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# Mesons in the Higher-Symmetry Reggeization (*).

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**Summary.** — Classification of the mesic states into the $SU_3$ multiplets occurring in the first two members of the most degenerate Feynman series is made. On the basis of the physical masses of the particles, a simple mass formula for the $SU_3$ multiplets in an $SU_6$ representation is suggested.

## 1. – Introduction.

We present here some evidence in support of the most degenerate Feynman series recently discussed by DELBOURGO, RASHID, SALAM and STRATHDEE [1] as the series of representations of $U_6 \times U_6$ containing the physical particles. This is achieved through the classification of the increasing number of meson resonances into the $SU_3$ multiplets of the two lowest representations of the series.

Mainly there are two ways of classifying meson states: one is the quark model, where these states are due to strong binding between quark and anti-quark through their mutual potential. The lowest states corresponding to the

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(1) R. DELBOURGO, M. A. RASHID, A. SALAM and J. STRATHDEE: *Reggeization of* $U_6 \times U_6$, Imperial College preprint ICTP/67/1; R. DELBOURGO, A. SALAM and J. STRATHDEE: *Reggeization of quark number*, IC/68/14, preprint. (These two are referred to as I in the text.) R. DELBOURGO: *Representation functions for the degenerate baryon series in* $U_6 \times U_6$, Imperial College preprint, ICTP/67/4. The Feynman series was first presented by Y. DOTHAN, M. GELL-MANN and Y. NE'EMAN (*Phys. Lett.*, **17**, 148 (1965)). Only in I above this series is used for physical application.

$l = 0$ states are the pseudoscalar and vector mesons, and higher-spin states are just the excited states of the quark-antiquark pair. With the basic quark triplet only nonets can ensue from this model. The other, less discussed way is to use the larger representations of $U_{6,6}$ (and consequently of $U_6 \times U_6$) which contain high-spin mesons. Unfortunately, the simplest of the representations of $U_6 \times U_6$ which contains spin-$2^+$ mesons also contains high $SU_3$ multiplets such as 10, $\overline{10}$ and 27. It is the purpose of this paper to show that many of the states in these representations are in fact filled up by the meson resonances recently reported. Also we suggest a simple mass formula for the physical particles belonging to an $SU_6$ representation which enables us to predict the mean mass of all the $SU_3$ multiplets contained in the 405-representation of $SU_6$. It is worth remarking that although the quark model is free from the embarrassing situation of having to predict too many physical particles, it lacks accomodation for higher $I$-spin and $Y$-states, some of which have been indicated in recent experiments.

As has been discussed in I, corresponding to this Feynman series of representations, namely,

$$(1, 1),\ (6, \bar{6}),\ (21, \overline{21}), \qquad \text{for bosons}$$

$$(56, 1),\ (126, \bar{6}), \qquad \text{for baryons}$$

in $U_6 \times U_6$ there is only a single Casimir label, the quark number $N$. There will be a single Regge trajectory given by a graph which connects all the integral $N$ in a plot of the mean squared multiplet mass *vs.* the quark number $N$ (Fig. 1). At the integral $N$ values the $U_6$ content of the corresponding $U_6 \times U_6$ multiplets can be found (Fig. 2).

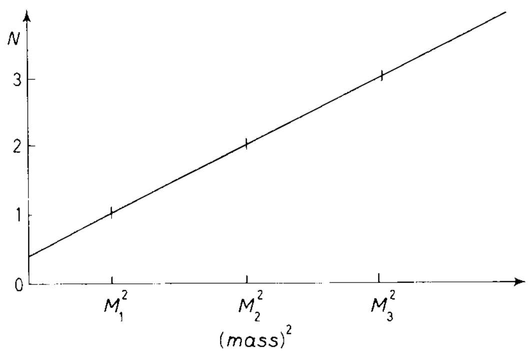


Fig. 1. – « The master boson trajectory ».

Next, at these $N$-integral values each $SU_6$ multiplet can be decomposed to obtain the $SU_3 \times \times SU_2$ content. This leads to sequences (generations) of Regge trajectories for the different $SU_3$ multiplets. A complete picture of the Regge trajectories of the higher-symmetry multiplets is obtained when we superimpose all the $SU_3$ multiplet Regge trajectories. This will then show all the properties mentioned in I (Fig. 3).

