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PHYSICOCHEMICAL MEASUREMENTS

MEASURABLE PROPERTIES

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A variety of properties may be divided into four classes: qualitative (discrete, continuous, similarity-ordered, and multidimensional), quantitative ones (non-Archimedean, scalar, and multidimensional quantities), space-time ones (quantitative and qualitative), and combined ones. Examples are used to show that there are measurable properties in all these classes, not only quantitative ones but also qualitative ones.
Key words: *measurable properties, qualitative, quantitative, spatiotemporal, combined.*

There is usually no doubt that properties can be measured because the property concept is used in the definitions of key metrological terms. There are various views on what properties can be measured. Extreme points of view are as follows: one can measure any properties; and one can measure only quantitative properties, namely physical quantities, i.e., quantities appearing in physics formulas. The definition of the bounds to metrology and that of the class of procedures defined as measurements are dependent on the position taken in this range of views. I examine the question by classifying properties and then discussing ways of using the various classes of property in metrology. In the general dictionary sense, a property is a quality, feature, or component of a distinctive feature of something (S. I. Ozhegov).

People's knowledge is based on models for objects (articles, fields, phenomena, processes, organisms). The significance of the models is revealed by describing their properties. One speaks of similarities or differences between properties of objects or of a single object in different states. The occurrences of any property form a set, whose elements have definite logical relationships, i.e., a property has a definite logical structure. The meanings of these basic concepts will be dealt with more fully in the subsequent text.

The proposed classification of properties has been compiled without any attempt at exhaustive completeness; it is of multilevel character and uses various classification features. At the first level, that of general meaning, properties can be divided into at least four classes (Fig. 1).

A qualitative property is described by its set of occurrences, which do not have any quantitative feature, i.e., any pair of elements in the set may have the logical relation of equivalence (nominal identity, indistinguishability) or else difference.

A quantitative property is described by the set of its quantitative occurrences, i.e., any pair of elements from that set are in the logical relation of order by size (one occurrence is greater or less than another) or else of equivalence. Space-time properties have been distinguished as a separate class because of the fundamental character of space and time in philosophical and scientific concepts: everything occurs in space and time. Many other properties have a spatiotemporal aspect (development, change, motion), and here we note at once that some spatiotemporal properties are also quantitative ones, i.e., those two classes partially intersect. Combined properties are qualities or features described by sets of distinct properties. We now transfer to the next level of discussion for these four classes.

Qualitative Properties. It is difficult to divide qualitative properties into subclasses, although there are certain classification features (Fig. 2): discreteness, continuity, disorder, order in similarity, and multidimensionality.

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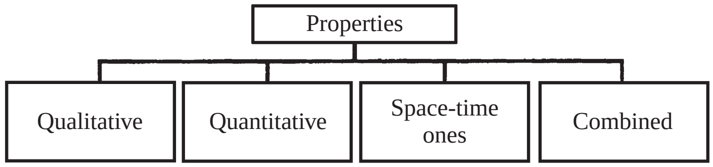

Fig. 1. General property classification.

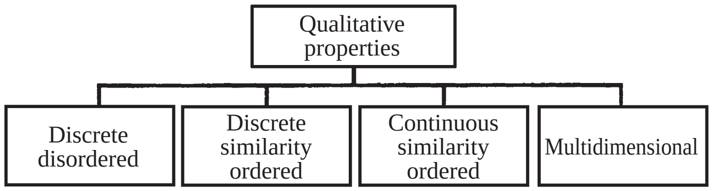

Fig. 2. Qualitative property classification.

For example, a qualitative property such as color (color sensation) has all these features in various models. Color is a qualitative feature because assertions such as the following are meaningless: some color is greater or less than another, or colors are either identical or distinct. In models where people are classified by their hair color (blonds, brunettes, graying, or red-heads), while animals such as horses may similarly be classed by color, color has a discrete disordered property. Sets of color-discrete objects (dyes, materials, light sources) may have some signs of similarity ordering, e.g., dark blue and purple are closer to light blue than they are to yellow. A rainbow represents a continuously ordered subset of spectrally pure colors. On the other hand, the set of all colors differing in hue and brightness possible for a person with normal color vision can be simulated by a continuous three-dimensional set (color space, model three-dimensional non-Euclidean space, in which there is no distance measure).

