Some problems of plane wedges of nonisotropic material.

Summary

Using complex functions some simple problems of an aeolotropic plane wedge used as a cantilever have been solved in this paper.

Indroductlon

The object of this paper is to solve a few problems of triangular wedges formed by two intersecting edges of thin orthotropic plates having constant thickness. The base of the triangular plate is assumed to be encastered while it is supposed to have either uniformly distributed load on the top horizontal edge or a concentrated load at the vertex. The method developed by the author (Sen, 1939) in a previous paper is found to be very useful, and it has been successfully employed in the solution of the problems discussed in this paper.

1. Stress-Strain relations and the expression for the stress function

Let the origin 0 be taken at the vertex and axis of x along the top horizental edge. The axis of y is drawn vertically downwards.

In the material considered we take

E_1, E_2 = Young's moduli in x and y directions respectively,

σ_1 = ratio of the contraction parallel to x-axis to the extension parallel to y-axis,

σ_2 = ratio of the contraction parallel to y-axis to the extension parallel to x-axis,

μ = Modulus of rigidity associated with the directions of x and y . . . . (1.1).

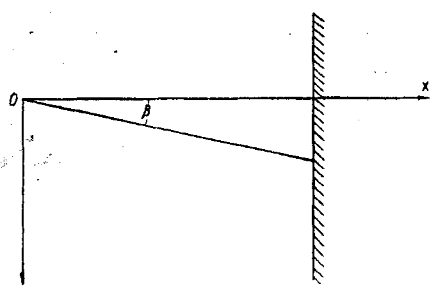

Fig. 1

We further assume that

$$\frac{\sigma_1}{E_2} = \frac{\sigma_2}{E_1} = k \text{ (say)} \quad . \; . \; . \; . \; . \; . \quad (1.2),$$

a relation which holds good in a materiallike wood.

The relations between the average stressses and average stains are

$$\left.\begin{aligned} e_{xx} &= E_1^{-1}\widehat{xx} - k\,\widehat{yy}\,, \\ e_{yy} &= -k\,\widehat{xx} + E_2^{-1}\widehat{yy}\,, \\ e_{xy} &= \mu \quad \widehat{xy} \end{aligned}\right\} \; . \; . \; . \; . \quad (3).$$

In the absence of body forces, equations of equilibrium are satisfied if we write

$$\widehat{xx} = \frac{\partial^2 X}{\partial y^2}\,, \qquad \widehat{yy} = \frac{\partial^2 X}{\partial x^2}\,, \qquad \widehat{xy} = -\frac{\partial^2 X}{\partial x\,\partial y} \quad (1.4).$$

From the relations (1.3), (1.4) and the compatibility equation

$$\frac{\partial^2 e_{yy}}{\partial x^2} + \frac{\partial^2 e_{xx}}{\partial y^2} = \frac{\partial^2 e_{xy}}{\partial x \partial y}\,,$$

we obtain the equation satisfied by X as

$$\frac{\partial^4 X}{\partial x^4} + 2\,M \frac{\partial^4 X}{\partial x^2\,\partial \eta^2} + \frac{\partial^4 X}{\partial \eta^4} = 0 \; . \; . \; . \quad (1.5),$$

in which

$$\left.\begin{aligned} 2\,M &= (E_1\,E_2)^{1/2}\,[\mu^{-1} - 2\,k]\,, \\ \eta &= \epsilon y\,, \qquad \epsilon = [E_1\,E_2^{-1}]^{1/4} \end{aligned}\right\} \; . \; . \; . \quad (1.6).$$

It may be mentioned here that for woods whose elastic constants are known M is positive and greater than unity. As a solution of (1.6) we can write

$$X = Re\,[F_1\,(x + i\,\alpha_1\,\eta) + F_2(x + i\,\alpha_2\,\eta)] \qquad (1.7),$$

where Re denotes the real part, $i = \sqrt{-1}$,

$$\left.\begin{aligned} \alpha_1 &= [M + (M^2 - 1)^{1/2}]^{1/2}\,, \\ \text{and} \qquad \alpha_2 &= [M - (M^2 - 1)^{1/2}]^{1/2} \end{aligned}\right\} \; . \; . \quad (1.8).$$

Finally putting

$$\left.\begin{aligned} &\gamma_1 = \alpha_1 \epsilon \quad \text{and} \quad \gamma_2 = \alpha_2 \epsilon\,, \\ \text{we get } X &= Re\,[F_1(x + i\,\gamma_1\,y) + F_2(x + i\,\gamma_2\,y] \\ &= Re\,[F_1\,(z_1) + F_2\,(z_2)] \end{aligned}\right\} \quad (1.9),$$

in which for brevity, we write

$$z_1 = x + i\,\gamma_1 y \quad \text{and} \quad z_2 = x + i\,\gamma_2 y \quad (1.10).$$

For materials like wood which are being considered here, γ_1 and γ_2 will be real positive quantities.

