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# On high fertility rates in developing countries: birth limits, birth taxes, or education subsidies?

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**Abstract** In this paper, we consider two types of population policies observed in practice: birth limits and birth taxes. We find that both achieve very similar equilibrium solutions if tax revenue finances lump-sum transfers. By reducing fertility and promoting growth, both birth policies may achieve higher welfare than conventional education subsidies financed by income taxes. A birth tax for education subsidies can achieve the first-best solution. The welfare gain of the first-best policy may be equivalent to a massive 10–50% rise in income, depending on the degree of human capital externalities and the elasticity of intertemporal substitution.

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## 1 Introduction

Owing to various factors such as human capital externalities in education, developing countries typically have high birth rates and little education for their populations. Several types of public policies on education and fertility have emerged in the past several decades to tackle this problem. For example, China has imposed strict *birth limits* since the late 1970s and has gradually switched toward *birth taxes*. From an economics perspective, we use a dynastic family model with human capital accumulation to analyze these two types of birth policies and seek an economic explanation for this policy switch. In addition to these population policies, we also consider conventional education subsidies financed by income taxes. As far as social welfare is concerned, we ask whether such birth policies can do better than the conventional education subsidy, and what is the best mixture of birth and education policies. Quantitatively, we also explore how large the welfare gains of these public policies can be through numerical simulations.

We find the following results. First, both types of population policies achieve very similar equilibrium solutions if birth tax revenue finances lump-sum transfers. Second, by reducing fertility and promoting growth, these policies on fertility may achieve higher welfare than the conventional education subsidies financed by income taxes. Third, using a birth tax to subsidize education spending can achieve the first-best solution. Numerically, the welfare gain of implementing the first-best policy may be equivalent to a massive 10–50% rise in income for every generation, depending on the degree of human capital externalities and the elasticity of intertemporal substitution.

Numerous empirical studies of cross-country growth performance find that standards of living and economic growth are negatively associated with population growth; see, e.g., Mankiw et al. (1992) and Barro and Sala-i-Martin (1995). As a major contributor to population growth, the fertility rate varies significantly between rich and poor countries. According to the World Bank (2002), the average total fertility rate of 52 high-income countries, with a real gross national income (GNI) per capita above US$9,266, is 1.7 children per woman in 2000, which is almost 20% lower than the replacement fertility rate of 2.1 children per woman. By contrast, the average fertility rate of 63 low-income countries, with a real GNI per capita below US$755, is 3.6 children per woman in 2000, which is 71% higher than the replacement rate. Also, there is much more state subsidization of education in developed countries than in developing countries according to the World Bank (2002). It appears that conventional education subsidies financed by income taxes may be an ideal means to resolve the problem of having very high birth rates and little education in developing countries. Indeed, cross-country evidence in Zhang and Casagrande (1998) indicates a statistically significant positive effect of education subsidies on the growth rate of per capita GDP. However, the effect of education subsidies on fertility is statistically insignificant in their cross-country evidence. Given this, we need to explore whether public policies on fertility can help poor countries reduce their birth rates and raise their education investment for faster economic growth.

There are also many theoretical studies of the negative relationship between fertility and the level, or the growth rate, of income. In particular, they ask why the western countries moved away from high-fertility to low-fertility regimes in their development process. Among them, Barro and Becker (1989) show a negative relationship between the level of output per capita and the fertility rate. With human capital accumulation, Becker et al. (1990) argue that a subsistence level of consumption per child can cause multiple development regimes concerning the trade-off between the quantity and quality of children. When initial human capital is low relative to subsistence consumption, the rate of return on human capital is low, and hence, parents choose a large number of children and make no education investment. When initial human capital is high, subsistence consumption becomes negligible, and hence, parents choose a small number of children and invest in their education, leading to sustainable economic growth. They also find that these two equilibria are stable. Thus, a "big push" by good luck is necessary in their model for poor economies to break away from the Malthusian poverty trap and move toward the sustainable growth path.

Morand (1999) and Tamura (2000) also provide similar multiple development regimes with a different mechanism to switch from one regime to the other. For example, in Morand's model with income inequality and subsistence consumption for each child, human capital externalities can trickle down human capital growth to poor households that have many children and make no investment in their education. Consequently, in the absence of any "big push," average human capital in poor economies can grow slowly to a threshold level associated with subsistence consumption. Beyond this threshold, the growth rate will be higher and the average fertility rate will be lower when most households make education investment and have fewer children.

In addition, the steady decline in mortality in most countries' development process may have contributed to lower fertility, according to Ehrlich and Lui (1991) and Zhang et al. (2001), among others. Lagerlof (2003) argues that the decline in fertility is a result of epidemic shocks. By linking young adult mortality negatively to average young adult human capital, Tamura (2006) shows that small initial differences in initial human capital could lead to a large difference in timing of the demographic transition to economic growth. In a related paper, Tamura (2002) shows that the West could have industrialized before China and India because of lower-productivity agriculture in Europe compared with Asia. Also, according to Galor and Weil (2000) and Galor and Moav (2002), the speed of technological progress is the key factor that causes the demographic transition in the process of development.

Furthermore, some institutional factors may have caused lower fertility in developed than in developing countries as well, including social security, as in Cigno and Rosati (1992) and Zhang and Zhang (2004); child labor regulations, as in Doepke (2004); financial development, as in Zhang (1999); and urbanization, as in Zhang (2002).[1] These institutional factors allow workers to arrange their retirement income security and, hence, reduce the role of children for old-age support. However, the trickle-down process, the technological progress, and all the institutional factors may take many generations to reduce fertility gradually, during which poverty remains as a major problem in developing countries.

As suggested by Sen (2000), it is important to ask whether public population policies should be used to achieve a sharper decline in fertility in developing countries. These public policies may use a collaborative approach that emphasizes a voluntary reduction in births. In practice, however, some developing countries also use punitive population policies. Facing huge population pressure and striving to promote economic growth, China and India – the two most populous countries in the world – have incorporated population policies into their development plans (since the early 1950s in India and the 1960s in China). While India has mainly adopted a collaborative approach mainly through family planning programs, China has enforced a birth-limit policy since the late 1970s.[2] According to the World Bank (2004), fertility has declined much more dramatically in China, from 5.78 in 1970 to 1.89 in 2000, than in India, from 5.77 in 1970 to 3.07 in 2000. However, the coercive population policy in China has been widely criticized. In recent years, the Chinese government has relaxed its birth-limit policy, allowing a couple with only one child in their family to pay for a second or third birth, which is essentially a birth tax.

Our finding that a fertility tax may raise welfare is not new. Mirrlees (1972) finds that family size should be taxed or subsidized depending on whether the marginal product of labor is smaller or larger than the average product. Cigno and Pettini (2002) find that family size must be taxed or subsidized according to whether household expenditure on children is decreasing or increasing in the mother's wage rate. However, our present paper differs from their work in that we have a different focus and a different mechanism. We focus on how developing countries can use various population policies to tip the trade-off between the quantity and welfare of children to the latter in the presence of human capital externalities.

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[1] In Zhang (2003), if altruistic bequests are operative, then a combination of government debt and education subsidies can improve welfare by reducing fertility and raising education investment in the presence of the human capital externality. However, there is no consensus with regard to what motivates bequests; see, e.g., Laitner and Thomas (1996) and Altonji et al. (1997). Our results in this paper do not depend on whether such bequests are operative. In addition, since developing countries do not have matured bonds markets for governments to borrow from the public, the application of the result in Zhang (2003) is very limited in these countries in dealing with the population issue for economic growth.

[2] After a 3-year severe famine in the early 1960s, the Chinese government launched a family planning program to educate the population to have fewer children. This program yielded a moderate decline in fertility in the urban population but little change in fertility in the rural population, which accounted for 80% of the total population. In 1979, the government launched the birth-limit project, see Banister (1987).

It is important to conduct an economic analysis of these different public policy instruments, since population growth is a key factor determining per capita income growth. In particular, it is important to compare birth limits with birth taxes because the former set a limit on the number of children parents can choose. This comparison can also reveal which policy is more effective in lowering fertility, which policy is more conducive to education investment and economic growth, and which policy improves social welfare. According to our findings mentioned earlier, it is indeed the case that the tax approach can outperform birth limits in all these dimensions provided that the tax revenue is used, at least partly, as education subsidies.

The rest of the paper proceeds as follows: Section 2 introduces the base model with log preferences. Section 3 presents equilibrium solutions in various cases and derives the results. Section 4 conducts sensitivity analysis by changing the elasticity of intertemporal substitution. Section 5 discusses policy implications, particularly for the case of China. The last section concludes.

## 2 The base model

Consider an economy with an infinite number of periods and overlapping generations of identical agents who live for two periods (childhood and adulthood, respectively). Children embody human capital through education and make no decision. Adults work, spend income on consumption and on investment in children's education, and choose the number of children. There is no distinction by gender. Each adult has one unit of labor time and devotes it to child rearing and working. As in Becker et al. (1990), raising a child requires $v$ fixed units of time where $0 < v < 1$, which sets an upper bound on the number of children, $1/v$.

Let subscript $t$ denote a period of time. The utility function of an altruistic parent, $V(t)$, depends on his/her own consumption $c_t$, the number of children $n_t$, and the average utility of children $\bar{V}(t+1)$. Since agents are identical, the utility of each child is the same, that is, $\bar{V}(t+1) = V(t+1)$. In our base model, the preferences of individuals are assumed to be logarithmic:

$$V(t) = \ln c_t + \rho \ln n_t + \alpha V(t+1), \quad \rho > 0, \quad 0 < \alpha < 1, \tag{1}$$

where $\rho$ is the taste for the number of children and $\alpha$ is the taste for per-child welfare or the subjective discounting factor. To guarantee a solution to the problem, we assume that the taste for the number of child is sufficiently strong relative the taste for the welfare of children:

**Assumption 1** $\rho > \frac{\alpha\delta}{1-\alpha}$.

The assumption of log utility will allow us to derive analytic results. However, it restricts the elasticity of intertemporal substitution to unity and may thus lead to unrealistic results. In Section 4, we shall allow for nonunitary elasticities of intertemporal substitution in order to see how sensitive the results are in this regard.

The human capital of each child, $H_{t+1}$, depends positively on parental education spending per child, $q_t$, parental human capital, $H_t$, and the economy-wide average human capital, $\bar{H}_t$. The education technology is Cobb–Douglas, as below:

$$H_{t+1} = Aq_t^{\delta}\left[H_t^{\beta}\bar{H}_t^{1-\beta}\right]^{1-\delta}, \quad A > 0, \;\; 0 < \beta < 1, \;\; 0 < \delta < 1, \tag{2}$$

where $\delta$ is the share parameter of education spending. Following Tamura (1991), we assume that average human capital has a positive external effect on education for $0 < \beta < 1$. A smaller $\beta$ means a stronger human capital externality. A log-linear version of Eq. 2 has been widely used in tackling the relationship between parental earnings and children's earnings in the empirical studies of intergenerational earnings mobility. As $q_t$ is expected to be proportional to parental earnings and, hence, to parental human capital in this model, the parental influence on children's earnings can be captured by the coefficient $\delta + \beta(1-\delta)$, which is also coined as the *persistent* coefficient. Recent studies using micro panel data indicate that this persistent coefficient is in the range of 0.4–0.6 in the USA; see, e.g., Solon (1999).[3] Given that education is less physical-input-intensive than final production, a plausible value of the share parameter $\delta$ associated with $q_t$ should be smaller than capital's share in production (0.3), e.g., $0.1 < \delta < 0.3$. Let us assume the midpoint $\delta = 0.2$. Putting together $0.4 < \delta + \beta(1-\delta) < 0.6$ and $\delta = 0.2$, we expect $0.25 < \beta < 0.5$, implying $0.75 \geq 1-\beta \geq 0.5$. On the other hand, using macro panel data across countries, the annual convergence rate is about 2%. For a 20-year generation length, the persistence coefficient might be 0.75, and hence, the spillovers might be closer to 0.25. For $\delta = 0.2$, this implies $\beta = 0.69$ or $1-\beta = 0.31$; for $\delta = 0.1$, this implies $\beta = 0.72$ or $1-\beta = 0.28$. With either the macro or micro data, the empirical evidence indicates an essential role for outside family factors, captured by average human capital in the parental generation, in the education process. Heckman (2003) suggests that human capital externalities in China may be large.

