ELSEVIER European Journal of Operational Research 113 (1999) 271–280

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

Solving multiple criteria choice problems by interactive trichotomy segmentation

A. Jaszkiewicz [a,*], A.B. Ferhat [b]

[a] *Institute of Computer Science, Poznan University of Technology, Piotrowo 3A, 60-965 Poznan, Poland*
[b] *Université de Paris Dauphine, place du Maréchal de Lattre de Tassigny 75775, Paris Cedex 16, France*

Abstract

The paper describes a method for multiple criteria choice problems with an explicitly given but relatively large set of alternatives. The approach can be placed between methods with a priori specification of the decision maker's (DM's) preferences and interactive procedures. The method presented includes some interaction with the DM, but is based on an assumption that, at the beginning of the solution process, he/she has already relatively well-defined preferences. The DM's preferences are modeled with an outranking relation in the neighborhood of a reference profile in the non-dominated set. This preference model is used in order to define a trichotomy of the non-dominated set in terms of accepting good alternatives, rejecting non-interesting ones and defining alternatives that can neither be accepted, nor rejected based on the available preference information. The method stops if the class of good alternatives is sufficiently small and if the DM is able to select one of them as the best compromise. Otherwise, the DM can modify the preferential information by changing either the reference profile, or the thresholds used to build the outranking relation. In other words, the DM defines dynamically the three categories. The description of the procedure is followed by presentation of an application.

Keywords: Multiple criteria choice; Aspiration levels; Outranking relation; Trichotomy segmentation

1. Introduction

Specifying desired values (aspiration levels) on particular criteria composing a *reference profile* in the criteria space and searching for alternatives similar or better than the reference profile is one of the most natural ways of solving multiple criteria choice problems (see e.g. Charnes and Cooper, 1961, 1977; Lewandowski and Wierzbicki, 1988). Of course, unless there exists an alternative giving the aspiration levels on all objectives, the knowledge of the reference profile is not enough to characterize a single alternative being the most interesting for the decision maker (DM).

A number of interactive procedures use reference profile as a parameter of a scalarizing function which optimized on the set of feasible alternatives gives a single non-dominated alternative. To this class of methods belong Reference Point method (Wierzbicki, 1980), Max–Min method (Posner and Wu, 1981) and Pareto Race

* Corresponding author.

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(Korhonen and Wallenius, 1988). The scalarizing functions include, however, some other parameters, e.g., the direction of projection, that are set automatically by the method. By changing the parameters one can obtain different alternatives for the same reference profile. So, the choice of a single alternative presented to the DM in a given iteration is rather arbitrary. In order to find the best compromise alternative the DM may be forced to change the reference profile even if his/her preferences are stable. In this case, the reference profile starts to play a technical role. It becomes a tool that allows the DM to control the search for the best compromise. The methods, seems to be most appropriate when the DM is interested in learning about the problem by free scanning of the non-dominated set (cf. Vanderpooten, 1989).

In the method presented, the reference profile is used in order to characterize a subset of alternatives being not worse than the reference profile – category C^+. The remaining alternatives are classified as either worse or incomparable to the reference profile – categories C^- and $C^?$, respectively. We propose to use an outranking relation, based on the additional preference information supplied by the DM, in order to compare non-dominated points to the reference profile z^{ref}. The outranking relation allows to take into account meaninglessness of small differences on criteria for the DM, as well, difficulties in comparing points having significantly different scores on particular objectives. So, the procedure allows the DM to concentrate on the most interesting region of the non-dominated set but does not select arbitrary a single solution.

In the decision phase, the DM can browse alternatives belonging to category C^+ and stop the procedure when he/she is able to find an alternative giving the best compromise. Otherwise he/she may interactively modify the reference profile as well as the parameters of the outranking relation.

