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Does Everybody Count? Reflections on Reforms in School Mathematics

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Philosophers and sociologists today raise significant questions about traditional epistemology. Feminists, postmodernists, and pragmatists have all challenged the Cartesian notion of a mind separated from the world it knows; they have questioned the unitariness of this mind—whether it is construed as an epistemological subject or as the workings of a particular, individual mind.[1] Universality has been challenged in both ethics and epistemology. Power has been described as a force in the social world, one residing more in structures, institutions, and language than in human agents.[2] We are not so sure any longer about a "constituting subject," a Cartesian notion that reached its heights in the writings of existentialists like Jean-Paul Sartre.[3] Many philosophers now speak of a "constituted subject." But this constituted subject is not the merely responding entity of behaviorist psychology. Its acts cannot be described in terms of causal chains. Rather, it is a subject largely constituted by its situation—by its temporality, language, and cultural, racial, religious, and gender identities.

Despite all this commotion in philosophy, math educators have rarely asked how they themselves are constituted—why they think as they do and what influences their recommendations. With some notable exceptions,[4] math educators fall in with the dominant ideology in public policy. In this paper, I will raise some questions about current policy talk and associated recommendations for reform of school mathematics. In particular, I want to question why we put such emphasis on mathematics for all students. As this question is explored, others will arise: Are there mathematical skills and concepts that everyone needs? Should the content of mathematics courses vary with the needs and interests of students? How might mathematics courses contribute more effectively to general education?

WHY DO WE INSIST ON MATH FOR EVERYONE?

Does everybody count? In *Everybody Counts,* we are led to believe, first, that almost all jobs today require mathematics and, second, that the mathematical

Note to readers: Superscripts refer to the Notes section at the end of the article and contain references which are not in American Psychological Association style format.

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education of our citizens is vital for the maintenance and enhancement of our national economy. Are these claims true, only partly true, or wholly false?[5] The authors of *Everybody Counts* claim that mathematics is the key to opportunity at every level: individual, corporate, and national. They write, "Over 75 percent of all jobs require proficiency in simple algebra and geometry, either as a prerequisite to a training program or as part of a licensure examination."[6] Notice that the writers do not claim that the jobs themselves require algebra and geometry. Clearly, most jobs do not. Access to training or licensure often requires proficiencies unnecessary for the work itself. One should not need a statistical study to be convinced that most people use virtually no algebra or geometry in their personal or work lives. Just drive or walk through the commercial area of a town or city. Consider the host of retail sellers, delivery drivers, police, mail persons, politicians, waiters, cooks, mechanics, cleaners, bartenders, social workers, English teachers, actors, journalists, childcare workers, security guards, and, yes, even physicians who rarely if ever use algebra or geometry. Indeed, if we consider retail sales, it is clear that clerks need less math today than they did several generations ago. They almost certainly need more on-the-job training than their predecessors to handle computers, electronic tags, charge cards, doubtful checks and the like, but they do not need more math. Similarly, physicians and healthcare workers depend heavily on charts and tables rather than their own solutions to various formulas. Further, many of the new jobs being created are actually low-skill and low-paid. It may look sophisticated to work at a computer terminal, but for many people it is just the contemporary equivalent of old-fashioned labor with or without the backache.

None of what I have said so far denies the basic truth of the claim in *Everybody Counts* that, in fact, applicants often must demonstrate some proficiency in algebra and geometry even if their eventual work requires neither. As educators and citizens, we could work toward establishing more realistic requirements for job access. But that sort of activity seems to most of us to be outside our domain of influence. We could and should ask, however, why we endorse the present state of things so heavily and whether there are educational alternatives to the present emphasis on math for everyone.

One admirable reason for introducing all students to algebra and geometry arises precisely because we, as educators, have so little control over the larger social world. We are powerless to insist that young people, regardless of their formal credentials, be given a chance at work and trained on the job. What we can do, we think, is to provide access to the school subjects that are used as gatekeepers. Responsible educators are keenly aware that certain groups of students have been systematically deprived of the mathematics that provides access to better paying jobs, and we want to eliminate that injustice. Thus, we set out to include all children in the mathematics once reserved for a few. We say, "All children can learn,"[7] and "Children *can* succeed in mathematics. If more is expected, more will be achieved."[8] With good will and a sense of justice, we set out to remove grievous inequalities.

