Lett Spat Resour Sci (2008) 1: 37–43
DOI 10.1007/s12076-008-0004-6

ORIGINAL PAPER

Rangeland management using cell grazing: a dynamic and stochastic analysis of the optimal temporal control

Amitrajeet A. Batabyal

Received: 22 October 2007 / Accepted: 31 March 2008 / Published online: 15 April 2008

Abstract We analyze the temporal control choice problem faced by a manager who uses cell grazing to manage a parcel of rangeland. Specifically, we first delineate the relevant dynamic and stochastic features of a stylized parcel of rangeland. Next, we use a renewal theoretic framework to derive the manager's long run expected net cost (*LRENC*) of management operations per unit time. Finally, we demonstrate that the optimal temporal control is the solution to the manager's *LRENC* minimization problem.

Keywords Cell grazing · Rangeland management · Renewal theory · Temporal control

JEL Classification Q24 · D81

1 Introduction

In contemporary times, renewable natural resources such as fisheries and rangelands are routinely managed using both spatial and temporal controls. For instance, Hartwick and Olewiler (1998) have pointed out that fisheries in the United States Pacific Northwest are commonly managed with a combination of season length specifications (one kind of temporal control) and geographic regulations (one kind of spatial control). Similarly, in the context of rangeland management, Holechek et al. (2001,

A.A. Batabyal thanks the Editor Yoshiro Higano and two anonymous referees for their helpful comments on a previous version of this note. In addition, he acknowledges financial support from the Gosnell endowment at RIT. The usual disclaimer applies.

A.A. Batabyal (✉)
Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA
e-mail: aabgsh@rit.edu

pp. 254–277) have noted that specialized grazing systems such as deferred rotation grazing, high intensity-low frequency grazing, rest rotation grazing, and particularly cell grazing[1] all have spatial and temporal control aspects to them. The recent work of Batabyal (2004a) tells us that these spatial and temporal control aspects are most pronounced in the case of cell grazing.

In a cell grazing system, the manager first divides the pertinent parcel of rangeland or *cell* into a number of fenced paddocks. Next, this manager brings his herd of animals into a particular paddock to graze for a specific period of time. Finally, upon the completion of this time period, the animals are moved to the next paddock and the manager continues this process in a sequential manner. The spatial control aspect of the management problem involves choosing the number of and the areas of the individual paddocks.[2] Once this has been done, the temporal aspect of the management problem kicks in. This involves selecting the length of *time* during which the manager's animals graze the various paddocks into which the cell under study has been divided. It is particularly important to select the temporal control optimally because overgrazing bears "little relationship to the number of animals but rather to the *time* plants [are] exposed to the animals" (Savory and Butterfield 1999, p. 46, emphasis in original).

Weltz and Wood (1986), Pluhar et al. (1987), and Manley et al. (1997) have all conducted empirical analyses of cell grazing and a perusal of the rangeland management literature concerning cell grazing tells us that there are, in fact, many such empirical analyses. In sharp contrast, there are virtually *no* theoretical analyses of either the spatial control aspect or the temporal control aspect of cell grazing. Therefore, in this note, we use renewal theory[3] to conduct a dynamic and stochastic analysis of the temporal control choice problem faced by a manager who uses cell grazing to manage a parcel of rangeland. Specifically, we first delineate the relevant dynamic and stochastic features of a stylized parcel of rangeland. Next, we use a renewal theoretic framework to derive the manager's long run expected net cost (*LRENC*) of management operations per unit time. Finally, we demonstrate that the optimal temporal control is the solution to the manager's *LRENC* minimization problem.

Referring to 21st century ranchers, Holechek et al. (2001, p. 412, emphasis added) note that the management of "climatic, biological, financial, and political *risks* will probably be much more important to their success than their capacity to increase output of livestock and other products." Holechek et al. (2001, p. 567, emphasis added) go on to point out that rangeland "economics is concerned with improving efficiency, increasing equity, and managing *risk*." More recently, Batabyal (2004b, p. 2) has observed that "a crucial point about range management is that it involves decision making under *uncertainty*." These three quotations from the extant literature tell us that rangeland economics involves dealing with risk and uncertainty. In addition, they together explain why we are conducting our analysis in this note from a stochastic perspective.

[1]Cell grazing is also known as rapid rotation grazing, as short duration grazing, and as time controlled grazing. For additional details on this point, see Holechek et al. (2001, p. 269).