The outranking relation has also been used as a preference model in the Cone Contraction interactive procedure proposed by Jaszkiewicz and Slowinski (1992), in Light Beam Search (LBS) interactive procedure proposed by Jaszkiewicz and Slowinski (1995), and in the AIM method proposed by Lotfi et al. (1992). In all these methods, however, the outranking relation is used to model DM's preferences with respect to some non-dominated points, while in our approach, it models preferences with respect to the reference profile. Furthermore, both LBS and AIM support free scanning of the non-dominated set and the outranking relation is used to extend the information presented to the DM at each iteration.

Trichotomic segmentation approach has already been used by Moscorola and Roy (1977) and Yu (1992) (see also Roy and Bouyssou, 1993). Their methods, however, concern the multiple criteria sorting problems, and furthermore, are based on a priori specification of DM's preferences, while the proposed method assumes interaction.

The paper is organized in the following way: after a formal statement of the problem and basic definitions, the main idea and the general scheme of the procedure will be outlined. Some characteristics of the procedure will be described more precisely in the Section 5. In Section 6, an application of the interactive procedure will be described. Finally, main features of the procedure will be summarized.

2. Problem statement and basic definitions

The general Multiple Criteria Choice (MCC) problem is formulated as

$$\begin{array}{ll} \max & \{f_1(a) = z_1, \ldots, f_J(a) = z_J\}, \\ \text{s.t.} & \qquad a \in A, \end{array} \tag{P1}$$

where A is a finite set of feasible alternatives, $f_1, \ldots, f_J$ are *criterion functions* (*objectives*). It is also assumed that set A is relatively large, i.e., its review needs a special guidance characteristic for interactive procedures. So, problem P1 belongs to the multiple-criteria choice problems with large sets of alternatives (MCCL) (see Jaszkiewicz and Slowinski, 1997a). The problem P1 can be the original decision problem, e.g., the problem of selecting the best candidate for a position from a large set of applicants. The set A can also be obtained by generating a relatively large number of solutions of a multiple objective mathematical programming problem, e.g., A may be obtained by generating a large number of schedules of a multiple objective scheduling problem.

An image of an alternative a in the criterion space is a *point* $\mathbf{z}^a = [z_1^a, \ldots, z_J^a]$, such that $z_j^a = f_j(a)$, $j = 1, \ldots, J$. An image of a set A in the criterion space is a set Z composed of points being images of feasible alternatives.

Problem (P1) can also be formulated more succinctly as

$$\begin{aligned} \max \quad & \{\mathbf{z}\} \\ \text{s.t.} \quad & \mathbf{z} \in Z. \end{aligned} \tag{P2}$$

Point $z' \in Z$ is *non-dominated* if and only if there is no $z \in Z$ such that $z_j \geqslant z'_j, \forall j$, and $z_i > z'_i$ for at least one i. Point $z' \in Z$ is *weakly non-dominated* if there is no $z \in Z$ *such that* $z_j > z'_j, \forall j$. The set composed of all non-dominated points constitutes the *non-dominated set* denoted by N.

The point z^* composed of the best attainable criteria function values is called the *ideal point*

$$z_j^* = \max\{f_j(a),\ a \in A,\ j = 1, \ldots, J\}.$$

The point z_* composed of the worst criteria function values in the non-dominated set is called the *nadir point*.

3. Main idea of the method

At each iteration the method assigns the non-dominated points to one of three mutually exclusive categories C^+, C^- and $C^?$. Category C^+ is composed of points not worse than the reference profile z^{ref}, C^- of points worse than z^{ref}, and $C^?$ of points incomparable to z^{ref}. Clearly, to category C^+ belong each non-dominated point z^a such that, $\forall j$, $z_j^a \geqslant z_j^{\text{ref}}$. In practice, however, small differences on a criterion are insignificant to the DM and he/she considers two points indifferent with respect to this criterion. The existence of an indifference region can be caused by an inability of the DM to perceive small differences on a criterion, as well as by imprecision and uncertainty of measurement, and/or of the mathematical model. This phenomenon can be modeled with the *indifference threshold* q_j which is, in general, a function of z_j. So, the assignment of a given non-dominated point z^a to the acceptance category C^+ remains justified if the inequalities $z_j^a \geqslant z_j^{\text{ref}}$ are slightly contradicted for some of the J criteria.