The final output a worker produces is assumed to be equal to his/her effective labor input, $(1 - vn_t)H_t$. The budget constraint of a parent is thus given by:

$$c_t = (1 - vn_t)H_t - n_tq_t. \tag{3}$$

[3]The type of externality we refer to is about how the average human capital of the parental generation affects the formation of human capital of individuals in the children's generation. This externality may arise from state funding of education, which is large relative to private spending on education given the fact that primary and secondary levels of education are heavily funded by the state. It differs from the other type of human capital externality in the production of final goods and services in another body of literature (e.g., Moretti 2004; Ciccone and Peri 2006). The latter type of externality considers how the average human capital of the labor force affects the earnings of individual workers. We abstract from the production externality in this paper since it also leads to under-investment in human capital.

When we consider taxes or subsidies later, the budget constraint will change accordingly.

## 3 Equilibrium and results in the base model

In this section, we begin with a competitive equilibrium without government intervention and compare it to the social planner's solution. We then derive competitive solutions with various forms of population policies or with a conventional education subsidy financed by income taxes. At the end of this section, we will also use a numerical approach to explore the quantitative implications of the base model.

### 3.1 Competitive equilibrium without government intervention

Substituting Eqs. 2 and 3 into Eq. 1, the household problem is formulated in the following Bellman equation:

$$V(H_t) = \max_{n_t, H_{t+1}} \left\{ \ln\left[(1 - vn_t)H_t - n_t \left(A^{-1}H_{t+1}\right)^{1/\delta} \left(H_t^{\beta}\bar{H}_t^{1-\beta}\right)^{-(1-\delta)/\delta}\right] + \rho \ln n_t + \alpha V(H_{t+1}) \right\}. \tag{4}$$

Differentiating Eq. 4 with respect to $H_t$ and $H_{t+1}$, respectively, we have:

$$\frac{n_t q_t}{\delta c_t H_{t+1}} = \frac{\alpha}{c_{t+1}} \left[\left(1 - vn_{t+1}\right) + \frac{\beta(1-\delta)n_{t+1}q_{t+1}}{\delta H_{t+1}}\right]. \tag{5}$$

This condition implies that the marginal utility forgone from giving up an additional unit of consumption to invest in children's education equals the marginal utility obtained from increasing the welfare of children through raising their human capital.

Differentiating Eq. 4 with respect to $n_t$ gives:

$$\frac{\rho}{n_t} = \frac{vH_t + q_t}{c_t}. \tag{6}$$

Here, the marginal utility forgone from giving up $vH_t + q_t$ units of consumption for one additional child is compensated by the marginal utility obtained from having this child.

Denote the fraction of income spent on consumption per capita as $\gamma_c$ and the fraction of income invested in a child's education as $\gamma_q$, where $\gamma_c = c_t/[(1 - vn_t)H_t]$ and $\gamma_q = q_t/[(1 - vn_t)H_t]$, respectively. In equilibrium, both fertility and proportional output allocations are expected to be constant over time under our assumptions of a logarithmic utility function and a Cobb–Douglas education technology.

Solving the first-order conditions and constraints in Eqs. 2, 3, 5, and 6 yields the competitive equilibrium solutions for fertility and for proportional income allocations:

$$\gamma_c^* = \frac{1-\alpha\beta(1-\delta)-\alpha\delta}{1-\alpha\beta(1-\delta)}, \tag{7}$$

$$\gamma_q^* = \frac{\alpha\delta}{n^*[1-\alpha\beta(1-\delta)]}, \tag{8}$$

$$n^* = \frac{\rho\left[1-\alpha\beta(1-\delta)-\alpha\delta\right]-\alpha\delta}{v(1+\rho)\left[1-\alpha\beta(1-\delta)-\alpha\delta\right]}. \tag{9}$$

As expected, both fertility and the proportional income allocations are constant over time. Also as expected, educational spending per child as a fraction of income is inversely related to fertility in Eq. 8, exhibiting a trade-off between the quantity and the quality of children. It is easy to verify that the solution $(n^*, \gamma_c^*, \gamma_q^*)$ indeed satisfies the first-order conditions and constraints in all periods and, hence, is valid on the entire equilibrium path.

The growth rate of human capital is:

$$\phi^* = H_{t+1}/H_t - 1 = A\left(\gamma_q^*\right)^\delta \left(1-vn^*\right)^\delta - 1. \tag{10}$$

According to Eq. 10, the growth rate of human capital depends positively on the fraction of income invested in education for each child, $\gamma_q^*$, and negatively on fertility, $n^*$. Since $1 - vn^*$ is constant over time, final output is proportional to human capital in this model. Therefore, the growth rate $\phi^*$ in Eq. 10 is also the growth rate of per capita income in this model. When the productivity parameter $A$ is large enough such that $\phi^* > 0$, the model has sustainable growth in per capita income. Note that the solution $(n^*, \gamma_c^*, \gamma_q^*, \phi^*)$ implies the solution for the sequence $(c_t, q_t, n_t)_{t=0}^\infty$, where $n_t = n^*$ for all $t \geq 0$.

However, due to the presence of $n_t q_t$ in the budget constraint, the feasible set of the household is not a convex set. Therefore, we need to derive carefully the sufficient condition for the solution from the first-order conditions to be optimal for the household. We state the sufficient condition for a unique optimal solution to the household problem below and relegate the proof to Appendix A.

**Lemma 1** *The sufficient condition for a unique optimal solution to the household problem is* $\rho > \alpha\delta/[1-\alpha\beta(1-\delta)-\alpha\delta]$.

This condition is guaranteed by Assumption 1, that is $\rho > \alpha\delta/(1-\alpha)$ implies $\rho > \alpha\delta/[1-\alpha\beta(1-\delta)-\alpha\delta]$, and hence, $n > 0$ for all $\beta \in (0, 1)$. In other words, when the taste for the number of children is sufficiently strong, there exists a unique optimal solution for fertility and, hence, for other choice variables.

In order to determine optimal public policies later, let us find the equilibrium solution for an individual's welfare level, given initial human capital $H_0$. In the equilibrium solution for identical agents, $H_0 = \bar{H}_0$ and the decision $(n^*, \gamma_c^*, \gamma_q^*)$ are the same across households. Substituting the solution for $(c_t, q_t, n_t)_{t=0}^{\infty}$ into Eq. 4 or setting $H_0 = \bar{H}_0$ and $(n_t, \gamma_{ct}, \gamma_{qt}) = (n^*, \gamma_c^*, \gamma_q^*)$ in the counterpart equation in Appendix A, the equilibrium solution for the welfare level at time 0 with initial human capital $H_0$ is:

$$V(0) = \frac{1}{1-\alpha}\left[\ln \gamma_c^* + \rho \ln n^* + \ln\left(1 - vn^*\right)\right] + \frac{\alpha}{(1-\alpha)^2}\left[\ln A + \delta \ln \gamma_q^* + \delta \ln\left(1 - vn^*\right)\right] + \frac{1}{1-\alpha} \ln H_0. \quad (11)$$

An individual's welfare in Eq. 11 is fully determined by his or her initial human capital $H_0$ and parameters in technologies and preferences via the solution $(n^*, \gamma_c^*, \gamma_q^*)$. Thus, Eq. 11 gives a reduced-form equilibrium solution for the welfare level.

### 3.2 Social planner's solution

Differing from individual parents, the social planner can internalize the human capital externality by setting $H_t = \bar{H}_t$ when maximizing utility in Eq. 4 subject to Eqs. 2 and 3. Obviously, the social planner's solution for $(\gamma_c, \gamma_q, n)$ is a special case of the competitive solution with $\beta = 1$:

$$\gamma_c^P = \frac{1-\alpha}{1-\alpha(1-\delta)}, \quad (12)$$

$$\gamma_q^P = \frac{\alpha\delta}{n^P[1-\alpha(1-\delta)]}, \quad (13)$$

$$n^P = \frac{\rho(1-\alpha) - \alpha\delta}{v(1+\rho)(1-\alpha)}, \quad (14)$$

where the superscript $P$ refers to the social planner. Assumption 1 $\rho > \alpha\delta/(1-\alpha)$ guarantees the sufficient condition for a unique solution to the social planner problem as in Lemma 1.

The growth rate of human capital, or that of output per capita, can be derived as:

$$\phi^P = A\left[\gamma_q^P(1 - vn^P)\right]^{\delta} - 1. \quad (15)$$

Since the social planner can internalize the external effect of average human capital in the society as a whole, the growth rate of human capital, or that of output per capita, is higher in the social planner's solution than in the competitive solution. This can be seen as follows: Comparing the competitive solution in Eqs. 7–9 with the social optimum in Eqs. 12–14, both fertility and the fraction of output spent on consumption are higher but the fraction of output spent on education is lower in the former solution than in the latter.

Intuitively, in the presence of the human capital externality, the private rate of return on education investment relative to that on having a child is lower than the social rate. Therefore, in the decentralized economy with the human capital externality, parents have more children and allocate more output to consumption and less to education investment than their social optimum. A stronger externality means a larger gap between the private and social rates of return and, hence, larger gaps in fertility, in the proportional allocation of output, and in the growth rate between the competitive solution and the social planner's. Specifically, differentiating the solution in Eqs. 7–9 in the decentralized economy with respect to $\beta$, we have $\partial\gamma_c^*/\partial\beta < 0$, $\partial\gamma_q^*/\partial\beta > 0$, and $\partial n^*/\partial\beta < 0$. That is, as $\beta$ becomes smaller, which is equivalent to a stronger human capital externality, both fertility and the fraction of income spent on consumption rise, but the fraction of income spent on each child's education falls. These consequences of the human capital externality conform to the stylized facts in less developed countries that have high fertility and little education spending per child.

It is thus interesting to consider whether government policies that affect parents' decisions on fertility and education spending can narrow the gap between the competitive equilibrium path and the social planner's optimal path. Through the trade-off between the quantity and quality of children, policies that lower fertility will raise private education investment for each child and will therefore accelerate human capital accumulation and income growth. In the rest of the paper, we consider two types of such policies that influence the decision on the number of children: one through birth limits and the other through birth taxes. The case with birth taxes will be split further into two scenarios, depending on whether the tax revenue is made as lump-sum transfers or as education subsidies. We will compare the respective effects of these different population policies on education investment, fertility, economic growth, and welfare. Finally, we will compare all these population policies to a conventional education subsidy financed by income taxes.

### 3.3 Birth limits

Consider a situation whereby the government sets an upper limit on the number of children for every family in a way to maximize the welfare of the representative household. For the purpose of this paper, we focus on the case that this upper limit is binding. The analysis involves two stages. In the first stage, parents choose their consumption and education investment for their children to maximize utility in Eq. 4, while taking the mandatory limit on the number of children $n^L$ as given, where the superscript $L$ refers to birth limits. Parental decisions in the competitive solution will be functions of the mandatory limit on births. In the second stage, the government can then maximize the welfare of the representative agent by choosing the level of the mandatory limit on births.

In equilibrium, $\gamma_c^L = [1-\alpha\beta(1-\delta)-\alpha\delta]/[1-\alpha\beta(1-\delta)]$ and $\gamma_q^L = \alpha\delta/\{n^L[1-\alpha\beta(1-\delta)]\}$. In order to determine the fertility level that maximizes individuals' welfare, we replace $(\gamma_c^*, \gamma_q^*, n^*)$ in $V(0)$ of Eq. 11 by $(\gamma_c^L, \gamma_q^L, n^L)$. In so doing, we obtain the solution for the welfare level at time 0 under government birth limits, denoted by $V_0^L$, as follows:

$$V(0)^L = \frac{1}{1-\alpha}\Big[\ln\gamma_c^L + \rho\ln n^L + \ln\left(1 - vn^L\right)\Big] \\ +\frac{\alpha}{(1-\alpha)^2}\Big[\ln A + \delta\ln\gamma_q^L + \delta\ln\left(1 - vn^L\right)\Big] + \frac{1}{1-\alpha}\ln H_0. \quad (16)$$

The optimal mandatory limit on births corresponds to the fertility rate that maximizes $V^L$. We thus have the following result:

**Proposition 1** *With* $0 < \beta < 1$, *the optimal birth-limit policy improves welfare by setting fertility at its social optimum:* $n^L = n^P = [\rho(1-\alpha) - \alpha\delta]/[v(1+\rho)(1-\alpha)]$.