We notice that for any $SU_3$ multiplet, except for the leading trajectory, there is a group of degenerate trajectories in different generations. The number of trajectories in a particular generation remains constant only after reaching the highest $j$ values with which the multiplet first appeared. For example, in the second-generation trajectory for the octet, there is only one trajectory for the spin-0 state, and three for the spin-1 state, where it remains constant

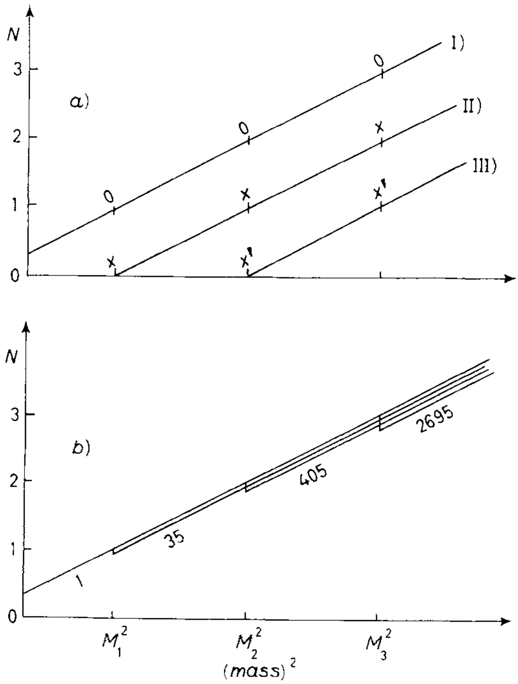


Fig. 2. – $a$) and $b$) represent two equivalent ways of looking at the $U_6\times U_6\to U_6$ decomposition at the integral $N$ values I), II), III) of $a$) label the first, second, third generation of the trajectories respectively.

and this is the highest spin with which the 8-representation first appeared. This leads us to the following general rule for finding the number of trajectories in a given generation. For an $SU_3$ representation $D$ which had highest spin $j^n$ in its first occurrence in the decomposition of the representations of the Feynman series, the number of trajectories in the $k$-th generation is the number of $D$ with spin $j^n$ occuring in the decomposition of the $U_6\times U_6$ multiplet characterized by $N=k$.

Another important point about the Feynman series is that there are far greater numbers of self-adjoint $SU_3$ representations appearing than the non-self-adjoint ones.

It is usual to associate physical particle states with each integral $J$ value. Realization of such an idea looks at first sight preposterous in view of the large number of particles predicted thereby. However, it can be argued (²)

(²) D. Horn, J. J. Coyne, S. Meshkov and J. C. Carter: *Phys. Rev.*, **147**, 980 (1966).

that the production cross-section of particles belonging to the higher $SU_3$ representations will be very small due to the large number of competing channels—also that the particles belonging to the higher rungs of the Feynman

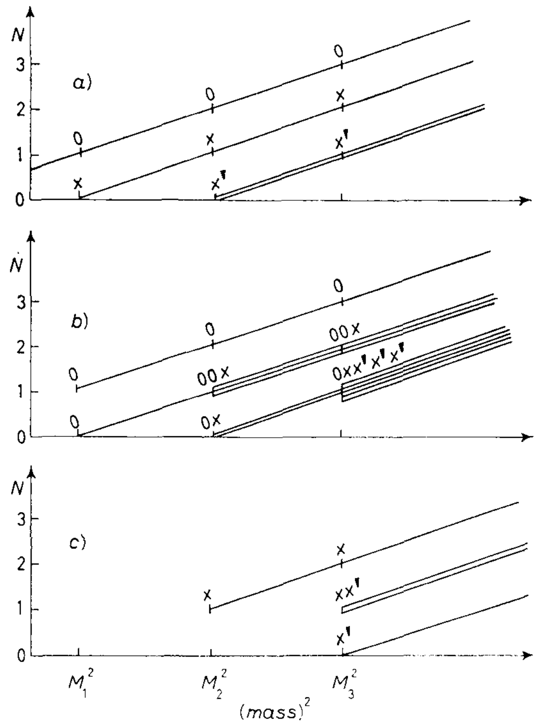

Fig. 3. – *a*) Regge trajectory for $\underline{1}$. *b*) Regge trajectory for $\underline{8}$. *c*) Regge trajectory for $10+\overline{10}$. In the Figure, the number of lines represents the total number of trajectories.

series are expected to be increasingly heavy, so that only with higher energy production can one expect to see them. Therefore the 10, 27, ..., states are not very easy to detect. Nevertheless we show below that with the recently reported mesons [3], most of the states in $(6, \overline{6})$ and $(21, \overline{21})$ representations which are experimentally accessible (that include spin states of 10's and 27's) can indeed be filled up.

