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Terrestrial Magnetism: Historical Perspectives and Future Prospects

"Baggage We Carry with Us"

David Gubbins

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Abstract This collection of reviews marks the state of the art of geomagnetic data collection, modelling, and interpretation at a time of unprecedented advances in all 3 facets of the subject. For the first time we have excellent satellite data with the prospect of more to come, vast improvements in laboratory techniques, and opportunities to use large scale computing to model the data. In the past, research has been conducted by the separate disciplines largely in isolation; we can hope the subject has now matured enough for progress to be made by genuine collaboration between theoreticians and experimentalists. The purpose of this chapter is to set the historical setting, and I have chosen a starting date of 1980, when vector satellite data first became available and stimulated many new advances in the subject. We can hope for a similar or better stimulus in the next decade.

Keywords Geomagnetism · Paleomagnetism · Dynamo theory

1 Introduction

Our story begins with the MAGSAT mission of 1980, the first satellite to measure all 3 components of the vector magnetic field at low altitude, one of many magnetometer missions led by the late Mario Acuna of Goddard Space Flight Center. Why choose this date? Two reasons. First, MAGSAT's comprehensive geographical coverage of vector data provided a baseline, a gold standard, by which older, less accurate measurements could be judged. It gave confidence in some shadowy features of the geomagnetic field only hinted at in earlier studies and provided some limited estimates of their evolution in time. MAGSAT did not fly for long enough to produce any very new information about the geomagnetic secular variation, but it did provide ground-truth for many magnetic observatories that monitor secular variation locally. Secondly, and in my view more importantly, it stimulated much

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new research in geomagnetism, with over 100 papers in the first few years after the mission (Langel 1985).

There have been many advances in geomagnetism since MAGSAT's launch. The discovery of the Bangui anomaly in central Africa arose directly from maps of the lithospheric magnetic field created from MAGSAT data. Magnetic mapping by the mineral and hydrocarbon industries has become popular once again, although satellite data has so far contributed rather little on the short length scales of interest. Maps of the magnetic field downward continued to the core surface were produced and largely believed as realistic by the community: with hindsight one might say that such maps could have been created earlier, but it took the accuracy of MAGSAT data to give them any validity. These led to maps of fluid flow at the top of the core using methods devised 15 years earlier. The dynamo problem, for so long stuck in the mathematicians' wardrobe of difficult and impenetrable puzzles, was finally brought to a level of sophisticated numerical simulations that produced magnetic fields sufficiently earth-like to make direct comparison with the observed geomagnetic field a worthwhile activity. Theory is now being bolstered and tested by sophisticated and ambitious laboratory experiments using liquid metals as the working fluid. While the last 2 decades have sadly seen a decrease in the number of ground observatories, paleomagnetic laboratory techniques have advanced apace, with dramatic improvements in measurement accuracy, repeatability and, most of all, volume of production of useable measurements, particularly of intensity.

Terrestrial magnetism has been dogged by an inability to bring theory and observation together in any meaningful way. This is mainly because the theory is inadequate (it cannot predict the observations) and partly because of our inability to measure what we would really like to measure (a continuous time series of paleomagnetic data and the magnetic field at the core-mantle boundary are two examples). This has led theorists and observers to progress their fields in something not unlike isolation: theories with little predictive power of interest to an observer, and data modelled with little or no recognition of the fundamental underlying physics. The best example that springs to mind is the paleomagnetic tests of the difference between the Earth's normal and reversed states: invariance under change of sign of the magnetic field is one of the few useful things about which dynamo theory is unequivocal, the test is of either the validity of Maxwell's equations or the assumptions made about the magnetic properties of rocks; results in favour of a difference are more likely a comment on inadequate geographical sampling.

Happily, much of this schism between theory and observation has vanished in the decades since MAGSAT, but we must remain vigilant that it does not return. The subtitle of my article contains the message: learn from history! We must admit the limitations of both theory and observation and put what we do have together to produce a proper theory of the terrestrial magnetic field, a thorough understanding of the established observed facts. I believe this will happen in the next 2 or 3 decades.

2 Observations and Their Interpretation: Direct and Paleomagnetic Measurements

After MAGSAT there followed nearly 20 years with no significant monitoring of the geomagnetic field by satellite. The Danish satellite Oersted was launched in 1999, ringing in a new era of geopotential research. Despite an initial remit to monitor the field for 14 months, Oersted continued to transmit useful measurements for an entire decade. Oersted was closely followed by CHAMP, launched in July 2000, with a planned mission of 5 years. CHAMP was launched into a circular, near-polar orbit at a height of 454 km, lower than Oersted and therefore yielding better resolution. SAC-C was also launched in 2000, also into low orbit. It also continues to return data. These satellites have produced datasets of unprecedented quality, as discussed in Chap. 3. Apart from giving high quality models of the magnetic field of the core and crust, they provided the first opportunity to study the change in the magnetic field at high resolution by comparing with MAGSAT. These 3 satellites have now flown for a sufficiently long time to deliver estimates of secular change, possibly the most significant advance over previous decades. The future of satellite monitoring looks bright, with the imminent launch of the European Space Agency's multi-satellite mission SWARM, in which 3 satellites flying at different altitudes will monitor the geomagnetic field.

We have also seen advances in ground and near-ground measurements, although sadly some critical magnetic observatories have closed. Apia, for example, provided a long run of data from a remote part of the globe, and as I write this another email arrives petitioning against the closure of Huancayo. However, the INTERMAGNET project (www.intermagnet.org) has been highly successful in developing standards of observatory measurements and disseminating digital data in a timely fashion, and aeromagnetic surveying has enjoyed a new burst of activity in the search for minerals and hydrocarbons.

2.1 Geomagnetic Field Modelling

We expect the larger wavelengths to originate in the core, the shorter wavelengths in the magnetised crust. These are called, respectively, the *main field* and *crustal* or *lithospheric field*. Separation of these two components has been a fundamental goal of geomagnetism for many years. It is impossible to perform the separation on the basis of wavelength (or spherical harmonic degree) alone, but the shape of the spectrum of the integrated surface energy, often called the Mauersberger-Lowes spectrum, approximates to two straight lines on a log-linear plot (Fig. 1), which is highly suggestive of two sources. The break between these lines is usually taken to be the cross-over from core to crustal sources, but history has seen this cross-over spherical harmonic degree, n_c, gradually increase, suggesting we are really looking at the cross-over between long wavelength sources and noise. Main field models have typically been represented by truncated spherical harmonic series ending near degree n_c, a justifiable procedure if n_c represents the point where noise dominates. Early IGRFs were truncated at degree 8, and close inspection of the spectrum in Fig. 1 shows degree 9 to be exceptionally weak. Now IGRFs, which are particularly conservative, are truncated near degree 13. Figure 1 shows n_c to be somewhat higher, and it is likely to go higher as data continues to improve.

Undoubtedly the most important advance in main field modelling during this period has been the use of *core-radius regularisation* introduced by Shure et al. (1982). The main field originates in the core and, if the mantle can be treated as an electrical insulator, can be represented by a gradient of a potential that satisfied Laplace's equation everywhere outside the core, $r > r_{\mathrm{c}}$. Upward continuation of a potential field imposes a severe low-pass filter, a geometrical factor of $(r_{\mathrm{c}}/r)^l$ on each geomagnetic coefficient of spherical harmonic degree l.

