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Adaptive backstepping control and synchronization of a modified and chaotic Van der Pol-Duffing oscillator

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Abstract: In this paper, we propose a backstepping approach for the synchronization and control of modified Van-der Pol Duffing oscillator circuits. The method is such that one controller function that depends essentially on available circuit parameters that is sufficient to drive the two coupled circuits to a synchronized state as well achieve the global stabilization of the system to its regular dynamics. Numerical simulations are given to demonstrate the effectiveness of the technique.

Keywords: Chaos; Synchronization; Back-stepping; Van der Pol-Duffing oscillator

1 Introduction

In 1990, the problem of chaos control [1] and synchronization [2] emerged as two exciting topics in nonlinear science that have promising applications. Pecora and Carroll [2] introduced a method to synchronize two identical chaotic systems evolving from different initial conditions, while Ott et al. [1] presented the OGY control algorithm. Chaos synchronization is closely related to the observer problem in control theory and recent studies deal with the synchronization problem based on control theory approach. Chaos synchronization has been intensively investigated in the context of many specific problems arising from physical [3, 4], chemical and ecological [5], and applications to secure communications [6∼8] to mention a few. Enormous research progress has been made in developing and understanding various types of synchronization schemes, such as adaptive control [9], active-backstepping design [10, 11], active control [12∼14], backstepping [15], and sliding mode control [16].

In most of the methods mentioned above, the controllers and the design approach are often very complex and could be difficult to achieve in practice. Thus, designing simple and available control inputs that can achieve global and stable synchronization of coupled oscillators is generally significant and of practical interest in view of the foreseen applications of chaos synchronization in circuits and lasers. This is an open challenge that has remained unresolved. In this paper, we propose an integrator and self-adaptive backstepping approach to deal with this problem using the modified van der Pol-Duffing oscillator circuit (MVDPD).

In 2005 and 2007, Fotsin et al. [9, 17] proposed both nonadaptive and adaptive approaches to the synchronization problem of the MVDPD oscillator. The controllers proposed by Fotsin et al. [9, 17] contains feedback gains that are not dependent on the system parameters. In the control simulations, the feedback gains were arbitrarily selected to satisfy some stability criteria. This is definitely a limitation for experimental applications. In this paper, we show that 1) the synchronization control input can be very simple relative to the systems being synchronized, and 2) the controller feedback gains could be strictly dependent on the system parameters and thus readily available for measurement. Furthermore, by considering the relationship between synchronization and control theory, we develop a nonlinear control scheme that is capable of driving the otherwise chaotic state to a regular one.

2 Modified Van der Pol-Duffing oscillator circuit

The modified Van der Pol-Duffing oscillator circuit that we study here is described by the following set of normalized nonlinear ordinary differential equations [9, 17]:

$$\begin{cases} \dot{x} = -m(x^3 - \alpha x - y + \mu), \\ \dot{y} = x - y - z, \\ \dot{z} = \beta y - \eta z. \end{cases} \tag{1}$$

The circuit representation of equation (1) is shown in Fig. 1. When the load resistance R_L placed in series with the inductor L shown in Fig. 1 is removed, the circuit reduces to the well-known Van der Pol-Duffing oscillator circuit described by the normalized equations:

$$\begin{cases} \dot{x} = -m(x^3 - \alpha x - y), \\ \dot{y} = x - y - z, \\ \dot{z} = \beta y. \end{cases} \tag{2}$$

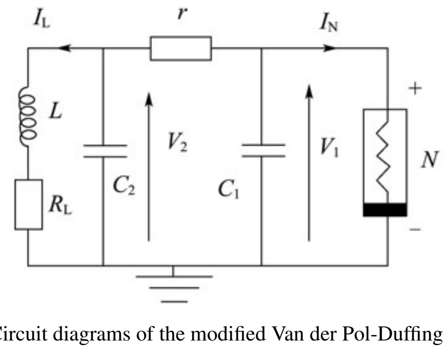

Fig. 1 Circuit diagrams of the modified Van der Pol-Duffing oscillator.

