

Fragmentation experiment and model for falling mercury drops

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Abstract

The experiment consists of counting and measuring the size of the many fragments observed after the fall of a mercury drop on the floor. The size distribution follows a power-law for large enough fragments. We address the question of a possible crossover to a second, different power-law for small enough fragments. Two series of experiments were performed. The first uses a traditional film photographic camera, and the picture is later treated on a computer in order to count the fragments and classify them according to their sizes. The second uses a modern digital camera. The first approach has the advantage of a better resolution for small fragment sizes. The second, although with a poorer size resolution, is more reliable concerning the counting of *all* fragments up to its resolution limit. Both together clearly indicate the real existence of the quoted crossover.

The model treats the system microscopically during the tiny time interval when the initial drop collides with the floor. The drop is modelled by a connected cluster of Ising spins pointing *up* (mercury) surrounded by Ising spins pointing *down* (air). The Ising coupling which tends to keep the spins segregated represents the surface tension. Initially the cluster carries an extra energy equally shared among all its spins, corresponding to the coherent kinetic energy due to the fall. Each spin which touches the floor loses its extra energy transformed into a thermal, incoherent energy represented by a temperature used then to follow the dynamics through Monte Carlo simulations. Whenever a small piece becomes disconnected from the big cluster, it is considered a fragment, and counted. The results also indicate the existence of the quoted crossover in the fragment-size distribution.

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In many instances, the fracture of solid objects or dense liquids leads to scaleless distributions of fragment sizes (see, for instance, Ref. [1]). For a recent study, as well as a nice theoretical analysis, see also Ref. [2]. This scaling phenomenon is observed among other experiments after the drop of objects on the floor. Objects with different shapes (cubes, rods, discs, plates, etc.) and made of several materials (various plastics, frozen potatoes, soap, etc.) were studied. The main result of each experiment is the distribution of fragment sizes,

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classified according to their masses, which follows a power-law

$$\frac{N}{m} \propto m^{-\beta},$$

where $N(m)$ is the number of fragments with masses larger than m . Plotted on a double-logarithmic scale, such a distribution shows a decreasing straight line, the slope of which gives the characteristic exponent β of that particular experiment, $\beta > 1$. This exponent, however, depends on various parameters, such as the geometry of the initial object, the material from which it is made, the falling height and others. It is not a universal index. For the largest fragment sizes, of course, an exponential decay bends the straight line downwards, but this finite size effect is out of the scope of the present work.

The same phenomenon is observed in many other systems within completely different size scales, including nuclear multifragmentation (see Ref. [3] and references therein), atomic cluster fragmentation [4], magnetic liquids [5], icebergs [6] and meteorite showers [7].

In many cases, instead of a single straight line denoting a simple power-law, one observes two straight lines with different slopes. There is a crossover from the large-fragment regime (exponent β already mentioned) to the small-fragment counterpart (with a smaller exponent $\tilde{\beta}$). This is the case, among others, of some meteorite showers, fitted in Ref. [7] by the sum of two power-laws.

We raise here the hypothesis that this behaviour, with two superimposed power-laws, can also be applied to the beautiful experiment carried-out by Sotolongo-Costa et al. [8] with the fall of mercury drops. They count and measure the fragment sizes on a microscope, find the expected power-law behaviour for large fragments, but their resolution is not enough to sample the supposed small-fragment regime. In order to probe this hypothesis, we repeat the same experiment with a larger mercury drop (about 0.7 cm diameter) and observe the result by taking pictures, instead of using a microscope. The results are shown in Fig. 1. Along the vertical axis we plot directly the (normalised) counting $N(m)$ (instead of N/m), thus the slopes correspond to $1 - \beta$ instead of $-\beta$.

