

# The generalized Coase Theorem and separable individual preferences: an extension

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## Abstract

In a recent article [Bernholz, P., 1997. Property rights, contracts, cyclical social preferences, and the Coase Theorem: A synthesis. *European Journal of Political Economy*, 13, 419–442] it was shown that the Coase Theorem can be generalized to all cases of assignments of rights, including political and bureaucratic rights. At the same time, however, additional assumptions besides the absence of transaction costs had to be introduced to assure the validity even of the original Coase Theorem limited to market phenomena. The assumptions are that external and internal contracts must be binding, that a finely divisible good like money must be present and be valued by all, and that individual preference orderings are separable. In the present extension this latter rather restrictive assumption will be removed. I will thus demonstrate the conditions under which the generalized Coase Theorem holds without the assumption of separable individual preference orderings. © 1999 Elsevier Science B.V. All rights reserved.

*Keywords:* Coase Theorem; Rights; Individual preference ordering

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## 1. Sketch of notations and assumptions

I show first by using an example that problems arise with the Coase Theorem if individual preference orderings are not separable. In doing so, I assume *strong* individual preference orderings, three issues, and a society consisting of three

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Table 1  
Members of society

	1	2	3
Strong	$x_3$	$x_2$	$x_6$
Individual	$x_1$	$x_1$	$x_4$
Preference	$x_4$	$x_6$	$x_2$
Orderings	$x_7$	$x_4$	$x_8$
over	$x_8$	$x_8$	$x_5$
Outcomes	$x_5$	$x_5$	$x_1$
	$x_6$	$x_7$	$x_7$
	$x_2$	$x_3$	$x_3$

individuals, i.e.,  $V = \{1,2,3\}$ . Individual preference orderings over the outcomes  $x_1, x_2, \dots, x_8$ , are given by Table 1.

Each outcome comprises three alternatives, one out of the three issues. The three issues  $M_1, M_2$  and  $M_3$  are represented by the three dimensions of Fig. 1, where each issue contains only two alternatives. Thus, for example, outcome  $x_8$  is different from  $x_1$  in the alternatives of all three issues,  $x_3$  only in those of the first issue  $M_1$ .

All rights refer to issues and are assigned to subsets of society. In the example the right to decide the first and the third issue is assigned to individuals 1 and 3, respectively. All individuals together have the right to decide  $M_2$ . Thus we have  $V_1 = \{1\} \subset V, V_2 = \{1,2,3\} = V, V_3 = \{3\} \subset V$ . We assume further that  $M_2$  is decided by simple majority vote.

It is obvious that externalities are present in the example. A decision by 1 to move from  $x_5$  to  $x_7$  implies negative externalities for 2 and 3. But 1 has the right to do so, since the first issue has been assigned to it. And it can do so, if  $x_5$  happens to be the status quo. Similarly externalities are present in other cases, for

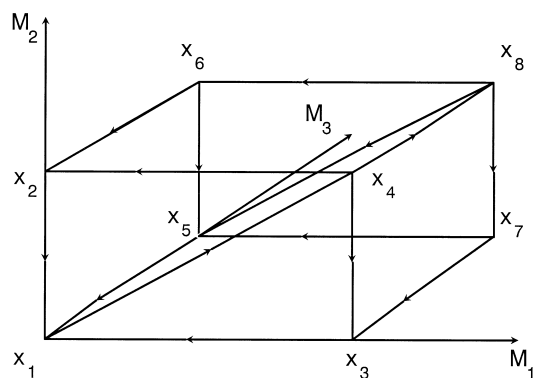


Fig. 1.

example if a majority  $\{2,3\}$  decides concerning issue 2 to move from  $x_1$  to  $x_2$ . For this implies a negative externality to the outvoted minority, voter 1. Note further that there are inseparabilities in individual preference orderings. Consider the preference ordering of individual 3 in Table 1. He prefers  $x_6$  to  $x_2$  and at the same time  $x_4$  to  $x_8$ . But the two outcomes of both pairs differ among each other only in the same alternatives of the same issue  $M_3$  (Fig. 1). Thus separability would require that the ordering between the outcomes of one of the pairs be reversed.

The arrows in Fig. 1 point from the dominating to the dominated outcome. Thus, since 1 has the right to decide  $M_1$ , the arrows in that dimension correspond to his preference ordering.

The right to decide  $M_2$ , however, has been assigned to  $V_2 = \{1,2,3\}$  who decide by simple majority. Now 2 and 3 form such a majority, for example, for the decision between  $x_4$  and  $x_3$ . As a consequence the arrow points to the latter, since it is worse for both of them. Finally, we have to mention that a move, for example, from  $x_5$  to  $x_8$  or  $x_4$  can only be accomplished with the help of a contract among  $V_1$  and  $V_2$  or  $V_1$ ,  $V_2$  and  $V_3$ , respectively, since rights refer only to issues.