But, perhaps, we should think more deeply on the problem. As we academics travel to conferences, lectures, and consulting engagements, we are served by many of the workers I mentioned earlier and a large number of others: hotel maids, airline attendants and ticket counter clerks, hairdressers, cooks, waitresses, truck drivers who transport our food, and farm workers who do the back-breaking work of harvesting it. Those jobs will not disappear if everyone studies algebra and geometry. It may be that all jobs—lower and higher—will be better distributed over racial and gender groups with equitable participation in school mathematics, but the jobs that do not require mathematics will still have to be filled. The work these people do is essential to our society, and it should be recognized as essential. No one who works conscientiously at an honest job should live in poverty, and yet many of these people do. These people *should count;* that is, in a moral sense, the well-being of these workers should matter to all of us.

In today's schools, we often add insult to injury. We suggest that students can escape the sort of labor their parents do by studying mathematics and, to a degree, this is true. It is true if not too many people take us up on it and succeed at it. The social problem remains; over 14 percent of our population now live in poverty, and that figure will in all likelihood grow as companies downsize and employ more part-time and temporary workers. The solution to poverty cannot be found in teaching everyone algebra and geometry. The solution lies more obviously in moral education than mathematical education, and I'll say more about this in a bit.

Everybody Counts, despite its generous spirit, is a seriously misleading document. The authors write, "Unless corrected, innumeracy and illiteracy will drive America apart."[9] They add, "Inadequate preparation in mathematics imposes a special economic hardship on minorities."[10] Educators should reflect deeply on such claims. The truth might well be almost the opposite of such statements. Innumeracy and illiteracy are almost certainly the products of divisions long present and maintained by practices largely outside the control of school. It is not illiteracy and innumeracy that doom people to an impoverished state; it is poverty that induces illiteracy and innumeracy. Further, as I have already pointed out, there is no morally or practically persuasive reason why lack of mathematical preparation should impose a debilitating economic hardship on minorities or on any of the individuals who contribute significant work to our society. Our culture *chooses* to pay such workers a poverty wage.

Many policy makers argue not only that mathematics is necessary for individual success but, also, that our national economy depends on a mathematically well-educated work force. Often the school system is blamed for the drop in national productivity and the loss of jobs. But, in fact, there is little or no evidence that American workers are less productive than their foreign counterparts. The relocation of manufacturing in other, usually third-world, countries is motivated by the desire for higher profits. It is true that we shamefully neglect the education of our noncollege bound students,[11] and this should change. But unless industry leaders pledge themselves to provide jobs here, there may be little incentive to increase the vocational sophistication of noncollege curricula. There are still compelling reasons to improve the education of this group along other lines, and I will discuss some of them in the next section.

If what I have said so far rings at all true, why do we educators endorse the party line so heartily? Many educators have undoubtedly accepted the prevailing arguments about the central importance of mathematics because they are concerned with equality. Others have thought through the issues in a critical manner and will offer rigorous arguments counter to the one I have presented here. But many are just caught up in the flow of events. Herbert Mehrtens has described convincingly how mathematicians in Germany were pressed by the social events leading to Nazism; self-preservation seemed to demand compromise.[12] Even before National Socialism, however, it was clear that self-interest governed much of what was decided in mathematical organizations. To maintain their productivity, mathematicians must have positions. Because most positions for mathematicians exist in universities, mathematicians must teach. Therefore, students of mathematics are required. The majority of math majors become teachers, and, so, if the supply of university and college students is to be maintained, there must be a need for pre-college teachers of mathematics and, thus, for hosts of high school students to take mathematics. Surely, the same conditions apply here and now. Many Ph.D.s in math and physics cannot find positions in which to use their highly developed skills, but there are positions in mathematics education, and the more teachers are required, the more numerous and secure those positions will be.

It is hard to accept the possibility that the arguments we are now making for more math for everyone are elaborate rationalizations, and I am not advancing such a hypothesis as a conspiracy theory. Rather, I am suggesting that we take seriously the ideas on power put forth by Foucault and others. We may indeed be constituted subjects—not wholly autonomous agents. But we can at least exercise our critical intelligence to examine the possibility and to consider what we might do if we become convinced that many current arguments for reform are largely artifacts of the organizational drive for self-preservation. If we believe something like this, or even suspect that we have been influenced a bit along such lines, we might ask ourselves what alternatives might be feasible.

Before considering what might be done differently as a result of honest reflection on our own motivations and the forces that work on us, I should say that, if I were once again teaching high school mathematics, I too would encourage students to take as much college-preparatory mathematics as they need to gain college entrance. Like other conscientious educators, I would want to do my part in enhancing the life-chances of as many students as possible. However, I would proceed with a somewhat heavy heart. What talents will go unnoticed and underdeveloped while I cram algebra and geometry into unwilling minds? What attitudes will my students develop toward work? What sense will they make of their world as a result of my teaching? Rejecting the role of Don Quixote, I would teach algebra and geometry, but I would also consider different forms of mathematics for different students, and I would add topics in the sociology and psychology of mathematics for all students.

