Mechanism and Machine Theory 45 (2010) 239–250

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Design of decoupled parallel manipulators by means of the theory of screws

Victor Glazunov *

Mechanical Engineering Research Institute, Russian Academy of Sciences, 121354, Kutuzov st. 2, ap. 402, Moscow, Russia

ARTICLE INFO

Article history:
Received 25 March 2009
Received in revised form 4 September 2009
Accepted 7 September 2009
Available online 30 September 2009

Keywords:
Parallel manipulator
Decoupled mechanism
The theory of screws
Twist
Wrench
Singularities
Screw groups

ABSTRACT

This paper focuses on a study of the decoupling of parallel manipulators. Decoupled parallel manipulators with three parallel kinematic chains are considered. The translation kinematic pairs are designed as planar four-bar parallelograms. The synthesis of these mechanisms is carried out by means of screw groups. This approach allows avoiding completed equations by synthesis and singularity analysis of these mechanisms.

1. Introduction

It is well known that the closed-loops of parallel manipulators cause high stiffness and payload capacity [1–7]. However, due to the coupling between kinematic chains, control of the motions of the moving platform becomes complicated. There exist different solutions of this problem [2–5]. One of them is to arrange for coincidence of centers of the spherical kinematic pairs of three identical kinematic chains S–P–S of the Gough–Stewart platform [8,9]. Another solution corresponds to the special architecture of the U–P–U kinematic chains of a 6 degrees of freedom (6-DOF) parallel manipulator in which three U-joints mounted on the moving platform (end-effector) are designed as a spherical mechanism [10–12]. However, these solutions do not allow the retention of constant orientation of the moving platform when only the actuators corresponding to translation motion are driven.

Another approach to a solution of this problem is applicable for a parallel manipulator with reduced degrees of freedom. For example, the well known Delta robot consists of three R–R–P–R kinematic chains (the P-pair is designed as a four-bar planar parallelogram) causing translation motions of the moving platform and of one R–U–P–U kinematic chain, causing rotation about the vertical axis [13]. This robot corresponds to Schoenflies motions besides, in this robot, three translation motions and one rotation motion are decoupled. Another well known robot with Schoenflies motions of the platform is PAMINSA [14]. In this manipulator one vertical motion is decoupled from the three planar motions.

Note that the translation kinematic pairs can be represented as planar four-bar parallelograms [10–13,15–17]. By this approach numerous families of decoupled and translation parallel mechanisms are obtained [15–17].

A 6-DOF manipulator with decoupled translation and rotation motions and with linear and rotating actuators situated on the base is synthesized by means of geometrical constraints [18–20]. This manipulator consists of three kinematic R–P–P–P–R–R

* Tel.: +7 (495) 624 00 28; fax: +7 (495) 446 30 07.
E-mail address: vaglznv@mail.ru

0094-114X/$ - see front matter
doi:10.1016/j.mechmachtheory.2009.09.003

chains and allows retention of a constant orientation of the moving platform when only the linear actuators corresponding to the translation motion are driven.

In this paper, we use the approach to synthesis of decoupled parallel manipulators based on closed screw groups [21,22] that include all the screw products of the main members of these groups. These groups describe motions of non-overconstrained mechanisms. Similar screw groups are considered from different viewpoints [23–26]. A new decoupled 6-DOF parallel manipulator is proposed which consists of three kinematic R–P–P–P–R–R chains where the P-pairs are represented as four-bar planar parallelograms. This allows transferring of rotations without sliding. Besides decoupled manipulators with three DOF, four DOF and six DOF are obtained.

With regard to the determination of the singularity the Jacobian matrices relating the input speeds to the output speeds can be applied [27]. Using this approach one needs to differentiate equations expressing the constraints imposed by kinematic chains. Therefore in this paper we use the screw groups approach to describe singularities [28] which allows avoidance of complicated mathematical equations. As the screw groups can be obtained without complicated equations the style of the article can be chosen like a textbook on the subject.

2. Description of closed screw groups

Let us consider the closed screw groups [21,22] corresponding to planar, spherical, and spatial mechanisms. These groups include all the screw products of their main members. Without loss of generality we can use the simplest representation of the main screws (twists) of these groups by Plücker coordinates. Besides which, we consider only 1-DOF kinematic pairs.

The closed screw groups are:

- One-member screw group which can be represented by Plücker coordinates $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0) or $\boldsymbol{\Omega}_1$ (1, 0, 0, p, 0, 0) or $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0). This group corresponds to one 1-DOF kinematic pair, rotation, screw, or prismatic. Here, p is the pitch.
- Two-member screw group which can be represented by Plücker coordinates of the main members $\boldsymbol{\Omega}_1$ (1, 0, 0, p, 0, 0) and $\boldsymbol{\Omega}_2$ (0, 0, 0, 1, 0, 0), where p is the pitch. This group corresponds to two 1-DOF kinematic pairs (one of them can be prismatic) whose axes are situated along the same line, this expresses motions along and around one axis.
- Two-member screw group which can be represented by Plücker coordinates of the main members $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0) and $\boldsymbol{\Omega}_2$ (0, 0, 0, 0, 1, 0). This group corresponds to two prismatic kinematic pairs whose axes are not parallel. These express planar mechanisms with translation motions.
- Three-member screw group which can be represented by Plücker coordinates of the main members $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_2$ (0, 0, 0, 0, 1, 0) and $\boldsymbol{\Omega}_3$ (0, 0, 0, 0, 0, 1). This group corresponds to three prismatic kinematic pairs whose axes are not coplanar, in particular perpendicular to each other. These express translation mechanisms.
- Three-member screw group which can be represented by Plücker coordinates of the main members $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{\Omega}_2$ (0, 1, 0, 0, 0, 0) and $\boldsymbol{\Omega}_3$ (0, 0, 1, 0, 0, 0). This group corresponds to three rotation kinematic pairs whose axes intersect at one point. These express spherical mechanisms.
- Three-member screw group which can be represented by Plücker coordinates of the main members $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_2$ (0, 0, 0, 0, 1, 0) and $\boldsymbol{\Omega}_3$ (0, 0, 1 0, 0, p). This group corresponds to one screw kinematic pair and two prismatic kinematic pairs whose axes are perpendicular to each other. If $p = 0$ then one kinematic pair gives rotation and these express planar mechanisms.
- Four-member screw group which can be represented by Plücker coordinates of the main members $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{\Omega}_3$ (0, 0, 0, 0, 0, 1) and $\boldsymbol{\Omega}_4$ (0, 0, 1, 0, 0, p). This group corresponds to one screw kinematic pair and three prismatic kinematic pairs whose axes are not coplanar, but perpendicular to each other. These express the mechanisms of Schoenflies motions. The pitch p can be equal to zero.
- Six-member screw group which can be represented by Plücker coordinates of the main members $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{\Omega}_2$ (0, 1, 0, 0, 0, 0), $\boldsymbol{\Omega}_3$ (0, 0, 1, 0, 0, 0), $\boldsymbol{\Omega}_4$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_5$ (0, 0, 0, 0, 1, 0) and $\boldsymbol{\Omega}_6$ (0, 0, 0, 0, 0, 1). This group corresponds to three rotation kinematic pairs and three prismatic kinematic pairs. These express all the motions in space.

Note that all the screw products of the main screws of these groups are members of the same group. If all the motions of a rigid body are described by one of these groups then after any finite displacement of this body the screw group corresponding to all its motions will be the same as before motion. It means that a rigid body can be connected to the base by any number of kinematic chains corresponding to one of the closed screw groups and the degree of freedom will be determined by the number of the main members of this group. Parallel mechanisms corresponding to closed screw groups can be synthesized on this basis [7,26].

3. Structural synthesis by using closed screw groups

Let us consider parallel manipulators corresponding to three-member and four-member screw groups. We use the notification (Fig. 1): (a) actuated prismatic pair (linear actuator), (b) actuated rotation pair (rotating actuator), (c) twist of zero pitch, (d) twist of infinite pitch, (e) wrench of zero pitch, and (f) wrench of infinite pitch.

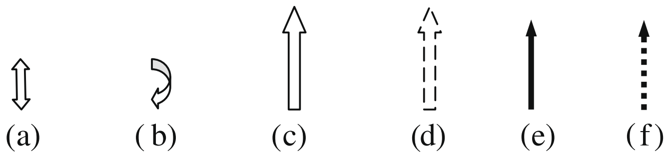

Fig. 1. Twists and wrenches.

Firstly we consider a translating parallel mechanism (Fig. 2a) which is analogous to one of the mechanisms represented in [15–17]. Each kinematic chain consists of one actuated prismatic pair (linear actuator) situated on the base and two prismatic kinematic pairs represented as four-bar parallelograms. The unit screws of the axes of these kinematic pairs have coordinates: $\boldsymbol{E}_{11}$ (0, 0, 0, 1, 0, 0), $\boldsymbol{E}_{12}$ (0, 0, 0, 0, e_{12y}, e_{12z}), $\boldsymbol{E}_{13}$ (0, 0, 0, 0, e_{13y}, e_{13z}), $\boldsymbol{E}_{21}$ (0, 0, 0, 0, 1, 0), $\boldsymbol{E}_{22}$ (0, 0, 0, e_{22x}, 0, e_{22z}), $\boldsymbol{E}_{23}$ (0, 0, 0, e_{23x}, 0, e_{23z}), $\boldsymbol{E}_{31}$ (0, 0, 0, 0, 0, 1), $\boldsymbol{E}_{32}$ (0, 0, 0, e_{32x}, e_{32y}, 0), $\boldsymbol{E}_{33}$ (0, 0, 0, e_{33x}, e_{33y}, 0).

All the screws are of infinite pitch. This mechanism is isotropic so that each actuator corresponds to one Cartesian coordinate x, y or z. All three kinematic chains impose the same constraints, so that one can insert other similar chains between the base and moving platform and the degree of freedom will remain equal to three. The wrenches of the constraints imposed by kinematic chains have coordinates (Fig. 2b): $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{R}_3$ (0, 0, 0, 0, 0, 1). All the twists of motions of the platform can be represented by the twists reciprocal to the wrenches of the imposed constraints (Fig. 2b): $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{\Omega}_3$ (0, 0, 0, 0, 0, 1). All three twists are of infinite pitch.

