# SOME PROPERTIES OF THE TRAJECTORIES OF PLASMA WAVES IN THE IONOSPHERIC PLASMA NEAR RESONANCE AT THE UPPER HYBRID FREQUENCY

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The problem of the refraction of plasma waves in a nonuniform plasma in the frequency region of the upper hybrid resonance is analyzed in the approximation of geometrical optics. It is shown that trajectories of the loop type are possible with quasitransverse propagation if the upper hybrid frequency is less than the second harmonic of the gyrofrequency. The criteria for the formation of such trajectories are obtained as a function of the plasma parameters and the wave characteristics. If the upper hybrid frequency is greater than the second harmonic of the gyrofrequency, then the trajectories have the ordinary "quasiparabolic" form. The results can be used for the interpretation of resonances observed on high-latitude ionograms with external sounding of the ionosphere.

Resonance effects which are observed in the external sounding of the ionosphere have been discussed in numerous experimental and theoretical reports (see [1-7], for example). A number of reports are connected with the necessity of explaining the greater durations of signals at the resonance frequencies $\omega_p$, $\omega_h$, $\omega_H$, and $n\omega_H$ ($\omega_H$ is the Langmuir frequency, $\omega_h$ is the upper hybrid frequency, $\omega_H$ is the electron gyrofrequency, and $n = 2, 3, \ldots$). The interpretation of these durations with the help of the refraction of packets of electrostatic waves in a nonuniform plasma has obtained wide distribution up to now [2-7]. Such an interpretation was first proposed by McAfee in [3], where the propagation of plasma waves with $\omega \approx \omega_p$ was discussed. In [4, 5] and a number of other articles the analogous problem was solved for longitudinal waves near the upper hybrid resonance $\omega \approx \omega_h$. The possibility of the occurrence of group trajectories with a return to the vicinity of the emitter after propagation in a nonuniform ionospheric plasma is analyzed in qualitative form in [4]. An analytical examination of this problem is contained in [5]. However, the results obtained in [5] can be used only for the equatorial region of the ionosphere, since it is assumed here that the propagation takes place strictly perpendicular to the magnetic field $\mathbf{H}_0$ and the vector $\nabla\omega_h$ lies in the plane perpendicular to $\mathbf{H}_0$. It is known that the resonance $\omega \approx \omega_h$ is also observed at middle and even at high latitudes, where the model used in [5] is not applicable.

The refraction of plasma waves in the region of frequencies $\omega \approx \omega_h$ with quasitransverse propagation will be analyzed in the present report. The use of the simplest geometry of the problem ($\nabla\omega_h \parallel \mathbf{H}_0$) makes it possible to obtain analytical expressions for the group trajectories and the group lag times. It is shown that the trajectories have different characters depending on whether the inequalities $\omega_h > 2\omega_H$ or $\omega_h < 2\omega_H$ are satisfied. The results obtained can be used for the interpretation of the resonance $\omega \approx \omega_h$ on high-latitude ionograms.

§1. The dispersion equation for electrostatic waves propagating in a plasma at an angle $\theta$ to the magnetic field $\mathbf{H}_0$ with allowance for the thermal motion of the electrons was first obtained in [8] (see also the monograph [9] or [10]). Since the resonance $\omega \approx \omega_h$ is transverse, we will consider the region of angles $\theta$ close to $\pi/2$, assuming that the inequality

$$\cos^2\theta \ll 1 \tag{1}$$

is satisfied. Then the dispersion equation for waves propagating in the region of frequencies $\omega \approx \omega_h$ can be obtained from the equation presented in [9] (p. 225) using the inequality (1), and it takes the form

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$$\frac{\omega^2-\omega_h^2}{\omega^2}k_x^2+\frac{\omega^2-\omega_H^2}{\omega^2}\frac{\omega^2-\omega_p^2}{\omega^2}k_z^2-3\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_x^4}{\omega^2-4\omega_H^2}=0, \tag{2}$$

where $k_x$ and $k_z$ are the components of the wave vector and $v_t$ is the electron thermal velocity. It is assumed that the propagation takes place in the xz plane and $\mathbf{H}_0$ is directed along the z axis. From (2) it is easy to obtain the expressions for the components of the group velocity:

$$v_{grx}=\frac{\omega}{k_x}\left(6\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_x^2}{\omega^2-4\omega_H^2}-\frac{\omega^2-\omega_h^2}{\omega^2}\right),$$
$$v_{grz}=-\frac{k_z\omega}{k_x^2}\frac{\omega^2-\omega_H^2}{\omega^2}\frac{\omega^2-\omega_p^2}{\omega^2}. \tag{3}$$

