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ORIGINAL PAPER

Predicting Hysteresis of the Water Retention Curve from Basic Properties of Granular Soils

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Abstract The water retention curve (WRC), which represents the relationship between volumetric water content (θ) and suction (ψ), is required to analyze the hydro-geotechnical response of unsaturated soils. The laboratory (or field) determination of the WRC can however be time consuming and difficult to conduct. A practical alternative, particularly useful at the preliminary stages of a project, is to estimate the WRC using a predictive model based on basic geotechnical properties that are easy to obtain. One common limitation of such predictive models is due to hysteresis effects, which are not taken into account by most of these models. The authors present in this paper an extended version of the Modified Kovács (MK) predictive model that incorporates hysteresis of the WRC along different paths, including the main wetting and drying curves and the wetting and drying scanning curves for granular soils. The model formulation is presented, and predictions are compared to experimental data obtained on different granular soils. The results show a good agreement for the main and scanning curves.

Keywords Water retention curve · Hysteresis · Predictive model · Laboratory tests

List of symbols

α	Parameter in the van Genuchten model
ψ	Suction head
θ	Volumetric water content
ψ_1	Suction head at the inversion point
$\theta_{1d}(\psi)$	Volumetric water content in the drying scanning curve
$\theta_{1w}(\psi)$	Volumetric water content in the wetting scanning curve
ψ_a	Air entry value (or AEV)
θ_a	Volumetric water content associated with the AEV
β_d	Contact angle during the drying process
$\theta_d(\psi)$	Volumetric water content in the main drying curve
$\theta_d(\psi_1)$	Volumetric water content in the main drying curve at the inversion point
ψ_{resMK}	Residual suction head (corresponding to the residual water content)
θ_r	Residual volumetric water content
θ_s	Saturated volumetric water content

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β_w Contact angle during the wetting process
$\theta_w(\psi)$ Volumetric water content in the main wetting curve
$\theta_w(\psi_1)$ Volumetric water content in the main wetting curve at the inversion point
a_c Adhesion coefficient
AEV Air entry value
C_U Coefficient of uniformity
D_{10} Diameter at 10 % passing on the cumulative grain-size distribution
D_{60} Diameter at 60 % passing on the cumulative grain-size distribution
e Void ratio
h_{co} Equivalent capillary rise
h_{cod} Equivalent capillary rise for the drying process
h_{cow} Equivalent capillary rise for the wetting process
k_u Hydraulic conductivity
m_{MK} Pore size distribution parameter in the MK model (−)
MDC Main drying curve
MK Modified Kovács model
MK_h Modified Kovács model with hysteresis effect
MWC Main wetting curve
n Porosity
DSC Drying scanning curve
PF Pedotranfer function
WSC Wetting scanning curve
r Channel radius
R Pore radius
S_a Adhesion component of the degree of saturation
S_c Capillary component of the degree of saturation
S_r Degree of saturation
WRC Water retention curve

1 Introduction

Description and prediction of water movement through unsaturated soils require detailed knowledge of their hydraulic properties. Water flow in a soil depends on the relationship between volumetric water content (θ) and hydraulic conductivity (k_u), which in turn depends on the water retention curve (WRC) relating θ with the negative pore water pressure (or suction) ψ. Laboratory and field measurements of the WRC can be slow, cumbersome, and difficult to conduct. Alternative procedures have been developed to estimate the WRC using routinely available data, such as particle size distribution, dry bulk density, and clay content.

These relationships are conventionally referred to as pedotransfer functions (PF)—(Bouma 1989) and can be quite convenient to estimate the WRC, especially early in a project when only basic information is available. However at the final design stage it is recommended that there should be measurements made of the WRCs for the soils that may affect the engineering design (Fredlund and Houston 2009).

Haverkamp and Reggiani (2002) grouped available PF into two categories: empirical and physical. With empirical models, the different WRC equations are related to one or more of the following parameters: dry bulk density, percent of sand, silt, organic matter, and other basic soil properties using statistical regression analyses (Gupta and Larson 1979; Rawls and Rakensiek 1982; Ahuja et al. 1985). More information on the use of soil texture in PF can be found in Nemes and Rawls (2004). With physically-based models, a fundamental relationship is postulated between the water retention curve and easily measurable properties of the soil, such as the cumulative particle-size distribution (PSD) and porosity (Haverkamp et al. 2002). Among the physical models, one can mention the Arya and Paris (1981) model, and other similar ones (e.g. Tyler and Wheatcraft 1989), which divide the particle size distribution curve into a number of fractions, assigning a pore volume and volumetric water content to each fraction. Haverkamp and Parlange (1986) also proposed a model that is based on the similarity between the cumulative PSD and WRC, using the Brooks and Corey (1964) equation. A physically-based model was also proposed by Kovács (1981), and it was later modified by Aubertin et al. (1998, 2003) for non-deformable soils and Mbonimpa et al. (2006) for deformable soils. The Modified Kovács (MK) model assumes that water retention results from the combined effect of capillary and adhesion forces, and that each component can be related to the pore space and specific surface of the particles, which can in turn be expressed from the porosity and PSD for granular soils.

