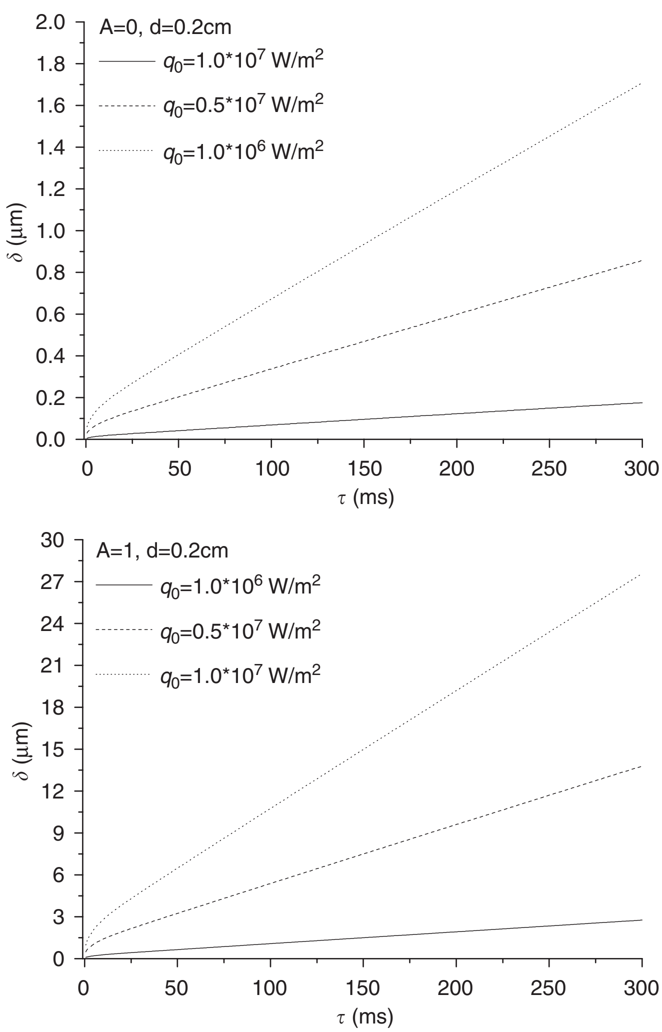

Fig. 3. Thermal deformation under different heat fluxes for pure silicon mirror.

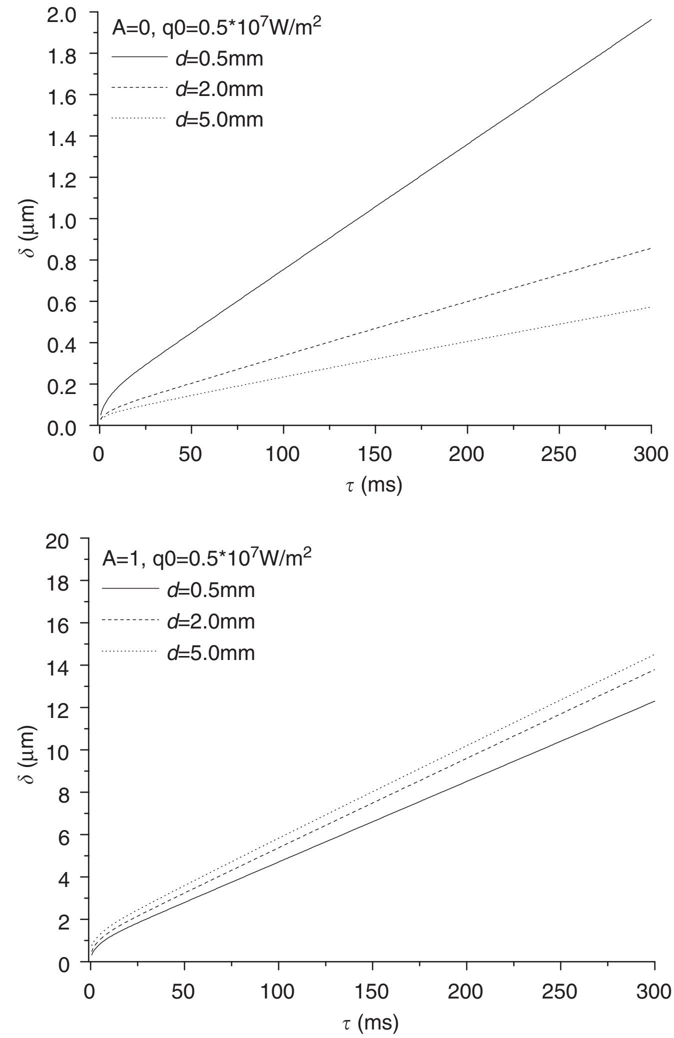

Fig. 4. Thermal deformation under different characteristic beam radii for pure silicon mirror.

