Journal of Process Control 19 (2009) 923–931

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Variable-weighted Fisher discriminant analysis for process fault diagnosis

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ARTICLE INFO

Article history:
Received 19 May 2008
Received in revised form 28 November 2008
Accepted 13 December 2008

Keywords:
Fault diagnosis
Fisher discriminant analysis
Variable weighting
Classification

ABSTRACT

Variable-weighted Fisher discriminant analysis (VW-FDA) is proposed to improve the fault diagnosis performance of the conventional FDA. VW-FDA incorporates the variable weighting into FDA. The variable weighting is used to find out each weight vector for all faults. After all fault data are weighted by the corresponding weight vectors, the summed fault data can be constructed to magnify each fault's local characteristics. Then, VW-FDA is performed on the summed fault data rather than the original fault data. It is helpful to extract discriminative features from overlapping fault data. Moreover, the partial *F*-values with the cumulative percent variation are used for exactly variable weighting, which is indispensable to VW-FDA. The proposed approach is applied to Tennessee Eastman process. The results demonstrate that VW-FDA shows better fault diagnosis performance than the conventional FDA.

1. Introduction

Multivariate statistic process monitoring (MSPM) methods such as principle component analysis (PCA), partial least square (PLS), Fisher discriminant analysis (FDA) and canonical variable analysis (CVA) have been widely applied to continuous and batch process monitoring for operating industrial processes more safely, efficiently and economically [1–4]. As multivariate projection techniques, they can extract the first few latent variables from highly correlated process data to capture the key process information. Typically, two monitoring indices: Hotelling's T^2 and the squared prediction error (SPE), are used to detect abnormal situations [3,4].

After a fault has been detected, fault diagnosis aims at determining the root cause of the fault [4]. As an important part of MSPM, fault diagnosis has attracted much attention. Various types of statistical approaches including, but not limited to, contribution plots [5,6], reconstruction-based approach [7,8], structured residuals [9], and pattern recognition [10,11], have been used for process fault diagnosis. Regardless of quantitative difference, a deterministic fault often exhibits qualitative similarity and is related to the same root cause [11]. Assume that different deterministic fault data sets have been gathered from historical data, plant tests or mechanistic knowledge. Then, the task of process fault diagnosis is to classify on-line out-of-control observations to the closest one of predefined fault classes [12]. In this sense, process fault diagnosis becomes a multi-classification problem. FDA is a classical technique for dimension reduction and pattern classification. It has been approved to outperform PCA-based [13] or PLS-based [14] diagnosis methods [15]. This is attributed to the fact that FDA seeks directions that are efficient for discrimination but PCA for representation [16,17]. However, the classification performance of FDA will degenerate as long as overlapping samples exist. One of the main reasons is that the Fisher criteria only considers the distance between the total mean and the each class mean, which leads to overlapped distribution on projection space [18]. So, many methods were proposed to improve the multi-classification performance of FDA [18–21]. Unlike classification problems in pattern recognition, process fault diagnosis can use a normal historical data set as a common benchmark. All fault data are relative to the normal data set. It can supply important classification information to improve the diagnosis performance of FDA.

Focusing on the characteristic of process fault diagnosis, variable-weighted FDA (VW-FDA) is developed in the paper for the multi-classification where fault data are overlapped. VW-FDA incorporates the variable weighting into FDA. The conventional FDA assumes the same contribution of each variable to the classification. When all variables are used in a same level, the fault data are masked with irrelevant information [22]. As a result, the classification problem suffers from more overlapping fault data and FDA is more difficult to extract the effective discriminant information from these fault data. The variable weighting is used to find out each fault's weight vector by making full use of the normal data information. One weight vector characterizes one fault. When all fault data are weighted by the corresponding weight vectors, all variables in each fault are used in different levels. Then, the summed fault data can be constructed to magnify the

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0959-1524/$ - see front matter © 2009 Published by Elsevier Ltd.
doi:10.1016/j.jprocont.2008.12.001

corresponding fault local characteristics. VW-FDA is performed on the summed fault data rather than the original fault data, which is useful for FDA to extract discriminative features from overlapping fault data. It should be noted that the exactly variable weighting is indispensable to VW-FDA. So, the variable weighting based on the partial *F*-values with the cumulative percent variation (CPV) is adopted. Compared with the fault direction, the standardized fault direction and the partial *F*-values, the proposed approach shows better variable weighting performance.

The paper is organized as follows. The concept of FDA is reviewed in Section 2. Then VW-FDA for process fault diagnosis is described in Section 3. Its basic characteristics and calculation procedures are discussed in detail in this section. Section 4 illustrates an application to Tennessee Eastman process (TEP), and Section 5 concludes the paper.

2. Basics of FDA

Let $\mathbf{X}_0 \in \Re^{n_0 \times m}$ scaled to zero mean and unit variance be the normal data set with n_0 samples (rows) and m variables (columns). $\mathbf{X}_k$ ($k = 1,\ldots,c$) is the kth fault data set with n_k samples and m variables. c is the number of fault classes. All fault data are scaled with the mean and variance of the normal data set. Stack all fault data sets into the matrix $\mathbf{X} \in \mathscr{R}^{n \times m}$. Thus, $\mathbf{X} = \bigcup_{k=1}^{c} \mathbf{X}_k$. The ith row of $\mathbf{X}$ represents the fault sample vector $\mathbf{x}_i$.

