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A theory of the spoils system*

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Abstract

In a spoils system, it is axiomatic that 'to the winners go the spoils.' This essay formalizes spoils systems as cooperative games, with winners given by a simple game structure, and represented by a fixed number of political appointments. We analyze the resulting spoils games by means of the non-transferable utility value, and we offer two practical applications of the results.

1. Introduction

Since the time of Senator Marcy, it has been axiomatic that in a spoils system, 'To the winner go the spoils.' This essay offers a model of spoils systems, based on cooperative games in which utility is not transferable. The solution concept applied is the non-transferable utility (NTU) value. Aumann (1985a) and Kern (1985) have recently axiomatized this solution concept. Aumann (1985b) contains a survey of applications of the NTU-value in other economic and political contexts. Indeed, we show here that the NTU-value of a spoils game is a natural generalization of the Shapley-Shubik index of political power in simple games.

To study a spoils system in action, one must specify who the winners are and what the spoils might be. Our model describes a simple game that identifies winners as winning coalitions in a power structure. Spoils, on the other hand, are a fixed number of political appointments available for a winning coalition to fill. These two features suffice to characterize a spoils system, at least of the simplest variety that we study here. The NTU-value of the resulting cooperative game measures the probability that a player in a spoils game will receive a spoils position. This probability depends in an obvious way on the power structure and the number of available appointments.

Although the essay's main thrust is to demonstrate the usefulness of the

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NTU-value for public-choice theory, the NTU-value also appears to have some potential for practical applications. We briefly consider two such applications. One concerns the distribution of municipal employment. The case studied is Atlanta before and after the election of its first black mayor. The other concerns the incidence of unemployment in the USSR. In this case, one considers the entire Soviet economy to be a single gigantic spoils system.

Section 2 lays out the basic model and assumptions, while section 3 discusses the NTU-value itself. Sections 4 and 5 study the NTU values for games with a finite number of players and a continuum of players, respectively. Section 6 contains the practical applications. The last section discusses the dual of a spoils systems, a bads system, in which 'To the loser go the bads.'

2. Model

Society consists of n agents, numbered 1 through n, in the set N. A coalition, S, is any subset of N. Each agent would like to get a political appointment. Appointments are indivisible (no appointment-sharing), and each agent can hold at most one appointment (no double-dipping). Each agent, i, has a non-Neumann-Morgenstern utility function u_i. Let x_i denote the probability that agent i is appointed. It follows immediately that

$$u_i(x_i) = x_i u_i(1) + (1-x_i)u_i(0). \tag{1}$$

Since von-Neumann-Morgenstern utility is unique up to a positive linear transformation, one has two degrees of freedom. We normalize so that $u_i(1) = 1$ and $u_i(0) = 0$ for every player i. Substituting these values in equation (1) gives us

$$u_i(x_i) = x_i, \tag{2}$$

for every agent i. Given the indivisibility and boundedness assumptions on appointments, the probability of appointment adequately reflects utility in the model, regardless of agents' attitudes toward risk.

To describe the workings of a spoils system, one must first specify who the winners are in such a system and what spoils they might receive. We assume that there is a fixed number of appointments, k, between 1 and n, which an agent could fill. A set of winning coalitions, W, describes the system's political structure. A winning coalition has the power to make appointments, but only among its own members.[1] W satisfies these conditions:

(i) if S in W, then N/S not in W
(ii) if S in W, and if T contains S, then T in W
(iii) N in W.

These conditions mean that we can represent the political power structure by a strong, proper, monotone-simple game, which we denoted by w, and which satisfies

$$w(S) = \begin{cases} 1 \text{ if S in W,} \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

A coalition is *minimum-winning* if it contains no proper subset that is winning. Let the *size* of a coalition be the number of its members. Then minimum-winning coalitions are the smallest winning coalitions. We denote by s the size of the smallest winning coalition; s plays an important role in one of the results to follow.

