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Slow-light solitons in three-level atomic systems modified by a microwave field

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Abstract. We study the modified effect of slow-light soliton in a resonant three-level atomic system via electromagnetically induced transparency (EIT) by utilizing a microwave field. We derive a high-order nonlinear Schrödinger equation by using a perturbation method of multiple-scales, and calculate the modification of soliton velocity and frequency shift. We find that in the presence of the microwave field an obvious decrease of propagating velocity of soliton can be realized, which provides an effective method to slow down optical solitons in EIT systems. We also find that the down shift of oscillating frequency of soliton in such system can be largely suppressed by the microwave field.

1 Introduction

Solitons describe a class of fascinating shaping-preserving wave propagation phenomena in nonlinear media [1]. Among various solitons studied so far, optical solitons have been extensively investigated because of their potential application in optical information processing and transmission. Due to far-off-resonance nature of excitation schemes, conventional methods of generating optical solitons require intense electromagnetic radiation and media with extended optical length. Optical solitons produced in such schemes travel with a velocity very closed to the speed of light in vacuum. Such high propagating velocity and requirement of extended optical length pose some significant obstacles for certain wave propagation devices which must have small form factor before being implemented into existing telecom systems.

In recent years, optics in coherent media has attracted much attention due to the finding of electromagnetically induced transparency (EIT) and related quantum coherence phenomena [2,3]. In addition to a large suppression of optical absorption, EIT effect can be used to substantially enhance the efficiency of nonlinear optical processes. Another noticeable effect of EIT is the significant change of dispersion property of optical media and the large reduction of the group velocity of optical wave packets [4,5], which can be used to design slow-light device, e.g. optical buffer, which is one of the most critically sought for components in optical communications and signal processing. Slow light under EIT conditions has been successfully observed in resonant multiple-level atoms [2,3] and semiconductor quantum wells and quantum dots [6]. Based on the enhancement of nonlinear optical processes, many remarkable phenomena, such as giant Kerr nonlinearity [7–9], four-wave mixing, and so on, have also been demonstrated [3,10–13].

Recently, the possibility of generating slow-light solitons in three-level EIT systems has been proposed [14,15]. Optical solitons are ideal information carriers because they are robust during propagation. However, although within an EIT configuration ultra-slow or even stopped light has been experimentally demonstrated, the slowing down of velocity of optical solitons in such systems are restricted by the formation condition of solitons, i.e. a balance between dispersion and nonlinearity. To acquire the balance, system parameters must be chosen and fixed, and hence velocity and frequency shift of the soliton can not be adjusted. It is desirable to find suitable methods to adjust slow-light solitons, which are needed by optimizing the properties of the solitons and even controlling them. We notice that some schemes for controlling linear pulse propagation in EIT systems [16,17] by using additional microwave fields have been proposed. It is natural to apply a similar method to manipulates nonlinear optical properties of EIT systems.

In this work, we present a study on the modification of a slow-light soliton in a three-level Λ-type atomic system by utilizing an additional microwave field. We derive a high-order nonlinear Schrödinger (NLS) equation, which includes correction terms of linear and differential absorptions, nonlinear dispersion, delay in nonlinear refractive

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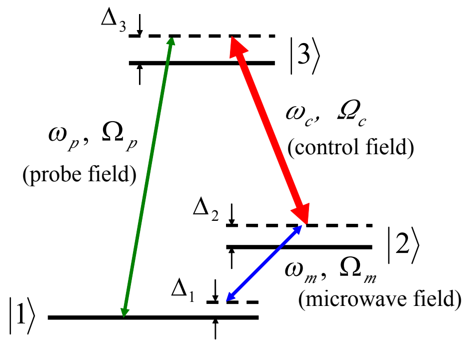

Fig. 1. (Color online) Energy-level diagram and excitation scheme of a three-level Λ-type atomic system. Ω_p (ω_p), Ω_c (ω_c), and Ω_m (ω_m) are half Rabi frequencies (center angular frequencies) of probe, control, and microwave fields, respectively. Δ_1, Δ_2, and Δ_3 are corresponding detunings.

index, third-order dispersion, and modulation induced by the microwave field. Using a perturbation theory of soliton we calculate the contribution of the correction terms to solitonlike pulse propagation. We find that the microwave field can be used to adjust and control the property of the slow-light soliton. In the presence of the microwave field an obvious decrease of propagating velocity of the soliton can be obtained, which provides an effective method to further slow down slow-light optical solitons in EIT systems. We find also that the down shift of oscillating frequency of the soliton in such system can be largely suppressed by the microwave field.

The paper is arranged as follows. In Section 2, we describe our theoretical model and give the derivation of the high-order NLS equation describing the evolution of the envelope of a probe pulse. In Section 3, we discuss the property of slow-light solitons and make a detailed comparison between the cases with and without the microwave field. Finally, a conclusion of the present work is given in the last section.

2 Nonlinear envelope equation

We consider a lifetime broadened three-level atomic system depicted in Figure 1. $|1\rangle$ and $|2\rangle$ are two hyperfine ground states, and $|3\rangle$ is an excited state. A weak probe field with central angular frequency ω_p and a continuous-wave controlling field with central angular frequency ω_c couple $|1\rangle - |3\rangle$ and $|2\rangle - |3\rangle$ via electric dipole transitions, respectively. A weak microwave field with central angular frequency ω_m couples the $|1\rangle$ and $|2\rangle$ via magnetic dipole transition, which are electric dipole transition forbidden states. The quantities $\Delta_3 = \omega_p - (\omega_3 - \omega_1)$, $\Delta_2 = \omega_p - \omega_c - (\omega_2 - \omega_1)$, and $\Delta_1 = \omega_p - \omega_c - \omega_m$ are one-, two-, and three-photon detunings, with $\hbar\omega_j$ the eigenenergy of the state $|j\rangle$ $(j = 1, 2, 3)$. In the absence of the microwave field, the system is reduced to a conventional EIT configuration that has been widely studied in literature. The electric field vector and magnetic induction field vector of the system are assumed to propagate along z axis and read respectively $\mathbf{E} = \sum_{l=p,c} \mathbf{e}_l \mathcal{E}_l(z,t) e^{i(k_l z - \omega_l t)}$+c.c. and $\mathbf{B} = \mathbf{e}_m \mathcal{B}_m(z,t) e^{i(k_m z - \omega_m t)}$+c.c., where $\mathbf{e}_l$ (k_l) is the unit polarization vector (wave number) of the electric-field component with the envelope $\mathcal{E}_l$ ($l = p, c$), and $\mathbf{e}_m$ (k_m) is the unit polarization vector of the magnetic field (wave number) with the envelope $\mathcal{B}_m$.

To investigate the time evolution of the system, it is more convenient to employ an interaction picture, which can be obtained by making the transformation $\sigma_{ij} = \rho_{ij} \exp\{i\{(k_j - k_i)z - [(\omega_j - \omega_i) - (\Delta_j - \Delta_i)]t\}\}$ $(i, j = 1, 2, 3)$, with ρ_{ij} being the density-matrix element of atoms and $k_1 = k_p - k_c - k_m$, $k_2 = k_p - k_c$, $k_3 = k_p$. The Hamiltonian of the system can be written as $\mathcal{H} = -\hbar[\Delta_1|1\rangle\langle 1| + \Delta_2|2\rangle\langle 2| + \Delta_3|3\rangle\langle 3| + (\Omega_m^*|1\rangle\langle 2| + \Omega_p^* e^{-i(k_1 z + \Delta_1 t)}|1\rangle\langle 3| + \Omega_c^*|2\rangle\langle 3| + \text{H.c.})]$, where $\Omega_{p(c)} = (\mathbf{e}_{p(c)} \cdot \mathbf{p}_{31(32)})\mathcal{E}_{p(c)}/\hbar$ is the half Rabi frequency of the probe (control) field, and $\Omega_m = (\mathbf{e}_m \cdot \mu_{21})\mathcal{B}_m/\hbar$ is the half Rabi frequency of the microwave field, with $\mathbf{p}_{ij}$ and μ_{ij} being the electric and magnetic dipole matrix elements associated with the transition from $|j\rangle$ to $|i\rangle$. The optical Bloch equations controlling the evolution of σ_{ij} read

