RESEARCH ARTICLE

Robust Stability Results for Nonlinear Markovian Jump Systems with Mode-Dependent Time-Varying Delays and Randomly Occurring Uncertainties

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This article is concerned with the robust stability analysis for Markovian jump systems with mode-dependent time-varying delays and randomly occurring uncertainties. Sufficient delay-dependent stability results are derived with the help of stability theory and linear matrix inequality technique using direct delay-decomposition approach. Here, the delay interval is decomposed into two subintervals using the tuning parameter η such that $0 < \eta < 1$, and the sufficient stability conditions are derived for each subintervals. Further, the parameter uncertainties are assumed to be occurring in a random manner. Numerical examples are given to validate the derived theoretical results. © 2015 Wiley Periodicals, Inc. Complexity 000: 00–00, 2015

Key Words: Markovian jump; randomly occurring uncertainties; direct delay-decomposition; Lyapunov–Krasovskii functional; mode-dependent time-varying delay

1. INTRODUCTION

In many practical applications, there are several factors such as time-delays, disturbances, uncertainties, and so forth, which can cause undesirable dynamic behaviors namely instability and poor performance. Mathematical modeling of many processes in the field of biology, medicine, chemistry, engineering, economics, and so forth, involve delays, which can cause severe mathematical complications in the stability analysis. Hence, systems involving delays have drawn great deal of attention and several research works have been carried out, for more details see [1–9] and references therein.

Markovian jump system (MJS) is a two-level hybrid system in which the first level is governed by a set of modes described by differential and/or difference equations and the second level coordinates the switching among the

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modes. Thus, the system admits continuous states which take values from a vector space and discrete states which take values from a discrete index set. These types of systems can be found in many real world applications such as chemical processes, transportation systems, and computer controlled systems (see [10] and [11]). Hence these kinds of systems received much attention in the recent decades (e.g., see [12–21], and references therein). In recent years, there has been an increasing interest in the investigation of stability analysis for MJSs with mode-dependent time-delays (for details, see [22–27]).

The behavior of the system can be affected by the presence of complexities such as nonlinearities and uncertainties which occur during the process of system modeling. These complexities are subject to random changes in environment circumstances, random failures, or repairs of components and sudden environment disturbances, and so forth, which may occur in the probabilistic way. Hence the concept of randomly occurring nonlinearities and uncertainties have received more attention in the recent years (e.g., see [28–40] and references therein). In this work, we are concerned with random occurring uncertainties only.

The problem of nonlinear stochastic systems with randomly occurring incomplete information has been discussed in [30]. The authors in [32] have studied the problem of robust sliding mode control for discrete-time stochastic systems with randomly occurring uncertainties and nonlinearities, whereas the authors in [34] have investigated the problem of synchronization of chaotic systems with randomly occurring uncertainties based on stochastic sampled-data control. Stochastic stability analysis of semi-MJSs with mode-dependent delays has been investigated in [23] and the authors in [25] have designed the robust H_∞ filter for the problem of uncertain discrete Markov jump singular systems with mode-dependent time-delays. In [40], the authors have investigated the problem of robust passive control for networked fuzzy systems with randomly occurring uncertainties. Also, there has been an increasing interest in the investigation of stability analysis of time-delay systems based on delay-decomposition approach (for details, see [41–43], and references therein). To the best of authors' knowledge, the problem of stability analysis of nonlinear MJSs with mode-dependent time-varying delays and randomly occurring uncertainties based on delay-decomposition approach has not been fully investigated in the existing literature.

Motivated by the above observations, in this article, the stochastic stability analysis of uncertain MJSs with mode-dependent time-varying delays based on direct delay-decomposition approach has been investigated. Robust stochastic stability analysis for the considered system has been derived with the condition that the parameter uncertainties are assumed to occur randomly. Sufficient stability conditions for the considered system are derived in terms of linear matrix inequalities (LMIs) by constructing suitable Lyapunov–Krasovskii functional (LKF). The derived stability conditions can be easily solved using MATLAB LMI solvers. Finally, numerical examples are illustrated to show the effectiveness of the proposed theoretical results. In addition, comparison results have been given to demonstrate the less conservativeness of the derived results.

The rest of this article is organized as follows. In section 2, problem description and preliminaries are given. Section 3 provides the stochastic stability conditions for mode-dependent nonlinear MJSs based on direct delay-decomposition approach. Section 4 deals with the robust stochastic stability conditions for nonlinear MJSs with randomly occurring uncertainties and mode-dependent time-varying delays. In section 5, numerical examples are given to demonstrate the effectiveness and less conservativeness of the derived results and section 6 concludes the article.

Notations: Throughout this article, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices. The superscript T denotes the transposition and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). I denotes the identity matrix with appropriate dimension and $\mathcal{C}([-\tau_2, 0], \mathbb{R}^n)$ denotes the family of continuously differentiable functions from $[-\tau_2, 0]$ to $\mathbb{R}^n$, where $\tau_2 > 0$. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. The notation $*$ always denotes the symmetric block in a symmetric matrix. Matrices, if not explicitly stated are assumed to have compatible dimensions.

2. PROBLEM DESCRIPTION AND PRELIMINARIES

Let $\{r(t), t \geq 0\}$, be a right-continuous Markov chain defined on the complete probability space taking values in a finite space $S=\{1, 2, \cdots, N\}$ with generator $\Pi=(\gamma_{ij})_{N\times N}$ given by

$$\Pr\{r(t+\delta t)=j|r(t)=i\}=\begin{cases}\gamma_{ij}\delta t+o(\delta t), & \text{if } i \neq j\\ 1+\gamma_{ii}\delta t+o(\delta t), & \text{if } i=j\end{cases}$$

where $\delta t > 0, \lim_{\delta t\to 0}\frac{o(\delta t)}{\delta t}=0$ and $\gamma_{ij} \geq 0$ is the known transition rate from i to j if $i \neq j$ where $\gamma_{ii}=-\sum_{j\neq i}\gamma_{ij}, i, j \in S$.

