

Influence of heterogeneity on fracture behavior in multi-layered materials subjected to thermo-mechanical loading

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ABSTRACT

Based on the heterogeneous characteristics of materials at mesoscopic level, the stress state and fracture behavior in multi-layered materials are numerically investigated by using RFPA^{2D}, a numerical code incorporated with thermo-mechanical-damage (TMD) model. We conduct this research through two logical steps. Firstly, we investigate the stress states between two adjacent fractures for a typical three-layer model. Our results show that the critical value of fracture spacing to layer thickness ratio is about 0.9 which is gently lower than that of value in latest publications. Secondly, we use the same three-layer model without pre-assigned fractures. An isothermal loading is applied on the model until the state of fracture saturation is achieved. In the stage of initiation, fractures are formed one after another; in the stage of infilling, new fractures are created sequentially between the earlier formed fractures; in the stage of saturation, the formed fractures are spaced so closely that no more new fractures can be filled even with decreasing the temperature. Numerical study show that both the fracture pattern and the critical value of fracture spacing to layer thickness ratio is strongly dependent on the heterogeneous characteristics of the central layer materials. For the cases with a relatively homogeneous central layer, more interface fractures occur and the interface delamination evidently influences the fracture saturation.

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1. Introduction

Fracture behavior in layered material has been intensely investigated in the past decades for its important contribution to civil engineers, mechanical engineers and material science [1–7]. Based on stress-transfer theory, Hobbs gave for the first time the theoretical explanation for the relationship between the fracture spacing and the fractured layer thickness [8]. To better understand the linear relation between fracture spacing and layer thickness, more scholars have investigated the stress distribution between two adjacent fractures as a function of the fracture spacing to layer thickness ratio using a three-layer elastic model with a fractured central layer. The results show that when the ratio changes from greater than to less than a critical value (approximately 1.0) the normal stress acting perpendicular to the fractures changes from tensile to compressive. This stress state transition precludes further infilling of fractures unless there are existing flaws and/or the fractures are driven by an internal fluid pressure or other mechanisms.

Although the mentioned explanation of fracture saturation has been widely accepted, few of the existing models can adequately reproduce the process of fracture nucleation, propagation, infilling and saturation, as observed experimentally. Furthermore, the analytical and numerical models considered so far tend to oversimplify the materials as a homogeneous medium, as pointed by previous studies [9–12]. We note that failure phenomena depend very strongly on the properties of material heterogeneity and fracture of heterogeneous solids typically initiates from scattered weak sites in the medium which nucleate fractures upon exposure to tensile stress.

In addition, the previous studies tend to conclude that the infilling fractures are driven by an internal fluid pressure or other mechanisms. But few researchers use the coupled thermo-mechanical-damage model to investigate the infilling fractures. Thermal mismatch plays an important role in the damage and failure process of the materials subjected to fire or in cold region, especially for heterogeneous materials. When temperature changes are taken into account, large differences in coefficients of thermal expansion (CTEs) which occur in local sites lead to severe thermal or residual stresses, and this type of stresses must be added to stresses induced by external mechanical loads. Thermal mismatch stress (TMS) can be so great that the elastic range

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of the materials is exceeded and permanent deformations occur in critical regions. In extreme problems, TMS will result in failure directly without external loads. Kingery pointed out, with a change in temperature, “no stresses arise providing that the body is homogeneous, isotropic and unrestrained (free to expansion)” [13]. In fact, heterogeneity is one of the factors affecting the rock failure when it exposures to high or low temperatures. It is important to find a way to describe the heterogeneous as the physical characters of materials [14,15]. Although some of approaches (fracture mechanics, damage mechanics or other numerical approaches) can predict where the crack will occur at first, they are not available as soon as cracks are initiated in materials.

To better understand the fracture behavior of layered materials, a numerical code, RFPA^{2D}, based on the coupled thermo-mechanical-damage (TMD) model, is employed to conduct numerical investigation. We apply the same three-layer model, but without pre-existing fractures, to investigate the progressive evolution of infilling fracture in the layered materials subjected to thermo-mechanical loading.

