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A study of radial velocity members of eight rich clusters of galaxies†

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Abstract On the basis of a radial velocity criterion for cluster membership, supplemented by a consideration of surface density distribution, we calculate the distribution parameters and membership probabilities of individual galaxies for 8 Abell clusters of galaxies. The membership determination is shown to be a reasonable and a feasible one, and the distribution of the radial velocity dispersion to be in agreement with the dynamical model. An elongated structure seems to be universal in rich clusters. An obvious relationship between N_A and σ_c is found from these clusters and the mass to light ratio of rich clusters is found to lie between 100 and $600h_{100}^{-1}$, with an average value of $350h_{100}^{-1}$.

Key words: clusters of galaxies—radial velocity—membership

1. INTRODUCTION

In the studies of clusters of galaxies, reasonable determination of cluster members is an all important link, for contamination by field galaxies will bring about most undesirable consequences in the research.

In recent years, following the accumulation of accurate redshift measurements of galaxies[1], attention has been turned on using radial velocity as membership criterion[2–5]. ZHAO Jun-liang et al.[7] first constructed a mathematical model similar to Sanders' model for star clusters. Later, they further improved the model by including a consideration of the surface number density, so making the model more rigorous, and have successfully applied it to the study of membership in the Coma and Virgo Clusters[8].

When studying a sample of several clusters, it is useful to adopt a simpler method. Therefore, in this paper, we take as basis the method described by equation (19) of Ref. [8], while taking into consideration departure from spherically symmetric distribution, and carried out a membership study and a parameter evaluation for 8 Abell clusters of galaxies. A preliminary discussion on the results will also be given.

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2. MATHEMATICAL MODEL FOR THE DETERMINATION OF DISTRIBUTION PARAMETERS AND MEMBERSHIP

2.1 Basic Expression for the Frequency Function

The frequency function for galaxy i in the region given by expression (19) of Ref. [8] can be simplified into the following form:

$$\Phi(i) = \Phi_c(i) + \sum_{j=1}^{K} \Phi_{fj}(i) = n_c \phi_c^r(i) \phi_c^v(i) + \sum_{j=1}^{K} n_{fj} \phi_{fj}^r(i) \phi_{fj}^v(i) \quad (i = 1, 2, \cdots, N) \tag{1}$$

Suffixes c and f refer to cluster and field galaxies, respectively, $j = 1, 2, \cdots, K$ indicates the fact that there are K field galaxies in the region. n_c and n_{fj} are the normalized numbers of cluster and field galaxies, they satisfy the equation

$$n_c + \sum_{j=1}^{K} n_{fj} = 1 \tag{2}$$

while ϕ^r and ϕ^v are the normalized distribution functions of galaxies in the surface number density space and the radial velocity space, respectively. For the region of the cluster we have

$$\begin{aligned}
\phi_c^r(i) &= \{\pi[1 + r^2(i)] \ln(1 + r_{\max}^2)\}^{-1} & (3)\\
\phi_c^v(i) &= (2\pi)^{-1/2} \sigma_c^{-1} \exp\{-0.5[v(i) - v_c]^{1/2} \sigma_c^{-1/2}\} & (4)\\
\phi_{fj}^r(i) &= (\pi r_{\max}^2)^{-1} & (5)\\
\phi_{fj}^v(i) &= (2\pi)^{-1/2} \sigma_{fj}^{-1} \exp\{-0.5[v(i) - v_{fj}]^{1/2} \sigma_{fj}^{-1/2}\} & (6)
\end{aligned}$$

where $r(i)$ is the distance of galaxy i from the cluster centre, $r_{\max}$ is the radius of the region under study, all expressed in units of the core radius of the cluster, r_c, which is one of the parameters to be determined. V_c and σ_c are the mean radial velocity and dispersion, and V_{fj}, σ_{fj} are the mean radial velocity and dispersion for the distribution of field galaxy j.

It is easily appreciated that the frequency distribution described by (1) gives a combined description of the distribution of cluster and field galaxies in the surface number density space and the radial velocity space. Therefore, this model can be regarded as a method of membership determination based on using simultaneously two parameters, surface density and radial velocity, as criteria.