Consider the nonlinear MJS with mode-dependent time-varying delays and randomly occurring uncertainties as follows

$$\begin{aligned}\dot{x}(t)=&A_{r(t)}(t)x(t)+B_{r(t)}(t)x(t-\tau_{r(t)}(t))+F_{1r(t)}f_1(t, x(t))\\ &+F_{2r(t)}f_2(t, x(t-\tau_{r(t)}(t))),\end{aligned} \tag{1}$$

$$x(t)=\phi(t), t \in [-\tau_2, 0],$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system, $\tau_2=\max\{\tau_{r(t)}(t), r(t) \in S\}$ and $\phi(t) \in \mathcal{C}([-\tau_2, 0], \mathbb{R}^n)$ denotes the

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initial condition. For notational convenience, denote $r(t)$ as i, $i \in S$. Hence system (1) can be written as

$$\dot{x}(t)=A_i(t)x(t)+B_i(t)x(t-\tau_i(t))+F_{1i}f_1(t,x(t)) \\ +F_{2i}f_2(t,x(t-\tau_i(t))), \tag{2}$$

where $A_i(t)=A_i+\alpha(t)\Delta A_i(t), B_i(t)=B_i+\beta(t)\Delta B_i(t)$. The matrices A_i and B_i are known constant matrices of appropriate dimensions. F_{1i} and F_{2i} are the coefficients of the nonlinear functions $f_1(t,x(t))$ and $f_2(t,x(t-\tau_i(t)))$, respectively, with appropriate dimensions. $\Delta A_i(t)$ and $\Delta B_i(t)$ are the time-varying parameter uncertainty matrices, which satisfy $[\Delta A_i(t) \quad \Delta B_i(t)]=LF_i(t)[E_{ai} \quad E_{bi}]$, where L, E_{ai}, and E_{bi} are known constant matrices of appropriate dimensions, $F_i(t) \in \mathbb{R}^{n\times n}$ is an unknown possibly time-varying real matrix with Lesbesgue measurable elements satisfying $F_i^T(t)F_i(t) \le I, \forall t > 0$. $\alpha(t)$ and $\beta(t)$ are the stochastic variables introduced to describe the random occurring phenomena and they are assumed to satisfy $\Pr\{\alpha(t)=1\}=\alpha, \Pr\{\alpha(t)=0\}=1-\alpha, \Pr\{\beta(t)=1\}=\beta, \Pr\{\beta(t)=0\}=1-\beta$ with $\alpha,\ \beta \in [0,1]$. $\tau_i(t)$ denotes the mode-dependent time-varying delays, which satisfy

$$0 \le \tau_i(t) \le \tau_{2i},\ \ \dot{\tau}_i(t) \le \mu_i,\ \ i \in S, \tag{3}$$

where τ_{2i} and μ_i are constants for any $i \in S$.

The following assumption, definition, and lemma are required to prove the main results.

Assumption 2.1

[44] *The nonlinear functions $f_l : \mathbb{R}^+\times\mathbb{R}^n \to \mathbb{R}^n (l=1,2)$ are continuous, satisfy $f_l(t,0) \equiv 0$ and the Lipschitz condition*

$$||f_l(t,x_0)-f_l(t,y_0)|| \le ||u_l(x_0-y_0)||, \forall\ t \text{ and } \forall x_0, y_0 \in \mathbb{R}^n$$

such that u_l are some known matrices of appropriate dimensions.

Definition 2.2

[25] *The system (2) is said to be stochastically stable, if there exists a scalar $M(r_0,\phi(\cdot))$ for every initial state $\phi(\cdot)$ and initial mode $r_0=i_0 \in S$, the condition*

$$\lim_{\bar{T}\to\infty} \mathbb{E}\left\{\int_0^{\bar{T}} ||x(t)||^2 dt | r_0, x(s)=\phi(s)\right\} < M(r_0,\phi(\cdot))$$

holds, where x(t) denotes the solution of system (2) under initial condition $\phi(\cdot)$ at time t.

Lemma 2.3

[3] *Given matrices $\bar{Q}=\bar{Q}^T, M, N$ with appropriate dimensions, then $\bar{Q}+MF_iN+N^TF_i^TM^T < 0 \forall F_i(t)$ satisfying $F_i^T(t)F_i(t) \le I$, if and only if there exists a scalar $\epsilon_i > 0$ such that $\bar{Q}+\epsilon_i^{-1}MM^T+\epsilon_i N^TN < 0, \forall i \in S$.*

3. STOCHASTIC STABILITY ANALYSIS OF MJSs

In this section, stochastic stability analysis for mode-dependent MJS without uncertainties will be addressed. Thus system (2) without uncertainties is given by

$$\dot{x}(t)=A_ix(t)+B_ix(t-\tau_i(t))+F_{1i}f_1(t,x(t))+F_{2i}f_2(t,x(t-\tau_i(t))) \tag{4}$$

with mode-dependent time-varying delay $\tau_i(t)$ satisfying condition (3). Here, the stability conditions are derived by using the delay-decomposition approach. The delay interval $[-\tau_2, 0]$ is decomposed into two subintervals namely $[-\tau_2, -\eta\tau_2]$ and $[-\eta\tau_2, 0]$ with the help of the tuning parameter η such that $0 < \eta < 1$. If $\eta=0.5$ then the length of subintervals will be equal otherwise it will be unequal.

