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Human Dental Arch Shape Evaluated by Euclidean-Distance Matrix Analysis

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ABSTRACT Form differences between biological structures can be evaluated using several approaches. When landmark data are available, a recently proposed method (euclidean-distance matrix analysis) seems to be able to differentiate between size and shape differences. This method also localizes those areas which differ most between the two structures. We have applied it to analyze the sexual dimorphism in dental arch form in a sample of 50 men and 45 women. Subjects ranged in age between 20 and 27 years, and had sound dentitions. Fourteen landmarks, corresponding to the centers of gravity (centroids) of the occlusal surfaces of all permanent teeth (right second molar to left second molar), were individualized on the dental casts of subjects. All the possible linear distances between pairs of teeth were computed, thus creating four mean form matrices (one for each arch within sex). Gender differences were tested by using euclidean-distance matrix analysis. No significant differences were demonstrated in the shape of arches, while male arches proved to be slightly bigger than female arches.

Form differences between biological structures can be evaluated using several approaches. The choice of approach depends on the nature of the investigated structure, and on the hypothesis being tested. Macroscopic differences can be appreciated by visual inspection, but such subjective evaluation cannot be quantified.

Since the beginning of this century, morphometrists have sought to objectively quantify the forms of biological objects, as well as their differences and changes (Bookstein, 1984; Lestrel, 1989). The form of any object can be viewed as a combination of size and shape. While size and its changes have been easily quantified in almost all settings, the quantification of shape is still unsatisfactory. This is clearly reflected by the large number of algorithms developed for this purpose. Besides the conventional metric approach based on linear distances, angles and ratios, two main categories of procedures have been developed so far: homologous-point representations and boundary representations (Lestrel, 1989).

Boundary representations deal with the outlines of the objects: a curve-fitting procedure (usually a harmonic analysis such as the Fourier series) is used to compute a mathematical function which will describe the object's profile, and which will be used to compare different objects, or the same object through time. Both classic Fourier descriptors and elliptic Fourier descriptors have been applied in the study of craniofacial structures (Ferrario et al., 1990, 1991b, 1992a; Halazonetis et al., 1991; Lu, 1965).

Homologous-point representations individualize a set of homologous landmarks on the forms to be compared, and then analyze how the relationships between these points

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change. These landmarks have the same definition for geometrical location, and are biologically homologous (Bookstein, 1984). Several algorithms have been developed: superimposition methods (Lele, 1991), finite-element analysis (Baluta and Lavelle, 1987; Corner and Richtsmeier, 1991; Lele, 1991; Lestrel, 1989; Moss et al., 1985; Richtsmeier et al., 1990), biorthogonal grids (Bookstein, 1982, 1983, 1984; Lestrel, 1989), and, more recently, Euclidean-distance matrix analysis (Corner and Richtsmeier, 1991; Lele, 1991; Lele and Richtsmeier, 1991).

Euclidean-distance matrix analysis (EDMA) compares the form of two objects (or of two groups of objects) defined by a group of homologous landmarks. EDMA first calculates all the possible euclidean distances between the selected landmarks on a single object, and then compares the two objects by calculating a matrix of ratios of corresponding linear distances measured on each object. EDMA has already been applied to the study of craniofacial morphology (Corner and Richtsmeier, 1991; Lele, 1991; Lele and Richtsmeier, 1991), but it may also be useful for the study of dental arch shapes.

Two main mathematical approaches have been used for dental arch shape classifications and comparisons: the fitting of a mathematical equation to the set of points individualized on occlusal surfaces (BeGole, 1980; Biggerstaff, 1972; Ferrario et al., 1991a,c; Jones and Richmond, 1989; Lu, 1966; Pepe, 1975; Richards et al., 1990; Sampson, 1981, 1983), and finite-element analysis (Baluta and Lavelle, 1987).

When dental arches are described by a mathematical equation, form differences are usually tested by comparing equation coefficients (Ferrario et al., 1991a; Richards et al., 1990). This approach is seldom satisfactory because it does not provide an indication of which areas contribute to the shape difference. On the contrary, EDMA provides not only an objective measurement of shape differences, but also localizes the sites of major variations by suggesting which landmarks are more interesting in the form difference (Corner and Richtsmeier, 1991; Lele and Richtsmeier, 1991). Moreover, it does not involve any prior assumptions about arch form.

