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Radiation from a microwave source in the intermediate zone

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The vertical distribution of the field radiated by a microwave transmitter in the proximity of a smooth earth is obtained by direct application of Huygens' principle. For points at either side of the horizon and on the horizon itself this new approach offers uniform results which are free from the interpolations which characterize the methods currently used.

STATEMENT OF THE PROBLEM

The problem of calculating the radiation of a microwave source over a smooth earth in the so-called intermediate zone, i.e., near the horizon of the transmitter, hitherto has not been satisfactorily solved. Yet the need to know the field distribution a few meters above the sea surface, at ranges of several tens of kilometers from a low-placed microwave transmitter, is a problem of considerable current interest.

The traditional approach to the problem of a dipole radiating above a smooth spherical earth is extensively treated by *Bremmer* [1949]. The solution is reached by applying Watson's transformation to a zonal harmonic expansion of the radiated field, and imposing the boundary conditions. In this form the problem is reduced to the evaluation of an integral in the complex plane. This integral can be identified with a series of residues around the infinite poles of the integrand. To each pole corresponds a spherical wave which is standing in the radial direction, and travelling in the angular direction, with respect to the earth's center. The natural quenching of these waves increases with the order of the poles. The number of terms needed to approximate the series at a point increases as the distance between the source and the point decreases. On the horizon of the source, the sum loses its convergence. At closer range a solution may be again obtained by means of the saddle point method.

In brief, while the radiated field can be computed within the horizon (interference zone), or beyond the horizon (diffraction zone), there exists an intermediate region for which no direct solution is available.

A procedure suggested by *Kerr* [1951] for determining the field-strength-versus-distance relationship in the intermediate zone consists of, in his words, "extending the plot from the region in which interference methods are valid through the intermediate region into the region of validity of the diffraction methods by bold interpolation." Obviously, the validity of this procedure is open to question.

OUTLINE OF THE NEW SOLUTION

The need for getting a satisfactory description of the field radiated over the sea and near the horizon by a microwave source has pushed us to seek a solution free from the previous limitations. According to Huygens' principle the field created at a point is the result of the interference of all the spherical waves radiated by every element of any surface surrounding that point.

Using the method to be described, we are building up the field using this principle, step by step, along the propagation path, from the source down to the horizon, and beyond.

In order to simplify our task, we have considered a scalar field, so that the algorithm of Huygens' principle is Kirchhoff's integral,

$$E(P) = -(k/4\pi) \iint_S (1 + \cos\theta) E(r) \cdot \{[\exp(-jkr)]/r\}\, dS' \qquad (1)$$

where $E(P)$ is the field at the observer's point, $E(r)$ the field distribution on the surface considered, r the distance between a surface element and P, θ the angle between the normal to the surface and the ray path, and k the wavenumber. The integral is extended over the whole surface surrounding P.

A further simplification frequently introduced in problems of this kind was adopted here by substituting the spherical earth with a cylindrical boundary. In this way, and assuming independence of the field from the axial coordinate, the problem is made two-dimensional.

The geometry used is shown in Figure 1. The plane of the figure represents the vertical plane containing the transmitter T. The circular arc OHF indicates the earth's surface. The tangent to this arc, passing through T, defines the point on the horizon, H. The origin of the coordinates is chosen at the intersection of the above-mentioned arc with the normal to the segment TH, drawn from T. The radius of the arc corresponds to the modified effective earth's radius [*Bean and Dutton,* 1966],

$$a = 1/[1/a_0 + (1/n)\, dn/dh] \qquad (2)$$

where $(1/n)\, dn/dh$ is the logarithmic gradient of the tropospheric index of refraction in T, and $a_0 = 6.371 \times 10^6$ m is the mean radius of the earth.

The vertical directivity of the transmitting antenna may be accounted for by assuming a finite distribution of line sources orthogonal to the figure along the plane $x = 0$.

On the basis of our geometry, we can now obtain the field at a point P_A, located on the plane $x = x_A$, by numerically integrating equation 1. To this purpose we must compute amplitudes and phases of all the waves radiated from each line source of which

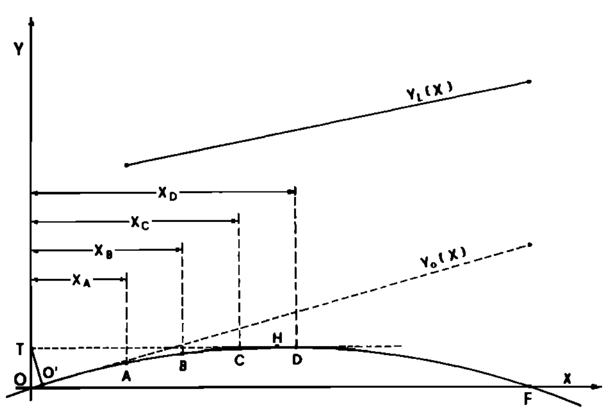

Fig. 1. Geometry of the problem.

the array is formed which reach point P_A both by direct path and by reflection upon the surface arc OA. The field intensity in P_A is given by the vectorial sum of all these components.

