ON THE EQUIVALENCE OF TWO APPROACHES IN THE EXCITON-POLARITON THEORY

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The polariton effect in the optical processes involving photons with energies near that of an exciton is investigated by the Bogoliubov diagonalization and Green's function approaches in a simple model of the direct band gap semiconductor with an electrical dipole allowed transition. Taking into account the nonresonant terms of the interaction Hamiltonian of the photon-exciton system, the Green's function approach derived by Nguyen Van Hieu is presented with the use of the Green's function matrix technique analogous with that suggested by Nambu in the theory of superconductivity. It is shown that with a suitable choice of the phase factors the renormalization constants are equal to the diagonalization coefficients. The dispersion of polaritons and the matrix elements of processes with the participation of polaritons are identically calculated by both methods. However the Green's function approach has an advantage in including the damping effect of polaritons.

I. INTRODUCTION

In recent years many optical processes in semiconductors have been intensively studied in the resonant region: resonant Raman scattering [1–3], resonant scattering of light by light [4–5], resonant electronic Raman scattering [6–7], high order harmonic generation at resonance frequencies [8–10].

Due to the photon-exciton interaction in semiconductors there arise new elementary excitations, which are mixtures of the photon and the exciton: excitonic polaritons, or briefly polaritons. The polariton effect plays a very important role in the resonance because in this region the dispersion of the polaritons is rather different from those of the photon and the exciton.

The polariton theory was developed by many authors: the method of Bogoliubov diagonalization [12–16], Green's function approach [17, 18] and other microscopic approaches (for example [19]). Recently a new Green's function approach to the scattering of the polaritons has been derived by Nguyen Van Hieu [20]. By the use of the renormalization recipe, this method allows one to construct the matrix element of all optical processes involving a photon at resonant energies. Moreover, in this Green's function approach to the theory of polaritons the effect of the damping of polaritons can be included in a rather elegant manner.

The purpose of this work is to compare two approaches to the polariton problem: the Bogoliubov diagonalization approach and the Green's function approach [20].

We show that these approaches lead to the same results, but the second has an advantage in including the damping effect of polaritons. For this purpose we take a simple model of the direct band gap semiconductor with the electrical dipole allowed transition and with non-degenerate valence and conduction bands whose extrema are located at the centre of the Brillouin zone. We suppose that the polarization states of all photons are given. Here only the $n = 1$ exciton state is taken into account.

In section II we give a review of the Bogoliubov diagonalization method. In section III the Green's function approach derived by Nguyen Van Hieu is presented with the use of Greens' function matrix technique analogous with that suggested by Nambu in the theory of superconductivity [21]. Section IV is devoted to the study of the amplitudes of some polariton interaction processes. We shall use the unit system with $h = c = 1$.

For simplicity we use following assumption for the Bloch function $\psi_{\boldsymbol{k}}(\boldsymbol{r})$

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = e^{i\boldsymbol{kr}}\, u_{\boldsymbol{k}}(\boldsymbol{r}) \approx e^{i\boldsymbol{kr}}\, u_0(\boldsymbol{r})$$

where $u_0(\boldsymbol{r}) = u_{\boldsymbol{k}=0}(\boldsymbol{r})$, $u_{\boldsymbol{k}}(\boldsymbol{r})$ is the periodic part.

II. BOGOLIUBOV DIAGONALIZATION APPROACH

In ref. [20] it was shown that the exciton-photon transition can be described by the following effective interaction Hamiltonian

$$\mathscr{H}^{\gamma - Ex}(t) = g \int A(r, t)\, B(r, t)\, d^3 r \tag{1}$$

with the effective coupling constant

$$g = \frac{e}{m} (2E_0)^{1/2}\, \psi(0) \Pi\,, \tag{2}$$

where e and m are the (free) electron charge and mass, $E_0 \equiv E(\boldsymbol{p} = 0)$ is the energy of the exciton with the total momentum $\boldsymbol{p} = 0$, $\psi(0) \equiv \psi(\boldsymbol{r} = 0)$ is the external wave function of the exciton at the point $\boldsymbol{r} = 0$. Π denotes the matrix element of the dipole transition between the valence and conduction bands

$$\Pi = i\langle c|\, (\xi \boldsymbol{P})\, |v\rangle = \frac{1}{\Omega} \int u_0^*(\boldsymbol{r})\, (\xi \nabla)\, v_0(\boldsymbol{r})\, d^3 r\,. \tag{3}$$

Ω is the elementary cell volume, ξ is the unit vector characterizing the polarization of the photon, $A(\boldsymbol{r}, t)$ and $B(\boldsymbol{r}, t)$ are the effective scalar quantum fields for describing the photons and excitons in the given polarization state. We write

$$\begin{aligned} A(\boldsymbol{r}, t) &= A^{(+)}(\boldsymbol{r}, t) + A^{(-)}(\boldsymbol{r}, t)\,, \\ B(\boldsymbol{r}, t) &= B^{(+)}(\boldsymbol{r}, t) + B^{(-)}(\boldsymbol{r}, t)\,, \end{aligned} \tag{4}$$

where $A^{(+)}(\boldsymbol{r}, t)$ and $A^{(-)}(\boldsymbol{r}, t)$, $B^{(+)}(\boldsymbol{r}, t)$ and $B^{(-)}(\boldsymbol{r}, t)$ are Hermitian conjugate to each other,

$$(5) \qquad A^{(+)}(\boldsymbol{r}, t) = \frac{1}{(2\pi)^{3/2}} \frac{\hat{a}(\boldsymbol{k}) \exp\{i[\boldsymbol{kr} - \omega(\boldsymbol{k})\, t]\}}{[2\epsilon_0 \omega(\boldsymbol{k})]^{1/2}}\, d^3k\,,$$

$$B^{(+)}(\boldsymbol{r}, t) = \frac{1}{(2\pi)^{3/2}} \frac{\hat{b}(\boldsymbol{p}) \exp\{i[\boldsymbol{pr} - E(\boldsymbol{p})\, t]\}}{[2E(\boldsymbol{p})]^{1/2}}\, d^3p\,.$$

$\hat{a}(\boldsymbol{k})$ and $\hat{b}(\boldsymbol{p})$ are the annihilation operators of the photons and the excitons, respectively, ϵ_0 is the background dielectric constant, $\omega(\boldsymbol{k})$ is the energy of the photon with momentum $\boldsymbol{k}$.

