

Fault Tree Analysis of Sequential Systems

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Fault tree analysis is a systems safety technique for determining the logical combinations of events which could cause a specific hazard to occur. Digraph models are proposed which describe sequential relationships between events. A computer program which automatically constructs fault trees from digraph models is illustrated for an air-drying process.

Introduction

Fault tree analysis has been used in the aerospace, electronics, nuclear, and chemical industries to aid in (1) the discovery and control of failures before they occur; (2) the analysis of accidents, and (3) the planning of maintenance activities (Fussell, 1974; Powers, 1974). In the use of the fault tree method many safety analysts have expressed concern over the ability of the method to handle sequentially dependent events.

Esary recently illustrated the problem of applying fault tree analysis to sequential systems (Esary and Ziehms, 1975). In his analysis he considered a phased-mission in which the status and function of the components within a system depend on the phase (time sequence) of the mission. Esary presented a fault tree for the phased mission which was composed of sub-trees, one for each phase in the mission. He also presented an excellent discussion of the difficulties involved in calculating the probability of system success given the sequential interdependence of events. In this paper we present a strategy for partially automating the synthesis of fault trees when sequential interdependencies are considered. The key to the

method is the development of cause and effect models which explicitly consider the possible sequential interactions between events. These models are used in an algorithm which automatically generates fault trees.

Digraph Models

In a previous paper (Lapp, 1977), we have developed the concept of a digraph model for the description of both the normal and failed behavior of components in interconnected systems. The models will be briefly reviewed here and extended to sequential situations.

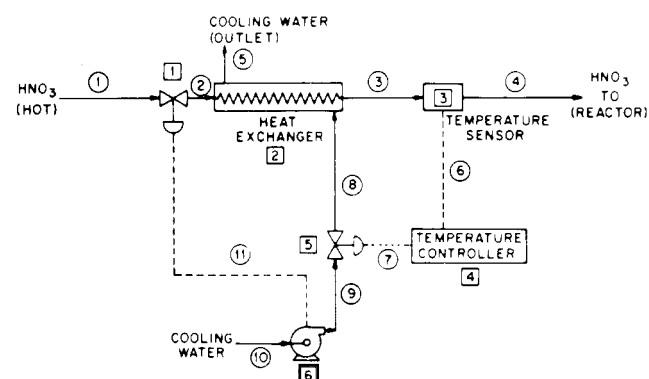


Figure 1. Flow diagram for part of a nitrification process. When low pump speed is sensed at the pump, valve 1 is closed.

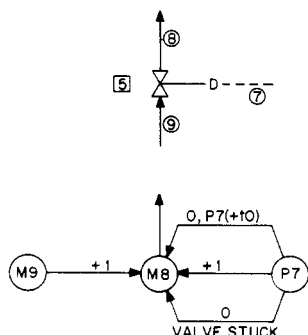


Figure 2. A cause-and-effect model for a control valve.

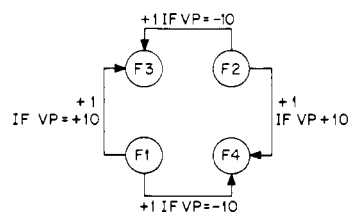
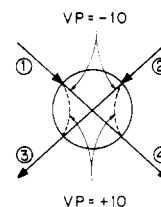


Figure 3. Cause-and-effect model for a four-way valve.

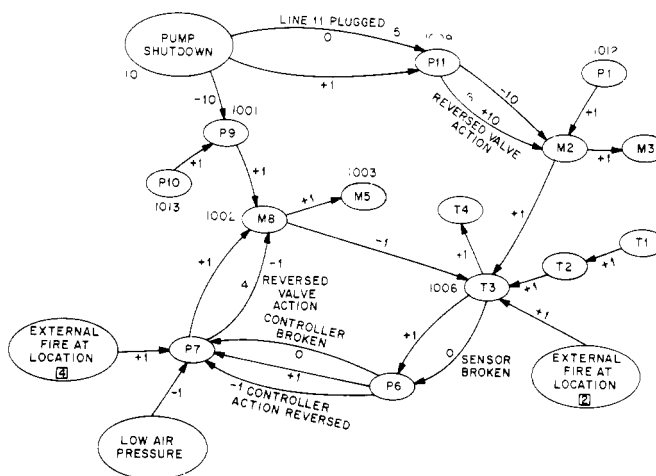


Figure 4. A partial cause-and-effect model for the output variable temperature in stream 4 (T_4) in Figure 1.

Table I. Operating Procedure for Fixed-Bed Drier System

Time period	Time since beginning of cycle, h	Value position (steam connections)			Bed status	
		3W	4WI	4WII	Bed I	Bed II
1	1 } 2 }	11→12	18→19 AND 22→23	20→21 AND 24→25	Regeneration	In service
2	3	11→18	18→19 AND 22→23	20→21 AND 24→25	Cooling	In service
3	4 } 5 }	11→12	18→23 AND 22→19	20→25 AND 24→21	In service	Regeneration
4	6	11→18	18→23 AND 22→19	20→25 AND 24→21	In service	Cooling
Return to Time Period 1						

In developing logic models for physical systems, it is necessary to capture the cause-and-effect nature of interactions which occur between the variables which describe the system

Table II. Sequential Dryer Process Events

Title event number	Probability	Text
1	2.00×10^{-3}	4-Way valve leaks across
2	5.00×10^{-8}	Fire at Bed I
3	1.20×10^{-4}	No alumina in Bed I, or channeling
4	7.15×10^{-4}	4-Way valve II motor failure
5	3.30×10^{-1}	Time = 1
6	1.67×10^{-1}	Time = 2
7	4.72×10^{-6}	4-Way valve II timer changes at wrong time
8	5.00×10^{-8}	Fire at Bed II
9	1.20×10^{-4}	No alumina in Bed II, or channeling
10	1.67×10^{-1}	Time = 4
11	4.72×10^{-6}	4-Way valve II timer fails
12	8.63×10^{-5}	4-Way valve II control line 29 cut
13	3.30×10^{-1}	Time = 3
14	5.00×10^{-4}	Inlet air flow up
15	5.00×10^{-4}	Proportionating valve pressure up (P10)
16	7.15×10^{-4}	4-Way valve I motor failure
17	7.99×10^{-5}	Heater leaks steam into air
18	5.00×10^{-4}	Inlet air water concentration up
19	7.15×10^{-4}	3-Way valve motor failure
20	4.72×10^{-6}	4-Way valve I timer changes at wrong time
21	4.72×10^{-6}	4-Way valve I timer fails
22	8.63×10^{-5}	4-Way valve I control line 28 cut
23	1.00×10^{-5}	Water separator trap clogged
24	2.10×10^{-3}	Valve 6 closed
25	5.00×10^{-8}	External fire at separator
26	4.72×10^{-6}	3-Way valve timer changes at wrong time
27	4.72×10^{-6}	3-Way valve timer fails
28	8.63×10^{-5}	3-Way valve control line 27 cut
29	5.00×10^{-4}	Cooling water flow down
30	5.00×10^{-4}	Cooling water temperature up
31	2.99×10^{-3}	Cooler fouled
32	5.00×10^{-8}	External fire at cooler
33	2.10×10^{-3}	Valve 4 closed
36	5.00×10^{-4}	Inlet Air pressure up

(i.e., temperatures, pressures, flow rates, concentrations, operator action, voltages, valve positions, etc.) and events which occur within the system (i.e., valve failure, fire, explosion, operator error, weather changes, etc.). For example, consider the flow sheet given in Figure 1. The system is composed of valves, pumps, heat exchangers, etc. In order to determine how failures or deviations in input variables propagate through the system, it is possible to construct digraphs whose nodes represent events or variables and whose edges represent the relationships between the nodes. Figure 2 illustrates a cause and effect model for valve 5 in this system. Note that the edges may be event dependent. That is, a relationship between two variables may be dependent on the value of other variables or events in the system. For example, the gain between the pressure on the valve actuator P7 and the mass flow rate leaving the valve M8 is normally +1. An increase in pressure opens the valve and allows a higher flow rate. However, if the pressure on the actuator is very high ($P7 = +10$) the gain is 0. The valve is wide open and further increases in pressure P7 do not give an increase in the flow rate M8.

If the edges in a digraph are made dependent on other events in the system, it is possible to capture in one digraph the sequential behavior of the complete system. Consider the four-way valve shown in Figure 3. When the valve is in position 1 (valve position = $VP = +10$) the flow is from stream 1 to stream 3 and from stream 2 to stream 4. In position 2 (valve position = $VP = -10$) the flow is from stream 2 to stream 3 and from stream 1 to stream 4. A digraph for this valve is given in Figure 3 where VP is the valve position. If, from another

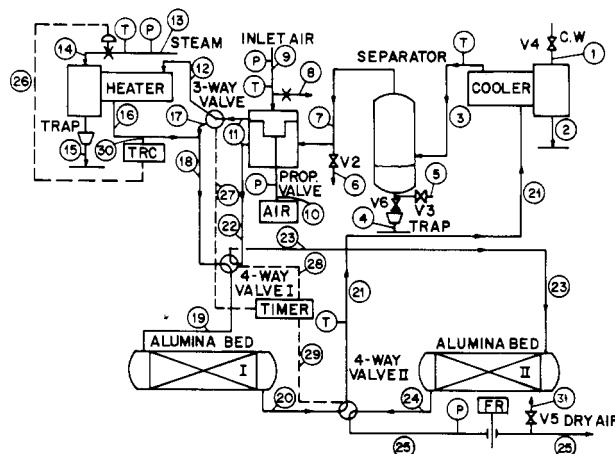
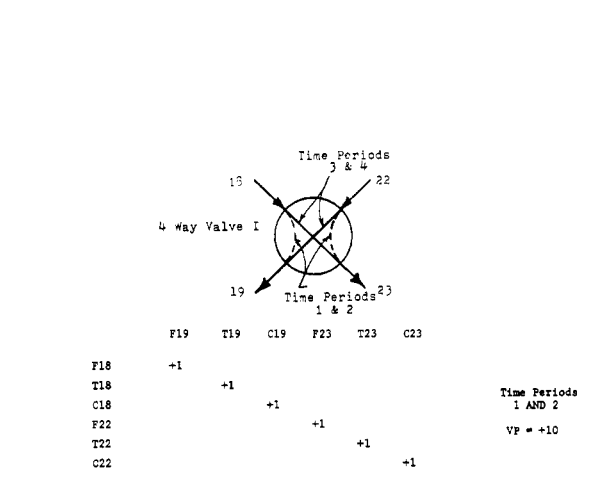
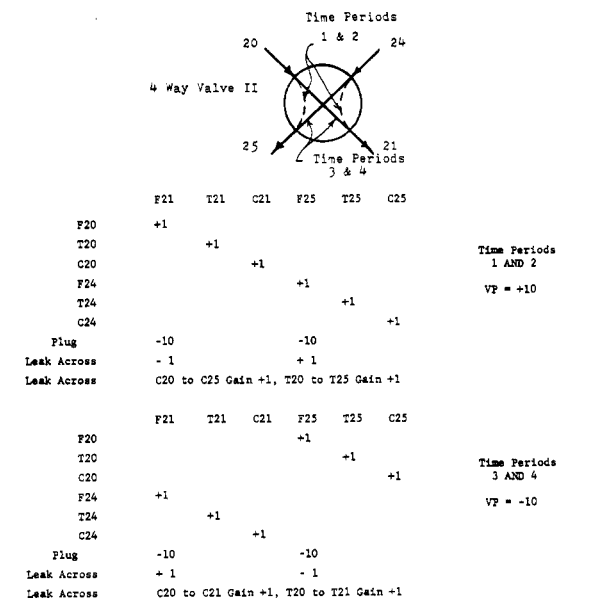


Figure 6. Flow diagram of a utility air drying process. After King (1970).



