Dust-acoustic solitary waves in an inhomogeneous magnetized hot dusty plasma with dust charge fluctuations

Amar P. Misra and A. Roy Chowdhury

Citation: Physics of Plasmas **13**, 062307 (2006); doi: 10.1063/1.2210928
View online: http://dx.doi.org/10.1063/1.2210928
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/13/6?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF PLASMAS 13, 062307 (2006)

Dust-acoustic solitary waves in an inhomogeneous magnetized hot dusty plasma with dust charge fluctuations

Amar P. Misra[a)]
Department of Mathematics, Siksha Bhabana, Visva-Bharati University, Santiniketan-731 235, India

A. Roy Chowdhury[b)]
High Energy Physics Division, Department of Physics, Jadavpur University, Kolkata-700 032, India

(Received 16 March 2006; accepted 16 May 2006; published online 19 June 2006)

Using a standard reductive perturbation theory, a Zakharov-Kuznnetsov (ZK) equation is derived in an inhomogeneous dusty plasma comprised of negatively charged dust grains of equal radii, Boltzmann distributed electrons and positive ions. The effects of dust thermal pressure, dust charge variation, dust-neutral collision, and the external magnetic field are taken into account. Either compressive or rarefective solitons are shown to exist depending on the critical value of the phase velocity which in turn, depends on the dust temperature and pressure. An analytic soliton solution is obtained and discussed. The behaviors of the soliton amplitude, width, and the Mach number are investigated numerically for different plasma parameters. © *2006 American Institute of Physics*.
[DOI: 10.1063/1.2210928]

I. INTRODUCTION

Charged dust particles or impurities frequently appear in most space[1–10] and laboratory plasmas.[8,11,12] Many low temperature technological plasmas are contaminated by highly charged dust or impurities, as they carry a considerable amount of negative charge of the plasma. In reality, the charge on the dust grain varies both with space and time due to the electron and ion current flow into or out of the dust grain, as well as other processes like secondary emission, photoemission of electrons etc. these lead to the dust charge fluctuations.

The charged dust grains not only modify the existing plasma wave spectra,[13–15] but also introduce new eigenmodes, viz., dust ion-acoustic (DIA),[16,17] dust-acoustic (DA),[18,19] dust lattice,[20–22] etc., in unmagnetized dusty plasma.

DIA waves observed in laboratory experiments[17,23] were first theoretically predicted by Shukla and Silin.[16] On the other hand, Rao *et al.*[18] first theoretically predicted the existence of extremely low phase velocity (in comparison with the electron and ion thermal ones) DA waves in an unmagnetized dust-electron-ion plasma where the charged dust grain provides the inertia and the pressures of inertialess electrons and ions (Boltzmann) provide the restoring force.

The linear properties of DIA, DA waves in a dusty plasma have been studied by a number of authors, e.g., Refs. 5, 8, 16, 18–20, and 24. The nonlinearities contribute to the localizations of waves and lead to different types of nonlinear structures, namely, solitons, shocks, vortices etc. which are each nowadays a topic of important research both from theoretical and experimental points of view.

It has been found that the harmonic generated nonlinearity gives rise to small amplitude DA solitary waves which are governed by the Kortewg–de-Vries (KdV) equations.[25] A number of theoretical investigations have been made recently to study the DA solitary waves and double layers, DA shock, DIA, and DA solitons[26–28] in a magnetized/unmagnetized dusty plasma. Most studies used charging currents independent of the magnetic field[28(b)] assuming that the dust grain radius is smaller than the electron gyroradius. For a plasma laboratory that uses weak magnetic fields, this condition may be valid. The presence of an external magnetic field makes a dusty plasma system anisotropic, i.e., charging currents to a spherical dust grain is different in different directions. However, in the presence of very strong magnetic field, the orbits of the magnetized plasma particles are confined to one dimension along the field lines, as if they are "glued" to the magnetic field lines. Hence, the perturbed field does not come into play and the problem of charging currents becomes independent of the magnetic field.[29].

However, the investigations are made for homogeneous plasmas where the equilibrium plasma quantities are taken as independent of space. Inhomogeneity exists widely in most equilibrium plasma states such as in space and laboratory discharges. For example, recent experiments on observations of the dust density perturbations in a dc glow discharge in neon revealed that the phase velocity of the density perturbations varies considerably along the plasma column.[30] Comparatively, little research has been done about the affect of spatial inhomogeneity on nonlinear phenomena in a dusty plasma system.[31–34] It has been shown that the soliton propagation exhibits peculiar properties quite different from the homogeneous case. In real situations, the inhomogeneity in plasma can occur either from the equilibrium dust density gradient or from equilibrium plasma density gradient. The former is on the slow dust relaxation time scale and the latter is on relatively quicker ion relaxation time scale. But, both of them can significantly affect the dynamics of the DA waves. As the dust grain charge is self-consistently determined by plasma charging currents that depend on plasma number den-

[a)]Also at High Energy Physics Division, Dept. of Physics, Jadavpur University, Kolkata-700 032. Electronic-mail: apmisra@gmail.com
[b)]Electronic-mail: asesh_r@yahoo.com

sities, the plasma density gradient can still influence the DA waves through the equilibrium dust charge in addition to acting as a neutralizing background even if the equilibrium dust density is kept uniform. In this case, the dust charge instead of dust number density becomes spatially nonuniform. The soliton propagation in such plasmas in pthe resence of external magnetic field and neutral dust collision have not yet been investigated.

On the other hand, Zakharov and Kuznetsov[35] theoretically first made an attempt to model a soliton structure in a three-dimensional (3D) magnetized plasma system and they obtained a differential equation which is known as the ZK equation.