2.2 Correction for Departure from Spherical Symmetry

As a matter of fact, the projected image of many galaxy clusters shows an obvious, elongated form[9,10], and description of the distribution by a simple core radius r_c would not be adequate. Therefore, we introduce ellipticity ϵ, angle between the major axis and the circle of right ascension, θ, and effective core radius, r_{ec}, related to the semi-major and minor axes of the ellipse, (a, b), by the relations

$$r_{ec}^2 = a \cdot b \tag{7}$$

$$\varepsilon = (a - b)/a \tag{8}$$

Correspondingly, let $r_e(i)$ be the effective distance of galaxy i from the centre, in units of r_{ec},

$$r_e(i) = [r_a^2(i) + r_b^2(i)]^{1/2} \tag{9}$$

where $r_a(i), r_b(i)$ are the projected distances along the major and minor axes, in units of a and b. If we replace the $r(i)$ in (3) by $r_e(i)$, then the frequency function will retain its form, while the meaning of its parameters and its applicability have been extended to non-spherical distributions.

2.3 Solution of the Parameters and Calculation of the Membership Probability

For the frequency distribution (1), the sampling likelihood function is

$$L = \prod_{i=1}^{N} \Phi(i, q_l), \tag{10}$$

where $q_l(l = 1, 2, \cdots, m) \equiv (r_{ec}, \epsilon, \theta, V_c, \sigma_C; V_1, \sigma_1, n_1; \cdots; V_K, \sigma_K, n_K)$ are adjustable parameters, numbering $m = 3K + 5$. The Principle of Maximum Likelihood will enable us to find their maximum likelihood estimates and uncertainties.

Having evaluated the distribution parameters of the cluster region, the membership probability of galaxy i is given uniquely by the expression,

$$P_c(i) = \frac{\Phi_c(i)}{\Phi(i)} \tag{11}$$

3. THE CLUSTER REGIONS AND THEIR DISTRIBUTION PARAMETERS

3.1 Radial Velocity Data and Selection of Clusters

From the clusters given in the 1989 ACO Catalogue[11] we selected those with measured radial velocities in the CfA redshift catalogue[12] and have 80 or more galaxies, as objects for the present study. There are 14 clusters satisfying these conditions. Of these, we picked out 8; the other 6 being more complex in showing evidence of double or multiple clustering, will be discussed in a separate paper. In order to ensure a sufficient number of field galaxies to participate in the maximum likelihood solution, we further extended the selected regions to three times the Abell radius ($\sim 4.5h_{100}^{-1}$ Mpc). The details of the 8 cluster regions are given in Table 1.

Table 1 Selected Cluster Regions

Cluster Name	α (1950) (h)	(min)	δ (1950) (°)	(′)	Abell Type	B-M Type	Richness Class	Distance Class	Count (N_A)	r_{max} (′)
A0194	01	23.0	−1	46	–	III	0	1	37	295
A0539	05	13.9	+6	24	R	III	1	2	50	256
A0548	05	44.9	−25	39	RI	I-II	1	1	79	131
A1060	10	34.3	−27	19	RI	III	1	0	50	420
A1367	11	41.9	+20	7	IR	II-III	2	1	117	245
A1656	12	57.4	+28	15	R	II	2	1	106	227
A2256	17	6.6	+78	47	R	II-III	2	3	88	91
S0463	04	28.0	−53	56	R	I-II	0	3	26	136

3.2 Distribution Parameters

Using the model described in the previous Section, we used the method of maximum likelihood to solve for the distribution parameters of the 8 rich clusters. The results are given in Table 2. The contents are 1) the name of the cluster or field; 2) the effective core radius r_{ec} of the cluster, expressed in both arcmin and h_{100}^{-1} kpc; 3) the apparent shape and orientation of the cluster as specified by the ellipticity ϵ and the angle θ between the long axis and the circle of right ascension; 4) the average radial velocity in the cluster or field, V, the dispersion σ, and the number of galaxies, N; 5) the number of field galaxies discarded before the solution because of gross departure from the mathematical model, N_p. For two of the clusters, A 0548 and A 2256, the result of the solution gives $n_f \rightarrow 0$, implying that, apart from those field galaxies discarded before the solution, all the galaxies are members, that is, $K = 0$. Because of limitation of space here, the individual membership probabilities will be published elsewhere.

