DIAGNOSTICS OF STELLAR EVOLUTION: THE OXYGEN ISOTOPES

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Abstract:

The oxygen isotope ratios provide a probe of the physics of stellar interiors. Among the phenomena that can be studied through the oxygen isotope ratios are the maximum depth of penetration of the convective envelopes on the giant branch, the efficiency of nonconvective mixing, as well as restrictions on the mass loss that occurs. The oxygen isotope ratios also provide an indicator of the nuclear processing that occurs during third dredge up and carbon star formation. In addition to the oxygen isotopes, we indicate the expected behavior of nitrogen and the light elements.

1. Introduction

Stellar evolution is one of the most quantitatively successful theories in astronomy. In its simplest form, it connects many different types of stars as an evolutionary sequence and links them to specific terminal states. Comparisons to observed properties like the stellar distribution in HR diagrams and luminosity functions of clusters provide physical support for this theory [Tinsley and Gunn 1986]. Its success gives us some confidence in using the predictions of stellar evolution in other studies. Examples of areas where the results of stellar-evolution calculations are used include the age of the universe [Vandenberg 1983; Janes and Demarque 1983; Fowler 1987], the primordial composition [Rood et al. 1976; Dearborn et al. 1986a, b], and the use of stellar models to restrict the properties of postulated particles like axions, or WIMPs [Fukugita et al. 1982a, b; Krauss et al. 1983; Faulkner and Gilliland 1985; Dearborn et al. 1986a, b; Raffelt and Dearborn 1987, 1988].

At best, numerical models are only as good as the physics included in them, and the standard theory of stellar evolution is known to have some weak areas. Questions about convective efficiency, convective overshoot, and nonconvective mixing can be significant in correctly understanding the evolution of stars. Rotation, or magnetic fields should act to drive such a mixing in stars but the magnitude of the effect is uncertain. The uncertainty of how to include these phenomena coupled with the success of the theory without their inclusion generally leads to their neglect.

In this paper, we show that the CNO composition structure of stars is sensitive to such mixing, offering us the opportunity to probe for its effect. The change in composition on the giant branch, caused when convection dredges up material that has been processed through the CNO cycle, provides us with an indicator of the mixing history of the observed stars. By considering a sample of red giants, we can provide additional tests and constraints on the theory of stellar evolution.

Because of the extreme temperature sensitivity of nuclear reactions, relatively sharp boundaries are formed where isotopes are either enhanced or destroyed. The most fragile species, ^{15}N and ^{18}O, are destroyed at a relatively shallow depth. Slightly deeper, ^{12}C is converted to ^{13}C, and deeper still, the ^{13}C transforms to ^{14}N. This results in a ^{13}C rich region which contains most of the ^{13}C in the star. At a higher temperature, ^{16}O is converted to ^{17}O, and these two isotopes may come to equilibrium in the core. When a star evolves to the giant branch, a deep convection zone develops that mixes the processed material to the surface, changing the observable composition. The magnitude of the change is indicative of where each of these boundaries were, and the depth reached by the convection zone. The depth of the convection zone and the existence of nonconvective mixing will each leave signs on the surface composition.

The carbon isotopes have been well observed [Wallerstein 1988; Pilachowski 1987], and show ^{12}C/^{13}C ratios at or below the values expected for stars after the first dredge up on the giant branch. Hypotheses to explain the lower values include nonconvective mixing while on the main sequence extending the ^{13}C rich region, or mass loss prior to the deep penetration of convection removing ^{12}C rich material. The low nitrogen enhancement observed favored a slow (on the timescale of the main sequence lifetime) mixing to a depth limited by a molecular weight gradient [Dearborn and Eggleton 1977], but uncertainty in observed C/N ratios limited the strength of this result.

Constraints in nonconvective mixing can be strengthened from observations of the oxygen isotope ratios which are very sensitive to internal mixing, and the depth of convection. They can be used with the C N isotopes to determine the depth actually reached by convection, and to constrain the amount of nonconvective mixing that has occurred. As a bonus, observations of the oxygen isotopes can reduce some of the uncertainty in the nuclear reaction rates used, and provide a thermometer for the base temperatures of the convection zones in certain types of carbon stars.

In the same manner that the carbon and oxygen isotope ratios are sensitive to nonconvective mixing in the deep interior of a star, the light elements will be affected by such mixing in the surface regions. These fragile elements are quickly destroyed everywhere but in the surface region. The dilution that occurs on the giant branch leads to a reduced surface abundance. Those stars which have a blue helium burning phase may suffer a second depletion as they approach the asymptotic giant branch. We have therefore included the reactions on lithium and beryllium.

In section 2 we review observations of the oxygen isotope ratios in various types of red giants. Section 3 begins with a description of stellar evolution code and calculations. We then discuss the uncertainties in the nuclear reaction rates, initial composition and mixing length. Section 4 compares our calculations to normal giants, and examines how the uncertainties discussed in section 3 translate into variations in the oxygen isotope ratios. Section 5 discusses constraints that can be placed on convective mixing; section 6 examines limitations that can be placed on convection during third dredge up, based on a comparison to carbon star observations. Finally, we present a summary and conclusions.

2. Observations

Confidence in the accuracy of stellar-evolution calculations must be based on comparison to observations. The carbon isotope ratios that have been available provide confidence that the calculated interiors are generally correct, but also indicated that some unaccounted for behavior was occurring in some stars. Without additional information on other abundance or isotope ratios, it is not possible to resolve the cause of the low $^{12}C/^{13}C$ ratios. The oxygen isotope ratios provide that information. These ratios have been determined in a number of red normal giants by observing CO features at 2 and 5 μm and a selection of such ratios is given in table 1.

