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What is the Meaning of the Physical Magnitude 'Work'?

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Abstract Usually, in physics textbooks, the physical magnitude 'work' is introduced as the product of a force multiplied by its displacement, in relation to the transfer of energy. In other words, 'work' is presented as an internal affair of physics theory, while its relation to the world of experience, that is its empirical meaning, is missing. On the other hand, in the history of its creation, 'work' was a concept that had empirical meaning from the start. It was constructed by engineers to measure the work (labor) of motor engines, men, and animals. Very soon however this initial meaning seems to vanish. In this article, it will be looked at how 'work' is presented in physics textbooks, what was its initial meaning in the history of its formulation, under what circumstances this initial meaning faded, and how elements from the history of its creation can be used in the classroom to teach it.

1 Introduction

Reading on 'work' in any textbook of physics (with a few exceptions), you have the impression that 'work' is a theoretical concept, which is defined by other theoretical concepts (i.e. force), and is related to some additional ones (i.e. changes of kinetic energy or transfer of energy), but has no direct relation to the world of experience, in other words, it has no empirical meaning.[1] You cannot also suspect that it had a mundane, down to earth, history. In fact, 'work' was created (in the beginning of the nineteenth century) to measure the work of motor engines, and additionally of men and animals. So, it was directly connected to the affairs of the world, and its meaning had a clear empirical component. In this study, it will be examined how 'work' is presented in physics textbooks, what was its initial meaning in the history of science, and under what circumstances this initial meaning vanished. Also, it will be looked at how the stages of the creation of 'work' as a measure of work, and its inclusion within the theory of mechanics, can be used to teach it in secondary education.

[1] On the empirical meaning or the empirical component of the meaning of a scientific concept see Baltas 1988.

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2 'Work' in Contemporary Physics Textbooks

Usually, in physics textbooks for the secondary education, the physical magnitude 'work' is presented in two ways. The one is to introduce it directly as a way to transfer energy from one body to another, and as the measure of this energy (see Dobson 1995; Chadha and Sang 2009 etc.). The other is to introduce it, in a more formal manner, as the product of force multiplied by displacement, without justifying its introduction. Only later, in the applications, 'work' is connected to the transfer of energy (see Tsokos 2010; Marshall and Jacobs 2004 etc.). In both ways 'work' is introduced without a direct anchor to the world of experience.

According to Chadha and Sang (2009) for example, the 'work' done by a force F acting on an object, which is calculated by $W = F \cdot x$, increases the kinetic energy or the gravitational potential energy of the object, and in fact is energy transferred. According to Tsokos (2010), 'work' done by a force is defined by the equation $W = F \cdot s \cdot cos\theta$, and later it is shown that $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, namely that 'work done by net force is equal to the change of kinetic energy'.

The second way to present 'work' is dominant among university textbooks (see Serway and Fauhn 2002; Ohanian and Markert 2007; Serway and Jewett 2010 etc.). In Serway and Jewett (2010), for example, 'work done by a constant force' is defined as $W \equiv F\Delta r cos\theta$. After that, 'work' is connected to kinetic energy in the 'work-kinetic energy theorem', that is $W_{ext} = K_f - K_i = \Delta K$, where W_{ext} is the total 'work' of the external forces, K_f the final kinetic energy, and K_i the initial one.

In other words, 'work' is presented as a rather internal matter of physics' theory, and its empirical meaning is missing. Only its systemic (within theory) meaning[2] is displayed, through its relation to the energy transfer and the changes of kinetic energy.

3 The Emergence of 'Work' in the History of Science

In the history of its creation, 'work' was a magnitude with empirical meaning from the beginning. It was created to measure the work (or the produced effect) of motor engines, men, and horses. The creation of 'work' as a magnitude of mechanics occurred in the beginning of nineteenth century, when two different eighteenth century practices were met. The first was carried out by philosophers and mathematicians, and was about theoretical calculations related to the 'living forces' (vires vivae). The second was undertaken by engineers, and concerned practical calculations related to the effectiveness of motor engines. In both practices, philosophers, mathematicians and engineers were using the products 'weight multiplied by height' or 'force multiplied by displacement'. The physical magnitude 'work' emerged from these practices and these products.

4 Calculating Vis Viva

Leibniz and his followers calculated the 'force' of a moving body ('vis viva' or 'living force') by the product 'weight multiplied by height' or more generally by 'force multiplied by displacement'. Leibniz published his first article on the subject in March of 1686 under

[2] According to Baltas, the meaning of a concept, apart from its empirical component, has also a systemic component, which is determined by the position this concept occupies within the conceptual framework and its relations with the other concepts of the system (Baltas 1990).

the title *‘A brief demonstration of a notable error of Descartes and others concerning a natural law’ (Brevis demostratio erroris memorabilis Cartesii et aliorum circa legem naturalem)*. He rejected the Cartesian view that the measure of a moving body’s ‘force’ was its ‘quantity of motion’. His reasoning was based on two assumptions:

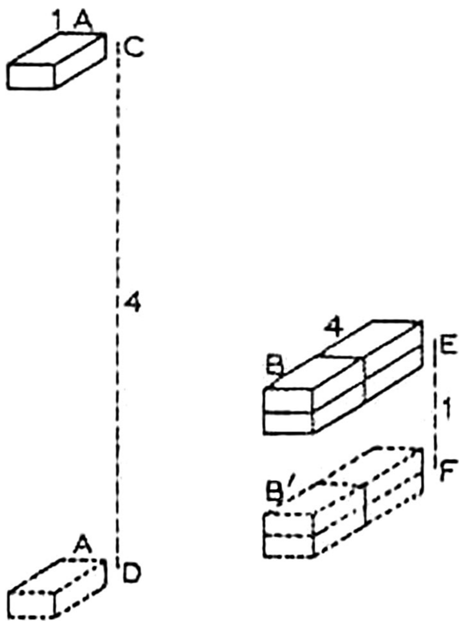
1. ‘A body falling from a certain altitude acquires the same force which is necessary to lift it back to its original altitude.’, and
2. ‘The same force is necessary to raise a body A (Fig. 1) of 1 pound [libra] to the height CD of 4 yards [ulnae] as is necessary to raise the body B of 4 pounds to the height EF of 1 yard’ (Leibniz 1989a, p. 296).

Using these assumptions, Leibniz proved that while the ‘forces’ of these two bodies were the same, their ‘quantities of motion’ were different. Consequently:

> … force is rather to be estimated from the quantity of the effect which it can produce; for example, from the height to which it can elevate a heavy body of a given magnitude and kind but not from the velocity which it can impress upon the body (Leibniz 1989a p. 297).

According to Leibniz, philosophers and mathematicians of his time knew and accepted his two assumptions. In fact, the 2nd assumption was the main principle of Descartes’ short treatise on statics (Descartes 1996). From a letter to Leibniz from the philosopher Arnauld, it seems that Leibniz was not aware of Descartes’ short treatise before *‘Brevis demonstratio’* (Freudenthal 1986). In a letter to Arnauld in November 1686, Leibniz remarked that in the meantime he had read Descartes’ work on statics, and pointed out that ‘if he [Descartes] had remembered this when he was writing his principles of physics, he might perhaps have avoided the errors into which he fell regarding the laws of nature’ (quoted in Freudenthal 1986, p. 219). Freudenthal underlines Leibniz’s experience from the silver mines of Harz. Leibniz was from 1676 at the service of the duke of Brunswick, who ruled Hanover, and he got involved in the affairs of the mines of Harz. In order to stabilize the supply of the water current, which was moving the mines’ water wheels, Leibniz proposed

Fig. 1 Leibniz: Descartes error



to use wind-mills in order to raise and stock water on high ground reservoirs (Freudenthal 1986; Antognazza 2009). Hence, he was probably aware of the water's 'force', and the significance of both the water's height and the water's weight to the magnitude of this 'force'.

For Leibniz, there were two kinds of 'force'. The first kind was the 'dead force' (vis mortua), which was present in cases where it did not exist motion but only a solicitation to motion. Such 'force' was the centrifugal force, the force of gravity, the force of a stretched elastic body etc. The other kind was the 'living force', the 'force' of bodies in motion. 'Living force arises from an infinite number of continuous impressions of dead force' (Leibniz 1989b, p. 438). The measurement of 'living force' was based on the metaphysical principle that the result is always equal to the cause which produces it (Leibniz 1989c, p. 444). The result must be equal to its cause; otherwise we would have mechanical perpetual motion, which is impossible (Leibniz 1973).

A body's 'living force' diminishes only to the degree that the body gives 'force' to other neighboring bodies. Even in the case of inelastic collisions, where 'living force' appear to vanish for good, 'living force' went to the constituent small parts of the bodies, and did not in fact diminish.

> I have affirmed that active forces are preserved in the world. The author [Clark] objects, that two soft or un-elastic bodies meeting together, lose some of their force. I answer, no. It is true, their wholes lose it with respect to their total motion; but their parts receive it, being shaken by the force of the concourse. And therefore that loss of force is only appearance. The forces are not destroyed, but scattered among the small parts. The bodies do not lose their forces; but the case here is the same, as when men change great money into small (Leibniz's fifth letter to Clarke, in Alexander 1998, pp. 87–88).

In the case of the vertical movement of a body, which was the only case Leibniz himself analyzed, 'living force' was calculated either from its cause (in a body's fall) or from the produced result (in the case of throwing a body upwards). In both cases what was measured was the product 'weight multiplied by height',[3] and 'living force' arose proportional to the body's mass and the square of its velocity. Johann Bernoulli, Leibniz's distinguished follower, in his '*Hydraulica*', calculated the 'living force' that a body acquires, when pushed by a stretched elastic, with the integral $\int pdx$, where p was the variable motive force of the elastic and dx the infinitesimal displacement, and proved that this 'living force' was also proportional to mv^2 (Bernoulli 1968, p. 354). These products or integrals, nonetheless, were not independent entities, expressions of a physical magnitude, but were only mathematical tools for calculations.

