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Calculation on X-ray scattering of 17.4 keV radiation and image degradation in mammography

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Abstract

We have calculated the angular distribution of radiation scattered in water samples at 17.4 keV (K_α-radiation of Mo) by using form factor and incoherent scattering factor approximations. It was shown that radiation scattered and leaving a plane object with thickness in the cm-region is mainly due to elastic scattering within an angle of $\pm 30°$. The calculations were compared with other theoretical and experimental values for mammography, showing the influence of multiple scattering. The results allow some conclusions to be drawn about collimators used in mammography.

1. Introduction

In diagnostic radiology the radiation scattered by the tissue degrades the image quality. In the forward direction elastic scattering dominates, while for larger angles Compton scattering becomes more important [1]. It was shown, that it is possible to calculate the scattered intensity in soft tissue even for large biological samples using water as phantom material [2]. In this paper predictions are given for the angular distribution of the scattered radiation at an X-ray energy of 17.4 keV. This energy is important in mammography, where K_α-radiation from Mo-anodes dominates the X-ray spectrum [3].

In conventional mammography the X-ray film is placed directly under the mamma. In this case the scattered flux is about 20 to 80% of the total radiation incident on the film, depending on the breast thickness [4,5]. This is too high to obtain a good image quality. Using moving grid collimators the scattered intensity is reduced by a factor of 3 to 4. In another method amplified images are obtained by increasing film distance from the object. This results in a smaller solid angle and a reduction of scattered radiation of about 30% in comparison with the conventional technique.

Previous attempts have been made to calculate the intensity of the scattered radiation. Several groups have worked on multiple Compton scattering, mainly on corrections of the line profile [6,7], although these papers are not related to medical X-ray applications. In another paper, interesting results have been obtained applying Monte Carlo methods to determine the ratio of the scatter-to-primary radiation in mammography [8]. We complement these results in the following aspects. Molecular interference effects, that dominate other processes at small angles, will be considered. These effects were already pointed out in Refs. [9–11] without calculations. The second aspect is to give a simpler description of the angular distribution of single scattering behind the object. It will be shown that single scattering contributes about 60 to 85% to the total scattered radiation, depending on the breast thickness.

Moreover, knowledge of the angular distribution of the scattered radiation provides useful information on the effectiveness of the collimators used in mammography.

2. Theory

To study the attenuation of X-rays in matter, it is necessary to consider scattering and absorption. In the

Table 1
Total cross sections for Compton and Rayleigh scattering and photoelectric absorption for water. The attenuation is theoretically given by the sum of the three cross sections. The values were obtained by linear interpolation at an energy of 17.4 keV [12]

Water cross section $[10^{-24}$ cm^2/molecule]				Attenuation coefficient μ/ρ [cm^2/g]		
σ_{inc}	σ_{coh}	σ_p	σ_{tot}	water	soft tissue	muscle
5.2	3.1	27	36	1.2	1.1	1.2

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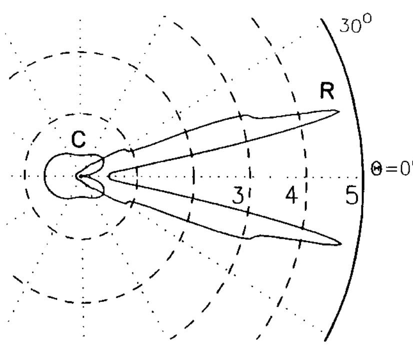

Fig. 1. Polar diagram of the differential cross section $d\sigma/d\Omega$ for Rayleigh (elastic) (R) and Compton (inelastic) (C) scattering at 17.4 keV calculated using Refs [1,13,9] in units of 10^{-24} cm^2.

energy range used in medical appliances, the radiation is scattered due to the Compton (incoherent) and Rayleigh (coherent) effects while the attenuation is mainly due to the photoelectric absorption. Table 1 shows the cross sections for scattering, photoelectric effect and total attenuation. In addition, the total attenuation coefficient is given.

Considering the integrated cross sections over the whole angular range, Compton effect is larger than Rayleigh scattering by a factor of 2 (Table 1). However, for angles lower than 40° Rayleigh scattering dominates and Compton scattering becomes negligible. The angular distribution of the scattered radiation can be calculated using the form factor and incoherent scattering factor theory [13]. At small scattering angles interference effects in Rayleigh scattering occur and molecular form factors have to be used for elastic scattering [9]. Fig. 1 shows the differential cross sections calculated from these theories.

In radiology the mamma is pressed between two parallel plates. Thus, in the calculations, the object can be represented by a parallel slab of thickness d (Fig. 2). It will be shown later that the area of the slab is not very important for the scattered intensity. According to Fig. 2 the scattered photon flux $d\phi^2(\theta)$ scattered by dx in a solid angle $d\Omega$ at an angle θ can be obtained from:

$$d^2\phi(\theta) = \phi_i\eta\frac{d\sigma(\theta)}{d\Omega}e^{-\mu x-\mu(d-x)/\cos\theta}\,dx\,d\Omega. \quad (1)$$

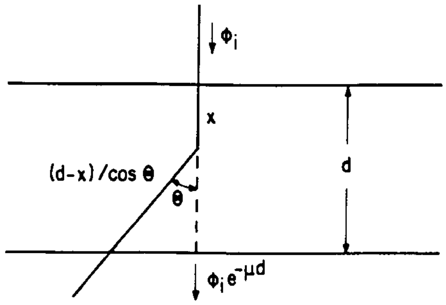

Fig. 2. Geometry used for calculation of the scattered flux leaving a slab object including absorption.

