31. J. Mandel, "Equations constitutives et directeurs dans les milieux plastiques et visco-plastiques," Int. J. Solids Struct., 9, No. 6, 725-740 (1973).
32. Y. F. Dafalias, "The plastic spin concept and a simple illustration of its role in finite plastic transformations," Mech. Mater., 3, No. 3, 223-233 (1984).
33. F. Sidoroff, "Incremental constitutive equation for large strain elastoplasticity," Int. J. Eng. Sci., 20, No. 1, 19-26 (1982).
34. V. N. Kukudzhanov and V. I. Kondaurov, "Numerical solution of problems of the dynamics of a hard deformable solid, which are not one-dimensional," Problems of the Dynamics of Elastoplastic Media [Russian translation], Mir, Moscow (1975), pp. 39-84.
35. V. A. Lubarda and E. H. Lee, "A correct definition of elastic and plastic deformation and its computational significance," Trans. Am. Soc. Mech. Eng.: J. Appl. Mech., 48, No. 1, 35-40 (1981).
36. L. B. Freund, "Constitutive equations for elastic-plastic materials at finite strain," Int. J. Solids Struct., 6, No. 8, 1193-1209 (1970).
37. V. I. Levitas, "Dissipation postulate for discrete and continual plastic systems," Dokl. Akad. Nauk UkrSSR, Ser. A, No. 6, 26-32 (1983).
38. N. V. Novikov and V. I. Levitas, "On experimental confirmation of the reinforced postulate of ideal plasticity under a monotonic load," Dokl. Akad. Nauk UkrSSR, Ser. A, No. 2, 42-46 (1984).
39. N. V. Novikov, V. I. Levitas, and O. A. Rozenberg, "On experimental confirmation of the reinforced postulate of ideal plasticity under a quasimonotonic load," Dokl. Akad. Nauk UkrSSR, Ser. A, No. 8, 31-34 (1985).

THEORY OF ROCK CUTTING

G. P. Cherepanov

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Cutting is one of the main production processes in the treatment of materials and drilling boreholes. The study of rock cutting is of particular importance for modern drilling in view of the invention and extensive use of the most promising cutter construction of the "stratapax" type in drilling bits [1]. Cutters of this type are distinguished by extremely high wear resistance; their front surface is coated with diamond particles applied by a special technology in a hard alloy layer (tungsten carbide).

Main Experimental Results. Let a certain absolutely hard sharp body (cutter) move along the surface of another hard deformable body (rock) causing surface fracture of the latter; N and T are applied normal and tangential forces respectively, applied to the cutter; force T is called the cutting resistance force.

Cutting in an elastoplastic regime (most typical for metals) is characterized by marked plastic deformation of the material beneath the cutter and a cutting resistance force steady with time. In this way visually a continuous shaving is observed (Fig. 1). In an elastoplastic steady cutting regime there are the following regularities [2, 3]:

$$T = \eta\sigma_s hH\sin\alpha;\qquad T = \mu N,$$

where h is cutting depth (shaving thickness); H is specimen thickness (along the normal to the diagram in Fig. 1a); σ_s is yield strength; α is slope angle of the cutter plane; η and μ are dimensionless coefficients depending on cutter configuration and cutting depth.

In a brittle regime (most typical for the majority of rocks) cutting is distinguished by marked oscillation in time of the resistance force operating on a moving cutter from the direction of the rock. Visually in this way a fine particle and not a shaving is observed.

Given in Fig. 2a is a typical oscillogram for the dependence of cutting resistance force on cutter displacement with a typical very marked regular oscillation of frequency ~10^3 Hz. As shown subsequently, these relaxation inertia-free oscillations are of the "stick-slip" type. This suggests realization of a steady random process with time; each oscillation is

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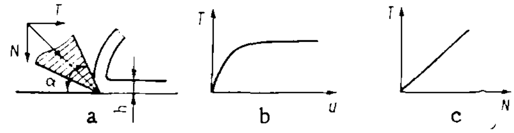

Fig. 1. Cutting in an elastoplastic regime: a) cutting scheme with a continuous shaving; b) diagram of the dependence of cutting resistance force T on cutter displacement u (the initial section of the T—u diagram characterizes the start of cutting); c) diagram of the dependence of cutting resistance force T on normal force N applied to the cutter in the steady cutting stage.

characterized by a maximum (T_{max}) and minimum (T_{min}) value of the cutting resistance force. With a constant cutting depth the average values $\langle T_{max}\rangle$ and $\langle T_{min}\rangle$ may be assumed to be independent of time and cutter displacement. In fact, in this sense it may indicate a steady cutting regime in the case given (values of T_{max} and T_{min} are assumed not to be dependent on cutter displacement). With a variable cutting depth the process may be assumed to be quasisteady. It is noted that the cutting depth for the prescribed cutter is clearly governed by the normal load on the cutter N. Cutter displacement proceeds in jumps (Fig. 2b).

Values of T_{max} and T_{min} in a steady cutting rgime are approximately proportional to the normal force operating on the cutter (Fig. 2c):

$$T_{max} \approx \mu_{max} N; \qquad T_{min} \approx \mu_{min} N. \tag{2}$$

Values of μ_{max} and μ_{min} depend markedly on cutter sharpness, i.e., on the radius of its curvature in the working area; the sharper the cutter, the greater are values of μ_{max} and μ_{min} (generally μ_{max} = 1-10 for different types of cutter configuration).

Given below are a theory for oscillations occurring during cutting of rocks and a theory for the resistance of rocks to cutting. In particular, for a brittle cutting schedule in the case of rock specimens found in a plane stressed state, the following relationship is derived:

$$T_{max} = HK_{IIc}\sqrt{2h}. \tag{3}$$

Here K_{IIc} is sliding toughness for the rock [2], which is a basic indicator of rock resistance to cutting.

