



Capital Structure and Debt Priority

Sami Attaoui and Patrice Poncet*

We study a defaultable firm's debt priority structure in a simple structural model where the firm issues senior and junior bonds and is subject to both liquidity and solvency risks. Assuming that the absolute priority rule prevails and that liquidation is immediate upon default, we determine the firm's interior optimal priority structure along with its optimal capital structure. We also obtain closed-form solutions for the market values of the firm's debt and equity. We find that the magnitude of the spread differential between junior and senior bond yields is positively, but not linearly related to the total debt level and the riskiness of assets. Finally, we provide an in-depth analysis of probabilities of default and the term structure of credit spreads.

In a seminal work, Merton (1974) derived the first structural model for corporate debt pricing. Relying on the contributions of Black and Scholes (1973) and Merton (1973), he obtained the price of a zero coupon defaultable bond in the context of a terminal default boundary and investigated the behavior of the ensuing credit spread term structure. Abundant literature has since been developed to investigate the pricing of corporate bonds in more general frameworks and, more broadly, to analyze capital structure decisions.¹

This is an important area of study. The simplifying assumption of a homogeneous debt structure made in the prior literature contradicts firms' actual practice of issuing debt of different priorities. For instance, Barclay and Smith (1995) find that most of the firms in their sample rely on senior and junior (subordinated) classes of bonds. Billett, King, and Mauer (2007) confirm that subordinated debt accounts for approximately 25% of the total debt issuance in their sample.² Rauh and Sufi (2010) observe that rated firms simultaneously issue debt with different types and priorities and that the number of debt classes depends markedly upon these firms' credit standing. Additionally, in a large data set of rated and unrated publicly listed firms, Colla, Ippolito, and Li (2013) report that the sample mean ratios for senior and subordinated bonds over total debt are approximately 40% and 10%, respectively. Moreover, a growing body of theoretical research acknowledges and attempts to explain debt heterogeneity and priority structure. One strong argument is that credit quality is a major force driving the firm's debt structure. For example, high quality firms can borrow directly from arm's length (junior) creditors and reduce the costs of (senior) bank debt associated with monitoring. Slightly more generally, the type and priority

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**Sami Attaoui is an Associate Professor of Finance in the Economics and Finance Department, at the Rouen Business School in Mont Saint Aignan, France. Patrice Poncet is a Distinguished Professor of Finance in the Finance Department at ESSEC Business School in Cergy-Pontoise, France.*

¹ See, for instance, Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), Briys and de Varenne (1997), Collin-Dufresne and Goldstein (2001), Leland (1994), Leland and Toft (1996), Goldstein, Ju, and Leland (2001), Ju et al. (2005) and Ju and Ou-Yang (2006). See also useful surveys by Bielecki and Rutkowski (2002), Uhrig-Homburg (2002), and François (2005).

² See their Table II on p. 705.

of debt is designed optimally to mitigate managerial and creditor agency issues (Diamond, 1991; Bolton and Freixas, 2000). The existence of senior and junior debt has also been attributed to covenants that prohibit firms from issuing further senior debt. However, recent empirical evidence contradicts this argument. For instance, Billet, King, and Mauer (2007) report only a few cases of debt issuance restriction in their sample. Senior debt issuance including restrictions on issuing further senior debt or subordinated debt is only observed in 0.4% and 0.7% of cases, respectively. When subordinated debt is issued, these percentages increase but remain marginal (4.5% and 22.4%, respectively).³ Overall, both the empirical evidence and the theoretical argument above make the optimality of the debt mixture outside of financial distress a relevant problem to study. In our paper, we consider two classes of bonds, senior and junior. We contribute to the literature in four related ways.

Our first contribution is an *interior* optimal priority structure. This results from the trade-off between bankruptcy costs and tax deductions. For a given level of total debt, increasing the proportion of (more costly) junior bonds increases the firm's probability of default hence the present value of bankruptcy costs, but it also increases the present value of tax deductions the larger coupons provide. The optimal debt priority structure is reached when these two effects cancel out at the margin.

Our second contribution is to account explicitly for two sources of risk. We are motivated by Davydenko (2010), who investigates the nature of the default boundary for distressed firms empirically. He finds that default is triggered either by a negative net worth at some principal payment date (the firm is insolvent) or by a cash flow shortage (the firm is illiquid). Thus, our model embeds these two risks in the following manner. First, default (or bankruptcy, the two terms being used in this paper interchangeably) occurs if at any date the firm does not generate enough cash flow to fund its debt coupons (*liquidity risk*). Then, provided that bankruptcy due to illiquidity has not occurred before the firm's debt maturity, default may still occur at maturity if the firm's assets are not sufficient to back the full amount of the debt principals (*solvency risk*). This distinction between liquidity and solvency risks has also been emphasized in theoretical models by Gryglewicz (2011) and von Thadden, Berglöf, and Roland (2010).⁴ The existence of these two kinds of risk has considerable bearing on default probabilities and, consequently, on bond values and yield spreads and on the firm's optimal financing policy.⁵

Our next contribution is the simultaneous analyses of the optimal debt priority mix and the optimal capital structure. The valuation of senior and junior debt has been investigated by Black and Cox (1976) and Geske (1977). Whereas Black and Cox (1976) analyze zero coupon bonds maturing on the same date, Geske's (1977) model allows for bonds with different maturities. However, these papers are silent as to the optimality of the exogenous priority structure. Our work is also related to two recent studies by Hackbarth, Hennessy, and Leland (2007) and Hackbarth and Mauer (2012), but differs in several respects. First, in both papers, the debt is infinitely lived. In contrast, our debts are (relatively) short-lived (from 2 to 20 years in the simulations) so that we can highlight the role of debt maturity in determining both the optimal priority structure and the optimal leverage. Second, Hackbarth, Hennessy, and Leland

³ See their Table III on p. 707.

⁴ For example, von Thadden, Berglöf, and Roland (2010) write "firms default either because they lack liquidity, but are fundamentally sound or because they have liquidity but little long-term value" (p. 2651).

⁵ In an earnings before interest and taxes (EBIT)-based setting with a homogeneous debt, Gryglewicz (2011) determines that the interaction between liquidity and solvency has an impact on leverage policies and credit spreads.

(2007) posit a strategic default setting to investigate the debt priority structure between a bank debt and a market debt, while Hackbarth and Mauer (2012) analyze the role that priority debt structure plays in resolving conflicts between stakeholders over investment policy. We abstract from these complications and assume immediate liquidation in the case of bankruptcy. In particular, both liquidity and solvency defaults are triggered by bond covenants and, as such, are considered to be passive, as opposed to strategic, defaults (Fan and Sundaresan, 2000). According to the absolute priority rule under Chapter 7 liquidation, junior debtholders are paid some amount upon default only if senior bondholders have received full payment. Empirical results obtained by Bris, Welch, and Zhu (2006) confirm the absence of violations of the absolute priority rule under Chapter 7. A final and crucial distinction from both Hackbarth, Hennessy, and Leland (2007) and Hackbarth and Mauer (2012) is that their firms face solvency risk only.

Our model predicts that low leverage firms rely more on junior debt than do more indebted firms. In addition, high leverage firms increase their proportion of senior debt as their assets become riskier. These results follow from the trade-off between the tax benefits the firm receives from the deduction of larger coupons on its junior debt and the expected bankruptcy costs generated by a higher probability of intermediate or terminal default. In addition, a weakly levered firm relies heavily on junior bonds, with a smaller role played by either the volatility of its operational income or the maturity of its debt. On the contrary, the high levered firm's optimal policy depends upon the riskiness of its assets. If the latter is low, the proportion of senior debt becomes significant only at long maturities and then rises sharply as the debt maturity increases even more. If asset return volatility is high, the proportion of senior debt becomes significant at much shorter maturities and rockets to 100% for maturities that are even slightly longer. Additionally, we obtain realistic values for the firm's leverage ratio, measured as D/V where D is the bonds' market value and V is the firm's market value. The ratio ranges from 49.9% (for very short debt maturity and low asset volatility) to 20.6% (for long-term debt and high asset volatility). For instance, D/V may be as small as 27.7% for a 30% asset volatility and a 10-year bond maturity. This result is consistent, in particular, with the prediction of Ju, Parrino, Potoshman, and Weisbach (2005) and the empirical findings of Faulkender and Petersen (2006). To achieve this optimal leverage, the firm relies essentially on junior debt to take full advantage of the tax deductions. The firm's optimal leverage decreases with debt maturity and asset return volatility, as the probability of premature or terminal default increases on both counts.

Finally, we contribute to the literature by determining the default probabilities for senior and junior debt and by analyzing the ensuing senior and junior credit spreads. Credit spreads for senior bonds range from zero to 52 basis points (bps) and widen as either the volatility of the assets or the proportion of the assets financed through debt increases. Credit spreads for junior bonds are naturally much higher and may reach 600 bps for highly volatile and strongly levered firms. We find that the spread differential is also positively related to the total amount of debt and the riskiness of the assets. These relationships, however, are not linear. We attribute this result to the differences in the expected recovery rates of the senior and junior bonds that stem from the debt priority structure, the level of leverage, and the type of default risk that may materialize.

The remainder of the paper is organized as follows. Section I introduces our model setting and assumptions. Section II provides the pricing formulas for both senior and junior bonds. Section III derives the firm and equity values. In Section IV, we perform a numerical analysis of the debt priority structure, the optimal capital structure, the debt values and credit spreads, and the probabilities of default. Section V provides our conclusions.

I. Model Setting and Assumptions

This section describes the model and discusses its main assumptions.

Assumption 1. *Uncertainty in our economy is formalized by the filtered complete probability space $(\Omega, \mathbb{F} = \{\mathcal{F}_t, 0 \leq t < \infty\}, \mathcal{F}, \mathbf{P})$.⁶ Trading takes place continuously in a dynamically complete financial market. Consequently, there is a unique probability measure, \mathbf{Q} , equivalent to the physical probability measure \mathbf{P} (Harrison and Kreps, 1979) under which discounted price processes are martingales. Thus, all prices will be obtained by using the risk-neutral measure \mathbf{Q} .*

Assumption 2. *As in Merton (1974), Black and Cox (1976), and Leland and Toft (1996), under the risk-neutral measure \mathbf{Q} , the pre-tax market value of the firm's assets, $A(t)$, obeys the following dynamics:*

$$dA(t) = A(t)[(r - \eta)dt + \sigma dW(t)], \quad A(0) = A > 0, \quad (1)$$

where r is the constant riskless rate of interest, η is the constant fraction of value paid out to all security holders and thus includes dividends and coupons, σ is the constant volatility of asset returns, and W is a standard Wiener process under measure \mathbf{Q} . As in \mathbf{Q} similar studies, η is not affected by changes in the debt/equity ratio.

Assuming a constant interest rate is a simplification motivated by results of Ju and Ou-Yang (2006), who find that the impact of stochastic interest rates is negligible, provided that the value retained for the riskless rate is its long-term average.

Assumption 3. *The firm pays income taxes at rate τ . The initial, pre-tax value of the firm's assets is equal to $A(0)$, denoted by A for simplicity. A is assumed to be strictly positive and can be interpreted as the initial pre-tax value of the (benchmark) all-equity financed firm.*

Assumption 4. *The firm's capital structure is currently composed of two coupon bonds and equity. The two bonds, although having the same finite maturity T , have different repayment priorities. The senior (high priority) bond has a nominal (principal) value P_S and a coupon rate c_S . The junior (or subordinated) bond has a nominal value P_J and a coupon rate c_J . Thus, the total debt principal is $P = P_S + P_J$. At time 0, the value of the firm's assets, A is larger than the total debt principal P .*

In the sequel, we will use for ease of exposition the debt priority parameter ω . The latter is the relative proportion of junior debt defined by $\omega = P_J/P_S$. At maturity T , the senior and junior bondholders receive the principals P_S and P_J , respectively, if two conditions are met. First, bankruptcy has not occurred before T (see Assumption 6 below). Additionally, the after-tax market value of existing assets at time T is large enough $((1 - \tau)A(T) \geq P)$. The following two assumptions provide the necessary details.

Assumption 5. *In the event that the first condition, but not the second is met, that is, $(1 - \tau)A(T) < P$, the firm is bankrupt at time T . This reflects the firm's solvency risk. A constant fraction γ of the after-tax assets' value $(1 - \tau)A(T)$ is lost as bankruptcy or liquidation costs. Therefore, the bondholders receive as final repayment $(1 - \gamma)(1 - \tau)A(T) \ll P$. In this case, the senior bondholders will receive the minimum between P_S and $(1 - \gamma)(1 - \tau)A(T)$. The junior*

⁶ Ω is a finite set, \mathcal{F} denotes a tribe, \mathbf{P} represents the physical probability measure and \mathcal{F}_t is the augmented filtration generated by $W(t)$ (see Appendix A in Duffie (2001) or Chapter 1 in Protter (2004) for further details).

bondholders will receive either a positive amount if $(1 - \gamma)(1 - \tau)A(T) - P_S > 0$, or nothing.⁷ According to the absolute priority rule, shareholders receive nothing.⁸

This assumption is consistent, in particular, with the empirical evidence documented by Bris, Welch, and Zhu (2006), who report no violations of the absolute priority rule under Chapter 7 liquidations.

Assumption 6. *If the firm defaults at any time $t \leq T$ on a coupon payment, it is not allowed to raise equity capital or sell assets, which is consistent with keeping η constant in Equation (1).⁹ Upon default, the bondholders file for the firm's bankruptcy, take control, and liquidate the assets at their market value less taxes and bankruptcy costs. Consequently, they receive the remaining value $(1 - \gamma)(1 - \tau)A(t)$ and share it according to the priority rule. This reflects the firm's liquidity risk.*

Thus, default on both classes of debt may be triggered by the nonpayment of either the senior or the junior coupon. This assumption is in line with the empirical evidence reported by Billett, King, and Mauer (2007) that cross-default provisions are found in 51% of sampled debt issues.¹⁰

Formally, the prior-to-maturity (or premature) default occurs when:

$$A(t) < P_S \frac{c_S + c_J \omega}{\eta} \equiv A_b, \quad (2)$$

where A_b is the default boundary. Since the firm is not initially bankrupt, the initial value of its assets is larger than the boundary ($A > A_b$). Therefore, bondholders file for bankruptcy if $A(t)$ hits A_b from above. We denote by t_b the first passage time of the process $A(t)$ through A_b , that is:

$$t_b = \inf \{t \geq 0 : A(t) \leq A_b\}. \quad (3)$$

Our assumption of the firm's immediate liquidation upon default is consistent with Chapter 7 of the US Bankruptcy Code. It is supported by the evidence reported by Bris, Ravid, and Sverdløve (2010) that 56% of the sampled firms that filed for Chapter 7 bankruptcy had both senior and junior debts. Also note that in Ju and Ou-Yang's (2006) model, the firm defaults (before the debt maturity T) the first time the expected time- T value of its assets becomes smaller than the debt principal P .¹¹ Here, by contrast, the firm defaults (before maturity) the first time its value hits from above the boundary A_b , which is higher or lower than the principal value of the senior debt P_S , depending upon the value of the $\frac{c_S + c_J \omega}{\eta}$ ratio. This point is further discussed in Section II below.

II. Corporate Bond Pricing

Upon default, the senior and junior bondholders receive various amounts depending upon the firm's predicament. The latter is dictated by the value of the senior debt relative to the remaining

⁷ Therefore, the junior bondholders are entitled to the option-like payoff $\max(0, (1 - \gamma)(1 - \tau)A(T) - P_S)$.

⁸ It is not difficult to accommodate the case where the absolute priority rule would not be enforced and the stockholders would receive part of the assets' remaining value.

⁹ As noted by Leland and Toft (1996), bond covenants often preclude the liquidation of existing assets.

¹⁰ See their Table III on p. 707.

¹¹ Another major difference is that their capital structure includes only a single (senior) type of debt.

value of the firm's assets and by its value relative to total debt. This leads us to four possible cases.

