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Particle Dynamics in a Bounded Region

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TABLE I. Charge exchange cross sections for D^+ in D_2. The conversion from α_I (cm^{-1}) to σ_{cx} (cm^2/atom) may be obtained from the relation $3.536 \times 10^{16} \sigma_{cx} = \alpha_I$ (see reference 4).

Energy (eV)	α_I (cm^{-1})	σ_{cx} (cm^2/atom)
0–50	0	0
65	0.1	0.0283×10^{-16}
80	0.5	0.141×10^{-16}
100	0.74	0.21×10^{-16}
150	0.86	0.243×10^{-16}
200	0.79	0.224×10^{-16}
250	0.81	0.229×10^{-16}
300	0.93	0.263×10^{-16}
400	1.05	0.297×10^{-16}

seem to support the above picture. The τ of a decaying deuterium plasma has been measured both by collecting ions on an end collector and by microwaves. The data yields a τ of about 20 msec at a gas pressure of 2×10^{-5} Torr. The initial plasma density is not yet known unambiguously, but lies in the range of 10^9–10^{11} ions/cm^3. Preliminary experimental results for helium indicate a much shorter τ value than for deuterium in agreement with theory.

Thus, there is some experimental data supporting the theoretical conclusion that operating a deuterium plasma in a mirror machine at about 70 eV ion energy leads to the longer plasma lifetime. The advantage of being able to operate a plasma research device at the relatively high pressure of 2×10^{-5} Torr is obvious.

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Particle Dynamics in a Bounded Region

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In a recent paper, Montgomery and Gorman[1] have devised a very simple method for studying the oscillations of a "collisionless" electron plasma which is confined to a finite geometry. Their technique consists essentially of replacing the actual bounded plasma by an infinite one; and the specular reflections of the particles by the confining walls are then automatically taken into account, provided only that the distribution function for the infinite plasma has certain periodicity properties. By making use of a slight generalization of these arguments, it is possible to include, in such an analysis, the effects of external force fields as well as to treat certain physical situations which cannot be described in terms of small perturbations from equilibrium.

To see this in detail, consider a collection of identical particles confined to motion in a one-dimensional box which is located on the interval $(0, L)$ and assume that this system can be described by a single-particle distribution function $f(x, v, t)$. As shown by Montgomery and Gorman, the extension to the three-dimensional case is straightforward and thus we shall only examine the one-dimensional case. This distribution function $f(x, v, t)$, of course, vanishes outside of the box, and its dynamical behavior inside is governed by some form of the Boltzmann equation which may include terms for external and self-consistent force fields, but whose detailed structure is not directly involved in the present considerations. It is convenient to define a new function $F(x, v, t)$ by the relation

$$F(x, v, t) = (2L)^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} a_n(v, t) \exp\left(\frac{in\pi}{L} x\right) \qquad (1)$$

with the coefficient $a_n(v, t)$ given in terms of $f(x, v, t)$ by

$$a_n(v, t) = (2L)^{-\frac{1}{2}}\left[\int_0^L dx\, f(x, v, t) \exp\left(-\frac{in\pi}{L} x\right) + \int_{-L}^0 dx\, f(-x, -v, t) \exp\left(-\frac{in\pi}{L} x\right)\right]. \qquad (2)$$

From this definition it follows that $F(x, v, t)$ is periodic in x with period $2L$, that on the interval $(0, L)$ it coincides with the actual distribution function $f(x, v, t)$ and that on the interval $(-L, 0)$, $F(x, v, t)$ is given by

$$F(x, v, t) = f(-x, -v, t); \qquad -L \le x \le 0. \qquad (3)$$

Finally, it is necessary to extend, to all values of x, the definition of any external force fields. In terms of $e(x, t)$, the force acting on the particles in the box, we define a new force $E(x, t)$ which again is periodic in x with a period $2L$, and on the interval $(0, L)$ it coincides with $e(x, t)$ while on the interval $(-L, 0)$ it is defined by

$$E(x, t) = -e(-x, t); \qquad -L \le x \le 0. \qquad (4)$$

Furthermore, it follows from Eqs. (1)–(3), that any self-consistent fields, and in particular the electric field, will automatically satisfy this oddness and periodicity property. And it is claimed that the dynamical behavior of $F(x, v, t)$—and thereby also that of $f(x, v, t)$—may be obtained by solving the Boltzmann equation for this infinite plasma but with complete disregard for the bounding walls at $x = 0, L$.

