

Growth and variation in maize¹⁾.

By Raymond Pearl and Frank M. Surface.

(Eingegangen 6. April 1914).

The investigation reported in this paper is an attempt to analyze normal „fluctuating“ variation in a particular case, from the standpoint of Entwicklungsmechanik. It is a direct outgrowth from, and indeed in some respects, a supplement to a study of variation and differentiation in *Ceratophyllum* made some years ago by one of the present writers (PEARL, '07). We have here attempted to approach the problem of inter-individual variation from the same point of view and with similar methods to those applied to the problem of intra-individual variation in the case of *Ceratophyllum*.

Our problem and our point of view may be most clearly defined by considering briefly certain well-known, indeed obvious, facts about variation. If one brings together a homogeneous group of individual plants or animals and measures the same character in each individual, there may be formed from the resulting data a characteristic variation curve for that group and character. The precise form of this curve, as well as the location in it of any particular individual, are functions of two basic variables. Of these one is the hereditary or germinal constitution of the individual. The other is the complex of environmental stresses and strains, which, each acting on the individual at some time during its ontogeny, have influenced the end result of the activity of the hereditary determiners or genes.

Now it is altogether usual in discussion of variation and heredity to take the two end terms of the ontogenetic series, the gene on the

¹⁾ Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 64.

one hand and the adult soma on the other hand, as things given. What comes between the two is neglected. But clearly what goes between is a part of the very essence of heredity itself.

In our group of adult individuals each one will show some particular variation, in the sense of a deviation from the typical condition of the group. We take it to be one of the final objects of investigations in genetics to find out why (in the sense of locating the essential causal factors involved) a particular individual A exhibits a particular variation a, and not some other variation out of the indefinitely large number of possibilities. Since the discovery of MENDEL's laws we have gone a long way towards being able to answer the question for many kinds of qualitative variations. For fluctuating or quantitative variations too, considerable progress has been made through the researches especially of East and his associates, following those of NILSSON-EHLE on colors determined by multiple factors. These studies have made it extremely probable that definite genes determine, for example, how tall a particular Indian corn plant shall be, or how large an ear it shall have. But in these researches only the end elements of the ontogenetic series have received attention.

What we have tried to do in the present investigation is, by studying the growth of the individual, to analyze the adult variation curve into its component elements. We have attempted, in other words, to make a beginning at an understanding of the developmental physiology of the genes concerned in the production of the characters studied. Specifically the angle from which this general problem has been approached in the present case is that of the analysis of the adult variation curve in terms of its component individuals. A given individual, at an early stage of its growth, may exhibit an exceptional condition of a character, as compared with other individuals of the same age or stage of growth. It may be, for example, very short, the shortest plant in the whole population at 3 weeks of age. Will this same individual be the shortest plant in the adult population, after all growth is completed? If not, where is it in the adult variation curve, and how did it get there? This example will give a concrete idea of the general mode of approach to the problem of variation followed in this paper. The specific problems will be set forth in detail in the sections where they are discussed

Materials and Methods.

The data for this work were collected during the summer of 1908. At that time an ear-to-the-row test of sweet corn was conducted at Farmington, Maine. The results of this test along with others have already been published (PEARL and SURFACE, '10). At the same place, data regarding the soil, fertilization and cultivation of the crop are given, so that it will not be necessary to repeat them here.

The corn used for the growth work was that described in the paper referred to as Type I. This is a white, small kerneled corn, very desirable from the standpoint of the packers. It makes a fairly rapid growth and matures fairly early. The plants are rather short, rarely reaching a height of more than 160 centimeters.

In 1908 the plot in which this corn was grown consisted of 100 rows of approximately 100 hills each. The rows were three feet apart, and the hills 18 inches in the row. Each row was planted from a single ear which had been selected in the field of the previous year. These ears had been especially selected on account of early maturity and in most cases because of a good yield of grain. This corn was planted on May 19, 1908, and was harvested for seed on September 6 to 12, 1908. Three kernels were planted in each hill but as soon as the plants were well started these were thinned to one stalk in order to secure a uniform stand.

For the growth work three series of plants were measured as follows: Sixty plants in each of two rows, Nos. 131 and 133¹⁾ were selected for study. The third series consisted of one plant from each of sixty rows viz., rows 101 to 160. Thus in the first two sets of measurements all the plants, in each case, were the descendants of a single mother ear. In the third set, sixty different mother ears were represented.

In choosing the plants to be measured no selection was practiced except that the plant should appear normal and vigorous. Thus in rows 131 and 133 the first sixty hills in each row were taken. In rows 101 to 160 the third hill from the end of the row was taken unless for some reason this hill appeared abnormal at the time the plants were selected. As was to be expected various accidents occurred from

¹⁾ The rows in this plot were numbered from 101 to 200 inclusive. The numbers used here are the same as those used in the paper referred to above.

time to time so that the number of plants completing the season was slightly less than sixty in each case (cf. tables 25 to 27).

The growth of the plant was recorded by measuring the total height. In order to do this it was necessary to have a base to measure from which would not shift with the growth of the plant or would not be altered by the cultivation. To do this a small stake, one inch square and fifteen inches long, was driven into the ground near each plant to be measured. These stakes were driven down until the tops were nearly level with the ground. This formed a solid base from which all measurements were taken.

The measurements were made with a meter stick to which a sliding arm, about eight inches in length, was attached. The measurements were recorded to the nearest millimeter.

From the nature of a young corn plant it is somewhat difficult to obtain a good character for measuring its height. After some experimenting it was decided to use the height to the tip of the tallest leaf when this was stretched up. This is by no means an ideal character. In the first place we are measuring the combined growth of two things viz., the leaf and the stalk internodes. The second difficulty lies in the fact that as each new leaf unfolds there comes a time when it exceeds in height the preceding leaf. This means that one is measuring the height, first to the tip of one leaf and then to the tip of another. These difficulties did not prove to be as serious as was at first expected. They are undoubtedly responsible for certain irregularities in the measurements. These will be discussed more in detail when we come to deal with the data.

It is realized that in certain respects the present data are unsatisfactory. A better measure of the growth would have been the total weight of the plant. Obviously, however, it is impossible to obtain such data at frequent intervals on the same plant. For the object we had in view, no better measure of growth suggested itself at the time these measurements were taken.

In addition to the observations on leaf height noted above, measurements of the tassel height were also taken as soon as the tassels appeared. The tassel forms an excellent character for measuring growth but it appears too late in the development of the plant to be of much use. During the latter part of the season both the leaf height and the tassel height of each plant were measured. The measurements are dealt with separately in every case.

All the plants were measured at intervals of three and one-half days beginning on June 11, 1908. It will be noted that this was twenty-three days after the corn had been planted. Circumstances over which we had no control prevented us from beginning the measurements earlier. However, with the cool springs of Maine the plants had not advanced very far by that time. The measurements were continued at the twice-a-week intervals until July 27, 1908. At that time the plants had practically completed their growth. Three later measurements of the tassel height were taken viz., on August 3, August 10, and August 24. As may be seen from Fig. 1 there was no increase in the average height after August 3. In fact the later measurements showed a slight decrease in height due to the weathering of the tassel tip.

There are thus six series of measurements available for discussion. In order to facilitate reference to these series they have been given the following designations.

Series A —	Leaf height of plants from Row 131.
Series B —	" " " " " 133.
Series C —	" " " " " 101—160.
Series D —	Tassel " " " " " 131.
Series E —	" " " " " 133.
Series F —	" " " " " 101—160.

The measurements in the field were all made by one person (F. M. SURFACE). The computations have been made at various times and by various people. A number of these were done by Dr. E. P. HUMBERT during his connection with the laboratory. In nearly all cases the computations have been checked several times, usually by different members of the laboratory staff.

Part I.

General Growth Curves.

The first step in the discussion of the growth data has been to form frequency distributions of the various series for each date. These frequency distributions are exhibited in tables 25 to 30 (pp. 172—183). In making these distributions the measurements have been arranged in groups of 25 millimeters. As will be seen this gives reasonably smooth distributions except that in some of the later distributions there are a

number of zero frequencies. This arises from the fact that the range of the observations is much greater towards the middle of the growing season. Considering the data as a whole it appears that no uniform grouping could be obtained which would give better distributions.

From these distributions the means, standard deviations and coefficients of variability have been calculated. These together with their probable errors are given in tables 1 to 6¹⁾.

Table 1.
Constants of Variation in Height of Plants for Series A.

Date of Measurement	Mean	Standard Deviation	Coefficient of Variation
June 12	139·35 ± 2·32	25·28 ± 1·64	18·14 ± 1·21
June 15	189·35 ± 3·49	38·07 ± 2·47	20·10 ± 1·36
June 19	228·24 ± 4·74	51·61 ± 3·35	22·61 ± 1·55
June 22	299·07 ± 4·79	52·14 ± 3·38	17·44 ± 1·17
June 26	331·48 ± 4·65	50·62 ± 3·29	15·27 ± 1·01
June 29	385·19 ± 4·08	44·49 ± 2·89	11·55 ± 0·76
July 3	441·67 ± 6·06	66·06 ± 4·29	14·96 ± 0·99
July 6	554·17 ± 7·20	78·43 ± 5·09	14·15 ± 0·94
July 10	689·81 ± 9·66	105·19 ± 6·83	15·25 ± 1·01
July 13	824·07 ± 12·15	132·42 ± 8·59	16·07 ± 1·07
July 17	974·07 ± 12·18	132·68 ± 8·61	12·62 ± 0·83
July 20	1103·24 ± 11·07	120·57 ± 7·83	10·93 ± 0·72
July 24	1193·52 ± 9·71	105·81 ± 6·87	8·87 ± 0·58
July 27	1215·28 ± 9·36	102·00 ± 6·62	8·39 ± 0·55

¹⁾ It will be noted from the tables that the measurements of the different series were not all made on the same day. Since it was not convenient to devote all of any one day to making these measurements, the convention was adopted of measuring only one set of plants on any half day. Thus the first measurements on Series B were made on the afternoon of June 11. Series A was measured on the forenoon of June 12 and Series C on the afternoon of the same day. The next measurement of Series B was on the forenoon of June 15, Series A on the afternoon of that day and Series C on the forenoon of June 16. In this way the interval between the measurements of each series was three and one-half days. For convenience in plotting, the dates of measurements have been grouped and supposed to center on one of these days. Thus in figures 1 to 3 the first measurements are plotted as if all were made on June 12, the second on June 15 and so on. This will serve to explain any apparent discrepancy between the tables and the figures.

Table 2.
Constants of Variation in Height of Plant for Series B.

Date of Measurement	Mean	Standard Deviation	Coefficient of Variation
June 11	121·30 ± 2·25	24·52 ± 1·59	20·21 ± 1·36
June 15	177·78 ± 2·93	31·97 ± 2·08	17·99 ± 1·20
June 18	209·26 ± 3·69	40·20 ± 2·61	19·21 ± 1·29
June 22	289·81 ± 4·55	49·54 ± 3·22	17·09 ± 1·14
June 25	347·69 ± 5·00	54·49 ± 3·54	15·67 ± 1·04
June 29	400·00 ± 4·43	48·23 ± 3·13	12·06 ± 0·79
July 2	418·98 ± 4·30	46·88 ± 3·04	11·19 ± 0·74
July 6	516·20 ± 6·22	67·73 ± 4·40	13·12 ± 0·87
July 9	652·31 ± 9·21	100·38 ± 6·51	15·39 ± 1·02
July 13	804·17 ± 10·85	118·22 ± 7·67	14·70 ± 0·97
July 16	953·70 ± 11·96	130·25 ± 8·45	13·66 ± 0·90
July 20	1110·19 ± 10·93	119·12 ± 7·73	10·73 ± 0·70
July 23	1232·41 ± 9·27	100·99 ± 6·55	8·19 ± 0·54
July 27	1306·02 ± 8·69	94·64 ± 6·14	7·25 ± 0·47

Table 3.
Constants of Variation in Height of Plant for Series C.

Date of Measurement	Mean	Standard Deviation	Coefficient of Variation
June 12	143·41 ± 2·32	25·53 ± 1·64	17·80 ± 1·18
June 16	212·50 ± 3·59	39·52 ± 2·5	18·60 ± 1·24
June 19	252·04 ± 4·43	48·71 ± 3·13	19·33 ± 1·29
June 23	327·06 ± 4·43	48·71 ± 3·13	14·89 ± 0·98
June 26	378·86 ± 4·57	50·21 ± 3·23	13·25 ± 0·87
June 30	422·50 ± 4·66	51·19 ± 3·29	12·13 ± 0·79
July 3	477·04 ± 5·66	62·23 ± 4·00	13·05 ± 0·85
July 7	622·50 ± 10·10	111·02 ± 7·14	17·83 ± 1·18
July 10	750·23 ± 11·26	123·82 ± 7·96	16·50 ± 1·09
July 14	916·14 ± 13·66	150·24 ± 9·66	16·40 ± 1·08
July 17	1026·59 ± 13·03	143·29 ± 9·22	13·96 ± 0·92
July 21	1170·23 ± 10·93	120·22 ± 7·73	10·27 ± 0·67
July 24	1263·41 ± 8·46	93·01 ± 5·98	7·36 ± 0·48
July 28	1297·50 ± 8·57	94·18 ± 6·06	7·26 ± 0·47

Table 4. Constants of Variation in Height of Plant for Series D.

Date of Measurement	Mean	Standard Deviation	Coefficient of Variation
July 6	359·62 ± 10·67	80·64 ± 7·54	22·42 ± 2·20
July 10	464·86 ± 9·76	105·29 ± 6·90	22·65 ± 1·56
July 13	613·89 ± 12·93	140·86 ± 9·14	22·95 ± 1·57
July 17	811·57 ± 15·23	165·88 ± 10·77	20·44 ± 1·38
July 20	1018·52 ± 14·92	162·50 ± 10·55	15·95 ± 1·06
July 24	1220·83 ± 11·51	125·44 ± 8·14	10·27 ± 0·67
July 27	1284·72 ± 10·86	118·28 ± 7·68	9·21 ± 0·60
Aug. 3	1301·85 ± 10·52	114·60 ± 7·44	8·80 ± 0·58
Aug. 10	1301·39 ± 10·58	115·21 ± 7·48	8·85 ± 0·58
Aug. 24	1301·39 ± 10·56	111·01 ± 7·46	8·84 ± 0·58

Table 5. Constants of Variation in Height of Plant for Series E.

Date of Measurement	Mean	Standard Deviation	Coefficient of Variation
July 6	340·44 ± 6·75	41·28 ± 4·77	12·13 ± 1·42
July 9	416·94 ± 7·25	72·15 ± 5·13	17·30 ± 1·27
July 13	553·07 ± 9·57	103·25 ± 6·76	18·67 ± 1·27
July 16	701·39 ± 11·84	128·96 ± 8·37	18·39 ± 1·23
July 20	904·17 ± 13·04	142·05 ± 9·22	15·71 ± 1·04
July 23	1132·41 ± 12·84	139·87 ± 9·08	13·35 ± 0·88
July 27	1317·59 ± 11·05	120·39 ± 7·81	9·14 ± 0·60
Aug. 3	1385·65 ± 11·39	124·13 ± 8·06	8·96 ± 0·59
Aug. 10	1385·19 ± 11·44	124·63 ± 8·09	9·00 ± 0·59
Aug. 24	1383·80 ± 11·39	124·13 ± 8·06	8·97 ± 0·59

Table 6. Constants of Variation in Height of Plant for Series F.

Date of Measurement	Mean	Standard Deviation	Coefficient of Variation
July 7	401·01 ± 7·57	68·29 ± 4·39	17·03 ± 1·37
July 10	505·87 ± 11·58	95·80 ± 8·19	18·94 ± 1·34
July 14	665·68 ± 13·11	144·19 ± 9·27	21·66 ± 1·46
July 17	805·23 ± 15·51	170·59 ± 10·97	21·18 ± 1·42
July 21	1032·95 ± 15·01	164·99 ± 10·61	15·97 ± 1·05
July 24	1233·41 ± 11·90	130·86 ± 8·42	10·61 ± 0·69
July 28	1333·86 ± 10·32	113·43 ± 7·29	8·50 ± 0·55
Aug. 4	1352·95 ± 10·96	120·53 ± 7·75	8·91 ± 0·58
Aug. 10	1357·50 ± 10·81	118·91 ± 7·65	8·76 ± 0·57
Aug. 24	1352·95 ± 10·89	119·77 ± 7·70	8·85 ± 0·57

Changes in the Mean Height.

The means of the several series as given in tables 1 to 6 are shown graphically in fig. 1.

From this figure it is noted that the plotted means give in each case a relatively smooth growth curve. It has been shown by a number of authors, particularly in the excellent papers by Donaldson and Hatai, that the growth of an organism can usually, if not always, be described by a logarithmic curve of the general type,

$$Y = a + b \log x.$$

There is no longer any question as to the value of such curves in describing growth data. Recently Hatai ('11) has been able to give a more definite physiological interpretation of the logarithmic curves. The present growth data have not been fitted with theoretical curves chiefly because we desired to devote our efforts to other growth problems. Further the character here measured (height of plant) does not take account of the whole of the growth of the corn plant. This fact introduces certain peculiarities into the data.

If we could have taken the weight of the entire plant at each date, the growth curves would undoubtedly have been quite different. As will be pointed out later, there is considerable evidence of cyclical periods of growth in the corn plant. Thus for some time after the corn comes through the ground it is probable that most of the growth occurs in the root system. This growth is not shown in our measurements. Again, as will be pointed out below, as soon as the tassel blooms the plant absolutely ceases to grow in height, although the ear continues to grow very rapidly. Thus our data show the growth to cease very suddenly about August 1, while as a matter of fact the plant continues to gain in weight for a long time after this. However, these facts in no way invalidate the present data for the purpose for which they were collected (cf. Part II).

Considering first the leaf measurements (Series A, B and C) the curves as they stand may be divided into two principal phases. The first phase extends from the beginning of the measurements to July 3 and the second from July 3 to July 27 at which time the plants have completed their growth, so far as leaf measurements are concerned. The first of these phases could be fitted reasonably well by a straight line although if we had measurements from the time the plants came through the ground we should probably find the curve tailing off

somewhat differently towards the zero point¹⁾). After July 3 it is evident that the plants in all the series are growing much more rapidly, and that this increased rate of growth is maintained until the final height is reached. Now by July 3 tassels were showing in the majority of plants although these were not sufficiently advanced to measure until July 6. It seems unquestionable that the increased rate of growth (as measured by the height of the plant) is associated with the formation of the tassel. The question naturally arises as to whether the approaching maturity of the sexual organs is in any way responsible for this change in the rate of growth. Thus we know that in many mammals, for example man, the growth curve changes very markedly before puberty leading to the so-called „pre-pubertal acceleration“. Further in certain plants such as the common dandelion there is a very rapid elongation of the seed stem with the ripening of the seed.

While this more rapid elongation of the internodes in the corn plant may be influenced by the approaching maturity of the male sexual elements the following facts should also be considered. At the time the tassel appears practically all the leaves of the corn plant are fully formed. Further, from general observation supported by a large number of measurements made by ourselves, it is certain that a corn leaf never grows after it is fully unfolded, and we believe that the actual growth ceases some time before this. That is, the final unfolding of the leaf is due to the lengthening of the internodes. Thus, with the completion of the leaves, by the time the tassel emerges the growth forces are directed into another channel viz., the lengthening of the internodes. This explanation is simpler and will probably account for many of the facts unless it can be demonstrated that the male sexual products in themselves are capable of accelerating the growth. This question will be referred to again in connection with the variability.

The curves obtained from the tassel heights (series D, E and F) show a still more rapid growth than the leaf measurements during the same period. This arises from the fact that in measuring the tassel height we are measuring only the length of the internodes, and further

¹⁾ It will be noted that there is a slight drop in all the curves at June 19. The reason for this is not clear. There were no meteorological conditions which should have affected the corn adversely at this time. We find from our notes, however, that the plots were given a hoeing on June 15 and 17. It is possible that the root pruning at this time may have caused the slight drop in the curves. This question will be discussed more fully in connection with the variation constants.

that the leaf measurements cannot take account of the growth of the internode above the last leaf. In the later periods this internode grows faster than any below it.

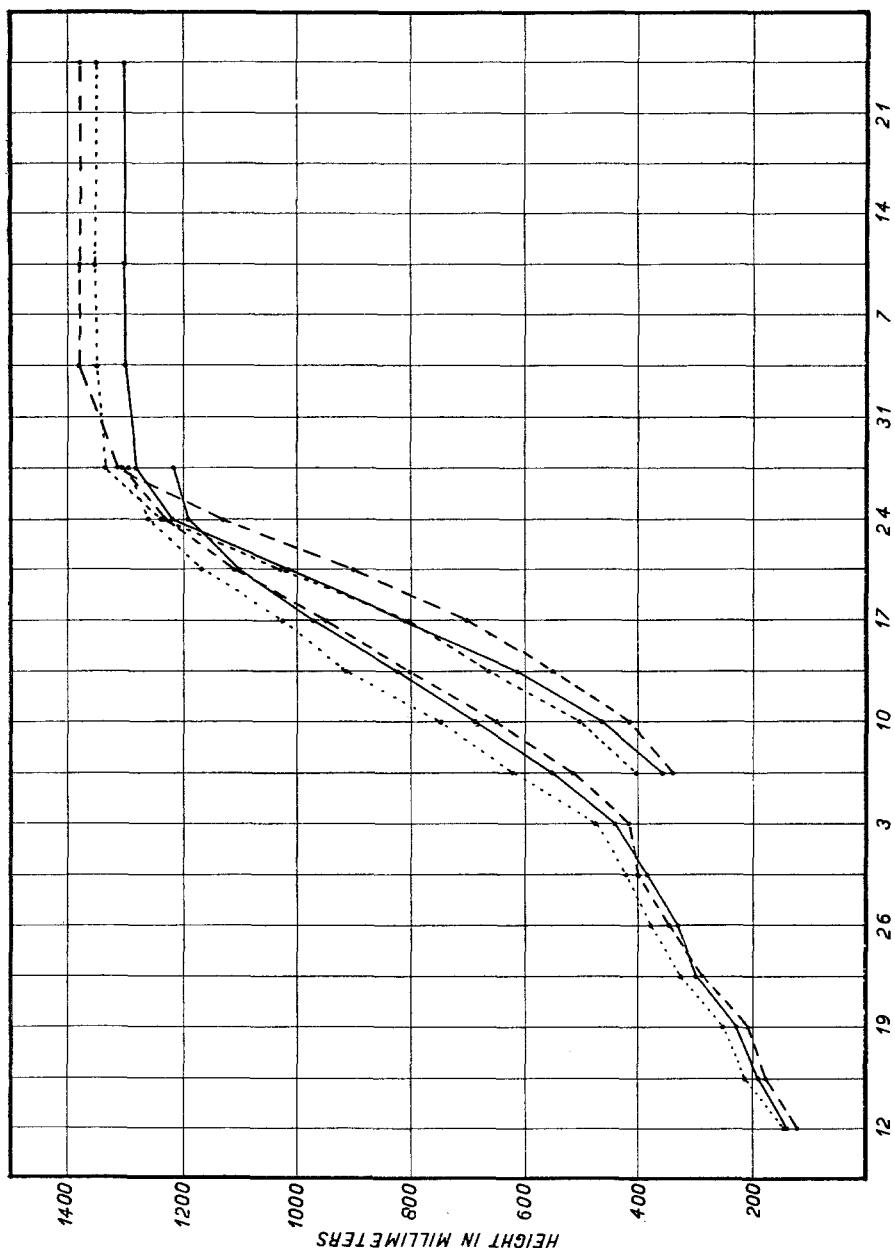


Fig. 1. Showing the mean height of plants in each of the six series and for each successive measurement.
 Series A and D —; Series B and E - - -; Series C and F

Fig. 1 also shows that the growth ceases rather abruptly. This abruptness is still more marked in the case of individual plants than in the means. Observations in the field indicate that this cessation of growth is very closely associated with the blooming of the tassel. These observations open the question as to the cause of the termination of growth in a corn plant. Stated in general terms, it seems most probable that after the fertilization of the ear the growth forces of the plant are directed towards the formation of the grain instead of the elongation of the internodes. The immediate causes of this change in the direction of growth present a physiological problem which will not be dealt with in the present paper.

Fig. 1 brings out certain differences between the several series of measurements which are of interest. Considering first the leaf measurements we note that the average height of series C which consists of plants from different mother ears is considerably higher than either of the other series. At the beginning of the measurements, June 12, this series showed an average height of only 4 millimeters above that of series A. Thus the subsequent difference in the height of series C was probably not due to the unconscious selection of more vigorous plants.

Series A and B run nearly parallel with the exception of the points near June 29 and near the end of the growth period (July 20 to 27). Now it so happened that there was a marked difference in the time of maturity of these two rows. Row No. 131 (series A) was the earliest maturing row in the field, while row No. 133 (series B) was one of the very latest rows. These differences are shown very clearly in the two curves. Thus the majority of the plants in series B tasseled later than those in series A. This accounts for the lag of the former curve at the first of July. At the end of the growing season the tassels matured much earlier in series A and these stopped growing while series B was growing at almost the same rate as before.

