

## TRUTH AT A WORLD IS A MODALITY

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Possible world semantics has formalized the intuitive "Leibnizian Notion" that what is necessary is what obtains at all worlds. Thus, for example, we think that necessarily  $2+2=4$  just in case  $2+2=4$  at all possible worlds, more generally;

Necessarily  $p$  if and only if  $p$  at all possible worlds.

In this paper I will investigate the nature of being *at a world*, more gracefully but more confusingly expressed as "being *true at a world*". I will argue that it is a *modality*, a mode or way of being true, in a sense to be made precise below, like necessity itself, and not a *relation* between truth bearers and worlds as it appears in formal possible world semantics, or as the use of the expression '*true at a world*' might suggest.

Why even try to analyze or reinterpret talk of truth at a world? Why not treat it as a well understood primitive or as already adequately analyzed by the recursive definition of truth in possible world semantics? I will argue that the nature of truth at a world is at the heart of several unresolved issues in modal metaphysics, and secondly, a metaphysical account of truth at a world is needed to supplement any semantic account.

Does the Leibnizian Notion provide an *analysis* of necessity? One might hold that necessity really just is truth at all worlds. Thus the Notion is a way of reducing intensional notions of necessity and possibility to an extensional language at the "cost" of increasing one's ontology to include possible worlds. Others, who have been termed "modalists," argue that the idea of necessity is prior to that of possible worlds. For them the Leibnizian Notion is true and useful for semantics but does not provide a *reduction* of necessity. Rather, modalists are likely to want to reduce talk of possible worlds to talk of modal facts such as what propositions are possibly true, or *would be* true if such and such were the case. Note that the dispute over modalism is independent of the issue of actualism versus realism about possible worlds, whether possible worlds are constructions out of actual entities like propositions or states of affairs or rather some *sui generis* entity. It is natural for modalists to be actualists, but the positions are logically independent.<sup>1</sup>

As usually understood necessity is a modality, a way of being true expressed in language with an operator (written ' $\Box$ ') which creates intensional contexts. Truth at a world, however, *seems* to be a relation between truth bearers and worlds expressed with a (metalinguistic) relation symbol, one which exists in a wholly extensional metalanguage. The reduction of an intensional language to an extensional metalanguage (which allows quantification over possible worlds) is meant to correspond with an analysis of modalities in terms of relations between truth bearers and possible worlds. Thus the debate over modalism is one issue where the nature of truth at a world is crucially important. A second is the existence of objects in different possible worlds, "transworld individuals."

David Lewis is one of those who sees the Leibnizian Notion as providing a *reduction* of necessity. What's more, he provides an *analysis* of truth at a world. Lewis' account of truth at a world arises in the context of his attack on cross-world individuals, objects that can exist in different possible worlds.<sup>2</sup> The real difficulty with allowing objects to exist in different possible worlds, Lewis suggests, comes not with saying what it is that makes the object the same in different possible worlds, thus causing a search for some sort of individual "essence". Rather, it appears as soon as one considers accidental or contingent properties. Suppose an object  $x$  has a property  $P$ , say being blue, only contingently. That means that  $x$  can exist in other possible worlds without having  $P$ . Now, asks Lewis, does  $x$  have  $P$  or not? It won't do, he argues, to say that it does and it doesn't, while muttering "at  $w$ " or "at  $w$ " under one's breath. That, he says, is incoherent, either  $x$  has  $P$  or it doesn't. This Lewis takes to be an argument against cross world individuals. His own view is that there must be different individuals that have  $P$  and lack it, his famous "counterparts". (His own analysis of ' $x$  has  $P$  at  $w$ ' is that simply ' $x$  has  $P$  and  $x$  is in  $w$ '. It is a counterpart  $x^*$  which lacks  $P$  and is in  $w$ .)

According to Lewis it is not coherent to say that an object has a property "at" one world but not "at" another. Why is this? Surely one might say, we relativize truth to an index all the time, as when we say that an object has a property at one time and not at another, as an account of change. This is not a persuasive reply. Indeed Lewis' qualms carry over equally well to having a property at a time, and can be illuminated by the example of change. What is the problem with relativizing the possession of a property? I think it is the non-relative nature of properties. Properties are "universal" precisely because they are the very same thing in each of their instances. The blue that an object here instantiates is the same as the blue instantiated by a blue