Certain quantitative features may have subsets of qualitative properties, e.g., identical colors may differ in brightness, which is a quantitative property.

The set of sounds audible to the human ear resembles the set of colors in being a multidimensional qualitative property: sounds differ in frequency, loudness, and modulation (vibration or interruptedness), as well as duration and timbre (the characteristic color of a sound). The timbre of a sound is described by qualitative features, while the frequencies of pure tones and the loudness (sound level) are quantitative. However, no general multidimensional model has yet been set up for perceived sounds analogous to the model described above for colors.

Taste and smell are also multidimensional qualitative properties. Qualitative taste features such as hotness, sweetness, salinity, acidity, and so on have certain quantitative features (for example, hot and even hotter). The variety of pleasant and unpleasant smells of various types also has the quantitative feature of strength (weak or strong). The model structure of these properties has been researched even less than that for sounds.

Qualitative properties are used in numerous identification and classification models: qualitative analysis, recognition, diagnosis, grading, and so on.

We particularly note that it is essentially impossible to transform qualitative properties into quantitative ones simply by improving the models for them even though one may introduce some quantitative elements. This is due to the general logical aspect of qualitative properties: the lack of a larger-smaller relation (it is in principle impossible to define this).

Quantitative Properties: Quantities. These properties are called quantities, and the various occurrences of some quantitative property are called its values. The meaning of the term quantity has undergone various extensions during advances in science, and on current views, quantities may be grouped into three subclasses (Fig. 3).

Non-Archimedean quantities: these are described by the logic relationships of equivalence and size order (larger–smaller), but the concept of proportionality is not applicable to them, i.e., it is not possible to obtain information on

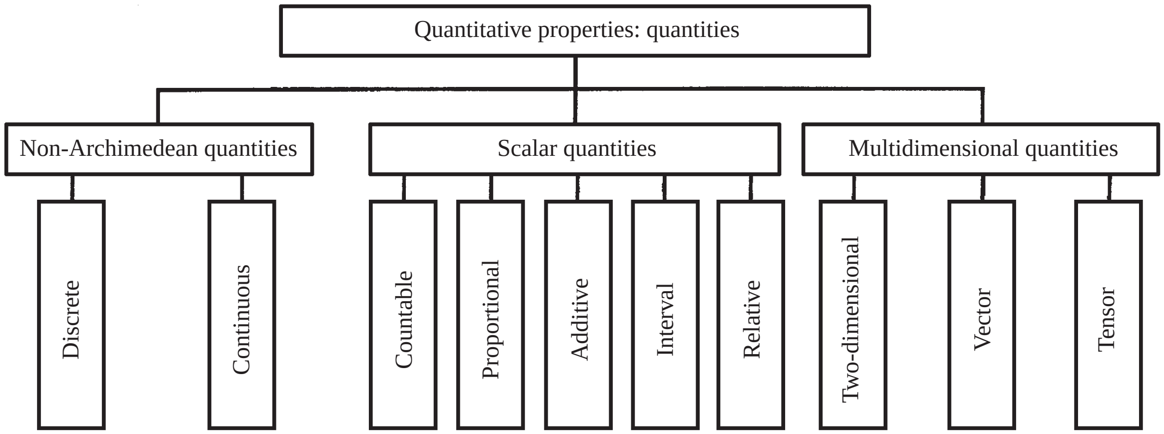

Fig. 3. Classification of quantitative properties.

by what factor one occurrence of a property is greater than or smaller than another occurrence. Also, a non-Archimedean quantity does not necessarily have a zero. In mathematics, this is expressed as follows: Eudoxius's axiom (Archimedes axiom) is not obeyed from non-Archimedean quantities: no matter what the quantities a and b of a given kind happen to be, there exists an integer n such that $a < nb$ for $a > b$. This lack of proportionality cannot be eliminated by any improvement in the models. One should not assume that those who formulate models have not attained the possible level of logical completeness. In some models, one encounters continuous and discrete non-Archimedean quantities.

It is often found difficult to understand non-Archimedean quantities because of the habit of operating only with arithmetic and algebraic concepts, so I explain them on detailed examples.

