

Letter

# On the strain rate sensitivity values of superplastic materials

R.A. Vasin, F.U. Enikeev \*, M.I. Mazurski

*Institute for Metals Superplasticity Problems, Khalturina, 39, Ufa, 450001, Russia*

Received 23 June 1997; received in revised form 22 June 1998

## Abstract

Some problems arising in experimental determination of the strain rate sensitivity of the flow stress,  $\sigma$ , on the strain rate,  $\dot{\epsilon}$ , are considered. It is pointed out that the strain rate sensitivity index,  $m$ , entering the standard power relation  $\sigma = K\dot{\epsilon}^m$ , is not identical to the slope  $M = \partial(\log \sigma)/\partial(\log \dot{\epsilon})$  of the sigmoidal superplastic curve. The difference between  $m$  and  $M$  is to be taken into consideration in treating the results of the mechanical tests. Published by Elsevier Science S.A.

**Keywords:** Superplasticity; Mechanical properties; Strain rate sensitivity

It is commonly recognized now that the most characteristic feature of the mechanical behavior of superplastic materials is the high strain rate sensitivity of the flow stress,  $\sigma$ , on the strain rate,  $\dot{\epsilon}$  [1–7]. In the technical literature, this sensitivity is usually defined as a derivative or a rate of change. Strain rate sensitivity (SRS) of stress is defined as an incremental change occurring in stress when the strain rate is changed incrementally:

$$M = \partial(\log \sigma)/\partial(\log \dot{\epsilon}) = \partial(\ln \sigma)/\partial(\ln \dot{\epsilon}) \quad (1)$$

At the same time the so-called strain rate sensitivity index,  $m$ , is often introduced in the literature [5,7]

$$\sigma = K\dot{\epsilon}^m \quad \text{or} \quad \dot{\epsilon} = K'\sigma^n \quad (2)$$

where  $K$  is a material constant;  $K' = 1/K^n$ ,  $n = 1/m$ .

It is evident that, in general,  $m = M$  only if  $m$  does not depend on strain rate. It is only in a very special case of power law creep or in the regime of a single deformation mechanism that one can assume  $m = M = \text{constant}$ . In this case the dependence (2) would be a straight line if plotted in logarithmic coordinates  $\log \sigma$  versus  $\log \dot{\epsilon}$ . The slope of this straight line would be equal to  $m = M$ . However, the corresponding experimental data can not all be fitted by the same straight line; in practice most superplastic materials shows a

sigmoidal variation of the flow stress with strain rate (Fig. 1a); the slope  $M$  of this curve depends upon  $\dot{\epsilon}$ , so that the standard  $M(\dot{\epsilon})$  curve has a specific dome-like shape (Fig. 1b). The maximum value of the SRS  $M_0$  corresponds to the optimum strain rate  $\dot{\epsilon}_0$ ; the pair  $\sigma_0$ ,  $\dot{\epsilon}_0$  being the point of inflection of the sigmoidal curve.

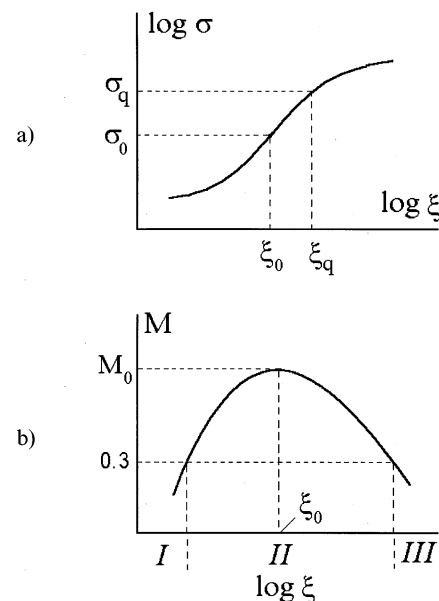


Fig. 1. Typical dependencies of the flow stress,  $\sigma$ , (a) and the slope of the sigmoidal curve,  $M$ , (b) on the strain rate,  $\dot{\epsilon}$ .

