(MALAGELADA and STANGHELLINI, 1985) and diabetes (MEARIN *et al.*, 1986). In our future papers we will deal with these topics.

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Technical note

Optimised algorithm to compute respiratory impedance by pseudorandom forced excitation

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1 Introduction

THE MECHANICAL impedance of the respiratory system Z_{rs} is used to characterise the dynamic properties of the system over a wide frequency band. Z_{rs} is generally computed from the auto- and cross-spectra of the pressure and flow signals recorded during an externally applied forced excitation (MICHAELSON *et al.*, 1975). Periodic (pseudorandom or sinusoidal) excitation signals are commonly used as in this case the excitation power is concentrated at the frequencies of interest leading to a higher signal-to-noise ratio (LANDSER *et al.*, 1976).

Another advantage of this kind of signal is that spectral estimation may be easily carried out with the fast Fourier transform (FFT) provided that the sampling rate is adequately synchronised with the excitation signal. Moreover, it is common practice to extend the length of the time recording to which the FFT is applied to increase the resolution in the frequency analysis and thus to reduce the

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artefacts due to the superposition of the noises induced by the subject's spontaneous breathing (DAROCZY and HANTOS, 1982; FARRÉ *et al.*, 1989; FRANKEN *et al.*, 1983; LANDSER *et al.*, 1976; PESLIN *et al.*, 1985).

Nevertheless, this usual procedure suffers from the disadvantage that considerable computation work is wasted because the FFT computes spectral components at many frequencies where there is no excitation signal. In this context, our aim was to implement an algorithm based on the widespread FFT routine to substantially reduce the computation cost in Z_{rs} measurements by pseudorandom excitation.

2 Algorithm

2.1 *Conventional algorithm*

The pseudorandom signal commonly used to measure Z_{rs} contains power at P frequencies $f_0, 2f_0, \ldots, Pf_0$ multiples of a fundamental frequency f_0. Therefore, both the input and output (flow and pressure) signals are periodic with a period $T_0 = 1/f_0$.

The conventional spectral analysis is based on the application of the discrete Fourier transform (DFT) to the sequence $x(0), x(1), \ldots, x(N-1)$ which corresponds to a time block of pressure or flow signal with a duration of L times the fundamental period of the forced excitation T_0. If the signal is sampled at a frequency f_s, the number of samples corresponding to the fundamental period is $N_0 = T_0 f_s$. Therefore, the sequence $x(0), x(1), \ldots, x(N-1)$ may be written as $x(0), \ldots, x(N_0-1), \ldots, x(2N_0-1), \ldots, x(LN_0-1)$. As shown in Fig. 1, the DFT of this sequence is the sequence $X(0), X(1), \ldots, X(N-1)$ where

$$X(m) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\exp(-2\pi jmr/N) \qquad m = 0, \ldots, N-1 \quad (1)$$

The frequency resolution (Δf) is in this case $\Delta f = f_s/N$ and may be expressed as $\Delta f = f_0/L$ because $f_s = N_0/T_0$ and $N = N_0 L$. Therefore, the values of the DFT corresponding to each of the P forced excitation frequencies $f_0, 2f_0, 3f_0, \ldots, Pf_0$ are given by the values $X(kL)$

$$X(kL) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\exp(-2\pi jrkL/N) \qquad k = 1, 2, 3, \ldots, P \quad (2)$$

Consequently, although the whole sequence $X(0), X(1), \ldots, X(N-1)$ is computed by the DFT, only some of the calculated values are effectively employed to compute impedance (Fig. 1). Taking into account that the computation effort in Z_{rs} estimation is mainly devoted to the FFT, the computational cost of impedance depends on $N \log_2 N$ (BRIGHAM, 1974).

$x(0), \ldots, x(N_0-1), \ldots, x(2N_0-1), \ldots, x(LN_0-1)$

↓ digital Fourier transform of length $N = LN_0$

$X(0), \ldots, X(N_0-1), \ldots, X(2N_0-1), \ldots, X(LN_0-1)$

↓ selection of data points

$X(L), X(2L), \ldots, X(PL)$

Fig. 1 *Conventional algorithm. $x(\cdot)$: sampled signal; $X(\cdot)$: digital Fourier transform of $x(\cdot)$; N_0: number of samples corresponding to the fundamental period; L: number of fundamental periods contained in the sampled signal; P: number of forced excitation frequencies*

2.2 *Optimised algorithm*

The algorithm proposed in this work is based on a property of the DFT which when used in this particular application allows a substantial reduction in computation time. This property may be derived by rearranging the order of summations in eqn. 2. After algebraic manipulation, the values $X(kL)$ effectively required to calculate impedance may be written as

$$X(kL) = \frac{1}{N_0}\sum_{r=0}^{N_0-1}\left[\frac{1}{L}\sum_{i=1}^{L} y_i(r)\right]\exp(-2\pi jrkL/N_0) \quad (3)$$

where

$$y_i(r) = x((i-1)N_0 + r) \quad (4)$$

For each $r = 0, \ldots, N_0 - 1$, the value inside brackets in eqn. 3 corresponds to the rth value of the mean $y(r)$ of the L sequences y_i. Thus, eqn. 3 becomes

$$X(kL) = \frac{1}{N_0}\sum_{r=0}^{N_0-1} y(r)\exp(-2\pi jrkL/N_0) \quad (5)$$

and taking into account the definition of the DFT, it follows that $X(kL)$, $k = 1, \ldots, P$ coincide with $Y(1), \ldots, Y(P)$ which are the P first terms of the DFT of the mean sequence $y(r)$, $r = 1, \ldots, N_0 - 1$.