It should be emphasized that a brittle cutting regime specifies a crack moving ahead of the cutter under conditions of considerable plastic deformation in the material beneath the cutter. Failure in this regime is not brittle or quasibrittle (therefore it is not possible to use the Griffiths–Irwin theory or any other theory of linear fracture mechanics).

In spite of the very extensive literature on cutting and the considerable volume of studies in this field, until now these phenomena do not have either quantitative theoretical description or qualitative schematization.

All of the experimental results provided in this work for rock cutting were obtained by M. I. Vorozhtsov (laboratory of rock fracture mechanics, All-Union Scientific-Research Institute on Drilling Technology). Some experimental work for oscillation during rock cutting [4, 5] and also for oscillation with dry friction [6-12] are noted (e.g., oscillations of the "stick-slip" type). For example, monographs [13-18] are devoted to rock cutting resistance.

<u>Oscillations during Cutting.</u> We shall consider a cutter as a certain concentrated pointed mass m moving along the surface of a rock specimen and pressed towards the surface with normal force N. The rock specimen is immobile. The cutter is joined with a certain massive elastic body (support), whose movement leads to cutter movement. Let a certain point O of this body be reckoning point for mutual displacements (Fig. 3). The elastic connection between point O and the cutter is modeled by a spring. We assume that point O moves with constant velocity v along the specimen surface with t > 0 (t is time). It is required to determine cutter movement under the action of the force of inertia, elastic reaction of the support, and resistance force T from the direction of the rock.

The elastic reaction of the support is

$$F_e = k\Delta u. \tag{4}$$

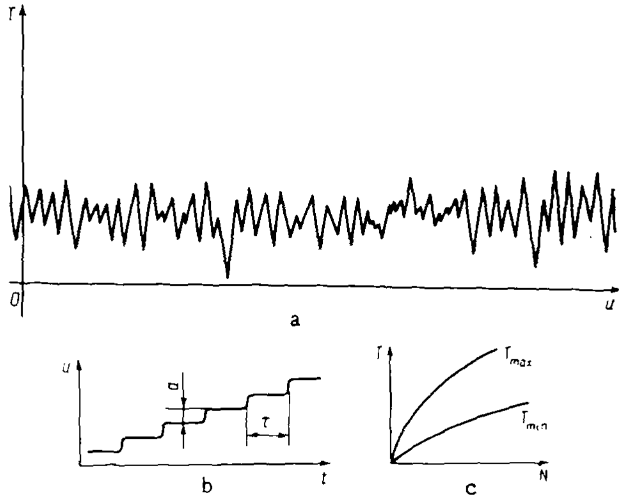

Fig. 2. Cutting in a brittle regime: a) typical oscillogram for the dependence of resistance force T on cutter displacement u; b) dependence of cutter displacement u on time t (*a* is the magnitude of the jump; τ is oscillation period); c) dependence of maximum T_{max} and minimum T_{min} cutting resistance force on normal force N applied to the cutter.

Here Δu is cutter displacement in relation to point O; k is spring rigidity coefficient, easily determined by experiment.

We ignore oscillations in a distributed support; i.e., in future, consideration is only given to oscillations with frequency $\omega \ll c/\ell$, where c is sound velocity in the spring; ℓ is typical linear spring dimension (if for example $\ell \sim 5$ cm, $c \sim 5 \cdot 10^5$ cm/sec, then $\omega \ll 10^5$ Hz).

At first with displacement of point O the cutter will be immobile relative to the specimen and the limiting condition will not yet be reached (preceding failure and separation), at which $T = T_{max}$ and

$$T_{max} = ku_0 \quad (vt = u_0), \tag{5}$$

where u_0 is cutter displacement relative to point O in the limiting condition at instant of time $t = t_0$.

Let us assume that with $t > t_0$ the cutting resistance force T will be less than T_{max} (e.g., as a result of local loss of stability and propagation of a local dynamic crack ahead of the cutter). Then apparently with $t > t_0$ cutter movement commences relative to the immobile rock specimen; as is normal, we designate in terms of u cutter displacement relative to the immobile specimen. In this way cutter displacement relative to point O will equal

$$\Delta u = vt - u = v(t - t_0) - u + u_0. \tag{6}$$

The equation for cutter movement with $t > t_0$ under the action of all forces on a basis of (4)-(6) takes the form

$$m\frac{d^2u}{dt^2} = k[v(t - t_0) - u + u_0] - T \tag{7}$$

or

$$m\frac{d^2u}{dt^2} = kv(t - t_0) - ku + T_{max} - T. \tag{8}$$

The value of T for a brittle cutting schedule of elastoplastic and brittle materials will be a certain periodic sawtooth function of u of the type shown in Fig. 3b. We take the following linear approximation of this function with $0 < u < a$, $\dot{u} \geq 0$:

$$T = T_{min} + \frac{T_{max} - T_{min}}{a}u. \tag{9}$$

It is noted that the condition for non-negativity of cutter velocity $\dot{u} \geq 0$ is a condition for cutter contact with the specimen; with reverse movement of the cutter contact is disrupted, and instead of (9) a condition for the absence of rock reaction should be fulfilled: T = 0 with $\dot{u} < 0$. Parameter a is previously unknown, it determines cutter displacement for one act of failure (fracture of the projection), and it is subject to determination during the solution (Fig. 3).

