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Radiative corrections to the quark–gluon-Reggeized quark vertex in QCD ☆

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Abstract

This paper is devoted to the calculation of quark–gluon-Reggeized quark effective vertex in perturbative QCD in the next-to-leading order. The case of QCD with massive quarks is considered. This vertex has a number of applications, in particular, the result can be used for determination of the next-to-leading correction to the massive Reggeized quark trajectory.

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1. Introduction

One of important and widely applied properties of QCD is the Reggeization of elementary particles. Contrary to QED, where only the electron Reggeizes [1] while the photon does not [2], in QCD both quarks and gluons are Reggeized [3,4]. The gluon Reggeization is the base of the BFKL approach to the description of so-called high-energy semihard processes in QCD. Such processes have two (hard) well separated energy scales so that one has to sum large energy logarithms in all orders of perturbation theory. Derived

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originally [5] in the Leading Logarithmic Approximation (LLA) the BFKL equation is presently known up to the next-to-leading order accuracy [6–8].

Basing on the quark Reggeization instead of the gluon one, the BFKL-like equation for amplitudes mediated by two (interacting) Reggeized quarks in t-channel was derived in [4] in the LLA. This equation could be obviously useful to study the high-energy behaviour of amplitudes with meson quantum numbers in t-channel. However, the LLA has a big disadvantage related to that no scale dependencies can be fixed there as it has been continuously pointed before (see Refs. [6–8], for example). So, exactly as in the BFKL case, a reliable theoretical description is impossible without knowledge of the structure of the radiative corrections.

To study these radiative corrections, one has to know, in particular, the interactions of Reggeized quarks with elementary particles in the Next-to-leading Logarithmic Approximation (NLA). This paper is devoted to the NLA calculation of quark–gluon-Reggeized quark effective vertex in QCD with massive quarks. Let us note, that quite a lot of information about Reggeized quarks in the NLA is already available in literature. Among this there is the result of Ref. [9] for the same vertex we consider but in the massless QCD case. Our calculation confirms the correctness of the Ref. [9]. The other important results are the two-loop Regge trajectory of the massless Reggeized quark [10] and the quasi-multi-Regge amplitudes with quark exchanges in crossing channels [11].

In order to reach our aim we consider the one-loop amplitude $q\bar{q} \to gg$ of the Fig. 1 with quark quantum numbers and positive signature in the t-channel in the Regge kinematics

$$\begin{aligned} &-u = -(p_A - p_{B'})^2 \approx s = (p_A + p_B)^2 \to \infty, \\ &t = q^2 = (p_A - p_{A'})^2 - \text{fixed}. \end{aligned} \tag{1.1}$$

Due to the quark Reggeization, the above amplitude $\mathcal{A}$ can be written as follows [4]

$$-\mathcal{A} = \overline{\Gamma}_B(q, s_0)(\not{q} - m)^{-1} \frac{1}{2}\left[\left(\frac{-s}{s_0}\right)^{\hat{\omega}(q)} + \left(\frac{-u}{s_0}\right)^{\hat{\omega}(q)}\right] \Gamma_A(q, s_0), \tag{1.2}$$

with $\overline{\Gamma}_B$ and Γ_A being the quark–gluon-Reggeized quark (QGR) effective interaction vertices which we are interested in here. The parameter s_0 is artificial and the amplitude (1.2) does not depend on it. The other notations in the relation above are: m for the quark mass

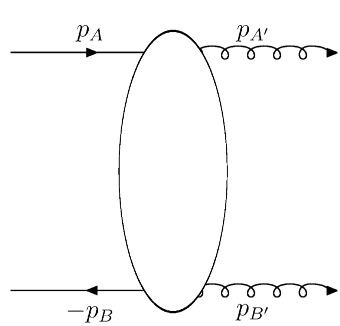

Fig. 1. Schematic representation of $q\bar{q} \to gg$ process.

and $\hat{\omega}(q)$ for the Reggeized quark trajectory. In the LLA it looks

$$\hat{\omega}^{(1)}(q) = \frac{g^2 C_F}{2\pi}(\not{q} - m)\int \frac{d^{D-2}k_\perp}{(2\pi)^{D-2}} \frac{\not{q}_\perp - \not{k}_\perp + m}{k_\perp^2[(q-k)_\perp^2 - m^2]}$$

$$= -g^2 C_F \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{(\not{q}+m)\not{q}}{q^2}\left(m^2 - q^2\right)\int_0^1 \frac{dx}{[(1-x)(m^2 - xq^2)]^{1-\epsilon}}, \quad (1.3)$$

where $\Gamma(z)$ is the Euler gamma-function, C_F is related to the standard notation for the $SU(N)$ colour group Casimir operator in the fundamental representation

$$t^a t^a = C_F I = \frac{N^2-1}{2N} I, \quad (1.4)$$

g is the gauge coupling constant and the integration is carried out over $(D-2)$-dimensional vector orthogonal to the initial particle momenta plane. Throughout all this note we use the dimensional regularization with the space–time dimension $D = 4 + 2\epsilon$.

To calculate the one-loop amplitude $\mathcal{A}$ (1.2) we follow the t-channel unitarity approach developed in Ref. [12] which allows to considerably simplify the calculation. To illustrate the idea of the approach, let us consider the kinematics where $m^2 \ll -t \ll s \to \infty$, which evidently is a part of the Regge kinematical region. There, in order to have a growing with s contribution, one has to have the t-behaviour like $(-s/t)^{1/2}$ since the mass-dependence is not stronger than the logarithmic one. Therefore, no polynomial in t is allowed at large t in the Regge asymptotics of the amplitude. This means that two analytic functions with the same t-channel singularities and physically correct mass-dependence coincide in the Regge kinematics so that only singular in t part of the amplitude is important in this limit. At the one-loop level singularities in t are given by the contributions of two t-channel intermediate states: quark–gluon and quark ones. In the next section we consider the former contribution. The one-quark pole contribution will be calculated in Section 3.