Case 1: $(1 - \gamma)(1 - \tau)A_b \leq P_S$

In the event of a coupon default, the senior bondholders receive, in agreement with the absolute priority rule, the total amount $(1 - \gamma)(1 - \tau)A_b$, and the junior bondholders are left with nothing. At maturity T , if the default threshold has not been reached before, the firm lives on if the after-tax total value of its assets is larger than the total principal repayment P , and is declared bankrupt otherwise. Therefore, we must distinguish between two sub-cases: $P_S \leq (1 - \gamma)P$ and $P_S > (1 - \gamma)P$.

Sub-case 1.1: $P_S \leq (1 - \gamma)P$

In this situation, three events can occur at maturity T . First, the after-tax value of assets $(1 - \tau)A(T)$ is larger than P implying no default on the firm's part. The total debt principal is then fully paid back to the creditors. Second, the after-tax value of assets is larger than $\frac{P_S}{1 - \gamma}$, but smaller than P . The firm is then bankrupt, but the senior bondholders still receive the full principal amount P_S , and the junior bondholders are paid the difference $(1 - \tau)(1 - \gamma)A(T) - P_S$. Third, $(1 - \tau)A(T)$ is smaller than $\frac{P_S}{1 - \gamma}$, the worst case of bankruptcy. Senior debtholders receive the value of the remaining assets $(1 - \tau)(1 - \gamma)A(T)$. Junior creditors will receive nothing.

The value of each bond is the expectation, under the risk-neutral measure \mathbf{Q} , of the discounted stream of cash flows on which the holder has a claim. Accordingly, the time-0 values of the senior and junior debts are given, respectively, by:

$$\begin{aligned} D_S^{(1,1)}(0, T) = & c_S P_S \int_0^T \mathbb{E}(e^{-ru} 1_{t_b > u}) du + (1 - \tau)(1 - \gamma) A_b \int_0^T \mathbb{E}(e^{-ru} 1_{t_b \in du}) du \\ & + e^{-rT} P_S \mathbb{E}(1_{t_b > T} 1_{(1-\tau)A(T) \geq P}) + e^{-rT} P_S \mathbb{E}(1_{t_b > T} 1_{\frac{P_S}{1-\gamma} \leq (1-\tau)A(T) < P}) \\ & + e^{-rT} (1 - \tau)(1 - \gamma) \mathbb{E}(A(T) 1_{t_b > T} 1_{(1-\tau)A(T) < \frac{P_S}{1-\gamma}}), \end{aligned} \quad (4)$$

and

$$\begin{aligned} D_J^{(1,1)}(0, T) = & c_J P_J \int_0^T e^{-ru} \mathbb{E}(1_{t_b > u}) du + e^{-rT} P_J \mathbb{E}(1_{t_b > T} 1_{(1-\tau)A(T) \geq P}) \\ & + e^{-rT} \mathbb{E}([(1 - \tau)(1 - \gamma)A(T) - P_S] 1_{t_b > T} 1_{\frac{P_S}{1-\gamma} < (1-\tau)A(T) < P}), \end{aligned} \quad (5)$$

where \mathbb{E} denotes the time-0 conditional expectation under \mathbf{Q} and $\mathbf{1}$ is the indicator function.

Sub-case 1.2: $P_S > (1 - \gamma)P$

In this case, only two situations are possible. Either $(1 - \tau)A(T)$ is equal to or larger than P (no default), and all bondholders are paid their respective principal amounts or $(1 - \tau)A(T)$ is smaller than P , in which event the firm defaults and P_S is larger than $(1 - \gamma)(1 - \tau)A(T)$. Accordingly, the senior bondholders receive the total amount $(1 - \tau)(1 - \gamma)A(T)$, and the junior debtholders get nothing. Therefore, the time-0 values of the senior and junior bonds are given, respectively, by:

$$\begin{aligned}
D_S^{(1,2)}(0, T) = & c_S P_S \int_0^T \mathbb{E} (e^{-ru} 1_{t_b > u}) du \\
& + (1 - \tau)(1 - \gamma) A_b \int_0^T \mathbb{E} (e^{-ru} 1_{t_b \in du}) du \\
& + e^{-rT} P_S \mathbb{E} (1_{t_b > T} 1_{(1-\tau)A(T) \geq P}) \\
& + e^{-rT} (1 - \tau)(1 - \gamma) \mathbb{E} (A(T) 1_{t_b > T} 1_{(1-\tau)A(T) < P}),
\end{aligned} \tag{6}$$

and

$$D_J^{(1,2)}(0, T) = c_J P_J \int_0^T e^{-ru} \mathbb{E}(1_{t_b > u}) du + e^{-rT} P_J \mathbb{E} (1_{t_b > T} 1_{(1-\tau)A(T) \geq P}). \tag{7}$$

Case 2: $(1 - \gamma)(1 - \tau)A_b > P_S$

When the barrier A_b is hit in the interval $[0, T]$, the firm is bankrupt. A fraction of the total after-tax assets is lost as liquidation costs and the bondholders receive the remaining assets' value. Given the priority structure of the debt, senior bondholders are sure to recover (at time $t \in [0, T]$) their principal since, by assumption in this case $P_S < (1 - \gamma)(1 - \tau)A_b$. Liquidity risk is borne by junior bondholders who will receive (at time t) the remaining value $(1 - \gamma)(1 - \tau)A_b - P_S$.

At maturity T , if the barrier A_b has not been breached before, senior bondholders face no solvency risk on their principal P_S , but junior bondholders may bear such a risk on P_J , depending upon the relative magnitude of their claim. Indeed, if the assets' value net of taxes is smaller than the total debt principal, $(1 - \tau)A(T) < P$, the firm is liquidated. Allowing for bankruptcy costs, bondholders are entitled to the amount $(1 - \tau)(1 - \gamma)A(T)$. In accordance with the absolute priority rule, senior bondholders always receive P_S in this case since, by continuity, $(1 - \tau)(1 - \gamma)A(T) \geq P_S$. Junior debtholders will receive the rest. The latter depends upon whether P_S is smaller or larger than $(1 - \gamma)P$. As in Case 1, we must distinguish between two sub-cases.

Sub-case 2.1: $P_S \leq (1 - \gamma)P$

When premature bankruptcy has not been triggered, junior bondholders will fully receive at time T their principal P_J if $(1 - \tau)A(T) > P$ (no terminal default), but will receive only $(1 - \gamma)(1 - \tau)A(T) - P_S$ otherwise.

Sub-case 2.2: $P_S > (1 - \gamma)P$

In absence of premature bankruptcy, the following relationships hold: $(1 - \tau)A(T) \geq (1 - \tau)A_b > \frac{P_S}{1 - \gamma} > P$. Therefore, both senior and junior principal amounts will be fully refunded to their respective holders.

Accordingly, in Sub-case 2.1, the time-0 values of the senior and junior debts are given (with obvious notation), respectively, by:

$$\begin{aligned}
D_S^{(2,1)}(0, T) = & c_S P_S \int_0^T \mathbb{E} (e^{-ru} 1_{t_b > u}) du \\
& + P_S \int_0^T \mathbb{E} (e^{-ru} 1_{t_b \in du}) du + e^{-rT} P_S \mathbb{E} (1_{t_b > T}),
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
 D_J^{(2,1)}(0, T) = & c_J P_J \int_0^T e^{-ru} \mathbb{E}(1_{t_b > u}) du + ((1 - \tau)(1 - \gamma)A_b - P_S) \\
 & \times \int_0^T e^{-ru} \mathbb{E}(1_{t_b \in du}) du + e^{-rT} P_J \mathbb{E}(1_{t_b > T} 1_{(1-\tau)A(T) \geq P}) \\
 & + e^{-rT} \mathbb{E}([(1 - \tau)(1 - \gamma)A(T) - P_S] 1_{t_b > T} 1_{\frac{P_S}{1-\gamma} < (1-\tau)A(T) < P}).
 \end{aligned} \tag{9}$$

In Sub-case 2.2, these values are given, respectively, by:

$$D_S^{(2,2)}(0, T) = D_S^{(1,1)}(0, T), \tag{10}$$

and

$$\begin{aligned}
 D_J^{(2,2)}(0, T) = & c_J P_J \int_0^T e^{-ru} \mathbb{E}(1_{t_b > u}) du \\
 & + ((1 - \tau)(1 - \gamma)A_b - P_S) \int_0^T e^{-ru} \mathbb{E}(1_{t_b \in du}) du + e^{-rT} P_J \mathbb{E}(1_{t_b > T}).
 \end{aligned} \tag{11}$$

The first term in Equations (8), (9), (10), and (11) is the discounted value of the coupons received until bankruptcy is triggered by a coupon payment default ($A(t) = A_b$), if such an event happens to occur. In that case, the bondholders receive the amount given by the second terms in the equations (the principal P_S for senior debt, and the remaining value of the assets for the junior bonds). If no default on the coupons has occurred, then bondholders will receive, at maturity T , amounts that depend upon whether the firm is bankrupt. In case of default, that is, if $(1 - \tau)A(T)$ is smaller than P , the senior debt is fully repaid, but junior bondholders receive only the remaining value of the assets after payment of the senior debt (fourth term in Equation (9)). If the firm is not bankrupt, the after-tax terminal value of assets being large enough, both types of bondholders collect their respective principals (the third term in the equations).

It is interesting to note that in both sub-cases, senior debtholders do not bear a solvency risk. In addition, although they are fully repaid in case of premature default, the *ex ante* impact of such a liquidity risk is (limited but) not zero and justifies a (small) credit spread. This is because being refunded early at time t ($< T$) may be detrimental to their wealth. As for junior debtholders, they may be refunded an amount higher than the principal P_J when a coupon incident has occurred, since the remaining assets' value $(1 - \tau)(1 - \gamma)A_b$ may be higher (with some small, but positive probability) than P .¹² Consequently, even though the junior bonds bear more liquidity and solvency risks than do the senior debt, we expect that the spread between their respective prices and yields may be relatively narrow if the volatility of asset returns is low and liquidation costs are small.

Case 1 is worse than Case 2 for both types of debtholders. The main differences between the two cases can be summarized as follows. First, in contrast to Case 2, in Case 1, senior creditors are only partly refunded and junior bondholders will lose everything should a coupon default occur

¹² Black and Cox (1976) had already observed that premature bankruptcy may be beneficial to junior debtholders.

(liquidity risk). This is because the after-tax value of the firm's assets, net of bankruptcy costs, is smaller than the principal P_S of the senior debt. Additionally, in the event that no coupon incident has occurred prior to maturity, senior bondholders still face solvency risk in Case 1 as the value of the firm's assets after taxes and bankruptcy costs may be smaller than the senior debt principal P_S . Consequently, junior bondholders also suffer more if the firm is eventually insolvent. In rough terms, Case 1 is more realistic for younger, less prevalent and more aggressively leveraged firms, while Case 2 is more relevant to well-established, mature and dominant companies ("cash cows").

To compute the conditional expectations present in Equations (4)-(11), we borrow from Harrison (1985) the following result:

$$\Pi(t, t_b) = \Phi(h_1(t)) + \left(\frac{A_b}{A}\right)^{\frac{2}{\sigma^2} \frac{r-\eta}{2} - 1} \Phi(h_2(t)), \quad (12)$$

$$\text{where } h_1(t) = \frac{\log(\frac{A_b}{A}) - (r-\eta - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}, \quad h_2(t) = \frac{\log(\frac{A_b}{A}) + (r-\eta - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}.$$

$\Pi(t, t_b)$ denotes the cumulative probability function that a default is triggered when operating cash flows are not large enough to meet coupon payments at time t , that is, t is the first passage time t_b , and $\Phi(\cdot)$ denotes the cumulative normal distribution.

In addition, let $\pi(t, t_b)$ be the probability density function corresponding to $\Pi(t, t_b)$. Reiner and Rubinstein (1991) show that the value $G(T) \equiv \int_0^T e^{-ru} \pi(u, t_b) du$ is equal to:

$$G(T) = \left(\frac{A}{A_b}\right)^{-\lambda_1 + \lambda_2} \Phi(h_3(T)) + \left(\frac{A}{A_b}\right)^{-\lambda_1 - \lambda_2} \Phi(h_4(T)), \quad (13)$$

$$\text{with } h_3(T) = \frac{\log(\frac{A_b}{A}) - \lambda_2 \sigma^2 T}{\sigma\sqrt{T}}, \quad h_4(T) = \frac{\log(\frac{A_b}{A}) + \lambda_2 \sigma^2 T}{\sigma\sqrt{T}}, \quad \lambda_1 = \frac{r-\eta}{\sigma^2} - \frac{1}{2}, \quad \lambda_2 = \frac{\sqrt{(\lambda_1 \sigma^2)^2 + 2r\sigma^2}}{\sigma^2}.$$

Moreover, we can show that if the threshold value A_b has not been hit in the time interval $[0, T]$, the probability of default at maturity T is given by:

$$\Psi(T, t_b) = \Phi(h_5(T)) - \Phi(h_6(T)) - \left(\frac{A_b}{A}\right)^{\frac{2}{\sigma^2} \frac{r-\eta}{2} - 1} [\Phi(h_7(T)) - \Phi(h_8(T))], \quad (14)$$

$$\text{with } h_5(T) = \frac{\log(\frac{A_b}{A}) + (r-\eta - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \quad h_6(T) = \frac{\log(\frac{(1-\tau)A}{P}) + (r-\eta - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$h_7(T) = \frac{\log(\frac{A_b}{A}) + (r-\eta - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \quad h_8(T) = \frac{\log(\frac{(1-\tau)A_b^2}{AP}) + (r-\eta - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

If the liquidity risk associated with the coupon payments does not materialize, the firm must still have sufficient assets to pay back its creditors at maturity T . The minimum value of the assets needed to do so is $\frac{P}{1-\tau}$. If $A(T)$ is below this minimum value, the firm is bankrupt. The following proposition provides the values of the senior and junior debts under Case 1, which realistically applies to more firms than Case 2. Analogous formulas for the latter case are given in Appendix A.

