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COMPONENT REGISTRATION IN TWO-DIMENSIONAL MODEL SEISMOLOGY

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INTRODUCTION

In model seismology piezoelectric receivers are generally used for component measurements. In 1956 Kato and Takagi [1] carried out such measurements along the edges of "two-dimensional" models, thereby simulating registration on the surface of the earth. For the component registration they placed the piezoelectric receiver into special right-angled cut-outs consecutively in two positions perpendicular to one another. Gupta and Kisslinger [2] used two capacitive detectors which were fixed perpendicular to one another in a cut-out immediately beneath the edge of the model. Strobach [3, 4] and Steinbeck [5] carried out component measurements "inside" "two-dimensional" models. They used multimorphous piezoelectric elements which they placed perpendicular to the surface of the model. Such measurements "inside" models permit a considerably deeper insight into the nature of wave propagation. Similar methods of registration were later described by Gutdeutsch and Guha [6], Sorge [7] and many other authors. Although registrations using multimorphous piezoelectric elements "inside" and on the "surface" of two-dimensional models have proved successful, the theory of this technique does not seem to have been exhaustively dealt with. The question arises, firstly, whether further information could be extracted by this method; secondly, how the results obtained from the models are transferable to natural conditions.

MEASUREMENT ON "TWO-DIMENSIONAL SEISMIC MODELS"

Two-dimensional models are plates whose thickness $2H$ must be sufficiently small compared with the used wavelength λ. Then we have approximately two-dimensional conditions of propagation which will here be defined by the premise for the generation of wave fronts of cylindrical symmetry. On an ideal two-dimensional model longitudinal plate waves can propagate with the velocity $a' = 2b\sqrt{(1 - b^2/a^2)}$, $a = \sqrt{[(\lambda + 2\mu)/\varrho]}$ and shear waves with the velocity $b = \sqrt{(\mu/\varrho)}$; μ, λ = Lamé's constants, ϱ = density. Both waves are polarized in the plane of the plate (1 – 2-plane) in Fig. 1). Their amplitudes are independent of x_3. The determination of orbital curves amounts to the measurement of two values u_1 and u_2 or of the direction of particle movement β and the horizontal amplitude $\sqrt{(u_1^2 + u_2^2)}$. With the aid of the theory of dispersion of plane waves on plates it can be shown that two-dimensional conditions of propagation according to the definition just given can only be obtained in the case of infinitely thin plates, i.e. $2H/\lambda = 0$ [8]. Although in practice $2H/\lambda$ is only small, it never, of course, vanishes. Deviations from the theory of two-dimensional models are therefore to be expected. For the models used here $2H/\lambda$ had a value of approximately 0·03. The deviations from the theory of two-dimensional wave-

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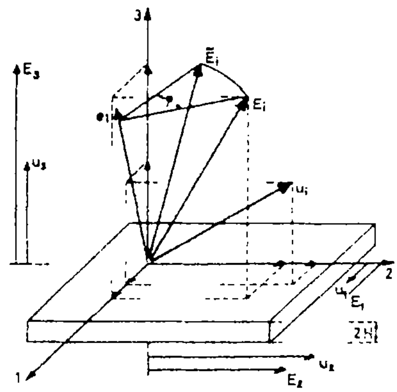
propagation which appear here can partly be estimated. Instead of the longitudinal plate wave a slightly dispersive wave of elliptical particle movement will be observed. Under the assumption that attenuation may be neglected [9] its phase velocity will be about 0·2% and its group velocity 0·3% smaller than a' [10]. The displacement vector will not lie in the 1 – 2-plane but will show a relatively small component in the 3-direction u_3. For $x_3 = \pm H$

$$(1) \qquad u_3 = 0{\cdot}36\dot{u}_h \quad (u_3 \text{ in mm}, \dot{u}_h \text{ in mm}/\mu\text{sec})$$

is approximately valid, or for sinusoidal u_h

$$(1') \qquad |u_3| = 0{\cdot}02|u_h| \,.$$

The deviations from the expected velocity a' are so small that they are negligible. The fact that u_3 does not vanish, as it actually should, but constitutes 2% of u_h must not be neglected. It must be remembered that the receiver is generally sensitive to u_h as well as u_3. The problem consists in finding a suitable method for eliminating the influence of the vertical components u_3 on the registration of the two-dimensional orbital curve $u_1(t)$, $u_2(t)$.