We have already investigated the morphology of dental arches in healthy young adults and in children (Ferrario et al., 1991a,c) using a classic mathematical interpolation of the buccal cusps. The equations provided a good description of arch form, but shape differences could not be appropriately tested. In this study EDMA is applied to analyze the gender differences in dental arch shape in a group of young adults in a narrow age range with sound dentitions.

MATERIALS AND METHODS

Sample

Fifty male and 45 female subjects aged 20–27 years (mean = 22) were screened from a group of 160 healthy white (Caucasian) dental students by a detailed questionnaire and verified through clinical examination. All subjects gave informed consent.

The following criteria were used in the selection: 1) absence of moderate or severe clinical mandibular disorders; 2) absence of extensive restorations, cast restorations, or cuspal coverage; 3) no previous or current orthodontic treatment; 4) absence of anterior or lateral cross-bite; 5) absence of pathologic periodontal condition; and 6) clinically normal arch shapes with minimal dental crowding.

Dental arches of the 95 subjects were reproduced by alginate models (Naturalgin, NaturalDent, Italy) cast in stone (Zeus Magicwhite, Zeus, Alessandria, Italy).

Digitization of arches

Standardized photographs of models were obtained using a Polaroid 3" CU-5 camera (frame 1:1) with Polaroid Instant Pack 665 ISO 80/20° black and white films (Polaroid Corporation, Cambridge, MA). A 13 cm camera-model distance was kept for all photographs. Moreover, models were photographed together with a ruler which allowed a careful control of magnification. Photographs were taken using a special box which allowed orthogonality between the camera and the occlusal surface. For each model two photographs were taken: a) maxillary occlusal surfaces; b) mandibular occlusal surfaces.

In both photographs the midpoints of incisal edges and canine cusps and the buccal

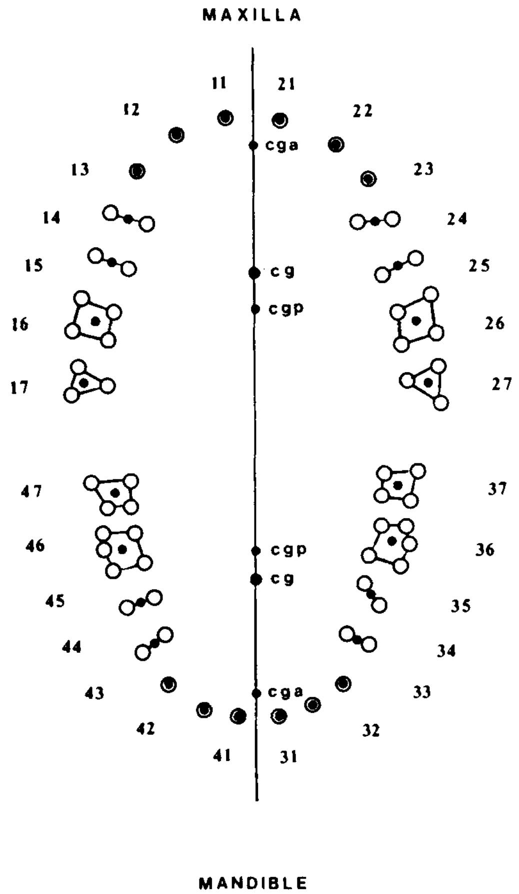

Fig. 1. Digitized points on the occlusal view of maxillary and mandibular dental arches. The open circles are the dental cusps, the small black dots are the dental centroids. The centers of gravity of arches (cg, total; cga, anterior teeth—incisors and canines; cgp, posterior teeth—premolars and molars) are also indicated. Each tooth is labelled with its two-digit WHO codification. The first digit localizes the quadrant (1 = upper right, 2 = upper left, 3 = lower left, 4 = lower right), the second digit identifies the tooth (1 = central incisor, 2 = lateral incisor, 3 = canine, 4 = first premolar, 5 = second premolar, 6 = first molar, 7 = second molar).

and lingual cusps of premolars, first and second molars were traced (Fig. 1). Coordinates of cusp tips were obtained using a semiautomatic image analyzer (MM 1201, Summagraphics Co., Fairfield, CT) interfaced with an AT computer (Olivetti M380, Olivetti, Ivrea, Italy). Tracing and digitization of cusps were performed by a single operator.

From these photographs, only the arches with a complete set of 14 teeth (right second molar to left second molar) were then chosen. This resulted in a subset of 38 male maxillary arches, 33 female maxillary arches, 35 male mandibular arches, and 30 female mandibular arches.

