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Heat and mass mixed convection for MHD visco-elastic fluid past a stretching sheet with ohmic dissipation

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ABSTRACT

In this study, an analysis has performed for heat and mass transfer of a steady laminar boundary-layer flow of an electrically magnetic conducting fluid of second-grade subject to suction and to a transverse uniform magnetic and electric field past a semi-infinite stretching sheet. Parameters $Gr, Gc, E1, M, Sc$ which are used to represent the dominance of the buoyant effect, magnetic effect, electric effect and mass transfer effect are present in governing equations, respectively. The novelty of the present work is considered the mass transfer phenomena to a coupled heat and mass transfer boundary-layer equations. The similar transformation and the finite-difference method are used to analyze the present problem. The numerical solutions of the flow velocity distributions, temperature profiles, the wall unknown values of $\theta'(0)$ and $\phi'(0)$ for calculating the heat and mass transfer of the similar boundary-layer flow are carried out as functions of the visco-elastic parameter E, the Prandtl number Pr, the buoyancy parameters Gr, Gc and mass transfer coefficient Sc. The value of E, Pr, $E1$, Gr, Gc and Sc parameter are important factors in this study. It will produce greater heat transfer efficiency with a larger value of those parameters, but the magnetic parameter M may reduce the heat transfer efficiency. On the other hand, for mass transfer, the value of Sc parameter is important factor in this study. It will produce greater heat transfer efficiency with a larger value of Sc.

1. Introduction

The study of visco-elastic fluids has become of increasing importance in the last few years. This is mainly due to their many applications in petroleum drilling, manufacturing of foods and paper and many other similar activities. Some of these fluids, which can be formulated by the model used in the present study, are termed second-grade fluids. It is a well-known fact in the studies of non-Newtonian fluid flows [1]. Thus, if we use a non-Newtonian fluid as the coolant of the cooling systems or heat exchangers might greatly reduce the required pumping power. Therefore, a fundamental analysis of the flow field of non-Newtonian fluids in a boundary layer adjacent to a stretching sheet or an extended surface is very important and is also an essential part in the area of the fluid dynamics and heat transfer. Srivatsava [2], and Rajeswari and Rathna [3] studied the non-Newtonian fluid flow near a stretching sheet. Mishra and Panda [4] analyzed the behavior of second-grade visco-elastic fluids under the influence of a side-wall injection in an entrance region of a pipe flow. Rajagopal et al. [5] studied a Falkner-Skan flow field of a second-grade visco-elastic fluid. Massoudi and Ramezan [6] studied a wedge flow with suction and injection along walls of a wedge by the similarity method and finite-difference calculations. Hsu et al. [7] also studied the flow and heat transfer phenomena of an incompressible second-grade visco-elastic fluid past a wedge with

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suction or injection. An excellent review of boundary layers in non-linear fluids was recently written by Rajagopal [8]. These are related studies to the present investigation about second-grade fluids. The visco-elastic nature of a second-grade fluid has found in some dilute polymer solutions or in polymer fluids. These fluids exhibit both the viscous and elastic characteristics. Same as Newtonian fluids, the viscous property is due to the transport phenomenon of the fluid molecules. The elastic property is due to the chemical structure and configuration of the polymer molecule. The term "elastic" means that the visco-elastic fluid "remembers" where it was. Macromolecules act as small rubber band and tend to snap back when the external forces have removed and hence produce "elastic recoil" of the fluid. Detailed information of visco-elastic fluid can be found in the books of Rheology. Rajagopal et al. [5] studied the Falkner-Skan flow of a fluid of second grade. They used the perturbation procedure and finite-difference method for simplify a non-linearization problem to a quasilinearization problem.

All of above are dealing with forced convection problems. Recently, Vajravelu and Soewono [9] had solved the fourth-order non-linear systems arising in combined free and forced convection flow of a second-order fluid over a stretching sheet. The stretching sheet flow of a non-Newtonian fluid is also one of important flow fields in real world, Garg and Rajagopal [10] had studied its flow fields and Raptis [11] had studied its heat transfer of a visco-elastic fluid.

