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Missing Plot Technique in Diallel Crosses for Griffing Method-III

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Abstract

The method of combining ability analysis of diallel crosses for Griffing method-3 for one and two missing observations has been presented. The formulae for estimating the parameters, the variances of different estimates and the sum of squares for various effects have been presented. The given analysis can easily be extended for the case of missing of more than two observations. The procedure of analysis has been illustrated through an example.

Key words: General combining ability effects, specific combining ability effects, maternal effects, maternal interaction effects, Bartlett missing plot technique, complete diallel crosses.

1. Introduction

Diallel crosses are commonly used by plant and animal breeders in their hybrid breeding programmes. Quite often, some of the observations are lost due to several natural hazards. The experimenters are unable to perform the combining ability analysis of diallel crosses in these cases because of the non-availability of the method of analysis. Hence they can not utilize the advantage of diallel crossing system. At present, the experimenters usually estimate the missing values by the standard formulae of the design used and then proceed with usual combining ability analysis without caring for the consequence of missing of observation on genetic parameters.

Missing plot technique has been quite commonly used in environmental designs but its use has been very restricted in mating designs. In the present paper, the missing plot technique for diallel cross analysis given by Kaushik and Puri (1984) for Griffing (1956) method-3, where F_1''s and reciprocals are taken but parents are ignored, has been presented. The estimates of genetic parameters and their standard errors have also been given when 1 or 2 observations are missing. The model under consideration is valid only for Griffing (1956) method 1 and 3 but not for method 2 and 4. The missing plot technique for method-3 is being discussed here and for method-1, it will be presented in a separate communication.

BARTLETT (1937) missing plot technique will be used for developing the diallel analysis for missing data. In this technique for 'n' missing observations the analysis of covariance with 'n' concomitant variables is considered. The value of a pseudo covariate (x) is assumed to be 1 if the observation is missing and Zero otherwise. The value of main variable 'y' for the missing case is taken to be zero.

2. When One Observation is Missing

Let $v(v-1)$ crosses are laid out in a Randomized Block Design (RBD) in r replications. The diallel table consisting of cross totals based on r replications are written in following form.

Table 1
Diallel Table

Females	Males 1	2	...	j'	...	v	Total	Mean
1	–	$y_{12.}$	...	$y_{1j'.}$	...	$y_{1v.}$	$y_{1..}$	$\bar{y}_{1..}$
2	$y_{21.}$	–	...	$y_{2j'.}$	...	$y_{2v.}$	$y_{2..}$	$\bar{y}_{2..}$
.	.	.	...	.	...	.	.	.
.	.	.	...	.	...	.	.	.
i'	$y_{i'1.}$	$y_{i'2.}$	...	$y_{i'j'.}$	...	$y_{i'v.}$	$y_{i'..}$	$\bar{y}_{i'..}$
.	.	.	...	.	...	.	.	.
.	.	.	...	.	...	.	.	.
v	$y_{v1.}$	$y_{v2.}$	...	$y_{vj'.}$	...	–	$y_{v..}$	$\bar{y}_{v..}$
Total	$y_{.1.}$	$y_{.2.}$	...	$y_{.j'.}$	...	$y_{.v.}$	$y_{...}$	
Mean	$\bar{y}_{.1.}$	$\bar{y}_{.2.}$	...	$\bar{y}_{.j'.}$	...	$\bar{y}_{.v.}$	$\bar{y}_{...}$	

Let $y_{i'j'l'}$-th observation on $(i'\times j')$-th cross (i'-th female mated to j'-th male) in l'-th block be missing. Clearly in above diallel table, all observations will be based on r replications except $y_{i'j'.}$ which will be based on $(r-1)$ replications as one observation on this cross is missing.

We shall assume the following model for combining ability analysis.

$$y_{ijl}=\mu+g_i+g_j+s_{ij}+m_i+n_{ij}+b_l+\beta(x_{ijl}-\bar{x})+e_{ijl} \qquad (2.1)$$
$$i,j=1,2,\ldots,v, \quad i\neq j; \quad l=1,2,\ldots,r$$

where μ is the general effect, $g_i(g_j)$ is the general combining ability (g.c.a.) effect of i-th (j-th) line; m_i is the maternal effect of i-th line; s_{ij} and n_{ij} are specific combining ability (s.c.a.) and maternal interaction effects of i-th and j-th line; x_{ijl} is the corresponding observation on the concomitant variable which takes the value 1 if the observation is missing and 0 otherwise and $\bar{x}$ is its overall mean. The value of y for the missing observation will be taken as zero. Here β is regression coefficient and e_{ijl} is the random error assumed to be normally and independently distributed with mean zero and variance σ^2.