Shukla *et al.*[36] studied the DIA, DA, and DL solitons in a uniform hot dusty plasma. Choi *et al.*[31] investigated the DIA solitary waves in a dusty plasma taking into account the effects of obliqueness, external magnetic field, and ion thermal pressure. Low frequency electromagnetic solitary and shock waves have ben studied by Shukla[34] in an inhomogeneous dusty magnetoplasma. Recently, El-Labany *et al.*[28] studied the DA solitary waves in a hot magnetized dusty plasma through the ZK equation in a homogeneous medium. Also, El-Taibany *et al.*[27] investigated DA solitary as well as double layers in the presence of magnetic field, dust charge variation, and nonthermal ions. But, equilibrium considerations and thus clarifying the problem of how even weak inhomogeneity in plasma structures and external magnetic field influence the DA waves. Though the magnetic field must influence the characteristics of charging of dust particles, because the path described by the electrons and ions is modified and in this we have cyclotron motion of electrons and ions around the magnetic field lines, as mentioned earlier, but, it has been shown by Chang and Spariosu[37] through numerical calculations that for the grain radius $a \ll \rho_G$ ($=\sqrt{\pi/2}r_{Le}$, with r_{Le} being the electron Larmor radius) the effect of the magnetic field on the charging of the dust particles can be neglected. So, in our plasma we have assumed that the orbital motion limited (OML) theory is still valid in a magnetized plasma.

In this article, we investigate the DA soliton propagation in an inhomogeneous 3D collisional plasma taking into account the effects of dust thermal pressure, neutral-dust collision, external magnetic field, and dust charge fluctuations. By using the standard reductive perturbation technique, a ZK equation is derived which governs the solitary wave propagation of 3D waves in a magnetized dusty plasma. The variation of the amplitude, width, and the soliton velocity with respect to the propagation distance are analyzed numerically.

II. BASIC EQUATIONS

We consider a three-component dusty plasma consisting of negatively charged dust grains, isothermal electrons and ions in the presence of an externally applied magnetic field $B_0\hat{x}$. We assume that the plasma has a density gradient $\partial n_{jo}/\partial x$ along the x axis $(j=e,i)$ and $v_{i0}=v_{i0}(x)$ as the drift velocity of ion. At equilibrium, we have

$$n_{i0}(x) = n_{e0}(x) + Z_{d0}(x)n_{d0}, \tag{1}$$

where e is the elementary charge, Z_{d0} is the equilibrium charge number on the dust grains and n_{d0} (assumed as constant) is the uniform equilibrium dust number density.

The basic normalized equations governing the dynamics of our dusty plasma system are the following:

$$\frac{\partial N_d}{\partial T} + \nabla\cdot(N_d\mathbf{V}_d) = 0, \tag{2}$$

$$\frac{\partial \mathbf{V}_d}{\partial T} + V_d\cdot\nabla\mathbf{V}_d + \frac{\sigma_d}{N_d}\nabla P = \beta Z\nabla\phi - \Gamma\mathbf{V}_d - Z\omega_{cd}\mathbf{V}_d\times\mathbf{B}, \tag{3}$$

$$\frac{\partial P}{\partial T} + \mathbf{V}_d\cdot\nabla P + \gamma P\nabla\cdot\mathbf{V}_d = 0, \tag{4}$$

$$\nabla^2\phi = \frac{1}{(1+\sigma\delta)}[(\delta-1)ZN_d + N_e - \delta N_i]. \tag{5}$$

On the dust-acoustic time scale, the electrons and ions are in local thermodynamic equilibrium and their number densities obey the Boltzmann distributions:

$$N_e(X) = N_{e0}(X)\exp(\Phi), \tag{6}$$

$$N_i(X) = N_{i0}(X)\exp(-\sigma\Phi), \tag{7}$$

where we have used the following normalizations:

$$T = \omega_{pd}t, \quad \omega_{pd} = (4\pi n_{d0}Z_{d0}^2(0)e^2/m_d)^{1/2},$$

$$X,Y,Z = x,y,z/\lambda_d,$$

$$\lambda = (T_{\text{eff}}/4\pi Z_{d0}(0)n_{d0}(0)e^2)^{1/2},$$

$$T_{\text{eff}} = \frac{Z_{d0}(0)n_{d0}(0)T_eT_i}{n_{e0}(0)T_i + n_{i0}(0)T_e},$$

$$\delta = n_{i0}(0)/n_{e0}(0), \quad \sigma = T_e/T_i, \quad \sigma_d = T_d/Z_{d0}(0)T_{\text{eff}},$$

$$\omega_{cd} = Z_{d0}(0)eB_0(0)/cm_d, \quad B = B(X)/B_0(0),$$

$$P = p/n_{d0}(0)T_d, \quad V_d = v_d/C_D, \quad C_D = \omega_{pd}\lambda_D,$$

$$N_\alpha = n_\alpha/n_\alpha(0), \quad Z = Z_d/Z_{d0}(0), \quad \Phi = e\phi/T_e,$$

where T_e, T_i, and T_d are the electron, ion, and dust temperatures, m_d is the grain mass, C_D is the DA speed, ϕ is the electrostatic potential, ω_{pd} is the dust plasma frequency, λ_D is the effective Debye length, v_d is the dust fluid velocity, p is the dust pressure, and B is the ambient magnetic field. In Eq. (3), $\beta=(1+\delta\sigma)/(\delta-1)$ and $\Gamma=\nu_{dn}/\omega_{pd}$ is the effective frequency of the dust-neutral collision normalized by the dust plasma frequency. Other collisions of the dust grains with electrons and ions being smaller are neglected ($\nu_{di}/\nu_{id} \sim m_i/m_d \sim 0^{-10}$). Also, $\gamma=(2+N)/N, N$ being the number of degrees of freedom. For the 3D case, $N=3$, so that $\gamma=5/3$.

Here we take into account the charging model of the dust particles in the presence of an external magnetic field assuming that the OML theory is still valid in a magnetized plasma. The equation for the charge current balance reads

$$\frac{\partial q_d}{\partial t} + \mathbf{v}_d \cdot \nabla q_d = I_e + I_i, \tag{8}$$

where $q_d=-Z_d e$ and electron and ion currents for spherical grains of radius a are[38,39]

$$I_e = - e\pi a^2 \bar{v}_e n_e \exp\left(- \frac{Z_d e^2}{aT_e}\right), \tag{9}$$

$$I_i = e\pi a^2 \bar{v}_i n_i \left[\frac{1}{2}\exp(-\zeta_0^2) + \left(\frac{1}{2} + \zeta_0^2 + \frac{Z_d e^2}{aT_i}\right)\frac{\sqrt{\pi}}{2}\frac{\mathrm{erf}(\zeta_0)}{\zeta_0}\right], \tag{10}$$

where $\zeta_0=v_{i0}/v_{Ti}$, where $v_{Ti}=\sqrt{2T_i/m_i}$ is the ion thermal speed, $\bar{v}_\alpha=\sqrt{8T_\alpha/\pi m_\alpha}(\alpha=e,i)$ and $\mathrm{erf}(x)$ is the error function.