To apply the Bogoliubov diagonalization method we write down the Hamiltonian of the photon-exciton system

$$(6) \qquad H = H_0 + H^{\gamma - E_x}\,,$$

$$(7) \qquad H_0 = \sum_{\boldsymbol{k}} \omega(\boldsymbol{k})\, \hat{a}^+(\boldsymbol{k})\, \hat{a}(\boldsymbol{k}) + \sum_{\boldsymbol{p}} E(\boldsymbol{p})\, \hat{b}^+(\boldsymbol{p})\, \hat{b}(\boldsymbol{p})\,.$$

According to (1) $H^{\gamma - E_x}$ is written in the form:

$$(8) \qquad H^{\gamma - E_x} = \sum_{\boldsymbol{k}} g_0(\boldsymbol{k}) \left[\hat{a}^+(\boldsymbol{k})\, \hat{b}(\boldsymbol{k}) + \hat{a}(\boldsymbol{k})\, \hat{b}^+(\boldsymbol{k}) + \hat{a}^+(\boldsymbol{k})\, \hat{b}^+(\boldsymbol{k}) + \hat{a}(\boldsymbol{k})\, \hat{b}(\boldsymbol{k})\right],$$

where

$$(9) \qquad g_0(\boldsymbol{k}) = \frac{g}{2[\epsilon_0\, E(\boldsymbol{k})\, \omega(\boldsymbol{k})]^{1/2}}\,.$$

By the Bogoliubov diagonalization method we can rewrite the Hamiltonian (6):

$$(10) \qquad H = \sum_{\nu, \boldsymbol{k}} \varepsilon_\nu(\boldsymbol{k})\, \hat{c}_\nu^+(\boldsymbol{k})\, \hat{c}_\nu(\boldsymbol{k})\,.$$

In this formula $\hat{c}_\nu(\boldsymbol{k})$ and $\hat{c}_\nu^+(\boldsymbol{k})$ are the annihilation and creation operators, respectively, of the polarization with momentum $\boldsymbol{k}$ in the branch ν; $\varepsilon_\nu(\boldsymbol{k})$ is the polariton energy and can be determined by the algebraic equation

$$(11) \qquad \frac{\omega^2(\boldsymbol{k})}{\varepsilon_\nu^2(\boldsymbol{k})} = 1 + \frac{1}{\varepsilon_\nu^2(\boldsymbol{k})} \frac{4g_0^2(\boldsymbol{k})\, \omega(\boldsymbol{k})\, E(\boldsymbol{k})}{E^2(\boldsymbol{k}) - \varepsilon_\nu^2(\boldsymbol{k})}\,.$$

It is easy to obtain the precise expression for $\varepsilon_\nu(\boldsymbol{k})$

$$(12) \qquad 2\varepsilon_{1,2}^2(\boldsymbol{k}) = E^2(\boldsymbol{k}) + \omega^2(\boldsymbol{k}) \pm \{[E^2(\boldsymbol{k}) - \omega^2(\boldsymbol{k})]^2 + 4g_0^2(\boldsymbol{k})\, E(\boldsymbol{k})\, \omega(\boldsymbol{k})\}^{1/2}\,.$$

The annihilation and creation operators of polaritons are connected with those of photons and excitons as follows

$$(13) \qquad \hat{c}_\nu(\boldsymbol{k}) = u_\nu^{\gamma *}(\boldsymbol{k})\, \hat{a}(\boldsymbol{k}) - v_\nu^{\gamma}(\boldsymbol{k}^*)\, \hat{a}^+(-\boldsymbol{k}) + u_\nu^{E_x}(\boldsymbol{k}^*)\, \hat{b}(\boldsymbol{k}) - v_\nu^{E_x}(\boldsymbol{k}^*)\, \hat{b}^+(-\boldsymbol{k})$$

with the transformation coefficients

$$u_\nu^\gamma(\boldsymbol{k}) = \frac{\omega(\boldsymbol{k}) + \varepsilon_\nu(\boldsymbol{k})}{2[\omega(\boldsymbol{k})\,\varepsilon_\nu(\boldsymbol{k})]^{1/2}}\,\mathscr{B}_\nu^\gamma(\boldsymbol{k})\,, \tag{14}$$

$$v_\nu^\gamma(\boldsymbol{k}) = \frac{\omega(\boldsymbol{k}) - \varepsilon_\nu(\boldsymbol{k})}{2[\omega(\boldsymbol{k})\,\varepsilon_\nu(\boldsymbol{k})]^{1/2}}\,\mathscr{B}_\nu^\gamma(\boldsymbol{k})\,,$$

$$u_\nu^{E_x}(\boldsymbol{k}) = \frac{g_0(\boldsymbol{k})}{\varepsilon_\nu(\boldsymbol{k}) - E(\boldsymbol{k})}\left[\frac{\omega(\boldsymbol{k})}{\varepsilon_\nu(\boldsymbol{k})}\right]^{1/2}\mathscr{B}_\nu^\gamma(\boldsymbol{k})\,,$$

$$v_\nu^{E_x}(\boldsymbol{k}) = \frac{-g_0(\boldsymbol{k})}{\varepsilon_\nu(\boldsymbol{k}) + E(\boldsymbol{k})}\left[\frac{\omega(\boldsymbol{k})}{\varepsilon_\nu(\boldsymbol{k})}\right]^{1/2}\mathscr{B}_\nu^\gamma(\boldsymbol{k})\,.$$

Here

$$\mathscr{B}_\nu^\gamma(\boldsymbol{k}) = \frac{|\varepsilon_\nu^2(\boldsymbol{k}) - E^2(\boldsymbol{k})|}{\{[\varepsilon_\nu^2(\boldsymbol{k}) - E^2(\boldsymbol{k})]^2 + 4g_0^2(\boldsymbol{k})\,E(\boldsymbol{k})\,\omega(\boldsymbol{k})\}^{1/2}}\,. \tag{15}$$

The annihilation operators of photons and excitons are expressed in terms of the annihilation and creation operators of polaritons by the inverse transformation

$$\hat{a}(\boldsymbol{k}) = \sum u_\nu^\gamma(\boldsymbol{k})\,\hat{c}_\nu(\boldsymbol{k}) + v_\nu^{\gamma *}(\boldsymbol{k})\,\hat{c}_\nu^+(-\boldsymbol{k})\,, \tag{16}$$

$$\hat{b}(\boldsymbol{k}) = \sum_\nu u_\nu^{E_x}(\boldsymbol{k})\,\hat{c}_\nu(\boldsymbol{k}) + v_\nu^{E_x *}(\boldsymbol{k})\,\hat{c}_\nu^+(-\boldsymbol{k})\,.$$

Using (16), one can construct a matrix element of each interaction process with the participation of polaritons from the corresponding processes involving photons or excitons. This method was applied to the study of resonant Raman scattering [14], absorption of light [16], scattering of light by light [4, 5], and nonlinear optical effect [15].