We substitute expression (9) in (8) and solve the linear equation obtained by using initial conditions u = 0, $\dot{u} = 0$ with $t = t_0$. With $t_0 \leqslant t < t_0 + \tau$ we have

$$u = \frac{T_{max} - T_{min}}{m\omega^2} + \frac{kv}{m\omega^2}(t - t_0) - \frac{kv}{m\omega^3}\sin\omega(t - t_0) - \frac{T_{max} - T_{min}}{m\omega^2}\cos\omega(t - t_0),$$

or in more compact form

$$u = \frac{kv}{m\omega^3}\left\{\operatorname{tg}\varphi + \omega(t - t_0) - \frac{\sin[\omega(t - t_0) + \varphi]}{\cos\varphi}\right\};$$ (10)

$$\dot{u} = \frac{kv}{m\omega^2}\left\{1 - \frac{1}{\cos\varphi}\cos[\omega(t - t_0) + \varphi]\right\}.$$

Here

$$\omega^2 = \frac{k}{m} + \frac{T_{max} - T_{min}}{am}; \qquad \operatorname{tg}\varphi = \omega\frac{T_{max} - T_{min}}{kv};$$

τ, ω, φ are oscillation period, frequency, and phase respectively.

In order to find the previously unknown values of a and ω use is made of the following periodicity conditions:

$$u(t_0 + \tau) - u(t_0) = a; \qquad \dot{u}(t_0 + \tau) = 0.$$ (11)

On the basis of (10) they take the form

$$a = \frac{kv}{m\omega^3}\left[\operatorname{tg}\varphi + \omega\tau - \frac{\sin(\omega\tau + \varphi)}{\cos\varphi}\right];$$ (12)

$$\cos(\omega\tau + \varphi) = \cos\varphi.$$

It is easy to show that the required condition $\dot{u} \geqslant 0$ is satisfied by a single solution of the second equation of (12):

$$\omega\tau = 2(\pi - \varphi).$$ (13)

By substituting the value of $\omega\tau$ obtained in the first equation of (12), we have

$$a = \frac{2kv}{m\omega^3}(\pi + \operatorname{tg}\varphi - \varphi).$$ (14)

Now we find parameter a from Eq. (10) governing frequency ω, and substitute it in (14); as a result of this we obtain the following transcendental equation for finding the oscillation frequency sought:

$$\frac{\lambda^3}{2(\lambda^2 - \alpha^2)} = \pi - \operatorname{Arctg}\lambda \times \lambda \quad \left(0 < \lambda < \infty,\ 0 < \operatorname{Arctg}\lambda < \frac{\pi}{2},\quad 0 < \alpha < \infty\right);$$ (15)

where λ and α are dimensionless parameters,

$$\lambda = \frac{\omega}{kv}(T_{max} - T_{min}); \qquad \alpha = \frac{T_{max} - T_{min}}{v\sqrt{km}}.$$ (16)

Equation (15) has a single positive root with any α. For the function $\lambda(\alpha)$ there are two asymptotes:

$$\lambda = \alpha + \alpha^3/(4\pi) \text{ for } \alpha \to 0;$$

$$\lambda = \sqrt{2}\alpha - \pi/4 + 0(1/\alpha) \text{ for } \alpha \to \infty.$$

Whence by means of (16) we determine the range of possible oscillation frequencies:

$$\omega_f \leqslant \omega \leqslant \sqrt{2}\omega_f \quad (\omega_f^2 = k/m),$$

where ω_f is frequency of free oscillations for pointed mass m on a spring with stiffness k; i.e., due to nonlinearity of the model, free oscillations of the cutter are transformed.

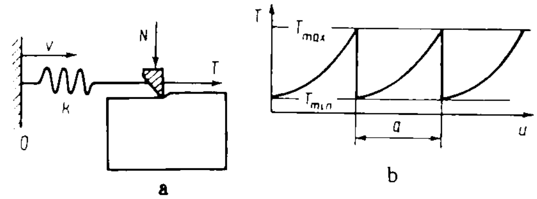

Fig. 3. Theoretical scheme for studying cutter movement during cutting in a brittle regime: a) cutting scheme taking account of cutter support elasticity; b) diagram for the dependence of resistance to cutting force T on cutter displacement u.

Thus, the periodic (for rate) oscillation cycle sought exists and it is unique. The form of its phase φ, frequency ω, wavelength a, and period τ are determined respectively by Eqs. (10), (15), (16), (14), (13). It is typical that in the model (9) considered with constant T_{max} and T_{min}, emergence into the periodic regime constructed is already accomplished in the first oscillation. The oscillations found will be called transformed free oscillations.

We determine the periodic (for rate) solution constructed for stability towards small disturbances. Let the frequency of disturbed movement be

$$\omega(t) = \omega_0 + \Delta(t) \quad (\max|\Delta(t)| \ll \omega_0), \tag{17}$$

where ω_0 is undisturbed cycle frequency; $\Delta(t)$ is a small disturbance.

By substituting frequency ω according to (17) in the solution of (10) for $u(\omega, t)$, we obtain a disturbance solution:

$$u = u_0(\omega_0, t) + g(t); \qquad g(t) = \Delta(t)\left(\frac{\partial u}{\partial \omega}\right)_{\omega=\omega_0} \\ (\max|g(t)| \ll |u_0|). \tag{18}$$

After substituting disturbance solution (18) in differential Eq. (9) we have

$$\ddot{g} + g\left[\omega_0^2 + \frac{2u_0\omega_0}{(\partial u/\partial \omega)_{\omega=\omega_0}}\right] = 0. \tag{19}$$

Direct computation indicates that with $t \to \infty$

$$\frac{2u_0}{(\partial u/\partial \omega)_{\omega=\omega_0}} = -\frac{2\omega_0 \cos\varphi}{2\cos\varphi + \cos(\omega_0 t + \varphi)}. \tag{20}$$

Solution of Eq. (19) with $t \to \infty$ is of interest for us, and therefore the coefficients of Eq. (19) may be substituted by their limiting values with $t \to \infty$. Finally we obtain the following Hill equation:

$$\frac{d^2 g}{d\xi^2} + \frac{\cos\xi}{2\cos\varphi + \cos\xi} g(\xi) = 0 \quad (\xi = \omega_0 t + \varphi, \\ 0 < \varphi < \pi/2). \tag{21}$$

If function $g(\xi)/\xi$ is limited with $\xi \to \infty$, then the initial disturbance solution will be assumed to be stable towards small disturbances. In the opposite case the undisturbed solution will be assumed to be unstable.

