

# Hirshfeld-E Partitioning: AIM Charges with an Improved Trade-off between Robustness and Accurate Electrostatics

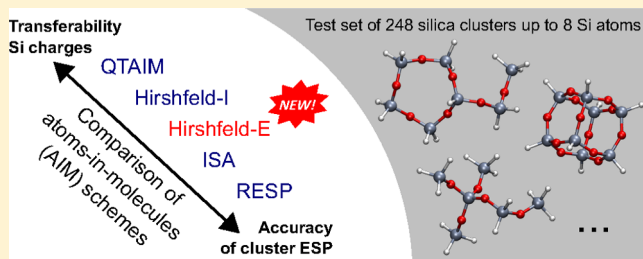
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## S Supporting Information

**ABSTRACT:** For the development of ab initio derived force fields, atomic charges must be computed from electronic structure computations, such that (i) they accurately describe the molecular electrostatic potential (ESP) and (ii) they are transferable to the force-field application of interest. The Iterative Hirshfeld (Hirshfeld-I or HI) scheme meets both requirements for organic molecules. For inorganic oxide clusters, however, Hirshfeld-I becomes ambiguous because electron densities of nonexistent isolated anions are needed as input. Herein, we propose a simple Extended Hirshfeld (Hirshfeld-E or HE) scheme to overcome this limitation. The performance of the new HE scheme is compared to four popular atoms-in-molecules schemes, using two tests involving a set of 248 silica clusters. These tests show that the new HE scheme provides an improved trade-off between the ESP accuracy and the transferability of the charges. The new scheme is a generalization of the Hirshfeld-I scheme, and it is expected that its improvements are to a large extent applicable to molecular systems containing elements from the entire periodic table.



## 1. INTRODUCTION

Force fields are the most computationally efficient models for approximating the Born–Oppenheimer potential energy surface of a molecular system. In computational material science and molecular biology, force fields are used to study molecular systems consisting of more than a million atoms<sup>1,2</sup> and allow one to follow the motions of macromolecules over a time scale of milliseconds.<sup>3</sup> These computational amenities come at a price, namely the empirical nature of a force-field model. Due to the large number of fitted parameters in a force field, statistical correlations restrict the transferability of these parameters to systems that are not included in the reference data.<sup>4–6</sup> Herein, we propose a new atoms-in-molecules (AIM) scheme, which is a huge step forward in the ongoing effort to develop methods for accurate and transferable ab initio force fields. Only the electrostatic electrostatic force-field term is considered in its simplest form, i.e., the energy of a set of fixed atomic point charges. One typically fits these charges to the ab initio electrostatic potential (ESP) of a set of reference molecules.<sup>7–9</sup> However, a large variety of population analysis or atoms-in-molecules schemes (other than ESP fitting) could be used instead. For the development of a force field, such a scheme must be optimal in two ways: (i) maximal *accuracy* of the ESP generated by the point charges and (ii) maximal *transferability* of the charges between similar atoms in a different molecular environment. An AIM scheme with a good trade-off between these two requirements is also of interest outside the scope of molecular mechanics force fields: it is a

helpful tool in any computational chemistry study to understand the electrostatic interactions,<sup>10</sup> and it is also beneficial in the field of conceptual density functional theory,<sup>11,12</sup> e.g., for the computation of condensed reactivity indicators.<sup>13–15</sup> Furthermore, Hirshfeld AIM methods are extensively used in dispersion corrections for DFT computations.<sup>16–19</sup>

ESP-fitted charges perform well (by definition) in terms of ESP accuracy but are weak in terms of transferability; even subtle conformational changes can lead to disproportional variations in the charges.<sup>20</sup> Quantum Theory of Atoms in Molecules (QTAIM) charges<sup>21</sup> have the opposite characteristics: the method is well-known for its transferability,<sup>22</sup> but one has to introduce long atomic multipole expansions to obtain a reasonable ESP.<sup>23</sup> The goal of this work is to propose a simple and transparent scheme that combines ESP accuracy with transferability. Several assessments based on test sets of organic molecules have shown that the Hirshfeld-I (HI) scheme<sup>24</sup> already combines accurate ESPs,<sup>25,26</sup> with different kinds of transferability: HI charges are robust with respect to the choice of basis set<sup>27</sup> and conformational changes,<sup>10,20,28</sup> and one has also proposed transferable parameters for the Electronegativity Equalization Method<sup>29</sup> (EEM) and the Split-charge Equilibration<sup>30</sup> (SQE) model that closely approximate HI charges.<sup>26,31,32</sup>

Unfortunately, there are several molecular and crystalline systems to which Hirshfeld-I cannot be applied in a

