Chemical Engineering Science, Vol. 41, No. 11, pp. 2925–2929, 1986.
Printed in Great Britain.

0009–2509/86 $3.00 + 0.00
Pergamon Journals Ltd.

ESTIMATION OF RATES AND ENHANCEMENT FACTORS IN GAS ABSORPTION WITH ZERO-ORDER CHEMICAL REACTION AND GAS-PHASE MASS-TRANSFER RESISTANCES

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(*Received* 21 *November* 1985)

Abstract The governing differential equation for gas absorption in laminar falling film with zero-order homogeneous chemical reaction and finite gas-phase mass transfer resistances is solved analytically by the method of separation of variables and subsequent solution with confluent hypergeometric function. The results for concentration profile, gas absorption rate, and enhancement factor are presented for various reaction rate and mass transfer resistance parameters.

INTRODUCTION

Most recently, Riazi *et al.* (1985) solved the governing equation for gas absorption with zero-order chemical reaction without gas-phase resistances. The main purpose of this work is to show the effect of finite resistance in the gas phase on absorption rate and enhancement factors in gas absorption with zero-order chemical reaction. The similar problem for the case of a first-order reaction was solved by Stepanek and Achwall (1976) and later by Pedersen and Prenosil (1981).

In our analysis we assume that the physical properties of the liquid, the liquid layer thickness, δ, and also the concentration of dissolving gas in the gas phase are all constant. The transport equation describing the concentration in a laminar liquid film with zero-order chemical reaction in appropriate dimensionless variables becomes:

$$(1-x^2)\frac{\partial C}{\partial z}=\frac{\partial^2 C}{\partial x^2}-\alpha. \tag{1}$$

The z-axis coincides with the film interfacial surface and it is directed down the flow. The x-axis is directed towards the wall. The corresponding boundary conditions for eq. (1) when there is a finite gas-phase mass-transfer resistance (R) are:

$$z=0; \quad 0\leqslant x\leqslant 1, \quad C=0 \tag{2}$$

$$x=0; \quad z>0, \quad \frac{\partial C}{\partial x}=\frac{1}{R}(C-1) \tag{3}$$

$$x=x^*; \quad \begin{cases} \text{if } x^*=1, & \dfrac{\partial C}{\partial x}=0 \\ \text{if } x^*<1, & C=\dfrac{\partial C}{\partial x}=0 \end{cases} \tag{4}$$

where x^* is the penetration depth of the dissolving gas and it can be determined by steady-state solution of eq. (1):

$$x^*=\begin{cases} 1 & \alpha\leqslant\alpha^* \\ [(2/\alpha)+R^2)^{1/2}]-R, & \alpha\geqslant\alpha^* \end{cases} \tag{5}$$

in which $\alpha^*=\dfrac{2}{1+2R}$.

Equation (1) can be solved by the method of separation of variables and subsequent solution with confluent hypergeometric function. The final result for $C(x, z)$ is in the following form:

$$C(x,z)=1-\alpha Rx^*-\alpha x^* x+\frac{\alpha}{2}x^2-\sum_{n=1}^{\infty}A_n\exp(-\lambda_n^2 z)X_n(x) \tag{6}$$

$$C(x,z)=0 \quad \text{if} \quad x\geqslant x^*$$

where eigenfunctions $X_n(x)$ are given by

$$X_n(x)=\lambda_n^{1/2}\exp\left(-\frac{\lambda_n}{2}x^2\right)\left[RM\left(\frac{1-\lambda_n}{4},\,1/2,\,\lambda_n x^2\right)+xM\left(\frac{3-\lambda_n}{4},\,\frac{3}{2},\,\lambda_n x^2\right)\right]. \tag{7}$$

Eigenvalues λ_n are zeros of the following characteristic equation:

$$-\lambda Rx^*M\left(\frac{1-\lambda}{4},\,\frac{1}{2},\,\lambda x^{*2}\right)+2\lambda Rx^*\left(\frac{1-\lambda}{2}\right)M\left(\frac{5-\lambda}{4},\,\frac{3}{2},\,\lambda x^{*2}\right)+(1-\lambda x^{*2})M\left(\frac{3-\lambda}{4},\,\frac{3}{2},\,\lambda x^{*2}\right)+\frac{\lambda(3-\lambda)}{3}x^{*2}M\left(\frac{7-\lambda}{4},\,\frac{5}{2},\,\lambda x^{*2}\right)=0. \tag{8}$$

Coefficients A_n can be determined by applying the boundary condition given by eq. (2) which yields:

$$A_n = \frac{\int_0^{x^*} (1 - \alpha R x^* - \alpha x^* x + \alpha/2 x^2)(1 - x^2) X_n(x)\,dx}{\int_0^{x^*} (1 - x^2) X_n^2(x)\,dx}. \tag{9}$$

