Optimization of Well Oil Rate Allocations in Petroleum Fields

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Currently, optimization of oil production operations in offshore fields is based mainly on heuristic rules and simulation-like tools with limited optimization features. In this work, we present a novel mixed-integer optimization formulation and an effective solution strategy for daily well operation and gas-lift allocation of integrated oil and gas production facilities in offshore fields considering the nonlinear reservoir behavior, the multiphase flow in pipelines, and surface capacity constraints. On the basis of (i) a degrees of freedom analysis, (ii) well oil rate upper and lower bounds, and (iii) the approximation of each well model with piecewise linear functions, an efficient approximate mixed-integer nonlinear program (MINLP) model is proposed. The resulting model is then solved as a sequence of MILP problems. The accuracy and robustness of the method is investigated by comparing the results of the exact and approximate models, while the performance of the algorithm is illustrated in a number of field examples. The results show that the proposed method achieves an increase in oil production up to 3% when compared with heuristic rules typically applied in practice.

Introduction

A typical offshore oilfield is shown in Figure 1 and consists of (i) the reservoir, which is defined as an accumulation of oil, gas, and water in porous permeable rock, (ii) the production wells, which transfer reservoir fluids to the manifold, (iii) the manifold, where the well streams are mixed, (iv) the surface flow lines, which connect the manifolds to the surface facilities, and (v) the surface facilities where the reservoir fluids are separated into oil, gas, and water and gas is compressed and injected back to the reservoir. Each well consists of two pipe segments: the well tubing and the well flow line. Between them, there is a valve, the choke, which is used to control the well flow rate. The region of the well inside the reservoir is called the well bore. The paper considers two types of production wells: (i) naturally flowing and (ii) gas lift. The first are able to provide flow naturally to the surface, while the second require an injection of high-pressure gas to reduce the pressure drop in the well tubing and therefore facilitate extraction. Finally, the wells, manifolds, flow lines, and surface facilities define the pipeline network, as shown in Figure 1.

In petroleum fields, oil production is often constrained by the reservoir conditions, flow characteristics of the pipeline network, and the capacity of the surface facilities.[1−3] Consequently, proper determination of the daily optimal operating conditions requires simultaneous consideration of the interactions between the reservoir, the wells, and the surface facilities.

Various methods for daily oil production optimization have been presented in the literature. These can be divided into three categories: (i) sensitivity analysis using simulation tools, (ii) heuristic methods, and (iii) mathematical programming methods.

Traditionally, the petroleum industry has applied NODAL[2] analysis,[4] which is a simulation method to determine the daily optimal operating policy by repetitively varying the optimization variables and simulating the underlying system. Therefore, NODAL analysis is by its trial and error nature limited to oilfields with a small number of wells.

The majority of publications[5−7] tackle the daily oil production optimization using heuristic rules that are incorporated in software tools known as well management routines.[8] Well management routines decompose the pipeline network into levels, usually (i) the well level, (ii) the manifold level, and (iii) the separator level as shown in Figure 1, and heuristic rules are applied sequentially. For instance, at the well level, rules such as close a well if it violates an upper bound in the gas−oil ratio (GOR), which is defined as the ratio of gas to oil volumetric flow rate, are applied. At the manifold level, upper bound constraints on oil, gas, and water flow rates are imposed. If any of the upper bounds is violated, the well production rate is scaled appropriately until the constraint is satisfied. At the separator level, oil production targets are imposed. If these are not satisfied, then decisions such as gas-lift initiation are enabled. It is obvious that well management routines, while accounting for network constraints, are formulated in an ad hoc manner and do not systematically address the maximization of oil production. The current major commercial reservoir simulators[9,10] are based on similar heuristic rules, and gas-lift allocation is considered separately from well rate optimization. One of the most widely applied heuristic rules for allocation of gas-to-gas-lift wells is known as the incremental GOR (IGOR) rule. IGOR is defined as the amount of gas required by a gas-lift well to produce an additional barrel of oil. It was proposed originally by Redden et al.,[11] but Weiss et al.[7] derived a necessary condition for optimal allocation of gas-to-gas-lift wells. The condition states that all wells tied to a common manifold must operate at the same IGOR. The IGOR heuristic rule has also been applied by Barnes et al.[12] and Stoisits et al.[13] in the Prudhoe Bay and Kuparuk River fields, respectively. However, it must be noted that the necessary

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10.1021/ie034171z CCC: $27.50 © 2004 American Chemical Society
Published on Web 03/24/2004

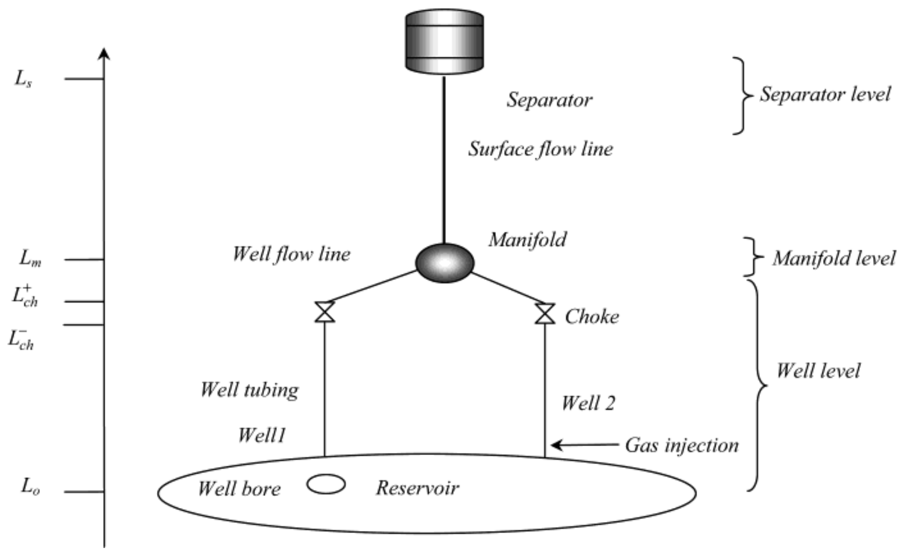

Figure 1. Two-well operation system.

condition was derived by analyzing a pipeline network where all of the wells are tied directly to a fixed-pressure separator and, therefore, interactions between the wells that share a common flow line are not considered.

Mathematical programming techniques have also been applied in production operations optimization. Carroll and Horne[14] applied gradient-based and polytope optimization techniques to a single well. Fujii and Horne[15] proposed a two-stage optimization strategy where a pipeline network simulator acted as an evaluation tool for a series of suggested values of control variables produced by an optimization algorithm. Martinez et al.,[16] Palke and Horn,[17] and Wang et al.[18] applied genetic algorithms (GAs). Although GAs are robust, they are especially computationally intensive when the optimization problem is subjected to nonlinear equality constraints. Fang and Lo[2] proposed a linear programming (LP) model to optimize gas lift subject to multiple nonlinear flow-rate constraints. Dutta-Roy and Kattapuram[19] analyzed a gas-lift optimization problem with two wells sharing a common flow line. They pointed out that when flow interactions among wells are significant, nonlinear optimization tools are needed. They coupled a pipeline network simulator with an optimization tool that is based on the sequential quadratic programming (SQP) method. However, because of commercial confidentiality, there is no information regarding the optimization model. Handley-Schahler et al.[20] developed a commercial optimization tool for optimization of production operations. The resulting optimization problem was solved using the sequential LP (SLP) method. Their tool was applied to a gas production network and to gas-lift wells that are tied directly to a separator. However, because of commercial confidentiality, there is no information available about the optimization model. Wang et al.[18] proposed a linear program (LP) and a mixed-integer LP (MILP) for optimization of production operations. The LP formulation was similar to that proposed by Lo et al.,[21] which is based on the premise that, for a pipeline network of naturally flowing wells, when the well chokes are fully open and the pipeline network is simulated, the resulting well oil rates are maximum. These well oil rates were used as upper bounds to the LP formulation that incorporated only mass balance and surface capacity constraints. The MILP formulation extended the work of Fang and Lo.[2] Both the LP and MILP formulations do not handle flow interactions among wells that share a common flow line. Recently, Wang et al.[22] proposed a SQP method that is able to take into account flow interactions among wells in a treelike structure pipeline network. Del Rio et al.[23] Applied spline interpolation to smooth the pressure gradient surface when mechanistic models[24] are used to predict the pressure drop in a pipeline network. Next, they applied polytope methods and GAs to determine the optimal tubing diameter and well oil rate of a single well.

Usually, in the petroleum industry, the pressure drop in a production network is computed by multidimensional hydraulic lookup tables,[29] which define the inlet pressure as a function of the outlet pressure and oil, gas, and water flow rates.[29] Therefore, there is a need to develop a robust and efficient optimization algorithm that simultaneously optimizes the well oil rates and gas-lift rates, is able to be integrated with measured data or hydraulic tables, and is applicable to any pipeline structure.

In sections 2 and 3, we present the problem statement and a model for the production operation optimization. Section 4 contains a motivating example showing the need for a more efficient solution method. In section 5, a novel approximate optimization model is presented. The resulting optimization model is based on a degrees of freedom analysis, well upper and lower bounds, and separability programming techniques and is solved as a sequence of MILP problems. In section 6, two motivating examples are used to demonstrate the accuracy and robustness of the proposed formulation. The performance of the algorithm is demonstrated in section 7 with three example problems. Conclusions are discussed in section 8.

2. General Problem Statement

The production optimization problem is based on a time period of 1 day and can be stated as follows: given is a reservoir, a set of naturally flowing wells, a set of gas-lift wells, a surface pipeline network, a set of separation facilities, and a compressor plant (see Figure 1). The problem involves the maximization of the profit from the sales of oil minus the cost of gas compression subject to a set of constraints. The constraints include mass and momentum balances in the wells, minimum and maximum pressure and flow-rate constraints at the inlet and outlet of the pipelines, maximum oil, gas, and water handling capacity constraints, and gas compression constraints. The following key decisions are considered: (i) how to control the well oil rates of naturally flowing wells by manipulating appropriately the well chokes and (ii) how to distribute gas among gas-lift wells.