object over there, and by one yesterday. An object either has a property or it lacks it, in more realist language, the property inheres in the object or it doesn't. What, then, is it to possess a property at a time or place or world? The strategy used with talk of being blue *at* a place is to distinguish *parts* of objects and say that being blue at a place is just for the part *in* or occupying that place to be blue, simpliciter. For having a property *at* a time one might distinguish temporal parts or stages, so being blue at *t* is just having the temporal parts which "occupies" *t* being blue. Blueness is the same in each instance; the objects instantiating it differ. Lewis just continues this strategy into having a property at a world. It is different "world slices" or "counterparts" which have the property or lack it. While no one wants to hold that having a property at a place is primitive, many do so for times and resist the temporal part analysis, and all who believe in cross-world individuals do so for having a property at a world. Can our intuitions in these cases be explained? In particular, what lies behind the reluctance to relativize properties to worlds? The universal nature of properties explains our feeling that an object either has a property or not when it comes to places. Why don't we all follow Lewis' intuition about having properties at a world?

Relativizing the having of properties to times, places or worlds seems different from other cases of relativizing. The usual response of a realist about universals to a relativized notion is to postulate a relation. Rather than think of Socrates as tall "with respect to" Simmias, or seeing "being taller than Simmias" as a primitive property, one acknowledges the need for a relation of being *taller than*, which holds between Socrates and Simmias. Likewise it might be thought, let us see being blue as a relation between a time and an individual, or a world and an individual. But that offends against the intuition that being blue is a monadic universal, a property, that something has or lacks, not a hidden relation.

Perhaps what we are confronted with is a pair of incompatible intuitions, that objects have or lack properties, and the intuition that something may be true in different ways, at one time and not another, at one world and not another. I think that the two intuitions are compatible. One can save intuitions about the "adicity" of universals and realize that properties may hold "at" different indices. Possible world semantics is, however, only a partial and inadequate way of reconciling these two. What is needed is not the "semantic ascent" provided by semantics, but rather a move to a more "material mode" of speech and explicit talk of what *makes* sentences true or false, facts and their modes. Some of this talk will be paraphrasable in the formal mode

for those who are ontologically squeamish, but I think the more blatantly realistic form of speech is more intuitively helpful.

The way that objects are blue at one time and not at another or at one world and not at another is different from the way that Socrates is taller than one person and not another. One might say that a thing being blue or Socrates being taller than Simmias is a *fact*, a fact about (or *of* or *in*) this world. That *x* is blue in this world is not a relation between an object, a property and a world, rather it is a fact about this world, that *x* is blue in it. The claim that *x* is *P* at *w* should be seen as a modification of *x*'s being *P*, or as qualifying *w*, rather than a relation between *x*, *P* and *w*. It has a structure one might represent by '*(x is P) at w*'. 'At *w*' expresses a way that *x* is *P*, a mode of being *P*. Holding at *w* is thus a modality, just as being necessarily true or contingent are modalities.

Here is a little argument to show that 'at' can't express a relation. Suppose that 'at' did indicate a three place relation between an object, a property, and a world in '*x is P at w*'. Suppose that *x is P at w* which is then a *fact* of a special "extra-worldly" sort, but still a fact. Is *that* fact true at a given *w*?<sup>3</sup> If *x is P at w at w*, then we face a dilemma. If 'at' expresses a relation then it expresses both a three place and a four place relation in the same sentence. That is incoherent, for relations have a fixed adicity. The alternative would seem to be to treat 'at *w*' as a predicate. But predicates must apply to individuals and *x is P at w* is not an individual. (I consider the claim that it is a disguised metalinguistic expression below). The only resolution of this dilemma, I claim, is to see 'at *w*' as an *operator*, a linguistic device that can be iterated but which does not express a property, or a relation, but rather a modality.

I have argued that both the issues of modalism and transworld individuals hinge on the nature of truth at a world. If truth at a world is not a modality then the Leibnizian Notion constitutes a threat to modalism. If truth at a world must be analyzed as Lewis suggests transworld individuals can't exist. I will suggest below that truth at a world is a modality not reducible to a relation, but more primitive than necessity. Two issues remain. What is wrong with the metalinguistic account of truth at a world, why doesn't it avoid the preceding argument? Secondly, just what is a modality?

One might think that the whole discussion so far has been marred by a use/mention confusion. Thus, one might say, the Leibnizian Notion should be formalized as:

' $\Box p$ ' is true if and only if '*p*' is true at *w*, for all *w*.

