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Atmospheric Refraction and Path Bending Delay†⋆

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Abstract Owing to the effect of refraction on the propagation of electromagnetic waves in the terrestrial atmosphere, the direction of propagation is changed. The path of propagation path becomes a curve with an increased path-length so increasing the propagation time. A simplified spherically symmetric atmospheric model is adopted to calculate the delay caused by the path bending, for different zenith distances.

Key words: astrometry—Earth: atmosphere

1. INTRODUCTION

When astronomical observations are made on the ground, the effect of the terrestrial atmosphere makes the propagation path of electromagnetic waves bend, causing changes both in the direction and length of propagation. As far back as 1906, Newcomb said, "In practical astronomy, perhaps there is not a branch like the problem of astronomical refraction on which so many papers have been published, while the state is still not so satisfactory." In the research on the atmospheric refraction delay before that, what people principally took into account was the delay produced by the change in the propagation velocity caused by the fact that the atmospheric density is not zero, and the delay caused by the bending of the propagation path was generally neglected because its effect is too small[1,2]. However, at present, with the appearance of various new space measurement techniques and the steady improvement in observing accuracy the influence of the effect of the bending needs to be taken into account. The effect is large particularly when the zenith distance is large.

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2. SPHERICALLY SYMMETIC ATMOSPHERIC MODEL

Variation in the atmospheric index of refraction results from the altitude-dependent atmospheric density. However, due to the action of the gravitational force the atmospheric density can be approximately regarded to exhibit a spherically symmetric distribution, then the equal refractive index surfaces can be expressed by some concentric surfaces[1]. Thus, the atmospheric layer can be divided into a large number of concentric shells each so thin that the atmospheric parameters in each shell can be considered as fixed constants. Light travels along a straight line within each shell and is refracted on the interface. Light travels along a straight line outside the atmosphere. For convenience of comparison with other works, the target light source is taken at $R = 26560$ km, the orbital radius of the GPS satellite.

In this paper we adopt, as basis of our calculations, the 1976 US standard atmospheric model[3]. The model is divided into 6 layers from the sea level up to 71 km, and the height ranges of these layers are 0—11, 11—20, 20—32, 32—47, 47—51 and 51—71 km, respectively. For each layer, the formulae for computing the atmospheric temperature, density and pressure in terms of their values at the seal level are listed in the model. In the calculations of this paper, the temperature and atmospheric pressure at the given altitude are firstly calculated according to the formulae and then the refractive index n and gravitational velocity g of the place are calculated by means of the following formulae:

$$10^6(n-1) = 77.6P/T\,, \tag{1}$$

where P is the pressure at the given altitude within the atmosphere, in units of N/n^2 and T is the absolute temperature there.

$$\frac{g_1}{g_2} = \frac{r_2^2}{r_1^2}\,, \tag{2}$$

where r is the geocentric distance at the given altitude within the atmosphere and the subscript refers to different points within the atmosphere.

Now, the refractive index n varies greatly at places close to the ground and varies little at places far above the ground. So, to improve the accuracy of calculation, when we divide the spherical layers, a layer corresponds to 1 km at the lower altitudes and to much larger distances at the higher altitudes. There are a total of 40 layers.

3. CALCULATION OF ATMOSPHERIC REFRACTION BENDING CORRECTION

As shown in Fig. 1, A is the observing point on the ground, S the target light source in vacuum, B, C and D are the divided layers of the atmosphere, respectively, and D the upper boundary of the atmosphere. From S to D light travels along the straight line in vacuum and travels along the curve $DCBA$ within the atmosphere.

In the calculation carried out in the present article, the bending correction is decomposed into two terms: The first term is the path length Δd_1 which is the difference between the curve $ABCD$ and the broken line AB, BC, CD, and the calculation of this term of correction is mainly that of the bending angle of light which is related to the calculation of astronomical refraction. The second term is the increased path length, Δd_2, of the broken line AB, BC, CD and DS over the straight path AS, and this is purely a geometric question.

3.1 Principle of Calculation of Δd_1

Based on the spherically symmetric atmospheric model adopted in this paper, the bending of light between two consecutive points is [1]

$$\mathrm{d}\gamma = \tan\theta \frac{\mathrm{d}n}{n} \,, \tag{3}$$

where $\mathrm{d}\gamma$ is the bending angle of light between consecutive points of the atmosphere, and θ the angle of incidence of light at a certain layer. Therefore, the total bending of light, i.e. the atmospheric refraction on the ground is

$$\gamma_0 = \int_{\ln n_1}^{\ln n_0} \tan\theta \mathrm{d}\ln n \,, \tag{4}$$

where n_0 and n_1 are the atmospheric refractive indices on the ground and at the atmospheric upper boundary layer, respectively, and we have $n_1 = 1$.

