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Detecting Roman land boundaries in aerial photographs using Radon transforms

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Abstract

Remotely sensed images are a valuable resource for archaeological landscape studies. Historical aerial photographs, in particular, often contain important data, especially where recent landscape change has drastically altered topography. Within the current study area, focussed upon the alluvial landscape surrounding the ancient city of Butrint in southern Albania, late 20th century agricultural intensification has obliterated topographical features relating to earlier land-use within the valley. However, through recourse to earlier aerial photographic images and the development of an image analysis procedure utilizing Radon transforms, a study of relic landscape divisions relating to the Roman settlement and land-use at Butrint has been possible. The analytical procedure described allows the rapid, automated detection of linear alignments relating to the orientation of the settlement over large spatial areas. The resulting data were imported into a GIS, allowing measurements between surviving linear elements to be made and regular patterns of land division or centuriation deduced. The study revealed the surviving remnants of a centuriation pattern composed primarily of 20 *actus* divisions, while a further possibly earlier pattern was also detected, based upon 16 by 24 *actus* divisions.

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1. Introduction

The use of remotely sensed images and aerial photographs, in particular, has become an essential part of archaeological landscape studies (e.g. [26], Chapter 3). Such images often reveal spatial or connective relationship between extant archaeological remains and other morphological features relating to the use and organization of the landscape in the past. Various methods of analyzing and quantifying aerial photographic images have therefore been developed, many of which are related to standard image processing techniques common within remote sensing and computer vision applications.

The current study utilizes a series of aerial photographs (APs) as part of an investigation of past land-use within the alluvial valley surrounding the ancient city of Butrint in southern

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Albania. While the urban topography of Roman Butrint is fairly well understood [13,14], less is known about the organization of the surrounding landscape at this time. The planned nature of the Roman remains at Butrint is reflected in the formal layout of the settlement to standard units of measure corresponding to the Roman actus (where one actus was equal to 120 Roman feet and measures ca. 35.4 m). Roman colonization was often accompanied by a rearrangement of the countryside, replacing earlier systems of land division, see [23]. Roman land surveyors, utilizing the actus as the basic unit of measure (which has agricultural origins), are therefore, likely to have divided the surrounding landscape into discrete land parcels. In many cases these were orientated with the principal axes (decumanus and kardo) of the settlement. The current study is focussed upon the detection within aerial photographic images of surviving features related to the organization of the landscape at the establishment of the Roman colony; detecting within the images linear landscape features following the same alignment as the decumani and kardines of the settlement.

The principal Roman land division is a 'century', commonly a square of 20 by 20 actus (200 iugera or 'yoke areas') after the Augustinian age, equivalent to 50.4 ha. A century should have originally been exactly 100 smallholdings or heredia ('inherited areas') of two iugera each, although the number of holdings in a century dwindled with time [8]. The usual Latin name for centuriation was limitatio, stressing the importance of the limites separating the centurae which when expressed in physical form, could be boundaries of any sort, including footpaths and roads [8]. It is the remnants of these features which often survive within modern landscapes.

The ancient city of Butrint is located slightly inland of the eastern shore of the Straits of Corfu (see Fig. 1), within a land-scape dominated by rugged limestone mountains running parallel to the Ionian coastline and rising to over 2000 m above sea-level. The walled city survives upon a low promontory situated on a bend in the Vivari Channel, draining from Lake Butrint to the northeast, which forms the remnant of a lagoonal embayment. To the south an extensive alluvial plain extends along a fault-bound valley, the result of continual sedimentation throughout the late Holocene by branches of the now canalized Pavllas River. It is along the distal margins of the

floodplain forming the Vrina Plain that the city of Butrint expanded during the Roman period. The intensity of modern land-use within the valley can be seen from the extensive network of drainage and irrigation channels, shown in Fig. 1.

