

## **ANALYSIS OF COUPLING BETWEEN TWO PARALLEL DIELECTRIC WAVEGUIDES**

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### **ABSTRACT**

The coupling between two parallel dielectric waveguides with a finite coupling length is investigated. The ends of the waveguides are tapered in order to reduce the scattering losses. It is shown that the taper sections can be utilized to achieve an effective coupling which is insensitive to the coupling length, thus, providing a much greater tolerance for the design and manufacturing of coupling devices.

### **INTRODUCTION**

We present here an analysis of the coupling between two parallel dielectric waveguides which overlap over a finite length. Such a structure has been employed for the design of directional couplers for millimeter

wave and integrated optical applications. Because of the finite coupling length, the presence of the discontinuities at the ends of the waveguides are inevitable and may cause considerable scattering losses as the energy is coupled from one waveguide to the other. Therefore, it is usually necessary to taper the ends of the waveguides to provide gradual transitions, so that the scattering losses may be reduced. The taper regions form two sections of varying coupling, and their effect to the effective coupling length remains to be assessed. The transition regions belong to the general class of nonuniform dielectric waveguides which have been previously studied. The purpose of this work is to quantify the scattering of waves by the nonuniform dielectric waveguides, with an aim at developing accurate criteria for the design of directional couplers.

The scattering of surface waves by nonuniform dielectric waveguides is not amenable to an exact analysis, even for simple geometrical profiles of the structure. Therefore, it is necessary to resort to approximate analysis. In this paper, we apply the method of staircase approximation, which has been previously employed for the analysis of general nonuniform dielectric waveguides.

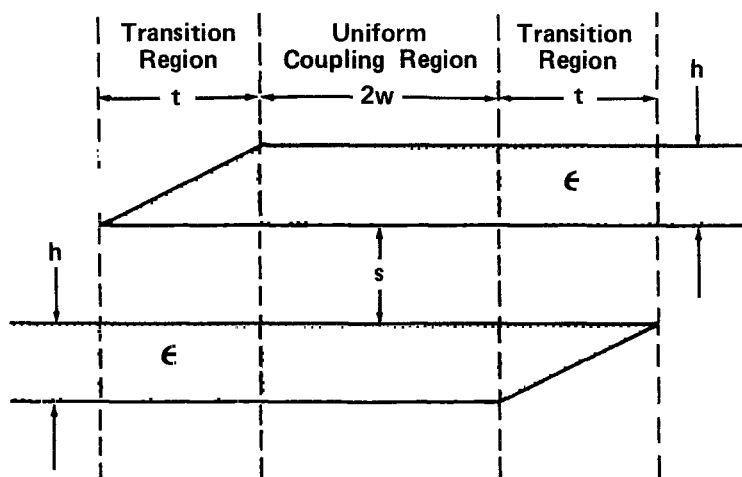


Fig. 1 Configuration of the coupler

## METHOD OF ANALYSIS

Fig.1 shows two parallel dielectric waveguides which are separated at a distance,  $s$ , and overlap over a finite length,  $w$ . Each of the two waveguides will be referred to as a basic waveguide, for simplicity; it consists of a uniform dielectric slab and a tapered end. The basic waveguide has a relative dielectric constant,  $\epsilon$ , and a height,  $h$ . The space surrounding the waveguides is assumed to be air and its relative dielectric constant is set to be unity. For the present study, the special case of two identical waveguides is considered, although the employed method of analysis is applicable to structures of more general configuration.

