Photoelastic Investigation of Stresses in Bars with Reinforced Semicircular Notches under Bending

Principal objective of investigation is to determine what type of reinforcement changes the stress distribution outside the notch to equal the conditions existing in a bar having no notch

by Heihachi Shimada

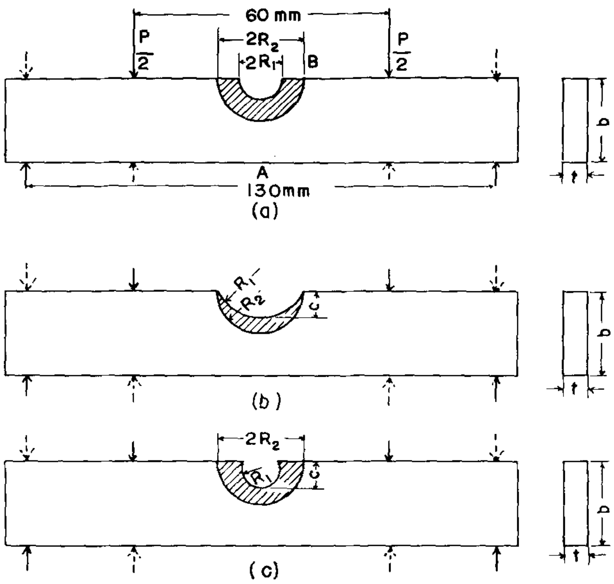

Fig. 1—Types of test pieces for (a) concentric reinforcement, (b) crescent reinforcement and (c) arch-type reinforcement

Nomenclature

b = breadth of bar
c = depth of inner notch (for concentric: $c = R_1$)
λ = R_2/b.
t = thickness of bar
R_1 = inner radius of reinforcement
R_2 = outer radius of reinforcement
τ_m = maximum shear stress
τ_{mm} = maximum value of τ_m
τ_n = nominal maximum shear stress = $3M/tb^2$
τ_0 = $\dfrac{Kb^2}{(b - R_2)^2}$
M = applied bending moment
K = stress-concentration factor of unreinforced semicircular notch
θ = angle of position on bonding boundary
θ_m = angle of position of τ_{mm}

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ABSTRACT—The object of the work described was to obtain information about the stresses in reinforced plastic bars. Tests were carried out under bending on a bar with a composite reinforced semicircular notch. The bars were of epoxy resin reinforced with duralumin. The reinforcements are the concentric, crescent and arch types. The relations between the maximum shear stress and breadth of bars are obtained. The most suitable reinforcements are discussed.

Introduction

A finite homogeneous bar having an unreinforced semicircular notch under bending has been the subject of several studies,[1–3] but in all of these no consideration was given to a composite model having a semicircular notch. Composite structures having a semicircular notch occur frequently in engineering. The photoelastic examination described in this paper centers around the most suitable type of reinforcement. A problem of some interest is to find what type of reinforcement renders the stress distribution outside the notch the same as that which would prevail if the notch did not exist; namely, under bending, the isochromatic fringe pattern is a system of longitudinal, parallel, uniformly spaced lines. Thus, bars with a reinforced semicircular notch in which reinforcements are the concentric, the crescent, and the arch types under bending are studied. Bars of epoxy resin are bonded to duralumin reinforcements by Araldite 121, using the techniques developed by Mylonas and others.[4–6] The observations are made in a simple polariscope.

Model Construction and Test Procedure

The resin used for the bar was epoxy resin KT-102 (made in Japan), the reinforcements were made of duralumin and the bonding medium was Araldite 121, which sets at room temperature. Very small initial bonding stresses were introduced. The shapes and dimensions of the models are shown in Fig. 1 and in Tables 1 to 3. To eliminate the error of each test piece, applied bending moments were subjected to one and reversed directions respectively. The