Table 2 Distribution Parameters in Eight Cluster Regions

Cluster Name		r_{ec} ($'$)	r_{ec} (kpc h_{100}^{-1})	ϵ	θ (°)	V (km s^{-1})	σ (km s^{-1})	N	N_P
A0194	Cluster	10.8 ±1.4	167	0.21±0.08	16±13	5338±39	400±28	126	4
	Field 1					1883±32	78±22	7±2	
	Field 2					11310±357	4193±256	139±9	
A0539	Cluster	2.2 ±0.6	57	0.17±0.12	69±23	8807±66	703±52	100	4
	Field 1					8440±478	3462±331	46±5	
A0548	Cluster	6.5±1.2	234	0.69±0.03	33±2	12423±80	870±49	128	2
A1060	Cluster	8.1±1.7	87	0.42±0.07	85±7	3671±60	605±37	130	2
	Field 1					2067±390	764±203	11±4	
	Field 2					11386±829	3054±682	14±4	
A1367	Cluster	7.0±1.3	133	0.15±0.07	158±22	6492±58	779±47	132	3
	Field 1					3344±50	178±41	17±4	
	Field 2					8409±504	4199±408	41±6	
A1656	Cluster	5.7±0.5	114	0.40±0.04	23±3	6879±34	955±32	316	9
	Field 1					922±99	294±79	10±2	
	Field 2					8984±532	4063±336	34±4	
	Field 3					23892±761	2177±671	10±2	
A2256	Cluster	1.7 ±0.4	87	0.29±0.13	161±15	17440±146	1348±107	88	3
S0463	Cluster	5.1±1.2	183	0.17±0.21	151±30	12312±79	688±59	92	3
	Field 1					1417±78	231±55	9±4	
	Field 2					15105±981	4371±706	23±6	

4. ANALYSIS AND DISCUSSION

4.1 Distribution of Member Galaxies

Fig. 1 gives the distributions in the $r_e - V$ space, of the galaxies in the eight cluster regions. Galaxies in three membership probability ranges are marked with different symbols. It can be seen that the cluster members (those with $P_0 \geq 0.9$, marked by circles) in each case are located in relatively tight ranges in both radial velocity and central distance. Specifically, (1) at a given radius, those galaxies with radial velocities closer to the mean radial velocity of the cluster have higher membership probabilities; (2) of galaxies with the same radial velocity, those closer to the centre of the cluster have higher membership probabilities. This result is obviously very reasonable and is consistent with our recognition that "cluster galaxies should constitute a dynamically bound system". Particularly Conclusion (2) is a consequence after the introduction of the surface density criterion in our improved method of membership determination, and so demonstrates the reasonableness and desirability of the improvement.

From the figure it is not difficult to see that the velocity dispersion is greater closer to the cluster center than further out. This is consistent with dynamical models of clusters such as the King-Michie model[4], and shows that these clusters have undergone dynamical evolution over some time. It is worth pointing out that in our method of member determination we assumed only a simple, one-dimensional normal distribution for the radial velocities of cluster members, and did not include any relation between the dispersion and the distance from the centre. This feature is in fact a consequence of a reasonable constraint in our mathematical model, namely, that the surface density of field galaxies be uniform (equation (5)). This once again shows the reasonableness of our method of member determination.

4.2 Apparent Shape of Clusters

Although our selected sample of clusters is not large, the results given in Table 2 Column 3 suffice to convince us that elongated structure is general among rich clusters of galaxies. When projection effect is taken into account, this inference is even firmer. This conclusion agrees completely with previous works[9,10].

Speaking as a whole, for the regular clusters (Abell morphology R) such as A 2256, S 0463, A 0539, the ellipticity is clearly smaller than for the other clusters (excepting the Coma and A 1367). This result should be obvious, for degree of symmetry in the surface distribution is itself one of the main criteria for classifying Abell morphology. Of course, ellipticity is not a unique measure for the degree of symmetry in the surface distribution, a loose structure, or a non-smooth surface distribution are also signs of lack of symmetry (A 1367 is one such example). On the other hand, a large ellipticity does not necessarily imply a poor degree of symmetry: the Coma cluster, for example, apart from a slight elongation in its surface distribution, is highly symmetrical.