$$\frac{\partial}{\partial t}\sigma_{11} = \Gamma_{12}\sigma_{22} + \Gamma_{13}\sigma_{33} + i\Omega_m^*\sigma_{21} - i\Omega_m\sigma_{21}^* + i\Omega_p^*\sigma_{31} - i\Omega_p\sigma_{31}^*, \tag{1a}$$

$$\frac{\partial}{\partial t}\sigma_{22} = -\Gamma_{12}\sigma_{22} + \Gamma_{23}\sigma_{33} - i\Omega_m^*\sigma_{21} + i\Omega_m\sigma_{21}^* + i\Omega_c^*\sigma_{32} - i\Omega_c\sigma_{32}^*, \tag{1b}$$

$$\frac{\partial}{\partial t}\sigma_{33} = -\Gamma_3\sigma_{33} - i\Omega_p^*\sigma_{31} + i\Omega_p\sigma_{31}^* - i\Omega_c^*\sigma_{32} + i\Omega_c\sigma_{32}^*, \tag{1c}$$

$$\frac{\partial}{\partial t}\sigma_{21} = id_{21}\sigma_{21} + i\Omega_m(\sigma_{11} - \sigma_{22}) - i\Omega_p\sigma_{32}^* + i\Omega_c^*\sigma_{31}, \tag{1d}$$

$$\frac{\partial}{\partial t}\sigma_{31} = id_{31}\sigma_{31} - i\Omega_m\sigma_{32} - i\Omega_p(\sigma_{33} - \sigma_{11}) + i\Omega_c\sigma_{21} = 0, \tag{1e}$$

$$\frac{\partial}{\partial t}\sigma_{32} = id_{32}\sigma_{32} - i\Omega_m^*\sigma_{31} + i\Omega_p\sigma_{21}^* - i\Omega_c(\sigma_{33} - \sigma_{22}) = 0, \tag{1f}$$

where the phase match condition $k_1 = 0$, $\Delta_1 = 0$ has been taken, and d_{ij} are defined as $d_{21} = \Delta_2 + i\gamma_{21}$, $d_{31} = \Delta_3 + i\gamma_{31}$, $d_{32} = (\Delta_3 - \Delta_2) + i\gamma_{32}$, respectively. The decay rate from the state $|i\rangle$ to state $|j\rangle$ is denoted by Γ_{ji}, the decay rate of off-diagonal density matrix elements is defined as $\gamma_{ij} = (1/2)(\Gamma_i + \Gamma_j) + \gamma_{ij}^{\text{col}}$, where $\Gamma_2 = \Gamma_{12}$, $\Gamma_3 = \Gamma_{13} + \Gamma_{23}$, with γ_{ij}^{col} the dipole dephasing rate. From the above equations we have $\Sigma_{j=1,2,3}\sigma_{jj} = 1$, i.e., the system is a closed one.

In the semi-classical approximation, the propagation of electromagnetic waves is described by the Maxwell equations

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\mathbf{P}, \tag{2a}$$

$$\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{B} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\mathbf{M}, \tag{2b}$$

where the electric and magnetic polarization densities read $\mathbf{P} = \mathcal{N}_a(\mathbf{p}_{13}\rho_{31} + \mathbf{p}_{23}\rho_{32} + \text{c.c.})$ and $\mathbf{M} = \mathcal{N}_a(\mu_{12}\rho_{21} + \text{c.c.})$ respectively, with $\mathcal{N}_a$ being the atomic concentration. Under the slow-varying envelope approximation, equation (2) is reduced to

$$i\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p + \kappa_{13}\sigma_{31} = 0, \tag{3a}$$

$$i\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_m + \kappa_{12}\sigma_{21} = 0, \tag{3b}$$

where $\kappa_{13} = \mathcal{N}_a\omega_p|\mathbf{p}_{13}|^2/(2\varepsilon_0 c\hbar)$ and $\kappa_{12} = \mathcal{N}_a\omega_m|\mu_{12}|^2/(2\varepsilon_0 c^3\hbar)$. When obtaining equations (3a) and (3b) we have assumed that the electric and magnetic fields are uniform in x and y directions.

We assume the initial population of the atoms is in the ground state $|1\rangle$. We are interested in weak excitations, i.e. both Ω_p and Ω_m are much less than Ω_c. In this case, the depletion of the ground state $|1\rangle$ is not significant and in linear approximation the system is closed to conventional EIT one. It is easy to show that even for small Ω_m the dynamics of the microwave field can be disregarded. This point can be seen clearly by considering the ratio κ_{12}/κ_{13}, which equals to $\frac{\omega_m}{\omega_p}\alpha^2$, where $\alpha \approx 1/137$ is the fine structure constant. Since both ω_m/ω_p and α^2 are very small, the second term on the right hand of equation (3b) can be neglected and hence Ω_m can be taken as undepleted. The physical reason is that $|1\rangle$ and $|2\rangle$ are electric-dipole transition forbidden, the transition probability between these two states induced by the microwave field via magnetic dipole transition is much smaller than the electric dipole transition between $|1\rangle$ and $|3\rangle$. In the following discussion, we disregard equation (3b).

The linear dispersion relation of the Maxwell-Bloch equations (1) and (3a) is given by

$$K(\omega) = \frac{\omega}{c} + \kappa_{13}\frac{(\omega + d_{21})}{|\Omega_c|^2 - (\omega + d_{21})(\omega + d_{31})}. \tag{4}$$

In most operation conditions $K(\omega)$ can be Taylor expanded into a rapidly conversion power series around the center frequency ω_p of the probe field, that is, $\omega = 0$. We thus have $K(\omega) = K_0 + K_1\omega + (1/2)K_2\omega^2 + (1/6)K_3\omega^3 + \ldots$, where $K_j = [\partial^j K(\omega)/\partial\omega^j]|_{\omega=0}$ $(j = 0, 1, 2, 3\ldots)$. These dispersion coefficients can be obtained analytically, where K_0 gives phase shift and absorption per unit length, $1/K_1$ relates to group velocity v_g, $1/K_2$ relates to group velocity dispersion, and K_3 relates to third-order dispersion of the system.

Because the system is a highly resonant one, the linear propagation of the probe pulse display strong dispersion, which results in pulse distortion. It is desirable to obtain a probe pulse that is robust during propagation. It is natural to utilize the nonlinear effect of the system to balance the dispersion and hence get a solitonlike probe pulse. For this aim, we apply a standard method of multi-scales to investigate the weak linear and nonlinear evolution of the probe field. We take the following asymptotic expansion

$$\sigma_{11} = 1 + \epsilon^2\sigma_{11}^{(2)} + \epsilon^3\sigma_{11}^{(3)} + \ldots, \tag{5a}$$

$$\sigma_{jj} = \epsilon^2\sigma_{jj}^{(2)} + \epsilon^3\sigma_{jj}^{(3)} + \ldots (j = 2, 3), \tag{5b}$$

$$\sigma_{j1} = \epsilon\sigma_{j1}^{(1)} + \epsilon^2\sigma_{j1}^{(2)} + \epsilon^3\sigma_{j1}^{(3)} + \ldots (j = 2, 3), \tag{5c}$$

$$\sigma_{32} = \sigma_{32}^{(0)} + \epsilon^2\sigma_{32}^{(2)} + \epsilon^3\sigma_{32}^{(3)} + \ldots, \tag{5d}$$

$$\Omega_p = \epsilon\Omega_p^{(1)} + \epsilon^2\Omega_p^{(2)} + \epsilon^3\Omega_p^{(3)} + \ldots, \tag{5e}$$

where ϵ is a dimensionless small parameter characterizing the small depletion of the ground state $|1\rangle$ (resulted from small but finite Ω_p). We assume $\Omega_m = \epsilon\Omega_m^{(1)}$ with $\Omega_m^{(1)}$ being a constant. To obtain a divergence-free expansion, all quantities (except for $\Omega_m^{(1)}$) on the right hand side of the expansion are considered as functions of the multi-scale variables

$$z_n = \epsilon^n z \quad (n = 0, 1, 2, 3), \tag{6a}$$

$$t_n = \epsilon^n t \quad (n = 0, 1). \tag{6b}$$

Substituting above expansion into the Bloch-Maxwell equations (1) and (3a), we obtain a series of equations of $\sigma_{ij}^{(l)}$ and $\Omega_p^{(l)}$ $(l = 1, 2, 3)$, which can be solved order by order in a systematic way.

