Ranking Using the Copeland Score: A Comparison with the Hasse Diagram

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This study concerns the problem of ranking objects (chemicals, projects, databases, etc.) when a number of indicators are available for these objects that convey different comparative information. There is no unique way to rank these objects while taking all indicators into account. Using the concept of partially ordered sets and the social choice theory, the Copeland score ranking methodology was applied outside of its usual political environment (voting) to rank objects in the sciences. This method avoids the disadvantages of the Hasse diagram and the linear extension usually used to resolve this issue. The ranking methodology was assessed using eight data sets, each with different numbers of objects and indicators. The results showed that the Copeland method appears to be an effective and stable tool for ranking objects, yielding results comparable to those of an evaluation by a Hasse diagram. Also, it has the advantage of facilitating the analysis of large partially ordered sets, which were practically impossible to handle using existing methods.

1. INTRODUCTION

Ranking, decision-support, and scoring systems can be used to focus attention and resources on the largest potential risk or gain concerning a set of objects and characterizing indicators. Examples of objects include projects, chemicals, databases, etc., and examples of indicators include prices, hazardous effects, sizes, etc. The decisions regarding the ranking of these objects may include which project to consider, which chemical is more hazardous, or which database conveys the most useful information.

Although numerous ranking and scoring systems have been or are being developed, there is currently no general consensus on the relative effectiveness of particular ranking methods. Chemical risk ranking has received the most attention, and several systems have been used, for example, to determine which chemical should be included in various regulatory pollutant lists. Davis et al.[1] thoroughly reviewed 51 chemical ranking and scoring systems; they presented characteristics including, among others, the system method or algorithm, the chemicals included, and the data selection approach together with literature resources for ranking chemicals. They indicated that chemical ranking and scoring systems are typically used as screening tools for the rapid assessment of relative chemical hazards. Other objects commonly considered in decision-making ranking include projects,[2] environmental databases,[3,4] pesticides,[5] sediment sites,[6] and fruits.[7]

Generally, different indicators are used to rank objects. The manner in which these indicators are combined to give an overall rank varies tremendously and significantly affects the final result. Indicators may be added, multiplied, divided, or not combined at all. Although most indicators for a certain problem are associated, they are neither comparable nor combinable.[8] Therefore, a strong urge arises to combine them into a single view, rank, or order without the need for developing new indicators by comprehensive aggregation.

In general, ranking methods can be classified as *relative ranking* or *categorical* methods. *Relative ranking* means that an overall rank or score is derived for the objects relative to one another, while *categorical* means that groups of objects are assigned high, medium, or low ranking or are determined to be selected or nonselected objects using different comparisons among their indicators. Examples of *relative ranking* are the Indiana Relative Chemical Hazard Score (IRCHS) from the CMTI Web site,[9] CHEMS-1,[10] and PROMETHEE,[2] and examples of *categorizing* methods are the Hasse diagram[11] and the method described by Ashby et al.[12]

Relative ranking has also been studied under the names "outranking" and "social choice theory". Outranking methods constitute a class of ordinal-ranking algorithms for multicriteria decision making; a good review of these methods can be found in Martel and Matarazzo.[13] The "outranking" methods uses pairwise comparison between similar indicators, and if a reference object was used for the pairwise comparisons, then the method is named Concordance Analysis; this method was introduced by Opperhuizen and Hutzinger[14] for chemical risk analysis. Social choice theory is a subfield of the political theory concerned with the logic of collective decision making, whether voting or multicriteria decision making. The original application of the social choice theory was in voting under a single-preference criterion but then has been extended to multidimensional decision making involving multiple decision criteria.[15]

Most ranking methods are parametric, i.e., a decision maker (external), information, or judgment is required to aggregate the different indicators in order to obtain the rank; a simple example is when weights are assigned to the indicators to convert them into a form suitable for aggregation into a single number, and thereafter a rank can be obtained. However, to the best of our knowledge, the Hasse diagram is the only nonparametric method used extensively for scientific (nonpolitical) ranking of objects, with many publications featuring simple mathematical concepts. The Hasse diagram is a technique that originated in discrete