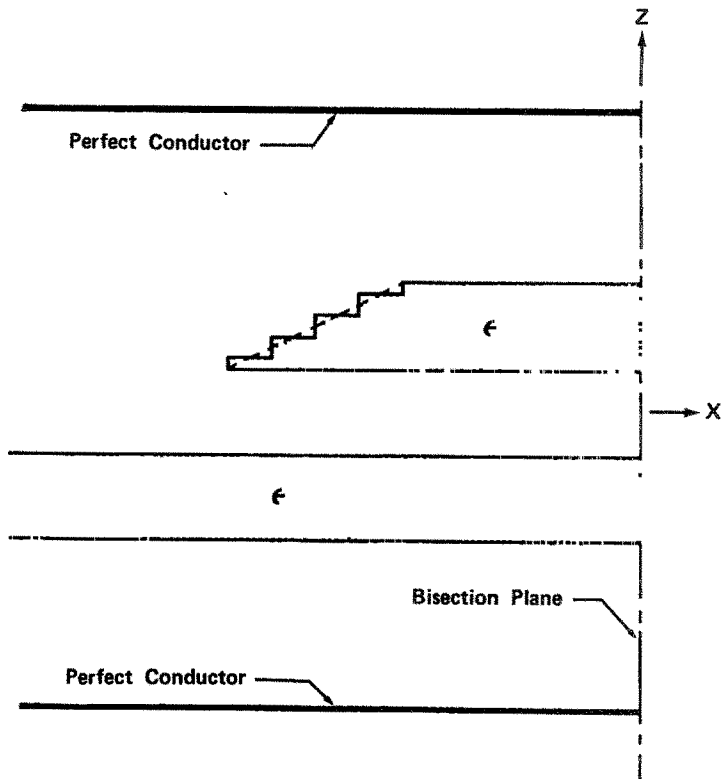


Fig. 2 Bisected coupler with staircase approximation

Such a special case possesses an interesting symmetry property, such that the structure remains invariant under the combined reflections with respect to the horizontal and vertical axes. With this symmetry property, only one half of the structure, with appropriate boundary conditions, needs to be considered and the analysis of wave scattering by the structure becomes simpler, as will be explained in detail later.

### 1. Equivalent network

In a nonuniform waveguide, the propagation of waves are quite complicated, because there exist multiple internal reflections and radiation caused by the nonuniformity. To analyze such a problem, we employ here the method of staircase approximation that replaces a continuous profile by a piecewise constant one, as shown in Fig.2, for one half of the structure. In addition, the entire structure is enclosed in an oversized parallel plate waveguide, so that the modal spectrum in each uniform region is discretized to facilitate the analysis. It is noted that the oversize waveguide must be arranged in such a way that the original symmetry of the structure is preserved. Thus, the structure can now be viewed as consisting of a series of uniform waveguides in junction. Each uniform region may be viewed as a partially filled parallel plate waveguide and the modes of such a waveguide are well known. Thus, the wave scattering by a nonuniform waveguide can be analyzed in terms of the scatterings of waves by the discrete discontinuities. In this approach, though approximate, the phenomenon of multiple internal reflections is accounted for by the multiple scatterings of waves by the series of discontinuities, and the radiation phenomenon is represented by the couplings, at the discontinuities, to higher order modes that carry away energy from the structure.

The electromagnetic fields in each uniform region can be represented by the complete set of waveguide modes for each region and are then required to satisfy the boundary conditions at each junction in the structure. This is known as the method of mode matching for a junction discontinuity. Since each mode in a uniform waveguide propagates independently, it can be represented by an independent transmission

line. The modes of two adjacent waveguides are coupled at the junction discontinuity. Such mode coupling had been characterized by an equivalent transformer that connects the transmission lines representing the modes on both sides of the discontinuity; a basic equivalent network, as depicted in Fig. 3, had been developed