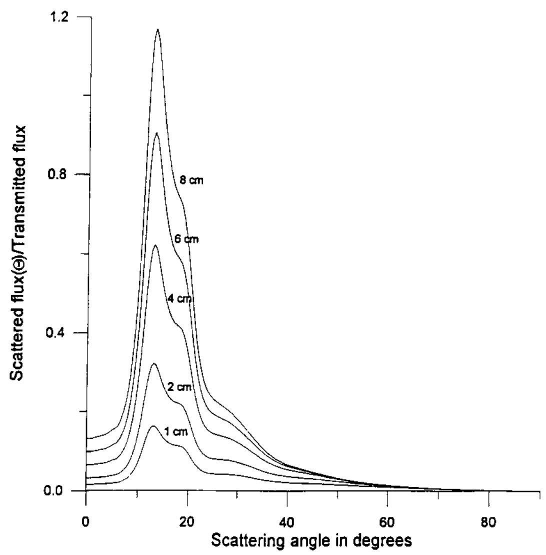

Fig. 3. Calculation of the angular distribution of the scattered radiation $R(\theta, d)$ leaving a slab object of thickness $d(=1, 2, 4, 6$, and 8 cm) using the cross sections of Fig. 1.

In Eq. (1) the absorption of the incident and scattered radiation is considered, assuming that no energy shift occurs. According to Figs. 1 and 3 this assumption is valid, because in the forward direction the scattering is mainly due to elastic processes. In addition, the maximum energy shift for Compton scattering is only about 0.6 keV. In the equation ϕ_i is the incident flux in photons per second, η is the density of the molecules ($=3.34\times10^{22}$ cm^{-3} for water), $d\sigma(\theta)/d\Omega$ is the differential cross section (Fig. 1), consisting of the sum of Rayleigh and Compton cross sections, and μ is the attenuation coefficient (Table 1). It is possible to integrate Eq. (1) obtaining the angular distribution scattered by the whole slab of thickness d:

$$\frac{d\phi(\theta)}{d\Omega} = \phi_i\eta\frac{d\sigma(\theta)}{d\Omega}e^{-\mu d/\cos\theta}\int_0^d e^{\mu(1-\cos\theta)x/\cos\theta}\,dx \quad (2)$$

The integration is easily performed. Dividing the result by the transmitted flux $\phi_i e^{-\mu d}$ one obtains the flux ratio $R(\theta, d)$, which is the scattered flux (leaving the object) per solid angle divided by the transmitted flux:

$$R(\theta, d) = \frac{d\phi(\theta)}{d\Omega}\frac{1}{\phi_i\,e^{-\mu d}} = \eta\frac{d\sigma(\theta)}{d\Omega}\frac{1}{Q}(1-e^{-Qd}) \quad (3a)$$

with

$$Q = \mu(1-\cos\theta)/\cos\theta. \quad (3b)$$

To estimate the flux ratio entering the X-ray film it is necessary to integrate over the solid angle $d\Omega$. If the film is in contact with a large object, integration has to be performed from $\theta = 0°$ to 90°. The upper integration limit θ_{max} is smaller when the distance between the object and the X-ray film is increased:

$$R'(\theta_{max}, d) = \frac{\phi(\theta_{max})}{\phi_i \, e^{\mu d}} = \int R(\theta, d) \, d\Omega = \int_0^{\theta_{max}} R(\theta, d) \, 2\pi \sin\theta \, d\theta. \quad (4)$$

For a given geometry, R' is usually called scatter-to-primary ratio (S/P). The results obtained for R for different angles and thicknesses (Eqs. (3a) and (3b)) are shown in Fig. 3. Based on these values, Eq. (4) was integrated by using numerical methods for various limits of θ_{max} and different breast thicknesses d. The results for R' are given in Fig. 4.

3. Results and discussion

Fig. 3 shows the ratio $R(\theta, d)$ between the angular distribution of the scattered photon flux leaving the object and the transmitted flux (Eqs. (3a) and (3b)). Different object sizes d were studied. In another work it is shown that results of measurements of scattering in tissue are nearly equal to those for water [2], so it is expected that Fig. 3 represents the behaviour of tissue. It can be seen that most of the radiation is scattered at angles lower than 40°. This is due to predominance of forward scattering, and the higher absorption for large angles due to the longer path in the object. Thus single scattering in mammography is mainly caused by Rayleigh scattering, in spite of the fact that the total cross section for Compton scattering is larger (Table 1).