Discrete non-Archimedean quantities are as follows: hardness numbers on the Mohs scale, earthquake strengths on the observed damage scale, and wind speeds on the Beaufort scale. Continuous non-Archimedean quantities are as follows: hardness numbers on the Brinell, Rockwell, Vickers, and Shore scales, or acid, bromine, and iodine numbers, octane and cetane numbers, sensitivities of photographic materials, etc. It is clear that these quantities are not additive, as it is meaningless to add them together or sum their intervals. It is also impossible to conceive of an experiment to answer the following question: by what factor is the hardness on a particular scale for one specimen greater than that for another? One can only determine the placing of those specimens on that hardness scale. This shows that it is in essence impossible to establish proportionality for non-Archimedean quantities, i.e., it is essentially impossible to transform them into ordinary scalar quantities represented by units of measurement by any model improvement.

Scalar quantities: these are a major subclass and are used for the quantitative description of models. The subclass is divided into countable, proportional, additive, interval, and relative quantities (Fig. 3). Countable quantities are discrete, since they are expressed as positive integers to define the number of objects in the given realization. The objects may be not only homogeneous (equivalent), such as electrons, neutrons, or apples, but also inhomogeneous, for example, the objects in a batch of baggage. Proportional quantities are expressed by continuous sets of positive real numbers beginning from zero. For these quantities, one can make a judgment on by what factor one is greater or less than another, and consequently one can apply the operations of arithmetic subtraction, multiplication, and division, but addition has no meaning. An example of a proportional quantity is the thermodynamic temperature: adding together thermodynamic temperatures is meaningless.

Additive quantities are also expressed in each case by a continuous set of positive real numbers. All arithmetic operations are applicable to them, including addition, e.g., the masses of objects may be added together even if the objects differ in nature. There are scalar quantities that have the features of proportional or additive ones in different situations. For example, electrical resistances are added together arithmetically if the resistors are connected in series (additive quantity), while on parallel connection, the rule for addition is quite different, and the total resistance in that case is a proportional quantity. Similarly, electrical capacitance in the parallel connection of capacitors is an additive quantity, while on series connection, it is a proportional one. The concepts of countable, proportional, and additive quantities contain the usual logical concept on zero: zero amount (zero value), i.e., absence of occurrence of that property.

Interval quantities differ in that there is no logically based definition of zero amount, namely zero, but intervals for such quantities have the logical structure of proportional or additive quantities and so contain zero. Interval quantities do not allow of addition, and it is impossible to define a nominal zero (adopted by convention) from which one reckons the nominally positive and negative values of the interval quantity. A good example of an interval quantity is the current time, which is reckoned from a nominal zero in the positive and negative directions, while of course time intervals may have zero value (simultaneous events). One can add together time intervals, but adding together dates is meaningless. Another interval quantity is the spatial extent along an infinite straight line, on which one can denote a nominal zero, while the intervals in the extent (distances or lengths) have values beginning with natural zero.

The arithmetic unit is a natural unit for measuring countable quantities. The units of measurement for proportional, additive, and interval quantities are chosen and set by convention. *Relative quantities* have not only the features of proportional or additive ones but also logically predefined units of measurement. This is because relative quantities are by definition ratios between two arbitrary (unspecified) quantitative occurrences (values) of the same property. Such relations are expressed by real numbers, for which the single-valued unit of measurement is the arithmetic unit. Relative quantities are represented by various coefficients (transmission, reflection, attenuation, amplification), or by modulation depth, quality factor, efficiency, probability, Mach number, Reynolds number, other such numbers, similarity criteria, and finally the sizes of planar and solid angles.

Multidimensional quantities may be two-dimensional, three-dimensional (vectors), with nine dimensions (tensors), or with other dimensions appropriate to the corresponding mathematical concepts. In that series, scalar quantities are to be called one-dimensional. Multidimensional quantities can also be considered to include other nonscalar quantities (various matrices, high-dimensional tensors, and so on). For multidimensional quantities, the logical relation of greater or less in general is meaningless; that relation is however meaningful for the moduli of certain two-dimensional and three-dimensional quantities. The operations of addition and multiplication for multidimensional quantities have specific meanings, e.g., the sum of several nonzero vectors may be zero, while the product of vectors may be a scalar or vector quantity. Examples of two-dimensional quantities representing detailed properties are as follows: impedance, which is a combination of resistive and reactive components (or modulus and phase); blood pressure, which is characterized by a set of upper and lower pressures, while three-dimensional quantities (vectors) are velocity and acceleration in space, force, electric field strength, and so on. Detailed examples of nine-dimensional quantities representing properties are the tensors for mechanical stresses, dielectric permeability, and refractive index for anisotropic media.