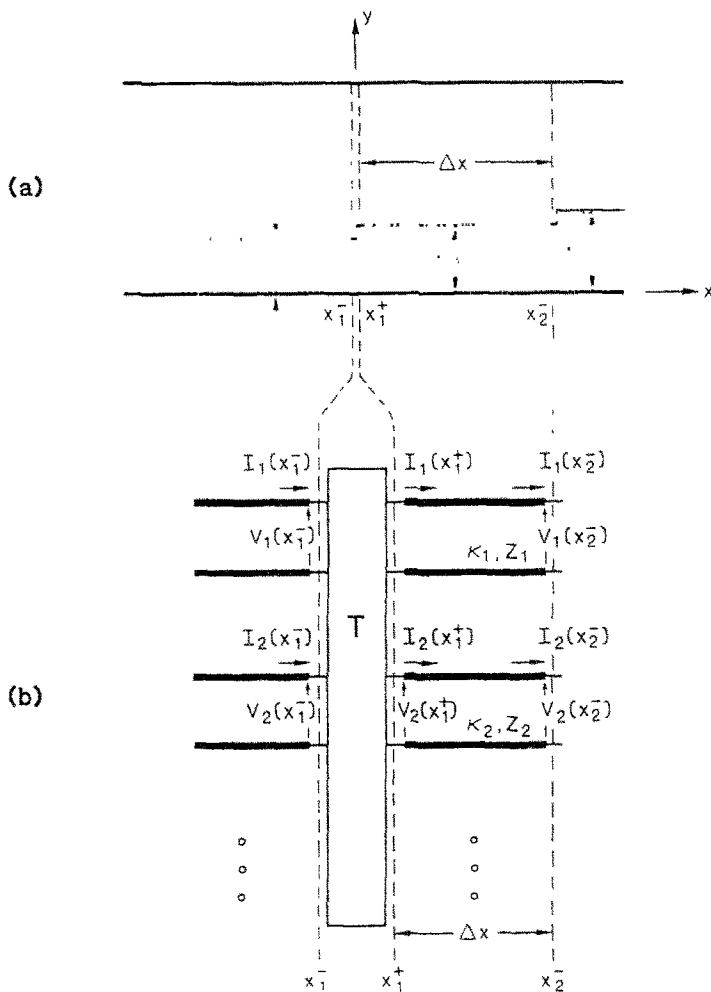


Fig. 3 Nonuniform dielectric waveguide with a piecewise constant profile.

(a) Basic unit. (b) Equivalent network

for a junction discontinuity between two different uniform dielectric waveguides.[1,2,3] The equivalent networks, each representing a step discontinuity and a uniform waveguide, can be put in cascade to form an overall network for the entire transition region. The scattering of surface waves by the nonuniform waveguide section can then be analyzed in terms of the cascaded network.

## 2. Symmetry and boundary conditions

The structure shown in Fig.1 possesses a symmetry property that remains invariant under the combined reflections with respect to the horizontal and vertical axes. In particular, the uniform coupling region is invariant under the reflection with respect to the horizontal axis alone. Such a simple reflection symmetry guarantees that the mode functions of the coupling region can be divided into two subsets: one is symmetric and the other antisymmetric; explicitly, these two subsets of mode functions satisfy, respectively, the conditions:

$$Q'_n(-z) = Q'_n(z) \quad (1)$$

$$Q''_n(-z) = -Q''_n(z) \quad (2)$$

for every integer  $n = 1, 2, 3, \dots$ . These two subsets of mode functions together form a complete set of mode functions which can be utilized for representing the fields in the region.

Referring to Fig.2, the  $y$  and  $z$  component of the electromagnetic fields in the uniform coupling region are given by:

$$E_y(x, z) = a \sum_n [Q'_n(z) V'_n(x) + Q''_n(z) V''_n(x)] \quad (3)$$

$$H_z(x, z) = a \sum_n [Q'_n(z) I'_n(x) + Q''_n(z) I''_n(x)] \quad (4)$$

On the other hand, if the incident wave is from the right, with the amplitude  $b$ , we have

$$E_y(x, z) = b \sum_n [Q'_n(z) V'_n(-x) - Q''_n(z) V''_n(-x)] \quad (5)$$

$$H_z(x, z) = -b \sum_n [Q'_n(z) I'_n(-x) - Q''_n(z) I''_n(-x)] \quad (6)$$

In general, when two waves of the amplitudes,  $a$  and  $b$ , are incident from both sides, the transverse components of the fields are:

$$E_y(x,z) = (a+b) \sum_n Q_n'(z) V_n'(x) + (a-b) \sum_n Q_n''(z) V_n''(x) \quad (7)$$

$$H_z(x,z) = (a-b) \sum_n Q_n'(z) I_n'(x) - (a+b) \sum_n Q_n''(z) I_n''(x) \quad (8)$$

