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Modelling dynamic niche and community model by type-2 fuzzy set☆

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Niche and community theory are the basis for much ecological research. The interrelationship between life and environment demands a dynamic niche model to assess where species habitats are preferential. In this paper, we propose a niche model with dynamic characteristics based on the “broadband” in type-2 fuzzy sets. The community model and its relatives, such as ecotone, similarity degree and separate degree of community are also established. Based on the proposed model, it is better to describe the dynamic processes between fundamental niche and realized niche, as well as inherent ecosystem uncertainties. Furthermore, the effect on community structure of the physical environment and of inter-specific competition can be identified. Simulation results indicate the niche could not only include the result by classic model, but also imply more information, i.e., the range of niche affected by environment. The model accurately locates the range distribution in natural secondary forests of *Phellodendron amurense* and *Tilia amurensis*.

1. Introduction

Niche theory has been widely used on much ecological research. Though there are amounts of niche’s definitions, the most significant ones are from Grinell, Elton and Hutchinson, which is regarded as spatial niche, functional niche and *n*-dimensional hyper-volume niche (Cao, 1995). Generally speaking, ecological niches are defined by the relationship between organisms and the physical and biological environment, taking into account both time and space (Chesson, 2000).

Community is a structure unit that every biotic population lives together in certain region or habitat. Generally, community has the following characteristics: all living things within a community connect with each other; community and its environment cannot be separated at any time; every individual within a community has a different ecological significance, they have certain dynamic character and distribution range; the boundary of community is ambiguity which depends on the division of habitat (Sun, 1988).

In 1965, Zadeh invented fuzzy set theory to describe uncertainty and fuzziness. Edward and Abraham (1983) introduced fuzzy set to ecosystem and studied the interspecific competition. Bosserman and Ragade (1982) used the fuzzy sets for analysis of ecosystem; Cao (1995) defined niche as α-cut set in a fuzzy set, stating that index α was a measure for competition. Wang et al. (2003) presented the model of niche and community by fuzzy set theory. They also discussed the measurement of niche based on the model. However, the above models lack of dynamic character, only treat them as static. Recently, the definition of type-2 fuzzy sets and type-2 fuzzy logic system is proposed, which open up a new way to describe

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higher uncertainties (Mendel, 2001, 2006). Li et al. (2006) proposed the niche width and niche overlap based on type-2 fuzzy set, which could better reflect the diversity of resources used by species or communities. We notice that the *broadband* effect of MF could better describe the dynamic process between fundamental niche and realized niche. As these issues are not independent, it is essential to find a means of considering them joint. Towards this goal, the dynamic niche model based on *broadband* effect inherent in membership function of type-2 fuzzy set is proposed here. Furthermore, we show how this model applies to the analysis of niche changes according environment, and use it to explain relationship between community and niche, which provides a method to describe complexity inherent in ecosystem. We also model the community model and its relatives, such as ecotone, similarity degree and separate degree of community, by the proposed model.

2. Introduction of type-2 fuzzy sets

2.1. *Introduction*

The concept of a type-2 fuzzy set was introduced by Zadeh (1975) as an extension of the concept of an ordinary fuzzy set (henceforth called a type-1 fuzzy set). A type-2 fuzzy set is characterized by a fuzzy membership function, i.e., the membership value (or membership grade) for each element of this set is a fuzzy set in [0,1], unlike a type-1 set where the membership value is a crisp number in [0,1]. Such sets are useful in the circumstances where it is difficult to determine the exact membership function for a fuzzy set; hence, they are useful for incorporating rule uncertainties, and they let us propagate such uncertainties through them (Mendel, 2001).

Fig. 1 shows a type-2 fuzzy set, where the membership grade for every point is a Gaussian type-1 set in [0,1]. We call such type-2 sets "Gaussian type-2 sets" and their membership grades "Gaussian type-1 sets".

2.2. *Notation and terminology*

Now we briefly introduce notation of type-2 sets and their associated membership functions. For details, please see Mendel (2001).

A type-2 fuzzy set in X is $\tilde{A}$, and the membership grade of $x \in X$ in $\tilde{A}$ is $\mu_{\tilde{A}}(x)$, which is a type-1 fuzzy set in [0,1]. The elements of the domain of $\mu_{\tilde{A}}(x)$ are called primary memberships of x in $\tilde{A}$, e.g., the vertical axes in Fig. 1(a), and the horizontal axes in Fig. 1(b). The memberships of the primary memberships in $\mu_{\tilde{A}}(x)$ are called secondary memberships of x in $\tilde{A}$, e.g., the vertical axes in Fig. 1(b). The membership grade of any $x \in X$ in $\tilde{A}$ can be presented as

$$\mu_{\tilde{A}}(x) = \int_{u \in [0,1]} f_x(u)/u, \; u \in J_x \subseteq [0,1] \tag{1}$$

where $u \in J_x$ indicate the primary memberships of x and $f_x(u) \in [0,1]$ indicate the secondary memberships (grades) of x. The integrals indicate logical union.