## 2. – Classification into $SU_3$ multiplets.

The classification of various mesons into the $SU_3$ multiplets of these two $U_6 \times U_6$ representations is presented in the Table I. There are a large number

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([3]) A. H. ROSENFELD, N. BARASH-SCHMIDT, A. BARBARO-GALTIERI, L. R. PRICE, P. SODING, C. G. WOHL, M. ROOS and W. J. WILLIS: *Data on particles and resonant states*, UCRL 8030, Jan. 1968.

TABLE I. – *Classification of the mesic states.*

| $U_6 \times U_6$ repn. | $SU_6$ repn. | $(SU_3;\, J^P)_C$ | Particles in order of increasing $Y$, $I$ | Meam squared mass $(\mathrm{GeV})^2$ | | |
|---|---|---|---|---|---|---|
| | | | | $SU_3$ physical | $SU_3$ unitary | $SU_6$ |
| $(6, \bar{6})$ | 1 | $(1, 0^-)_+$ | $\eta'(x^0)$ (958) | 0.92 | 0.90 | 0.90 |
| | 35 | $(8, 0^-)_+$ | $\eta$ (549), $\pi$ (137), K (495) | 0.17 | 0.17 | |
| | | $(1, 1^-)_-$ | $\varphi$ (1019) | 1.04 | 0.78 | 0.64 |
| | | $(8, 1^-)_-$ | $\omega$ (783), $\rho$ (765), K* (890) | 0.70 | 0.73 | |
| $(21, \overline{21})$ | 1 | $(1, 0^+)_+$ | $X^{00}$ | 3.19 (*) | $\underline{3.12}$ | $\underline{3.12}$ |
| | | $(8, 0^+)_+$ | $\eta_v$ (1070), $\pi_v$ (1016), $K_v$ (1080) | 1.11 | 1.12 | |
| | 35 | $(1, 1^+)_-$ | $\mathcal{N}\bar{\mathcal{N}}$ (2380) | 5.66 | 5.35 | 2.96 |
| | | $(8, 1^+)_-$ | $\rho$ (1700), $\eta_A$ (1830), $K_A$(1800) | 3.63 | 3.28 | |
| | | $(1, 0^+)_+$ | $\varepsilon$ (730) | 0.53 | 0.31 | |
| | | $(1, 2^+)_+$ | f′ (1514) | 2.29 | 1.76 | |
| | | $(8, 0^+)_+$ | $\sigma$ (410), $\delta$ (963), $\varkappa$ (725) | 0.63 | 0.66 | |
| | | $(8, 1^+)_-$ | H (990), B (1220), $K_c$ (1230) | 1.43 | 1.22 | |
| | 405 | $(8, 1^+)_+$ | D (1285), $A_1$ (1070), $K_A$ (1320) | 1.50 | 1.39 | |
| | | $(8, 2^+)_+$ | f (1260), $A_2$ (1300), $K_v$ (1420) | 1.84 | 1.91 | |
| | | $(10+\overline{10}, 1^+)$ | $\pi_A$ (1640), $K_A$ (1280) | | | 1.98 |
| | | | $K^*_{I=\frac{3}{2}}$(1265), KK (1470) | 2.00 | 1.72 | |
| | | $(27, 0^+)_+$ | — | — | $\underline{1.81}$ | |
| | | $(27, 1^+)_-$ | $\Phi_A$ (1830), $K_sK_s$(1440) | | | |
| | | | $A_{I=2}$ (1320), K* (1651) | | | |
| | | | $K^*_{I=\frac{3}{2}}$(1170), KK $\underline{(1297)}$ | 1.87 | 2.02 | |
| | | $(27, 2^+)_+$ | — | — | $\underline{2.46}$ | |

The underlined masses are predicted masses.
(*) Predicted from singlet-octet mixing with a mixing angle equal to that for $\eta$-$\eta'$ mixing.

of identical isotopic-spin multiplets and assignment of these multiplets to different $SU_3$ multiplets would cursorily seem to be quite arbitrary. Fortunately, there are a number of restrictions that guide us almost uniquely to the particular choice in each $SU_3$ multiplet. These restrictions are mainly due to selection rules and mass formulae. We discuss below how they limit our choice in an $SU_3$ multiplet.