From above, provide the motivation for the present study to deal with the heat and mass transfer in an incompressible second-grade fluid which caused by a stretching sheet with visco-elasticity on flow with heat and mass transfer characteristics for mixed convection phenomena. The paper presents a similar analysis Rajagopal et al. [5] method to solve the non-linear problem. Lately, relative researches in connection with mixed convection almost all were working for Newtonian fluid [12,13]. On the other hand, researches in connection with visco-elastic fluid or second grade non-Newtonian fluids, but there are not the mixed convection flow [14–16]. Recently, Sanjayanand et al. [17], Cortell, Rafael [18] and Seddeek [19] had studied the heat and mass transfer problems about the visco-elastic boundary-layer flow over a stretching sheet with magnetic effect, but not consider the mixed convection with magnetic effect. Abel et al. [20–23] had studied the related heat and mass transfer problems in many important effects, such as hydromagnetic effect, heat sink and source, etc, but still not consider present problem. Recently, Wang et al. [24] have developed a lattice Boltzmann algorithm for fluid–solid conjugate heat transfer for a new generalized heat generation, which insures the temperature and heat flux continuities at the interface. The new scheme agrees well with the classical CFD method for predictions of flow and heat transfer in a heated thick-wall microchannel with less mesh number and less computational costs. Yang et al. [25] have developed another novel approach by using the commercially available software ANSYS for an air convection heat transfer enhancement problem, and application to the magnetic quadrupole field. MHD flow with Radiation effect were studied by Aydin and Kaya [26], and MHD flow with non-uniform heat source/sink were studied by Abel and Nandeppanavar [27]. Ahmed [28] studied similarity solution in MHD effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. Hsiao and Chen [29–32] had studied conjugate heat transfer problems about a second-grade fluid adjacent to a stretching sheet or to a fin, but still had not toward the heat and mass transfer for electrical conducting magnetic mixed convection past a stretching sheet.

Therefore, in the present investigation, a study for heat and mass transfer problem has undertaken to provide results for the mixed convection flow of a second-grade fluid past a stretching sheet. So that, the novelty of the present work is considered the mass transfer phenomena to a coupled heat and mass transfer boundary-layer equations. A similarity derivation technique has been used and the resulting similar equations have been solved by using the numerical method of finite difference. The effects of the visco-elastic parameter E, the buoyancy parameter Gr and Gc, the Prandtl number Pr, viscous dissipation coefficient Ec, the electric parameter E_1, mass transfer coefficient Sc and magnetic parameter M to the heat and mass transfer on the stretching sheet have been used in the present study.

2. Theory and analysis

An incompressible, homogeneous, non-Newtonian, second-grade fluid having a constitutive equation based on the postulate of gradually fading memory suggested by Rivlin–Ericksen [33] is used for the present flow. The model equation is express as follows:

$$\mathbf{T} = -P\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \tag{1}$$

where $\mathbf{T}$ is the stress tensor, p is the pressure, μ is the dynamic viscosity, α_1 and α_2 are first and second normal stress coefficients that are relate to the material modulus and for the present second-grade fluid

$$\mu \geqslant 0, \quad \alpha_1 > 0, \quad \alpha_1 + \alpha_2 = 0 \tag{2}$$

The kinematic tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^T \tag{3}$$

$$\mathbf{A}_2 = \frac{d\mathbf{A}_1}{d\mathbf{t}} + \mathbf{A}_1(\nabla\mathbf{V}) + (\nabla\mathbf{V})^T\mathbf{A}_1 \tag{4}$$

where $\mathbf{V}$ is velocity and $d/d\mathbf{t}$ is the material time derivative. As mentioned in Markovitz and Coleman [34], Acrivos [35], Bear and Walters [36], this model is applicable to some dilute polymers. In the present analysis we consider the flow of a second-grade fluid obeying Eq. (1) adjacent to a stretching sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces have applied along the x-axis (a positive x-axis has taken vertically and parallel to the direction of gravity). The geometric model is shown in Fig. 1. The steady two-dimensional boundary-layer equations for this flow, heat transfer and mass transfer, in usual notations, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = v\frac{\partial^2 v}{\partial y^2} + k_1\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] + g_r\beta^*(T - T_\infty) + g_c\beta^{**}(C - C_\infty) + \frac{\sigma E_0 B_0}{\rho} - \frac{\sigma B_0^2}{\rho}u \tag{6}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + (uB_0)^2\sigma - (E_0)^2\sigma \tag{7}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{8}$$

