# THE HORIZONTAL VARIATION OF TEMPERATURE IN THE LOW SOLAR PHOTOSPHERE

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**Abstract.** Observations of the rms intensity fluctuations in the continuum obtained by Pravdjuk *et al.* (*Solnechnye Dannye*, No. 2, p. 70, 1974) from white-light photographs made with the Soviet Stratospheric Solar Observatory are analyzed to obtain a horizontal temperature-fluctuation amplitude as a function of depth. The results indicate that temperature fluctuations increase with depth monotonically from a small value at $\tau_{5000} \cong 0.5$ (cf. Figure 2). The initial rise of $\Delta T$ appears quite steep, having a slope of approximately 20 K $\mathrm{km}^{-1}$. The model of Wilson (*Solar Phys.* **9**, 303, 1969) is incompatible with the data. Convective flux in the present model is approximately 6% of the total flux at $\tau_{5000} = 1$.

## 1. Introduction

The horizontal variation of temperature with depth, $\Delta T(z)$, in the photosphere is an important quantity for determining the convective flux in the visible layers of the sun and thus provides a check on models of convection. A widely used model for $\Delta T(z)$ has been that of Wilson (1969), which is based on the reduction of Stratoscope data (Schwarzschild, 1959) by Edmonds (1962). However, a recent study by Altrock and Musman (1976) has called that model into question. They find a monotonic increase of $\Delta T$ with depth with an amplitude of approximately 140 K at 14 km above $\tau_{5000} = 1$, vs the maximum $\Delta T$ of approximately 900 K found by Wilson (1969) at that height. A theoretical study by Musman and Nelson (1976) is also in qualitative agreement with Altrock and Musman (1976).

In an attempt to resolve this disagreement, I have analyzed the data of Pravdjuk *et al.* (1974) (hereinafter referred to as PKA). These data are rms intensity fluctuations in the continuum from the center to the limb, $\delta I(\mu)$, where $\mu \equiv \cos\theta$, and are taken from white-light pictures made with the Soviet Stratospheric Solar Observatory. Thus, the data are obtained in a manner similar to the Stratoscope data, and the method of analysis I will follow will be similar to that utilized by Wilson (1969).

## 2. The Data

PKA tabulate values of $\delta I(\mu)$ at $\lambda \cong 4600$ Å from $\theta = 0$ to 85°. Only the best frames were used. The data were corrected for film grain and measurement errors but not instrumental smearing. The values presented are taken from two different flights, and the results in the range $\theta = 30$ to 50° (from the later flight) have been scaled (divided by 1.33) to the results outside that range from the earlier flight.

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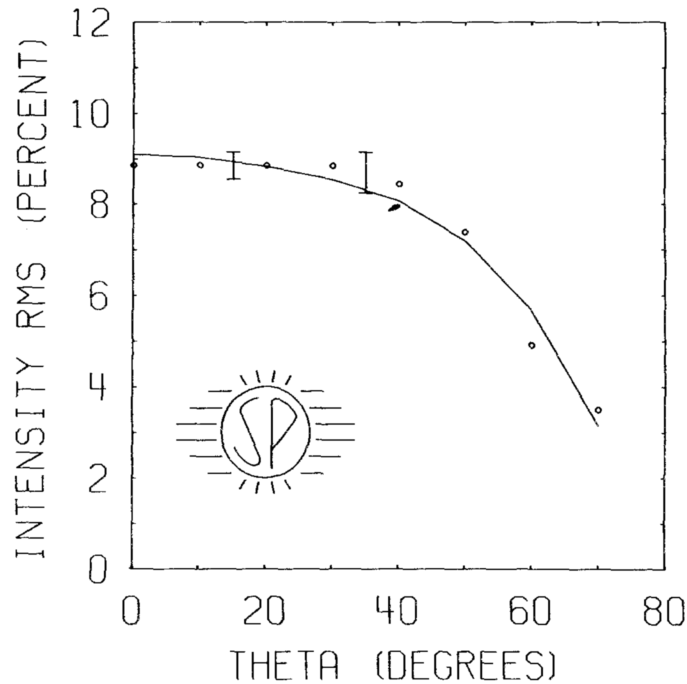


Fig. 1. rms intensity fluctuations as a function of disk-position angle. Circles: data of Pravdjuk *et al.* (1974); solid line: 'best-fit' solution of this paper. Uncertainty bars are given for data (see text for discussion).

Thus, there appears to be a systematic uncertainty in the amplitude of all values of $\delta I$ of at least 33% of the local value.