Furthermore, experience indicates that, usually, there is no precise limit between indifference and strict preference, but there exists an intermediary region where the DM hesitates between both while comparing two alternatives a and b. This corresponds to the situation of *weak preference* between the two alternatives. It can be modeled by the *preference threshold* p_j which is, in general, a function of z_j, such that

$$p_j(z_j) \geqslant q_j(z_j) \geqslant 0.$$

Criterion z_j involving indifference and preference thresholds is called a *pseudo-criterion*. If z_j is to be maximized, then:

$a\ I_j\ b$, i.e., a and b are *indifferent* with respect to $z_j \iff -q_j(z_j^a) \leqslant z_j^a - z_j^b \leqslant q_j(z_j^b)$,

$a\ Q_j\ b$, i.e., a is *weakly preferred to* b with respect to $z_j \iff q_j(z_j^b) \leqslant z_j^a - z_j^b \leqslant p_j(z_j^b)$,

$a\ P_j\ b$, i.e., a is *strictly preferred to* b with respect to $z_j \iff p_j(z_j^b) \leqslant z_j^a - z_j^b$.

From the other side, often even significant gains on some criteria, may not compensate losses on other criteria. The *veto threshold* v_j allows to take into account the possible difficulties of comparing two alternatives when one is significantly better than the other on a subset of criteria, but much worse on at least one other criterion. In general, the veto threshold is also a function of z_j. Alternative a cannot be considered not worse than b if there exists a criterion j such that $z_j^b \geqslant z_j^a + v_j(z_j^a)$ regardless of the other criteria. If neither a can be considered not worse than b nor b can be considered not worse than a, the two alternatives are qualified as incomparable.

So, the preference model used to compare a non-dominated point z^a with the reference profile z^{ref} should take into account the four following situations: indifference I, weak preference Q, strict preference P and incomparability? For this purpose, the concept of an outranking binary relation S (Roy, 1985) is used. The outranking relation is defined as a grouped relation of I, Q and P. In other words, $a\ S\ b$ means that a is not worse than b. Using the outranking relation, one can express all preference situations with respect to two alternatives a and b:

$a\ S\ b$ and not $(b\ S\ a) \Rightarrow a\ P\ b$ or $a\ Q\ b$,

$a\ S\ b$ and $b\ S\ a \Rightarrow a\ I\ b$,

not $(a\ S\ b)$ and not $(b\ S\ a) \Rightarrow a\ ?\ b$.

Construction of the outranking relation S is essentially based on two concepts called *concordance* and *discordance* tests. The goals of these tests are, respectively, to:

- characterize a group of criteria considered to be in concordance with the affirmation $a\ S\ b$ and assess the relative importance of this criteria group compared to the remainder criteria, and
- characterize, among the criteria which are not in concordance with the affirmation being studied, the ones whose opposition is strong enough to reduce the credibility of $a\ S\ b$, which would result from taking into account just concordance.

In the proposed method, the preference model is used to assign the non-dominated points to one of three categories:

z^a is assigned to $C^+ \iff \mathbf{z}^a\ P\ \mathbf{z}^{\text{ref}}$ or $\mathbf{z}^a\ Q\ \mathbf{z}^{\text{ref}}$ or $\mathbf{z}^a\ I\ \mathbf{z}^{\text{ref}}$, i.e., $\mathbf{z}^a\ S\ \mathbf{z}^{\text{ref}}$,

z^a is assigned to $C^- \iff \mathbf{z}^{\text{ref}}\ P\ \mathbf{z}^a$ or $\mathbf{z}^{\text{ref}}\ Q\ \mathbf{z}^a$, i.e., $\mathbf{z}^{\text{ref}}\ S\ \mathbf{z}^a$ and not $(\mathbf{z}^a\ S\ \mathbf{z}^{\text{ref}})$,

z^a is assigned to $C^? \iff \mathbf{z}^a ? \mathbf{z}^{\text{ref}}$, i.e., not $(\mathbf{z}^a\ S\ \mathbf{z}^{\text{ref}})$ and not $(\mathbf{z}^{\text{ref}}\ S\ \mathbf{z}^a)$.

