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Thermoeconomic optimization of a two stage combined refrigeration system: a finite-time approach

Bahri Sahin[a,*], Ali Kodal[b]

[a]Department of Naval Architecture, Yildiz Technical University, Besiktas, 80750, Istanbul, Turkey
[b]Department of Aeronautical Engineering, Istanbul Technical University, Maslak, 80626, Istanbul, Turkey

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Abstract

A finite-time thermoeconomic performance analysis based on a new kind of optimization criterion has been carried out for a two-stage endoreversible combined refrigeration cycle model. The optimal performances and design parameters that maximize the objective function (cooling load per total cost) are investigated. In this context, the optimal temperatures of the working fluids, the optimum performance coefficient, the optimum specific cooling load and the optimal distribution of the heat exchanger areas are determined in terms of technical and economical parameters. The effects of the economical parameter that characterizes the investment and energy consumption costs on the general and the optimal performances have been discussed. © 2002 Elsevier Science Ltd and IIR. All rights reserved.

Keywords: Refrigeration system; Two-stage system; Performance; Cost; Parameter; Optimization; Thermodynamics

Optimisation thermoéconomique d'un système frigorifique hybride biétagé : approche par intervalle de temps défini

Mots clés : Système frigorifique ; Système biétagé ; Performance ; Coût paramètre ; Optimisation ; Thermodynamique

1. Introduction

A significant fraction of industrial refrigeration plants operate with large difference between evaporating and condensing temperatures. Two-stage or multi-stage combined refrigeration systems are often employed when the span of the temperatures between the condensing and evaporating is large [1,2]. In the last years, many optimization studies for vapor-compression refrigeration systems based on the endoreversible and irreversible models have been carried out by considering finite-time and finite-size constraints [2–20]. Although there have been performance optimization works focused on the performance of single-stage vapor-compression refrigerators, only a few efforts on the performance optimization of a two-stage and multi-stage refrigerator systems have been made by using finite-time thermodynamics [2,17–20]. Chen and Wu [2] investigated the optimal performance of a two-stage endoreversible refrigeration system by considering the coefficient of performance as an objective function under a given specific cooling load (cooling load per total heat-transfer area). Chen and Yan [17] examined a class of endoreversible combined refrigeration cycles and obtained optimal configuration and a relation between the optimal coefficient of performance and the cooling load by minimizing the

* Corresponding author. Tel.: +90-212-258-2157; Fax: +90-212-261-6659.
E-mail address: sahinb@yildiz.edu.tr (B. Sahin).

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Nomenclature

a	investment cost parameter for the heat exchangers (ncu year^{-1} m^{-2})
A	heat-transfer area (m^2)
b_1	investment cost parameter for the compressor and its driver (ncu year^{-1} kW^{-1})
b_2	energy consumption cost parameter (ncu year^{-1} kW^{-1})
b	b_1+b_2 (ncu year^{-1} kW^{-1})
C	cost (ncu year^{-1})
COP	coefficient of performance
F	objective function (kW ncu^{-1} year)
k	a/b (kW m^{-2})
ncu	national currency unit
$\dot{Q}$	rate of heat transfer (kW)
$\dot{q}$	specific cooling load (kW m^{-2})
S	entropy (kJ K^{-1})
T	temperature (K)
$\dot{W}$	power input (kW)
U	overall heat-transfer

Subscripts

e	energy consumption
H	heat sink
i	investment
L	cooled space
max	maximum
X	warm working fluid of the first cycle
W	warm working fluid of the second cycle
Y	cold working fluid of the first cycle
Z	cold working fluid of the second cycle