A similar difference is to be noted in the curves for tassel heights. Thus series E, from the same plants as series B, shows a marked lag behind the other two series until near the end of July. At this time the curve for series E crosses both the other curves and results in plants whose final average height is considerably greater than that of the other series. This of course, comes about from the fact that the plants in series E had a longer growing period. The difference between series D and E is shown very nicely in the curves, and indicates a genotypic difference in these several respects. Series D corresponded

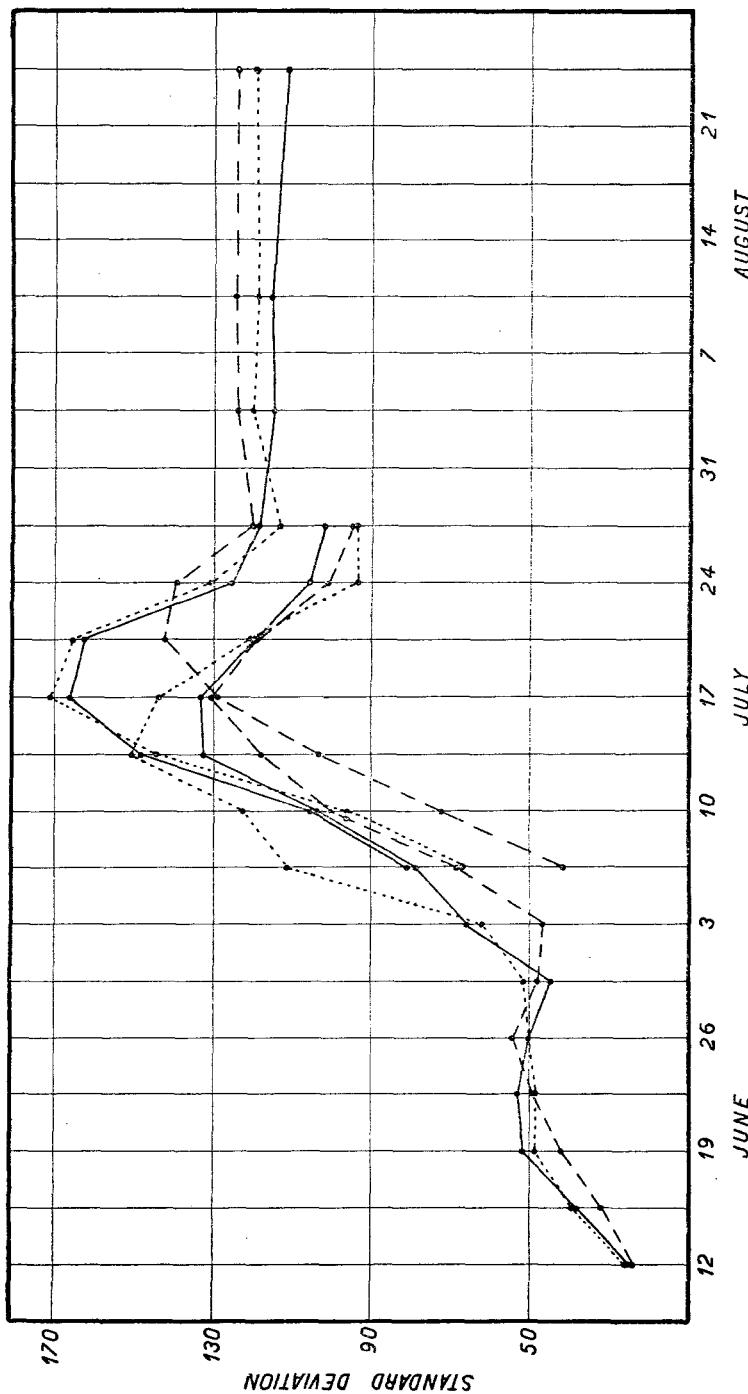


Fig. 2. Showing the changes in the standard deviation for the height of plants in each of the six series and for each successive measurement. Series A and D ——; series B and E - - - ; series C and F

with the random sample of plants of this variety (series F) in respect to final height. But as clearly shown by the curves, the plants of series D began to mature much earlier than those of the other series.

Changes in the Standard Deviation.

Tables 1—6 give the standard deviations and the coefficients of variation for the different dates and for each series. Fig. 2 shows the standard deviations for all six series. The standard deviation is a measure of the absolute dispersion, and in cases like the present where the mean is changing so rapidly, it cannot be taken as a good measure for comparing the variability of the successive growth stages. Still, certain points of interest can be brought out by a study of these constants.

From Fig. 2 we note that there is a gradual increase in the standard deviation in the early portion of the curves. From about July 1 to 15 there is a marked increase in the size of this constant. This increase is undoubtedly associated with the tasseling of the plants. Those plants on which the tassels appear first, grow faster and in this way increase the dispersion of the individuals. After the middle of July all tassels have appeared and the individuals become less scattered about their means.

In the above discussion it should be remembered that the rapid increase in the size of the standard deviation is associated with a rapid increase in the mean of the observations. In general the larger the size of the thing measured the larger its absolute variability will be. The changes in the relative variability at this period will be shown in the discussion of the coefficients of variation.

Changes in the Coefficient of Variation.

The changes in the coefficients of variation for the successive measurements and for all six series are displayed graphically in Fig. 3. It will be noted that on the whole the relative variability decreases and is, on the average, about ten percent lower in the matured plants than at the beginning of the season. Such a decrease in the relative variability is characteristic of growth curves. This is shown in the statistics of the growth of children (*vide infra* p. 114). MINOT ('91) first discussed it in connection with the growth of guinea pigs and PEARL ('07) has developed the same idea in the case of intraindividual growth of the leaf whorls on *Ceratophyllum*. This law of diminishing variability appears to be of fundamental significance.

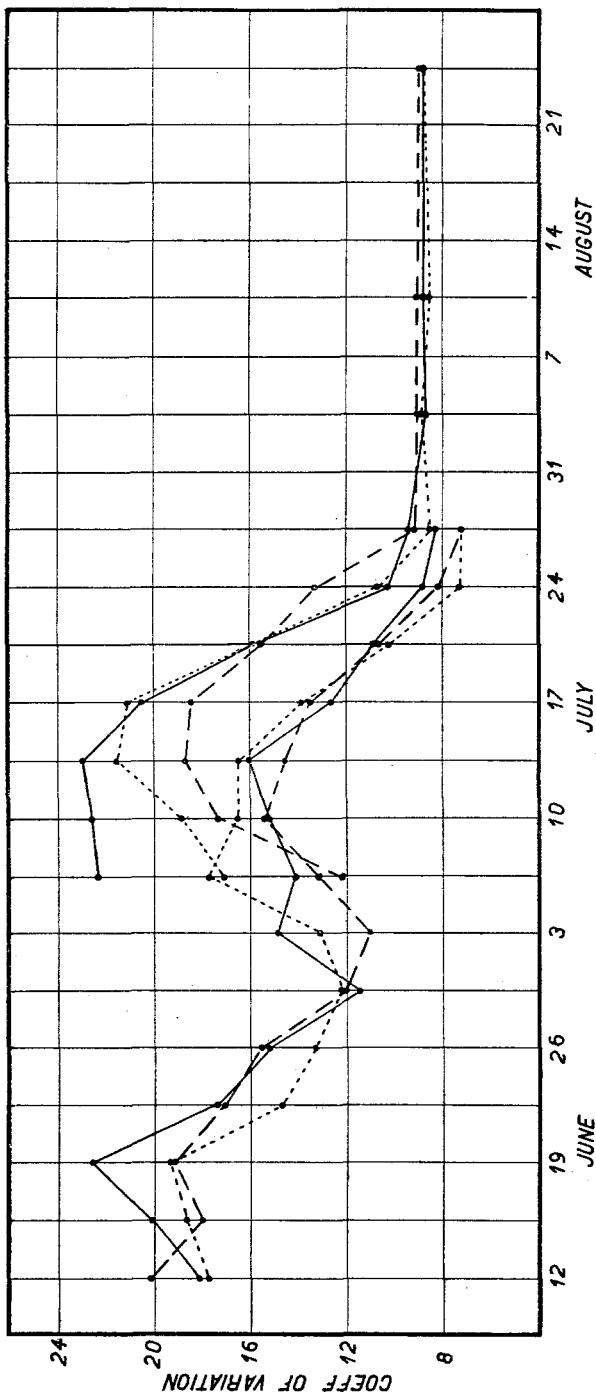


Fig. 3. Showing the changes in the coefficient of variation of the height of plants at the successive measurements, in each of the six series. Series A and D ——; series B and E - - - ; series C and F ······.

An examination of Fig. 3 will make it clear that in the growth of the corn plant the decrease in variability is not equal in all periods of growth. In other words the change in variability cannot be represented by a straight line. In general, there is a marked increase in variability during the early stages of growth, i. e., up to June 19. This is followed by a very rapid decrease up to June 29. During the tasseling period there is another very marked increase in variability and this is again followed by a gradual decline until a stable condition is reached at maturity.

The cause of the increase in variability up to June 19 is not clear. Series A and C show a consistant increase in variability up to this date, while series B shows a decrease at the second measurement, but again increases at June 19. In discussing the growth curves (p. 106 foot note) it was noted that there was a slight sag in the curves at June 19. It was suggested that this may have been caused by the hoeing which the corn received a few days previous. Such a disturbance would also tend to increase the absolute variability. Referring to Fig. 2 it is seen that the standard deviation increases up to June 19 and then remains practically constant until the plants begin to tassel. If the increased variability were due to hoeing or other environmental circumstances acting for a short time only we should not expect to get such a gradual rise in these curves. It seems, therefore, that some biological significance is to be attached to this increased variability in the early growth period.

It is well known that in the growth of animals there is a period of increased variability following birth. This is to be explained by the fact that the young animals are becoming adjusted to a new mode of life. Some become adjusted earlier than others and tend to grow faster. Later when all have become accustomed to the new environment they tend to grow at the same rate and hence maintain the same variability. In the early growth stages of the corn plant the chief growth must occur in the root system. Plants must vary considerably, in the development of the roots and in the supply of food material obtained. Plants which develop their root systems earliest will grow faster than the others and hence increase the variability. After about the middle of June all the plants have developed their root systems and tend to grow at the same rate. This appears to be the most plausible explanation of the increase in variability at the beginning of the season.

After June 19 there is a period of rapid decrease in the relative variability. This decrease continues until the plants begin to tassel or about the first of July. This decrease in the coefficient of variation is probably an expression of the general growth law of diminishing variability referred to above. It will be noted from Fig. 2 that during this period the absolute variability remains nearly constant indicating that the plants are all growing at about the same rate. This would indicate that plants retarded during the first growth period do not make up for their loss during this second period.

With the exception of series B the plants began to tassel about July 3, and there is no noticeable increase in the average rate of growth (Fig. 1) before this time. However, the effect on the variability is noticeable earlier than this. Thus series A, which was the first to tassel, showed only three tassels on July 3, yet at this date it exhibits a marked increase in its variability, both absolute and relative, over that of the preceding measurement (June 29). During the period in which the tassels make their appearance, July 3 to 10, there is a very rapid increase in variability. This means that certain plants, having a tendency to tassel earlier than the others, grow in height much faster than their neighbors during this period. After the tassels have appeared on all the plants, the variability, both absolute and relative, begins to decrease. This decrease is quite rapid and continues until all the plants have attained their final height. This decrease in absolute variability indicates that here those plants which tasseled first and grew very rapidly grow much slower as they approach maturity, while the late tasseling plants grow very rapidly and thus tend to decrease the dispersion about the mean. In the later measurements (July 20 and 24) some of the plants have already attained their final height, while others continue to grow.

It is of interest to note that in the tassel measurements (series D, E and F) the variability, both absolute and relative, reached a greater height than in the case of the leaf measurements (series A, B and C). Further the decrease in variability begins much earlier in the leaf measurement series than in the tassel series. These facts are in agreement with those previously discussed, and shown in Fig. 1, namely that the plant reaches its final height as shown by leaf measurements much earlier than in the case of the tassel height. Thus the upper node, of which the tassel is a part, continues to grow after the lower nodes have ceased. The changes in variability show that the growth

stimulus due to tasseling continues to exert itself as long as the upper node is growing.

The effect of tasseling upon the growth and variability of the corn plant is not unlike the effect of puberty upon the growth of children. To show the similarity in the form of these variation curves we have calculated the coefficients of variation for the stature of school children from the age of five and one-half years up to eighteen and one-half. These coefficients were calculated from the means and standard deviations given by BOAS ('97). The particular data used were the growth of American boys and girls given on pages 1555 and 1566 of his paper. These coefficients are shown in Fig. 4.

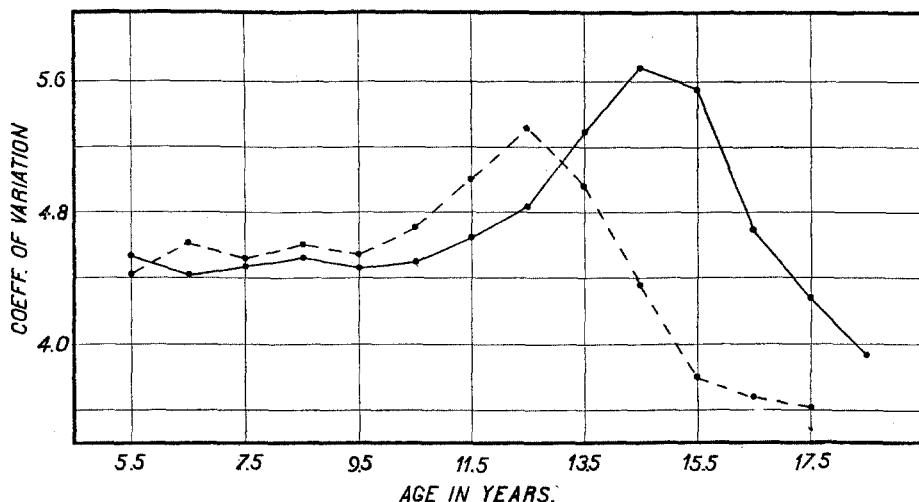


Fig. 4. Coefficients of variation for the stature of children; —— boys; - - - girls.

From this figure it is seen that the onset of puberty greatly increases the variability. After the age of about thirteen for girls and fifteen for boys there is a very rapid decrease in the variability until it reaches a point considerably lower than its value before the beginning of puberty.

In these respects the effect of puberty is not unlike the effect of tasseling upon the growth of the corn plant. However, this does not mean that puberty and tasseling are necessarily analogous. We have already indicated that one of the reasons for the increased growth of the corn plant after tasseling is that the leaves are all formed by this time and hence the growth energies are directed towards the elongation

of the internodes. Whether the approaching maturity of the sexual organs affects in any way the growth of the plant is a physiological subject worth investigating, but upon which we have no exact data. It would appear that the rapid elongation of the flower stalk in many plants or of the seed stalks, as in the common dandelion, was in some way dependent upon the reproductive organs. It is possible that the same is true with the corn plant in which case the effect of tasseling would be quite analogous to that of puberty in mammals.

The facts brought out by these growth data strongly suggest that there are certain well defined cycles in the growth of the corn plant. Owing to the nature of the data, and to the fact that all of the early growth is not included in these measurements, it is not possible to present definite proof of all of these cycles. However, it will be worth while to indicate their existence, and to show the evidence upon which our conclusions rest.

In all, we can distinguish four such cycles. The first of these may be called the root growth cycle. This may be described as extending from the beginning of growth until about June 19 in the present data. This period is characterized by the rapid growth of the root system. The evidence for the existence of this cycle is indirect and is based chiefly upon the behavior of the variation constants during this period. It was pointed out above that there was a marked increase in the relative variability up to June 19. These changes are probably caused by variation in the time of the establishment of the root system by the various plants.

The second growth period may be called the leaf cycle. It is characterized by the development and growth of the leaves. It extends from about June 19 to the beginning of tasseling, about July 1. Naturally these growth cycles overlap each other to some extent. The periods given are those particularly characteristic. This cycle is characterized by a steady but moderate increase in height. The absolute variability remains practically constant, but there is a very marked decrease in the relative variability with reference to the height of plant. During this period the main growth energies of the plant are directed towards the development of the leaf system, and growth in height is a relatively secondary matter.

The third growth period is the tassel cycle, and is characterized by the rapid elongation of the upper internodes of the plant. This cycle extends from the beginning of tasseling, about July 1, until the blooming

of the tassel and the resulting fertilization of the ear, about August 1. During this time there is a very rapid increase in the height of the plant. Also during the first part of this cycle there is a rapid increase in both absolute and relative variability. This is followed in the second portion of the period by a decrease in variability, so far as the height of the plants is concerned.

This period is followed by the fourth or ear growth cycle. This extends from the time of fertilization until maturity. During this period there is no increase in the height of the plant. Since we have no data regarding the growth of the ear itself this cycle cannot be discussed in detail.

While there is much in the literature to indicate that growth often takes place in cycles, it appears from the present data that these cycles are particularly well marked in the corn plant.

Summary of Part I.

Measurements were made at twice-a-week intervals of the height of three series of maize plants. The heights were measured to the tip of the tallest leaf. In addition to these, separate sets of measurements were also made of the tassel height as soon as the tassels appeared.

The growth curves obtained by plotting the mean height at each measurement are relatively smooth. (Fig. 1.).

After July 3, the time of tasseling, the plants grow in height much faster than before. Growth in height ceases entirely as soon as the tassel blooms.

The absolute variability (Fig. 2) shows a marked increase up to about June 19. From this until the time of tasseling it remains nearly constant but shows a very great increase at the time of tasseling. After all the plants have tasseled the absolute variability decreases somewhat.

The relative variability considered for the whole season (Fig. 3) shows a marked progressive diminution. It thus follows the general growth law of diminishing variability. Considered in detail, however, the relative variability first shows an increase. After June 19 there is a rapid decrease until the time of tasseling. During the period of tasseling there is a very rapid increase in the relative variability. This is followed by an equally rapid decrease. A stable condition is finally reached which is some ten percent lower than the variability at the beginning of the season.

It is interesting to note that the relation of tasseling to the growth and variability of the height of corn plants is not unlike the relation of puberty to the growth of children. This does not mean that the two processes are necessarily analogous.

From the data presented it appears probable that the corn plant grows in a series of cycles. Each cycle is characterized by the special development of one set of organs. They are, in order, the root cycle, the leaf cycle, the tassel cycle and the ear cycle. The reasons for the postulation of these cycles is given in the text.

Part II.

The Relation of Variation to Growth.

In Part I of this paper we have discussed the general growth curves and the question of variability for the height of corn plants at the various growth stages. In the present portion we shall deal with a phase of the growth problem which has hitherto received but little attention.

What it is desired to ascertain is: how individual plants or groups of plants having the same relative size at one stage are distributed in the remaining growth stages? In other words, is there a definite tendency for plants relatively small at one stage of their growth to be relatively small in the other growth stages? Or again, is there a tendency for certain individual plants to maintain a certain relative size throughout the growth period, indicating perhaps a genotypic difference in this respect? Or taking the other side of the question, is there a tendency for plants relatively large at an early stage to grow slower, and for plants relatively small at an early stage to grow faster, so that in the end, plants of both kinds will, on the average, occupy a mean position? What is the extent or degree of any of these tendencies which may be found to exist?

In the main such problems have been neglected by students both of growth and of genetics. It seems, however, that some points of interest may be gained by considering in detail the individual plants which go to make up both the curve of growth and the frequency (or variation) curve. Such methods have been of great value in the study of genetics, and it seems probable that they may be of value in other related physiological problems. The present paper attempts to make a beginning in this direction.

Quintile Distributions.

It would be possible to gain some ideas regarding the problems discussed above by plotting the several frequency distributions, and then noting the position of individual plants in each successive distribution. Such a method, however, would give us but little idea of the general tendency of any group of plants. The best method of attacking these problems has seemed to be, first to divide each frequency distribution into percentiles, and then to follow the groups of plants which fall in these several areas. In this way one can study the relative variability of individual plants in the successive stages entirely apart from the change in absolute size.

While all plants increase in absolute size with the advance in the season, there are always some plants which are relatively smaller, for example, than the remainder. If it is determined, for each measurement, which plants fall in the lower one-fifth of that frequency distribution, these will be the relatively small plants in each case. Now is it the same plants which occupy this lower one-fifth of each distribution? Or are the plants which are at this end of the first distribution just as likely to lie somewhere else in the succeeding distributions?

Considering the number of plants in each series it has seemed best to divide each distribution into five parts or quintiles. In doing this each frequency distribution was divided at such points that each of the resulting five parts has the same area. In other words one-fifth of the number of plants in the given series fell in each quintile. The quintiles are numbered I, II, III, IV and V, beginning at the lower end of the range. In any distribution the relatively small plants are in quintile I and the relatively large ones in quintile V.

The exact meaning of a quintile will be made clearer by the accompanying diagram (Fig. 5). In this diagram a normal curve has been divided into five equal areas by the four perpendicular lines. The frequency in any one of these areas is equal to the frequency in any other.

The ungrouped data have been used in determining the quintile limits. Even with ungrouped measurements it has been impossible to make all the areas exactly equal. This is always likely to be the case when one is dealing with discrete variates and not large numbers. Thus, in two of the series there are fifty-four plants. If these are divided into

five equal parts there would be 10·8 plants in each part. Yet for present purposes it is obviously impossible to consider that part of a plant falls in one quintile, and the remainder in another, at a given measurement. In such cases the best division that can be effected is to have 11 plants in each of four quintiles and 10 plants in the fifth. In other cases two or more plants having the same measurement may fall on the dividing line between two quintiles. In such cases it is necessary to include all of these plants in one or the other quintile. In such cases as this the so-called quintiles do not have equal areas. However, with the ungrouped data, there are very few cases where

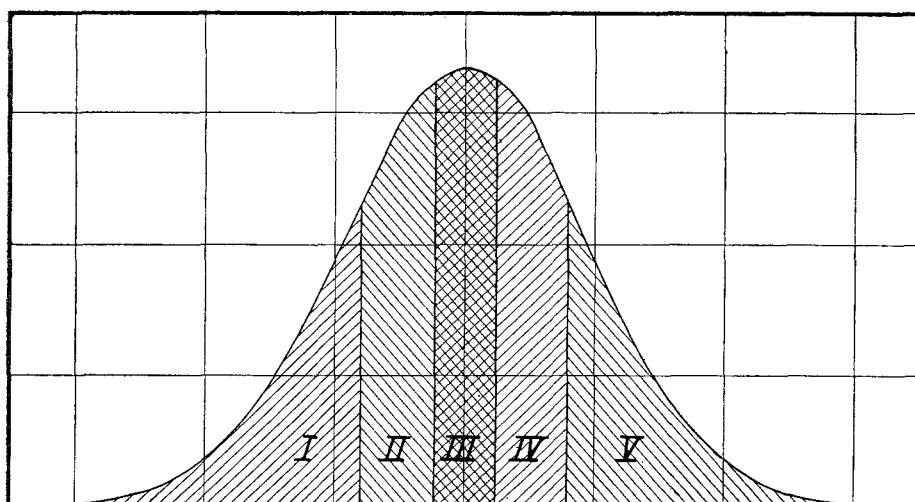


Fig. 5. Diagram to illustrate the quintile division of a frequency curve.

more than one plant falls on a quintile boundary line. In series A and B, each of which contain 54 plants, the convention of allowing 11 plants in each of the first four quintiles and 10 plants in the fifth has been adopted. Thus the fifth quintile is not equal to the others in area but in this way the successive distributions are made comparable one with another. In series C there are fifty-five plants so that each quintile contains 11 plants in nearly every distribution. Wherever it has been possible allowance has been made for these defects in the quintile distributions (cf. p. 123).

In order to get the data before us for discussion it will be convenient to give tables showing for the three series the quintile distri-

butions for the successive measurements, first of the plants which start in given quintiles and second of the plants which end in given quintiles. These distributions are given in tables 31—60. In these tables there are also given the mean quintile position, and the standard deviation of the quintile distribution, of the plants for each measurement. It will be more convénient to discuss these constants after certain other features of these distributions have been considered.

The quintile deviations of plants starting in a given quintile.

In tables 31—45 there is given in each case the total number of observations which fall in a given quintile for the whole season, exclusive of the measurements at the beginning of the season. In Table 31, for example, there were 11 plants which were in quintile I of the frequency curve resulting from the first measurement, June 12. On June 15 seven of these plants were again in quintile I of that distribution, three were in quintile II and one was in quintile III. Now if we exclude the measurement on June 12, because, by hypothesis, all these plants then fell in quintile I, there are thirteen distributions remaining, each containing eleven observations, or in all 143 observations. From the sums at the foot of this table it is seen that of these 143 observations, 69 fell in quintile I, 41 in quintile II, 20 in quintile III, 13 in quintile IV and none in quintile V. Or, transferring to percentage figures, 48·25 percent of the observations made on these plants fell in quintile I, 28·67 percent in quintile II, and so on. Stating it in another way, it may be said that the eleven plants starting in quintile I spent half their time (48·25 percent) in that same quintile, and only one-fourth their time (28·67 percent) in quintile II, and still less in any one of the remaining quintiles.

There is, then, a marked tendency for the plants which were relatively small at the beginning of the season to have remained, on the average, relatively small throughout most of the season. In order to bring these facts for the several tables into a more condensed form, as well as to show certain other relations, we have collected these figures for plants starting in each quintile, and for each series in tables 7, 8 and 9. In these tables there are given both the actual and percentage frequencies of the observations falling in the several quintiles, for plants starting in the given quintile. Thus the first row of table 7 shows for series A, the number and the percent of observations falling in the different quintiles for the plants which start in quintile I.

The second row shows the same for plants which start in quintile II and so on.

Table 7.

Series A.

Showing the number and percent of the measurements falling in the several quintiles for plants starting in a given quintile.

Quintile in which plants start	Quintile number										Total number	Root-mean-square deviation	Standard deviation of the root-mean-square deviation			
	I		II		III		IV		V							
	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent						
I	69	48·25	41	28·67	20	13·99	13	9·09	0	0	143	16·34	3·43			
II	24	16·78	39	27·27	37	25·87	30	20·98	13	9·09	143	5·94	3·43			
III	7	4·49	24	15·38	39	25·00	44	28·21	42	26·92	156	9·43	3·29			
IV	26	20·00	25	19·23	30	23·08	27	20·77	22	16·92	130	1·55	3·59			
V	19	14·62	14	10·77	17	13·08	27	20·77	53	40·77	130	11·64	3·59			
Total	145		143		143		141		130		702					
Theoretical mean		20·66		20·37		20·37		20·08		18·52						

Table 8.

Series B.

Showing the number and percent of the measurements falling in the several quintiles for plants starting in a given quintile.

