ELSEVIER

Electrical Power & Energy Systems Vol. 18, No. 1, pp. 55–64, 1996

0142-0615(95)00005-4
0142-0615/96 $15.00+0.00

Sliding mode control of a static VAR controller for synchronous generator stabilization

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The paper deals with the design and evaluation of a variable-structure stabilizer for static VAR compensators using a sliding mode control technique. The static VAR system plays an important role as a stability aid for small and large signal transient disturbances in an interconnected power system. A systematic procedure for selection of switching hyperplanes in the design of variable-structure controllers is developed by using a geometric approach known as projector theory. The sliding mode control of static VAR controllers improves the transient response of the power system and provides significant damping to the electromechanical oscillations of the synchronous generator. Computer simulation results for a typical power system shows the effectiveness of a VSS static VAR stabilizer.

Keywords: computer simulation, variable structure systems, sliding mode control, static VAR compensators

I. Introduction

Static VAR systems play an important role as a stability aid for dynamic and transient disturbances in an interconnected power system[1–3]. For any SVS scheme, the firing angle control of the thyristor banks determines the equivalent shunt admittance presented to the power system. The input signals to the SVC (static VAR controller) are usually the bus voltage incremental change along with auxiliary signals such as machine speed and bus frequency change. Another important signal for providing significant damping to the electromechanical oscillations of the generator, when the SVC is located at a bus other than the generator bus, is the phase angle deviation between the generator and the SVC bus. Recently a technique combining the bang-bang control of the phase angle loop and the linear optimal control of the voltage regulator loop has been presented in Reference 4. This method provides rapid damping for different system configurations and loading conditions.

In many cases linear control of the SVC is very effective in solving the dynamic stability problem caused by small disturbances. Damping of large power system oscillations, however, requires a much larger control effort and bang-bang control results in optimum utilization of the available SVC rating and fast damping. However, this form of discontinuous control does not guarantee the disturbance rejection and insensitivity to plant parameter variation, which are very important in the context of transient stabilization of power systems. Further, the optimization of the bang-bang controller is mathematically involved.

This paper examines another form of control for static VAR controllers for power system stabilization using sliding mode (variable structure control) theory[5–8]. Variable structure systems theory has been applied to the stabilization of a synchronous machine by Chan and Hsu[9], but this study was limited to small perturbations of the system. In this paper a variable structure SVC is designed using a fourth-order linearized system model of the synchronous machine connected to the infinite bus, and the performance of the stabilizer is evaluated on a nonlinear model under various disturbance conditions. The SVC uses a linear control for the voltage regulator loop and a sliding mode control for the auxiliary signals for damping.

The transient performance of the system is examined for two alternate locations of static VAR controller, i.e. one at the generator terminal and the other at the load bus between the sending and receiving ends. The simulation studies have shown that the sliding mode control gives

Received 20 January 1994; revised 27 September 1994; accepted 19 December 1994

significantly improved performance in terms of system damping, transient stability, and post-fault recovery of the terminal voltage.

II. Variable structure static VAR compensator

II.1 *Case I*

The system considered consists of a synchronous generator connected to a large power system through a transmission line, as shown in Figure 1. The synchronous generator is described by a fourth-order nonlinear mathematical model incorporating an exciter/AVR and a static VAR compensator at the generator bus. The VAR compensator provides damping to the electromechanical oscillations of the generator and comprises a fixed capacitor and variable inductor. The block diagram of the static VAR controller is shown in Figure 2.

The static VAR controller has two anti-parallel thyristors and the firing angles are adjusted according to the variations in terminal voltage V_t. To provide damping, the VAR controller is provided with supplementary control signals through a variable structure stabilizer using a sliding mode. The susceptance B_s presented to the synchronous generator-transmission system is thus regulated using a VSS controller.

VSS are a special class of systems characterized by a discontinuous control action which changes structure upon reaching a set of switching hyperplanes. During the sliding motion, the system has invariance properties yielding motion which is independent of system parameters and disturbances and the system behaves like a linear system. The variable structure control law using projector theory[8] is given in the Appendix.

For the VSS design, the power system is represented by a fourth-order linearized model and the state equation is written as (assuming the excitation voltage e'_q is equal to its initial value e'_{q0} and the dynamics of e'_q is neglected:

$$x = Ax + Bu \tag{1}$$

where $x = [\Delta\delta \;\; \Delta\omega \;\; \Delta E_{fd} \;\; \Delta B_s]^T$ is the state vector.

The parameters of the A and B matrices (for an operating point $P = 0.9$ p.u., $Q = 0.4$ p.u. are

$$A = \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ -14.903 & 0 & -11.785 & -8.387 \\ 1006.451 & 0 & -415.371 & -1118.279 \\ 25.161 & 0 & -9.884 & -37.957 \end{bmatrix} \tag{2}$$

$B = [0 \;\; 0 \;\; 0 \;\; 100]^T$, u = auxiliary stabilising signal for the SVC.

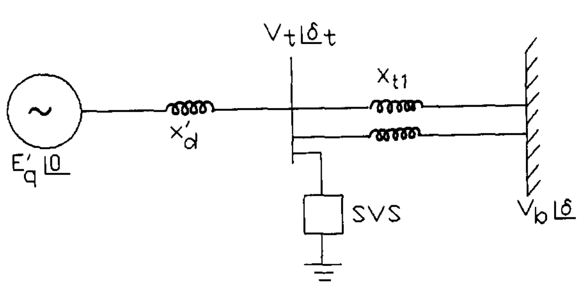

Figure 1. SVS at generator bus

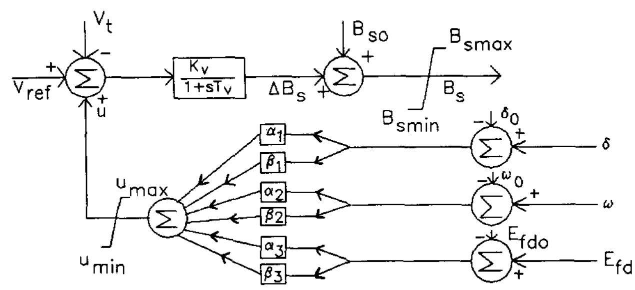

Figure 2. SVS control at generator bus (variable structure)

The design objective is to determine the switching vector C and the gains α_i and β_i using projector theory (Appendix II) and conditions for the existence of the sliding mode. The vector C for the best performance for nonlinear simulation studies is obtained by assigning eigenvalues to the A matrix. Matrix J and the row vector L are chosen as

$$J = \begin{bmatrix} -2.8 & 0 & 0 \\ 0 & -3.2 & 0 \\ 0 & 0 & -0.05 \end{bmatrix}, \quad L = [1 \;\; 1 \;\; 1] \tag{3}$$

During sliding the poles of the reduced system matrix (AI.14) is influenced by the choice of J.