The equilibrium dust charge number is given by the current balance equation $I_e+I_i=0$, which gives

$$J_e N_{e0}\exp(- z_0 Z_0) = J_i N_{i0}(1 + AZ_0), \tag{11}$$

where $z_0=Z_{d0}(0)e^2/aT_e$ is the normalized dust charge number evaluated at $x=0$, $A=2\sigma z_0/[1+2\zeta_0^2+2\zeta_0\exp(-\zeta_0^2)/\sqrt{\pi}\mathrm{erf}(\zeta_0)]$ and the normalized electron, ion flux evaluated at $Z_d=0$ are the following:

$$J_e = \pi a^2 \bar{v}_e n_{e0}(0)/\omega_{pd} Z_{d0}(0), \tag{12}$$

$$J_i = \frac{\pi a^2 \bar{v}_i n_{i0}(0)}{\omega_{pd} Z_{d0}(0)}\left[\frac{1}{2}\exp(-\zeta_0^2) + \frac{\sqrt{\pi}}{4}(1+2\zeta_0^2)\frac{\mathrm{erf}(\zeta_0)}{\zeta_0}\right]. \tag{13}$$

III. DERIVATION OF THE ZK EQUATION

Following the standard procedure[40] we introduce the stretched variables as

$$\xi = \epsilon^{1/2}\left(\int_0^X \frac{d\tilde{X}}{U(\tilde{X})} - T\right), \quad \eta = \epsilon^{1/2} Y,$$
$$\zeta = \epsilon^{1/2} Z, \quad \tau = \epsilon^{3/2} X, \tag{14}$$

where ϵ is the small ordering parameter directly reflects to the DA wave and $U(X)$ being a slowly varying function of X because of inhomogeneity, is the velocity of the moving frame to be determined from the linear dispersion relation. As the equilibrium quantities are time independent, we can assume that $\partial/\partial\xi=0$ for them. All the dynamical variables expanded in power series of ϵ as

$$N_d = 1 + \epsilon N_{d1} + \epsilon^2 N_{d2} + \cdots,$$
$$Z = Z_0(X) + \epsilon Z_1 + \epsilon^2 Z_2 + \cdots,$$
$$P = P_0 + \epsilon P_1 + \epsilon^2 P_2 + \cdots,$$
$$B = B_0 + \epsilon^{3/2} Z_1 + \epsilon^2 B_2 + \cdots, \tag{15}$$
$$\Phi = \epsilon\Phi_1 + \epsilon^2\Phi_2 + \cdots,$$
$$V_{dX} = \epsilon V_{dX1} + \epsilon V_{dX2}^2 + \cdots,$$
$$V_{dY,Z} = \epsilon^{3/2} V_{dY1,Z1} + \epsilon^2 V_{dY2,Z2} + \cdots.$$

Note that the transverse velocity components $V_{dY,Z}$ in Eq. (15) appear at higher order in ϵ than does the parallel velocity component V_{dx}. This anisotropy is introduced by the influence of a strong magnetic field. In this approximation, the fluid gyromotion is treated as a higher order effect. Similar approximation is also used in Refs. 27 and 28. For simplicity, we have assumed that the equilibrium dust density and dust pressure are independent of X (this does not affect the dynamics of the DA waves).

We substitute the expressions from Eq. (15) in the basic set of equations (2)–(10) and collect the coefficients of different powers of ϵ. The lowest order of ϵ yields the following equations:

$$N_{e1} = N_{e0}\Phi_1, \tag{16}$$

$$N_{i1} = - \sigma N_{i0}\Phi_1, \tag{17}$$

$$N_{d1} = \frac{V_{dX1}}{U}, \tag{18}$$

$$V_{dX1} = - \frac{1}{U}(\beta Z_0 \Phi_1 + \sigma_d P_1), \tag{19}$$

$$V_{dY1} = - \frac{1}{\omega_{cd} B_0}\left(\beta\frac{\partial\Phi_1}{\partial\zeta} - \frac{\sigma_d}{Z_0}\frac{\partial P_1}{\partial\zeta}\right), \tag{20}$$

$$V_{dZ1} = \frac{1}{\omega_{cd} B_0}\left(\beta\frac{\partial\Phi_1}{\partial\eta} - \frac{\sigma_d}{Z_0}\frac{\partial P_1}{\partial\eta}\right), \tag{21}$$

$$P_1 = \frac{\gamma P_0}{U} V_{dX1}, \tag{22}$$

$$N_{e1} + (\delta - 1)(Z_0 N_{d1} + Z_1) = \delta N_{i1}, \tag{23}$$

$$J_e^0 N_{e1} - J_i^0 N_{i1} = (J_e^0 z_0 N_{e0} + J_i A N_{i0}) Z_1, \tag{24}$$

where

$$J_e^0 = J_e \exp(- z_0 Z_0), \quad J_i^0 = J_i(1 + AZ_0). \tag{25}$$

From the set of first order equations we have the linear dispersion relation:

$$U^2 = \frac{5}{3}\sigma_d P_0 + \left[\frac{1}{\beta Z_0^2}\left(B + \frac{N_{e0} + \sigma\delta N_{i0}}{\delta - 1} \right) \right]^{-1}, \tag{26}$$

where

$$B = \frac{(1 + AZ_0)(1 + \sigma)}{(1 + AZ_0)z_0 + A}.$$

In the absence of the pressure term ($\sigma_d = 0 = P_0$) we can recover the same dispersion relation as in Ref. 33. Equation (26) shows that the expression for U which is normalized by the DA speed reflects the correction for the phase velocity by the dust pressure, dust charge variation and plasma inhomogeneity.

