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RELATIVISTIC QUANTUM THEORY DAMPING EQUATION IN THE HAMILTONIAN FORMULATION OF QUANTUM FIELD THEORY

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The fundamental equation of Sokolov–Heitler quantum damping theory [1-3] is widely used to describe various quantum-mechanical processes. This equation relates the amplitude on the energy surface to matrix elements of the Hermitian Wigner K-matrix. If the K-matrix is found by some appropriate method, preserving its Hermitian form, then the amplitude obtained by solving the quantum theory damping equation automatically satisfies the unitary condition.

The Sokolov–Heitler integral equation has yet another unique feature — its two-dimensionality, as a result of which the corresponding partial relationships are ordinary algebraic equations.

A relativistic generalization of the quantum theory damping equation was obtained in [4] using a one-dimensional formulation of the two-body problem in quantum field theory. This equation is evaluated and its application to problems in the theory of strong interactions is discussed in [5].

In the present study the relativistic quantum theory damping equation will be obtained within the framework of the Hamiltonian formulation of quantum field theory [6]. The Hamiltonian approach is convenient because it permits construction of a relativistic formalism which is closely related to the apparatus of nonrelativistic quantum mechanics [7-10]. The approach proposed herein will produce new diagram technique rules for construction of the integrand of the relativistic quantum theory damping equation — elements of the U-matrix.

We write the scattering S-matrix in the form

$$S=1+\mathrm{i}R=\left(1+\frac{\mathrm{i}}{2}K\right)\cdot\left(1-\frac{\mathrm{i}}{2}K\right)^{-1}. \tag{1}$$

The Hermitian property of the Wigner K-matrix guarantees a unitary S-matrix.

The perturbation theory series for the R and K operators appear as follows [6]:

$$R=\sum_{n=1}^{\infty}(-1)^n \mathrm{i}^{n-1}\int \mathrm{d}x_1\ldots\mathrm{d}x_n H(x_1)\,\theta(x_1^0-x_2^0)\,H(x_2)\ldots\theta(x_{n-1}^0-x_n^0)\,H(x_n); \tag{2}$$

$$K=\sum_{n=1}^{\infty}(-1)^n(\mathrm{i}/2)^{n-1}\int \mathrm{d}\,x_1\ldots\mathrm{d}\,x_n H(x_1)\;\varepsilon(x_1^0-x_2^0)\,H(x_2)\;\ldots\varepsilon(x_{n-1}^0-x_n^0)\,H(x_n), \tag{3}$$

where $H(x)$ is the Hamiltonian and $\theta(x_0)$ and $\varepsilon(x_0)$ are step and sign functions, respectively.

The series of Eqs. (2), (3) may be considered as successive iterations of the integral equations

$$R(\lambda\tau)=-\tilde{H}(\lambda\tau)-\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\tau'\tilde{H}(\lambda\tau-\lambda\tau')\,\frac{R(\lambda\tau')}{\tau'-\mathrm{i}0}\,, \tag{4}$$

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$$K(\lambda\tau) = -\tilde{H}(\lambda\tau) - \frac{P}{2\pi}\int_{-\infty}^{\infty} d\tau' \tilde{H}(\lambda\tau - \lambda\tau') \frac{K(\lambda\tau')}{\tau'}, \tag{5}$$

given the condition that R(0) = R and K(0) = K.

In Eqs. (4), (5), λ is a unit timelike four-vector: $\lambda^2 = \lambda_0^2 - \boldsymbol{\lambda}^2 = 1$, $\lambda_0 \geq 0$, the symbol P indicates calculation of the integral in the sense of the main value, and $\tilde{H}(\lambda\tau)$ is the Fourier transform of the Hamiltonian H(x).

Equation (4) leads to a spurion diagram technique differing from Feinman's. According to its rules, the momenta of all particles, even those in intermediate states, belong to the mass surface and the diagram contains additional quasiparticle-spurion lines, with a spurion with momentum $\lambda\tau$ corresponding to the propagator $[2\pi(\tau - i0)]^{-1}$.

Equation (5) also permits constructing an opeator $K(\lambda\tau)$ using perturbation theory, with new diagram technique rules. In this case the quasiparticle propagator corresponds to the function $P/2\pi\tau$.

From Eqs. (4), (5) we obtain the following operator relationship:

$$R(\lambda\tau) = K(\lambda\tau) + \frac{i}{2} K(\lambda\tau)\cdot R(0). \tag{6}$$

In the nonrelativistic limit at $\tau = 0$, by calculating the matrix element with Eq. (6) between two-particle initial and final scattering states, we arrive at the Sokolov–Heitler equation with Hermitian integrand $K^{+}(0) = K(0)$.

