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TOPICAL ISSUE

The Use of Analytical Predictive Models in Control Systems of Flying Vehicles and in Flight Trainers

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Abstract—Questions are are considered as to the use of analytical solutions for predictive models of motion of a flying vehicle in the development of algorithmic software that is efficient in the computational respect of airborne complexes of control of flying vehicles and flight trainers.

1. INTRODUCTION

The scientific school of Academician A.A. Krasovskii features, along with the development of questions of the theory of control, the solution of a wide range of applied problems, including the works on the verification of the principles of development and algorithmic software of piloting-navigational complexes of flying vehicles (FV) and flight trainers. The fundamental idea in those works is the optimization of control processes in nonclassical quality functionals—functionals of the generalized operation (FGO). The practical implementation of the obtained schemes of synthesis of optimal control in real time appreciably depends on the required amount of computations. One of the ways of the development of algorithms efficient in the computational respect is the use, in the course of synthesis of the control, of analytical solutions of predictive models of motion of a FV. Similar schemes can also be used with advantage in the development of mathematical software of flight trainers. In this work, we consider analytical solutions of equations for motion of a FV in body-axis and trajectory systems of coordinates and their applications to problems of optimal control of motion, loops of the automatic learning of flight trainers and distributed networks of trainers.

2. MINIMIZATION OF THE FUNCTIONAL OF THE GENERALIZED OPERATION

The use of functionals of the nonclassical type in the problems of optimization of control of mobile objects, namely, the functionals of the generalized operation, enables us to solve the basic problem of optimal automatic control—the problem of the formation in real time of control actions in order to achieve the main final objective of the current stage of motion of an object.

Let a controllable object be described by differential vector equations of the form

$$\begin{aligned} \dot{x} &= f[x(t), y(t), t], \\ \dot{y}(t) &= u(t), \end{aligned} \tag{1}$$

where (x, y) is the component state vector of the object and u is the control vector. In what follows, the subvector y will be called the control factor.

The objective functional of the generalized operation with quadratic expenditures for control, which expresses the aims of the current stage of functioning of the object, can be written in the form

$$I = V_{pr}(x(t_{\mathrm{f}}), y(t_{\mathrm{f}}), t_{\mathrm{f}}) + \int\limits_{t_0}^{t_{\mathrm{f}}} Q_{\mathrm{pr}}(x(\tau), y(\tau), \tau) d\tau$$

$$+0.5 \int\limits_{t_0}^{t_{\mathrm{f}}} \left(u^{\mathrm{T}}(\tau)\mathrm{K}^{-1}u(\tau) + u_{\mathrm{op}}^{\mathrm{T}}(\tau)\mathrm{K}^{-1}u_{\mathrm{op}}(\tau)\right) d\tau, \tag{2}$$

where V_{pr} is the prescribed positive definite function that formalizes the requirements for components of the state vector of the object in the specified finite instant of time t_{f}; Q_{pr} is the prescribed positive definite function that formalizes the requirements for components of the state vector (the trajectory) in motion from the current instant of time t_0 to the finite instant of time t_{f}; K is the prescribed symmetric nonsingular matrix of coefficients of the penalty for the expenditure of the control at the stage of motion under consideration; and u_{op} is the control in an optimal closed system, which is unknown until the solution of the synthesis problem.

We assume that with the assignment of initial conditions $x(t_0)$, $y(t_0)$ equal to current values of $x(t)$, $y(t)$ and with the zero control $u = 0$, there exists a unique solution of Eqs. (1), which is given in the form $X_{\mathrm{M}}(x(t_0), y(t_0), t_0, t)$.

The basic theorem of the principle of minimum of the generalized operation is stated in the following way [1]. The optimal control that minimizes the functional of the generalized operation (2) for the object (1) is defined by the expression

$$u_{\mathrm{op}} = -\mathrm{K}\frac{\partial^{\mathrm{T}}}{\partial y}\left\{V_{\mathrm{pr}}\left(X_{\mathrm{M}}(x(t_0), y(t_0), t_0, t), y, t_{\mathrm{f}}\right) + \int\limits_{t_0}^{t_{\mathrm{f}}} Q_{\mathrm{pr}}\left(X_{\mathrm{M}}(x(t_0), y(t_0), t_0, \tau)y(\tau), \tau\right) d\tau\right\}$$

