Exact black holes and universality in the backreaction of non-linear sigma models with a potential in (A)dS$_4$.

Andrés Anabalón

Departamento de Ciencias, Facultad de Artes Liberales y,
Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez,
Viña del Mar, Chile

E-mail: andres.anabalon@uai.cl

ABSTRACT: The aim of this paper is to construct accelerated, stationary and axisymmetric exact solutions of the Einstein theory with self interacting scalar fields in (A)dS4. To warm up, the backreaction of the (non)-minimally coupled scalar field is solved, the scalar field equations are integrated and all the potentials compatible with the metric ansatz and Einstein gravity are found. With these results at hand the non-linear sigma model is tackled. The scalar field Lagrangian is generic; neither the coupling to the curvature, neither the metric in the scalar manifold nor the potential, are fixed ab initio. The unique assumption in the analysis is the metric ansatz: it has the form of the most general Petrov type D vacuum solution of general relativity; it is a a cohomogeneity two Weyl rescaling of the Carter metric and therefore it has the typical Plebanski-Demianski form with two arbitrary functions of one variable and one arbitrary function of two variables. It is shown, by an straightforward manipulation of the field equations, that the metric is completely integrable without necessity of specifiying anything in the scalar Lagrangian. This results is that the backreaction of the scalar fields, within this class of metrics, is universal. The metric functions generically show an explicit dependence on a dynamical exponent that allows to smoothly connect this new family of solutions with the actual Plebanski-Demianski spacetime. The remaining field equations imply that the scalar fields follow geodesics in the scalar manifold with an affine parameter given by a non-linear function of the space-time coordinates and define the on-shell form of the potential plus a functional equation that it has to satisfy. To further find the exact form of the potential the simplest case associated to a flat scalar manifold is taken. The most general potential compatible with the Einstein theory and the metric ansatz is constructed in this case and it is shown that it has less symmetry than the maximal compact subgroup of the coset construction. Finally, the most general family of (A)dS4 static hairy black holes is explicitly constructed and its properties are outlined.

Contents

1 Introduction

The fact that asymptotically flat black holes are described by a small set of parameters in four dimensions is what makes them a fundamental object of study in general relativity and, it is indeed of interest, to see what happens with this situation when the cosmological constant is included. Actually, to summarize this paper briefly, within the class of metrics described below, the most general backeraction of scalar fields is found and all the possible potentials compatible with that form of the metric are constructed. It turns out that this backreaction is described by a small set of parameters, without any specification of the scalar model and without mentioning the existence of a black hole. Within the asumptions of this work, it is proved that the most general potential for minimally coupled scalar fields compatible with the Einstein theory is the sum of six exponentials. When the scalar field is non-minimally coupled, the most general potential can take the form of the sum of powers of the scalar field. Therefore, an interesting consequence of this paper is that if some other physical requirement would fix the form of the potential, the coupling to the curvature will be automatically fixed.

In particular, it is shown that the most general cohomogeneity one, static, black holes that can be constructed within four dimensional Einstein gravity with uncharged scalar fields have the following form (in the Einstein frame):

$$ds^2 = S(r)(-F(r)dt^2 + \frac{dr^2}{F(r)} + d\Sigma_k), \tag{1.1}$$

$$S(r) = \frac{\nu^2 \eta^{\nu-1} r^{\nu-1}}{(r^\nu - \eta^\nu)^2}, \tag{1.2}$$

$$F(r) = \left(\nu^{-1} \left(1 + \frac{2\eta^\nu r^{-\nu}}{\nu - 2} \right) r^2 - \frac{\eta^2}{\nu - 2} \right) k$$
$$+ \left(\left(\eta^\nu - \frac{r^\nu}{\nu + 2} + \frac{\eta^{2\nu} r^{-\nu}}{\nu - 2} \right) r^2 - \frac{\nu^2 \eta^{\nu+2}}{\nu^2 - 4} \right) 6M - \frac{\Lambda}{3} \tag{1.3}$$

here k is the constant curvature of the surface $d\Sigma_k$, normalized to be $k = \pm 1$, 0. The normalization of r is the simplest that I found to write the solution. It is straightforward to see that the black holes are asymptotically (A)dS around $r = \eta$.

This result holds for any kind of scalar fields and non-linear sigma models with possitive kinetic energy. The reader can now go to the next section or read some of the physical motivation behind this study.

1.1 Motivation

It is well known that the introduction of a cosmological constant allow the black holes to support scalar hair [1–8]. In these cases, the scalar fields can be considered as a dynamical form of the cosmological constant, such that when the asymptotic region is reached the scalar field potential defines the constant value of the curvature. When the cosmological constant is negative and the asymptotic form of the metric is in the conformal equivalence class of $R \times S^2$, it has been argued that all the potentials that follow from a class of superpotentials do not give place to regular hairy black holes [9]. In this work it is proved that is possible to be exhaustive with the potentials within a certain class of metrics. The interest in being exhaustive is rooted in that is also physically relevant to clarify what are the possible hairy configurations when the cosmological constant is positive and when the asymptotic form of the metric is in any conformal equivalence class when Λ is negative.

1.1.1 $\Lambda > 0$

The standard cosmological model includes a scalar field with a potential [10]. It is therefore a fair question to ask what the stationary and axisymmetric configurations are, that the backreaction of this scalar field would generate. This seems to be more than an academic question since the black hole that is located in the center of our galaxy, Sagittarius A*, is suffering a close scrutiny [11–13]. In particular, it has been noted that its angular and quadrupolar momentum, J and Q respectively, can be determined by the orbital precession of stars very near to it, therefore allowing to check the relation $Q = -\frac{J^2}{Mc}$ that follows from the Kerr solution [14, 15]. Due to the uniqueness and ho hair theorems of asymptotically flat, four dimensional, general relativity this would actually test whether

gravity is described by the Hilbert Lagrangian and also whether some other fields play an important role in the description of strong gravitational effects, as has been already studied for the Einstein-dilaton-Gauss-Bonnet case in [16]. If the Einstein equations are guiding the dynamics of the metric in this situation, it could very well be that the inclusion of a cosmological constant can have a non-linear effect and a new hairy rotating black hole can be found to describe SgrA*.