The total dimensionless rate of absorption, $\dot{m}$, is:

$$\dot{m} = -\int_0^z \left.\frac{\partial C}{\partial x}\right|_{x=0} dz. \tag{10}$$

After substitution of eq. (6) into eq. (10), the following relation for the absorption rate can be obtained:

$$m = \alpha x^* z + \sum_{n=1}^{\infty} B_n (1 - e^{-\lambda_n^2 z}) \tag{11}$$

where

$$B_n = A_n/\lambda_n^{3/2}.$$

In order to compare the absorption rates evaluated with the combined influence of gas-phase mass-transfer resistances and finite film thicknesses, we introduce the absorption rate found from the simple penetration model as discussed by Best and Hoerner (1979):

$$\dot{m}_0 = \sqrt{\left(\frac{4}{\pi}\right) z}. \tag{12}$$

An enhancement factor, E, is defined for the absorption system as the ratio of absorption rates using eqs (11) and (12):

$$E = \frac{\dot{m}}{\dot{m}_0} = \frac{\alpha x^* z + \sum_{n=1}^{\infty} B_n (1 - e^{-\lambda_n^2 z})}{\sqrt{\left(\frac{4}{\pi}\right) z}}. \tag{13}$$

RESULTS AND DISCUSSION

In Table 1, values of α^* and x^* defined in eq. (6) are given for different values of the resistance number, R, and the reaction rate parameter α. The eigenvalues λ_n and coefficients A_n and B_n have been numerically evaluated for the same values of R and α given in Table 1 and are shown in Table 2.

The enhancement factor, E, is plotted in Fig. 1 as a function of the Hatta number $\sqrt{M'} = \sqrt{(\pi/4)\alpha z}$ [as used by Best and Hoerner (1979)] for the case of no resistance in the gas phase when $R = 0$. Figure 2 shows the effect of resistance R on the enhancement factor E for $\alpha = 0.1$. As the resistance parameter R increases, the enhancement factor E decreases. The resistance parameter R indicates the ratio of the rate of diffusion

Table 1. Values of α^* and x^* for different values of α and R

α	$R = 0$	0.1	0.5	1	10
	$\alpha^* = 2.0$	1.667	1.0	0.667	0.095
0	1.0	1.0	1.0	1.0	1.0
0.1	1.0	1.0	1.0	1.0	0.954
1	1.0	1.0	1.0	0.732	0.1
10	0.447	0.358	0.171	0.095	0.01

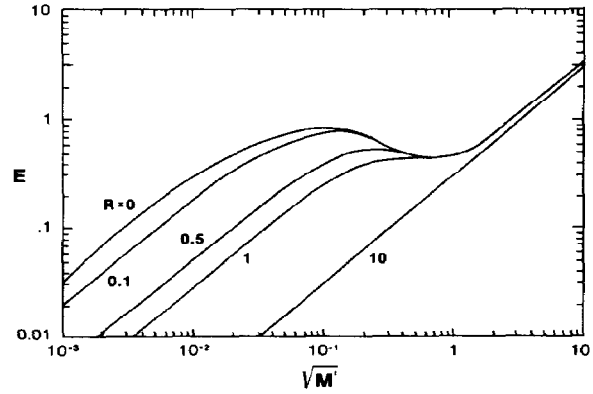

Fig. 1. Enhancement factor, E, as a function of $\sqrt{M'}$ for $R = 0$ with α as the parameter.

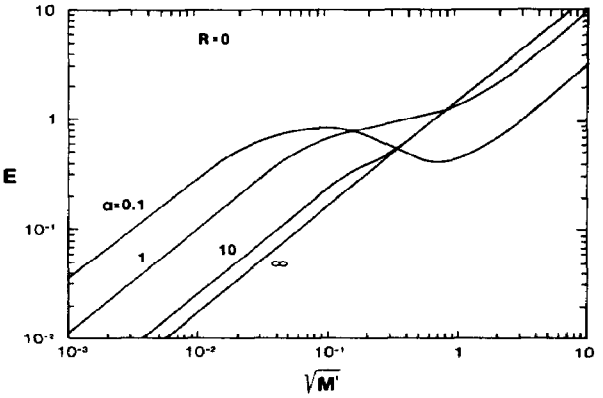

Fig. 2. Enhancement factor, E, as a function of $\sqrt{M'}$ for $\alpha = 0.1$ with R as the parameter.