IS THERE SOMETHING ABOUT MATHEMATICS THAT EVERYONE SHOULD KNOW?

Mathematics educators have rightly recommended that students today should learn at least some basic material on statistics, computers, and calculators in addition to the fundamental knowledge traditionally associated with elementary mathematics. They are also deeply and properly concerned with the development of pedagogical methods that will engage students and give them some sense of their own power as thinkers and problem solvers. These concerns seem reasonable.

But methods and ideas that are entirely reasonable at the elementary level may not be so at the high school level and beyond. Here, it seems to me, we are overly reluctant to face the fact that human interests vary widely and that many highly intelligent people are just not attracted to mathematics. Others, a smaller number, are fascinated by mathematics and will adopt it as a lifetime occupation and/or recreation. In between, there are those who will need mathematics for further study and those who will actually use it in eventual careers. Should all of these students be compelled to take the same courses?

In other places, I have argued that students should not be *required* to take any mathematics courses in high school.[13] By the time students reach high school age, the mathematics they study should actually be real-world; that is, they should use mathematics and learn new mathematics in the context of work they are considering or have already chosen. Vocational programs should be designed in partnership with local companies, and they should be exciting and rigorous. Colleges should demand college-preparatory mathematics only for courses of study that actually require it, and even those courses that are taught as college-preparatory should vary with the interests and aims of students.

The objection usually thrown up against such a plan is that high school students are too young to know how to choose wisely and that they may change their minds. Some readers may even draw on personal experience and argue, "If I hadn't been forced to take algebra, I wouldn't be where I am today." It is indeed possible that young people will make choices that are unstable. They may in fact change their minds. Lots of people today change their careers even in their forties and fifties. The important thing is that, while they are working energetically on a line of study they have chosen, they will learn; if the courses are exciting and well-taught, they will learn how to learn, gain confidence in carrying out their own plans, feel a part of a significant community, and, with participation in a community, develop civic consciousness and fellow feeling. If, after a few months or years, they want to try something different, a society that talks about life-long learning should make it feasible for them to continue study and preparation for new lines of work.

But, you may protest, mathematics prepares students for so many opportunities. Does it really harm kids to force them to take a couple years of mathematics? I do not know the answer to that, but it is a question we should study seriously. I don't know what talents and interests are lost under such coercion, what levels of confidence are eroded, what nervous habits develop, what rationalizations are concocted, or what evils are visited on the next generation as a result of our benevolent insistence. Alice Miller's work is relevant in this connection.[14] Her description of "poisonous pedagogy" includes all those things we do to children for their "own good" against their own wills and interests and, then, forbid them to complain about. To be effective, poisonous pedagogy must be successful in convincing children that what their teachers and parents did was somehow good for them. Is the coerced study of mathematics poisonous in this sense? Maybe nothing really bad happens and the forced study of math is as healthful as, and no more painful than, forced immunizations or trips to the dentist. But taking mathematics is not like taking a shot or getting one's teeth repaired. "Taking" mathematics does not guarantee learning mathematics. When we force students to take mathematics, we can ensure only that they will have certain credentials to present; we cannot ensure that they have learned the mathematics required for the next level. They may or may not learn skills, concepts, and ways of thinking. As John Dewey argued so persuasively, people are far more likely to learn when they are thoroughly occupied and engaged in what they are doing and when they have participated in the construction of their own learning objectives.[15]

What many of us fear, I suspect, is that, if we allow teenagers to make significant choices, our own child may choose an "inferior" course of study. We want *our* children to go to college and get good jobs—perhaps even more important, to have high status. As fair people, then, we also want other children to have a similar opportunity. What I am arguing, of course, is that there should be no "inferior" course of study, that "tracks" should not be lower and higher but different. Every course of study offered should be enticing to those with relevant interests, and the community should grant all its students and workers the respect and dignity they earn in doing honest work.

At this point, I will insert a paragraph that would not have appeared ten years ago in a policy analysis—certainly not in a scholarly paper. I am going to answer (briefly) the question, "Who is speaking?" Postmodern thinkers, feminist epistemologists, and black feminists especially have argued strongly that this is an epistemologically relevant question.[16] Indeed, when I present my argument for differentiated courses of study in public lectures, members of the audience often ask, "And what about your children? Did *they* go to college?" The answer is that some did and some did not. One boy became a diesel mechanic and now holds a civil service job at considerably higher than average pay. He will be eligible for retirement at a very early age and plans to open a repair shop or sales agency of his own, but he may change his mind on this. Another son studied at a well known culinary academy and hopes, after a lengthy apprenticeship, to be a chef. In fact, my experience in raising a large, diverse family is as important in motivating my recommendations as my experience in mathematics teaching and philosophical analysis.