In the next order of ϵ we obtain the second order quantities:

$$N_{e2} = N_{e0}\left(\frac{1}{2}\Phi_1^2 + \Phi_2 \right), \tag{27}$$

$$N_{i2} = N_{i0}\left(\frac{1}{2}\sigma^2\Phi_1^2 - \sigma\Phi_2 \right), \tag{28}$$

$$- \frac{\partial N_{d2}}{\partial \xi} + \frac{1}{U}\frac{\partial V_{dX2}}{\partial \xi} + \frac{1}{U}\frac{\partial (N_{d1}V_{dX1})}{\partial \xi} + \frac{\partial V_{dX1}}{\partial \tau} + \frac{\partial V_{dY2}}{\partial \eta} + \frac{\partial V_{dZ2}}{\partial \zeta} = 0, \tag{29}$$

$$- \frac{\partial V_{dX2}}{\partial \xi} + \frac{1}{U}V_{dX1}\frac{\partial V_{dX1}}{\partial \xi} = \frac{\beta Z_0}{U}\left[U\frac{\partial \Phi_1}{\partial \tau} + \left(\frac{Z_1}{Z_0} + N_{d1} \right)\frac{\partial \Phi_1}{\partial \xi} + \frac{\partial \Phi_2}{\partial \xi} \right] - \tilde{\Gamma} V_{dX1} - \sigma_d\left(\frac{1}{U}\frac{\partial P_2}{\partial \xi} + \frac{\partial P_1}{\partial \tau} \right), \tag{30}$$

$$V_{dY2} = - \frac{1}{\omega_{cd}B_0 Z_0}\frac{\partial V_{dZ1}}{\partial \xi}, \tag{31}$$

$$V_{dZ2} = \frac{1}{\omega_{cd}B_0 Z_0}\frac{\partial V_{dY1}}{\partial \xi}, \tag{32}$$

$$- \frac{\partial P_2}{\partial \xi} + V_{dX1}\left(\frac{1}{U}\frac{\partial P_1}{\partial \xi} \right) + \gamma P_0\left(\frac{1}{U}\frac{\partial V_{dX2}}{\partial \xi} + \frac{\partial V_{dY2}}{\partial \eta} + \frac{\partial V_{dZ2}}{\partial \zeta} + \frac{\partial V_{dX1}}{\partial \tau} \right) + \frac{\gamma P_1}{U}\frac{\partial V_{dX1}}{\partial \xi} = 0, \tag{33}$$

$$\frac{\partial^2 \Phi_1}{\partial \xi^2} = \frac{U^2}{1 + \sigma\delta}[N_{e2} + (\delta - 1)(Z_0 N_{d2} + Z_2 + Z_1 N_{d1}) - \delta N_{i2}], \tag{34}$$

$$J_e^0\left[(N_{e2} - z_0 Z_1 N_1) + N_{e0}\left(\frac{1}{2}z_0^2 Z_1^2 - z_0 Z_2 \right) \right] - J_i^0 N_{i2} - J_i A(N_{i1}Z_1 + N_{i0}Z_2) = 0, \tag{35}$$

where $\tilde{\Gamma} = \epsilon^{3/2}\Gamma$. Eliminating the previous second order quantities using the dispersion relation as well as the relations for the first order quantities we ultimately obtain the ZK equation as

$$\frac{\partial \Phi_1}{\partial \tau} + C_2\Phi_1\frac{\partial \Phi_1}{\partial \xi} + C_3\Phi_1 + C_1\frac{\partial^3 \Phi_1}{\partial \xi^3} + C_0\frac{\partial}{\partial \xi}\left(\frac{\partial^2 \Phi_1}{\partial \eta^2} + \frac{\partial^2 \Phi_1}{\partial \zeta^2} \right) = 0, \tag{36}$$

where the coefficients are

$$C_0 = \frac{U}{2(\omega_{cd}B_0 Z_0)^2},$$

$$C_1 = \frac{(1 + \delta\sigma)\left(U^2 - \frac{5}{3}\sigma_d P_0 \right)^2}{2\beta Z_0^2(\delta - 1)U^5},$$

$$C_2 = - \left[\left(\frac{2\beta Z_0}{U^2 - \frac{5}{3}\sigma_d P_0} \right)^2\left(1 - \frac{2}{3}\frac{\sigma_d P_0}{U^2} \right) + \frac{\left(U^2 - \frac{5}{3}\sigma_d P_0 \right)(N_{e0} - \delta\sigma^2 N_{i0})}{U^2 Z_0(\delta - 1)} - \frac{3BP}{U^2} + \frac{\left(U^2 - \frac{5}{3}\sigma_d P_0 \right)A^2B^2}{U^2 Z_0(1 + AZ_0)(A + z_0(1 + AZ_0))} \right]\frac{\left(U^2 - \frac{5}{3}\sigma_d P_0 \right)}{2\beta Z_0 U},$$

$$C_3 = \left[\frac{\partial}{\partial \tau}\left(\frac{UZ_0}{U^2 - \frac{5}{3}\sigma_d P_0} \right) + \frac{5}{3}\frac{\sigma_d P_0}{U}\frac{\partial}{\partial \tau}\left(\frac{Z_0}{U^2 - \frac{5}{3}\sigma_d P_0} \right) + \frac{Z_0\tilde{\Gamma}}{U^2 - \frac{5}{3}\sigma_d P_0} \right]\frac{\left(U^2 - \frac{5}{3}\sigma_d P_0 \right)}{2Z_0 U}.$$

The coefficients C_1 and C_2 are well known to be the dispersion and nonlinear coefficients. The coefficient C_3 arises due to the inhomogeneity of the equilibrium dust charge Z_0 and linear DA phase velocity U as well as the collisional effect Γ. The coefficient C_0 stands for the combined effects of the transverse perturbation and magnetic field. In the absence of dust temperature and pressure, the coefficient C_2 (except the negative sign outside the square brackets) and C_3 become same as in Ref. 33. In the absence of inhomogeneity and collisional effects we can recover the similar ZK equation as in Refs. 27 and 28. Without collision, inhomogeneity, transverse perturbations, and magnetic field C_0, C_3 will not appear and in this case Eq. (36) reduces to the usual KdV equation, which is well known to have hyperbolic secant soliton solution.[40]