*Proof* Differentiating $V_0^L$ in Eq. 16 with respect to $n^L$ yields:

$$\frac{dV(0)^L}{dn^L} = \left(\frac{1}{1-\alpha}\right)\left[\frac{\rho}{n^L} - \frac{v}{1-vn^L} - \frac{\alpha\delta}{(1-\alpha)n^L} - \frac{\alpha\delta v}{(1-\alpha)(1-vn^L)}\right]. \quad (17)$$

Setting $dV(0)^L/dn^L = 0$ in Eq. 17 leads to the claimed value of $n^L$. The second-order condition holds as below:

$$\frac{d^2V(0)^L}{d(n^L)^2} = -\frac{1}{(1-\alpha)^2}\left\{\frac{\rho(1-\alpha)-\alpha\delta}{(n^L)^2} + \frac{v^2[1-\alpha(1-\delta)]}{(1-vn^L)^2}\right\} < 0,$$

under Assumption 1, $\rho > \alpha\delta/(1-\alpha)$. The result follows. □

Notice that the optimal level of fertility under birth limits is the same as that chosen by the social planner. The reason for this equality between the two fertility rates is that both the government and the social planner can choose the fertility rate to internalize the human capital externality. Unlike the social planner, however, the government in the case with birth limits does not decide how to allocate income to consumption and to children's education. Thus, there should be under-investment in education in the equilibrium solution with birth limits in the presence of the human capital externality. Specifically, the ratio of total education spending to output in the competitive solution with birth limits, $n^L\gamma_q^L = \alpha\delta/(1-\alpha\beta(1-\delta)$, is the same as that in the competitive solution without birth limits, which is below its social optimum, $n^P\gamma_q^P = \alpha\delta/(1-\alpha(1-\delta)$. This is because the unitary elasticity of intertemporal substitution in the base model implies equal percentage changes in fertility and human capital investment per child in opposite directions.

Further, since birth limits reduce fertility without changing the ratio of total education spending to output, education investment per child as a fraction of output per worker is higher in the equilibrium solution with birth limits than without. On the other hand, since birth limits lead to the same fertility rate but a lower ratio of total education investment to output than the social optimum, education investment per child as a fraction of output is lower in the competitive solution with birth limits than in the social planner's solution. In sum, we have $\gamma_q^* < \gamma_q^L < \gamma_q^P$.

The growth rate of human capital, or that of output, under birth limits is $\phi^L = A[\gamma_q^L(1 - vn^L)]^\delta - 1$. Since $\gamma_q^* < \gamma_q^L < \gamma_q^P$, and $n^* > n^L = n^P$, the growth rate $\phi^L$ in the competitive solution with birth limits must be higher than that in the competitive solution without government intervention, but it must be lower than that in the social planner's solution.

The welfare loss of altruistic parents from having fewer children will be compensated by the dynamic welfare gains arising from more education investment per child and a subsequent rise in productivity. Starting with too many children and too little education in the presence of the externality, the welfare level is maximized by setting fertility at $n^L = n^P$ as given in Proposition 1 under birth limits. However, since there is still under-investment in education with birth limits compared to the social planner's solution, the birth-limit policy cannot reach the first-best solution.

Summing up the analysis above, we have the following results:

**Proposition 2** *For* $0 < \beta < 1$, *the birth-limit policy increases education investment per child relative to output and the growth rate of output. However, the ratio of education spending to output and the growth rate are still lower than their social optimum. Thus, the birth-limit policy cannot reach the first-best solution.*

*Proof* To measure the net gain in per capita income growth, we determine the ratio of the gross growth rates, $(1 + \phi^L)/(1 + \phi^*) \equiv R_\phi^{L*}$, between the birth-limit case and the no-government case below:

$$R_\phi^{L*} = \left\{ \frac{A\left[\gamma_q^L(1 - vn^P)\right]}{A\left[\gamma_q^*(1 - vn^*)\right]} \right\}^\delta = \left(\frac{n^*}{n^P}\right)^\delta \left(\frac{1 - vn^P}{1 - vn^*}\right)^\delta > 1,$$

since $n^L = n^P < n^*$. Similarly, we determine the ratio of the gross growth rates, $(1 + \phi^L)/(1 + \phi^P) \equiv R_\phi^{LP}$, between the birth-limit case and the social planner's solution below:

$$R_\phi^{LP} = \left\{ \frac{A[\gamma_q^L(1 - vn^P)]}{A[\gamma_q^P(1 - vn^P)]} \right\}^\delta = \left(\frac{\gamma_q^L}{\gamma_q^P}\right)^\delta = \left[\frac{1 - \alpha(1 - \delta)}{1 - \alpha\beta(1 - \delta)}\right]^\delta < 1,$$

since $n^L = n^P$. The other results follow our earlier analysis. □

Though the birth limit policy lowers fertility and improves growth and welfare, the decision on the number of children is not made voluntarily by individual parents. We now consider a birth tax policy as an alternative means, which allows parents to choose the number of children. The birth tax revenue may finance a lump-sum transfer or an education subsidy. We first consider the case in which the tax revenue is made as transfers.

### 3.4 Birth taxes for lump-sum transfers

In this case, the government imposes a tax on each additional birth in excess of a threshold number of children, $n_{\text{tax}}$, and makes the tax revenue as lump-sum transfers. The use of lump-sum transfers can allow us to focus on the substitution effect of the birth tax. In practice, the birth tax in China does not apply to the first birth. That is, in the real world, it is natural to have $n_{\text{tax}} \geq 1$ for a couple. In our model with a single type of gender, this corresponds to $n_{\text{tax}} \geq 1/2$. The household budget constraint becomes

$$c_t = (1 - vn_t)H_t - q_t n_t - (n_t - n_{\text{tax}})T_t B + \Pi_t, \tag{18}$$

where $T_t$ is the birth tax, $\Pi_t$ is the lump-sum transfer per worker, and $B$ is an indicator variable, which equals 1 for $n_t \geq n_{\text{tax}}$ and equals 0 for $n_t < n_{\text{tax}}$. A balanced government budget constraint requires $\Pi_t = (n_t - n_{\text{tax}})T_t B$. Since optimal birth-tax policies do not emerge in the situation with $n_t < n_{\text{tax}}$, we will suppress $B$ in our analysis for brevity.

The first-order condition with respect to education investment is the same as in Eq. 5. The first-order condition with respect to fertility becomes:

$$\frac{\rho}{n_t} = \frac{vH_t + q_t + T_t}{c_t}. \tag{19}$$

It is clear that the birth tax increases the marginal loss in utility of having a child on the right-hand side of Eq. 19. Therefore, we expect a negative effect of the birth tax on fertility.

The solution for the ratio of total education investment to income is the same as in the preceding cases without government intervention or with birth limits, that is, $\gamma_q^T n^T = \gamma_q^L n^L = \gamma_q^* n^* = \alpha\delta/[1 - \alpha\beta(1-\delta)]$, where the superscript $T$ stands for the case where the birth tax finances lump-sum transfers. Clearly, the ratio of total education spending to income is independent of the birth tax due to the unitary elasticity of intertemporal substitution. Consequently, $\gamma_c^T = \gamma_c^L = \gamma_c^*$, which is also independent of the birth tax. Define $\tau = T_t/\bar{H}_t$, as we expect the birth tax to be proportional to average income or average human capital on the equilibrium growth path.

The solution for fertility is indeed a decreasing function of the birth tax:

$$n^T = \frac{\rho[1 - \alpha\beta(1-\delta) - \alpha\delta] - \alpha\delta}{v(1+\rho)[1 - \alpha\beta(1-\delta) - \alpha\delta] + \tau[1 - \alpha\beta(1-\delta)]}. \tag{20}$$

The negative effect of birth taxes on fertility is consistent with empirical evidence in Boyer (1989) and Whittington et al. (1990). In Boyer's work, child allowances under the old poor law had a positive effect on birth rates in 1826–1830 in southeastern England, as argued by Malthus (1807, 1872). In the work of Whittington et al., personal exemptions for dependants have a positive effect on birth rates in the USA. Both the child allowances and the personal exemptions for dependants in the tax system are like birth subsidies, which are the opposite of birth taxes.

The solution for the welfare level has the same expression as in the cases without government intervention or with birth limits. We then have the following result:

**Proposition 3** *For* $0 < \beta < 1$*, the optimal birth tax financing lump-sum transfers is:* $0 < n_{\text{tax}} < n^P$ *and*

$$\tau^* = \frac{\alpha^2 \delta v(1+\rho)(1-\delta)(1-\beta)}{[\rho(1-\alpha)-\alpha\delta][1-\alpha\beta(1-\delta)]}.$$

*It achieves exactly the same equilibrium solution as that under the optimal birth limit.*

*Proof* Substituting $(n^T, \gamma_c^T, \gamma_q^T)$ into the solution for welfare in Eq. 11 to replace $(n^*, \gamma_c^*, \gamma_q^*)$, the welfare level becomes a function of the tax rate via $n^T$, denoted by $V(0)^T$. Differentiating $V(0)^T$ with respect to the tax rate $\tau$, we have the following first-order condition:

$$\frac{dV(0)^T}{d\tau} = \frac{dn^T}{d\tau}\left(\frac{1}{1-\alpha}\right)\left[\frac{\rho}{n^T} - \frac{v}{1-vn^T} - \frac{\alpha\delta}{n^T(1-\alpha)} - \frac{\alpha\delta v}{(1-\alpha)(1-vn^T)}\right] = 0.$$

Since $dn^T/d\tau < 0$, we have:

$$\frac{\rho}{n^T} - \frac{v}{1-vn^T} - \frac{\alpha\delta}{n^T(1-\alpha)} - \frac{\alpha\delta v}{(1-\alpha)(1-vn^T)} = 0.$$

The second-order condition holds as in the proof of Proposition 1. From these conditions and the solution for fertility in Eq. 20, we obtain the solution for the optimal tax rate $\tau^*$. Substituting $\tau^*$ into Eq. 20, it is easy to verify that $n^T = n^L = n^P$. Combining this with $\gamma_q^T n^T = \gamma_q^L n^L$, we have $\gamma_q^T = \gamma_q^L$. Now, it is also easy to verify that both the growth rate and the welfare level are the same as those with the optimal birth-limit policy. From the government budget constraint and the solution $(n^T, \tau) = (n^P, \tau^*)$, we have $\Pi_t = (n^P - n_{\text{tax}})\tau^* H_t$. We can thus choose any value of $n_{\text{tax}}$ as long as $0 < n_{\text{tax}} < n^P$ because the lump-sum transfer can vary to balance the government budget. □

Intuitively, the birth tax reduces fertility by increasing the cost of having a child. When the tax revenue is made as lump-sum transfers, the birth tax has no effect on the ratio of total education investment to output because of the unitary elasticity of intertemporal substitution. Combining these two results together, the birth tax must raise education investment per child as a fraction of output, and hence, it must raise the growth rate of output. Since total education investment as a fraction of output is lower than its social optimum as in the case with birth limits, the optimal birth tax financing lump-sum transfers cannot reach the first-best solution. Also, like the optimal birth-limit policy, the optimal birth tax financing lump-sum transfers has a lower growth rate of output than the social optimum. To implement this tax, the government can choose a pair of the threshold number of children and the amount of transfers such that $0 < n_{\text{tax}} < n^P$. Next, we explore the case when the birth tax revenue is used to subsidize education.