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The parameters m, α, and β are the circuit parameters and are chosen to ensure the chaotic behaviours of equations (1) and (2). μ is the offset term, and η is the parameter arising from the addition of the resistance R_L in the modified circuit. The cubic term in equations (1) and (2) is derivable from nonlinear function of the nonlinear resistor N. x, y, and z correspond to the rescaled form of the voltage across C_1, the voltage across C_2, and the current through L, respectively. System (1) exhibit the one-scroll chaotic attractor shown in Fig. 2 (a) when $\alpha = 0.35$, $\beta = 300$, $\mu = 0.035$, $\eta = 0.2$, and $m = 100$, while system (2) exhibits a double-scroll chaotic attractor when $\alpha = 0.35$, $\beta = 300$, and $m = 300$, as shown in Fig. 2 (b).

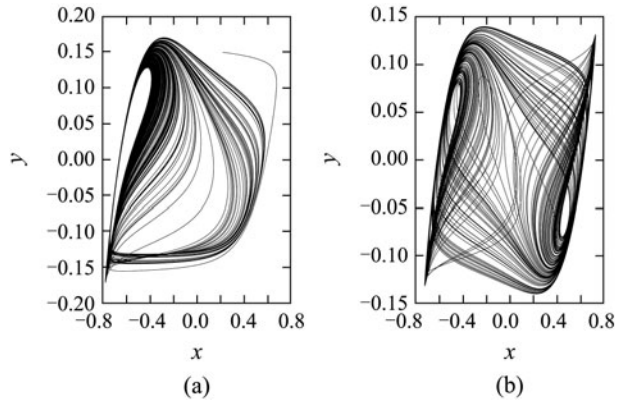


Fig. 2 Phase space of the Van der Pol-Dufing oscillators. (a) Double-scroll attractor of the Van der Pol-Dufing oscillators, (b) One-scroll attractor of the modified Van der Pol-Dufing oscillators with off-set term.

3 Synchronization control design

In the synchronization controller design, it is convenient to transform the modified Van der Pol-Duffing oscillator equation (1) by redefining the variables as follows: $x = x_3, y = x_2$ and $z = x_1$; expressing (1) in a drive-response configuration is given respectively as

$$\begin{cases} \dot{x}_1 = \beta x_2 - \eta x_1, \\ \dot{x}_2 = x_3 - x_2 - x_1, \\ \dot{x}_3 = -m(x_3^3 - \alpha x_3 - x_2 + \mu), \end{cases} \tag{3}$$

for the driver and

$$\begin{cases} \dot{x}'_1 = \beta x'_2 - \eta x'_1, \\ \dot{x}'_2 = x'_3 - x'_2 - x'_1, \\ \dot{x}'_3 = -m(x'^3_3 - \alpha x'_3 - x'_2 + \mu) - U(t), \end{cases} \tag{4}$$

for the responding system, where $U(t)$ is a nonlinear control input. We define the error system as the difference between the signals from drive and the response system as $e_1 = x_1 - x'_1$, $e_2 = x_2 - x'_2$; and $e_3 = x_3 - x'_3$. By considering the time derivative of the error signals together with equations (3) and (4), we obtain the error dynamics system:

$$\begin{cases} \dot{e}_1 = \beta e_2 - \eta e_1, \\ \dot{e}_2 = e_3 - e_2 - e_1, \\ \dot{e}_3 = m(x'^3_3 - x_3^3) + m\alpha e_3 + m e_2 + U(t). \end{cases} \tag{5}$$

With error dynamics (5), the synchronization problem is equivalent to that of realizing asymptotic stability of the zero solution to (5). In the absence of the controller $U(t)$, equation (5) would have an equilibrium at $(0, 0, 0)$, so that if appropriate $U(t)$ is chosen such that the equilibrium $(0, 0, 0)$ is unchanged, then asymptotic stabilization would be realized, and hence, the synchronization between two systems (3) and (4) would be globally stable. To achieve this goal, we employ a self-adaptive procedure that involves three steps, wherein each step would require the stabilization of each subsystem in (5) using appropriately defined Lyapunov function. To start with, let V_1 be a Lyapunov function for the $\dot{e}_1$ subsystem given as

$$V_1 = \frac{1}{2}e_1^2, \tag{6}$$

and then,

$$\dot{V}_1 = e_1\dot{e}_1 = \beta e_1 e_2 - \eta e_1^2. \tag{7}$$

Suppose that e_2 is a virtual control, then to stabilize e_1 subsystem, let $e_{2\mathrm{d}} = -e_1$, where $e_{2\mathrm{d}}$ implies a desired controller for e_2 required to stabilize e_1 subsystem. Thus, we obtain

