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LETTERS TO THE EDITOR

Low-temperature operational regime of an adiabatic Brownian motor

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The motion of a Brownian particle in two cyclically-switching spatially-periodic asymmetric potentials with the cycle period longer than the relaxation time is considered. It is shown that at low temperatures a directed movement occurs in the case when the positions of the absolute maxima and minima of the two switching potentials alternate or an asymmetric potential is cyclically switched ON and OFF. The obtained results elucidate the role of diffusion processes and force effects arising from the potential switching in the operational mechanisms of Brownian motors. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4876230]

Brownian motors represent a model of the processes where the directed motion of nanoparticles emerges in the absence of average applied forces and concentration gradients. Such processes may be caused by non-equilibrium effects of various origins. These effects are described by the stochastic or deterministic time dependence of the particle potential energy which arises due to either changes in the parameters of the particle itself (conformational state, charge, and multipole moments) or external forces with zero mean value.[1,2] Brownian motors, similar to conventional motors, can perform useful work (against the loading or friction forces), the ratio of which to the external energy supplied to the system determines the efficiency of their operation.[3] The most efficient are the Brownian motors, in which the change in the particle potential energy is accompanied by a shift of their extrema.[4–6]

The interest in Brownian motors has been historically originated from the studies of the properties of protein motors.[7,8] Initially, the models of muscle contraction have been used in which the shift of a myosin molecule relative to actin filaments arises directly due to ATP hydrolysis (tight mechanochemical coupling).[9] The power stroke of this protein motor is not associated with diffusive (or thermally activated) particle motion.[10] That is why for a long time such power-stroke motors were not considered Brownian. Later it turned out that the concept of a Brownian motor is equally well applicable to the processes with both weak and strong mechanochemical coupling which generates directional movement.[11] In the first case, it suffices to consider the fluctuations of the potential energy with the unchanged positions of the extrema, while in the second case they are changing.[12]

The low-temperature limit of the mean velocity of a Brownian motor allows to distinguish the processes where diffusion plays a dominant role and those where it is not important. Since the diffusive motion vanishes in the limit of low temperatures, the analysis of low-temperature behavior of a Brownian motor provides the key to the separation of the two processes. In this report, a simple explicit expression for the mean velocity of an adiabatic Brownian motor was obtained, which implies several nontrivial conclusions about how the alternation of the extremum positions of the fluctuating potential provides functionality of the motor and controls the direction of its movement. Note that a similar analysis of the adiabatic regime in the low-temperature limit was conducted for the problem of motion of a vortex in a superconductor with a periodic pinning potential.[13,14]

Let us consider the motion of a Brownian particle, the potential energy of which fluctuates between two periodic (with a period L) potential landscapes $U_\sigma(x)(\sigma = \pm 1)$ with the lifetimes τ_σ sufficiently long to establish the thermodynamic equilibrium (the adiabatic approximation). In the adiabatic approximation, the distinction of stochastic and deterministic switching between the potential landscapes disappears,[15] therefore the fluctuation process can be considered as cyclic with a period of $\tau = \tau_1 + \tau_{-1}$. The particle velocity $\langle v \rangle$, averaged over the period τ of the potential energy fluctuations is expressed as a simple analytical formula:[12,16,17]

$$\langle v \rangle = \frac{L}{\tau}\Phi, \quad \Phi = \int_0^L dx[q_1(x) - q_{-1}(x)]\int_0^x dy[\rho_1(y) - \rho_{-1}(y)], \tag{1}$$

where

$$\rho_\sigma(x) = \frac{\exp[-\beta U_\sigma(x)]}{\int_0^L dx \exp[-\beta U_\sigma(x)]},$$

$$q_\sigma(x) = \frac{\exp[\beta U_\sigma(x)]}{\int_0^L dx \exp[\beta U_\sigma(x)]}, \quad \beta = (k_B T)^{-1}, \tag{2}$$

k_B is the Boltzmann constant and T is the absolute temperature.