2. The branch-point contribution

The quark–gluon t-channel discontinuity of the amplitude $\mathcal{A}$ is given by an ordinary Cutkosky cut of the contributing diagrams as it is depictured in the Fig. 2. After this cut we come to the consideration of the convolution of two on-mass-shell Born amplitudes related to the upper (A) part of the Fig. 2 and the lower one (B). Because of the on-mass-shellness and since the external gluons are physical there is the invariance of the A and B under gauge transformations of the intermediate gluon's polarization and one is allowed to sum up over this polarization in an arbitrary gauge. We choose the Feynman gauge to perform this sum, so that we use $-g_{\mu\nu}$ for the polarization tensor. Then, after this convolution one is allowed to perform the loop integration with the complete propagators instead of the on-mass-shell δ-functions. The amplitude obtained in this way has the same t-channel singularities as the complete one that is enough to restore the correct Regge asymptotics according to the conclusions of the Ref. [12]. So we have to consider

$$\mathcal{A} = \int \frac{d^D p}{i(2\pi)^D} \frac{\sum(-)BA}{(k^2 + i\delta)(p^2 - m^2 + i\delta)}, \quad k = p - q, \quad (2.1)$$

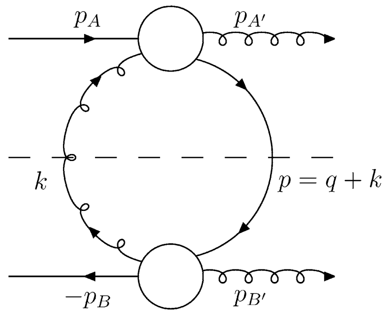

Fig. 2. Schematic representation of the t-channel quark–gluon intermediate state.

where the convolution is performed on-mass-shell and the notations for relevant momenta are given in Fig. 2.

The amplitude A has a form

$$A = -ig^2\bar{u}(p)\Big\{\big(t^c t^{A'}\big)_{iA}\Big[\not{e}(\not{q}-m)^{-1}\not{e}^*_{A'} - \frac{2}{s_1}\big(\not{e}^*_{A'}(ep_{A'}) + \not{e}(e^*_{A'}k) + (\not{q}-m)(ee^*_{A'})\big)\Big] + \big(t^{A'}t^c\big)_{iA}\Big[(\not{q}-m)\Big(\frac{\not{e}^*_{A'}\not{e}}{u_1-m^2} + \frac{2(ee^*_{A'})}{s_1}\Big) + 2\not{e}^*_{A'}\Big(\frac{(ep_A)}{u_1-m^2} + \frac{(ep_{A'})}{s_1}\Big) + 2\not{e}(e^*_{A'}k)\Big(\frac{1}{u_1-m^2} + \frac{1}{s_1}\Big)\Big]\Big\}u_A, \tag{2.2}$$

and we have an analogous expression for the B. We introduce the notations e, c $(u(p), i)$ for the intermediate gluon (quark) spin wave function and colour index, respectively. Other notations for the external particles spin wave functions are evident and for the external gluons we use the light-cone gauges

$$(e_{A'}p_{A'}) = (e_{B'}p_{B'}) = (e_{A'}p_{B'}) = (e_{B'}p_{A'}) = 0, \tag{2.3}$$

which mean, in other words, that the final result in general gauges will be given by the replacements

$$e_{A'} \to e_{A'} - \frac{p_{A'}(e_{A'}p_{B'})}{(p_{A'}p_{B'})}, \qquad e_{B'} \to e_{B'} - \frac{p_{B'}(e_{B'}p_{A'})}{(p_{A'}p_{B'})}. \tag{2.4}$$

We also introduce the intermediate invariants according to as follows

$$u_1 = (k+p_A)^2, \qquad s_1 = (k-p_{A'})^2, \qquad u_2 = (k-p_B)^2, \qquad s_2 = (k+p_{B'})^2. \tag{2.5}$$

Looking at the expression of the Eq. (2.2) for the amplitude A, one can realize that its convolution with the B is simple, but rather long, nevertheless. For this reason we explain simplifications we use on a relatively simple example of the convolution of so-called "asymptotic" parts [12] of the A and B which we choose in the form

$$A_{\text{as}} = -ig^2\big(t^c t^{A'}\big)_{iA}\bar{u}(p)\Big(\not{e} + (\not{q}-m)\Big[\frac{(ep_A)}{u_1-m^2} - \frac{(ep_{A'})}{s_1}\Big]\Big)(\not{q}-m)^{-1}\not{e}^*_{A'}u_A,$$

$$B_{\mathrm{as}} = -ig^2\left(t^{B'}t^c\right)_{Bi}\bar{v}_B \not{e}^*_{B'}(\not{q}-m)^{-1}\left(\not{e}^* - (\not{q}-m)\left[\frac{(e^*p_B)}{u_2-m^2} - \frac{(e^*p_{B'})}{s_2}\right]\right)u(p). \tag{2.6}$$

This terminology "asymptotic" is because the amplitudes A_{as} and B_{as} give the asymptotics of the complete amplitudes A and B in their Regge limits $|u_1| \approx |s_1| \gg |t|$ and $|u_2| \approx |s_2| \gg |t|$, respectively. They are invariant under gauge transformations of the intermediate gluon's polarization as well as the complete amplitudes (let us remind that the prescription (2.4) is supposed in the relations (2.6)).

The convolution of the asymptotic parts has a form

$$\begin{aligned}
\sum(-)B_{\mathrm{as}}A_{\mathrm{as}} = -g^4C_F\left(t^{B'}t^{A'}\right)_{BA}\bar{v}_B\Big\{&\not{e}^*_{B'}(\not{q}-m)^{-1}\gamma^\mu(\not{p}+m)\gamma_\mu(\not{q}-m)^{-1}\not{e}^*_{A'} \\
&+ \not{e}^*_{B'}(\not{q}-m)^{-1}\left[\frac{\not{p}_A}{u_1-m^2} - \frac{\not{p}_{A'}}{s_1}\right](\not{p}+m)\not{e}^*_{A'} \\
&- \not{e}^*_{B'}(\not{p}+m)\left[\frac{\not{p}_B}{u_2-m^2} - \frac{\not{p}_{B'}}{s_2}\right](\not{q}-m)^{-1}\not{e}^*_{A'} \\
&- \not{e}^*_{B'}(\not{p}+m)\not{e}^*_{A'}\frac{s}{2}\left[\frac{1}{(u_2-m^2)(u_1-m^2)} + \frac{1}{s_2s_1}\right. \\
&\left.- \frac{1}{(u_2-m^2)s_1} - \frac{1}{s_2(u_1-m^2)}\right]\Big\}u_A.
\end{aligned} \tag{2.7}$$