## 3.5 Birth taxes for education subsidies

Now, we assume that the government subsidizes education spending for every child at a rate $s$, using the revenue collected from the birth tax. Assume that the birth-tax payment is increasing in the number of children in a simple form: $(n_t - n_{\text{tax}})^\theta T_t$, where $1 \geq \theta > 0$. We will see that the restriction on $\theta$ has subtle implications for a proper choice of $n_{\text{tax}}$, when the tax revenue finances education subsidies. Also, $T_t = \tau \bar{H}_t$, as in the preceding case. The household's budget constraint Eq. 3 becomes:

$$c_t = (1 - vn_t)H_t - (1 - s)q_t n_t - (n_t - n_{\text{tax}})^\theta T_t, \tag{21}$$

and the government's budget constraint is:

$$(n_t - n_{\text{tax}})^\theta T_t = n_t s q_t. \tag{22}$$

The first-order condition with respect to human capital investment now takes the following form:

$$\frac{(1-s)n_t q_t}{\delta c_t H_{t+1}} = \frac{\alpha}{c_{t+1}}\left[(1 - vn_{t+1}) + \frac{\beta(1-\delta)(1-s)n_{t+1}q_{t+1}}{\delta H_{t+1}}\right]. \tag{23}$$

In this condition, the subsidy rate reduces the cost of education investment proportionately (the left-hand side), while it reduces the benefit of education less than proportionately (the right-hand side). As a result, the education subsidy tends to raise education investment.

The first-order condition with respect to fertility is

$$\frac{vH_t + (1-s)q_t + \theta(n_t - n_{\text{tax}})^{\theta-1}T_t}{c_t} = \frac{\rho}{n_t}. \tag{24}$$

In this condition, the birth tax increases the cost of having a child and, hence, tends to reduce fertility. On the other hand, the education subsidy reduces the cost of having a child less than proportionately, compared to its proportionate effect on the cost of education mentioned above.

These first-order conditions and the budget constraints of households and the government lead to:

$$\gamma_c^E = \frac{(1-s)[1-\alpha\beta(1-\delta)] - \alpha\delta}{(1-s)[1-\alpha\beta(1-\delta)]}, \tag{25}$$

$$\gamma_q^E = \frac{\alpha\delta}{n^E(1-s)[1-\alpha\beta(1-\delta)]}, \tag{26}$$

$$\rho\gamma_c^E(1 - vn^E) = vn^E + (1-s)\gamma_q^E n^E(1 - vn^E) + \tau n^E\theta\left(n^E - n_{\text{tax}}\right)^{\theta-1}. \tag{27}$$

The superscript $E$ stands for the case with the birth tax financing education subsidies. Note that the solution $(n^E, \gamma_c^E, \gamma_q^E)$ is a function of policies chosen by the government $(s, \tau, n_{\text{tax}})$.

Like the birth tax for lump-sum transfers, the birth tax for education subsidies raises the cost of having a child and, hence, reduces fertility. On the other hand, using the birth tax revenue to subsidize education investment raises the rate of return on education investment and, hence, induces parents to spend more on education and less on consumption. Because of this, the birth tax for education subsidies is expected to have a positive effect on the growth rate of income. Both the decline in fertility and the rise in education investment, caused by the birth tax for education subsidies, help to narrow the gaps between the competitive solution and the social planner's solution, in the presence of the human capital externality. We thus expect this policy to outperform birth limits and the birth tax for lump-sum transfers in terms of social welfare. We give the results below and relegate the proof to Appendix B:

**Proposition 4** *With $0 < \beta < 1$, the optimal birth tax financing education subsidies obtains the first-best solution, $(n^E, \gamma_c^E, \gamma_q^E) = (n^P, \gamma_c^P, \gamma_q^P)$, by setting*

$$s^* = \frac{\alpha(1-\delta)(1-\beta)}{[1-\alpha\beta(1-\delta)]},$$

*and*

$$n^*_{\text{tax}} = n^P(1-\theta),$$

$$\tau^* = \frac{s^* \gamma_q^P n^P (1 - \upsilon n^P)}{\left(n^P - n^*_{\text{tax}}\right)^{\theta}}.$$

*We have $s^* > 0$, $n^*_{\text{tax}} < n^P$, and $\tau^* > 0$. Also, if $0 < \theta < 1$, then $n^*_{\text{tax}} > 0$; if $\theta = 1$, then $n^*_{\text{tax}} = 0$.*

It is interesting to note that the threshold number of births to face the birth tax depends on the value of $\theta$. When $\theta = 1$, this threshold number of children is equal to zero, and hence, there is a flat tax $T_t$ on all children. In this case, the total tax payment is proportional to the number of children. When $0 < \theta < 1$, this threshold number of children is positive and below the socially optimal rate of fertility. In this last case, the total tax payment is a more complicated function of the number of children. When the number of children rises just from the tax-free threshold level $n^*_{\text{tax}}$, there is a jump from a zero to a positive tax payment. Beyond this, a further rise in the number of children, which gains diminishing marginal utility, leads to a less-than-proportional rise in the tax payment for $\theta \in (0, 1)$. In our numerical experiment later, we will explore how the value of $\theta$ can determine the threshold number of children $n_{\text{tax}}$.

It is also interesting to compare the various forms of public population policies to a conventional education subsidy financed by income taxes in the presence of the human capital externality. In a similar model, Zhang and Casagrande (1998) focus on how an education subsidy financed by income taxes affects fertility and education investment without taking the human capital externality into account and without considering the welfare consequence. Here, with the human capital externality, we will also consider how such a policy affects welfare. In particular, we will make welfare comparisons between this conventional policy on the one hand and the various forms of population policies we have analyzed on the other.

### 3.6 Education subsidies financed by incomes taxes: a comparison

With an income tax at a rate $\tau$ and an education subsidy at a rate $s$, the household budget constraint becomes:

$$c_t = (1 - vn_t)(1 - \tau)H_t - (1 - s)q_t n_t, \tag{28}$$

and the government budget balance requires:

$$\tau(1 - vn_t)H_t = sn_t q_t.$$

The first-order condition with respect to education investment is:

$$\frac{(1 - s)n_t q_t}{\delta c_t H_{t+1}} = \frac{\alpha}{c_{t+1}}\left[(1 - vn_{t+1})(1 - \tau) + \frac{\beta(1 - s)(1 - \delta)n_{t+1}q_{t+1}}{\delta H_{t+1}}\right]. \tag{29}$$

The first-order condition with respect to the number of children is:

$$\frac{v(1 - \tau)H_t + (1 - s)q_t}{c_t} = \frac{\rho}{n_t}. \tag{30}$$

According to these first-order conditions, the income tax and the education subsidy have opposing effects on the net marginal benefits of both education investment and the number of children, respectively. The education subsidy tends to raise the rate of return on education investment, whereas the income tax does the opposite. Comparing the left-hand sides of both equations above, the education subsidy reduces the cost of education proportionately, while it reduces the cost of having a child less than proportionately. As a result, the education subsidy tends to tip the trade-off between the quality and the quantity of children towards the former. However, the income tax reduces the time cost of having a child, and hence tends to raise fertility.

The solution for fertility turns out to be the same as that in the competitive equilibrium without government intervention:

$$n^S = n^* = \frac{\rho\left[1 - \alpha\beta(1 - \delta) - \alpha\delta\right] - \alpha\delta}{v(1 + \rho)\left[1 - \alpha\beta(1 - \delta) - \alpha\delta\right]}, \tag{31}$$

where the superscript $S$ stands for this conventional education subsidy. That is, using an income tax to subsidize children's education has no effect on the fertility rate, which is consistent with empirical evidence from cross-country data in Zhang and Casagrande (1998).

Further, as expected, the education subsidy induces parents to spend less on consumption and more on children's education:

$$\gamma_c^s = \frac{(1-s)[1-\alpha\beta(1-\delta)-\alpha\delta]}{(1-s)[1-\alpha\beta(1-\delta)]+s\alpha\delta}, \tag{32}$$

$$n^s\gamma_q^s = \frac{\alpha\delta}{(1-s)[1-\alpha(1-\delta)]+s\alpha\delta}. \tag{33}$$

We now provide the optimal subsidy rate and the optimal income tax rate below:

**Proposition 5** *For* $0<\beta<1$, *the optimal education subsidy financed by an income tax is:*

$$s^* = \frac{\alpha(1-\delta)(1-\beta)}{1-\alpha\beta(1-\delta)-\alpha\delta}; \qquad \tau^* = \frac{\alpha^2\delta(1-\delta)(1-\beta)}{[1-\alpha(1-\delta)][1-\alpha\beta(1-\delta)-\alpha\delta]}.$$

*It improves welfare by raising the ratio of education investment to output. However, it has no effect on fertility. Thus, it cannot reach the first-best solution.*

*Proof* By Eq. 31, the fertility rate with the income tax and the education subsidy is the same as that without government intervention. In other words, fertility is independent of the subsidy/tax rates under a balanced government budget. Substituting the solution for $(n^s, \gamma_c^s, \gamma_q^s)$ in Eqs. 31–33 into the solution for welfare and maximizing welfare by choosing $s$, we can obtain the claimed optimal subsidy rate $s^*$. Substituting $s^*$ into Eqs. 32 and 33, we have the same proportional output allocations to consumption and to education as those in the social planner's solution:

$$\gamma_c^s = \gamma_c^P = \frac{1-\alpha}{1-\alpha(1-\delta)},$$

$$n^s\gamma_q^s = n^P\gamma_q^P = \frac{\alpha\delta}{1-\alpha(1-\delta)}.$$

Thus, the ratio of consumption to output here is less than that in the case without government intervention, while the ratio of total education spending to output is greater. However, because the fertility rate is greater than its social optimum, the resultant education spending per child as a fraction of output is lower than its social optimum. Thus, the income-tax-financed eduction subsidy cannot reach the first-best solution. Using the government budget balance plus the solution $n^s\gamma_q^s = \alpha\delta/[1-\alpha(1-\delta)]$ and $\gamma_c^s = (1-\alpha)/[1-\alpha(1-\delta)]$, we can then find $\tau^*$. □

Comparing all the different policies, the education subsidy financed by an income tax can increase education investment to its social optimum, while birth limits or the birth tax for lump-sum transfers can lower fertility to its social optimum. However, none of them can achieve both. It can be shown that birth limits or the birth tax for lump-sum transfers may or may not achieve higher welfare than the education subsidy financed by an income tax. The first-best policy option analyzed in this model is the birth tax for education subsidies. Finally, we present numerical results to illuminate the quantitative implications of this model.

### 3.7 Numerical results

For plausible parameterizations, it is interesting to look at whether the equilibrium solutions can differ significantly across all these cases in terms of fertility, education investment, economic growth and welfare. Since there is a wide range of possible values about the strength of the human capital externality according to macro and micro data in the literature, we vary $\beta$ widely from 0.1 to 0.9. The discount factor $\alpha$ is set at 0.75, which is plausible in an overlapping-generations model. Other parameters are chosen to generate realistic values of fertility and the growth rate. Also, we normalize initial human capital to unity, and regard the length of one period in this model as 20 years to convert the growth rate $\phi$ into its annual rate $g$.

The numerical results are presented in Table 1. The first column of Table 1 indicates the various cases: no government intervention, birth limits (or the birth tax for transfers), the conventional education subsidy through income taxes, and the birth tax for education subsidies (the social optimum). Among these cases, we regard the first-best birth tax for education subsidies as the benchmark case. From the second to the last column of Table 1, we give our numerical results for the fertility rate $n$, the ratio of education investment per child to output $\gamma_q$, the annual growth rate of output per worker $g$, the welfare level $V(0)$, the tax rate, the subsidy rate, and the equivalent payment, respectively. The equivalent payment measures the percentage change in income we should add to a nonbenchmark case in every period, $\mu Y_t = \mu(1 - vn)H_t$, so as to reach the same welfare level in the benchmark case:

$$V(0)^{\text{nonbenchmark}} + \frac{1}{1-\alpha}\ln(1+\mu) = V(0)^{\text{benchmark}}.$$

This is to add $\sum_{t=0}^{\infty} \alpha^t \ln(1+\mu) = [\ln(1+\mu)]/(1-\alpha)$ to the welfare solution in Eq. 11 for a nonbenchmark case. In other words, the equivalent payment indicates the gain in welfare in terms of a percentage change in income in every period when we move from a nonbenchmark case to the benchmark case.

