CORRESPONDENCE

Air Quality Control Using Minimum Pollution Dispatching Algorithm

Sir: The paper summarizing the ongoing research by the authors [R. L. Sullivan and D. F. Hackett, *Environ. Sci. Technol.*, **17** (11), 1019–22 (1973)] is an interesting and timely contribution to an important problem, and we merely wish to add some additional concepts and ideas which extend the usefulness of their models. Particularly, we would like to elaborate on the authors' closing statements, that minimization techniques other than those presented should be considered.

To begin with, we wish to use a somewhat simpler notation as follows: Let there be J sources of power, $j = 1, 2, 3, \ldots, J$, and I receptor or control points, $i = 1, 2, 3, \ldots, I$. Let a_{ij} = ground level concentration of a pollutant at a point i from a unit of production at source j. Furthermore, let the production at point j be limited by lower and upper bounds—i.e., for all j, $l_j \leq P_j \leq u_j$, and let the SO_2 emission be related to power production by the function, $Q_j = \alpha_j + \beta_j P_j + \gamma_j P_j^2$, and let the total power requirement be P.

1. Consider now the case where we wish to minimize the ground level concentration for a particular point, say point 1. [The authors' terminology of "environmental cost" is somewhat awkward and inappropriate; their objective function (*1*) is the ground level concentration at a specific control point.] The mathematical programming problem then is,

$$\text{Minimize: } \sum_{j=1}^{J} a_{ij}(\alpha_j + \beta_j P_j + \gamma_j P_j^2)$$

$$\text{Subject to: } l_j \leq P_j \leq u_j \qquad j = 1, 2, 3, \ldots, J$$

$$\sum_{j=1}^{J} P_j \leq P$$

This type of problem is known as the quadratic programming problem—that is, to find the minimum of a quadratic function subject to linear constraints. There are several algorithms and computer programs available to solve this type of problem (*1, 2*). These algorithms have great advantages over the Lagrangian multiplier approach. While a Lagrangian multiplier approach and the Kuhn-Tucker condition identify a global optimum, they are not a readily implementable procedure to find the optimal solution. For a discussion of the problems with this approach, see Charnes and Cooper (*3*). To find a global optimum, the only requirement is that the function must be convex, which is the case here, since the function is a positive semidefinite quadratic form.

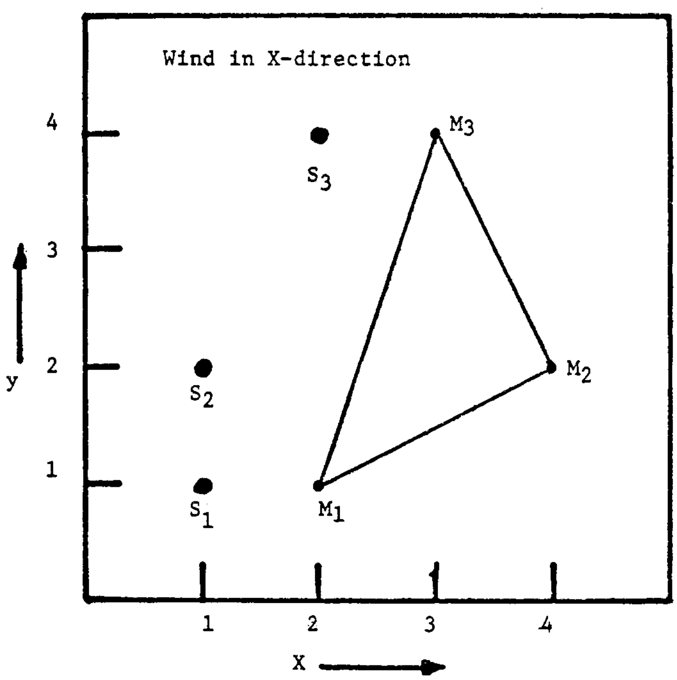

Figure 1. Three point sources of pollution

2. Consider now the case where one wishes to minimize the cost of producing power subject to the constraint that the ground level concentration at a particular point, say k, must be below some limiting value. Assume that the cost of producing power is a convex function of the form $f_j(P_j)$, then we have the problem,

$$\text{Minimize: } \Sigma f_j(P_j)$$

$$\text{Subject to: } l_j \leq P_j \leq u_j$$

$$\sum_{j=1}^{J} P_j \geq P$$

$$\sum_{j=1}^{J} a_{kj}(\alpha_j + \beta_j P_j + \gamma_j P_j) \leq C_k$$

Again, this is a convex programming problem with one additional quadratic constraint. But, it is a convex constraint, and can be handled by the penalty function approach by adding the constraint to the objective function.

3. Consider now the case where the maximum allowable ground level concentration is specified not for a single control point, but for an entire area. The basic problem then is to determine where to locate the, for example, control points, such that these points will be representing the areas of maximum concentration. One approach is to put a fine grid over the area of interest, and specify a maximum ground level concentration for every point, This, of course, leads to a large number of constraints and a significant computational burden. A more efficient approach is to exploit the properties of the dispersion model equation. This function is a unimodal function reaching its maximum value at a certain distance downwind from the source. The ground level concentration of any point is the summation of the contributions from each source, and this function is a multipeak surface obtained by summing the individual unimodal functions. It is not known, a priori, which of the peaks is the greatest, nor where the peaks actually occur. However, since all the individual functions are of the unimodal type, the maxima (peaks) must occur on a line connecting the maxima of the individual functions. It is therefore sufficient to write the constraints for a number of points along these lines. If the constraints are met at these points, they will be met at every other point in the region (*4*).