When the first term in the curly brackets of the above relation is put into Eq. (2.1) as an integrand, the result of integration can be expressed through the vector q only. Therefore, performing the projection we conclude that the integration momentum p in this term can be replaced by

$$p \to q\frac{m^2+q^2}{2q^2}, \tag{2.8}$$

where we have applied the on-mass-shellness of the intermediate particles too. Therefore, we obtain

$$\begin{aligned}
&\not{e}^*_{B'}(\not{q}-m)^{-1}\gamma^\mu(\not{p}+m)\gamma_\mu(\not{q}-m)^{-1}\not{e}^*_{A'} \\
&\quad = -\not{e}^*_{B'}(\not{q}-m)^{-1}\left((1+\epsilon)(\not{q}-m)\frac{\not{q}}{q^2}(\not{q}-m) - 2m\right)(\not{q}-m)^{-1}\not{e}^*_{A'}
\end{aligned} \tag{2.9}$$

for the first term of Eq. (2.7). Analogously, the integration momentum in the numerator of the second term of Eq. (2.7) is expressed in terms of the vectors $p_{A'}$ and q (see Eq. (2.5)). The projection gives

$$p \to \frac{1}{m^2-q^2}\left[\frac{(m^2-q^2)(m^2+q^2) - 2(u_1-m^2)q^2}{m^2-q^2}p_{A'} + \left(u_1-m^2\right)q\right], \tag{2.10}$$

and this term gets a form

$$\not{e}^*_{B'}(\not{q}-m)^{-1}\left[\frac{\not{p}_A}{u_1-m^2} - \frac{\not{p}_{A'}}{s_1}\right](\not{p}+m)\not{e}^*_{A'}$$

$$
\begin{aligned}
&= \not{e}^*_{B'}(\not{q}-m)^{-1}\Bigg\{(2e^*_{A'}q)\Bigg[\bigg(\frac{2m^2\not{q}}{m^2-q^2}+m-\not{q}\bigg)\frac{1}{u_1-m^2} \\
&\quad + \frac{\not{q}-m}{s_1}-\frac{2m^2\not{q}}{(m^2-q^2)^2}\Bigg]-\Bigg[\frac{q^2+2m^3(\not{q}-m)^{-1}}{m^2-q^2} \\
&\quad + \big(m\not{q}+m^2-q^2-2mq^2(\not{q}-m)^{-1}\big)\frac{1}{u_1-m^2}\Bigg]\not{e}^*_{A'}\Bigg\}. \qquad (2.11)
\end{aligned}
$$

The consideration of the third term in the curly brackets of Eq. (2.7) repeats the previous one with evident changes. The corresponding relations are

$$
\begin{aligned}
&p \to \frac{1}{m^2-q^2}\Bigg[-\frac{(m^2-q^2)(m^2+q^2)-2(u_2-m^2)q^2}{m^2-q^2}p_{B'}+\big(u_2-m^2\big)q\Bigg], \\
&\not{e}^*_{B'}(\not{p}+m)\Bigg[\frac{\not{p}_B}{u_2-m^2}-\frac{\not{p}_{B'}}{s_2}\Bigg](\not{q}-m)^{-1}\not{e}^*_{A'} \\
&= -\Bigg\{(2e^*_{B'}q)\Bigg[\bigg(\frac{2m^2\not{q}}{m^2-q^2}+m-\not{q}\bigg)\frac{1}{u_2-m^2}+\frac{\not{q}-m}{s_2}-\frac{2m^2\not{q}}{(m^2-q^2)^2}\Bigg] \\
&\quad - \not{e}^*_{B'}\Bigg[\frac{q^2+2m^3(\not{q}-m)^{-1}}{m^2-q^2}+\big(m\not{q}+m^2-q^2-2mq^2(\not{q}-m)^{-1}\big)\frac{1}{u_2-m^2}\Bigg]\Bigg\} \\
&\quad \times (\not{q}-m)^{-1}\not{e}^*_{A'}. \qquad (2.12)
\end{aligned}
$$

Finally, the integration momentum p in the numerator of the last term of Eq. (2.7) is expressed through the complete basis of the problem $p_{A'}$, $p_{B'}$ and q

$$
p \to\approx \frac{u_1-m^2}{s}\bigg(\frac{m^2-q^2}{2q^2}q+p_{B'}\bigg)+\frac{u_2-m^2}{s}\bigg(\frac{m^2-q^2}{2q^2}q-p_{A'}\bigg)+\frac{m^2+q^2}{2q^2}q, \qquad (2.13)
$$

where we have kept the terms which survive (after the integration in Eq. (2.1)) in the Regge limit (1.1) only. Taking into account the previous relation we get

$$
\begin{aligned}
&\not{e}^*_{B'}(\not{p}+m)\not{e}^*_{A'}\frac{s}{2}\Bigg[\frac{1}{(u_2-m^2)(u_1-m^2)}+\frac{1}{s_2s_1}-\frac{1}{(u_2-m^2)s_1}-\frac{1}{s_2(u_1-m^2)}\Bigg] \\
&= -\not{e}^*_{B'}(\not{q}-m)^{-1}\bigg(\frac{1}{u_1-m^2}-\frac{1}{s_1}\bigg)\bigg((\not{q}-m)(2e^*_{A'}q)+\frac{m^2-q^2}{2q^2}\big(q^2+m\not{q}\big)\not{e}^*_{A'}\bigg) \\
&\quad - \bigg((2e^*_{B'}q)(\not{q}-m)+\not{e}^*_{B'}\big(q^2+m\not{q}\big)\frac{m^2-q^2}{2q^2}\bigg)\bigg(\frac{1}{u_2-m^2}-\frac{1}{s_2}\bigg)(\not{q}-m)^{-1}\not{e}^*_{A'} \\
&\quad - \not{e}^*_{B'}(\not{q}-m)^{-1}\frac{m^2-q^2}{4q^2}\big(q^2+m\not{q}\big)s\bigg(\frac{1}{u_2-m^2}-\frac{1}{s_2}\bigg)\bigg(\frac{1}{u_1-m^2}-\frac{1}{s_1}\bigg)\not{e}^*_{A'}. \qquad (2.14)
\end{aligned}
$$