Finding the main field from measurements is an inverse problem for a model, the main field, from data, the measurements. The number of measurements is necessarily finite and incomplete, the model a continous vector function defined by an infinity of parameters; the problem is ill-posed and further regularisation is required to obtain a solution. Regularisation applied at the Earth's surface is arbitrary and ineffective: note that truncating a spherical harmonic series at some point is a form of regularisation, as is a less drastic tapering of the series above some degree. This procedure is satisfactory if all one wants to do is represent

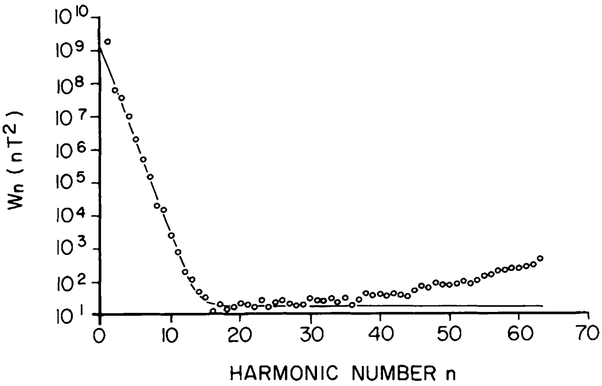

Fig. 1 The geomagnetic spectrum typical of models derived from MAGSAT. The steep fall of the low order harmonics is attributed to geometric attenuation from the core surface, the flatter high order spectrum is attributed to crustal magnetisation. The break at harmonic number (spherical harmonic degree) $n_c \approx 15$ has increased, and will probably continue to increase, as data improves, indicating that some of the intermediate wavelength signal is noise. From Cain et al. (1989)

the long wavelength part of the field, for mapping purposes say, as is the case for the IGRF, but scientific studies of the core field require something better.

Damping the model at the core-mantle boundary (CMB) is equivalent to damping the geomagnetic coefficients by a factor $(r_E/r_c)^l$ (squared in the normal equations), where r_E is Earth's radius: this is a very steep increase in damping with rising degree l. Some form of damping must be applied to the core field. It is possible to make physical arguments to limit the short wavelengths at the CMB based on considerations of heat flux from the core (Gubbins 1983). This extra damping usually takes the form of powers of l, say $l^3(r_E/r_c)^l$. It makes little difference to the surface field because the rise with l is small compared to the geometrical factor. For example, consider the change in damping factor from $l = 10$ to $l = 11$. The geometrical factor gives an increase of $r_E/r_c = 1.828$ but l^3 increases only by a factor 1.33, the difference being even smaller at high degree.

These bounds are good enough to provide satisfactory model solutions that converge at the Earth's surface, and could be used to provide Bayesian estimates of errors on the geomagnetic coefficients, although this has never been done. Further assumptions and more severe damping are required to produce a converged main field model at the CMB: see Backus (1988). It does, however, provide a self-consistent way to produce maps of the geomagnetic field at the core surface for further interpretation, and these have been a great help in understanding core processes. Core-radius regularisation has been used to produce self-consistent maps of main field for epochs dating back to the 17th century (Bloxham et al. 1989; Hutcheson and Gubbins 1990); the similarity of even small features between these independent models is quite remarkable. An important further development was to introduce time dependence into the modelling (Bloxham and Jackson 1989), culminating in the latest model *gufm* of Jackson et al. (2000), which involved extensive researching of the archives to uncover many thousands of historical measurements. More recently, time-dependent models have been produced for older eras using paleomagnetic and archeomagnetic data (Korte and Constable 2003, 2005; Korte et al. 2009).

Movies of the magnetic field dating back to AD1590 have been used by many authors for interpretation of the geodynamo and core processes, and further use is beginning to be made of the paleomagnetic models. The problem with time-dependent modelling is the need

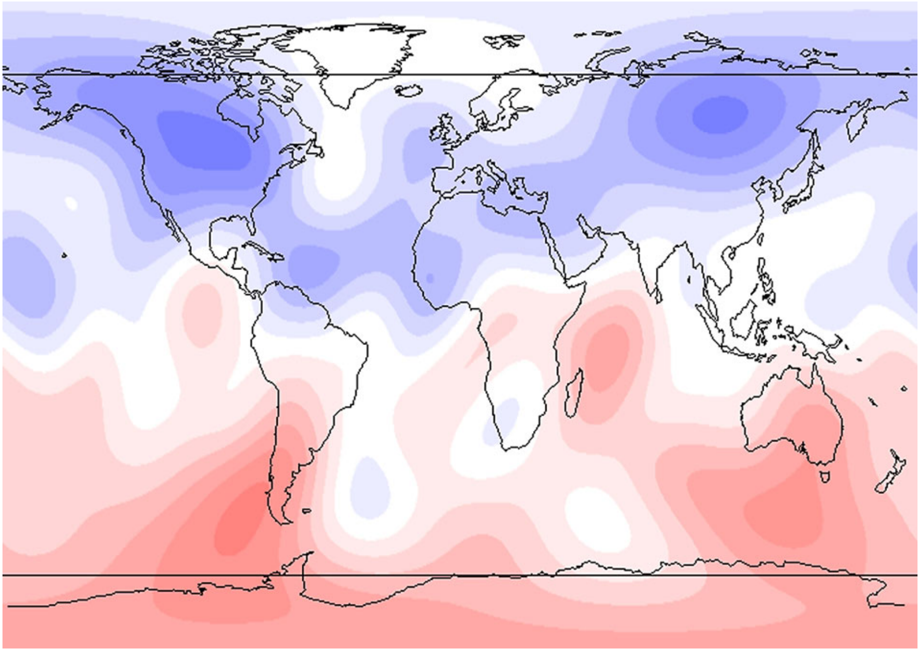

Fig. 2 The vertical component of the magnetic field at epoch AD1750 based on model *gufm*, plotted on the core surface. The four main lobes and absence of flux over the poles seem to be the main feature of the geomagnetic field in historical times. The Southern Hemisphere has suffered major disruption subsequently, possibly associated with the fall in dipole moment

for additional damping in time. Not only is the form of this damping necessarily arbitrary, it also introduces a confusion between the two styles of regularisation. Technically, cross-resolutions are involved that make the true resolution difficult to understand: single epoch models do not suffer this problem, and when the same feature turns up in two independent epochs its reality is clear: it cannot be an artefact arising from temporal smoothing.

Core-radius regularisation has never been used for the IGRF or DGRF series of models, although there is some discussion ongoing for the IGRF to change from its established truncated spherical harmonic series. Clearly some form of regularisation that incorporates the prior information that the field originates in the core would be an improvement over present practice, but the IGRF's use in mapping imposes a lot of inertia on the methodology. The IGRF is always a prediction because it is made from data taken at times earlier than the epoch of the model. The DGRF is constructed later and is intended to provide a definitive record of the geomagnetic field's history. The DGRF would also benefit from modern analysis procedures and should be a time-dependent model updated by data assimilation. This is not even being considered yet.

Secular variation (SV) has normally been determined by differencing measurements at the same site at different times. Only recently have satellites flown for sufficiently long times for SV to be estimated directly. Magnetic observatories occupy the same site for long periods of time and are the only places where SV may be measured with any degree of accuracy. Repeat stations, or reoccupied temporary sites, can also give SV but rather little use has been made of them, mainly because they lack sufficient accuracy through contamination from the varying external magnetic field and unknown changes to the magnetic environment between occupations. Ordinary survey measurements suffer from the large signal from the crustal field—typically several hundred nanoTeslas, equivalent in many places to a decade or more of SV. Measurements can be used in a global model survey but they must be weighted down according to the crustal "error". In a magnetic observatory measurement the constant crustal component can be subtracted out using a site correction; the difference of measurements then typically contributes several hundred times more information than an uncorrected survey measurement if least squares is used. These considerations determine the accuracy of SV models derived by differentiating one of the time-dependent main field models such as *gufm*.

The fundamental impossibility of separating the core field from the crustal field on the basis of wavelength alone means the main field models inevitably contain some components of the long wavelength crustal field. Some progress is being made in identifying the crustal component by *ab initio* forward modelling, the simple process of estimating the total magnetisation of the crust and calculating its magnetic field at, for example, satellite altitude for comparison with observation. Initial attempts (Meyer et al. 1983) were not followed up but the development of GIS techniques and large scale models of various properties of the crust has made possible more meaningful models of crustal magnetisation that reproduce many aspects of the intermediate wavelength signal in satellite field models (Hemant and Maus 2005). Figure 3 shows a comparison of the field generated from a magnetisation model of one ongoing study with a satellite model.