The hyperplane coefficients c_1, c_2, c_3, c_4 are obtained in a vector form as outlined in Appendix II:

$$C = [-0.00914, \; -0.0000317, \; -0.00366, \; -0.01]^T$$

The VSS gains α_is and β_is are obtained from nonlinear simulation studies for the best performance (minimizing a performance index J described in Section V) and the supplementary stabilizing signal is

$$u = -\sum_{i=1}^{3} \psi_i x_i \tag{4}$$

where

$$\psi_i = \begin{cases} \alpha_i \text{ if } \sigma x_i > 0 \\ \beta_i \text{ if } \sigma x_i < 0 \end{cases} \quad i = 1, 2, 3 \tag{5}$$

and $x_1 = \Delta\delta$, $x_2 = \Delta\omega$, and $x_3 = \Delta E_{fd}$.

The following values of switching gains are obtained for the best performance

$$\begin{aligned} \alpha_1 &= -0.14203, & \beta_1 &= -0.16203, \\ \alpha_2 &= -0.005278, & \beta_2 &= -0.025278, \\ \alpha_3 &= 0.028109, & \beta_3 &= -0.008109. \end{aligned} \tag{6}$$

II.2 *Case II*

The second case considered in this paper pertains to the location of the static VAR compensator at a mid-point bus between the generator terminal and the infinite bus as shown in Figure 3. In this case, a static VAR compensator with only a voltage regulator control cannot provide significant damping to the electromechanical oscillations of the generator as the auxiliary speed feedback signal will not be available at this location. Two alternative auxiliary stabilizing signals, namely the bus voltage change and the bus phase angle with respect to the infinite bus, are found to provide significant damping. The SVC control block diagram is shown in Figure 4.

For an operating point $P = 0.9$ p.u., $Q = 0.4$ p.u. (active and reactive powers, respectively) the system matrices are

$$A = \begin{bmatrix} -100.0 & 0 & -9.017 & 0 & 2.935 & 9.315 \\ 0 & -100.0 & 97.96 & 0 & 3.673 & -.457 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -20.12 & 0 & -11.785 & -2.898 \\ 0 & 0 & 246.86 & 0 & -106.97 & -132.489 \\ -400.0 & 0 & 0 & 0 & 0 & -5.0 \end{bmatrix} \tag{7}$$

$$B = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 400]^{\mathrm{T}}$$

and the state vector is

$$x = [\Delta V_{\mathrm{s1}} \quad \Delta\delta_{\mathrm{s1}} \quad \Delta\delta \quad \Delta\omega \quad \Delta E_{\mathrm{fd}} \quad \Delta B_{\mathrm{s}}]^{\mathrm{T}} \tag{8}$$

and the control is

$$u = -\psi_1 \Delta V_{\mathrm{s1}} - \psi_2 \Delta\delta_{\mathrm{s1}} \tag{9}$$

$$\psi_1 = \begin{cases} \alpha_1, & \text{if } \sigma\Delta V_{\mathrm{s1}} > 0 \\ \beta_1, & \text{if } \sigma\Delta V_{\mathrm{s1}} < 0 \end{cases}, \quad \psi_2 = \begin{cases} \alpha_2, & \text{if } \sigma\Delta\delta_{\mathrm{s1}} > 0 \\ \beta_2, & \text{if } \sigma\Delta\delta_{\mathrm{s1}} < 0 \end{cases} \tag{10}$$

and the signals ΔV_{s1} and $\Delta\delta_{\mathrm{s1}}$ (changes in the load bus filtered voltage and angle) are obtained as

$$\Delta V_{\mathrm{s1}} = \frac{\Delta V_{\mathrm{s}}}{1 + sT_f}, \quad \Delta\delta_{\mathrm{s1}} = \frac{\Delta\delta_{\mathrm{s}}}{1 + sT_f} \tag{11}$$

Using projector theory and the inequality constraints for the existence of the sliding mode the hyperplane coefficients and the switching gains are obtained as

$$C = [0.0388 \quad -0.00429 \quad -0.00378 \quad -0.000081 \quad 0.00395 \quad 0.0025]^{\mathrm{T}}$$
$$\alpha_1 = -2.007, \ \beta_1 = -2.4078,$$
$$\alpha_2 = 0.333, \ \beta_2 = -.0664. \tag{12}$$

In this example, however, the control strategy becomes a suboptimal one, as the optimal VSS requires all the state variables for feedback. At the mid-point bus, load angle, speed, and field voltage signals are not available for control and thus there is a necessity of an alternate approach for the construction of the hyperplane and sliding mode control.

III. Alternate approach

The following describes a simplified approach for the construction of the hyperplane and the conditions for the existence of the sliding modes.