The well-known Boussinesq approximation is used to represent the buoyancy mixed term. Where u, v are the velocity components in the x and y directions, T is the temperature, g is the magnitude of the gravity, v is the kinematic viscosity, $k_1 = \alpha_1 B/\mu$ is the visco-elastic parameter, β^* is the coefficient of thermal expansion, β^{**} is the concentration coefficient, T_∞ is the temperature of the ambient fluid, ρ is the density, c_p is the specific heat at constant pressure, k is the conductivity, σ is the electrical conductivity, E_0 is the electric field factor, B_0 is the magnetic field factor, q is the specific heat generation rate, D is mass diffusivity, respectively. The boundary conditions to the problem are

$$u = Bx, \quad v = v_w = -(B\upsilon)^{1/2}(m - 1/m)$$
$$y = 0,\ B > 0,\ u \to 0,\ \partial u/\partial y \to 0 \quad \text{at } y \to \infty$$
$$T = T_w = T_\infty + A\left(\frac{x}{L}\right) \quad \text{at } y = 0$$

where T_w and T_∞ are constant wall temperature and ambient fluid temperature, A and B is the proportional constant, $v_w = -(B\upsilon)^{1/2}(m - 1/m)$, and L is the characteristic length, respectively. It should be noted that $m > 1$ corresponds to suction $(v_w < 0)$. Where $m < 1$ corresponds to blowing $(v_w > 0)$. In the case when the parameter $m = 1$, the stretching sheet is impermeable. In this study, set all of parameter $m = 1$ simplified the problem in the conjugate heat transfer. A similarity solution for velocity will be obtained if introduce a set of transformations, such that

$$u = \mathrm{Bx}f'(\eta), \quad v = -(Bv)^{1/2}f(\eta)$$
$$\eta = (B/v)^{1/2}y \tag{9}$$

Eq. (9) has satisfied the continuity equation (5), Substituting (9) into (6), we have

$$f'^2 - ff'' = f''' + E(2f'f'') + E(2f'f''' - f''^2 - ff^{IV}) + G_r\theta + G_c\phi + ME1 - Mf' \tag{10}$$

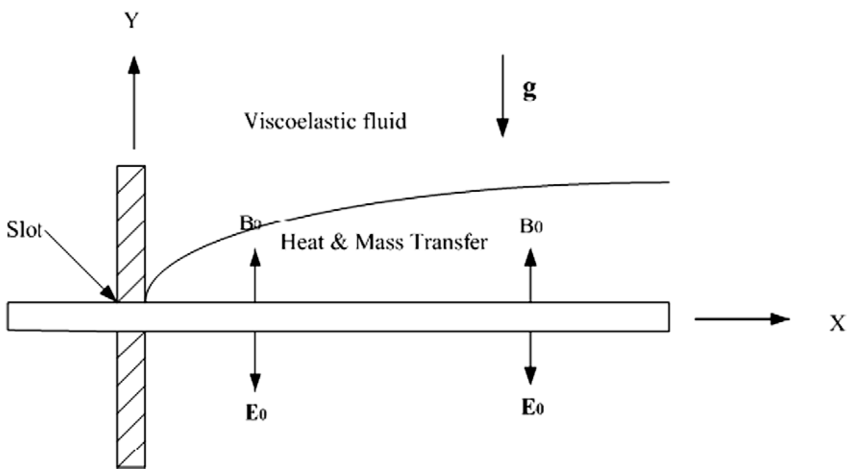

Fig. 1. A sketch of the physical model for mixed convection heat and mass transfer past a stretching sheet with magnetic and electric effects.

where $E = \alpha_1 B/\mu$ is the visco-elastic parameter and $G_r = g_x\beta^*(T_w - T_\infty)/B^2x$ is the free-convection parameter, $G_c = g_x\beta^{**}(\phi_w - \phi_\infty)/B^2x$ is the free mass convection parameter, $E1 = E_0/B_0Bx$ is the electric parameter and $M = \sigma B_0/\rho B$ is the magnetic parameter. Where L is the wall thickness of the stretching sheet. The corresponding boundary conditions become

$$\begin{aligned} &f = 0,\ f' = 1 \quad \text{at } \eta = 0 \\ &f' \to 0,\ f'' \to 0 \quad \text{at} \quad \eta \to \infty \end{aligned} \tag{11}$$

For the prescribed surface temperature. We introduce the dimensionless temperature $\theta(\eta)$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{12}$$

And combine the transformations from Eq. (9), the energy equation becomes

$$\theta'' + Pr(f\theta' - 2f'\theta) + Pr\,E_c[(f'')^2 + M(E1)^2 - 2E1f' - M(f')^2] = 0 \tag{13}$$

where $Pr = \mu c_p/k$ is the Prandtl number and $E_c = B^2t^2/c_p$ is the Eckert number. The corresponding thermal boundary conditions are

$$\begin{aligned} &\theta = 1 \quad \text{at } \eta = 0 \\ &\theta \to 0 \quad \text{as } \eta \to \infty \end{aligned}$$