Superscripts

*	optimum conditions

power input for given cycle time and cooling load. Goktun [18] studied the effects of thermal resistances and internal irreversibilities on the performance of a two-stage combined refrigerator. Chen et al. [19] determined a relation between cooling load and coefficient of performance for a general combined refrigerator cycle by optimizing the specific rate of refrigeration. Goktun [20] investigated the optimum performance of an irreversible heat-driven combined vapor-compression and absorption refrigerator system under the conditions of maximum power output of a heat engine. In the above referenced studies, the objective function chosen for the optimization is usually cooling load, coefficient of performance, power input, exergy output or specific cooling load. In these studies, one of the performance characteristics is chosen as an objective function while taking the rest as constraints. Some of these optimization criteria are related to the investment costs and some of them are related to the energy consumption costs. For a more realistic optimization, the objective function desired to be optimized, must include both of the investment and the energy consumption costs. In this context, Sahin and Kodal [21] have very recently introduced a new optimization criterion which is defined as the cooling load per total cost (total of the investment and the energy consumption costs). Using this new criterion, they performed finite-time thermoeconomic performance analysis for endoreversible [21] and irreversible [22] refrigerators to find the design parameters under the optimal conditions.

In this study, finite-time thermoeconomic optimization work carried out by Sahin and Kodal [21] for a single stage refrigerator model is extended to a two-stage endoreversible combined refrigeration model. The optimal performance and design parameters that maximize the thermoeconomic objective function are investigated. The optimization results obtained in this analysis represent the economical design conditions.

2. Performance optimization

The model of the two-stage combined endoreversible refrigerator cycle and its T-S diagram are shown in Fig. 1. The endoreversible model is a modified Carnot cycle in which the irreversibility due to finite rate heat transfer are taken into account. In the model, two cycles have to two different working fluids and operate in different temperature ranges. They are combined with a heat exchanger between them such that the heat released from the second refrigeration cycle is absorbed by the first cycle. Both cycles have two isothermal and two isentropic processes. The combined cycle operates continuously in a steady state operation and only the irreversibility of finite-rate heat transfer is considered. In the shown model, $\dot{Q}_{\rm H}$ is the rate of heat transfer from the first cycle to the heat sink at temperature T_H, $\dot{Q}_{\rm W}$ is the rate of heat transfer from the second cycle to the first cycle, $\dot{Q}_{\rm L}$ is the rate of heat transfer from the cooled space at temperature T_L to the second cycle, and $\dot{W}$ is the power input of the combined system. T_X and T_Y are the temperatures of the working fluid in the first cycle, and T_W and T_Z are the temperatures of the working fluid in the second cycle of the respective isothermal processes.

When heat transfer obeys Newton's cooling law, one has

$$\dot{Q}_{\rm L} = U_{\rm L}A_{\rm L}(T_{\rm L} - T_{\rm Z}) \tag{1}$$

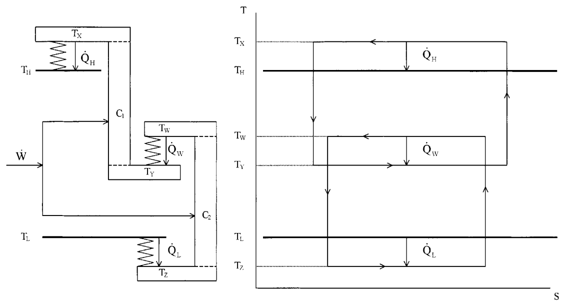

Fig. 1. Two-stage combined endoreversible refrigerator model and its T-S diagram.

$$\dot{Q}_H = U_H A_H (T_X - T_H) \tag{2}$$

$$\dot{Q}_W = U_W A_W (T_W - T_Y) \tag{3}$$

where U_L and A_L are the overall heat-transfer coefficient and area between the second cycle and the cooled space, U_H and A_H are the overall heat-transfer coefficient and area between the heat sink and the first cycle, U_W and A_W are the overall heat transfer coefficient and area between the two cycles. The first law of thermodynamics requires the power input to the refrigerator to be

$$\dot{W} = \dot{Q}_H - \dot{Q}_L. \tag{4}$$

From the second law of thermodynamics for an endoreversible combined cycle, the change in the entropies of the working fluid for the heat addition and heat removing isothermal processes yields

$$\dot{Q}_H / T_X = \dot{Q}_W / T_Y \tag{5}$$

$$\dot{Q}_L / T_Z = \dot{Q}_W / T_W. \tag{6}$$

From Eqs. (1)–(6), we obtain the coefficient of performance, COP and the specific cooling load, $\dot{q}_L$ of the combined cycle, respectively as

$$COP = \frac{\dot{Q}_L}{\dot{W}} = \frac{1}{T_X T_W / (T_Z T_Y) - 1}, \tag{7}$$

$$\dot{q}_L = \frac{\dot{Q}_L}{A} = \left[\frac{1}{U_L (T_L - T_Z)} + \frac{T_W}{U_W T_Z (T_W - T_Y)} + \frac{T_W T_X}{U_H T_Z T_Y (T_X - T_H)} \right]^{-1} \tag{8}$$

where the total heat transfer area A is equal to the sum of A_H,A_W and A_L.

