Computer Science in Economics and Management **4**: 117–133, 1991.

Qualitative Economic Reasoning: a Disequilibrium Perspective

KUAN-PIN LIN
Department of Economics, Portland State University, Portland, OR 97207, U.S.A.

and

ARTHUR M. FARLEY
Department of Computer and Information Science, University of Oregon, Eugene, OR 97403, U.S.A.

(Received: 18 September 1990)

Abstract. Extending a qualitative simulation of a market-based equilibrium model, this paper demonstrates the technical applicability of partial simulation from a more realistic disequilibrium viewpoint, where a market cannot be cleared promptly and it is subject to quantity constraints of spillover from other imbalanced markets. Some well-known observations of disequilibrium economics such as the effectiveness of policy change in relation with disequilibrium regime are proven to be correct in qualitative reasoning and simulation.

Key words: Comparative statics, disequilibrium economics, partial simulation, qualitative calculus, qualitative reasoning, qualitative simulation.

1. Introduction

In the absence of complete and accurate quantitative information about the structure and behavior of complex economic systems, economists have long relied on the qualitative techniques of confluence, causal ordering, and comparative statics to explain and predict economic effects and events. In particular, techniques of qualitative matrices are not new among economics theorists in dealing with solvability and stability of economic systems (see Allingham and Morishima, 1973; Quirk, 1981; Ritschard, 1983). Recently, qualitative modeling has emerged as a powerful technique for reasoning about physical systems (see Bobrow, 1985; Weld and de Kleer, 1990, for individual contributors in this area). Under the title *Qualitative Physics*, qualitative abstraction is applied both to the value domains of a model's parameters and variables and to the forms of constraints that describe component and system behaviors. There are at least four different theories and implementations for qualitative reasoning about physical systems: *Confluences* of de Kleer and Brown (1984), *Qualitative Process Theory* of Forbus (1984), *Qualitative Simulation* of Kuipers (1986), and *Causal Ordering* of Iwasaki and Simon (1986). Unfortunately, the adaptation of qualitative modeling and simulation techniques for economic analysis is far more complicated than those designed for reasoning about mechanical or electrical devices. Problems in economics tend to be conceptually complex and poorly (mathematically) defined. Furthermore, the economic system can not be perceived directly. Not only the ideas of advocates of a particular theory involved, but also the concerns of consistency with economic reality, make the implementation of qualitative reasoning difficult.

Along the line of Samuelson's qualitative calculus for economic analysis, qualitative solvability and stability are powerful theoretical properties. However, conditions for them to hold are very strict and unlikely to be fulfilled, except for very small systems. Techniques of qualitative economic reasoning inspired from recent development of qualitative physics focus on the understanding of the cognitive process of economic theory. Previous works on qualitative economic reasoning are primarily theoretical (Bourgine and Raiman (1986), Farley (1986), Berndsen and Daniels (1989), Farley and Lin (1990a). While Bourgine and Raiman (1986) discussed the use of order of magnitude information as a means for removing ambiguity from the process of qualitative macroeconomics, the contribution of Farley and Lin (1990a) lies in the formalization of qualitative market models and of a simulation paradigm suited to market-based economic reasoning, including techniques for realization and application of comparative statics.

The basic element of our qualitative economic model is a market, as specified by a set of causal relations and a tatonnement adjustment process. More complex economic theories can be constructed by combining instances of this simple market model. Since modular markets are constructed one at a time and linked together with connections, simulation about the whole economic system consists of sequential updating of markets via these connections. To solve the problem of embedded ambiguity for a large size model with many interrelated markets, the technique of partial simulation has proven useful (see Farley and Lin, 1990b). By applying *ceteris paribus* assumptions, we simulate one market at a time given other relevant stays intact. When all changes are made within the market, it passes changes through connections to perturb other related markets (again one at a time) until no further market updates are possible. By assuming that stable equilibrium is the likely state of a market following from some changes in the environment, the qualitative technique of comparative statics can be performed to combine changes in all final circumstances of partial simulation.

Extending our earlier study of qualitative economic reasoning (Farley and Lin, 1990a,b), in this paper we consider situations of multi-market disequilibrium. First, the initial state of a market model may not be in equilibrium at all. Further, the existence of disequilibrium may persist following market perturbation. The central feature of a disequilibrium model is its rigidity in prices due to the lack of a full or complete adjustment in the market. Indeed, a freely adjusted price mechanism is not a realistic economic consideration for market-based simulation. Instead of assuming a flexible price adjustment, each market is affected by quantity 'spillover' from related markets in disequilibrium. Along this line, studies of disequilibrium economics of Barro and Grossman (1976); Portes (1977), Ito (1980), Benassy (1982), and Quandt (1988) are important extensions of Keynes' *General Theory* (1935).

