Russian Physics Journal, Vol. 47, No. 2, 2004

COSMOLOGICAL MODELS WITH ROTATION

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A stationary cosmological model with rotation is constructed for the Ozsvath–Schucking metric where perfect fluid which is not comoving with the system is a source of the gravitational field. A nonstationary cosmological model for the Bianchi metric of type IX is also developed. This is characterized by expansion, rotation and acceleration. A co-moving with the system anisotropic liquid is a source of the gravitational field in this model.

Nowadays, the question on possible rotation of the Universe and its relationship with the rotation of galaxies remains topical, hence the interest in the construction and investigation of cosmological models with rotation.

We have developed the following stationary cosmological model with rotation for the Ozvath–Schucking metric [1]:

$$ds^2 = (dt)^2 + R\sqrt{1-2k^2}\,\omega^3 dt - \left(\frac{R}{2}\right)^2 \left\{(1-k)(\omega^1)^2 + (1+k)(\omega^2)^2 + (1+2k^2)(\omega^3)^2\right\}, \tag{1}$$

where $\omega^1 = \cos x^3 dx^1 + \sin x^1 \sin x^3 dx^2$; $\omega^2 = -\sin x^3 dx^1 + \sin x^1 \cos x^3 dx^2$; $\omega^3 = \cos x^1 dx^2 + dx^3$; $0 \le x^1 \le \pi$; $0 \le x^2$; $x^3 \le 2\pi$; $R, k = \text{const}$, $|k| < 1/2$.

Note that in [1], a cosmological model with rotation with metric (1) filled with dust, with a cosmological term was developed. In our model, a perfect fluid which is not comoving with the system and whose energy-impulse tensor $T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu - pg_{\mu\nu}$, where $\varepsilon > 0$ and $u_\mu u^\mu = 1$, is a source of the gravitational field. The velocity-vector components are as follows: $u_0 \ne 0$, $u_1 = 0$, $u_2 = \tilde{u}_2\sqrt{1-2k^2}\cos x^1$, and $u_3 = \tilde{u}_3\sqrt{1-2k^2}$. The Einstein gravitational constant is a unity for our model. Then nonzero Einstein equations for metric (1) are written as

$$\frac{8k^2-5}{(k^2-1)R^2} = (\varepsilon + p)u_0^2 - p\,, \tag{2}$$

$$\frac{8k^2-1}{2R(k^2-1)} = (\varepsilon + p)u_0\tilde{u}_2 - p\frac{R}{2}\,, \tag{3}$$

$$\frac{8k^2-1}{2R(k^2-1)} = (\varepsilon + p)u_0\tilde{u}_3 - p\frac{R}{2}\,, \tag{4}$$

$$\frac{1}{(k^2-1)} = pR^2\,, \tag{5}$$

Perm State University. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 2, pp. 19–21, February, 2004. Original article submitted September 4, 2003.

 1064-8887/04/4702-0128

$$-\frac{1}{(k^2-1)}=-pR^2, \tag{6}$$

$$\frac{-(16k^4-2k^2-1)}{4(k^2-1)}=(\varepsilon+p)\tilde{u}_2\tilde{u}_3(1-2k^2)+p\frac{R^2(1+2k^2)}{4}, \tag{7}$$

$$\frac{-(16k^4-2k^2-1)}{4(k^2-1)}=(\varepsilon+p)\tilde{u}_3^2(1-2k^2)+p\frac{R^2(1+2k^2)}{4}, \tag{8}$$

$$\begin{gathered}\frac{\cos 2(x^3)\sin^2(x^1)k+\sin^2(x^1)(16k^4-2k^2)-16k^4+2k^2+1}{4(k^2-1)}=\\ =(\varepsilon+p)\tilde{u}_2^2(1-2k^2)\cos^2(x^1)+p\frac{R^2(1+k\sin^2(x^1)\cos 2(x^3)+2k^2\cos^2(x^1))}{4}.\end{gathered} \tag{9}$$

It follows from Eqs. (3) and (4) that $\tilde{u}_2=\tilde{u}_3$, in which case Eqs. (7) and (8) are similar. Using Eqs. (5) and (6), we get $p=(R^2(k^2-1))^{-1}$. Substituting p in Eqs. (2), (3), (7), and (9), we get the following set of equations:

$$(\varepsilon+p)u_0^2=\frac{8k^2-4}{R^2(k^2-1)}, \tag{10}$$

$$(\varepsilon+p)u_0\tilde{u}_2=\frac{4k^2}{R(k^2-1)}, \tag{11}$$

$$(\varepsilon+p)\tilde{u}_2^2=\frac{-4k^4}{(k^2-1)(1-2k^2)}. \tag{12}$$

Provided $u_\mu u^\mu=1$, we have

$$u_0^2\frac{(2k^2+1)}{2}+2u_0\tilde{u}_2\frac{(1-2k^2)}{R}-2\tilde{u}_2^2\frac{(1-2k^2)}{R^2}=1. \tag{13}$$