For the solutions of heat and mass transfer equations, it can be defined non-dimensional temperature and concentration variables as

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

This leads to the non-dimensional form of temperature and concentration equations as follows:

$$\phi'' + S_cf\phi' - S_cf'\phi = 0 \tag{14}$$

where $Sc = v/D$ is the Schmidt number. The corresponding boundary conditions are

$$\begin{aligned} &\phi = 1 \quad \text{at } \eta = 0 \\ &\phi = 0 \quad \text{as } \eta \to \infty \end{aligned}$$

The parameters $k,\ E,\ Pr,\ Ec,\ G_r,\ G_c$ and Sc control variations of the flow and the heat and mass transfer characteristics. In the present study, isothermal condition is considered along the sheet, and it does not admit similar solutions. In terms of similarity parameters and dimensionless quantities defined by Eq. (9), the heating rate on the wall is

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{15}$$

Once we know the $f(\eta)$ and its derivatives, one can calculate the values of the local skin friction at the surface from the following relations:

$$\tau_x = \frac{-(\partial u/\partial y)_{y=0}}{\mu Bx\sqrt{B/v}} = -\frac{1}{\mu}f''(0) \tag{16}$$

In addition, the local Nusselt number Nu_x is defined by

$$Nu_x = \frac{hx}{k} = \frac{q_w}{T_w - T_\infty}\frac{x}{k} \tag{17}$$

This expression can be written as

$$Nu_x = \frac{hx}{k} = \frac{q_w}{T_w - T_\infty}\frac{x}{k} = -\theta'(0)[G_r^{1/4} + G_c^{1/4}] \tag{18}$$

The Sherwood number is defined by

$$Sh_x = \frac{hc}{C_w - C_\infty}\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\sqrt{B/v}\phi'(0) \tag{19}$$

3. Numerical technique

In the present problem, the set of similar equations (13)–(17) are solved by a finite-difference method. These ordinary differential equations are discretized by an accurate central difference method, and a computer program has been developed to solve these equations. To avoid errors in discretization and calculation processing and to ensure the convergence of numerical solutions, some conventional numerical procedures have been applied in order to choose a suitable grid size $\Delta\eta = 0.01-0.05$, a suitable η range and a direct Gauss elimination method with Newton's method [37] is used in the computer program to obtain solutions of these difference equations. The finite difference formulas are divided to forward finite-difference for the boundary-layer inner edge $\eta = 0$, backward finite-difference formula for the boundary-layer outer edge $\eta = \infty$, and centered finite-difference formula for the internal points.

(1) forward finite-difference formulas for first derivative to fourth derivative

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$
$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$$
$$f'''(x_i) = \frac{f(x_{i+3}) - 3f(x_{i+2}) + 3f(x_{i+1}) - f(x_i)}{h^3}$$
$$f''''(x_i) = \frac{f(x_{i+4}) - 4f(x_{i+3}) + 6f(x_{i+2}) - 4f(x_{i+1}) + f(x_i)}{h^4}$$

(2) backward finite-difference formulas for first derivative to fourth derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$$
$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}$$
$$f'''(x_i) = \frac{f(x_i) - 3f(x_{i-1}) + 3f(x_{i-2}) - f(x_{i-3})}{h^3}$$
$$f''''(x_i) = \frac{f(x_i) - 4f(x_{i-1}) + 6f(x_{i-2}) - 4f(x_{i-3}) + f(x_{i-4})}{h^4}$$

(3) centered finite-difference formulas for first derivative to fourth derivative

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$
$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$
$$f'''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + 2f(x_{i-1}) - f(x_{i-2})}{2h^3}$$
$$f''''(x_i) = \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4}$$

The present finite-difference method is a similar to Box method [37], but directly applied above finite-difference formulas into the similarity boundary-layer equations, and solves them by using the Gauss elimination method with Newton's method [43]. Vajravelu et al. [38–40] and Hsiao [41,42] are also using analytical and numerical solutions to solve the related problems. So, some numerical technique methods will be applied to the same area in the future. In this study, the program to compute finite-difference approximations of derivatives for equal spaced discrete data. The code employ centered differences of $O(h^2)$ for the interior points and forward and backward differences of $O(h)$ for the first and last points, respectively. See Chapra and Canale, Numerical Methods for Engineers [43]. To ensure the convergence of the numerical solution to exact solution, the step sizes $\Delta\eta$ and have been optimized and the results presented here are independent of the step sizes at least up to the fourth decimal place.