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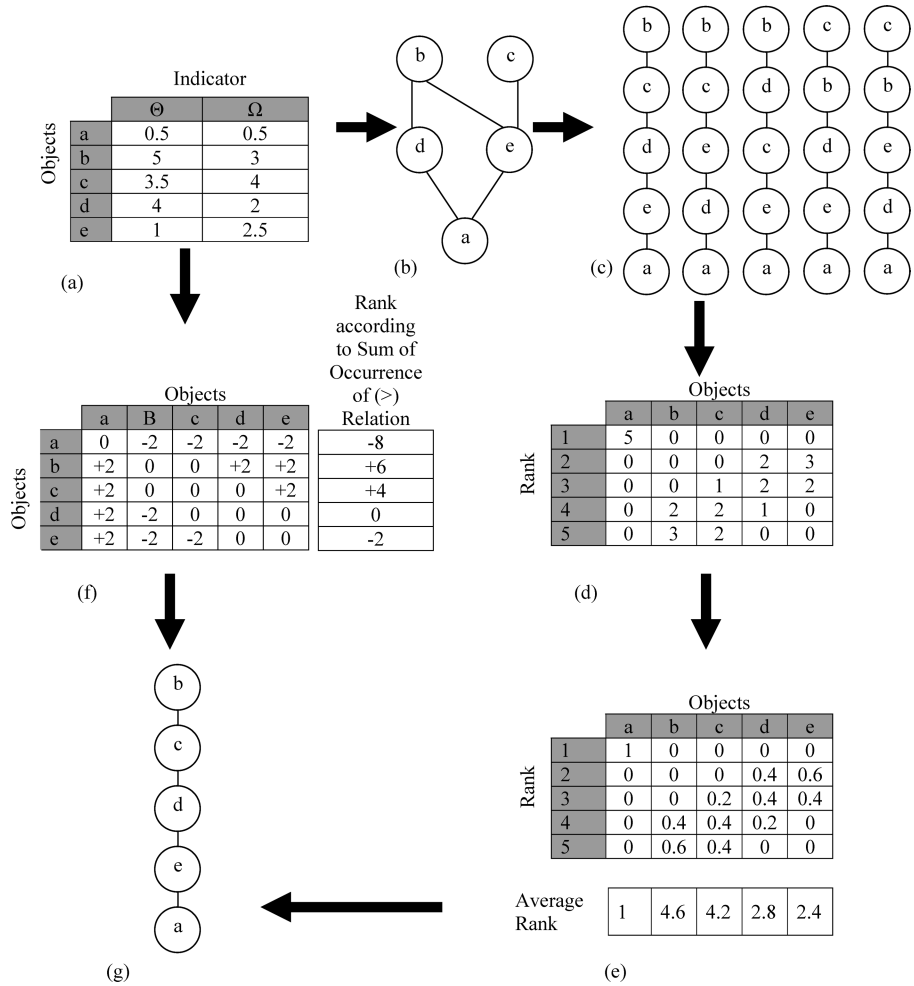

Figure 1. Ranking process starting from partially ordered data to the final total order or rank using Copeland score and Hasse diagram methods. (a) Data matrix; (b) Hasse diagram; (c) linear extensions; (d) occurrence of ranks; (e) rank probability; (f) *comparison matrix* (+1 occurrence of (>), −1 occurrence of (<), and 0 otherwise); and (g) final total order of objects (final rank).

mathematics related to partially ordered sets. It is a multicriteria evaluation applicable to decision making. In this work, we compare the Hasse method with one of the social choice methods, the Copeland score. The Copeland score method is also a simple nonparametric ranking tool, but it is usually used in the political field (voting) to rank candidates.

2. THEORY

Partial-order theory is one of the tools used for ranking objects using multiple indicators that convey different comparative information. The method of partial-order ranking has been used in many areas for a variety of purposes as an attractive way of handling multiple types of information. This method also allows a visual representation of a partially ordered set using the Hasse diagram, named after Helmut Hasse[16] for his effective use of the diagram.

