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Anomalous diagonal resistivity and soliton lattice in bilayer quantum Hall systems

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Abstract

In the bilayer quantum Hall (QH) system at the filling factor $\nu = 1$, the parallel magnetic field $B_{\|}$ penetrates between the two layers as sine-Gordon solitons, creating a soliton lattice for $B_{\|} > B_{\|}^{*}$ with $B_{\|}^{*}$ a certain transition point. This regime is called the incommensurate (IC) phase. The diagonal resistance R_{xx} has been observed experimentally to exhibit an anomalous peak just inside the IC phase though the Hall resistivity R_{xy} shows a well-developed plateau. We interpret this phenomenon as an evidence of the soliton lattice formation: A nontrivial transmission coefficient arises when an electron collides with thermally fluctuating solitons, as leads to an anomalous increase of the diagonal resistance R_{xx} in the QH regime at finite temperature.

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The bilayer quantum Hall (QH) system closely resembles the superconductor Josephson junction at the filling factor $\nu = 1$, where the interlayer phase coherence develops spontaneously [1]. However, there exists no Meissner effect. Indeed, the parallel magnetic field $B_{\|}$ penetrates between the two layers homogeneously when it is small. This regime is called the commensurate (C) phase [2]. However, for $B_{\|} > B_{\|}^{*}$ with $B_{\|}^{*}$ a certain transition point, it has been argued [1,3] to penetrate between the two layers as sine-Gordon solitons, creating a soliton lattice. This regime is called the incommensurate (IC) phase [2]. The C–IC phase transition was first discovered [4] experimentally by way of an anomalous decrease of the activation energy as a function of $B_{\|}$.

In this work we argue how to observe the formation of a soliton lattice experimentally. At the zero temperature the soliton lattice is a perfectly periodic system, where electrons propagate as a Bloch wave without reflection. However, the periodicity is broken at finite temperature, as would lead to a nontrivial transmission coefficient. Hence, we expect an anomalous increase of the diagonal resistance R_{xx} due to the backscattering of electrons against thermally fluctuating solitons in the IC phase. We propose this as the interpretation of the recent experimental result [5] on the anomalous increase of R_{xx} just beyond the phase transition point $B_{\|}^{*}$: See Fig. 1.

The electron field acquires an Aharanov–Bohm phase $x\delta_{\mathrm{m}}$ with $\delta_{\mathrm{m}} \equiv edB_{\|}/\hbar$ in the presence of $B_{\|}$ with the layer separation d. The effective Hamiltonian is [1,2]

$$\mathscr{H} = \frac{J_{\mathrm{s}}^{d}}{2} \int \mathrm{d}x \left((\partial_x \varphi)^2 + \frac{2}{\lambda_J^2} [1 - \cos(\varphi - x\delta_{\mathrm{m}})] - \delta_{\mathrm{m}}^2 \right), \tag{1}$$

where $\varphi(x)$ is the interlayer phase field; J_{s}^{d} is the interlayer pseudospin stiffness; $\lambda_J = \sqrt{2J_{\mathrm{s}}^{d}/\Delta_{\mathrm{SAS}}\rho_0}$ is the penetration depth with the tunneling gap Δ_{SAS} and the total electron density ρ_0.

Provided δ_{m} is small, $\delta_{\mathrm{m}} < \delta_{\mathrm{m}}^{*} \equiv 4/\pi\lambda_J$, the ground state is given by $\varphi = x\delta_{\mathrm{m}}$, with which $\mathscr{H} = 0$. This is the C phase. When $\delta_{\mathrm{m}} > \delta_{\mathrm{m}}^{*}$, there arises a soliton lattice solution,