Fig. 1 shows two different experiments, the first corresponding to the open circles. Pictures were taken with a traditional camera, using films ORWO with 50 ASA. This fine grain film has a good resolution, allowing us

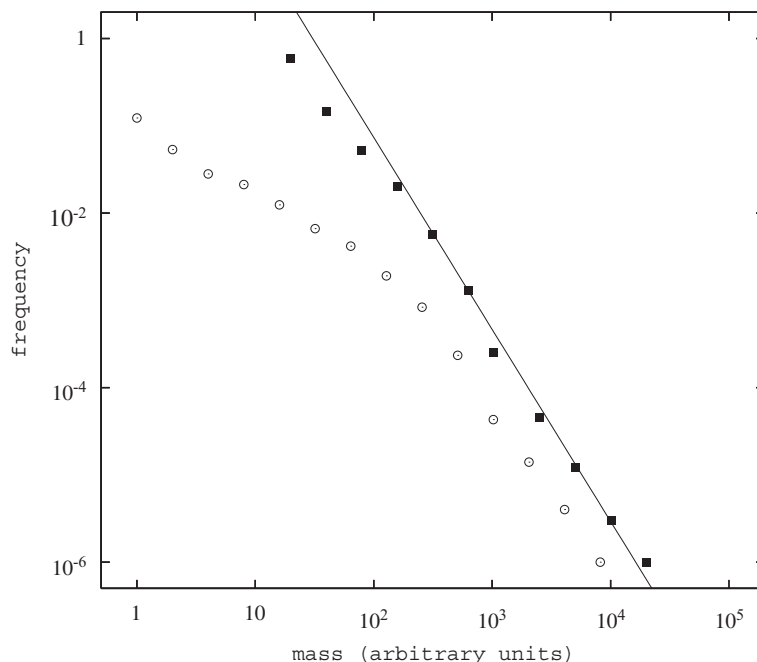


Fig. 1. Experimental results for the falling mercury drop. Circles correspond to a traditional camera with good resolution film. Squares were obtained with a digital camera with greater statistics (a 10-fold larger number of realisations of the fall experiment). Thus, the largest fragment observed among all digital pictures is larger than that on the photographic films.

to probe a much larger range of mercury fragments, their masses covering four orders of magnitude. (We adopt the smallest mass as unit, in Fig. 1). The expected two regimes appear with the second, less-slanted straight line on the left describing the smaller fragments. The crossover point falls near mass $m = 10^2$, two orders of magnitude smaller than the largest fragment. The resolution, however, is enough to probe also fragments two further orders of magnitude smaller. The plot corresponds to 10 realisations of the experiment, for an initial drop height of 2.2 m, all 10 data sets superimposed. Other realisations with a height of 2.4 m confirm the same behaviour, again with the film camera. This two-slopes behaviour was also verified with other heights, with a digital camera, as shown later. Based on these data (circles in Fig. 1) one would be tempted to conclude in favour of the real existence of the crossover. Prudence is better. Although presenting good resolution, the film pictures could in principle suffer from a drawback, as follows. The light coming from each mercury fragment impresses a cluster of neighbouring grains on the film surface. The smallest mercury fragments correspond to the smallest clusters of grains. With a small number of grains to be impressed by the light, the failure probability of the whole cluster maybe large enough to artificially increase the undercounting on the region of the smallest fragments. Remember that the chemical blow-up of a single grain is not free of correlations concerning the blow-up of neighbouring grains on the film.

Because of this possible drawback, we decided to repeat the experiment once more, now taking pictures with a digital camera. Its resolution is 3.2 Megapixels. Different heights present always two slopes. As an example, the results for the same height of 2.2 m are shown by the full squares in the same Fig. 1. Now, the data correspond to 100 realisations, thus the largest fragment is larger than the previous one pictured by the traditional camera with only 10 realisations. The mass ratio between these two largest fragments is used in order to properly plot both data sets on the same figure with the same mass unit. The continuous line is a fit for masses larger than $m = 10^2$. The digital image has a lower resolution than the photographic film, and then misses fragments smaller than $m = 10$. However, neighbouring pixels are free of correlations with other pixels because they are independently treated by the electronic mechanism. Therefore, the probability of undercounting the smaller mercury fragments is negligible, and we can trust the data in between $m = 10$ and 10^2 . Although small, the bend is also visible. We can safely conclude that the observation of two distinct power-law exponents is not an artefact of the measurement process. Note also that both experiments show the same position for the crossover point, near $m = 10^2$.

Possibly the same qualitative behaviour would appear with liquids other than mercury, with different densities and surface tension properties. Following Ref. [8], we used mercury because the relatively high surface tension keeps the fragments spherical, allowing the experimental determination of their masses from the two-dimensional projection, i.e., the pictures. Another, more complicated experimental approach would be necessary for other liquids. However, within our simulational model described below, one can vary the (equivalent of the) surface tension and height at will.