F18	+1					
T18		+1				
C18			+1			
F22				+1		
T22					+1	
C22						+1

Failures: Plug -10, Leak Across -1, Leak Across C18 to C23 Gain +1, T18 to T23 Gain +1



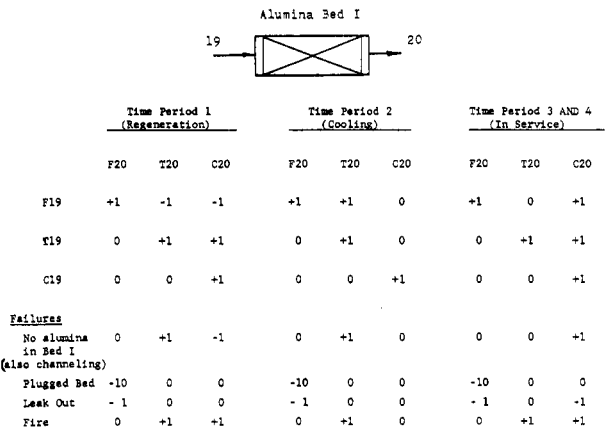
F20	+1					
T20		+1				
C20			+1			
F24				+1		
T24					+1	
C24						+1

Failures: Plug -10, Leak Across -1, Leak Across C20 to C25 Gain +1, T20 to T25 Gain +1



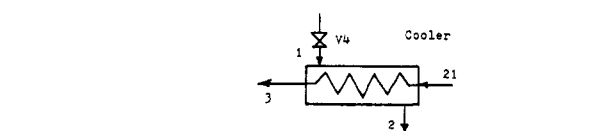
F12	+1					
T12		+1				
C12			+1			
F17				+1		
T17					+1	
C17						+1

Failures: Valve Leaks Across -1, Plugged Valve -10



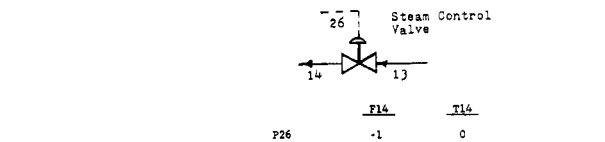
F19	+1	-1	-1	+1	+1	0	+1	0	+1
T19	0	+1	+1	0	+1	0	0	+1	+1
C19	0	0	+1	0	0	+1	0	0	+1

Failures: No alumina in Bed I (also channeling) 0, Plugged Bed -10, Leak Out -1, Fire 0



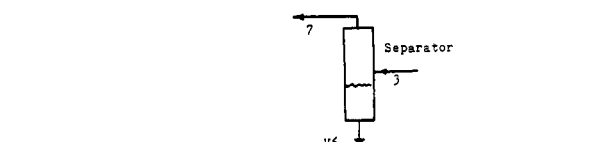
F23	+1	0	+1	+1	-1	-1	+1	+1	0
T23	0	+1	+1	0	+1	+1	0	+1	0
C23	0	0	+1	0	0	+1	0	0	+1

Failures: No alumina in Bed II (channeling) 0, Plugged Bed -10, Leak Out -1, Fire 0



F1	+1	-1	0	-1	-1
T1	0	+1	0	+1	+1
F21	0	+1	+1	+1	+1
T21	0	+1	0	+1	+1
C21	0	0	0	0	+1

Failures: External Fire 0, Fouling 0, V4 Open Wide +10, V4 Closed -10



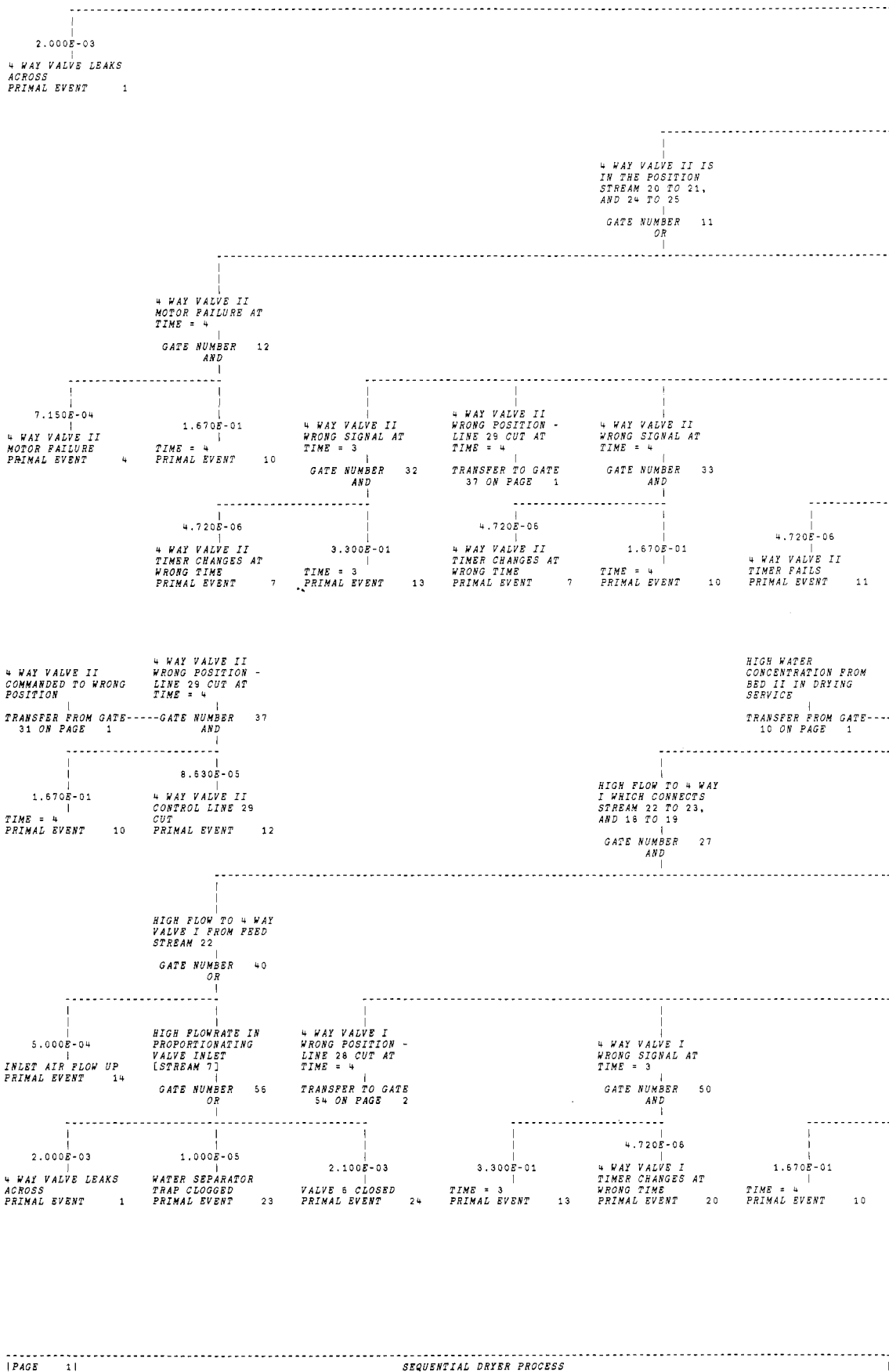
F14	-1	0
T14	+1	0
F13	0	+1
T13	-10	0

Failures: Line 13 plugged -10, Valve Reversed -10, Valve Fails Closed +10, Valve Fails Open 0



F7	+1	0	0	+1	0
T7	0	+1	+1	0	+1
C7	0	0	+1	0	0

Failures: Trap Plugged +1, Fire 0, V6 Closed +1



HIGH WATER
CONCENTRATION IN
OUTLET AIR (STREAM
25)