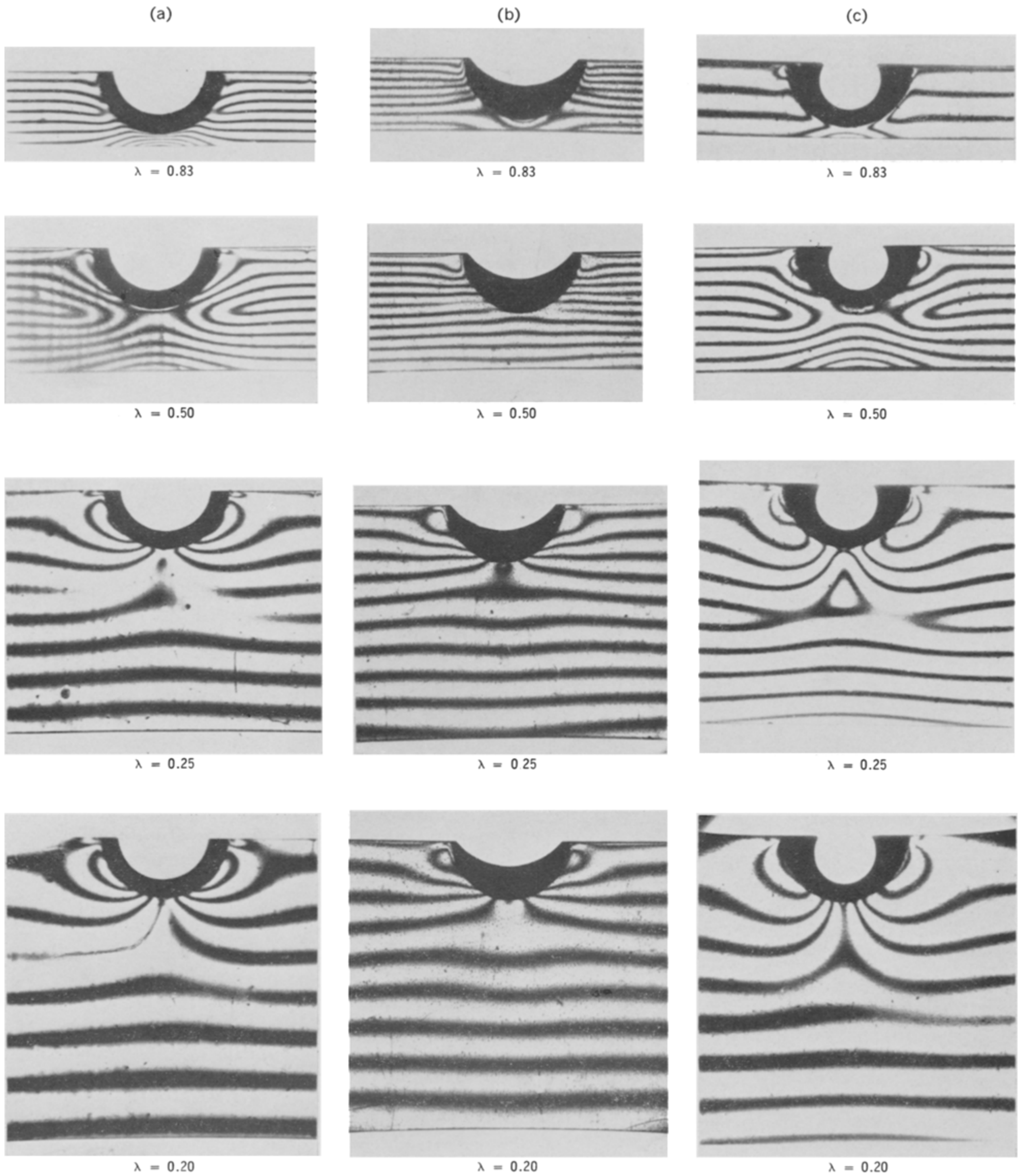

Fig. 2—Isochromatic fringes for (a) concentric reinforcement, $R_2/R_1 = 1.33$, (b) crescent reinforcement, $R_2/c = 2.0$, and (c) arch-type reinforcement, $R_2/c = 1.25$

apparatus for applying the loads and the test procedure selected to determine the influence of breadth on the reinforced semicircular notch were shown in author's previous paper.[4] After the first test, the breadth of test piece is machined to a smaller size. Thus, the test piece is tested first with maximum breadth and then with successively smaller values of breadth. Figure 2 shows part of the isochromatic fringe patterns of the test pieces in the light field. The relative retardations were measured from the isochromatic fringes, with fractional measurements by the Tardy method where necessary.