$$= -\mathrm{K}\left\{\frac{\partial X_{\mathrm{M}}^{\mathrm{T}}(x(t_0), y(t_0), t_0, t_{\mathrm{f}})}{\partial y}\frac{\partial V_{\mathrm{pr}}^{\mathrm{T}}(X_{\mathrm{M}}(x(t_0), y(t_0), t_0, t), y, t_{\mathrm{f}})}{\partial X_{\mathrm{M}}(x(t_0), y(t_0), t_0, t_{\mathrm{f}})}\right.$$

$$+\frac{\partial V_{\mathrm{pr}}^{\mathrm{T}}(X_{\mathrm{M}}(x(t_0), y(t_0), t_0, t), y, t_{\mathrm{f}})}{\partial y} + \int\limits_{t_0}^{t_{\mathrm{f}}}\left[\frac{\partial Q_{\mathrm{pr}}^{\mathrm{T}}(X_{\mathrm{M}}(x(t_0), y(t_0), t_0, \tau)y(\tau), \tau)}{\partial y}\right.$$

$$\left.\left.+\ \frac{\partial X_{\mathrm{M}}^{\mathrm{T}}(x(t_0), y(t_0), t_0, \tau)}{\partial y}\frac{\partial Q_{\mathrm{pr}}^{\mathrm{T}}(X_{\mathrm{M}}(x(t_0), y(t_0), t_0, \tau)y(\tau), \tau)}{\partial X_{\mathrm{M}}(x(t_0), y(t_0), t_0, \tau)}\right] d\pi\right\}, \tag{3}$$

where $X_{\mathrm{M}}(x(t_0), y(t_0), t_0, t)$ is the solution of Eqs. (1) under the above-stated conditions.

It follows from (3) that the optimal control represents a derivative of the main part of the functional of the generalized operation with respect to the control factor, which is estimated in the "free" motion of the object, i.e., under conditions of the virtual predictive motion from the current point of the trajectory with the zero control. The solution of equations of the free motion received the name "prediction" and the corresponding algorithm of optimal control came to be known as the algorithm with a predictive model [2].

By now, an appreciable number of versions of the algorithm with the predictive model are worked out. Among these are the following algorithms [2–4]: (1) the algorithm with numerical differentiation; (2) the modified algorithm; (3) the algorithm with a sensitivity matrix; (4) the algorithm with an analytical solution; (5) the algorithm with a synchronous differentiation; and (6) the algorithm with a physically predictive model. These algorithms differ in the methods of calculating the solution of the free motion $X_{\mathrm{M}}(x(t_0), y(t_0), t_0, t)$ and in the differentiation with respect to the control factor of the main part of the functional of the generalized operation (FGO), which is estimated in this solution.

The most economical version of the algorithm in the computational respect is the algorithm with the predictive model and the analytical solution, but the use of this version is limited by the condition of existence of nontrivial analytical solutions of the equations for motion of the object (1). We will examine the questions of their implementation.

3. ANALYTICAL SOLUTIONS OF THE EQUATIONS OF MOTION OF A FV

For the problems of the trajectory optimization of motion of a FV, Krasovskii suggested an analytical solution of the equation of motion of the FV, which is written in the body-axis system of coordinates [5].

We will represent the equations of motion of a FV in the following way:

(1) kinematics of the rotary motion

$$\dot{\varepsilon}^{\mathrm{T}} = \Omega\varepsilon^{\mathrm{T}}, \quad \Omega = \begin{bmatrix} 0 & \omega_{\mathrm{z}} & -\omega_{\mathrm{y}} \\ -\omega_{\mathrm{z}} & 0 & \omega_{\mathrm{x}} \\ \omega_{\mathrm{y}} & -\omega_{\mathrm{x}} & 0 \end{bmatrix}, \tag{4}$$

where ε is the matrix of direction cosines of the transition from the body-axis to the normal terrestrial system of coordinates and Ω is the Poisson matrix;