The convergence criteria based on the relative difference between the current and previous iteration values of the velocity, temperature and concentration gradients at wall have been employed. When the difference reaches less than 10^{-6} for the

Table 1
Values of $-f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for different values of physical parameters.

E	Gr	Gc	M	Pr	Ec	$E1$	Sc	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.5	0.5	0.0	1.0	0.0	0.1	0.1	0.3907	1.4979	0.3328
0.1	0.0	0.5	0.0	1.0	0.0	0.1	0.1	0.5564	1.4754	0.3274
0.1	0.5	0.5	0.0	1.0	0.1	0.1	0.5	0.5240	1.4619	0.7646
0.1	0.0	0.5	0.5	1.0	0.1	0.2	0.5	0.9172	1.4149	0.7175
0.1	0.2	0.5	0.5	1.0	0.0	0.2	1.0	0.9041	1.3922	1.0658

flow fields, the solution has assumed to converge and the iterative process has terminated. The sequence of equations of above was expressed in difference form using central difference scheme in η-direction. In each iteration step, the equations were then reduced to a system of linear algebraic equations.

4. Results and discussion

The objective of the present analysis is to study the heat and mass transfer of a stretching sheet cooled or heated by a high or low Prandtl number, second-grade visco-elastic fluid with various parameters. The model for grade-two fluids is used in the momentum equations. The effects of dimensionless parameters, the Prandtl number (Pr), the electric parameter ($E1$), the magnetic parameter (M), the elastic number (E), the free-convection parameter (Gr), the free-convection mass transport parameter (Gc), and mass transfer coefficient (Sc) are main interests of the study. Flow and temperature fields of the stretching sheet flow are analyzed by utilizing the boundary-layer concept to obtain a set of coupled momentum equation, energy

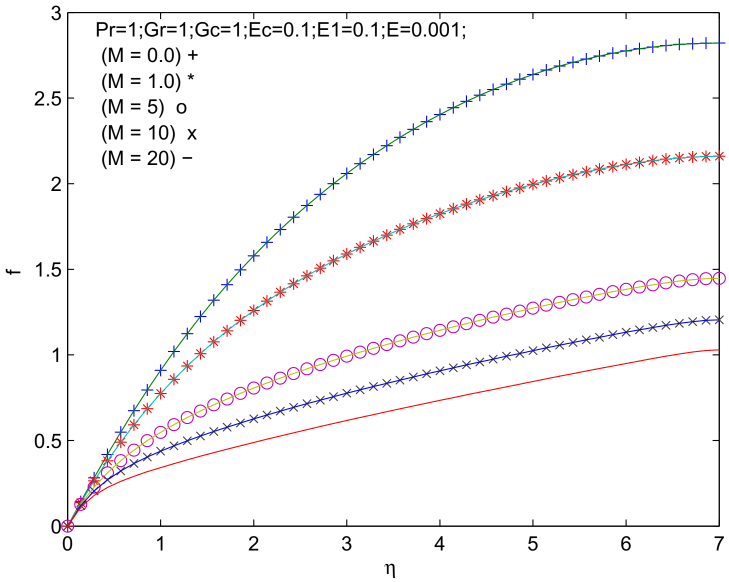

Fig. 2. Dimensionless velocity profiles f vs. η as $Pr = 1.0,\ Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E1 = 0.1,\ E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$.

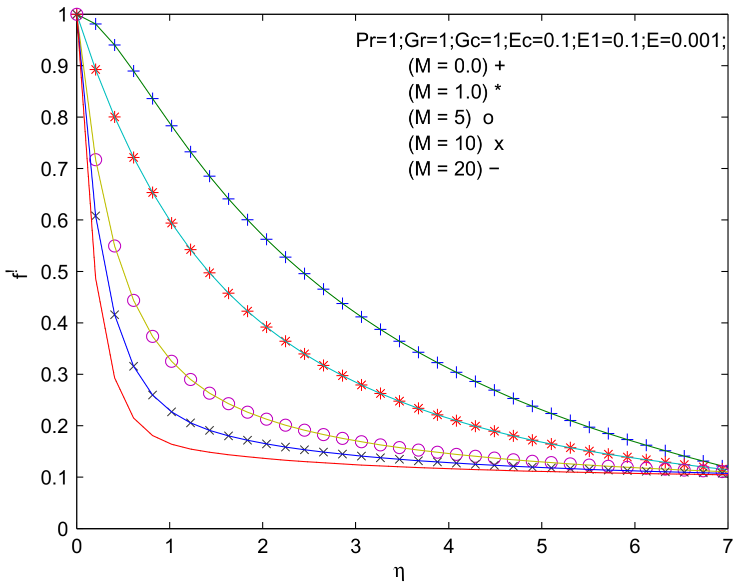

Fig. 3. Dimensionless velocity gradient profiles f' vs. η as $Pr = 1.0,\ Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E1 = 0.1,\ E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$.