There was a strong opposition to Leibniz's views both by Cartesians and by Newtonians. Cartesians stated that the proper measure of a moving body's 'force' was its 'quantity of motion', whereas Newtonians insisted that 'force', which was taken either as 'momentum' (which, as it was conceived, could be destroyed in collisions) or as 'impulsive force' (or 'blow'), could not be a conserved quantity (Desaguliers 1723a; Maclaurin 1748). A central point of the dispute was how the result of this 'force' had to be measured, and what magnitude had to be employed for this measurement: the space traveled by the moving body, or the time passed? As de Mairan, later secretary of the French Académie des Sciences, had put it: a body A thrown upwards, with twice as much velocity than a body B, produces four times greater results, but in a time twice as much. If we will calculate these results in the unit of time, they will arise only two times greater in the body A, and a body's 'force' will come out to be proportional to its velocity, and not to

[3] Leibniz did not express it explicitly as product. The multiplication of two quantities that were not pure numbers was in the 17th century mathematically forbidden (Ravetz 1961).

the square of the velocity (de Mairan 1728). Behind these arguments, we can see the persistence of the opponents of the 'living forces' to the instant action of the bodies' 'forces', whereas the supporters of the 'living forces' were insisting to calculate the overall result of these 'forces' in a depth of time.

Experimental investigations on the moving bodies' 'force' were accomplished by Giovanni Poleni in Padua, Willem's Gravesande in Leiden, and later by Desaguliers (1744) in London. Poleni's experiments were published in 1718, in his book *'De Castellis per quae Derivantur Fluviorum Aqua Habentibus Latera Convergentia Liber'*. Poleni examined experimentally non elastic collisions, supposing that the produced deformations were proportional to the bodies' loss of 'force'. He let hard balls to fall upon soft materials, and found that the magnitude of the produced cavities, and consequently the bodies' 'force' (since all 'force' was lost) was proportional to the balls' weight and the height of their fall (Maffioli 1994). The description of the experiments was translated in English by Desaguliers as follows (by Desaguliers' own words and grammar):

> I took a vessel, that had in it congeal'd tallow six inches deep, and fix'd it to a level floor, in such manner, that the surface of the tallow, which was flat, should every where be equally distant from the floor. I had caused to be made two balls of equal bigness, the one of lead, the other of brass, the last of which was a little hollow in the middle, that it might weigh but one pound, whilst the other weigh'd two. Suspending these balls from the ceiling by threads, in such manner, that the lighter ball hung over the surface of the tallow, from twice the height that the heavier ball did, I cat the threads, and the balls falling perpendicularly upon the tallow, by their fall made pits in the tallow, that were precisely equal: the ball of one pound, from the beginning of its fall, till it came to rest, going through a space express'd by the number two, produc'd an effect equal to that, which the two pound ball did produce, in falling thro' a space express'd by the number one. It follows therefore, that we may look upon it as a settled truth, that the active forces (vires vivas) of falling bodies are then equal, when their proper weights are in a reciprocal ratio of the spaces, which the said bodies describe by their fall. And because these spaces are in the same ratio, as the squares of the numbers expressing the velocities; it appears by the experiment, that the active force (vim vivam) of the falling body is that which is made up of the body itself, multiplied into the space described in the fall, or into the square of the number, that expresses the velocity of the body, at the end of the motion. The experiment I did not only make once, but several times, changing the balls, the distances, and the body on which they fell, as for example, making use of clay, or soft wax: and notwithstanding these various ways of trying the experiments, the effects were constantly the same; which made me easily conclude, that there was always the same reason in nature for this phenomenon (Desaguliers 1723b, pp. 285–286).

's Gravesande had undertaken similar experiments, and published them in 1722, seeking to find what was the measure a moving body's force: its quantity of motion ($m \cdot v$) or its 'living force' ($m \cdot v^2$)? He was measuring the 'action of the force', since 'the action of the force is equal to the force that the body had lost by this action' ('s Gravesande 1774 p. 227). In the most known of his experiments, hollow copper balls of the same diameter, but with different weight due to different fillings, were let to fall upon soft clay, and the action of their 'force' was measured by the cavity on the clay.'s Gravesande found that the 'force' of a ball increased as the height of its fall, and concluded that the 'force' of a moving body was proportional to the square of its velocity ('s Gravesande 1774, pp. 228–229).

Although there was no disapproval to the experiments, there were many objections to their interpretation. Most adversaries focused on the time of action and to its effect on the outcome of collision. For example, according to Desaguliers, 'when two bodies move with equal forces but different velocities, that, which moves the swiftest, must make the deepest impression' because '[the clay's] parts have so much less time to oppose this body's motion' (Desaguliers 1723b, pp. 286–287). As also stated by Samuel Clarke:

> When a body projected with double velocity, enters deeper into snow or soft clay, or into a heap of springy or elastic parts, than in proportion of its velocity;'tis not because the force is more than proportional to the velocity; but because the depth it penetrates into a soft medium, arises partly from the degree of the force or velocity, and partly from the time wherein the force operates before it be spent[4] (Clarke 1727, p. 387).