From Eqs. (2) and (3) one can establish that

$$\boldsymbol{k}\boldsymbol{v}_{gr}=\frac{3\omega}{k_x^2}\frac{\omega_p^2}{\omega^2}\frac{v_t^2}{\omega^2-4\omega_H^2}, \tag{4}$$

from which it follows that waves near $\omega\approx\omega_h$ are direct waves with the condition that $\omega_h>2\omega_H$ ($kv_{gr}>0$). If the plasma parameters are such that $\omega_h<2\omega_H$, then reverse waves propagate ($kv_{gr}<0$).

§2. Suppose that the medium is plane-layered with the vector $\nabla\omega_h$ being directed parallel to $H_0$, i.e., along the z axis.* Then Snell's law takes the form

$$k_x=k_{x0}=\text{const}. \tag{5}$$

From (3), with allowance for (5), we obtain the equation

$$k_z^2=\left(3\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_{x0}^4}{\omega^2-4\omega_H^2}-\frac{\omega^2-\omega_h^2}{\omega^2}k_{x0}^2\right)\left(\frac{\omega^2-\omega_H^2}{\omega^2}\frac{\omega^2-\omega_p^2}{\omega^2}\right)^{-1}. \tag{6}$$

From (6) it is easy to establish the conditions for transparency to waves propagating in a nonuniform medium in the given direction with $\omega\approx\omega_h$, namely:

$$\omega^2-\omega_h^2>0,\quad \frac{\omega^2-\omega_h^2}{\omega^2}<3\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_{x0}^2}{\omega^2-4\omega_H}\qquad(\omega>2\omega_H); \tag{7}$$

$$\omega^2-\omega_h^2<0,\quad \left|\frac{\omega^2-\omega_h^2}{\omega^2}\right|>3\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_{x0}^2}{|\omega^2-4\omega_H^2|}\qquad(\omega<2\omega_H). \tag{8}$$

The coordinates of the point of reflection are found from the condition $k_z^2=0$, i.e., in accordance with (6) from the equality

$$3\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_{x0}^2}{\omega^2-4\omega_H^2}=\frac{\omega^2-\omega_h^2}{\omega^2}. \tag{9}$$

It follows from (9) that the waves determined by the conditions (7) and (8) both have a point of reflection.

From Eqs. (3) and (8) it is seen that when $\omega_h<2\omega_H$ and $k_z>0$ we have

$$v_{grz}<0,\qquad v_{grx}>0,$$

if

$$3\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_{x0}^2}{\omega^2}\left|\frac{\omega^2}{\omega^2-4\omega_H^2}\right|<\left|\frac{\omega^2-\omega_h^2}{\omega^2}\right|<6\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_{x0}^2}{\omega^2}\left|\frac{\omega^2}{\omega^2-4\omega_H^2}\right|, \tag{10}$$

and

$$v_{grz}<0,\qquad v_{grx}<0,$$

if

$$\left|\frac{\omega^2-\omega_h^2}{\omega^2}\right|>6\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_{x0}^2}{\omega^2}\left|\frac{\omega^2}{\omega^2-4\omega_H^2}\right|. \tag{11}$$

Moreover, we note that $v_{grx}=0$ with the condition that

$$\frac{\omega^2-\omega_h^2}{\omega^2}=6\frac{\omega_p^2}{\omega^2}\frac{v_t^2k_{x0}^2}{\omega^2-4\omega_H^2}. \tag{12}$$

*The assumption that $\nabla\omega_h \parallel H_0$ is valid only in the high-latitude region of the ionosphere.