For a symmetrical excitation,  $a=1/2$  and  $b=1/2$ , the fields at the bisection plane are:

$$E_y(0,z) = \sum_n V_n'(0) Q_n'(z) \quad (9)$$

$$H_z(0,z) = \sum_n I_n''(0) Q_n''(z) \quad (10)$$

Eq.(9) shows that the voltages of all the antisymmetric modes at the bisection plane  $x=0$  are identically equal to zero, whereas Eq.(10) shows that the current of all the symmetric modes at the bisection plane  $x=0$  are identically equal to zero. In other words, at the bisection plane,  $x=0$ , a symmetric mode has an open circuit termination and an antisymmetric mode has a short circuit termination. Explicitly, we may impose the boundary conditions on the voltages and currents:

$$V_n''(0) = 0 \quad (11)$$

$$I_n'(0) = 0 \quad (12)$$

for the symmetric excitation case. On the other hand, in the case of antisymmetric excitation,  $a=1/2$  and  $b=-1/2$ , (7) and (8) become:

$$E_y(0,z) = \sum_n V_n''(0) Q_n''(z) \quad (13)$$

$$H_z(0,z) = \sum_n I_n'(0) Q_n'(z) \quad (14)$$

(13) means that the voltages of the symmetric modes are identically equal to zero and so are the currents of the antisymmetric modes. In other words, we have a short circuit for a symmetric mode and an open circuit for an antisymmetric. Explicitly, the boundary conditions on the voltages and currents are:

$$V_n'(0) = 0 \quad (15)$$

$$In''(0) = 0 \quad (16)$$

these conditions permit us to consider only one half of the structure and the analysis can be thus simplified considerably.

### NUMERICAL RESULTS

Referring to Fig.1, we assume that the fundamental mode of the lower waveguide is incident from the left and we shall be interested in the coupling of power to the fundamental mode of the upper waveguide. In this paper, we shall restrict ourselves to the case of TE mode incidence. Our approach is to analyze the coupling problem in terms of the scattering of the guided wave by the nonuniform coupling region. the dimensions of dielectric waveguide are chosen such

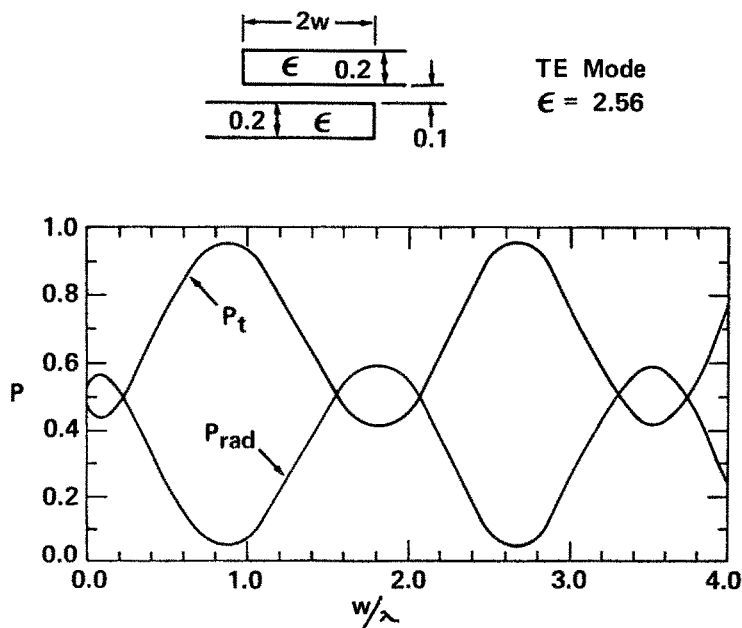


Fig. 4 Transmitted and radiated powers vs. the coupling length  $w$ . ( $s = 0.1\lambda$ )



that they support only the fundamental surface wave mode. Practically, we are also interested in the reflected power of the fundamental mode, but it is usually too small to be of significant and will be omitted in the discussion here. In the present approach, the radiated power is represented by the total power of the higher order modes of the partially filled parallel plate waveguide. Since the structure under consideration is lossless, the total power must be conserved, and such a conservation law is used as a gauge of numerical accuracy.

