ANALYSIS OF STEADY-STATE FLOW TO MULTISCREENED WELLS UNDER NATURAL VERTICAL FLOW CONDITIONS

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ABSTRACT: Water levels measured at multiscreened wells in unconfined aquifers may not coincide, in general, with the elevation of the water table. The presence of vertical gradients (as often is the case in recharge areas) or the existence of confining layers may cause the water levels to differ from local hydraulic heads in the aquifer. In these cases, a misinterpretation of water levels may lead to the erroneous conclusion that observed drawdowns are provoked by overpumping. In this paper, we analyze the effect that a natural vertical gradient has on water levels in wells screened over their entire saturated thickness. As one would expect, it is observed that, even without pumping, the water level in the well lies below the water table. Type curves relating the steady-state drawdown to the vertical gradient and to the hydraulic conductivity anisotropy are presented. These curves were obtained using a groundwater flow numerical model (FREESURF: Neuman and Witherspoon, 1970). The theoretical results are checked with field data from deep wells in the detrital Madrid aquifer. In this particular aquifer, it is observed that the effect of vertical gradients is important both in terms of drawdowns and flow rates.

RÉSUMÉ: Les niveaux piézométriques mesurés dans des puits à obturations multiples forés dans des aquifères non captifs peuvent ne pas coincider avec le niveau de la surface de la nappe. L'existence de gradients verticaux (comme c'est souvent le cas dans les zones de recharge) ou l'existence de niveaux captifs peut provoquer des écarts entre les niveaux piézométriques et les charges hydrauliques locale dans l'aquifère. Dans ce cas, une erreur d'interprétation des niveaux peut conduire à la conclusion erronée que les descentes observées seraient dues à un pompage. Dans ce travail, nous analysons l'effet d'un gradient vertical naturel sur les niveaux d'eau dans des puits obturés sur toute l'épaisseur de la zone saturée. Sont données les courbes type reliant la descente en régime permanent au gradient vertical et à l'anisotropie de la permeabilité. Ces courbes ont été fournies par un modèle numérique d'écoulement souterrain (FREESURF, Neuman et Witherspoon, 1970). Les résultats théoriques ont été vérifiés sur la terrain, dans des puits profonds de l'aquifère détritique de Madrid. D'après les observations, les gradients verticaux dans cet aquifère ont un effet important à la fois sur la descente et sur les vitesses d'écoulement.

INTRODUCTION

The water level measured in a deep well with a long screen provides the average piezometric head of the aquifer. In the absence of a vertical hydraulic gradient, flow is purely horizontal and the head is constant along the vertical. The water level is the same regardless of the depth of the well. In many cases, however, groundwater flow has a predominant vertical component. Vertical hydraulic gradients are common in recharge and discharge areas of regional aquifers. In recharge areas, piezometric heads decrease with depth so that deep wells record heads which are lower than those measured with shallow wells. Under these conditions, the depth to the water table is best defined by the water level measured in a shallow well. The measured hydraulic head in a deep well is below the water table. This difference between the measured head and the true water table (which always exists in the presence of a vertical gradient) must be carefully recognized before attributing this apparent drawdown to other reasons, such as overpumping.

It is well known that, under vertical flow conditions, the water level measured in a deep well screened over its entire saturated thickness is equal to the vertical average of the heads in the aquifer. The well creates a bypass for groundwater flow from the upper to the lower part of the aquifer. This bypass may constitute a fast pathway for a contaminant to reach deep zones of the aquifer. Although a very large number of studies have addressed the theory of flow to wells (i.e, Lohman, 1972), only a few have considered the vertical water movement in the well. The numerical study of Cooley and Cunningham (1979) showed that the inflow into a pumping well is generally influenced by the vertical water flow in the well. Different vertical velocities in the well due to different pumping rates can cause different inflow rates from the aquifer layers. More recently, Kaleris (1989) also studied the effect of vertical movement in the well on the concentrations measured in monitoring wells with long screens. To the best of our knowledge, however, no general quantitative results are available in the hydrologic literature for relating the apparent drawdown in the well to aquifer and well parameters. Samper et al. (1989, 1991) were the first to obtain quantitative results for the transient and steady-state drawdown as a function of both horizontal and vertical hydraulic conductivities, vertical hydraulic head gradient and well depth. These authors prepared drawdown type-curveswhich can be used to either (a) compute actual drawdowns for given aquifers and well parameters, or (b) determine aquifer parameters from drawdown data. In this paper, we summarize their results and provide a more complete set of steady-state drawdown type-curves for multiscreened wells under natural vertical flow conditions.

