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Comparative performance of locally nonlinear matched filters and of optimal tradeoff filters

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Abstract

A comparison of the performance of Locally Nonlinear Filters (LNMFs) and Optimal Tradeoff Filters (OTFs) for the recognition of single images has been carried out. The comparison is done using several performance criteria such as Signal to Noise Ratio, Peak to Correlation Energy Ratio and Optical Efficiency. A strong similarity between the whitening strategies of both filter families has been found when the Optimal Tradeoff Filters for peak sharpness and noise robustness are compared with LNMFs. Our tests show a marginal superiority of OTFs over LNMFs.

1. Introduction

Optical Correlation has proven to be a powerful method for pattern recognition. Since the introduction of filters matched to the target [1], much effort has been directed towards improving correlation results. The ideal correlation output should yield a sharp peak for each target in the input scene. In addition, the correlation process should have a strong immunity against noise and an acceptable optical efficiency, which means that light attenuation in the optical correlator has to be kept to a moderate level.

One of the key aspects in optical correlation is related to filter design. A matched filter designed from the Fourier spectrum of the target can be whitened in several ways so that the correlation peak parameters are modified to obtain a particular behavior. The Phase-Only Filter [2] is a good example of how a whitened matched filter can lead to very narrow correlation peaks, maximum optical efficiency and medium noise resistance. The importance of whitening has been pointed out for several filter designs such as the Optimal Trade-off Filter (OTF) [3,4] and the Locally Nonlinear Matched Filter (LNMF) [5]. These two filter designs make the phase of the filter equal to that of the Fourier spectrum of the target and suggest an amplitude distribution that depends on a set of parameters; this allows tuning of some of the properties of the filter response. OTFs and LNMFs offer alternatives in addition to those of matched filters and phase-only filters, when a particular pattern recognition problem is proposed, allowing for a better adaptation to specific conditions.

In this paper, the results of a numerical comparison of the performance of LNMFs and of OTFs are presented. A number of well-known performance criteria have been evaluated for a representative sample of the two filter families.

2. Performance measures

A performance comparison between two filters can involve several aspects that are important to the corre-

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lation process. In this work, performance criteria for measuring noise resistance, peak sharpness and optical efficiency have been considered [6,7]. For the following definitions, $H(u, v)$ denotes a filter function, $F(u, v)$ denotes the Fourier spectra of the input scene and $P_n(u, v)$ denotes the power spectral density of the noise at the input. It is also assumed that the target is located at the origin of the input plane.

2.1. Signal-to-noise ratio

The output Signal-to-Noise Ratio (SNR) can be defined as

$$\mathrm{SNR} = C_0^2/\mathrm{MSE}\,, \tag{1}$$

with

$$C_0^2 = \left| \iint F(u, v)\ H^*(u, v)\ \mathrm{d}u\, \mathrm{d}v \right|^2 \tag{2}$$

and the Mean Squared Error (MSE) is

$$\mathrm{MSE} = \iint P_n(u, v)\ |H(u, v)|^2\ \mathrm{d}u\, \mathrm{d}v\,. \tag{3}$$

Filters with higher values of the SNR are better because this implies more robustness against input noise. When only the SNR is considered, it is easy to show the optimality of the matched filter for additive white Gaussian noise. Although the SNR definition of Eq. (1) is different from classical SNR definitions, it is preferred because only one correlation is required to have a SNR estimation.

2.2. Peak-to-correlation energy ratio

The Peak-to-Correlation Energy ratio measures the sharpness of the correlation peak according to the relation

$$\mathrm{PCE} = C_0^2/\mathrm{CPE}\,, \tag{4}$$

where

$$\mathrm{CPE} = \iint |F(u, v)|^2\ |H(u, v)|^2\ \mathrm{d}u\, \mathrm{d}v\,. \tag{5}$$

It may be shown that the PCE values are $\in [0, 1]$, and that the inverse filter yields its maximum value. The PCE has been found to be well suited for measuring peak sharpness. For targets such as those used in our simulations, PCE values are of the order of 10^{-3} for a matched filter and of 10^{-2} for a phase-only filter.