3. Mathematical Model

The mathematical model of the system depends on (i) the type of reservoir such as dry gas and oil and gas reservoirs and (ii) the type of well such as naturally flowing and gas lift. In this section, we present the mathematical model of the two-well network shown in Figure 1, which consists of a naturally flowing and a gas-lift well. The two-well network model contains all of the components that allow modeling of more complex pipeline networks. The model is based on the following assumptions:

(i) The system is under steady-state conditions because the dynamics of the reservoir is in order of weeks, that of the pipeline network is in order of minutes, and the time horizon of the production operation problem is 1 day.

(ii) The thermodynamic description of the reservoir fluids is based on the black oil model,[24] an empirical approach widely applied in the petroleum industry.

(iii) A continuous and differentiable multiphase pressure drop model was applied.[25]

(iv) The energy balance in the pipeline network is considered, assuming linear temperature profiles along the pipes.[4]

For simplicity in the presentation of the paper, we describe in the following the optimization model of a two-well network problem as shown in Figure 1.

(i) A well inflow model that describes the multiphase fluid flow from the reservoir to the well bores of a production well. In this work, Peaceman's[26] well model was applied. This model assumes steady-state radial flow and can be expressed as

$$q_{o,i} = \mathrm{PI}_i[P_{R,i} - P_i(L_0)], \quad \forall\ i \in I \tag{1}$$

where I is the set of wells i, $q_{o,i}$ is the well volumetric oil rate under stock tank conditions, PI_i is the productivity index of the well, $P_{R,i}$ is the reservoir block pressure that contains the well and can be assumed constant for a time period of 1 day, and $P_i(L_0)$ is the bottom hole pressure of the well i. For naturally flowing wells, the gas flow rate $q_{g,i}$ under stock tank conditions is given by the relation

$$q_{g,i} = \mathrm{GOR}_i q_{o,i}, \quad \forall\ i \in N \tag{2}$$

where N is the set of naturally flowing wells. For gas-lift wells, the gas flow rate is given by

$$q_{g,i} = q_{g,i}^{\mathrm{inj}} + \mathrm{GOR}_i q_{o,i}, \quad \forall\ i \in G \tag{3}$$

where G is the set of gas-lift wells and $q_{g,i}^{\mathrm{inj}}$ is the gas injection rate. The water $q_{w,i}$ and liquid $q_{L,i}$ flow rates are given by

$$q_{w,i} = \mathrm{WOR}_i q_{o,i}, \quad \forall\ i \in I \tag{4}$$

$$q_{L,i} = q_{o,i} + q_{w,i}, \quad \forall\ i \in I \tag{5}$$

where WOR_i is the water oil ratio of well i defined as the ratio of water to oil flow rate.

The productivity index PI_i, GOR_i, and WOR_i are generally nonlinear functions of the well oil flow rate $q_{o,i}$. However, they may be assumed constant for a period of 1 day, with their values given from a reservoir simulator.

(ii) The well tubing model simulates the multiphase fluid flow from the well completion to the wellhead of a production well and is described by the following ordinary differential equation (ODE), which is valid at $L \in [L_0, L_{ch}]$:

$$\frac{\mathrm{d}P_i}{\mathrm{d}L} = f_P(P_i, q_{o,i}, q_{g,i}, q_{w,i}), \quad \forall\ i \in I,\ L \in [L_0, L_{ch}] \tag{6}$$

(iii) The well choke model is used to control the well flow rate. The characteristic of the choke valve is the transition between the critical and subcritical flow regimes. A generic choke model is as follows:

$$q_{L,i} = f_c[d_i, P_i(L_{ch}^-), P_i(L_{ch}^+), y_i, \mathrm{GOR}_i, \mathrm{WOR}_i], \quad \forall\ i \in I \tag{7}$$

$$y_i = \max(y_c, P_i(L_{ch}^-)/P_i(L_{ch}^+)), \quad \forall\ i \in I \tag{8}$$

where d_i is the choke valve diameter, $P_i(L_{ch}^-)$ and $P_i(L_{ch}^+)$ are the pressures upstream and downstream of the choke, y_i is the well choke pressure ratio, and y_c is the critical pressure ratio, which depends on upstream conditions. In this paper, the choke model proposed by Sachdeva et al.[27] is used because it satisfies continuity at the transition point. The choke model is not differentiable because of the existence of the max operator, which is approximated using the smoothing function proposed by Samsatli et al.[28]

(iv) The well flow-line model simulates the multiphase fluid flow from the wellhead to the manifold and is described by an ODE similar to eq 6 but valid at $L \in [L_{ch}, L_m)$. The combination of the well inflow, well tubing, choke, and well flow-line models will be refered to in the rest of the paper as the well model.

(v) The manifold model is where the well streams are mixed. The mass balance for each phase p is given by the linear equation

$$\sum_{i \in I_m} q_{p,i} = q_{p,m}, \quad \forall\ m \in M,\ p \in \{\mathrm{o, g, w, L}\} \tag{9}$$

where I_m is the set of the wells tied to the manifold m and $q_{p,m}$ is the volumetric flow rate of phase p in manifold m. Moreover, any network point must have unique pressure. Therefore,

$$P_i(L_m) = P_m(L_m), \quad \forall\ m \in M,\ i \in I_m \tag{10}$$

where $P_i(L_m)$ is the pressure of the well i at the manifold level and $P_m(L_m)$ is the manifold pressure.

(vi) The surface flow-line model simulates the multiphase flow from the manifold to the surface facilities and is described by the following ODE valid at $L \in [L_m, L_s]$:

$$\frac{\mathrm{d}P_m}{\mathrm{d}L} = f_P(P_m, q_{o,m}, q_{g,m}, q_{w,m}), \quad \forall\ m \in M,\ L \in [L_m, L_s] \tag{11}$$

Table 1. Consistent Initialization of the First Stage of a Naturally Flowing Well

equation	unknown
1. $q_{o,i} = \mathrm{PI}_i(P_{R,i} - P_i(L_0))$	$q_{o,i}$, $P_i(L_0)$
2. $q_{g,i} = \mathrm{GOR}_i q_{o,i}$	$q_{g,i}$
3. $q_{w,i} = \mathrm{WOR}_i q_{o,i}$	$q_{w,i}$
4. $q_{L,i} = q_{o,i} + q_{w,i}$	$q_{L,i}$
5. $\dfrac{dP_i(L_0)}{dL} = f_P(P_i(L_0),q_{o,i},q_{w,i},q_{g,i})$	$\dfrac{dP_i(L_0)}{dL}$
no. of equations: 5 degrees of freedom: 1	no. of unknowns: 6

(vii) The separator s, which operates at pressure P_s and has capacity $C_{p,s}$ for each phase p

$$P_m(L_s) = P_s, \quad \forall\ s \in S,\ m \in M_s \tag{12}$$

$$\sum_{m\in M_s} q_{p,m} \le C_{p,s}, \quad \forall\ p, s \in S \tag{13}$$

where M_s is the set of manifold flow lines m that are connected to separator s, S is the set of separators, and $P_m(L_s)$ is the pressure of the surface flow line at the separator level.

(viii) Operational constraints involve (a) well oil rate lower bounds to satisfy stable flow rate,[4] (b) well oil rate upper bounds to prevent sand production, (c) lower bound pressure constraints at the inlet of the flow lines to prevent development of erosional velocities, (d) choke diameter design constraints, and (e) upper bound constraints on gas-lift availability.

$$q_{o,i}^{L} \le q_{o,i} \le q_{o,i}^{U}, \quad \forall\ i \in I \tag{14}$$

$$P_m^{L} \le P_m(L_m), \quad \forall\ m \in M \tag{15}$$

$$0 \le d_i \le 1, \quad \forall\ i \in I \tag{16}$$

$$\sum_{i\in G} q_{g,i}^{\mathrm{inj}} \le C \tag{17}$$

(ix) An objective function is defined as maximization of the profit from the sales of oil minus the cost of gas compression

$$\max\ w_o \sum_{i\in I} q_{o,i} - w_g \sum_{i\in G} q_{g,i}^{\mathrm{inj}} \tag{18}$$

where the coefficients w_o and w_g are the price of oil and gas compression, respectively.

The mathematical model of the system involves (i) ODEs that apply over a particular length interval or stage[30] and (ii) boundary conditions between the stages. Considering, for instance, the two-well network of Figure 1, then the first stage of the system is defined by the well tubing model (eq 6) with boundary conditions described by the well inflow model (eqs 1−5). The second stage of the system is defined by the well flow-line model (eq 6) with boundary conditions imposed by the choke model (eqs 7 and 8). Finally, the third stage of the system is described by the surface flow-line model (eq 11) with boundary conditions imposed by the separator pressure (eq 12). Consistent initialization of the system of ODEs that describe the first stage of a naturally flowing well involves the solution of the nonlinear equations shown in Table 1. When the number of equations and unknowns are counted in Table 1, it is observed that the first stage of a naturally flowing well has only 1 degree of freedom. When the well oil rate $q_{o,i}$ is selected as the degree of freedom and its value is specified, the nonlinear system of equations in Table 1 becomes well posed and its solution provides initial conditions for the integration of the ODE for the first stage. Therefore, the first stage of a naturally flowing well model can be replaced by the algebraic equation

Table 2. Initialization of the Second Stage of a Naturally Flowing Well[a,b]

equation	unknown
$q_{L,i} = f_c[d_i,P_i(L_{ch}^-),P_i(L_{ch}^+),y_i,\mathrm{GOR}_i,\mathrm{WOR}_i]$	d_i, $P_i(L_{ch}^+)$, y_i
$y_i = \max[y_c,P_i(L_{ch}^-)/P_i(L_{ch}^+)]$	
$\dfrac{dP_i(L_{ch}^+)}{dL} = f_P[P_i(L_{ch}^+),q_{o,i},q_{w,i},q_{g,i}]$	$\dfrac{dP_i(L_{ch}^+)}{dL}$
no. of equations: 3 degrees of freedom: 1	no. of unknowns: 4

[a] $q_{o,i}$, $q_{w,i}$, $q_{g,i}$, and y_c were calculated from initialization of the first stage. [b] $P_i(L_{ch}^-)$ was calculated from integration of the first stage.

