1) 
$$ds^2 = (h - p^2 \cos^2 \kappa u^3) du^{0^2} + 2du^0 du^1 + 2(n - p \cos^2 \kappa u^3) \times du^0 du^2 - (\cos \kappa u^3 du^2)^2 - du^{3^2},$$
 $A_0 = (pl/\kappa) \sin \kappa u^3, \quad A_1 = \alpha u^0, \quad A_3 = (l/\kappa) \sin \kappa u^3 + u^3 + u^2 u^0, \quad A_3 = 0, \quad \kappa^2 = -2\Lambda, \quad \kappa (\alpha^2 + l^2) = \kappa^2,$ 
 $\gamma_3 = 1/\cos^2 \kappa u^3, \quad \beta_3 = n/\cos^2 \kappa u^3, \quad \alpha_3 = h - 2np + n^2/\cos^2 \kappa u^3,$ 
 $\rho_3 = (ln/\kappa) \sin \kappa u^3/\cos^2 \kappa u^3 + nt/\cos^2 \kappa u^3,$ 
 $\tau_3 = (l/\kappa) \sin \kappa u^3/\cos^2 \kappa u^3 + t/\cos^2 \kappa u^3, \quad \gamma_0 = 0,$ 
 $t_0 = 1, \quad \alpha_0 = \alpha u^0, \quad \tau_0 = p\alpha u^0;$ 

2)  $ds^2 = (\alpha u^{3^2} + pu^3 + c - (bu^3 + n)^2/\gamma_0) du^{0^2}/t_0^2 + u^3 + 2du^0 du^1 + 2(bu^3 + n)/\gamma_0 du^0 du^2 - t_0^2/\gamma_0 du^{2^2} - t_0^2 du^{3^3},$ 
 $A_0 = u^3 (h - bq_0), \quad A_1 = 0, \quad A_2 = q_0, \quad A_3 = 0,$ 
 $\gamma_0/\gamma_0 - 3\gamma_0^2/2\gamma_0^2 - 4t_0/t_0 + 2t_0\gamma_0/t_0\gamma_0 - b^2/\gamma_0^2 t_0^4 = u^2\gamma_0 q_0/t_0^2 + 2(n - bq_0)^2/t_0^6,$ 
 $\gamma_3 = 0, \quad \alpha_3 = \alpha u^{3^2} + pu^3 + c, \quad \beta_3 = bu^3, \quad \tau_3 = \alpha bu^3,$ 
 $\rho_3 = -\alpha au^{3^2} + hu^3, \quad \alpha_0 = 0, \quad \tau_0 = \gamma_0 q_0.$ 

We note that for the metrics and potentials considered here, the full separation of variables is possible also in the Hamilton-Jacobi and Klein-Fock equations.

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## ONE-LOOP DIVERGENCES OF THE EINSTEIN THEORY WITH

## A NONMINIMALLY INTERACTING SCALAR FIELD

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We calculate the one-loop divergences of Einstein gravity with a nonlinear non-minimally coupled scalar field. The conformal transformation of the field variables allows us to reduce the Lagrangian to the minimal form which significantly simplifies the application of the Schwinger-DeWitt generalized technique. In the appropriate limit, the end result completely agrees with known results.

At present, there is a large number of works devoted to the calculation and analysis of the one-loop divergences and anomalies in various models of quantum field theory in a curved space—time [1-10, 11 and references therein] (a wide list of literature on this subject is in [12]).

In addition to the problem of the theoretical analysis of renormalization of these models, the results of such works have significant applications. In particular, in cosmological problems the interest along such lines of research is due to the fact that the one-loop divergences can be used to search for quantum corrections of the dynamics of the Universe, especially in the early stages of its development.

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In spite of the fact that the above-mentioned works cover a wide spread of all possible models, from the simplest theories to supergravity, it appears that, until the present, there was no analysis of the simple model that includes Einstein gravity with the  $\Lambda$ -term nonminimally coupled to a self-interacting scalar field.

The analysis of this problem is now of special interest due to the latest intensive progress in the inflation theory of the Universe with an inflationary scalar field nonminimally coupled with the space—time curvature [13, 14]. Therefore, finding the one-loop divergences and anomalies allows us to solve a whole complex of problems applied to this theory, i.e., to determine the low-energy finite part of the one-loop effective action that is responsible for the quantum dynamics, to construct an effective potential and analyze the phase transitions at the initial stage of the evolution of the Universe, and to define the one-loop high-energy scaling of the theory that is necessary for analyzing the condition for normalizing the quantum state of the inflationary Universe and for finding the probability for inflation in quantum cosmology [15].