IV. ANALYTIC SOLUTION

To obtain an analytical solution of the ZK equation [Eq. (36)] we write

$$\Phi_1 = \bar{\Phi}\Psi, \tag{37}$$

where $\bar{\Phi}$ is the amplitude factor defined as

$$\bar{\Phi} = \frac{1}{\sqrt{U}}\left(\frac{U^2 - \frac{5}{3}\sigma_d P_0}{Z_0}\right)^{\lambda} \exp\left(-\int_0^X \frac{\Gamma}{2U} d\tilde{X}\right), \tag{38}$$

with

$$\lambda = \frac{1}{2} + \frac{5}{6}\frac{\sigma_d P_0}{U^2}.$$

Substituting it into Eq. (36) and making use of the transformations

$$\xi_1 = (\tilde{C}_2/C_1)^{1/2}\xi, \quad \tau_1 = \int_0^{\tau} (\tilde{C}_2^3/C_1)^{1/2} d\tilde{\tau},$$

$$\eta_1 = (\tilde{C}_2/C_0)^{1/2}\eta, \quad \zeta_1 = (\tilde{C}_2/C_0)^{1/2}\zeta, \quad \tilde{C}_2 = C_2\bar{\Phi}.$$

We get the reduced equation as

$$\frac{\partial\Psi}{\partial\tau_1} + \Psi\frac{\partial\Psi}{\partial\xi_1} + \frac{\partial^3\Psi}{\partial\xi_1^3} + \frac{\partial}{\partial\xi_1}\left(\frac{\partial^2\Psi}{\partial\eta_1^2} + \frac{\partial^2\Psi}{\partial\zeta_1^2}\right) = 0. \tag{39}$$

Now, to find the stationary solution of Eq. (39) we substitute $\tilde{\xi} = l_x\xi_1 + l_y\eta_1 + l_z\zeta_1 - V\tau_1$ into it, where l_x, l_y, l_z are the direction cosines of the wave vector along the axes. Integrating twice and using the boundary conditions $\Psi \to (0)\Psi_0$ as $\tilde{\xi} \to \infty(-\infty)$, $d\Psi/d\tilde{\xi} \to 0$ as $|\tilde{\xi}| \to \infty$, the localized soliton solution of Eq. (39) is given by

$$\Psi = \Psi_m^{(1)}\mathrm{sech}^2(\tilde{\xi}/W), \tag{40}$$

where $\Psi_m^{(1)} = 3V$ is the amplitude and $W = 2/\sqrt{V}$ is the width.

To express the soliton solution according to the real coordinates, we transform the stretched variables back into ordinary X, Y, Z, and T. Noting that the real electrostatic potential associated with the soliton, $\Phi = \Sigma\epsilon^j\Phi_j \approx \epsilon\Phi_1$ we have

$$\Phi = \frac{3}{2}l_x\Phi_0\tilde{\Phi}\mathrm{sech}^2\left[\left(\frac{2}{l_x}\right)^{3/2}\sqrt{\frac{C_2\tilde{\Phi}\Phi_0}{C_1}}\left(T - \int_0^X \frac{d\tilde{X}}{M_1} - \frac{1}{M_2} - \frac{1}{M_3}\right)\right], \tag{41}$$

where

$$\frac{1}{M_1} = \frac{1}{U} + \frac{1}{2}\Phi_0\tilde{\Phi}C_2 + \frac{d}{dX}\sqrt{\frac{C_1}{C_2\tilde{\Phi}\Phi_0}}\int_0^X \sqrt{\frac{(C_2\tilde{\Phi}\Phi_0)^3}{C_1}}d\tilde{X}, \tag{42}$$

$$M_2 = \frac{l_x}{l_y}\sqrt{\frac{C_0}{C_1}}, \quad M_3 = \frac{l_x}{l_z}\sqrt{\frac{C_0}{C_1}}, \tag{43}$$

Φ_0 is the amplitude factor of the potential at $X=0$, and $\tilde{\Phi} = \bar{\Phi}(X)/\bar{\Phi}(0)$. For weak inhomogeneity the last term in Eq. (42) can be neglected, so that the Mach number (soliton speed) is given by

$$\mathrm{M}^2 = \frac{U^2}{\left(1 + \frac{1}{2}U\Phi_0\tilde{\Phi}C_2\right)^2} + \frac{l_x^2(1-l_x^2)}{l_y^2 l_z^2}\frac{C_0}{C_1} \tag{44}$$

and the width of the soliton is

$$\Delta = \left(\frac{l_x}{2}\right)^{3/2}\sqrt{\frac{C_1}{C_2\tilde{\Phi}\Phi_0}}. \tag{45}$$

Note that the second term in Eq. (44) is due to the combined effects of the magnetic field as well as the transverse perturbations. The expression for the Mach number in the one-dimensional inhomogeneous case without dust pressure and magnetic field is the same as in Ref. 33. The soliton speed not only proportional to its amplitude but also depends on the parameters C_0, C_1, C_2, and U. From relation (45) we find that the width of the soliton is inversely proportional to the square root of the amplitude and in addition, square root of the ratio C_1/C_2.

The propagation of compressive or rarefactive solitons depends on the sign of the nonlinear coefficients of the ZK equation C_2. Thus, the DA solitary waves are compressive or rarefactive according to $C_2 >$ or <0. This transition can occur through some critical points. But, it can be proved that the dispersion coefficient C_1 is always positive for $\delta > 1$. So, the presence of the critical behavior is due to the nonlinear coefficient C_2 which vanishes at $U_c = \sqrt{5/3\sigma_d P_0}$ and the ZK equation fails to describe in this case.

V. NUMERICAL RESULTS

We numerically investigate the different characteristic properties of the DA waves in a typical dusty plasma. We consider the following plasma parameters: $B_0(0) \sim 10^{-3}$ T, $T_e \sim 4$ eV, $T_i \sim 1$ eV, $T_d \sim 0.6$ eV, $n_{i0} \sim 10^9\,\mathrm{cm}^{-3}$, $n_{e0} \sim 5 \times 10^8\,\mathrm{cm}^{-3}$, $a \sim 1\ \mu$m, $Z_{d0}(0) \sim 10^2$. We estimate $\Gamma \sim 10^{-3}$, $\omega_{cd} \sim 0.042$. The equilibrium ion density is assumed to be of the form $N_{i0}(X) = \exp(-X/L)$, L being the density scale length taken as 200. The equilibrium electron density and dust charge number are then derived from the quasineutrality and charging balance equations. These equilibrium numbers are then plotted in Fig. 1 with respect to X. All N_{e0}, N_{i0}, and Z_0 decrease with X, but the decrease of N_{e0} is more quicker than those of N_{i0} and Z_0. Note that here the behavior of Z_0 is similar to N_{i0} quite different from the case studied in Ref. 33.