The proof of this statement follows directly by use of Eqs. (1)–(4). Consider a typical particle inside the box at the point x and suppose that at a given instant of time it is moving to the right with a velocity v and is acted upon by a force which for the sake of definiteness is also assumed to be directed towards the right. From the definition of F in Eq. (1), it follows that at this same instant at the point $2L - x$, there will be a particle which moves to the left with the same velocity v and is acted upon by force of the same strength but directed towards the left. Thus, the particle originaly at x will reach the wall at $x = L$ from the left at the same instant and with the identical velocity that the particle originally at $2L - x$ reaches the wall from the right. And furthermore, at the instant that the original particle crosses the wall at $x = L$, the other one crosses this boundary from the left and thereby enters the interior of the physical box. Thus, we see that in this way, the reflection of the particles by the walls at $x = 0, L$ is automatically taken into account provided only that one calculates the dynamical behavior of the plasma in terms of the distribution function F as defined in Eqs. (1)–(3) without regard to the reflections at the physical walls.

As a very simple application of this method to a nonequilibrium situation consider an ideal gas confined to the left half of a one-dimensional container of length L, and suppose that at $t = 0$, a partition at $x = \frac{1}{2}L$ is removed so that the gas enters the entire chamber. If c is the thermal velocity of the gas, a simple calculation shows that the coefficient $a_n(v, t)$ in Eq. (2) is given by

$$a_n(v, t) = (4\pi Lc^2)^{-\frac{1}{2}} \frac{1}{n\pi} \sin \tfrac{1}{2}n\pi \exp\left(-\frac{v^2}{2c^2} - \frac{in\pi}{L} vt\right).$$

On substituting this formula back into Eq. (1), one finds that such a description of this nonequilibrium situation agrees with the intuitively expected results. In particular, for example, in a time of order L/c, the gas attains a uniform density throughout the container. To the extent that the mirror points in a magnetic bottle may be replaced by reflecting walls, this nonequilibrium analysis may be extended and used to describe the injection of charged particles into such a geometry.

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Relativistic Virial Theorem

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The virial theorem[1] has been generalized by Chandrasekhar and Fermi[2] to include static electromagnetism. Their result was applied by Schmidt[3] to demonstrate that plasmoids cannot exist—a plasmoid being a self-confined bundle of electromagnetic and material energy. In this note denial of plasmoid existence is extended to relativistic systems.

The notation and definitions of Landau and Lifshitz[1] are used in the following analysis. The divergence of the total stress-energy tensor, for matter plus electromagnetic field,

$$\partial T_{ik}/\partial X_k = 0, \tag{1}$$

is multipled by X_i^2 and integrated, using Gauss' theorem, over a cylindrical hypervolume Ω which is coaxial with the X_4 axis and is bounded by two infinitesimally separated, spacelike hypersurfaces S_4, plus a connecting cylindrical surface Λ, yielding

$$\int X_i^2 \frac{\partial T_{ik}}{\partial X_k} d\Omega = -2 \int X_k T_{ik}\, d\Omega + \int X_i^2 T_{i4}\, dS_4 + \int X_i^2 T_{i\alpha}\, d\Lambda = 0. \tag{2}$$

The index i is set equal to 4. Upon recognition that

$$d\Omega = dS_4\, dX_4, \qquad d\Lambda = dS_\alpha\, dX_4, \tag{3}$$

where S_α is an ordinary surface, Eq. (2) is cast into the form

$$\frac{d}{dX_4} \int X_i^2 T_{44}\, dS_4 = 2 \int X_k T_{4k}\, dS_4 - \int X_i^2 T_{4\alpha}\, dS_\alpha. \tag{4}$$

A similar treatment of the first moment of Eq. (1) reveals that

$$\frac{d}{dX_4} \int X_k T_{4k}\, dS_4 = \int T_{ii}\, dS_4 - \int X_k T_{k\alpha}\, dS_\alpha. \tag{5}$$

Equations (4), (5) are combined after Eq. (4) is differentiated with respect to X_4, so that

$$\frac{d^2}{dX_4^2} \int X_i^2 T_{44}\, dS_4 = 2 \int T_{ii}\, dS_4 - 2 \int X_k T_{k\alpha}\, dS_\alpha - \frac{d}{dX_4} \int X_i^2 T_{4\alpha}\, dS_\alpha. \tag{6}$$