We consider the optimization of the cooling load per total cost in order to account both investment and energy consumption costs. The function to be optimized is defined as

$$F = \dot{Q}_L / (C_i + C_e) \tag{9}$$

where C_i and C_e refer to annual investment and energy consumption costs, respectively. For the investment cost, we may consider the investment costs of the main system components which are the heat exchangers and the compressors together with their prime movers. The investment cost of the heat exchangers is assumed to be proportional to the total heat-transfer area. On the other hand, the investment cost due to compressors and their drivers is assumed to be proportional to their compression capacities or the required power input. Thus the annual investment cost of the system can be given as

$$\begin{aligned} C_i &= a(A_H + A_L + A_W) + b_1 \dot{W} \\ &= a(A_H + A_L + A_W) + b_1 \left(\dot{Q}_H - \dot{Q}_L \right) \end{aligned} \tag{10}$$

where the proportionality coefficient for the investment cost of the heat exchangers, a is equal to the capital recovery factor times investment cost per unit heat transfer area and its dimension is ncu/(year m^2), and the proportionality coefficient for the investment cost of the compressors and their drivers, b_1 is equal to the capital recovery factor times investment cost per unit power and its dimension is ncu/(year kW). The unit ncu stands for the national currency unit. The initial investment cost is converted to equivalent yearly payment using the capital recovery factor [23,24]. Annual energy consumption cost is proportional to the power input, i.e

$$C_e = b_2 \dot{W} = b_2 \left(\dot{Q}_H - \dot{Q}_L \right) \tag{11}$$

where the coefficient b_2 is equal to the annual operation hours times price per unit energy and its dimension is ncu/(year kW). The energy price is thought as levelized price value over economical life of the system, therefore C_e is considered as the levelized annual energy consumption cost. Substituting Eqs. (10) and (11) into Eq. (9), we get

$$F = \frac{\dot{Q}_L}{a(A_H + A_L + A_W) + b\left(\dot{Q}_H - \dot{Q}_L \right)}, \tag{12}$$

where

$b = b_1 + b_2.$

Using Eqs. (1)–(6) in Eq. (12), we obtain

$$bF = \frac{1}{\left(1+\frac{k}{U_{\mathrm{H}}(T_{\mathrm{X}}-T_{\mathrm{H}})}\right)\frac{T_{\mathrm{X}}T_{\mathrm{W}}}{T_{\mathrm{Z}}T_{\mathrm{Y}}}+\frac{kT_{\mathrm{W}}}{U_{\mathrm{W}}T_{\mathrm{Z}}(T_{\mathrm{W}}-T_{\mathrm{Y}})}+\frac{k}{U_{\mathrm{L}}(T_{\mathrm{L}}-T_{\mathrm{Z}})}-1}, \tag{13}$$

where k a/b is the economical parameter. The parameter k emphasizes the relative investment cost of the heat exchangers to the cost due to investment and energy consumption of compressors. The dimensionless objective function in Eq. (13) can be maximized with respect to T_{X}, T_{W}, T_{Y} and T_{Z}. The results are

$$T_{\mathrm{X}}{}^{*} = T_{\mathrm{H}} + \sqrt{kT_{\mathrm{H}}/U_{\mathrm{H}}}, \tag{14}$$

$$T_{\mathrm{Z}}{}^{*} = \frac{T_{\mathrm{L}}}{1+\left[\sqrt{U_{\mathrm{L}}}\left(\sqrt{T_{\mathrm{H}}/k}+1/\sqrt{U_{\mathrm{H}}}+1/\sqrt{U_{\mathrm{W}}}\right)\right]^{-1}} \tag{15}$$

$$\left(\frac{T_{\mathrm{W}}}{T_{\mathrm{Y}}}\right)^{*} = 1+\frac{1}{\sqrt{U_{\mathrm{W}}}\left(\sqrt{T_{\mathrm{H}}/k}+1/\sqrt{U_{\mathrm{H}}}\right)} \tag{16}$$