Case I. — If $\widehat{xy} = 0$ when $y = 0$, we can write

$$X = Re\,[\gamma_2\,F(z_1) - \gamma_1\,F(z_2)]\,,$$

so that on putting $f(z_1)$ and $f(z_2)$ for the second derivatives of $F(z_1)$ and $F(z_2)$ with respect to z_1 and z_2 respectively, we obtain

$$\left.\begin{aligned} \widehat{xy} &= Re\,[i\,\gamma_1\,\gamma_2\,\{f(z_2) - f(z_1)\}]\,, \\ \widehat{yy} \quad & Re\,[\gamma_2 f(z_1) - \gamma_1 f(z_2)]\,, \\ \widehat{xx} &= Re\,[\gamma_1\,\gamma_2^2\,f(z_2) - \gamma_2\,\gamma_1^2\,f(z_1)] \end{aligned}\right\} \quad (1.11).$$

Case II. If $\widehat{yy} = 0$ when $y = 0$, we can put

$$X = Re\,[G(z_1) - G(z_2)]\,.$$

In this case, writing $g(z_1)$ and $g(z_2)$ for the second derivatives of $G(z_1)$ and $G(z_2)$ with respect to z_1 and z_2 respectively, we get

$$\left.\begin{aligned} \widehat{yy} &= Re\,[g(z_1) - g(z_2)]\,, \\ \widehat{xx} &= Re\,[\gamma_2^2\,g(z_2) - \gamma_1^2\,g(z_1)]\,, \\ \widehat{xy} &= Re\,[i\,\{\gamma_2\,g(z_2) - \gamma_1\,g(z_1)\}] \end{aligned}\right\} \; . \; . \quad (1.12).$$

2. The wedge loaded uniformly on the top Horizontal edge

Let w be the load per unit length on the upper horizontal edge of the triangular plate. Then putting

$$X = Re\left[\frac{A\,z_1^2}{2} + \frac{B\,z_2^2}{2} + \gamma_2\,F(z_1) - \gamma_1\,F(z_2)\right] \; (2.1),$$

(where A and B are real constants), and writing as before $f(z_1)$ and $f(z_2)$ for $F''(z_1)$ and $F''(z_2)$ respectively, we have

$$\left.\begin{aligned} \widehat{xx} &= Re\,[-(A\gamma_1^2 + B\gamma_2^2) + \gamma_1\gamma_2^2 f(z_2) - \gamma_2\gamma_1^2 f(z_1)]\,, \\ \widehat{xy} &= Re\,[i\,\gamma_1\,\gamma_2\,\{f(z_2) - f(z_1)\}]\,, \\ \widehat{yy} &= Re\,[(A + B) + \gamma_2 f(z_1) - \gamma_1 f(z_2)] \end{aligned}\right\} \; (2.2).$$

If the x component and the y component of the tractions across the line Θ = constant at a point having the polar co-ordinates (r, Θ) be denoted by $\widehat{\Theta x}$ and $\widehat{\Theta y}$, we have

$$\left.\begin{aligned} r\,\widehat{\Theta x} &= -\,y\,\widehat{xx} + x\,\widehat{xy} = (A\,\gamma_1^2 + B\,\gamma_2^2)\,y \\ &\quad + Re\,[i\,\gamma_1\,\gamma_2\,\{z_2 f(z_2) - z_1 f(z_1)\}]\,, \\ \widehat{\Theta y} &= -\,y\,\widehat{xy} + x\,\widehat{yy} = (A + B)\,x \\ &\quad + Re\,[\gamma_2\,z_1 f(z_1) - \gamma_1\,z_2 f(z_2)] \end{aligned}\right\} \quad (2.3).$$