The ratio between scattered and transmitted flux $R'(\theta_{max}, d)$ is presented in Fig. 4a as a function of the object thickness d and some values of the maximum scattering angle θ_{max}. In conventional mammography the X-ray film is placed directly under the object. In this case the maximum integration angle is $\theta_{max} = 90°$. For single scattering the scatter-to-primary ratio is about 20 to 40% for a breast thickness between 2 and 8 cm. It can be seen, that the scattering rate increases only slightly in the region $40° < \theta_{max} < 90°$. Consequently the scattered flux is nearly independent of the diameter of the object, in so far as it is larger than the thickness d. About 85% of the scattered radiation is directed forward within an angle of $\pm 40°$.

In Fig. 4b our calculations are compared with other experimental and theoretical results. It was already mentioned that the 90°-curve of Fig. 4 represents the scatter-to-primary ratio in normal mammography. This curve is compared with Monte Carlo calculations of Ref. [8], which includes multiple scattering. It may be concluded that more than 70% of the scattered radiation is due to single scattering. Multiple scattering should not be very important for small breast thickness, because the mean distance between two scattering events is about $1/\eta\mu_{scattering}$, which is about 3 cm (Table 1). In addition experimental curves are shown for the scattered radiation in mammography with

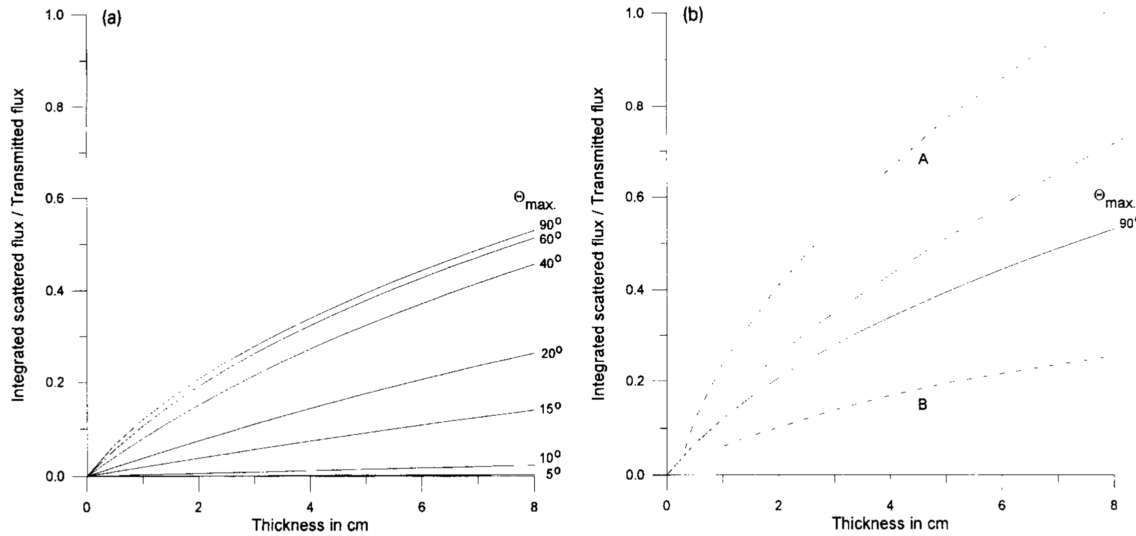

Fig. 4. (a) Integrated scattered flux for θ_{max}(= 0°, 5°, 10°, 20°, 40°, 60°, and 90°) divided by the transmitted flux in dependence of object thickness d at 17.4 keV ($R'(\theta_{max}, d) = S/P$). (b) Calculated scattered flux (solid line) in mammography (90°-curve of Fig. 4) in comparison with Monte Carlo results (dashed line) of Ref. [8]. Experimental scattering data for mammography (small dashed line A) and mammography using a moving collimator (small dashed line B) are included [4].

and without a grid collimator [4]. The measurements without a collimator are higher than the calculation. This may be due to scattering in the material surrounding the X-ray tube.

The collimators for mammography currently in use have about 30 strips per cm consisting of 16 μm lead foils. The ratio between height and distance of the foils is 5 : 1 and they have a maximum aperture of about $\pm 20°$. Figs. 3 and 4 show that the collimators are expected to be rather inefficient for single scattered radiation. For multiple scattered radiation, assuming an isotropic angular distribution they could be effective. It seems reasonable that the experimental values of Fig. 4b for the scattered radiation using a collimator are similar to our 20° calculation of Fig. 4a. If theory and experiments are correct, a reduction of the aperture of the collimator from 20° to 10° should reduce the scattered radiation from 20% to some percent. Calculations including multiple scattering and molecular interference in the forward direction are planned. In addition the propriety of using water as a basis for the calculations should be more tested. Some measurements indicate, that breast components with high fat content have its scattering peak at slightly lower angles [11].

Another interesting conclusion from Figs. 3 and 4 is that the scattered radiation leaves the object mainly at angles smaller than 30°. Thus, in general, an increase in the distance of the X-ray film will not change significantly the flux of single scattered photons.

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