1) The self-adjoint representations 1, 8, 27 etc., have definite $C$-parity defined in the usual way and therefore the $G$-parity of the $Y=0$ states in each of these multiplets is determined. This imposes a powerful constraint; for example, B(1220) with $I^G=1^+$ and D(1285) with $I^G=0^+$ cannot belong

to the same octet because they have opposite $C$-parity. Most of the $Y=0$ states in 1, 8, and 27 are fixed by the $G$-parity alone.

2) Decays of $(27, 1^+) \to 1^- 0^-$ are forbidden by $SU_3$ and charge-conjugation invariance (4).

3) Decays of

$$(27, 1^+) \to 0^- 0^-, \; (27, 2^+) \to 1^- 0^-, \; (27, 2^+) \to 0^- 0^-, \; (10 + \overline{10}; 1^+) \to 0^- 0^-$$

are all forbidden by $SU_{6w}$ selection rules (5).

4) Decays of any member of an octet, where the neutral ($Y=0$, $I=0$) state is odd under $CP$, into two pseudoscalar mesons are forbidden (6).

The $2^+$ octet in the 405-representation of $SU_6$ is readily recognized once the spin parity of the particles are known. Also with the known spin parity assignment the $G$-parity determines the $J=0$ states of the two octets with opposite $C$-parity belonging to the 405-representation. The complicated situation with the $Y=1$, $I=\frac{1}{2}$ states in these different spin-1 octets can be resolved quite unambiguously. Out of all such states observed, only $K_A(1320)$ has the known decay into two pseudoscalar mesons. Using 3) and 4) this particle can be clearly placed in the $(8, 1^+)$ multiplet with positive $C$-parity. Finally to determine the particle assignments to the remaining states in the other multiplets we have to invoke the Gell-Mann–Okubo mass formula for the octet and decuplet. However, as this mass formula is inadequate for the 27-multiplet, it is reasonable to generalize it by including terms in the mass operator that transform as the $I=0$, $Y=0$ member of the 27-representation. Inclusion of such additional terms in the mass operator leads to the following second-order mass formula (7):

$$M = a + b\left[\frac{Y^2}{4} - I(I+1)\right] + cY^2 + d\left[\frac{Y^2}{4} - I(I+1)\right]^2 \tag{1}$$

for the mesons. The following examples illustrate how these considerations work in assigning the different mesic states to the $SU_3$ multiplets:

*a*) Considering the unresolved states $K_c(1230)$, $K_A(1280)$ and $K_A(1800)$, these can be accommodated in any of the two $(8, 1^+)$'s or the $(10 + \overline{10}; 1^+)$.

(4) D. Horn, H. Lipkin and S. Meshkov: *Phys. Rev. Lett.*, **17**, 1200 (1966).

(5) These results follow from the results of R. Delbourgo: *Phys. Lett.*, **15**, 347 (1965), using the $(21, \overline{21})$ representation in place of the $(15, \overline{15})$.

(6) A. Levinson, H. J. Lipkin and S. Meshkov: *Nuovo Cimento*, **32**, 1376 (1964).

(7) S. Okubo: *Phys. Lett.*, **4**, 14 (1963), we have used squares of the masses in all calculations involving mass.

With $\rho(1700)$ and $\eta_A(1830)$, $K_A(1800)$ forms on octet that satisfies the Gell-Mann–Okubo mass formula approximately. Similar considerations also force us to place $K_c(1230)$ in the octet with H(990) and B(1220). $K_A(1280)$ then goes to the $(10+\overline{10};\,1^+)$ along with K(1265) $I=\frac{3}{2}$ and $\pi_A(1640)$ forming an almost degenerate decuplet with the following prediction for the mass of the remaining $I=0$, $Y=2$ state:

$$M(I=0,\ Y=2)=1470\ \text{MeV}\ .$$

There is no evidence for any such $(K^+K^0)$-like resonance.

