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Roles of Ion Motion and of Return Currents in Transverse Instability of Colliding Plasmas

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Proper account of the ion dynamics is shown to lead to a significant change in the extent of the unstable range of transverse modes. However, no very high growth rates can be produced by the ions. For relativistic electron beams in a cold plasma, the role of return currents induced in the plasma and of plasma collisions are analyzed. The return currents do not influence the transverse instability except for the extreme relativistic region $\gamma n_b/n_p > 1$, where the plasma electrons "take over" in maintaining the growth rate high. Collisions are unimportant unless their frequency, $\nu \geq \omega_p^2$ (plasma)$/\omega_p$(beam), where transverse oscillations cease to be electrically neutralized, so that pinching becomes impossible and the growth rate of the instability goes to zero as $1/\nu$.

I. INTRODUCTION

Transverse instabilities of colliding plasmas have recently received much attention.[1–4] This is due to interest in such collisions both in astrophysics and in laboratory-produced electron beams interacting with plasma. In both contexts, it is of some importance to deal with the problem taking into account the presence of an ambient magnetic field, so that the plasma is anisotropic both because of this field and because of the relative streaming of its components. The derivation of a dispersion relation

$$f(\mathbf{k}, \omega) = 0,$$

then proceeds along the way outlined, e.g., in Chap. 10 of Ref. 5, Eq. (10.35).

Two specific questions connected with these instabilities have attracted some attention,[3,6–8] but cannot yet be considered as answered by now, namely;

(a) In case not only electrons, but also ions, take part in the relative motion, do these have an important role in the existence of instabilities and on the growth rate of disturbances?

(b) When an electron beam traverses a plasma, a neutralizing return current may be produced by the plasma electrons.[9] Does the presence of this alter the dispersion relation considerably?

In the following, these two problems will be dealt with in the simplest ways possible, namely, by treating the case of no ambient magnetic field, and the ion problem in the nonrelativistic limit. For completeness, a simple derivation of the transverse two-beam instability is given to start with.

II. SIMPLE EXAMPLE OF TRANSVERSE INSTABILITY

Let us consider a collisionless plasma filling the whole space and consisting of two groups of particles at relative motion directed along the z axis.

Purely transverse modes of interst to us are those having their electric field along the z axis, magnetic field along the y axis, and $\mathbf{k}$, the wavenumber vector along the x axis. This corresponds to the pinching mode of an anisotropic plasma,[10] or to the ordinary mode of Refs. 2 and 3.

Let F_{0j} be the distribution function of the jth component (j is electrons or ions) of the undisturbed plasmas. Let f_j be the change in the distribution function due to a disturbance of wavenumber vector $\mathbf{k}$. The linearized nonrelativistic Vlasov equation for f_i is

$$\frac{\partial f_j}{\partial t} + \mathbf{v}\cdot\nabla_x f_j + \frac{e_j}{m_j}(\mathbf{E}+c^{-1}\mathbf{v}\times\mathbf{B})\cdot\nabla_v F_{0j} = 0. \quad (1)$$

$\mathbf{E}$, $\mathbf{B}$, and f_j are assumed to vary as

$$\exp[i(kx-\omega t)].$$

Using Maxwell's equations

$$\nabla\cdot\mathbf{E} = 4\pi\rho, \qquad \nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{j} + c^{-1}\frac{\partial E}{\partial t},$$

the self-consistent fields may be expressed in terms of the perturbation f_j by

$$i\mathbf{k}\cdot\mathbf{E} = 4\pi\sum_j e_j \int f_j(\mathbf{v})\, d^3v = 0, \quad (2)$$

$$i\mathbf{k}\times\mathbf{B} = (4\pi/c)\sum_j e_j \int \mathbf{v} f_j(\mathbf{v})\, d^3v - (i\omega/c)\mathbf{E}. \quad (3)$$

$\mathbf{B}$ may be eliminated by Maxwell's equation

$$\nabla\times\mathbf{E} = -c^{-1}\frac{\partial B}{\partial t} \quad \text{or} \quad \frac{i\omega}{c}\mathbf{B} = i\mathbf{k}\times\mathbf{E}. \quad (4)$$