Table 3. Consistent Initialization of the First Stage of a Gas-Lift Well

equation	unknown
$q_{o,i} = \mathrm{PI}_i[P_{R,i} - P_i(L_0)]$	$q_{o,i}$, $P_i(L_0)$
$q_{g,i} = \mathrm{GOR}_i q_{o,i} + q_{g,i}^{\mathrm{inj}}$	$q_{g,i}$, $q_{w,i}$
$q_{w,i} = \mathrm{WOR}_i q_{o,i}$	$q_{g,i}$
$q_{L,i} = q_{o,i} + q_{w,i}$	$q_{L,i}$
$\dfrac{dP_i(L_0)}{dL} = f_P[P_i(L_0),q_{o,i},q_{w,i},q_{g,i}]$	$\dfrac{dP_i(L_0)}{dL}$
no. of equations: 5 degrees of freedom: 2	no. of unknowns: 7

$$P_i(L_{ch}^-) = n(q_{o,i}), \quad \forall\ i \in N \tag{19}$$

where $n(\cdot)$ is a functional relation whose closed form is unknown but whose value can be determined after initialization and integration of the corresponding ODE.

Initialization of the second stage involves the system of nonlinear equations shown in Table 2. When the number of equations and the number of unknowns are counted, it is observed that the second stage adds 1 extra degree of freedom to the system. The choke diameter d_i is selected as the degree of freedom because, in practice, this is the control variable of a well. In summary, when $q_{o,i}$ and d_i are selected as the degrees of freedom, the ODEs in the first and second stages can be integrated from the well bore up to the manifold. Then, the pressure of the well at the manifold level $P_i(L_m)$ is calculated from the following algebraic equation:

$$P_i(L_m) = n(q_{o,i},d_i), \quad \forall\ i \in N \tag{20}$$

Consistent initialization of the system that describes the first stage of a gas-lift well involves the solution of the nonlinear equations shown in Table 3. When the number of equations and unknowns in Table 3 are counted, it is observed that the first stage of gas-lift well has 2 degrees of freedom. Consistent initialization of the second stage involves the solution of the following

Table 4. Initialization of the Third Stage

equation	unknown
$\frac{dP_m(L_m)}{dL} = f_P[P_m(L_m),q_{o,m},q_{w,m},q_{g,m}]$	$\frac{dP_m(L_m)}{dL}$, $P_m(L_m)$, $q_{o,m}$, $q_{w,m}$, $q_{g,m}$
no. of equations: 1	no. of unknowns: 5
degrees of freedom: 4	

system of nonlinear equations:

$$q_{L,i} = f_c[d_i,P_i(L_{ch}^-),P_i(L_{ch}^+),y_i,\mathrm{GOR}_i,\mathrm{WOR}_i], \quad \forall\, i \in G \tag{21}$$

$$y_i = \max[y_c,P_i(L_{ch}^-)/P_i(L_{ch}^+)], \quad \forall\, i \in G \tag{22}$$

$$\frac{dP_i(L_{ch}^+)}{dL} = f_P[P_i(L_{ch}^+),q_{o,i},q_{w,i},q_{g,i}], \quad \forall\, i \in G \tag{23}$$

The unknowns of the system of equations (21)−(23) are d_i, $P_i(L_{ch}^+)$, y_i, and $dP_i(L_{ch}^+)/dL$. However, gas-lift wells either do not have choke valves or, if they have the choke, do not operate, and therefore their settings (d_i) are fixed. In both cases, the second stage of a gas-lift well does not add any extra degrees of freedom. In summary, a gas-lift well model has 2 degrees of freedom: one can select the well oil rate and the other the gas injection rate. Thus, the pressure of a gas-lift well at the manifold level $P_i(L_m)$ is calculated by the following algebraic equation:

$$P_i(L_m) = g(q_{o,i},q_{g,i}^{inj}), \quad \forall\, i \in G \tag{24}$$

Initialization of the third stage involves the solution of the system of nonlinear equations shown in Table 4. When the number of equations and the number of unknowns are counted, it is observed that the third stage has 4 degrees of freedom. Selection of fluid flow rates $q_{p,m}$ and manifold pressure $P_m(L_m)$ allows the integration of the ODEs for the third stage from the manifold to the separator. Then, the pressure of the surface flow line at the separator level is calculated from the following relation:

$$P_m(L_s) = f[P_m(L_m),q_{o,m},q_{g,m},q_{w,m}] \tag{25}$$

When the above developments are taken into account, the optimization formulation for the two-well network can be written as follows:

$$\max\ w_o\sum_{i\in N} q_{o,i} - w_g\sum_{i\in G} q_{g,i}^{inj}$$

s.t.

$$\begin{aligned}
&P_m(L_s) = f[P_m(L_m),q_{o,m},q_{g,m},q_{w,m}], \quad \forall\, m \in M\\
&q_{p,m} = \sum_{i\in I_m} q_{p,i}, \quad \forall\, p,\, m \in M\\
&P_m(L_m) = P_i(L_m), \quad \forall\, m \in M,\, i \in I_m\\
&P_m(L_s) = P_s, \quad \forall\, s \in S,\, m \in M_s\\
&P_m^L \le P_m(L_m), \quad \forall\, m \in M\\
&\sum_{i\in G} q_{g,i}^{inj} \le C\\
&P_i(L_m) = g(q_{o,i},q_{g,i}^{inj}), \quad \forall\, i \in G\\
&q_{w,i} = \mathrm{WOR}_i q_{o,i}, \quad \forall\, i \in I\\
&q_{L,i} = q_{o,i} + q_{w,i}, \quad \forall\, i \in I\\
&q_{g,i} = \mathrm{GOR}_i q_{o,i}, \quad \forall\, i \in N\\
&q_{g,i} = \mathrm{GOR}_i q_{o,i} + q_{g,i}^{inj}, \quad \forall\, i \in G\\
&q_{o,i}^L \le q_{o,i} \le q_{o,i}^U, \quad \forall\, i \in I\\
&0 \le d_i \le 1, \quad \forall\, i \in I
\end{aligned} \tag{P1}$$

However, formulation (P1) can cause severe computational difficulties when the choke operates in the critical flow regime because in this case the choke upstream pressure $P_i(L_{ch}^+)$ becomes independent of the well flow rate and the nonlinear system of equations in Table 2 becomes infeasible. Mathematically, this implies that the Jacobian matrix of the nonlinear system of equations in Table 2 becomes rank deficient. A way to avoid this difficulty is to replace the choke model with a positive pressure drop. The rational behind this is the following. The choke valve is used to control the well flow rate by creating a suitable pressure drop in the well tubing. Therefore, the choke upstream pressure is the new degree of freedom that can replace the choke diameter. With this, (P1) becomes

$$\max\ w_o\sum_{i\in N} q_{o,i} - w_g\sum_{i\in G} q_{g,i}^{inj}$$

s.t.

$$\begin{aligned}
&P_m(L_s) = f[P_m(L_m),q_{o,m},q_{g,m},q_{w,m}], \quad \forall\, m \in M\\
&q_{p,m} = \sum_{i\in I_m} q_{p,i}, \quad \forall\, p,\, m \in M\\
&P_m(L_m) = P_i(L_m), \quad \forall\, m \in M,\, i \in I_m\\
&P_m(L_s) = P_s, \quad \forall\, s \in S,\, m \in M_s\\
&P_m^L \le P_m(L_m), \quad \forall\, m \in M\\
&\sum_{i\in G} q_{g,i}^{inj} \le C\\
&P_i(L_m) = n[q_{o,i},P_i(L_{ch}^+)], \quad \forall\, i \in N\\
&P_i(L_m) = g(q_{o,i},q_{g,i}^{inj}), \quad \forall\, i \in G\\
&q_{w,i} = \mathrm{WOR}_i q_{o,i}, \quad \forall\, i \in I\\
&q_{g,i} = \mathrm{GOR}_i q_{o,i}, \quad \forall\, i \in N\\
&q_{g,i} = \mathrm{GOR}_i q_{o,i} + q_{g,i}^{inj}, \quad \forall\, i \in G\\
&q_{o,i}^L \le q_{o,i} \le q_{o,i}^U, \quad \forall\, i \in I
\end{aligned} \tag{P2}$$

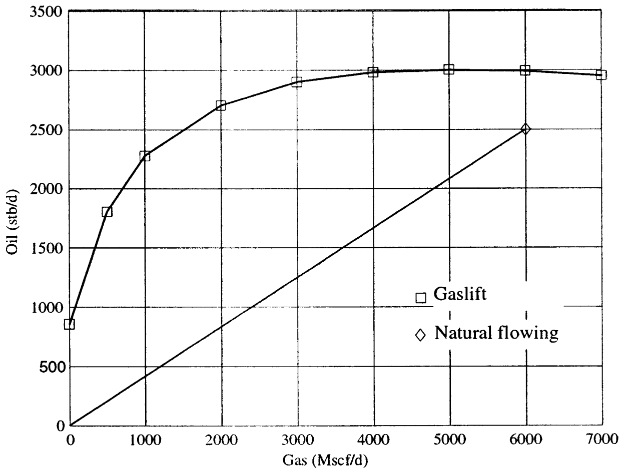

Figure 2. Oil−gas relation for a naturally flowing and gas-lift well.

Formulation (P2) may again not be robust for the following reasons:

(i) When a well is weak and cannot flow given the manifold pressure selected by the optimizer, the well integration becomes infeasible and (P2) will fail to converge because the objective function and the constraints cannot be evaluated.

(ii) Some gas-lift wells known as noninstantaneous cannot flow without the appropriate amount of gas lift. In this case, (P2) will fail to converge because the well integration becomes infeasible.