equation and mass equation. A similarity transformation is then used to convert the nonlinear, coupled partial differential equations to a set of nonlinear, coupled ordinary differential equations. A generalized derivation is used to analyze a stretching sheet flow have been studied. A second-order accurate finite-difference method is used to obtain solutions of these equations. Fig. 1 is a sketch of the physical model for the conjugate heat transfer of a stretching sheet past a second-grade viscoelastic fluid. Table 1 shows that the different values of $-f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for different values of physical parameters. It is a numerical calculation result by this study.

Fig. 2 depicts dimensionless velocity profiles f vs. η as $Pr = 1.0$, $Gr = 1.0$, $Gc = 1.0$, $Ec = 0.1$, $E1 = 0.1$, $E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$. The dimensionless velocity profiles f are satisfied the boundary conditions and varies by a smooth cure for different M values. When the M is larger and the f curve is lower, so that the dimensionless velocity becomes lower, so that the momentum for the fluid becomes lower too.

Fig. 3 depicts dimensionless velocity gradient profiles vs. η as $Pr = 1.0$, $Gr = 1.0$, $Gc = 1.0$, $Ec = 0.1$, $E1 = 0.1$, $E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$. The dimensionless velocity gradient profiles f' are satisfied the boundary conditions and varies

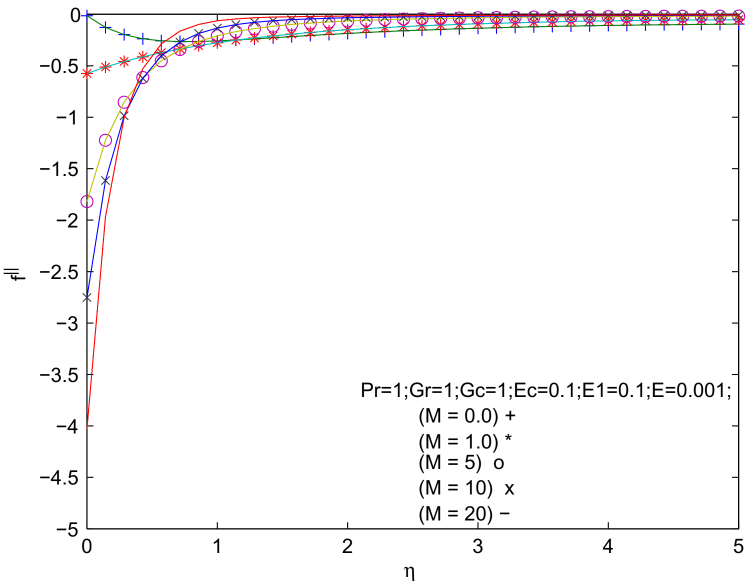

Fig. 4. Dimensionless velocity second gradient profiles f'' vs. η as $Pr = 1.0$, $Gr = 1.0$, $Gc = 1.0$, $Ec = 0.1$, $E1 = 0.1$, $E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$.

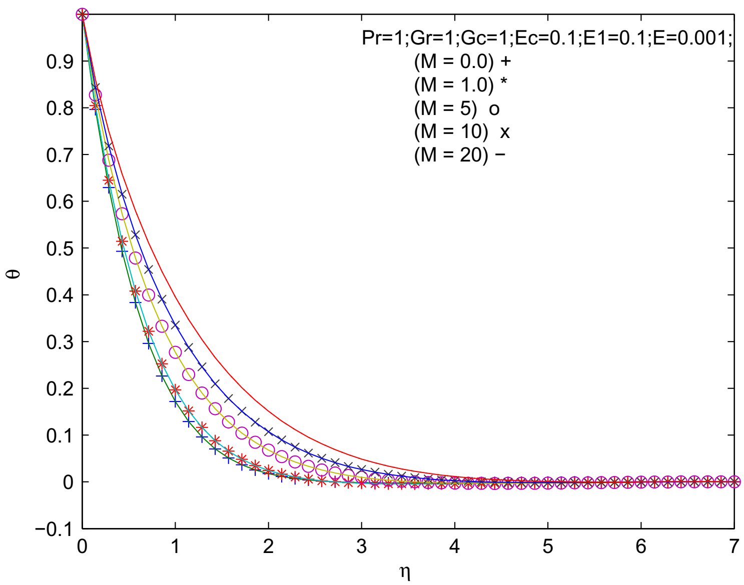

Fig. 5. Dimensionless temperature profiles θ vs. η as $Pr = 1.0$, $Gr = 1.0$, $Gc = 1.0$, $Ec = 0.1$, $E1 = 0.1$, $E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$.

by a smooth cure for different M values. When the M is larger and the f' curve is lower, so that the velocity gradient becomes lower too.