2. Numerical method description

In order to simulate the fracture process of material subjected to thermal and external loading, the heterogeneity of mesoscopic structures of material must be considered. The Realistic Failure Process Analysis (RFPA), a numerical code based on FEM, gives an effective description to the microscopic damage mechanism and macroscopic failure of heterogeneous materials subjected to thermo-mechanical loading. In RFPA modeling, the material is numerically represented with mesoscopic elements with same size and different mechanical properties. The mesoscopic element is assumed homogeneous and isotropic, whose stress–strain relationship meets with the specific elastic damage constitutive law. In order to capture the heterogeneity of quasi-brittle materials at meso-level, the mechanical parameters of materials, including the Young's modulus, strength and Poisson's ratio are assumed to conform to Weibull distribution as defined in the following probability density function:

$$f(u) = \frac{m}{u_0} \left(\frac{u}{u_0} \right)^{m-1} \exp \left(- \frac{u}{u_0} \right)^m \quad (1)$$

where u is the parameter of element (such as strength or elastic modulus); the scale parameter u_0 is related to the average value of parameters and the m defines the shape of the distribution function. The parameter m defines the degree of material homogeneity, is called homogeneity index. The details about RFPA code can be referred to some published papers [16–18]

3. Numerical simulation of fracture infilling and discussion

3.1. Numerical model

The following analyses have been carried out to study fracture saturation in multiple layered materials. We use a three-layer model put forth by Bai and others [1]. The mesh for the model contains $480 \times 120 = 57,600$ elements with geometry of 240×60 mm. Thickness of the central layer (T_c) is 10 mm. The overall thickness of the model (T) is 60 mm, which can be defined as $T = T_c + T_b + T_t$. As shown in Fig. 1, the right and left boundary are fixed in x direction, both the bottom and top boundary are free. Plain strain is assumed for all calculations. The three layers are subjected to a uniform temperature decline from 0°C , at a decline rate of 0.5°C per step. The Young's modulus (E_b), compressive strength (σ_b) and Poisson's ratio (ν_b) are same for adjacent layers, $E_b = 60$ GPa, $\sigma_b = 500$ MPa, $\nu_b = 0.25$, and those for the central layer are

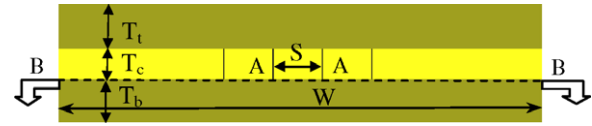


Fig. 1. The configuration of three-layered model. (The thickness of central layer and neighbor layers are indicated with T_c , T_t and T_b , respectively. The space of adjacent fractures is denoted S , the whole width is denoted with W).

20 GPa, 150 MPa and 0.3, respectively. In addition, the thermal expansion coefficient for the adjacent layers and central layer is $1.0 \times 10^{-5}/^\circ\text{C}$ and $2.0 \times 10^{-5}/^\circ\text{C}$, respectively.

3.2. Numerical investigation on the stress field

Firstly, we focus on the stress distribution in the model. Four fractures are pre-assigned in the central layer and they are equally-spaced along the central layer, perpendicular to the long axis and fully transect the layer height (T_c). With this defined model, the evolution of the stress distribution is examined as a function of the fracture spacing to the fractured layer thickness ratio. To clearly understand the transition of horizontal stress (σ_x) along the line A–A in Fig. 1 for a homogeneous medium, the homogeneity index is set to represent a uniform material with m large enough to represent a very homogeneous material.

Fig. 2 is the stress field numerically obtained with RFPA. It is shown that there is evident stress concentration on the tip of the pre-assigned fractures. In order to give quantitative analysis, we use the stress along the line A–A represents the stress between adjacent fractures. Numerical results (Fig. 3) show that the critical spacing to fractured layer thickness ratio is about 0.9. This critical value determines the stress state between the adjacent fractures: when the ratio is below this critical value the stress along line A–A is compressive, while above the critical value the stress is tensile. In the study, the loading pattern is different to those employed in previous studies and the deformation of central layer is controlled by both the TMS effect and its own cooling shrinkage.