Laboratory and field measurement results show that the WRC is not unique, as the θ–ψ relationship may depend on the wetting or drying path applied during the test (Haines 1930, Miller and Miller 1956, Poulovassilis 1962, Davis et al. 2009, Mualem and Beriozkin 2009). Such hysteresis effects can induce a considerable difference in the θ values for a given suction, depending on water movement history during the wetting or drying processes. This effect is not taken into consideration by the above mentioned PF predictive models.

Considering the relative simplicity of the MK model and its ability to predict the WRC during the drying process for different types of materials (Aubertin et al. 2003, Mbonimpa et al. 2006), a study was initiated to include hysteresis effects into this model for granular soils.

This paper begins with background information on hysteresis effects and their causes, followed by a description and inclusion of the hysteresis effects into the predictive MK model; model validation is then presented. Finally the paper provides a discussion and conclusion.

2 Background on Hysteresis Effects and Their Causes

2.1 Hysteresis Effects

As stated above, laboratory and field test measurements show that the WRC is not unique, as the θ–ψ relationship may depend on the wetting or drying path applied during the test (see Fig. 1). Observations indicate that when the volumetric water content is monotonically increased from a relatively dry state, such as during an infiltration episode, the WRC can be described by a unique function. Similarly, when the water content is monotonically decreased during evaporation and/or gravity drainage, the WRC can also be described by another unique function. However, these two functions can be quite different: for a given suction, the volumetric water content value is typically smaller during the wetting process than during the drying process.

Experimental evidences also show that the value of θ for a given ψ is a function of the initial condition of the soil (see Fig. 1). The WRC obtained during the first wetting of the dry soil (with a negligible volume of entrapped air) corresponds to the main wetting curve (MWC). The drainage of a fully saturated soil gives the main drying curve (MDC). When a wetting or drying process is reversed while following the two main hysteresis curves, the path tends to follow a wetting scanning curve (WSC) or drying scanning curve (DSC) for a wetting or drying process respectively. Secondary and higher-order (scanning) curves can also result from additional reversals (e.g. Jaynes 1992, Kool and Parker 1987).

Many authors (Kaluarachchi and Parker 1989, Whitmore and Heinen 1999, Si and Kachanoski 2000, Zhang et al. 2009) have shown the importance of hysteresis effects when analyzing problems

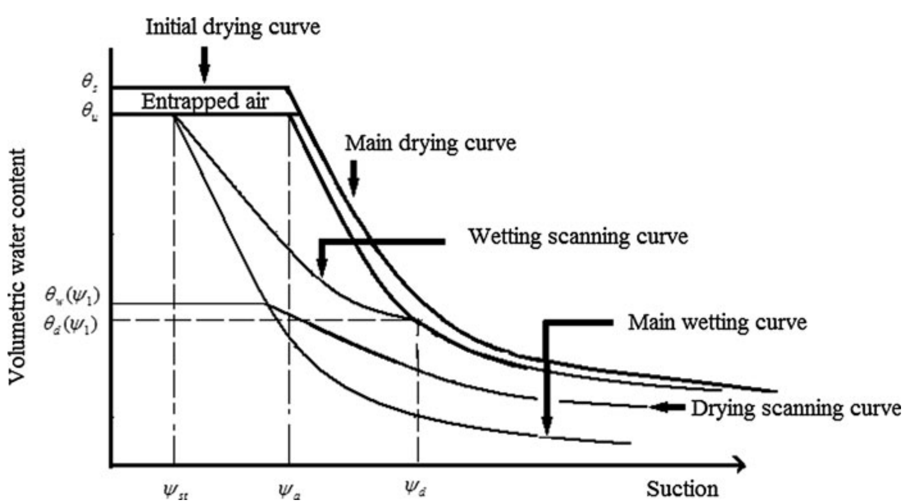

Fig. 1 Schematic view of the hysteresis of the WRC (adapted from Haverkamp et al. 2002); ψ_a and ψ_{st} are respectively the air entry value and the wetting pressure scale parameters of the Brooks and Corey (1964) equation; ψ_1 is the suction at the inversion point from the main drying curve to the scanning wetting curve

involving water transfer, solute transport, multiphase flow and/or microbial activities. Neglecting hysteresis effects in calculations can lead to significant discrepancies between predicted and measured results (Gillham et al. 1976, Hoa et al. 1977, Kool and Parker 1987, Kaluarachchi and Parker 1989, Mitchel and Mayer 1998).

2.2 Hysteresis Causes

The main causes of hysteresis of the WRC include (Hillel 1980, Jury et al. 1991, Klausser 1991, Vereecken et al. 1989, Don Scott 2000, Delage and Cui 2000, Flynn et al. 2004, Naasz et al. 2008): (1) geometric non-uniformity of individual pores, resulting in the so called "ink bottle" effect; (2) liquid–solid contact angle difference between wetting and drying processes; (3) aggregate effects from phenomena such as swelling and shrinking; (4) air entrapment effect; and (5) capillary condensation effect.