Theorem 3.1

For given scalars $\tau_2 > 0$, μ_i, and $0 < \eta < 1$, and the time-varying delays satisfying condition (3), system (4) is stochastically stable if there exist symmetric positive-definite matrices P_i, Q_1, Q_2, Q_3, R_1, R_2, positive scalars $\epsilon_{1i}, \epsilon_{2i}$ such that the following symmetric LMIs hold $\forall i \in S$

$$\begin{bmatrix} \Omega^i & A_{ci}^TY \\ * & -Y \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} \Pi^i & A_{ci}^TY \\ * & -Y \end{bmatrix} < 0, \tag{6}$$

where $\Omega^i=(\Omega^i_{p,q})_{6\times6}, \Pi^i=(\Pi^i_{p,q})_{6\times6}$ with

$$\Omega^i_{1,1}=P_iA_i+A_i^TP_i+\sum_{j=1}^{N}\gamma_{ij}P_j+Q_1+Q_3+\eta\tau_2\bar{\gamma}Q_3+\epsilon_{1i}u_1^Tu_1 \\ -\frac{1}{\eta\tau_2}R_1, \quad \Omega^i_{1,2}=P_iB_i+\frac{1}{\eta\tau_2}R_1,$$

$$\Omega^i_{1,5}=P_iF_{1i}, \quad \Omega^i_{1,6}=P_iF_{2i}, \quad \Omega^i_{2,2} \\ =-(1-\mu_i)Q_3+\epsilon_{2i}u_2^Tu_2-\frac{2}{\eta\tau_2}R_1, \quad \Omega^i_{2,3}=\frac{1}{\eta\tau_2}R_1,$$

$$\Omega^i_{3,3}=-Q_1+Q_2-\frac{1}{\eta\tau_2}R_1-\frac{1}{\tau_2(1-\eta)}R_2, \quad \Omega^i_{3,4}=\frac{1}{\tau_2(1-\eta)}R_2, \\ \Omega^i_{4,4}=-Q_2-\frac{1}{\tau_2(1-\eta)}R_2,$$

$$\Omega^i_{5,5}=-\epsilon_{1i}I, \quad \Omega^i_{6,6}=-\epsilon_{2i}I,$$

$$\Pi^i_{1,1}=P_iA_i+A_i^TP_i+\sum_{j=1}^{N}\gamma_{ij}P_j+Q_1+Q_3+\tau_2(1-\eta)\bar{\gamma}Q_3 \\ +\epsilon_{1i}u_1^Tu_1-\frac{1}{\eta\tau_2}R_1, \quad \Pi^i_{1,2}=P_iB_i,$$

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$$\Pi^i_{1,3}=\frac{1}{\eta\tau_2}R_1,\quad \Pi^i_{1,5}=\Omega^i_{1,6},\quad \Pi^i_{1,6}=\Omega^i_{1,6},$$
$$\Pi^i_{2,2}=-(1-\mu_i)Q_3+\epsilon_{2i}u_2^Tu_2-\frac{2}{\tau_2(1-\eta)}R_2,$$

$$\Pi^i_{2,3}=\Pi^i_{2,4}=\frac{1}{\tau_2(1-\eta)}R_2,\quad \Pi^i_{3,3}=\Omega^i_{3,3},\quad \Pi^i_{4,4}=\Omega^i_{4,4},$$
$$\Pi^i_{5,5}=\Omega^i_{5,5},\quad \Pi^i_{6,6}=\Omega^i_{6,6},$$

$$\bar{\gamma}=\max\{-\gamma_{ii}, i\in S\},\quad \tau_2=\max\{\tau_{2i}, i\in S\},$$
$$A_{ci}=[A_i\ B_i\ 0\ 0\ F_{1i}\ F_{2i}],\quad Y=\eta\tau_2R_1+\tau_2(1-\eta)R_2$$

and the remaining terms are zero.

Proof

To derive the stochastic stability conditions for system (4), consider the LKF as follows

$$\begin{aligned}V(t,x(t),i)=&x^T(t)P_ix(t)+\int_{t-\eta\tau_2}^{t}x^T(s)Q_1x(s)ds\\&+\int_{t-\tau_2}^{t-\eta\tau_2}x^T(s)Q_2x(s)ds+\int_{t-\tau_i(t)}^{t}x^T(s)Q_3x(s)ds\\&+\int_{-\eta\tau_2}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R_1\dot{x}(s)dsd\theta\\&+\int_{-\tau_2}^{-\eta\tau_2}\int_{t+\theta}^{t}\dot{x}^T(s)R_2\dot{x}(s)dsd\theta\\&+\bar{\gamma}\int_{-\eta\tau_2}^{0}\int_{t+\theta}^{t}x^T(s)Q_3x(s)dsd\theta,\end{aligned}\tag{7}$$

where P_i, Q_1, Q_2, Q_3, R_1, and R_2 are unknown symmetric positive-definite matrices with appropriate dimensions to be determined for each $i\in S$.

Let $\mathcal{L}$ be the weak infinitesimal generator of the random process $\{(x(t),i), t\geq 0\}$ along the solution of system (4), then from LKF (7), it can be obtained that

$$\begin{aligned}\mathcal{L}V(t,x(t),i)\leq\ &x^T(t)[P_iA_i+A_i^TP_i+\sum_{j=1}^{N}\gamma_{ij}P_j+Q_1+Q_3\\&+\eta\tau_2\bar{\gamma}Q_3]x(t)+x^T(t)P_iB_ix(t-\tau_i(t))+x^T(t)P_iF_{1i}f_1(t,x(t))\\&+x^T(t)P_iF_{2i}f_2(t,x(t-\tau_i(t)))+x^T(t-\eta\tau_2)[-Q_1+Q_2]x(t-\eta\tau_2)\\&-x^T(t-\tau_2)Q_2x(t-\tau_2)-(1-\mu_i)x^T(t-\tau_i(t))Q_3x(t-\tau_i(t))\\&+\dot{x}^T(t)[\eta\tau_2R_1+\tau_2(1-\eta)R_2]\dot{x}(t)+\sum_{j=1}^{N}\gamma_{ij}\int_{t-\tau_j(t)}^{t}x^T(s)Q_3x(s)ds\\&-\bar{\gamma}\int_{t-\eta\tau_2}^{t}x^T(s)Q_3x(s)ds-\int_{t-\eta\tau_2}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds\\&-\int_{t-\tau_2}^{t-\eta\tau_2}\dot{x}^T(s)R_2\dot{x}(s)ds.\end{aligned}\tag{8}$$