Space-Time Properties. Only some of these are quantities, and mainly they are properties that cannot be called quantities, but which it would be illogical to consider as usual qualitative properties because of their distinctive features. I therefore call them spatial, time, and space-time properties (Fig. 4).

Spatial properties are particularly varied, because real space is three-dimensional and unbounded. Spatial quantities (scalars) characterizing the sizes of objects are as follows: distance (length), area, and volume, whose general definitions are given in mathematics. These usual quantities relate to lines, surfaces, and three-dimensional figures. The distance between two points in space is considered as an interval (segment) in something of unbounded extent and not having a zero point, namely a straight line passing through these points. One can determine the length of any curved line between two points on that basis, e.g., the distance between inhabited points on a railroad or the length of the segment on the evolvent line on a gear tooth. Similarly, one defines the area of a surface bounded by a closed contour of any shape in a plane or by a surface of any shape, or the volume of space (in a body) bounded by a closed surface of any shape.

The spatial properties of position, direction, orientation, shape, symmetry, structure, and polarization are not quantities because the larger or smaller relation is meaningless for them. Position properties relate to points, lines, surfaces, figures, and combinations of them; points may occur on lines, surfaces on one or both sides of a surface (plane), inside or outside a closed surface, and so on; lines may intersect and cross in various ways or be parallel or lie in the same plane or be on the same surface; surfaces may be variously disposed one relative to another and may touch, intersect, be one within another, and so on; figures may also be variously disposed in space one relative to another (nose to nose, back to back, side by side, in a column, in an array, or chaotically), and they may touch, intersect, or be one within another. The properties of direction