In order to illustrate the effect of the transition regions, we consider first the extreme case where the transition regions are absent, as depicted in the inset. Fig.4 shows the variations of the transmitted and radiated powers versus the coupling length. The incident power is normalized to unity and the transmitted power,  $P_t$ , and the radiated power,  $P_{rad}$ , depend periodically on the coupling length  $w$ . In the coupling region, the power is coupled back and forth between the waveguides; therefore, the dependence of the transmitted power from the lower waveguide to the upper one is expected to vary periodically with respect to the coupling length. In contrast, the radiation is caused by the step discontinuities at the entrance and exit ends. The radiation caused by the entrance end should be independent of the coupling, but the radiation caused by the exit end depends on the field strength at the discontinuity. When the transmitted power is at its minimum, the power is coupled back to the lower waveguide at the open end. In other words, the strong fields appear at the discontinuity, resulting in a strong radiation via the coupling to higher order modes; hence, the radiation is at its maximum, as confirmed by the results here.

From Fig.4, we observe that under the optimum condition, about 95% of the incident power is transmitted and about 5% is radiated. The reflected power is practically equal to zero. This means that the discontinuities at the two ends cause the radiation but not reflection. An explanation of such a phenomenon is that at the entrance end, the incident energy is confined in the lower waveguide and only the evanescent fields appear at the end of the upper waveguide. At

the exist end, the energy is confined in the upper waveguide and only the evanescent fields reach the end of the lower one. Consequently, the perturbations caused by the discontinuities are small, as exhibited by the results. on the other hand, the minimum transmitted power is at about 40%; and the radiated power reaches a maximum of 57%. incidently, though not shown, the reflected power is at about 3%. While the effect of the entrance discontinuity remains practically unchanged, the energy is now coupled back to the lower waveguide at its open end. In this case, the perturbation of the discontinuity is directly on the portion of strong fields. Thus, the strong radiation and reflection occur from an open end of the lower waveguide.

We have illustrated the effects of the discontinuities at the entrance and exit ends. To minimize the effect on the maximum transmission, we may increase

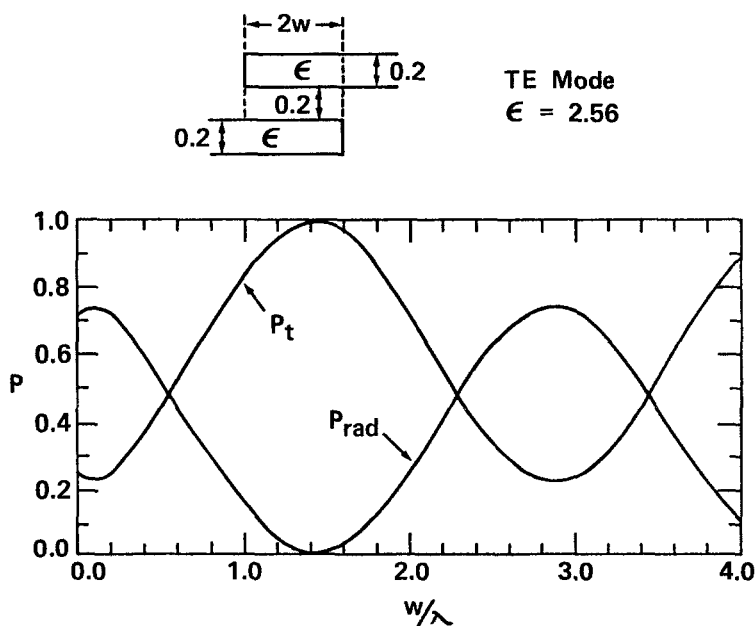


Fig. 5 Transmitted and radiated powers vs. the coupling length  $w$ . ( $s = 0.2\lambda$ )

the separation between the two waveguides. As shown in Fig.5, the maximum transmitted power can reach almost 100% of the incident power, for a larger separation distance,  $s=0.2\lambda$ , where  $\lambda$  is the free space wavelength. This means that the two end discontinuities have practically no effect on the scattering process, because the energy is almost totally confined in the uniform parts of the individual waveguides. Furthermore, the increase in the separation also increase the discontinuity at the open ends. For this larger separation, the minimum transmitted power is at about 22%, while the radiated power is increased to about 73%. This leaves the reflected power at about 5%.