The limits for the oxygen isotope ratios were set by Harris and his collaborators by determining the ratio for which a synthetic spectrum could no longer represent the observed features. As such they are not simple one or three sigma limits. The agreement that we find between observations and calculations, as well as the consistency in behavior found in the observations themselves, suggests that

Table 1

Star	Mass	$^{12}C/^{13}C$	$^{16}O/^{17}O$ min	$^{16}O/^{17}O$ max	$^{16}O/^{18}O$ min	$^{16}O/^{18}O$ max	Ref[a]
Alpha Ari	1.0	22	420	640	360	560	[3]
Alpha Boo	1.1	7	800	1500	425	700	[2]
Beta Gem	1.3	18	190	300	420	620	[3]
Alpha Ser	1.4	16	150	450	200	600	[3]
Beta Umi	1.5	12	440	600	370	530	[3]
Alpha Tau	1.5	9	450	900	350	675	[2]
Beta Peg	1.7	7	825	1550	350	575	[2]
Gamma Dra	2.0	13	225	400	350	700	[2]
Mu Gem	2.0	13	250	475	350	675	[2]
Beta And	2.5	11	125	205	350	575	[2]
Alpha UMa	3.0	19	280	400	475	750	[3]
Alpha Her	5.0	17	100	260	375	775	[2, 4]
Alpha Ori	15.0	7	400	775	525	1000	[1]
Alpha Sco	15.0		550	1400	300	800	[1]

[a] [1] [Harris and Lambert 1984a]; [2] [Harris and Lambert 1984b]; [3] [Harris et al. 1988]; [4] [Tsuji 1985].

these limits are stronger than one sigma. However, as the authors caution, systematic errors could be present.

The masses listed are adopted from Harris and Lambert, and in most cases are based on the assumption that the star has a helium burning core. As such they are generally overestimates of the mass. This mass uncertainty limits our comparison, and we must be satisfied with simple consistency for the $^{16}O/^{17}O$ ratio in low mass stars. Determining the oxygen isotope ratios in a group of stars with well known masses, like the Hyades giants, would provide an improved test of the maximum depth to which convection penetrates (including overshoot).

Table 2 lists the results of Harris et al. [1985] and Harris et al. [1987] for the $^{17}O/^{18}O$ ratios in a group of chemically peculiar stars. These stars all show evidence of the products of helium burning in the surface. This is presumably the result of a deeply penetrating convection zone following a thermal pulse. Subsequent (post-pulse) nuclear processing is dependent on conditions at the base of this convective region. We will use the $^{16}O/^{17}O$ ratio as a probe measuring the temperature there during a thermal pulse. This issue is important to understand the ultimate nucleosynthetic yield of low and intermediate mass stars.

Table 2

Star	$^{16}O/^{17}O$	$^{17}O/^{18}O$ min	$^{17}O/^{18}O$ max	Neutron exposure	Type
30 Her	675	1.0	1.7	0.13	MS
HR6702	850	1.3	1.7	0.09	MS
o Ori	925	1.0	1.9	0.16	MS
RS Cnc	1275	0.7	0.9	0.28	MS
HR8062	1850	1.5	2.0	0.30	MS
HR1105	2250	1.7	2.5	0.32	S
HR8714	2400	0.9	1.8	0.30	S
HR 363	3000	0.7	3.5	0.33	S
VA ql	1225	0.6	1.8		N
UU Aur	1125	0.7	1.6	0.31	N
ST Cam	1850	0.5	1.1	0.19	N
RT Cap	825	0.8	2.0		N
W CMa	750	0.8	1.8		N
X Cnc	550	0.9	2.1	0.47	N
RV Cyg	1750	0.5	1.3		N
V460 Cyg	1525	1.3	1.8		N
UX Dra	1650	1.1	1.8		N
TU Gem	725	0.8	1.5	0.22	N
U Hya	1425	0.7	1.4	0.35	N
Y Hya	675	1.0	2.7		N
BL Ori	1475	0.9	1.5	0.53	N
W Ori	975	0.7	2.1		N
TX Psc	1950	0.5	1.1	0.69	N
Z Psc	1750	1.0	1.9	0.41	N
Y Tau	2675	0.6	1.1	0.52	N
VY UMa	725	0.9	2.7	0.24	N
U Cam	1275	>0.3			N
V Hya	4100	>0.2			N
TW Oph	1200	>0.2			N
WZ Cas	400	2.8	6.6		J
VX And	400	>0.2			J
Y CVn	575	>1.4			J
RY Dra	350	>1.7			J
T Lyr	850	>0.4			J

3. Calculation and rates

Our stellar-evolution code is derived from that of Eggleton [1967, 1968, 1971], and uses a technique to solve the structure equations similar to that of Henyey et al. [1959]. Regions of convective instability are determined from the Schwarzschild criterion, and the structure through these regions is nearly adiabatic (as determined from a mixing length approximation). Mixing is treated with a diffusion approximation, and the rate of diffusion (in convective regions) is set proportional to the square of the difference between the adiabatic gradient, and the radiative gradient.

We evolved models of twelve different masses ranging from 0.8 to 25.0 solar masses (fig. 1). These models used 150 mesh zones and Pop I abundances ($X = 0.70$, $Z = 0.02$). Structure changes resulting from different compositions (opacities) can modify the expected surface abundances. As the observed stars represent a range of ages (and compositions), our conclusions must be insensitive to the effects of a reasonable range of composition. To explore this sensitivity, we include a set of models with a higher initial hydrogen content ($X = 0.75$, $Z = 0.02$), and Pop II abundances ($X = 0.75$, $Z = 0.004$). The opacities expected from these compositions were interpolated in the opacity tables by Cox and Tabor [1976]. We also included runs with mixing lengths ranging from 1.4 to 1.65. The depth of convection in low mass main sequence stars is sensitive to the mixing length, but the maximum depth of convective penetration on the giant branch is not. The composition changes that resulted from the use of different mixing lengths is too small to show on the figures that we present.

