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Role of the interface in stratified slurry flow

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Abstract

Many heterogeneous slurry flows can be modelled using the two-layer approach, wherein the particles' suspended weight is supported mostly by fluid turbulence in the upper portion of the pipe, and by inter-granular contact in the lower portion. The approach requires that the frictional force acting between the two layers be correctly evaluated. In early work, this friction was based on an equivalent roughness proportional to particle size. Recent experimental evidence indicates that for a wide range of flows, the interface between the two zones is several grain diameters in thickness, and the friction acting between the zones scales with this thickness, which is directly proportional to the shear stress. The shear-stress-dependent roughness allows calculation of more accurate values for the pressure gradient and the velocity at the limit of stationary deposition.

Keywords: Slurry-flow; Friction; Bed-load; Deposition; Shear-layer; Roughness; Freight-pipelines

1. Introduction

Two mechanisms of particle support are active in heterogeneous slurry flow — turbulent suspension and intergranular contact. Near the bottom of the pipe the concentration of solids is large and intergranular contact is important; but with increasing height the solids concentration diminishes and turbulent suspension becomes the dominant mechanism for particle support. The resulting flow tends to be stratified, with a particle-rich zone near the bottom of the pipe and a leaner zone above it. This type of behaviour is now commonly modelled mathematically by the two-layer approach, which sets up force balances for the two zones mentioned above. To apply this model, it is necessary to evaluate the interfacial friction acting between the two zones.

Experiments, some of which will be discussed below, showed that the interface between the zones is usually not abrupt, but typically forms a ''ramp'' of concentration (and also of velocity) that links the values in the upper and lower zones. Known as the ''shear layer'', this ramp has a thickness, δ_s, which usually amounts to several grain diameters. The calculation of interfacial friction, mentioned above, is based on some characteristic length associated with the interface. This length is called the ''equivalent roughness'' of the interface. It was thought initially that the interface would behave as a hydraulically-rough boundary, with the equivalent roughness proportional to the size of the solid grains. More recent work has shown that such is not the case. As described below, the equivalent roughness of the interface depends directly on the shear-layer thickness, and can be very much larger than the grain size.

2. Limit of stationary deposition

The limiting case of stratified flow comprises flow with a stationary deposit, i.e., with the lower zone not moving. Although this configuration is a convenient one for the experimental investigation of interfacial friction, it is generally avoided in industrial practice, where the typical requirement is to evaluate the limit of deposition. (The freight pipeline designer needs to know this limit in order to avoid a stationary bed of solids.)

The force-balance approach can also be applied in calculating the mean flow velocity at the deposition limit, V_{sm}. It should be noted that, for the designer's convenience, mean flow velocity is defined as discharge divided by total pipe cross-section, i.e., $4Q/\pi D^2$, where D is internal pipe diameter. For a specified pipe size, fluid type, and given solid properties and concentration, there will be a certain mean flow velocity beyond which all particles

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will be set in motion. If the solids concentration is varied, say working up from a small initial value, the velocity at which all particles are set in motion increases initially, reaches a maximum, and then decreases slowly with further increases in concentration. This maximum is known as the deposition-limit velocity V_{sm}.

In the 1970s, the two-layer model was applied to the deposition limit, on the basis that the lower layer must be set in motion when the driving forces just exceed the resisting force. The resisting force is produced by mechanical friction between the solid particles and the bottom of the pipe, and thus is proportional to mechanical friction coefficient μ_s and the submerged unit weight of the solids in the bed layer. For aqueous slurries the latter quantity varies with $(S_s - 1)$, where S_s is the specific gravity of the solids.