Extension of the process to many points of equal abscissa will yield the field distribution along $x = x_A$.

Our next step consists of calculating the field at a new point P_B, having another abscissa $x = x_B$, further elongated from the source. To this purpose we may again apply Huygens' concept. However, now we will assume that the radiation, instead of coming from the true array on $x = 0$, originates in the distribution of the fictitious sources calculated on the ordinate $x = x_A$.

The process previously described is thus repeated. The field in P_B is obtained as the vectorial sum of all the wavelets leaving the various points considered on $x = x_A$ and reaching P_B both directly and by means of reflection on the surface arc AB.

It is legitimate to inquire how many points are needed in order to approximate the field satisfactorily. Notwithstanding that the convergence is asymptotic [*Baker and Copson,* 1953], the sum can be performed in most cases with a reduced number of addends, as will be shown later.

The field distribution along the ordinate $x = x_B$ is obtained in this way by choosing a sufficient number of points P_B at various heights. The previous procedure can be repeated at pleasure. Thus, the field distribution on $x = x_C$ may be derived from the distribution on $x = x_B$, the field distribution on $x = x_D$ from the distribution on $x = x_C$, and so on, proceeding step by step toward increasing abscissae, up to the horizon and beyond.

The applicability of this method is only limited by the need of a reasonable economy in computer time. Abscissae separation and height intervals on a chosen ordinate are the result of a compromise between required overall precision and the number of operations involved.

We have noticed that direct calculation from the array source located on $x = 0$ and the ordinate considered offers a field distribution which is correct for all heights above a certain minimum. Underneath this minimum the shadowing effect of the earth becomes important. Thus, it appears possible to build up the field distribution at any abscissa by the fusion of two different parts, one obtained by a one-step computation from the emitting plane, the other obtained from the field distribution on a closer plane by means of the multi-step method previously described. The suture point lies approximately on the geometric horizon of point 0 (see Figure 1). Here the two solutions differ only by a normalization factor. The results obtained by this method represent a considerable saving in effective computation time.

The practical feasibility of the method outlined in this section will be proved later.

ALGEBRA OF THE SOLUTION

The reader is referred to Figure 2. A typical step is described, valid for getting the field distribution on any abscissa x_B from the known field distribution at a preceding abscissa x_A. Let

$$y_{n_A} = y_c(x_A) + n_A \Delta y_A \tag{3}$$

be the ordinates of the $N_A + 1$ selected points of abscissa x_A, in which the real and imaginary parts of the field $E(x_A, y_{nA})$ are assumed to have already been calculated and stored. In (3) n_A is any integer between 0 and N_A, Δy_A is a constant interval of coordinate, and y_C is the ordinate of the point at the earth's surface.

Analogously, $N_B + 1$ points are selected for the abscissa x_B, having ordinates

$$y_{n_B} = y_c(x_B) + n_B \Delta y_B \tag{4}$$

Integral (1) is adequately evaluated by Simpson approximation. Thus we write,

$$E(x_B, y_{n_B}) = (\Delta y_A/3) \sum_{k=1,3,5,\cdots}^{k=N_A} [(D_k + 4D_{k+1} + D_{k+2}) + (R_k + 4R_{k+1} + R_{k+2})] \tag{5}$$

where

$$D_k = E(x_A, y_{k_A}) \exp(-j2\pi r_{AB}/\lambda)/r_{AB} \tag{6}$$

represents a wavelet moving directly from a point of the distribution A to P_B, and,

$$R_k = E(x_A, y_{k_A})\Gamma_S \exp[-j(2\pi S_{AB}/\lambda + \phi_S)]/S_{AB} \tag{7}$$

represents a wavelet reaching P_B through reflection, $\Gamma_S \exp(-j\, \phi_S)$ being the reflection coefficient. The remaining symbols are explained in Figure 2.