(2) dynamics of the translational motion of the center of mass of the FV

$$\begin{bmatrix} \dot{V}_{\mathrm{kx}} \\ \dot{V}_{\mathrm{ky}} \\ \dot{V}_{\mathrm{kz}} \end{bmatrix} = \Omega \begin{bmatrix} V_{\mathrm{kx}} \\ V_{\mathrm{ky}} \\ V_{\mathrm{kz}} \end{bmatrix} + g\left(\begin{bmatrix} n_{\mathrm{x}} \\ n_{\mathrm{y}} \\ n_{\mathrm{z}} \end{bmatrix} - \begin{bmatrix} \varepsilon_{21} \\ \varepsilon_{22} \\ \varepsilon_{23} \end{bmatrix}\right), \tag{5}$$

where $V_{\mathrm{k}} = [V_{\mathrm{kx}}\ V_{\mathrm{ky}}\ V_{\mathrm{kz}}]^{\mathrm{T}}$ is the vector of the terrestrial speed of the FV in the projections on the axis of the body-axis coordinate system; $n = [n_x\ n_y\ n_z]^{\mathrm{T}}$ is the vector of an overload of the FV in the projections on the axis of the body-axis coordinate system; and $\varepsilon_2 = [\varepsilon_{21}\ \varepsilon_{22}\ \varepsilon_{23}]^{\mathrm{T}}$ is the second column of the transposed matrix of the direction cosines ε;

(3) kinematics of the translational motion of the center of mass

$$\begin{bmatrix} \dot{x}_{\mathrm{g}} \\ \dot{y}_{\mathrm{g}} \\ \dot{z}_{\mathrm{g}} \end{bmatrix} = \varepsilon \begin{bmatrix} V_{\mathrm{kx}} \\ V_{\mathrm{ky}} \\ V_{\mathrm{kz}} \end{bmatrix}, \tag{6}$$

where $s = [x_{\mathrm{g}}\ y_{\mathrm{g}}\ z_{\mathrm{g}}]^{\mathrm{T}}$ is the vector of rectangular normal terrestrial coordinates of the center of mass of the FV.

For the subvector y of the system (1) it is suggested that the vectors of an overload and an angular velocity of the FV be used in projections on the axis of the body-axis coordinate system [5]. Then, under the condition of the steadiness of these vectors, the following analytical solution[1] can be written, which goes below under the name of the "spiral prediction:"

—for the equations of kinematics of the rotary motion

$$\varepsilon^{\mathrm{T}}(t) = \Phi(t)\varepsilon^{\mathrm{T}}(0), \tag{7}$$

where $\Phi(t) = E_{3\times3} + \dfrac{\sin|\omega|t}{|\omega|}\Omega + \dfrac{1-\cos|\omega|t}{|\omega|^2}\Omega^2$, $E_{3\times3}$ is the identity matrix of dimension 3, and $\omega = [\omega_{\mathrm{x}}\ \omega_{\mathrm{y}}\ \omega_{\mathrm{z}}]^{\mathrm{T}}$ is the angular velocity vector that is constant according to the condition;

[1] The analytical solution presented below differs from the solution given in [5] in that it has no particular point and is found by the author.

—for the equations of dynamics of the translational motion

$$\begin{bmatrix} V_{\rm kx}(t) \\ V_{\rm ky}(t) \\ V_{\rm kz}(t) \end{bmatrix} = \Phi(t)\left(\begin{bmatrix} V_{\rm kx}(0) \\ V_{\rm ky}(0) \\ V_{\rm kz}(0) \end{bmatrix} - gt\begin{bmatrix} \varepsilon_{21}(0) \\ \varepsilon_{22}(0) \\ \varepsilon_{23}(0) \end{bmatrix}\right) + g\Phi_1(t)\begin{bmatrix} n_{\rm x} \\ n_{\rm y} \\ n_{\rm z} \end{bmatrix}, \tag{8}$$

where $\Phi_1(t) = tE_{3\times 3} + \dfrac{1-\cos|\omega|t}{|\omega|^2}\Omega + \dfrac{1}{|\omega|^2}\left(t - \dfrac{\sin|\omega|t}{|\omega|}\right)\Omega^2$;