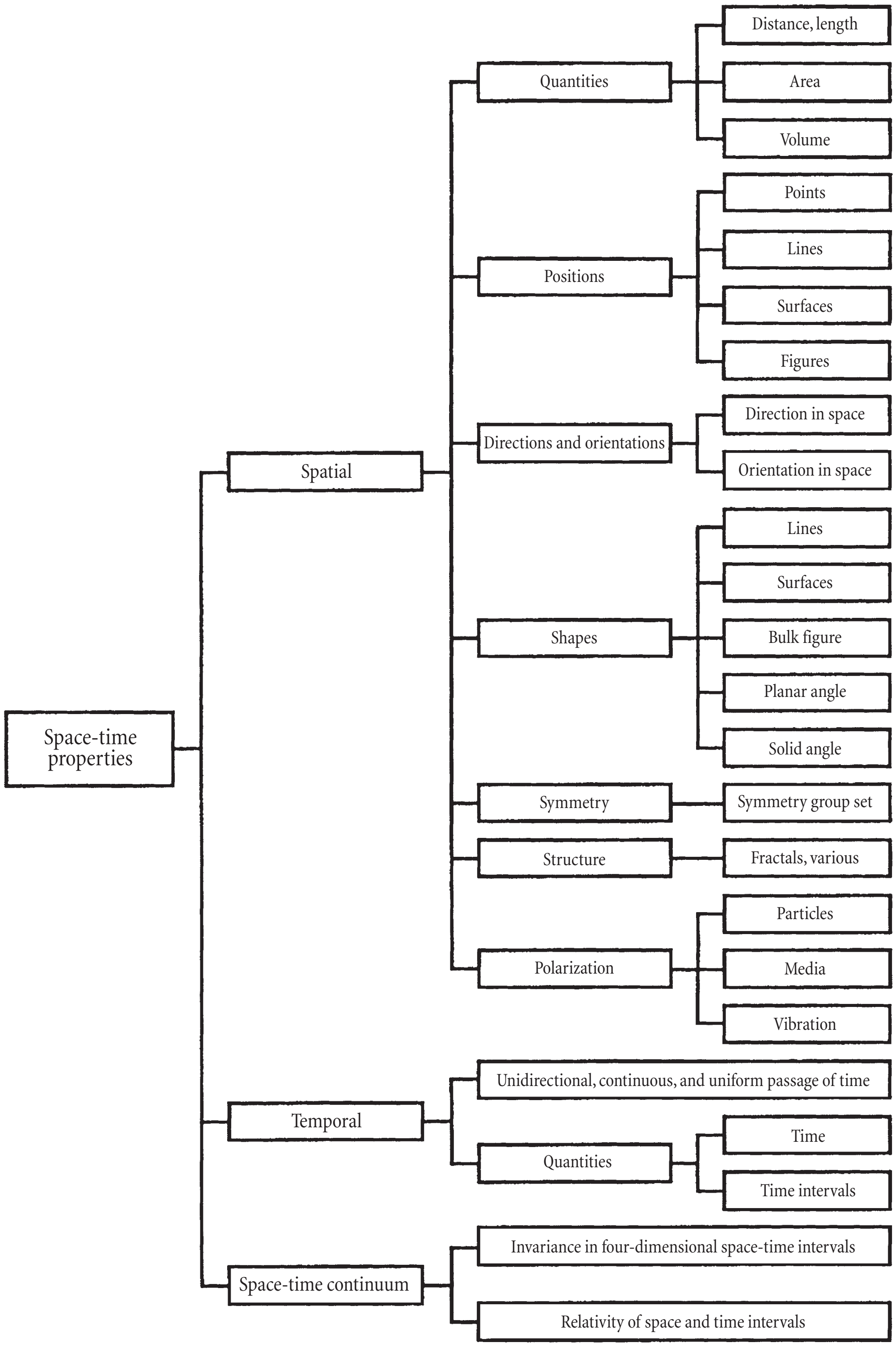

Fig. 4. Classification of space-time properties.

and orientation in space make themselves felt in relation to other objects: the set of directions along the vertical or in a horizontal plane, azimuths at various points on the Earth and in the atmosphere, and the direction to the Pole Star; that one also has the orientation of the rotation axis for a gyroscope, the orientation of the Earth's axis of rotation with respect to the plane of the ecliptic, or orientation of dipoles along lines of force in a field.

The property of shape (in space) occurs in lines, surfaces, three-dimensional figures, planar angles, and solid angles. Lines may be straight, curved, kinked, lie in a plane or in a surface, of any shape in space, closed, circular, oval, etc. Surfaces may be flat, spherical, elliptical, parabolic, ribbed, corrugated, closed, singly connected, or any shape. Three-dimensional figures may be spherical, ellipsoidal, oval, egg-shaped, multifaced, hollow, acicular, or arbitrary shape. The term planar angle (or simply angle) has two meanings: a geometrical figure in the form of two rays arising from a single point, or a quantity characterizing the aperture of that geometrical figure. In the first sense, one can speak of qualitative differences in the shape of angles: acute, obtuse, or complementary to an acute angle, or straight, bent, or of full angle corresponding to the set of all directions in a plane arising from a point. A solid angle can have the form of a circular or any other cone, a dihedral angle, a three-faced or multifaced cone, or the complete solid angle corresponding to the set of all directions in space arising from a point. Also, surfaces and figures may have similarity relations, and sets of points may be disposed in a similar way.

The property of spatial symmetry is fundamental in nature and reflects codimensionality, identity in the disposition of the parts of something on opposite sides of a point, line, or plane. There are also more complicated forms of symmetry. The logical structure of this property is indicated by classification elements such as center of symmetry, symmetry axis of order N, the corresponding rotations around the symmetry axes through angles of $360°/N$, mirror planes and glide reflection planes, right-handed and left-handed screw axes of various orders, one-dimensional translations applicable to polar and nonpolar patterns, ornaments, strips, and other symmetry elements. Combinations of symmetry elements form sets of one-dimensional, two-dimensional and three-dimensional symmetry groups. A three-dimensional object may be symmetrical or asymmetric (with lacking or disrupted symmetry), or else antisymmetric. The point symmetry groups describe the shapes of crystals (form and number of faces, edges, and vertices), while spatial symmetry groups indicate the disposition of particles (atoms, ions, molecules) in a crystal. There are 32 point symmetry groups and 230 space symmetry groups for crystals. The particles constituting a crystal can be arranged in space only at those points allowed by the symmetry elements of the given space group. There are only seven symmetry groups for border patterns. Symmetry also occurs in physical phenomena and laws. For example, the symmetry of a crystal predetermines the spatial symmetry of its refractive indicatrix (and the corresponding tensors), as well as for the dielectric permeability, magnetic susceptibility, elastic stresses, and strains. Electric and magnetic fields have differing symmetries, because the strength of an electric field is described by a polar vector, while that of a magnetic field is described by an axial one. Various complicated forms of symmetry occur in animals and plants. Living forms also show dissymmetry, i.e., the figure cannot be brought into coincidence with its mirror image by simple superposition. A good example of dissymmetric figures is provided by the right and left hands.

