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The $B \to D^*\ell\nu$ Form Factor at Zero Recoil

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We describe a model independent lattice QCD method for determining the deviation from unity for $h_{A_1}(1)$, the $B \to D^*\ell\nu$ form factor at zero recoil. We extend the double ratio method previously used to determine the $B \to D\ell\nu$ form factor. The bulk of statistical and systematic errors cancel in the double ratios we consider, yielding form factors which promise to reduce present theoretical uncertainties in the determination of $|V_{cb}|$. We present results from a prototype calculation at a single lattice spacing corresponding to $\beta = 5.7$.

1. Introduction

The form factor $h_{A_1}(1)$ parameterizes hadronic matrix elements in $B \to D^*\ell\nu$ decays. Its theoretical determination is necessary in order to extract the CKM matrix element $|V_{cb}|$ from the experimental decay rate[1], extrapolated to zero recoil,

$$\lim_{\omega\to 1} \frac{1}{(\omega^2-1)^{1/2}} \frac{d\Gamma(B \to D^*\ell\nu)}{d\omega} = \frac{G_f^2}{4\pi^3}(m_B - m_{D^*})^2 m_{D^*}{}^3 |V_{cb}|^2 |h_{A_1}(1)|^2 . \quad (1)$$

Heavy quark symmetry constrains this form factor[2]. Up to radiative corrections, it has deviations from unity beginning at order $1/m_Q^2$ in an expansion in inverse powers of the quark masses[3]. It is exactly one in the infinite mass limit. We write:

$$h_{A_1}(1) = \eta_A \left[1 - \delta_{1/m_Q^2} + \mathcal{O}(1/m_Q^3)\right] , \quad (2)$$

where the heavy quark expansion in Heavy Quark Effective Theory (HQET) is within brackets, and η_A denotes radiative corrections in the HQET-to-QCD matching[4].

Previous calculations of δ_{1/m_Q^2} have relied on quark models or sum rules estimates[5]. We determine δ_{1/m_Q^2}, in principle model independently, using lattice QCD. Our method extends our previous work in determining the zero-recoil form factor in $B \to D\ell\nu$ decays[6].

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2. Procedure

Consider double ratios,

$$\begin{aligned} R_{J_\mu}^{B\to D}(t) \quad &\equiv \quad \frac{C^{DJ_\mu B}(t) C^{BJ_\mu D}(t)}{C^{BJ_\mu B}(t) C^{DJ_\mu D}(t)} \\ &\xrightarrow[T/2\gg t\gg 0]{} \quad \frac{\langle D|J_\mu|B\rangle \langle B|J_\mu|D\rangle}{\langle D|J_\mu|D\rangle \langle B|J_\mu|B\rangle} \end{aligned} \quad (3)$$

of lattice three-point functions, $C^{DJ_\mu B}(t) = \left\langle \chi_D(T/2)\, J_\mu(t)\, \chi_B^\dagger(0) \right\rangle$. Double ratios are constructed to be identically one when the "charm" and "bottom" quarks are of equal mass. The bulk of statistical and systematic uncertainties cancel in such ratios[6].

We need three double ratios:

$$\overline{\rho}_{V_0} \sqrt{R_{V_0}^{B\to D}} \quad \to \quad |h_+(1)| \,/\, \eta_V \quad (4)$$

$$\overline{\rho}_{V_0} \sqrt{R_{V_0}^{B^*\to D^*}} \quad \to \quad |h_1(1)| \,/\, \eta_V \quad (5)$$

$$\overline{\rho}_{A_j} \sqrt{R_{A_j}^{B\to D^*}} = \sqrt{\frac{h_{A_1}^{BD^*}(1)\; h_{A_1}^{DB^*}(1)}{h_{A_1}^{DD^*}(1)\; h_{A_1}^{BB^*}(1)} \frac{\eta_A^{DD^*}\eta_A^{BB^*}}{\eta_A^2}} . \quad (6)$$

The lattice-to-HQET matching coefficients, $\overline{\rho}_{J_\mu}$, are near unity for typical lattice spacings[7]. Right-hand expressions in the equations above are to be interpreted within HQET. Known normalizations for elastic vector-current matrix elements were used to simplify the first two ratios. These normalizations are obtained nonperturbatively in our numerical work.