—for the equations of kinematics of the translational motion

$$\begin{bmatrix} x_{\rm g}(t) \\ y_{\rm g}(t) \\ z_{\rm g}(t) \end{bmatrix} = \begin{bmatrix} x_{\rm g}(0) \\ y_{\rm g}(0) \\ z_{\rm g}(0) \end{bmatrix} + \varepsilon(0)\left(t\begin{bmatrix} V_{\rm kx}(0) \\ V_{\rm ky}(0) \\ V_{\rm kz}(0) \end{bmatrix} - g\frac{t^2}{2}\begin{bmatrix} \varepsilon_{21}(0) \\ \varepsilon_{22}(0) \\ \varepsilon_{23}(0) \end{bmatrix} + g\Phi_2(t)\begin{bmatrix} n_{\rm x} \\ n_{\rm y} \\ n_{\rm z} \end{bmatrix}\right), \tag{9}$$

where $\Phi_2(t) = \dfrac{t^2}{2}E_{3\times 3} - \dfrac{1}{|\omega|^2}\left(t - \dfrac{\sin|\omega|t}{|\omega|}\right)\Omega + \dfrac{1}{|\omega|^2}\left(\dfrac{t^2}{2} - \dfrac{1-\cos|\omega|t}{|\omega|^2}\right)\Omega^2.$

The use of the spiral prediction in the problems of motion optimization by the criterion of a minimum of the functional of the generalized operation makes it possible to obtain economical computational procedures of the synthesis and, in a number of cases, analytical control laws. This follows from the existence of analytical expressions for derivatives of the state vector of the FV with respect to the control factor:

$$\begin{aligned}
&\frac{\partial\varepsilon^{\rm T}(t)}{\partial n} = 0, \quad \frac{\partial\varepsilon^{\rm T}(t)}{\partial\omega} = \frac{\partial\Phi(t)}{\partial\omega}, \\
&\frac{\partial V(t)}{\partial n} = g\Phi_1(t), \quad \frac{\partial V(t)}{\partial\omega} = \frac{\partial\Phi(t)}{\partial\omega}\{V(0) - gt\varepsilon_2(0)\} + g\frac{\partial\Phi_1(t)}{\partial\omega}n, \\
&\frac{\partial\dot{s}(t)}{\partial n} = g\varepsilon(0)\Phi_1(t), \quad \frac{\partial\dot{s}(t)}{\partial\omega} = g\varepsilon(0)\left\{\frac{\partial\Phi^{\rm T}(t)}{\partial\omega}\Phi_1(t) + \Phi^{\rm T}(t)\frac{\partial\Phi_1(t)}{\partial\omega}\right\}n, \\
&\frac{\partial s(t)}{\partial n} = g\varepsilon(0)\Phi_2(t), \quad \frac{\partial s(t)}{\partial\omega} = g\varepsilon(0)\frac{\partial\Phi_2(t)}{\partial\omega}n.
\end{aligned} \tag{10}$$

The spiral prediction (7)–(9) and expressions (10) provided the basis for the development of algorithmic software of on-board complexes of control of a FV for various stages of the flight, including the route flight, the low-altitude flight, the landing, and the control of motion of a spacecraft [6–9]. Characteristic for these stages of the flight is the direct inclusion in the functions $Q_{\rm pr}$ and $V_{\rm pr}$ of the objective functional of the components of vectors $V_{\rm k}$, s, and the components of the matrix ε, which comprise the state vector of a FV.

A further development of the spiral prediction was the analytical solution of the equations for motion of a FV in the trajectory coordinate system $OX_{\rm f}Y_{\rm f}Z_{\rm f}$. The trajectory coordinate system is the most natural one for the description of the trajectory of motion of a FV, and the requisite solution is effective in the problems of synthesis of the optimal control of the FV at the stages of the special maneuvering, group interaction, etc.

To obtain this solution, we will transform the appropriate equations for motion of a FV in the trajectory coordinate system to the vector-matrix form of the notation, which is analogous to the notation of equations in the body-axis system of coordinates:

(1) the equations of kinematics of the angular motion

$$
\begin{aligned}
&\dot{\varepsilon}_{\mathrm{f}}^{\mathrm{T}} = \Omega_{\mathrm{f}} \varepsilon_{\mathrm{f}}^{\mathrm{T}},\\
&\varepsilon_{\mathrm{f}} = \begin{bmatrix} \cos\psi\cos\theta & -\cos\psi\sin\theta & \sin\psi \\ \sin\theta & \cos\theta & 0 \\ -\sin\psi\cos\theta & \sin\psi\sin\theta & \cos\psi \end{bmatrix},\\
&\Omega_{\mathrm{f}} = \begin{bmatrix} 0 & \omega_{\mathrm{zk}} & -\omega_{\mathrm{yk}} \\ -\omega_{\mathrm{zk}} & 0 & \omega_{\mathrm{xk}} \\ \omega_{\mathrm{yk}} & -\omega_{\mathrm{xk}} & 0 \end{bmatrix},\\
&\omega_{\mathrm{xk}} = \dot{\psi}\sin\theta, \quad \omega_{\mathrm{yk}} = \dot{\psi}\cos\theta, \quad \omega_{\mathrm{zk}} = \dot{\theta},
\end{aligned}
\tag{11}
$$

where ε_{k} is the matrix of direction cosines of the transition from the trajectory system to the normal system of coordinates, Ω_{k} is an appropriate Poisson matrix, ψ is the track angle of the FV, and θ is the angle of inclination of the trajectory of the FV;

