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Dynamic stability of a human standing on a balance board

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ABSTRACT

The neuromuscular system used to stabilize upright posture in humans is a nonlinear dynamical system with time delays. The analysis of this system is important for improving balance and for early diagnosis of neuromuscular disease. In this work, we study the dynamic coupling between the neuromuscular system and a balance board—an unstable platform often used to improve balance in young athletes, and older or neurologically impaired patients. Using a simple inverted pendulum model of human posture on a balance board, we describe a surprisingly broad range of divergent and oscillatory CoP/CoM responses associated with instabilities of the upright equilibrium. The analysis predicts that a variety of sudden changes in the stability of upright postural equilibrium occurs with slow continuous deterioration in balance board stiffness, neuromuscular gain, and time delay associated with the changes in proprioceptive/vestibular/visual-neuromuscular feedback. The analysis also provides deeper insight into changes in the control of posture that enable stable upright posture on otherwise unstable platforms.

1. Introduction

Research into the instability mechanisms of upright standing posture is of great relevance for the improved rehabilitation and fall-prevention among the elderly (Kannus et al., 2005), athletes (Emery et al., 2005), persons suffering from neuromuscular diseases such as Parkinson's (Ashburn et al., 2001; Blaszczyk et al., 2007; Stolze et al., 2004) and multiple sclerosis (Corradini et al., 1997), and people impaired due to stroke (Kannus et al., 2005) or cancer treatment (Winters-Stone et al., 2011). In daily life, individuals maintain upright posture in dynamically evolving environments where the balance system must interact with an external dynamical system. Therefore, it is important to examine the stability of upright posture under various environmental conditions, especially those that are likely to cause instability. A balance board—an inherently unstable platform that pivots about a fulcrum with its center of mass located above the pivot—provides a simple environmental manipulation which results in instability. This manipulation is especially important because the balance board has been used to prevent injury in young athletes (Aaltonen et al., 2007; Emery and Meeuwisse, 2010) and to improve stability in balance-compromised populations (e.g. de Bruin et al., 2009; Godard et al., 2004; Hinman, 2002; Nordt et al., 1999). Although balance boards are present in many clinics, the mechanisms behind balance improvements are not yet clear (Zech et al., 2010). For example, improved balance after training on an unstable surface can stem from a variety of factors including an improved ability to rapidly process and act on sensory information, adopting more appropriate levels of muscle stiffness, or strength increases which allow more joint torque to be produced so that body perturbations can be more effectively attenuated.

From a dynamical systems point of view a human attempting to balance upright on an unstable balance board represents the coupling of two dynamical systems, the human balance system with neuromuscular feedback supported on the balance board (an inverted pendulum). The coupling of these two dynamical systems, with time delay and nonlinearities, creates an ideal setting for the emergence of complex postural behavior and unanticipated interactions between the individual, task, and the external dynamical system. Thus, applying a dynamical systems perspective should provide important insights into the study of stability on a balance board. Recently such an approach has been applied to the study of upright sitting posture on an unstable surface (Cholewicki et al., 2000; Reeves et al., 2006; Tanaka et al., 2010). However, to the best of our knowledge mathematical models and their nonlinear dynamic analysis of *standing* postural balance on balance boards are not available.

In order to understand the dynamic stability of a human standing upright on an unstable balance board we present a simple mathematical model that couples the standard inverted pendulum posture (Asai et al., 2009; Barauskas and Krusinskiene, 2007; Corradini et al., 1997; Fukuoka et al., 2001; Hur et al., 2010; Iqbal and Roy, 2004; Ishida et al., 1997; Johansson et al., 1988;

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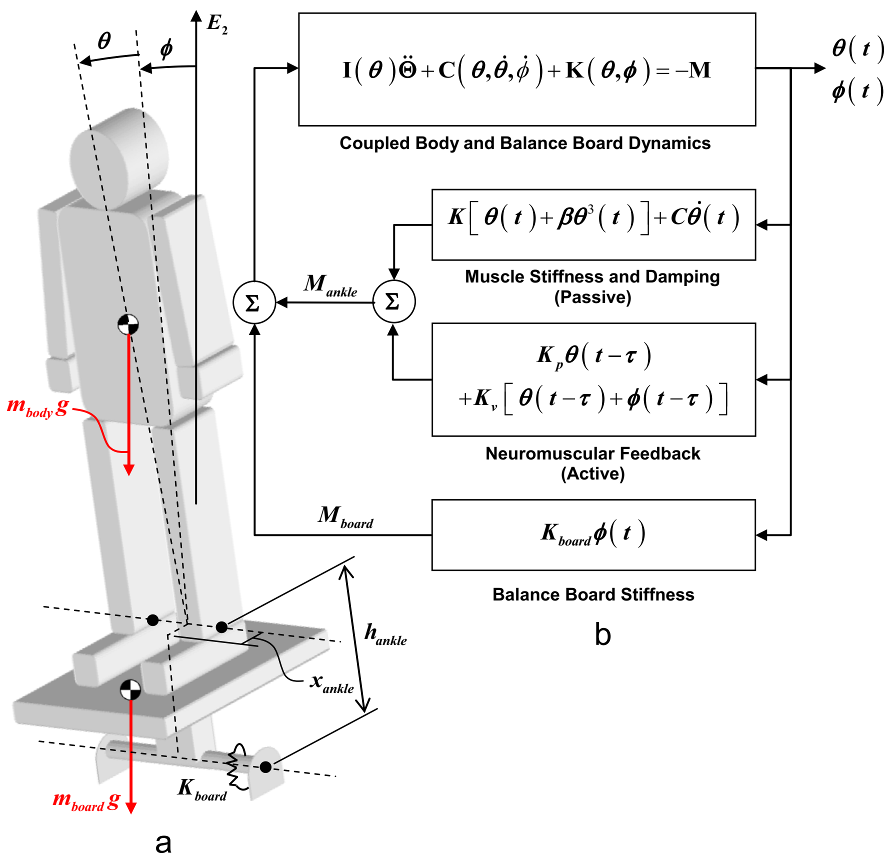

Fig. 1. (a) Diagram of posture on a 1-DOF balance board with forces and correcting moments along with system geometric parameters, and (b) a block diagram of the postural control system on a 1-DOF balance board.

Loram et al., 2005; Masani et al., 2003, 2006; Maurer and Peterka, 2005; Milton et al., 2009; Peterka, 2000, 2002, 2003; Ting et al., 2009; Verdaasdonk et al., 2004; Vette et al., 2010; Yao et al., 2001) to a one degree of freedom (1-DOF) balance board with torsional stiffness. We analyze the bifurcations and stability inherent in this simple coupled system with nonlinear muscle stiffness, large sway nonlinearities, and a time delay in neuromuscular feedback. Bifurcations have been studied within mathematical models of posture (Asai et al., 2009; Verdaasdonk et al., 2004; Yao et al., 2001), but they have not been studied for humans coupled to balance boards. We identify a variety of bifurcations (i.e. Hopf, pitchfork, and saddle-node) which suggests a possible control strategy that is used while maintaining posture on a balance board, as well as how that strategy may be adjusted as the neuromuscular system degrades.