Combining the relations (2.9), (2.11), (2.12), (2.14) according to Eq. (2.7) we come to the following equality

$$
\begin{aligned}
&\sum(-)B_{\mathrm{as}}A_{\mathrm{as}} \\
&\quad= -g^4 C_F (t^{B'}t^{A'})_{BA}\bar{v}_B \Biggl(\not{e}^*_{B'}(\not{q}-m)^{-1} \Biggl\{ \left(\frac{m^2-q^2}{u_1-m^2}-1\right)\frac{2m^2\not{q}}{(m^2-q^2)^2}(2e^*_{A'}q) \\
&\qquad + \Biggl[(1+\epsilon)\frac{\not{q}(\not{q}+m)}{2q^2} - \epsilon + m(\not{q}-m)^{-1} + m^2(\not{q}-m)^{-2} \\
&\qquad + \bigl(\not{q}+m^2(\not{q}-m)^{-1}\bigr)\frac{\not{q}+m}{u_1-m^2} + \left(\frac{1}{u_1-m^2}-\frac{1}{s_1}\right)\frac{m^2-q^2}{2q^2}\not{q}(\not{q}+m)\Biggr]\not{e}^*_{A'}\Biggr\} \\
&\qquad + \Biggl\{ (2e^*_{B'}q)\frac{2m^2\not{q}}{(m^2-q^2)^2}\left(\frac{m^2-q^2}{u_2-m^2}-1\right) + \not{e}^*_{B'}\Biggl[(1+\epsilon)\frac{(\not{q}+m)\not{q}}{2q^2} - \epsilon \\
&\qquad + m(\not{q}-m)^{-1} + m^2(\not{q}-m)^{-2} + \frac{\not{q}+m}{u_2-m^2}\bigl(\not{q}+m^2(\not{q}-m)^{-1}\bigr) \\
&\qquad + (\not{q}+m)\not{q}\frac{m^2-q^2}{2q^2}\left(\frac{1}{u_2-m^2}-\frac{1}{s_2}\right)\Biggr]\Biggr\}(\not{q}-m)^{-1}\not{e}^*_{A'} \\
&\qquad + \not{e}^*_{B'}(\not{q}-m)^{-1}\frac{m^2-q^2}{4q^2}\not{q}(\not{q}+m)s\left(\frac{1}{u_2-m^2}-\frac{1}{s_2}\right) \\
&\qquad \times \left(\frac{1}{u_1-m^2}-\frac{1}{s_1}\right)\not{e}^*_{A'}\Biggr)u_A. \qquad (2.15)
\end{aligned}
$$

Let us note, that the Dirac equations as well as the conditions (2.3) were also used in order to express the convolution in the above form. This convolution is to be put into Eq. (2.1) as the integrand and only scalar loop integrals appear there.

We see that in the t-channel unitarity approach the amplitude $\mathcal{A}$ (1.2) is naturally expressed through a set of independent scalar loop integrals. The algebra used for this purpose is strongly simplified by the on-mass-shellness of the intermediate particles and by the particular (Regge) kinematics we are interested in. We have shown that for the simple contribution of the asymptotic parts of the intermediate amplitudes A and B but evidently the same can be applied to the complete amplitude $\mathcal{A}$. Here we skip the details of the complete consideration and just quote the result

$$
\begin{aligned}
\mathcal{A}_2 &= -g^2\bigl(t^{B'}t^{A'}\bigr)_{BA}\bar{v}_B\not{e}^*_{B'}(\not{q}-m)^{-1}\frac{\hat{\omega}(q)}{2}\left(\ln\left(\frac{-s}{s_0}\right)+\ln\left(\frac{-u}{s_0}\right)\right)\not{e}^*_{A'}u_A \\
&\quad - g^4 C_F\bigl(t^{B'}t^{A'}\bigr)_{BA}\bar{v}_B\Biggl(\not{e}^*_{B'}(\not{q}-m)^{-1}\Biggl\{-\Biggl[m(\not{q}-m)^{-1}\not{e}^*_{A'} \\
&\quad + \left(2-(1+\epsilon)\frac{(\not{q}-m)\not{q}}{2q^2}\right)\not{e}^*_{A'} \\
&\quad - \left(2m+(1+\epsilon)\frac{m^2-q^2}{q^2}\not{q}+(1+2\epsilon)(\not{q}-m)\right)\not{e}^*_{A'}(\not{q}+m)^{-1} \\
&\quad - \bigl(1+\epsilon-\not{q}(\not{q}+m)^{-1}\bigr)\frac{\not{q}\not{e}^*_{A'}(\not{q}+m)^{-1}}{C_A C_F}\Biggr]\bigl(I_1-I_1\bigl(q^2=m^2\bigr)\bigr) \\
&\quad + m(\not{q}+m)\not{e}^*_{A'}I_1'\bigl(q^2=m^2\bigr)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\not{q}-m}{C_AC_F}\left[\frac{m^2-q^2}{2q^2}\not{q}\not{e}^*_{A'}-m\big((1-\epsilon)m-\epsilon\not{q}\big)\not{e}^*_{A'}(\not{q}+m)^{-1}\right]I_2\\
&+\left(2+\frac{1}{C_AC_F}\right)\frac{m^2-q^2}{2q^2}(\not{q}-m)\not{q}\not{e}^*_{A'}I_3\\
&+\frac{m^2-q^2}{8q^2}(\not{q}+m)\not{q}\not{e}^*_{A'}\left[\left(2+\frac{1}{C_AC_F}\right)^2I_4+\frac{I_5}{C_A^2C_F^2}\right.\\
&\left.\left.-\frac{2I_6}{C_AC_F}\left(2+\frac{1}{C_AC_F}\right)\right]\right\}\\
&+\left\{-\left[m\not{e}^*_{B'}(\not{q}-m)^{-1}+\not{e}^*_{B'}\left(2-(1+\epsilon)\frac{\not{q}(\not{q}-m)}{2q^2}\right)-(\not{q}+m)^{-1}\not{e}^*_{B'}\right.\right.\\
&\times\left(2m+(1+\epsilon)\frac{m^2-q^2}{q^2}\not{q}+(1+2\epsilon)(\not{q}-m)\right)\\
&\left.-\frac{(\not{q}+m)^{-1}\not{e}^*_{B'}\not{q}}{C_AC_F}\big(1+\epsilon-(\not{q}+m)^{-1}\not{q}\big)\right]\big(I_1-I_1\big(q^2=m^2\big)\big)\\
&+m\not{e}^*_{B'}(\not{q}+m)I_1'\big(q^2=m^2\big)\\
&+\left[\not{e}^*_{B'}\not{q}\frac{m^2-q^2}{2q^2}-m(\not{q}+m)^{-1}\not{e}^*_{B'}\big((1-\epsilon)m-\epsilon\not{q}\big)\right]\frac{\not{q}-m}{C_AC_F}I_2\\
&+\left(2+\frac{1}{C_AC_F}\right)\frac{m^2-q^2}{2q^2}\not{e}^*_{B'}\not{q}(\not{q}-m)I_3+\not{e}^*_{B'}\not{q}(\not{q}+m)\frac{m^2-q^2}{8q^2}\\
&\left.\times\left[\left(2+\frac{1}{C_AC_F}\right)^2I_4+\frac{I_5}{C_A^2C_F^2}-\frac{2I_6}{C_AC_F}\left(2+\frac{1}{C_AC_F}\right)\right]\right\}\\
&\left.\times(\not{q}-m)^{-1}\not{e}^*_{A'}\right)u_A,
\end{aligned}
\tag{2.16}
$$