(2) dynamics of the translational motion of the center of mass of the FV

$$
\begin{bmatrix} \dot{V}_{\mathrm{f}} \\ 0 \\ 0 \end{bmatrix} = g \begin{bmatrix} n_{\mathrm{xk}} \\ 0 \\ 0 \end{bmatrix} - gA \begin{bmatrix} \varepsilon_{\mathrm{k21}} \\ \varepsilon_{\mathrm{k22}} \\ \varepsilon_{\mathrm{k23}} \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{12}
$$

where n_{xk} is the longitudinal trajectory overloading;

(3) kinematics of the translational motion of the center of mass of the FV

$$
\begin{bmatrix} \dot{x}_{\mathrm{g}} \\ \dot{y}_{\mathrm{g}} \\ \dot{z}_{\mathrm{g}} \end{bmatrix} = \varepsilon_{\mathrm{f}} \begin{bmatrix} V_{\mathrm{f}} \\ 0 \\ 0 \end{bmatrix}. \tag{13}
$$

A requisite analytical solution can be found under the assumptions similar to those made for the receiving of the spiral prediction—the constancy of vectors of the angular velocity and the longitudinal trajectory of the overloading of the FV:

—for Eqs. (11)

$$
\begin{aligned}
&\varepsilon_{\mathrm{f}}^{\mathrm{T}}(t) = \Phi(t)\varepsilon_{\mathrm{f}}^{\mathrm{T}}(0), \qquad (14)\\
&\Phi(t) = E_{3\times 3} + \frac{\sin|\omega_{\mathrm{f}}|t}{|\omega_{\mathrm{f}}|}\Omega_{\mathrm{f}} + \frac{1-\cos|\omega_{\mathrm{f}}|t}{|\omega_{\mathrm{f}}|^2}\Omega_{\mathrm{f}}^2,
\end{aligned}
$$

—for Eqs. (12)

$$
\begin{aligned}
&\begin{bmatrix} V_{\mathrm{f}}(t) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} V_{\mathrm{f}}(0) \\ 0 \\ 0 \end{bmatrix} + g\left(\begin{bmatrix} n_{\mathrm{xk}} \\ 0 \\ 0 \end{bmatrix} t - A\Phi_1(t)\begin{bmatrix} \varepsilon_{\mathrm{k21}}(0) \\ \varepsilon_{\mathrm{k22}}(0) \\ \varepsilon_{\mathrm{k23}}(0) \end{bmatrix}\right),\\
&\Phi_1(t) = tE_{3\times 3} + \frac{1-\cos|\omega_{\mathrm{f}}|t}{|\omega_{\mathrm{f}}|^2}\Omega_{\mathrm{f}} + \frac{1}{|\omega_{\mathrm{f}}|^2}\left(t - \frac{\sin|\omega_{\mathrm{f}}|t}{|\omega_{\mathrm{f}}|}\right)\Omega_{\mathrm{f}}^2,
\end{aligned}
\tag{15}
$$

—for Eqs. (13)

$$
\begin{aligned}
s(t) = s(0) &+ \varepsilon_{\mathrm{f}}(0)\int_0^t \Phi^{\mathrm{T}}(\tau)d\tau V_{\mathrm{f}}(0) + g\varepsilon_{\mathrm{f}}(0)\int_0^t \tau\Phi^{\mathrm{T}}(\tau)d\tau n(0)\\
&- g\varepsilon_{\mathrm{f}}(0)\int_0^t \Phi^{\mathrm{T}}(\tau)A\Phi_1(\tau)d\tau\varepsilon_{\mathrm{k2}}^{\mathrm{T}}(0).
\end{aligned}
\tag{16}
$$

The analytical expressions for appropriate integrals in (16) are cumbersome, and so they are not given in this work.