Now let E and B be the only components of $\mathbf{E}$ and of $\mathbf{B}$, in the z and y directions, respectively. Maxwell's fourth equation $\nabla\cdot\mathbf{B}=0$, is automatically satisfied (since $\mathbf{k}$ is in the x direction). Combining Eqs. (1)–(4) we get a homogeneous equation for E, which has a nonzero solution only if

$$\omega^2 - k^2c^2 - \sum_j \omega_{pj}^2\left(1 + \int_0 \frac{v_z^2 F_{0j}}{(v_x-\omega/k)^2}\, d^3v\right) = 0. \quad (5)$$

In Eq. (5) it is understood that the integration is performed below the singularity $v_x = \omega/k$ in the complex v_x plane.[5] Also, we used the definitions

$$\int F_{0j}\, d^3v = 1$$

and

$$\omega_{pj}^2 = 4\pi N_j e_j^2/m_j.$$

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To understand the meaning of Eq. (5), let us first take the simplest example, namely, that of immovable ions and a "resonance" distribution function for the electrons:

$$F_{0e}=F_z(v_z)\,\frac{1}{\pi^2}\,\frac{V_T}{v_x^2+V_T^2}\,\frac{V_T}{v_y^2+V_T^2}\,, \tag{6}$$

where V_T is a characteristic thermal velocity of a plasma electron perpendicular to the direction of relative motion (i.e., the z axis). Define

$$\bar{u}^2=\int_{-\infty}^{\infty} v_z^2F_z(v_z)\,dv_z; \tag{7}$$

$\bar{u}^2$ is larger than V_T^2 in two cases:

(a) We are dealing with a beam-plasma problem, with the beam velocity not small compared with the thermal velocity, or

(b) we are not dealing with a beam problem at all, but with plasma with an anisotropic temperature, where

$$\bar{u}\approx V_{T\|}, \qquad V_T\approx V_{T\perp}, \qquad T_{\|}>T_{\perp}.$$

Thus, the treatment given here is a unified treatment of the transverse-mode beam instability and the pinching instability of anisotropic plasmas.[10]

The integration in Eq. (5) may be performed analytically to give (for $\mathrm{Im}\omega/k>0$),

$$\omega^2-k^2c^2-\omega_{pe}^2\left(1-\frac{k^2\bar{u}^2}{(kV_T-i\omega)^2}\right)=0. \tag{8}$$

This equation has two distinct solutions:

(a) $$\omega^2\approx\omega_{pe}^2+k^2c^2.$$

This propagates almost like an ordinary transverse wave and does not concern us here.

(b) ω is purely imaginary and $|\omega|\ll kc$. In that case,

$$\omega\approx i[\omega_{pe}k\bar{u}/(\omega_{pe}^2+k^2c^2)^{1/2}-kV_T]. \tag{9}$$

These are standing waves which are damped if

$$V_T\geq\bar{u}\ \text{(stable region)}, \tag{10}$$

but are growing if

$$V_T<\bar{u}\quad\text{for}\quad |k|<(\omega_{pe}/c)[(\bar{u}^2/V_T^2)-1]^{1/2}\qquad\text{(unstable region)}. \tag{11}$$

The maximum growth rate in (9) is

$$\delta_{\max}=(\omega_{pe}/c)(\bar{u}^{2/3}-V_T^{2/3})^{3/2}. \tag{12}$$

For Maxwellian distribution, V_T in Eqs. (10)–(12) must be replaced by V_T', where $V_T\leq V_T'\leq 1.36V_T$.

The physical explanation of the instability is as follows: Assume density fluctuations along the x axis, so that bunches of particles streaming in the $+z$ direction are formed. Between two such bunches, there is a bunch of particles streaming in the $-z$ direction (this is not necessary for the instability, and anyway it depends on the choice of a frame of reference). A magnetic field in the y direction is produced by the currents. The force it applies on the moving particles is such as to bunch them even further (as in the pinch effect). The currents are thus growing with time, so the magnetic field is time dependent, and an electric field is produced. This is found to be parallel to the z axis, and decelerates the particles going in the $+z$ direction if they are inside their appropriate bunch. Similarly, the electric field decelerates the particles in the $-z$ bunch in that direction, thus providing some stabilization, which is, however, insufficient. All this happens, in the nonrelativistic limit, without the apperance of charge fluctuations which would produce restoring electric forces in the x direction.