A fundamental set of solutions for Eqs. (21) will be the following:

$$g_1(\xi) = \cos\xi + 2\cos\varphi;$$

$$g_2(\xi) = g_1(\xi)\int_{\xi_*}^{\xi} \frac{d\xi}{(g_1(\xi))^2},$$

where ξ_* is any number with which the integral converges.

We quote the final result of the research which was obtained by O. A. Akhanchenok: with $0 < \varphi < \pi/3$ the undisturbed solution is stable towards small disturbances, and with $\pi/3 \leq \varphi < \pi/2$ it is unstable.

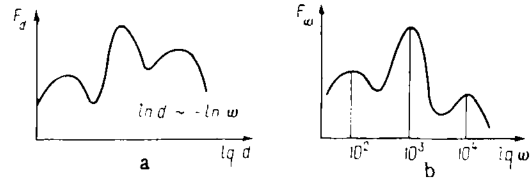

Fig. 4. Diagram of the conformity of tool oscillation frequency ω during cutting of rocks to a fraction size d of the broken material ($\ln d \sim -\ln \omega$): a) distribution curve for the size of broken rock fractions; b) cutter oscillation spectrum.

We cite as an example calculation of oscillations occurring during cutting of specimens in test equipment KIPR-R [19]. Starting data are: strain dynamometer stiffness $k = 2.5 \cdot 10^5$ N/mm; weight of the oscillating holder with the cutter m = 0.3 kg; free oscillation frequency $\omega_f = 2.9 \cdot 10^4$ Hz; frequency of transformed oscillations $4 \cdot 10^4$ Hz $> \omega > 2.9 \cdot 10^4$ Hz. In this case with v = 1 m/sec we have $a \approx 0.1$ mm; $\tau \approx 10^{-4}$ sec; max $\dot{u} = v$.

As can be seen, the cutting oscillations considered have low intensity and very high frequency, close to the frequency of tool free oscillations: they are not picked up by the measuring instrument and they lead to formation of the finest fractions of failed material, therefore in pure form they can only be realized with a very small cutting depth $h \sim a$.

With an increase in cutting depth to a value of h comparable with grain size d, microinhomogeneity of the rock becomes marked; the cutting resistance force becomes quasiperiodic with period d/v, where v is cutting rate. Grains of rock will consist of failed material fractions. Oscillation frequency for the cutting force corresponding to these fractions is of the order of

$$\omega = \frac{v}{d}. \tag{22}$$

Oscillations of this nature will have a stochastic character due to the random sizes of particles. In this case model (9) is inapplicable, and oscillations and their frequency range are calculated elementarily by Eq. (22). In particular, for Koelginsk marble $d \sim 0.5$ mm, and with v = 0.5 m/sec we have $\omega \sim 10^3$ Hz; a frequency of this order (from 800-1300 Hz) was recorded in tests for cutting Koelginsk marble in KIPR-R equipment [19] with a natural frequency of the oscillograph sweep of 2300 Hz.

With a further increase of depth of the cutter into the rock ($h \gg d$) the size a of the rock particles broken off is determined by the condition of instability for rectilinear development of a crack and the start of its curvature. From physical considerations and experimental data it follows that $a \sim (1\text{-}3)h$. Oscillation frequency for the cutting force corresponding to these fractions is of the order (the value of a in this case is also random to a significant degree)

$$\omega = \delta \frac{v}{h} \quad (\delta = 0,3 \ldots 1). \tag{23}$$

In cutting Koelginsk marble in KIPR-R equipment (h = 4 mm; v = 0.5 m/sec) low-frequency oscillations were observed with a frequency of about 50-150 Hz, which is close to the frequency of 100 Hz determined by Eq. (23). In this case model (9) is also evidently unsuitable.

Thus, after fracture of a projection of height $h \gg d$ during a gradual increase of depth for the cutter into the rock there is a uniform increase in the size of the broken fractions (from $2\pi v/\omega_f$ to d) and a gradual reduction in cutter oscillation frequency (from ω_f to v/d), then finally the cutter stops and the resistance force T increases uniformly to a value T_{max}, after which there is separation of the projection and the whole process is repeated (with a frequency of $\delta v/h$). This is the picture of tool oscillation during cutting of rocks. It is noted that oscillations of the last two types (relating to particle sizes $a \sim d$ and $a \sim (1\text{-}3)h$) are inertia-free relaxation oscillations of the stick-slip type (in contrast to the transformed free oscillations considered above).

On the basis of the analysis carried out we obtain the following procedure for determining the tool oscillation spectrum during cutting of rocks from the curve for distribution of broken material fraction sizes (Fig. 4).