When the secondary membership functions (MF) of a type-2 fuzzy set are type-1 one interval sets, we call such set an interval type-2 set. Next, we give some definitions of type-2 fuzzy, most of which could be found in Mendel (2001); but they are essential for this paper.

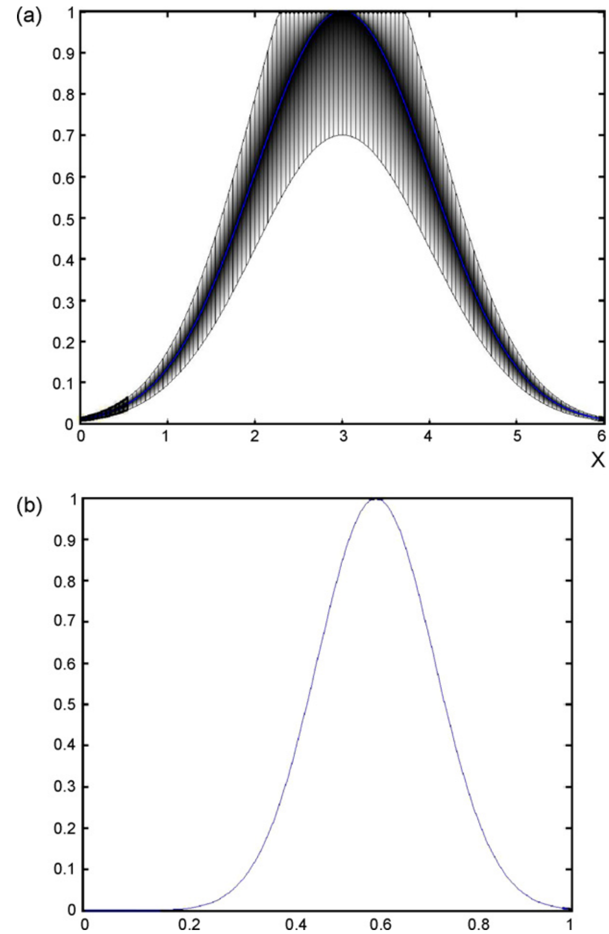

Fig. 1 – (a) A Gaussian type-2 set and (b) the secondary memberships corresponding to (a).

Definition 1 *(Footprint of uncertainty of a type-2 MF or broadband effect of MF; Mendel, 2001:)*. Uncertainty in the primary membership grades of a type-2 $\tilde{A}$ consists of a bounded region that we call the footprint of uncertainty (FOU) of a type-2 $\tilde{A}$ (e.g., the shade region in Fig. 1(a), also called broad effect inherent in membership of type-2 fuzzy set). It is the union of all primary membership grades, i.e.

$$\mathrm{FOU}(\tilde{A}) = \bigcup_{x \in X} J_x \tag{2}$$

The term of FOU is very useful, because it not only focuses our attention on uncertainties inherent in a specific type-2 membership function, whose shape is a direct consequence of the nature of these uncertainties, but it also provides a very convenient verbal description of the entire domain of support for all the secondary grades of a type-2 membership function (Mendel, 2001, 2006).

Definition 2 *(Upper and lower MFs; Mendel, 2001:)*. An upper MF (UMF) and a lower MF (LMF) of $\tilde{A}$ are two type-1 MFs that bound for the FOU of atype-2 fuzzy set $\tilde{A}$. The UMF is associated with the upper bound of FOU($\tilde{A}$), and is denoted as $\bar{\mu}_{\tilde{A}}(x)$. The LMF is associated with the lower bound of FOU($\tilde{A}$), and is denoted as $\underline{\mu}_{\tilde{A}}(x)$, i.e., $\forall x \in X$

$$\bar{\mu}_{\tilde{A}}(x) \equiv \overline{\mathrm{FOU}(\tilde{A})}, \qquad \underline{\mu}_{\tilde{A}}(x) \equiv \underline{\mathrm{FOU}(\tilde{A})} \tag{3}$$

Example 1 *(Gaussian Primary MF with Uncertain Mean; Liang and Mendel, 2000:)*. Consider the case of a Gaussian Primary MF having a fixed standard deviation σ and an uncertain mean that takes on value in (m_1, m_2), i.e.

$$\mu_{\tilde{A}}(x) = \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right], \quad m \in [m_1, m_2] \tag{4}$$

The UMF $\bar{\mu}_{\tilde{A}}(x)$ is (see Fig. 2)

$$\bar{\mu}_{\tilde{A}}(x) = \begin{cases} N(m_1, \sigma; x), & x < m_1 \\ 1, & m_1 \le x \le m_2 \\ N(m_2, \sigma; x), & x > m_2 \end{cases} \tag{5}$$

where, for example

$$N(m_1, \sigma; x) = \exp\left[-\frac{1}{2}\left(\frac{x-m_1}{\sigma}\right)^2\right] \tag{6}$$

The LMF $\underline{\mu}_{\tilde{A}}(x)$ is (see Fig. 2)

$$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} N(m_2, \sigma; x), & x \le \dfrac{m_1+m_2}{2} \\ N(m_1, \sigma; x), & x > \dfrac{m_1+m_2}{2} \end{cases} \tag{7}$$

2.3. *Operation on type-2 sets*

As introduced above, the membership grades of type-2 set are type-1 sets; therefore, we should perform operation like union and intersection on type-2 sets.

