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Comparative modeling of stand development in Scots pine dominated forests in Estonia

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Abstract

In general, forests in Estonia are characterized by great variability, not only in protected areas but in commercial forests as well. The data needed for the derivation and calibration of growth models can be obtained by continuous observation of permanent growth plots (also known as longitudinal studies) or by establishing chronosequences with temporary plots distributed over a wide range of growing sites, densities and ages (also known as cross-sectional studies). A compromise may be achieved by a system of ‘‘interval plots’’ (also known as a short-time series: series which covers a short time). Since the measurement interval is a period of undisturbed growth, it is possible to measure change rates as in a longitudinal study and at the same time cover a wide range of initial conditions as in a cross-sectional study. Numerous models of stand growth have been derived from re-measured sample plots. This study, which uses the data of 142 five-year intervals from 134 unmanaged Scots pine stands, compares six different model combinations involving algebraic difference equations and fixed time-step increment equations. New stand-level diameter and basal area increment equations and a tree survival model which showed close correspondence with the existing stand-level model for Estonia were developed. The main advantage of the use of algebraic difference equations over the fixed-step increment equations is the ability to use flexible time steps. However, the projection intervals should not deviate too much from the time steps of the measurement data. An important constraint when using the algebraic difference equations is to avoid long-term predictions in one projection sequence.

Keywords: Forest management planning; Short-term growth modeling; Interval plots; Algebraic difference equation; Fixed-step increment equation

1. Introduction

In many regions of the world, people depend on forests for their livelihood and well-being. Forests represent an important renewable reservoir of raw materials for the wood processing industry and a remnant wilderness of high recreational and spiritual value in urbanized societies. To meet the demands of society, foresters have been developing silvicultural treatment schedules which are assumed to be optimal for a given set of site and market conditions. Changes in human populations, cultures and attitudes can rapidly shift the effect of human intervention on the natural processes. These shifts alter the patterns of anthropogenic disturbance and lead to changes in patterns of natural disturbance which became apparent many decades later (Oliver and Larson, 1996).

Forest dynamics is affected by many processes at different levels of ecosystem regulation. The growth and change in the number of trees belonging to various age and size classes has been a classic approach to describing forest dynamics. However, the long-term processes include great variation in the factors affecting the dynamic patterns of forest development. The patterns of change shows considerable variation on the temporal and spatial scale: a great number of trees can be removed from a stand within a short period (natural disturbances, cuttings) or the number of trees may decrease gradually by natural mortality or gap formation (Kangur et al., 2005).

There has been considerable debate about empirical modeling of stand growth and yield processes. The purpose of using a growth model is to make reasonable predictions about tree growth and stand development, which may be achieved in different ways and at varying levels of detail, depending on the data available about the trees and the growing site. Models based on stand variables have been used for more

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Nomenclature

$c_1, \ldots, c_4$	regression coefficients
D_t, D_{t_1}, D_{t_2}	stand quadratic mean diameter (cm) at the age of t, t_1 and t_2, respectively
D_{t+5}	stand quadratic mean diameter for the next 5-year period (cm)
ΔD_{t+5}	5-year stand quadratic mean diameter increment (cm per 5-year period)
G_t, G_{t_1}, G_{t_2}	stand basal area (m^2/ha) at age t, t_1, and t_2, respectively
G_{t+5}	stand basal area (m^2/ha) at the end of the 5-year period
ΔG_{t+5}	stand basal area increment in 5-year periods (m^2/ha per 5 years)
H_t, H_{t_1},H_{t_2}	stand mean height (m) at the age of t, t_1 and t_2, respectively
H_{100}	site index (stand mean height (m) at the age of 100 years)
HF	form height (m)
L_t	stand sparsity (average distance between trees (m)) at age t
L_{t+5}	stand sparsity at the end of the 5-year period (m)
ΔL_{t+5}	5-year stand sparsity change (m)
N_t, N_{t_1}, N_{t_2}	stand density (the number of trees per hectare) at age t, t_1, and t_2, respectively
N_{t+5}	stand density (the number of trees per hectare) at the end of the 5-year prediction
OHOR	thickness of soil organic layer (cm)
P_{t+5}	tree survival probability after the 5-year period
R	correlation coefficient
RMSE	root mean square error
RD$_t$	degree of stocking at age t
V_t, V_{t_1}, V_{t_2}	stand volume (m^3) at the age t, t_1 and t_2, respectively
Δy	observed change
$\Delta \hat{y}$	predicted change

Greek letter

ε	error component

practical purposes. Single-tree modeling leads to greater flexibility in attempting to use details of spacing (Hägglund, 1981; Söderberg, 1986; Nabuurs and Päivinen, 1996; Hynynen et al., 2002; Pretzsch et al., 2002; Hasenauer, 2005).

Since most growth and yield models are developed on the basis of existing empirical data, the most appropriate modeling technique is determined by the level of detail of the available data and the level of resolution of the projection. The data needed for development and calibration of growth models can be obtained by continuous observation of permanent growth plots (also known as longitudinal studies) or by establishing chronosequences with temporary plots distributed over a wide range of growing sites, densities and ages (also known as cross-sectional studies). A compromise may be achieved by a system of "interval plots" (also known as a short-time series: series which covers a short-time). Since the measurement interval is a period of undisturbed growth, it is possible to measure change rates as in a longitudinal study and at the same time cover a wide range of initial conditions as in a cross-sectional study (Glenn-Lewin and van der Maarel, 1992; Gadow and Hui, 1999).

Forest growth and yield tables have traditionally been used in Estonia to offer predictions of forest growth for decision-making in forest management planning. In general, the forests in Estonia are characterized by wide variability in tree species composition and stand structure, both in protected areas and commercial forest (Pärt et al., 2006). The great natural diversity of our forests in combination with the growing role of environmental and socio-cultural values in forest management planning has created a situation in which traditional growth and yield tables do not meet these new requirements (Nilson, 1996, 1999).