III. GREEN'S FUNCTION APPROACH

It is essential to notice that due to the presence of the terms

$$\int A^{(+)}(\boldsymbol{r}, t)\, B^{(+)}(\boldsymbol{r}, t)\, \mathrm{d}^3 r \quad \text{and} \quad \int A^{(-)}(\boldsymbol{r}, t)\, B^{(-)}(\boldsymbol{r}, t)\, \mathrm{d}^3 r$$

in the Hamiltonian (1) corresponding to the nonresonant interaction between exciton and photon we must apply the matrix – Green's function analogous to that used by Nambu [21] in the theory of superconductivity.

First we define the matrices of free field operators of photon and exciton

$$\tilde{A}(\boldsymbol{r}, t) = \begin{pmatrix} A^{(+)}(\boldsymbol{r}, t) \\ A^{(-)}(\boldsymbol{r}, t) \end{pmatrix}, \quad \tilde{A}^+(\boldsymbol{r}, t) = (A^{(-)}(\boldsymbol{r}, t), A^{(+)}(\boldsymbol{r}, t))\,. \tag{17}$$

$$\tilde{B}(\boldsymbol{r}, t) = \begin{pmatrix} B^{(+)}(\boldsymbol{r}, t) \\ B^{(-)}(\boldsymbol{r}, t) \end{pmatrix}, \quad \tilde{B}^+(\boldsymbol{r}, t) = (B^{(-)}(\boldsymbol{r}, t), B^{(+)}(\boldsymbol{r}, t))\,.$$

We denote by $\hat{\tau}$ the following matrix

$$\hat{\tau} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{18}$$

Then we can rewrite the Hamiltonian (1) in the form

$$\mathscr{H}^{\gamma - E_x}(t) = \tfrac{1}{2}g \int \tilde{A}^+(\boldsymbol{r}, t)\, \hat{\tau}\, \tilde{B}(\boldsymbol{r}, t)\, \mathrm{d}^3 r + \tfrac{1}{2}g \int \tilde{B}^+(\boldsymbol{r}, t)\, \hat{\tau}\, \tilde{A}(\boldsymbol{r}, t)\, \mathrm{d}^3 r\,. \tag{19}$$

Let us introduce the matrix of Green's functions of free fields

$$\begin{aligned} \tilde{D}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) &\equiv \mathrm{i}\langle 0| T\{\tilde{A}(\boldsymbol{r}_1, t_1)\, \tilde{A}^+(\boldsymbol{r}_2, t_2)\}\, |0\rangle = \\ &= \begin{pmatrix} D_{11}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) & D_{12}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) \\ D_{21}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) & D_{22}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) \end{pmatrix}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} D_{11}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) &\equiv \mathrm{i}\langle 0|\, T\{A^{(+)}(\boldsymbol{r}_1, t_1)\, A^{(-)}(\boldsymbol{r}_2, t_2)\}\, |0\rangle \\ D_{12}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) &\equiv \mathrm{i}\langle 0|\, T\{A^{(+)}(\boldsymbol{r}_1, t_1)\, A^{(+)}(\boldsymbol{r}_2, t_2)\}\, |0\rangle \\ D_{21}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) &\equiv \mathrm{i}\langle 0|\, T\{A^{(-)}(\boldsymbol{r}_1, t_1)\, A^{(-)}(\boldsymbol{r}_2, t_2)\}\, |0\rangle \\ D_{22}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) &\equiv \mathrm{i}\langle 0|\, T\{A^{(-)}(\boldsymbol{r}_1, t_1)\, A^{(+)}(\boldsymbol{r}_2, t_2)\}\, |0\rangle\,. \end{aligned} \tag{20'}$$

Analogously we define

$$\tilde{G}(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) \equiv \mathrm{i}\langle 0|\, T\{\tilde{B}(\boldsymbol{r}_1, t_1)\, \tilde{B}^+(\boldsymbol{r}_2, t_2)\}\, |0\rangle\,. \tag{21}$$

Here we denote by $|0\rangle$ the vacuum state of the noninteracting photon-exciton system. We also denote by $|\gamma(\boldsymbol{k})\rangle$ or $|E_x(\boldsymbol{p})\rangle$ the one-photon or one-exciton state with definite energies $\omega(\boldsymbol{k})$ or $E(\boldsymbol{p})$ and momenta $\boldsymbol{k}$ or $\boldsymbol{p}$, respectively, in the interaction representation. In the Heisenberg representation we have the interacting field operators determined as follows

$$\begin{aligned} A_H^{(\pm)}(\boldsymbol{r}, t) &= S(0, t)\, A^{(\pm)}(\boldsymbol{r}, t)\, S(t, 0) \\ B_H^{(\pm)}(\boldsymbol{r}, t) &= S(0, t)\, B^{(\pm)}(\boldsymbol{r}, t)\, S(t, 0)\,, \end{aligned} \tag{22}$$

where

$$S(t, t_0) = \sum_{n=0}^{\infty} \frac{(-\mathrm{i})^n}{n!} \int_{t_0}^{t} \dots \int_{t_0}^{t} T\{\mathscr{H}^{\gamma - E_x}(t_1) \dots \mathscr{H}^{\gamma - E_x}(t_n)\}\, \mathrm{d}t_1 \dots \mathrm{d}t_n \tag{23}$$

and the quantum interacting (dressed) field state

$$\begin{aligned} |\gamma^H(\boldsymbol{k})\rangle &= S(0, -\infty)\, |\gamma(\boldsymbol{k})\rangle \\ |E_x^H(\boldsymbol{k})\rangle &= S(0, -\infty)\, |E_x(\boldsymbol{k})\rangle\,. \end{aligned} \tag{24}$$

In a similar manner we define the matrices of the interacting field operators of phonon and exciton

$$\tilde{A}_H(\boldsymbol{r}, t) = \begin{pmatrix} A_H^{(+)}(\boldsymbol{r}, t) \\ A_H^{(-)}(\boldsymbol{r}, t) \end{pmatrix}, \quad \tilde{A}_H^+(\boldsymbol{r}, t) = (A_H^{(-)}(\boldsymbol{r}, t), A_H^{(+)}(\boldsymbol{r}, t)) \tag{25}$$

$$\tilde{B}_H(\boldsymbol{r}, t) = \begin{pmatrix} B_H^{(+)}(\boldsymbol{r}, t) \\ B_H^{(-)}(\boldsymbol{r}, t) \end{pmatrix}, \quad \tilde{B}_H^+(\boldsymbol{r}, t) = (B_H^{(-)}(\boldsymbol{r}, t), B_H^{(+)}(\boldsymbol{r}, t))$$

and the matrices of Green's functions of interacting fields

$$\tilde{D}_H(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) \equiv \mathrm{i}\langle\phi_0|\, T\{\tilde{A}_H(\boldsymbol{r}_1, t_1)\, \tilde{A}_H^+(\boldsymbol{r}_2, t_2)\}\, |\phi_0\rangle \tag{26}$$

$$\tilde{G}_H(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) \equiv \mathrm{i}\langle\phi_0|\, T\{\tilde{B}_H(\boldsymbol{r}_1, t_1)\, \tilde{B}_H^+(\boldsymbol{r}_2, t_2)\}\, |\phi_0\rangle\,.$$

Here $|\phi_0\rangle$ denotes the vacuum state of the photon-exciton system in the Heisenberg representation.

