Relative dating of Late Quaternary deposits using cluster and discriminant analysis, Audubon Cirque, Mt. Audubon, Colorado Front Range

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BOREAS

Dowdeswell, Julian A. 19820601: Relative dating of Late Quaternary deposits using cluster and discriminant analysis, Audubon Cirque, Mt. Audubon, Colorado Front Range. *Boreas*, Vol. 11, pp. 151–161. Oslo. ISSN 0300-9483.

Holocene and Late Quaternary talus, lobate rock glaciers, and moraines within Audubon Cirque, Colorado Front Range, were assigned relative ages using the following age-dependent criteria: fresh-weathered ratio, pitting, weathering rind development, angularity, and surface oxidation of boulders, together with lichen cover and largest lichen diameter. Principal Components Analysis was followed by Cluster and Discriminant Analyses on the first principal component scores, yielding four major groups of deposits (relative age units C, R, E and V, from youngest to oldest). Tentative correlation with other Colorado Late Quaternary sequences suggests that unit C is of Gannett Peak age (100–300 years B.P.), unit R of Audubon age (950—1850 years B.P.), unit E of Early Neoglacial age (3000–5000 years B.P.), and unit V of Late Pinedale age (about 10,000 years B.P.). Correlation is problematic due to differences in operational definitions of relative dating parameters between workers, and because climatic and lithologic variations between areas may confound the data.

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Relative dating has been used by a number of workers to establish a relative chronology for Late Quaternary deposits in Colorado (e.g. Benedict 1967, 1968, 1973; Birkeland 1973; Mahaney 1973a, 1973b; Williams 1973; Carroll 1974). Several studies have assigned relative ages to groups of deposits on the basis of a qualitative appraisal of the data (e.g. Birkeland 1973; Mahaney 1973a), whereas others (Carroll 1974; Miller 1979) have used multivariate statistical methods based on information and graph theory to define groupings of sites.

The aims of this paper are twofold. First, to demonstrate the use of Cluster and Discriminant Analysis as an alternative method of grouping relative age data, after standardization and orthogonalization of the raw data matrix using Principal Components Analysis. Second, to present data based on a multi-parameter approach using weathering and lichenometric techniques, to determine the relative ages of deposits within Audubon Cirque, Colorado Front Range.

Once relative age units have been defined, correlations are made with other Late Quaternary sequences in Colorado to provide tentative absolute ages for the surficial deposits in Audubon Cirque.

Study area

Audubon Cirque lies on the eastern flank of Mt. Audubon in the Indian Peaks region of the Colorado Front Range (Fig. 1). The cirque is open to the southeast, with floor elevations between 3470 and 3570 m. Bedrock is mainly medium grained monzonite and quartz monzonite (Gable & Ma-

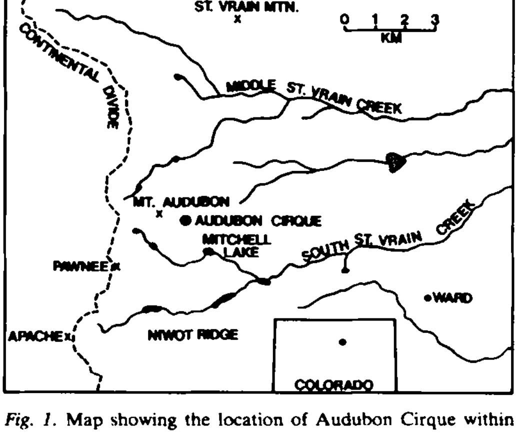

Fig. 1. Map showing the location of Audubon Cirque within the South St. Vrain drainage basin, and the study area within Colorado.

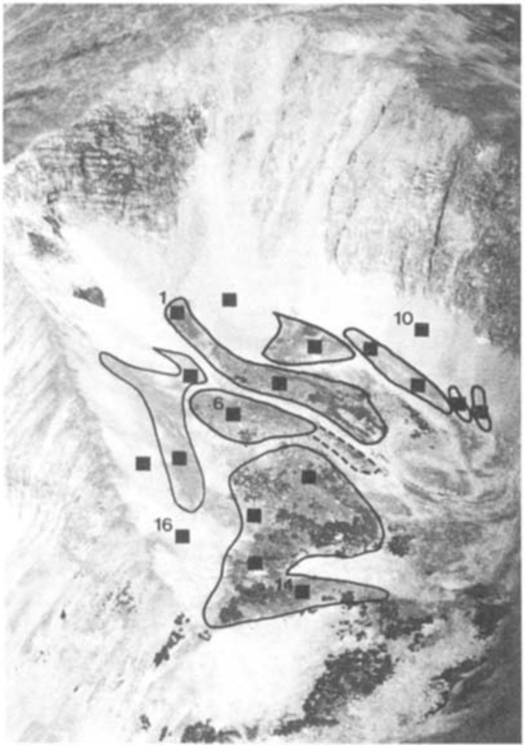

Fig. 2. Air photograph of the surficial deposits in Audubon Cirque. Only five sites are numbered to avoid obscuring details. Comparison should be made with Fig. 3.