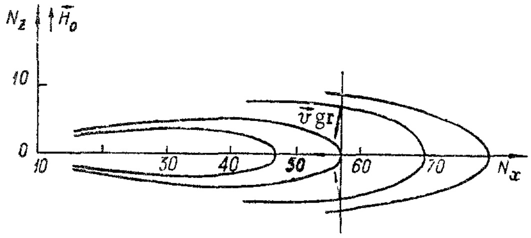


Fig. 1

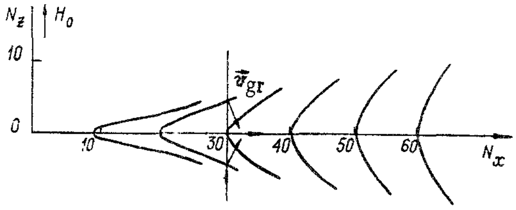


Fig. 2

Fig. 1. Surfaces of index of refraction $N(\theta)$ for $\omega_h < 2\omega_H$ [the parameters $\omega_p = 4.5$ MHz, $\omega_H = 3.9$ MHz, and $v_t \sim 3 \cdot 10^2$ km/sec are taken from [2] (ionogram on p. 783, Fig. 7)].

Fig. 2. Surfaces of index of refraction $N(\theta)$ for $\omega_h > 2\omega_H$ [the parameters $\omega_p = 7.2$ MHz, $\omega_H = 3.0$ MHz, and $v_t \sim 3 \cdot 10^2$ km/sec are taken from [1] (ionogram on p. 275, Fig. 21)].

The relations (10), (11), and (12) mean that for the case of $\omega_h < 2\omega_H$ with a given detuning $\Delta = \omega - \omega_h$ there exists a certain critical angle of emergence $\theta_{cr}$ at which $v_{gr\,x} = 0$; $v_{gr\,x} > 0$ for angles less than $\theta_{cr}$ and $v_{gr\,x}$ becomes negative for angles greater than $\theta_{cr}$. However, the relation $\mathbf{k}\mathbf{v}_{gr} < 0$ remains satisfied everywhere in this case; i.e., the waves are reverse. The form of the surfaces of the index of refraction $N(\theta)$, presented in Fig. 1 for $\omega_h < 2\omega_H$, confirms what was said above. In fact, the group velocity is directed along the normal to the surface $N(\theta)$ [11], and when $\theta \approx \theta_{cr}$ the component $v_{gr\,x}$ is reduced to zero.

When $\omega_h > 2\omega_H$ and $k_z > 0$ it follows from (3) and (7) that

$$v_{grz} < 0, \quad v_{grx} > 0, \qquad \text{if} \quad \frac{\omega^2 - \omega_h^2}{\omega^2} < 3\,\frac{\omega_p^2}{\omega^2}\,\frac{v_t^2 k_{x0}^2}{\omega^2 - 4\omega_H^2}; \tag{13}$$

i.e., $v_{gr\,x} > 0$ and $v_{gr\,z} < 0$ for any angles of emergence satisfying the condition of transparency (7), with the waves being direct. The surfaces of the index of refraction for this case are presented in Fig. 2.

§3. Let us consider the case when the properties of the medium vary by a linear law, namely,

$$\omega_h^2 = \omega_{h0}^2 + \beta z, \tag{14}$$

where $\omega_{h0}$ is the value of the hybrid frequency at the level of the emitter ($z = 0$) and $\beta = (\partial\omega_h^2/\partial z)_{z=0}$.* We will further assume that the medium is smoothly nonuniform, so that $|\beta z| \ll \omega_{h0}^2$.

With allowance for (14) it follows from Eqs. (2), (3), (6), and (12) that the components $v_{gr\,z}$ and $v_{gr\,x}$ of the group velocity can be reduced to zero at the levels $z = z_{max}$ and $z = z_1$, respectively:

$$z_{\max} = \frac{1}{\beta}\left[(\omega^2 - \omega_{h0}^2) - 3\,\frac{\omega_{p0}^2}{\omega^2}\,v_t^2 k_{x0}^2\,\frac{\omega^2}{\omega^2 - 4\omega_H^2}\right]; \tag{15}$$

$$z_1 = \frac{1}{\beta}\left[(\omega^2 - \omega_{h0}^2) - 6\,\frac{\omega_{p0}^2}{\omega^2}\,v_t^2 k_{x0}^2\,\frac{\omega^2}{\omega^2 - 4\omega_H^2}\right]. \tag{16}$$

From (8), (10), (11), (15), and (16) it follows that $|z_{max}| > |z_1|$. The ray equation has the form

$$\frac{dx}{v_{grx}} = \frac{dz}{v_{grz}}. \tag{17}$$

Substituting the values of $v_{gr\,x}$ and $v_{gr\,z}$ from (3) into (17) with allowance for (6) and (14), after integration we obtain

$$x = \left[\frac{2}{\beta}\sqrt{\beta z + b}\left(c - \frac{1}{3}\beta z\right) - d\right]\gamma^{-1}, \tag{18}$$

where

$$c = a + \frac{2}{3}b, \qquad d = \frac{2\sqrt{b}}{\beta}\left(a + \frac{2}{3}b\right),$$

*Since $\partial\omega_h^2/\partial z = (\partial\omega_p^2/\partial z) + (\partial\omega_H^2/\partial z)$, one can assume that $\partial\omega_h^2/\partial z \approx \partial\omega_p^2/\partial z$; i.e., the gradient of the magnetic field can be ignored (we have in mind the ionospheric plasma).