Proposition 1. *Given the dynamics under the risk-neutral measure \mathbf{Q} of the firm's asset value (Equation (1)), the time-0 prices of defaultable senior (D_S) and junior (D_J) coupon bonds are equal to:*

Case 1: $(1 - \gamma)(1 - \tau)A_b \leq P_S$:

Sub-case 1.1: $P_S \leq P(1 - \gamma)$:

$$\begin{aligned}
 D_S^{(1,1)}(0, T) = & \frac{c_S P_S}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)] + (1 - \tau)(1 - \gamma) A_b G(T) \\
 & + e^{-rT} P_S \left(\Phi \left(d_{1, \frac{P}{1-\tau}} \right) - \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} - 1} \Phi \left(d_{2, \frac{P}{1-\tau}} \right) \right) \\
 & + e^{-rT} P_S \left(\Phi \left(d_{1, \frac{P_S}{(1-\tau)(1-\gamma)}} \right) - \Phi \left(d_{1, \frac{P}{1-\tau}} \right) + \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} - 1} \right. \\
 & \times \left. \left[\Phi \left(d_{2, \frac{P}{1-\tau}} \right) - \Phi \left(d_{2, \frac{P_S}{(1-\tau)(1-\gamma)}} \right) \right] \right) + (1 - \tau)(1 - \gamma) \\
 & \times e^{-\eta T} A \left(\Phi \left(d_{3, A_b} \right) - \Phi \left(d_{3, \frac{P_S}{(1-\tau)(1-\gamma)}} \right) + \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} + 1} \right. \\
 & \times \left. \left[\Phi \left(d_{4, \frac{P_S}{(1-\tau)(1-\gamma)}} \right) - \Phi \left(d_{4, A_b} \right) \right] \right),
 \end{aligned} \tag{15}$$

and

$$\begin{aligned}
 D_J^{(1,1)}(0, T) = & \frac{c_J P_J}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)] \\
 & + e^{-rT} P_J \left(\Phi \left(d_{1, \frac{P}{1-\tau}} \right) - \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} - 1} \Phi \left(d_{2, \frac{P}{1-\tau}} \right) \right) \\
 & + (1 - \tau)(1 - \gamma) e^{-\eta T} A \left(\Phi \left(d_{3, \frac{P_S}{(1-\tau)(1-\gamma)}} \right) - \Phi \left(d_{3, \frac{P}{1-\tau}} \right) \right. \\
 & + \left. \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} + 1} \left[\Phi \left(d_{4, \frac{P}{1-\tau}} \right) - \Phi \left(d_{4, \frac{P_S}{(1-\tau)(1-\gamma)}} \right) \right] \right) \\
 & - e^{-rT} P_S \left(\Phi \left(d_{1, \frac{P_S}{(1-\tau)(1-\gamma)}} \right) - \Phi \left(d_{1, \frac{P}{1-\tau}} \right) \right. \\
 & + \left. \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} - 1} \left[\Phi \left(d_{2, \frac{P}{1-\tau}} \right) - \Phi \left(d_{2, \frac{P_S}{(1-\tau)(1-\gamma)}} \right) \right] \right).
 \end{aligned} \tag{16}$$

Sub-case 1.2: $P_S > P(1 - \gamma)$:

$$\begin{aligned}
 D_S^{(1,2)}(0, T) = & \frac{c_S P_S}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)] + (1 - \tau)(1 - \gamma) A_b G(T) \\
 & + e^{-rT} P_S \left(\Phi \left(d_{1, \frac{P}{1-\tau}} \right) - \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} - 1} \Phi \left(d_{2, \frac{P}{1-\tau}} \right) \right) \\
 & + (1 - \tau)(1 - \gamma) e^{-\eta T} A \left(\Phi \left(d_{3, A_b} \right) - \Phi \left(d_{3, \frac{P}{1-\tau}} \right) \right. \\
 & + \left. \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} + 1} \left[\Phi \left(d_{4, \frac{P}{1-\tau}} \right) - \Phi \left(d_{4, A_b} \right) \right] \right),
 \end{aligned} \tag{17}$$

and

$$D_J^{(1,2)}(0, T) = \frac{c_J P_J}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)] \\ + e^{-rT} P_J \left(\Phi\left(d_{1, \frac{P}{1-\tau}}\right) - \left(\frac{A_b}{A}\right)^{\frac{r-\eta}{\sigma^2}-1} \Phi\left(d_{2, \frac{P}{1-\tau}}\right) \right), \quad (18)$$

with

$$d_{1,H} = \frac{\log\left(\frac{A}{H}\right) + \left(r - \eta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \quad d_{2,H} = \frac{\log\left(\frac{A_b^2}{HA}\right) + \left(r - \eta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\ d_{3,H} = \frac{\log\left(\frac{A}{H}\right) + \left(r - \eta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \quad d_{4,H} = \frac{\log\left(\frac{A_b^2}{HA}\right) + \left(r - \eta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\ H = \left\{ A_b, \frac{P_S}{(1-\tau)(1-\gamma)}, \frac{P}{1-\tau} \right\}.$$

Equations (15) to (18) (along with Equations (A-1) to (A-4) in Appendix A) are novel, closed-form solutions for the prices of senior and junior risky coupon bonds when default may be triggered either by a cash flow shortage (liquidity risk) or a negative net worth (solvency risk). Equations (A-1) and (A-3) are less complicated than Equations (15) and (17), as senior bondholders are certain to be repaid the full amount of their claim. This case also applies to junior debtholders in Sub-case 2.2 (Equation A-4), where a full repayment of the debt principal P_J at maturity is ensured as long as the liquidity default has not been triggered.

In Case 1, the firm has more leverage than in Case 2. As such, even its senior debt is exposed to our two types of risk. As demonstrated by Proposition 1, the second sub-case (1.2) entails more solvency risk for senior bondholders than Sub-case 1.1. What distinguishes the two situations is the relative proportion of senior and junior bonds. Since $P = P_S + P_J$, the separating condition $P_S \leq (1-\gamma)P$ is equivalent to $P_S \leq ((1-\gamma)/\gamma)P_J$. Under this form, it is clear that given total indebtedness P , raising the share of senior bonds increases solvency risk for their holders (and decreases it for junior bonds). Note, however, that this higher risk is mitigated by the reduction in liquidity risk brought about by smaller coupon commitments.

Equations (15) to (18) embed two special cases. The first involves ignoring liquidity risk. Setting the default boundary A_b to zero, Equations (15) to (18) reduce, respectively, to:

$$D_S^{(1,1)}(0, T) = \frac{c_S P_S}{r} [1 - e^{-rT}] + e^{-rT} P_S \Phi\left(d_{1, \frac{P_S}{(1-\tau)(1-\gamma)}}\right) + (1-\tau)(1-\gamma) \\ \times e^{-\eta T} A \left(1 - \Phi\left(d_{3, \frac{P_S}{(1-\tau)(1-\gamma)}}\right) \right), \quad (19)$$

$$D_J^{(1,1)}(0, T) = \frac{c_J P_J}{r} [1 - e^{-rT}] + e^{-rT} P_J \Phi\left(d_{1, \frac{P}{1-\tau}}\right) + (1-\tau)(1-\gamma) \\ \times e^{-\eta T} A \left(\Phi\left(d_{3, \frac{P_S}{(1-\tau)(1-\gamma)}}\right) - \Phi\left(d_{3, \frac{P}{1-\tau}}\right) \right) \\ - e^{-rT} P_S \left(\Phi\left(d_{1, \frac{P_S}{(1-\tau)(1-\gamma)}}\right) - \Phi\left(d_{1, \frac{P}{1-\tau}}\right) \right), \quad (20)$$

$$D_S^{(1,2)}(0, T) = \frac{c_S P_S}{r} [1 - e^{-rT}] + e^{-rT} P_S \Phi\left(d_{1, \frac{P}{1-\tau}}\right) + (1 - \tau) \times (1 - \gamma) e^{-\eta T} A \left(1 - \Phi\left(d_{3, \frac{P}{1-\tau}}\right)\right), \quad (21)$$

$$D_J^{(1,2)}(0, T) = \frac{c_J P_J}{r} [1 - e^{-rT}] + e^{-rT} P_J \Phi\left(d_{1, \frac{P}{1-\tau}}\right). \quad (22)$$

The difference between the prices given by Equations (19) to (22) and the price of a default-free coupon bond, namely

$$D_k^{\text{df}}(0, T) = \frac{c_k P_k}{r} [1 - e^{-rT}] + e^{-rT} P_k, \quad k = \{S, J\}, \quad (23)$$

is due to the probability that no sufficient assets would be available to repay the senior debt principal (inducing the second and third terms in Equations (19) and (21)) and the junior debt principal after paying the senior one (inducing all terms but the first one in Equations (20) and (22)). In such events, the firm's default is triggered and the ensuing bankruptcy costs are deducted from the assets' value.

The second, even more special case occurs if we further eliminate solvency risk. Formally, letting $A \rightarrow \infty$ implies $\Phi(d_{1, \frac{P}{1-\tau}}) = \Phi(d_{1, \frac{P_S}{(1-\tau)(1-\gamma)}}) = \Phi(d_{3, \frac{P}{1-\tau}}) = \Phi(d_{3, \frac{P_S}{(1-\tau)(1-\gamma)}}) = 1$, directly leading to Equation (23).

III. Firm and Equity Values

This section is devoted to the derivation of the firm and equity values. The firm's market value, $V(0)$, is the sum of the after-tax value of its assets and the value of the tax shield associated with the coupons on its debts, denoted by $TS(0, T)$, less the value of bankruptcy costs denoted by $BC(0, T)$. Hence, the time-0 market value of the firm, $V(0)$, is equal to:

$$V(0) = A^* + TS(0, T) - BC(0, T), \quad (24)$$

where $A^* = (1 - \tau)A$ is the initial after-tax value of the all-equity firm. The tax shield is obtained from the deductible coupon payments up to the time the barrier A_b is hit or to the debt maturity T if the barrier has not been breached. Thus, its time-0⁺ value of the tax shield is given by:

$$TS(0, T) = \tau (c_S P_S + c_J P_J) \mathbb{E} \left(\int_0^T e^{-ru} 1_{t_b > u} du \right). \quad (25)$$

Explicit computation yields:

$$TS(0, T) = \tau \frac{c_S P_S + c_J P_J}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)]. \quad (26)$$

Proportional bankruptcy costs are computed on the basis of the after-tax value of the firm's assets. Bankruptcy is triggered either by a default on a coupon (first term in Equation (27)) or at maturity by an after-tax asset value that is lower than the total debt principal P (second term in

Equation (27)). The time-0⁺ value of these costs is given by:

$$BC(0, T) = \gamma \left(\frac{(1 - \tau) A_b \int_0^T (e^{-ru} 1_{t_b \in du}) du + e^{-rT}}{(1 - \tau) \mathbb{E} (A(T) 1_{t_b > T} 1_{(1-\tau)A(T) < P})} \right). \quad (27)$$

Explicit computation when $\frac{P}{1-\tau} \geq A_b$ leads to:¹³

$$BC(0, T) = \gamma \left((1 - \tau) A_b G(T) + e^{-\eta T} (1 - \tau) A \left(\Phi(d_{3, A_b}) - \Phi(d_{3, \frac{P}{1-\tau}}) + \left(\frac{A_b}{A} \right)^{2\frac{r-\eta}{\sigma^2} + 1} [\Phi(d_{4, \frac{P}{1-\tau}}) - \Phi(d_{4, A_b})] \right) \right), \quad (28)$$

where d_{3, A_b} , $d_{3, \frac{P}{1-\tau}}$, $d_{4, \frac{P}{1-\tau}}$ and d_{4, A_b} are given in Proposition (1).

The market value of the firm's equity, denoted by $E(0)$, immediately after the issuance of its debts and simultaneous refunding of its shareholders in due proportion, is given by:

$$E^{(i,j)}(0) = V(0) - D_S^{(i,j)}(0, T) - D_J^{(i,j)}(0, T), \quad i, j = 1, 2, \quad (29)$$

where the various $D_S^{(i,j)}(0, T)$ and $D_J^{(i,j)}(0, T)$ are given in Proposition (1) and in Appendix A.

One of our objectives is to identify the optimal proportions of senior and junior debts in the total debt amount that maximizes the firm's market value. Since $P_J = \omega P_S$ and $P = P_J + P_S$, then we have $P_S = \frac{1}{1+\omega} P$ and $P_J = \frac{\omega}{1+\omega} P$. When ω is equal to 1, total debt is evenly spread between the junior and senior debt. As ω increases, the proportion of senior debt decreases. The firm's optimal market value is obtained for ω such that:

$$\frac{\partial v(0)}{\partial \omega} = 0. \quad (30)$$

IV. Numerical Analysis

Exploiting our closed-form solutions for the senior and junior corporate coupon bond values and the firm and equity values, this section investigates four different, yet related, issues. First, we examine the debt priority structure and, in particular, the way the optimal mix of senior and junior bonds is impacted by changes in the base case parameters. Additionally, we analyze the leveraged firm's value and study the behavior of the optimal debt ratio. Moreover, we discuss the level and the shape of the term structure of credit spreads generated by the model. Finally, we apply the same analysis to the term structure of probabilities of default.

The base case parameter values used in the numerical analysis are as follows: $A^* = 100$, $r = 4.5\%$, $c_J = 5.5\%$, $c_S = 5\%$ and $\eta = 5\%$.¹⁴ These appear to be realistic pre-crisis parameters. The corporate tax rate τ is, as in Ju, Parrino, Poteshman, and Weisbach (2005), equal to 34%. We also investigate the impact of a lower tax rate of 15%. The fraction of firm's assets lost to

¹³ In the case where $\frac{P}{1-\tau} < A_b$, $BC(0, T)$ is simply equal to $\gamma(1 - \tau)A_b G(T)$.

¹⁴ Since junior bondholders bear more risk, it is natural for the junior coupon rate to be higher than the senior rate. This is also consistent with the empirical evidence reported by Roberts and Viscione (1984). Occasionally, we set c_J equal to 9% for static comparative analysis.

bankruptcy costs, γ , is set equal to 35%.¹⁵ To analyze the effect of these costs, we also consider, as in Hackbarth, Hennessy, and Leland (2007), a higher value of 50%. The assets' volatility σ is set at 20% or 30%. When it is not optimized, but given exogenously, the total debt principal, P , is, in general, equal to either 25 or 50 corresponding to a low leverage of 25% or a high leverage of 50%, respectively. The values of the volatility and leverage are similar to those estimated by Schaefer and Strebulaev (2008). They report a mean volatility of assets equal to 22% and mean leverage ratios equal to 25% when investment-grade bonds are involved and to 58% when speculative bonds are issued.¹⁶

Our base case for the long-term debt maturity, T , is 10 years. This choice is motivated by the market practice of considering the ten-year maturity as a benchmark for credit yield spreads and by the fact that the extant literature focuses on this maturity (see, for instance, Ju, Parrino, Poteshman, and Weisbach (2005)). For completeness, however, we also discuss the cases of shorter ($T=2$) and longer ($T=20$) maturities.

A. Debt Priority Structure

First, we examine the debt priority structure. As discussed further below, the trade-off existing between tax benefits and bankruptcy costs determines the optimal debt structure, in much the same way it determines the firm's optimal capital structure (see the next subsection). For the sake of the argument, let us suppose that the total level of indebtedness has been optimized, although the reasoning that follows is general. A shift away from senior bonds toward more junior debt has two opposite effects on the firm's value. The first is negative. Given Equation (2), a larger proportion of ω rises the threshold of bankruptcy, which makes more likely a default event due to a liquidity shortage. This, in turn, increases the present value of future bankruptcy costs. The second effect is positive. As junior coupon rate is larger than the senior, the present value of the tax deductions it provides is greater. The optimum debt structure is reached, and the firm's total value maximized, when these two effects exactly cancel out for a marginal change in ω . This is the key trade-off that produces an *interior* optimal priority structure. Since neither effect is linear – the first one because the value of a barrier option is not linear with the level of the barrier, and the second one because premature default would prevent the firm from reaping the full value of the tax shield – the relative merits of the two types of debt depend upon the actual value of the firm's total indebtedness. Changes in the optimal priority structure when the level of total debt is modified are due to the increasing concave and increasing convex patterns, respectively, of tax benefits and bankruptcy costs.

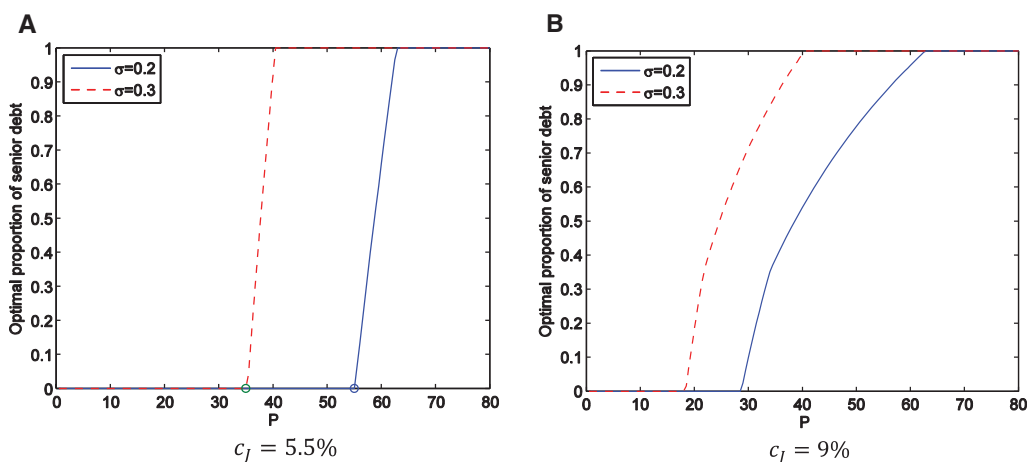
Figure 1 plots the optimal proportions of senior debt against total debt values for two levels of asset volatility σ (20% and 30%) and two values of junior coupon c_J (5.5% and 9%). Some interesting patterns emerge. First, the relationship is highly nonlinear. This was expected given the barrier option feature of our framework. A low-levered firm relies almost exclusively on more expensive junior bonds to optimize its tax benefits. At some level of total debt, indicated by an "o" in Figure 1, the firm substitutes its junior debt, progressively or rapidly, for senior debt. Above some level of leverage, only senior bonds are issued as the burden of interest payments becomes heavy to the firm. Second, the switch level "o" depends dramatically upon the firm's asset volatility, which conditions the probability of default and the junior coupon rate on which

¹⁵ This level is an average value between the one estimated empirically by Andrade and Kaplan (1998) (20%) and the one adopted in various papers, such as those of Ju, Parrino, Poteshman, and Weisbach (2005) and Hackbarth, Hennessy, and Leland (2007) (50%). Hackbarth and Mauer (2012) used 25%.