Carrying out the integration by parts of the above equation and letting ln $n_1 = 0$, one may have

$$\gamma_0 = \ln n_0 \tan\theta_0 - \int_{\theta_1}^{\theta_0} \ln n \sec^2\theta \mathrm{d}\theta \,, \tag{5}$$

where

$$\int \ln n \sec^2\vartheta = -\frac{1}{2}\int \sec^2\theta \tan\theta \mathrm{d}(\ln n)^2 - \int \sec^2\theta \tan\theta \ln n \mathrm{d}\ln|\boldsymbol{s}| \,. \tag{6}$$

The formula of refraction can be written as

$$\gamma_0 = \ln n_0 \tan\theta_0 + [\frac{1}{2}(\ln n_0)^2 - I] \sec\theta_0 \tan\theta_0 \,, \tag{7}$$

where θ_0 is the incidence angle of light on the ground, i.e. the apparent zenith distance, and

$$I = \frac{\beta_d p_0}{|\boldsymbol{s}_0 \boldsymbol{g}_0|}(1 + I_1) + \frac{v_0 w_\mathrm{m}}{|\boldsymbol{s}_0|} \,. \tag{8}$$

The second term of the above equation is the term of the wet component, which can be neglected because its value is very small for the visible light wave band, and it is all we are concerned with in the present article. The values of I_1 and β_d are given by the following empirical formulae:

$$\beta_d = 2.2255 \times 10^{-4} + 1.249 \times 10^{-6}\lambda^{-2} + 1.012 \times 10^{-8}\lambda^{-4} + 1.3 \times 10^{-10}\lambda^{-6} \,, \tag{9}$$

where λ is the wavelength of the incident light and

$$I_1 = -0.4504 - 3.8 \times 10^{-5}(T_0 - 273.15) \,, \tag{10}$$

where T_0 is the absolute temperature on the ground.

Substituting the above equations into Eq.(7), we obtain the bending angle of light of every layer and hence the bending angle $\gamma_{\rm i}$ of light between two consecutive layers by subtraction.

With $\gamma_{\rm i}$, the path length increased from the broken line to the curve can be calculated:

$$\Delta d_1 = \sum (1 - \cos\gamma_{\rm i}) \sec Z_{\rm i} \Delta r \,, \tag{11}$$

where Z is the zenith distance of light with respect to the given layer within the atmosphere (see Fig.2).

3.2 Principle of Calculation of Δd_2

As shown in Fig. 2, O is the earth's barycenter, A the position of an observer on the ground, B, C and D are the incident points of various layers, respectively. For the triangle OAB consisting of a segment of broken line and two geocentric distances, we let the location of A be the i-th layer and from the cosine formula the length $l_{\rm i}$ of the broken line is

$$l_{\rm i} = \sqrt{r_{\rm i+1}^2 + r_{\rm i}^2 \sin^2 Z_{\rm i}} - r_{\rm i} \cos Z_{\rm i} \,, \tag{12}$$

and then from the sine formula the central angle $\alpha_{\rm i}$ corresponding to each segment of the broken line can be calculated:

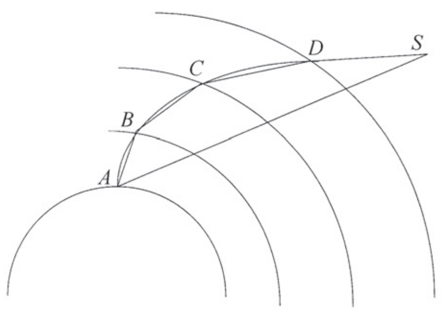

Fig. 1 Schematic diagram of the atmospheric layers and propagation of light in every layer

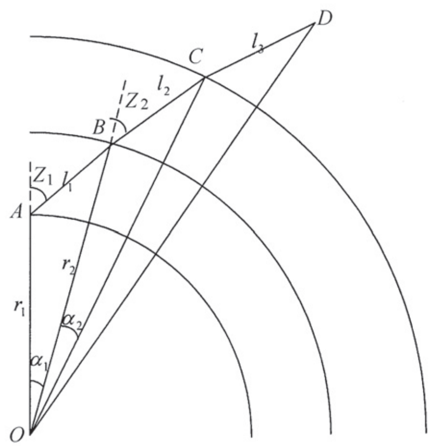

Fig. 2 Calculation of the rectilinear propagation distance of light in every layer

$$\alpha_{\rm i} = \arcsin(l_{\rm i} \sin Z_{\rm i} / r_{\rm i+1}) \,. \tag{13}$$

The zenith distance of the next layer is

$$Z_{\rm i+1} = Z_{\rm i} + \gamma_{\rm i} - \alpha_{\rm i} \,, \tag{14}$$

and the distance D between the observer and the light source is

$$D = \sqrt{r_0^2 + r_n^2 - 2 r_0 r_n \cos(\sum \alpha_{\rm i})} \,. \tag{15}$$

Therefore, we have

$$\Delta d_2 = \sum l_{\mathrm{i}} - D\,. \tag{16}$$

Finally, the bending correction of light can be obtained

$$\Delta d = \Delta d_1 + \Delta d_2\,. \tag{17}$$

4. CALCULATED RESULTS

Substituting the data of the standard atmospheric model into the above formulae (11) and (16), we obtain the calculated results.