For nonlinear functions $f_l(\cdot)$, given $\epsilon_{1i}>0, \epsilon_{2i}>0$, we have

$$-\epsilon_{1i}f_1^T(t,x(t))f_1(t,x(t))+\epsilon_{1i}x^T(t)u_1^Tu_1x(t)\geq 0,\tag{9}$$

$$\begin{aligned}&-\epsilon_{2i}f_2^T(t,x(t-\tau_i(t)))f_2(t,x(t-\tau_i(t)))\\&+\epsilon_{2i}x^T(t-\tau_i(t))u_2^Tu_2x(t-\tau_i(t))\geq 0.\end{aligned}\tag{10}$$

Now, to evaluate the last two terms of the inequality (8), we have two cases of time-delay namely

- $0\leq\tau_i(t)\leq\eta\tau_2$,
- $\eta\tau_2\leq\tau_i(t)\leq\tau_2$.

Considering the case when $\tau_i(t)\in[0,\eta\tau_2]$ and using Jensen's inequality in [3], one can obtain

$$\begin{aligned}&-\int_{t-\eta\tau_2}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds-\int_{t-\tau_2}^{t-\eta\tau_2}\dot{x}^T(s)R_2\dot{x}(s)ds\\&=-\int_{t-\tau_i(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds-\int_{t-\eta\tau_2}^{t-\tau_i(t)}\dot{x}^T(s)R_1\dot{x}(s)ds\\&\quad-\int_{t-\tau_2}^{t-\eta\tau_2}\dot{x}^T(s)R_2\dot{x}(s)ds\\&\quad\leq-\frac{1}{\eta\tau_2}[x(t)-x(t-\tau_i(t))]^TR_1[x(t)-x(t-\tau_i(t))]\\&\quad-\frac{1}{\eta\tau_2}[x(t-\tau_i(t))-x(t-\eta\tau_2)]^TR_1[x(t-\tau_i(t))-x(t-\eta\tau_2)]\\&\quad-\frac{1}{\tau_2(1-\eta)}[x(t-\eta\tau_2)-x(t-\tau_2)]^TR_2[x(t-\eta\tau_2)-x(t-\tau_2)].\end{aligned}\tag{11}$$

Noting that from $\gamma_{ij}\geq 0$, for $j\neq i$ and $\gamma_{ii}\leq 0$, then one can obtain

$$\begin{aligned}&\sum_{j=1}^{N}\gamma_{ij}\int_{t-\tau_j(t)}^{t}x^T(s)Q_3x(s)ds\\&=\sum_{j\neq i}\gamma_{ij}\int_{t-\tau_j(t)}^{t}x^T(s)Q_3x(s)ds+\gamma_{ii}\int_{t-\tau_i(t)}^{t}x^T(s)Q_3x(s)ds\\&\leq-\gamma_{ii}\int_{t-\eta\tau_2}^{t}x^T(s)Q_3x(s)ds\\&=\bar{\gamma}\int_{t-\eta\tau_2}^{t}x^T(s)Q_3x(s)ds.\end{aligned}\tag{12}$$

Combining inequalities (8)–(12), one can obtain

$$\mathcal{L}V(t,x(t),i)\leq\xi^T(t)\Omega_1^i\xi(t)<0,\tag{13}$$

where $\xi^T(t)=[x^T(t)\quad x^T(t-\tau_i(t))\quad x^T(t-\eta\tau_2)\quad x^T(t-\tau_2)\quad f_1^T(t,x(t))\quad f_2^T(t,x(t-\tau_i(t)))]$ and $\Omega_1^i=\Omega^i+A_{ci}^T[\eta\tau_2R_1+\tau_2(1-\eta)R_2]A_{ci}$. Hence from inequality (13), it can be seen that

$$\Omega^i+A_{ci}^T[\eta\tau_2R_1+\tau_2(1-\eta)R_2]A_{ci}<0\tag{14}$$

and also $\mathcal{L}V(t,x(t),i)\leq-\lambda||x(t)||^2$, where $\lambda=\min_{i\in S}(-\Omega_1^i)>0$. Applying Schur complement lemma

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in [3] to inequality (14), one can get LMI (5). Taking expectation on both sides of inequality (13) and using Dynkin's formula, it is true that for all $t \geq \eta\tau_2$,

$$\mathbb{E}\{V(t)\} - \mathbb{E}\{V(\eta\tau_2)\} \leq -\lambda\mathbb{E}\left\{\int_{\eta\tau_2}^{t} ||x(s)||^2 ds\right\}.$$

From the above relation, it follows that

$$\mathbb{E}\left\{\int_{\eta\tau_2}^{t} ||x(s)||^2 ds\right\} \leq \frac{1}{\lambda}\mathbb{E}\{V(\eta\tau_2)\}.$$

Hence from system (4) and using the similar proof of [14], it is clear that there exists a scalar $\rho > 0$ such that

$$\mathbb{E}\left\{\int_0^{\eta\tau_2} ||x(s)||^2 ds\right\} \leq \rho \sup_{s\in[-\eta\tau_2,0]} \mathbb{E}\{||\phi(s)||^2\}.$$