The starting action of the theory under discussion has the form\*

$$S[g, \Phi] = \int d^4x g^{1/2} \left\{ \frac{1}{k^2} (R - 2\Lambda) - \frac{1}{2} g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} - \frac{1}{2} m^2 \Phi^2 - \frac{1}{2} \xi R \Phi^2 - \frac{\lambda}{4!} \Phi^4 \right\}.$$
 (1)

The main difficulty in calculating the one-loop divergences of the theory (1) is related to the technical complexities of reducing the operator in the quadratic part of the action to the minimal form. Apparently, this has been the main obstacle for obtaining the one-loop divergences of the present theory. However, in the present paper we will show that applying the method of conformal transformations allows us to carry out the minimization procedure in the simplest way and significantly reduce the calculations. Let us note that this idea was first discussed in the work [3], but was not carried out there.

Thus, we first convert to new field variables and then directly calculate the one-loop divergences of the theory (1).

The one-loop effective action for gauge theories in the condensed DeWitt notation [16], whose divergent part will be calculated here, has the following general form

$$iW_{1-\text{loop}} = -\frac{1}{2} \operatorname{Tr} \ln \frac{\delta^2 S[\varphi]}{\delta \varphi^A \delta \varphi^B} + \operatorname{Tr} \ln Q_\alpha^\beta, \tag{2}$$

where  $\phi^A$  is the full set of fields in the theory,  $Q^3_{\gamma} \equiv \nabla^A_{\alpha} \frac{\delta \chi^3}{\delta \varphi^A}$  is a ghost operator,  $\nabla^A_{\alpha}$  are the

generators of the gauge transformations of the field variables  $\phi A$ ,  $\chi \beta$  are the calibration conditions, and Tr represents the functional trace.

It appears that calculating (2) is simplest in application to (1) when this theory is transformed from  $(g_{\mu\nu}, \Phi)$  to new variables. The reparametrization we are introducing is based on the conformal transformation of the initial metric  $g_{\mu\nu}$ :

$$G_{\mu\nu} = \Omega^2 g_{\mu\nu}, \ \Omega^2 = 1 + b\Phi^2, \ b = -\frac{1}{2} k^2 \xi.$$
 (3)

Then, taking into account the corresponding transformation of the scalar curvature

$$\Omega^{-2}R = R(G) + 6\Omega^{-1} \square \Omega - 12\Omega^{-2}G^{\mu\nu}\Omega_{,\mu}\Omega_{,\nu}$$
(4)

(R(G) is the scalar curvature with respect to the metric  $G_{\mu\nu}$ ;  $\overline{\Box} = G^{\mu\nu} \overline{\phantom{a}}_{\nu\mu} \overline{\phantom{a}}_{\nu\nu}$ , and  $\overline{\phantom{a}}_{\mu\nu}$  is the covariant derivative with respect to  $G_{\mu\nu}$ ), one can easily derive for the action S[g,  $\Phi$ ], in terms of the new metric  $G_{\mu\nu}$ , the following expression:

$$S[g,\Phi] = \int d^4x G^{1/2} \left\{ \frac{1}{k^2} R(G) - \frac{1}{2} \Omega^{-4} (1 - a\Phi^2) G^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} - \Omega^{-4} \left[ \frac{2\Lambda}{k^2} + \frac{1}{2} m^2 \Phi^2 + \frac{\lambda}{4!} \Phi^4 \right] \right\}, \tag{5}$$

<sup>\*</sup>Our notation: sign  $g_{\mu\nu}=2$ ;  $g=|\det g_{\mu\nu}|$ ;  $R=g^{\mu\nu}$   $R^{\alpha}_{,\mu\alpha\nu}=g^{\mu\nu}$   $(\partial_{\alpha}\Gamma^{\alpha}_{\mu\nu}-...)$ ;  $\partial_{i}\partial x_{\alpha}=\partial_{\alpha}=_{,\alpha}$ 

where a  $\equiv (1/2)k^2\xi(1-6\xi)$ . However, in the kinetic term of the Lagrangian in (5), there now appears a significant nonlinearity with respect to the scalar field  $\Phi$ . To remove it, we introduce a new scalar field  $\Phi$ , which is connected to the old one by the differential relation

$$Q^{-4}(1-a\Phi^2)G^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu} = \alpha G^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}, \tag{6}$$

where  $\alpha = \text{sign}[\Omega^{-4}(-a\Phi^2)]$ . Further, we confine ourselves to  $\alpha = +1$  since the opposite sign corresponds to the scalar ghost fields.