Stresses on Bonding Boundaries of Concentric Reinforcements

Figure 3 shows τ_m/τ_n on the bonding boundary for

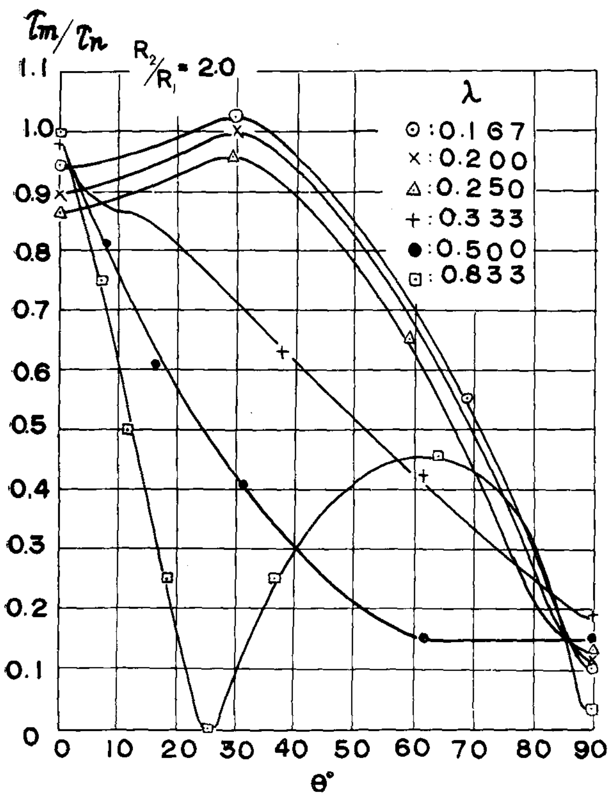

Fig. 3—τ_m/τ_n on bonding boundary of concentric reinforcement, $R_2/R_1 = 2.0$

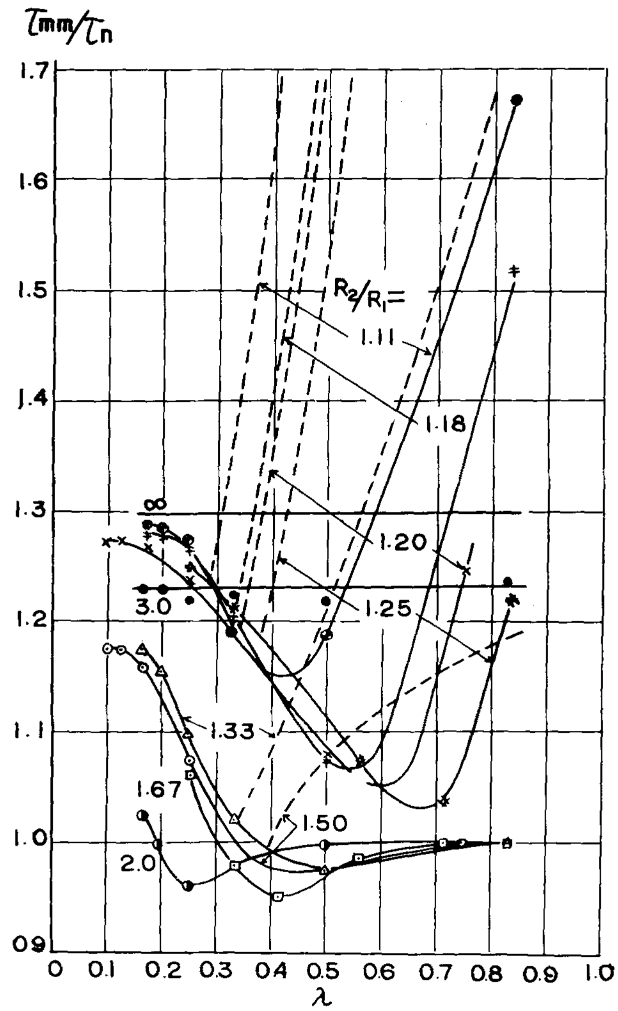

Fig. 4—τ_{mm}/τ_n of concentric reinforcement

$R_2/R_1 = 2$. The figure is very different from the bars with the unreinforced semicircular notches, in which maximum stress concentration occurs at the root of the notch. Figure 4 shows τ_{mm}/τ_n vs. λ, and dotted lines are τ_m/τ_n of point A (Fig. 1). The position observed of τ_{mm}/τ_n on the boundary is shown in Fig. 7. There it is seen that for $R_2/R_1 = 1.25$, $\theta_m > 0°$, and τ_{mm}/τ_n is larger than unity over the range $0 \leqq \lambda < 1.0$, and for $1.33 \leqq R_2/R_1 \leqq 2.0$, $\theta_m > 0°$ when λ is small, and $\theta_m = 0°$ when λ is comparatively large, and $\tau_{mm}/\tau_n > 1$ to small values of λ, and $\tau_{mm}/\tau_n \leqq 1.0$ to large values of λ, for $R_2/R_1 = 3$, $\theta_m = 0°$ (point B) over the full range of λ, and τ_{mm}/τ_n is the constant large value.