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straightforward manner. For example in silica, the oxygen charge comes close to  $-2e$ , and hence the scheme requires the density of an nonexistent isolated oxygen dianion as input. The computation of such reference atoms can only be carried out in an artificial way for which several procedures were proposed. Originally, the problematic anions were computed in a finite basis which forces the extra electron onto the atom.<sup>24</sup> In the DDEC schemes one surrounds the anion by a shell of positive point charges at an empirically determined radius to bind the excess electron.<sup>33,34</sup> More recently one has proposed to compute the reference atoms in a periodic box with plane waves, neglecting the part of the electron density that escapes from the anions.<sup>35</sup> However, it remains discouraging that the Hirshfeld-I scheme can only be applied consistently to well-behaved organic systems. When a system includes more diverse chemical elements, one must introduce workarounds in the Hirshfeld-I scheme, and yet there is no consensus on the preferred approach.<sup>36,37</sup>

The *original Hirshfeld-I procedure* yields charges that severely overestimate the polarity of inorganic oxide systems.<sup>32–34,38</sup> In this work, we illustrate this undesirable behavior with a small silicate molecule. (This weakness is not only present in periodic systems.) The more recent alternative approaches<sup>33–35</sup> are more promising in this regard, but their implementation is not as straightforward. Therefore, a new variation of the HI scheme is proposed, namely Hirshfeld-E (HE), where E stands for “Extended,” to overcome the ambiguity of HI. This scheme combines several conceptual advantages of other HI variants: (i) it is mathematically simple, (ii) it precludes the use of unrealistic atomic references, (iii) it fully exploits the transferability of the isolated atom densities, and (iv) the method does not depend on empirical parameters.

The paper is structured as follows. The details of the DFT computations used to test the new HE scheme are discussed in the next section. Section 3 shows one of the weaknesses of the original Hirshfeld-I procedure with a simple example and introduces the Hirshfeld-E method. Two tests involving the entire set of test molecules are given in section 4. The last section summarizes the new findings.

## 2. COMPUTATIONAL DETAILS

Using a test set of 248 silica clusters, the performance (in terms of ESP accuracy and transferability) of the new HE scheme is compared with a selection of other popular population analysis schemes. The test set contains all possible silicon–oxide clusters with two to eight silicon atoms that fulfill the following criteria: (i) each silicon atom is coordinated by four atoms (either oxygen or hydrogen), (ii) the oxygen atoms form a bridge between two silicon atoms, (iii) the hydrogen atoms terminate the cluster, (iv) rings in the bond graph with less than four silicon atoms are not allowed, and (v) the optimized geometry should not contain Si–O bonds that deviate more than 10% from the average Si–O bond length in silica, i.e., 1.65 Å. The last criterion is necessary to exclude two unrealistic structures with eight silicon atoms. The chemical formula of these clusters is  $\text{Si}_x\text{O}_{2x-y}\text{H}_{2y}$ . Geometries were optimized with suitable density functional theory methods (PBE0 functional,<sup>39</sup> 6-311+(d,p) basis for Si and H, 6-311+(2df,p) basis for O<sup>40,41</sup>) with Gaussian 09.<sup>42</sup> Images and Cartesian coordinates of all clusters are included in the Supporting Information. Isolated atoms and ions for the Hirshfeld-I and Hirshfeld-E partitioning are computed at the same level of theory, and all-electron densities are used throughout this work.

## 3. THE HIRSHFELD-E SCHEME

The general ansatz in stockholder partitioning (e.g., Hirshfeld,<sup>43</sup> Hirshfeld-I,<sup>24</sup> and ISA<sup>44</sup>) is the definition of atomic weight functions based on pro-atomic densities,<sup>43,45,46</sup>  $\rho_A^0$ :

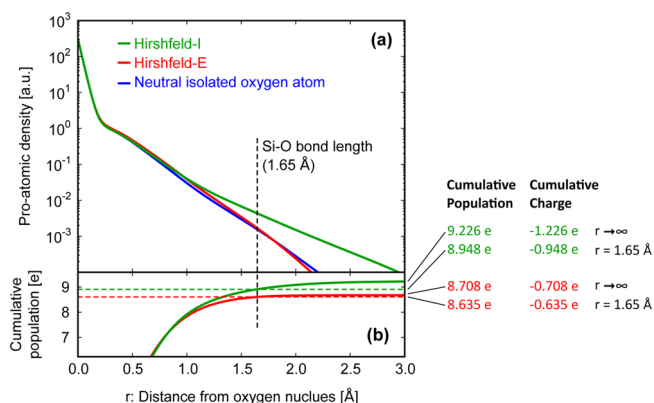
$$w_A(\mathbf{r}) = \frac{\rho_A^0(\mathbf{r})}{\sum_B \rho_B^0(\mathbf{r})} \quad (1)$$

