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Acoustic band gaps in two-dimensional rectangular arrays of liquid cylinders

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Abstract

In this paper, we present the acoustic band gap results for a new geometry of two-dimensional arrays of water (mercury) cylinders with circular cross-section in mercury (water) host, arranged in rectangular lattice. The results show that when the lattice constants $a_x = a_y$, which correspond to the square lattice, for the systems with either water cylinders in mercury or mercury cylinders in water, the widest range of filling fraction is achieved for the gap appearance. The filling fraction range shrinks as a_x/a_y increases. It also shows that wider gap can be obtained in rectangular system other than square system in some occasions. © 2002 Elsevier Science Ltd. All rights reserved.

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In recent years, the propagation of acoustic (AC) or elastic (EL) wave in artificial periodic structures of elastic materials—the phononic crystals, has received a great deal of attention [1–28]. Of particular interest is the existence of forbidden frequency bands in which no sound and vibration are allowed. The motivation for these studies is to better understand the propagation and localization of sound and vibrations in inhomogeneous media [29,30], as well as their numerous engineering applications such as frequency filters, vibrationless environments for high-precision mechanical systems or the design of new transducers.

The appearance of the elastic (acoustic) wave band gap is determined mainly by the density and velocity contrast of the components of the composite, the filling fraction, the lattice structure and the topology [24–26]. Several methods such as the plane-wave expansion (PWE) method [5–11], the finite difference time domain (FDTD) method [12–19] and the multiple-scattering theory (MST) [1–4, 20–22], have been developed to study the elastic (acoustic) band structures in various structures. Acoustic band gaps (ABG) had been found in three-dimensional (3D) systems with face-centered cubic (fcc), body-centered cubic (bcc), and simple-cubic (sc) lattices, and all kinds of solid, fluid component [22–26]. In two-dimensional (2D) phononic crystals, complete ABG were also obtained in the systems of square, triangular, center rectangular, hexagonal and boron nitride lattices [5–7,21]. In this paper, we present the results on the ABG for a new geometry of 2D phononic crystals for rectangular arrays of infinite water (with longitudinal velocities $c_l = 1.48$ km/s and density $\rho = 1.0 \times 10^3$ kg/m^3) or mercury ($c_l = 1.45$ km/s, $\rho = 13.5 \times 10^3$ kg/m^3) cylinders immersed in a mercury or water host. To be practical, some latex materials could be used to contain the liquid within the cylinders [5–7,10,11], since the mass density and speed of rubber are comparable to those of water, and the effect of this thin latex film can be neglected.

In the periodic liquid composite media, only the longitudinal waves can propagate, the acoustic wave

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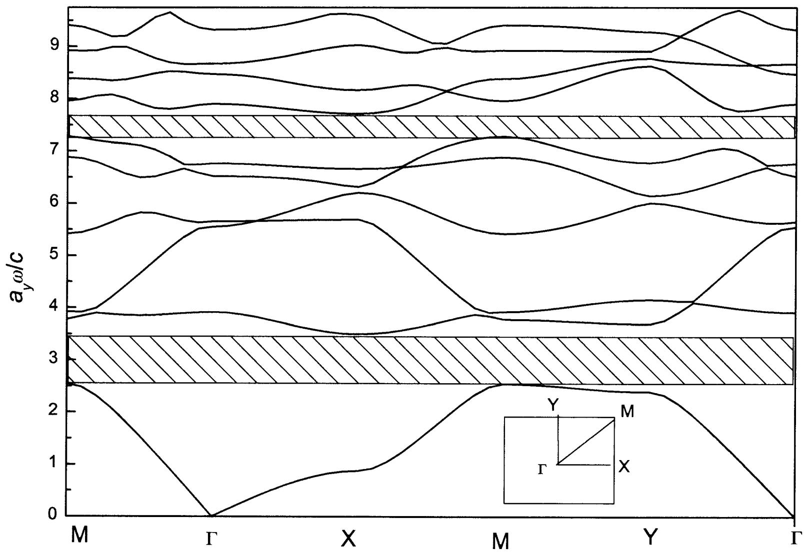

Fig. 1. Acoustic band structure for water in mercury system. The ratio of lattice constants $a_x/a_y = 1.4$ and filling fraction $F = 0.567$. The hatched region represents complete band gap.

equation can be simplified as follows [5–7]

$$\frac{\partial^2 p}{\partial t^2} = \lambda \nabla\left(\frac{\nabla p}{\rho}\right), \tag{1}$$

where $\lambda(\vec{r})$, $\rho(\vec{r})$ and p are the bulk modulus, the mass density and pressure of the fluid.