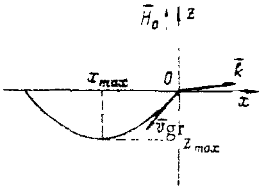


Fig. 3

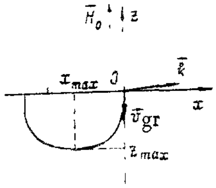


Fig. 4

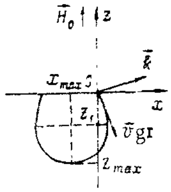


Fig. 5

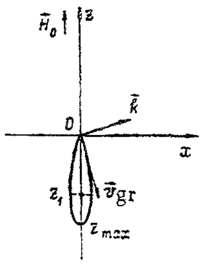


Fig. 6

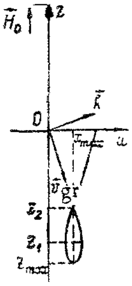


Fig. 7

$$a = (\omega^2 - \omega_{h0}^2) - 6\frac{\omega_{p0}^2}{\omega^2} k_{x0}^2 v_t^2 \frac{\omega^2}{\omega^2 - 4\omega_{H0}^2},$$

$$b = 3\frac{\omega_{p0}^2}{\omega^2} v_t^2 k_{x0}^2 \frac{\omega^2}{\omega^2 - 4\omega_{H0}^2} - (\omega^2 - \omega_{h0}^2),$$

$$\gamma = \left[(\omega^2 - \omega_{H0}^2)\frac{\omega^2 - \omega_{p0}^2}{\omega^2}\right]^{1/2}.$$

From Eq. (18) for the trajectory one can find the coordinate $x = x_{max}$, which together with (15) gives the position of the point of reflection:

$$x_{max} = -d/\gamma. \tag{19}$$

From (18) and (19) it follows that the coordinate $x_{max}$ corresponds not only to the point of reflection $z = z_{max}$, but also to the point of intersection $z = z_2$, where

$$z_2 = 3c/\beta. \tag{20}$$

If $|z_{max}| > |z_2| > 0$, then the trajectory has a loop form.

Using Eqs. (15), (16), (18), (19), and (20), one can obtain the criteria for the formation of loop trajectories as a function of the angle of emergence. We introduce the parameter $\psi$, the value of which depends only on the angle of emergence for given plasma parameters ($\omega_{p0}$, $\omega_{H0}$, $\omega_{h0}$, $v_t$) and a fixed wave frequency, namely,

$$\psi = \frac{(\omega^2 - \omega_{h0}^2)/\omega^2}{(\omega_{p0}^2/\omega^2)[v_t^2 k_{x0}^2/(\omega^2 - 4\omega_{H0}^2)]}. \tag{21}$$

We assume that the inequality $\omega_{h0} < 2\omega_{H0}$ is satisfied. Then the condition of transparency (8) has the form $\psi > 3$. The coordinate of the point of reflection $z_{max} < 0$. If the angle of emergence is increased but the parameter $\psi$ varies in the range of $3 < \psi < 6$, then the trajectory has the ordinary "quasiparabolic" form, since $z_{max} < 0$, $x_{max} < 0$, and $z_1 > 0$ (see Fig. 3). When $\psi = 6$ and $z_1 = 0$, i.e., when the angle of emergency satisfies the condition $\psi = 6$, the group velocity is antiparallel to $H_0$ at the level of the emitter (Fig. 4). If the angle of emergence is such that $6 < \psi < 12$, then the point of reversal $z_1 < 0$, the point of intersection $z_2 > 0$, $z_{max} < 0$, and the trajectory has a horseshoe form (see Fig. 5). When $\psi = 12$ and $0 < |z_1| < |z_{max}|$ the point of intersection $z_2 = 0$ and $x_{max} = 0$; the points of emergence and incidence of the ray coincide (Fig. 6). With a further increase in the angle of emergence the inequality $\psi > 12$ is satisfied. With this condition $0 < |z_2| < |z_1| < |z_{max}|$, $x_{max} < 0$, and the trajectory has a loop form (see Fig. 7).