From (20) and (21) we have the expression for the Fourier transform of each element of the matrix of Green's function of free field

$$D_{11}(\boldsymbol{k}, \omega) = \frac{1}{2\epsilon_0\, \omega(\boldsymbol{k})\, [\omega(\boldsymbol{k}) - \omega]} \tag{27}$$

$$D_{22}(\boldsymbol{k}, \omega) = \frac{1}{2\epsilon_0\, \omega(\boldsymbol{k})\, [\omega(\boldsymbol{k}) + \omega]}$$

$$D_{12}(\boldsymbol{k}, \omega) = D_{21}(\boldsymbol{k}, \omega) = 0$$

$$G_{11}(\boldsymbol{k}, \omega) = \frac{1}{2E(\boldsymbol{k})\, [E(\boldsymbol{k}) - \omega]} \tag{28}$$

$$G_{22}(\boldsymbol{k}, \omega) = \frac{1}{2E(\boldsymbol{k})\, [E(\boldsymbol{k}) + \omega]}$$

$$G_{12}(\boldsymbol{k}, \omega) = G_{12}(\boldsymbol{k}, \omega) = 0\,.$$

We denote

$$D(\boldsymbol{k}, \omega) = \sum_{i,j=1,2} D_{ij}(\boldsymbol{k}, \omega) = \frac{1}{\epsilon_0[\omega^2(\boldsymbol{k}) - \omega^2]} \tag{29}$$

$$G(\boldsymbol{k}, \omega) = \sum_{i,j=1,2} G_{ij}(\boldsymbol{k}, \omega) = \frac{1}{E^2(\boldsymbol{k}) - \omega^2}\,.$$

From (19), (22), (23), and the perturbative expansions of Green's functions of the interacting fields it is easy to verify that the Fourier transforms of these matrices of Green's functions must satisfy the linear matrix equations

$$\tilde{D}_H(\boldsymbol{k}, \omega) = \tilde{D}((\boldsymbol{k}, \omega) + g^2\, \tilde{D}(\boldsymbol{k}, \omega)\, \hat{\tau}\, \tilde{G}(\boldsymbol{k}, \omega)\, \hat{\tau}\, \tilde{D}_H(\boldsymbol{k}, \omega) \tag{30}$$

$$\tilde{G}_H(\boldsymbol{k}, \omega) = \tilde{G}(\boldsymbol{k}, \omega) + g^2\, \tilde{G}(\boldsymbol{k}, \omega)\, \hat{\tau}\, \tilde{D}(\boldsymbol{k}, \omega)\, \hat{\tau}\, \tilde{G}_H(\boldsymbol{k}, \omega)\,.$$

From (30) we obtain

$$D^H_{12}(\mathbf{k}, \omega) = D^H_{21}(\mathbf{k}, \omega) = \frac{g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)}{4\epsilon_0\, \omega^2(\mathbf{k}) \left[1 - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)\right]} \tag{31}$$

$$D^H_{11}(\mathbf{k}, \omega) = \frac{4\epsilon_0\, \omega^2(\mathbf{k})\, D_{11}(\mathbf{k}, \omega) - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)}{4\epsilon_0\, \omega^2(\mathbf{k}) \left[1 - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)\right]}$$

$$D^H_{22}(\mathbf{k}, \omega) = \frac{4\epsilon_0\, \omega^2(\mathbf{k})\, D_{22}(\mathbf{k}, \omega) - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)}{4\epsilon_0\, \omega^2(\mathbf{k}) \left[1 - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)\right]}$$

$$D^H(\mathbf{k}, \omega) = \sum_{i,j=1,2} D^H_{ij}(\mathbf{k}, \omega) = \frac{D(\mathbf{k}, \omega)}{1 - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)}$$

and similarly

$$G^H_{1,2}(\mathbf{k}, \omega) = G^H_{21}(\mathbf{k}, \omega) = \frac{g^2\, D(\mathbf{k}, \omega)\, G(\mathbf{k}, \omega)}{4E^2(\mathbf{k}) \left[1 - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)\right]} \tag{32}$$

$$G_{11}(\mathbf{k}, \omega) = \frac{4E^2(\mathbf{k})\, G_{11}(\mathbf{k}, \omega) - g^2\, D(\mathbf{k}, \omega)\, G(\mathbf{k}, \omega)}{4E^2(\mathbf{k}) \left[1 - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)\right]}$$

$$G^H_{22}(\mathbf{k}, \omega) = \frac{4E^2(\mathbf{k})\, G_{22}(\mathbf{k}, \omega) - g^2\, D(\mathbf{k}, \omega)\, G(\mathbf{k}, \omega)}{4E^2(\mathbf{k}) \left[1 - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)\right]}$$

$$G^H(\mathbf{k}, \omega) \equiv \sum_{i,j=1,2} G^H_{ij}(\mathbf{k}, \omega) = \frac{G(\mathbf{k}, \omega)}{1 - g^2\, G(\mathbf{k}, \omega)\, D(\mathbf{k}, \omega)}\,.$$

Each polariton corresponds to a pole of the Green's function $G^H_{ij}(\mathbf{k}, \omega)$ or $D^H_{ij}(\mathbf{k}, \omega)$. From (31) and (32) we can see that the poles of the Green's functions satisfy the equation

$$1 - g^2\, D(\mathbf{k}, \omega)\, G(\mathbf{k}, \omega) = 0\,. \tag{33}$$

Its solutions are

$$2\, \varepsilon_\nu(\mathbf{k}) = E^2(\mathbf{k}) + \omega^2(\mathbf{k}) \pm \{[E^2(\mathbf{k}) - \omega^2(\mathbf{k})]^2 + g^2/\epsilon_0\}^{1/2}\,. \tag{34}$$

By a comparison of the expressions (12) and (34) we conclude that the dispersions of polaritons derived by the Green's function approach are coincident with the result obtained in the Bogoliubov diagonalization approach.

