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Downside/upside price spillovers between precious metals: A vine copula approach

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ABSTRACT

We studied downside and upside price spillovers between four precious metals (gold, silver, platinum and palladium), characterizing the multivariate dependence structure using a vine copula model and computing downside and upside value-at-risk and conditional value-at-risk. We found that the dependence structure differed across precious metals, all of which displayed different average and tail dependence features. Gold and silver prices were highly dependent except at the upper tail, whereas silver prices were integrated with those for platinum and palladium except at the upper tail. The gold market was very little integrated with the platinum and palladium markets. We document asymmetric downside and upside price spillover effects that differed in magnitude across precious metals; silver, in particular, had a greater downside and upside price impact on gold. Our results, indicating that precious metals do not behave as a single asset class, have implications for risk management, trading and hedging strategies for portfolios that include precious metals.

1. Introduction

In recent years, global economic uncertainty and high equity and bond market volatility have spurred investor interest in including gold and other precious metals in their portfolios. Precious

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metals are a useful hedging device as they have low dependence with other financial assets; when there is extreme market movement in financial markets, precious metals may display safe-haven asset characteristics.[1] In this context, a logical question is whether investors should include one or several precious metals in their portfolio for risk-management purposes. The answer crucially depends on co-movement between different precious metal prices, that is, on whether precious metals behave as a single asset class or not. The aim of this paper is to study the multivariate dependence structure of prices for certain precious metals (gold, silver, platinum and palladium) in order to disentangle the nature of precious metal price dependence and, in particular, to assess the possible existence of spillover effects of extreme upward or downward price movements in one precious metal on other precious metals. Such information has important implications in terms of risk management, trading and hedging strategies for portfolios that include precious metals.

Extant empirical studies have featured precious metal price dynamics from different perspectives. One strand of the literature has focused on the role of the main macro variables in determining precious metal prices – including among others, inflation, interest rates, the US dollar (USD) exchange rate and oil prices – finding that these variables are relevant in determining prices, mainly of gold (see, e.g., Batten, Ciner, & Lucey, 2010; Christie-David, Chaudhry, & Koch, 2000; Sari, Hammoudeh, & Soytas, 2010; Wang & Chueh, 2013). Other studies have focused on modelling and forecasting precious metal volatility using different generalized autoregressive conditional heteroskedastic (GARCH) specifications, finding evidence of volatility dynamics, asymmetric effects, persistence in volatility and volatility breaks in precious metal prices (Arouri, Hammoudeh, Lahiani, & Nguyen, 2012; Baur, 2012; Cochran, Mansur, & Odusami, 2012; Hammoudeh & Yuan, 2008; Tully & Lucey, 2007; Vivian & Wohar, 2012). Another strand of the literature has examined conditional volatility and correlation dependency between four major precious metals as well as spillover effects from precious metals to currency markets, finding evidence of high correlation between precious metals (Sensoy, 2013), of volatility spillovers between precious metals (Hammoudeh, Yuan, McAleer, & Thompson, 2010) and of spillover effects from precious metals to currency and crude oil prices (Antonakakis & Kizys, 2015). Finally, more recent empirical studies have focused on modelling and forecasting value-at-risk (VaR) of precious metals (Cheng & Hung, 2011; Demiralay & Ulusoy, 2014; Hammoudeh, Malik, & McAleer, 2011; Hammoudeh, Araújo Santos, & Al-Hassan, 2013) and on studying the hedge and safe-haven properties of precious metals for financial assets (see, e.g., Baur & McDermott, 2010; Baur & Lucey, 2010; Reboredo, 2013a, 2013b; Reboredo & Rivera-Castro, 2014b; and references therein). All the above-mentioned empirical studies provide useful information on precious metal price dynamics, correlation and downside risk. However, little is still known on the multivariate dependence structure, specifically, non-linear and asymmetric dependence between precious metal prices and possible transmission of extreme upward or downward price movements in one precious metal to other precious metals. This paper fills this gap and contributes to the existing literature on precious metal price relationship in three ways.

First, we characterize multivariate dependence structure between four precious metals (gold, silver, palladium and platinum) using vine copula models. Vine copulas (Joe, 1996) allow high-dimensional joint distributions to be modelled through a hierarchical structure composed of a set of bivariate copulas (called pair copulas) that capture dependence between two variables. Thus, the vine copula model shows the conditional hierarchical dependence structure between precious metals and, in particular, indicates whether or not a particular precious metal is directly or indirectly related to other precious metals. Also, by allowing different pair copula specifications with different symmetric and asymmetric structures and static and time-varying dependence structures, vine copulas provide useful information for diversification practices, given that asymmetric dependence requires specific compensation by investors (Ang, Chen, & Xing, 2006; Cromwell, Taylor, & Yoder, 2000). Furthermore, vine copulas allow marginal distributions and multivariate dependence structure to be modelled separately, so it is possible to capture specific volatility dynamics and asymmetries in the univariate prices of precious metals regardless of the dependence structure.

[1] Jensen, Johnson, and Mercer (2002), Draper, Faff, and Hillier (2006), Canover, Jensen, Johnsos, and Mercer (2009) and Reboredo and Rivera-Castro (2014a) have provided evidence on the benefits of including precious metals in financial and monetary asset portfolios.

Second, on the basis of the multivariate conditional dependence structure we quantify spillover effects of extreme downward or upward price movements from one precious metal to other precious metals by computing the downside and upside conditional VaR (CoVaR). The CoVaR (Adrian & Brunnermeier, 2011; Girardi & Ergün, 2013) is a systemic risk measure that captures spillover effects in precious metal prices by providing the VaR of one precious metal price conditional on the fact that another precious metal price is experiencing extreme movements as measured by its own VaR. Furthermore, we tested for the existence of spillover effects by considering significant differences between CoVaR and VaR values using the Kolmogorov–Smirnov (KS) bootstrapping test as proposed by Abadie (2002). We also tested for the asymmetric impact of extreme downward or upward price movement spillovers.

Finally, for the period October 2001 to January 2015 our evidence indicates that a multivariate dependence structure is given by a D-vine hierarchical structure, indicating that no metal price predominated in determining the structure of conditional dependence. Also, only some precious metal prices highly co-moved on average, displaying asymmetric tail dependence in the form of lower tail dependence and upper tail independence; other precious metals – like gold with platinum or gold with palladium – displayed low average dependence and tail independence. We also found evidence of downside price spillover effects, given that CoVaR values were significantly lower than VaR values, corroborating the fact that downward movements in any precious metal price has a quantitative impact on any other precious metal prices that does differ in size across metals. This result shows that, although there was one-way downside contagion in precious metal prices, the intensity of contagion when the market was bearish differed across metals. Hence, the precious metals did not behave as a single asset class. When considering upside price movements, we also found that spillover effects existed, were different in size across precious metals and were of lesser intensity than downside spillovers. Hence, precious metals cannot be considered as a single asset class in a bullish market.

Our results have implications for both investors and manufacturers. The asymmetric downside and upside co-movement between precious metal prices implies that investors who want to protect their portfolios using precious metals should carefully select the best precious metals when they manage downside and/or upside risk; moreover, they need to adopt different risk management strategies in each case, given that precious metals do not behave as a single asset class and have different price spillover effects. Also, jewellery, electronics, medicine and other manufactures that use precious metals in their production processes should consider the asymmetric impact on their financial results of price movements for precious metals not used in their production processes, especially when prices are at the low end of the market.