FDA is a well-known linear technique for reducing dimensions and pattern classification. It determines a set of Fisher optimal discriminant vectors by maximizing the scatter between the classes while minimizing the scatter within each class [23]. The transformation matrix consisting of these discriminant vectors is denoted by $\mathbf{A} \in \mathscr{R}^{m \times d}$ where $d \leqslant c-1$ is the dimension of the projection subspace. Then, a fault sample vector $\mathbf{x}_i$ is transformed to the projection subspace by

$$\mathbf{z}_i = \mathbf{A}^T \mathbf{x}_i. \tag{1}$$

The transformation matrix can be obtained by maximizing the Fisher criterion $J_F(\mathbf{A})$:

$$J_F(\mathbf{A}) = \mathrm{trace}\{(\mathbf{A}^T\mathbf{S}_w\mathbf{A})^{-1}(\mathbf{A}^T\mathbf{S}_b\mathbf{A})\}, \tag{2}$$

where $\mathbf{S}_b = \sum_{k=1}^{c} n_k(\mathbf{b}_k - \mathbf{b})(\mathbf{b}_k - \mathbf{b})^T$ and $\mathbf{S}_w = \sum_{k=1}^{c}\sum_{\mathbf{x}_i \in \mathbf{X}_k}(\mathbf{x}_i - \mathbf{b}_k)(\mathbf{x}_i - \mathbf{b}_k)^T$ are the between-class scatter matrix and the within-class scatter matrix, respectively. Here, $\mathbf{b}$ is the total mean vector of all fault samples and $\mathbf{b}_k$ is the mean vector for fault class k.

If S_w is nonsingular, the optimization problem becomes a conventional eigenvalue problem, that is, finding the eigenvectors of $\mathbf{S}_w^{-1}\mathbf{S}_b$ corresponding to the d largest eigenvalues. Specially, the Fisher optimal discriminant vector is $\mathbf{S}_w^{-1}(\mathbf{b}_0 - \mathbf{b}_k)$ for the two-class problem: the normal data set $\mathbf{X}_0$ and the fault data set $\mathbf{X}_k$. $\mathbf{b}_0$ is the mean vector for the normal data set.

The optimal dimension of the projection subspace can be obtained by maximizing Akaike's information criterion [3]:

$$f_{\mathrm{FDA}} = s(d) - \frac{d}{n_{avg}}, \tag{3}$$

where $s(d)$ is the cross validation classification success rate at the dimension d and n_{avg} is the average number of observation per class.

3. VW-FDA for process fault diagnosis

VW-FDA integrates the variable weighting into the conversional FDA. It includes three main procedures. Firstly, the variable weighting based on the partial *F*-values with CPV is used to find out each weight vector for all faults. Then, after all fault data are weighted by the corresponding weight vectors, the summed fault data can be constructed. They magnify the corresponding fault local characteristics and supply important classification information to the conventional FDA. Finally, VW-FDA is performed on the summed fault data.

3.1. Variable weighting based on partial F-values with CPV

Different variables usually have different contributions to a fault. The variable weighting aims to weight all variables with continuous values according to their responsibilities for a fault. PCA or PLS-based contribution plots are well-known variable weighting approaches [3–6]. Although they can easily be calculated with no prior knowledge, they can not explicitly reveal the cause of an abnormal condition due to 'smearing' effect [4].

As far as discriminant analysis is concerned, the variable weighting is used to find out a discriminant vector by maximizing the separation between the normal and fault data sets. Each element of the discriminant vector represents the corresponding variable's responsibility for the fault. He et al. [24] proposed the variable weighting based on the fault direction in pair-wise FDA. Moreover, they made a summarization that the method provides better variable weighting performance than PCA or PLS-based contribution plots. For a fault direction $\boldsymbol{\alpha}_k$, $\boldsymbol{\alpha}_k = [\alpha_k(1), \alpha_k(2), \ldots, \alpha_k(i), \ldots, \alpha_k(m)]^T$, the ith element $\alpha_k(i)$ is the contribution to the fault k from the ith variable. In fact, $\boldsymbol{\alpha}_k$ is a Fisher optimal discriminant vector obtained by performing pair-wise FDA on the normal data set $\mathbf{X}_0$ and the fault data set $\mathbf{X}_k$. However, the fault direction is considered difficult to correctly evaluate each variable's contribution because of the fact that between-class variability is assessed relative to within-class variability. Large elements of the fault direction may be caused by either large between-class variability or small within-class variability [25]. By an additional standardization, the standardized discriminant vector, here named the standardized fault direction, makes all variables comparable and improves the interpretation of the fault direction [26]. Then, the standardized fault direction of the fault k can be calculated as:

$$\boldsymbol{\alpha}_{std,k} = \mathrm{diag}(\mathbf{S}_w)^{1/2}\boldsymbol{\alpha}_k. \tag{4}$$

The partial *F*-values assess each variable's contribution to Hotelling's T^2 for class separation. Trendafilov et al. [27] argued that the partial *F*-values provide a better way to interpret each variable's contribution than the fault direction and the standardized fault direction. In the case of two-classes: X_0 and $\mathbf{X}_k$, the partial F value of the variable i is given by [26]

$$F(i) = (\nu - m + 1)\frac{T_m^2 - T_{m-1,i}^2}{\nu + T_{m-1,i}^2}, \quad i = 1,\ldots,m, \tag{5}$$

where $\nu = n_0 + n_k - 2$, T_m^2 is the two-class Hotelling's T^2 with all m variables. It is defined as:

$$T_m^2 = \frac{n_0 n_k}{n_0 + n_k}(\mathbf{b}_0 - \mathbf{b}_k)^T\mathbf{S}_k^{-1}(\mathbf{b}_0 - \mathbf{b}_k), \tag{6}$$

where $\mathbf{S}_k$ is the covariance matrix of the stacked matrix $\mathbf{X}_{0+k} = \begin{bmatrix}\mathbf{X}_0 \\ \mathbf{X}_k\end{bmatrix}$. $T_{m-1,i}^2$ is Hotelling's T^2 with all variables except for the variable i. The F statistic is distributed as $F_{1,\nu-m}$. If $F(i) \geqslant F_{\delta,1,\nu-m}$, the variable provides useful information for the classification. Here, $F_{\delta,1,\nu-m}$ is the critical value with a significance level δ.