Following the PWE methodology [5–7,10,11,24–26], Eq. (1) can be derived in the form of a standard eignvalue problem at hand:

$$\sum_{\vec{G}'}\left[\omega^2 \lambda^{-1}_{\vec{G}-\vec{G}'} - \rho^{-1}_{\vec{G}-\vec{G}'}(\vec{k}+\vec{G})\cdot(\vec{k}+\vec{G}')\right]p_{\vec{G}'} = 0. \tag{2}$$

The reciprocal vectors $\vec{G}$ can be expressed as $\vec{G} = 2\pi(n_x\vec{i}/a_x + n_y\vec{j}/a_y)$ for the rectangular lattice, where a_x, a_y stand for lattice constants along x and y directions, and n_x, n_y are integers. The filling fraction is thus $F = \pi r^2/a_x a_y$, where r is the radius the cylinders. Eq. (2) is an infinite set of linear equations. In practice, only a finite number of $\vec{G}$ vectors are used in the numerical calculation. In the present paper, 361 reciprocal vectors (i.e. the integers n_x, n_y in $\vec{G}$ are ranged from -9 to $+9$) are used in the calculation, which show a very good convergence.

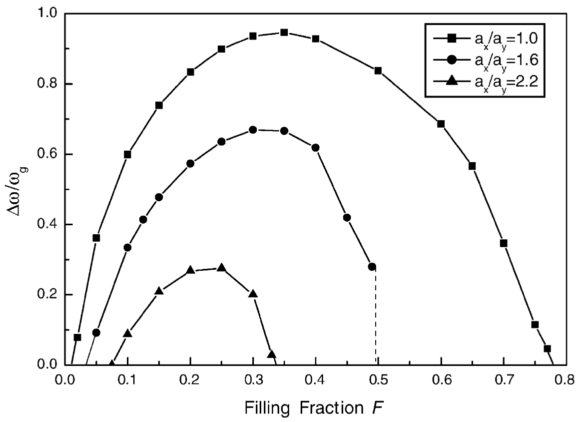

Fig. 2. For water in mercury system, the dependence of the gap/midgap of the lowest gap on filing fraction F for different ratios a_x/a_y.

First, we investigate the band structures for the rectangular lattice of water cylinders in mercury host. Fig. 1 shows the first ten bands for ratio $a_x/a_y = 1.4$ and filling fraction $F = 0.567$. The plots are rendered in terms of dimensionless frequency $\Omega = a_y\omega/c$ (c is the longitudinal speed in mercury) vs dimensionless Block vector $\vec{k} = \vec{K}a_y/2\pi$, letting $\vec{k}$ scan the five principal symmetry directions $M\Gamma XMY\Gamma$ of the Brillouin zone (see the inset in Fig. 1). We obtain two band gaps, one is between the first and second band, and another is a narrower band gap between the sixth and the seventh band. The lower edge of the lower gap appears at the M points of the first band while the higher edge appears at the X point of the second band. Further investigation shows that for any system of water in mercury, the upper edge of the gap appears at the X point of the second band when $a_x/a_y > 1.0$, and shift to the Y point of the second band when $a_x/a_y < 1.0$, while the lower edge always remains at the M point of the first band.

