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Simple method of designing centralized PI controllers for multivariable systems based on SSGM

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ABSTRACT

A method is given to design multivariable PI/PID controllers for stable and unstable multivariable systems. The method needs only the steady state gain matrix (SSGM). The method is based on the static decoupler design followed by SISO PI/PID controllers design and combining the resulted decoupler and the diagonal PI(D) controllers as the centralized controllers. The result of the present method is shown to be equivalent to the empirical method proposed by Davison EJ. Multivariable tuning regulators: the feed-forward and robust control of general servo-mechanism problem. IEEE Trans Autom Control 1976;21: 35–41. Three simulation examples are given. The performance of the controllers is compared with that of the reported centralized controller based on the multivariable transfer function matrix.

1. Introduction

Design of PI controllers for multi-input multi-output (MIMO) processes is difficult when compared to that of the single-input single output (SISO) processes due to the interaction between the input/output variables. The MIMO processes can be controlled by decentralized or decoupled controllers or by centralized PI controllers. For mild interacting MIMO processes, design of decentralized PI controllers based on the diagonal processes (based on proper pairing) is carried out with a suitable detuning method. The detuning step involves in decreasing the controller gains by F, and multiplying integral times by F, and decreasing derivative times by F. Here, F is the detuning factor which may vary from 1.5 to 4, depending on the extent of interactions, dictated by the relative gain array (RGA). For systems with large interactions, the decoupler (D) is designed so as to make the MIMO processes into n SISO processes. The PI controllers (G_c) are designed for the resulting SISO processes. The overall control system is the combined decoupler and the diagonal controllers ($=DG_c$). We can also design straight away the centralized PID controllers. The methods include synthesis method [1], decoupler based centralized controllers [2–6], IMC method [7], and gain and phase margin method [8]. Reviews on the design of multivariable controllers for MIMO systems are given by Maciejowski [9], Skogestad and Postlethwaite [10] and Wang et al. [8,11].

Tanttu and Lieslehto [12] have discussed the simple methods of tuning PI controllers for a stable transfer function matrix of a system. Davison's method [12,13] makes use of only the steady state gain matrix of the system for the design of centralized PI controllers. The steady state gain matrix of the multivariable system can be obtained easily than that of identifying all the dynamic model parameters (time constants, time delays, numerator dynamics, and steady-state gains). The method is shown to give a satisfactory response for many case studies of transfer function matrix models. However, there is no derivation available for Davison's method.

Katebi [14] gives a summary of these simple design methods. Subramaniam et al. [15] have compared the performances of the centralized multi-variable PI controllers (designed by Davison's method) by simulation of the stable nonlinear model equations of a nuclear reactor. Reddy et al. [16] have compared the performance of centralized PID controllers for a MSF desalination plant. Reddy et al. [17] applied these simple centralized controllers for a nonminimum phase stable multivariable system. Sarma and Chidambaram [18] have extended the method to design centralized controllers for a stable non-square system. In all the above methods [12–18], the centralized PI controllers designed by Davison's method is shown to give a satisfactory performance. As stated earlier, the steady state gain matrix of the multivariable system can be obtained easily than that of identifying the dynamic model parameters and hence, it is easier to tune the controllers. Dhanyaram et al. [19] extended Davison's method [13] to unstable

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Nomenclature

D	transfer function matrix of the decoupler
G_P	transfer function of the system
G_V	disturbance transfer function matrix
K_C	proportional gain of the centralized controller
K_I	integral gain matrix of the centralized controller
$K_{c,d}$	proportional gain matrix (diagonal) for the decoupled system
$K_{I,d}$	integral gain matrix (diagonal) for the decoupled system
$k_{c,p}$, $k_{I,P}$	elements of $K_{c,d}$ and $K_{I,d}$ respectively
$k_{c,ij}$, $k_{I,ij}$	elements of the centralized controller (K_c and K_I)
n	number of inputs or outputs
P	desired (diagonal) transfer function matrix of the decoupled system
P_{11}, P_{22}	elements of the diagonal P matrix
s	Laplace variable
t	time
U	manipulated variable vector
V	disturbance variable vector
Y	output variable vector

Greek letters

$\tau_{1,d}$, $\tau_{2,d}$	time constants of the desired decoupled system P
$\tau_{d1,d}$, $\tau_{d2,d}$	time delays of the desired decoupled system P
δ_1	tuning parameter for the proportional gain of the centralized controller
δ_2	tuning parameter for the integral gain of the centralized controller
δ_3	tuning parameter for the derivative gain of the centralized controller

MIMO systems. In the present work, derivation for a simple method of designing multivariable PI controllers is proposed, based on steady-state gain matrix of the system and the result of the present method is shown to be similar to that of empirical Davison's method.

2. Multivariable system

The multivariable system (refer to Fig. 1) is assumed to be of the form

$$Y(s) = G_P(s)\,U(s) \tag{1}$$

where Y is the $(n \times 1)$ vector of the output variables, and U is the $(n \times 1)$ vector of the manipulated variables. $G_P(s)$ is the $n \times n$ transfer function matrix of the system. The centralized controllers are assumed as

$$G_c(s) = K_C + (K_I)/s \tag{2}$$

where K_C and K_I are matrices of size $n \times n$:

$$K_C = [k_{c,ij}] \tag{3}$$

$$K_I = [k_{c,ij}/\tau_{I,ij}], \quad i = 1, \ldots n \text{ and } j = 1, \ldots, n \tag{4}$$

$k_{c,ij}$ is the ijth element of the proportional gain matrix (K_C) and $\tau_{I,ij}$ is the corresponding integral time of the controllers.