A chemical process is often characteristic of large scale and process data masked with irrelevant information, which results in the difficulty of variable weighting [22]. Although the partial *F*-values show better interpretation of the single discriminant vector than the fault direction and the standardized fault direction, they still suffer from irrelevant variables and low computation efficiency. CPV based on each variable's equivalent variation is proposed to determine candidate variables [28]. These candidate variables are sufficient to express all process change information of the abnormal behavior. Then, the partial F-values can be performed on these candidate variables rather than all variables. It not only reduces the computational complexity but also eliminates the redundant variables to improve the variable weighting performance.

For Hotelling's T^2, generally, the mean and the covariance matrix changes are two essential factors representing the process change from the normal situation to some fault situation [29]. Each variable's contribution to the process change comes from two parts: the mean or variance change of the variable itself and the relation with other variables responsible for the process change. So, we define the equivalent variation of the ith variable for the k th fault as:

$$\Delta V_k(i) = \sum_{j=1}^{m} |R_k(j,i)\Delta b_k(i)^2| + \|\Delta \mathbf{S}_k(:,i)\|_1, \tag{7}$$

where $\Delta\mathbf{b}_k$ and $\Delta\mathbf{S}_k$ are the mean vector and the covariance matrix changes between the normal data set $\mathbf{X}_0$ and the kth fault data set $\mathbf{X}_k$, respectively. Moreover, $\Delta\mathbf{b}_k = \mathbf{b}_k - \mathrm{b}_0$ and $\Delta\mathbf{S}_k = \mathbf{S}_k - \mathrm{S}_0$. $\Delta\mathbf{b}_k(i)$ is the ith element of the vector $\Delta\mathbf{b}_k$, and $\Delta\mathbf{S}_k(:,i)$ is the ith column of the matrix $\Delta\mathbf{S}_k$ $\|\cdot\|_1$ denotes the 1-norm of a vector. $R_k(j,i)$ is the jth row and the ith column element of the correlation matrix of $\mathbf{X}_k$. $\sum_{j=1}^{m}|R_k(j,i)\Delta\mathbf{b}_k(i)^2|$ shows the equivalent mean change of the ith variable, which includes its own mean change and the mean change induced by other variables. Similarly, $\|\Delta\mathbf{S}_k(:,i)\|_1$ represents the equivalent variance change of the ith variable. Each variable's equivalent variation $\Delta V_k(i)$ comes from the mean and variance changes of either the variable itself or the relative other variables. When these variables neither experience their mean or variance change nor have the relation with other contribution variables, they have no contribution to the two-class separation and their partial F values are directly set to be zero. For the partial F-values, finding out these irrelevant variables and then eliminating them not only increase the computation efficiency but also improve the variable weighting performance.

After all variable's equivalent variations are obtained, we can rank them. Then, CPV can be calculated by

$$CPV(l_k) = \frac{\sum_{i=1}^{l_k}\Delta V_k(i)}{\sum_{i=1}^{m}\Delta V_k(i)} \times 100\%. \tag{8}$$

CPV is a measure of the percent variation captured by the first l_k candidate variables. The number of these candidate variables can be determined when CPV reaches a predetermined limit, such as 85%.

The partial F-values with CPV are performed on normal data and each class of fault data. Let $\mathbf{F}_k$ be the weight vector for the fault k, $\mathbf{F}_k = [F_k(1), F_k(2), \ldots, F_k(i), \ldots, F_k(m)]^T$. The ith element $F_k(i)$ is the F value of the ith variable and represents the corresponding variable contribution to Hotelling's T^2 statistic. This process is repeated until all faults are analyzed.

3.2. VW-FDA for fault diagnosis

After all weight vectors $\mathbf{F}_k$ ($k = 1, \ldots, c$) are obtained for c faults though the variable weighting based on the partial F-values with CPV. Moreover, $F_k(i)$ is set to be zero when $F(i) < F_{\delta,1,\nu-m}$ and $\mathbf{F}_k$ is normalized to $\overline{\mathbf{F}}_k$. Then, the original fault data sets are weighted by the corresponding weight vectors. The weighted fault sample $\tilde{\mathbf{x}}_i$ and the weighted fault data set $\tilde{\mathbf{X}}_k$ are, respectively [30]

$$\tilde{\mathbf{x}}_i = \mathrm{diag}(\overline{\mathbf{F}}_k)\mathbf{x}_i, \mathbf{x}_i \in \mathbf{X}_k \quad \text{and} \quad k = 1, \ldots, c, \tag{9}$$

$$\tilde{\mathbf{X}}_k = \mathbf{X}_k \mathrm{diag}(\overline{\mathbf{F}}_k), \quad k = 1, \ldots, c. \tag{10}$$

Each weight vector characterizes the corresponding fault. These weighted fault data sets $(\tilde{\mathbf{X}}_1, \ldots, \tilde{\mathbf{X}}_k, \ldots, \tilde{\mathbf{X}}_c)$ contain local characteristics of the corresponding fault. To keep $\tilde{\mathbf{X}}_k$ column full rank, define the summed fault sample $\hat{\mathbf{x}}_i$ and the summed fault data set $\hat{\mathbf{X}}_k$ as:

$$\hat{\mathbf{x}}_i = \mathbf{x}_i + \gamma\tilde{\mathbf{x}}_i, \mathbf{x}_i \in \mathbf{X}_k, \tilde{\mathbf{x}}_i \in \tilde{\mathbf{X}}_k \quad \text{and} \quad k = 1, \ldots, c, \tag{11}$$

$$\hat{\mathbf{X}}_k = \mathbf{X}_k + \gamma\tilde{\mathbf{X}}_k, \quad k = 1, \ldots, c, \tag{12}$$

where $\gamma \geqslant 0$ is a fusion coefficient. It controls the participation degree of the weighted fault data. The larger γ, the more contribution the weighted fault data set $\tilde{\mathbf{X}}_k$. When $\gamma = 0$, $\tilde{\mathbf{X}}_k$ has no active effect to improve FDA classification performance and VW-FDA becomes the conventional FDA. The summed fault data sets $\hat{\mathbf{X}}_k$ combines the weighted fault data set $\tilde{\mathbf{X}}_k$ and the original fault data set $\mathbf{X}_k$. It emphasizes the corresponding fault characteristics and involves more local characteristic information than the original fault data set. VW-FDA is performed on these summed fault data sets $(\hat{\mathbf{X}}_1, \ldots, \hat{\mathbf{X}}_k, \ldots, \hat{\mathbf{X}}_c)$. It is helpful to extract discriminative features from overlapping fault data. Fig. 1 illustrates the two-class classification procedure using VW-FDA.