Two driving forces must be considered, one proportional to the pressure gradient in the pipe (denoted i_m, and expressed in m water/m pipe), and the other based on the shear stress at the interface. In early applications of the two-layer model, the interfacial shear stress was calculated using an equivalent roughness proportional to the size of the solid grains, as mentioned above. Thanks to the nomographic expertise of the late Prof. F.M. Wood, a large volume of computer output was condensed into a single chart, reproduced here as Fig. 1 (see Refs. [4,5]). This figure is set up for the convenience of the designer. The left-hand panel is based on sand-weight aqueous slurries $[(S_s - 1) = 1.65]$ with narrow particle grading and μ_s constant at about 0.4. The internal pipe diameter D is entered on the left vertical axis, and the particle size d is located on the curved ''demi-McDonald''. These points are joined by straight-line and projected onto the central vertical axis, at which V_{sm} is read. This value of V_{sm} applies directly to sand-weight materials (i.e., specific gravity equal to 2.65); for solids with specific gravity other than 2.65, the value of S_s is entered on the sloping axis in the right-hand panel of the chart, joined by straight-edge to sand-weight value of V_{sm} on the central vertical axis, and then projected to the right-hand vertical axis, which gives V_{sm} for the actual value of S_s.

It should be noted that the two branches of the demi-McDonald reflect different mechanisms by which the stationary bed is eliminated. The right-hand branch pertains to beds of small particles that can be eroded away by turbulent suspension; for the left-hand branch the particles are larger, and the initiation of motion can be sliding of the bed en bloc. For the assumed variation of effective interfacial roughness with particle size, larger particles give rise to larger interfacial shear, implying that the bed will be set

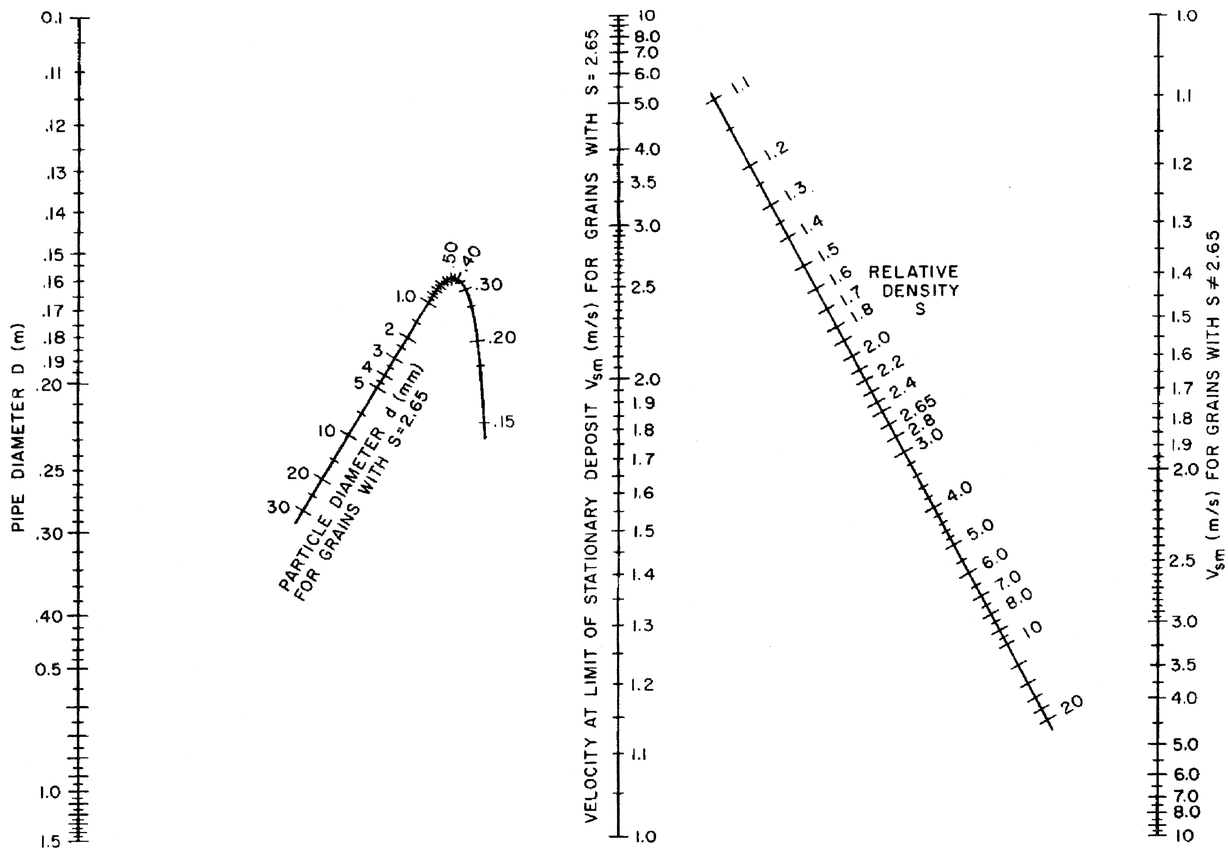

Fig. 1. Nomographic chart, from Ref. [5].