The estimates of genetic parameters obtained after putting the restrictions

$$\sum_i g_i = \sum_i s_{ij} = \sum_j s_{ij} = \sum_i n_{ij} = \sum_j n_{ij} = 0$$

are given below.

(2.2)
$$\hat{g}_i = [r\,(v-1)\,(\bar{y}_{i..} + (v-1)\,\bar{y}_{.i.} - v\bar{y}_{...}) + a_i^{(g)}\beta]/rv\,(v-2)$$

$$\hat{m}_i = [r\,(v-1)\,(\bar{y}_{i..} - \bar{y}_{.i.}) + \beta a_i^{(m)}]/rv$$

$$\hat{s}_{ij} = [(v-2)\,(\bar{y}_{ij.} + \bar{y}_{ji.}) - (v-1)\,(\bar{y}_{i..} + \bar{y}_{j..} + \bar{y}_{.i.} + \bar{y}_{.j.}) + 2v\bar{y}_{...} + a_{ij}^{(s)}\beta/r\,(v-1)]/2\,(v-2)$$

$$\hat{n}_{ij} = [vr\,(\bar{y}_{ij.} - \bar{y}_{ji.}) - r\,(v-1)\,(\bar{y}_{i..} + \bar{y}_{j..} + \bar{y}_{.i.} + \bar{y}_{.j}) + a_{ij}^{(n)}\beta]/2vr\,,$$

$$\beta = E_{xy}/E_{xx}\,,$$

where

$$E_{xy} = [y_{...} - ry_{..l'} - v\,(v-1)\,y_{i'j'.}]/rv\,(v-1) \quad \text{and}$$

$$E_{xx} = (r-1)\,(v\,(v-1)-1)/rv\,(v-1)\,,$$

$a_i^{(g)}$, $a_i^{(m)}$, $a_{ij}^{(s)}$ and $a_{ij}^{(n)}$ are the constants as defined below.

$a_i^{(g)} = 0$ if i-th line is involved in the missing observation as a female, i.e. $i = i'$
$= -(v-2)$ if i-th line is involved in the missing observation as a male i.e. $i = j'$
$= 1$ otherwise

$a_i^{(m)} = -1$ if i-th line is involved in the missing observation as a female, i.e. $i = i'$
$= 1$ if i-th line is involved in the missing observation as a male, i.e. $i = j'$
$= 0$ otherwise

$a_{ij}^{(s)} = -(v-2)\,(v-3)$ if both i-th and j-th parents are involved in the missing observation, i.e. $i = i'$, $j = j'$
$= (v-3)$ if either i-th or j-th parent is involved in the missing observation, i.e. $i = i'$ or $j = j'$
$= -2$ otherwise

$a_{ij}^{(n)} = -(v-2)$ if i-th female and j-th male are involved in the missing observation, i.e. $i = i'$, $j = j'$, $i' \neq j'$
$= (v-2)$ if i-th male and j-th female are involved in the missing observation, i.e. $i = j'$, $j = i'$, $i' \neq j'$
$= 1$ if either i-th female or j-th male is involved in the missing observation, i.e. $i = i'$ or $j = j'$, $i' \neq j'$
$= -1$ if i-th male or j-th female is involved in the missing observation, i.e. $i = j'$ or $j = i'$, $i' \neq j'$
$= 0$ otherwise

The estimates of genetic parameters are obtained after substituting the appropriate values of a's in formulae (2.2). The estimate of block effects are not given here as they are of no interest to a geneticist or breeder.

It is clear from above, there will be three possible values each of $\hat{g}_i$ and $\hat{m}_i$ depending on whether i-th female is involved in the missing observation or i-th male is involved in the missing observation or neither i-th male nor i-th female is involved in the missing observation. Similarly $\hat{s}_{ij}$ will have three values depending on whether both i-th and j-th parents or neither of the two is involved in the missing observation. Similarly $\hat{n}_{ij}$ will have 5 values.