The *ink-bottle effect* is due to the irregular shape and size of individual and interconnected pores (Hillel 1998). A pore consists of a relatively wide void of radius R, bounded by narrow channels of radius $r < R$. During the drying process, the pore drainage requires a suction exceeding the value of $1/r$; in the wetting process, the pore rewets only if the suction is greater than $1/R$. Hence, the drying process depends mainly on the narrow radius of the connected channels while the wetting process depends more significantly on the maximum radius of the large pores.

The *liquid–solid contact angle* varies according to the direction in which the water meniscus moves. The contact angle is larger when pores are filling (wetting process) than when they are emptying (Morrow 1975, Araujo et al. 1995, Ustohal et al. 1998, Bachman et al. 2002, Bustos and Toledo 2003, Suciu et al. 2005).

S*welling and shrinking*, which result in differential changes of soil structure, depend on the wetting and drying history of a sample (Hillel and Mottes 1966, Peng and Horn, 2007). The volume changes alter the pore size distribution by rearranging particles and aggregates.

Air entrapment occurs during the wetting process, when 'bubbles' can become trapped inside some of the larger pores. Given sufficient time, the entrapped air will however dissolve in water.

C*apillary condensation* is a phenomenon by which the water content in a porous medium can increase due to condensation of humidity in the pores (Coasne et al. 2005). Capillary condensation appears only at relatively low water content (i.e. near the residual water content).

During a wetting process, the ink-bottle effect induces a delay that is observable at the beginning when suctions are rather high (i.e. for water content near residual saturation). Considering that the median curve part is the most important part of the water retention curve, and because the ink-bottle effect occurs mostly at higher suction, this factor can often be considered as negligible. Swelling and shrinking effects are particularly important for clayey soils, but can be neglected for typical granular soils with low plasticity. Air entrapped effect during the wetting process and under typical experimental conditions (where sufficient time is given to reach equilibrium—as it is the case for most laboratory tests, e.g. Maqsoud et al. 2002, 2004) can also be neglected; A similar assumption was used by Simunek et al. (1998) for estimating hysteresis in the WRC. The effect of capillary condensation occurs at very low saturation and the impact on the hysteresis of WRC is considered non-significant for relatively coarse soils. The effect of the contact angle is the most important cause of hysteresis for many soils, particularly those with a relatively coarse texture. This assumption is in accordance with the findings of Moseley and Dhir (1996), Wang et al. (2004), Likos and Lu (2002) and Ishakoglu and Baytas (2004).

3 MK Model and Hysteresis Effects

3.1 Basic Equations of the MK Model for Granular Soils

The MK (Aubertin et al. 1998, 2003; Mbonimpa et al. 2000) is a full predictive model, where a set of equations developed to predict the water retention curve (WRC) are derived from the Kovács (1981) model. In the MK model, the degree of saturation (S_r) includes two components acting jointly: one created by capillary forces (S_c) whose contribution is more important at relatively low suction, and one associated with adhesive forces (S_a) which mainly contributes at higher suction. Both components can be evaluated from basic (and generally available) material properties, including the effective diameter D_{10}, the void ratio e and uniformity coefficient C_U for coarse-grained materials. These properties are first used to define the equivalent capillary rise h_{co}, which constitutes the central parameter in the MK model.

The set of equations used to predict the WRC contains three parameters required for model application: the residual suction ψ_{resMK}, the pore size distribution parameter m_{MK}, and the adhesion coefficient a_c. In the case of granular materials, a relationship has been developed between ψ_{resMK} and basic geotechnical properties. The two other parameters, m_{MK} and a_c are also expressed from basic geotechnical properties.

The basic equation of the model can be written as follows:

$$S_r = [1 - \langle 1 - S_a \rangle (1 - S_c)]. \tag{1}$$

S_c and S_a correspond to the capillary and adhesive components of the total degree of saturation S_r (=θ/n where θ is the volumetric water content and n is the porosity);

< > represents the Macauley brackets ($<y> = 0.5(y+|y|)$).

The contributions of S_c and S_a are functions of the equivalent capillary rise h_{co} (cm) and suction ψ (cm):

$$S_c = 1 - [(h_{co}/\psi)^2 + 1]^{m_{MK}} \exp[-m_{MK}(h_{co}/\psi)^2] \tag{2}$$

$$S_a = a_c\left(1 - \frac{\ln(1+\psi/\psi_{resMK})}{\ln(1+\psi_0/\psi_{resMK})}\right) \frac{(h_{co}/\psi_n)^{2/3}}{e^{1/3}(\psi/\psi_n)^{1/6}} \tag{3}$$

where

$$h_{co} = \frac{0.75 \cos \beta_d}{eD_{10}(1.17 * \log(C_U) + 1)} \tag{4}$$

$$\psi_{resMK} = 0.11\left(\frac{C_U}{e * D_{10}}\right) \tag{5}$$

$$C_U = \frac{D_{60}}{D_{10}} \tag{6}$$

$$m_{MK} = \frac{1}{C_U} \tag{7}$$

In Eq. (2), m_{MK} (−) is a pore size coefficient that controls capillary saturation that can be estimated using Eq. (7) for granular soils.

