SHACKLE'S Φ FUNCTION AND GAMBLER INDIFFERENCE MAP

by B. R. Williams, Keele (U. K.)

SUMMARY: §1. Need to test Shackle's theory by plausibility of his assumptions §2 Shackle's four assumptions about psichology-§3 Diagram used in exposition-§4 Values of Φ when surprise scale is numerical §5-6 Gambler indifference map unnecessary, and-§7-8 Reasons for abandoning it.-§9 « Focus gain-loss pair » assumption not general-§10 Crucial nature of numerical surprise scale.

1. - The theory of expectation expounded by Professor G. L. S. Shackle in *Expectation in Economics* is based on both explicit and implicit hypotheses about the psychology of decision-making. Proof of the validity of Shackle's theory is dependent on demonstrating (*a*) that his theorems (deductions from hypotheses) are logically correct and (*b*) that his hypotheses about psychology are correct — whether we etablish this correctness by a direct test of the hypotheses or by the (provisional) indirect test of the coincidence between the facts and the predictions from a (logically correct) theory. It is not immediately possible to test the hypotheses, but the stimulus to find ways of testing will be greatly increased if it can be shown that the theorems are correct and that the « predictions » of the theory are *plausible*. Certain predictions of the Shackle theory concerning e. g. fluctuation in investment and the sensitivity of asset prices to small happenings, are plausible. But as alternative « explanations » are also plausible, we may expect economists to give immediate attention to what they judge to be the plausibility of the hypotheses about psychology as well as to the relation between the logic and the hypotheses.

2. - Shackle makes three and possibly four significant assumptions about the psychology of decision making.

(i) That subjective stability or reposefulness is necessary to call forth from the individual the mental and nervous energy required to devise and try to execute a plan of any consequence. « To attain this subjective stability, the individual will defer repeatedly the crystallization of any plan, waiting continually for a situation where he can feel that the number of unanswered questions is at a minimum and likely to increase on balance... Only at such a time of freedom from impending fresh knowledge will he consent in important matters to decide and act » (pp. 111-112) (1).

---

(1) Page references to Second Edition of *Expectation in Economics*, Cambridge, 1952.

(ii) That the intensity of enjoyment of a given hypothetical outcome by imagining it in advance is (dominantly) « an increasing function of the desirability of the outcome in question and a decreasing function of the degree of potential surprise associated with it » (p. 17). That is to say, the stimulus of an hypothetical outcome, whether agreeable or disagreeable, is scaled down as potential surprise, or degree of disbelief in the realism of the hypothesized outcome, rises above zero.

(iii) That individuals simplify the problem of decision making by reducing any uncertainty situation to the simplicity of an ordinary bet in which only two possible outcomes are considered, the one of which is a definite amount of gain, and the other a definite amount of loss (p. 18).

Professor Shackle derives this third hypothesis from the first, arguing that the desire for stability carries with it the need to represent the outcome by a small number of dominant hypotheses. But an argument stemming from this that the individual will, therefore, focus on *two* outcomes, involves a disjunction. Consequently, the third hypothesis is shown as an independent hypothesis.

(iv) The possible fourth assumption is that there are two stimulation functions — a focussing stimulation function and an action stimulation function (Appendix *A* to Chapter 2). The first, the Φ function, is concerned with the process by which a person is mentally stimulated to focus his attention on a single gain and a single loss; the second, implicit in Shackle's gambler indifference map, is concerned with the decision to undertake a project. The gambler indifference curves deal with the relation between the opposing gain and loss stimuli.