Now we plot the ratio of the gap width to midgap frequency $\Delta\omega/\bar{\omega}$ of the lower band gap as a function of filling fraction F for three different ratios a_x/a_y in Fig. 2. Considering the similarity between the cases of $a_x/a_y > 1.0$ and $a_x/a_y < 1.0$, we only give the results of the $a_x/a_y \geq 1.0$ case. One can easily see that the gap only appears in a certain range of the filling fraction. The range narrows gradually as a_x/a_y increases, and disappear when $a_x/a_y >$

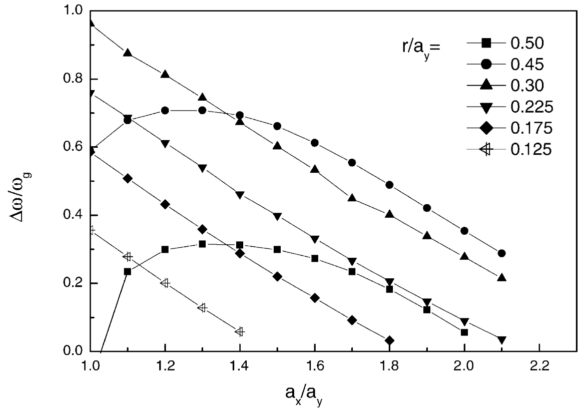

Fig. 3. The gap/midgap for water in mercury systems as the function of the ratio of the lattice constants a_x/a_y for different ratios r/a_y.

2.7. For $a_x/a_y = 1.0$, 1.6 and 2.2, the gap opens at $F = 0.01$, 0.03 and 0.07, and closes at $F = 0.77$, 0.49 and 0.33, respectively. Noticeable is the fact that, for $a_x/a_y = 1.6$, the gap remains finite for close-packing, but for $a_x/a_y = 1.0$ and 2.2, there is no gap for close packing (the close-packing for $a_x/a_y = 1.0$, 1.6 and 2.2 corresponds to $F = 0.785$, 0.49 (dashed line) and 0.357, respectively). The maximum gaps for the three a_x/a_y values are obtained at $F = 0.35$, 0.30 and 0.25, and the corresponding gap/midgap ratios are $\Delta\omega/\bar{\omega} = 0.946$, 0.669 and 0.275, respectively. For any filling fraction, the gap width for $a_x/a_y = 1.0$ is the largest, comparing to those for $a_x/a_y = 1.6$ and 2.2.

But for the same value r/a_y, the gap width for square structure ($a_x/a_y = 1.0$) is not always the largest. Fig. 3 shows the dependence of the gap/ midgap ratio on a_x/a_y for six different values $r/a_y = 0.125$, 0.175, 0.225, 0.300, 0.450, 0.500, respectively. It can be seen that the maximum gaps are located at $a_x/a_y = 1.0$ when $r/a_y = 0.125$, 0.175, 0.225, 0.300, and shift to $a_x/a_y > 1.0$ when $r/a_y = 0.45$ and 0.50.

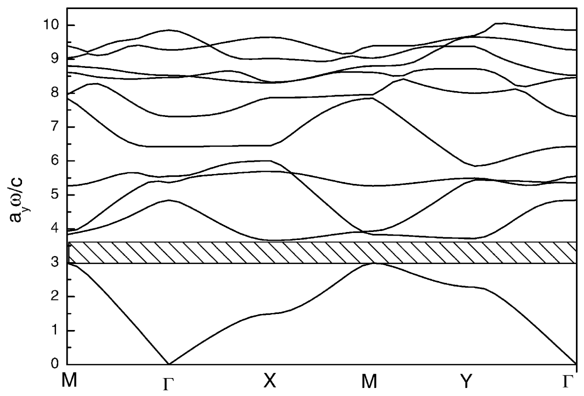

Fig. 4. Acoustic band structures for mercury in water system. The lattice is $a_x/a_y = 1.2$ and filling fraction $F = 0.53$. The hatched region represents complete band gap.

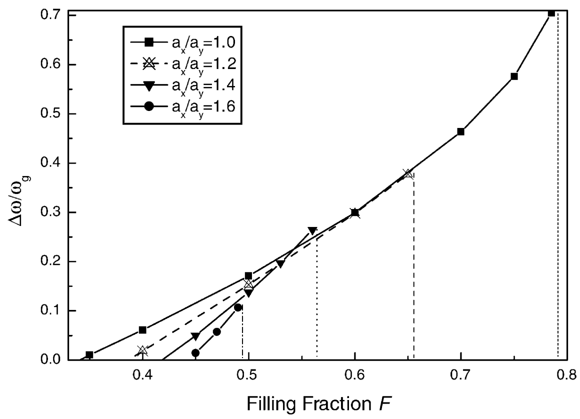

Fig. 5. For mercury in water system, the dependence of the gap/midgap of the lowest gap on filing fraction F for different ratios a_x/a_y.