An agent, i, has a *veto*, if he belongs to every winning coalition. For example, if the grand coalition, N, is the only winning coalition, then every player has a veto. A player is a *dictator*, if by himself he is a winning coalition. A player is *pivotal*, if his departure from a winning coalition turns it into a losing coalition. Player i is pivotal whenever there is an S satisfying the equation

$$1 = w(S \cup \{i\}) - w(S). \tag{5}$$

The Shapley-Shubik index, denoted Ψw, for the simple game w, computes the probability that each player is pivotal:

$$\Psi w = (\Psi w(\{1\}), \ldots, \Psi w(\{n\})), \tag{6}$$

such that w(i) is the probability that (5) holds for player i if S is a coalition drawn at random. For example, $\Psi w(\{i\}) = 1$ if i is a dictator; whereas $\Psi w(\{i\}) = 1/n$ under majority rule.

We now turn to the spoils game itself, which we denote by v. Suppose, first, that coalition S is losing. Then it has no spoils to distribute, and v(S) is simply a vector of 0's, one for each member of S. If S is winning, then the situation is more complicated, because now S must distribute the spoils among its members. Here two possibilities arise. First, if there are at least as many appointments available as members of S, then every member of S gets an appointment and v(S) is a vector of 1's. Otherwise, there are not enough appointments to go around, and they must be rationed among the members of S. We lose no generality by assuming that rationing takes the form of a lottery:

$$0 \leq x_i \leq 1, \text{ for every i in S: } \sum_{\text{i in S}} x_i \leq k. \tag{7}$$

In this case v(S) is the set of utility vectors that satisfy (7).

The following representation of the spoils game, v, proves useful in the sequel. Let $\lambda = (\lambda_1, \ldots, \lambda_n)$ be a vector of positive numbers. Let f(S) be the maximum of $\Sigma\, \lambda_i\, x_i$, subject to (7); and define v_λ as

$$v_\lambda(S) = f(S)w(S). \tag{8}$$

If we interpret λ as a vector of distributional weights, one for each player, then v_λ is the social-welfare function corresponding to game v. At the same time, we can think of v_λ as a game in its own right. Indeed, we study v_λ to find the NTU-value of v, a matter to which we now turn.

3. The NTU-value

Let v_λ be a game with transferable utility. Define player i's *marginal product* to be the difference between what a coalition can do with player i and what it can do without him. The Shapley value of player i in the game v_λ, written $\Psi w_\lambda(\{i\})$, is the expected value of i's marginal product, such that we take the expectation over a random ordering of the set of players. For example, the Shapley-Shubik index is a special case of the Shapley value, defined for simple games, in which a player's marginal product has either the value 1 (as in equation (5)) or 0.

The NTU-value extends the Shapley value to non-transferable utility games. $u = (u_1, \ldots, u_n)$ is a NTU-value of the spoils game, v, if there exists a nonnegative nonzero vector λ, such that

$$\lambda_i u_i = \Psi v_\lambda(\{i\}) \tag{9}$$

holds for every player, i, with v_λ given by equation (8). The idea behind equation (9) is to pretend that utility is transferable according to distributional weights, λ, and then to compute the Shapley value, Ψv_λ. This accomplished, we rescale utility according to equation (9) to recapture a utility value that is meaningful in the original game, v. Of course, we must find a λ that satisfies equation (9), to find an NTU-value. Kern (1985) proves the existence of a strictly positive λ that satisfies equation (9) for polyedral games, such as the spoils game. The only NTU-values that we study are of this variety. In particular, we then have a precise characterization of the NTU-values:

$$u_i = \Psi v_\lambda(\{i\})/\lambda_i \qquad \text{for all i.} \tag{10}$$

4. Spoils games with a finite number of players

This section considers the NTU-values of a spoils games with a finite number of players. First, we show that a positive Shapley-Shubik index leads to a positive probability of appointment.

Proposition 1. If player i has a positive Shapley-Shubik idex, then $u_i > 0$.
Proof. We construct a lower bound for u_i. By definition, player i is pivotal with probability $\Psi w(\{i\}) > 0$. If player i is pivotal, this marginal product is worth at least $\lambda_i > 0$, for instance, if he appoints himself. Thus, for any $k \geq 1$,

$$\Psi v_\lambda(\{i\}) \geq \lambda_i \Psi w(\{i\}). \tag{11}$$

Dividing both sides by λ_i and using equation (10), we have

$$u_i \geq \Psi w(\{i\}), \tag{12}$$

the desired lower bound.