The following expressions are easily found. Coordinates of the point on the horizon:

$$x_H = (2ah_T + h_T^2)^{1/2}, \quad y_H = a - (a^2 - x_H^2)^{1/2} \tag{8}$$

Equation of the reflecting surface:

$$y_c(x) = (2x_H x - x^2)/2a - [(x_H - x)^4 - x_H^4]/8a^3 \tag{9}$$

Height of a point of coordinates x, y:

$$h(x, y) = [(x_H - x)^2 + (a - y_H + y)^2]^{1/2} - a \tag{10}$$

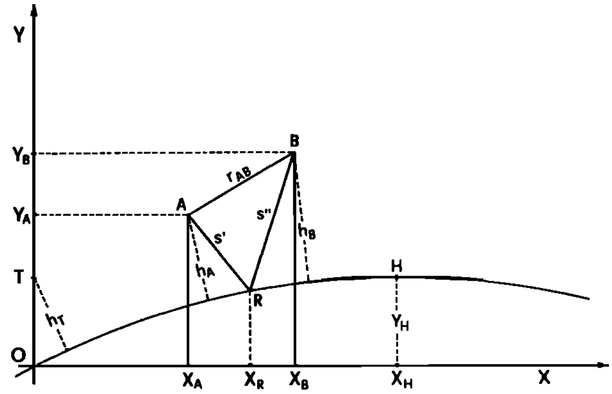

Fig. 2. Graphic visualization of a typical step.

Direct distance between point A and point B:

$$r_{AB} = [(x_B - x_A)^2 + (y_B - y_A)^2]^{1/2} \tag{11}$$

Length of the path between A and B by means of reflection:

$$S_{AB} = \{(x_R - x_A)^2 + [y_A - y_c(x_R)]^2\}^{1/2} + \{(x_B - x_R)^2 + [y_B - y_c(x_R)]^2\}^{1/2} \tag{12}$$

where the approximate abscissa of the reflecting point is

$$x_R = x_A + h_A(x_B - x_A)/(h_A + h_B) \tag{13}$$

The choice of the abscissa separation $x_B - x_A$, and of the ordinate intervals Δy_A and Δy_B is determined by the criterion of keeping the phase difference between adjacent wavelets within a given limit, typically $\pi/4$. The basic formulas to be used are,

$$\varphi_r = 2\pi r_{AS}/\lambda$$
$$\varphi_S = 2\pi S_{AB}/\lambda \tag{14}$$
$$\Delta\varphi = \partial\varphi/\partial y_A \leq \pi/4$$

The choice of N_A is determined by the need that the sum (5) converges to integral (1). This can be determined by means of the following extrapolating procedure.

It has been pointed out that sum (5) involves two components, one related to a process of direct propagation, the other to a process of propagation through reflection over the earth. Imagine that we display these components in the complex plane, while the number of adding terms is gradually increased. The resulting plots are curves which, after a few initial wigglings, soon assume the shape of a Cornu spiral, wrapping around its asymptote. Figures 3 and 4 offer a typical example of the case.

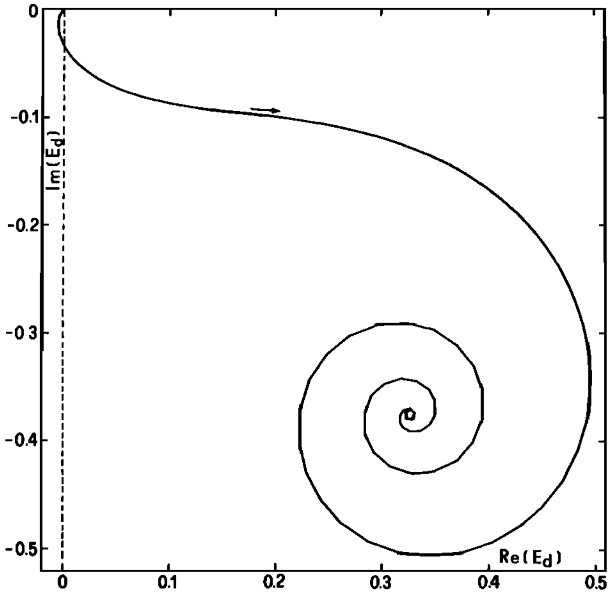

Fig. 3. Plot in the complex plane of the partial sum approximating the field, as the number of addends increases, direct component.

