A Design Parameter for Multicomponent Tray Design Estimates[1]

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This report develops computational techniques that are useful in multicomponent distillation. A design parameter, $(\partial n/\partial N)_e$, is introduced which is numerically equal to the number of theoretical plates required to do the work of one total reflux tray at the bottom of the enriching section. For a minimum sized tower, $(\partial n/\partial N)_e$ is less than 3.5. Economic conditions may permit values as high as 6.

The ratio of the number of theoretical trays, n, to the minimum number, N, for the combined enriching and stripping sections, including reboiler and condenser, and the ratio of the reflux leaving the enriching section, L_e, to the minimum reflux, L_{M_e}, can be found from:

$$n/N = \frac{2.303(\partial n/\partial N)_e \log_{10}(\partial n/\partial N)_e}{(\partial n/\partial N)_e - 1}$$

$$L_e/L_{M_e} = \frac{(\partial n/\partial N)_e}{(\partial n/\partial N)_e - 1}$$

A total of 36 sets of minimum reflux data in multicomponent systems were correlated with an average error of −4% in the ratio n/N by these equations.

Since $(\partial n/\partial N)_e$ is related to the relative volatility and the compositions of the counterflowing liquid and vapor streams at the bottom of the enriching section, the reflux L_e can be found from a feed tray balance. A procedure is provided for evaluating L_e after a value for the design parameter has been selected.

When a designer has selected a reflux ratio and made a tray calculation, he frequently has little idea of the kind of selection he has made and may be forced into an involved economic balance in order to find out. Even when such a balance has been made, he still has little idea where he stands in relation to the distillation process itself. A logical measure of the desirability of a design choice is to compare it with the minimum number of trays and the minimum reflux for the same separation. A more convenient and flexible measure is to introduce a design parameter which gives the number of theoretical operating trays required to do the work of one total reflux tray at any point in the tower.

Basic Equations To compare the number of operating trays with the number of total reflux trays it is useful to place the operating line equations in the same form as the equilibrium relationship by introducing a composition ratio ϕ, such that[1]

$$Y_{n+1} = \phi_n X_n \quad (1)$$

Then, along with the equilibrium relationship

$$Y_n = a_n X_n \quad (2)$$

there results for a single tray

$$Y_n/Y_{n+1} = a_n/\phi_n \quad (3)$$

In logarithmic form

$$\log Y_n - \log Y_{n+1} = \log(a_n/\phi_n) \quad (4)$$

For differential numbers of trays, the following procedure is a short cut to the lengthy derivation of Equation (71) in Appendix B. Differential trays are discussed under Mathematical Conventions in Appendix C.

Using Taylor's series for the finite difference in Equation (4),

$$\log(a/\phi)_n = \sum_{k=1}^{\infty} \frac{(-)^k d^k \log Y_{n+1}}{k!\, dn^k} \quad (5)$$

Differentiating,

$$\frac{d}{dn}\log(a/\phi)_n = -\sum_{k=2}^{\infty} \frac{(-)^k k d^k \log Y_{n+1}}{k!\, dn^k} \quad (6)$$

Combining,

$$\log(a/\phi)_n + \tfrac{1}{2}\frac{d}{dn}\log(a/\phi)_n + \frac{d}{dn}\log Y_{n+1} = \sum_{k=3}^{\infty} \frac{(-)^k(1 - k/2) d^k \log Y_{n+1}}{k!\, dn^k} \quad (7)$$

Rearranging and neglecting third order and higher terms,

$$\log(a/\phi)_n\, dn + \tfrac{1}{2} d \log(a/\phi)_n + d \log Y_{n+1} = 0 \quad (8), (71)$$

At total reflux, ϕ is equal to unity and the minimum number of trays is given by

$$\log a_N dN + \tfrac{1}{2} d \log a_N + d \log Y_{N+1} = 0 \quad (9)$$

Equations (8) and (9) were found to have an error less than 4% in the number of trays. (See discussion of Equation (76) in Appendix B.)

The component distribution at total reflux is different from that at some other reflux ratio. However, for the keys, a_N can be taken equal to a_n when Y_{N+1} equals Y_{n+1}. Equations (8) and (9) then yield

$$dn = \frac{\log a_n}{\log(a/\phi)_n} dN + \tfrac{1}{2}\frac{d \log \phi_n}{\log(a/\phi)_n} \quad (10)$$

Equation (10) holds for any pair of components where there is little difference between a_n and a_N. In what follows it will be written only for the key components.

Equation (10) shows that the differential of n has been written as a function of N and ϕ, i.e.,

$$dn = (\partial n/\partial N) dN + (\partial n/\partial \phi) d\phi \quad (11)$$

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Inspection shows that

$$(\partial n/\partial N) = \frac{\log\alpha_n}{\log(\alpha/\phi)_n} \qquad (12)$$

Rearranging,

$$\frac{\log\alpha_n}{\log\phi_n} = \frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1} \qquad (13)$$

Differentiating and neglecting a term in $d \log\alpha$ which is not justified by subsequent approximate methods of integration,

$$d \log\phi_n = \frac{\log\alpha_n}{(\partial n/\partial N)^2} d(\partial n/\partial N) \qquad (14)$$

Making the substitutions in Equation (10)

$$dn = (\partial n/\partial N)dN + \tfrac{1}{2} d \log(\partial n/\partial N) \qquad (15)$$

Integration of Equations (8) and (15) can be accomplished by taking $\log Y_n - \log Y_{n+1}$ linear in $\log Y_{n+1}$[2], and by setting $1/(\partial n/\partial N)$ proportional to N. Then,

$$-\Delta n \cdot \frac{\Delta \log(Y_n/Y_{n+1})}{\Delta \log\log(Y_n/Y_{n+1})} = \Delta \log Y_{n+1} + \tfrac{1}{2}\Delta \log(Y_n/Y_{n+1}) \qquad (16)$$

and,

$$\Delta n \cdot \frac{\Delta\left(\dfrac{1}{(\partial n/\partial N)}\right)}{\Delta \log\left(\dfrac{1}{(\partial n/\partial N)}\right)} = \Delta N - \tfrac{1}{2}\Delta\left(\frac{1}{(\partial n/\partial N)}\right) \qquad (17)$$

where

$$-\Delta N \cdot \frac{\Delta \log\alpha_N}{\Delta \log\log\alpha_N} = \Delta \log Y_{N+1} + \tfrac{1}{2}\Delta \log\alpha_N \qquad (18)$$

At first glance, there would appear to be no particular advantage of Equation (17) over (16). However, the differential coefficient defined by Equation (12) is a useful design parameter which gives the ratio of the operating trays to the total reflux trays at any point in the tower. The linear approximation used in arriving at Equation (16) was found to yield accuracies of from 5 to 10% in the number of trays. The equations are applicable to the stripping or enriching sections but require a feed plate match before they can be applied to the overall tower.

Feed Plate Match A feed plate match can be made by reducing the multicomponent system to an equivalent binary system and by introducing feed at the intersection of the operating lines. In the new basis, the operating line in the enriching section takes the form

$$V_{n+1}(y_l + y_h)_{n+1}(Y/1 + Y)_{n+1} = L_n(x_l+x_h)_n (X/1+X)_n + D(x_l+x_h)_D (X/1+X)_D; \quad 0 \leq n \leq e \qquad (19)$$

where

$$(Y/1 + Y)_{n+1} = (\phi X/1 + \phi X)_n \qquad (20)$$

$$(Y/1 + Y)_n = (\alpha X/1 + \alpha X)_n \qquad (21)$$

Similar expressions can be written for the stripping section for $n \geq (f-1)$.