Substituting Eqs. (14)–(16) into Eq. (13), we obtain

$$(bF)_{\max} = \frac{T_{\mathrm{L}}}{k\left[\sqrt{T_{\mathrm{H}}/k}+1/\sqrt{U_{\mathrm{H}}}+1/\sqrt{U_{\mathrm{W}}}+1/\sqrt{U_{\mathrm{L}}}\right]^{2}-T_{\mathrm{L}}}. \tag{17}$$

Substituting Eqs. (14)–(16) into Eqs. (7) and (8), we can obtain the optimum coefficient of performance and the optimum specific cooling load, respectively, as

$$COP^{*} = \frac{T_{\mathrm{L}}}{T_{\mathrm{H}}+\sqrt{kT_{\mathrm{H}}}\left(1/\sqrt{U_{\mathrm{H}}}+1/\sqrt{U_{\mathrm{W}}}+1/\sqrt{U_{\mathrm{L}}}\right)-T_{\mathrm{L}}} \tag{18}$$

$$\dot{q}_{\mathrm{L}}^{*} = \frac{T_{\mathrm{L}}}{\left(1/\sqrt{U_{\mathrm{H}}}+1/\sqrt{U_{\mathrm{W}}}+1/\sqrt{U_{\mathrm{L}}}\right)\left(\sqrt{T_{\mathrm{H}}/k}+1/\sqrt{U_{\mathrm{H}}}+1/\sqrt{U_{\mathrm{W}}}+1/\sqrt{U_{\mathrm{L}}}\right)}. \tag{19}$$

The optimal distributions of the heat exchanger areas for a given total heat exchanger area (i.e. $A = A_{\mathrm{H}} + A_{\mathrm{W}} + A_{\mathrm{L}}$) can be found using Eqs. (1)-(3), (5)–(6) and (14)–(16) as

$$A_{\mathrm{L}}^{*} = A/\left(1+\sqrt{U_{\mathrm{L}}/U_{\mathrm{W}}}+\sqrt{U_{\mathrm{L}}/U_{\mathrm{H}}}\right), \tag{20}$$

$$A_{\mathrm{W}}^{*} = A/\left(1+\sqrt{U_{\mathrm{W}}/U_{\mathrm{H}}}+\sqrt{U_{\mathrm{W}}/U_{\mathrm{L}}}\right), \tag{21}$$

$$A_{\mathrm{H}}^{*} = A/\left(1+\sqrt{U_{\mathrm{H}}/U_{\mathrm{W}}}+\sqrt{U_{\mathrm{H}}/U_{\mathrm{L}}}\right). \tag{22}$$

3. Discussion

The variation of the objective function given in Eq. (13) with respect to the coefficient of performance given in Eq. (7) for various economical parameter (k) values is shown in Fig. 2. From the figure, one can observe the effect of k on the global and optimal performance. It should be noted that k emphasizes the relative investment cost of the heat exchangers to the cost due to investment and energy consumption of compressors. As k decreases, the global and optimal performances increase. From Eq. (7), we can see that when there is no finite-rate heat transfer irreversibilities, i.e. $T_{\mathrm{X}} = T_{\mathrm{H}}$, $T_{\mathrm{Z}} = T_{\mathrm{L}}$ and $T_{\mathrm{Y}} = T_{\mathrm{W}}$, COP is equal to $\mathrm{COP}_{\max} = T_{\mathrm{L}}/(T_{\mathrm{H}} - T_{\mathrm{L}})$, the performance coefficient of the reversible Carnot refrigeration cycle. In this case, the objective function becomes zero as shown in Fig. 2. Therefore, the upper bounds of the performance coefficient ($COP_{\max}$) do not have very large instructive significance for practical two-stage refrigerator. It is also seen from Fig. 2 that very large difference exists between the optimal performance coefficient (COP^{*}) and the maximum performance coefficient. However, this difference reduces with decreasing k. The objective function can be plotted with respect to the specific cooling load ($\dot{q}_{\mathrm{L}}$) for various k values, as shown in Fig. 3. When we examine Figs. 2 and 3 together, we see that the effect of k on performances decreases as COP and $\dot{q}_{\mathrm{L}}$ approach to their