2.1. Davison method

Davison [13] has proposed an empirical method of tuning a multivariable PI control system, where the matrices K_C and K_I in Eq. (2) are given by

$$K_C = \delta_1[G_P(s=0)]^{-1} \tag{5}$$

$$K_I = \delta_2[G_P(s=0)]^{-1} \tag{6}$$

Here, $[G_P(s=0)]^{-1}$ is called the rough tuning parameters. The rough tuning matrix is the inverse of steady-state gain matrix. The tuning parameters range is usually 0–1 and the recommended values are 0.1–0.5 for δ_1 and 0.05–0.2 for δ_2. Methods are available to estimate the SSGM of multivariable systems. For stable and integrating systems, we can substitute K_P for $G_P(s=0)$.

3. Derivation of controllers design

Consider a MIMO system as presented in Fig. 2. G_P is the process transfer function matrix. It is assumed that the pairing based on relative gain array (RGA) is carried out and the recommended pairing is already considered as shown in Fig. 2. We have to select the decoupler [D(s)] such that the resultant system of the combined process and the decoupler becomes a diagonal system

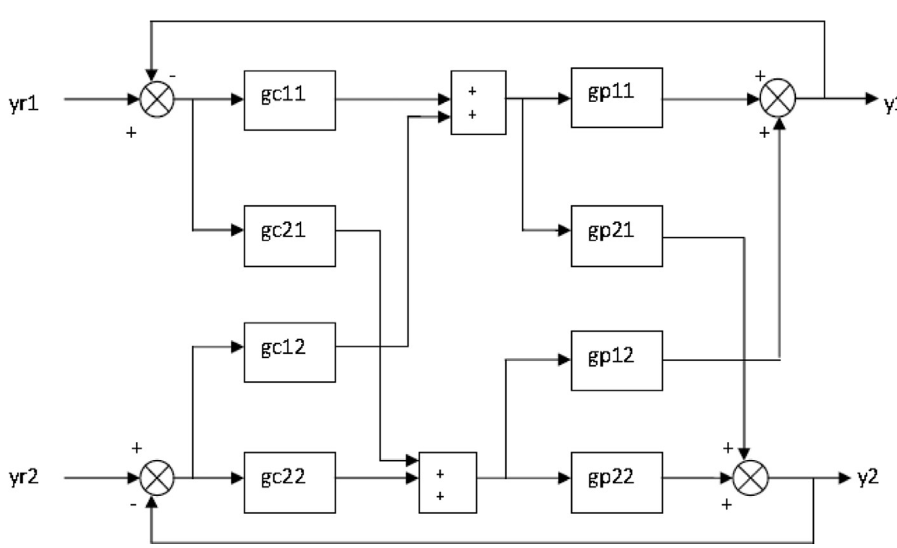

Fig. 1. Centralized control system of a TITO process.

P(s) [20]:

$$G_P(s)\,D(s) = P(s) \tag{7}$$

where P(s) is the desired transfer function matrix (diagonal matrix) of the decoupled system. For example, if D(s) is selected such that it is equal to $[G_P(s)]^{-1}$, then P(s)=I. This is an ideal situation. Instead, let us consider

$$P_{11} = \exp(-\tau_{d1,d}s)/(\tau_{1,d}s+1) \tag{8}$$

$$P_{22} = \exp(-\tau_{d2,d}s)/(\tau_{2,d}s+1) \tag{9}$$

Here, $\tau_{d1,d}$ and $\tau_{1,d}$ are the time delay and time constant of the decoupled system, P_{11}. Similarly we can define for other terms such as $\tau_{d2,d}$ and $\tau_{2,d}$. The values of time constants ($\tau_{1,d}$, $\tau_{2,d}$) and time delays ($\tau_{d1,d}$, $\tau_{d2,d}$) of the decoupled system are slightly greater than that of the open loop system so as to take into account the interactions among the loops.

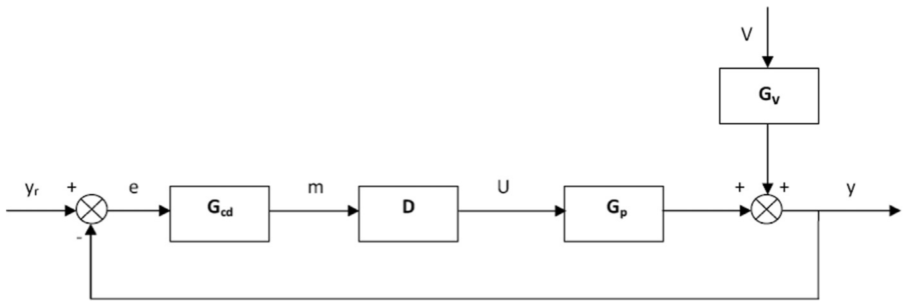

Fig. 2. Decoupler and controller for the multivariable systems D—decoupler matrix; G_p—process matrix; G_{cd}—controller matrix for the decoupled system; G_p D=P; P is a diagonal matrix; G_{cd}—diagonal matrix; G_p, D—full matrix; G_c=(D G_{cd}) as shown in Fig. 1 is a full matrix; V—disturbance variable vector; G_V is the disturbance transfer function matrix.