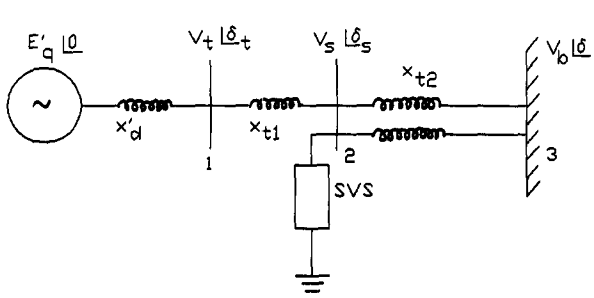

Figure 3. SVS at the mid point bus (bus no. 2)

The linearized system state equations are given by (only the measurable states like the SVS bus voltage and phase angle deviations ΔV_{s} and $\Delta\delta_{\mathrm{s}}$ respectively and the susceptance change ΔB_{s})

$$\dot{x}_1 = a_1 x_1 + a_2 x_3 + a_3 x_5 + a_4 x_6$$
$$\dot{x}_2 = a_5 x_2 + a_6 x_4 + a_7 x_5 + a_8 x_6 \tag{13}$$
$$\dot{x}_6 = a_9 x_1 + a_{10} x_6 + bu$$

The coefficients $a_1, a_2, \ldots, a_{10}$, etc. and b can be obtained from A and B matrix values given in (7), and $x_1 = \Delta V_{\mathrm{s1}}$, $x_2 = \Delta\delta_{\mathrm{s1}}$, $x_3 = \Delta\delta$,

$x_4 = \Delta\omega$, $x_5 = \Delta E_{\mathrm{fd}}$, $x_6 = \Delta B_{\mathrm{s}}$

The equation for the switching surface σ is given by

$$= c_1 x_1 + c_2 x_2 + x_6 \tag{14}$$

where the parameters c_1 and c_2 are to be designed to achieve the specified system performance. The chosen form of the sliding surface is not a unique one, being a suitable path to bring the system state to the origin of the phase plane. For example, a nonlinear combination of state variables can also be used, provided that the desired dynamic performance can be secured with the chosen form.

Existence of a sliding mode on the switching surface σ requires[7] in the vicinity of $\sigma = 0$, that

$$\sigma\dot{\sigma} < 0 \tag{15}$$

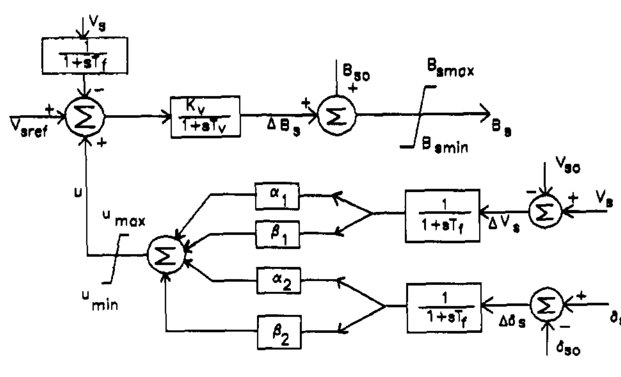

Figure 4. SVS control at a bus other than the generator bus

where $\dot\sigma$ is the time derivative of σ. The expression for $\sigma\dot\sigma$ is

$$\begin{aligned}\sigma\dot\sigma = {}& (c_1a_2 - c_1^2a_4 - c_1c_2a_8 + a_9 - a_{10} - b\psi_1)\sigma x_1 \\ & + (-c_1c_2a_4 + c_2a_5 - c_2^2a_8 - c_2a_{10} - b\psi_2)\sigma x_2 \\ & + (c_1a_2 + c_2a_6)\sigma x_3 + (c_1a_3 + c_2a_7)\sigma x_5 - bK_1|\sigma|\end{aligned} \tag{16}$$

and the control is

$$u = -\psi_1x_1 - \psi_2x_2 - K_1\,\mathrm{sgn}(\sigma)$$

From (16) ignoring the dynamics of the state x_3, we obtain

$$c_1 = -c_2\frac{a_6}{a_2} \tag{17}$$

Further for $\sigma\dot\sigma < 0$,

$$K_1 > \left[\frac{(c_1a_3 + c_2a_7)x_5\,\mathrm{sgn}(\sigma)}{b}\right] \tag{18}$$

The value of x_5 in equation (18) is taken as the upper limit of the field voltage deviation to compute the value of K_1. Hence, the feedback gain parameters are chosen as:

$$\begin{aligned}\psi_1 &= \begin{cases}\alpha_1, & \text{if } x_1\sigma > 0\\ \beta_1, & \text{if } x_1\sigma < 0\end{cases}\\ \psi_2 &= \begin{cases}\alpha_2, & \text{if } x_2\sigma > 0\\ \beta_2, & \text{if } x_2\sigma < 0\end{cases}\end{aligned} \tag{19}$$

where α_1, β_1, α_2, β_2 are obtained from (16) as

$$\begin{aligned}\alpha_1 &> \frac{1}{b}(c_1a_1 - c_1^2a_4 - c_1c_2a_8 + a_9 - a_{10})\\ \beta_1 &< \frac{1}{b}(c_1a_1 - c_1^2a_4 - c_1c_2a_8 + a_9 - a_{10})\\ \alpha_2 &> \frac{1}{b}(c_2a_5 - c_1c_2a_4 - c_2^2a_8 - a_{10}c_2)\\ \beta_2 &< \frac{1}{b}(c_2a_5 - c_1c_2a_4 - c_2^2a_8 - a_{10}c_2)\end{aligned} \tag{20}$$

Thus, if the inequality constraints in equation (20) are satisfied, the existence of a sliding mode on the chosen surface is guaranteed.

However, in order to optimize the gains α_is and β_is ($i = 1, 2$) satisfying equation (20), a performance index J (equation (22)) is used in the computer simulation. In the calculation of J, the two variables taken are c_2 and yx. yx is defined as:

$$\alpha_1(\beta_1) = \frac{1}{b}[c_1a_1 - c_1^2a_4 - c_1c_2a_8 + a_9 - a_{10}] + (-)yx$$

and,

$$\alpha_2(\beta_2) = \frac{1}{b}[c_2a_5 - c_2^2a_8 - c_1c_2a_4 - a_{10}c_2] + (-)yx$$

IV. Computer simulation results

The variable structure static VAR stabilizer presented in the preceding sections is based on a linearized model. Thus there is a need to test its effectiveness on a nonlinear synchronous generator model subjected to severe transient disturbances. The following case studies have been undertaken to evaluate the performance of the variable