The low mass models ($M > 2M_\odot$) were evolved to the point of helium flash, and then restarted from static core helium burning models having the same core and envelope masses. These models were then evolved until they had carbon–oxygen cores with masses over $0.8M_\odot$. Intermediate mass models were evolved directly to this stage without the necessity of restarting. Our higher mass stars were evolved into the carbon burning phase.

We have used nuclear reaction rates from the tables of Fowler et al. [1975], and Harris et al. [1983] who we will collectively refer to as FCZ. Coulomb screening is included, but is a relatively small correction for the CNO nucleosynthesis. The stellar-evolution code currently follows only those elements necessary for calculating the energy production, or solar neutrino rate. The structures

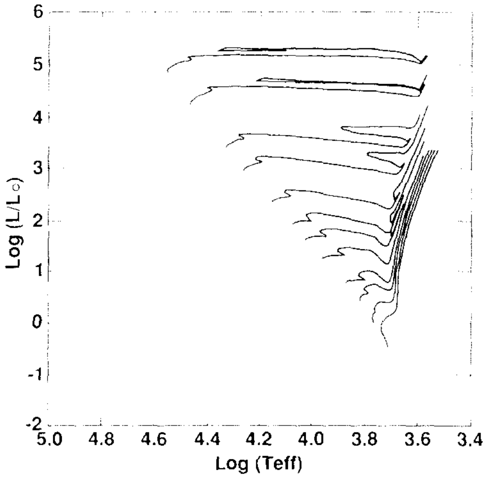

Fig. 1. Luminosity versus temperature tracks for models with mass 0.8, 1.0, 1.3, 1.5, 2.0, 2.5, 3.0, 4.0, 6.0, 8.0, 15.0, and 25.0 solar masses.

produced by this code are stored and then used with a post-processor containing a nucleosynthesis network to calculate the detailed composition structures expected for each model. In this manner, we determined the evolution of the surface abundances, and the yields of each star. The network includes the PP chain (with reactions on lithium and beryllium included), CNO tri-cycle, Na–Ne cycle, Mg–Al cycle, helium burning to carbon and oxygen, and several neutron producing (and absorbing) reactions, but for the purpose of this paper, we are only interested in the CNO reactions.

Some of the rates have considerable uncertainties. FCZ weigh the uncertain terms in their formulae by including a factor which can range from 0 (low) to 1 (high). Normally we follow the recommendation of FCZ using a value of 0.1 (intermediate) for this weight. For the purpose here, however, we have examined the ^{18}O(P, Alpha)^{15}N and ^{17}O(P, Alpha)^{14}N rates using the extreme (high and low) values. The changes in the expected surface values resulting from this are substantial, and comparing our calculations to the values observed in massive red giants sets some constraints on the nuclear rates.

4. Results

Typical results of our calculations are given in figs. 2 and 3, where we show the composition structures of low ($1.5M_{\odot}$) and high ($15M_{\odot}$) mass models near the end of their main sequence lives. In both cases, there is a region where the ^{13}C is distinctly enhanced. Above this region, the ^{13}C has its original abundance. Below it, there is a low ^{12}C/^{13}C ratio, but the abundance of both of these nuclei is low. As the convective mixing penetrates entirely through this ^{13}C rich region, the surface value of the carbon isotope ratio is not sensitive to the ultimate depth of convective penetration. The ^{18}O retains its original value down to a location near the ^{13}C rich region, and then is rapidly destroyed. The ^{17}O abundance becomes substantially enhanced at a temperature achieved below the ^{13}C rich region, but in the more massive models, ^{17}O destruction becomes significant and limits the enhancement expected in ^{17}O. The effect on the composition structure of using the high and low reaction rates for ^{17}O, ^{18}O, and

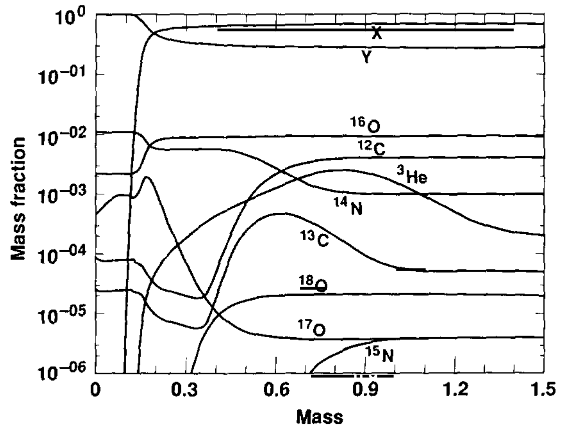

Fig. 2. The composition structure for a $1.5M_{\odot}$ model near the end of its main sequence life. The ^{13}C rich region is near 0.6 solar masses. Inside of this region, ^{17}O is enhanced, and ^{18}O is destroyed.