It can be seen that the reinforcements near $\theta = 0°$ have bad effects on stresses of the bonding boundary for $R_2/R_1 > 2$; then, if the depth of the reinforcements near $\theta = 0°$ is small (namely, crescent type), stresses will be reduced.

Stresses on Bonding Boundaries of Crescent Reinforcements

From the results of the concentric reinforcement, the crescent reinforcements are considered. Figure

TABLE 1—DIMENSIONS AND CONSTANTS OF BARS WITH CONCENTRIC REINFORCEMENT

No.	R_1, mm	R_2, mm	R_2/R_1	t, mm	Ee,* kg/mm²	Ed,† kg/mm²	1/λ
1	4.50	5.00	1.11	5.84	340	7050	6, 5, 4, 3, 2, 1.2
2	4.25	5.00	1.18	5.83	340	7050	6, 5, 4, 3, 2, 1.2
3	2.50	3.00	1.20	5.85	320	7050	10, 8, 4, 2, 1.33
4	4.00	5.00	1.25	5.90	340	7050	4, 3, 2.4, 1.8, 1.4, 1.2
5	3.75	5.00	1.33	5.90	340	7050	6, 5, 4, 3, 2, 1.2
6	2.00	3.00	1.50	5.84	320	7050	10, 8, 4, 2, 1.33
7	3.00	5.00	1.67	5.95	340	7050	4, 3, 2.4, 1.8, 1.4, 1.2
8	2.50	5.00	2.00	5.94	340	7050	6, 5, 4, 3, 2, 1.2
9	1.67	5.00	3.00	5.94	320	7050	6, 5, 4, 3, 2, 1.2
10	1.00	3.00	3.00	5.90	340	7050	10, 8, 4, 2, 1.33
11	0	5.00	..	5.90	340	7050	4, 3, 2.4, 1.8, 1.4, 1.2
12	0	5.00	..	5.84	340	7050	6, 5, 4, 3, 2, 1.2

* Ee = Young's Modulus of epoxy resin.
† Ed = Young's Modulus of duralumin.