A considerable number of growth and yield tables (Krigul, 1969; Kiviste, 1988), several stand growth equation systems (Tappo, 1982; Kiviste, 1999a,b) and some single-tree models (Jõgiste, 1998) have already been developed for stand growth prediction in Estonia. Most of those models are available for public use and have been incorporated into the Database of Forest Management Models (ForMod) which provides open access through an internet-based information system (Sims, 2003, 2005). In principle, the existing growth and yield tables and growth equation systems in Estonia have been created for long-term general growth and yield prediction for practical forest management planning at landscape level.

The concept of adaptive forest management planning has been elaborated in Estonia by Nilson (1996). The idea of this method is adaptive planning of the cutting age for every stand depending on its individual characteristics. The full exploitation of this method requires more detailed growth model systems for stand-level modeling than we have today. An Estonian network of permanent forest growth plots for modeling stand variables and construction of stand growth simulators, which require individual tree growth measurement series, was established in 1995 (Kiviste and Hordo, 2003).

For predicting the main stand variables (height, diameter, density, basal area and volume) requires a complete set of models including both growth and static models. One of the objectives of the current study was the development of short-term stand growth models based on interval plot data for analyzing the performance of various sets of growth models to predict Scots pine dominated stand development during a period of undisturbed growth. The individual model components may be combined in many ways to predict the growth of previously disturbed stands as a whole using the changed initial state of stands functioning as growth predictors as the indirect indicators of previous disturbances. We might thus expect to see great differences when using different types of model with different combinations of predictor variables. Accordingly, the second objective of this study was to analyze various simulation combinations to find the best set of model components for describing short-term Scotch pine stand growth.

2. Material and methods

This section introduces the Estonian network of permanent forest growth and yield plots. We also describe the type of growth model used.

2.1. Interval measurement data

For the study of growth and yield of Estonian forests, we used 5-year interval measurement data provided by the Estonian network of permanent forest growth and yield monitoring plots This network was established in 1995 and was designed using experience of Finnish studies (Gustavsen et al., 1988) to provide empirical data for developing forest growth and yield models (Kiviste and Hordo, 2003). The 679 permanent growth and yield monitoring plots were distributed randomly in 2–10 plots clusters over the entire land surface of Estonia, mainly following the grid of ICP Forest level I monitoring plots (Karoles et al., 2000). The spatial distribution of the plot cluster locations appears in Fig. 1.

The network of permanent plots covers the main forest types and the age range of typical commercial forests in Estonia, re-measurements on the permanent plots being carried out at 5-year intervals. The plots are of circular shape with varying radius, containing at least 100 upper storey trees. The polar coordinates and breast height diameters of all trees are assessed on each plot. In addition, the total tree height and crown length of selected sample trees are measured (Kiviste and Hordo, 2003). The dataset from the re-measurements of 679 plots includes almost 190,000 single-tree measurements.

For this study, Scots pine dominated plots were selected from the database. The stand was considered as pine dominated if the proportion of pine volume exceeded 50%. Since most of the plots selected were pure pine stands (93% of trees on selected plots being pines) all other tree species in the main storey included in model development and for stand mean variables (*H*, *D*, *N*, *G*, *V*) calculation were considered as Scots pine. The 5-year changes in stand height (*H*), quadratic mean diameter (*D*), density (*N*), basal area (*G*) and volume (*V*), at various measurement intervals appear in Fig. 2.

The current study used 142 growth intervals from 134 unmanaged Scots pine dominated plots (Table 1), the average size being 0.1456 ha. Since no silvicultural treatments took place in these plots during the monitoring period, the data represents the stand growth over a 5-year growth interval, undisturbed by forest management. Unfortunately, since there are no reliable records available about the previous management in these plots, we applied a modeling technique "without memory" in which the prediction is a function of the initial system state.

2.2. Model components

Several model components are presented in this section, including difference models based on Estonian forest inventory data and an alternative set of growth models based on interval plot data.

2.2.1. Difference models based on Estonian forest inventory data

The algebraic difference models of Kiviste (1999a,b) are being employed as general growth and yield prediction functions in practical forest management planning in Estonia. These models were developed from Cieszewski and Bella type stand growth equations (1989). The model parameters were estimated using the data of the state forest inventory in Estonia in 1984–1993 (Kiviste, 1995, 1997). The average height, quadratic mean diameter at breast height, and volume of

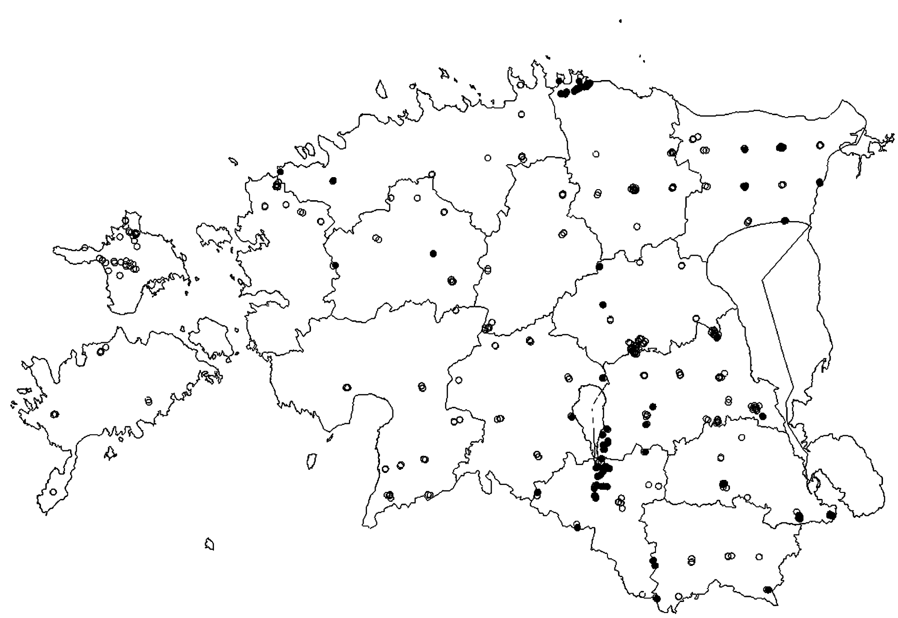

Fig. 1. Geographic location and spatial distribution of Estonian network of permanent forest growth and yield monitoring plots. Each circle on the map presents a cluster of 2–10 sample plots. On the map the circles represent all monitoring areas and the filled dots Scots pine dominated sample plots used in this study.