$$\dot{V}_1 = -(\beta + \eta)e_1^2, \tag{8}$$

which is negative definite. This implies that the subsystem, $\dot{e}_1$ in (5) is fully stabilized. To stabilize the second subsystem, in (5), let the error state between e_2 and $e_{2\mathrm{d}}$ be f_2, i.e.,

$$f_2 = e_2 - e_{2\mathrm{d}} = e_2 + e_1. \tag{9}$$

Introducing another Lyapunov function V_2 given as

$$V_2 = V_1 + \frac{1}{2}f_2^2, \tag{10}$$

and then,

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + f_2\dot{f}_2 \\ &= \dot{V}_1 + f_2[e_3 + (\beta - 1)e_2 - e_1(1 + \eta)]. \end{aligned} \tag{11}$$

Suppose that e_3 is a virtual control in (11), and then, to stabilize the subsystem e_2 and e_1, let

$$e_{3\mathrm{d}} = e_1(1 + \eta) - (\beta - 1)e_2. \tag{12}$$

Thus,

$$\dot{V}_2 = \dot{V}_1 = -(\beta + \eta)e_1^2, \tag{13}$$

which is negative definite. Let the error state between e_3 and $e_{3\mathrm{d}}$ be f_3, that is,

$$f_3 = e_3 - e_{3\mathrm{d}}. \tag{14}$$

Introducing the third Lyapunov function V_3 and its time derivative expressed as

$$\begin{cases} V_3 = V_2 + \dfrac{1}{2}f_3^2, \\ \dot{V}_3 = \dot{V}_2 + f_3\dot{f}_3. \end{cases} \tag{15}$$

$$\begin{aligned} \dot{V}_3 = \dot{V}_2 + f_3[&m(e_3 + \alpha e_3 + x'^3_3 - x_3^3) + U(t) \\ &- (\beta e_2 - \eta e_1)(1+\eta) + (\beta - 1)(e_3 - e_2 - e_1)]. \end{aligned} \tag{16}$$

If the controller $U(t)$ is chosen such that

$$U(t) = k_0 + k_1 e_1 + k_2 e_2 + k_3 e_3, \tag{17}$$

where the feedback gains are defined by

$$\begin{cases} k_0 = m(x_3^3 - x'^3_3), \\ k_1 = (\beta - 1) - \eta(1 + \eta), \\ k_2 = (\beta - 1) + (1 + \eta)\beta - m, \\ k_3 = (1 - \beta) - m\alpha, \end{cases} \tag{18}$$

then,

$$\dot{V}_3 = -(\beta + \eta)e_1^2 \tag{19}$$

is negative definite, so that the two systems (3) and (4) are globally asymptotically synchronized. Note that the linear feedback gains (k_1, k_2, k_3) in controller (17) are essentially dependent on the circuit parameters and thus can be determined directly without rigorous numerical simulations. On the other hand, the nonlinear component of $U(t)$, namely, k_0 depends on the nonlinear resistor (N) in Fig. 1, which is also available, implying that our design control can be experimentally realized. Significantly, the nonlinearity of k_0 imposes some advantages on the control input (17) because in practical physical systems, nonlinearities arise due to physical limitations, and their presence usually causes serious degradation of system performance and decrease in speed of response time. In some cases, they might cause chaotic perturbation to original regular behaviour if the controller is not well designed. This implies that the effect of nonlinearity cannot be completely ignored in the design and analysis of control inputs.

4 Stabilization control design

The goal of this section is to design a control input based on the backstepping scheme presented in the previous section that would stabilize the MVDPD oscillator unto its periodic orbit. To achieve this goal, we first reexpress the variables in equation (1) such that, $x = z$, $y = y$, $z = x$, and $\mu = 0$. Thus, the controlled MVDPD oscillator is rewritten as

$$\begin{cases} \dot{x} = \beta y - \eta x, \\ \dot{y} = z - y - x, \\ \dot{z} = -m(z^3 - \alpha z - y) + u(t), \end{cases} \tag{20}$$

where $u(t)$ is a control function to be determined, and the offset term has been neglected. Considering the first equation in (20) and assume that y is a virtual control, we design a stabilizing function $\alpha_1(x)$ to make the time-derivative of the Lyapunov function

$$V_1(x) = \frac{x^2}{2} \tag{21}$$

negative definite. Suppose that $\alpha_1(x) = -x$, then,

$$\dot{V}_1(x) = -(\eta + \beta)x^2. \tag{22}$$

Let $\bar{y} = y - \alpha_1(x)$. Then, we obtain the $(x, \bar{y})$ subsystem

$$\begin{cases} \dot{x} = \beta\bar{y} + (\beta - \eta)x, \\ \dot{\bar{y}} = z + (\beta - 1)\bar{y} + [(\beta - 1) - (\eta + 1)]\,x. \end{cases} \tag{23}$$