On putting $f(z) = C\,i\log z$ (where C is a real constant), and remembering that $z_1 = x + i\gamma_1 y$ and $z_2 = x + i\gamma_2 y$, we find

$$r\,\widehat{\Theta x} = x\left[(A\gamma_1^2 + B\gamma_2^2)\tan\Theta + C\gamma_1\gamma_2\{(\log\sec\Theta_1 - \log\sec\Theta_2) + \tan\Theta(\gamma_2\Theta_2 - \gamma_1\Theta_1)\}\right] \quad (2.4),$$

$$r\,\widehat{\Theta y} = x\left[(A+B) + C\{(\gamma_1\Theta_2 - \gamma_2\Theta_1) + \gamma_1\gamma_2\tan\Theta(\log\sec\Theta_2 - \log\sec\Theta_1)\}\right] \quad (2.5),$$

in which

$$y = x\tan\Theta,\quad \gamma_1 y = x\tan\Theta_1,\quad \text{and}\quad \gamma_2 y = x\tan\Theta_2 .$$

When $y = 0$, we have $\Theta = \Theta_1 = \Theta_2 = 0$ and hence

$$\widehat{\Theta x} = 0,\quad \text{and}\quad \widehat{\Theta y} = A + B .$$

From the given condition, therefore, we get

$$A + B = -w \quad (2.6).$$

When $\Theta = \beta$ (constant), we have $y = x\tan\beta$. Assuming the corresponding values of Θ_1 and Θ_2 to be β_1 and β_2 respectively, we have $\tan\beta_1 = \gamma_1\tan\beta$, and $\tan\beta_2 = \gamma_2\tan\beta$.

Thus we find that on the edge $\Theta = \beta$ the stresses will be zero, if

$$C\gamma_1\gamma_2[(\log\sec\beta_1 - \log\sec\beta_2) + \tan\beta(\gamma_2\beta_2 - \gamma_1\beta_1)] = -(A\gamma_1^2 + B\gamma_2^2)\tan\beta \quad (2.7),$$

and

$$C[(\gamma_1\beta_2 - \gamma_2\beta_1) + \gamma_1\gamma_2\tan\beta(\log\sec\beta_2 - \log\sec\beta_1)] = -(A+B) = w \quad (2.8).$$

From (2.7) and (2.8) we get

$$\left.\begin{aligned} C &= \frac{w}{\triangle},\\ A &= \frac{w\gamma_2}{(\gamma_1^2 - \gamma_2^2)\triangle}[\beta_1(\gamma_1^2 - \gamma_2^2) + Q\gamma_1(\cot\beta + \gamma_2^2\tan\beta)],\\ B &= \frac{w\gamma_1}{(\gamma_1^2 - \gamma_2^2)\triangle}[-\beta_2(\gamma_1^2 - \gamma_2^2) - Q\gamma_2(\cot\beta + \gamma_1^2\tan\beta)] \end{aligned}\right\} \quad (29),$$

where

$$\left.\begin{aligned} Q &= \log\sec\beta_2 - \log\sec\beta_1\\ \triangle &= (\gamma_1\beta_2 - \gamma_2\beta_1) + Q\gamma_1\gamma_2\tan\beta \end{aligned}\right\} \quad (2.10).$$

3. A concentrated load at the vertex of the wedge

Let us now suppose that there is a concentrated load at the vertex of the wedge whose components parallel to the x and y-axes are X_1 and Y_1 respectively. If we put $f(z) = D\,i/z$ in (1.11) and $g(z) = E/z$ in (1.12) where D and E are real constants, we have by combining the two system of stresses, the resultant stress components $\widehat{xx}, \widehat{xy}$ and $\widehat{yy}$ given by

$$\left.\begin{aligned} \widehat{xx} &= D\gamma_1\gamma_2 y\left[\frac{\gamma_2^2}{r_2^2} - \frac{\gamma_1^2}{r_1^2}\right] + Ex\left[\frac{\gamma_2^2}{r_2^2} - \frac{\gamma_1^2}{r_1^2}\right],\\ \widehat{xy} &= -D\gamma_1\gamma_2 x\left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right] + Ey\left[\frac{\gamma_2^2}{r_2^2} - \frac{\gamma_1^2}{r_1^2}\right],\\ \widehat{yy} &= D r_1 r_2 y\left[\frac{1}{r_1^2} - \frac{1}{r_2^2}\right] + Ex\left[\frac{1}{r_1^2} - \frac{1}{r_2^2}\right] \end{aligned}\right\} \quad (3.1)$$

in which $r_1^2 = x^2 + \gamma_1^2 y^2$ and $r_2^2 = x^2 + \gamma_2^2 y^2$.