Quintile in which plants start	Quintile number										Total number	Root-mean-square deviation	Standard deviation of the root-mean-square deviation			
	I		II		III		IV		V							
	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent						
I	90	57·69	34	21·79	18	11·54	12	7·69	2	1·28	156	19·57	3·29			
II	39	27·27	46	32·17	32	22·38	17	11·89	9	6·29	143	9·00	3·43			
III	8	6·15	31	23·85	32	24·62	47	36·15	12	9·23	130	10·74	3·50			
IV	7	4·90	27	18·88	42	29·37	30	20·98	37	25·87	143	8·77	3·43			
V	0	0	6	4·62	18	13·85	36	27·69	70	53·84	130	20·08	3·50			
Total	144		144		142		142		130		702					
Theoretical mean		20·51		20·51		20·23		20·23		18·52						

Table 9.

Series C.

Showing the number and percent of the measurements falling in the several quintiles for plants starting in a given quintile.

Quintile in which plants start	Quintile number										Total number	Root-mean-square deviation	Standard deviation of the root-mean-square deviation			
	I		II		III		IV		V							
	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent						
I	78	54·54	35	24·48	21	14·69	8	5·59	1	0·70	143	19·08	3·35			
II	17	11·89	51	35·67	37	25·87	23	16·08	15	10·49	143	9·49	3·35			
III	10	6·99	14	9·79	36	25·17	43	30·07	40	27·97	143	9·64	3·35			
IV	32	22·38	22	15·38	29	20·28	39	27·27	21	14·68	143	4·58	3·35			
V	6	4·20	21	14·68	20	13·99	31	21·68	65	45·45	143	13·94	3·35			
Total	143		143		143		144		142		715					
Theoretical mean		20·00		20·00		20·00		20·14		19·86						

From these tables it is clear that there is, in most instances, a tendency for the larger proportion of the observations to fall in or near the particular quintile in which the plant started. This tendency is to be expected on general grounds. However, it is well worth while to obtain a measure of it and to study its changes in the several cases.

In order to obtain such a measure it is necessary to consider certain aspects of the theory of probability. Let us take first the simplest case which is nearly realized in series C (Table 9). Here it will be remembered there were 55 plants thus making 11 in each quintile¹⁾. Each plant was measured 14 times, but since the plants have been grouped according to the quintile in which they started, we must exclude this initial measurement if we wish to preserve an analogy with the conditions of simple sampling. Consequently there are for consideration 13 measurements of each plant, or a total of 143 measurements for the 11 plants starting in each quintile (Table 9). Now if there were no influence except that of chance acting upon these corn plants it is clear that the observations would tend to fall in equal

¹⁾ In one distribution of the series there were 12 plants in quintile IV and 10 in quintile V so that there is a slight irregularity in these two quintiles.

numbers in each quintile. That is, since p , the chance of success, is $1/5$, and n is 143, the mean number falling in each quintile would be $np = 28.6$ observations, or 20 percent.

In order to make the following discussion clear we may consider its relation to throws of dice. If only chance were acting on the corn plants the results would be exactly comparable with those obtained by making 11 throws of thirteen unbiased 5-faced dice. The expectation here for the mean number of successes of each face would be 28.6 or 20 percent as given above.

Further the standard deviation of this result would be

$$\begin{aligned}\sigma &= \sqrt{npq} \\ &= \sqrt{143 \times \frac{1}{5} \times \frac{4}{5}} \\ &= 4.78 \text{ or } 3.34 \text{ percent.}\end{aligned}$$

All or practically all the observations ought to fall within a range of six times the standard deviation. Thus, if no influence other than chance were acting, no observation above 30 percent and none below 10 percent would be expected with very few deviating so far as these figures from the mean.

It is, of course, possible to observe the actual deviation of these corn plants from this theoretical mean value. The theoretical mean furnishes a base from which can be measured the tendency of plants to remain relatively small, or relatively large, as the case may be, throughout their life histories.

Before discussing these deviations however it is necessary to note that in the majority of these quintile distributions p is not $1/5$, but differs slightly from this value. Thus in Table 7 there are in all 702 observations. Of these, 145 fall in quintile I. Thus the chance of success, p , for observations falling in quintile I is $145/702$, which reduced to percentage gives 20.66 percent, instead of 20.00 percent. Again in quintile V there are only 130 observations which gives a theoretical mean of 18.52 percent. As already stated, these deviations are due to the fact that all the quintiles are not of exactly equal areas. This method of calculating the probability makes allowance for these deviations. In Tables 7 to 9 the figures in the percentage columns in the last row of each table give these theoretical mean percentages on the basis of pure chance.

The standard deviation

$$\sigma = \sqrt{npq}$$

has been calculated for each case and found to vary from 3·2 to 3·5 percent, with an average of about 3·34 percent. Since this deviation is comparatively slight it has not been thought necessary to encumber the tables with these figures. All the necessary data are given for the calculation of these constants if anyone desires them.

The data given in Tables 7 to 9 are shown graphically in Fig. 6.

From these figures it is at once noted that the plants starting in quintiles I and V show very marked deviations from the theoretical means. The greatest positive deviations are, in each of these cases, in the quintile in which the plants start. The greatest negative deviations occur in the quintile farthest removed from that in which the plants start. This of course indicates a strong tendency for plants which are relatively small at the beginning of the season to remain relatively small throughout their growth. Likewise relatively large plants tend to remain relatively large.

An idea as to the significance of these deviations can be obtained by comparing them with the limits of 3σ shown by the dotted lines in the figure. If the quintile distributions had been governed by the conditions of simple sampling none, or very few, of the observations would deviate so far from the mean value as 3σ .

Turning now to plants starting in quintiles II, III and IV, it is evident that, in no instances, are there such wide deviations from the theoretical mean. In some cases, for example plants starting in quintile IV for series A and C, the deviations are little if any greater than might be expected on the theory of probability alone. In some instances, especially the plants starting in quintile II, there is a tendency for a large percentage of the observations to fall in this quintile. In other cases there does not appear to be any greater tendency for the measurements to fall in the quintile in which the plants started than in a neighboring quintile.

In general then there is not such a marked tendency for medium sized plants to remain medium sized throughout the season as has been noted for the extremes. Instead some of the medium sized plants become relatively large and some become relatively small, so that for the whole season there is a much more even distribution of their measurements than of the very large or the very small plants. The significance of this fact will be considered after we have discussed a method of measuring these deviations.

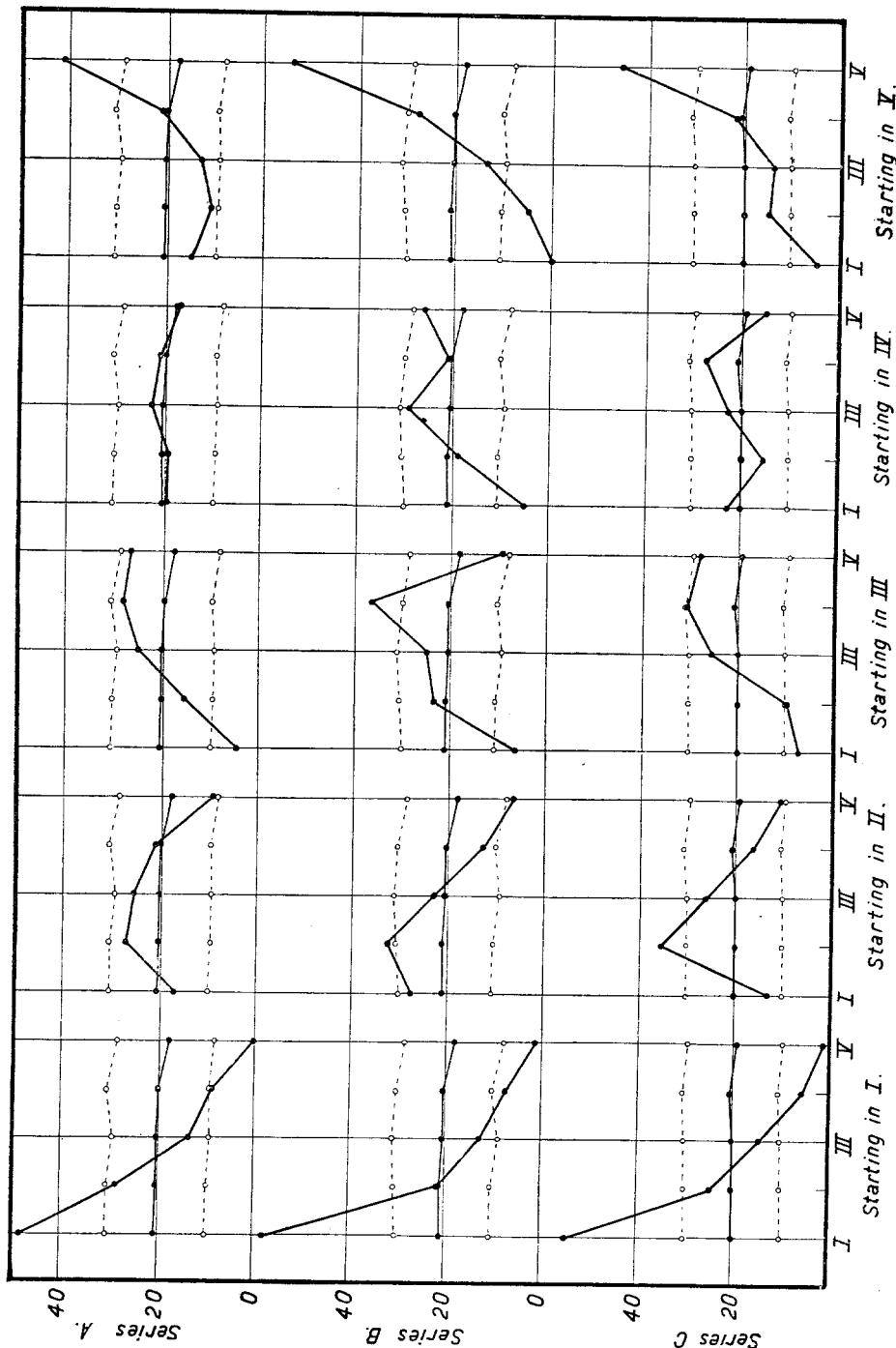


Fig. 6. Showing, for plants starting in a given quintile, the percentage of the total number of observations which fall in each quintile. The theoretical mean percentages on a basis of chance are also plotted. The broken lines show the limits of 3σ measured from these theoretical means.

From the method of obtaining the theoretical mean percentages (p. 122) as given in Tables 7 to 9, it is evident that the area of the frequency polygon obtained by plotting these mean values must be the same as the area of the polygon obtained by plotting the observed percentage values given in any row of the table. Thus, if, in the upper left hand diagram of Fig. 6, we draw perpendiculars from the ends of the line representing the values of the theoretical means, the area of the enclosed polygon will be the same as that of the polygon formed in a like manner from the line joining the observed percentage values. In other words, the sum of the positive deviations from the theoretical means is equal to the sum of the negative deviations from those means. It therefore follows that the sum of the squared deviations is a minimum in every case. The root-mean-square deviation will then form an excellent measure of the actual deviation of the observed values from the values to be expected on the theory of probability.

The root-mean-square deviation expresses in one constant the amount by which all of the plants starting in a given quintile deviate from what the mean value would have been under the conditions of simple sampling. In other words, it measures the effect of whatever influences other than chance are acting upon the given group of plants, in respect to the determination of their place in the variation curve of the population at different growth stages.

The values of the root-mean-square deviations for the plants starting in each quintile are given in the next to the last column of tables 7 to 9. These constants for the three series are shown graphically in Fig. 8.

In order to estimate the significance of these root-mean-square deviations it is necessary to know how widely such constants may fluctuate under the conditions of simple sampling. If, for example, we had thrown five-faced dice instead of measuring corn plants the percentage of successes would have been close to the theoretical mean of 20 percent. However, unless the number of throws had been very large the observed values would not have exactly coincided with the theoretical values. The amount of the deviation which can be expected in any instance is measured by the standard deviation of the theoretical mean. Now it will have been noticed that the root-mean-square deviation discussed above is the standard deviation of the observations taken about the theoretical mean, because the sum of the positive and negative deviations from these means is zero. Therefore the standard

deviation of simple sampling for the five series of observation may be compared directly with the corresponding root-mean-square deviations, and the significance of the latter estimated.

This theoretical standard deviation may be derived from the following relationship. If σ is the standard deviation of a whole series of observations, and $\sigma_1, \sigma_2 \dots \sigma_r$ are the standard deviations of the respective component series, whose means diverge from the mean of the whole series by amounts $d_1, d_2 \dots d_r$, then

$$N\sigma^2 = \Sigma(N_m\sigma_m^2) + \Sigma(N_m d_m^2),$$

where m is any subscript and $N = \Sigma(N_m)$ ¹.

If the observed root-mean-square deviation (standard deviation) exceeds the theoretical standard deviation calculated from the above equation, it is certain that some influence other than chance determined the distribution of the observations. Of course the greater the excess of the root-mean-square deviation the greater is the influence affecting the observations.

Returning now to a discussion of the results shown in Fig. 8, it is noted that in every instance except one, the root-mean-square deviation is greater than the standard deviation of simple sampling. The one exception is that for plants starting in quintile IV in series A. In this instance the distribution of the plants does not diverge farther from the theoretical mean than might be expected on the theory of simple sampling. In the case of plants starting in quintiles I and V the values of the root-mean-square deviations are very large, ranging from four to six times the theoretical standard deviations. It should be remembered that here the comparison is between two standard deviations. The conditions are quite different from those in Fig. 6 where a mean was compared with its standard deviation. If, as in the present case, the observed standard deviation exceeds the theoretical constant, it means that the distribution of the observations was determined by some factor other than chance. The greater the excess the greater was this extraneous influence.

Considering now the changes in the root-mean-square deviations for plants starting in the different quintiles it is seen from Fig. 8 that

¹) In calculating the standard deviations from the above equation the actual standard deviation for each quintile is needed, but as was stated above, these constants have not been tabulated. Since they do not vary greatly in any case from the value 3.34 percent, no significant change would be made in the constants by using that value throughout. However, in calculating the standard deviations given in the last columns of tables 7-9, the actual values of the individual standard deviations were used.

the general form of these polygons is W-shaped with the outer wings very much elongated. The fact that the values for the end quintiles (i. e., for very large and very small plants) are so much greater than the values for the middle quintiles is of some significance. As shown in the figures, there is nothing in the theory of probability which would tend to make the standard deviations for the ends of the series larger than for the middle. Hence it is clear that the non-random influence affecting plants which are very small or very large at the beginning, is much greater than that affecting the medium sized plants.

Having established this fact it may next be inquired as to what this influence is which affects the extremely large or extremely small plants. The first explanation that suggests itself is that of nutrition. Thus one plant may lie in an exceptionally fertile spot of soil. Its root system becomes developed early, and this extra amount of food supply is continued during the whole season. Naturally under such conditions the plant would continue to be large during the entire season. Again plants lying in exceptionally poor spots of soil would tend to remain small during the entire season.

Opposed to this explanation is the fact that the field on which this corn was grown is an extremely even piece of intervalle ground. There were no spots in which the soil appeared to be better or worse than other places. The manure and fertilizer were evenly distributed over the field and well mixed with the soil. Further, considering the nature of the root system of a corn plant, it does not seem probable that the above explanation would account for the great differences observed in adjacent plants. Thus, as will be pointed out later, it often happened that of adjacent plants only eighteen inches apart, one remained very small throughout the season and the other remained very large. Such constant differences can hardly be due entirely to the food supply or to other environmental conditions.

A second explanation might be found in genotypic or hereditary differences in the growth factors of the different plants. Within the past few years students of genetics have clearly shown that the inheritance of quantitative morphological characters may be explained upon a Mendelian factor basis. For example, EMERSON and EAST ('13) have shown that the final height of corn plants is in a large measure determined by their germinal constitution. It is not inconceivable, therefore, that the steps by which a plant reaches its final height is also determined in the same manner. It seems probable that the greater

divergence of the very large and very small plants from the theoretical mean may be due to genotypic differences. If it be assumed that there are several separate hereditary factors influencing growth, then the very large and the very small plants would tend to be more nearly homozygous for these conditions, and consequently tend to maintain the same type of growth. Under such conditions these plants would tend to diverge farther from the theoretical mean.

It is not our intention to discuss the relation of genetic factors to growth at this point. This question will be discussed more in detail after other facts have been brought out. It is sufficient to point out here the possibility of such a theory.

The quintile deviations of plants ending in a given quintile.

In the preceding section we have discussed the quintile variation of the measurement of plants starting in a given quintile. We shall now consider, but much more briefly, the same kind of data for plants which end in the given quintiles. The fundamental data for this discussion are given in tables 46 to 60 inclusive. These data have been collected and reduced to percentages in tables 10, 11 and 12. These tables correspond in all particulars to tables 7 to 9, which have already been discussed.

Table 10.

Series A.

Showing the number and percent of the measurements falling in the several quintiles for plants ending in a given quintile.

Quintile in which plants end	Quintile number										Total number	Root-mean-square deviation	Standard deviation of the root-mean-square deviation			
	I		II		III		IV		V							
	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent						
I	53	33·97	42	26·92	32	20·51	15	9·62	14	8·97	156	9·16	3·29			
II	45	34·61	44	33·85	14	10·77	13	10·00	14	10·77	130	11·23	3·59			
III	25	17·48	17	11·89	39	27·27	31	21·68	31	21·68	143	5·33	3·43			
IV	17	11·89	25	17·48	26	18·18	47	32·87	28	19·58	143	7·17	3·43			
V	4	3·08	16	12·31	33	25·38	34	26·15	43	33·08	130	11·36	3·59			
Total	144		144		144		140		130		702					
Theoretical mean		20·51		20·51		20·51		19·95		18·52						

Table 11.

Series B.

Showing the number and percent of the measurements falling in the several quintiles for plants ending in a given quintile.

Quintile in which plants end	Quintile number										Total number	Root-mean-square deviation	Standard deviation of the root-mean-square deviation			
	I		II		III		IV		V							
	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent						
I	55	38·46	51	35·66	23	16·08	13	9·09	1	0·70	143	14·16	3·45			
II	49	34·27	30	20·98	32	22·37	16	11·19	16	11·19	143	8·08	3·45			
III	21	14·69	24	16·78	26	18·18	43	30·07	29	20·28	143	5·49	3·45			
IV	18	12·59	13	9·09	30	20·98	40	27·97	42	29·37	143	8·66	3·45			
V	2	1·54	26	20·00	30	23·08	31	23·84	41	31·54	130	10·58	3·61			
Total	145		144		141		143		129		702					
Theoretical mean		20·66		20·51		20·08		20·37		18·38						

Table 12.

Series C.

Showing the number and percent of the measurements falling in the several quintiles for plants ending in a given quintile.

Quintile in which plants end	Quintile number										Total number	Root-mean-square deviation	Standard deviation of the root-mean-square deviation			
	I		II		III		IV		V							
	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent	No.	Per- cent						
I	61	42·66	25	17·48	19	13·29	14	9·79	24	16·78	143	11·67	3·35			
II	20	13·98	31	21·68	25	17·48	43	30·07	24	16·78	143	5·54	3·35			
III	23	16·08	37	25·87	40	27·97	16	11·19	27	18·88	143	6·24	3·35			
IV	26	18·18	23	16·08	20	13·99	44	30·77	30	20·98	143	5·81	3·35			
V	13	9·09	27	18·88	39	27·27	27	18·88	37	25·87	143	6·49	3·35			
Total	143		143		143		144		142		715					
Theoretical mean		20·00		20·00		20·00		20·14		19·86						

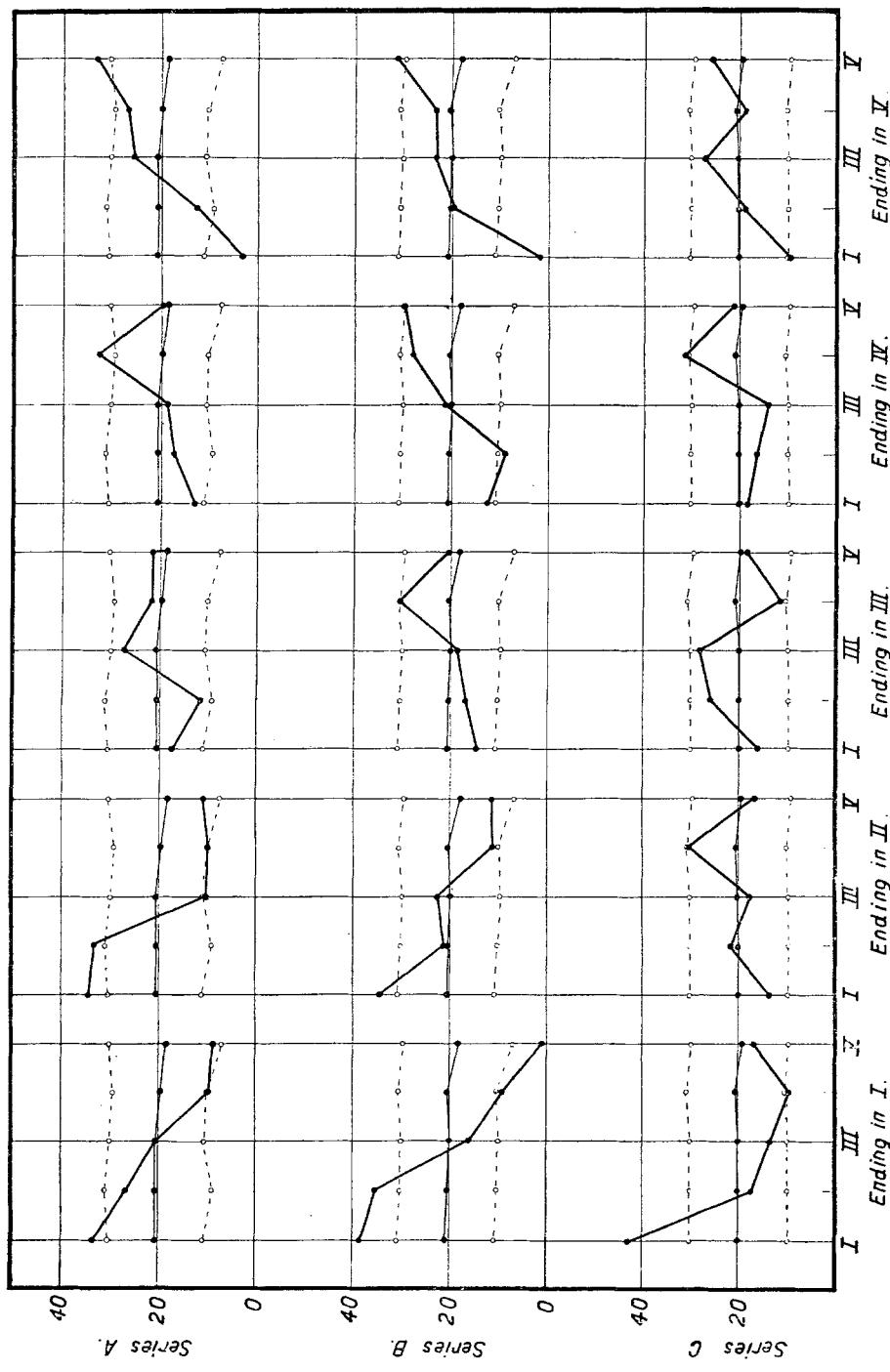


Fig. 7. Showing for plants ending in a given quintile, the percent of the total number of observations which fall in each quintile.
The theoretical mean percentages are also plotted, together with the limits of 3σ measured from these means.

The values given in the percentage columns of these tables are shown graphically in Fig. 7. This figure corresponds to Fig. 6 in all particulars.

Fig. 7 shows in general the same kind of facts noted in Fig. 6. In most instances a larger proportion of the observations fall in the quintile in which the plants end than in any other. On the other hand the deviations are in no case so great as in the corresponding diagram in Fig. 6. In the majority of cases the observations do not deviate from the theoretical mean more than three times the standard deviation. The greatest deviations are again to be found in the case of very large and very small plants.

These differences can best be comprehended in the form of root-mean-square deviations. These have been calculated in the same way as for tables 7—9. These constants are tabled in the next to the last column of tables 10—12. They are shown graphically in Fig. 9. Figs. 8 and 9 have been placed next to each other to facilitate comparison between them.

Fig. 9 shows in a more exact manner the differences which we have already noted from Fig. 7. In every case these root-mean-square deviations are significantly greater than the standard deviations of simple sampling. On the other hand, the deviations in quintiles I and V are not nearly so great as the corresponding ones in Fig. 8. In Series B and C the greatest deviations occur in quintile I. Series C does not show any greater deviation in quintile V than in any of the three preceding quintiles.

On the whole the deviations shown in Fig. 9 are considerably smaller than those shown in Fig. 8. From this it follows that plants relatively large at the end of the season are less likely to have been relatively large during the entire season than are the plants which are large at the beginning of the season. The same is true of small plants. Thus the relative position of the plants at the beginning of the season is a much better criterion of their average relative size for the whole season than is their relative position at the end of the season. Or again, on the hypothesis suggested earlier, the relative position of the plants at the beginning of the season is a better criterion of their probable genotypic constitution, so far as height is concerned, than is their relative position at the end of the season. The reason for this difference will be discussed at another place in this paper.