In this mechanism, singularities corresponding to loss of one degree of freedom exist if three screws $\boldsymbol{E}_{i1}$, E_{i2} and $\boldsymbol{E}_{i3}$ (i = 1, 2, 3) are linearly dependent, which is possible if any two screws $\boldsymbol{E}_{i2}$, E_{i3} are parallel. In particular, if $\boldsymbol{E}_{22}$ (0, 0, 0, 1, 0, 0) = $\boldsymbol{E}_{23}$ (0, 0, 0, 1, 0, 0) (Fig. 2c) then there exist four wrenches of constraints imposed by kinematic chains: $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{R}_3$ (0, 0, 0, 0, 0, 1) and $\boldsymbol{R}_4$ (0, 0, 1, 0, 0, 0) and only two twists of motion of the platform

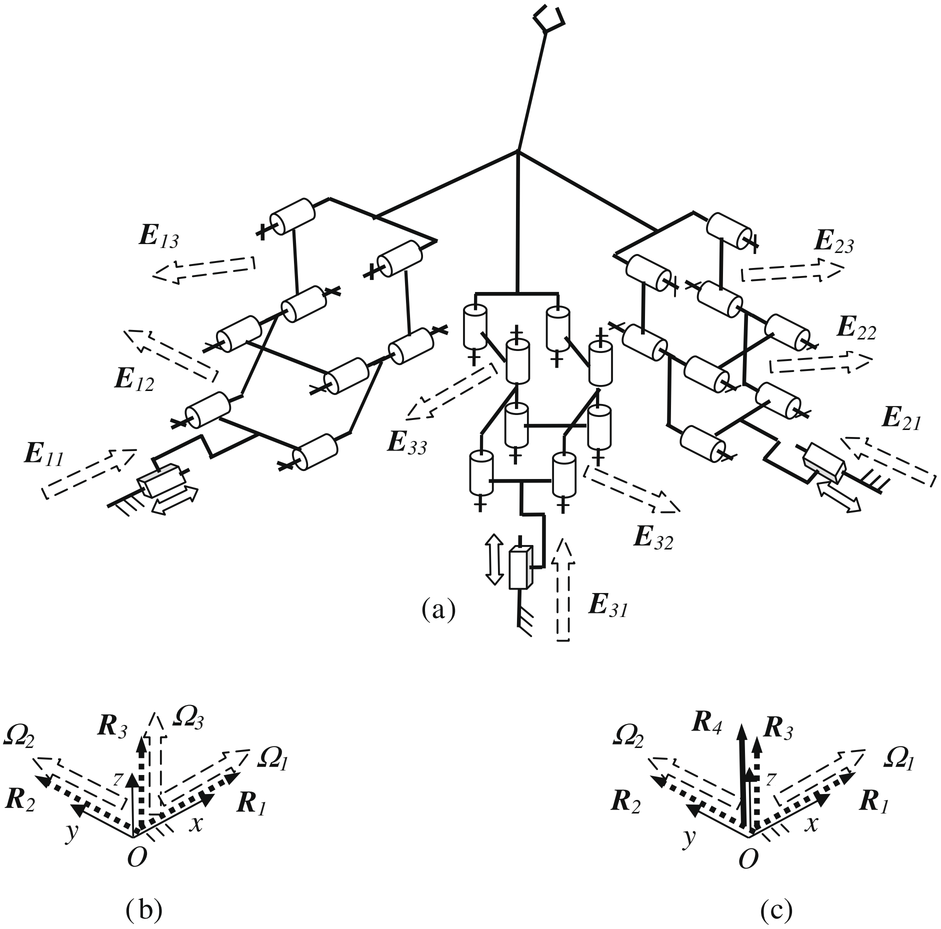

Fig. 2. Translational parallel mechanism.

reciprocal to these wrenches $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0) and $\boldsymbol{\Omega}_2$ (0, 0, 0, 0, 1, 0). If the parallelograms in each kinematic chain are replaced by general prismatic kinematic pairs then this singularity does not exist.

If the actuators are fixed then there exist six wrenches imposed by kinematic chains: $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{R}_3$ (0, 0, 0, 0, 0, 1), $\boldsymbol{R}_4$ (1, 0, 0, 0, 0, 0), $\boldsymbol{R}_5$ (0, 1, 0, 0, 0, 0), $\boldsymbol{R}_6$ (0, 0, 1, 0, 0, 0). The wrenches $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are of zero pitch.

Note that the parallelograms in each kinematic chain can be changed by three rotation pairs with axes parallel to the axis of the corresponding linear actuator. In this case the isotropic parallel mechanism will be obtained [3,15].

Now let us consider a spherical parallel mechanism (Fig. 3a). Each kinematic chain consists of one actuated rotation pair (rotating actuator) situated on the base and two passive rotation kinematic pairs. The unit screws of the axes of these kinematic pairs have coordinates (note that the origin of the coordinate system is the point O in which the axes of all the pairs intersect): $\boldsymbol{E}_{11}$ (1, 0, 0, 0, 0, 0), $\boldsymbol{E}_{12}$ (e_{12x}, e_{12y}, e_{12z}, 0, 0, 0), $\boldsymbol{E}_{13}$ (e_{13x}, e_{13y}, e_{13z}, 0, 0, 0), $\boldsymbol{E}_{21}$ (0, 1, 0, 0, 0, 0), $\boldsymbol{E}_{22}$ (e_{22x}, e_{22y}, e_{22z}, 0, 0, 0), $\boldsymbol{E}_{23}$ (e_{23x}, e_{23y}, e_{23z}, 0, 0, 0), $\boldsymbol{E}_{31}$ (0, 0, 1, 0, 0, 0), $\boldsymbol{E}_{32}$ (e_{32x}, $e_{32\,y}$, e_{32z}, 0, 0, 0), $\boldsymbol{E}_{33}$ (e_{33x}, e_{33y}, e_{33z}, 0, 0, 0).

All the screws are of zero pitch. All three kinematic chains impose the same constraints, so that one can insert other similar chains between the base and moving platform and the degree of freedom will remain equal to three. The wrenches of the constraints imposed by kinematic chains have coordinates (Fig. 3 b): $\boldsymbol{R}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{R}_2$ (0, 1, 0, 0, 0, 0), $\boldsymbol{R}_3$ (0, 0, 1, 0, 0, 0), these wrenches are of zero pitch. All the twists of motions of the platform can be represented by the twists reciprocal to the wrenches of the imposed constraints (Fig. 3b): $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{\Omega}_2$ (0, 1, 0, 0, 0, 0), $\boldsymbol{\Omega}_3$ (0, 0, 1, 0, 0, 0). All three twists are of zero pitch.

In this mechanism singularities expressed by loss of one degree of freedom exist if any three screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$ (i = 1, 2, 3) are linearly dependent which is possible if they are coplanar (they are situated in the same plane). In particular if the unit screws $\boldsymbol{E}_{11}$ (1, 0, 0, 0, 0, 0), $\boldsymbol{E}_{12}$ (e_{12x}, e_{12y}, $e_{12z,}$ 0, 0, 0), $\boldsymbol{E}_{13}$ (e_{13x}, e_{13y}, e_{13z}, 0, 0, 0) are coplanar (Fig. 3 c) then there exist four wrenches of constraints imposed by kinematic chains: $\boldsymbol{R}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{R}_2$ (0, 1, 0, 0, 0, 0), $\boldsymbol{R}_3$ (0, 0, 1, 0, 0, 0) and $\boldsymbol{R}_4$ (0, 0, 0, 0, r_{4y}, r_{4z}) and only two twists of motion of the platform reciprocal to these wrenches $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0) and $\boldsymbol{\Omega}_2$ (ω_{2x}, ω_{2y}, ω_{2z}, 0, 0, 0), these twists are of zero pitch. The wrench $\boldsymbol{R}_4$ is of infinite pitch, it is perpendicular to the axes $\boldsymbol{E}_{11}$, $\boldsymbol{E}_{12}$, $\boldsymbol{E}_{13}$.

If the actuators are fixed then there exist six wrenches imposed by kinematic chains: $\boldsymbol{R}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{R}_2$ (0, 1, 0, 0, 0, 0), $\boldsymbol{R}_3$ (0, 0, 1, 0, 0, 0), $\boldsymbol{R}_4$ (0, 0, 0, r_{4x} , r_{4y}, r_{4z}), $\boldsymbol{R}_5$ (0, 0, 0, r_{5x}, r_{5y}, r_{5z}) and $\boldsymbol{R}_6$ (0, 0, 0, r_{6x}, r_{6y}, r_{6z}). The wrenches $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are of infinite pitch. Singularities corresponding to non-controlled infinitesimal motion of the moving platform (end-effector) exist if the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are linearly dependent which is possible if the wrenches $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are coplanar. In this case the twist of zero pitch $\boldsymbol{\Omega}$ (ω_x, ω_y, ω_z, 0, 0, 0) exists which is perpendicular to the axes of the wrenches $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ and therefore reciprocal to all the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$.