*b*) With K*(1170) and $A_2(1320)$ as the $I=\frac{3}{2}$ and $I=2$ states respectively of the $(27,1^+)$ multiplet, the mass formula (1) forces us to choose $K_sK_s(1440)$ and $\Phi_A(1830)$ as the $I=1$ and $I=0$ states respectively from the observed resonances with correct $J^P$, $I^G$ assignments. Solving for the unknown parameters of the mass formula, we predict the following masses for the remaining states of the 27-multiplet [8]:

$$M(Y=1,\ I=\tfrac{1}{2})=1650\ \text{MeV}\ ,$$

$$M(Y=2,\ I=1)=1295\ \text{MeV}\ .$$

For the $I=\frac{1}{2}$ state, there is the reported $K_v(1660)$ [8a] resonance with nearly the correct mass. It is interesting that some evidence for a $K^+K^+$ $(Y=2,\ I=1)$ enhancement at 1280 MeV was observed by a group at CERN [9]; however, other groups have failed to see such enhancement.

There are experimental indications of two scalar octets [9a], $\eta_v(1070)$, $\pi_v(1016)$, $K_v(1080)$ and $\sigma(410)$, $\varkappa(725)$, $\delta(963)$ and a scalar singlet $\varepsilon(730)$. As will be shown below, considerations of the general mass formula for $SU_6$ and octet-singlet mixing favour the identification of $\sigma(410)$, $\varkappa(725)$, $\delta(963)$ as the scalar octet belonging to the 405-multiplet. Naturally, the other octet will belong to the 35-multiplet.

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[8] This assignment depends critically upon the choice $J^P=1^+$ for the $K_sK_s(1440)$ state.

[8a] The spin-parity of this particle ($K_v(1660)$) has not yet been firmly determined, so that it is misleadingly called a vector particle.

[9] G. GOLDHABER: *Proceedings of the XIII International Conference on High-Energy Physics* (1966), p. 136.

[9a] For experimental indications of these scalar mesons we refer to the following review talks. G. GOLDHABER: *Proceedings of the XIII International Conference on High-Energy Physics* (1966), p. 103; I. BUTTERWORTH: *Proceedings of the Heidelberg International Conference on Elementary Particles* (1967), p. 11; L. MONTANET: *Meson resonances*, CERN lecture notes, CERN-67-23.

### 3. – Mixing of $SU_3$ multiplets.

Even the classification of these particles into different $SU_3$ representations has been rendered doubly difficult by the mixing of similar isospin states belonging to various $SU_3$ representations. Mixing occurs on two levels, namely, mixing between $SU_6$ multiplets belonging to the same $U_6 \times U_6$ representation and that between different $SU_3$ multiplets belonging to the same $SU_6$ representation. In Table II we have calculated the mixing angles for the octet and

TABLE II. – *Mixing parameters for the unitary octets and singlets.*

| $(SU_3; J^P)_c$ multiplets | Mixing angle | $m_8^2$ (GeV)$^3$ | $m_1^2$ (GeV) | Mean unitary squared mass (GeV)$^2$ (mixing effect) |
|---|---|---|---|---|
| $\left.\begin{matrix}(8, 0^-)_+\\(1, 0^-)_+\end{matrix}\right\}$ η(549), π(137), K(495), η(X⁰)(958) | ~80° | 0.32 | 0.90 | 0.20 |
| $\left.\begin{matrix}(8, 1^-)_-\\(1, 1^-)_-\end{matrix}\right\}$ ω(783), ρ(765), K*(890), φ(1019) | ~40° | 0.86 | 0.78 | 0.75 |
| $\left.\begin{matrix}(8, 0^+)_+\\(1, 0^+)_+\end{matrix}\right\}$ σ(410), δ(963), ϰ(725), ε(730) | ~38° | 0.39 | 0.31 | 0.66 |
| $(8, 1^+)^a_-$ H (990), B (1220), $K_c$ (1230) | — | — | — | 1.22 |
| $(8, 1^+)^b_+$ D(1285), $A_1$(1070), $K_A$(1320) | — | — | — | 1.39 |
| $\left.\begin{matrix}(8, 2^+)_+\\(1, 2^+)_+\end{matrix}\right\}$ f(1260), $A_2$(1300), $K_v$(1420), f′(1514) | ~30° | 2.13 | 1.76 | 1.91 |
| $\left.\begin{matrix}(8, -^+)_-\\(1, 1^+)_-\end{matrix}\right\}$ ρ(1700), $\eta_A$(1830), $K_A$(1800), $\mathcal{N}\bar{\mathcal{N}}$(2380) | ~70° | 3.20 | 5.35 | 3.26 |

