SEPARATION OF ISOCHROMATICS AND ISOCLINICS USING FOURIER TRANSFORM

by Y. Morimoto, Y. Morimoto, Jr. and T. Hayashi

In photoelasticity, the image obtained in the field of a plane polariscope scope consists of isochromatics and isoclinics.[1] In analysis of stress, the basic method is to obtain the difference of the principal stress and the principal stress direction using the isochromatic fringes and the isoclinics fringes.[2-5] However, the isochromatic fringes obtained in a circular polariscope have the error due to the ellipticity of the circular polarized incident light and so on.[5] The positions of the isoclinics obtained in a plane polariscope are not so accurate because of the wide width of the isoclinic lines and the accuracy of quarter wave plates. Moreover, photoelastic stress analysis of the whole field of a specimen is tedious and time consuming. Recently, image processing is widely used to analyze the images obtained in photoelasticity.[2-9] Umezaki *et al.*[8] proposed a method to separate the isoclinic lines from many images obtained by rotating the crossed polaroids in a plane polariscope. Image processing allows high speed and more accurate analysis. The ability of computers has been remarkably progressed in memory space and processing speed. By using this ability, the authors[9] developed a software of 3-D image processing, and applied it to the analysis of the 3-D (x, y, θ) image data consisted with the spatial coordinates (x, y) and the angle θ of the crossed polaroids. In the method of Ref. (9), the isochromatics is obtained by adding up all the brightness values on each (x, y) of the (x, y, θ) 3-D image. The distribution of principal stress direction is obtained by detecting the angle θ when the brightness value is the maximum on each (x, y). The detected angle θ of the polaroids is the same as the principal stress direction on each (x, y). By using the 3-D image processing, various analysis became easier and faster than using 2-D one.

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In this paper, a new method is proposed to separate the isochromatics and the isoclinics using Fourier transform on the 3-D image processing.

THEORY OF SEPARATION OF ISOCHROMATICS AND ISOCLINICS USING FOURIER TRANSFORM

The light intensity obtained in a dark-field plane polariscope, shown in Fig. 1, is expressed by

$$i(\theta) = a^2 \sin^2\{2(\phi - \theta)\} \cdot \sin^2(m\pi) \tag{1}$$

$$= \frac{a^2}{2}\{1 - \cos 4(\phi - \theta)\} \cdot \sin^2(m\pi) \tag{2}$$

where a is the amplitude of the incident polarized light, ϕ is the angle of the principal stress and θ is the angle of the analyzer and m is the isochromatic fringe order. The fringe order m is expressed as follows:

$$m = (C/\lambda)d(\sigma_1 - \sigma_2) \tag{3}$$

where C is the photoelastic constant, d is the thickness

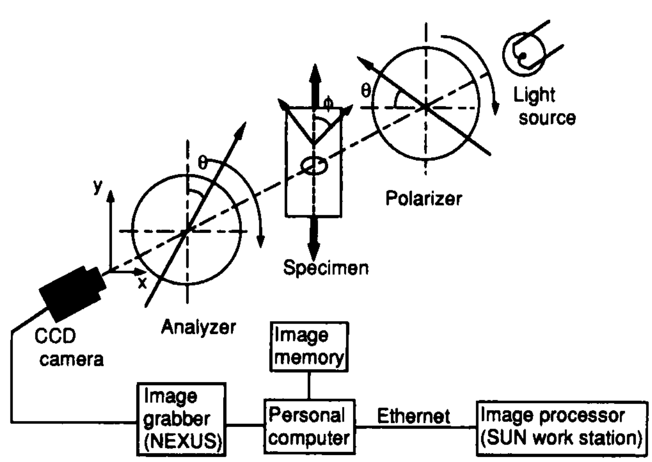

Fig. 1—*Schematic diagram of polariscope and image processor*

of the specimen, λ is the wavelength and $(\sigma_1 - \sigma_2)$ is the principal stress difference of the specimen.