2.2 Core Motions

Secular variation is caused by fluid motion near the top of the core. Evolution of the magnetic field is governed by the induction equation, a combination of Maxwell's equations in the non-relativistic approximation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{1}$$

where η is the magnetic diffusivity. One might try to find $\mathbf{u}$ by applying this equation at the core surface and using measurements of main field and SV downward continued to the core mantle boundary. The problem is the radial derivatives of B_r in the diffusion term: $\mathbf{B}$ may be estimated on the surface, its horizontal gradients may also be estimated (in principle at least), and even the radial derivative of B_r from the solenoidal condition $\nabla \cdot \mathbf{B} = 0$, but higher radial derivatives remain unknown.

Roberts and Scott (1965) were first to point out that diffusion might be weak and therefore ignored on the short timescale of SV. They never extended their work to an actual estimation, although others did. A subsequent landmark paper by Backus (1968) showed that, even if diffusion were dropped from (1), it is still impossible to find $\mathbf{u}$ uniquely even given perfect data. He quantified the ambiguity in determining $\mathbf{u}$ precisely. This result, plus the difficulty of downward continuing the measurements to the core surface, an unstable calculation, effectively squashed any further attempts to determine core motions from SV until MAGSAT data became available.

Good satellite data gave more credence to the maps of core-surface fields, and further effort was put into determining core flows. Solution requires additional regularisation assumptions, and many have been tried. "Blind" regularisation, simple damping of small scale motions, is doomed to failure because it suppresses the whole null space defined by Backus when there is no reason to suppose it to be small. Additional physical constraints are more convincing and have been applied with some success. These include toroidal-only motions, which would occur if the upper core were density stratified (Whaler 1980), tangentially

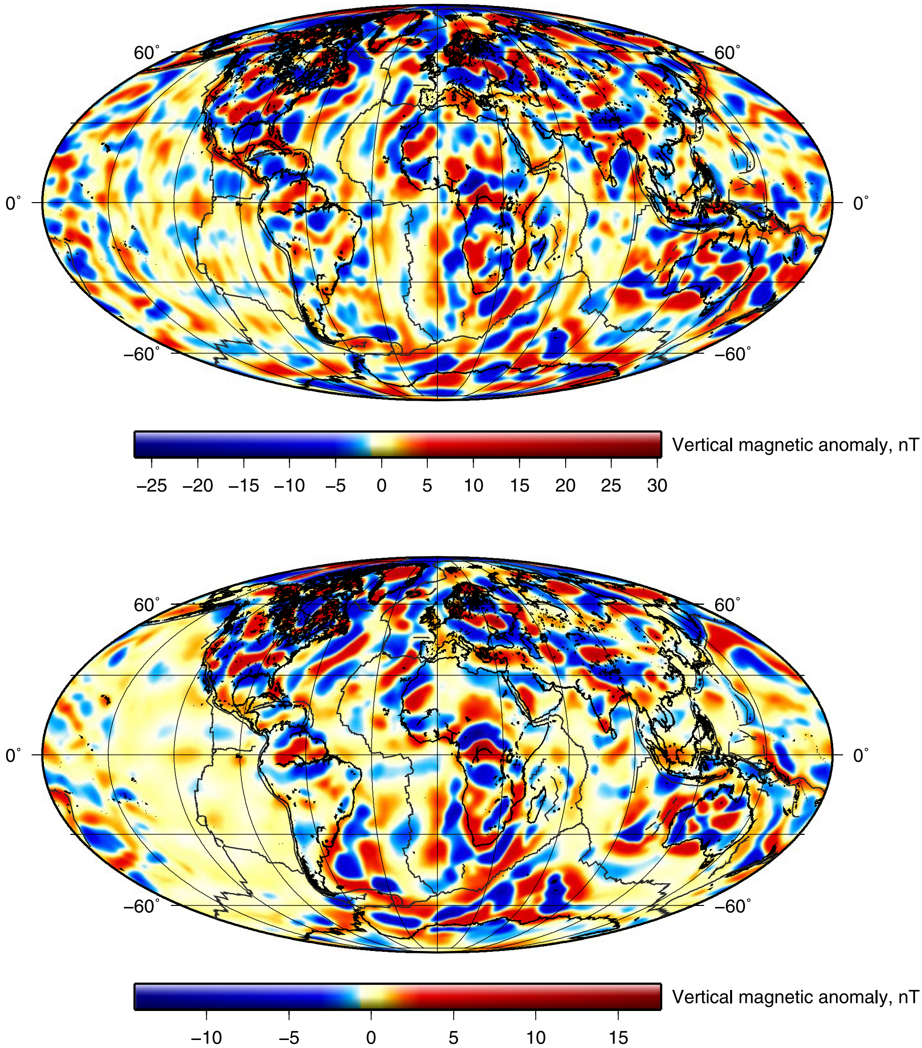

Fig. 3 Comparison of a model of the crustal field derived by *ab initio* methods (*lower panel*, from work in progress, S. Masterton, University of Leeds, Ph.D. Thesis) and a field model derived from satellite data (*upper panel*, POMME-6)

geostrophic flows, which would occur magnetic forces are weak at the top of the core (LeMouël et al. 1985), and steady flow (Voorhies and Backus 1985). The first two conditions restrict the class of flows capable of explaining the data but still do not provide uniqueness, the last requires additional knowledge of secular acceleration or, equivalently, the main field at 3 distinct epochs rather than 2.

The case of toroidal motion illustrates the problem with such severe non-uniqueness: it only allows determination of one component of the flow at any one point, the one perpendicular to contours of B_r (extrema and saddle points of B_r are special cases). An undamped inversion is dominated by the null space and flows appear parallel to contours of B_r; as damping is increased this component is gradually eliminated, as far as the topology of the contours allows, until the flow appears everywhere orthogonal to the contours. Simply by changing the damping constant we rotate the flow vectors through 90°!

Ambiguities notwithstanding, core flow studies have yielded a number of stimulating maps of surface core motion. They tend to show westward drift across low latitudes in the equatorial Atlantic region and little flow in the Pacific. The Atlantic flow may either return at high latitudes, forming a pair of gyres, or may upwell around Indonesia and sink beneath Central America (e.g. Bloxham and Jackson 1991).

If diffusion is removed, the magnetic field lines become frozen to the fluid, this is Alfvén's theorem (Roberts 2007). Field lines become tracers, allowing some estimation of the flow. The ambiguity arises because it is impossible to label all the field lines uniquely (consider the example of a uniform field, it is impossible to tell how fast the surface is moving). Backus (1968) showed that one component of flow could be determined on what he called *null-flux curves*, where $B_r = 0$. These curves are uniquely labelled because there are no field lines penetrating the surface there, however the flow along the contour $B_r = 0$ still cannot be found.

Backus (1968) also showed that integrals of the flux over patches of the core surface bounded by null flux lines cannot change except by diffusion. This provided a test for the presence of diffusion. There is good evidence that the flux through some patches in the Southern Hemisphere have changed significantly in the last 200 years. This should not be a surprise: the decay time of the dipole is about 25,000 years by diffusion and the decay time of a feature increases approximately with spherical harmonic degree squared, so a patch of size comparable with degree 10 will have a diffusion time of around 250 years—perhaps much less if the radial length scale is shorter, defined by a thin boundary layer for example.

In fact the problem of diffusion is more serious than one might expect from simple numerical considerations. The limit of zero diffusion is a singular limit, so setting the diffusivity $\eta = 0$ may give different results from taking the limit $\eta \to 0$. Analysis of the limit shows the small parameter involved is in fact (η/ω), where ω is the frequency of the magnetic variation (SV), and not simply η (Gubbins 1996). The accuracy of the approximation depends not only on the magnetic diffusivity but also the time scale involved, as originally noted by Roberts and Scott (1965). This renders the steady motion uniqueness theorem (Voorhies and Backus 1985) invalid: steady motion leads ultimately to steady fields, which necessarily involves diffusion (Gubbins and Kelly 1996). For frozen flux theory to be useful in determining core motions we require rapid magnetic field changes with relatively steady flows; if the flow varies too rapidly we will not be able to determine it because it takes a finite time to determine SV accurately, if the flow varies too slowly the SV will be too slow in relation to diffusion. Perhaps the existence of geomagnetic jerks, rapid accelerations in the field, are just what we need to provide meaningful estimates of core surface flow!