In the case of $k = 1$, every player attains this lower bound. Suppose not. Then for some player, j, $u_j > \Psi w(\{i\})$. Summing across equation (12) over all i, we have

$$\Sigma\, u_i > \Sigma\, \Psi w(\{i\}) = 1, \tag{13}$$

which contradicts condition (7).

For a number of appointments greater than 1, the probability of being appointed cannot be less, and usually is greater than, a player's Shapley-Shubik index. The clearest instance of this property occurs if there are enough appointments for everyone, $k = n$. Then it follows immediately from conditions (7), that $u_i = 1$ for all i. For an intermediate number of appointments, one expects that u_i lies in between these two extremes. A nice instance of this case corresponds to a power structure such as one person, one vote, majority rule, in which every player has the same Shapley-Shubik index.

Proposition 2. Suppose that for all i, $\Psi w(\{i\}) = 1/n$. Then the NTU-value $u_i = k/n$ for all i.
Proof. We show first that there is a symmetric solution to equation (10). Set $\lambda_i = 1$ for all i. Since w is symmetric, so is v_λ, following from equation (8).

Then equation (10) requires that u_i be the same for all i. Finally, from condition (7), we have that $u_i = k/n$.

Next, we suppose that there is an asymmetric solution to equation (10). Suppose that some player does better than the rest. $u_i > u_j$, for all $j \neq i$. Then it must be that $\lambda_i > \lambda_j$. It follows from equation (7) that either $u_j = 0$ for some j (contradicting proposition 1), or that $u_i = 1$. Let λ' be the largest of the λ_j's. An upper bound on the value of i's marginal product is given by

$$\lambda_i + (k-1)\lambda',$$

and since j is pivotal with probability 1/n,

$$\lambda_i + (k-1)\lambda'/n \geq \Psi v_\lambda(\{i\}) = \lambda_i u_i = \lambda_i, \tag{14}$$

which is a contradiction.

Once asymmetries in the power structure occur, they will show up in the spoils-appointment probabilities. Recall that s is the size of the smallest winning coalition. A very strong sort of asymmetry is the possession of veto power.[2]

Proposition 3. If player i has veto power, then $u_i = 1$, for all $k \geq s$.
Proof. Recall first three properties of the Shapley-Shubik index, namely that all veto players have the same index, that this common index $\geq 1/s$, and that their index is greater than that of any non-veto player.

If player j lacks a veto, then an argument like that of Proposition 2 shows that $\lambda_i > \lambda_j$. A lower bound on the value of i's marginal product is

$$(s-1)\lambda_j + \lambda_i.$$

One thus has

$$\begin{aligned} \lambda_i u_i = \Psi v_\lambda(\{i\}) &\leq [(s-1)\lambda_j + \lambda_i]\ \Psi w(\{i\}) \\ &\leq [(s-1)\lambda_j + \lambda_i] 1/s \\ &\leq s\lambda_i/s \\ &= 1. \end{aligned}$$

The bound $k \geq s$ in proposition 3 need not to be strict. Table 1 contains the NTU-values of some asymmetric 3- and 4-player spoils games. Notice, for instance, that in the 4-player game, in which only player 1 has veto power and $s = 3$, $u_1 = 1$ even when $k = 2$.

A general picture emerges from Table 1. Starting with $k = 1$, the probability of getting a spoils appointment increases with k. Once veto players are appointed (certainly not later than $k = 1$), appointment probabilities increase somewhat faster for the less powerful players. Finally, at $k = n$, all members are appointed with probability 1.

Spoils games with a continuum of players

This section considers spoils games played in mass polities. Such situations occur, for instance, if the party winning the mayor's office of a large city distributes spoils, or if the only legal party in a one-party state does so. We can closely approximate games with a large number of players by games with a continuum of players. Here, then, we take the player set, N, to be the unit interval [0, 1].

We consider the case in which there are two identifiable classes of players. In a municipal election, for instance, these could be the party affiliations of members of the electorate. These also might be personal characteristics, such as race or class. For any group S, S belonging to N, define $\mu(S)$ to be the percentage of all players belonging to S, and z(S), the percentage of all votes belonging to S. Let N_i, $i = 1, 2$, be the two sets consisting of each player class. Let $\mu(S \cap N_i)$ measure how many players there are in S of class i. Thus, one has

$$\mu(S) = \sum_i \mu(S \cap N_i). \tag{15}$$

We can characterize every group of players in terms of its size, composition, and voting strength. Lorenz curves are a natural device for depicting such a situation. Figure 1, for instance, shows a case in which one class is both more numerous and stronger in terms of votes. The smaller class had 38% of the population but only 17% of the vote.