This fourth assumption is not an explicit assumption. In *Expectation in Economics* Shackle writes: « Φ is something assigned to a pair of values (x, y) in this pair's own right, independently of other pairs, and varying in degree from point to point of a y curve. We conceive the enterpriser to pass these points in review, and in this process to pause upon one only out of the hypotheses of gain and there to fix his *effective* attention; and to do the same amongst the hypotheses of loss... The concept of focus outcomes leads us at once to that of an indifference map of uncertainties, from which, given the standardized focus gains of any two investment opportunities or speculative outlays of money, and their standardized focus losses, we can read off which of these two opportunities the individual whose map it is will prefer. Such a map will summarize comprehensively everything about the attitude to uncertainty of one particular person, in his circumstances of a given moment, that is relevant for determining his actions » (p. 29). Professor Shackle comes closer to an explicit formulation of the fourth assumption in « Three Versions of the Φ-Surface » [1]. There he argues that the shape of the gambler

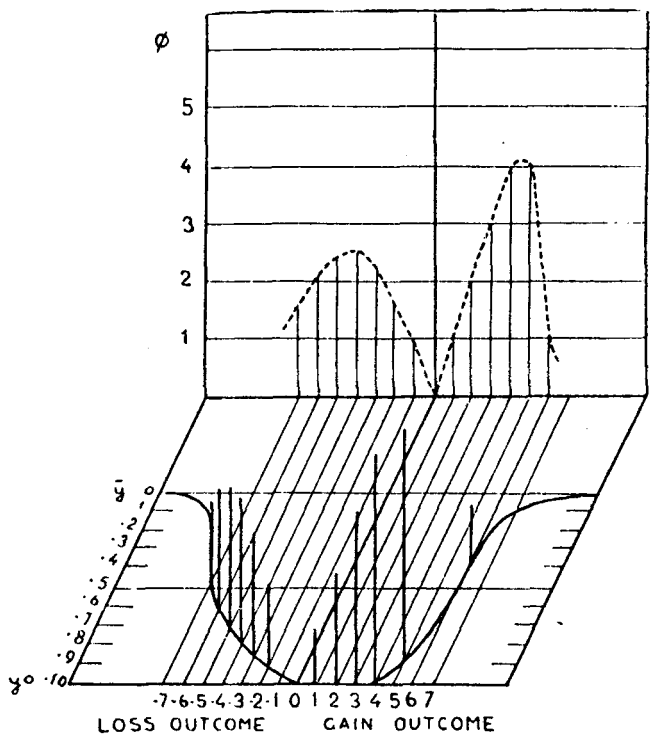
[1] « Review of Economic Studies », vol. XVIII (2), No. 46.

indifference map is not formally and rigidly related to that of the Φ-surface, though it may well be psychologically so related. Even here however the formulation is not definite, and we are left with a possibility that the gambler indifference curve is used not because of an assumption about the psychological distinctness of the « focussing-decision » and the « action-decision », but because, given the form of the Shackle Φ function, it is analytically necessary to introduce an indifference map of uncertainties.

3. - Consideration of Shackle's theory will be related to the following diagrams.

In Diagram 1 hypothetical outcomes of a project are measured on the x axis. Degrees of potential surprise are measured on the y

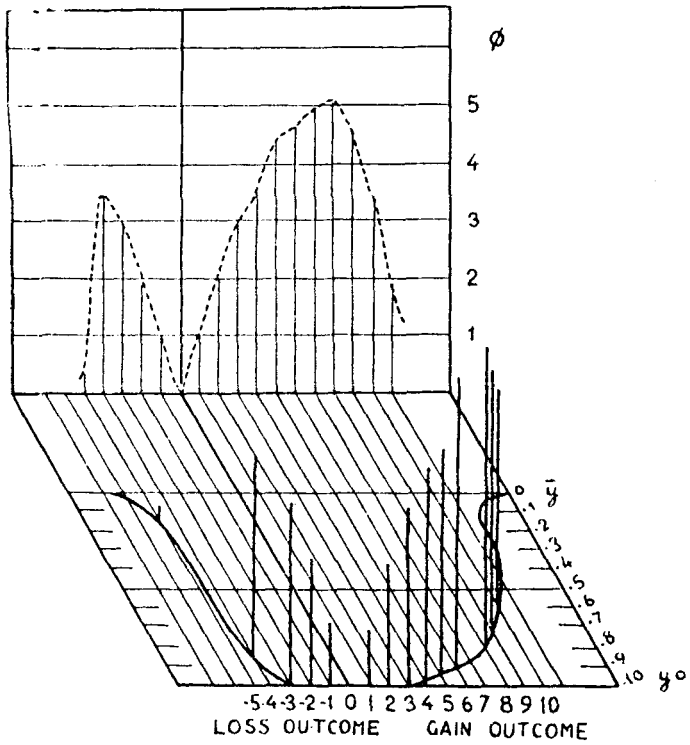
DIAGRAM 1 (A)