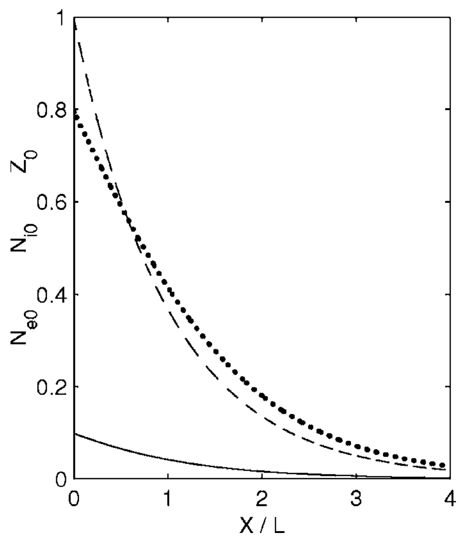

FIG. 1. The distributions of the equilibrium plasma quantities for $\delta=3$, $\sigma=14$, and $\sigma_d=0.014$. The solid, dashed, and dotted lines correspond to N_{e0}, N_{i0}, and Z_0, respectively.

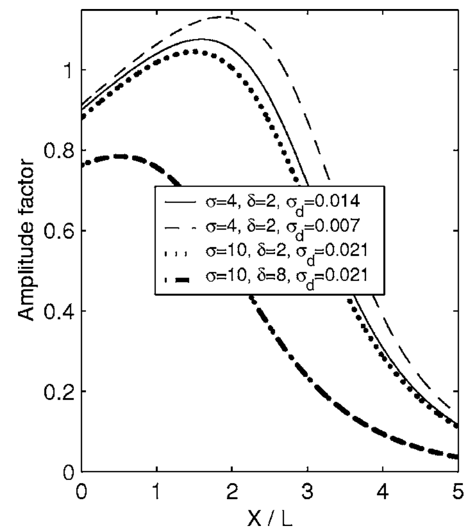

FIG. 3. The variations of the amplitude factor ($\tilde{\Phi}$) with respect to X/L.

Figure 2 shows the variation of the phase velocity U with respect to X for different plasma parameters δ,σ,σ_d. For small value of dust temperature σ_d, U decreases rapidly with X. This is because when σ_d is small ($\sigma_d \approx 0$), U is proportional to Z_0, it decreases with X and tends to become zero at long X.[33] On the other hand, from the expression for U [Eq. (26)] it is clear that U never tends to zero at large X unless $\sigma_d=0$, rather, it becomes more or less constant at large X and comparatively large value of σ_d, which is shown by the solid and dashed lines. Moreover, the increase in plasma density and temperature ratio (δ,σ) respectively decreases and increases the value of U at small X. Similar behavior of U with the increase of δ,σ has been reported in Ref. 27 where the homogeneous case is studied. The variation of the amplitude factor $\tilde{\Phi}$ is shown in Fig. 3 for different plasma parameters.

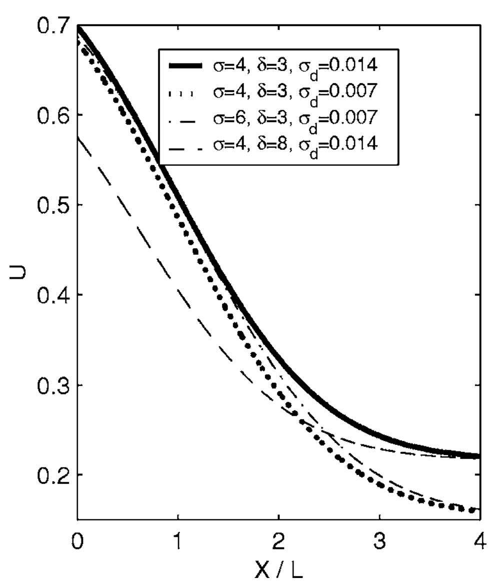

FIG. 2. The variations of the phase velocity (U) with respect to distance X/L.

From Eq. (38) it is found that the amplitude factor except the exponential factor is somewhat modified by the dust temperature and pressure (compared with the previous result[33]). It mainly depends on two factors: one Γ is due to the neutral-dust collision, and other factor is the inhomogeneity in both Z_0 and U. In the absence of dust temperature the first factor becomes $\sqrt{U/Z_0}$ which does not change the behavior of $\tilde{\Phi}$, because Z_0 follows mainly the profile of U.[33] In this case, first when the inhomogeneity exceeds the collisional effect, $\tilde{\Phi}$ increases with X, attains its maximum value and then ultimately decreases with X because of the dominance of the collisional effect over the inhomogeneity. The presence of dust temperature and pressure changes the situation significantly. The amplitude once again has the increasing behavior at large $X/L \approx 20$ (not shown in Fig. 3). This is due to the critical behavior of the phase velocity. At large X, though both U and Z_0 decrease with X, but the ratio $(U^2 - 5\sigma_d P_0/3)/Z_0$ increases exponentially and dominates over the collisionality. As σ_d increases the maximum value of $\tilde{\Phi}$ decreases, whereas the same also decreases with the increase of δ for fixed values of σ and σ_d. The soliton width is shown in Fig. 4. It increases with X and attains largest maximum when $\sigma_d=P_0=0$. Because, Δ is proportional to C_1 which in turn, is inversely proportional to U for $\sigma_d=0$ and U decreases with X. This results conform with the characteristic width studied in Ref. 33 in a cold nonuniform dusty plasma. In the presence of dust temperature, Δ first increases, reaches the maximum value and then decreases with X. Actually, for nonzero σ_d and P_0, C_1 increases or decreases with X according as $U<$ or $>U_c$. We find that as the dust temperature decreases, the maximum value for soliton width increases. Also the temperature ratio σ decreases the same for a fixed δ and σ_d. Thus, the peculiar behavior of Δ is somewhat modified by the dust temperature. Fig. 5 displays the soliton speed (M) with respect to X. From the expression for M [Eq. (44)] it is clear that for small amplitude wave $\Phi_0 \ll 1$ the first term is proportional to U, whereas the second term which arises due to transverse perturbations and external magnetic field is

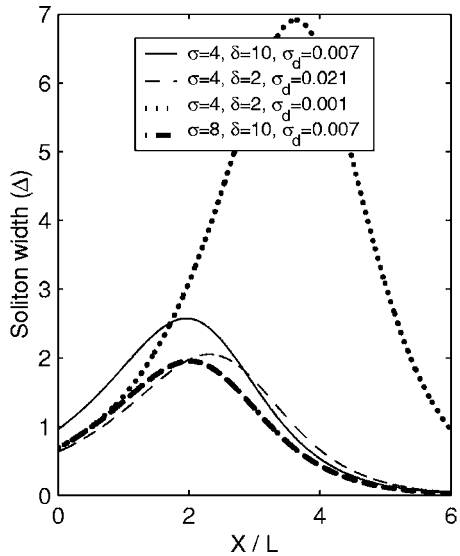

FIG. 4. The typical variations of the characteristic width (Δ) with respect to distance X/L.