[2]The number of paddocks choice problem has recently been analyzed by Batabyal (2001a).

[3]See Ross (2003, pp. 401–473) and Tijms (2003, pp. 33–79) for textbook accounts of renewal theory.

The two papers that are most closely related to the analysis in this note are Batabyal (2001b, 2004c). Like this note, Batabyal (2001b, 2004c) also provides a theoretical analysis of the temporal control choice problem. However, the theoretical frameworks used in the two previous studies by Batabyal differ from the theoretical framework used in this note. In particular, in both these previous studies, Batabyal works with what he calls the "operative lifetime of a paddock" to conduct his analysis. In contrast, we use the well known Poisson process[4] to conduct our dynamic and stochastic analysis. In addition, although the optimal temporal control in Batabyal (2001b, 2004c) is difficult to characterize generally, because of our use of the Poisson process and our net cost assumptions (on which more below), we are able to not only characterize the optimal temporal control in a straightforward manner but we are also able to conduct simple comparative statics exercises with this optimal temporal control.

The rest of this note is organized as follows. First, in Sect. 2.1, we describe the renewal-reward theorem. Second, in Sect. 2.2, we use this renewal-reward theorem to derive our rangeland manager's *LRENC* of management operations per unit time. Third, in Sect. 2.3, we show that the optimal temporal control we seek is the solution to our rangeland manager's *LRENC* minimization problem. Finally, in Sect. 3, we conclude and then we discuss potential extensions of the research delineated in this note.

2 The theoretical framework

2.1 Preliminaries

The textbook by Ross (2003, pp. 416–425) tells us that a stochastic process $\{Z(t) : t \geq 0\}$ is a counting process if $Z(t)$ represents the total number of counts that have taken place by time t. Clearly, since $Z(t-1)$, $Z(t)$, $Z(t+1)$, etc. are stochastic, the time between any two counts $Z(t)$ and $Z(t-1)$ is also stochastic. This time between any two counts is called the interarrival time. A counting process for which the interarrival times have a general cumulative probability distribution function is a renewal process.

Consider a renewal process $\{Z(t) : t \geq 0\}$ with interarrival times $X_z, z \geq 1$ which have a cumulative probability distribution function $G(\bullet)$. In addition, assume that a monetary reward R_z is earned when the zth renewal is completed. Let $R(t)$, the total reward earned by time t, be $\sum_{z=1}^{Z(t)} R_z$, let $E[R_z] = E[R]$, let $E[X_z] = E[X]$, and $E[.]$ is the expectation operator. The renewal-reward theorem—see Ross (2003,

[4] A stochastic process $\{N(t) : t \geq 0\}$ is said to be a counting process if $N(t)$ represents the total number of counts or events that occur by time t. The counting process $\{N(t) : t \geq 0\}$ is said to be a Poisson process with parameter or rate $\beta > 0$ if (i) $N(0) = 0$, (ii) this process has what is known as independent increments, and (iii) the number of counts or events in any time interval of length t is Poisson distributed with mean βt. For textbook accounts of the Poisson process, the reader should consult Ross (2003, pp. 288–348) and Tijms (2003, pp. 1–32).

p. 417) or Tijms (2003, p. 41)—tells us that if $E[R]$ and $E[X]$ are finite, then with probability one,

$$\lim_{t\to\infty}\{E[R(t)]/t\} = E[R]/E[X]. \tag{1}$$

In words, (1) is telling us that if we think of a cycle being completed every time a renewal occurs, then the long run expected reward—the left-hand-side (LHS) of (1)—is the expected reward in a cycle or $E[\mathit{reward\ per\ cycle}] = E[R]$ divided by the expected amount of time it takes to complete that cycle or $E[\mathit{length\ of\ cycle}] = E[X]$. The reader should note that the renewal-reward theorem holds for positive rewards such as profits *and* for negative rewards such as costs. Let us now derive our rangeland manager's *LRENC* of management operations per unit time.

2.2 LRENC of management operations

Consider a parcel of rangeland or cell that has been divided into a number of fenced paddocks of equal grazing capacity.[5] Because the individual paddocks are of equal grazing capacity, the question of determining the optimal amount of time that the rangeland manager's animals ought to spend on any given paddock will generally be similar for all the concerned paddocks. Therefore, in what follows, we focus on the choice problem confronting our manager for a *representative* paddock.