dole 1976), and relative weathering data were collected only on boulders of these rock types. Because the cirque is above tree-line today, and probably has been throughout the Late Quaternary (Legg & Baker 1980), the problem of boulder surface rejuvenation by spalling is not present (Birkeland 1973).

Eighteen sites were examined within the cirque (Fig. 2). Preliminary selection of sites was based on tonal and morphological differences revealed in air photographs of the cirque. Field reconnaissance ensured that all sites were located on topographic high points, rather than in possibly waterlogged hollows. Four sites were placed on talus deposits, six on lobate rock glaciers (Wahrhaftig & Cox 1959; Outcalt & Benedict 1965), and eight on moraines (Fig. 3). Rock glacier identification was based solely on morphological criteria, since no information on internal structure or movement was available.

Relative dating methods

Relative dating is based on the assumption that certain weathering parameters are time dependent, and that changes over time may be used to group sites into units of different age (Burke & Birkeland 1979). Relative dating techniques are particularly useful for dating surficial Quaternary sequences in the Colorado Front Range, because there is a scarcity of radiometric dating control. Blackwelder (1931) and Burke & Birkeland (1979) stress the need for a multi-parameter approach to relative dating, since it reduces the possibility of errors in subdivision and correlation of units due to non-temporal variations in any single parameter. This study represents such a multi-parameter approach.

Birkeland (1973), Carroll (1974), and Burke & Birkeland (1979) provide summaries of relative dating methods. However, no completely standard sampling techniques and definitions of weathering criteria have been adopted by all workers. To facilitate comparability between studies, and the replicability of any single study, it is therefore necessary to state explicitly the precise nature of the techniques used.

In this study, data for 13 relative dating parameters were collected at each site (Table 1). Observations at each site were conducted within a 10×10 m square.

(1) Fresh to weathered ratio: At each site 50 boulders greater than 50 cm in diameter are randomly chosen. A weathered boulder is rough to the touch and exhibits single grain mineral relief, whereas a fresh surface is smoother (Birman 1964; Carroll 1974). Two measures of weathering are employed: (a) where more than 10% of the exposed surface of 50 boulders is so weathered; and (b) where more than 50% of the exposed surface of 50 boulders is so weathered.

(2) Pitting: A boulder is pitted if it has one or more closed depressions formed as a result of grain-by-grain disintegration rather than by spalling (Carroll 1974). Three measures of pitting are used: (a) The percentage of pitted boulders in a random sample of 50 boulders greater than 50 cm in diameter; (b) The depth of the largest single pit on each of the first 25 boulders is measured from the pit base to a reconstructed upper surface. The largest pit at each site is recorded in Table 1; and (c) The mean of the maximum pit depth on each of the first 25 boulders is calculated.

(3) Weathering rinds: A rind is a zone of weathered rock, recognized by a discolouration, that parallels the outer surface of boulders (Nelson 1954; Burke & Birkeland 1979). Three weathering rind criteria are used: (a) 25 boulders are broken open and the percentage with weathering rinds is calculated; (b) The thickest single weathering rind at each site is noted; and (c) The mean of 25 rind thicknesses (including the zeros) is calculated. Porter (1975) suggests that although a reasonably constant relationship exists between maximum and mean rind thickness, there is a danger of miscorrelation if either the single largest rind at a site is missed or a stone with an anomalously thick rind is found.

(4) Angularity: Boulders in the cirque are initially angular, and as they weather their edges become more rounded. A visual comparison chart is used to classify 50 boulders according to Powers' (1953) roundness index. Boulders are defined as angular, sub-angular, sub-rounded or rounded.

(5) Surface oxidation: 50 boulders are classified on the basis of surface discolouration. Unoxidized boulders exhibit no surface discolouration, partly oxidized boulders are only partly discoloured, and oxidized boulders show full discolouration.

(6) Lichen cover: The percentage lichen cover (all species) is estimated on 25 boulders at each site, using a visual comparison chart (Locke *et al.* 1980), because Beschel (1961) states that lichen cover increases with time. Two measures are used: (a) The single largest lichen cover percentage is recorded for each site; and (b) The mean percentage cover for 25 boulders is calculated.