Alternatives or objects are said to be partially ordered when it is impossible to find mutual relationships of ≥ or ≤ for all the objects considered. This situation happens with objects having different properties or indicators, some of which decrease and increase, and, therefore, conveying different comparative information. A total order, in contrast, ensures that all objects can be compared clearly even with different indicators. In the case of totally ordered objects, ranking and decision making is straightforward, while in the case of partially ordered objects, the ranking is based on different methods, and many investigations are under way for the efficient transfer of problems from partial to total ordering.

Partial-order ranking, which is based on element−pair comparisons and on the visualization of the results using the Hasse diagram, can be considered to be a nonparametric method. Details of these concepts are presented elsewhere;[11] only a simple example is presented in this work. The main idea of the Hasse diagram is to perform ranking decisions using the available indicators rather than creating new indicators or using simple aggregation. A given number of objects (e.g., chemicals) are usually taken for evaluation, and different indicators of these objects are also collected (e.g., flammability limit or LD) to capture the different aspects of their effect, as shown by the example in Figure 1a. The ranking of objects starts by comparing the different data pairs in terms of ≤ or ≥; if every indicator of a chemical is ≥ to all indicators of another chemical, then it is ranked higher (object *b* is greater than object *e*) and vice versa for the ≤ relation; in both cases, a line is drawn between them. It is important to note that if an object has some indicators higher and some lower than another object, the two objects are termed incomparable (object *b* is incomparable with object *c*). Thereafter, the objects are visualized in a graph, the Hasse

diagram, where each object is represented by a circle and a line is drawn between *comparable* chemicals. Figure 1b shows the Hasse diagram for the data. The important information derived from the graph is that it groups the objects into categories. In the example of Figure 1b, objects *b* and *c* are both in the top category, and object *a* is in the lowest category. Categories are associated with the indicators used, for example, a category can present a hazard effect or a high recommendation. Note, however, that the location of objects in different categories does not imply comparability and that no unique total order exists.

Hasse diagrams can be plotted using some decision analysis softwares, such as, PyHasse[17] and DART.[18] PyHasse[17] is a newly developed software and is available from Rainer Brüggemann, and DART[18] was developed by Talete srl and is made freely available as a service to scientific researchers from the Institute of Health and Consumer Protection Web site. The main advantage of ranking with the help of the Hasse diagram technique is that different pieces of information are not merged, which means that a consensus function is not needed;[19] i.e., it assumes neither linearity nor any numerical relationship among indicators and is, therefore, defined as a nonparametric method. Moreover, beyond sorting, many conclusions can be drawn from Hasse diagrams because they represent a well-defined mathematical structure,[6] the most general and least subjective[20] among ranking methods and relatively robust to uncertainties in the input data.[21] However, a Hasse diagram will yield a ranking of objects only in category form or levels, where objects may be high, medium, or low, i.e., hierarchies. It fails to provide a total ordering of the objects, and it ranks two objects as incomparable even if most of the indicators of one object are higher than the other. Another drawback of the Hasse diagram is that sometimes it consists of several parts, which are not connected. In such cases, the interpretation becomes troublesome or even impossible; Brüggemann et al.[19] used a lattice theory to help in such cases; however, this theory is not easy to implement, especially for large data sets. Although the mathematical structure behind the Hasse diagram is very simple, drawing the diagram requires considerable programming skills. Moreover, diagrammatic visualization must be interpreted by the user, and this interpretation may be incomplete.[5]

The Hasse diagram leaves users with the need to agree on how to break incomparabilities and resolve ambiguities, thus creating a total ordering of the objects in a way that cannot be manipulated to the advantage of any particular object. One method is to use a set of linear extensions to provide ranking probabilities. The method involves applying combinatorial enumeration to find all the possible ranking positions of the object from which a rank probability and an average rank are calculated. An example of the linear extension and probability method is shown in Figure 1c−e. All possible ranks of the objects are indicated, and then rank probabilities were calculated. The number of possible ranking positions increases dramatically with the number of incomparable objects. Therefore, even when using a supercomputer, it is practically impossible to identify all possibilities for partial order with more than 20 objects. This problem was tackled by studying an efficient way to implement only part of the linear extension based on random selection[22] or by a Monte Carlo−Markov Chain method;[23] recently De Loof et al.[24,25] developed a method based on lattice theory. Another way for obtaining a total ordering from the Hasse diagram was the derivation of an approximate analytical expression for an average rank and a ranking probability; Brüggemann et al.[26] derived an expression for an average rank depending on the number of successor objects, the total number of objects, and the number of incomparable objects. Finally, obtaining a unique decision on the rank can be achieved after the Hasse diagram by subsequent steps of aggregation and weighting procedures proposed by Voigt and Brüggemann.[27] Another disadvantage of the Hasse diagram is that it gives levels of ranking without any indication of how quantitatively different the categories are; for example, is the difference between the first and second categories the same as that between the second and third?

