PLASTICITY CONDITION OF ORTHOTROPIC METAL MATERIALS PRODUCED BY PRESSURE TREATMENT

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The plasticity condition based on the loading surface concept has been proposed in [1] for solids with general-type strain anisotropy resulting from hardening of the material during the plastic deformation process. This condition was checked experimentally in the range of small normal and shearing strain values (of the order of 3-5%), produced by uniaxial and biaxial tensile or torsional loads [3]. The use of the condition presented in [1] to describe the anisotropy produced by technological straining of the material in the case of one type of pressure treatment – rolling – is investigated with regard to three types of aluminum alloys that are widely used in technology. The investigations were carried out by using tubular specimens (the gauge length, the diameter, and the wall thickness were equal to 100, 29.5, and 0.75 mm, respectively), made of AMg-6, D16T, and V95 aluminum alloys, which, at delivery, had the shape of rods with diameters of 40-45 mm. The specimens are tested in a TsDMU-30t machine according to the method described in [3], the load consisting of an axial force and internal pressure. As a result, a biaxial stressed state develops in the specimen's wall, characterized by the axial σ_z and peripheral σ_θ stresses (the radial stress σ_r is low and is, therefore, neglected). The loading trajectory consists of the straight lines $\sigma_z = n\sigma_\theta$, where the n coefficient assumes the values 0, 0.5, 1, 2, and ∞.

Preliminary tests under uniaxial tensile stress in the direction of the rod and in the transverse direction have shown that the materials under investigation are anisotropic. The degree of anisotropy can be estimated with respect to the ratio of the elastic limits σ_{zT} and $\sigma_{\theta T}$ or with respect to the ultimate strength values $\sigma_{z\,ult}$ and $\sigma_{\theta\,ult}$ in the axial and tangential directions, respectively, which are given in Table 1 (in megapascal units).

Using the test data for these specimens, we estimated the scalar and vector characteristics of the materials under investigation in relation to the loading trajectory. The scalar properties are characterized by the strain diagram, which represents the stress intensity $\sigma_i = \left(\frac{3}{2}S_{ij}S_{ij}\right)^{1/2}$, $i,j=1,2,3$ as a function of the strain intensity $\varepsilon_i = \left(\frac{2}{3}e_{ij}e_{ij}\right)^{1/2}$ (where S_{ij} and e_{ij} are the components of the stress and strain deviators, respectively), while the vector characteristics are determined by the angle between the strain vector $\mathfrak{I}$ and the rectilinear loading trajectory, represented in Il'yushin's five-dimensional vector space $\{S_i\}$ [2]. We found that the $\sigma_i(\varepsilon_i)$ diagrams depend essentially on the type of the stressed state. The strain vector $\mathfrak{I}$ virtually coincides with the loading trajectory in the elastic range, while it somewhat deviates from this trajectory in the elastoplastic range.

The maximum value of the angle between the $\mathfrak{I}$ vector and the loading trajectory does not exceed 10°. This suggests that, for the materials investigated, the stress deviator $\{S_{ij}\}$

TABLE 1

Alloy	σ_{zT}	$\sigma_{\theta T}$	$\sigma_{z\,ult}$	$\sigma_{\theta\,ult}$
AMg-6	270	220	415	315
D16T	425	328	550	400
V95	560	470	620	490

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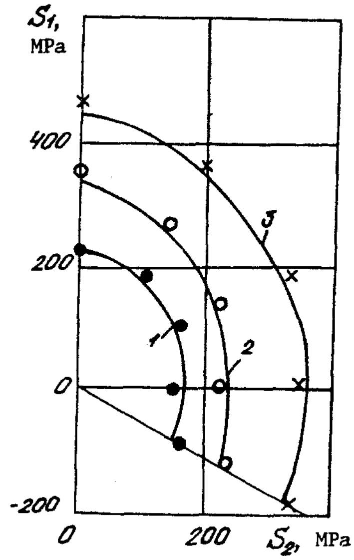