where the large energy logarithms responsible for the quark Reggeization were explicitly written in the first term of the $\mathcal{A}_2$. The subscript 2 in the notation $\mathcal{A}_2$ is to say that this part of the complete amplitude $\mathcal{A}$ has the correct t-channel singularities related to the two-particle t-channel intermediate state. As for the pole singularity contribution appeared "by accident" at the above consideration, it was completely removed from the $\mathcal{A}_2$ in order to be correctly restored from the one-particle t-channel unitarity relation. That will be done in the next section. Also, the amplitude $\mathcal{A}_2$ was already projected on the positive signature and quark colour quantum numbers in the t-channel since we are interested only in these quantum numbers as it was mentioned above. The definitions of the six independent scalar integrals entering Eq. (2.16) are the following

$$
\begin{aligned}
I_1&=\int\frac{d^Dp}{i(2\pi)^D}\frac{1}{((p-q)^2+i\delta)(p^2-m^2+i\delta)},\\
I_2&=\int\frac{d^Dp}{i(2\pi)^D}\frac{1}{((p-q)^2+i\delta)(p^2-m^2+i\delta)((p+p_{A'})^2-m^2+i\delta)},
\end{aligned}
$$

$$I_3 = \int \frac{d^D p}{i(2\pi)^D} \frac{1}{((p-q)^2+i\delta)(p^2-m^2+i\delta)((p-p_A)^2+i\delta)},$$
$$I_4 = \int \frac{d^D p}{i(2\pi)^D} \frac{s}{((p-q)^2+i\delta)(p^2-m^2+i\delta)((p-p_A)^2+i\delta)((p+p_B)^2+i\delta)} - \omega(q^2)\ln\left(\frac{-s}{s_0}\right),$$
$$I_5 = \int \frac{d^D p}{i(2\pi)^D} \times \frac{s}{((p-q)^2+i\delta)(p^2-m^2+i\delta)((p-p_{B'})^2-m^2+i\delta)((p+p_{A'})^2-m^2+i\delta)} - \omega(q^2)\ln\left(\frac{-s}{s_0}\right),$$
$$I_6 = \int \frac{d^D p}{i(2\pi)^D} \times \frac{u}{((p-q)^2+i\delta)(p^2-m^2+i\delta)((p-p_{B'})^2-m^2+i\delta)((p-p_A)^2+i\delta)} - \omega\big(q^2\big)\ln\left(\frac{-u}{s_0}\right), \tag{2.17}$$

with $\omega(q^2)$ defined by Eq. (1.3) and

$$\hat{\omega}(q) = g^2 C_F \frac{m^2-q^2}{q^2}(\not{q}+m)\not{q}\,\omega\big(q^2\big). \tag{2.18}$$

As it is clear from the LLA results of Ref. [4] the terms with $\omega(q^2)$ cancel the large energy logarithms in the box integrals I_4–I_6. The results of integration in Eq. (2.17) are listed in Appendix A of this paper.

3. The pole contribution

The calculation of the pole contribution is much simpler than the previous one. One has to convolute two amplitudes of a real gluon emission by an on-mass-shell quark. Diagrammatically such the amplitude is presented in Fig. 3 where the diagrams related to the external lines renormalization are not explicitly shown. The calculation of this amplitude is done according to the ordinary rules and is very simple due to the on-mass-shellness. Because of the simplicity we just present below the result for the renormalized amplitude skipping the details of this calculation

$$A^{\lambda}_{\mathrm{QQG}} = -ig\gamma^{\lambda}t^c\left(1 + g^2C_F\left[\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\sum_f (m_f^2)^{\epsilon}}{3\epsilon C_F} + \left(1+\frac{1}{2C_AC_F}\right) \times\left(\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{(m^2)^{\epsilon}}{\epsilon} + (3+2\epsilon)I_1\big(q^2=m^2\big) + 4m^2 I_1'\big(q^2=m^2\big)\right)\right]\right), \tag{3.1}$$