Table 2

ANOVA Table when one observation is missing

S.V.	d.f.	Adjusted sum of squares
Blocks	$r-1$	$R_{yy}+\hat{\beta}E_{xy}-y^2_{i'j'.}/r\,(r-1)$
Crosses	v^2-v-1	$C_{yy}+\hat{\beta}E_{xy}-y^2_{..l'}/(v^2-v)\,(v^2-v-1)$
Parental Effects	$v-1$	$P_{yy}+\hat{\beta}E_{xy}-[(y_{i'..}+y_{.i'.}+y_{j'..}+y_{.j'.})/2\,(v-2)\,r-y_{..l'}/v\,(v-1)$ $-y_{i'j'.}/r-y_{...}/r\,(v-1)\,(v-2)]^2//(r-1)\,(v^2-v-1)$ $+(v-1)/rv\,(v-1)]$
Parental Interactions	$v\,(v-3)/2$	$PI_{yy}+\hat{\beta}E_{xy}-[(y_{i'j'.}+y_{j'i'.})/2r$ $-(y_{i'..}+y_{.i'.}+y_{j'..}+y_{.j'.})/2\,(v-2)\,r-y_{..l'}/v\,(v-1)$ $-y_{i'j'.}/r+2y_{...}/rv\,(v-2)]^2/[2\,(r-1)\,(v^2-v-1)$ $+v\,(v-3)/2rv\,(v-1)]$
Maternal Effects	$v-1$	$M_{yy}+\hat{\beta}E_{xy}-[(y_{i'..}-y_{.i'.}-y_{j'..}+y_{.j'.})/2vr-y_{..l'}/v\,(v-1)$ $-y_{i'j'.}/r+y_{...}/v\,(v-1)\,r]^2/[(r-1)\,(v^2-v-1)+(v-1)/rv\,(v-1)]$
Maternal Interactions	$(v-1)\,(v-2)/2$	$MI_{yy}+\hat{\beta}E_{xy}-[(y_{i'j'.}-y_{j'i'.})/2r-(y_{i'..}-y_{.i'.}-y_{j'..}+y_{.j'.})/2vr$ $-y_{..l'}/v\,(v-1)-y_{i'j'.}/r+y_{...}/v\,(v-1)\,r]^2/[2\,(r-1)\,(v^2-v-1)$ $+(v-1)\,(v-2)/2rv\,(v-1)]$
Error	$(v^2-v-1)\,(r-1)-1$	$E_{yy}-\hat{\beta}E_{xy}$
Total	$v\,(v-1)\,r-2$	

The adjusted sum of squares due to parameters are obtained as usual by method of fitting constants and are presented in Table 2. In this table, T_{yy}, R_{yy}, P_{yy} etc. have the following meanings:

(2.3)
$$R_{yy}=[\sum_l y^2_{..l}-y^2_{...}/r]/v\,(v-1)$$

$$C_{yy}=[\sum_{i,j} y^2_{ij.}-y^2_{...}/v\,(v-1)]/r$$

$$P_{yy}=[\sum_i (y_{i..}+y_{.i.})^2-4y^2_{...}/v]/2r\,(v-2)$$

$$PI_{yy}=[\sum_{i,j}(y_{ij.}+y_{ji.})^2-\sum (y_{i..}+y_{.i.})^2/(v-2)+2y^2_{...}/(v-1)\,(v-2)]/2r$$

$$M_{yy}=\sum_i (y_{i..}-y_{.i.})^2/2vr$$

$$MI_{yy}=[\sum_{ij} (y_{ij.}-y_{ji.})^2-\sum_i (y_{i..}-y_{.i.})^2/v]/2r$$

$$E_{yy}=\sum_{i,j,l} y^2_{ijl}-\sum_{ij} y^2_{ij.}/r-\sum_l y^2_{..l}/v\,(v-1)+y^2_{...}/v\,(v-1)\,r$$

After computing the values of R_{yy}, C_{yy} etc. from these formulae, the adjusted sum of square can be very easily obtained after substituting the totals from the existing data of diallel Table 1. The variances of the estimates of the parameters for the case of one missing observation are summarized below.

$$V(\hat{g}_i)=[(v-1)^2/rv^2\,(v-2)+(v-1)\,(a_i^{(g)})^2/rv\,(r-1)\,(v-2)^2\,(v^2-v-1)]\,\sigma^2$$
$$V(\hat{m}_i)=[2\,(v-1)/rv^2+(v-1)\,(a_i^{(m)})^2/rv\,(r-1)\,(v^2-v-1)]\,\sigma^2$$
$$V(\hat{s}_{ij})=[(v-3)/2r\,(v-1)+v(a_{ij}^{(s)})^2/4r\,(r-1)\,(v-1)\,(v-2)^2\,(v^2-v-1)]\,\sigma^2$$
$$V(\hat{n}_{ij})=[(v-2)/2vr+(v-1)\,(a_{ij}^{(n)})^2/4rv\,(r-1)\,(v^2-v-1)]\,\sigma^2$$

It is clear from above that the variances of the estimates are obtained after substituting the appropriate values of $a_i^{(g)}$, $a_i^{(m)}$, $a_{ij}^{(s)}$ and $a_{ij}^{(n)}$ in the respective formulae. The values of these constants depend upon whether i-th and/or j-th line is involved in the missing observation or not.