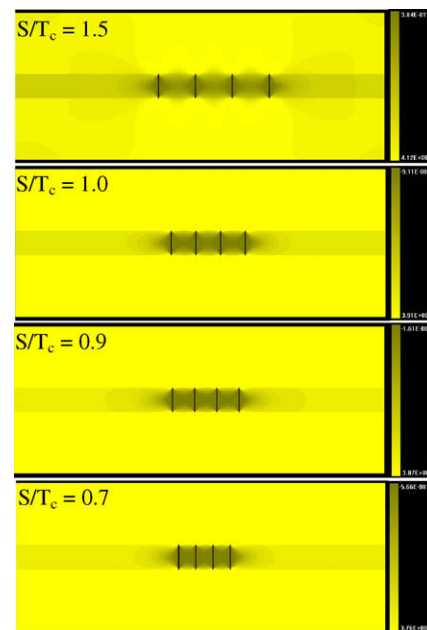


Fig. 2. Distribution of stress σ_x in the models with different fracture spacing to layer thickness ratio S/T_c . (The different grey scales represent different values of stress).

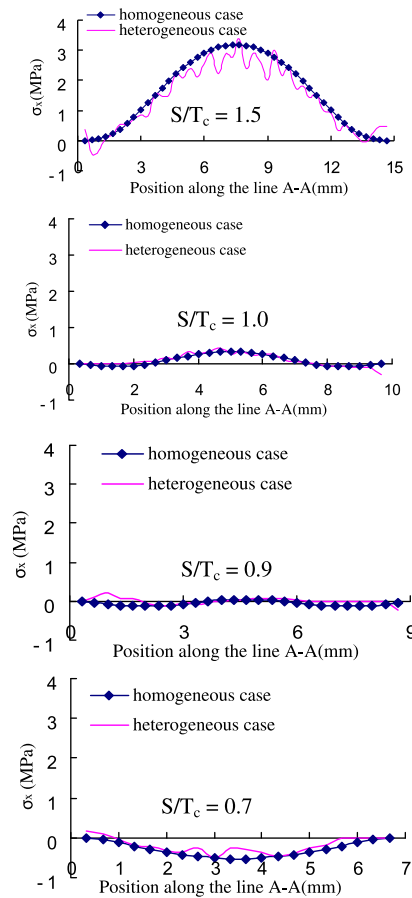


Fig. 3. The stress variation along the line A–A between adjacent fractures as the fracture spacing to layer thickness ratio S/T_c decreases. (Positive sign represents tensile stress and the negative sign represents compressive stress. It indicates that 0.9 is the critical spacing to layer thickness ratio for stress transition from tension to compression).

Therefore the critical spacing to fractured layer thickness ratio obtained in this study is gently lower than 1.0.

Real materials, including both natural and manmade materials, are never homogeneous for it contains different natural ingredients of various scales. Here we assumed the homogenous index of central layer materials is 4.0. As shown in Fig. 3, because of heterogeneity, the horizontal stress (σ_x) along the line A–A is fluctuant and the peak value does not arise accurately at the middle point of the line, which may be more approximate to the stress field in real materials. But it is important that the peak value will arise in certain range around the middle point. Although the distribution of stress σ_x along the line A–A obtained from heterogeneous materials is obviously different from that for homogeneous materials, both of them have the similar varying trends as a function of spacing to layer thickness ratio. The following numerical results regarding fracture pattern for heterogeneous materials show that during the process of fracture infilling the new fracture does not always initiate at the middle point between the earlier formed fractures. Instead, the new fracture initiates at a point where the local element stress reaches its failure strength.