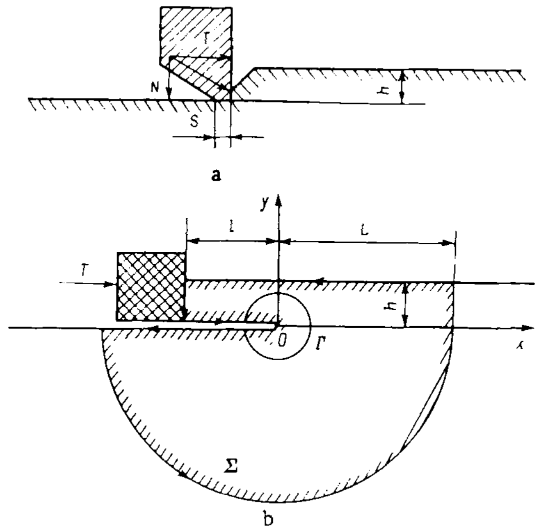

Fig. 5. Actual (a) and idealized (b) scheme for the development of fracture during cutting in a brittle regime.

First we plot an empirical curve for distribution $F_d(d)$ (Fig. 4a) from test results with a constant cutting rate ($F_d(d)\Delta d$ is the relative volume fraction of particles of broken rock with a size from d to d + Δd in relation to all particle sizes from 0 to ∞, and particle size is determined as the cubic root of its volume V, i.e., $d = V^{1/3}$). Methods for sorting particles according to their weight are well developed. Then by replotting the distribution curve $F_d(d)$ in Fig. 4a to a new variable $\omega = v/d$, we obtain the spectrum sought $F_\omega(\omega)$ for cutter oscillation during cutting (Fig. 4b).

On the $F_\omega(\omega)$ curve there will be three maxima relating to inertia-free quasistatic cutter oscillations (with frequencies of about $\delta v/h$ and v/d) and transformed free cutter oscillations with a frequency of about $\omega_f \gg v/d \gg \delta v/h$. Conversely, by this method it is possible from the measured experimental spectrum $F_\omega(\omega)$ to determine the distribution curve $F_d(d)$.

We now compare the transformed free cutter oscillations with relaxation oscillations of the stick-slip type with dry friction. According to [6] in the latter case there will be

$$a = \frac{T_{max} - T_{min}}{k}; \quad \tau = \frac{a}{v}; \qquad\qquad (24)$$
$$\omega = \frac{v}{a} = \frac{vk}{T_{max} - T_{min}}.$$

For example, in KIPR-R equipment with v = 1 m/sec and using a standard G41 cutter, dry friction (with shaving of surface roughness) occurs with $T_{max} \lessapprox 10^3$ N. In this case, assuming that $T_{max} - T_{min} \approx 10^3$ N, $k \approx 2.5 \cdot 10^5$ N/mm, we obtain $a \approx 4 \cdot 10^{-3}$ mm, $\omega \approx 2.5 \cdot 10^5$ Hz, i.e., the frequency of inertial oscillations of the stick-slip type is an order of magnitude greater than the frequency of transformed free oscillations.

Mathematical Model of Cutting. Let a specimen of rock with thickness H be in a plane stressed state and be quite large compared with the cutting depth h, so that it is possible to present it in the form of a plane layer with thickness H occupying at first a half plane $y < h$ (Fig. 5). The cutter is assumed to have the shape of a right prism and to be absolutely rigid; the height of the prism is greater than H; during cutting a layer $0 < y < h$ is removed from the specimen surface (Fig. 5a).

Cutting is presented simply as a repeated process of brittle fracture of the projection of thickness h and length ℓ with a flat absolutely rigid and smooth indentor (cutter) as a result of the development of a plane crack along section $y = 0$, $-\ell < x < 0$ (Fig. 5b). The indentor gives at the boundary of the projection with $x = -\ell$, $0 < y < h$ force T in the direction of axis x. The rest of the specimen boundary (with $x < -\ell$, $y = 0$ and with $x > -\ell$, $y = h$) we assume to be free from external loads.

Stresses σ_y and τ_{xy} at the slip crack edges are also assumed to be zero. This assumption is valid only in the limiting condition, at the instant immediately preceding unstable slip crack development (i.e., separation of the projection). With an increase in force T the slip crack (at first a knife-shaped plastic zone) develops steadily from the angle of the projection along axis x; the start of its unsteady growth and a change in direction correspond to fracture of the projection. In fact, this development of the prefailure process was observed in experiments with cutting of rocks (Fig. 5a). In addition, directly beneath the cutter with the action of compression a significant plastic zone forms [20, 21].

As direct experimental measurements indicate, the value of vertical force N operating on the cutter at instant of projection fracture is approximately proportional to the resistance force T (force N as it were builds on force T in order to provide a constant cutting depth h, i.e., the steady cutting condition). The coefficient of proportionality markedly depends on cutter sharpness: the sharper the cutter, i.e., the smaller the contact area S, the smaller the coefficient of proportionality.

Thus, the theoretical scheme in Fig. 5b was checked strictly by experiment for quite sharp cutters; however, it is always suitable when there is periodic separation of projections (and not their crumpling). Normal force N during cutting plays a secondary role in the process of unstable local failure at the end of a slip crack. There appears to be little importance in the situation that with cyclic repeated fracture the shape of the upper face of the specimen ahead of the cutter may differ markedly from the suggested straight line $y = h$, $x > -\ell$ (see Fig. 5a and b). In addition, during subsequent theoretical calculation it is assumed that $\ell \gg h$. However, all of these assumptions are valid as a result of the marked concentration of the shear stress field close to the crack end and the exponentially decreasing error of the theoretical solution with an increase in distance from the start of the coordinates with $0 < y < h$ and $x \to -\infty$.