¹⁶ See their Table 7 on page 10. They actually estimated a quasi-market leverage ratio, defined as the debt book value divided by the sum of the debt book value and the equity market value.

Figure 1. Debt Priority Structure

The figure plots the optimal proportion of senior debt for two levels of σ (20% and 30%). Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $\tau = 34\%$, $\gamma = 35\%$, $c_S = 5\%$, and $T = 10$.



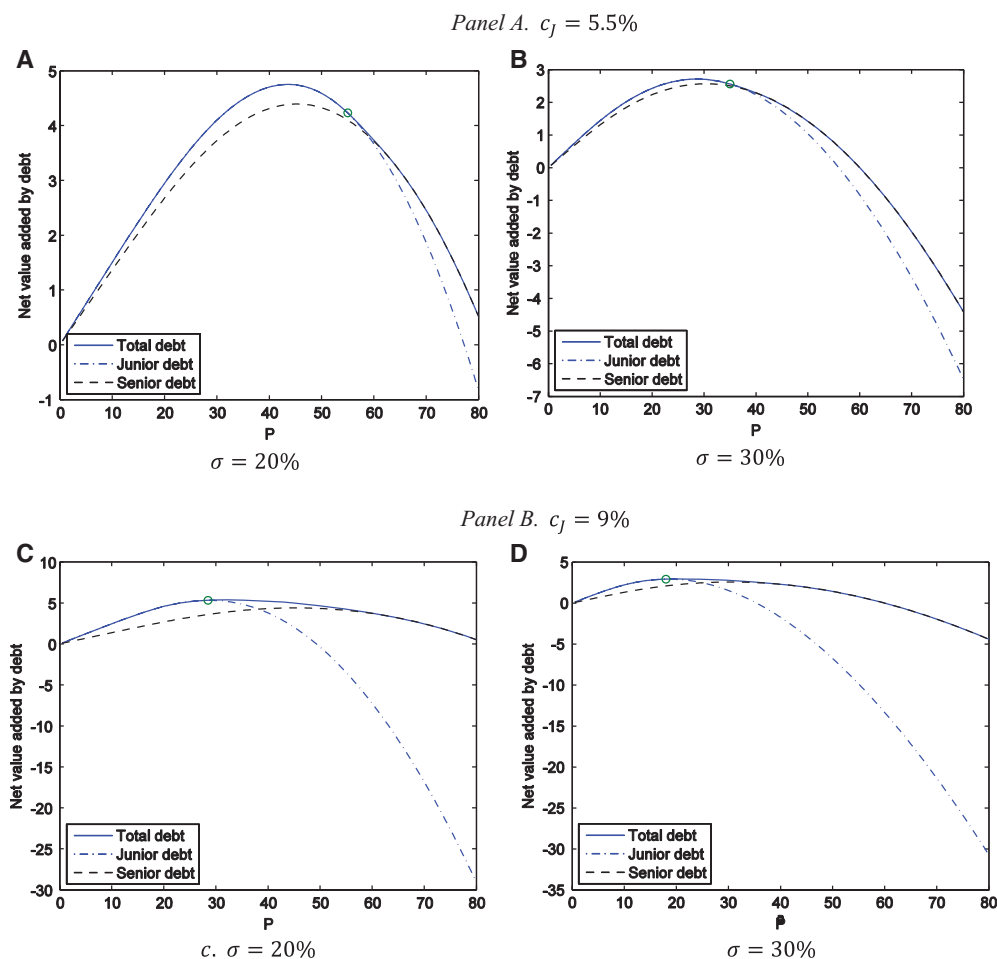
tax benefits depend. When σ jumps from 20% to 30%, the switch to senior bonds is made at a smaller level of debt and the slope of the curve becomes steeper as the substitution from almost fully junior debt to fully senior is swifter. Moreover, this change depends markedly on the junior coupon bond. When c_J is relatively small, the proportion of senior bonds increases dramatically, and almost linearly, with total debt as the junior coupon advantage is thin. When c_J is large, the proportion of senior debt increases much more progressively as the firm slowly trades off the (large) tax advantage for less default risk and bankruptcy costs. To summarize, the firm issues homogeneous debt at low and high levels of leverage, but spreads its debt priority structure with both senior and junior bonds at intermediate levels.

Our results are partially consistent with the empirical findings of Colla, Ippolito, and Li (2013) and Rauh and Sufi (2010). This evidence suggests, as our model predicts, that firms with low and medium credit quality (large junior coupon, high asset volatility, intermediate leverage) spread their debt priority. In addition, firms presenting a large probability of default due to very high leverage specialize their debt structure. However, in contrast to Colla, Ippolito, and Li (2013) and Rauh and Sufi (2010), who find that high credit quality firms largely favor senior bonds, our results indicate that as credit quality (measured by total debt and/or asset volatility) improves, the firm relies more on subordinated debt. One plausible explanation for this discrepancy is that in both empirical studies, firms use other forms of debt, including bank loans, inducing a strategic debt service that is ignored in this paper.

The trade-off between the increased value of the expected tax benefits and the larger value of the expected bankruptcy costs associated with issuing more junior debt is illustrated more precisely in Figure 2, which plots the net value added by debt as a function of total debt depending upon its composition. The net value added by debt is the difference between the present value of future tax deductions and that of future bankruptcy costs. Two junior coupon rates (5.5% and 9%) and two asset volatilities (20% and 30%) are considered. In each panel, three curves are displayed. The

Figure 2. Net Value Added by Debt

This figure plots the increase in the firm's value due to the optimal use of junior debt for different levels of total debt P . Two junior coupon rates and two asset volatilities are considered. The net value added by debt is the difference between the tax shield value and the present value of future bankruptcy costs. The solid line (—) depicts the net value added by debt obtained with the optimal priority mix. The dashed line (---) plots the value obtained with senior debt only. The dot-dashed line (-.-) displays the value obtained when increasing total debt from point “o”, but freezes the junior debt at the optimal level achieved at this point. Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $\tau = 34\%$, $\gamma = 35\%$, $c_S = 5\%$, and $T = 10$.



solid line represents the case where total debt respects the optimal priority mix. The dashed line plots the net value added by debt obtained when only senior bonds are issued. The dot-dashed line starting from point “o” displays the value obtained when we increase total debt from point “o” on, but freezes the amount of junior debt at the (optimal) level achieved at this point. In short, to the left of point “o”, the dashed line corresponds to too much senior debt, while to the right, the dot-dashed line corresponds to too much junior debt.

Incidentally, the net value added by debt exhibits an inverted U-shape that conforms to the trade-off theory and to the empirical evidence reported by van Binsbergen, Graham, and Yang (2010).¹⁷ More specifically, Figure 2 illustrates the loss of value, measured by the distance between the solid and the dashed lines, suffered by the firm when it issues too much senior debt. Although by doing so the firm lowers the likelihood of premature default and, hence, the value of bankruptcy costs, it does not fully benefit from the larger tax shield provided by the junior bonds and this effect dominates. In contrast, for higher debt levels and larger default probabilities, the loss, materialized by the difference between the solid and the dot-dashed lines, is due to the firm issuing too much junior debt. This unduly increases the present value of liquidation costs since the probability of premature default sharply increases with ω , while the compensating gain in tax benefits becomes increasingly smaller as the tax deduction is lost on bankruptcy. In addition, Figure 2 underlines the substantial impact of the volatility in asset returns, particularly when the junior coupon rate is large. An increase in the firm's business risk significantly reduces the net benefit of debt by raising the probability of hitting the default threshold, thereby tipping the optimal debt priority mix toward more senior debt. Beyond some leverage level, only senior debt is optimally issued. This result is supported empirically by Colla, Ippolito, and Li (2013), who find that firms have a high degree of debt specialization when the volatility of their operational cash flows is high.

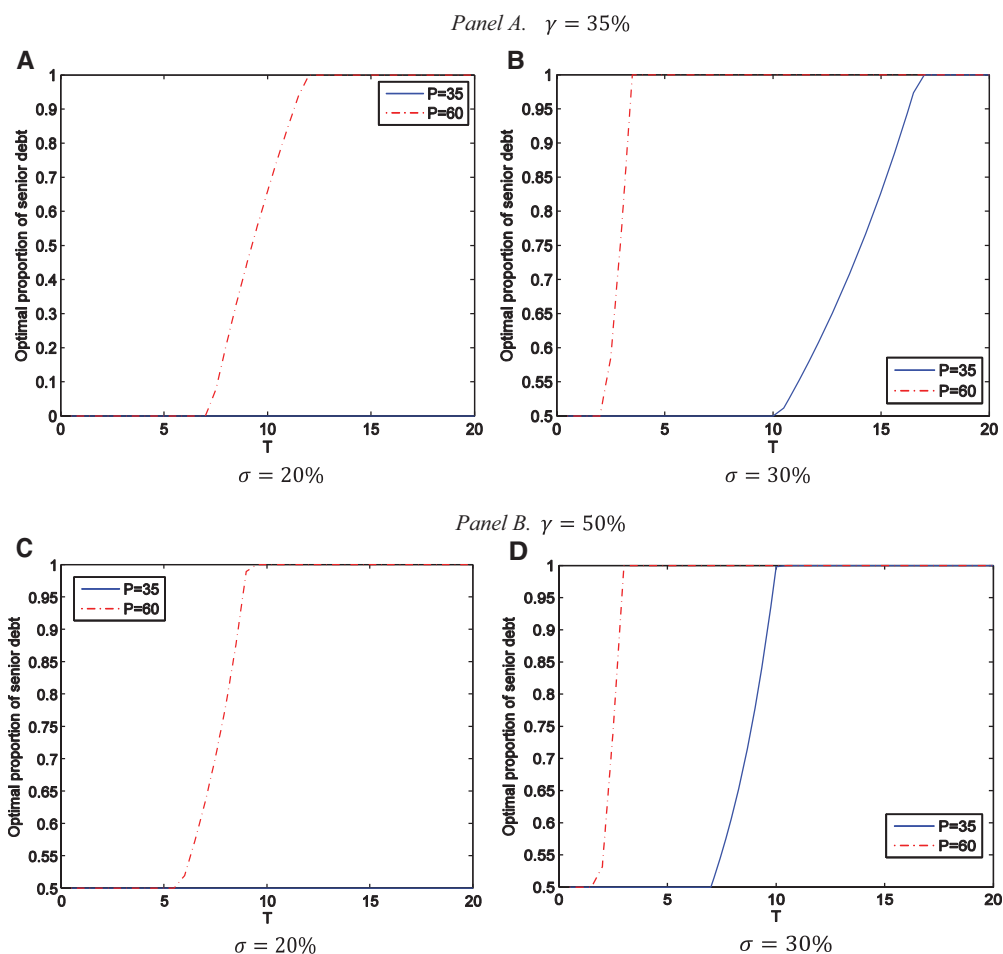
As to the impact of debt maturity on the optimal priority structure, Figure 3 provides the optimal fraction of senior bonds for different maturities and for given levels of total debt principal ($P=35$ or 60), asset volatility (20% or 30%) and bankruptcy costs (35% or 50%). The general pattern that emerges is that the optimal proportion of senior debt increases rapidly with debt maturity. This feature is more pronounced when the firm has more leverage, has riskier assets, and suffers from potentially heavier costs of liquidation. In these cases, the firm's senior debt also becomes significant at shorter maturities. All this is consistent with the trade-off explanation provided above. Only when the firm has little or medium leverage and enjoys rather stable asset returns ($P=35$ and $\sigma=20\%$) does it issue junior bonds, regardless of maturity, to reap the full tax benefits.

We have performed other comparative static analyses to assess the impact of various parameters on the optimal fraction of senior debt for a relatively high-levered firm ($P = 50$). Table I reports the results. First, the sensitivity of the optimal proportion of senior debt to the fraction of value paid out to all stakeholders (η) is negative and very strong, in particular, for long debt maturities and strong asset volatility. For instance, for $\sigma = 30\%$ and $T = 10$, the percentage of senior debt rises from 53.6% to 100% when η decreases from 6% to 5% or less. This is because a smaller η leads to an increase in the default threshold A_b and, as such, in the default probability, which implies that bankruptcy costs outweigh the tax benefits. Additionally, a decrease in the interest rate entails an increase in the proportion of senior debt for long maturity debt. This is due to the negative effect of the drift ($r - \eta$) on the market value of assets $A(t)$ increasing the probability of hitting the default barrier. Moreover, a small increase in the junior coupon rate leads to little change in the debt structure, as the higher tax benefits and the larger expected bankruptcy costs tend to cancel out. In contrast, and as previously explained, a major increase in that rate dramatically shifts the structure toward more senior debt as the probability of default surges and becomes determinant. For instance, for an asset volatility of 20%, increasing c_J from

¹⁷ See also the discussion in the next subsection. Moreover, our base case optimal net values (Figure 2a) are aligned with those estimated in van Binsbergen, Graham, and Yang (2010) and Korteweg (2010), as the median of their net values ranges from 4% to 5.5% of the total asset value. Neither paper, however, investigates the impact of the debt structure on its net value.

Figure 3. Term Structure of Optimal Proportion of Senior Debt

The figure plots the optimal proportion of senior debt (relative to total debt) that maximizes the firm's value across maturities. Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $\tau = 34\%$, $c_S = 5\%$, and $c_J = 5.5\%$.



5.5% to 9%, results in a proportion of senior bonds jumping from 0.2% to 77.8% when $T = 10$ and from 47.6% to 93.5% when $T = 20$. Furthermore, even a small increase in the coupon rate differential between the bonds may lead to a substantially higher proportion of junior debt if asset volatility is low and debt maturity is long. In that case, the probability of default is small and the tax benefits may be large, which favors issuing the more expensive debt. For example, if c_S is set to 5% instead of 5.25%, with c_J remaining equal to 5.5%, the proportion of senior bonds sharply declines from 95.4% to 47.6% for $\sigma = 20\%$ and $T = 20$. Finally, either a decrease in the tax rate or an increase in bankruptcy costs leads, in general, to more senior debt being optimally issued.

Table I. Comparative Statics for Optimal Priority Debt Structure

This table reports comparative statics for the optimal proportion of senior debt. Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $\tau = 34\%$, $c_S = 5\%$, $c_J = 5.5\%$, $\gamma = 35\%$, and $P = 50$.

	$\sigma = 20\%$			$\sigma = 30\%$		
	$T = 2$	$T = 10$	$T = 20$	$T = 2$	$T = 10$	$T = 20$
Base case	0.2%	0.2%	47.6%	0.2%	100.0%	100.0%
$\eta = 4.25\%$	0.2%	100.0%	100.0%	54.4%	100.0%	100.0%
$\eta = 6\%$	0.2%	0.2%	0.2%	0.2%	53.6%	100.0%
$r = 4\%$	0.2%	0.2%	79.8%	0.2%	100.0%	100.0%
$r = 5\%$	0.2%	0.2%	14.6%	0.2%	100.0%	100.0%
$c_J = 6\%$	0.2%	11.2%	73.8%	0.2%	100.0%	100.0%
$c_J = 9\%$	40.3%	77.8%	93.5%	69.8%	100.0%	100.0%
$c_S = 5.25\%$	0.2%	0.2%	95.4%	0.2%	100.0%	100.0%
$\tau = 15\%$	15.4%	100.0%	100.0%	100.0%	100.0%	100.0%
$\gamma = 50\%$	0.2%	0.2%	100.0%	0.2%	100.0%	100.0%

B. Optimal Capital Structure

We now examine the issue of capital structure. Optimal market leverage, computed as the ratio of the market value of total debt over the firm's total market value, $\frac{D_S^{(i,j)}(0,T) + D_J^{(i,j)}(0,T)}{V(0)} = \frac{D}{V}$, $i, j = 1, 2$, is achieved at the firm's maximum market value given an optimal debt priority structure. Table II displays, for our base case parameters, the optimal leverage for different maturities, asset volatilities, and bankruptcy costs.