Equation (1) is expressed in the Fourier series with respect to θ, because it is a periodic function with a period $T = \pi/2$. That is

$$i(\theta) = \sum_{n=-\infty}^{\infty} c_n \exp(jn\omega_0\theta) \tag{4}$$

where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} i(\theta)\cdot\exp(-jn\omega_0\theta)d\theta \tag{5}$$

and

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi/2} = 4 \tag{6}$$

The Fourier transform of eq (4) is

$$\begin{aligned} I(\omega) &= \int_{-\infty}^{\infty} i(\theta)\cdot\exp(-j\omega\theta)d\theta \\ &= \sum_{n=-\infty}^{\infty} c_n \int_{-\infty}^{\infty} \exp(jn\omega_0\theta)\cdot\exp(-j\omega\theta)d\theta \\ &= \sum_{n=-\infty}^{\infty} 2\pi\cdot c_n\delta(\omega - n\omega_0) \end{aligned} \tag{7}$$

where j is the imaginary unit, ω is frequency and δ is the Dirac delta function.

Equation (7) indicates the discrete frequency spectrum which appears only on the frequency components $n\omega_0(n = 0, \pm 1, \pm 2 \ldots)$. By extracting the delta function $2\pi c_0\delta(\omega)$ and substituting eq (2) and eq (5) into eq (7) when $n = 0$, $2\pi c_0(n = 0)$ is obtained.

$$\begin{aligned} 2\pi c_0 &= \frac{2\pi}{T}\int_{-T/2}^{T/2} i(\theta)d\theta \\ &= 4\int_{-\pi/4}^{\pi/4} \frac{a^2}{2}\{1 - \cos 4(\phi - \theta)\}\cdot\sin^2(m\pi)d\theta \\ &= 2a^2\sin^2(m\pi)\int_{-\pi/4}^{\pi/4}\{1 - \cos 4(\phi - \theta)\}d\theta \\ &= \pi a^2\sin^2(m\pi) \end{aligned} \tag{8}$$

$2\pi c_0 = \pi a^2\sin^2(m\pi)$ is obtained from the amplitude at $\omega = 0$ of the Fourier spectrum of the θ directional brightness distribution. Equation (8) indicates that the isoclinics are eliminated. That is, the image of $2\pi c_0$ shows the isochromatics in the whole field of the specimen.

In the same way, from eq (2), eq (5) and eq (7) where $n = -1$, $2\pi c_{-1}$ can be written as

$$\begin{aligned} 2\pi c_{-1} &= \frac{2\pi}{T}\int_{-T/2}^{T/2} i(\theta)\cdot\exp(j\omega_0\theta)d\theta \\ &= 4\int_{-\pi/4}^{\pi/4}\frac{a^2}{2}\{1 - \cos 4(\phi - \theta)\}\cdot\sin^2(m\pi) \\ &\quad\cdot\exp(j4\theta)d\theta \\ &= 2a^2\sin^2(m\pi)\Bigg[\int_{-\pi/4}^{\pi/4}\exp(j4\theta)\,d\theta \\ &\quad - \frac{1}{2}\int_{-\pi/4}^{\pi/4}[\exp\{j4(\phi - \theta)\} \\ &\quad + \exp\{-j4(\phi - \theta)\}]\cdot\exp(j4\theta)d\theta\Bigg] \\ &= -a^2\sin^2(m\pi)\Bigg[\int_{-\pi/4}^{\pi/4}\exp(j4\phi)d\theta \\ &\quad + \int_{-\pi/4}^{\pi/4}\exp\{-j4(\phi - 2\theta)\}d\theta\Bigg] \\ &= -\frac{\pi}{2}a^2\sin^2(m\pi)\cdot\exp(j4\phi) \end{aligned} \tag{9}$$

The argument 4ϕ is obtained by calculating the arctangent of the ratio of the imaginary and real parts of $2\pi c_{-1}$:

$$4\phi = \tan^{-1}\left\{\frac{I_m(2\pi c_{-1})}{R_e(2\pi c_{-1})}\right\} \tag{10}$$

