Cis and Trans Links in Natural Channel Networks

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Abstract. An explanation of the excess of trans links over cis links in natural channel networks is proposed. The basic assumptions are (1) channel networks develop by headward growth, (2) tributaries are generated independently on the two sides of the main channel, and (3) once a tributary has been established, the probability of other tributaries developing further downstream on the same side is greatly diminished. Computer simulations of a simplified version of this model produce networks with about 58% trans links.

The concept of cis and trans links was introduced into channel network geomorphology by *James and Krumbein* [1969]. A main channel link is called a cis link if the tributaries at the upstream and downstream ends enter the main channel from the same side, and a trans link if they enter from opposite sides. (When two channels meet, the main channel is the one of higher magnitude and the tributary channel is the one of lower magnitude.) A priori, it would appear that trans and cis links should occur with equal frequencies, but James and Krumbein found when studying stream networks in eastern Kentucky that a sample of 485 interior links had 293 trans links and 192 cis links (60.4% trans). The probability of this or a more unequal distribution occurring if the target population actually contained equal proportions of the two kinds is less than 0.00001. Smart (unpublished data, 1969) studied several networks of magnitudes about 200 and found proportions of trans links ranging between 54 and 68%.

James and Krumbein suggested that the networks initially develop with equal numbers of cis and trans links, and that the observed difference is because of subsequent channel adjustments, in particular the elimination of very short cis links by lateral erosion. Although there is no reason to doubt that readjustments do occur in channel networks after the drainage development is essentially complete, it also seems doubtful if such changes could account for the magnitude of the observed difference. To explain the James and Krumbein data, one-third of the original cis links would have to be eliminated by one cause or another.

In this paper we propose an alternative explanation of the prevalence of trans links. Our explanation is based on three general assumptions: (1) channel networks develop by headward growth, (2) tributaries are generated independently on the two sides of the main channel [*James and Krumbein*, 1969, p. 554], and (3) once a tributary has been established at a given point on the mainstream, the available cis downstream drainage area is curtailed, so that the probability of other tributaries developing further downstream on the same side is greatly diminished. This last assumption is perhaps most appropriate for an area with a generally uniform regional slope.

We have not succeeded in obtaining an analytic treatment of this model but various versions of it can be simulated by computer. *Smart and Moruzzi* [1971*a*] have programed a random walk, headward growth model of drainage development on homoclinal ridges in which the probability of growth is determined by the amount of upland area available for contributing runoff. Thus the Smart–Moruzzi model explicitly incorporates assumptions (1) and (3) above and a study of the rules indicates that assumption (2) is implicitly guaranteed. Drainage networks generated by this method show a preponderance of trans links. Moreover, drainage networks produced by an earlier model [*Smart and Moruzzi,* 1971*b*], in which the probability of growth is determined by a random choice independent of existing stream configurations, have approximately equal numbers of cis and trans links.

The essential features of our proposed explanation are somewhat obscured by the details of the Smart–Moruzzi model, and can be seen more easily in the simplified version illustrated in Figure 1. The simulation is performed on an array of squares with L rows and three columns. The central column represents the course of a mainstream that grows in successive steps from the first to the Lth row. At each stage in the mainstream development, certain sites in the left column are identified as active sites, where tributaries can be established, and similarly for the right column. When the mainstream is extended from the $(i - 1)$th row to the ith row, the left and right sites in the ith row are added to the left and right active site lists, respectively. The two lists are then combined and one site is selected at random. A decision about generating a tributary at this point is made by comparing a random number drawn from a uniform distribution between 0 and 1 with a probability p specified at the start of the simulation. If a tributary is not allowed, the mainstream moves into the $(i + 1)$th row and the process is repeated. If a tributary is created, the outside sites in the ith row are removed from their respective active site lists. Also all active sites downstream from the new tributary and on the same side as the new tributary are removed from the appropriate active site list. Then the mainstream is extended to the $(i + 1)$th row.