3.3. Modeling of fracturing processes

In the following analysis, we use the same three-layer model without pre-assigned fractures and the homogeneity index m of central layer is chose to be 4.0. The process of fracture saturation is clearly shown in Fig. 4. According to the order of the fracture for-

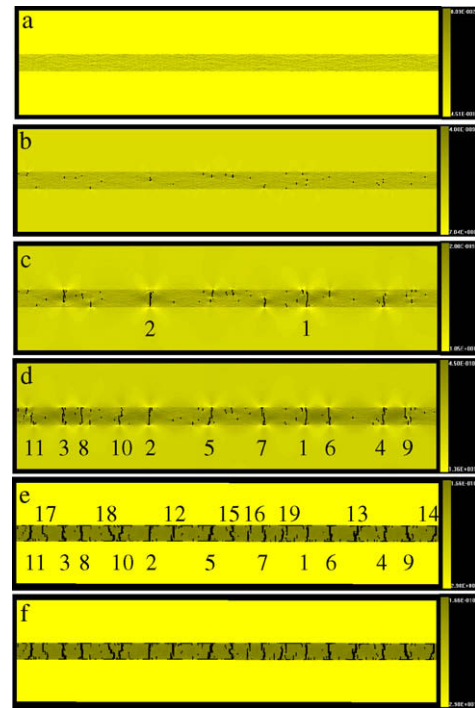


Fig. 4. The process of fracture saturation simulated with RFPA^{2D}. (The number from '1' to '19' indicates the sequence of fracture initiation).

mation, we can divide the whole process into three stages. In the first stage (Fig. 4a and b), fractures form in the central layer at its weak points where the tensile stress reaches its tensile strength, during which the tensile stress is mainly from cooling shrinkage of central layer itself. In this stage, the initiated fractures are isolated and do not interact. The following stage (Fig. 4c–e) is the process of fracture infilling, in which new fractures infill continuously between the earlier formed fractures, and the spacing to fractured layer thickness ratio decreases. In this process, the stress field build-up between the fractures is due to the stress transfer from adjacent layers and the constrain from the adjacent segments in the same layer the stress, for the fractures can not transfer stress when they are opened. In the last stage, the fractures are spaced so closely that no more new fractures can infill, even with decreasing temperature. As shown in Fig. 4f, the infilled fractures become more opened. Furthermore more interface fractures between adjacent layers begin to initiate and propagate. From the results of numerical simulation of the model, we can obtain that the number of fractures keeps 19 in the condition of saturation. As pointed by Wierzbicki that fracture criterion play an important role in the study of the problem of the crack population formation [19]. Our numerical simulation involves the calculation of the stress acting on the elements and the mechanical property change of the damaged elements according to the constitutive laws and strength criterion [17,18]. The statistical distribution of the breakdown thresholds is a material property and is described by Eq. (1). Under a quasi-statically decreasing temperature, the stress of the elements is given by the solution of the FEM. If the stress of an element attains its breakdown strength, the element fails irreversibly, and its elastic constant is changed according to its post-failure law. Iterating the procedure leads to fracture propagation, where fractures are defined by groups and alignments of failed elements.

No localized zone or crack nucleation is observed before the macro-fractures come into being in the central layer, and thus it is very difficult to predict where the nucleation will begin.

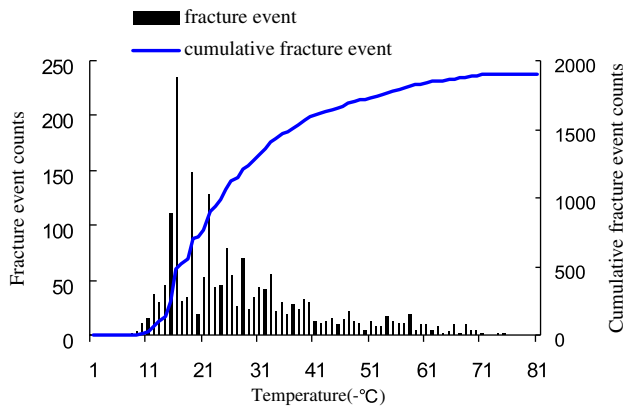


Fig. 5. Numerically obtained fracture event counts during the cooling process.

Immediately after a certain stress is reached, zones of intense fracture event activity are identified, and nucleation sites rapidly evolve into a macro-fracture that cuts across the central layer. Fig. 5 is the temporal distribution of fracture events that occurred during the cooling process. When a new fracture forms, the fracturing process is accompanied by a rapid increase in the fracture events count. While in saturation state, the magnitude of fracture events is also small and the cumulative fracture event counts almost keep constant. This indicates that no more new fractures form in the saturation state.