$2H \ll \lambda$

u_i = Vector of displacement

E_i = Vector of sensitivity

e_i = Vector of the axis of rotation

Fig. 1.

DESCRIPTION OF THE RECEIVER

The effective receiving element is a multimorphous piezoelectric barium titanate ceramic in the shape of a rectangular rod measuring 0·3 × 18·0 × 1·2 mm. It is constructed in such a way that it gives a relatively high voltage at its electrodes, particularly when being bent. For this reason it is specially suited for measurements of displacements vertical to its longest side.

Figure 2 shows the receiver with which measurements were carried out. The system consists of a tripod, two of its legs being regulating screws and the third leg being a brass tip producing the contact with the piezoelectric element. The receiving system is placed in a brass tube turnable about its axis by 360° by means of a lever. The tube contains a long brass cylinder in order to ensure the damping of the system. The lever is prolonged to form a pointer with a plexiglass scale divided into degrees attached to it. The brass mass rests on the tip in order to ensure constant load. The location of the piezoelectric element in the receiving system is shown in Fig. 3. When the system is to be predominantly sensitive to displacements vertical to its axis of rotation, the probe shown on the left can conveniently be used. The element is cast in araldite into the concentric boring of the brass cylinder so that its axis is nearly parallel to the axis of rotation of the receiving system. When the displacement perpendicular as well as parallel to the axis of rotation is to be observed, it is preferable to use the probe depicted on the right. Here the element is placed horizontally and is coupled to the model by a brass tip. The sensitivity to displacements parallel to the axis E_3 and perpendicular to the axis E_h is a function of the angle δ. It should be emphasized that the brass tip coupling the receiver to the model must stand exactly upon the axis of rotation,

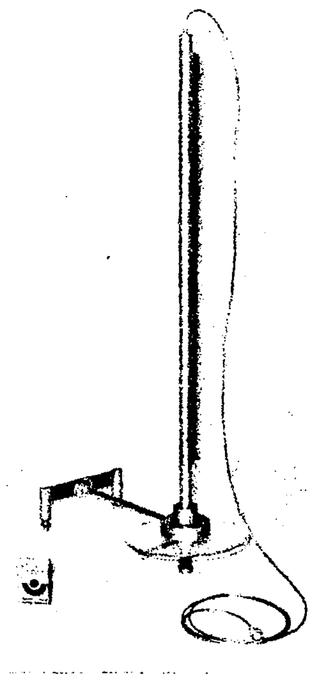


Fig. 2.

as otherwise the position of the reception will be altered by rotation. But this requirement can easily be satisfied as the probe was made of one solid piece using a lathe. Brass was chosen for the working material as the acoustic impedance between brass and barium titanate is very small. Therefore good damping is ensured. On the other hand, we have good protection against inductive disturbances. The two probes have the same diameter. They may be employed in the same receiver and can be exchanged as required. The probe shown on the left has proved much more sensitive than the one on the right.

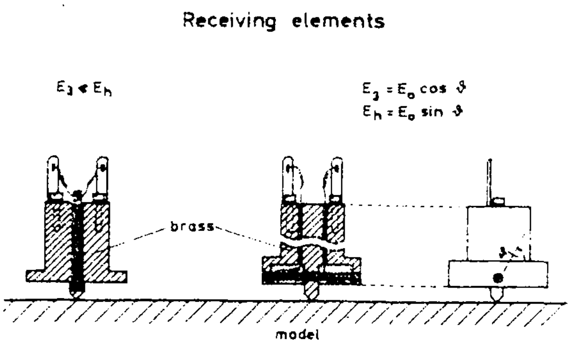


Fig. 3.

FUNDAMENTAL MEASUREMENTS

First of all it is necessary to become acquainted with the function of the receiver in the two methods of observation in use to-day. On the one hand, several authors [1, 2] place the receiver upon the edge of the model. On the other hand, some authors [3–5] place it upon the surface of the model, i.e. with the axis of the receiver in the 3-direction. The receiver described here was employed for both types of observation. Figure 4 shows the experimental set-up. The object of observation was the longitudinal plate wave in a 3 mm plate of plexiglass. The arrows with roman numbers mark the direction of the axis of rotation of the receiver in the positions concerned.