3. When Two Observations are Missing

Now the above technique is extended to the case of two missing observations. Let observations on $(i'\times j')$-th cross in l'-th block and $(u\times t)$-th cross in m-th block be missing. The additional covariate z assumed for second missing observation will be introduced in model (2.1). The z will take value 1 if the observation is missing and 0 otherwise. Now the model can be written as.

$$y_{ijl}=\mu+g_i+g_j+s_{ij}+m_i+n_{ij}+b_l+\beta_1\,(x_{ijl}-\bar{x})+\beta_2\,(z_{ijl}-\bar{z})+e_{ijl}\,, \quad i,j=1,2,\dots,v,\quad i\neq j;\quad l=1,2,\dots,r\,. \tag{3.1}$$

where β_2 is the regression coefficient of z on y and other parameters are same as mentioned in model (2.1). The estimates of parameters obtained by least square procedure are given below.

$$\hat{g}_i=[r\,(v-1)\,(\bar{y}_{i..}+(v-1)\,\bar{y}_{.i.}-v\bar{y}_{...})+(a_i^{(g)}\beta_1+c_i^{(g)}\beta_2)]/rv\,(v-2) \tag{3.2}$$
$$\hat{m}_i=[r\,(v-1)\,(\bar{y}_{i..}-\bar{y}_{.i.})+(a_i^{(m)}\beta_1+c_i^{(m)}\beta_2)]/rv$$
$$\hat{s}_{ij}=[(v-2)\,(\bar{y}_{ij.}+\bar{y}_{ji.})-(v-1)\,(\bar{y}_{i..}+\bar{y}_{j..}+\bar{y}_{.i.}+\bar{y}_{.j.})+2v\bar{y}_{...} +(a_{ij}^{(s)}\beta_1+c_{ij}^{(s)}\beta_2)/r\,(v-1)]/2\,(v-2)$$
$$\hat{n}_{ij}=[rv\,(\bar{y}_{ij.}-\bar{y}_{ji.})-r\,(v-1)\,(\bar{y}_{i..}+\bar{y}_{j..}+\bar{y}_{.i.}+\bar{y}_{.j.})+(a_{ij}^{(n)}\beta_1 +c_{ij}^{(n)}\beta_2]/2vr\,.$$

where $c_i^{(g)}$, $c_i^{(m)}$, $c_{ij}^{(s)}$ and $c_{ij}^{(n)}$ take the same values as $a_i^{(g)}$, $a_i^{(m)}$, $a_{ij}^{(s)}$ and $a_{ij}^{(n)}$ as mentioned in last section. The a's relate to $(i'\times j')$-th missing observation while c's relate to $(u\times t)$-th missing observation.

For estimating g_i, each of $a_i^{(g)}$ and $c_i^{(g)}$ will take 3 values and hence there will be 9 combinations of a's and c's, which on substituting in the formulae will give 9 possible estimates of g_i's. Similarly there will be 9 possible estimates of m_i and s_{ij}

and 25 of $n_{ij.}$. The estimates are to be picked up depending upon the applicability of the case. The estimates of β_1 and β_2 will be obtained as under.

$$(3.3)\qquad \beta_1=(E_{zz}E_{xy}-E_{xz}E_{zy})/(E_{xx}E_{zz}-E_{xz}^2),\quad \beta_2=(E_{xx}E_{zy}-E_{xz}E_{xy})/(E_{xx}E_{zz}^2-E_{xz}^2)\,,$$

where

$$E_{xx}=E_{zz}=(r-1)\,(v^2-v-1)/rv\,(v-1),\quad E_{xy}=(y_{...}-ry_{..l'}-v\,(v-1)\,y_{i'j'.})/rv\,(v-1)$$

$$E_{xz}=1/rv\,(v-1)\quad\text{and}\quad E_{zy}=(y_{...}-ry_{..m}-v\,(v-1)\,y_{ut.})/rv\,(v-1)\,.$$

Here E_{xx} and E_{zz} are error sum of squares for x and z, E_{xy} and E_{zy} are the error sum of products of y with x and z respectively and E_{xz} is the error sum of product of x with z.

The sum of squares due to parameters are obtained by method of fitting constants and are given in table 3.

