APPROXIMATE CALCULATION OF THE INTERACTION OF SUPERSONIC FLOW WITH A LAMINAR LAYER IN THE FLOW SEPARATION REGION

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The distributions of the pressure, friction, and heat transfer coefficients in the separation zone of a laminar boundary layer ahead of a wedge on a plate are calculated approximately. The integral method of Cohen and Reshotko is used to calculate the flow in the boundary layer.

NOTATION

x is the distance along the plate; y is the surface ordinate; s is the distance along the wedge; β is the wedge angle; δ is the boundary layer thickness; δ^* is the displacement thickness; δ^{**} is the momentum thickness; u is the velocity; M is the Mach number; R is the Reynolds number; ρ is the density; p is the pressure; τ is the friction; C_p is the pressure coefficient; C_f is the friction coefficient; α is the heat transfer coefficient.

SUBSCRIPTS

0 is the beginning of interaction region; 1 is the separation of the boundary layer; 2 is the vertex of wedge; 3 is the reattachment of separated layer; 4 is the end of interaction region; w is the flow parameters at the wall.

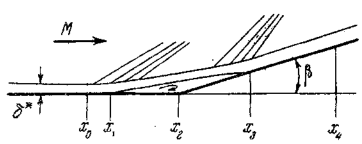

Fig. 1

1. With the interaction of shocks of sufficient intensity with a laminar boundary layer, regions of flow separation arise. So far there are no exact methods for calculating the flow in such regions, and therefore use is made of the approximate integral methods. One of the very first successful attempts to calculate the interaction of a nearly isentropic supersonic flow with a boundary layer, which leads to a separated flow region, was presented in [1]. In this work it is assumed that the pressure distribution in the outer flow is induced by the growth of the boundary layer displacement thickness and is described approximately by the equation for the Prandtl-Meyer flow. The flow in the boundary layer is described by an integral equation of motion. A single-parameter family of profiles is used to solve this problem. An average velocity profile, whose connection with the displacement thickness is given by a semi-empirical method, is introduced as a parameter. This creates considerable inconvenience in the use of the method of [1]. It is shown in [2] that the interaction problem may be solved by the use of more conventional single-parameter families of profiles in which the pressure gradient defines uniquely the relationship between the integral characteristics of the boundary layer, for example, by the family of "similar" solutions of the Falkner-Skan equation for separated and nonseparated flows in the boundary layer. However, in [1] the problem of the interaction was solved in simplified fashion; it was assumed that in the interaction region only the pressure p and the displacement thickness vary, while the Mach number M, the density, velocity, and momentum thickness are constant. This leads to overstatement of the pressure in the flow region behind the separation point. Velocity profiles in the form of polynomials were studied in [3, 4]. The single-parameter family of "similar" profiles was used in [5], but the relationships between the integral boundary layer characteristics were determined not by the local pressure gradient but by the solution of the integral equation for the first moment of the equation of motion.

2. The present study uses a method of the type used in [2], but without the simplifying assumptions made there. The following system of equations is used to describe the flow in the interaction region:

$$\frac{d}{ds}[\rho u^2\delta^{**}] + \delta^*\rho\frac{dp}{ds} = \tau_w, \tag{2.1}$$

$$\frac{d\theta}{ds} = \frac{\sqrt{M^2-1}}{p\gamma M^2}\frac{dp}{ds}, \tag{2.2}$$

$$\frac{dY}{ds} = \theta\frac{dx}{ds} \quad (Y = y(x) + \delta^*). \tag{2.3}$$

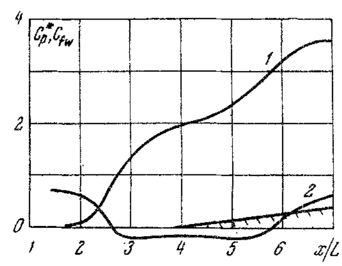

Fig. 2

Equation (2.1) describes the flow in the boundary layer, Eq. (2.2) is the equation for the Prandtl-Meyer flow, Eq. (2.3) determines the connection between the outer flow deviation angle and the boundary layer displacement thickness. In the practical performance of the calculations, Eq. (2.1) was solved using the Cohen and Reshotko method [6]. For the solution of the Cauchy problem, beginning at some section $x = x_0$, we must specify at this section δ^{**}, δ^*, and θ, in addition p and M must be specified at this section. The quantity dp/ds at this section is uniquely determined by the quantities δ^{**} and $H = \delta^*/\delta^{**}$, if a single-parameter family of profiles is given, and if the local pressure gradient is used as a parameter. In using the Cohen-Reshotko method it is clearly difficult to express explicitly the dependence of dp/ds on the quantities H and δ^{**}; therefore the quantity dp/ds at each section was determined by iterations. To do this Eq. (2.2) was solved for dp/ds, and the iterations were continued until the given value of dp/ds was in agreement with the value calculated from (2.2). The parameters δ^{**}, δ^* and θ at the section $x = x_0$ were determined from the solution of (2.1) for given p = const and M = const in the region $x < x_0$. It is shown in [1, 5] that if the boundary layer for $x < x_0$ develops in a strongly accelerated flow or on a strongly cooled wall ("supercritical layer"), then at the section $x = x_0$ the quantities θ, p, M, H, dp/ds undergo a discontinuity. In the present study the calculation was made for the case of a thermally insulated surface and the flow for $x < x_0$ developed with p = const.