The model was placed horizontally and the receiver containing the probe shown on the left side of Fig. 3 was put consecutively into positions I and II on the surface of the model immediately at the edge of the model. Here the axis of rotation pointed into the 3-direction. The angle of rotation φ of the pointer*) against the negative 1-axis was changed at intervals of 15°.

*) The pointer has been gauged and regulated in the direction of maximum sensitivity of the bimorphous piezoelectric element.

Pos. I: The direction of greatest amplitude should theoretically be $\varphi = 30{\cdot}6°$, the angles of vanishing amplitude should be $\varphi \pm 90° = \varphi_1 = 120{\cdot}6°$ and $\varphi_2 = 300{\cdot}6°$, but $\varphi_1 = 121{\cdot}0°$ and $\varphi_2 = 299{\cdot}8°$ were measured.

Pos. II: It can be seen that the observed wave is polarized approximately in the 1-direction, i.e. $\varphi = 180°$. It should be expected that vertically to this the amplitude passes zero at the angles $\varphi_1 = 90°$ and $\varphi_2 = 270°$. These angles can be measured very accurately, namely $\varphi_1 = 90{\cdot}2°$, $\varphi_2 = 269{\cdot}6°$. Deviations from the expected value cannot be reduced to errors of measurement.

The model was held vertically and the receiver placed consecutively into positions III and IV perpendicular to the edge of the model (axis of rotation in the 1-direction).

Pos. III: The points at which the amplitudes vanish were measured as $\varphi_1 = 6{\cdot}0°$ and $\varphi_2 = 183{\cdot}0°$.

Pos. IV: In this position the rotation of the receiver has hardly any effect on the amplitude which is very small and varies from receiver to receiver.

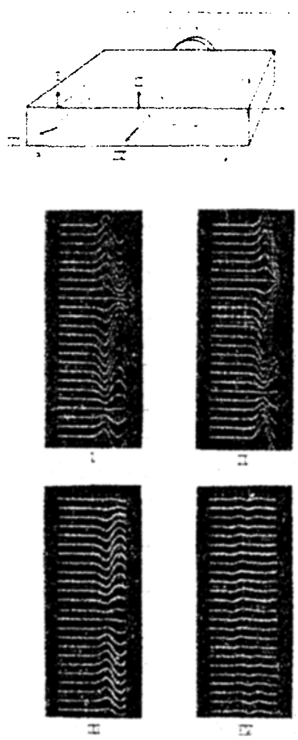


Fig. 4.

The following conclusions can be drawn from these measurements:

1. The sensitivity to movements in the direction of the receiver axis E_v is much smaller than the sensitivity E_h to movements perpendicular to it; (E_v/E_h) is much smaller than unity. The exact value differs from receiver to receiver.

2. The direction of particle movement of the observed wave is not perpendicular to the direction of the zero point of the amplitudes.

3. The measurement positions I and II are to be preferred to positions III and IV when particle movement is to be determined.

BASIC PRINCIPLES FOR MEASURING ORBITAL CURVES

The observed frequencies are far below the single effective resonance frequency of the receiver. Therefore the occurring wavelength is far greater than the dimension of the bimorphous piezoelectric element but far smaller than the whole receiver including the damping mass. Under such conditions, the element reacts nearly statically, as is also shown by Steinbeck's experiments. This justifies a simplified description of its function. Thus the voltage V is approximately proportional to the displacement vector u_i in the model. The receiver should respond to a displacement

u_1 in the 1-direction with the voltage $U_1 = E_1 u_1$. E_1 indicating the sensitivity to displacement in the 1-direction. Accordingly, $U_2 = E_2 u_2$ and $U_3 = E_3 u_3$ are found for the other directions. Then the sum of the voltage at the electrodes is $U = U_1 + U_2 + U_3 = E_i u_i$. The sensitivity E_1 depends on the piezoelectric properties of the element and its dimensions.