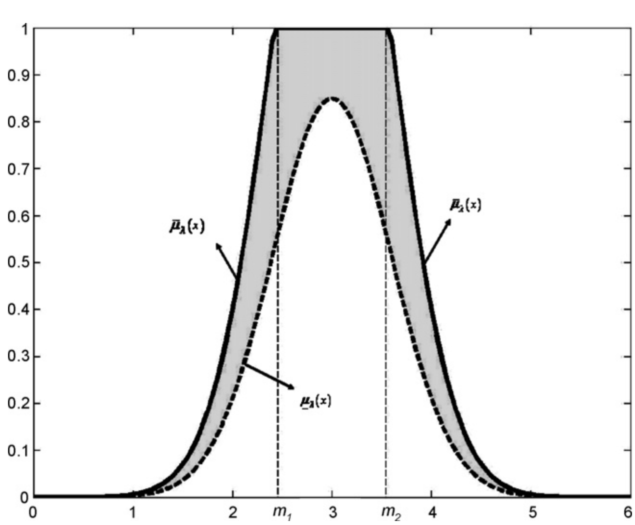

Fig. 2 – Gaussian primary MF with uncertain mean. The thick solid lines denote upper MFs and the thick dashed lines denote lower MFs. The shaded regions are the footprint of uncertainty for interval secondaries.

Consider two type-2 fuzzy set, $\tilde{A}$ and $\tilde{B}$, in a universe X. Let $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ be the membership grades (fuzzy sets in $J_x \subseteq [0,1]$) of these two sets, represented, for each x we have

$$\tilde{A} = \int_u \frac{f_x(u)}{u} \quad \text{and} \quad \tilde{B} = \int_w \frac{g_x(w)}{w}$$

where $u, w \in J_x$ indicate the primary memberships of x and $f_x(u)$ $g_x(w) \in [0,1]$ indicate the secondary memberships (grades) of x.

Using Zadeh's Extension Principle, the membership grades for union, intersection and complement of type-2 fuzzy set $\tilde{A}$ and $\tilde{B}$ as follows (Mendel, 2001, 2006):

$$\mu_{\tilde{A}}(x) \hat{\cup} \mu_{\tilde{B}}(x) = \int_u \int_w \left(f_x(u) \star g_x(w)\right) / (u \vee v) \tag{8}$$

$$\mu_{\tilde{A}}(x) \hat{\cap} \mu_{\tilde{B}}(x) = \int_u \int_w \left(f_x(u) \star g_x(w)\right) / (u \star v) \tag{9}$$

$$\neg \mu_{\tilde{A}}(x) = \int_u \frac{f_x(u)}{1-u} \tag{10}$$

where $\vee$ represents the max t-norm and $\star$ represents a t-norm. Then, the *join*, *meet* and *negation* of type-2 fuzzy sets can be defined as follows (Karnik and Mendel, 2001):

$$(\tilde{A} \cup \tilde{B})(x) = \mu_{\tilde{A}}(x) \hat{\cup} \mu_{\tilde{B}}(x) \tag{11}$$

$$(\tilde{A} \cap \tilde{B})(x) = \mu_{\tilde{A}}(x) \hat{\cap} \mu_{\tilde{B}}(x) \tag{12}$$

$$(\tilde{A}^c)(x) = \neg \mu_{\tilde{A}}(x) \tag{13}$$

Definition 3 *(Mendel, 2001)*. Let $\tilde{A}$ and $\tilde{B}$ be two type-2 fuzzy sets, denote

$$\tilde{A} \circ \tilde{B} = \left\| \hat{\bigcup_{x \in X}} (\mu_{\tilde{A}}(x) \hat{\cap} \mu_{\tilde{B}}(x)) \right\| \tag{14}$$

$$\tilde{A} \hat{\circ} \tilde{B} = \left\| \hat{\bigcap_{x \in X}} (\mu_{\tilde{A}}(x) \hat{\cup} \mu_{\tilde{B}}(x)) \right\| \tag{15}$$

We call $\tilde{A} \circ \tilde{B}$ and $\tilde{A} \hat{\circ} \tilde{B}$ the inner product and outer product of interval type-2 fuzzy sets. Note that the membership functions of Interval type-2 fuzzy sets are interval fuzzy sets, so $\|\cdot\|$ be the interval norm, i.e.,

$$\text{For} \quad I_{[0,1]} = \{a = [\underline{a}, \bar{a}] \,|\, 0 \le \underline{a} \le \bar{a} \le 1, \underline{a}, \bar{a} \in R\}, \|I_{[0,1]}\| = \left|\bar{a} - \underline{a}\right| \le 1 \tag{16}$$

Given interval type-2 fuzzy set $\tilde{A}$, let interval type-2 fuzzy set $\tilde{B}$ approach $\tilde{A}$. We will find the inner product increase, while the outer product decreases. Contrariwise, when $\tilde{A} \circ \tilde{B}$ is great and $\tilde{A} \hat{\circ} \tilde{B}$ to be smaller, $\tilde{A}$ and $\tilde{B}$ is close. As a result, we combine the inner product with outer product to describe the degree of similarity for interval type-2 fuzzy sets.