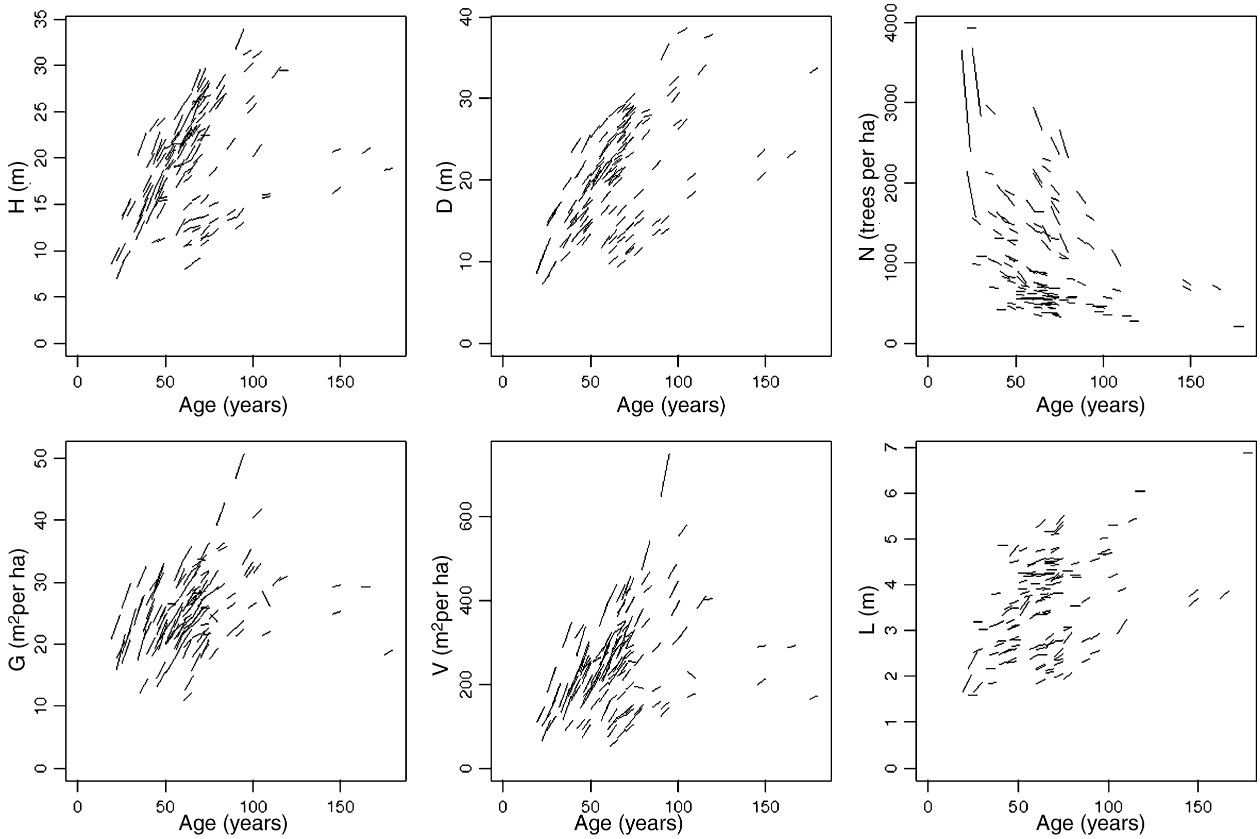

Fig. 2. Change in height (H), diameter (D), density (N), basal area (G) and volume (V) over age and LD relationship in Scots pine dominated stands on the Estonian network of permanent forest growth and yield monitoring plots.

423,919 stands were grouped by forest site type, dominant tree species, stand origin (naturally regenerated or cultivated), and stand age-class (using 5-year intervals). This grouping produced a total of 171 age-series of height, diameter, and volume. Data from young stands (under 20 years for coniferous and hardwood, and 10 years for deciduous forests), over-mature stands and outliers were excluded before the calculation.

Kiviste's difference models were developed from stands in which both natural and anthropogenic disturbances were included. The presumed maximum stand age in these models was fitted with the optimal rotation period of the dominant tree species. In pine-dominated stands this age is 120 years. Future stand mean height (H_{t_2}) at the desired age (t_2) was calculated from the initial age (t_1) and height (H_{t_1}) as follows:

$$H_{t_2} = \frac{H_{t_1} + \mathrm{d}H + \mathrm{r}H}{2 + 4\beta H(t_2{}^{-1.58})/H_{t_1} - \mathrm{d}H + \mathrm{r}H} \tag{1}$$

where $\beta H = 8319 - 493\ \ln(\mathrm{OHOR} + 1)$, $\mathrm{d}H = \beta H/50^{1.58}$ and $\mathrm{r}H = \sqrt{(H_{t_1} - \mathrm{d}H)^2 + 4\beta H H_{t_1}/t_1{}^{1.58}}$.

The stand quadratic mean diameter (D_{t_2}) at the desired age (t_2) was calculated as follows:

$$D_{t_2} = \frac{D_{t_1} + \mathrm{d}D + \mathrm{r}D}{2 + 4\beta H(t_2{}^{-1.33})/D_{t_1} - \mathrm{d}D + \mathrm{r}D} \tag{2}$$

where $\beta D = 6051 - 306\ \ln(\mathrm{OHOR} + 1)$, $\mathrm{d}D = \beta D/50^{1.33}$, $\mathrm{r}D = \sqrt{(D_{t_1} - \mathrm{d}D)^2 + 4\beta D D_{t_1}/t_1{}^{1.33}}$.