to the rate of mass transfer in the gas phase in relation to the thickness of the film. Thus a small value of R could be due to a very low rate of diffusion in the liquid, a very high rate of mass transfer in the gas phase, or a very thick liquid film. In all cases, the enhancement is less than that observed in the absence of resistance in the gas phase. The effect is most pronounced at low values of $\sqrt{M'}$. At large values of $\sqrt{M'}$ where it is greater than 2, the enhancement factor E can be predicted from the following simplified equation with errors of less than 2%:

$$\sqrt{M'} > 2, \quad E = \begin{cases} \alpha^{1/2}\sqrt{M'}, & \alpha \leqslant \alpha^* \\ \alpha^{1/2}\left[\left(\frac{2}{\alpha} + R^2\right)^{1/2} - R\right]\sqrt{M'}, & \alpha \geqslant \alpha^*. \end{cases} \tag{14}$$

Table 2. Eigenvalues λ_n and coefficients A_n and B_n for different values of α and R

	λ_n	A_n	B_n		λ_n	A_n	B_n
$R = 0$	2.2631092072	1.7504205704	0.5141434669	$R = 0.1$	1.9951744080	1.6200199127	0.5748429894
$\alpha = 0$	6.2977790833	0.9457654357	0.0598415136	$\alpha = 0$	5.6373319626	0.7011763453	0.0523862503
	10.3077039719	0.6850752234	0.0507044960		9.3471527100	0.3682686090	0.0128868259
	14.3127698898	0.5342614651	0.0098666213		13.1390151978	0.2012571692	0.0042257868
	18.3159027100	0.4285235405	0.0054667927		16.9917907715	0.1132090092	0.0016163019
$R = 0$	2.2631092072	1.6887512207	0.4960295558	$R = 0.1$	1.9951744080	1.5500831604	0.5500268340
$\alpha = 0.1$	6.2977790833	0.9477198124	0.0599651746	$\alpha = 0.1$	5.6373319626	0.7033987045	0.0525522865
	10.3077039719	0.6827884316	0.0206320956		9.3471527100	0.3665308952	0.0128260180
	14.3127698898	0.5347718596	0.0098760463		13.1390151978	0.2018160820	0.0042375214
	18.3159027100	0.4278833270	0.0054586232		16.9917907715	0.1129676700	0.0016128563
$R = 0$	2.2631092072	1.1337356567	0.3330072165	$R = 0.1$	1.9951744080	0.9206314683	0.3266741037
$\alpha = 1$	6.2977790833	0.9653742909	0.0610822253	$\alpha = 1$	5.6373319626	0.7234310508	0.0540489405
	10.3077039719	0.6622081399	0.0200102106		9.3471527100	0.3508828282	0.0122784451
	14.3127698898	0.5393675566	0.0099609196		13.1390151978	0.2068448663	0.0043431111
	18.3159027100	0.4221180081	0.0053850748		16.9917907715	0.1107940674	0.0015818235
$R = 0$	3.7165431976	0.4296758175	0.0599697568	$R = 0.1$	3.5629329681	0.3169710636	0.0471311547
$\alpha = 10$	18.2082366943	0.3978564262	0.0051206420	$\alpha = 10$	11.0843973160	0.2944829559	0.0007979823
	32.7502441406	0.3068409562	0.0016371608		19.3086242676	0.1758402586	0.0020724873
	47.3220825195	0.0775337219	0.0002381745		27.8878173828	0.0581454560	0.0003948153
	61.9855957031	−0.0747544169	−0.0001531796		36.6285400391	0.0141297914	0.0000637392
$R = 0.5$	1.4105405807	1.0771751404	0.0429954171	$R = 1$	1.0970897675	0.7549554706	0.6569887996
$\alpha = 0$	4.7896308899	0.1494035721	0.0142530650	$\alpha = 0$	4.5609941483	0.0478455201	0.0049119294
	8.5945749283	0.0365768224	0.0014516758		8.4534711838	0.0090595931	0.0003686007
	12.5123405457	0.0122974142	0.0002778473		12.4128980637	0.0023530105	0.0000538040
	16.4683074951	0.0044669770	0.0000668406		16.3920440674	0.0003718399	0.0000056028

Table 2. (*Contd.*)