Faced with the difficulties of the Hasse diagram, a ranking method derived from the social choice theory is proposed. This method is the simple Copeland score method,[28] a more than half-century old voting procedure based on pairwise comparisons of candidates. It is one of many vote aggregation systems that social choice theorists have invented in their attempts to determine the most appropriate systems for a variety of voting situations. The Copeland rule selects the alternative with the largest Copeland score, which is the number of times an alternative beats other alternatives minus the number of times that alternative loses to other alternatives, when the alternatives are considered in pairwise comparisons. The Copeland method was proposed by A. H. Copeland in an unpublished seminar at the University of Michigan on applications of mathematics to the social sciences and has been used extensively in evaluating election results after voting. In two papers, Saari and Merline[29,30] provide an exhaustive investigation into the properties and flaws of the Copeland method. The major arguments against this method are that it puts too much emphasis on the quantity of pairwise victories and defeats rather than their magnitudes and, therefore, may lose some information during the aggregation. To use this method outside of the voting field, candidates are replaced by objects, and votes are replaced by indicators; the modified Copeland method is presented next.

This method is based on comparing one indicator after another for each pair of objects and counting the number of $\geq$ results between indicators (add $+1$) and the number of $\leq$ (add -1). The total sum of comparisons is set as an element in a *comparison matrix*. An example is shown in Figure 1f; this *comparison matrix* will be a square matrix where the element in row *i* and column *j* is the count of comparisons between similar indicators for objects *i* and *j*. The first number in the matrix shown in Figure 1f is 0 because the number of $\geq$ between the indicators of object a (the first object) compared to itself is always 0. The next number is -2 because the two indicators for object *a* are $\leq$ the indicators of object *b*, and so on for the other elements in the matrix. After evaluating all the matrix elements, the sum of the rows forms the Copeland score for each object, which is the total count of comparisons between an object and all other objects. Consequently, the total order of the objects is evaluated, as shown in Figure 1g.

The *comparison matrix* has the characteristic of having all zeros as the diagonal elements; also the element in any row *i* and column *j* is the negative of the element in row *j*

Table 1. Environmental Indicators[a]