In the leading order $(l = 1)$, we obtain

$$\Omega_p^{(1)} = F\exp(i\theta) + \Omega_c\Omega_m^{(1)}/d_{21}, \tag{7}$$

with $\theta = K(\omega)z_0 - \omega t_0$, $F = F(z_1, z_2, z_3, t_1)$ being a yet to be determined envelope function. Notice that the term $\Omega_c\Omega_m^{(1)}/d_{21}$ appeared on the right hand side of expression (7) is due to the contribution of the microwave field.

In the next order $(l = 2)$, a divergence-free solution for $\Omega_p^{(2)}$ requires

$$i\left(\frac{\partial F}{\partial z_1} + \frac{1}{v_g}\frac{\partial F}{\partial t_1}\right) = 0, \tag{8}$$

i.e. the envelope function F travels with group velocity v_g.

In the third order $(l = 3)$, the solvability condition yields a modified NLS equation

$$i\frac{\partial F}{\partial z_2} - \frac{1}{2}K_2\frac{\partial^2 F}{\partial t_1^2} - We^{-2\bar{\alpha_0}z_2}|F|^2F - \left|\Omega_m^{(1)}\right|^2 S_1^{(3)}F = 0, \tag{9}$$

where $\bar{\alpha_0} = \epsilon^{-2}\alpha_0$ $(\alpha_0 \equiv \text{Im}K_0)$, W is the nonlinear coefficient related to self-phase modulation of the probe pulse, the term $|\Omega_m^{(1)}|^2\ S_1^{(3)}F$ is the cross-phase modulation contributed from the microwave field.

Since the system under study is a highly resonant system, the evolution property of the probe-field envelope is very sensitive to the pulse duration τ_0 (see Fig. 2 below). The high-order dispersion and nonlinear effects play significant role even when τ_0 has only a small increase or decrease. Thus it is necessary to consider the correction from the next order approximation. In the forth order $(l = 4)$,

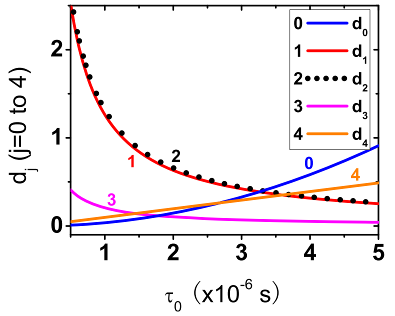

Fig. 2. (Color online) Coefficients d_j $(j = 0, 1, 2, 3, 4)$ of equation (13) as functions of pulse duration τ_0. Parameters are given in the text.

the solvability condition results in

$$i\frac{\partial F}{\partial z_3} - i\frac{K_3}{6}\frac{\partial^3 F}{\partial t_1^3} - i\beta_1 e^{-2\bar{\alpha}_0 z_2}\frac{\partial\left(|F|^2 F\right)}{\partial t_1} + i\beta_2 e^{-2\bar{\alpha}_0 z_2} \times F\frac{\partial\left(|F|^2\right)}{\partial t_1} - i\left|\Omega_m^{(1)}\right|^2 S_1^{(4)}\frac{\partial F}{\partial t_1} = 0. \quad (10)$$

Combining equations (8)–(10), we obtain

$$i\left(\frac{\partial}{\partial z} + \alpha_0\right)U - \frac{1}{2}K_2\frac{\partial^2 U}{\partial \eta^2} - W|U|^2U - i\left[\beta_1\frac{\partial}{\partial \eta}\left(|U|^2U\right) - \beta_2 U\frac{\partial}{\partial \eta}\left(|U|^2\right) + \frac{1}{6}K_3\frac{\partial^3 U}{\partial \eta^3}\right] - |\Omega_m|^2\left(S_1^{(3)} + iS_1^{(4)}\frac{\partial}{\partial \eta}\right)U = 0 \quad (11)$$

after returning to original variables, where $\eta = t - z/v_g$ and $U = \epsilon F\exp(-\alpha z)$. The detailed derivation of above equations and the explicit expressions of W, $S_1^{(3)}$, β_1, β_2, and $S_1^{(4)}$ are given in the Appendix.

Above equation is a modified high-order NLS equation controlling the spatial-temporal evolution of the probe-field envelope. It has properly included the effects of three-order dispersion, static and dynamic Kerr nonlinearities, and the cross-phase modulation contributed by the microwave field Ω_m.

3 Dynamics of slow-light optical solitons modified by the microwave field

3.1 A preliminary discussion of equation (11)

The modified high-order NLS equation (11) is a Ginzburg-Landau equation with high-order correction terms, in which the coefficients are complex. Such a equation is generally not integrable, which means a solitonlike solution may be highly unstable and undergo serious distortion. However, if a realistic set of system parameters can be found so that the imaginary part of these coefficients can be much smaller than their corresponding real part, it is possible to get a shape-preserving, localized solution that can propagate a rather long distance without significant distortion. We shall show that the present system possesses indeed this property and hence a soliton is possible, as will be shown below.

When the imaginary part of the coefficients are small, equation (11) can be approximated by

$$i\left(\frac{\partial}{\partial z} + \alpha_0\right)U - \frac{1}{2}\tilde{K}_2\frac{\partial^2 U}{\partial \tau^2} - \tilde{W}|U|^2U + \mathrm{Im}\left(K_1\right)\frac{\partial}{\partial \tau}U - \left[\tilde{\beta}_1\frac{\partial}{\partial \tau}\left(|U|^2U\right) - \tilde{\beta}_2 U\frac{\partial}{\partial \tau}\left(|U|^2\right) + \frac{1}{6}\tilde{K}_3\frac{\partial^3 U}{\partial \tau^3}\right] - |\Omega_m|^2\left(\tilde{S}_1^{(3)} + i\tilde{S}_1^{(4)}\frac{\partial}{\partial \tau}\right)U = 0, \quad (12)$$

where we have defined $\Omega_p = U\exp(i\tilde{K}_0 z)$, $\tau = t - z/\tilde{v}_g$, with the symbol tide denoting the corresponding real part of each coefficient.

The key step in solving equation (12) is to estimate the relative importance of various terms appearing in it. For this aim and for the convenience of the following discussion, we rewrite equation (12) into the dimensionless form using new variables $\sigma = \tau/\tau_0$, $s = -z/(2L_D)$ and $U = U_0 u$. Then we have

$$i\frac{\partial}{\partial s}u + \frac{\partial^2 u}{\partial \sigma^2} + 2|u|^2u = +\left(id_0 - d_0'\right)u - i\left[d_1\frac{\partial}{\partial \sigma}\left(|u|^2u\right) - d_2 u\frac{\partial}{\partial \sigma}\left(|u|^2\right) + d_3\frac{\partial^3 u}{\partial \sigma^3}\right] - \left(d_4 + id_4'\right)\frac{\partial}{\partial \sigma}u. \quad (13)$$