Given a debt maturity of ten years, the market leverage D/V is roughly equal to 42.4% for low asset volatility and small bankruptcy costs (Panel A). It drops sharply to 27.7% when asset volatility increases from 20% to 30% (Panel B). The corresponding volatilities for equity are 32% and 39%, respectively. This level of market leverage is in line with the empirical value of 28.4% reported in Faulkender and Petersen (2006) for firms that have access to public debt markets.¹⁸ The negative relationship predicted by our model between asset volatility and market leverage is shared with virtually all structural debt pricing models (Leland and Toft, 1996; Leland, 1998; Ju, Parrino, Poteshman, and Weisbach, 2005; Hackbarth and Mauer, 2012). It is also supported empirically by Faulkender and Petersen (2006) and Frank and Goyal (2009). As assets become riskier, the probability of default surges and the firm adjusts its leverage downward to mitigate the effect.

The impact of bankruptcy costs on optimal leverage is obviously negative. When these costs increase from 35% to 50%, the optimal leverage declines from 42.5% to 37.3% for $\sigma = 20\%$ and to 22.5% for $\sigma = 30\%$. The level our model predicts is consistent with the evidence reported by Ju, Parrino, Poteshman, and Weisbach instead of Ju et al. (2005), who find that the median debt to total capital ratio is 22.6%. This result illustrates the significant impact of risk transfer from shareholders to bondholders that takes place when these costs increase.

The priority structure reported in Table II indicates an extremely low proportion of senior debt in our base case as the value of the tax shield associated with the higher junior coupon rate is large relative to bankruptcy costs. The firm's equity value (E) increases (and, as such, leverage decreases) with asset volatility σ . According to Table II, for a 10-year debt maturity, E rises

¹⁸ They also report, for these firms, a mean asset volatility of 18.89%.

Table II. Optimal Leverage

This table reports the optimal value of market leverage (D/V) when the debt priority is optimal. D , V , and E denote the market values of debt, firm, and equity, respectively. BC and TS denote the values of bankruptcy costs and tax shields, respectively. $P_S/P(\%)$ represents the optimal percentage of senior debt relative to total debt. D^{df} is the risk-free value of the total debt and $\frac{E}{A^*-P}$ denotes the market-to-book ratio. Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $\tau = 34\%$, $c_S = 5\%$, and $c_J = 5.5\%$.

	P	D	BC	TS	$P_S/P(\%)$	V	$D/V(\%)$	E	$\frac{E}{A^*-P}(\%)$	$\frac{D}{P}(\%)$	$\frac{D^{df}}{D}(\%)$
<i>Panel A. $\gamma = 35\%$ and $\sigma = 20\%$</i>											
$T = 2$	50	50.74	0.17	1.79	0.20	101.62	49.93	50.88	101.76	101.48	100.42
$T = 10$	44	44.39	1.65	6.40	0.23	104.75	42.38	60.36	107.78	100.89	107.09
$T = 20$	43	42.82	2.59	9.40	0.23	106.81	40.09	63.99	112.27	99.57	113.66
<i>Panel B. $\gamma = 35\%$ and $\sigma = 30\%$</i>											
$T = 2$	38	38.41	0.22	1.36	0.26	101.14	37.98	62.73	101.18	101.08	100.82
$T = 10$	29	28.46	1.39	4.10	0.34	102.71	27.71	74.25	104.58	98.13	110.09
$T = 20$	28	26.56	2.19	5.74	0.36	103.54	25.65	76.98	106.92	94.86	119.30
<i>Panel C. $\gamma = 50\%$ and $\sigma = 20\%$</i>											
$T = 2$	48	48.73	0.16	1.72	0.21	101.56	47.98	52.83	101.59	101.52	100.38
$T = 10$	38	38.83	1.40	5.61	0.26	104.21	37.26	65.39	105.47	102.17	105.74
$T = 20$	37	37.21	2.52	8.39	0.27	105.87	35.15	68.66	108.98	100.58	112.52
<i>Panel D. $\gamma = 50\%$ and $\sigma = 30\%$</i>											
$T = 2$	35	35.44	0.18	1.25	0.29	101.07	35.06	65.64	100.98	101.26	100.65
$T = 10$	23	23.00	1.07	3.34	0.43	102.27	22.49	79.26	102.94	100.02	108.01
$T = 20$	22	21.15	1.96	4.77	0.45	102.81	20.57	81.66	104.69	96.13	117.71

from 60.36 to 74.25 when σ increases from 20% to 30% (with bankruptcy costs equal to 35%), and from 65.39 to 79.26 (when such costs reach 50%). It is tempting to find this result obvious, keeping in mind the option-like nature of equity. However, this conclusion is erroneous. First, as stated in Leland and Toft (1996), the existence of tax benefits (and their potential loss in case of default) and the presence of bankruptcy costs imply that debt and equity holders do not share the whole claim on the underlying assets' value. This, in turn, makes equity a more complex claim than a call option. Second, and much more importantly, the comparison between the two levels of asset volatility is misleading. As optimal debt sharply decreases with σ , while the firm's total value is barely affected, E increases sharply (and D/V plummets). What is relevant is the market-to-book equity ratio ($E/(A^* - P)$), which decreases with σ . For instance, for $T = 10$, the ratio falls from 107.8 to 104.6 when σ increases from 20% to 30% (with 35% bankruptcy costs) and from 105.5 to 102.9 (with 50% costs). This is due to the level of optimal debt and that of the tax shield being markedly smaller (the decline in bankruptcy costs is too small to fully compensate).

In addition, our results indicate that the optimal market leverage is inversely related to debt maturity and, all the more so because asset volatility is high and bankruptcy costs are large. These results are in line with those of Ju and Ou-Yang (2006) and with the empirical findings of Johnson (2003).¹⁹ When a firm settles for short-term debt, it is optimal to issue more bonds to

¹⁹ Johnson (2003) relates the negative relationship between leverage and debt maturity to the underinvestment problem that the firm faces. As issuing short-term debt aggravates underinvestment, the firm tends to increase its overall leverage to compensate.

maximize tax benefits. However, when the debt maturity lengthens, the tax benefits accumulate over a longer period. Thus, the firm can afford to issue fewer bonds so that the probability of default is reduced. Note that the market-to-book equity ratio increases with the debt maturity due to larger tax benefits, despite the fact that optimal debt is reduced. However, this increase is mitigated as asset volatility σ and/or liquidation costs γ increase, as explained above. Moreover, as the optimal debt's market value decreases with the debt maturity (due to a higher probability of default), our model predicts a negative relationship between the optimal market leverage and the market-to-book-equity ratio. This result is supported by the empirical evidence reported by Baker and Wurgler (2002) and Liu (2009).

Table III reports the influence of various other parameters on optimal leverage for the 10-year debt benchmark. The impact of total payout rate η on optimal leverage D/V is monotonously positive for all levels of asset volatility and bankruptcy costs that fall in the ranges we have assumed. This is because a larger η leads to a decrease in the default threshold A_b and, as such, in the default probability implying that debt is less risky.²⁰ The firm's market-to-book ratio increases with total payout. A higher value of η leads to a larger amount of dividends received by the shareholders, other things being equal (the Modigliani-Miller irrelevance theorem on dividend policy does not apply in the presence of bankruptcy costs). An increase in the level of the risk-free rate influences variously the optimal market leverage (D/V), depending upon the level of bankruptcy costs. With such small costs, the optimal leverage decreases with r , while the reverse is true with larger costs. This can be explained as follows. A rise in r enhances the firm's optimal debt level, as assets become, on average, more profitable. Greater indebtedness has a smaller positive impact on the tax benefits and an even smaller negative influence on bankruptcy costs, so that the firm's market value slightly increases in all cases. The impact of the rise in r on the debt market value is more complex. The induced increase in the cost of debt has a positive effect on the firm's probability of default (higher threshold), but the induced increase in the assets' profitability (drift of Equation (1)) affects it negatively. When liquidation costs are low, the market value of debt is (relatively) large and the negative effect of discounting it at a higher rate r dominates the positive effect of an increased debt level, leading the net impact to be negative (e.g., optimal D decreases from 44.04 to 43.89 when r rises from 4% to 5%. See Panel A). When bankruptcy costs are high, it is the second effect that dominates and the net impact is positive (e.g., optimal D increases from 38.27 to 39.30. See Panel C).

Table III also indicates that the senior coupon rate, so long as it remains strictly below the junior coupon rate, has no impact on optimal leverage and equity. This is because, for a 10-year debt maturity, the rate has no bearing on the debt priority structure. In contrast, a sizeable increase in the junior bond rate leads the firm to decrease its total issuance of debt (P) as liquidity risk soars. This is tempered by a huge shift toward more senior (cheaper) debt and less junior debt, leading the debt market value (D) to decrease much less than P . The firm's market value (V) increases only slightly as the tax shield associated with the cheaper debt is smaller than the one generated by the junior debt. Then, as D decreases and V increases, the optimal market leverage ratio (D/V) is negatively related to the junior coupon rate.

As in the standard Modigliani-Miller world but to a much lesser extent due to both liquidity costs and the variation in the optimal level of debt, the optimal leverage and the firm's value (relative to that of its all equity benchmark) depend positively upon the tax rate because of the fiscal benefit associated with issuing debt. This result is consistent with empirical findings

²⁰ While an increase in η lowers the early default boundary (and thus the premature default probability), it also increases the terminal default probability. Therefore, the net impact of η on optimal leverage might not be monotonous for all ranges of parameter values, in spite of the fact that the premature default probability is usually much larger than the terminal default one.

Table III. Comparative Statics for Optimal Leverage

This table reports the comparative statics for optimal leverage. D , V and E denote the market values of debt, firm and equity, respectively. BC and TS denote the values of bankruptcy costs and tax shield, respectively. $P_S/P(\%)$ represents the optimal percentage of senior debt relative to total debt. D^{df} is the risk-free value of the total debt and $\frac{E}{A^*-P}$ denotes the market-to-book ratio. Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $\tau = 34\%$, $c_S = 5\%$, $c_J = 5.5\%$, and $T = 10$.

	P	D	BC	TS	P_S/P (%)	V	D/V (%)	E E	$\frac{E}{A^*-P}$ (%)	$\frac{D}{P}$ (%)	$\frac{D^{\text{df}}}{D}$ (%)
<i>Panel A. $\gamma = 35\%$ and $\sigma = 20\%$</i>											
$\eta = 4.25\%$	39	40.25	1.22	5.67	0.26	104.46	38.54	64.20	105.25	103.22	104.67
$\eta = 6\%$	46	45.45	2.00	6.75	0.22	104.75	43.39	59.30	109.81	98.80	109.36
$r = 4\%$	42	44.04	1.63	6.26	0.24	104.63	42.09	60.59	104.47	104.85	107.16
$r = 5\%$	45	43.89	1.53	6.41	0.22	104.87	41.85	60.98	110.88	97.53	106.56
$c_J = 6\%$	41	43.32	1.48	6.49	0.24	105.01	41.26	61.69	104.55	105.67	106.05
$c_J = 9\%$	29	37.89	1.44	6.76	2.41	105.32	35.97	67.43	94.97	130.64	103.69
$c_S = 5.25\%$	44	44.39	1.65	6.40	0.23	104.75	42.38	60.36	107.78	100.90	107.09
$\tau = 15\%$	26	27.64	0.36	1.71	0.38	101.35	27.27	73.71	99.61	106.31	101.63
<i>Panel B. $\gamma = 35\%$ and $\sigma = 30\%$</i>											
$\eta = 4.25\%$	24	24.31	1.03	3.43	0.42	102.40	23.74	78.09	102.75	101.30	106.65
$\eta = 6\%$	33	31.31	1.71	4.66	0.30	102.95	30.41	71.64	106.93	94.89	113.86
$r = 4\%$	28	28.49	1.40	4.05	0.36	102.65	27.75	74.16	103.00	101.75	110.42
$r = 5\%$	30	28.41	1.37	4.15	0.33	102.78	27.64	74.37	106.24	94.69	109.75
$c_J = 6\%$	27	27.75	1.34	4.15	0.37	102.81	27.00	75.06	102.82	102.79	109.00
$c_J = 9\%$	19	24.16	1.31	4.23	7.37	102.92	23.47	78.77	97.24	127.14	105.29
$c_S = 5.25\%$	29	28.46	1.39	4.10	0.34	102.71	27.71	74.25	104.58	98.14	110.09
$\tau = 15\%$	14	14.75	0.29	0.91	0.71	100.62	14.66	85.87	99.85	105.36	102.53
<i>Panel C. $\gamma = 50\%$ and $\sigma = 20\%$</i>											
$\eta = 4.25\%$	35	36.27	1.13	5.16	0.29	104.03	34.87	67.76	104.24	103.63	104.25
$\eta = 6\%$	39	39.22	1.70	5.79	0.26	104.09	37.68	64.87	106.34	100.57	107.43
$r = 4\%$	36	38.27	1.35	5.45	0.28	104.09	36.77	65.82	102.85	106.32	105.68
$r = 5\%$	40	39.30	1.43	5.77	0.25	104.34	37.67	65.03	108.39	98.26	105.77
$c_J = 6\%$	37	39.27	1.44	5.93	0.27	104.49	37.58	65.22	103.52	106.14	105.58
$c_J = 9\%$	26	33.86	1.33	6.15	2.69	104.82	32.31	70.96	95.89	130.24	103.94
$c_S = 5.25\%$	38	38.83	1.40	5.61	0.26	104.22	37.26	65.39	105.46	102.18	105.74
$\tau = 15\%$	23	24.51	0.29	1.52	0.43	101.23	24.21	76.72	99.64	106.56	101.38
<i>Panel D. $\gamma = 50\%$ and $\sigma = 30\%$</i>											
$\eta = 4.25\%$	20	20.39	0.87	2.91	0.50	102.04	19.99	81.65	102.06	101.97	105.94
$\eta = 6\%$	26	25.32	1.38	3.78	0.38	102.39	24.72	77.08	104.16	97.37	110.96
$r = 4\%$	22	22.86	1.07	3.27	0.45	102.21	22.37	79.35	101.73	103.90	108.12
$r = 5\%$	25	23.94	1.21	3.54	0.40	102.33	23.39	78.39	104.52	95.75	108.53
$c_J = 6\%$	22	22.91	1.10	3.47	0.45	102.37	22.38	79.46	101.88	104.13	107.60
$c_J = 9\%$	17	20.74	1.12	3.61	22.35	102.49	20.24	81.75	98.49	122.01	105.76
$c_S = 5.25\%$	23	23.01	1.07	3.34	0.43	102.27	22.5	79.26	102.94	100.03	108.01
$\tau = 15\%$	11	11.66	0.19	0.72	0.91	100.53	11.59	88.88	99.86	105.96	101.94

Table IV. Joint Optimality of the Capital and the Debt Priority Structures

This table reports optimal proportions of the total debt, P , and the senior debt, P_S/P , across τ and γ . Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $T = 10$, $c_S = 5\%$, and $c_J = 5.5\%$.

γ	$\tau = 15\%$		$\tau = 25\%$		$\tau = 34\%$		$\tau = 45\%$	
	P	P_S/P (%)	P	P_S/P (%)	P	P_S/P (%)	P	P_S/P (%)
<i>Panel A. $\sigma = 20\%$</i>								
15%	34	0.29	46	0.22	58	0.17	77	0.13
25%	29	0.34	39	0.26	49	0.20	66	0.15
35%	26	0.38	35	0.29	44	0.23	57	0.18
45%	24	0.42	32	0.31	40	0.25	51	0.20
50%	23	0.43	31	0.32	38	0.26	48	0.21
<i>Panel B. $\sigma = 30\%$</i>								
15%	21	0.48	32	0.31	44	0.23	61	0.16
25%	16	0.63	25	0.40	34	0.29	50	0.20
35%	14	0.71	25	0.40	34	0.29	50	0.20
45%	12	0.83	18	0.56	25	0.40	36	0.28
50%	11	0.91	17	0.59	23	0.43	34	0.29

of Frank and Goyal (2009), who report a positive correlation between tax rate and market leverage.