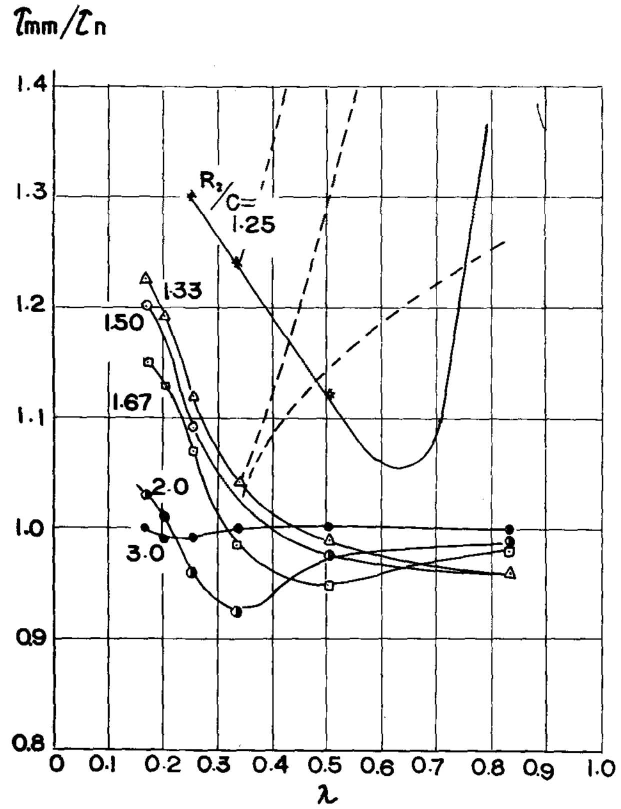

Fig. 5—τ_{mm}/τ_n of crescent reinforcement

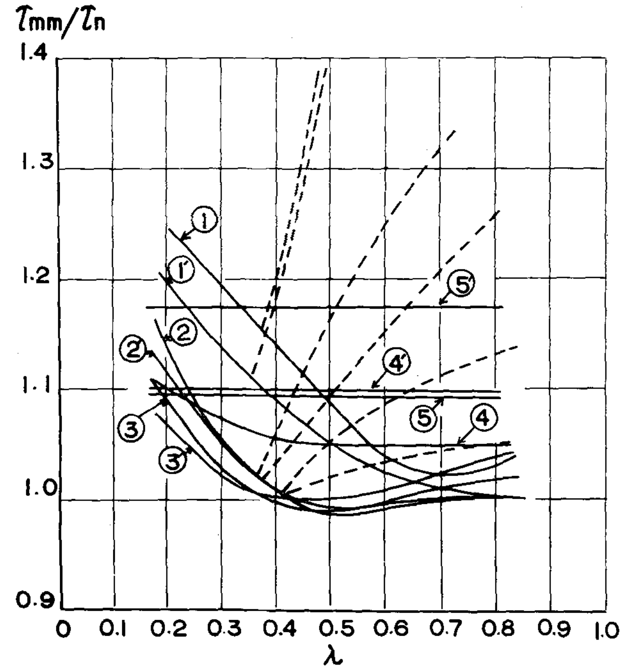

Fig. 6—τ_{mm}/τ_n of arch-type reinforcement, where

①: $R_2/c = 1.25$, $R_2/R_1 = 1.42$
②: $R_2/c = 1.33$, $R_2/R_1 = 1.43$
③: $R_2/c = 1.50$, $R_2/R_1 = 1.65$
④: $R_2/c = 1.67$, $R_2/R_1 = 1.97$
⑤: $R_2/c = 2.00$, $R_2/R_1 = 2.44$
①: $R_2/c = 1.25$, $R_2/R_1 = 2.00$
②: $R_2/c = 1.33$, $R_2/R_1 = 2.07$
③: $R_2/c = 1.50$, $R_2/R_1 = 2.76$
④: $R_2/c = 1.67$, $R_2/R_1 = 3.00$
⑤: $R_2/c = 2.00$, $R_2/R_1 = 3.45$

5 shows τ_{mm}/τ_n vs. λ for variable R_2/c, and the dotted curves are τ_m/τ_n of the point A. θ_m on the bonding boundary is shown in Fig. 8.

Stresses on Bonding Boundaries of Arch-type Reinforcements

From the results of the concentric and the crescent reinforcements, the arch-type reinforcements are considered. Figure 6 shows τ_{mm}/τ_n vs. λ for variable R_2/c, where the light arch reinforcement is nearer the crescent reinforcement, the heavy arch reinforcement has large values of R_2/R_1, and the same value of R_2/c. θm on the bonding boundary is shown in Fig. 9.

Comparison Between Concentric, Crescent, and Arch-type Reinforcements

From the results of Figs. 4, 5 and 6, when R_2/c = 1.25, the order of high stresses is the crescent, the concentric, the light-arch and the heavy-arch reinforcements over the full range of λ. When $1.33 \leqq R_2/c \leqq 1.67$, the order of high stresses is the crescent, the concentric, the light-arch and heavy-arch reinforcements for the range of small values of λ; the heavy-arch, the light-arch, the concentric and the crescent reinforcements are for the range of large values of λ. When $R_2/c \geqq 2.0$, the order of high stresses is the heavy-arch, the light-arch, the concentric and the crescent reinforcements over the full range of λ. And then, when $\theta_m = 0°$ for the concentric reinforcement, the order of high stresses is the heavy arch, the light arch, the concentric and the crescent. When θ_m is near 0° for the concentric