The center of gravity (CG, or centroid) of each tooth was then calculated from cusp tip coordinates. In the anterior teeth (central incisor to canine), the CG of each tooth corresponds to the digitized points (midpoints of incisal edge and canine cusp); in the posterior teeth (premolars and molars), it was calculated from the two to five digitized cusps (depending on dental morphology). This gave a total of 14 landmarks, which were later used in the EDMA calculations (Fig. 1).

The dental centroids were based on the cusp tips, and differ from those delineated from the occlusal surface boundary. This definition was chosen because tracing and digitization are easier for cusp tips than for the boundary of the occlusal surface. Moreover, if morphology has to explain function, the mandibular dynamics could be described by the analysis of occlusal contacts which are more linked to the dental cusps than to the occlusal surface.

Euclidean distance matrix (form matrix)

For each subject, all possible linear distances between pairs of landmarks were computed from the coordinates of the 14 dental CGs (Lele, 1991; Lele and Richtsmeier, 1991). This resulted in 35 male mandibular form matrices of 91 distances [14 * (14 − 1)/2], 30 female mandibular matrices, 38 male maxillary matrices, and 33 female maxillary matrices.

Form matrices were then averaged within sex and arch, thus obtaining four mean form matrices.

Form difference matrix

Like linear distances from the male and the female maxillary matrices were paired, and a ratio was computed for each linear distance. Linear distances from the male sample served as the numerator, while female distances appeared in the denominator.

TABLE 1. Form difference matrix for the comparison of male maxillary arches with female arches sorted from lowest to highest

	Teeth[1]	Ratio[2]		Teeth[1]	Ratio[2]		Teeth[1]	Ratio[2]
1	23-24	0.9912	32	21-27	1.0367	63	14-11	1.0487
2	15-14	0.9954	33	16-15	1.0380	64	13-26	1.0495
3	23-25	0.9967	34	17-12	1.0381	65	15-23	1.0497
4	21-22	0.9971	35	11-26	1.0382	66	24-27	1.0497
5	24-25	1.0154	36	14-22	1.0393	67	15-24	1.0499
6	21-24	1.0175	37	24-26	1.0402	68	17-23	1.0500
7	12-22	1.0193	38	11-23	1.0402	69	16-24	1.0501
8	23-26	1.0199	39	13-22	1.0403	70	13-27	1.0504
9	12-24	1.0201	40	14-24	1.0404	71	14-26	1.0514
10	15-13	1.0206	41	13-24	1.0409	72	14-27	1.0517
11	21-25	1.0211	42	22-27	1.0417	73	13-21	1.0520
12	15-12	1.0224	43	16-11	1.0421	74	17-24	1.0527
13	11-22	1.0237	44	15-11	1.0423	75	13-12	1.0554
14	22-25	1.0245	45	17-13	1.0423	76	25-26	1.0576
15	16-14	1.0258	46	11-27	1.0424	77	13-11	1.0580
16	12-25	1.0260	47	17-14	1.0428	78	17-15	1.0588
17	22-24	1.0277	48	15-22	1.0429	79	15-25	1.0603
18	11-24	1.0286	49	16-21	1.0430	80	16-25	1.0614
19	16-12	1.0288	50	16-22	1.0430	81	25-27	1.0633
20	21-26	1.0299	51	15-21	1.0440	82	17-25	1.0633
21	14-12	1.0301	52	14-13	1.0446	83	15-27	1.0642
22	12-23	1.0303	53	14-23	1.0451	84	15-26	1.0644
23	16-13	1.0309	54	14-21	1.0460	85	26-27	1.0655
24	12-26	1.0320	55	13-25	1.0465	86	16-26	1.0666
25	11-25	1.0324	56	17-21	1.0467	87	16-27	1.0671
26	21-23	1.0332	57	11-21	1.0468	88	17-26	1.0681
27	23-27	1.0335	58	17-22	1.0468	89	17-27	1.0684
28	12-21	1.0337	59	17-11	1.0470	90	17-16	1.0695
29	22-26	1.0341	60	13-23	1.0478	91	22-23	1.0716
30	12-11	1.0357	61	16-23	1.0481			
31	12-27	1.0361	62	14-25	1.0483			

[1] Teeth are codified as shown in Figure 1 and are the endpoints of each linear distance.
[2] Mean distance between the two teeth in the male sample divided by the corresponding distance in the female sample.