The stand volume (V_{t_2}) at the desired age (t_2) was calculated as follows:

$$V_{t_2} = \frac{V_{t_1} + \mathrm{d}V + \mathrm{r}V}{2 + 4\beta H(t_2^{-1.93})/V_{t_1} - \mathrm{d}V + \mathrm{r}V} \tag{3}$$

Table 1
Summary statistics of 142 growth intervals of unmanaged Scots pine dominated monitoring plots

Variable	Minimum	0.25 quantile	Mean	0.75 quantile	Maximum
Stand age (years)	19.0	50.0	63.5	70.0	175.0
Basal area (m^2/ha)	10.8	20.8	24.5	28.4	46.7
Quadratic mean diameter (cm)	7.2	14.4	19.9	24.7	38.0
Stand density (stems/ha)	212.2	551.3	1039.0	1387.0	3930.0
Stand height (m)	6.9	13.9	19.0	22.9	31.7
Stand volume (m^3/ha)	53.0	157.3	231.0	284.8	647.0

where $\beta V = 380,540 - 54,348 \ln(\mathrm{OHOR} + 1)$, $\mathrm{d}V = \beta V / 50^{1.93}$ and $\mathrm{r}V = \sqrt{(V_{t_1} - \mathrm{d}V)^2 + 4\,\beta V V_{t_1} / t_1{}^{1.93}}$.

2.2.2. Growth models based on interval plot data

The predictor variables (quadratic mean diameter, stand density, basal area and sparsity) should be selected as closely as possible to the originally measured variables for reducing the error propagation, collinearity and variance inflation generated during derivation. The model forms should be selected according to the principles of model simplicity (i.e., parameter parsimony) (Burkhart, 2003) and biological realism (Gadow, 1996; Schmidt et al., 2006). The following equations were selected to describe the change in stand variables (growth and survival). The following model was used for stand basal area growth:

$$\Delta G_{t+5} = c_1\,\mathrm{e}^{-c_2 D_t} + c_3 H_{100} + c_4 G_t + \varepsilon \tag{4}$$

where ΔG_{t+5} is the stand basal area increment in a 5-year period (m^2/ha per 5 years), D_t the stand quadratic mean diameter (cm) at the beginning of the 5-year period, H_{100} the site index (stand mean height (m) at the age of 100 years), G_t the stand basal area (m^2/ha) at the beginning of the 5-year period, $c_1, \ldots, c_4$ the regression coefficients and ε is the error component. Stand basal area at the end of the 5-year period (G_{t+5}) can be calculated as follows:

$$G_{t+5} = G_t + \Delta G_{t+5} \tag{5}$$

The following regression equation was applied to estimate the stand quadratic mean diameter increment ΔD_{t+5}:

$$\Delta D_{t+5} = c_1 + c_2 D_t + c_3 H_{100} + c_4 G_t + \varepsilon \tag{6}$$

Stand quadratic mean diameter at the end of the 5-year period (D_{t+5}) can be calculated as follows:

$$D_{t+5} = D_t + \Delta D_{t+5} \tag{7}$$

A classic approach to predicting stand density at the end of a 5-year period is to estimate the probability of tree survival (P_{t+5}) during the prediction interval (Vanclay, 1994). The logistic equation with logit-transformation was used for this purpose:

$$P_{t+5} = \frac{\mathrm{e}^x}{1 + \mathrm{e}^x} \tag{8}$$

with $x = c_1 + c_2\,\mathrm{RD}_t + c_3 D_t + c_4 H_{100} + \varepsilon$, where RD_t is degree of stocking at age t. The number of trees per hectare at the end of the period (N_{t+5}) is calculated as follows:

$$N_{t+5} = N_t P_{t+5} \tag{9}$$

Hart (1928) proposed calculating the average distance between the trees in a forest with N stems per hectare as the square root of the growing space $L = \sqrt{10,000/N}$. This approach assumes regular spacing of trees. The variable L is known as the sparsity of a stand (average distance between trees (m) at time t). The linear dependence between stand mean diameter and stand sparsity has been shown by earlier studies (Nilson, 1973, 2005). Following Nilson, we fitted the separate tree distance based regression equation for estimating the development of variable L:

$$\Delta L_{t+5} = c_1 + c_2 D_t + c_3 H_{100} + G_t + \varepsilon \tag{10}$$

where ΔL_{t+5} is the 5-year stand sparsity change. Stand sparsity at the end of the 5-year period (L_{t+5}) can be calculated as follows:

$$L_{t+5} = L_t + \Delta L_{t+5} \tag{11}$$

The difference equation for tree survival published by Gurjanov et al. (2000) was used:

$$N_{t_2} = 1000 \times \left[\left(\frac{N_{t_1}}{1000}\right)^{c_1} + c_2\left(H_{t_2}^{c_3} - H_{t_1}^{c_3}\right)\right]^{1/c_1} + \varepsilon \tag{12}$$

where N_{t_1} is the number of trees per hectare at the beginning of the prediction period and N_{t_2} is the number of trees per hectare at the end of the prediction period.

For describing basal area growth, the difference equation presented by Gadow and Hui (1999) was applied:

$$G_{t_2} = G_{t_1} N_{t_2}^{1 - c_1 H_{t_2}^{c_2}} N_{t_1}^{c_1 H_{t_1}^{c_2} - 1} \left(\frac{H_{t_2}}{H_{t_1}}\right)^{c_3} + \varepsilon \tag{13}$$

where G_{t_2} is stand basal area (m^2/ha) at the end of the prediction period.

For comparison of model fit, we calculated the predicted change ($\Delta\hat{y} = \hat{y}_2 - y_1$) and observed change ($\Delta y = y_2 - y_1$) during the period for every model. The correlation coefficient (R) and root mean square error (RMSE) between predicted $\Delta\hat{y}$ and observed Δy were calculated to analyze the model residuals.