(*a*) All the states in this multiplet can mix with states of $(27, 1^+)$ having similar quantum numbers. But since $K_c$(1230) can mix also with similar states of $(8, 1^+)_+$, and $(10+\overline{01}, 1^+)$, it is assumed that disagreement in the GMO mass formula is due to this state. See footnote ([10a]).

(*b*) Only $K_A$(1320) can mix with $I=\frac{1}{2}, I=1$ states of $(8, 1^+)_-$ and $(27, 1^+)_-$, $(10+10, 1^+)$ whereas $A_1$, D are pure octet states.

In all other cases only singlet-octet mixing of the neutral states is supposed to be dominant.

([10a]) The most complicated mixing in $(21, \overline{21})$ representation occurs for the state $I=\frac{1}{2}$, $Y=1$ with $J^P=1^+$. As it is not an eigenstate of $G$-parity, these states belonging to the octets of opposite $C$-parity will mix with each other and also with those occurring in $(27, 1^+)$ and $(10+\overline{10}, 1^+)$. On the other hand, since there is only one octet with $J^P=1^+$ and $C=+$, it is natural to assume that these $I=0, 1$ states are pure octet states and any disagreement with the Gell-Mann–Okubo mass formula is caused by the $I=\frac{1}{2}$, $Y=1$ state.

singlet representations of $SU_3$ belonging to various $SU_6$ representations ([10]). The following features are evident from the Table:

*a*) The mixing between $SU_3$ states belonging to different $SU_6$ representations is «small» and is given by the $\eta$-$\eta'$ mixing angle. Assuming it to be roughly the same order of magnitude for all such cases, we can predict the $SU_6$ singlet mass belonging to the $(21, \overline{21})$ to be approximately equal to 1770 MeV.

*b*) The octet and singlet belonging to the same $SU_6$ representations has a mixing angle roughly equal to that of $\varphi$-$\omega$ mixing.

The results obtained for various $SU_3$ multiplets of $(21, \overline{21})$ can be consistently accounted for by these two basic mixing angles. It is of course quite natural and realistic to extend this mixing to all other isospin states but such a detailed consideration becomes highly complicated ([10a]).

## 4. – A simple mass formula.

Looking at the mean masses of the unitary multiplets some direct, simple variation of mass with spin and unitary spin becomes evident. We suggest the following simple-minded mass formula for the $SU_3$ multiplets contained in an $SU_6$ multiplet:

$$M = M_0 + M_1 C_2^{(2)} + M_2 C_2^{(3)} + M_3 C_2^{(2)} C_2^{(3)} , \tag{2}$$

where $M_0$, $M_1$, $M_2$, $M_3$ are parameters characteristic of the $SU_6$ representation, and $C_2^{(2)}$, $C_2^{(3)}$ are the second-order Casimir operators of $SU_2$ (spin) and $SU_3$ groups respectively, $C_2^{(2)} = J(J+1)$ for a spin-$J$ multiplet and $C_2^{(3)} = \frac{2}{3}[p^2 + q^2 + 3(p+q) + pq]$ for an $SU_3$ multiplet $D(p, q)$ ([11]). Using the mean

([10]) The mixing angle $\theta$ is defined as usual (see for example, M. GOLDBERG: in *Symposium on the present status of $SU_3$ for particle couplings and reactions*, Argonne National Laboratory) such that for $\theta = 0$ the lighter isosinglet would always be the pure unitary singlet. This explains the high mixing angles for $\eta$-$\eta'$ case, which would have been «small» if we had defined $\theta$ such that for $\theta = 0$ the lighter isosinglet is the pure unitary octet state.