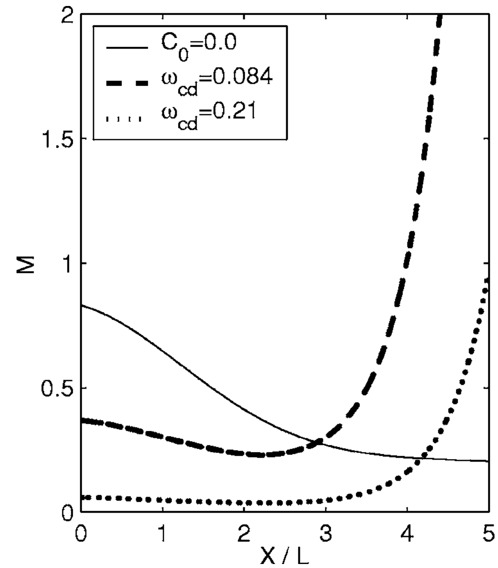

FIG. 6. The variations of the same (M) as in Fig. 5 with respect to distance X/L but for $C_0=0$ (no magnetic field and transverse perturbation), $\omega_{cd}=0.084, 0.21$. The scales for M along the vertical axis for dashed and dotted lines are each $\mathrm{M} \sim M/100$.

inversely proportional to C_1. Also, C_1 is inversely proportional to U which clearly decreases with X for $\sigma_d=0$. Therefore, for $\sigma_d=0$, M follows the profile of U and decreases with X.[33] On the other hand, for $\sigma_d=0.014$, M follows the critical behavior of U i.e., it first decreases, reaches a minimum and ultimately then increases with X (opposite behavior to Δ), because, in this case M is inversely proportional to C_1). Thus the presence of magnetic field and dust temperature changes the soliton speed significantly. Figure 6 shows clearly the same. Here the Mach number increases monotonically with the increase of the dust cyclotron frequency (the scales for M along vertical axis for the dashed and dotted lines are each $\mathrm{M} \sim M/100$). In the absence of the magnetic field and transverse perturbations the second term in Eq. (44) will not appear and M decreases in this case with X (solid line). Figure 7 is a plot of the soliton shapes at different positions $X/L=0, 1.0, 2.0$. The initial amplitude of the soliton is taken as $\Phi_0=-0.1$, so that $C_2\Phi_0>0$. It shows that as the soliton propagates from $X=0$ to $X=2L$ its shape flattens, both width and height increase significantly instead of becoming narrow. This typical behavior of the soliton is quite different from the magnetized homogeneous[27,28] or unmagnetized inhomogeneous case.[32]

VI. DISCUSSIONS AND CONCLUSIONS

In this article, the propagation DA solitons in a magnetized inhomogeneous dusty plasma with transverse plane perturbations is investigated taking into account the effects of dust charge variation, dust thermal pressure, neutral-dust collision. The equilibrium plasma number densities, dust

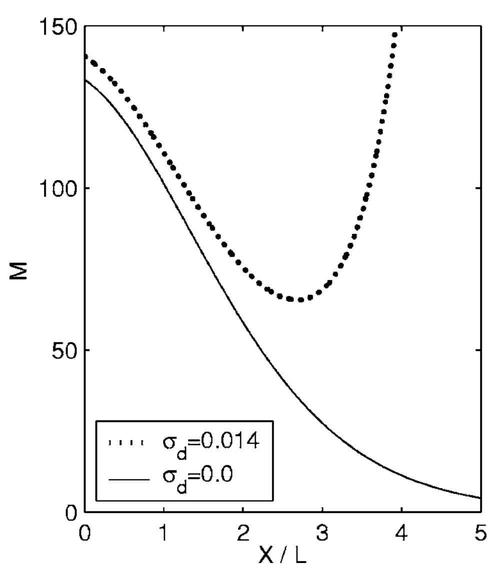

FIG. 5. The typical profile of the Mach number (M) with respect to X/L for $\sigma_d=0, 0.014$, $P_0=1.0$, and $\Phi_0=-0.001$.

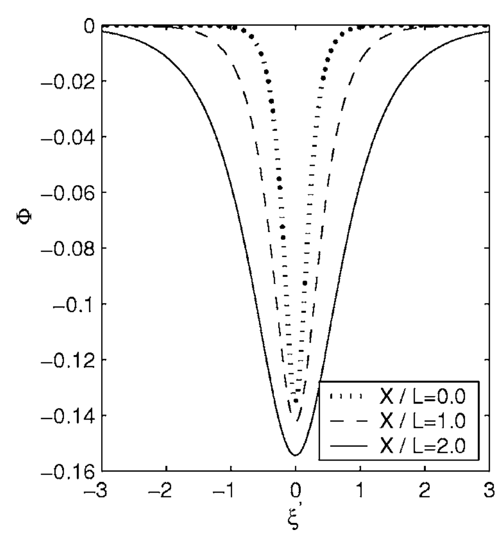

FIG. 7. The typical soliton profile (Φ) vs the propagation coordinate $[\xi'=T-\int_0^X(1/M_1)d\tilde{X}-1/M_2-1/M_3]$ for $X/L=0, 1.0, 2.0$, $\Phi_0=-0.1$, $P_0=1.0$, and $\sigma_d=0.014$.