From (5), we have the equation

$$\left(\frac{d\varphi}{d\Phi}\right)^2 = \Omega^{-4} (1 - a\Phi^2),\tag{7}$$

whose integration gives a nontrivial dependence  $\phi = \phi(\Phi)$  for the new field  $\phi$  with respect to the old  $\Phi$ :

$$\varphi = \begin{cases} -\frac{\sqrt{a}}{b} \arcsin(\sqrt{a}\Phi) - \frac{\sqrt{-(a+b)}}{2b} \ln\frac{1+z}{1-z} \\ \text{for } 0 \leqslant \xi \leqslant 1/6, \ z = \Phi \sqrt{-(a+b)/(1-a\Phi^2)}; \\ \frac{\sqrt{|a|}}{b} \arcsin(\sqrt{|a|}\Phi) - \frac{\sqrt{|a|-b}}{2b} \ln\frac{1+z}{1-z} \\ \text{for } \xi \leqslant 0, \ \xi \geqslant 1/6, \ z = \Phi \sqrt{(|a|-b)/(1-|a|\Phi^2)}, \end{cases}$$
(8)

where the constant of integration is defined by the condition  $\phi(\Phi)|_{\Phi=0}=0$ . From the solution (8), it is obvious that it is impossible to analytically express the old scalar field  $\Phi$  in terms of the new one  $\phi$  (i.e., to find the function  $\Phi=\Phi(\phi)$ ), and, thus, in principle an explicit form of the potential  $V(\phi)$  in terms of the new field  $\Phi$  is unknown. However, this is not relevant for the calculation of the one-loop divergences of the theory (1), since we do not require an explicit expression for the potential  $V(\phi)$ , which is formally defined as follows:

$$V(\varphi) = \Omega^{-4} \left( \frac{2\Lambda}{k^2} + \frac{1}{2} m^2 \Phi^2 + \frac{\lambda}{4!} \Phi^4 \right) \Big|_{\Phi = \Phi(\varphi)}. \tag{9}$$

Finally, taking into account Eqs. (5), (6), and (9), the initial action (1) in terms of the field variables  $G_{\mu\nu}$  and  $\phi$  is reduced to the form

$$S[G, \varphi] = \int d^4x G^{1/2} \left\{ \frac{1}{k^2} R(G) - \frac{1}{2} G^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} - V(\varphi) \right\}. \tag{10}$$

For us, this functional is the starting point for calculating the divergent part of the one-loop effective action  $W_{1-loop}^{div}$  for the Einstein theory of gravitation having a cosmological term that is nonminimally coupled with a self-interacting scalar field. To find  $W_{1-loop}^{div}$ , we use the method of background fields and the Schwinger-DeWitt generalized technique [12, 16, 17], which already have become standard (all calculations in detail are in [18]).

The one-loop divergences of the theory (1) in terms of the new metric  $G_{\mu\nu}$  and the new scalar field  $\phi$  are defined by the expression

$$W_{1-\text{loop}}^{\text{div}} = \frac{1}{32\pi^{2} (2 - \boldsymbol{w})} \int d^{4}x G^{1/2} \left\{ \frac{43}{60} R_{\alpha\beta}^{\circ} (G) + \frac{1}{40} R^{2} (G) + \frac{1}{40} R^{2}$$

To write Eq. (11) in terms of the old field variables  $g_{\mu\nu}$  and  $\Phi,$  we use the transformation relations

$$R(G) = \Omega^{-2}R - 6\Omega^{-6}b\Phi_{,\alpha}\Phi^{,\alpha} - 6\Omega^{-4}b\Phi\Box\Phi,$$

$$\int d^4x G^{1/2} \left[ \frac{43}{60} R_{\alpha\beta}^2 (G) + \frac{1}{40} R^2 (G) \right] = \int d^4x g^{1/2} \left[ \frac{43}{60} R_{\alpha\beta}^2 + \frac{1}{40} R^2 - \frac{19}{6} b\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} - \frac{19}{6} b\Omega^{-2} R\Phi\Box\Phi + \frac{19}{2} \Omega^{-8} b^2 (\Phi_{,\alpha}\Phi^{,\alpha})^2 + \frac{19}{6} b\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} - \frac{19}{6} b\Omega^{-2} R\Phi\Box\Phi + \frac{19}{2} \Omega^{-8} b^2 (\Phi_{,\alpha}\Phi^{,\alpha})^2 + \frac{19}{6} b\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} - \frac{19}{6} b\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} - \frac{19}{6} b\Omega^{-2} R\Phi\Box\Phi + \frac{19}{2} \Omega^{-8} b^2 (\Phi_{,\alpha}\Phi^{,\alpha})^2 + \frac{19}{6} b\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} - \frac{19}{6} b\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} - \frac{19}{6} b\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} - \frac{19}{6} b\Omega^{-2} R\Phi\Box\Phi + \frac{19}{2} \Omega^{-8} b\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} - \frac{19}{6} B\Omega^{-4} R\Phi_{,\alpha}\Phi^{,\alpha} -$$