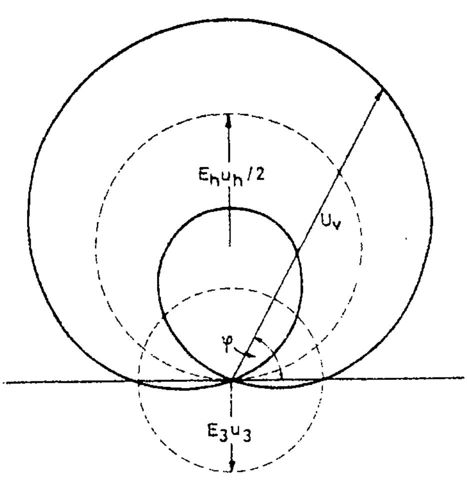
In order to find the direction of particle movement in the plane of the model the receiving element is rotated about the angle φ, the voltage $U(\varphi)$ being observed simultaneously. The situation is illustrated in Fig. 1. e_i represents the one-vector in the direction of the axis of rotation. By rotating about the angle φ the "vector of sensitivity" E_i comes into the new position $E_i = E_j d_{ji}$ ($d_{ji} = e_i e_j + (\delta_{ij} - e_i e_j) \cos \varphi - \varepsilon_{ijh} e_h \sin \varphi$ = tensor of rotation [11]). Then the voltage obtained is

$$(2) \qquad U(\varphi) = e_i e_j u_i E_j + \sqrt{[(u_i E_i - e_i e_j u_i E_j)^2 + (\varepsilon_{ijh} e_i u_j E_h)^2]} \, . \cos(\varphi - \beta)$$

with $\tan \beta = (\varepsilon_{ijh} e_i E_j u_h)(u_i E_i - e_i e_j u_i E_j)$. Experiments have shown that it is best to place the receiver upon the model so that its axis of rotation is perpendicular to it, and therefore we shall investigate the function of the receiver in this position, i.e. $e_i = \delta_{i3}$. Then Eq. (1) is simplified thus

$$(3) \qquad U_v(\varphi) = u_3 E_3 + u_h E_h \cos(\varphi - \beta)\,,$$

where $u_h = \sqrt{(u_1^2 + u_2^2)}$ = horizontal component of displacement, $E_h = \sqrt{(E_1^2 + E_2^2)}$ = horizontal component of sensitivity, $\beta = \arccos [(u_1 E_1 + u_2 E_2) : u_h E_h]$ = angle between the vectors $(E_1, E_2, 0)$ and $(u_1, u_2, 0)$ = angle between the direction of greatest horizontal sensitivity and the direction of horizontal displacement. Figure 5 shows an example in which $U_v(\varphi)$ is plotted in polar coordinates as a function of φ. In agreement with the experiments, the example shows that the direction of maximum obtained voltage U_v is not perpendicular to the direction of vanishing amplitude. Therefore we cannot deduce the direction of particle movement from the direction of vanishing amplitude by adding $\frac{1}{2}\pi$. This is a result of the vertical displacement u_3 [see Eq. (1)].



Polar diagram (calculated)

$U_v = E_3 u_3 + E_h u_h \cos(\varphi - \beta)$

$(\beta = 90°, \; E_3 u_3 / E_h u_h = 0{,}36)$

Fig. 5.

It follows from Eq. (2) and (3) that:

1. equations (2) and (3) are of similar structure. Using a method in which φ only is variable (i.e. it is rotated about one

axis only) the influence of incorrect vertical orientation cannot be eliminated,

2. as E_3 and E_h are unknown, the absolute values of u_3 and u_h cannot be determined.

METHODS OF EVALUATION FOR DETERMINING $u_h E_h$, β AND $u_3 E_3$

The system of coordinates is placed so that the vector of sensitivity has no 2-component, i.e. $E_i = (E_1, 0, E_3)$. In this case, β gives the angle of horizontal displacement u_h against the positive 1-axis which is connected to the receiving system. The two angles, φ_1 and φ_2, for which the voltage $U_v(\varphi)$ vanishes, are measured. Thus from

$$\varphi_1 = \beta - 90^\circ + \arcsin\left(E_3 u_3 / E_h u_h\right), \tag{4}$$

$$\varphi_2 = \beta + 90^\circ - \arcsin\left(E_3 u_3 / E_h u_h\right) \tag{5}$$

we obtain for the required angle

$$\beta = \tfrac{1}{2}(\varphi_1 + \varphi_2). \tag{6}$$

We use the result for β from Eq. (6) and bring the receiver consecutively into the position $\varphi = \beta$ and $\varphi = 180^\circ + \beta$. For these positions $\cos(\varphi - \beta)$ is either $+1$ or -1; in the first case the voltage has a maximum, in the second case a minimum. We call the measured values $U_{v\max}$ and $U_{v\min}$.