	λ_n	A_n	B_n		λ_n	A_n	B_n
$R = 0.5$	1.4105405807	0.9907150269	0.5913850069	$R = 1$	1.0970897675	0.6573227644	0.5720253587
$\alpha = 0.1$	4.7896308899	0.1516969800	0.0144718550	$\alpha = 0.1$	4.5609941483	0.0496011823	0.0050921701
	8.5945749283	0.0361844897	0.0014361048		8.4534711838	0.0089658387	0.0003647862
	12.5123405457	0.0126400404	0.0002855884		12.4128980637	0.0026227653	0.0000599722
	16.4683074951	0.0045387968	0.0000679153		16.3920440674	0.0004876989	0.0000073486
$R = 0.5$	1.4105405807	0.2125674486	0.1268873215	$R = 1$	1.1686534882	0.0696053505	0.0550952107
$\alpha = 1$	4.7896308899	0.1723362803	0.0164408423	$\alpha = 1$	4.6753129959	0.0584045723	0.0057773814
	8.5945749283	0.0326585621	0.0012961663		8.5241432190	0.0147225931	0.0005915731
	12.5123405457	0.0157237202	0.0003552609		12.4626197815	0.0038035950	0.0000864530
	16.4683074951	0.0051848441	0.0000775823		16.4301757813	0.0020402642	0.0000306354
$R = 0.5$	3.2556552887	0.0301477350	0.0051321238	$R = 1$	3.1911230087	0.0031029557	0.0005443921
$\alpha = 10$	9.8997726440	0.0268292108	0.0008613334	$\alpha = 10$	9.4322271347	0.0029165589	0.0001006802
	19.1082916260	0.0204839297	0.0002452340		18.6700134277	0.0020605912	0.0000255648
	25.3806762695	0.0096746497	0.0000756625		24.2130432129	0.0013424407	0.000011263
	35.0996246338	0.0021055911	0.0000101256		33.2863006592	0.0002303256	0.0000011991
$R = 10$	0.3827895522	0.1577504873	0.6660864353	$R = 10$	1.0025072098	0.0009999900	0.0009962145
$\alpha = 0$	4.3165597916	0.0003352202	0.0000373787	$\alpha = 1$	4.4394960403	0.0000347035	0.0000037090
	8.3190031052	0.0001499212	0.0000043409		8.3837451935	0.0000295434	0.0000012170
	12.3209276199	0.0001390586	0.0000032154		12.4425344467	0.0000197625	0.0000004528
	16.3222961426	0.0001017571	0.0000020101		16.3900909424	0.0000094061	0.0000001417
$R = 10$	0.3833905458	0.0029963467	0.0126220435	$R = 10$	3.1617984772	−0.0567727052	−0.0100980774
$\alpha = 1$	4.3344650269	0.0008013588	0.0000888023	$\alpha = 10$	8.9558515549	−0.0337069482	−0.0012576494
	8.3610992432	0.0000856632	0.0000035432		17.9992065430	−0.0240080096	−0.0003143954
	12.3909912109	0.0000410574	0.0000009413		23.0939331055	−0.0213930011	−0.0001927634
	16.3630981445	0.0000100534	0.0000001519		30.8634643555	−0.0188581236	−0.0001099845

In order to check the reliability of our results, first we compared our analytical solution for the case of no reaction and no gas-phase mass-transfer resistance with the analytical solution of Olbricht and Wild (1969), and exact agreement was observed. In addition, when PDE (1) was solved by a numerical implicit technique, the numerical result for the concentration profile and the absorption rate were within 0.5% of the analytical values.

Acknowledgement—The computer time for this work was provided by The School of Engineering at Wright State University.

NOTATION

A, B coefficients
C dimensionless concentration of dissolved gas in the liquid phase ($= \bar{C}/\bar{C}^*$)
$\bar{C}^*$ dimensional concentration of dissolved gas at the interface
D liquid-phase diffusion coefficient
E enhancement factor ($= m/m_o$)
k zero-order reaction rate constant
k_G mass-transfer coefficient in the gas phase in terms of liquid concentration
M confluent hypergeometric function
$\sqrt{M'}$ a form of Hatta number ($= \sqrt{(\pi/4)\alpha z}$)
$\dot{m}$ dimensionless absorption rate without reaction given by eq. (11)
$\dot{m}_o$ dimensionless absorption rate without reaction given by eq. (12)
R dimensionless resistance parameter ($= D/k_G\delta$)
$\bar{u}$ dimensional liquid velocity
$\bar{u}_o$ dimensional liquid velocity at interface
X eigenfunction given by eq. (7)
x dimensionless distance from the interface towards the wall ($= \bar{x}/\delta$)
x^* dimensionless maximum penetration depth (Δ/δ) given by eq. (5)
z dimensionless axial distance ($= \bar{z}D/\bar{u}_o\delta^2$)

Greek letters

α dimensionless reaction parameter ($= k\delta^2/DC^*$)
α^* critical value of α defined in eq. (5) where the maximum penetration depth for the dissolving gas is less than the liquid film thickness
Δ dimensional maximum penetration depth
δ liquid film thickness
λ_n n-th eigenvalue

Superscript

$-$ indicates dimensional quantities

Subscript

n values corresponding to n-th eigenvalue

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