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Mathematical separation of directly driven and unloading components in the ionospheric equivalent currents during substorms

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Abstract. This paper attempts to separate objectively the directly driven and unloading components in substorm processes by applying the method of natural orthogonal components (MNOC). A time series of the ionospheric equivalent current function with time resolution of 5 min during March 17-19, 1978 is calculated on the basis of six meridian chains magnetometer data during the International Magnetospheric Study in order to obtain the fundamental orthogonal basis set. The first and second natural components of the set thus obtained dominate over the rest of the natural components. The first natural component is found to have a two-cell pattern, which is well known to be associated with global plasma convection in the magnetosphere. It is enhanced during the growth phase and expansion phase of substorms and decays during the recovery phase of substorms. Further, it is in fair correlation to the ε parameter with time lag of 20-25 min. This can be identified as the directly driven component. The second natural component reveals itself as an impulsive enhancement of the westward electrojet around midnight between 65° and 70° latitude during the expansion phase only. It is much less correlated with the ε parameter than the first one. Thus, as a first approximation, we identify it as the unloading component. It is shown that the directly driven component tends to dominate over the unloading component except for a brief period soon after substorm onset. This is the first clear determination of the time profile of the unloading component.

1. Introduction

It was long believed during the earliest days of substorm research in the 1960s that the magnetospheric substorm was nothing more that a spontaneous, sudden conversion of magnetic energy that had been continuously accumulated and stored in the magnetotail [cf. *Axford,* 1967; *Siscoe and Cumming,* 1969]. However, many researchers found in the 1970s that the north-south component (B_Z) of the interplanetary magnetic field plays a vital role in causing substorms [cf. *Foster et al.,* 1971; *Arnoldy,* 1971; *Burton et al.,* 1975; *Perreault and Akasofu,* 1978] on the basis of a high correlation between individual substorms and individual southward turnings of the B_Z component. Thus *Akasofu* [1979] proposed that the magnetospheric substorm consists of two processes, the directly driven component and the unloading component. This concept of a two-component process was recognized by *Rostoker et al.* [1987]. The two components can be described as follows:

The driven component is the one in which time variations in the rate of energy derived from the solar wind are approximately the same as time variations of the sum of energies deposited directly in the ionosphere, ring current and elsewhere with appreciable time delay. The equivalent current pattern for the driven system features two vortices and has been referred in the past by the terms *DS* and S_q^p and it likely incorporates the high latitude portion of *DP* 2.

The unloading component refers to deposition of the energy stored in the magnetotail into the auroral ionosphere and into the ring current. The equivalent current pattern associated with the unloading has a single vortex involving a longitudinally confined westward ionospheric electrojet located in the midnight sector. This equivalent system has been referred to as *DP* 1.

The equivalent current is a mathematical representation of ground magnetic disturbances and is assumed to flow only in a spherical shell, the ionosphere [*Chapman,* 1935]. The *DP* 1 and *DP* 2 current system [*Akasofu et al.,* 1965; *Nishida,* 1968] are the equivalent current representation of two components of magnetic disturbances in the polar region. *Clauer and Kamide* [1985] tried to separate the *DP* 1 and *DP* 2 current systems by means of a differential technique, accomplished by choosing a time interval $T = T2 - T1$. The current at *T*1 is assumed to remain constant during the interval *T*, while an additional current system develops during the same interval and is estimated by subtracting the effects of the currents at time *T*1 from the total currents. Thus the separation of the *DP* 1 and *DP* 2 in this method had to assume that the current at *T*1 is kept constant during the interval. They found that both *DP* 1 and *DP* 2 currents develop during the course of the substorm activity. Recently, *Kamide and Kokubun* [1996] suggested that the two components are as the result of the relative changes of electric fields and conductivities in the auroral electrojet. However, this method of separation requires the knowledge of both electric field and conductivity over the entire polar region as a function of time.