'Living forces', due to the attacks of Cartesians and Newtonians, were being put at the margin of physical philosophy until the end of the eighteenth century, and were only used in special cases, i.e. elastic collisions (Iltis 1970, p.140). Great eighteenth century mathematicians, however, connected the acquired 'living forces' of a system of bodies to the summary of the products 'force multiplied by displacement', in a kind of broadened 'conservation of living forces' (d'Alembert 1758, p. 252–258; Lagrange 1788, pp. 206–227).[5]

5 Calculating the Effectiveness of Motor Engines

5.1 Analyzing Water Wheels

A systematic study of water-wheels' function was published by the French mathematician-engineer Antoine Parent at the beginning of the eighteenth century. Parent focused on the impact of the water current against the wheel's blades, and the impact forces the water exerted on the blades, and overlooked the direct action of the water's weight on the wheel, due to gravity (Parent 1704, pp. 325–333). The majority of water-wheel studies in the eighteenth century, following Parent's mode of analysis, were examining water-wheels statically, focusing on the momentary actions of the forces and the water's collision on the blades, and were not calculating the wheel's accumulated result in a given time interval.[6] The French engineer and mathematician Henri Pitot, for example, in a memoir on water wheels, published in 1725, wrote:

> In all engines, what is produced by the moving force, or (for engines moved by water) the force of the water's impulsion against the blades multiplied by the speed of the blades, is always equal to the product of the weight moved by the engine multiplied by its speed. So, if we call x the speed of the blades, t the force of the impact, P the weight moved by the engine, and v its speed, we will have $t \cdot x = P \cdot v$ (Pitot 1725, p. 79).

In Britain, the engineer John Smeaton performed a series of experiments on models of water-wheels and wind-mills, and checked his results (which contested Parent's calculations) with measurements on real engines. Smeaton measured the water-wheel's produced 'power' by the product of the raised weight and its height of elevation. And, for an overshot wheel, he measured the 'power' supplied by the water's fall with the product of the water's weight and the height of its fall (Smeaton 1759). In 1776, he presented another series of experiments that examined the relation of the 'mechanical power' given to a body (measured by the weight of a descending body multiplied by the vertical height of its descent) and the speed which it could acquire (Smeaton 1776). Although he never used the words 'living force', Smeaton took implicitly the part of the 'living forces' on the *vis viva*

4 This is a typical case of disrupted communication. They were using the same word ('force'), but they were meaning different things. For the advocates of 'momentum', 'force' meant instantaneous action, and time counted, whereas the supporters of the 'living force' were interested in the total outcome of motion, and time was irrelevant.

5 See also Kanderakis (2010).

6 See for example Pitot (1725), Desaguliers (1734), Triewald (1734), Hutton (1796), and Belidor (1819).

controversy, concluding that 'mechanical power' is proportional to the square of the speed it produces (Smeaton 1776). Smeaton's experiments were highly respected by the British engineers, and became for decades the standard reference on water wheels' studies (Reynolds 1973, pp. 317–318, 340–346). In France, Smeaton's articles were published with great delay in 1810, in a book titled *'Recherches Expérimentales sur l'Eau et le Vent'*, and they were immediately incorporated into the French technical literature (Reynolds 1973, pp. 389–390).

Meanwhile, in France, the engineer Jean Charles Borda used 'living forces' and their conservation to analyze water wheels, and calculated the accumulated result of the engines by the product 'weight multiplied by height' (Borda 1770a, b). His work, however, was ignored for the whole eighteenth century, probably because he used the disrespectful 'living forces' (Reynolds 1973, pp. 487–505).

5.2 Steam-Engines and their 'Duty'

During the eighteenth century, steam-engines were multiplying in Britain in order to drain mines. Engineers were measuring their 'duty' (effectiveness) by the number of pounds (lbs) of water they could raise one foot high, in a minute or with the consumption of one bushel[7] of coal. Another measure of the steam-engine's 'duty' was the 'horse', which, according to the value fixed by Watt, was equivalent to 33,000 lbs raised 1 ft high in 1 min.[8] These measures were originally connected to the raising of water, that is the engine's product, and were not universal measures for the steam-engine's work. Towards the end of the century, steam-engines were being used to move other engines, such as machinery in textile mills, but the engineers kept the previous quantities as measures of the steam-engines' work. The intermediate for this transfer of use was probably the 'horse'. Steam-engines were replacing horses in textile and other factories, and their capacities, quite naturally, were estimated by 'horses', and consequently by foot-pounds per minute.

6 The General Theory of the Engines in Motion and the Formulation of the Physical Magnitude 'Work'

'Work' acquired the status of a physical magnitude, incorporated in the theory of mechanics, in France, during the first decades of the nineteenth century, when rational mechanics met practical engineering to build a general theory of the moving engines. The main persons who created this theory were Navier, Coriolis and Poncelet,[9] polytechniciens and teachers in French higher engineering schools.

6.1 Navier and the Footnotes on Belidor

Louis Navier, engineer and teacher of Applied Mechanics at the École des Ponts et Chaussées, edited in 1819 a new publication of Belidor's *'Architecture Hydraulique'*, and sketched a general theory for the moving engines in its footnotes and additions. His main magnitudes were the 'living force' and the 'quantity of action'. The latter measured the

[7] A unit of capacity (volume) for dry cargo. 1bushel = 8 gallons (Hutton 1796, entry MEASURE).

[8] See for example Cardwell (1967, 1971), Hills and Pacey (1972), and Hills (1989).