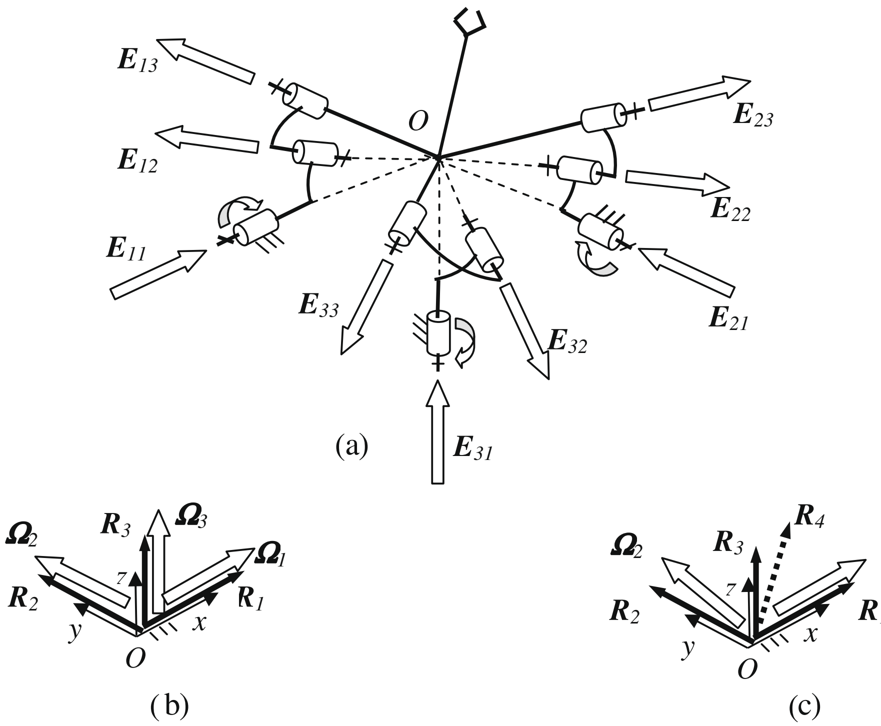

Fig. 3. Spherical parallel mechanism.

Moreover singularities exist corresponding both to loss of one degree of freedom and to non-controlled motion of the moving platform. By this any three screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$ ($i = 1, 2, 3$) and the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are linearly dependent.

Now let us consider a planar parallel mechanism (Fig. 4a). Each kinematic chain can consist of one rotation kinematic pair and two prismatic kinematic pairs (the axis of the rotation pair is perpendicular to the axes of the prismatic pairs), or of two rotation kinematic pair and one prismatic kinematic pair (the axes of the rotation pairs are parallel to each other and are perpendicular to the axis of the prismatic pair), or of three rotation kinematic pairs with parallel axes. In our mechanism two kinematic chains consist of three rotation kinematic pairs (one of them is actuated and situated on the base) and one kinematic chains consists of one actuated rotation kinematic pair situated on the base (rotating actuator) and two prismatic kinematic pairs represented as four-bar parallelograms. The unit screws of the axes of these kinematic pairs have coordinates: $\boldsymbol{E}_{11}$ (0, 0, 1, 0, 0, 0), $\boldsymbol{E}_{12}$ (0, 0, 1, e_{12x}, e_{12y}, 0), $\boldsymbol{E}_{13}$ (0, 0, 1, e_{13x}, e_{13y}, 0), $\boldsymbol{E}_{21}$ (0, 0, 1, 0, 0, 0), $\boldsymbol{E}_{22}$ (0, 0, 1, e_{22x}, e_{22y}, 0), $\boldsymbol{E}_{23}$ (0, 0, 1, e_{23x}, e_{23y}, 0), $\boldsymbol{E}_{31}$ (0, 0, 1, 0, 0, 0), $\boldsymbol{E}_{32}$ (0, 0, 0, e_{32x}, e_{32y}, 0), $\boldsymbol{E}_{33}$ (0, 0, 0, e_{33x}, e_{33y}, 0).

The screws $\boldsymbol{E}_{32}$ and $\boldsymbol{E}_{33}$ are of infinite pitch. All other screws are of zero pitch. All three kinematic chains impose the same constraints, so that one can insert other similar chains between the base and moving platform and the degree of freedom will remain equal to three. The wrenches of the constraints imposed by kinematic chains have coordinates (Fig. 4b): $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{R}_3$ (0, 0, 1, 0, 0, 0). All the twists of motions of the platform can be represented by the twists reciprocal to the wrenches of the imposed constraints (Fig. 4b): $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{\Omega}_3$ (0, 0, 1, 0, 0, 0). The twists $\boldsymbol{\Omega}_1$ and $\boldsymbol{\Omega}_2$ are of infinite pitch, the twist $\boldsymbol{\Omega}_3$ is of zero pitch.

In this mechanism singularities corresponding to loss of one degree of freedom exist if three screws $\boldsymbol{E}_{i1}$, E_{i2} and $\boldsymbol{E}_{i3}$ ($i = 1, 2, 3$) are linearly dependent which is possible if three screws $\boldsymbol{E}_{i1}$, E_{i2} and E_{i3} ($i = 1, 2$) are situated in the same plane or if two screws $\boldsymbol{E}_{32}$, $\boldsymbol{E}_{33}$ are parallel. In particular if $\boldsymbol{E}_{32} = \boldsymbol{E}_{33}$ (Fig. 4 c) then there exist four wrenches of constraints imposed by the kinematic chains: $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{R}_3$ (0, 0, 1, 0, 0, 0), $\boldsymbol{R}_4$ (r_{4x}, r_{4y}, 0, 0, 0, 0) and only two twists of motion of the platform reciprocal to these wrenches $\boldsymbol{\Omega}_1$ (0, 0, 0, v_{1x}, v_{1y}, 0) and $\boldsymbol{\Omega}_2$ (0, 0, 1, 0, 0, 0). Note that $\boldsymbol{R}_4$ is perpendicular to $\boldsymbol{E}_{32}$ and $\boldsymbol{E}_{33}$, and $\boldsymbol{\Omega}_1$ is parallel to them.

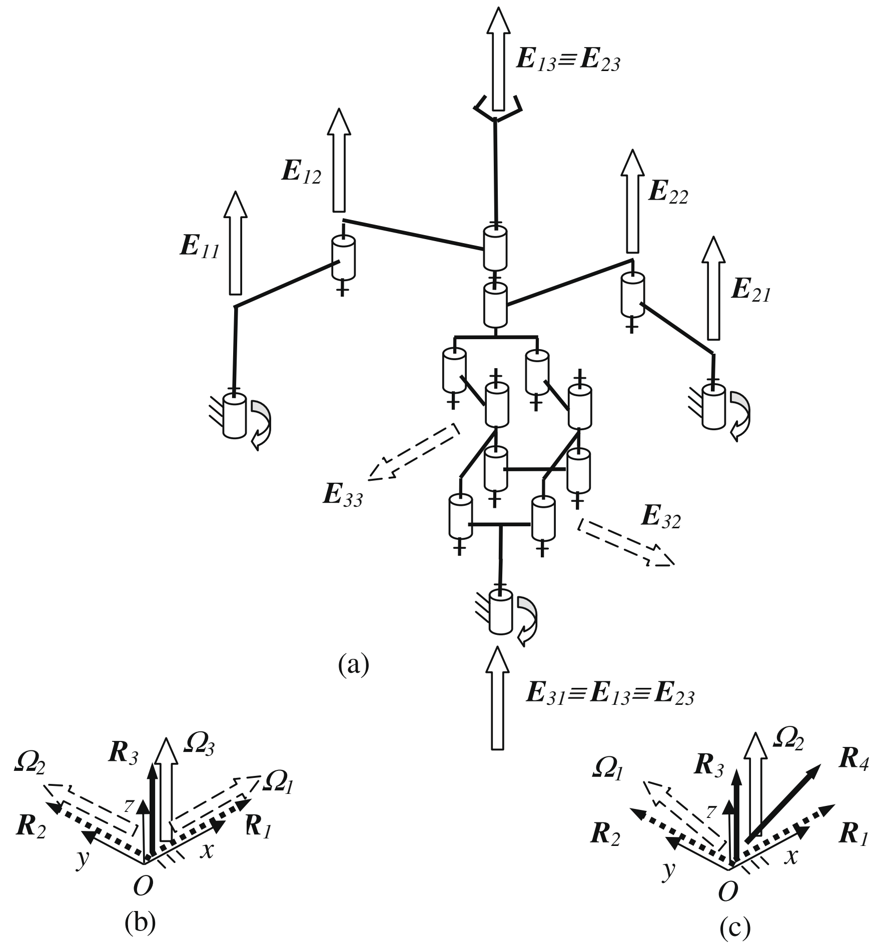

Fig. 4. Planar parallel mechanism.

If the actuators are fixed then there exist six wrenches imposed by the kinematic chains: $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{R}_3$ (0, 0, 1, 0, 0, 0), $\boldsymbol{R}_4$ (r_{4x}, r_{4y}, 0, 0, 0, 1), $\boldsymbol{R}_5$ (r_{5x}, r_{5y}, 0, 0, 0, 1) and $\boldsymbol{R}_6$ (0, 0, 0, 0, 0, 1). The wrenches $\boldsymbol{R}_4$ and $\boldsymbol{R}_5$ are of zero pitch, they are situated along the axes of the links connecting passive rotation pairs of the first and the second kinematic chains. $\boldsymbol{R}_6$ is of infinite pitch. Singularities corresponding to non-controlled infinitesimal motions of the moving platform exist if the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are linearly dependent which is possible if the wrenches $\boldsymbol{R}_4$, and $\boldsymbol{R}_5$ coincide. In this case the twist of infinite pitch $\boldsymbol{\Omega}$ (0, 0, 0, v_x, v_y, 0) exists which is perpendicular to the axes of the wrenches $\boldsymbol{R}_4$ and $\boldsymbol{R}_5$ and therefore reciprocal to all the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$.

Note that singularities exist corresponding both to loss of one degree of freedom and to non-controlled infinitesimal motion of the moving platform. By this any three screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$ (i = 1, 2, 3) and the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are linearly dependent.

This mechanism is particularly decoupled. The matter is that in the third kinematic chain the input link of the first parallelogram and the output link of the second parallelogram are connected correspondingly to the rotating actuator ant to the end-effector in the middles of these links, and the output link of the first parallelogram and the input link of the second parallelogram coincide. It causes that the first and the second actuators drive the position of the end-effector. The third actuator drives the orientation of the end-effector.