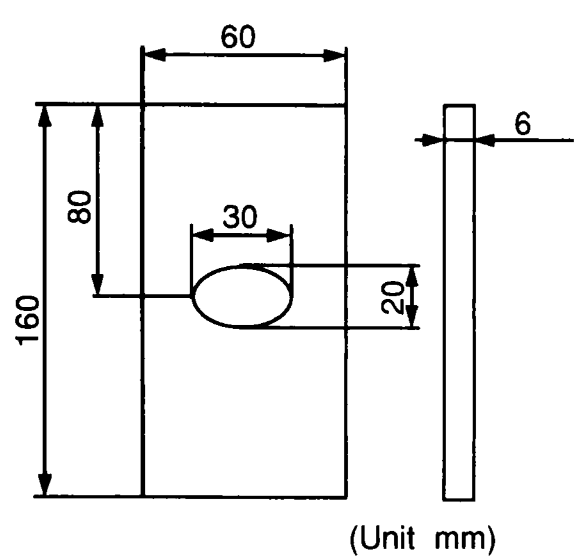

Fig. 2—*Specimen*

Equation (10) indicates that one fourth of the argument is equal to the direction of principal stress.

In this method, by calculating the Fourier transform of the sequential images captured by rotating the angle θ of the crossed polaroids. The isochromatic image is obtained from only the intensity data of the frequency where $\omega = 0$. The direction of principal stress on each point is computed from only the argument where $\omega = -\omega_0$. Other frequency components, in other words, experimental noise is completely eliminated.

EXPERIMENTAL RESULTS AND DISCUSSION

The photoelastic experiment system with the crossed plane polariscope is shown in Fig. 1. Figure 2 shows the specimen made of epoxy resin. The rectangular plate has an elliptic hole at the center. The constant tensile load along the y axis yielding on the specimen is 650 N. The polaroids are rotated in the crossed position.

Ninety sequential 2-D images recorded at every 1 deg angle of θ from -45 deg to 44 deg with a CCD camera are stored into a hard disk. The sequential 2-D images are shown in Fig. 3. The size of the 2-D images is 128 (width) × 120 (height) pixels. One mm corresponds to 1.98 pixels. The ninety 2-D images are treated as a 3-D image as shown in Fig. 4. The size of the 3-D images is 128 (width) × 120 (height) × 90 (depth) pixels. In ordinary 2-D image processing programs, the 2-D image data are stored as a file of 1-D sequential data in computer memory. In this case, ninety files of 1-D sequential data are connected sequentially as a large file of 1-D sequential data. We regard these connected 1-D sequential data as one (x, y, θ) 3-D image. The 3-D image data are directly processed using the 3-D image processing program including Fourier transform.

By calculating the mixed radix fast Fourier transform[10] (MRFFT) in the θ direction of the (x, y, θ) 3-D image data, the Fourier spectrum shown in Fig. 5 is obtained as a (x, y, ω) 3-D image data. In this figure, the 2-D image at $\omega = 0$ shows the isochromatics. Figure 6 is the isochromatics. Figure 7 shows the isochromatics obtained by a circular polariscope. The positions of the isochromatics coincide well with each other. Figure 6 has less noise than Fig. 7.

By calculating the arctangents of the ratios of the imaginary parts and the real parts of the data in the frequency at $\omega = -\omega_0$, the directions of principal stresses are determined in the whole field of the specimen as shown in Fig. 8. Figures 9 and 10 show directions along lines