charge number are considered nonuniform, but the dust number density, dust pressure are uniform. Using the standard reductive perturbation technique, a ZK equation governing the dynamics of the DA solitary waves is derived. It is shown that in the presence of dust thermal pressure there exists a critical value of the phase velocity $U_c = 5\sigma_d P_0/3$ quite distinct from the magnetized homogeneous case[27,28] for which either compressive or rarefactive solitons can propagate according as $U <$ or $> U_c$. In our numerical simulations only the latter ones are shown to exist.[28] Due to the presence of dust thermal pressure there exists an extra term in the expression for U for which the phase velocity never vanishes, rather it becomes more or less constant at larger X. The interplay between the collisional damping and spatial inhomogeneity in Z_0 and U as well as the critical behavior of U due to σ_d and P_0, the soliton amplitude can be subject to a transition from increasing to decreasing and then again to increasing nature. Due to critical nature of the phase velocity, the width of the soliton does not follow the usual monotonic behavior as in the cold inhomogeneous case. It is seen that the magnitude of the external magnetic field has no effect on the amplitude and width of the soliton profile. However, it does have an effect on the speed (M) of the same. The soliton speed increases significantly with the increase of the magnitude of the magnetic field. The dust thermal pressure, on the other hand, makes the characteristic width more spiky. It is to be mentioned that for describing the evolution of the present system at the critical situation, one has to seek another evolution equation suitable for describing the same. This is beyond scope of the present investigation and will be reported later elsewhere. However, our present analysis should be useful for understanding different nonlinear features of localized electrostatic disturbances in a number of astrophysical dusty plasma system, viz. planetary rings, cometary environments, and the interstellar medium.

ACKNOWLEDGMENTS

The authors wish to thank the referee for valuable criticisms and comments which improved the original manuscript in the present form. One of the autors (A.P.M.) is grateful to SAP (Special Assistance Program), Government of India, for support.

[1] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).

[2] D. A. Mendis, *Advances in Dusty Plasmas*, edited by P. K. Shukla, D. A. Mendis, and T. Desai (World Scientific, Singapore, 1997), pp. 3–19.

[3] D. A. Mendis and M. Horanyi, *Cometary Plasma Processes*, edited by A. J. Johnston (Geophys, Washington, D.C., 1991), p.17.

[4] U. de Angelis, Phys. Scr. **45**, 465 (1992).

[5] P. K. Shukla, *The Physics of Dusty Plasmas* (World Scientific, Singapore, 1996), pp. 107–121.

[6] T. Nakano, Astrophys. J. **494**, 587 (1998).

[7] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer, Dordrecht, The Netherlands, 2000).

[8] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (IOP, Bristol, 2002).

[9] D. A. Mendis, Physiol. Rev. **11**, A219 (2002).

[10] P. K. Shukla, *Dust Plasma Interaction in Space* (Nova, New York, 2002).

[11] J. Winter, Plasma Phys. Controlled Fusion **40**, 1201 (1998).

[12] C. Hollenstein, Plasma Phys. Controlled Fusion **42**, R93 (2000).

[13] P. V. Bliokh and V. V. Yaroshenko, Astron. Zh. **29**, 330 (1995).

[14] U. de Angelis, V. Formisan, and M. Giordano, J. Plasma Phys. **40**, 399 (1998).

[15] P. K. Shukla and L. Stenflo, Astrophys. Space Sci. **190**, 23 (1992).

[16] P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).

[17] R. L. Merlino, A. Barkan, C. Thompson, and N. D' Angelo, Phys. Plasmas **5**, 1607 (1998).

[18] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).

[19] A. Barkan, R. L. Merlino, and N. D' Angelo, Phys. Plasmas **2**, 3563 (1995).

[20] F. Melandsø, Phys. Plasmas **3**, 3890 (1996).

[21] A. Homann, A. Melzer, S. Peters, and A. Piel, Phys. Rev. E **56**, 7138 (1997).

[22] B. Farokhi, P. K. Shukla, and N. L. Tsintsadze, Phys. Lett. A **264**, 318 (1999).

[23] A. Barkan, N. D' Angelo, and R. L. Merlino, Planet. Space Sci. **44**, 239 (1996).

[24] A. P. Misra, A. R. Chowdhury, and K. R. Chowdhury, Phys. Lett. A **323**, 110 (2004).

[25] A. P. Misra and A. R. Chowdhury, Eur. Phys. J. D **37**, 105 (2006).

[26] S. K. El-Labany and E. F. El-Shamy, Phys. Plasmas **12**, 042301 (2005).

[27] W. F. El-Taibany and R. Sabry, Phys. Plasmas **12**, 082302 (2005).

[28] S. K. El-Labany, W. M. Moslem, W. F. El-Taibany *et al.*, Phys. Plasmas **11**, 03303 (2004); L.-P. Zhang and J.-K. Xue, *ibid.* **12**, 042304 (2005).

[29] M. Salimullah, I. Sandberg, and P. K. Shukla, Phys. Rev. E **68**, 0274023 (2003).

[30] Λ. Samaryan, Λ. Chernyshev, O. Petrov *et al.*, JETP **92**, 0454 (2001).

[31] C. R. Choi, C.-M. Ryu, N. C. Lee *et al.*, Phys. Plasmas **12**, 072301 (2005); S. V. Singh and N. N. Rao, *ibid.* **6**, 03157 (1999); Y. F. Li, J. X. Ma, and J. J. Li, *ibid.* **11**, 01366 (2004).

[32] Y.-f. Li, J. X. Ma, and J.-J. Li, Phys. Plasmas **11**, 01366 (2004).

[33] D.-L. Xiao, J. X. Ma, and Y.-f. Li, Phys. Plasmas **12**, 052314 (2004).

[34] P. K. Shukla, Phys. Plasmas **10**, 04907 (2003).

[35] V. E. Zakharov and E. Λ. Kuznetsov, Sov. Phys. JETP **39**, 0285 (1974).

[36] P. K. Shukla and A. A. Mamun, New J. Phys. **5**, 17.1 (2003).

[37] J. S. Chang and K. Spariosu, J. Phys. Soc. Jpn. **62**, 097 (1993).

[38] E. C. Whipple, Rep. Prog. Phys. **44**, 01197 (1981).

[39] T. G. Northrop, Phys. Scr. **45**, 0475 (1992).

[40] R. C. Davidson, *Methods in Nonlinear Plasma Theory* (Academic, New York, 1972), Chap. 2.