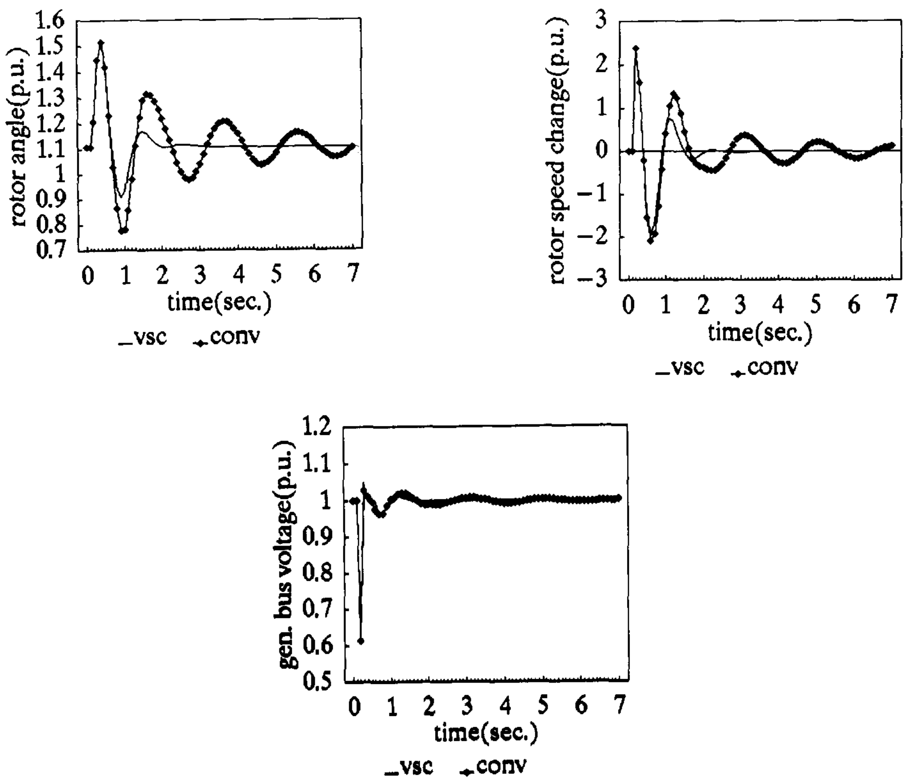

Figure 5. Comparison of transient performance for a three-phase fault at infinite bus with SVC at generator bus

structure static VAR compensator. The performances are compared with that with a conventional stabilizer described in Appendix I.

IV.1 *Case 1: SVS at the generator terminal*

A three-phase short circuit is applied at the infinite bus and the fault is cleared after 0.1 s. The transient performance of the synchronous generator for an operating point $P = 0.9$ p.u., $Q = 0.4$ p.u. for this type of fault is shown in Figure 5. The transient oscillations in rotor angle and speed exhibit slow damping with the conventional stabilizer. In the case of variable structure supplementary stabilizing input, the rotor angle oscillations, speed and terminal voltage are damped very fast in the case of a three-phase short-circuit at infinite bus.

A similar pattern of damped oscillations is seen in the case of a sudden change of turbine input power of 20% (Figure 6).

A disturbance of 5% change in the reference voltage of the variable structure stabilizer is applied for 0.5 s and the transient response of the power system is shown in Figure 7. The performance of the saturating control (obtained from the variable structure system theory) is

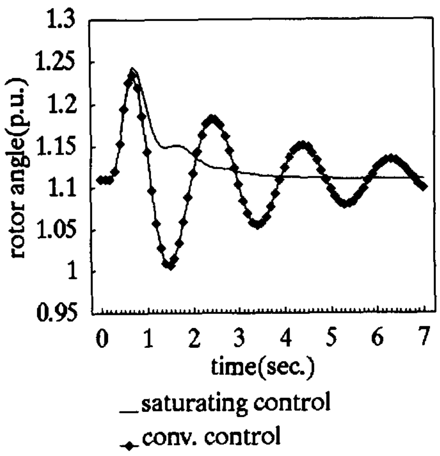

Figure 7. Comparison of saturating control and the conventional control for 5% change in V_{ref} with SVC at generator bus ($\epsilon = 0.001$, $K = 0.0001$)

found to be superior to the transient response with conventional control.

To test the robustness of the variable structure control, a 20% increase in the magnitude of the voltage regulator gain of the SVC controller is used. The transient response of the power system with nominal and increased value of voltage regulator gain shown in Figure 8 reveals the robustness of the variable structure control. Although a single case has been presented here, it suggests that the variable structure control is less sensitive to some of the plant parameter variations.

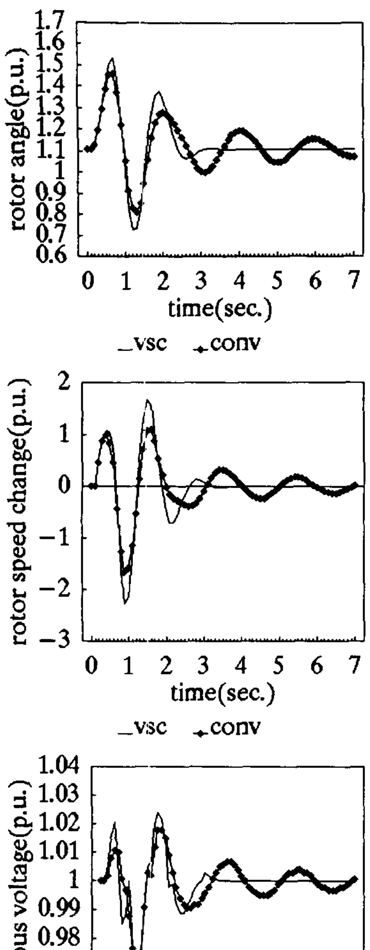

Figure 6. Comparison of transient performance for 20% increase in turbine input with SVC at generator bus

IV.2 *Case 2: SVC at the middle of the transmission line*

When the static VAR controller is located at any bus other than the generator terminal, the rotor angle and speed stabilizing signals are not available for control. In cases, the tie-line power measurements are undertaken to estimate the bus angle δ_s and its derivative. The bus angle δ_s is related to the load angle as

$$\delta_s = \delta - \tan^{-1}\{P_s/[Q_s + V_s^2/(x_d' + x_{t1})]\} \tag{21}$$

where P_s and Q_s are the active and reactive powers flowing into the SVC bus. The phase angle calculation for a realistic power system is documented in Reference 4.