(7) Lichen size: Beschel (1961) showed that the diameter of *Rhizocarpon geographicum* (*sensu lato*, due to taxonomic uncertainty) increases with time. The five largest lichens at each 10 × 10 m site are recorded, and the mean largest lichen diameter is calculated (Matthews 1974, 1975). Diameter is measured to the nearest 1 mm along the shortest axis of lichens with long-to-short axis ratios less than 3:2.

Results

Both Birkeland (1973) and Carroll (1974) point out that relative dating techniques are subjective,

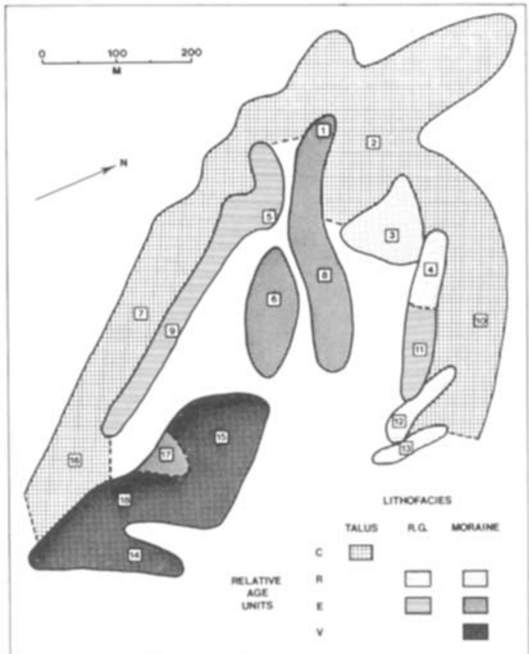

Fig. 3. Map of deposits in Audubon Cirque, showing both relative age and deposit type or lithofacies. Dashed lines represent areas where there is a gradation rather than a clear boundary between different deposits.

and dependent on the definitions used to describe the state of rock weathering. However, Carroll (1974) stresses that different workers can obtain the same relative age differentiation within their data even though their operational definitions, and therefore the absolute values they obtain, may be different. In recognition of this problem, the same observer gathered data on a given relative dating parameter at all sites, in order to standardize definitions throughout the study period.

Cluster Analysis used to group Late Quaternary deposits

Carroll (1974) and Miller (1979) used the computer programs CHARANAL and GRAPH (Andrews & Estabrook 1971) to group field sites that are similar in terms of relative dating criteria. CHARANAL and GRAPH require the use of nominal or ordinal scale data, whereas interval or ratio scale data must be input to the CLUSTR routine used in this study (Davis 1973). Most relative age data are collected at the

Table 1. Relative age data collected for 13 variables at 18 sites in Audubon Cirque, Colorado Front Range. The sites are located in Figs. 2 and 3.

Site	Weathered		Pitting (cm)			Rinds (mm)			Ang.	Ox.	Lichen Cover		Lichen Dia.	Type of	Age
Fig. 2	>10%	>50%	% Pit	Max Pit	Mean Pit	% Rind	Max Rind	Mean Rind	% Ang	% Ox	Max %	Mean %	$\bar{x}$ 5 Largest	Deposit	Unit
1	46	4	10	1	0.2	60	3	0.9	74	4	70	28	39	M	C?
2	44	0	0	0	0	44	3	0.9	80	8	15	4	10	T	C
10	30	20	10	2	0.1	70	3	1.1	92	20	15	5	30	T	C
7	46	42	20	2	0.3	88	6	3.2	86	18	40	9	36	T	C
16	60	55	48	4	0.9	75	5	2.6	48	20	10	3	40	T	C
3	82	54	44	6	1.6	76	12	4.2	36	26	85	46	39	M	R
4	90	76	36	5	1.3	88	10	4.7	84	10	70	35	52	RG	R
13	98	66	62	5	1.6	88	9	4.0	34	30	30	11	48	RG	R
12	96	64	62	4	1.4	76	9	2.9	74	42	60	25	44	RG	R
5	98	76	58	5	1.5	100	14	6.7	78	50	65	41	67	RG	E
6	98	96	88	5	2.3	100	10	5.6	32	10	85	56	56	M	E
8	100	70	74	7	2.0	100	9	4.3	46	12	90	61	41	M	E
11	98	88	80	5	1.7	88	5	2.0	46	45	80	46	70	RG	E
9	100	98	78	8	2.6	100	13	6.1	48	46	95	47	43	RG	E
17	100	94	90	10	4.3	100	14	7.0	40	30	70	43	42	M	E
14	100	100	100	11	4.4	100	18	7.9	12	60	90	60	60	M	V
15	100	100	100	8	4.3	100	17	8.8	16	44	80	59	59	M	V
18	100	100	100	12	4.9	100	15	7.8	20	30	80	53	55	M	V