Equation (1) is valid when the time for establishing the equilibrium Maxwell distribution in the phase space of velocities $\tau_v = m/\zeta$ (where m is the mass of the particle and ζ is the friction coefficient) is the shortest characteristic time of the system, while the period τ is the longest. Several characteristic times of the system can be noted: the diffusion time $\tau_D = \zeta L^2/k_B T$ through the period L of the potential, the time $\tau_s = \zeta L^2/V$ required for a particle to roll down from the maximum of the potential landscape with the amplitude V, and the time $\tau_b = \tau_s \exp(\Delta V/k_B T)$ required to overcome the local barriers ΔV, if they are present in the potential profile. The low-temperature approximation assumes that $k_B T \ll V$ and $\tau_s \ll \tau_D$, so, the movement in a given potential profile dominates over the diffusion motion. Since a dichotomous process contains two switching potential landscapes with different amplitudes, we should distinguish between the general case of a nonzero landscape when the low-temperature approximation is justified when $\tau_v \ll \tau_s < \tau_b \ll \tau$, and a special case of the absence of the potential relief in one of the states of the dichotomous process (so called on-off ratchet[1]), in which $\tau_v \ll \tau_D \ll \tau$. These two cases are discussed separately below.

Let a_σ and b_σ ($a_\sigma, b_\sigma \in (0,L)$) denote the coordinates of the smallest minimum and the largest maximum of the potential landscape $U_\sigma(x)$, respectively (Fig. 1). Then, in the low-temperature limit, the functions $\rho_\sigma(x)$ and $q_\sigma(x)$, involved in Eq. (1), can be approximated by delta functions: $\rho_\sigma(x) = \delta(x - a_\sigma)$, $q\sigma(x) = \delta(x - b_\sigma)$. Their substitution into Eq. (1) leads to the following result:

$$\Phi = \sum_{\sigma=\pm 1} [\theta(b_\sigma - a_\sigma) - \theta(b_\sigma - a_{-\sigma})], \tag{3}$$

where $\theta(x)$ is the theta function (equal to 1 for $x > 0$, 1/2 at $x = 0$, and -1 for $x < 0$).

The presence or absence of movement in the low-temperature limit and its direction are determined by the relative positions of the four extrema a_σ and b_σ ($\sigma = \pm 1$). First, we note that the symmetry of each potential imposes certain conditions on its extrema, $|a_\sigma - b_\sigma| = L/2$, which leads to vanishing of the average velocity, as it should be. For the potentials with a fluctuating amplitude, the positions of the extrema for the two potentials coincide: either $a_1 = a_{-1}$ and $b_1 = b_{-1}$, when the positions of minima and maxima coincide in the both potentials, or $a_1 = b_{-1}$ and $b_1 = a_{-1}$ when the minimum of one potential becomes the maximum of the other and vice versa. It is easy to verify that in these cases $\Phi = 0$. Mean speed is nonzero only in the case of alternating the positions of maxima and minima of the fluctuating potentials (see the circular diagrams in Fig. 1). Positive and negative directions of movement (left and right side of Fig. 1) correspond to looping through the extrema clockwise and counterclockwise, respectively. Analysis of trajectories (arrows along and across the potential energy profiles) showed that the net direction of movement is determined by the direction from the maximum to the minimum of a potential between which a maximum of another potential is located. With this arrangement of the extrema, a particle moves to the minimum point in each potential landscape, bypassing the barriers of another potential due to the switching between the two potentials.

Let us consider the case when there is no potential landscape in one of the states of the dichotomous process (on-off ratchet): $U_\sigma(x) = (1/2)(\sigma + 1)V(x)$ and $V(x) = V(x + L)$ (Fig. 2). The absence of the potential at $\sigma = -1$ provides purely diffusive motion in this state and the uniform equilibrium distribution $\rho_{-1}(x) = q_{-1}(x) = L^{-1}$. Using this uniform distribution in Eq. (1) and the distributions $\rho_1(x) = \delta(x - a)$ and $q_1(x) = \delta(x - b)$ for the state $\sigma = 1$ in the low-temperature limit (where a and b are the coordinates the lowest minimum and the highest maximum of $V(x)$), we obtain