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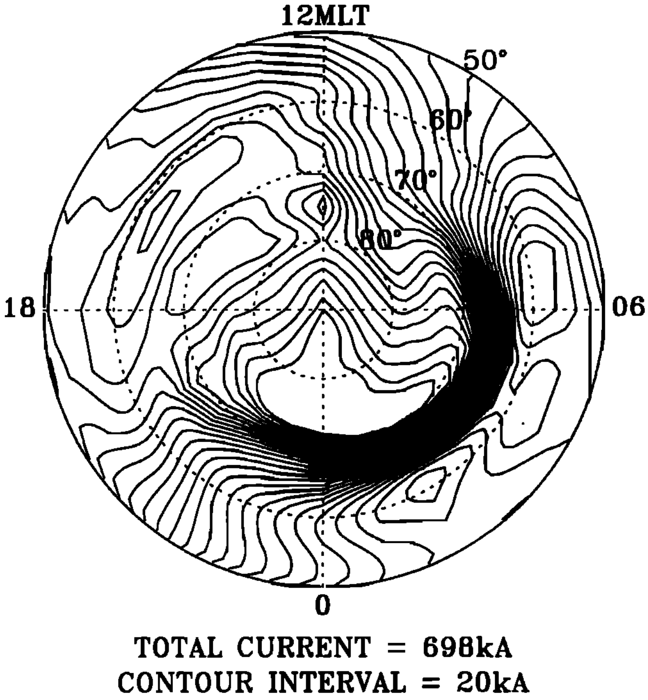

Figure 1. The ionospheric equivalent current function at 0230 UT on March 18, 1978. The total current is the integrated current over all contour intervals.

Many other authors examined also both the directly driven and the unloading components. *Baker et al.* [1981] found that the correlation of the *AE* index to the ε function and interplanetary electric field (*VBs*) has a peak value of 0.54 and 0.6 with a time lag of 40 min. *Bargatze et al.* [1985] discussed the magnetospheric impulse responses measured by the *AL* index to the solar wind parameter *VBs* and found that two response pulses have 20 min and 60 min time lag, respectively. They considered that the 20-min pulse represents the directly driven component in the solar wind-magnetosphere coupling process and the 60-min pulse represents unloading component. *Blanchard and McPherron* [1995] analyzed the linear response function relating *AL* to *VBs* and suggested that the *AL* index is controlled by two distinct processes for most of substorms, both proportional to *VBs* and describable as the directly driven component. *Blanchard et al.* [1996] made the linear correlation between the reconnection electric field in the magnetotail and *VBs* and found that the correlation coefficient is 0.46 with a time delay of 70 min

In the present paper we introduce the method of natural orthogonal components (MNOC) in analyzing the equivalent current function during substorms and obtain the fundamental orthogonal basis set. We show that the first natural component and the second natural component are dominant and are found to have well-known characteristics of the directly driven component and the unloading component, respectively. As a data set we use magnetic records that were extensively collected during the International Magnetospheric Study (IMS) from a total of 71 stations [*Kamide et al.*,1982a]. The equivalent current functions during March 17-19, 1978 with time resolution of 5 min are calculated on the basis of the IMS data [*Kamide, et. al.*, 1982b]. Figure 1 shows an example of the equivalent current function at 0230 UT on March 18, 1978. The contour interval is 20 kA. The total current is the integrated current over all contour intervals. The total of 864 patterns of the equivalent current function during 3 days is used as input data for analysis of the natural orthogonal components.

2. Description of the Method of Natural Orthogonal Components

MNOC [*Kendall and Stuart*, 1976] is often used to separate the structure of the geomagnetic field into longer and shorter wavelength parts [*Frynberg*, 1975; *Pushkov et al.*, 1976, *Rotanova N. M. et al*, 1982]. MNOC was also used to separate geomagnetic field variation into quiet and disturbed components [*Golovkov et al.*, 1978]. The separated *Sq* variation was applied to improve the calculation of the *K* indices at a local station [*Papitashvili et al.*, 1992].