2. Methods

2.1. Model of human posture on a 1-DOF balance board

Prior mathematical models of human posture on a rigid surface include proposed passive proportional-integral-derivative (PID) controllers (Asai et al., 2009; Barauskas and Krusinskiene, 2007; Johansson et al., 1988), active PID controllers with time delay (Masani et al., 2003, 2006; Peterka, 2000, 2003), combinations of passive and active PID controllers (Asai et al., 2009; Maurer and Peterka, 2005; Peterka, 2002, 2003; Vette et al., 2010), and complex controllers such as hysteresis (or bang–bang) controllers (Asai et al., 2009). Furthermore it has long been recognized that upright body sway may involve more than 1-DOF owing to the contribution of other joints (Gunther et al., 2009; Kuo and Zajac, 1993; Pinter et al., 2008; Sasagawa et al., 2009). Nonetheless, the 1-DOF model with PID control has been shown to reliably model the movement of the CoM (Maurer and Peterka, 2005). In what follows we present a mathematical model that couples this 1-DOF balance model with the dynamics of a 1-DOF balance board. While more advanced mathematical models for postural control may need to be developed to fully understand this system, our goal is to study the simplest coupled system model in terms of the inherent nonlinear dynamics as a proof-of-concept of the emergent phenomena inherent in such a coupled system.

For this purpose we model human posture on a balance board by coupling the 1-DOF inverted pendulum model to a 1-DOF inverted pendulum balance board controlled by ankle torque M_{ankle} and a torque between the balance board and the ground M_{board} (Fig. 1a), where the system dynamics are described by the coupled equations

$$\begin{bmatrix} I_{11}(\theta) & I_{12}(\theta) \\ I_{21}(\theta) & I_{22}(\theta) \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{Bmatrix} + \begin{Bmatrix} C_{11}(\theta,\dot{\theta},\dot{\phi}) \\ C_{21}(\theta,\dot{\theta},\dot{\phi}) \end{Bmatrix} + \begin{Bmatrix} K_{11}(\theta,\phi) \\ K_{21}(\theta,\phi) \end{Bmatrix} = -\begin{Bmatrix} M_{ankle} \\ M_{board} \end{Bmatrix}. \tag{1}$$

Details of terms in Eq. (1) can be seen in Appendix. The human body has a mass m_{body} whose center of mass (CoM) is assumed to be a constant distance of h_{body} from the ankle joint and the sway angle θ is measured relative to the balance board in the anterior-posterior (AP) direction (Fig. 1a). The sway angle of the balance board is ϕ in the AP direction. The ankle joint is assumed to be shifted a distance of x_{ankle} from the balance board's axis-of-rotation. The balance board and foot have a lumped mass m_{board} that is assumed to be centered at a constant distance of h_{board} from the balance board axis-of-rotation. The balance board also has a torsional spring connected at its hinge which applies a torque to the board as follows

$$M_{board}(t) = K_{board}\phi(t), \tag{2}$$

where K_{board} represents the torsional spring constant.

We model the corrective ankle torque applied by the neuromuscular system as the sum of a passive and an active torque. The passive torque arises from the stiffness and damping due to muscle stretching while the active torque is applied in response to the motion sensed by the neuromuscular system (Peterka, 2002). Because passive torque only acts as the ankle angle changes, the corrective passive torque can be modeled as the nonlinear controller,

$$M_{ankle,passive}(t) = K[\theta(t)+\beta\theta^3(t)]+C\dot{\theta}(t), \tag{3}$$

where K represents the linear muscle stiffness and β represents the extent of nonlinearity in force-extension/compression response of the muscle groups involved in postural control. Specifically, β is the ratio of passive cubic-nonlinear muscle stiffness to passive linear muscle stiffness, and C represents the linear muscle damping (Barauskas and Krusinskiene, 2007; Fukuoka et al., 2001; Maurer and Peterka, 2005; Peterka, 2002, 2003; Vette et al., 2010).

Table 1
Comparison of commonly used postural parameters from similar models.

Author	K^{cr} [N m]	k $\left[\frac{\text{N m}}{\text{rad}}\right]$	C $\left[\frac{\text{N m s}}{\text{rad}}\right]$
Asai et al.(2009)	**588.60**	**470.88**	**4.00**
Maurer and Peterka(2005)	**648.64**	**584.43**	**171.89**
Peterka (2002)	**732.19**	**91.67**	**24.64**
Vette et al. (2010)	**713.81**	**521.00**	**5.00**
This Paper	**659.92**	**593.93**	**131.98**

Active torque on a rotational board needs to be modeled differently than passive torque. The proprioceptive sensory system provides information of muscle length and joint angle, acting as a corrective term for changes to ankle angle *relative* to the board. On the other hand, the vestibular and visual systems provide information of body angle in the absolute reference frame, acting as a corrective term for changes to body angle in the *absolute* reference frame. Thus, active control on a balance board is modeled as the nonlinear time delayed controller

$$M_{ankle,active}(t) = K_p\theta(t-\tau) + K_v[\theta(t-\tau)+\phi(t-\tau)], \quad (4)$$

where K_p is the proprioceptive feedback gain, K_v is the combined vestibular/visual feedback gain, and $\tau \in (0,\infty)$ s is the time delay of neuromuscular feedback. A block diagram of the postural control system can be seen in Fig. 1b.

2.2. Model parameters

Before studying the coupled system model Eqs. (1–4), it is necessary to identify the appropriate values of the model parameters. All postural control gains are referred to as ratios of the critical muscle stiffness (K^{cr}) required to stabilize the upright equilibrium on a rigid surface with no additional feedback ($K^{cr} = m_{body}gh_{body}$). Board stiffness gain is referred to as a ratio of the effective critical stiffness (K^{cr}_{eff}) required to stabilize the upright equilibrium of the human inverted pendulum fused to the 1-DOF balance board with no additional feedback ($K^{cr}_{eff} = m_{board}gh_{board} + m_{body}g(h_{body}+h_{ankle})$).

In studying the stability of upright posture we observe the variable space of equilibrium positions (θ^* and ϕ^* which are introduced in Section 2.3) in the parameter space of the active control parameters (K_p, K_v, and τ), board stiffness (K_{board}), and ankle position (x_{ankle}) while the passive control parameters (K, β, and C) are held constant. K is assumed to be 90% of the critical muscle stiffness (Loram and Lakie, 2002). C is assumed to be 20% of the critical muscle stiffness resembling an over-damped system. This value is chosen by matching frequency responses of experimental data used in Chagdes et al. (2009) to theoretical data simulated by Eq. (1) of posture on a balance board with very high stiffness representing a rigid surface. β was chosen by matching amplitudes of sway angles of theoretical data simulated in Eq. (1) with the board stiffness infinitely large to mimic posture on a rigid surface and experimental data seen in patients with Parkinson's disease from Schmit et al. (2006). By changing β the theoretical limit cycle amplitude is matched with the experimental sway amplitude of Schmit et al. (2006) and was found to be a value if 1000. The values of the constant parameters are listed and compared to the parameters used in similar models of posture on a rigid surface (Table 1).

2.3. Computational techniques

Before describing the results, it is pertinent to review some standard terminology from dynamical systems theory which will be used to describe the findings. First of all, an equilibrium position refers to the positions of the board and inverted pendulum for which $\dot{\theta} = \ddot{\theta} = \dot{\phi} = \ddot{\phi} = 0$ and all external forces and moments are balanced. The upright equilibrium position refers to an equilibrium position where $\theta = \phi = 0$. Second, in this work we refer to stability of an equilibrium position as asymptotic stability. An equilibrium position is said to be asymptotically stable if all trajectories that start out near the equilibrium converge to the equilibrium (Guckenheimer and Holmes, 1983; Jordan and Smith, 1977; Troger and Steindl, 1991). In addition, an asymptotically stable equilibrium position is said to be globally stable if every trajectory converges to the equilibrium position; otherwise it is locally stable (Guckenheimer and Holmes, 1983; Jordan and Smith, 1977; Troger and Steindl, 1991). An equilibrium position is said to be unstable if at least one eigenvalue has a positive real part. The equilibrium position(s) of the body angle relative to the board (θ^* and ϕ^*, where "$*$" represents the static equilibrium), and their stability are calculated using the DDE-BIFTOOL MATLAB package (Engelborghs et al., 2001) and verified using MATLAB (The MathWorks, Inc.) simulations[1] of the system (Eq. 1).