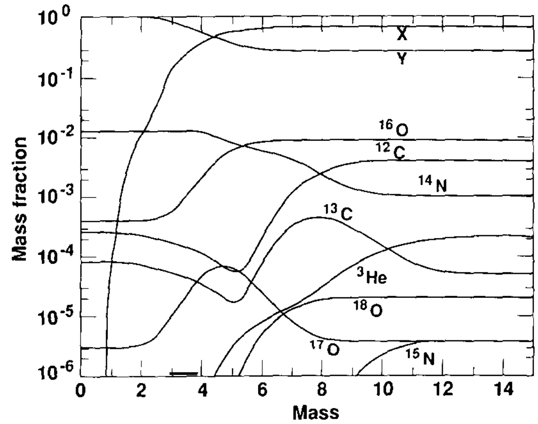

Fig. 3. The composition structure of a $15M_{\odot}$ model. The ^{17}O is enhanced below the ^{13}C rich region as it tries to come to equilibrium. At higher temperatures, equilibrium is achieved, and both the ^{16}O and ^{17}O are converted to nitrogen.

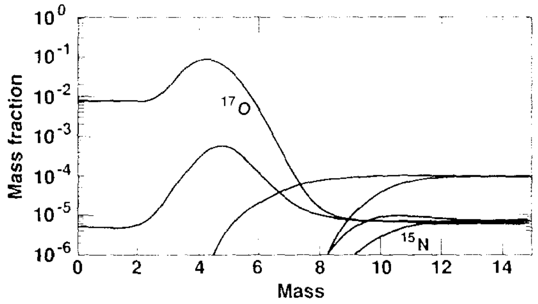

Fig. 4. The composition structure for ^{15}N, ^{17}O, and ^{18}O in a $15M_\odot$ model using the high and low destruction rates of FCZ. Shifting from the low rate of destruction to the high rate for ^{18}O, shifts the location where it is destroyed from inside of the ^{13}C peak to outside of the ^{13}C peak.

^{15}N is shown in fig. 4. The relative positions of the ^{18}O depleted, and the ^{13}C enhanced regions are sensitive to the ^{18}O destruction rate.

Convection on the giant branch penetrates into a region that has a substantial ^{17}O enhancement. Because ^{13}C and ^{18}O are both depleted at this depth, variations in the depth of convection merely change the dilution of these isotopes as they are mixed through the convection zone. Therefore, the ^{12}C/^{13}C and ^{16}O/^{18}O ratios are relatively insensitive to the maximum depth of penetration reached by the convective envelope. However, in low mass stars, the abundance gradient of ^{17}O is quite steep in the region of maximum convective penetration. As a result, the ^{17}O abundance is very sensitive to the depth reached. Finally, as discussed above, the ^{17}O enhancement in high mass stars is limited by the increased efficiency of ^{17}O destruction. In these stars, the ^{16}O/^{17}O ratio becomes sensitive to the reaction rate used.

Table 3 gives the final values for the oxygen isotopes as well as the carbon and nitrogen isotopes. For reasons discussed below, we present results using the high destruction rate for ^{17}O and the low rate for ^{18}O. The primary topic of this paper is the use of the oxygen isotopes to test the adequacy of the physics included in the normal stellar-evolution theory. Because the nitrogen isotopes could provide additional constraints on nonconvective mixing models, we have included our results for the ^{14}N/^{15}N ratio (using the intermediate destruction rate for ^{15}N), and hope that observations of this isotope ratio will become

Table 3

	Ratio				Destruction factor		
Mass	^{12}C/^{13}C	^{14}N/^{15}N	^{16}O/^{17}O	^{16}O/^{18}O	^{6}Li	^{7}Li	^{9}Be
initial	90	270	2625	480	1	1	1
0.8	32	475	2550	500	$10^{[illegible]}$	26	8
1.0	26	720	1950	520	10^{3}	25	11
1.3	24	950	760	560	290	30	14
1.5	24	1050	345	580	75	32	16
2.0	23	1250	115	610	43	33	18
2.5	23	1400	90	620	44/56	34/39	20
3.0	22	1450	115	625	45/56	33/39	20
4.0	22	1700	140	640	36/45	27/33	16/19
6.0	21	2300	210	655	27/47	20/36	12/22
8.0	22	2350	265	650	28/49	21/35	13/21
15.0	19	2250	485	640	46/160	31/85	22/42
25.0	20	2500	615	580	48/110	37/80	25/50

available in the future. Additionally, we have included the factor by which the surface lithium is depleted in our models. As we began our evolution with a static zero-age main sequence model, we do not include any destruction of lithium and beryllium from the premain sequence phase. Therefore, these destruction factors are lower limits, especially in the models with masses below $1.3M_{\odot}$. The dual values listed for the lithium destruction represent the destruction factors reached on the first and second ascent of the giant branch. The large difference in the $15M_{\odot}$ and $25M_{\odot}$ models is due to additional lithium destruction during a blue loop.

In fig. 5a, we see that the predicted value of the $^{16}O/^{18}O$ isotope ratio is only weakly dependent on mass. The upper line results from using FCZ's high rate for ^{18}O destruction, and is clearly too high. The observed values for the K and M giants listed in table 1 are consistent with the low destruction rate for ^{18}O as was suggested in work by Harris and Lambert [1984a]. This low rate is consistent with that of Champagne and Pitt [1986]. Figure 5b shows the calculations for low mass models with different compositions. The difference in $^{16}O/^{18}O$ ratios resulting from the reaction rates is much more significant than uncertainties introduced by composition variations.

The $^{16}O/^{17}O$ predictions in fig. 6a show results of the high and low destruction rates for ^{17}O. The massive models are clearly consistent with the high destruction rate of FCZ. The extreme sensitivity of this ratio to mass (in the low mass region) is a direct result of the convective envelope penetrating further up the ^{17}O gradient seen in fig. 2. Figure 6b concentrates on the low mass region, and shows the effects of composition variations. The scatter of the $^{16}O/^{17}O$ values in these low mass stars is attributable to the uncertainties in determining the masses of field stars. As we stated above, the masses that were adopted were more likely to be high than low, and there appears a systematic shift of the observed values to the high mass side of the predictions.