Definition 4. Let $\tilde{A}$ and $\tilde{B}$ be two type-2 fuzzy sets, denote

$$N(\tilde{A}, \tilde{B}) = (\tilde{A} \circ \tilde{B}) \wedge (\tilde{A} \hat{\circ} \tilde{B})^C \tag{17}$$

where $\wedge$ represents min t-norm.

Obviously, $\forall \tilde{A}, \tilde{B}, \tilde{C}$ be interval type-2 fuzzy set, N satisfied

(1) $N(\tilde{A}, \tilde{B}) = N(\tilde{B}, \tilde{A})$
(2) $N(\tilde{A}, \tilde{A}) = 1, N(\tilde{X}, \tilde{\emptyset}) = 0$
(3) $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C} \Rightarrow N(\tilde{A}, \tilde{C}) \leq N(\tilde{A}, \tilde{B}) \wedge N(\tilde{B}, \tilde{C})$

where $\tilde{X}$ represents the interval type-2 fuzzy set whose primary and secondary grades is 1; $\tilde{\emptyset}$ represents the interval type-2 fuzzy set whose primary and secondary grades is 0.

Therefore, $N(\tilde{A}, \tilde{B})$ is called the lattice degree of similarity for $\tilde{A}$ and $\tilde{B}$.

3. The definition of niche by type-2 fuzzy set theory

According to Shelford's law of tolerance, each species within ecosystem has its tolerance range. It would show adaptive ability according to the disturbance from outside, which could be regarded as the dynamic character. We use the *broadband* effect inherent in membership function of type-2 fuzzy sets corresponding to the range of tolerance for the ecological factors of species. Also the footprint of uncertainty of type-2 MF corresponding to the dynamic process from realized niche to fundamental niche, such model could better express the exploitation and utilization for bounds under influence of environmental factors. To begin with, only one-dimensional resources are considered here.

Definition 5. Let x-axis be one resource axis of certain species, $\tilde{A}$ is the type-2 fuzzy set of resource curve. The niche of a species on resource axis is defined as the footprint of uncertainty of $\tilde{A}$ on resource axis (Li et al., 2006).

From Fig. 3(a), every point x' on the resource axis means certain status of the resource, the niche related to it is $[\underline{\mu}_{\tilde{A}}(x'), \bar{\mu}_{\tilde{A}}(x')]$, where $\underline{\mu}_{\tilde{A}}(x')$ and $\bar{\mu}_{\tilde{A}}(x')$ is the LMF and UMF of $\mu_{\tilde{A}}(x)$.

As we known, the fundamental niche of a species includes the total range of environmental conditions that are suitable for existence without the influence of interspecific competition or predation from other species. The realized niche describes that part of the fundamental niche actually occupied by the species. And the ideal niche denotes the niche under ideal environment, i.e., the minimal niche space to maintain the species to survive. According to the proposed model, we denote fundamental niche, realized niche and ideal niche as V_F, V_R and V_I, obviously we have $V_I \subseteq V_R \subseteq V_F$. According to the definition of niche in this paper, V_I corresponds to $\underline{\mu}_{\tilde{A}}(x')$, V_F corresponds to $\bar{\mu}_{\tilde{A}}(x')$ and V_R varies in $[\underline{\mu}_{\tilde{A}}(x'), \bar{\mu}_{\tilde{A}}(x')]$ (i.e., $[V_I, V_F]$), which intuitively illuminates niche changes occurring between V_F and V_I as the environment changes.

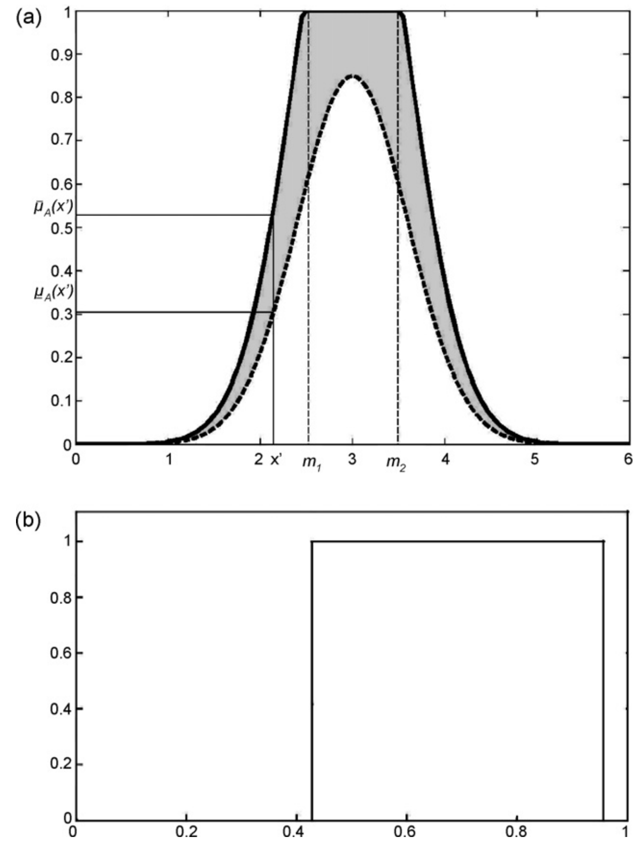

Fig. 3 – (a) Type-2 MF and (b) the secondary MF corresponding to (a).