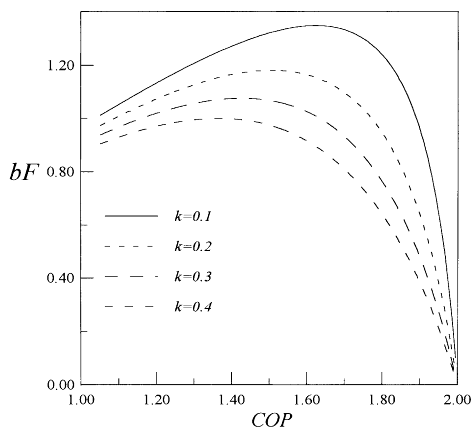

Fig. 2. Variation of the objective function with respect to β for various k values ($T_{\mathrm{H}} = 300$ K, $T_{\mathrm{L}} = 200$ K, $U_{\mathrm{H}} = U_{\mathrm{L}} = U_{\mathrm{W}} = 0.5$ kW/m^2 K).

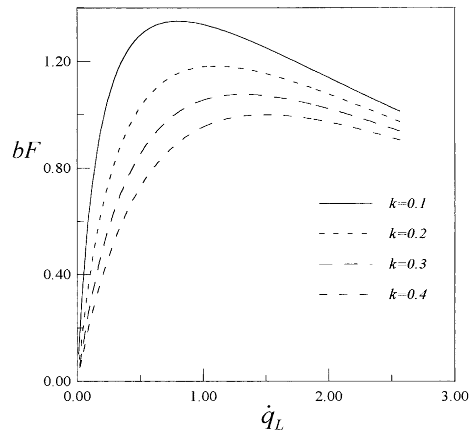

Fig. 3. Variation of the objective function with respect to $\dot{q}_L$ for various k values ($T_H = 300$ K, $T_L = 200$ K, $U_H = U_L = U_W = 0.5$ kW/m^2 K).

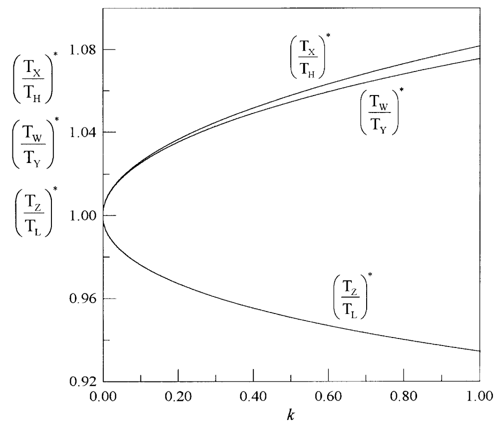

Fig. 5. Variations of the optimal temperatures with respect to k ($T_H = 300$ K, $T_L = 200$ K, $U_H = U_L = U_W = 0.5$ kW/m^2 K).

minimum and maximum values. The variations of the optimum coefficient of performance given in Eq. (18) and the optimum specific cooling load given in Eq. (19) with respect to k are shown in Fig. 4. From the figure, we observe that the optimal specific cooling load increases while the optimal performance coefficient decreases with increasing k. This result indicates that for a specified cooling load ($\dot{Q}_L$), increasing k leads to the result of decreasing optimal total heat transfer area but decreasing optimal performance coefficient. It should be noted that when $k = 0$ the optimum performance coefficient reaches its upper bound (COP_{max}) but optimal specific cooling load becomes zero. In real applications k is always greater than zero and its value should be determined according to economical conditions of a country. From Eqs. (14)–(16), we can obtain the curves of the optimal temperatures of the working fluids varying with k as shown in Fig. 5. It is seen from the figure that the optimal temperature differences in the heat exchangers ($T_X{}^* - T_H$, $T_W{}^* - T_Y{}^*$, $T_L - T_Z{}^*$) increase with increasing k. As a result, the optimum total heat transfer area decreases with increasing k. Also from Fig. 5, we observe that when $k = 0$, the optimal temperature differences in the heat exchangers become zero. In such a case, finite-rate heat transfer irreversibilities disappear and the performance become equal to that of a reversible Carnot refrigeration cycle operating in the same temperature range. The optimal distributions of the heat transfer areas of the heat exchangers are given in Eqs. (20)–(22). We see that the optimal distributions of the heat transfer areas are independent of the economical parameter, k. It is very interesting to note that