To derive the closed equation for the relativistic two-particle amplitude, we introduce the operator describing projection onto the two-particle state $\Pi_2 = |2\rangle\langle 2|$ and define the operator

$$U(\lambda\tau) = K(\lambda\tau)\cdot\left[1 - \frac{i}{2}(1 - \Pi_2) K(0)\right]^{-1}. \tag{7}$$

It is evident that the operator U(0) is not Hermitian due to the specifics of quantum field theory — the contribution of multiparticle states:

$$\langle m|(1 - \Pi_2)\cdot K|n\rangle = \sum_{l \neq 2} \langle m|l\rangle\langle l|K|n\rangle.$$

From Eqs. (6) and (7) we obtain

$$R(\lambda\tau) = U(\lambda\tau) + \frac{i}{2} U(\lambda\tau)\cdot\Pi_2\cdot R(0). \tag{8}$$

With equal scattering particle masses and $\tau = 0$ in the system of the center of inertia Eq. (8) leads to

$$\langle \boldsymbol{p}|T|\boldsymbol{\kappa}\rangle = \langle \boldsymbol{p}|U|\boldsymbol{\kappa}\rangle + \frac{i\pi}{8}\frac{|\boldsymbol{p}|}{p_0}\int d\omega_q \langle \boldsymbol{p}|U|\boldsymbol{q}\rangle\langle \boldsymbol{q}|T|\boldsymbol{\kappa}\rangle, \tag{9}$$

where $d\omega_q$ is the integration measure on a sphere, $p_0 = \sqrt{\boldsymbol{p}^2 + m^2} = q_0$, and

$$\langle p_1, p_2|R(0)|\kappa_1, \kappa_2\rangle = 2\pi\delta(E_p - E_\kappa)\delta^{(3)}(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2)\langle \boldsymbol{p}|T|\boldsymbol{\kappa}\rangle/\sqrt{2p_1^0 2p_2^0 2\kappa_1^0 2\kappa_2^0}; \tag{10}$$

$$\langle p_1, p_2|U(0)|\kappa_1, \kappa_2\rangle = 2\pi\delta(E_p - E_\kappa)\delta^{(3)}(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{\kappa}_1 - \boldsymbol{\kappa}_2)\langle \boldsymbol{p}|U|\boldsymbol{\kappa}\rangle/\sqrt{2p_1^0 2p_2^0 2\kappa_1^0 2\kappa_2^0}. \tag{11}$$

Equation (9) is the relativistic generalization of the quantum theory damping equation.

Using spurion diagram technique, Eq. (9) can provide a graphic interpretation and establish rules for finding the U-matrix elements from diagrams. For this purpose, we note that the spurion propagator can be represented in the form

$$[2\pi(\tau - i0)]^{-1} = P/2\pi\tau + \frac{i}{2}\delta(\tau). \tag{12}$$

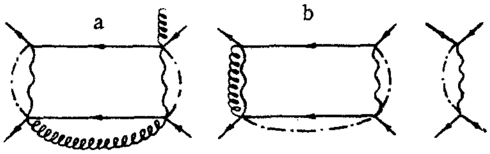

Fig. 1

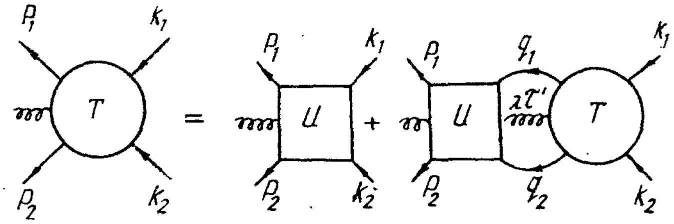

Fig. 2

The set of diagrams with full propagator (12) specifies an R-matrix, while the set of diagrams containing the first term of Eq. (12) as the spurion propagator defines the K-matrix.

We will now introduce graphic notation. A line corresponding to a particle will be denoted by a continuous line, a line corresponding to a spurion with propagator $P/2\pi\tau$, by a dash-dot line, and a line corresponding to a spurion with propagator $i/2\ \delta(\tau)$ by a spiral line. The set of diagrams which are irreducible in the sense of sections along two particle lines and one spiral line will be denoted by a rectangle. Figure 1 presents reducible (a) and irreducible (b) diagrams. Denoting the complete set of diagrams by a circle, the relativistic quantum theory damping equation can be represented in the form shown in Fig. 2.

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