The remainder of the paper is laid out as follows: in Section 2 we outline the vine copula methodological approach to characterizing conditional multivariate dependence and the CoVaR; in Section 3 we present our data; in Section 4 we discuss the results; and finally, Section 5 concludes the paper.

2. Research method

2.1. Multivariate dependence modelling with vine copulas

Consider a four-dimensional random vector of precious metal price returns $x = (x_1, x_2, x_3, x_4)$ with joint density function $f(x_1, x_2, x_3, x_4)$ and distribution function $F(x_1, x_2, x_3, x_4)$. According to the Sklar's (1959) theorem, the distribution function of those returns can be expressed in terms of a copula function as:

$$F(x_1, x_2, x_3, x_4) = C(u_1, u_2, u_3, u_4), \tag{1}$$

where $C()$ is a copula function and $F_i(x_i) = u_i$ is the marginal distribution function of variable x_i, for $i = 1, \ldots, 4$. Also, the converse of Eq. (1) holds, so any multivariate distribution can be represented in terms of its marginals and a copula function. Assuming that F_i and C are differentiable, the joint density function f can be decomposed as the product of the marginal densities $f_i(x_i)$ and the multivariate copula density:

$$f(x_1, x_2, x_3, x_4) = f_1(x_1) f_2(x_2) f_3(x_3) f_4(x_4)\, c\,[F_1(x_1), F_2(x_2), F_3(x_3), F_4(x_4)], \tag{2}$$

where each f_i captures the behaviour of the marginals and the density copula given by:

$$c(u_1, u_2, u_3, u_4) = \frac{\partial^d C(u_1, u_2, u_3, u_4)}{\partial u_1 \partial u_2 \partial u_3 \partial u_4}, \tag{3}$$

which, in turn, captures the dependence structure among each x_i. Different copula specifications account for different symmetric and asymmetric dependence structures.

One way to attain more modelling flexibility in multivariate dependence is to factorize the multivariate copula density in terms of a successive mixing of $4(4-1)/2$ bivariate linking copulas – computationally more tractable and accounting for more specific bivariate dependence features than copulas with higher dimensions. This decomposition is referred to as vine copulas (see Aas, Czado, Frigessi, & Bakken, 2009; Bedford & Cooke, 2001, Bedford & Cooke, 2002; Joe, 1997; Kurowicka & Cooke, 2006). Since there is no unique copula density factorization, there are different types of vine copulas specifications with specific kinds of variables that allow modelling of bivariate dependence at different levels. These can be graphically represented through a hierarchical structure, such that, in a first tree level, four nodes are connected by edges that represent dependence between two variables, and in successive tree levels, nodes are obtained from the edge set of the previous tree level. In our study, we considered the C-vine, D-vine and R-vine copulas with different hierarchical tree structures.

The four-dimensional density function of the C-vine model is given by:

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = & \prod_{k=1}^{4} f_k(x_k) \prod_{h=2}^{4} c_{1,h}(F_1(x_1), F_h(x_h)) \\ & \times \prod_{j=2}^{4-1} \prod_{i=1}^{4-j} c_{j,j+i|1,\ldots,j-1}\left(F\left(x_j|x_1,\ldots,x_{j-1}\right), F\left(x_{j+i}|x_1,\ldots,x_{j-1}\right)\right), \end{aligned} \tag{4}$$

where $c_{j,j+i|1,\ldots,j-1}$ is the conditional copula and where the conditional distribution function of the x_i variable, given the variable x_j, is given by (Joe, 1997):

$$F_{i|j}(x_i|x_j) = \frac{\partial C_{ij}\left(F_i(x_i), F_j(x_j)\right)}{\partial F_j(x_j)}. \tag{5}$$

Fig. 1 represents the C-vine dependence decomposition in a hierarchical tree structure, where each tree (T) has a star structure. In this structure, one variable plays a pivotal role as the dependence of the remaining variables is measured with respect to this pivotal variable, using bivariate copulas as indicated by the second term in Eq. (4). The key variable that governs dependence between variables is identified as the one that maximizes the sum of pairwise dependencies as measured by Kendall's tau. The tree is successively expanded in such a way that the nodes of the respective trees are configured

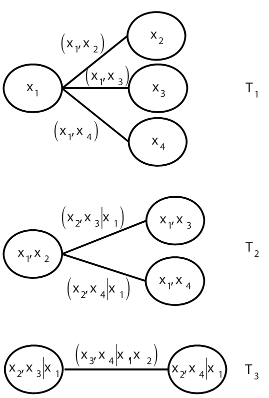

Fig. 1. Hierarchical tree structure for the C-vine copula with four variables.

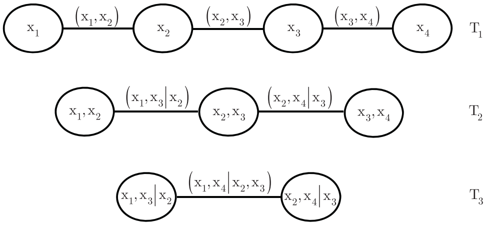

Fig. 2. Hierarchical tree structure for the D-vine copula with four variables.

by the edges of the previous tress and dependence in the different level structures is given by the third term in Eq. (4), and as represented in Fig. 1.

The four-dimensional density function of the D-vine model is given by:

$$f(x_1, x_2, x_3, x_4) = \prod_{k=1}^{4} f_k(x_k) \prod_{h=1}^{4-1} c_{h,h+1}\left(F_h(x_h), F_{h+1}(x_{h+1})\right)$$
$$\times \prod_{j=2}^{4-1} \prod_{i=1}^{4-j} c_{i,i+j|i+1,\ldots,i+j-1}\left(F\left(x_i|x_{i+1},\ldots,x_{i+j-1}\right), F\left(x_{i+j}|x_{i+1},\ldots,x_{i+j-1}\right)\right). \quad (6)$$

Fig. 2 represents the hierarchical dependence of the D-vine copula, where variables are treated equally and the order of the variables in the first tree determines the bivariate dependency in the remaining trees. Order in the first tree is selected with the aim of capturing as much dependence as possible (see Nikoloulopoulos, Joe, & Li, 2012).

Finally, the four-dimensional density function of the R-vine model is given by:

$$f(x_1, x_2, x_3, x_4) = \prod_{k=1}^{4} f_k(x_k) \prod_{i=1}^{4-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}\left(F\left(x_{j(e)}|x_{D(e)}\right), F\left(x_{k(e)}|x_{D(e)}\right)\right), \quad (7)$$

where nodes are denoted by E_i and where $x_{D(e)}$ is the subvector of x, indicated by the indices contained in the conditional set $D(e)$. The appropriate R-vine structure is chosen using the maximum spanning tree that solves the following optimization problem for each tree:

$$\max \sum_{\text{edges } e=\{i,j\} \text{ in spanning tree}} \left|\hat{\tau}_{ij}\right|, \quad (8)$$

where $\hat{\tau}_{ij}$ denotes the pairwise empirical Kendall's tau and a spanning tree is a tree on all nodes.[2]

2.2. Marginal and copula modelling

Marginal and bivariate copula models required for multivariate dependence modelling with vine copulas are described as follows.