Note: Maximum and mean pit and rind measurements refer to depths. Ang = Angularity. Ox = Oxidation. T = Talus, M = Moraine RG = Rock Glacier.

interval or ratio scale (e.g. Carroll 1974; Burke & Birkeland 1979; Miller 1979). Therefore, the unquantified loss of information resulting from the need to convert the raw relative age data matrix (Table 1) to the ordinal scale, and the arbitrary choice of ordinal classes, are avoided here. Interval and ratio scale data have the additional advantage of being amenable to Discriminant Analysis, which may be used to test the statistical significance of groupings of sites subjectively derived from CLUSTR. CLUSTR has previously been applied to Quaternary deposits in Baffin Island by Nelson (1978, 1980). However, routines requiring the input of nominal or ordinal scale data are useful for the analysis of relative age data (e.g. moraine morphology) which may be collected at a lower level of measurement (Andrews & Estabrook 1971).

CLUSTR has two options for input to the clustering operation: the matrix of between site distances or the matrix of between site correlations. Davis (1973) suggests that distance matrices usually cluster more successfully than correlation matrices, because they yield higher cophenetic correlations (i.e. lower distortion due to averaging together cluster members and treating them as a single new object). The distance option is therefore used here, similarity between sites being measured as a Euclidean distance coefficient (D), which is computed by the equation:

$$d_{ij} = \sqrt{\frac{\sum_{k=1}^{m} (X_{ik} - X_{jk})^2}{m}} \qquad \text{(Equation 1)}$$

where X_{ik} is the kth component score on object (site) i, X_{jk} is the kth component measured on site j, m is the number of components used in the analysis, and d_{ij} is the distance between sites i and j (Davis 1973).

Prior to computing distance measures, the raw data (Table 1) were standardized as z scores and subject to a Principal Components Analysis (Table 2). Standardization ensures that each variable is weighted equally (with mean zero and variance one), and is required because some variables were measured in percentage terms whereas others were in millimetres and centimetres. Principal Components Analysis is necessary in addition to standardization because each derived component is orthogonal to, and therefore independent of, all other components. Cluster Analysis uses Euclidean distance measures (Equation 1) to form groupings of sites, and if several variables or components are used in the derivation of these clusters it is important that all distances are computed on a uniform coordinate system. Orthogonalization ensures that coordinate axes are 90 degrees from one another in *n* dimensional space. Principal component scores are therefore used as the input to CLUSTR. If the raw data set were standardized using *z* scores, and subject directly to Cluster Analysis without the use of Principal Components Analysis, the results would be unintentionally weighted due to the non-orthogonality of the Euclidean coordinate system.

Principal Components Analysis yielded only one component with an eigenvalue above 1.0 (Table 2), and this component alone is used in subsequent analysis. The reason for ignoring the other components is that they contain less information than any one of the original standardized variables, which all have a variance of 1.0. Nonetheless, this decision is a subjective one on the part of the investigator (Daultrey 1976). Component 1 accounts for 74 % of the information in the raw data set, and therefore 26 % of the statistical information in the original data is lost (Table 2). However, some of this discarded information may be 'noise', and beyond the resolution of the relative dating techniques employed.

Study of the eigenvector loadings (Gould 1967) on the more important components (often the first three; Table 2), aids in interpreting the substantive importance or otherwise of the progressively smaller amounts of information contained in discarded components. The eigenvector loadings on component 1 show that it is an 'average' variable (Johnston 1978), in that loadings on all the original variables (Table 1) are of similar magnitude. Component 2 exhibits high loadings on the two lichen cover measures, whereas component 3 loads high on lichen diameter. Therefore, although component 1 is an average variable, it is weighted towards relative weathering rather than lichenometric evidence simply because 10 of the 13 original variables concern the former. This implies that the subjective decision

Table 2. Eigenvalues and cumulative percentage of relative age data variability explained by each of the first three Principal Components from Principal Components Analysis.