The observed $^{16}O/^{17}O$ ratios are entirely consistent with the predictions, but this is possibly a result of the uncertainty in the masses. The lowest ratio observed is approximately 175 (Beta And). However, our calculations would lead us to expect values as low as 100 for masses between 2 and 3 solar masses. The predictions can be made consistent with the observations by using a slightly higher destruction rate,

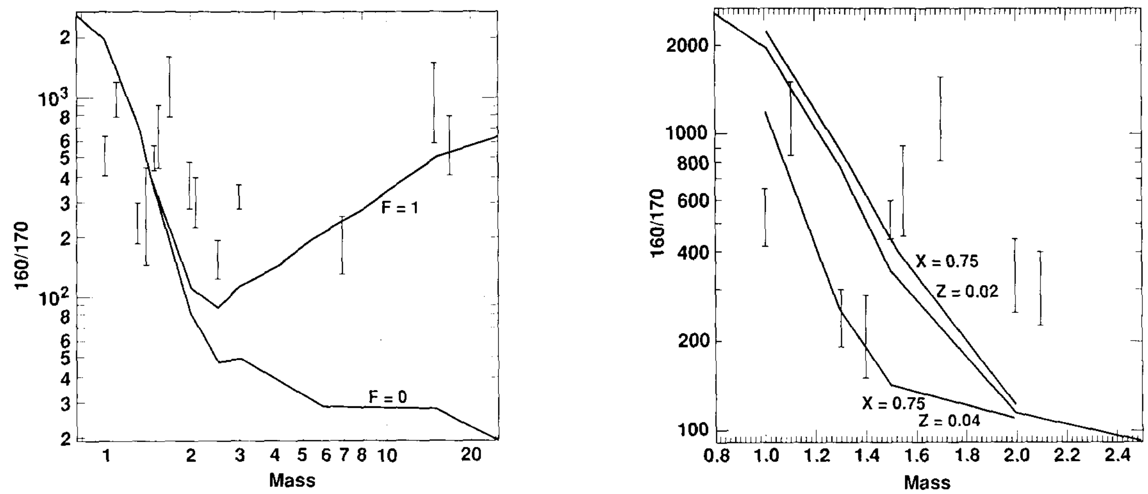

Fig. 5. (a) A comparison of the observed $^{16}O/^{17}O$ ratios of various mass stars from table 1 with the calculations using both the high ($f = 1$) and low ($f = 0$) destruction rates. The high mass stars clearly indicate that a high rate is appropriate. (b) Variations in the composition lead to some expected scatter in the $^{16}O/^{17}O$ ratios. The masses adopted here are likely to be overestimates, and the observations are consistent with the results of the calculations with no additional mixing.

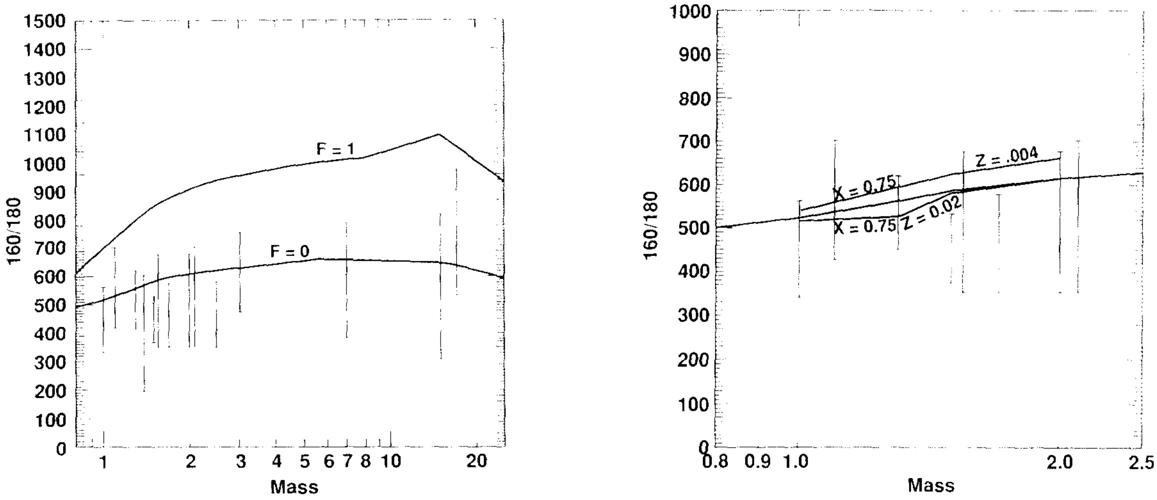

Fig. 6. (a) A comparison of the observed $^{16}O/^{18}O$ ratios for the stars of table 1 with calculations. Here, the low rate of FCZ gives agreement. (b) The uncertainty in the ^{18}O destruction rate causes much greater variation in the calculated $^{16}O/^{18}O$ ratio than any reasonable variations in the composition or mixing length.

or by reducing the depth of penetration of convection. To increase the predicted value for two solar-mass stars to 175 it is necessary to increase the destruction rate above the high rate of FCZ by a factor of 3 to 5 at 20 million kelvin or to reduce the depth of convective penetration by 0.05 solar masses, or 2.5%. Any significant mixing caused by overshoot at the base of this convection zone would aggravate this disagreement. However, the steep entropy gradient at the base of the convection zone in a star where convection has penetrated to its maximum depth leads one to expect little overshoot.