The aerial photographic imagery used within the current study consists of 10 vertical APs taken by the RAF in 1943 (sortie number GB 0172), covering an area of 42 km² extending from the village of Mursia northwards to the settlement at Butrint. In order to obtain geographically corrected images of the required accuracy for the current study, a large number of GPS derived ground control points were utilized within a polynomial transformation model. As no camera information was readily available for the photo set, it was not possible to create planimetrically true images of the surface. While a more robust orthorectification model might be desirable (incorporating calibrated camera information and a digital terrain model), the topographically uniform nature of the terrain over areas of interest (alluvial floodplain) meant the rectification method adopted gave satisfactory results. The contrast of the rectified images was then unified by matching the color map of each image to a specified histogram (see [26], Chapter 4), allowing a mosaic of AP images to be combined into a single layer

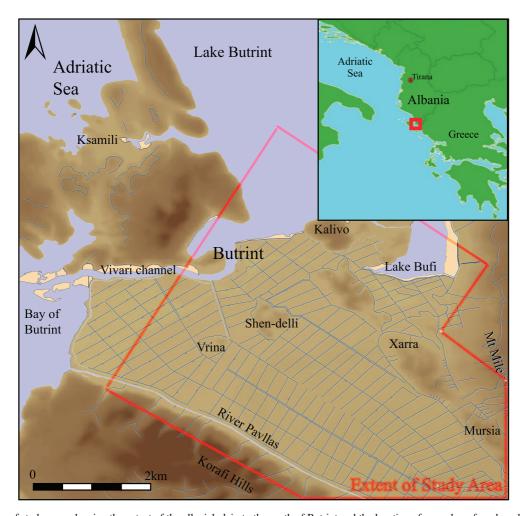


Fig. 1. Location map of study area, showing the extent of the alluvial plain to the south of Butrint and the location of a number of modern drainage features. The extent of the current study area is also shown.

within the GIS, resulting in the composite image shown in Fig. 2. The image reveals a complex network of field systems and land divisions, representing an accumulation of land-use strategies spanning at least two millennia, and indicates the intensity of past land-use within the valley predating the large-scale agricultural engineering works initiated since the 1960s.

Land divisions visible within the image are for the most part characterized by hedgerows and scrub which mark the boundaries of fields, while many linear divisions are also marked by the line of roads and trackways, again visible as changes in ground cover within the image. Other, more ephemeral linear boundary features such as ditches or ridges

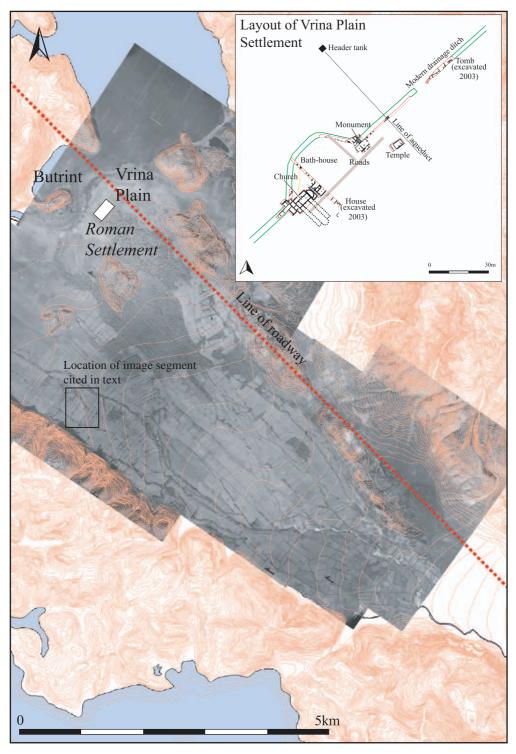


Fig. 2. Composite AP image of the study area overlaid onto topographical data within the GIS. Top right insert shows a plan of the Roman settlement remains on the Vrina Plain. The alignment of the former roadway along the valley is also shown.

may also be present, picked out by shadow. In this instance, there is the possibility of an anisotropy effect as a result of the illumination geometry causing some distortion in visible features at given orientations.