Finally, we provide two robustness checks regarding the firm's capital and debt priority structures.²¹ First, Table IV reports the optimal proportions of total debt and senior debt across realistic values of the tax rate τ (ranging from 15% to 45%) and of the bankruptcy cost rate γ (ranging from 15% to 50%) for a debt maturity T equal to 10 years. The table indicates that the optimal debt priority structure is achieved, across the board, for a very low proportion of senior debt ($P_S/P = \frac{1}{1+\omega}$), when the capital structure is optimal. As expected, this optimal proportion slightly increases with γ , as the firm should lower its probability of default when bankruptcy costs are larger, and slightly decreases with τ , as junior debt provides more tax benefits. In addition, the volatility of assets has a positive, but minor impact on this proportion as, other things being equal, the firm's probability of default increases.

Second, we have computed and reported in Table V the optimal proportion of senior debt, P_S/P , when total debt is sub-optimal. Sub-optimality is arbitrarily obtained by decreasing (under-levered case; Panel A) and increasing (over-levered case; Panel B) by 20% the level of optimal total debt displayed in Table IV.²² The optimal proportion of senior debt increases in both cases (except for the highest values of γ and τ in Panel B). However, this increase is generally much more substantial when the firm is over-levered because as the default threshold rises, expected bankruptcy costs sharply increase, compelling the firm to rely more heavily on senior debt. Alternatively, when the firm's leverage is too low, the incurred loss of tax benefits is greater than the decrease in expected bankruptcy costs. Consequently, the firm must compensate for the difference by slightly increasing the proportion of senior debt. Moreover, as intuition suggests, the effect of higher asset return volatility on the optimal proportion P_S/P is more substantial when the firm is over-levered.

²¹ We thank the anonymous referee for suggesting these checks.

²² Our results are robust to changes in these percentages.

Table V. Optimal Debt Priority Structure When the Capital Structure is Suboptimal

This table reports the optimal proportion of senior debt, P_s/P , when the total debt, P , is suboptimal. Suboptimality is achieved by decreasing (under levered, Panel A) and increasing (over levered, Panel B) the total debt by 20%. Optimal values of P are given in Table IV. Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $T = 10\%$, $c_S = 5\%$, and $c_J = 5.5\%$.

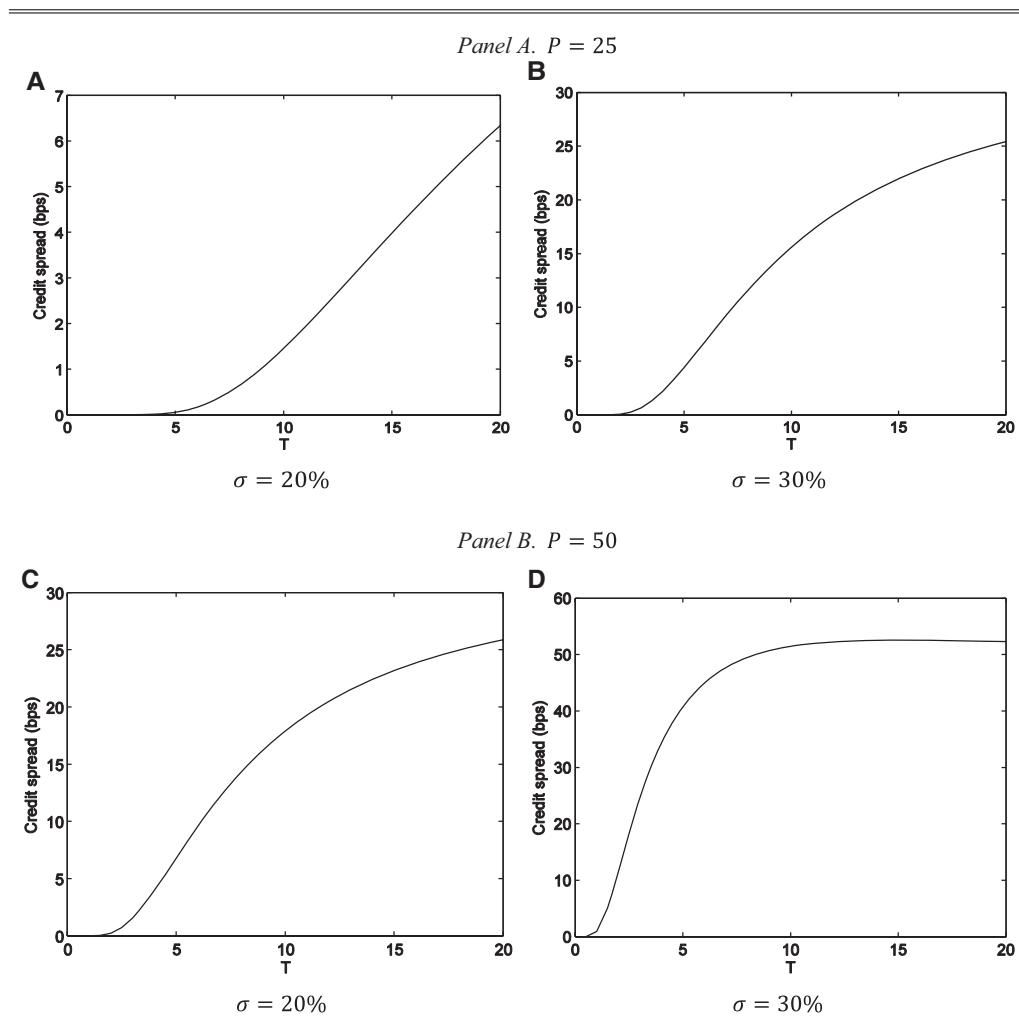
γ	$\tau = 15\%$		$\tau = 25\%$		$\tau = 34\%$		$\tau = 45\%$	
	P	P_s/P (%)	P	P_s/P (%)	P	P_s/P (%)	P	P_s/P (%)
<i>Panel A. Under Levered</i>								
<i>Panel A.1. $\sigma = 20\%$</i>								
15%	27.2	0.37	36.8	0.27	46.4	0.22	61.6	0.16
25%	23.2	0.43	31.2	0.32	39.2	0.26	52.8	0.19
35%	20.8	0.48	28.0	0.36	35.2	0.28	45.6	0.22
45%	19.2	0.52	25.6	0.39	32.0	0.31	40.8	0.25
50%	18.4	0.54	24.8	0.40	30.4	0.33	38.4	0.26
<i>Panel A.2. $\sigma = 30\%$</i>								
15%	16.8	0.60	25.6	0.39	35.2	0.28	48.8	0.20
25%	12.8	0.78	20.0	0.50	27.2	0.37	40.0	0.25
35%	11.2	0.89	16.8	0.60	23.2	0.43	33.6	0.30
45%	9.6	1.04	14.4	0.69	20.0	0.50	28.8	0.35
50%	8.8	1.14	13.6	0.74	18.4	0.54	27.2	0.37
<i>Panel B. Over Levered</i>								
<i>Panel B.1. $\sigma = 20\%$</i>								
15%	40.8	92.16	55.2	67.75	69.6	59.20	92.4	73.59
25%	34.8	81.90	46.8	33.55	58.8	0.17	79.2	0.13
35%	31.2	73.08	42.0	19.05	52.8	0.19	68.4	0.15
45%	28.8	67.01	38.4	4.17	48	0.21	61.2	0.16
50%	27.6	59.06	37.2	2.15	45.6	0.22	57.6	0.17
<i>Panel B.2. $\sigma = 30\%$</i>								
15%	25.2	99.60	38.4	74.22	52.8	78.98	73.2	78.42
25%	19.2	81.25	30.0	49.67	40.8	8.58	60.0	7.67
35%	16.8	95.24	25.2	34.52	34.8	0.29	50.4	0.20
45%	14.4	72.22	21.6	9.26	30.0	0.33	43.2	0.23
50%	13.2	49.24	20.4	3.92	27.6	0.36	40.8	0.25

C. Debt Values and Credit Spreads

The credit spread, or yield spread, is defined as the difference between the yield of a defaultable bond and that of a default-free bond having the same characteristics. Our model generates realistic results. First, it is clear from Proposition (1) that bond prices are decreasing functions of bankruptcy costs. Therefore, credit spreads increase with γ . Second, the term structure of credit spreads for both senior and junior bonds is, in general, upward-sloping and mostly concave as illustrated in Figures 4 and 5, respectively, for two levels of total nominal debt, $P = 25$ (Panel A) and $P = 50$ (Panel B). This result is in line with the empirical evidence reported by Huang and Zhang (2008). They find that over 80% of investment-grade (senior and junior) and high

Figure 4. Credit Spread Term Structure for Senior Bonds

Base case parameters are $A^* = 100$, $\omega = 1$, $r = 4.5\%$, $\eta = 5\%$, $c_S = 5\%$, $c_J = 5.5\%$, $\gamma = 35\%$, and $\tau = 34\%$.

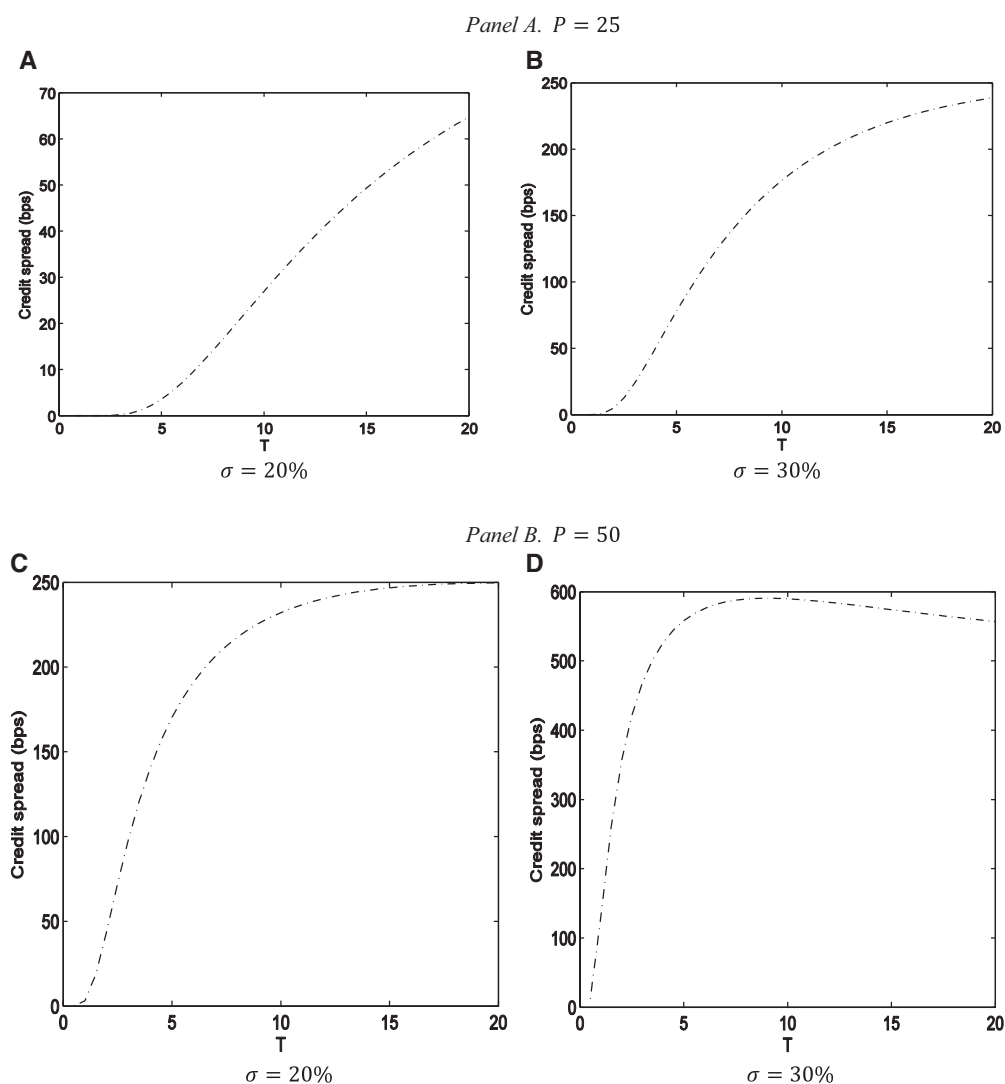


yield coupon-bearing bonds exhibit an upward-sloping shape.²³ The shape of the credit curve can be explained as follows. At issuance, a longer maturity implies a larger number of coupons the firm has to pay and a greater probability of defaulting on any payment. Therefore, as maturity increases, the bond becomes riskier prompting investors to demand a higher yield. However, beyond a certain intermediate date up to which no default has occurred, investors may consider

²³ Huang and Zhang (2008) extend the analysis of Helwege and Turner (1999) to investment-grade bonds and confirm their findings for speculative-grade bonds.

Figure 5. Credit Spread Term Structure for Junior Bonds

Base case parameters are $A^* = 100$, $\omega = 1$, $r = 4.5\%$, $\eta = 5\%$, $c_S = 5\%$, $c_J = 5.5\%$, $\gamma = 35\%$, and $\tau = 34\%$.



the firm to be likely to survive until the bond's maturity. In addition, the remaining number of coupons to be paid is reduced. Therefore, the yield curve flattens toward an asymptote (and may even slightly decrease).

Typically, the spread for senior bonds ranges from 0 to 55 bps and that for subordinated debt ranges from a few to 600 bps depending upon the asset volatility and the overall leverage.

Table VI. Expected Recovery Rates for Senior and Junior Bonds

This table reports, for senior and junior bonds, the recovery rates in the case of premature (prior-to-maturity) bankruptcy due to a cash flow shortage (Panel A) and the expected terminal recovery rates when bankruptcy occurs only at maturity (Panel B).

Panel A. Premature Recovery Rates				
	Case 1		Case 2	
	Senior Debt	Junior Debt	Senior Debt	Junior Debt
Sub-case 1	$\frac{(1-\tau)(1-\gamma)A_b}{P_S}$	0	$\frac{P_S}{P_S}$	$\frac{(1-\tau)(1-\gamma)A_b - P_S}{P_J}$
Sub-case 2	$\frac{(1-\tau)(1-\gamma)A_b}{P_S}$	0	$\frac{P_S}{P_S}$	$\frac{(1-\tau)(1-\gamma)A_b - P_S}{P_J}$
Panel B. Expected Terminal Recovery Rates				
	Case 1			
	Senior Debt	Junior Debt		
Sub-case 1	$\frac{P_S E \left(\mathbf{1}_{t_b > T} \mathbf{1}_{\frac{P_S}{1-\gamma} \leq (1-\tau)A(T) < P} \right)}{P_S}$ $+ \frac{(1-\tau)(1-\gamma)E \left(A(T) \mathbf{1}_{t_b > T} \mathbf{1}_{(1-\tau)A(T) < \frac{P_S}{1-\gamma}} \right)}{P_S}$	$E \left(\frac{[(1-\tau)(1-\gamma)A(T) - P_S] \mathbf{1}_{t_b > T} \mathbf{1}_{\frac{P_S}{1-\gamma} < (1-\tau)A(T) < P}}{P_J} \right)$		
Sub-case 2	$\frac{(1-\tau)(1-\gamma)E(A(T) \mathbf{1}_{t_b > T} \mathbf{1}_{(1-\tau)A(T) < P})}{P_S}$	0		
	Case 2			
	Senior Debt	Junior Debt		
Sub-case 1	$\frac{P_S}{P_S}$	$E \left(\frac{[(1-\tau)(1-\gamma)A(T) - P_S] \mathbf{1}_{t_b > T} \mathbf{1}_{\frac{P_S}{1-\gamma} < (1-\tau)A(T) < P}}{P_J} \right)$		
Sub-case 2	$\frac{P_S}{P_S}$	0		

These values are in line with those found in Avramov, Jostova, and Philipov (2007) and Bedendo, Catheart, and El-Jahel (2007). Avramov, Jostova, and Philipov (2007) report credit spreads ranging from 0 to 1290 bps, and Bedendo, Catheart, and El-Jahel (2007) find credit spreads ranging from 0 to 700 bps.²⁴ Figures 4 and 5 illustrate substantial differences in the pattern of credit spreads between senior and junior bonds. An increase in the total amount of debt leads to an increase in the spread differential. The latter is all the more pronounced as the volatility of assets is higher and the total amount of debt is larger. These results are partially consistent with those in John, Ravid, and Reisel (2010), who find that the spread differential is positive for high-grade bonds and negative for low-grade bonds. The authors attribute their findings to the rating agencies' notching policy. Our results are explained by the differences in the expected recovery rates of both types of bonds. The recovery rate is defined as the proportion of the bond face value received by its holder upon bankruptcy. As demonstrated in Table VI, upon both premature and terminal defaults, a junior debtholder generally recovers a lower percentage of her initial investment than

²⁴ Neither paper, however, makes the distinction between senior and junior bonds.