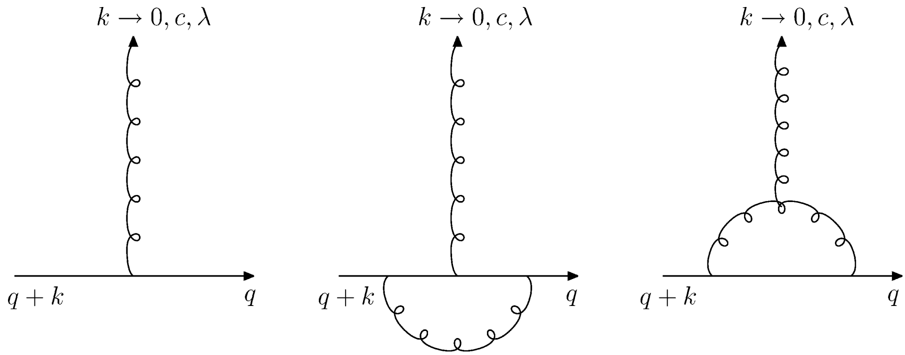

Fig. 3. The diagrams for a gluon emission by a quark.

where the integral I_1 was defined by Eq. (2.17) and the sum is over active quark flavours (including the quark with the mass m by which the gluon is emitted). The notation γ^λ in the above relation is for the Dirac matrix and the other notations there are clear from the Fig. 3.

Now one has to perform the convolution of the two above on-mass-shell amplitudes in the intermediate t-channel quark quantum numbers, multiply it with the external particles wave functions and then, exactly as in the previous section, replace the on-mass-shell δ-function of the intermediate quark by the complete propagator. Doing so one gets the amplitude

$$
\begin{aligned}
\mathcal{A}_1 = {} & -g^2\big(t^{B'}t^{A'}\big)_{BA}\bar{v}_B\Bigg(\not{e}^*_{B'}(\not{q}-m)^{-1}\not{e}^*_{A'} \\
& + \not{e}^*_{B'}(\not{q}-m)^{-1}g^2C_F\Bigg[\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\sum_f (m_f^2)^\epsilon}{3\epsilon C_F} \\
& + \left(1+\frac{1}{2C_AC_F}\right)\left(\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{(m^2)^\epsilon}{\epsilon} + (3+2\epsilon)I_1\big(q^2=m^2\big)\right. \\
& \left. + 4m^2 I_1'\big(q^2=m^2\big)\right)\Bigg]\not{e}^*_{A'} + g^2C_F\not{e}^*_{B'}\Bigg[\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\sum_f (m_f^2)^\epsilon}{3\epsilon C_F} \\
& + \left(1+\frac{1}{2C_AC_F}\right)\left(\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{(m^2)^\epsilon}{\epsilon} + (3+2\epsilon)I_1\big(q^2=m^2\big)\right. \\
& \left. + 4m^2 I_1'\big(q^2=m^2\big)\right)\Bigg](\not{q}-m)^{-1}\not{e}^*_{A'}\Bigg)u_A
\end{aligned}
\tag{3.2}
$$

which restores the correct pole (and only pole) singularity of the complete amplitude Fig. 1 due to the quark t-channel intermediate state.

Now we sum Eqs. (2.16) and (3.2) and obtain the complete Regge asymptotics $\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2$ which is to be compared with the one-loop expansion of Eq. (1.2) in order to find the quark–gluon-Reggeized quark NLA effective vertices we are interested in. This gives

$$
\begin{aligned}
\Gamma_A(q,s_0) = gt^{A'}\Bigg(&\not{e}^*_{A'} + g^2 C_F\Bigg\{\Bigg[\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\sum_f (m_f^2)^\epsilon}{3\epsilon C_F} \\
&+ \left(1+\frac{1}{2C_AC_F}\right)\left(\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{(m^2)^\epsilon}{\epsilon} + (3+2\epsilon)I_1\big(q^2=m^2\big)\right)\Bigg]\not{e}^*_{A'} \\
&+ \left(3+\frac{1}{C_AC_F}\right)2m^2 I_1'\big(q^2=m^2\big)\not{e}^*_{A'} \\
&- \Bigg[m(\not{q}-m)^{-1}\not{e}^*_{A'} + \left(2-(1+\epsilon)\frac{(\not{q}-m)\not{q}}{2q^2}\right)\not{e}^*_{A'} \\
&- \left(2m + (1+\epsilon)\frac{m^2-q^2}{q^2}\not{q} + (1+2\epsilon)(\not{q}-m)\right)\not{e}^*_{A'}(\not{q}+m)^{-1} \\
&- \big(1+\epsilon-\not{q}(\not{q}+m)^{-1}\big)\frac{\not{q}\not{e}^*_{A'}(\not{q}+m)^{-1}}{C_AC_F}\Bigg]\big(I_1 - I_1\big(q^2=m^2\big)\big) \\
&+ \frac{\not{q}-m}{C_AC_F}\left[\frac{m^2-q^2}{2q^2}\not{q}\not{e}^*_{A'} - m\big((1-\epsilon)m-\epsilon\not{q}\big)\not{e}^*_{A'}(\not{q}+m)^{-1}\right]I_2 \\
&+ \left(2+\frac{1}{C_AC_F}\right)\frac{m^2-q^2}{2q^2}(\not{q}-m)\not{q}\not{e}^*_{A'}I_3 \\
&+ \frac{m^2-q^2}{8q^2}(\not{q}+m)\not{q}\not{e}^*_{A'}\Bigg[\left(2+\frac{1}{C_AC_F}\right)^2 I_4 + \frac{1}{C_A^2C_F^2}I_5 \\
&- \frac{2}{C_AC_F}\left(2+\frac{1}{C_AC_F}\right)I_6\Bigg]\Bigg\}\Bigg)u_A
\end{aligned}
\tag{3.3}
$$