It is easy to see that

$$D_{11}(\mathbf{k}, -\omega) = D_{22}(\mathbf{k}, \omega)\,, \quad G_{11}(\mathbf{k}, -\omega) = G_{22}(\mathbf{k}, \omega)\,, \tag{35}$$
$$D^H_{11}(\mathbf{k}, -\omega) = D^H_{22}(\mathbf{k}, \omega)\,, \quad G^H_{11}(\mathbf{k}, -\omega) = G^H_{22}(\mathbf{k}, \omega)\,.$$

From (31), (32) and (34) we have

$$D^H_{11}(\mathbf{k}, \omega) = \sum_\nu \frac{|\mathscr{B}^\gamma_\nu(\mathbf{k})|^2}{8\epsilon_0\, \omega^2(\mathbf{k})\, \varepsilon_\nu(\mathbf{k})} \left[\frac{\varepsilon_\nu(\mathbf{k}) + \omega(\mathbf{k})}{\varepsilon_\nu(\mathbf{k}) - \omega} + \frac{\varepsilon_\nu(\mathbf{k}) - \omega(\mathbf{k})}{\varepsilon_\nu(\mathbf{k}) + \omega}\right] \tag{36}$$

$$G^H_{11}(\boldsymbol{k}, \omega) = \sum_\nu \frac{|\mathscr{B}^{E_x}_\nu(\boldsymbol{k})|^2}{8E^2(\boldsymbol{k})\,\varepsilon_\nu(\boldsymbol{k})}\left[\frac{\varepsilon_\nu(\boldsymbol{k}) + E(\boldsymbol{k})}{\varepsilon_\nu(\boldsymbol{k}) - \omega} + \frac{\varepsilon_\nu(\boldsymbol{k}) - E(\boldsymbol{k})}{\varepsilon_\nu(\boldsymbol{k}) + \omega}\right],$$

where $\mathscr{B}^\gamma_\nu(\boldsymbol{k})$ is determined as in (15) and

$$|\mathscr{B}^{E_x}_\nu(\boldsymbol{k})|^2 = \frac{[\varepsilon^2_\nu(\boldsymbol{k}) - \omega^2(\boldsymbol{k})]^2}{[\varepsilon^2_\nu(\boldsymbol{k}) - \omega^2(\boldsymbol{k})]^2 + g^2/\epsilon_0}. \tag{37}$$

From (37) and (15) we have the identity

$$|\mathscr{B}^{E_x}_\nu(\boldsymbol{k})|^2 = \frac{g^2}{\epsilon_0}\,|G(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))|^2\,|\mathscr{B}^\gamma_\nu(\boldsymbol{k})|^2. \tag{38}$$

In order to calculate the matrix elements of the interaction processes with the participation of polaritons we define the matrix-wavefunctions of bare (non-interacting) photons and excitons as the matrix elements

$$\langle 0|\,\tilde{A}(\boldsymbol{r}, t)\,|\gamma\rangle = \tilde{u}(\boldsymbol{r}, t) = \frac{1}{(2\pi)^4}\int \exp\left[i(\boldsymbol{q}\boldsymbol{r} - \varepsilon t)\right]\tilde{u}(\boldsymbol{q}, \varepsilon)\,d^3q\,d\varepsilon\,, \tag{39}$$

$$\langle 0|\,\tilde{B}(\boldsymbol{r}, t)\,|E_x\rangle = \tilde{v}(\boldsymbol{r}, t) = \frac{1}{(2\pi)^4}\int \exp\left[i(\boldsymbol{q}\boldsymbol{r} - \varepsilon t)\right]\tilde{v}(\boldsymbol{q}, \varepsilon)\,d^3q\,d\varepsilon\,.$$

The matrix-wavefunctions of the dressed (interacting) photons and excitons are determined in a similar manner

$$\langle\phi_0|\,\tilde{A}_H(\boldsymbol{r}, t)\,|\gamma^H\rangle = \tilde{u}^H(\boldsymbol{r}, t) = \frac{1}{(2\pi)^4}\int \exp\left[i(\boldsymbol{q}\boldsymbol{r} - \varepsilon t)\right]\tilde{u}^H(\boldsymbol{q}, \varepsilon)\,d^3q\,d\varepsilon \tag{40}$$

$$\langle\phi_0|\,\tilde{B}_H(\boldsymbol{r}, t)\,|E^H_x\rangle = \tilde{v}^H(\boldsymbol{r}, t) = \frac{1}{(2\pi)^4}\int \exp\left[i(\boldsymbol{q}\boldsymbol{r} - \varepsilon t)\right]\tilde{v}^H(\boldsymbol{q}, \varepsilon)\,d^3q\,d\varepsilon\,.$$

The matrix-wavefunctions $\tilde{u}^H(\boldsymbol{q}, \varepsilon)$ and $\tilde{v}^H(\boldsymbol{q}, \varepsilon)$ satisfy the equations

$$\tilde{u}^H(\boldsymbol{q}, \varepsilon) = \tilde{u}(\boldsymbol{q}, \varepsilon) + g^2\,\tilde{D}(\boldsymbol{q}, \varepsilon)\,\hat{\tau}\,\tilde{G}(\boldsymbol{q}, \varepsilon)\,\hat{\tau}\,\tilde{u}^H(\boldsymbol{q}, \varepsilon) \tag{41}$$

$$\tilde{v}^H(\boldsymbol{q}, \varepsilon) = \tilde{v}(\boldsymbol{q}, \varepsilon) + g^2\,\tilde{G}(\boldsymbol{q}, \varepsilon)\,\hat{\tau}\,\tilde{D}(\boldsymbol{q}, \varepsilon)\,\hat{\tau}\,\tilde{v}^H(\boldsymbol{q}, \varepsilon)\,.$$

Expressions (41) show again that the energies and momenta of the dressed photon and exciton must obey the dispersion equations of the polaritons, i.e. they are polaritons.