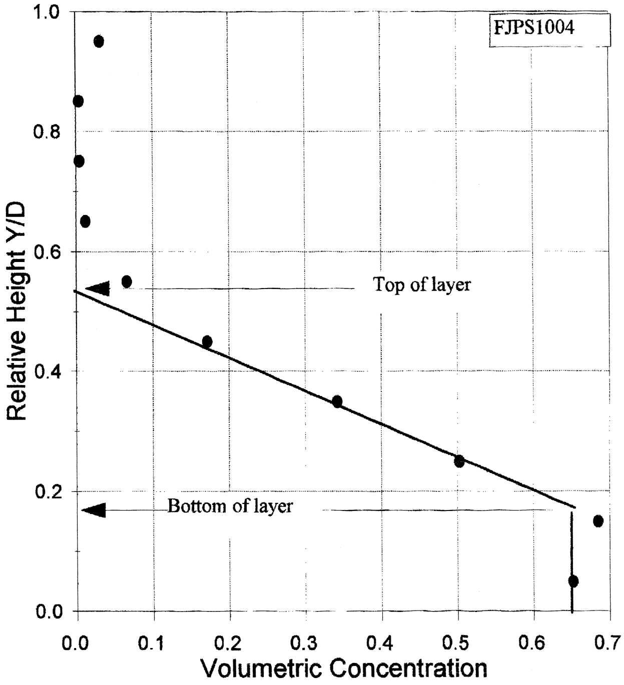

Fig. 2. Concentration profile, data from Ref. [2].

in motion at a lower value of V_{sm}. An example using Fig. 1 with $D = 0.30$ m and particle diameters 2 and 10 mm demonstrates this effect. A straight line passing through $D = 0.30$ and $d = 2$ mm projects onto a deposition velocity of 3.2 m/s. Projecting a straight line from $D = 0.30$ through $d = 10$ mm yields a significantly lower deposition velocity of about 2.2 m/s.

Experimentally-determined concentration and velocity profiles have now shown that there is a considerable range of conditions for which the effective interfacial roughness does not depend on particle size. This is the case when the interface is not sharp, but forms a ‘ramp’ in the concentration profile that has a height of many grain diameters. Early ramped profiles of this type were obtained by Shook and Daniel [3], and more recently one of the present authors [2] carried out a number of experimental runs with sands and with bakelite ($S_s = 1.53$). The concentration profile for one of the sand runs is shown on Fig. 2. This run was performed using 0.56 mm silica ($S_s = 2.65$) and water at 14.9°C in a 10-cm pipe. The mean flow velocity was 1.5 m/s. The concentration profile was obtained using a Caesium-137 γ-ray density profiler. The profiler's source and detector were attached to a rigid steel hoisting structure equipped with an electronic scale and elevated by a hydraulic cylinder. For each relative height indicated on Fig. 2, a chord-averaged density was determined from a count of the impacts on the detector. On this figure the stationary deposit in the lower portion of the pipe corresponds to the loose-poured particle concentration of about 0.65 (by volume). Above that is the interfacial ramp, approximating a linear decrease of concentration with height. In the top portion of the flow the concentration is rather small, and the profile is curved, as would be expected for particles supported by turbulent eddies. Fig. 2 indicates how a measure of the height of the ramp (also known as the shear-layer thickness δ_s) can be obtained from the near-linear portion of the concentration profile.