Truth at, then, is simply a two place relation between a sentence and a world. The complete account is a little more complicated. Possible worlds semantics also gives truth conditions for iterated modalities

such as ' $\Box \Box p$ '. A standard way of proceeding is as follows; truth simpliciter is defined as truth at a designated world @ the actual world. A recursive definition then defines 'A is true at w' (written  $\models_w A$ ) for arbitrary A and w. If A is of the form  $\Box B$ , then the necessity clause is invoked;

$\models_w \Box B$  if and only if  $\models_{w'} B$  for all  $w'$ , such that ... $w'$ ...

(where ' $\dots w'$ ' expresses some property of  $w'$  such as being "accessible from" w. Below I will follow the " $S_5$ " definition which leaves out that restriction). The account of iterated modalities is then as follows:  $\models_w \Box \Box B$  if and only if  $\models_{w'} \Box B$  for all  $w'$ , if and only if  $\models_{w''} B$  for all  $w''$ . The puzzle about iterating 'at w' does not arise in possible world semantics because one is using a metalanguage which contains expressions such as ' $\models_w A$ ' which do not occur in the object language where ' $\Box A$ ' does. One simply can't ask whether " $\models_w \models_{w'} A$ ". That's not well-formed. Problems only arise when one treats the necessity clause as a potential *analysis* of modal discourse, one that we can use to restate everything we might want to say about necessary truths. In ordinary language the ability to ask of any given claim whether it is necessary or not is on a par with asking of the proposed analysis whether it is necessary or not, and consequently whether the proposed analysis holds of the analyzed claim, whether, that is, p is true at w at  $w'$  or not. Once one introduces talk of such "semantic facts" into a language, in this case truth at a world, then one cannot appeal to a language/metalanguage distinction to reject talk about those facts.<sup>4</sup> Possible world semantics is thus expressively inadequate.

One might think that the full definition of truth at a world, like Tarski's definition of truth, provides an elimination of semantic terms so that the weakness of being unable to express ' $\models_w \models_{w'} A$ ' is only apparent, that there are no semantic facts whose status must be evaluated, and so when fully analyzed the expression "'p' is true at all worlds w" will not include any talk of truth at a world. Not so. The recursive definition of truth at a world provides an account of truth at a world for arbitrary sentences as a result of giving recursive clauses for operators and connectives and basis clauses for atomic sentences. In addition to the "necessity clause" there will be others such as ' $\models_w A \& B$ ' if and only if  $\models_w A$  and  $\models_w B$ '. What do the atomic clauses look like? Consider an atomic sentence 'Ft'. The truth definition will include an account of the denotation of a singular term as a function of worlds, say with a function  $\text{den}(t, w)$  and of the satisfaction of predicates by objects as a function of worlds, with a relation,  $x \text{ sats } F, w$ . These basis clauses will differ crucially from those in the Tarski truth definition which does not relativize denotations or satisfaction to worlds or anything else. As

such Tarski was able to make use of the fact that languages contain finitely many constants and predicates to remove the last remaining use of semantic expressions by the recursive process. He could define satisfaction for primitive predicates with clauses like the following:

x satisfies 'blue' if and only if x is blue.

What basis clause will we have for modal semantics? It could only be:

x sats 'blue', w, if and only if x is blue at w.

('w' occurs here as a variable, not an abbreviation for a description of a possible world, or in some way schematically.) The right hand side of this clause is precisely the expression that is at issue. It is not a metalinguistic expression. All the language in it is used, not mentioned. As such, I claim, it states a sort of fact whose truth at worlds can itself be considered. It is up to one who objects to show what is illegitimate about such questions. The appeal to the object/metalinguage distinction has been a long detour leading back to the very same issue with which it began. (Notice, however, that by analyzing 'x is blue at w' into 'x is blue' and 'x is in w', Lewis *can* make use of the simple Tarskian account of primitive satisfaction, supplemented by an account of the relation 'x is in w', for him a part/whole relation explained in terms of spatio-temporal connections.)

What then is truth at a world? The syntax of the expression 'true at w' is that of a sentential operator. I have argued that it is not a predicate or part of a relation symbol since it is not preceded by a name of a truth bearer and can be iterated. 'True at w' behaves syntactically just like 'necessarily'. *Semantically*, 'true at w' expresses a modality, a way of being true, in the same way that 'necessarily' does. There is a way of presenting the semantics of operators as functions of propositions which will be useful with the case of truth at a world. Some notation borrowed from categorial grammar will help. One can assign syntactic categories to expressions by viewing them as functions, taking as arguments the piece of language to which they "apply" and as values the syntactic result of that application. With basic categories T for singular terms and S for sentences one can represent predicates as functions of the class  $T \rightarrow S$ . *Operators* like 'necessarily' apply to sentences yielding further sentences and so are of the class  $S \rightarrow S$ . (that the output category is the same as the input indicates that they can be iterated.) I suggest that 'true at w' is also of that class. If one sees the 'w', the variable ranging over worlds, as a syntactic component, then it will be an operator of class  $S \times W \rightarrow S$ , taking a sentence and a term for a world (of category W) and yielding a further sentence.