PROBLEM DEFINITION

We consider groundwater flow to a multiscreened well drilled in an unconfined aquifer where flow is primarily vertical, either downwards or upwards. In these conditions, the water level in the well does not coincide with the water table. Let P, P_w, and P_{wt} be the depths of the well, the water in the well, and the water table, respectively. In a vertically downwards flowing aquifer, water in the well lies below the water table, that is, $P_w > P_{wt}$. This drawdown, s, is related to the penetration p of the well below the water table (see Figure 1).

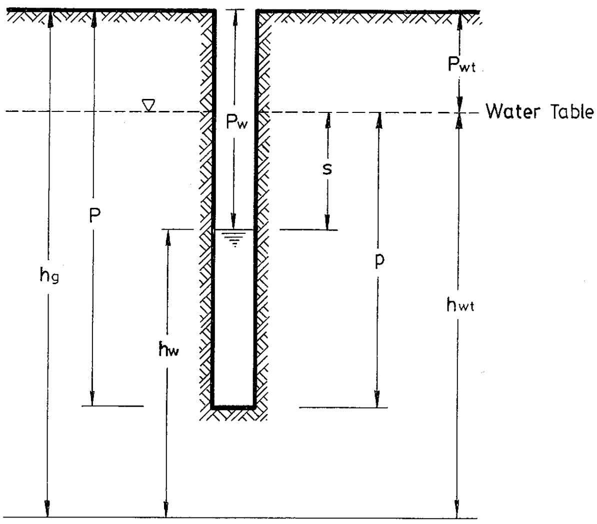

Figure 1. Definition of terms.

These four variables are related through:

$$P_w = s + P_{wt} \tag{1}$$

$$P = p + P_{wt} \tag{2}$$

The ratio s/p is then given by:

$$\frac{s}{p} = \frac{P_w - P_{wt}}{P - P_{wt}} \tag{3}$$

Let h_{wt} be the water table level, h_w the head at the well, and h_g the height of the ground surface. In terms of heads, s/p is given by:

$$\frac{s}{p} = \frac{h_{wt} - h_w}{h_{wt} - h_g + P} \tag{4}$$

and the depth to the water table is given by:

$$P_{wt} = \frac{P_w - P\ s/p}{1 - s/p} \tag{5}$$

Inasmuch as $P_{wt} \leq P$, it follows that:

$$\frac{s}{p} \leq = \frac{P_w}{P} \tag{6}$$

This equation provides an upper limit for the ratio s/p in terms of measurable quantities (P_w and P).

Consider an unbounded aquifer having a constant recharge rate q per unit area. The condition at the free surface for a flat water table is:

$$q = -K_z \frac{\partial h}{\partial z} \Big|_{z=0} \tag{7}$$

where K_z is the vertical hydraulic conductivity. The vertical hydraulic gradient β is constant and equal to q/K_z. This flow regime changes drastically in the presence of a multiscreened well. As shown by Figure 2, the water level at the well lies below the water table. After a transient regime, a new steady-state flow pattern is attained in which the well acts as a bypass for groundwater flow from the upper to the lower part of the aquifer.