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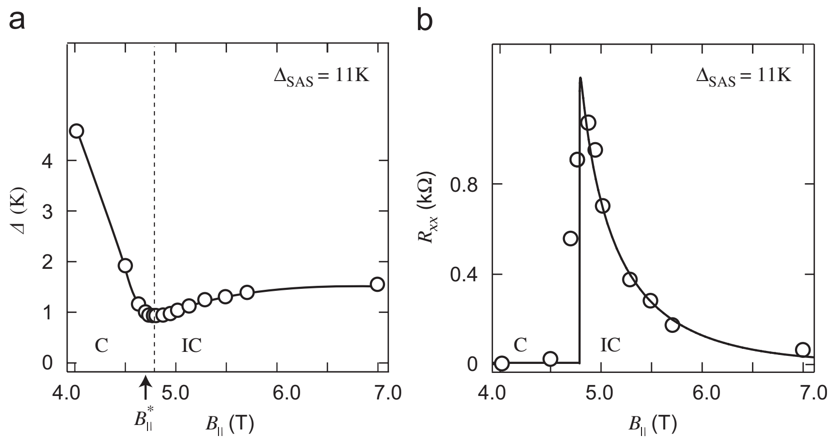

Fig. 1. (a) The activation energy Δ shows an anomalous decrease as $B_\parallel$ increases towards the C–IC transition point $B_\parallel^*$. (b) The diagonal resistance R_{xx} yields an anomalous peak in $B_\parallel$ when it crosses $B_\parallel^*$, though there exists a well-developed Hall plateau. The data, taken from Terasawa et al. [5], are fitted by the theoretical formula (6) with (8). The parameter values are $\Delta_{SAS} = 11\,$K, $d = 23.1\,$nm, $\rho_0 = 0.86 \times 10^{15}\,$m^{-2} and $T = 60\,$mK.

for which we derive the energy density

$$\mathscr{E}_{SL}(\kappa, \delta_m) = \frac{4J_s^d}{\lambda_J^2}\left[\frac{1}{2} + \frac{1}{\kappa^2}\frac{E(\kappa^2)}{K(\kappa^2)} - \frac{1}{2\kappa^2} - \frac{1}{\kappa K(\kappa^2)}\frac{\delta_m}{\delta_m^*}\right] \tag{2}$$

after an explicit integration of (1), where κ is an integration constant; $K(\kappa^2)$ and $E(\kappa^2)$ are the complete elliptic integral of the first kind and the second kind, respectively. It is easy to see that $\mathscr{E}_{SL}(\kappa, \delta_m) > 0$ for $\delta_m < \delta_m^*$, where no soliton lattice is formed (C phase). On the other hand, for $\delta_m > \delta_m^*$, $\mathscr{E}_{SL}(\kappa, \delta_m)$ takes one unique minimum value at $\kappa = \overline{\kappa}(\delta_m)$ given by

$$E(\bar{\kappa}^2)/\bar{\kappa} = \delta_m/\delta_m^*, \tag{3}$$

and is negative, $\mathscr{E}_{SL}(\bar{\kappa}, \delta_m) < 0$. Hence, a soliton lattice is formed (IC phase). The soliton density is given by

$$\rho_{SL}(\delta_m) = \frac{\pi\delta_m^*}{8\bar{\kappa}K(\bar{\kappa}^2)}. \tag{4}$$

It increases rapidly beyond the C–IC phase transition point $\delta_m = \delta_m^*$, where $\bar{\kappa} = 1$.

A soliton is a topological excitation. Let us focus on an electron in the QH current. An electron is initially in the symmetric state, and moves to a higher energy state within a soliton since the antisymmetric component is mixed. It is reasonable to simulate this as a one-dimensional potential problem, where an electron makes a ballistic scattering against a potential barrier made by a soliton. Let us set the transmission coefficient as $T_{SL} = 1 - t_{SL}$ with $0 < t_{SL} \ll 1$. When there are N solitons placed at random, the transmission coefficient increases as $T_{SL}^N \simeq 1 - Nt_{SL}$. According to the 4-terminal Landauer formula we would conclude an increase of the diagonal resistance as

$$R_{xx} = \frac{\pi\hbar}{e^2}\frac{1 - T_{SL}^N}{T_{SL}^N} \simeq N\frac{\pi\hbar}{e^2}t_{SL} \equiv NR_{SL}. \tag{5}$$

Thus one soliton gives a contribution $R_{SL} = \pi\hbar t_{SL}/e^2$ to the resistance.