To construct the fault diagnosis model, VW-FDA is performed on these summed fault data sets. The Fisher criterion is rewritten as:

$$J_F(\mathbf{A}) = \mathrm{trace}\{(\mathbf{A}^T\mathbf{S}_w\mathbf{A})^{-1}(\mathbf{A}^T\hat{\mathbf{S}}_b\mathbf{A})\}, \tag{13}$$

where $\hat{\mathbf{S}}_b = \sum_{k=1}^{c} n_k(\hat{\mathbf{b}}_k - \mathbf{b})(\hat{\mathbf{b}}_k - \mathbf{b})^T$ and $\hat{\mathbf{b}}_k$ is the mean vector of $\hat{\mathbf{X}}_k$.

In the phase of model construction, two parameters: the dimension of the projection subspace d and the fusion coefficient γ should be determined. After d is selected by maximizing fitness function, i.e. Eq. (3), a stepwise selection strategy is adopted to determine the fusion coefficient γ. Given a new fault sample $\overline{\mathbf{x}}_{new}$ scaled with the mean and variance of the normal data, a summary of the on-line VW-FDA diagnosis procedure is given below:

(1) Repeat Step 1 to 2 for $k = 1, \ldots, c$.
Step 1: Calculate the summed sample $\hat{\mathbf{x}}_{new}$ according to Eq. (11).
Step 2: Calculate the on-line FDA score vector by substituting $\hat{\mathbf{x}}_{new}$ into Eq. (1).

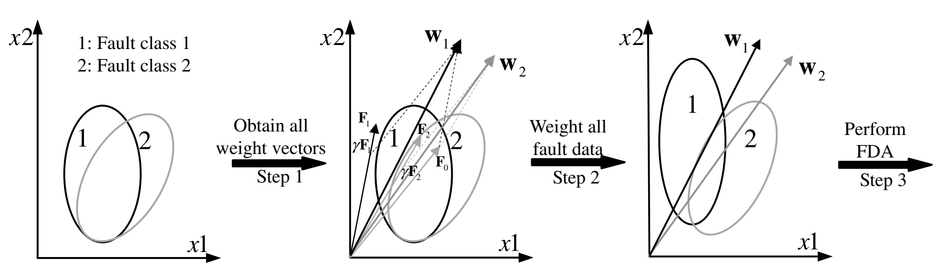

Fig. 1. Schematic diagram of the two-class classification procedure using VW-FDA for 2-dimensional fault data. $\mathbf{w}_1$ and $\mathbf{w}_2$ are the weight vectors of the summed fault data for Faults 1 and 2, respectively. $\mathbf{F}_1$ and $\mathbf{F}_2$ are respectively the weight vectors of the fault data for Faults 1 and 2 using the partial F-values with CPV. $\mathbf{F}_0$ is the vector with all ones. γ is the fusion coefficient.

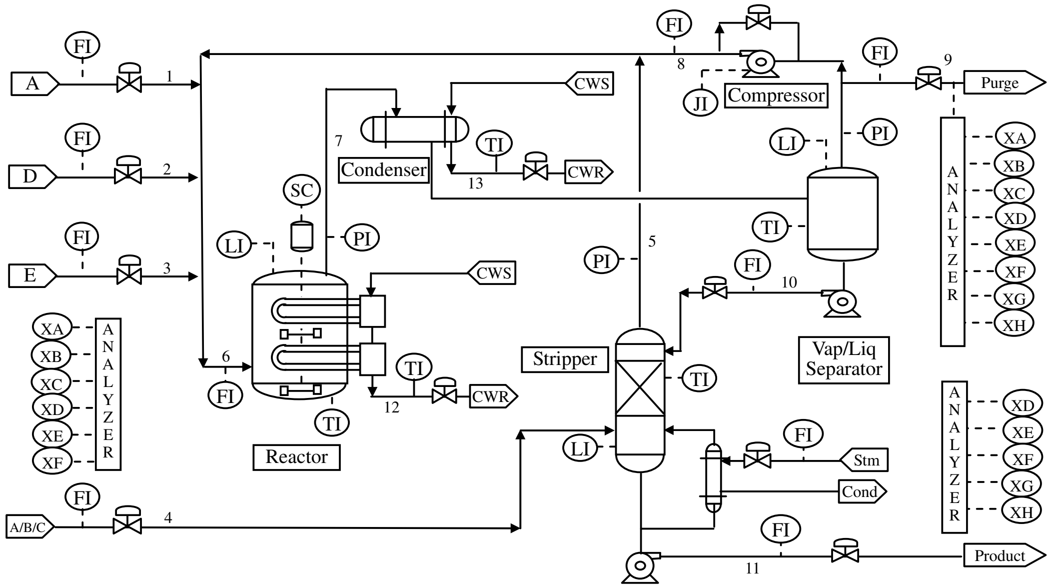

Fig. 2. A flow sheet of the Tennessee Eastman process.

(2) Make a diagnosis decision using minimum Mahalanobis distance classifier.