In Eq. (3), a_c (−) is the adhesion coefficient (a_c = 0.01 for granular soils, when suctions are in cm), ψ_{resMK} is the residual suction (cm) at residual water content that can be estimated using Eq. (5) (obtained from grain size distribution); ψ_{resMK} is also equal to the water entry value when no distinction is made between drainage and wetting path, and ψ_n is a unit consistency parameter (ψ_n=1 cm when ψ and h_{co} are given as head, in cm).

In Eq. (4), D_{10} is the diameter (cm) corresponding to 10 % passing on the cumulative grain-size distribution, β_d corresponds to the contact angle (taken as zero for a drainage condition), C_U is the coefficient of uniformity that can be estimated using Eq. (6) (where D_{60} is the diameter in cm, corresponding to 60 % passing on the cumulative grain-size distribution), and e is the void ratio (e = $n/(1-n)$).

The water content is forced to be equal to zero when ψ reaches a limit imposed by thermodynamic equilibrium (θ = 0 at $\psi = \psi_0 = 10^7$ cm of water, corresponding approximately to complete dryness).

The MK model was originally developed to predict the main drying curve and more details on this model are presented by Aubertin et al. (1998, 2003) and Mbonimpa et al. (2000).

Despite its good predictive capability, the modified Kovács model does not take into account the influence of hysteresis on the WRC. It thus has to be modified to include this important effect.

3.2 Inclusion of Hysteresis into the MK Model

The approach taken in this study was to lump all of the possible mechanism for hysteresis into one variable, the contact angle, which has been shown by at least several researchers to be the primary mechanism for hysteresis of the WRC for granular soils. In other words, even if other mechanisms play a role, they have been incorporated, empirically, into contact angle assumption.

The MK model is thus adapted by using different values of contact angle, based on results from the literature. For drying processes, the contact angle is commonly assumed to be equal to 0° (e.g. Marshall et al. 1996). For a wetting process, the contact angle is typically much larger. For sandy soils, experimental studies have shown that the wetting angle can vary between 30° and 80° (e.g. Letey et al. 1962, Kumar and Malik 1990, Bradford and Leij 1995). The actual value of the contact angle (β_w) to be used in the MK model for a wetting process is assessed below, based on specific characteristics of water retention curves measured under drying and wetting paths.

3.2.1 *Relationship Between the Water Entry Value and Equivalent Capillary Rise*

The equivalent capillary rise h_{co}, given in Eq. (4) for granular materials, constitutes a central parameter for the MK model application (Aubertin et al. 2003). The proportionality between h_{co} under drying and wetting conditions is used here to quantify the hysteresis of the water retention curve, according to:

$$h_{cod}/h_{cow} = \varepsilon. \tag{8}$$

By combining Eqs. (4) and (8) (with β_d = 0° for drying), one obtains:

$$\cos\beta_w = 1/\varepsilon \tag{9}$$

As the value of h_{cod} of a soil is related (almost linearly) to its air entry value (ψ_a) (Kovács, 1981, Aubertin et al. 1998, 2003), the value of ε can also be estimated by comparing ψ_a (i.e. the suction at which the largest pores start to desaturate on a drying path) with the suction at satiation ψ_{st} (i.e. the pressure at which water has completely infiltrated the entire porosity of the material, assuming that there is no air entrapment). Bouwer (1966) suggested that the value of ψ_a is typically twice that of ψ_{st}. Similar ratios ψ_a/ψ_{st} (= ε) were also reported by Gupta and Larson (1979), Kool and Parker (1987), Wang et al. (1997) and Haverkamp et al. (2002). Based on typical values taken from the literature, it can be established that the ratio ψ_a/ψ_{st} = ε is typically close to 2, so the contact angle β during a wetting process is expected to be close to 60° for granular soils.

Therefore, the equivalent capillary rise used in the MK$_h$ model (MK with hysteresis) to predict the MWC is the same equation as Eq. (4) except that the contact angle for a wetting path takes a value of 60°, based on various WRCs measured on granular soils (as will be shown below).

A contact angle of 60° for a wetting process is in accordance with experimental results obtained by Letey et al. (1962) and Kumar and Malik (1990) on sandy soils; the same value was used by Huang et al. (2011) in the infiltration and drainage processes in multi-layered coarse soils. The other equations of the MK model remain the same for the wetting and drying paths.