axis, with y_0 representing zero potential surprise (or unit degree of belief), and $\bar{y}$ representing maximum potential surprise (or zero degree of belief). The stimulus Φ attached to any xy pair is measured in the vertical dimension. Φ values are shown projected on to the $x\Phi$ plane, and used as the basis of stimulus curves.

Diagram 2 shows the simplification used by Professor Shackle. If we prepare a table showing the Φ value for every point on the xy plane, we can pick out all the successive x values which, with increas-

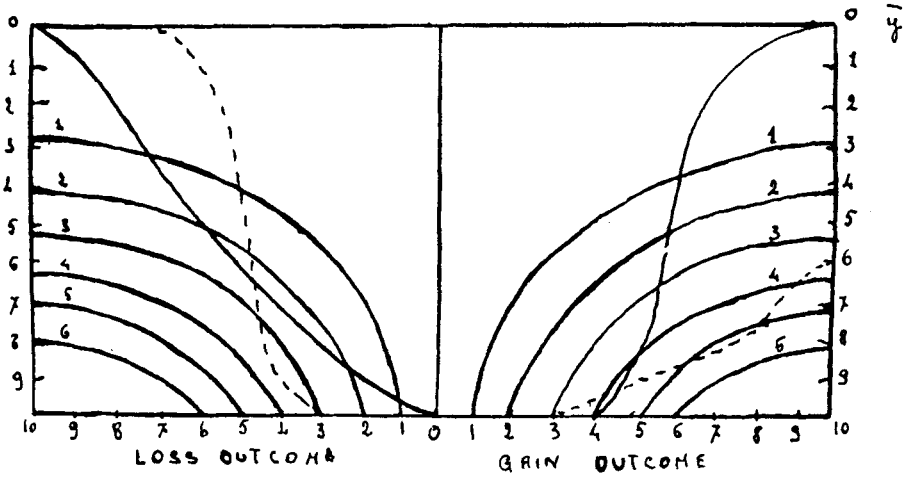
DIAGRAM 1 (B)



ing potential surprise, have a Φ value of 1. Thus if $\Phi = xy^2$, x will have a Φ value of 1 when x is 1 and degree of belief 1, when x is 2 and the square of the degree of belief is .5, when x is 3 and the square of the degree of belief is 1/3. In this way we derive the contour line labelled 1. We proceed in the same fashion for $\Phi = 2$ to get the contour line labelled 2. These contour lines may be superimposed on the xy plane, and the point of intersection of the y curve and a contour line will enable us to read off the Φ value of the appropriate xy pair. Thus, with the broken y curve, when $x = 4$ and $y = 1$, the Φ value is 4, and when $x = 5$ with an accompanying $y = .9$ the Φ value is slightly more than 4. If the xy plane were filled with contour lines we could read off the precise Φ value of any xy pair. But even without preparing an accurate family of contour lines, which in fact may not be worth the trouble of accurate scale drawing, the use of contour lines has the advantage of making it visually easy to judge

the effect of a change in the shape or position of the y curve. In diagram 2 the solid y curve is that shown in diagram 1 for project B, and the broken y curve that of project A.