The categories are mutually exclusive and $A = C^+ \cup C^- \cup C^?$. Let us note that in the proposed method, the outranking relation models the local preferences of the DM with respect to the reference profile. Thus, a single value of each threshold is needed for a given reference profile. Specifying such values is much easier for the DM than explicitly defining functional thresholds $p_j(z_j), q_j(z_j)$, and $v_j(z_j)$, which have to be defined if the outranking relation is used as a global preference model (cf. Roy and Bouyssou, 1993).

4. General scheme of the interactive procedure

The following is a general scheme of the proposed procedure presented in a Pascal like form:

Find the ideal z^*, and nadir z_* points; present them to the DM.
Ask the DM to specify a reference profile z^{ref}.
Repeat
  Get the (updated) preferential information (the indifference and optionally the preference and/or veto thresholds).
  Calculate the three categories (C^+, C^- and $C^?$).
If $\{C^+ \neq \emptyset\}$ Then
    Present to the DM points belonging to this category.
  EndIf
If $\{C^? \neq \emptyset\}$ Then
    Present (optionally) to the DM points incomparable to the reference profile which are contained in category $C^?$, with possibility of presenting only a given number of representative points.
EndIf
If $\{C^- \neq \emptyset\}$ Then
    Present (optionally) to the DM points belonging to this category.
EndIf
If the DM wants to store the reference profile Then
    Add it to the set of "stored points".
If the DM cannot select the final solution from C^+ Then he/she can either:
  ∘ attempt to reduce the number of points belonging to C^+ by defining an *optimistic reference profile*, i.e., specifying a new profile dominating the current one,
  ∘ attempt to increase the number of points belonging to C^+ by defining a *pessimistic reference profile*, i.e., specifying a new profile dominated by the current one,
  ∘ define a new reference profile non-dominated with respect to the current one in order to scan a new non-dominated region,
  ∘ update the preferential information used to build the outranking relation,
  ∘ return to one of the stored points and consider it as a reference profile.
EndIf
Until The DM' feels satisfied with a point found during the interactive process or decides that there is non acceptable solution (Fig. 1).

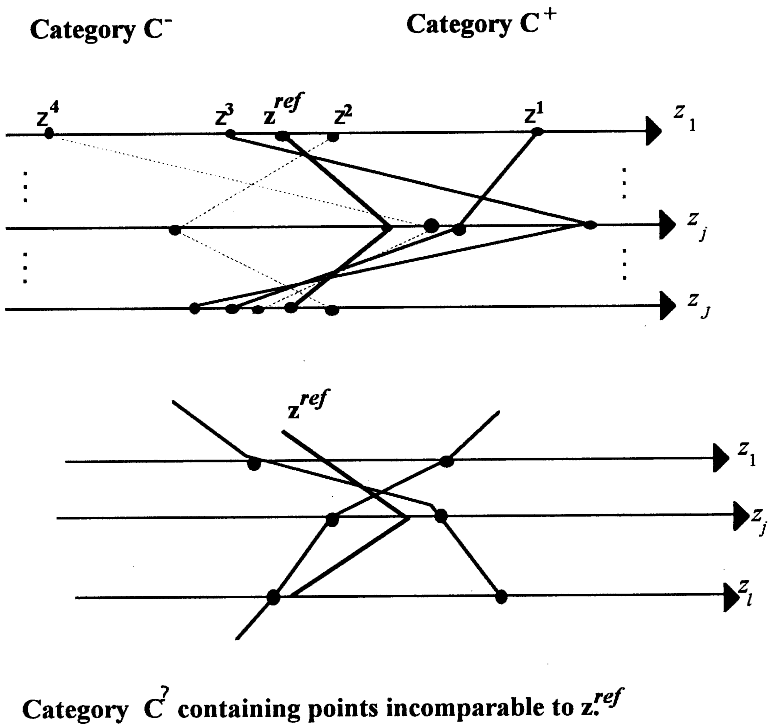

Fig. 1. Trichotomic segmentation of a finite sample A of alternatives.