We now move to the theoretical model presented in Fig. 5b. We consider a closed contour Σ shown by arrows and isolated by hatching (it is assumed that $L \gg h$, $\ell \gg h$). According to the theory of invariant Γ-integrals there is the following equality [2]:

$$\int_{\Sigma} (U n_1 - \sigma_{ij} n_j u_{i,1})\, d\Sigma = 0 \quad (i = 1, 2), \tag{25}$$

where U is elastic potential of a unit volume; n_1 and n_2 are components of the single external normal to the contour Σ; σ_{ij} and u_i are stresses and displacements in a Cartesian coordinate system Ox_1x_2, moving together with the crack end ($x_1 \leftrightarrow x_2$, $x_2 \leftrightarrow y$).

This equality is valid for arbitrary closed contour Σ in two cases:

1) rock behaves as a nonlinearly elastic body;

2) rock is arbitrary in its rheological properties (ductility, toughness, etc.), but the process is steady in the coordinate system Ox_1x_2. (In the second case U is deformation energy.)

In order to cut rocks the first case is realized since in a single act of loading and separation of the projection, loading at all points of the rock is close to simple, and the rock reaction may be worked out from elastoplastic deformation theory.

Integral (25) along the specimen boundary $y = 0$, $-L < x < -\ell$ and $y = \ell$, $-\ell < x < L$ equals zero in view of the conditions $n_1 = 0$, $\sigma_y = \tau_{xy} = 0$ at this boundary. As a result of the same conditions fulfilled in the limiting condition of maximum rock resistance, this integral equals zero along the crack edges with $y = 0$, $-\ell < x < 0$. The value of integral (25) for the closed small contour surrounding the crack end is designated in terms of $-H\Gamma$ (Γ is energy dissipation at the crack end related to two units of its area). Integral (25) along section $x = L$, $0 < y < h$ and along circle $x^2 + y^2 = L^2$, $y < 0$ is negligibly small (with $L \gg h$), since stresses on them are of the order of $1/L$ (as from a concentrated force at the boundary of the half plane). With $x = -\ell$, $0 < y < h$, where $\ell \gg h$, the specimen material may be assumed to be elastic; the elastic field will be as follows:

$$\sigma_x = -T/(hH); \quad \sigma_y = \tau_{xy} = 0; \quad u_{x,x} = -T/(EhH);$$
$$U = T^2/(2Eh^2H^2) \quad (n_1 = -1,\ n_2 = 0). \tag{26}$$

By means of expressions (25) and (26) we obtain

$$\Gamma = \frac{T^2}{2EhH^2}. \tag{27}$$

This is the basic theoretical result for the given model. We now consider some practical consequences emerging from it.

1. Maximum Cutting Resistance Force. In the limiting condition the start of dynamic instability for shear crack development is characterized by a certain value $\Gamma = \Gamma_c$, which it is natural to consider as the failure energy for a given brittle rock during cutting [2]. Whence by using relationship (27) we obtain the maximum cutting resistance force:

$$T_{\max} = H\sqrt{2E\Gamma_c h}. \tag{28}$$

The cutting scheme being considered (prismatic cutter and a flat specimen in a plane stressed state), and also Eq. (28), may be used in order to measure indirectly during a test the failure energy for a given rock during cutting.

2. Scale of Rock Resistance to Cutting. In Eq. (28) there is implicitly sliding toughness K_{IIc}, which is [2]

$$K_{IIc} = \sqrt{E\Gamma_c}. \tag{29}$$

This is a unique rock physical constant on which the maximum cutting resistance force depends:

$$T_{\max} = HK_{IIc}\sqrt{2h}. \tag{30}$$

Therefore, it is natural to measure rock resistance to cutting by the value of K_{IIc}, determined in the same test by means of Eq. (30). The qualitative scale obtained in this way for the resistance of different rocks to cutting has obvious and undoubted advantages over the scales suggested previously [13-18]. Comparison of this scale at the present along basic lines is carried out in the rock fracture mechanics laboratory of All-Union Scientific-Research Institute of Drilling Technology. It is recalled that the size of K_{IIc} equals force divided by the length to a power of two-thirds.

3. Index of Cutter Shape Efficiency. In drilling practice use is made of various cutters of complex spatial configuration. In view of the varied parameters there is as yet no single index characterizing cutter shape efficiency; obtaining such an index is one of the most important problems of drilling bit construction theory.

Equation (30) suggests a scientifically based method for selecting cutter shape efficiency index. For this purpose we introduce the value q as the average volume of rock broken by the cutter per unit time (v is average cutter movement velocity). Under test conditions with a flat specimen (Fig. 5), according to (30) we obtain

$$q = vhH; \qquad T_{\max} = \frac{q}{v} K_{IIc}\sqrt{\frac{2}{h}}. \tag{31}$$

On the basis of simple physical expressions and analysis of the sizes for cutters of complex spatial configuration moving over the surface of a comparatively large specimen, there will be a similar equation, but with a dimensionless coefficient η depending on cutter shape and on the ratio of the cutter size to cutting depth:

$$T_{\max} = \frac{1}{\eta}\cdot\frac{q}{v} K_{IIc}\sqrt{\frac{2}{h}}. \tag{32}$$

We present it in the following form:

$$\eta = \frac{\sqrt{2}qK_{IIc}}{vT_{\max}\sqrt{h}}. \tag{33}$$

Coefficient η is an index of cutter shape efficiency at the instant of time being considered; the greater the value of η, the greater the volume broken by a cutter of the corresponding configuration per unit of time. Determination of this index is necessary for comparative evaluation of cutter constructions.