2.3. Designing simulation combinations

Six different simulation combinations were used to analyze model predictions of five important stand variables (height, quadratic mean diameter, density, basal area and volume) at the end of a 5-year prediction period. Table 2 shows the calculations of projected stand variables in these combinations. In simulation combinations, some stand variables were calculated using growth models (Eqs. (1)–(13)) while other variables were calculated using static models (Eqs. (14)–(16)). The calculations of stand variables differ in simulation combinations in the use of different formulas or different calculation sequences. The simulations were carried out on the data on 142 intervals from the Estonian network of permanent growth and yield sample plots. Stand volume, form height and degree of stocking in the simulations were calculated according to the Estonian forestry inventory practice:

$$V_t = \mathrm{HF}_t\, G_t \tag{14}$$

$$\mathrm{HF}_t = H_t \times \left(-0.0309 + \frac{2.5936}{H_t} + -0.0617\sqrt{H_t} + 0.2107\ln(H_t) \right) \tag{15}$$

Table 2
The sequence of modeling steps and equations used in simulation combinations showing the sequence of calculation of projected variables with corresponding formula or formula reference of each simulation

Simulation combination	Sequence of model components in the six simulation combinations				
	1	2	3	4	5
$D \times G$	H_{t_2} = Eq. (1)	D_{t_2} = Eq. (7)	G_{t_2} = Eq. (5)	$N_{t_2} = \frac{40{,}000}{\pi}\frac{G_{t_2}}{D_{t_2}^2}$	V_{t_2} = Eq. (14)
$D \times L$	H_{t_2} = Eq. (1)	D_{t_2} = Eq. (7)	$N_{t_2} = \left(\frac{100}{\text{Eq. (11)}}\right)^2$	$G_{t_2} = \frac{\pi N_{t_2} D_{t_2}^2}{40{,}000}$	V_{t_2} = Eq. (14)
$D \times N$	H_{t_2} = Eq. (1)	D_{t_2} = Eq. (7)	N_{t_2} = Eq. (9)	$G_{t_2} = \frac{\pi N_{t_2} D_{t_2}^2}{40{,}000}$	V_{t_2} = Eq. (14)
$G \times N$	H_{t_2} = Eq. (1)	G_{t_2} = Eq. (5)	N_{t_2} = Eq. (9)	$D_{t_2} = \sqrt{\frac{40{,}000}{\pi}\frac{G_{t_2}}{N_{t_2}}}$	V_{t_2} = Eq. (14)
Dif	H_{t_2} = Eq. (1)	N_{t_2} = Eq. (12)	G_{t_2} = Eq. (13)	$D_{t_2} = \sqrt{\frac{40{,}000}{\pi}\frac{G_{t_2}}{N_{t_2}}}$	V_{t_2} = Eq. (14)
Est	H_{t_2} = Eq. (1)	V_{t_2} = Eq. (3)	D_{t_2} = Eq. (2)	$G_{t_2} = \frac{V_{t_2}}{\text{Eq. (15)}}$	$N_{t_2} = \frac{40{,}000}{\pi}\frac{G_{t_2}}{D_{t_2}^2}$

$$\mathrm{RD}_t = \frac{V_t}{-30.5946 + 16.6305 H_t + 0.0254 H_t^2} \tag{16}$$

The root mean square errors (RMSE) were calculated for each stand variables in all simulation combinations.

3. Results and discussion

The growth and yield models routinely used in decision support systems for forest management planning in general lack sensitivity to the interactions of successional dynamics over various ages. They are stand development curves fitted to observed stand growth data describing the net production of a stand of trees. The trend in forest management planning towards ecosystem-based forest management principles has created a need to apply more sophisticated decision support systems. Several more detailed modeling approaches, such as individual tree models using competition indices, hybrid and process based models have already been developed and applied. One way of strengthening the traditional growth and yield modeling approach is to construct compatible tree and distribution models in addition to stand-level models (Richardson et al., 2006).

3.1. Model parameter estimates

Table 3 shows the parameter estimates for the growth models both developed and calibrated from the data on 142 intervals of unmanaged interval plots dominated by Scots pine. The fit statistics R and RMSE of these models are not comparable in the case of different dependent variables, but different equations for the same dependent variable are comparable (e.g., Eq. (4) with Eq. (13) for basal area prediction and Eq. (8) with Eq. (12) for survival prediction). The fit statistics (R = 0.655 and RMSE = 0.952) of the basal area growth model (Eq. (4)) for 5-year growth projection show better results than those for the basal area difference model (Eq. (13)). Similarly, the fit statistics (R = 0.530 and RMSE = 0.045) of the survival probability model (Eq. (8)) perform better than those of the survival difference model (Eq. (12)).

An important aspect of increment functions is that because the actual increment rates are estimated directly from the observed data, the functions are based on a limited set of independent variables (Hasenauer, 2005), but this also restricts prediction of stand variables for a given time interval (usually 5 years, depending on the calibration data measuring interval). The prediction of stand development using algebraic difference

Table 3
Parameter estimates and fit statistics for growth models based on unmanaged Scots pine interval plot data

Model	Parameter estimates				R	RMSE
	c_1	c_2	c_3	c_4		
Basal area growth model (Eq. (4))	12.0422	0.1712	0.1039	−0.0506	0.655	0.952
Diameter growth model (Eq. (6))	1.1909	−0.0256	0.0403	−0.0198	0.585	0.336
Survival probability model (Eq. (8))	3.7012	−0.0210	−2.5612	0.0997	0.530	0.045
Stand sparsity model (Eq. (10))	0.1103	−0.0033	−0.0004	0.0017	0.246	0.075
Gurjanov et al. survival diff. model (Eq. (12))	−2.1023	0.0002	2.5313		0.363	0.055
Gadow and Hui basal area diff. model (Eq. (13))	0.8747	−0.0340	0.9732		0.640	1.134

R: correlation coefficient between observed Δy and predicted $\Delta\hat{y}$; RMSE: root mean square error.

Table 4
RMSE values of projected vs. observed stand variables at the end of the 5-year prediction period according to different simulation combinations

Simulation combination	*H*	*D*	*N*	*G*	*V*
$D \times G$	0.617	0.336	98.232	0.952	21.252
$D \times L$	0.617	0.336	105.634	0.935	21.107
$D \times N$	0.617	0.336	99.434	0.986	21.138
$G \times N$	0.617	0.340	99.434	0.952	21.252
Dif	0.617	0.584	94.850	1.132	24.585
Est	0.617	0.395	160.204	1.695	24.194

models offers more flexibility in terms of prediction interval length.

3.2. Comparing different combinations of model components

The RMSE has been calculated for every stand variable in the simulations (Table 4). In all simulation combinations, the stand height was the first projected variable in the calculation sequence obtained by the same stand height model (Eq. (1)) and where the same RMSE value (0.617 m) occurred. The set of RMSE values showed negligible difference for the first four simulation combinations in Table 4, where projected variables were calculated based on increment equations.