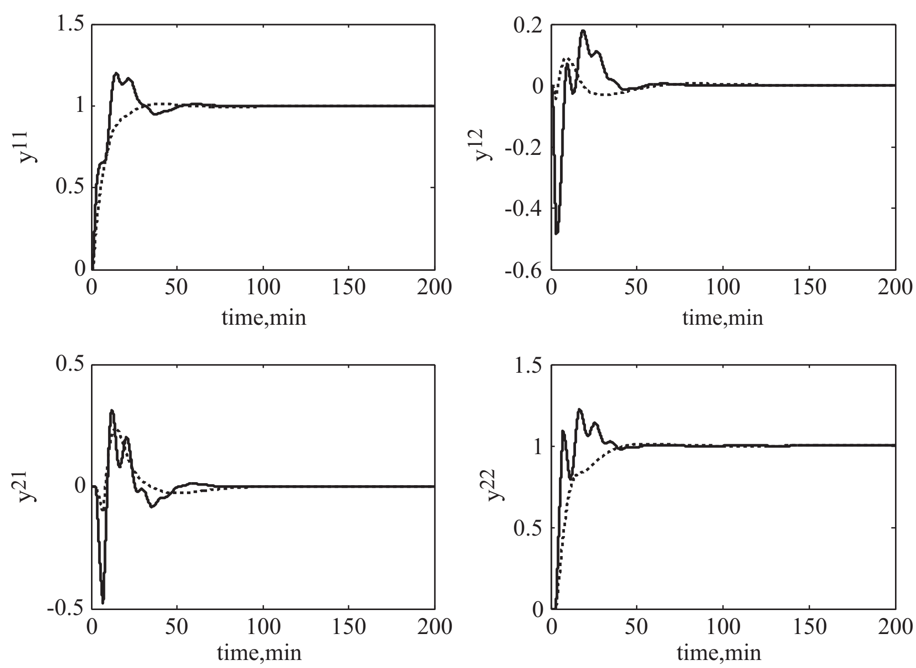

Fig. 3. Servo response comparison of Wood and Berry. Solid line—present method with δ_1=2and δ_2=0.3; dotted line—synthesis method [1].

Table 1
(a) IAE values for the closed loop system (example-1).

Eg:1	IAE values					**Sum**		
	Method	y_{11}	y_{21}	y_{12}	y_{22}	**Overall**	**Main action**	**Interaction**
Servo	Proposed	8.103	5.403	4.53	7.866	25.902	15.97	9.93
	Kumar et al. [1]	8.031	4.046	1.903	11.32	25.3	19.35	5.95
Regulatory	Proposed	55.5	37.32	87.67	89.37	269.86	144.87	124.99
	Kumar et al. [1]	97.08	48.53	141.6	174.9	462.11	271.98	190.13

Overall: $y_{11}+y_{21}+y_{12}+y_{22}$.
Main action: $y_{11}+y_{22}$.
Interaction: $y_{21}+y_{12}$.

Let us consider the static decoupler for the system as

$$D = [G_P(s=0)]^{-1} \quad (10)$$

For the resulting decoupled system (P), suitable PI controllers need to be designed as

$$G_{c,d}(s) = (K_{c,d}) + [(K_{I,d})/s] \quad (11)$$

Here, $K_{c,d}$ and $K_{I,d}$ are diagonal matrices. Hence, the overall controller matrix to be implemented on the process (combination of the decoupler and the diagonal PI controllers) is given by

$$G_c(s) = [G_P(s=0)]^{-1}[G_{c,d}(s)] \quad (12)$$

Let the controllers ($k_{c,p}$, $k_{I,p}$) be designed for the worst case of a FOPTD model among the diagonal elements of P (i.e. with larger delay, smaller time constant) so that same PI controller setting be used

$$G_{cd}(s) = [(k_{c,p}) + (k_{I,p}/s)]\,[I] \quad (13)$$

Hence, the overall controller system is given by

$$G_c(s) = [G_P(s=0)]^{-1}[(k_{c,p}) + (k_{I,p}/s)][I] \quad (14)$$

Here, $G_c(s)$ is the full matrix and is called a centralized controller. The above equation can be written as

$$G_c(s) = \delta_1[G_P(s=0)]^{-1} + (\delta_2/s)[G_P(s=0)]^{-1} \quad (15)$$

The reason for introducing the new parameters δ_1 $(=k_{c,p})$ and δ_2 $(=k_{I,p})$ is that, the relation to Davison's method can be understood. Eq. (15) can be rewritten as

$$G_c = \delta_1[K_P]^{-1} + (\delta_2/s)[K_P]^{-1} \quad (16a)$$

$$G_c = K_C + (K_I/s) \quad (16b)$$

where

$$K_C = \delta_1[K_P]^{-1} \text{ and } K_I = \delta_2[K_P]^{-1} \quad (16c)$$

Here, K_C is the centralized controller gain matrix and K_I is the centralized integral gain matrix. Let us now focus on the tuning of the PI or PID controllers for the resulting SISO–FOPTD systems. For the decoupled scalar system given by Eq. (8) or Eq. (9), the value of $k_{c,p}$ is given by $0.9(\tau_{1,d}/\tau_{d1,d})$ and $\tau_{I,p}$ as $3.3\tau_{d1,d}$ [21]. For typical range of values of model parameters, we can assume the range of δ_1 as 0.1–3. The range of values for δ_2 is 0.05–1.5. Similarly for a PID controller, the value of $k_{c,p}$ is given by $1.2(\tau_{1,d}/\tau_{d1,d})$ and $\tau_{I,p}$ as $2\tau_{d1,d}$