Therefore, from definition of $V(t,x(t),i)$ and $x(t)$, there always exists a scalar $\bar{\rho} > 0$ such that

$$\lim_{\bar{T}\to\infty} \mathbb{E}\left\{\int_0^{\bar{T}} ||x(t)||^2 dt\right\} \leq \bar{\rho} \sup_{s\in[-\eta\tau_2,0]} \mathbb{E}\{||\phi(s)||^2\} < M(r_0,\phi(\cdot)) \tag{15}$$

which implies that system (4) is stochastically stable by Definition 2.2 for the case when time-varying delay $\tau_i(t) \in [0,\eta\tau_2]$. Next, we prove the theorem for case (ii), that is, $\tau_i(t) \in [\eta\tau_2,\tau_2]$. To derive the stochastic stability conditions under case (ii) modify LKF (7) by replacing the last term with $\bar{\gamma}\int_{-\tau_2}^{-\eta\tau_2}\int_{t+\theta}^{t} x^T(s)Q_3x(s)dsd\theta$. In this case, the last two terms of inequality (8) can be treated as follows

$$\begin{aligned}
&-\int_{t-\eta\tau_2}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-\tau_2}^{t-\eta\tau_2} \dot{x}^T(s)R_2\dot{x}(s)ds \\
&\quad = -\int_{t-\eta\tau_2}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-\tau_i(t)}^{t-\eta\tau_2} \dot{x}^T(s)R_2\dot{x}(s)ds \\
&\qquad -\int_{t-\tau_2}^{t-\tau_i(t)} \dot{x}^T(s)R_2\dot{x}(s)ds \\
&\quad \leq -\frac{1}{\eta\tau_2}[x(t)-x(t-\eta\tau_2)]^T R_1[x(t)-x(t-\eta\tau_2)] \\
&\qquad -\frac{1}{\tau_2(1-\eta)}[x(t-\eta\tau_2)-x(t-\tau_i(t))]^T R_2[x(t-\eta\tau_2)-x(t-\tau_i(t))] \\
&\qquad -\frac{1}{\tau_2(1-\eta)}[x(t-\tau_i(t))-x(t-\tau_2)]^T R_2[x(t-\tau_i(t))-x(t-\tau_2)].
\end{aligned} \tag{16}$$

Combining the inequalities (8)–(10), 16, and using the fact that $\sum_{j=1}^{N}\gamma_{ij}\int_{t-\tau_j(t)}^{t} x^T(s)Q_3x(s)ds \leq \bar{\gamma}\int_{t-\tau_2}^{t-\eta\tau_2} x^T(s)$ $Q_3x(s)ds$ and following the similar procedure as in case (i), one can easily prove that system (4) is stochastically stable under case (ii). Hence the nonlinear MMJS with mode-dependent time-varying delays is stochastically stable which completes the proof of this theorem. ■

Remark 3.2

When the nonlinear functions are not considered, and the time-delays are considered to be constant and mode-independent, system (4) becomes

$$\dot{x}(t) = A_ix(t) + B_ix(t-\tau_2). \tag{17}$$

Stability conditions for the above system can be determined by considering the following LKF

$$\begin{aligned}
\bar{V}(t,x(t),i) = {} & x^T(t)P_ix(t) + \int_{t-\eta\tau_2}^{t} x^T(s)Q_1x(s)ds \\
& + \int_{t-\tau_2}^{t-\eta\tau_2} x^T(s)Q_2x(s)ds \\
& + \int_{-\eta\tau_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\
& + \int_{-\tau_2}^{-\eta\tau_2}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta
\end{aligned} \tag{18}$$

and using the similar procedure as in Theorem 3.1. The corresponding delay-dependent stochastic stability conditions are summarized in the following corollary.

Corollary 3.3

For given scalars $\tau_2 > 0$ and $0 < \eta < 1$, system (17) is stochastically stable if there exist symmetric positive-definite matrices P_i, Q_1, Q_2, R_1, R_2 such that the following LMIs hold $\forall i \in S$

$$\begin{bmatrix} \bar{\Omega}^i & A_{ci}^T Y \\ * & -Y \end{bmatrix} < 0, \tag{19}$$

where $\bar{\Omega}^i = (\bar{\Omega}^i_{a,b})_{3\times 3}$ with

$$\bar{\Omega}^i_{1,1} = P_iA_i + A_i^TP_i + \sum_{j=1}^{N}\gamma_{ij}P_j + Q_1 - \frac{1}{\eta\tau_2}R_1, \bar{\Omega}^i_{1,2} = \frac{1}{\eta\tau_2}R_1, \bar{\Omega}^i_{1,3} = P_iB_i,$$

$$\bar{\Omega}^i_{2,2} = -Q_1 + Q_2 - \frac{1}{\eta\tau_2}R_1 - \frac{1}{\tau_2(1-\eta)}R_2, \bar{\Omega}^i_{2,3} = \frac{1}{\tau_2(1-\eta)}R_2,$$
$$\bar{\Omega}^i_{3,3} = -Q_2 - \frac{1}{\tau_2(1-\eta)}R_2,$$

$$\bar{A}_{ci} = [A_i \quad 0 \quad B_i]$$

and the remaining terms are zero.

Remark 3.4

In Theorem 3.1, stability results of system (4) are established by decomposing the delay interval into two

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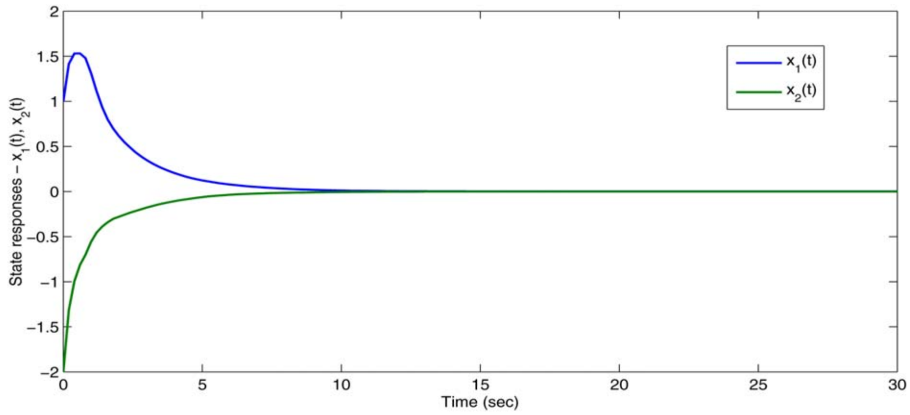
FIGURE 1



State trajectories of Example 5.1 with initial conditions $x_1(0)=1$ and $x_2(0)=-2$ for subsystem 1.

subintervals by introducing the tuning parameter η instead of decomposing it into "m" subintervals. In the case of decomposing, the delay interval into "m" subintervals, the increase in the number of subintervals automatically increases the number of decision variables, which results in computation complexity. Hence the results proposed in this article present improved result than those in the existing literature.