The correct velocity boundary condition at the core surface is no-slip: strictly speaking there is no flow there. Core motion results therefore apply at some depth below the CMB. Many authors have discussed the correct location for the flows determined by the frozen flux hypothesis. Roberts and Scott (1965) decided it was immediately below the free stream, which is only a few centimetres below the CMB if the Ekman layer thickness is determined by molecular diffusion. Backus (1968) is undecided but puts the layer thicker (see also Backus 1991). In practice the inversion for core flow is kinematic, as the title of Backus' early paper emphasises, and dynamics has nothing to do with the result. Analysis of the inversion procedure including diffusion shows the result to be an average through a layer at the top of the core whose thickness is determined by the electromagnetic skin depth based on the *observation time*. This seems odd at first sight, how can the result depend on how long we observe for? Consider the equivalent thermal inversion. Heat diffuses up to the surface, and the longer we wait the more we can infer about the temperature deeper down. Magnetic diffusion also determines the depth at which the deduced core motions apply. Diffusion also removes any issue of discontinuity in the horizontal field across any boundary layer at the top of the core (see Roberts and Scott 1965; Backus 1968; Hide and Stewartson 1972): when diffusion is included the horizontal component becomes redundant, it adds no information whatsoever (Gubbins 1996).

Diffusion has been incorporated in recent studies of core motions (Amit and Olson 2004), as well as the admission that core-surface maps may conceal many small scale features that give the verisimilitude of diffusion through their averaging. Other studies have performed frozen flux inversions on output from numerical geodynamo simulations (e.g. Rau et al. 2000), generally concluding that frozen flux yields good estimates of the surface flow—although this probably results from the over-simplified nature of the dynamo models.

Core flow studies during the period under consideration has been dominated by use of the frozen flux hypothesis. Earlier work involving diffusion had been largely ignored, including that of Allan and Bullard (1966), who provide a mechanism that nicely explains the present changes to flux patches in the Southern Hemisphere (Bloxham 1986), and Braginsky (1984), who provides a dynamical theory that avoids the problems of the singular limit of the frozen flux hypothesis. Now that we can determine SV from satellite measurements alone over a short time period and have some detailed numerical models of the geodynamo, perhaps it is time to revisit these and other theories that include diffusion.

2.3 Paleomagnetism

Recent decades have seen huge advances in the technically difficult subject of paleomagnetism. Automation of laboratory procedures had enabled processing of large numbers of samples and production of datasets that are meaningful for geomagnetic studies. Quality control has become more stringent, making many older measurements unacceptable for new studies. Databases such as `EARTHREF` are making new observations readily available for general use. Paleomagnetic directions are typically good to one or two degrees. Most of the recent technical developments have been directed towards determination of paleointensity, which remains problematic. Paleointensities are good at best to 10% but there are often systematic errors that can destroy any attempt to enhance a signal by data processing, (stacking for example), whether in global modelling or by oversampling a site.

Dating of samples is also a problem: it is usually restricted to magnetostratigraphy unless archeological or radiometric dates are available. Examples of time-series quality data are rare indeed, exceptions being on Hawaii where a great many ^{14}C dates have been made, and parts of Europe where there are good archeological dates. Poor dating has also limited the identification of excursions, but recently many excursions have been identified during the Brunhes and even the late Matuyama (Langereis et al. 1997; Lund et al. 2006; Laj and Channel 2007). Excursions now seem to be quite common and it is likely that more of these large SV events will be firmed up in the near future.

Dating errors also restrict the type of analysis that can be done. Estimating the *time average* requires no age dates and has occupied much attention. The *geocentric axial dipole*, or GAD, has served us well, but can we do better? This is a basic question for paleomagnetism that many believe remains unresolved. The Earth's magnetic field certainly has no business being an exact axial dipole, it should show some of the characteristics expected from the geodynamo: low flux over the pole inside the tangent cylinder, and a non-cosine variation with colatitude.

Departures from axisymmetry in the time average require, and will be evidence of, influence of the solid overlying mantle. Some studies claim to see lateral variations resembling those in the modern field (Gubbins and Kelly 1993; Johnson and Constable 1995), while others have claimed the evidence for departures from axisymmetry is controversial (e.g. Carlut and Courtillot 1998). Many dynamo studies using the temperature of the lower mantle as boundary conditions have now been found to generate magnetic fields with similar lateral variations (Bloxham 2002; Olson and Christensen 2002; Kutzner and Christensen 2004) and it is becoming increasingly difficult to ignore the paleomagnetic evidence. Some dynamo models (e.g. Davies et al. 2008) have longitudinal variations that are so strong they may swamp efforts to find departures from the GAD in the axisymmetric part of the field. It is to be hoped that future paleomagnetic efforts will be directed towards detecting longitudinal variations.

The study of time variations, *paleosecular variation* (PSV), is more problematic because it requires age dating. Considerable advances have been made recently for holocene SV as mentioned earlier, and we can expect further improvements in both the data collection and modelling. However, most studies lack accurate dates and analyses must rely on the scatter in the data, making it much more difficult to detect anomalies in PSV. Much effort has gone into SV in the Pacific region, which has been exceptionally low in historical periods. This low SV has existed for at least several thousand years, but it remains difficult to make a strong case for longer periods (Merrill et al. 1996). Hawaii has shown little variation in declination but some change in inclination. However, even this change in inclination, which is itself about half the change at other longitudes over just a few hundred years, is slow and indicative of a completely different physical mechanism from that responsible for, say, European PSV. The slow change might well be of convective origin, the rapid change might result from wave motion. Rapidity of change cannot be determined from scatter alone, and therefore scatter cannot differentiate between these distinct physical mechanisms.

The *geomagnetic reversal time scale* relevant to the ocean floor was well established by 1980, the time of MAGSAT. Subsequent advances have included the recording of *transitions*, catching the Earth's magnetic field in the act of switching poles, and documenting older reversals and *superchrons*, intervals when there have been very few or no reversals. A great many reversal transitions have now been documented, the data usually being presented in the form of *virtual geomagnetic pole* (VGP) paths and, when available, intensity plots. While we would not expect the geomagnetic field to be dipolar during a transition, and there is good evidence that it is not, these VGP paths have been remarkably instructive and have stimulated a number of new ideas, some of which gain support from theory. The VGP paths have respectively been seen to pass through the site (indicating an axisymmetric transition field), on a path 90° from the site, and to follow preferred longitudes: they can't all be right! The latter can be explained if magnetic flux remains on the same longitudes as the present concentrations (see Fig. 2). The reversal could then appear similar to the 11-year reversal on the sun, in which toroidal flux is expelled in sunspots that migrate and turn the field inside out. The last transition, Matuyama-Brunhes, fits the model rather well (Gubbins and Love 1998). This could be another example of mantle control of the geodynamo, and some evidence has already appeared in numerical simulations (Kutzner and Christensen 2004).

Superchrons are also likely caused by mantle effects, the timescale being too long for any credible core dynamics. Two explanations have been offered, either the mantle suffered a large change in heat throughput at the onset and end of each superchron (Larson and Olson 1991), or changes in the pattern of lower mantle heat flux changed the regime in the geodynamo (Glatzmaier et al. 1999). The Cretaceous Normal Superchron (CNS) is well documented and recorded on the ocean floor. The older Kiaman superchron is now quite well established, with even some intensity measurements (e.g. Cottrell et al. 2008); there is a recent claim of an even older superchron (Rodionov et al. 2001). We can expect further observations and more numerical geodynamo studies addressing the issue of superchrons and what defines the boundary between reversing and non-reversing regimes.