Suppose that a coalition is winning if its total vote exceeds a fixed quota, q (greater than or equal to 1/2). Then the power structure of the model describes the simple game

$$w(S) = \begin{cases} 1 \text{ if } z(S) \geq q \\ 0 \text{ otherwise} \end{cases} \tag{16}$$

For games such as (16), the Shapley-Shubik index coincides with the voting distribution:

$$\Psi w(S) = z(S). \tag{17}$$

Notice that it no longer makes sense to talk about the power of a single

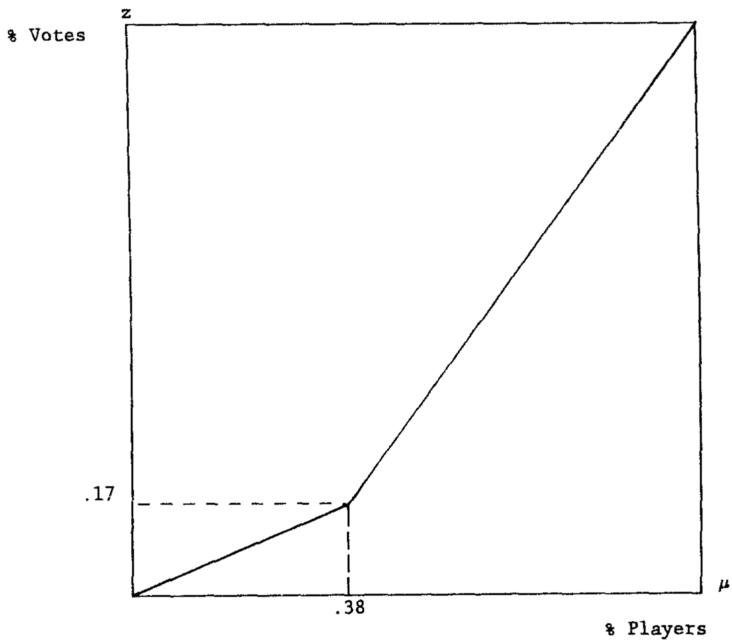

Figure 1. Lorenz curve for a model with 2 player classes.

player, because μ and w are continuous distributions.

We now represent spoils by a spoils rate k, between 0 and 1, measuring the proportion of the player set that can be appointed. Let $\lambda = (\lambda_1, \lambda_2)$ be distributional weights for each player class and x be a distribution of appointments. For instance, $x(N_i)$ represents the total appointments going to player class i. We can now define the game v_λ as

$$v_\lambda(S) = \begin{cases} \max\limits_{x} \sum\limits_{i} \lambda_i x(S \cap N_i), & \text{if S is winning;} \\ \text{subject to: } \begin{array}{l} x(S \cap N_i) \le \mu(S \cap N_i), \quad i = 1, 2; \\ \sum\limits_{i} x(S \cap N_i) \le k, \end{array} & \\ \text{otherwise} & \end{cases} \tag{18}$$

The NTU-value of the spoils game, v, satisfies this analogue of (10):

$$u(N_i) = \Psi v_\lambda(N_i)/\lambda_i, \qquad (10)'$$

such that u is the distribution of appointments at the NTU-value. The NTU-value in general depends on the six parameters $\mu(N_1)$, $\mu(N_2)$, $z(N_1)$, $z(N_2)$, q, and k, which characterize the model. Call the ratio $z(N_i)/\mu(N_i)$ the *relative strength* of player class i. If the relative strengths of the two player classes are the same, the Lorenz curve is a 45° line, and the two classes are indistinguishable. The NTU-value is the symmetric solution, just as in Proposition 2:

$$u(N_i) = k\mu(N_i), \qquad i = 1, 2. \tag{19}$$

Otherwise, an asymmetric solution occurs. The precise kind of asymmetry depends on the relative strengths and the availability of appointments. We enumerate these below.