To add generality, besides an arbitrary potential, the survey presented here also let the coupling to the curvature arbitrary. Indeed, it is known that to encompass a Higgs like potential with an inflationary scenario the Higgs field should be non-minimally coupled to the curvature [17].

Static solutions with scalar fields conformally coupled to the curvature have been explored and there is a family of spherically symmetric and accelerated black holes. They have a non-trivial potential, whose generalization give rise to wormholes, regular black holes and bouncing de Sitter cosmologies [18]. Since all these solutions have been found within assumptions of the form of the potential and the coupling to the curvature, this work also intend to explore how generic these solutions are.

Therefore, starting with an arbitrary potential and an arbitrary coupling to the curvature all the assumptions goes in the symmetries that the problem should have. Since this work is looking for deviations from Kerr-de Sitter; a metric that is a cohomogeneity two Weyl rescaling of the Carter metric is taken as the ansatz. Remarkably, the whole system can be integrated without any further assumption. All the scalar field potentials compatible with this metric can be on-shell found. Their off shell construction is explicitly done in the minimally coupled case while the construction is outlined in the remaining cases. In the minimally coupled case it is show that the most general potential is the sum of six exponentials.

With these results at hand the construction of the backreaction of an arbitrary non-linear sigma model is also done. In this case the scalar fields follow geodesics in the scalar manifold and the potential satisfy a functional constraint. In order to solve this constraint, it is necessary to pick a metric in the scalar manifold and to solve the geodesic motion on it. Since this is a simple exercise, when doable, it is done just in the minimally coupled case.

The solutions in general have the Plebański-Demiański form with a non-trivial dynamical exponent that (locally) smoothly connect the geometry with Kerr-de Sitter. This dynamical exponent could be useful to describe the slow fall off that the galactic rotation curves have at therefore at the end of the work the spherically symetric black holes are explicitly written. The same exponent, generically, preclude the interpretation of the metric as a rotating black hole. However, there is a special case, when the scalar is conformally coupled, that it does allow for a physically sensible rotating hairy black hole in (A)dS. This case is briefly discussed here and will be reported with more details in another publication.

1.1.2 $\Lambda < 0$

Dynamical exponents play a central role in the very interesting fact that four dimensional gravity with a negative cosmological constant and an adequate matter content is dual to strongly coupled condensed matter phenomena [19], for a recent review see [20]. The simplest setting where the situation can be described is for an extremal, planar, U(1)

charged, black hole. In this case the boundary is in the conformal equivalence class of R^3 and it represents the UV behavior of the theory. The radial coordinate r sets the energy scale. The system is seen to enjoy a different scale invariance in the UV and the IR. Indeed, at the boundary, located at $r = 0$, the configuration is invariant under the rescaling $x^\mu \to \lambda x^\mu$. On the other hand, since the near horizon geometry of the spacetime is $\mathrm{AdS}_2 \times R^2$, the configuration in the IR turns out to be invariant under radial and time rescalings while the remaining coordinates remain unchanged. This can be seen as the limit when the dynamical exponent goes to infinity of the anisotropic scalings in space and time that arise in condense matter systems due to the presence of a Fermi surface [21]. To make the system undergone a second order phase transition, and to have a finite dynamical exponent, it is necessary to include charged matter in the gravitational Lagrangian. Either in the form of spin 0 or spin 1/2 particles [22, 23]. The class of systems where four dimensional holography is under control, in the sense of their ultraviolet completion is the ABJM class [24, 25] (for a more recent work see for instance [26]) and the Lifshitz solutions of supergravity [27–29].

The solutions constructed here are independent of the scalar model and they do have a dynamical exponent, therefore they should correspond as the uncharged limit of black holes with charged scalars that must exist within the AdS/CM program. It is interesting to note, that the solutions are asymptotically AdS in the Fefferman-Graham sense. Thus, it is very likely that its charged extensions have a non-trivial scaling behavior at the horizon instead of at infinity as is in the Lifshitz case.

1.2 Outline

The outline of the paper is as follows: in the second section, the metric ansatz is discussed and its physical relevance recalled. In the third section the Einstein equations are exactly solved for this ansatz in the presence of a stationary and axisymetric scalar field with an arbitrary coupling to the curvature. The most general on-shell potential, compatible with this ansatz, is found, as well as the most general scalar field that can exist as an exact solution of this system. Two subsections follow to it; one where the minimally coupled case is discussed and the most general potential reconstructed and other when the conformally coupled case is considered and some comments are made about the existence of a rotating solution. The fourth section is devoted to non-linear sigma models in general and the same integrability properties are shown to arise. The problem of finding the exact backreaction of non-linear sigma models is reduced to the finding of geodesics on the scalar manifold and a functional equation on the scalar potential. To give a simple example on how this mechanism work the lineal sigma model is solved. The fifth section briefly outline the large family of static (A)dS black holes that arise as exact solutions of this system. The last section discuss the connection of these solutions with higher dimensional black holes and gauged supergravity.

The notation follows [30]. The conventions of curvature tensors are such that an sphere in an orthonormal frame has positimve Riemann tensor and scalar curvature. The metric signature is taken to be $(-, +, +, +)$. Greek letters are in the coordinate tangent space and capital latin letters in the scalar manifold, $8\pi G = \kappa$ and the units are such that $c = 1 = \hbar$.