[9] Lazare Carnot had also created a theory for the moving engines (Carnot 1783), but his theory was obscure, contained almost no applications, and had little impact on the engineers' work (Darrigol 2001).

work or the produced result of an engine, and was equal to the product of the force applied by the engine multiplied by the distance traveled. This measure of work was based on the vertical raising of a weight, the model work for every other work (Belidor 1819, pp. 376–395).

> In the action of an engine, there is always an effort or pressure acting on a point while a distance is covered by this point. This remark leads us naturally to recognize that the kind of work the most proper to serve the evaluation of every other work is the vertical elevation of a heavy body. Indeed, … we always can substitute the work brought about by a given engine, whatever the nature of this work be, by the elevation of a weight. Not only in thought, by an abstraction of the mind, but in reality. Because we can dispense with the resistance, and tie a thread in the direction of its action, which passes around a diversion pulley, in the edge of which we suspend a weight equal to the effort or the pressure that the resistance exerts… The elevation of this weight will consequently represent the work of the engine (Belidor 1819, p. 377).

'Quantity of action', according to Navier, is of the same kind and the same nature with 'living force'. Let's suppose, he wrote, an 'effort or pressure' P of a force acting on a point that covers a distance p in the direction of P. If the force acts on a resisting obstacle, it will expend 'quantity of action' Pp. If, however, it acts on a mass m which recedes freely, the mass, covering the distance p, will acquire a velocity U and a 'living force' mU^2. 'Living force' and 'quantity of action' will be connected by the equation[10] $mU^2 = 2Pp$.

> We see that work done by an engine represents in reality the quantity of the living force created by the forces acting on the engine, if, instead of this, they had acted on freely receding bodies. This approach leads us to appreciate the value of Montgolfier's saying: *living force is what we pay* (ibid, p. 380).

According to Chatzis (1997), Navier seems to give priority to the notion of 'living force', and to subordinate 'quantity of action' to 'living force'. 'Quantity of action' was involved in Navier's reasoning not as an entity with autonomous physical significance, but as a mathematical intermediary useful for calculations.

6.2 Coriolis and the Engines in Motion

In 1829, Gaspard Coriolis, engineer and teacher at the École Polytechnique and then at the École Centrale des Arts et Manufactures, presented his ideas on motor engines and their work in the textbook *'Du Calcul de l'Effet des Machines'*.[11] The purpose of the book, as we can read in its preface, was to clear up what it was called 'force' or 'mechanical impulse', to give a supplement to the lessons of mechanics at the École Polytechnique and the Écoles d'Application (higher engineering schools), and to develop a theory for the motor engines which could be incorporated into rational mechanics (Coriolis 1829).

Coriolis named 'work' ('travail') the magnitude $\int Pds$, where P was the acting force and ds the infinitesimal displacement, in order to connect it with the everyday use of the term 'work'. 'Work' was used to measure the work of motor engines, people, and animals.

> We give actually to the word *work*, in this sense, the idea of an effort acted and a distance traveled in the same time. Because, we do not say that work is produced, when there is only a force acting on a stationary point, as in a machine in equilibrium. Also, we do not use the name work in a displacement which is done without a resistance to overcome. Consequently, this name is suitable to signify the union of these two elements: displacement and force (Coriolis 1829, p. 17).

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[10] Navier had already proved this equation in a long footnote on the basic theory of mechanics (Belidor 1819 pp. 103–106).

[11] A great part of the book was being circulated among his colleagues in a manuscript of 1819 (Grattan-Guinness 1984).

In 1826, a commission, consisted by Laplace, Navier and Fourier, and named by the Academy of the Sciences, accepted officially the name ‘travail’ for this magnitude (Darrigol 2001).

According to Coriolis, ‘work’ was the tool to evaluate motor engines and study its most economical use. Within a given time interval, moving ‘forces’ (water, wind, animal, or coal) give restricted amount of ‘work’, and machines aim at the most economical use of it, without increasing it.

> This quantity is the main tool to evaluate motor engines in commerce. Work is what we seek to economize. This quantity is mainly related to all questions on economy as regards the use of motor engines. We produce everything necessary for our needs, displacing bodies or changing their form. These can be done,… only when we overcome resistances and exert certain efforts in the direction of the movement. Consequently, the faculty to produce displacement in this way, accompanied by force in the direction of the displacement, is not but the faculty to produce the quantity we have called work[12] (ibid, pp. 27–28).

Coriolis modified the magnitude of the ‘living force’ from $m \cdot v^2$ to $\frac{1}{2} m \cdot v^2$, in order to be exactly equivalent to ‘work’ and not simply proportional to it, and to give priority to ‘work’ rather than to *vis viva*. This modification, according to Coriolis, simplified the expression of the principles of mechanics (Coriolis 1829, pp. 18–19).

In a vertical displacement of a weight, Coriolis calculated the ‘work’ done by the product of the weight moved multiplied by the height difference, but criticized those who calculated the ‘work’ done in a horizontal displacement by the same product—mainly Coulomb (1799) and Navier (Belidor 1819 pp. 395–396). In this case, the weight is perpendicular to the displacement, and it is proportional but not equal to the resistance, and consequently to the needed motive force. The same ‘work’, Coriolis remarked, could be related to vertical forces with very different magnitude. A horse, for example, can drag, depending on the circumstances, from a small carriage to a big boat. Hence, ‘work’ is proportional to the product of the vertical force and the distance covered, but not equal to it.