One of the advantages of the numerical simulation is that detailed information about the stress distribution can be obtained, including data on failure-induced stress redistribution. Fig. 6 gives the tensile stresses variation across section B–B in different cooling stage. The figure shows that, although the stress distribution at the initial loading stage is statistically homogeneous on a macro-scale, it varies on a micro-scale due to the micro-scale heterogeneity of the model. Because of the fracture initiation, propagation, and coalescence, a high-stress concentration is induced. When large fracture zones develop, highly non-uniform stress distributions also develop, especially when the fracture zone is not immediately stress free (for example, at -20°C as shown in the Fig. 6). With the cooling process, stress distribution gradually becomes smooth on a macro-scale, which also indicates that both fractures in the central layer and the interface fractures between the adjacent layers are all well-developed completely.

In order to study the influence of heterogeneity on the fracture patterns, six different values of the homogeneity index, $m = 2.0$,

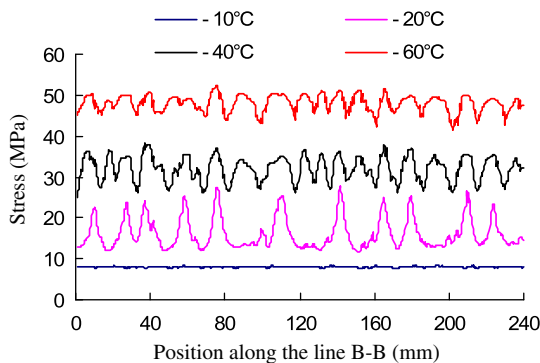


Fig. 6. Stress variation along the line B–B for different cooling stage. (In a lower stress level, the stress fluctuation is mainly influenced by the micro-scale heterogeneity of the model. In a higher stress level, the stress fluctuation is mainly influenced by newly formed fracture propagation. Once the fractures are all well-developed completely, stress distribution gradually becomes uniform and smooth on a macro-scale).

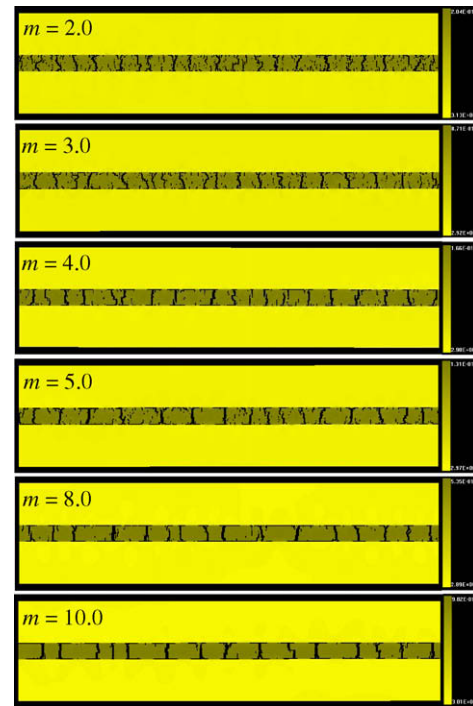


Fig. 7. Fracture pattern at saturation for the six model with different homogeneity index, $m = 2.0, 3.0, 4.0, 5.0, 8.0, 10.0$. (It shows that the number of vertical fractures in central layer decreases and more interface fractures occur with increasing homogeneity index on the whole).

3.0, 4.0, 5.0, 8.0 and 10.0, for the central layer is chosen to setup models representing relatively heterogeneous to relatively homogeneous materials, while keeping other parameters constant.

Fig. 7 shows the final stage of the modeling for all six models after the fractures are saturated. It is found that fractures did not form by crack propagation for highly heterogeneous material model, but rather formed by the coalescence of independent cracks. However the fractures tend to form through the propagation of newly nucleated small cracks in lines more or less perpendicular to the layer. With the influence of heterogeneity of material, new fractures are observed to form at locations that not always in the middle between the existing adjacent fractures. The failure modes are sensitive to the local disorder feature of the central layer material. The fracture path of a homogeneous case is smoother than that of a heterogeneous case.