2.3. Optical efficiency

The Horner efficiency measures the proportion of the input energy that is available at the correlator output:

$$\mathrm{OE} = \mathrm{CPE} \left(\iint |F(u, v)|^2\ \mathrm{d}u\, \mathrm{d}v \right)^{-1}. \tag{6}$$

A modified Horner efficiency measure substitutes in the CPE in the numerator of Eq. (6) for the energy C_0^2 in the correlation peak. The OE for matched filters is usually about 35% while for inverse filters it is lower than 0.001%, and phase-only filters guarantee maximum OE.

3. LNMFs background

Inverse filters, phase-only filters and matched filters can be obtained from the following definition

$$H(u, v) = |F(u, v)|^m \exp[\mathrm{i}\Phi(F(u, v))]\,, \tag{7}$$

when m is equal to -1, 0 and 1, respectively. Fractional power filters [6] can also be obtained from the expression above, when m takes real values $\in [-1, 1]$.

The LNMF is based on the idea of combining in a single filter the interesting properties of an inverse filter, a phase-only filter, and a matched filter. The proposed heuristic is that the amplitude values of the LNMF should be taken from an inverse filter for low frequencies, from a phase-only filter for medium frequencies and from a matched filter for high frequencies. However, the transition from one region into another can be smooth and several functions could be used to model how the whitening of the spectrum is done.

LNMFs are defined by the expression

$$H(u, v) = |F(u, v)|^{1-m(\rho)} \exp[\mathrm{i}\Phi(F(u, v))]\,, \tag{8}$$

with $\rho = \sqrt{u^2 + v^2}$.

The function $m(\rho)$ was originally suggested to be a Gaussian function, but other forms of dependence can also be considered. In the case of a Gaussian function

$$m(\rho) = C \exp(-k\rho^2) , \tag{9}$$

where C and k are the adjusting parameters of the LNMF; particular filter forms for the inverse filter, the phase-only filter and the matched filter can be obtained with (C, k) equals to $(-2, 0)$, $(-1, 0)$ and $(0, 0)$, respectively.

4. OTF background

The PCE and the SNR are complementary performance criteria: to increase the PCE a filter should look for a tighter adaptation to object details whereas to increase the SNR the filter should have a more flexible adaptation. First generation OTFs optimized this compromise between the PCE and the SNR by using the Lagrange multipliers method. The Horner efficiency was later included in the optimization process, leading to the OTF for peak sharpness, noise resistance and Horner efficiency. The latter is obtained by minimizing an objective function that is a linear combination of the MSE, the CPE and the energy (C_0^2) in the correlation peak, constraining the filter module to be less than one. After solving the optimization problem, the OTF obtained is

$$\mathrm{OTF}(u, v) = \sigma_\lambda \left[\frac{F(u, v)}{\mu S(u, v) + (1-\mu)\,|F(u, v)|^2} \right], \tag{10}$$

where $S(u, v)$ is the covariance matrix of the noise and $F(u, v)$ is the Fourier spectrum of the target. The parameters λ and μ can be adjusted to obtain a particular behavior of the OTF, and they must satisfy the condition that $\mu \in [0, 1]$ while $\lambda \in [0, \infty]$. The function σ_λ is defined as,

$$\sigma_\lambda(x) = \begin{cases} \lambda x, & \text{if } |x| \leqslant 1/\lambda \\ \mathrm{phase}(x), & \text{otherwise} . \end{cases} \tag{11}$$

Similarly to the LNMF case, the inverse filter, the phase-only filter, and the matched filter can also be obtained as particular cases of an OTF by taking the parameters (λ, μ) as $(\epsilon_0, 0)$, $(\infty, 0)$ and $(\epsilon_0, 1)$, respectively (ϵ_0 is a positive constant as close to zero as possible).

The OTF is optimized in two steps. In the first step, a trade-off between the SNR and the PCE is set up by means of the parameter μ; the nearer to unity is μ, the higher is the SNR obtained. After this trade-off is done, the optical efficiency can be adjusted using the parameter λ in the following way: the modulus of all the components of the filter are multiplied by λ and those that remain higher than one are set to one.