where the dimensionless coefficients are defined as $d_j = 2L_D/L_j$ $(j = 0$ to $4)$, with $L_0 = 1/\alpha_0$ (characteristic linear absorption length), $L_1 = \tau_0/(\tilde{\beta}_1 U_0^2)$ (characteristic nonlinear dispersion length), $L_2 = \tau_0/(\tilde{\beta}_2 U_0^2)$ (characteristic delay length in nonlinear refractive index), $L_3 = 6\tau_0^3/\tilde{K}_3$ (characteristic third-order dispersion length) and $L_4 = \tau_0/\mathrm{Im}(K_1)$ (characteristic differential absorption length). Here τ_0 is the duration of the probe-field pulse, $L_D = \tau_0^2/\tilde{K}_2$ is the characteristic dispersion length at which the group velocity dispersion becomes important, $L_{NL} = 1/(U_0^2\tilde{W})$ is the effective length that characterize the influence of the nonlinearity, with $U_0 = (1/\tau_0)(\tilde{K}_2/\tilde{W})^{1/2}$ being the typical Rabi frequency of the probe field. We have also assumed $L_D = L_{NL}$, i.e., the balance of the dispersion and the nonlinearity in order to favor the formation of soliton. We have assumed $\Delta_{21(31)} > 0$ so that both $\tilde{K}_2$ and $\tilde{W}$ are positive, which corresponds to a bright-soliton condition [1]. Explicit expressions of d_j are given by $d_0 = 2\alpha\tau_0^2/\tilde{K}_2$, $d_1 = 2\tilde{\beta}_1/(\tau_0\tilde{W})$, $d_2 = 2\tilde{\beta}_2/(\tau_0\tilde{W})$, $d_3 = \tilde{K}_3/(3\tau_0\tilde{K}_2)$ and $d_4 = 2\mathrm{Im}(K_1)\tau_0/\tilde{K}_2$. The contribution of the microwave field is reflected by the dimensionless coefficients $d_0' = 2\tau_0^2|\Omega_m|^2\tilde{S}_1^{(3)}/\tilde{K}_2$ and $d_4' = 2\tau_0|\Omega_m|^2\tilde{S}_1^{(4)}/\tilde{K}_2$.

[1] A dark soliton is also possible but we focus on only bright solitons here.

We now present a practical example to show that for a realistic cold atomic system the imaginary parts of the coefficients in equation (11) can be indeed much less than their corresponding real parts. We consider a typical alkali system where the decay rates are $\Gamma_2 = 500.0\ \mathrm{s}^{-1}$, $\Gamma_3 = 0.5\times 10^7\ \mathrm{s}^{-1}$. Other parameters are taken as $\kappa_{13} = 1.0\times 10^9\ \mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, $\Omega_c = 1.6\times 10^7\ \mathrm{s}^{-1}$, $\Delta_{21} = 8.0\times 10^5\ \mathrm{s}^{-1}$, and $\Delta_{31} = 4.0\times 10^7\ \mathrm{s}^{-1}$. With the parameters given here, we obtain $K_0 = (3.57 + i0.03)\ \mathrm{cm}^{-1}$, $K_1 = (5.11 + i0.09)\times 10^{-6}\ \mathrm{cm}^{-1}\,\mathrm{s}$, $K_2 = (1.89+i0.17)\times 10^{-12}\ \mathrm{cm}^{-1}\,\mathrm{s}^2$, $K_3 = (1.16+i0.17)\times 10^{-18}\ \mathrm{cm}^{-1}\,\mathrm{s}^3$, $W = (3.94+i0.05)\times 10^{-14}\ \mathrm{cm}^{-1}\,\mathrm{s}^2$, $\beta_1 = (7.45 + i0.16)\times 10^{-20}\ \mathrm{cm}^{-1}\,\mathrm{s}^3$, $\beta_2 = (7.78 + i0.89)\times 10^{-20}\ \mathrm{cm}^{-1}\,\mathrm{s}^3$, $S_1^{(3)} = -(2.31 + i0.19)\times 10^{-13}\ \mathrm{cm}^{-1}\,\mathrm{s}^2$, $S_1^{(4)} = (4.88+i0.12)\times 10^{-18}\ \mathrm{cm}^{-1}\,\mathrm{s}^3$, which justify the validity of replacing the high-order Ginzburg-Landau equation (11) by equation (12).

The property of the solutions of equation (13) is controlled by coefficients d_j. In Figure 2, we have plotted values of d_j ($j = 0$ to 4) as functions of the pulse width τ_0, using the parameters given above. From Figure 2 we see that the behavior of coefficients d_j are quite different in different pulse duration region. d_1, d_2 and d_3 are large when the pulse width τ_0 is small; d_0 and d_4 become large when τ_0 is large. In the central region all coefficients are small quantities and hence the right hand side of equation (13) can be treated as perturbation of the standard NLS equation $i\partial u/\partial s + \partial^2 u/\partial\sigma^2 + 2|u|^2 u = 0$. In the present work, we concentrate on this central region (i.e. τ_0 is around 3.5×10^{-6} s), and use the perturbation theory of solitons to study the dynamic of solitons.

3.2 Slow-light solitons modified by the microwave field

When τ_0 is chosen around the central region of Figure 2, all d_j can be taken as small quantities. Notice that in this region the third-order dispersion length L_3 are much lager than other typical lengths, i.e., $d_3 \ll d_0, d_1, d_2, d_4$. So the term proportional to d_3 can be neglected safely. If we choose the intensity of the microwave field, $|\Omega_m|^2$, in a proper range so that $d'_{0(4)}$ are much smaller than unity, equation (13) can be expressed as

$$i\frac{\partial}{\partial s}u + \frac{\partial^2 u}{\partial\sigma^2} + 2|u|^2 u = iR[u], \tag{14}$$

which is a perturbed NLS equation with the perturbation term $iR[u] = (id_0 - d'_0)u - i[d_1\partial(|u|^2u)/\partial\sigma - d_2 u\partial(|u|^2)/\partial\sigma] - (d_4 + id'_4)\partial u/\partial\sigma$.

Equation (14) can be solved by using a standard perturbation theory of solitons [18]. The procedure is to use the soliton solution of equation (14) with $R[u] = 0$ as an initial input. Equation (14) with $R[u] = 0$ has the general four-parameter bright-soliton solution

$$\begin{aligned} u_0 &= 2\beta\,\mathrm{sech}\left[2\beta\left(\sigma - \sigma_0 + 4\alpha s\right)\right] \\ &\quad\times \exp\left[-2i\alpha\sigma - 4i\left(\alpha^2 - \beta^2\right)s - i\phi_0\right], \end{aligned} \tag{15}$$

where[2] α, β, σ_0 and ϕ_0 are real free parameters which determine the propagating velocity, amplitude (as well as width), initial position, and initial phase of the soliton, respectively. When the perturbation $R[u] \neq 0$ the soliton will undergo a deformation and generate a continuous-wave radiation. Based on the soliton perturbation theory, we take the solution of equation (14) as $u = u_0 + u_1$ [18,19]. The parameters α, β, σ_0 and ϕ_0 may be changed during propagation, representing the deformation of the soliton; u_1 represents the continuous-wave radiation generated from the soliton. By rewriting $u_0 = 2\beta\exp(-i\phi)\mathrm{sech}(w)$ with $w = 2\beta(\sigma - \xi)$ and $\phi = 2\alpha(\sigma - \xi) + \delta$ and following the procedure of reference [18], we obtain the equations of motion for α, β, ξ and δ:

$$\frac{d\alpha}{dz'} = -\frac{4}{3}\left(d_4 + id'_4\right)\beta^2, \tag{16a}$$

$$\frac{d\beta}{dz'} = -2(d_0 + id'_0)\beta - 4\left(d_4 + id'_4\right)\alpha\beta, \tag{16b}$$

$$\frac{d\xi}{dz'} = 4\alpha - \frac{4}{3}\left(3d_1 - 2d_2\right)\beta^2, \tag{16c}$$

$$\frac{d\delta}{dz'} = 4\left(\alpha^2 + \beta^2\right) + \frac{16}{3}d_2\alpha\beta^2, \tag{16d}$$

where s has been replaced by $z' = z/(2L_D)$. We assume that the soliton is input at $z = 0$, then the initial condition of the above set of equations are $\beta_0 = 1/2$ and $\alpha_0 = \xi_0 = \delta_0 = 0$. We consider the case that $d_4 + id'_4$ are much less than $d_0 + id'_0$, equation (A.13a) yields $\beta(z') = \exp[-2(d_0 + id'_0)z']$. Then we obtain the analytical solution