TABLE 2—DIMENSIONS AND CONSTANTS OF BARS WITH CRESCENT REINFORCEMENT

No.	R_1, mm	R_2, mm	c, mm	R_2/c	t, mm	Ee, kg/mm²	Ed, kg/mm²	1/λ
13	5.12	5.00	4.00	1.25	6.00	340	7050	6, 5, 4, 3, 2, 1.2
14	5.15	5.00	3.75	1.33	6.00	340	7050	6, 5, 4, 3, 2, 1.2
15	5.42	5.00	3.33	1.50	6.01	340	7050	6, 5, 4, 3, 2, 1.2
16	5.50	5.00	3.00	1.67	6.01	340	7050	6, 5, 4, 3, 2, 1.2
17	6.00	5.00	2.50	2.00	6.00	340	7050	6, 5, 4, 3, 2, 1.2
18	7.92	5.00	1.67	3.00	6.00	340	7050	6, 5, 4, 3, 2, 1.2

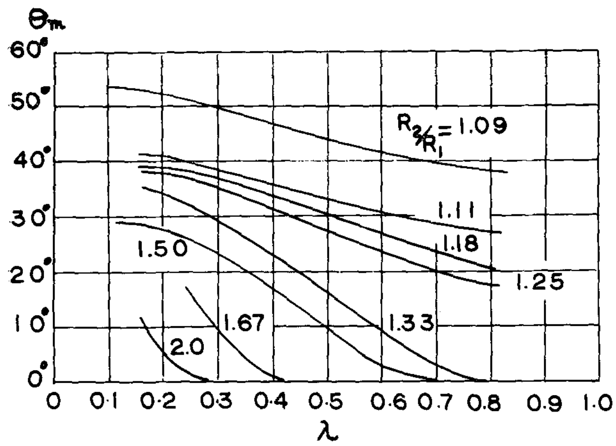

Fig. 7—Observed position of τ_{mm} of concentric reinforcement

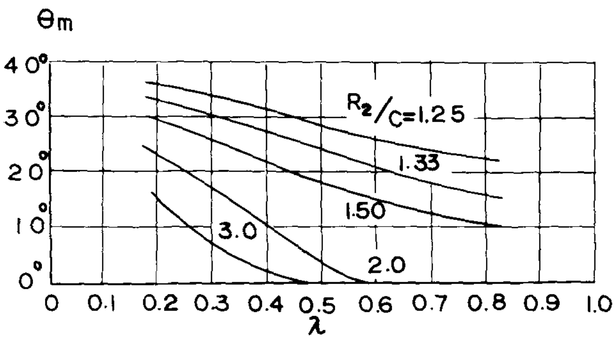

Fig. 8—Observed position of τ_{mm} of crescent reinforcement

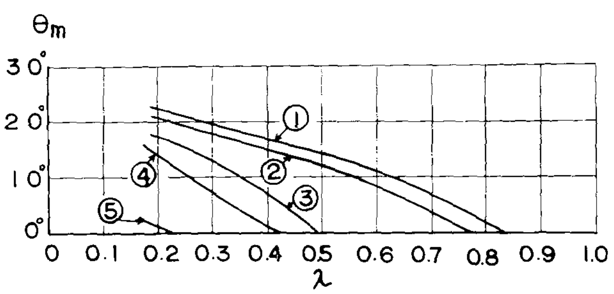

Fig. 9—Observed position of τ_{mm} of arch-type reinforcement, where

(1): $R_2/c = 1.25$, $R_2/R_1 = 1.42$
(2): $R_2/c = 1.33$, $R_2/R_1 = 1.43$
(3): $R_2/c = 1.50$, $R_2/R_1 = 1.65$
(4): $R_2/c = 1.67$, $R_2/R_1 = 1.97$
(5): $R_2/c = 2.00$, $R_2/R_1 = 2.44$