The marginal densities for precious metal returns are assumed to be given by an autoregressive moving average (ARMA) model with p and q lags specified as:

$$r_t = \phi_0 + \sum_{j=1}^{p} \phi_j r_{t-j} + \sum_{h=1}^{q} \varphi_j \varepsilon_{t-h} + \varepsilon_t, \quad (9)$$

[2] Further discussion regarding vine copula model selection can be found in Czado, Brechmann, and Gruber (2013).

where $\varepsilon_t = \sigma_t z_t$, with σ_t accounting for the conditional standard deviation that has dynamics described by an asymmetric power GARCH (APGARCH) specification as proposed by Ding, Granger, and Engle (1993):

$$\sigma_t^\delta = \omega + \sum_{k=1}^{r} \beta_k \sigma_{t-k}^\delta + \sum_{h=1}^{m} \alpha_h \left(\left| \varepsilon_{t-h} \right| - \lambda_h \varepsilon_{t-h} \right)^\delta, \tag{10}$$

where ω denotes a constant, β and α are the GARCH and autoregressive conditional heteroskedasticity (ARCH) parameters, respectively, λ captures the leverage effect for $\lambda \neq 0$ and $\delta > 0$ is the parameter for the power term. This model nests a number of standard ARCH and GARCH specifications (see Ding et al., 1993; Hentschel, 1995); specifically, for $\delta = 2$ and $\lambda = 0$ we have the standard GARCH model. z_t is a stochastic variable that follows a Hansen's (1994) skewed-t density distribution given by:

$$f(z_t; \upsilon, \eta) = \begin{cases} bc\left(1 + \frac{1}{\upsilon - 2}\left(\frac{bz_t + a}{1 - \eta}\right)^2\right)^{-(\upsilon+1)/2} & z_t < -a/b \\ bc\left(1 + \frac{1}{\upsilon - 2}\left(\frac{bz_t + a}{1 + \eta}\right)^2\right)^{-(\upsilon+1)/2} & z_t \geq -a/b \end{cases}, \tag{11}$$

where υ and η are the degrees of freedom parameter ($2 < \upsilon < \infty$) and the symmetric parameter ($-1 < \eta < 1$), respectively. The constants a, b and c are given by $a = 4\eta c\left(\upsilon - 2/\upsilon - 1\right)$, $b^2 = 1 + 3\eta^2 - a^2$ and $c = \Gamma\left(\upsilon + 1/2\right)/\sqrt{\pi(\upsilon - 2)}\Gamma\left(\upsilon/2\right)$. For the particular case of $\eta = 0$ and as $\upsilon \to \infty$, the skewed-t density converges to the Gaussian density, whereas when $\eta = 0$ and υ is finite, the skewed-t density converges to the symmetric Student-t distribution.

We used five different bivariate copula specifications to capture different dependence characteristics such as tail independence (Gaussian), symmetric tail dependence (Student-t) and asymmetric tail dependence (Gumbel, Rotated Gumbel and BB7). Table 1 summarizes the main features of the static and dynamic copula functions used in the empirical analysis.

We estimated the parameters of the marginal and copula models through maximum likelihood in two steps (Joe & Xu, 1996). First, we estimated the parameters of the four marginal distributions as given by Eqs. (9)–(11). Second, using the probability integral transformation of the standardized residuals for the marginals as copula pseudo-sample observations, we estimated the static and time-varying bivariate copula parameters for the first tree in the vine copula, where, for the time-varying copulas, we considered that the copula parameter changes over the sample period according to the dynamic equation contained in the note to Table 1; for the remaining trees we sequentially recomputed the pseudo-sample observations through the copulas estimated for the previous trees using Eq. (5). This sequential copula parameter estimation procedure was introduced by Aas et al. (2009) and later examined in Hobæk Haff (2013). The number of lags in the mean and variance equations of the marginal models was selected according to the Akaike information criteria (AIC) and the different copula models were evaluated using the AIC adjusted for small-sample bias, as in Breymann, Dias, and Embrechts (2003) and Reboredo (2013a).

2.3. Downside and upside precious metal price spillovers

We account for price spillover effects by measuring how the upside or downside VaR of one precious metal is impacted by the upside or downside VaR, respectively, of another precious metal.

VaR provides information on the maximum loss for a given time period and a confidence level $(1 - \alpha)$; i.e., it is the α-quantile of the price return distribution, $Pr(x_t \leq \mathrm{VaR}_{\alpha,t}) = \alpha$. It is computed from the marginal model as $VaR_{\alpha,t} = \mu_t + t_{\upsilon,\eta}^{-1}(\alpha)\sigma_t$, where μ_t and σ_t are the conditional mean and standard deviation of price returns obtained from Eqs. (9) and (10), respectively, and where $t_{\upsilon,\eta}^{-1}(\alpha)$ denotes the

Table 1
Copulas specifications.

Name	Copula	Parameter	Structure dependence
Gaussian	$C_N(u, v; \rho) = \Phi\left(\Phi^{-1}(u), \Phi^{-1}(v)\right)$	ρ	No tail dependence: $\lambda_U = \lambda_L = 0$
T-student	$C_{ST}(u, v; \rho, \upsilon) = T(t_\upsilon^{-1}(u), t_\upsilon^{-1}(v))$	ρ,υ	Symmetric tail dependence: $\lambda_U = \lambda_L = 2t_{\upsilon+1}\left(-\sqrt{\upsilon+1}\sqrt{1-\rho}/\sqrt{1+\rho}\right)$
Gumbel	$C_G(u, v; \delta) = \exp\left(-\left((-\log u)^{\delta} + (-\log v)^{\delta}\right)^{1/\delta}\right)$	$\delta \geq 1$	$\lambda_L = 0, \quad \lambda_U = 2 - 2^{1/\theta}$
Rotated Gumbel	$C_{RG}(u, v; \delta) = u + v - 1 + C_G(1-u, 1-v; \delta)$	$\delta \geq 1$	$\lambda_L = 2 - 2^{1/\theta}, \quad \lambda_U = 0$
BB7	$C_{BB7}(u, v; \delta, \theta) = 1 - \left(1 - \left[\left(1-(1-u)^{\theta}\right)^{-\delta} + \left(1-(1-v)^{\theta}\right)^{-\delta} - 1\right]^{-1/\delta}\right)^{1/\theta}$	$\theta \geq 1, \delta > 0$	$\lambda_L = 2^{-1/\delta}, \quad \lambda_U = 2 - 2^{1/\theta}$

Note: $\lambda_u(\lambda_u)$ denotes upper (lower) tail dependence. We captured time-varying parameter (TVP) dependence by assuming that copula parameters change over time. For the Gaussian and the Student-t copulas, we adopted an ARMA(1,q)-type process (Patton, 2006) for the linear dependence parameter ρ_t: $\rho_t = \Lambda_1\left(\psi_0 + \psi_1\rho_{t-1} + \psi_2\frac{1}{q}\sum_{j=1}^{q}\Phi^{-1}(u_{t-i})\cdot\Phi^{-1}(v_{t-i})\right)$, where $\Lambda_1(x) = (1-e^{-x})(1+e^{-x})^{-1}$ is the modified logistic transformation that keeps the value of ρ_t in $(-1,1)$, $\Phi^{-1}(x)$ is standard normal quantile function. For the Student-t copula, $\Phi^{-1}(x)$ is replaced by $t_\upsilon^{-1}(x)$. We considered the TVP for the Gumbel copula, the rotated Gumbel copula and the BB7 copula by assuming that the parameters follow the dynamics given by the following equation: $\delta_t = \bar{\omega} + \bar{\beta}\delta_{t-1} + \bar{\alpha}\frac{1}{q}\sum_{j=1}^{q}|u_{t-i} - v_{t-i}|$ and $\theta_t = \omega + \beta\theta_{t-1} + \alpha\frac{1}{q}\sum_{j=1}^{q}|u_{t-j} - v_{t-j}|$.