and

$$
\begin{aligned}
\overline{\Gamma}_B(q,s_0) = \bar{v}_B g t^{B'}\Bigg(&\not{e}^*_{B'} + g^2 C_F\Bigg\{\not{e}^*_{B'}\Bigg[\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{\sum_f (m_f^2)^\epsilon}{3\epsilon C_F} \\
&+ \left(1+\frac{1}{2C_AC_F}\right)\left(\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{(m^2)^\epsilon}{\epsilon} + (3+2\epsilon)I_1\big(q^2=m^2\big)\right)\Bigg] \\
&+ \not{e}^*_{B'}\left(3+\frac{1}{C_AC_F}\right)2m^2 I_1'\big(q^2=m^2\big) - \Bigg[m\not{e}^*_{B'}(\not{q}-m)^{-1} \\
&+ \not{e}^*_{B'}\left(2-(1+\epsilon)\frac{\not{q}(\not{q}-m)}{2q^2}\right) \\
&- (\not{q}+m)^{-1}\not{e}^*_{B'}\left(2m + (1+\epsilon)\frac{m^2-q^2}{q^2}\not{q} + (1+2\epsilon)(\not{q}-m)\right) \\
&- \frac{(\not{q}+m)^{-1}\not{e}^*_{B'}\not{q}}{C_AC_F}\big(1+\epsilon-(\not{q}+m)^{-1}\not{q}\big)\Bigg]\big(I_1 - I_1\big(q^2=m^2\big)\big) \\
&+ \left[\not{e}^*_{B'}\not{q}\frac{m^2-q^2}{2q^2} - m(\not{q}+m)^{-1}\not{e}^*_{B'}\big((1-\epsilon)m-\epsilon\not{q}\big)\right]\frac{\not{q}-m}{C_AC_F}I_2
\end{aligned}
$$

$$+\left(2+\frac{1}{C_AC_F}\right)\frac{m^2-q^2}{2q^2}\not{e}^*_{B'}\not{q}(\not{q}-m)I_3$$
$$+\not{e}^*_{B'}\not{q}(\not{q}+m)\frac{m^2-q^2}{8q^2}\left[\left(2+\frac{1}{C_AC_F}\right)^2 I_4+\frac{1}{C_A^2C_F^2}I_5\right.$$
$$\left.\left.\left.-\frac{2}{C_AC_F}\left(2+\frac{1}{C_AC_F}\right)I_6\right]\right\}\right). \tag{3.4}$$

The vertices (3.3) and (3.4) (of course, they are related each to other by an evident symmetry), together with the list of integrals of Appendix A are the results of this paper.

It was mentioned in the introduction that the effective vertices in the massless QCD case were already calculated in Ref. [9]. Using the list of massless integrals given in Appendix A, one can easily obtain from Eq. (3.3) the following

$$\Gamma_A(q,s_0)|_{m_f=0}=gt^{A'}\left(\not{e}^*_{A'}-\frac{g^2C_F(-q^2)^\epsilon}{(4\pi)^{2+\epsilon}}\frac{\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{\epsilon\Gamma(1+2\epsilon)}\left\{\left[-\ln\left(\frac{-t}{s_0}\right)\right.\right.\right.$$
$$\left.+\frac{2}{\epsilon}-\frac{3-\epsilon}{2(1+2\epsilon)}+\psi(1)+\psi(1-\epsilon)-2\psi(1+\epsilon)\right]\not{e}^*_{A'}$$
$$\left.\left.+\left(1+\frac{1}{C_AC_F}\right)\left(\frac{\not{e}^*_{A'}}{\epsilon}+\frac{\epsilon}{1+2\epsilon}\frac{\not{q}\not{e}^*_{A'}\not{q}}{q^2}\right)\right\}\right)u_A \tag{3.5}$$

and the analogous expression for $\overline{\Gamma}_B(q,s_0)|_{m_f=0}$, which are in full correspondence with the results of Ref. [9] when $s_0=-t$ as it is adopted there. Let us note that in the massless case in our approach one does not need to consider the pole contribution at all—it is completely given by the Born amplitude and the radiative corrections can contribute to the branch point singularity only.

4. Discussion

We have calculated here the NLA quark–gluon-Reggeized quark effective interaction vertices for the case of QCD with massive quarks. They are to be applied for the determination of two-loop Regge trajectory of massive Reggeized quark, for instance. For massless QCD such vertices are known and our results in this case reproduce the corresponding expressions of the Ref. [9].

We also note that throughout all this paper we worked with the bare coupling g instead of the renormalized one g_μ. In order to remove the ultraviolet divergences from our results (3.3) and (3.4) it is enough to re-express them in terms of the renormalized coupling (in $\overline{\text{MS}}$)

$$g=g_\mu\mu^{-\epsilon}\left[1+\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\frac{g_\mu^2N\Gamma(1-\epsilon)}{2\epsilon(4\pi)^{2+\epsilon}}\right]. \tag{4.1}$$

At the moment such renormalization does not look so sensible to perform since our results are in any case intermediate and contain infrared poles in ϵ which would cancel only in final physical results. As for the quark mass which enters the results, in our approach it appears automatically as the renormalized pole quark mass because we always used the unitarity relations with the renormalized intermediate amplitudes. Of course, it is very easy to express back through the bare mass (or through the mass in any other scheme).

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Appendix A

Here we present the results of integration in Eqs. (2.17). Since in the massive quark case the integrals cannot be explicitly calculated without the ϵ-expansion we perform only those integrations over Feynman parameters which can be done exactly and leave the others untouched.

$$I_1 = -\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{1}{\epsilon}\int_0^1 dx\big[(1-x)\big(m^2-xq^2\big)\big]^{\epsilon}, \tag{A.1}$$

$$I_1\big(q^2=m^2\big) = -\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{m^{2\epsilon}}{\epsilon(1+2\epsilon)}, \tag{A.2}$$

$$I_1'\big(q^2=m^2\big) = \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{(m^2)^{\epsilon-1}}{2\epsilon(1+2\epsilon)}, \tag{A.3}$$

$$I_2 = -\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{1}{\epsilon(m^2-q^2)}\int_0^1 \frac{dx}{x}\Big(\big[(1-x)\big(m^2-xq^2\big)\big]^{\epsilon}-\big[(1-x)m\big]^{2\epsilon}\Big), \tag{A.4}$$

$$I_3 = -\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{1}{2\epsilon}\int_0^1 \frac{dx}{[(1-x)(m^2-xq^2)]^{1-\epsilon}}, \tag{A.5}$$