For a partially vaporized feed, a component balance gives

$$F(z_l + z_h)(Z/1 + Z) = F_L(x_l+x_h)_F (X/1+X)_F + F_V(y_l+y_h)_F (Y/1+Y)_F \qquad (22)$$

$$Y_F = \alpha_F X_F \qquad (23)$$

The operating lines for stripping and enriching are coupled by

$$L_{f-1}(x_l + x_h)_{f-1} - L_e(x_l + x_h)_e = F_L(x_l + x_h)_F \qquad (24a)$$

$$V_{e+1}(y_l + y_h)_{e+1} - V_f(y_l + y_h)_f = F_V(y_l + y_h)_F \qquad (24b)$$

The intersection of the operating lines is defined by

$$\phi_s = Y_{s+1}/X_s \qquad (25)$$

Rearranging Equations (24a & b) with the aid of a component balance and allowing e or $f-1$ to approach the intersection,

$$\frac{-F_L(x_l + x_h)_F}{F_V(y_l + y_h)_F}\left[\frac{(X/1 + X)_F - (X/1 + X)_s}{(Y/1 + Y)_F - (Y/1 + Y)_{s+1}}\right] =$$

$$\frac{L_e(x_l+x_h)_e}{V_{e+1}(y_l+y_h)_{e+1}}\left[\frac{(X/1+X)_s - (X/1+X)_e}{(Y/1+Y)_{s+1} - (Y/1+Y)_{e+1}}\right]_{s=f-1} \qquad (26a)$$

$$= \frac{L_{f-1}(x_l+x_h)_{f-1}}{V_f(y_l+y_h)_f}\left[\frac{(X/1+X)_s - (X/1+X)_{f-1}}{(Y/1+Y)_{s+1} - (Y/1+Y)_f}\right]_{s=e} \qquad (26b)$$

The right hand sides of Equations (26a & b) tend toward unity so that

$$\frac{-F_L(x_l+x_h)_F}{F_V(y_l+y_h)_F} = \frac{(Y/1+Y)_F - (Z/1+Z)}{(X/1+X)_F - (Z/1+Z)} \approx \frac{(Y/1+Y)_{s+1} - (Z/1+Z)}{(X/1+X)_s - (Z/1+Z)} \qquad (27)$$

Equation (27) also applies when feed is flashed on an equilibrium feed tray, and

$$X_f = X_{e+1} = X_F; \quad Y_f = Y_{e+1} = Y_F \qquad (28)$$

For an all liquid feed at its bubble point,

$$X_s = X_e = X_{f-1} = X_F; \quad F_V = 0; \quad Z = X_F \qquad (29)$$

For an all vapor feed at its dew point,

$$Y_{s+1} = Y_{e+1} = Y_f = Y_F; \quad F_L = 0; \quad Z = Y_F \qquad (30)$$

Using these feed conventions, and setting $(\partial n/\partial N)$ equal to unity at X_D and X_B, Equation (17) takes the form, for the overall tower: (See Appendix D.)

$$(n/N) = \frac{(\partial n/\partial N)_s \log(\partial n/\partial N)_s}{(\partial n/\partial N)_s - 1} \qquad (31)$$

It is assumed in the above that

$$\alpha_{f-1} = \alpha_s = \alpha_e$$

so that, for a saturated liquid or vapor feed,

$$(\partial n/\partial N)_e = (\partial n/\partial N)_s = (\partial n/\partial N)_{f-1}$$

Figure 1 shows the intersection of the operating lines.

A certain amount of caution should be exercised in the use of Equation (31). Refluxing of excessive light and heavy components about the feed tray may cause a reverse fractionation which can result in the operating lines intersecting on the opposite side of the equilibrium curve with $\phi > \alpha$. This situation frequently occurs at minimum reflux[3]. Theoretically, refluxing of heavy non-product components in the fractionation section can be carried out until all of the overhead product components have been displaced from the reflux. Then, since $V = L + D$, the product components appear in the vapor stream in the same ratio as in the overhead product. Inspection of the particular case at hand will show if it is advantageous to assume in (31) that all non-product components are absent from the enriching and stripping sections when the design parameter is determined and then to allow for the refluxing effect around the feed tray[4].

Effect of Reflux Ratio When the design parameter is constant, the ratio of the actual number of trays to the minimum number of trays for the overall tower is equal to the parameter itself. To obtain, for example, a system requiring three times the minimum number of trays with $(\partial n/\partial N)$ constant at 3, it is only necessary to draw in the total reflux steps on a McCabe-Thiele diagram in the equivalent binary system, and then, between the first and second steps on the 45° diagonal, begin two more total reflux constructions. The diagram is now composed of three interlacing total reflux steps with compositions given by their intersections. (Note that $X_n = Y_{n+3}$.) The operating line is now a curve drawn nearly one third the way from the equilibrium curve to the 45° diagonal, as in Figure 2.

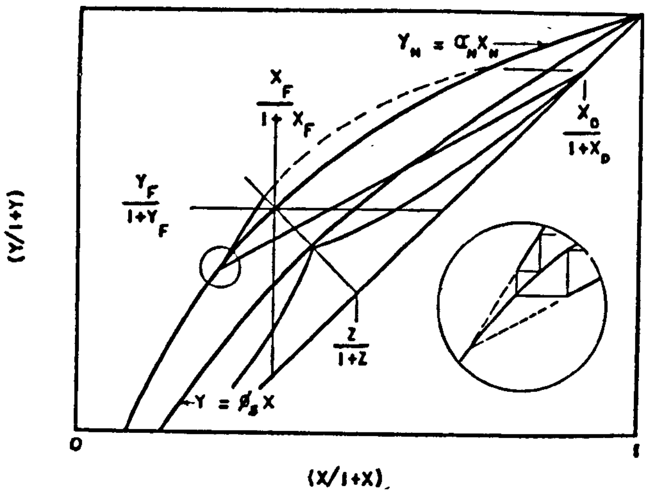

Figure 1—Intersection of the operating lines and extrapolation of minimum reflux in the equivalent binary system.

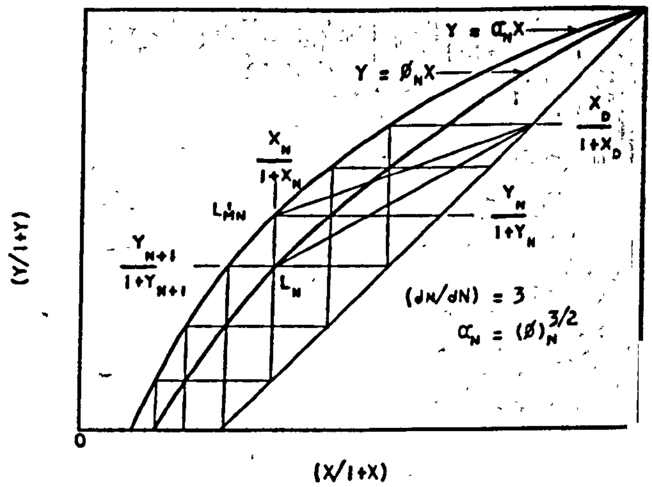

Figure 2—McCabe-Thiele diagram in the equivalent binary system when $(\partial n/\partial N)$ equals 3.