$$\alpha\left(z'\right) = \frac{d_4 + id'_4}{12\left(d_0 + id'_0\right)}\left[e^{-4\left(d_0 + id'_0\right)z'} - 1\right], \tag{17a}$$

$$\begin{aligned} \xi\left(z'\right) &= -\frac{d_4 + id'_4}{3\left(d_0 + id'_0\right)}z' + \frac{1}{12\left(d_0 + id'_0\right)} \\ &\quad\times\left[3d_1 - d_2 - \frac{d_4 + id'_4}{d_0 + id'_0}\right]\left[e^{-4\left(d_0 + id'_0\right)z'} - 1\right], \end{aligned} \tag{17b}$$

$$\begin{aligned} \delta\left(z'\right) &= \frac{1}{36}\left(\frac{d_4 + id'_4}{d_0 + id'_0}\right)^2 z' - \frac{d_4 + id'_4}{72\left(d_0 + id'_0\right)^2} \\ &\quad\times\left[d_2 + \frac{d_4 + id'_4}{d_0 + id'_0}\right]\left[e^{-8\left(d_0 + id'_0\right)z'} - 1\right] \\ &\quad - \frac{1}{4\left(d_0 + id'_0\right)}\left[1 - \frac{d_4 + id'_4}{18\left(d_0 + id'_0\right)}\right. \\ &\quad\times\left.\left(2d_2 + \frac{d_4 + id'_4}{18\left(d_0 + id'_0\right)}\right)\right]\left[e^{-4\left(d_0 + id'_0\right)z'} - 1\right]. \end{aligned} \tag{17c}$$

With these results one can obtain explicit expressions of the shape, velocity, and oscillating frequency of the soliton under the action of $R[u]$.

[2] We use standard symbols in the perturbation theory of NLS solitons [18]. Here α is a soliton parameter, not the fine structure constant used in Section 2.

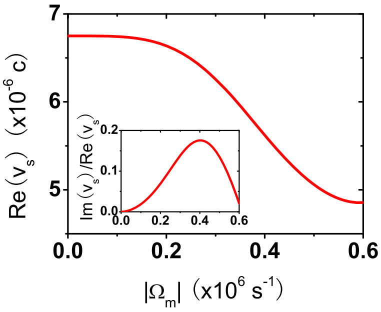

Fig. 3. (Color online) Soliton velocity as a function of $|\Omega_m|$ after propagating to the distance $z = L_D$. The inset is the ratio $\mathrm{Im}(v_s)/\mathrm{Re}(v_s)$. Parameters are given in the text.

The propagating velocity of the soliton is given by

$$v_s = \frac{\tilde{v}_g}{G} \tag{18}$$

with

$$G = 1 - \frac{\tilde{v}_g \tau_0}{6L_D} \times \left[\frac{d_4 + id_4'}{d_0 + id_0'} + \left(3d_1 - d_2 - \frac{d_4 + id_4'}{d_0 + id_0'} e^{-4\left(d_0 + id_0'\right)z'}\right)\right],$$

where $\tilde{v}_g$ is the propagating velocity of the soliton in the absence of the perturbation $R[u]$. With the parameters given above, we have $\tilde{v}_g = 6.52 \times 10^{-6}c$, which is much slower than the light velocity in vacuum (*slow-light soliton*). If the microwave field is absent (i.e. $|\Omega_m|^2 = 0$), $3d_1 - d_2 > 0$ in most operation condition, and hence the soliton velocity acquires a increase due to the influence of $R[u]$. For large z', v_s approaches the limit $\tilde{v}_g/[1 - \tilde{v}_g \tau_0 d_4/(6L_D d_0)]$, which is around $1.03\tilde{v}_g$ for the parameters given in the text.

However, when the microwave field is present (i.e. $|\Omega_m|^2 \neq 0$) the soliton velocity can be decreased largely. Shown in Figure 3 is the soliton velocity as a function of $|\Omega_m|$ after propagating to $z = L_D = 3.3$ cm. One sees that a significant decrease of the soliton velocity can be realized by increasing the microwave field intensity. With the system parameters given here, the soliton velocity in the presence of the microwave field can acquire a decrease of 30% in comparison with that in the absence of the microwave field. The inset of Figure 3 is the ratio of $\mathrm{Im}(v_s)/\mathrm{Re}(v_s)$, which shows $\mathrm{Im}(v_s)$ is much smaller than $\mathrm{Re}(v_s)$.

Figure 4 shows the change of the soliton velocity as a function of z. We see that in the presence (curve 1) of the microwave field (with $|\Omega_m| = 0.6 \times 10^6$ s^{-1}) the soliton velocity decreases to a minimum value when propagating to distance L_D; while in the absence of the microwave field (curve 2) the change of velocity is negligibly small. With the parameters plotting Figure 4, we have $\mathrm{Im}(v_s)/\mathrm{Re}(v_s) = 0.03$, which means that the change in absorption due to the microwave field can be neglected safely.

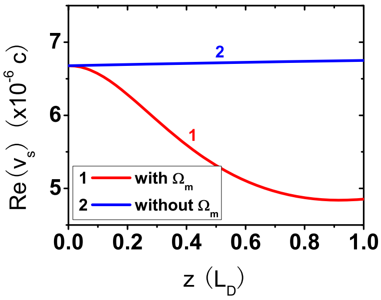

Fig. 4. (Color online) Soliton velocity as a function of z in the presence (curve 1) and absence (curve 2) of the microwave field. Parameters are given in the text.

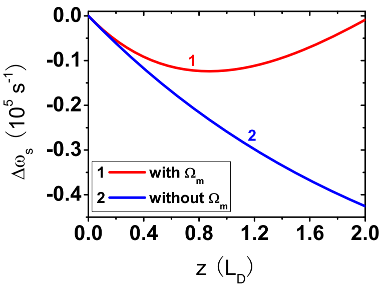

Fig. 5. (Color online) Frequency shift $\Delta\omega_s$ of the soliton with (curve 1) and without (curve 2) the microwave field. Parameters are given in the text.

The real part of $\alpha/2$ (Eq. (17a)) gives the shift of the oscillating frequency of the soliton. When the microwave field is absent, the differential absorption (i.e. the term proportional to d_4 in Eq. (13)) results in a down shift of the oscillating frequency of the soliton, for large z' the down shift reaches the maximum value of $d_4/(6d_0\tau_0)$. In this case, a frequency downshift due to the high-order perturbation is unavoidable. However, the application of the microwave field $|\Omega_m|$ contributes a upshift of the oscillating frequency. Shown in Figure 5 is the soliton frequency shift $\Delta\omega_s$ as a function of z. The curve 2 is for the case of absence of the microwave field. We see that in this case a significant frequency downshift occurs. But when the microwave field is present, the downshift is stopped and a recovery to its initial value for large z is realized (curve 1).

Above results show that the application of the microwave field provides a very effective method to slow down the propagating velocity and suppress the frequency shift of the soliton.

In order to confirm how the microwave field make the soliton slowdown, we have numerically integrated the Bloch-Maxwell equations (1) and (3a) by using a fourth-order Runge-Kutta method combined with a finite difference method. We take $\tau_0 = 2.5 \times 10^{-6}$ and other parameters are same as those given above, with which we have $L_D = 3.3$ cm. In our numerical simulation the soliton solution (15) is taken as an initial condition, with $\beta = 0.65$ and $\alpha = \sigma_0 = \phi_0 = 0$. Then the initial pulse takes the form $\Omega_p(0,t)\tau_0 = 8.77\mathrm{sech}(1.3t/\tau_0)$. Shown in Figure 6 is the result of the numerical simulation. Curve 1 in the figure is the initial pulse; curve 2 (curve 3) is the pulse after

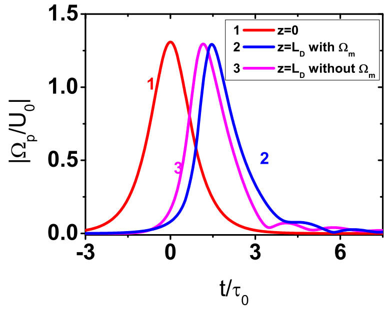

Fig. 6. (Color online) The result of numerical simulation starting directly from equations (1) and (3a) by taking the soliton solution (15) as initial condition. Curve 1 is the shape of the input probe pulse $\Omega_p(0,t)\tau_0 = 8.77\text{sech}(1.3t/\tau_0)$. Curve 2 (curve 3) is the probe pulse after propagating to $z = L_D = 3.3$ cm for the case of the presence (absence) of the microwave field.

propagating to $z = L_D = 3.3$ cm for the case of the presence (absence) of the microwave field. We see that there is indeed a slowing down of the soliton when the microwave field is applied, and in both cases the probe pulse suffers no serious distortion during the propagation.