4.1 Calculation of Δd_1 and Δd_2

First, the bending angle of light in every spherical layer with respect to the outer atmosphere is calculated. The difference between them is just the bending angle of light between the two spherical layers, and Δd_1 can be calculated by substituting the difference into Eq.(11). The quantity γ in Eq.(11) is just the bending angle of light, and the γ on the ground or the total bending angle γ_0 in Eq.(7) is the astronomical refraction. We calculate γ_0 for every 5°of the zenith distances from 5°to 75°, obtaining 15 values. These were compared with the standard astronomical refraction given in Chinese Astronomical Almanac[4]: the minimum relative error is 6.3%, the maximum is 6.6% and the average is 6.5%. Considering that our model is a simplified model, the relative errors of such size seem to be allowable. Especially, the relative error does not increase for large zenith distances, which shows the reliability of our model. This comparison can be taken as a sort of test on the reliability of the final results obtained here.

Then Δd_2 is calculated according to Eq.(16). Our calculated results are given in Table 1.

Table 1 Calculated results of the corrections

zenith distance	Δd_1	Δd_2	Δd
5	0.000016	0.001468	0.001483
10	0.000065	0.006035	0.006100
15	0.000152	0.014197	0.014349
20	0.000288	0.026926	0.027215
25	0.000491	0.045817	0.046308
30	0.000787	0.073470	0.074257
35	0.001223	0.114199	0.115422
40	0.001877	0.175245	0.177122
45	0.002886	0.269402	0.272288
50	0.004504	0.420369	0.424874
55	0.007239	0.675246	0.682485
60	0.012184	1.136061	1.148245
65	0.022008	2.050582	2.072590
70	0.044303	4.122645	4.166948
75	0.106287	9.864349	9.970636
80	0.349321	32.174803	32.52412
85	2.224764	192.987997	195.212761

It can be seen from Table 2 that when the zenith distance is below 60°, the bending correction is less than 1 mm and can be neglected, while when the zenith distance exceeds 70°, the bending correction increases abruptly. Thus, the influence of the effect should be considered when the zenith distance is large.

4.2 Comparison of the Result of This Paper with the Other Obtained Results

A comparison of the results of this paper with those already obtained in the 3 papers of Yan Hao-jian[5], Li Yan-xing et al. [6] and Mao Wei et al. [7] is given in Table 2.

Table 2 Comparison with the results in Refs. [5, 6, 7]

Z	60°	65°	70°	75°	80°	85°
This paper Δd	1.148	2.073	4.167	9.971	32.524	195.21
Ref. [5] ΔS	0.288	0.521	1.055	2.560	8.689	65.786
Ref. [6] ΔS	1.229	2.213	4.431	10.522	33.658	198.301
Ref. [7] ds_1	0.949	1.716	3.457	8.318	27.611	190.230
Ref. [7] ds_2	1.013	1.831	3.690	8.878	29.470	203.030

5. CONCLUSIONS

The spherically symmetric atmospheric model is adopted in the calculations carried out in the present article. The computing formula of Δd_1 is derived on the basis of this model and the derivation of Δd_2 is based on a strict geometric reasoning, both with very high confidence. Also, our calculation makes use of the 1976 US standard atmospheric model, which so far is still the most precise model. Rather than the 19-layer stratification of the atmosphere, (of which only 14 layers have refractive indices greater than 1) often used in general literature, we adopted a 40-layer stratification in our recalculations using the primary formulae, thereby improving the accuracy of calculation.

As pointed out before, the path bending correction should be taken into account when the zenith distance is large. However, near the horizon the propagation of light in the atmosphere may be in a chaotic state, the basic Snell refraction formula needs be corrected. For this reason, the corrections for zenith distances above 80°were not calculated in this paper.

It can be seen from Table 2 that the result calculated in this present article is very close to that in Ref.[6], which adopted the same 1976 US standard atmospheric model, and the difference is within 7%. Also, our result agrees quite well with that of Ref. [7], which adopted a different model. But the discrepancy between our result and that of Ref. [5] is comparatively large.

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