From the expansions (5) it follows that for the one-photon or one-exciton state with the definite energy $\omega(\boldsymbol{k})$ or $E(\boldsymbol{p})$ and momentum $\boldsymbol{k}$ or $\boldsymbol{p}$ we have

$$\tilde{u}(\boldsymbol{q}, \varepsilon) = \frac{\delta[\varepsilon - \omega(\boldsymbol{k})]}{[2\epsilon_0\,\omega(\boldsymbol{k})]^{1/2}}\,\delta^3(\boldsymbol{k} - \boldsymbol{q})\begin{pmatrix}1\\0\end{pmatrix}, \quad \tilde{v}(\boldsymbol{q}, \varepsilon) = \frac{\delta[\varepsilon - E(\boldsymbol{k})]}{[2E(\boldsymbol{k})]^{1/2}}\,\delta^3(\boldsymbol{k} - \boldsymbol{q})\begin{pmatrix}1\\0\end{pmatrix}. \tag{42}$$

Similarly for the polariton branch $\omega = \varepsilon_\nu(\boldsymbol{k})$, $\nu = 1, 2$ we have

$$\tilde{u}_\nu^H(\boldsymbol{q}, \varepsilon) = \frac{\delta[\varepsilon - \varepsilon_\nu(\boldsymbol{k})]}{\sqrt{2\epsilon_0\, \omega(\boldsymbol{k})}}\, \delta^3(\boldsymbol{k} - \boldsymbol{q}) \begin{pmatrix} Z_\nu^\gamma(\boldsymbol{k}) \\ W_\nu^\gamma(-\boldsymbol{k}) \end{pmatrix} \tag{43}$$

$$\tilde{v}_\nu^H(\boldsymbol{q}, \varepsilon) = \frac{\delta[\varepsilon - \varepsilon_\nu(\boldsymbol{k})]}{\sqrt{2E(\boldsymbol{k})}}\, \delta^3(\boldsymbol{k} - \boldsymbol{q}) \begin{pmatrix} Z_\nu^{E_x}(\boldsymbol{k}) \\ W_\nu^{E_x}(-\boldsymbol{k}) \end{pmatrix}$$

with the renormalization constants $Z_\nu^\gamma(\boldsymbol{k})$, $W_\nu^\gamma(-\boldsymbol{k})$, $Z_\nu^{E_x}(\boldsymbol{k})$, $W_\nu^{E_x}(-\boldsymbol{k})$. From the spectral representation of the Green's function we have

$$D_{11}^H(\boldsymbol{k}, \omega) = \frac{1}{\epsilon_0} \sum_{\nu=1,2} \left\{ \frac{|Z_\nu^\gamma(\boldsymbol{k})|^2}{2\omega(\boldsymbol{k})\,[\varepsilon_\nu(\boldsymbol{k}) - \omega]} + \frac{|W_\nu^\gamma(\boldsymbol{k})|^2}{2\omega(\boldsymbol{k})\,[\varepsilon_\nu(\boldsymbol{k}) + \omega]} \right\} \tag{44}$$

$$G_{11}^H(\boldsymbol{k}, \omega) = \sum_{\nu=1,2} \left\{ \frac{|Z_\nu^{E_x}(\boldsymbol{k})|^2}{2E(\boldsymbol{k})\,[\varepsilon_\nu(\boldsymbol{k}) - \omega]} + \frac{|W_\nu^{E_x}(\boldsymbol{k})|^2}{2E(\boldsymbol{k})\,[\varepsilon_\nu(\boldsymbol{k}) + \omega]} \right\}.$$

By a comparison of equations (44) and (36) we obtain

$$|Z_\nu^\gamma(\boldsymbol{k})|^2 = \frac{[\varepsilon_\nu(\boldsymbol{k}) + \omega(\boldsymbol{k})]^2}{4\omega(\boldsymbol{k})\, \varepsilon_\nu(\boldsymbol{k})}\, |\mathscr{B}_\nu^\gamma(\boldsymbol{k})|^2 \tag{45}$$

$$|W_\nu^\gamma(\boldsymbol{k})|^2 = \frac{[\varepsilon_\nu(\boldsymbol{k}) - \omega(\boldsymbol{k})]^2}{4\omega(\boldsymbol{k})\, \varepsilon_\nu(\boldsymbol{k})}\, |\mathscr{B}_\nu^\gamma(\boldsymbol{k})|^2$$

$$|Z_\nu^{E_x}(\boldsymbol{k})|^2 = \frac{[\varepsilon_\nu(\boldsymbol{k}) + E(\boldsymbol{k})]^2}{4E(\boldsymbol{k})\, \varepsilon_\nu(\boldsymbol{k})}\, |\mathscr{B}_\nu^{E_x}(\boldsymbol{k})|^2$$

$$|W_\nu^{E_x}(\boldsymbol{k})|^2 = \frac{[\varepsilon_\nu(\boldsymbol{k}) - E(\boldsymbol{k})]^2}{4E(\boldsymbol{k})\, \varepsilon_\nu(\boldsymbol{k})}\, |\mathscr{B}_\nu^{E_x}(\boldsymbol{k})|^2 .$$

From (14), (15), (37) and (45) it follows that with a suitable choice of the phase factors of the renormalization constants $Z_\nu^\gamma(\boldsymbol{k})$, $W_\nu^\gamma(\boldsymbol{k})$, $Z_\nu^{E_x}(\boldsymbol{k})$, $W_\nu^{E_x}(\boldsymbol{k})$ they are exactly coincident with the Bogoliubov transformation coefficients $u_\nu^\gamma(\boldsymbol{k})$, $v_\nu^\gamma(\boldsymbol{k})$, $u_\nu^{E_x}(\boldsymbol{k})$, $v_\nu^{E_x}(\boldsymbol{k})$. If we use the phase factor as in eqs. (14) we have the following identities

$$\frac{1}{[\epsilon_0\, \omega(\boldsymbol{k})]^{1/2}} \begin{pmatrix} Z_\nu^\gamma(\boldsymbol{k}) \\ W_\nu^\gamma(-\boldsymbol{k}) \end{pmatrix} = \frac{g}{[E(\boldsymbol{k})]^{1/2}}\, \tilde{D}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\, \hat{\tau} \begin{pmatrix} Z_\nu^{E_x}(\boldsymbol{k}) \\ W_\nu^{E_x}(-\boldsymbol{k}) \end{pmatrix} \tag{46a}$$

$$\frac{1}{[E(\boldsymbol{k})]^{1/2}} \begin{pmatrix} Z_\nu^{E_x}(\boldsymbol{k}) \\ W_\nu^{E_x}(-\boldsymbol{k}) \end{pmatrix} = \frac{g}{[\varepsilon_0\, \omega(\boldsymbol{k})]^{1/2}}\, \tilde{G}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\, \hat{\tau} \begin{pmatrix} Z_\nu^\gamma(\boldsymbol{k}) \\ W_\nu^\gamma(-\boldsymbol{k}) \end{pmatrix}. \tag{46b}$$

They will be used in section IV.