According to our numerical results in Table 1, there are substantial differences in the fertility rate, the ratio of education spending per child to output per worker, and the growth rate of income per worker. For example, when the externality has medium strength with $\beta = 0.5$, the fertility rate is 1.111 in cases with birth limits or with birth taxes for either transfers or education subsidies, while it is 2.929 in cases without any population policy (education subsidies financed by income taxes or no intervention). Interestingly, the ratio of education spending per child to output and the growth rate of output per worker are much higher in the cases with birth limits or with birth taxes than in the case with the education subsidy financed by income taxes. Also,

**Table 1** Numerical results in the base model

| $\alpha = 0.75, \delta = 0.2, \rho = 0.8, \upsilon = 0.1, A = 2.2978, \theta = 0.5, H_0 = 1.0$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| Cases[a] | Fertility rate | Per child education/output | Growth rate (%) | Welfare level | Tax rate | Subsidy rate | Equivalent payment[b] (%) |
| | $\beta = 0.1$ | | | | | | |
| No government | 3.390 | 0.047 | 0.692 | 3.211 | – | – | 59.6 |
| Birth limit/birth tax | 1.111 | 0.144 | 2.124 | 4.214 | 0.31 | – | 24.2 |
| Income tax & subsidy | 3.390 | 0.111 | 1.556 | 4.077 | 0.26 | 0.68 | 28.5 |
| Birth tax & subsidy | 1.111 | 0.338 | 3.000 | 5.080 | 0.26 | 0.58 | Benchmark |
| | $\beta = 0.3$ | | | | | | |
| No government | 3.201 | 0.057 | 0.916 | 3.561 | – | – | 46.2 |
| Birth limit/birth tax | 1.111 | 0.165 | 2.263 | 4.429 | 0.28 | – | 17.7 |
| Income tax & subsidy | 3.201 | 0.117 | 1.643 | 4.212 | 0.24 | 0.63 | 24.2 |
| Birth tax & subsidy | 1.111 | 0.338 | 3.000 | 5.080 | 0.23 | 0.51 | Benchmark |
| | $\beta = 0.5$ | | | | | | |
| No government | 2.929 | 0.073 | 1.205 | 3.963 | – | – | 32.2 |
| Birth limit/birth tax | 1.111 | 0.193 | 2.425 | 4.652 | 0.23 | – | 11.3 |
| Income tax & subsidy | 2.929 | 0.128 | 1.773 | 4.391 | 0.21 | 0.55 | 18.8 |
| Birth tax & subsidy | 1.111 | 0.338 | 3.000 | 5.080 | 0.19 | 0.43 | Benchmark |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $\beta = 0.7$ | | | | | | |
| No government | 2.507 | 0.103 | 1.613 | 4.429 | – | – | 17.7 |
| Birth limit/birth tax | 1.111 | 0.233 | 2.618 | 4.871 | 0.17 | – | 5.4 |
| Income tax & subsidy | 2.507 | 0.150 | 1.991 | 4.638 | 0.16 | 0.42 | 11.7 |
| Birth tax & subsidy | 1.111 | 0.338 | 3.000 | 5.080 | 0.14 | 0.31 | Benchmark |
| | $\beta = 0.9$ | | | | | | |
| No government | 1.756 | 0.186 | 2.309 | 4.930 | – | – | 3.8 |
| Birth limit/birth tax | 1.111 | 0.294 | 2.856 | 5.046 | 0.07 | – | 0.9 |
| Income tax & subsidy | 1.756 | 0.214 | 2.452 | 4.964 | 0.07 | 0.19 | 2.9 |
| Birth tax & subsidy | 1.111 | 0.338 | 3.000 | 5.080 | 0.06 | 0.13 | Benchmark |

[a]"No government": competitive solution without government intervention; "birth limit/birth tax": competitive solutions with either the birth limit or the birth tax for lump-sum transfers; "income tax & subsidy": competitive solution with education subsidies financed by income taxes; "birth tax & subsidy": socially optimal competitive solution with birth taxes for education subsidies.

[b]"Equivalent payment": percentage change in income we should add to a nonbenchmark case in every period so as to reach the same welfare level in the first-best benchmark case.

the welfare level is higher with birth limits or with birth taxes than with the education subsidy financed by income taxes. Moreover, it is interesting to note that the first-best case (the birth tax for education subsidies) has a much higher ratio of education spending per child to output and a much higher growth rate than all the other cases. In particular, replacing birth limits by a birth tax for education subsidies can achieve a much higher growth rate of income and a much higher ratio of education spending per child to income but has the same fertility rate, suggesting a promising reform direction for the Chinese population policy. This observation suggests that there are large dynamic gains through internalizing the human capital externality.

Indeed, there are considerable gains in welfare from a nonbenchmark case to the benchmark in terms of percentage changes in income, particularly for small values of $\beta$ (strong human capital externalities). First, the gain in welfare by moving from the no government case to the benchmark case with birth taxes for education subsidies is equivalent to a 59.6% gain in income for $\beta = 0.1$, or a 32% gain in income for $\beta = 0.5$. Second, the gain in welfare by moving from the birth-limit case to the benchmark is equivalent to a 24.2% gain in income for $\beta = 0.1$ or an 11.3% gain in income for $\beta = 0.5$. Third, the gain in welfare by moving from the conventional education subsidy through income taxes to the benchmark is equivalent to a 28% gain in income for $\beta = 0.1$ or an 18.8% gain in income for $\beta = 0.5$. Even for $\beta$ at a value abound 0.7 according to the macro data in the literature mentioned earlier, the welfare gains are still very large in Table 1.

In the case of the first-best birth tax for education subsidies, what is the quantitative relationship between the threshold number of children to face the birth tax and the parameter $\theta$ in the tax function? In Proposition 4, the value of $\theta$ does not affect the equilibrium solution for income allocations, fertility, and the optimal subsidy rate, provided that we have $1 \geq \theta > 0$. The relevance of the value of $\theta$ is to determine an implementable threshold $n_{\text{tax}} \geq 1/2$ in this single-gender model, together with the tax rate. From the same parameterization as that in Table 1, we set $\theta = 0.1$, 0.2, 0.3, 0.4, and 0.5, respectively. The corresponding values of $n^*_{\text{tax}}$ are 1.00, 0.89, 0.78, 0.67, and 0.56, all of which are above 1/2 and, hence, implementable. For larger values of $\theta$, the corresponding values of $n^*_{\text{tax}}$ are below 1/2 and, hence, are not implementable. The tax rate in the benchmark case with the birth tax for education subsidies varies from over 20% of average output to below 10%, increasing with the threshold level of fertility and with the strength of the externality.

## 4 Sensitivity analysis

In the base model, the assumption of the log preferences restricts the elasticity of intertemporal substitution to unity. Though this has helped greatly in deriving analytical results with desired clarity in the previous sections, it may lead to unrealistic results. We now explore how sensitive the results are to variations in the value of this elasticity parameter, assuming a more general utility function below:

$$V(H_0) = \sum_{t=0}^{\infty} \alpha^t \frac{\left(c_t^{1-\rho} n_t^{\rho}\right)^{\sigma}}{\sigma}, \quad \sigma < 1, \quad \sigma \neq 0. \tag{34}$$

For $\sigma = 0$, we assume a log utility function, a case we have analyzed in the previous sections. Letting $\sigma = 1 - \eta$ with $\eta > 0$, $1/\eta$ is the measure of the elasticity of intertemporal substitution across generations in this dynastic family model. The log utility assumed at $\sigma = 0$ corresponds to $\eta = 1$ or $1/\eta = 1$. In the macroeconomic applications of intertemporal models, a range of the elasticity of intertemporal substitution in $[1/2, 1)$, i.e., $0 > \sigma \geq -1$ or $2 \geq \eta > 1$, is widely accepted; see, e.g., Greenwood et al. (1988). However, it measures elasticities of intertemporal substitution across periods in lifetime rather than across generations in a dynasty. In a dynastic family model of Becker, Murphy, and Tamura with $u(c) = c^{\sigma}/\sigma$, the elasticity of intertemporal substitution is restricted in $(1, \infty)$ under their assumption $0 < \sigma < 1$. Since there is little consensus in the literature about the exact size of this elasticity parameter across generations, we shall vary it from below to above unity in the sensitivity analysis.

With nonunitary elasticities of intertemporal substitution, the model has only implicit solutions. For example, in the case without government intervention, the implicit solution for fertility is given by

$$(\rho - vn)^{1-\sigma\delta(1-\rho)} n^{\sigma\delta(1-\rho)} / A^{\sigma(1-\rho)} = \alpha[\delta(1 - vn) + \beta(1 - \delta)(\rho - vn)].$$

One can then solve for the proportional allocations $\gamma_q = (\rho - vn)/[n(1 - vn)]$ and $\gamma_c = (1 - \rho)/(1 - vn)$ and the growth rate $\phi = H_{t+1}/H_t - 1 = A[\gamma_q(1 - vn)]^{\delta}$, and find the implicit solution for the welfare level:

$$V(H_0) = \sum_{t=0}^{\infty} \alpha^t \frac{\left(c_t^{1-\rho} n^{\rho}\right)^{\sigma}}{\sigma} = \frac{[(1 - \rho)H_0]^{\sigma(1-\rho)} n^{\sigma\rho}}{\sigma[1 - \alpha(1 + \phi)^{\sigma(1-\rho)}]}.$$

See Appendix C for derivations. In cases with the various government policies such as the birth limit, the birth tax, or education subsidies, one can derive implicit solutions as functions of government policy variables. Additionally, one can then differentiate the implicit solution for the welfare level with respect to each government policy variable so as to maximize social welfare. However, it is difficult to make analytical comparisons across different cases with different government policies by using these implicit solutions. Thus, we use numerical methods in this section for sensitivity analysis. In computing the equivalent payment between a nonbenchmark case and the benchmark case, the formula changes into

$$V(0)^{\text{nonbenchmark}} (1 + \mu)^{\sigma(1-\rho)} = V(0)^{\text{benchmark}}$$

according to the more general utility function in this section.

In Tables 2–4, we report numerical results with $\sigma = 0.2$, $\sigma = -0.25$, and $\sigma = -1.0$, respectively. With $\sigma = 0.2$ in Table 2, the elasticity of intertemporal substitution exceeds unity that was used in the base model of Table 1. The potential welfare gains from government policies measured by the equivalent payments in Table 2 are much larger than in Table 1. The intuition lies in the mechanism of the model: the trade-off between the quantity of children and parental consumption today on the one hand and the welfare of children tomorrow via human capital investment on the other hand. The stronger willingness of intertemporal or intergenerational substitution means a larger efficiency loss caused by externalities and a larger response by parents to government policies. Conversely, with $\sigma = -0.25$ in Table 3, the elasticity of intertemporal substitution falls below unity and the potential welfare gains become smaller than in Table 1 but remain substantial by size. With $\sigma = -1.0$ in Table 4, the elasticity of intertemporal substitution falls further, and so do the potential welfare gains.

Among these results differentiated by the elasticity value, one would like to know which one is a plausible or implausible parameterization. The answer may hinge on the gap in fertility rates between cases with or without government intervention in comparison with those observed in different development regimes in the real world. In other words, government intervention may aim at closing the gap in fertility between developed and developing countries. This needs fertility to fall by half or more. Accordingly, the growth rate should rise substantially. Quantitatively, the results in Table 4 with $\sigma = -1.0$ fail to generate the needed significant decline in fertility since even the biggest gap in fertility is about 30%, which occurs at the extreme value $\beta = 0.1$. The room for improvement in the growth rate is also very limited as well in Table 4. We may thus regard $\sigma = -1.0$ as implausible. By contrast, the results with $\sigma = 0.2$ in Table 2 or $\sigma = -0.25$ in Table 3 provide the needed room for a 50% decline in the fertility rate and a large increase in the growth rate. If we take $\sigma \in [-0.25, 0.2]$ as the plausible range, then the result that government policy can achieve a substantial welfare gain tends to be robust.