Whether my disclosure adds anything to the argument I've been trying to make depends on one's view of epistemology. The argument might well stand even if all my children went to college. (It might be, for example, that all of them really wanted to go; it might even be that all were interested in math, etc.) And the argument might hold even if I had had no children of my own. What such disclosure often accomplishes is the creation of trust. Listeners or readers feel that the speaker "has been there" and that her or his words carry the authority of lived experience.[17] But personal experience has many aspects. Our family had the knowledge, will, and means to support children in a wide variety of endeavors. We could help them select training programs that had excellent reputations. As an academic and as a citizen, that is what I am arguing for: excellent programs for all our children. Let's differentiate by interests and try to reduce differentiation by traditional status hierarchies.

To accomplish such a program, teachers would have to assume a counseling function. The psychology of learning mathematics would become part of the mathematics curriculum. Instead of using psychology *on* students to motivate and manipulate them, teachers would use psychology *with* students to help them understand their own motives, working styles, fears, hopes for the future, and how to capitalize on their strengths. With students who are considering (or are already in) strong vocational tracks, teachers could discuss options within the vocation and how an interest in mathematics might bear on those options. With students taking math simply because they must to get into college, teachers could work on test anxiety and teach students all the tricks they know about tests. We should not hesitate, with such students, to concentrate on helping them get respectable grades and high test scores. That does not mean that we should never share challenging problems with them or give them opportunities to achieve a deep understanding of certain topics. We might even joke with them occasionally and say, "Aha! I caught you. You *are* interested in math!" But, on the whole, our attitude should be one of respect for their choices, of understanding and sympathy for the trials our culture visits on them, and of steady commitment to helping them achieve their goals.

With students genuinely interested in math, we might share Poincaré's wonderful account of mathematical creation[18] and Hadamard's study of mathematical invention.[19] We might discuss intuition and elegance. With these students, too, we would work for deeper self-understanding, discuss the loneliness that sometimes accompanies extended intellectual work and the joy that emerges from successful encounters with mathematics. We would help them to understand, also, that their gifts are not "higher" than others—just wonderfully different. Nor, we would assure them, are they odd because they would rather do math than something else.

Sharing the psychology of mathematics learning with students is at least hinted at in several important approaches to mathematics today. Alan Schoenfeld, for example, wants students to adopt the mathematician's world view and learn to understand and monitor their own cognition. He wants them to attend to metacognition.[20] My suggestions are to some degree compatible with his. I would help students who are deeply interested in math to learn something about the way mathematicians look at the world. But I would not push all students to "think like a mathematician." They should learn to use math for their own purposes.

Similarly, the cognitive apprenticeship model recommended by John Seely Brown and his colleagues is not generous enough in its thinking.[21] Many of the techniques they suggest—for example, demonstrating, fading, coaching—are fine pedagogical practices, but the word apprenticeship is misleading. Apprentices lived with their masters; they learned not just skills but a whole way of life. With good masters, they acquired manners appropriate to their occupation and both excellences and moral virtues. In contrast, today's mathematics students typically spend less than an hour a day for less than a year with a given mathematics teacher, and the teacher, faced with more than one hundred students, rarely takes responsibility for the intellectual, emotional, and moral growth of particular students. Further, it is the special task of teachers, in contrast to masters, to encourage students to develop their own talents and styles directed toward their own ends—not to produce people who think and act in the mode of the master. Of course, there should be modeling, and there will be emulation, and teachers must be conscious of what they model and help students to reflect on patterns they emulate, but teachers should not suppose that the thinking and techniques they share must necessarily be adopted. Teaching—at least until graduate school and maybe even then—is properly more like the parent-child relationship than that of master-apprentice. The parenting metaphor has its risks and limits, too. By using it, I do not mean to suggest a relationship of control and dependency, but, rather, one in which the parent is committed to the growth and growing autonomy of the child.

Both the Schoenfeld program and the cognitive apprenticeship model are aimed at providing something of value for all students. My argument is that mathematical thinking and activity cannot and should not be the same for all students. A third movement that attempts to improve the curriculum for all students concentrates on "real-world" problems. This program, like the others, has attractive features.[22] But a particular problem that is "real-world" in the sense that adult human beings actually grapple with it may not be "real" at all in the school setting. Some students may be interested in traffic patterns, models of finance, or actuarial calculations, but many will not be. A problem is real for actual people when they need to or want to solve it. Problems that are real enough in their original or regular settings become strange and unreal in the school setting. Conversely, school problems can be very real to students who identify themselves with the school culture. I am not arguing against problems that are real in the sense that they require formulation, gathering and sorting information, decisions with respect to both means and ends, and attention to alternative solutions. But if students are asked to spend extended periods of time on complex problems, then they ought to have some choice among topics. Slaving away at someone else's real-life problem can be as deadly as doing a set of routine exercises and a lot more difficult.