Table 3
ANOVA – Table when two observations are missing

S.V.	d.f.	Adjusted S.S.
Blocks	$r-1$	–
Crosses	v^2-v-1	$C_{yy}+\beta_1E_{xy}+\beta_2E_{zy}-\beta_1^{(c)}(E_{xy}+C_{xy})-\beta_2^{(C)}(E_{zy}+C_{zy})$
Parental Effects	$v-1$	$P_{yy}+\beta_1E_{xy}+\beta_2E_{zy}-\beta_1^{(P)}(E_{xy}+P_{xy})-\beta_2^{(P)}(E_{zy}+P_{zy})$
Parental Interaction	$v\,(v-3)/2$	$PI_{yy}+\beta_1E_{xy}+\beta_2E_{zy}-\beta_1^{(PI)}(E_{xy}+PI_{xy})-\beta_2^{(PI)}(E_{zy}+PI_{zy})$
Maternal Effects	$v-1$	$M_{yy}+\beta_1E_{xy}+\beta_2E_{zy}-\beta_1^{(M)}(E_{xy}+M_{xy})-\beta_2^{(M)}(E_{zy}+M_{zy})$
Maternal Interaction	$(v-1)\,(v-2)/2$	$MI_{yy}+\beta_1E_{xy}+\beta_2E_{zy}-\beta_1^{(MI)}(E_{xy}+MI_{xy})-\beta_2^{(MI)}(E_{zy}+MI_{zy})$
Error	$(v^2-v-1)\,(r-1)-2$	$E_{yy}-\beta_1E_{xy}-\beta_2E_{zy}$

Table 4
Table of T_{xx}, T_{zz}, T_{xy} and T_{zy}

Components	$T_{xx}(T_{zz})$	T_{xy}	T_{zy}
Crosses	$(v^2-v-1)/vr\,(v-1)$	$y_{i'j'.}/r-y_{...}/v\,(v-1)\,r$	$y_{ut.}/r-y_{...}/v\,(v-1)\,r$
Parental Effect	$1/rv$	$(y_{i'..}+y_{.i'.}+y_{j'..}+y_{.j'.})/2r\,(v-2)-2y_{...}/rv\,(v-2)$	$(y_{u..}+y_{.u.}+y_{t..}+y_{.t.})/2r\,(v-2)-2y_{...}/rv\,(v-2)$
Parental Interaction	$(v^2-4v+2)/2r\,(v-1)\,(v-2)$	$(y_{i'j'.}+y_{j'i'.})/2r-(y_{i'..}+y_{.i'.}+y_{j'..}+y_{.j'.})/2r\,(v-2)+y_{...}/r\,(v-1)\,(v-2)$	$(y_{ut.}+y_{tu.})/2r-(y_{u..}+y_{.u.}+y_{t..}+y_{.t.})/2r\,(v-2)+y_{...}/r\,(v-1)\,(v-2)$
Maternal Effect	$1/vr$	$(y_{i'..}-y_{.i'.}-y_{j'..}+y_{.j'.})/2vr$	$(y_{u..}-y_{.u.}-y_{t..}+y_{.t.})/2vr$
Maternal Interaction	$(v-2)/2vr$	$(y_{i'j'.}-y_{j'i'.})/2r-(y_{i'..}-y_{.i'.}-y_{j'..}+y_{.j'.})/2vr$	$(y_{ut.}-y_{tu.})/2r-(y_{u..}-y_{.u.}-y_{t..}+y_{.t.})/2vr$
Error	$(r-1)\,(v^2-v-1)/vr\,(v-1)$	$y_{...}/vr\,(v-1)-y_{i'j'.}/r-y_{..l'}/v\,(v-1)$	$y_{...}/vr\,(v-1)-y_{ut.}/r-y_{..m}/v\,(v-1)$

Table 5
Table of T_{xz}.