The spatial structure property makes itself felt as regular or chaotic alternation of elements with the same or different shapes. Sets with extremely irregular branches or dissected structure are called fractals, which differ in nature and fractional dimensions. Almost all natural structures are fractal with signs of self-similarity. The various ways in which the property of spatial polarization occurs relate to particles, media, transverse vibrations, and so on. For example, the polarization of an atom is the displacement of the electrons in the shells of the atom relative to the nucleus on account of an external electric field, while the polarization of an insulator involves a relative displacement of opposite charged particles in the atoms or molecules in response to an electric field. Another example of particle polarization is the preferential orientation of elementary-particle spins with respect to their directions of motion or the direction of a magnetic field. Planar polarization of a ray of light means that the oscillations of the electric vector at all points in the ray occur in a single plane passing through the ray direction. There are also circular and elliptical forms of polarization for light.

Various mathematical models and theories are used to describe spatial properties that are not quantities: spatial coordinate systems, vector and tensor calculus, analytical geometry, group theory, fractal theory, special vectors and matrices (four-component Stokes vector, 2×2 Jones matrix) for a state of polarization in light, and so on.

Time models are unbounded in the past and future and are unidirectional (the arrow of time), with instants of time not repeatable, uniform time flow representing a scalar additive quantity in an inertial frame of reference. The calculus of

TABLE 1. Measurable Properties of Various Classes

Class	Measurable property	Means of measurement used
1	2	3
Qualitative multidimensional property	Color	Three-channel colorimeters for measurement on the three-dimensional color coordinate scale set up by the International Illumination Commission. Color comparators and standard color specimens (a state standard and test scheme exist)
Non-Archimedean quantities	Metal hardness	Various types of hardness tester for measuring hardness on the Brinell, Rockwell, Vickers, and Shore scales (there are state standards and test schemes)
	Gasoline predetonation resistance	Authorized octane meters: instruments for measuring octane number by a standardized method
	Baking quality of flour	Authorized special instruments for measuring the drop number characterizing flour quality by a standard method
Countable quantities	Number of items	Authorized item counters used in accounting operations (the authorized units for counting are entered into the all-Russia units of measurement classification)
Proportional quantities	Density	Density meters, areometers, spirit meters, oil density meters, etc.
	Pressure	Manometers, barometers, etc.
	Viscosity	Viscometer
Additive quantities	Mass	Balances and weighing machines
	Energy	Electricity meters and so on
	Intensity of illumination	Luxmeters
	Time intervals	Meters for working time, telephone conversations, timers, and so on
	Extent intervals (lengths)	Measuring tools, length gauges, tapes, and so on
Interval quantities	Current time	Chronometers, clocks, as checked from precision time signals provided by the state standard for the national time scale
	Distance from capital along railroads	Geodesic instruments, instruments for measuring distance (length) from a nominal zero point
	Temperature on the Celsius scale	Thermometers
Relative quantities	Light transmission coefficient	Photometers
	Signal modulation depth	Modulation meters
	Planar angle size	Angle-measuring instruments
Multidimensional quantities	Impedance	Instruments for measuring the resistive and reactive components (modulus and phase) of impedance: two-dimensional quantity
	Mechanical force	Three-component force-measuring instruments for measuring the three components of a force along the axes of a chosen coordinate system or the modulus of the force and two planar angles characterizing its direction in the coordinate system
	Flying vehicle speed	Navigational instruments for measuring the modulus of the velocity and the planar angles characterizing the direction of motion in the coordinate system
Spatial additive quantities	Surface area	Geodesic instruments
	Liquid volume	Instruments for measuring a volume, measuring vessels, liquid flow rate meters, and so on