3 Theory

The theory of the geomagnetic field falls naturally into three parts: the energy supply, the fluid or magnetohydrodynamics, and the numerical simulations that lead to results that can be related to the observed field. While all three are related, they require rather different approaches. There have been major advances in all 3 fields since 1980.

3.1 Energy Supply

The driving mechanism for the geodynamo was established in all its essential parts by Braginsky's original paper (Braginsky 1963). In what has become the *standard model*, the core is supposed to cool at a rate controlled by the mantle while convecting with sufficient vigour to mix the liquid to near-uniform composition. The pressure remains close to hydrostatic and the mixing ensures the temperature remains close to adiabatic. The inner core boundary (ICB) is a melting transition and the liquid core freezes from the bottom up, causing the inner core to grow. The temperature of the core may be estimated from the melting temperature of iron, or suitable iron mixtures, and the adiabatic temperature gradient for the liquid mixture. As the liquid freezes onto the inner core surface, some lighter elements are left behind in the liquid. These are buoyant and rise to drive further convection, to be ultimately mixed throughout the outer core. The inner core grows slowly in size while the outer core liquid becomes gradually diluted with light elements. Convection is driven by a combination of thermal and chemical convection.

This much was agreed by about 1980; since then improvements in our knowledge of the properties of liquid iron mixtures at high pressure have narrowed the range of allowed thermal histories for the core. In particular, the adiabatic gradient is known to be steep, making the cooling rates high and inner core growth fast. This led Labrosse et al. (2001) to argue that the inner core is a relatively young feature, forming about a billion years ago. The dynamo must be maintained prior to inner core formation by thermal convection alone. It is difficult, but not impossible, to derive scenarios in which the Earth cools fast enough for this to happen. The only way to retain an ancient inner core with presently accepted values of the relevant parameters is if the core contains significant amounts of radiogenic heating. Potassium is the favoured element, whose isotope ^{40}K could contribute the required heating if sufficient amounts are present in the core (Nimmo et al. 2004). Whether such large amounts of potassium could have entered the core is controversial.

While the dynamo is ultimately driven by cooling, heat determines rather little of the generated magnetic energy. This is partly because of the way in which a dynamo generates magnetic field, partly because much heat is lost by conduction down the adiabat and partly because convection does not participate directly in generating magnetic field. Compositional convection, on the other hand, stirs the core directly: its potential energy is converted to heat by frictional processes that are dominated by magnetic dissipation. Some light elements are conducted away by the process of barodiffusion, but this is weaker than the loss of heat down the adiabat. Consideration of entropy, rather than energy, make clear the balance between sources of buoyancy and dissipation. Adding radiogenic heating to the liquid core keeps the core warm and prolongs the life of the inner core but does little to aid magnetic field generation because heat is released uniformly throughout the liquid, an inefficient way to change the entropy.

Whatever the scenario, the heat budget for the geodynamo is tight. This is perhaps not surprising when we look at our nearest neighbours. Mars and Venus. Neither has a magnetic field at present, although Mars almost certainly had a dynamo in the past and Venus may have lost evidence of its ancient field through a resurfacing event. Both are likely to have at least partially liquid cores; Mars appears to have cooled too rapidly and its core is no longer convecting, while Venus is not cooling fast enough to provide the necessary vigorous convection. Earth, like the little bear's porridge, is just right!

Many of the improvements in our knowledge of core properties has come from theory, relatively little from experiment. With the widespread availability of diamond anvil cells we can expect this situation to change in the near future. Of all the relevant properties, thermal and electrical conductivities remain the least certain. They are probably the most important quantities when it comes to estimating the heat budget because the first determines how much heat is lost down the adiabat while the second determines how much electrical heating is generated by the magnetic field, effectively determining how much energy is required to generate a given magnetic field. The two are usually assumed to be linked through the Wiedeman-Franz law—another law ripe for testing at high pressure by experiment and theory—which assumes heat and electricity are carried exclusively by free electrons. If this law applies then increasing thermal conductivity will increase the heat lost down the adiabat, decreasing it will decrease the electrical conductivity and therefore increase the heat required to power the dynamo.

Current work is addressing possible anomalous regions in the outer core. The lowermost 150 km may be stably stratified (Souriau and Poupinet 1991), a curious state of affairs in a region where light elements are being released as the main driver of core convection. This may be a two-phase boundary layer (Gubbins et al. 2008) or it could result from lateral variations in the heat flux at the ICB causing melting in places. Melting would release heavy liquid into the bottom layer that fails to mix completely, remaining near the ICB to form a stable layer (Gubbins et al. 2010).

The uppermost core is also receiving attention. The seismic evidence for departures from neutral stability is less convincing here (but see Eaton and Kendall 2006; Helffrich and Kaneshima 2004), such a layer would certainly exist if light elements continue to pass across the CMB into the core, as recently suggested by Asahara et al. (2007). This would lead to the development of a thin stratified layer at the top of the core (Buffett and Seagle 2010). Stratification of the upper core is probably needed to allow for the effects of lateral variations in heat flux across the CMB inferred from geomagnetic measurements, as discussed in Sect. 2. Stratification allows the heat flux anomalies to penetrate into the main body of the core liquid by conduction, thus influencing the whole dynamo; vigorous convection at the top of the core appears to sweep away any effects of the boundary anomalies and render them ineffective (Sreenivasan and Gubbins 2008).

3.2 Convection and Dynamo Theory

Early work on dynamo theory divided it into studies of the kinematic dynamo, in which the fluid velocity is specified and no account is taken of the back-reaction of magnetic forces on the flow, and convection studies with or without a magnetic field. By 1980 plane layer models of convection had been developed to the point of acting as dynamos, and in at least one case these had been put into spherical geometry (Busse 1975). These models are limited mainly to weak magnetic forces, and have received relatively little (perhaps too little) attention from geomagnetists. The first of the modern generation of "numerical simulations of the geodynamo" was by Glatzmaier and Roberts (1995), in which magnetic forces are large although viscosity remains unrealistically large. Most subsequent simulations have followed this lead.

The full dynamo problem involves solving the equations of momentum, heat conduction, and induction simultaneously under the solenoidal conditions on magnetic field and fluid velocity:

$$\frac{E}{qPr}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\times(\nabla\times\mathbf{u})\right) + \mathbf{z}\times\mathbf{u} = -\nabla P + qRaT\mathbf{r} + (\nabla\times\mathbf{B})\times\mathbf{B} + E\nabla^2\mathbf{u}, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T = q\nabla^2 T, \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \nabla^2\mathbf{B}, \tag{4}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{5}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{6}$$

where $\mathbf{u}$ is the fluid velocity, $\mathbf{B}$ is the magnetic field, T is the temperature deviation from the basic state temperature, P is the pressure, and ρ is the density. These equations are in dimensionless form, E is the Ekman number, Pr and q are Prandtl numbers, and Ra is the Rayleigh number. Most authors have confined themselves to thermal driving and included the effects of compositional convection through codensity variations that incorporate the buoyancy effects of both, though not the more subtle effects arising from double diffusion. Boundary conditions have been, variously, constant temperature or heat flux, no-slip or stress free, and insulating or perfectly conducting. Several calculations have incorporated lateral variations in heat flux at the upper boundary, and most now allow the inner core to rotate if required. Equations (2)–(6) are the Boussinesq equations; many studies use a somewhat more sophisticated anelastic approximation that allows for some variation in basic density apart from that entering the buoyancy force.

The very small values of the molecular thermal diffusion and viscosity make numerical solution of these equations impossible. Some assumption about the nature of the turbulence must be made to render them soluble. The most common assumption is that turbulence tends to equalise the effects of the diffusion parameters on the larger scales, in which case the thermal diffusivity and kinematic viscosity are set equal to, or nearly equal to, the much larger magnetic diffusivity. The viscous term in (2) remains too small to allow numerical solution, requiring further artificial increases in viscosity (through the Ekman number in the dimensionless equations). Some authors use hyperdiffusivity, in which the viscosity becomes a function of the wavelength of the flow, increasing for smaller scale flows. This has been roundly criticised (Zhang and Jones 1997; Grote et al. 2000) because of the strong effect it has on the small scales, which are critical in determining the nature of the convection and dynamo action. Hyperviscosity effectively allows one to claim a small "headline" viscosity or Ekman number when in reality viscous effects are large.