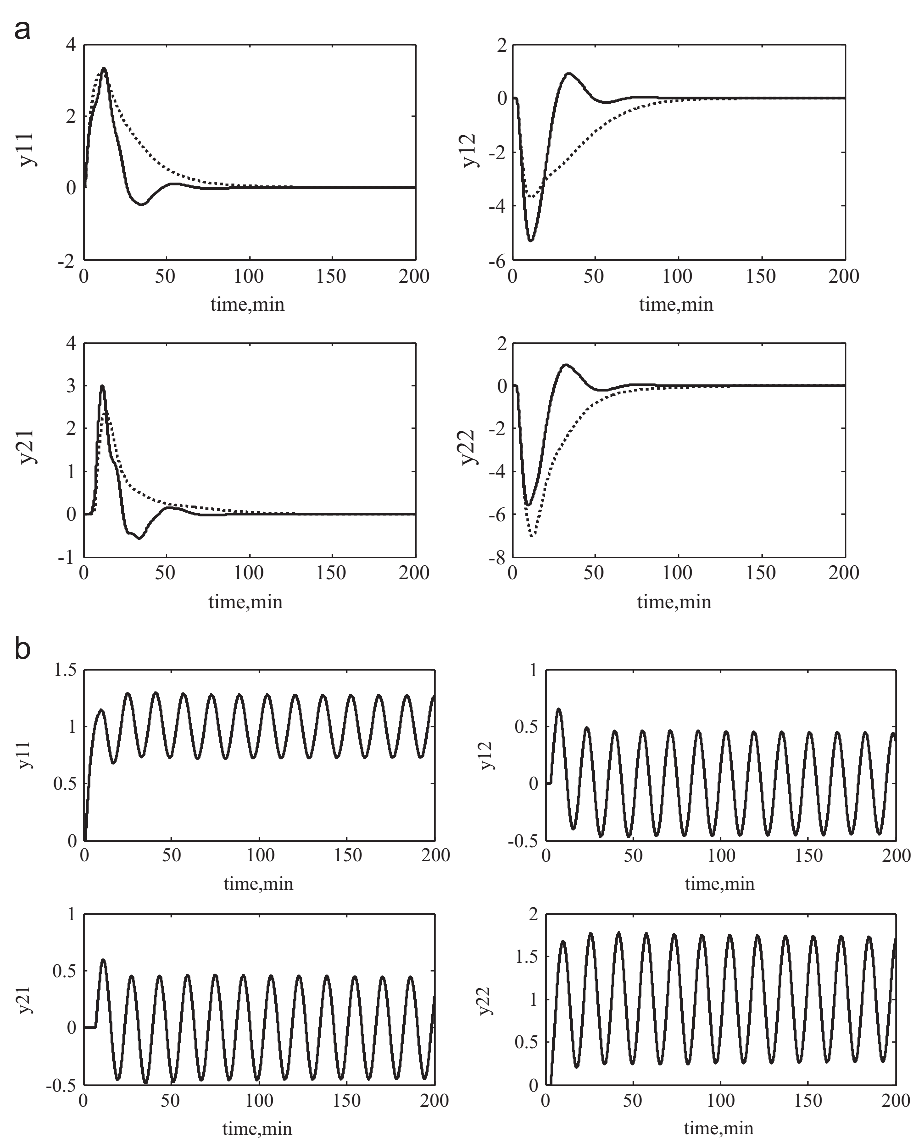

Fig. 4. (a) Regulatory response comparison for example-1. Solid line—present method with $\delta_1=2$and $\delta_2=0.3$; dotted line—synthesis method [1]. (b) Servo response for example-1 using only the diagonal elements of the proposed controller settings without the decouplers.

and $\tau_{D,p}$ as $0.5\tau_{d1,d}$ [21]. For typical values of these parameters we get the range values for δ_1 as 0.1–3 and δ_2 as 0.05–1.5 and δ_3 as 0.05–0.5

$$G_C = \delta_1[K_P]^{-1} + (\delta_2/s)[K_P]^{-1} + (\delta_3 s)[K_P]^{-1} \quad (17)$$

In case, we would like to consider the decoupler as the inverse of the system at ω_c rather than at $\omega=0$, we then get the resulting control structure as that given by Maciejowski [9]. However, this method requires the transfer function matrix of the process.

4. Simulation studies

4.1. Example-1: Wood and Berry column

The Wood and Berry distillation column plant is a multivariable system that has been studied extensively [1,22]. Transfer function matrix of WB column is given by