In MNOC the fundamental orthogonal basis set is obtained during the calculation procedure, so that it is different from the spherical harmonic and Fourier analysis in which the fundamental orthogonal basis set is set previously. We suppose that the equivalent current function $E(t_i,r_j)$ is a function of time t_i (i=1,2,...,m) and location r_j (j=1,2,...,n). In the present calculation, the period of time is 3 days with a resolution $\Delta t = t_{i+1} - t_i$ = 5 min, hence m=864. In the pattern of the equivalent current function as shown in Figure 1 the latitude range is 50° to 90° with 2° interval, and the longitude range is 0° to 360° with 15° interval. Thus the number of space grid points $n = 20 \times 24 = 480$.

An expansion is sought in terms of functions $X^k(r_i)$, k=1,2,...,h $\leq n$, i.e.,

$$E_{ij} = \sum_{k=1}^{h} T_i^k X_j^k \tag{1}$$

where T_i^k is a function of time and $X^k(r_i)$ values are orthogonal over the data locations,

$$\sum_{j=1}^{n} X_j^l X_j^p = 0 \qquad \text{for } l \neq p \tag{2}$$

To find X_j^k and T_i^k minimize

$$\delta_n = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[E_{ij} - \sum_{k=1}^{h} T_i^k X_j^k\right]^2 \tag{3}$$

with respect to X_j^k and T_i^k. This procedure yields

$$T_i^k = \frac{\sum_{j=1}^{n} E_{ij} X_j^k}{\sum_{j=1}^{n} (X_j^k)^2} \tag{4}$$

and

$$\sum_{p=1}^{n} X_p^k \sum_{i=1}^{m} E_{il} E_{ip} = X_l^k \sum_{i=1}^{m} (T_i^k)^2 \sum_{p=1}^{n} (X_p^k)^2 \tag{5}$$

If we define

$$A_{lp} = \frac{1}{m} \sum_{i=1}^{m} E_{il} E_{ip} \tag{6}$$

$$\lambda_k = \frac{1}{m} \sum_{i=1}^{m} (T_i^k)^2 \sum_{p=1}^{n} (X_p^k)^2 \tag{7}$$

$$\mathbf{X}^k = (X_1^k, X_2^k, ..., X_n^k)^T \tag{8}$$

where the coefficient $1/m$ is introduced for normalization, then equation (5) becomes

$$\mathbf{A}\mathbf{X}^k = \lambda_k \mathbf{X}^k \tag{9}$$

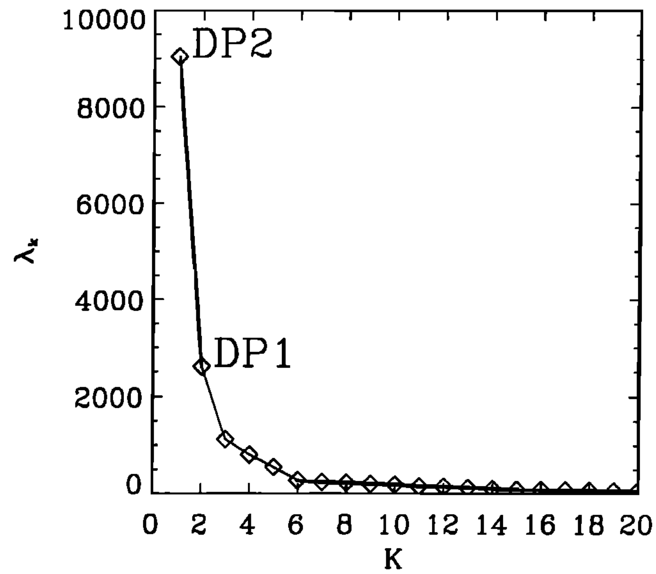

Figure 2. The value of λ_k as a function of the order number k for $k = 1,2,...,20$.