α-quantile of the skewed Student-t distribution in Eq. (11). We can define and compute the upside VaR in a similar way by considering $\Pr(x_t \geq \mathrm{VaR}_{1-\alpha,t}) = \alpha$.

Downside price spillover from one precious metal, named 2, to another precious metal, named 1, can be formally stated from the four precious metals dependence structure as:

$$Pr(x_{1t} \leq \mathrm{CoVaR}^1_{\beta,t} | x_{2t} \leq \mathrm{VaR}^2_{\alpha,t}, x_{3t}, x_{4t}) = \beta, \tag{12}$$

where CoVaR^1 denotes the conditional VaR of variable 1, which, in statistical terms, is the β-quantile of a conditional distribution, and where VaR^2 denotes the VaR of variable 2. Note that price return spillovers are closely related to the propagation of failures from one precious metal market to another, given that such spillovers provide a quantitative measure of the impact of distress in one market on the performance of other markets under extreme circumstances, as proposed in the systemic risk literature (see e.g., Billio, Getmansky, Lo, & Pelizzon, 2012; Bisias, Flood, Lo, & Valavanis, 2012). Likewise, we can measure upside price spillover as:

$$Pr(x_{1t} \geq \mathrm{CoVaR}^1_{\beta,t} | x_{2t} \geq \mathrm{VaR}^2_{1-\alpha,t}, x_{3t}, x_{4t}) = \beta, \tag{13}$$

where CoVaR^1 and VaR^2 now denote the upside conditional VaR and the upside VaR of variables 1 and 2, respectively.

CoVaR values can be represented and computed from copulas (Reboredo, 2015; Reboredo & Ugolini, 2015a, 2015b) as follows. Note that Eq. (12) can be written as:

$$\frac{F_{x_{1t},x_{2t}|x_{3t},x_{4t}}\left(\mathrm{CoVaR}^1_{\beta,t}, \mathrm{VaR}^2_{\alpha,t}\right)}{F_{x_{2t}|x_{3t},x_{4t}}\left(\mathrm{VaR}^2_{\alpha,t}\right)} = \beta. \tag{14}$$

From the copula representation of the distribution function, Eq. (14) can be rewritten as:

$$C_{x_{1t},x_{2t}|x_{3t},x_{4t}}\left(F_{x_{1t}|x_{3t},x_{4t}}\left(\mathrm{CoVaR}^1_{\beta,t}\right), F_{x_{2t}|x_{3t},x_{4t}}\left(\mathrm{VaR}^2_{\alpha,t}\right)\right) = \alpha\beta. \tag{15}$$

Similarly, Eq. (13) can be written as:

$$\begin{aligned} &1 - F_{x_{1t}|x_{3t},x_{4t}}\left(\mathrm{CoVaR}^1_{\beta,t}\right) - F_{x_{2t}|x_{3t},x_{4t}}\left(\mathrm{VaR}^2_{1-\alpha,t}\right) \\ &\quad + C_{x_{1t},x_{2t}|x_{3t},x_{4t}}\left(F_{x_{1t}|x_{3t},x_{4t}}\left(\mathrm{CoVaR}^1_{\beta,t}\right), F_{x_{2t}|x_{3t},x_{4t}}\left(\mathrm{VaR}^2_{1-\alpha,t}\right)\right) = \alpha\beta. \end{aligned} \tag{16}$$

Hence, CoVaR in Eqs. (15) and (16) can be obtained from the vine copula model using a three-step procedure:

(1) Given the significance levels for the VaR and CoVaR (α and β, respectively) and for specific forms of the copula function we can solve Eq. (15) or Eq. (16) in order to obtain the value of $F_{x_{1t}|x_{3t},x_{4t}}\left(\mathrm{CoVaR}^1_{\beta,t}\right)$.
(2) From the value computed in the above step and from the conditional copula model we extract information on $F_{x_{1t}}\left(\mathrm{CoVaR}^1_{\beta,t}\right) = u_1$.
(3) From u_1 we obtain CoVaR as the quantile of the distribution of x_1, with a cumulative probability equal to u_1, by inverting the marginal distribution function of x_1: $\mathrm{CoVaR}^1_{\beta,t} = F_1^{-1}(u_1)$.

To test for the significance of downside (upside) price spillovers, we compared the cumulative distribution for downside (upside) CoVaR and VaR, using the KS bootstrapping test, introduced by Abadie (2002) and applied by Bernal, Gnabo, and Guilmin (2014) to compare CoVaR values. This test measures the difference between two cumulative quantile functions relying on the empirical distribution function but without considering any underlying distribution function. It is defined as:

$$KS_{mn} = \left(\frac{mn}{m+n}\right)^{\frac{1}{2}} \sup_x \left|F_m(x) - G_n(x)\right|, \tag{17}$$

where $F_m(x)$ and $G_n(x)$ are the cumulative CoVaR and VaR distribution functions, respectively, and n and m are the size of the two samples. With this statistic we tested the hypothesis of no systemic impact between two precious metal markets as:

$$\mathrm{H}_0: \quad \mathrm{CoVaR}^1_{\beta,t} = \mathrm{VaR}^1_{\beta,t}.$$

3. Data

We empirically characterized the dependence structure between gold, silver, platinum and palladium using weekly spot price data from 26 October 2001 to 9 January 2015. Our database covered the period of the subprime/global financial crisis and the European debt crisis, so we could better disentangle the dynamics of co-movement and spillovers among major precious metals during financial crises, when precious metals are demanded for financial as well as industrial purposes. Prices, measured in USD per troy ounce, were sourced from Bloomberg. Fig. 3 displays temporal dynamics for precious metal prices, providing visual evidence of high co-movement between all precious metal prices and of a drop in prices around the onset of the global financial crisis, mainly for platinum. Precious metal prices gradually recovered thereafter – except for silver which showed a sharply downward trend – to maintain an upward trend that slightly reversed from the beginning of 2013 when global uncertainty began to gradually remit.

Descriptive statistics for precious metal price returns, computed on a continuous compounded basis, are described in Table 2. Weekly returns had average values close to zero and differing high standard deviations, indicating dispersion in volatility behaviour across markets. Silver and palladium showed higher volatilities than gold and platinum. Silver and palladium also showed more extreme maxima and minima than gold and platinum. All series were skewed and exhibited significant kurtosis. The evidence provided by the Jarque–Bera (JB) test indicated that all series had non-normal distributions. Likewise, the Ljung-Box (LB) statistic confirmed the absence of serial correlation, except for platinum, while the autoregressive conditional heteroskedasticity-Lagrange multiplier (ARCH-LM) statistic clearly indicated the presence of ARCH effects in all series. Finally, the evidence reported by the

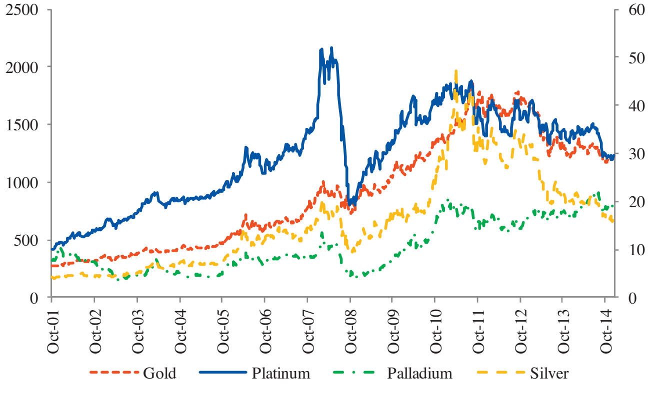

Fig. 3. Time series plots for weekly precious metal prices for the period October 2001 to January 2015 (silver prices indicated on the right axis).