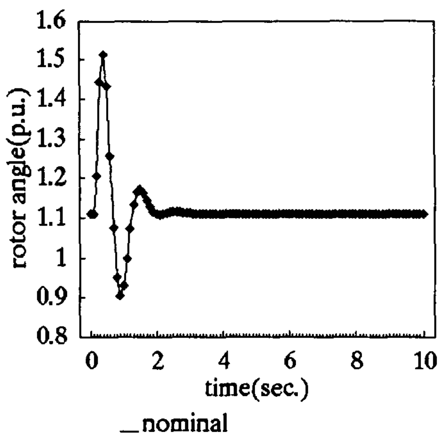

Figure 8. Test of robustness of VSS for 20% change in K_V with three-phase fault at infinite bus

IV.2.1 *Projector theory*
To find the stabilizing control u, it is required to construct the hyperplane with measurable signals like δ_s, V_s, and B_s (VAR susceptance) at the mid-point bus and the gains are obtained using projector theory as described in Appendix II.

Figure 9 shows the comparative transient performance of the conventional and variable structure auxiliary controllers for the SVC for a three-phase fault at infinite bus. It is seen from the figure that the damping of electromechanical oscillations of the generator is achieved

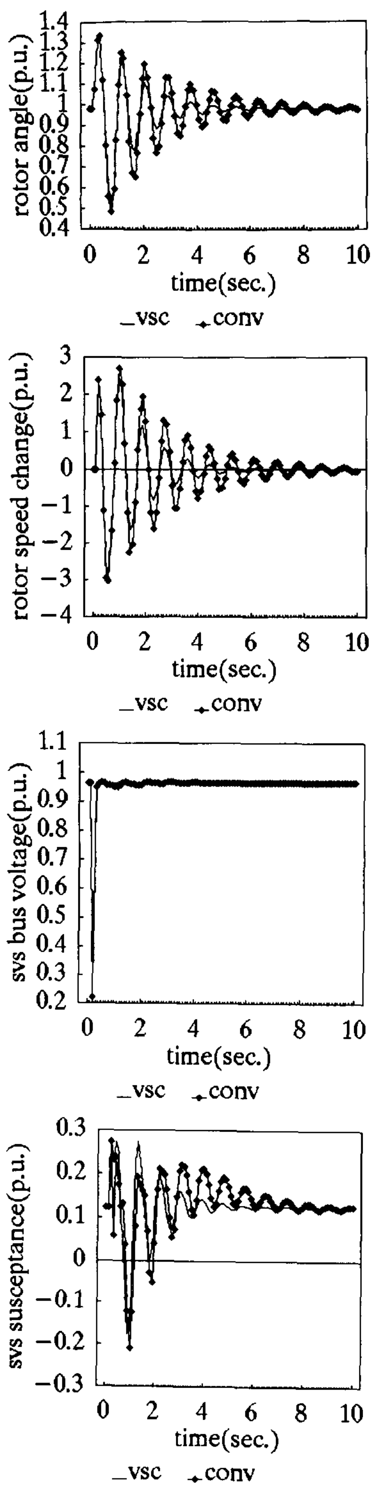

Figure 9. Comparison of transient performance for the three-phase fault at infinite bus with SVC at mid-point bus (projector theory)

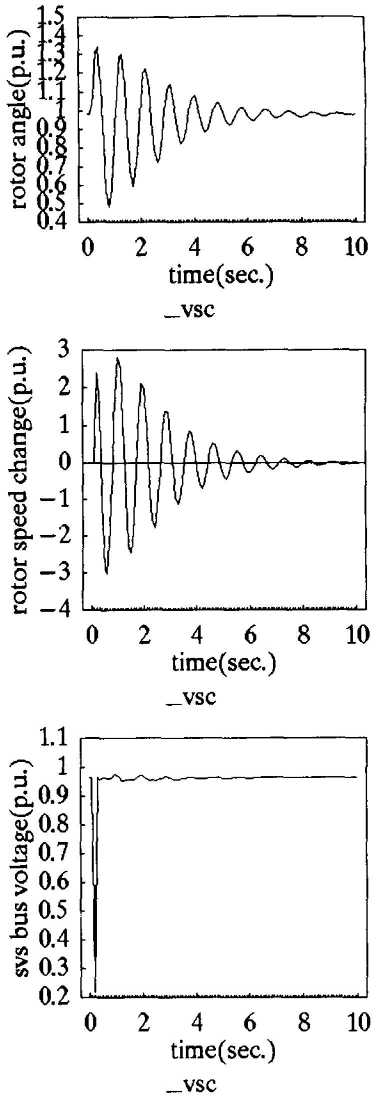

Figure 10. Transient performance for a three-phase fault at infinite bus with SVC at mid-point bus (alternate approach)

within 5 s which is faster than the conventional stabilizer at the mid-point bus.

IV.2.2 *Alternate approach*
As mentioned earlier in Section IV.2.1, the variable structure supplementary stabilizer is obtained using a simplified approach for calculating the hyperplane coefficients c_1, $c_2, \ldots,$etc. Using this simplified approach, the sliding mode control of the SVS (static VAR system) is obtained and its performance is shown in Figures 10 and 11 for a three-phase fault and 50% line switching, respectively. A comparison of Figures 9 and 10 indicates that both the approaches (projector theory and alternate approach) to obtain the control law yield almost similar responses.