Solving the set of equations (10) – (12) and taking into account Eq. (13), we find

$$\varepsilon=\frac{8k^2-3}{R^2(k^2-1)},\quad u_0^2=\frac{2(2k^2-1)}{4k^2-1},\quad \tilde{u}_2^2=\frac{2k^4R^2}{(2k^2-1)(4k^2-1)}.$$

Conditions $\varepsilon>0$, $u_0^2>0$, $\tilde{u}_2^2>0$ are satisfied for $|k|<1/2$.

Expansion, shift, and acceleration are not accounted for in the model, and rotation is nonzero for $k\neq 0$, and we get $\omega=\dfrac{2\sqrt{2}k^2}{R\sqrt{1-4k^2}\sqrt{1-k^2}}$.

We have constructed another cosmological model. Let us consider the Bianchi metric of type IX adopted in [2]

$$ds^2=(dt+A\omega^1)^2-(B\omega^1)^2-C^2((\omega^2)^2+(\omega^3)^2), \tag{14}$$

where A, B, and C are the time-dependent functions and ω^1, ω^2, and ω^3 are 1-forms satisfying structural Bianchi relations of type IX.

We represent our metric in a tetrad form. Use is made of a Lorentzian tetrad with nonzero components

$$e_0^{(0)} = 1,\ \ e_1^{(0)} = -A\sin x^3,\ \ e_2^{(0)} = A\sin x^1 \cos x^3,$$

$$e_1^{(1)} = -B\sin x^3,\ \ e_2^{(1)} = B\sin x^1 \cos x^3, \tag{15}$$

$$e_1^{(2)} = C\cos x^3,\ \ e_2^{(2)} = C\sin x^1 \sin x^3,$$

$$e_2^{(3)} = C\cos x^1,\ \ e_3^{(3)} = C.$$

We consider the case where $A = kC$ and $B = \alpha C$ (k, α = const). In this model, anisotropic liquid comoving with the system with the energy-impulse tensor written in a tetrad form is a source of the gravitational field.

$$T_{ab} = (\varepsilon + \pi)u_a u_b + (\sigma - \pi)\chi_a \chi_b - \pi\eta_{ab},$$

where

$$u_a u^a = 1;\ \ \chi_a \chi^a = -1;\ \ \chi^a u_a = 0;\ \ \varepsilon > 0,\ \ \sigma > \pi. \tag{16}$$

Assume $u^a = \delta_0^a$ and $\chi^a = \delta_1^a$. The tetrad Einstein equations are written in the following form:

$$\frac{-(8CC''k^2 + 4C'^2k^2 - 12C'^2\alpha^2 - 3k^2\alpha^2 + \alpha^4 - 4\alpha^2)}{4C^2\alpha^2} = \varepsilon, \tag{17}$$

$$\frac{k(4CC'' - 4C'^2 - \alpha^2)}{2C^2\alpha} = 0, \tag{18}$$

$$\frac{-(8CC''\alpha^2 - 12C'^2k^2 + 4C'^2\alpha^2 + k^2\alpha^2 - 3\alpha^4 + 4\alpha^2)}{4C^2\alpha^2} = \sigma, \tag{19}$$

$$\frac{8CC''k^2 - 8CC''\alpha^2 + 4C'^2k^2 - 4C'^2\alpha^2 + k^2\alpha^2 - \alpha^4}{4C^2\alpha^2} = \pi. \tag{20}$$

Using Eq. (18), we find $C = \dfrac{\alpha}{2H}\mathrm{ch}(Ht)$ and $(H = \text{const})$.

We derive the following relations for ε, π, and σ:

$$\varepsilon = \frac{-12C'^2(k^2 - \alpha^2) + k^2\alpha^2 + 4\alpha^2 - \alpha^4}{4C^2\alpha^2},$$

$$\sigma = \frac{12C'^2(k^2 - \alpha^2) - k^2\alpha^2 - 4\alpha^2 + \alpha^4}{4C^2\alpha^2},$$

$$\pi = \frac{12C'^2(k^2 - \alpha^2) + 3k^2\alpha^2 - 3\alpha^4}{4C^2\alpha^2}.$$

To satisfy Eq. (16), we impose the condition $k^2 + 1 < \alpha^2 < k^2 + 4$.

The kinematic model parameters are calculated for this metric. For the expansion $\theta = \frac{3C'}{C}$, rotation $\omega = \frac{k}{2C}$, and acceleration $a = \frac{C'k}{C\alpha}$, there is no shift.

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