The OTF optimality is guaranteed in the following sense: given an OTF filter with a set of values for some performance criteria ($E_1, E_2, \ldots, E_n$) there is no other filter that can improve the value of one of the criteria without decreasing one of the others.

5. Simulation

Comparison of two filter designs is difficult, especially when three performance criteria are involved. In this case there is an additional difficulty, given that the numerical values of the performance criteria cannot be set during the filter design stage, by calculating one filter of each family with equal values for two performance criteria and then comparing the performance on the third criteria. The process must be developed backwards: a filter is calculated and then the values of the performance criteria are estimated.

The tests went as follows. First a good number of LNMFs and OTFs were computed according to the

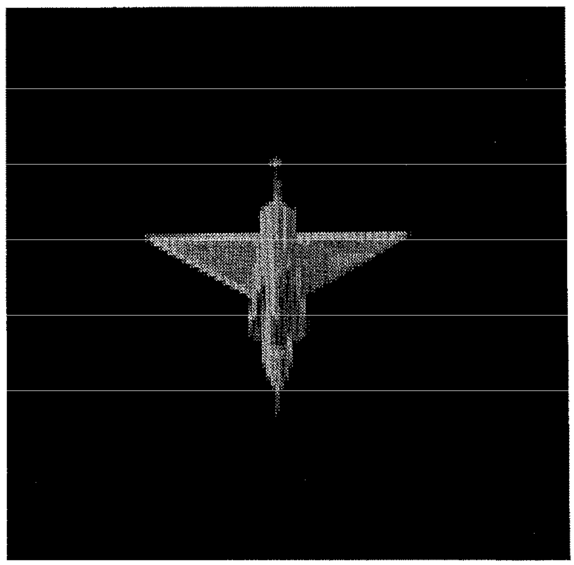

Fig. 1. Target image sampled at 256 gray levels over 128×128 pixels.

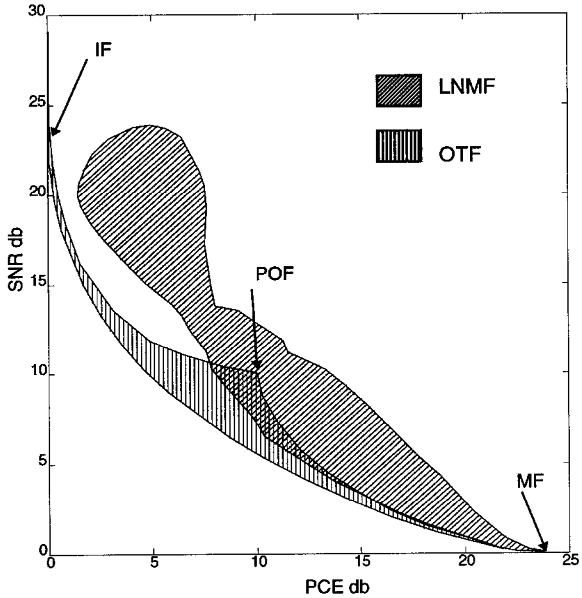

Fig. 2. Possible values of the peak sharpness and noise resistance that LNMFs and OTFs can yield.

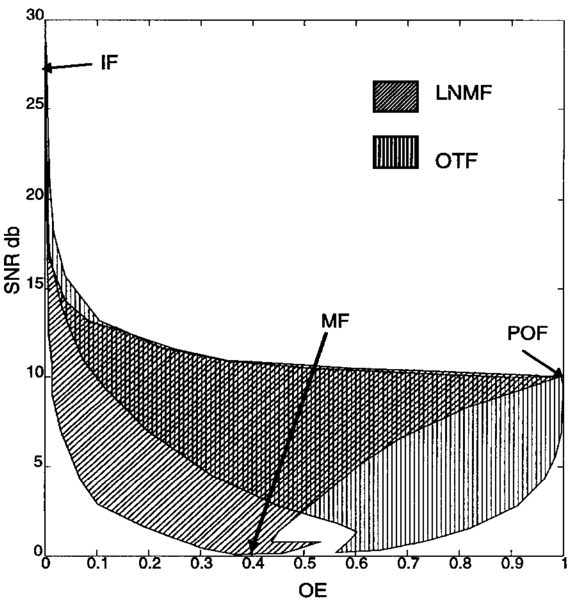

Fig. 3. Possible values of the optical efficiency and noise resistance that LNMFs and OTFs can yield.