Investigations of the Roman remains on the Vrina Plain have revealed the principal alignment of the settlement to be 46° northeast (see Fig. 2). An analysis of topographic features and extant archaeological remains has revealed that the line of an existing ancient roadway is likely to have been the controlling feature [3] – symbolically linking the settlement with Rome. The line of the proposed roadway, constrained by the topography of the surrounding terrain, runs along the edge of the valley over elevated ground formed by a series of relic terraces, shown in Fig. 2 overlain onto the AP image. From Xarra southwards, this line closely follows that of the modern road. It seems likely then that the decumanus of the planned settlement upon the Vrina Plain was parallel to this principal roadway and is most likely to be a road leading from the settlement into the pre-existing walled city to the northwest, bridging the channel to the north alongside the aqueduct. The principal kardo of the Roman settlement, running perpendicular, appears to be defined by a long string of building remains recorded within the section of a modern drainage ditch, defining the main settlement axis (Fig. 2). The greater extent of these building has been recorded by past geophysical surveys (see [4,15]), while a small section of road recorded within the southwest portion of the settlement confirms this alignment.

2. Detecting linear features using the Radon transform

It can be seen from Fig. 2 that a dense and complex pattern of field boundaries existed within the valley and that a semiquantitative manual analysis of boundary alignments would be a time consuming process, making an automated analytical approach desirable. Similar lineation detection and analysis type problems occur within several other remote sensing and computer vision contexts, including the detection and quantification of geomorphological lineations forming glacial landforms and the characterization of geological structures (e.g. [6,20,21]) and in more general pattern recognition applications (e.g. [18,28]). A number of numerical techniques have been employed for solving these problems, from the use of simple edge filtering procedures, geometrical analysis [9], segment tracing algorithms [16] and the application of two-dimensional Fourier transforms [20]. Detecting periods (relating to the spacing of land divisions) using spectral analysis have been found to be more problematic, with difficulty in deriving statistically significant results (see [20,22]). The application of Fourier analysis to the investigation of Roman land divisions in East Anglia, UK, was undertaken by Peterson [22] and found to be effective for identifying periods within a 20 actus interval, the corresponding boundaries of which being identifiable on the ground.

In a departure from the techniques outlined above, the current study utilizes the parametric Radon transform to detect landscape features which follows the orientation of the Roman

settlement. Radon transforms, an integral transform and close relative of the Hough transform (see [2]), have found uses in a wide range of applications including tomography, seismology and Synthetic Aperture Radar (SAR) [5,11,17,29]. The transform effectively represents an image as projections of image intensity along a number of radial lines. In general, the projection of a two-dimensional function, f(x, y), is a line integral of f parallel to the y'-axis, which can be computed along any angle, θ (see [7,11]). The geometry of the Radon transform is shown in Fig. 3 and can be defined by:

$$R_{\theta}(x') = \int_{-\infty}^{+\infty} f(x' \cos \theta - y' \sin \theta, x' \sin \theta + y' \cos \theta) dy', \qquad (1)$$

where

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

During the analysis of an image, collinear elements following a particular angle of orientation, θ , will lead to a relative maxima or minima of R. The Radon transform can effectively be used either to determine the most commonly occurring alignment orientation within an image, by sweeping incrementally between θ and $\theta + \pi/2$ or to detect known alignments by specifying their orientation θ .