Table 2
Descriptive statistics.

	Gold	Silver	Platinum	Palladium
Mean	0.002	0.002	0.002	0.001
Maximum	0.090	0.153	0.109	0.171
Minimum	−0.093	−0.305	−0.151	−0.197
Std. Dev.	0.025	0.047	0.031	0.047
Skewness	−0.400	−1.262	−0.815	−0.348
Kurtosis	4.152	9.337	6.114	4.905
JB	55.750*	1318.228*	350.106*	116.518*
LB(20)	20.914	28.187	32.304	23.126
	[0.402]	[0.105]	[0.040]	[0.283]
ARCH	7.072	4.987	6.370	3.582
	[0.000]	[0.000]	[0.000]	[0.000]
Correlation matrix				
Gold	1			
Silver	0.75	1		
Platinum	0.56	0.58	1	
Palladium	0.40	0.51	0.63	1

Notes: Weekly data for the period 26 October 2001 to 9 January 2015. JB denotes the Jarque–Bera statistic for normality; an asterisk (*) indicates rejection of the null hypothesis at the 5% level. LB(20) is the Ljung-Box statistics for the serial correlation in returns computed with 20 lags. ARCH denotes Engle's LM test for heteroskedasticity computed using 20 lags. For those tests, p values are reported in square brackets.

unconditional linear Pearson correlation indicated that all series were highly dependent, but especially gold and silver.

4. Results

4.1. Marginal model results

The empirical estimates of the marginal models in Eqs. (9)–(11) for the four precious metal price returns are reported in Table 3. Model parameters were selected for a range of values between 0 and 2, taking as the best fitting model the one that minimized the AIC value. The empirical evidence shows that average returns do not display serial dependence given that no autoregressive (AR) or moving average (MA) coefficients were significant in any series. Volatility estimates indicated that ARCH and GARCH components were significant in all series and that volatility persisted across precious metal markets. Likewise, we observed no leverage effects except for gold, indicating that precious metal markets responded symmetrically to informational shocks. The estimated value of the δ parameter indicates that the standard GARCH was unable to yield the best fit in all precious metal price returns. Asymmetry and degrees-of-freedom parameter estimates for the skewed Student-t distribution indicated that error terms were well characterized by a distribution with asymmetries and fat tails, with the exception of palladium, where symmetry reflected the best characterization.

The results of different goodness-of-fit tests applied to our marginal models are reported in the last rows of Table 3. The Ljung-Box test indicated that no serial correlation was evident in either the residual series or the squared residual series. We also tested for structural changes in the GARCH models using the cumulative sum test, but found no evidence of change. The evidence provided by the ARCH statistic indicated that no GARCH effects remained in the model residuals. We also provided evidence on the goodness-of-fit of the skewed-t distribution by checking whether the distribution of the standardized model residuals was uniform (0,1). To that end, we compared the empirical and theoretical distributions of the standardized residuals using the KS, Cramér-von Mises (CvM) and Anderson–Darling (AD) tests. The last three rows of Table 3, reporting the p values for these tests, provides evidence in favour of the null hypothesis of correct specification of the distribution model. In short, our goodness-of-fit tests indicate that there was no misspecification problems in our marginal models for precious metal price returns.

Table 3
Parameter estimates for marginal models.

	Gold	Silver	Platinum	Palladium
Mean				
ϕ_0	0.003*	0.002	0.002*	0.002
	(2.868)	(0.555)	(2.091)	(1.173)
Variance				
ω	0.005	16.689	0.362	0.762
	(0.376)	(0.816)	(0.464)	(0.524)
α_1	0.071*	0.057*	0.087*	0.108*
	(3.557)	(2.044)	(2.561)	(2.487)
β_1	0.857*	0.938*	0.907*	0.873*
	(31.350)	(31.390)	(25.760)	(17.630)
λ	−0.236*	−0.550	−0.172	−0.142
	(−3.180)	(−1.893)	(−1.298)	(−1.636)
δ	3.000*	0.800*	1.760*	1.924*
	(4.143)	(2.160)	(3.391)	(3.970)
Asymmetry	−0.180*	−0.162*	−0.171*	−0.101
	(−3.821)	(−2.453)	(−3.585)	(−1.826)
Tail	19.174	5.389	5.999	6.298
LogLik	1589.52	1206.46	1491.47	1173.69
LJ	10.685	18.804	25.221	25.782
	[0.95]	[0.53]	[0.19]	[0.17]
LJ2	26.553	19.594	14.624	23.386
	[0.09]	[0.30]	[0.69]	[0.18]
ARCH	1.244	1.004	0.667	1.178
	[0.21]	[0.45]	[0.86]	[0.27]
KS	[0.59]	[0.68]	[0.64]	[0.83]
CvM	[0.71]	[0.91]	[0.72]	[0.86]
AD	[0.84]	[0.94]	[0.76]	[0.89]

Notes: The table reports estimates and z-statistics (in brackets) for the parameters of the marginal models described in Eqs. (9)–(11). LogLik, LJ, LJ2 denote the log-likelihood value and the Ljung-Box statistic for serial correlation in the residual model and in the squared residual model calculated with 20 lags. Engle's LM test for the ARCH effect in residuals up to 20th order are denoted by ARCH. KS, CvM and AD denote the Kolmogorov–Smirnov, Cramér-von Mises and Anderson–Darling tests, where p values (in square brackets) below 0.05 indicate rejection of the null hypothesis. An asterisk (*) indicates significance at 5%.

4.2. Copula model results

As pseudo-sample observations for the copula, we used the probability integral transform of the standardized residuals for each of the marginal models reported in Table 3 and estimated the three vine copula models using the static and time-varying bivariate copulas displayed in Table 1. According to the AIC values, the best copula fit was provided by the D-vine copula model, represented in Fig. 4 along with information on the best pair-copula specification for each series pair. Accordingly, the

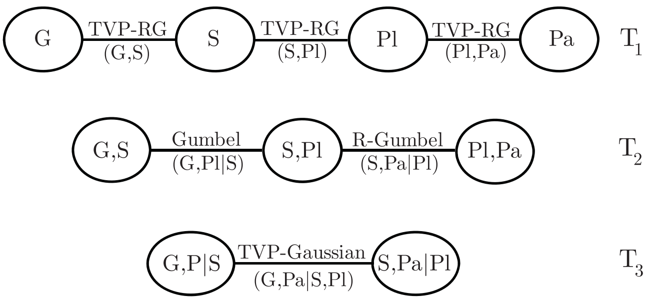

Fig. 4. D-vine copula model for gold (G), silver (S), platinum (Pl) and palladium (Pa).

dependence structure between precious metals was not dominated by any specific precious metal, a result that contrasts with the leading role that gold is usually assumed to play regarding contagion between precious metals (Sensoy, 2013). Our result is also consistent with the fact that there is no substantial difference in industrial and financial uses of precious metals in terms of price leadership.