The methodology of the use of the analytical solutions of Eqs. (14)–(16) of FV motion in the trajectory coordinate system is similar to that developed for the spiral prediction (7)–(10).

4. MATHEMATICAL SOFTWARE OF FLIGHT TRAINERS

The analytical models of the spiral and the trajectory prediction, which are worked out for the solution of the problems of optimization of the trajectory motion of a FV, provide a basis for the development of mathematical software of perspective flight trainers.

The use of these models involves a few aspects. First, using the equations of the spiral prediction, it is possible to develop a two-channel method of the numerical integration of the equation of dynamics of the flight [5, 10, 11]. This method can be economical in the computational respect and the single-step one by the structure of accounting for control actions and can be considered in the development of mathematical software of simulators of dynamics of the flight of trainers.

The second important application of the analytical solutions relates to the development of loops of the automatic learning of flight trainers and, in particular, the loop of an optimal electronic instructor [10].

The conception of the optimal electronic instructor in the composition of a flight trainer presupposes the solution of a complex of problems by a certain computational loop in parallel with the pilot. These problems make up the essence of the occupational activity of the pilot. The output parameters of the optimal electronic instructor are the solutions that ensure the achievement of the aims of the current stage of the flight (Fig. 1).

These solutions must be accessible for their implementation by an operator; by their structure, they must be similar to the actions of pilots (experts). To ensure these requirements, it is necessary to shape up requisite solutions on the basis of an adequate model of the professional activity of the pilot.

The development of simulation models of the professional activity of the pilot is one of the most complex problems of the psychophysiology and engineering psychology. The basic requirements for a model is the functional adequacy and constructiveness, i.e., the possibility of obtaining results on the basis of the available means and methods not in the description form, but in the form suitable for engineering calculations. These features display the predictive-optimizing model of the activity of an operator [12], which is suggested by Krasovskii and based on the works of Academicians N.A. Bernshtein and P.K. Anokhin, the known scientists in the field of neurophysiology.

The basic features of the predictive-optimizing model of the activity of an operator are the following:

—the foresight (prediction) of motion of a controllable man-machine system with the fixed position of rudder controls (i.e., the “free” motion of a FV in our case);

—the motivation and optimization, i.e., the decision-making of control on the basis of the comparison of the given prediction of the free motion with the specified plan of motion;

—the adaptation to the current conditions of the flight performed.

The formalization of the predictive-optimizing model of the activity of an operator can be performed in the framework of the approach of the nonclassical theory of optimization according to the criterion of a minimum of the functional of the generalized operation in the version with the analytical solution. For the analytical solution of the free motion of a FV, it is expedient to use the trajectory prediction (14)–(16) or the spiral prediction (7)–(9) depending on the aims of the current stage of the flight and the set of parameters defining this stage of the flight. The adaptation to the current conditions of the flight performed is carried out by way of the adjustment of the