	When	two missing	observations	pertain	to
Component	Same Cross	Reciprocal Crosses	Same male or female	Crosses with male in one the same as female in other	Crosses with no common parent
Crosses	$(v^2-v-1)/rv\,(v-1)$	$-1/v\,(v-1)\,r$	$-1/v\,(v-1)\,r$	$-1/v\,(v-1)\,r$	$-1/v\,(v-1)\,r$
Parental Effect	$1/vr$	$1/vr$	$(v-4)/2rv\,(v-2)$	$(v-4)/2rv\,(v-2)$	$-2/rv\,(v-2)$
Parental Interaction	$(v-3)/2r\,(v-1)$	$(v-3)/2r\,(v-1)$	$(3-v)/2r\,(v-1)\,(v-2)$	$(3-v)/2r\,(v-1)\,(v-2)$	$1/r\,(v-1)\,(v-2$
Maternal Effect	$1/vr$	$-1/vr$	$1/2vr$	$-1/2vr$	0
Maternal Interaction	$(v-2)/2vr$	$-(v-2)/2vr$	$-1/2vr$	$1/2vr$	0
Error	$E_{zz}=-(r-1)/vr\,(v-1)$		if two observations are missing in same replication.		
	$=-(v^2-v-1)/vr\,(v-1)$		if two missing observations are of same cross.		
	$=1/vr\,(v-1)$		otherwise.		

In ANOVA table 3, $\beta_1^{(c)}, \beta_2^{(c)}, \beta_1^{(p)}, \beta_2^{(p)}$ etc. are obtained by replacing Error sum of squares and error sum of products in formulae (3.3) by Treatment (T) plus error sum of squares and sum of products, where T is C (Crosses), P (Parental effect), PI (Parental Interaction effect), M (Maternal effect) or MI (Maternal Interaction effect). The value of T_{xx}, T_{zz}, T_{xy} and T_{zy} are obtained from Table 4 and that of T_{xz} from table 5.

The variances due to estimates are obtained and given below:

$$V(\hat{g}_i)=[(v-1)^2/rv^2\,(v-2)+(v-1)\left((a_i^{(g)})^2+(c_i^{(g)})^2\right)/rv\,(r-1)\,(v-2)^2\,(v^2-v-1)]\,\sigma^2$$

$$V(\hat{m}_i)=[2\,(v-1)/rv^2+(v-1)\,(a_i^{(m)})^2+\left((c_i^{(m)})^2\right)/rv(r-1)\,(v^2-v-1)]\sigma^2$$

$$V(\hat{s}_{ij})=[(v-3)/2r\,(v-1)+v\left((a_{ij}^{(s)})^2+(c_{ij}^{(s)})^2\right)/4r\,(r-1)\,(v-1)\,(v-2)^2\,(v^2-v-1)]\,\sigma^2$$

$$V(\hat{n}_{ij})=[(v-2)/2vr+(v-1)\left((a_{ij}^{(n)})^2+(c_{ij}^{(n)})^2\right)/4vr\,(r-1)\,(v^2-v-1)]\,\sigma^2$$

The values of the variances of these estimates are obtained after substituting the appropriate values of a's and c's as mentioned in section 2. The above method of analysis can be very easily extended to the case when more than two observations are missing.

4. Illustrative Example

We shall now illustrate the procedure of analysis through an example. Consider a 4×4 diallel cross excluding parents laid out in RBD in 3 replications on mustored crop (*Brassica juncea* czern and Coss) at H.A.U. farm during 1982. The data recorded on days to first flowering is given in Appendix. The data is first analysed by the method given by KAUSHIK and PURI (1982) when no observation is missing and the computed mean squares are given in Table 11. To illustrate the method then the data is analysed by assuming (i) one observation missing and (ii) Two observations as missing. The computations are given below.

Case–I: When one observation is missing

Let us assume that observation on cross (1×2) in second replication is missing, i.e., $i'=1$, $j'=2$, $l'=2$. The values of the coefficients $a_i^{(g)}$, $a_i^{(m)}$, $a_{ij}^{(s)}$ and $a_{ij}^{(n)}$ to be used in formulae (2.2) will be as given in table 6.

Table 6

Table of $a_{ij}^{(s)}$ (below diagonal), $a_{ij}^{(n)}$ (above diagonal), $a_i^{(g)}$ (marginal row) and $a_i^{(m)}$ (marginal column).

	$i \backslash j$	$a_{ij}^{(n)}$ 1	2	3	4	$a_i^{(m)}$
	1	–	−2	1	1	−1
$a_{ij}^{(s)}$	2	−2	–	−1	−1	1
	3	1	1	–	0	0
	4	1	1	−2	–	0
	$a_i^{(g)}$	0	−2	1	1	

It may be noted here that $a_{ij}^{(s)}=a_{ij}^{(s)}$ and $a_{ij}^{(n)}=-a_{ji}^{(n)}$. Here $E_{xy}=-40.5833$, $E_{xx}=0.6111$ and $\beta=E_{xy}/E_{xx}=-66.4102$. By putting $v=4$, $r=3$, $\beta=-66.4102$ and after substituting the appropriate values of a's and corresponding 'y' means, the estimates of parameters can be obtained. For instance,

$$\hat{g}_1=3\ (60.111+3\times 65.778-4\times 63.833)/8+0=.7924$$

and

$$\hat{m}_1=3\ (60.111-65.778)/4+66.410/12=1.2839$$

Similarly other estimates of parameters are calculated and given in Table 7.