$$+19^{2-6}b^{2}\Phi_{,\alpha}\Phi^{,\alpha}\Phi\Box\Phi+\frac{19}{2}2^{-4}b^{2}\Phi^{2}(\Box\Phi)^{2},$$
(13)

$$\int d^{4}x G^{1/2} (G^{\alpha\beta} \, \varphi_{,\alpha} \, \varphi_{,\beta})^{2} = \int d^{4}x g^{1/2} \, \Omega^{-8} (1 - a \Phi^{2})^{2} (\Phi_{,\alpha} \, \Phi^{,\alpha})^{2},$$

$$\int d^{4}x G^{1/2} R (G) G^{\alpha\beta} \, \varphi_{,\alpha} \, \varphi_{,\beta} = \int d^{4}x g^{1/2} (1 - a \Phi^{2}) \left[ \Omega^{-4} R \Phi_{,\alpha} \, \Phi^{,\alpha} - \Phi^{,\alpha} \Phi^{,\alpha} \right]$$

$$- 6 \Omega^{-8} b (\Phi_{,\alpha} \Phi^{,\alpha})^{2} - 6 \Omega^{-6} b \Phi_{,\alpha} \Phi^{,\alpha} \Phi \square \Phi$$

$$(15)$$

and the fact that

$$V(\varphi)|_{\varphi=\varphi(\Phi)} \equiv V(\varphi)|_{\Phi} \equiv \widetilde{V}(\Phi) = \Omega^{-4} \left( \frac{2\Lambda}{k^2} + \frac{1}{2} m^2 \Phi^2 + \frac{\lambda}{4!} \Phi^4 \right), \tag{16}$$

and that the derivatives  $(\partial V/\partial \varphi)|_{\varphi = \varphi(\Phi)} = (\partial V/\partial \varphi)|_{\Phi}$  and  $(\partial^2 V/\partial \varphi^2)|_{\varphi = \varphi(\Phi)} = (\partial^2 V/\partial \varphi^2)|_{\Phi}$  can be calculated in the implicit form

$$\frac{\partial V(\varphi)}{\partial \varphi}\Big|_{\Phi} = \left(\frac{\partial \tilde{V}(\Phi)}{\partial \Phi}\right) \left(\frac{\partial \varphi}{\partial \Phi}\right)^{-1} = 2^{-4}\Phi \frac{(-8b\Lambda/k^2) + m^2 + (\lambda/6)\Phi^2 - m^2b\Phi^2}{V[-a\Phi^2]}; \qquad (17)$$

$$\frac{\partial^2 V(\varphi)}{\partial \varphi^2}\Big|_{\Phi} = \frac{\frac{\partial^2 \tilde{V}(\Phi)}{\partial \Phi^2} \frac{\partial \varphi}{\partial \Phi} - \frac{\partial \tilde{V}(\Phi)}{\partial \Phi} \frac{\partial^2 \varphi}{\partial \Phi}}{(\partial \varphi/\partial \Phi)^3} - \frac{2^{-4}}{(1 - a\Phi^2)^2} \times \left(\frac{8b\Lambda}{k^2} + m^2 + \Phi^2 \left(24b^2\Lambda/k^2 + \frac{1}{2}\lambda - 6m^2b\right) + \Phi^4 \left(-32ab^2\Lambda/k^2 - \frac{1}{6}\lambda b - \frac{1}{3}\lambda a + m^2b^2 + 6abm^2\right) + \Phi^6 \left(\frac{1}{3}ab\lambda - 2ab^2m^2\right). \qquad (18)$$