In addition, the sensitivity analysis can help check whether other results in the base model with the log preferences are robust. From Tables 2–4, the birth limit and the birth tax for lump-sum transfers yield almost identical results, supporting the claim in the base model in Proposition 3. Under the two types of policies, the fertility rate is no longer equal to the fertility rate chosen by the social planner compared to the result in Proposition 1. However, the fertility rate under these policies is much closer to the social planner solution compared to the competitive solution without government intervention. Further, the income-tax-funded education subsidies have small effects on fertility, which only differs slightly from the claim of a zero effect of this policy on fertility in Proposition 5. In Appendix C, we also provide the first-best birth tax and the first-best education subsidy rate that are similar to those in Proposition 4.

**Table 2** Numerical results: sensitivity analysis $\sigma = 0.2$

| | $\alpha = 0.73, \delta = 0.14, \rho = 0.47, v = 0.1, A = 3.8, \theta = 0.5, H_0 = 1.0$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Cases[a] | Fertility rate | Per child education/ output | Growth rate (%) | Welfare level | Tax rate | Subsidy rate | Equivalent payment[b] (%) |
| | $\beta = 0.1$ | | | | | | |
| No government | 3.984 | 0.030 | 3.94 | 25.627 | – | – | 80.8 |
| Birth limit | 1.543 | 0.079 | 4.89 | 26.261 | – | – | 43.6 |
| Birth tax | 1.542 | 0.079 | 4.90 | 26.261 | 0.26 | – | 43.6 |
| Income tax & subsidy | 3.969 | 0.093 | 4.77 | 26.452 | 0.28 | 0.76 | 34.1 |
| Birth tax & subsidy | 1.176 | 0.340 | 6.00 | 27.288 | 0.41 | 0.69 | Benchmark |
| | $\beta = 0.3$ | | | | | | |
| No government | 3.837 | 0.037 | 4.10 | 25.865 | – | – | 65.7 |
| Birth limit | 1.489 | 0.096 | 5.05 | 26.465 | – | – | 33.5 |
| Birth tax | 1.486 | 0.097 | 5.05 | 26.465 | 0.25 | – | 33.5 |
| Income tax & subsidy | 3.819 | 0.097 | 4.82 | 26.528 | 0.27 | 0.72 | 30.5 |
| Birth tax & subsidy | 1.176 | 0.340 | 6.00 | 27.288 | 0.38 | 0.63 | Benchmark |
| | $\beta = 0.5$ | | | | | | |
| No government | 3.613 | 0.047 | 4.31 | 26.158 | – | – | 49.0 |
| Birth limit | 1.421 | 0.123 | 5.23 | 26.697 | – | – | 22.5 |
| Birth tax | 1.415 | 0.124 | 5.24 | 26.697 | 0.24 | – | 22.5 |
| Income tax & subsidy | 3.589 | 0.104 | 4.89 | 26.639 | 0.24 | 0.65 | 25.5 |
| Birth tax & subsidy | 1.176 | 0.340 | 6.00 | 27.288 | 0.33 | 0.55 | Benchmark |

**Table 2** continued

| Cases[a] | Fertility rate | Per child education/ output | Growth rate (%) | Welfare level | Tax rate | Subsidy rate | Equivalent payment[b] (%) |
|---|---|---|---|---|---|---|---|
| | $\alpha = 0.73$, $\delta = 0.14$, $\rho = 0.47$, $v = 0.1$, $A = 3.8$, $\theta = 0.5$, $H_0 = 1.0$ | | | | | | |
| | $\beta = 0.7$ | | | | | | |
| No government | 3.224 | 0.068 | 4.62 | 26.536 | – | – | 30.2 |
| Birth limit | 1.332 | 0.169 | 5.48 | 26.959 | – | – | 12.1 |
| Birth tax | 1.324 | 0.170 | 5.48 | 26.959 | 0.20 | – | 12.1 |
| Income tax & subsidy | 3.193 | 0.118 | 5.03 | 26.811 | 0.20 | 0.53 | 18.1 |
| Birth tax & subsidy | 1.176 | 0.340 | 6.00 | 27.288 | 0.25 | 0.42 | Benchmark |
| | $\beta = 0.9$ | | | | | | |
| No government | 2.356 | 0.130 | 5.19 | 27.045 | – | – | 8.81 |
| Birth limit | 1.218 | 0.260 | 5.80 | 27.221 | – | – | 2.35 |
| Birth tax | 1.212 | 0.262 | 5.81 | 27.221 | 0.11 | – | 2.35 |
| Income tax & subsidy | 2.321 | 0.167 | 5.38 | 27.111 | 0.11 | 0.29 | 6.33 |
| Birth tax & subsidy | 1.176 | 0.340 | 6.00 | 27.288 | 0.12 | 0.20 | Benchmark |

[a] "No government": competitive solution without government intervention; "birth limit" or "birth tax": competitive solutions with either the birth limit or the birth tax for lump-sum transfers; "income tax & subsidy": competitive solution with education subsidies financed by income taxes; "birth tax & subsidy": socially optimal competitive solution with birth taxes for education subsidies.

[b] "Equivalent payment": percentage change in income we should add to a nonbenchmark case in every period so as to reach the same welfare level in the first-best benchmark case.

**Table 3** Numerical results: sensitivity analysis $\sigma = -0.25$

| | $\alpha = 0.78, \delta = 0.14, \rho = 0.32, v = 0.102, A = 2.5, \theta = 0.5, H_0 = 1.0$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Cases[a] | Fertility rate | Per child education/ output | Growth rate (%) | Welfare level | Tax rate | Subsidy rate | Equivalent payment[b] (%) |
| | $\beta = 0.1$ | | | | | | |
| No government | 2.331 | 0.046 | 2.27 | −14.387 | – | – | 23.9 |
| Birth limit | 0.943 | 0.111 | 3.02 | −14.122 | – | – | 11.0 |
| Birth tax | 0.943 | 0.111 | 3.02 | −14.122 | 0.20 | – | 11.0 |
| Income tax & subsidy | 2.350 | 0.107 | 2.87 | −14.107 | 0.16 | 0.65 | 10.3 |
| Birth tax & subsidy | 1.032 | 0.233 | 3.54 | −13.873 | 0.24 | 0.57 | Benchmark |
| | $\beta = 0.3$ | | | | | | |
| No government | 2.197 | 0.056 | 2.42 | −14.279 | – | – | 18.5 |
| Birth limit | 0.964 | 0.125 | 3.10 | −14.056 | – | – | 8.0 |
| Birth tax | 0.964 | 0.125 | 3.10 | −14.056 | 0.17 | – | 8.0 |
| Income tax & subsidy | 2.218 | 0.113 | 2.92 | −14.069 | 0.15 | 0.59 | 8.6 |
| Birth tax & subsidy | 1.032 | 0.233 | 3.54 | −13.873 | 0.21 | 0.51 | Benchmark |
| | $\beta = 0.5$ | | | | | | |
| No government | 2.012 | 0.072 | 2.61 | −14.150 | – | – | 12.3 |
| Birth limit | 0.986 | 0.143 | 3.19 | −13.989 | – | – | 5.0 |
| Birth tax | 0.986 | 0.143 | 3.19 | −13.989 | 0.14 | – | 5.0 |
| Income tax & subsidy | 2.036 | 0.122 | 2.99 | −14.021 | 0.13 | 0.50 | 6.4 |
| Birth tax & subsidy | 1.032 | 0.233 | 3.54 | −13.873 | 0.18 | 0.42 | Benchmark |

**Table 3** continued

| | $\alpha = 0.78, \delta = 0.14, \rho = 0.32, v = 0.102, A = 2.5, \theta = 0.5, H_0 = 1.0$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Cases[a] | Fertility rate | Per child education/ output | Growth rate (%) | Welfare level | Tax rate | Subsidy rate | Equivalent payment[b] (%) |
| | $\beta = 0.7$ | | | | | | |
| No government | 1.743 | 0.099 | 2.87 | −14.016 | – | – | 6.22 |
| Birth limit | 1.009 | 0.167 | 3.31 | −13.927 | – | – | 2.3 |
| Birth tax | 1.009 | 0.167 | 3.31 | −13.927 | 0.10 | – | 2.3 |
| Income tax & subsidy | 1.766 | 0.139 | 3.11 | −13.959 | 0.09 | 0.37 | 3.7 |
| Birth tax & subsidy | 1.032 | 0.233 | 3.54 | −13.873 | 0.13 | 0.31 | Benchmark |
| | $\beta = 0.9$ | | | | | | |
| No government | 1.327 | 0.161 | 3.25 | −13.898 | – | – | 1.10 |
| Birth limit | 1.028 | 0.205 | 3.45 | −13.881 | – | – | 0.34 |
| Birth tax | 1.028 | 0.205 | 3.45 | −13.881 | 0.04 | – | 0.34 |
| Income tax & subsidy | 1.340 | 0.180 | 3.33 | −13.891 | 0.04 | 0.15 | 0.77 |
| Birth tax & subsidy | 1.032 | 0.233 | 3.54 | −13.873 | 0.05 | 0.13 | Benchmark |

[a] "No government": competitive solution without government intervention; "birth limit" or "birth tax": competitive solutions with either the birth limit or the birth tax for lump-sum transfers; "income tax & subsidy": competitive solution with education subsidies financed by income taxes; "birth tax & subsidy": socially optimal competitive solution with birth taxes for education subsidies.

[b] "Equivalent payment": percentage change in income we should add to a nonbenchmark case in every period so as to reach the same welfare level in the first-best benchmark case.

**Table 4** Numerical results: sensitivity analysis $\sigma = -1.0$

| | $\alpha = 0.78, \delta = 0.14, \rho = 0.30, v = 0.13, A = 2.0, \theta = 0.5, H_0 = 1.0$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Cases[a] | Fertility rate | Per child education/ output | Growth rate (%) | Welfare level | Tax rate | Subsidy rate | Equivalent payment[b] (%) |
| | $\beta = 0.1$ | | | | | | |
| No government | 1.730 | 0.056 | 1.28 | −3.138 | – | – | 8.07 |
| Birth limit | 1.126 | 0.082 | 1.62 | −3.062 | – | – | 4.35 |
| Birth tax | 1.127 | 0.082 | 1.62 | −3.062 | 0.10 | – | 4.35 |
| Income tax & subsidy | 1.768 | 0.102 | 1.70 | −3.030 | 0.10 | 0.54 | 2.80 |
| Birth tax & subsidy | 1.213 | 0.139 | 1.99 | −2.972 | 0.11 | 0.48 | Benchmark |
| | $\beta = 0.3$ | | | | | | |
| No government | 1.657 | 0.065 | 1.39 | −3.087 | – | – | 5.57 |
| Birth limit | 1.150 | 0.090 | 1.68 | −3.034 | – | – | 2.99 |
| Birth tax | 1.151 | 0.090 | 1.68 | −3.034 | 0.08 | – | 2.99 |
| Income tax & subsidy | 1.696 | 0.105 | 1.73 | −3.016 | 0.08 | 0.47 | 2.12 |
| Birth tax & subsidy | 1.213 | 0.139 | 1.99 | −2.972 | 0.10 | 0.42 | Benchmark |
| | $\beta = 0.5$ | | | | | | |
| No government | 1.565 | 0.077 | 1.53 | −3.040 | – | – | 3.29 |
| Birth limit | 1.173 | 0.099 | 1.75 | −3.008 | – | – | 1.74 |
| Birth tax | 1.175 | 0.099 | 1.75 | −3.008 | 0.06 | – | 1.74 |
| Income tax & subsidy | 1.602 | 0.109 | 1.77 | −3.002 | 0.07 | 0.38 | 1.45 |
| Birth tax & subsidy | 1.213 | 0.139 | 1.99 | −2.972 | 0.08 | 0.34 | Benchmark |