$$U_{v\max} = E_3 u_3 + E_h u_h, \quad U_{v\min} = E_3 u_3 - E_h u_h, \tag{7), (8}$$

$$E_h u_h = \tfrac{1}{2}(U_{v\max} - U_{v\min}). \tag{9}$$

Measurements have shown that $u_3 E_3$ is much smaller than $u_h E_h$. Since, moreover, the measurements of amplitude (7) and (8) are subject to an accidental error of measurement of approximately 10%, it is unsuitable to calculate $u_3 E_3$ merely from the data of eq. (7) and (8). A far more reliable method is made possible by measurement of the angles (5) and (4). The determination of the angles φ_1 and φ_2 is much safer than that of the amplitudes $U_{v\max}$ and $U_{v\min}$. Using Eq. (4), (5) and (9) we obtain

$$E_3 u_3 = -\tfrac{1}{2}(U_{v\max} - U_{v\min}) \cos \tfrac{1}{2}(\varphi_1 - \varphi_2). \tag{10}$$

The orbital curve can be recorded free of the vertical component when the received signal is processed for a sufficiently close sequence of moments t_n, using the above method. Then we obtain the magnitude of the two-dimensional vector $E_h u_h$ and its angle against the 1-axis β for each moment t_n. Also the time sequence $E_3 u_3(t_n)$, i.e. the seismogram free of the horizontal component, can be obtained by evaluating the signal of the vertical component for a sufficiently close sequence of time values.

ERRORS DUE TO INCORRECT VERTICAL ORIENTATION

The fact that deviations due to incorrect vertical orientation occurring in this method cannot be eliminated by suitable measurements demands higher precision for model and receiver. Difficulties appear because local unevenness on the surface of the model (e.g. indentation due to the load of the receiver etc.) immediately at the point of measurement causes errors of inclination which cannot be detected well enough by a level placed next to the receiver. A plane and smooth surface of the model is therefore the first condition for proper measuring. Errors of inclination could be caused by differences in the thickness of the model. Tests made at random on several models enable us to estimate the upper limit of a possible error of inclination for plexiglass and aluminium models with $\pm 0{\cdot}2°$. It is therefore no use forcing the accuracy of vertical orientation much above $0{\cdot}2°$. An approximation shows that the error due to incorrect vertical orientation of the direction of particle movement β is of the order of the error of inclination. Hence under optimal conditions the error of the measurement of direction of particle movement can be reduced to $\Delta\beta = \pm 0{\cdot}2°$.

EXAMPLES

Measurements were carried out in order to prove the method of evaluation. Figure 6 shows the experimental arrangement and the seismograms of the three components of the displacement vector calculated by the method discussed above. The seismograms represent signals of the longitudinal plate wave ($P1$) and the shear wave (S) generated at the source. The components u_1 and u_2 lie in the plane of the model, the first longitudinal and the second transversal to the wave front. The $P1$-wave has not only a component u_1 but also a small component u_3 perpendicular to the plane of the model. According to the theoretical approximation (1), u_3 is proportional to $\dot{u}_1$. The onset of the S-wave cannot be recognized on the seismogram u_3. This fact agrees well with the theory of two-dimensional models, which deduces $u_3 = 0$ for the case of an S-wave. The seismogram u_3 contains signals of a higher frequency than the seismograms u_1 and u_2. The reason may be found, on the one hand, in the fact that u_3 can be approximated by the time derivative of u_1. On the other hand, it must be taken into account that $E_3 u_3$ is very small and only slightly higher than the noise level. Therefore it is not possible to obtain highly detailed information of the recording of u_3.

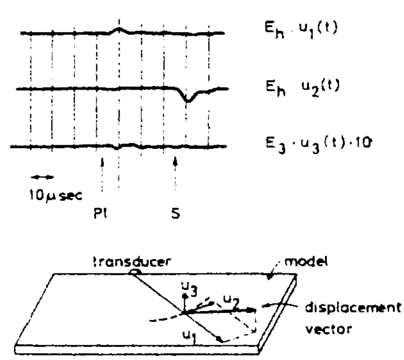


Fig. 6.