III. ROLE OF ION MOTION

For Maxwellian distribution,

$$F_{0j}=\pi^{-3/2}V_{\|j}^{-1}V_{\perp j}^{-2}\exp(-v_\perp^2/V_{\perp j}^2)\times\exp[-(v_z-u_j)^2/V_{\|j}^2]. \tag{13}$$

Equation (5) becomes

$$\omega^2-k^2c^2-\sum_j\omega_{pj}^2[1+\bar{u}_j^2V_{\perp j}^{-2}Z'(\omega/kV_{\perp j})]=0, \tag{14}$$

where

$$\bar{u}_j^2=V_{\|j}^{-1}\pi^{-1/2}\int_{-\infty}^{\infty}v_z^2\exp\left(-\frac{(v_z-u_j)^2}{V_{\|j}^2}\right)dv_z$$

and the plasma dispersion function[11]

$$Z(\mu)=\pi^{-1/2}\int_{-\infty}^{\infty}\frac{\exp(-x^2)\,dx}{x-\mu},\qquad (\mathrm{Im}\mu>0).$$

The Z function has the following properties:

$$|\mu|\gg1,\qquad Z(\mu)\approx-\mu^{-1}, \tag{15}$$

$$|\mu|\ll1,\qquad Z(\mu)\approx i\pi^{1/2}-2\mu. \tag{16}$$

Let us deal separately with the following three regions for ω/k:

(a) $|\omega/k|\gg V_{\perp e}$,

(b) $V_{\perp e}\gg|\omega/k|\gg V_{\perp i}$,

(c) $V_{\perp i}\gg|\omega/k|$.

Using the approximations (15) and (16), the dispersion relation (14) becomes

$$\omega^2-k^2c^2-\sum_j\omega_{pj}^2[1+(\bar{u}_j^2k^2/\omega^2)]=0\qquad\text{(region }a\text{)}. \tag{17}$$

Since $\omega_{pi}\ll\omega_{pe}$, this coincides with the low-temperature limit of Eq. (8), and the role of the ions here is negligible.

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In region b,

$$\omega^2-k^2c^2-\omega_{pi}^2[1+(\bar{u}_i^2k^2/\omega^2)] -\omega_{pe}^2[1-(2\bar{u}_e^2/V_{\perp e}^2)]=0. \quad (18)$$

For k going from 0 to ∞, Imω falls from

$$\{\omega_{pe}^2[(2\bar{u}_e^2/V_{\perp e}^2)-1]-\omega_{pi}^2\}^{1/2} \quad \text{to} \quad \omega_{pi}\bar{u}_i/c.$$

In reality, not all of this dependence belongs to region b. The part which does, corresponds to the portion BC of curve 3, Fig. 1. For region c,

$$\omega^2-k^2c^2-\omega_{pi}^2[1-(2\bar{u}_i^2/V_{\perp i}^2)] -\omega_{pe}^2[1-(2\bar{u}_e^2/V_{\perp e}^2)]=0. \quad (19)$$

Since

$$\omega_{pi}^2 \ll \omega_{pe}^2, \qquad \omega_{pi}^2/V_{\perp i}^2 \approx (T_{\perp e}/T_{\perp i})(\omega_{pe}^2/V_{\perp e}^2),$$

Eq. (19) can be written

$$\omega^2-k^2c^2+k_D^2c^2=0, \quad (20)$$

where

$$k_D^2c^2=\omega_{pe}^2\{(2\bar{u}_e^2/V_{\perp e}^2)[1+(T_{\perp e}/T_{\perp i})]-1\} \quad (21)$$

and we have assumed $\bar{u}_e=\bar{u}_i$.