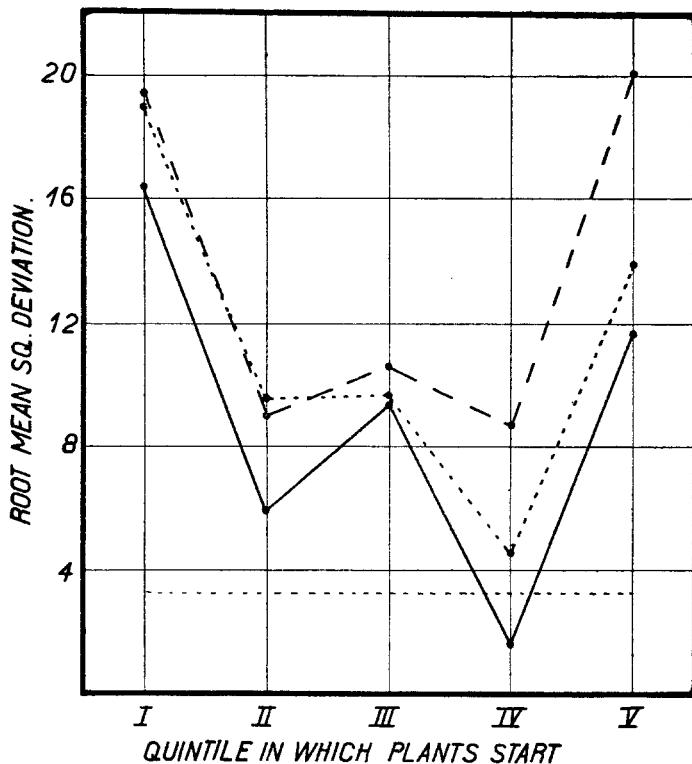


Fig. 8.

Changes in the root-mean-square deviation as measured from the theoretical mean. For plants starting in a given quintile.

Series A ——;
" B - - -;
" C

The lower dotted line represents the average standard deviation of simple sampling.

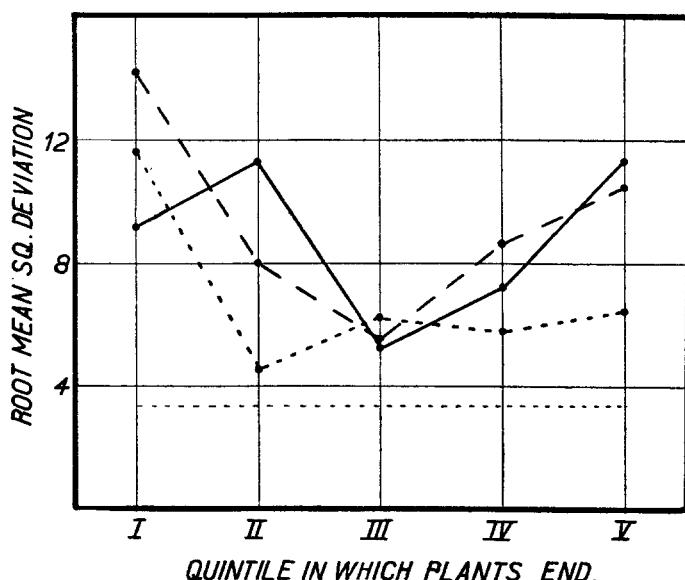


Fig. 9.

Changes in the root-mean-square deviation as measured from the theoretical mean. For plants ending in a given quintile.

Series designated as in Fig. 8.

Changes in the mean quintile position in the successive stages of growth.

a) Plants Starting in a Given Quintile.

In the preceding section of this paper we have discussed the quintile distribution of groups of plants taking the season as a whole. In this portion of the paper an attempt will be made to carry the analysis a step farther. It has been shown, for example, that there is a tendency for plants relatively small at the beginning of the season to be relatively small for the season as a whole. This result may mean, on the one hand, that such a group of plants tended to remain in or near quintile I during the whole season with only random fluctuation from this value. Or, on the other hand, it may mean that there was a gradual regression towards the mean of the population with the advance of the season. Thus it is conceivable that plants relatively small at the beginning of the season become, gradually, relatively larger until at the end of the season they are on the average not far different from the mean of the population. Similarly the relatively large plants might become relatively smaller until they too are, on the average, not far from the mean of the population. Either of these conditions would give similar results with the methods used in the preceding section.

In order to study this question we have determined the mean quintile position, at each successive measurement, for the plants starting in a given quintile. These mean quintile positions are given in tables 31 to 45. These values were determined from the distributions in the respective rows of these tables. Both the mean and the standard deviation of these means were calculated by the method of moments in the usual way.

Referring, for example, to table 31 it is noted that in the distribution of June 12 all eleven plants were, by hypothesis, in quintile I. The mean quintile position is of course 1·0 and the standard deviation zero. In the distribution of June 15 three of these eleven plants fell in quintile II and one in quintile III, while seven remained in quintile I. The mean quintile position here is 1·45 and the standard deviation 0·66.

The change in these constants can best be seen when displayed graphically. Figs. 10, 11 and 12 show the mean quintile positions for the several groups of plants from each series.

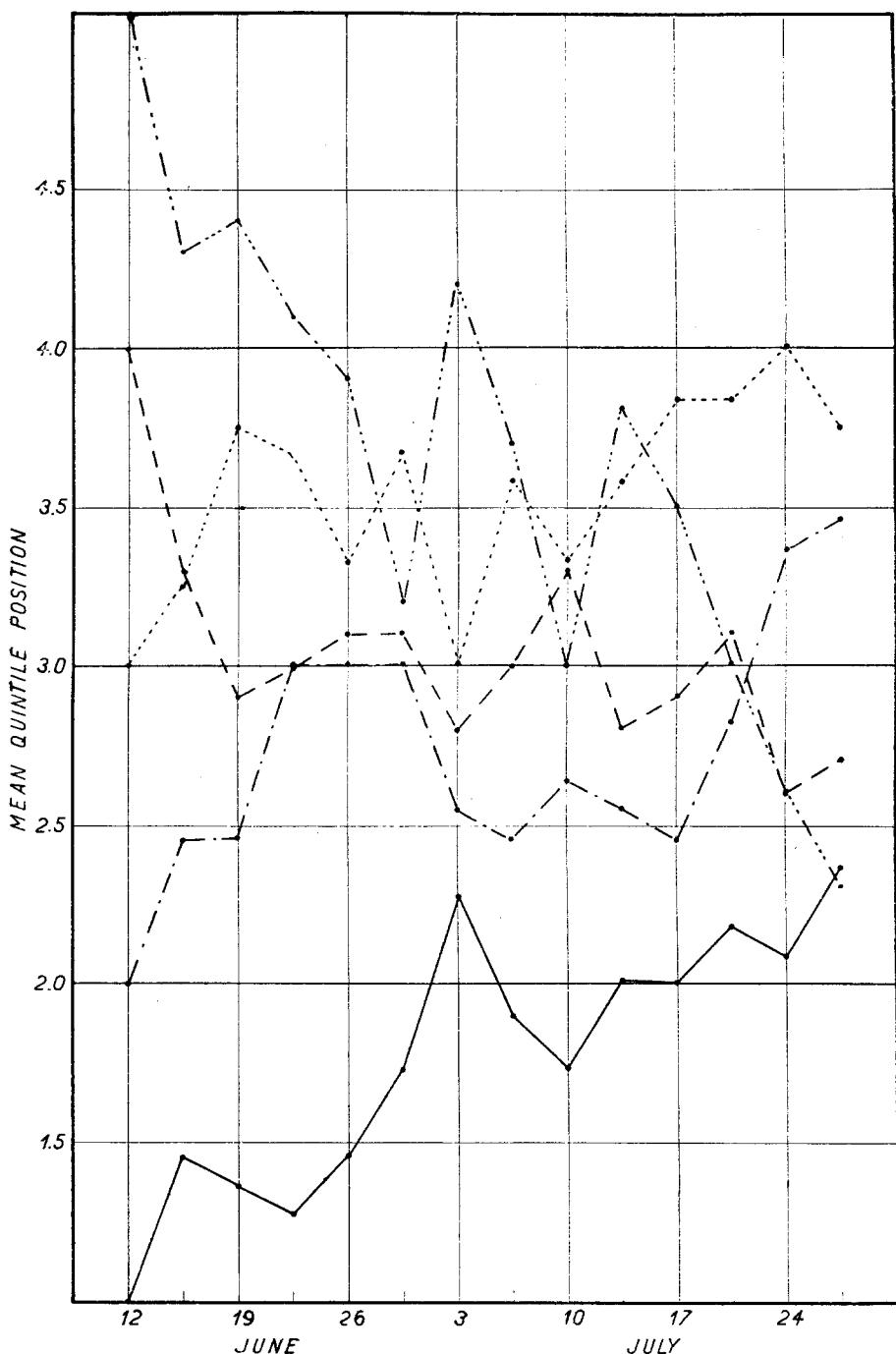


Fig. 10. Plants from series A. Changes in the mean quintile position in the successive measurements of plants starting in each quintile. Plants starting in quintile I —, II - . - . - . -, III , IV - - - , V - - - - - .

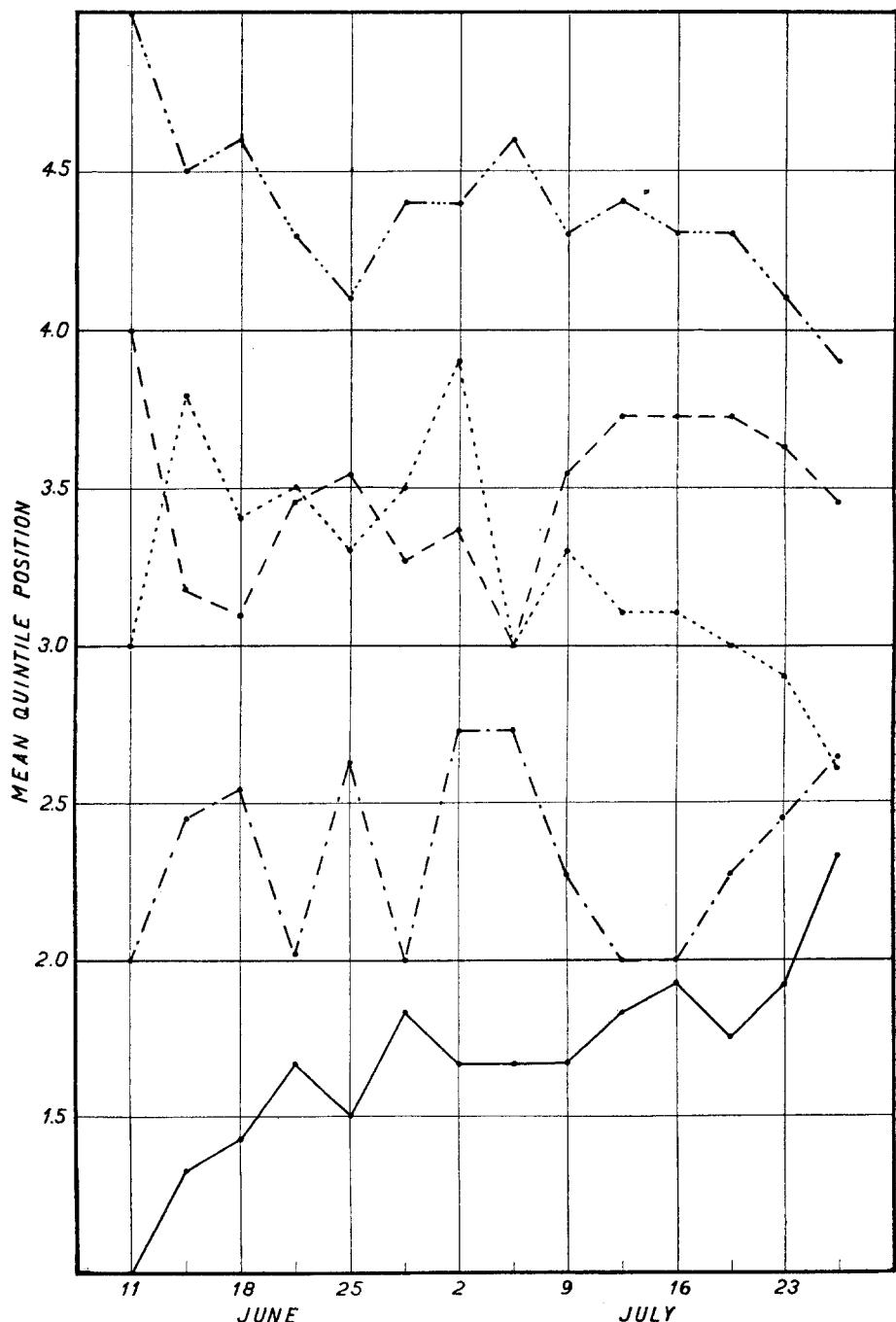


Fig. 11. Plants from series B. Legend same as for Fig. 10.

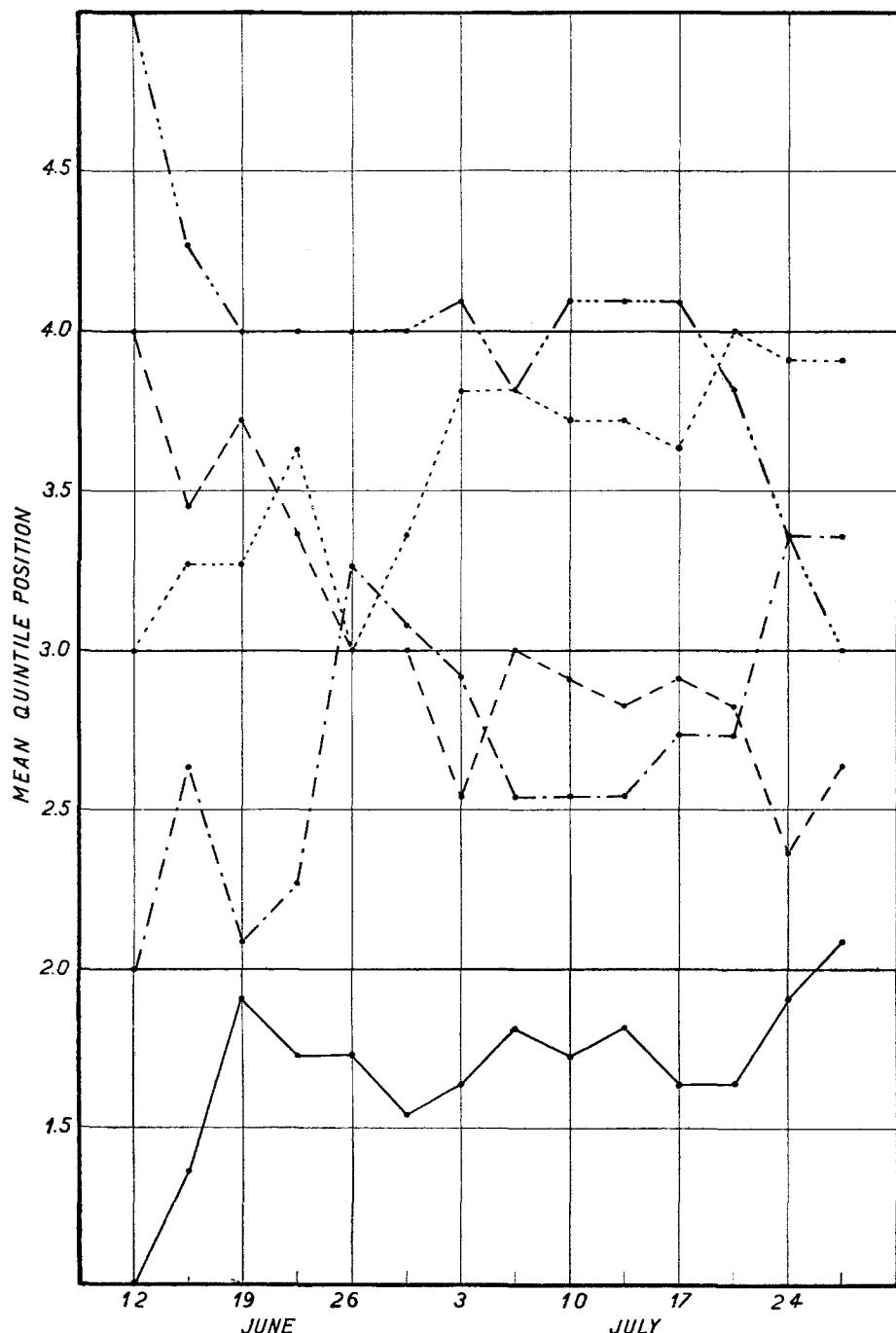


Fig. 12. Plants from series C. Legend same as for Fig. 10.

It is clear from the method of obtaining the quintile distributions that the mean quintile position of the whole population of plants at any moment will be 3·0. That is, if we add together the quintile distributions at any measurement of all the plants in a series the mean of this total distribution will tend to be at the center of the third or middle quintile. Further it is clear that if there were no influence other than chance affecting the plants starting in a given quintile the mean quintile position would be very nearly 3·0 at every measurement except the first. It has been shown above that there is some other influence determining the height of the plants in these several groups and this effect is clearly shown in the above figures.

From these figures it is noted that for plants starting in quintile I there is a gradual rise in the mean quintile position with the advance of the season. Likewise the relatively large plants starting in quintile V gradually come to lie nearer the mean of the population.

It would be possible to get a measure of these tendencies by fitting straight lines to these means for each group of plants. The slope of these lines from the horizontal would indicate the amount of the regression towards the mean. These lines have not been fitted to the data because it was believed that all essential points could be seen without them. Further since there is only a small number of plants in each group there are a number of irregularities which would unduly influence the result, although on the whole these irregularities are perhaps meaningless.

Plants starting in quintiles II, III and IV show a number of irregularities in the different series. Thus in Fig. 10 it is seen that plants starting in quintile IV fluctuate very closely about the mean of the population for the remainder of the season. This is in accord with the result for this group of plants noted above and shown in Fig. 8. In Fig. 10 the plants starting in quintile III, instead of remaining near this value for the remainder of the season, give the subsequent means all above 3. Similar irregularities are noted in the other series.

The most important point brought out by these figures is that there is, on the average, a tendency for the plants small at the beginning of the season to become relatively larger as the season advances. This change tends to take place gradually so that the mean quintile position of the last measurement is in most cases nearer the general mean than any of the others. In all instances the mean quintile positions of these small plants lie well below the general mean. Thus,

while it is true that on the average, the relatively small plants tend to become larger as the season advances, it is equally true, as brought out in the preceding section (cf. p. 120), that such small plants tend to remain smaller than the average for the season as a whole. Very similar statements may be made for plants which were relatively large at the beginning of the season.

In several of these figures there is still another condition worth noting. Thus, as is most clearly seen in Fig. 12, there is a period during the middle of the season in which the mean quintile position of plants starting in quintile I and V remain practically constant. In these cases there is a sharp change in the mean quintile position at the beginning of the season and a similar change at the end of the season. A similar, although less marked, condition is seen in Fig. 11, while in Fig. 10 the conditions are quite different. The data are not sufficient to discuss the possible meaning of these variations in the series.

In tables 31 to 46 there is given in addition to the mean quintile position the standard deviation of this mean position. It may be seen from these constants that in general there is an increase in this standard deviation with the advance of the season. The fluctuation in these constants due to the small number of individuals in each table are so great that it is not possible to draw further conclusions from them with certainty. The constants have been included in the tables because they give some idea of the probable error to which the mean are subject.

b) Plants Ending in a Given Quintile.

In tables 46—60 there are given the mean quintile positions and standard deviations for plants ending in the given quintiles. These may be examined very briefly and by the same methods employed in the preceding paragraphs. Fig. 13, 14 and 15 show the graphs of the means for the several groups of plants in each series.

Comparing these figures with Figs. 10 to 12 it is seen that in general there is a much greater tendency here for the mean quintile position of each group of plants to lie near the mean of the population, viz., 3·0. This is particularly marked in Fig. 15. Here, with the possible exception of plants ending in quintile I, these values fluctuate about this mean value with only slightly greater deviations than might be expected on the theory of chance alone. In Figs. 13 and 14 the fluctuations are somewhat farther from the mean but in general they are not so far as in the corresponding Figs. 10 and 11.

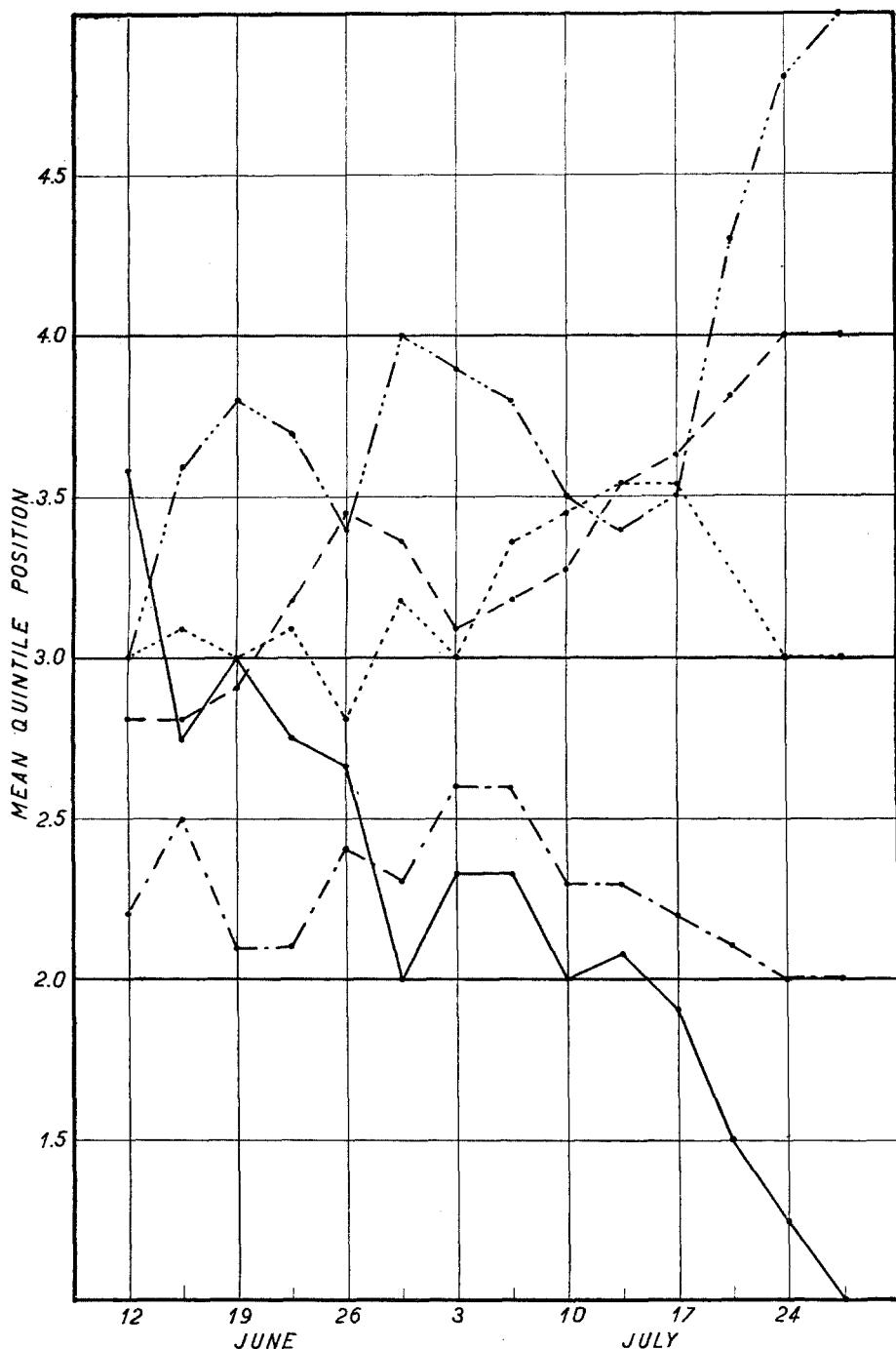


Fig. 13. Plants from series A. Changes in the mean quintile position in the successive measurements of plants ending in each quintile. Plants ending in quintile I —, II —·—·—·—, III , IV ———, V —··—··—··—

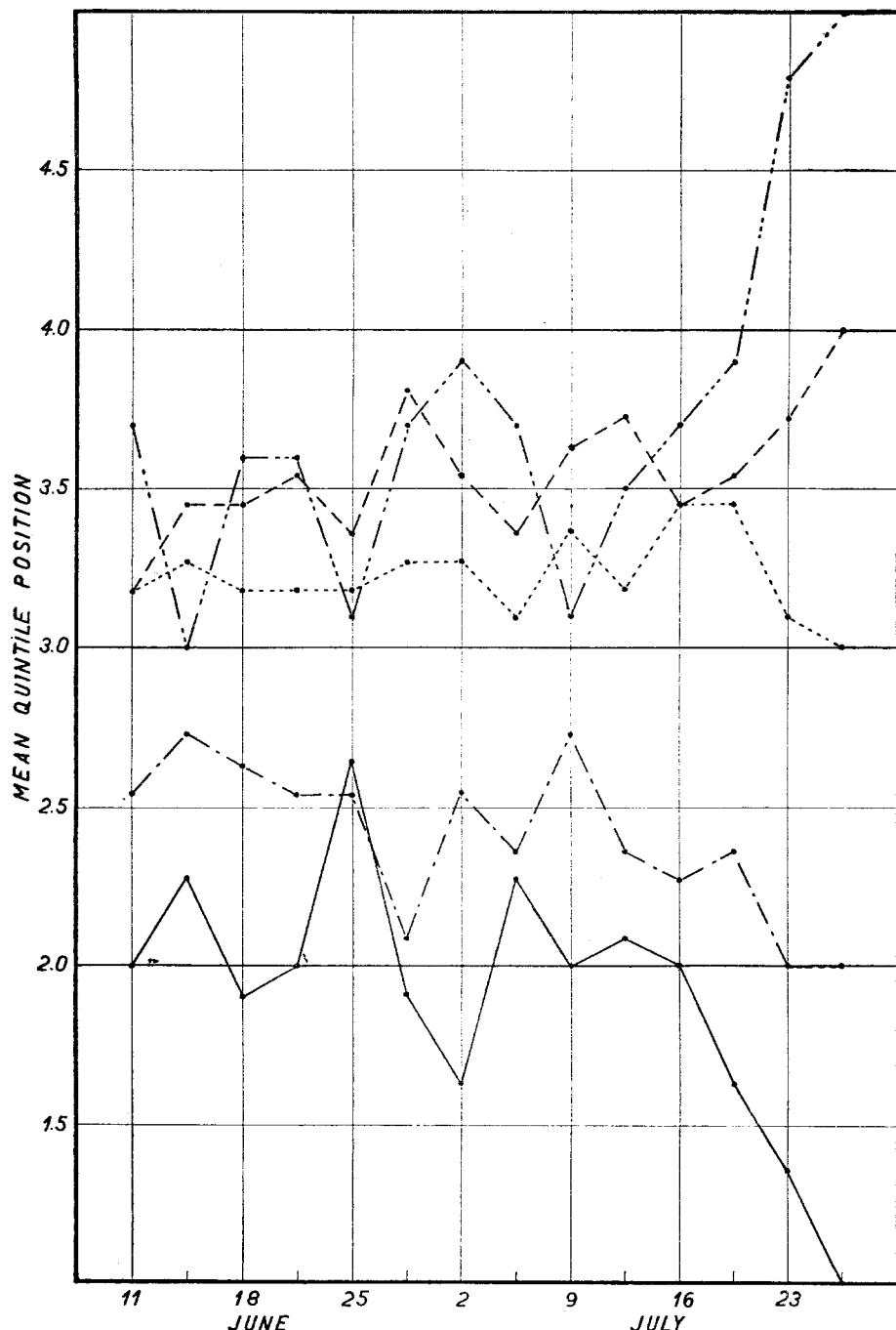


Fig. 14. Plants from series B. Legend the same as for Fig. 13.

