A Computer Simulation of Cold Plasma Effects on the Whistler Instability for Geostationary Orbit Plasma Parameters
2. The Case of Enhancement of the Maximum Rate of Growth

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Cuperman and Landau (1974) have analytically investigated the modifications of parallel propagating whistler waves in an infinite uniform warm (w) anisotropic plasma in the presence of various amounts of cold (c) plasma. The authors found that the addition of cold plasma increases the instability range in k space and increases (decreases) the maximum rate of growth if $P \equiv \beta A(A + 1)^2 < 1$ (>1). Here $\beta \equiv 4\pi n_w K T_{\parallel,w}/B_0^2$ and $A \equiv (T_\perp/T_\parallel)_w - 1$. Cuperman et al. (1973) have performed a computer simulation experiment and confirmed the analytical predictions for the case $P > 1$. This paper presents the results of a computer simulation for the regime $P << 1$. The analysis of the time behavior of the total electromagnetic activity developed confirms the basic analytical predictions for the possible enhancement of the whistler mode instability in the presence of cold plasma.

In a previous paper, *Cuperman et al.* [1973] presented the results of a computer simulation experiment designed to study the characteristics of the whistler instability in an infinite warm anisotropic plasma in the presence of various amounts of cold plasma. Warm plasma parameters defined by the quantity $P \equiv \beta A \cdot (A + 1)^2 = 3.2$ (where $\beta = 4\pi n_w K T_{\parallel,w}/B_0^2$ and $A = T_{\perp,w}/T_{\parallel,w} - 1 = 1$) were selected for the simulation experiment. The main results were found to be in good agreement with the linear analytical predictions of *Cuperman and Landau* [1974] for the selected plasma conditions; that is, the larger the relative cold plasma density the wider the instability range in k space, the larger the k value corresponding to the maximum wave amplitude (maximum growth rate), and the lower the total electromagnetic energy built up. Similar computer simulations of the whistler instability have been performed by *Ossakow et al.* [1972, 1973*a*, *b*] in the regime $P \geq 0.6$. However, no significant enhancement is theoretically predicted for $P \gtrsim 0.5$. This can easily be seen in Figure 1, the analytical prediction for the maximum enhancement $F \equiv \gamma_{max}([n_c/n_w]_{optim})/\gamma_{max}(n_c/n_w = 0)$ as a function of P for $A = 0.5$ [*Cuperman and Landau*, 1974].

Unlike the case where $P \gtrsim 1$ the analytical treatment predicts that for warm plasma parameters defined by $P << 1$ the maximum growth rate increases with the increase in relative cold plasma density. The small growth rates characteristic of $P \ll 1$ conditions imply computation times much longer than those characteristic of $P > 1$ and therefore require modifications in the simulation formulation. *Cuperman et al.* [1973] thus limited their simulation experiment to the case $P > 1$, where the instability growth and relaxation rates are relatively fast.

This paper presents the results of the computer simulation experiment for the regime $P << 1$ for various cold plasma densities. The warm plasma parameters selected included a bi-Maxwellian particle distribution function for the warm electrons with $KT_{\parallel,w} = 5$ keV and $KT_{\perp,w} = 7.5$ keV ($T_\perp/T_\parallel = 1.5$), a particle density $n_w = 1$ cm^{-3}, and a static magnetic field of 10^{-3} G. Consequently, $\beta \simeq 0.1$, $A = 0.5$, and $P \equiv \beta A(A + 1)^2 = 0.11$ ($<<1$). These plasma parameters are more consistent with those observed in the distant equatorial plane of the magnetosphere ($L = 6.6$) than with those selected in the previous simulation experiment ($P = 3.2$). Therefore this experiment has a more direct relevance to the general ideas put forth by *Kennel and Petschek* [1966], *Brice and Lucas* [1971], and *Cornwall et al.* [1970] for the relationship between natural cold plasma distributions and the spatial and temporal behavior of the energetic particle populations and also a more direct relevance to the controlled cold plasma injection experiments described by *Brice* [1970], *Williams* [1971], and *Cornwall and Schulz* [1971].

In the next section we present a summary of the analytical linear predictions for parallel propagating whistler waves in an infinite uniform warm anisotropic plasma in the presence of various amounts of cold plasma. Then we discuss those technical aspects of the integration method that are specifically required for the present experiment, and we give the results of the computer simulation and their discussion.