4. The model of dissipation structure of community

4.1. *The dynamic type-2 fuzzy model of community*

Community is a structure unit that every biotic population lives together in certain region or habitat. Here, V_{λ_i} denotes the type-2 niche of species. Taking into account of the expression theorem of type-2 fuzzy set (Mendel, 2001), the hierarchic structure of biotic community can be expressed by type-2 fuzzy set.

Definition 6. $\tilde{A} = \bigcup_{\lambda_i \in \Lambda} (\lambda_i / V_{\lambda_i})$, denoted as a biotic community, is a structure unit that every biotic population lives together in certain region or habitat Here, λ_i denotes certain ecological factor, V_{λ_i} is an interval type-2 fuzzy set, i.e., which denotes the type-2 niche on this factor. Here, Λ expresses the set of all ecological factors. In practice, λ_i is measurement of ecological factor, which indicate the membership degree.

4.2. *The hierarchic structure of community*

Generally, community can be determined strictly and distinguished from each other. But when a clear cut within community habitat is not showed in space, a clearer boundary may be the results of interaction of community itself. In the dynamic type-2 fuzzy model for community, $\tilde{A} = \bigcup_{\lambda_i \in \Lambda} (\lambda_i / V_{\lambda_i})$ denoted as a biotic community, taken the discrete or continuous value for λ_i, by the definition of the corresponding type-2 niche, we may know the followings (shown in Fig. 4(a–d)):

(1) When ecological factors λ_i is taken the discrete value and the discrete degree is very large, the hierarchy of its corresponding niche V_{λ_i} is more distinct and clearly show discontinuity of niche, which may lead to the stronger discontinuity of community.
(2) When λ_i is taken continuous value, the hierarchy of niche V_{λ_i} is relatively fuzzy so as to the obscure of habitat boundary of the community.
(3) Hierarchic phenomena are obvious characters of the spatial structure of the community. That is to say, $\tilde{A} = \bigcup_{\lambda_i \in \Lambda}(\lambda_i/V_{\lambda_i})$. Clearly, the hierarchic degree of the community is determined by the set Λ.
(4) The interaction of all subsystem (different hierarchy of λ_i) in community is nonlinear, which also show the effect of restrict and promote to each other.

4.3. *The relationship between niche and community*

Community is combined with uncountable intersection individuals. Therefore, the function of community is union of individual's function and reflects the fitness of individuals. However, the relationship between niche and community has not been an issue. Based on the proposed model, we can express the relation with the help of the expression theorem of type-2 fuzzy set theory. Given certain niche V_{λ_i}, a community can be expressed as $\tilde{A} = \bigcup_{\lambda_i \in \Lambda}(\lambda_i/V_{\lambda_i})$. Meanwhile, we can have the followings:

(1) The expression $\tilde{A} = \bigcup_{\lambda_i \in \Lambda}(\lambda_i/V_{\lambda_i})$ is a structure unit. The primary MF of species disciplinary behaved as Gaussian, expressing the distinct hierarchic structure.
(2) λ_i varies on discrete or continuous value under different environmental factors, there distributed different species, even as species distributed in nature.
(3) According to the decomposed theorem of fuzzy sets, for a community represented by a type-2 fuzzy set $\tilde{A}$, there exists a interval type-2 fuzzy set V_{λ_i}, satisfying

$$\tilde{A} = \bigcup_{\lambda_i \in \Lambda}(\lambda_i/V_{\lambda_i}) \tag{18}$$

(4) The relationship between niche and community can be clearly expressed by the expression and decomposed theorem of type-2 fuzzy sets.
(5) The interaction of all subsystem (different hierarchic of λ_i) in community is nonlinear. As a result, the model given by expression and decomposed theorem of type-2 fuzzy set also shows stronger nonlinearity of ecosystem.