## 2. Problems with the Coase Theorem given non-separable individual preference orderings

It follows directly from looking at Fig. 1 that if contracts are not allowed,  $x_4$  is the only stable outcome. But 1 and 2 prefer  $x_1$ , and 1 controls the first and they both together as a majority control the second issue. Thus they conclude a binding *external* and *internal* contract to bring about  $x_1$ . But in spite of the fact that the contract is *binding* for them, this is not a stable outcome. For 3 is not a partner to the contract, and he prefers not only  $x_5$  but can realize it, since he has the right to decide issue  $M_3$ . But this outcome is Pareto-inferior, since all members of society prefer  $x_8$  to it and can realize this outcome by a binding contract implying the first two issues. But it again is Pareto-inferior to  $x_4$ , which can be brought about by 3. As a consequence a cycle is present. We have cyclical ‘social preferences’. No stable Pareto-optimal outcome exists. Or, put differently, the existing *negative externalities* cannot be removed even by *binding contracts*. The latter statement is directly related to the fact that *cyclical social preferences implying two or more issues* can only occur if *negative externalities* are present (Bernholz, 1997).

The reason for the existence of cyclical social preferences and no stable Pareto-optimal solution as asserted by the Coase Theorem is clearly the *existence of non-separable individual preference orderings*. In the example the cycle mentioned would, for example, not exist, if individual 3 had a separable preference ordering, so that  $x_8$  would be preferred by him to  $x_4$  (the general proof is given in Bernholz, 1997). Now cyclical social preferences would not be of any concern if they contained only Pareto-optimal outcomes. But as shown by the

example, this is clearly not the case. On the other hand, the assumption of *separable individual preference orderings* is rather strong and restrictive. We intend thus to show subsequently that the validity of the generalized Coase Theorem can be maintained in the absence of this assumption under certain conditions.

### 3. Proof of the generalized Coase Theorem with non-separable individual preference orderings present

Let us first consider the cycle mentioned for the above example. We have seen that of the four outcomes in the cycle, two are not Pareto-optimal, whereas  $x_1$  and  $x_4$  are. Can this situation be solved with the help of *binding contracts*? This is indeed the case in the sense that all Pareto-inferior outcomes can be excluded.

If the cycle were the only one, all its outcomes would recur regularly with a certain frequency or probability, where we may call the probability with which  $x_i$  occurs  $p_i$ . With only the above cycle considered, we might reasonably set all  $p_i$  equal to  $1/4$ . Now consider first  $x_8$ , which is dominated by Pareto-optimal  $x_4$ . Since we have assumed strong individual preferences, the latter is better than the former for all three members of society. They could thus conclude a profitable contract agreeing that they would no longer bring about  $x_8$ , but instead limit themselves to move to the other outcomes in the former cycle with probabilities  $p_1 = p_5 = 1/4$  and  $p_{4'} = 1/2$ . The frequency of  $x_8$  would thus be added to that of the Pareto-superior outcome. But since  $x_4$  is also Pareto-superior to  $x_5$ , a similar reasoning applies to these two outcomes. Consequently the *binding* contract will state that its partners will bring about the two Pareto-optimal outcomes of the former cycle with frequencies  $p_{4''} = 3/4$  and  $p_1 = 1/4$ . But then the cycle is broken, negative externalities are removed and a stable Pareto-optimal solution alternating between  $x_1$  and  $x_4$  is reached.

It is obvious that a similar result can be easily proved for the general case with many issues and members of society and with many, possibly, interlocking cycles. For assume that there exist one or more possibly interlocking social preference cycles. Then the system might become stuck in one of them. In this case assign the observed frequencies  $p_i$  with  $\sum p_i = 1$  to all outcomes  $x_i$  in the (interlocking) cycle(s). Next select all Pareto-inferior outcomes  $x_j$  within the (interlocking) cycle(s) and substitute for them in the contract to be agreed on Pareto-superior outcomes from within or without the cycle(s). Add to the frequencies of these outcomes those of the outcomes for which they have been substituted. Note that for Pareto-superior outcomes selected from outside the cycle(s) the original frequency is zero. It follows from this argument that the *binding* contract concluded contains only Pareto-optimal outcomes which will be brought about by the partners with different frequencies. Thus a stable solution comprising only Pareto-optimal outcomes emerges. Finally, since the argument holds true for any

(possibly connected) social preference cycle(s) in which the system may become stuck, such an outcome will always be reached by a binding contract.

#### 4. Concluding remarks

The argument can be easily extended to the case of weak individual preference orderings by assuming that at least one finely divisible good is present, which is positively valued by all individuals (see Bernholz, 1997).

More importantly, however, the above proof shows that, without the assumption of separable individual preference orderings, the generalized and even the limited Coase theorem is only valid, if distributional considerations are taken into account when concluding a contract of the type discussed above. In the example, member 3 would only agree to the contract if the Pareto-optimal outcome  $x_4$  would be brought about three times as often as  $x_1$  by the partners.

It follows that potential partners who have the right to decide issues or are necessary members of a decisive coalition  $C_i \subset V_i$  can insist on higher frequencies for the Pareto-optimal outcomes favouring their interests.

In this paper I have, moreover, neglected bargaining problems which may arise because there can exist several outcomes which are Pareto-preferred to a given Pareto-inferior outcome in a cycle. For in this case there may arise a conflict because of different individual preference orderings about which of these outcomes to substitute. We have thus implicitly assumed that such conflicts are always solved because of the interest of all partners to move to a better solution.

#### References

- Bernholz, P., 1997. Property rights, contracts, cyclical social preferences, and the Coase Theorem: a synthesis. *European Journal of Political Economy* 13, 419–442.