2 The metric anzats

It was Carter [31] who, requiring the separability of the Klein-Gordon and Hamilton-Jacobi equations on a background, found the generalization of the Kerr-Newman metric to the case when the cosmological constant is present, furthermore adding a NUT parameter to this solution. This metric was discovered in parallel by Plebański [32] who wrote it in the Wick rotated double Kerr-Schild form. The Carter-Plebański metric, also known as Kerr-Newman-TAUB-NUT-de Sitter metric contains 7 parameters: the mass, the NUT parameter, the electric charge, the magnetic charge, the cosmological constant, the angular momentum and an extra discrete parameter.

The Carter-Plebański metric has a very important generalization namely the Plebański-Demiański spacetime [34]. These spacetimes contain one further parameter related with a conical singularity which produce that the two black holes present in the maximal extension of the spacetime accelerate apart or collapse one against the other, depending whether there is a conical defect or excess [35–37]. The non-rotating version of the Plebański-Demiański metric was discovered, just a few years after the Schwarschild solution, by Levi-Civita in 1918, in the forthcoming years it was rediscovered and analyzed at least three times (see [38] for references). Its was baptized after the name C-metric by Ehlers and Kundt in 1968. Most of its physical life comes from the interpretation of Kinnersely and Walker [38] who showed that it is the general relativistic analogue of the Born solution in electrodynamics. The election of a good parametrization and analytic extension of the spacetime allowed them to show that it can be interpreted as two black holes being accelerated apart. When the cosmological constant vanishes, the issue of radiation was settled by Bičák who proved that the Bondi news are non-trivial for this spacetime, implying that there is a flux of gravitational radiation through null infinity [39].

In five dimensions a Wick rotation of a metric of the Plebański-Demiański type has been used as the base space of a fibration to construct a large family of Ricci flat (and charged) black holes that contains as special limits the Myers-Perry black hole and also the Black-Ring [40]. The generalization of the Plebański-Demiański spacetime and its static limit, the C-metric, to higher dimensions is currently unknown, since the obvious generalization fails (see the discussion in the appendix of [41]).This class of metrics have been found to be solution of the four dimensional Einstein-Maxwell-Conformally coupled scalar field with a quartic potential in [42].

The ansatz that allows to obtain the Plebański-Demiański spacetime in vacuum general relativity is the starting point of this paper:

$$ds^2 = \frac{S(q,p)}{-6+\xi\kappa\phi^2} \left(\frac{1+p^2q^2}{Y(q)}dq^2 + \frac{1+p^2q^2}{X(p)}dp^2 - \frac{Y(q)}{1+p^2q^2}\left(p^2d\tau+d\sigma\right)^2 + \frac{X(p)}{1+p^2q^2}(d\tau-q^2d\sigma)^2 \right),$$
(2.1)

where

$$\phi = \phi(q,p).$$
(2.2)

The choice of the form of the conformal factor in the metric ansatz allows to integrate the problem when the scalar field is non-minimally coupled.

3 The single scalar field case

To understand how the calculation goes in the most complicated case of the non-linear sigma model it is useful to do the simpler exercise of the single scalar field first. It also allows to understand how the non-minimal coupling to the curvature and it is connected with the allowed form of the potentials that follow from the metric (2.1) and the backreaction of Einstein gravity.

The action principle is:

$$S(g, \phi) = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\xi}{12} \phi^2 R - V(\phi) \right], \tag{3.1}$$

with field equations:

$$E_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \kappa T_{\mu\nu}, \tag{3.2}$$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial \phi)^2 - g_{\mu\nu} V(\phi) + \frac{\xi}{6} \left(g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu + R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \phi^2 \tag{3.3}$$

$$\Box \phi = \frac{\xi}{6} R \phi + \frac{\partial V}{\partial \phi}, \tag{3.4}$$

where $V(\phi)$ is arbitrary. When $\xi = 0$ the scalar field is minimally coupled and when $\xi = 1$ it is conformally coupled. The ansatz for the metric is (2.1) and the scalar field is taken to respect the symmetry of the metric:

$$\phi = \phi(q, p) \tag{3.5}$$

Let us consider the tensor (3.2). The equations E_σ^τ and E_τ^σ are linear in $S(q, p)$ and its partial derivatives. It turns out that they can be combined to obtain the form of the $\partial_p \ln S$ and $\partial_q \ln S$ in terms of of X, Y, and its derivatives. Therefore, using the fact that $[\partial_p, \partial_q] \ln S = 0$ it is possible to obtain a relation between X, Y and its derivatives. Since these are functions of independent variables the integration of this relation is straightforward. The most general solution is:

$$X(p) = C_0 + C_2 p^2 + C_4 p^4 + C_1 p^{-\nu+2} + C_3 B_3 p^{\nu+2}, \tag{3.6}$$

$$Y(q) = C_4 - C_2 q^2 + C_0 q^4 + C_3 C_1 q^{-\nu+2} + B_3 q^{\nu+2}. \tag{3.7}$$