Now let us consider a parallel mechanism of Schoenflies motions (Fig. 5a). The first and the second kinematic chains consist of one actuated prismatic pair (linear actuator) situated on the base, two prismatic kinematic pairs represented as four-bar parallelograms and one rotation kinematic pair (the axes of rotation pairs of these two chains coincide). The third kinematic chain consists of one actuated rotation pair (rotating actuator) situated on the base, one actuated prismatic pair (linear actuator) (the axes of rotating and linear actuators coincide) and two prismatic kinematic pairs represented as four-bar parallelograms. The unit screws of the axes of these kinematic pairs have coordinates: $\boldsymbol{E}_{11}$ (0, 0, 0, 1, 0, 0), $\boldsymbol{E}_{12}$ (0, 0, 0, 0, e_{12y}, e_{12z}), $\boldsymbol{E}_{13}$ (0, 0, 0, 0, e_{13y}, e_{13z}), $\boldsymbol{E}_{14}$ (0, 0, 1, 0, 0, 0), $\boldsymbol{E}_{21}$ (0, 0, 0, 0, 1, 0), $\boldsymbol{E}_{22}$ (0, 0, 0, e_{22x}, 0, e_{22z}), $\boldsymbol{E}_{23}$ (0, 0, 0, e_{23x}, 0, e_{23z}), $\boldsymbol{E}_{24}$ (0, 0, 1, 0, 0, 0), $\boldsymbol{E}_{31}$ (0, 0, 1, 0, 0, 0), $\boldsymbol{E}_{32}$ (0, 0, 0, 0, 0, 1), $\boldsymbol{E}_{33}$ (0, 0, 0, e_{33x}, e_{33y}, 0), $\boldsymbol{E}_{34}$ (0, 0, 0, e_{34x}, e_{34y}, 0).

The screws $\boldsymbol{E}_{11}$, $\boldsymbol{E}_{12}$, $\boldsymbol{E}_{13}$, $\boldsymbol{E}_{21}$, $\boldsymbol{E}_{22}$, $\boldsymbol{E}_{23}$, $\boldsymbol{E}_{32}$, $\boldsymbol{E}_{33}$ and $\boldsymbol{E}_{34}$ are of infinite pitch. All other screws are of zero pitch. All three kinematic chains impose the same constraints, so that one can insert other similar chains between the base and moving platform and the degree of freedom will remain equal to four. The wrenches of the constraints imposed by kinematic chains have coordinates (Fig. 5b): $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0). All the twists of motions of the platform can be represented by the twists reciprocal to the wrenches of the imposed constraints (Fig. 5b): $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{\Omega}_3$ (0, 0, 0, 0, 0, 1), $\boldsymbol{\Omega}_4$ (0, 0, 1, 0, 0, 0). The twists $\boldsymbol{\Omega}_1$, $\boldsymbol{\Omega}_2$ and $\boldsymbol{\Omega}_3$ are of infinite pitch, the twist $\boldsymbol{\Omega}_4$ is of zero pitch.

In this mechanism singularities corresponding to loss of one degree of freedom exist if four screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, E_{i3} and $\boldsymbol{E}_{i4}$ (i = 1, 2, 3) are linearly dependent which is possible if any two screws $\boldsymbol{E}_{12}$ and $\boldsymbol{E}_{13}$, or $\boldsymbol{E}_{22}$ and $\boldsymbol{E}_{23}$, or $\boldsymbol{E}_{33}$ and $\boldsymbol{E}_{34}$ are parallel. In particular if $\boldsymbol{E}_{22}$ (0, 0, 0, 1, 0, 0) = $\boldsymbol{E}_{23}$ (0, 0, 0, 1, 0, 0) (Fig. 5c) then there exist three wrenches of constraints imposed by the kinematic chains: $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0) and $\boldsymbol{R}_3$ (0, 0, 1, 0, 0, 0) and only three twists of motion of the platform reciprocal to these wrenches $\boldsymbol{\Omega}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_1$ (0, 0, 0, 0, 1, 0) and $\boldsymbol{\Omega}_3$ (0, 0, 1, 0, 0, 0). Note that $\boldsymbol{R}_3$ is perpendicular to $\boldsymbol{E}_{22}$ and $\boldsymbol{E}_{23}$, and $\boldsymbol{\Omega}_1$ is parallel to them.

If the actuators are fixed then there exist six wrenches imposed by the kinematic chains: $\boldsymbol{R}_1$ (0, 0, 0, 1, 0, 0), $\boldsymbol{R}_2$ (0, 0, 0, 0, 1, 0), $\boldsymbol{R}_3$ (1, 0, 0, 0, 0, 0), $\boldsymbol{R}_4$ (0, 1, 0, 0, 0, 0), $\boldsymbol{R}_5$ (0, 0, 0, 0, 0, 1) and $\boldsymbol{R}_6$ (0, 0, 1, 0, 0, 0). The wrenches $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_6$ are of zero pitch, the wrench $\boldsymbol{R}_5$ is of infinite pitch.

This mechanism is particularly decoupled and isotropic. Each linear actuator controls the motion of the platform along one Cartesian coordinate. In the third kinematic chain the input link of the first parallelogram and the output link of the second parallelogram are connected correspondingly to the rotating actuator ant to the end-effector in the middles of these links, and the output link of the first parallelogram and the input link of the second parallelogram coincide. It causes that the rotating actuator drives the orientation of the end-effector. The linear actuators drive the position of the end-effector.

These mechanisms will be used for synthesis of 6-DOF parallel decoupled manipulators.

4. Structural synthesis of 6-DOF decoupled parallel mechanisms

Let us consider 6-DOF parallel decoupled manipulators synthesized by using of the mechanisms represented above. The condition of decoupling is that the linear actuators control only translation motions and rotating actuators control only orientation motions. If linear actuators are fixed then the position of the moving platform is fixed. If rotating actuators are fixed then the orientation of the moving platform is fixed. The approach which we use is combining of 3-DOF parallel translation and orientation mechanisms.

Firstly we consider 6-DOF parallel mechanism (Fig. 6a) 3 P–P–P–R–R–R. Each kinematic chain consists of one actuated prismatic pair (linear actuator) situated on the base, two prismatic kinematic pairs represented as four-bar parallelograms, one actuated rotation pair (rotating actuator) and two passive rotation pairs. The axes of all the rotation pairs intersect in the same point O which is the origin of the coordinate system. This point O is movable but the directions of the coordinate axes are constant.

The unit screws of the axes of these kinematic pairs have coordinates: $\boldsymbol{E}_{11}$ (0, 0, 0, 1, 0, 0), $\boldsymbol{E}_{12}$ (0, 0, 0, 0, e_{12y}, e_{12z}), $\boldsymbol{E}_{13}$ (0, 0, 0, 0, e_{13y}, e_{13z}), $\boldsymbol{E}_{14}$ (1, 0, 0, 0, 0, 0), $\boldsymbol{E}_{15}$ (e_{12x}, e_{12y}, e_{12z}, 0, 0, 0), $\boldsymbol{E}_{16}$ (e_{16x}, e_{16y}, e_{16z}, 0, 0, 0), $\boldsymbol{E}_{21}$ (0, 0, 0, 0, 1, 0), $\boldsymbol{E}_{22}$ (0, 0, 0, e_{22x}, 0, e_{22z}), $\boldsymbol{E}_{23}$ (0, 0, 0, e_{23x}, 0, e_{23z}), $\boldsymbol{E}_{24}$ (0, 1, 0, 0, 0, 0), $\boldsymbol{E}_{25}$ (e_{25x}, e_{25y}, e_{25z}, 0, 0, 0), $\boldsymbol{E}_{26}$ (e_{26x}, e_{26y}, e_{26z}, 0, 0, 0), $\boldsymbol{E}_{31}$

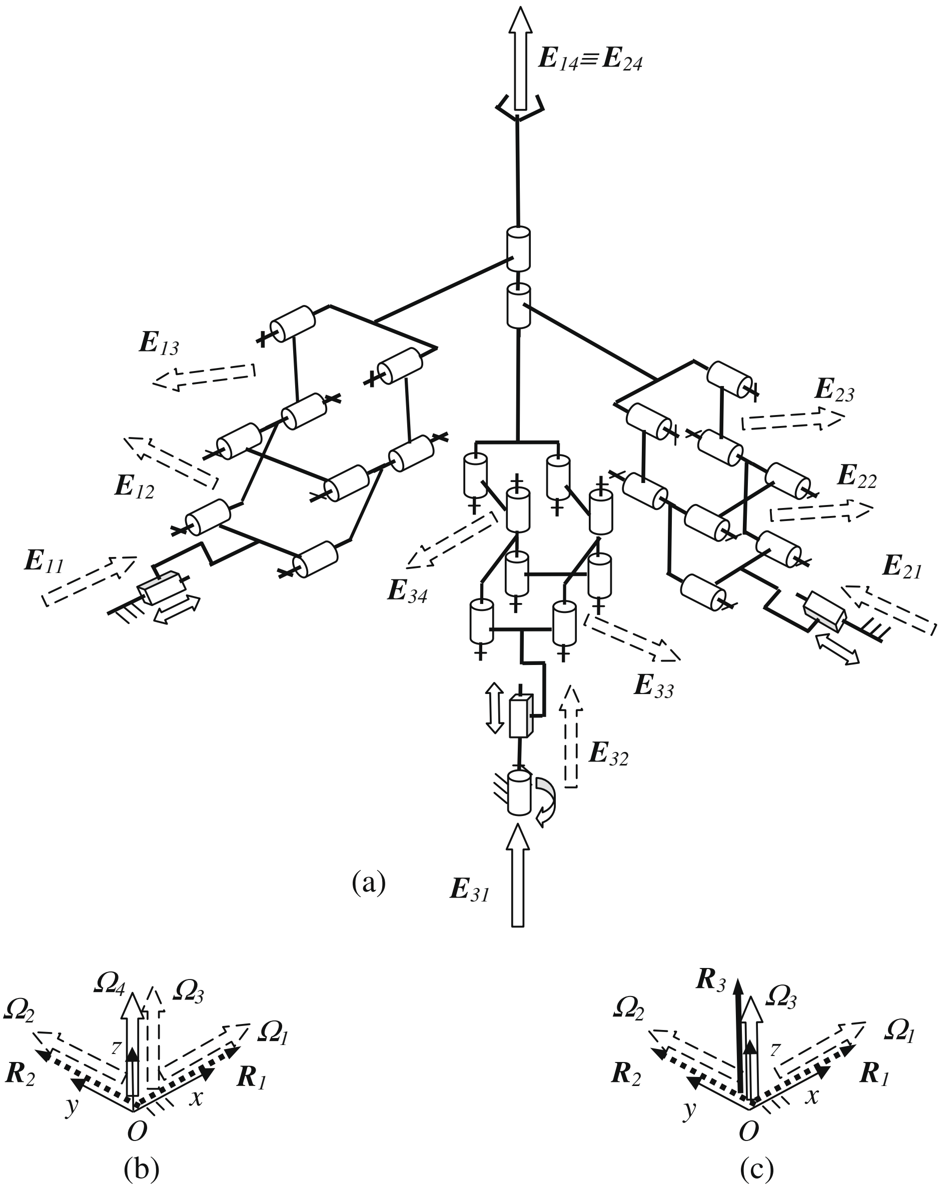

Fig. 5. Schoenflies motion parallel mechanism.