1	2	3
Spatial position class	Positions of points, lines, surfaces, and figures in an object	Three-coordinate measuring machines fitted with means of processing measurements on the coordinates of points on objects
	Positions of points on the Earth's surface	Geodesic and satellite instruments for measuring geodesic coordinates in an agreed coordinate system
Spatial direction and orientation class	Direction of vertical at a particular point on the Earth	Instruments for measuring angles of deviation from the vertical in building structures and gyroscopic vertical indicators
	Horizontal position of a plane at a particular point on the Earth	Level gauges, gyroscopic horizon indicators, and other instruments for measuring angles of deviation from the horizontal plane
	Flying vehicle orientation	Navigational instruments, gyroscopes
Spatial shape class	Surface planarity	Monitoring instruments for measuring deviations from a plane
	Sphericity of surfaces and optical items	Standard test glasses used in the observation and counting of interference fringes when superimposed on test surfaces
	Evolvent shape of spur gear surfaces	Special monitoring instruments
Spatial symmetry class	Crystal symmetry groups	Instruments for measuring crystallographic parameters
Spatial polarization class	Polarization of light	Polariscopes, polarimeters, spectropolarimeters, polarization analyzers, compensators, etc.

event chronology can occur only from a conventionally agreed zero. In a space-time model, the theory of relativity operates with a single space-time continuum, which has special properties: relativity of the intervals of spatial extent (length) and time intervals in a moving system, but with invariance of the four-dimensional space-time intervals corresponding to two events.

Combined Properties. It is meaningless to divide combined properties into subclasses because they individually may link various types of qualitative, quantitative, and space-time properties in the production of some characteristic property for example. For example, the consumer's appreciation property for bakery products is determined by the composition of the flour and additives, the techniques used in making the dough and in the baking, the color, taste, smell, shape, hardness, chewability, baking date, price, and so on. The state of the weather is characterized by a set of parameters: pressure, temperature, humidity, wind speed and direction, type and extent of cloud cover, aerosol type and concentration, amount and form of precipitation, etc. One's state of health is determined by body temperature, blood pressure, vision parameters, hearing, respiration, psychological reactions, personal feelings about one's state, and many other factors. Such multidimensional combined properties cannot be quantities, so one should classify them as qualitative properties even if quantities (quantitative properties) appear in the set of parameters. Thus in principle one can term them a subclass of qualitative properties. Combined properties essentially cannot be reduced to one-dimensional quantities without distorting their logical structure.

Property Measurement. Measuring or estimating some property is possible only when a convention has been established for the scale of measurement, which reflects the logical structure of the model for the property in a system of numbers or other characteristics. In essence, establishing a measurement scale for a property means that there is a convention on the system for encoding the primary measurement information on the occurrences of this property in various objects. On measurement scale theory, qualitative properties correspond to name scales, which may be disordered, ordered, or multidimensional. Non-Archimedean quantities have discrete or continuous ranking scales, while scalar and spatial quantities, time intervals, and current time have metrical scales (proportional and additive scales for ratios or differences), and may also have discrete (countable) and continuous absolute scales. Multidimensional quantities have multidimensional scales, which link the scales for scalar quantities, while spatial properties that are not quantities may have discrete or continuous multidimensional name scales, and combined properties may have multidimensional name scales consisting of a set of scales of different types in each case. The very fact that there is knowledge about all the properties in the classification indicates that there are fairly definite measurement scales applicable for them.

Not all ways of obtaining knowledge, information, or data can be called measurements. It is usually preferable not to give the name measurements to any study, research, identification, evaluation, or determination of properties, particularly if those actions are performed without the use of experiments and special technical devices. Of course, far from all measurement scales are used in measurements with tested or calibrated meters. However, the content of applied metrology shows that almost all classes and subclasses of properties to some extent are covered by measurement engineering. Table 1 demonstrates this on some examples.

These numerous Table 1 examples show that applied metrology is not restricted to measurements only on physical quantities. Instruments are available to measure not only ordinary proportional and additive quantities but also non-Archimedean and relative quantities, multidimensional ones, and properties that cannot be called quantities (direction, shape, position), and even qualitative properties. We have the necessary means of supporting unified measurements, although ways of describing the accuracy of such measurements are not always set out in the usual forms for estimating errors or uncertainties. In fact, such measurements enter into the state system for providing unified measurements. It appears that the sphere of legal metrology will expand, particularly in connection with advances in information technology. There is already a need for extending metrological principles to the evaluation of algorithms and uncertainties, e.g., in ecological forecasts, diagnostic procedures, and identification methods. A reasonably general metrological basis for such research is provided by measurement scale theory (representative measurement theory) together with suggestions made here on classifying properties.