From Eq. (20) it is seen that ω is imaginary as long as $k<k_D$. Equation (21) thus determines the location of point D (the end of the instability region) in Fig. 1. Thus, we conclude that the participation of the ions in the motion considerably affects the extent of the instability region for transverse waves (as already pointed by Bünemann[8]), but does not produce higher growth rates than the electrons alone.

Some comments about the extreme growth rates reported by Lee and Armstrong[3] are in order here. Analysis of the numerical example given by them shows that

(a) The presence of a magnetic field is not of any importance for the attainment of high growth rates (this is also clear from their choice of $\Omega_e/\omega_{pe}=0.1$).

(b) At zero temperature, their dispersion relation becomes

$$\omega^2-c^2k^2-\sum_j \omega_{pj}^2[1-(T_{\|j}/T_{\perp j})-2(u^2/v_{\perp j}^2)]=0,$$

which gives infinite frequencies in the limit of a small external magnetic field $\Omega\to 0$.

(c) The dispersion relation derived in Ref. 3 is also in conflict with that derived in Ref. 4.

IV. ROLE OF RETURN CURRENTS

In this section, we deal exclusively with the problem of relativistic electron beam interaction with plasma. We adopt the reasoning, as well as the notation, of Watson, Bludman, and Rosenbluth.[12,13] As these authors note, there is a minor inconsistency in their model, since the unperturbed configuration is that of a stream of electrons in a stationary plasma with no magnetic fields present, which is in conflict with Maxwell's equations.

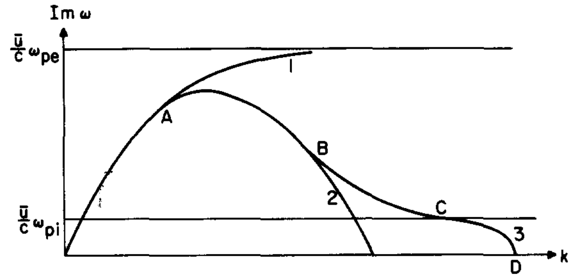

FIG. 1. Growth rates in various approximations. 1. Cold plasma. 2. Hot electrons, no ions. 3. Ions participating in relative motion. At point A, the electron temperature becomes important:

$$k_A \approx c^{-1}\omega_{pe}[(0.01\bar{u}^2/V_T^2)-1]^{1/2} \text{ [Notation of Eq. (9)].}$$

At point B, the ions start to influence:

$$k_B \approx k_0-(V_{\perp i}^2/V_T^2)k_0, \quad \text{where} \quad k_0=\omega_{pe}/c[(\bar{u}^2/V_T^2)-1]^{1/2} \text{ [See Eq. (11)].}$$

At point C, only the ions support the instability:

$$c^2k^2{}_C=\omega_{pi}^2[(\bar{u}^2/c^2)+(\bar{u}^2/V_{\perp i}^2)-1]-\omega_{pe}^2(1-2\bar{u}^2/V_{\perp e}^2).$$

At point D, the instability disappears. K_D is given in Eq. (21).

One way out of this is to attack the problem of a beam of finite radius, and take into account the nonlinear electron motion in the unperturbed magnetic field.[14] This is undoubtedly the correct procedure for perturbations of long wavelength. For short wavelengths, however, there is interest in the problem of a space-filling beam, in which case the above-mentioned inconsistency is released by assuming a homogeneous return current to flow in the plasma. In certain circumstances, this seems to correspond to physical reality.[9]

The authors of Ref. 13 state that they do not find a transverse electromanetic instability. However, such an instability exists just as in the nonrelativistic case. In fact, the dispersion relation (4.16) of Ref. 13 is simply solved to yield

$$\omega=k_zv_0\pm i\epsilon,$$

$$\epsilon=[kv_0\sin\theta/(\omega_p^2+c^2k_x^2)^{1/2}](\omega_{b\perp}^2\sin^2\theta+\omega_{b\|}^2)^{1/2} \quad (22)$$

which shows instability. [The reason for the misunderstanding is as follows. In the absence of the beam, the number of degrees of freedom is determined by the number of possible orientations of the electric field. When the beam is present, new degrees of freedom appear. To see this, one can, for instance, imagine such perturbations of the beam-plasma system which raise no electric field at all. Formally, one continues to find the dispersion relation from the determinant of an equation for the electric field, so one may be led to the conclusion that the number of solutions is unchanged. However, it is the new solutions which are the unstable ones, and only to order $O(\omega_b/\omega_p)$. The statement