6.3 Poncelet and the Industrial Mechanics

Jean Victor Poncelet was an engineer and teacher at the École de l’Artillerie et du Génie at Metz, where he was lecturing on ‘Mécanique Appliquée aux Machines’. In parallel to his official lectures, Poncelet gave a more simple series of lectures to the artisans and workers of Metz, which were published in 1829 into a lithographed book, and printed in 1870 under the title *‘Introduction a la Mécanique Industrielle Physique ou Expérimentale’* (Chatzis 1998).

Poncelet defined ‘work’[13] as the product of the applied force multiplied by the displacement in the force’s direction. Working on mechanics, he wrote, means to destroy or to overcome, in a given time, resistances which are constantly renewed, as for example the cohesion forces, the force of gravity, the force of the spring, or the inertia of matter.[14]

[12] There are elements in the writings of Coriolis (as well as Navier) that could be related to the concept of ‘value’ in economics. François Vatin examines the economic dimension of their work (and Coulomb’s research), and tries to connect the concept of ‘work’ with the concept of ‘value’ in economic theories. According to Vatin, Coriolis had been influenced by the economic theories of Jean-Baptiste Say. Although it is doubtful that Coriolis had read Say’s work, Say was teaching ‘Industrial Economy’ at the Conservatoire des Arts et Métiers, and his views were probably known among the Parisian engineers (Vatin 1993).

[13] Poncelet used Coriolis’ term (Poncelet 1870, p. 2).

[14] For Poncelet inertia was a real resisting force.

'Mechanical work' has to do with a resistance which is vanquished continuously along a distance.

> Mechanical work does not simply presuppose a resistance which is outdone once and for all, or remains in equilibrium with a motive force, but a resistance which is constantly destroyed along a distance, traveled by its point of application… In order to raise a piece of material with the help of a tool, … not only an effort is needed, directly opposite to the resistance raised by the piece of material, but the point of application of the tool has also to be pushed forward in the direction of the resistance.… Consequently, work done in each instant increases with the intensity of the effort, in the proper direction, and the length of the distance covered. An analogous reasoning is applicable to all industrial works undertaken with the help of tools or machines (Poncelet 1870, p. 64).

When the only resistance is the moving body's inertia, 'work' is transformed into 'living force'. When a body with a weight P falls along a height H, the 'work' produced by the weight will be $P \times H$, the resulting 'living force' will be $\frac{P}{g} \times V^2$, and $P \times H = \frac{1}{2}\frac{P}{g} \times V^2$. Reciprocally, a moving body can, in its turn, be a source of 'work'. When, for example, a body is thrown vertically upwards, his velocity raises it to a certain height, and the inertia of matter can restore the quantity of 'work' spent initially to overcome it. Similarly, molecular forces ('forces moleculaires') in an elastic spring can restore the 'work' spent to stretch it (Poncelet 1870, pp. 117–119).

> In a word, inertia, just as springs, tends to store up mechanical work, transforming it to living force and vis versa. Hence, living force is in reality an *available work* (ibid, p. 119).

Due to lectures and books of Navier, Coriolis and Poncelet, and lectures and books of their followers (e.g. Bélanger 1864), the new ideas on the moving engines were spread between engineers and mathematicians, and finally were incorporated within the corpus of theoretical mechanics (Darrigol 2001).

6.4 Motor Engines' Theory Become British

In Britain, the emergence of 'work' as a theoretical magnitude, useful to calculate the efficiency of motor engines, took place through two different ways: by the evolution of the empirical practices the engineers were using for years, and by assimilating and disseminating the French theories for the moving engines.[15]

British engineers, for many decades, were using the amount of pounds raised one foot high per minute to measure the effectiveness of their steam-engines. Moreover, many of them, in contrast to natural philosophers and mathematicians, were aware of Smeaton's experimental work and appreciated it. In 1805, William Wollaston, secretary of the Royal Society, gave a lecture on the moving body's force, and his lecture was published next year (Wollaston 1806). Wollaston tried to show that 'momentum' (or 'quantitas motus') and 'impetus' (living force) were both useful to study percussions, and that they were measuring different things.[16] 'Impetus', however, and its cause 'quantitas mechanica' (force exerted through space), were more useful on matters of everyday work and on motor engines:

> The former conception of a quantity dependent on the continuance of a given *vis motrix* for a certain *time* may have its use, when correctly applied in certain philosophical considerations: but the latter idea of a quantity resulting from the same force exerted through a determinate *space* is of greater practical utility, as it occurs daily in the usual occupations of men: since any quantity of work performed is always appreciated by the extent of effect resulting from their exertions: for it is well

[15] See for example Smith and Wise (1989), Smith (1998), and Wise (1989 and 1990) .

[16] See also the section on the vis viva controversy, in this article.

known that raising any great weight 40 feet would require 4 times as much labour as would required to raise an equal weight to the height of 10 feet, and that in its slow descent the former would produce 4 times the effect of the latter in continuing the motion of any kind of machine (Wollaston 1806, p. 15).