In the next section we revisit the basic framework of qualitative calculus for economic reasoning. Building up from a simple demand-supply market model, in section III, a forward-chaining scheme of qualitative simulation is described and extended to multi-market interactions. In particular we present a PROLOG (Clocksin and Mellish, 1984)* implementation of partial simulation to deal with propagational ambiguity associated with feedbacks in multi-market equilibrium system. Section IV considers the technical applicability of partial simulation from a more realistic disequilibrium viewpoint. It is a straightforward and natural extension of our qualitative simulation paradigm to provide validation of disequilibrium theory in economics. Our implementation of qualitative reasoning is powerful, not only in proving consistency with the existing model but also in providing flexible frameworks for alternative extended or new theories to be tested and simulated. Some conclusions of disequilibrium qualitative simulation are given in Section 5.

2. Qualitative Calculus Revisited

Since the first development of qualitative calculus in economics by Samuelson (1947, pp. 23–28), economists are used to reason qualitatively about changes in economic condition due to policy influence. Often with limited or no quantitative information they need to predict the course of economic fluctuation. The so-called *Calculus of Qualitative Relations* is simple but powerful. In the following we briefly review the operations of arithmetic and derivative rules of qualitative calculus. The former defines the static arithmetic operations of qualitative values, while the later concerns with the direction of change or dynamic behavior of the system.

Qualitative calculus is calculus of qualitative variables with values taken from the domain $\{-, 0, +\}$. An economic entity X is said to have qualitative value + (−) if and only if it is a positive (negative) real number. The value 0 is typically used to indicate a 'quiescent' or normal state of the model. Using the notations +, −, 0 to represent qualitative values, the qualitative algebraic operations (+), (−), (*), (/) are defined as follows:

* PROLOG stands for PROgramming in LOGic. Logic programming is based on the idea that statements in first-order predicate logic, cast in Horn clause form, can be used directly as a programming language. In this paper, Edinburgh syntax of Prolog is used for implementation.

(+)	+	−	0
+	+		+
−		−	−
0	+	−	0

(−)	+	−	0
+		−	−
−	+		+
0	+	−	0

(*)	+	−	0
+	+	−	0
−	−	+	0
0	0	0	0

(/)	+	−	0
+	+	−	0
−	−	+	0

Note that blanks in the above tables are interpreted as the undefined or ambiguous cases. The derivative rules govern the direction of change of an entity X, denoted ∂X. With the same qualitative domain $\{-, 0, +\}$ representing its direction of change, ∂X is interpreted as decreasing $(-)$, steady (0), or increasing $(+)$ of a specified change of X respectively. Quantitatively, for changes of algebraic combination of two entities X and Y, we have:

$$\partial(X + Y) = \partial X + \partial Y$$
$$\partial(X - Y) = \partial X - \partial Y$$
$$\partial(X * Y) = Y * \partial X + X * \partial Y$$
$$\partial(X\ /\ Y) = (Y * \partial X - X * \partial Y)\ /\ (Y * Y)$$

The qualitative changes of algebraic combination can be defined accordingly. A purely qualitative analysis relies only on the manipulation of this set of qualitative calculus. For most economic studies, we deal with non-negative finite real numbers as the legitimate range of quantitative values. Unless otherwise specified, the rules of qualitative change are defined for non-negative finite values only in our implementation discussed below. It is understood that the larger or the more complex system, the more ambiguous of the qualitative analysis will result. Due to the nature of inherent ambiguity in economic analysis, qualitative reasoning about economic systems rely upon this set of qualitative rules is doomed to be limited. However, by distinguishing the structural and propagational ambiguities in an economic market system, Farley and Lin (1990b) demonstrate a technique of partial simulation to extract qualitative information of comparative statics in multi-market models. For a qualitative system, instead of simulating through all possible traversal orderings, we keep track of paths visited in each simulation and thus avoid redundant updatings. Under the assumption that the overall model is qualitatively stable, the methods of partial simulation and its associated comparative analysis clearly demonstrate the power of qualitative calculus in reasoning changes of a complex economic system.

3. Qualitative Reasoning about Economic Systems

In this section, we summarize the techniques of qualitative simulation to reason about market-based economic system as described elsewhere in Farley and Lin (1990a,b). Applying *ceteris paribus* assumptions, economists typically reason about one market of a complex system at a time, propagating interactions between interrelated markets in a piecemeal fashion. By assuming that an equilibrium is restored after perturbation, variable changes in the system are then realized through the use of qualitative methods of confluence and comparative statics.