This type of construction can be repeated for any rational value of n/N equal to $(\partial n/\partial N)$. The geometry of the problem is such that the distance between the equilibrium curve and the 45° diagonal is broken up into nearly equal parts totalling $(\partial n/\partial N)$ in number. The distance between the equilibrium curve and the 45° diagonal is given by

$$(Y/1+Y)_{n+1} - (X/1+X)_{n+1} \;\&\; (Y/1+Y)_n - (X/1+X)_n$$

while the distance from the operating line to the 45° line is in turn

$$(Y/1+Y)_{n+1} - (X/1+X)_n$$

The ratio of the two distances is then

$$\left[\frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1}\right]_Y = \frac{(Y/1+Y)_{n+1} - (X/1+X)_{n+1}}{(Y/1+Y)_{n+1} - (X/1+X)_n} \quad \ldots\ldots (32a)$$

$$\left[\frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1}\right]_X = \frac{(Y/1+Y)_n - (X/1+X)_n}{(Y/1+Y)_{n+1} - (X/1+X)_n} \quad \ldots\ldots (32b)$$

A little manipulation gives

$$\left[\frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1}\right]_Y = \frac{\phi_n + Y_{n+1}}{\alpha_{n+1} + Y_{n+1}} \cdot \frac{\alpha_{n+1} - 1}{\phi_n - 1} \quad \ldots\ldots (33a)$$

$$\left[\frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1}\right]_X = \frac{1 + \phi_n X_n}{1 + \alpha_n X_n} \cdot \frac{\alpha_n - 1}{\phi_n - 1} \quad \ldots\ldots (33b)$$

Equations (33a) and (33b) are at best approximations. They yield the same value for the design parameter when lines of constant ϕ and α are parallel. For small values of relative volatilities they reduce to Equation (13). They will be useful in determining the effect of reflux ratio on the design parameter.

Define L'_{Mn} as the reflux that obtains when the operating line in the enriching section intersects the equilibrium curve. L'_{Mn} is not a true minimum reflux but a pseudo minimum reflux that is obtained from a geometrical construction with an equilibrium curve that has already been established and not from material and equilibrium balances.

When L'_{Mn} intersects the equilibrium curve at $Y = Y_{n+1}$, a condition analagous to an all vapor feed at its dew point, Equation (19) yields

$$\frac{L_n(x_l+x_h)_n}{L'_{Mn}(x'_l+x'_h)_{Mn}} = \frac{(Y/1+Y)_{n+1} - (X/1+X)_{n+1}}{(Y/1+Y)_{n+1} - (X/1+X)_n} \quad \ldots\ldots (34)$$

$$= \left[\frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1}\right]_Y \quad \ldots\ldots (35)$$

When L'_{Mn} intersects the equilibrium curve at $X = X_n$, a condition analagous to an all liquid feed at its bubble point, then

$$\frac{L_n(x_l+x_h)_n}{L'_{Mn}(x'_l+x'_h)_{Mn}} = \frac{(X/1+X)_D - (Y/1+Y)_{n+1}}{(X/1+X)_D - (Y/1+Y)_n} \cdot \frac{(Y/1+Y)_n - (X/1+X)_n}{(Y/1+Y)_{n+1} - (X/1+X)_n} \quad \ldots (36)$$

$$= \frac{(X_D - \phi_n X_n)(1 + \alpha_n X_n)}{(X_D - \alpha_n X_n)(1 + \phi_n X_n)} \left[\frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1}\right]_X \quad \ldots\ldots (37)$$

In the fractionation section, components lighter than the light key component soon reach constant compositions[5], and

$$y_{i,n+1} = y_{in} = K_{in} x_{in} \quad \ldots\ldots (38)$$

$$= (L_n/V_{n+1}) x_{in} + (D/V_{n+1}) x_{iD} \quad \ldots\ldots (39)$$

and

$$L_n x_{in} = \frac{D x_{iD}}{\left[\frac{K_{in}}{(L_n/V_{n+1})} - 1\right]}; \; n \leq e \quad \ldots\ldots (40)$$

During the refluxing of heavy components in the fractionation section, the heavy key component goes through a maximum. A maximum implies a region of constant composition, however small. Assume Equation (38) applies. Designating u as the tray at which the maximum occurs and making use of the fact that the overhead product is small for the heavy key component, Equation (40) reduces to

$$K_{hu} \geq L_u/V_{u+1}; \; u < e \quad \ldots\ldots (41)$$

A similar treatment in the stripping section for the components heavier than the heavy key component yields

$$L_n x_{jn} = \frac{B x_{jB}}{\left[1 - \frac{K_{jn}}{(L_n/V_{n+1})}\right]}; \; n \leq f - 1 \quad \ldots\ldots (42)$$

while for the light key component at its point of maximum concentration

$$K_{lw} \leq L_w/V_{w+1}; \; w > f - 1 \quad \ldots\ldots (43)$$

Relative volatilities can be substituted in Equation (40) and in Equation (42) when they are written for n equal to u and n equal to w.

Correlation of Minimum Reflux Except for $n = e$, all values of the true minimum reflux, L_{Mn}, are essentially the same as the pseudo minimum reflux. Since heavy components are fractionated out as n grows large, and lighter components reach

TABLE 1

COMPARISON OF RESULTS OF EQUATIONS (31) & (44)

Source of Data	Test No.	Feed Cond.	N	$(L/D)_M$	L/L_M	n/N	Calculated n/N	Actual $n/N \div$ Calc.
(A)	Ex. 2	B.P.	8.64	0.937	1.25	2.11	2.00	1.06
	3	60% Vap.	"	*	"	2.00	"	1.00
	5	B.P.	"	*	"	2.06	"	1.03
	6	"	"	*	"	2.08	"	1.04
	7**	"	"	*	"	1.99	"	0.996
(B)	Ia	8% Vap.	26.85	5.32	6.08	1.11	1.09	1.02
	b	"	"	"	3.57	1.19	1.17	1.02
	c	"	"	"	2.50	1.29	1.28	1.01
	IIa	"	25.65	4.37	5.50	1.14	1.10	1.04
	b	"	"	"	3.04	1.27	1.21	1.05
	c	"	"	"	2.06	1.47	1.37	1.07
	III	72% Vap.	"	3.25	2.77	1.33	1.24	1.07
	IV	B.P.	18.4	1.83	2.06	1.47	1.37	1.07
	V	"	10.44	2.54	2.89	1.34	1.22	1.10
	VI	50% Vap.	13.3	0.942	3.18	1.28	1.20	1.07
	VII	B.P.	29.45	7.00	1.36	1.77	1.80	0.983
(C)	II	B.P.	19	5.2	1.92	1.42	1.41	1.01
		B.P.	"	"	1.35	1.84	1.82	1.01
	III	"	11.2	0.98	2.04	1.52	1.38	1.10
	IV-A	***	9.4	0.53	3.77	1.38	1.15	1.20†
	IV-B	***	11.2	0.76	2.63	1.52	1.26	1.21†
	VII	B.P.	9.0	1.25	1.28	2.00	1.94	1.03
		"	"	"	1.68	1.67	1.52	1.10
		"	"	"	4.11	1.22	1.14	1.07
	VIII	"	43.1	6.0	2.0	1.35	1.39	0.97
		"	"	"	3.16	1.21	1.21	1.00
	IX	"	29.0	7.0	2.86	1.21	1.22	0.99
		"	"	"	5.00	1.10	1.15	0.96
	X	"	20.2	2.3	2.17	1.43	1.34	1.07
		"	"	"	6.52	1.09	1.09	1.00
	XI	"	11.8	2.26	1.14	2.35	2.4	0.98
	XII	"	7.7	1.5	3.78	1.23	1.16	1.06
		"	"	"	6.00	1.14	1.09	1.05
	XIII	"	8.6	1.85	4.86	1.22	1.12	1.09
	XIV	"	13.0	0.9	3.33	1.31	1.19	1.09
	XV	68% Vap.	3.4	0.78	1.15	2.94	2.32	1.27†
		"	"	"	2.56	1.65	1.27	1.30†
	XVI	B.P.	19.7	3.3	1.21	2.09	2.11	0.99
		"	"	"	1.82	1.59	1.45	1.09
		"	"	"	3.03	1.30	1.22	1.06

* Not reported.

** Reverse fractionation in the stripping section(4).

*** Cold feed.

† Rejected.

(A) Murdoch, P. G., Holland, C. D., Chem. Eng. Progr., **48**, 254 (1952).

(B) Brown, G. G., Martin, H. Z., Trans. Am. Inst. Chem. Engrs., **35**, 679 (1939).