The 91 ratios were then sorted from lowest to highest, and the statistic T was calculated as T = maximum ratio/minimum ratio.

This statistic was used to test the null hypothesis of similarity of forms. If the two forms are equal, the form difference matrix is a matrix of constants (if only the shape is similar then the constants are equal to some number different from one; if also the size is similar then this number should be one). Therefore, if the null hypothesis is true, the statistic T should be close to one.

The null distribution of T was estimated using a bootstrap procedure with replacements as detailed by Corner and Richtsmeier (1991) and Lele and Richtsmeier (1991), which was repeated 1,000 times.

The position of the observed value of T relative to the T null distribution was then analyzed. The null hypothesis (no differences between male and female samples) was rejected if the area of the T null distribution at the right of the observed T value was equal to or less than 5% ($P \leq 0.05$). The procedure was repeated with the mandibular arch matrices. All the calculations were performed on a personal computer (AST Bravo 286/16, AST Research Inc., Irvine, CA), using original programs written by one of the authors (V.F.F.).

RESULTS

Maxillary arch

The form difference matrix for the maxillary arch is presented in Table 1. The 91 ratios between like distances computed in 38 male and 35 female subjects were sorted from lowest to highest. In order to describe the form difference, two values were considered: the maximum ratio/minimum ratio, and the median ratio (Corner and Richtsmeier, 1991). The ratio of extreme ratios represents the total range of shape difference between the two samples. The median

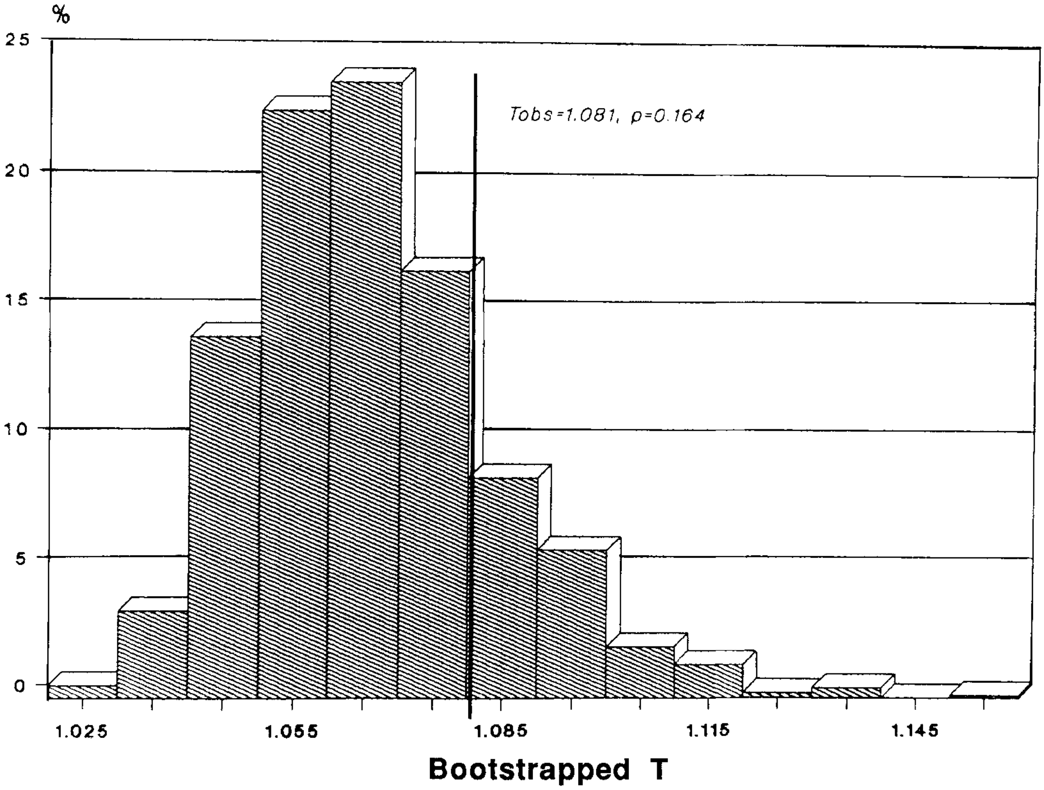

Fig. 2. Null distribution of T for the comparison of maxillary arches in males and females. The observed T = 1.081 is also indicated. It leaves 16.4% of bootstrapped Ts at its right (P = 0.164).

ratio may be considered a measure of the general size difference represented by the form difference matrix.