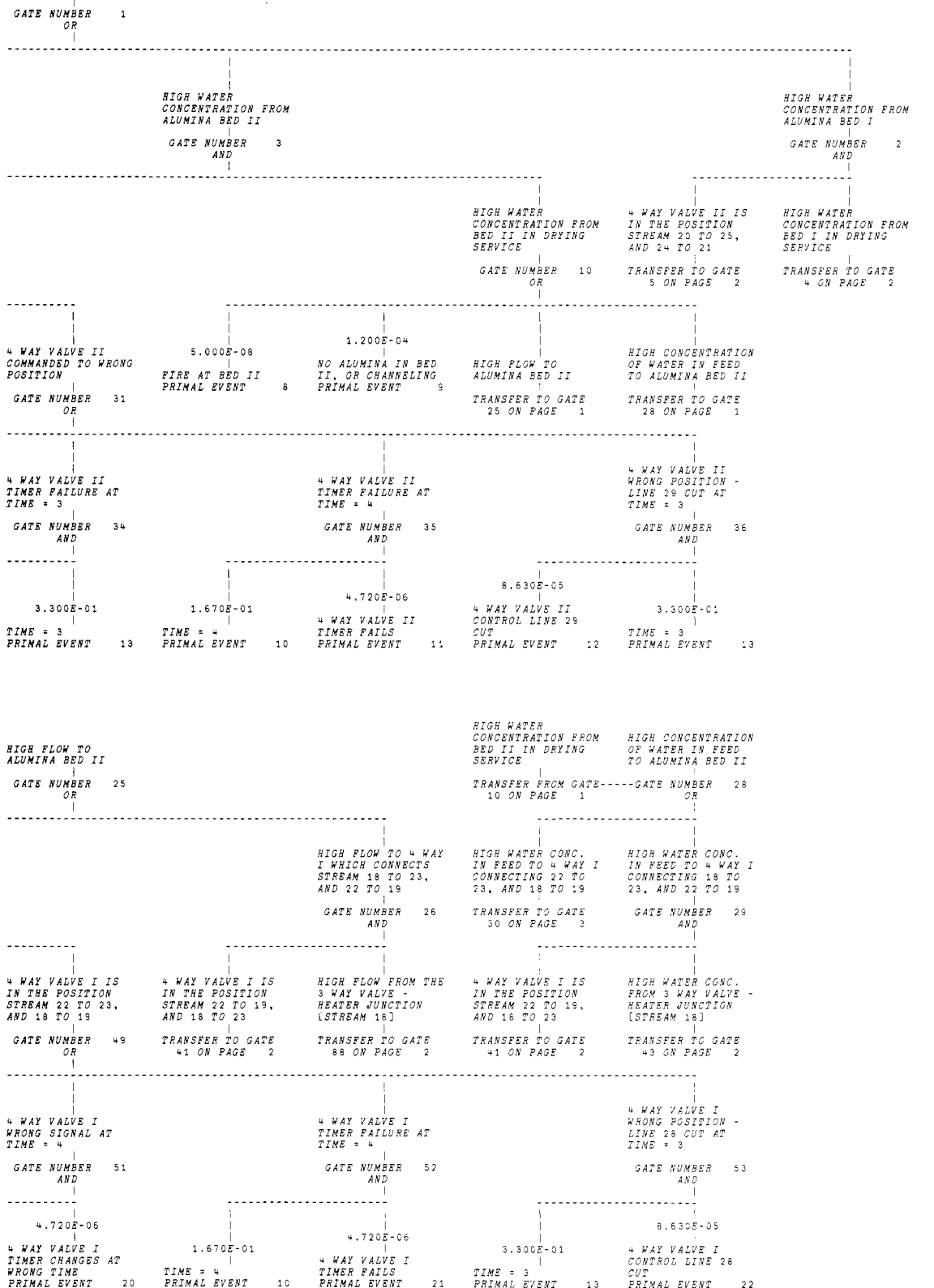
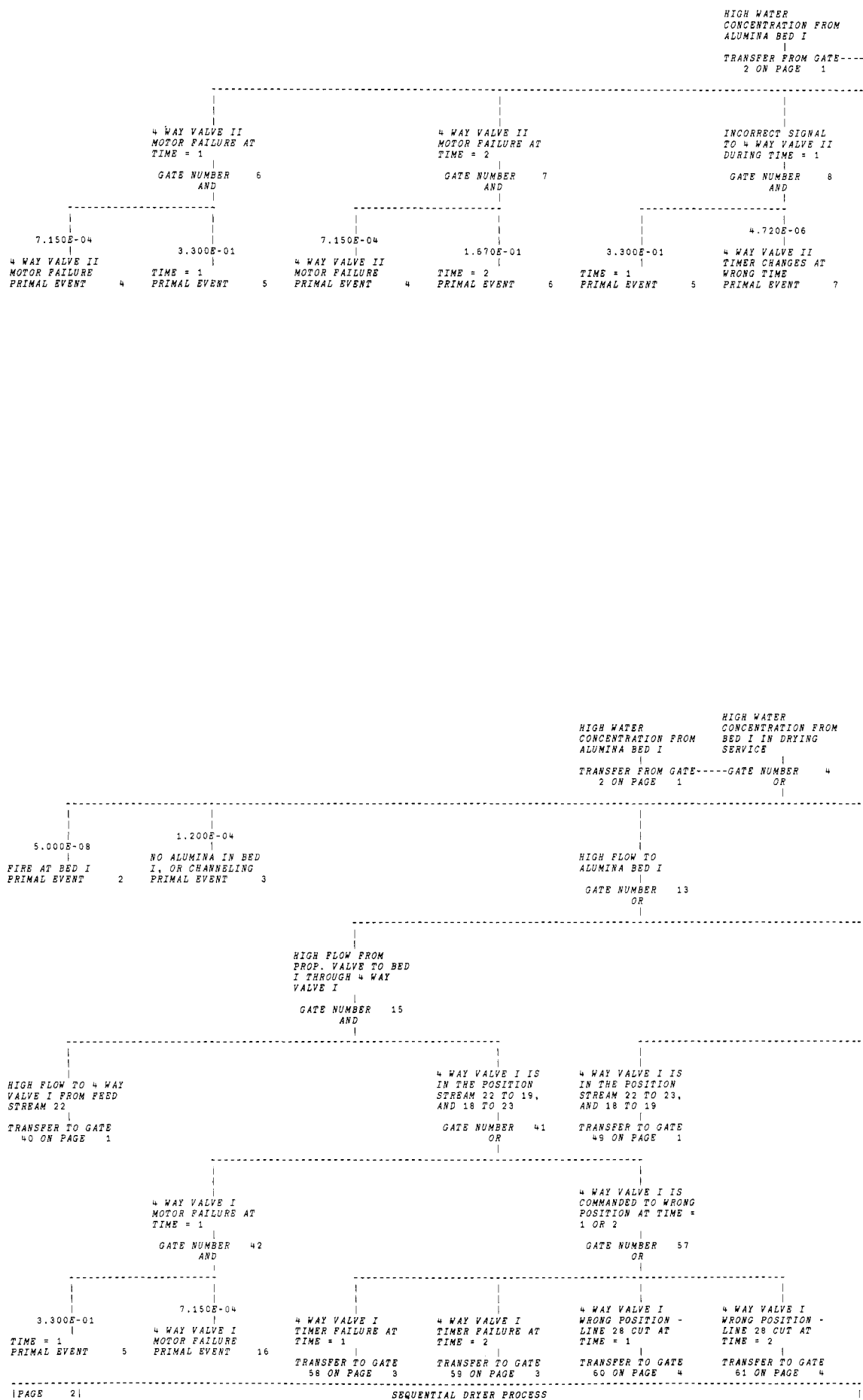


Figure 9 continues



4 WAY VALVE II IS
IN THE POSITION
STREAM 20 TO 25,
AND 24 TO 21

-GATE NUMBER 5
OR

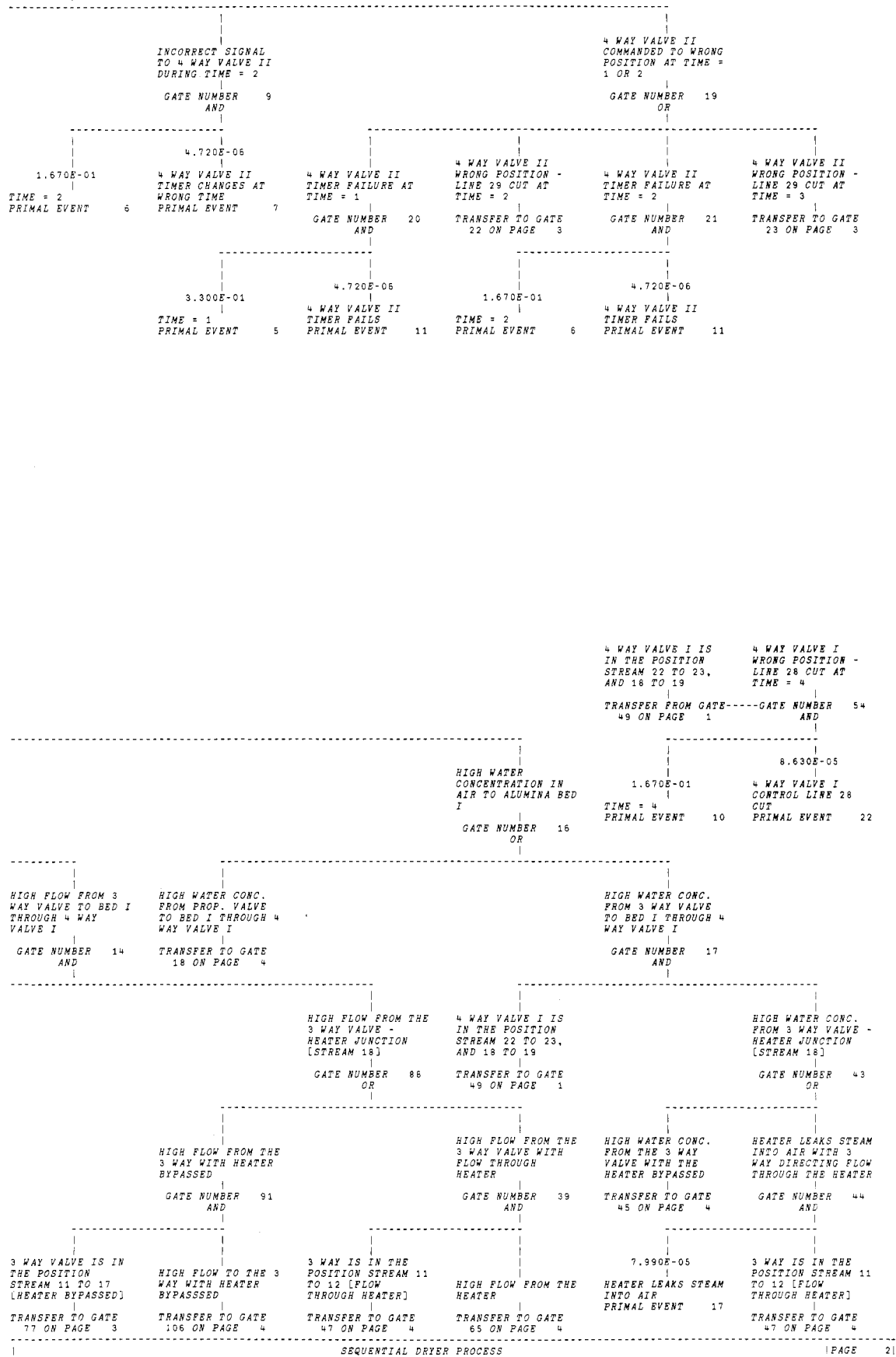
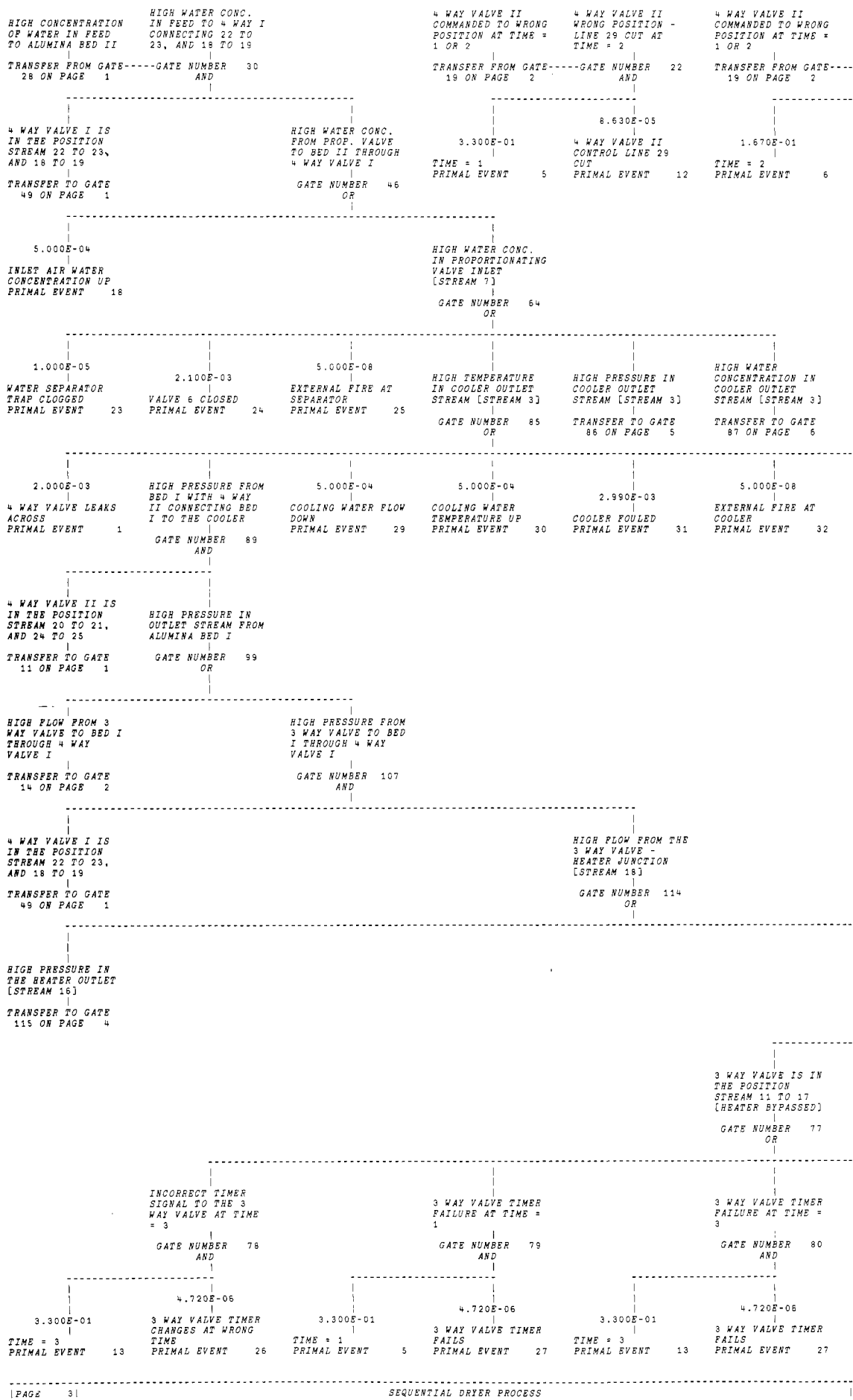