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in Ref. 13 that only solutions which make the last term of their Eq. (4.16) a small perturbation are sought in the weak beam limit is not true. However, the statement that the old solutions cannot become unstable for an infinitesimally dilute beam is, of course, true.] It is the purpose of the present section to find out what happens to this transverse instability when the return current is taken into account.

If the unperturbed state of the plasma contains current flow, then the conductivity of the plasma becomes a tensor, like that of the beam (since there is a preferred direction present).

The linearized equations of motion for the plasma electrons and ions are [instead of Eq. (2.7) of Ref. 13]

$$-i(\omega-\mathbf{k}\cdot\mathbf{v}_1)n_0m\mathbf{v}'=en_0[\mathbf{E}'+\omega^{-1}\mathbf{v}_1\times(\mathbf{k}\cdot\mathbf{E}')]-\nu[n_0mN_0M/(n_0m+N_0M)](\mathbf{v}'-\mathbf{V}'), \quad (23)$$

$$-i(\omega-\mathbf{k}\cdot\mathbf{V}_1)N_0M\mathbf{V}'=-eN_0[\mathbf{E}'+\omega^{-1}\mathbf{V}_1\times(\mathbf{k}\cdot\mathbf{E}')]+\nu[n_0mN_0M/(n_0m+N_0M)](\mathbf{v}'-\mathbf{V}'). \quad (24)$$

The notation here is identical to that of Ref. 13, except for $\mathbf{v}_1$ and $\mathbf{V}_1$ which were introduced to denote the unperturbed plasma electron and ion velocities producing the return current. Also, ω and k are defined to have the opposite sign compared with that in Ref. 13.

Because of the complexity of this set of equations we are going to drop the ion degrees of freedom:

$$M\rightarrow\infty, \qquad \mathbf{V}_1=\mathbf{V}'=0.$$

In analogy to the expression $-4\pi i\omega\mathbf{j}_b'=\mathsf{I}\cdot\mathbf{E}'$ introduced for the beam current in Ref. 13 Eq. (2.1), let us write the pertubation plasma current

$$-4\pi i\omega\mathbf{j}_p'=\mathsf{J}\cdot\mathbf{E}'.$$

The components of J are found by solving Eqs. (23) and (24):

$$J_{xx}=J_{yy}=\omega_e^2[\omega'/(\omega'+i\nu)] \qquad J_{xy}=J_{yx}=0,$$

$$J_{xz}=J_{zx}=\omega_e^2[v_1k_x/(\omega'+i\nu)],$$

$$J_{zz}=\omega_e^2[\omega^2+(k_xv_1)^2]/[\omega'(\omega'+i\nu)], \quad (25)$$

where $\omega'=\omega-\mathbf{k}\cdot\mathbf{v}_1=\omega-k_zv_1$.

The dispersion relation now becomes

$$\det[\mathbf{1}(c^2k^2-\omega^2)+\mathsf{I}+\mathsf{J}-c^2\mathbf{kk}]=0. \quad (26)$$

Here, I is the beam conductivity tensor, as in Ref. 13. The x-z part of this is

$$\omega^4-\omega^2(c^2k^2+I_{xx}+J_{xx}+I_{zz}+J_{zz})+(c^2k_z^2+I_{xx}+J_{xx})\cdot(c^2k_x^2+I_{zz}+J_{zz})-(-c^2k_xk_z+I_{xz}+J_{xz})^2=0. \quad (27)$$

Let us introduce the current neutralization condition

$$\mathbf{j}_b^0+\mathbf{j}_p^0=0,$$

or, using Eq. (3.3) of Ref. 12 and (2.4) of Ref. 13,

$$\gamma\omega_{b\perp}^2v_0=-\omega_e^2v_1. \quad (28)$$

From Eqs. (25) and (28), it may be seen that the nondiagonal elements of J are of opposite sign to, and larger than, those of I.