3.3 Model Validation

3.3.1 *Origin of the Data*

Data from different sources are used to evaluate the validity of the Modified Kovács with hysteresis (MK$_h$) model. The following information had to be available to apply the model: the wetting and drying WRCs, the grain-size distribution parameters D_{10} and D_{60}, and the void ratio e of the tested specimens. Overall, results obtained on seven different soils are used here. Most of the experimental data were taken from three independent soil databases: Mualem (1976), Unsoda (Leij et al. 1996), and Grizzly (Haverkamp et al. 1998), and from our experimental tests (Maqsoud et al. 2002). In other cases, data were extracted directly from the published papers. Note that the air entrapment is not taken into consideration in this application of the MK$_h$ model (i.e. wetting can proceed until full saturation of the soil).

The materials tested have a void ratio e between 0.37 and 0.70, a D_{10} between 0.0075 and 0.023 cm, and a D_{60} between 0.032 and 0.054 cm (see Table 1). The values of h_{cod} calculated with Eq. (4) (for the drying curve) are between 40.26 and 96.93 cm.

3.3.2 *Application of the MK$_h$ Hysteresis Model*

Two approaches are used here to apply and validate the MK$_h$ model. When measured MDC and MWC are available (soils 1 and 2, in Table 1), experimental results are compared directly with the predicted WRCs using the model equations for wetting and drying conditions. When experimental MWC data are not available (soils 3–5 in Table 1), the experimental wetting scanning curve (WSC; see Fig. 1) is compared with the predicted WSC. For these particular cases, the WSC ($\theta_{1w}(\psi)$) is predicted from the MDC ($\theta_d(\psi)$) and MWC ($\theta_w(\psi)$) (calculated with MK$_h$) using the following Mualem (1984) model equation:

$$\theta_{1w}(\psi) = \theta_d(\psi_1) + \frac{[\theta_s - \theta_d(\psi_1)]}{[\theta_s - \theta_w(\psi_1)]}[\theta_w(\psi) - \theta_w(\psi_1)] \tag{10}$$

In Eq. (10), ψ_1 is the suction at the inversion point from the drying to the wetting process on the MDC (see Fig. 1), $\theta_d(\psi_1)$ is the volumetric water content on the MDC at ψ_1, $\theta_w(\psi_1)$ is the volumetric water content

Table 1 Soils identification, textural classes, sources, available information, and basic geotechnical parameters

No	Sample	Authors	Available data	Void ratio (e)	D_{10} (cm)	D_{60} (cm)	h_{cod} (cm)
1	Fine Sand	Maqsoud et al. (2002)	MDC, MWC, GSD	0.3698	0.0229	0.034	40.26
2	Fine sand	Yang et al. (2004)	MDC, MWC, GSD	0.6978	0.017	0.035	46.25
3	Sand Grizzly 10	Haverkamp and Parlange (1986)	MDC, WSC, GSD	0.3698	0.0152	0.0318	96.93
4	Sand 4107	Mualem (1976)	MDC, WSC, GSD	0.4388	0.02	0.054	47.58
5	Sand Grizzly 12	Haverkamp and Parlange (1986)	MDC, WSC, GSD	0.4500	0.014	0.035	43.65
6	Sand	Gillham et al. (1976)	MDC, WSC, DSC	0.4286	0,02 (fitted)	0,04 (fitted)	64.71
7	Sand Unsoda 4108	Mualem (1976)	MDC, WSC	0.4245	0.0075 (Fitted)	0.012 (Fitted)	190.16

on the main wetting curve at the inversion point, and θ_s is the saturated volumetric water content considered here equal to the porosity n (see Fig. 1).

3.3.2.1 Validation Using the MWC The main water retention curves obtained for both paths (wetting and drying) are presented in Fig. 2a and b for soils 1 and 2, together with the predicted results using the MK$_h$ model equations. The emphasis here is on the MWC since the capability of the MK model to predict the main drying curve (MDC) has been addressed previously (Aubertin et al. 2003). For soil 1, there is a fairly good agreement between predicted and measured water retention curves, for both the wetting and drying processes, using data coming from the authors experiments (see Fig. 2a). For soil 2, it is seen that there is a difference between some of the experimental results and the predicted MWC, especially near ψ_{st}, using data taken from Yang et al. (2004) (see Fig. 2b). The main tendencies are nevertheless relatively well represented by the model for both the wetting and drying processes. Considering the relative precision of suction and volumetric water content measurements in such coarse-grained soils, it can be said that the MK$_h$ model appears to give a good estimate of the MWC for these soils.

3.3.2.2 Validation Using the WSC The main wetting curves are not frequently measured because of experimental difficulties. The wetting scanning curves WSC (starting from the MDC) are measured more regularly. For this reason, much of the previous work on hysteresis of the WRC relates to WSCs. Some experimental WSCs were selected to apply the proposed MK$_h$ model. Results obtained on these soils have also been used previously to validate other hysteresis models (Haverkamp and Parlange 1986, Haverkamp et al. 2002).