The collective behaviour of the many microscopic components of the initial mercury drop, during the tiny time interval when the crash occurs, should be responsible for the observed fragment distribution. The kinetic energy acquired during the fall is distributed among these many components, partially breaking the surface tension which glues them together in a single drop before the crash. The consequence is to separate it into many different fragments.

We decide to treat this behaviour through a simple dynamic lattice drop model already adopted in many other instances (for a review, see Ref. [9]). The model consists in treating a drop, with its characteristic surface tension, as a cluster of neighbouring Ising spins pointing *up* surrounded by spins pointing *down*. These spins occupy a square or cubic lattice, and do not move. They can only flip their *up* or *down* orientations. In most cases the total drop mass should be conserved, then any *up* \rightarrow *down* flip is necessarily followed by a *down* \rightarrow *up* flip performed on another lattice site. The Ising interaction applies only to neighbouring spin pairs: if they are parallel, no energy contribution from this pair is counted; otherwise, if they point one *up* and the other *down*, then the pair contributes with an energy unit to the whole system. Thus, the Ising energy is distributed exclusively along the drop surface, i.e., the interface between the cluster of spins *up* and the sea of spins *down* outside, and characterises the surface tension. Because of that feature, during the Monte Carlo dynamics, we toss to flip only pairs of non-neighbouring spins which are currently positioned along this interface, one pointing *up* and the other *down*. The interaction neighbourhood we adopt for each spin corresponds to its 8 (18) nearest neighbours on the square (cubic) lattice.

This lattice drop model was introduced [10] within a static version in order to study the shape of water drops hanging from a vertical wall. Later, it was generalised [11] to study a dynamic phenomenon: the time interval series of the successive drops falling from a leaky faucet. It was also applied to other dynamic phenomena such as nuclear multifragmentation [12], magnetic hysteresis [13] and annealing phenomena in magnetic multilayers [14]. In each case, further particular ingredients describing the physical system are introduced, besides the surface tension common to all cases, represented by the Ising interaction. Here, we will describe only the present case of the mercury drop.

We start the computer simulation at the very moment when the initial mercury ball touches the floor. It is represented by a big round cluster of N spins *up*, and the floor by an inert flat boundary. Our first parameter E corresponds to the initial kinetic energy accumulated during the fall, therefore this parameter E represents the falling height. We assign the share $e = E/N$ to each pixel of the drop. The initial Monte Carlo temperature is $T = 0$, because no random motion exists at this moment.

From this starting configuration, we simulate the collision with the floor as follows:

1. the n pixels currently touching the floor are transferred to random positions along the drop free surface, and the drop as a whole is moved down one lattice row;
2. the energy e carried by each of these n pixels is set to zero, because they just lost their coherent initial kinetic energy;
3. the same energy total amount $\Delta E = ne$ is transformed into incoherent, thermal form, by increasing the Monte Carlo temperature $T \rightarrow T + \Delta E$;
4. the drop shape is then allowed to relax, by performing r movements on its free surface, where the number r is our second parameter defining the relaxation time scale. Each movement consists in flipping two non-neighbouring spins, one pointing *up* and positioned along the inner free surface, the other pointing *down* and located along the outer surface. The positions of both are randomly chosen, and the double flipping is performed or not according to Metropolis algorithm: if the total Ising energy decreases as a consequence of this movement, then it is performed with certainty; otherwise, if the total Ising energy would increase by an amount δE , then the movement is performed only with probability $\exp(-\delta E/T)$.

These four steps are repeated iteratively. At some moment during this process, some piece of the drop becomes disconnected from the main part still touching the floor. It is considered a fragment. If the total energy of the whole system increases due to this fragmentation, then the fragment is discarded and counted in the statistics. Also in this case, the Monte Carlo temperature is decreased $T \rightarrow T - kE_f$, where E_f is the surface Ising energy carried out by the fragment, and k is our third constant parameter. It can be varied in order to simulate different surface tensions. The series of r relaxations proceeds with the remainder drop. The process stops when the drop vanishes, or when the Monte Carlo temperature is low enough to forbid any further fragmentation.

The whole dynamics is repeated again and again, always starting from the same initial drop with the same initial kinetic energy E , and the fragments are classified according to their masses. We have performed a lot of tests with different values for our parameters E , r and k , for both the square and cubic lattices. In all cases we obtained the same kind of fragment distribution, showing the already quoted crossover from large to small size regimes. Figs. 2 and 3 are typical examples. The bend is always present. The two slopes, however, depend slightly on the parameters, characterising their non-universality generally observed in many other fragmentation problems. Of course, we have a much better statistics than the experimental set up, thus these plots present a much greater number of points without need of dividing the mass axis in bins.