Figure 9 continues



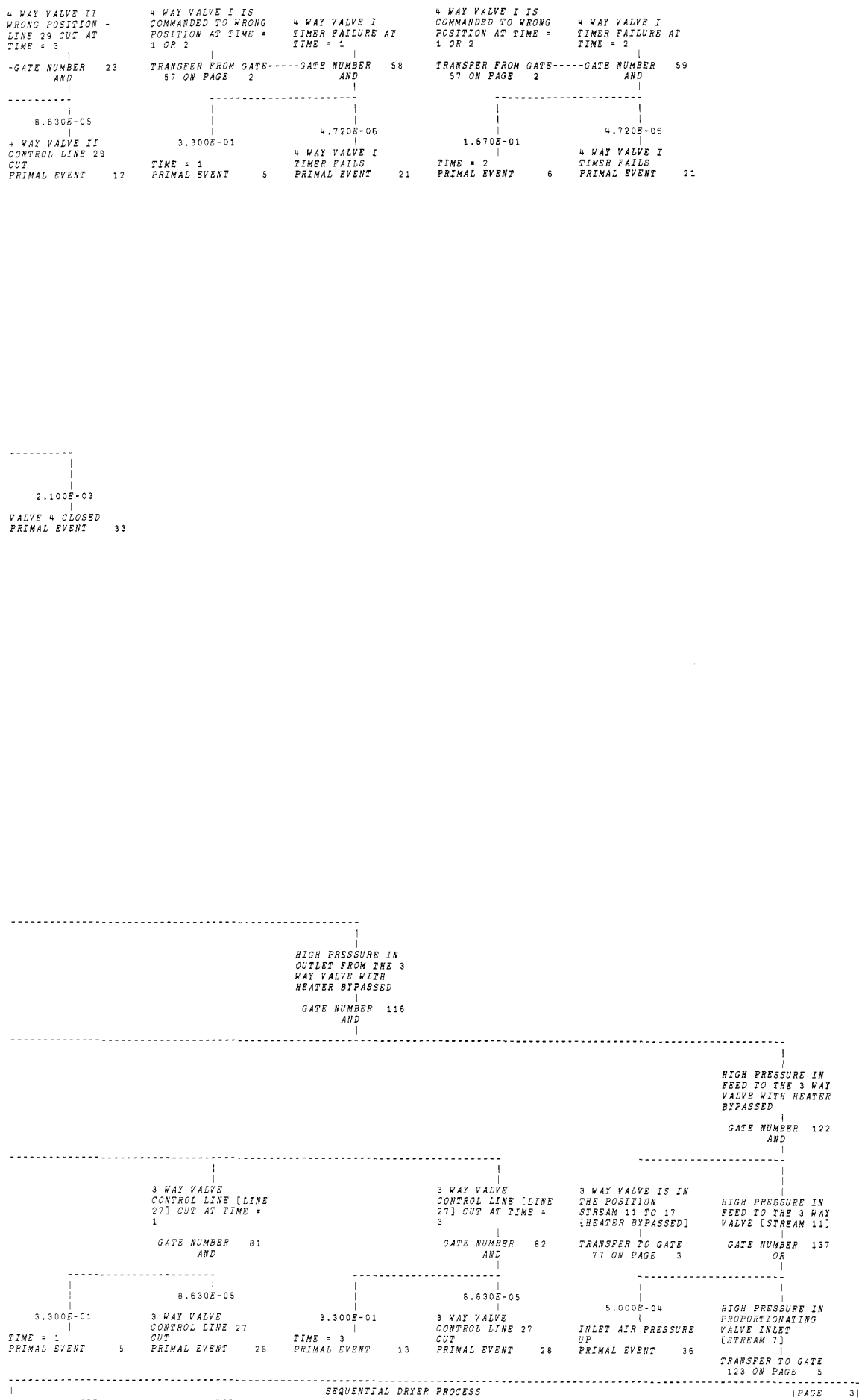
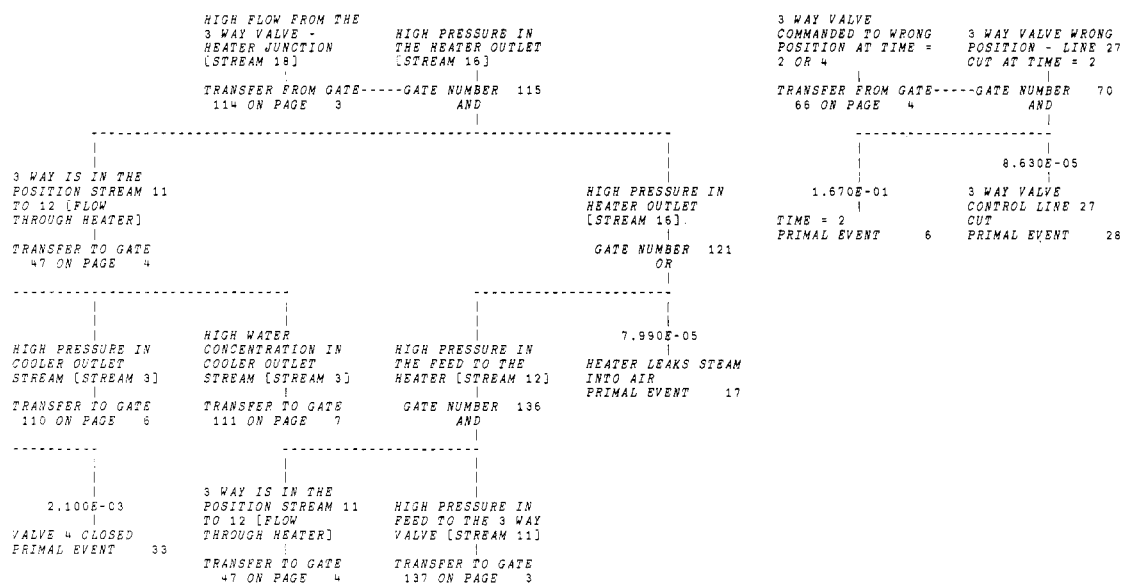
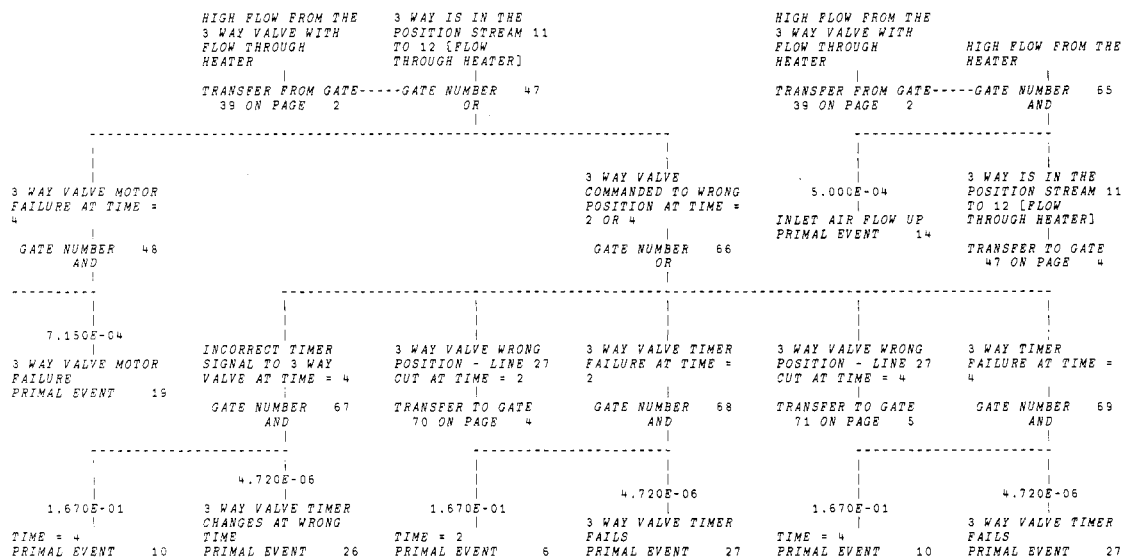