where the denominator sums over all pro-atomic densities in a molecule and is therefore called the pro-molecular density. The AIM densities and charges are then derived from the molecular density,  $\rho_{\text{mol}}$ , and the weight functions as follows:

$$\rho_A(\mathbf{r}) = w_A(\mathbf{r}) \rho_{\text{mol}}(\mathbf{r}) \text{ and } q_A = Z_A - \int \rho_A(\mathbf{r}) \, d\mathbf{r} \quad (2)$$

Each stockholder scheme has a slightly different definition for the pro-atomic density. In the original work of Hirshfeld, the pro-atoms are spherically averaged densities of isolated neutral atoms. In the Hirshfeld-I scheme, the pro-atoms are linear interpolations between isolated ion densities such that the AIM populations and the pro-atomic populations coincide. In systems with very negative atoms ( $-2 < q_A < -1$ ), this leads to pro-atoms that interpolate between isolated anions and dianions. However, no isolated atomic dianion is stable. In the original Hirshfeld-I procedure, the two excess electrons were always bound due to basis set limitations.<sup>24</sup> (The energy of the oxygen atom as a function of the population is discussed in the Supporting Information.) The original procedure is used in this work because it leads to robust charges<sup>10,20,25,26,28</sup> that can be reproduced with transferable parameters.<sup>25,26,31,32</sup> However, for very negative atoms, such Hirshfeld-I pro-atoms are clearly unrealistic, potentially leading to undesirable artifacts.

Figure 1 illustrates a typical artifact in the HI partitioning of the disiloxane molecule ( $\text{SiH}_3\text{--O--SiH}_3$ ), the smallest molecule



**Figure 1.** (a) The HI and HE pro-atomic density of oxygen in disiloxane compared to the spherically averaged density of an isolated oxygen atom. (b) The HI and HE electron population of the oxygen in disiloxane within a given distance from the nucleus.

in our test set. Figure 1a shows the Hirshfeld-I pro-atom for oxygen, which has a net charge of  $-1.226e$  and a slowly decaying electron-density tail compared to the neutral oxygen. This pro-atom is artificial because it contains contributions from an unrealistic  $\text{O}^{2-}$  density. Even if one would be able to properly compute the isolated  $\text{O}^{2-}$  density, such a highly delocalized ion would not be representative of the oxygen in a ceramic system. The large tail will lead to an increase of the Hirshfeld-I weight function of the oxygen atom at long

distances. The implications of this long tail are shown in Figure 1b, which is a plot of the following integral:

$$N_A(r) = \int_{|\mathbf{r}-\mathbf{R}_A|<r} \rho_A(\mathbf{r}) d\mathbf{r} \quad (3)$$

where  $\rho_A$  is the AIM density of the oxygen atom in disiloxane and  $\mathbf{R}_A$  is the position of the oxygen nucleus. This function, hereafter referred to as *cumulative population*, counts the number of electrons in the oxygen within a sphere with radius  $r$  centered at the nucleus. Within the Hirshfeld-I scheme, the cumulative population of oxygen converges to 9.226e, of which 0.318e stems from contributions at distances further away than the neighboring silicon nuclei. This means that a considerable part of the oxygen charge is determined by very nonlocal contributions. This is detrimental for the quality of the ESP, as such nonlocal contributions prohibit a rapidly converging multipole expansion of the AIM densities.

The principal idea of Hirshfeld-E is to generalize the definition of the pro-atom as follows:

$$\rho_A^{0,\text{HE}}(\mathbf{r}) = \sum_{k=1}^K c_k \langle f_k^{\text{iso}} \rangle (|\mathbf{r} - \mathbf{R}_A|) \quad (4)$$

where  $\langle f_k^{\text{iso}} \rangle$  is a spherically averaged Fukui function<sup>47,48</sup> of an isolated atom. The expansion is limited to Fukui functions obtained from a difference between two stable (not basis-set bound) ions of element A; i.e., the computed energy of the ions must decrease as electrons are added.<sup>49</sup> With this ansatz, negative pro-atoms can be constructed that do not contain contributions from artificial isolated anions. The expansion coefficients,  $c_k$ , are determined iteratively, in analogy with Hirshfeld-I:

1. The molecule is partitioned with the original Hirshfeld scheme.
2. The coefficients  $c_k$  are derived from a least-squares fit of each pro-atom to the corresponding spherically averaged AIM density, with constraints to keep the coefficients positive and to fix the charge of the pro-atom at the AIM charge.
3. The molecular density is partitioned with the new pro-atoms to obtain updated AIM densities.