(iii) Multiple solutions exist in multiphase network problems, and not all of them are stable.

The above limitations can be avoided by implementing a preprocessing step where weak, noninstantaneous gas-lift wells and unstable solutions are identified and treated appropriately prior to the solution of the optimization problem.

4. Separable Programming for Optimization of Production Operations

Fang and Lo[2] applied separability programming techniques to the optimization of production operations. To simplify the optimization problem, they assumed that (i) the manifold pressure is constant and known, (ii) the gas-lift wells respond instantaneously to gas lift, (iii) the GOR and WOR of each well are constant, and (iv) the multiphase flow network model has a unique solution.

Because a naturally flowing well has 2 degrees of freedom, when the manifold pressure is fixed and the choke is fully open, the well model equations can be solved from the well bore up to the manifold. Moreover, the resulting oil flow rate is the maximum because the choke was set fully open and any oil flow rate less than the maximum can be achieved by reducing the choke diameter. In the case of gas-lift wells, when the manifold pressure is fixed, the oil flow rate becomes a function of the gas injection rate, as can be seen by eq 24. Figure 2 shows a graphical representation of the oil and gas flow-rate relations for a naturally flowing and gas-lift well. Fang and Lo[2] approximated the gas−oil relation for gas-lift wells with piecewise linear functions. First, the gas injection rate was discretized at j points for each well i, $q_{g,i,j}^{inj,d}$, and the oil flow rate $q_{o,i,j}^{d}$ was calculated after simulation of each well for each discrete value of the gas injection rate. Then the gas−oil relation was approximated with piecewise linear functions using the following constraints:[31]

$$\left.\begin{aligned} &q_{o,i} = \sum_j \lambda_{i,j} q_{o,i,j}^{d} \\ &q_{g,i}^{inj} = \sum_j \lambda_{i,j} q_{g,i,j}^{inj,d} \\ &\sum_j \lambda_{i,j} = 1 \\ &\lambda_{i,j} \geq 0 \\ &\lambda_{i,j},\ \mathrm{SOS} \end{aligned}\right\} \quad \forall\ i \in G \qquad (26)$$

where J is the set of discrete points j, $|J|$ is the cardinality of the set J, and $\lambda_{i,j}$ are positive variables, known as a special order set (SOS), where at most two must be adjacent. The adjacency condition is imposed with the binary variables $y_{i,j}$ (see Appendix A).

Because of the concavity property of the gas−oil relations as shown in Figure 2, the adjacency condition can be automatically satisfied without using binary variables $y_{i,j}$ and the problem can be formulated as a LP problem:[2]

$$\max\ w_o \sum_{i \in I} q_{o,i} - w_g \sum_{i \in G} q_{g,i}^{inj}$$

s.t.

$$\sum_i q_{p,i} \leq C_p, \quad \forall\ p$$

$$\left.\begin{aligned} &q_{o,i} = \sum_j \lambda_{i,j} q_{o,i,j}^{d} \\ &q_{g,i}^{inj} = \sum_j \lambda_{i,j} q_{g,i}^{inj,d} \\ &\sum_j \lambda_{i,j} = 1 \\ &\lambda_{i,j} \geq 0 \\ &q_{g,i} = \mathrm{GOR}_i q_{o,i} + q_{g,i}^{inj} \end{aligned}\right\} \quad \forall\ i \in G$$

$$\left.\begin{aligned} &q_{o,i} \leq q_{o,i}^{max} \\ &q_{g,i} = \mathrm{GOR}_i q_{o,i} \end{aligned}\right\} \quad \forall\ i \in N \qquad (P3)$$

$$\left.\begin{aligned} &q_{w,i} = \mathrm{WOR}_i q_{o,i} \\ &q_{L,i} = q_{w,i} + q_{o,i} \\ &q_{o,i}^{L} \leq q_{o,i} \leq q_{o,i}^{U} \end{aligned}\right\} \quad \forall\ i \in I$$

However, the oil−gas relation for a noninstantaneous gas-lift well is not concave as shown in Figure 3, where the well starts flowing only after a certain amount of gas is injected. In this case, the adjacency condition must be imposed using binary variables and the formulation (P3) becomes a MILP problem, which can be solved by a standard branch and bound method.[32] (P3) ignores the network constraints from the manifold to the surface facility, which could lead to suboptimal or even infeasible solutions depending on the value selected for the manifold pressure. Next, we propose an optimization formulation, which extends the work of Fang and Lo,[2] takes into account the interactions among wells when allocating well oil rates and gas-lift rates,

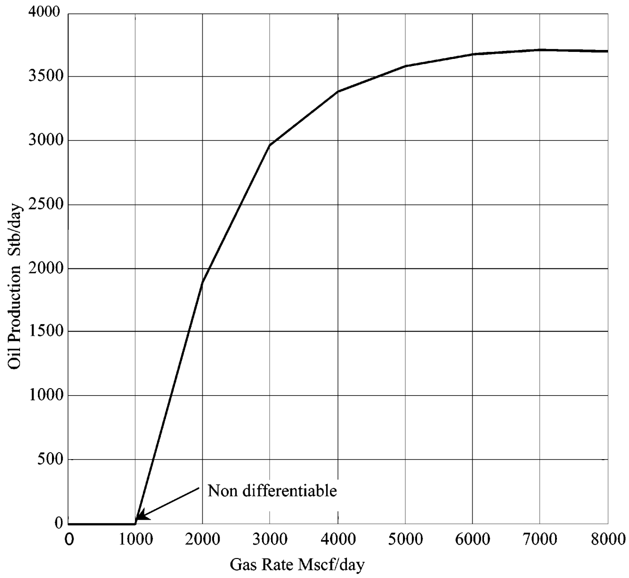

Figure 3. Oil−gas relation of a noninstantaneous gas-lift well.

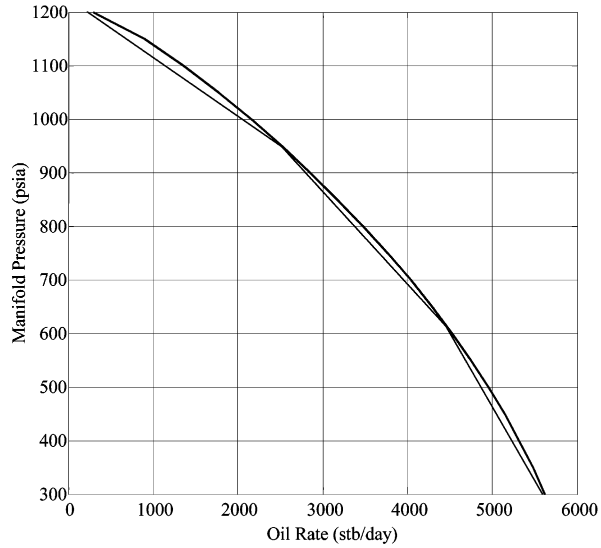

Figure 4. Piecewise linear approximation of the maximum well oil rate in naturally flowing wells.

and overcomes the computational difficulties of formulation (P2). The algorithm is based on (i) well oil rate upper and lower bounds and (ii) the approximation of each well with piecewise linear functions.

4.1. Well Oil Rate Upper and Lower Bounds and Piecewise Linear Approximation. Because a naturally flowing well has 2 degrees of freedom, when the manifold pressure is discretized at j points $P^{\mathrm{d}}_{m,j}$ and the choke of each well i that is connected to the manifold m is fully open, the maximum well oil rate $q^{\max,\mathrm{d}}_{\mathrm{o},i,j}$ is determined for each discrete value of the manifold pressure after solution of the well model. If for a given value of manifold pressure $P^{\mathrm{d}}_{m,j}$ the well model is infeasible, $q^{\max,\mathrm{d}}_{\mathrm{o},i,j}$ is equal to zero. The resulting flow rate is the maximum because the well was set fully open and any flow rate $q_{\mathrm{o},i}$ can be achieved by reducing the choke diameter. Figure 4 shows a typical schematic representation of the maximum well oil rate $q^{\max}_{\mathrm{o},i}$ as a function of the manifold pressure. The maximum well oil rate $q^{\max}_{\mathrm{o},i}$ can be approximated with piecewise linear functions by applying the following constraints:

$$\left.\begin{aligned}
&q^{\max}_{\mathrm{o},i} = \sum_j \lambda_{m,j} q^{\max,\mathrm{d}}_{\mathrm{o},i,j}\\
&P_m = \sum_j \lambda_{m,j} P^{\mathrm{d}}_{m,j}\\
&P^{\mathrm{L}}_m \le P_m \le P^{\mathrm{U}}_m\\
&q_{\mathrm{o},i} \le q^{\max}_{\mathrm{o},i}\\
&\sum_j \lambda_{m,j} = 1\\
&\lambda_{m,j},\ \mathrm{SOS}
\end{aligned}\right\}\quad \forall\, m, \forall\, i \in I_m \tag{27}$$

Constraint equation (27) sets an upper bound on the well flow rate for any manifold pressure. Next, well oil rate lower bounds for stable flow are determined. Lea and Tighe[33] proposed the following criterion for stability:

$$\frac{\partial P_i(L_{\mathrm{o}})}{\partial q_{\mathrm{o},i}} \ge 0,\quad \forall\, i \in I \tag{28}$$

