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Symmetry Correlations in Nuclear Reactions.

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The exclusion principle, applied to a system of nucleons, amounts to certain restrictions on the symmetry of the wave function with respect to the exchange of the spatial, spin and isotopic spin coordinates of the nucleons. A convenient way to classify such symmetries is to introduce a set of five quantum numbers: P, P', P'' (the partition numbers of WIGNER (1)), S (the total intrinsic spin) and T (the total isotopic spin).

Of course, not all of them are independent, nor necessarily good quantum numbers: P, P', P'' are good ones if the nuclear forces are only of Wigner or Majorana type: S of Wigner, Majorana, Bartlett and Heisenberg type: T of charge independent character.

However, in any case, also when such quantum numbers are not good, it is by no means useless to ask what is the mixture of the different pure states (i.e. states specified by a single set of the above five quantum numbers). In the same sense that it is not irrelevant to know what is the mixture of S and D states in the ground state of the deuteron, although L is not a good quantum number.

The purpose of the present letter is to stress the importance of such a knowledge for the study of nuclear forces, and to point out a method to get such an information, which seems not too far from the range of the present experimental possibilities. We want to prove in fact that a correlation exists between the spin and isotopic spin polarization of the particles emitted in a nuclear reaction, and that this polarization is in general different for different states of symmetry. So, if one is able to perform such experiments, one can draw some information on the mixture of different symmetries in the final state and therefore on the strength of the forces which tend to spoil the goodness of the above quantum numbers.

To be definite let us take only a simple example. Let an He^3 atom be split as a consequence of a nuclear process (for instance the absorption of a γ-ray) into two protons and a neutron. Assume we have an experimental device to determine the nature of the particle emitted (proton p or neutron n) as well as its state of polarization with respect to a

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(1) E. P. WIGNER: *Phys. Rev.*, **51**, 106 (1937); E. P. WIGNER and E. FEENBERG: *Rep. Prog. Phys.*, **8**, 274 (1941).

fixed z-axis: in the following we shall indicate the two spin states of a nucleon by a and b. Three such devices are now put in three different places, which we call A, B, C, to detect the decay products of He^3.

We count the coincidence $(Apa)(Bpa)(Cna)$; $(Apa)(Bpa)(Cnb)$; $(Apa)(Bpb)(Cna)$: where a symbol like $(Apa)(Bpa)(Cnb)$ means that the particle in A is a proton in the polarization state a: the particle in B is a proton in the polarization state a: the particle in C is a neutron in the polarization state b, and similarly.

The a priori possible states for the final system are 24 (a proton may go in each of the 6 states Aa, Ab, Ba, Bb, Ca, Cb; if it goes in Ca, for instance, the only states for the other proton are the four Aa, Ab, Ba, Bb; then only two states are left for the neutron. Therefore $6\cdot4\cdot2=48$; but in this way each state is counted twice, since proton 1 going in Aa and proton 2 in Cb is the same as proton 1 in Cb and proton 2 in Aa; so the number is actually 24). These states may be arranged in the different pure states, which are listed in the first column of table I, where we have denoted them both with the numbers P, P', P'', S, T and with the corresponding Young tableaux, for those among the readers more familiar with this other kind of notations (²). It is quite elementary to write the wave function of these states and to deduce therefrom the relative probabilities of the different events. The results are summarized in Table I.

Of course, one can modify the experiment at will by displacing detector C or simply dropping it. In the former case one measures the probabilities at different energies, account being taken of the energy-momentum relation; in the latter case one gets the probability of the processes $\alpha+\beta$ compared with the probability of the two events in γ.

One more example, just to show how apparently different phenomena may be encompassed in a single scheme. Consider the decay of a τ-meson into three π. It was shown by DALITZ (³) that, if charge independence holds, the ratio between the two competitive processes $\tau^+\to2\pi^++\pi^-$ and $\tau^+\to2\pi^0+\pi^+$ must be between 1 and 4. It is easily proved in fact that this ratio is 4 if the wave function of the three decaying π is symmetric in the space coordinates, whereas it is one, if the special wave function belongs to the two dimensional representation of the symmetric permutation group on three variables. In this example, since the spin of the π is zero, the correlation is directly between the symmetry of space and isotopic spin coordinates of the wave function.

We shall not enter into the question of the significance of such information, which is actually information on the final state. So if there are no conservation laws, one can hardly say anything about the initial state. But anyhow *it is an information* which must be taken into account to get a more complete set of data as regards the process. And one may think that such kind of information is perhaps of more profound physical significance than other data, which are usually taken into account and which depend critically on the *form* (Yukawa, Gaussian, etc.) and not only on the *type* (of Wigner, Heisenberg, etc.) of the potential.

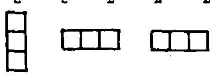
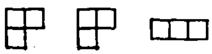
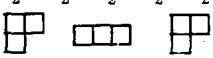
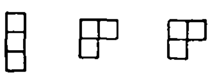
After this work was completed a paper by SIMON and WELTON (⁴) came our notice, in which these authors deal

(²) A. GAMBA, R. MALVANO and L. A. RADICATI: *Phys. Rev.*, **87**, 441 (1952).

(³) R. H. DALITZ: *Proc. Phys. Soc.*, A. **66**, 710 (1953).

(⁴) A. SIMON and T. A. WELTON: *Phys. Rev.*, **90**, 1036 (1953); A. SIMON: *Phys. Rev.*, **92**, 1051 (1953).

TABLE I. – *Relative probabilities* ($\alpha:\beta:\gamma$) *of the different events in each state of symmetry.*

P, P', P'', S, T and the corresponding Young tableaux for space, spin and isotopic spin	α $(Apa)(Bpa)$ (Cna)	β $(Apa)(Bpa)$ (Cnb)	γ $(Apa)(Bpa)(Cna)$ or $(Apa)(Bpb)(Cnb)$	Number of states of each symmetry
$\frac{3}{2}$, $\frac{3}{2}$, $\frac{3}{2}$, $\frac{3}{2}$, $\frac{3}{2}$ 	3	1	1	4
$\frac{3}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{2}$ 	0	1	1	4
$\frac{3}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{1}{2}$ 	3	1	1	8
$\frac{1}{2}$, $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$ 	0	0	1	2
$\frac{3}{2}$, $\frac{3}{2}$, $\frac{3}{2}$, $\frac{1}{2}$, $\frac{1}{2}$ 	0	4	1	2
$\frac{3}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$ 	0	1	1	4

with the polarization of particles emitted in nuclear reactions. They treat the problem with the standard technique of RACAH as a generalization of the usual problem of angular correlation. What has escaped their attention, however, is the fact that in reactions of the type we considered the polarization effects are completely independent from the azimuthal quantum numbers l (apart from the trivial consequences of conservation of total angular momentum) and depend only on the symmetries of the wave function with respect to permutations. To put it in a different way, as RACAH [5] pointed out to the earlier investigators of angular correlation phenomena that the natural development of the correlation function was not a development in power of cosines, but in a series of spherical harmonics, in the same way we point out that the natural way to deal with the polarization for certain reactions is not to use spherical harmonics but to take account of the symmetries with respect to permutations.

[5] G. RACAH: *Phys. Rev.*, **84**, 910 (1951).