(C) Gilliland, E. R., Ind. Eng. Chem., **32**, 1220 (1940).

constant compositions, the sum of the combined keys obtained by the intersection of the operating line and equilibrium curve on the equivalent binary McCabe-Thiele diagram differs slightly from the result calculated from an equilibrium balance because of changes in relative volatility. The component distributions at the operating and minimum refluxes are not the same. The essential difference at $n = e$ between the pseudo minimum reflux and the true minimum reflux is the quantity of heavy material that enters the enriching section from the feed tray. The sum of the combined keys remains nearly the same. Further, a feed tray analysis will reveal if there is any appreciable difference between the percentages of combined keys at the true minimum reflux and the operating reflux. As a first approximation, Equations (35) and (37) can be written as

$$(L/L_M)_n = (\partial n/\partial N)/[(\partial n/\partial N) - 1]; \; n = e, s \ldots\ldots\ldots\ldots (44)$$

Table 1 shows the results obtained with Equations (31) and (44).

Design Procedure To arrive at a tower design, select a value of $(\partial n/\partial N)$, for use in Equation (31). From Equation (13), obtain ϕ, and then by means of Equations (22) through (30), find

$$X_s = L_s x_{ls}/L_s x_{hs}$$

Since Y_{s+1} and X_s are known, the sum of the combined keys,

$$L_s(x_{ls}+x_{hs}) = V_{s+1}(y_{ls+1}+y_{hs+1}) - D(x_{lD}+x_{hD})$$

can be readily found from Equation (19), by eliminating the vapor term.

The sum of a distributed component and the heavy key component can be obtained in the same manner using a value of the design parameter equal to that used for the light and heavy key components. Components lighter than the light key and heavier than the heavy key can be found from Equations (40) and (42) and a feed tray balance. Sufficient information is now available to arrive at L_s. (See Appendix E).

Relative volatilities can be used as a first trial in Equations (40) and (42). If a trial value of L_s is used, it can be set equal to L_n in (41). Using the same value of the design parameter and the same value of n at the intersection of the operating lines when fractionating between the distributed component and the heavy key component is equivalent to saying that n/N is the same as it is for the light and heavy key components. If desired, the feed tray balance can be reworked and $(\partial n/\partial N)_s$ can be calculated and compared with that originally used.

Design Conditions In an economic study with the aid of a computer, Albright[6] found that reflux ratios of 1.20 to 1.30 times the minimum were optimum for his conditions, while Happel[7] presents a correlation which prescribes reflux ratios that are close to the minimum. Substituting these reflux factors in Equations (44) and (31) yields the following:

$(L/L_M)_e$	n/N	$(\partial n/\partial N)_e$
1.2	2.14	6.0
1.25	2.00	5.0
1.3	1.9	4.33

These values should be compared with those employed in the sample problems.

In this connection it is interesting to note that when the relative volatility is sufficiently small, then Equation (44), or (53), predicts the results of Cohen[8]: An enriching section has a minimum size, or volume, when the reflux ratio is twice the minimum at all points and when twice the minimum number of trays are employed. Going a step further, an enriching section with a constant reflux ratio has a minimum size when:

$(\partial n/\partial N)_e$	3.5	3.4	3.0	1.0
n/N	1.75	1.73	1.65	1.00
N_e	∞	8.73	1.54	0
$(L/L_M)_e$	1.4	1.42	1.50	∞

The combined enriching and stripping sections is a minimum when $(\partial n/\partial N)_e$ is less than 3.5.

Discussion

The design parameter introduced herein has certain advantages:

(1) It permits a rapid estimate of the total number of trays from the minimum number of trays.

(2) It permits an estimate of the required reflux ratio from a feed tray balance and eliminates the need for a minimum reflux calculation.

(3) It allows the designer to select a set of design conditions which, if not the optimum, are none the less desirable.

The use of the pseudo minimum reflux is based upon the geometry of the McCabe-Thiele diagram[9] alone. It supposes that the equilibrium curve has been determined by the separation under consideration.

Conventional methods are employed in making a feed tray balance[5]. Location of the feed tray is a free choice and for computational convenience it is taken at the intersection of the operating lines in the equivalent binary system. The question of optimum feed tray location is left to the consideration of the particular case at hand. For "normal" systems, the optimum location occurs at the intersection of the operating lines in the equivalent binary system.

The use of Equation (44) in the optimization studies in the Appendix A is, admittedly, an over simplification. Its use was felt justified because of the results obtained in Table 1. The data of Table 1 could be represented with an average error of -4% in the ratio (n/N), by means of Equations (44) and (31). For the systems represented in Table 1, the key components have relative volatilities of 3.0 or less. Binary systems are not included in the data of Table 1.

Referring to Figure 1, it should be noted that crossing the equilibrium curve during the refluxing of non-product components, particularly at minimum reflux, forces the operating line to change direction within a single step. With an infinite number of steps at the crossing during minimum reflux, this results in a discontinuity of slope of the operating line. An infinity of steps at or near equilibrium at the point of crossing suggests that the operating line becomes tangent to the equilibrium curve at this point. Thus, if the intersection of the operating lines in the equivalent binary system is known for the minimum reflux, the pinch points in the fractionating and stripping sections can be found approximately, simply by drawing tangents to the equilibrium curve from the point of intersection.

Use of the linear approximation in the integration of Equations (8) and (15) follows from the fact that the operating line for enriching can be put into the form

$$[V(y_l+y_k)(Y-1)/(Y+1)]_{n+1} = [L(x_l+x_k)(X-1)/(X+1)]_n + D(x_l-x_k)_D$$

In terms of logarithms,

$$\log Y_{n+1} = \frac{L_n(x_l+x_k)_n \log X_n + 2D(x_l-x_k)_D}{V_{n+1}(y_l+y_k)_{n+1}} + 2g$$

where

$$g \equiv \sum_{k=1}^{\infty} \frac{1}{1+2k}\left[\left(\frac{Y-1}{Y+1}\right)_{n+1}^{1+2k} - \frac{L_n(x_l+x_k)_n}{V_{n+1}(y_l+y_k)_{n+1}}\left(\frac{X-1}{X+1}\right)_n^{1+2k}\right]$$

When $Y_{n+1} < 2$ and $X_n > \frac{1}{2}$, then g becomes small.

When the operating lines in the equivalent binary system are straight, then the slope of the $\log Y_{n+1}$ vs. $\log X_n$ curve goes through a maximum in the enriching section and use of the linear approximation in the integration of Equations (8) and (15) can result in values of n somewhat larger than actual. Usually, $L_n(x_l+x_k)_n$ increases with increasing n for $n < e$, and then decreases as the heavy components begin to appear. If the intersection of the operating lines is determined from the maximum flows, then the linear approximation will yield values of n less than actual. At constant total molal downflow, the maximum flows for the combined keys are given by:

$$L_n(x_l+x_k)_{n,\max} \leqq L_e(x_l+x_k)_e + \sum_{i>h} V_{e+1}\, y_{i,e+1}\,;\ n<e \quad \text{........(45)}$$

$$\leqq L_{f-1}(x_l+x_k)_{f-1} + \sum_{i<l} L_{f-1}\, x_{i,f-1};\ n>f-1$$

A straight operating line in the equivalent binary sytem permits the use of the coordinate transformation of Stoppel[10] in Equation (9) by replacing Y_{N+1} with

$$\frac{(Y/1+Y)_{n+1} - (\underline{Y}/1+\underline{Y})}{(\overline{Y}/1+\overline{Y}) - (Y/1+Y)_{n+1}} = \frac{(X/1+X)_n - (\underline{X}/1+\underline{X})}{(\overline{X}/1+\overline{X}) - (X/1+X)_n}$$

where $(\underline{X}, \underline{Y})$ and $(\overline{X}, \overline{Y})$ are the left-hand and right-hand intersections of the operating line and equilibrium curve.