Figure 2 shows a plot of the fraction of trans links f_T as a function of p. The points indicate mean values of f_T for 10 simulations, each generating about 500 links. The standard deviations range between 0.012 and 0.027, generally increasing with decreasing p. The results for small p, where the discrete properties of the model have less effect, would be expected to correspond most closely to natural networks. For $p < 0.5$, f_T is approximately 0.58, to be compared with the value of 0.604 measured by James and Krumbein.

The reason for the excess of trans links over cis links in our model can be ascertained by a detailed look at the steps involved in the simulation. In most instances, whenever a new tributary is created, either the number of cis links increases by one or the number of trans

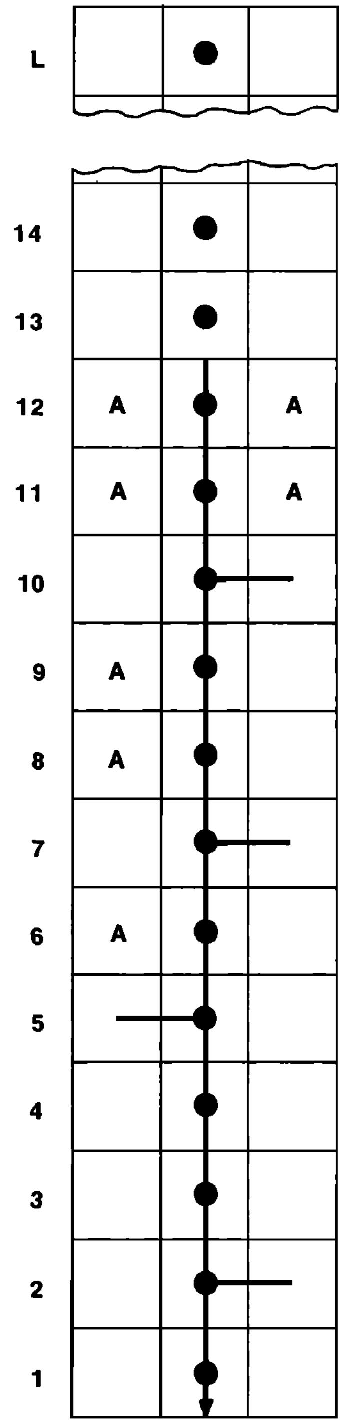

Fig. 1. Mainstream and tributary configuration at 12th step in a simulation with $p = 0.4$. Active sites are indicated by A.

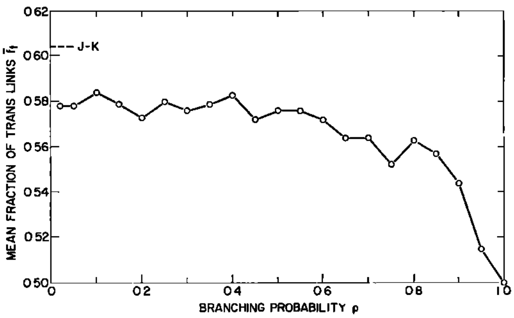

Fig. 2. Fraction of trans links as a function of branching probability. Plotted points are mean values of f_T for 10 simulations, each generating about 500 links. J-K indicates James-Krumbein observation.

links increases by one. An important exception occurs when there is a cis link (or chain of cis links) at the upstream end, as shown in Figure 1. If a left-branching tributary is created in either row 8 or row 9, the number of trans links is increased by two and the number of cis links is decreased by one. A step-by-step trace of several simulations indicates that this particular event accounts approximately for the difference in numbers of cis and trans links, and that all other tributary assignments produce cis and trans links in nearly equal frequencies.

James and Krumbein found that the observed cis and trans link lengths were differently distributed, the most noticeable discrepancy being a dearth of very short cis links. Statistical studies of link lengths for our model indicate that the two types of links have approximately the same length distribution. It should be noted that our model is not incompatible with the James and Krumbein suggestion of readjustments in the late stages of network development. In particular, the elimination of very short cis links would shift both the trans–cis ratio and the link length distribution toward the observed values.

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