Book Review

Communicated by Dr. Takacz

Marchuk GI: *Mathematical models in environmental problems.* Studies in mathematics and its applications, North Holland, Amsterdam 1986, 218 p, Dfl 140.00.

Environmental problems are becoming steadily more important and more urgent for all developed societies, and this is true independently of the respective economical system, though the economical and political background has its own special influence in this process. The apparent need to cope with this environmental problems requires more and more insight into adequate models of (parts of) the environment. Due to the complex, dynamic, non-linear character of many interesting phenomena in this field the formulation of suitable mathematical models is based on deep and elaborate theories such as system theory (partial differential equations, stochastic processes), decision theory and optimization. The present book of G. I. Marchuk has to be seen against this background. It presents a number of interesting environmental models, though special in nature (essentially deterministic diffusion models representable by systems of differential equations), and with a perspective that apparently is derived from the regulatory needs and possibilities of economics with central control.

To go more into detail, the main objective of Marchuk's monograph is the mathematical modelling of the influence of atmospheric pollution by industrial plants on neighbouring regions. Thus the author starts with a deduction of the fundamental partial differential equations describing the transport and diffusion processes that lead to the spreading of particles or noxious gas components in the air. While the pure transport process can be modelled by a first order differential equation, the inclusion of diffusion phenomena leads to parabolic partial differential equations on a cylindrical domain with non-trivial boundary conditions for the concentration of the aerosol. Provided that the air drift vector field (wind field) is known (and fixed) one may start a detailed study of the air pollution using the classical methods for solving partial differential equations.

The author explains the behaviour of the solutions by some simple examples. Since he is aware of the fact that detailed information on the air drift vector field is not available in many cases, he also studies time averages (called the *"statistical model"*). Particle fall-out as well as secondary chemical reactions of aerosols are included in the respective mathematical models. Marchuk gives some hints for the practical measurement of essential parameters for the differential equations such as diffusion coefficients. A short appendix about *mesometeorological* and *mesooceanic processes* is helpful in getting wind field information.

In the second chapter the author deduces *adjoint partial differential equations* for the original parabolic equations in a suitable duality context. It turns out that adjoint equations are not only theoretically interesting but contribute substantially to the solution of basic problems (of some – though restricted – practical importance) such as finding regions in which a pollutant emitting source is permissible under the restriction that a given upper bound for the air pollution at a given point x should not be exceeded. For readers who prefer an approach to adjoint equations via the Lagrange identity for differential operators the author shows the equality of the resulting partial differential equations with those developed by his own methods.

The major part of Marchuk's book starting from Chapter 3 is concerned with various questions concerning the solutions of the deduced partial differential equations. The author first explains the *numerical machinery* used. The basic ideas are those of *splitting theory* originally developed by G. Douglas and D. Peaceman. As a simple example consider the evolution equation

$$\frac{\partial \varphi}{\partial t} + A\varphi = f, \quad \varphi(0) = \varphi_0$$

with a positive semidefinite (differential) operator $A = A_1 + A_2$ for two positive semidefinite operators A_1, A_2. Using difference operator approximations for the differential operators and a suitable alternating Cranck-Nickolson scheme one can reduce the complexity of the original problem considerably. Marchuk explains the techniques by various examples and supplies *stability* proofs. He also shows how *finite element methods* are correlated to splitting techniques.

In Chapter 4 the author applies the mathematical tools explained before to *location problems* of pollution emittors. Given several regions of different requirements for maximal and total annual pollution he tries to determine a zone where an industrial plant can be located so that the global and local restrictions will be observed. The mathematical modelling of these questions leads to non-linear optimization problems. For sufficiently simplified special cases Marchuk shows how to provide solutions for location problems using adjoint equations.

It should be pointed out that the questions addressed here might be seen as a first step towards more sophisticated models and solution procedures. In particular, a proper incorporation of *stochastic aspects* (e.g. w.r.t. the drift) and a full modelling of a given decision situation within the framework of modern *multi-attribute decision theory* seem to be required here. Such a broader approach would also lend itself to more flexible ways to cope with pollution restrictions, characteristic for the way such problems are addressed in economic systems without strong central control.

Chapter 5 of Marchuk's book is a short detour on a (still quite special) multicriteria decision problem, viz. the minimization of environment deterioration caused by loss of populations and biomass of different kind. Again the author restricts to air pollution problems which are now modelled by systems of partial differential equations – one for each relevant aerosol component. Using cost coefficients for each component including expenditures for restoration of biological resources optimal or merely admissible locations of pollutant emittors (in the described *limited meaning* of these notions) are looked for. The author uses *graphical methods* to give solutions in simple cases.

The problem of optimizing emissions of several working industrial units with given locations under restrictions to annual maximal permissible doses of aerosols in specified regions is also considered. Here the contrast of economic and ecological effects has to be taken into account. Since the reduction of emissions is expensive in general, one can ask (as a very first step) for minimal total costs arising from pollution prevention thereby observing contamination bounds in sensitive regions. Starting again from transport and diffusion equations, Marchuk reduces a special version of this problem to linear optimization. In all these considerations, however, again stochastic aspects and general tools from multi-attribute decision theory need to be incorporated in the long run, as should be all kinds of *interdependences* arising from certain patterns of combining different working industrial units.

Another field of applications for drift and diffusion equations is finally outlined by the author. *Dispersion processes* in water lead to different boundary conditions caused by the solid shore influences. As in the case of air pollution the author outlines numerical algorithms for solving the differential equations, this time concentrating on *finite element methods*.

Undoubtably Marchuk's book is a detailed mathematical contribution to drift and diffusion problems and correlated questions in environmental protection. In this sense, its importance for getting expertise in modelling environmental phenomena has to be acknowledged. The book is also rather useful from a numerical point of view, although some questions remain open in practical applications. As indicated above already, the title of the book is, however, obviously too promising for those who expect to find a wide range of mathematical models for all kinds of ecological problems. Moreover, also for the questions treated, the need of the incorporation of additional aspects from *stochastical* and *decision theoretical modelling* seems to be obvious. Sometimes, the specifics of centrally controlled economical systems are apparent in the text, as is the almost exclusive restriction to Soviet literature (translations included). Keeping these aspects in mind the book can be recommended for scientific model builders in the field, for applied mathematicans in general and also for students with a good background in partial differential equations, optimization and numerical mathematics.

Passau *K. Donner* and *F. J. Radermacher*