4.4. *Ecotone in community*

Ecotone is where two or more communities not only meet but integrated. Ecotone arises in the transition zone between two communities that exhibit a shift in dominance. The variety and density of life are often greatest in and about edges and ecotones. This phenomenon is called edge effect (Robert, 1995). Assuming that $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_k$ represent k different com-

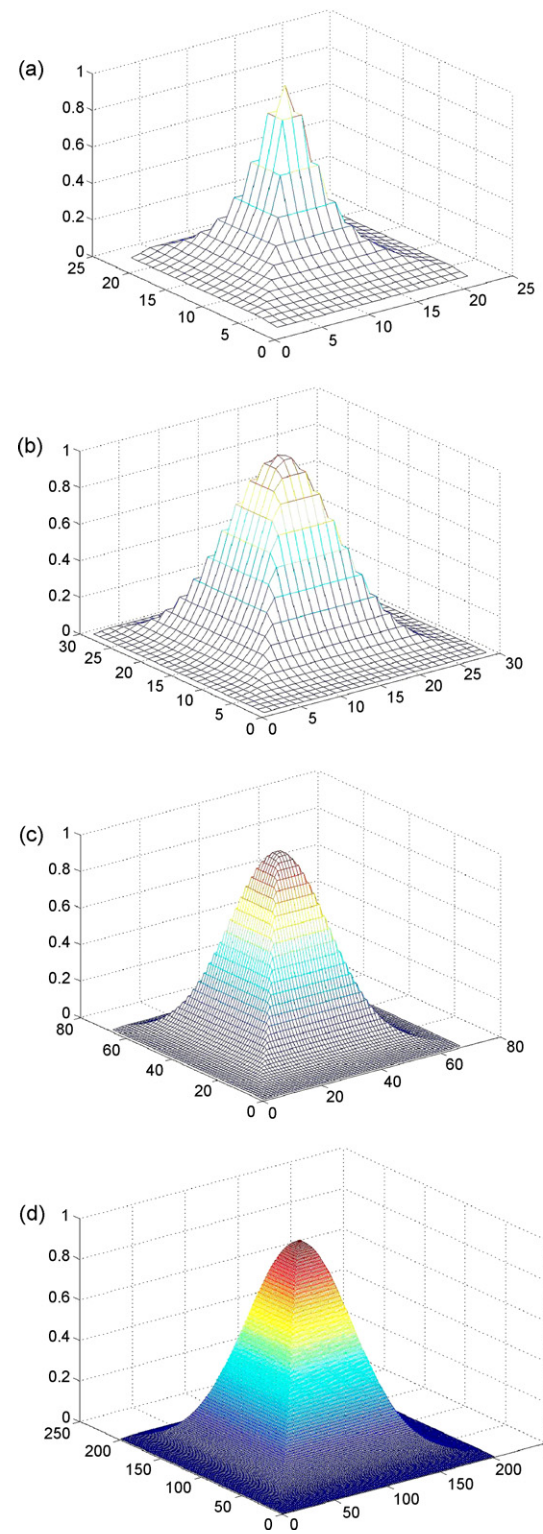

Fig. 4 – According to the discrete and continuous varieties of λ_i, the primary MF of species regularly behaved as Gaussian, expressing the distinct hierarchic structure. (a) λ_i takes the discrete value and the discrete degree is very large, (b) λ_i takes the discrete value and the discrete degree is less, (c) λ_i takes the discrete value and the discrete degree is quite small and (d) λ_i takes the continuous value and the hierarchic of niche is relatively fuzzy.

munities, their ecotone, according to fuzzy set theory, can be expressed as

$$\mathrm{EcoT} = \bigcap_{i=1}^{K} \tilde{A}_i = \bigcap_{i=1}^{K} \left(\bigcup_{\lambda_j \in \Lambda} \left(\frac{\lambda_j}{V_{\lambda_j}} \right) \right) \tag{19}$$

From the expression, we can conclude that ecotone is an overlap of all the community on each environment gradients.

4.5. *The similarity of community*

Note that an interval type-2 fuzzy set $\tilde{A}$ can represent a community. Therefore, the similarity degree of $\tilde{A}, \tilde{B}$ means the similarity of communities in ecology.

Definition 7. Suppose $\tilde{A}$ and $\tilde{B}$ be two different communities,

$$\mathrm{Sim}(\tilde{A}, \tilde{B}) = N(\tilde{A}, \tilde{B}) = (\tilde{A} \circ \tilde{B}) \wedge (\tilde{A} \hat{\circ} \tilde{B})^C \tag{20}$$

is defined as the similarity degree of community $\tilde{A}$ and $\tilde{B}$. Here $\tilde{A} \circ \tilde{B}$ represent the degree of which the population is subordinate to the ecotone, and $\tilde{A} \hat{\circ} \tilde{B}$ represent the degree that the population is subordinate to the outside ecotone. Obviously, when inner product is small, the product of ecotone is more small, the ecotone of community $\tilde{A}$ and $\tilde{B}$ is great, and the outside ecotone that the population scattered is more small. The community $\tilde{A}$ and $\tilde{B}$ is more similar.

4.6. *Separate degree of community*

Definition 8. Suppose $\tilde{A}$ and $\tilde{B}$ are two different communities,

$$\mathrm{Sep}(\tilde{A}, \tilde{B}) = 1 - \mathrm{Sim}(\tilde{A}, \tilde{B}) \tag{21}$$

is called the separate degree of community.