Condition equation (28) states that the partial derivative of the bottom hole pressure of the well tubing with respect to the well oil flow rate must be nonnegative. A typical form of the bottom hole pressure as a function of the well oil rate is shown in Figure 5. The stability condition is considered by adding a lower bound in the well oil rate. Because a gas-lift well i has 2 degrees of freedom, by discretization of the manifold pressure at j points $P^{\mathrm{d}}_{m,j}$ and the gas injection rate of each well at k points $q^{\mathrm{inj,d}}_{\mathrm{g},i,k}$, the well oil flow rate $q^{\mathrm{d}}_{\mathrm{o},i,j,k}$ can be calculated by solving the well model equations for each grid point (j, k). If the simulation of the gas-lift well is infeasible, then the corresponding well oil rate $q^{\mathrm{d}}_{\mathrm{o},i,j,k}$ is set equal to zero. Therefore, the well oil flow rate $q_{\mathrm{o},i}$ for each gas-lift well can be approximated by the following piecewise linear relations:

$$\left.\begin{aligned}
&q_{\mathrm{o},i} = \sum_j \sum_k \mu_{i,j,k} q^{\mathrm{d}}_{\mathrm{o},i,j,k}\\
&q^{\mathrm{inj}}_{\mathrm{g},i} = \sum_j \sum_k \mu_{i,j,k} q^{\mathrm{inj,d}}_{\mathrm{g},i,j,k}\\
&P_m = \sum_j \sum_k \mu_{i,j,k} P^{\mathrm{d}}_{m,j}\\
&\sum_j \sum_k \mu_{i,j,k} = 1\\
&\eta_{i,j} = \sum_k \mu_{i,j,k}\\
&\xi_{i,k} = \sum_j \mu_{i,j,k}\\
&\zeta_{i,t} = \sum_j \mu_{i,j,j+t}\\
&n_{i,j}, \xi_{i,k}, \zeta_{i,t},\ \mathrm{SOS}
\end{aligned}\right\}\quad \forall\, m, \forall\, i \in I_m \tag{29}$$

Figure 6 depicts the oil flow rate as a function of the manifold pressure and gas injection rate for a gas-lift well. When the above developments are taken into account, the mathematical programming formulation for optimization of production operations can be written as follows:

$$\max\ w_{\mathrm{o}}\sum_{i\in N} q_{\mathrm{o},i} - w_{\mathrm{g}}\sum_{i\in G} q_{\mathrm{g},i}^{\mathrm{inj}}$$

s.t.

$$P_m(L_s) = f[P_m(L_m), q_{\mathrm{o},m}, q_{\mathrm{g},m}, q_{\mathrm{w},m}]$$

$$q_{p,m} = \sum_{i\in I_m} q_{p,i}, \quad \forall\ p$$

$$q_{p,m} \le C_{p,m}, \quad \forall\ p,\ m$$

$$P_m(L_s) = P_s, \quad \forall\ s,\ m$$

$$P_m^{\mathrm{L}} \le P_m(L_m), \quad \forall\ m$$

$$\sum_{i\in G} q_{\mathrm{g},i}^{\mathrm{inj}} \le C$$

$$\left.\begin{aligned}
& q_{\mathrm{o},i}^{\max} = \sum_j \lambda_{m,j} q_{\mathrm{o},i,j}^{\max,\mathrm{d}} \\
& P_m = \sum_j \lambda_{m,j} P_{m,j}^{\mathrm{d}} \\
& q_{\mathrm{o},i} \le q_{\mathrm{o},i}^{\max} \\
& \sum_j \lambda_{m,j} = 1 \\
& \lambda_{m,j},\ \mathrm{SOS} \\
& q_{\mathrm{o},i} \le q_{\mathrm{o},i}^{\max} \\
& q_{\mathrm{g},i} = \mathrm{GOR}_i q_{\mathrm{o},i}
\end{aligned}\right\} \forall\ m, \forall\ i \in N$$

$$\left.\begin{aligned}
& q_{\mathrm{o},i} = \sum_j \sum_k \mu_{i,j,k} q_{\mathrm{o},i,j,k}^{\mathrm{d}} \\
& q_{\mathrm{g},i}^{\mathrm{inj}} = \sum_j \sum_k \mu_{i,j,k} q_{\mathrm{g},i,j,k}^{\mathrm{inj,d}} \\
& P_m = \sum_j \sum_k \mu_{i,j,k} P_{m,j}^{\mathrm{d}} \\
& \sum_j \sum_k \mu_{i,j,k} = 1 \\
& \eta_{i,j} = \sum_k \mu_{i,j,k} \\
& \xi_{i,k} = \sum_j \mu_{i,j,k} \\
& \zeta_{i,t} = \sum_j \mu_{i,j,j+t} \\
& n_{i,j},\ \xi_{i,k},\ \zeta_{i,t},\ \mathrm{SOS}
\end{aligned}\right\} \forall\ m, \forall\ i \in G$$

$$\left.\begin{aligned}
& q_{\mathrm{w},i} = \mathrm{WOR}_i q_{\mathrm{o},i} \\
& q_{\mathrm{L},i} = q_{\mathrm{o},i} + q_{\mathrm{w},i} \\
& q_{\mathrm{o},i}^{\mathrm{L}} \le q_{\mathrm{o},i} \le q_{\mathrm{o},i}^{\mathrm{U}}
\end{aligned}\right\} \quad \forall\ i \in I \qquad \text{(P4)}$$

(P4) does not ignore any constraint of the original formulation (P1), and the manifold pressure (P_m) is an optimization variable and is not fixed to an arbitrary value.[2,18] Thus, (P4) takes into account the interactions among wells tied to a common manifold. However, the cost for taking into account the interactions among wells tied to a common manifold is the approximation of a gas-lift well model by a 2D curve as shown in Figure 6 instead of a 1D curve as shown in Figure 2. The mathematical programming formulation (P4) involves nonlinear equations and 0−1 binary variables; therefore, it belongs to the class of MINLP[34] problems. Moreover, it has two characteristics: (i) the number of nonlinear equality constraints is equal to the number of surface flow lines and independent of the number of wells, and (ii) the binary variables are used to linearize nonlinear constraints. The most efficient way to solve (P4) is a sequence of MILP problems following an SLP method.

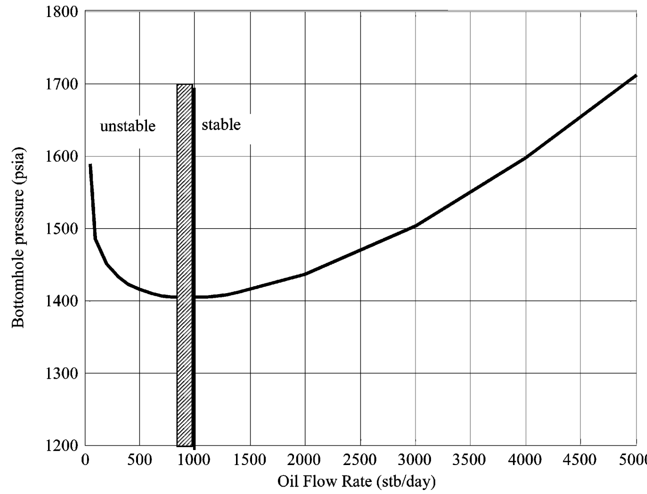

Figure 5. Well stable region.

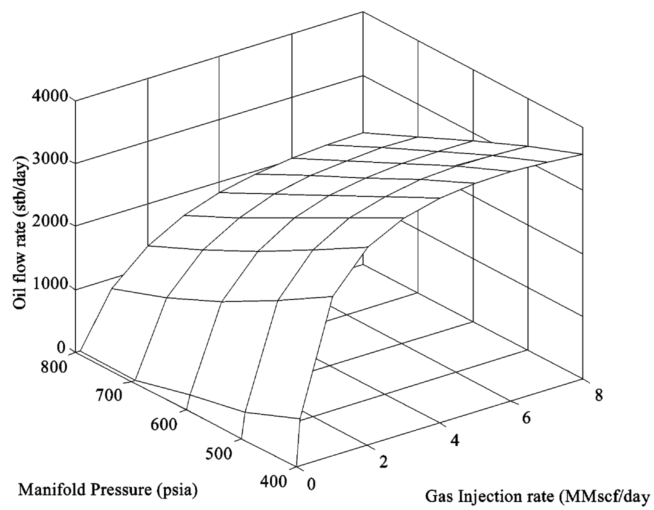

Figure 6. Piecewise linear approximation of a gas-lift well.

5. Solution Procedure

SLP methods solve nonlinear optimization programming problems via a sequence of LP problems. Since their introduction by Griffith and Stewart,[35] many variants of SLP algorithms have appeared in the literature.[36−38] In this work, the penalty successive LP algorithm of Zhang et al.[37] was applied. The formulation (P4) can be written in compact form as follows (P5):

$$\max\ f(\mathbf{x}) \tag{30}$$

s.t.

$$h(\mathbf{x}) = 0 \tag{31}$$

$$A\mathbf{x} + B\mathbf{y} \le c \tag{32}$$

$$\mathbf{x}^{\mathrm{L}} \le \mathbf{x} \le \mathbf{x}^{\mathrm{U}} \tag{33}$$

where $\mathbf{x}$ is the vector of continuous variables, $\mathbf{y}$ is the vector of binary variables, $f(\mathbf{x})$ is the linear objective function of (P4), $h(\mathbf{x})$ is the nonlinear equality constraints of (P4), which represent the surface flow-line model, and constraint (32) represents the mixed-integer linear constraints, which approximate the well models. The algorithm of Zhang et al.[37] involves the following steps:

1. Initialization. Set iteration counter $l = 1$ and select a starting point x_l after deleting the nonlinear constraints $[h(\mathbf{x})]$ in (P5) and solving the corresponding MILP problem. Select a large positive weight ϕ and scalars $0 < \rho_0 < \rho_1 < \rho_2 < 1$ (typically $\rho_0 = 10^{-6}$, $\rho_1 = 0.25$, and $\rho_2 = 0.75$). Let $\mathbf{\Delta}_l$ be a bound vector on continuous variables and $\Delta_{\mathrm{LB}} > 0$ its lower bound.

2. MILP Subproblem. Solve the following MILP to obtain a new point $\mathbf{x}$:

$$\max f(\mathbf{x}) - \phi[\sum_r (z_r^+ - z_r^-)]$$

s.t.

$$z_r^+ - z_r^- = h_r(\mathbf{x}_l) + \nabla h_r(\mathbf{x}_l)\,(\mathbf{x} - \mathbf{x}_l)$$

$$A\mathbf{x} + B\mathbf{y} \le c$$

$$\mathbf{x}^{\mathrm{L}} \le \mathbf{x} \le \mathbf{x}^{\mathrm{U}}$$

$$-\mathbf{\Delta}_l \le \mathbf{x} - \mathbf{x}_l \le \mathbf{\Delta}_l$$

$$z_r^+, z_r^- \ge 0 \tag{P6}$$

where r is the number of nonlinear equality constraints in (P4) and z_r^+ and z_r^- are nonnegative variables.