Principal Component	Cumulative proportion of total variance	Eigenvalue
1	74 %	9.56
2	81 %	0.91
3	87 %	0.86

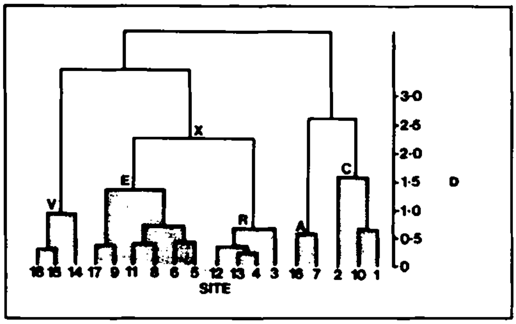

Fig. 4. Dendrogram produced by Cluster Analysis of the scores on the first principal component of relative age data from Audubon Cirque. D is a Euclidean distance coefficient of similarity. C, A, R, E and V represent groups of sites derived by visual inspection of the dendrogram. The pivotal point X is discussed in the text.

about which variables measured in the field to include in quantitative analysis influences the component structure yielded from Principal Components Analysis. The justification for including more than one measure of a parameter, such as pit depth or lichen cover, is that a single (often maximum) value for any parameter at a given site may be unrepresentative, and should be mediated by measures on more than one boulder or lichen at any site (e.g. Matthews 1973, 1974).

Scores on the first principal component for each site were input to CLUSTR, which groups sites into a hierarchy using a weighted pair-group sorting strategy with arithmetic averages (Sokal & Sneath 1963). Sites with the highest mutual similarity are placed closest together. Then clusters of sites are linked with other groups they most closely resemble, and so on until all sites have been placed into a complete classification scheme (Davis 1973). The dendrogram or tree diagram is the most common way of displaying the results of CLUSTR (Fig. 4). The smaller the value of the distance coefficient (Equation 1), the greater the similarity between a pair of sites.

However, dendrograms must be interpreted with care because initially they represent no more than a topological network of similarities. For example, in Fig. 4 sites 3, 4, 13 and 12 may be pivoted about point X to fit between sites 17 and 14, with the maintenance of topological equivalence. Reference to the original relative age data (Table 1) has therefore been made to ensure that the *clusters* of sites identified by shading in Fig. 4 represent a relative age contin-

Table 3. Two *a priori* groupings of sites to be tested for statistical significance using Discriminant Analysis. The third grouping represents the relative age units actually defined in this study, and mapped in Fig. 3. Where units C and A are combined, the new group is referred to as C.

	Units of increasing relative age				
	C	A	R	E	V
A priori site classification 1	1(?), 2, 10	16, 7	3, 4, 13, 12	5, 6, 8 11, 9, 17	14, 15, 18
A priori site classification 2	1(?), 12, 10, 16, 7		3, 4, 13, 12	5, 6, 8 11, 9, 17	14, 15, 18
Derived relative age units	1(?), 12, 10, 16, 7		3, 4, 13, 12	5, 6, 8 11, 9, 17	14, 15, 18

uum from youngest on the right to oldest on the left. Even so, the position of sites *within* a single cluster remains merely topological because at this scale the interpretation of relative age is in danger of going beyond the resolution of the raw data (Birkeland *et al.* 1979).

Discriminant Analysis used to test the significance of groupings of sites

Davis (1973) points out that no direct tests of statistical significance are available for this method of clustering, and clusters are therefore defined subjectively. Visual inspection of the dendrogram based on scores on the first principal component (Fig. 4) indicates the presence of several groups of sites (Table 3). The statistical significance of these groups may be assessed using Discriminant Analysis (Johnston 1978). Discriminant Analysis requires an *a priori* classification of sites into groups, which in this study are derived from subjective study of the dendrogram. The groupings calculated from Discriminant Analysis are then compared with the *a priori* clusters, and the statistical significance of the subjective groupings is assessed. However, a general problem of significance testing is present: that is, the *choice of significance level* (α)at which a group of sites is accepted as meaningful. Here, the significance level $\alpha = 0.05$ is conventionally but arbitrarily selected. Discriminant Analysis also indicates the number of 'misclassified' sites in the *a priori* classification relative to the groups defined through Discriminant Analysis.

Discriminant Analysis on the first principal component alone is similar to Analysis of Variance (ANOVA). Discriminant Analysis rather than ANOVA is preferred in this study because it is more widely applicable to classificatory problems in that, unlike ANOVA, it can be used to test the significance of groupings when more than two variables are input to the analysis. This situation may occur in relative dating studies concerning periods longer than the Holocene, where some relative age variables would be expected to lose their sensitivity, while others would become relevant over longer time intervals. In this situation several principal components would probably be included in subsequent analysis, and only Discriminant Analysis could be used to test for significance. This work therefore provides a brief introduction to Discriminant Analysis in the context of Late Quaternary relative dating studies.