We represent the qualitative state of a market by its equilibrium condition and a list of parameters. Since our method of partial simulation considers only one 'reasoning path' at a time, we use a feature of dynamic database of PROLOG to maintain and update qualitative states of each market for all simulation paths*. First, a parameter is an external determined influence of the market, and it is represented by

```
parameter(Market, Parameter, Change, Direction,
Markets).
```

Where `Market` is the identifier of the market under consideration, while `Markets` (the last entry) is used to keep track of simulation path of all markets visited so far. For single market simulation, the entries `Market` and `Markets` are not necessary. We have kept our representation general to cope with the complexity of multi-market simulation described later. Initially `Markets` = [] is an empty list, and at the end of simulation `Markets` contains all markets have been visited during a partial simulation. `Parameter` is the symbolic name of the parameter. `Change` is its accumulated changes through the visitation recorded in `Markets`, and `Direction` indicates its direction of undergoing change. Both accumulated change and direction of change are elements of qualitative domain $\{-, 0, +\}$.

Next, equilibrium condition defines the algebraic combination of variables and parameters describing the state of the market. That is,

```
equilibrium(Market, Expression, Value, Direction,
Markets).
```

* Built-in predicates `assert` and `retract` are used to keep track of changes in market states during simulation.

For a demand-supply market-based model, the difference between quantity demanded and supplied is a typical expression of the equilibrium condition. The qualitative `Value` of equilibrium `Expression` indicates that market is in excess demand (+), in excess supply (−), or in balance (0). For most equilibrium analysis, `Value` = 0 is used to denote the 'quiescent' or normal state of the market. Given the current value of equilibrium expression, the `Direction` of change in equilibrium condition can be increasing (+), decreasing (−), or steady (0).

Given a new direction of change, new equilibrium value is computed by summing the `Value` with `Direction` of the change. By assuming continuity of changes, the value of an equilibrium expression cannot invert its sign directly. That is, when changing from (+) to (−) or vise versa it must become (0) first. For computing comparative statics, we are interested in changes between equilibrium states. As such, we introduce the following states of *asymptotic equilibrium*:

```
equilibrium(Market, Expression, +, -, Markets).
equilibrium(Market, Expression, -, +, Markets).
```

These definitions imply that, after perturbation, an equilibrium state is attainable or possible if sufficient information (qualitatively and quantitatively) is readily available. This is equivalent to say that a qualitative stable equilibrium exists. Given the nature of inherent ambiguity in qualitative economics, the best we can expect is to characterize a possible equilibrium state from a given parameter perturbation. The existence of asymptotic equilibrium assumes away problems of propagational ambiguity in the sense that feedbacks cannot be the dominant effects. Therefore redundant updatings of equilibrium conditions are trimmed from the simulation, as described in Farley and Lin (1990b).

In addition to the database specification of parameter and equilibrium condition, there are functional relations that describe the internal influences of market activities. Such influences can be represented by a market's causal relations and adjustment process among entities. There are two different types of variable changes in a market. First, causal relation is expressed as:

```
causal-relation(Market, Effect, Cause, Direction).
```

This functional relation specifies the cause-effect relationship between symbolic `Effect` (variable) and `Cause` (other variable or parameter) with the given `Direction` of change. This relation is consistent with the concept of *qualitative proportionalities* of Forbus (1984), and that of monotone function M^+ or M^- of Kuipers (1986) with explicit causal constraint. We note that there could be multiple causes affecting the same `Effect` variable. The predicate `causal-relation` is used to specify each of these relationships. Of course, structural ambiguity is a potential problem when we combine multiple causal relations for the effected variable.

Given causal restrictions among entities in a market, their interaction could be simulated in conjunction with one or more adjustment processes. This second type of variable change is defined by:

```
adjustment(Market, Target, Source, Direction).
```

The `Target` variable changes in accordance with the qualitative value of `Source`. For market simulation, the `Target` is usually the price change while the `Source` of an adjustment is the equilibrium condition of the market under consideration. In particular, the price adjustment process reflects the pressure of price increase when the market demand exceeds its supply. It is exactly the qualitative representation of 'the law of demand and supply' in general equilibrium theory. This definition of tatonnement adjustment process is similar to the idea of *influence* operator I^+ or I^- of Forbus (1984), in which the directional change of `Target` variable is related to the qualitative value of the `Source`.

For multi-market simulation, markets are connected via connections defined as

```
connection(Market1, Market2, Link).
```

Here `Link` must be a variable in `Market1` and a parameter in `Market2` so that it is capable of propagating changes from `Market1` to `Market2`. These markets interact during qualitative simulation through the activation of connections. A sequential visitation of the markets, following active connections and performing individual market updates during the visits, produces overall model behavior.

The forward-chaining scheme of qualitative simulation for a market-based equilibrium model works like this:

1. Given a parameter change, identify the affected markets and perform market simulation:

1.1. For a single market simulation, decompose the market equilibrium expression into components of parameters, adjustments, and causal relations. Update changes of parameters, adjustments, and causal relations respectively. Update equilibrium condition by summing all these component updates.