(0, 0, 0, 0, 0, 1), $\boldsymbol{E}_{32}$ (0, 0, 0, e_{32x}, e_{32y}, 0), $\boldsymbol{E}_{33}$ (0, 0, 0, e_{33x}, e_{33y}, 0). $\boldsymbol{E}_{34}$ (0, 0, 1, 0, 0, 0), $\boldsymbol{E}_{35}$ (e_{35x}, e_{35y}, e_{35z}, 0, 0, 0), $\boldsymbol{E}_{36}$ (e_{36x}, e_{36y}, e_{36z}, 0, 0, 0). The screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$ are of infinite pitch, the screws $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ are of zero pitch (i = 1, 2, 3). This mechanism is particularly isotropic so that each linear actuator corresponds to one Cartesian coordinate x, y or z.

All six twists of motions of the platform can be represented as: $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{\Omega}_2$ (0, 1, 0, 0, 0, 0), $\boldsymbol{\Omega}_3$ (0, 0, 1, 0, 0, 0), $\boldsymbol{\Omega}_4$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_5$ (0, 0, 0, 0, 1, 0), $\boldsymbol{\Omega}_6$ (0, 0, 0, 0, 0, 1).

In this mechanism singularities expressed by loss of one or more degrees of freedom exist if any six screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ (i = 1, 2, 3) are linearly dependent which is possible if any two screws $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$ are parallel or if any three screws $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ are coplanar. In particular if $\boldsymbol{E}_{22}$ (0, 0, 0, 1, 0, 0) = $\boldsymbol{E}_{23}$ (0, 0, 0, 1, 0, 0) then there exist one wrench of the constraint imposed by the second kinematic chain: $\boldsymbol{R}$ (0, 0, 1, 0, 0, 0) and only five twists of motion of the platform reciprocal to this wrench $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{\Omega}_2$ (0, 1, 0, 0, 0, 0), $\boldsymbol{\Omega}_3$ (0, 0, 1, 0, 0, 0), $\boldsymbol{\Omega}_4$ (0, 0, 0, 1, 0, 0) and $\boldsymbol{\Omega}_5$ (0, 0, 0, 0, 1, 0). If the unit screws $\boldsymbol{E}_{14}$ (1, 0, 0, 0, 0, 0), $\boldsymbol{E}_{15}$ (e_{15x}, e_{15y}, e_{15z}, 0, 0, 0), $\boldsymbol{E}_{16}$ (e_{16x}, e_{16y}, e_{16z}, 0, 0, 0) are coplanar then there exist one wrench of constraint imposed by the first kinematic chain: $\boldsymbol{R}$ (0, 0, 0, 0, r_y, r_z) and only five twists of motion of the platform reciprocal to this wrench $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{\Omega}_2$ (ω_{2x}, ω_{2y}, ω_{2z}, 0, 0, 0), $\boldsymbol{\Omega}_3$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_4$ (0, 0, 0, 0, 1, 0), $\boldsymbol{\Omega}_5$ (0, 0, 0, 0, 0, 1). The wrench $\boldsymbol{R}$ is of infinite pitch, it is perpendicular to the axes $\boldsymbol{E}_{14}$, $\boldsymbol{E}_{15}$, $\boldsymbol{E}_{16}$.

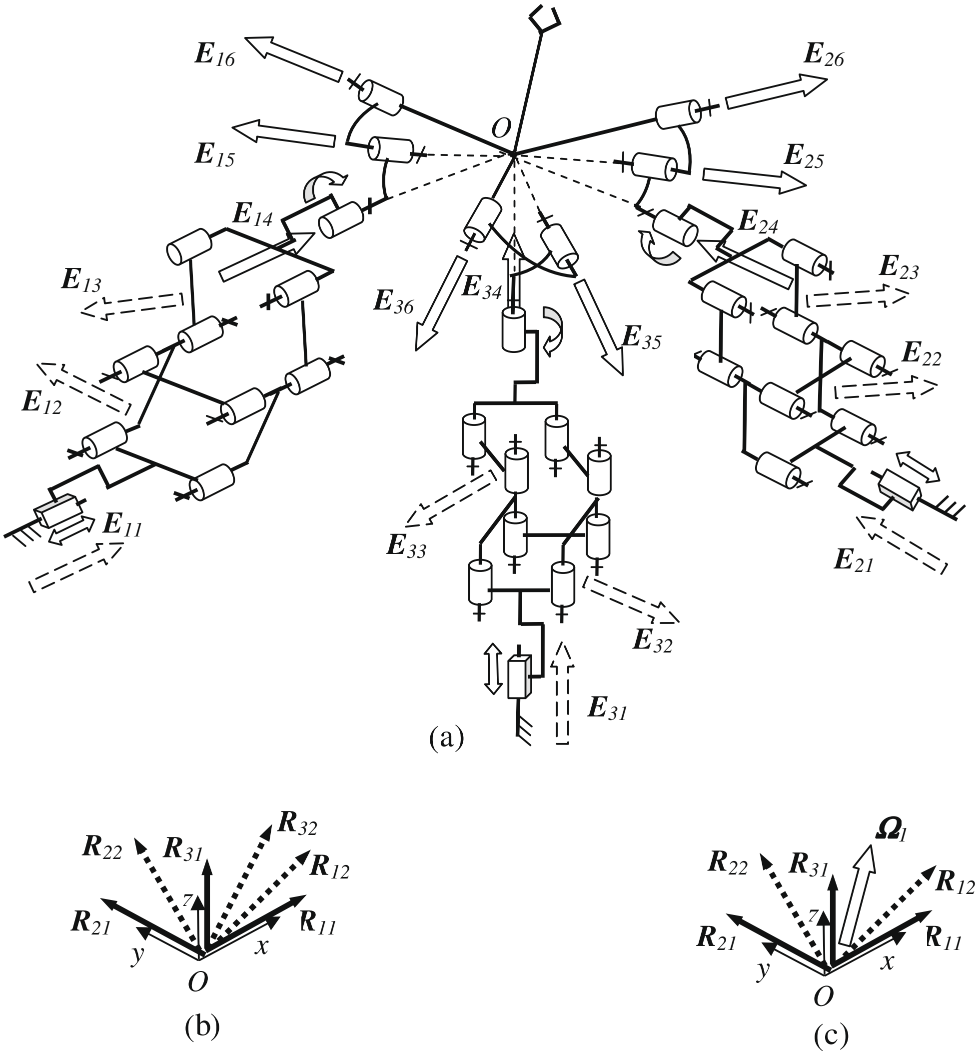

Fig. 6. 6-DOF parallel mechanism 3 P–P–P–R–R–R.

If the actuators are fixed then the wrenches of the constraints imposed by the kinematic chains have coordinates (Fig. 6b): $\boldsymbol{R}_{11}$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{R}_{12}$ $(0, 0, 0, r_{12x}, r_{12y}, r_{12z})$, $\boldsymbol{R}_{21}$ $(0, 1, 0, 0, 0, 0)$, $\boldsymbol{R}_{22}$ $(0, 0, 0, r_{22x}, r_{22y}, r_{22z})$, $\boldsymbol{R}_{31}$ $(0, 0, 1, 0, 0, 0)$, $\boldsymbol{R}_{32}$ $(0, 0, 0, r_{32x}, r_{32y}, r_{32z})$. The wrenches $\boldsymbol{R}_{i1}$ and $\boldsymbol{R}_{i2}$ are imposed by the ith kinematic chain. The wrenches $\boldsymbol{R}_{i1}$ are of zero pitch, the wrenches $\boldsymbol{R}_{i2}$ are of infinite pitch ($i = 1, 2, 3$). Singularities corresponding to non-controlled infinitesimal motion of the moving platform exist if the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{22}$, $\boldsymbol{R}_{31}$, $\boldsymbol{R}_{32}$ are linearly dependent. It is possible if the wrenches $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{22}$, $\boldsymbol{R}_{32}$ are coplanar (Fig. 6c). In this case the twist of zero pitch $\boldsymbol{\Omega}$ $(\omega_x, \omega_y, \omega_z, 0, 0, 0)$ exists which is perpendicular to the axes of the wrenches $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{22}$, $\boldsymbol{R}_{32}$ and therefore reciprocal to all the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{22}$, $\boldsymbol{R}_{31}$, $\boldsymbol{R}_{32}$.

In this mechanism singularities exist corresponding both to loss of one degree of freedom and to non-controlled infinitesimal motion of the moving platform. By this any six screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ (i = 1, 2, 3) and the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are linearly dependent. The corresponding conditions are represented above.

Now let us consider 6-DOF mechanism 3 R–R–R–P–P–P in which the rotating actuators are situated on the base and the linear actuators are situated in moving kinematic pairs (Fig. 7a). Each kinematic chain consists of one actuated rotation pair (rotating actuator), two passive rotation pairs, one actuated prismatic pair (linear actuator) and two prismatic kinematic pairs represented as four-bar parallelograms. The axes of all the rotation pairs intersect in the same point O which is the origin of the coordinate system.