In what follows we use bifurcation theory to explain how slow changes in system parameters (i.e. changes in neuromuscular system) lead to sudden changes in stability of the upright equilibrium. The bifurcations which we discuss are the Hopf, pitchfork, and saddle-node bifurcations (Guckenheimer and Holmes, 1983; Jordan and Smith, 1977; Troger and Steindl, 1991). A supercritical (subcritical) Hopf bifurcation occurs when a stable (unstable) equilibrium position becomes unstable (stable) with the introduction of a stable (unstable) limit cycle around the equilibrium position. A supercritical (subcritical) pitchfork bifurcation occurs when a single stable (unstable) equilibrium position becomes unstable (stable) with the introduction to two stable (unstable) equilibrium positions symmetrically around the unstable (stable) equilibrium position. A saddle-node bifurcation refers to a situation where a stable and unstable pair of equilibrium solutions are either created or destroyed at a given point in phase space as the system parameters are varied.

[1] The MATLAB code for the model may be obtained by request.

3. Results

3.1. Effect of combined vestibular and visual feedback

To study the effect of combined vestibular/visual feedback of the 1-DOF balance board system the proprioceptive gain was held at zero ($K_p = 0$) while K_v was varied in Eq. (4) for a fixed value of neuromuscular time delay of $\tau = 0.1$. The complementary case of $K_v = 0$ while K_p is varied is presented in Section 3.3. We also initially placed the ankle joint directly in line with the balance board's axis of rotation. The local stability of the upright equilibrium position was calculated in the continuous parameter space (K_{board}, K_v) (Fig. 2).

The range of K_v required for stabilizing the upright equilibrium is much greater at large values of K_{board} (Fig. 2). In this region, where static upright posture is stable, there co-exists an unstable limit cycle: for small perturbations from the upright position, the CoM would return to the upright position. However, larger perturbations would result in CoM oscillations that grow uncontrollably beyond a critical value (Fig. 2c). When K_v is at low values, decreasing K_{board} results in a loss of stability of the upright equilibrium via a pitchfork bifurcation and two stable (forward/backward) leaning positions emerge (Fig. 2a). On the other hand, when K_v is at large values, decreasing K_{board} the upright static equilibrium destabilizes via a subcritical Hopf bifurcation (red line) (Fig. 2b). In Section 4 we will discuss how these results inform the possible mechanisms behind improved balance following balance board training.

3.2. Effect of ankle positioning

Here we consider how the inclusion of ankle position misaligned with the axis of rotation of the balance board changes the bifurcation diagram of the system. All other system parameters are identical to those in Section 3.1. When the ankle joints are directly in line with the balance board's axis of rotation the equilibrium positions are symmetrical about the upright equilibrium (Fig. 3); whereas, the misalignment of the ankle joints with the balance board's axis of rotation breaks the mirror symmetry of the static equilibrium positions (Fig. 4). The pitchfork bifurcation of the symmetric problem (Fig. 3) unfolds into a saddle-node bifurcation (Fig. 4) so that the equilibrium position begins to deviate from the upright position as K_{board} decreased (Fig. 4a, note that this is still referred to as the upright position even though $\theta^* \neq 0°$). The leaning is caused by a torque generated from the offset position of the body's CoM when the ankle joints are out of line with the balance board's axis of rotation. The remaining results are qualitatively similar to those for the symmetric problem (Fig. 3).

When $K_v = 0.2K^{cr}$ the upright equilibrium positions are stable for infinite board stiffness, and destabilize as K_{board} decreases (Figs. 3a and 4a). This results in an unstable, upright equilibrium position and the emergence of two unstable (forward/backward) leaning equilibrium positions (Figs. 3a and 4a). As K_{board} deceases further a supercritical Hopf bifurcation arises causing the emergence of a stable limit cycle (around the forward or backward equilibrium position) (Figs. 3a and 4a).

Once K_{board} is below a value at which the static equilibrium position deviates significantly from the upright position and/or

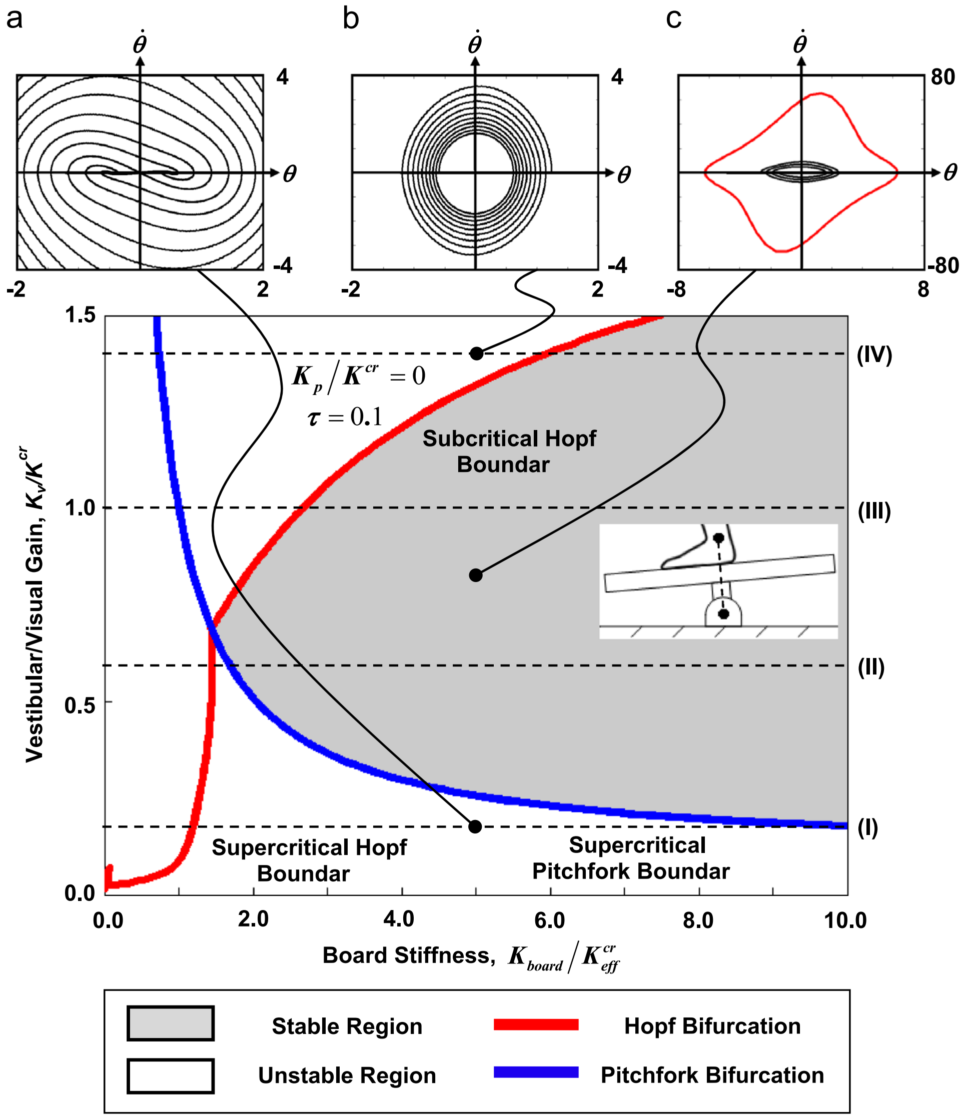

Fig. 2. Upright equilibrium ($\theta^* = 0°$) stability regions of posture on a 1-DOF balance board with ankle joints directly in line with the balance board axis of rotation (seen on right) for neuromuscular time delay of 100 ms ($\tau = 0.1$ s) with no proprioceptive feedback ($K_p = 0$). Stable regions (gray) and unstable regions (white) are bounded by a supercritical pitchfork bifurcation (blue line) and a subcritical and supercritical Hopf bifurcation (red line). Above the graph are phase portraits of the different stability regions showing the changes in system dynamics where limit cycle trajectories are shown in red. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