IV.2.3 *Optimization of the gains α and β of the VSS*
It is extremely important to obtain optimum values of α and β so that the sliding mode control of the VAR stabilizer provides the best dynamic performance. The integral squared error technique (ISE) is used for obtaining optimum values of c_1 and c_2 (equation (14)) which are used for gain calculations considering a performance index

$$J = \int_0^{\alpha} [(\Delta V_s)^2 + (\Delta B_s)^2]\, dt \qquad (22)$$

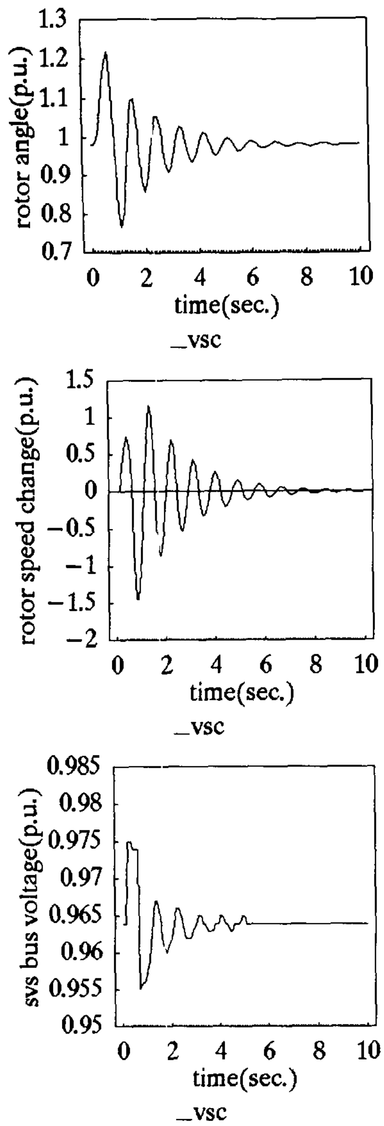

Figure 11. Transient performance for 50% line switching with SVC at mid-point bus (alternate approach)

Figure 12 shows the variation of J with yx, which is used for calculating α_1, α_2, β_1 and β_2 for different values of c_2, and c_1 is a constant multiplied by c_2. J is found to be minimum for $c_2 = 0.11$ and $yx = 0.001$ for the mid-bus SVC location.

If there is more than one SVC in the system it necessitates the construction of a hyperplane for each SVC auxiliary control signal. Further, the technique of decentralized sliding mode control can be used in the case of the SVC controller having a weak interaction with a generator in the multimachine power system. A future paper will present these results.

From Figures 10 and 11, it is quite evident that the oscillations are damped within 5 to 6 s, which is comparatively less than the results in previous works.

From the figures it is observed that the sliding mode control of the susceptance of the VAR controller is quite effective in damping the electromechanical oscillations of the generator and the voltage deviations both at the terminal of the generator and the mid-point bus. It can also be observed in Figure 6 that, though the voltage deviation is not damped well, compared with the conventional control at the initial stage, the sliding mode control subsequently takes over the conventional one, indicating

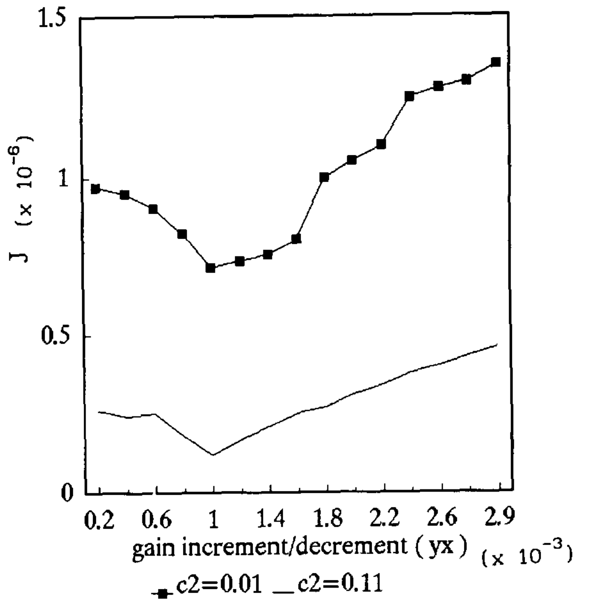

Figure 12. Performance index curve

its efficiency. This discontinuous control is different from the bang-bang one and is less sensitive to plant parameter variations and guarantees the generation of sliding mode.

V. Conclusions

A systematic approach has been presented for the design of a variable structure controller for static VAR systems operating in the sliding mode. Simulation results reveal that the VSS concept of designing a VAR controller provides significant improvement in damping and transient stability performance. The post-fault recovery of the generator terminal or SVC bus voltage is significantly faster than most of the SVC case studies presented earlier using speed stabilizing signals. An integral-square-error criterion has been used to optimize the gain settings of the SVC supplementary stabilizer. The sliding mode control of the SVC produces a robust controller and is insensitive to variations of some of the parameters of the SVC controller or the power system.

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Appendix I

Synchronous machine model

A third-order synchronous machine model is used for this study and the equations are

$$\dot{\delta} = \Delta\omega$$

$$\Delta\dot{\omega} = \frac{1}{M}[P_m - e'_q i_q - (x_q - x'_d) i_d i_q] \quad \text{(A.1)}$$

$$\dot{e}'_q = \frac{1}{T'_{d0}}(E_{fd} - e'_q - (x_d - x'_d(i_d))$$

AVR

The excitation system comprises a simple voltage regulator as

$$\dot{E}_{fd} = K_e(V_{ref} - V_t)/(1 + sT_e) \quad \text{(A.2)}$$

Static VAR compensator

The magnitude of the SVC admittance $B_s(\alpha)$ is a function of the firing angle α and is obtained as using a fixed capacitor and variable reactor combination

$$B_s = \frac{1}{X_c} - B_L(\alpha) \quad \text{(A.3)}$$

$$B_L(a) = \frac{2\pi - 2\alpha + \sin 2\alpha}{2\pi X_s}$$

for $0 \le \alpha \le \pi$, X_c = capacitive reactance of the SVC. and X_s = maximum reactance of the SVC.

The SVC differential equation is written as

$$\dot{B}_s = \frac{B_s - B_{s0}}{T_v} + \frac{K_v}{T_v}(V_{sref} - V_s) + \frac{K_v}{T_v}u \quad \text{(A.4)}$$

for the SVC at mid-bus and

$$\dot{B}_s = \frac{B_s - B_{s0}}{T_v} + \frac{K_v}{T_v}(V_{ref} - V_t) + \frac{K_v}{T_v}u$$

for the SVC at the generator bus where u = auxiliary damping control signal.