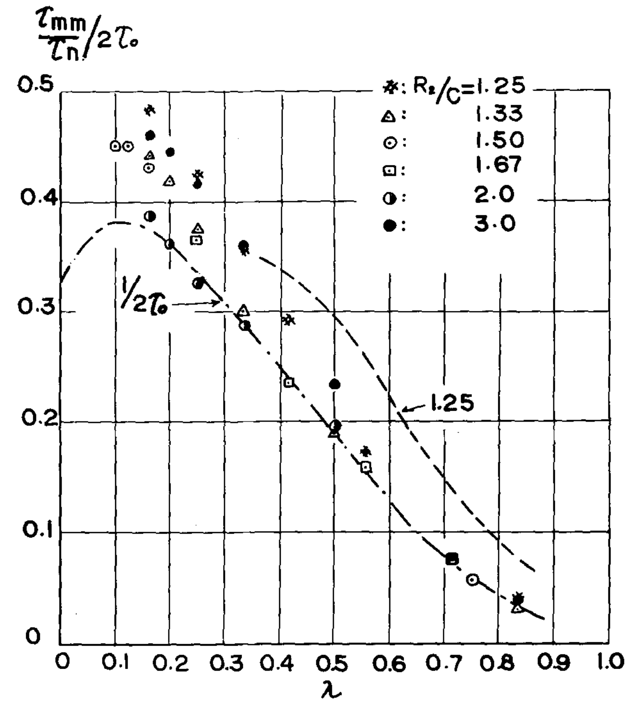

Fig. 10—Effect of concentric reinforcement vs. λ, for $R_2/R_1 = 1.25 \sim \infty$

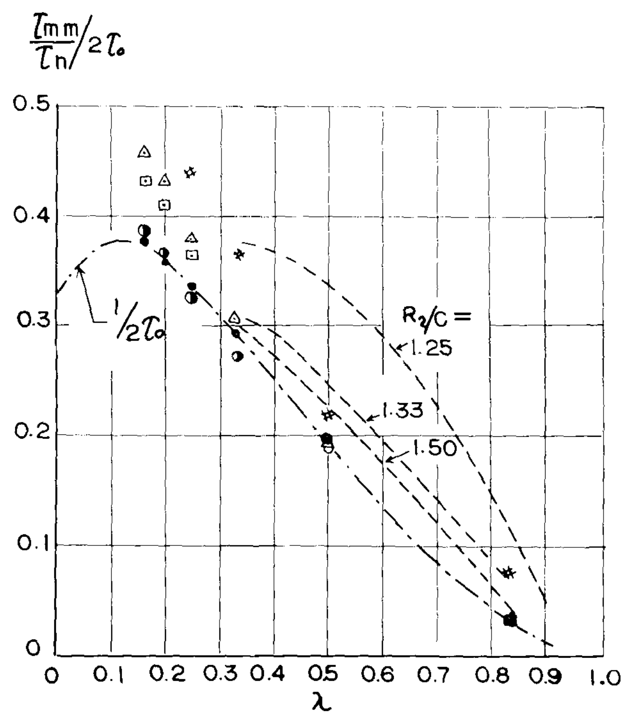

Fig. 11—Effect of crescent reinforcement vs. λ, for $R_2/c = 1.25 \sim \infty$

reinforcement, the order of small stresses is not clear. When $\theta_m \gg 0°$ for the concentric reinforcement, the order of high stresses is the crescent, the concentric, the light arch and the heavy arch.

Effect of Reinforcements

Figures 10, 11 and 12 show $(\tau_{mm}/\tau_n)/2\tau_0$ vs. λ, at $R_2/c = 1.25 \sim \infty$, respectively, for the concentric, the crescent and the arch-type reinforcements. These figures indicate "the effect of the reinforcement," and the broken curve is $^1/_2\ \tau_0$ (the base line), and $2\ \tau_0$ is obtained; $2\tau_0 = [K(b - R_2)^2]/b^2$. The