([11]) Clearly this mass-formula is not adequate as it cannot distinguish between the two $1^+$ octets of opposite $C$-parity which have considerable mass difference. Applying similar considerations we obtain the following mass formula for the isospin multiplets within an $SU_3$ representation (with $B = 0$): $M = a + b[I(I+1)]$. This gives the relation $M_{\frac{1}{2}} = \frac{5}{8} M_0 + \frac{3}{8} M_1$ where the subscripts denote the isospin of the multiplets. This is satisfied within 10% in all the octets considered here which is an indication that our mass formula (2) for $SU_6$ is not unreasonable.

squared unitary mass [12] of the $(8, 2^+)$, $(8, 1^+)$, $(1, 2^+)$ and $(1, 0^+)$, the parameters of the mass formula (2) came out with the values

$$M_0 = 0.31,\; M_1 = 0.24,\; 12M_2 = 1.13,\; 24M_3 = -0.83$$

and consequently, we can make the following predictions for approximate mass value of the remaining $SU_3$ multiplets of the $\underline{405}$ representation:

$$m^2(27, 2^+) = 2.46\ (\text{GeV})^2, \qquad m^2(27, 1^+) = 2.02\ (\text{GeV})^2,$$
$$m^2(27, 0^+) = 1.81\ (\text{GeV})^2, \qquad m^2(10 + \overline{10};\, 1^+) = 1.72\ (\text{GeV})^2,$$
$$m^2(8, 0^+) = 0.87\ (\text{GeV})^2.$$

The mean squared masses for the $(27, 1^+)$ multiplet and the decuplet as obtained with the particles shown in Table I is in fair agreement with the predicted masses. Concerning the scalar nonet, we have the following possibilities:

*a*) The octet with $\eta_v(1070)$, $\pi_v(1016)$, $K_v(1080)$ has the mean unitary square mass $= 1.12\ (\text{GeV})^2$, compared with the predicted value given by the mass formula, $0.87\ (\text{GeV})^2$. Since the scalar particles occur in this representation in the same way as the $2^+$ particles, the mixing effects are expected to be similar. Allowing for an octet-singlet mixing of $\simeq 40°$, the scalar singlet mass can be predicted to be $\sim 1230$ MeV. However, at present there is no indication of such a scalar particle, but scalar particles are notoriously difficult to observe.

*b*) The octet with $\sigma(410)$, $\varkappa(725)$, $\delta(963)$ has the mean squared mass very close to the mass predicted by the mass formula. Also, with $\varepsilon(730)$ which is a $J = 0$, $I = 0$ particle, the singlet-octet mixing angle is found to be $\simeq 38°$ which is very close to the mixing angle for the $2^+$ nonet. These considerations lead us strongly to place this octet and the singlet in the $\underline{405}$-representation.

With the heavier scalar octet $\eta_v$, $\pi_v$, $K_v$ in the $\underline{35}$-representation, we can compute the mass of the $SU_6$ singlet of $(21, \overline{21})$. Assuming the mixing angle to be roughly equal to that obtained in $\eta$-$\eta'$ (pseudoscalar singlet-octet) mixing, the mass of the unitary singlet is predicted to be $\simeq 1800$ MeV and the physical mass comes out to be $\simeq 1770$ MeV. In this mass region there are indications of a number of enhancements, but it has not yet been possible to determine their $J^P$, $I^G$ correctly.

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[12] It is expected that the $M_3$ term in eq. (2) should be small compared with the other terms. Applying this mass formula without this term to the 35 representation of $SU_6$ containing the pseudoscalar and vector mesons, the following values are obtained for the constants: $M_0 = 0.23$, $M_1 = 0.27$, $M_2 = 0.01$ which shows reasonable similarity with these for the 405-representation.

## 5. – Conclusion.

Our results for the octets and singlets can be summarized by their Regge trajectories (Fig. 4). Comparing with the ideal trajectories of Fig. 3 *a*) and 3 *b*), a mass shift for the lower-spin multiplet is observed. Within experimental

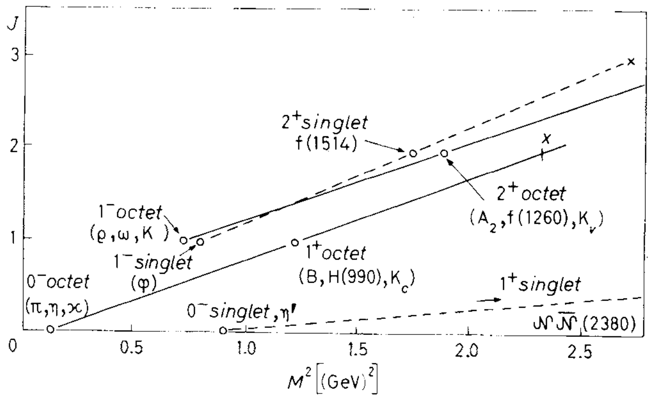


Fig. 4. – Regge trajectories for octets and singlets using the mean squared unitary masses. ——— octet, – — – singlet.