The suggested procedure consists of checking the behavior of these components and of determining the asymptote's coordinates as soon as the spiralling starts to take place. This is obtained by fixing the center of the circle osculating the spiral. More specifically, the sensitive element used in this operation is the phase change $\Delta\varphi$ occurring between the last two terms of the sum. When $\Delta\varphi$ becomes equal to or greater than $\pi/8$, the summing process is arrested, and the sum is taken to coincide with the coordinates of the center of the circle passing through the last three points evaluated.

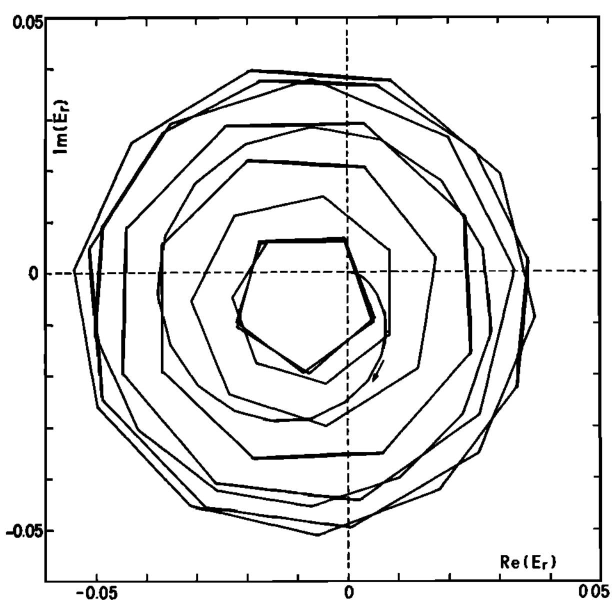

Fig. 4. Same as Figure 3, reflected component.

NUMERICAL EXAMPLE

The feasibility of the method outlined in the previous sections was tested by applying it to a situation simulating the case of a shipborne radar. The assumed parameters are:

height of the transmitter, $h_T = 10$ m
wavelength, $\lambda = 3$ cm
antenna vertical pattern, approximated by the Gaussian distribution,

$$E(0, y) = \exp\left[-0.092(y - y_T)^2\right] \tag{15}$$

maximum distance, $x_M = 26$ km
perfectly reflecting earth, i.e., $\Gamma_S = 1$ and $\phi_S = \pi$
equivalent radius of the earth, $a = 4\, a_0/3$ which means $(1/n)\, dn/dh = -1/4a_0$.

At this point the choice of the operating parameters must be made, having in mind the need for a reasonable economy in computer time. First, we should check the ordinate at which the less elevated minimum of the multipath pattern falls at the maximum range, x_M. This can be found using the simplified formula,

$$y_{mM} = x_M[\lambda/2h_T + y_C'(0)] \tag{16}$$

where the prime indicates the derivative with respect to x. In our case we get $y_{mM} = 75$ m.

Next, we have to choose the width of the abscissa interval Δx. This choice is related to that of the ordinate interval Δy. We want to be sure that (a) the phase shift between adjacent wavepaths does not exceed $\pi/4$, and (b) the path between every couple of chosen points in each step is unshadowed. These two requirements are expressed by the formulas,

$$\Delta y \leq \lambda\, \Delta x/16 y_{mM}, \quad \Delta y \leq (\Delta x/2)^2/a \tag{17}$$

The results of our selection are $\Delta x = 3$ km, and $\Delta y = 15$ cm.

The subsequent steps of the procedure consist of:

(a) Determining the ordinate above which direct computation of the field is possible, for every abscissa. This limit ordinate, which lies on the horizon of 0, is obtainable from the expression

$$y_0(x) = xy_C'(0) \tag{18}$$

(b) Calculating the direct radiation of the array at $x = 0$ for each point of the chosen abscissae, starting from $y_0(x)$ and moving upward to a limit ordinate $y_L(x)$ which was chosen to be the multiple of Δy closest to

$$y_L(x) = y_1 + [(y_{mM} - y_1)/(x_M - x_1)](x - x_1) \tag{19}$$

where x_1 is the abscissa of the first step and y_1 is arbitrary.

(c) Calculating the lower part of the vertical distribution, between $y_C(x)$ and $y_0(x)$, by making use of the closest field distribution. Here the convergence of the sum requires use of the Cornu spiral concept. The normalization factor is determined by setting the second solution equal to the first for $y = y_0(x)$.

(d) Calculating the squared field intensities, and plotting them as a function of height.

A copy of the computer program can be obtained from the authors.