The ratio of maximum ratio to minimum ratio corresponds to the statistic $T = 1.0716/0.9912 = 1.081$. The null distribution of T was estimated using a bootstrap procedure which was repeated 1,000 times. Figure 2 shows the T null distribution and the observed T value: 16.4% of the bootstrapped Ts are higher than 1.081. Male and female maxillary arches therefore do not have a significantly different shape.

Male arches are slightly bigger than female arches: the median ratio is 1.0423. Size differences are not conspicuous, and the first four ratios in Table 1 (distances between left canine and left first premolar, left canine and left second premolar, right premolars, left incisors) are lower than 1: these distances are slightly longer in women than in men.

Even if the statistical analysis yielded no significant gender differences, the form difference matrix could be further analyzed to determine the areas where the shape variation between male and female arches is greater. Molars and premolars are involved in many of the extreme ratios. Women have longer arches in the premolar area: the ratios of the distances between premolars (landmarks 15-14-25-24) within quadrant in a mesiodistal direction are at the minimum end of the ratio matrix.

The ratios of molar distances (landmarks 16-17-26-27) are all grouped at the maximum end of the form difference matrix: men have longer and wider arches in the molar area than women both within quadrant (mesiodistal) and between quadrants (right-left). Mesiodistal different ratios are probably determined by larger molar dental crowns in males. The ratios between antimeres could be different because the male

TABLE 2. Form difference matrix for the comparison of male mandibular arches with female arches sorted from lowest to highest

	Teeth[1]	Ratio[2]		Teeth[1]	Ratio[2]		Teeth[1]	Ratio[2]
1	31-42	0.9820	32	31-47	1.0256	63	45-47	1.0339
2	41-42	0.9901	33	34-44	1.0257	64	37-35	1.0345
3	35-34	0.9947	34	33-32	1.0259	65	41-44	1.0356
4	44-45	0.9948	35	35-31	1.0263	66	36-32	1.0362
5	31-41	0.9964	36	42-46	1.0264	67	37-41	1.0367
6	32-42	0.9962	37	33-43	1.0267	68	41-46	1.0373
7	35-42	1.0066	38	36-34	1.0272	69	36-31	1.0375
8	31-43	1.0082	39	37-43	1.0274	70	33-47	1.0375
9	33-42	1.0124	40	31-46	1.0276	71	37-33	1.0377
10	35-44	1.0142	41	34-43	1.0278	72	34-46	1.0379
11	35-43	1.0163	42	43-46	1.0280	73	33-41	1.0392
12	34-42	1.0169	43	37-36	1.0284	74	37-47	1.0400
13	32-43	1.0173	44	42-44	1.0288	75	36-45	1.0403
14	31-44	1.0176	45	32-47	1.0289	76	37-46	1.0412
15	31-45	1.0182	46	33-44	1.0297	77	33-46	1.0413
16	42-45	1.0182	47	32-41	1.0298	78	33-31	1.0417
17	44-46	1.0194	48	35-47	1.0299	79	35-33	1.0426
18	44-47	1.0196	49	34-45	1.0300	80	36-41	1.0436
19	32-44	1.0205	50	37-32	1.0314	81	42-43	1.0437
20	37-42	1.0209	51	41-45	1.0315	82	36-33	1.0442
21	43-45	1.0211	52	32-46	1.0315	83	34-31	1.0457
22	35-45	1.0214	53	37-31	1.0316	84	36-47	1.0459
23	32-45	1.0225	54	35-41	1.0316	85	34-32	1.0469
24	41-43	1.0231	55	32-31	1.0316	86	34-41	1.0469
25	46-47	1.0237	56	36-44	1.0318	87	45-46	1.0469
26	42-47	1.0240	57	35-46	1.0320	88	43-44	1.0491
27	36-42	1.0244	58	36-43	1.0328	89	35-46	1.0493
28	37-44	1.0254	59	41-47	1.0330	90	36-35	1.0504
29	35-32	1.0255	60	33-45	1.0331	91	34-33	1.0878
30	37-34	1.0255	61	37-45	1.0331			
31	43-47	1.0256	62	34-47	1.0336			

[1] Teeth are codified as shown in Figure 1 and are the endpoints of each linear distance.
[2] Mean distance between the two teeth in the male sample divided by the corresponding distance in the female sample.

arch becomes wider distally (from the first molars). Also the ratios of the distances between molars and contralateral second premolars (landmarks 15 and 25) are at the maximum end of the form difference matrix.