3. Implementation

Within the context of the current study, the intensity of past land-use has led to complex network of superimposed field boundaries, with no clearly predominant landscape alignment (see Fig. 2). Medieval field alignments dominate the pre-1960s

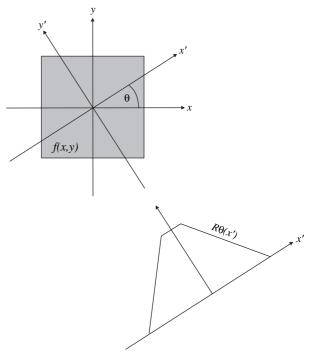


Fig. 3. Diagram showing the geometry of the Radon transform [19].

landscape, and tend to follow a number of differing ad hoc orientations. Since the alignment of the planned Roman settlement upon the plain is known, a search was made for surviving Roman landscape divisions which might follow a similar alignment. By calculating the Radon transform for segments of the image corresponding to the alignment of the settlement i.e. θ and $\theta + \pi/2$, where $\theta = 44^{\circ}$ (measured anticlockwise from the *x*-axis), the function effectively acts like a filter, detecting only alignments of potential interest. The

spatial position of high intensity values of R along the x'-axis, corresponding to a detected linear feature, can then be plotted onto the analyzed image.

The implementation of the Radon transform procedure is illustrated below using a small, sampled section of the AP image (see Fig. 4(a)). Within the image segment selected, field boundaries are in fact aligned at 30° northeast, which while being different from that hypothesized for Roman Butrint, reflecting a later land-use pattern, the clarity and completeness

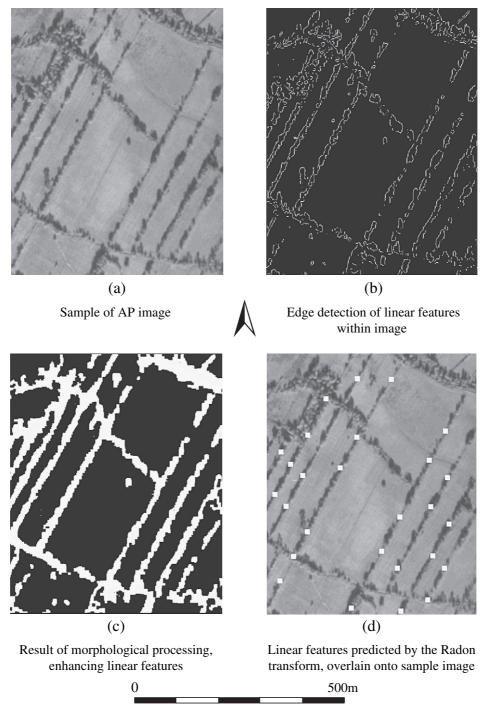


Fig. 4. Edge-detection, morphological image processing and predicted linear features for small test sample of image. Note that the alignment of linear field boundaries here is ca. 60°.

of these linear boundaries within this segment better illustrate the functioning of the procedure. The computational routine, which performs the automated processing of a specified input image, producing an output file of point map coordinates of detected linear features, was written using MATLAB programming script.

All linear features within the composite AP image are first highlighted by applying a neighborhood-based, edge-detection filter; locating horizontal and vertical edges using the Sobel approximation to the derivative (see [10]). The result is a binary image, where 1's represent extracted edges with 0's elsewhere (see Fig. 4(b)). Determining an output threshold value for the filter can potentially be difficult as a result of illumination effects (e.g. [16]), although in this instance, the equalized image presented no real problems. Linear features such as field boundaries, resulting from topographical changes and irregularities, are clearly identifiable as shadows cast as a result of the relatively low angle of the sun in the west. It can be seen that the filter has successfully delineated linear features within the sample image, although their relative width has led to a 'double line' representation within the binary image. This effect can easily be rectified by further remedial processing using an operation known as morphological closing. Here the dilation of image elements is performed followed by erosion, using a rectangular structuring element orientated with the features of interest (see [12,27]). The operation effectively removes the gap or 'holes' along the center line of the linear feature, ensuring that it is represented as cleanly as possible. The results of this operation are shown in Fig. 4(c).