Conventional auxiliary control

$$u = K_s\left[\frac{sT_\omega}{1 + sT_\omega}\right]\left[\frac{1 + sT_1}{1 + sT_2}\right]u_s \quad \text{(A.5)}$$

where $u_s = \Delta\omega$ for the SVC at the generator bus, and $u_s = \Delta\delta_s$, for the SVC at the mid-point bus of the transmission line.

The parameters a_1 to a_{10} in equation (13) for the power system with the SVC located at the midpoint bus are:

$$a_1 = -1/T_f$$

$$a_2 = A_6/T_f$$

$$a_3 = A_7/T_f$$

$$a_4 = A_8/T_f$$

$$a_5 = -1/T_f$$

$$a_6 = G_4/T_f$$

$$a_7 = G_5/T_f$$

$$a_8 = G_6/T_f$$

$$a_9 = -K_v/T_v$$

$$a_{10} = -1/T_v$$

$$b = K_v/T_v$$

$$A_6 = \frac{V_{sd0}D_8}{V_{s0}} + \frac{V_{sq0}A_3}{V_{s0}}$$

$$A_7 = \frac{V_{sq0}A_4}{V}$$

$$A_8 = \frac{V_{sd0}D_9}{V_{s0}} + \frac{V_{sq0}A_5}{V_{s0}}$$

$$G_4 = \frac{1}{\sec^2\delta_{s0}}(G_1 + G_2D_1 + G_3D_3)$$

$$G_5 = \frac{1}{\sec^2\delta_{s0}}G_3D_2$$

$$G_6 = \frac{1}{\sec^2\delta_{s0}}(G_2A_2 + G_3D_4)$$

$$A_3 = \frac{-V_{b0}\sin\delta_0}{1 - x_{t2}B_{s0}} + \frac{x_{t2}D_3}{1 - x_{t2}B_{s0}}$$

$$A_4 = \frac{x_{t2}D_2}{1 - x_{t2}B_{s0}}$$

$$A_5 = \frac{x_{t2}V_{b0}\cos\delta_0 + X_{t2}^2 i_{d0}}{(1 - x_{t2}B_{s0})^2} + \frac{x_{t2}D_4}{1 - x_{t2}B_{s0}}$$

$$D_1 = \frac{V_{b0}\cos\delta_0}{(x_{t1} + x_d)(1 - x_{t2}B_{s0}) + x_{t2}}$$

$$D_2 = \frac{1 - x_{t2}B_{s0}}{(x_{t1} + x_d)(1 - x_{t2}B_{s0}) + x_{t2}}$$

$$D_3 = \frac{V_{b0}\sin\delta_{s0}}{(x_{t1} + x_d)(1 - x_{t2}B_{s0}) + x_{t2}}$$

$$D_4 = \frac{x_{t2}(x_1 + x_2)[E_{fd0}(1 - x_{t2}B_{s0}) - V_{b0}\cos\delta_0]}{[(x_{t1} + x_d)(1 - x_{t2}B_{s0}) + x_{t2}]^2} - \frac{x_{t2}E_{fd0}}{(x_{t1} + x_d)(1 - x_{t2}B_{s0}) + x_{t2}}$$

$$A_2 = \frac{V_{b0}\sin\delta_0(x_{t1} + x_q)x_{t2}}{[(x_{t1} + x_q)(1 - x_{t2}B_{s0}) + x_{t2}^2]}$$

$$D_8 = \frac{V_{b0}\cos\delta_0}{1 - x_{t2}B_{s0}} - \frac{x_{t2}D_1}{1 - x_{t2}B_{s0}}$$

$$D_9 = \frac{X_{t2}(V_{b0}\sin\delta_0 - x_{t2}i_{q0})}{(1 - x_{t2}B_{s0})^2} - \frac{x_{t2}A_2}{1 - x_{t2}B_{s0}}$$

$$G_1 = \frac{V_{b0}\cos\delta_0}{V_{b0}\cos\delta_0 + x_{t2}i_{d0}} + \frac{(V_{b0}\sin\delta_0 - x_{t2}i_{q0})V_{b0}\sin\delta_0}{(V_{b0}\cos\delta_0 + x_{t2}i_{d0})^2}$$

$$G_2 = -\frac{x_{t2}}{V_{b0}\cos\delta_0 + x_{t2}i_{d0}}$$

$$G_3 = \frac{(V_{b0}\sin\delta_0 - x_{t2} - i_{q0})x_{t2}}{(V_{b0}\cos\delta_0 + x_{t2}i_{d0})^2} \quad \text{(A.6)}$$

The suffix '0' denotes the initial value of the quantity.

Appendix II

AII.1 *Design of variable structure systems in the sliding mode*

Variable structure systems (VSS) are a special class of nonlinear systems characterized by a discontinuous control action which changes structure upon reaching a set of switching hyperplanes.

For a linear time-invariant system:

$$x = Ax + Bu \tag{A.7}$$

where x is an $n \times 1$ state vector, u is a scalar control input, A an $n \times n$ matrix and B an $n \times 1$ vector, the switching hyperplane is specified by the equation

$$\sigma = C^{\mathrm{T}}x = 0 \tag{A.8}$$

where C is a constant vector of appropriate dimensions. The control has the form

$$u = -\psi^{\mathrm{T}}x = -\sum_{i=1}^{n-1} \psi_i x_i \tag{A.9}$$

where each element of ψ^{T} is changed according to a preassigned rule

$$\psi_\iota = \begin{cases} \alpha_i \text{ if } \sigma x_i > 0 \\ \beta_i \text{ if } \sigma x_i < 0 \end{cases} \tag{A.10}$$

The condition for the reachability of the sliding mode is

$$\sigma\dot{\sigma} < 0 \tag{A.11}$$

for all x in the neighbourhood of the hyperplane, where $\dot{\sigma}$ is the time-derivative of σ. In the sliding mode, the system satisfies equations

$$\sigma = 0, \ \dot{\sigma} = 0 \tag{A.12}$$

The conditions for the existence of a sliding mode on the switching hyperplane $\sigma = 0$ are

$$\alpha_i > \frac{1}{B^{\mathrm{T}}C}[a_i c_i - c_n^{-1} c_i a_n^{\mathrm{T}} C], \quad i = 1, 2, \ldots, n-1 \tag{A.13}$$

$$\beta_i < \frac{1}{B^{\mathrm{T}}C}[a_i c_i - c_n^{-1} c_i a_n^{\mathrm{T}} C]$$

where a_i is the ith column of the A matrix. Sliding motion can be obtained by suitably choosing α_i and β_i from the inequality constraints (A.13).