Fig. 1

TABLE 2

Alloy	a_1	a_2	R
AMg-6	11	- 41	213
D16T	8	-104	350
V95	2	-94	458

and the strain deviator $\{e_{ij}\}$ are proportional to each other regardless of the direction of the loading trajectory. The yield boundary, which is represented in the $\{S_i\}$ space, is approximated by a sphere in the general case of a complex stressed state [1]

$$(S_i - a_i)(S_i - a_i) = R^2, \quad i = 1,2,3,4,5 \tag{1}$$

with the radius R and the center coordinates a_i. The S_i components are related to the S_{ij} components by the well-known expressions [2]

$$\begin{gathered} S_1 = \sqrt{\frac{3}{2}}\, S_{11}; \quad S_2 = \frac{\sqrt{2}}{2}(S_{22} - S_{33}); \\ S_3 = \sqrt{2}\, S_{12}; \quad S_4 = \sqrt{2}\, S_{13}; \quad S_5 = \sqrt{2}\, S_{31}. \end{gathered} \tag{2}$$

In our case of a biaxial stressed state ($\sigma_r = \sigma_{z\theta} = \sigma_{\theta r} = \sigma_{rz} = 0$), considering that the strain in rolling occurs due to normal stresses (in which case, $a_3 = a_4 = a_5 = 0$), the sphere (1) passes, with an allowance for (2), into a circle,

$$(S_1 - a_1)^2 + (S_2 - a_2)^2 = R^2. \tag{3}$$

The parameters a_1, a_2, and R figuring in the above expression are found by processing the experimental values of the points belonging to the yield boundary, using the method of least squares. The quantities sought are determined by solving the system of equations

$$\begin{gathered} \sum_{j=1}^{N} [(S_{1j} - a_1)^3 + (S_{2j} - a_2)^2 (S_{1j} - a_1) - R^2 (S_{1j} - a_1)] = 0; \\ \sum_{j=1}^{N} [(S_{1j} - a_1)^2 (S_{2j} - a_2) + (S_{2j} - a_2)^3 - R^2 (S_{2j} - a_2)] = 0; \\ \sum_{j=1}^{N} [(S_{1j} - a_1)^2 + (S_{2j} - a_2)^2 - R^2 = 0. \end{gathered} \tag{4}$$

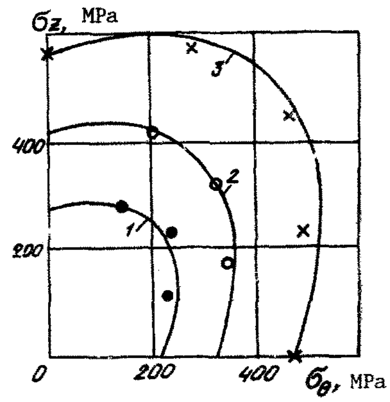

Fig. 2

Here, S_{1j} and S_{2j} are the coordinates of the experimentally determined points of the yield boundary (they correspond to the allowance for plastic strain, equal to 0.2%) in the S_1-S_2 plane of the $\{S_i\}$ space under loading based on different trajectories, $\sigma_z = n\sigma_\theta$. The thus determined yield boundaries for AMg-6, D16T, V95 alloys are shown in Fig. 1.

The solid curves represent the circle (3), the parameters of which are found by solving system (4). The points correspond to the experimental values. The numbers 1, 2, and 3 denote the yield boundaries of AMg-6, D16T, and V95 alloys. The numerical values of a_1, a_2, and R are given in Table 2 (in megapascal units). It is evident from Fig. 1 and Table 2 that $a_1 \ll a_2$ for all the materials investigated that are characterized by anisotropy of the same type. Assuming that $a_1 = 0$ as a small quantity and passing to the $\sigma_z-\sigma_\theta$ plane, the plasticity condition (3), with an allowance for (2), assumes the following form:

$$\frac{2}{3}(\sigma_z^2 + \sigma_\theta^2 - \sigma_z\sigma_\theta) - \sqrt{2}\,a_2\sigma_\theta + a_2^2 - R^2 = 0. \tag{5}$$