Intersections of the operating line and the equilibrium curve are obtained by assuming a to be constant outside the region of immediate interest. Such regions should not include irregular variations of volatility such as that which may occur across a condenser. Thus, in the enriching section, for example:

$$\overline{Y} = a_1\overline{X};\ \underline{Y} = a_s\underline{X};\ 1 \leqq n \leqq s$$

$$\frac{L_n(x_l+x_k)_n}{V_{n+1}(y_l+y_k)_{n+1}} = \frac{(\underline{Y}/1+\underline{Y}) - (X/1+X)_D}{(\underline{X}/1+\underline{X}) - (X/1+X)_D} = \frac{(\overline{Y}/1+\overline{Y}) - (X/1+X)_D}{(\overline{X}/1+\overline{X}) - (X/1+X)_D}$$

It should be noted that $\overline{X}$ can be negative, passing from $+\infty$ to $-\infty$ when $\overline{X}/1+\overline{X}$ is unity.

If the relative volatility is constant, then a_N can be replaced by

$$\frac{1 + (a_n-1)(\overline{X}/1+\overline{X})}{1 + (a_n-1)(\underline{X}/1+\underline{X})}$$

If a_N is not constant, then the transformed volatility is calculated in the obvious way from known quantities. With suitable assumptions, a final form for n, analagous to Equation (18), is obtained.

Nomenclature

B = Bottoms product, moles
D = Overhead product, moles
e = Theoretical plate number at the bottom of the enriching section
$f-1$ = Theoretical plate number at the top of the stripping section
F = Total feed, moles
F_L = Liquid portion of the feed, moles
F_V = Vapor portion of the feed, moles
k = Index of demarkation
K_{in} = Equilibrium constant for component i at the temperature and pressure of plate n
log = Natural logarithm
L_{Me} = Minimum reflux leaving the bottom tray of the enriching section obtained from a feed tray balance, moles
L_{Mn} = Minimum reflux leaving plate n obtained from an equilibrium balance, moles
L_{Ms} = Minimum reflux leaving the enriching section at the intersection of the operating lines given by Equation (44), moles
L_n = Reflux leaving plate n, moles
L_s = Reflux leaving the enriching section at the intersection of the operating lines, moles
L'_{Mn} = Pseudo minimum reflux leaving plate n obtained by a geometrical construction, moles; $n<e$
n = Theoretical plate number counting from the top of the distillation tower. A partial condenser with condensate to reflux is taken as the first plate, and an equilibrium reboiler as the last. See Appendix D
N = Total reflux trays for the key components computed in terms of vapor compositions
$(\partial n/\partial N)_s$ = Differential coefficient defined by Equations (11) and (12) and evaluated at the intersection of the operating lines in Equation (31)
s = Value of n at the intersection of the operating lines
u = Theoretical plate in the enriching section where the heavy key component reaches its maximum concentration
V_{n+1} = Vapor stream entering plate n, moles.
V_{s+1} = Vapor entering the enriching section at the intersection of the operating lines, moles
w = Theoretical plate number in the stripping section where the light key component is at its maximum concentration
x_{iF} = Mole fraction of component i in the liquid portion of the feed
x_{in} = Mole fraction of component i in the liquid leaving plate n
X = Mole ratio: moles of light key component per mole of heavy key in the liquid
X_B = Mole ratio in the bottoms product
X_D = Mole ratio in the overhead product
X_F = Mole ratio in the liquid portion of the feed
X_n = Mole ratio in the liquid leaving plate n
X_s = Mole ratio in the liquid at the intersection of the operating lines
y_{iF} = Mole fraction of component i in the vapor portion of the feed
y_{in+1} = Mole fraction of component i in the vapor entering plate n
Y = Mole ratio: moles of light key component per mole of heavy key in the vapor
Y_F = Mole ratio in the vapor portion of the feed
Y_{n+1} = Mole ratio in the vapor entering plate n
Y_{s+1} = Mole ratio in the vapor at the intersection of the operating lines
z_i = Mole fraction of component i in the total feed
Z = Moles of light key component per mole of heavy key component in the total feed

Greek Letters

α_{ijn} = Volatility of component i relative to the volatility of component j at the temperature and pressure of plate n
Δ = Final value minus the initial value of the quantity to which it is applied over the region of interest concerned
ϕ_{ijn} = Composition ratio defined by Equation (1)

Superscripts

$'$ = Pseudo minimum reflux
$''$ = Maximum flows

Subscripts

h = Heavy key component
l = Light key component
M = Minimum reflux

In general the subscripts l and h are not used unless it is necessary to distinguish between components other than the light and heavy keys.

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APPENDIX A: OPTIMIZATION STUDIES

Since the reflux loading in a tower determines to a large extent the diameter, the size of an enriching section can be taken as proportional to the integral

$$P = \int_0^e (L_n/D)dn$$

To find a minimum sized plant it is necessary to determine the conditions under which the integral P is stationary.

Variable Reflux Case. Inspection of Figure 2 shows that the reflux loading $L_n(x_{ln} + x_{hn})$ can be gotten in terms of $(\partial n/\partial N)$ and Y_{n+1}, or with Equations (9) and (15), in terms of n and $(\partial n/\partial N)$. In a region where Equations (40) and (41) are applicable, the percentage of combined keys has a value which depends only upon L_n when α does not vary. The functional dependence of the reflux can then be indicated by

$$(L_n/D) = L/D\,[(\partial n/\partial N), n] \qquad (46)$$

According to the Calculus of Variations[11], the integral P will be stationary when

$$\frac{\partial(L_n/D)}{\partial(\partial n/\partial N)} - \frac{d\left(\dfrac{\partial(L_n/D)}{\partial p}\right)}{dn} = 0 \qquad (47)$$

Reference to (46) shows that $p \equiv \dfrac{d(\partial n/\partial N)}{dn}$ does not appear and Equation (47) reduces to

$$\frac{\partial(L_n/D)}{\partial(\partial n/\partial N)} = 0 \qquad (48)$$

Equation (48) requires that the design parameter remain constant, hence,

$$dn = (\partial n/\partial N)dN \qquad (49)$$

Rewriting the integral P

$$P = (\partial n/\partial N)\int_0^{N_e}(L_n/D)dN \qquad (50)$$

Differentiating with respect to the design parameter and setting the result equal to zero

$$\frac{dP}{d(\partial n/\partial N)} = 0 = \int_0^{N_e}\left(\frac{L_n}{D}\right)dN + (\partial n/\partial N)\int_0^{N_e}\frac{\partial(L_n/D)}{\partial(\partial n/\partial N)}dN \qquad (51)$$

Making use of Equation (1) and the fact that $(x_l'/x_h')_{Mn} = X_{n+1}$ in Equations (35) and (33a) results in

$$\frac{L_n}{L'_{Mn}} = \frac{\alpha_{n+1} - 1}{\phi_n - 1}\cdot\frac{x'_{lMn}}{x_{ln}} \qquad (52)$$

with $x'_{lMn} \geqq x_{ln+1}$. With α sufficiently small and of little variation, Equations (52) and (13) yield

$$\frac{L_n}{L'_{Mn}} \cong \frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1} \quad (53)$$

Substituting (53) in (51) results in

$$\left[\frac{(\partial n/\partial N)}{(\partial n/\partial N) - 1} - \frac{(\partial n/\partial N)}{[(\partial n/\partial N) - 1]^2}\right]\int_0^{N_e}(L'_{Mn}/D)dN = 0 \quad (54)$$