1.1.1. To update a parameter change, modify the parameter database describing the qualitative state of the market to reflect such change.

1.1.2. To update the target variable of an adjustment, find the associated equilibrium condition and apply its value to the target change.

1.1.3. To update the effect of a causal relation, update the change of its cause first. Depending on the cause being a parameter, an adjustment, or a causal relation, update it according to (1.1.1.), (1.1.2.), and (1.1.3.) recursively.

1.2. Repeat (1.1.) until no further update is possible. The result should be either an asymptotic equilibrium or a structural ambiguity within one of component updates.

1.3. Activate market connections, and simulate each connected market through the active parameter change as in (1.1.) and (1.2.). By keeping track of sequential market visitation, repeat activation of the same market is excluded.

2. Collect all changes of qualitative states in each partial simulation, and compute comparative statics. Report the results of qualitative consistency or ambiguity of each variable in the model.

The implementation of partial simulation utilizes extensively the automatic backtracking facility of PROLOG language. In the following, we report a single market simulation as well as a 'gross substitutes' three-market partial simulation. More complex examples of Keynesian macroeconomic models can be found in Farley and Lin (1990a).

First, consider a single market model 1 with the following causal relations of demand d and supply s:

```
causal_relation(1, d, p, -).
causal_relation(1, d, y, +).
causal_relation(1, s, p, +).
```

Where p stands for the price and y for income, respectively. The tatonnement adjustment of price is specified as follows:

```
adjustment(1, p, d-s, +).
```

Given the initial qualitative state of parameter y, and the market equilibrium condition defined as the excess demand d-s:

```
parameter(1, y, 0, 0, []).
equilibrium(1, d-s, 0, 0, []).
```

The qualitative simulation of an increase in y produces a sequence of changes in equilibrium condition d-s, in price p, and in demand d and supply s. The final state of the market is:

```
parameter(1, y, +, 0, [1]).
equilibrium(1, d-s, +, -, [1]).
```

The specification of equilibrium condition with positive but decreasing excess demand implies that equilibrium could be restored asymptotically. By assuming a stable final equilibrium for the market, we compare initial and final equilibrium states and compute changes in adjustment process and causal relations: p increase, and so are d and s. Qualitative simulation produces a correct solution of comparative statics in a single market environment.

As specified earlier, the multi-market system model is a component-based construction with connections linking relevant market components. By adding new market modules 2 and 3, to the previous market 1 and postfixing each variable and parameter with market identifier, we have the following complete specification of three-market 'gross substitutes' demand-supply system in which all other goods are substitutes for one another (see Hicks, 1939):

```
parameter(1, y, 0, 0, [ ]).
parameter(1, p2, 0, 0, [ ]).
parameter(1, p3, 0, 0, [ ]).
parameter(2, y, 0, 0, [ ]).
parameter(2, p1, 0, 0, [ ]).
parameter(2, p3, 0, 0, [ ]).
parameter(3, y, 0, 0, [ ]).
parameter(3, p1, 0, 0, [ ]).
parameter(3, p2, 0, 0, [ ]).

equilibrium(1, d1-s1, 0, 0, [ ]).
equilibrium(2, d2-s2, 0, 0, [ ]).
equilibrium(3, d3-s3, 0, 0, [ ]).

connection(1, 2, p1).
connection(1, 3, p1).
connection(2, 3, p2).
connection(2, 1, p2).
connection(3, 1, p3).
connection(3, 2, p3).

causal_relation(1, d1, y, +).
causal_relation(1, d1, p1, -).
causal_relation(1, d1, p2, +).
causal_relation(1, d1, p3, +).
causal_relation(1, s1, p1, +).
causal_relation(2, d2, y, +).
causal_relation(2, d2, p2, -).
causal_relation(2, d2, p1, +).
causal_relation(2, d2, p3, +).
causal_relation(2, s2, p2, +).
causal_relation(3, d3, y, +).
causal_relation(3, d3, p3, -).
causal_relation(3, d3, p1, +).
causal_relation(3, d3, p2, +).
causal_relation(3, s3, p3, +).

adjustment(1, p1, d1-s1, +).
adjustment(2, p2, d2-s2, +).
adjustment(3, p3, d3-s3, +).
```

These market descriptions should be self-explanatory. In particular, the property of 'gross substitutes' is expressed in terms of positive causal relations of cross prices with demand in all markets*.

Note that `income` is a parameter for all three markets. When this parameter changes, all markets are 'shocked' and need to adjust simultaneously. To create overall model behavior, we simulate one market at a time and propagate interactions through connections. Our qualitative paradigm of market-based economic reasoning produces the following results of comparative statics with an initial perturbation of increase in income: price and supply (and therefore demand) increase in all markets. A correct solution is generated for the case of 'gross substitutes'. It follows from combining 33 final qualitative states of all partial simulations enlisted in the Appendix.