THEORY

The analytical results of *Cuperman and Landau* [1974] have recently been substantiated by numerical solution of the exact linear dispersion relation [*Cuperman and Salu*, 1974]. Figure 2 shows the results of both analytical and numerical calculation of the wave growth rate γ_k and the frequency ω_{0k}, both normalized to the electron gyrofrequency $|\Omega_-|$ for the conditions of the present computer simulation experiment and for several values of the ratio of cold to warm plasma density (indicated on each curve). Notice that the maximum value of γ_k occurs for a nonzero value of n_c/n_w. Specifically, γ_k maximizes for $n_c/n_w \simeq 8$ at a value of k given by $\bar{k} \equiv k v_{th,\parallel}/|\Omega_-| \simeq 0.45$.

An important feature of the whistler instability for small values of the parameter P is that while it exhibits a relative enhancement of the maximum rate of growth in the presence

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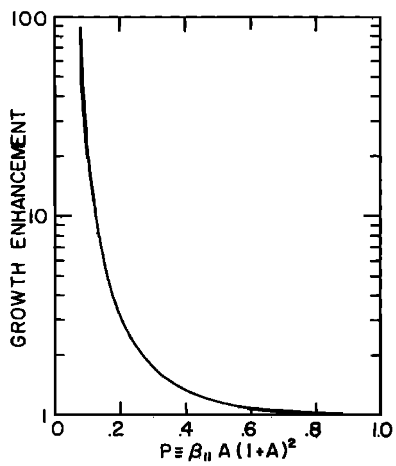

Fig. 1. The maximum enhancement of the growth rate for the parallel whistler instability corresponding to the addition of the optimum amount of cold plasma as a function of the parameter $P \equiv \beta_{\|}A(1 + A)^2$.

of a cold plasma population, the magnitude itself of γ_{max} is small (much smaller than that for $P > 1$). Thus the buildup of the electromagnetic field (in the linear stage) and the relaxation of the system in general are relatively slow.

Integration Method

The linear growth rates of the instability for the selected plasma conditions $P \ll 1$ are rather small. As can be seen from Figure 2, the maximum growth rate $\gamma_{max} \simeq 5 \times 10^{-4}|\Omega_-|$ for $n_c = 0$, and $\gamma_{max} \simeq 10^{-2}|\Omega_-|$ for $n_c/n_w = 8$; therefore only a slow evolution of the initially unstable system should occur. Consequently, the background numerical noise must be kept at a level significantly below that of the electromagnetic activity developed during the instability. Thus in this work we used a nonradiative (inductive) integration scheme as developed by *Morse and Nielson* [1970]. This code is in the same spirit as that developed by *Haber et al.* [1970] and used in the whistler simulations conducted by *Ossakow et al.* [1972, 1973*a*, *b*]. In this scheme the field equation for the transverse magnetic wave field $B_\perp$ is obtained by dropping the radiative term $\partial^2\mathbf{A}/\partial t^2$ in the wave equation for the vector potential $\mathbf{A}$. (A complete description of the code is given by *Forslund et al.* [1972].) This integration method appears to satisfy the requirement mentioned above for the case $P \ll 1$ treated in this work.

After a relatively large number of trial experiments in which the parameters of the integration method were varied, the following conditions have been selected: 5000 simulation particles for the warm electron component and consequently 40,000 simulation particles for the cold electrons on a 1/100 cell grid ($L = 8\pi\omega_p^{-1}c$) and a time step of 1/100 of the electron gyroperiod. The good agreement shown in Figure 3 for a case with 20,000 warm particles and one with 5000 warm particles confirms that 5000 warm particles are sufficient to simulate the behavior of the warm plasma component accurately. The thermal noise represented less than 10^{-3} of the total energy of the system.

Results

In this section we present the results of the computer simulation of the whistler instability for energetic plasma parameters characterized by the quantity $P \ll 1$ and for relative cold plasma densities $n_c/n_w = 0$ (case I) and $n_c/n_w = 8$ (case II).

The linear prediction that a larger value of the maximum rate of growth (and therefore of the total RF activity developed) is obtained for case II than for case I is fully confirmed by our computer experiment. In fact, while there is almost no change from the initial total electromagnetic energy density for case I, as was expected, a definite enhancement is observed in case II.

This can be seen in Figure 3, in which the temporal growth of the magnetic wave energy $W_B(t) \equiv \sum_k B_k^2(t)/8\pi$ is shown for the two cases. After eight electron gyroperiods, $W_{B,\mathrm{II}}/W_{B,\mathrm{II}}(t = 0) \simeq 1.7$, which is very close to the value predicted by the linear theory, namely, 1.7. This last value has been obtained by calculating the quantities $W_B \equiv (1/8\pi)\int B_k^2\,dk$, where $B_k^2(t) = B_k^2(t = 0)\cdot e^{2\gamma_k t}$ with γ_k given by the expression (33) of *Cuperman and Landau* [1974]. (Note that $E_k \ll B_k$ for this case.)