As discussed above, the $^{16}O/^{17}O$ ratio is very sensitive to the maximum depth of convective penetration in low mass stars. This depends mainly on the size of the helium core developed prior to the star's becoming a giant. Therefore it is dependent on the central opacities, and the opacity at the base of the convection zone. Lower opacities delay the onset of the expansion to the giant branch, and allow a slightly larger core to form. The strength of the opacity effect is weak because lower opacity also results in higher central temperatures, and the region where nucleosynthesis occurs moves out, compensating for the reduced penetration. An opacity decrease will result from either a decrease in the metallicity, or the hydrogen mass fraction. The maximum depth of penetration is not sensitive to the mixing length assumed as it is in a main sequence star. Models evolved with mixing lengths of 1.4 and 1.65, show no effect visible on figs. 5b or 6b.

5. Carbon isotopes and nonconvective mixing

While the oxygen isotope ratios seem to be reasonably consistent with the standard predictions, it is well known that the carbon isotope ratios range from the expected value of about 20 down to near the equilibrium value. Ratios as low as 7 and 10 are observed in many of the stars in our sample which show normal (expected) oxygen isotope ratios (table 1). Whatever mechanism is responsible for lowering the $^{12}C/^{13}C$ ratio must do so without excessive destruction of ^{18}O or production of ^{17}O.

While mass loss has been suggested as a solution to this reduced ratio [Dearborn et al. 1976; Guzik et al. 1987] the presence of lithium in normal red giants [Luck and Lambert 1982] as well as low mass main sequence stars [Boesgaard 1987; Boesgaard et al. 1988; Hobbs and Pilachowski 1988] argues against such mass loss, until a means of producing it on the main sequence can be demonstrated. Such mass loss may also lead to a problem with the ^{18}O abundance. We have not evolved a two solar-mass star with the mass loss rate proposed by Guzik et al., but since the composition profiles are substantially determined early on the main sequence, we may approximate the result by removing the outer one solar mass from our two solar-mass star. Doing this at the end of its main sequence life results in a ^{12}C/^{13}C ratio of 5, and an ^{18}O/^{17}O ratio of 900. This is higher than the upper limit observed for any low mass star. This same procedure applied to a $2M_\odot$ model at an age of 0.6 Gy also yields inconsistent results.

The most likely cause of the low carbon isotope ratios is a slow nonconvective mixing [Dearborn and Eggleton 1977] which increases the ^{13}C production. A number of mechanisms have been proposed to account for such a mixing [Schatzman 1977; Hubbard and Dearborn 1980; Tassoul and Tassoul 1983, 1984a], but because such mechanisms are inherently multidimensional, they cannot be evaluated without significant assumptions. Additionally, they are often dependent on the initial angular momentum and its evolution. Lowering the ^{12}C/^{13}C ratio while maintaining the oxygen isotope ratios in an acceptable range places a strong constraint on the nature of any such mixing mechanism.

To examine the general nature of the effect, we have incorporated a simple model (following Fricke and Kippenhahn [1972]) of meridional mixing into our nucleosynthesis network, and applied it to our $1M_\odot$ and $2M_\odot$ models. Such mixing must be constrained by a molecular weight barrier, and we used a condition allowing nonconvective-only mixing when $\chi_r > \mathrm{d}u/\mu$; where μ is the mean molecular weight, and χ_r = centrifugal force/gravity, given by $\chi_{r0}(r/R)^3$ for solid body rotation where χ_{r0} is the surface value. The rate of diffusion was then taken as

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}r}\, lV_{\mathrm{mix}}\, \frac{\mathrm{d}X}{\mathrm{d}r} = 4\pi r^2 \rho l \,\frac{\mathrm{d}}{\mathrm{d}m}\, 4\pi r^2 \rho V_{\mathrm{mix}}\, \frac{\mathrm{d}X}{\mathrm{d}m}\,.$$

From Kippenhahn we have

$$V_{\mathrm{mix}} = \frac{\nabla_{\mathrm{ad}}}{(\mathrm{d}\ln P/\mathrm{d}\ln T)_P(\nabla_{\mathrm{ad}} - \nabla)}\; \frac{L_r \chi_r}{M_r}\; \frac{3}{4\pi r^2 6\rho} \approx \frac{5\chi_r R}{\tau_{\mathrm{KH}}}$$

or

$$\frac{\mathrm{d}X}{\mathrm{d}t} \approx \frac{5m^2}{\tau_{\mathrm{KH}}}\, \frac{\mathrm{d}}{\mathrm{d}m}\, \chi_r\, \frac{\mathrm{d}X}{\mathrm{d}m}\,.$$

This is a simple approximation to the rate of such nonconvective mixing, and does not account for the transport of angular momentum. However, it should be adequate for the purpose of testing how effectively the molecular weight barrier inhibits the change in the oxygen isotope ratios while allowing a ^{13}C enhancement. A more accurate set of approximations might be used for the rate of mixing, but the accuracy of such a result will remain sensitive to assumptions of sphericity (mixing will be quite different in the direction of the pole and equator in a rotating object), the initial angular momentum distribution assumed, and the transport of that angular momentum (see Tassoul and Tassoul [1984b] for a discussion of the evolution of χ_r).