Since the separate degree and similarity degree of community are two similar concepts, we may have the following conclusions:

(1) If $0 < \mathrm{Sep}(\tilde{A}, \tilde{B}) < 1$, there will be overlap on resource axis. The competition between $\tilde{A}$ and $\tilde{B}$ happened within ecotone.
(2) If $\mathrm{Sep}(\tilde{A}, \tilde{B}) = 1$, i.e., $\mathrm{Sim}(\tilde{A}, \tilde{B}) = 0$. The degrees of similarity is 0, thus $\tilde{A}$ and $\tilde{B}$ are separated, there is almost no competition on this gradient.
(3) If $\mathrm{Sep}(\tilde{A}, \tilde{B}) = 0$, there exists certain population being the similar species of community $\tilde{A}$ and $\tilde{B}$.
(4) If $\mathrm{Sep}(\tilde{A}, \tilde{B})$ is larger, the hierarchy of niche overlap is large too. This would results in enlarging the ecotone of $\tilde{A}$ and $\tilde{B}$. Whereas, $\mathrm{Sep}(\tilde{A}, \tilde{B})$ is smaller, the hierarchy of niche overlap is smaller.
(5) Considering two community $\tilde{A}$ and $\tilde{B}$, whose membership function is

$$\mu_{\tilde{A}}(x) = \exp\left[-\frac{1}{2} \left(\frac{x - m}{\delta_1} \right)^2 \right], \quad m \in [m_1, m_2] \tag{22}$$

Table 1 – Environmental factors value in sample area

Sample area	Water contained in soil (%)	Height (m)
1	21.9	380
2	37.8	400
3	16.0	430
4	25.0	450
5	17.4	470
6	14.1	520
7	13.5	550

$$\mu_{\tilde{B}}(x) = \exp\left[-\frac{1}{2} \left(\frac{x - m'}{\delta_2} \right)^2 \right], m' \in [m'_1, m'_2] \tag{23}$$

where δ_1, δ_2 are the degree of decentralization in community. If δ_1, δ_2 are smaller, the distribution curves of community are more precipitous (Wang et al., 2004). The number and density of the population within the ecotone are greater than the neighbor one. This increased density tendency is called the edge effect of community.

5. Applications

Example 2. The field experiment of the research is carried out on experimental forestry centre of Northeast Forest University. The place locates at long. 127°30′–127°34′, lat. 45°20′–45°25′. The centre is 2040 m above sea level, the average temperature is 2.7 °C and the amount of rain is 720 mm. The area is a typical Natural Secondary Forests, which is composed with *Phellodendron amurense*, *Tilia amurensis*, *Populus davidiana*, *Betula platyphylla*, *Ulmus macroarpa*, *Acer mono* and so on. In typical forest community area, we choose continuous topography that has obvious habitual gradual. We set sample area according to height above sea level, each with area 20 m × 30 m, listed in Tables 1 and 2 (Ren, 1998).

The relative MF (Chen, 1993) on different latitude for each species is $\mu_{Ph}(x)$, $\mu_T(x)$, $\mu_{Pd}(x)$, $\mu_B(x)$, $\mu_U(x)$ and $\mu_A(x)$, i.e.,

$$\mu_{Ph}(x) = \exp\left[-\frac{1}{2} \left(\frac{x - m_1}{13.7653} \right)^2 \right] \quad m_1 \in [440, 460], \quad x \in [380, 550] \tag{24}$$

$$\mu_T(x) = \exp\left[-\frac{1}{2} \left(\frac{x - m_2}{8.1007} \right)^2 \right] \quad m_2 \in [450, 460], \quad x \in [380, 550] \tag{25}$$

$$\mu_{Pd}(x) = \exp\left[-\frac{1}{2} \left(\frac{x - m_3}{15.6108} \right)^2 \right] \quad m_3 \in [460, 480], \quad x \in [380, 550] \tag{26}$$

$$\mu_B(x) = \exp\left[-\frac{1}{2} \left(\frac{x - m_4}{11.6180} \right)^2 \right] \quad m_4 \in [460, 480], \quad x \in [380, 550] \tag{27}$$

$$\mu_U(x) = \exp\left[-\frac{1}{2}\left(\frac{x-m_5}{12.7989}\right)^2\right] \quad m_5 \in [460, 480],$$
$$x \in [380, 550] \tag{28}$$

$$\mu_A(x) = \exp\left[-\frac{1}{2}\left(\frac{x-m_6}{5}\right)^2\right] \quad m_6 \in (460, 480),$$
$$x \in [380, 550] \tag{29}$$

Then we get the upper and lower MF from Example 1 for each species on height. As for $\mu_{Ph}(x)$, we have

$$\bar{\mu}_{Ph}(x) = \begin{cases} \exp\left[-\frac{1}{2}\left(\frac{x-440}{13.7653}\right)^2\right], & x < 440 \\ 0.87, & 440 \le x \le 460 \\ \exp\left[-\frac{1}{2}\left(\frac{x-460}{13.7653}\right)^2\right], & x > 460 \end{cases},$$

$$\underline{\mu}_{Ph}(x) = \begin{cases} \exp\left[-\frac{1}{2}\left(\frac{x-460}{13.7653}\right)^2\right], & x \le 450 \\ \exp\left[-\frac{1}{2}\left(\frac{x-440}{13.7653}\right)^2\right], & x > 450 \end{cases}$$

Similarly, we can obtain the upper and lower MF for other species. Then, we may calculate the type-2 niche for every species on height. We may calculate the niche on 470 m for every species as following:

$$V_{\text{Ph-470}} = [0.093, 0.7681], \qquad V_{\text{T-470}} = [0.047, 0.4667],$$
$$V_{\text{Pd-470}} = [0.6342, 0.95], \qquad V_{\text{B-470}} = [0.036, 0.6904],$$
$$V_{\text{U-470}} = [0, 0], \qquad V_{\text{A-470}} = [0.6065, 1].$$

5.1. *Discussion*

Ren (1998) used the niche model by Cao and Hutchinson to calculate the niche for the above species. We list their results and those obtained in this paper in Table 3.