Figure 9 continues

4 WAY VALVE I IS COMMANDED TO WRONG POSITION AT TIME = 1 OR 2	4 WAY VALVE I WRONG POSITION - LINE 26 CUT AT TIME = 1	4 WAY VALVE I IS COMMANDED TO WRONG POSITION AT TIME = 1 OR 2	4 WAY VALVE I WRONG POSITION - LINE 26 CUT AT TIME = 2	HIGH FLOW FROM THE 3 WAY WITH HEATER BYPASSED	HIGH FLOW TO THE 3 WAY WITH HEATER BYPASSED
TRANSFER FROM GATE-----GATE NUMBER 57 ON PAGE 2 AND 60	TRANSFER FROM GATE-----GATE NUMBER 57 ON PAGE 2 AND 61	TRANSFER FROM GATE-----GATE NUMBER 57 ON PAGE 2 AND 61	TRANSFER FROM GATE-----GATE NUMBER 57 ON PAGE 2 AND 61	TRANSFER FROM GATE-----GATE NUMBER 51 ON PAGE 2 AND 106	TRANSFER FROM GATE-----GATE NUMBER 51 ON PAGE 2 AND 106
3.300E-01	8.630E-05	1.670E-01	8.530E-05	5.000E-04	3 WAY VALVE IS IN THE POSITION STREAM 11 TO 17 [HEATER BYPASSED]
TIME = 1	4 WAY VALVE I CONTROL LINE 28 CUT	TIME = 2	4 WAY VALVE I CONTROL LINE 28 CUT	INLET AIR FLOW UP	TRANSFER TO GATE 77 ON PAGE 3
PRIMAL EVENT 5	PRIMAL EVENT 22	PRIMAL EVENT 6	PRIMAL EVENT 22	PRIMAL EVENT 14	
					1.670E-01
					TIME = 4
					PRIMAL EVENT 10

HIGH WATER CONC. FROM 3 WAY VALVE - HEATER JUNCTION [STREAM 18]	HIGH WATER CONC. FROM THE 3 WAY VALVE WITH THE HEATER BYPASSED	HIGH WATER CONCENTRATION IN AIR TO ALUMINA BED 1	HIGH WATER CONC. FROM PROP. VALVE TO BED I THROUGH 4 WAY VALVE I
TRANSFER FROM GATE-----GATE NUMBER 43 ON PAGE 2 AND 45	TRANSFER FROM GATE-----GATE NUMBER 43 ON PAGE 2 AND 45	TRANSFER FROM GATE-----GATE NUMBER 16 ON PAGE 2 AND 18	TRANSFER FROM GATE-----GATE NUMBER 16 ON PAGE 2 AND 18
3 WAY VALVE IS IN THE POSITION STREAM 11 TO 17 [HEATER BYPASSED]	HIGH WATER CONCENTRATION FROM THE 3 WAY OUTLET [STREAM 17]	4 WAY VALVE I IS IN THE POSITION STREAM 22 TO 19, AND 18 TO 23	HIGH WATER CONC. IN PROPORTIONATING VALVE INLET [STREAM 7]
TRANSFER TO GATE 77 ON PAGE 3	GATE NUMBER 63 AND	TRANSFER TO GATE 41 ON PAGE 2	GATE NUMBER 72 OR
5.000E-04	3 WAY VALVE IS IN THE POSITION STREAM 11 TO 17 [HEATER BYPASSED]	1.000E-05	5.000E-06
INLET AIR WATER CONCENTRATION UP	TRANSFER TO GATE 77 ON PAGE 3	WATER SEPARATOR TRAP CLOGGED	EXTERNAL FIRE AT SEPARATOR
PRIMAL EVENT 18		PRIMAL EVENT 23	PRIMAL EVENT 25
2.000E-03	HIGH PRESSURE FROM BED II WITH 4 WAY II CONNECTING BED II TO THE COOLER	5.000E-04	5.000E-04
4 WAY VALVE LEAKS ACROSS	GATE NUMBER 74 AND	COOLING WATER FLOW DOWN	COOLING WATER TEMPERATURE UP
PRIMAL EVENT 1		PRIMAL EVENT 29	PRIMAL EVENT 30
4 WAY VALVE II IS IN THE POSITION STREAM 23 TO 25, AND 24 TO 21	HIGH PRESSURE IN OUTLET STREAM FROM ALUMINA BED II		
TRANSFER TO GATE 5 ON PAGE 2	GATE NUMBER 75 OR		
HIGH FLOW TO 4 WAY I WHICH CONNECTS STREAM 18 TO 23, AND 22 TO 19	HIGH PRESSURE FROM 3 WAY VALVE TO BED II THROUGH 4 WAY VALVE I		
TRANSFER TO GATE 26 ON PAGE 1	TRANSFER TO GATE 83 ON PAGE 5		
[PAGE 4]			

SEQUENTIAL DRYER PROCESS



SEQUENTIAL DRYER PROCESS (PAGE 4)

Figure 9 continues

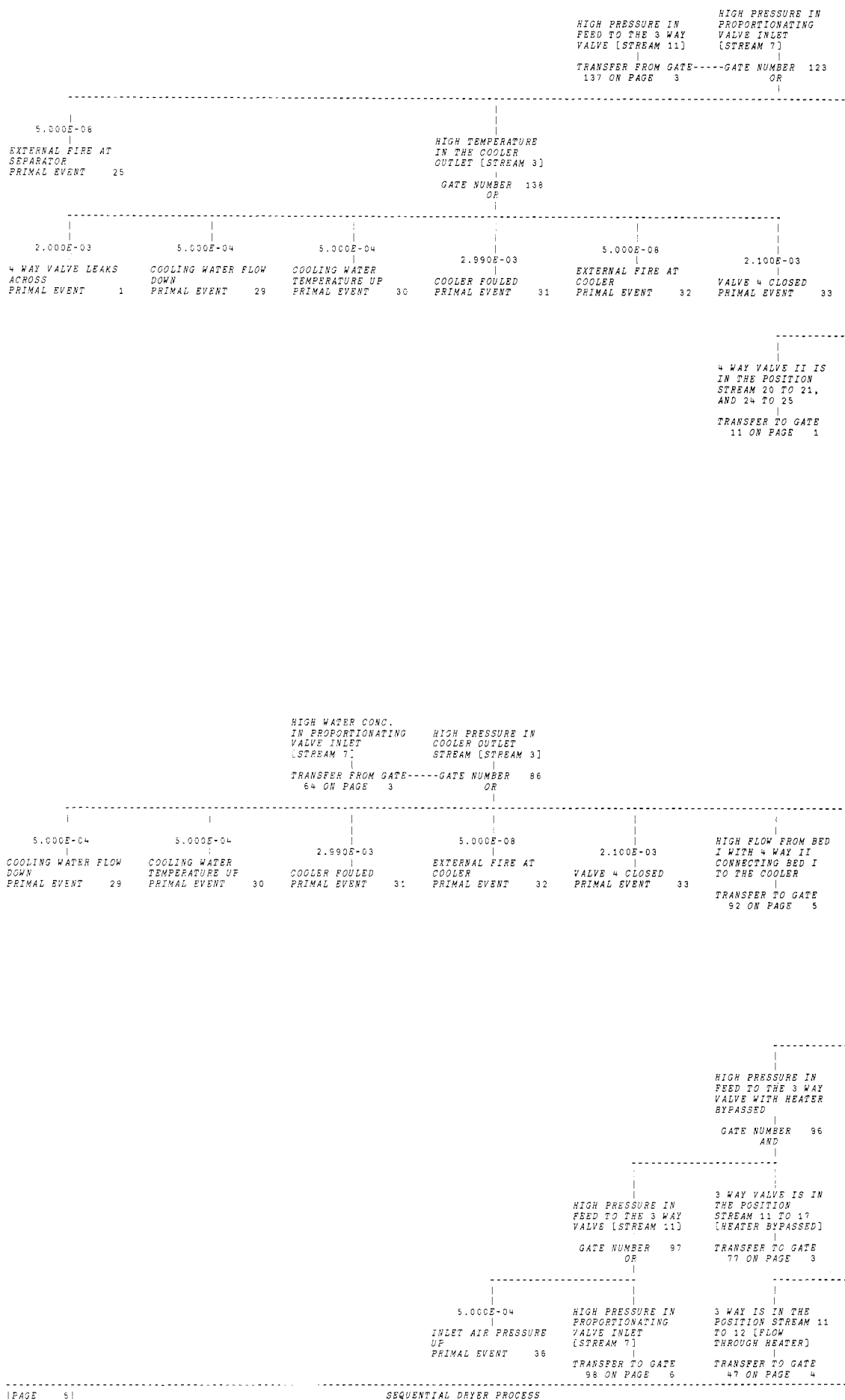
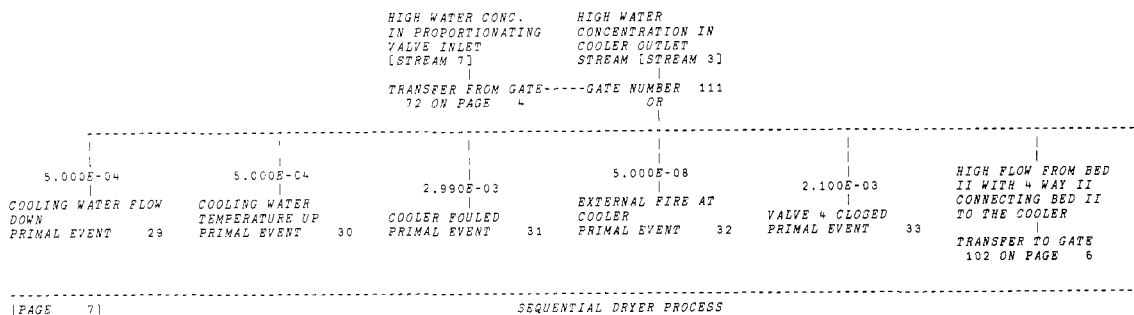
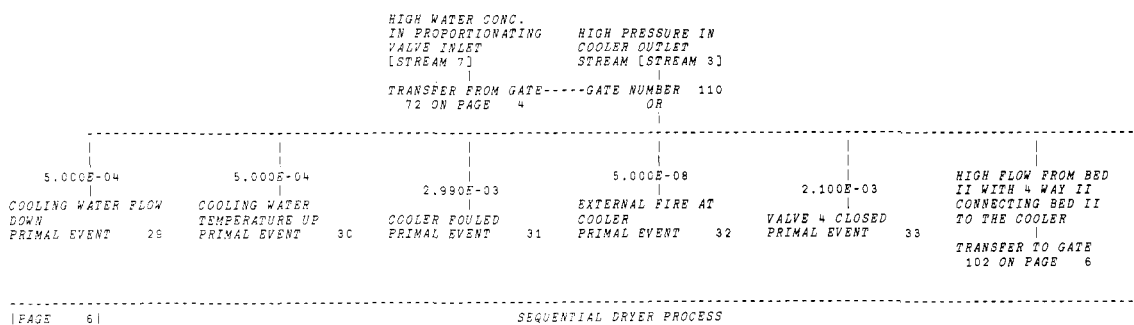
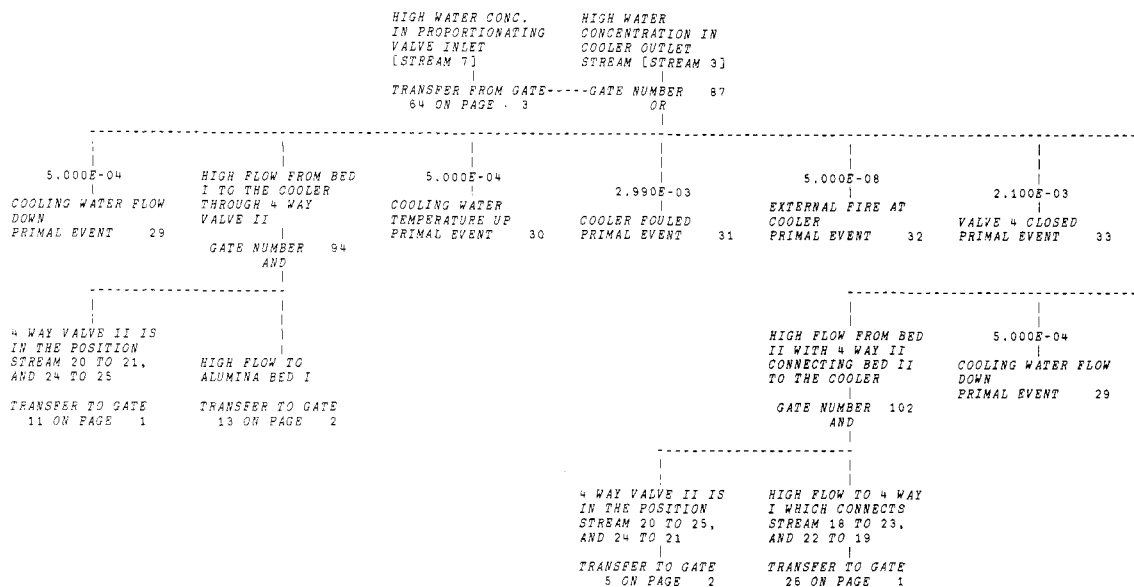