5. Detailed description of particular steps

The method starts by filtering the non-dominated points from the set A. Then the ideal z^* and nadir z_* points are found and presented to the DM. In the first step, the starting reference profile (reference point) z^{ref} is specified by the DM. Then, the DM is asked to give the preferential information for each criterion z_j (i.e., the indifference and, optionally, the preference and veto thresholds). At this stage, the DM should decide if he/she wants to specify the preference and/or veto thresholds, however, he/she is allowed to change these settings at each step of the procedure.

The definition of the outranking relation used in this method is based on that proposed in the LBS-Discrete method (Jaszkiewicz and Slowinski, 1997b). This definition does not use criteria weights, but assumes that none of the criteria is more important than all the others together. In order to test if a point z^a outranks a reference point z^b, the following numbers are used:

$m_Q(\mathbf{z}^a, \mathbf{z}^b)$: the number of criteria for which point z^a is weakly preferred to z^b,
$m_P(\mathbf{z}^a, \mathbf{z}^b)$: the number of criteria for which point z^a is strictly preferred to z^b,
$m_V(\mathbf{z}^a, \mathbf{z}^b)$: the number of criteria being in a strong opposition to the assertion $z^b\ S\ z^a$,

i.e., card $\{j{:}\, \mathbf{z}^a_j \geqslant z^b_j + v_j \quad (j = 1\ldots, J)\}$.

The construction of the outranking relation depends on the type of preferential information supplied by the DM. If he/she has specified all the thresholds, the following definition of the outranking relation is used:

$$\mathbf{z}^a S^I \mathbf{z}^b \iff \begin{cases} m_V(\mathbf{z}^b, \mathbf{z}^a) = 0 \text{ and} \\ m_P(\mathbf{z}^b, \mathbf{z}^a) \leqslant 1 \text{ and} \\ m_Q(\mathbf{z}^b, \mathbf{z}^a) + m_P(\mathbf{z}^b, \mathbf{z}^a) \\ \leqslant m_Q(\mathbf{z}^a, \mathbf{z}^b) + m_P(\mathbf{z}^a, \mathbf{z}^b) \\ \quad + m_v(\mathbf{z}^a, \mathbf{z}^b). \end{cases}$$

If the DM has decided not to specify the veto thresholds, the following definition of the outranking relation is used:

$$\mathbf{z}^a\ S^{II}\ \mathbf{z}^b \iff \begin{cases} m_P(\mathbf{z}^b,\mathbf{z}^a) = 0 \text{ and} \\ m_Q(\mathbf{z}^b,\mathbf{z}^a) \leqslant m_Q(\mathbf{z}^a,\mathbf{z}^b) \\ \quad + m_P(\mathbf{z}^a,\mathbf{z}^b). \end{cases}$$

If the DM has decided not to specify the preference threshold, the following definition of the outranking relation is used:

$$\mathbf{z}^a\ S^{III}\ \mathbf{z}^b \iff \begin{cases} m_V(\mathbf{z}^b,\mathbf{z}^a) = 0 \text{ and} \\ m_P(\mathbf{z}^b,\mathbf{z}^a) \leqslant 1 \text{ and} \\ m_P(\mathbf{z}^b,\mathbf{z}^a) \leqslant m_P(\mathbf{z}^a,\mathbf{z}^b) \\ \quad +m_v(\mathbf{z}^a,\mathbf{z}^b). \end{cases}$$

Finally, if the DM has decided to specify only the indifference thresholds, the following definition of the outranking relation is proposed:

$$\mathbf{z}^a\ S^{IV}\ \mathbf{z}^b \iff m_P(\mathbf{z}^b,\mathbf{z}^a) = 0.$$

Observe that, for a given set of values $q_j, p_j, v_j (j = 1,\ldots,J)$ the following inclusions hold: $S^{IV} \subseteq S^{II} \subseteq S^I$ and $S^{IV} \subseteq S^{III} \subseteq S^I$. Moreover, $S^{II} = S^I$ if $p_j = v_j$, $\forall j$, $S^{III} = S^I$ if $q_j = p_j$, $\forall j$, $S^{IV} = S^I$ if $q_j = p_j = v_j$, $\forall j$.