The data are presented in Figure 1. The uncertainty bar at $\theta = 15°$ represents $\pm\sigma$, where $\sigma$ is the standard deviation of the values of $\delta I$ given by PKA for nine areas with $\theta < 30°$. The other uncertainty bar indicates the range in values for the two values presented at that approximate value of $\theta$. An 'eyeball' estimate of the scatter beyond $\theta = 50°$ (from Figure 3*a* of PKA) indicates that the value of $\sigma$ continues to increase with $\theta$. A smooth curve has been drawn through the observed data points by PKA. I do not present data (or the results of the analysis) beyond $\theta = 70°$. This is because both Edmonds (1962) and PKA indicate that beginning at approximately $\theta = 80°$ the nature of the inhomogeneities seen in the continuum undergoes a fundamental change. The granules begin to disappear, and large-scale, diffuse non-uniformities begin to account for the measured intensity fluctuations. Although the nature of these non-uniformities is not clear, it appears that they represent a different class of structures from granules and should not be included in a study of granules.

Finally, it is of interest to note that the brightness distribution found by PKA is distinctly asymmetric and implies that the ratio of areas of bright and dark elements is 0.80 at the center of the disk. This confirms the results of Parvey and

Musman (1971) and also implies that the asymmetries in $\Delta T(z)$ between hot and cold elements found by Altrock and Musman (1976) are real (cf. also Musman and Nelson, 1976, in this respect).

## 3. Analysis

I have modified the program utilized by Altrock and Cannon (1975) to compute the electron pressure $P_e$ following Altrock and Musman (1976). Thus, at each point on the solar disk, a horizontal sinusoidal variation in temperature with an amplitude, $\Delta T(z)$, that is a function of depth is added to the basic one-dimensional atmosphere. The basic atmosphere assumed is the Harvard-Smithsonian Reference Atmosphere (HSRA: Gingerich *et al.* 1971). Pressure equilibrium is assumed with the HSRA at all points in the atmosphere, and $P_e$ is computed as a function of $(T+\Delta T, P)$ at each point in the two-dimensional atmosphere. The importance of including fluctuations in electron pressure (opacity) has been discussed by Musman and Nelson (1976). I have assumed a horizontal wavelength of 1160 km (Allen, 1973, p. 188). The emergent intensity is computed at a number of horizontal points along this temperature fluctuation.

Having obtained a one-dimensional horizontal variation of intensity, I fit it with the function

$$I(\mu, x) = \bar{I}(\mu) + \Delta I(\mu) \cos 2\pi(x - x_0)/l, \tag{1}$$

corresponding to the assumed temperature fluctuation function,

$$\Delta T(x, z) = \Delta T(z) \cos 2\pi x/l, \tag{2}$$

where $l$ is the horizontal wavelength. I then assume that the solar surface projected on the line of sight at that value of $\mu$ is covered by a *two-dimensional* pattern,

$$I(\mu, x, y) = \bar{I}(\mu) + \Delta I(\mu) \cos \frac{2\pi x}{\mu l} \cos \frac{2\pi y}{l}, \tag{3}$$

where $\bar{I}(\mu)$ and $\Delta I(\mu)$ are obtained from Equation (1). The $x$-axis is taken perpendicular to the limb. It may be shown that the rms deviation of the intensity is then given by

$$\delta I(\mu) = \frac{\Delta I(\mu)}{2}. \tag{4}$$

I have confirmed that the assumption of Equation (2) does in fact lead to the sinusoidal intensity distribution of Equation (1) to a high degree of precision. In fact, the value of $\delta I(\mu)$ computed numerically from the individual values of $I(\mu, x)$ differs from that computed from the one-dimensional analog of Equation (4), $\delta I(\mu) = \Delta I(\mu)/\sqrt{2}$, by less than 0.0001 $\bar{I}(\mu)$.

The granulation pattern on the sun is smeared by passage of the light through the telescope. This has the effect of lowering the observed values of $\delta I$, particularly as one approaches the limb and foreshortening becomes important. In order to mimic this effect in my calculations, I smeared the emergent intensity profile, $I(x, y)$, with the image of a point source as observed through a 50 cm aperture telescope, such as used in the early SSSO flights (cf. Krat, 1971). The two-dimensional Fourier transform of this smearing function, $S(\rho)$, is derived by a number of authors (e.g. Goodman, 1968, pp. 102–120). It may be shown that the effect of smearing the intensity distribution of Equation (4) with $S(\rho)$, is equivalent to the operation

$$\Delta I^s(\mu) = S(\rho_l)\Delta I(\mu) \tag{5}$$

where $\rho_l^2 = (1/\mu l)^2 + (1/l)^2$, and the superscript $s$ denotes the smeared value of the amplitude. The value of $\delta I^s(\mu)$ is then determined by substituting $\Delta I^s(\mu)$ into Equation (4).