Fig. 4 depicts dimensionless velocity second gradient profiles f'' vs. η as $Pr = 1.0,\ Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E1 = 0.1,\ E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$. The dimensionless second velocity gradient profiles f'' are satisfied the boundary conditions at $\eta \to \infty$ and varies by a smooth cure for different M values. When the M is larger and the f'' curve is lower, so that the second velocity gradient becomes lower too.

Fig. 5 depicts dimensionless temperature profiles θ vs. η as $Pr = 1.0,\ Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E1 = 0.1,\ E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$. The dimensionless temperature profiles θ are satisfied the boundary conditions and varies by a smooth cure for different M values. When the M is larger and the θ curve is higher, so that the heat convective efficiency is lower for a larger M.

Fig. 6 depicts dimensionless temperature gradient profiles θ' vs. η as $Pr = 1.0,\ Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E1 = 0.1,\ E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$. The figure shows that $-\theta'(0)$ is an important factor for $-\theta'$ at $\eta = 0$ and -$\theta'(0)$ is lower when the magnetic parameter M is larger, so that the heat transfer efficiency is lower at a larger magnetic parameter.

Fig. 7 depicts dimensionless concentration profiles θ vs. η as $Pr = 1.0,\ Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E1 = 0.1,\ E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$. The dimensionless concentration profiles ϕ are satisfied the boundary conditions and varies by a smooth cure for different M values. When the Sc is larger and the ϕ curve is lower, so that the mass transfer efficiency is higher for a larger Sc.

Fig. 8 depicts dimensionless concentration gradient profiles ϕ' vs. η as $Pr = 1.0, M = 0.1;\ Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E = 0.001$ and $Sc = 0.0, 1.0, 5.0, 10, 20$. The figure shows that-$\phi'(0)$ is larger when the Schmidt number Sc is larger, so that the mass transfer efficiency is larger at a larger Schmidt number parameter.

5. Conclusion

A heat and mass transfer with magnetic and electric effects for a steady two-dimensional mixed convection of an incompressible second-grade fluid past a stretching sheet has been studied. Dimensionless heat transfer important factor $-\theta'(0)$ are related to the values of $E,\ Pr,\ E1,\ M,\ Gr$ and Gc, so that the value of $E,\ Pr,\ E1,\ Gr$ and Gc parameters are the important factors in this study. Present study is especially toward the both magnetic parameter effect and mass transfer effect, it will produce lower heat transfer efficiency with a larger value of M, so that the magnetic parameter may reduce the heat transfer efficiency. On the other hand, for mass transfer, the value of Sc parameter is an important factor in this study. It will produce greater mass transfer efficiency with a larger value of Sc. So that, the novelty of the present work is considered the mass transfer phenomena to a coupled heat and mass transfer boundary-layer equations. At last, there are two important results for this study obtained as:

(1) It has found that from Figs. 2–4, when magnetic parameter M increased, the dimensionless fluid velocity decreased. However it is observed that the effect of momentum in the boundary layer, which causes the momentum to decrease, which results in decreasing the fluid velocity. On the other hand, even to the dimensionless fluid velocity first and sec-

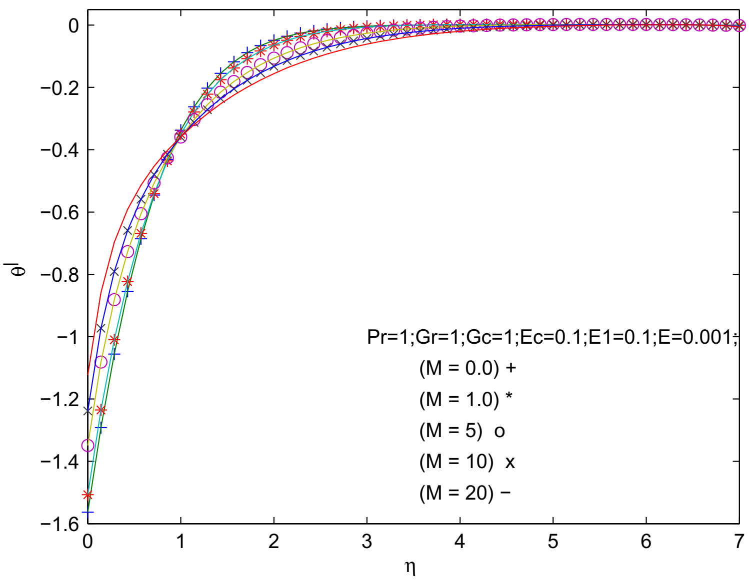

Fig. 6. Dimensionless temperature gradient profiles θ' vs. η as $Pr = 1.0,\ Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E1 = 0.1,\ E = 0.001$ and $M = 0.0, 1.0, 5.0, 10, 20$.