4. Application to TEP

4.1. Simulation data

In this section, Tennessee Eastman process is used to evaluate the new fault diagnosis approach with VW-FDA. TEP is based on an industrial process where the components, kinetics, and operating conditions are modified for proprietary reasons [3,31]. It has been a well-known benchmark process for evaluating process monitoring methods [3]. TEP consists of five main units: a reactor, a compressor, a stripper, a separator and a condenser. Fig. 2 shows the process flowsheet for TEP. There are four gaseous reactants (A, C, D, and E), two liquid products (G and H), a byproduct F and an inert B. The gaseous reactants are fed to the reactor where the liquid products are formed. More detailed information for TEP is provided in Refs. [3] and [31]. The TEP simulator used in our study can be downloaded from http://www.brahms.scs.uiuc.edu. It can simulate normal operating condition and 21 faulty conditions. Every sample contains 52 process variables. The plant-wide control structure recommended by Lyman and Georgakis [32] was implemented to generate the closed loop simulated process data for each fault.

Nine faults including Faults 2, 4, 5, 8, 9, 11, 12, 13 and 14 are considered. They not only cover all fault types in the TEP simulation, but also are good representation of overlapping fault data. These faults are divided into two cases of fault combination. Table 1 lists the selected faults' description in each fault combination. Each of these faults runs for 60 h. At the beginning of the simulation, there are no faulty conditions. Every faulty condition is introduced after 30 h. It is noted that the steady state fault data after the fault occurrence is considered to characterize the fault condition. The initial fault signatures and the transient behaviors are not included for all fault cases. The sampling interval is set to be 3 minutes. The observations from 1 to 600 are the normal data. The observations 801–1200 are used to represent the steady state fault data and the first 200 fault data involving the initial fault signatures and the transient behaviors are not adopted for all faults. The first 300 fault data (observations 801–1100) and 600 normal data are formed into the weighting data set to find out the weight vector for every fault. The training data set including the first 300 fault data for each fault is used to construct the diagnosis model. The testing data set includes other 100 fault data for each fault (observations 1101–1200) and is used to validate the model.

4.2. Results and discussion

4.2.1. Case 1: Faults 4, 9 and 11

Fault 4 and Fault 11 are associated with same fault variables, but they have different fault types. The fault variables for Fault 9 are different from those for Faults 4 and 11. These three faults are good representation of overlapping data [12]. A graphical analysis of the proposed method is carried out in this section.

Fault 4 is a step change in the reactor cooling water inlet temperature. When Fault 4 occurs at $t = 30$ h (observations 601), a step

Table 1
Selected faults for two fault combination cases.

	ID	Fault description	Type
Case 1	Fault 9	D feed temperature	Random variation
	Fault 4	Reactor cooling water inlet temperature	Step change
	Fault 11	Reactor cooling water inlet temperature	Random variation
Case 2	Fault 2	B composition, A/C ratio constant	Step change
	Fault 5	Condenser cooling water inlet temperature	Step change
	Fault 8	A, B, C feed composition	Random variation
	Fault 12	Condenser cooling water inlet temperature	Random variation
	Fault 13	Reactor kinetics	Slow drift
	Fault 14	Reactor cooling water valve	Sticking

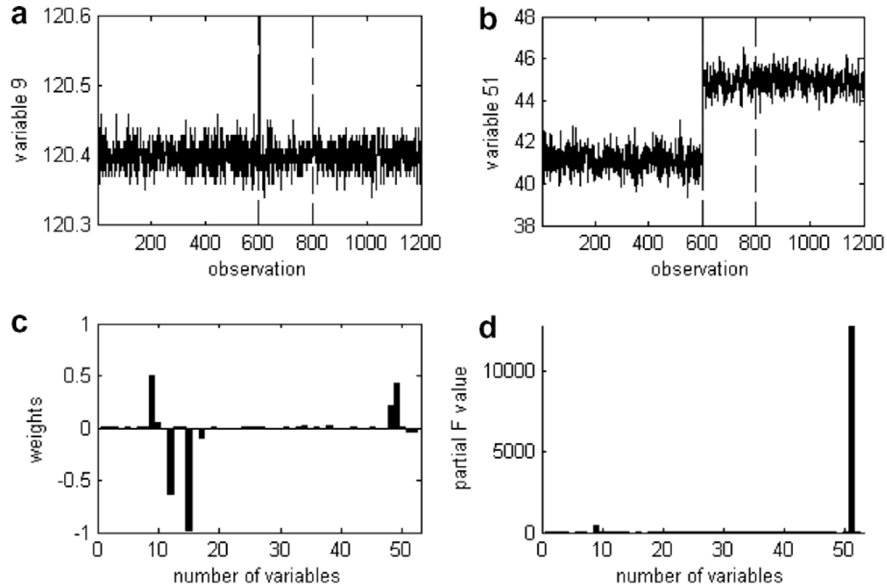

Fig. 3. Variable weighting of Fault 4: (a) the time-series plot of variable 9, (b) the time-series plot of variable 51, (c) the weight plot based on the fault direction, and (d) the weight plot based on the partial *F*-values with CPV.

change in the reactor cooling water flow (variable 51) is induced, which results in a sudden increase of the reactor temperature (variable 9). The other 50 variables show normal operating condition behaviors. Fig. 3a and b shows the time-series plots for the variables 9 and 51, respectively. The weight plot based on the fault direction is shown in Fig. 3c. The fault direct incorrectly provides the variable 12, 15, 47 and 48 with large weight values. By contraries, the fault variable 51 has too small weight value. The proposed variable weighting scheme based on the partial *F*-values with CPV is applied to the weighting data set. The variable weighting result is shown in Fig. 3d. The weight vector exactly weights the fault variables with large weight values. Compared with the fault direction, the partial *F*-values with CPV show better variable weighting performance.

Fault 11 is basically similar to Fault 4 since they are both associated with the reactor cooling water inlet temperature. But Fault 11 has a different fault type, i.e., random variation. After Fault 11 occurs, the variables 9 and 51 go through large variance changes (see Fig. 4a and b). The other 50 variables have no obvious changes. Fig. 4c and d illustrate the weight plots based on the fault direction and the partial *F*-values with CPV for Fault 11, respectively. The fault direction does not provide the fault variables 9 and 51 but the normal variables 15, 49, 1 and 38 with large weight values. Unlike the fault direction, the weight vector based on the partial *F*-values with CPV exactly represents the corresponding variables' weight values for Fault 11.