Call a solution *interior* if the constraints $x(S \cap N_i) \leq \mu(S \cap N_i)$ in (18) are not binding. At an interior solution, the distributional weights λ_i are equal for both player classes. We lose no generality by setting this common distributional weight equal to 1. Two kinds of interior solutions are possible. The first arises if appointments are relatively scarce:

Proposition 4. Suppose $k \leq q$, and the relative strength of each player class is bound above by $1/k$. Then the NTU-value satisfies

$$u(S) = z(S)k. \tag{20}$$

Proof. We offer only a heuristic argument here. A rigorous proof along the lines found in Gardner (1981, 1984) is possible.

Consider a small player, dt, centered at the point t along the unit interval. This player has a positive marginal product only if he is pivotal. In this case his marginal product equals k, from equation (18). This player is pivotal with probability z(dt), from equation (17). Therefore, his expected marginal product in the game v_λ is kz(dt). Integrating gives us equation (20).

At this interior solution, appointments are relatively scarce. If appointments are rather more common, then the solution is somewhat more complicated.

Proposition 5. Suppose $k > q$, and the relative strength of each player class is bounded above by $(1-k+q)/q$. Then the NTU-value satisfies

$$u(S) = z(S)q + \mu(S)(k-q). \tag{21}$$

Proof. Again consider a small player, dt, centered at the point t. This player has a positive marginal product in two instances. The first occurs if he is pivotal, in which case his marginal product is q, from equation (18). The second occurs if he joins a coalition that is already winning, but that cannot fill all appointments. In this second event, his marginal product is 1 dt, the appointment he fills. Weighting these events by their respective probabili-

Table 1. NTU-values for various 3- and 4-person spoils games

	k = 1	k = 2	k = 3	Power structure
i	λ_i, u_i	λ_i, u_i	λ_i, u_i	
1	1,2/3	2,1		Minimal winning coalitions
2	1,1/6	1,1/2		{1,2} {1,3}
3	1,1/6	1,1/2		s = 2
				Player 1 has a veto
1	1,3/4	3,1	5,1	Minimal winning coalitions
2	1,1/12	1,1/3	1,2/3	{1,2} {1,3} {1,4}
3	1,1/12	1,1/3	1,2/3	s = 2
4	1,1/12	1,1/3	1,2/3	player 1 has a veto
1	1,1/2	1,1	2,1	Minimal winning coalitions
2	1,1/16	1,1/3	1,2/3	{1,2,3} {1,2,4} {1,3,4}
3	1,1/6	1,1/3	1,2/3	s = 3
4	1,1/6	1,1/3	1,2/3	Player 1 has a veto
1	1,5/12	1,5/6	5/2,1	Minimal winning coalitions
2	1,5/12	1,5/6	5/2,1	{1,2,3} {1,2,4}
3	1,1/12	1,1/6	1,1/2	s = 3
4	1,1/12	1,1/6	1,1/2	Players 1,2 have veto
1	1,7/12	7/5,1	3,1	Minimal winning coalitions
2	1,3/12	1,3/5	4/3,1	{1,2} {1,3,4}
3	1,1/12	1,1/5	1,1/2	s = 2
4	1,1/12	1,1/5	1,1/2	Player 1 has a veto
1	1,1/3	1,2/3	1,11/12	Minimal winning coalitions
2	1,1/3	1,2/3	1,11/12	{1,2} {1,3,4} {2,3,4}
3	1,1/6	1,1/3	1,7/12	s = 2
4	1,1/6	1,1/3	1,7/12	No veto player

ties, we arrive at the expected marginal product $qz(dt) + dt(k-q)$. Integrating this expression gives us equation (21).

In these results we require the conditions on relative strength to keep the solution interior. Indeed, if one player class is considerably stronger than the other, we will have a *corner solution*, at which one player class is certain to be appointed. As in Proposition 3, the player class that is relatively stronger is the one that is certain to be appointed at a corner solution. Without loss of generality, suppose that player class 1 is relatively stronger. Then any of these parameter values will lead to a corner solution favoring player class 1:

$$k \le q, \quad z(N_1)/\mu(N_1) > 1/k; \qquad (22)$$
$$k \ge q, \quad z(N_1)/\mu(N_1) > (1-k+q)/q.$$

6. Applications

This section considers two applications of spoils games, involving the two kinds of interior-solution studied in Section 5.