In fact, with the passage of time t the cutter wears and its local index η decreases. For cutting conditions varying with time, the volume of rock broken by the cutter in time t_0 with constant cutting rate and depth equals

$$Q = \int_0^{t_0} q(t)\,dt = \frac{v\sqrt{h}}{\sqrt{2}K_{IIc}} \int_0^{t_0} \eta(t)\,T_{\max}(t)\,dt. \tag{34}$$

We introduce dimensionless coefficient η_G, which is

$$\eta_G = \frac{\sqrt{2} K_{IIc} Q}{v \sqrt{h} T_{\max}(0)} = \int_0^\infty \eta(t) \frac{T_{\max}(t)}{T_{\max}(0)} dt. \quad (35)$$

It is apparent that the value of η_G is the most objective global index of cutter quality, its wear, and operating efficiency. Normally in drilling practice use is made of another index, i.e., cutter operating time, which however depends markedly on the type of rock and cutting conditions: depending on these factors it may vary by a factor of tens and hundreds.

4. Comparative Method for Determining Sliding Toughness and the Rock Cutting Resistance Scale. Procedures for determining sliding toughness and rock cutting resistance described in Parts 1 and 2 have the following drawbacks. They require specific flat specimens (in order to provide a plane stressed state), sufficient thickness and length (so that it is possible to ignore edge effects of rock separation close to the free surface). Special cutters in the shape of a prism are necessary (the cutting face is the side edge of the prism).

Meanwhile, such a method for obtaining sliding toughness follows from the procedure given in Part 3 which is easily realized in any specimen with a flat (easily ground) face.

We choose the most typical cutter configuration (including its precise dimensions and shape) as a standard, and by Eq. (33) we determine its efficiency index η in any of the rocks studied. It is natural to take as an example the standard cutter G41. By using the standard cutter for cutting other rocks, their sliding toughness according to (33) is found by the equation

$$K_{IIc} = \eta \frac{v T_{\max} \sqrt{h}}{\sqrt{2} q}. \quad (36)$$

It is remembered that a necessary and apparently satisfactory external indicator of a brittle cutting regime is the absence of a continuous shaving and marked oscillations in the cutting resistance force. It is possible to discern these oscillations by means of low-inertia recording instruments (the sweep of an oscillograph). For typical cutting conditions (e.g., in a KIPR-R device) the frequency of these oscillations is about 100-1000 Hz. Many researchers have not recorded oscillations due to using sweeps with considerable inertia, and in view of this a record was made of the average data for cutter resistance and displacement. Naturally these data are unsuitable for our model.

Some Experiments. We now introduce the results of typical experiments carried out on specimens of Koelginsk marble. Three specimens 10 × 55 × 155 mm in size were tested. The cutter was fastened rigidly to a massive holder joined through a lever with an elastic strain gauge in order to measure the cutting resistance force by means of an oscillograph N 117/1. The specimen was fastened to the platform of a KIPR-R test device, which moved relative to the cutter [19]. Cutting was carried out at a low angle $\alpha = 1.9°$ to the specimen surface, which made it possible to obtain the whole curve for the dependence of cutting force resistance on cutting depth. In addition the same specimen may be subjected repeatedly to cutting under different regimes, thus recording a whole series of curves for it. In order to fasten the specimen to the moving platform use was made of clamps with thin rubber spacers or Wood's metal.

Presented in Fig. 6 are the dependences of maximum resistance force on cutting depth (obtained by simple conversions from the oscillogram) for three different Koelginsk marble specimens. Along the ordinate axis is value $T_{\max}/(H\sqrt{2h})$, and along the absissa axis is cutting depth h. It is noted that only some characteristic points (in fact each curve relates to several tens of points) are given on each empirical curve.

Since oscillation of resistance force with a frequency of about 100 Hz on the oscillogram corresponds to one act of projection separation, statistical treatment of all of the acts of separation (~300) recorded by oscillograms were subjected directly from the oscillogram to statistical treatment. The value of sliding toughness K_{IIc}, determined in this way by Eq. (30), appeared to be 115 ± 3 $N \cdot mm^{-3/2}$. As can be seen, tests confirmed the theory (relative error less than 4%). If the value of $E = 5 \cdot 10^4$ MPa, then by means of relationship (29) we find the failure energy during cutting of Koelginsk marble $\Gamma_c = 2.51 \pm 0.09$ $N \cdot cm^{-1}$.

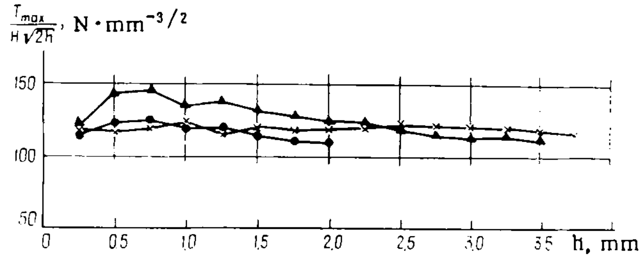

Fig. 6. Experimental dependences of maximum cutting resistance force T_{max} on cutting depth h for three different specimens of Koelginsk marble (H is specimen thickness).

Given below are experimental results for measuring sliding toughness K_{IIc} of some materials, $N \cdot mm^{-3/2}$:

Limestone (block)	61.8 ± 11.3
Argillite	81.7 ± 6.6
Anhydrite	159.5 ± 64
Limestone (plutonic)	205.5 ± 60
Dolomite	215.2 ± 37
Granite	782.5 ± 87
Optical glass	228 ± 20
Marble (Koelginsk)	115 ± 4

These data were obtained by the comparative method given above. Koelginsk marble was taken as a standard. As can be seen, this method is less accurate than the method of flat specimens.