Fault 9 is a random variation of the D feed temperature. The fault data have little differences with the normal data in the means and variances of all variables [3]. Hence, the variable weighting isn't carried out. The weight value of each variable is set to be one.

After all weight vectors are normalized, the diagnosis model can be constructed by off-line learning. Fig. 5a shows the bivariate

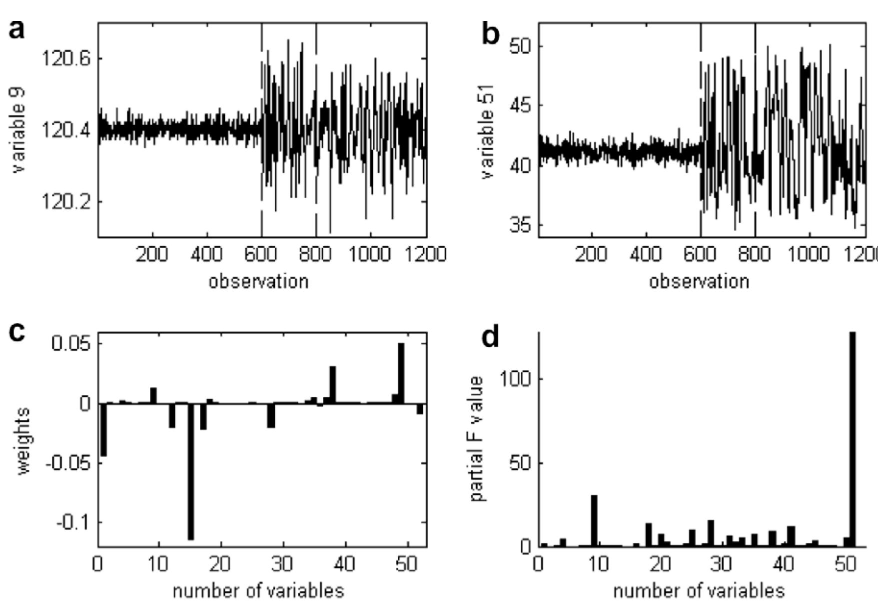

Fig. 4. Variable weighting of Fault 11: (a) the time-series plot of variable 9, (b) the time-series plot of variable 51, (c) the weight plot based on the fault direction, and (d) the weight plot based on the partial *F*-values with CPV.

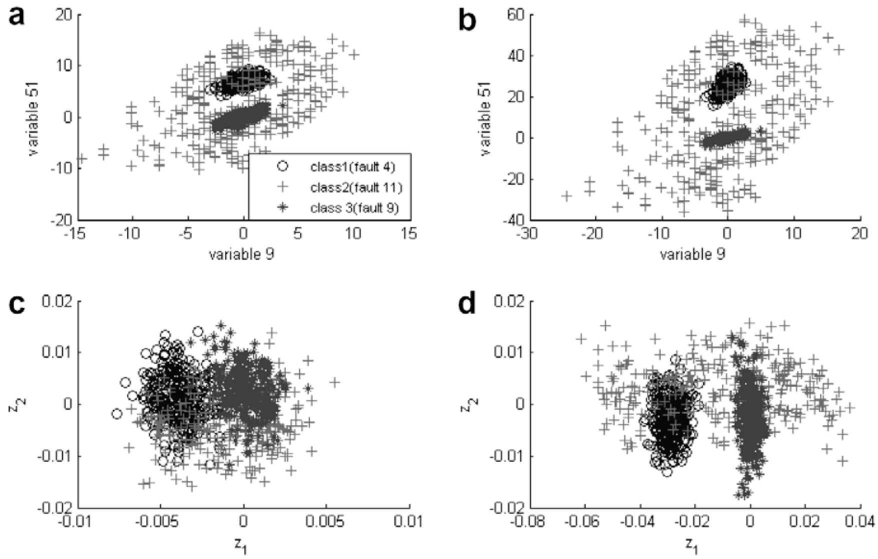

Fig. 5. Comparison among the scatter plots of the training data sets of three fault classes (Faults 4, 11 and 9): (a) the bivariate scatter plot of variables 9 and 51 for the original training data sets, (b) the bivariate scatter plot of variables 9 and 51 for the summed training data sets, (c) the scatter plot of classification result for the original training data sets, and (d) the scatter plot of classification result for the summed training data sets.

scatter plot of the training data sets (variables 9 and 51). The training data sets of Faults 9 and 4 are separable. But these two data sets overlap with the data set of Fault 11. After the training data sets are weighted by the corresponding weight vectors, the summed fault data combine the original fault data and the weighted fault data. As shown in Fig. 5b, the summed fault data set of Fault 4 shrinks to the mean and has the better separation from the summed fault data set of Fault 9. At the same time, the summed fault data set of Fault 11 expands away from the summed fault data sets of Faults 4 and 9. The summed fault data emphasize the local characteristics of Faults 4 and 11. Compared with the original training data, they offer important supplemental classification information. Fig. 5c and d illustrates the classification results for the original training data sets and the summed data sets, respectively. FDA is performed on the summed fault data sets, which is helpful to exact discriminative features from overlapping data. In our study, 95% confidence limit $F_{0.05,1,850} = 3.85$.

After the diagnosis model based on VW-FDA is constructed, on-line diagnosis is performed on the testing data sets according to the on-line diagnosis procedure discussed in Section 3.2. VW-FDA shows better classification performance than FDA. The total classification success rate for the testing data sets using VW-FDA increases to 73.7% from 66.0% in comparison with one using FDA. Moreover, Fault 4 stays in a same classification success rate: 90%, but the classification success rates of Faults 9 and 11 show obvious increases by 12% and 11%, respectively. A summary of the classification results for VW-FDA and FDA is listed in Case 1 of Table 2.