To investigate the influence of the return currents, let us take the purely transverse ($k_z=0$), collisionless ($\nu=0$, $1/\tau=0$), cold ($\theta_{||}=\theta_\perp=0$), dilute beam ($\omega_b\ll\omega_e$) limit. The conductivity tensors become

$$I_{xx}=\omega_{b\perp}^2, \qquad I_{xz}=I_{zx}=\omega_{b\perp}^2(kv_0/\omega),$$

$$I_{zz}=\omega_{b||}^2+\omega_{b\perp}^2(kv_0/\omega)^2, \qquad J_{xx}=\omega_e^2,$$

$$J_{xz}=J_{zx}=\omega_e^2(kv_1/\omega), \qquad J_{zz}=\omega_e^2+\omega_e^2(kv_1/\omega)^2.$$

From Eqs. (27) and (28), the dispersion relation is now

$$\omega^4-\omega^2\left[c^2k^2+\omega_\perp^2+\omega_{||}^2+\omega_{b\perp}^2\left(\frac{kv_0}{\omega}\right)^2\left(1+\gamma^2\frac{\omega_{b\perp}^2}{\omega_e^2}\right)\right]+\omega_\perp^2\left[c^2k^2+\omega_{||}^2+\omega_{b\perp}^2\left(\frac{kv_0}{\omega}\right)^2\left(1+\gamma^2\frac{\omega_{b\perp}^2}{\omega_e^2}\right)\right]-\omega_{b\perp}^4\left(\frac{kv_0}{\omega}\right)^2(1-\gamma)^2=0.$$

Here $\omega_\perp^2=\omega_e^2+\omega_{b\perp}^2$, $\omega_{||}^2=\omega_e^2+\omega_{b||}^2$.

Let us define $\beta=v_0/c$ and the new variables

$$\xi=c^2k^2, \qquad \eta=\omega^2.$$

The dispersion relation can now be written[15]

$$\xi=\eta[(\eta-H_1)(\eta-H_2)/(\eta-H_3)(\eta-H_4)], \quad (29)$$

where

$$H_1=\omega_\perp^2, \qquad H_2=\omega_{||}^2,$$

$$H_3=\omega_\perp^2+O(\omega_{b\perp}^4/\omega_\perp^2),$$

$$H_4=-\beta^2\omega_{b\perp}^2[1+(2\gamma-1)(\omega_{b\perp}^2/\omega_e^2)+O(\omega_{b\perp}^4/\omega_e^4)].$$

A plot of Eq. (29) is given in Fig. 2. An instability exists if η is found to be negative (or complex) for positive ξ. Thus, a branch of unstable solutions exists as a result of H_4 being negative. In the extreme relativistic limit, $(2\gamma-1)(\omega_b^2/\omega_e^2)$ becomes greater than unity, and the return current assumes a more important role in making H_4 negative than the beam current. Except for this unrealistic limit, the role of the return current is minor.

It may be noted that a different type of instability could have arisen if H_3 were greater than H_1. In that case, the two curves approaching one another near H_3 would turn around the other way as they do in Fig. 2, and as a result, for some range of ξ, only one real η would exist, instead of three. This means that there would be two complex solutions for η, and therefore for ω [Eq. (29) is a third-order real equation for η for given ξ]. This situation is shown in Fig. 3. It can, however, be

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shown from Eqs. (28) and (29), that

$$H_1-H_3=\frac{\beta^2\omega_{b\perp}^4(\gamma-1)^2}{H_3+\beta^2\omega_{b\perp}^2[1+\gamma^2(\omega_b^2/\omega_e^2)]}$$

which is always positive, so that this type of instability does not occur.

In summary, the effect of the return current on the beam stability is quite weak, except in the extreme relativistic region of beam mass density higher than that of the plasma electrons. Note that, in this section, the only account of plasma temperature was taken by introducing collisions into the equations of motion, (23), (24). A broader investigation of the temperature effects might be of importance.