Having these metric functions it is possible to integrate $S(q, p)$ to obtain

$$S(q, p) = C \frac{p^{\nu-1} q^{\nu-1}}{(C_3 p^\nu + q^\nu)^2}. \tag{3.8}$$

Next, to solve the scalar field we give a look to $E_\tau^\tau - E_q^q$ and $E_\sigma^\sigma - E_p^p$, and E_p^q from these equations it can be extracted the value of $(\partial_p \phi)^2$ and $(\partial_q \phi)^2$ and $(\partial_q \phi)(\partial_p \phi)$. Using (3.6)–(3.8)

it is possible to check that:

$$E_\tau^\tau - E_q^q = 0 \Leftrightarrow (\partial_q \phi)^2 = \frac{\nu^2 - 1}{12 q^2 \kappa} \frac{\left(\xi \kappa \phi^2 - 6\right)^2}{\left(\kappa \xi (\xi - 1) \phi^2 + 6\right)}, \tag{3.9}$$

$$E_\sigma^\sigma - E_p^p = 0 \Leftrightarrow (\partial_p \phi)^2 = \frac{\nu^2 - 1}{12 p^2 \kappa} \frac{\left(\xi \kappa \phi^2 - 6\right)^2}{\left(\kappa \xi (\xi - 1) \phi^2 + 6\right)}, \tag{3.10}$$

$$E_p^q = 0 \Leftrightarrow \partial_q \phi \partial_p \phi = -\frac{\nu^2 - 1}{12 pq \kappa} \frac{\left(\xi \kappa \phi^2 - 6\right)^2}{\left(\kappa \xi (\xi - 1) \phi^2 + 6\right)}. \tag{3.11}$$

It follows from (3.9)–(3.11) that:

$$q \partial_q \phi + p \partial_p \phi = 0 \Leftrightarrow \phi = F\left(\frac{q}{p}\right) \equiv F(z). \tag{3.12}$$

In this case $E_\tau^\tau - E_q^q$, $E_\sigma^\sigma - E_p^p$ and E_p^q reduce to a single equation for the scalar field:

$$\left(F'\right)^2 = \frac{\left(\nu^2 - 1\right)\left(\xi \kappa F^2 - 6\right)^2}{12 \kappa z^2 \left(\kappa \xi (\xi - 1) F^2 + 6\right)} \tag{3.13}$$

where $F' = \frac{dF}{dz}$. This equation can be exactly integrated and gives a not very illuminating result for arbitrary ξ, of the form:

$$z = z(\phi, \xi, h, \nu) \tag{3.14}$$

where h is an integration constant. At this stage all the diagonal components of E_ν^μ are equal while the non-diagonal are zero. The only remaining equation is the one that determines V which is

$$V(\phi, z) = (12\kappa C)^{-1}[C_0(\nu-1)(\nu-2)z^{\nu+1} + C_4(\nu+1)(\nu+2)z^{\nu-1} - 4C_3(\nu^2-1)(C_0 z + C_4 z^{-1})$$
$$+ C_3^2 C_0(\nu+1)(\nu+2)z^{-\nu+1} + C_3^2 C_4(\nu-1)(\nu-2)z^{-\nu-1}](-6 + \xi \kappa \phi^2)^2 \tag{3.15}$$

Therefore, replacing (3.14) in (3.15) the potential is written uniquely in terms of ϕ. It follows that, by explicit construction, all the potentials and its backreaction on the class of metrics (2.1) have been determined. Note that when $\nu^2 = 1$ the scalar field is constant, the potential too and the metric becomes the accelerated version of Kerr-NUT-AdS found by Plebański-Demiański [34], which reduces to Kerr-(A)dS when $C_3 = 0$ and $C_1 = 0$ ($\nu = 1$). It follows that the class of metrics studied here are (locally) continuos deformations of Kerr-NUT-AdS. This is not true globally since the topology is still free at this point.

The static limit and the form of the potentials for the minimally coupled and conformally coupled cases are described in more detail in the next subsections.

3.1 The static limit

For further reference it is worth to understand the static limit of the metric (2.1). It is achieved in the scaling limit $x^\mu \to \lambda x^\mu$ and $\lambda \to 0$. Note that the conformal factor transforms homogenously. Indeed, from (3.8) it follows that $S(\lambda q, \lambda p) = \lambda^{-2} S(q, p)$. Since the objective is to retain the largest possible number of integration constants in the limit

$\lambda \to 0$, it follows that the constant in the metric functions (3.6) and (3.7) should transform as $C_2 \to \lambda^{-2}C_2$, $C_1 \to \lambda^{\nu-2}C_1$ and $B_3 \to \lambda^{-2-\nu}B_3$. With these rescalings the $\lambda \to 0$ limit takes the following form:

$$ds^2 = \frac{S(q,p)}{-6 + \xi\kappa\phi^2}\left(\frac{dq^2}{Y(q)} + \frac{dp^2}{X(p)} - Y(q)d\tau^2 + X(p)d\sigma^2\right), \tag{3.16}$$

$$X(p) = C_0 + C_2 p^2 + C_1 p^{-\nu+2} + C_3 B_3 p^{\nu+2}, \tag{3.17}$$

$$Y(q) = C_4 - C_2 q^2 + C_3 C_1 q^{-\nu+2} + B_3 q^{\nu+2}. \tag{3.18}$$

While the scalar field and the potential remains unchanged.

3.2 Minimally coupled case ($\xi = 0$)

In this case the differential equation for the scalar field (3.13) gives:

$$\phi = \pm\sqrt{\frac{\nu^2 - 1}{2\kappa}}\ln(h\frac{q}{p}) \tag{3.19}$$

where h is a would be hairy integration constant. The potential can be written in terms of the coefficients $\alpha_+ = \sqrt{2\kappa}\frac{\nu+1}{\sqrt{\nu^2-1}} = \sqrt{2\kappa}\sqrt{\frac{\nu+1}{\nu-1}}$ and $\alpha_- = \sqrt{2\kappa}\frac{\nu+1}{\sqrt{\nu^2-1}} = \sqrt{2\kappa}\sqrt{\frac{\nu-1}{\nu+1}}$ as follows:

$$V(\phi) = \frac{3}{\kappa C}\left[\frac{C_0(\nu-1)(\nu-2)}{h^{\nu+1}}e^{\alpha_+\phi} + \frac{C_4(\nu+1)(\nu+2)}{h^{\nu-1}}e^{\alpha_-\phi} - 4C_3(\nu^2-1)\frac{C_0}{h}e^{\frac{\alpha_+-\alpha_-}{2}\phi}\right.$$
$$\left. + \frac{C_3^2 C_0(\nu+1)(\nu+2)}{h^{-\nu+1}}e^{-\alpha_-\phi} + \frac{C_3^2 C_4(\nu-1)(\nu-2)}{h^{-\nu-1}}e^{-\alpha_+\phi} - 4C_3(\nu^2-1)\frac{C_4}{h^{-1}}e^{\frac{\alpha_--\alpha_+}{2}\phi}\right]. \tag{3.20}$$