The results of our calculations are displayed in Figure 5. In the figure the abscissae and ordinates correspond to Figure 1. For each selected abscissa the vertical distribution of the squared field intensities is plotted on an arbitrary db scale, as indicated, relative to the largest maximum, which is located on the corresponding abscissa. At abscissa 0 the field is Gaussian; at all other abscissae the field passes through a succession of maxima and minima, as the effect of the multipath interference. The resulting system of lobes is, in turn, framed by an envelope which reveals the vertical pattern of the transmitting antenna.

The maxima and minima are bound by the equations,

$$\begin{aligned} E_M(x) &= f_d E_0(x)[1 + \Gamma_z(y_M)] \\ E_m(x) &= f_d E_0(x)[1 - \Gamma_z(y_m)] \end{aligned} \tag{20}$$

where $E_0(x)$ is the average field intensity, f_d and f_r are the pattern factors of the antenna for the direct ray and for the reflected ray, respectively, and $\Gamma = D\, \rho\, f_r/f_d$ in which D is the divergence factor and ρ is the intrinsic reflection coefficient of the surface material.

If it is assumed that for a maximum and a minimum close together the reflection coefficients in (20) are practically equal, the related system of equations becomes immediately solvable. In par-

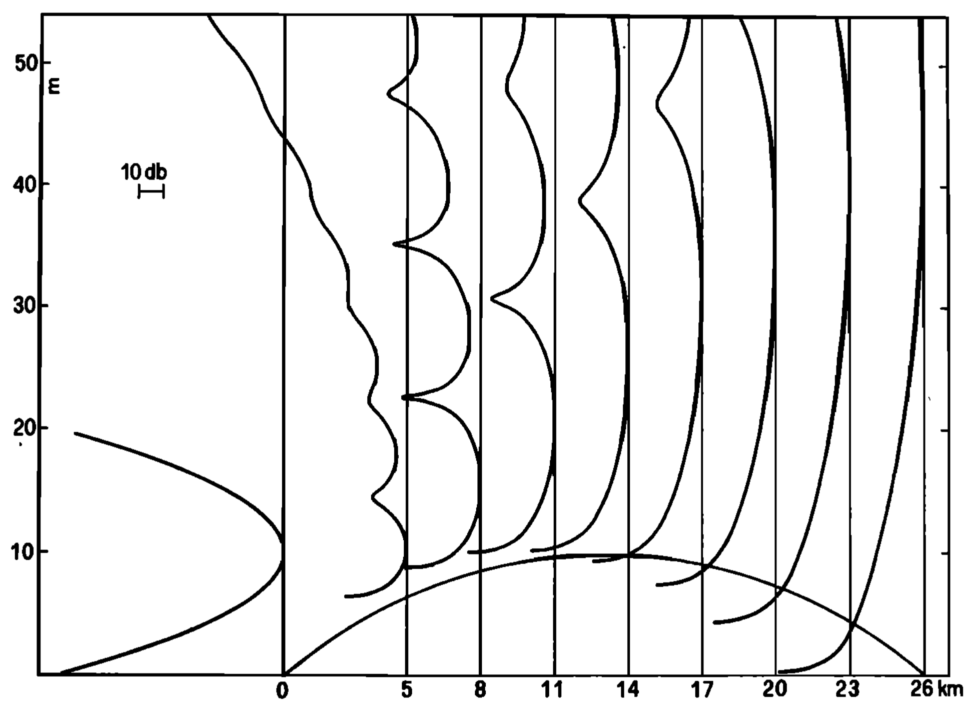

Fig. 5. Display of the vertical distribution at 3-km intervals, from 5 to 26 km from the transmitter (10 m high). Power density on an arbitrary db scale, as indicated. The largest maximum is located on the corresponding abscissa.

ticular, if we choose for a given abscissa the largest maximum, and its close minimum, we may write the approximate relation:

$$E_0(x) = (E_M/f_{r_M} + E_m/f_{r_m})/(f_{d_M}/f_{r_M} + f_{d_m}/f_{r_m}) \tag{21}$$

where f_{d_M}, f_{r_M}, f_{d_m}, f_{r_m} are the pattern factors for the direct and reflected rays relative to the maximum and minimum, respectively.

In this case $E_0^2(x)$ is tied to the parameters of the transmitter by the fundamental equation,

$$E_0^2(x) = P_0 G \eta_0 / 4\pi x^2 \tag{22}$$

where η_0 is the wave impedance of free space, P_0 the power fed into the transmitting antenna, and G the antenna gain. By combining (21) with (22) a relationship may be established among the field intensities of the various abscissae in Figure 5.