In the computation phase of the procedure, three categories C^+, C^- and $C^?$ delimited by an outranking profile given by the DM are calculated. Each point belonging to the set N is compared to the reference profile $\mathrm{z}^{\mathrm{ref}}$, the ones which outrank $\mathrm{z}^{\mathrm{ref}}$ are classified in the acceptance category C^+, corresponding to the sub-region containing points which are not worse than the profile, i.e., outrank the profile. The ones which are outranked by $\mathrm{z}^{\mathrm{ref}}$ and do not outrank $\mathrm{z}^{\mathrm{ref}}$ are classified in the rejection category C^-, and the ones which are incomparable to $\mathrm{z}^{\mathrm{ref}}$ are classified in the incomparability category $C^?$. The three categories C^+, C^- and $C^?$ are calculated in the following way:

For each non dominated point z^a belonging to the non-dominated set N:
If $\mathrm{z}^a\ S\ \mathrm{z}^{\mathrm{ref}}$ then $\mathrm{z}^a \in C^+$
else
If $(\mathrm{z}^{\mathrm{ref}} S \mathrm{z}^a)$ then $\mathrm{z}^a \in C^-$
else $\mathrm{z}^a \in C^?$.

In the decision phase, if the acceptance category C^+ is not empty, then it is presented to the DM. If he/she thinks that the number of points belonging to this category is too large, it is possible to present to him/her the so called *characteristic points* of the acceptance category and the *locally ideal* point $\bar{\mathbf{z}}^*$. The characteristic points are points $\mathbf{z}^j$, $j = 1,\ldots,J$, that maximize particular criteria within this category. The locally ideal point $\bar{\mathbf{z}}^*$ is defined from the characteristic points: $\bar{z}^*_j = z^j_j, j = 1,\ldots,J$.

The DM can also see the points belonging to incomparable category $C^?$. Presentation of incomparable points can help the DM in defining a new reference profile non-dominated with respect to the previous one. In the case where the number of points in $C^?$ is too large, the DM may specify the number of representative points that he/she would like to see. Selecting a given number of the most representative points consists in solving the maximal dispersion problem (cf. White, 1991). Because finding the optimal solution of this problem is prohibitively time consuming, it is advised to use one of the heuristic filtering techniques (Steuer, 1986, Ch. 11; Törn, 1980). Optionally, the DM can see the rejection category C^-.

If the acceptance category C^+ is not empty but contains too many points, the DM has the possibility to define interactively an *optimistic reference profile* by increasing successively or simultaneously the performances of the reference profile $\mathrm{z}^{\mathrm{ref}}$. Otherwise, if the category C^+ is empty or contains too few points, the DM has, as above, the possibility to define interactively a pessimistic reference profile by decreasing successively or simultaneously the performances of the reference profile $\mathrm{z}^{\mathrm{ref}}$. Moreover, he/she can scan another sub-region of the non-dominated set by defining another reference profile non-dominated with respect to the current one. The procedure stops if one of the points presented is satisfactory for the DM on all criteria.

Before continuing the scanning, the DM can store the reference profile. He/she is allowed to go back to one of the stored points at any time. Finally, the DM is able to modify the preferential information given for each criterion, i.e., the values of indifference, preference and veto thresholds.

He/she can also change the type of the outranking relation.