Where the positive branch of (3.19) was taken. The negative branch can be obtained changing $\phi \to -\phi$ everywhere.

3.3 Conformally coupled case and a rotating solution ($\xi = 1$)

In this case the scalar field is:

$$\phi = \pm\sqrt{\frac{6}{\kappa}\frac{q^\mu - p^\mu + h(q^\mu + p^\mu)}{q^\mu + p^\mu + h(q^\mu - p^\mu)}}, \qquad \mu^2 = \frac{\nu^2 - 1}{3}. \tag{3.21}$$

and the potential is a rational function of ϕ obtained replacing

$$z = \left[\frac{h-1}{h+1}\frac{\phi\sqrt{\kappa}+6}{\phi\sqrt{\kappa}-6}\right]^{\frac{1}{\mu}} \tag{3.22}$$

in (3.15). As before, the negative branch correspond to the transformation $\phi \to -\phi$ in the potential. It can be seen that when $\nu^2 = 4$ then $\mu^2 = 1$. In this case ($\nu = 2$, $\mu = 1$):

$$\phi = \pm\sqrt{\frac{6}{\kappa}\frac{q - p + h(q + p)}{q + p + h(q - p)}}, \tag{3.23}$$

and the denominator of the conformal factor of the metric is:

$$\frac{S(q,p)}{-6 + \xi\kappa\phi^2} = \frac{C(p + q - hp + hq)^2}{24(h^2 - 1)(C_3 p^2 + q^2)^2} \tag{3.24}$$

Which is regular at $p = 0$. Allowing its interpretation as a rotating black hole [43].

4 An arbitrary sigma model

The procedure to solve the field equations in section one have explicitly shown that, whenever $T_\sigma^\tau = 0 = T_\tau^\sigma$, the integration of the metric is independent of the form of the energy momentum tensor. For, locally, stationary and axisymmetric scalars it is always the case that $T_\sigma^\tau = 0 = T_\tau^\sigma$, independently of the number of scalar fields and whether is a linear or a non-linear sigma model. This is not the case for locally stationary and axisymmetric spin one fields since its energy momentum tensor is, in general, such that $T_\sigma^\tau \neq 0 \neq T_\tau^\sigma$. The classical work where non-linear sigma models where studied, without a potential, in four dimensions is [44].

It is therefore interesting to extend the complete integrability of the system to appreciate how a pattern appears. Therefore, the theory under consideration is:

$$S(g, \phi^C) = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} - \frac{1}{2} G_{AB}(\phi) \partial_\mu \phi^A \partial_\nu \phi^B g^{\mu\nu} - V(\phi) \right], \tag{4.1}$$

where the number of scalars is arbitrary, $G_{AB}(\phi)$ depends in all the ϕ^C and $V(\phi)$ should, in principle, inherits the symmetries of G_{AB} but is left arbitrary at this point.

The field equations are:

$$E_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \kappa T_{\mu\nu}, \tag{4.2}$$

$$T_{\mu\nu} = G_{AB}(\phi) \partial_\mu \phi^A \partial_\nu \phi^B - \frac{1}{2} g_{\mu\nu} G_{AB}(\phi) \partial_\alpha \phi^A \partial_\beta \phi^B g^{\alpha\beta} - g_{\mu\nu} V(\phi) \tag{4.3}$$

$$G_{CB} \Box \phi^B = -\frac{1}{2} (\partial_A G_{CB} + \partial_B G_{CA} - \partial_C G_{AB}) \partial_\alpha \phi^A \partial_\beta \phi^B g^{\alpha\beta} + \partial_C V, \tag{4.4}$$

Since the scalar fields are minimally coupled, the metric ansatz is slightly different:

$$ds^2 = S(q, p) \left(\frac{1+p^2q^2}{Y(q)} dq^2 + \frac{1+p^2q^2}{X(p)} dp^2 - \frac{Y(q)}{1+p^2q^2} \left(p^2 d\tau + d\sigma \right)^2 + \frac{X(p)}{1+p^2q^2} \left(d\tau - q^2 d\sigma \right)^2 \right) \tag{4.5}$$

The scalar fields respect the symmetry of the metric:

$$\phi^A = \phi^A(q, p) \tag{4.6}$$

As in the previous section it is possible to completely integrate the metric functions, therefore obtaining (3.6)–(3.8). Using this information, the following equalities can be extracted from the field equations:

$$E_\tau^\tau - E_q^q = 0 \Leftrightarrow G_{AB} \partial_q \phi^A \partial_q \phi^B = \frac{\nu^2 - 1}{2q^2\kappa}, \tag{4.7}$$

$$E_\sigma^\sigma - E_p^p = 0 \Leftrightarrow G_{AB} \partial_p \phi^A \partial_p \phi^B = \frac{\nu^2 - 1}{2p^2\kappa}, \tag{4.8}$$

$$E_p^q = 0 \Leftrightarrow G_{AB} \partial_p \phi^A \partial_q \phi^B = -\frac{\nu^2 - 1}{2pq\kappa}. \tag{4.9}$$