Discriminant Analysis was performed on two *a priori* groupings of the sites in Audubon Cirque (Table 3). Classification 1 is based solely on visual perception of clusters within the dendrogram, whereas classification 2 takes into account pre-existing work conducted in nearby areas of the Front Range, specifically that of Benedict (1967, 1968, 1973). The results of both Discriminant Analyses produced no misclassified sites, and all units were significantly different from one another at $\alpha = 0.05$. Site 1 is included in unit C in Tables 1 and 3, but is not considered in subsequent discussion because it is unusually young when compared with other moraines. This is because it is the site nearest to the cirque headwall, where talus is transgressing an older substrate, thereby giving an anomalously young age to the moraine facies. Site 1 is on the same moraine ridge as site 8 (Fig. 2), and is therefore mapped as unit E in age (Fig. 3).

Interpretation and relative age assignments

Birkeland *et al.* (1979) suggest the use of letter designations in order to keep relative age unit nomenclature informal (e.g. Crandell 1967), and this policy is followed here. It is clear from the relative age data in Table 1 and the field relationships between sites (Figs. 2, 3) that unit C is the youngest unit and unit V the oldest. Intermediate units are also arranged in a relative age continuum (Table 3) after reference to the raw data.

A fundamental problem in interpreting the output of Cluster and Discriminant Analysis is that of the number of units present in the data. Discriminant Analysis goes some way towards validating the groupings subjectively derived from CLUSTR through its ability to test for *statistical* significance, but there remains the difficulty of whether four or five relative age units are *substantively* meaningful. At this stage interpretation is necessarily subjective, and is based largely on a knowledge of local field relationships and the sensitivity of the relative dating methods used. Pre-existing literature concerning Front Range Quaternary stratigraphy also provides information, but here there is an obvious danger of perpetuating the existing model of events (e.g. Benedict 1973).

The sites comprising units C and A (Table 3), excluding site 1, are all located on talus (Figs. 2, 3). Sites 2 and 10 (Unit C) are south facing, whereas sites 7 and 16 (Unit A) face north. Reference to Table 1 shows that relative dating variables are 'younger' on the former pair of sites. However, it is possible that south facing sites on talus stabilize more slowly than those facing north because of greater meltwater availability, and thus the apparent chronological differences between units C and A (Table 3) may be no more than the effect of aspect. Units C and A are therefore combined, and will be referred to as unit C from now on. There is no statistical justification for this decision, which is based largely on the similarity in lithofacies between sites 2, 10, 7 and 16 (Fig. 3). This interpretation is not extended to units R and E even though all four sites in unit R face south (Fig. 3), because the resolution of relative dating techniques decreases with increasing age, implying that progressively greater time intervals are required to produce observable differences in weathering and lichenometric parameters. It is unlikely that time differences between deposition and stabilization due to aspect are of sufficient length to produce the observed differences between units R and E (which are tentatively correlated with Audubon and Early Neoglacial aged deposits respectively in a later section).

Table 4. Stratigraphic nomenclature in the Colorado Front Range.

Years B.P.	Stratigraphic nomenclature		
	Gannett Peak	Arapaho Peak	Gannett Peak
0			
1000			
2000	Temple Lake 'b'	Arikaree	Audubon
3000			
4000	Temple Lake 'a'	Triple Lakes	Early Neoglacial
5000			
6000			
7000			
8000			
9000	Pinedale		
10,000		Satanta Peak	Late Pinedale
	Richmond (1960)	Benedict (1968)	This study; Meierding & Birkeland (1980)

Units R, E and V are therefore retained in the form derived from Cluster and Discriminant Analysis, and together with unit C provide a four group classification of relative age units within Audubon Cirque (Table 3; Fig. 3). The definition of four groups of sites leads to the generation of the hypothesis that these units may be comparable with those found by Benedict (1967, 1968, 1973) in Arapaho Cirque, 9 km to the south in the Front Range. This hypothesis will be tentatively tested through correlation with evidence from other glaciated areas of Colorado.

Stratigraphic nomenclature in the Front Range

A varied stratigraphic nomenclature has been used for Holocene deposits in the Front Range (Table 4). Richmond (1960) applied the nomenclature of the Wind River Range, Wyoming, to the Holocene deposits in Rocky Mountain National Park. Benedict (1967, 1968, 1973) preferred to adopt local names for Front Range units because of the problems involved in long distance correlation.

Since Benedict's (1967, 1968, 1973) work the stratigraphic nomenclature has evolved considerably. The term 'Arikaree' was pre-empted in the

Table 5. Comparison between the percentage of boulders with less than 10 % of their exposed surface weathered for different relative age units at Audubon Cirque (this study) and Arikaree Cirque (Carroll 1974).