Table VII. Comparative Statics of Credit Spreads

This table reports comparative statics results of credit spreads for senior and junior bonds. Base case parameters are $A^* = 100$, $r = 4.5\%$, $\eta = 5\%$, $c_S = 5\%$, $c_J = 5.5\%$, $y = 35\%$, $\tau = 34\%$, $\omega = 1$, $P = 25$ and $\sigma = 30\%$.

	Senior Bond			Junior Bond		
	$T = 2$	$T = 10$	$T = 20$	$T = 2$	$T = 10$	$T = 20$
Base case	0	16	25	5	176	239
$\eta = 4.25\%$	0	2	7	4	179	235
$\eta = 6\%$	0	32	53	6	182	239
$r = 4\%$	0	19	34	5	192	261
$r = 5\%$	0	13	18	4	162	218
$c_J = 6\%$	0	11	18	5	183	247
$c_J = 9\%$	0	4	10	5	200	257
$c_S = 5.25\%$	0	14	25	5	181	244
$\omega = 0.5$	1	52	73	6	180	237
$\omega = 1.5$	0	2	7	4	159	215

does a senior bondholder. One notable exception occurs in Case 2 where, with some small, but positive probability, junior debtholders might recover an amount higher than the principal P_J when a coupon incident has occurred (premature default). In that case, the remaining assets' value $(1 - \tau)(1 - \gamma)A_b$ could be higher than P for small values of τ and γ .²⁵

It is interesting to note that the recovery rates in the case of premature default are independent of the volatility of assets (see Panel A of Table VI), but dependent upon the degree of overall leverage (e.g., compare Cases 1 and 2). In contrast, recovery rates when terminal default occurs also depend upon the volatility of assets (see Panel B of Table V). This outcome was expected as the latter is effectively crucial for the probability distribution of the firm's terminal value.

Table VII reports the sensitivities of credit spreads to various parameters. Our first finding is that yield spreads are inversely related to the interest rate level. This result can also be found in Longstaff and Schwartz (1995) and Leland and Toft (1996), where a single class of debt is considered. The recent empirical study by Avramov, Jostova, and Philipov (2007) vindicates this prediction. When considering the spot rates of various maturities (2, 5, 10, and 30 years), Avramov, Jostova, and Philipov (2007) find that changes in these rates significantly and negatively affect credit spreads. Our analysis continues as follows. Recall that r is the (risk neutral) rate of return on the firm's assets. A smaller (higher) level thus implies an increase (a decrease) in the probability of not having enough assets at maturity to ensure full repayment of the bonds. Additionally, the effect is more pronounced in absolute terms for junior debt, particularly at intermediate and long maturities, but the reverse is true in relative terms as senior bonds enjoy rather low spreads in the base case. Increasing the junior coupon rate has a positive impact on the junior credit spread and a negative effect on the senior spread.²⁶ This can be explained as follows. An increase in the coupon rate raises the default threshold, which entails two opposite effects. On the one hand, the probability of hitting the barrier from above is larger, which is detrimental to junior bondholders due to the priority rule. On the other hand, the value of the assets left to debtholders when the barrier is reached is relatively high implying that senior bondholders will hardly be hurt.

²⁵ In this particular and unlikely case, the spread differential may even become negative.

²⁶ This is also true for an increase in the senior coupon rate, but to a slightly lesser extent.

Our second prediction is that an increase in the total payout rate η has a positive impact on the spreads, which is more pronounced for senior bonds.²⁷ This is because the default threshold is lower. Thus, the value of the remaining assets upon bankruptcy is small. In addition, as the drift of the asset value process is smaller, the probability of hitting the barrier prior to maturity or at maturity tends to increase in spite of the lower barrier.

Third, increasing ω from 1.0 to 1.5 (i.e., decreasing the proportion of senior debt from 50% to 40%) leads to a decrease in the spreads of both types of debt.²⁸ The decrease in the senior bond spread has a straightforward interpretation since in Sub-case 2.1, the senior bond is riskless and its holder has a 100% recovery rate (see Table VI) unlike what she can expect in Sub-case 1.1. The subordinated bond spread also decreases because, upon bankruptcy, junior bondholders will receive more when senior debt is small. When ω decreases to 0.5 (producing Sub-case 1.2), the proportion of senior debt rises to 67% and senior bondholders expect a lower recovery rate (see Table VI). This significantly increases the credit spread (except for very short maturities where the spread is negligible). The junior credit spread is essentially unaffected because, in this unfavorable sub-case, junior bondholders receive nothing upon bankruptcy whether it occurs prematurely or at the maturity date.

Finally, Figure 6 plots the senior and junior debt values as functions of the firm's asset value when the total debt principal is equal to 50 and $\omega = 1$ (same nominal value of 25 for each type of debt). We first note that, in contrast to what is reported by Black and Cox (1976), senior and junior bond values generally exhibit similar patterns. They are, as expected, concave increasing with total asset value and decreasing with asset volatility.²⁹ As the after-tax value of total assets increases, the probabilities of (premature and terminal) default obviously decrease and the bond market values rise toward their corresponding default-free values (Equation (21)). Additionally, since the absolute priority rule is enforced, the price of senior bonds reaches its default-free level at lower after-tax asset values than does the price of the subordinated bonds. Moreover, when asset returns are more volatile and thus liquidity and solvency risks are more severe, the prices of (both senior and junior) intermediate and long-term defaultable bonds increase at a slower rate toward their corresponding default-free levels as the after-tax value of assets increases (compare Figures (6e) and (6f) with (6b) and (6c), respectively). Finally, a salient feature of Figure 6 is that junior debts may be more valuable than senior ones. When asset volatility is low and the debt maturity is short, the market values of both types of bonds tend toward their respective risk-free levels (the spreads tend to zero with the probability of default) as the value of total assets (and thus of equity) becomes larger. Since the coupon on junior debt is greater than that on senior bonds, the result follows. This can also be true either for high asset volatility and short maturities or for low volatility and medium and long maturities, provided the initial value of the assets is large relative to the total debt principal. For high asset volatility and medium and long maturities, senior debt is more valuable irrespective of the initial value of the firm's assets (but relatively less and less so as leverage decreases) due to its lower probability of default see Figures (6e) and (6f)).

D. Default Probabilities

The valuation model allows us to associate the physical (historical) probabilities of default with credit spreads provided we assume as given the level of the market risk premium. In the sequel, we investigate the shape and level of the term structure of premature and terminal default

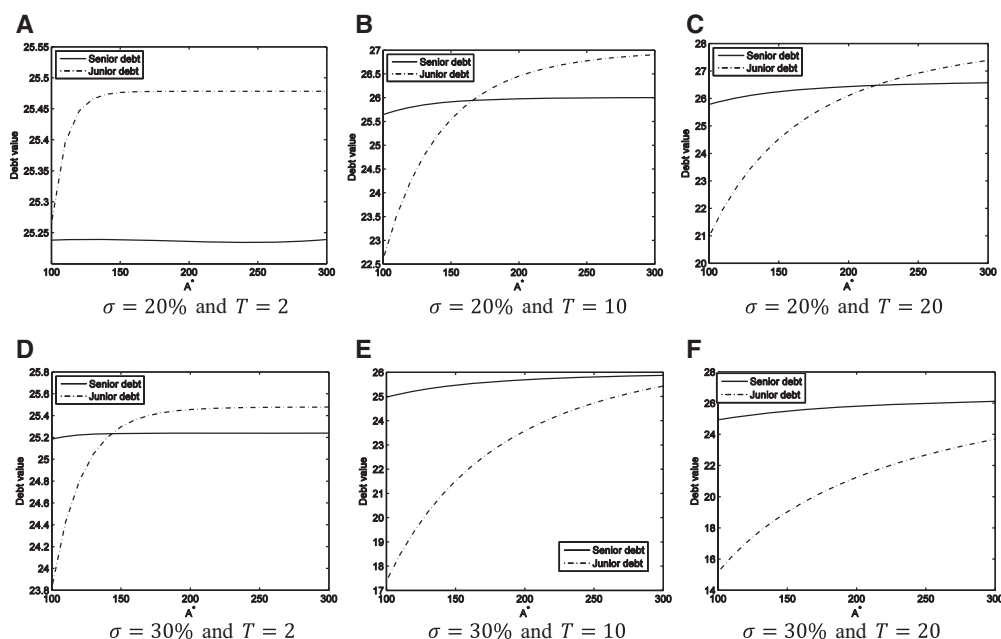
²⁷ Results not reported here find that the impact of η on the junior credit spread is not monotonous.

²⁸ The base case (with $\omega = 1$) corresponds to Sub-case 1.1, and $\omega = 1.5$ corresponds to Sub-case 2.1.

²⁹ The concave shape of the senior bond is more pronounced for large bankruptcy costs and high asset volatility.

Figure 6. Senior and Junior Debt Value Pattern as a Function of Total Assets

Base case parameters are $P = 50$, $\omega = 1$, $r = 4.5\%$, $\eta = 5\%$, $c_S = 5\%$, $c_J = 5.5\%$, $\gamma = 35\%$, and $\tau = 34\%$.



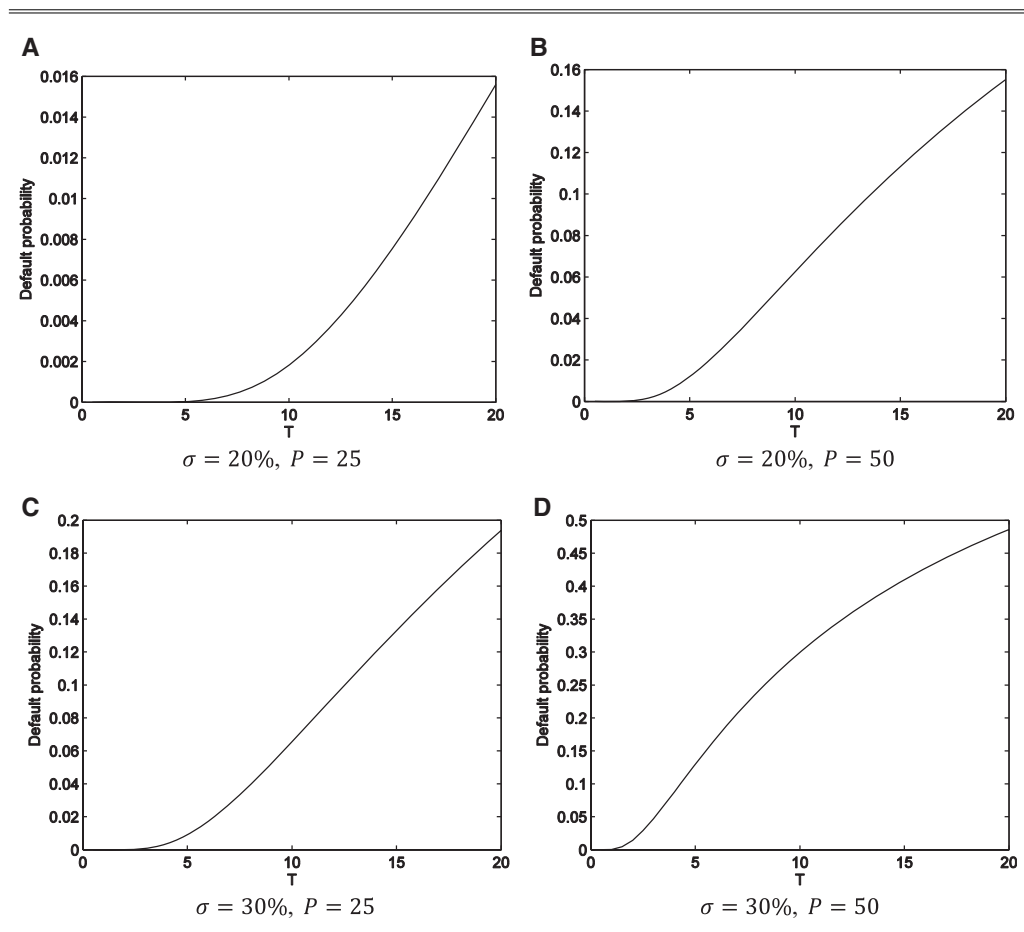
probabilities generated by our setting (see Equations (12) and (14), respectively). We assume, as do Ju, Parrino, Potoshman, and Weisbach (2005), an equity risk premium of 6% making the drift of the equity value process, μ_E , equal to 10.5% as the baseline risk-free rate is 4.5%. Consequently, the drift of the firm's assets, μ_A , is equal to 9.4% for our baseline parameters with $\omega = 1$, $T = 10$ and $P = 25$.³⁰ Similarly, μ_A is equal to 8.5% for $P = 50$.

We examine the physical prior to maturity default probabilities first. Figure 7 plots the term structure of these probabilities for different values of asset volatility, σ , and total debt, P . These probabilities are quite substantial (and may reach 48%), except for low asset volatility and smaller leverage. Figure 7 illustrates, as expected, that the term structure has a monotonous positive slope due to the large number of coupons that the firm has to cash out and that the probability of default also sharply increases with both σ and P . For instance, increasing P , all else being equal, raises the default threshold A_b . Therefore, the probability that the value $A(t)$ of assets hits the barrier increases. Furthermore, for low σ and P , the term structure of default probability is convex increasing. The pattern switches progressively to concavity as the firm becomes riskier and highly levered (see Figure 7d). This pattern reflects two phenomena. First, the uncertainty associated with the coupon payments increases rapidly as the maturity and thus the number of coupons increase. Therefore, this generates convexity. Second, the rate at which uncertainty increases

³⁰ The formal relationship between μ_A and μ_E is derived in Appendix C.

Figure 7. Term Structure of Default Probabilities

This figure depicts the term structure of physical prior-to-maturity default probabilities. Base case parameters are $A^* = 100$, $\mu_E = 10.5\%$, $\eta = 5\%$, $\omega = 1$, $\tau = 34\%$, $c_S = 5\%$, and $c_J = 5.5\%$.