The ‘‘Dif’’ simulation combination showed considerably higher RMSE values for quadratic mean diameter, basal area and volume calculations. It has a higher RMSE, because the stand density and height were predicted independently, and especially since the height model was calculated using a different data set. Both height and stem number already contain a prediction error. The results of the ‘‘Est’’ combination showed the highest RMSE values in stand basal area and density calculations. Both variables were calculated via volume and form height (HF_t, Eq. (14)) and were therefore dependent on the prediction error of the form height model. Fig. 3 shows poorer performance between the observed and predicted basal area of the ‘‘Dif’’ and ‘‘Est’’ simulation combinations in comparison with other simulation combinations. The solid line in Fig. 3 generated by Kernel regression indicates the bias between the observed and predicted basal areas.

The simulation combinations in which diameter was predicted directly using growth equation (7) showed the lowest RMSE values for quadratic mean diameter. The simulations with different combinations of diameter, basal area and tree survival models conducted in the current study showed that the ‘‘$D \times G$’’, ‘‘$D \times N$’’, ‘‘$D \times L$’’ and ‘‘$G \times N$’’ combinations give almost as good or better results than the difference equations.

3.3. Long-term simulation of stand development

We can only predict growth and survival if the interval is a period of undisturbed growth. However, to evaluate different management scenarios, we must be able to model the disturbance events as well as the growth. A basic assumption with interval plots is that the interval is a period of undisturbed

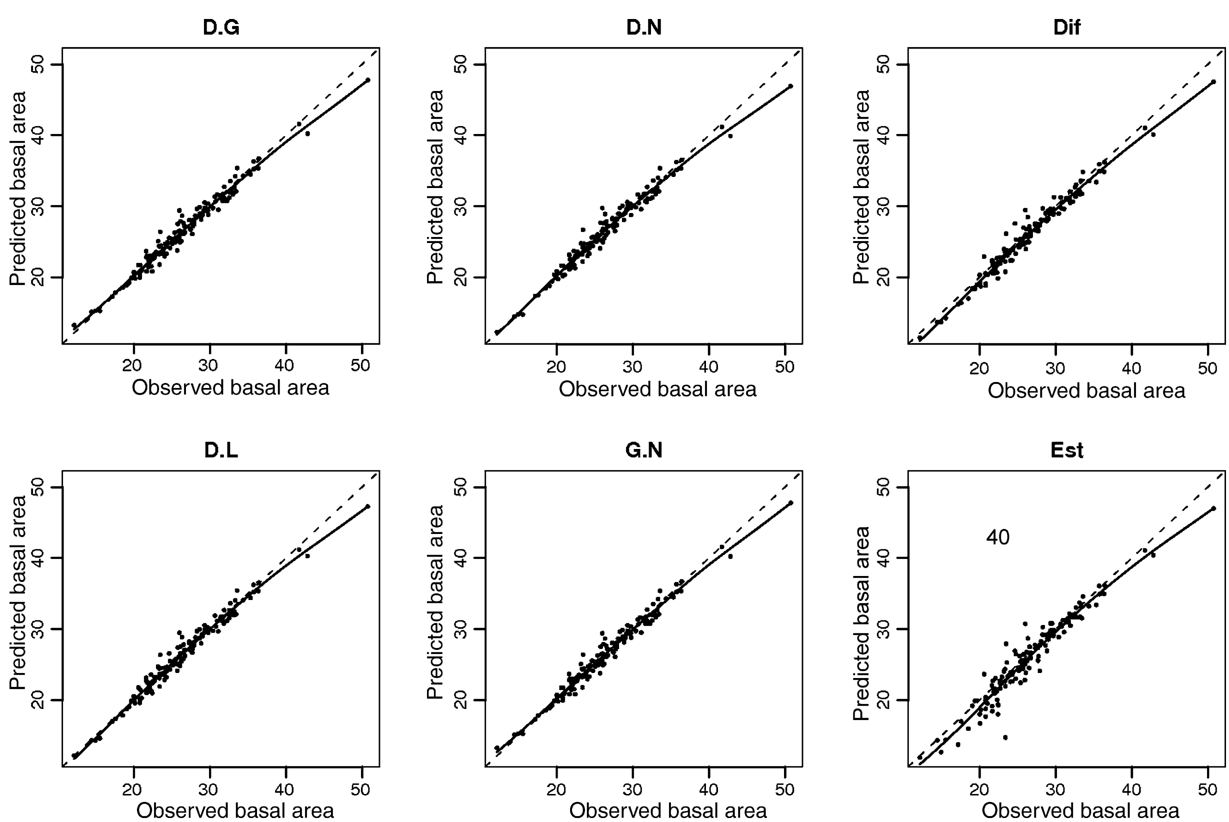

Fig. 3. Observed vs. predicted basal area for various simulation combinations. Note the systematic underestimation in stands with very large basal areas.

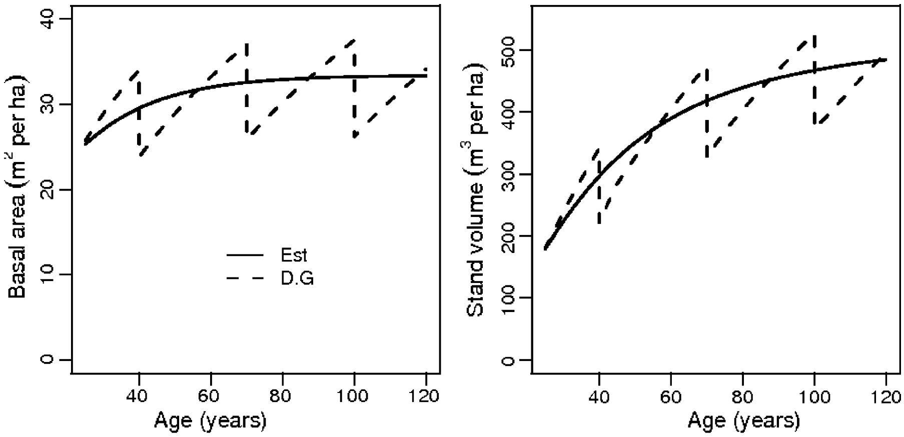

Fig. 4. An example of long-term prediction with the "Est" and "$D \times G$" simulations. The initial data from one sample plot was used (A = 25 years, D = 14.4 cm, H = 13.3 m, G = 25.7 m^2/ha, V = 179 m^3/ha).

growth. All models in this study, except the Kiviste difference equations (Kiviste, 1999a,b), were developed or calibrated on the undisturbed interval plot data. The growth models include natural growth and natural single-tree survival but they do not include anthropogenic interference as can be expected in the case of commercial forests. These models therefore allow us to predict stand growth in commercial forests between harvest events in the long run.