In practical matters, what was worth was 'quantitas mechanica', the force exerted combined with the space traveled.

In practical mechanics, however, it is at least very rarely that the *momentum* of bodies is in any degree an object of consideration: the strength of machinery being in every case to be adapted to the *quantitas motrix*, and the extent and value of the effect to be produced depending upon *quantitas mechanica* of the force applied, or in other words to the space through which a given *vis motrix* is exerted (ibid, p. 18).

In 1808, Peter Ewart, engineer and vice-president of Manchester's Literary and Philosophical Society, had defended Smeaton's and Wollaston's views before the Society, and his influential lecture was published in 1813. Ewart remarked that, although British natural philosophers had rejected Smeaton's views on the measure of moving bodies' force, these views had not been forgotten by the British engineers (Ewart 1813). Davies Gilbert, engineer that later would become president of the Royal Society, in an article of 1827, examined the two measures of the moving body's force, and the 'efficient power' of the motor engines (Gilbert 1827). According to Gilbert, although 'momentum' (mv) and 'impetus' (mv^2) signified different properties and were not incompatible, neither of them measured the 'efficient power' of ordinary machines. The quantity that measured this power was the force exerted multiplied by the distance traveled.

… neither impetus nor momentum have usually much to do with action of ordinary machines; which is undoubtedly true; since neither of these functions measures directly their efficient power. The criterion of their efficiency is force multiplied by the space through which it acts ($f \times s$); and the effect which they produce, measured in the same way, has been denominated *duty*, a term first introduced by Mr. Watt (Gilbert 1827, p. 26).

On the other hand, in the decades of 1830 and 1840, the French way to do physics, part of which was the French way to study engines, was transmitted to British universities (Crosland and Smith 1978; Cardwell 1990). An event that supported this transmission, at least on the part of the engines' theory, was that British engineering got the status of an academic discipline.

William Whewell, professor of mineralogy (and later moral philosophy) at the University of Cambridge, in his book *'The mechanics of engineering'*, assimilated the French studies on the theory of motor engines, and accommodated it to the British style. The magnitude that measured the engines' work, which Whewell called 'labouring force', was defined by the product of the 'resistance overcome and the space through which it is overcome' (Whewell 1841). Using this magnitude he managed to study different motor engines, and the various kinds of steam-engines. Whewell observed that 'labouring force' was what we pay for, not only for the labor of men but for the labor of machinery as well.

Labouring Force is the labor we pay for. In many cases the work to be done may be performed by various agencies; by men, by horses, by water, by wind, by steam. In these cases, that is the cheapest mode of doing the work which gives us the requisite laboring force at the smallest expense: and the price men are willing to pay, and customarily do pay, is proportional to the quantity of labouring force which they purchase. Labouring force enters the prices of articles produced by man; … But wages of labour are paid, not only for man's labour, but for the labouring force when arising from machinery. In our towns in which large manufactories exist, such establishments often generate by their machinery more labouring force than they need; and the surplus … is hired by other persons, and employed for the purposes of the most various kinds of work. … The cost is proportional to the quantity of labouring force so bought and sold (Whewell 1841, pp. 148–149).

In 1843, Henry Moseley, professor of natural philosophy and astronomy in King's College London, in his book *'The mechanical principles of engineering and architecture'*, presented Poncelet's version of industrial mechanics, including the physical magnitude 'work' and the relation between 'work' and 'vis viva' (Moseley 1856).

In 1840, an engineering chair was founded in Glasgow College, the first engineering chair in a British university, and Lewis Gordon, who had studied at the Royal Mining Academy of Freiburg and at the École Polytechnique in Paris, occupied the chair. In his lectures and research, Gordon was using the new French ideas on the study of engines, and his main magnitude was 'mechanical effect', the product of the 'effort' and the 'distance through which it was exerted' (Smith and Wise 1989, pp. 30 and 291).

Eventually, the new mechanics of 'work' (and the 'living forces') came to the hands of James Joule, William and James Thomson, Macquorn Rankine et al., and gave them the conceptual tools to formulate (with their colleagues in continental Europe) the concept of 'energy' and the new science of thermodynamics (Smith and Wise 1989; Smith 1998).

7 Discussion

'Work' was constructed by engineers as the measure of work, mainly of the motor engines, but also of men and animals, and was connected to the everyday conception of work. Consequently, its meaning had an empirical component from the beginning. During the development of mechanics, however, through its various theoretical reformulations and transformations, this initial meaning of 'work' seems to vanish.

In France, many university textbooks of the last decades of the nineteenth century introduced 'work' as a purely theoretical magnitude, without connecting it to the work of motor engines or men (see Sturm 1883; Boussinesq 1889). For example Sturm defined 'work' with the integral $\int Tds$, where ds was the arc covered by a mobile under the action of a force P during the instant dt, and T was the tangential component of the force P. 'Work' was connected then to the changes of the 'living force' by the equation $mv^2 - mk^2 = 2\int_{s_0}^{s_1} Tds$, where m was the mobile's mass, k was its initial velocity, and v its final one.

