

## A CORRECTION TO "PARTIAL DERIVATIVE OF LOVE NUMBERS"

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### Abstract

*A small error in the computations of Okubo and Saito has been corrected. The correction removes certain previously peculiar results and slightly affects partials within the inner core.*

Okubo and Saito (1983) (subsequently referred to as OS) presented partial derivative of Love numbers with respect to the earth's structural parameters about three independent sets of solutions (tide, load, shear) for model 1066A (Gilbert and Dziewonski 1975). Recently, one of us discovered a coding error in the program used in their work. The error occurred in the calculation of  $y$ -values ( $y_i(r)$ ,  $i = 1, 2, \dots, 6$ ) within the inner core. So the partial derivative which is related to  $y$ -values within the inner core should be revised. In the mantle and outer core, the  $y$ -values are correct and the partials there are not affected. The most evident effect of the correction is with reference to the delta-function parts of the partials at the bottom of the liquid core which are given in *Table 1*. The original *Table 5* of OS showed that the partials were zero except those of  $h_2$  and  $k_2$  of a tide solution. It was rather strange and led to the error's discovery.

The delta-function parts of the partials at the bottom of the liquid core are determined by (1)  $y_2$  (a radial function for normal stress) of three independent solutions, (2) density, and (3) gravity at the bottom of the liquid core. Because OS equated  $y_2$  at the bottom of the liquid core with that at the top of inner core, the partials are severely distorted. From *Table 1*, partials other than those of  $h_2$  and  $k_2$  of a tide solution are one or two orders of magnitudes smaller than those. That is why the error escaped notice through the numerical tests. We calculate the Love number changes due to density perturbation at the bottom of the liquid core in two ways. One is obtained from direct subtractions of the Love numbers for the original 1066A model and the perturbed 1066A model. The other is obtained by integrating partials. These two methods give identical results within first-order infinitesimal errors as shown in *Table 2*.

The partials excluding the delta-function parts also should be corrected in the inner core. But the effects were very small and it is not necessary to revise *Fig. 1* of OS. It is due to the fact that Love number changes due to perturbation of the structure of the inner core come mainly from the surface gravity change which stems from density perturbation within the inner core, while contributions from  $y$ -variations due to perturbation of the structural parameters there are much smaller.

Table 1

Coefficient of  $\delta(r - c^+)$  in the normalized partials for model 1066A

	$\rho \frac{\partial H_2}{\partial \rho}$	$\rho \frac{\partial L_2}{\partial \rho}$	$\rho \frac{\partial K_2}{\partial \rho}$
Tide	6.116E*4	5.538E-5	6.919E-4
Load	7.098E-5	6.427E-6	8.029E-5
Shear	4.895E-5	4.433E-6	5.538E-5

\* E-n means  $10^{-n}$  $r = c^+$  designates the bottom of liquid core.

Table 2

Relative discrepancy of Love number changes due to density perturbation at the bottom of liquid core by 10 % in two ways ; (I) direct subtraction of Love numbers (II) integration of partials.  $\Delta H_2$  denotes  $(\Delta h_2^I - \Delta h_2^{II})/\Delta h_2^I$  and this rule holds for other Love numbers

	$\Delta H_2$	$\Delta L_2$	$\Delta K_2$
Tide	7E-2	7E-2	7E-2
Load	7E-2	7E-2	7E-2
Shear	7E-2	8E-2	7E-2

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## REFERENCES

- F. GILBERT and A.M. DZIEWONSKI : An application of normal mode theory to the retrieval of structural parameters and source mechanisms from seismic spectra, *Phil. Trans. Roy. Soc. London A*, 278, pp. 187-269, 1975.
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