Using (4.7)–(4.9) it follows that:

$$G_{AB} \left(q \partial_q \phi^A + p \partial_p \phi^A \right) \left(q \partial_q \phi^B + p \partial_p \phi^B \right) = 0. \tag{4.10}$$

If G_{AB} is Euclidean and invertible (4.10) has a unique solution:

$$q\partial_q\phi^A + p\partial_p\phi^A = 0 \Leftrightarrow \phi^A = F^A\left(\frac{q}{p}\right) \equiv F^A(z), \tag{4.11}$$

in which case (4.7)–(4.9) become the same equation:

$$G_{AB}\partial_z F^A \partial_z F^B = \frac{\nu^2 - 1}{2z^2\kappa}. \tag{4.12}$$

Using this equation the on-shell form of the potential can be found:

$$V(z) = -(2\kappa C)^{-1}[C_0(\nu-1)(\nu-2)z^{\nu+1} + C_4(\nu+1)(\nu+2)z^{\nu-1} - 4C_3(\nu^2-1)(C_0 z + C_4 z^{-1})$$
$$+ C_3^2 C_0(\nu+1)(\nu+2)z^{-\nu+1} + C_3^2 C_4(\nu-1)(\nu-2)z^{-\nu-1}] \tag{4.13}$$

and coincide with the expression (3.15) when $\xi = 0$ and rescaling $C \longrightarrow -6C$. Remarkably, the on-shell form of the potential is universal, namely, independent of the sigma model.

At this point, all the Einstein equations has been completely integrated, reducing them to the on-shell potential (4.13) and to the constraint (4.12).

The equation (4.12) implies that the scalar fields follows geodesics in the scalar manifold with affine parameter $\omega = \ln(z)$. Indeed, defining $F^A(z) \equiv H^A(\omega)$ and deriving with respect to ω the expression (4.12) it becomes:

$$G_{CB}\left(\partial_\omega^2 H^B\right) + \gamma_{CAB}\partial_\omega H^A \partial_\omega H^B = 0. \tag{4.14}$$

where $\gamma_{CAB} = \frac{1}{2}(\partial_A G_{CB} + \partial_B G_{CA} - \partial_C G_{AB})$. Replacing (4.14) in (4.4) a constraint on the form of the potential is obtained:

$$G_{CB}\left(\frac{2z^2\kappa}{(\nu^2-1)}\frac{dV}{dz}\partial_z F^B\right) - \partial_C V = 0. \tag{4.15}$$

To further extract information from the system is necessary to pick a metric in the scalar manifold. To gain some insight the linear sigma model case is worked out in the next section.

4.1 $G_{AB} = \delta_{AB}$

The geodesic equation (4.14) is in this case:

$$\partial_\omega^2 H^A = 0 \Leftrightarrow F^A = c^A \ln(z) + \left(\delta_{CB}c^C c^B\right) h^A \tag{4.16}$$

Where the form of the integration constants h^A has been chosen for further transparency. The integration constants, c^A satisfy the constraint (4.12):

$$\delta_{AB}c^A c^B = \frac{\nu^2 - 1}{2\kappa} \tag{4.17}$$

The equation for the potential (4.15) is:

$$\left(\frac{2\kappa}{(\nu^2-1)}c^B c_A - \delta_A^B\right)\partial_B V = 0. \tag{4.18}$$

Using (4.17) it follows that if the scalar manifold has dimension D then the matrix $\left(\frac{2\kappa}{(\nu^2-1)}c^B c_A - \delta_A^B\right)$ has one null eigenvalue and $D-1$ non null eigenvalues. The potential is thus constrained to be of the form

$$V = V(c_A \phi^A) = V(c_A F^A). \tag{4.19}$$

It can be checked that (4.19) solves all the constraints (4.18). The explicit form of the potential can now be obtained from

$$c_A \phi^A = c_A c^A \ln(z) + \left(c_B c^B\right) c_A h^A \Leftrightarrow z = \exp\left(\frac{2\kappa}{\nu^2-1}c_A \phi^A - c_A h^A\right) \tag{4.20}$$

and therefore the off-shell potential has the same structure than (3.20). Note that the combination $c_A \phi^A$ is invariant under the transformations that let the vector c_A invariant, namely $\text{SO}(D-1)$.

5 The cohomogeneity one black holes

To gain some insight in the physical meaning of the solutions just constructed it is convenient to consider its cohomogeneity one limit. To obtain it from the metric (4.5) with metric functions (3.6)–(3.8) a non symmetric scaling limit should be taken. It is then possible to obtain the following family of black holes:

$$ds^2 = S(r)(-F(r)dt^2 + \frac{dr^2}{F(r)} + d\Sigma_k), \tag{5.1}$$

$$S(r) = \frac{\nu^2 \eta^{\nu-1} r^{\nu-1}}{(r^\nu - \eta^\nu)^2}, \tag{5.2}$$

$$F(r) = \left(\nu^{-1}\left(1 + \frac{2\eta^\nu r^{-\nu}}{\nu-2}\right)r^2 - \frac{\eta^2}{\nu-2}\right)k$$
$$+ \left(\left(\eta^\nu - \frac{r^\nu}{\nu+2} + \frac{\eta^{2\nu} r^{-\nu}}{\nu-2}\right)r^2 - \frac{\nu^2 \eta^{\nu+2}}{\nu^2-4}\right)6M - \frac{\Lambda}{3} \tag{5.3}$$

here k is the constant curvature of the surface $d\Sigma_k$, normalized to be $k = \pm 1, 0$. The limit $k = 0$ can be taken after solving either N or M from the equation $M(\nu+2) - N(\nu-2) = k\lambda$. The normalization of r is the simplest that I found to write the solution. It is straightforward to see that the black holes are asymptotically (A)dS around $r = 1$.