DIAGRAM 2



4. - To begin, suppose that the surprise scale is fully numerical. The stimulus $\Phi = \Phi(x, y)$. Thus, given x we need a conversion factor to enable us to scale down the stimulus of an hypothetical outcome as degree of belief in the outcome falls below unity. This conversion factor will reflect the re-action of an individual to a « long shot » as compared with a « sure thing ». The greater the « risk aversion » [1] the greater the scale-down of the stimulus of an hypothetical outcome as potential surprise increases. We may suppose for one individual that $\Phi = xy^2$, and measure degree belief from 1 ($= y_0$) to 0 ($= y$).

The potential surprise curves relating to hypothetical ventures may vary in shape — they may rise slowly or sharply from the y_0 range, and they may be distributed differently between the gain and loss sectors on the xy plane. Thus in project A the potential surprise curve rises more steeply in the gain section from y_0 than in project B, with the consequence that in B the peak of the gain stimulus curve is reached well outside the inner range (of x values with zero potential surprise); and conversely for losses. In project B the inner range spreads further into the loss sector than in project A. In consequence of this and of the differing slopes of the y curve, the greatest loss Φ value is greater in B than in A, though the primary focus loss value of x is not greater in B than A.

---

[1] risk used here as in « risk capital ».

From the values of x and y it can be seen that the greatest loss Φ value in project A is 2.5 corresponding to an x value of — 4. This is equivalent to the stimulus from an x value of — 2.5 with zero potential surprise. Thus Shackle's standardized (loss) focus outcome is $x = -2.5$ The maximum gain Φ value is 4.05, corresponding to an x value of 5, which is equivalent to the stimulus from an x value, with zero potential surprise, of 4.05.

For project B Φ values are 3.6 and 5.12 respectively, for x values of — 4 and 8. This gives standardized focus outcomes of $x = -3.6$ and $x = 5.12$. That is to say the x values of the standardized focus outcomes are equal to the Φ values of the primary focus outcomes (when we measure degree of belief between 1 and 0 and use $\Phi = xy^2$ or some such similar equation).

Professor Shackle transfers these standardized focus gains and losses $x = -2.5$, $x = 4.05$, $x = -3.6$, $x = 5.12$ — to a gambler indifference map. The basis of this gambler indifference map is a table of equivalents — of focus gains that *compensate* focus losses. Now so long as a focus loss is not compensated by an equal focus gain and so long as the Φ function does not distinguish between the significance for action of gains and losses, this use of a gambler indifference map is necessary to enable us to see whether action on a particular project is worthwhile or which project is preferable. But given the table of equivalents there is no analytical reason why we should not dispense with the gambler indifference map. For on the basis of the table of equivalents we can divide the Φ function into its gain and loss components (1). We could write, e. g. $\Phi_g = xy^2$ and $\Phi_e = a\,(xy)^2$, a greater than 1, and judge projects directly from the net Φ values (2).

5. - In « Three Versions of the Φ-Surface » Professor Shackle objects to the form of the Φ-surface proposed by Mr. Mars, in that it makes it possible to dispense with the gambler indifference map. Shackle argues that this « procedure is not acceptable, for it means that when a scale for Φ is given, and a particular form is specified for the Φ surface with respect to that scale, the shape of every gambler indifference curve is thereby also specified, and, in consequence, the shapes of the gambler indifference curves are not independent of each other, they have no freedom to express that aspect of the individual's temperament which it is the purpose of the gambler indifference map to express... The gambler indifference map is free to reflect a different aspect of the individual's attitude to uncertainty from that reflected by the Φ surface » (3). Where in the last paragraph I said that analytically the gambler

(1) Cf. SHACKLE: *Expectation in Economics*,, p. 42.

(2) Values in the loss sectors of project A and B would thus be greater than those shown in Diagram 1 and for the same reason the contour lines in Diagram 2 would rise more steeply towards y in the loss sector.

(3) *Three Versions of the Φ-Surface*, pp. 119-121.

indifference map could be dispensed with, I did not argue that the shape of every gambler indifference curve should be specified by the Φ scale, but that form of the Φ function should be determined by the individual's attitude to risk, from which attitude the gambler indifference map is also drawn. It remains, therefore, to consider whether there is any ground in psychology to maintain the gambler indifference map. Unless there is such ground, what I have listed in Section 2 as Shackle's fourth assumption should be deleted, and the use of the gambler indifference map becomes simply a question of expository or aesthetic advantage or disadvantage.