To stabilize the $(x, \bar{y})$ subsystem (23), we design a stabilizing function $\alpha_2(x, \bar{y})$ for the virtual controlled variable z. By considering the following Lyapunov function for (23) defined as

$$V_2(x, \bar{y}) = V_1(x) + \frac{1}{2}\bar{y}^2, \tag{24}$$

and its corresponding time-derivative given by

$$\begin{aligned} \dot{V}_2(x, \bar{y}) = {} & [z + (\beta - 1)\bar{y} + [(\beta - 1) - (\eta + 1)]\,x] \\ & -(\beta + \eta)x^2, \end{aligned} \tag{25}$$

and if we assume that

$$\alpha_2(x, \bar{y}) = [(\eta + 1) - (\beta - 1)]\,x - (\beta - 1)\bar{y},$$

then we can make $\dot{V}_2(x, \bar{y})$ negative definite:

$$\dot{V}_2(x, \bar{y}) = \dot{V}_1(x) = -(\beta + \eta)x^2. \tag{26}$$

Suppose that the error state for the virtual control input $\alpha_2(x, \bar{y})$ is $\bar{z} = z - \alpha_2(x, \bar{y})$; then, we stabilize the full $(x, \bar{y}, \bar{z})$ system given by

$$\begin{cases} \dot{x} = \beta\bar{y} + (\beta - \eta)x, \\ \dot{\bar{y}} = z + (\beta - 1)\bar{y} + \beta_\eta x, \\ \dot{\bar{z}} = -m\{z^3 - \alpha\bar{z} + (\beta_\eta\alpha - 1)x \\ \qquad +[\alpha(\beta - 1) - 1]\bar{y}\} + u(t), \end{cases} \tag{27}$$

where $\beta_\eta = [(\beta - 1) - (\eta + 1)]$. Note that in (27), we can leave the cubic product z^3, which can also be expressed in terms of the variables $x, \bar{y}, \bar{z}$. Let the Lyapunov function for the full system (27) be given by

$$V_3(x, \bar{y}, \bar{z}) = V_2(x, \bar{y}) + \frac{1}{2}\bar{z}^2. \tag{28}$$

Then, its time derivative

$$\dot{V}_3(x, \bar{y}, \bar{z}) = \dot{V}_2(x, \bar{y}) + \bar{z}\dot{\bar{z}}$$

is negative definite if $\dot{\bar{z}} = 0$. This condition is fully satisfied if we choose the control input $u(t)$ as follows:

$$u(t) = k_0 + k_1 x + k_2 y + k_3 z, \tag{29}$$

where

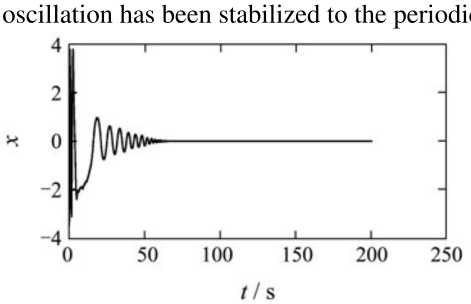
$$\begin{cases} k_0 = mz^3, \\ k_1 = (\beta - 1) - \eta(1 + \eta), \\ k_2 = (\beta - 1) + \beta(1 + \eta) - m, \\ k_3 = (1 - \beta - m\alpha). \end{cases} \tag{30}$$

Note that the control and the parameters in the controller (29) have the same form as that of equation (17). Whereas the feedback in (17) is provided by the error states, the feedback in (29) is provided by the state variables.

