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UNSTEADY MHD SQUEEZING FLOW BETWEEN TWO PARALLEL ROTATING DISCS

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Introduction

The study of the squeezing flow between two planes is of special interest for application to bearings with liquid-metal lubrications. The interaction of the flowing liquid-metal lubricant with the applied magnetic field can be used to increase the total load which the rotor can support and reduce the viscous drag on the rotor. Studies on the effects of magnetic field in lubrication were made by Hughes and Elco [1], Kuzma et al. [2], Krieger et al.[3] and Kamiyama [4]. In these investigations they considered the magnetic force term but neglected some or all the inertia terms in the Navier-Stokes equations. Hamza [5] studied the squeezing flow between two discs in the presence of a magnetic field acting perpendicular to the discs by taking into account all the inertia terms. In a subsequent paper [6] he obtained a similarity solution where the axial magnetic field is assumed to be of particular time-dependent form.

The study of similarity solutions for the steady motion of an incompressible viscous fluid contained between two rotating discs was initiated by Batchelor [7]. Using von Karman similarity transformation the governing equations were reduced to two coupled fourth-order diffferential equations. The solutions were obtained by employing a shooting method by Lance and Rogers [8]. Subsequently, Holodniok [9] used finite differencing and Newton's iteration for obtaining solutions at higher Reynolds number. A detailed review of these studies was made by Zandbergen

[10]. When the angular velocities of the discs are time-dependent, the Navier-Stokes equations for the governing flow can be reduced to a pair of coupled non-linear partial differential equations. Hamza and Macdonald [11] considered the case where two parallel discs, which were rotating with time-dependent angular velocities, had a velocity component in a direction perpendicular to their plane and obtained a similarity solution. His solution requires that at time t, the separation of the discs must be proportional to $(1-\alpha t)^{1/2}$ and the angular velocities of the discs are proportional to $(1-\alpha t)^{-1}$, where α^{-1} denotes a representative time.

In the present study, we considered the motion of an electrically conducting viscous fluid film between two parallel discs, in which the lower one is rotating with an arbitrary time-dependent angular velocity. The effect of a uniform axial magnetic field is also included. Numerical solutions of the governing non-linear parabolic partial differential equations are obtained through a fourth-order accurate Hermitian finite-difference scheme. The normal forces and torques which the fluid exerts on the discs are obtained for various values of Reynolds number and magnetic field parameter at various non-dimensional times. A considerable increment in the load due to normal force exerted on the upper disc is found by increasing the magnetic force. The load also increases with decrease of the gap between the discs. The torque on the lower disc increases with increase in the magnetic field as well as the angular velocity of the lower disc.

Formulation

We consider cylindrical polar co-ordinates (r, θ, z) and denote by u, v and w, the velocity components in the radial, tangential and axial directions respectively. Two parallel discs occupying the planes $z=0$ and $z=d(t^*)$, where $d(t^*)$ is a function of the non-dimensional time t^*, contain an electrically conducting viscous incompressible fluid (Figure 1). The lower disc $(z=0)$ rotates with a time-dependent angular velocity $\Omega\phi(t^*)$ in its own plane and the upper disc approaches the lower one with a constant velocity W_d. A magnetic field B_0 is applied perpendicular to the two discs. Initially the discs were at a distance H apart and the lower disc was rotating with a uniform angular velocity Ω. This implies that $d(0)=H$ and $\phi(0)=1$.