In the case of $\omega_{h0} > 2\omega_{H0}$ the condition of transparency (7) has the form $\psi < 3$. In this case $z_{max} < 0$ and the wave is reflected as before from the region where the electron concentration is less than at the level of the emitter. However, the points $z_1$ and $z_2$ lie in the region of opacity ($|z_1|$, $|z_2| > |z_{max}|$), $x_{max} > 0$, and therefore only ordinary trajectories of the "quasiparabolic" type are possible.

All the foregoing pertains to the case of $k_{x0} > 0$. When $k_{x0} < 0$ the trajectories will have a form analogous to that presented in Figs. 3-7 with reversal of the direction of the x axis. The criteria for the existence of the different types remain as before. If waves with $k_{x0} > 0$ and $k_{x0} < 0$ are emitted simultaneously, then there can be two rays at the point of reception when $\omega_h < 2\omega_H$ (one trajectory of the loop type and the other ordinary). This result can be used for the interpretation of beats observed near the resonance $\omega \approx \omega_h$ in external sounding of the ionosphere [2, 4].

From what was presented above it follows that in the case of $\omega_h < 2\omega_H$ the types of trajectories possible near the upper hybrid resonance are analogous to those examined earlier in the region of frequencies $\omega \approx \omega_p$ if $\omega_p > \omega_H$ [3, 12]. An important difference is that reverse waves propagate when $\omega \approx \omega_h$ and $\omega_h < 2\omega_H$, while the waves are direct when $\omega \approx \omega_p$ and $\omega_p > \omega_H$. Trajectories of the loop type are possible in both cases [3, 7, 12], however, and the criteria for the formation of such trajectories are analogous.

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## LITERATURE CITED


1. Proc. IEEE, 57, No. 6 (1969).
2. D. B. Muldrew, Radio Sci., 7, 779 (1972).
3. J. R. McAfee, J. Geophys. Res., 74, 802 (1969).
4. J. R. McAfee, J. Geophys. Res., 74, 6403 (1969).
5. B. N. Gershman, A. E. Krupina, and Yu. Ya. Yashin, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 16, No. 7, 981 (1973).
6. B. N. Gershman, A. E. Krupina, and Yu. Ya. Yashin, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 17, No. 10, 1461 (1974).
7. M. Petit, J. Bitoun, R. Feldstein, P. Graff, and B. Higel, Ann. Telecom., 27 (1972).
8. B. N. Gershman, Zh. Éksp. Teor. Fiz., 24, 659 (1953).
9. V. L. Ginzburg, Propagation of Electromagnetic Waves in a Plasma [in Russian], Nauka, Moscow (1967).
10. V. P. Silin and A. A. Rukhadze, Electromagnetic Properties of a Plasma and of Plasmalike Media [in Russian], Atomizdat, Moscow (1961).
11. K. G. Budden, Radio Waves in the Ionosphere, Cambridge University Press, Cambridge (1961).
12. A. E. Krupina and V. A. Yashnov, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 20, No. 2, 183 (1977).


# RESONANCE EFFECTS IN THE EARTH – IONOSPHERE CAVITY

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Natural oscillations of an electrical type in the earth – ionosphere cavity are analyzed with allowance for the height profiles of the plasma parameters of the lower ionosphere. Besides the well-known branch of natural frequencies corresponding to Schumann resonances (ones and tens of hertz) new resonance frequencies are obtained in the range of ones of kilohertz. The problem of forced oscillations of the cavity is solved within the framework of the same model of the ionosphere. The resonance parameters of the energy and cross spectra are compared with allowance for the suppression of interference.

The spherical cavity formed by the earth's surface (conductance on the order of $10^{10}$) and the lower ionosphere (conductance on the order of $10^4$) represents an electromagnetic resonator in a wide range of frequencies from ones of hertz to ones of kilohertz.

The resonance cavity is bounded below by the sphere $r = a$ at which the jump in conductance reaches values on the order of $10^{10}$. The upper diffuse boundary is formed by the plasma of the lower ionosphere in which the particle concentration increases with height. The ionosphere is in the constant magnetic field of the earth and represents, generally speaking, a medium with double refraction [1, 2].

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