WINCKLER–VON MISES TYPE INEQUALITIES

In 1866, Winckler [3] derived a large number of inequalities relating to cumulative distribution functions (CDFs) of distributions of random variables (X) possessing a continuous CDF, which is unimodal and symmetrical. Similar inequalities, under less restrictive conditions, were obtained by von Mises* [2] in 1938. More recently, these inequalities have been refined by Beesack [1].

Typical inequalities of this kind are of form: “For any real a, and under certain conditions on the CDF,

$$\Pr[|X-a| \geqslant x] \leqslant \left(\frac{r}{r+1}\right)^r \frac{{}_a\nu_r}{x^r}$$
$$\text{if } x \geqslant \frac{r}{r+1}\{(r+1)_a\nu_r\}^{1/r}$$

and

$$\Pr[|X-a| \geqslant x] \leqslant 1 - \frac{x}{\{(r+1)_a\nu_r\}^{1/r}}$$
$$\text{if } x < \frac{r}{r+1}\{(r+1)_a\nu_r\}^{1/r},$$

where ${}_a\nu_r = E[|X-a|^r]$ is the rth absolute moment* of X about a.”

If $x = r(r+1)^{-1}\{(r+1)_a\nu_r\}^{1/r}$, both upper bounds are equal to $(r+1)^{-1}$. See Beesack [1] for further details.

REFERENCES

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2. von Mises, R. (1938). *Bull. Sci. Math. (2)*, **62**, 68–71.
3. Winckler,A. (1866). *Sitzungsber. Math.- Natur. Kl. Kon. Akad. Wiss. Wien, Zweite Abt.*, **53**, 6–41.

See also Inequalities on Distributions: Bivariate and Multivariate and Probability Inequalities for Sums of Bounded Random Variables.