becomes unstable, it can be brought closer to the upright position and stabilized by increasing K_v (Figs. 3b–d and 4b–d). This increase in K_v however shifts the Hopf bifurcations to a higher value of K_{board}, thus destabilizing the equilibrium at a larger value of K_{board}. As with the symmetrical case (Fig. 3) the nature of the Hopf bifurcation changes from super to subcritical with an increased value of K_v (Fig. 4).

3.3. Effect of proprioceptive feedback

To study the effect of proprioceptive feedback the combined vestibular/visual gain is held at zero ($K_v = 0$) while the K_p is varied in Eq. (4) for a fixed value of neuromuscular time delay of $\tau = 0.1$. We placed the ankle joint directly in line with the balance board's axis of rotation. The local stability of the upright equilibrium position was calculated in the continuous parameter space (K_{board}, K_p) (Fig. 5).

For low values of K_p, decreasing K_{board} destabilizes the upright equilibrium via a pitchfork bifurcation (Fig. 5) resulting in the emergence of two stable (forward/backward) leaning equilibrium positions (Fig. 5a). In fact, at high values of K_{board} the pitchfork bifurcation resembles that of the pitchfork bifurcation in Fig. 2. This is because at high values of K_{board} the relative body angle (affecting proprioceptive feedback) is almost identical to absolute body angle (affecting vestibular/visual feedback). Besides this subtle difference, qualitatively the stability diagram and the stable/unstable behaviors therein are similar to those described in Section 3.1 where the effect of combined vestibular/visual gain was studied.

3.4. Effect of time delay in neuromuscular system

Neuromuscular time delay is a key parameter that changes with age and disease. In the previous sections we have kept the neuromuscular time delay (τ) constant while studying stability in either the (K_{board}, K_v) space or the (K_{board}, K_p) space. In order to study now the effect of τ, we chose to plot the stability boundaries in the (K_{board}, K_v) space (i.e. $K_p = 0$) for a larger value of τ (300 ms, compared to 100 ms studied earlier). We have also performed these studies in the (K_{board}, K_p) space (i.e. $K_v = 0$) but those results

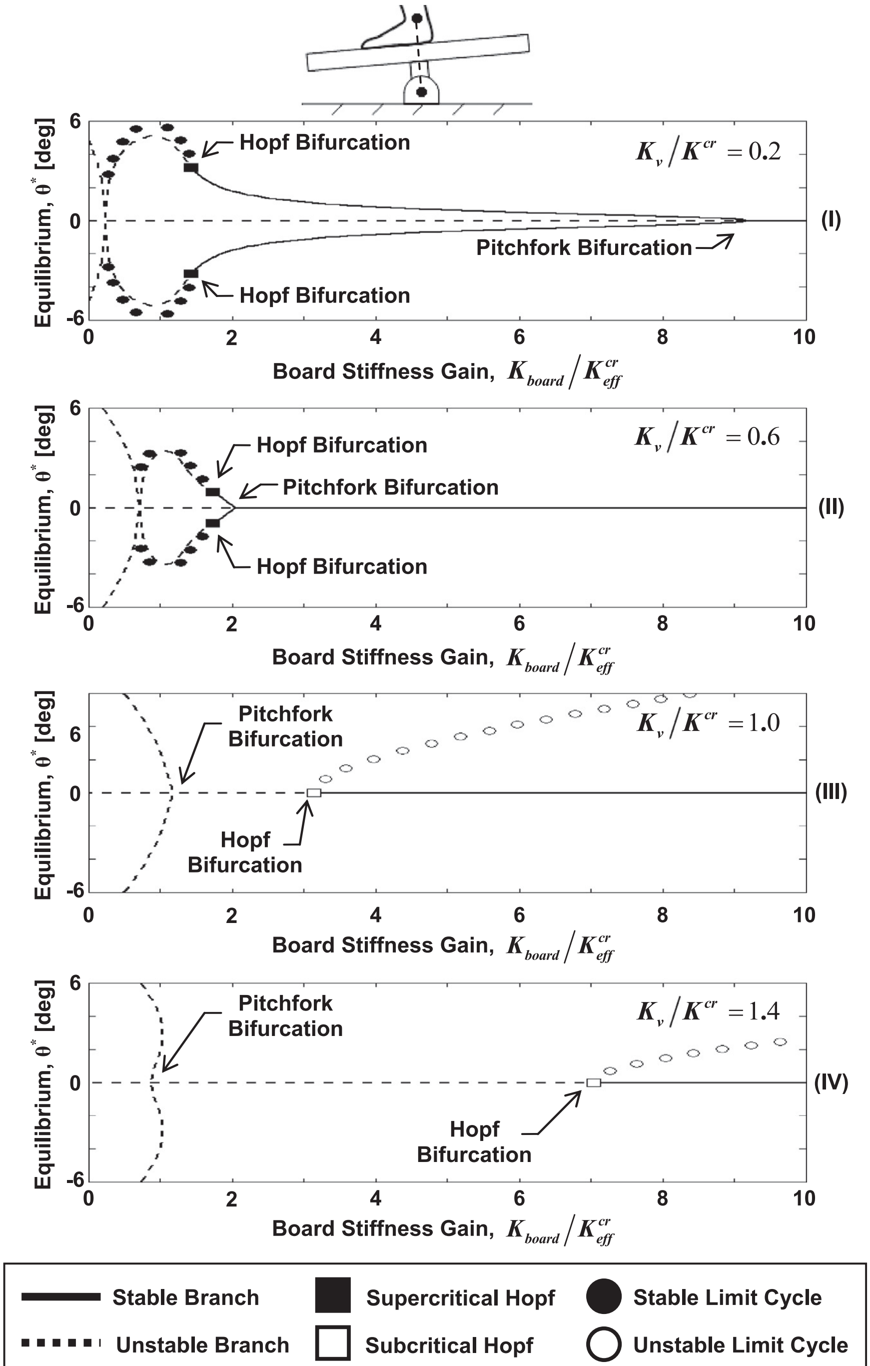

Fig. 3. Equilibrium positions (θ^*) as a function of balance board stiffness (K_{board}) for posture on a 1-DOF balance board with ankle joints directly in line with the balance board axis of rotation (seen on top) for neuromuscular time delay of 100 ms ($\tau = 0.1$ s) with no proprioceptive feedback ($K_p = 0$) and vestibular/visual feedback gains (K_v) of (I) 20%, (II) 60%, (III) 100%, and (IV) 140% of the critical muscle stiffness. Solid lines and solid circles represent stable equilibrium positions and stable limit cycles respectively where dashed lines and unfilled circles represent unstable equilibrium positions and unstable limit cycles respectively.

are qualitatively the same and thus not shown here. The ankle joints were chosen to be directly in line with the balance board's axis of rotation. The local stability of the upright equilibrium position for $\tau = 0.3$ s was calculated in the continuous parameter space (K_{board}, K_v) (Fig. 6). In addition, the static equilibrium positions were calculated as a function of K_{board} for $\tau = 0.3$ s and $K_v = 0.6K^{cr}$ (Fig. 7b) and compared to the case of $\tau = 0.1$ s (Fig. 7a). Figs. 6 and 7 clearly show that increased time delay in the neuromuscular system dramatically shrinks the range of K_v required to stabilize upright posture on the balance board.