Although the fracturing pattern is very complex with the influence of heterogeneity of material, the length scale of fracture spacing shows an overall scaling behavior closely related to that in

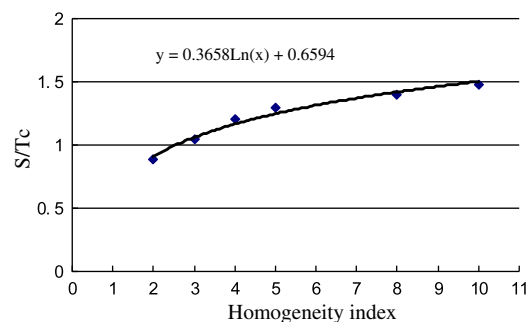


Fig. 8. Numerically obtained critical ratio of fracture spacing to layer thickness as a function of homogeneity index. (It shows that the critical spacing to fractured layer thickness ratio is lower than 1.0 if the interface layer is perfectly bonded).

homogeneous materials. However, quantitatively, difference in length scale of fracture spacing is found between the heterogeneous and homogeneous models. The critical spacing to layer thickness ratio is non-linearly related to the homogeneity index. As shown in Fig. 8, for the case $m = 2.0$, the critical fracture spacing to layer thickness ratio is closed to 0.9, which is consistent with the value provided by stress analysis above. For other cases, the critical fracture spacing to layer thickness ratio are gently higher than 1.0. It is that the interface delamination and through-going fracturing (fracture nucleation and propagation along the interface between layers) reduce the TMS effect between layers. Increasing heterogeneity of the layer increases the variation magnitude of the local stress concentration, which result in more fractures forming in a lower stress level. For a relatively heterogeneous case, the fractures reach the saturation state before interface delamination occurs. In a higher stress level, the interface cracks begin to initiate and delamination begin to occur. The delamination could change the local stress state significantly. One of the most important effects is that the delamination will stop the transition of stress from the neighboring layers to the central layer, and, in turn, prevent further infilling of new fractures between existing fractures in the central layer.

4. Conclusions

A quantitative analysis using numerical method with a three-layer model subjected to thermo-mechanical loading has been conducted to study the stress redistribution and the process of fracture saturation in multi-layered materials, from which we can draw the following conclusions:

- (1) Stress analysis shows that the critical spacing to fractured layer thickness ratio obtained is lower than 1.0 in the multi-layered materials subjected to thermo-mechanical loading because the deformation of layered material is controlled by both the TMS effect and its own cooling shrinkage. And fracture process modeling further validate that the critical spacing to fractured layer thickness ratio obtained is lower than 1.0 if the interface layer is perfectly bonded.
- (2) The heterogeneity of materials and newly formed fractures directly govern the stress redistribution. Although the stress distribution in a relatively lower stress level is statistically homogeneous on a macro-scale, it varies on a micro-scale due to the micro-scale heterogeneity of the model. When large fracture zones develop in a relatively higher stress level, highly non-uniform stress distributions develop, especially when the fracture zone is not immediately stress free. Once the fractures are all well-developed completely, stress distribution gradually becomes uniform and smooth on a macro-scale.
- (3) Heterogeneity is a key factor that influences the fracture pattern in modeling. For highly heterogeneous materials, the path of the infilling fractures is irregular and the location

is isolated. In saturation state, we obtain that the critical spacing to fracture thickness ratios is lower than the magnitude of 1.0. The fracture path of a homogeneous case is smoother than that of a heterogeneous case, and the critical spacing to fracture thickness ratio is gently higher than 1.0. The reason is that the interface delamination and through-going fracturing reduce the TMS effect between layers. The delamination will stop the transition of stress from the neighboring layers to the central layer. As a result the tensile stress between adjacent existing fractures is bigger than zero but it is not big enough to offer the tensile stress for new fracture formation.

Although the simulations here are two-dimensional, the reproduction of these phenomena in a numerical simulation is significant and the phenomenological approach shows in great detail the propagation of the fracture zone and the interaction of the cracks with the structure of multi-layered materials. Numerical simulation provides abundant information on the stress distribution and fracture-induced stress redistribution, which will help us to make further progress in the field of multi-layered materials mechanics.

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