definitions of Eqs. (8) and (10); the image of an airplane (see Fig. 1), sampled at 256 gray levels of 128×128 pixels was used as a target; the parameters, (C, k) for the LNMFs and (λ, μ) for the OTFs were adequately chosen so that the resulting filters covered well all the possibilities that each design offers. Then

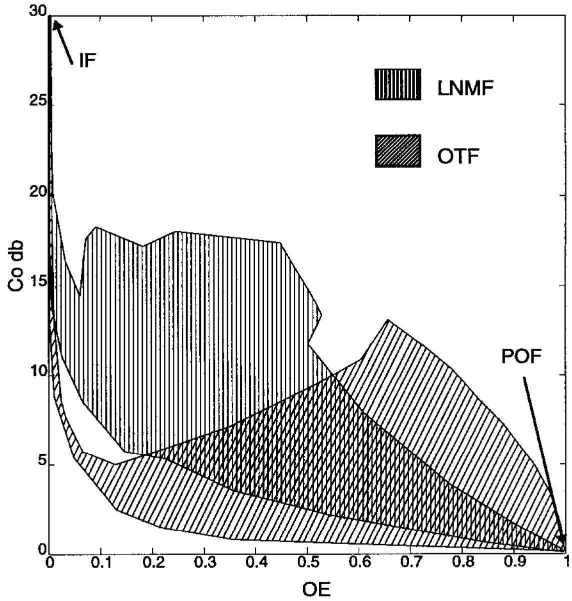

Fig. 4. Relationship between the correlation peak energy and the optical efficiency for the two filter families.

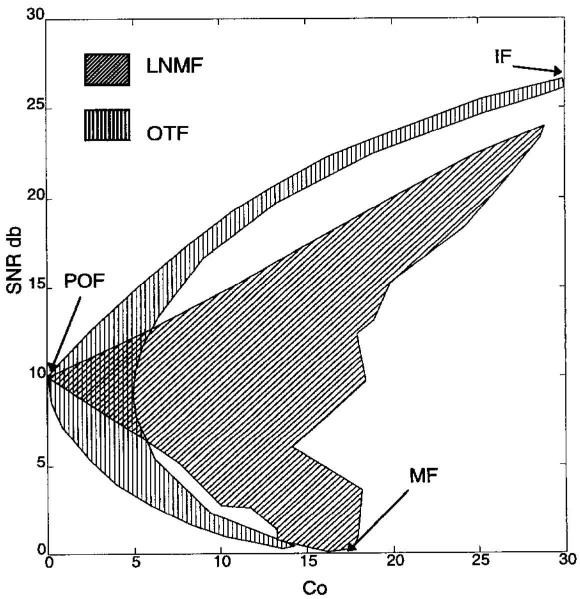

Fig. 5. Possible values of the correlation peak energy and noise resistance that LNMFs and OTFs can yield.

the correlations between the target image and each filter and the values of the three performance criteria were evaluated. Finally, several figures were designed to clearly show the differences between the filter performances. The same test was then repeated with several other images and similar behavior was observed in the results although the numerical values were different. The mean value of the intensity of the target was subtracted before the tests to obtain a more uniform energy distribution in the Fourier plane. SNR values have been estimated assuming white Gaussian noise with zero mean and a variance equal to one.

The values for some of the figures were taken in dB, using as a reference the maximum possible value for each criterium:

$$\mathrm{SNR\ dB} = 10\ \log\left[\frac{\mathrm{SNR_{MF}}}{\mathrm{SNR}_X}\right], \tag{12}$$

$$\mathrm{PCE\ dB} = 10\ \log\left[\frac{\mathrm{PCE_{IF}}}{\mathrm{PCE}_X}\right] = 10\ \log\left[\frac{1}{\mathrm{PCE}_X}\right], \tag{13}$$

$$C_0\ \mathrm{dB} = 10\ \log\left[\frac{C_{0\mathrm{POF}}}{C_{ox}}\right], \tag{14}$$

in the above expressions, the subscript X holds for the filter that is being considered.