4 Discussion and summary

Usually, for a closed-loop system the total relative phase affects optical response of the system (see Refs. [21,22] and references therein). It seems that the total relative phase in our system $\phi = \phi_p - \phi_c - \phi_m$ (where ϕ_p, ϕ_c and ϕ_m are the initial phase of the probe, control and microwave field, respectively) should play a significant role for the dynamics of the slow-light soliton in the system. However, notice that in the regime we have considered, Ω_m is a weak field and can be taken as being undepleted (see the discussion below Eq. (3b)). Therefore, in the leading order approximation the system has a standard configuration of Λ-type EIT, in which closed-loop effect does not exist, and hence a good EIT effect occurs under the condition $|\Omega_c|^2 \gg \gamma_{21}\gamma_{31}$. Consequently, the effect due to the total relative phase ϕ is a high-order perturbation. In this work, we have assumed a zero total relative phase for simplicity. A non-zero total relative phase will modify and even undo the formation condition of the soliton, which will be discussed elsewhere.

In summary, we have investigated the modified effect of slow-light soliton in a resonant three-level atomic system via electromagnetically induced transparency (EIT) by utilizing a microwave field. We have derived a high-order nonlinear Schrödinger equation by using a perturbation method of multiple-scales, and calculated the modification of soliton velocity and frequency shift. We have found that in the presence of the microwave field an significant decrease of propagating velocity of the soliton can be obtained, which provides an effective method to slow down optical solitons in EIT systems. We have found also that the down shift of oscillating frequency of the soliton in such system can be largely suppressed by using the microwave field. The results presented in this work may be useful for the application of optical information processing and transmission.

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Appendix: Detailed derivation of the modified high-order NLS equation (11)

To derive the nonlinear envelope equation for the probe field, we apply the standard method of multiple-scales [20] to solve the Bloch-Maxwell equations (1) and (3a). By making the asymptotic expansion (5) and using the multi-scale variables (6), equations (1) and (3a) become

$$i\left(\frac{\partial}{\partial z_0} + \frac{1}{c}\frac{\partial}{\partial t_0}\right)\Omega_p^{(l)} = M^{(l)} - \kappa_{13}\sigma_{31}^{(l)}, \tag{A.1a}$$

$$\left(i\frac{\partial}{\partial t_0} + d_{21}\right)\sigma_{21}^{(l)} = N^{(l)} - \Omega_m^{(l)} - \Omega_c^*\sigma_{31}^{(l)}, \tag{A.1b}$$

$$\left(i\frac{\partial}{\partial t_0} + d_{31}\right)\sigma_{31}^{(l)} = P^{(l)} - \Omega_p^{(l)} - \Omega_c\sigma_{21}^{(l)}, \tag{A.1c}$$

$$\left(i\frac{\partial}{\partial t_0} + d_{32}\right)\sigma_{32}^{(l)} = Q^{(l)} - \Omega_c\left(\sigma_{11}^{(l)} - 2\sigma_{22}^{(l)}\right), \tag{A.1d}$$

$$i\left(\frac{\partial}{\partial t_0} + \Gamma_{13}\right)\sigma_{11}^{(l)} = Y^{(l)} - i\left(\Gamma_{13} - \Gamma_{12}\right)\sigma_{22}^{(l)}, \tag{A.1e}$$

$$i\left(\frac{\partial}{\partial t_0} + \Gamma_{12} + \Gamma_{23}\right)\sigma_{22}^{(l)} = Z^{(l)} - i\Gamma_{23}\sigma_{11}^{(l)} - \Omega_c^*\sigma_{32}^{(l)} + \Omega_c\sigma_{32}^{*(l)}. \tag{A.1f}$$

The explicit expressions of $M^{(l)}$, $N^{(l)}$, $P^{(l)}$ ($l = 1$ to 4) and $Q^{(l)}$, $Y^{(l)}$ and $Z^{(l)}$ ($l = 1$ to 3) can be systematically obtained, which are omitted here. The above equations (A.1a)–(A.1f) can be solved order by order.

(i) *First-order approximation.* The leading-order ($l = 1$) gives the linear solution of the system. In this order one obtains the linear dispersion relation (4), and the leading-order solution given by

$$\Omega_p^{(1)} = Fe^{i\theta} + \frac{\Omega_c\Omega_m^{(1)}}{d_{21}}, \tag{A.2a}$$

$$\sigma_{31}^{(1)} = \frac{(\omega + d_{21})}{D}Fe^{i\theta}, \tag{A.2b}$$

$$\sigma_{21}^{(1)} = -\frac{\Omega_c^*}{D}Fe^{i\theta} - \frac{\Omega_m^{(1)}}{d_{21}}, \tag{A.2c}$$

where $D = |\Omega_c|^2 - (\omega + d_{21})(\omega + d_{31})$, $F = F(z_1, z_2, z_3, t_1)$ is a yet to be determined envelope function depending on the indicated slow variables.

(ii) *Second-order approximation.* In the second order ($l=2$), a divergence-free solution for Ω_p requires

$$i\left(\frac{\partial F}{\partial z_1}+\frac{1}{v_g}\frac{\partial F}{\partial t_1}\right)=0, \tag{A.3}$$

with v_g being the complex group velocity of the envelope F. The second-order approximation solution reads

$$\Omega_p^{(2)}=0, \tag{A.4a}$$
$$\sigma_{31}^{(2)}=\frac{i}{\kappa_{13}}\left(\frac{1}{v_g}-\frac{1}{c}\right)\frac{\partial}{\partial t_1}Fe^{i\theta}, \tag{A.4b}$$
$$\sigma_{21}^{(2)}=-i\frac{\Omega_c^*}{D^2}\left(2\omega+d_{21}+d_{31}\right)\frac{\partial}{\partial t_1}Fe^{i\theta}, \tag{A.4c}$$

$$\sigma_{11}^{(2)}=a_{11b}^{(2)}|F|^2e^{-2\bar{\alpha}_0z_2}+\Omega_m^{*(1)}a_{11n1}^{(2)}Fe^{i\theta}+\Omega_m^{(1)}a_{11n2}^{(2)}F^*e^{-i\theta^*}+\left|\Omega_m^{(1)}\right|^2a_{11n3}^{(2)}, \tag{A.4d}$$
$$\sigma_{22}^{(2)}=a_{22b}^{(2)}|F|^2e^{-2\bar{\alpha}_0z_2}+\Omega_m^{*(1)}a_{22n1}^{(2)}Fe^{i\theta}+\Omega_m^{(1)}a_{22n2}^{(2)}F^*e^{-i\theta^*}+\left|\Omega_m^{(1)}\right|^2a_{22n3}^{(2)}, \tag{A.4e}$$
$$\sigma_{32}^{(2)}=a_{32b}^{(2)}|F|^2e^{-2\bar{\alpha}_0z_2}+\Omega_m^{*(1)}a_{32n1}^{(2)}Fe^{i\theta}+\Omega_m^{(1)}a_{32n2}^{(2)}F^*e^{-i\theta^*}+\left|\Omega_m^{(1)}\right|^2a_{32n3}^{(2)}, \tag{A.4f}$$