Another apparently gender-specific characteristic is the position of the left canine (landmark 23) relative to the contiguous teeth: while the ratio 23-24 is the lowest in the form difference matrix (females have a greater distance), the ratio 22-23 is the highest (males have a greater distance). This tooth seems to be in a more distal position in men than in women.

Mandibular arch

The procedure followed for the maxillary arch was repeated for the 30 women and 33 men with a complete mandibular dentition (14 permanent teeth from right second molar to left second molar). The form difference matrix is presented in Table 2. The arches do not evince a sexual dimorphism: $P = 0.19$ when the statistic T ($T = 1.0878/0.982 = 1.108$) is compared to a null distribution estimated by a bootstrap procedure (Fig. 3).

More information can be obtained by a careful examination of the form difference matrix. As already found in the maxillary arch, size differences are small. The median ratio is 1.030: male lower arches are somewhat bigger than female arches.

The statistical analysis did not demonstrate significant shape differences between the two samples, but ratios located in the extreme ends of the matrix provide some indications of gender-specific characteristics. The first six ratios are lower than 1; the corresponding distances are bigger in women than in men. These ratios are scattered in the premolar, canine, and incisive areas. Men seem to have smaller incisors than women, and the distance between canines and first premolars seems to be

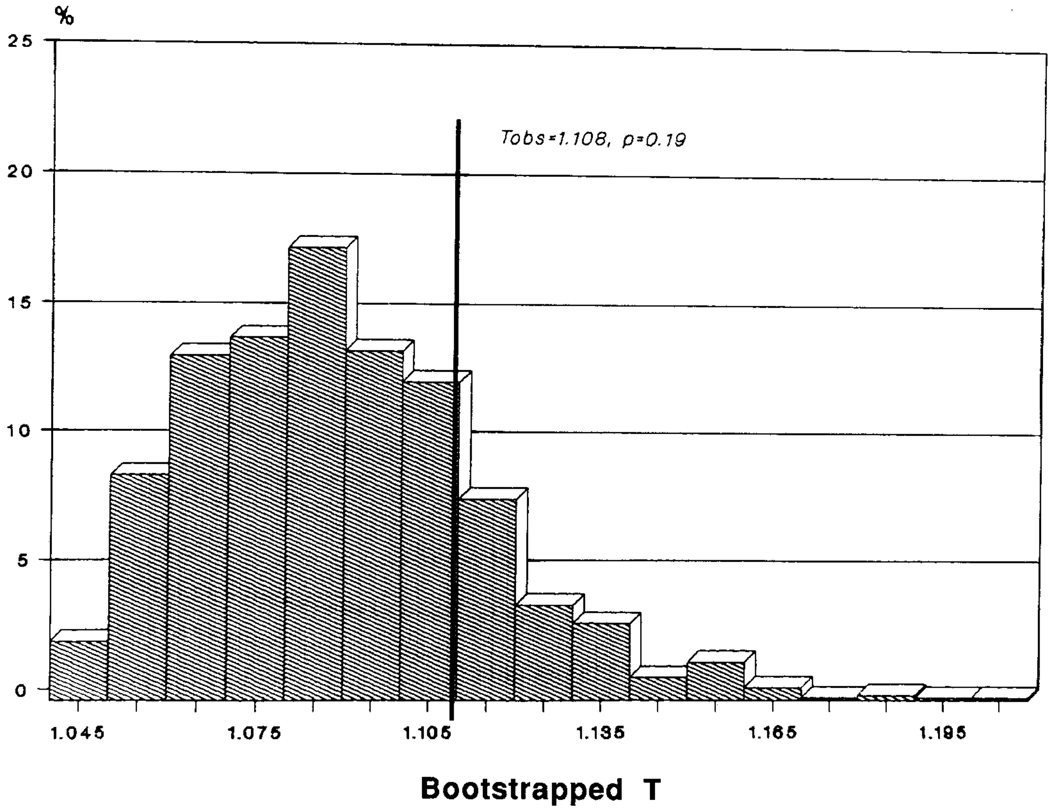

Fig. 3. Null distribution of T for the comparison of mandibular arches in males and females. The observed T = 1.108 is also indicated. It leaves 19% of bootstrapped Ts at its right ($P = 0.19$).

smaller in men than in women (distances 43-44 and 23-24).