The "apparent angle of incidence" $e = \arctan (u_2/u_1)$ (Fig. 7) of a plate wave arriving under the angle i_p on a free surface (in this case the edge of the model) is a function of i_p and the velocities of the plate wave a' and the shear wave b. In a model experiment these three magnitudes can be measured very accurately. Hence the "apparent angle of incidence" can be calculated in advance with great accuracy when the angle of incidence i_p is known. In order to test the method of measurement, the receiver was placed immediately at the edge of the model upon the model surface and the orbital curve of a plate wave was measured. The distance from the transducer was 6 wavelengths. This result is also found in Fig. 7. The orbital curve is a rather extended ellipse from which the direction of greatest ground amplitude (angle between the 1-axis and the line connecting the coordinate origin and maximum amplitude e_1) and the main direction of particle movement (angle between 1-axis and major half-axis of the ellipse e_2) can be read quite accurately.

Angle of incidence	$i_p = 59{\cdot}1° \pm 0{\cdot}5°$.
Apparent angle of incidence (calculated)	$e = 30{\cdot}5° \pm 0{\cdot}5°$.
Apparent angle of incidence (measured: angle of greatest ground amplitude)	$e_1 = 30{\cdot}0 \pm 0{\cdot}5$.
Apparent angle of incidence (measured: angle of direction of main particle movement)	$e_2 = 31{\cdot}4° \pm 0{\cdot}5°$.

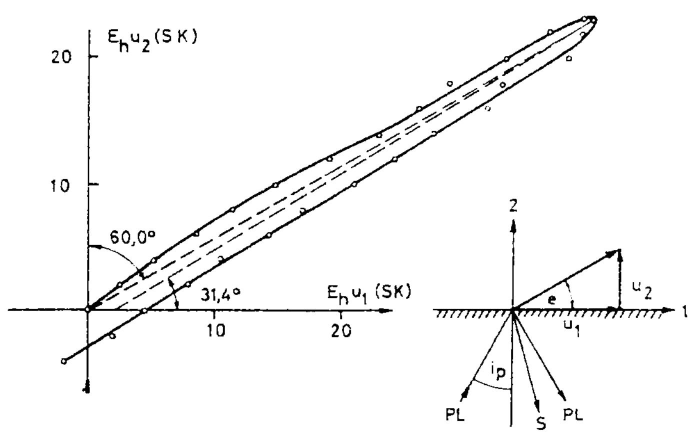


Fig. 7.

For the "apparent angle of incidence" we have the choice of two measured values, the angles e_1 and e_2. Both values differ only slightly from the theoretical value. The angle e_1, though, seems to agree better with the theoretical value. The deviation lies within the order given as the tolerance boundary for our method of measurement. The elliptic polarization could, on the one hand, be caused by the influence of the near field; on the other hand, the possibility of the generation of Rayleigh waves has to be considered, particularly as the angle of incidence of the plate wave is rather large.

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Резюме

РЕГИСТРАЦИЯ СОСТАВЛЯЮЩИХ СМЕЩЕНИЯ В ДВУХМЕРНЫХ СЕЙСМИЧЕСКИХ МОДЕЛЯХ

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Обсуждаются методы регистрации составляющих в двухмерных моделях, применяемые с 1956 г. Однако, модели не строго двухмерные, и поэтому следует ожидать, что наблюденные амплитуды будут отличаться от вычисленных, соответствующих двухмерной волне. В целях особого рассмотрения этого факта был создан пьезоэлектрический датчик. По измерениям можно было судить о поведении приемника, которое оказалось возможным описать весьма простой теорией. Эта теория основана на экспериментальном факте, что полученное напряжение U приблизительно пропорционально вектору смещения u_i модели в точке наблюдения. Отсюда получилась методика наблюдений, при которой в каждой точке навлюдения имеются три величины: составляющие u_1 и u_2 смещения в плоскости модели и составляющая u_3 в перпендикулярном направлении. Приводятся примреы наблюдений.

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Discussion

Gilbert: Have you calibrated your system so that you can obtain true amplitudes?

Gutdeutsch: We did not calibrate the system, we compared only the form of the signal obtained with the form of the signal recorded by a capacity detector which gave a voltage proportional to the displacement on the model; the agreement was quite good.