International Journal of Fracture Mechanics, Vol. 6, No. 2, June 1970
Wolters-Noordhoff Publishing–Groningen
Printed in the Netherlands

On Fracture Path Stability in the Compact Tension Test

B. COTTERELL

Department of Mechanical Engineering, University of Sydney, Sydney.

(Received Oct. 17, 1969)

ABSTRACT
A simple criterion based on the engineers' theory of bending is suggested for the determination of the stability of fracture path in the compact tension test.

1. Introduction

The compact tension specimen (see Fig. 1) is gaining acceptance for fracture toughness measurements. It is desirable that the fracture path follow essentially the prolongation of the crack starter and not deviate strongly to one side. In similar specimens used to measure the fracture toughness of plastics where the arms were long, the fracture path deviated unless a compressive stress was induced along the fracture path [1] or the specimen was deeply grooved to control the fracture [2]. A theoretical criterion to determine whether a path is stable has been developed [3]. The stress near the crack tip can be expanded as a power series

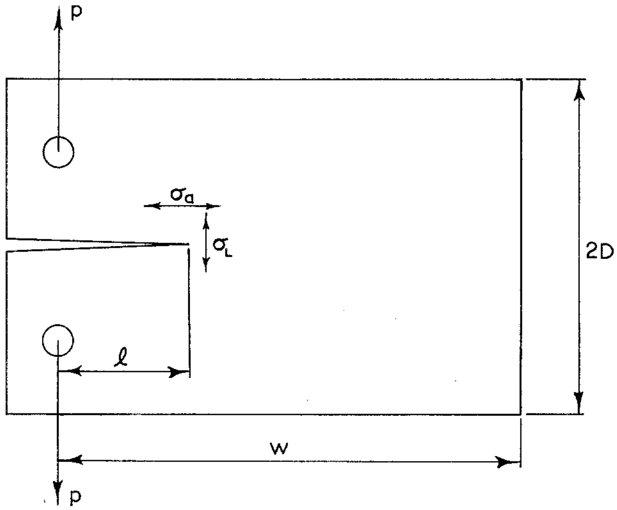

Figure 1. The compact tension specimen.

$$\sigma_\theta = \frac{a_1}{4}\left(\frac{l}{r}\right)^{\frac{1}{2}}\left[3\cos\theta/2+\cos\frac{3\theta}{2}\right] + a_2\sin^2\theta + \frac{a_3}{4}\left(\frac{r}{l}\right)^{\frac{1}{2}}\left[5\cos\theta-\cos\frac{5\theta}{2}\right] + 0\left(\frac{r}{l}\right). \tag{1}$$

If a small excursion from the ideal path is made because of some local inhomogeneity the path is stable if the next most probable path is directed back towards the ideal one (in Fig. 2 $d\phi > d\theta$). Retaining a higher order term than was in the previous work [3], the relationship between $d\theta$ and $d\phi$ becomes

$$d\phi = \left\{1 - \left(\frac{S}{l}\right)^{\frac{1}{2}}\left[\frac{4}{\pi}\left(\frac{a_2}{a_1}\right) + \left(\frac{a_3}{a_1}\right)\left(\frac{S}{l}\right)^{\frac{1}{2}}\right]\right\} d\theta\,, \tag{2}$$

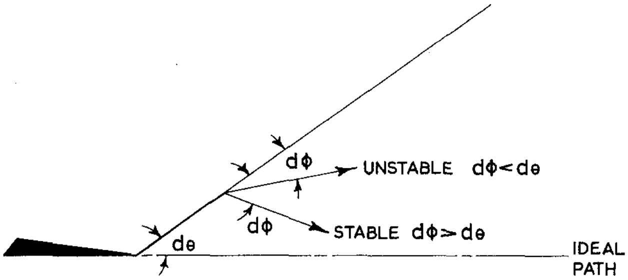

Figure 2. Stability of the crack path.

where S is the length of the excursion from the ideal path. It is reasonable to suppose that S will be comparable to the plastic zone size at the tip of the crack. Thus in a very brittle material the stability of the fracture path will be determined by the sign of (a_2/a_1). If this ratio is positive the path will be unstable.

2. Discussion of Stability of the Fracture Path

In tests on the A.S.T.M. compact tension specimen manufactured from a maraging steel (yield strength 230 k.s.i.) the fracture path was predominantly stable* [4]. For this particular specimen a_2/a_1 is positive, but a_3/a_1 is negative (see Table 1), thus the stability of the path

TABLE 1

The coefficients of the power series expansion of the stress distribution for the compact tension specimen $D/W=0.6$

$\frac{l}{W}$	$\frac{a_1}{(P/D)}$	$\frac{a_2}{(P/D)}$	$\frac{a_3}{(P/D)}$	$\frac{a_4}{(P/D)}$
0.2	2.542	0.728	−1.704	0.156
0.3	2.554	1.444	−2.826	0.186
0.4	2.775	2.093	−4.340	0.182
0.5	3.260	2.774	−6.851	0.180
0.6	4.210	3.564	−12.15	0.448
0.7	6.167	5.296	−26.86	2.250

depends on the size of the excursion S from the ideal path. Equation (2) holds strictly for a crack of length l in an infinite sheet and cannot be used to determine the critical length S quantitatively for the A.S.T.M. specimen. However, since it is likely that the actual excursion from the ideal path will be roughly proportional to the plastic zone size, tests were made on glass specimens made to the same dimensions as the A.S.T.M. specimens. These specimens also exhibited stable crack paths. Thus Eq. (2) seems to fail to give a reasonable prediction of the stability of the path. Perhaps the failure is because the equation only indicates when the path may deviate and the probability that it does actually deviate increases with the length of the fracture. A finite crack growth is required before the fracture deviates. The A.S.T.M. specimen is short and the crack would come under the influence of the boundary of the specimen before it is highly probable that the fracture deviates. When the fracture approaches the boundary of the specimen, Eq. (2) ceases to be accurate.