Flow in the upper part is convergent while it diverges in the lower part. The boundary between both zones is located at a level h_s. Between these zones, there is a stagnant zone where the hydraulic gradient and water velocity are small. The area influenced by the well has a radius R, defined so that the total recharge in the influence zone is equal to the flow rate through the well Q_w, that is:

$$R = \sqrt{\frac{Q_w}{\pi q}} \tag{8}$$

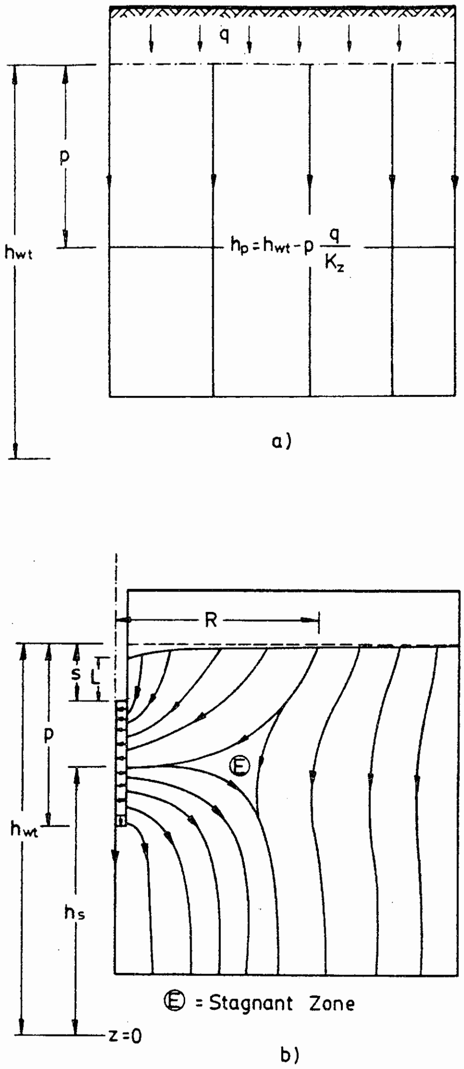

Figure 2. Flow patterns (a) prior to, and (b) after completion of a multiscreened well.

The water level in the well h_w is lower than the water table h_{wt} and greater than the head in the aquifer at a depth p below the water table. Therefore:

$$h_{wt} - p\frac{q}{K_z} \leq h_w \leq h_{wt} \tag{9}$$

In terms of the drawdown $s = h_{wt} - h_w$, this condition states that:

which provides another upper limit for the ratio s/p.

The head in the well h_w can be written as a linear convex combination of its maximum (h_{wt}) and minimum value (that corresponding to the head in the aquifer at a depth p below the water table):

$$h_w = \lambda h_{wt} + (1-\lambda)(h_{wt} - p\beta) \tag{11}$$

where λ is a coefficient that can take values between zero and one. The closer λ is to 1, the closer gets h_w to h_{wt} and, conversely, values of λ close to zero mean that h_w is close to the head at the bottom of the well. In terms of λ, s/p is given by:

$$\frac{s}{p} = (1 - \lambda)\beta \tag{12}$$

It will be shown later that λ is usually less than 0.5 and decreases when β increases.

Good practice calls for casings to prevent cascading waters. In the absence of casing, a seepage face may develop above the water level in the well.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider an anisotropic, homogeneous unbounded medium in which a well of radius r_w is drilled. Given the cylindrical symmetry, the equation governing the flow is:

$$K_r \frac{\partial^2 h}{\partial^2 r} + \frac{K_r}{r}\frac{\partial h}{\partial r} + K_z \frac{\partial^2 h}{\partial^2 z} = S_s \frac{\partial h}{\partial t} \tag{13}$$

where h(r,z,t) is the head at time t at a radial distance r and at a depth z (measured from the initial water table), K_r and K_z are the horizontal and vertical hydraulic conductivities, respectively, and S_s is the storativity coefficient. Assuming a flat horizontal water table with a negligible drawdown, the boundary conditions are:

(a) Free surface (z = 0):

$$K_z \frac{\partial h}{\partial z}\Big|_{z=0} = -q \tag{14}$$

(b) Well interface ($r = r_w$):

$$h(r_w,z,t) = h_w(t) \quad \text{for} \quad h_{wt} - h_w(t) \le z \le p \tag{15}$$

$$h(r,p,t) = h_w(t) \quad \text{for} \quad 0 \le r \le r_w \tag{16}$$

(c) Seepage face:

$$h(r_w,z,t) = h_{wt} - z \quad \text{for} \quad 0 \le z \le h_{wt} - h_w(t) \tag{17}$$

(d) Flow rate from the aquifer into the well, Q_a:

$$Q_a = \int_0^p 2\pi r_w K_r \frac{\partial h}{\partial r}\Big|_{r=r_w} dz + \int_0^{r_w} 2\pi r K_z \frac{\partial h}{\partial z}\Big|_{z=p} dr \tag{18}$$

where again it has been assumed as a first approximation that the water table remains horizontal, and the drawdown in the well is small compared to the total depth of the well.

The water level in the well $h_w(t)$ changes with time according to the following ordinary differential equation:

$$\pi r_w^2 \frac{\partial h_w}{\partial t} = Q_a - Q_p \tag{19}$$

where Q_p is the pumping rate, and Q_a is the flow coming into the well from the aquifer (given by Equation 18).

The initial conditions in the aquifer correspond to those shown in Figure 2a, in which flow is perfectly vertical with a hydraulic gradient β. After a "sufficiently large" period of time, a steady-state regime is reached. The governing flow equation is similar to Equation (13) except for the righthand term that cancels out. The boundary conditions are given by Equations (14)-(18). Notice that, under steady-state conditions, neither h nor h_w are functions of time. For no pumping ($Q_p = 0$), the mass balance equation in the well becomes:

$$Q_a = 0 \tag{20}$$

For the sake of preparing type curves of drawdown, it is convenient to define the following dimensionless variables:

$$r_D = \frac{r}{r_w} \quad \text{Dimensionless radial distance}$$

$$z_D = \frac{z}{p} \quad \text{Dimensionless depth}$$

$$\beta = \frac{q}{K_z} \quad \text{Vertical gradient}$$

$$\alpha = \frac{K_r}{K_z}\frac{p^2}{r_w^2} \quad \text{Anisotropy parameter}$$

$$R_D = \frac{R}{r_w} \quad \text{Dimensionless radius of influence}$$

$$s_D = \frac{s}{p\beta} \quad \text{Dimensionless drawdown} \qquad (21)$$

In terms of dimensionless variables, the formulation of the steady-state flow regime is as follows:

$$\alpha \frac{\partial^2 s_D}{\partial r_D^2} + \frac{\alpha}{r_D}\frac{\partial s_D}{\partial r_D} + \frac{\partial^2 s_D}{\partial z_D^2} = 0 \qquad (22)$$

$$\frac{\partial s_D}{\partial z_D} = 1 \quad \text{for} \quad z_D = 0 \qquad (23)$$

$$s_D(1,z_D) = s_{D_w} \quad \text{for} \quad s_{D_w}\beta \le z_D \le \beta$$

$$s_D\,(r_D,z_D) = s_{D_w} \quad \text{for} \quad 0 \le r_D \le 1 \qquad (24)$$

$$s_D\,(1,z_D) = z_D/\beta$$

$$\text{for} \quad 0 \le z_D \le \beta s_{D_w} \qquad (25)$$

$$\alpha \int_0^1 \frac{\partial s_D}{\partial r_D}\Big|_{r_D=1} dz_D$$

$$+ \int_0^1 r_D \frac{\partial s_D}{\partial z_D}\Big|_{z_D=1} dr_D = 0 \qquad (26)$$

where:

$$s_{D_w}$$

is the dimensionless drawdown in the well, defined as:

$$s_{D_w} = \frac{h_{wt} - h_w}{p\beta} \qquad (27)$$

This drawdown depends on dimensionless parameters α and β. The flow rate through the well Q_a is given by:

$$Q_a = 2\pi p^2 K_r \beta \left(\int_{s_{Ds}}^1 \frac{\partial s_D}{\partial r_D}\Big|_{r_{D=1}} dz_D\right.$$

$$\left. + \frac{1}{\alpha}\int_0^1 r_D \frac{\partial s_D}{\partial z_D}\Big|_{z_{D=1}} dr_D\right) \qquad (28)$$

where:

$$s_{D_s}$$

is the drawdown corresponding to level h_s, that is:

$$s_{D_s} = \frac{h_{wt} - h_s}{p\beta} \qquad (29)$$

From Equation (28), it follows that the dimensionless flow rate:

$$Q_{D_w}$$

is given by:

$$Q_{D_w} = \frac{Q_w}{2\pi p^2 K_r \beta} \qquad (30)$$

The dimensionless radius of influence R_D is obtained from Equations (8) and (30):

$$R_D = \sqrt{2\alpha Q_{D_w}} \qquad (31)$$

Finally, parameter λ (in Equation 11) is given by:

$$\lambda = 1 - s_{D_w} \qquad (32)$$

APPROXIMATE SOLUTION

As stated above, the steady-state flow to a multiscreened well under vertical flow conditions is a nonlinear, three-dimensional flow problem that cannot be solved analytically. Under simplifying assumptions, it is possible, however, to find closed-form analytic solutions for asymptotic cases. One of the solutions corresponds to the assumption of pure horizontal flow with a nearly horizontal water table. Assuming the validity of the Thiem Equation for steady-state radial flow to a well, the drawdown s(z) at a given depth z below the water table should be proportional to the flow rate Q(z) at such depth. The incoming flow in the upper part of the well must be equal to the outgoing flow in the lower part so that the net flow rate through the well is equal to zero. This means that the integral of Q(z) along the well should vanish and so should the integral of s(z). Therefore:

$$\int_s^p s(z)\, dz = 0 \qquad (33)$$

The drawdown s(z) is given by:

$$s(z) = s - \beta z \qquad (34)$$

Notice that the drawdown (the difference between the heads before and after well completion) is positive in the upper part where water flows into the well and is negative in the lower part where water exits the well.

Substituting Equation (34) into Equation (33), integrating and solving the resulting equation for s, one obtains the dimensionless drawdown in the well $s/p\beta$ given by:

$$\frac{s}{p\beta} = \frac{1}{2-\beta} \qquad (35)$$

which indicates that, for small β, this drawdown is approximately equal to 0.5 and increases slightly when β increases. This approximate solution is only valid for pure horizontal flow (i.e., for low values ofa). As shown later, it agrees well with the more accurate numerical solution which is described next.

NUMERICAL FORMULATION FOR STEADY-STATE FLOW

Given the lack of analytical solutions for the problem stated previously, a numerical solution has been developed. The program FREESURF (Neuman and Witherspoon, 1970) is suitable for this purpose because it can handle free surface problems involving internal or external seepage faces. Even though flow is three-dimensional, the cylindrical symmetry allows it to be reduced to a two-dimensional problem. A vertical cross-section of the aquifer is shown in Figure 3. The inner boundary coincides with the axis of the well and the upper boundary is the free surface. Both the lower and the outer boundaries are set far enough away from the well so that they have a negligible effect. Both the inner and the outer boundaries are assumed impermeable. At the free surface, a recharge q is imposed. Because the lower boundary is located far enough from the well, it can be treated as a constant head boundary with a head value equal to that existing at such depth before the construction of the well.

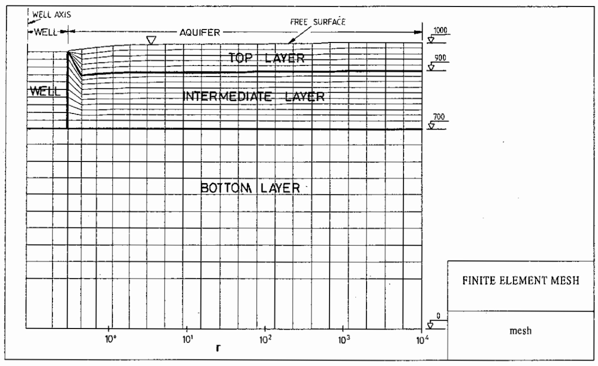

Figure 3. Cross section of the aquifer.