Equation (54) reduces to

$$(\partial n/\partial N) = 2$$

and $\quad (L/L_M)_n \cong (L/L'_M)_n = 2; \; n < e$

Constant Reflux. Setting L_n equal to L_e in the integral P,

$$P = nL_e \quad (55)$$

Rewriting Equation (55) for $n = e$ with the aid of (44) and (17), differentiating with respect to $(\partial n/\partial N)_e$ and setting the result equal to zero,

$$N_e = \frac{\frac{1}{2}(\log m - m + 1)}{(m/m - 1)(m - 1 - 2\log m)}; \; m \equiv (\partial n/\partial N)_e \quad (56)$$

The solution of (56) yields the following for a minimum sized enriching section with constant reflux:

N_e	$(\partial n/\partial N)_e$
0.00	1.00
1.00	2.82
1.54	3.00
2.02	3.10
8.73	3.40
16.10	3.45
∞	3.50

For the stripping section, P is equal to $(n - s)(L_e + F_L)$, while for the combined enriching and stripping sections, P is given by $nL_e + (n - s)F_L$. The overall tower is a minimum when $L_{Ms}N/F_L$ is equal to

$$\frac{(m - 1)^2(1/2m) + (N - N_s)(\log m - m + 1)}{(m/m - 1)(m - 1 - 2\log m)}; \; m \equiv (\partial n/\partial N)_s$$

In order for P to be a minimum for the overall tower, $(\partial n/\partial N)_s$ has a maximum value of 3.5.

APPENDIX B: DERIVATION OF EQUATIONS

The basic differential equations of multicomponent distillation have been arrived at by resorting to a standard technique and employing Taylor's formula. To find the differential change in n, expand n at the point $\log Y_{n+1}$, $\log X_n$, and $\log\alpha_n$ by the Taylor formula, retaining only first order terms. Then,

$$dn = (\partial n/\partial \log X_n)_{Y,\alpha}\, d\log X_n + (\partial n/\partial \log Y_{n+1})_{X,\alpha}\, d\log Y_{n+1} \quad (57)$$
$$+ (\partial n/\partial \log\alpha_n)_{X,Y}\, d\log\alpha_n$$

Making the substitutions

$$d\log X_n = d\log Y_{n+1} - d\log\phi_n \quad (58)$$

and

$$d\log\alpha_n = d\log(\alpha/\phi)_n + d\log\phi_n \quad (59)$$

Equation (57) becomes

$$dn = [(\partial n/\partial \log X_n)_{Y,\alpha} + (\partial n/\partial \log Y_{n+1})_{X,\alpha}]\, d\log Y_{n+1} \quad (60)$$
$$+ [\partial n/\partial \log\alpha_n)_{X,Y} - (\partial n/\partial \log X_n)_{Y,\alpha}]\, d\log\phi_n$$
$$+ (\partial n/\partial \log\alpha_n)_{X,Y}\, d\log(\alpha/\phi)_n$$

Since

$$(\partial n/\partial \log\alpha_n)_{X,Y} - (\partial n/\partial \log X_n)_{Y,\alpha} =$$
$$[\partial n/\partial \log(\alpha/\phi)_n)_{Y,\phi} - (\partial n/\partial \log(\alpha/\phi)_n)_{Y,\alpha} \quad (61)$$

symmetry suggests that this term be zero, so that as a function of Y and α/ϕ, the differential of n becomes

$$dn = [(\partial n/\partial \log X_n)_{Y,\alpha} + (\partial n/\partial \log Y_{n+1})_{X,\alpha}]\, d\log Y_{n+1}$$
$$+ (\partial n/\partial \log\alpha_n)_{X,Y}\, d\log(\alpha/\phi)_n \quad (62)$$

Justification for setting (61) equal to zero can be found by referring to a $\log X$ versus $\log Y$ diagram. On this type of plot, lines of constant α and constant ϕ are parallel and of unit slope. A little manipulation will show that if $\log Y_n - \log Y_{n+1}$ changes from one tray to the next, then it makes no difference if this change is accomplished by a change in α or ϕ, or both, the end result will be the same.

If n had been expanded by means of the Taylor formula about $\log\alpha_{n+1}$, and if Y_{n+1} had been eliminated from the resulting expansion, there would be the term

$$(\partial n/\partial \log\alpha_{n+1})_{X,Y} + (\partial n/\partial \log Y_{n+1})_{X,\alpha} =$$
$$[\partial n/\partial \log(\alpha_{n+1}/\phi_n)]_{X,\phi} - [\partial n/\partial \log(\alpha_{n+1}/\phi_n)]_{X,\alpha} \quad (63)$$

Following the same sort of reasoning that was employed in setting (61) equal to zero, this term can also be set equal to zero. There are then, the pair of equations

$$[(\partial n/\partial \log\alpha)_{X,Y} - (\partial n/\partial \log X_n)_{Y,\alpha}]_{\alpha=\alpha_n} = 0 \quad (64)$$

$$[(\partial n/\partial \log\alpha)_{X,Y} + (\partial n/\partial \log Y_{n+1})_{X,\alpha}]_{\alpha=\alpha_{n+1}} = 0 \quad (65)$$

The difference in sign that appears here is due to the fact that $\Delta\log\alpha$ plays a dual role. Near $X = X_{n+1}$, $\Delta\log\alpha$ behaves as $-\Delta\log X$, while near $Y = Y_n$, it behaves as $+\Delta\log Y$. Writing (64) and (65) for the same value of relative volatility

$$|(\partial n/\partial \log\alpha)_{X,Y}| = |(\partial n/\partial \log X_n)_{Y,\alpha}| = |(\partial n/\partial \log Y_{n+1})_{X,\alpha}| \quad (66)$$

and

$$(\partial n/\partial \log X_n)_{X,\alpha} = (\partial n/\partial \log Y_{n+1})_{X,\alpha} \quad (67)$$

Equation (67) simply states that total condensation of a part of the vapor stream and total vaporization of a part of the liquid stream yield the same results.

Substituting (64) in (62), the total differential for n becomes

$$dn = [(\partial n/\partial \log X_n)_{Y,\alpha} + (\partial n/\partial \log Y_{n+1})_{X,\alpha}]\, d\log Y_{n+1} +$$
$$(\partial n/\partial \log X_n)_{Y,\alpha}\, d\log(\alpha/\phi)_n \quad (68)$$

When α/ϕ is constant, Equation (4) becomes

$$\log(Y_{n+1}/Y_1) = -n\log(\alpha/\phi) \quad (69)$$

and differentiation yields

$$(\partial n/\partial \log Y_{n+1})_{\alpha/\phi} = -1/\log(\alpha/\phi)_n \quad (70)$$

Making the necessary substitutions in Equation (68)

$$-\log(\alpha/\phi)_n\, dn = d\log Y_{n+1} + \tfrac{1}{2}d\log(\alpha/\phi)_n \quad (71)$$

This is Equation (8) that was previously developed.

In terms of liquid compositions, the companion equation is

$$-\log(\alpha_{n+1}/\phi_n)\, dn = d\log X_n - \tfrac{1}{2}d\log(\alpha_{n+1}/\phi_n) \quad (72)$$

If the equilibrium relationship had been written in terms of the equilibrium constant

$$y_n = K_n x_n \quad (73)$$

there would be the pair of equations:

$$-\log(K_n x_n/y_{n+1})\, dn = d\log y_{n+1} + \tfrac{1}{2}d\log(K_n x_n/y_{n+1}) \quad (74)$$

$$-\log(K_{n+1}x_n/y_{n+1})\, dn = d\log x_n - \tfrac{1}{2}d\log(K_{n+1}x_n/y_{n+1}) \quad (75)$$

The derivation of these equations parallels that of (71) since the geometry of the $\log y$ vs. $\log x$ diagram is the same as the $\log Y$ vs. $\log X$ diagram.