The dominant terms in the momentum equation (2) is between buoyancy, Coriolis, and magnetic forces, the so-called *magnetogeostrophic* balance. Other terms are small but essential: the viscous term for satisfying the boundary conditions and the inertia terms for providing the rapid transfer of momentum required to establish and maintain equilibria. Early work on convection by Chandrasekhar showed that both rotational and magnetic forces inhibit convection when acting separately but can promote convection (through lowering the critical Rayleigh number) when acting together. An optimal balance is possible giving a minimum Rayleigh number for convection in a plane layer (Chandrasekhar 1961). This led to the idea that the geodynamo operated near this regime, because magnetic forces would grow and make convection stronger initially, only to equilibrate near or beyond the point where the balance with rotational forces led to convection occurring most readily. A distinction was made between those dynamos in which an initial magnetic field inhibited convection and prevented further growth immediately, the so-called *weak field* dynamos, and those in which the initial magnetic field rendered the convection stronger, leading to runaway and eventual equilibrium with much stronger magnetic forces, the so-called *strong-field* dynamos. The Earth was generally supposed to have a strong-field dynamo, a state that was not achieved until relatively late on by the Braginsky model-Z dynamo (Braginsky and Roberts 1987) and the Glatzmaier-Roberts numerical simulations (Glatzmaier and Roberts 1995). However, Zhang and Jones (1996) pointed out that these ideas are based on convection studies with imposed *force-free* magnetic fields, which do not occur in spherical geometry with an insulating boundary layer. No minimum critical Rayleigh number is found when the imposed magnetic field exerts a force on the flow. Instead, the critical Rayleigh number continues to decrease through zero, when magnetic instabilities that draw their energy from the applied field take over. In a dynamo this would drain the magnetic energy: the point at which the critical Rayleigh number approaches zero may therefore be a more appropriate indicator of where the Earth's dynamo would equilibrate. The finding makes the distinction between weak- and strong-field dynamos redundant. Zhang and Gubbins (2000) have suggested this regime might lead to interesting instabilities related to excursions and reversals.

J.B. Taylor proved an interesting early result (Taylor 1963), that any magnetic field generated by a dynamo operating in the magnetogeostrophic regime will exert zero torque on cylinders aligned with the spin axis; any departure from this would result in rapid time variations involving inertial and viscous effects that would return the dynamo to what is now called a *Taylor state*. Much effort has gone into finding dynamos in a true Taylor state, all of them more or less unsuccessful. Numerical simulations have not yet reached low enough levels of the viscosity to be in a Taylor state, claims to the contrary notwithstanding. Taylor (1963) gave a recipe for solving the equations in the magnetostrophic approximation, but to date no solutions have been found, and it may be that no such solutions exist.

The rapid variations that result from departures from a Taylor state involve oscillations along whole cylinders are called *torsional oscillations*. They are Alfvén waves, the only form of oscillation that produces no restoring Coriolis force (those influenced by both magnetic and Coriolis forces are called MAC, or slow magnetohydrodynamic, waves). Torsional oscillations have been detected in the Earth (Zatman and Bloxham 1997) and may be responsible for geomagnetic jerks and the decade variations in the length of day (Jackson et al. 1993).

By 1980 there were virtually no experimental studies of dynamo action, and very few experiments with electrically conducting fluids. This situation has changed drastically, and we may expect many exciting experimental results to emerge in the future. The problem with a laboratory dynamo is the size of the magnetic Reynolds number (ten or greater); this is the product of flow speed, size, and conductivity. The flow speed is increased by pumping: there is little hope of convective-driven flows self-generating magnetic fields in the laboratory because they are too slow. The size is obviously limited by cost, and the conductivity by the available working fluids (most experiments use sodium or gallium). These are severe challenges if we want the experiment to act as a self-exciting dynamo.

The Riga experiment (Gailitis et al. 2001) was based on pumped flows in a configuration shown theoretically by Ponomarenko to generate magnetic field. The flow is particularly simple, a spiral flow in a cylinder. The Karlsruhe dynamo experiment (Stieglitz and Mueller 2001) demonstrated successful generation of electric current using fluid pumped through a series of pipes twisted into a configuration that was known to act as a kinematic dynamo from numerical calculations. It was an extension of the earlier experiments of Gailitis that demonstrated the existence of an alpha-effect. The dynamos behaved as predicted by theory except for the saturation levels of each dynamo, which cannot be predicted from laminar theory. The VKS experiment (Aumaître et al. 2008) produced quite strange results. Two rotating propellers produced flow in a cylinder intended to mimic the axisymmetric flow known to produce a field led by a dipole field with axis perpendicular to the axis of the cylinder, but it produced a dipole axis parallel to that of the cylinder! Furthermore, dynamo action was only achieved when the propellers were made of iron, introducing a magnetic susceptibility into the apparatus. This is reminiscent of the original Lowes-Wilkinson dynamo, a laboratory model of the Herzenberg dynamo, in which iron rotors in an iron block generated a magnetic field (Lowes 2007), where the large magnetic susceptibility was also responsible for raising the magnetic Reynolds number to the point where dynamo action occurred. An ambitious program of research is underway in Grenoble; it has already yielded important experimental data on MHD behaviour in non-dynamo experiments (e.g. Schmitt et al. 2008). A large sphere is under construction in Dan Lathrop's lab that may be the most ambitious dynamo experiment yet: see http://complex.umd.edu.

Dynamo experiments make headlines when they self-generate, but they are necessarily very different from the Earth's dynamo and present only limited opportunities for interpretation when they work—indeed, it may be argued that the VKS experiment was the most stimulating of these experiments through the surprise of generating a dipole in an unexpected direction! I believe much can still be learned from magnetodyrodynamics experiments at lower magnetic Reynolds numbers, below the dynamo threshold but still in regions that are inaccessible to numerical or analytical methods. This includes the tricky question of turbulence in the core. We can expect even greater impact from experimental work in the next few decades than we have in the last.

3.3 Numerical Dynamo Models

Numerical geodynamo simulations cannot yet be run with the correct parameters for the Earth and are unlikely to reach those values in the foreseeable future. However, existing numerical models reproduce many features of the observed field: dipole-dominated fields; westward drift of magnetic features; and complete polarity reversals (Glatzmaier and Roberts 1995; Kuang and Bloxham 1997; Christensen et al. 1998; Kutzner and Christensen 2002). Several geodynamo computer simulations have incorporated lower mantle seismic shear wave velocity as a proxy for heat flux out of the core (Glatzmaier et al. 1999; Bloxham 2000; Olson and Christensen 2002; Christensen and Olson 2003; Kutzner and Christensen 2004; Aubert et al. 2007); these generate magnetic fields with some of the features seen in the geomagnetic field that could be caused by thermal core-mantle interaction: a non-axisymmetric time-averaged field, low SV in the Pacific, and preferred longitudes for transitional VGPs.

Problems remain with achieving more realistic parameters, principally the very low Ekman number. This controls the wavenumber of the underlying convection and flow in the main body of the core, probably scaling as $O(E^{-1/3})$. Even for a simple turbulent viscosity: $E = 10^{-9}$, the length scale is about a kilometre. Estimates of the computational power required to achieve this Ekman number with a pseudospectral code is 31 years on a computer cluster with 54,000 nodes for a single run of one magnetic diffusion time (Davies et al. 2010): not a realistic option in the foreseeable future. Improved codes using larger numbers of nodes may, however, be possible.

The other outstanding issue is the true nature of turbulence in the core. Most calculations use simple homogeneous eddy diffusivities based on ideas from other fields, notably atmospheric shear flow. There is good evidence that turbulence in magnetohydrodynamic flows is different (Braginsky and Meytlis 1990). For a recent review see Buffett and Matsui (2007). Is it wishful thinking to hope that new theories of turbulence will produce a simpler numerical problem?