It is common usage to express the ratio $(E/E_0)^2$ in terms of db below, or above, the free-space propagation level. This is done in Figure 6 where the data of Figure 5, normalized using the above-mentioned procedure, are plotted in db with respect to free propagation as a function of range, for five heights above sea level (1, 2, 5, 10, and 25 m). The dots appearing in the figure represent points obtained using our method, while the solid lines were obtained using the familiar procedure due to Kerr. The fact that the agreement is good in the diffraction region, where Kerr's solution better approximates the original integral, while our method involves a maximum accumulation of computational steps, appears to us a satisfactory proof of the reliability of the method presented in this paper.

CONCLUDING REMARKS

The previous sections have introduced a new method for evaluating the field of a microwave transmitter in the intermediate zone. The method offers the following advantages: (a) a solution to the earth's diffraction problem which is straightforward, and consistent with physical intuition; (b) a computation of the field which grants uniform results

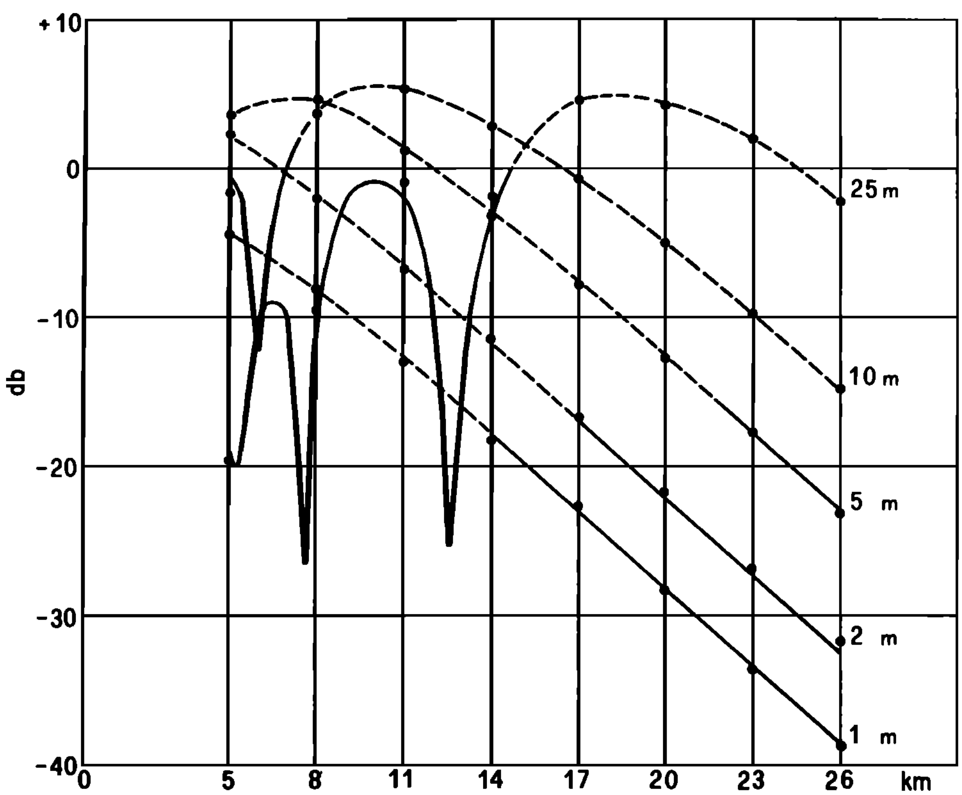

Fig. 6. Plots of power density relative to free-space propagation, in db versus range at fixed heights (elevation of the transmitter 10 m). The solid lines are computed using Kerr's method; the dots are points obtained with the method presented in this paper.

before, on, and beyond the horizon of the transmitter; and (c) an output which displays directly the whole vertical distribution of the field near the earth's surface.

The last feature seems to us to be particularly rewarding if utilized in connection with measurements of microwave propagation over the sea. In current practice, since the method used yields an intensity versus range computational datum, propagation tests are generally performed at constant height and variable range from the transmitter (see, for example, *Pidgeon* [1970]). Unfortunately, a considerable degree of inaccuracy is inherent in this practice. Conversely, with the method proposed here the computational data to be compared with the experiments require a vertical sounding of the field at a fixed range. Because of the small excursion of heights considered, these measurements could be done in a very short time, and hence with great accuracy. As a consequence, a new experimental technique may be expected to develop, capable of better evaluating the physical parameters affecting the microwave propagation over the sea.

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