6. Results

Fig. 2 shows the possible combinations of (SNR, PCE) obtained with many LNMFs and OTFs. As pointed out by several authors, these two criteria are mutually exclusive because one of them cannot be increased without a decreasing of the other. Overspe-

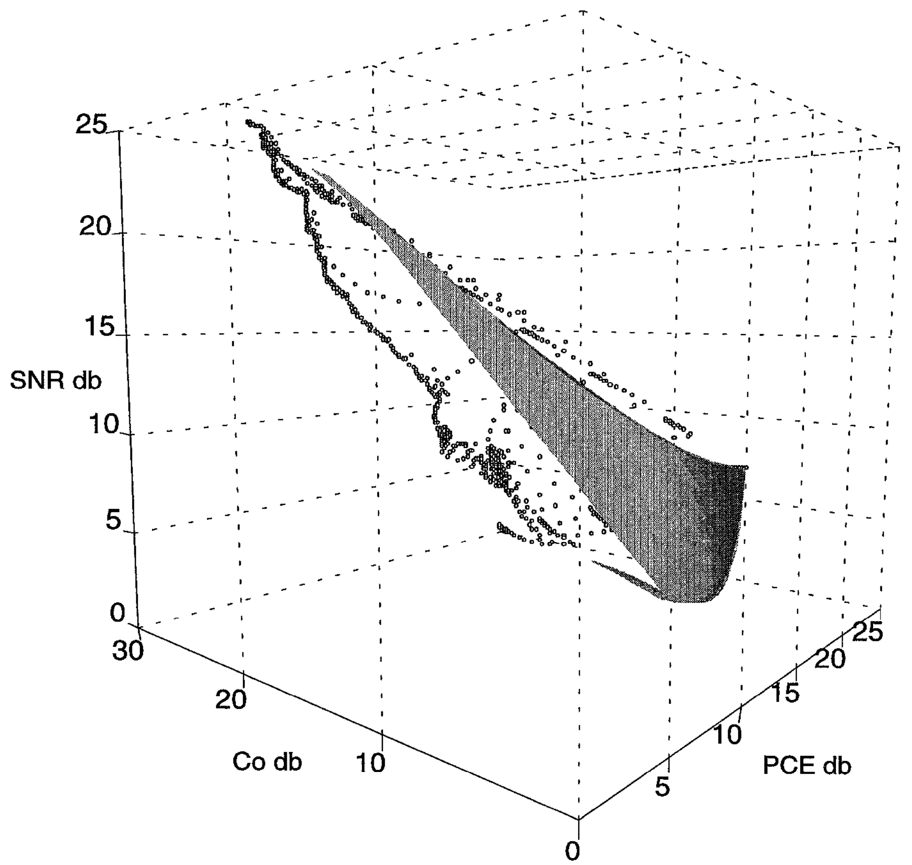

Fig. 6. 3D representation of the three performance criteria for both filter families. The shaded surface corresponds to OTFs and the points correspond to LNMFs.

cialization of the matched filter and of the phase-only filter is clearly observed because the corresponding points for those filters are at the extreme of one axis. A filter that could simultaneously maximize the SNR and the PCE should be located at the origin of the figure, so in a certain way, the distance to the origin can be considered a measure of the degree of specialization of the filter. This reasoning leads to the conclusion that in general, OTFs perform better than LNMFs. However, the OE has not been considered in this case, and the filters near the origin could be unacceptable for optical implementation if their OE is too low.

In Fig. 3, the possible (OE, SNR) values are plotted for several LNMFs and OTFs. As in Fig. 2, the distance of one of the filters to the point (1, 0), measures the specialization of the filter in the sense of OE and SNR.

Fig. 4 shows the dependence between the optical efficiency and the energy of the correlation peak. The figure shows that a given level of optical efficiency could be associated with strong differences in the correlation peak height. This means that it matters not only that a good amount of energy cross over the filtering plane, but that it be redirected to form correlation peaks.