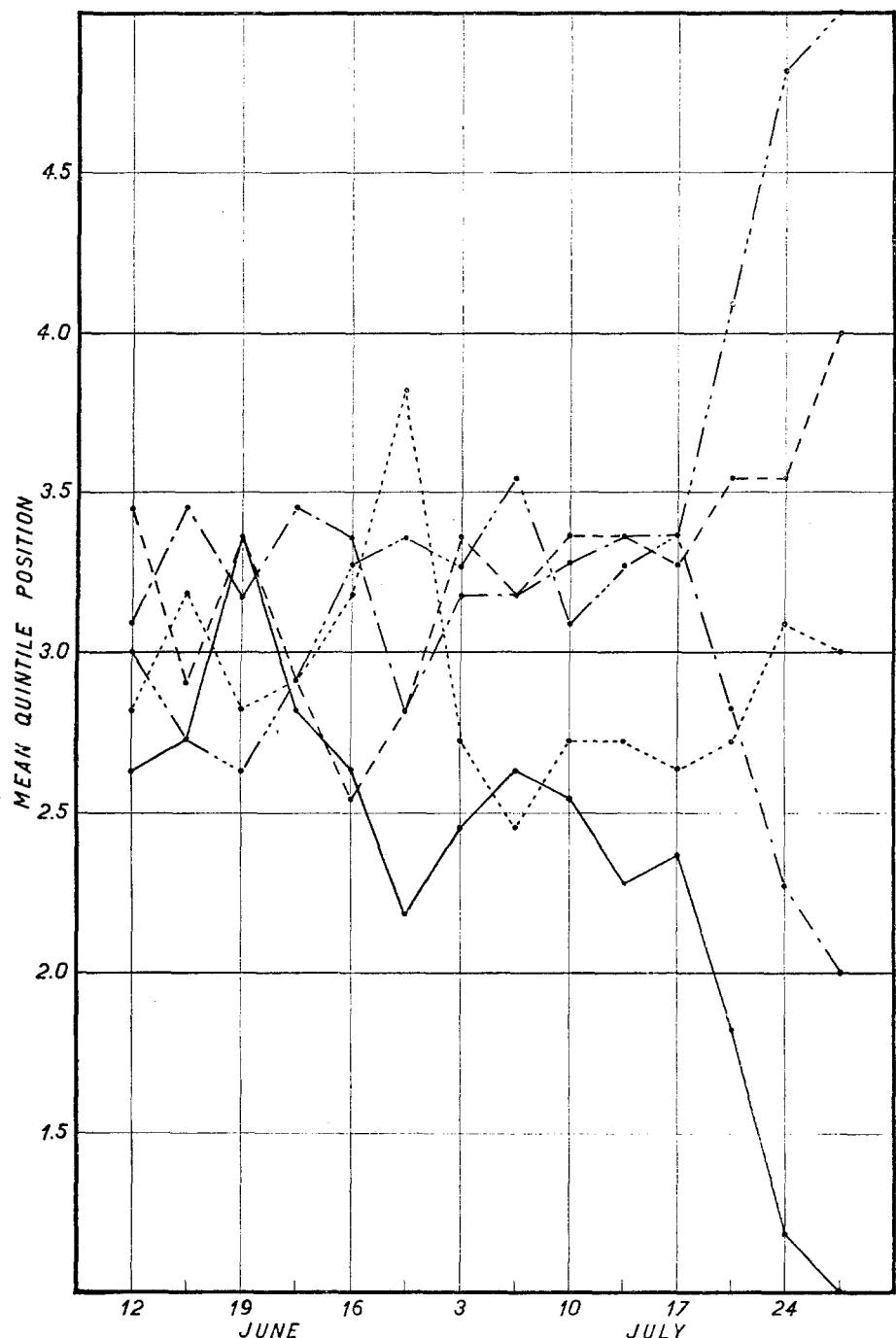


Fig. 15. Plants from series C. Legend the same as for Fig. 13.

In general Figs. 13 to 15 do not show such a gradual change from the relatively small or relatively large plants at one end of the season towards the mean of the population at the other as was noted in Figs. 10 to 12. Instead there is a tendency to maintain about the same relative position until near the end of the season. Then there is a rather sudden change in the mean quintile position. An exception to the above statement is found in the plants ending in quintile I in Fig. 13. In this case there has been a gradual change from a condition near the mean to relatively small plants at the end of the season.

It was pointed out above that the quintile in which a plant ends is not as good a measure of its average relative size for the season as is the quintile in which it started. A comparison of these two sets of figures aids us in getting at some of the causes of this. Probably the chief reason lies in the fact that certain plants which are relatively large, or at least of medium size, throughout the season cease growing earlier than other plants of the same size. Consequently in the final measurements these plants are relatively small compared with the remainder. Likewise some plants have a longer growing season and, on that account, become relatively large in the last measurements, although for the remainder of the season they were relatively small or of medium size.

These facts undoubtedly account for many of the peculiarities in Figs. 13 to 15.

The mean quintile position of individual plants.

It is next proposed to carry the analysis of the growth curves one step farther and to consider the quintile distribution of the measurements of each individual plant. The quintile distributions for each plant in each series are shown in tables 61, 62 and 63. The plants have been arranged in these tables for a special purpose, but since the individual number of each plant is given, they can serve as the tables of fundamental distributions. The mean quintile position¹⁾ and its standard deviation have been calculated for each plant in these distributions. These constants were calculated by the method of mo-

¹⁾ It should be noted that the 'mean quintile position' of an individual plant is entirely different from the term 'mean quintile position' used in the preceding section of this paper. As used in the remainder of this paper the term 'mean quintile position' will refer to the individual plant unless expressly stated to the contrary.

Table
Series

Showing the mean quintile position and standard deviation to mean

Plant No.	1·00—1·80		Plant No.	1·80—2·60		Plant No.	Mean quintile position
	Mean quintile position	Standard deviation		Mean quintile position	Standard deviation		Mean quintile position
	1·5714	0·7284	6	2·0000	0·7559	3	2·7143
14	1·2857	0·6685	9	1·8571	0·7284	4	2·7857
17	1·5712	1·1836	10	2·0714	0·7620	12	3·3572
19	1·0714	0·2575	13	2·0000	0·5345	15	3·0714
22	1·1429	0·5151	26	2·2857	0·6999	16	2·8571
25	1·3571	0·4791	30	2·2857	1·0973	28	3·0000
29	1·7857	1·1450	35	2·4286	0·9793	31	3·2143
37	1·1429	0·3499	45	2·2857	1·3848	36	2·7143
46	1·7143	0·8806	56	2·2857	0·7954	38	2·9285
48	1·5000	0·5000				40	2·7857
						41	2·6429
						52	2·6429
						53	2·9286
						55	2·7857
						60	2·7143
Total	14·1426	6·6577		19·4999	7·7375		43·1429
Average	1·4143	0·6658		2·1667	0·8597		2·8762
No. of Plants	10			9			15

ments, and the square roots were taken with sufficient accuracy from BARLOW's tables.

The mean quintile position of an individual plant gives a measure of its relative size for the whole season. Thus a plant that has tended to remain small throughout the season will have most of its measurements in or about quintile I and consequently its mean quintile position will approach 1·0. Those which have tended to remain large will have a value near 5·0.

The standard deviation of these means measures the scatter of the observations. Thus for example there are a few cases in which all the measurements of a plant fall in one quintile. The standard

13.

A.

deviation of each plant in Series A arranged according to quintile classes.

tile classes.

3·40 Standard deviation	Plant No.	3·40—4·20		Plant No.	4·20—5·00	
		Mean quintile position	Standard deviation		Mean quintile position	Standard deviation
1·2775	2	3·5000	1·5385	7	4·3571	0·6102
1·1450	20	4·0714	0·8835	11	4·2857	0·7954
0·8183	23	3·6429	0·8113	32	4·4286	0·7284
0·8535	24	3·6429	0·8950	33	4·7857	0·4102
1·2454	27	3·7857	0·9395	34	4·7143	0·5890
0·9285	39	3·8571	0·7423	42	4·5714	0·9035
1·0124	51	4·0000	1·0690	44	4·5000	0·7819
1·0973	54	4·0714	0·7986	47	4·6429	0·7178
1·5337	57	3·8571	0·8330	49	4·2143	0·8601
0·8601	58	3·7857	0·9395			
0·9715	59	4·0000	1·0686			
0·7178						
0·9610						
0·7726						
1·3850						
15·5796		42·2142	10·5188		40·5000	6·3365
1·0386		3·8377	0·9563		4·5000	0·7041

11

9

deviation in such a case is zero. In a number of cases all fourteen of the measurements of a plant fall in two contiguous quintiles in which case the standard deviation cannot be greater than 0·5 of a quintile. The largest standard deviation that could be obtained would be given by a distribution in which seven observations fall in quintile I and seven in quintile V. The standard deviation would then be 2·0. As a matter of fact no plant gave a distribution even approaching this form. There are no standard deviations greater than 1·55 (cf. tables 13—15) which means that the measurements of such a plant were but little more widely scattered than if they had been equally distributed in all five quintiles.

Table
Series

Showing the mean quintile position and standard deviation to mean

Plant No.	1·00—1·80		Plant No.	1·80—2·60		Plant No.	Mean quintile position
	Mean quintile position	Standard deviation		Mean quintile position	Standard deviation		Mean quintile position
4	1·0000	0·	1	2·1429	0·6389	3	2·8571
5	1·5000	0·7319	11	2·2857	0·8806	6	3·2857
13	1·1429	0·3499	22	2·3571	1·2944	16	2·9286
14	1·2857	0·5890	25	2·2143	0·7726	20	3·2857
15	1·5000	0·6268	27	2·0714	0·7986	23	2·7857
17	1·4286	0·6227	35	2·0000	1·1948	28	3·0714
18	1·7143	0·5890	47	2·2143	0·7726	30	2·8571
24	1·4286	0·4948	49	2·0000	1·1954	38	3·0714
32	1·2857	0·4517	53	2·0714	0·8835	56	2·8571
43	1·0000	0·				57	3·0000
58	1·2857	0·5890				60	2·8571
Total	14·5715	5·0448		19·3571	8·4312		32·8569
Average	1·3247	0·4586		2·1508	0·9368		2·9870
No. of Plant		11		9			11

The mean quintile position and its standard deviation measure definite characteristics of a plant. The constants give at a glance a large amount of information about the growth of any particular plant. Thus the mean quintile position of a plant tells whether, for the season as a whole, the plant has been relatively large or small or medium. Further, the standard deviation tells whether the plant has maintained this relative size throughout the season or whether it has fluctuated from one relative size to another. These are important quantitative characters of a plant. We hope to show that these characters are of a fundamental importance for the analysis of growth and particularly for the relation of growth to underlying genetic factors. For this reason these characters will be discussed in detail in the following pages.

14.

B.

deviation of each plant in Series B arranged according to quintile classes.

tile classes.

—3·40 Standard deviation	Plant No.	3·40—4·20		Plant No.	4·20—5·00	
		Mean quintile position	Standard deviation		Mean quintile position	Standard deviation
0·9147	8	3·7142	0·5890	9	5·0000	0·
1·1606	10	3·5000	1·1180	12	4·2857	0·7954
1·0798	21	4·0000	0·9258	19	4·7143	0·4517
0·5890	29	3·7143	0·7954	31	4·5000	0·8237
1·0124	36	3·6429	0·8950	33	4·2143	0·6739
1·1000	37	3·6429	0·8950	34	4·3571	0·6102
0·9147	39	3·8571	1·2454	41	4·3571	0·8113
1·2525	40	3·9286	0·8835	42	4·6429	0·4792
1·3178	44	3·4286	1·2934	48	4·5000	0·6267
1·0691	45	3·4286	0·7284	54	4·5000	1·0280
1·3499	46	3·6429	0·7178			
	51	3·7857	0·9395			
	59	3·4286	1·2433			
11·7654		47·7144	12·2695		45·0714	6·3001
1.0696		3·6704	0·9438		4·5071	0·6300
		13			10	

This discussion will be begun with a study of the mean quintile positions of the individual plants. The data can best be presented in the form of frequency distributions of these means. Since there are not more than 55 plants in any series it is necessary to choose rather large class units to obtain an approach to a smooth distribution. Further since the range is strictly limited, equal to four quintiles (cf. Fig. 5), it is necessary to choose a class unit which will divide the range equally. We have used a class unit of 0·8 of a quintile since smaller units would give very few observations in some classes.

The frequency distributions of these means are given in tables 13, 14 and 15. In these tables there are also given the numbers of the

Table
Series

Showing the mean quintile position and standard deviation to mean

Plant No.	1·00—1·80		Plant No.	1·80—2·60		Plant No.	Mean quintile position
	Mean quintile position	Standard deviation		Mean quintile position	Standard deviation		Mean quintile position
21	1·2857	0·7954	1	2·2857	1·1600	3	3·0000
22	1·6428	0·4792	4	1·9286	1·1627	5	3·2143
25	1·7857	1·0807	9	2·5714	1·1168	13	2·8571
30	1·0000	0·	14	1·9286	0·7034	15	2·7143
31	1·1429	0·3499	16	2·2857	0·7954	17	2·8571
38	1·0000	0·	19	2·5714	0·9035	18	3·3571
52	1·3571	0·7178	24	2·5714	0·9035	26	3·1429
55	1·3571	0·4812	28	1·8571	0·9898	34	2·8571
			32	1·8571	0·7423	37	3·2857
			33	1·9286	0·7423	40	2·8571
			36	1·9286	0·9610	41	3·3571
						44	3·2857
						47	3·0000
						49	3·0000
						50	3·3571
						56	2·6429
						57	2·9286
Total	10·5713	3·9042		23·7142	10·1418		51·7141
Average	1·3214	0·4880		2·1558	0·9220		3·0420
No. of plants		8		11			17

individual plants falling in the given class, together with the mean quintile position and the standard deviation of each. In the next to the last row of the table there is given the average mean quintile position and the average standard deviation of the plants in each class. Finally in the last row of the tables there is given the number of plants in each class.

The frequency distributions given in the last rows of these tables will be discussed first. For convenience these distributions together with their constants have been brought together in Table 16.

15.

C.

deviation of each plant in Series C arranged according to quintile classes.

tile classes.

—3·40 Standard deviation	Plant No.	3·40—4·20		Plant No.	4·20—5·00	
		Mean quintile position	Standard deviation		Mean quintile position	Standard deviation
1·3091	2	3·7143	0·6999	10	4·7857	0·5579
0·5579	8	3·8571	1·1865	12	4·3571	0·8113
1·3015	11	4·0714	1·0320	42	4·7857	0·4104
1·4910	20	4·1429	0·9147	43	4·8571	0·3499
0·9897	27	4·1429	0·9768	45	4·7857	0·4102
0·8112	29	3·6249	0·8112	46	4·7143	0·5990
0·6388	35	3·5000	1·0521	48	4·2143	1·4230
1·3015	39	3·6429	0·9715	54	4·2857	0·6389
1·0973	51	4·0714	1·2226			
0·9147	53	3·6428	0·9715			
0·9715	60	3·8571	0·9948			
1·2775						
1·3626						
1·0000						
0·9721						
0·9715						
1·1628						
18·1307		42·2677	10·8336		36·7856	5·2006
1·0665		3·8425	0·9849		4·5982	0·6501

It will be noted at once that these distributions present certain rather remarkable peculiarities. Thus for example in series B the plants are distributed very nearly equally in each of the five classes. In series A and C there is a more or less pronounced mode falling in the middle class. However even in these series there is a marked tendency for the distribution to spread out, so that the end classes are only slightly smaller than the mode. These facts can best be seen from the standard deviations, and for this purpose the standard deviation has been given in table 16 in class units as well as in quintiles, i. e., after

Table 16.
Frequency Distribution of the Mean Quintile Position of
Individual Plants.

Se- ries	Mean Quintile Position					Mean	Standard Deviation in Quintiles	Standard Deviation in Class Units
	1·0—1·8	1·8—2·6	2·6—3·4	3·4—4·2	4·2—5·0			
A	10	9	15	11	9	$3\cdot0000 \pm 0\cdot0979$	$1\cdot0667 \pm 0\cdot0692$	$1\cdot3333 \pm 0\cdot0865$
B	11	9	11	13	10	$3\cdot0296 \pm 0\cdot1028$	$1\cdot1205 \pm 0\cdot0727$	$1\cdot4006 \pm 0\cdot0909$
C	8	11	17	11	8	$3\cdot0000 \pm 0\cdot0910$	$1\cdot0004 \pm 0\cdot0643$	$1\cdot2504 \pm 0\cdot0804$
Total	29	29	43	35	27	$3\cdot0098 \pm 0\cdot0562$	$1\cdot0638 \pm 0\cdot0397$	$1\cdot3292 \pm 0\cdot0497$
Percent	17·8	17·8	26·4	21·5	16·6			

being multiplied by the unit of grouping (0·8 quintile in this case). The standard deviation in class units represents the variation without reference to the grouping. It is the same as if the classes had the value of 1, 2, 3, 4, and 5.

Now it can be shown by simple algebra that if the variates are equally distributed among the classes the standard deviation in class units will be the same as for the first n natural members, in which case

$$\sigma^2 = \frac{n^2 - 1}{12}$$

When n is equal to 5,

$$\sigma = \sqrt{\frac{1}{2}} = 1\cdot4142.$$

This would be the standard deviation if the observations were equally distributed in all five classes. By comparing this value with the constants in the last column of table 16 it is seen that the observed values are rather close to the theoretical constant. For series B the agreement is very close, the difference being only about one-seventh of the probable error. In no case does the observed value deviate from the theoretical value as much as three times its probable error. Thus, none of the observed standard deviations are certainly significantly different from the standard deviation of a distribution in which the variates are equally distributed among the classes. However, all the observed values deviate from the theoretical in the same direction. This fact is probably significant and indicates that there is some fundamental cause which produced a frequency

distribution in which the variates are nearly equally distributed in all classes but in which there is a tendency towards a slight hump near the middle class.

Now, it can be shown that this is not at all the form of a distribution which would be obtained if the observations were distributed according to the laws of chance. It will be remembered that in the above paragraph we are discussing the distribution of the mean quintile position and not the distribution of the direct observations.

The discussion can most readily be put into terms of probability if we suppose ourselves to be dealing with throws of dice in place of the corn measurements. Each corn plant was measured fourteen times. Each measurement could fall in any one of five classes, i. e., in quintile I, II, III, etc. These conditions are strictly analogous to a throw of fourteen five-faced dice. The chance of success, p , for any face is $1/5$, and the mean number of dice showing a given face at any throw will tend to be np or $14/5$. It is clear that if the dice are unbiased this mean will be the same for each of the five faces. Thus the observations tend to be distributed equally among the several classes.

Such a distribution is strictly analogous to the quintile distributions of the individual plants as given in tables 62 to 64. What has been termed the mean quintile position would be analogous to the mean number of spots on the fourteen dice at any throw. This mean can be obtained from the frequency distribution given above in which each of the five classes have equal frequency. In a distribution where the frequency in each of n classes is the same, the mean in terms of class units is equal to the mean of the first n natural number or the

$$\text{mean} = \frac{n+1}{2}.$$

When $n = 5$ the mean is equal to 3.

It has been shown above that the standard deviation of the first n natural numbers is given by

$$\sigma^2 = \frac{n^2-1}{12}.$$

If $n = 5$, then $\sigma = \sqrt{2} = 1.4142$.

Thus the average number of spots per die showing at any throw will tend to be 3 and the standard deviation of this mean will tend to be $\sqrt{2}$.

Now supposing we had a series of such throws each with a mean approaching 3 but tending to deviate by a standard deviation of $\sqrt{2}$.

What would be the mean and standard deviation of such a distribution of means? It is clear that such a distribution is exactly analogous to the distribution of mean quintile position given in table 16.

If we have a series of n frequency distributions with means $a_1, a_2, a_3, \dots, a_n$ the mean, m , of the sum of these distributions is

$$m = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}.$$

In the present instance $a_1 = a_2 = a_3 = a_n = 3.0$

$$m = \frac{3 \cdot n}{n} = 3.0.$$

This it will be seen corresponds with the value of the means given in table 16.

Now the standard deviation, σ_s , of the sum of a series of n frequency distributions whose standard deviations $\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_n$ are measured from the same mean is

$$\sigma_s = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots + \sigma_n^2}{n}}$$

$$\text{But } \sigma_1 = \sigma_2 = \sigma_3 = \sigma_n = \sqrt{2}.$$

$$\text{Hence } \sigma_s = \sqrt{\frac{2 \cdot n}{n}} = \sqrt{2}.$$

For series A and B in table 16, $n = 54$ and for series C, $n = 55$.

Substituting these values

$$\sigma_s = 0.1925 \text{ and } 0.1907$$

respectively.

These values are comparable with the standard deviations given in the next to the last column of table 16. It is evident that they are of a very different order of magnitude. Roughly these theoretical standard deviations are only about one-eighth as large as the observed values. Thus in the above theoretical distributions the means of the separate distributions do not deviate very far from the mean of the sum of the distributions. In other words the distribution of the theoretical means gives a very peaked curve while the observed means give a very flat curve.

In order to make the differences between these two curves clear Fig. 16 has been prepared. This figure shows the graphs of two normal curves each having the same area and mean but different standard deviations. The flat topped curved has a standard deviation of

1.0667 corresponding to the observed distribution of the means in series A (Table 16). The peaked curve on the other hand has a standard deviation of 0.1925 corresponding to the theoretical curve having

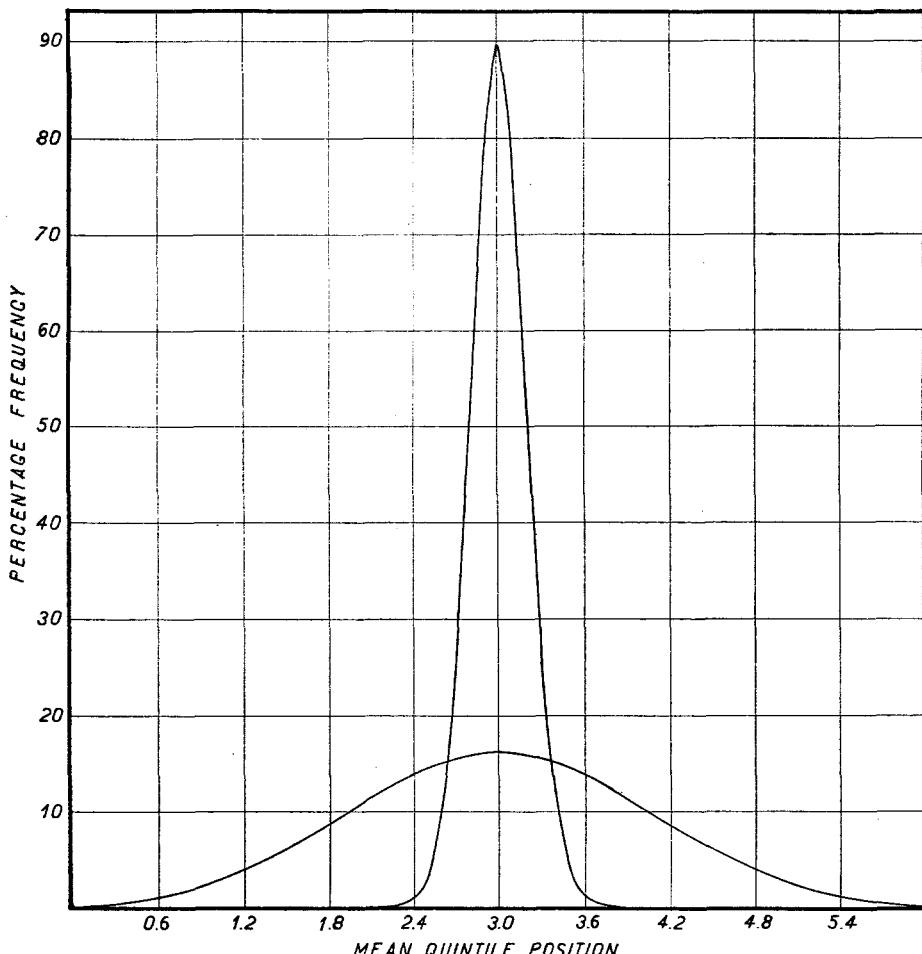


Fig. 16. Diagram showing two normal curves with the same area and mean. The flat curve has a standard deviation corresponding to the distribution of the observed mean quintile positions. The peaked curve has the standard deviation of the theoretical distributions of means on the assumption that the observations were distributed at random.

the same area and mean as that for series A. The difference between these two curves is striking and shows that the laws which govern their formation are quite different.