The Navier-Stokes equations for the governing unsteady axi-symmetric flow can be written as

$$\frac{\partial\zeta}{\partial t}+u\frac{\partial\zeta}{\partial r}+w\frac{\partial\zeta}{\partial z}-\frac{u\zeta}{r}-2\frac{v}{r}\frac{\partial v}{\partial z}=\nu\left(\nabla^2\zeta-\frac{\zeta}{r^2}\right)-\frac{1}{\rho}\frac{\partial}{\partial z}(\sigma B_0^2u) \tag{1}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\frac{1}{\rho}\nu\left(\nabla^2 v-\frac{v}{r^2}\right)-\sigma B_0^2 v \tag{2}$$

where ζ is the circumferential component of vorticity given by

$$\zeta=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial r}$$

and the stream function ψ is defined as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial z},\ w=-\frac{1}{r}\frac{\partial\psi}{\partial r}$$

Thus

$$\zeta = \frac{1}{r}\left(\nabla^2\psi - \frac{2}{r}\frac{\partial\psi}{\partial r}\right) \tag{3}$$

where the Laplacian operator, ∇^2 is given by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.$$

In writing (1) and (2), the effect of induced magnetic field on the flow is neglected and this is justified for flow at small magnetic Reynolds number. This is indeed true for flow of liquid metals e.g., mercury or liquid Sodium.

We now introduce the following non-dimensional functions

$$\eta = z/d(t^*), \beta(t^*) = d(t^*)/H, t^* = \Omega t$$

$$f(\eta, t^*) = \psi/r^2\Omega H\phi(t^*)$$

$$\Gamma(\eta, t^*) = \zeta H/\Omega r\phi(t^*)$$

$$g(\eta, t^*) = v/r\Omega\phi(t^*), \tag{4}$$

Using these the equations (1) - (3) transform to

$$Re^{-1}\Gamma_{\eta\eta} + 2\phi\beta\left(f\Gamma_\eta + gg_\eta\right) + \beta\beta_{t^*}\eta\Gamma_\eta - \beta^2\left(\Gamma_{t^*} + \Gamma\phi_{t^*}\phi^{-1}\right) - M\beta^2\Gamma = 0 \tag{5}$$

$$Re^{-1}g_{\eta\eta} + 2\phi\beta\left(fg_\eta - gf_\eta\right) + \beta\beta_{t^*}\eta g_\eta - \beta^2\left(g_{t^*} + g\phi_{t^*}\phi^{-1}\right) - M\beta^2 g = 0 \tag{6}$$

$$f_{\eta\eta} - \beta^2\Gamma = 0 \tag{7}$$

The boundary conditions (for $t > 0$) are

$$\eta = 0 : f = 0, f' = 0, g = 1, \Gamma = \frac{f_{\eta\eta}}{\beta^2}$$

$$\eta = 1 : f = \epsilon/\left[2\phi(t^*)\right], f' = 0, g = 0, \Gamma = \frac{f_{\eta\eta}}{\beta^2} \tag{8}$$

Here f ,Γ and g are non-dimensional functions of η and non-dimensional time t^*. Further W_d, the velocity with which the upper disc approaches the lower one, is taken as $\Omega H\epsilon$, where ϵ is a positive constant.Hence,

$$-W_d = w(1, t^*) = -2\Omega\phi(t^*)Hf(1, t^*).$$

This explains the boundary condition on f at $\eta = 1$ given by (8). Further it is clear that $\beta(t^*) = 1 - \epsilon t^*$. In (5) and (6) $Re (= H^2\Omega/\nu)$ is the Reynolds number, where ν is the kinematic viscosity and Ω is the angular velocity with which the lower disc was rotating initially ($t^* = 0$) and $M\ (= B_0^2\sigma/\rho\Omega)$ is the magnetic interaction parameter which measures the strength of the electromagnetic body force relative to the Coriolis force. Here σ and ρ denote the electrical conductivity and density of the fluid respectively.