In this case, as mentioned above, the two main curves (MDC and MWC) are first predicted using the MK$_h$ model. Then, the WSC is predicted using Eq. (10). This equation takes into account the volumetric water content corresponding to the suction at the inversion point for the MDC and MWC.

Experimental data and predicted results for soils 3, 4 and 5 are presented in Fig. 2c, d and f, respectively. For soil 3 (soil 10 in the GRIZZLY database—Haverkamp et al. 1998), the agreement between experimental and predicted results is excellent (see Fig. 2c). For soil 4

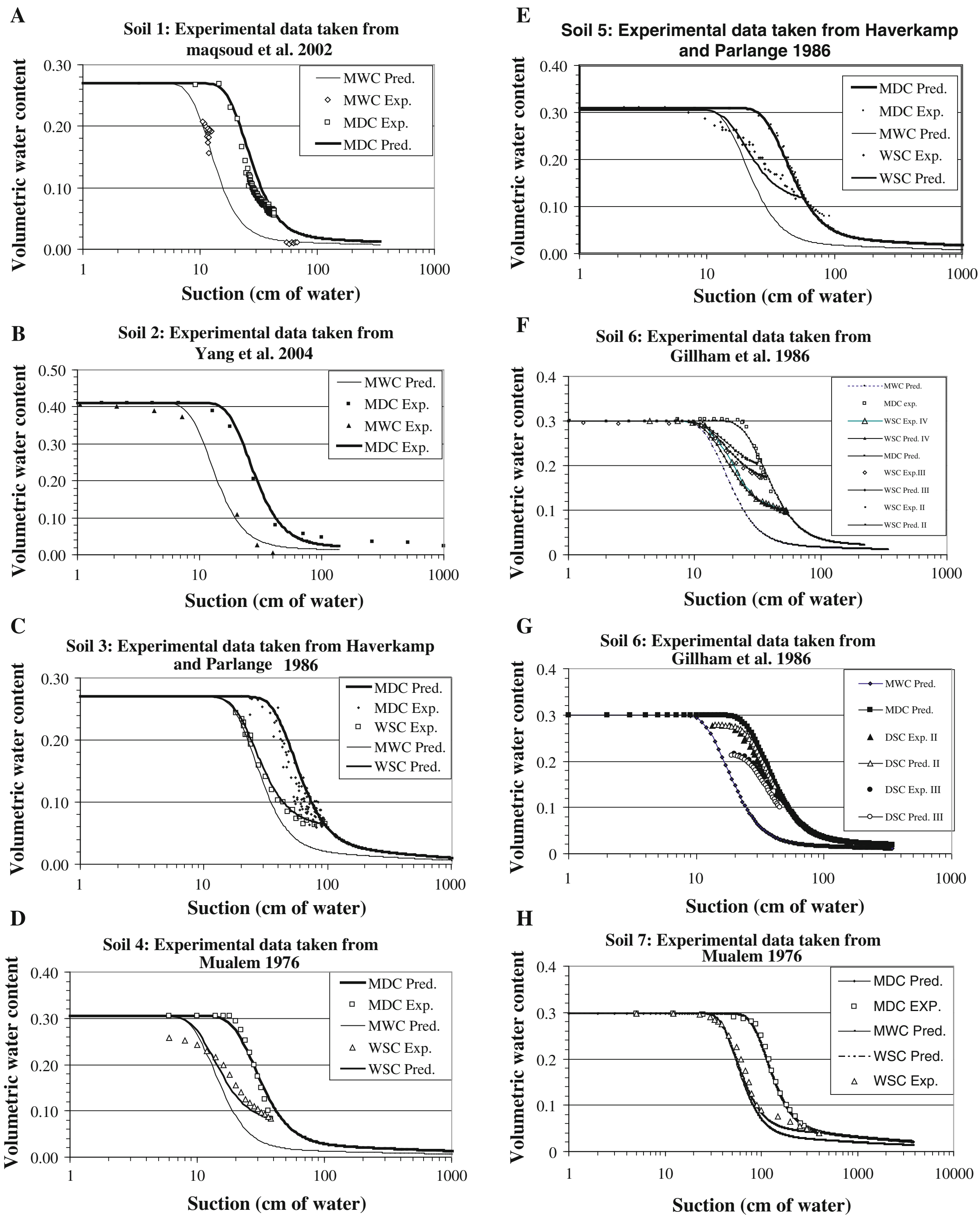

Fig. 2 Experimental and predicted wetting and drying branches of water retention curve: **a** Soil 1; **b** Soil 2; **c** Soil 3; **d** Soil 4; **e** Soil 5; **f** soil 6; **g** Soil 6; **h** Soil 7 (*Pred.* predicted data, *Exp.* Experimental data, *MDC* Main drying curve, *MWC* Main wetting curve, *WSC* Wetting Scanning curve and *DSC* Drying scanning curve)

(soil 12 in the GRIZZLY database) and soil 5 (soil 4,107 from the Mualem 1976 data), there is also a relatively good correlation between measured and predicted data for the WSC (see Fig. 2d and e). A difference between experimental and predicted values can nonetheless be observed for these two soils, particularly for volumetric water contents between saturation and ψ_{st}. This difference could be attributed to the testing procedure (i.e. low water flux for soil imbibitions and insufficient time to reach equilibrium) and/or to the effect of air entrapment which prevents the soil from reaching complete water saturation. Nonetheless, the results are encouraging and indicate that MK_h provides an acceptable estimate of the WRC along the main drying and wetting paths.