Progress will continue in comparing the output of numerical dynamo simulations with geomagnetic and paleomagnetic data, drawbacks of the modelling notwithstanding. Simpler numerical simulations can be run on small clusters available to everyone and the codes are widely distributed, making it possible to explore very extensive parameter ranges and compare the results against the improving satellite-generated geomagnetic fields as well as improving paleomagnetic data.

4 Conclusions

Will the next decade bring the sort of advances we saw in the decade after MAGSAT? We should hope so, since the current satellite program is producing an advance in data quality at least equal to that of the first low altitude vector mission. Many new models have already been published based on these data, including the first high-quality map of SV based on satellite data alone and the highest resolution lithospheric field model ever obtained. Interpretation and refinement of these results are just beginning.

References

D.W. Allan, E.C. Bullard, The secular variation of the Earth's magnetic field. Proc. Camb. Philos. Soc. **62**, 783–809 (1966)

H. Amit, P. Olson, Helical core flow from geomagnetic secular variation. Phys. Earth Planet. Int. **147**(1), 1–25 (2004)

Y. Asahara, D.J. Frost, D.C. Rubie, Partitioning of FeO between magnesiowustite and liquid iron at high pressures and temperatures: Implications for the composition of the Earth's outer core. Earth Planet. Sci. Lett. **257**, 435–449 (2007)

J. Aubert, H. Amit, G. Hulot, Detecting thermal boundary control in surface flows from numerical dynamos. Phys. Earth Planet. Int. **160**, 143–156 (2007)

S. Aumaître, M. Berhanu, M. Bourgoin, et al., The VKS experiment: turbulent dynamical dynamos. C. R. Phys. **9**, 689–701 (2008)

G.E. Backus, Kinematics of the secular variation. Philos. Trans. R. Soc. Lond. **263**, 239–266 (1968)

G.E. Backus, Bayesian inference in geomagnetism. Geophys. J. **92**, 125–142 (1988)

G.E. Backus, *Current Meters in the Core of the Earth, 91–24*. Scripps Institute of Oceanography Reference Series (Univ. California at San Diego, San Diego, 1991)

J. Bloxham, The expulsion of flux from the Earth's core. Geophys. J. R. Astron. Soc. **87**, 669–678 (1986)

J. Bloxham, Sensitivity of the geomagnetic axial dipole to thermal core-mantle interactions. Nature **405**, 63–65 (2000)

J. Bloxham, Time-independent and time-dependent behaviour of high-latitude flux bundles at the core-mantle boundary. Geophys. Res. Lett. **29**, 1854 (2002)

J. Bloxham, A. Jackson, Simultaneous stochastic inversion for geomagnetic main field and secular variation—2. 1820–1980. J. Geophys. Res. **94**, 15753–15769 (1989)

J. Bloxham, A. Jackson, Fluid-flow near the surface of the Earth's outer core. Rev. Geophys. **29**, 97–120 (1991)

J. Bloxham, D. Gubbins, A. Jackson, Geomagnetic secular variation. Philos. Trans. R. Soc. Lon. **329**, 415–502 (1989)

S.I. Braginsky, Structure of the F layer and reasons for convection in the Earth's core. Dokl. Akad. Nauk. SSSR Engl. Trans. **149**, 1311–1314 (1963)

S.I. Braginsky, Short-period geomagnetic secular variation. Geophys. Astrophys. Fluid Dyn. **30**, 1–78 (1984)

S.I. Braginsky, V.P. Meytlis, Local turbulence in the Earth's core. Geophys. Astrophys. Fluid Dyn. **55**, 71–87 (1990)

S.I. Braginsky, P.H. Roberts, A model-z geodynamo. Geophys. Astrophys. Fluid Dyn. **38**, 327–349 (1987)

B.A. Buffett, H. Matsui, Core turbulence, in *Encyclopedia of Geomagnetism and Paleomagnetism*, ed. by D. Gubbins, E. Herrero-Bervera (Springer, Dordrecht, 2007), pp. 101–104

B.A. Buffett, C.T. Seagle, Stratification at the top of the core due to chemical interactions with the mantle. J. Geophys. Res. **115**(4), B04407 (2010). doi:10.1029/2009JB006751

F.H. Busse, A model of the geodynamo. Geophys. J. R. Astron. Soc. **42**, 437–459 (1975)

J.C. Cain, Z.G. Wang, D.R. Schmitz, J. Meyer, The geomagnetic spectrum for 1980 and core crustal separation. Geophys. J. Int. **97**, 443–447 (1989)

J. Carlut, V. Courtillot, How complex is the time-averaged geomagnetic field over the past 5 Myr? Geophys. J. Int. **134**, 527–544 (1998)

S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Clarendon Press, Oxford, 1961)

U. Christensen, P. Olson, G.A. Glatzmaier, A dynamo model interpretation of geomagnetic field structures. Geophys. Res. Lett. **25**, 1565–1568 (1998)

U.R. Christensen, P. Olson, Secular variation in numerical geodynamo models with lateral variations of boundary heat flow. Phys. Earth Planet. Int. **138**, 39–54 (2003)

R.D. Cottrell, J.A. Tarduno, J. Roberts, The Kiaman reversed polarity superchron at Kiama: Toward a field strength estimate based on single silicate crystals. Phys. Earth Planet. Int. **169**(1–4), 49–58 (2008). doi:10.1016/j.pepi.2008.07.041

C.J. Davies, D. Gubbins, A.P. Willis, P.K. Jimack, Time-averaged paleomagnetic field and secular variation: Predictions from dynamo solutions based on lower mantle seismic tomography. Phys. Earth Planet. Int. **169**, 194–203 (2008)

C.J. Davies, D. Gubbins, P.K. Jimack, Scalability of pseudospectral methods for geodynamo simulations. Concurr. Comput. Pract. Exp. (2010). doi:10.1002/cpe.1593

D.W. Eaton, J.M. Kendall, Improving seismic resolution of outermost core structure by multichannel analysis and deconvolution of broadband SmKS phases. Phys. Earth Planet. Int. **155**, 104–109 (2006)

A. Gailitis, O. Lielausis, E. Platacis, S. Dement'ev, A. Cifersons, Magnetic field saturation in the Riga dynamo experiment. Phys. Rev. Lett. **86**, 3024–3027 (2001)

G.A. Glatzmaier, P.H. Roberts, A three-dimensional convective dynamo solution with rotating and finitely conducting inner core and mantle. Phys. Earth Planet. Int. **91**, 63–75 (1995)

G.A. Glatzmaier, R.S. Coe, L. Hongre, P.H. Roberts, The role of the Earth's mantle in controlling the frequency of geomagnetic reversals. Nature **401**, 885–890 (1999)

E. Grote, F.H. Busse, A. Tilgner, Effects of hyperdiffusivities on dynamo simulations. Geophys. Res. Lett. **27**, 2001–2004 (2000)

D. Gubbins, Geomagnetic field analysis I: Stochastic inversion. Geophys. J. R. Astron. Soc. **73**, 641–652 (1983)

D. Gubbins, A formalism for the inversion of geomagnetic data for core motions with diffusion. Phys. Earth Planet. Int. **98**, 193–206 (1996)

D. Gubbins, P. Kelly, Persistent patterns in the geomagnetic field during the last 2.5 Myr. Nature **365**, 829–832 (1993)

D. Gubbins, P. Kelly, A difficulty with using the frozen flux hypothesis to find steady core motions. Geophys. Res. Lett. **23**, 1825–1828 (1996)

D. Gubbins, J. Love, Geomagnetic reversal transition fields: a test of 4-fold symmetry. Geophys. Res. Lett. **25**, 1079–1082 (1998)

D. Gubbins, G. Masters, F. Nimmo, A thermochemical boundary layer at the base of Earth's outer core and independent estimate of core heat flux. Geophys. J. Int. **174**, 1007–1018 (2008)