which can be solved for $\mathbf{X}^k$ and λ_k by the standard method. Once $\mathbf{X}^k$ is found, T_i^k can be computed by equation (4). It is not difficult to prove that the time function T_i^k is also a fundamental orthogonal basis set:

$$\sum_{i=1}^{m} T_i^f T_i^g = 0 \qquad \text{for } f \neq g \tag{10}$$

Note that the natural components $\mathbf{X}^k$ are normalized in solving equation (9), i.e.,

$$\sum_{j=1}^{n} (X_j^k)^2 = 1 \tag{11}$$

3. Results

3.1. Separation of Directly Driven and Unloading Components

Figure 2 shows values of λ_k for $k = 1,2,...,20$ that indicate the importance of the corresponding $\mathbf{X}^k$ in $E(t_i,r_j)$ One can easily see that the first two natural components dominate over the rest. The sum of the contributions of the first natural component and the second natural component ($T_i^1 X_j^1 + T_i^2 X_j^2$) is, on the average, 83% of the total current ($E_{ij} = \sum_{k=1}^{n} T_i^k X_j^k$) for the 3 days. Therefore we believe that the first two natural components represent the most important and common properties of substorms during 3 days. Certainly, any individual substorm has its temporal and local variation, which can be discussed in detail by combining all natural components. Hereafter we consider only the first two natural components. Figure 3 shows the patterns of first two normalized natural components ($\mathbf{X}^1$, $\mathbf{X}^2$). It can be seen that the first natural component has a two-cell convection pattern, while the second natural component indicates a westward electrojet around the midnight sector between 65° and 70° latitude. Thus the patterns of the first two natural components resemble the equivalent current patterns of *DP* 2 and *DP* 1, respectively.

Figure 4 shows the *AE* index for a substorm event during 0900-1300 UT on March 19, 1978. A slow increase of the *AE* index occurred from 1000-1135 UT, followed by a sudden intensification at 1140 UT. Figure 5 shows the current function and the contribution from the first natural component ($T_i^1\,\mathbf{X}^1$) and the second natural component ($T_i^2\,\mathbf{X}^2$) at two particular epochs, 1100 UT and 1210 UT (i = 708 and 712) on March 19, 1978, as indicated by two vertical lines in Figure 4 . The former was during the growth phase of the substorm, while the latter was during the expansion phase. One can see that the first natural component shows clearly the two-cell convection pattern during