4.3 Core Radius

The results given in Table 2 Column 2 show that there is a large dispersion among the 8 effective core radii we found, the values range from 57 to $234h_{100}^{-1}$ kpc, with an average of $133h_{100}^{-1}$ kpc. This is in basic agreement with the view of Bahcall et al.[12,13] that the core radii of regular clusters are mostly around $250h_{50}^{-1}$ kpc. Also, the average value for the 4 regular clusters in our sample is relatively small, being $\sim 111h_{100}^{-1}$ kpc. This is also consistent with our basic understanding of galaxy clusters, that is, the dynamical evolutionary time of regular clusters is longer, which means, in general, a greater concentration towards the centre, or a smaller core.

4.4 Correlation between N_A and σ_c

Our method of member determination is based on using radial velocity as criterion and so discards effectively field galaxies with abnormal radial velocities. Hence, we may say that among the distribution parameters, the radial velocity dispersion σ_c (averaged over the whole sky region) is one of the most valuable parameters resulting from our method.

A careful analysis of the values of σ_c in Table 2 show that they are clearly correlated

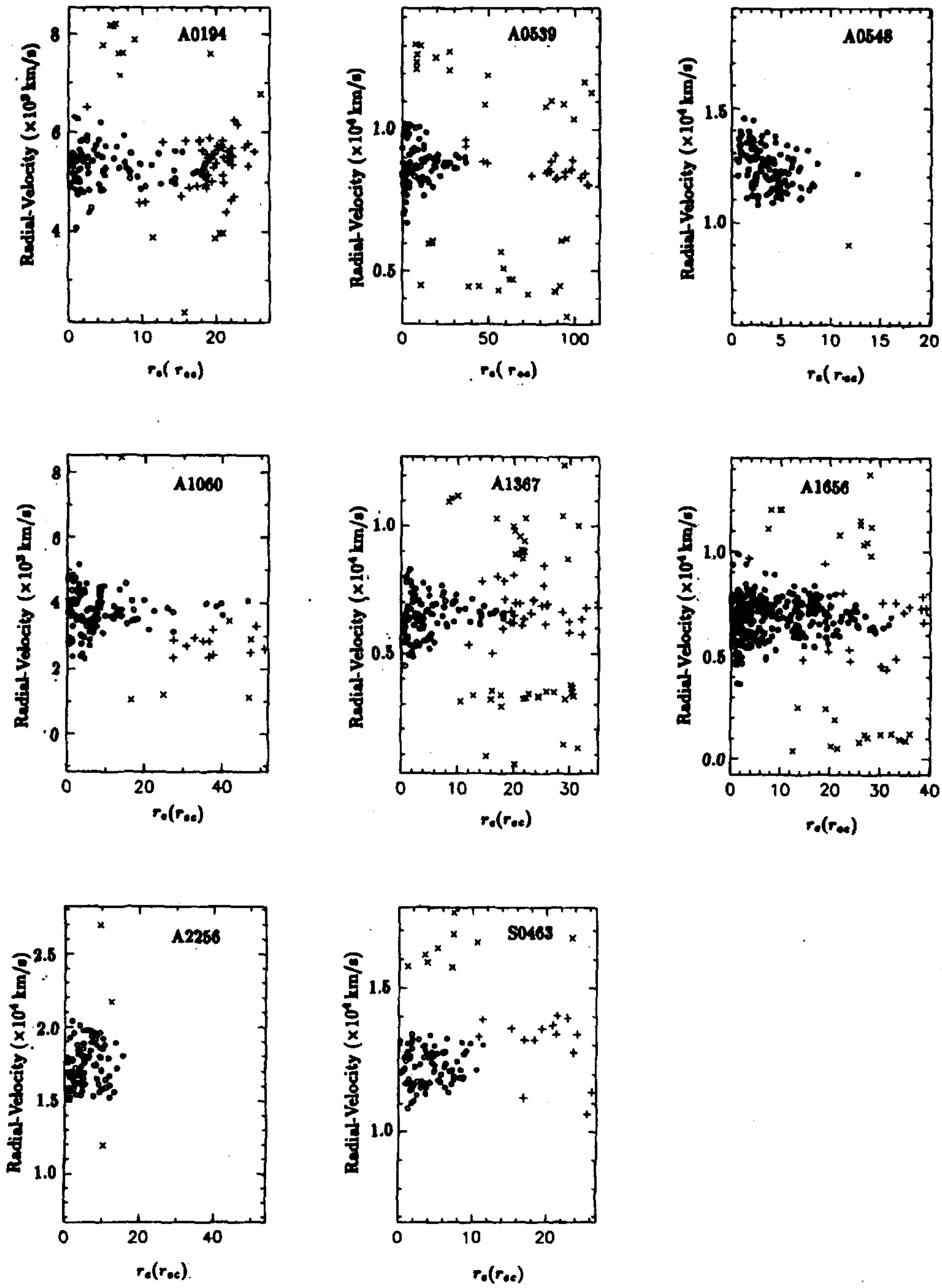

Fig. 1 Distribution of galaxies in the central distance - radial velocity plane. Cluster membership probabilities less than 0.1, between 0.1 and 0.9, greater than 0.9, are represented by crosses, plus signs, and circles, respectively