My argument so far has been that, at least by high school, we should provide different mathematics courses for students with different interests. This recommendation, I said, suggests attention to the psychology of creating and learning mathematics. But we should not use psychological strategies on our students to get them to do things they do not want to do. Rather, it requires us to do psychology (as well as mathematics) *with* them. The object is to educate persons who can make well-informed choices and take increasing responsibility for the direction of their own lives.

In addition to the psychology of mathematics, students need to learn something about the sociology and politics of math. Philip Davis writes:

> It is of vital importance to give some account of mathematics as a human institution, to arrive at an understanding of its operation and at a philosophy consonant with our experience with it and, on this basis, to make recommendations for future mathematical education.[23]

Davis wants to take this thinking in two directions: toward a new philosophy of mathematics and into new approaches to math education. Here we are interested in the latter. What does Davis have in mind? He is concerned with both the benefits and dangers of mathematicization. Remarking on oppressive and constraining features of mathematics, he writes:

> The subconscious modalities of mathematics and of its applications must be made clear, must be taught, watched, argued. Since we are all consumers of mathematics, and since we are both beneficiaries as well as victims, all mathematicizations ought to be opened up in public forums where ideas are debated. These debates ought to begin in the secondary school.[24]

I agree. Students should know what it means socially and politically to live in a mathematicized world: how their futures are partly determined by models of testing, how their insurance rates are affected by mathematicizations of where they live—not how well they drive, how their tastes are shaped by consumer preference models. They may be amazed to learn that their desire for a particular brand of shoe or jeans is not entirely a product of either rational choice or genuine personal desire.

Some years ago, when I was serving as a public school administrator, an insurance investigator called at our office one day and asked to see the records of a boy who had recently died in an accident. At first I thought he might be trying to find out whether the boy was a reckless or irresponsible sort and thus perhaps culpable in the accident. But the truth was simpler and more horrifying. His company wanted to put a "fair price" on the boy's life. From the boy's grades and the courses he took, the company would project how much he might have earned and, therefore, what his life was worth. Students should also learn about the benefits of mathematicization—how air and automobile travel have been made more safe and comfortable, how agricultural output has been increased, and a host of other benefits.

In addition to topics in the psychology and sociology of mathematics, historical and biographical material should be made available. Students should learn that mathematicians have interests beyond mathematics. Many students have spiritual or religious interests, and they may find the theological interests of mathematicians quite fascinating: Descartes' well known proof of God's existence (can students find a flaw in it?), Pascal's criticism of Descartes and his own "wager" on God's existence, Leibniz's attempt to exonerate God from complicity in evil (his *Theodicy* and its discussion of possible worlds), Newton's belief that theology is more important than mathematics, Euler's use of a nonsense proof of God's existence to humiliate Diderot, Bertrand Russell's scathing denunciations of all religions—especially Christianity—and the exploration of contemporary mathematicians into the connections between God and the infinite.[25] Throughout all of these discussions, opportunities will arise to move in a variety of directions. Some students will want to know more about the social customs and politics of particular times, some may want to explore topics in logic and philosophy more deeply, some may engage in personal spiritual quests, and some may not be interested at all.

None of the material discussed so far on the psychology, sociology, history, and biographies of mathematics should become a tightly specified, required part of the curriculum. There is no guarantee that a given student will necessarily be more interested in these topics than in any others. But texts should include such material, and teachers should be prepared to lead discussions when the topics arouse interest. The more connections that teachers can make with other subjects and with the personal lives of their students, the more likely that each student will find something of interest.

So far I have said nothing about the conventional topics of mathematics and which ones are essential for all students to know. Beyond basic arithmetic I would hesitate to name anything as essential, but this is an area in which continued research might be very useful. How *do* people use mathematics in their jobs and everyday lives? Are their mathematical intuitions and skills increased or decreased by school knowledge? This last is not a foolish question. Nunes has shown how relatively unschooled young people can figure things out informally but become horribly confused when asked to use the standard algorithms taught in schools.[26] Similarly, I have seen many intelligent students draw the following invalid conclusion after elementary instruction in logic:

1) All fish can swim.
2) I can swim.
3) *I am a fish.*

Before instruction in logic, these students would not have announced to the world that they were fish! There is much to learn about both the harms and the benefits of our teaching.