**Table 4** continued

| | $\alpha = 0.78$, $\delta = 0.14$, $\rho = 0.30$, $\upsilon = 0.13$, $A = 2.0$, $\theta = 0.5$, $H_0 = 1.0$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Cases[a] | Fertility rate | Per child education/ output | Growth rate (%) | Welfare level | Tax rate | Subsidy rate | Equivalent payment[b] (%) |
| | $\beta = 0.7$ | | | | | | |
| No government | 1.449 | 0.095 | 1.69 | −3.001 | – | – | 1.40 |
| Birth limit | 1.194 | 0.112 | 1.83 | −2.987 | – | – | 0.72 |
| Birth tax | 1.196 | 0.112 | 1.83 | −2.987 | 0.04 | – | 0.72 |
| Income tax & subsidy | 1.479 | 0.116 | 1.83 | −2.986 | 0.20 | 0.53 | 0.67 |
| Birth tax & subsidy | 1.213 | 0.139 | 1.99 | −2.972 | 0.06 | 0.24 | Benchmark |
| | $\beta = 0.9$ | | | | | | |
| No government | 1.301 | 0.121 | 1.88 | −2.975 | – | – | 0.14 |
| Birth limit | 1.209 | 0.129 | 1.93 | −2.974 | – | – | 0.10 |
| Birth tax | 1.210 | 0.128 | 1.93 | −2.974 | 0.01 | – | 0.10 |
| Income tax & subsidy | 1.314 | 0.129 | 1.92 | −2.974 | 0.02 | 0.10 | 0.10 |
| Birth tax & subsidy | 1.213 | 0.139 | 1.99 | −2.972 | 0.02 | 0.09 | Benchmark |

[a]"No government": competitive solution without government intervention; "birth limit" or "birth tax": competitive solutions with either the birth limit or the birth tax for lump-sum transfers; "income tax & subsidy": competitive solution with education subsidies financed by income taxes; "birth tax & subsidy": socially optimal competitive solution with birth taxes for education subsidies.

[b]"Equivalent payment": percentage change in income we should add to a nonbenchmark case in every period so as to reach the same welfare level in the first-best benchmark case.

## 5 Policy implications and applications

We now apply our model to China's population policies, especially the "one-child" policy. When the Chinese government started family planning programs in the 1960s, it attempted mainly to educate the population to have later marriages, fewer children, and longer intervals between births. As a result, there was a moderate decline in the birth rate of the urban population. However, the birth rate of the rural population, which accounted for 80% of the total population, remained high. Under mounting population pressure in both urban and rural areas, the Chinese government introduced a strict population policy using quotas to limit the size of families in 1979. There are basically three categories of quotas: urban parents cannot have more than one child, rural parents can have a second child if the first one is a girl, and ethnic minority groups can have more than two children. Based on the quotas and the shares of these respective population groups in the total population, the average "policy-targeted" fertility rate of China is approximately 1.5 per woman, which is much lower than the replacement fertility rate; see Guo et al. (2003). Since then, the average total fertility rate in China fell to 1.89 in 2000, as mentioned earlier.[4] This fertility rate is close to those in developed countries.

As was observed in industrialized countries, the dramatic decline in fertility has been accompanied by remarkable economic growth in China. The GDP growth rate has been, on average, over 8% per year since 1978, and the growth rate of GDP per capita has been over 7%. The sharp decline in birth rates appears to have contributed, among other factors, to such remarkable per capita GDP growth, as our model suggests.

As opposed to birth limits by the government, our model provides an alternative policy combination: birth taxes and education subsidies. The birth tax raises the cost of having children, while the education subsidy reduces the cost of education. Consequently, blending them together can achieve an ideal balance between the number and the education of children. It leads to more education investment, faster economic growth and the same fertility rate, compared to the case with birth limits. If the human capital externality is indeed a key factor causing too many children and too little education in poor countries, this alternative policy also leads to higher social welfare than the birth-limit policy does. Indeed, the Chinese government has recently relaxed its birth-limit policy by allowing a couple that has only one child in the family to pay for a second or third birth, a partial move toward the birth tax. A more complete transition to the first-best birth tax will include phasing out the mandatory birth quotas and spending the birth tax revenue on education and health care for all children. Also, when the first birth is tax-free, as in China,

[4] Scotese and Wang (1995) suggest that birth limits may not have a persistent effect unless there is a change in the preferences for fewer children. Whyte and Gu (1987) argue that most Chinese parents still prefer having at least two children to having just one, despite the sharp drop in fertility as a result of the government's one-child policy.

we call for more caution in determining the tax payment for each additional birth. As described in Proposition 4, in order to achieve the first-best outcome, a further rise in the number of children exceeding the threshold level requires a less-than-proportional rise in birth tax payments per family.

## 6 Conclusion

In this paper, with a trade-off between the quantity and the quality of children in an endogenous growth model, we have studied why developing countries have many children and little education and what policies governments can use to tip the trade-off toward the quality of children for economic growth and social welfare. In the presence of a human capital externality, we have shown that education investment and the growth rate of income are lower and fertility is higher in the competitive solution than their socially optimal levels, a highly relevant situation in developing countries. Concerning government policies, we have shown that, although a birth-limit policy can improve welfare by reducing fertility, it cannot raise education investment to its social optimum. Collecting a birth tax for lump-sum transfers achieves a very similar equilibrium solution to that with the birth-limit policy. Taxing income to subsidize education investment improves welfare by promoting education investment and economic growth but cannot reduce fertility to its social optimum. In addition, the growth rate of output with this conventional education subsidy through income taxes is lower than the socially optimal rate and may be lower than the level with birth limits or birth taxes for lump-sum income transfers. Such comparisons across the various policies are interesting and not obvious at all without thorough analysis. The most desirable policy in this model is to tax births and use the tax revenue to subsidize education.

Concerning policy implications, our results offer an economic explanation for China's move from strict birth limits to birth taxes and education subsidies. Our numerical results indicate that such a move can achieve a much higher growth rate of per capita income and a much higher ratio of education spending to income. We have also paid careful attention to relevant features of the birth tax structure concerning how to design an implementable threshold number of children to start paying the tax. Numerically, the welfare gain of such a policy change is found to be very large.

Given the modest purpose and limited focus in this paper, our model is not intended to be a theory to explain the demographic transition for economic growth. That is, our model is not intended to explain the switch between development regimes without government policies. Nevertheless, our results do suggest an active role for governments in developing countries to reduce fertility and raise growth rates.

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## Appendix A

*Proof of Lemma 1* The derivation of the sufficient condition involves a few steps. First, we reformulate the household problem as a choice of proportional allocations and then solve it again by the guess-and-verify approach. The resultant welfare function of the household will be used to derive the sufficient condition. We guess that, at time $t$, the value function takes the form $V(H_t, \bar{H}_t) = E + \Psi_1 \ln H_t + \Psi_2 \ln \bar{H}_t$, where $E$, $\Psi_1$, and $\Psi_2$ are parameters to be determined. In this reformulation, we distinguish between individual and economy-wide average human capital. We substitute it into the Bellman equation with $\gamma_{ct} = 1 - n_t\gamma_{qt}$, with $H_{t+1}$ linked to $H_t$ and $\bar{H}_t$ in Eq. 2, and with $\bar{H}_{t+1}/\bar{H}_t = A(\gamma_q^*)^\delta(1 - vn^*)^\delta$ in Eq. 10:

$$\begin{aligned}
E + \Psi_1 \ln H_t + \Psi_2 \ln \bar{H}_t &= \ln \gamma_{ct}(1 - vn_t)H_t + \rho \ln n_t \\
&\quad + \alpha\left(E + \Psi_1 \ln H_{t+1} + \Psi_2 \ln \bar{H}_{t+1}\right) \\
&= \ln(1 - n_t\gamma_{qt}) + \ln(1 - vn_t) + \ln H_t + \rho \ln n_t + \alpha E \\
&\quad + \alpha\Psi_1\big[\ln A + \delta \ln \gamma_{qt}(1 - vn_t) + \delta \ln H_t \\
&\qquad + \beta(1-\delta)\ln H_t + (1-\beta)(1-\delta)\ln \bar{H}_t\big] \\
&\quad + \alpha\Psi_2\big[\ln A + \delta \ln \gamma_q^*(1 - vn^*) + \ln \bar{H}_t\big]
\end{aligned}$$

where $\gamma_q^*$ and $n^*$ are decisions made in the economy as a whole and thus taken as given by the individual household we focus on. We thus have

$$\Psi_1 = \frac{1}{1 - \alpha\delta - \alpha\beta(1-\delta)}$$

$$\Psi_2 = \frac{\alpha\Psi_1(1-\beta)(1-\delta)}{1-\alpha}$$

$$\begin{aligned}
E &= \frac{1}{1-\alpha}[\ln(1 - n_t\gamma_{qt}) + \rho \ln n_t + \ln(1 - vn_t)] \\
&\quad + \frac{\alpha\delta\Psi_1}{1-\alpha}[\ln \gamma_{qt} + \ln(1 - vn_t)] + \frac{\alpha\delta\Psi_2}{1-\alpha}\big[\ln \gamma_q^* + \ln(1 - vn^*)].
\end{aligned}$$

Clearly, $\Psi_1$ and $\Psi_2$ are coefficients with $\Psi_1 + \Psi_2 = 1/(1-\alpha)$. By contrast, $E$ is a function of individual variables $n_t$ and $\gamma_{qt}$ and economy-wide variables $n^*$ and $\gamma_q^*$. The sufficient condition of the solution to the individual's problem takes economy-wide variables as given. It proceeds as follows: The first-order conditions are

$$E_n \equiv \frac{\partial E}{\partial n_t} = \frac{1}{1-\alpha}\left(\frac{-\gamma_{qt}}{1 - n_t\gamma_{qt}} + \frac{\rho}{n_t} - \frac{v}{1 - vn_t} - \frac{\alpha\delta\Psi_1 v}{1 - vn_t}\right) = 0,$$

$$E_q \equiv \frac{\partial E}{\partial \gamma_{qt}} = \frac{1}{1-\alpha}\left(\frac{-n_t}{1 - n_t\gamma_{qt}} + \frac{\alpha\delta\Psi_1}{\gamma_{qt}}\right) = 0.$$

It is easy to verify that these first-order conditions lead to the same solutions for $n_t$ and $\gamma_{qt}$ as in Eqs. 8 and 9. The second derivatives are thus as below:

$$E_{nn} = \frac{-1}{1-\alpha}\left[\frac{\gamma_{qt}^2}{(1-n_t\gamma_{qt})^2} + \frac{\rho}{n_t^2} + \frac{v^2}{(1-vn_t)^2} + \frac{\alpha\delta\Psi_1 v^2}{(1-vn_t)^2}\right] < 0,$$

$$E_{qq} = \frac{-1}{1-\alpha}\left[\frac{n_t^2}{(1-n_t\gamma_{qt})^2} + \frac{\alpha\delta\Psi_1}{\gamma_{qt}^2}\right] < 0,$$

$$E_{nq} = \frac{-1}{1-\alpha}\left[\frac{1}{1-n_t\gamma_{qt}} + \frac{n_t\gamma_{qt}}{(1-n_t\gamma_{qt})^2}\right] < 0.$$

The first principal minor of the Hessian matrix is negative. The second principal minor is signed below:

$$\begin{aligned} E_{nn}E_{qq} - E_{nq}^2 = {} & \frac{\alpha\delta\Psi_1 + \rho}{(1-\alpha)^2(1-n_t\gamma_{qt})^2} + \frac{\rho\alpha\delta\Psi_1}{(1-\alpha)^2 n_t^2\gamma_{qt}^2} \\ & + \frac{(1+\alpha\delta\Psi_1)v^2 n_t^2}{(1-\alpha)^2(1-vn_t)^2(1-n_t\gamma_{qt})^2} + \frac{\alpha\delta\Psi_1(1+\alpha\delta\Psi_1)v^2}{(1-\alpha)^2(1-vn_t)^2\gamma_{qt}^2} \\ & - \frac{1}{(1-\alpha)^2(1-n_t\gamma_{qt})^2} - \frac{2n_t\gamma_{qt}}{(1-\alpha)^2(1-n_t\gamma_{qt})^2}. \end{aligned}$$