Smearing is also introduced by the scanning aperture of the microphotometer. However, since the width of this aperture at $\mu = 1$ is 66 km on the Sun, and the width of the Airy disk of the telescope at $\mu = 1$ is 142 km on the Sun, I ignored this source of smearing as being small relative to the telescope smearing.

As there is no generally accepted functional form for $\Delta T(z)$ in the Sun, I experimented with forms consisting of a number of intersecting straight lines. The final form adopted was a single straight line having a given slope with a limiting constant value in the mid-photosphere (cf. Figure 2).

## 4. Results

The analysis proceeds by varying the three $\Delta T$-variables empirically in such a way as to reproduce the observed $\delta I(\mu)$. The final result is a three-dimensional atmosphere having a horizontal sinusoidal variation in temperature whose amplitude is determined by fitting the observations of rms intensity fluctuations and which is in pressure-equilibrium with the HSRA and has a self-consistent electron pressure. The "best-fit" result is shown in Figure 1. The amplitude of the horizontal temperature fluctuation that produced the result of Figure 1 is shown in Figure 2. The functional form of the 'best-fit' temperature amplitude is $\Delta T(z) = 690 + 20\,z$ for $z \geqslant -34$ km and $\Delta T(z) = 0$ for $z < -34$ km. The average of the six solutions obtained by Altrock and Musman (1976) in a lower resolution (1″) study is also shown in Figure 2.

The results are sensitive to the assumed value of the horizontal wavelength of $\Delta T$, to the full width at half maximum (FWHM) of the assumed instrumental smearing function and to the assumed functional form of $\Delta T(x)$. To test these effects I experimented on variations to a best-fit solution (not shown) utilizing only one-dimensional smearing (i.e., ignoring the y-variation by adopting Equation (1) and smearing the transform with a $\mu$-dependent Gaussian of FWHM = 142 km at $\mu = 1$).

For example, if the horizontal wavelength is increased by a factor of two, the rms increases by 0.35% $\bar{I}(\mu)$ at $\theta = 0$ and 2.65% $\bar{I}(\mu)$ at $\theta = 70°$. Thus, if the

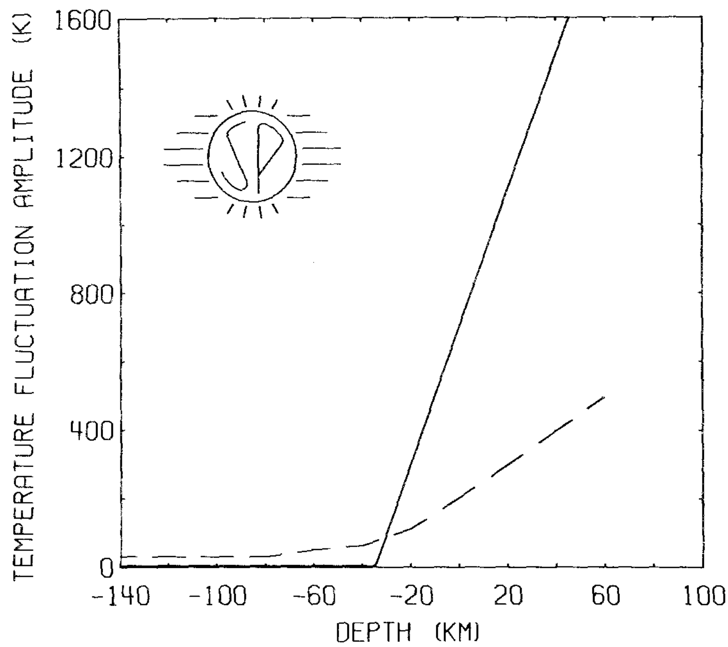


Fig. 2. Solutions for the *amplitude* of the horizontal temperature fluctuations. Solid line: 'best-fit' solution; dashed line: average of $|\Delta T|$ from Altrock and Musman (1976). Values of rms of $\Delta T$ may be obtained from the values given in this graph by dividing by two [cf. Equation (4)].

wavelength is larger than 1160 km, somewhat smaller values of $\Delta T$ will be required than are shown in Figure 2. Variations in the assumed value of FWHM of the smearing function have a similar effect. If the size of the Airy disk is increased (an unwarranted assumption, as Krat *et al.* (1972) report measuring solar features as small as 140 km on their spectrograms) a higher value of $\Delta T$ is required to fit the observed rms.

