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# Interpretation of electrostatic energy analyzer data of a flowing plasma

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Extracting correct plasma parameters from the experimental data from a retarding field electrostatic energy analyzer in a flowing plasma requires proper interpretation of the effect of plasma flow on the collector current. In this paper, relationships for computing plasma parameters directly from the analyzer data are described for any type of velocity distribution of particles.

## I. INTRODUCTION

Retarding field electrostatic energy analyzers[1] have been used extensively in plasma experiments. For example, Andersen *et al.*,[2] Andersen *et al.*,[3] Guillemot *et al.*,[4] Ikezi and Taylor,[5] Taylor and Coroniti,[6] Kiwamoto,[7] and Mau[8] report useful application of retarding field electrostatic energy analyzers. In these works, although extensive data have been used, none contains a comprehensive discussion of how the correct velocity distribution of sampled particles or the effective temperature are obtained from the data in a flowing plasma. Only in Refs. 5–7 was the effect of particle acceleration on the experimental data briefly discussed. However, electrostatic acceleration is not the only experimental situation where the sampled particles have a significant mean velocity.

The present paper is intended to illuminate the proper interpretation of the analyzer data in obtaining the plasma parameters, regardless of how the flow has come about and the type of the velocity distribution of the particles.

In this paper we also discuss the case of a retarding field energy analyzer moving at a constant velocity in a stationary plasma (i.e., with zero mean velocity). From this analysis, considerable insight is gained for the interpretation of data obtained in experimentally feasible situations.

## II. BASIC EQUATIONS

We consider the case of a retarding field energy analyzer sampling the axial component $V_\parallel$ of the velocity distribution of particles. Then, to be collected by the analyzer, a particle must possess a finite positive axial velocity component. However, since we are interested in plasmas with substantial mass flow, the number of particles with negative velocities is insignificant.

For the configuration of Fig. 1,

$$(1/2)m\,V_\parallel^2 = e\varphi,$$

where $\varphi$ is the retarding potential.

$$V_\parallel(\varphi) = +(2e\varphi/m)^{1/2}$$

$$dV_\parallel = +(e/2m\varphi)^{1/2}\,d\varphi.$$

We define the particle velocity distribution function $f(V_\parallel)$ as

$$\langle n\rangle = \int_{-\infty}^{\infty} f(V_\parallel)\,dV_\parallel,$$

where $\langle n\rangle$ is the average particle number density. In general, the velocity distribution function would be a function of the position $r$ and it should be defined as

$$\langle n(r)\rangle = \int_{-\infty}^{\infty} f(V_\parallel, r)\,dV_\parallel.$$

Assuming a collector area $A$ and an overall geometric transmission coefficient $\tau$ for the grids, the collector current for $\varphi$ is

$$I_c(\varphi) = A\tau e\int_{V_\parallel(\varphi)}^{\infty} V_\parallel f(V_\parallel)\,dV_\parallel = A\tau e\int_{\varphi}^{\infty} \times\left(\frac{2e\varphi'}{m}\right)^{1/2}\left(\frac{e}{2m\varphi'}\right)^{1/2} f[V_\parallel(\varphi')]\,d\varphi',$$

and the collector characteristic is

$$I_c(\varphi) = \frac{A\tau e^2}{m}\int_{\varphi}^{\infty} f[V_\parallel(\varphi')]\,d\varphi'. \qquad (1)$$

Equation (1) is the analytic form of the collector characteristic for any distribution. Furthermore, the "experimental" particle distribution function is given by

$$dI_c/d\varphi = (A\tau e^2/m)f[V_\parallel(\varphi)]. \qquad (2)$$

Equation (2) has been stated without derivation in a footnote to the paper by Ikezi and Taylor.[5] This result has been used in many plasma measurements, although

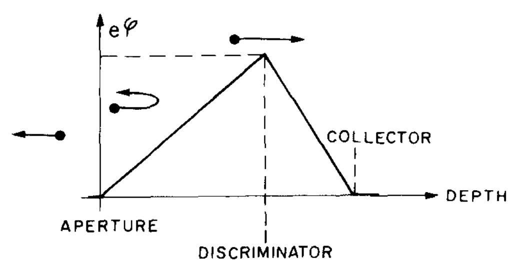


FIG. 1. Electrodes and potential configuration in the analyzer.