The applicability of these equations is readily determined by evaluating the logarithmic composition terms, $\log(Y_n/Y_{n+1})$, $\log(y_n/y_{n+1})$, etc., for adjacent trays and then dividing the arithmetic average of the two by the logarithmic average. The result gives the fraction of the actual trays accounted for by these equations at that point. This can be shown from Equation (16) when $\log(\alpha_n/\phi_n) \equiv Q_n$ is linear in $\log Y_{n+1}$. Then, for one tray,[12]

$$\Delta n = \frac{\frac{1}{2}(Q_{n+1} + Q_n)}{(Q_{n+1} - Q_n)}\log(Q_{n+1}/Q_n) \quad (76)$$

gives the calculated number of trays. When Q_{n+1}/Q_n lies between 0.5 and 2, then Δn is less than 1.04.

APPENDIX C: MATHEMATICAL CONVENTIONS

A theoretical tray or plate is considered here as the difference operator E of the calculus of finite differences. E is such that $E\log Y_n = \log Y_{n+1}$. The subscript n records the number of times the operator E occurs. Since E is related to the differential operator d of the differential calculus by the Taylor series expansion[18]: $E = \exp(d/dn)$; there is no difficulty with differential theoretical trays. Accordingly, the differential tray is taken as the differential operator such that $\log Y_{n+dn} = \log Y_n + d \log Y_n$. For a single theoretical tray it follows easily that

$$(E - 1) \log Y_n = \int_n^{n+1} d \log Y_n = \log(Y_{n+1}/Y_n).$$

With these conventions, differential trays and differential numbers of theoretical trays are permissible.

APPENDIX D: TRAY NUMBERING PROCEDURE

The theoretical plate number, n, can assume all values greater than zero and its integral values need not coincide with a physical boundary. However, n itself represents an arbitrary boundary or reference plane. Thus, the quantity and properties of the reflux leaving a plate and crossing a theoretical plate boundary bear the number of the plate as a subscript. Correspondingly, the counter-flowing vapor stream entering the plate carries the theoretical plate number increased by unity as a subscript. In effect, functional dependence upon n is indicated by a subscript even though n is not a true index of demarkation.

In this report, the counting of theoretical plates begins in the enriching section at $X_D = Y_1 = X_0$, and ends in the stripping section at X_B on the 45° diagonal of the McCabe-Thiele diagram in the equivalent binary system. The enriching section includes the condenser and the stripping section includes the reboiler.

Equation (31) determines the theoretical plate requirements for a separation from X_D to X_B. To determine the number of theoretical plates in the tower proper, it is necessary to deduct the equivalent number of plates for the reboiler and condenser. In the case of a total condenser and total reboiler the product compositions coincide with the terminal points of the tower operating lines and there is no correction. In the case of a partial condenser, equilibrium reboiler, and thermosyphon reboiler, the operating lines terminate before they reach the product compositions at X_D and X_B. It is necessary to extrapolate the operating lines to evaluate the auxiliary equipment. In Equation (31), this extrapolation was in effect accomplished by and evaluated by Equation (17). If correction is to be made to the total number of plates obtained from Equation (31), it should be done by means of Equation (17). Customarily, the extrapolations are performed at constant L/V ratio using the terminal flows at the top or bottom of the tower itself.

The number of theoretical plates at the bottom of the enriching section, e, and the number of plates at the top of the stripping section, $f-1$, are not necessarily equal unless the compositions of liquid and vapor at these points are equal. This is the case in Equations (29) and (30) where saturated liquid and vapor feeds are introduced at the intersection of the operating lines.

In the case of a partially vaporized feed in Equation (26), the operating lines terminate before they intersect. A separation effect results from the introduction of feed which is considered as a part of the distillation process and is included in the estimation of tray requirements. In Equation (31), this composition difference is evaluated along extrapolations of the operating lines near their intersections. A number of different mechanisms can be suggested for the coupling of the operating lines and the introduction of partially vaporized feed. The simplest situation occurs when the compositions of the liquid and vapor portions of the feed and the compositions at the intersection of the operating lines are the same and lie on the diagonal joining the equilibrium point (X_F, Y_F) and the point Z on the 45° diagonal on the McCabe-Thiele diagram in the equivalent binary system.

APPENDIX E: SAMPLE CALCULATIONS

Data: Edmister, W. C., "Hydrocarbon Absorption and Fractionation Process Design Methods", Tables 55, 58, 59. Reprinted from the Petrol. Eng.

Comp	F_s	Dx_D	Bx_B	F_Lx_F	F_Vy_F	α_F	α_D	α_B
C_1	26	26		1	25	20.6		
C_2	9	9		1	8	5.09		
C_3	25	24.6	0.4	7	18	2.06	3.12	1.86
C_4	17	0.3	16.7	8	9	1.00	1.00	1.00
C_5	11		11.0	7	4	0.429		
C_6	12		12.0	10	2	0.206		

Basis: Assume $(\partial n/\partial N)_s = 2.90$; $s = e$

Equation (13)

$$\frac{\log_{10} 2.06}{\log_{10} \phi_s} = 2.90/1.90; \ \phi_s = 1.61$$

Equation (27)

$$\frac{\dfrac{1.61X_s}{1 + 1.61X_s} - \dfrac{25/17}{1 + 25/17}}{\dfrac{X_s}{1 + X_s} - \dfrac{25/17}{1 + 25/17}} = \frac{-(8 + 7)}{18 + 9}$$

$$X_s = 1.09; \ Y_{s+1} = 1.75$$

Equation (19)

$$\frac{\dfrac{24.6/0.3}{1 + 24.6/0.3} - (1.75/2.75)}{(1.75/2.75) - (1.09/2.09)} = \frac{L_s(x_{3s} + x_{4s})}{24.6 + 0.3}$$

$$L_s(x_{3s} + x_{4s}) = 76.2$$

Equation (40)

$$(Lx_1)_s = 26.0/(20.6 - 1) = 1.3$$
$$(Lx_2)_s = 9.0/(5.09 - 1) = 2.2$$

Equation (42) & Feed Balance

$$(Lx_5)_s = 11.0/(1 - 0.429/2.06) - 7 = 6.9$$
$$(Lx_6)_s = 12/(1 - 0.206/2.06) - 10 = 3.3$$

Reflux leaving the enriching section:

	C_1	C_2	C_3	C_4	C_5	C_6
$(Lx_i)_s$	1.3	2.2	39.7	36.5	6.9	3.3
Actual	1.34	2.41	37.0	37.7	8.13	3.42

Equation (18) (Enriching & Stripping Stepwise Application.)

$$-\Delta N = \frac{\left[\log_{10}\left(\dfrac{1.75}{24.6/0.3}\right) + \tfrac{1}{2}\log_{10}(2.06/3.12)\right]}{\dfrac{2.303 \log_{10}\left(\dfrac{\log_{10}2.06}{\log_{10}3.12}\right)}{\log_{10}(2.06/3.12)}} + \frac{\left[\log_{10}\left(\dfrac{0.4/16.7}{1.75}\right) + \tfrac{1}{2}\log_{10}(1.86/2.06)\right]}{\dfrac{2.303 \log_{10}\left(\dfrac{\log_{10}1.86}{\log_{10}2.06}\right)}{\log_{10}(1.86/2.06)}}$$

$$N = 10.91 \text{ (Enriching + Stripping)}$$

Equation (31)

$$n/N = 2.303\,(2.90/1.90) \log_{10}2.90 = 1.62$$
$$n \text{ (including reb. \& cond.)} = 17.7; \text{ actual} = 17$$

Assume maximum flows are given by Equation (45):

Equation (45)

$$L_n(x_l + x_h)_{n,\,max} = 76.2 + 3.3 + 6.9 = 86.4; \ n < e$$
$$L_n(x_l + x_h)_{n,\,max} = 76.2 + (7+8) + (1.3+1) + (2.2+1) = 96.7; \ n > f - 1$$

Equation (19)

$$(Y/1 + Y)''_{n+1} = \frac{86.4(X/1 + X)'_n + 24.6}{86.4 + 24.6 + 0.3}; \ n < e$$
$$(Y/1 + Y)''_{n+1} = \frac{96.7(X/1 + X)'_n - 0.4}{96.7 - 16.7 - 0.4}; \ n > f - 1$$

Intersection of the operating lines for maximum flows yields:

$$Y''_{s+1} = 1.63; \ X''_s = 1.05; \ \phi''_s = 1.55$$

Equations (13) and (31) yield:

$$n''/N = 1.53; n'' = 16.7$$

Inclusion in Equation (31) of the term neglected in the differentiation of Equation (14) results in:

Equation (31a)

$$n = \frac{N(\partial n/\partial N)_s \log(\partial n/\partial N)_s}{(\partial n/\partial N)_s - 1} + \tfrac{1}{2}\int_0^n [(\partial n/\partial N) - 1]\, d \log \log \alpha_n$$

The correction term in Equation (31a) amounts to −0.32 in the first case and −0.24 in the second.