\* Corresponding author. Fax.: +7 3472 253759; e-mail: enikeev@bash.ac.ru

Therefore Eq. (2) is often considered as a local approximation of the sigmoidal curve, which is applicable only at narrow strain rate interval, where the hypothesis  $m \approx \text{constant}$  can be adopted. As it was indicated in [1,3,8], one should keep in mind the difference between  $m$  and  $M$ . Unfortunately, this difference has usually not been taken into consideration in the literature; the same symbol 'm' has been used to denote the exponent  $m$  on the right of Fig. 2 as well as the slope of the sigmoidal curve  $M(\xi)$  [1,2,5–7]. It is evident, that the function  $M(\xi)$  can be determined unambiguously during standard mechanical tests. Thus, one can measure unambiguously the SRS  $M_q$  for some arbitrary point of the sigmoidal plot  $\xi = \xi_q$ . Let the slope of the sigmoidal plot  $M$  at  $\xi = \xi_q$  be equal to, e.g.  $M_q = 0.5$ . The following question arises then—is it possible to conclude that the value of the flow stress is proportional to the square root of the strain rate within close vicinity of the point  $\xi = \xi_q$ ? In order to answer this question let us consider the following problem—let the function  $M = M(\xi)$  be known. It is necessary to determine the corresponding function  $m = m(\xi)$  entering the right side of Eq. (2). It is usually assumed in the literature that the value of the material constant  $K$  does not depend upon strain rate [1–7]. However, for the sake of generality let us suppose at first that  $K = K(\xi)$ . Such hypothesis leads immediately to the following consequence—one can let the value of  $m$  be equal to some arbitrary constant value  $m^*$ :  $m(\xi) = m^* = \text{constant}$  by means of the following choice of the function  $K = K(\xi)$ :

$$K(\xi) = \sigma(\xi)/\xi^{m^*} \quad (3)$$

where  $\sigma(\xi)$  is the experimental stress-strain rate curve (some arbitrary function  $m = m(\xi)$  may also be substituted at the right side of Eq. (3) instead of  $m^* = \text{constant}$ ). Thus, we have obtained the following conclusion—the value of the strain rate sensitivity index,  $m$ , may be fitted to any given constant value  $m(\xi) = m^* = \text{constant}$  by means of an appropriate choice of the function  $K = K(\xi)$  in accordance with Eq. (3). In order to eliminate such arbitrariness one has to assume  $K = \text{constant}$  in Eq. (2). In this case it is easy to show that the unknown function  $m = m(\xi)$  can be found as follows:

$$m(\xi) = [\log \sigma - \log K]/\log \xi \quad (4)$$

The value of  $K = \text{constant}$  entering the right side of Eq. (4) is, in general, unknown. Thus, one can fit the value of  $m$  to any given value  $m(\xi_q) = m^*$  at the point under consideration  $\xi = \xi_q$  by means of an appropriate choice of the constant  $K$  on the right side of Eq. (4). Thus, now we are ready to give an answer for the above-formulated question. In principle, it is not possible to give an unequivocal answer, since one can choose the value of  $K$  so that it will be  $m(\xi_q) = 1$  or, say,  $m(\xi_q) = 0.1$ . In other words it is possible, in principle, to refer the slope

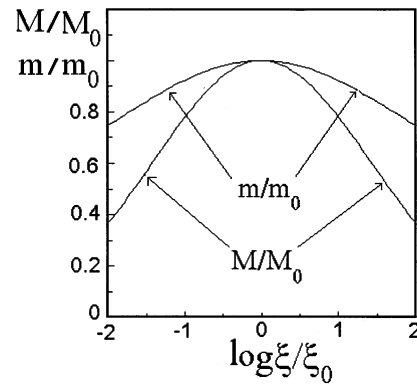


Fig. 2. Universal superplastic curve: dependencies  $M(\xi)$  and  $m(\xi)$ , calculated according to Eqs. (5) and (6), respectively.

$M(\xi_q) = 0.5$  to a Newtonian viscous liquid ( $m = 1$ ) as well as to a non-superplastic material ( $m = 0.1$ ). It is clear that such situation is not acceptable in practice, therefore the value of  $K$  is to be determined from some additional considerations. One possible way to eliminate the above-described lack of uniqueness is suggested below. As found in [9] the dependence of the SRS,  $M$  on the strain rate,  $\xi$  in normalized coordinates  $M/M_0$  versus  $\xi/\xi_0$  is common for a wide range of materials. This unique curve can be described, e.g. by the following expression:

$$M(\xi) = M_0 \exp[-a^2 \log^2(\xi/\xi_0)] \quad (5)$$

where  $a^2 \approx 0.25$  [9]. The concrete values of  $M_0$  and  $\xi_0$  characterize superplastic properties of a given material at a given temperature. Eq. (5) may be used to construct a constitutive equation of a superplastic material. After integrating Eq. (5) one finds:

$$\log \frac{\sigma}{\sigma_0} = \int_0^{\log(\xi/\xi_0)} M_0 \exp(-a^2 x^2) dx = m(\xi) \log \frac{\xi}{\xi_0} \quad (6)$$

where  $m(\xi)$  is the clearly defined function. In Fig. 2 the dependence  $M(\xi)$  calculated in accordance with Eq. (5) is compared with the corresponding dependency  $m(\xi)$  calculated in accordance with Eq. (6). One can see in Fig. 2, that  $m(\xi) \geq M(\xi)$ . The equality  $M = m$  takes place at the inflection point of the sigmoidal curve  $\xi = \xi_0$ . Recently the behavior of different combinations of non-linear viscous elements during active loading was analyzed in [10]. The dependencies  $m(\xi)$  and  $M(\xi)$  for the mixed combination suggested in [10] in order to describe the universal curve is shown to be in a good agreement with those shown in Fig. 2. Besides, it was shown, that there is a notable difference between  $m(\xi)$  and  $M(\xi)$  curves for parallel and sequential combinations.

The material constant  $K$  may be found from the condition  $m_0 = m(\xi_0) = M(\xi_0) = M_0$ . There are the following arguments in favor of such choice. 1. In the vicinity of the inflection point the following equalities

hold:  $M(\xi) \equiv m(\xi) = \text{constant}$ . It may be shown that in this case  $dm/d \log \xi = 0$  at  $\xi = \xi_0$ . Thus, the curve  $m(\log \xi)$  will have an extremum at  $\xi = \xi_0$ . Therefore the curve  $m(\xi)$  is expected to have a specific dome-like shape, similar to that of  $M(\xi)$  shown in Fig. 1b.

Consider now the load relaxation test [1,3,8,11]. The procedure to determine the SRS by means of load relaxation is based upon the use of Eq. (2) along with the assumption  $m = \text{constant}$ . Therefore the load relaxation test enables the value of  $m$  instead of  $M$  to be measured. This circumstance may be used, e.g. in explaining the discrepancy between measured values of  $m$  for Pb–Sn eutectic alloy [11]: actually, the value of  $m$ , measured by load relaxation should not inevitably be equal to the value of  $M$ , measured by means of standard procedures. On the other hand, it was shown recently [12] that the constitutive laws similar to that of Eq. (2) fail in describing the mechanical response of superplastic material under active and passive loading conditions by using the same set of material constants. Therefore this problem requires further investigations.

Thus, the difference between  $m(\xi)$  and  $M(\xi)$  is to be taken into account in treating the mechanical response of a superplastic material. It is necessary to keep in mind this difference when comparing the results obtained from different methods of evaluating SRS, e.g. in comparing the results obtained during step strain rate tests with those obtained during load relaxation tests.

## Acknowledgements

Authors thank Russia Foundation of Fundamental Investigations for financial support (Grant No. 96-01-01317).

## References

- [1] M.V. Grabski, Structural Superplasticity of Metals, Metallurgy Publishing House, Moscow, 1975.
- [2] O.M. Smirnov, Pressure Metals Treatment in Superplastic State, Mashinostroyenie, Moscow, 1979.
- [3] K.A. Padmanabhan, G.J. Davies, Superplasticity, Springer, Berlin, 1980.
- [4] J.W. Edington, Met. Trans. A13 (1982) 703–715.
- [5] T.G. Langdon, Met. Trans. A13 (1982) 689–701.
- [6] O.A. Kaibyshev, Superplasticity of Alloys, Intermetallides and Ceramics, Springer, Berlin, Heidelberg, 1992.
- [7] J. Pilling, N. Ridley, Superplasticity in Crystalline Solids, The Institute of Metals, Camelot, London, 1989.
- [8] J. Hedworth, M.J. Stowell, J. Mater. Sci. 6 (1971) 1061–1069.
- [9] A.A. Sirenko, M.A. Murzinova, F.U. Enikeev, J. Mat. Sci. Lett. 14 (1995) 773–774.
- [10] R.A. Vasin, F.U. Enikeev and M.I. Mazurski, “Mechanical modelling of the universal superplastic curve”, Proc. Int. Seminar on Microstructure, Micromechanics and Processing of Superplastic Materials (IMSP-97), 8-9 August, 1997 Mie University, Tsu, Japan (in press).
- [11] F.U. Enikeev, M.I. Mazurski, Scri. Meta. 32 (1995) 1–6.
- [12] R.A. Vasin, F.U. Enikeev, M.I. Mazurski, Mat. Sci. Forum 170–172 (1994) 675–680.