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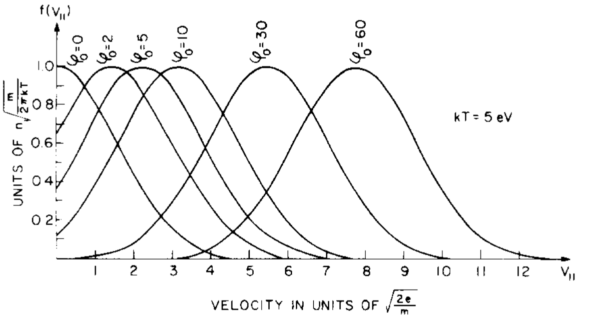


FIG. 2. Parallel velocity distributions for different velocities. $\varphi_0$ is the potential corresponding to the parallel velocity at the peak of the distribution.

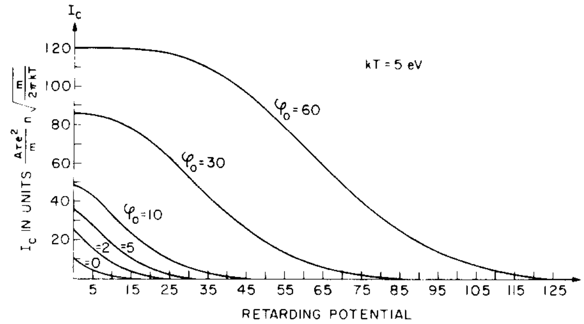


FIG. 3. Collector characteristics corresponding to different velocities.

not derived or discussed in the paper describing these measurements.

In an actual experiment, the differentiation would be performed electronically,[4] or graphically. Once the "experimental" particle distribution function (i.e., $f[V_\parallel(\varphi)]$) has been obtained, the various plasma parameters can be computed (here we let $f[V_\parallel(\varphi)] = g(\varphi)$ for convenience):

$$\langle n \rangle = \int_{-\infty}^{\infty} f(V_\parallel)\, dV_\parallel = \left(\frac{e}{2m}\right)^{1/2} \int_0^\infty \frac{g(\varphi)}{\varphi^{1/2}}\, d\varphi, \quad (3)$$

$$\langle nV_\parallel \rangle = \int_{-\infty}^{\infty} V_\parallel f(V_\parallel)\, dV_\parallel = \frac{e}{m}\int_0^\infty g(\varphi)\, d\varphi, \quad (4)$$

$$\left\langle \frac{mV_\parallel^2 n}{2} \right\rangle = \int_{-\infty}^{\infty} \frac{mV_\parallel^2}{2} f(V_\parallel)\, dV_\parallel$$

$$= e\left(\frac{e}{2m}\right)^{1/2} \int_0^\infty \varphi^{1/2} g(\varphi)\, d\varphi; \quad (5)$$

hence,

$$\langle V_\parallel \rangle = \langle nV_\parallel \rangle / \langle n \rangle$$

$$= \left(\frac{2e}{m}\right)^{1/2} \left(\int_0^\infty g(\varphi)\, d\varphi\right) \left(\int_0^\infty \frac{g(\varphi)}{\varphi^{1/2}}\, d\varphi\right)^{-1}, \quad (6)$$

$$\langle mV_\parallel^2/2 \rangle = \langle mV_\parallel^2 n/2 \rangle / \langle n \rangle$$

$$= e\left(\int_0^\infty \varphi^{1/2} g(\varphi)\, d\varphi\right) \left(\int_0^\infty \frac{g(\varphi)}{\varphi^{1/2}}\, d\varphi\right)^{-1}, \quad (7)$$

and the mean random energy associated with axial motion $\langle E_\parallel \rangle$ would be

$$\langle E_\parallel \rangle = \langle mV_\parallel^2/2 \rangle - (m/2)\langle V_\parallel \rangle^2$$

$$= e\left\{\left(\int_0^\infty \varphi^{1/2} g(\varphi)\, d\varphi\right)\left(\int_0^\infty \frac{g(\varphi)}{\varphi^{1/2}}\, d\varphi\right)^{-1} - \left[\left(\int_0^\infty g(\varphi)\, d\varphi\right)\left(\int_0^\infty \frac{g(\varphi)}{\varphi^{1/2}}\, d\varphi\right)^{-1}\right]^2\right\}. \quad (8)$$

If the particles have a Maxwellian distribution, $\langle E_\parallel \rangle$ would relate to the parallel temperature $T_\parallel$ as

$$\langle E_\parallel \rangle = kT_\parallel/2.$$

In practice, once $g(\varphi)$ is obtained experimentally, it can be digitized and the various averages can be calculated using a computer.