The first application considers urban municipal employment, in particular Atlanta in the period 1970–1978. Eisinger (1982) finds that black municipal employment rises dramatically during this period, and that black gains in political authority contribute significantly to this rise. Given that voting is by majority rule ($q = 1/2$) and that municipal jobs are only a small fraction of total employment, we have $k < q$. Let N_1 = blacks, and N_2 = others. During this period blacks are roughly one-half the population: $\mu(N_1) = 0.5$. We can apply the result of proposition 4 to this case. It follows from equation (20) that

$$u(N_1)/k = z(N_1), \tag{23}$$

such that $u(N_1)/k$ is the percentage of black municipal employment. This percentage rose from 38.1% in 1970 to 55.6% in 1978. In terms of the spoils game, then, it appears that black political influence rose on the order of fifteen to twenty percent during the 1970s. Of course, the most visible feature of this rise is the election of a black mayor. A substantial amount of the increase may reflect affirmative-action programs.[3] Nevertheless, it seems safe to conclude that in a city such as Atlanta, blacks now have a relative strength roughly equal to that of other groups.

The second application considers industrial employment in the Soviet Union during the period 1925–1928. Unfortunately, appropriate data are not available for more recent times. Here one thinks of the entire economy as a gigantic spoils system, in which the winners are given by those forces that come to control the Communist Party. Although this would appear to be an unlikely place to apply the concepts of this essay, such applications have succeeded before[4] (see Gardner, 1984). In any event, it is worth seeing what the NTU-value has to say. Here, we set N_1 = members of the Party; N_2 = others. As Rigby (1968) shows, $\mu(N_1)$ has been about .1 throughout the Soviet régime. In this case, since k is economy-wide, one can assume that q represents the percentage of people that must accept the Soviet régime, for it to rule. Somewhat surprisingly, an interior solution exists in this context, according to data from Nove (1969) and Rigby (1968). In 1925 overall unemployment was eight percent, while unemployment among Party members was only one percent. In 1928, the numbers were eleven percent and four percent, respectively. We can apply the result of Proposition 5 to this case. It follows from (21) that

$$u(N_1)/\mu(N_1) = z(N_1)q/\mu(N_1) + (k-q). \tag{24}$$

The employment rate among Party members depends linearly on their relative strength and on the total employment rate. If total employment falls by three percent, so does the employment among Party members. Moreover, one can estimate the relative strength of a Party member, given a value of q. For instance, with $q = 0.5$, $k = 0.92$, and $u(N_1)/z(N_1) = 0.99$ (the 1925 values), one has a relative strength of 1.14. The estimate of relative strength falls as the value of q rises. Being in the Party, then, would appear to make someone about ten percent more influential politically than being outside the Party.

Both these examples appear to support received assessments of political strength. To say that blacks in Atlanta have a relative strength comparable to that of other groups in the city, or that Communist Party members have a relative strength greater than that of non-Party members in the Soviet Union, is not very controversial. What is surprising is that so simple a model, which portrays an extremely complicated situation with a handful of parameters, works at all. In some sense, a cooperative game model is like a macroeconomic model. An enormous amount of strategic detail is suppressed in order to focus on a few major features of the game, in the same way that an enormous amount of microeconomic detail is suppressed in order to focus on a few major macro relationships. Such a process works, if at all, only when representative features of the situation survive the aggregation process. In the present two instances, the spoils model may simply have been lucky. There are plenty of other naturally occurring spoils systems which would be worth a closer look, from this or some other game-theoretic standpoint.

7. Extensions and conclusion

The models considered so far all concern the distribution of a good by means of a spoils system. But people can use a system to distribute a bad. We call a system that distributes bads according to the principle, 'To the loser go the bads,' a *bads system.*

Analysis of spoils systems applies equally to bads systems. For concreteness, suppose that the bad being distributed is involuntary military service.[5] Let x_i now be the probability of not being drafted. Then equation (2) holds under this reiterpretation. Let there be a draft quota, $n-k$, which a winning coalition is responsible for fulfilling. At the same time, a winning coalition also gets to run the draft; in particular, it can draft players not belonging to it to serve. Suppose resistance to the draft is futile, in that the utility of resistance is below zero.[6] Then one can interpret the characteristic function, v, as showing to what extent the members of a winning coalition can avoid being drafted. It is easy to check that the corresponding weighted utility game, v_λ is identical to equation (8). Hence, the spoils system with k appointments and the bads system with $n-k$ bads have the same NTU-value. The probability of being appointed to a spoils position is the same as the probability of avoiding the draft.