At the bottom of the ramp, the particles are only barely moving, and, as noted by Bagnold [1], there will be a Coulombic proportionality between the shear stress τ_s and the submerged weight of the overlying contact-load solids: $\tau_s \propto \rho_f\, g(S_s - 1)\delta_s$, where δ_s is the height of the shear layer. It follows that the height of the shear layer will be proportional to the shear stress. It has been proposed earlier [6,7], that the velocity difference across the shear layer U_δ should be proportional to the shear velocity U^*_{bed} (i.e., the square root of interfacial shear stress divided by fluid density). The experiments mentioned above [2] involved simultaneous measurements of pressure gradients and both concentration and velocity profiles. Thus, both U^*_{bed} and U_δ could be determined. The ratios of these quantities are shown on Fig. 3. It can be seen that U_δ/U^*_{bed}

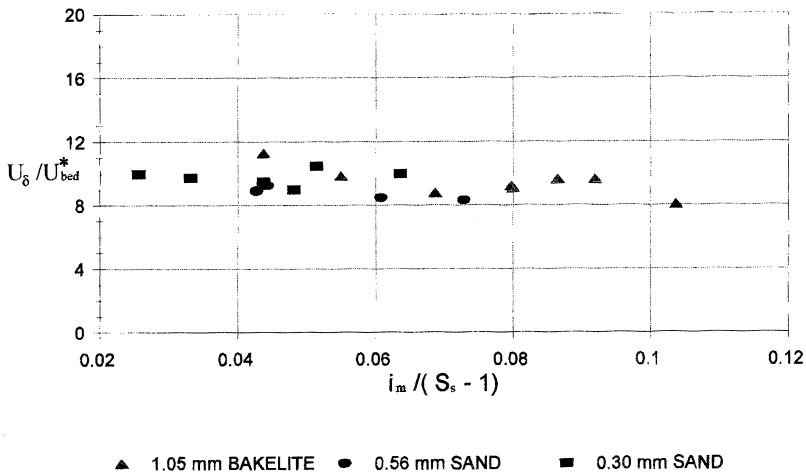

Fig. 3. Velocity difference ratio across shear layer, data from Ref. [2].

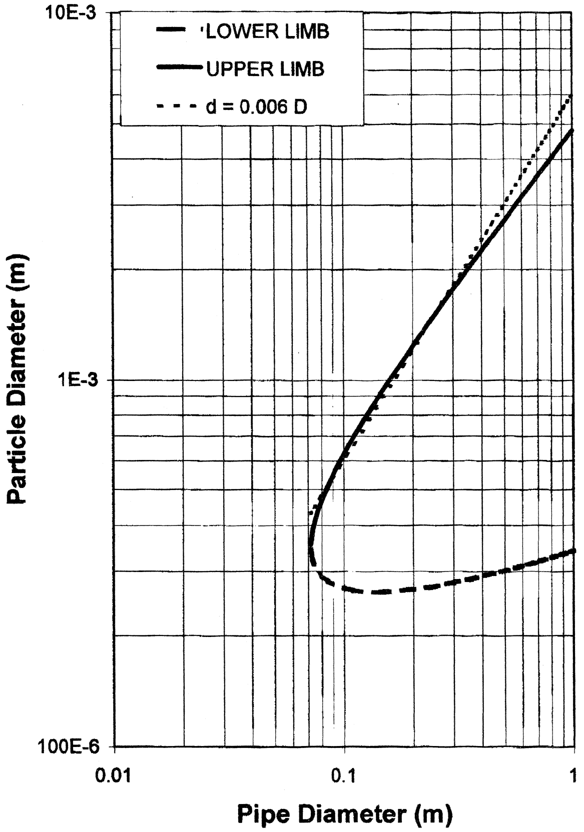

Fig. 4. Deposition locus.

is essentially constant (about nine) for all three solids investigated, across a broad range of values of the abscissa.