IV. APPLICATION AND DISCUSSION

Let us aplly the above results to the study of interaction processes with the participation of the polaritons. To explain the method we shall consider two simple examples. As the first example we consider the process

$$\text{(I)} \qquad P_\nu(\boldsymbol{k}) + a \to b + c + \dots$$

where, a, b, c denote some elementary excitations different from polaritons, $P_\nu(\boldsymbol{k})$ is the polariton in the branch ν with the momentum $\boldsymbol{k}$. This process has been considered in ref. [20], and we will not go into details here. We just want to note that besides the processes

$$\text{(Ia)} \qquad \gamma(\boldsymbol{k}) \;\; + a \to b + c + \dots$$

$$\text{(Ib)} \qquad E_x(\boldsymbol{k}) + a \to b + c + \dots$$

which give the main contribution to the matrix element of the process (I) there are two other processes which can also give some contributions

$$\text{(Ic)} \qquad a \to \gamma(-\boldsymbol{k}) \;\; + b + c + \dots$$

$$\text{(Id)} \qquad a \to E_x(-\boldsymbol{k}) + b + c + \dots.$$

This contribution is negligible in comparison with those of (Ia) and (Ib) in the resonant domain and is often neglected in the resonant approximation. Let the matrix elements of these processes in the absence of the photon-exciton interaction be T_{Ia}, T_{Ib}, T_{Ic}, T_{Id}, respectively.

From the expression (46) we have

$$(47) \qquad \frac{Z_\nu^\gamma(\boldsymbol{k})}{[\epsilon_0\,\omega(\boldsymbol{k})]^{1/2}} = \frac{g}{[E(\boldsymbol{k})]^{1/2}}\, D_{11}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\,[Z_\nu^{Ex}(\boldsymbol{k}) + W_\nu^{Ex}(-\boldsymbol{k})]$$

$$\frac{W_\nu^\gamma(-\boldsymbol{k})}{[\epsilon_0\,\omega(\boldsymbol{k})]^{1/2}} = \frac{g}{[E(\boldsymbol{k})]^{1/2}}\, D_{22}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\,[Z_\nu^{Ex}(\boldsymbol{k}) + W_\nu^{Ex}(-\boldsymbol{k})]$$

$$\frac{Z_\nu^{Ex}(\boldsymbol{k})}{[E(\boldsymbol{k})]^{1/2}} = \frac{g}{[\varepsilon_0\,\omega(\boldsymbol{k})]^{1/2}}\, G_{11}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\,[Z_\nu^\gamma(\boldsymbol{k}) + W_\nu^\gamma(-\boldsymbol{k})]$$

$$\frac{W_\nu^{Ex}(-\boldsymbol{k})}{[E(\boldsymbol{k})]^{1/2}} = \frac{g}{[\epsilon_0\,\omega(\boldsymbol{k})]^{1/2}}\, G_{22}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\,[Z_\nu^\gamma(\boldsymbol{k}) + W_\nu^\gamma(-\boldsymbol{k})]\,.$$

In a manner analogous with that in ref. [20] we obtain the following exact matrix element of process (I)

$$(48) \qquad T_\nu^{\mathrm{pol}}(\boldsymbol{k}) = Z_\nu^\gamma(\boldsymbol{k})\,\Big[T_{\mathrm{Ib}} + g\left(\frac{E(\boldsymbol{k})}{\epsilon_0\,\omega(\boldsymbol{k})}\right)^{1/2} G_{11}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\,T_{\mathrm{Ib}} +$$

$$+ g\left(\frac{E(\boldsymbol{k})}{\epsilon_0\,\omega(\boldsymbol{k})}\right)^{1/2} G_{22}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\,T_{\mathrm{Id}}\Bigg] +$$

$$+ W_\nu^\gamma(-\boldsymbol{k})\left[T_{\mathrm{Ic}} + g\left(\frac{E(\boldsymbol{k})}{\epsilon_0\,\omega(\boldsymbol{k})}\right)^{1/2} G_{11}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\,T_{\mathrm{Ib}} + g\left(\frac{E(\boldsymbol{k})}{\epsilon_0\,\omega(\boldsymbol{k})}\right)^{1/2} G_{22}(\boldsymbol{k}, \varepsilon_\nu(\boldsymbol{k}))\,T_{\mathrm{Id}}\right].$$

According to (47) $T_\nu^{\mathrm{pol}}(\boldsymbol{k})$ can be rewritten in an equivalent form

$$T_\nu^{\mathrm{pol}}(\boldsymbol{k}) = Z_\nu^\gamma(\boldsymbol{k})\,T_{\mathrm{Ia}} + Z_\nu^{E_x}(\boldsymbol{k})\,T_{\mathrm{Ib}} + W_\nu^\gamma(-\boldsymbol{k})\,T_{\mathrm{Ic}} + W_\nu^{E_x}(-\boldsymbol{k})\,T_{\mathrm{Id}}\,. \tag{49}$$

Before considering the second example, we formulate the recipe for the construction of the matrix element of each interaction process with the participation of polaritons in the initial and final states. From the above lines of reasoning we see that for this purpose it is enough to write down all the lowest order matrix elements of this interaction process with the direct absorption and/or emission of photon or exciton without exciton-photon transition. The sum of these lowest order matrix elements which are multiplied by the corresponding renormalization constants ($Z_\nu^\gamma(\boldsymbol{k})$ for an external photon and $Z_\nu^{E_x}(\boldsymbol{k})$ for an external exciton) will be the main contribution to the total matrix element of the polarization interaction process. To obtain the nonresonant terms of the matrix element of this process we must write down all those lowest order matrix elements, but instead of the absorption (or emission) of one photon (or exciton) we have the emission (or absorption) of this photon (or this exciton) with the momentum in the inverse direction. Each matrix element is then multiplied by the corresponding renormalization constants $W_\nu^\gamma(-\boldsymbol{k})$ for a substituted external photon and $W_\nu^{E_x}(-\boldsymbol{k})$ for a substituted external exciton. The sum of these matrix elements will be the nonresonant contribution to the total matrix element of this polariton interaction process.

Now we consider the following process

$$P_{\nu_1}(\boldsymbol{k}_1) + a \to P_{\nu_2}(\boldsymbol{k}_2) + b + c + \ldots. \tag{II}$$

According to the above recipe we have the following lowest order processes which give the main contribution to the total matrix element

$$\gamma(\boldsymbol{k}_1) + a \to \gamma(\boldsymbol{k}_2) + b + c + \ldots \tag{IIa}$$

$$E_x(\boldsymbol{k}_1) + a \to \gamma(\boldsymbol{k}_2) + b + c + \ldots \tag{IIb}$$

$$\gamma(\boldsymbol{k}_1) + a \to E_x(\boldsymbol{k}_2) + b + c + \ldots \tag{IIc}$$

$$E_x(\boldsymbol{k}_1) + a \to E_x(\boldsymbol{k}_2) + b + c + \ldots \tag{IId}$$

and the other lowest order processes which give the nonresonant contribution

$$\gamma(-\boldsymbol{k}_2) + a \to \gamma(-\boldsymbol{k}_1) + b + c + \ldots \tag{IIe}$$

$$\gamma(-\boldsymbol{k}_2) + a \to E_x(-\boldsymbol{k}_1) + b + c + \ldots \tag{IIf}$$