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Categorical grammars yield a natural *semantic* interpretation in terms of functions as well. There is a familiar way of interpreting operators on sentences as expressing functions on propositions. Let 'P' be the symbol for the class of propositions, then sentential operators are of the form  $P \rightarrow P$ , taking propositions, say *that*  $2+2=4$ , into other propositions, eg. *that necessarily*  $2+2=4$ .

This functional semantics is not quite what is needed for truth at a world, however. Propositions, while not sentences, can still be seen as truth *bearers*, things that must correspond with facts in the world in order to be true. The puzzle about being blue at  $w$  cannot be resolved by simply pointing out that 'x is blue' expresses one proposition and 'x is blue at  $w$ ' expresses another, the value of the former under a certain function. Being blue at  $w$  is rather something which *makes* propositions and sentences true or false.

It is for this reason that I speak of truth at a world as a *modality*, a way of obtaining or being true, which is a primitive feature or category of the world, on the order of objects, universals and facts. Modalities are to facts as propositional operators are to propositions. Using the expression 'F' for facts and 'W' for worlds (not *variables* ranging over worlds as before), one might say that a *modality* can be represented by a function of category  $F \rightarrow F$  or  $F X W \rightarrow F$ .

In response to Lewis, then, one can agree that it is of the nature of objects and properties that properties simply apply to objects or not. Yet an object's *having* a property is a *fact* which may be necessary or not, true at a world  $w$  or not. That Lewis should resist this is understandable, for it requires an increase of basic ontological categories beyond objects and even universals to facts and modalities. In analyzing 'x is P at  $w$ ' as 'x is P *and* x is in  $w$ ' Lewis can be seen to be affecting a reduction of modalities, getting by without them by in effect reducing them to relations between objects and worlds. For an object to have a property in a certain way, the manner of a world  $w$ , is for it simply to be in that world (and to have the property). Each object only has properties in one world mode, the mode of the world it is in.

What of the status of the Leibnizian Notion on my account, does it *reduce* necessity to truth at a world? Truth at a world is a modality of the same category as necessity. As a result the Leibnizian Notion does *not* reduce modalities to something else, say a relation between worlds and something else. It does not allow one to eliminate an intensional operator in an extensional metalanguage as possible world semantics does. What then does it do? The Leibnizian Notion reduces one coarse modality to a more fine grained modality, necessity is just truth at *all*

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worlds. All the alethic modalities of necessity, possibility, contingency, etc., can be reduced to truth at various worlds.

Responding to Lewis' qualms about being P at a world requires an increase in basic ontological categories. Thus one can see the appeal of interpreting the Leibnizian Notion in some sort of actualist way rather than keeping necessity as analyzed into other modalities and not eliminated, and having truth at a world as a primitive. If one sees worlds themselves as certain maximal propositions, and treats truth of a proposition as a primitive as well as necessity, then p is true at w just in case if w were true, p would be. This is to retreat from a discussion of what it is that *makes* propositions true, such as facts or modes. Lewis' realist account avoids the proliferation of ontological categories at a cost. It is understandable then why the alternative to Lewis' realism is usually one of the actualist alternatives, a genuinely realistic alternative to Lewis is costly. I think it is worth the cost.

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NOTES

- <sup>1</sup> Modalism, Realism and Actualism are discussed under different names in Peter van Inwagen, "Two Concepts of Possible Worlds", in *Midwest Studies in Philosophy*, Vol. XI, P.A. French, T.E. Uehling and H.K. Wettstein, eds. University of Minnesota Press, Minneapolis, 1986, 185–214.
- <sup>2</sup> See David Lewis, *On the Plurality of Worlds*, Basil Blackwell, Oxford, 1986, p. 200.
- <sup>3</sup> Most will say yes, because it is true at any other world, following Plantinga, who says that being snub-nosed in @, the actual world, is an essential property of Socrates, one that he has in all worlds. A. Plantinga, *The Nature of Necessity*, Oxford University Press, Oxford, 1974, p. 55.
- <sup>4</sup> See Jon Barwise and John Etchemendy, *The Liar*, Oxford University Press, Oxford, 1987. Their semantic facts, however, consist of propositions having properties, which differs from my account.