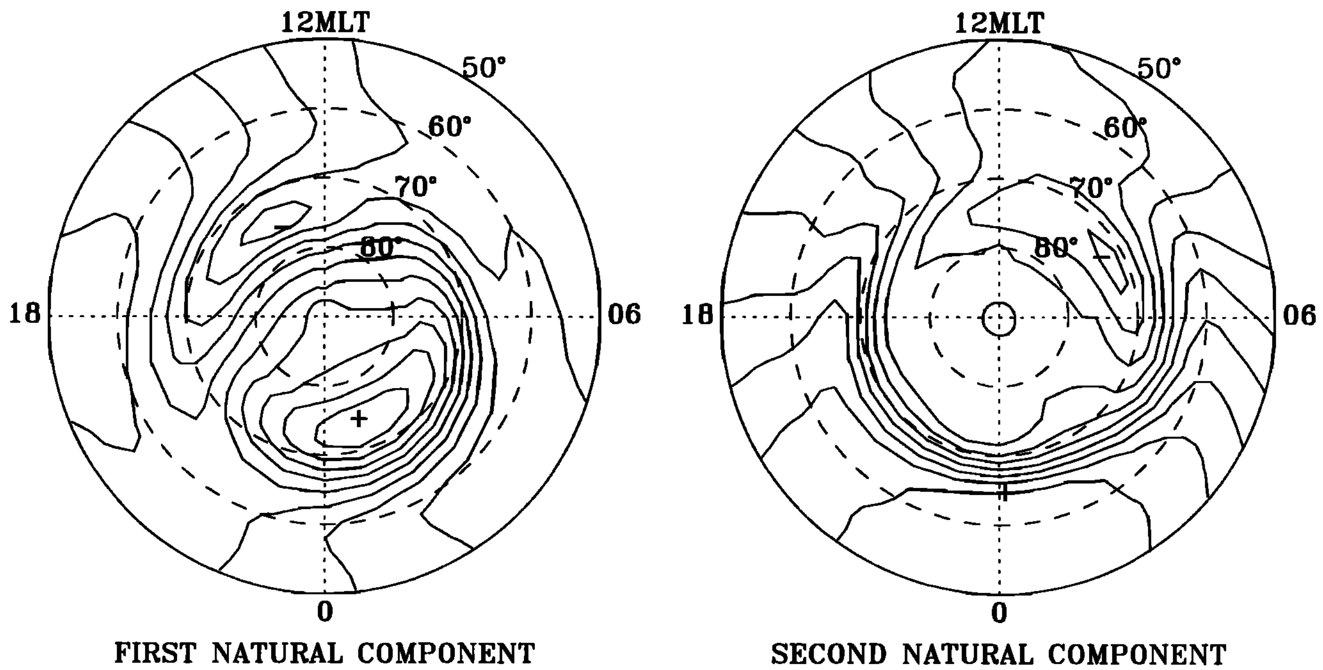

Figure 3. Patterns of the first and second normalized natural components ($\mathbf{X}^1$, $\mathbf{X}^2$) for March 17-19, 1978.

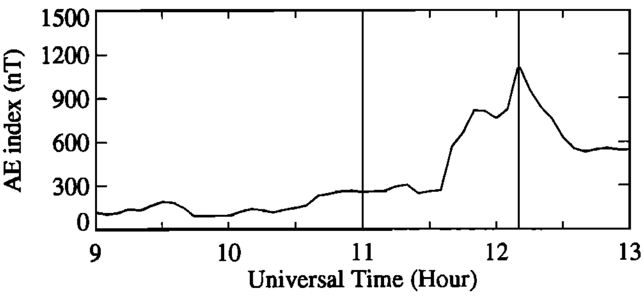

Figure 4. *AE* index for the substorm during 0900-1300 UT on March 19, 1978. The two vertical lines indicate the time corresponding to the growth phase and the peak of the expansion phase.

the growth phase. Further, it becomes enhanced during the expansion phase. On the other hand, the second natural component shows little indication of its presence during the growth phase but is greatly enhanced during the expansion phase.

3.2. Cross Correlation Between ε Parameter and First Two Natural Components

Figure 6 (top) shows the solar wind-magnetosphere coupling function ε on March 18 and 19, 1978, calculated on the basis of the solar wind and interplanetary magnetic field data observed by the satellite IMP 8. The solid curve shows variations of the ε function with 5-min resolution, and the shaded area shows the hourly mean values of the ε function. Panel 2 shows the *AE* index and the total current. The bottom two panels show time variations of the contribution from the first and second natural components (T_i^1, T_i^2), respectively. One can see that there is fair correlation between ε function and the first natural component. On the other hand, the second natural component tends to be impulsive and large mainly at about substorm onset of three substorms at 0900 UT on March 18 and 2200 UT and 1100 UT on March 19.

Note that relatively smaller negative values occur in the second natural component, although the sum of all the natural component values must equal the total current. In a mathematical analysis of a physical phenomenon, solutions with no physical meaning are not adopted. The simplest example is some solutions of a quadratic equation. Fourier analysis of a positive change might contain some negative values. However, these do not invalidate the analyzed result. Thus the negative values in the second natural component without physical meaning will not be adopted.