What different aspect of the individual's uncertainty attitude could be relevant to a choice of projects but irrelevant (and distorted if included in the Φ function) to the epitomizing process? I lack sufficient knowledge of the psychology of choice to answer this question with any confidence and will instead go through (in part repetitively) what I take to be Shackle's answer, and try to bring out the main questions of fact and analysis involved.

In the epitomizing process the individual considers gains and losses separately. « The mental experiences derived respectively from thoughts of gain and from thoughts of loss are different in *quality* or tone, so that although we say that of two rival (i. e. mutually exclusive) hypotheses of gain (or loss) the weaker will be dismissed and attention confined to the more powerful, we cannot suppose that thoughts of a venture's possibilities of gain will enable the enterpriser to dismiss from his mind thoughts of its possibilities of loss, or vice versa » (p. 42). From contemplation of hypothetical gains (and losses) and their associated potential surprise, stimuli will be imparted to the individual's mind, and attention will be focussed on the gain (and loss) outcome with the greatest Φ value. These two outcomes will be standardized (to make possible comparison with focus outcomes of other hypothetical ventures) and then paired — brought face to face — for the first time. This « pairing » takes place on a gambler indifference map — a sort of « black box » which, constructed to the individual's action uncertainty-attitude, will present, and rank, worth while projects on « agenda » platforms representing higher and lower orders of desirability.

6. - In appraising this account we are faced with three questions of fact on which the validity of Shackle's answer depends. First, do individuals focus on a single gain-loss hypothesis? If so, second, do individuals focus on gains and losses in isolation, and then confront the two? If so, third, does something significant happen as the result of this pairing — that is to say, do individuals, when faced with a pair of focus outcomes, in some sense *refocus*, so that a net stimulus is received from the gain-loss pair different from that received when considering them unpaired?

Ignoring for the moment the first question, which will be examined in Section 9, it can be seen that it is the last that is crucial. For

even if individuals do first focus and then pair the focus outcomes, we do not need a gambler indifference map to represent different aspects of individual's uncertainty attitude unless they *refocus*. Now it may be that when faced with a gain-loss pair one or the other in some sense becomes magnified — as the possibility of action approaches the individual may get timid or bold. On Shackle's scheme this would show itself in the shapes of the gambler indifference curves. This I think is a plausible hypothesis. If we accept it, then the argument that we can work from net Φ values would be invalid if the individual concerned has no determinate tendency to get bold or timid as the time for deciding action approaches. If there is no such determinate tendency then a gambler indifference map is needed. But in this case the gambler indifference map would not exist!

Moods of optimism and pessimism are not inconsistent with working from net Φ values. Moods of optimism and pessimism will presumably affect the shape and position of the potential surprise curve and the rate at which gain and loss hypotheses are discounted with increasing surprise. In this way the focus loss that compensates the focus gain may be influenced, and so the relation of the gain stimulus function to the loss stimulus fonction.

7. - There is I think one advantage in working from net Φ values in that, in not (apparently) abandoning the Φ values once the focus outcomes are determined, it is easier to control the « instinctive desire to give some importance to the dispersion of stimuli about the mode » (1). Shackle is surely right in maintaining that the hypothetical outcomes are rivals jostling each other for attention, and that therefore they are non-additive. But the shape and position of the potential surprise curve is significant. A potential surprise curve in the project B gain sector is such that similar stimuli are clustered together. This happens because of the existence of a relatively wide range of hypothetical outcomes in which potential surprise increases slowly. An isolated stimulus « peak » on the other hand is due to a rapid increase of potential surprise outside the zero range. If we work from net Φ values, so that we keep in our mind the stimulus rising out of an xy point, I do not think that there would be an « instinctive desire » to count twice that estimate of surprise at various outcomes which determines the shape of the y curve.