4. Discussion

The bifurcation analysis demonstrates that the dynamic stability of upright stance on a balance board changes as a function of proprioceptive feedback gain (K_p), combined vestibular/visual feedback gain (K_v), neuromuscular time delay (τ), board stiffness (K_{board}), and ankle position (x_{ankle}). Here we discuss the possible implications of the results.

As expected, balance is lost with decreased board stiffness if a person has too low proprioceptive gain or vestibular/visual gain via a

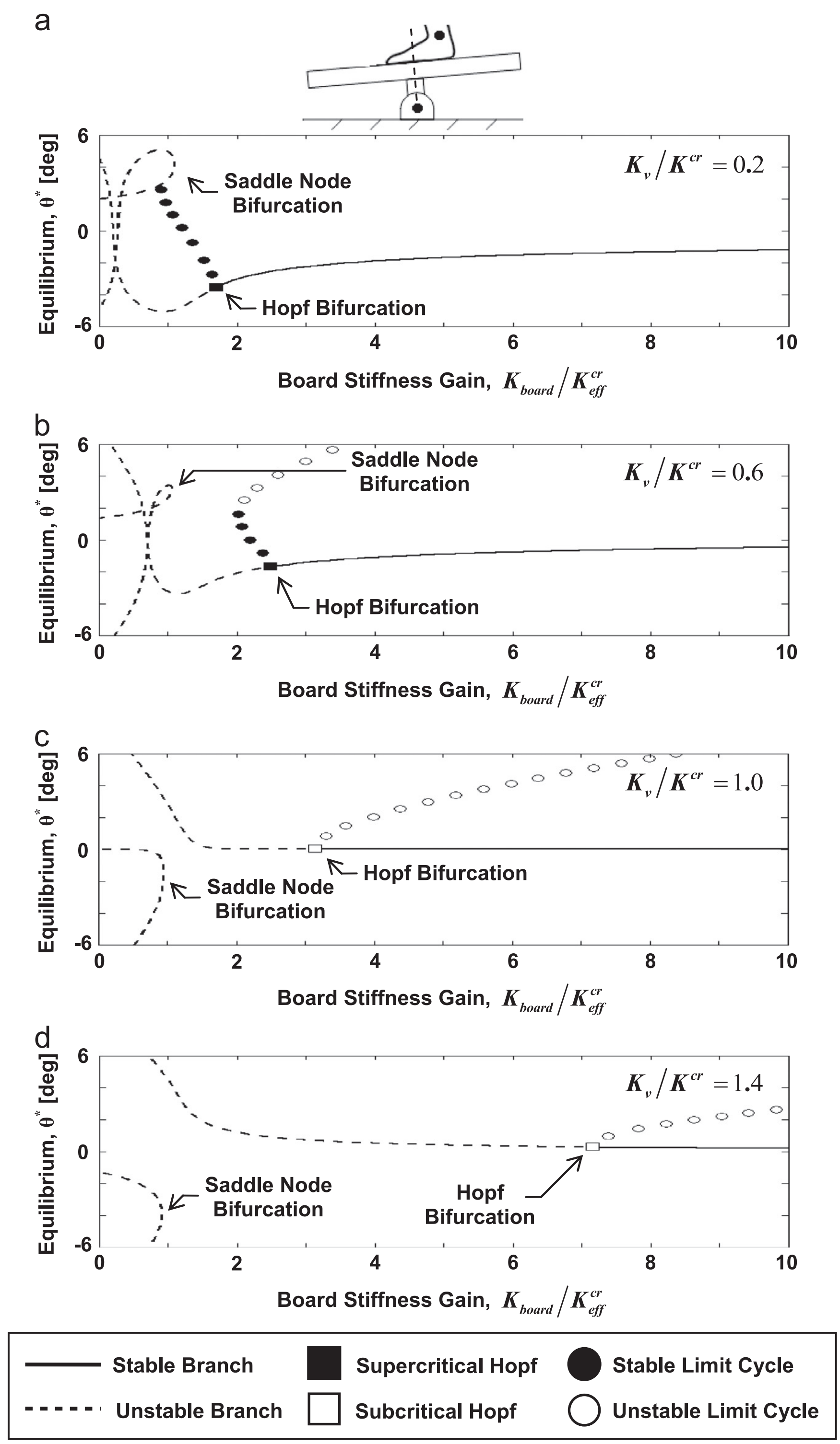

Fig. 4. Equilibrium positions (θ^*) as a function of balance board stiffness (K_{board}) for posture on a 1-DOF balance board with ankle joints out of line with the balance board axis of rotation (seen on top) for neuromuscular time delay of 100 ms ($\tau = 0.1$ s) with no proprioceptive feedback ($K_p = 0$) and vestibular/visual feedback gains (K_v) of (I) 20%, (II) 60%, (III) 100%, and (IV) 140% of the critical muscle stiffness. Solid lines and solid circles represent stable equilibrium positions and stable limit cycles respectively where dashed lines and unfilled circles represent unstable equilibrium positions and unstable limit cycles respectively.

pitchfork bifurcation. Therefore an individual can stabilize their upright posture on a balance board by increasing proprioceptive gain or vestibular/visual gain and thus eliminating the two leaning positions and stabilizing upright position (Figs. 2 and 5). On the other hand, we have seen that if an individual has too large of proprioceptive gain or vestibular/visual gain the upright equilibrium will become unstable via a Hopf bifurcation, but by decreasing proprioceptive gain or vestibular/visual gain upright posture can be stabilized.

However, it is incorrect to conclude that stabilization of upright posture on a balance board via adjustment of proprioceptive gain as a balance strategy is somehow equivalent to the adjustment of vestibular/visual gain. Rather the stability boundaries encountered while adjusting proprioceptive gain are quite different from those