$$G_p(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \quad (18)$$

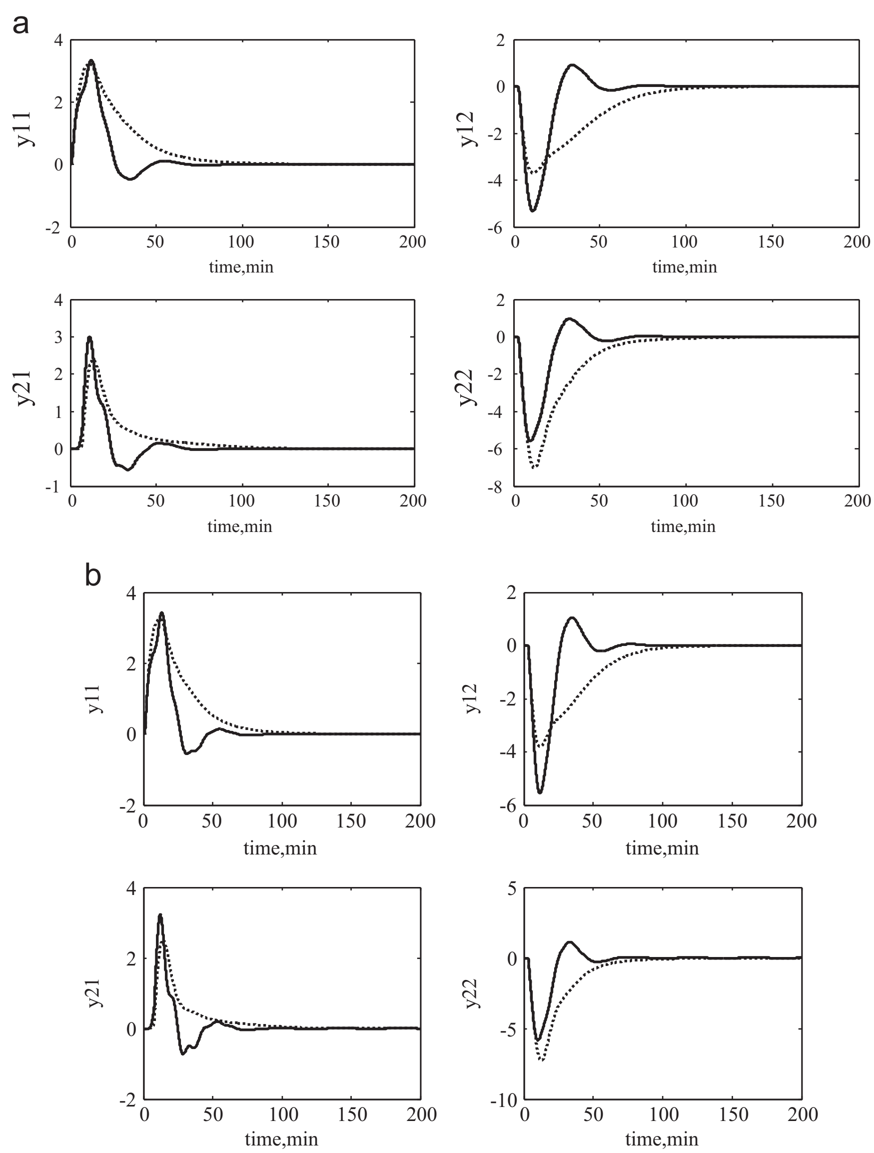

Fig. 5. (a) Robustness comparison (+10% perturbation in process gain) of regulatory response for example-1. Solid line—present method with $\delta_1=2$ and $\delta_2=0.3$; dotted line—synthesis method [1]. (b) Robustness comparison (+10% perturbation in process delay) of regulatory response for example-1. Solid line—present method with $\delta_1=2$ and $\delta_2=0.3$; dotted line—synthesis method [1].

Table 1
(b) IAE values for robustness of the closed loop system—regulatory problem (10% perturbation in each gain and time delay) for example-1.

Uncertainty	IAE values					**Sum**		
	Method	y_{11}	y_{21}	y_{12}	y_{22}	**Overall**	**Main action**	**Interaction**
Process gain	Proposed	55.25	37.46	86.87	88.78	268.36	144.03	124.33
	Kumar et al. [1]	97.08	48.54	141.6	174.9	462.12	271.98	190.14
Process delay	Proposed	57.73	39.77	91.46	93.36	282.32	151.09	131.23
	Kumar et al. [1]	97.08	48.53	141.6	174.9	462.11	271.98	190.13

The elements of RGA are found to be $\lambda_{11}=\lambda_{22}=2.01$ and $\lambda_{12}=\lambda_{21}=-1.01$.The centralized controller matrix based on synthesis method [1] is given by

$$G_C(s)=\begin{bmatrix}0.1697+\frac{0.0173}{s} & -0.0172-\frac{0.0140}{s}\\ 0.0161+\frac{0.0048}{s} & -0.0723-\frac{0.0096}{s}\end{bmatrix} \tag{19}$$

The inverse of the steady-state gain matrix is obtained as

$$K_P^{-1}=\begin{bmatrix}0.1570 & -0.1529\\ 0.0534 & -0.1036\end{bmatrix} \tag{20}$$

For the present work, the PI controllers settings are calculated using different values of the tuning parameter δ_1 and δ_2 and the closed loop performances are evaluated. The settings $\delta_1=2.0$ and $\delta_2=0.30$ give a better closed loop performance. The resulting centralized control system is given by

$$G_c(s)=\begin{bmatrix}0.3140+\frac{0.0471}{s} & -0.3058-\frac{0.04587}{s}\\ 0.1068+\frac{0.01602}{s} & -0.2072-\frac{0.03108}{s}\end{bmatrix} \tag{21}$$

The servo response for a unit step change in the set point of y_{r1} is evaluated and the response in y_1 and the interaction in y_2 are shown in Fig. 3 (left side of Fig. 3). Similarly, the servo response for a unit step change in the set point of y_{r2} is evaluated and the response in y_2 and the interaction in y_1 are shown in Fig. 3 (right side of Fig. 3). Fig. 3 compares the responses of the present method with the centralized PI control systems designed based on the synthesis method, using the transfer function matrix (i.e., values of delays, time constants are known), reported by Kumar et al. [1]. Kumar et al. [1] have shown their method is better than the recently reported methods. Table 1a shows that the sum of the IAE values for the main responses for the servo problem is lesser for the present method and whereas, the interaction is larger. Fig. 4(a) shows the performances of the two methods for the regulatory problems for a unit step change in the load variable (assuming the disturbances transfer function matrix is the same as that of the process transfer function matrix as shown in Fig. 2). An improved performance is obtained for the proposed method. Table 1a shows that the sum of the IAE values for the main responses and also the sum of the IAE values for the interactions are also lesser for the regulatory problem. If the decouplers are removed, and the same diagonal PI controllers only are used, then oscillatory responses are obtained for the servo problem as shown in Fig. 4(b).

The controller settings given by Eq. (21) are based on the SSGM and the tuning parameters are selected based on simulation of the closed loop system. The performance of the controllers for perturbation in each gain (10% increase to that of the nominal value) in the process is studied and the regulatory responses for a unit step change in the load variable are shown in Fig. 5(a). The same values for the tuning parameters ($\delta_1=2.0$ and $\delta_2=0.30$) are used. The IAE values are presented in Table 1b. The sum of the IAE values for the main responses and also the sum of IAE values for the interactions are lesser for the present method. Similar results are obtained for the variation in each time delay (10% increase to that of the nominal value) and the regulatory responses are shown in Fig. 5(b). The same values for the tuning parameters ($\delta_1=2.0$ and $\delta_2=0.30$) are used. As seen from Table 1b, a robust