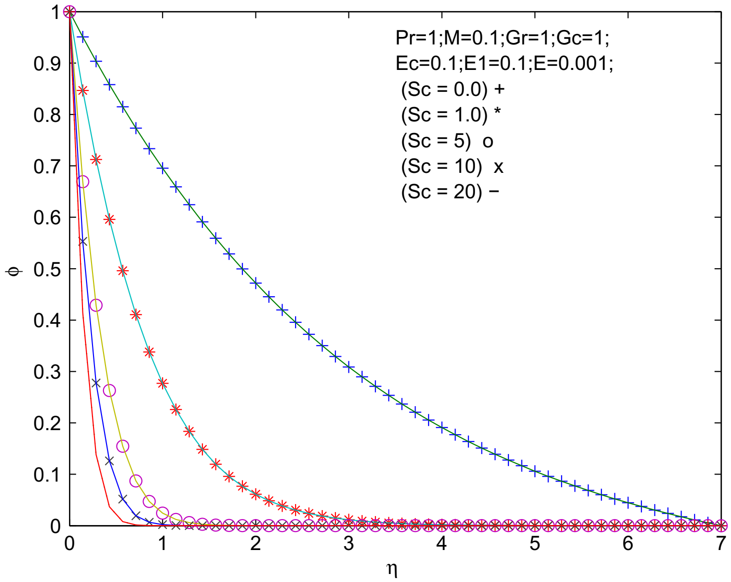

Fig. 7. Dimensionless concentration profiles ϕ vs. η as $Pr = 1.0, M = 0.1; Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E = 0.001$ and $Sc = 0.0, 1.0, 5.0, 10, 20$.

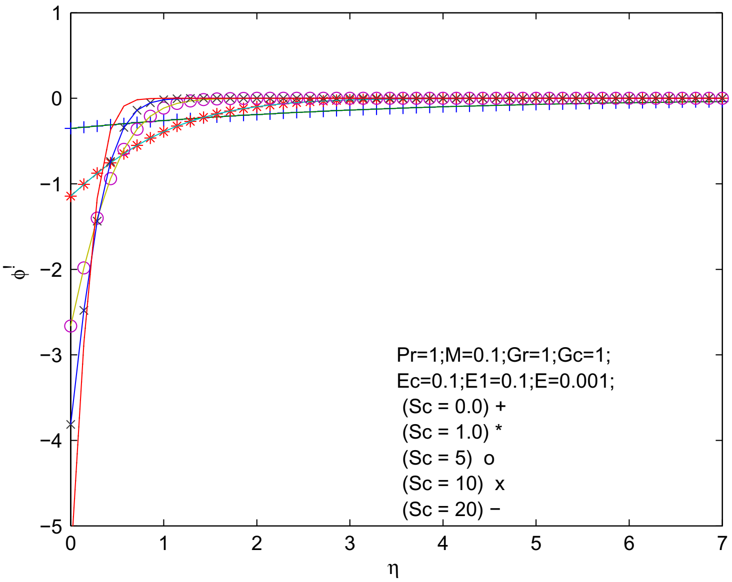

Fig. 8. Dimensionless concentration gradient profiles ϕ' vs. η as $Pr = 1.0, M = 0.1; Gr = 1.0,\ Gc = 1.0,\ Ec = 0.1,\ E = 0.001$ and $Sc = 0.0, 1.0, 5.0, 10, 20$.

ond gradient f' and f'' have the same result to reduce the momentum for the higher magnetic parameter M. It has also found that from Figs. 5 and 6, when the magnetic parameter M is larger, so that the heat transfer effect is lower at a larger magnetic parameter.

(2) It has found that from Figs. 7 and 8, the effect of Schmidt number Sc on mass transfer process may show that the increase of value of Schmidt number Sc results in the decrease of concentration distribution as a result of decrease of the concentration boundary-layer thickness with the increased values of Sc.

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