3.4 Additional Results

The MK model equations can also be used in a descriptive manner by fitting parameters a_c and m to the measured WRCs (Mbonimpa et al. 2006). Moreover, when the main drying and wetting and drying scanning curves are available, but without the grain-size distribution (as is the case for many soils in the Mualem 1976 and Leij et al. 1996), the MK model can be used to estimate D_{10} and C_U by fitting predicted to the measured WRC. Optimal values for D_{10} and C_U can be determined using a fitting procedure with Microsoft EXCEL Solver. The parameters obtained can then be incorporated into MK_h to predict the MWC.

It is important to recall that this procedure is not ideal but was taken to complete the model validation because no complete data was available in our database and in the literature.

This procedure was applied with experimental data taken from Gillham et al. (1976) and Mualem (1976). Figure 2f presents the results for soil 6 from Gillham et al. (1976), illustrating the MDC and three WSCs that started from the main drying curve at different inversion points (corresponding to different volumetric water content). Two DSCs (from the same soil—Gillham et al. 1976) that started from the main wetting curve at different inversion points are presented in Fig. 2g. Finally, the MDC and WSCs of soil 7 (corresponding to soil 4,108 in Mualem 1976), are presented in Fig. 2h.

By fitting the main drying curves with the MK model, it is possible to evaluate D_{10} and C_U for soil 6 (Gillham et al. 1976) and soil 7 (soil 4,108 in Mualem 1976); results are presented in Table 1. These parameters were then used to predict the MWC, and the WSCs using Eq. (10) for soils 6 and 7. DSCs were also predicted for soil 6 using the following equation from Mualem (1984):

$$\theta_{1d}(\psi) = \theta_w(\psi_1) - \frac{[\theta_s - \theta_d(\psi)]}{[\theta_s - \theta_w(\psi)]}[\theta_w(\psi_1) - \theta_w(\psi)] \tag{11}$$

In this equation, ψ_1 is the suction at the inversion point from wetting to drying processes on the MWC (see Fig. 1), $\theta_w(\psi_1)$ is the volumetric water content on the MWC at ψ_1, and θ_s is the saturated volumetric water content (considered equal to the porosity n; see Fig. 1).

Results of these predictions using Eqs. (10) and (11) are presented in Fig. 2f, g, and h. These figures show that the agreement between experimental and predicted results is excellent for all wetting and drying scanning curves. Despite the slight difference between experimental and predicted data close to inversion points, it can be said that the results are encouraging. These results tend to indicate that the MK_h model provides an acceptable estimate of the WRCs along wetting and drying scanning paths when combined with the hysteresis model of Mualem (1984).

4 Discussion

Comparisons between experimental and predicted hysteresis curves (see Fig. 2) show a good agreement (at least qualitatively) for the main wetting and wetting and drying scanning curves. To further analyse the differences between experimental and predicted curves, statistical tools were used.

The coefficient of determination R^2 is a well-known statistical measure to evaluate of how well the regression line approximates the real data points. R^2 can be calculated using the following equation:

$$R^2 = \left[\frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2}\sqrt{\sum(y_i - \bar{y})^2}}\right]^2 \tag{12}$$

The calculated R^2 values for nine sets of data (results not presented here) show that the coefficients of determination are between 0.96 and 1 respectively for soil 5 and 6. These coefficients of determination confirm quantitatively the good agreement between measured and predicted values of volumetric water content.

Measured versus predicted volumetric water contents are plotted for a given suction in Fig. 3a and b for soils 5 and 6, respectively. These figures show two examples: the extreme cases of good and bad correlations between measured and predicted (using MK_h) volumetric water content for different suctions. The regression line with its R^2 calculated using eq. (12) is also shown, along with two lines representing the upper and the lower boundaries of the 95 % confidence interval. The choice of the 95 % tail is based on Tchebysheff's theorem that says that at least ¾ of the measurements will fall in the interval ± 2 standard deviations around the mean, which is defined as the 95 % 2-tailed test for a normal distribution function. Figure 3a and b show that all predicted values are included within the confidence interval of 95 %.