6. Application

In order to illustrate the work with the procedure, a nurse scheduling problem is used. The problem is described in Jaszkiewicz and Lewandowska (1996) and Jaszkiewicz (1997). Because of limited space a brief description only is given here. The data used come from the surgery unit of the hospital Przemienienia Panskiego in Poznan. The precise description of the problem is available from the authors upon request.

The problem consists in finding a feasible schedule, i.e. in selecting a working shift (morning, afternoon or night) or a day-off for each nurse and each day in the scheduling period such that all the constraints are satisfied. The scheduling period is one month and 15 nurses are scheduled. The following goals are taken into account:

- Grouping of working days in which a nurse works on the same shift. In order to achieve this goal the average number of working days pairs in which a nurse works on different shifts (C_1) is minimized.
- Grouping of working days. This goal is achieved by minimizing the average deviation of the length of each sequence of working days form a desired length (C_2).
- Grouping of free days. In order to achieve this goal the average length of each sequence of free days (C_3) is maximized.
- Uniform distribution of types of working shifts over the personnel. This goal is achieved by minimizing the average deviation of the number of each type of the working shifts from the numbers of shifts resulting from totally uniform distribution (C_4).
- Uniform distribution of nurses on morning shifts. In order to achieve this goal the average deviation of the number of person during each morning shift from such a number resulting from totally uniform distribution is minimized (C_5).

All the above criteria are averaged over the set of all nurses.

The problem defined above belongs to the class of multiple objective combinatorial optimization problems. In order to solve it a two stages procedure is proposed. In the first stage a sample of potentially efficient solutions approximating the whole set of efficient solutions is generated by the Pareto-Simulated Annealing metaheuristic procedure (Czyzak and Jaszkiewicz, 1998). In result of the first stage 544 different solutions were obtained. In the second stage the trichotomy based procedure was used to support the decision maker in selecting the best compromise schedule. An exemplary interactive session is presented below. The ideal z^* point is as follows:

$$C_1 = 0.93, C_2 = 6.28, C_3 = 2.20, C_4 = 1.74,$$
$$C_5 = 1.710,$$

while the nadir z_* is:

$$C_1 = 6.50, C_2 = 18.43, C_3 = 1.29, C_4 = 10.55,$$
$$C_5 = 2.097.$$

The above two points are presented to the DM, then he/she specifies the following reference profile z^{ref} and thresholds q_j, p_j, v_j:

$$C_1 = 2.5, C_2 = 9.5, C_3 = 2, C_4 = 4, C_5 = 1.8.$$

Criterion	C_1	C_2	C_3	C_4	C_5
Indifference q_j	0.2	0.2	0.1	0.5	0.1
Preference p_j	0.5	0.5	0.2	1	0.2
Veto v_j	1.5	1.5	0.5	3	0.5

In the computation phase, the three categories $C^+, C^-, C^?$ are obtained. Number of points in each category is given below:

Category	C^+	C^-	$C^?$
Number of points	0	430	114

None of the solutions can be considered not worse than z^{ref} because the aspiration levels specified by the DM are too demanding and the preferential thresholds too strict. So, one of options that DM may consider is to define a more pessimistic reference profile by giving on all or some objectives worse values of the aspiration levels. The DM is, however, interested in seeing the $C^?$ category. Because the number of points in $C^?$ is too large, he/she decides to see 16 representative points, which are given below:

Criterion	C_1	C_2	C_3	C_4	C_5
z^1	4	6.64	1.87	3.72	1.97
z^2	4.14	6.57	1.95	4.21	1.84
z^3	2	15.36	1.46	3.35	1.84
z^4	5.64	7	2.01	3.22	1.71
z^5	4	7.36	1.94	4.21	1.84
z^6	6.07	8.57	2.15	3.51	1.71
z^7	5	9.5	2.14	2.88	1.84
z^8	1.93	14.93	1.49	2.88	1.97
z^9	6.36	7.93	1.96	2.95	1.71
z^{10}	4.21	6.36	1.93	4.14	1.84
z^{11}	4.36	6.29	1.89	3.26	1.90
z^{12}	4.79	9	2.1	2.88	1.9
z^{13}	3.86	7.36	1.82	4.14	1.84
z^{14}	4.29	6.64	1.87	3.26	1.9
z^{15}	4.79	6.36	1.98	4.01	1.77
z^{16}	4	6.29	1.89	3.9	1.9