3. The trust region method[39] is used to determine whether the new point $\mathbf{x}$ should be accepted or rejected. First, $\Delta F_{\mathrm{E},l}$ and $\Delta F_{\mathrm{EL},l}$ are calculated

$$\Delta F_{\mathrm{E},l} = F_{\mathrm{E}}(\mathbf{x}_l) - F_{\mathrm{E}}(\mathbf{x}) \tag{34}$$

$$\Delta F_{\mathrm{EL},l} = F_{\mathrm{E}}(\mathbf{x}_l) - F_{\mathrm{EL}}(\mathbf{x}) \tag{35}$$

where

$$F_{\mathrm{E}}(\mathbf{x}_l) = f(\mathbf{x}_l) + \phi[\sum_r |h_r(\mathbf{x}_l)|] \tag{36}$$

$$F_{\mathrm{EL}}(\mathbf{x}) = f(\mathbf{x}_l) + \nabla f(\mathbf{x}_l)\,(\mathbf{x} - \mathbf{x}_l) + \phi \sum_r |h_r(\mathbf{x}_l) + \nabla h_r(\mathbf{x}_l)\,(\mathbf{x} - \mathbf{x}_l)| \tag{37}$$

$F_{\mathrm{E}}(\mathbf{x})$ is an exact penalty function, and $F_{\mathrm{EL}}(\mathbf{x})$ is a first-order Taylor approximation of $F_{\mathrm{E}}(\mathbf{x})$.

If $\Delta F_{\mathrm{EL},l} = 0$ or

$$|f(\mathbf{x}) - f(\mathbf{x}_l)| < \epsilon(1 + |f(\mathbf{x}_l)|) \tag{38}$$

stop; the optimum is found. Otherwise, compute the ratio $R_l = (\Delta F_{\mathrm{E},l}/\Delta F_{\mathrm{EL},l})$.

(i) If $R_l < \rho_0$, reject the current solution x and shrink $\mathbf{\Delta}_{l+1}$ to $0.5\mathbf{\Delta}_l$.

(ii) If $\rho_0 < R_l < \rho_1$, accept the current solution x, shrink $\mathbf{\Delta}_{l+1}$ to $0.5\mathbf{\Delta}_l$, and set counter $l = l + 1$.

(iii) If $\rho_1 < R_l < \rho_2$, accept the current solution $\mathbf{x}$ and set counter $l = l + 1$.

(iv) If $R_l > \rho_2$, accept the current solution x, $\mathbf{\Delta}_{l+1} = 2\mathbf{\Delta}_l$, and set counter $l = l + 1$.

Go to step 2.

Remarks

1. The MILP subproblems were solved in ILOG CPLEX 8.1[42] on an Intel 4 1.8 GHz machine.

2. To find a near-global optimal solution, the relative optimality criterion of MILPs was set 0.0001%, while the convergence criterion for the SLP, which is defined by eq 38, was set equal to $\epsilon = 10^{-3}$.

3. The adjacency condition can also be enforced in a Simplex type LP solver[39] with restricted basis entry rule, which allows only adjacent SOS variables to enter the Simplex tableau.

4. A similar algorithm was proposed by Bullard and Biegler[38] but in the context of solving nonsmooth nonlinear systems of equations, where the nonsmooth equations were handled using binary variables and the nonlinear system of equations was solved with an SLP method.

The solution procedure for the optimization of production operations involves the following steps:

(i) Preprocessing step, where the reservoir block pressure $P_{\mathrm{R},i}$, the productivity index PI_i, the GOR_i, and the WOR_i of each well are extracted from a reservoir simulator and each well model is approximated by piecewise linear functions. The following method is developed to construct piecewise linear approximations of each well model. The method utilizes hydraulic look-up tables. First, the choke valve model of each naturally flowing well is set fully open and the well tubing model, choke model, and well flow-line model are integrated for different discrete values of the manifold pressure, oil rate, gas rate, water rate, and gas injection rate, and the bottom hole pressure is stored in tabular form. Then using linear interpolation between the well inflow model and the tabulated bottom hole pressure, the maximum well oil rate is calculated as a function of the manifold pressure (eq 27) for the case of naturally flowing wells, while for the case of gas-lift wells, the oil rate is calculated for each discrete value of the manifold pressure and gas injection rate for gas-lift wells (eq 29). The well oil rate bound for stable flow can be extracted from the bottom hole table. It must be noted that the cost of constructing each well model is minimal. However, the accuracy of the approximation depends on the number of discrete points used to construct look-up tables. The computational cost of constructing look-up tables is not included in the optimization run because a preprocessing tool has been used for their construction.

(ii) Processing step, which involves the solution of (P4) as described previously.

(iii) Postprocessing step, which involves the determination of each well choke setting. The well choke setting is determined by fixing the manifold pressure, the well oil rate, and the gas injection rate at the values calculated from step ii and solving the corresponding well model.

6. Accuracy and Robustness of the Formulation

The accuracy and robustness of the optimization formulation (P4) has been investigated by comparing it with the exact optimization formulation (P2) in two example problems. The first example involves the two-well network shown in Figure 1, where the reservoir fluid is a dry gas. The corresponding exact optimization

Table 5. Reservoir, Well, and Separator Data for the Two-Well Dry Gas Network

	well 1	well 2
reservoir pressure (psia)	3660	3000
a, *b* coefficients[a]	793, 0.004	700, 0.003
well vertical length (ft)	5000	5000
well horizontal length (ft)	2000	2000
flow-line length (vertical) (ft)	2000	

	pressure (psia)	capacity (lbm/s)
separator	1500	10

[a] *a* and *b* reservoir deliverability coefficients.

Table 6. Interpolation Data for the Two-Well Dry Gas Network

manifold pressure (psia)	well 1 (lbm/s)	well 2 (lbm/s)
1800	4.97	2.80
1900	4.65	2.38
2000	4.30	1.92
2100	3.92	1.43
2200	3.52	0.91
2300	3.10	0.35

Table 7. Choke Settings for the Two-Well Dry Gas Network

	choke setting (% open)		choke setting (% open)
well 1	100	well 2	100

Table 8. Optimal Values of Both Methods in the Two-Well Dry Gas Network

	approximate optimization	exact optimization
objective value	6.04	6.04
manifold pressure (psia)	2017.4	2017.1
well 1 flow rate (lbm/s)	4.22	4.23
well 2 flow rate (lbm/s)	1.82	1.84

Table 9. Computational Performances of Both Methods in the Two-Well Dry Gas Network

	function evaluations	iterations
exact optimization	180	30
proposed algorithm	12	4

formulation is given in Appendix B. The reservoir block pressure, the productivity index, the well design parameters, and the separator pressure and capacity are summarized in Table 5. The proposed algorithm was applied. First, the manifold pressure was discretized from 1800 to 2300 psia every 100 psia and the corresponding maximum gas well rates are summarized in Table 6. The objective function was maximization of gas production, and the resulting MINLP problem involved one nonlinear equation because the system has one flow line and five binary variables. The MILP subproblems were solved using CPLEX,[42] and the optimality criterion was set equal to 0.0001%. Finally, the well choke settings were calculated by simulating each well after fixing the manifold pressure and the well gas rate to the optimal values. The choke settings are presented in Table 7, where it is observed that both wells have their chokes fully open. Then the exact optimization formulation (P2) was solved using the SQP solver of MATLAB.[40] The values of the objective function, manifold pressure, and gas well rates calculated from application of both methods are summarized in Table 8. The results suggest that the proposed method is very accurate. In Table 9, the computational performance, of both methods in terms of the number of iterations

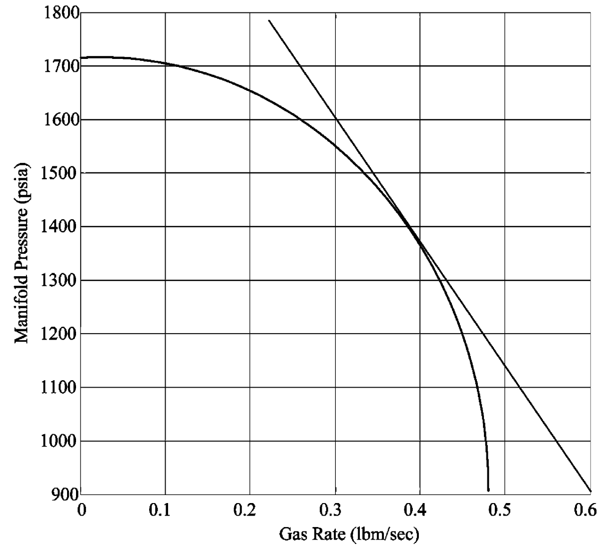

Figure 7. Linearization of a gas well manifold curve.

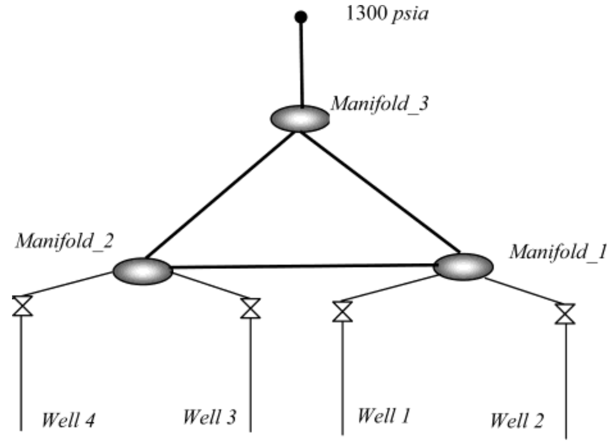

Figure 8. Four-well dry gas model.

and function evaluations are reported. The results suggest that the SQP method requires more iterations and function evaluations compared to the proposed method. This is due to the fact that linearization of highly nonlinear equations is likely to be poor even relatively close to the linearization point. This leads to successive solutions being very close together, which potentially slows the convergence. The case is depicted in Figure 7 where the manifold pressure is linearized. In addition, the increased number of function evaluations can be explained by the fact that the exact optimization formulation has three nonlinear constraints while the approximate formulation has only one.