4. Qualitative Disequilibrium Analysis

Although the building block of our qualitative reasoning scheme is the traditional market equilibrium model, the adjustment process in the market may be incomplete or 'retarded' within the period of interest. A fix-price or quantity constrained equilibrium seems to be a more realistic formulation for market simulation. In the following, we extend our method of qualitative simulation to reason about multi-market systems from the disequilibrium perspective viewpoint.

The term 'disequilibrium' is used to represent situations where markets do not promptly clear. The central feature of a disequilibrium economic model is its rigidity in prices. As a consequence of price rigidity, transaction can occur only on the short-side of the market. The unfulfilled demand or supply may further restrict the activities in other markets. When a market is constrained and cannot adjust freely to restore equilibrium, demand and supply are said to be 'rationed'. Other interrelated markets must take into account the 'spillover' of excess demand or supply from this imbalanced market. In the extreme, all markets interact through quantity spillovers and not price adjustments. This representation of disequilibrium models is in line with the original ideas of *General Theory* (Keynes, 1935), further pursued by Barro and Grossman (1976), Benassy (1982), Ito (1980), and Portes (1977), among others.

Market demand and supply with spillovers are called 'effective' demand and supply. They coincide with the traditional 'notional' demand and supply if quantity constraints of other markets were not encountered. There are at least three different specifications of spillover effects in the disequilibrium literature (see Benassy, 1982; Portes, 1977; Ito, 1980; Barro and Grossman, 1976). As Quandt (1988) pointed out, 'it is not easy to see how any of the above formations could claim preeminence on theoretical ground'. His study (Quandt, 1988, Ch. 5)

* There are many other qualitatively specified cases of demand-supply system in the literature. For example, the famous Morishima case where 'substitutes of substitutes and complements of complements are substitutes whereas substitutes of complements and complements of substitutes are complements' can be represented and simulated similarly.

focused on the 'coherency' condition of spillovers to ensure the solvability and stability of a disequilibrium system. For qualitative simulation, we use the version of 'effective' spillover of Benassy (1982) and Portes (1977). In particular, the causal effect of effective demand (supply) due to spillover is measured as a proposition of the difference between actual transactions and effective supply (demand) in the other markets. Only when the demand or supply is constrained will the market be affected by the spillovers. The qualitative value (− or +) of a spillover effect depends on which side of disequilibrium (excess demand or excess supply) the related market is residing.

To develop a qualitative representation of spillover effects, we consider a two-market demand-supply system with special reference to product (identifier 1) and labor (identifier 2) markets. We adopt the same notation as used in previous sections, with the understanding that all demand and supply now represent the effective concepts. The effect of spillover from other markets depends on the equilibrium condition (qualitative value of equilibrium position and its direction of change) of those markets. First, the spillover effect can only be 'fired' when a concerned or related market is in disequilibrium. For example, demand in (product) market 1 will be rationed if there is unemployment in (labor) market 2. The positive effect of spillover on demand in (product) market 1 from excess supply in (labor) market 2 indicates that its effective demand must be lower than that of unconstrained or notional one. Similarly, supply in (product) market 1 is negatively related with the excess demand of (labor) market 2. Spillover effect is a kind of causal relation dependent upon the equilibrium condition of other related markets. These spillover effects are represented by the following conditional causal relations.*

```
causal_relation(1, d1, d2-s2, +): -
  equilibrium(2, d2-s2, -, _, _), ! .
causal_relation(1, s1, d2-s2, -): -
  equilibrium(2, d2-s2, +, _, _), ! .
causal_relation(2, d2, d1-s1, +): -
  equilibrium(1, d1-s1, -, _, _), ! .
causal_relation(2, s2, d1-s1, -): -
  equilibrium(1, d1-s1, +, _, _), ! .
```

Assuming the extreme case of no price adjustment, price is treated as a parameter and cannot be a valid connection between markets. The interaction of markets is now solely specified by the excess demand and excess supply through spillover effects. For our two-market example,

```
connection(1, 2, d1-s1) .
connection(2, 1, d2-s2) .
```

* An unspecified or anonymous variable is denoted by an underline character. The 'cut' or ! is used to pick up the most recent equilibrium condition of a market under consideration.