Before concluding this section, I want to make it clear that I am *not* saying, "Math is unnecessary for most people. Throw it out!" Rather, I am urging open-minded skepticism and criticism of current recommendations for reform. We should not allow ourselves to be pushed into politically correct nonsense about uniform and universal capabilities and, from there, into well-meaning recommendations that coerce students into studies that waste their real talents and demean their genuine interests. Clearly, we are often pressed into such positions by a legitimate concern for poor and minority students, but we might respond to this concern in more healthy ways than to exercise more universal coercion. Robert Davis and Carolyn Maher have raised excellent questions concerning the problems minority children experience with school mathematics.[27] They, too, urge that we look beyond the drills, skills, and concepts of ordinary school mathematics to ask: Who has fun with mathematics and where (in school or outside?)? Do these young people envision a future for themselves? What sort? How do these students look at work? at play? How do they evaluate their own work? How do they look at errors? Do they help one another? Questions of this sort are more fundamental than those that proceed from abstract models drawn from the mathematical experience of a few and hastily proclaimed as mathematics for everyone.

However, some students *are* interested in mathematics for its own sake. These are the kids who enjoy the unfolding patterns of hard factoring problems and trigonometric identities, do logic puzzles for fun, and appreciate the beauty of mathematical proof. Here I agree with Professor Wu (this issue) when he expresses concern about the effects of our current de-emphasis on proof and the lack of depth in some exploratory exercises. Surely some students should have mathematical experience that includes proof and depth. Students need not be separated into classes explicitly or implicitly labeled poor, fair, good, and excellent in order to provide appropriate mathematical experiences for all of them. As I said earlier, some students may need no formal mathematics in high school, although I think they should be exposed to psychological, sociological, and cultural aspects of mathematics somewhere. Many others should learn the mathematics they need in real contexts, not in formal math classes. Still others can profit from considerable formal study without great emphasis on proof or, even, deep mathematical understanding. And there are those who are passionately interested in mathematics. Instead of calling classes for these students "honors classes," we might call them "classes for the passionately interested" and allow any students to enter who are willing to devote the required time and energy.

As I read Professor Wu's papers, I fell to thinking about the difficult and powerful work some of my geometry students were able to do years ago. In a unit on altitudes, medians, and bisectors, many of them were able to complete three kinds of proofs for several of the theorems: a traditional demonstrative proof, one using analytic geometry, and one using vectors. The purpose of working in such depth was to help students appreciate freedom and choice in mathematics. Placement of the coordinate grid could make the solution easy or difficult, clumsy or elegant. Similarly, the notation used for a vector proof could facilitate or impede progress toward a proof. Further, even when students could not complete a proof using a particular method, they often said, "Thank goodness there's another way." It seems a shame to deprive interested students of such encounters just because the majority of students neither want or need them.

I will express one final note of skepticism on the current reform movement. Teachers all over the country are being urged to use forms of cooperative learning in mathematics classes. Some states even mandate the use of such methods. By and large, working together makes great sense. Reasonable teachers twenty and thirty years ago encouraged their students to work together. The present emphasis on a community of mathematical thought is also laudable. But there are two issues that should induce critical thinking. One is the proliferation of competing approaches to cooperative learning, each presenting more and more sophisticated and precise methods requiring considerable training. To define cooperative learning too narrowly and precisely is to risk implementation failures and the eventual loss of the entire movement. Teachers, like students, need to be encouraged to create and adapt, not just follow instructions and implement. Second, we should not get carried away with cooperative learning. Much of the hardest and most lasting mathematical learning is done in solitary concentration, and there is a certain joy, or at least satisfaction, that comes from working a problem through on one's own.

Perhaps the guiding principle is that we should care for and respect all our students and that we do this best by working with them closely enough to know what their interests are and the sorts of futures they envision. Of course, we should guide and shape those visions and interests, but we should not force them into one model, nor should we suppose that a mathematics program, however good it may be for some, can meet the purposes of all.

A POSTSCRIPT ON TEACHER EDUCATION

Every curriculum project or framework engenders teacher training programs, and the failure of such projects is often blamed on faulty implementation. Teachers, for whatever reasons, just cannot or will not do things according to the new prescriptions. What chance has any reform if teachers are incompetent or, at least, recalcitrant?

Surely, if high school math curricula are to include psychological, historical, political, and sociological explorations, teachers will need a different kind of preparation from the one they receive now. They will need mathematical preparation especially designed for teachers.