Table 4 reports estimates for the static and time-varying bivariate copulas in the different trees of the vine hierarchical structure. The first tree accounts for the gold–silver, silver–platinum and platinum–palladium dependence relationship. For these pairs, parameter estimates provide evidence of high positive dependence that is consistent with the unconditional correlation values reported in Table 2. On comparing different copula specifications, we observed that the time-varying parameter (TVP) rotated Gumbel copula yielded the best fit for all pairs included in the first tree of the vine. This would corroborate the fact that precious metal prices are coupled on average and in bearish markets, but decoupled in bullish markets. This evidence of asymmetric tail behaviour may be due to the fact that, during financial market upturns, investor interest in preserving portfolio value decreases, so the financial demand for certain precious metals, especially gold, is negatively affected; in contrast, the industrial demand for certain metals remains constant or even increases.

The second tree in the D-vine model accounts for the gold–platinum and silver–palladium dependence relationship conditional on silver and on platinum, respectively. For these pairs, our empirical pair-copula estimations indicate average positive dependence for the two pairs, even though conditional dependence was weaker than unconditional dependence, as reported by the correlation coefficients reported in Table 2. According to the AIC test, the static versions of the Gumbel and rotated Gumbel copulas characterized the conditional dependence between gold and platinum and between silver and palladium, respectively. Thus, gold displayed upper tail dependence and lower tail independence with platinum, while silver displayed static lower tail dependence and upper tail independence with palladium. Finally, the last tree of the D-vine copula accounts for dependence between gold and palladium conditional on silver and platinum. The best-fitting copula characterizing this dependence was the Gaussian TVP, whose correlation coefficient near zero indicates conditional average and tail independence between gold and palladium.

To sum up, the evidence on the multivariate dependence structure provided by the D-vine model indicates dependence and independence between precious metal prices as follows: (a) positive and high average dependence between all prices, with the exception of gold with palladium; (b) lower tail dependence between all prices, with the exception of gold with platinum and gold with palladium; and (c) upper tail independence between all prices, with the exception of gold and platinum. Gold is the only precious metal that has a specific conditional relationship with platinum and palladium in times of extreme market movements, probably due to fact that gold plays a more specific financial role as an asset than the other precious metals. In other words, demand for gold is more affected by financial issues than is demand for the other metals, for which demand and prices are mainly driven by industrial uses.

4.3. Spillover effects between precious metal prices

Using the univariate marginal distribution functions and the best pair-copula fit estimates from the D-vine hierarchical structure, we computed the downside/upside VaR and CoVaR values at the 95% confidence level ($\beta = 0.05$, $\alpha = 0.05$)[3] following the procedures described above.

Fig. 5 reports graphical evidence on the size and dynamics of downside/upside VaR and CoVaR values throughout the sample period. Descriptive statistics and hypothesis test results are reported in Table 5. For all four precious metals, downside and upside VaR values were roughly symmetric and displayed similar temporal dynamics, although gold and palladium experienced more abrupt upward and downward extreme movements than silver and platinum. Regarding the impact of precious metal prices, CoVaR values significantly differed from VaR values and those differences were reflected in different magnitudes across the precious metals. We could corroborate that the systemic impact of silver regarding gold was considerably greater than the systemic impact of the other two precious

[3] Results at the 99% confidence level are available on request.

Table 4
Pair-copula estimates for the D-vine copula model for gold (G), silver (S), platinum (Pl) and Palladium (Pa).

Copula	T_1			T_2		T_3
	(G,S)	(S,Pl)	(Pl,Pa)	(G,Pl|S)	(S,Pa|Pl)	(G,Pa |S,Pl)
Panel A: Parameter estimates for time-invariant copulas						
Gaussian						
ρ	0.752*	0.540*	0.607*	0.225*	0.188*	−0.044
	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.01)
AIC	−562.179	−232.529	−309.907	−33.358	−22.586	0.683
Student-*t*						
ρ	0.755*	0.546*	0.609*	0.225*	0.188*	−0.040
	(0.02)	(0.03)	(0.02)	(0.04)	(0.04)	(0.04)
υ	15.134	8.084*	14.253*	12.329	12.599	17.135
	(9.82)	(1.99)	(1.34)	(9.95)	(7.04)	(12.62)
AIC	−565.163	−241.810	−311.531	−35.405	−25.251	0.147
Gumbel						
δ	2.017*	1.479*	1.573*	1.158*	1.106*	1.005*
	(0.06)	(0.04)	(0.05)	(0.03)	(0.03)	(0.01)
AIC	−493.566	−182.087	−235.257	**−37.342**	−15.382	1.698
Rotated Gumbel						
δ	2.099*	1.574*	1.703*	1.136*	1.127*	1.000*
	(0.07)	(0.05)	(0.05)	(0.03)	(0.03)	(0.04)
AIC	−550.266	−263.533	−338.731	−23.943	**−27.662**	2.006
BB7						
θ	1.654*	1.152*	1.179*	1.165*	1.040*	1.009*
	(0.10)	(0.06)	(0.07)	(0.05)	(0.04)	(0.13)
δ	1.330*	0.906*	1.088*	0.109*	0.208*	0.001
	(0.11)	(0.08)	(0.09)	(0.05)	(0.05)	(0.96)
AIC	−533.291	−258.023	−323.337	−33.718	−26.036	3.507
Panel B: Parameter estimates for time-varying copulas						
TVP-Gaussian						
ψ_0	2.056	−0.127	−0.344	0.531	0.198	−0.006
	(2.05)	(0.07)	(1.34)	(1.54)	(0.12)	(0.01)
ψ_1	0.153	0.057	0.034	0.044	−0.159	−0.062*
	(0.13)	(0.03)	(0.11)	(0.20)	(0.10)	(0.03)
ψ_2	−0.266	2.438*	2.858	−0.398	1.169*	1.906*
	(2.79)	(0.15)	(2.56)	(6.93)	(0.55)	(0.07)
AIC	−559.977	−241.931	−317.231	−29.412	−22.140	**−0.168**
TVP-Student						
ψ_0	2.963	−0.112	3.220*	0.848*	0.731*	−0.145
	(1.64)	(0.06)	(0.26)	(0.20)	(0.15)	(0.13)
ψ_1	0.194*	0.057*	0.108	−0.029	0.168*	0.093
	(0.09)	(0.03)	(0.09)	(0.08)	(0.05)	(0.13)
ψ_2	−1.493	2.404*	−3.071*	−1.702*	−2.000*	−1.306
	(2.19)	(0.15)	(0.22)	(0.62)	(0.07)	(0.73)
υ	18.350	9.028*	15.589*	12.418	13.170*	17.885
	(10.77)	(3.41)	(3.96)	(7.11)	(0.97)	(39.59)
AIC	−565.857	−250.473	−310.521	−31.487	−26.903	3.513
TVP-Gumbel						
$\bar{\omega}$	1.877*	−0.093	1.680*	1.868*	1.088	5.000
	(0.22)	(0.10)	(0.21)	(0.07)	(1.86)	(43.22)
$\bar{\beta}$	−0.212*	0.566*	−0.469*	−1.226*	−0.704	−4.740
	(0.08)	(0.05)	(0.15)	(0.12)	(1.71)	(3.12)
$\bar{\alpha}$	−2.594*	−0.262	−0.927*	−0.144	0.069	−0.212
	(0.52)	(0.14)	(0.40)	(0.14)	(0.34)	(77.61)
AIC	−519.210	−186.811	−237.880	−34.785	−11.389	3.946