where $a_{jjn1}=a_{jjn2}^*$ $(j=1,2)$ and the coefficients are

$$a_{11b}^{(2)}=\frac{-X_1^{(2)}\left[(\omega+d_{21})/D-\text{c.c.}\right]+X_2^{(2)}}{-\Gamma_{12}\Gamma_3-i\left(\Gamma_{12}+\Gamma_{13}\right)\left(d_{32}^{-1}-\text{c.c.}\right)}, \tag{A.5a}$$
$$a_{11n1}^{(2)}=\frac{X_3^{(2)}\left[1-(\omega+d_{21})/d_{21}^*\right]\Omega_c^*/D-X_4^{(2)}}{\omega^2+i\left(\Gamma_{12}+\Gamma_3\right)\omega-\Gamma_2\Gamma_3-X_5^{(2)}}, \tag{A.5b}$$
$$a_{11n3}^{(2)}=-\frac{X_6^{(2)}\left(d_{21}^{-1}-c.c\right)-X_7^{(2)}}{\Gamma_{12}\Gamma_3+X_8^{(2)}}, \tag{A.5c}$$
$$a_{22b}^{(2)}=i\frac{\left[(\omega+d_{21}/D)-c.c\right]+i\Gamma_{13}a_{11b}^{(2)}}{\Gamma_{13}-\Gamma_{12}}, \tag{A.5d}$$
$$a_{22n1}^{(2)}=i\frac{\left(1-\frac{\omega+d_{21}}{d_{21}^*}\right)\frac{\Omega_c^*}{D}-\left(\omega+i\Gamma_{13}\right)a_{11n1}^{(2)}}{\Gamma_{13}-\Gamma_{12}}, \tag{A.5e}$$
$$a_{22n3}^{(2)}=-i\frac{\left(d_{21}^{-1}-c.c\right)-i\Gamma_{13}a_{11n3}^{(2)}}{\Gamma_{13}-\Gamma_{12}}, \tag{A.5f}$$
$$a_{32b}^{(2)}=\frac{\Omega_c}{d_{32}}\left[D^{*-1}-\left(a_{11b}^{(2)}+2a_{22b}^{(2)}\right)\right], \tag{A.5g}$$
$$a_{32n1}^{(2)}=\frac{\left(d_{21}^{*-1}+\frac{\omega+d_{21}}{D}\right)-\Omega_c\left(a_{11n1}^{(2)}+2a_{22n1}^{(2)}\right)}{\omega+d_{32}}, \tag{A.5h}$$
$$a_{32n2}^{(2)}=\frac{\Omega_c^2/\left(D^*d_{21}\right)-\Omega_c\left(a_{11n2}^{(2)}+2a_{22n2}^{(2)}\right)}{-\omega+d_{32}}, \tag{A.5i}$$
$$a_{32n3}^{(3)}=\frac{\Omega_c}{d_{32}}\left[|d_{21}|^{-2}-\left(a_{11n3}^{(2)}+2a_{22n3}^{(2)}\right)\right], \tag{A.5j}$$

with

$$X_1^{(2)}=i\left(\Gamma_{12}+\Gamma_{23}\right)-2\left|\Omega_c\right|^2\left(d_{32}^{-1}-\text{c.c.}\right),$$
$$X_2^{(2)}=-i\left(\Gamma_{13}-\Gamma_{12}\right)\left|\Omega_c\right|^2\left[\left(d_{32}D^*\right)^{-1}-\text{c.c.}\right],$$
$$X_3^{(2)}=\left[\omega+i\left(\Gamma_{12}+\Gamma_{23}\right)\right]-2\left|\Omega_c\right|^2\left[\left(\omega+d_{32}\right)^{-1}-\text{c.c.}\right],$$
$$X_4^{(2)}=i\left(\Gamma_{13}-\Gamma_{12}\right)\Omega_c^*\left[-1-\left(\omega+d_{32}\right)^{-1}\times\left(Dd_{21}^{-1}+\omega+d_{21}\right)+\left|\Omega_c\right|^2\left(-\omega+d_{32}^*\right)^{-1}d_{21}^{*-1}\right],$$
$$X_5^{(2)}=\left[2\omega+i\left(\Gamma_{12}+\Gamma_{13}\right)\left|\Omega_c\right|^2\left[\left(\omega+d_{32}\right)^{-1}-\text{c.c.}\right]\right],$$
$$X_6^{(2)}=i\left(\Gamma_{12}+\Gamma_{23}\right)-2\left|\Omega_c\right|^2\left(d_{32}^{-1}-\text{c.c.}\right),$$
$$X_7^{(2)}=i\left(\Gamma_{13}-\Gamma_{12}\right)\left[\left(d_{21}^{-1}-\text{c.c.}\right)-\left|\Omega_c/d_{21}\right|^2\left(d_{32}^{-1}-\text{c.c.}\right)\right]$$

and

$$X_8^{(2)}=i\left(\Gamma_{12}+\Gamma_{13}\right)\left|\Omega_c\right|^2\left(d_{32}^{-1}-\text{c.c.}\right).$$

(iii) *Third-order approximation.* In the third order ($l=3$), the solvability condition requires

$$i\frac{\partial F}{\partial z_2}-\frac{1}{2}\frac{\partial^2K}{\partial\omega^2}\frac{\partial^2F}{\partial t_1^2}-W|F|^2Fe^{-2\alpha z_0}-\left|\Omega_m^{(1)}\right|^2S_1^{(3)}F=0, \tag{A.6}$$

where W is the static Kerr coefficient, giving a self-phase modulation of the probe field, which reads

$$W=-\kappa_{13}\frac{\Omega_ca_{32}^{*(2)}+\left(\omega+d_{21}\right)\left(2a_{11}^{(2)}+a_{22}^{(2)}\right)}{\left|\Omega_c\right|^2-\left(\omega+d_{21}\right)\left(\omega+d_{31}\right)}. \tag{A.7}$$

The coefficient $S_1^{(3)}$ is given by

$$S_1^{(3)}=-\left(K-\frac{\omega}{c}\right)\left(\frac{\Omega_c}{\omega+d_{21}}N_1^{(3)}-P_1^{(3)}\right), \tag{A.8}$$

with $N_1^{(3)}=-[(a_{11n1}^{(2)}-a_{22n1}^{(2)})-a_{32n3}^{*(2)}-\Omega_ca_{32n2}^{*(2)}/d_{21}]$, $P_1^{(3)}=-[\Omega_c(2a_{11n1}^{(2)}+a_{22n1}^{(2)})/d_{21}+(2a_{11n3}^{(2)}+a_{22n3}^{(2)})-a_{32n1}^{(2)}]$. The third-order approximation solution reads

$$\Omega_p^{(3)}=J_1e^{-i\theta^*}+J_2+J_3e^{2i\theta}+J_4e^{-2\bar{\alpha}_0z_2}, \tag{A.9a}$$
$$\sigma_{31}^{(3)}=\sigma_{31b}^{(3)}+\Omega_m^{(1)2}a_{31n2}^{(3)}F^*e^{-i\theta^*}+\Omega_m^{*(1)}a_{31n4}^{(3)}F^2e^{2i\theta}+\Omega_m^{(1)}a_{31n5}^{(3)}|F|^2e^{-2\bar{\alpha}_0z_2}, \tag{A.9b}$$
$$\sigma_{21}^{(3)}=\sigma_{21b}^{(3)}+\left|\Omega_m^{(1)}\right|^2a_{21n1}^{(3)}Fe^{i\theta}+\Omega_m^{(1)2}a_{21n2}F^*e^{-i\theta^*}+\Omega_m^{(1)}\left|\Omega_m^{(1)}\right|^2a_{21n3}^{(3)}+\Omega_m^{*(1)}a_{21n4}^{(3)}F^2e^{2i\theta}+\Omega_m^{(1)}a_{21n5}^{(3)}|F|^2e^{-2\bar{\alpha}_0z_2}, \tag{A.9c}$$
$$\sigma_{11}^{(3)}=\sigma_{11b}^{(3)}+\left(i\Omega_m^{*(1)}a_{11n1}^{(3)}\frac{\partial}{\partial t_1}Fe^{i\theta}+c.c\right), \tag{A.9d}$$
$$\sigma_{22}^{(3)}=\sigma_{22b}^{(3)}+\left(i\Omega_m^{*(1)}a_{22n1}^{(3)}\frac{\partial}{\partial t_1}Fe^{i\theta}+c.c\right), \tag{A.9e}$$
$$\sigma_{32}^{(3)}=\sigma_{32b}^{(3)}+\left(i\Omega_m^{*(1)}a_{32n1}^{(3)}\frac{\partial}{\partial t_1}Fe^{i\theta}+c.c\right). \tag{A.9f}$$