This experimental finding verifies the earlier proposal that the equivalent roughness of this type of interface is proportional to the interfacial shear stress, implying that the interfacial friction factor depends on the ratio $i_m/(S_s - 1)$, and not on the particle size. Computer modelling on this basis [8] had already indicated that the resulting limit of deposition could be approximated by the equation:

$$V_{sm}/\left[2gD(S_s-1)\right]^{0.5} = \left[0.018/f_w\right]^{0.13} \quad (1)$$

where f_w is the Darcy–Weisbach friction factor $(2gDi/V^2)$ for water flowing in the pipe. If the value of $(V_{sm})_{max}$ determined from Eq. (1) is less than that found from the nomographic chart, then the value from Eq. (1) should be used. The locus where the two methods give the same value of the deposition-limit velocity is displayed on Fig. 4, which has as its axes the pipe and particle diameters. The lower limb of the locus, with particle diameter generally less than 0.3 mm, corresponds to the equivalence of Eq. (1) to the right-hand portion of the demi-McDonald, and similarly for the upper limb of the locus and the left portion of the demi-McDonald, with larger particle diameters.

As shown on Fig. 4, the upper limb of the locus can be approximated by the simple equation:

$$d = 0.006D \quad (2)$$

In words, if the particle diameter d exceeds $0.006D$, the nomographic chart should be used to obtain V_{sm}, but if d is less than $0.006D$ (but greater than 0.3 mm) then Eq. (1) should be used.

3. Flows with stationary deposits

Although pipelines operating with stationary deposits of solids are generally avoided by design engineers, it sometimes happens that such operations are found in practice. The common opinion is that stationary deposits lead inexorably to instabilities and ultimate line blockage, but it certainly appears that some lines do operate with a deposit. An example of how this situation can arise is given in Chapter 13 of Ref. [8]. For conditions where Eq. (1) is applicable, the pressure gradient i_m for flows with stationary beds can be modelled in terms of three parameters. One is the gradient ratio $i_m/(S_s - 1)$ (which was mentioned in the paragraph before Eq. (1), and forms the abscissa of Fig. 3). The second, denoted F_D, is the Durand velocity parameter $V_m/[2gD(S_s - 1)]^{0.5}$ which is a generalisation of the left-hand side of Eq. (1), using the throughput velocity V_m (i.e., $4Q/\pi D^2$) instead of the deposition velocity. The third parameter is the delivered volumetric concentration of solids, C_{vd}.

In presenting the results of the computations it is convenient to use the Durand velocity parameter as the abscissa and the gradient ratio as the ordinate, plotting the results for various values of the delivered solids concentration. A representative plot is given on Fig. 5, showing that C_{vd} has a considerable effect on the gradient ratio, whereas the Durand velocity parameter has a relatively small influence.

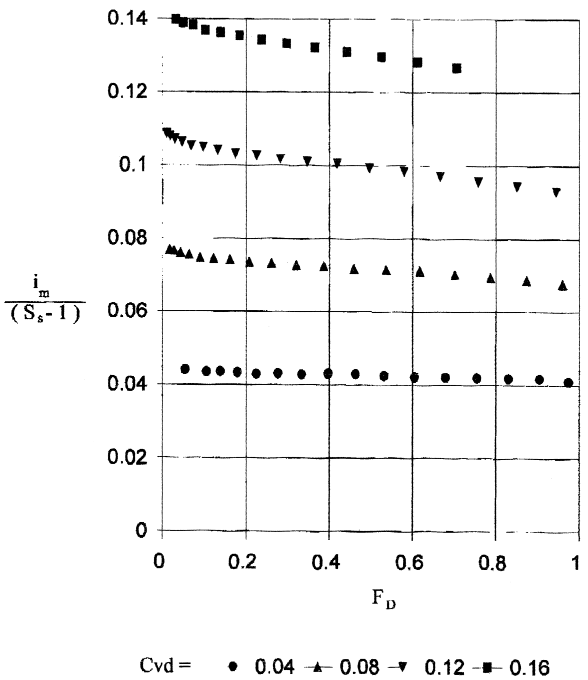

Fig. 5. Modelled results for pressure–gradient ratio.