The on-shell potential has the same structure than in the cohomogenity two case:

$$V_\nu(\phi(r)) = \frac{\alpha}{2}\left(\frac{\nu-1}{\nu+2}\eta^{-\nu-1}r^{1+\nu} + \frac{\nu+1}{\nu-2}\eta^{\nu-1}r^{1-\nu}\right) - \frac{\alpha}{2}\left(\frac{\nu+1}{\nu-2}\eta^{1-\nu}r^{\nu-1} + \frac{\nu-1}{\nu+2}\eta^{1+\nu}r^{-1-\nu}\right)$$
$$+ 2\alpha\frac{\nu^2-1}{\nu^2-4}\left(\eta^{-1}r - \eta r^{-1}\right) + \frac{\Lambda(\nu^2-4)}{6\kappa\nu^2}\left(\frac{\nu+1}{\nu-2}(r\eta^{-1})^{(\nu-1)} + \frac{\nu-1}{\nu+2}(r\eta^{-1})^{-(1+\nu)}\right)$$
$$+ \frac{2\Lambda}{3\kappa\nu^2}(\nu^2-1)(r\eta^{-1})^{-1}. \tag{5.4}$$

The scalar fields satisfy:

$$G_{AB}\partial_r F^A \partial_r F^B = \frac{\nu^2-1}{2r^2\kappa} \tag{5.5}$$

plus the field equation:

$$G_{CB}\Box F^B = -\frac{1}{2}(\partial_A G_{CB} + \partial_B G_{CA} - \partial_C G_{AB})\partial_r F^A \partial_r F^B g^{rr} + \partial_C V, \qquad (5.6)$$

which therefore constraint the potential to have a given form as in the cohomogeneity two case.

When there is only one minimally coupled scalar field it has the following form:

$$\phi = \pm \left(\frac{\nu^2 - 1}{2\kappa}\right)^{1/2} \ln(r) + h$$

where h is a would be hairy integration constant; it follows that the off shell form of the potential is the sum of six exponentials. The metric (5.1)–(**??**) is assymptotically (A)dS at $r = 1$. Therefore, the scalar field is everywhere regular as well as the back hole (5.1)–(**??**) outside the horizons.

These black holes are very interesting in many regards:

- They represent the most general black holes that can be constructed in the presence of stationary and axisymmetric uncharged scalars. These gravitational fields can be used to describe the departure from the r^{-1} Newtonian behavior.

- For each solution there are two different black holes. This follows from the fact that there is a black hole when $r \in [0, 1]$ and other when $r \in [1, \infty]$. When $\Lambda < 0$ this observation implies the existence of phase transitions.

- When $\Lambda < 0$ it can be seen that there are black holes with two horizons and therefore extremal black holes for some values of the parameters.

- In this section only the Einstein frame was discussed. Any other frame correspond to a simple field redefinition of it.

  These issues will be discussed in a forthcoming publication.

6 What's next

Although the construction of this paper is interesting due to its generality it is somehow incomplete due to the lack of inclussion of gauge fields in it and I would like to describe its inclussion and its connection with gauged supergravity as well as with higher dimensional black holes.

It is very likely that the fact that the problem posed by the integration of the backreaction of the general class of scalar fields considered here can be solved due to the hidden symmetries that the metric (4.5) has. This class of metrics are Petrov type D and support a rank two, irreducible conformal Killing tensor. The requirement that the metric support a non-trivial rank two conformal Killing tensor seems to not be very restrictive. On one hand, all the Petrov type D metrics in vacuum were constructed by Kinnersley [45]. On the other hand, the most general metric admitting an irreducible rank two Killing tensor

and $D-2$ Killing vectors was kinematically constructed by Benenti and Francaviglia [46]. The restriction of this family of metrics to be of Petrov type D, plus a further restriction of some functions, indeed coincide with the form of the Carter-Plebański metric in four dimensions. Therefore a cohomegeneity two Weyl rescaling of it has the Plebański-Demiański form, cf. (4.5). When the Ricci flat condition is imposed on it gives rise to all the metrics classified by Kinnersley as special limits [47]. It follows that all the Petrov type D, Ricci flat, stationary and axisymmetic spacetimes admit a (possibly conformal) Killing tensor.

These are, however, not the most general Petrov type D metrics that can be constructed in four dimensions in presence of matter. It also exist other solutions when U(1) vector fields are considered [48]. These solutions, however, do not reduce to the class studied here in the limit of vanishing vectors and therefore it should exist a family of solutions that encompass them all. It is worth mentioning that very recently, an extremely interesting approach has been introduced in the literature of exact solutions [49]. It allows to construct general classes of exact solutions to the Einstein equations with scalars and vector fields in (A)dS, in such a way that the Hamilton-Jacobi equation is separable for a charged particle. It can be that the canonical form of the metric thus constructed can be generalized to admit solutions of the Plebański-Demiański form by Weyl rescaling it.