The second example involves the pipeline network shown in Figure 8. The network consists of four dry gas wells and four flow lines, and the gas deliverability pressure was set equal to 1300 psia. The reservoir and well data are summarized in Table 10. The objective function was maximization of gas production, and the resulting MINLP problem involved 4 nonlinear equality constraints and 20 binary variables. The optimality condition for the MILP subproblems was set equal to 0.0001% and was solved with CPLEX.[42] The proposed method converged to the optimal solution after five iterations. The optimal point is presented in Table 11, where it is observed that well 4 is closed. The exact optimization formulation (P2) failed to converge because well 4 could not flow to the manifold pressure selected

Table 10. Reservoir, Well, and Separator Data for the Four-Well Dry Gas Model

	well 1	well 2	well 3	well 4	flow lines
reservoir pressure (psia)	3660	3000	2500	2300	
a, *b* coefficients[a]	793, 0.004	700, 0.003	650, 0.005	600, 0003	
well vertical length (ft)	5000	5000	5000	5000	
well horizontal length (ft)	2000	2000	2000	2000	8000
diameter (in.)	2.5	2.5	2.5	2.5	3.0
			pressure (psia)		
separator			1300		

[a] *a* and *b* reservoir deliverability coefficients.

Table 11. Optimal Values of the Four-Well Dry Gas Model

	well flow rate (lbm/s)		pressure (psia)
well 1	4.680	manifold 1	1922.1
well 2	2.446	manifold 2	1817.5
well 3	0.982	manifold 3	1526.9
well 4	0.0	manifold 4	1300.0

by the optimizer. It must be noted that the accuracy of the method has been shown in a number of examples including naturally flowing and gas-lift wells.

7. Examples

7.1. Naturally Flowing Wells. A field consisting of 10 naturally flowing wells and 2 separators is analyzed. The pipeline network is depicted in Figure 9, the reservoir and well data are given in Table 12, and the operating pressure and capacity of each separator are given in Table 13. The objective function is maximization of oil production and the MILP subproblem involves 20 binary variables, 942 constraints, and 535 continuous variables. The problem is solved using CPLEX[42] with optimality criterion equal to 0.0001%. The optimal fluid flow rates in the separators are given in Table 13. The results of Table 13 suggest that the gas capacity of separator A is active and the wells with the higher GOR are choked back, as can be observed in Table 14, where the GOR and WOR of each well and the corresponding optimal choke settings are reported. Similarly, the water capacity of separator B is the bottleneck of the network as shown by Table 13. The wells with the highest WOR are operated with choke restrictions, as can be observed from the results summarized in Table 14. Then the LP method proposed by Wang et al.[18] was applied to solve the naturally flowing well network. Their algorithm consists of the following steps:

(i) The well chokes are set fully open, and the corresponding network is simulated without considering the separator capacity limits.

(ii) The oil flow rates calculated from the previous step become upper bounds, which along with separator handling limits and mass balances on the node constitute the LP formulation whose solution determines the optimal operating policy.

The results of the LP method are summarized in Table 13. The results of Table 13 suggest that the proposed method produces 94 more barrels of oil per day compared to the LP method. The reason is that setting the well chokes fully open does maximize oil production.

7.2. Gas-Lift Well Example. A field consisting of 13 gas-lift wells and 2 separators was analyzed. The pipeline network is depicted in Figure 10, and the reservoir, well, and separator data are given in Table 15. The gas−oil relation for each gas-lift well was constructed by discretizing the manifold pressure between 300 and 800 psia every 100 psia and the gas injection rate at 0, 250, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, and 8000 Mscf/day. The computational time for the preprocessing step, which involves the approximation of all wells, is 7.2 s on an Intel 4 1.8 GHz machine The objective function was maximization

Table 12. Reservoir and Well Data for the Naturally Flowing Wells Example

	reservoir pressure (psia)	PI (stb/psia)	horizontal (ft)	vertical (ft)	diameter (in.)
well 1	1720	8.0	2000	3400	4.0
well 2	1700	8.0	2000	3000	3.5
well 3	1800	6.0	2000	3000	3.5
well 4	2000	6.0	2500	4000	4.0
well 5	2100	10.0	2500	4000	4.0
well 6	1200	7.0	2500	4000	3.5
well 7	1100	6.9	2500	4000	4.0
well 8	1000	6.0	2000	3000	3.5
well 9	1000	5.0	2000	3000	4.0
well 10	1100	6.0	2500	4000	4.0
flow line A				5000	5.0
flow line B				5000	5.0

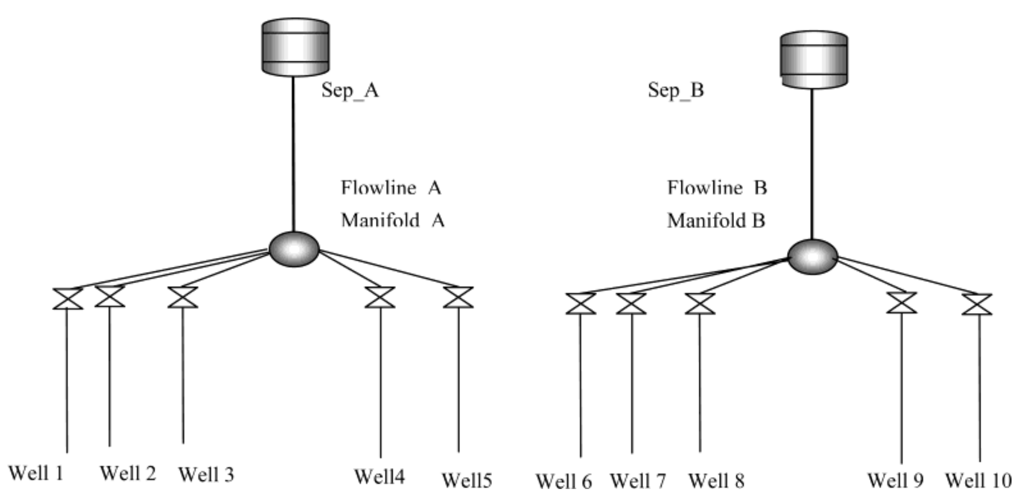

Figure 9. Naturally flowing well example.

Table 13. Separator Data and Optimal Separator Flow Rates

	Sep_A capacity	approximate method	LP method	Sep_B capacity	approximate method	LP method
oil (barrel/day)	15000	12047.4	11989.1	10000	6738.6	6702.4
water (barrel/day)	5000	2629.3	2446.8	2000	2000	2000
gas (MMscf/day)	25000	25000	25000	18000	8561.8	8658
pressure (psia)	600			300		

Table 14. Reservoir and Optimal Well Choke Settings of the Naturally Flowing Well Example

	GOR (Mscf/stb)	WOR (stb/stb)	choke settings (% open)
well 1	2000	0.3	100
well 2	2500	0.1	27.1
well 3	2000	0.7	68.3
well 4	3000	0.3	11.4
well 5	2000	0.01	100
well 6	1000	0.01	100
well 7	1500	0.5	49.5
well 8	2000	0.5	29.4
well 9	1000	0.3	100
well 10	1500	0.5	30.0

Table 15. Reservoir and Well Data for Gas-Lift Well Example

	reservoir pressure (psia)	PI (stb/psia)	horizontal (ft)	vertical (ft)	diameter (in.)
well 1	1920	13	1000	1000	4
well 2	1900	9	1000	2000	4
well 3	1800	6	2500	2700	4
well 4	1720	10	3000	2000	4
well 5	1600	5	2500	2000	4
well 6	1820	3	2500	2500	4
well 7	1420	4	2000	2000	4
well 8	1320	2	2000	3000	4
well 9	1450	3.5	2000	3000	4
well 10	1550	5.5	1500	2000	4
well 11	1720	10	2000	4000	4
well 12	1720	10	2000	1500	4
well 13	1520	8	3000	5000	4
flow line A				5000	6.0
flow line B				5000	6.0

	pressure (psia)	oil capacity (stb/day)	gas capacity (Mscf/day)	water capacity (stb/day)
separator A	300	15000	25000	6000
separator B	300	15000	15000	4000

of the profit (eq 18), where the price of the oil was 10$/barrel and the cost of gas compression was 0.1$/MMscf. The MILP subproblem involved 348 binary variables, 912 continuous variables, and 1160 constraints, and the optimality criterion was set equal to 0.0001%. The algorithm converges to the optimal solution after five iterations. The optimal well oil rate and gas injection rate for each well are summarized in Table 16. The results indicate that the field will produce at total oil rate of 19904.4 stb/day.

Table 16. Comparison of the Approximate and IGOR Heuristic Rule in the Gas-Lift Example

	approximate method		IGOR method	
	gas-lift rate (Mscf/day)	oil rate (stb/day)	gas-lift rate (Mscf/day)	oil rate (stb/day)
well 1	4000	2833.7	2500	2750.2
well 2	3000	2651.1	3000	2734.0
well 3	3000	1539.7	2530	1463.2
well 4	1966	1979.0	2000	1987.2
well 5	2750	1535.33	2500	1550.2
well 6	631	948.4	1200	1153.1
well 7	1500	869.6	1330	761.9
well 8	750	353.2	650	240.87
well 9	500	485.1	1082	710.9
well 10	2250	1584.7	2082	1486.4
well 11	663	1759.3	430	1613.2
well 12	2250	2971.7	1910	2792.6
well 13	500	393.6	560	238.8
total		19904.4		19482.6

Then the problem is solved by applying the IGOR heuristic rule as follows:

(i) A realistic manifold pressure is selected, and the IGOR curve for each manifold is constructed. The pressure of each manifold was set equal to 575 psia, and the manifold IGOR curve was constructed as follows. First, the gas−oil relation for each gas-lift well is constructed, and then by variation of the value of IGOR, the manifold oil rate as a function of the gas flow rate is plotted as shown in Figure 11.