with the richness class of the cluster, the greater the richness class, the greater σ_c. For the richness classes 0, 1, 2, the mean dispersions are 544, 726 and 1027 km/s, respectively. If we further use the Abell count N_A as a more quantitative measure of richness, then we find a clear correlation between N_A and σ_c. See Fig. 2.

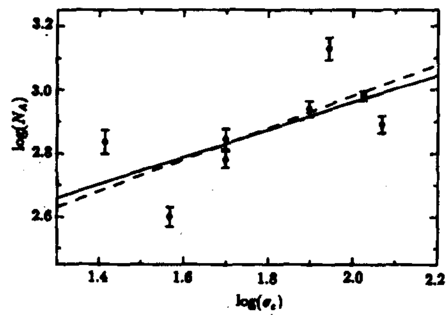

Fig. 2 The $N_A - \sigma_c$ correlation among galaxy clusters

This phenomenon is not difficult to explain. The Abell count N_A is defined as the total number of galaxies not more than 2 magnitudes fainter than the third brightest galaxy in the region and located within a radius of $1.5h_{100}^{-1}$ Mpc about the cluster centre, and so is a rather objective measure of the cluster size. The N_A given in the ACO Catalogue have been corrected to some degree for contamination[11] and are independent of other factors such as the distance of the cluster from us. If we admit that there is some generally applicable luminosity function for galaxy clusters, then N_A will be simply proportional to the total luminosity of the cluster, hence to its luminosity mass[14]. On the other hand, the dynamical mass of the cluster is proportional to the square of the velocity dispersion (or radial velocity dispersion)[15]. Thus, the correlation between N_A and σ_c can be understood as a correlation between the two kinds of mass.

A linear fit to the data in Table 2 gives the following relation (the solid line in Fig. 2):

$$\lg \sigma_c = 2.10(\pm 0.13) + 0.43(\pm 0.20) \lg N_A \tag{12}$$

The gradient, 0.43 ± 0.20, is close to the value 0.5, corresponding to the square of the dispersion. If we fix the gradient at 0.5, then the best fit (dotted line in Fig. 2) is

$$\lg \sigma_c = 1.98(\pm 0.12) + 0.5 \lg N_A \tag{13}$$

In either case, the correlation coefficient is greater than 0.90, but the dispersion about the linear regression is far greater than due to measuring errors in the radial velocity. Hence we come to the following conclusions: (1) There is a clear correlation between N_A and σ_c. (2) The dispersion about the regression line is mainly due to some intrinsic property of the clusters, e.g., difference in the mass-to-light ratio (or virial coefficient) among individual clusters.

4.5 Mass-to-Light Ratio of Galaxy Clusters

If we assume that the clusters have the same luminosity function and mean mass-to-light ratio, while the virial mass of the cluster can be calculated from σ_c^2, then the above correlation is a reflection of the average mass-to-light ratio of the clusters and its dispersion.

Using the value $330h_{100}^{-1}$ for the best researched Come cluster[16] as calibration, we find that for the 8 clusters studied here, the average mass-to-light is $\sim 350h_{100}^{-1}$, and its dispersion is about 70% of that value, that is, the mass-to-light ratio of galaxy clusters is mainly distributed in the range $100 \sim 600h_{100}^{-1}$.

5. CONCLUDING REMARK

In sum, the application of our method of member determination to rich clusters of galaxies has been successful. A preliminary analysis has yielded much valuable data such as the ellipticities of the clusters, their core radii, the correlation between their N_A and σ_c and their mass-to-light ratios, which will be useful in deeper investigations. The present sample is, however, rather small and incomplete, if more data on cluster galaxies are available, especially radial velocity data, then we can expect even more reliable results.

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