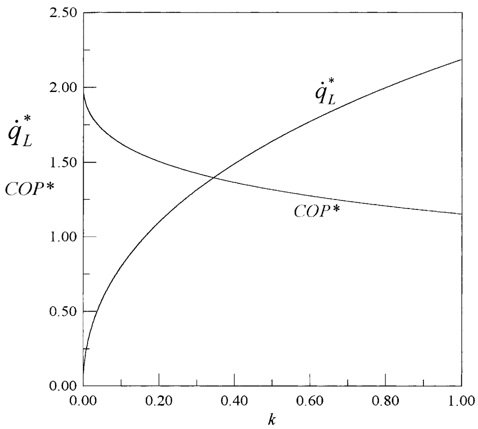

Fig. 4. Variations of the optimum performance coefficient and the optimum specific cooling load with respect to k ($T_H = 300$ K, $T_L = 200$ K, $U_H = U_L = U_W = 0.5$ kW/m^2 K).

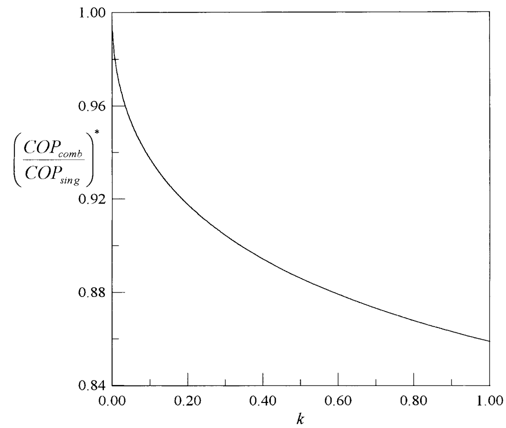

Fig. 6. Comparison of the optimal performance coefficients for the single and the two-stage refrigeration cycles with respect to k ($T_H = 300$ K, $T_L = 200$ K, $U_H = U_L = U_W = 0.5$ kW/m^2 K).

the optimal relations given in Eqs. (20)–(22) are the same as those of the results obtained by Chen and Wu [2] who used maximum coefficient of performance criterion in their analysis. It is seen that as U_W approaches infinity, the relations obtained for two-stage endoreversible combined refrigeration cycle is identical with that of a single-stage endoreversible one operating in the same temperature range [21]. Practically U_W is finite, so that the optimal coefficient of performance of a two-stage endoreversible combined refrigeration system is smaller than that of a single-stage endoreversible refrigeration system for the same temperature range (Fig. 6). However, when the difference of temperatures between the cooled space and the heat sink is large, two-stage combined refrigeration systems have to be employed. As it can be seen from Fig. 6, as k decreases, the optimum coefficient of performance of the two-stage cycle approaches to that of the single cycle and when $k=0$, they will equalize.

4. Conclusion

We have presented a finite-time thermoeconomic performance analysis in order to determine the optimal operation and design parameters based on a new kind of thermoeconomic criterion for the two-stage endoreversible combined refrigerators. We have derived analytically the optimal temperatures of the working fluids, the optimum performance coefficient, the optimum specific cooling load and the optimal distribution of the heat exchanger areas based on the considered thermoeconomic objective function. It is demonstrated that the effects of the investment and the energy consumption costs on the general and the optimal performances can be characterized by a economical parameter, k. The obtained results showed that the economical parameter has a great influence on the optimal operating and design parameters. Since the value of k depends on the economical conditions of a country, the optimal design of the refrigerators may differ for different countries. In this study, the reported optimization results represent the economical design conditions and may provide a general theoretical tool for the optimal design of two-stage combined refrigeration systems.

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