The manager's animals enter the representative paddock in accordance with a stationary Poisson process with parameter $\beta > 0$. Once the animals have entered this paddock, they graze the paddock for a length of time denoted by T. Upon the completion of time T, the manager moves his animals to the next paddock on the cell and, as noted in Sect. 1, he continues this process in a sequential manner. We assume that the representative paddock has no animals in it at time $t = 0$.

Every time our manager moves his animals from the representative paddock, he incurs both costs and benefits that are fixed and variable in nature. The fixed costs stem from things like the cost of fencing the representative paddock. The fixed benefits arise from things like animal weight gain. We denote the fixed net costs of management operations by $F > 0$. The variable costs stem from things like the deleterious impacts of the hoof actions of the manager's animals on water infiltration on the paddock. The variable benefits flow from things like forage recovery on the paddock. We denote the variable net costs of management operations per animal by $V > 0$. Put differently, for each animal there is a variable net cost of $V > 0$ for each time unit that this animal grazes the representative paddock. Our task now is to derive the rangeland manager's *LRENC* of management operations per unit time.

To this end, note that given the way in which we have modeled the optimal temporal control choice problem, the stochastic process describing the number of animals in the representative paddock "renews" or "regenerates" itself every time the manager removes his animals from this paddock. To see this clearly, note that we are using the fact that for the stationary Poisson process, the times between successive entries of animals—the interarrival times—are exponentially distributed and hence

[5] This "equal grazing capacity" assumption has been employed previously by Batabyal (2001b, 2004c).

memoryless.[6] Let us now consider a grazing cycle to be the time interval between two successive animal removals from the representative paddock. Then, it is clear that the expected length of a grazing cycle is

$$E[\textit{length of grazing cycle}] = T. \tag{2}$$

To determine the expected net cost incurred by our manager in a grazing cycle, we will first need an expression describing the total grazing time of all the manager's animals that enter and graze the representative paddock during one cycle. Mathematically, we want the expected total grazing time in the time interval $(0, T)$. From the discussion in Ross (2003, p. 312), we infer that this expectation is

$$E[\textit{total grazing time during } (0, T)] = (\beta T^2)/2. \tag{3}$$

Equation (3) and some thought together tell us that the expected total net cost incurred by our manager in a grazing cycle is

$$E[\textit{net cost per grazing cycle}] = F + (\beta V T^2)/2. \tag{4}$$

Having computed the expected net cost incurred by our manager in a grazing cycle and the expected length of this grazing cycle, a straightforward application of the renewal-reward theorem—see (1)—gives us the manager's *LRENC* of management operations per unit time. Specifically, we get

$$\begin{aligned} \textit{LRENC} &= E[\textit{net cost per grazing cycle}]/E[\textit{length of grazing cycle}] \\ &= F/T + (\beta V T)/2, \end{aligned} \tag{5}$$

with probability one. Inspection of (5) tells us that our rangeland manager's *LRENC* depends fundamentally on the fixed and the variable net costs of management operations (F, V), the parameter of the stationary Poisson process (β), and the expected length of a grazing cycle (T). The reader will note that in the special case in which there are no fixed net costs $(F = 0)$ and the variable net costs equal unity $(V = 1)$, our rangeland manager incurs net costs at rate k when there are k animals grazing the representative paddock. In addition, in this special case, the expected net cost of management operations per unit time gives us the expected number of animals grazing the representative paddock. We now use (5) and discuss our rangeland manager's *LRENC* minimization problem.

2.3 The optimization problem

Our rangeland manager's objective is to choose the temporal control T to minimize the *LRENC* of management operations given in (5). Mathematically, the optimization problem of interest is

$$\min_{\{T \geq 0\}}[F/T + (\beta V T)/2]. \tag{6}$$

[6] See Ross (2003, pp. 272–275) and Tijms (2003, pp. 440–441) for more on the memoryless property of exponentially distributed random variables.

Differentiating this expression for the *LRENC* in (6) with respect to T and then setting the resulting derivative equal to zero gives us the optimal temporal control we seek. Specifically, we get

$$T^* = \sqrt{(2F)/\beta V}. \tag{7}$$

In words, (7) tells us that the optimal temporal control is given by the square root of the ratio of twice the net fixed costs $(2F)$ to the product of the Poisson parameter and the net variable costs (βV). To see how the optimal temporal control is affected by changes in F, β, and V, let us undertake three simple comparative statics exercises. In these exercises, we differentiate the right-hand-side (RHS) of (7) with respect to F, β, and V. We get $\partial T^*/\partial F > 0$, $\partial T^*/\partial \beta < 0$, and $\partial T^*/\partial V < 0$. This tells us that the optimal temporal control (T^*) is increasing in the net fixed costs (F) and decreasing in the Poisson process parameter (β) and in the net variable costs (V). From an economic perspective, note that while an increase in fixed costs (F) raises the optimal control, an increase in variable costs (V) lowers this same optimal temporal control. These dissimilar impacts of F, β, and V on the optimal temporal control can be explained by the different ways in which these three terms enter the expected net cost per grazing cycle expression in (4) and hence the long run expected net cost expression in (5).[7]