With an allowance for the boundary conditions

$$\sigma_z = \sigma_{zT} \quad \text{for} \quad \sigma_\theta = 0;$$
$$\sigma_\theta = \sigma_{\theta T} \quad \text{for} \quad \sigma_z = 0$$

the parameters a_2 and R figuring in the above equation can be expressed, according to (5), in terms of the yield points σ_{zT} and $\sigma_{\theta T}$ as follows:

$$a_2 = \frac{\sqrt{2}}{3}\frac{\sigma_{\theta T}^2 - \sigma_{zT}^2}{\sigma_{\theta T}}, \quad R = \sqrt{\frac{2}{3}\sigma_{zT}^2 + a_2^2}. \tag{6}$$

After the parameters a_2 and R, calculated by means of expressions (6) on the basis of data from Table 1, are substituted in (5), the yield boundaries of AMg-6, D16T, and V95 alloys are described by the following equations, respectively:

$$\begin{aligned}
&\frac{2}{3}(\sigma_z^2 + \sigma_\theta^2 - \sigma_z\sigma_\theta) + 73{,}3\,\sigma_\theta - 4{,}8\cdot 10^4 = 0;\\
&\frac{2}{3}(\sigma_z^2 + \sigma_\theta^2 - \sigma_z\sigma_\theta) + 146{,}6\,\sigma_\theta - 12{,}0\cdot 10^4 = 0;\\
&\frac{2}{3}(\sigma_z^2 + \sigma_\theta^2 - \sigma_z\sigma_\theta) + 129{,}7\,\sigma_\theta - 20{,}9\cdot 10^4 = 0.
\end{aligned} \tag{7}$$

The yield boundaries of AMg-6, D16T, and V95 alloys, plotted on the basis of conditions (7), are shown by solid curves in Fig. 2 and are denoted by the numbers 1, 2, and 3, respectively.

As in Fig. 1, the points denote the experimental values. On the basis of Fig. 2, we reach the conclusion that the plasticity condition in the form (5) is in satisfactory agreement with experimental data. Using the parameters a_1, a_2, and R given in Table 2, we can also determine the components S_{ij} of the stress deviator and the plastic strain tensor ε_{ij}^P of the material during the rolling process. According to [3], we have

$$S_{11} = \sqrt{\frac{2}{3}}\,\frac{a+R}{a}a_1;$$
$$S_{22} = \frac{(a+R)(\sqrt{3}\,a_2 - a_1)}{\sqrt{6}\,a}; \tag{8}$$
$$S_{33} = \frac{(a+R)(a_1 + \sqrt{3}\,a_2)}{\sqrt{6}\,a}.$$

where $a = (a_1{}^2 + a_2{}^2)^{1/2}$.

Moreover, the parameters a_1 and a_2 are related to the plastic strain values ε_z^P and ε_θ^P by the expressions

$$a_1 = \frac{\varepsilon_z^p}{\varepsilon_i^p}a; \quad a_2 = \frac{1}{\sqrt{3}}\frac{\varepsilon_z^p + 2\varepsilon_\theta^p}{\varepsilon_i^p}a, \tag{9}$$

where ε_i^P is the plastic strain intensity.

Hence, with an allowance for the incompressibility condition

$$\varepsilon_z^p + \varepsilon_\theta^p + \varepsilon_r^p = 0,$$

we find

$$\varepsilon_z^p = \frac{a_1}{a}\varepsilon_i^p; \quad \varepsilon_\theta^p = (\sqrt{3}\,a_2 - a_1)\frac{\varepsilon_i^p}{2a};$$
$$\varepsilon_r^p = -(a_1 + \sqrt{3}\,a_2)\frac{\varepsilon_i^p}{2a}. \tag{10}$$

Thus, we can say that the onset of the yield of orthotropic materials in the form of round rolled stock under biaxial tensile stress is determined by the plasticity condition given by (5), which is a particular case of the more general condition (1). The hardening which produces the anisotropy is described by shifting the loading surface in the direction of the normal to the rod axis. The data from similar investigations of AMg-6 and D16T alloys, provided in [4], indicate that plasticity conditions of the Coulomb and Mises–Hill types can also be used for these materials.

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