Thus, taking into account the above transformations, the expression for the one-loop divergences of the theory (1) in terms of the initial variables has the form

$$W_{1-\text{loop}}^{\text{div}}[g, \Phi] = \frac{1}{32\pi^{2}(2-w)} \int d^{4}x g^{1/2} \left\{ \frac{43}{60} R_{\alpha\beta}^{2} + \frac{1}{40} R^{2} + \frac{1}{40} R^{2} + \frac{1}{9} b^{2} + \frac{5}{4} k^{4} (1-a\Phi^{2})^{2} + 2k^{2}b (1-a\Phi^{2}) \right] \Omega^{-8} (\Phi_{,x}\Phi^{,a})^{2} + \left[ 19b^{2} + 2k^{2}b (1-a\Phi^{2}) \right] \Omega^{-6} \Phi_{,x}\Phi^{,x}\Phi \Box \Phi + \frac{19}{2} \Omega^{-4} b^{2}\Phi^{2} (\Box \Phi)^{2} - \frac{19}{6} b\Omega^{-2} R\Phi \Box \Phi + \left[ -\frac{19}{6} b - \frac{1}{3} k^{2} (1-a\Phi^{2}) \right] \Omega^{-4} R\Phi_{,x}\Phi^{,x} + \right.$$

$$\left. + b \left[ 26k^{2}V(\varphi) |_{\Phi} + \frac{\partial^{2}V(\varphi)}{\partial \varphi^{2}} |_{\Phi} \right] \Phi \Box \Phi + k^{2}\Omega^{-2} V(\varphi) |_{\Phi} [k^{2} (1-a\Phi^{2}) + 26b] \Phi_{,x}\Phi^{,x} + 2^{-2} \frac{\partial^{2}V(\varphi)}{\partial \varphi^{2}} |_{\Phi} \left[ -2k^{2} (1-a\Phi^{2}) + b \right] \Phi_{,x}\Phi^{,x} + + 2^{2} \left[ -\frac{13}{3} k^{2}V(\varphi) |_{\Phi} - \frac{1}{6} \frac{\partial^{2}V(\varphi)}{\partial \varphi^{2}} |_{\Phi} \right] R + \Omega^{4} \left[ 5k^{4}V^{2}(\varphi) |_{\Phi} - 2k^{2} ((\partial V(\varphi)/\partial \varphi)) |_{\Phi})^{2} + \frac{1}{2} ((\partial^{2}V(\varphi)/\partial \varphi^{2}) |_{\Phi})^{2} \right].$$

$$(19)$$

From Eqs. (16)-(19), it is clear that the one-loop divergences have a sufficiently complex structure (the counterterms do not even have a polynomial form with respect to the scalar field  $\Phi$ ).

In conclusion, let us consider the limiting case when

$$\Lambda = \lambda = m = 0. \tag{20}$$

In this case, a = b = 0,  $\Omega^2 = 1$ , which identifies the new and old variables:

$$G_{\mu\nu} \equiv g_{\mu\nu}, \quad \varphi \equiv \Phi. \tag{21}$$

Then, for the action (10) we have

$$S[g, \Phi] = \int d^4x g^{1/2} \left\{ \frac{1}{k^2} R - \frac{1}{2} g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} \right\},$$

and as can be easily shown, the one-loop divergences (19) (or (11)) are defined by the expression

$$W_{1-\text{loop}}^{\text{div}} = \frac{1}{32\pi^{2}(1-w)} \int d^{4}x g^{1/2} \left\{ \frac{43}{60} R_{\alpha\beta}^{2} + \frac{1}{40} R^{2} + \frac{5}{4} k^{4} (g^{\alpha\beta} \Phi_{,\alpha} \Phi_{,\beta})^{2} - \frac{1}{3} k^{2} g^{\alpha\beta} \Phi_{,\alpha} \Phi_{,\beta} R \right\}, \tag{22}$$

which on the mass shell, i.e., taking into account the equations of motion

$$\Box \Phi = 0, R = \frac{1}{2} k^2 g^{\alpha\beta} \Phi_{,\alpha} \Phi_{,\beta}, \tag{23}$$

agrees with the known result of t'Hooft and Veltman [1].

Further, as expected, the obtained expression (19) for the divergent part of the one-loop effective action, in the case  $k^2 \rightarrow \infty$ , m = 0,  $\xi = 1/6$  has the conformal invariance

$$W_{1-\text{loop}}^{\text{div}}[g, \Phi] = \frac{1}{32\pi^{2}(1-w)} \int d^{4}x g^{1/2} \left( \frac{43}{60} \left( R_{23}^{2} - \frac{1}{3} R^{2} \right) + \frac{19}{2} \Phi^{-4} \left( \Phi \Box \Phi - \frac{1}{6} R \Phi^{2} \right)^{2} - \frac{79}{6} \lambda \left( \Phi \Box \Phi - \frac{1}{6} R \Phi^{2} \right) + \frac{91}{72} \lambda^{2} \Phi^{4} \right)$$
(24)

as a consequence of the explicit availability of conformally invariant structures.

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