Consider, for example, three sources of pollution, arranged in an area as shown in Figure 1. Assume a wind direction that is out of the west (in X direction), and further assume for simplicity the dispersion model to be of the form

$$C_1(x,y) = [1/(x - 1)]\, e^{-[(x-1)^2 + (y-1)^2]}$$

$$C_2(x,y) = [1/(x - 1)]\, e^{-[(x-1)^2 + (y-2)^2]}$$

$$C_3(x,y) = [1/(x - 2)]\, e^{-[(x-2)^2 + (y-4)^2]}$$

The individual maximum ground level concentrations of the three sources will occur at the points M_1, M_2, and M_3. It is therefore sufficient to write the constraints for the ground level concentrations at points along the lines connecting these three points.

Finally, we wish to point out that the authors might want to consider the operation of the generators at the power stations as an on-off proposition—that is, a generator is either operating or not. Under this assumption, the problems may be reformulated as integer programming problems.

Literature Cited

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Sir: The authors (R. L. Sullivan and D. F. Hackett) would like to thank Deininger and Cohen (see above) for their interesting response to the authors' paper on ambient air quality control [*Environ. Sci. Technol.*, **17** (11), 1019–221 (1973)]. Since they made several points concerning various aspects of the paper, each point will be addressed separately.

As Deininger and Cohen correctly pointed out, the basic problem could be formulated as a quadratic programming problem subject to either linear or quadratic constraints, where solution techniques for solving such problems are readily available. The Lagrange multiplier and Kuhn-Tucker approach was used in the paper in spite of the computational difficulties, which can be circumvented (*1*), because of the inherent valuable control information associated with the various multipliers.

For the power system dispatcher to make a control decision, it is often convenient and necessary to have sensitivity information for each of the independent parameters. The Lagrange multiplier, λ, for example, provides the dispatcher with sensitivity information relating the change in ground level concentrations to the change in the amount of imported or exported power (*2*)—i.e.,

$$\lambda = -\frac{\Delta L}{\Delta P_T} \text{ or specifically}$$

$$\lambda = -\frac{P_{\text{Load}} + P_T + \sum_{i=1}^{G} \frac{\beta_i}{\gamma_i}}{\sum_i \frac{1}{M_i K_i \gamma_i}} \tag{1}$$

$$L = \sum_{i=1}^{G} \epsilon_i = \sum_{i=1}^{G} M_i Q_i = \sum_i^G M_i k_i \left(\alpha_i + \beta_i P_{Gi} + \frac{\gamma_i}{2} P_{Gi}^2\right)$$

$$P_T = \Sigma P_{Gi} - P_{\text{Load}}$$

L = ground level SO_2 concentration in ppm
M_i = meteorological coefficient containing all meteorological parameters
k_i = conversion factor converting quadratic cost curves into quadratic emission rate curves

From Equation 1, we have

$$\Delta L = -\lambda \ \Delta P_T \text{ or } \Delta P_T = -\frac{\Delta L}{\lambda}$$

With this information and incremental cost information for the two systems involved, and operator could determine the cost of decreasing the ground level concentration by amount ΔL. Obviously, it is also possible to utilize the Kuhn-Tucker dual variables in the same manner.

Another advantage of the approach taken by the authors is that if no inequality constraints are violated, then an optimal linear control law defined by the following equation can be developed:

$$\Delta \begin{vmatrix} P_{G1} \\ \vdots \\ P_{GG} \end{vmatrix} = K \begin{vmatrix} \Delta P_{L1} \\ \vdots \\ \Delta P_{LN} \end{vmatrix} \tag{2}$$

ΔP_{Gi} = change in output of generator i
ΔP_{Lj} = change in load at bus j

The optimal control strategy defined by Equation 2 facilitates changing the generator outputs automatically as *small* changes in load occurs, thus ensuring continuous minimum pollution dispatching (*1*).

Deininger and Cohen also suggest a problem formulation that is amenable to integer programming techniques. Unfortunately the authors feel that the use of integer programming is inappropriate in this case since the requirement that the generators must be on or off is implied. In reality, generators operate, once they are on-line, over their complete capability range in a continuous fashion, although certainly the range of operation could be discretized to allow only integer operating levels. Integer programming could be used, however, to determine which generators to commit to service, since in this case the generators are either committed or not committed—a point worth further investigation.

To conclude this discussion, the authors would like to point out that this basic approach to ambient air quality control should be classified as an on-line approach only. It does not, for example, suggest how a given utility with a certain amount of low-sulfur fuel could best utilize that fuel, taking into consideration generator forced outages and maintenance schedules. The authors are presently investigating a technique that is basically a probabilistic planning tool for determining minimum pollution fuel usage strategies and maintenance schedules.

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