Table 4

Mass	χ_{r0}	$^{12}C/^{13}C$	$^{16}O/^{17}O$	$^{16}O/^{18}O$	Mass	χ_{r0}	$^{12}C/^{13}C$	$^{16}O/^{17}O$	$^{16}O/^{18}O$
1.0	0.000	27	1950	520	2.0	0.000	23	115	610
1.0	0.001	19	1000	600	2.0	0.001	9	112	615
1.0	0.010	6	500	1500	2.0	0.010	4	110	1000
1.0	0.100	3	75	>10000	2.0	0.100	3	90	>10000

Table 4 shows the effect of allowing this nonconvective mixing with different rotational mixing parameters (χ_{r0}). As this parameter increases, mixing can penetrate more deeply against the molecular weight boundary, and fragile nuclei like ^{18}O are destroyed. This results in ^{18}O ratios that are unobserved in any star. We find that for mixing parameters in the range $\chi_{r0} < 0.01$, we can lower the $^{12}C/^{13}C$ ratios without undue change in the oxygen isotope ratios. The ability to lower the $^{12}C/^{13}C$ ratio without excessive destruction of the ^{18}O is a result of using the lower destruction rate. If we had used the high destruction rate, it would have been virtually impossible to mix out significant ^{13}C while avoiding the destruction of too much ^{18}O.

For mixing parameters in the range $0.01 < \chi_{r0} < 0.10$, part of the destruction that we see in this calculation occurs after the star reaches the giant branch, and the hydrogen burning shell has reached out to a mass beyond the composition discontinuity caused by the deepest penetration of the convection zone. After this, the molecular weight barrier outside of the burning shell is small, and continued mixing moves the ^{18}O to a temperature where it is destroyed. A large enough mixing parameter is even capable of moving material to temperatures where ^{17}O will become enhanced. Such nonconvective mixing on the giant branch was proposed by Sweigart and Mengel [1979] to explain observations suggesting the evolution of the CNO isotopes in globular cluster stars ascending the giant branch [Butler et al. 1975; Bell et al. 1979]. Clearer evidence of continuous chemical evolution on the giant branch comes from Pilachowski's [1986] lithium observations of NGC 7786. Using the mixing model described here, values of $\chi_{r0} > 0.1$ completely destroyed the lithium, and resulted in unacceptable oxygen isotope ratios.

The mixing rates that we used to enhance the ^{13}C abundances resulted in the complete destruction of the surface lithium during the early main sequence evolution. Of course, a different assumption on the initial angular momentum, and a proper accounting of its evolution could certainly allow the lithium to survive, or be destroyed more slowly. Such mixing has been proposed as an explanation of the lithium depletion observed in low mass stars [Pinsonneault et al. 1989]. However, the observed lithium abundance in the Hyades (and other clusters) shows a well behaved trend with effective temperature. Any model for nonconvective mixing must explain how to obtain large variations in the $^{12}C/^{13}C$ ratio, and avoid large fluctuations in the lithium.

There are reasonable mechanisms for forcing a nonconvective mixing in main sequence stars, and as a result of such mixing, lowering $^{12}C/^{13}C$ ratios. These mechanisms cannot be strong enough to drive mixing against a significant molecular weight barrier or excessive destruction of ^{18}O will occur. The ^{18}O observations are more than adequate to show that such barriers are quite effective on both the main sequence and giant branch of most stars. The absence of any normal giant in our list showing ^{18}O destruction beyond that expected in the absence of nonconvective mixing strongly constrains the effects that such mixing may be postulated as having. It is unlikely that such mixing could bring a fresh supply of hydrogen to the core, extending the life of a star.

6. Carbon stars and hot bottoms

As stars ascend the asymptotic giant branch, and the mass between the hydrogen and helium burning shells becomes small, thermal pulses begin [Iben and Renzini 1983]. This results in the dredge up and appearance of the products of helium burning (most notably carbon), as well as enhancements of the S process elements at the surface of the star. A subset of these stars also shows lithium enhancements, probably produced as beryllium at the base of a warm convection zone. The temperature history of the base of the convection zone during dredge up is a critical issue for the ultimate yields of lithium, carbon, nitrogen, and possibly ^{26}Al. High temperatures could result in considerable processing of the carbon to nitrogen and a low ^{12}C/^{13}C ratio as is seen in J type carbon stars. Higher temperatures still are necessary to produce ^{26}Al.

Lifetimes for carbon stars as suggested by the observed numbers are typically a million years [Claussen et al. 1987]. Converting a substantial fraction of the envelopes from carbon to nitrogen in this time requires temperatures in excess of 25 million kelvin. At such temperatures the ^{18}O will be destroyed at a rate comparable to or slightly faster than the carbon. Significant production of ^{17}O and ^{26}Al on this timescale requires temperatures in excess of 40 million kelvin.

From table 3 it can be seen that the ^{17}O/^{18}O ratio increases from 0.2 to 6 as the stellar mass goes from $0.8M_\odot$ to $2.5M_\odot$, and has a value of 1 near 1.4 $M_\odot$. The S and N type stars listed in table 2 show ^{17}O/^{18}O ratios in the range 0.7 to 3, corresponding to a mass range between $1.2M_\odot$ and $1.7M_\odot$. This is no surprise, as it matches the masses expected from kinematic arguments. There is little evidence of ^{18}O destruction among the S and N type stars, indicating rather little CNO processing during the dredge up.

While the ^{17}O/^{18}O ratio shows no evidence of modification, Harris et al. [1985, 1987] find some indication in both N and S type stars that both the ^{16}O/^{17}O and ^{16}O/^{18}O ratios decrease with neutron exposure by factors of up to 3. This can result from either an enhancement of the ^{16}O (along with the ^{12}C), or a simultaneous destruction of ^{17}O and ^{18}O. Even in high temperature regions where hydrogen burns to completion, the ^{17}O is only weakly depleted (see fig. 3). Incomplete burning, in fact, enhances the ^{17}O, as it must first be brought into equilibrium with the ^{16}O. Therefore, a hot bottom convection zone will quickly destroy the ^{18}O, but the immediate response of the ^{17}O abundance will be to increase. Only as the entire envelope becomes substantially processed (helium rich) will the ^{17}O decrease, and this will occur long after the ^{18}O is gone. Periodic infusions of material from a region where the hydrogen has been completely converted to helium could lower the ^{17}O and ^{18}O, but the ^{18}O would drop more quickly. Additionally, changing the ^{16}O/^{18}O ratio by factors of 3 would require converting much of the envelope to helium.