The position of this point O is constant but the directions of the coordinate axes rotate corresponding to the rotations of the axes of the linear actuators. The unit screws of the axes of these kinematic pairs have coordinates: $\boldsymbol{E}_{11}$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{E}_{12}$ $(e_{12x}, e_{12y}, e_{12z}, 0, 0, 0)$, $\boldsymbol{E}_{13}$ $(e_{13x}, e_{13y}, e_{13z}, 0, 0, 0)$, $\boldsymbol{E}_{14}$ $(0, 0, 0, 1, 0, 0)$, $\boldsymbol{E}_{15}$ $(0, 0, 0, 0, e_{15y}, e_{15z})$, $\boldsymbol{E}_{16}$ $(0, 0, 0, 0, e_{16y}, e_{16z})$, $\boldsymbol{E}_{21}$ $(0, 1, 0, 0, 0, 0)$, $\boldsymbol{E}_{22}$ $(e_{22x}, e_{22y}, e_{22z}, 0, 0, 0)$, $\boldsymbol{E}_{23}$ $(e_{23x}, e_{23y}, e_{23z}, 0, 0, 0)$, $\boldsymbol{E}_{24}$ $(0, 0, 0, 0, 1, 0)$, $\boldsymbol{E}_{25}$ $(0, 0, 0, e_{25x}, 0, e_{25z})$, $\boldsymbol{E}_{26}$

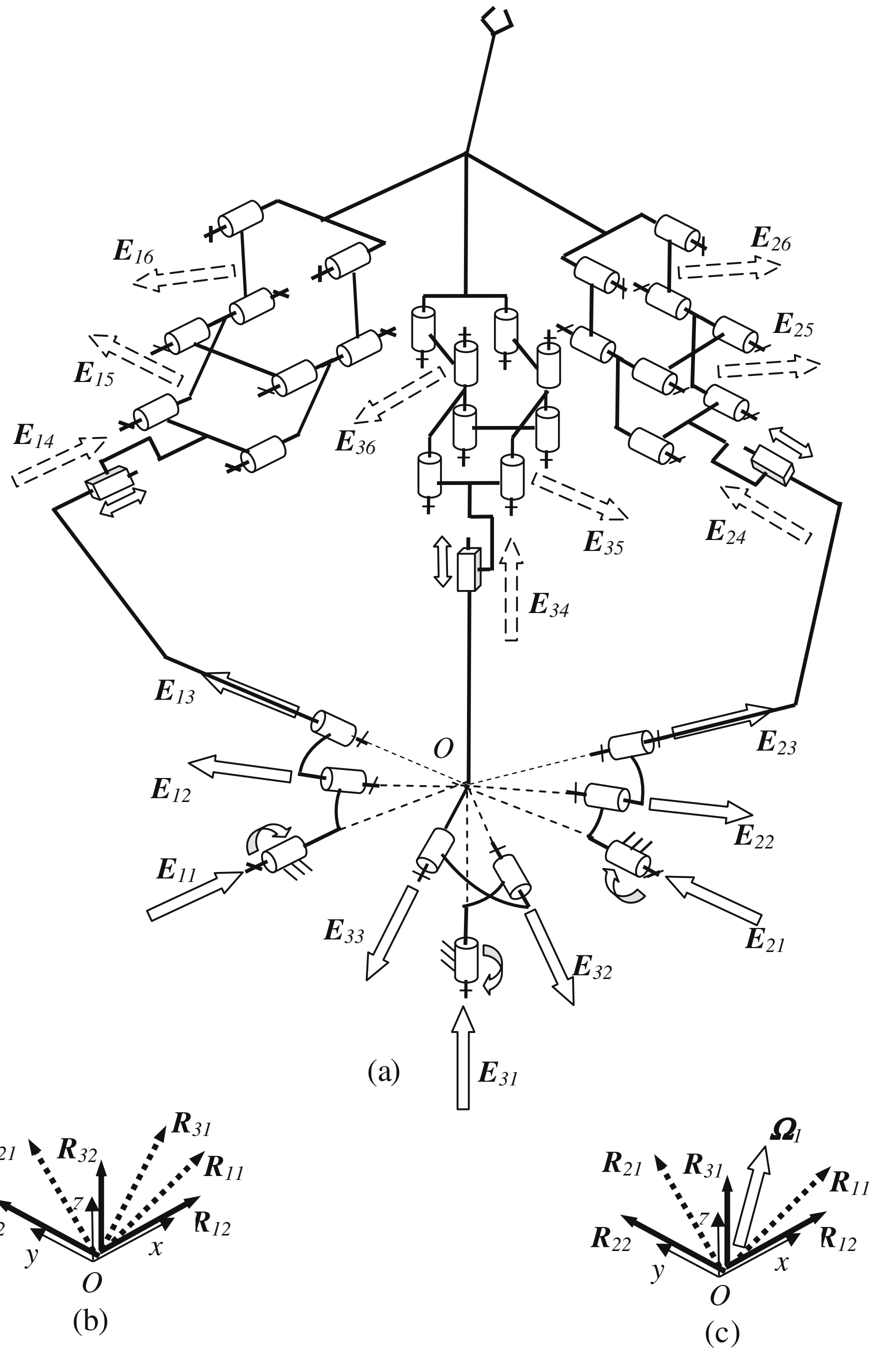

Fig. 7. 6-DOF parallel mechanism 3 R–R–R–P–P–P.

$(0, 0, 0, e_{26x}, 0, e_{26z})$, $\boldsymbol{E}_{31}$ $(0, 0, 1, 0, 0, 0)$, $\boldsymbol{E}_{32}$ $(e_{32x}, e_{32y}, e_{32z}, 0, 0, 0)$, $\boldsymbol{E}_{33}$ $(e_{33x}, e_{33y}, e_{33z}, 0, 0, 0)$, $\boldsymbol{E}_{34}$ $(0, 0, 0, 0, 0, 1)$, $\boldsymbol{E}_{35}$ $(0, 0, 0, e_{35x}, e_{35y}, 0)$, $\boldsymbol{E}_{36}$ $(0, 0, 0, e_{36x}, e_{36y}, 0)$. The screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$ are of zero pitch, the screws $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ are of infinite pitch ($i = 1, 2, 3$). This mechanism is particularly decoupled as the position of the point O is constant by any motions in linear actuators.

All six twists of motions of the platform can be represented as: $\boldsymbol{\Omega}_1$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{\Omega}_2$ $(0, 1, 0, 0, 0, 0)$, $\boldsymbol{\Omega}_3$ $(0, 0, 1, 0, 0, 0)$, $\boldsymbol{\Omega}_4$ $(0, 0, 0, 1, 0, 0)$, $\boldsymbol{\Omega}_5$ $(0, 0, 0, 0, 1, 0)$, $\boldsymbol{\Omega}_6$ $(0, 0, 0, 0, 0, 1)$.

In this mechanism singularities expressed by loss of one or more degrees of freedom exist if any six screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ ($i = 1, 2, 3$) are linearly dependent which is possible if any two screws $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ are parallel or if any three screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$ are coplanar. In particular if $\boldsymbol{E}_{25}$ $(0, 0, 0, 1, 0, 0) = \boldsymbol{E}_{26}$ $(0, 0, 0, 1, 0, 0)$ then there exist one wrench of the constraint imposed by the second kinematic chain: $\boldsymbol{R}$ (0, 0, 1, 0, 0, 0) and only five twists of motion of the platform reciprocal to this wrench $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{\Omega}_2$ (0, 1, 0, 0, 0, 0), $\boldsymbol{\Omega}_3$ (0, 0, 1, 0, 0, 0), $\boldsymbol{\Omega}_4$ (0, 0, 0, 1, 0, 0) and $\boldsymbol{\Omega}_5$ (0, 0, 0, 0, 1, 0). If the unit screws $\boldsymbol{E}_{11}$ (1, 0, 0, 0, 0, 0), $\boldsymbol{E}_{12}$ ($e_{12x}, e_{12y}, e_{12z}$, 0, 0, 0), $\boldsymbol{E}_{13}$ ($e_{13x}, e_{13y}, e_{13z}$, 0, 0, 0) are coplanar then there exist one wrench of the constraint imposed by the first kinematic chain: $\boldsymbol{R}$ (0, 0, 0, 0, r_y, r_z) and only five twists of motion of the platform reciprocal to this wrench $\boldsymbol{\Omega}_1$ (1, 0, 0, 0, 0, 0), $\boldsymbol{\Omega}_2$ ($\omega_{2x}, \omega_{2y}, \omega_{2z}$, 0, 0, 0), $\boldsymbol{\Omega}_3$ (0, 0, 0, 1, 0, 0), $\boldsymbol{\Omega}_4$ (0, 0, 0, 0, 1, 0), $\boldsymbol{\Omega}_5$ (0, 0, 0, 0, 0, 1). The wrench $\boldsymbol{R}$ is of infinite pitch, it is perpendicular to the axes $\boldsymbol{E}_{11}$, $\boldsymbol{E}_{12}$, $\boldsymbol{E}_{13}$.

If the actuators are fixed then the wrenches of the constraints imposed by kinematic chains have coordinates (Fig. 7b): $\boldsymbol{R}_{11}$ (0, 0, 0, $r_{11x}, r_{11y}, r_{11z}$), $\boldsymbol{R}_{12}$ (1, 0, 0, 0, 0, 0), $\boldsymbol{R}_{21}$ (0, 0, 0, $r_{21x}, r_{21y}, r_{21z}$), $\boldsymbol{R}_{22}$ (0, 1, 0, 0, 0, 0), $\boldsymbol{R}_{31}$ (0, 0, 0, $r_{31x}, r_{31y}, r_{31z}$), $\boldsymbol{R}_{32}$ (0, 0, 1, 0, 0, 0). The wrenches $\boldsymbol{R}_{i1}$ and $\boldsymbol{R}_{i2}$ are imposed by the ith kinematic chain. The wrenches $\boldsymbol{R}_{i1}$ are of infinite pitch, the wrenches $\boldsymbol{R}_{i2}$ are of zero pitch (i = 1, 2, 3). The wrenches $\boldsymbol{R}_{i1}$ are perpendicular to the screws $\boldsymbol{E}_{i2}$ and $\boldsymbol{E}_{i3}$.