(IIg) $E_x(-\boldsymbol{k}_2) + a \to \gamma(-\boldsymbol{k}_1) + b + c + \ldots$

(IIh) $E_x(-\boldsymbol{k}_2) + a \to E_x(-\boldsymbol{k}_1) + b + c + \ldots$

(IIk) $\gamma(\boldsymbol{k}_1) + \gamma(-\boldsymbol{k}_2) + a \to b + c + \ldots$

(III) $a \to \gamma(-\boldsymbol{k}_1) + \gamma(\boldsymbol{k}_2) + b + c + \ldots$

(IIm) $\gamma(-\boldsymbol{k}_2) + E_x(\boldsymbol{k}_1) + a \to b + c + \ldots$

(IIn) $a \to \gamma(\boldsymbol{k}_2) + E_x(-\boldsymbol{k}_1) + b + c + \ldots$

(IIp) $E_x(-\boldsymbol{k}_2) + \gamma(\boldsymbol{k}_1) + a \to b + c + \ldots$

(IIq) $a \to E_x(\boldsymbol{k}_2) + \gamma(-\boldsymbol{k}_1) + b + c + \ldots$

(IIr) $E_x(\boldsymbol{k}_1) + E_x(-\boldsymbol{k}_2) + a \to b + c + \ldots$

(IIs) $a \to E_x(\boldsymbol{k}_2) + E_x(-\boldsymbol{k}_1) + b + c + \ldots$

Then the total matrix element can be written

$$
\begin{aligned}
(50)\qquad T^{\mathrm{pol}\ \ \mathrm{pol}}_{\nu_1(\boldsymbol{k}_1)\,\nu_2(\boldsymbol{k}_2)} = {} & Z^{\gamma}_{\nu_1}(\boldsymbol{k}_1)\, Z^{\gamma}_{\nu_2}(\boldsymbol{k}_2)\, T_{\mathrm{IIa}} + Z^{E_x}_{\nu_1}(\boldsymbol{k}_1)\, Z^{\gamma}_{\nu_2}(\boldsymbol{k}_2)\, T_{\mathrm{IIb}} + \\
& + Z^{\gamma}_{\nu_1}(\boldsymbol{k}_1)\, Z^{E_x}_{\nu_2}(\boldsymbol{k}_2)\, T_{\mathrm{IIc}} + Z^{E_x}_{\nu_1}(\boldsymbol{k}_1)\, Z^{E_x}_{\nu_2}(\boldsymbol{k}_2)\, T_{\mathrm{IId}} + W^{\gamma}_{\nu_2}(-\boldsymbol{k}_2)\, W^{\gamma}_{\nu_1}(-\boldsymbol{k}_1)\, T_{\mathrm{IIe}} + \\
& + W^{\gamma}_{\nu_2}(-\boldsymbol{k}_2)\, W^{E_x}_{\nu_1}(-\boldsymbol{k}_1)\, T_{\mathrm{IIg}} + W^{E_x}_{\nu_2}(-\boldsymbol{k}_2)\, W^{\gamma}_{\nu_1}(-\boldsymbol{k}_1)\, T_{\mathrm{IIh}} + W^{E_x}_{\nu_2}(-\boldsymbol{k}_2)\, W^{E_x}_{\nu_1}(-\boldsymbol{k}_1)\, T_{\mathrm{IIi}} + \\
& + Z^{\gamma}_{\nu_1}(\boldsymbol{k}_1)\, W^{\gamma}_{\nu_2}(-\boldsymbol{k}_2)\, T_{\mathrm{IIk}} + Z^{\gamma}_{\nu_2}(\boldsymbol{k}_2)\, W^{\gamma}_{\nu_1}(-\boldsymbol{k}_1)\, T_{\mathrm{IIl}} + Z^{E_x}_{\nu_1}(\boldsymbol{k}_1)\, W^{\gamma}_{\nu_2}(-\boldsymbol{k}_2)\, T_{\mathrm{IIm}} + \\
& + Z^{\gamma}_{\nu_2}(\boldsymbol{k}_2)\, W^{E_x}_{\nu_1}(-\boldsymbol{k}_1)\, T_{\mathrm{IIn}} + Z^{\gamma}_{\nu_1}(\boldsymbol{k}_1)\, W^{E_x}_{\nu_2}(-\boldsymbol{k}_2)\, T_{\mathrm{IIp}} + Z^{E_x}_{\nu_2}(\boldsymbol{k}_2)\, W^{\gamma}_{\nu_1}(-\boldsymbol{k}_1)\, T_{\mathrm{IIq}} + \\
& + Z^{E_x}_{\nu_1}(\boldsymbol{k}_1)\, W^{E_x}_{\nu_2}(-\boldsymbol{k}_2)\, T_{\mathrm{IIr}} + Z^{E_x}_{\nu_2}(\boldsymbol{k}_2)\, W^{E_x}_{\nu_1}(-\boldsymbol{k}_1)\, T_{\mathrm{IIs}}\,.
\end{aligned}
$$

The matrix elements of processes (I) and (II) can also be calculated by the Bogoliubov diagonalization method. From the expression (16) we can transform the interaction Hamiltonian of photons and excitons with other elementary excitations into the interaction Hamiltonian of polaritons with them. Then by the use of this interaction Hamiltonian we easily obtain the same expressions (49) and (50) for the $T^{\mathrm{pol}}_{\nu(\boldsymbol{k})}$ and $T^{\mathrm{pol}\ \ \mathrm{pol}}_{\nu_1(\boldsymbol{k}_1)\,\nu_2(\boldsymbol{k}_2)}$ but instead of the renormalization constants $Z^{\gamma}_{\nu}(\boldsymbol{k})$, $Z^{E_x}_{\nu}(\boldsymbol{k})$, $W^{\gamma}_{\nu}(-\boldsymbol{k})$, $W^{E_x}_{\nu}(-\boldsymbol{k})$ we have the diagonalization constants $u^{\gamma}_{\nu}(\boldsymbol{k})$, $u^{E_x}_{\nu}(\boldsymbol{k})$, $v^{\gamma}_{\nu}(\boldsymbol{k})$, $v^{E_x}_{\nu}(\boldsymbol{k})$. It has been shown above that with a suitable choice of the phase factors these constants are exactly coincident. The dispersions of polaritons derived by the two approaches are also coincident. We conclude that these two approaches are equivalent for the simple model of a direct band gap semiconductor. Our result can be generalized to more complicated cases.

In conclusion we emphasize once more that the Green's function approach which was derived in ref. [20] can include the effect of the damping of polaritons. The results of the consideration of the damping effect of polaritons will be published elsewhere.

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