It can be observed that only two of the points are better on C_1 than the corresponding aspiration level's value. The same two points are the only ones that are worse of criterion C_2 than the corresponding aspiration level's value. So, the DM decided to be less demanding on C_1 and more demanding on C_2. He/she tries to achieve it by defining a new reference profile.

$$C_1 = 5, C_2 = 8, C_3 = 2, C_4 = 4, C_5 = 1.8.$$

This changes the assignment of points to $C^+, C^-, C^?$ categories. Number of points in each category is given below.

Category	C^+	C^-	$C^?$
Number of points	86	197	261

The DM decides to see a representation of the C^+ category composed of 10 points which are given below:

Criterion	C_1	C_2	C_3	C_4	C_5
z^1	4.86	8.93	2.13	2.88	1.9
z^2	4.21	9.43	2.17	3.93	1.9
z^3	5.21	8.93	2.13	2.97	1.84
z^4	4.93	6.36	1.98	3.58	1.77
z^5	4.14	9.21	2.14	3.41	1.9
z^6	4.36	9.43	2.18	3.41	1.9
z^7	4.29	6.79	2.03	3.79	1.71
z^8	4.29	6.36	1.98	4.14	1.84
z^9	4.43	6.79	2	3.6	1.71
z^{10}	4.07	7.14	2	3.83	1.71

Analyzing the points the DM sees a possibility of finding solutions giving relatively good results on C_1, C_2 and C_4. So, he/she decides to specify an optimistic reference profile by being more demanding of the three criteria and update the preferential thresholds. He/she specifies the following reference profile:

$$C_1 = 4, C_2 = 7, C_3 = 2, C_4 = 3, C_5 = 1.8.$$

The update thresholds are as follows:

Criterion	C_1	C_2	C_3	C_4	C_5
Indifference q_j	0.2	0.2	0.1	0.5	0.1
Preference p_j	0.4	0.4	0.2	0.7	0.2
Veto v_j	1	1	0.5	1.5	0.5

Observe, that the DM has decreased values of preference and veto thresholds on C_1, C_2 and C_4. This means that at this moment smaller differences of C_1, C_2 and C_4 result in strict preference, and he/she smaller worsening of values of these criteria results in rejection of a given alternative. Number of points in each category is given below:

Category	C^+	C^-	$C^?$
Number of points	4	354	186

The following points belong to the C^+ category:

Criterion	C_1	C_2	C_3	C_4	C_5
z^1	4.14	6.79	2.01	3.83	1.71
z^2	4.79	6.79	1.97	3.46	1.71

z^3	4.86	6.79	1.97	3.34	1.71
z^4	4.14	6.57	1.95	4.21	1.84

Finally, the DM feels satisfied with point z^1. So, the best compromise gives the following values of objectives:

$$C_1 = 4.14, C_2 = 6.79, C_3 = 2.01, C_4 = 3.83, C_5 = 1.71.$$

7. Conclusion

A procedure for solving the multiple-criteria choice problems with large sets of alternatives (MCCL) is proposed. It models the preferences of the DM with respect to a reference profile by using outranking relation. The preference model is used in order to distinguish the class C^+ of alternatives that can be considered not worse than the reference profile, the class C^- of alternatives being worse than the reference profile and class $C^?$ of alternatives incomparable to the reference profile. During the interaction, the procedure helps the DM in concentrating on the most interesting region of the non-dominated set and in reducing the size of the C^+ class such the he/she can select the best alternative from this class.

In the decision phase, many non-dominated points may be presented to the DM. Both numerical and graphical forms of presentation with statistical data (like the percent of non-dominated points belonging to each category) should be used to help the DM in evaluating large amounts of information.

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