It may further be noted here that $\sum g_i=\sum m_i=0$ and also $\sum_i s_{ij}=\sum_j s_{ij}=\sum_i n_{ij}=$ $=\sum_j n_{ij}=0$. Further $s_{ij}=s_{ji}$ and $n_{ij}=-n_{ji}$.

For computing sum of squares, C_{yy}, P_{yy}, PI_{yy}, M_{yy}, MI_{yy} and E_{yy} are computed using (2.3) and are as under.

$C_{yy}=1749.6667$, $P_{yy}=850.6667$, $PI_{yy}=364.6666$, $M_{yy}=149.8333$, $MI_{yy}=$ $=1749.6666$ and $\beta E_{xy}=2695.1451$.

Table 7
Table of $\hat{s}_{ij}$ (below diagonal), $\hat{n}_{ij}$ (above diagonal) $\hat{g}_i$ (marginal row) and $\hat{m}_i$ (marginal column)

$i \backslash j$	$\hat{n}_{ij}$ 1	2	3	4	$\hat{m}_i$
$\hat{s}_{ij}$ 1	–	.0342	.7330	– .7671	1.2839
2	–.8111	–	–1.0663	1.1006	–3.4514
3	.3215	.4880	–	–0.3331	.7500
4	.4878	.3213	– .8113	–	1.4168
$\hat{g}_i$	.7924	–1.1734	–2.1416	2.5253	

In ANOVA table 2, by putting $v=4$, $r=3$, $i'=1, j'=2$, $l'=2$ and taking $y_{..2}=$ $=729$, $y_{12.}=131$, $y_{1..}=541$, $y_{.1.}=592$, $y_{2..}=496$, $y_{.2.}=528$, $y_{...}=2298$ and after substituting the values of β, E_{xy} and C_{yy}, P_{yy}, PI_{yy} etc., the sum of squares can be obtained and are given in table 11.

The variances due to the estimates of parameters are obtained after substituting $v=4$, $r=3$, $\hat{\sigma}^2=1.382$ and the values of a's as given above. For example,

$$\mathrm{v}(\hat{g}_i)=9\times 1.382/96=.1295$$

The variances due to other estimates are computed similarly and are given in Table 8.

Table 8
Variances of $\hat{n}_{ij}$ (above diagonal), $\hat{s}_{ij}$ (below diagonal), $\hat{g}_i$ (marginal row) and $\hat{m}_i$ (marginal column)

$i \backslash j$	1	2	V($\hat{n}_{ij}$) 3	4	V($\hat{m}_i$)
V($\hat{s}_{ij}$) 1	–	.1309	.1191	.1191	.1885
2	.0838	–	.1191	.1191	.1885
3	.0785	.0785	–	.1152	.1727
4	.0785	.0785	.0768	–	.1724
V($\hat{g}_i$)	.1295	.1453	.1335	.1335	

Case–II: When Two Observations are Missing

Now let us consider that two observations on cross (1×2) and cross (2×3) in second replication are missing i.e. $i'=1, j'=2$, $u=2$, $t=3$ and $l'=m=2$. For estimating the parameters, the values of a's will be same as mentioned in last case and the values of c's i.e. $c_i^{(g)}$, $c_i^{(m)}$, $c_{ij}^{(s)}$ and $c_{ij}^{(n)}$ are as given in Table 9.

Further $E_{xx}=E_{zz}=0.6111$, $E_{xz}=0.0556$, $E_{xy}=-37.4167$, $E_{zy}=-33.0833$ and consequently $\beta_1=-66.7029$ and $\beta_2=-60.2034$. After substituting the above values the estimates of parameters are obtained as given in Table 10.

It may be verified here that $\sum \hat{g}_i=\sum \hat{m}_i=\sum_i \hat{s}_{ij}=\sum_j \hat{s}_{ij}=\sum_i \hat{n}_{ij}=\sum_i \hat{n}_{ij}=\sum_j \hat{n}_{ij}=0$ and further $\hat{n}_{ij}=-\hat{n}_{ji}$ and $\hat{s}_{ij}=\hat{s}_{ji}$.