Table 2
Comparison among diagnosis success rates using different approaches.

		Diagnosis success rate (%)	
		VW-FDA	FDA
Case 1	Fault 9	90	78
	Fault 4	90	90
	Fault 11	41	30
	Total	73.7	66.0
Case 2	Fault 2	100	100
	Fault 5	100	100
	Fault 8	99	98
	Fault 12	70	66
	Fault 13	72	44
	Fault 14	90	95
	Total	88.5	83.8

4.2.2. Case 2: Faults 2, 5, 8, 12, 13 and 14

In this section, six faults including different fault types are considered. On the one hand, Faults 2 and 5 are used to make a comparison among the variable weighting performance of the fault direction, the standardized fault direction, the partial *F*-values and the partial *F*-values with CPV. On the other hand, the multi-classification performance of VW-FDA is analyzed in comparison to the conversional FDA.

Fault 2 is a step change of gaseous inert B in Stream 4. Due to the close loop reactor, the inert B composition in Stream 6, 9 and 11 returns to the normal situation after experiencing a transient increase. The byproduct F composition decreases in those streams. Total 16 variables including the variables 47, 43, 34, 10, 50, 19 and so on, show abnormal behaviors. Fig. 6a and b shows the weight plots based on the fault direction and the standardized fault direction for Fault 2, respectively. Both of them incorrectly determine the variables 12 and 48 as the key variables responsible for the fault and give them large weight values. As shown in Fig. 6c, although the 16 fault variables are represented by relatively large weight values, the partial *F*-values incorrectly weight other 6 variables such as the variables 17 and 52. Fig. 6d shows the weight plot based on the partial *F*-values with CPV. The variable weighting approach proposed in the paper exactly weights the 16 fault variables. When the predetermine limit is set to be 85%, 37 candidate variables are selected from total 52 measured variables by CPV. It not only reduces the computation complexity but also effectively eliminates the redundant information and improves the variable weighting performance of the partial *F*-values.

Fault 5 is a step change in the unmeasured condenser cooling water inlet temperature. When Fault 5 occurs, a step change in the reactor cooling water flow (variable 52) is directly induced. As a result, more than 34 variables experience transient behaviors because of control loops. They take about 8 h to reach their steady states. Fig. 7a and b shows the weight plots based on the fault direction and the standardized fault direction for Fault 5, respectively. Both of them incorrectly weight the variables 15 and 49. Fig. 7c shows the weight plot based on the partial *F*-values. The partial *F*-values exactly provide the fault variables 17 and 52 with

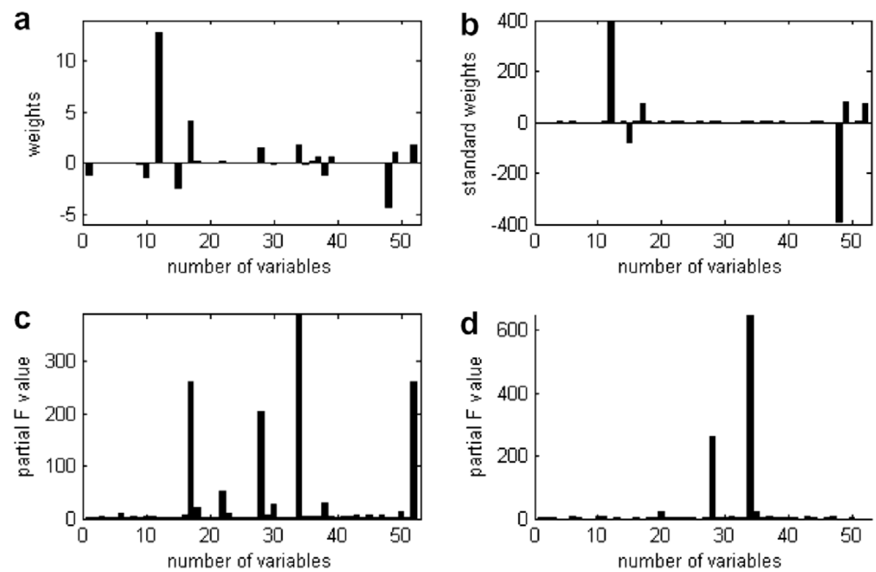

Fig. 6. Comparison among the weight plots based on different variable weighting methods for Fault 2: (a) the fault direction, (b) the standardized fault direction, (c) the partial *F*-values, and (d) the partial *F*-values with CPV.

very large weight values. However, it need calculate all partial *F* values of the measured variables. When the predetermine limit is set to be 85%, 39 candidate variables are selected. It indicates that other variables cannot supply useful information for classifying the fault data set and the normal data set. As shown in Fig. 7d, the proposed approach based on the partial *F*-values with CPV also weights the fault variables exactly. Furthermore, it effectively increases the computation efficiency.

Faults 8 and 12 have a same fault type: random variation, but they are associated with different fault variables. For Fault 8, there is a random change of A, B and C feed composition at $t = 30$ h, which causes 36 variables to experience obvious changes. For the weighting data set, the partial *F*-values with CPV weight all these variables. Fig. 8a shows the bar plot of the weight vector. Fault 12 is a random variation in the condenser cooling water inlet temperature. A total of 28 variables show the different behaviors. The variable weighting based on the partial *F*-values with CPV exactly weights these variables, such as 50, 18, 19, 20, 38, and so on, with relatively large weight values. Fig. 8b shows the bar plot of the weight vector. Fault 13 is a slow drift of the reactor kinetics. Fault 14 is a sticking of the reactor cooling water valve. Their bar plots of the weight vectors are illustrated in Fig. 8c and d, respectively.