Table 4 (*Continued*)

Copula	T_1			T_2		T_3
	(G,S)	(S,Pl)	(Pl,Pa)	(G,Pl|S)	(S,Pa|Pl)	(G,Pa |S,Pl)
TVP-Rotated Gumbel						
$\bar{\omega}$	1.897*	0.002	0.001	1.944*	0.751	0.000
	(0.30)	(0.08)	(0.07)	(0.09)	(3.01)	(1.00)
$\bar{\beta}$	−0.230*	0.519*	0.516*	−1.312*	−0.323	0.000
	(0.10)	(0.04)	(0.03)	(0.14)	(2.68)	(1.00)
$\bar{\alpha}$	−2.146*	−0.306*	−0.228	−0.237	−0.096	0.000
	(0.63)	(0.12)	(0.12)	(0.14)	(0.34)	(1.00)
AIC	−**566.514**	−**274.266**	−**349.522**	−23.264	−23.717	6.041
TVP-BB7						
$\bar{\omega}_\theta$	1.746*	2.717*	1.953*	0.016	0.832	0.215
	(0.34)	(0.73)	(0.21)	(5.28)	(7.63)	(2.14)
$\bar{\beta}_\theta$	0.447	−0.059	−0.671	−0.126	0.566	0.126
	(1.47)	(0.13)	(0.49)	(0.37)	(2.23)	(5.95)
$\bar{\alpha}_\theta$	−0.626*	−1.574*	−1.146*	−0.324	−0.790	−0.346*
	(0.13)	(0.11)	(0.19)	(4.50)	(7.03)	(0.05)
$\bar{\omega}_\delta$	1.935*	0.896	0.672*	1.006*	0.818*	−0.008
	(0.37)	(2.26)	(0.05)	(0.11)	(0.24)	(3.81)
$\bar{\beta}_\delta$	−2.530	−0.455	−0.375*	−1.933*	−0.367	0.016
	(2.15)	(2.27)	(0.14)	(0.18)	(0.34)	(7.12)
$\bar{\alpha}_\delta$	−0.261*	0.220	0.404*	−1.408*	−1.017*	−0.013
	(0.07)	(1.20)	(0.02)	(0.13)	(0.22)	(1.02)
AIC	−538.649	−202.841	−337.827	−31.912	−22.121	11.501

Notes: The table reports the maximum likelihood (ML) estimates for the different pair-copulas in Table 1 for the D-vine model represented in Fig. 4. Standard errors are given in brackets. The number of lags in the time-varying parameter (TVP) copulas was set to 10. The minimum AIC value (in bold) indicates the best copula fit. An asterisk (*) indicates significance at 5%.

metals, and also that the systemic downward impact on price was greater in magnitude than the upward systemic impact. This can be attributed to high co-movement between gold and silver at the lower tail and low dependence at the upper tail. The price spillover effect of platinum and palladium regarding gold was also significant, even though the systemic impact was of lesser intensity than in the case of silver. This is consistent with the fact that platinum and palladium were much less coupled with gold than with silver. In fact, gold and palladium were independent at the tails, so, not surprisingly, average upside and downside CoVaR and VaR values were of a similar size, even though CoVaR value volatility differed widely from VaR value volatility. This would explain why the KS test rejected the null hypothesis of equality between CoVaR and VaR.

The empirical evidence for silver indicated that the downward systemic impact of all precious metals regarding silver was significant and of a similar magnitude. This result is consistent with the fact that silver was coupled with other precious metals at the lower tail and they all thus behaved as a single asset. However, in considering upside systemic risk we observed that precious metals were not equally coupled and also that there were differences in the size of the systemic impact: gold had a greater impact than the other precious metals. In all cases, the null hypothesis of equality between upside VaR and CoVaR was rejected, even though differences between those values were lower than VaR and CoVaR differences in the lower tail.

Regarding platinum, we found evidence of a significant systemic price impact of all precious metals regarding platinum, although the magnitude differed. Thus, we could confirm that silver and palladium had a similar systemic impact on platinum, in contrast with gold, which had a significant but smaller impact, consistent with low tail dependence between gold and palladium. Considering upside systemic risk, we found that all the precious metals had a significant systemic impact according to the KS test. This impact was of a similar size across metal prices, although the systemic impact of gold was more volatile than that for silver or palladium.

Finally, the evidence on downside/upside price spillovers for palladium indicated that these were significant in both types of spillover. We found evidence of price spillovers from silver and platinum of a similar magnitude, in contrast with spillovers from gold, which were smaller in magnitude and

Table 5
Summary downside/upside VaR and CoVaR statistics and hypothesis test results.

	Downside VaR	Downside CoVaR	H_0:VaR = CoVaR H_1:VaR > CoVaR	Upside VaR	Upside CoVaR	H_0:VaR = CoVaR H_1:VaR < CoVaR
Gold	−0.075 (0.02)			0.071 (0.02)		
Gold|Silver		−0.144 (0.04)	0.829 [0.00]		0.110 (0.03)	0.549 [0.00]
Gold|Platinum		−0.090 (0.03)	0.231 [0.00]		0.084 (0.02)	0.274 [0.00]
Gold|Palladium		−0.074 (0.03)	0.097 [0.00]		0.078 (0.03)	0.206 [0.00]
Silver	−0.039 (0.01)			0.037 (0.01)		
Silver|Gold		−0.097 (0.03)	0.957 [0.00]		0.067 (0.02)	0.807 [0.00]
Silver|Platinum		−0.095 (0.03)	0.950 [0.00]		0.058 (0.02)	0.691 [0.00]
Silver|Palladium		−0.087 (0.03)	0.878 [0.00]		0.044 (0.01)	0.288 [0.00]
Platinum	−0.047 (0.02)			0.045 (0.01)		
Platinum|Gold		−0.054 (0.02)	0.221 [0.00]		0.072 (0.03)	0.524 [0.00]
Platinum|Silver		−0.110 (0.04)	0.854 [0.00]		0.069 (0.02)	0.628 [0.00]
Platinum|Palladium		−0.112 (0.04)	0.857 [0.00]		0.072 (0.02)	0.653 [0.00]
Palladium	−0.074 (0.02)			0.072 (0.02)		
Palladium|Gold		−0.085 (0.06)	0.187 [0.00]		0.081 (0.04)	0.231 [0.00]
Palladium|Silver		−0.154 (0.05)	0.799 [0.00]		0.075 (0.03)	0.071 [0.03]
Palladium|Platinum		−0.159 (0.04)	0.844 [0.00]		0.117 (0.03)	0.581 [0.00]

Notes: The table reports descriptive VaR and CoVaR statistics (percentages) at the 95% confidence level using the vine copula model. Standard deviations are reported in round brackets and p values are reported in square brackets.

much more volatile. Regarding upside price spillovers, the magnitudes differed across metals, but were greater for platinum – given that platinum and palladium prices co-move closely – and smaller for gold and silver – given that gold and silver show lower average dependence and upper tail independence with palladium.