Young's modulus, v is the Poisson's ratio, and α_0, E_0, v, k_0, β, γ and δ are material constants. The exponential rules usually make the problem easier to solve and can actually provide a much better approximation than some other rules, e.g., linear functions, especially for the mechanical properties, but they do not give real representation for material properties, expect for the substrates of FGM.

To observe preliminarily the tendency of thermal deformation under laser heating, we assume that the thermophysical and mechanical properties of FGMs are temperature independent and vary linearly in one Cartesian coordinate. Thus, the impact of thermal dependence of these properties on thermal stress distributions can be neglected, and the linear approximation transforms the heterogeneous problems to known homogeneous problems for simple boundary element method [17]. In the present work, the thermal conductivity and the volumetric heat capacity are chosen to have the same functional variation so that the thermal diffusivity is constant, i.e.

$$k = k_0(1 + mz), \quad \rho c_E = \rho_0 c_{E0}(1 + mz), \tag{7}$$

where m, k_0, ρ_0 and c_{E0} are constants. The assumption of constant thermal diffusivity, $k_0/(\rho_0 c_{E0})$, in FGMs provides benchmark solutions to finite element method and can provide valuable insight into the thermal behavior of FGMs. Similarly, we assume that the related mechanical properties vary linearly in one Cartesian coordinate as follows:

$$\lambda = \lambda_0(1+nz), \quad \mu = \mu_0(1+nz), \quad \alpha = \alpha_0(1+pz), \tag{8}$$

where n, p, λ_0, μ and α_0 are constants. This assumption is based on the works of Tanigawa. et al. [18,19], who have studied a class of thermoelastic problems in semi-infinite medium with linear variation of mechanical properties, i.e., $\alpha = \alpha_0(1+\beta z)$, $G = G_0(1+\beta z)$, where G is the shear modulus and G_0 are constant. The assumption can be realized through gradation composition with multilayers that vary from the thermal-resistant ceramic substrate to the rigid metal or nonmetallic substance.

Four types of FGMs are designed, among which the type one represents actually for the pure materials (silicon and copper). Only one layer of gradation is concerned between the substrates. The values of m, n and p are given in Table 2. To reduce the deformation of mirror, the constants for mechanical properties are assumed to be negative.

Figs. 5 and 6 debate the effects of slope of linear functions on the thermal deformation of the silicon and copper mirrors. From these results, one can find that, the thermophysical properties of FGMs affect sharply on the mechanical behavior, and, during the same laser irradiation time, the magnitude of thermal deformation of silicon mirror decreases obviously from about 1.0 to 0.2 μm for doughnut-shaped laser and from about 3.0 to 0.7 μm for Gaussian-type laser with the same value of q_0 ($= 0.5\times10^7\,\mathrm{W/m^2}$) when the slope of functions for representation of thermal conductivity and volumetric heat capacity increase. The magnitude of thermal deformation of copper mirror also decreases obviously from about 2.0 to 0.6 μm for $A = 0$ and from about 9.0 to 2.0 μm for $A = 1$ when increasing m from 0 to 800. While, compared with the dramatic effects of the thermophysical properties of FGM materials on the mechanical behavior, the variation of n and p change little of the deformation at the surface centers for both silicon and copper mirrors. It can also be pointed out that the copper mirror distorts several times than that of silicon mirror, which is consistent with the results of Herrit et al. [8]. The temperature of heated mirror of type three falls to about 600 and 330 K for silicon and copper mirrors respectively under Gaussian laser irradiation with maximum heat flux $q_0 = 0.5\times10^7\,\mathrm{W/m^2}$ at the same irradiation time as

Table 2
Constants of FGMs

Material type	m	n	p
Type 0	0	0	0
Type 1	400	−40	−100
Type 2	400	−60	−200
Type 3	800	−40	−100