To examine the correlation in more detail, we choose two periods to make the cross correlation analysis between the coupling function ε and the total current, the first natural component, and the second natural component because of the intermittent nature of the interplanetary data with 5-min resolutions during 3 days. Figures 7a and 7b show the cross correlation during 0900-2100 UT March 18 and 1500-2400 UT March 19, respectively. The top panels show the cross correlation between the total current and the ε function. It can be

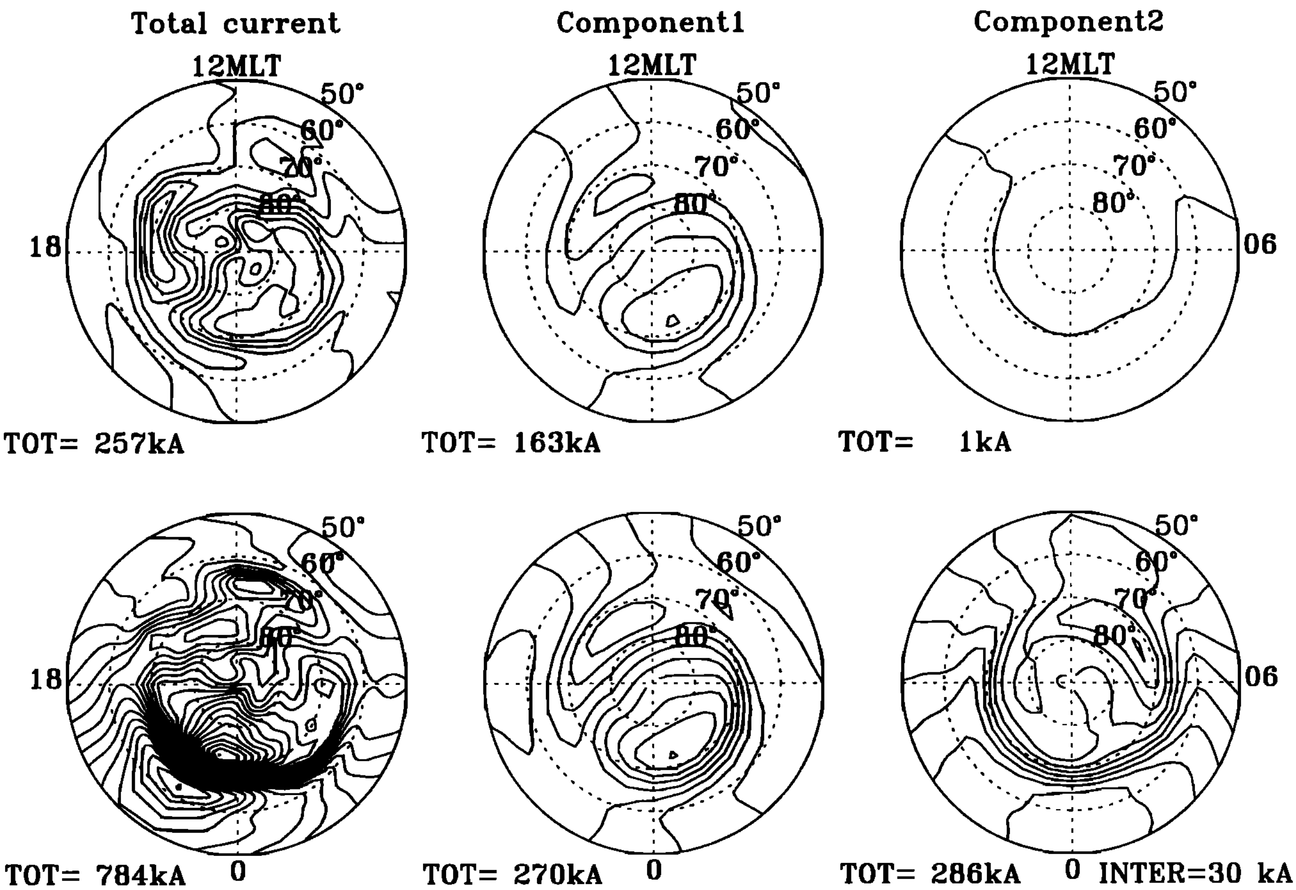

Figure 5. (top) Equivalent current functions and first and second natural components at 1100 UT, March 19, 1978, for the growth phase of substorms. (bottom) Similar patterns at 1200 UT for the expansion phase.