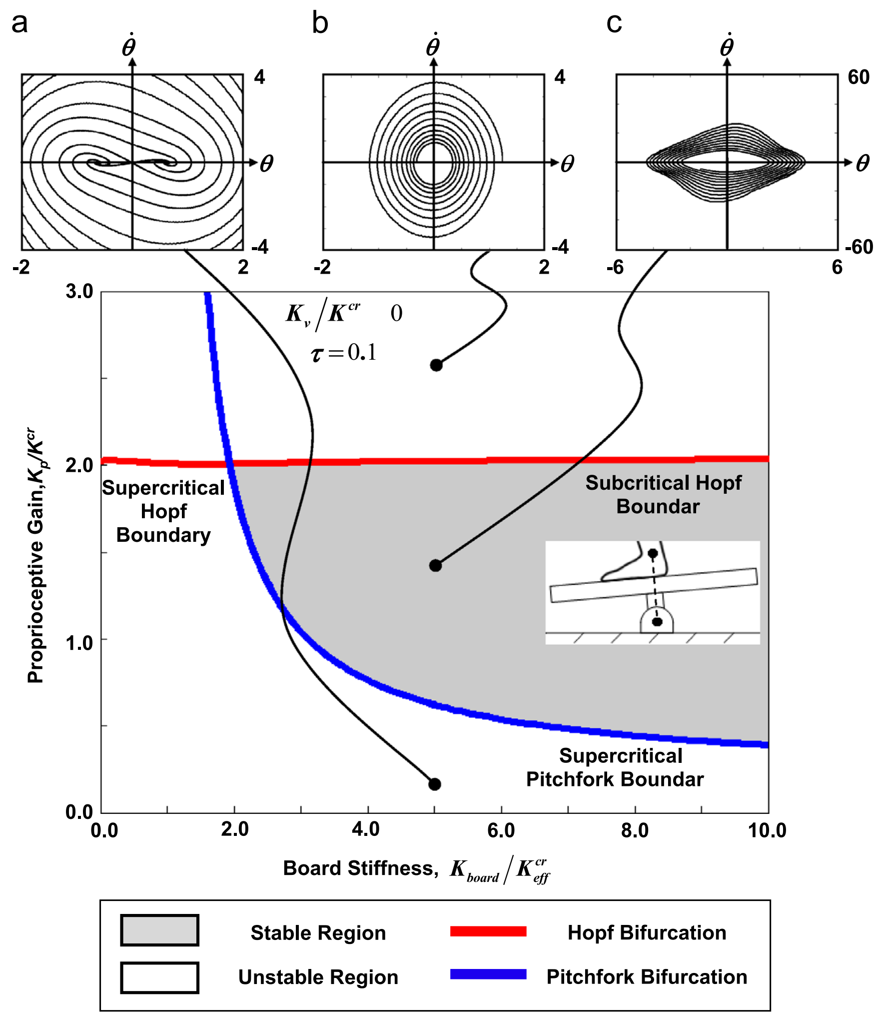

Fig. 5. Upright equilibrium ($\theta^* = 0°$) stability regions of posture on a 1-DOF balance board with ankle joints directly in line with the balance board axis of rotation (seen on right) for neuromuscular time delay of 100 ms ($\tau = 0.1$ s) with no vestibular/visual feedback ($K_v = 0$). Stable regions (gray) and unstable regions (white) are bounded by a supercritical pitchfork bifurcation (blue line) and a subcritical and supercritical Hopf bifurcation (red line). Above the graph are phase portraits of the different stability regions showing the changes in system dynamics where limit cycle trajectories are shown in red. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

encountered while adjusting vestibular/visual gain (see pitchfork boundaries (red lines) and Hopf boundaries (red lines) in Figs. 2 and 5).

Comparing the case when neuromuscular time delay is 100 ms ($\tau = 0.1$ s) (Fig. 2) and 300 ms ($\tau = 0.3$ s) (Fig. 6), the stability and bifurcations of the upright equilibrium are similar; however, when time delay is 300 ms there is a smaller range of parameter values for which upright posture is stable. This observation is consistent with the intuitive expectation that a longer time delay will increase difficulty of achieving stable upright equilibrium. Moreover, the locations of the equilibrium positions do not change with an increase in neuromuscular time delay (Fig. 7a and b). This is expected since at equilibrium $\theta(t) = \theta(t-\tau)$; thus, the static equilibrium positions do not depend on the time delay. Similarly, the pitchfork bifurcation occurs at the same value of board stiffness for time delays of both 100 and 300 ms (Fig. 7a and b). On the other hand, since the Hopf bifurcation is time delayed induced, the Hopf bifurcation occurs at different values of board stiffness (Fig. 7a and b). The increase in neuromuscular time delay shifts the Hopf bifurcation to larger board stiffness values, thus destabilizing equilibrium positions for all lower board stiffness (Fig. 7b). This suggests that if an individual tends to wobble or oscillate (suffer a limit cycle) about the unstable upright equilibrium they must decrease their neuromuscular time delay to stabilize the upright equilibrium position. Thus, balancing upright on the balance board requires an individual to "fine-tune" their intrinsic neuromuscular time delay and neuromuscular feedback gains. These findings support the hypothesis that the mechanisms behind improved balance from balance board training include improved ability to modify the visual and vestibular gains as a function of environmental demand and improved time delays. However, many controlled experiments need to be performed to validate this hypothesis. In addition, the results show that when the ankle joints are in line with the balance board axis of rotation, the Hopf bifurcation occurs after the pitchfork bifurcation. On the other hand, when the ankle joints are out of line with the balance board axis of rotation, the Hopf bifurcation occurs before the saddle-node bifurcation. This critical difference between the two cases shows how a balance strategy changes depending on the positioning of the ankles while standing on a balance board.

4.1. Future applications

Finally we discuss the possible use of our results for better design of balance board based treatments. By measuring bifurcations as board parameters are varied, rehabilitation systems like the Biodex Balance System SD® could be used to identify intrinsic postural control parameters. This information in turn could be used to develop a treatment plan based on a person's unique postural parameters.

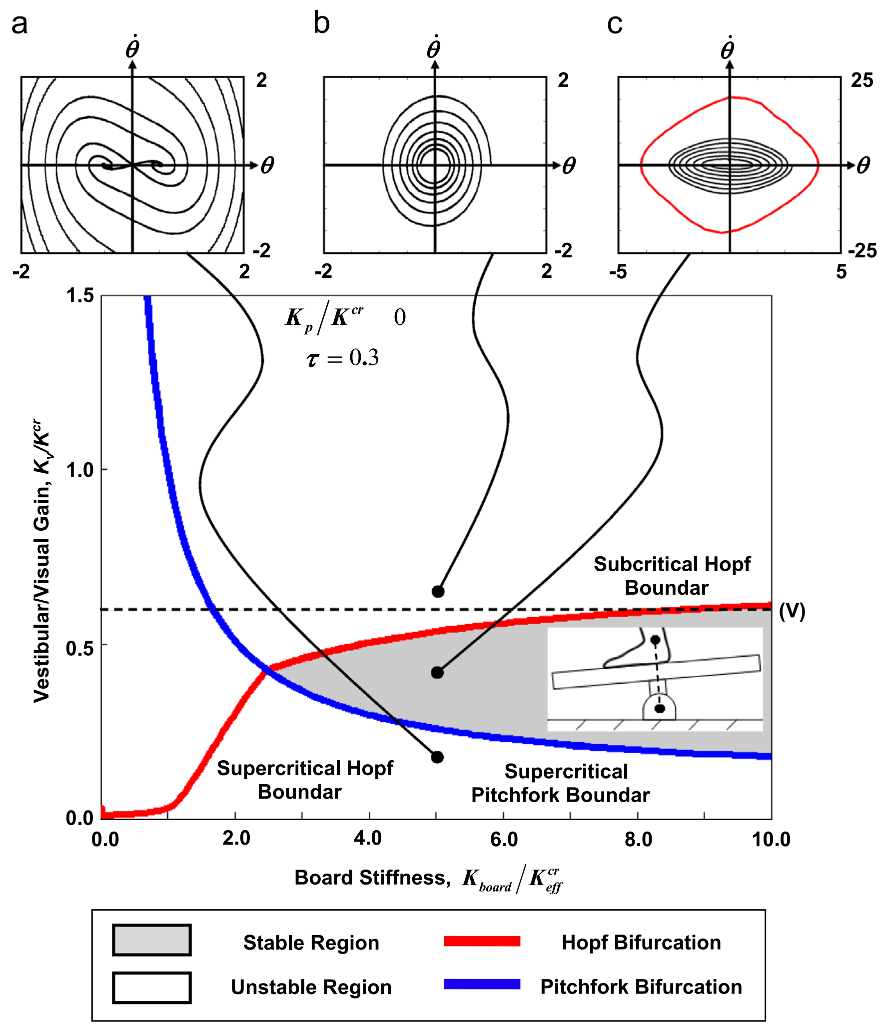

Fig. 6. Upright equilibrium ($\theta^* = 0^\circ$) stability regions of posture on a 1-DOF balance board with ankle joints directly in line with the balance board axis of rotation (seen on right) for neuromuscular time delay of 300 ms ($\tau = 0.3$ s) with no proprioceptive feedback ($K_p = 0$). Stable regions (gray) and unstable regions (white) are bounded by a supercritical pitchfork bifurcation (blue line) and a subcritical and supercritical Hopf bifurcation (red line). Above the graph are phase portraits of the different stability regions showing the changes in system dynamics where limit cycle trajectories are shown in red.