The finite element mesh is made up of quadrilateral elements. These elements are arranged in three quasihorizontal layers, the limits of which coincide with the elevation of the bottom of the well and the elevation of the water in the well. The bottom layer, located below the bottom of the well, has N_3 elements. The intermediate layer has a thickness equal to the height of the water column in the well. The number of elements in a given column of this layer is N_2. The top layer includes the part of the aquifer located above the water level in the well. Each column of this layer has N_1 elements.

The well can be treated in several different ways. One possibility calls for specifying the head at all nodes inside the well. However, the head value to be assigned is unknown (in fact it is what we are after). To overcome this difficulty, the water in the well is considered as a permeable material having a "very large" conductivity and a storativity equal to 1. In this way, a second free surface has to be considered for the water in the well. Between the free surface in the aquifer and that in the well, a seepage face may develop.

In order to accurately model the flow near and inside the well, several columns (N_{cw}) of elements are considered inside the well radius. Between the well surface and the outer boundary, N_{ce} columns are arranged so that their size increases with the logarithm of the radial distance. This allows having smaller elements near the well and larger ones far away.

The finite element grid is automatically generated with a preprocesor which also assigns the proper boundary conditions, material properties, etc. The mesh was refined until the results did not change significantly. This was achieved by using 5 elements per column in the bottom layer ($N_3 = 5$) and 10 elements per column in the intermediate and top layers.

For the purpose of preparing the type curves, a well radius of 0.5 was considered. The outer boundary was set at a distance of 10,000. The bottom boundary was assigned the zero level, and the elevation of the water table was set at level 1,000. The well penetration was made equal to 300. The units are arbitrary and can be meters, feet, etc.

RESULTS

Before presenting the drawdown type-curves, some results corresponding to a hypothetical situation are presented which illustrate the hydrogeological effects of multiscreened wells in aquifers with vertical flow.

The results shown in Table 1 correspond to an aquifer where the elevation of the water table is 1000 m. The well has a radius of 0.5 m and penetrates 300 m below the water table. Horizontal and vertical conductivities are equal to 0.416 m/day and 1.5 10^{-3} m/day, respectively. For these parameter values, a is equal to 10^8. Parameter β takes on values between 0.1 and 0.4. From theresults shown in Table 1, one can see that:

(a) The values of s/p are always less than ß and greater than 0.5 β. The dimensionless drawdown in the well s_{Dw} increases with increasing ß from 0.558 for β = 0.1 to 0.634 for β = 0.5. Parameter γ is always less than 0.5, and its value decreases with β.

(b) The elevation of the separation of convergent and divergent flow zones, h_s, remains fairly constant.

(c) The seepage face is negligible for values of β smaller than 0.1. Its thickness is only about 10% of the drawdown s for β = 0.2. For β = 0.4, however, the thickness of the seepage face is more than 83% of the drawdown. It is clear that the seepage face becomes important when β is greater than 0.2.

(d) The flow rate through the well Q_a increases with increasing β in a manner similar to what it does the ratio s/p. This can be taken as an indication that the flow rate is proportional to the drawdown. The dimensionless flow rate Q_{Dw}, however, remains nearly constant.

(e) The radius of influence R decreases slightly with increasing β. The ratio R/p decreases from 1.5 for β = 0.1 to 1.384 for β = 0.4.

According to Samper (1988), changing the radius from 0.5 to 0.25 has no effect on the drawdown in the well, although the shape of the water table changes slightly.

Table 1. Numerical results corresponding to α = 10^8 and different values of the vertical hydraulic gradient β ranging from 0 to 0.4.