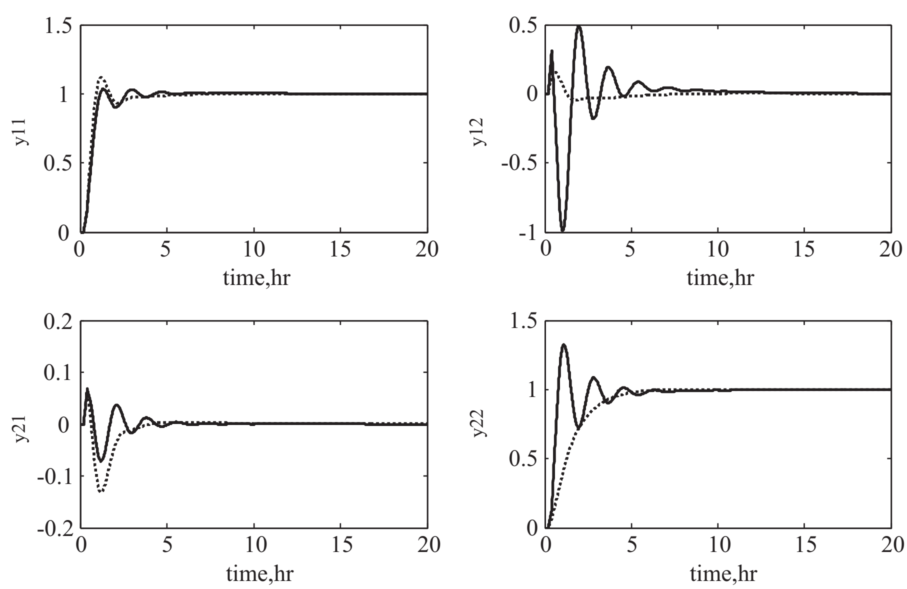

Fig. 6. Servo response comparison for example-2. Solid line: present method with $\delta_1=5$ and $\delta_2=1.5$; dotted line: synthesis method.

Table 2
IAE values for the example-2.

Eg:2	IAE values					Sum		
	Method	y_{11}	y_{21}	y_{12}	y_{22}	Overall	Main action	Interaction
Servo	Proposed	0.7993	0.1078	1.3437	1.068	3.3188	1.8673	1.4515
	Kumar et al. [1]	0.7594	0.1806	0.2634	1.628	2.8314	2.3874	0.444
Regulatory	Proposed	15.26	3.121	7.717	3.852	29.95	19.112	10.838
	Kumar et al. [1]	14.29	10.14	7.213	8.001	39.644	22.291	17.353

Overall: $y_{11}+y_{21}+y_{12}+y_{22}$.
Main action: $y_{11}+y_{22}$.
Interaction: $y_{21}+y_{12}$.

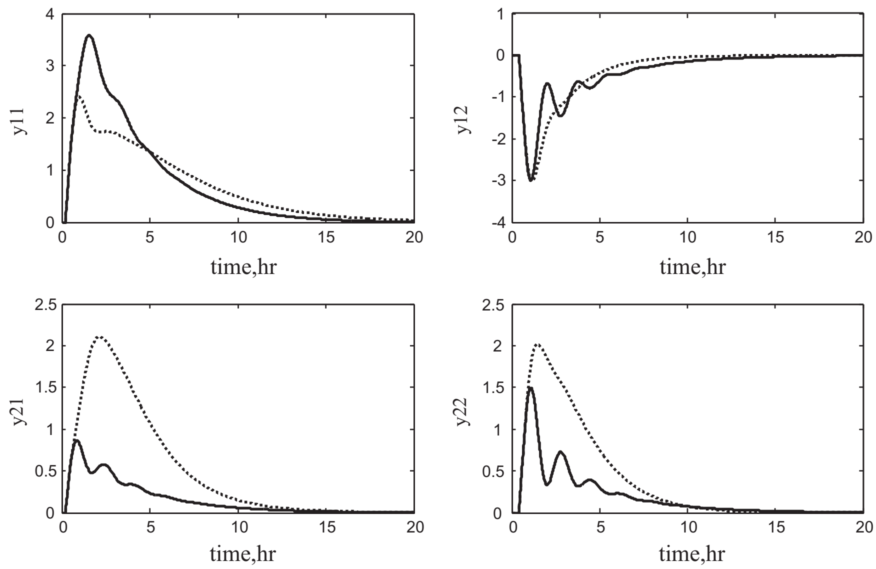

Fig. 7. Regulatory response comparison for example-2. Solid line: present method with $\delta_1=5$ and $\delta_2=1.5$; dotted line: synthesis method.

performance is obtained for the present method under uncertainty in the model parameters (gain and delay).

4.2. *Example-2: Industrial-Scale Polymerization (ISP) reactor*

The transfer function matrix for the system is given by [1,23]

$$G_p(s)=\begin{bmatrix}\frac{22.89e^{-0.2s}}{4.572s+1} & \frac{-11.64e^{-0.4s}}{1.807s+1}\\ \frac{4.689e^{-0.2s}}{2.174s+1} & \frac{5.8e^{-0.4s}}{1.801s+1}\end{bmatrix} \tag{22}$$

The elements of RGA are found to be $\lambda_{11}=\lambda_{22}=0.71$ and $\lambda_{12}=\lambda_{21}=0.29$. The centralized controllers are designed by the synthesis method [1] as

$$G_C(s)=\begin{bmatrix}0.1644+\frac{0.0424}{s} & 0.1403+\frac{0.0383}{s}\\ -0.1922-\frac{0.0538}{s} & 0.0843+\frac{0.0764}{s}\end{bmatrix} \tag{23}$$

The inverse of the steady-state gain matrix is given by

$$K_p^{-1}=\begin{bmatrix}0.0310 & 0.0621\\ -0.0250 & 0.1222\end{bmatrix} \tag{24}$$

For the present work, the PI controllers settings are calculated using different values of the tuning parameters δ_1 and δ_2 and the closed loop performances are evaluated. Earlier it was specified that the range of δ_1 is from 0.1 to 2, but in this example, the tuning parameters giving the best responses are found as to be $\delta_1=5$ and $\delta_2=1.5$. The resulting centralized PI control system is given by

$$G_c(s)=\begin{bmatrix}0.155+\frac{0.0465}{s} & 0.3105+\frac{0.09315}{s}\\ -0.125-\frac{0.0375}{s} & 0.611+\frac{0.1833}{s}\end{bmatrix} \tag{25}$$

Fig. 6 shows the servo responses of the present method along with that of Kumar et al. [1]. The present method works well. The IAE values for the responses and the interactions are presented in Table 2 for both the methods. The present method gives lesser IAE values for the main responses. The interactions are slightly higher. Fig. 7 shows the performance of the two methods for the regulatory problems for a unit step change in the load variable (assuming the disturbances transfer function matrix is the same as that of the process transfer function matrix as shown in Fig. 2). As seen from Table 2, for regulatory problems, both the main responses and also the interactions of the closed loop system are found to be better for the present method than that controller design proposed by Kumar et al. [1].