		accumulation indicators[b]				inverse degradation indicators[c]		
no.	chemical name	acc. 1	acc. 2	acc.3	acc. 4	$(\text{deg. } 1)^{-1}$	$(\text{deg. } 2)^{-1}$	$(\text{deg. } 3)^{-1}$
1	toluene	380	94	0.1	1900	3802×10^{-5}	11 900	10.0
2	bromobenzene	190	48	0.7	1500	2874×10^{-5}	5076	3.3
3	4-isopropylnitrobenzene	120	190	0.4	240	5	6803	5.0
4	2,4-dichloronitrobenzene	150	80	0.8	310	10	8929	10.0
5	pentachloronitrobenzene	3100	1140	0.4	4500	10	2326	10.0
6	benzoic acid	6	5.1	0.1	1300	1529×10^{-5}	9804	10.0
7	4-bromobenzoic acid	25	6.5	1.3	11	0.769	2747	10.0
8	2,6-dichlorobenzoic acid	9	1	0.3	9	10	35 700	10.0
9	aniline	7	5	0.5	500	4878×10^{-5}	2151	1.25
10	4-chloroaniline	260	13	4.9	280	4405×10^{-5}	3610	10.0
11	phenylenediamine	450	6	0.8	460	0.263	1862	10.0
12	benzidine	850	83	0.7	1,210	10	2451	10.0
13	2,4,6-trichloroaniline	260	330	0.7	870	2.5	2532	10.0
14	3,3′-dichlorobenzidine	940	610	0.9	3100	0.370	2427	10.0
15	phenol	200	20	0.3	2190	2415×10^{-5}	3077	5.0
16	3-cresol	40	17	0.2	1120	2809×10^{-5}	2985	5.0
17	hydroquinone	40	40	0.5	870	0.133	1742	10.0
18	4-nitrophenol	20	51	0.3	30	2.0	2532	10.0
19	pentachlorophenol	1250	260	1.0	1100	5.0	2000	10.0
20	2,6-di-*tert*-butylphenol	880	660	2.9	2600	0.286	3390	10.0
21	2,4-dichlorophenol	260	100	0.2	340	0.357	1984	10.0
22	2,4,6-trichlorophenol	50	310	7.8	60	3.33	1520	10.0
23	biphenyl	540	280	0.4	2600	6579×10^{-5}	10 500	10.0
24	2,2′-dichlorobiphenyl	2690	2420	0.1	6300	10.0	28 600	10.0
25	2,4′-dichlorobiphenyl	6720	3550	0.6	9800	10.0	34 500	10.0
26	2,5,4′-trichlorobiphenyl	8950	8850	3.2	32 000	2.0	16 900	10.0
27	2,4,6,2′-tetrachlorobiphenyl	18 300	3150	2.7	6500	10.0	25 600	10.0
28	2,4,6,2′,4′-pentachlorobiphenyl	11 500	2320	39.8	27 800	3.33	19 200	10.0
29	benzene	30	10	1.5	1700	3425×10^{-5}	6579	1.67
30	chlorobenzene	50	72	0.8	1700	3175×10^{-5}	5405	5.00
31	1,4-dichlorobenzene	100	50	0.4	560	10.0	19 600	10.0
32	1,2,4-trichlorobenzene	250	490	1.0	1400	10.0	10 200	10.0
33	pentachlorobenzene	4500	3000	2.0	14 300	10.0	50 000	10.0
34	hexachlorobenzene	24 800	2600	55.7	35 000	10.0	66 700	10.0

[a] Ref 31. [b] Accumulation Indicators: acc. 1 ≡ bioaccumulation in algae; acc. 2 ≡ bioaccumulation in fish; acc. 3 ≡ % retention in rats; and acc. 4 ≡ bioaccumulation in activated sludge. [c] Degradation Indicators: deg. 1 ≡ mineralization rates by activated sludge (%CO_2); deg. 2 ≡ photoirradiation $\times 10^{-5}$ (%CO_2); and deg. 3 ≡ photoirradiation (% organic fragments).

and column *i*. The rank found from this method is a relative rank to the other objects, and a negative rank does not mean that it is not important or, for example, not risky.

The Copeland ranking method is similar to other methods, such as linear extension or the average rank by Brüggemann et al.,[26] in that it does not guarantee that the final ranking will result in all objects having different ranks; it may result in some objects having the same rank. This is theoretically and practically acceptable in the sciences, as some objects are actually similar in their effects within the domain of a particular study.

It is interesting to note that the Copeland method can be applied where it is desired to rank objects into categories. The categories can be obtained by recognizing large differences between the calculated ranks. To elaborate further, the objects from Figure 1f and g are *b*, *c*, *d*, *e*, and *a* with ranks +6, +4, 0, −2, and −8, respectively; a large difference in the ranks is observed between +4 and 0 and between −2 and −8. This leads to having objects *b* and *c* in the first category, *d* and *e* in the second category, and *a* in the last category. These categories are the same as those obtained from the Hasse diagram (Figure 1b). Note that the procedure for categorizing the results of the Copeland score is not fixed because the judgment of what constitutes "large differences" between rank is highly related to the decision maker's considerations.