In Fig. 5 is shown the behavior of the SNR for the different correlation peak energies obtained in the simulations. As in Fig. 1, the distance to the origin of coordinates gives an idea of the degree of specialization of the filter.

Finally, in Figs. 6 and 7, a 3D plot with the three performance criteria in dB along the axes, is used to show the performance of the two kinds of filters. Figs. 2 and 4 correspond to the sideview and the front view of those 3D plots. The OTF family trend is drawn as a surface and LNMFs are plotted as independent points; with the 3D perspective of the figure, the LNMFs are

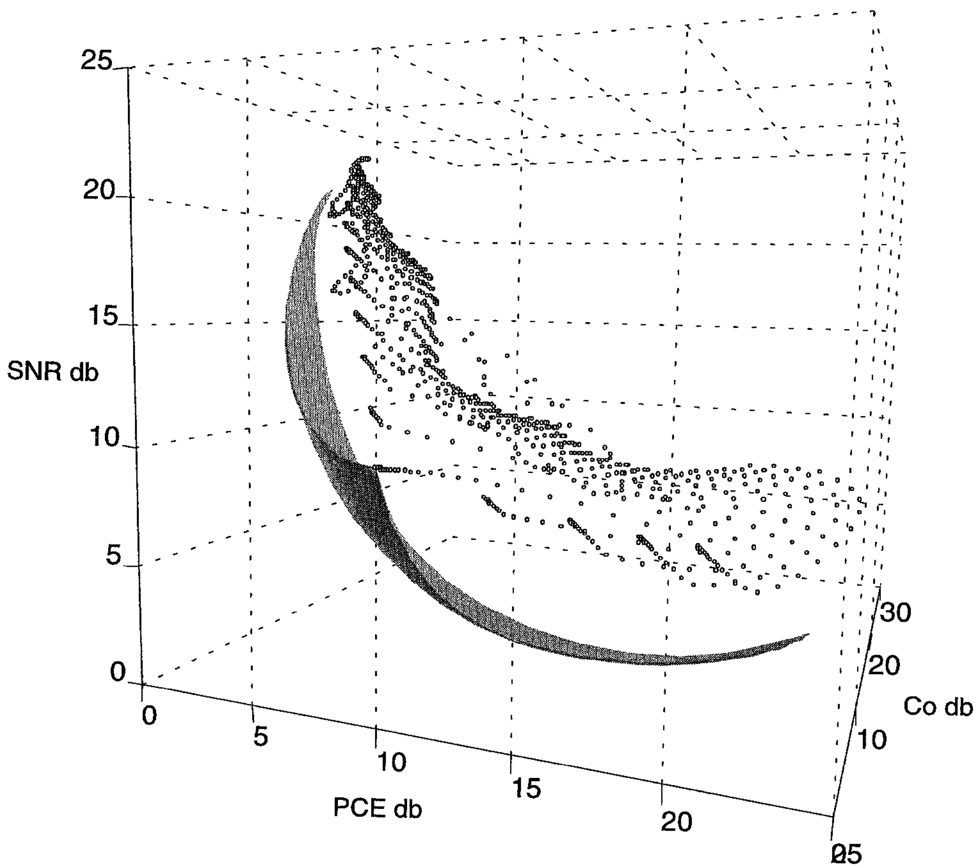

Fig. 7. The same surface of figure 6 but vertical axis has been turned counterclockwise to improve seeing.

behind the surface of the OTFs, and hence always farther from (0, 0, 0) or the ideal filter representation.

7. Discussion

We pointed out at the beginning of this paper that when the recognition of a single image is considered, OTFs and LNMFs are different approaches for whitening a target filter. However Eqs. (8) and (10) show the strong difference in their whitening strategies: OTFs whiten in terms of the filter modulus while LNMFs whiten in terms of the spatial frequency.