In the first example, the interaction is significant as shown by the RGA (elements of RGA is calculated as $\lambda_{11}=\lambda_{22}=2.01$ and $\lambda_{12}=\lambda_{21}=-1.01$). The main responses of the present method are better than that of the synthesis method which is based on the transfer function matrix. In the second example, the interaction is not significant as shown by the RGA ($\lambda_{11}=\lambda_{22}=0.71$ and $\lambda_{12}=\lambda_{21}=0.29$). For example 2, for the regulatory problems, the main responses and as well the interactions are better for the proposed method. For the servo problem, the interactions are also less for the present method.

4.3. *Example-3: Ogunnaike and Ray column [24,25]*

The transfer function matrix of a binary ethanol–water system of a pilot plant distillation column proposed by Ogunnaike et al. [24] is considered:

$$G_p(s)=\begin{bmatrix}\frac{0.66\,e^{-2.6s}}{6.7s+1} & \frac{-0.61\,e^{-3.5s}}{8.64s+1} & \frac{-0.0049\,e^{-s}}{9.06s+1}\\ \frac{1.11\,e^{-6.5s}}{3.25s+1} & \frac{-2.36\,e^{-3s}}{5s+1} & \frac{-0.01\,e^{-1.2s}}{7.09s+1}\\ \frac{-34.68\,e^{-9.2s}}{8.15s+1} & \frac{46.2\,e^{-9.4s}}{10.9s+1} & \frac{0.87(11.61s+1)\,e^{-s}}{(3.89s+1)(18.8s+1)}\end{bmatrix} \tag{26}$$

The elements of RGA are found as

$$\Lambda=\begin{bmatrix}2.0084 & -0.7220 & -0.2864\\ -0.6460 & 1.8246 & -0.1786\\ -0.3624 & -0.1026 & 1.4650\end{bmatrix} \tag{27}$$

The centralized controller matrix based on the Xiong et al. method [25] is given by

$$G_C(s)=\begin{bmatrix}1.2266\left(1+\frac{1}{6.7s}\right) & -0.0716\left(1+\frac{1}{3.25s}\right) & 0.0017\left(1+\frac{1}{8.15s}\right)\\ 0.5758\left(1+\frac{1}{8.64s}\right) & -0.2219\left(1+\frac{1}{5.0s}\right) & 4.7035e^{-004}\left(1+\frac{1}{10.9s}\right)\\ 61.1085\left(1+\frac{1}{9.06s}\right) & 13.9406\left(1+\frac{1}{7.09s}\right) & 2.8540\left(1+\frac{1}{12.4150s}\right)\end{bmatrix} \tag{28}$$

The inverse of the steady-state gain matrix is obtained as

$$[G_P(s=0)]^{-1}=\begin{bmatrix}3.0430 & -0.5820 & 0.0104\\ 1.1836 & -0.7731 & -0.0022\\ 58.4481 & 17.8564 & 1.6839\end{bmatrix} \tag{29}$$

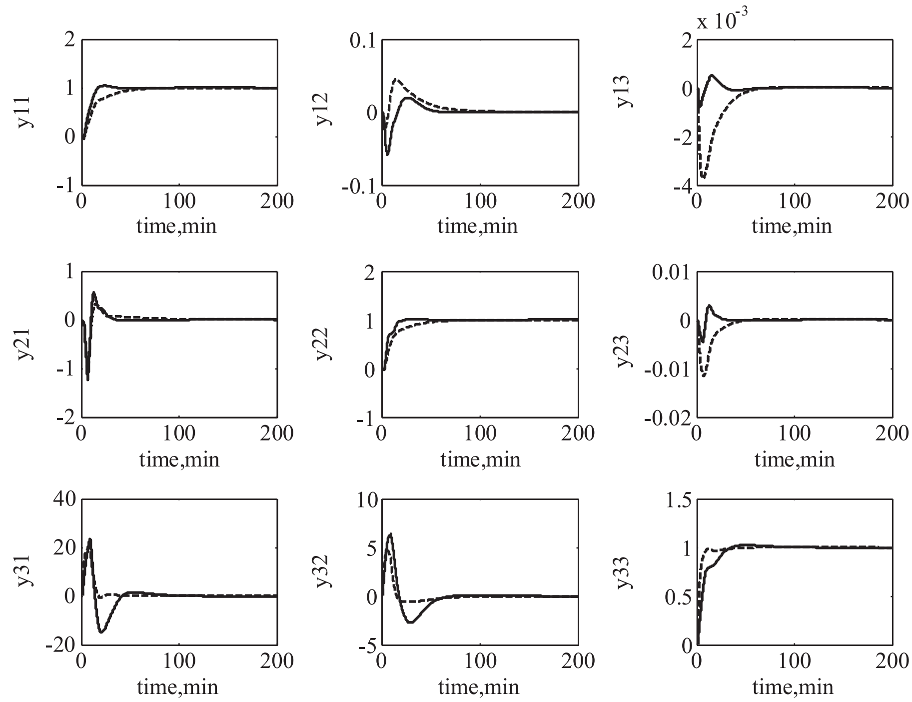

Fig. 8. Servo response comparison for example 3. Solid line—present method with $\delta_1=0.5$ and $\delta_2=0.125$; dotted line—Xiong et al. [25].