In this note, we used the renewal-reward theorem (1) to derive our rangeland manager's *LRENC* of management operations. The renewal-reward theorem is a *long run* result. Therefore, by choosing the temporal control in the manner described in this note, our manager will ensure that the parcel of rangeland or cell under study is used in a sustainable manner. This concludes our discussion of the rangeland manager's *LRENC* minimization problem.

3 Conclusions

In this note, we used renewal theory to analyze the way in which the temporal control ought to be chosen optimally by an individual who uses cell grazing to manage a parcel of rangeland. Specifically, from the standpoint of a rangeland manager, we first derived the *LRENC* of management operations per unit time. We then showed that the optimal temporal control is the solution to a particular minimization problem in which the objective function is the above derived *LRENC* function.

The analysis in this note can be extended in a number of directions. Here are two suggestions for extending this note's research on rangeland management using cell grazing. First, it would be useful to analyze the temporal control choice problem when the outcome of the choice of the optimal spatial control results in paddocks

[7] Note that the theoretical results of this note cannot be directly compared to the results of extant empirical studies on cell grazing. This is because in most of these extant studies that we are aware of—see Holechek et al. (2001, pp. 269–271) for one example—typically no attempt is made to determine what the temporal control optimally ought to be. Instead, a particular value of the temporal control is taken as given and then the analyst proceeds to study an issue such as water infiltration rates into the soil. Having said this, it should be clear to the reader that based on the analysis in this note, one can certainly compute a numerical value for the optimal temporal control once one is given specific numerical values of F, β, and V.

that are *not* of equal grazing capacity. Second, it would also be useful to study a scenario in which temporal controls are used jointly with controls on the *number* of grazing animals, to manage a particular cell. Studies of the optimal management of rangelands using cell grazing that incorporate these features of the problem into the analysis will provide additional insights into a management function that has salient economic and ecological implications.

References

Batabyal, A.A.: On the optimal number of paddocks in short duration grazing. J. Econ. Res. **6**, 159–169 (2001a)

Batabyal, A.A.: Some theoretical aspects of short duration grazing. Int. J. Ecol. Environ. Sci. **27**, 215–220 (2001b)

Batabyal, A.A.: Stochastic Modeling in Range Management. Nova Science Publishers, Hauppauge (2004a)

Batabyal, A.A.: Introduction. In: Stochastic Modeling in Range Management. Nova Science Publishers, Hauppauge (2004b)

Batabyal, A.A.: An approach to the choice of the optimal temporal control in short duration grazing. In: Stochastic Modeling in Range Management. Nova Science Publishers, Hauppauge (2004c)

Hartwick, J.M., Olewiler, N.D.: The Economics of Natural Resource Use, 2nd edn. Addison-Wesley, Reading (1998)

Holechek, J.L., Pieper, R.D., Herbel, C.H.: Range Management, 4th edn. Prentice-Hall, Upper Saddle River (2001)

Manley, W.A., Hart, R.H., Samuel, M.J., Smith, M.A., Waggoner, J.W. Jr., Manley, J.T.: Vegetation, cattle, and economic responses to grazing strategies and pressure. J. Range Manag. **50**, 638–646 (1997)

Pluhar, J.J., Knight, R.W., Heitschmidt, R.K.: Infiltration rates and sediment production as influenced by grazing systems in the Texas rolling plains. J. Range Manag. **40**, 240–244 (1987)

Ross, S.M.: Introduction to Probability Models, 8th edn. Academic Press, San Diego (2003)

Savory, A., Butterfield, J.: Holistic Management, 2nd edn. Island Press, Washington (1999)

Tijms, H.C.: A First Course in Stochastic Models. Wiley, Chichester (2003)

Weltz, M., Wood, M.K.: Short-duration grazing in central New Mexico: effects on infiltration rates. J. Range Manag. **39**, 365–368 (1986)