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Table 1
Ratios and their $1/m_Q^2$ coefficients.

Ratio	coefficient $c_i^{(2)}$
$\bar{\rho}_{V_0}\sqrt{R_{V_0}^{B\to D}}$	$\ell_P/4$
$\bar{\rho}_{V_0}\sqrt{R_{V_0}^{B^*\to D^*}}$	$\ell_V/4$
$\bar{\rho}_{A_j}\sqrt{R_{A_j}^{B\to D^*}}$	$(\ell_P+\ell_V+\Delta)/8$

All three ratios have quark mass dependence

$$\frac{1-\bar{\rho}\sqrt{R_i}}{\Delta_{m_Q}^2} = c_i^{(2)} - c_i^{(3)}\left(\frac{1}{am_c}+\frac{1}{am_b}\right)+\cdots \quad (7)$$

where, $\Delta_{m_Q} \equiv \left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)$. Table 1 displays our notation for the $c_i^{(2)}$. Note that the $B\to D^*\ell\nu$ coefficient contains a linear combination of the other two coefficients. The form of this coefficient is derived by substituting the mass dependence[5],

$$\delta_{1/m_Q^2} = \Delta_{m_Q}\left(\frac{\ell_V}{2m_c}-\frac{\ell_P}{2m_b}\right)-\frac{\Delta}{4m_c m_b}, \quad (8)$$

into the expression for $R_{A_j}^{B\to D^*}$ shown in Eq. 6.

Our procedure for determining $h_{A_1}(1)$ in $B\to D^*\ell\nu$ decays is: Extract $c_i^{(2)}$ by studying the mass dependence of the double ratios. Solve for ℓ_P, ℓ_V and Δ. Substitute these values and the values we determine for m_c and m_b into the expression for δ_{1/m_Q^2} shown in Eq. 8. We then match to QCD and determine $h_{A1}(1)$ as in Eq. 2.

3. Prototype Calculation at $\beta = 5.7$

Many of the numerical details in this study are common to our study of $B\to D\ell\nu$ matrix elements[6]. We note in particular:

- We use a subset of 200 $\beta = 5.7$ quenched gauge configurations on a $12^3\times 24$ lattice.
- We use the Sheikholeslami-Wohlert quark action with a tadpole-improved tree-level coefficient, $c_{SW} = 1.57$. Results are interpreted within the Fermilab heavy quark formalism[8].
- We study double ratios for nine combinations of heavy quarks with bare masses corresponding to $\kappa_h \in \{0.125, 0.119, 0.110, 0.100, 0.089, 0.062\}$. Quark masses range around both the charm and bottom masses.
- Statistical errors were obtained using a single-elimination jackknife procedure.
- The physical tree-level charm and bottom quark masses, m_c and m_b, were determined by adjusting bare mass inputs and demanding that calculated meson kinetic masses match the physical D and B meson masses.
- Matching factors $\bar{\rho}_{J_\mu}$ are only known to one loop order. For consistency, η_A is truncated to one loop order. Matching factors are computed using the V scheme coupling. We use BLM matching scales which account for $\beta_0\alpha_s^2$ contributions.

Results in this paper have the spectator quark mass fixed near the strange quark mass. In our $B\to D\ell\nu$ study, we checked the dependence upon the spectator mass for $R_{V_0}^{B\to D}$. Values in the chiral limit were consistent with those for the strange quark, but with statistical errors which were twice as large. We anticipate similar chiral behavior for the other two ratios we use in this study. Hence, we expect insignificant differences in $c_j^{(2)}$ in the chiral limit, and similar increases in statistical errors. Here, we account for the uncertainty of not performing the chiral extrapolation by doubling our statistical errors.