From the evidence so far brought forth it is clear that the quintile distribution of the measurements of a plant is not determined according to chance. On the other hand it is equally evident that the mean quintile positions of the individual plants are distributed as if these means were determined by chance. Thus, it was pointed out above that the standard deviations of the distributions of the observed means were not far different from the standard deviation of a distribution in which the frequency of each class was equal. Now it has also been shown that a distribution with equal frequency in each class is the result of the operation of the law of chance in which the probability of success in any class is equal to that in any other class. The fact that the standard deviation of the observed distribution tends to be slightly less than that of the theoretical, indicates that the chance of success was not quite equal in all classes.

The fact that the means are distributed as if they were determined by the law of probability is a matter of fundamental importance. A moment's consideration will make it clear that if a series of means are distributed according to a law of probability it necessarily follows that the causes underlying these means are distributed according to the same law of chance. Thus, if a number of plants remain relatively small throughout their growth this result is not due to chance but to some definite cause or causes.

Likewise if more plants remain relatively large than is to be expected on the theory of chance this result must be due to some underlying cause. If the number of small plants and medium plants and large plants are distributed according to some law, i. e., if they form a smooth distribution or if several independent series show the same tendency, it implies that the causes of small, medium and large plants are distributed according to the same law. Thus, for example, if, in a random sample, the number of small plants, medium plants and large plants are equal it means that the causes of these three kinds of plants are equally distributed among such plants. Further these causes are distributed at random according to the laws of probability.

Now it is just such a phenomenon which underlies the Mendelian interpretation of heredity. It is assumed that the 'factors' are distributed according to the laws of probability. Thus in the ordinary Mendelian F_2 generation with one pair of allelomorphic characters the frequency of each of the four classes **AA**, **Aa**, **aA**, and **aa** is equal. The fact that the mean quintile positions of these plants are distributed

with such regularity in the three independent series strongly suggests that the manner of growth of these plants is dependent upon internal factors which are distributed in much the same manner as Mendelian factors. This possibility has already been referred to (cf. p. 128). The present facts strongly support this view.

Before discussing this phase of the subject in detail it will be advantageous to consider the standard deviations of the mean quintile positions of individual plants. Thus if our supposition that the manner of growth of a plant is dependent upon Mendelian factors is true, it would follow that those plants which were relatively very large or very small (i. e., the extremes), would be more nearly homozygous, on the average, than plants occupying an average quintile position. Thus if we may assume for the moment that growth is dependent upon four pairs of factors the extreme plants would tend to be of the zygotic constitution of **AABBCCDD** and **aabbccdd** respectively. Some would be heterozygous for one factor or another but they would tend to approach this type. If that is the case, and we assume that dominance is absent, these end plants ought to be less variable than the medium sized plants. It will be worth while to determine this point.

It has been pointed out above that the standard deviation of the mean quintile position of a plant measures a definite characteristic of that plant. Thus a plant which maintains the same relative size throughout the whole season will have a small standard deviation, even being zero in some cases. On the other hand a plant which is relatively large at one time and relatively small at another will have a larger standard deviation. It has been noted above that the largest possible standard deviation of fourteen measurements distributed in five classes would be 2·00 and that if the measurements were equally distributed in all five classes the standard deviation would be 1·4142.

In tables 13, 14 and 15 the standard deviation of each individual plant is given. Owing to the small number of observations these standard deviations are subject to a rather large probable error, about 13 percent. For this reason it is not possible to draw very definite conclusions from individual standard deviations. However, it is entirely possible to treat these constants by statistical methods and to draw definite conclusions from the general trend of a series of such constants. Thus, although the fact that any individual standard deviation is small is not by itself significant, the fact that all the standard deviations of a series are small may be of very great significance.

An examination of tables 13 to 15 shows that these standard deviations range from zero to 1.54. Only two plants (table 13) show a standard deviation of 1.5 or over, and only two other standard deviations are as high as 1.4 (table 15). This shows at once that the measurements of most of these plants are not as widely scattered about their means as would be the case if the observations were equally distributed among all five quintiles.

In order to get at the general trend of these constants, frequency distributions of the standard deviations for each series may be made. Such distributions together with their means and standard deviations are given in table 17.

Table 17.
Frequency Distributions of the Standard Deviations of the Mean Quintile Positions of Individual Plants.

Series	Classes								Mean Standard Deviation	Standard Deviation of the Mean Standard Deviation
	0-0.2	0.2-0.4	0.4-0.6	0.6-0.8	0.8-1.0	1.0-1.2	1.2-1.4	1.4-1.6		
A	0	2	6	16	16	8	4	2	0.8556 ± 0.0246	0.2685 ± 0.0174
B	3	1	9	14	11	9	7	0	0.7741 ± 0.0291	0.3250 ± 0.0211
C	2	2	7	9	17	10	6	2	0.8673 ± 0.0281	0.3092 ± 0.0199

From this table it is noted that the distributions are fairly smooth considering the small number of observations and also that they are reasonably symmetrical. The means of these distributions range from 0.77 to 0.87. Now it was shown above that if the observations were distributed among the quintiles at random the average standard deviation would be 1.4142. The difference between this and the largest observed mean standard deviation is more than 25 times the probable error.

This small mean standard deviation and the uniformity with which it occurs in all three series shows conclusively that there is a strong tendency for the measurement of any individual plant to be closely grouped about its mean. That is, this shows again the point which has been previously brought out from other evidence, viz., that there is a strong tendency for plants relatively small at one stage to remain relatively small throughout their growth and similarly for medium sized plants and for large plants.

The question of most importance in this connection is in regard to the variation of the quintile distributions of plants with different mean quintile positions. For example, will plants which are relatively small for the entire season be more or less variable than medium sized plants?

In tables 13 to 15 there is given the average standard deviation of the plants whose mean quintile position falls in the same class. Thus from table 13 it is seen that the plants whose mean quintile position is between 1·0 and 1·8 have an average mean quintile standard deviation of 0·6658. The values of these mean standard deviations for the different series are brought together in table 18. It will be noted that these are the average standard deviations for these plants, obtained by averaging the standard deviations of the plants in a class. This average standard deviation is entirely different from the standard deviation of the combined frequency distributions of the plants in a class. This latter standard deviation will be referred to later.

Table 18.
Average Mean Quintile Standard Deviations of Plants Having
Different Mean Quintile Positions.

Series	Mean Quintile Classes				
	1·0—1·8	1·8—2·6	2·6—3·4	3·4—4·2	4·2—5·0
A	0·6658	0·8597	1·0386	0·9563	0·7041
B	0·4586	0·9368	1·0696	0·9438	0·6300
C	0·4880	0·9220	1·0665	0·9849	0·6501

From this table it is seen at once that the average standard deviation of the end classes, that is of the very small and the very large plants, are much smaller than the standard deviations of the plants in the three middle classes. Also the standard deviation of the plants in the middle class is larger than that of any other class. There is thus an apparent difference in the variability of the large and small plants on the one hand and the medium sized plants on the other.

However, before drawing definite conclusions regarding these variabilities, it must be remembered that plants with small or large means cannot have as large standard deviations as the medium sized plants. A plant which has a mean of 1·8 cannot have its observations so widely scattered as a plant with a mean of 3·0. This can best be

shown by examples. Thus a plant with a mean quintile position not greater than 1·8 cannot have its fourteen observations more widely distributed than the following example.

Quintiles	I	II	III	IV	V
Frequency	11	0	0	1	2

The mean of this distribution is 1·786 and the standard deviation is 1·520. On the other hand the widest possible distribution of the measurement of a plant having a mean of 3·0 is given by

Quintiles	I	II	III	IV	V
Frequency	7	0	0	0	7

The mean of this distribution is 3·0 and the standard deviation is 2·00.

It is thus evident that the plants which are medium sized may, because of this very fact alone, have larger standard deviations than either the small plants or the large plants. The question with which we are concerned is whether the difference between the variabilities of the extreme plants and the medium sized plants is greater than can be accounted for by the differences in their means. That is are the plants in the end classes actually less variable after making allowance for the difference in the means?

What we wish to know then is the most probable standard deviation of groups of plants whose means fall within the classes of 1·0 to 1·8, 1·8 to 2·6, etc. One way of determining this is to write down every possible combination of 14 figures which when distributed in five classes or less will give a mean value between 1·0 and 1·8. An example will make this clear.

Quintiles	I	II	III	IV	V
11	0	0	1	2	
10	1	1	0	2	
11	0	1	0	2	
9	3	0	0	2	
	etc., etc.				

The mean of each of these distributions is less than 1·8. It requires some time to write out all the possible combinations but it is not difficult to do. Similarly the various combinations having means between 1·8 and 2·6 and those between 2·6 and 3·4 may be written out. The values in the last two classes will be the reverse of those in the first two classes.

It would be possible to determine the standard deviation of each of these distributions and the average of these for each class could be compared with the average standard deviation given in table 18. Since there are several thousand possible combinations this would involve a great amount of work. It will be simpler to sum all the distributions in a given class and then obtain the standard deviations from these total distributions.

Table 19 gives the total number of possible observations falling in each quintile, distributed with respect to the classes of mean quintile position. The mean and standard deviation of each distribution is also given.

Table 19.

Showing the quintile distribution of the total number of possible observations in all the separate combinations of fourteen observations of which the individual mean would fall within the class given at the left of the table:

Mean Quintile Classes	Number of possible observations in quintile					Total	Mean	Standard Deviation
	I	II	III	IV	V			
1·0—1·8	1111	327	136	75	45	1 694	1·5927	0·9932
1·8—2·6	3593	3135	1887	1175	808	10 598	2·2895	1·2502
2·6—3·4	3011	3856	4522	3856	3011	18 256	3·0000	1·3198
3·4—4·2	808	1175	1887	3135	3593	10 598	3·7106	1·2502
4·2—5·0	45	75	136	327	1111	1 694	4·4073	0·9932
Total	8568	8568	8568	8568	8568	42 840	—	—

In order to obtain constants which are comparable with these means and standard deviations it will be necessary to go back to the quintile distributions of the individual plants and obtain the total frequency on each quintile for plants having their mean quintile positions in the same class. The distributions of the individual plants and the totals for each class are given in tables 61, 62 and 63.

The means and standard deviations of each of these total distributions may be obtained and compared with the theoretical constants from table 19. The means and standard deviations for each series together with the theoretical constants taken from tables 19 are given in table 20.

Table

Showing the mean and standard deviation of the total quintile distributions in the

Series	Mean quin				
	1·0—1·8		1·8—2·6		2·6—
	Mean	Standard Deviation	Mean	Standard Deviation	Mean
A	1·4143 ±·0432	0·7558 ±·0305	2·4127 ±·0495	0·8240 ±·0850	2·8762 ±·0504
B	1·3247 ±·0303	0·5569 ±·0214	2·1508 ±·0562	0·9349 ±·0397	2·9801 ±·0571
C	1·3214 ±·0419	0·6576 ±·0296	2·1558 ±·0469	0·8627 ±·0332	3·0420 ±·0489
Ave- verages ¹⁾	1·3535 ±·0219	0·6567 ±·0155	2·2398 ±·0292	0·8739 ±·0207	2·9661 ±·0298
Theo- retical	1·5927 ±·0285	0·9932 ±·0202	2·2895 ±·0082	1·2502 ±·0058	3·0000 ±·0066
Diffe- rence	-0·2402 ±·0359	-0·3465 ±·0255	-0·0497 ±·0303	-0·3763 ±·0215	-0·0399 ±·0305

In the third from the last row of this table the constants for the three series are averaged. There is no essential difference in respect of these constants between the three series so that the average of the three will give a better value than to take each series separately.

From this table it is noted that in general the average means are not widely different from the theoretical means. This is especially true of the middle classes where the differences between the average and theoretical means are but little greater than the probable errors. In the end classes the differences are several times the probable errors and hence are undoubtedly significant. It is further noted that in the case of small plants the observed means are smaller than the theoretical while with large plants the reverse is true.

In regard to the standard deviations the differences are much more marked. In every case these differences are from ten to fifteen times their probable error. Further in every case the observed standard deviations are lower than the theoretical. Thus the measurements of the corn plants are much less scattered about their means than the most probable distributions on the theory of chance.

¹⁾ The probable errors of these averages are calculated by taking $n =$ the sum of the variates in the three series.

20.

bution of all the plants in each series which have their mean quintile classes indicated.

tile classes				
3·4	3·4—4·2		4·2—5·0	
Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
1·0840 ± 0·0357	3·9026 ± 0·0550	1·0114 ± 0·0389	4·5000 ± 0·0446	0·7427 ± 0·0316
1·0506 ± 0·0404	3·6703 ± 0·0481	0·9614 ± 0·0340	4·5071 ± 0·0412	0·7221 ± 0·0291
1·1182 ± 0·0346	3·8442 ± 0·0554	1·0201 ± 0·0392	4·5804 ± 0·0230	0·3607 ± 0·0163
1·0843 ± 0·0211	3·8057 ± 0·0295	0·9676 ± 0·0209	4·5292 ± 0·0211	0·6085 ± 0·0149
1·3198 ± 0·0047	3·7106 ± 0·0082	1·2502 ± 0·0058	4·4073 ± 0·0285	0·9932 ± 0·0202
— 0·2355 ± 0·0217	+ 0·0951 ± 0·0306	— 0·2826 ± 0·0217	+ 0·1219 ± 0·0355	— 0·3847 ± 0·0251

As was pointed out above the most interesting question in connection with the standard deviations is whether the end classes are less variable than the middle classes when account is taken of the differences in the size of the means. An examination of the above table will make it clear that there is a marked difference in this respect. Thus in the small plants having their mean quintile positions between 1·0 and 1·8, the average observed standard deviation is 0·35 less than the standard deviation of the theoretical distribution. This difference is some fourteen times its probable error. In the plants whose means fall in the middle class, i. e., between 2·6 and 3·4, the observed deviation is only 0·24 less than the theoretical. This difference is more than ten times its probable error. Thus the extremely small plants have a standard deviation nearly fifty percent lower than the medium sized plants after all allowances are made for the differences in the size of the means. A similar but even more marked difference is shown between the very large and the medium sized plants.

On the assumption which we made that the manner of growth was determined by Mendelian factors this is exactly the result which would be expected. The extreme classes would tend to be more nearly homozygous for all factors and hence would be less variable than their heterozygous neighbors. On this assumption the two end classes would tend to be equally homozygous and ought to show about the same

variability. Similarly the classes lying next to the end classes ought to be similar in this respect. There appears to be some difference in the two latter classes as shown in table 62. However this difference is probably due to the rather small number of variates in each series. In order to test our assumption and to obtain somewhat smoother series it will be permissible to average the standard deviation of the two end classes and also the two classes lying next to these end classes. We can then compare these two averages with each other and with the deviation of the middle class. Table 21 shows these averages for the observed standard deviation and for the theoretical and the differences between the two.

Table 21.

Showing the average standard deviation of similar classes and the differences between the observed and theoretical constants.

	Plants having their mean quintile position between		
	1·0—1·8 and 4·2—5·0	1·8—2·6 and 3·4—4·2	2·6—3·4
Average observed standard deviation	0·6326 ± ·0108	0·9208 ± ·0147	1·0843 ± ·0212
Average theoretical standard deviation ¹⁾	0·9932 ± ·0081	1·2502 ± ·0041	1·3198 ± ·0047
Difference	-0·3606 ± ·0168	-0·3294 ± ·0152	-0·2355 ± ·0217

From this table it is seen; (1) that the difference between the observed and theoretical standard deviations is much greater in the end classes than elsewhere. This means that the observed standard deviations are on the average much lower relatively in the end classes than elsewhere.

2. The difference between these standard deviations is about fifty percent greater in the end classes than in the middle class. The probable errors are so small that there can be no question as to the significance of these differences.

3. The difference between the theoretical and observed standard deviations in the classes next to the extremes is slightly less than the difference in the end classes. It is, however, much greater than the difference between these constants in the middle class.

¹⁾ The probable errors of these averages have been calculated by taking $n =$ the sum of all the variates entering the average.

Summing up the evidence from these mean quintile standard deviations it may be said that the standard deviations of all the classes are much smaller than would be expected if the observations were distributed according to pure chance. Further the standard deviations of the extremes, i. e. the very large and the very small plants, are relatively much smaller than the deviation of the middle classes. The standard deviation of the middle class is relatively much greater, i. e., it approaches nearer to the theoretical standard deviation than any of the classes on either side of it.

It has already been pointed out that these results are what would be expected if the growth was determined by Mendelian factors. In the following discussion the facts brought out in this paper will be summarized and an attempt made to show their relation to other facts of growth and genetics.

Discussion and Results.

The question which next concerns us is whether it is possible to present a plausible theory to account for these observed facts. Let us keep clearly in mind the main fact that is to be accounted for, namely, that in the frequency distribution of the average relative size for the whole season, the number of plants is practically equal in each class. Further there is a strong tendency for individual plants, especially those in the extreme classes, to maintain the same relative size throughout the season. Two general lines of explanation suggest themselves. First, the differences in individual plants may be due to the action of environmental influences and, second, they may be due to the action of internal factors. These two possible explanations have already been suggested but they will be discussed more in detail here.

At first sight it appears quite probable that these differences in individual plants are due to environmental influences. Thus plants which are in better soil or which are less crowded will grow better than plants under less favorable conditions. These conditions would on the whole remain nearly constant throughout the season and hence might produce the effects noted. It has already been pointed out that the field in which these plants grew was extremely uniform. So far as general observation went there were no places where the soil appeared to be better or worse than at any other place. The manure was evenly spread over the ground and then plowed under. The fertilizer was distributed by a machine which secured a uniform distribu-

tion. Only one plant was allowed to remain in each hill so that no plant was crowded more than another.

Finally it can be shown that the large and small plants were evenly distributed throughout the rows. There were no patches where all the plants were very large and others where they were all small. In tables 13, 14 and 15 the mean quintile position of each individual plant is given together with the number of the plant. The plants were numbered consecutively from one end of the row to the other. From these tables it is possible to follow the distribution of consecutive plants. Some idea of the random nature of these distributions can be obtained by making a frequency distribution of the mean quintile positions of each group of ten consecutive plants. Such a distribution from series A is given in table 22. The other series show similar distributions. It will be remembered that all of the original sixty plants did not complete the season so that some classes have less than ten plants.

Table 22.
Frequency distributions of the mean quintile positions in
each group of ten consecutive plants in series A.

Plant Nos.	Mean quintile position				
	1·0—1·8	1·8—2·6	2·6—3·4	3·4—4·2	4·2—5·0
1—10	1	3	2	1	1
11—20	3	1	3	1	1
21—30	3	2	1	3	0
31—40	1	1	4	1	3
41—50	2	1	1	0	4
51—60	0	1	4	5	0
Total	10	9	15	11	9

From this table it is seen that within each group of ten plants the distribution is entirely random. We could hardly expect throws of dice to show a more random distribution than that given in the table if we take account of the difference in the probability of each class.

The fact that the position in the field has no effect upon the size of the plant can also be seen by studying the distribution of the plants as given in tables 13 to 15. A single example may be cited

here although many others showing the same thing can be found. These plants are taken from series A table 13.

Plant No.	44	45	46	47	48	49
Mean Quintile Position	4.50	2.28	1.71	4.64	1.50	4.21

Here within a distance of some seven feet there are three very small plants and three very large ones. It is improbable that the difference in the soil or other conditions could account for the great difference between plant No. 48 and those standing eighteen inches to either side of it. Many other examples showing the same thing could be picked out of these tables.

It, therefore, is highly improbable that the differences in the growth of individual plants are to be accounted for on the basis of environmental factors.

At several places in this paper occasion has been taken to point out that the facts could be interpreted by assuming that growth is controlled by internal factors. Within the past few years it has been clearly shown by Nilsson-Ehle, East and Hayes, Emerson and East and others that the inheritance of quantitative characters may be explained upon a Mendelian factor basis. Thus Emerson and East (:13) present a considerable amount of evidence to show that the height of corn plants is inherited in a Mendelian fashion. If this is true it is not inconceivable that the height at various growth stages or, in fact, the manner of growth is also dependent upon similar internal factors.

The theory developed, especially by Emerson and East, to account for the so-called 'blended' inheritance of quantitative characters may be stated briefly as follows. It is assumed that the character, height of plant for example, may be dependent upon several pairs of independent factors and that these factors, each allelomorphic to its absence, are cumulative in their effect upon the plant. It is also assumed that dominance is absent and consequently that the homozygous condition of a factor will have twice the effect of the heterozygous condition.

To illustrate this theory we may borrow the hypothetical case given by Emerson and East. They assume that a plant genotypically 12 inches tall is crossed with a plant genotypically 28 inches tall. The difference is 16 inches. If this difference is due to one pair of factors then the large plant would have the constitution AA and the small plant aa. Under the above theory the presence of each A will add 8 inches to the height of the plant. The heterozygous Aa would then be 20 inches tall, providing of course there was no fluctuating varia-

tion due to environment. In the F_2 generation twenty-five percent of the plants would be AA or 28 inches tall, fifty percent would be Aa or 20 inches tall and twenty-five percent aa or 12 inches tall. If more than one pair of factors are present the number of F_2 classes will be greater and the percentage frequency will be different.

Let us see if a somewhat similar theory will interpret the growth facts brought out in this paper. Let us assume first that there are two pairs of independent factors; that dominance is absent and that the effect of each factor is cumulative. The corn plant is open fertilized so that a random sample of corn plants at any time would have the constitution of a stable Mendelian population in which all possible matings take place at random. Pearson (:04) has shown that this constitution will be the same as that of the individuals in the segregating generation.

A random sample of corn plants would then have the constitution obtained by mating at random,

$$\text{♂♂ gametes} \left\{ \begin{array}{l} AB \\ Ab \\ aB \\ ab \end{array} \right. \text{with ♀♀ gametes} \left\{ \begin{array}{l} AB \\ Ab \\ aB \\ ab \end{array} \right.$$

By the usual recombination this gives the combinations and proportions given in columns one and two of table 23.

Table 23.

Showing the zygotic constitution of a random sample of corn plants together with the theoretical mean quintile position of each class of plants under each of two assumptions for the values of the factors concerned.

Zygotic formulae	Frequency	Mean quintile position if $A = B = 1$ quintile	Mean quintile position if $A = 0.5$ quintile and $B = 1.5$ quintile
AABB	1	5.0	5.0
AABb	2	4.0	3.5
AaBB	2	4.0	4.5
AaBb	4	3.0	3.0
AAbb	1	3.0	2.0
Aabb	2	2.0	1.5
aaBB	1	3.0	4.0
aaBb	2	2.0	2.5
aabb	1	1.0	1.0

We may assume that plants of the constitution **aabb** will remain relatively small throughout the season, i. e., that they will have a mean quintile position of 1·0. Further the **AABB** plants will remain relatively large or will have a mean quintile value of 5·0. If we assume that each factor has the same value in increasing the mean quintile value of a plant, then the addition of any one factor will tend to raise the mean quintile position of the plant one quintile. Each type of plant will then have the theoretical mean quintile position shown in column three of table 23.

If a frequency distribution of these mean quintile positions is made using the same class units as those in the observed data it is found that the frequency of each class occurs in the ratio of 1 : 4 : 6 : 4 : 1. In percentage figures the ratios are 6·25 : 25·0 : 37·5 : 25·0 : 6·25. Now the observed frequencies taking all the series together as given in table 16 are, for the same classes, 29 : 29 : 43 : 35 : 27. In percentages these are in the ratio 17·8 : 17·8 : 26·4 : 21·5 : 16·6. It is clear from the two percentage ratios that they are not at all related and that our supposition does not fit the facts.

However, there is no a priori reason why each pair of factors should have the same effect upon the plants. It is entirely conceivable that the precent of the factor A, for example, has only one-half or one-third the effect of the presence of the factor B. Applying this principle it has been assumed as before that plants **aabb** would have a mean quintile position of 1·0. If one **A** were present it would increase the mean quintile position 0·5 of a quintile or to 1·5 quintile. Two **A**'s would of course have twice the effect of one. If one **B** were present the mean quintile position would be increased 1·5 quintiles or to a value of 2·5 quintiles. According to this scheme the theoretical mean quintile position of each class of plants is that given in the last column of table 23. Putting these into a frequency distribution as before we get the theoretical distribution given in table 24. The observed percentage frequencies are also given in this table. These latter frequencies are the total percentage frequencies for all three series as given in table 16.

From this table it is seen that the two percentage distributions correspond very well indeed. As a matter of fact if Elderton's test for the goodness of fit is applied to these two distributions it is found that the probability, $P > 0\cdot9626$. This means that in a series of random samples, from material following the theoretical law, we could expect

to get less than four series out of one hundred which would agree as closely or more closely with the theoretical distribution than does the observed distribution. Thus the agreement is exceedingly good.

Table 24.

Showing the frequency distribution of the theoretical mean quintile positions obtained from the values given in the last column of table 23. Also the observed percentages taken from table 16.

	Mean quintile classes				
	1·0—1·8	1·8—2·6	2·6—3·4	3·4—4·2	4·2—5·0
Theoretical frequencies	3	3	4	3	3
Theoretical percentage frequencies	18·75	18·75	25·0	18·75	18·75
Observed percentage frequencies	17·8	17·8	26·4	21·5	16·6

This close agreement does not necessarily mean that there are only two pairs of factors concerned in the growth of these plants. It might be possible to get a series of weighings for three, four or more factors which would give as good or better fit than that obtained above. The breeding data necessary to determine the probable number of factors in these plants is not available. For this reason it is not desirable to make any more complicated assumptions than are necessary to give a reasonable explanation of the facts.

Two alternative conceptions of the action of these factors may be postulated. In the first place it may be conceived that all the pairs of factors affect the growth of the plant throughout the season. On the other hand it may be assumed that certain factors affect the growth only during a certain stage of its growth. It is somewhat suggestive to imagine that each of the four cycles into which the growth curve can be divided is controlled by separate factors. The present data are not sufficient to give evidence for or against such a supposition.

Summary.

Part I of this paper gives a description of the data used and deals with the general growth curves and the constants of variation for each series as a whole. The reader is referred to pages 116—117 for a summary of the results of this portion of the paper.