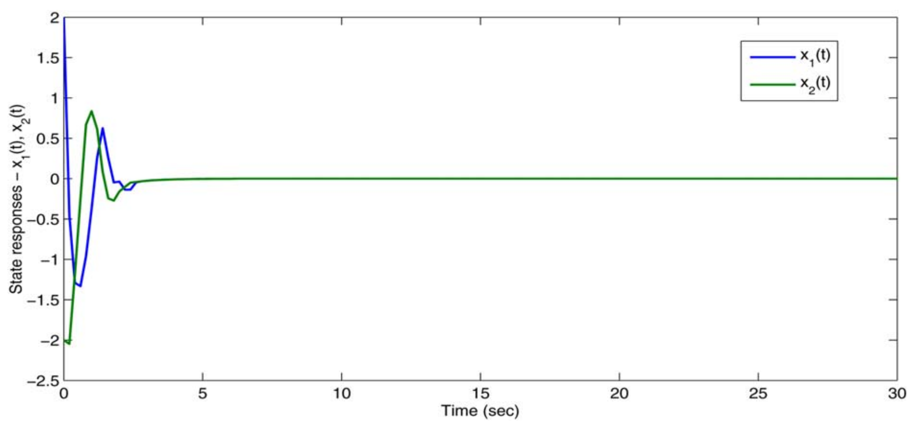
4. STOCHASTIC STABILITY ANALYSIS OF UNCERTAIN MJSs

In this section, we extend the results obtained in Theorem 3.1 to uncertain MJSs. In the presence of randomly occurring uncertainties, stability conditions of system (2) are derived by using the similar procedure as followed in Theorem 3.1 and using Lemma 2.3. The following theorem presents sufficient delay-dependent stability conditions.

Theorem 4.1

For given scalars $\tau_2 > 0$, μ_i, and $0 < \eta < 1$, α, β, system (2) is stochastically stable for all time-varying delays satisfying condition (3) if there exist symmetric positive-definite matrices P_i, Q_1, Q_2, Q_3, R_1, R_2, positive scalars $\epsilon_{1i}, \epsilon_{2i}, \epsilon_{3i}, \epsilon_{4i}$ such that the following LMIs hold $\forall i \in S$

FIGURE 2



State trajectories of Example 5.1 with initial conditions $x_1(0)=2$ and $x_2(0)=-2$ for subsystem 2.

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TABLE 1

Comparison Results of MAUB for Example 5.2

Methods	[12]	[21]	[22]	Corollary 3.3
τ_2	1.23	1.33	1.43	2.57

$$\begin{bmatrix} \bar{\Omega}^i & A_{ci}^T Y & \bar{P}_i L & 0 \\ * & -Y & 0 & YL \\ * & * & -\epsilon_{3i} I & 0 \\ * & * & * & -\epsilon_{4i} I \end{bmatrix} < 0, \quad \begin{bmatrix} \bar{\Pi}^i & A_{ci}^T Y & \bar{P}_i L & 0 \\ * & -Y & 0 & YL \\ * & * & -\epsilon_{3i} I & 0 \\ * & * & * & -\epsilon_{4i} I \end{bmatrix} < 0, \tag{20}$$

where $\bar{\Omega}_{p,q} \neq \Omega_{p,q}$ *and* $\bar{\Pi}_{p,q} \neq \Pi_{p,q}$ *when* $((p,q) \neq (1,1),(1,2),(2,2))$, *where*

$$\bar{\Omega}^i_{1,1} = \Omega^i_{1,1} + (\epsilon_{3i}+\epsilon_{4i})\alpha^2 E_{ai}^T E_{ai}, \bar{\Omega}^i_{1,2} = \Omega^i_{1,2} + (\epsilon_{3i}+\epsilon_{4i})\alpha\beta E_{ai}^T E_{bi},$$

$$\bar{\Omega}^i_{2,2} = \Omega^i_{2,2} + (\epsilon_{3i}+\epsilon_{4i})\beta^2 E_{bi}^T E_{bi}, \bar{\Pi}^i_{1,1} = \bar{\Omega}^i_{1,1}, \bar{\Pi}^i_{1,2} = \bar{\Omega}^i_{1,2}, \bar{\Pi}^i_{2,2} = \bar{\Omega}^i_{2,2}$$

and for the remaining entries, the matrices $\bar{\Omega}^i = \Omega^i$ *and* $\bar{\Pi}^i = \Pi^i$, $\bar{P}_i = [P_i\ 0\ 0\ 0\ 0\ 0]^T$, *and the remaining terms are defined as in Theorem 3.1.*

Proof

The proof of this theorem is obtained from Theorem 3.1 and Lemma 2.3 by replacing A_i and B_i by $A_i + \alpha(t)\Delta A_i(t)$ and $B_i + \beta(t)\Delta B_i(t)$, respectively. ■

Remark 4.2

When the Markovian jumping parameters, nonlinearities, and randomness of the uncertainties in system (2) are absent, then system (2) can be rewritten as

$$\dot{x}(t) = A_1(t)x(t) + B_1(t)x(t-\tau_2). \tag{21}$$

Robustly asymptotic stability results for the above system can be obtained using the similar procedure as in Theorem 4.1 and the corresponding results are summarized in the following corollary. From the numerical example, it can be seen that the results obtained in this article is less conservative than those obtained in the existing literature.