For example, if a person was found to have a long neuromuscular time delay a treatment plan may be to balance at specific board stiffness so a range of ideal combined vestibular/visual gain is targeted. Without a solid mathematical model as a basis it would not be easy to identify suitable board stiffness for people with unique circumstances. In addition, the symmetry breaking that occurs when ankle joints are out of line with the balance board is an important finding for clinicians who use a balance board during therapy. The ability to adapt gain may be improved by training with the ankles in various positions relative to the fulcrum, rather than using the same position on the platform as is common practice. However, further work is needed to validate these hypotheses which are the subject of ongoing research.

4.2. Limitations

It is worthwhile emphasizing the limitations of the theoretical results:

a. We have used the assumption that all rotation of the body takes place about the ankle joints. Although this assumption is widely accepted for capturing the movements of the CoM for quiet stance on a rigid surface, it has not been validated for stance on a balance board.
b. The PID law used contains only the most general terms such as proportional feedback on angular position and velocity of the body at an instantaneous time as well as at a delayed time. Although this PID law is widely used, the accuracy of this assumption must be taken into account due to the complexity of the neuromuscular system.

5. Conclusions

In summary, we have presented a proof-of-concept systematic study of the nonlinear dynamics inherent in the simplest model of human balance coupled to an external unstable dynamical system —in this case an unstable balance board. A variety of postural behaviors have been found and sudden changes in these behaviors are predicted even with slow, continuous deterioration of neuromuscular system and balance board parameters. The prediction of sub- and super-critical Hopf bifurcations, pitchfork bifurcations and their symmetry breaking can help explain a control strategy that is used to maintain posture on a non-rigid surface such as an unstable balance board. This analysis suggests a balance strategy (i.e. fine-tuning feedback gains to remain within the "stable region") undertaken by humans on an unstable platform and

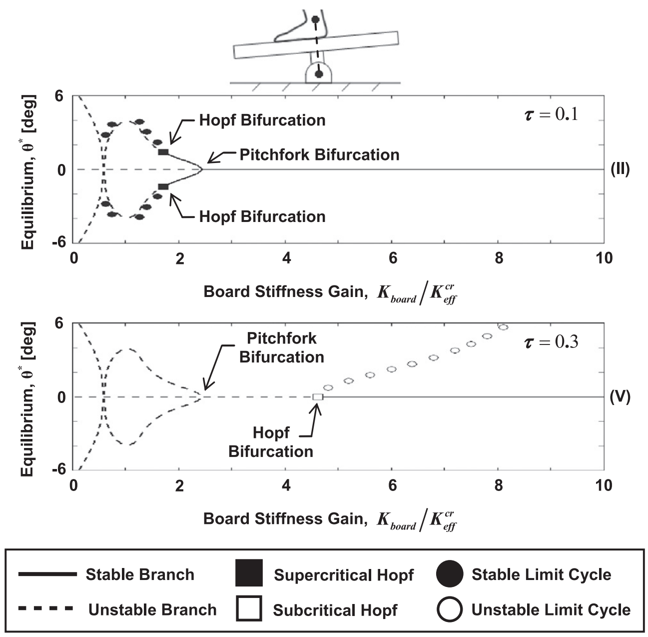

Fig. 7. Equilibrium positions (θ^*) as a function of balance board stiffness (K_{board}) for posture on a 1-DOF balance board with ankle joints directly in line with the balance board axis of rotation (seen on top) for neuromuscular time delay of (II) $\tau = 0.1$ s and (V) $\tau = 0.3$ s with no proprioceptive feedback ($K_p = 0$) and vestibular/visual feedback gain 60% of the critical muscle stiffness ($K_v = 0.6K^{cr}$). Solid lines and solid circles represent stable equilibrium positions and stable limit cycles respectively where dashed lines and unfilled circles represent unstable equilibrium positions and unstable limit cycles respectively.

how balance improvement devices such as the Biodex Balance System SD® may also be used for assessing balance and balance disorders.

Conflict of interest statement

The authors declare that no conflict of interest is associated with the present study.

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Appendix A. Supplemantary amterial

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jbiomech.2013.08.012.

References

Aaltonen, S., Karjalainen, H., Heinonen, A., Parkkari, J., Kujala, U.M., 2007. Prevention of sports injuries—systematic review of randomized controlled trials. Archives of Internal Medicine 167, 1585–1592.

Asai, Y., Tasaka, Y., Nomura, K., Nomura, T., Casadio, M., Morasso, P., 2009. A model of postural control in quiet standing: robust compensation of delay-induced instability using intermittent activation of feedback control. Plos One 4, 14.

Ashburn, A., Stack, E., Pickering, R.M., Ward, C.D., 2001. Predicting fallers in a community-based sample of people with Parkinson's disease. Gerontology 47, 277–281.

Barauskas, R., Krusinskiene, R., 2007. On parameters identification of computational models of vibrations during quiet standing of humans. Journal of Sound and Vibration 308, 612–624.

Blaszczyk, J.W., Orawiec, R., Duda-Kiodowska, D., Opala, G., 2007. Assessment of postural instability in patients with Parkinson's disease. Experimental Brain Research 183, 107–114.

Chagdes, J.R., Rietdyk, S., Haddad, J.M., Zelaznik, H.N., Raman, A., Rhea, C.K., Silver, T.A., 2009. Multiple timescales in postural dynamics associated with vision and a secondary task are revealed by wavelet analysis. Experimental Brain Research 197, 297–310.

Cholewicki, J., Polzhofer, G.K., Radebold, A., 2000. Postural control of trunk during unstable sitting. Journal of Biomechanics 33, 1733–1737.

Corradini, M.L., Fioretti, S., Leo, T., Piperno, R., 1997. Early recognition of postural disorders in multiple sclerosis through movement analysis: a modeling study. IEEE Transactions on Biomedical Engineering 44, 1029–1038.

de Bruin, E.D., Swanenburg, J., Betschon, E., Murer, K., 2009. A randomised controlled trial investigating motor skill training as a function of attentional focus in old age. BMC Geriatrics 9, 15–24.

Emery, C.A., Cassidy, J.D., Klassen, T.P., Rosychuk, R.J., Rowe, B.H., 2005. Effectiveness of a home-based balance-training program in reducing sports-related injuries among healthy adolescents: a cluster randomized controlled trial. Canadian Medical Association Journal 172, 749–754.

Emery, C.A., Meeuwisse, W.H., 2010. The effectiveness of a neuromuscular prevention strategy to reduce injuries in youth soccer: a cluster-randomised controlled trial. British Journal of Sports Medicine 44, 555–562.

