A CALCULATOR FOR AIDING MATRIX CALCULATIONS *

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An apparatus is described in which a number of standard calculators are arranged in a battery, their individual movements being so ganged as to make possible the simultaneous multiplication of coefficients set on each machine by a common factor. This materially assists in carrying out matrix calculations of various types.

The machine described in this paper was designed primarily to facilitate the solution of linear simultaneous equations. It has been used in the calculation of inverse matrices of order up to twelve and for substitutions in an inverted matrix to obtain solutions for sets of simultaneous equations. The machine may be used in assisting in calculations of many types which involve manipulation of matrices.

In the course of matrix calculations, it soon becomes obvious that the majority of operations consists of multiplying sets of coefficients by a single factor and then algebraically adding these results to a similarly multiplied set of coefficients, this process being repeated many times in the course of a computation. Using a simple desk calculator, it can be seen that one keeps repeatedly setting in the same coefficients time after time. Many attempts have been made to reduce the

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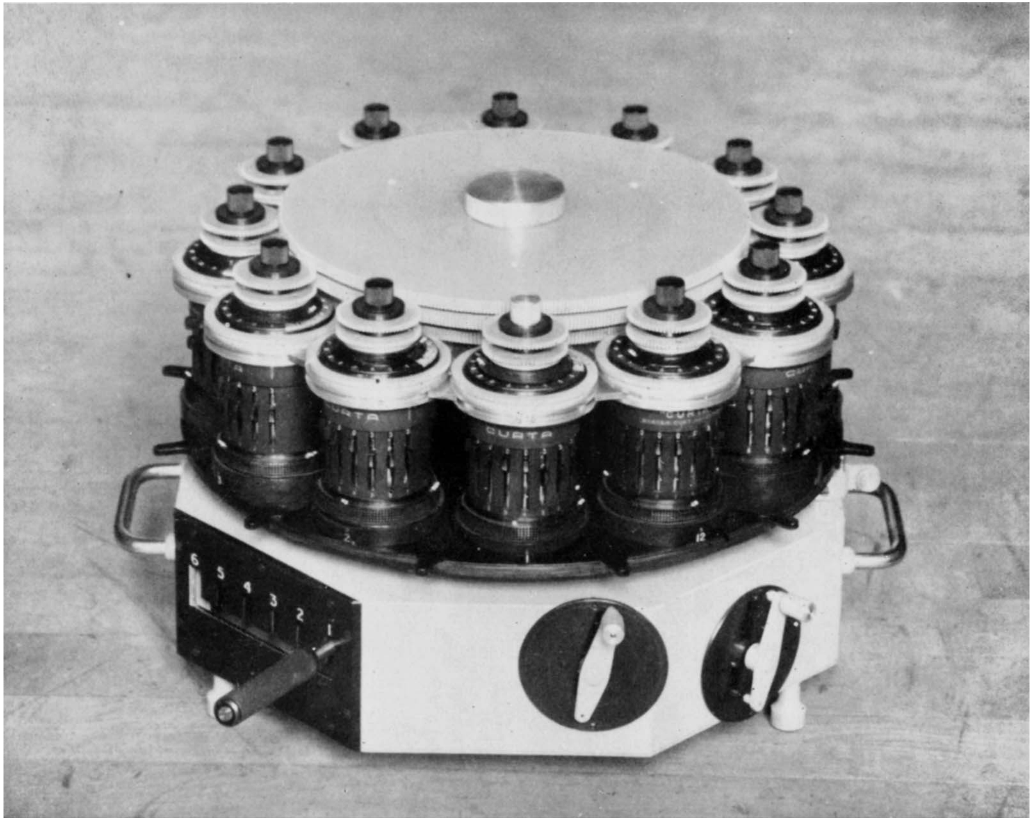

FIG. 1.—General view of completed machine.

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labour of such calculations and these have led to the now familiar electrical analogue type of computor. However, this type of machine suffers in general from having a precision governed by the quality and stability of the electrical components which are used in its construction and it is often with difficulty that a precision of 0·1 % can be achieved. For many purposes, this is not adequate and it would seem desirable that the precision of a mechanical calculator should be utilized in a similar kind of way.