A very simple minded approach to the problem of the stability of the path in the case of the compact tension specimen seems to work. The engineers theory of bending can be used to calculate the maximum nominal bending stress in the ligament and arm of the specimen.

* Only 2 out of 11 specimens tested showed strong deviation of the fracture path from the centre line. In a further 3 specimens there was some tendency to deviate.

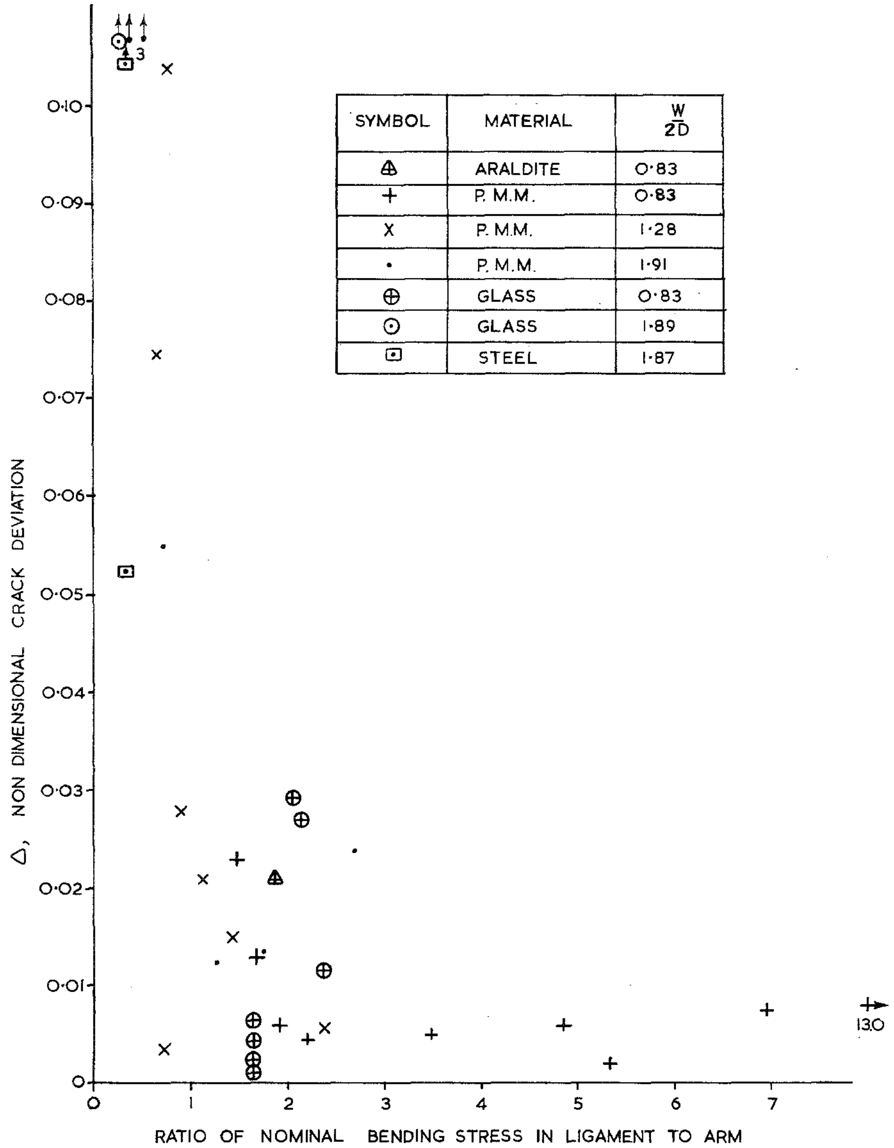

Figure 3. Crack deviation as a function of the nominal bending stress at the crack tip.

$$\sigma_A = \frac{6Pl}{D^2}\,; \quad \sigma_L = \frac{3P(W+l)}{(W-l)^3} \tag{3}$$

where P is the force per unit thickness of the specimen. It is tempting to suppose that the crack will deviate from the straight path if σ_A is very much greater than σ_L. In Fig. 3 the crack deviation measured at the edge of the specimen, non-dimensionalised by the length of the crack path $(W-l)$ is plotted against the ratio of the nominal bending stress in the ligament to that in the arm for various materials. It is seen that provided $\sigma_L/\sigma_a > 1$ the crack deviation is small, if $\sigma_L/\sigma_a < 1$ a large crack deviation is probable.

3. Acknowledgements

Dr. Srawley is thanked for drawing attention to this problem and providing Table 1.

This work was financed by the Australian Research Grants Committee.

REFERENCES

[1] J. J. Benbow, Stable Crack Propagation in Plastics, *Proceedings of the Physical Society*, 78 (1961) 970.
[2] J. P. Berry, Determination of Fracture Surface Energies by the Cleavage Technique, *Journal of Applied Physics*, 34 (1963) 62.
[3] B. Cotterell, Notes on the Paths and Stability of Cracks, *International Journal of Fracture Mechanics*, 2 (1966) 526.
[4] J. E. Srawley, private communication.

RÉSUMÉ
Un critère simple, basé sur l'équation d'équarissage, est proposé pour la détermination de la stabilité du parcours que suit une rupture au cours d'un essai de traction sur éprouvette compacte.

ZUSAMMENFASSUNG
Zur Bestimmung der Stabilität des Fortpflanzungspfades einer Rißes beim Zugversuch an kompakten Proben, wird ein einfacher, von der Ingenieurtheorie für Biegung abgeleiteter Kennwerk vorgeschlagen.

Int. Journ. of Fracture Mech., 6 (1970) 189–192