'Work' was presented in a similar way in many textbooks for the French Lycée (see Vieille 1867 and Combette 1882). Combette, for example, introduced the 'work of a constant force, whose point of application was moving in the direction of the force', as the product of the force by the distance traveled (Combette 1882, p. 600), and then calculated 'work' in various theoretical situations (many forces, variable force etc.). All his applications were on the 'theorem of the living forces': the sum of 'works' of all the forces acting on a system of bodies is equal to the half of the total change of the 'living force'.

In the English speaking world, many textbooks on mechanics were insisting for decades to connect theoretical knowledge to its applications on machines and industrial production (see for example Peck 1870). In the end, however, the abstract theoretical way prevailed. In Thomson and Tait's *'Elements of Natural Philosophy'*, the first textbook containing energy concepts, only some hints on the initial empirical meaning of 'work' were given:

> A force is said to *do work* if its place of application has a positive component motion in its direction; and the work done by it is measured by the product of its amount into this component motion. Generally, unit of work is done by unit force acting through unit space. In lifting coals from a pit, the amount of work done is proportional to the weight of the coals lifted; that is, to the force overcome in raising them; and also to the height through which they are raised. The unit for the measurement of work adopted in practice by British engineers, is that required to overcome a force equal to the

> weight of a pound through the space of a foot; and is called a *Foot-Pound*. In purely scientific measurements, the unit of work is not the foot-pound, but the kinetic unit force acting through unit of space (Thomson and Tait 1879, pp. 67–68).

The writers, still, were simultaneously establishing the standard abstract theoretical way to deal with energy concepts. After having defined kinetic and potential energy, for example, they could express the ‘law of energy’ as follows:

> The whole work done in any time, on any limited material system, by applied forces, is equal to the whole effect in the forms of potential and kinetic energy produced in the system, together with the work lost in friction (ibid, pp. 82–83)

Maxwell, on the other hand, in his ‘Matter and motion’, although he was trying to be understandable to almost everyone, introduced ‘work’ in a rather theoretical manner, without any connection to motor engines or industry.

> WORK is the act of producing a change of configuration in a system in opposition to a force which resists that change.
> ENERGY is the capacity of doing work (Maxwell 1920, p. 54).

And in a footnote:

> The work done is a quantitative measure of the effort expended in deranging the system, in terms of the consumption of energy that is required to give effect to it.
> The idea of work implies a fund of energy, from which the work is supplied (ibid p. 54).

According to François Vatin, when new concepts are created in the history of science, we often find pragmatic questions behind them, concerning human life and material production. The subsequent theoretical elaboration, however, tends to eliminate the earthly conditions under which the new scientific concepts were formulated, and bare them from their human and handy elements. A typical example is thermodynamics (Vatin 1993). Hence, it is not unusual in the history of physics, the physics scientific community, in order to lend validity and coherence to its theories, to try to give them logical-deductive structure, eliminating the initial empirical meaning of their concepts.[17] Mining this original meaning, however, can offer useful ideas to science education research, and lead to teaching proposals that help students to understand the relevant concepts.

8 Educational Implications

In order to teach ‘work’ as a measure of work for high school physics classes, a teaching sequence is designed, which follows more or less (but not exactly) the steps of the creation of ‘work’ in the history of science (Kanderakis & Danili: *Teaching ‘work’ as a measure of work*, in preparation). During the first step, students are raising weights at different heights, and an initial aspect of ‘work’ that measures the students’ work is constructed. It is easily inferred that ‘work’ is proportional to the raised weight and the height of elevation. In the second step, this measure is extended to cover new applications: first, to measure human (student) work to transport horizontally a weight, and second, to measure the work done by a motor engine (a toy car) in order to transport horizontally a small box. In the course of the third step, students generalize their findings, and are guided to a general formula to calculate the quantity of the work done or ‘work’ ($W = F{\cdot}s$).

[17] According to Kuhn, after a scientific revolution in a certain scientific field, a similar rewriting of its history usually takes place (Kuhn 1996, pp. 136–143).

The teaching sequence functions well in the classroom, but it does not connect 'work' with the other concepts of mechanics (i.e. 'kinetic energy'), and leaves it without systemic meaning. A theoretical concept, apart from its empirical meaning, in order to be part of a theory or a conceptual system and not an independent practical measure, needs also to have some systemic meaning—relations with other concepts of the system. An important relation, at least in mechanics, is between 'work' and the changes of 'kinetic energy'. In order to teach this relation,[18] some version of Poleni's and's Gravesande's experiments can be used, with some modifications especially in their goal.[19] The experiments can be conducted with balls of equal volume but of different materials and different weights, falling upon plasticine or soft clay. The magnitude of the final 'kinetic energy' can be estimated by the produced deformation of the soft material.

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[18] Meanwhile, students have to get familiar with the concept of 'kinetic energy'.

[19] Poleni and's Gravesande showed experimentally that a falling body's 'force' was proportional to the height of fall. Using Galileo's law of fall (as they mentioned it), that is, that the height of the body's fall was proportional to the square of its final velocity, they concluded that its 'force' was proportional to the square of its velocity. On the other hand, students of today can be guided in some way to construct the magnitude of 'kinetic energy', and then, doing the experiments, to infer that 'kinetic energy' is proportional to the body's weight and its height of fall (and consequently to 'work').

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