$$\begin{aligned}
I_4 &= -\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\int_0^1 \frac{dx}{[(1-x)(m^2-xq^2)]^{1-\epsilon}} \\
&\quad\times\left[\frac{1}{\epsilon}+\psi(1)+\psi(1-\epsilon)-2\psi(1+2\epsilon)+\ln\left(\frac{s_0}{(1-x)(m^2-xq^2)}\right)\right] \\
&= -\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\left[\frac{1}{\epsilon}+\psi(1)+\psi(1-\epsilon)-2\psi(1+2\epsilon)-s_0^{\epsilon}\frac{d}{d\epsilon}s_0^{-\epsilon}\right] \\
&\quad\times\int_0^1 \frac{dx}{[(1-x)(m^2-xq^2)]^{1-\epsilon}},
\end{aligned} \tag{A.6}$$

$$I_5 = I_4 + \frac{2\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \int_0^1 \frac{dx \ln(x/(1-x))}{[(1-x)(m^2 - xq^2)]^{1-\epsilon}}, \tag{A.7}$$

$$I_6 = I_4 + \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \int_0^1 \frac{dx \ln(x/(1-x))}{[(1-x)(m^2 - xq^2)]^{1-\epsilon}}, \tag{A.8}$$

where $\psi(z)$ is the logarithmic derivative of the Euler gamma-function.

In the case $m = 0$ the above integrals can be expressed in terms of gamma-functions:

$$\begin{aligned} I_1 &= -\frac{\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)} \frac{(-q^2)^\epsilon}{\epsilon(1+2\epsilon)}, \\ I_2 = I_3 &= -\frac{\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)} \frac{(-q^2)^{\epsilon-1}}{\epsilon^2}, \\ I_4 = I_5 = I_6 &= -\frac{\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)} \frac{2(-q^2)^{\epsilon-1}}{\epsilon} \\ &\quad \times \left[-\ln\left(\frac{-t}{s_0}\right) + \frac{2}{\epsilon} + \psi(1) + \psi(1-\epsilon) - 2\psi(1+\epsilon) \right]. \end{aligned} \tag{A.9}$$

Also, for $m \to 0$ we have to take $I_1(q^2 = m^2) \to 0$ and $m^2 I_1'(q^2 = m^2) \to 0$ in the framework of the dimensional regularization.

Let us note that the relation (see Eqs. (A.6)–(A.8))

$$I_4 + I_5 - 2I_6 = 0, \tag{A.10}$$

valid for the Regge asymptotics of the above box integrals, is not accidental and can be proved without calculation of the integrals. Let us introduce the Sudakov decomposition for the integration momentum

$$p = \beta p_{A'} + \alpha p_{B'} + p_\perp. \tag{A.11}$$

For the other momenta of the problem such decompositions look as follows

$$\begin{aligned} &p_A = (1-\alpha_R)p_{A'} + \alpha_R p_{B'} + q_\perp, \qquad p_B = (1-\alpha_R)p_{B'} + \alpha_R p_{A'} - q_\perp, \\ &q = p_A - p_{A'} = p_{B'} - p_B = \alpha_R(p_{B'} - p_{A'}) + q_\perp, \qquad \alpha_R \simeq \frac{m^2 + \vec{q}^{\,2}}{s}, \end{aligned} \tag{A.12}$$

where, as usually, the vector notation is used for the transverse momenta. The matter is that for the combination (A.10) the relevant to the Regge asymptotics integration region is

$$|\alpha|, |\beta| \sim \alpha_R, \qquad \vec{p}^{\,2} \sim \vec{q}^{\,2}. \tag{A.13}$$

The integration in (A.10) in this region takes the form

$$\begin{aligned} &I_4 + I_5 - 2I_6 \\ &\quad \simeq -i \int \frac{d^{D-2}p}{2(2\pi)^{D-2}} \frac{s^2}{(\vec{p}-\vec{q})^2(\vec{p}^{\,2}+m^2)} \end{aligned}$$

$$\times \int_{-\infty}^{+\infty} \frac{d\alpha}{2\pi i} \left(\frac{1}{s(\alpha - \alpha_R) + (\vec{p} - \vec{q})^2 - i\delta} - \frac{1}{s\alpha - \vec{p}^{\,2} - m^2 + i\delta} \right)$$
$$\times \int_{-\infty}^{+\infty} \frac{d\beta}{2\pi i} \left(\frac{1}{s(\beta + \alpha_R) - (\vec{p} - \vec{q})^2 + i\delta} - \frac{1}{s\beta + \vec{p}^{\,2} + m^2 - i\delta} \right)$$
$$- 2\omega\big(q^2\big) \ln\left(\frac{-s}{-u} \right), \tag{A.14}$$

where $\omega(q^2)$ is defined by the relations (1.3) and (2.18). The integrations over longitudinal Sudakov variables are factorized in Eq. (A.14) and both are performed by taking the residues, that leads to the pure imaginary result. This means, that energy dependence of the combination (A.10) could only correspond to the negative t-channel signature, i.e., could only be

$$I_4 + I_5 - 2I_6 \sim \ln(-s) - \ln(-u) \tag{A.15}$$

with some energy-independent coefficient.

From other side, the combination (A.10) is proportional to the purely nonasymptotic contribution (compare with Eq. (2.1))

$$\mathcal{A}_{\mathrm{na}}^{(3,+)} = \left(\int \frac{d^D p}{i(2\pi)^D} \frac{\sum(-) B_{\mathrm{na}} A_{\mathrm{na}}}{(k^2 + i\delta)(p^2 - m^2 + i\delta)} \right)^{(3,+)} \tag{A.16}$$

to the Regge asymptotics of the amplitude $\mathcal{A}$ with quark colour quantum numbers and positive signature in the t-channel. In the above relation we have introduced $A_{\mathrm{na}} = A - A_{\mathrm{as}}$, where the amplitude A and its Regge asymptotics A_{as} are given by Eqs. (2.2) and (2.6), respectively, and analogously for the B_{na}. Therefore, both (A.10) and (A.16) must vanish (see also Ref. [12]). In other words, the nonasymptotic contribution is of the negative signature for the quark colour quantum numbers in the t-channel.

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