In short, we have measured a crossover between two different power-laws from large to short fragments, within a mercury ball crashing on the floor, Fig 1. We raise here the proposal that it is a real phenomenon, not a simple undercounting artefact. We also introduced a dynamic stochastic model to simulate this phenomenon in computers, which reproduces again the same crossover in both two and three dimensions, Figs. 2 and 3.

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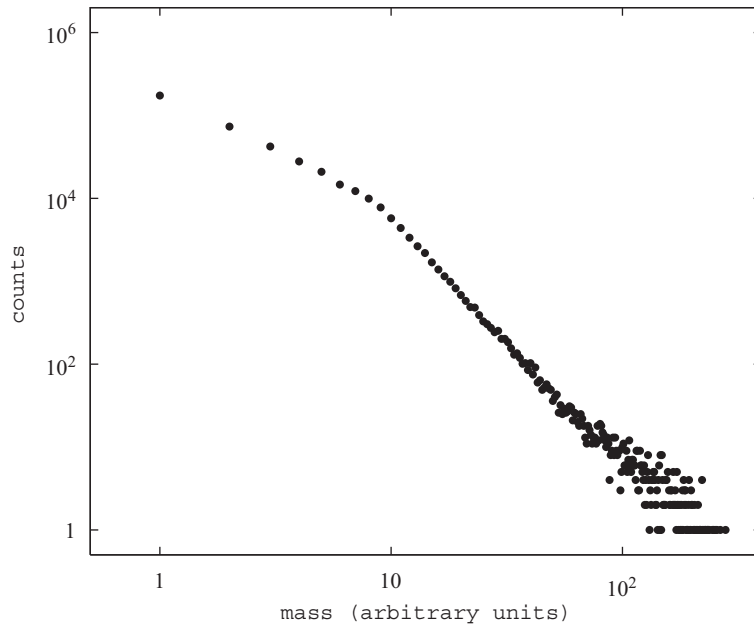


Fig. 2. Simulation results in two dimensions. The initial ball has 321 pixels and parameters are $E = 2000$, $r = 80$ and $k = 0.2$ (see text). All fragments obtained from 5000 independent falls were superimposed. The same behaviour is obtained, with slightly different slopes, for many other combinations of these parameters.

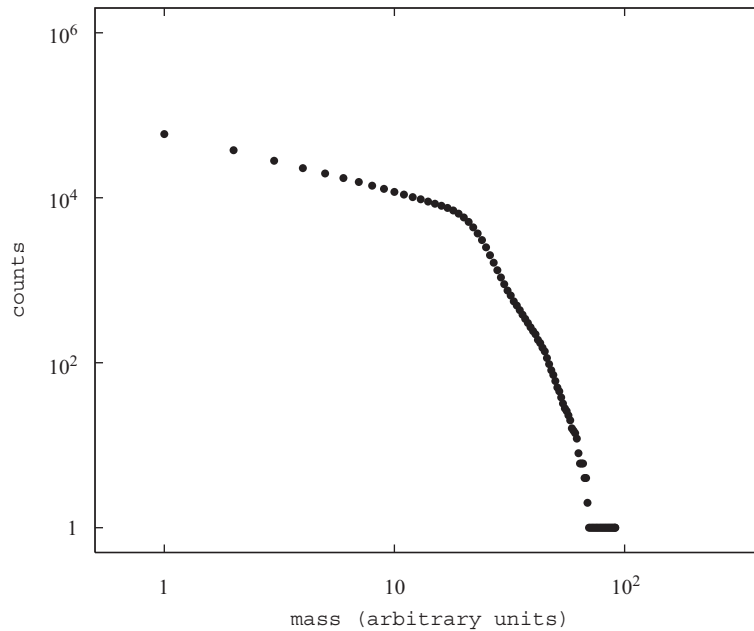


Fig. 3. Simulation results in three dimensions. The initial ball has 179 pixels and parameters are $E = 1$, $r = 50$ and $k = 0.9$, and 10 000 independent falls were superimposed. Again, the same behaviour with slightly different slopes is obtained for many other combinations of these parameters.

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