CONCLUSIONS

1. Theoretical analysis of oscillations during cutting of rocks detected the presence of three maxima in the oscillation spectrum (located in the ranges 10^2, 10^3, and 10^4 Hz respectively for typical cutting conditions). The first two maxima relate to inertia-free relaxation oscillations of the stick-slip type, occurring as a result of projection separation and rock microinhomogeneity; the oscillation frequencies corresponding to them are of the order v/h and v/d (v and h are cutting rate and depth; d is grain size).

2. The Γ-integral is invariant with cutting in a brittle regime, but under conditions of considerable plastic deformation it appeared to be a very effective tool since it made it possible to establish the main characteristic of cutting resistance (K_{IIc} or Γ_c), to calculate the maximum cutting resistance T_{max}, or to determine the cutter efficiency index.

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LITERATURE CITED

1. W. Maurer, Advanced Drilling Techniques, Petroleum Publ. Co., Tulusa (1980).
2. G. P. Cherepanov, Mechanics of Brittle Fracture, McGraw-Hill, New York (1979).
3. G. P. Cherepanov, "Configuration forces in solid deformed body mechanics," Zh. Prikl. Mat. Mekh., 49, No. 4, 593-603 (1985).
4. A. L. Krylovskii, "Experimental study of the frequency properties of cutting force during rock breaking by RKS cutters," in: Machine and Equipment for Mining [in Russian], TsNIITÉI-tyazhmash, Moscow (1979).
5. L. B. Glatman, L. B. Shilov, and Ya. B. Gitman, "Determination of the amplitude-frequency characteristics of vibration of RKS-1 rotating cutters," Probl. Provedeniya Gornykh Vyrabotok, No. 216, 3-5 (1983).
6. J. C. Jaeger and N. G. W. Cook, Fundamentals of Rock Mechanics, Chapman and Hall, London (1977).
7. F. P. Bowden and L. Leben, "The nature of sliding and the analysis of friction," Proc. R. Soc., A169, 371-391 (1939).

8. F. Morgen, M. Muskat, and D. W. Reed, "Friction phenomena and the stick-slip process," J. Appl. Phys., 12, No. 2, 743-752 (1941).
9. J. D. Byerlee, "Theory of friction based on brittle fracture," J. Appl. Phys., 38, No. 12, 2928-2934 (1967).
10. A. L. Ruina, "Theory of rate and state dependent friction," J. Geophys. Res., 88, NB 12, 10359-10370 (1983).
11. J. E. Gu, J. R. Rice, A. L. Ruina, and S. T. Tse, "Slip motion and stability of a single degree of freedom elastic system with rate and state dependent friction," J. Mech. Phys. Sol., 32, No. 3, 167-196 (1984).
12. J. H. Dieterich, "Mechanical behavior of crusted rocks," Geophys. Monograph Series, American Geophysical Union, No. 24, 103-120 (1981).
13. A. I. Beron, A. S. Kazanskii, B. M. Leibov, and E. Z. Pozin, Coal Cutting [in Russian], Gostekhizdat, Moscow (1962).
14. V. D. Abezgauz, Cutting Tools of Milling Type Machines for Developing Rocks and Soils [in Russian], Mashinostroenie, Moscow (1965).
15. V. I. Vrublevskii, Resistance of Rocks to Failure [in Russian], Tekhnika, Kiev (1964).
16. I. A. Istroushko, Drilling of Hard Rocks [in Russian], Nedra, Moscow (1966).
17. E. I. Il'nitskaya, R. I. Teder, E. S. Vatolin, and M. F. Kuntysh, Properties of Rocks and Methods of Determining Them [in Russian], Nedra, Moscow (1969).
18. "Failure of rocks during drilling," Proc. of the Third All-Union Sci.-Tech. Conf., Vols. 1 and 2 [in Russian], Minnefteprom, Moscow (1983).
19. M. I. Vorozhtsov, E. I. Korol'ko, G. F. Perlov, et al., "Experimental device for studying processes of rock failure by cutting with different deep drilling conditions," Trudy VNII Burovoi Tekhniki, No. 37, 217-223 (1975).
20. V. V. Bundaev and V. S. Nikiforovskii, "Behavior of the failed zone in rock with a change in cutting angle and failed layer thickness," Fiz.-Tekh. Probl. Razrab. Polezn. Iskop., No. 5, 56-59 (1984).
21. V. S. Nikiforovskii and E. I. Shemyakin, Dynamic Failure of Solids [in Russian], Nauka, Novosibirsk (1979).

STUDY OF THE STRESS–STRAIN STATE OF CYLINDRICAL SHELLS DURING HEAT TREATMENT

A. S. Tsybenko, N. G. Krishchuk, and V. V. Palenyi

UDC 539.3

To improve the service characteristics of thick-walled cylindrical shells made of aluminum alloys, it has been proposed that they be provided with a heat treatment which will produce "useful" residual stresses and strains.

In accordance with the shell manufacturing technology in use, the shell is placed in a cylindrical steel band and heated together with the yoke in an oil bath to the specified temperature T_k over a period of 1.5-2 h. The heating creates interference between the inside surface of the band and the outside surface of the shell, leading to residual plastic strains in the shell. The magnitude and distribution of these strains depend on the heating program and the temperature T_k. After heating, the structure is cooled slowly in oil to the initial temperature T_0 over a period of about 10 h. A gap is created between the shell and band during the cooling, so that the shell can be easily removed from the band. The strength properties of the shell are improved considerably as a result of the thermal work-hardening which took place.

To determine efficient regimes for the manufacturing operation, we developed an efficient method of theoretically studying the nonisothermal thermoelastoplastic stress–strain state (SSS) of the shell-band system. The finite element method (FEM) was used to perform the calculations [1].

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