Figure 9 continues



	HIGH PRESSURE IN FEED TO THE 3 WAY VALVE [STREAM 11]	HIGH PRESSURE IN PROPORTIONATING VALVE INLET [STREAM 7]	
	TRANSFER FROM GATE-----GATE NUMBER 98		
	97 ON PAGE 5	OR	
HIGH PRESSURE FROM BED I WITH 4 WAY II CONNECTING BED I TO THE COOLER	HIGH PRESSURE IN THE COOLER OUTLET [STREAM 3]	HIGH TEMPERATURE IN THE COOLER OUTLET [STREAM 3]	
TRANSFER TO GATE	GATE NUMBER 101	TRANSFER TO GATE	
89 ON PAGE 3	OR	138 ON PAGE 5	
5.000E-04	2.980E-03	5.000E-06	2.100E-03
COOLING WATER TEMPERATURE UP	COOLER FOULED	EXTERNAL FIRE AT COOLER	VALVE 4 CLOSED
PRIMAL EVENT 30	PRIMAL EVENT 31	PRIMAL EVENT 32	PRIMAL EVENT 33

|
|
HIGH PRESSURE FROM
BED II WITH 4 WAY
II CONNECTING BED
II TO THE COOLER
|
TRANSFER TO GATE
74 ON PAGE 4

|
SEQUENTIAL DRYER PROCESS
|

|
PAGE 6
|

|
|
HIGH PRESSURE FROM
BED II WITH 4 WAY
II CONNECTING BED
II TO THE COOLER
|
TRANSFER TO GATE
74 ON PAGE 4

|
SEQUENTIAL DRYER PROCESS
|

|
PAGE 7
|

Figure 9 continues

GATE TABLE OF CONTENTS

TITLE:

SEQUENTIAL DRYER PROCESS

GATE NUMBER	GATE TYPE	PROBABILITY	DEVELOPED ON PAGE	TEXT	
1	OR		1	HIGH WATER	CONCENTRATION IN OUTLET AIR [STREAM 25]
2	AND		1	HIGH WATER	CONCENTRATION FROM ALUMINA BED I
3	AND		1	HIGH WATER	CONCENTRATION FROM ALUMINA BED II
4	OR		2	HIGH WATER	CONCENTRATION FROM BED I IN DRYING SERVICE
5	OR		2	4 WAY VALVE II IS	IN THE POSITION STREAM 20 TO 25, AND 24 TO 21
6	AND		2	4 WAY VALVE II	MOTOR FAILURE AT TIME = 1
7	AND		2	4 WAY VALVE II	MOTOR FAILURE AT TIME = 2
8	AND		2	INCORRECT SIGNAL	TO 4 WAY VALVE II DURING TIME = 1
9	AND		2	INCORRECT SIGNAL	TO 4 WAY VALVE II DURING TIME = 2
10	OR		1	HIGH WATER	CONCENTRATION FROM BED II IN DRYING SERVICE
11	OR		1	4 WAY VALVE II IS	IN THE POSITION STREAM 20 TO 21, AND 24 TO 25
12	AND		1	4 WAY VALVE II	MOTOR FAILURE AT TIME = 4
13	OR		2	HIGH FLOW TO	ALUMINA BED I
14	AND		2	HIGH FLOW FROM 3	WAY VALVE TO BED I THROUGH 4 WAY VALVE I
15	AND		2	HIGH FLOW FROM	PROP. VALVE TO BED I THROUGH 4 WAY VALVE I
16	OR		2	HIGH WATER	CONCENTRATION IN AIR TO ALUMINA BED I
17	AND		2	HIGH WATER CONC.	FROM 3 WAY VALVE TO BED I THROUGH 4 WAY VALVE I
18	AND		4	HIGH WATER CONC.	FROM PROP. VALVE TO BED I THROUGH 4 WAY VALVE I
19	OR		2	4 WAY VALVE II	COMMANDED TO WRONG POSITION AT TIME = 1 OR 2
20	AND		2	4 WAY VALVE II	TIMER FAILURE AT TIME = 1
21	AND		2	4 WAY VALVE II	TIMER FAILURE AT TIME = 2
22	AND		3	4 WAY VALVE II	WRONG POSITION - LINE 29 CUT AT TIME = 2
23	AND		3	4 WAY VALVE II	WRONG POSITION - LINE 29 CUT AT TIME = 3
25	OR		1	HIGH FLOW TO	ALUMINA BED II
26	AND		1	HIGH FLOW TO 4 WAY I WHICH CONNECTS	STREAM 18 TO 23, AND 22 TO 19
27	AND		1	HIGH FLOW TO 4 WAY I WHICH CONNECTS	STREAM 22 TO 23, AND 18 TO 19
28	OR		1	HIGH CONCENTRATION OF WATER IN FEED	TO ALUMINA BED II
29	AND		1	HIGH WATER CONC.	IN FEED TO 4 WAY I CONNECTING 18 TO 23, AND 22 TO 19
30	AND		3	HIGH WATER CONC.	IN FEED TO 4 WAY I CONNECTING 22 TO 23, AND 18 TO 19
31	OR		1	4 WAY VALVE II	COMMANDED TO WRONG POSITION
32	AND		1	4 WAY VALVE II	WRONG SIGNAL AT TIME = 3
33	AND		1	4 WAY VALVE II	WRONG SIGNAL AT TIME = 4
34	AND		1	4 WAY VALVE II	TIMER FAILURE AT TIME = 3
35	AND		1	4 WAY VALVE II	TIMER FAILURE AT TIME = 4
36	AND		1	4 WAY VALVE II	WRONG POSITION - LINE 29 CUT AT TIME = 3
37	AND		1	4 WAY VALVE II	WRONG POSITION - LINE 29 CUT AT TIME = 4
39	AND		2	HIGH FLOW FROM THE 3 WAY VALVE WITH	FLOW THROUGH HEATER
40	OR		1	HIGH FLOW TO 4 WAY VALVE I FROM FEED	STREAM 22
41	OR		2	4 WAY VALVE I IS	IN THE POSITION STREAM 22 TO 19, AND 18 TO 23
42	AND		2	4 WAY VALVE I	MOTOR FAILURE AT TIME = 1
43	OR		2	HIGH WATER CONC.	FROM 3 WAY VALVE - HEATER JUNCTION [STREAM 18]
44	AND		2	HEATER LEAKS STEAM INTO AIR WITH 3	WAY DIRECTING FLOW THROUGH THE HEATER
45	AND		4	HIGH WATER CONC.	FROM THE 3 WAY VALVE WITH THE HEATER BYPASSED
46	OR		3	HIGH WATER CONC.	FROM PROP. VALVE TO BED II THROUGH 4 WAY VALVE I
47	OR		4	3 WAY IS IN THE	POSITION STREAM 11 TO 12 [FLOW THROUGH HEATER]
48	AND		4	3 WAY VALVE MOTOR	FAILURE AT TIME = 4
49	OR		1	4 WAY VALVE I IS	IN THE POSITION STREAM 22 TO 23, AND 18 TO 19
50	AND		1	4 WAY VALVE I	WRONG SIGNAL AT TIME = 3
51	AND		1	4 WAY VALVE I	WRONG SIGNAL AT TIME = 4
52	AND		1	4 WAY VALVE I	TIMER FAILURE AT TIME = 4
53	AND		1	4 WAY VALVE I	WRONG POSITION - LINE 28 CUT AT TIME = 3
54	AND		2	4 WAY VALVE I	WRONG POSITION - LINE 28 CUT AT TIME = 4
56	OR		1	HIGH FLOWRATE IN	PROPORTIONATING VALVE INLET [STREAM 7]
57	OR		2	4 WAY VALVE I IS	COMMANDED TO WRONG POSITION AT TIME = 1 OR 2
58	AND		3	4 WAY VALVE I	TIMER FAILURE AT TIME = 1
59	AND		3	4 WAY VALVE I	TIMER FAILURE AT TIME = 2
60	AND		4	4 WAY VALVE I	WRONG POSITION - LINE 28 CUT AT TIME = 1
61	AND		4	4 WAY VALVE I	WRONG POSITION - LINE 28 CUT AT TIME = 2
63	AND		4	HIGH WATER	CONCENTRATION FROM THE 3 WAY OUTLET [STREAM 17]
64	OR		3	HIGH WATER CONC.	IN PROPORTIONATING VALVE INLET [STREAM 7]
65	AND		4	HIGH FLOW FROM THE HEATER	