Singularities corresponding to non-controlled infinitesimal motion of the moving platform exist if the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{22}$, $\boldsymbol{R}_{31}$, $\boldsymbol{R}_{32}$ are linearly dependent. It is possible if the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{31}$ are coplanar (Fig. 7c). In this case the twist of zero pitch $\boldsymbol{\Omega}$ ($\omega_x, \omega_y, \omega_z$, 0, 0, 0) exists which is perpendicular to the axes of the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{31}$ and therefore reciprocal to all the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{22}$, $\boldsymbol{R}_{31}$, $\boldsymbol{R}_{32}$.

Note that in this mechanism also singularities exist corresponding both to loss of one degree of freedom and to non-controlled infinitesimal motion of the moving platform. By this any six screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ (i = 1, 2, 3) and the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are linearly dependent. The corresponding conditions are represented above.

Obviously it is more preferable to situate the actuators as close to the base as possible. One can direct the axes of the linear actuator and of the rotating actuator along the same line. Let us consider 6-DOF mechanism 3 R–P–P–P–R–R in which the rotating actuators are situated on the base and the axes of linear actuators coincide with the axes of rotating actuators. (Fig. 8a). Each kinematic chain consists of one actuated rotation pair (rotating actuator) situated on the base, one actuated

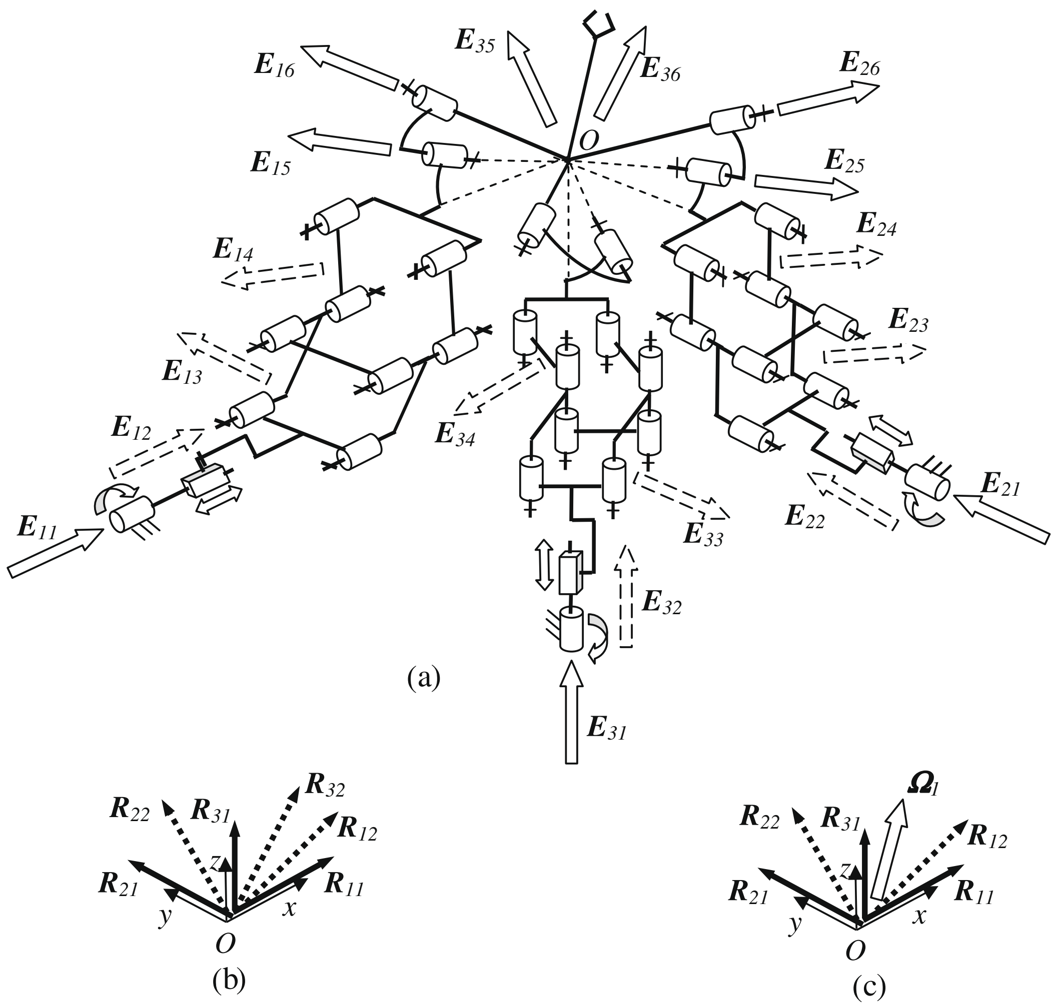

Fig. 8. 6-DOF parallel mechanism 3 R–P–P–P–R–R.

prismatic pair (linear actuator), two prismatic kinematic pairs represented as four-bar parallelograms and two passive rotation pairs.

The axes of all the passive rotation pairs intersect in the same point O which is the origin of the coordinate system. This point O is movable but the directions of the coordinate axes are constant. The unit screws of the axes of these kinematic pairs have coordinates: $\boldsymbol{E}_{11}$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{E}_{12}$ $(0, 0, 0, 1, 0, 0)$, $\boldsymbol{E}_{13}$ $(0, 0, 0, 0, e_{13y}, e_{13z})$, $\boldsymbol{E}_{14}$ $(0, 0, 0, 0, e_{14y}, e_{14z})$, $\boldsymbol{E}_{15}$ $(e_{15x}, e_{15y}, e_{15z}, 0, 0, 0)$, $\boldsymbol{E}_{16}$ $(e_{16x}, e_{16y}, e_{16z}, 0, 0, 0)$, $\boldsymbol{E}_{21}$ $(0, 1, 0, 0, 0, 0)$, $\boldsymbol{E}_{22}$ $(0, 0, 0, 0, 1, 0)$, $\boldsymbol{E}_{23}$ $(0, 0, 0, e_{23x}, 0, e_{23z})$, $\boldsymbol{E}_{24}$ $(0, 0, 0, e_{24x}, 0, e_{24z})$, $\boldsymbol{E}_{25}$ $(e_{25x}, e_{25y}, e_{25z}, 0, 0, 0)$, $\boldsymbol{E}_{26}$ $(e_{26x}, e_{26y}, e_{26z}, 0, 0, 0)$, $\boldsymbol{E}_{31}$ $(0, 0, 1, 0, 0, 0)$, $\boldsymbol{E}_{32}$ $(0, 0, 0, 0, 0, 1)$, $\boldsymbol{E}_{33}$ $(0, 0, 0, e_{33x}, e_{33y}, 0)$, $\boldsymbol{E}_{34}$ $(0, 0, 0, e_{34x}, e_{34y}, 0)$, $\boldsymbol{E}_{35}$ $(e_{35x}, e_{35y}, e_{35z}, 0, 0, 0)$, $\boldsymbol{E}_{36}$ $(e_{36x}, e_{36y}, e_{36z}, 0, 0, 0)$. The screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ are of zero pitch, the screws $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$ are of infinite pitch ($i = 1, 2, 3$).

All six twists of motions of the platform can be represented as: $\boldsymbol{\Omega}_1$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{\Omega}_2$ $(0, 1, 0, 0, 0, 0)$, $\boldsymbol{\Omega}_3$ $(0, 0, 1, 0, 0, 0)$, $\boldsymbol{\Omega}_4$ $(0, 0, 0, 1, 0, 0)$, $\boldsymbol{\Omega}_5$ $(0, 0, 0, 0, 1, 0)$, $\boldsymbol{\Omega}_6$ $(0, 0, 0, 0, 0, 1)$. If rotating actuators are fixed then the linear actuators drive translation motions of the end-effector analogously to the mechanism drown in the Fig. 2. By this the kinematic pairs corresponding to the screws $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$ are used. If linear actuators are fixed then the rotating actuators drive orientation motions of the end-effector analogously to the mechanism drown in the Fig. 3. By this the kinematic pairs corresponding to the screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ are used but the rotations are transferred by the parallelogram analogously to the thirds kinematic chains of the mechanisms drown in the Figs. 4 and 5.

In considered mechanism (Fig. 8a) singularities expressed by loss of one or more degrees of freedom exist if any six screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ ($i = 1, 2, 3$) are linearly dependent. Analogously to previous cases it is possible if any two screws $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$ are parallel or if any three screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$ are coplanar. In particular if $\boldsymbol{E}_{23}$ $(0, 0, 0, 1, 0, 0) = \boldsymbol{E}_{24}$ $(0, 0, 0, 1, 0, 0)$ then there exists one wrench of the constraint imposed by the second kinematic chain: $\boldsymbol{R}$ $(0, 0, 1, 0, 0, 0)$ and only five twists of motion of the platform reciprocal to this wrench $\boldsymbol{\Omega}_1$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{\Omega}_2$ $(0, 1, 0, 0, 0, 0)$, $\boldsymbol{\Omega}_3$ $(0, 0, 1, 0, 0, 0)$, $\boldsymbol{\Omega}_4$ $(0, 0, 0, 1, 0, 0)$ and $\boldsymbol{\Omega}_5$ $(0, 0, 0, 0, 1, 0)$. If the unit screws $\boldsymbol{E}_{11}$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{E}_{15}$ $(e_{15x}, e_{15y}, e_{15z}, 0, 0, 0)$, $\boldsymbol{E}_{16}$ $(e_{16x}, e_{16y}, e_{16z}, 0, 0, 0)$ are coplanar then there exists one wrench of the constraint imposed by the first kinematic chain: $\boldsymbol{R}$ $(0, 0, 0, 0, r_y, r_z)$ and only five twists of motion of the platform reciprocal to this wrench $\boldsymbol{\Omega}_1$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{\Omega}_2$ $(\omega_{2x}, \omega_{2y}, \omega_{2z}, 0, 0, 0)$, $\boldsymbol{\Omega}_3$ $(0, 0, 0, 1, 0, 0)$, $\boldsymbol{\Omega}_4$ $(0, 0, 0, 0, 1, 0)$, $\boldsymbol{\Omega}_5$ $(0, 0, 0, 0, 0, 1)$. The wrench $\boldsymbol{R}$ is of infinite pitch, it is perpendicular to the axes $\boldsymbol{E}_{11}$, $\boldsymbol{E}_{15}$, $\boldsymbol{E}_{16}$.