The maintenance of this intimate relation, so to speak, between y and Φ also makes it easy to grasp the solution to the Harrod paradox — that on Shackle's theory the individual guides his conduct by reference to outcomes that he expects not to happen. For if an individual regard anythings outside the y_0 range as something he expects not to happen then the potential surprise curve would rise very steeply. But if the individual does think in terms of *degrees* of surprise concerning all hypothetical outcomes (as supposed in drawing the

(1) C. F. CARTER: *Expectation in Economics*, « Economic. Journal », March 1950, p. 104.

y curves for projects A and B) then the x outcomes outside the y_0 range could produce the major stimulus unless risk-aversion was such that the rate of discounting stimuli rose very steeply with surprise, as, e. g. when $\Phi = xy^4$.

8. - There is, I think, another advantage in working from net Φ values. Suppose we are comparing two projects. We could for each project find the present discounted values of the streams of surpluses through time based on a variety of assumptions about the future, attach to each value a degree of potential surplus, derive from the Φ function the pairs of standardized outcomes and, then, from the gambler indifference map the preferred project, and whether it would be worthwhile undertaking. But in the present net value focus outcomes there might lie buried vital information about the time distribution of losses and gains within the hypothetical loss and gain outcomes. This time distribution may vary between projects and the individual may not be indifferent to it.

By including in the loss Φ function the essential information carried in the gambler indifference map about the focus gain that compensates focus loss it is possible to overcome this difficulty. For instead of using the hypothetical present x values (with profits and losses included in this summary figure as varying only in sign) we could then sum the (discounted) net annual stimuli in the focus gain hypothesis (which may include losses in one or more years), and in the focus loss hypothesis. The two projects could be validly compared from the net Φ values.

9. - Whether individuals may reasonably be expected to focus on a single gain-loss hypothesis, is surely a question of the context in which an individual forms his expectations. There are cases where it is reasonable to suppose that the dominant hypotheses will be in the gain sector, and cases where it is reasonable to suppose that there will be three or four dominant hypotheses. (What I regard as a reasonable supposition is of course related to « what I would do in the circumstances »!).

Suppose an individual is considering the purchase of equities because he is certain that a capital gain can be made. From the basis of the present price of equity K he makes an estimate of possible prices a year hence. Call the purchase price x_0 and the considered range of possible prices x_1 to x_7. Suppose that the y curve rises steeply from a zero surprise inner range of x_3 to x_6, giving Φ values for x_1 to x_7 of .25, 1, 3, 4, 5, 6, 3.5 respectively. Suppose for equity J that the considered range of possible prices is x_2 to x_8, that surprise is zero for prices x_4 and x_5, and that Φ values are respectively from x_2 to x_8 : .5, 1.5, 4, 5, 4.5, 3.5, 2. The greatest K Φ value of 6 is for outcome x_6. The greatest J Φ value of 5 is for outcome x_5. In neither case is there a loss Φ to offset against the gain Φ, so that net Φ = gain Φ unless we treat the cost of purchase and sale (potential surprise zero)

as the loss. Analytically however this would be best treated as a constant that would fix the lowest Φ value consistent with a decision to purchase. In the case chosen the rise in potential surprise is such that the greatest Φ value in each case is for the x value at the upper limit of the inner range. Of course for an individual with a lower capacity for surprise or a lower rate of stimulus-discount this need not happen.

Where there is not a focus gain-loss pair, the Shackle method has no significant advantage over the conventional method of analysis. But although the Shackle type of analysis has no advantage in this case, it can be used. It is capable of being used as a *general* analysis.