From Table 3 we can see that whichever algorithms are used, every result is contained in the results by the model proposed in this paper. Different from the former method, the result of proposed model is an interval value, which contains

Table 3 – Value of niche calculated using different model

Species	Cao	Hutchinson	Model by type-2 fuzzy sets
P. amurense	0.563	0.472	[0.093,0.7681]
T. amurensis	0.243	0.355	[0.047,0.4667]
Populus davidiana	0.876	0.790	[0.6342,0.95]
Betula platyphylla	0.311	0.412	[0.036,0.6904]
Ulmus macroarpa	0	0	[0,0]
Acer mono	0.901	0.836	[0.6065,1]

more information. We also notice that the niche for *Populus davidiana* and *Acer mono* is quite large, and the corresponding right-hand point of interval result is nearly toward 1, which indicates that the available resources are abundant for them and they have ability to utilize resources. Moreover, it reflected the range of niche affected by environment, as well as the communication between environment and species. Such kind of variety has the character of dynamic and medi-transition.

From the Definition 6, $\tilde{A}$ can be expressed as $\bigcup_{\lambda_i \in \Lambda} (\lambda_i/V_{\lambda_i})$. Therefore, the natural secondary forests in this example can be expressed as

$$\tilde{A} = \left(\frac{\lambda_{Ph}}{V_{Ph}}\right) \cup \left(\frac{\lambda_T}{V_T}\right) \cup \left(\frac{\lambda_{Pd}}{V_{Pd}}\right) \cup \left(\frac{\lambda_B}{V_B}\right) \cup \left(\frac{\lambda_U}{V_U}\right) \cup \left(\frac{\lambda_A}{V_A}\right) \cup \cdots$$

where $\lambda_{Ph}, \ldots, \lambda_A$ are the membership grades on niche interval; V_{Ph}, V_T, V_{Pd}, V_B, V_U, V_A are the type-2 niche of species.

In this example, natural secondary forests $\tilde{A}$ can be expressed as

$$\tilde{A} = \left(\frac{0.55}{[0.08, 0.87]}\right) \cup \left(\frac{0.6}{[0.03, 0.93]}\right) \cup \left(\frac{0.47}{[0.12, 0.95]}\right)$$
$$\cup \left(\frac{0.375}{0.03, 0.76}\right) \cup \left(\frac{0.42}{[0.01, 0.83]}\right) \cup \left(\frac{0.46}{[0.17, 1]}\right) \cup \cdots \tag{30}$$

From the above formula, we notice that the λ_{Ph} and λ_T is larger then others, while the corresponding niche is also larger. Such results indicate that the species *P. amurense* and *T. amurensis* has range distribution and they have a better adaptation to the environmental factors in this community, which is similar to the results in Ren's paper. Actually, the species *P. amurense* and *T. amurensis* have large niche width, which indicate the available resources, is abundant for them. Similarly, for the Artificial Forests community $\tilde{B}$, it can be expressed as the formation like $\tilde{A}$ by the Definition 6. Then, we obtain the ecotone Cr of natural secondary forests community $\tilde{A}$ and

Table 2 – Individual amounts in each sample area

Sample area	*Phellodendron amurense*	*Tilia amurensis*	*Populus davidiana*	*Betula platyphylla*	*Ulmus macroarpa*	*Acer mono*
1	0	0	0	0	10	0
2	0	0	0	0	14	0
3	2	2	4	13	8	6
4	5	12	11	15	2	13
5	2	5	19	12	0	16
6	1	1	4	0	0	3
7	0	0	0	0	0	0

artificial forests community$\tilde{B}$:

$$\mathrm{Cr} = \tilde{A} \cap \tilde{B} = \left(\bigcup_{\lambda_i \in \Lambda} \left(\frac{\lambda_i}{V_{\lambda_i}} \right) \right) \cap \left(\bigcup_{\lambda'_i \in \Lambda'} \left(\frac{\lambda'_i}{V_{\lambda'_i}} \right) \right) \tag{31}$$

6. Conclusion

In this paper, the dynamic model of niche is established by the broadband effect inherent in membership function of type-2 fuzzy set. We also proposed the model of hierarchic structure of community, ecotone, degree of similarity and separate. Under this model, the relationship between biotic community and niche could be clearly expressed. Via this model, it would better describe uncertainties and complexities inherent in ecosystem. The results of the experiment indicate the ones under classic model, but with more information.

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