"Est" simulation models have been developed on the basis of forest inventory data, which contain both natural mortality (gap phase disturbances) and thinnings and can be used for long-term prediction. The use of growth models developed on interval plot data in long-term prediction necessitates including both natural and anthropogenic disturbances. An example of the long-term prediction of stand basal area and volume development simulated with the "$D \times G$" simulation combination in comparison with "Est", which represents the average development of Estonian stands, appears in Fig. 4. In the "$D \times G$" model combination (Table 2), calculations were repeated with 5-year intervals up to 120 years, and the degree of stocking was calculated for every step with the Eq. (16) model. When the degree of stocking exceeded a value of 0.9, the basal area and number of trees was then reduced by 30%, following the Estonian thinning instructions. In spite of different performance in short-term prediction, both simulation combinations showed quite comparable performance in long-term projections.

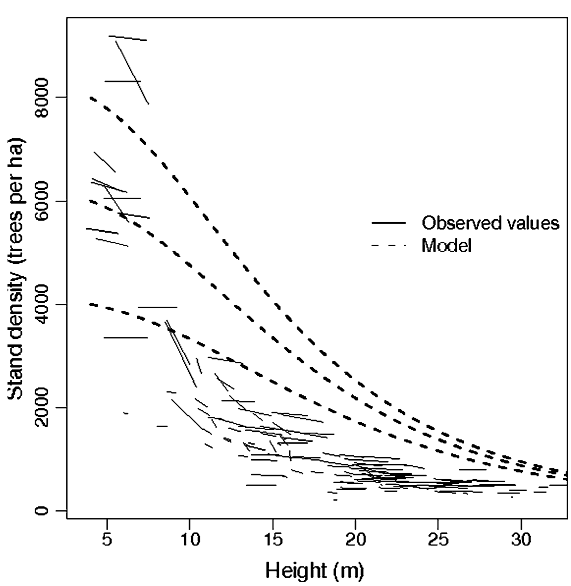

Fig. 5. The algebraic difference models prediction trajectories with three different initial states in comparison with the observed data from the Estonian network of permanent forest growth and yield monitoring plots.

The main advantage of the use of algebraic difference equations over the fixed-step increment equations is the ability to use flexible time steps. However, experience has shown that the projection intervals should not deviate too much from the time steps of the measurement data. An important constraint when using the algebraic difference equations is to avoid long-term predictions in one prediction sequence. Fig. 5 shows an example of the long-term projection of stand density in three different initial densities. A considerable variance in model predictions in comparison of observed values can be seen. This typically happens when only non-overlapping intervals have been used in the model parameter estimation. It is often advisable to use all possible intervals, but even in that case one has to be careful with long-term projections in one sequence.

4. Conclusions

This study compares the results of stand simulation using fixed interval increment functions and algebraic difference functions with variable interval lengths. To compare the flexibility of two different types of empirical model in stand-level prediction, we tested the performance of: (a) increment equations developed in the current study on the basis of interval plot data (Eqs. (4), (6), (8) and (10)), (b) algebraic difference equations calibrated on interval plot data (Eqs. (12) and (13)) and (c) algebraic difference equations developed on Estonian forest inventory data (Eqs. (1)–(3)). The model tests were carried out by comparing the projections of five stand variables (height, quadratic mean diameter, basal area, survival and volume) in different combinations.

The results do confirm the assumption that using different types of model for obtaining projected stand variables makes the projections differ, but not the assumption that considerable differences can be expected in projections when using different combinations of predictor variables in calculation sequences for obtaining projected variables. In this regard, the 5-year projections of dependent stand variables with simulation combinations using increment equations showed negligible difference from each other, but considerable difference from difference equations.

Stand density development is one of the most important but still complicated aspects of forest modeling. The algebraic difference models allow us to predict the average long-term stand development in accordance with a given initial state. The growth models developed in the current study predict growth by 5-year intervals and are therefore inconvenient for the end-user to apply. On the other hand, they are more flexible when taking the limiting line of self-thinning into account. The stands on interval plots used for model parametrizations have not reached the self-thinning state yet and show relatively high basal area and diameter growth. Improving the prediction abilities of these models requires longer intervals of undisturbed development.

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References

Burkhart, H.E., 2003. Suggestions for choosing an appropriate level for modeling forest stands. In: Amaro, A., Reed, D., Soares, P. (Eds.), Modelling Forest Systems. CAB International, pp. 3–10.

Cieszewski, C.J., Bella, I.E., 1989. Polymorphic height and site index curves for lodgepole pine in Alberta. Can. J. Forest Res. 19, 1151–1160.

Gadow, K.V., 1996. Modelling growth in managed forests—realism and limits of lumping. Sci. Total Environ. 183, 167–177.

Gadow, K.V., Hui, G.Y., 1999. Modelling Forest Development. Kluwer Academic Publisher, Dordrecht, 213 pp.

Glenn-Lewin, D.C., van der Maarel, E., 1992. Patterns and processes of vegetation dynamics. In: Glenn-Lewin, D.C., Peet, R.K., Veblen, T.T. (Eds.), Plant Succession: Theory and Prediction. Chapman & Hall, London.

Gurjanov, M., Sánchez Orois, S.U., Schröder, J., 2000. Grundflächenmodelle für gleichaltrige Fichtenreinbestände-Eine vergleichende Analyse. Centralblatt für das gesamte Forstwesen. 117. Jahrgang. Heft 3/4, 187–198.

Gustavsen, H.G., Roiko-Jokela, P., Varmola, M., 1988. Kivennäismaiden talousmetsien pysyvät (INKA ja TINKA) kokeet. Suunnitelmat, mittausmenetelmät ja aineistojen rakenteet. Metsäntutkimus-laitoksen Tiedonantoja, 292. Helsinki, 212 pp. (in Finnish).