We have applied the theory of spoils to municipal employment in a large city and to industrial employment in a one-party state. Certain other potential applications come to mind. Any selection process that involves entry-restriction on the part of a governing body (medical licensing boards, for example) would be strategically quite analogous to a spoils system. The spoils game could incorporate testing procedures and thus approach the form of a model of civil service.

One somewhat restrictive assumption in our analysis is that all positions are the same. We could allow for heterogeneous positions, in which case the winners must also solve an assignment problem, to match up their members best with the available slots. A model like this might prove useful in analyzing cabinet selection in the United States or the distribution of ministerial portfolios in Western Europe. Another extension concerns the one-man-one-position assumption. There are cases of spoils systems where an individual may hold more than one position. The model can be extended in this direction, and the symmetric solutions result (Proposition 2) continues to hold.

Shepsle's analysis (1978) of Committee assignments in the U.S. House of Representatives is an intriguing example of both these lines of generalization. First, although the rules are silent about this, the percentage of committee assignments going to each party is almost exactly equal to the percentage of seats held by each party.[7] This is just what one would expect in a situation where political power is proportional to the number of seats held. Second, the composition of assignments by committee varies widely. On three committees (Appropriations, Rules, Ways and Means) the majority party always holds an extraordinary majority, exceeding three-fifths, while on the Standards of Official Conduct assignments to the two parties are equal. A detailed game-theoretic analysis of this example might prove to be very rewarding.

This essay argues that, regardless of the environment in which they appear, all spoils systems share a common strategic nature. The resulting non-transferable utility cooperative games, analyzed from the standpoint of the NTU-value, yield quantitative results on the relationship between an index of political power (the Shapley-Shubik index) and the distribution of spoils. 'To the winner go the spoils,' far from being a mere truism, is a proposition with empirical content.

NOTES

1. Alternatively, we could assume that a winning coalition could fill appointments with players outside the coalition. A study along these lines would lead to the Harsanyi value of the game.
2. This result provides an interesting contrast to the core. In the core, as soon as k is as great as the number of veto players, every veto player is certain to be appointed.
3. The legal details of affirmative action are much more complicated that the game model can capture. Title VII legislation, requiring nondiscriminatory hiring, did not apply to cities until 1972. Before this date, there was no legal limit upon patronage. In 1973, a group of black police officers on the Atlanta police force filed suit against the city, charging discrimination by the then white major in hiring and promotion. Later, white officers from the force, represented by the Fraternal Order of Police, intervened in the suit. By this time, the lawsuit was three-sided – mayor, black officers, and white officers – and the mayor was now black. The lawsuit was resolved by a consent decree in December 1979, calling for racial quotas in the hiring and promotion process. After administering a standard test, the quotas would be filled from the top down on separate white and black eligibility lists. Besides the affirmative action case, there may have been other legal factors at work in the municipality that affected the racial composition of the municipal work force. I am grateful to Charles Shanor of the Emory Law School for bringing this material to my attention.
4. There is no reason in principle that game theoretic models should not apply to totalitarian political systems like that of the Soviet Union. The concepts of political power and winning coalition transcend differences among political systems. For the present application, it suffices to think of the entire industrial sector of the Soviet Union as being under the control of the Communist Party, a view which is not so far-fetched. The Party was somewhat successful in preventing migration from the countryside to the cities, for example by instituting internal passports during this period. Moreover, Party members were already firmly entrenced in industrial management positions. However, the present model does ignore the power struggle within the Politburo for control of the Party leadership, a struggle ultimately won by Stalin. Indeed, Stalin suspended publication of data on unemployment, party composition, and other relevant data for the next twenty-five years.
5. In a one-party state, involuntary reeducation or other terms of forced labor would provide other examples.
6. If resistance to the draft if not futile, then the strategic situation is much more complicated. For one possible way of modelling this possibility, see Gardner, 1981.
7. Table 6.1, p. 110, in Shepsle (1978) shows that the difference never exceeded 2% in the period 1958–1974. Table E6, p. 274, gives data on committee composition.

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