Engelborghs, K., Luzyanina, T., Samaey, G., 2001. DDE-BIFTOOL v. 2.00: a Matlab package for bifurcation analysis of delay differential equations. In: Department of Computer Science, K.U.L. (Ed.), Technical Report TW-330. Department of Computer Science, K.U.Leuven, Leuven, Belgium, Leuven, Belgium.

Fukuoka, Y., Nagata, T., Ishida, A., Minamitani, H., 2001. Characteristics of somatosensory feedback in postural control during standing. IEEE Transactions on Neural Systems and Rehabilitation Engineering 9, 145–153.

Godard, M.P., Whitman, S.A., Richmond, S.R., 2004. Effects of balance training on stability and neuromuscular drive in older women. Activities, Adaptation & Aging 28, 59–73.

Guckenheimer, J., Holmes, P., 1983. Nonlinear oscillations, dynamical systems, and bifurcations of verctor fields. Springer-Verlag, New York.

Gunther, M., Grimmer, S., Siebert, T., Blickhan, R., 2009. All leg joints contribute to quiet human stance: a mechanical analysis. Journal of Biomechanics 42, 2739–2746.

Hinman, M.R., 2002. Comparison of two short-term balance training programs for community-dwelling older adults. Journal of Geriatric Physical Therapy 25, 10–20.

Hur, P., Duiser, B.A., Salapaka, S.M., Hsiao-Wecksler, E.T., 2010. Measuring robustness of the postural control system to a mild impulsive perturbation. IEEE Transactions on Neural Systems and Rehabilitation Engineering 18, 461–467.

Iqbal, K., Roy, A., 2004. Stabilizing PID controllers for a single-link biomechanical model with position, velocity, and force feedback. Journal of Biomechanical Engineering-Transactions of the Asme 126, 838–843.

Ishida, A., Imai, S., Fukuoka, Y., 1997. Analysis of the posture control system under fixed and sway-referenced support conditions. IEEE Transactions on Biomedical Engineering 44, 331–336.

Johansson, R., Magnusson, M., Akesson, M., 1988. Identification of human postural dynamics. IEEE Transactions on Biomedical Engineering 35, 858–869.

Jordan, D.W., Smith, P., 1977. Nonlinear Ordinary Differential Equations. Clarendon Press, Oxford.

Kannus, P., Sievanen, H., Palvanen, M., Jarvinen, T., Parkkari, J., 2005. Prevention of falls and consequent injuries in elderly people. Lancet 366, 1885–1893.

Kuo, A.D., Zajac, F.E., 1993. Human standing posture—multijoint movement strategies based on biomechanical constraints. Progress in Brain Research 97, 349–358.

Loram, I.D., Lakie, M., 2002. Direct measurement of human ankle stiffness during quiet standing: the intrinsic mechanical stiffness is insufficient for stability. Journal of Physiology-London 545, 1041–1053.

Loram, I.D., Maganaris, C.N., Lakie, M., 2005. Human postural sway results from frequent, ballistic bias impulses by soleus and gastrocnemius. Journal of Physiology-London 564, 295–311.

Masani, K., Popovic, M.R., Nakazawa, K., Kouzaki, M., Nozaki, D., 2003. Importance of body sway velocity information in controlling ankle extensor activities during quiet stance. Journal of Neurophysiology 90, 3774–3782.

Masani, K., Vette, A.H., Popovic, M.R., 2006. Controlling balance during quiet standing: proportional and derivative controller generates preceding motor command to body sway position observed in experiments. Gait & Posture 23, 164–172.

Maurer, C., Peterka, R.J., 2005. A new interpretation of spontaneous sway measures based on a simple model of human postural control. Journal of Neurophysiology 93, 189–200.

Milton, J., Cabrera, J.L., Ohira, T., Tajima, S., Tonosaki, Y., Eurich, C.W., Campbell, S.A., 2009. The time-delayed inverted pendulum: implications for human balance control. Chaos 19, 12.

Nordt, W.E.R., Sachatello, S.A., Plotkin, E.S., Dintino, K., 1999. The effects of single-axis balance board intervention on balance parameters in the elderly. American Journal of Orthopedics 28, 447–450.

Peterka, R.J., 2000. Postural control model interpretation of stabilogram diffusion analysis. Biological Cybernetics 82, 335–343.

Peterka, R.J., 2002. Sensorimotor integration in human postural control. Journal of Neurophysiology 88, 1097–1118.

Peterka, R.J., 2003. Simplifying the complexities of maintaining balance. IEEE Engineering in Medicine and Biology Magazine 22, 63–68.

Pinter, I.J., van Swigchem, R., van Soest, A.J., Rozendaal, L.A., 2008. The dynamics of postural sway cannot be captured using a one-segment inverted pendulum model: a Pca on segment rotations during unperturbed stance. Journal of Neurophysiology 100, 3197–3208.

Reeves, N.P., Everding, V.Q., Cholewicki, J., Morrisette, D.C., 2006. The effects of trunk stiffness on postural control during unstable seated balance. Experimental Brain Research 174, 694–700.

Sasagawa, S., Ushiyama, J., Kouzaki, M., Kanehisa, H., 2009. Effect of the hip motion on the body kinematics in the sagittal plane during human quiet standing. Neuroscience Letters 450, 27–31.

Schmit, J.M., Riley, M.A., Dalvi, A., Sahay, A., Shear, P.K., Shockley, K.D., Pun, R.Y.K., 2006. Deterministic center of pressure patterns characterize postural instability in Parkinson's disease. Experimental Brain Research 168, 357–367.

Stolze, H., Klebe, S., Zechlin, C., Baecker, C., Friege, L., Deuschl, G., 2004. Falls in frequent neurological diseases. Journal of Neurology 251, 79–84.

Tanaka, M.L., Ross, S.D., Nussbaum, M.A., 2010. Mathematical modeling and simulation of seated stability. Journal of Biomechanics 43, 906–912.

Ting, L.H., van Antwerp, K.W., Scrivens, J.E., McKay, J.L., Welch, T.D.J., Bingham, J.T., DeWeerth, S.P., 2009. Neuromechanical tuning of nonlinear postural control dynamics. Chaos 19, 12.

Troger, H., Steindl, A., 1991. Nonlinear Stability and Bifurcation Theory: An Introduction for Scientists and Engineers. Springer-Verlag, London.

Verdaasdonk, B.W., Koopman, H., van Gils, S.A., van der Helm, F.C.T., 2004. Bifurcation and stability analysis in musculoskeletal systems: a study in human stance. Biological Cybernetics 91, 48–62.

Vette, A.H., Masani, K., Nakazawa, K., Popovic, M.R., 2010. Neural-mechanical feedback control scheme generates physiological ankle torque fluctuation during quiet stance. IEEE Transactions on Neural Systems and Rehabilitation Engineering 18, 86–95.

Winters-Stone, K.M., Torgrimson, B., Horak, F., Eisner, A., Nail, L., Leo, M.C., Chui, S., Luoh, S.-W., 2011. Identifying factors associated with falls in postmenopausal breast cancer survivors: a multi-disciplinary approach. Archives of Physical Medicine and Rehabilitation 92, 646–652.

Yao, W., Yu, P., Essex, C., 2001. Delayed stochastic differential model for quiet standing. Physical Review E 6302. (art. nono. 021902).

Zech, A., Hubscher, M., Vogt, L., Banzer, W., Hansel, F., Pfeifer, K., 2010. Balance training for neuromuscular control and performance enhancement: a systematic review. Journal of Athletic Training 45, 392–403.