Table 3
IAE values for the closed loop system of example-3.

	Method	y_{11}	y_{21}	y_{31}	y_{12}	y_{22}	y_{32}	y_{13}	y_{23}	y_{33}
Serv.	Proposed	9.031	10.08	446.2	0.842	8.321	132.6	0.0115	0.0479	9.786
	Xiong et al. [25]	16.64	11.16	223.5	1.447	15.45	66.04	0.0744	0.1722	3.958
Reg.	Proposed	5.819	9.902	385.7	5.719	19.63	508.8	0.4294	0.0835	7.494
	Xiong et al. [25]	7.034	10.51	294.3	4.434	27.81	354	0.0176	0.0490	4.557

Serv.: servo response; Reg.: regulatory response.

Table 4
IAE values for the closed loop system of example-3.

	Method	Main action	Interaction
Serv.	Proposed	27.138	589.7814
	Xiong et al. [25]	36.048	302.3936
Reg.	Proposed	32.943	910.6339
	Xiong et al. [25]	39.401	663.3106

Main action: $y_{11}+y_{22}+y_{33}$.
Interaction: $y_{21}+y_{31}+y_{12}+y_{32}+y_{13}+y_{23}$.

For the present work, the PI controllers settings are calculated using different values of the tuning parameters δ_1 and δ_2, and the closed loop performances are evaluated. The settings giving the best responses are obtained for $\delta_1=0.5$ and $\delta_2=0.125$. The obtained controller settings are given by

$$G_c(s)=\begin{bmatrix} 1.5215+\frac{0.3804}{s} & -0.291-\frac{0.0727}{s} & 0.0052+\frac{0.0013}{s} \\ 0.5918+\frac{0.1479}{s} & -0.38655-\frac{0.0966}{s} & -0.0011-\frac{0.0003}{s} \\ 29.2240+\frac{7.3060}{s} & 8.9282+\frac{2.2320}{s} & 0.84195+\frac{0.2105}{s} \end{bmatrix} \tag{30}$$

Fig. 8 shows the servo responses of the present method with that of Xiong et al. [25]. The IAE values for the responses and the interactions are presented in Table 3 for both the methods. Table 4 shows the sum of the IAE values for the main action $(y_{11}+y_{22}+y_{33})$ and sum of the IAE values for the interaction $(y_{21}+y_{31}+y_{12}+y_{13}+y_{31}+y_{32})$ for servo and regulatory problems respectively. Fig. 9 shows the performance of the two methods for the regulatory problems (assumed the disturbances entering along with the manipulated variables). From Table 4, it can be seen that sum of IAE values of servo response for the main actions is lesser for the proposed method when compared with that of the Xiong method and whereas, the interaction is higher. Similar results are obtained for the regulatory problems also. The present method even though is based on SSGM, the method works good. The method is simple to use and the main responses of the closed loop system are found to be better than the method than that based on the full transfer function matrix.

5. Conclusions

Based on the steady state gain matrix, a simple method is given to tune the centralized PI controllers. The basic idea is to use the static decoupler followed by the design of SISO PI controllers. The derived equations are shown to be equivalent of the empirical method proposed by Davison. The main responses of the proposed controllers are shown to be better than the centralized PI controllers reported recently in the literature based on the full system transfer function matrix. This is illustrated with three simulation examples. For the present method, knowledge of the steady state gain matrix is only needed rather than on the full dynamics (gain, time delay, and time constant). The obtained main responses are good, signifying

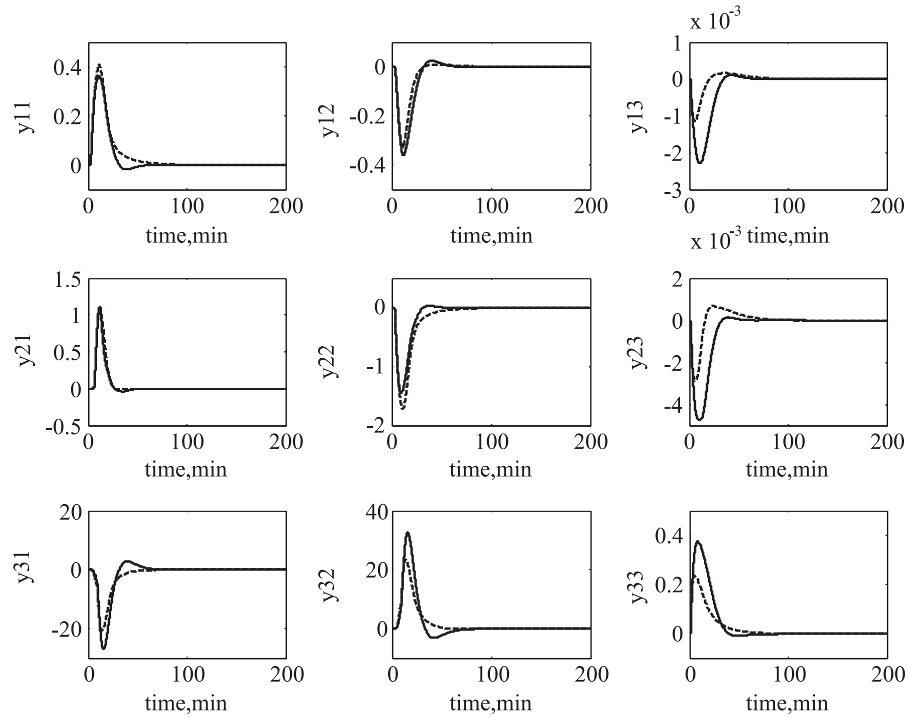

Fig. 9. Regulatory response comparison for example-3. Solid line: present method with $\delta_1=0.5$ and $\delta_2=0.125$; dotted line—Xiong et al. [25].

that the method can be recommended for interacting multivariable systems.

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