The analysis of the processed binary image using the Radon transform is performed by first partitioning or segmenting the image into discrete square parcels. Optimal segment size is governed by the required precision of the angular measurement and the resolution of the input image. For angular alignments to be resolved at a theoretical accuracy of 1° (for 1 pixel wide lineations) the minimum segment size is going to be ca. 60 by 60 pixels. The maximum segment size is perhaps less critical, although if this becomes too large, there is a danger that the position of short, discontinuous linear features within the segment will not be accurately recorded. The resolution of the sample image is ca. 1.2 pixels per meter, so a segment size of 60 by 60 pixels covers a spatial extent of 72 m². This was found to allow the discrimination and accurate plotting of significant linear features within the image.

The Radon transform shown in Eq. (1) is applied to each segment sequentially (where $\theta=60^{\circ}$ to detect alignments within the sample image shown in Fig. 4). The routine also computes the Radon transform for the corresponding orthogonal angle. If θ corresponds to the angular alignment of a linear feature within the segment being processed, a high image intensity value will result. As the detection angle θ is inclined in relation to the orientation of the segment bounding box, there will be a drop in the sensitivity to linear features located at the extremities of x' that exceed the dimensions of the segment (see Fig. 3). However, with an optimally chosen segment size, such features should be adequately detected within adjacent segments. The resulting intensity values are also subject

to a threshold to eliminate noise. The locations of projected intensity values along x' exceeding the threshold value can then be plotted onto the input image.

Alignments detected by the routine for the sample image are shown in Fig. 4(d). It can be seen that the routine is successful at detecting alignments where the length of the lineation is sufficient to cause an intensity value that exceeds the threshold. Intensity values derived from more diffuse patterns of noise within the image, such as those relating to woodland cover, fall below the threshold level.

4. Roman land division at Butrint

The locations of alignments detected by the routine within the valley surrounding Butrint were imported into the GIS and overlain onto the original AP image. Interconnecting lines were drawn through points which obviously related to discrete linear features. Tools available within the GIS were then used to perform spatial measurements between detected features, allowing adjacent features separated by distances divisible by 1 actus (35.4 m) to be highlighted. Relatively few linear features were detected within the AP image conforming to the specified alignment of 46° and 136° (i.e. corresponding to the 44° northeast and perpendicular alignments within the settlement), making the analysis of the predicted results within the GIS relatively straightforward. A more complex pattern might have made it necessary to adopt an automated process of spatial analysis. This could be simply accomplished by devising a correlation filter, designed to remove predicted points that do not conform to the 1 actus relationship, while further statistical analysis of the spatial distribution of detected features could also be undertaken.

It was found that many of the detected features appeared to be separated by distances of 20 *actus*, while others seemed to be separated by distances of 12 *actus* along the southwest—northeast orientation and 16 *actus* along the northwest—southeast orientation. To test whether these patterns related to possible centuriation, two hypothetical grid layouts were created within the GIS and superimposed onto each pattern, the results of which are shown in Figs. 5 and 6.

The first pattern of 12 by 16 actus divisions (Fig. 5) does not seem to relate clearly to the settlement remains upon the Vrina Plain in terms of distances divisible by the actus unit, i.e. the hypothetical decumanus and kardo of the settlement do not correlate with the limites of this centuriation pattern. It is, however, possible that this land division relates to an earlier, Republican organization of the landscape, predating the establishment of the colony, since the roadway through the valley appears to form the decumanus maxiums for the centuriation. Similar divisional units have been recorded at Corinth, where there is evidence of at least two systems of land division [24,25].

By contrast, the 20 by 20 *actus* grid shown in Fig. 6 does correlate with the settlement in terms of divisible distances; the identified northwest—southeast *kardo* within the settlement (Fig. 2) falling upon the 20 *actus* interval of the grid. The line of the aqueduct appears to be built within a 1 *actus* corridor,



Fig. 5. Alignments detected within the study area (dark lines) conforming to a possible 12 by 16 actus grid layout (light lines).