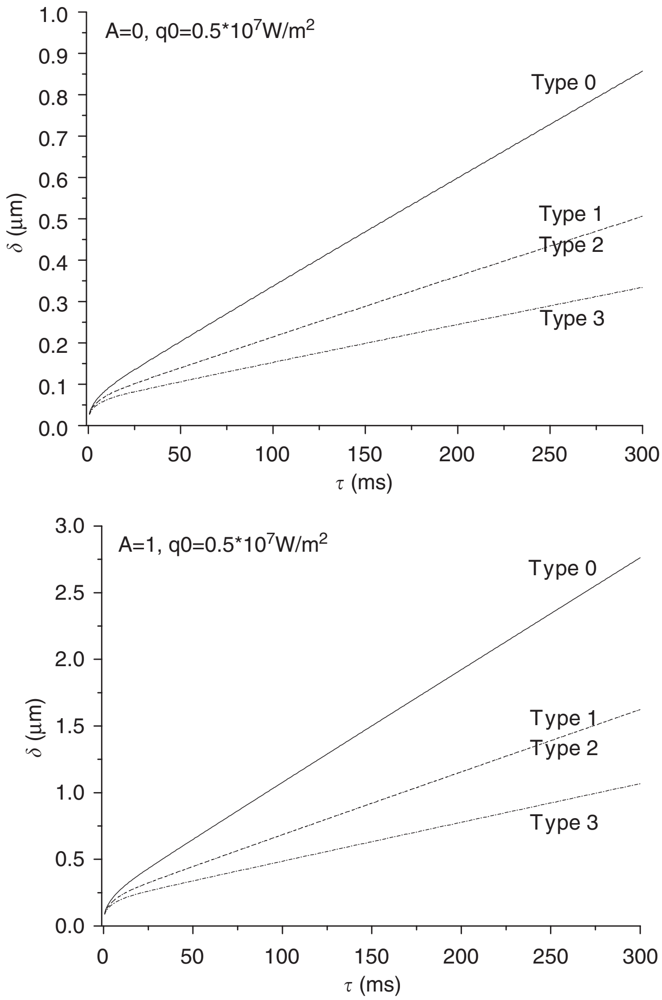

Fig. 5. Thermal deformations of different FGMs based on silicon substrate.

in pure materials. According to the above results, the positive variation of heat conductive property and negative variation of mechanical properties from the irradiative surface will reduce thermal deformation. And the bigger variation ratio will lead to the dramatic reduction.

4. Discussions

Since the comparative study of different mirror materials by Anthony et al. [3], who had decided that silicon or copper are the most competitive materials, variant cooling techniques were designed for both silicon and copper mirrors in high-power laser system. For instance, phase-change cooling of mirror surfaces can drop in temperature of mirrors directly. However, liquid pressure induced deformation will be generated. Changes of structures, including cavity, annular channel, honeycomb, can reduce thermal deformation and lower the temperature of mirror simultaneously. Experimental research on copper mirror shows that the annular channel and honeycomb structure present greater potential for reducing the thermal deformation than the cavity structure, since the liquid pressure

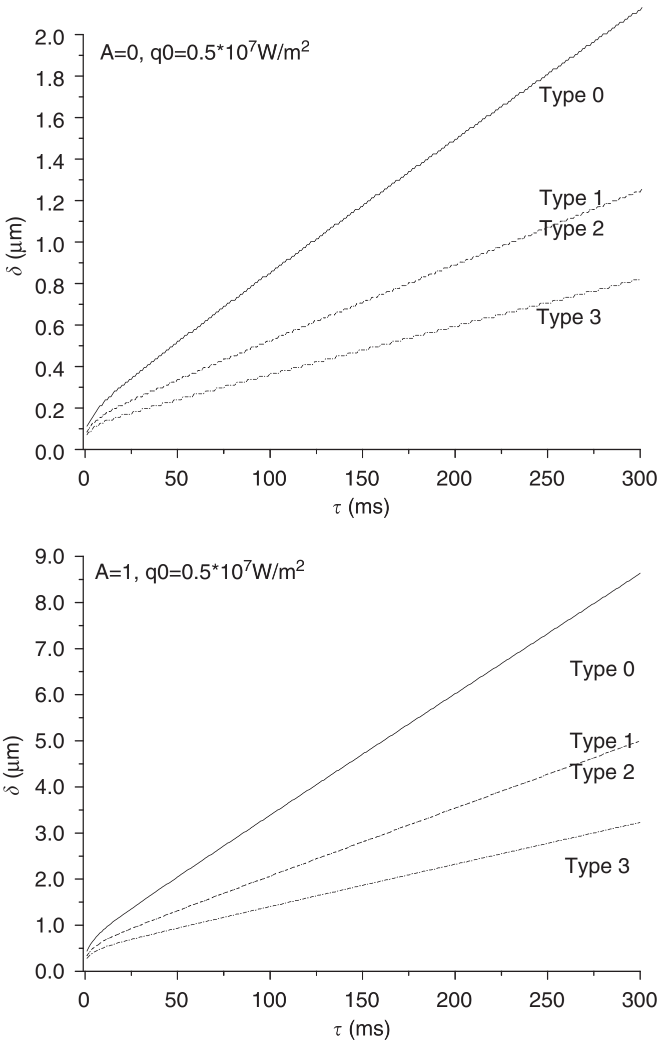

Fig. 6. Thermal deformations of different FGMs based on copper substrate.