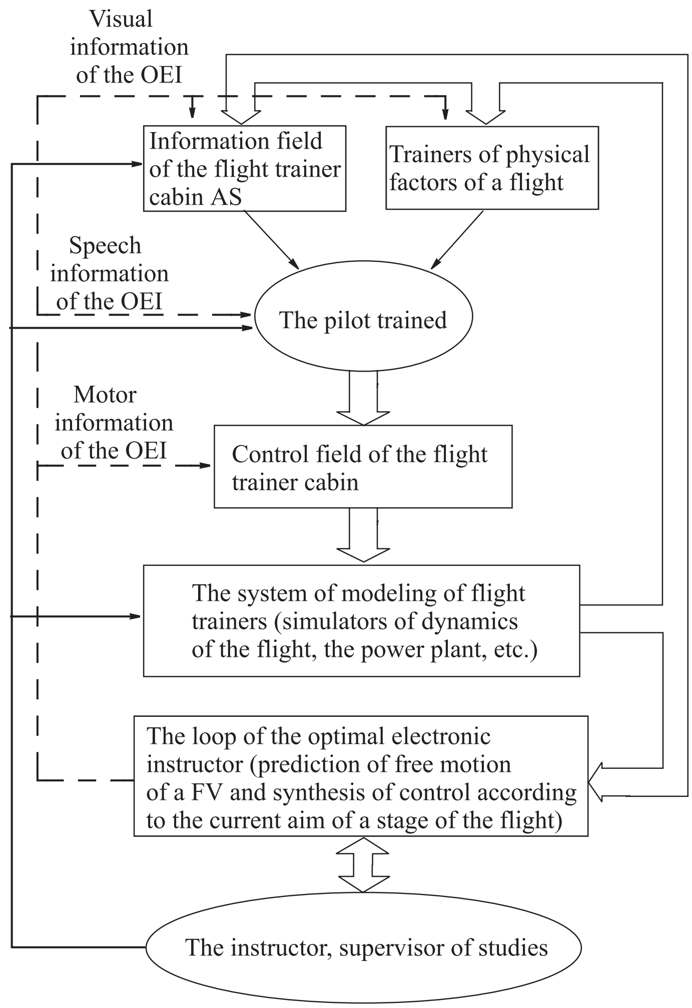

Fig. 1. The block diagram of a trainer with the loop of an optimal electronic instructor (OEI).

time of prediction and the coefficients of the objective functions $Q_{\rm pr}$ and $V_{\rm pr}$ of the functional of the generalized operation with the use of the technology of expert systems [13].

5. DISTRIBUTED NETWORKS OF TRAINERS AND PREDICTIVE MODELS

The modern concepts of the development and use of trainers call for the support of the technology of distributed interactive simulation (DIS). In accordance with this technology, objects of various kinds (real, "virtual" ones—the trainers proper, "constructive" ones—the systems of modeling) are united into a single computer network for the purpose of the joint simulation of operators–members of the "crews" of these objects in the framework of the synthesized, professionally oriented, virtual world. The information interaction of objects of the distributed simulation network is defined by the DIS interface [14].

One of the basic aspects of the DIS interface is the transmission of the information by each object—the node of the distributed simulation network—to all of the remaining objects (nodes). This is the information not only on the current angular and the spatial position, but also on the parameters of motion and the type of analytical predictive model approximately describing the current character of motion of an object. In accordance with this information, each node of the network can approximate in future instants of time the parameters of the angular and the spatial position of those objects (nodes of the distributed network) which are found to be in its