3. CASE STUDIES

The comparison between the Copeland and Hasse methods began with a case study. Freitag et al.[31] provided environmental accumulation and degradation indicators for a number of chemicals belonging to several classes, i.e., benzene, benzoic acid, aniline, phenol derivatives, chlorobiphenyl, and chlorobenzene congeners. Halfon and Reggiani[11] modified the data by averaging indicators of repeated chemicals and by providing consistency in presenting the environmental hazard (i.e., taking the reciprocal of degradation indicators); the data for 34 chemicals are shown in Table 1. Halfon and Reggiani[11] used the Freitag et al.[31] data to display the chemicals in a Hasse diagram. The 34 chemicals were ranked into 4 levels starting with the most hazardous level (each number is a chemical defined in Table 1):

Level 1 : 27, 33, 34, 25.
Level 2 : 26, 24, 28, 32.
Level 3 : 1, 23, 12, 20, 13, 30, 14, 19, 5, 4.
Level 4 : 6, 2, 3, 7, 29, 9, 10, 22, 8, 15, 16, 17, 11, 18, 21, 31.

Note again that the location of chemicals in different categories does not imply comparability or relative order; the reader should refer to the Hasse diagram, shown in Halfon and Reggiani,[11] for exact comparability lines between chemicals.

Chemical	Copeland Score
1	-17
2	-75
3	-56
4	7
5	56
6	-111
7	-73
8	-57
9	-158
10	-31
11	-65
12	17
13	-3
14	39
15	-94
16	-132
17	-90
18	-93
19	19
20	60
21	-70
22	-25
23	21
24	89
25	135
26	142
27	159
28	144
29	-74
30	-60
31	0
32	64
33	159
34	175

Most Environmentally Hazardous

34
33
27
28
26
25
24
32
20
5
14
23
19
12
4
31
13
1
22
10
3
8
30
11
21
7
29
2
17
18
15
6
16
9

Least Environmentally Hazardous

Figure 2. Rank of chemicals (chemical number is related to Table 1).

To apply the Copeland method, a program was written in MATLAB[32] mathematical software, and the results gave ranks for the chemicals as shown in Figure 2. Looking at the results of the Copeland and Hasse diagrams, it was clear that none of the relations in the Hasse diagram were violated in the Copeland ranking (Figure 2), i.e., any line in the Hasse diagram (shown in Halfon and Reggiani)[11] indicating a $\leq$ relation between two chemicals also exists in the new ranking. In technical terms, the Copeland method preserves the order relations found in the Hasse diagram.

To study the suitability of the ranks evaluated from the Copeland method, the relative hazardous effects of some chemicals were investigated. In the Copeland ranking, 2,4,6-trichlorophenol was ranked higher (more hazardous) than phenol; this is consistent with several previous reports such as the *objective concentration* ranks of these chemicals recommended by the Ministry of Environment in Ontario, which were 0.66 and 40 ppm, respectively.[33] Also, 1,4-dichlorobenzene was ranked as more hazardous than chlorobenzene, and this agrees with the generally noted dependence of toxicity on the degree of chlorination.[34,35]

Ranks obtained from the Copeland method were compared with the Hasse diagram, using further cases with different data set sizes. Seven more cases were taken from different sources with the requirement that clear data and a Hasse diagram were provided. However, as the Hasse ranking gives categories and the Copeland method gives a total rank with a different scale, normalized ranks were used for comparison.

Table 2. Cases Studied

case	reference no.	number of objects	number of indicators	average correlation coefficient PPMC
1	11	34	7	0.9877
2	3	15	5	0.9550
3	36	19	6	0.8895
4	5	50	4	0.9978
5	19	5	9	0.9325
6	6	20	5	0.9852
7	20	12	4	0.9741
8	7 Case 3	31	3	0.9216

Data for the eight cases are presented in Table 2, and the comparison plots are in Figure 3. The plots clearly show that the two methods give consistent rankings, as indicated by the consistent peaks in the plots. This consistency shows the performance and quality of the proposed simple Copeland method, as its results are comparable to the well-established Hasse diagram.