Now, we introduce transition regions at the two waveguide ends. For simplicity, we consider only the case of linear taper profile of length  $t$ . Fig.6 shows the dependence of the transmitted power on the coupling length, for various transition lengths. We see that the variations become smaller, when the transition length are increased. Eventually, the transmitted power will reach almost 100%, regardless of the length of the coupling region. Alternatively,

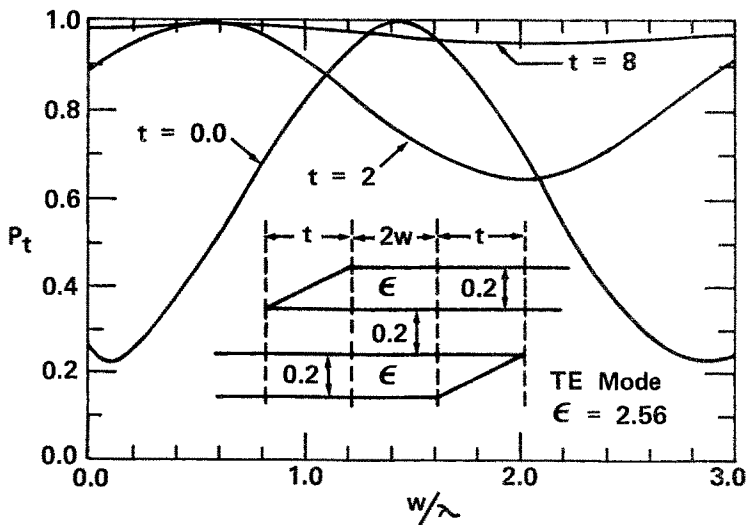


Fig. 6 Dependence of the transmitted power on the coupling length  $w$ .

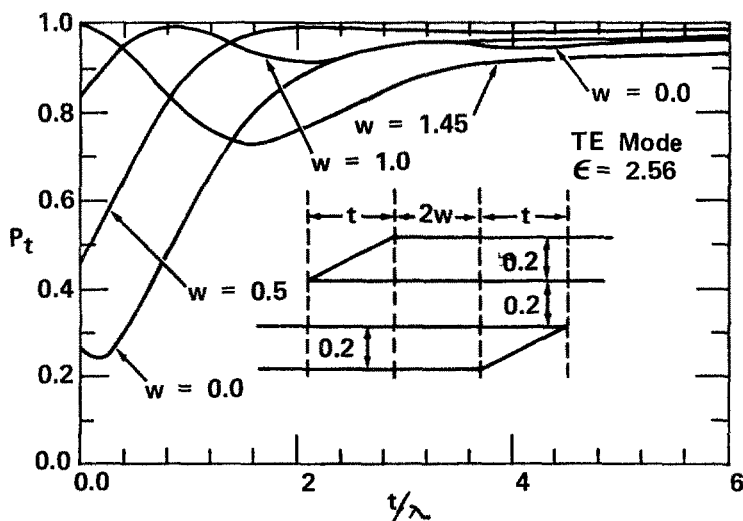


Fig. 7 Effect of tapers on transmission characteristics

Fig.7 shows the dependence of the transmitted power on the transition length  $t$ , with the coupling length  $w$  as a parameter. Evidently, with sufficient transition lengths, the transmitted power is almost independent of the coupling length. Such an insensitivity to the coupling length can afford a greater tolerance in the design and manufacturing of coupling devices.

#### REFERENCES

- [1] S. T. Peng, "Transitions in Millimeter Waveguides," SPIE Millimeter Wave Technology Conference, Arlington, VA, April 8-12, 1985.
- [2] S. T. Peng, S. J. Xu and F. K. Schwering, "Scattering of Surface Waves by Nonuniform dielectric Waveguides," IEEE MTT International Microwave Symposium Digest, PP.627-630, St.

Louis, MO, june, 1985.

- [3] S. T. Peng and A. Oliner, "Guidance and Leakage Properties of a Class of Open Dielectric Waveguides," IEEE Trans. MTT-29, pp.843-855, Sept. 1981.