Using the solution of $n_t$ for $vn/(1-vn)$ and linking $n\gamma_q$ to $(1-n\gamma_q)$ via $E_q = 0$, we can rearrange the second principal minor as

$$\begin{aligned} & \frac{[1-\alpha\beta(1-\delta)]\{\rho[1-\alpha\delta-\alpha\beta(1-\delta)]-\alpha\delta\}}{(1-\alpha)^2(1-vn_t)^2[1-\alpha\delta-\alpha\beta(1-\delta)][1-\alpha\beta(1-\delta)]^2\alpha\delta}\{[1-\alpha\beta(1-\delta)]^2 \\ & \quad + \{\rho[1-\alpha\delta-\alpha\beta(1-\delta)]-\alpha\delta\}[2-\alpha\delta-\alpha\beta(1-\delta)]\} \end{aligned}$$

whose sign is positive if and only if $\rho[1-\alpha\delta-\alpha\beta(1-\delta)]-\alpha\delta > 0$. This condition implies a negative semidefinite Hessian matrix. □

## Appendix B

*Proof of Proposition 4* Substituting the solution $(\gamma_c^E, \gamma_q^E, n^E)$ into Eq. 11, the first-order condition with respect to $\tau$ is given by:

$$\frac{\partial V^E}{\partial \tau} = \left\{\frac{\rho(1-\alpha)-\alpha\delta}{n^E(1-\alpha)^2} - \frac{[1-\alpha(1-\delta)]v}{(1-\alpha)^2(1-vn^E)}\right\}\frac{\partial n^E}{\partial \tau} = 0,$$

which leads to $n^E = n^P = [\rho(1-\alpha)-\alpha\delta]/[v(1+\rho)(1-\alpha)]$ since $\partial n^E/\partial\tau \neq 0$ according to Eq. 27.

The first-order condition with respect to $s$ is:

$$\frac{\partial V^E}{\partial s} = \frac{-[1-\alpha\beta(1-\delta)]}{(1-\alpha)\{(1-s)[1-\alpha\beta(1-\delta)]-\alpha\delta\}} + \frac{1-\alpha(1-\delta)}{(1-s)(1-\alpha)^2} + \left\{\frac{\rho(1-\alpha)-\alpha\delta}{(1-\alpha)^2 n^E} - \frac{[1-\alpha(1-\delta)]v}{(1-\alpha)^2(1-vn^E)}\right\}\frac{\partial n^E}{\partial s} = 0,$$

where the large coefficient on $\partial n^E/\partial s$ is equal to zero from the above first-order condition $\partial V^E/\partial\tau = 0$. Thus, these two first-order conditions above imply the claimed value of $s^*$. Substituting the solution for $s^*$ into the solution for $(\gamma_c^E, \gamma_q^E)$ in Eqs. 25 and 26 yields $(\gamma_c^E, \gamma_q^E) = (\gamma_c^P, \gamma_q^P)$. Consequently, the growth rate and welfare will be the same as those in the social planner solution. The remaining task is to find $\tau^*$ and $n^*_{\text{tax}}$.

From the government budget constraint and $(n^E, \gamma_c^E, \gamma_q^E) = (n^P, \gamma_c^P, \gamma_q^P)$, we have

$$s^*\gamma_q^P n^P(1-vn^P) = \left(n^P - n_{\text{tax}}\right)^\theta \tau.$$

Combining this with Eq. 27 gives

$$n^*_{\text{tax}} = \frac{n^P\left[\rho\gamma_c^P(1-vn^P) - vn^P - (1-s^*+\theta s^*)\gamma_q^P n^P(1-vn^P)\right]}{\rho\gamma_c^P(1-vn^P) - vn^P - (1-s^*)\gamma_q^P n^P(1-vn^P)}$$

$$= n^P\left[1 - \frac{\theta s^*\gamma_q^P n^P(1-vn^P)}{\rho\gamma_c^P(1-vn^P) - vn^P - (1-s^*)\gamma_q^P n^P(1-vn^P)}\right].$$

Substitute the solutions for $s^*$, $n^P$, $\gamma_c^P$ and $\gamma_q^P$ into the following ratio and simplify:

$$\frac{s^*\gamma_q^P n^P(1-vn^P)}{\rho\gamma_c^P(1-vn^P) - vn^P - (1-s^*)\gamma_q^P n^P(1-vn^P)} = 1.$$

Substituting this back into the expression for $n^*_{\text{tax}}$ leads to

$$n^*_{\text{tax}} = n^P(1-\theta).$$

Thus, $n^*_{\text{tax}} = 0$ if $\theta = 1$; $n^*_{\text{tax}} > 0$ if $\theta < 1$. From the government budget constraint and the solution for $(s^*, n^*_{\text{tax}})$, we have $\tau^* = s^*\gamma_q^P n^P(1-vn^P)/(n^P - n^*_{\text{tax}})^\theta$. □

## Appendix C

*Derivation of solution with the CES utility function.* Consider the case without government intervention. The Bellman equation changes into

$$V(H_t) = \max_{n_t, H_{t+1}}\left\{\left[(1-vn_t)H_t - n_t\left(A^{-1}H_{t+1}\right)^{1/\delta} \times \left(H_t^\beta \bar{H}_t^{1-\beta}\right)^{-(1-\delta)/\delta}\right]^{\sigma(1-\rho)} n_t^{\sigma\rho}/\sigma + \alpha V(H_{t+1})\right\}. \quad (35)$$

Differentiating Eq. 35 with respect to $H_t$ and $H_{t+1}$, respectively, and canceling common factors gives

$$n_t q_t c_t^{\sigma(1-\rho)-1} n_t^{\sigma\rho} = \alpha[\delta(1 - vn_{t+1})H_{t+1} + \beta(1-\delta)n_{t+1}q_{t+1}]c_{t+1}^{\sigma(1-\rho)-1} n_{t+1}^{\sigma\rho}. \quad (36)$$

Differentiating Eq. 35 with respect to $n_t$ and simplifying the result yields

$$\rho c_t = (1-\rho)(vH_t + q_t)n_t \quad (37)$$

Equation 37 and the budget constraint $c_t = (1 - vn_t)H_t - n_t q_t$ imply $c_t = (1-\rho)H_t$ or $\gamma_{ct} = (1-\rho)/(1 - vn_t)$. Consequently, $n_t q_t = (\rho - vn_t)H_t$ or $\gamma_{qt} = (\rho - vn_t)/[n_t(1 - vn_t)]$.

In equilibrium with $\bar{H} = H$, the growth rate is $\phi_t \equiv H_{t+1}/H_t - 1 = A(\rho - vn_t)^{\delta}/n_t^{\delta}$, and the first-order conditions Eqs. 36 and 37 imply

$$\begin{aligned}(\rho - vn_t)^{1-\sigma\delta(1-\rho)} n_t^{\sigma\delta(1-\rho)} A^{-\sigma(1-\rho)} n_t^{\sigma\rho} \\ = \alpha[\delta(1 - vn_{t+1}) + \beta(1-\delta)(\rho - vn_{t+1})]n_{t+1}^{\sigma\rho}.\end{aligned}$$

According to this, the determination of $n_t$ is independent of the state variable $H_t$. Due to this and due to the recursive structure, one expects $n_t = n_{t+1}$, leading to a time-invariant implicit solution for $n_t = n^*$. It is easy to observe that the implicit solution from $(\rho - vn)^{1-\sigma\delta(1-\rho)} n^{\sigma\delta(1-\rho)} A^{-\sigma(1-\rho)} = \alpha[\delta(1 - vn) + \beta(1-\delta)(\rho - vn)]$ can satisfy all the equilibrium conditions (the first-order conditions, the budget constraint, and the technology). Thus, it is the equilibrium solution at all times. When the solution for $n$ is constant over time, so are the proportional allocations $(\gamma_c, \gamma_q)$ and the growth rate $\phi$. With the solution for $(\gamma_c, \gamma_q, \phi, n)$, the solution for the optimal welfare level is

$$\begin{aligned}V(0) &= \sum_{t=0}^{\infty} \alpha^t \left[\gamma_c(1 - vn)H_0(1+\phi)^t\right]^{\sigma(1-\rho)} n^{\sigma\rho}/\sigma \\ &= \frac{[\gamma_c(1 - vn)H_0]^{\sigma(1-\rho)} n^{\sigma\rho}}{\sigma[1 - \alpha(1+\phi)^{\sigma(1-\rho)}]}.\end{aligned}$$

The cases with government policies can be solved similarly for implicit solutions with one extension: the derivative of $V(0)$ with respect to the policy variable is equal to zero.

Since the gap between the social planner and the competitive equilibrium solutions is only caused by the externality in this model, setting $\beta = 1$ in Eq. 36 leads to the social planner solution $(\gamma_c^P, \gamma_q^P, n^P)$ implicitly. We can use this observation to determine the first-best birth tax rate for education subsidies. In this case, the first-order conditions are

$$\begin{aligned}(1-s)n_t q_t c_t^{\sigma(1-\rho)-1} n_t^{\sigma\rho} = \alpha\big[\delta(1 - vn_{t+1})H_{t+1} \\ + (1-s)\,\beta(1-\delta)n_{t+1}q_{t+1}\big]\, c_{t+1}^{\sigma(1-\rho)-1} n_{t+1}^{\sigma\rho}, \quad (38)\end{aligned}$$

$$\rho c_t = (1-\rho)n_t\left[vH_t + (1-s)q_t + \theta(n_t - n_{\text{tax}})^{\theta-1}T_t\right]. \quad (39)$$

Equation 38 becomes the same as that faced by a social planner, i.e., $n_t q_t c_t^{\sigma(1-\rho)-1} n_t^{\sigma\rho} = \alpha[\delta(1 - vn_{t+1})H_{t+1} + (1-\delta)n_{t+1}q_{t+1}]c_{t+1}^{\sigma(1-\rho)-1} n_{t+1}^{\sigma\rho}$, if $\delta(1-vn_{t+1})H_{t+1}/(1-s)+\beta(1-\delta)n_{t+1}q_{t+1}=\delta(1-vn_{t+1})H_{t+1} + (1-\delta)n_{t+1}q_{t+1}$. This condition corresponds to

$$q_{t+1} = \frac{s\delta(1 - vn_{t+1})H_{t+1}}{n_{t+1}(1-s)(1-\beta)(1-\delta)}.$$

Thus, $\gamma_q = \gamma_q^P$ if $s\delta/[n(1-s)(1-\beta)(1-\delta)] = \gamma_q^P$, implying the first-best subsidy rate below

$$s^* = \frac{\gamma_q^P(1-\beta)(1-\delta)n^P}{\delta + \gamma_q^P(1-\beta)(1-\delta)n^P}.$$

Substituting the government budget constraint $sn_t q_t = T_t(n_t - n_{\text{tax}})^\theta$ in Eq. 39 yields

$$\rho c_t = (1-\rho)n_t[vH_t + (1-s)q_t + \theta s n_t q_t/(n_t - n_{\text{tax}})],$$

which becomes the same as that faced by the social planner, i.e., $\rho c_t = (1-\rho)n_t[vH_t + q_t]$, if $1 - s + \theta s n_t/(n_t - n_{\text{tax}}) = 1$. This immediately implies $n_{\text{tax}}^* = (1-\theta)n^P$. Finally, from $\tau \equiv T_t/H_t$ and the results above, the first-best tax rate is equal to $\tau^* = n^P s^* q_t/[(n^P - n_{\text{tax}}^*)^\theta H_t] = n^P s^* \gamma_q^P(1 - vn^P)/[n^P(1-\theta)]^\theta$. These first-best policies are similar to those in Proposition 4. □

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