Data: Amundson, N. R., Pontinen, A. J., Ind. Eng. Chem., **50**, 730 (1958). Table IV, 15th iteration. Feed is vaporized on the feed tray: $X_F = X_f$; $Y_F = Y_f$. Cross plots of the data show that the intersection of the operating lines lies near the bottom of the feed tray. A check will be made of the number of trays

Item	z	x_B	x_D	x_1	y_2	x_f	y_{f+1}
C_3	0.200	0.009	0.853	0.910	0.903	0.233	0.367
C_4	0.370	0.473	0.0165	0.042	0.0387	0.495	0.509
α		2.00	3.00	2.38		2.00	
Moles	100	77.4	22.6				

(x_1 and y_2 are compositions of liquid leaving and vapor entering the condenser.)

$$Y_{s+1} \cong Y_{f+1} = 0.367/0.509 = 0.722$$
$$X_s \cong X_f = 0.233/0.495 = 0.470$$
$$\phi_s \cong 0.722/0.470 = 1.53$$
$$(\partial n/\partial N)_s = (\log_{10}2.00)/\log_{10}(2.00/1.53) = 2.60$$

$$-\Delta N = \left[\log_{10}\left(\frac{0.903/0.0387}{0.853/0.0165}\right) + \tfrac{1}{2}\log_{10}(2.38/3.00)\right] \frac{2.303 \log_{10}\left(\frac{\log_{10}2.38}{\log_{10}3.00}\right)}{\log_{10}(2.38/3.00)} + \left[\log_{10}\left(\frac{0.722}{0.903/0.0387}\right) + \tfrac{1}{2}\log_{10}(2.00/2.38)\right] \frac{2.303 \log_{10}\left(\frac{\log_{10}2.00}{\log_{10}2.38}\right)}{\log_{10}(2.00/2.38)} + \left[\log_{10}\left(\frac{0.009/0.473}{0.722}\right)\right]/\log_{10}2.00$$

$N = 10.75$ (Partial Condenser + Enriching + Stripping)

$n/N = 2.303(2.60/1.60)\log_{10}2.60 = 1.55$

n (including reb. & cond.) $= 16.7$; actual $= 16$

In the above example, heat of vaporization of the feed came from sensible heat in the feed. However, if the top tray and the feed tray act as heat transfer trays and heat effects are considerable, then stepwise application of Equation (17) may be necessary to avoid erratic results. Note that Equation (18) has been applied across the condenser itself because of the large change in relative volatility.

The sum of the combined keys has a maximum of 152.0 mol./hr. in the liquid leaving the top tray, and a maximum of 154 mol./hr. in the liquid leaving the third tray beneath the feed tray. The operating lines for these flows are:

$$(Y/1 + Y)''_{n+1} = \frac{152.0(X/1 + X)''_n + 22.6(0.853)}{152.0 + (0.853 + 0.0165)22.6};\ n < e$$

$$(Y/1 + Y)''_{n+1} = \frac{154.0(X/1 + X)''_n - 77.4(0.009)}{154.0 - (0.009 + 0.473)77.4};\ n > f - 1$$

Intersection of the operating lines yields, for the maximum flows:

$$\phi''_s = 1.46; n''/N = 1.44; n'' = 15.5$$

The calculated values of n properly straddle the actual value. The correction term in Equation (31a) amounted to −0.2 in both cases. Some of the slopes of the operating lines in the equivalent binary system in arithmetic and logarithmic coordinates are shown below:

n	$\frac{\Delta(Y/1 + Y)_{n+1}}{\Delta(X/1 + X)_n}$	$\frac{\Delta \log Y_{n+1}}{\Delta \log X_n}$
1 (cond.)	0.845	0.917
2 (top)	0.886	0.923
e	0.841	
s	0.848	0.909

Let $\log(\alpha/\phi)_n = Q_n - p_n$ where Q_n is linear in $\log Y_{n+1}$ and $p_n > 0$ when the operating line lies to the left of the linear approximation and zero otherwise. Let $\log(\alpha/\phi)_n = Q_n + q_n$ where $q_n > 0$ when the operating line lies to the right. Then the operating line and the linear approximation will yield the same value of n when

$$\int_0^n \left(\frac{1}{Q_n - p_n} - \frac{1}{Q_n}\right) d \log Y_{n+1} = \int_0^n \left(\frac{1}{Q_n} - \frac{1}{Q_n + q_n}\right) d \log Y_{n+1} \quad (83)$$

Hence,

$$\int_0^n \left|\frac{p_n dQ_n}{Q_n^2}\right| \leq \int_0^n \left|\frac{q_n dQ_n}{Q_n^2}\right| \quad (84)$$

In general, when $p_n/Q_n^2 > q_n/Q_n^2$ then the linear approximation will yield less than the actual number of trays. This is the situation which exists when the intersection of the operating lines lies to the left of the linear approximation in logarithmic coordinates. (It is understood in the above that dQ_n changes sign at $\log Y_{s+1}$.) It is difficult to show that n'' will always be less than actual when ϕ_s'' is obtained from the maximum flows, although this appears to be the case. However, correction to n'' for the linear approximation can be made.

Data: Peiser, A. M., Chem. Eng., **67**, No. 14, 129 (1960). Sufficient information is not available for a tray check. Design conditions will be analyzed.

Comp.	x_F	y_F	x_B	x_D	x_s	y_f
7	0.0592	0.0137	0.0002	0.1871	0.0379	0.0844
8	0.1689	0.2243	0.0416	0.4351	0.3186	0.3877
9	0.2759	0.1712	0.3963	0.0380	0.4440	0.3191
10	0.2076	0.0706	0.2999	0.0003	0.1239	0.0797
Mols	831.20	168.80	614.70	385.30	660.62	885.13

Basis: Components 8 and 9 will be taken as the keys.

$Z = 0.750$

$Y_F = 1.312$	$Y_{s+1} = 1.223$	$Y_f = 1.215$
$X_F = 0.678$	$X_s = 0.718$	$X_{f-1} = 0.70$
$\alpha_F = 1.93$	$\phi_s = 1.705$	$\phi_{f-1} = 1.735$

The intersection of the operating lines is estimated with Equation (27):

$$Y_{s+1} = 1.185$$
$$X_s = 0.689$$
$$\phi_s = 1.72 = \tfrac{1}{2}(1.705 + 1.735)$$

Using Equations (12), (31) and (44):

$$(\partial n/\partial N)_s = \log_{10}(1.93)/\log_{10}(1.93/1.72) = 5.70$$
$$n/N = 2.303(5.70/4.70)\log_{10}(5.70) = 2.1$$
$$(L/L_M)_s = 5.70/4.70 = 1.22$$

Reverse fractionation in the stripping and enriching sections is avoided by preselection of the point of intersection of the operating lines in the equivalent binary system.

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