If the actuators are fixed then the wrenches of the constraints imposed by the kinematic chains have coordinates (Fig. 8b): $\boldsymbol{R}_{11}$ $(0, 0, 0, r_{11x}, r_{11y}, r_{11z})$, $\boldsymbol{R}_{12}$ $(1, 0, 0, 0, 0, 0)$, $\boldsymbol{R}_{21}$ $(0, 0, 0, r_{21x}, r_{21y}, r_{21z})$, $\boldsymbol{R}_{22}$ $(0, 1, 0, 0, 0, 0)$, $\boldsymbol{R}_{31}$ $(0, 0, 0, r_{31x}, r_{31y}, r_{31z})$, $\boldsymbol{R}_{32}$ $(0, 0, 1, 0, 0, 0)$. The wrenches $\boldsymbol{R}_{i1}$ and $\boldsymbol{R}_{i2}$ are imposed by the ith kinematic chain. The wrenches $\boldsymbol{R}_{i1}$ are of infinite pitch, the wrenches $\boldsymbol{R}_{i2}$ are of zero pitch ($i = 1, 2, 3$). The wrenches $\boldsymbol{R}_{i1}$ are perpendicular to the screws $\boldsymbol{E}_{i5}$ and $\boldsymbol{E}_{i6}$.

Singularities corresponding to non-controlled infinitesimal motion of the moving platform exist if the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{22}$, $\boldsymbol{R}_{31}$, $\boldsymbol{R}_{32}$ are linearly dependent. It is possible if the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{31}$ are coplanar (Fig. 8c). In this case the twist of zero pitch $\boldsymbol{\Omega}$ $(\omega_x, \omega_y, \omega_z, 0, 0, 0)$ exists which is perpendicular to the axes of the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{31}$ and therefore reciprocal to all the wrenches $\boldsymbol{R}_{11}$, $\boldsymbol{R}_{12}$, $\boldsymbol{R}_{21}$, $\boldsymbol{R}_{22}$, $\boldsymbol{R}_{31}$, $\boldsymbol{R}_{32}$.

Note that in this mechanism also singularities exist corresponding both to loss of one degree of freedom and to non-controlled infinitesimal motion of the moving platform. By this any six screws $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$, $\boldsymbol{E}_{i3}$, $\boldsymbol{E}_{i4}$, $\boldsymbol{E}_{i5}$, $\boldsymbol{E}_{i6}$ ($i = 1, 2, 3$) and the wrenches $\boldsymbol{R}_1$, $\boldsymbol{R}_2$, $\boldsymbol{R}_3$, $\boldsymbol{R}_4$, $\boldsymbol{R}_5$, $\boldsymbol{R}_6$ are linearly dependent. The corresponding conditions are represented above.

This mechanism is decoupled and particularly isotropic. Each linear actuator controls the motion of the platform along one Cartesian coordinate. In each kinematic chain the input link of the first parallelogram and the output link of the second parallelogram are connected correspondingly to the linear actuator ant to the passive rotation kinematic pair in the middles of these links, and the output link of the first parallelogram and the input link of the second parallelogram coincide. It causes that only the rotating actuators drive the orientation of the end-effector. The linear actuators drive the position of the end-effector.

Note that the axes of the screws $\boldsymbol{E}_{i1}$ and $\boldsymbol{E}_{i2}$ can be not coinciding but the axes of the screws $\boldsymbol{E}_{i3}$ and $\boldsymbol{E}_{i4}$ must be perpendicular to the axis of the screw $\boldsymbol{E}_{i1}$.

This mechanism is similar to the mechanisms considered in [18–20] but here the prismatic kinematic pairs are represented as four-bar planar parallelograms as in [11–13]. It causes an advantage that the rotation motions are translated without the sliding motions in prismatic kinematic pairs. Therefore the mechanism in Fig. 8 is more practically applicable than the mechanisms represented in [18–20].

5. Conclusion

In this work, the approach to a synthesis of 6-DOF decoupled parallel manipulators is considered. The approach is based on the closed screw groups that include all the screw products of the main members of these groups. Synthesized manipulators consist of three parallel kinematic chains in that prismatic kinematic pairs are designed as planar four-bar parallelograms. The synthesis of these mechanisms and the singularity analysis is carried out by using of Plücker coordinates of twists and wrenches corresponding to the kinematic chains. The originality of the paper is determined by using of screw groups. It allows obtaining all the twists of the moving platform and all the wrenches of the constraints imposed by kinematic chains without any equations. Besides it allows avoiding the complications of Jacobian analysis by considering of the singularities. The decoupled manipulators with three DOF (Fig. 4), four DOF (Fig. 5) and six DOF (Figs. 6–8) obtained in this article are original.

References

[1] K. Hunt, Structural kinematics of in-parallel-actuated robot arms, ASME Journal of Mechanisms, Transmissions, and Automation in Design 105 (4) (1983) 705–712.
[2] J.-P. Merlet, Parallel Robots, second ed., Springer, 2006.
[3] X. Kong, C. Gosselin, Type Synthesis of Parallel Mechanisms, Springer, 2007.
[4] M. Ceccarelli, Fundamentals of Mechanics of Robotic Manipulations, Kluwer Academic Publishers, 2004.
[5] L. Tsai, Robot Analysis: The Mechanics of Serial and Parallel Manipulators, John Wiley & Sons, New York, 1999.
[6] Z. Huang, The Kinematics and type synthesis of lower-mobility parallel robot manipulators, in: Proceedings, XI World Congress in Mechanism and Machine Science, Tianjin, China, 2004, pp. 65–76.
[7] V.A. Glazunov, A.S. Koliskor, A.F. Kraynev, Spatial Parallel Structure Mechanisms (in Russian), Nauka, Moscow, 1991.
[8] C. Innocenti, V. Parenti-Castelli, Direct kinematics of the 6-4 fully parallel manipulator with position and orientation uncoupled, in: European Robotics and Intelligent Systems Conf., Corfou, 23–28 Juin, 1991.
[9] A.Sh. Koliskor, The l-coordinate approach to the industrial robot design, V IFAC/IFIP/IMACS/IFORS Symposium, Suzdal, URSS, April 22–25, 1986, pp. 108–115.
[10] K. Mianovski, Singularity analysis of parallel manipulator POLMAN 3×2 with six degrees of freedom, in: 12th IFToMM World Congress, Besançon (France), June 18–21, 2007.
[11] K. Mianowski, On Some Properties of POLMAN-3L Parallel Manipulator to the use as a 3-D Measuring System for Industrial/Educational Applications, in: Proceedings of the Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators, Quebec City, Canada, 2002.
[12] K. Mianowski, Dynamically decoupled parallel manipulator with six degrees of freedom for fast assembly, in: Proceedings of the International Conference on ISOM'2002, Chemnitz, 2002, pp. 274–280.
[13] R. Clavel, Device for Displacing and Positioning an Element in Space, Patent WO 87/03528, 1987.
[14] V. Arakelian, S. Guegan, S. Briot, Static and dynamic analysis of the PAMINSA, ASME 2005, in: International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Long Beach, California, USA, 2005, pp. 24–28.
[15] G. Gogu, Structural synthesis of fully-isotropic translational parallel robots via theory of linear transformations, European Journal of Mechanics, A/Solids 23 (2004) 1021–1039.
[16] G. Gogu, Structural Synthesis of Parallel Robots, Part 1: Methodology, Springer, Dordrecht, 2008.
[17] G. Gogu, Structural Synthesis of Parallel Robots, Part 2: Translational Topologies with Two and Three Degrees of Freedom, Springer, Dordrecht, 2009.
[18] Yan Jin, I-Ming Chen, Guilin Yang, Structure synthesis and singularity analysis of a parallel manipulator based on selective actuation, in: Proceedings of the 2004 IEEE International Conference on Robotics and Automation, New Orleans, LA, April 2004, pp. 4533–4538.
[19] Yan Jin, I-Ming Chen, Guilin Yang, Mobility and singularity analysis of a selectively actuated parallel mechanism, theory and practice of robots and manipulators (ROMANSY), in: Proceedings of XV CISM-IFToMM Symposium, Montreal, 2004.
[20] Yan Jin, I-Ming Chen, Guilin Yang, Structure Synthesis of 6-DOF 3-3 Decoupled Parallel Manipulators, in: 12th IFToMM World Congress, Besançon (France), June 18–21, 2007.
[21] F.M. Dimentberg, The Screw Calculus and its Applications in Mechanics, Nauka, Moscow, 1965. (English translation: AD680993, Clearinghouse for Federal Technical and Scientific Information, Virginia).
[22] F.M. Dimentberg, The Theory of Screws and Its Applications (in Russian), Nauka, Moscow, 1978.
[23] J. Herve, The Lie group of rigid body displacements, a fundamental tool for mechanism design, Mechanism and Machine Theory 34 (8) (1991) 719–730.
[24] J. Angeles, The qualitative synthesis of parallel manipulators, Journal of Mechanical Design 126 (2004) 617–624.
[25] K. Sugimoto, Existence criteria for overconstrained mechanisms: an extension of motor algebra, Transactions of ASME, Journal of Mechanical Design (3) (1990) 295–298.
[26] V.A. Glazunov, Principles of the construction and analysis of spatial parallel structure mechanisms, Journal of Machinery Manufacture and Reliability (1) (1995) 10–15.
[27] C.M. Gosselin, J. Angeles, Singularity analysis of closed loop kinematic chains, IEEE Transactions on Robotics and Automation 6 (3) (1990) 281–290.
[28] V.A. Glazunov, A.Sh. Koliskor, A.F. Krainev, B.I. Model, Classification principles and analysis methods for parallel-structure spatial mechanisms, Journal of Machinery Manufacture and Reliability (1) (1990) 30–37.