Figure 1 shows the heavy quark mass dependence we find for $\sqrt{R_{A_j}^{B\to D^*}}$. The quality of these results are representative of our results for the other two ratios. The two points to the left in the figure have large statistical errors. These decays involve the heaviest quark masses and suffer from well-known signal-to-noise problems. A fit to the functional form given in Eq. 7 is shown in the figure. Its y-intercept shows this $c^{(2)}$ determination has greater than 4σ significance.

We find this and the other two coefficients are of $\mathcal{O}(\Lambda_{QCD})$, as expected. The values we obtain are broadly consistent with previous estimates[5]. Quantitative comparisons may be misleading, however, since uncertainties in previous estimates are difficult to ascertain.

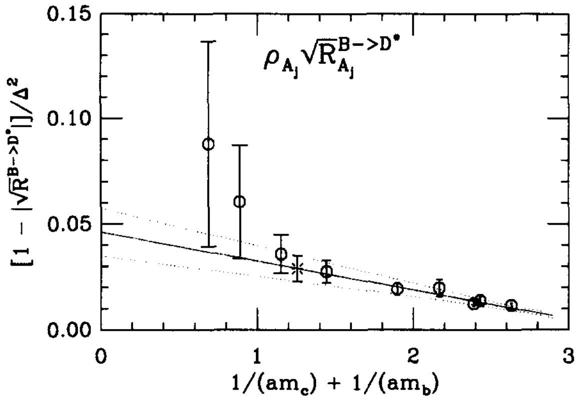

Figure 1. Calculated mass dependence for $\sqrt{R_{A_j}^{B\to D^*}}$ at $\beta = 5.7$. The solid line is a fit to the expected mass dependence given in Eq. 7. Dashed lines show 1-σ statistical errors from the fit. The burst indicates the fitted ratio at the physical combination of quark masses.

4. Determination of $\mathbf{h_{A_1}(1)}$

We present a *preliminary* determination of $h_{A_1}(1)$ using our prototype $\beta = 5.7$ study to illustrate the precision we expect in a complete study:

$$h_{A_1}(1) = 0.935 \pm 0.022(^{+0.008}_{-0.011}) \pm 0.008 \pm 0.020.$$

Sources of uncertainties are, respectively:

- Statistical. We double statistical errors to account for not having extrapolated to the down quark mass. We must still check the chiral behavior of all three double ratios used in the determination of h_{A_1}.
- Tuning of quark masses m_c and m_b. We estimate 10% and 13% uncertainties in our charm and bottom masses.
- Unknown radiative corrections beyond 1-loop. We estimate this uncertainty by varying the 1-loop coefficients by 20%.
- Undetermined $\mathcal{O}(1/m_Q^3)$ corrections to $h_{A_1}(1)$. We use the relative sizes of the $1/m_Q^3$ terms determined in our fits to estimate neglected power corrections.

Two important sources of systematic uncertainty remain to be evaluated fully:

- Lattice spacing dependence. In our studies of the decay constants f_D and f_B and of $B \to \pi\ell\nu$ matrix elements we find lattice artifacts are under control[9, 10]. We anticipate no large lattice artifacts in this study. We adjust the quark action and currents to match $1/m_Q$ terms of the QCD heavy quark expansion[8]. Contributions to δ_{1/m_Q^2} arise solely from these terms in the double ratio method[7]. We note that any remaining cutoff dependence may be removed by repeating the calculation for additional lattice spacings and taking the continuum limit.
- Uncertainty due to the quenched approximation. A full quantitative estimate of the error due to quenching must await an unquenched determination of h_{A_1}. Note, however, the uncertainty due to quenching affects the deviation of the form factor from unity. The quenching uncertainty in this deviation is commonly believed to be perhaps 20%.

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