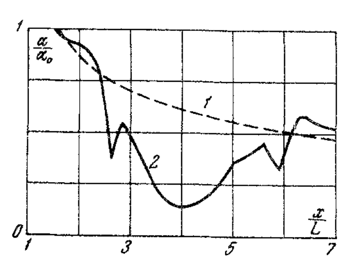

Fig. 3

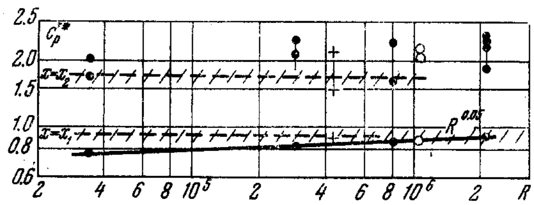
2.5
$c_p'^*$
2.0
1.5
1.0
0.8
0.6
$x=x_2$
$x=x_1$
$R^{0.05}$
2
4
6
8
10^5
2
4
6
8
10^6
2
R

Fig. 4

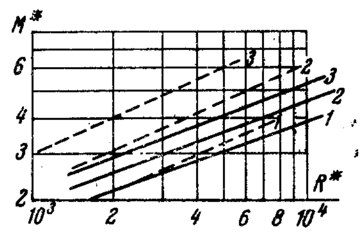
M^*
6
4
3
2
R^*
10^3
2
4
6
8
10^4

Fig. 5

It was assumed that the weak separation-free interaction in the regions $x < x_0$ and $x > x_4$ may be neglected, and as the initial conditions for the solution of the system (2.1)–(2.3) in the region $x_0 < x < x_4$ we can in the first approximation use the results of the solution of (2.1) with p = const in the region $x < x_0$. It was assumed that at the end of the interaction region the outer flow is parallel to the wedge surface, the pressure gradient is equal to zero, the pressure on the wedge corresponds to the pressure in the stream which has been turned isentropically through the angle β, the relationships between the integral parameters of the boundary layer correspond to flow on a flat plate. For each given value of x_0 the conditions at the end of the interaction region were satisfied by selection of the magnitude of the wedge angle β. Figure 1 shows the flow pattern.

3. Figure 2 shows the calculated distributions of the parameter $C_p^* = C_p\sqrt[4]{(M_0^2 - 1)R^1}$ (curve 1) and the friction coefficient C_{fw} (curve 2) for the case of flow past a wedge with angle $\beta = 0.128$ rad of an air stream with $M_0 = 2$ and $R_0 = 0.26 \cdot 10^6$. Figure 3 shows the distribution of the quantity α/α_0 which characterizes the thermal flux to the surface (curve 1 corresponds to the heat exchange on the plate without the wedge, and curve 2 corresponds to the heat exchange on the plate with wedge). Comparison of the results of the calculation with the experimental data of [7–10] shows that the calculation reflects quite accurately the flow pattern in the interaction region. Figure 4 shows the values of the quantities C_{p1}^* and C_{p2}^*, calculated at the points x_1 of separation and x_2 of the break in the flow surface for various wedge angles β. The dashed curves show the averaged experimental values of these quantities as a function of the Reynolds number R_1, and the shading shows the approximate limits of the scatter of the experimental data. At the point of separation the experimental and calculated values of the quantity C_{p1}^* agree to an accuracy of about 15%. At the surface breakpoint x_2 the quantity C_{p2}^* agrees with the experimental data with an accuracy of 15% only up to values of the angle β corresponding to $C_{p4}^* \approx 4.3$ (C_{p4}^* is the coefficient C_p^* at the end of the interaction region). For large values of β the quantities C_{p2}^* become larger than the experimentally measured values. It should be noted that after the point of reattachment x_3 the pressure continues to increase, and the difference of the pressure coefficients ($c_{p4} - c_{p3}$) is approximately equal to c_{p1} as a result of the symmetry of the flow up to the separation point and after the reattachment point.

It is of definite interest to clarify the conditions for the occurrence of separation of the laminar layer. An experimental study was made in [9] of these conditions on cylinders with a conical skirt. The Mach number M^* was determined for given values of R^* and β (M^* is the Mach number at which separation of the boundary layer is first observed; R^* is the Reynolds number calculated using the boundary layer thickness and the local parameters of the outer supersonic flow ahead of the beginning of the interaction region; β is the angle of deviation of the generator of the skirt surface from the direction of the cylinder generator). If we consider that the flow at the base of the skirt is approximately plane, then we can compare the calculated and experimental conditions for the occurrence of separation. Figure 5 shows the comparison of the calculated and experimental dependences of M^* on R^* for various values of β. The solid lines 1, 2, 3 correspond to the experimentally determined boundary of occurrence of separated flow for values of β = 0.0873, 0.1309, 0.1745 rad respectively. The dashed lines show the calculated results. We see that for $\beta = 0.0873$ rad the experimental and calculated values are in good agreement, although for large R^* the calculated separation occurs earlier, i.e., for larger M^*, than noted in the experiments. For large values of β the calculated separation also takes place somewhat earlier than found experimentally. It is possible that the experimental method of determining the occurrence of separation for small R^* used in [9] is not sufficiently sensitive.

The results of the calculations show that the length of the separated flow zone increases with increase of R^* and increases almost exponentially with increase of the quantity C_{p4}^*.

Figure 3 shows an example of the calculation of the quantity α/α_0 in the separated flow region, where α is the local heat transfer coefficient and α_0 is the heat transfer coefficient at the point x_0 of beginning of the interaction. The dashed curve shows the distribution of the ratio α/α_0 on a plate without a wedge (curve 1). The nature of the distribution of the ratio α/α_0 is in qualitative agreement with the experimentally measured distributions of the thermal fluxes in the separated flow zone ahead of a wedge [11].

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