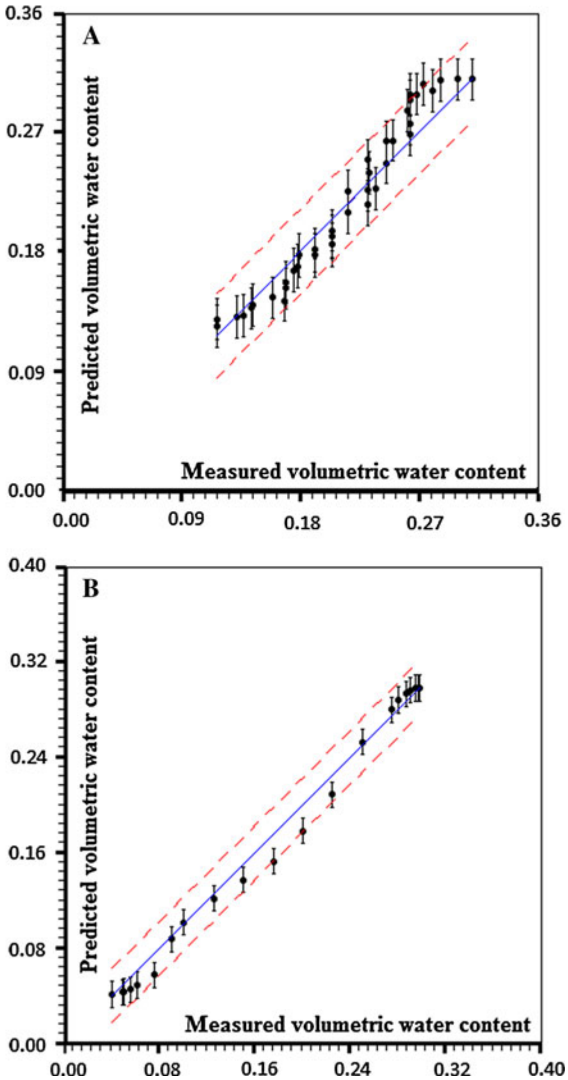

Fig. 3 **a** Comparison between measured and calculated volumetric water content value with *upper* and *lower* confidence interval of 95 % for soil 4. **b** Comparison between measured and calculated volumetric water content value with *upper* and *lower* confidence interval of 95 % for soil 5

The validation of MK_h was performed above using a single contact angle equal to 60° for the wetting path. However, experimental studies, and particularly those for sandy soils, show that the actual wetting contact angle can vary widely (e.g. Letey et al. 1962, Kumar and Malik 1990, Bradford and Leij 1995). It is thus useful to evaluate the impact of a contact angle variation on the prediction of the WRC for the wetting process. Two extreme values presented in the literature, 30° and 80°, are considered for this assessment. Results of these calculations are presented in Fig. 4 for soil 1 (a similar behavior was observed for the other soils). By using a contact angle equal to 30° (see Fig. 4), one can observe that the surface included between the two main curves (wetting and drying) is reduced (compared with a value of 60°), and the predicted main wetting curve and/or the wetting scanning curve are closer to the main drying curve. Consequently, by using a contact angle equal to 30°, the hysteresis effect is underestimated. Results obtained using a contact angle of 80° show that the surface included between the two main curves (wetting and drying) is amplified. Consequently, the use of a contact angle of 80° tends to exaggerate hysteresis

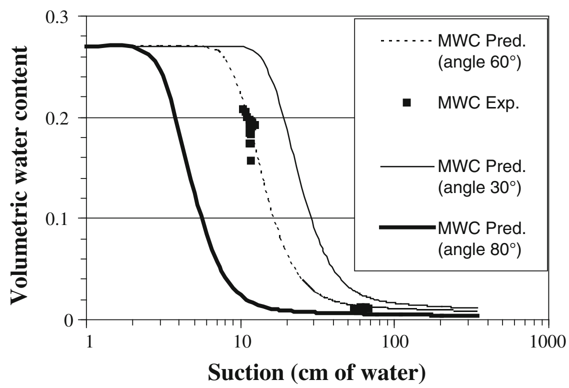

Fig. 4 Experimental and predicted main wetting curve (MWC Pred.) for soil 1 where the contact angle is equal to 30°, 60° and 80°

effect of the WRC. For the soils tested, a contact angle of 60° seems appropriate and gives predictions close to laboratory measurements.

5 Conclusion

This paper presents the basis for the inclusion of hysteresis effects into the MK model, which predicts the WRC from basic geotechnical properties of coarse grained soils. The changes made to the model are based on a variation of the contact angle, and lead to realistic estimates of the main wetting and wetting and drying scanning curve for different granular soils. The liquid–solid contact angle is estimated at 60° during the wetting process in the granular soils used. In all cases considered in this study, the MK model with hysteresis (MK_h) gave a relatively good representation of the experimental hysteresis of the WRC. Because of its relatively simple formulation, the hysteresis MK_h model is convenient at the first stage of a project to evaluate the influence of hysteresis effects on water movement and distribution into granular soils. However, the model does not replace actual measurements, and it is recommended to use experimental tests (Fredlund and Houston 2009) to evaluate hysteresis effects for the final design of a project. More work is presently underway to extend the MK_h model to fine-grained soils, and to include volume changes in the predicted WRC.

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