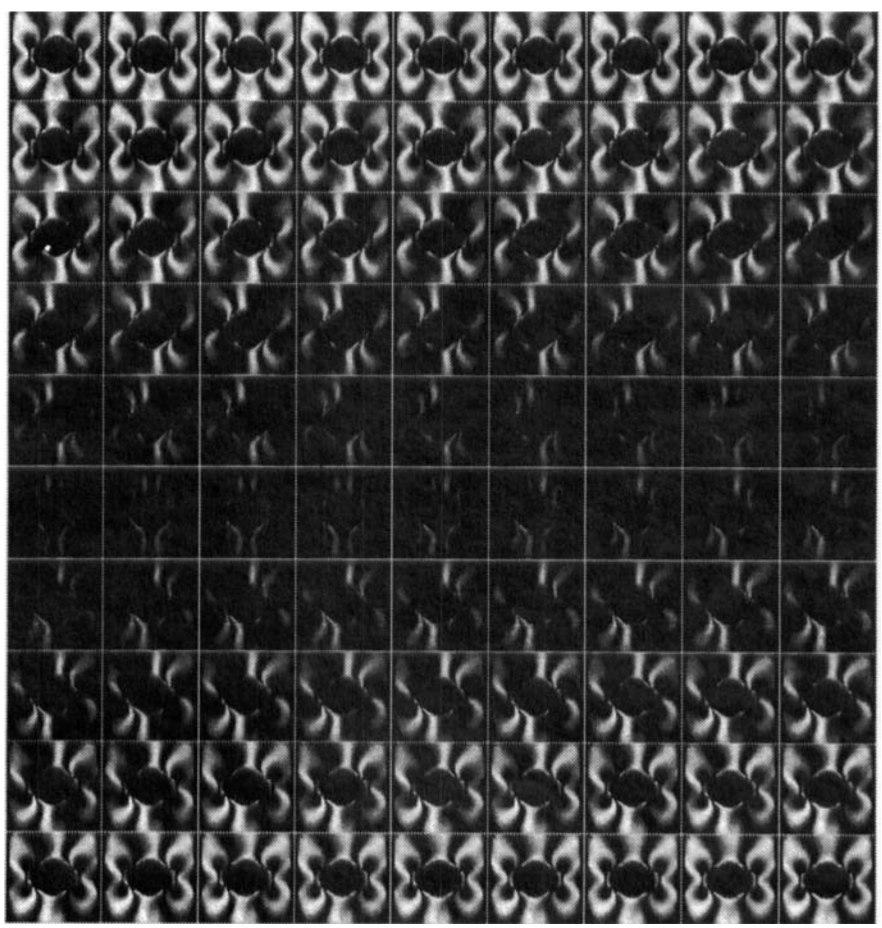

Fig. 3—*Sequential images obtained by changing angle of polarizer and analyzer*

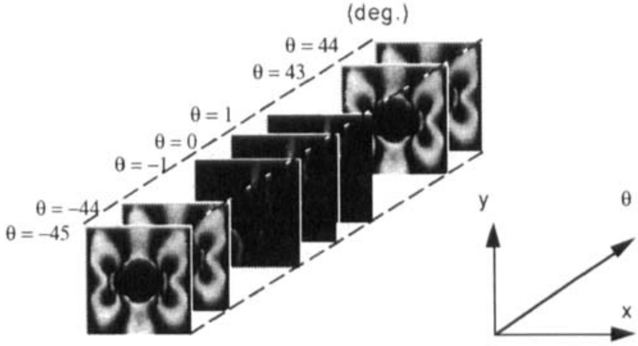

Fig. 4—*Three-dimensional image data*

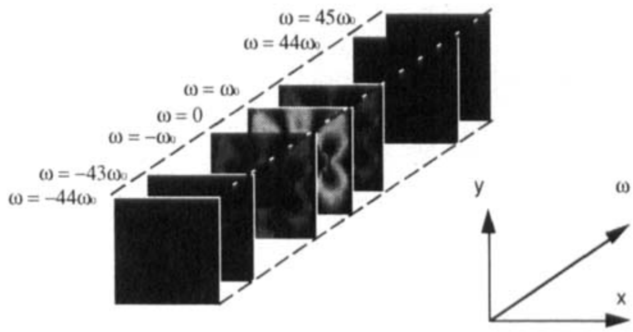

Fig. 5—*θ directional frequency spectrum of image data of Fig. 3*

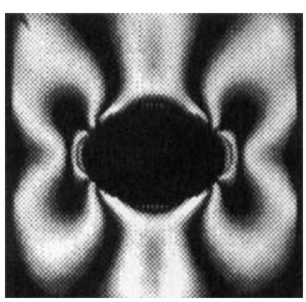

Fig. 6—*Isochromatics obtained by Fourier transform method*

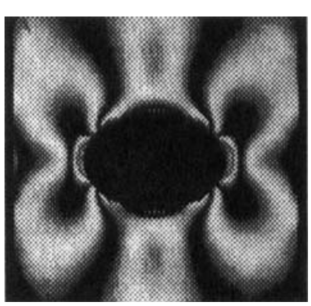

Fig. 7—*Isochromatics obtained by circular polariscope*

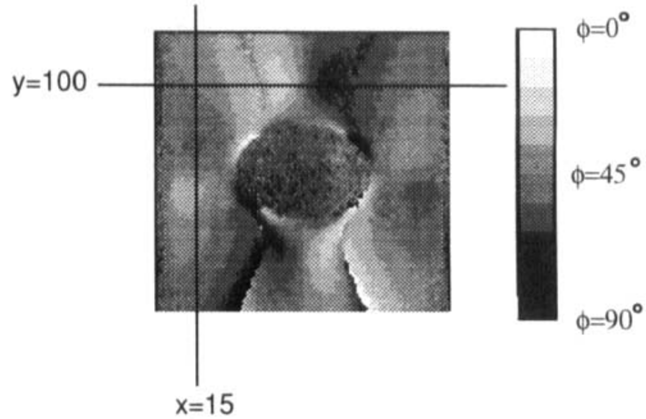

Fig. 8—*Distribution of principal stress direction*

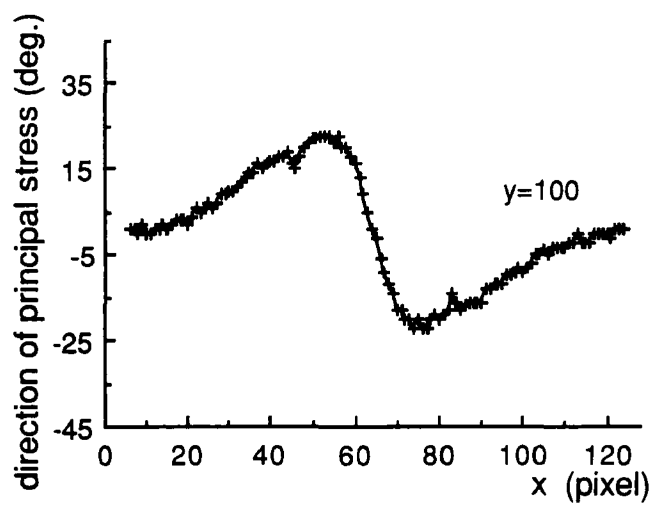

Fig. 9—*Distribution of principal stress direction along* **y** = *100*

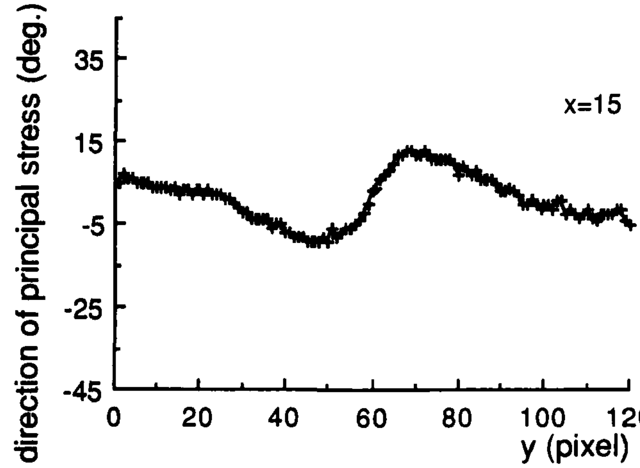

Fig. 10—*Distribution of principal stress direction along* **x** = *15*

y = 100 and x = 15 respectively, although the curve of Fig. 8 has error on the dark isochromatic lines as well as in the conventional method. The error is estimated from the viewpoint that the principal stress direction may be smooth, regardless of the isochromatics. The error is about 3 degrees on the centers (x = 48 and 85) of the isochromatic lines in Fig. 9. It will be improved by combining the images obtained in a different load.

This method is insensitive of high frequency noise, because the high frequency components are eliminated using Fourier filter. The positions and angles may have finer resolution than those in the conventional method, because the data obtained using Fourier transform is naturally smooth.

CONCLUSIONS

A new method using Fourier transform has been proposed to separate the isochromatics and the isoclinics obtained in the plane polariscope. The isochromatics and the isoclinics are separated almost automatically from the 3-D image data obtained by rotating the crossed polaroids.

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