On this topic, too, we have to exercise some postmodern skepticism. Why, for example, do so many of us buy into current recommendations for professionalization? Why do many of us insist that preparation for teaching should be conducted at the graduate level?[28] What we would like, of course, is for teaching to be a "real" profession, one like law and medicine. But it is obvious that most would-be teachers are not going to invest in three or four years of graduate school to enter a "profession" in which they have little autonomy, poor pay, and low status. Not so obvious is the possibility that medicine and law are not the examples we should emulate. Neither rates very high today in public credibility. A careful analysis of the contemporary movement toward professionalization culminates in the unsettling possibility that it is only about status, that it has little if anything to do with the actual improvement of teaching.[29]

If we were to take seriously the ideas I have sketched here, we might think of designing programs for teachers that more nearly resemble those of engineers instead of physicians and lawyers. College students would select education and their teaching major in their sophomore year. A math major would then study mathematics from the perspective outlined above. All or nearly all of the courses would treat psychological, historical, biographical, sociological, theological, and political aspects of mathematics. There would be references to music, art, and architecture, to war, insurance, taxes, and medical care. Would there be time for "real" mathematics, if all this must be done? Of course there would, but it would be real mathematics for teaching. Those preparing to teach need to know the high school mathematics curriculum very well, but they do not need the standard courses taken by those planning to do graduate mathematics, theoretical science, actuarial work, or engineering. They need broad and rigorous courses *designed for teachers.*

The present emphasis on subject matter majors for all teachers (even elementary school teachers) is, on one level, a highly conservative recommendation. It assumes that the problems of teaching boil down to status and that, if we can show that our people are educated in exactly the same way as those who enter more prestigious fields, we will at one and the same time improve the status of teachers and gain respect for our own professional programs. After all, graduates would enter these programs from the same undergraduate base as other prospective professionals and, presumably, their choice would be a real choice, not one for "something easy." This thinking, as I said, is conservative because it bolsters a system that has little to commend it. Undergraduate liberal education needs massive transformation, not acceptance and emphasis.

On another level, educators' recommendations that all teaching candidates present a standard undergraduate major is somewhat insulting. It demeans what could be a proud and distinctly knowledgeable profession. If teachers had the sort of preparation we've been discussing here, there would be no question whether teachers know something that others do not know, and we would not have to discuss teacher knowledge merely in terms of classroom management, modes of evaluation, and alternative methods of presenting a topic. None of these is unimportant, but the full knowledge of mathematics as a social enterprise is more important, and that cannot be achieved in a single intensive year—especially not when that year follows four impoverished undergraduate years.

Now, of course, the objection we encountered earlier to differentiated high school programs arises in full force. Suppose young people, forced to choose teaching early in their college careers, change their minds? Won't all that preparation be wasted? The answer has to be two-fold: First, even if they change their minds, the education they will have undergone will be a fine one, and they will have had the wonderful experience of working on material of vital interest—material central to the interests they had at the time of choice. Second, we should *not* use the fact that people change their minds to excuse our failure to prepare adequately those who do not change their minds. Surely, it is better to be thoroughly prepared for a field one plans to enter (even if one's mind changes) than to be ill-prepared in the name of a nebulous "general" preparation. The traditional response to this line of thinking is that a broadly defined general ("liberal") education can prepare a wide variety of students for a multitude of varying occupations or further studies. I think it is time to question this assumption.

Could a program of the sort I envision be established in our colleges and universities? Asked this question, we come full circle to the concerns with which we started. Why do we make the recommendations we hear ourselves utter? Why do we not stand back and criticize our own enterprise? Who would teach the courses I have suggested? Why would university mathematics teachers resist such courses? Why have university teachers generally resisted designing courses for teachers, referring to them as "watered-down" math courses?

Turning the postmodern critical eye on myself and my own recommendations, I have to admit that my views, like all others, spring from a set of situations and conditions: from parenting that saw some of my children thrive and more of them suffer under irrelevant school experience, from a fairly good theoretical mathematical training, from academic experience that emphasized philosophy, from teaching experience that allowed me time and again to teach the same students for several years, from a habit of omnivorous reading, from an unfulfilled and yet oddly enriching fascination with religion, from sources I have not even identified. But the recommendations and arguments, whatever their source, now take their place beside other recommendations and arguments. If they ring true for others—students, teachers, parents, policymakers—then the voice of dissent from the current reform movement will grow. At least, perhaps, more of us will ask deeper and harder questions about what we are doing and why.

Notes

1. See, for example, Richard Bernstein, *The New Constellation* (Cambridge: MIT Press, 1992); Louise M. Antony and Charlotte Witt, Eds., *A Mind of One's Own: Feminist Essays on Reason and Objectivity* (Boulder, CO: Westview Press, 1993).