It is clear that persistent disequilibrium may prevail in a market with fix-price formulation. Equilibrium becomes only accidental. The qualitative states of a disequilibrium model based on combination of equilibrium conditions in each market can be classified into four regimes: (K) Keynesian unemployment, (C) classical unemployment, (U) under consumption, and (R) repressed inflation. Under consumption may not be an interesting case for a model without inventory accumulation. For completeness, the initial state of the model will consist one of the following four market regimes represented as:

(K) Keynesian unemployment;

```
equilibrium(1, d1-s1, -, 0, [ ]).
equilibrium(2, d2-s2, -, 0, [ ]).
```

(C) Classical unemployment:

```
equilibrium(1, d1-s1, +, 0, [ ]).
equilibrium(2, d2-s2, -, 0, [ ]).
```

(U) Underconsumption:

```
equilibrium(1, d1-s1, -, 0, [ ]).
equilibrium(2, d2-s2, +, 0, [ ]).
```

(R) Repressed inflation:

```
equilibrium(1, d1-s1, +, 0, [ ]).
equilibrium(2, d2-s2, +, 0, [ ]).
```

and the following list of parameters:

```
parameter(1, y, 0, 0, [ ]).
parameter(1, p1, 0, 0, [ ]).
parameter(1, p2, 0, 0, [ ]).
parameter(1, d2-s2, 0, 0, [ ]).
parameter(2, y, 0, 0, [ ]).
parameter(2, p1, 0, 0, [ ]).
parameter(2, p2, 0, 0, [ ]).
parameter(1, d1-s1, 0, 0, [ ]).
```

The qualitative partial simulation paradigm for equilibrium model can be modified easily to take account of spillovers as parts of causal relations and market connections. The modification concentrates on the connection simulation whenever a spillover is in effect. It checks for the equilibrium condition of the market which 'spills' and updates causal relations in the affected markets. To analyze the external perturbation from one market only, we consider a positive shifting parameter a in the demand of (product) market 1:

```
parameter(1, a, 0, 0, [ ]).
causal_relation(1, d1, a, +).
```

The qualitative simulation of an increase in parameter a produces the following final state of the affected parameters independent of initial market regimes:

```
parameter(1, a, +, 0, [1]).
parameter(1, d2-s2, +, 0, [1,2,1]).
parameter(2, d1-s1, +, 0, [2, 1]).
```

On the other hand, the final state of equilibrium condition depends crucially on the initial regime classification. The following table lists and compares the initial and final states of equilibrium conditions for all four possible market regimes in our example of two-market simulation due to an increase in parameter a of market 1:

Regime	Initial State	Final State
(K)	equilibrium (1, d1–s1, –, 0, [ ]).	equilibrium (1, d1–s1, –, +, [1]).
	equilibrium (2, d2–s2, –, 0, [ ]).	equilibrium (2, d2–s2, –, +, [2,1]).
(C)	equilibrium (1, d1–s1, +, 0, [ ]).	equilibrium (1, d1–s1, +, 0, [1]).
	equilibrium (2, d2–s2, –, 0, [ ]).	equilibrium (1, d1–s1, +, 0, [1,2,1]).
		equilibrium (2, d2–s2, –, +, [2,1]).
(U)	equilibrium (1, d1–s1, –, 0, [ ]).	equilibrium (1, d1–s1, –, +, [1]).
	equilibrium (2, d2–s2, +, 0, [ ]).	equilibrium (2, d2–s2, +, 0, [1]).
(R)	equilibrium (1, d1–s1, +, 0, [ ]).	equilibrium (1, d1–s1, +, 0, [1]).
	equilibrium (2, d2–s2, +, 0, [ ]).	equilibrium (1, d1–s1, +, 0, [1,2,1]).
		equilibrium (2, d2–s2, +, 0, [2,1]).
(W)	equilibrium (1, d1–s1, 0, 0, [ ]).	equilibrium (1, d1–s1, +, 0, [1]).
	equilibrium (2, d2–s2, 0, 0, [ ]).	equilibrium (1, d1–s1, +, 0, [1,2,1]).
		equilibrium (2, d2–s2, +, 0, [2,1]).

Since the demand shifting occurs in market 1 and this market only, the changes of market 2 are always the induced changes as recorded in the simulation path from 1 to 2. It is interesting to note that shifting of demand in market 1 stimulates changes in equilibrium conditions of all related markets. Consider the effect of an expansionary policy to promote consumption (e.g. public works, tax-cut, etc.). Depending on the initial state of equilibrium condition, different extent of final equilibrium can be accounted for. For Keynesian unemployment, this type of expansionary policy seems to be effective in restoring (full employment) equilibrium. However, on the other hand, the expansionary policy alone cannot change the state of repressed inflation. If the model were perturbed from its initial Walrasian (full) equilibrium by an expansionary demand shifting, the model-

'stuck' in the regime of repressed inflation demonstrating the policy ineffectiveness in such environment. Nevertheless it shows partial effects in cases of classical unemployment and under consumption. Our qualitative simulation scheme produces not only an acceptable interpretation of disequilibrium theory, but also information about regime change due to policy shifting.