The last ratios involve distances between premolars within side in a mesiodistal direction (ratios 44-45 and 34-35): male values are higher than female ones. The first premolars therefore seem to be located in a more distal position in men than in women.

DISCUSSION

EDMA overcomes some of the problems associated with other methods for analyzing differences in biological forms. It is obviously superior to all the conventional metric approaches which analyze separate distances, angles, and ratios (Bookstein, 1984; Lestrel, 1989) because it takes the whole structure into account at one time. It is more objective than superimposition methods (Lele, 1991), and its application and interpretation are easier than those of finite-element analysis. This last method, together with the biorthogonal grids developed by Bookstein (1982, 1983, 1984), requires that the change be uniformly distributed within a triangular boundary given by three landmarks. This homogeneity is probably never achieved in biology (Moss et al., 1985), even if interesting results have been obtained in the analysis of craniofacial and dental forms (Baluta and Lavelle, 1987; Corner and Richtsmeier, 1991; Lestrel, 1989; Moss et al., 1985; Richtsmeier et al., 1990).

The shapes of dental arches have been extensively classified using different curve-fitting mathematical models, recently reviewed by Jones and Richmond (1989). Several algorithms have been used: parabolas (Ferrario et al., 1991a,c; Jones and Richmond, 1989), semi-ellipses (Ferrario et al., 1991a,c), catenary curves (Pepe, 1975), conic sections (Biggerstaff, 1972; Sampson, 1981, 1983), cubic spline curves (BeGole, 1980), and second- to eighth-order polynomials (Ferrario et al., 1991a,c; Lu, 1966; Pepe, 1975; Richards et al., 1990). Differences between samples have been evaluated by comparing equation coefficients (Biggerstaff, 1972; Ferrario et al., 1991a; Richards et al., 1990), but the results are often contradictory, and moreover are not informative about the areas of major variations. Besides, the conclusions depend upon the mathematical algorithms employed, i.e., on prior assumptions made about arch form.

The form of adult male and female faces is significantly different from both qualitative and quantitative points of view (Enlow, 1990; Ferrario et al., 1992c; Scheideman et al., 1980). The difference involves both hard and soft tissue elements (Halazonetis et al., 1991). A sexual dimorphism in dental arch form is therefore expected, having male teeth to occupy jaws with a different form.

This investigation showed that the form difference between adult male and female dental arches is just a size difference and not a shape difference. Our results confirm that female arches are smaller than male arches (Merz et al., 1991). As to shape, the present results contrast with a previous finding of a sexual dimorphism in the morphology of the contact surfaces of incisors and canines (Ferrario et al., 1992b). The two investigations are not incompatible: the present analysis dealt with a strictly two-dimensional representation of the occlusal view of arches, but dental surfaces should be analyzed in all their three-dimensional, spatial aspects. Further investigations could usefully apply EDMA to sets of dental three-dimensional data (Lele and Richtsmeier, 1991).

The analysis of the form difference matrices (Tables 1 and 2) showed that some gender-specific variations in arch form are located in different areas in the maxillary and mandibular arches. Nevertheless, these gender variations are not conspicuous and cannot be considered sex-linked "differences," but only areas with a higher gender variability.

The results of this analysis underscore the importance of global quantitative evaluations of dental arch form whenever orthodontic or extensive prosthetic treatments are to be made. Male and female patients cannot be treated using the same reference standards, since the different dental areas do not always behave in the same way in the two sexes. Some hypotheses explaining the observed differences may be proposed: the position of the maxillary left canine in men seems to reflect, at a minor degree, the well-known primate sexual dimorphism in the upper arch; larger maxillary molar crowns in men could be related to both larger body weight (Anderson et al., 1977) and longer dental development before eruption (Ash, 1986).

Both genetic and environmental influences could shape the form of dental arches. The scarce variability observed in our sample seems to indicate that genetic determinants might have a more important role than epigenetic factors. Nevertheless, it has to be mentioned that the criteria used in the selection of our subjects reduced external influences to a minimum (e.g., no orthodontic treatment, no extensive restorative or prosthetic work). The relative contribution of environment and genotype in the general population could be different (Enlow, 1990).

In conclusion, EDMA proved suitable for the analysis of sexual dimorphism in human dental arches. We also feel that other future investigations dealing with dental landmarks could usefully apply the same method to test age or arch differences.

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