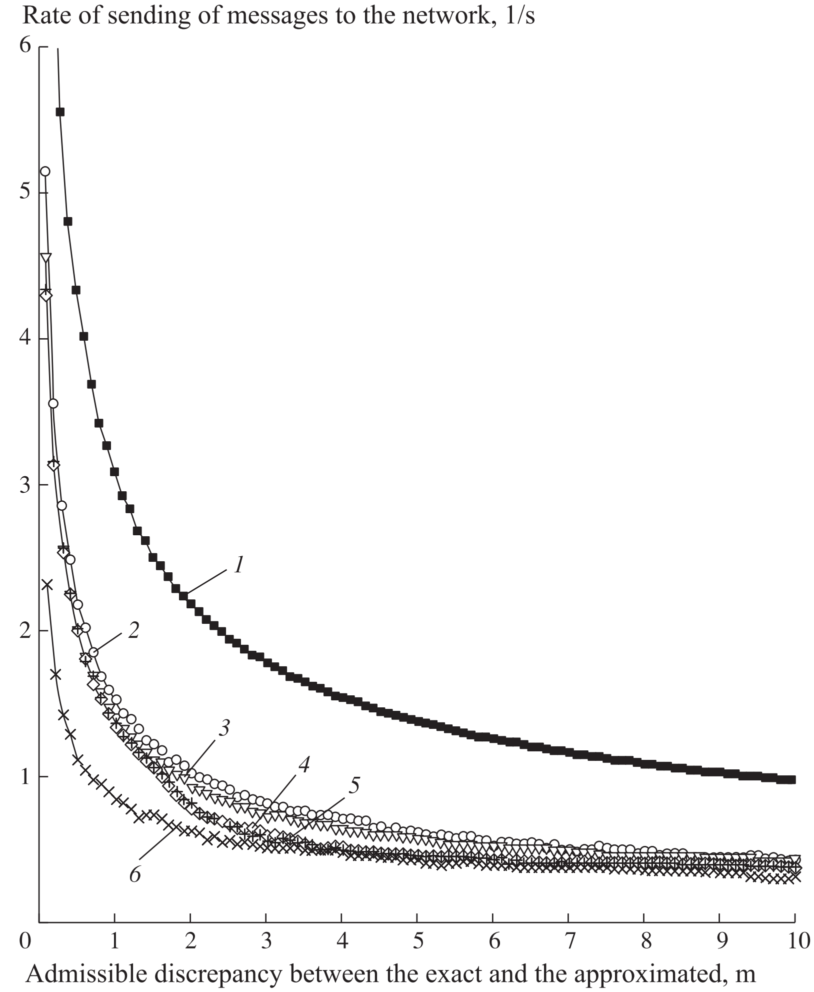

Fig. 2. Effectiveness of predictive models of motion with the spatial "maneuvering" of an object—a flying vehicle in the network of distributed simulation: (*1*) uniform motion in the Earth's axes; (*2*) uniformly accelerated motion in the Earth's axes; (*3*) uniformly accelerated motion in body axes; (*4*) spiral prediction; (*5*) spiral prediction without slip; (*6*) trajectory prediction.

neighborhood (the zone of "visibility"). An object—a transmitter of a message—"defines" both its own exact coordinates and approximate ones in accordance with the indicated model, which are sent to the remaining participants of the network. The object must give out to the network a new message as to its state in accordance with the condition of the excess of the discrepancy between the exact and the approximate solution of the specified admissible error of the approximation of the trajectory of motion. The implementation of this situation permits us to reduce considerably the intensity of information flows in the communication lines of the distributed network and to offer the prospect of the synthesis of the virtual (deductive) world with a great number of participants.

The record of the DIS interface [14] represents a collection of unified analytical predictive models describing the motion of a solid body under various assumptions for the parameters of its motion. These models are applicable for various types of objects—terrestrial, flying, above-water ones, etc., for which reason they do not take into account the specific features of motion of these objects. At the same time, FVs are the most dynamic objects of the virtual world synthesized and their messages largely determine the intensity of information flows. This permits us to suggest supplementing the set of models of the DIS record [14] by analytical models of motion of FVs, for example, the model of the spiral prediction (7)–(9) and the model of the trajectory prediction (14)–(16) [15].

To estimate the effectiveness of the inclusion of the spiral and the trajectory prediction in the set of predictive models, the static numerical modeling was made on the complete nonlinear mathematical model of dynamics of the flight of a trainer [10]. In the course of the modeling, the rate was estimated of the sending of messages as to the state of the object to the distributed network of trainers in accordance with the specified value of the admissible error of the approximation.

In the established modes of the flight and the modes of the unintensive maneuvering, the spiral and the trajectory prediction offer no advantage over the unified models of the DIS interface. But at the stages of the intensive maneuvering (the bypass and flyoff of the relief of a terrain at low altitudes, the oblique loop, etc.), the models (7)–(9) and (14)–(16) offer the possibility of markedly reducing the rate of the sending of messages by the "airborne" objects of the distributed simulation network.

In Fig. 2 are shown plots of the statistical estimates of the rates of the sending of messages for three models of the DIS record [14] (the uniform or the uniformly accelerated translational motion in the terrestrial or body-axis systems of coordinates, respectively), the spiral prediction (7)–(9), the modified spiral prediction ((7)–(9) with the additional supposition as to the zeroizing by the pilot of the lateral load factor), and the trajectory prediction (14)–(16) in the fulfillment of the three-dimensional maneuver of the type of "oblique loop." The value of the prescribed admissible error of the approximation of the trajectory of motion is laid off on the abscissa axis. The presented results confirm that at the stages of the intensive group maneuvering, which imposes the most rigid requirements on the value of the specified admissible error of the approximation of the trajectory of motion, the use of the spiral and the trajectory prediction can markedly reduce the network traffic and thus improve the probability estimates of the quality factor of functioning of the network of the distributed simulation.

6. CONCLUSIONS

In the works of Krasovskii, considerable attention is given to the algorithms economical in the computational respect for the solution of problems of the optimal control of motion of a flying vehicle and the algorithms for the loops of the automatic learning of flight trainers. In this work, we examine a new analytical solution of the equations of motion of a flying vehicle in the trajectory system of coordinates (trajectory prediction) and questions of the use of the spiral and the trajectory prediction both for the solution of problems of optimizing the motion of a FV and developing the loop of the optimal electronic instructor of the flight trainer (FT) and for the solution of problems in the new branch of the development of interfaces in the technology of distributed interactive simulation.

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