We could enlarge the above two-market product-labor model by the addition of price adjustment processes which depend on the market disequilibrium level. The well-known inflationary effect of a demand expansion policy is easy to simulate when price and wage are allowed to adjust freely in the model. However, for successfully simulating a disequilibrium model with flexible prices, we need to prioritize the effect of price adjustments and quantity spillovers qualitatively or quantitatively. Reasoning with order of magnitude information is beyond the scope of this paper. Further investigation is called for on the dynamics of regime change in qualitative and quantitative disequilibrium analysis, as well.

5. Conclusions

Qualitative reasoning about economic systems, typically for analysis of equilibrium perturbation, has been extended to include disequilibrium considerations in which markets need not be cleared and may be persistent in one-side transaction. Showing consistency with the existing disequilibrium theory, our implementation of partial simulation demonstrates the following conclusions pertaining effective demand-shifting policy in disequilibrium economics:

(1) Given the initial state of Walrasian (full employment) equilibrium, a demand-shifting leads to a disequilibrium state of repressed inflation or Keynesian unemployment depending on an expansionary or a contractionary policy change, respectively.
(2) Given the initial state of Keynesian unemployment, an expansionary demand-shifting will boot up output demand and scale back labor supply. This leads eventually to a possible state of equilibrium. Similarly a contractionary demand policy will be useful to eliminate repressed inflation and to achieve equilibrium asymptotically.
(3) However, for the cases of classical unemployment and under consumption, demand-shifting policy alone can produce at most partial effects in one market but leaving persistent disequilibrium in the other.

These results raise open questions about the qualitative dynamics of regime change in disequilibrium. From our best knowledge there is no existing theory or empirical evidence to explain and predict the evolutionary process of disequilibrium regime change. The challenge of qualitative reasoning in constructing and testing a dynamic theory of disequilibrium economics will be taken up in future research.

Appendix

In this appendix we list all 33 final parameter and equilibrium states resulting from partial simulation of an income increase in a three-market 'gross substitutes' demand-supply system as described in section III. Comparative statics can be carried out by summing all changes in parameters and variables via adjustments and causal relations:

```
parameter(3, p2, +, 0, [3,2,3]).
parameter(3, p1, +, 0, [3,1,2,3]).
parameter(2, p1, +, 0, [2,1,2,3]).
parameter(1, p2, +, 0, [1,2,3]).
parameter(2, p3, +, 0, [2,3]).
parameter(3, p1, +, 0, [3,1,3]).
parameter(3, p2, +, 0, [3,2,1,3]).
parameter(1, p2, +, 0, [1,2,1,3]).
parameter(2, p1, +, 0, [2,1,3]).
parameter(1, p3, +, 0, [1,3]).
parameter(3, y, +, 0, [3]).
parameter(2, p3, +, 0, [2,3,2]).
parameter(3, p1, +, 0, [3,1,3,2]).
parameter(2, p1, +, 0, [2,1,3,2]).
parameter(1, p3, +, 0, [1,3,2]).
parameter(3, p2, +, 0, [3,2]).
parameter(2, p3, +, 0, [2,3,1,2]).
parameter(1, p3, +, 0, [1,3,1,2]).
parameter(3, p1, +, 0, [3,1,2]).
parameter(2, p1, +, 0, [2,1,2]).
parameter(1, p2, +, 0, [1,2]).
parameter(2, y, +, 0, [2]).
parameter(3, p2, +, 0, [3,2,3,1]).
parameter(1, p2, +, 0, [1,2,3,1]).
parameter(2, p3, +, 0, [2,3,1]).
parameter(1, p3, +, 0, [1,3,1]).
parameter(3, p1, +, 0, [3,1]).
parameter(2, p3, +, 0, [2,3,2,1]).
parameter(1, p3, +, 0, [1,3,2,1]).
parameter(3, p2, +, 0, [3,2,1]).
parameter(1, p2, +, 0, [1,2,1]).
parameter(2, p1, +, 0, [2,1]).
parameter(1, y, +, 0, [1]).

equilibrium(3, d3-s3, +, -, [3,2,3]).
equilibrium(3, d3-s3, +, -, [3,1,2,3]).
```

```
equilibrium(2, d2-s2, +, -, [2,1,2,3]).
equilibrium(1, d1-s1, +, -, [1,2,3]).
equilibrium(2, d2-s2, +, -, [2,3]).
equilibrium(3, d3-s3, +, -, [3,1,3]).
equilibrium(3, d3-s3, +, -, [3,2,1,3]).
equilibrium(1, d1-s1, +, -, [1,2,1,3]).
equilibrium(2, d2-s2, +, -, [2,1,3]).
equilibrium(1, d1-s1, +, -, [1,3]).
equilibrium(3, d3-s3, +, -, [3]).
equilibrium(2, d2-s2, +, -, [2,3,2]).
equilibrium(3, d3-s3, +, -, [3,1,3,2]).
equilibrium(2, d2-s2, +, -, [2,1,3,2]).
equilibrium(1, d1-s1, +, -, [1,3,2]).
equilibrium(3, d3-s3, +, -, [3,2]).
equilibrium(2, d2-s2, +, -, [2,3,1,2]).
equilibrium(1, d1-s1, +, -, [1,3,1,2]).
equilibrium(3, d3-s3, +, -, [3,1,2]).
equilibrium(2, d2-s2, +, -, [2,1,2]).
equilibrium(1, d1-s1, +, -, [1,2]).
equilibrium(2, d2-s2, +, -, [2]).
equilibrium(3, d3-s3, +, -, [3,2,3,1]).
equilibrium(1, d1-s1, +, -, [1,2,3,1]).
equilibrium(2, d2-s2, +, -, [2,3,1]).
equilibrium(1, d1-s1, +, -, [1,3,1]).
equilibrium(3, d3-s3, +, -, [3,1]).
equilibrium(2, d2-s2, +, -, [2,3,2,1]).
equilibrium(1, d1-s1, +, -, [1,3,2,1]).
equilibrium(3, d3-s3, +, -, [3,2,1]).
equilibrium(1, d1-s1, +, -, [1,2,1]).
equilibrium(2, d2-s2, +, -, [2,1]).
equilibrium(1, d1-s1, +, -, [1]).
```