As a comparison, we also calculated the band structures for the systems with mercury cylinders in water host. Fig. 4 shows the band structures for the crystal with $a_x/a_y = 1.2$ and $F = 0.53$. Only one band gap between the first and the second band can be seen. The width of the band gap is determined by both the minimum of the second band at the X point, and the maximum of the first band at the M point.

Fig. 5 shows the gap width $\Delta\omega/\bar{\omega}$ as a function of filling fraction F for four different ratios $a_x/a_y = 1.0$, 1.2, 1.4, 1.6. Similar to the case of water in mercury, the gap only appears in a certain range of the filling fraction F, and the range gradually shrinks as the ratio a_x/a_y increases, finally disappears at $a_x/a_y > 1.7$. The gaps open at $F = 0.342$, 0.388, 0.42 and 0.45, respectively, and close at close-packing with $F = 0.785$, 0.650, 0.560, 0.490. Meanwhile, the gap width increases gradually as filling fraction F increases, and to maxima at corresponding close-packing limits with $\Delta\omega/\bar{\omega} = 0.705$, 0.375, 0.265 and 0.107, respectively. But being different from the case of water in mercury, the gap width in square lattices is not always the largest. The gap width for $a_x/a_y = 1.6$ is larger than for the square lattice at the filling fraction F near the close packing value. Fig. 6 gives the results of the gap/midgap as the function of a_x/a_y for given values $r/a_y = 0.425$, 0.450, 0.475 and 0.500, respectively. The gap/midgap increases linearly as a_x/a_y decreases for both $a_x/a_y < 1.0$ and $a_x/a_y > 1.0$, and all maxima appear at the close packing limits. For any value of a_x/a_y, we find the gap/midgap increase monotonously as r/a_y decreases.

In a brief summary, we investigate the dependence of the ABG on the lattice parameters a_x/a_y and r/a_y for 2D rectangular arrays of water (mercury) cylinders in mercury (water) host. The results show that filling fraction range for gap appearance narrows as the a_x/a_y increases for both systems with water in mercury and mercury in water. It also shows that the wider gap can be obtained in rectangular lattice other than in the square lattice in some occasions. For

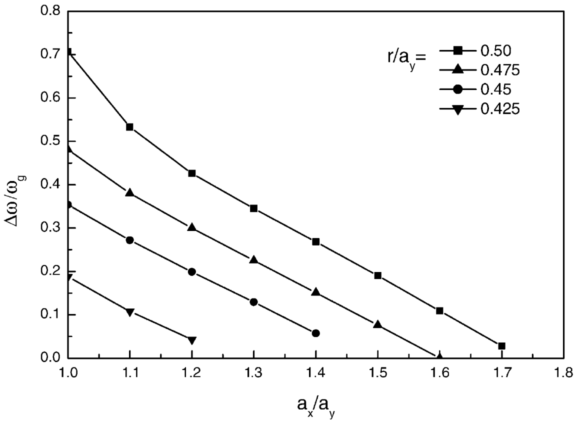

Fig. 6. The gap/midgap for mercury in water system as the function of the ratio of the lattice constants a_x/a_y for the given ratios r/a_y.

the system with water in mercury, this means that low-density cylinders in a high-density host, the square lattice always results in the widest gap in the whole filling fraction range. But the maximum of gap/midgap appears at $a_x/a_y = 1.0$ when $r/a_y < 0.30$, shifts to $a_x/a_y > 1.0$ within $0.30 < r/a \leq 0.50$. For the system with mercury in water for any a_x/a_y the gap/midgap ratio increases as the filling fraction increases, the gap width for a rectangular lattice can be bigger than that of square lattice. For any r/a_y the gap/midgap also increases as the ratio a_x/a_y decreases, and reaches maxima when cylinders become touching in the *X* or *Y* direction. For a given a_x/a_y, the square lattice is favorable for a wider gap.

Acknowledgments

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