The sum of squares P_{yy}, PI_{yy}, M_{yy} etc. are computed as in earlier case and are obtained as under:

$$C_{yy}=3369.4167,\quad P_{yy}=1653.4167,\quad PI_{yy}=279.1666$$
$$M_{yy}=344.5833,\quad MI_{yy}=1092.2500\quad \text{and}\quad E_{yy}=4510.1667$$

Table 9

Values of $c_{ij}^{(s)}$ (below diagonal), $c_{ij}^{(n)}$ (above diagonal) $c_i^{(g)}$ (marginal row) and $c_i^{(m)}$ (marginal column)

$i \setminus j$		$c_{ij}^{(n)}$ 1	2	3	4	$c_i^{(m)}$
	1	–	−1	1	0	0
$c_{ij}^{(s)}$	2	1	–	−2	1	−1
	3	1	−2	–	−1	1
	4	−2	1	1	–	0
$c_i^{(g)}$		1	0	−2	1	

Table 10

Table of $\hat{s}_{ij}$ (below diagonal) $\hat{n}_{ij}$ (above diagonal), $\hat{g}_i$ (marginal row) and $\hat{m}_i$ (marginal column)

$i \setminus j$		1	$\hat{n}_{ij}$ 2	3	4	$\hat{m}_i$
	1	–	−.8236	1.2775	−.4535	.6571
$\hat{s}_{ij}$	2	−1.0748	–	−1.8409	1.0180	−3.2860
	3	.6852	.3901	–	−.5644	1.2122
	4	.3894	.6844	−1.0743	–	1.4167
$\hat{g}_i$		1.0227	−1.8007	−2.2911	3.0690	

Further $E_{zz}+P_{zz}=E_{xx}+P_{xx}=25/36$, $E_{xz}+P_{xz}=-1/18$, $E_{xy}+P_{xy}=-44.4167$ and $E_{zy}+P_{zy}=-42.50$, consequently $\beta_1^{(p)}=-69.2947$, $\beta_2^{(p)}=-66.7393$ and $\beta_1^{(p)}(E_{xy}+P_{xy})+\beta_2^{(p)}(E_{zy}+P_{zy})=5914.2622$. Adjusted sum of squares for parental effect is then obtained as 226.6810 which gives the mean sum of squares as 75.5613. Likewise all adjusted mean squares are computed and given in Table 11. Now the test of significance can be carried out as suggested by KAUSHIK and PURI (1982).

Table 11

Mean sum of squares

S.V.	d.f.	No observation missing	One observation missing	Two observations missing
Replication	2	2.0278	–	–
Crosses	11	38.171	38.068	30.310
Parental Effect	3	98.667	95.1339	75.561
Parental Interaction	2	8.111	5.525	6.883
Maternal Effect	3	28.611	29.576	24.969
Maternal Interaction	3	7.278	7.185	5.158
Error	22*	1.4823	1.382	1.318

* Error d.f. will be 21 in Case I, when one observation is missing, 20 in Case 2, when two observations are missing.

The variances due to estimates of parameters are obtained and are given in Table 12.

Table 12

Table of variances due to $\hat{n}_{ij}$ (above diagonal), $\hat{s}_{ij}$ (below diagonal), $\hat{g}_i$ (marginal row) and variances due to $\hat{m}_i$ (marginal column)

$i \backslash j$	1	2	3	4	$V(\hat{m}_i)$
	$V(\hat{n}_{ij})$				
1	–	0.1104	0.1007	0.0975	0.1543
$\hat{V}(\hat{s}_{ij})$ 2	0.0700	–	0.1104	0.1007	0.1672
3	0.0657	0.0700	–	0.0975	0.1543
4	0.0700	0.0657	0.0700	–	0.1415
$V(\hat{g}_i)$	0.1093	0.1190	0.1222	0.1125	

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Appendix

Data on Days to first flowering

Cross	Replications R_1	R_2	R_3	Total ($y_{ij.}$) No. of missing observation 0	1	2
1×2	67	64	64	195	131	131
1×3	66	67	67	200	200	200
1×4	71	70	69	210	210	210
2×3	59	57	59	175	175	118
2×4	63	66	66	195	195	195
3×4	65	66	65	197	197	197
4×3	66	68	67	201	201	201
4×2	68	69	66	203	203	203
3×2	64	66	64	194	194	194
4×1	71	74	70	215	215	215
3×1	64	65	65	194	194	194
2×1	61	61	61	183	183	183
Total ($y_{..l}$)						
No. of	0 785	793	784	2362		
missing	1 785	729	784	2298	2298	2241
observation.	2 785	672	784	2241		

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