After all weight vectors are obtained and normalized, the summed fault data sets can be calculated according to Eq. (12). Then VW-FDA is performed on the summed fault data sets in order to construct the diagnosis model. The parameters d and γ are selected to be 5 and 0.27, respectively. On-line diagnosis is performed on the testing data sets according to the on-line diagnosis procedure. Apart from Fault 14, all faults have an improvement of fault diagnosis performance. Especially, the diagnosis success rate of Fault 13 shows an increase to 72% from 44%. The total classification success rate for the testing data sets using

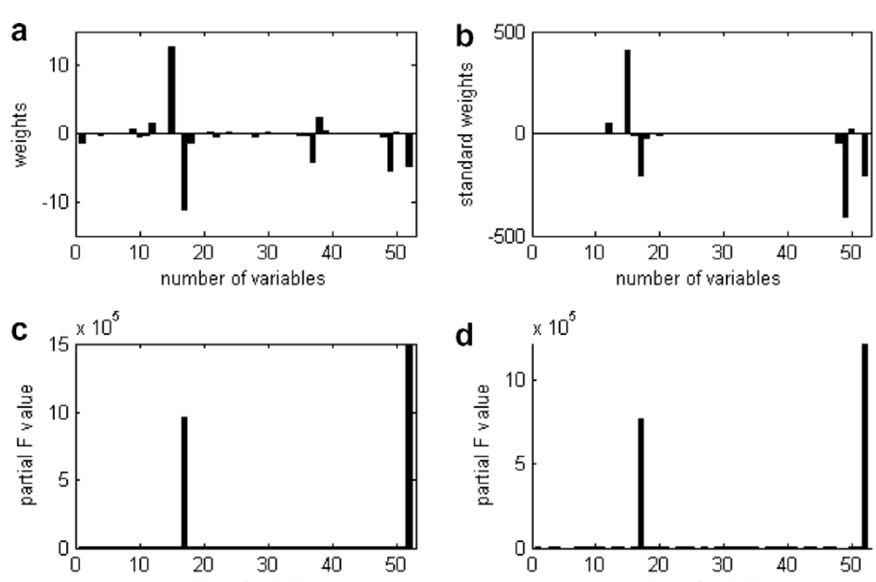

Fig. 7. Comparison among the weight plots based on different variable weighting methods for Fault 5: (a) the fault direction, (b) the standardized fault direction, (c) the partial *F*-values, and (d) the partial *F*-values with CPV.

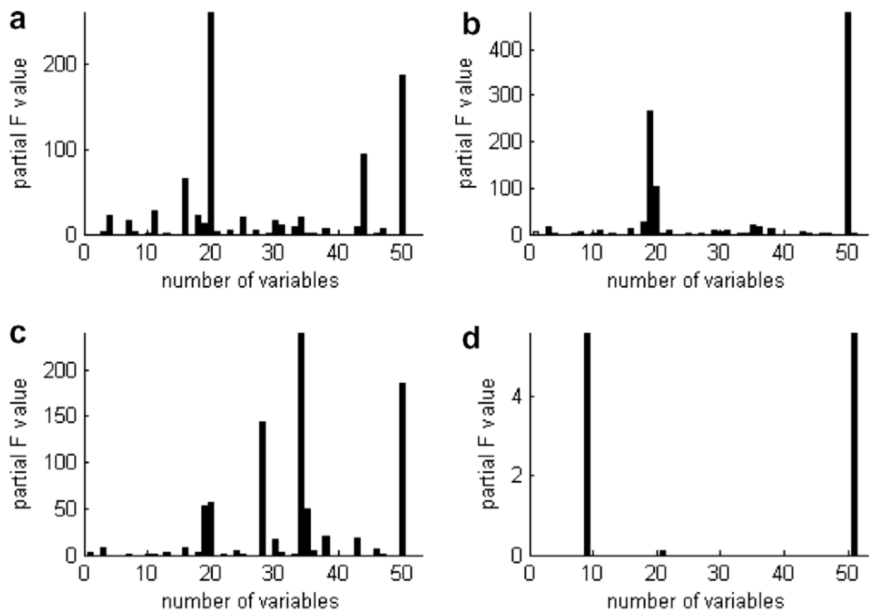

Fig. 8. The weight plots based on the partial *F*-values with CPV for: (a) Fault 8, (b) Fault 12, (c) Fault 13, and (d) Fault 14.

VW-FDA increases to 88.5% from 83.8% in comparison with one using FDA. A summary of the classification results for VW-FDA and FDA is listed in Case 2 of Table 2.

5. Conclusions

This paper proposed a new fault diagnosis approach with VW-FDA for the multi-classification where fault data are overlapped. The method incorporates the variable weighting based on the partial *F*-values with CPV into FDA. The variable weighting is used to find out the weight vector of each fault by maximizing the separation between the normal and fault data sets. Candidate variables can be selected from all variables by CPV. The partial *F* values are calculated for only these candidate variables rather than all variables. It not only deduces the computation burden but also eliminates irrelevant variables and improves the variable weighting performance. When all fault data are weighted by the corresponding weight vectors, all variables in every fault are used in different levels. The summed fault data can be constructed by combining the weighted fault data and the original fault data. VW-FDA is performed on the summed fault data. The summed fault data emphasize the corresponding fault characteristics and involve more local characteristic information than the original fault data. As a result, VW-FDA extracts discriminative features more effectively than the conventional FDA from overlapping fault data. The proposed approach is applied to Tennessee Eastman process. The results demonstrate that VW-FDA shows better fault diagnosis performance than the conventional FDA.

It should be noted that VW-FDA or FDA for fault diagnosis requires that different deterministic fault data sets have been gathered. How to obtain enough fault data sets is a challenge. On the one hand, more fault data sets should been gathered from historical data, plant tests or mechanistic knowledge. On the other hand, new fault data sets should keep gathering in the future so as to enrich the fault classes in VW-FDA for fault diagnosis.

Acknowledgements

This work was supported by National Basic Research Program of China under Grant 2004CB720703. We are grateful to the anonymous reviewers for their helpful comments and constructive suggestions to this paper.

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