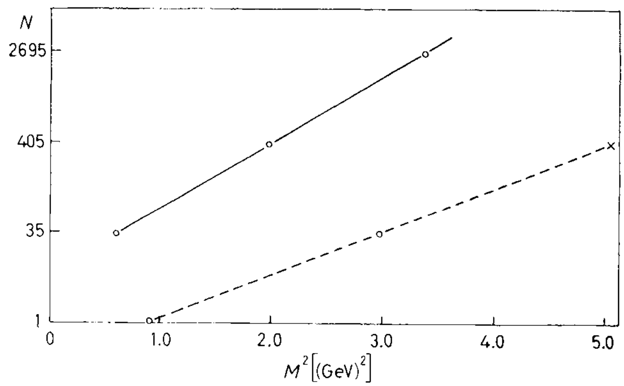


Fig. 5. – Leading (———) and second-generation (— — —) trajectories for $SU_6$ multiplets.

error, these mass shifts in the octets are equal for $(6, \bar{6})$ and $(21, \overline{21})$ representations and as such these trajectories are roughly parallel.

Only the leading trajectory is known for the singlet and its position relative to the octet trajectory suggests that the singlet in the leading trajectory will have the lowest mass of all the $SU_3$ multiplets in this leading trajectory. Also it is clear that except at the leading trajectory, *the picture of almost degenerate nonet structure of mesons as given by the quark model will no longer remain true.* This follows from the extrapolation of the second-generation singlet and octet trajectories which predict the mean squared masses at 2.26 and 8.90 $(\text{GeV})^2$ for the $2^-$ octet and singlet respectively.

The particles listed in the Table I exhaust nearly all the mesic states reported to have been observed, except for the E(1420) $\rho_v(1650)$ and the small-width resonances $S$, $T$, $U$. These particles may belong to higher representations in the leading trajectory if their spin parity are found to be the same as has been conjectured [13]. E(1420) and $\rho_v(1650)$ must certainly belong to the next representations of the Feynman series. It is interesting to note that the extrapolation of the two octet trajectories predict masses for the next $2^-$ and $3^-$ octets which are very close to these E(1420) and $\rho_v(1650)$ masses respectively [14]. Unfortunately we cannot yet make precise predictions in the region $>1600$ MeV as the mass distribution is « badly entangled ». Conclusive evidence for the high-isospin states used in the classification is urgently needed, and so also the spin-parity determination of many states.

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([13]) G. GOLDHABER: *Proceedings of the XIII International Conference on High-Energy Physics*, p. 134.

([14]) The g meson is reported to have the quantum numbers $J^P = 3^-$, $I = 1^+$ T. F. JOHNSTON, J. D. PRENTICE, N. R. STEENBERG, T. S. YOON, A. G. GARFINKEL, R. MORSE, B. V. OH and W. D. WALKER: *Phys. Rev. Lett.*, **20**, 1414 (1968).

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## RIASSUNTO (*)

Si classificano gli stati mesonici nei multipletti di $SU_3$ che si riscontrano nei primi due membri della serie di Feynman più degenerata. Sulla base delle masse fisiche delle particelle, si suggerisce una semplice formula di massa per i multipletti di $SU_3$ in una rappresentazione del gruppo $SU_6$.

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(*) *Traduzione a cura della Redazione.*

### Мезоны в реджеизации при высоких энергиях.

**Резюме** (*). — Проводится классификация мезонных состояний в $SU_3$-мультиплетах, встречающихся в первых двух членах наиболее вырожденного фейнмановского ряда. На основе физических масс частиц предлагается простая массовая формула для $SU_3$-мультиплетов в $SU_6$-представлении.

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(*) *Переведено редакцией.*