In Part II we have attempted, by the study of the growth of individual plants, to analyze the adult variation curve into its component elements. Specifically, we have attempted to follow individual plants and groups of plants, having the same relative size at one stage of their growth, through the remaining growth stages. We have endeavored to ascertain how such plants are distributed as to relative size in the successive growth stages and to discover some reasons for, or the laws governing the distributions.

To study these questions it is necessary to have a measure of the relative size of the plants at each growth stage. For this, each distribution was divided into five equal parts or quintiles. In any distribution the relatively small plants are in quintile I and the relatively large ones in quintile V. Fig. 5 illustrates the exact meaning of a quintile.

The problem was first approached by studying the quintile distribution of all the measurements, throughout the season, of a group of plants starting in a given quintile.

From this it has been shown that there is a strong tendency for the plants to remain in or near the quintile in which they started. As a measure of this tendency we used the root-mean-square deviation of these observed distributions from the most probable distribution of such measurements on the theory of chance. These root-mean-square constants are given in tables 7 to 9 and are shown graphically in Fig. 8.

From these constants it is seen that in every case (except one) the deviation from the theoretical mean is very much greater than would occur on the basis of chance.

The deviation of the very small and the very large plants (quintiles I and V) are much greater than any of the others. Thus there is a much more marked tendency for the extreme plants to remain in the extreme classes than for the medium sized plants to remain in a particular medium sized class.

Tables 10 to 12 and Fig. 9 show, by means of the same constants, that there is a similar, though less marked, tendency for the

plants ending in a given quintile to have remained in or near that quintile. In this case also there is, on the whole, a greater tendency for the extreme plants to have maintained their same relative size. However, this tendency is much less marked than in the cases cited above.

On the whole, the relative size (quintile position) of a plant at the beginning of the season is a much better criterion of its probable relative size for the whole season than is its relative size at the end of the season.

The second step in this analysis has been to study the mean quintile position of each group of plants in the successive growth stages. As shown in Figures 10—12, the group of plants starting in a given quintile show more or less regression towards the mean of the population with the advance of the season. Owing in part, at least, to the rather small number of individuals in each group there are a number of irregularities in some of these plotted lines.

However the main conclusion is entirely clear. Extreme variants at the beginning of the season tend strongly, on the whole, to remain extreme variants during the whole season. At the same time such extreme variants do tend somewhat to regress towards the general population mean as growth continues. This second tendency is, however, by no means so strong as the first.

The third step in the analysis of these variation curves has been a study of the average relative size (mean quintile position) of the individual plants and of the variability of individual plants with respect to relative size. The conclusions drawn from a study of these data are:

1. The observed difference in the manner of growth of individual plants and of groups of plants cannot be explained as the effect of external, environmental factors.
2. These differences are rather to be looked upon as the effect of internal factors.
3. The distribution of the average relative size (mean quintile position) of individual plants is such as to suggest the random distribution of these factors among the plants. The same thing is brought out by the distribution of the relative measurements of plants starting or ending with a given relative size (quintile).
4. The simplest method of explaining these facts is to regard the differences in the manner of growth as due to independent Mendelian factors which are distributed at random in any population of open fer-

tilized maize plants. These factors would occur in the proportions found in a stable Mendelian population mating at random.

5. By assuming the presence of two independent growth factors and weighting each with the proper value, it is possible to obtain a theoretical distribution agreeing very closely with the observed distribution. It is possible that by using more factors even a better fit might be obtained.

6. The interpretation of the growth of these plants by Mendelian factors is strongly supported by the distribution of the standard deviations of the plants with different relative sizes. Thus it has been shown that the extreme plants which would be more nearly homozygous and for this reason less variable are, as a matter of fact, some fifty percent less variable than the plants in the middle class after all allowance has been made for the difference in the size of the means.

Bibliography.

- BOAS, F., 1897: The Growth of Toronto Children. Rept. Commissioner of Education, 1896—97, Vol. II, pp. 1541—1599.
- and WISSLER, C., 1905: Statistics of Growth. U. S. Bur. of Education, 1905, Chap. II, pp. 25—132.
- DONALDSON, H. H., 1906: A Comparison of the White Rat With Man in Respect to the Growth of the Entire Body. Boas Anniversary Volume, New York, pp. 5—26.
- , 1908: A Comparison of the Albino Rat With Man in Respect to the Growth of the Brain and Spinal Cord. Jour. Comp. Neur. Psych., Vol. 18, pp. 346—392.
- , 1909: On the Relation of the Body Length to the Body Weight and to the Weight of the Brain and of the Spinal Cord in the Albino Rat (*Mus norvegicus var. albus*). Jour. Comp. Neur. Psych., Vol. 19, pp. 157—167.
- and HATAI, S., 1911: A Comparison of the Norway Rat with the Albino Rat in Respect to Body Length, Brain Weight, Spinal Cord Weight, and the Percentage of Water in Both the Brain and the Spinal Cord. Jour. Comp. Neur., Vol. 21, pp. 417—458.
- EAST, E. M. and HAYES, H. K., 1911: Inheritance in Maize. Conn. Agr. Exp. Station Bulletin 167, pp. 1—141.
- , 1912: Heterozygosis in Evolution and in Plant Breeding. U. S. Dept. of Agr., Bur. of Pl. Ind., Bull. 243, pp. 1—58.
- ELDERTON, W. PALIN: Tables for Testing the Goodness of Fit of Theory to Observation. Biometrika, Vol. I, pp. 155—164.
- EMERSON, R. A. and EAST, E. M., 1913: The Inheritance of Quantitative Characters in Maize. Nebraska Agr. Exp. Sta. Research Bulletin, No. 2, pp. 1—120.
- HATAI, S., 1909: Note of the Formulas Used for Calculating the Weight of the Brain in the Albino Rats. Jour. Comp. Neur. Psych., Vol. 19, No. 2, pp. —,
- , 1911a: An Interpretation of Growth Curves from a Dynamical Standpoint. Anat. Rec., Vol. 5, pp. 373—382.

- , 1911b: The Mendelian Ratio and Blended Inheritance. Amer. Naturalist, Vol. 45, pp. 99—106.
- MINOT, C. S., 1891: Senescence and Rejuvenation. First paper: on the Weight of Guinea-pigs. Jour. of Physiol., Vol. 12, pp. 97—153.
- , 1907: The Problem of Age, Growth, and Death. Popular Science Monthly, Vol. 71.
- NILSSON-EHLE, H., 1908: Einige Ergebnisse von Kreuzung bei Hafer und Weizen. Botan. Notiser, pp. 257—294.
- , 1909: Kreuzungsuntersuchungen an Hafer und Weizen. Lunds Universitets Års-skrift n. s. sec. 2, Vol. 5, No. 2, pp. 1—122.
- , 1911: Kreuzungsuntersuchungen an Hafer und Weizen II. Lunds Universitets Års-skrift n. s. sec. 2, Vol. 7, No. 6, pp. 1—82.
- PEARL, R., 1907: Variation and Differentiation in *Ceratophyllum*. Carnegie Institution Publication Nr. 58, pp. 1—135.
- and SURFACE, F. M., 1910: Experiments in Breeding Sweet Corn. Ann. Rpt. Me. Agr. Exp. Sta. 1910, pp. 249—307.
- PEARSON, K., 1904: On a Generalized Theory of Alternative Inheritance with Special Reference to Mendel's Laws. Phil. Trans. Roy. Soc. Ser. A., Vol. 203, pp. 53—86.

Table 25.
Frequency Distributions of Plants in Series A.

Height of Plants (Millimeters)	Date of Measurement													
	June						July							
	12	15	19	22	26	29	3	6	10	13	17	20	24	27
75—99	1													
100—124	14	1												
125—149	26	7	2											
150—174	9	12	7	1										
175—199	2	16	10	1	1									
200—224	1	9	8	1	0									
225—249	1	6	8	9	4									
250—274		0	10	7	2	1								
275—299		3	3	5	5	2	1							
300—324			4	11	10	3	0							
325—349			1	9	12	9	2							
350—374			1	7	11	9	5							
375—399				3	5	11	3							
400—424					2	18	11	3						
425—449						2	2	11	1					
450—474							3	8	3					
475—499							0	6	10					

Height of Plants (Millimeters)	Date of Measurement													
	June						July							
	12	15	19	22	26	29	3	6	10	13	17	20	24	27
500—524						1	4	6	2	2				
525—549						0	4	1	0					
550—574						0	6	3	0					
575—599						0	5	7	0					
600—624						2	4	3	0					
625—649						0	4	5	1	1				
650—674						1	4	7	2	1				
675—699						2	4	1	0					
700—724						2	6	9	1					
725—749						0	2	0						
750—774						5	3	0						
775—799						2	7	0		1				
800—824						2	1	1		0				
825—849						3	5	4	1					
850—874						0	3	4	0					
875—899						1	4	6	1					
900—924						1	1	2	1					
925—949						2	1	6	0					
950—974						5	2	4	1	1				
975—999						2	2	5	3	3	3			
1000—1024						0	6	2	2	1				
1025—1049						1	2	4	0	0	0			
1050—1074						1	3	2	1	1	1			
1075—1099						3	1	5	4	4	1			
1100—1124									2	2	3	2		
1125—1149									4	4	5	7		
1150—1174									2	4	3	4		
1175—1199									3	5	2	5		
1200—1224									1	3	6	4		
1225—1249										4	7	3		
1250—1274										3	3	4		
1275—1299										2	3	7		
1300—1324										1	6	5		
1325—1349											3	3		
1350—1374											2	1		
1375—1399												3		

Table 26.
Frequency Distributions of Plants in Series B.

Height of Plants (Millimeters)	Date of Measurements													
	June						July							
	11	15	18	22	25	29	2	6	9	13	16	20	23	27
50—74	3													
75—99	7													
100—124	17	2	1											
125—149	23	10	2	1										
150—174	3	11	9	1										
175—199	1	21	9	0	1									
200—224	6	15	1	1										
225—249	2	10	9	0										
250—274	2	6	7	3	1									
275—299	0	11	6	0										
300—324	2	10	6	3										
325—349		11	6	5	4									
350—374		1	14	8	7									
375—399		1	9	6	9	4								
400—424		1	5	13	8	2								
425—449			2	10	13	3								
450—474			1	6	7	4	1							
475—499				2	3	10	2							
500—524					2	6	4							
525—549					1	10	5							
550—574						2	2	2						
575—599						6	3	1						
600—624						6	3	0						
625—649						0	6	2						
650—674						0	7	4						
675—699						1	4	1	1					
700—724							2	5	2					
725—749							5	4	1	1				
750—774							3	4	2	0				
775—799							2	3	3	0				
800—824							3	5	0	0				
825—849							1	5	2	1				
850—874							1	1	3	0				
875—899							3	7	0					

Table 27.
Frequency Distributions of Plants in Series C.

Height of Plants (Millimeters)	Date of Measurements													
	June						July							
	12	16	19	23	26	30	3	7	10	14	17	21	24	28
75—99	1													
100—124	14													
125—149	18	2												
150—174	17	7	3											
175—199	3	16	2											
200—224	2	10	11	2										
225—249		10	14	3	1									
250—274		4	12	2	1									
275—299		6	5	7	2									
300—324			3	15	3	2								
325—349			1	6	8	2								
350—374			3	7	10	7	2							
375—399			1	12	8	6	5							
400—424				1	13	12	4	1						
425—449					6	10	7	2						
450—474					2	8	12	1	1					
475—499					1	4	5	2	0					
500—524						2	8	4	0					
525—549						2	7	7	2	1				
550—574							0	4	1	0				
575—599							4	5	2	1				
600—624							0	6	1	1	1			
625—649							0	2	5	0	0			
650—674							1	4	6	0	0			
675—699								3	3	1	1			
700—724								4	2	3	0			
725—749								1	3	2	1			
750—774								3	8	1	0			
775—799								2	4	2	0			
800—824								0	3	3	1	1		1
825—849								2	0	2	5	0		
850—874								2	3	1	1	1		
875—899								4	4	1	1	0		

Height of Plants (Millimeters)	Date of Measurements													
	June						July							
	12	16	19	23	26	30	3	7	10	14	17	21	24	28
900—924									2	2	0	0		
925—949								3	8	3	1			
950—974							0	6	3	0				
975—999							0	3	1	3				
1000—1024								1	1	3	2			
1025—1049								1	1	6	3	1		
1050—1074									3	7	0	0		
1075—1099									1	7	0	1		
1100—1124									5	1	5	2		
1125—1149									1	3	3	3	3	
1150—1174									0	3	3	1	2	
1175—1199									1	1	10	3	3	
1200—1224									1	1	5	7	5	
1225—1249										3	5	11	8	
1250—1274										1	4	4	4	
1275—1299										1	2	2	6	
1300—1324											3	5	1	
1325—1349											1	5	7	
1350—1374											1	3	4	
1375—1399											1	3	2	
1400—1424											1	2	5	
1425—1449												0	2	
1450—1474												1	1	
1475—1499												1	1	
1500—1524													0	
1525—1549													1	

Table 28.
Frequency Distributions of Plants in Series D.

Height of Plants (Tassel) (Millimeters)	Date of Measurement										
	July							August			
	3	6	10	13	17	20	24	27	3	10	24
175—199											
200—224											
225—249		1									
250—274		4	1	1							
275—299		2	0	0							
300—324		2	2	0							
325—349	2	4	2	0							
350—374	1	4	5	0							
375—399		2	6	1	1						
400—424		2	5	1	0						
425—449		1	6	2	0						
450—474		0	5	1	0						
475—499		2	5	3	1	1					
500—524		1	3	6	0	0					
525—549		1	3	3	1	0					
550—574			2	7	1	0					
575—599			3	3	0	0					
600—624			1	3	2	0					
625—649			1	7	0	0					
650—674			0	0	3	0					
675—699			0	2	5	1					
700—724			0	3	4	0	1				
725—749			3	1	1	1	0				
750—774				4	7	1	0				
775—799				1	4	1	0				
800—824				2	0	0	0				
825—849				0	3	2	0				
850—874				0	2	0	0				
875—899				0	1	5	0				
900—924				1	4	1	0				
925—949				0	3	4	0				
950—974				1	1	6	0	1			
975—999				0	3	2	0	0			

Height of Plants (Tassel) (Millimeters)	Date of Measurement										
	July							August			
	3	6	10	13	17	20	24	27	3	10	24
1000—1024				1	1	5	0	0			1
1025—1049					2	1	2	1	1	1	0
1050—1074					1	2	3	0	0	0	0
1075—1099					0	2	2	2	2	2	1
1100—1124						1	2	3	1	1	2
1125—1149						1	2	2	2	2	2
1150—1174						0	5	4	3	3	4
1175—1199						0	4	6	1	2	1
1200—1224							1	1	3	2	2
1225—1249								2	6	5	4
1250—1274								1	2	3	3
1275—1299								1	6	7	7
1300—1324									1	4	1
1325—1349									1	8	8
1350—1374									2	2	5
1375—1399									4	2	2
1400—1424										1	3
1425—1449									0	4	3
1450—1474									0	0	0
1475—1499									1	3	2
1500—1524										1	3

Table 29.
Frequency Distribution of Plants in Series E.

Height of Plants (Tassel) (Millimeters)	Date of Measurement									
	July							August		
	6	9	18	16	20	23	27	3	10	24
175—199										
200—224										
225—249										
250—274										
275—299	4									
300—324	3	5	1							
325—349	3	3	1							
350—374	3	7	0							
375—399	2	4	0	1						
400—424	2	9	3	1						
425—449	4	5	0							
450—474	3	2	1							
475—490	3	4	0							
500—524	4	6	1							
525—549	0	4	3							
550—574	1	8	4	1						
575—599	2	2	0	1						
600—624		3	2	1						
625—649		2	4	0						
650—674		5	5	1						
675—699		4	5	0	1					
700—724		0	6	2	0					
725—749		1	3	2	1					
750—774		1	1	2	0					
775—799		1	1	2	0					
800—824			4	2	1					
825—849			7	2	0					
850—874			1	4	0					
875—899			2	4	0					
900—924			0	7	1					
925—949			1	2	1	1				
950—974			0	6	0	0				
975—999			1	2	3	0				

Height of Plants (Tassel) (Millimeters)	Date of measurement									
	July							August		
	6	9	13	16	20	23	27	3	10	24
1000—1024					2	1	1	1	1	1
1025—1049					3	4	0	0	0	0
1050—1074				1	2	0	0	0	0	0
1075—1099				3	2	0	0	0	0	0
1100—1124					2	3	1	0	0	0
1125—1149				0		4	0	0	0	0
1150—1174				0		8	0	0	0	0
1175—1199				1		7	3	2	2	2
1200—1224					1	4	1	1	1	1
1225—1249						2	3	1	1	1
1250—1274						3	5	2	3	3
1275—1299						1	6	4	3	4
1300—1324						2	6	8	8	7
1325—1349						1	5	0	0	1
1350—1374						0	6	7	8	7
1375—1399						2	4	7	6	6
1400—1424							0	1	1	1
1425—1449							3	4	4	4
1450—1474							4	6	5	5
1475—1499							1	2	2	2
1500—1524							2	4	5	5
1525—1549							0	0	0	0
1550—1574							1	1	2	2
1575—1599								1	0	0
1600—1624								0	0	1
1625—1649								1	1	0
1650—1674								0	0	0
1675—1699								0	0	0
1700—1724								0	0	0
1725—1749								0	0	0
1750—1774								0	0	0
1775—1799								0	0	0
1800—1824								1	1	1

Table 30.
Frequency Distributions of Plants in Series F.

Height of Plants (Tassel) (Millimeters)	Date of Measurement										
	July							August			
	7	10	14	17	21	24	28	4	10	24	
175—199											
200—224											
225—249											
250—274											
275—299	1										
300—324	3	1									
325—349	5	1									
350—374	8	1	2								
375—399	4	2	0								
400—424	5	4	1								
425—449	1	7	2	2							
450—474	3	4	1	0							
475—499	3	6	1	0							
500—524	2	7	2	0							
525—549	1	1	1	2							
550—574	1	3	3	3							
575—599		2	5	0							
600—624		4	1	1	1						
625—649		2	7	2	1						
650—674		2	5	0	0						
675—699		0	3	2	0						
700—724		0	5	4	1						
725—749		2	1	3	1						
750—774			2	2	1						
775—799			3	7	3						
800—824			1	6	0						
825—849			3	1	1						
850—874			2	2	1			1			
875—899			1	2	0			0			
900—924			1	3	1			1			
925—949			0	1	3			0			
950—974			1	1	3			1			
975—999			0	2	3			0			

Table 31. Series A.
 Quintile distribution for each successive measurement of the plants starting in quintile I.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	11	0	0	0	0	1·0000	0
15	7	3	1	0	0	1·4545	0·6555
19	7	4	0	0	0	1·3636	·4810
22	8	3	0	0	0	1·2727	·4453
26	8	1	2	0	0	1·4545	·7820
29	6	3	1	1	0	1·7273	·9621
July 3	4	2	3	2	0	2·2727	1·1357
6	5	3	2	1	0	1·9091	·9958
10	6	3	1	1	0	1·7273	·9626
13	4	4	1	2	0	2·0909	1·0829
17	6	1	2	2	0	2·0000	1·2061
20	4	3	2	2	0	2·1818	1·1135
24	3	5	2	1	0	2·0909	·8999
27	1	6	3	1	0	2·3636	·7714
Total excluding June 12	69	41	20	13	0		

Table 32. Series A.
 Quintile distribution for each successive measurement of the plants starting in quintile II.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	11	0	0	0	2·0000	0
15	1	5	4	1	0	2·4545	0·7820
19	2	3	5	1	0	2·4545	·8907
22	1	3	3	3	1	3·0000	1·1281
26	1	3	3	3	1	3·0000	1·1281
29	1	3	3	3	1	3·0000	1·1281
July 3	2	4	3	1	1	2·5454	1·1573
6	4	2	2	2	1	2·4545	1·3723
10	2	3	4	1	1	2·6363	1·1497
13	3	3	2	2	1	2·5455	1·3046
17	2	5	1	3	0	2·4545	1·0321
20	2	2	4	2	1	2·8182	1·1920
24	1	2	2	4	2	2·3636	1·2407
27	2	1	1	4	3	3·4545	1·4377
Total excluding June 12	24	39	37	30	13		

Table 33. Series A.
Quintile distribution for each successive measurement of the plants starting in quintile III.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	0	12	0	0	3·0000	0
15	1	2	3	5	1	3·2500	1·0907
19	0	1	4	4	3	3·7500	0·9242
22	0	2	4	4	2	3·6667	1·0272
26	0	4	3	2	3	3·3333	1·1779
29	1	0	4	4	3	3·6667	1·1054
July 3	1	3	3	4	1	3·0833	1·1148
6	0	3	3	2	4	3·5833	1·1874
10	1	3	1	5	2	3·3333	1·2470
13	1	1	3	4	3	3·5833	1·1874
17	0	1	5	1	5	3·8333	1·0672
20	0	2	3	2	5	3·8333	1·1424
24	0	2	1	4	5	4·0000	1·0700
27	2	0	2	3	5	3·7500	1·4213
Total excluding June 12	7	24	39	44	42		

Table 34. Series A.
Quintile distribution for each successive measurement of the plants starting in quintile IV.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	0	0	10	0	4·00	0
15	2	1	2	2	3	3·30	1·4866
19	1	3	2	4	0	2·90	1·0440
22	2	2	2	2	2	3·00	1·4177
26	2	2	1	3	2	3·10	1·4457
29	1	4	1	1	3	3·10	1·4457
July 3	3	2	1	2	2	2·80	1·5362
6	1	1	4	2	2	3·00	1·3416
10	2	1	4	1	2	3·30	1·1874
13	2	2	3	2	1	2·80	1·2490
17	2	2	2	3	1	2·90	1·3000
20	2	3	1	2	2	3·10	1·4457
24	3	1	4	1	1	2·60	1·2806
27	3	1	3	2	1	2·70	1·3454
Total excluding June 12	26	25	30	27	22		

Table 35. Series A.
 Quintile distribution for each successive measurement of the plants starting in quintile V.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	0	0	0	10	5·0	0
15	0	0	2	3	5	4·3	0·7810
19	1	0	0	2	7	4·4	1·2000
22	0	1	2	2	5	4·1	1·0440
26	1	0	2	3	4	3·9	1·2206
29	2	1	3	1	3	3·2	1·4697
July 3	1	0	1	2	6	4·2	1·2490
6	0	2	0	4	4	3·7	1·4177
10	1	2	0	2	5	3·0	1·0954
13	1	1	2	1	5	3·8	1·4000
17	1	3	0	2	4	3·5	1·5000
20	3	1	1	3	2	3·0	1·5492
24	4	1	2	1	2	2·6	1·5621
27	4	2	2	1	1	2·3	1·3454
Total including June 12	19	14	17	27	53		

Table 36. Series B.
 Quintile distribution for each successive measurement of the plants starting in quintile I.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	12	0	0	0	0	1·0000	0
15	8	4	0	0	0	1·3333	0·4714
18	8	3	1	0	0	1·4167	.6401
22	7	2	3	0	0	1·6667	.8433
25	7	4	1	0	0	1·5000	.6455
29	7	3	0	1	1	1·8333	1·2802
July 2	7	3	1	1	0	1·6667	.9428
6	8	1	2	1	0	1·6667	1·0700
9	7	3	1	1	0	1·6667	.9428
13	6	3	2	1	0	1·8333	.9861
16	6	3	1	2	0	1·9167	1·1140
20	7	2	2	1	0	1·7500	1·0146
23	7	1	2	2	0	1·9167	1·1870
27	5	2	2	2	1	2·3333	1·3744
Total excluding June 11	90	34	18	12	2		

Table 37. Series B.
Quintile distribution for each successive measurement of the plants starting in quintile II.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	0	11	0	0	0	2·0000	0
15	3	3	2	3	0	2·4545	1·1172
18	3	3	2	2	1	2·5454	1·3042
22	4	4	1	1	1	2·1818	1·2620
25	4	2	1	2	2	2·6364	1·5584
29	3	5	3	0	0	2·0000	.7385
July 2	2	3	3	2	1	2·7273	1·2128
6	2	2	4	3	0	2·7273	1·0521
9	3	3	4	1	0	2·2727	.9621
13	4	3	4	0	0	2·0000	.8528
16	4	3	4	0	0	2·0000	.8528
20	3	4	2	2	0	2·2727	1·0521
23	2	6	1	0	2	2·4545	1·3046
27	2	5	1	1	2	2·6364	1·3661
Total excluding June 11	39	46	32	17	9		

Table 38. Series B.
Quintile distribution for each successive measurement of the plants starting in quintile III.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	0	0	10	0	0	3·00	0
15	0	1	2	5	2	3·80	0·8718
18	0	2	3	4	1	3·40	.9165
22	0	2	2	5	1	3·50	.9220
25	0	2	4	3	1	3·30	.9000
29	0	2	3	3	2	3·50	1·0247
July 2	1	3	3	2	1	3·90	1·1358
6	1	2	3	4	0	3·00	1·0000
9	0	3	1	6	0	3·30	.9000
13	0	4	2	3	1	3·10	1·0440
16	0	4	2	3	1	3·10	1·0440
20	1	3	2	3	1	3·00	1·1832
23	2	2	2	3	1	2·90	1·3000
27	3	1	3	3	0	2·60	1·2000
Total excluding June 11	8	31	32	47	12		

Table 39. Series B.
 Quintile distribution for each successive measurement of the plants starting in quintile IV.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	0	0	0	11	0	4·0000	0
15	0	2	6	2	1	3·1818	0·8332
18	0	4	4	1	2	3·0909	1·0831
22	0	2	4	3	2	3·4545	·9875
25	0	3	2	3	3	3·5454	1·1571
29	1	1	5	2	2	3·2727	1·1354
July 2	1	2	3	2	3	3·3636	1·2984
6	1	4	2	2	2	3·0000	1·2791
9	1	1	4	1	4	3·5454	1·3046
13	1	1	2	3	4	3·7273	1·2855
16	1	1	2	3	4	3·7273	1·2855
20	0	2	3	2	4	3·7273	1·1354
23	0	2	3	3	3	3·6364	1·0677
27	1	2	2	3	3	3·4545	1·3046
Total excluding June 11	7	27	42	30	37		