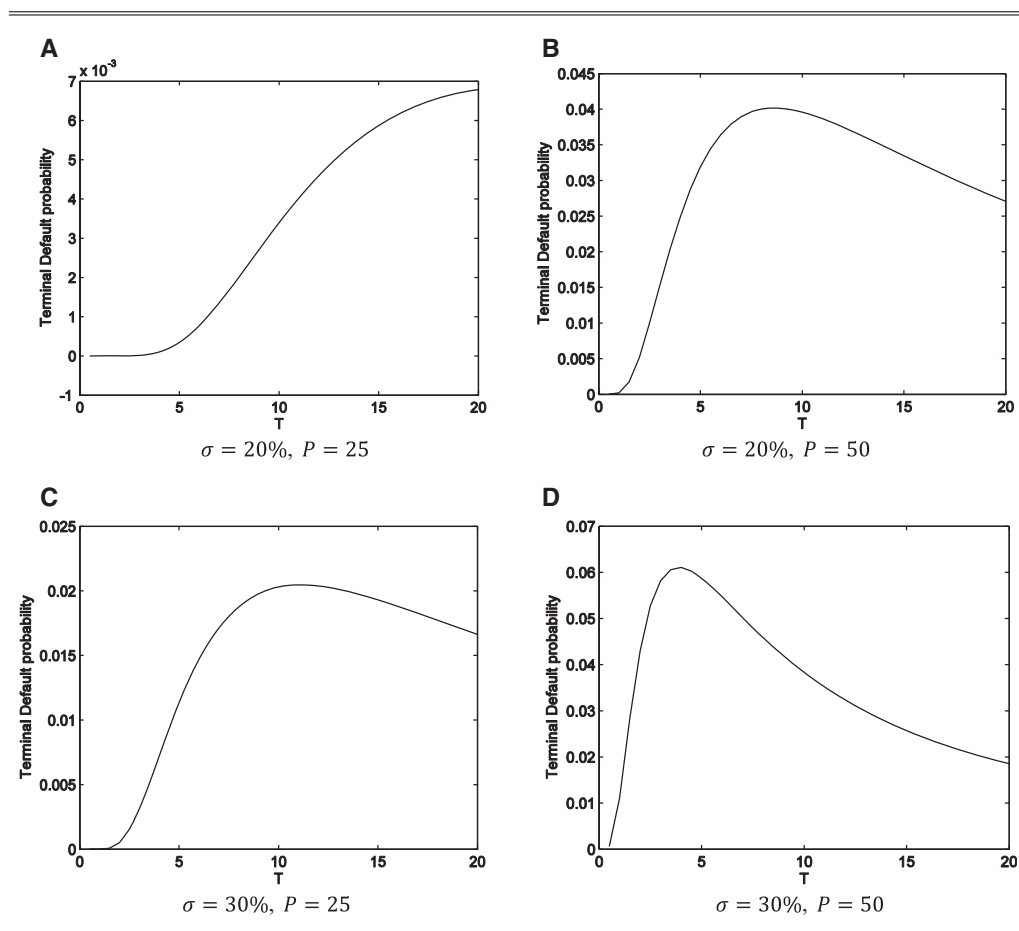
declines because, as time goes by, fewer coupons are left due creating concavity in the default probability structure.

Moreover, from Equation (2) defining the default threshold and the identity $P_S(1 + \omega) = P$, it is clear that the threshold is strictly increasing in c_S and c_J and decreasing in η . Therefore, the default probability increases with c_S and c_J and declines with η , as economic intuition suggests.

The term structure of physical probabilities of terminal default is depicted in Figure 8. This structure exhibits a hump-shaped pattern, except for low asset volatility and low overall leverage. The hump becomes more pronounced as the firm's riskiness and/or the total amount of debt increases. In addition, the terminal default probabilities are substantially lower, in particular for intermediate and long maturities, than the premature default probabilities regardless of the assets' risk and overall leverage. This is because, for the firm not to have experienced early default between dates 0 and T , the cash flows generated by the assets, hence the very value of these assets,

Figure 8. Term Structure of Terminal Default Probabilities

The figure depicts the physical default probabilities at debt maturity when the default has not occurred before. Base case parameters are $A^* = 100$, $\mu_E = 10.5\%$, $\eta = 5\%$, $\omega = 1$, $\tau = 34\%$, $c_S = 5\%$, and $c_J = 5.5\%$.



must have been large enough. If so, the probability of a terminal value of assets being too small to cover the bondholders' claims must be tiny. The (unreported) overall default probabilities (i.e., the sum of the terminal and premature default probabilities) are encouragingly realistic and aligned on the values reported, for instance, by Emery, Ou, Tennant, Kim, and Cantor (2008).

V. Conclusion

We derive closed-form solutions for the market values of a defaultable firm's debt and equity under heterogeneous debt priority, the absolute priority rule, and immediate liquidation of the firm upon bankruptcy. We analyze the priority structure that maximizes the firm's value and the joint optimal junior-senior bond mix and overall leverage in a setting where the firm's stakeholders

face a liquidity risk prior to the bonds' maturity and a solvency risk at maturity. Liquidity risk has a much higher probability of materializing than solvency risk and contributes more heavily to the derived credit spreads. The spread differentials between senior and junior bonds depend upon the debt priority structure, the riskiness of the firm's assets, the firm's overall leverage, and the debt maturity. They take on realistic values for plausible values of the exogenous parameters. Our setting could be extended in two potentially valuable directions. One would be to account for possible debt restructuring in the context of Chapter 11, for instance in the spirit of François and Morellec (2004). The other would be to allow for potential conflicts of interest that may arise between the firm's various stakeholders in the case of premature default due to liquidity reasons. This is left to future research.

Appendix

0. Preliminaries

Let W be a standard Brownian motion under a given probability measure. Consider the stochastic process $(X(t))_{t \geq 0}$, the dynamics of which is governed by the following SDE:

$$X(t) = \mu t + \sigma W_X(t). \quad (0-1)$$

Let $m^X(t) = \min_{s \in [0, t]} X(s)$ be the minimum value reached by $X(t)$ on the time interval $[0, t]$. We state the following well known Lemmas (see for instance Musiela and Rutkowski (2005)):

Lemma 1. $\forall x, y \in \mathbb{R}$ such that $y \leq 0$ et $y \leq x$,

$$\mathbb{P}(X_t \geq x, m_t^X \geq y) = \Phi\left(\frac{-x + \mu t}{\sigma \sqrt{t}}\right) - e^{2\mu y \sigma^{-2}} \Phi\left(\frac{2y - x + \mu t}{\sigma \sqrt{t}}\right). \quad (0-2)$$

$$\forall y \leq 0,$$

$$\mathbb{P}(m_t^X \geq y) = \Phi\left(\frac{-y + \mu t}{\sigma \sqrt{t}}\right) - e^{2\mu y \sigma^{-2}} \Phi\left(\frac{y + \mu t}{\sigma \sqrt{t}}\right). \quad (0-3)$$

Also, we have by complementarity:

$$\mathbb{P}(m_t^X \geq y) = \mathbb{P}(X_t \geq x, m_t^X \geq y) + \mathbb{P}(X_t \leq x, m_t^X \geq y). \quad (0-4)$$

A. Proposition 1 – Case 2

Proposition 1. *Given the dynamics under the risk-neutral measure \mathbf{Q} of the firm's asset value (Equation (1)), the time 0 prices of defaultable senior (D_S) and junior (D_J) coupon bonds are equal to:*

Case 2: $(1 - \gamma)(1 - \tau)A_b > P_S$:

Sub-case 2.1: $P_S \leq P(1 - \gamma)$:

$$D_S^{(2,1)}(0, T) = \frac{c_S P_S}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)] + P_S G(T) + e^{-rT} P_S (1 - \Pi(T, t_b)), \quad (A-1)$$

and

$$\begin{aligned}
 D_J^{(2,1)}(0, T) = & \frac{c_J P_J}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)] + ((1 - \tau)(1 - \gamma)A_b - P_S) G(T) \\
 & + e^{-rT} P_J \left(\Phi(d_{1, \frac{P}{1-\tau}}) - \left(\frac{A_b}{A} \right)^{2\frac{r-\eta}{\sigma^2}-1} \Phi(d_{2, \frac{P}{1-\tau}}) \right) + (1 - \tau)(1 - \gamma) \\
 & \times e^{-\eta T} A \left(\Phi(d_{3, A_b}) - \Phi(d_{3, \frac{P}{1-\tau}}) + \left(\frac{A_b}{A} \right)^{2\frac{r-\eta}{\sigma^2}+1} [\Phi(d_{4, \frac{P}{1-\tau}}) - \Phi(d_{4, A_b})] \right) \\
 & - e^{-rT} P_S \left(\Phi(d_{1, A_b}) - \Phi(d_{1, \frac{P}{1-\tau}}) + \left(\frac{A_b}{A} \right)^{2\frac{r-\eta}{\sigma^2}-1} [\Phi(d_{2, \frac{P}{1-\tau}}) - \Phi(d_{2, A_b})] \right).
 \end{aligned} \tag{A-2}$$

Sub-case 2.2: $P_S > P(1 - \gamma)$:

$$\begin{aligned}
 D_S^{(2,2)}(0, T) = D_S^{(1,1)}(0, T) = & \frac{c_S P_S}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)] \\
 & + P_S G(T) + e^{-rT} P_S (1 - \Pi(T, t_b)),
 \end{aligned} \tag{A-3}$$

and

$$\begin{aligned}
 D_J^{(2,2)}(0, T) = & \frac{c_J P_J}{r} [e^{-rT} (\Pi(T, t_b) - 1) + 1 - G(T)] + ((1 - \tau)(1 - \gamma)A_b - P_S) G(T) \\
 & + e^{-rT} P_J (1 - \Pi(T, t_b)),
 \end{aligned} \tag{A-4}$$

with

$$\begin{aligned}
 d_{1,H} &= \frac{\log\left(\frac{A}{H}\right) + \left(r - \eta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
 d_{2,H} &= \frac{\log\left(\frac{A_b^2}{HA}\right) + \left(r - \eta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
 d_{3,H} &= \frac{\log\left(\frac{A}{H}\right) + \left(r - \eta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
 d_{4,H} &= \frac{\log\left(\frac{A_b^2}{HA}\right) + \left(r - \eta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \\
 H &= \left\{ A_b, \frac{P_S}{(1 - \tau)(1 - \gamma)}, \frac{P}{1 - \tau} \right\}.
 \end{aligned}$$

Note that when $A_b > \frac{P}{1-\tau}$, the third and fourth terms in Equation (A-2) become nil.

B. Proof of Proposition 1

Let $X(T) = (r - \eta - \frac{1}{2}\sigma^2)T + \sigma W(T)$. Applying Ito's lemma to Equation (1) in the text yields:

$$A(T) = Ae^{X(T)}. \quad (\text{B-1})$$

Define $\hat{X}(T) = (r - \eta + \frac{1}{2}\sigma^2)T + \sigma \hat{W}(T)$, where \hat{W} is a standard Brownian motion under the probability measure $\hat{\mathbf{Q}}$ defined by the following Radon-Nikodym derivative:

$$\frac{d\hat{\mathbf{Q}}}{d\mathbf{Q}} = \exp \left(-\frac{1}{2}\sigma^2 t + \sigma W(t) \right). \quad (\text{B-2})$$

Then, Girsanov's theorem states that:

$$d\hat{W}(t) = dW(t) - \sigma dt. \quad (\text{B-3})$$

In the following, we compute various conditional expectations:

$$\mathbb{E} \left(1_{t_b > T} 1_{A(T) \geq H} \right) = \mathbb{P}(t_b > T, A(T) \geq H), \quad (\text{B-4})$$

$$= \mathbb{P} \left(m^X(T) \geq \log \left(\frac{A_b}{A} \right), X(T) \geq \log \left(\frac{H}{A} \right) \right), \quad H = \{P_S, P\}. \quad (\text{B-5})$$

Applying Lemma (1), Equation (0-2), we obtain:

$$\begin{aligned} \mathbb{E} \left(1_{t_b > T} 1_{A(T) \geq H} \right) &= \Phi \left(\frac{\log \left(\frac{A}{H} \right) + \left(r - \eta - \frac{1}{2}\sigma^2 \right) T}{\sigma \sqrt{T}} \right) \\ &\quad - \left(\frac{A_b}{A} \right)^{2\frac{r-\eta}{\sigma^2}-1} \Phi \left(\frac{\log \left(\frac{A_b^2}{HA} \right) + \left(r - \eta - \frac{1}{2}\sigma^2 \right) T}{\sigma \sqrt{T}} \right). \end{aligned} \quad (\text{B-6})$$

From Equation (0-4),

$$\mathbb{P}(t_b > T, A(T) < H) = \mathbb{P}(t_b > T) - \mathbb{P}(t_b > T, A(T) \geq H). \quad (\text{B-7})$$

$$\mathbb{E} \left(A(T) 1_{t_b > T} 1_{A(T) < H} \right) = Ae^{(r-\eta)T} \hat{\mathbb{P}}(t_b > T, A(T) < H), \quad (\text{B-8})$$

thus, using Equation (0-4), we obtain:

$$\hat{\mathbb{P}}(t_b > T, A(T) < H) = \hat{\mathbb{P}}(t_b > T) - \hat{\mathbb{P}}(t_b > T, A(T) \geq H). \quad (\text{B-9})$$

Using Lemmas (0-3) and (0-2) leads to:

$$\hat{\mathbb{P}}(t_b > T, A(T) < H) = \Phi \left(\frac{\log \left(\frac{H}{A} \right) - \left(r - \eta + \frac{1}{2}\sigma^2 \right) T}{\sigma \sqrt{T}} \right)$$

$$\begin{aligned}
& - \Phi \left(\frac{\log \left(\frac{A_b}{A} \right) - \left(r - \eta + \frac{1}{2} \sigma^2 \right) T}{\sigma \sqrt{T}} \right) \\
& + \left(\frac{A_b}{A} \right)^{2 \frac{r-\eta}{\sigma^2} + 1} \left[\Phi \left(\frac{\log \left(\frac{A_b^2}{HA} \right) + \left(r - \eta + \frac{1}{2} \sigma^2 \right) T}{\sigma \sqrt{T}} \right) \right. \\
& \quad \left. - \Phi \left(\frac{\log \left(\frac{A_b}{A} \right) + \left(r - \eta + \frac{1}{2} \sigma^2 \right) T}{\sigma \sqrt{T}} \right) \right],
\end{aligned} \tag{B-10}$$

with $H = \{A_b, \frac{P_S}{(1-\tau)(1-\gamma)}, \frac{P}{1-\tau}\}$. Furthermore, we have:

$$\begin{aligned}
\mathbb{E} \left(1_{t_b > T} 1_{\frac{P_S}{(1-\tau)(1-\gamma)} < A(T) < \frac{P}{1-\tau}} \right) &= \mathbb{P} \left(t_b > T, A(T) < \frac{P}{1-\tau} \right) \\
&\quad - \mathbb{P} \left(t_b > T, A(T) < \frac{P_S}{(1-\tau)(1-\gamma)} \right),
\end{aligned} \tag{B-11}$$

and

$$\begin{aligned}
\mathbb{E} \left(A(T) 1_{t_b > T} 1_{\frac{P_S}{(1-\tau)(1-\gamma)} < A(T) < \frac{P}{1-\tau}} \right) &= A e^{(r-\eta)T} \hat{\mathbb{P}} \left(t_b > T, A(T) < \frac{P}{1-\tau} \right) \\
&\quad - \hat{\mathbb{P}} \left(t_b > T, A(T) < \frac{P_S}{(1-\tau)(1-\gamma)} \right).
\end{aligned} \tag{B-12}$$

C. The Expressions of σ_E and μ_A

Given the dynamics of $A(t)$ under the physical probability measure we apply Ito's lemma on $E(A, t)$:

$$\begin{aligned}
dE(A, t) &= \frac{\partial E}{\partial t} dt + \frac{\partial E}{\partial A} dA + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} dA^2 \\
&= \left[\frac{\partial E}{\partial t} + \frac{\partial E}{\partial A} A(t) (\mu_A - \eta) + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A^2(t) \sigma^2 \right] dt + \frac{\partial E}{\partial A} A(t) \sigma dW(t).
\end{aligned} \tag{C-1}$$

The dynamics of the market equity value is given by:

$$dE(t) = E(t) [(\mu_E - \eta_E) dt + \sigma_E dW(t)], \tag{C-2}$$

where η_E is the payout rate. By identification between Equations (C-1) and (C-2), we obtain:

$$\mu_A = \eta + \frac{E(t) (\mu_E - \eta_E) - \frac{\partial E}{\partial t} - \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A^2(t) \sigma^2}{\frac{\partial E}{\partial A} A(t)}. \tag{C-3}$$

$$\sigma_E = \frac{\partial E}{\partial A} \frac{A(t)}{E(t)} \sigma. \tag{C-4}$$

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