TABLE 3—DIMENSIONS AND CONSTANTS OF BARS WITH ARCH-TYPE REINFORCEMENT

No.	R_1, mm	R_2, mm	c, mm	R_2/c		t, mm	Ee, kg/min²	Ed, kg/mm²	1/λ
19	3.53	5.00	4.00	1.25	L*	6.00	340	7050	6, 5, 4, 3, 2, 1.2
20	2.50	5.00	4.00	1.25	H†	6.01	340	7050	6, 5, 4, 3, 2, 1.2
21	3.51	5.00	3.75	1.33	L	6.01	340	7050	6, 5, 4, 3, 2, 1.2
22	2.41	5.00	3.75	1.33	H	6.01	340	7050	6, 5, 4, 3, 2, 1.2
23	3.04	5.00	3.33	1.50	L	6.01	340	7050	6, 5, 4, 3, 2, 1.2
24	1.81	5.00	3.33	1.50	H	6.01	340	7050	6, 5, 4, 3, 2, 1.2
25	2.54	5.00	3.00	1.67	L	6.00	340	7050	6, 5, 4, 3, 2, 1.2
26	1.67	5.00	3.00	1.67	H	6.00	340	7050	6, 5, 4, 3, 2, 1.2
27	2.05	5.00	2.50	2.00	L	6.00	340	7050	6, 5, 4, 3, 2, 1.2
28	1.45	5.00	2.50	2.00	H	6.00	340	7050	6, 5, 4, 3, 2, 1.2

* L = Light arch-type reinforcement.
† H = Heavy arch-type reinforcement.

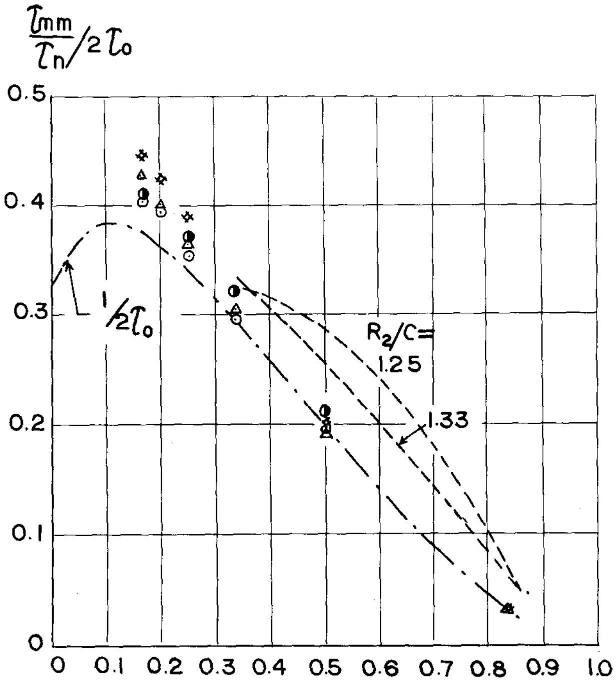

Fig. 12—Effect of arch-type reinforcement vs. λ, for $R_2/c = 1.25 \sim \infty$

values of K were known already.[1-3] The values of $^1/_2\ \tau_0$ mean the ratio of the stresses of a bar without notch to that with a notch. The values of $(\tau_{mm}/\tau_n)/2\ \tau_0$ mean the ratio of the stresses of a bar with the reinforced notch to that with the unreinforced notch for same λ, and are not beyond $^1/_2$ for full range of λ. In these figures, when λ is zero, $R_2 \neq 0$ and $b = \infty$.

From these figures, $(\tau_{mm}/\tau_n)/2\ \tau_0$ are given by the following empirical equations by which the reader may calculate (τ_{mm}/τ_n):

(1) for arch-type reinforcements

$$\frac{1}{2\tau_0} \leqq \left(\frac{\tau_{mm}}{\tau_n}\right)\Big/2\tau_0 \leqq 0.1 + \frac{1}{2\tau_0} \qquad \text{if } R_2/c \geqq 1.25$$

(2) for concentric reinforcements

$$\frac{1}{2\tau_0} \leqq \left(\frac{\tau_{mm}}{\tau_n}\right)\Big/2\tau_0 \leqq 0.125 + \frac{1}{2\tau_0} \qquad \text{if } R_2/c \geqq 1.25$$

(3) for crescent reinforcements

$$\frac{1}{2\tau_0} \leqq \left(\frac{\tau_{mm}}{\tau_n}\right)\Big/2\tau_0 \leqq 0.15 + \frac{1}{2\tau_0} \qquad \text{if } R_2/c \geqq 1.25$$

Consequently, the values of τ_{mm}/τ_n for the bars with the reinforced semicircular notch tend to approach the values of bars without the semicircular notch, when the most suitable reinforcement is used; and these do not go beyond the half values of bars with the unreinforced semicircular notch for the range of $R_2/c \geqq 1.25$.

Acknowledgments

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