The initial conditions (i.e., at $t^* = 0$) are governed by the solution of the corresponding steady-state similarity equations for the flow due to uniform rotation of the lower disc. The steady-state equations can be obtained from equations (5) - (7) by setting

$$\beta(0) = \phi(0) = 1 \text{ and } \frac{\partial}{\partial t^*} = 0 \tag{9}$$

By considering the axial momentum equation it can be shown that $\frac{\partial^2 p}{\partial\eta\partial r} = 0$, i.e., the radial pressure gradient $\frac{\partial p}{\partial r}$ is independent of η . It can be shown that

$$\frac{1}{r}\frac{\partial p}{\partial r} = \rho\Omega^2\phi\left[\frac{Re^{-1} f_{\eta\eta\eta}(0,t^*)}{\beta^3} + \phi g^2(0,t^*)\right] \tag{10}$$

If the upper surface is assumed to be a disc of finite radius a and of negligible thickness, the load or the normal force exerted on this disc is given by

$$L^* = 2\pi\int_0^a r\left[P(r,1,t^*) - P^*(r,1,t^*)\right] dr$$

where,

$$P(r,1,t^*) = p(r,1,t^*) - p(a,1,t^*)$$

Further $P^*(r,1,t^*)$ is defined in a similar way and corresponds to the conditions on the upper side of the disc. We assume that $\frac{\partial p^*}{\partial r}(r,1,t^*) = 0$

Hence from (10) we get

$$L^* = -\frac{\pi\rho\Omega^2 a^4}{4}\left[\frac{Re^{-1} f_{\eta\eta\eta}(0,t^*)}{\beta^3} + g^2(0,t^*)\phi\right]\phi$$

We define the non-dimentionalised load as

$$L = -\left[\frac{Re^{-1} f_{\eta\eta\eta}(0,t^*)}{\beta^3} + g^2(0,t^*)\phi\right]\phi$$

Similarly, assuming that the discs are of finite radius a, the torque which the fluid exerts on the lower disc ($\overline{T}_l$) is

$$\overline{T}_l = \int_0^a 2\pi r^2\mu\frac{\partial v}{\partial z}\Big|_{z=0}\, dr$$

The non-dimensional form of the torque experienced by the lower disc is given by

$T_l = \phi(t^*)g_\eta(0,t^*)/\beta(t^*)$

It may be noted that the equations (5) - (7) for the steady-state ($t^* = 0$) non-magnetic (M = 0) case correspond to the set of equations considered by Rogers and Lance [8] and equations (5) - (7) correspond to the equations considered by Hamza [5] in the absence of rotation (g = 0).

Numerical Method

The non-linear initial boundary value problem governed by equations (5) - (7) are solved by first differencing in the t^* - direction and averaging the other terms. An higher order accurate compact difference scheme are used to discretised the derivatives in η. This method considers as unknowns at each discretized point η_i not only the value of the function f_i itself but also of its first and second derivatives f_i' and f_i''. The system is closed by considering a fourth order accurate relationships between the function and its derivatives in three successive discretization points. A detailed description of the method is given in Bhattacharyya et al. [12].

Results and Discussion

In order to assess the accuracy of our method , we have compared our steady-state results (f, f_η, g) with M = 0 for various values of the Reynolds number (Re = 25, 81) and the rotation ratios of the discs ($\epsilon = 0, 0.5$) with those of Rogers and Lance [8] and found them in excellent agreement (see figure 2).

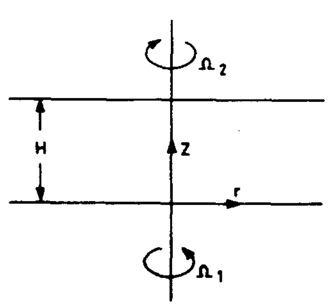

FIG. 1 Schematic diagram for the flow between two co-axial rotating discs.

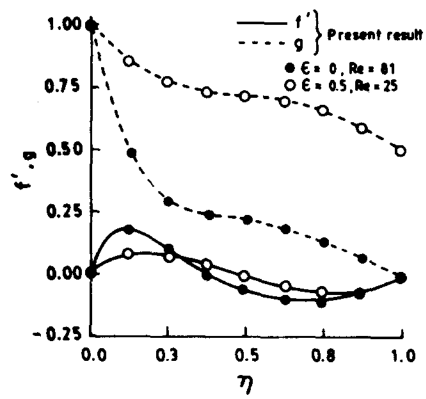

FIG. 2 Comparison of f_η, g with those of Rogers and Lance [8] for $M = 0$.