If the trend of the ^{16}O/^{17}O and ^{16}O/^{18}O ratios with neutron exposure suggested by the data of Harris et al. is not a result of some systematic effect in the model atmospheres, it seems more likely that sufficient ^{16}O is emerging to enhance this isotope by a factor of 3. This increases to a factor of 10 the carbon enhancement necessary to produce a carbon star, and requires ^{12}C/^{13}C ratios ranging from 100 to 200 depending on the initial value. While the production of some ^{16}O during thermal pulses seems, at least, possible, the observations of Lambert et al. [1988] show no such enhancement compared to iron. This leaves the trend suggested for the ^{16}O/^{17}O ratios with no acceptable solution. More recent observations of S stars [Smith and Lambert 1990] lead to a narrower range of neutron exposures than were used before (near $0.3\,\mathrm{mb}^{-1}$) with little relation to the oxygen isotope ratios. This and the theoretical difficulties, indicate that the suggested relationship was coincidental. Lambert (private communication) has suggested that the high values seen in the oxygen isotope ratios of some carbon rich stars result from a high primordial value of ^{16}O.

The J type carbon stars do show evidence of ^{18}O depletion as well as near-equilibrium carbon isotope ratios, and as discussed above this requires temperatures of at least 25 million kelvin. The $^{16}O/^{17}O$ values given by Harris et al. [1987] for the J type stars are systematically higher than for the N type stars. This could indicate that the J type stars represent a slightly higher mass population (by about $0.3M_{\odot}$) and are not a phase passed through by all carbon stars. Alternatively, it is possible that temperatures are reached capable of enhancing ^{17}O. If J type stars represent a phase of carbon star evolution, then they are a subset of carbon stars, the time available to destroy the ^{18}O and enhance the ^{17}O is less than a million years. This requires temperatures over 40 million K and would be accompanied by ^{26}Al production from proton capture on ^{25}Mg.

7. Discussion and conclusions

The agreement between the predicted behavior of the oxygen isotope ratios for various mass stars, and the observed values given in table 1 is quite good. These ratios are in quantitative agreement when the calculations used FCZ's low destruction rate for ^{18}O, and their high destruction rate for ^{17}O (as determined from the more massive stars, $M > 3M_{\odot}$). The minimum values observed for the $^{16}O/^{17}O$ ratio are not quite as low as the predicted values for stars between $2M_{\odot}$ and $3M_{\odot}$. This difference can be resolved with either an additional increase in the ^{17}O destruction rate (a factor of 3 to 5 at 25 million K) or a slight reduction (2.5%) in the depth of penetration of the convection zone. These two possibilities could be distinguished by observing giants with well determined masses, preferably less than $2M_{\odot}$. For such stars, the ^{17}O destruction is negligible even with the higher rate, and the $^{16}O/^{17}O$ ratio would provide a direct test of the depth of convection.

Nonconvective mixing is probably responsible for the low carbon isotope ratios observed in many stars, including some of the stars used here. The fact that stars showing ^{13}C enhancements show little or no modification of the oxygen isotopes places a constraint on the mechanism responsible for the change. The molecular weight barriers must be adequate to suppress mixing below the ^{13}C rich region, or we would see substantial destruction of the ^{18}O. Even with this condition, the ^{18}O can survive in adequate amounts only for the low destruction rate. While such nonconvective mixing is a likely candidate for ^{13}C enhancement, it remains to be determined if such mixing can simultaneously cause the variation observed in the ^{13}C, while maintaining the continuous, systematic behavior shown by the lithium abundance with effective temperature in the Hyades and other clusters.

Because the mixing cannot operate significantly deeper than the ^{13}C rich region, it cannot affect the amount of hydrogen available to the cores in most stars, and the ages calculated should not be changed. If complete mixing were responsible for the evolution of certain objects (e.g., blue stragglers), we would expect those objects to contain little to no ^{18}O and large enhancements of ^{17}O. The ratios observed in the 15 normal stars and 26 S and N type stars (for which both ratios exist) suggest that such evolution is exceedingly uncommon.

The $^{17}O/^{18}O$ ratio in S and N type stars indicates that little if any CNO processing occurs at the base of the convective envelope during the third dredge up. However, the deficiency of ^{18}O in J type carbon stars is consistent with the burning that must occur to produce the low, near equilibrium, $^{12}C/^{13}C$ ratios seen in those objects. Additionally, the observations suggest a systematically lower $^{16}O/^{17}O$ ratio than is seen in the N type stars. If this is due to ^{17}O enhancement (as opposed to a more massive progenitor population), the temperatures are adequate to produce ^{26}Al from ^{25}Mg. This would be accompanied by an increase in the $^{24}Mg/^{25}Mg$ ratio.

The oxygen isotope ratios have provided a valuable probe of the physics of stellar interiors. The maximum depth of penetration of the convective envelopes on the giant branch is very close to that calculated in our models, but observations of giants with known masses could provide an additional test of this. The nitrogen isotope ratios could also provide corroboration of the depth of mixing in the lower mass stars. Nonconvective mixing can cause the carbon isotope variation without disturbing the oxygen isotope ratios, but mass losing models have difficulty. Finally, the oxygen isotope ratios serve as an indicator of the processing that occurs during third dredge up and carbon star formation.

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