All the coefficients in the above equations can be explicitly calculated from the solutions (A.2) and (A.4), which are not written down explicitly. Here, we list the expressions of $\sigma_{31b}^{(3)}$, $\sigma_{21b}^{(3)}$, $\sigma_{11b}^{(3)}$, $\sigma_{22b}^{(3)}$ and $\sigma_{33b}^{(3)}$, which read

$$\sigma_{31b}^{(3)} = -\frac{i}{\kappa_{13}}\frac{\partial}{\partial z_2}Fe^{i\theta}, \tag{A.10a}$$

$$\sigma_{21b}^{(3)} = a_{21b1}^{(3)}\frac{\partial^2}{\partial t_1^2}Fe^{i\theta} + a_{21b2}^{(3)}|F|^2Fe^{-2\bar{\alpha}_0 z_2}e^{i\theta}, \tag{A.10b}$$

$$\sigma_{11b}^{(3)} = ia_{11b1}^{(3)}\frac{\partial}{\partial t_1}\left(|F|^2\right)e^{-2\bar{\alpha}_0 z_2} + i\left(a_{11b2}^{(3)}F^*\frac{\partial}{\partial t_1}Fe^{-2\bar{\alpha}_0 z_2} + c.c\right), \tag{A.10c}$$

$$\sigma_{22b}^{(3)} = ia_{22b1}^{(3)}\frac{\partial}{\partial t_1}\left(|F|^2\right)e^{-2\bar{\alpha}_0 z_2} + i\left(a_{22b2}^{(3)}F^*\frac{\partial}{\partial t_1}Fe^{-2\bar{\alpha}_0 z_2} + c.c\right), \tag{A.10d}$$

$$\sigma_{32b}^{(3)} = ia_{22b1}^{(3)}\frac{\partial}{\partial t_1}\left(|F|^2\right)e^{-2\bar{\alpha}_0 z_2} + ia_{22b2}^{(3)}F^*\frac{\partial}{\partial t_1}Fe^{-2\bar{\alpha}_0 z_2} + ia_{22b3}^{(3)}F\frac{\partial}{\partial t_1}F^*e^{-2\bar{\alpha}_0 z_2}, \tag{A.10e}$$

with $a_{jjb2}^{(3)} = a_{jjb3}^{*(3)}$ $(j = 1, 2)$, and coefficients are

$$a_{21b1}^{(3)} = \frac{\Omega_c^*}{D^3}\left[|\Omega_c|^2 + (\omega + d_{21})^2 + (\omega + d_{31})^2 + (\omega + d_{21})(\omega + d_{31})\right], \tag{A.11a}$$

$$a_{21b2}^{(3)} = \frac{1}{\omega + d_{21}}\left(a_{32b}^{*(2)} + \frac{\Omega_c^* W}{\kappa_{13}}\right), \tag{A.11b}$$

$$a_{11b1}^{(3)} = \frac{X_1^{(3)}a_{11b}^{(2)} - i\left(\Gamma_{13} - \Gamma_{12}\right)a_{22b}^{(2)} + X_2^{(3)}}{\Gamma_{12}\Gamma_3 + i\left(\Gamma_{12} + \Gamma_{13}\right)|\Omega_c|^2\left(d_{32}^{-1} - c.c\right)}, \tag{A.11c}$$

$$a_{11b2}^{(3)} = \frac{X_1\kappa_{13}^{-1}\left(v_g^{-1} - c^{-1}\right) + X_3^{(3)}}{\Gamma_{12}\Gamma_3 + i\left(\Gamma_{12} + \Gamma_{13}\right)|\Omega_c|^2\left(d_{32}^{-1} - c.c\right)}, \tag{A.11d}$$

$$a_{22b1}^{(3)} = i\frac{a_{11b}^{(2)} + i\Gamma_{13}a_{11b1}^{(3)}}{\Gamma_{13} - \Gamma_{12}}, \tag{A.11e}$$

$$a_{22b2}^{(3)} = i\frac{\kappa_{13}^{-1}\left(v_g^{-1} - c^{-1}\right) + i\Gamma_{13}a_{11b2}^{(3)}}{\Gamma_{13} - \Gamma_{12}}, \tag{A.11f}$$

$$a_{32b1}^{(3)} = -\frac{a_{32b}^{(2)} + \Omega_c\left(a_{11b1}^{(3)} + 2a_{22b1}^{(3)}\right)}{d_{32}}, \tag{A.11g}$$

$$a_{32b2}^{(3)} = -\frac{\Omega_c\left(a_{11b2}^{(3)} + 2a_{22b2}^{(3)}\right)}{d_{32}}, \tag{A.11h}$$

$$a_{32b3}^{(3)} = -d_{32}^{-1}\left[\frac{\Omega_c}{D^{*2}}\left(2\omega + d_{21}^* + d_{31}^*\right) + \Omega_c\left(a_{11b3}^{(3)} + 2a_{22b3}^{(3)}\right)\right]. \tag{A.11i}$$

(iv) *Fourth-order approximation*. In the fourth order ($l = 4$), the solvability condition reads

$$i\frac{\partial F}{\partial z_3} - i\frac{1}{6}\frac{\partial^3 K}{\partial\omega^3}\frac{\partial^3}{\partial t_1^3}F - i\beta_1\frac{\partial}{\partial t_1}\left(|F|^2F\right)e^{-2\bar{\alpha}_0 z_2} + i\beta_2 F\frac{\partial}{\partial t_1}\left(|F|^2\right)e^{-2\bar{\alpha}_0 z_2} - i\left|\Omega_m^{(1)}\right|^2 S_1^{(4)}\frac{\partial}{\partial t_1}F = 0, \tag{A.12}$$

where the coefficients are given by

$$\beta_1 = \frac{\kappa_{13}}{D}\left\{\Omega_c\left(a_{21b2}^{(3)} - a_{32b2}^{*(3)} + a_{32b3}^{*(3)}\right) + (\omega + d_{21}) \times\left[\frac{W}{\kappa_{13}} - \left(2a_{11b2}^{(3)} + a_{22b2}^{(3)}\right) + \left(2a_{11b3}^{(3)} + a_{22b3}^{(3)}\right)\right]\right\}, \tag{A.13a}$$

$$\beta_2 = \frac{\kappa_{13}}{D}\left\{-\Omega_c\left(a_{32b1}^{*(3)} + 2a_{32b2}^{*(3)} - a_{32b3}^{*(3)}\right) + (\omega + d_{21}) \times\left[\left(2a_{11b1}^{(3)} + a_{22b1}^{(3)}\right) - \left(2a_{11b2}^{(3)} + a_{22b2}^{(3)}\right) + \left(2a_{11b2}^{(3)} + a_{22b2}^{(3)}\right)\right]\right\}, \tag{A.13b}$$

$$S_1^{(4)} = -\frac{\kappa_{13}}{D}\left[\Omega_c N_1^{(4)} - (\omega + d_{21})P_1^{(4)}\right], \tag{A.13c}$$

with $N_1^{(4)} = -[a_{21n1}^{(3)} - \Omega_c a_{32n2}^{*(3)}/d_{21} + (a_{11n1}^{(3)} - a_{22n1}^{(3)})]$, and $P_1^{(4)} = a_{32n1}^{(3)} - \Omega_c(2a_{11n1}^{(3)} + a_{22n1}^{(3)})/d_{21}$.

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