The variation of load (L) with magnetic field and Reynods number with time ($t^* = 0.25$) is also compared with the perturbation solution obtained by Hamza [5] and it is found that the maximum percentage difference is about 2 % (see Table 1). In Table 1 results within the paranthesis correspond to those obtained by Hamza [5].

TABLE 1

t^*	Re	M	L	M	L	M	l
0.25	0.1	0	143.70	0.25	144.208	0.5	145.713
			(144.127)		(144.627)		(146.180)
	0.5	0	29.799	0.25	29.899	0.5	30.201
			(30.344)		(30.449)		(30.749)
	1.0	0	15.641	0.25	15.691	0.5	15.842
			(16.127)		(16.177)		(16.327)
	2.0	0	8.699	0.25	8.803	0.5	8.999
			(9.016)		(9.041)		(9.116)

Figure 3 shows the variation of load L with time t^*. Graphs are drawn for various values of the squeezing parameter ϵ with $\phi(t^*) = 1 - 0.3t^{*2}$ including the case of the squeezing flow where the lower disc is stationary ($\Omega = 0$). As time elapses the load increases monotonically. The results shown in figure 3 also illustrate the fact that the load L increases the decrease of the gap between the two discs (i.e., increase of t^*). It is also found that at a fixed time the load increases with increase in ϵ (i.e.,the speed of approach of the upper disc). For a slight squeezing ($\epsilon = 0.05$), the load L remains almost invariant with time. The variation in the time-dependent part of the lower disc angular velocity i.e., $\phi(t^*)$ produces a slight effect on L.

The effects of magnetic field M on load at $t^* = 1$ and $Re = 25$ are described in figure 4. The axial magnetic field produces a large increment in load and the increment is higher for higher values of M. It is also evident from figure 4 that the load increases slightly as the lower disc rotates faster.

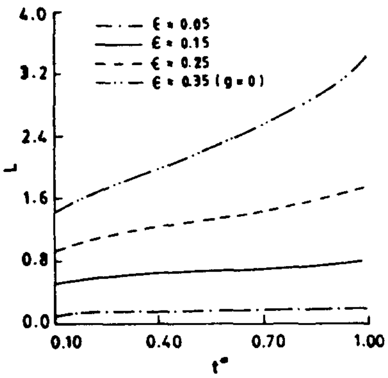

FIG. 3 Variation of load (L) with t^* when $Re = 5, M = 5, \epsilon = 0.05, 0.15, 0.25,$ $0.35(g = 0), \phi = 1 - 0.3t^{*2}$.

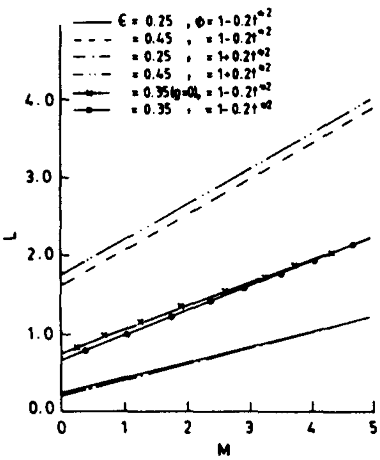

FIG. 4 Variation of load (L) with M when $Re = 25, t^* = 1, \epsilon =$ 0.25 , 0.45, 0.35, $0.35(g = 0), \phi = 1 \pm 0.2t^{*2}$.

The load decreases as the Reynolds number increases for a fixed value of M. Figure 5 shows that for M = 5, the variation of load L with the Reynolds number Re at time $t^* = 1$ for different values of the squeezing parameter ϵ and $\phi(t^*) = 1 - 0.4t^{*2}$. The increase in Re $(= \Omega H^2/\nu)$ can be made through the increase of initial ($t^* = 0$) angular velocity (Ω) of the lower disc. It is evident from figure 5 that the effect of squeezing (ϵ) on the load (L) is much prominent than the effect due to increase of angular velocity (Re), (with $Re > 10$) of the lower disc.