Unit	Audubon Cirque	Arikaree Cirque
Unit C/G.P.	30– 60	0
Units R,E/ENG	82–100	19–82
Unit V/LPD	100	77–94

Note: G.P. = Gannett Peak, ENG = Early Neoglacial, LPD = Late Pinedale.

literature, and Mahaney (1972) suggested the alternative name 'Audubon'. The type Triple Lakes till could be older than Benedict (1968, 1973) originally thought, perhaps as old as the type Satanta Peak (Birkeland & Shroba 1974; Davis & Waterman 1979). Because of this problem deposits considered to be 3000–5000 years old are termed Early Neoglacial (Table 4; Meierding & Birkeland 1980). The name Gannett Peak (Richmond 1960) is kept for the youngest Holocene deposits, and deposits about 10,000 years old are named Late Pinedale.

Absolute age assignments

The nearest radiocarbon date to Audubon Cirque comes from the basal peat in a bog near Mitchell Lake, 0.6 km southeast of Audubon Cirque (Fig. 1). The deposit gives a minimum date of 7690 ± 115 years B.P. for local ice-free conditions (Madole 1976). However, Meierding & Birkeland (1980) point out that interpretations have varied as to how closely a radiocarbon assay dates an associated deposit. Because no bracketing dates are available, this date provides no maximum limit to the time of Late Pinedale ice recession from the Mitchell-Audubon area.

The assignment of absolute ages to the relative age units defined above must therefore rely on correlation with other areas within Colorado (e.g. Benedict 1968, 1973; Birkeland 1973; Mahaney 1973a; Miller 1973; Williams 1973; Carroll 1974). However, correlation with other relative weathering and lichenometric studies is not easy, because the precise techniques of investigation are often not reported in detail (Mahaney 1973b). This is compounded by a lack of consistency in the definition of ostensibly similar surface weathering criteria. For these reasons, and because of variations in lithology and microclimate, correlation is necessarily difficult, and precise correspondence should not be expected.

The following sections test the hypothesis that unit C is equivalent to Gannett Peak deposits (100–300 years B.P.), unit R is similar to Audubon deposits (950–1850 years B.P.), unit E correlates with Early Neoglacial deposits (3000–5000 years B.P.) and unit V with Late Pinedale deposits (about 10,000 years B.P.). Estimates of absolute ages are derived from Benedict (1968, 1973) and Birkeland (1973).

Correlation of relative weathering data

(1) Fresh to weathered ratio: Comparison with Carroll's (1974) data from Arikaree Cirque (Table 5) shows that consistently higher percentages of weathered boulders are found in Audubon Cirque. The differences become less significant with increasing deposit age, and may be due either to differences in the operational definition of boulder surface weathering, or to the way rocks of different lithology weather.

(2) Pitting: Carroll's (1974) measurements of pit depth fit reasonably well with those of the present study, although again Audubon Cirque values are higher (Table 6). Williams' (1973) values

Table 6. Comparison of pit depths for different relative age units at three localities in the Colorado Front Range.

Unit	Depth of pitting (cm)			
	This study Max.pit	Carroll (1974) Max.pit	Williams (1973) Unspecified	This study Mean Pit
Unit C/ G.P.	0– 4	0	–	0–0.9
Unit R / AUD	4– 6	0– 5.2	0.3–0.7	1.3–1.6
Unit E / ENG	5–10		0.6–1.4	1.5–4.3
Unit V / LPD	8–12	3.3–10.5	0.7–1.4	4.3–4.9

Note: G.P. = Gannett Peak, AUD = Audubon, ENG = Early Neoglacial, LPD = Late Pinedale.

Table 7. Comparison of maximum weathering rind thicknesses for different age units at various localities in Colorado.

Unit	Maximum rind thickness (mm)					
	This study Audubon Cirque	Birkeland (1973) Ferguson Ranch W.	Carroll (1974) Arikaree Cirque	Carroll (1974) Mitchell Lake	Mahaney (1973a) Fourth of July Cirque	Williams (1973)
Unit C / G.P.	3–6	-	0	-	0	-
Unit R / AUD	9–12	-	0–12	-	2–3	-
Unit E / ENG	5–14	2–14		-	5–10	0–3
Unit V / LPD	15–18	6–45	0–21	14	-	0–9

Note: G.P. = Gannett Peak, AUD = Audubon, ENG = Early Neoglacial, LPD = Late Pinedale.

Table 8. Comparison between the percentage of angular boulders for different relative age units at Audubon Cirque (this study) and Arikaree Cirque (Carroll 1974).

Unit	Audubon C. % Angular	Arikaree C. % Angular
Unit C / G.P.	48–92	100
Units R,E / ENG	32–84	10–68
Unit V / LPD	12–20	0–4

Note: G.P. = Gannett Peak, ENG = Early Neoglacial, LPD = Late Pinedale.

for pitting (maximum or mean depth unspecified) are much lower than estimates of mean pit depth for Audubon Cirque.