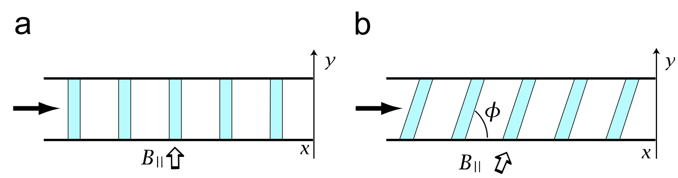

Fig. 2. (a) The parallel magnetic field $B_\parallel$ penetrates between the two layers to form a soliton lattice in the IC phase. An electron (arrow) travels over a soliton lattice as a Bloch wave. There arises no resistivity at the zero temperature. However, a resistance arises at finite temperature since the periodicity is broken by thermal fluctuations. (b) The resistance R_{xx} decreases as the angle ϕ increases, where ϕ is the angle between the parallel magnetic field $B_\parallel$ and the QH current.

We now argue the mechanism how the dissipation ($R_{xx} \neq 0$) occurs in the soliton lattice regime (Fig. 2). We start with an analysis at the zero temperature, where the soliton lattice makes a perfect periodic potential for electrons. The wave function of electrons spreads over the whole system and makes a Bloch state. There is no dissipation since there is no backscattering of electrons in a perfect periodic system. We next consider the system at a higher temperature. Since the soliton lattice fluctuates thermally, the perfect periodicity is lost. Backscattering must occur by individual solitons fluctuating thermally and incoherently. Such a fluctuation around its mean place may be modeled by a harmonic oscillator. Then, after quantization, the fluctuation probability is the thermo-active type with a certain gap energy Δ_{SG}. In a sample where an angle ϕ is made between the direction of the applied parallel magnetic field and the direction of the Hall current (Fig. 2(b)), the resistance behaves as

$$R_{xx} = (1 - \cos 2\phi)\rho_{SL}(\delta_m)\exp\left[-\frac{\Delta_{SG}(\delta_m)}{k_BT}\right]R_{SL}. \tag{6}$$

The Hall plateau should develop even if $R_{xx} \neq 0$ since electron–soliton scattering does not affect the incompressibility of the system.

We make a rough estimation of $\Delta_{SG}(\delta_m)$, simulating the fluctuation of a soliton by the harmonic oscillator,

$$H = -\frac{\hbar^2}{2M}\left(\frac{\partial}{\partial x}\right)^2 + \frac{1}{2}k(\delta_m)x^2. \tag{7}$$

Here, x represents the fluctuation of one soliton about the mean point taken at the origin. The gap is given by $\Delta_{SG}(\delta_m) = \hbar\sqrt{k(\delta_m)/M}$. We assume the mass M to be a phenomenological parameter. The elasticity $k(\delta_m)$ is determined by examining how rigid the soliton lattice is. It is quantitatively given by the energy increase of the soliton lattice when we change the lattice spacing $\bar{\ell}_{SL} = 1/\rho_{SL}(\bar{\kappa})$ by x. In this way we calculate the gap to be

$$\Delta_{SG}(\delta_m) = \hbar\sqrt{\frac{2J_s^d}{M\lambda_J^3}\frac{\sqrt{1-\bar{\kappa}^2}}{\bar{\kappa}^2}}\sqrt{\frac{\delta_m^*}{\delta_m}}. \tag{8}$$

We have fitted the experimental data [5] of the diagonal resistivity by the formula (6) at $\phi = 90^\circ$ in Fig. 1(b), where we have taken the soliton mass $M = 28m_e$ with m_e the electron rest mass in vacuum.

Our theoretical formula (6) with (8) explains quite well the anomalous behavior of the diagonal resistivity R_{xx} observed experimentally [5], though it seems to yield the C–IC phase transition point slightly higher than the observed value. In our analysis the electron–soliton transmission coefficient T_{SL} and the soliton mass M are phenomenological parameters. We wish to make a study of them in a future work.

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