It is easy in principle to see how this might be achieved by taking a number of hand calculators and so ganging their individual controls that the mechanical movements of multiplication may be performed on all simultaneously. The drawbacks to such a scheme lie in making such an apparatus as compact and cheap as possible, without sacrificing flexibility or scope. Further, it must be possible in a relatively simple way to link the movements precisely and positively.

Quite recently, a calculating element has become available which fits the requirements perfectly, namely, the Curta calculator, manufactured by Contina. It is cylindrical in shape and has the mechanism for setting the multiplicand on the side of the cylinder. At the top of the cylinder is a knurled carriage. This carriage bears the multiplier register and the product register and by raising and rotating the carriage, the appropriate position can be selected for multiplying by units, tens, hundreds, and so on. The carriage also carries a disc which can be rotated by a ring, one complete rotation clearing the registers. Above this is the multiplying handle, one complete rotation of which gives multiplication by 1 unit, 10 and so on, depending on the position of the carriage. Multiplication by a negative number is achieved by pulling the handle upwards along the axis of the machine and turning in the normal way. This adds the product into the register in a negative sense. The capacity of the machine is $8 \times 6 \times 11$ digits.

A further feature of this machine which adds considerably to its suitability in this connection is the fact that all the calculator operations are performed by rotary motions about the central axis of the cylinder. As will be appreciated, this leads to a great saving in space when the machines are ganged and, further, simplifies the design problems.

CONSTRUCTION DETAILS

The apparatus described here comprises twelve calculating units in the gang, although in principle there is no need to limit the number. Fig. 1 shows a general view of the finished machine.

The calculators are mounted on a platform A, fig. 2, which can be rotated manually on a main central bearing fitted to the octagonal base. This platform is fitted with 12 small projecting arms to facilitate the rotation and also with a spring-loaded catch which accurately locates the main platform so that any one machine can be brought to the front position. As will be seen below, this exact location of the platform enables a locking device to be incorporated to ensure that the operating handles may be used only when the machines are in the correct position.

The octagonal cast duralumin base accommodates the mechanisms for rotating the main multiplying shafts of the calculators, clearing the registers of the machines, raising the carriages of the calculators and rotating the carriages to the appropriate multiplying positions. The handle on the right, fig. 1, is the multiplying handle, that in the centre is the clearing handle, and that on the left raises and rotates the carriages into any one of the six multiplying positions. The carriages are raised by depressing this lever and rotated by a sideways motion, finally being located and lowered by raising the handle into the appropriate slot.

The constructional details are best explained by reference to fig. 2 and 3. The clearing handle N is mounted on a horizontal spindle passing through the octagonal base. This spindle runs through journal ball races in a housing and is terminated in a bevel gear B, which rotates on a vertical shaft (2/1 step down ratio) and is fixed to a pinion, meshing with a larger pinion, spur gear 4. This pair have a step-down ratio of 3/1. This larger pinion is mounted on a hollow shaft running on taper bearings through the top of the base, the platform and the central block mounted on the platform. The upper end of this hollow shaft carries spur gear 2 which meshes with 12 pinions C, one fastened to the clearing disc of each machine. This ratio is 6/1 step-up so that the net result is that one turn of the handle corresponds to one turn of the clearing discs.

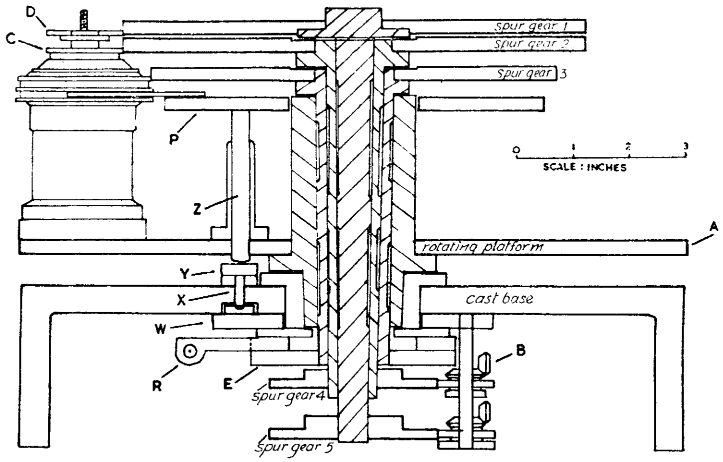

FIG. 2.—Diagram illustrating constructional details.