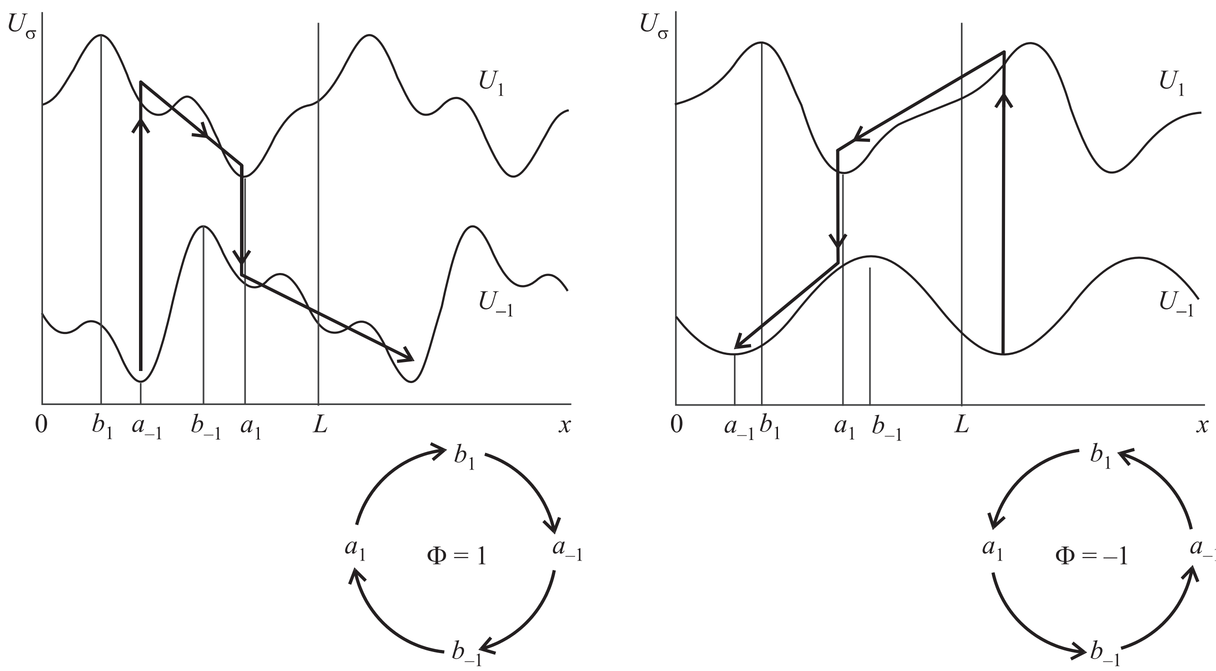

FIG. 1. Mechanism of the directional movement to the right and to the left (left and right panels) upon switching between the potentials. Motion at low temperatures becomes possible when the positions of maxima and minima of the switching potentials alternate.

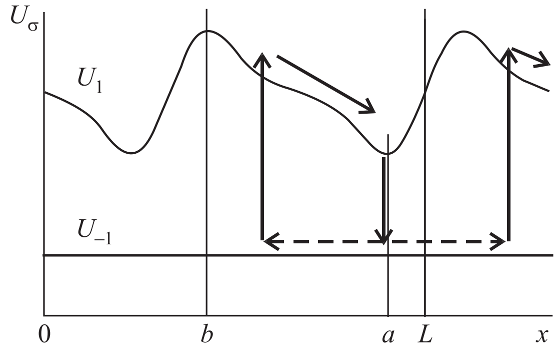

FIG. 2. Low-temperature mechanism of the directional movement upon switching an asymmetric potential ON and OFF.

$$\Phi = \left(\frac{|a-b|}{L} - \frac{1}{2}\right)\mathrm{sgn}(a-b), \quad \mathrm{sgn}\, x = 2\theta(x) - 1. \tag{4}$$

As a result of the diffusion stage of the movement in the state $\sigma = -1$ (dashed lines with arrows in Fig. 2) the particle may end up in the minima of the potential $U_1(x)$, belonging to various periods L. The effective mean particle displacement occurs in the direction from the minimum to the nearest maximum of the potential $U_1(x)$. It is assumed that the potential $U_1(x)$ is asymmetric, i.e., $|a - b| \neq L/2$.

Thus, the operation of adiabatic Brownian motors at low temperatures is possible only for the specific location of extreme points of the fluctuating potential profiles. Directional movement occurs upon alternating the positions of their absolute maxima and minima and whenever one of the profiles is absent and the second is asymmetric. Since the characteristic relaxation times of the system depend on the shape of the potential landscape and increase with decreasing temperature and the adiabaticity of the process is achieved due to sufficiently long lifetimes for the both potential profiles (exceeding all the relaxation times), the average velocity of directional movement in this mode will not be very small only for the profiles with one maximum and minimum per period. The conducted analysis of the low-temperature behavior of adiabatic Brownian motors reveals the role of diffusion processes and force effects which occur upon switching the potentials for the functioning mechanisms of Brownian motors.

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