References

Allingham, M. and Morishima, M. (1973) Qualitative Economics and Comparative Statics, M. Morishima *et al.* (eds.), *Theory of Demand: Real and Monetary*, Oxford University Press, Oxford.

Barro, R. J. and Grossman, H. I. (1976) *Money Employment and Inflation*, Cambridge University Press, London.

Banassy, J.-P. (1982) *The Economics of Market Disequlibrium*, Academic Press, New York.

Bobrow, D. G. (ed) (1985) *Qualitative Reasoning about Physical Systems*, MIT Press, Cambridge, Mass.

Berndsen, R. and Daniels, H. (1989) Sequential causality and qualitative reasoning in economics, *Expert Systems in Economics, Banking and Management*, L. F. Pau et al. (eds.), North-Holland, Amsterdam.

Bourgine, P. and Raiman, O. (1986) Economics as reasoning on a qualitative model, *Proceedings of the First International Conference on Economics and Artificial Intelligence*, Aix-en-Provence, France, September, pp. 185–189.

Clocksin, W. F. and Mellish, C. S. (1984) *Programming in Prolog*, 2nd edn., Springer-Verlag, New York.

de Kleer, J. and Brown J. S. (1984) A qualitative physics based on confluences, *Artificial Intelligence* **24**, 7–83.

Farley, A. M. (1986) Qualitative modeling of economic systems, *Proceedings of the First International Conference on Economics and Artificial Intelligence*, Aix-en-Provence, France, September, 61–64.

Farley, A.M. and Lin, K.-P. (1990a) Qualitative reasoning in economics, *J. Economic. Dynamics Control* **14**, 465–490.

Farley, A. M. and Lin, K.-P. (1990b) Qualitative economic reasoning: dealing with propagational ambiguity, *Proceedings of the Second International Conference on Economics and Artificial Intelligence*, Paris, France, July 4–6.

Forbus, K. D. (1984) Qualitative process theory, *Artificial Intelligence* **24**, 85–168.

Hicks, J. R. (1939) *Value and Capital*, Oxford University Press, Oxford.

Ito, T. (1980) Methods of estimation for multimarket disequilibrium, *Econometrica* **48**, 97–126.

Iwasaki, Y. and Simons H. A. (1986) Causality in device behavior, *Artificial Intelligence* **29**, 3–32.

Keynes, J. M. (1935) *The General Theory of Employment, Interest, and Money*, Harcourt Brace, New York.

Kuipers, B. (1986) Qualitative simulation, *Artificial Intelligence* **29**, 289–338.

Portes, R. (1977) Effect demand and spillovers in empirical two-market disequilibrium model, Harvard Institute of Economic Research, Discussion Paper No. 595.

Quandt, R. E. (1988) *The Econometrics of Disequilibrium*, Basil Blackwell, New York.

Quirk, J. (1981) Qualitative stability of matrices and economic theory: a survey article, in H. Greenberg and J. Maybee (eds.), *Computer-Assisted Analysis and Model Simplification*, Academic Press, New York.

Ritschard, G. (1983) Computable qualitative-comparative static techniques, *Econometrica* **51**, 1145–1168.

Samuelson, P. A. (1947) *Foundations of Economic Analysis*, Harvard University Press, Cambridge, Mass.

Weld, D. S. and de Kleer, J. (1990) *Readings in Qualitative Reasoning about Physical Systems*, Morgan Kaumann, San Mateo, Cal.