(ii) The optimal IGOR is the one that maximizes oil production while satisfying separator gas capacity constraints. From Figure 11, the optimal IGOR is calculated.

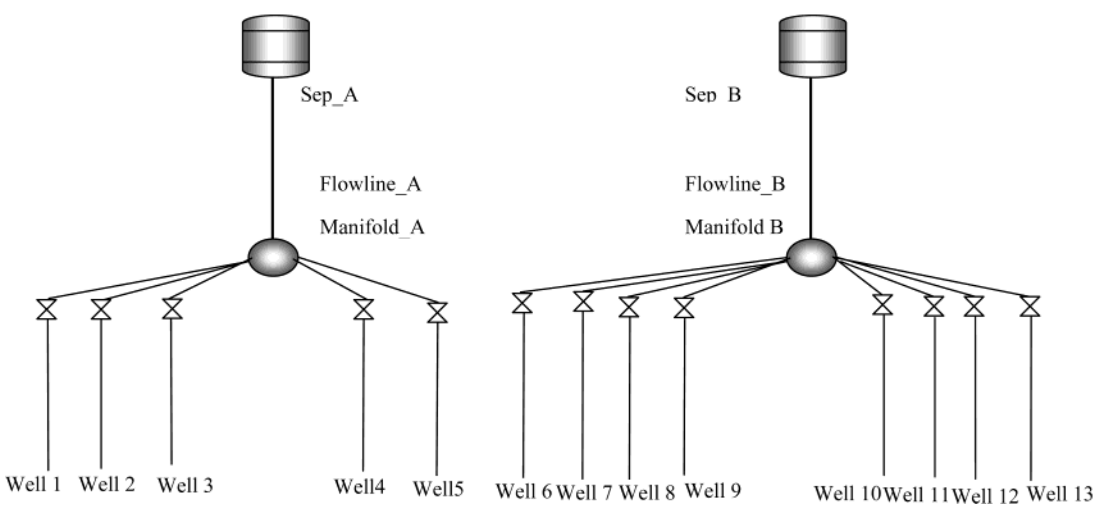

Figure 10. Gas-lift well pipeline network.

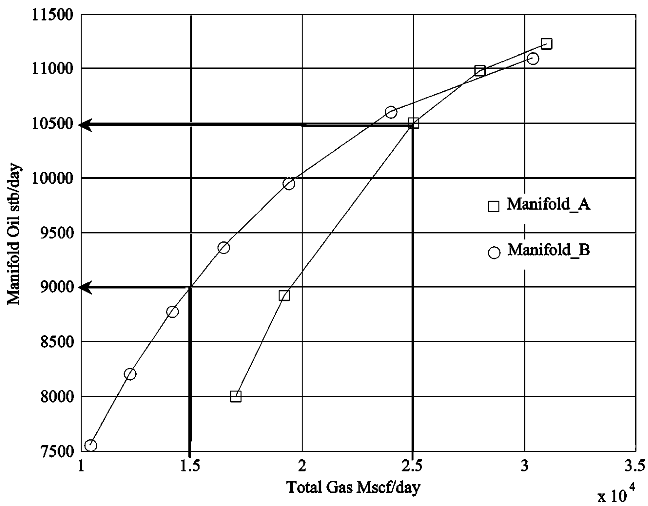

Figure 11. Manifold IGOR curves.

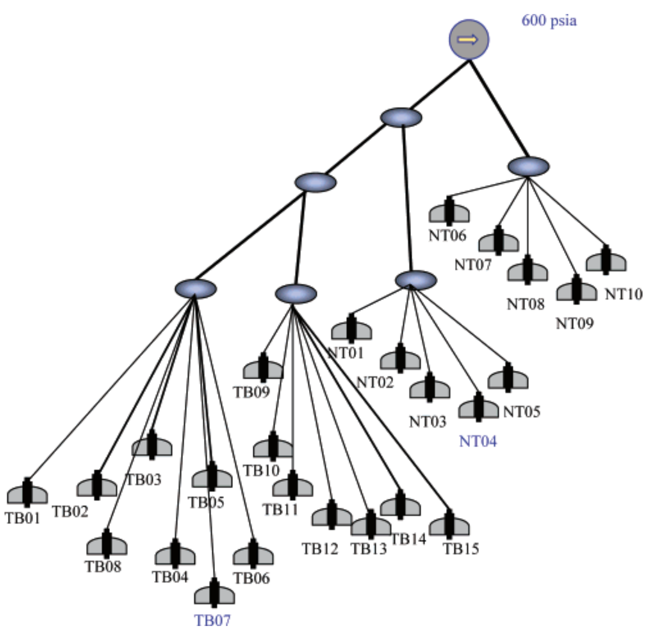

Figure 12. Treelike structure pipeline network.

(iii) From the optimal value of the IGOR, the gas-lift rates are calculated by fixing the IGOR of each well and the manifold pressure.

(iv) Using the gas-lift rates calculated from the previous step, the network is simulated and the optimal well oil rates are determined.

The well oil rates resulting from application of the IGOR heuristic rule are summarized in Table 16, where a difference of 422 barrels of oil per day is observed in favor of the proposed method. The result can be explained by the fact that the IGOR curves are constructed by neglecting the interactions between wells sharing a common flow line because the manifold pressure was fixed to a rather arbitrary value.

7.3. Treelike Structure Pipeline Network. To evaluate the computational performance of the algorithm, the pipeline network shown in Figure 12 was solved using the proposed method. The network consists of 25 wells, where 15 of them are gas-lift wells (TB01, ..., TB15) and the remaining 10 wells are naturally flowing (NT01, ..., NT10). The delivery pressure was set equal to 600 psia. Two scenarios of the problem were solved. In the first scenario, there is an upper bound on the total gas production, while in the second, there is an upper bound in the total water production. The objective in both cases was maximization of oil production, and unlimited gas-lift availability was assumed. The problem involved 612 binary variables, 1660 continuous variables, and 2133 constraints. The CPU time for approximating well models is 11.25 s. The total oil production, the number of iterations, and the CPU time on an Intel 4 1.8 GHz machine are reported in Table 17. It must be noted that the cost of solving each MILP subproblem was between 3 and 5 s, while the rest of the time was consumed in function evaluations of the pipeline network. In addition, as can be seen from the results of Table 17, the number of iterations required by the algorithm to converge was on the same order of magnitude as that for the previous examples.

Table 17. Computational Statistics of a Treelike Structure Pipeline Network

constraint	oil production (stb/day)	iterarations	CPU (s)
q_g < 110 MMscf/day	55 428	7	47.6
q_w < 10 000 stb/day	50 226	5	33.8

8. Conclusions

In this paper, a new formulation has been proposed for the optimization of oil and gas production operations. The new formulation simultaneously optimizes well rates and gas-lift allocation, is able to handle flow interactions among wells, and can be applied to difficult situations where some wells are too weak to flow to the manifold or require a certain amount of gas lift to flow. The accuracy, efficiency, and robustness of the formulation have been established by comparison with an exact optimization formulation that was solved using an SQP method in a number of examples. The algorithm is applicable both to treelike structure pipeline networks and to pipeline networks with loops. Because of the ability of the algorithm to account for flow interactions, it will always propose superior operating policies compared to the heuristic rules typically applied in practice. The proposed optimization method can be used for real-time production control because all of the variables required for the construction of a well model can be measured and the discrete data can be directly incorporated in the formulation, a subject currently under further investigation.

Appendix A

To impose the adjacency condition in the SOS variables, the following binary variables are introduced:[31]

$$\lambda_{i,1} \le y_{i,1} \tag{A.1}$$

$$\lambda_{i,j} \le y_{i,j-1} + y_{i,j}, \quad j = 2, ..., |J| - 1 \tag{A.2}$$

$$\lambda_{i,J} \le y_{i,J-1} \tag{A.3}$$

$$\sum_j y_{i,j} = 1 \tag{A.4}$$

where $y_{i,j}$ are binary variables.

Appendix B: Dry Gas Exact Optimization Formulation

The dry gas exact optimization formulation is as follows:

$$\max q_{g,m}$$

s.t.

$$P_m(L_s) = f(P_m, q_{g,m})$$

$$P_m(L_s) = P_s$$

$$\sum_i q_{g,i} = q_{g,m}$$

$$q_{g,m} \le C$$

$$P_i(L_m) = g[q_{g,i}, P_i(L_{ch}^{+})], \quad \forall\ i$$

$$P_{R,i}{}^2 - P_i{}^2(L_0) = a_i q_{g,i}{}^2 + b_i q_{g,i}, \quad \forall\ i$$

$$0 \le q_{g,i} \le q_{g,i}^{U}, \quad \forall\ i$$

where $q_{g,i}$ is the well gas flow rate and a_i and b_i are known as reservoir deliverability coefficients.[41]

Nomenclature

I = set of wells i
N = set of naturally flowing wells i
G = set of gas-lift wells i
M = set of manifolds m
S = set of separators s
d_i = diameter of choke i
GOR_i = gas−oil ratio of well i
$q_{p,i}$ = flow rate of phase p in stock tank conditions
$q_{g,i}^{inj}$ = gas injection rate of gas-lift well i
$P_{R,i}$ = reservoir block pressure of well i
$P_i(L_0)$ = well bore pressure of well i
$P_m(L_m)$ = pressure of the manifold m
P_s = separator pressure
PI_i = productivity index of well i
WOR_i = water oil ratio
y_i = pressure ratio of choke i
y_c = critical pressure ratio
w_o, w_g = weighting coefficient

Units

stb/day = barrels per day
scf/day = standard cubic feet per day

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Received for review October 8, 2003
Revised manuscript received January 9, 2004
Accepted January 13, 2004

IE034171Z