Hart, H.M.J., 1928. Stamtal en Dunning-een Orienteerend Onderzoek naar de Beste Plantwijdte en Dunningswijze voor den Djati. H. Venman & Zonen, Wageningen.

Hasenauer, H., 2005. Concepts within tree growth modeling. In: Hasenauer, H. (Ed.), Sustainable Forest Management. Growth Models for Europe. Springer, Berlin, pp. 3–17.

Hägglund, B., 1981. Forecasting growth and yield in established forests. An outline and analysis of the outcome of a subprogram within the HUGIN project. Swedish University of Agricultural Sciences, Report 31, Umeå, 145 pp.

Hynynen, J., Ojansuu, R., Hökka, H., Siipilehto, J., Salminen, H., Haapala, P., 2002. Models for predicting stand development in MELA system. The Finnish Forest Research Institute. Research Papers, 835. Vantaa Research Center, 116 pp.

Jõgiste, K., 1998. Productivity of mixed stand of Norway spruce and birch affected by population dynamics: a model analysis. Ecol. Model. 106, 77–91.

Kangur, A., Korjus, H., Jõgiste, K., Kiviste, A., 2005. A conceptual model of forest stand development based on permanent sample-plot data in Estonia. Scand. J. Forest Res. 20 (Suppl. 6), 94–101.

Karoles, K., Õunap, H., Pilt, E., Terasmaa, T., Kivits, H., 2000. Forest condition in Estonia 1988–1999, defoliation and forest damages on Level I sample points. Air pollution and forests in industrial areas of north-east Estonia. Forestry Studies XXXIII, 209–216.

Kiviste, A., 1988. Forest Growth Functions, Estonian Agricultural Academy, Tartu, 108 (in Russian).

Kiviste, A., 1995. Eesti riigimetsa puistute kõrguse, diameetri ja tagavara sõltuvus puistu vanusest ja kasvukohatingimustest 1984–1993. a. metsakorralduse takseerkirjelduste andmeil. Eesti Põllumajandusülikooli teadustööde kogumik, vol. 181, Tartu, pp. 132–148.

Kiviste, A., 1997. Eesti riigimetsa puistute kõrguse, diameetri ja tagavara vanuseridade diferentsmudel 1984–1993. a. metsakorralduse takseerkirjelduste andmeil [Difference equations of stand height, diameter and volume depending on stand age and site factors for Estonian state forests]. Eesti Põllumajandusülikooli teadustööde kogumik, vol. 189, Tartu, pp. 63–75.

Trans. Fac. For. Estonian Agric. Uni. 32, 28–36 (in Estonian).

Kiviste, A., 1999b. Site index change in the 1950s–1999s according to Estonian forest inventory data. In: Karjalainen, T., Spiecker, H., Laroussinie, O. (Eds.), Causes and Consequences of Accelerating Tree Growth in Europe. EFI Proceedings, vol. 27. pp. 87–100.

Kiviste, A., Hordo, M., 2003. The network of permanent sample plots for forest growth modeling in Estonia. In: Markevica, A., Lypsik, A., Leep, R. (Eds.), Research for Rural development 2003. International Scientific Conference Proceedings, Jelgava, pp. 174–177.

Krigul, T., 1969. Metsataksaatori Teatmik. Eesti Põllumajandus Akadeemia, Tartu, 139 pp.

Nabuurs, G.J., Päivinen, R., 1996. Large scale forestry scenario models—a compilation and review. European Forest Institute. Working Paper 10, Joensuu, 174 pp.

Nilson, A., 1973. Hooldusraiete arvutusliku projekteerimise teooriast (On the theory of programming of thinning). EPA teaduslike tööde kogumik 89, 136–142 (in Estonian with summary in Russian and English).

Nilson, A., 1996. Role of environmental values in forest management planning in the Baltic countries. In: Hyttinen, P., Nilson, A. (Eds.), Integrating Environmental Values into Forest Planning—Baltic and Nordic Perspectives. EFI Proceedings, vol. 13. pp. 23–34.

Nilson, A., 1999. Pidev metsakorraldus—mis see on. [Continuous forest inventory—what is it?]. In: *Pidev metsakorraldus*. Transactions of the Faculty of forestry, Estonian Agricultural University, vol. 32, pp. 4–13 (in Estonian).

Nilson, A., 2005. Fitness of allometric equation $N = aD^b$ and equation $N = (a + bD)^{-2}$ for modelling the dependence of the number of trees N on their mean diameter D in yield tables. Forestry Studies 1406-995443, 159–172.

Oliver, C.D., Larson, B.C., 1996. Forest Stand Dynamics (update edition). John Wiley & Sons, Inc., New York.

Pretzsch, H., Biber, P., Dursky, J., 2002. The single tree-based stand simulator SILVA: construction, application and evaluation. Forest Ecol. Manage. 162, 3–21.

In: Year book Forest 2005. .

Richardson, B., Watt, M.S., Mason, E.G., Kriticos, D.J., 2006. Advances in modelling and decision support systems for vegetation management in young forest plantations, Forestry, 79(1), 29–42.

Schmidt, M., Nagel, J., Skovsgaard, J.P., 2006. Evaluating individual tree growth models. In: Hasenauer, H. (Ed.), Sustainable Forest Management. Growth Models for Europe. Springer, Berlin, pp. 151–165.

Sims, A., 2003. The database of forestry models. Estonian Agricultural University. Master Thesis, 217 pp.

Sims, A., 2005. The database of forest management models. Forestry Studies 43, 124–131.

Söderberg, U., 1986. Funktioner för skogliga produktionsprognoser. Tillväxt och formhöjd för enskilda träd av inhemska trädslag i Sverige. Sveriges lantsbrukuniversitet, Rapport 14, Umeå, 251 pp. (in Swedish).

Tappo, E., 1982. Eesti NSV puistute keskmised takseertunnused puistu enamuspuuliigi, boniteedi ja vanuse järgi, 72 pp. (in Estonian).

Vanclay, J.K., 1994. Modelling Forest Growth and Yield: Applications to Mixed Tropical Forests. CAB International, Wallingford, UK, xvii + 312 pp.