The multiplying transmission mechanism is exactly similar. In this case, the shaft is driven by spur gear 5 and revolves on taper bearings within the hollow shaft of the clearing transmission. Spur gear 1 meshes with 12 pinions D, fitted on bosses, one on the multiplying shaft of each calculating machine. Multiplication in a negative sense is achieved by pulling up the multiplying spindle by means of the knob at the top of each machine. This causes the product to be added negatively into the register.

The most complicated part of the transmission is that operating the carriage lift and rotation since it is highly desirable that these two different motions should be performed by one operating handle. The further complication lies in the fact that the lifting motion of the carriages has to be transmitted from the base through the platform which has also to be capable of being rotated. Fig. 3 shows diagrammatically the underside of the base.

The carriage lever Q which pivots at R (R being a bearing on a plate rotating freely about the axis), passes over a rod S and also carries a fork which passes under S. This rod is fixed at each end to two similar cranked plates T, firmly braced together and mounted on a common spindle U about which they can rock. The other end of one of the cranked plates is connected by a carefully fitted system of three links made from square section brass V to a disc W, so arranged as to rotate about the axis. Thus, the up-and-down motion of Q results in a rotary motion of W. The plate W carries three small cams (inclined planes), so arranged that rotation of W causes these cams to push three rods X upwards through the base. These three rods are fixed to a ring Y which surrounds the main central column. Normally clear of this ring are mounted six symmetrically placed rods Z, which pass through the platform in sliding-fit guides. The upper ends of these rods are fitted to a plate P bearing forks which engage in grooved rings on the carriages of the 12 machines. Thus the up-and-down movement of handle Q is converted to a down-and-up movement of the carriages.

The rotation of the carriages in order to select multiplication by units, tens, hundreds, and so on, is obtained as follows. When the lever Q is depressed to raise the carriages, the toe of the lever engages in 1 of 12 slots in a brass disc E fitted on the lower end of a hollow shaft. As before, this shaft is constructed with taper bearings so that it can rotate easily by a sideways movement of the lever Q. At the upper end of the shaft is carried spur gear 3. This gear wheel is arranged to engage with 12 toothed rings, one being fitted to the carriage of each calculator. The ratio of these gears is so arranged that raising the lever Q into any one of the 6 slots on the face-plate through which the lever passes, corresponds exactly to the appropriate position of the carriages in which the multiplying operation is to be carried out.

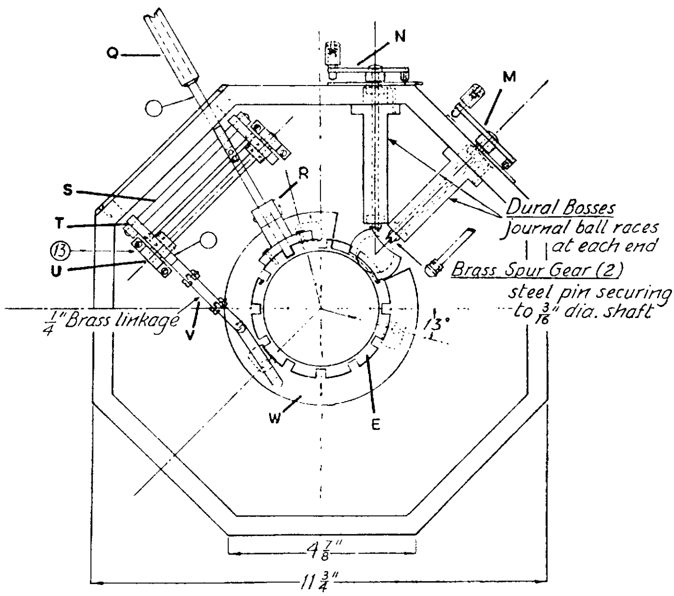

FIG. 3.—Diagram of underside of base.