4. SENSITIVITY ANALYSIS

It was of interest to determine how sensitive the Copeland method was to variation or uncertainty in the data; i.e., what influence small variations have on the final ranking of objects. This stability problem is of major importance because decision makers usually cannot correctly fix the

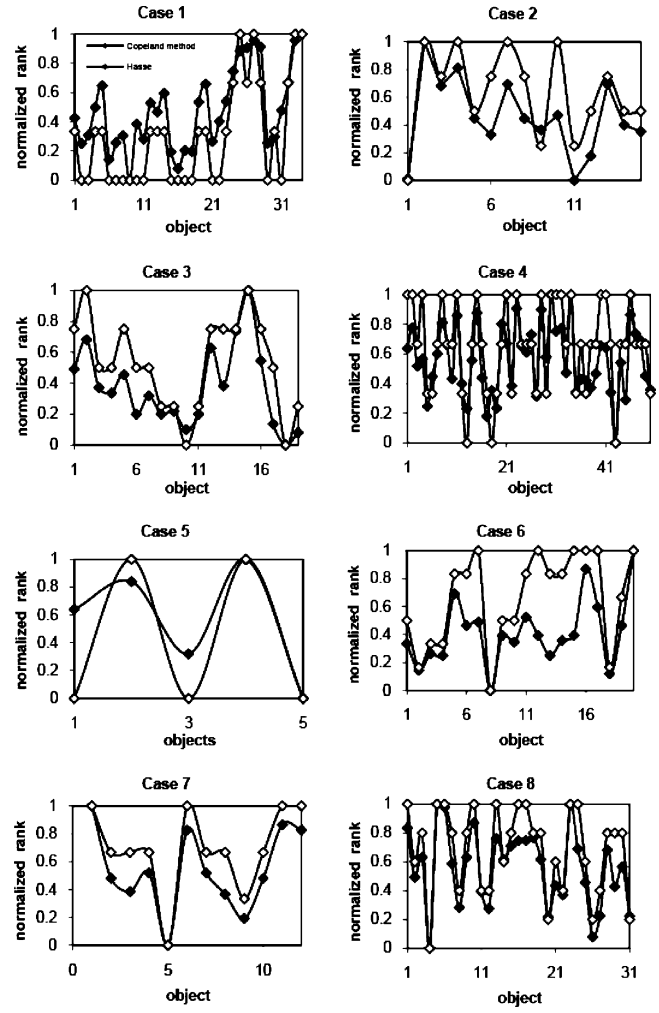

Figure 3. Copeland method and Hasse results for different cases.

values of the indicators; therefore, sensitivity analysis appears to be a necessary condition for a good ranking method. The PyHasse[17] software can perform sensitivity analysis to identify the most important object or indicator having the largest impact on the order relations. In this work, a simple Monte Carlo sensitivity test was applied; small variations in the indicators were randomly considered using MATLAB.[32] The test was 50 random variations for each case (a total of 400) in the range of −10 to 10%. New ranks were calculated and compared with the original ranks using the Pearson product moment correlation Coefficient (PPMC), and the results are shown in the last column of Table 2. The average PPMC coefficients were all close to 1, indicating a good association between the rankings and, hence, the high stability of this new ranking method.

5. CONCLUSIONS

Mathematical techniques can serve as valuable tools to evaluate scientific data in the area of decision-making based on ranking. Utilizing nonparametric methods, the data are ranked objectively, thus allowing rank-based decisions to be made without the inclusion of additional information or subjective preferences in the ranking process. The partially ordered set is a concept that has a wide application in decision-making studies. Although the concept is clear, the tools required to transform it into a totally ordered set are relatively complicated and in some cases impractical.

A clear ranking of objects according to their indicators is better evaluated when the data is totally ordered. The use of the Copeland political ranking method was proposed in this paper as a possible substitute for Hasse ranking, or as a simple nonparametric method, to obtain a relative or categorized order for objects in scientific applications. Although it has several weaknesses, such as the loss of some information during the aggregation, it has the following advantages:

(i) It is a rapid nonparametric ranking tool for identifying best or worst objects.
(ii) It is systematic and easily computerized even for very large data set size.
(iii) It is transparent, clear to the user, and flexible enough to be adapted for many purposes in terms of the number of objects or indicators.
(iv) It is based on a scientifically justified framework.
(v) Expert judgment can be used to add weights to the indicators, if needed.
(vi) It has proven to be stable to variations in the data.
(vii) It is readily comparable to the Hasse diagram method but has simpler mathematical requirements and allows for the possibility of providing *relative* or *categorical ranking*.

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