Finally, in considering the effect of non-sinusoidal temperature distributions I examined the functional form $\Delta T \alpha \cos^{1/n} x$, where $n$ varies from 1 to $\infty$ (two-column structure) by odd values. This yields progressively squarer distributions of $\Delta T(x)$. The results indicate that the rms increases progressively as $n$ goes from 1 to $\infty$, reaching a value of 12.4% $\bar{I}$ at $\mu = 1$. Thus, a two-column model would require a somewhat lower $\Delta T(z)$. By varying the size of bright elements relative to dark elements from the value unity assumed up to now, I found that the rms decreases in both directions from unity. By allowing $N$(bright)/$N$(dark), where $N$ is the number of points along a line through the two-column structure, to vary from 3.0 to 0.33 (cf. 0.80 found by PKA for $A$(Bright)/$A$(dark), where $A$ is the area), I found that the rms decreased by 2.7% and 1.3% $\bar{I}$, respectively, at $\theta = 0$ from the value 12.4%. Changes at $\theta = 70°$ are smaller than the changes at $\theta = 0$.

Thus, reasonable variations in the horizontal wavelength, Airy disk and functional form of $\Delta T(x)$ lead to small changes in $\delta I(\mu)$ that would be reflected in changes in the details of $\Delta T(z)$, but not the overall conclusions, to be stated in the next section.

The theoretical values of $\delta I(\mu)$ shown in Figure 1 have a further uncertainty due to the fact that the HSRA begins to become optically thin at points of maximum negative $\Delta T$. To estimate this uncertainty I extrapolated $T_e$ and $\rho$ logarithmically to $z = +120$ km in the HSRA and computed $P_e$ self-consistently. The maximum uncertainty appears to be approximately 0.4% $\bar{I}(\mu)$.

A further uncertainty may be ascribed to the specific geometrical model of $I(x, y)$ utilized. For example, going from a one-dimensional sinusoid [Equation (1)] to a two-dimensional sinusoid [Equation (3)] requires higher values of $\Delta T(z)$. In particular, for 'best-fit' solutions to these two cases, the increase in $\Delta T(z)$ at depth zero is 430 K. Thus, it is important to take account of the two-dimensional nature of the observations.

The solution is only marginally sensitive to the shape of the $\Delta T(z)$ function below $z = +60$ km. For example, if the value of $\Delta T(z)$ is kept constant at depths below $z = +60$ km, the maximum decrease in the computed rms is only 0.3% $\bar{I}(\mu)$.

## 5. Conclusions

I have analyzed the observations of rms intensity fluctuations in the continuum (Figure 1) obtained by Pravdjuk *et al.* (1974) (PKA) from the Soviet Stratospheric Solar Observatory (SSSO). Utilizing the most-likely values for horizontal wavelength of granulation and instrumental smearing, I obtained the 'best-fit' solution for horizontal temperature fluctuations in granulation as a function of depth, $\Delta T(z)$, shown in Figure 2.

The results are consistent with those of Altrock and Musman (1976) if account is taken of the lower spatial resolution of the previous study. I thus confirm their conclusion that the convective flux is small at $\tau_{5000} = 1$. For example, if a convective velocity of 1 km s$^{-1}$ is assumed at $z = 0$, the convective flux from the present model is 6% of the total flux. A similar calculation by Altrock and Musman (cf. their Section V.c) led to a value of less than 1% at the same height. They pointed out this was an understimate due to their low resolution.

The present results for $\Delta T(z)$ are not consistent with those derived by Wilson (1969) from the data of Edmonds (1962). Wilson (1969) obtains a functional form of $\Delta T(z)$ entirely different from that found here and a maximum *amplitude* of $\Delta T(z)$ equal to 900 K at $\tau_{5000} \cong 0.7$, whereas this study and that of Altrock and Musman (1976) obtain $\leq 350$ K at that depth. Wilson (1975) comments that the data of PKA and Edmonds (1962) are both somewhat unsatisfactory due to the 33% arbitrary correction (cf. Section 2) applied to the SSSO data in the region $30 \leq \theta \leq 50°$ and the fact that in this same region Edmonds' (1962) data are based on only one frame. The qualitative corrobation by Altrock and Musman (1976) of the results of this paper, however, appears to favor the PKA data. If the data in the range $30 \leqslant \theta \leqslant 50°$ were deleted from the present study, the results would change little.

Edmonds (1974) has utilized Wilson's (1969) values of $\Delta T(z)$ to obtain the convective flux at $\tau_{5000} = 1.3$. He finds a value of approximately 60% of the total flux at that height and compares this with the predictions of several theoretical models of convection. The much lower value of 6% at $\tau_{5000} = 1$ found here utilizing data of similar high resolution suggests that Edmonds' (1974) conclusions should be re-examined.

The results for $\Delta T(z)$ of this study also differ from those presented by PKA in their Figure 3a. Their values are brightness temperatures based on the assumption that $\Delta T$ does not change with height, whereas the present study does allow $\Delta T$ to be a function of height.

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