D. Gubbins, B. Sreenivasan, S. Rost, J. Mound, Is the inner core melting? Phys. Earth Planet. Inter. (2010, submitted)

G. Helffrich, S. Kaneshima, Seismological constraints on core composition from Fe-O-S liquid immiscibility. Science **306**, 2239–2242 (2004)

K. Hemant, S. Maus, Geological modeling of the new CHAMP magnetic anomaly maps using a geographical information system technique. J. Geophys. Res. **110**, B12103 (2005). doi:10.1029/2005003837

R. Hide, K. Stewartson, Hydromagnetic oscillations of the Earth's core. Rev. Geophys. **10**, 579–598 (1972)

K. Hutcheson, D. Gubbins, A model of the geomagnetic field for the 17th century. J. Geophys. Res. **95**, 10769–10781 (1990)

A. Jackson, J. Bloxham, D. Gubbins, Time-dependent flow at the core surface and conservation of angular momentum in the coupled core-mantle system, in *Dynamics of Earth's Deep Interior and Earth Rotation*, ed. by J.L. Le Mouël, D.E. Smylie, J. Herring. AGU Geophys. Monogr., vol. 72 (IUGG, Paris, 1993), pp. 97–107

A. Jackson, A.R.T. Jonkers, M.R. Walker, Four centuries of geomagnetic secular variation from historical records. Philos. Trans. R. Soc. Lond. **358**, 957–990 (2000)

C. Johnson, C. Constable, The time-averaged geomagnetic field as recorded by lava flows over the past 5 Myr. Geophys. J. Int. **122**, 489–519 (1995)

M. Korte, C. Constable, Continuous global geomagnetic field models for the past 3000 years. Phys. Earth Planet. Int. **140**, 73–89 (2003)

M. Korte, C.G. Constable, Continuous geomagnetic field models for the past 7 millennia: 2. cals7k. Geochem. Geophys. Geosyst. **6** (2005)

M. Korte, F. Donadini, C.G. Constable, Geomagnetic field for 0–3 ka: 2. A new series of time-varying global models. Geochem. Geophys. Geosyst. **10**(6), 24 (2009). doi:10.1029/2008GC002297

W. Kuang, J. Bloxham, An Earth-like numerical dynamo model. Nature **389**, 371–374 (1997)

C. Kutzner, U. Christensen, From stable dipolar towards reversing numerical dynamos. Phys. Earth Planet. Int. **131**, 29–45 (2002)

C. Kutzner, U.R. Christensen, Simulated geomagnetic reversals and preferred virtual geomagnetic pole paths. Geophys. J. Int. **157**, 1105–1118 (2004)

S. Labrosse, J.P. Poirier, J.L.L. Mouël, The age of the inner core. Earth Planet. Sci. Lett. **190**, 111–123 (2001)

C. Laj, J.E.T. Channel, Geomagnetic excursions, in *Treatise on Geophysics*, vol. 5.10 (2007), pp. 373–416

R.A. Langel, Special issue—a perspective on MAGSAT results—introduction. J. Geophys. Res. **90**, 2441–2444 (1985)

C.G. Langereis, M.J. Dekkers, G.J. de Lange, M. Paterne, P.J.M. van Santvoort, Magnetostratigraphy and astronomical calibration of the last 1.1 Myr from an eastern Mediterranean piston core and dating of short events in the Brunhes. Geophys. J. Int. **129**, 75–94 (1997)

R.L. Larson, P. Olson, Mantle plumes control magnetic reversal frequency. Earth Planet. Sci. Lett. **107**, 437–447 (1991)

J.L. LeMouël, C. Gire, T. Madden, Motions of the core surface in the geostrophic approximation. Phys. Earth Planet. Int. **39**, 270–287 (1985)

F.J. Lowes, Dynamo, Lowes-Wilkinson, in *Encyclopedia of Geomagnetism and Paleomagnetism*, ed. by D. Gubbins, E. Herrero-Bervera (Springer, Dordrecht, 2007), pp. 173–174

S. Lund, J.S. Stoner, J.E.T. Channell, G. Acton, A summary of Brunhes paleomagnetic field variability recorded in ocean drilling program cores. Phys. Earth Planet. Int. **156**, 194–204 (2006)

R.T. Merrill, M.W. McElhinny, P.L. McFadden, *The Magnetic Field of the Earth: Paleomagnetism, the Core, and the Deep Mantle* (Academic Press, San Diego, 1996), p. 527

J. Meyer, J.H. Hufen, M. Siebert, A. Hahn, Investigations of the internal geomagnetic field by means of a global model of the earth's crust. J. Geophys. Z. Geophys. **52**, 71–84 (1983)

F. Nimmo, G. Price, J. Brodholt, D. Gubbins, The influence of potassium on core and geodynamo evolution. Geophys. J. Int. **156**, 1407–1414 (2004)

P. Olson, U.R. Christensen, The time-averaged magnetic field in numerical dynamos with non-uniform boundary heat flow. Geophys. J. Int. **151**, 809–823 (2002)

S. Rau, U. Christensen, A. Jackson, J. Wicht, Core flow inversion tested with numerical dynamo models. Geophys. J. Int. **141**, 485–497 (2000)

P.H. Roberts, Alfvén's theorem and the frozen flux approximation, in *Encyclopedia of Geomagnetism and Paleomagnetism*, ed. by D. Gubbins, E. Herrero-Bervera (Springer, Dordrecht, 2007), pp. 7–11

P.H. Roberts, S. Scott, On the analysis of the secular variation. A hydromagnetic constraint: I. Theory. J. Geomagn. Geoelectr. **17**, 137–151 (1965)

V.P. Rodionov, V.E. Pavlov, Y. Gallet, Magnetic polarity structure of the stratotype section of the mid-Ordovician Kirenskii-Kudrinskii and Chertovskii horizons (up the Lena river, above the town of Kirensk) in relation to the Ordovician geomagnetic superchron. Izv. Phys. Solid Earth **37**, 498–502 (2001)

D. Schmitt, T. Alboussiere, D. Brito, N. Gagniere, D. Jault, H.C. Nataf, Rotating spherical couette flow in a dipolar magnetic field: experimental study of magneto-inertial waves. J. Fluid Mech. **604**, 175–197 (2008)

L. Shure, R.L. Parker, G.E. Backus, Harmonic splines for geomagnetic modelling. Phys. Earth Planet. Inter. **28**, 215–229 (1982)
A. Souriau, G. Poupinet, The velocity profile at the base of the liquid core from PKP(BC+Cdiff) data—an argument in favor of radial inhomogeneity. Geophys. Res. Lett. **18**, 2023–2026 (1991)
B. Sreenivasan, D. Gubbins, Dynamos with weakly convecting outer layers: implications for core-boundary locking. Geophys. Astrophys. Fluid Dyn. **102**, 395–407 (2008)
R. Stieglitz, U. Mueller, Experimental demonstration of a homogeneous two-scale dynamo. Phys. Fluids **13**, 561 (2001). doi:01.1063/11331315
J.B. Taylor, The magneto-hydrodynamics of a rotating fluid and the Earth's dynamo problem. Proc. R. Soc. **274**, 274–283 (1963)
C.V. Voorhies, G.E. Backus, Steady flows at the top of the core from geomagnetic field models: The steady motions theorem. Geophys. Astrophys. Fluid Dyn. **32**, 163–173 (1985)
K.A. Whaler, Does the whole of the Earth's core convect? Nature **287**, 528–530 (1980)
S. Zatman, J. Bloxham, Torsional oscillations and the magnetic field within the Earth's core. Nature **388**, 760–763 (1997)
K. Zhang, D. Gubbins, Is the geodynamo process intrinsically unstable? Geophys. J. Int. **140**, 1–4 (2000)
K. Zhang, C.A. Jones, On small Roberts number magnetoconvection in rapidly rotating systems. Proc. R. Soc. **452**, 981–995 (1996)
K. Zhang, C.A. Jones, The effect of hyperviscosity on geodynamo models. Geophys. Res. Lett. **24**, 2869–2872 (1997)