Table 40. Series B.
 Quintile distribution for each successive measurement of the plants starting in quintile V.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	0	0	0	0	10	5·00	0
15	0	2	0	1	7	4·50	0·9220
18	0	0	0	4	6	4·60	·4899
22	0	1	1	2	6	4·30	1·0050
25	0	0	3	3	4	4·10	·8307
29	0	0	1	4	5	4·40	·6633
July 2	0	0	1	4	5	4·40	·6633
6	0	1	0	1	8	4·60	·9165
9	0	1	1	2	6	4·30	1·0050
13	0	0	1	4	5	4·40	·6633
16	0	0	2	3	5	4·30	·7810
20	0	0	2	3	5	4·30	·7810
23	0	0	3	3	4	4·10	·8307
27	0	1	3	2	4	3·90	1·0440
Total excluding June 11	0	6	18	36	70		

Table 41. Series C.
 Quintile distribution for each successive measurement of the plants starting in quintile I.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	11	0	0	0	0	1.0000	0
16	7	4	0	0	0	1.3636	0.4810
19	6	2	2	0	1	1.9091	1.2394
23	6	3	1	1	0	1.7273	.9621
26	6	3	1	1	0	1.7273	.9621
30	7	3	0	1	0	1.5455	.8907
July 3	7	2	1	1	0	1.6364	.9791
7	6	2	2	1	0	1.8182	1.0284
10	7	1	2	1	0	1.7273	1.0523
14	6	2	2	1	0	1.8182	1.0284
17	6	3	2	0	0	1.6364	.7714
21	6	3	2	0	0	1.6363	.7714
24	4	4	3	0	0	1.9091	.7925
28	4	3	3	1	0	2.0909	.9958
Total excluding June 12	78	35	21	8	1		

Table 42. Series C.
 Quintile distribution for each successive measurement of the plants starting in quintile II.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	11	0	0	0	2.0000	0
16	1	4	4	2	0	2.6364	0.7768
19	5	2	2	2	0	2.0909	1.1641
23	3	4	3	0	1	2.2727	1.1354
26	0	5	1	2	3	3.2727	1.2856
30	1	2	4	3	1	3.0909	1.0832
July 3	0	5	4	0	2	2.9091	1.0832
7	2	3	4	2	0	2.5455	.9875
10	1	6	2	1	1	2.5455	1.0756
14	0	7	2	2	0	2.5454	.7821
17	1	5	2	2	1	2.7273	1.1354
21	1	4	3	3	0	2.7273	.9621
24	1	2	3	2	3	3.3636	1.2984
28	1	2	3	2	3	3.3636	1.2984
Total excluding June 12	17	51	37	23	15		

Table 43. Series C.
Quintile distribution for each successive measurement of the plants starting in quintile III.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	0	11	0	0	3·0000	0
16	1	1	4	4	1	3·2727	1·0521
19	0	3	4	2	2	3·2727	1·0521
23	0	1	4	4	2	3·6364	.8814
26	2	1	4	3	1	3·0000	1·2080
30	1	2	3	2	3	3·3636	1·2985
July 3	0	1	3	4	3	3·8182	.9360
7	0	1	3	4	3	3·8182	.9360
10	0	1	4	3	3	3·7273	.9621
14	1	0	3	4	3	3·7273	1·1353
17	1	1	2	4	3	3·6364	1·2263
21	1	1	0	4	5	4·0000	1·2790
24	1	1	1	3	5	3·9091	1·3111
28	2	0	1	2	6	3·9091	1·5048
Total excluding June 12	10	14	36	43	40		

Table 44. Series C.
Quintile distribution for each successive measurement of the plants starting in quintile IV.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	0	0	11	0	4·0000	0
16	1	1	3	4	2	3·4545	1·1571
19	0	1	3	5	2	3·7273	.8624
23	2	1	2	3	3	3·3636	1·4317
26	3	1	3	1	3	3·0000	1·5374
30	1	3	3	3	1	3·0000	1·1284
July 3	4	2	1	3	1	2·5455	1·4377
7	2	3	1	3	2	3·0000	1·4142
10	2	3	1	4	1	2·9091	1·3111
14	4	0	3	2	2	2·8182	1·5265
17	3	0	4	3	1	2·9091	1·3111
21	3	1	3	3	1	2·8182	1·3360
24	4	3	1	2	1	2·3636	1·3668
28	3	3	1	3	1	2·6364	1·3668
Total excluding June 12	32	22	29	39	21		

Table 45. Series C.
Quintile distribution for each successive measurement of the plants starting in quintile V.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	0	0	0	11	5·0000	0
16	1	1	0	1	8	4·2727	1·3544
19	0	3	0	2	6	4·0000	1·2783
23	0	2	1	3	5	4·0000	1·1282
26	0	1	2	4	4	4·0000	·9535
30	1	1	1	2	6	4·0000	1·3484
July 3	0	1	2	3	5	4·0909	·9958
7	1	2	1	1	6	3·8182	1·4659
10	1	0	2	2	6	4·0909	1·2397
14	0	2	1	2	6	4·0909	1·1639
17	0	2	1	2	6	4·0909	1·1639
21	0	2	3	1	5	3·8182	1·1921
24	1	1	3	5	1	3·3636	1·0677
28	1	3	3	3	1	3·0000	1·0871
Total excluding June 12	6	21	20	31	65		

Table 46. Series A.
Quintile distribution for each successive measurement of the plants ending in quintile I.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	1	2	2	3	4	3·5833	1·3199
15	3	1	5	2	1	2·7500	1·2329
19	3	2	2	2	3	3·0000	1·5274
22	3	2	3	3	1	2·7500	1·2988
26	3	4	1	2	2	2·6666	1·4833
29	4	5	2	0	1	2·0000	·9129
July 3	4	3	3	1	1	2·3333	1·2470
6	3	5	2	1	1	2·3333	1·1747
10	5	3	3	1	0	2·0000	1·0000
13	4	3	5	0	0	2·0833	·8620
17	4	5	3	0	0	1·9167	·7592
20	7	4	1	0	0	1·5000	·6455
24	9	3	0	0	0	1·2500	·4830
27	12	0	0	0	0	1·0000	0
Total excluding July 27	53	42	32	15	14		

Table 47. Series A.
 Quintile distribution for each successive measurement of the plants ending in quintile II.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	6	1	0	1	2	2·2	1·6613
15	3	4	0	1	2	2·5	1·5000
19	4	4	0	1	1	2·1	1·3000
22	4	4	0	1	1	2·1	1·3000
26	4	2	1	2	1	2·4	1·4283
29	4	2	2	1	1	2·3	1·3454
July 3	3	2	2	2	1	2·6	1·3565
6	3	2	3	0	2	2·6	1·4283
10	3	4	1	1	1	2·3	1·2689
13	3	4	1	1	1	2·3	1·2689
17	4	3	1	1	1	2·2	1·3266
20	3	4	2	1	0	2·1	·8307
24	1	8	1	0	0	2·0	·4472
27	0	10	0	0	0	2·0	0
Total excluding July 27	45	44	14	13	14		

Table 48. Series A.
 Quintile distribution for each successive measurement of the plants ending in quintile III.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	3	1	2	3	2	3·0000	1·4768
15	3	0	4	1	3	3·0909	1·5047
19	2	2	3	2	2	3·0000	1·3486
22	2	3	1	2	3	3·0909	1·5047
26	4	1	2	1	3	2·8182	1·6413
29	2	2	2	2	3	3·1818	1·4660
July 3	2	3	2	1	3	3·0000	1·4769
6	1	2	3	2	3	3·3636	1·2984
10	2	1	2	2	4	3·4545	1·4990
13	1	2	1	4	3	3·5454	1·3046
17	1	0	4	4	2	3·5454	0·9875
20	1	0	5	5	0	3·2727	0·8624
24	1	0	8	2	0	3·0000	0·7385
27	0	0	11	0	0	3·0000	0
Total excluding July 27	25	17	39	31	31		

Table 49. Series A.
Quintile distribution for each successive measurement of the plants ending in quintile IV.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	1	4	3	2	1	2·8181	1·1132
15	2	4	0	4	1	2·8181	1·3362
19	2	2	3	3	1	2·9091	1·2397
22	2	0	5	2	2	3·1818	1·2651
26	1	1	3	4	2	3·4545	1·1608
29	1	2	4	1	3	3·3636	1·2651
July 3	2	3	0	4	2	3·0909	1·4429
6	2	1	2	5	1	3·1818	1·2651
10	2	2	1	3	3	3·2727	1·4826
13	1	2	1	4	3	3·5454	1·3046
17	1	2	0	5	3	3·6363	1·2981
20	0	2	2	3	4	3·8182	1·1132
24	0	0	2	7	2	4·0000	·6030
27	0	0	0	11	0	4·0000	0
Total excluding July 27	17	25	26	47	28		

Table 50. Series A.
Quintile distribution for each successive measurement of the plants ending in quintile V.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	3	5	1	1	3·0	0·8944
15	0	2	3	3	2	3·6	·9695
19	0	1	3	3	3	3·8	·9798
22	0	2	2	3	3	3·7	1·1000
26	0	2	4	2	2	3·4	1·0198
29	0	0	2	6	2	4·0	·6325
July 3	0	0	4	3	3	3·9	·8307
6	1	1	1	3	4	3·8	1·3266
10	0	2	3	3	2	3·5	1·0247
13	2	0	3	2	3	3·4	1·4283
17	1	2	2	1	4	3·5	1·4318
20	0	1	1	2	6	4·3	1·0050
23	0	0	0	2	8	4·8	·4000
27	0	0	0	0	10	5·0	0
Total excluding July 27	4	16	33	34	43		

Table 51. Series B.
 Quintile distribution for each successive measurement of the plants ending in quintile I.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	5	2	3	1		2·0000	1·0444
15	4	2	3	2		2·2727	1·1354
18	4	5	1	1		1·9091	.8999
22	4	4	2	1		2·0000	.9535
25	3	3	1	3	1	2·6464	1·3674
29	4	4	3	0		1·9091	.7925
July 2	7	2	1	1		1·6363	.9791
6	3	3	4	1		2·2727	.9621
9	3	6	1	1		2·0000	.8528
13	3	5	2	1		2·0909	.8999
16	3	6	1	1		2·0000	.8528
20	5	5	1	0		1·6364	.6428
23	7	4	0	0		1·3636	.4810
27	11	0	0	0		1·0000	0
Total excluding July 27	55	51	23	13	1		

Table 52. Series B.
 Quintile distribution for each successive measurement of the plants ending in quintile II.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	2	5	1	2	1	2·5454	1·2320
15	3	3	1	2	2	2·7273	1·4825
18	3	3	1	3	1	2·6364	1·3668
22	4	2	2	1	2	2·5454	1·4993
25	4	2	2	1	2	2·5454	1·4993
29	4	2	5	0	0	2·0909	.8999
July 2	1	6	2	1	1	2·5454	1·0756
6	5	1	3	0	2	2·3636	1·4936
9	4	0	3	3	1	2·7273	1·4199
13	6	0	2	1	2	2·3636	1·6109
16	5	1	3	1	1	2·2727	1·3546
20	4	2	3	1	1	2·3636	1·2985
23	4	3	4	0	0	2·0000	.7273
27	0	11	0	0	0	2·0000	0
Total excluding July 27	49	30	32	16	16		

Table 53. Series B.
Quintile distribution for each successive measurement of the plants ending in quintile III.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	2	1	3	3	2	3.1818	1.3360
15	2	1	3	2	3	3.2727	1.4179
18	2	2	2	2	3	3.1818	1.4662
22	1	3	2	3	2	3.1818	1.2660
25	2	1	3	3	2	3.1818	1.3360
29	2	2	1	3	3	3.2727	1.4826
July 2	1	2	3	3	2	3.2727	1.2128
6	2	3	0	4	2	3.0909	1.4429
9	2	1	2	3	3	3.3636	1.4318
13	2	2	1	4	2	3.1818	1.4025
16	2	1	1	4	3	3.4545	1.4374
20	1	2	1	5	2	3.4545	1.2333
23	0	3	4	4	0	3.0909	.7925
27	0	0	11	0	0	3.0000	0
Total excluding July 27	21	24	26	43	29		

Table 54. Series B.
Quintile distribution for each successive measurement of the plants ending in quintile IV.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	2	1	3	3	2	3.1818	1.3360
15	2	1	1	4	3	3.4545	1.4374
18	2	0	3	3	3	3.4545	1.3726
22	2	0	2	4	3	3.5455	1.3726
25	2	1	2	3	3	3.3636	1.4318
29	1	2	0	3	5	3.8182	1.4021
July 2	2	1	2	1	5	3.5455	1.5588
6	2	1	2	3	3	3.3636	1.4318
9	1	1	2	4	3	3.6366	1.2491
13	0	2	2	4	3	3.7273	1.0521
16	1	1	4	2	3	3.4545	1.2332
20	1	1	4	1	4	3.5454	1.3045
23	0	1	3	5	2	3.7273	.8624
27	0	0	0	11	0	4.0000	0
Total including July 27	18	13	30	40	42		

Table 55. Series B.
 Quintile distribution for each successive measurement of the plants ending in quintile V.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 11	1	2	0	3	4	3.7	1.4177
15		5	2	1	2	3.0	1.1832
18		2	3	2	3	3.6	1.1136
22		2	3	2	3	3.6	1.1136
25		4	3	1	2	3.1	1.1358
29		1	3	4	2	3.7	.9000
July 2		0	3	5	2	3.9	.7000
6		2	2	3	3	3.7	1.1000
9		1	3	0	3	3.1	1.3748
13		2	4	1	3	3.5	1.1180
16		2	2	3	3	3.7	1.1000
20		1	2	4	3	3.9	.9434
23		0	0	2	8	4.8	.4000
27		0	0	0	10	5.0	0
Total excluding July 27	2	26	30	31	41		

Table 56. Series C.
 Quintile distribution for each successive measurement of the plants ending in quintile I.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	4	1	2	3	1	2.6364	1.4317
16	4	1	3	0	3	2.7273	1.6006
19	4	0	2	2	3	3.3636	1.2264
23	4	1	2	1	3	2.8182	1.6413
26	3	2	3	2	1	2.6364	1.2984
30	5	2	2	1	1	2.1818	1.3357
July 3	5	2	0	2	2	2.4545	1.6160
7	3	4	1	0	3	2.6364	1.4006
10	4	3	1	0	3	2.5454	1.6160
14	6	1	1	1	2	2.2727	1.6006
17	5	2	1	1	2	2.3636	1.4006
21	5	4	1	1	0	1.8182	.9359
24	9	2	0	0	0	1.1818	.3857
28	11	0	0	0	0	1.0000	0
Total excluding July 28	61	25	19	14	24		

Table 57. Series C.
 Quintile distribution for each successive measurement of the plants ending in quintile II.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	3	2	0	3	3	3.0909	1.6210
16	1	3	0	4	3	3.4545	1.3727
19	1	3	1	5	1	3.1818	1.1921
23	0	3	2	4	2	3.4545	1.0756
26	1	3	1	3	3	3.3636	1.2984
30	3	1	4	1	2	2.8182	1.4025
July 3	2	2	2	2	3	3.1818	1.4659
7	2	2	1	4	2	3.1818	1.4025
10	3	0	1	5	2	3.2727	1.4827
14	1	2	1	6	1	3.3636	1.1497
17	1	2	2	4	2	3.3636	1.2263
21	1	2	6	2	0	2.8182	.8382
24	1	6	4	0	0	2.2727	.3802
28	0	11	0	0	0	2.0000	0
Total excluding July 28	20	31	25	43	24		

Table 58. Series C.
 Quintile distribution for each successive measurement of the plants ending in quintile III.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	3	3	1	1	3	2.8182	1.5847
16	2	2	2	2	3	3.1818	1.4659
19	2	2	5	0	2	2.8181	1.2661
23	3	1	3	2	2	2.9091	1.4425
26	1	2	4	2	2	3.1818	1.1920
30	0	2	1	5	3	3.8182	1.0280
July 3	2	3	4	0	2	2.7273	1.2850
7	3	4	2	0	2	2.4545	1.3726
10	2	3	4	0	2	2.7273	1.2580
14	1	5	3	0	2	2.7273	1.2128
17	2	4	3	0	2	2.6364	1.2979
21	2	4	2	1	2	2.7273	1.3545
24	0	2	6	3	0	3.0909	.6681
28	0	0	11	0	0	3.0000	0
Total excluding July 28	23	37	40	16	27		

Table 59. Series C.
 Quintile distribution for each successive measurement of the plants ending in quintile IV.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	1	2	2	3	3	3·4545	1·3045
16	2	3	2	2	2	2·9091	1·3784
19	1	2	2	4	2	3·3636	1·2263
23	2	4	1	1	3	2·9091	1·5045
26	4	2	2	1	2	2·5455	1·4992
30	2	4	2	0	3	2·8182	1·4659
July 3	2	1	1	5	2	3·3636	1·3668
7	3	0	2	4	2	3·1818	1·4659
10	2	1	1	5	2	3·3636	1·3668
14	2	1	2	3	3	3·3636	1·4317
17	2	1	2	4	2	3·2727	1·3545
21	2	1	1	3	4	3·5455	1·4992
24	1	1	0	9	0	3·5455	·9875
28	0	0	0	11	0	4·0000	0
Total excluding July 28	26	23	20	44	30		

Table 60. Series C.
 Quintile distribution for each successive measurement of the plants ending in quintile V.

Date of measurement	Quintile number					Mean quintile position	Standard deviation
	I	II	III	IV	V		
June 12	0	3	6	1	1	3·0000	0·8528
16	2	2	4	3	0	2·7273	1·0521
19	3	4	1	0	3	2·6364	1·5534
23	2	2	3	3	1	2·9091	1·2397
26	2	2	1	3	3	3·2727	1·4827
30	1	2	2	4	2	3·3636	1·2263
July 3	0	3	4	2	2	3·2727	1·0521
7	0	1	5	3	2	3·5455	·8907
10	0	4	4	1	2	3·0909	1·0830
14	1	2	4	1	3	3·2727	1·2850
17	1	2	3	2	3	3·3636	1·2979
21	1	0	1	4	5	4·0909	1·1642
24	0	0	1	0	10	4·8182	·5749
28	0	0	0	0	11	5·0000	0
Total excluding July 28	13	27	39	27	37		

Table 61.

Series A.

Quintile distribution of the individual plants arranged according to their mean quintile classes.

Mean Quintile Class 1·0—1·8.

Quintile number	Plant numbers										Total	Percent
	1	14	17	19	22	25	29	37	46	48		
I	8	11	10	13	13	9	9	12	8	7	100	71·43
II	4	2	2	1	0	5	1	2	2	7	26	18·57
III	2	1	1	0	1	0	2	0	4	0	11	7·86
IV	0	0	0	0	0	0	2	0	0	0	2	1·43
V	0	0	1	0	0	0	0	0	0	0	1	0·71
											Total	140
												100·00

Mean Quintile Class 1·8—2·6. .

Quintile number	Plant numbers										Total	Percent
	6	9	10	13	26	30	35	45	56			
I	4	3	4	2	2	5	3	6	2	31	24·60	
II	6	10	6	10	6	2	4	3	7	54	42·86	
III	4	1	3	2	6	5	5	1	4	31	24·60	
IV	0	0	1	0	0	2	2	3	1	9	7·14	
V	0	0	0	0	0	0	0	1	0	1	0·79	
										Total	126	99·99

Mean Quintile Class 2·6—3·4.

Quintile number	Plant numbers														Total	Percent	
	3	4	12	15	16	28	31	36	38	40	41	52	53	55	60		
I	3	1	0	1	3	0	0	2	3	1	1	0	1	1	5	22	10·48
II	4	6	2	2	2	6	4	4	4	4	7	7	4	3	1	60	28·57
III	2	4	6	6	4	2	5	5	2	6	2	5	4	8	1	62	29·52
IV	4	1	5	5	4	6	3	2	1	3	4	2	5	2	7	54	25·71
V	1	2	1	0	1	0	2	1	4	0	0	0	0	0	0	12	5·71
															Total	210	99·99

Mean Quintile Class 3·4—4·2.

Mean Quintile Class 4·2—5·0.

Quintile number	Plant numbers									Total	Percent
	7	11	32	33	34	42	44	47	49		
I	0	0	0	0	0	0	0	0	0	0	0
II	0	0	0	0	0	1	0	0	0	1	0·79
III	1	3	2	0	1	1	2	2	4	16	12·70
IV	7	4	4	3	2	1	3	1	3	28	22·22
V	6	7	8	11	11	11	9	11	7	81	64·29
									Total	126	100·00

Table 62.

Series B.

Quintile distribution of the individual plants arranged according to their mean quintile classes.

Mean Quintile Class 1·0—1·8.

Mean Quintile Class 1·8—2·6.

Quintile number	Plant numbers									Total	Percent
	1	11	22	25	27	35	47	49	53		
I	2	2	4	3	3	6	2	8	4	34	26·98
II	8	8	3	5	8	5	8	0	6	51	40·48
III	4	2	5	6	2	1	3	4	3	30	23·81
IV	0	2	2	0	1	1	1	2	1	10	7·94
V	0	0	0	0	0	1	0	0	0	1	·79
									Total	126	100·00

Mean Quintile Class 2·6—3·4.

Quintile number	Plant numbers											Total	Percent
	3	6	16	20	23	28	30	38	56	57	60		
I	0	1	1	0	0	1	0	1	4	1	1	10	6·49
II	6	2	5	1	8	3	6	3	1	4	5	44	28·57
III	5	6	2	8	2	6	5	5	3	4	4	50	32·47
IV	2	2	6	5	3	2	2	4	5	4	3	38	24·68
V	1	3	0	0	1	2	1	1	1	1	1	12	7·79
										Total	154	100·00	

Mean Quintile Class 3·4—4·2.

Quintile number	Plant numbers												Total	Percent	
	8	10	21	29	36	37	39	40	44	45	46	51	59		
I	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0·55
II	0	3	1	1	2	1	3	1	3	1	1	1	3	21	11·54
III	5	5	3	4	3	6	3	3	3	7	4	5	4	55	30·22
IV	8	2	5	7	7	4	1	6	3	5	8	4	5	65	35·71
V	1	4	5	2	2	3	7	4	4	1	1	4	2	40	21·98
											Total	182	100·00		

Mean Quintile Class 4·2—5·0.

Quintile number	Plant numbers										Total	Percent
	9	12	19	31	33	34	41	42	48	54		
I	0	0	0	0	0	0	0	0	0	0	0	0
II	0	0	0	1	0	0	0	0	0	2	3	2·14
III	0	3	0	0	2	1	3	0	1	0	10	7·14
IV	0	4	4	4	7	7	3	5	5	1	40	28·57
V	14	7	10	9	5	6	8	9	8	11	87	62·15
										Total	140	100·00

Table 63.

Series C.

Quintile distribution of the individual plants arranged according to their mean quintile classes.

Mean Quintile Class 1·0—1·8.

Quintile number	Plant numbers								Total	Percent
	21	22	25	30	31	38	52	55		
I	12	5	8	14	12	14	11	9	85	75·89
II	1	9	3	0	2	0	1	5	21	18·75
III	0	0	1	0	0	0	2	0	3	2·68
IV	1	0	2	0	0	0	0	0	3	2·68
V	0	0	0	0	0	0	0	0	0	0·00
								Total	112	100·00

Mean Quintile Class 1·8—2·6.

Quintile number	Plant numbers												Total	Percent
	1	4	9	14	16	19	24	28	32	33	36			
I	4	8	3	4	2	1	2	7	4	4	4	43	27·92	
II	5	1	3	7	7	7	4	3	9	7	9	62	40·26	
III	3	3	6	3	4	3	6	3	0	3	0	34	22·08	
IV	1	2	1	0	1	3	2	1	1	0	0	12	7·79	
V	1	0	1	0	0	0	0	0	0	0	1	3	1·95	
												Total	154	100·00

Mean Quintile Class 2·6—3·4.

Quintile number	Plant numbers															Total	Percent		
	3	5	13	15	17	18	26	34	37	40	41	44	47	49	50	56	57		
I	2	0	2	4	0	0	0	1	2	2	0	2	3	0	0	2	1	21	8·82
II	3	1	5	3	6	3	2	7	1	1	3	1	2	5	4	4	5	56	21·43
III	5	9	2	3	6	3	8	2	2	8	5	5	3	6	2	5	4	78	32·35
IV	1	4	3	1	0	8	4	1	9	3	4	3	4	1	7	3	2	58	27·73
V	3	0	2	3	2	0	0	3	0	0	2	3	2	2	1	0	2	25	9·66
															Total	238	99·99		

Mean Quintile Class 3·4—4·2.

Quintile number	Plant numbers										Total	Percent	
	2	8	11	20	27	29	35	39	51	53			
I	0	1	0	0	0	0	1	0	1	0	0	3	1·95
II	1	1	2	0	2	1	2	1	1	2	2	15	9·74
III	3	2	1	5	0	5	1	7	1	4	2	31	20·13
IV	9	5	5	2	6	6	9	2	4	5	6	59	38·31
V	1	5	6	7	6	2	1	4	7	3	4	46	29·87
											Total	154	100·00

Mean Quintile Class 4·2—5·0.

Quintile number	Plant numbers								Total	Percent	
	10	12	42	43	45	46	48	54			
I	0	0	0	0	0	0	2	0	2	1·78	
II	0	0	0	0	0	0	0	0	0	0·00	
III	1	3	0	0	0	1	1	2	8	7·14	
IV	1	3	3	2	3	2	1	8	23	20·54	
V	12	8	11	12	11	11	10	4	79	70·54	
									Total	112	100·00