The last two steps are repeated until the expansion coefficients converge.

The new method is called Extended Hirshfeld because the ensemble conditions of Hirshfeld-I are extended; i.e., eq 4 includes all possible Hirshfeld-I pro-atoms but also allows more general linear combinations of atomic densities. The Hirshfeld-E pro-atoms no longer follow the strict prescriptions of Ensemble DFT<sup>50</sup> as in Hirshfeld-I.<sup>24</sup> One could also interpret Hirshfeld-E as a variant of the GISA method,<sup>28</sup> in which the pro-atom is expanded in a basis of spherically averaged atomic Fukui functions instead of s-type Gaussian functions.

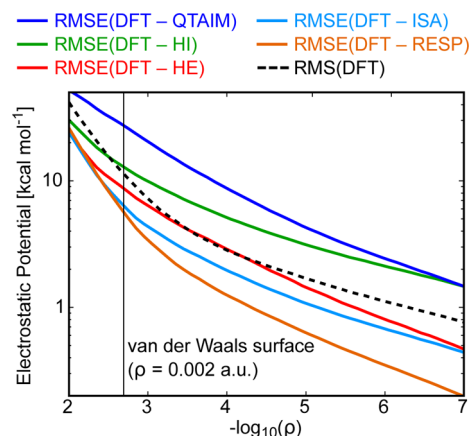
The Hirshfeld-E partitioning of the disiloxane molecule leads to an oxygen charge of  $-0.708e$  (cfr.  $-1.226e$  for HI). (All charges and pro-atom expansion coefficients are given in the Supporting Information.) This result is encouraging as it was found that the Hirshfeld-I charges were larger in absolute value than ESP fitted charges.<sup>32</sup> Figure 1a shows that the tail of the HE pro-atom decays faster and has an overall shape that is very similar to the neutral oxygen atom. Only 0.073e of the oxygen electron population comes from regions further away than the neighboring silicon nuclei in Hirshfeld-E.

## 4. RESULTS AND DISCUSSION

The ESP accuracy and transferability of the new HE scheme is compared in the two tests below with a selection of population analysis schemes: QTAIM,<sup>21</sup> HI,<sup>24</sup> ISA,<sup>44</sup> and RESP.<sup>51</sup> Both tests perform a statistical analysis on the results obtained for the entire test set of 248 silica clusters. QTAIM charges were computed with the AIMALL code.<sup>52</sup> RESP charges were derived with the RESP program from the Antechamber package.<sup>53</sup> HI, ISA, and HE charges were computed with our in-house program HiPart.<sup>54</sup>

First, the ESP accuracy of the atomic charges is tested. For this purpose, the DFT ESP and electronic density ( $\rho$ ) from the 248 silica clusters are computed on rectangular grids with a spacing of 0.3 Å. The boundaries of the grids are far enough from the molecule, such that all possible grid points with a density above  $10^{-7}$  au are included. On the same grids, also the ESPs predicted by the point charges (obtained with each scheme) are computed. For the statistical analysis, the grid points (from all 248 silica clusters) are divided in sets. The set  $S_i$  contains all the grid points that satisfy  $i/10 < -\log_{10}(\rho \text{ in au}) \leq (i+1)/10$ . Such sets are constructed for integer  $i$  going from 20 to 69, corresponding to densities varying from  $10^{-2}$  au to  $10^{-7}$  au. Within each set and for each population analysis scheme, the root-mean-square error (RMSE) between the DFT ESP and a point charge ESP is computed. This analysis is designed to show how well the point charges reproduce the electrostatic potential at different degrees of penetration into the electronic density.

The results of the first test are visualized in Figure 2. Obviously, the errors on the ESP decrease in regions of lower



**Figure 2.** Root-mean-square errors (RMSEs) of the point charge ESP versus the DFT ESP, as a function of the electron density. The root-mean-square value (RMS) of the DFT ESP is plotted as a black dashed line.

electron density, i.e., further outside the molecule. On the one hand, this is due to the convergence of the atomic multipole expansion at larger distances. On the other hand, the effect of penetration in the electronic density (which is not accounted for by point charges) vanishes at lower densities.<sup>55</sup> The RESP scheme performs best in this test, followed by ISA, HE, HI, and QTAIM in order of increasing error. The root-mean-square value (RMS) of the DFT ESP is included in the same figure as a reference. It shows that QTAIM and HI have errors on the ESP that are larger than the actual ESP data itself. At a density of 0.002 au, which is often used as an approximation of the van



der Waals surface,<sup>56</sup> none of the schemes reaches *chemical accuracy*, i.e., errors below 1 kcal mol<sup>-1</sup>. One must use higher atomic multipoles and/or delocalized charge distributions in order to reach chemical accuracy at the van der Waals surface.