The system dynamics in the sliding mode are given by (with $\sigma = 0$)

$$\dot{x} = A_{\mathrm{eq}}x \tag{A.14}$$

where $A_{\mathrm{eq}} = [I - B(C^{\mathrm{T}}B)^{-1}C^{\mathrm{T}}]A$.

The switching vector C is to be chosen so that A has m zero-valued eigenvalues and $(n-m)$ eigenvalues specified by the designer. During sliding the equivalent system matrix, A_{eq}, is of reduced order.

By applying projector theory[8] to the variable-structure systems in the sliding mode, the first row of $[B \ \ W]^{-1}$ can be taken as C^{T}, where W is a closed-loop eigenvector matrix. The eigenvector matrix W is obtained by solving the equation

$$AW - WJ = BL \tag{A.15}$$

where L is an arbitrary $m \times (n-m)$ matrix chosen to provide linear combinations of the columns of B. The matrix J depends on the assigned eigenvalues.

The matrix W satisfies

$$(A + BK')W = WJ \tag{A.16}$$

and K' is a state feedback matrix.

The generalized inverse B^{g} is obtained as

$$[B|W]'^{-1} = \begin{bmatrix} B^{\mathrm{g}} \\ W^{\mathrm{g}} \end{bmatrix} \tag{A.17}$$

where $B^{\mathrm{g}}B = I_m$, $W^{\mathrm{g}}W = I_{n-m}$.

The hyperplane matrix C is obtained as $C = NB^{\mathrm{g}}$, where N is an $m \times m$ matrix selected as I_m. A discussion of the sliding mode properties of parameter insensitivity and disturbance rejection will conclude this section. For ease of exposition, consider the presence of a disturbance f so that

$$\dot{x} = Ax + Bu + Df \tag{A.18}$$

where D is a compatibly dimensioned matrix. Differentiating the sliding mode condition (A.8), the value of $\dot{\sigma}$ becomes

$$\dot{\sigma} = C^{\mathrm{T}}\dot{x} = C^{\mathrm{T}}[Ax + Bu + Df] \tag{A.19}$$

For the reaching law $\sigma\dot{\sigma} < 0$ to be satisfied

$$C^{\mathrm{T}}[Ax + Bu + Df] = -K\,\mathrm{sgn}(\sigma) \tag{A.20}$$

Hence the value of u is obtained as

$$u = -(C^{\mathrm{T}}B)^{-1}C^{\mathrm{T}}(Ax + Df) - (C^{\mathrm{T}}B)^{-1}K\,\mathrm{sgn}(\sigma) \tag{A.21}$$

and $K > 0$.

Substituting the control effort into the system equation (A.18)

$$\dot{x} = [I_n - B(C^{\mathrm{T}}B)^{-1}C^{\mathrm{T}}][AX + Df] - B(C^{\mathrm{T}}B)^{-1}K\,\mathrm{sgn}(\sigma) \tag{A.22}$$

where I_n is an $n \times n$ unit matrix.

From equation (A.22), it is evident that the disturbances influence the sliding motion of the state trajectory. To make the sliding motion of the system completely insensitive to the uncertain disturbances, the following equation is to be satisfied:

$$[I_n - B(C^{\mathrm{T}}B)^{-1}C^{\mathrm{T}}]Df = 0 \tag{A.23}$$

If the columns of D are linear combinations of the input matrix B, then

$$Df = Be$$

for suitably chosen e.

The value of $\mathrm{sgn}(\sigma)$ is given by

$$\mathrm{sgn}(\sigma) = \begin{cases} 1 \text{ if } \sigma > 0 \\ -1 \text{ if } \sigma < 0 \\ 0 \text{ if } \sigma = 0 \end{cases} \tag{A.24}$$

The sgn function can be further replaced by a saturating function to reduce the chatter when the control is switched:

$$\mathrm{sat}(\sigma) = \begin{cases} \mathrm{sgn}(\sigma) \text{ if } |\sigma| > \epsilon \\ |\sigma|/\epsilon \text{ if } |\sigma| \le \epsilon \end{cases} \tag{A.25}$$

and ϵ is the width of the boundary layer to be suitably chosen by the designer.

Appendix III

AIII.1 *System data*

SVS at the generator terminal

$x_d = 2.0$ p.u., $x_q = 2.0$ p.u., $x_d' = 0.271$ p.u., $x_{t1} = 0.325$ p.u., $B_{s0} = 0.0$ p.u., $K_v = 10.0$ p.u., $T_v = 0.1$ s, $K_e = 200.0$, $T_e = 0.05$ s, $B_{smax} = 0.275$ p.u., $B_{smin} = -0.275$ p.u., $M = 0.03$ p.u., $T'_{d0} = 5.9$ s, $E_{fdmax} = 6.0$ p.u., $E_{fdmin} = -6.0$ p.u., $u_{max} = 0.1$ p.u., $u_{min} = -0.1$ p.u.

SVS at the mid-bus

$x_{t1} = x_{t2} = 0.1$ p.u., $B_{s0} = 0.125$ p.u., $B_{smax} = 0.275$ p.u., $B_{smin} = -0.275$ p.u., $K_v = 80.0$ p.u., $T_v = 0.2$ s, $T_f = 0.01$ s, $K_e = 120.0$, $T = 0.1$ s, $u_{max} = 0.01$ p.u., $u_{min} = 0.01$ p.u.

Conventional stabilizer

$K_s = 0.1$, $T_\omega = 2.0$ s, $T_1 = 0.05$ s, $T_2 = 0.5$ s.