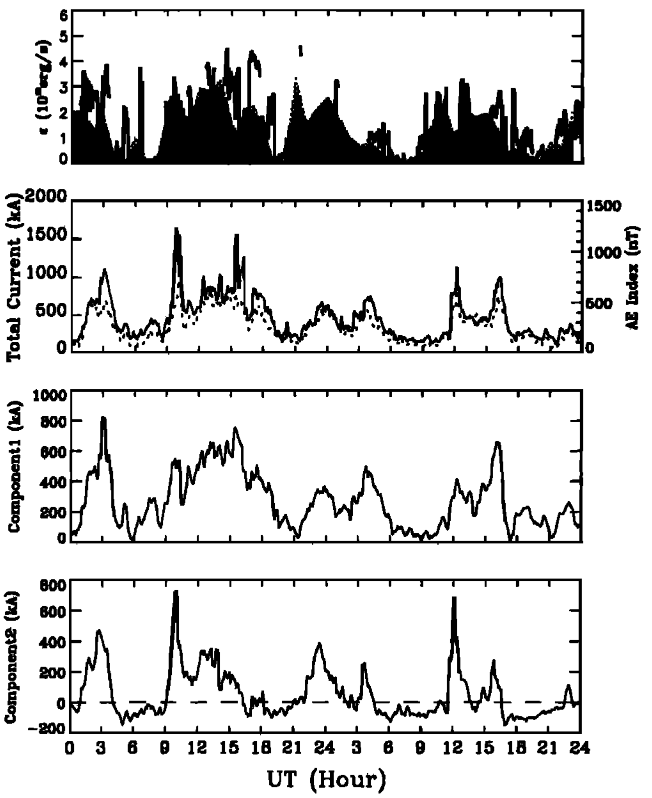

Figure 6. (top to bottom) The ε coupling function as a function of time during March 18-19, 1978, with 5-min time resolution; time series of the total current (solid curve) and the *AE* index (dotted curve); and contributions of the first and the second natural components ($T_i^1\, \mathbf{X}^1$ and $T_i^2\, \mathbf{X}^2$) during the same period.

seen clearly that the main peak value of the cross-correlation coefficient is 0.6-0.7 with time lag of 20-25 min. This suggests that the directly driven component lags behind the driving force by about 20-25 min. The second peak value of the cross correlation coefficient is about 0.4 with a time lag of about 70-80 min. This can be considered to be the effect from the unloading component. This result is consistent with that obtained by *Bargatze et al.* [1985]. The results state that both of the directly driven and the unloading components are driven by the solar wind, but the directly driven component has much higher correlation with the solar wind than does the unloading component. The middle panels in Figures 7a and 7b show the cross correlation between the ε function and the first natural component. It is interesting to note that they have only one main peak value of the cross correlation coefficient, which is about 0.7 with the time lag of 20-25 min. This result can be considered as another identification that the first natural component represents the directly driven component, in addition to having the two-cell pattern. The bottom panels in Figure 7a and 7b show the cross correlation between the ε function and the second natural component. There are peak values of 0.3-0.4 in the correlation coefficients with a time lag of 70-75 min.

4. Summary

In the present paper we introduced a mathematical method called the natural orthogonal components to analyze the equivalent current function during substorms and to obtain the fundamental orthogonal basis set. As is shown in equations (2) and (10), both T_i^k and X_j^k are the fundamental orthogonal basis set. This result indicates that all factors ($F^k = T_i^k X_j^k$), which contribute to the equivalent current, are completely independent, or are not correlated with each other, whether in space or in time.