To sum up, our results on downside/upside price spillovers between the four precious metals indicated that they do not behave as a single asset: the impact of extreme price movement in any given precious metal on any other precious metal differed depending on the kind of metal considered. An exception was the downside price impact of any precious metal price on downward movement in silver prices, which did not differ across precious metals. Our results are consistent with the fact that the structure of dependence differs across precious metals, which display different degrees of co-movement and different dependence intensities at lower and upper distribution tails. We observed that gold and silver markets were highly integrated, whereas silver markets were more integrated with platinum and palladium markets. Meanwhile, the gold market was barely integrated with the platinum and palladium markets; hence, the high unconditional Pearson correlation coefficient values (see Table 1) can be explained by silver–platinum, silver–palladium and silver–gold dependence. Differences in the conditional dependence relationship and in downside/upside price spillovers can be explained by different financial and industrial uses for precious metals. Gold and silver prices, but most

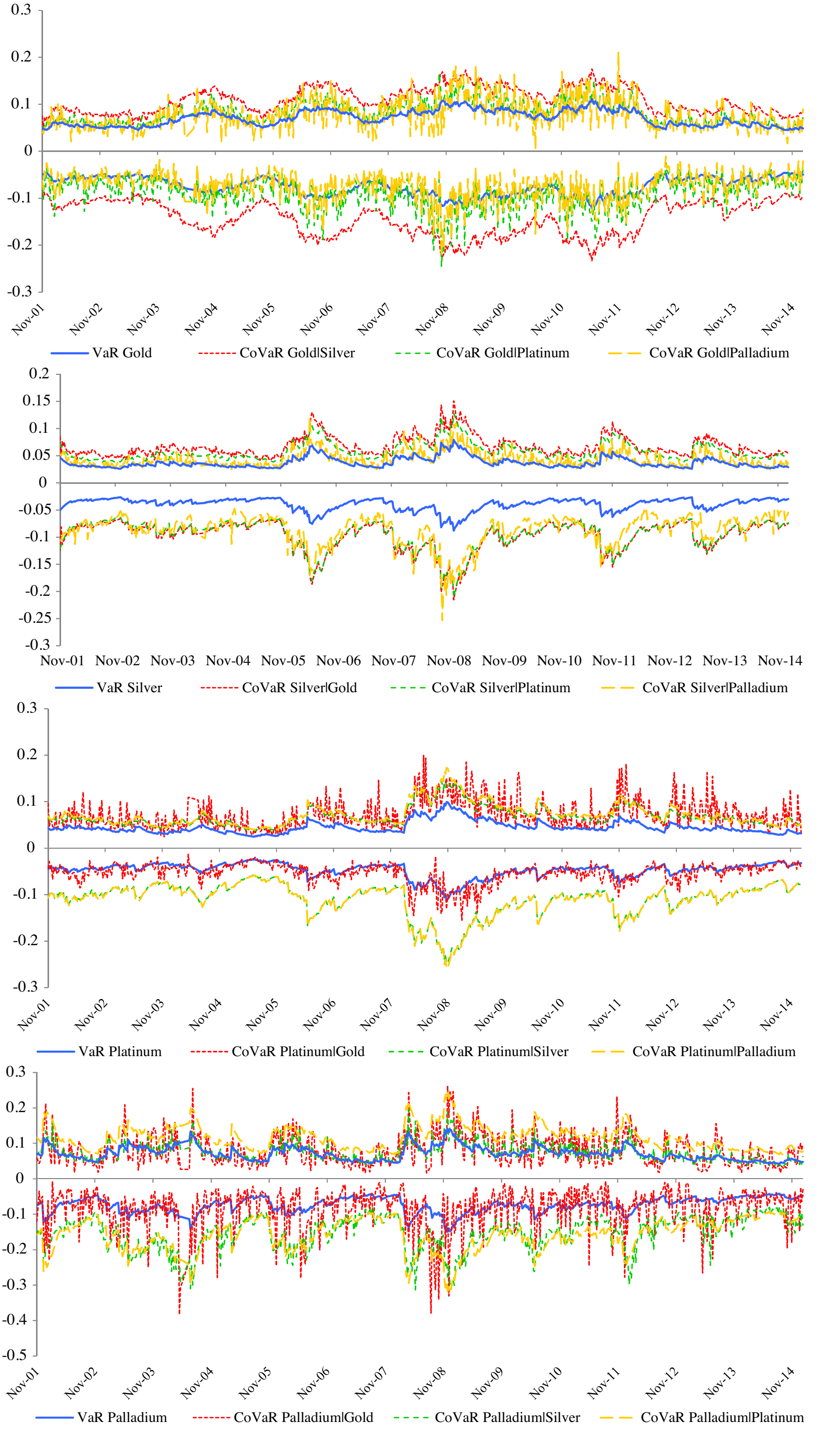

Fig. 5. Time series plots of downside/upside Var and CoVaR precious metal prices.

especially gold, are more driven by financial uses, while platinum and palladium are more dependent on industrial demand.

Our results have implications for both investors and manufacturers. The fact that precious metals do not behave as a single asset class has implications for investors who seek to manage portfolio risk by including precious metals. More specifically, the asymmetric co-movement between precious metal prices and differences in the size of spillovers imply that investors who want to protect their portfolios using precious metals should be highly selective when managing upside risk, given that upside market movements in one precious metal will be transmitted to other precious metals in a portfolio. This issue is even more crucial in managing downside risk, as spillover differences are more exaggerated than for upside risk. Finally, jewellery, electronics and medical instrumentation manufacturers should take into account the price impact of certain precious metals they do not use on the price of precious metals they do use, given that production costs can be affected. Specifically, our results indicate those effects are procyclical and asymmetric, with more significant effects in downturns than in upturns.

5. Conclusions

We examined the multivariate dependence structure between four precious metals (gold, silver, palladium and platinum) using a vine copula model, assessing the price spillover effects between them by computing the CoVaR measure from the vine copula model. This analysis is of interest for investors who include precious metals in their portfolios with the aim of hedging and safeguarding against extreme market movements in financial markets. In this context, a selection strategy for one or several precious metals crucially depends on the dependence structure between individual precious metals and on how price shocks in one precious metal market may be transmitted to other precious metal markets.

Using weekly prices for the period October 2001 to January 2015, our empirical results indicate that the dependence structure is characterized by a D-vine hierarchical structure in which no metal price predominates in determining the conditional dependence structure. Moreover, bivariate dependence differs across precious metals as these display different average and tail dependence features. Precious metal pairs like gold–silver, silver–platinum and platinum–palladium displayed high time-varying average and lower tail dependence with upper tail independence, whereas other precious metal pairs like gold–platinum and gold–palladium displayed low average dependence and tail independence. Our results on downside/upside price spillovers indicated CoVaR values to be significantly lower than VaR values; hence, a downward price movement for any precious metal has a quantitative impact on downward price movements for other precious metals; this effect also differed in magnitude across precious metals. Similarly, we found that upside spillover effects existed, although of less intensity and also differing in magnitude across precious metals. Thus, our evidence on dependence is consistent with the observation that precious metals do not behave as a single asset. For this reason, both investors and manufacturers should consider the existence of spillover effects and asymmetries in dependence when including precious metals in their financial portfolios or acquiring them for industrial activities.

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