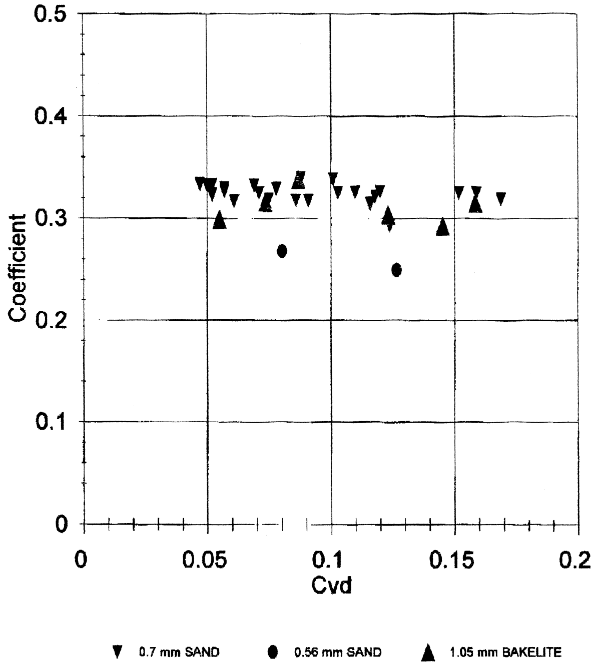

Fig. 6. Pressure–gradient coefficient for Eq. (4).

It was found that a reasonable approximation to the modelled results could be obtained by a power-law approximation of the type:

$$i_m/(S_s - 1) = a(C_{vd})^b(F_D)^c \quad (3)$$

where a is a coefficient and b and c are exponents. These could be estimated from the modelled results, but the use of experimental data was considered preferable. The solids employed were sands of median diameter 0.56 mm and 0.70 mm and bakelite of 1.05 mm. The range of Durand velocity parameter was 0.4 to 1.0, and that of C_{vd} from 0.05 to 0.16 [2]. The fit values of the exponents b and c were 0.6 and -0.1, respectively. Values for the coefficient, a, were then obtained for each data point by back-calculation. These are plotted on Fig. 6. The mean value of a is found to be 0.32, with a rather small deviation (6% on a root-mean-square basis). Substitution into Eq. (3) gives the following expression for pressure gradient (m of water/m of pipe):

$$i_m = 0.32(S_s - 1)^{1.05}(C_{vd})^{0.6}\left(V_m/[2gD]^{0.5}\right)^{-0.1}. \quad (4)$$

Eq. (4) is considered to give appropriate results for pipeline flows with a stable stationary deposit. However, deposition is often associated with instabilities; as noted in Ref. [8] it is recommended that pilot-plant testing be conducted before implementing any proposed design involving stationary-bed operation.

4. Conclusion

Many slurry flows exhibit stratified behaviour, with a lower particle-rich zone and an upper particle-lean one. The two-layer model of this configuration has shown the importance of determining the friction at the interface between the two zones. In the past it was believed that the interface had an equivalent roughness proportional to particle size, but it is now known that the equivalent roughness varies with the thickness of the interfacial ramp or shear layer, and hence with the interfacial shear stress itself.

Experimental study of the interface is facilitated if the particles below it form a stationary deposit. The analysis of the interface, confirmed by the experimental findings, has produced new mathematical models, which in turn have now been approximated by simple fit equations. These give methods for estimating the deposition-limit velocity for the shear-layer configuration, together with its range of applicability, and for calculating the pressure gradient for cases where stable stationary deposits occur.

5. List of symbols

a	coefficient in Eq. (3)
b	exponent in Eq. (3)
C_{vd}	delivered volumetric concentration of solids
c	exponent in Eq. (3)
D	internal pipe diameter
d	median particle diameter
F_D	Durand velocity parameter
f_w	Darcy–Weisbach friction factor for flow of water
g	gravitational acceleration
i_m	pressure gradient, expressed in m water/m pipe
Q	volumetric flow rate
S_s	specific gravity of solids
U^*_{bed}	interfacial shear velocity
U_δ	velocity difference across shear layer
V_m	mean flow velocity, $4Q/\pi D^2$
V_{sm}	deposition-limit value of V_m
Y	height from bottom of pipe
δ_s	thickness of shear layer
μ_s	mechanical friction coefficient

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