2. Michel Foucault is the prime-mover on the subject of power. See David Hoy, Ed., *Foucault: A Critical Reader* (Oxford: Basil Blackwell, 1986); also Michel Foucault, *Discipline and Punish: The Birth of the Prison:* trans. Alan Sheridan (New York: Vintage, 1979).

3. Jean-Paul Sartre, *Essays in Existentialism,* ed. Wade Baskin (Secaucus, NJ: Citadel Press, 1977).

4. For some essays that take the new epistemologies seriously, see Sal Restivo, Jean Paul Van Bendegem, and Roland Fischer, Eds., *Math Worlds* (Albany: State University of New York Press, 1993); Paul Ernest, too, at least raises the question whether the traditional cognizing subject should be accepted as unproblematic. See Ernest, "Constructivism, The Psychology of Learning and the Nature of Mathematics: Some Critical Issues," *Science and Education,* 1993, 2(1): 87–93.

5. See National Research Council, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Washington, D.C.: National Academy Press, 1989).

6. Ibid., p. 4.

7. For criticisms of this slogan, see Nel Noddings, "Excellence as a Guide to Educational Conversation," in *Philosophy of Education* 1992, ed. Henry Alexander (Urbana, IL: Philosophy of Education Society, 1992) pp. 5–16; also *Teachers College Record,* 1993, 94(4): 730–743.

8. *Everybody Counts,* p. 2.

9. Ibid., p. 14.

10. Ibid., p. 21.

11. William Brock, former secretary of labor, pointed out forcefully how shamefully we neglect our noncollege bound in an interview, July 23, 1990, with *Time,* pp. 12, 14.

12. See Herbert Mehrtens, "The Social System of Mathematics and National Social-

ism: A Survey," in *Math Worlds*, ed. Sal Restivo, Jean Paul Van Bendegem, and Roland Fischer (Albany: SUNY, 1993), pp. 219–246.

13. See Nel Noddings, *The Challenge to Care in Schools* (New York: Teachers College Press, 1992).

14. For an analysis and documentation of the unhappy results of coercion and rigidity in parenting, see Alice Miller, *For Your Own Good,* trans. Hildegarde and Hunter Hannun (New York: Farrar•Strauss•Giroux, 1983).

15. See John Dewey, *Experience and Education* (New York: Collier Books, 1963; original published 1938).

16. See the discussion in Patricia Hill Collins, "The Social Construction of Black Feminist Thought," SIGNS, 1989, 14(4): 745–773; see also Collins, *Black Feminist Thought: Knowledge, Consciousness, and the Politics of Empowerment* (Boston: Unwin Hyman, 1990).

17. See ibid.

18. Henri Poincaré, "Mathematical Creation," in *The World of Mathematics,* ed. James R. Newman (New York: Simon & Schuster, 1956) pp. 2041–2050.

19. Jacques Hadamard, *The Psychology of Invention in the Mathematical Field* (New York: Dover, 1954).

20. See Alan Schoenfeld, "Learning to Think Mathematically: Problem Solving, Metacognition, and Sense making in Mathematics," in *Handbook of Research on Mathematics Teaching and Learning,* ed. Douglas A. Grouws (New York: Macmillan, 1992) pp. 334–370.

21. See John S. Brown, A. Collins, & P. Duguid, "Situated Cognition and the Culture of Learning," *Educational Researcher,* 1989, 18(1): 32–42.

22. See the essays in Robert B. Davis and Carolyn A. Maher, Eds., *Schools, Mathematics, and the World of Reality* (Boston: Allyn and Bacon, 1993).

23. Philip J. Davis, "Applied Mathematics as Social Contract," in *Math Worlds,* p. 182.

24. Ibid., p. 189.

25. See Nel Noddings, *Educating for Intelligent Belief or Unbelief* (New York: Teachers College Press, 1993).

26. Terezinha Nunes, "Learning Mathematics: Perspectives from Everyday Life," in *School, Mathematics, and Reality,* pp. 61–78; see, also, Geoffrey B. Saxe, "Candy Selling and Math Learning," *Educational Researcher,* 1988, 17(6): 14–21.

27. Robert B. Davis and Carolyn A. Maher, "What are the Issues?" in *Schools, Mathematics, and Reality,* pp. 9–34.

28. See Holmes Group, *Tomorrow's Teachers* (East Lansing: Holmes Group, 1986).

29. See Nel Noddings, "Professionalization and Mathematics Teaching," in *Handbook of Research on Mathematics Teaching and Learning,* pp. 197–208.