V. EFFECT OF COLLISIONS

In this section we discuss the effect of dissipation in the plasma, in the form of electron–ion collisions, on the transverse instability. Collisions with the beam electrons will be neglected.

According to the collision frequency ν, four regions can be outlined:

1. $\nu\ll\omega_{b\perp}$. Here, collisions are unimportant, and the results of the previous sections hold.
2. $\omega_{b\perp}\ll\nu\ll\omega_e$. Return currents are possible in this region.[9,16] The transverse instability still exists, as discussed in the present section.
3. $\omega_e\ll\nu\ll\omega_e^2/\omega_{b\perp}$. No return currents are possible, but the instability nevertheless appears, and is due to pinching of the beam electrons only.
4. $\omega_e^2/\omega_{b\perp}\ll\nu$. In this region, oscillations with frequency ω_b, characteristic of the transverse mode, are no longer electrically neutralized by the plasma electrons. Pinching becomes impossible as the electric forces get as large as the magnetic ones, and the growth rate of the instability goes to zero.

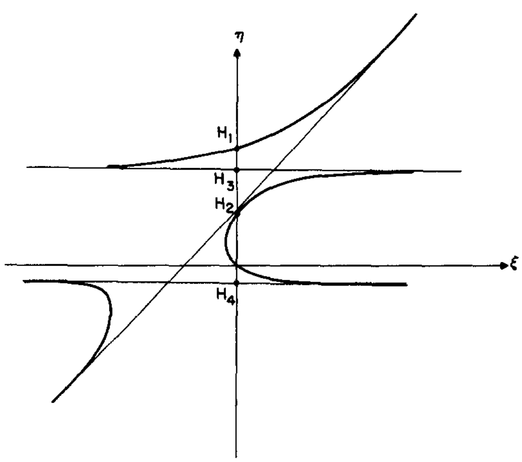

FIG. 2. ξ–η relation from Eq. (29). For each $\xi>0$ there are three values of η.

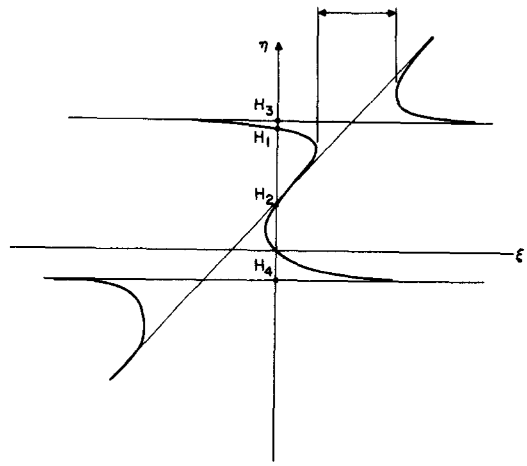

FIG. 3. Hypothetical ξ–η relation leading to complex η for real ξ in the region marked by the arrow.

The dispersion relation (27) becomes a complex equation when the plasma conductivity tensor, Eq. (25), is inserted. However, for the variable

$$\epsilon=-i\omega$$

it is a real equation

$$\epsilon^4+\epsilon^2\left(\xi+\omega_{b\perp}^2+\omega_{b\|}^2-\omega_{b\perp}^2\beta_0^2\xi/\epsilon^2+\omega_e^2\frac{\epsilon}{\epsilon+\nu}+\omega_e^2\frac{\epsilon^2-\beta_1^2\xi}{\epsilon(\epsilon+\nu)}\right)+\left(\omega_{b\perp}^2+\omega_e^2\frac{\epsilon}{\epsilon+\nu}\right)\times\left[\xi-\omega_{b\perp}^2\frac{\beta_0^2\xi}{\epsilon^2}+\omega_{b\|}^2+\omega_e^2\frac{\epsilon^2-\beta_1^2\xi}{\epsilon(\epsilon+\nu)}\right]+[\omega_{b\perp}^2\beta_0/\epsilon-\omega_e^2\beta_1/(\epsilon+\nu)]^2\xi=0. \quad (30)$$

Here, $\xi=c^2k^2$, $k_z=0$, $\beta_0=v_0/c$, and $\beta_1=-v_1/c$.