Besides the discussions already mentioned in the body of this work, the study of metrics admitting a conformal Killing tensor can be of utmost interest for the construction of higher dimensional black holes with non-trivial topologies. Remarkably enough, the rather non-trivial metric of the double spinning black ring is such that the four dimensional base of its Kaluza-Klein fibration has a conformal Killing tensor. This base space is not of Petrov type D as follows from the fact that it does not admit a conformal Killing-Yano tensor unless the single spinning limit is taken [50]. Indeed, starting from an analytic continuation of the Ricci flat solution of [40], is possible to obtain the following four dimensional solution

$$ds^2 = \frac{a_0 qp}{(1-\alpha q^2 p^2)^2} \left[\frac{q^2+p^2}{Y(q)} dq^2 + \frac{q^2+p^2}{X(p)} dp^2 - \frac{Y(q)\left(d\tau - p^2 d\sigma\right)^2}{q^2+p^2} + \frac{X(p)\left(d\tau + q^2 d\sigma\right)^2}{q^2+p^2} \right],$$

(6.1)

$$X(p) = -a_0^2 \alpha^2 p^6 + a_4 \alpha p^4 + a_2 p^2 + a_3 - a_0^2 p^{-2}, \quad Y(q) = a_0^2 \alpha^2 q^6 - a_3 \alpha q^4 - a_2 q^2 - a_4 + a_0^2 q^{-2},$$

(6.2)

$$A = \frac{d\tau}{q^2 p^2} + \frac{q^2-p^2}{q^2 p^2} d\sigma, \qquad \Phi = \ln(a_0 qp).$$

(6.3)

Where the normalization of the fields is given by the action principle

$$S(g, A, \Phi) = \int d^4 x \sqrt{-g} \left[R - \frac{1}{4} e^{3\Phi} F^2 - \frac{3}{2} (\partial \Phi)^2 \right],$$

(6.4)

with field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{3}{2} \left(\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} (\partial \Phi)^2 \right) + \frac{e^{3\Phi}}{2} \left(F_{\mu\alpha} F_\nu^{\cdot \alpha} - g_{\mu\nu} F^2 \right),$$

(6.5)

$$\partial_\mu \left(\sqrt{-g} e^{3\Phi} F^{\mu\nu} \right) = 0, \qquad \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi \right) - \frac{1}{4} e^{3\Phi} F^2 = 0.$$

(6.6)

the non-standard normalization of the scalar field in (6.4) is to avoid the appearance of $\sqrt{3}$ factors in the solution of the field equations. When the solution (6.1)–(6.3) is oxidized back to five dimensions the metric contains, in different limits, the Myers-Perry black hole, the Emparan-Reall black ring, the static and stationary black lens, the Kaluza-Klein monopole and the Schwarzschild-Tangherlini black hole.

When the configurations are compactified, they can be extended to be a solution of the four dimensional $U(1)^4$ gauged supergravity, which is a truncation of the $\mathcal{N} = 8$, SO(8) gauged supergravity; it is $\mathcal{N} = 2$ supergravity coupled to 3 abelian vector multiplets. When the axions are set to zero and only one of the U(1) gauge fields is retained a further consistent truncation can be obtained

$$S(g, A, \Phi) = \int d^4x \sqrt{-g} \left[R - \frac{1}{4} e^{3\Phi} F^2 - \frac{3}{2} (\partial \Phi)^2 + \frac{6}{l^2} \cosh(\Phi) \right], \tag{6.7}$$

with field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{3}{2} \left(\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \left((\partial \Phi)^2 - \frac{4}{l^2} \cosh(\Phi) \right) \right) + \frac{e^{3\Phi}}{2} \left(F_{\mu\alpha} F_\nu^{\cdot\alpha} - g_{\mu\nu} F^2 \right), \tag{6.8}$$

$$\partial_\mu \left(\sqrt{-g} e^{3\Phi} F^{\mu\nu} \right) = 0, \qquad \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi \right) + \frac{2}{l^2} \sinh(\Phi) - \frac{1}{4} e^{3\Phi} F^2 = 0. \tag{6.9}$$

It is clear that the theory (6.7) coincides with the Einstein-Maxwell-dilaton theory (6.4) when the cosmological constant vanishes. A trick in the construction of solutions in gauged supergravity arise from the observation that for metrics with the Carter form the field equations for the U(1) gauge fields are actually independent of the metric structure functions (X, Y), therefore the ansatz to extend a solution from the ungauged to the gauged case is to retain the matter fields and integrate the metric structure functions with the expectation that a solution in the gauged case can be obtained. This technique has been proven to be very successful in a large number of cases [48].

Indeed, the theory (6.7) admit a solution of the form:

$$ds^2 = \frac{a_0 qp}{\left(1 - a_0^2 q^2 p^2\right)^2} \left[\frac{q^2 + p^2}{Y(q)} dq^2 + \frac{q^2 + p^2}{X(p)} dp^2 - \frac{Y(q) \left(d\tau - p^2 d\sigma\right)^2}{q^2 + p^2} + \frac{X(p) \left(d\tau + q^2 d\sigma\right)^2}{q^2 + p^2} \right], \tag{6.10}$$

$$X(p) = -a_0^6 p^6 + a_4 a_0^2 p^4 + a_2 p^2 + a_3 - a_0^2 p^{-2},$$
$$Y(q) = a_0^6 q^6 - a_3 a_0^2 q^4 - a_2 q^2 - a_4 + a_0^2 q^{-2} + \frac{1}{4l^2 a_0^2} \left(1 + a_0^2 q^4\right), \tag{6.11}$$

$$A = \frac{d\tau}{q^2 p^2} + \frac{q^2 - p^2}{q^2 p^2} d\sigma, \qquad \Phi = \ln(a_0 qp). \tag{6.12}$$

Which coincides with the configuration (6.1)–(6.3) when $\frac{1}{l^2} = 0$ and $\alpha = a_0^2$. This provides an embedding of all the five dimensional black Ricci flat black holes in gauged supergravity and coincide with the class of metrics studied in this paper when the gauge field vanishes and a the dynamical exponent, ν is set to $\nu = \pm 2$. In a forthcoming work the most general solution with gauge fields and arbitrary ν will be reported.

Acknowledgments

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