In general, zero discriminator potential need not correspond to zero particle momentum; the discriminator potential corresponding to zero particle momentum has to be calibrated prior to any measurements. An example of such calibration is described in the paper by Andersen *et al.*,[2] where the authors note that a group of relatively cold charge exchange ions serve as a marker on the energy scale (see Fig. 3 of that paper) to determine the discriminator potential corresponding to approximately zero particle momentum. In experiments where no such process takes place, electrons from an accurate gun may serve the same purpose.

## III. CASE OF ANALYZER MOVING IN A UNIFORM, STATIONARY MAXWELLIAN PLASMA

Several experimental situations where the particles' directed velocity is comparable to or greater than the particle random velocity have been reported. A typical instance where the directed ion velocity is comparable to the ion random velocity along the magnetic field lines is discussed by Korn, Marshall, and Schlesinger,[9] who note the complications when the ion saturation current of a small probe is used for deducing the number density. Still another situation is reported in the paper [Fig. 9(c)] by Roth *et al.*,[10] who report that the plasma ions have substantial drift velocity. A situation with a probe mounted on a satellite, moving with a substantial velocity compared to the ion random velocity in the ionosphere, is reported by Bettinger and Chen.[11] Equations (3)–(8) are applicable in such situations. However, due to the implicit form of $g(\varphi)$, the effect of substantial particle directed velocity is not apparent.

To simulate a situation similar to one of the above,

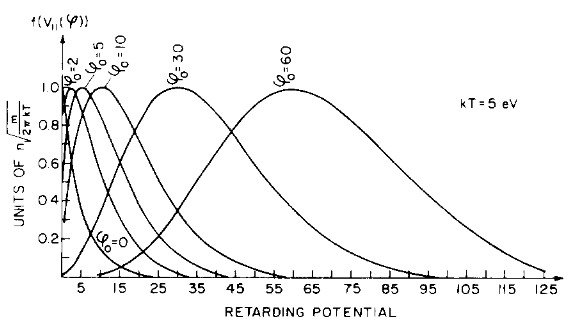


FIG. 4. "Experimental" distribution functions corresponding to different velocities.

we consider a retarding field analyzer moving with constant velocity in a stationary plasma whose particles are known to have Maxwellian distribution (with $kT = 5$ eV). In the coordinate system of the analyzer, the parallel velocity distribution corresponding to six different velocities would be as shown in Fig. 2.

We use Eqs. (1) and (2) for a Maxwellian distribution,

$$f(V_{\parallel}) = n(m/2\pi kT)^{1/2} \exp[-(m/2)(V_{\parallel} - \bar{V}_{\parallel})^2/kT],$$

where

$$\bar{V}_{\parallel} = (2e\varphi_0/m)^{1/2}$$

for $\varphi_0$ = potential corresponding to the kinetic energy of the mean velocity. Then

$$f[V_{\parallel}(\varphi)] = n(m/2\pi kT)^{1/2} \exp [-e(\varphi^{1/2} - \varphi_0{}^{1/2})^2/kT], \quad (2')$$

and

$$I(\varphi) = \frac{A\tau e^2}{m} n \left(\frac{m}{2\pi kT}\right)^{1/2} \int_{\varphi}^{\infty} \exp \left(\frac{-e(\varphi'^{1/2} - \varphi_0{}^{1/2})^2}{kT}\right) d\varphi'. \quad (1')$$

Now Eqs. (1′) and (2′) correspond to the collector characteristic and the "experimental" particle distribution function, respectively. In Figs. 3 and 4, Eqs. (1′) and (2′) have been plotted with $\varphi_0$ treated as a parameter.

## ACKNOWLEDGMENT

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