The lever Q can be depressed only when the platform carrying the machines is correctly located by the stop referred to above. This is achieved by fitting three pins to the outside edge of ring Y. In the lowered position of Y, these pins are just clear of the platform, but immediately the lever Q starts to be depressed, these pins either strike the platform and prevent further movement of Q, or, if the platform is correctly located, these pins slide into suitably placed holes of the platform, so permitting the further depression of Q. This has the further safeguard that it is impossible to rotate the platform while the carriages are raised, thus preventing damage to the calculators.

The handles M and N are fitted with spring loaded plungers in the knob and are so arranged that if a turn of the handle is made with the knob pressed in, the plunger will automatically lock into a depression on the face plate when exactly one turn has been completed.

The operations of the machine are as follows. Depression of lever Q raises the carriages. Movement of Q sideways rotates the carriages. Raising Q into a slot lowers the carriages in the corresponding position. With Q in the up position, the handle on the right can be rotated to make multiplying turns. With Q in the down position and at the extreme right of its limit of travel, the central clearing handle may be rotated and the registers on the carriages of the calculating machines returned to zero. With Q in the up position, the whole platform may be rotated, so bringing each machine in turn to the front for convenient access in setting the multiplicand. By pulling up the knob at the top of each machine, the product may be entered in the register in a negative sense.

OPERATION

The function of this machine can be stated quite simply as providing a means of multiplying up to 12 numbers simultaneously by a common factor.

For example, suppose we wish to perform the following operation :

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & \cdots & A_{1,12} \\ A_{2,1} & A_{2,2} & A_{2,3} & \cdots & A_{2,12} \\ A_{3,1} & A_{3,2} & A_{3,3} & \cdots & A_{3,12} \\ \vdots & \vdots & \vdots & & \vdots \\ A_{12,1} & A_{12,2} & A_{12,3} & \cdots & A_{12,12} \end{bmatrix} \times \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_{12} \end{bmatrix}$$

then the terms $A_{1,1}, A_{2,1}, A_{3,1} \ldots A_{12,1}$ are set, one on each machine, and then each is simultaneously multiplied by X_1. Then $A_{1,2}, A_{2,2}, A_{3,2} \ldots A_{12,2}$ are set on the respective machines and simultaneously multiplied by X_2, these products being automatically added algebraically to the products previously recorded on the register. This is continued until after finally setting on $A_{1,12}$, $A_{2,12}, A_{3,12} \ldots A_{12,12}$ and multiplying by X_{12}, the 12 coefficients of the final answer are displayed on their respective machines. This operation can be carried out on a matrix of order (12×12), having elements on the average of four figures in a time of 17 min.

A further feature of this instrument lies in the fact that it permits one to set in complete rows or columns of a matrix, thus avoiding the necessity of selecting specific figures from alternate matrices as would be necessary in calculating with only one machine. This decreases the possibility of operator errors.

This machine has also been used for obtaining the inverse of a matrix and for a matrix of order (12×12), this can be done in about 8 h. The method employed is that described by Crout [1] and also by Milne.[2] With 12 machines, such a matrix can be inverted and at the same time it is possible to run two check columns throughout the work. The only operation other than multiplication and addition required in this case is the division of the result of a number of accumulated products by one of the accumulated products. This can be done in two ways. The numbers on the product registers are set in their respective machine keyboards and either multiplied simultaneously by the reciprocal of the divisor, which is obtained on a separate machine, or, alternatively, the product register of the machine carrying the divisor on the keyboard is made up to unity by suitable multiplication. This latter operation automatically multiplies the other coefficients by the required reciprocal of the divisor.

These examples are illustrative of only two applications of the machine but there are many types of calculation to which it can be applied.

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[1] Crout, *Trans. Amer. Inst. Elec. Eng.*, 1941, **60**, 1235.
[2] Milne, *Numerical Calculus* (Princeton University Press, 1949), p. 17.