GATE TABLE OF CONTENTS

TITLE: SEQUENTIAL DRYER PROCESS

GATE NUMBER	GATE TYPE	PROBABILITY	DEVELOPED ON PAGE	TEXT
66	OR		4	3 WAY VALVE COMMANDED TO WRONG POSITION AT TIME = 2 OR 4
67	AND		4	INCORRECT TIMER SIGNAL TO 3 WAY VALVE AT TIME = 4
68	AND		4	3 WAY VALVE TIMER FAILURE AT TIME = 2
69	AND		4	3 WAY TIMER FAILURE AT TIME = 4
70	AND		4	3 WAY VALVE WRONG POSITION - LINE 27 CUT AT TIME = 2
71	AND		5	3 WAY VALVE WRONG POSITION - LINE 27 CUT AT TIME = 4
72	OR		4	HIGH WATER CONC. IN PROPORTIONATING VALVE INLET [STREAM 7]
73	OR		4	HIGH TEMPERATURE IN COOLER OUTLET STREAM [STREAM 3]
74	AND		4	HIGH PRESSURE FROM BED II WITH 4 WAY II CONNECTING BED II TO THE COOLER
75	OR		4	HIGH PRESSURE IN OUTLET STREAM FROM ALUMINA BED II
77	OR		3	3 WAY VALVE IS IN THE POSITION STREAM 11 TO 17 [HEATER BYPASSED]
78	AND		3	INCORRECT TIMER SIGNAL TO THE 3 WAY VALVE AT TIME = 3
79	AND		3	3 WAY VALVE TIMER FAILURE AT TIME = 1
80	AND		3	3 WAY VALVE TIMER FAILURE AT TIME = 3
81	AND		3	3 WAY VALVE CONTROL LINE [LINE 27] CUT AT TIME = 1
82	AND		3	3 WAY VALVE CONTROL LINE [LINE 27] CUT AT TIME = 3
83	AND		5	HIGH PRESSURE FROM 3 WAY VALVE TO BED II THROUGH 4 WAY VALVE I
84	OR		5	HIGH PRESSURE FROM THE 3 WAY VALVE - HEATER JUNCTION [STREAM 18]
85	OR		3	HIGH TEMPERATURE IN COOLER OUTLET STREAM [STREAM 3]
86	OR		5	HIGH PRESSURE IN COOLER OUTLET STREAM [STREAM 3]
87	OR		6	HIGH WATER CONCENTRATION IN COOLER OUTLET STREAM [STREAM 3]
88	OR		2	HIGH FLOW FROM THE 3 WAY VALVE - HEATER JUNCTION [STREAM 18]
89	AND		3	HIGH PRESSURE FROM BED I WITH 4 WAY II CONNECTING BED I TO THE COOLER
91	AND		2	HIGH FLOW FROM THE 3 WAY WITH HEATER BYPASSED
92	AND		5	HIGH FLOW FROM BED I WITH 4 WAY II CONNECTING BED I TO THE COOLER
94	AND		6	HIGH FLOW FROM BED I TO THE COOLER THROUGH 4 WAY VALVE II
95	AND		5	HIGH PRESSURE IN OUTLET FROM THE 3 WAY VALVE WITH HEATER BYPASSED
96	AND		5	HIGH PRESSURE IN FEED TO THE 3 WAY VALVE WITH HEATER BYPASSED
97	OR		5	HIGH PRESSURE IN FEED TO THE 3 WAY VALVE [STREAM 11]
98	OR		6	HIGH PRESSURE IN PROPORTIONATING VALVE INLET [STREAM 7]
99	OR		3	HIGH PRESSURE IN OUTLET STREAM FROM ALUMINA BED I
101	OR		6	HIGH PRESSURE IN THE COOLER OUTLET [STREAM 3]
102	AND		6	HIGH FLOW FROM BED II WITH 4 WAY II CONNECTING BED II TO THE COOLER
103	AND		5	HIGH PRESSURE IN THE HEATER OUTLET [STREAM 16]
104	OR		5	HIGH PRESSURE IN THE HEATER OUTLET [STREAM 16]
105	AND		5	HIGH PRESSURE IN THE FEED TO THE HEATER [STREAM 12]
106	AND		4	HIGH FLOW TO THE 3 WAY WITH HEATER BYPASSED
107	AND		3	HIGH PRESSURE FROM 3 WAY VALVE TO BED I THROUGH 4 WAY VALVE I
110	OR		6	HIGH PRESSURE IN COOLER OUTLET STREAM [STREAM 3]
111	OR		7	HIGH WATER CONCENTRATION IN COOLER OUTLET STREAM [STREAM 3]
114	OR		3	HIGH FLOW FROM THE 3 WAY VALVE - HEATER JUNCTION [STREAM 18]
115	AND		4	HIGH PRESSURE IN THE HEATER OUTLET [STREAM 16]
116	AND		3	HIGH PRESSURE IN OUTLET FROM THE 3 WAY VALVE WITH HEATER BYPASSED
121	OR		4	HIGH PRESSURE IN HEATER OUTLET [STREAM 16]
122	AND		3	HIGH PRESSURE IN FEED TO THE 3 WAY VALVE WITH HEATER BYPASSED
123	OR		5	HIGH PRESSURE IN PROPORTIONATING VALVE INLET [STREAM 7]
136	AND		4	HIGH PRESSURE IN THE FEED TO THE HEATER [STREAM 12]
137	OR		3	HIGH PRESSURE IN FEED TO THE 3 WAY VALVE [STREAM 11]
138	OR		5	HIGH TEMPERATURE IN THE COOLER OUTLET [STREAM 3]
139	OR		5	HIGH PRESSURE IN THE COOLER OUTLET [STREAM 3]

Figure 9 continues

EVENT TABLE OF CONTENTS				
TITLE: SEQUENTIAL DRYER PROCESS				
EVENT NUMBER	PROBABILITY	DEVELOPED ON PAGE	TEXT	
1	2.000E-03	1	4 WAY VALVE LEAKS	ACROSS
2	5.000E-08	2	FIRE AT BED I	
3	1.200E-04	2	NO ALUMINA IN BED	I, OR CHANNELING
4	7.150E-04	1	4 WAY VALVE II	MOTOR FAILURE
5	3.300E-01	2	TIME = 1	
6	1.670E-01	2	TIME = 2	
7	4.720E-06	1	4 WAY VALVE II	TIMER CHANGES AT WRONG TIME
8	5.000E-08	1	FIRE AT BED II	
9	1.200E-04	1	NO ALUMINA IN BED	II, OR CHANNELING
10	1.670E-01	1	TIME = 4	
11	4.720E-06	1	4 WAY VALVE II	TIMER FAILS
12	8.630E-05	1	4 WAY VALVE II	CONTROL LINE 29 CUT
13	3.300E-01	1	TIME = 3	
14	5.000E-04	1	INLET AIR FLOW UP	
16	7.150E-04	2	4 WAY VALVE I	MOTOR FAILURE
17	7.990E-05	2	HEATER LEAKS STEAM INTO AIR	
18	5.000E-04	3	INLET AIR WATER	CONCENTRATION UP
19	7.150E-04	4	3 WAY VALVE MOTOR	FAILURE
20	4.720E-06	1	4 WAY VALVE I	TIMER CHANGES AT WRONG TIME
21	4.720E-06	1	4 WAY VALVE I	TIMER FAILS
22	8.630E-05	1	4 WAY VALVE I	CONTROL LINE 28 CUT
23	1.000E-05	1	WATER SEPARATOR	TRAP CLOGGED
24	2.100E-03	1	VALVE 6 CLOSED	
25	5.000E-08	3	EXTERNAL FIRE AT	SEPARATOR
26	4.720E-06	3	3 WAY VALVE TIMER	CHANGES AT WRONG TIME
27	4.720E-06	3	3 WAY VALVE TIMER	FAILS
28	8.630E-05	3	3 WAY VALVE	CONTROL LINE 27 CUT
29	5.000E-04	3	COOLING WATER FLOW DOWN	
30	5.000E-04	3	COOLING WATER	TEMPERATURE UP
31	2.990E-03	3	COOLER FOULED	
32	5.000E-08	3	EXTERNAL FIRE AT	COOLER
33	2.100E-03	3	VALVE 4 CLOSED	
36	5.000E-04	3	INLET AIR PRESSURE UP	

Figure 9. The fault tree for the event water concentration too high from the utility air dryer process.

the structure of the digraph. It is not necessary for the analyst to preordain the logic (AND, OR, etc.) of the interactions between variables.