It can be easily verified that for all values of Θ

$$\left.\begin{aligned} r\,\widehat{\Theta x} &= -y\,\widehat{xx} + x\,\widehat{xy} = 0,\\ r\,\widehat{\Theta y} &= -y\,\widehat{xy} + x\,\widehat{yy} = 0 \end{aligned}\right\} \quad (3.2).$$

The unknown constants D and E can now be found from the conditions

$$\int_0^\beta r\,\widehat{rx}\,d\Theta = \int_0^\beta (x\,\widehat{xx} + y\,\widehat{xy})\,d\Theta = -X_1 \quad (3.3),$$

$$\int_0^\beta r\,\widehat{ry}\,d\Theta = \int_0^\beta (x\,\widehat{xy} + y\,\widehat{yy})\,d\Theta = -Y_1 \quad (3.4).$$

Using the notations of the previous section, we have for the determination of D and E, two equations

$$\left.\begin{aligned} &D\gamma_1\gamma_2 Q + E(\gamma_2\beta_2 - \gamma_1\beta_1) + X_1 = 0\\ \text{and}\quad &D(\gamma_2\beta_1 - \gamma_1\beta_2) + EQ + Y_1 = 0 \end{aligned}\right\} \quad (3.5)$$

These give

$$\left.\begin{aligned} D &= [Y_1(\gamma_2\beta_2 - \gamma_1\beta_1) - X_1 Q]\,U,\\ E &= [X_1(\gamma_2\beta_1 - \gamma_1\beta_2) - Y_1\gamma_1\gamma_2 Q]\,U,\\ &\text{where}\\ U &= [\gamma_1\gamma_2 Q^2 - (\gamma_2\beta_1 - \gamma_1\beta_2)(\gamma_2\beta_2 - \gamma_1\beta_1)]^{-1} \end{aligned}\right\} \quad (3.6).$$

It may be mentioned here that the problems of isolated forces acting along the internal bisector of the vertical angle of — an aeolotopic wedge and in a direction at right angles to it when these directions coincide with the axes of elastic symmetry were solved by Green (1939) in a different manner.

References

Green, A. E. 1939. Proc. Roy Soc. A. 173, p. 173.
Sen, B. 1939. Phil. Mag. Ser. 7, 27, p. 596.

Pilani, India. Bibhuti Bhusan Cafen.

Ein Inversionszeichengerät für das Rechnen in der komplexen Ebene.

Die Berechnung des frequenzabhängigen Widerstandes linearer Netzwerke ist oft sehr umständlich. Einfacher erscheint es, die Ortskurve des Widerstandes in der komplexen Ebene zu konstruieren, um aus der Ortskurve den Widerstand für jede Frequenz ablesen zu können. Zur Konstruktion der Ortskurve ist es meistens notwendig, teils in der Widerstandsebene und teils in der Leitwertebene zu zeichnen. Daher ist bei komplizierten Schaltungen die Kurve mindestens einmal zu invertieren. Die Inversion ist dann sehr einfach, wenn es sich nur um Geraden und Kreise handelt. Für Netzwerke, die aus Widerständen und Leitwerten zusammengesetzt sind, ist man fast immer gezwungen, die Inversion punktweise durchzuführen und dann die Kurve als Verbindungszug der invertierten Punkte zu ziehen.

Da man für die Widerstandsebene nur den 1. und 4. Quadranten gebraucht, ist es zweckmäßig die Leitwertebene in den 2. und 3. Quadranten zu legen. Auf der negativen reellen Achse der komplexen Ebene werden dann positive Leitwerte aufgetragen. Diese Verabredung der Aufteilung der komplexen Ebene hat den Vorteil, daß die invertierten Werte immer auf Geraden liegen, die durch den Nullpunkt der komplexen Ebene gezogen sind. Es liegt nun nahe, für die Inversion ein

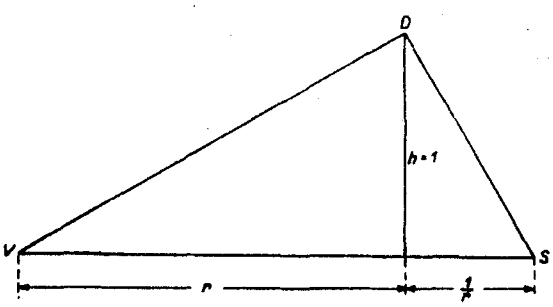

Bild 1