Variables	$\beta = 0.$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.4$
h_w	1000.	983.28	965.61	946.68	926.94
$s = h_{wt} - h_w$	0.	16.72	34.39	53.32	73.06
s/p	0.	0.0558	0.114	0.177	0.243
s_{Dw}	0.	0.558	0.570	0.590	0.607
$\lambda = 1 - s_{Dw}$	-	0.442	0.430	0.410	0.393
Q_a	0.	95.30	180.40	264.70	325.17
Q_{D_w}	0.	4.04 10^{-3}	3.82 10^{-3}	3.74 10^{-3}	3.45 10^{-3}
R	0.	449.72	437.51	432.71	415.34
R_D	0.	899.5	875.0	865.4	830.6
h_s	-	819.17	817.13	823.92	817.19
s_{Ds}	-	6.02	3.04	1.95	1.52
L	-	0.00	3.57	10.16	60.72
L/s	-	0.00	0.103	0.190	0.831

APPLICATION TO THE MADRID AQUIFER

The previous results can be considered representative of a large sedimentary basinsurrounding Madrid in central Spain. The Madrid basin is 160 km long, 30-60 km wide, and as much as 3,000 m deep. The basin is a graben filled with arkosic sands containing variable amounts of silt and clay deposited as alluvial fans between the Miocene and Pleistocene ages. Measured horizontal hydraulic conductivities range between 0.4 and 2.8 m/d. The vertical conductivity is apparently smaller by two or more orders of magnitude due to the presence of sand lenses embedded within a less permeable clay-rich matrix. Groundwater modeling and hydrochemical studies reveal the presence of shallow flow systems near rivers and creeks together with intermediate and regional flow components which extend up to more than 100 km. Recharge takes place mainly by infiltration of precipitation in the interfluves at an annual rate of 40 to 100 mm. Discharge occurs along rivers and swampy lowlands (Carrera and Neuman, 1982; Llamas et al., 1982).

For a vertical hydraulic conductivity K_z of 1.5 x 10^{-3} m/day, the values of β listed in Table 1 correspond to recharge rates varying from 54.7 to 219 mm/year. For this range of β values, the results in Table 1 indicate that the drawdown can range from 16.7 up to 73 m. This clearly indicates that the water level measured at a multiscreened well located in the recharge area of this aquifer may be significantly different from the true water table level. The seepage face that can develop in these wells may have a thickness as high as 60 m. The amount of water flowing through the well may range from 95.3 m^3/day for β = 0.1 to 325.1 m^3/day for β = 0.4. For such a relatively low-permeable medium, these flow rates are really significant. The radius of the area affected by the well ranges from 415 to almost 450 m.

TYPE CURVES

The type curves of βs_D (which, according to Equation 21, coincides with s/p) for different values of β are shown in Figure 4. One can see that βs_D is always smaller than β and greater than β/2. The type curves of s_D for different values of β (Figure 5) tend to coincide for values of a greater than 10^{11}.

In addition, the values of s/p for a number of deep wells located in a recharge area of the Madrid basin where the water table is about 40 m deep are illustrated in Figure 4. These data suggest that the vertical hydraulic gradient β is on the range 0.15-0.25. This result is coherent with the mean values of the ratio P_w/P (depth to water well depth) which, according to data from Rubio (1984) and FernándezUría (1984), are approximately equal to 0.18.

The type curves shown in Figure 5 can be used to determine the values of α and β from measured drawdown data s_i corresponding to wells of different depths p_i. Plotting the values of $(s_i/p_i,\ p^2_i/r^2_{wi})$ in semilog paper and matching the type curves, one can determine the value of β by selecting the curve that best fits the data. One then selects a match point having a value of α* and $(p^2/r_w^2)^*$. From these values, the anisotropy ratio can be determined from:

$$\frac{K_r}{K_z} = \frac{\alpha^*}{\left(\frac{p^2}{r_w^2}\right)^*} \tag{36}$$

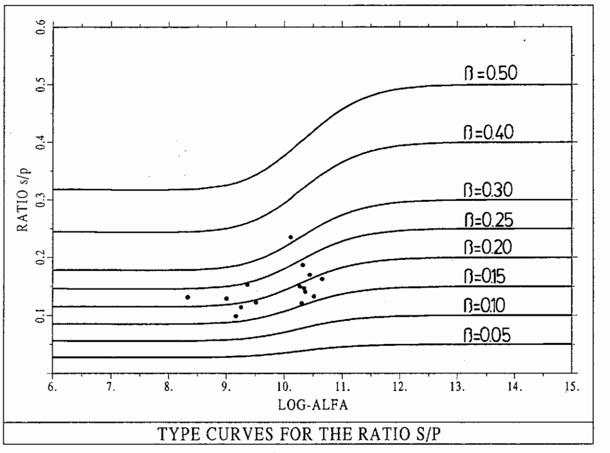

Figure 4. Type curves of β_{sd} for different values of β. (See text for meaning of solid circles.)

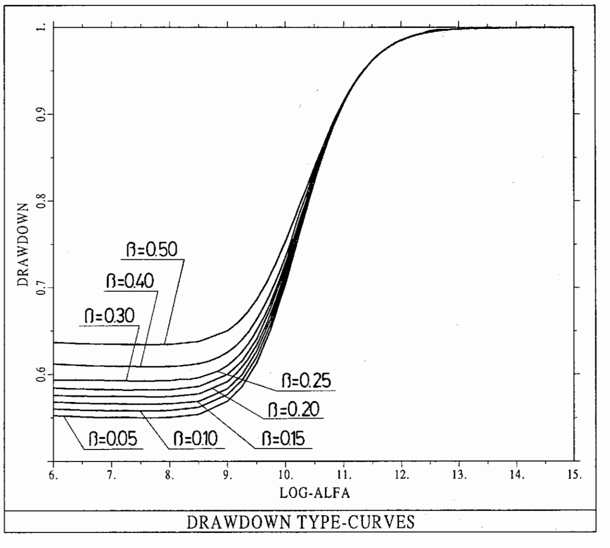

Figure 5. Type curves of dimensionless drawdown (S_D) for different values of β.

CONCLUSIONS

The steady-state flow into a multiscreened well under natural vertical flow conditions has been analyzed. A multiscreened well of depth P drilled in a recharge area with a vertical hydraulic head gradient ß records a water level which is below the water table. This apparent drawdown s is found to be a function of radial (K_r) and vertical (K_z) hydraulic conductivities, well radius (r_w), vertical hydraulic gradient, and well penetration (p) below the water table. The mathematical formulation of the problem in terms of dimensionless drawdown S_D is a function of β and a second parameter α given by:

$$\alpha = \frac{K_r}{K_z} \frac{p^2}{r_w^2}$$

The nonlinear nature of the problem prevents finding analytical expressions for s_D. An approximate assymptotic analytical solution has been obtained for a nearly flat water table and small values of α corresponding to pure horizontal flow. Under these conditions, s_D depends on β according to:

$$s_D = \frac{1}{2-\beta}$$

Drawdown type-curves have been obtained using a numerical solution based on the computer code FREESURF by Neuman and Witherspoon (1970). From the analysis of these curves, it can be concluded that:

(1) Drawdowns increase with the vertical gradient β;

(2) For a given gradient, s_D is constant and approximately equal to the previous asymptotic value for $\log_{10} \alpha$ less than 8. For $\log_{10} \alpha > 8$, s_D increases and reaches a value of 1 for $\log_{10}$ agreater than 13;

(3) The curves for different values of β merge into a single one for values of $\log_{10} \alpha$ greater than 11.

The curves have been used to estimate the vertical hydraulic gradient β in the recharge area of a large sedimentary basin in Spain using drawdown data from deep multicreened wells. For this particular low-permeability aquifer, drawdowns may range from 16 to 73 m for wells that penetrate 300 m below the water table. Vertical flow rates throughthe well range from 95 to 325 m^3/d.

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