With this type of model all foreseeable interactions may be described. What remains is to interconnect the digraph model for each piece of equipment in the system to obtain a model for the complete system. Figure 4 shows a partial digraph for the heat exchanger system shown in Figure 1. From the digraph model, fault trees may be directly deduced. The algorithm for this deduction has been described previously (Lapp, 1977). Briefly, the procedure involves starting at the node in the digraph which denotes the top event. The negative feed-forward and feedback loops through a node determine how it should be logically related to its inputs. The algorithm has over 30 different logical expansions of a node. The input nodes are logically expanded in a similar manner until the complete fault tree is obtained. The consistency of intermediate events and variables is maintained during the generation of the fault tree.

After generation of the fault tree, the tree is listed, "drawn" on a line printer, and put in minimal cut-set form. The minimal cut-sets of a Boolean equation are the sets of events which are sufficient to cause the top event and do not contain any other sufficient sets of events. The fault tree for the event temperature in stream 4 (T4) too high is given in Figure 5.

The following example illustrates the application of this strategy to a sequential process for drying air.

Example: Fixed Bed Alumina Air Dryers. Figure 6 illustrates a process for drying air. Ambient air which contains water vapor enters in stream 9. The air passes through a bed of alumina (Bed I) where the water vapor is adsorbed. The dried air passes out of the process in stream 25. This process has been used by Professor C. J. King of the Department of Chemical Engineering, University of California, Berkeley, Calif., as a case study in process design.

In order to maintain a continuous supply of dry air, two beds of alumina are employed. When one bed is removing water from the inlet air, the other bed is being regenerated. Regeneration involves passing hot air through a bed which has been loaded to capacity with water. The hot air strips the water from the alumina. The hot air leaving the regenerating bed is passed through a condenser where water is removed. The air is reheated and passed through the operating dryer. The regenerated bed is then cooled with inlet air and switched back into service. The same procedure is followed for the other bed. Table I gives the sequence of operations for a complete cycle.

If the outlet air from the process contains too much water a number of pieces of valuable equipment downstream may be destroyed. What could cause the water concentration in stream 25 to be too high? One way to answer this question is to construct a fault tree for the event concentration of water too high in stream 25 (C (+1) Stream 25).

Input-output models for several of the pieces of equipment

Table III. Minimal Cut Sets

PROBLEM ID: SEQUENTIAL DRYER PROCESS

162 MINIMAL CUT SETS GENERATED.

TOP EVENT PROBABILITY= 2.0001E-03

MINIMAL CUT SET NO. 1	ITS PROBABILITY: 2.00E-03	MINIMAL CUT SET NO. 2	ITS PROBABILITY: 2.83E-08
EVENT 1(2.00E-03) 4 WAY VALVE LEAKS ACROSS		EVENT 3(1.20E-04) NO ALUMINA IN BED I, OR CHANNELING	
		EVENT 4(7.15E-04) 4 WAY VALVE II MOTOR FAILURE	
		EVENT 5(3.30E-01) TIME = 1	
MINIMAL CUT SET NO. 3	ITS PROBABILITY: 1.43E-08	MINIMAL CUT SET NO. 4	ITS PROBABILITY: 1.43E-08
EVENT 3(1.20E-04) NO ALUMINA IN BED I, OR CHANNELING		EVENT 4(7.15E-04) 4 WAY VALVE II MOTOR FAILURE	
EVENT 4(7.15E-04) 4 WAY VALVE II MOTOR FAILURE		EVENT 9(1.20E-04) NO ALUMINA IN BED II, OR CHANNELING	
EVENT 6(1.67E-01) TIME = 2		EVENT 10(1.67E-01) TIME = 4	
MINIMAL CUT SET NO. 5	ITS PROBABILITY: 3.42E-09	MINIMAL CUT SET NO. 6	ITS PROBABILITY: 3.42E-09
EVENT 3(1.20E-04) NO ALUMINA IN BED I, OR CHANNELING		EVENT 9(1.20E-04) NO ALUMINA IN BED II, OR CHANNELING	
EVENT 5(3.30E-01) TIME = 1		EVENT 12(8.63E-05) 4 WAY VALVE II CONTROL LINE 29 CUT	
EVENT 12(8.63E-05) 4 WAY VALVE II CONTROL LINE 29 CUT		EVENT 13(3.30E-01) TIME = 3	
MINIMAL CUT SET NO. 7	ITS PROBABILITY: 1.73E-09	MINIMAL CUT SET NO. 8	ITS PROBABILITY: 1.73E-09
EVENT 9(1.20E-04) NO ALUMINA IN BED II, OR CHANNELING		EVENT 3(1.20E-04) NO ALUMINA IN BED I, OR CHANNELING	
EVENT 10(1.67E-01) TIME = 4		EVENT 6(1.67E-01) TIME = 2	
EVENT 12(8.63E-05) 4 WAY VALVE II CONTROL LINE 29 CUT		EVENT 12(8.63E-05) 4 WAY VALVE II CONTROL LINE 29 CUT	
MINIMAL CUT SET NO. 9	ITS PROBABILITY: 5.04E-10	MINIMAL CUT SET NO. 10	ITS PROBABILITY: 3.54E-10
EVENT 4(7.15E-04) 4 WAY VALVE II MOTOR FAILURE		EVENT 4(7.15E-04) 4 WAY VALVE II MOTOR FAILURE	
EVENT 5(3.30E-01) TIME = 1		EVENT 5(3.30E-01) TIME = 1	
EVENT 16(7.15E-04) 4 WAY VALVE I MOTOR FAILURE		EVENT 16(7.15E-04) 4 WAY VALVE I MOTOR FAILURE	
EVENT 31(2.99E-03) COOLER FOULED		EVENT 24(2.10E-03) VALVE 6 CLOSED	
MINIMAL CUT SET NO. 11	ITS PROBABILITY: 3.54E-10	MINIMAL CUT SET NO. 12	ITS PROBABILITY: 1.87E-10
EVENT 4(7.15E-04) 4 WAY VALVE II MOTOR FAILURE		EVENT 3(1.20E-04) NO ALUMINA IN BED I, OR CHANNELING	
EVENT 5(3.30E-01) TIME = 1		EVENT 5(3.30E-01) TIME = 1	
EVENT 16(7.15E-04) 4 WAY VALVE I MOTOR FAILURE		EVENT 11(4.72E-06) 4 WAY VALVE II TIMER FAILS	
EVENT 33(2.10E-03) VALVE 4 CLOSED			
MINIMAL CUT SET NO. 13	ITS PROBABILITY: 1.87E-10	MINIMAL CUT SET NO. 14	ITS PROBABILITY: 1.87E-10
EVENT 3(1.20E-04) NO ALUMINA IN BED I, OR CHANNELING		EVENT 9(1.20E-04) NO ALUMINA IN BED II, OR CHANNELING	
EVENT 5(3.30E-01) TIME = 1		EVENT 11(4.72E-06) 4 WAY VALVE II TIMER FAILS	
EVENT 7(4.72E-06) 4WAY VALVE II TIMER CHANGES AT WRONG TIME		EVENT 13(3.30E-01) TIME = 3	
MINIMAL CUT SET NO. 15	ITS PROBABILITY: 1.87E-10	MINIMAL CUT SET NO. 16	ITS PROBABILITY: 9.46E-11
EVENT 7(4.72E-06) 4WAY VALVE II TIMER CHANGES AT WRONG TIME		EVENT 7(4.72E-06) 4WAY VALVE II TIMER CHANGES AT WRONG TIME	
EVENT 9(1.20E-04) NO ALUMINA IN BED II, OR CHANNELING		EVENT 9(1.20E-04) NO ALUMINA IN BED II, OR CHANNELING	
EVENT 13(3.30E-01) TIME = 3		EVENT 10(1.67E-01) TIME = 4	
MINIMAL CUT SET NO. 17	ITS PROBABILITY: 9.46E-11	MINIMAL CUT SET NO. 18	ITS PROBABILITY: 9.46E-11
EVENT 9(1.20E-04) NO ALUMINA IN BED II, OR CHANNELING		EVENT 3(1.20E-04) NO ALUMINA IN BED I, OR CHANNELING	
EVENT 10(1.67E-01) TIME = 4		EVENT 6(1.67E-01) TIME = 2	
EVENT 11(4.72E-06) 4 WAY VALVE II TIMER FAILS		EVENT 11(4.72E-06) 4 WAY VALVE II TIMER FAILS	
MINIMAL CUT SET NO. 19	ITS PROBABILITY: 9.46E-11	MINIMAL CUT SET NO. 20	ITS PROBABILITY: 8.44E-11
EVENT 3(1.20E-04) NO ALUMINA IN BED I, OR CHANNELING		EVENT 4(7.15E-04) 4 WAY VALVE II MOTOR FAILURE	
EVENT 5(1.67E-01) TIME = 2		EVENT 5(3.30E-01) TIME = 1	
EVENT 7(4.72E-06) 4WAY VALVE II TIMER CHANGES AT WRONG TIME		EVENT 16(7.15E-04) 4 WAY VALVE I MOTOR FAILURE	
		EVENT 30(5.00E-04) COOLING WATER TEMPERATURE UP	

in the system are given in Figure 7. Note the time dependent nature of the three-way valve, four-way valves, and the timer. The input-output models were interconnected to give a digraph model for the dryer system. The complete digraph for this system contained 67 nodes and 439 edges. A reduced version of the digraph is shown in Figure 8. Only the main concentration and flow interactions are shown.

The fault tree generation algorithm required 30 s of IBM 360/67 time to generate the fault tree for this system. The tree contains 143 gates and is shown in Figure 9. This tree is different from the usual fault tree in that common events such as time periods are considered.

Probability data were gathered and estimated for the events included in the tree. Table II presents the data. Over 100 cut-sets were computed for the tree. The first twenty are presented in Table III.

An analysis of the cut-sets for this system indicates the importance of leaking of the four-way valve. The results of this fault tree analysis in conjunction with economic considerations of the dryer operation and other possible design or maintenance corrections can be used to decide an appropriate action.

Conclusions

With digraph models that contain edges that depend on other variables and events, it is possible to include common sequential behavior in a system digraph. This allows the generation of a fault tree that contains events (like the sequence of valve operations) that are normally true. The analysis of the minimal cut-sets that result from this fault tree allows the analyst to focus attention on the important parts of the system.

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