The second test probes the transferability of the charges in each scheme. Only silicon atoms are considered because they are (in our test set) always hard, nonpolarizable, and positively charged atoms, for which one expects transferable charges. Because the Si–O bond is more polar than the Si–H bond, the charge on a silicon atom will depend on the number of oxygens it binds to. Therefore, the silicon atoms are grouped into four atom types: Si1, Si2, Si3, and Si4, where the number equals the number of oxygen atoms bonded to the silicon. The spread on the charges within one set of silicon atoms is then used as an *inverse* measure for transferability.

Average charges and standard deviations for each type of silicon are given in Table 1. The last column in the table

**Table 1. Average Values and Standard Deviations of the Four Types of Silicon Charges Obtained with Each Scheme**

scheme	average charge				(mean) <sup>a</sup>
	Si1	Si2	Si3	Si4	
QTAIM	2.87	3.03	3.16	3.29	3.09
HI	1.72	2.20	2.61	3.05	2.40
HE	1.19	1.43	1.66	1.96	1.56
ISA	1.05	1.32	1.54	1.82	1.43
RESP	0.86	1.00	1.13	1.29	1.07
scheme	standard deviation				(mean)
	Si1	Si2	Si3	Si4	
QTAIM	0.006	0.003	0.003	0.006	0.005
HI	0.014	0.015	0.018	0.022	0.017
HE	0.011	0.020	0.027	0.034	0.023
ISA	0.007	0.021	0.029	0.066	0.031
RESP	0.062	0.080	0.106	0.137	0.096

<sup>a</sup>The last column represents the mean over the four types of silicon.

contains the mean value over the first four columns. The average silicon charge correlates well with the ESP accuracy in the first test: when a scheme assigns a more positive charge to the silicon atom, it also performs worse in the ESP test in Figure 2. The standard deviation follows exactly the opposite trend; i.e., QTAIM yields the most transferable charges, followed by (in order) HI, HE, ISA, and RESP. For the schemes considered in this paper, an increase in ESP accuracy implies a decrease in transferability. Table 1 also shows that the HE scheme is only slightly worse in transferability than HI, while it is a factor of 3–5 more accurate in the ESP test. This confirms that we have succeeded in our original goal, i.e., to design a new population analysis scheme that combines transferability (close to Hirshfeld-I) with accurate ESPs (close to ISA).

The two tests above only consider a set of silica molecules. We expect that Hirshfeld-E will perform similarly in ESP accuracy and transferability for other (e.g., organic) systems. The difference between Hirshfeld-I and Hirshfeld-E is mostly relevant for the AIM analysis of molecules with very negatively charged atoms. Further assessment studies, with more general sets of molecules that contain elements from the entire periodic table, are needed to confirm this hypothesis. Unfortunately, testing the transferability of atomic charges on more general sets is not straightforward. In this work, we were able to

attribute the fluctuations on the silicon charges entirely to methodological artifacts. In the case of more polarizable or softer atoms, e.g. the oxygen atoms in our test set, the atomic charges may also change due to polarization effects.<sup>32</sup> The spread on the atomic charge is therefore, in general, not a suitable *inverse* measure for transferability. For soft atoms, it is more appropriate to test the transferability of the parameters of an electronegativity equalization model (e.g., EEM<sup>29</sup> or SQE<sup>30</sup>) that reproduce atomic charges obtained with a given scheme.

## 5. CONCLUSIONS

A new Extended Hirshfeld (Hirshfeld-E or HE) population analysis scheme is proposed that is similar to Iterative Hirshfeld (Hirshfeld-I or HI), with an extended definition of the pro-atom. An assessment of several schemes on a test set of 248 isolated silica clusters shows that the HE charges yield significantly improved electrostatic potentials while their transferability is only slightly worse than HI charges. Therefore, the HE scheme is an attractive new method for the development of accurate and transferable electrostatic force-field terms, especially for polar systems.

## ■ ASSOCIATED CONTENT

### Supporting Information

Figures and XYZ files of the test set of isolated silica clusters. An illustration of basis-set bound oxygen anions. Detailed results for the Hirshfeld-E partitioning of disiloxane. This material is available free of charge via the Internet at <http://pubs.acs.org/>.

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### Notes

The authors declare no competing financial interest.

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