A detailed analysis of the differences between the whitening strategies of the OTF and the LNMF can be done only when a particular form of a Fourier spectrum is considered. Some interesting conclusions can be obtained if the Fourier spectrum of a one dimensional target (in a strong simplification) is assumed to take the form of a negative exponential:

$$F(u) = e^{-su}. \qquad (15)$$

To obtain an explicit equivalence between the possible OTFs and LNMFs for this kind of target is not very easy when the OTF optimizes the three performance criteria: SNR, PCE and OE; however, if OTF_1 that optimizes only the PCE and the SNR is considered:

$$\mathrm{OTF}_1(u, v) = \frac{F(u, v)}{\mu S(u, v) + (1-\mu)\,|F(u, v)|^2}, \qquad (16)$$

we can find an equivalent LNMF by equaling eqs. (8) and (16) which yields

$$m(\rho) = \frac{\log(\mu + (1-\mu)\,e^{-2s\rho})}{s\rho}. \qquad (17)$$

The above expression shows the basis of the OTF in terms of a whitening exponent of the LNMFs. The function $m(\rho)$ is illustrated in Fig. 8 for several values of μ. It may be observed how the whitening is hard at the center and softer at the outside of the spectrum for almost any value of μ, as the heuristic used for the LNMFs proposes. A more general conclusion that could be extracted from Fig. 8 is that OTF_1s whitens more the areas of the spectrum that have high modulus values. On the other hand, the LNMF behavior is obtained due to a whitening that is stronger for low spatial frequencies. The similarity of those two principles comes from the fact that common objects usually have a spectrum with higher modulus values at the center. The reason for the superiority of OTFs is per-

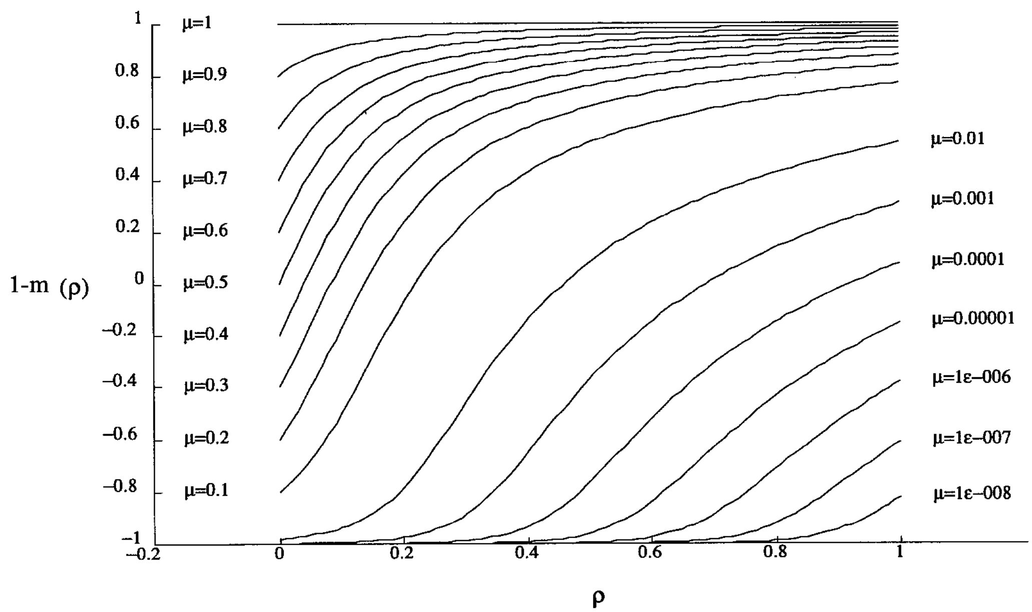

Fig. 8. Whitening performed with OTFs over a spectrum with modulus distributed as an inverse exponential. Several curves for different μ values are depicted.

haps because LNMFs assume a circular symmetry of the spectrum that the former does not. It could be said that an OTF effects a whitening that is more adapted to a given spectrum than does a LNMF.

8. Conclusion

A comparison of the performance of LNMFs and OTFs for the recognition of single images has been carried out. Several aspects of two different whitening strategies become clear when the first generation of OTFs is compared with LNMFs. The similarity between the OTF and the LNMF comes from the fact that for common objects the energy is concentrated at low spatial frequencies. According to the performance parameters suggested in the formulation of OTFs, the latter have a marginal superiority over LNMFs. The generality of the results presented in this paper was verified using several targets different from that of Fig. 1, and similar behavior was observed.

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