There may be more than two dominant hypotheses on occasions when a person is in doubt about the *general* state of future markets. Consider a project in an industry that seems certain to reap specially high profits from inflation or war. A person thinking inflation or war to be quite possible but not certain could be thought of as having two potential surprise curves — one based on the hypotheses that there will not be inflation (and not deflation), the other based on the hypothesis that there will be. Imagine a second y curve superimposed further to the right than the existing y curve on the xy plane for project A or B. This would give three or four relevant Φ peaks on the $x\Phi$ plane, and, if the new y curve were of the same shape as the old, the new Φ gain peak would be higher than the old. But it does not follow that this peak Φ value would provide the focus gain to the exclusion of the lower Φ value peak. For the Φ values attached to the two potential surprise curves are not rivals jostling for attention in the sense that Φ values derived from the same y curve are. The « focus » gains, derived from the superimposed y curves, are in some sense additive. But even when with differently shaped y curves the peak Φ value rises out of the non-inflation y curve, the existence of the second (lower) peak may induce an individual to choose a project which, though not preferable on the basis of the non-inflation y curve, is preferable on the basis of the inflation y curve. The existence of Φ values arising out of the inflation y curve may also be relevant in that it may induce an individual to instal the type of project that can be readily extended if the inflation assumption proves correct.

10. So far I have assumed that the surprise scale is fully numerical. If the degree of belief index is only ordered, then various Φ values may be associated with given hypothetical outcomes [1]. Outcomes 3, 4, 5 in project B would carry Φ values of 3.0, 3.6, 4.5 if the degrees of belief are 1, .95, .9 respectively, but 3, 3.2, 2.5 if degrees of belief are 1, .9, .7. Thus though the ordering of belief is not changed the maximum Φ value is.

It is necessary here to distinguish between two different criticisms of Shackle's procedure: (1) that it is unrealistic to represent potential

(1) See C. F. CARTER: *A note on the Theory of Expectation*, « Economic Journal », vol. LXVIII, No. 252, December, 1953.

surprise by a curve, and (2) that it is possible to say that one degree of belief is greater or smaller than another, but not by how much. These are distinct criticisms — the second does not follow necessarily from the first.

Consider the first criticism in isolation. If potential surprise is not a continuous variable — if attention is concentrated on a few typical outcomes — then on the x y plane there will be y values for outcomes, say, -6, -4, -2, 3, 5, 8, 10. The y curve would then simply be the theorist's picture of the situation if the individual did not simplify a very difficult problem by selecting outcomes to be considered. There may be significant differences between individuals in this respect, but the analysis is easily adapted (at any rate with the net Φ method) to the case of individuals who do hypothesise only a small number of outcomes.

The second criticism is a much more serious one. For if it is possible to say that one would be more surprised by this than by that, but not by how much, then the Φ values become quite indeterminable outside the y_0 range. In this case the business man would presumably make up his mind about difficult alternatives only by restricting considered outcomes to the inner range — by distinguishing in effect between substance and dream and rejecting as dreams those stimuli which arise from a result that would surprise him if it occurred [1] — or by tossing up or «backing a hunch».

Professor Shackle writes that within the y_0 to y range «each one of us will find in his own past experience particular occurrences or types of occurrences each of which has caused him some degree of surprise, the memory of which remains with him vividly enough to serve, in conjunction with the memory of the event which caused it, as one of a series of fixed levels, not necessarily evenly spaced, providing together a scale of potential surprise» (p. 10). Shackle then, by interpolating between these fixed levels, treats surprise as a continuous variable. Professor Carter in consequence of rejecting numerical degree of belief proposes that we rank outcomes in broad groups — the «perfectly possible», the «just possible» and so on. So far, apart from the interpolation between fixed levels, there is no conflict with Shackle. But if we can only rank these broad groups, how can we choose between difficult alternatives? And how decide whether a risky project is worth while? Professor Carter provides no solution to this, other than that mentioned at the end of the last paragraph.

Whether degrees of belief are in fact numerical is thus a crucial question, and further advance will depend on an answer to it. My own impression is that it is not unreal to postulate fixed levels of surprise but that it is unreal to postulate surprise as a continuous variable. That is one of the reasons why I think the net-Φ method is to be preferred to Professor Shackle's method.

---

[1] C. F. CARTER: *Op. cit.*, «Economic Journal», March 1950, pag. 103.