(3) Weathering rinds: The wide scatter of values in Table 7 makes correlation of rind data difficult. Significantly, Carroll (1974) obtains a rind value comparable with those in Audubon Cirque for a Late Pinedale deposit near Mitchell Lake (Fig. 1). Both Carroll's (1974) Arikaree Cirque data and those of Birkeland (1973) from Mt. Sopris are similar in magnitude to the rind thicknesses in this study. Mahaney (1973a) and Williams (1973), working in Fourth of July Cirque, obtain much lower rind thickness values, but even their results for the same cirque are not compatible (Table 7).

(4) Angularity: Carroll's (1974) data show less angularity than those from Audubon Cirque for any assumed age, but this could reflect difficulties in the interpretation of Powers' (1953) charts (Table 8).

(5) Oxidation: Little data are available with which to compare the values from Audubon Cirque.

Correlation of lichenometric evidence

Lichen size and percentage cover are discussed with reference to units C, R and E only, because over periods longer than 3000–5000 years lichen growth tends to slow considerably and make distinctions between deposits of different age difficult (Locke *et al.* 1980).

(1) Percentage lichen cover: Audubon Cirque data fits reasonably well with results from a number of studies in the Colorado Rockies (Table 9). Again, the discrepancies between Mahaney (1973a) and Williams (1973) illustrate the problems involving the operational definitions adopted by different workers in the same cirque.

(2) Lichen size: Lichen diameters from several studies correlate relatively well (Table 10). Carroll (1974) discusses the problem of snow kill of

Table 9. Comparison of mean percentage lichen cover for different age units at various sites in Colorado.

Unit	Mean percentage lichen cover					
	This study	Benedict (1968)	Miller (1973)	Mahaney (1973a)	Williams (1973)	Carroll (1974)
Unit C / G.P.	4–9	0–5	0–5	0–5	5	0
Unit R / AUD	11–46	10–40	10–65	25–40	50	15–30
Unit E / ENG	41–61	80–95	75–90	50–80	60	

Note : G.P. = Gannett Peak, AUD = Audubon, ENG = Early Neoglacial.

Table 10. Comparison of largest lichen diameters for different age units at various localities in Colorado.

Unit	Largest lichen diameter (mm)					
	This study $\bar{x}$ 5 largest	Benedict (1968) 1 largest	Miller (1973) 1 largest	Mahaney (1973a) 1 largest	Williams (1973) 1 largest	Carroll (1974) 1 largest
Unit C / G.P.	29	20	20	15	35	0
Unit R / AUD	39–52	42–71	41–77	41	60	13–75
Unit E / ENG	41–70	107 +	110 +	102–125	80–85	

Note: G.P. = Gannett Peak, AUD = Audubon, ENG = Early Neoglacial.

lichens, and this may explain the relatively low values for unit E lichen diameters at Audubon Cirque. If snow kill has affected lichen growth in the present study, direct interpolation of absolute age from Benedict's (1967) calibrated lichen growth curve will not be very meaningful.

Interpretation

The hypothesis proposed above, concerning a correlation between the relative age units in Audubon Cirque and chronologies elsewhere in Colorado, is accepted with the following caveats. First, the difficulties involved in correlating the results of several relative dating studies means that the above discussion can provide no more than tentative conclusions. Second, relative age units are specifically defined using a nomenclature unrelated to absolute age in order to stress the uncertain nature of absolute age designations, and to provide flexibility if additional absolute age data should become available.

Conclusions

Three conclusions may be drawn from the above study:

(1) Cluster and Discriminant Analysis of relative age data, which have been standardized and orthogonalized using Principal Components Analysis, are useful multivariate statistical tools for differentiating groups of sites of similar relative age. Subjectivity, nonetheless, remains in the interpretation of these analyses.

(2) Precise definition of field methods, including sample size and area, are important if comparability between studies and data reproducibility are required. A multi-parameter approach is useful due to the possible presence of unusual values for any single relative dating criterion.

(3) Four relative age units are defined for Audubon Cirque. A tentative correlation of these units with the absolute age units defined by Benedict (1968, 1973) is suggested. However, correlation with other work in the Colorado Front Range is difficult, because of the large range of values within each data set, the different operational definitions in use, and variations in microclimate and lithology between areas.

Acknowledgements. – Dr. Peter W. Birkeland introduced me to relative dating techniques, and kindly read and criticized several drafts of this paper. Drs. John T. Andrews and Nel Caine also commented on an early manuscript. Nel Caine supplied the photograph of Audubon Cirque. I am grateful to Barbara Buffington, Bruce Carey and Michael McAlpine for their assistance in collecting some of the data.

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