We identify the first two natural components as the *DP* 2 and the *DP* 1 current by their patterns and correlation to the ε coupling function. The first natural component has a two-cell pattern, which is indeed similar to the *DP* 2 current: this pattern is enhanced during the growth phase and the expansion phase and decays during the recovery phase of substorms. On the other hand, the variation of the first natural component is in fair correlation to the ε parameter with time lag of 20-25 min. Therefore we can conclude that the first natural component represents, as a first approximation, the directly driven

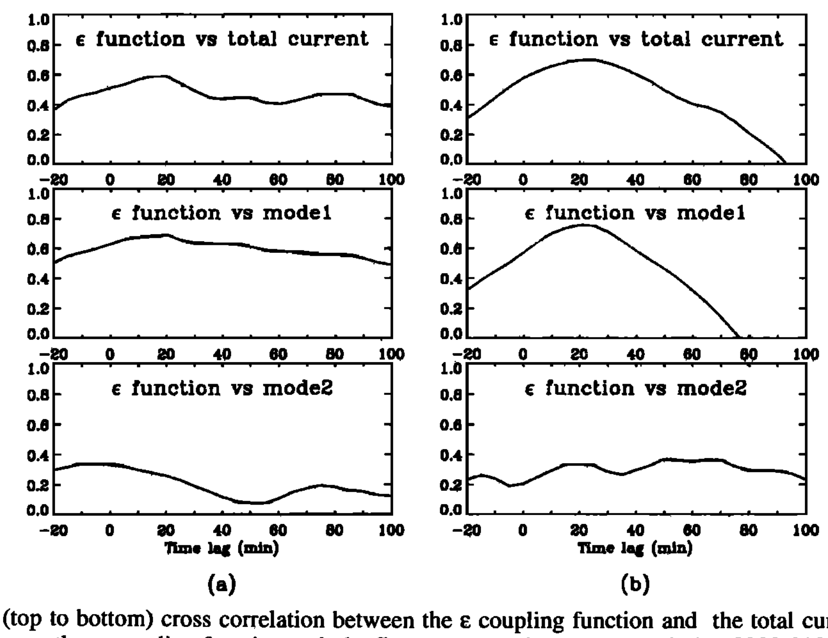

Figure 7. (a) (top to bottom) cross correlation between the ε coupling function and the total current and cross correlation between the ε coupling function and the first two natural components during 0900-2100 UT on March 18, 1978. (b) Same as Figure 7a except for the event during 1500-2400 UT on March 19, 1978.

component during substorms. The pattern of the second natural component displays a westward electrojet around the midnight sector between 65° and 70° latitude and has an impulsive positive value during the expansion phase only of substorms; it is indeed characteristic of the *DP* 1 current. The second natural component is much less correlated with the ε parameter. Thus we can conclude that the second natural component represents, as a first approximation, the unloading component.

It is of great interest that the particular mathematical method we employed in this paper identified the two major natural components, whose characteristics agree with the known directly driven component and unloading component, respectively. It is most interesting that the identification of the two components by *Baumjohann* [1983] is in good agreement with our results. Strictly speaking, it may not be possible to separate the two components. For example, an enhanced conductivity caused by the unloading process could enhance also the directly driven component. Thus, although the first natural component may be mixed by a small part of the second natural component, we believe that we have succeeded in separating roughly the two components. Our results suggest that the direct effect of the unloading component manifests itself impulsively (~1 hour) at substorm onset. Thus, knowing the characteristics and its time profile, we can now discuss physics of the unloading component and its cause. Although such a characteristic has been suggested theoretically for a long time, this is the first attempt to obtain its time profile observationally. The impulsive nature of the unloading component suggests that the stress caused during the growth phase is limited to a relatively short distance in the X coordinate, perhaps where the thin current sheet forms. Thus the stress can be released in a relatively short period. It remains to be seen whether or not the duration of the unloading coincides approximately with the time span during which the front of the expanding bulge reaches the highest latitude.

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