As in the previous section, we rearrange Eq. (30) in the form

$$\xi=N(\epsilon)/D(\epsilon),$$

where

$$D(\epsilon)=(\epsilon^2+\omega_b^2)\epsilon(\epsilon+\nu)-\omega_e^2[\omega_b^2(\beta_0+\beta_1)^2-(1-\beta_1^2)\epsilon^2]. \quad (31)$$

Instabilities are found as follows: For $\nu=0$, we had an unstable branch in the ξ–η plane corresponding to $H_4<0$ [Eq. (29)]. H_4 corresponds to two roots of $D(\epsilon)$, of equal magnitude and opposite sign. The positive root is the one leading to purely growing perturbations. For $\nu\neq0$, we look for roots of $D(\epsilon)$ graphically (Fig. 4). As is easily seen from Eq. (31) or Fig. 4, there is always a positive and a negative root. The positive root, leading to the instability, tends to zero as ν grows, and for large ν,

$$\epsilon_+\approx\omega_e^2\beta_0^2\gamma^2/\nu \qquad \text{for } \nu\geq\gamma^2\beta_0(\omega_e^2/\omega_b). \quad (32)$$

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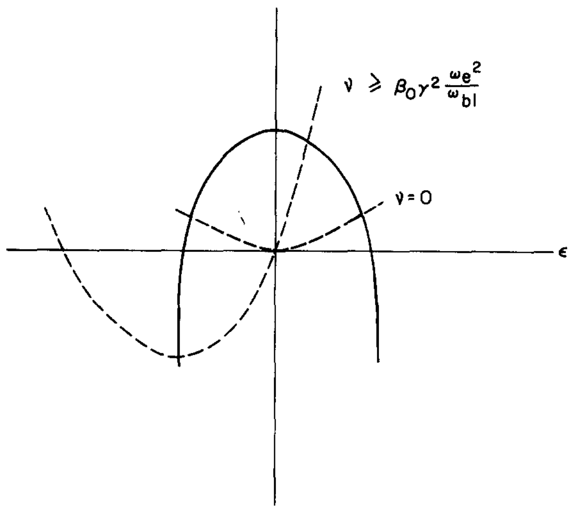

FIG. 4. Graphical solution to $D(\epsilon)=0$, Eq. (31). Solid line: $\omega_e^2[\omega_{b\perp}^2(\beta_0+\beta_1)^2-(1-\beta_1^2)\epsilon^2]$. Broken line: $(\epsilon^2+\omega_{b\|}^2)\epsilon(\epsilon+\nu)$.

The damped, negative root, is seen to grow in magnitude as ν grows.

VI. SUMMARY

The roles of the various components of two colliding plasmas in the transverse, or pinch, instability were analyzed under simplifying conditions. The growth rate of transverse perturbations is of the order of $\beta\omega_{be}$, where β is the relative velocity of the plasmas (in units of c) and ω_{be} is the electron plasma frequency of one of the plasmas (the dilute one, in case of a "beam" and a "quiescent plasma"). Where the thermal motion of the electrons may negate their pinching tendency, i.e., at high $\mathbf{k}$, the ions may still sustain the instability. Then, the growth rate falls to the order of $\beta\omega_i$, where ω_i is the ion plasma frequency. At still higher $\mathbf{k}$, the instability disappears.

If return currents are flowing in the plasma, carrying the same current density as an incoming electron beam, but with much higher electron density than the beam, and correspondingly lower streaming velocities with respect to the ions, it is shown that the beam particles have the dominant role in determining the instability region. This is shown to be so as long as the plasma electrically neutralizes the charge fluctuations due to the perturbation under study. If electron–ion collisions become more frequent than ω_{pe}^2/ω_b, this is no more the case, and the instability disappears, due to the self-electric restoring forces of the beam.

The following aspects of the problem were not studied in the present paper: the effect of an ambient magnetic field, return current-plasma ions interaction, and the role of plasma temperature in the dilute beam limit (except for the effect of collisions). These may be of special importance for such applications as plasma heating with electron beams.

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