The torques which the fluid exerts on the discs are also of interest in rotating flow problems. Figure 6 describes the variation of the torque T_l on the lower disc with time t^*. Graphs are depicted for flow due to accelerating $\left(\phi(t^*) = 1 + 0.3t^{*2}\right)$ as well as decelerating $\left(\phi(t^*) = 1 - 0.3t^{*2}\right)$ lower disc with $Re = 5$ and $\epsilon = 0.25$. The effect of the magnetic parameter M (=0, 2, 4, 6) is also illustrated in figure 6. For an accelerating lower disc, the magnitude of the torque on the lower disc T_l increases monotonically with time but decreases with time when the angular velocity of the lower disc decreases with time $\left(\phi(t^*) = 1 - 0.3t^{*2}\right)$. This results is plausible on physical grounds. For an accelerating disc, its angular velocity increases with time. Consequently the torque required to maintain such angular velocity of the disc also increases with time. Just the reverse is true for a decelerating disc. The magnetic field in the axial direction produces a huge increment in the magnitude of the torque T_l exerted by the fluid on the lower disc.This can be explained physically as follows. It is welknown that a magnetic field imparts some rigidity to the conducting fluid. Thus with increase in the magnetic field, greater effort will be necessary to maintain the rotation of the disc and this implies increase in T_l with increase in M.

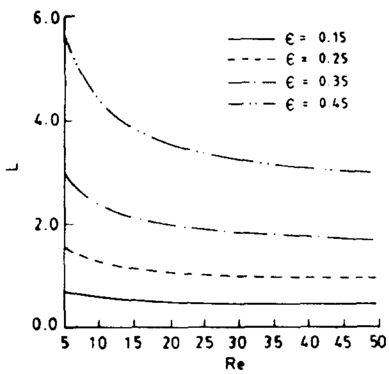

FIG. 5 Variation of load (L) with Re when $M = 4, t^* = 1, \epsilon = 0.15, 0.25, 0.35, 0.45, \phi = 1 - 0.4t^{*2}$.

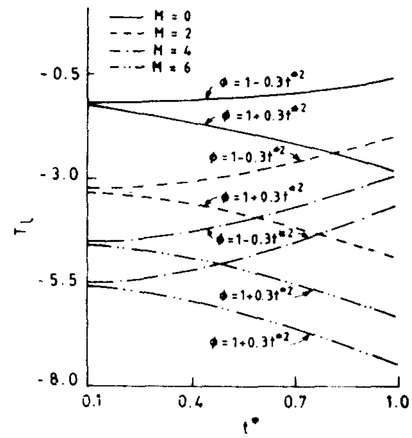

FIG. 6 Variation of T_l with time t^* when $Re = 25$, $M = 0, 2, 4, 5$, $\epsilon = 0.25$, $\phi = 1 \pm 0.3t^{*2}$.

The variation of the torque T_l with the squeezing parameter ϵ for different Reynolds number Re is shown in figure 7. The magnitude of the torque T_l increases with increase of the squeezing parameter ϵ for a fixed value of Re. Torque on the lower disc increases also with increase in Re i.e.,the increase in rotation of the lower disc.

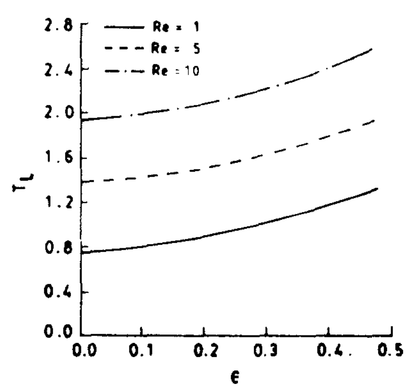

FIG. 7 Variation of T_l at time $t^* = 1$ with ϵ when $Re = 1, 5, 10$, $M = 2$.

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