induced deformation is very small due to the special structures. Compared with the cavity structure, the honeycomb structure reduces the deformation from about 5 to 0.1 μm for a copper mirror with radius of 3.4 cm and laser irradiation power of 350 W under the cooling pressure of 0.2 MPa [20]. But researchers still devotes their efforts to new designs because of the complexity of manufacture and application for these structures. Multilayers with PCMs is one of the new designs. The experimental results show that the thermal deformation of the silicon mirror is 0.25, 0.33 and 0.37 μm for paraffin/carbon, paraffin/aluminum and paraffin/copper powder as PCMs, respectively, when the laser irradiation power is 2 kW and the total time of laser irradiation is 3 s [9]. This kind of mirrors still need forced convection during time-out of laser irradiation to reconvert the latent heat of phase change material.

The concept of the novel design of mirror is proposed in this present work. The results in section three have illustrated the effects of FGMs on reducing the thermal deformation of mirror, from which one can seek for a new way of mirror design in high-power laser systems. For example, the thermal deformation of silicon mirror with thickness $L = 0.5$ cm and radius $R = 2$ cm decreases from about 1.0 to 0.2 μm under doughnut-shaped laser irradiation with maximum heat flux $q_0 = 0.5 \times 10^7\,\mathrm{W/m^2}$, using the FGM method. From the evolution of mirror design, as Fig. 7 has exhibited, the FGM method tracks and represents for the trend, of which stability, quietness, manipuility and high efficiency are required. Other than the finite element analysis proposed in this paper, one can determine the components of FGMs and volume fractions of the constituents that satisfy the variation of physical properties through optimum design in computer-aided design (CAD) systems [21]. Based on the numerical analysis and optimum design of mirror materials, we will fabricate mirrors that the temperature does not excess the melting point of silicon or copper after being heated by high-power laser and relax smoothly the thermal stress to meet the requirement of low deflection in high-power laser systems. Our future work will focus on deformation measurements of these mirrors and comparisons with the numerical analysis and related literatures for feedback on optimum design and manufacture.

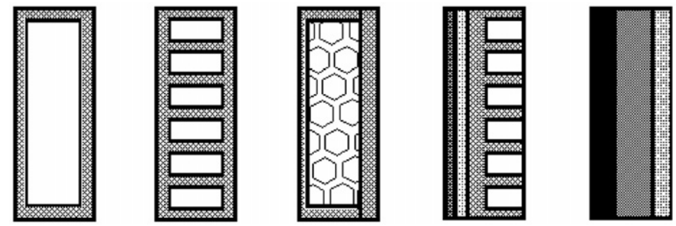

Fig. 7. The evolution of mirror structure design in high power-laser systems. From left to right: cavity; annular channel; honeycomb; multilayers with PCM; and FGM.

5. Conclusions

In this paper, we propose a novel method for mirror design in high-power laser systems, using functionally graded materials to vary smoothly the thermophysical and mechanical properties of mirrors and hence to reduce the thermal deformation of them. For mirrors with thickness $L = 0.5$ cm and radius $R = 2$ cm, the thermal deformation of silicon mirror decreases from about 1.0 to 0.2 μm for doughnut-shaped laser and from about 3.0 to 0.7 μm for Gaussian-type laser with the same value of maximum heat flux ($q_0 = 0.5 \times 10^7\,\mathrm{W/m^2}$) when the slope of the linear functions for thermal conductivity and volumetric heat capacity increases from 0 to 800. The thermal deformation of copper mirror decreases from about 2.0 to 0.6 μm for $A = 0$ and from about 9.0 to 2.0 μm for $A = 1$ when the slope of the linear functions for representation of thermal conductivity and volumetric heat capacity increases from 0 to 800. The proposed method provides an effective means for designing mirror structure and analyzing the thermal deformation of mirrors in high-power laser systems. Based on the numerical analysis and optimum design of mirror materials, we will fabricate mirrors to meet the requirement of low deflection in high-power laser systems. Our future work will focus on deformation measurements of these mirrors and comparisons with the numerical analysis and related literatures for feedback on optimum design and manufacture.

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