Therefore, the computations involved in eqn. 2 may be implemented according to the algorithm described in Fig. 2. First, the original sequence of length $N = N_0 L$ is converted into L sequences of length N_0. Secondly, the mean sequence of these L sequences is computed. And thirdly,

$x(0), \ldots, x(N_0-1), \ldots, x(2N_0-1), \ldots, x(LN_0-1)$

↓ construction of sequences $y_i(.)$ by partitioning the original sequence $x(.)$

$y_1(.)$: $x(0), \ldots, x(N_0-1)$

$y_i(.)$: … $x((i-1)N_0), \ldots, x(iN_0-1)$

$y_L(.)$: … $x((L-1)N_0), \ldots, x(LN_0-1)$

↓ construction of sequence $y(.)$ by averaging sequences $y_i(.)$

$y(0), y(1), \ldots, y(N_0-1)$

↓ digital Fourier transform of length N_0

$Y(0), Y(1), \ldots, Y(N_0-1)$

Fig. 2 *Designed algorithm. $x(\cdot)$: sampled signal; $X(\cdot)$: digital Fourier transform of $x(\cdot)$; $Y(\cdot)$: calculated digital Fourier transform. N_0: number of samples corresponding to the fundamental period; L: number of fundamental periods contained in the sampled signal*

the DFT is applied to this mean sequence. Therefore, in contrast with the conventional algorithm where an FFT of length $N = LN_0$ is required, the application of the proposed property only needs an FFT of length N_0 to obtain exactly the same results. Therefore, in this case the computational cost of impedance depends on $N_0 \log_2 N_0$.

3 Discussion

The main advantage of using this algorithm is the reduction in the computation cost of Z_{rs} when applied in the common conditions of pseudorandom forced excitation. This reduction is indicated by the ratio R between the estimated costs with the conventional algorithm and with the one proposed in this work: $R = (N \log_2 N)/(N_0 \log_2 N_0)$. In most Z_{rs} measurements the sampling frequency is $f_s = 128$ Hz, the fundamental frequency is $f_0 = 2$ Hz ($N_0 = 64$) and the duration of the time blocks to which the FFT is applied varies with the different authors: 2 s (i.e. $N = 256$) (PESLIN *et al.*, 1985), 4 s (i.e. $N = 512$) (FARRÉ *et al.*, 1991; FRANKEN *et al.*, 1983) or 8 s (i.e. $N = 1024$) (LANDSER *et al.*, 1976). In other cases, for instance in high-frequency measurements (FARRÉ *et al.*, 1989), $N = 1024$ and $N_0 = 128$. Table 1 depicts the values of the reduction factor R corresponding to these examples taken from the literature. This table shows that with the proposed algorithm the computation cost in Z_{rs} measurements may be reduced by a factor ranging from $R \approx 5$ (PESLIN *et al.*, 1985) to as much as $R \approx 27$ (LANDSER *et al.*, 1976).

Table 1 Reduction factor R in the computation cost

Reference	N	N_0	$N \log_2 N$	$N_0 \log_2 N_0$	R
PESLIN *et al.*, 1985	256	64	2048	384	5·33
FRANKEN *et al.*, 1983	512	64	4608	384	12·00
LANDSER *et al.*, 1976	1024	64	10240	384	26·67
FARRÉ *et al.*, 1989	1024	128	10240	896	11·43

N: total length of the block; N_0: reduced length after the algorithm

In practice, this would permit a substantial decrease in the time required for Z_{rs} measurements. For instance, the application of the present algorithm to the particular conditions described in FARRÉ *et al.* (1989) would reduce the time of each impedance computation from 5·7 min to 38 s, which is even shorter than the duration of the data acquisition. It is noteworthy that the computation cost of impedance can be reduced by an additional factor of 2 if the FFT of the two real signals involved (pressure and flow) are computed simultaneously (BRIGHAM, 1974).

The algorithm proposed in this work has two additional advantages when compared with the conventional one. First, the total memory size required to store data can be reduced (from $N = LN_0$ to N_0) because the different L blocks may be averaged immediately after sampling. Secondly, the round-off errors in the computation may be decreased because of the reduction in multiplications. Moreover, because the proposed algorithm strictly concerns the FFT calculation step, it may be applied regardless of the particular data-processing procedure adopted (filtering, overlapping, windowing, impedance estimator etc.) (DAROCZY and HANTOS, 1982; FARRÉ and ROTGER, 1991; NAVAJAS *et al.*, 1988) in input as well as in transfer impedance measurements (PESLIN *et al.*, 1985).

Alternative ways to reduce the computation cost in this application could be based on the modification of the Sande-Tukey FFT routine (decimation in frequency) (BRIGHAM, 1974). Nevertheless, this would prevent the use of the standard FFT routines commonly available and would make its application difficult for many users. By contrast, one of the features of the proposed algorithm is that it is based on the use of any general-purpose FFT routine. Therefore, because the algorithm allows a substantial reduction in computer time with an easy implementation, it is of interest in respiratory impedance measurements using the widespread pseudorandom forced excitation technique.

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