10 actus southwest of the line of the main valley roadway to the northeast, possibly marking the decumanus of the settlement and likely to be associated with the roadway crossing into the old city of Butrint. The northeasterly boundary of the 20 by 20 actus centuriation pattern within the valley is also offset from the line of the valley road by 10 actus, and probably forms the decumanus maximus of the centuriation, aligning with the decumanus of the settlement. It would appear, therefore, that the division of the landscape within the valley into 20 by 20 actus plots was contemporary with the planning of the Colonial settlement upon the Vrina Plain. The similar establishment of the Augustan colony at Nicopolis in northern Greece, was also accompanied by a cadastral organization of the peninsula south of the town into centuries of 20 by 40 actus [1, p. 139].

It appears from the analysis that the best preserved divisions run northwest—southeast along the axis of the valley (see Fig. 6) and it is possible that these have survived as track ways for communication and travel, whereas those at right angles spanning the shorter distance across the valley have tended to disappear. Public rights of way would have been

planned into the centuriation network, although the relatively small scale of the centuriated area within the valley may have only led to the construction of narrow tracks along major *limites*. The modern road running southeast from Vrina appears to follow one of these divisions for a short distance, before diverging along the line of one of the major collector channels connected with the modern drainage and irrigation network.

The main 'century' divisions are likely to have been further sub-divided into smaller plots. Internal *limites* within a century or *limites intercisivi*, literally 'baulks which cut inside' often consist of a single division in each direction, dividing the century into four equal parts [8]. Division into a series of strips is also fairly common, as seen at Padua and Asolo, where centuries are divided into 20 by 5 *actus* strips [8]. A brief field investigation of the area immediately southeast of the village of Vrina, which appears to be bound by a century square (Fig. 6), was undertaken in 2004. Several of the narrow parallel field divisions visible within the AP image are still discernable, surviving as low, denuded banks with occasional hedge plants. Although a similar organization of arable land seems to have

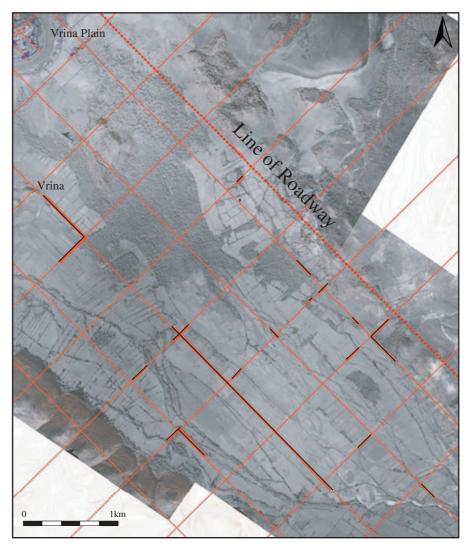


Fig. 6. Alignments detected within the study area (dark lines) conforming to a 20 by 20 actus grid layout (light lines). Note the possible survival of 20 by 5 actus strips of southwest Vrina.

existed during the medieval period (evident from the AP images), it is possible that within this area, land divisions survive more or less intact from the Roman period.

5. Conclusions

The study of the Roman landscape at Butrint using aerial photographic data and computational routines utilizing the Radon transform was found to offer a fast and efficient procedure for identifying linear landscape features conforming to specified alignments, within a spatially extensive and complex data set. The study has led to the identification of two systems of landscape organization associated with the Roman settlement. The first, likely to predate the establishment of the colony, divides the landscape into 12 by 16 *actus* units, orientated with the existing roadway flanking the low lying hills along the northeastern edge of the valley. The second phase of organization, which seems to be associated with the establishment of the settlement upon the Vrina Plain is orientated upon the

same alignment, using land divisions of 20 by 20 *actus* and relates directly to the layout of the settlement street grid. This type of investigation highlights the potential of examining the vestiges of agricultural field systems preserved within historical aerial photographic data and a large quantity of similar AP images, dating from the Second World War and the cold war periods, are potentially available within archives for many areas of the globe.

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Appendix A. Supplementary information

Supplementary information for this manuscript can be downloaded at doi:10.1016/j.jas.2005.10.012.

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