Corollary 4.3

For given scalars $\tau_2 > 0$ *and* $0 < \eta < 1$, *system (21) is robustly asymptotically stable if there exist symmetric positive-definite matrices* P_1, Q_1, Q_2, R_1, R_2, *positive scalars* ϵ_{31}, ϵ_{41} *such that the following LMI holds*

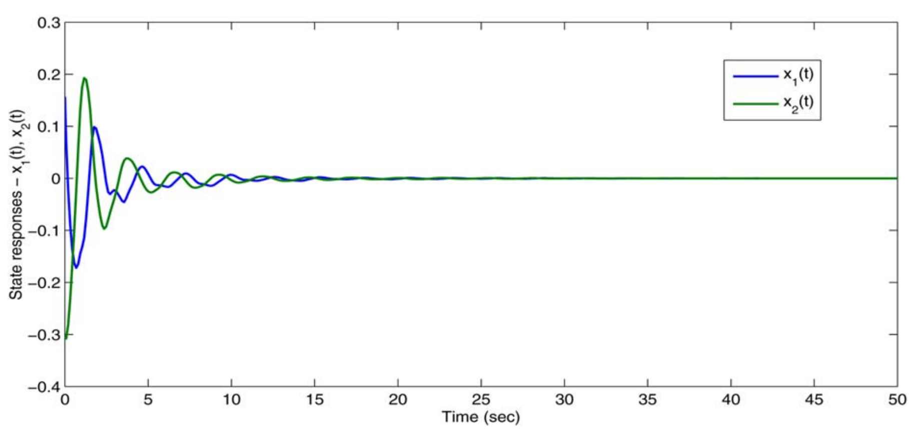
$$\begin{bmatrix} \hat{\Omega} & \bar{A}_c^T Y & \hat{P} L & 0 \\ * & -Y & 0 & YL \\ * & * & -\epsilon_{31} I & 0 \\ * & * & * & -\epsilon_{41} I \end{bmatrix} < 0, \tag{22}$$

where $\hat{\Omega} = (\hat{\Omega}_{c,d})_{3\times 3}$ *with*

$$\hat{\Omega}_{1,1} = P_1 A_1 + A_1^T P_1 + Q_1 - \frac{1}{\eta\tau_2} R_1 + (\epsilon_{31}+\epsilon_{41}) E_{a1}^T E_{a1}, \hat{\Omega}_{1,2} = \frac{1}{\eta\tau_2} R_1,$$

$$\hat{\Omega}_{1,3} = P_1 B_1 + (\epsilon_{31}+\epsilon_{41}) E_{a1}^T E_{b1}, \hat{\Omega}_{2,2} = -Q_1 + Q_2 - \frac{1}{\eta\tau_2} R_1 - \frac{1}{\tau_2(1-\eta)} R_2,$$

FIGURE 3



State trajectories of Example 5.3 with initial conditions $x_1(0)=0.2$ and $x_2(0)=-0.3$.

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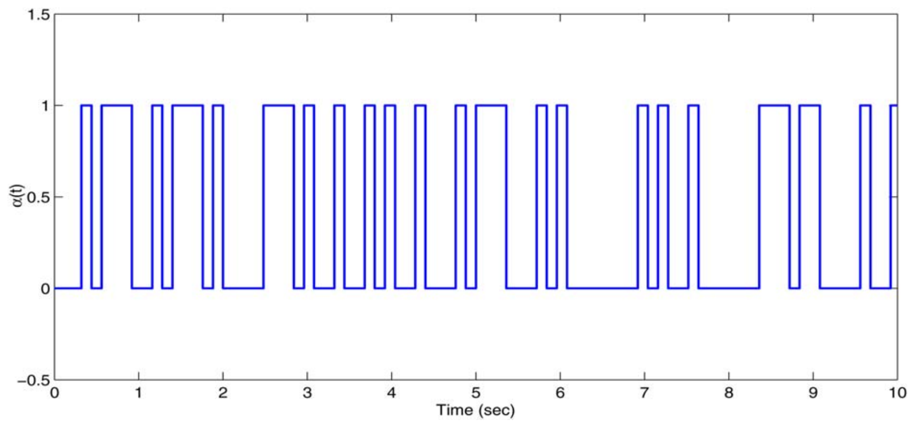
FIGURE 4



Simulations of the stochastic variable $\alpha(t)$ for Example 5.3.

$$\hat{\Omega}_{3,3}=-Q_2-\frac{1}{\tau_2(1-\eta)}R_2+(\epsilon_{31}+\epsilon_{41})E_{b1}^TE_{b1},\hat{P}=[P_1\ 0\ 0]^T$$

and the remaining terms are defined as in Theorem 4.1.

5. NUMERICAL EXAMPLES

In this section, we shall present numerical examples to demonstrate the effectiveness and less conservativeness of the derived theoretical results. Hybrid systems can be viewed as a class of dynamical systems, which evolve according to the mode-dependent continuous/discrete dynamics, and these systems can experience transitions between modes that are triggered by events. One such example is an automobile with a manual gearbox [10]. Consider the motion of a vehicle that travels along a fixed path which is characterized by two continuous variables namely velocity and position. In this case, system has two inputs namely throttle angle and engaged gear. In each mode, system dynamics evolve in a continuous manner according to some differential equation. Taking the above views into consideration, we have considered the following examples.