5 Numerical simulations

In what follows, we now present numerical simulation results to verify the effectiveness of controllers (17) and (29). In all cases, we select the circuit parameters ($m = 100, \alpha = 0.35, \beta = 300, \eta = 0.2, \mu = 0.035$) such that the chaotic state in Fig. 2 (a) is maintained and choose the following initial conditions for the drive-response system $x_1 = 0.72$, $y_1 = 0.04$, $z_1 = -0.592$, $x_2 = -0.85$, $y_2 = -0.05$, and $z_2 = -0.585$. Moreover, from (18), using the above circuit parameters, we have $(k_1, k_2, k_3) = (298.76, 559, -334)$. In Fig. 3, we illustrate the performance of the controller given by equation (17) when activated at $t \geqslant 30$. The error dynamics is found to convergence to the zero solution as $t \to \infty$, implying that the synchronization between systems (3) and (4) has been achieved.

Turning to the problem of stabilizing the dynamics to periodic solution, we activate controller (29) on the system (20). In Fig. 4, we illustrate the case in which the control is activated at $t \geqslant 30$. Clearly, we see that the otherwise chaotic oscillation has been stabilized to the periodic orbit.



(a)

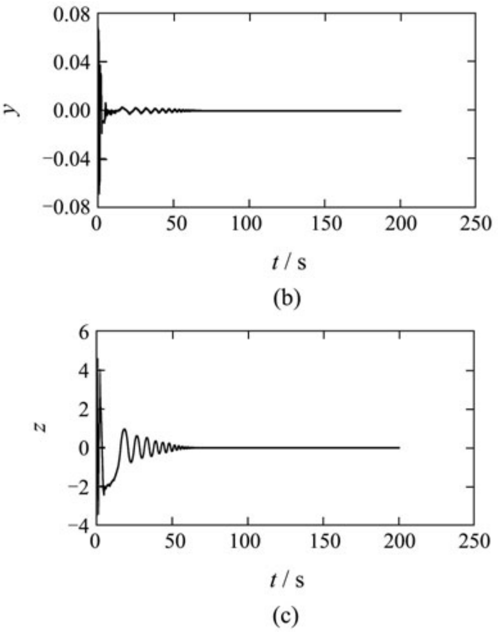


Fig. 3 Error dynamics for the modified Van-der Pol Duffing oscillators when the control has been activated at $t \geqslant 0$.

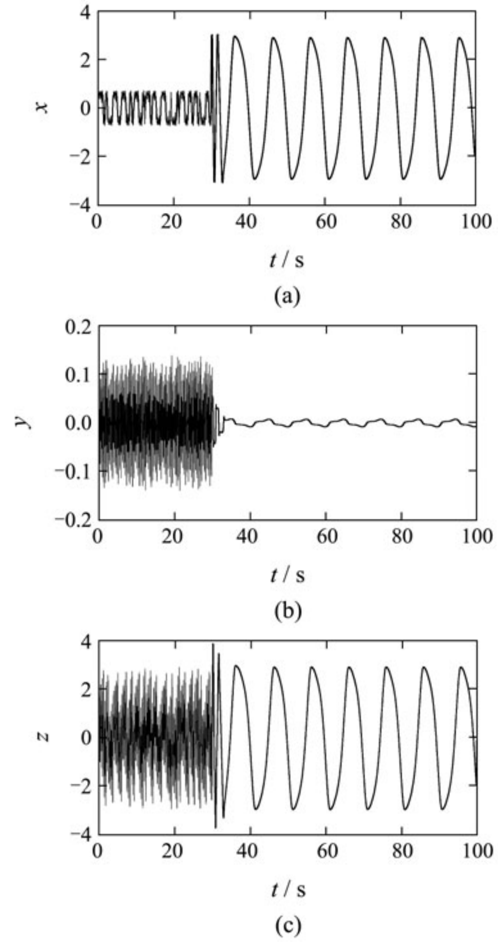


Fig. 4 Stabilization of the state space to periodic solutions. Controller (19) is activated at $t \geqslant 30$.

6 Conclusions

In this paper, we have presented a simple self-adaptive synchronization technique that ensures global synchronization of identical chaotic systems consisting of a driven-response system of modified Van der Pol-Duffing circuit oscillators. The method was also employed to drive the chaotic system to its regular dynamics. The proposed controller is essentially dependent on the measurable circuit parameters. Numerical simulations have been employed to verify the effectiveness of the proposed controller. The experimental realization of our proposed scheme remains to be presented, and we hope to report on this in the nearest future.

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