The Fourier analysis of the electromagnetic activity developed in case II indicates that (1) the width of the unstable wave spectrum in k space is confined between the modes $\bar{k} = 4$ and $\bar{k} = 9$, in agreement with the analytical prediction of Figure 2, (2) in the initial (linear) stage the unstable modes grow in agreement with Figure 2, and (3) although the initial thermal anisotropy is only little changed, after about five electron gyroperiods some of the modes (e.g., $\bar{k} = 6$, 8) decay or undergo significant periodic changes; this indicates that even at low level (amplitude), interaction between various electromagnetic waves is possible and important.

Inspection of Figure 3 indicates that after about $8\tau_{ce}$ there is little change in the total electromagnetic wave energy. This is also an indication that the system has become close to relaxation (from the collective point of view).

The small isotropization observed in these low P value experiments (at the end of the run, the ratio $0.5W_{\perp,w}/W_{\|,w}$ decreased by less than 1.5%) is consistent with the correspond-

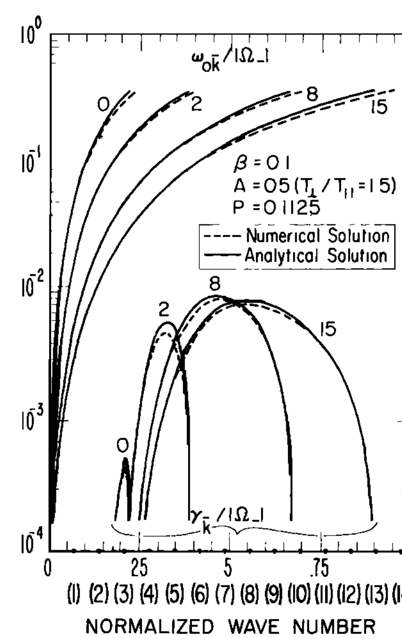

Fig. 2. The rates of growth $\gamma/|\Omega_-|$ and the real frequency $\omega_0/|\Omega_-|$ for the parallel whistler instability for several values of the ratio of cold to warm plasma density (indicated on each curve). The external parameters are $\beta = 0.1$ and $A = ½$. Two different normalizations for the wave numbers are used: the higher line of numbers (0.00, 0.25, · · ·) and ticks refers to $\bar{k} \equiv kv_{th\|}/|\Omega_-|$; the lower line of numbers in parentheses refers to a slightly different but more convenient normalization: $\bar{k} \equiv k/k_{min}$, and the corresponding divisions are indicated by heavy points.

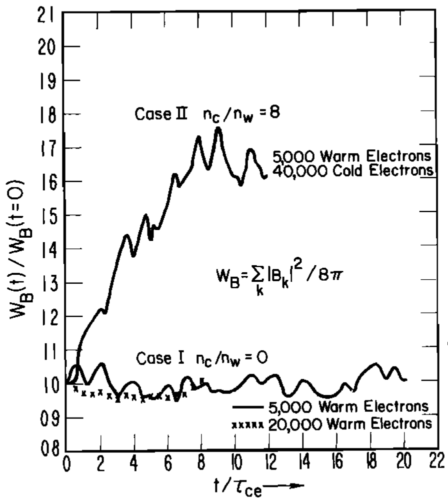

Fig. 3. The temporal behavior of the relative magnetic wave energy density for cases I and II. In both cases, $\beta = 0.1$ and $A = 0.5$. In case I (no cold plasma), 5000 warm simulation particles are used. For comparison, the results obtained with 20,000 warm simulation particles are also indicated (by discrete points). In case II ($n_c/n_w = 8$), 5000 warm simulation particles and 40,000 cold simulation particles are used.

ing results obtained in previous experiments: the smaller the initial value of the parameter $P \equiv \beta A(A + 1)^2$ the weaker the electromagnetic fields developed during the instability and the smaller the isotropization. This is illustrated by *Cuperman and Salu* [1972] and *Ossakow et al.* [1972, 1973*a*, b]. This result strongly indicates that the nonlinear effects quenching the instability are much more important in the low P value cases for which the experiment described in this paper represents a low limit case. This is presumably due to the establishment of a nonlinear quasi-stable equilibrium state in which the electromagnetic waves developed play an important role.

Finally, as an additional nonlinear effect one should mention the nonresonant heating by the waves of the cold electrons in the system. The energy involved is, however, very small, and the accuracy of the experiment is not sufficient to enable relevant conclusions on this aspect.

SUMMARY

We have confirmed in a computer simulation experiment the basic ideas and the linear prediction of the enhancement of the growth rate of the whistler instability in the presence of an additional cold plasma population for an energetic plasma characterized by the quantity $P \equiv \beta A(A + 1)^2 << 1$.

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