Example 5.1

Consider the nonlinear MJS (4) with two modes $S=\{1,2\}$ described by

Subsystem 1

$$A_1=\begin{bmatrix}-0.8 & -1.5\\ 0 & -2.4\end{bmatrix},B_1=\begin{bmatrix}-0.6 & -0.5\\ 0.1 & 0.4\end{bmatrix},$$
$$F_{11}=\begin{bmatrix}-0.89 & -0.08\\ -0.96 & -0.75\end{bmatrix},F_{21}=\begin{bmatrix}1.20 & -0.79\\ -0.03 & 0.92\end{bmatrix},$$

Subsystem 2

$$A_2=\begin{bmatrix}-4.2 & 1.9\\ 1.7 & 0.8\end{bmatrix},B_2=\begin{bmatrix}-0.5 & 0.4\\ 0.9 & -0.3\end{bmatrix},$$
$$F_{12}=\begin{bmatrix}-0.65 & -0.25\\ -0.77 & -0.92\end{bmatrix},F_{22}=\begin{bmatrix}1.49 & -0.88\\ -0.09 & 0.86\end{bmatrix},$$

and the mode switching is governed by Markov process with its generator is given by $\Pi=\begin{bmatrix}-7 & 7\\ 3 & -3\end{bmatrix}$.

By solving the LMIs (5) and (6) obtained in Theorem 3.1, the maximum allowable upper bound (MAUB) for nonlinear MJS (4) is obtained as 0.8 for $\eta=0.12,\mu_1=0.36,\mu_2=0.44,u_1=\mathrm{diag}\{0.1,0.1\}$ and $u_2=\mathrm{diag}\{0.5,0.5\}$. For numerical simulation purpose, choose $\tau_1(t)=0.36(1+\sin(t)),\tau_2(t)=0.44(1+\sin(t)),f_1=|\sin(x(t))|$ and $f_2=|\sin(x(t-\tau_j(t)))|$. Figure 1 shows the state responses $x_1(t)$ and $x_2(t)$ for subsystem 1 of system (4) with the initial condition $x_1(0)=1$ and $x_2(0)=-2$, whereas Figure 2 shows the state responses $x_1(t)$ and $x_2(t)$ for subsystem 2 of system (4) with the initial condition $x_1(0)=2$ and $x_2(0)=-2$. From Figures 1 and 2, it can be seen that both subsystems of system (4) are stable.

Example 5.2

Consider the Markovian jump time-delay system (17) with parameters described by

$$A_1=\begin{bmatrix}0.5 & -1\\ 0 & -3\end{bmatrix},B_1=\begin{bmatrix}0.5 & -0.2\\ 0.2 & 0.3\end{bmatrix},A_2=\begin{bmatrix}-5 & 1\\ 1 & 0.2\end{bmatrix},$$
$$B_2=\begin{bmatrix}-0.3 & 0.5\\ 0.4 & -0.5\end{bmatrix},\Pi=\begin{bmatrix}-7 & 7\\ 3 & -3\end{bmatrix},$$

$$\eta=0.99.$$

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For the above parameters, MAUB is calculated by solving the LMIs obtained from Corollary 3.3 and is listed in Table 1. The following table shows comparison with the results obtained from Corollary 3.3 to that of results in [12], [21], and [22]. Hence from Table 1, it is evident that the delay-dependent stability conditions obtained from Corollary 3.3 provide less conservative results than [12], [21], and [22]. From Corollary 3.3, one can see that the results depend on the parameter value η and the value of MAUB can be further increased by decreasing the value of η.

Example 5.3

Consider the nonlinear system (2) with single mode described as follows

$$A_1=\begin{bmatrix}-2.2 & 1.5\\ -1.5 & -0.8\end{bmatrix}, B_1=\begin{bmatrix}-0.5 & 0.4\\ 0.9 & -0.3\end{bmatrix},$$
$$F_{11}=\begin{bmatrix}-0.65 & -0.25\\ -0.77 & -0.92\end{bmatrix}, F_{21}=\begin{bmatrix}1.49 & -0.88\\ -0.09 & 0.86\end{bmatrix},$$
$$F_1(t)=\begin{bmatrix}\sin(t) & 0\\ 0 & \sin(t)\end{bmatrix}, L=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix},$$
$$E_{a1}=\begin{bmatrix}0.4 & 0\\ 0 & 0.4\end{bmatrix}, E_{b1}=\begin{bmatrix}0 & -0.3\\ 0.3 & 0\end{bmatrix}.$$

By choosing $\tau_1(t)=\frac{e^t}{1+e^t}$, $\Delta A_1(t)=L\text{diag}\{\sin(t),\sin(t)\}E_{a1}$, $\Delta B_1(t)=L\text{diag}\{\sin(t),\sin(t)\}E_{b1}$ *and assuming that the stochastic parameters $\alpha(t)$ and $\beta(t)$ are equal, the state trajectories of system (2) are shown in Figure 3 with the initial conditions $x_1(0)=0.2$ and $x_2(0)=-0.3$. From Figure 3, it is evident that system (2) is asymptotically stable. Figure 4 shows the random behavior of the stochastic variable $\alpha(t)$.*

Remark 5.1

From the numerical simulations, it can be seen that the results proposed in this article yield less conservative results than those in the existing literature. Also, the MAUBs can be further increased by decreasing the value of η. Further, the derived conditions involve less number of decision variables and hence the computation time will be less. This was achieved mainly due to the utilization of direct delay-decomposition approach. The results obtained in this article can be extended to systems with stochastic perturbations, neutral time-delays, filtering problems, and so forth. Further, the results presented in this article can be generalized for interval nondifferentiable time-varying delays by following the similar procedure in [1].

6. CONCLUSION

The problem of stochastic stability analysis of uncertain MJSs with mode-dependent time-varying delays has been investigated. Sufficient delay-dependent stability conditions have been derived using direct delay-decomposition approach and LMI techniques. Further, these results have been extended to MJSs with random occurring uncertainties. Finally, the derived theoretical results have been verified by suitable numerical examples. Also, comparison results have been provided to show the effectiveness and less conservativeness of the proposed results.

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