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Few-Body Systems

Three-body scattering in Poincaré-invariant quantum mechanics*

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Abstract The relativistic three-nucleon problem is formulated by constructing a dynamical unitary representation of the Poincaré group on the three-nucleon Hilbert space. Two-body interactions are included that preserve the Poincaré symmetry, lead to the same invariant two-body S-matrix as the corresponding non-relativistic problem, and result in a three-body S-matrix satisfying cluster properties. The resulting Faddeev equations are solved by direct integration, without partial waves for both elastic and breakup reactions at laboratory energies up to 2 GeV.

1 Introduction

Energy scales near a GeV are interesting in nuclear physics because of the relevance of sub-nuclear degrees of freedom. Some experimental manifestations of these degrees of freedom are the appearance of baryon resonances and the opening of meson production channels. Poincaré-invariance is required for a consistent treatment of the dynamics at these energies. For a limited energy range the number of relevant degrees of freedom should be small enough for a few-body treatment. A first step in modeling few-body systems at these energies is to demonstrate the feasibility of solving Faddeev equations in a Poincaré-invariant quantum theory and understanding the differences in the predictions of the relativistic and non-relativistic theories.

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In Poincaré-invariant quantum mechanics the dynamics is given by a dynamical representation of the Poincaré group [1]. The construction of this representation is done in four steps. The first step is to choose a basis for single particle irreducible representations of the Poincaré group. In this work the irreducible representation space is taken as the space of square integrable functions of the linear momentum and z-component of the canonical spin. The second step is to construct two- and three-body representations by taking tensor products of one-body irreducible representations. The third step is to use Poincaré Clebsch-Gordan coefficients in the chosen basis to reduce the tensor product of irreducible representations to a direct integral of irreducible representations. The final step is to add an interaction to the mass Casimir operator of the non-interacting irreducible representation that commutes with and is independent of the linear momentum and z-component of the canonical spin. Simultaneous eigenstates of the mass, linear momentum, spin, and z-component of the canonical spin are constructed by diagonalizing the interacting mass operator in the free-particle irreducible basis. These eigenstates are complete and transform irreducibly, and thus define the dynamical representation of the Poincaré group.

The invariance principle implies the equality of two-body wave operators constructed with the non-relativistic Hamiltonian $H = P^2/4M + k^2/m + v_{NN}$ and the relativistic mass square operator $M^2 = 4(k^2 + m^2) + 4mv_{NN}$ provided the relative momentum, k, obtained by transforming the single-particle momentum to the rest frame with a Galilean boost is identified with the relative momentum obtained by transforming the single-particle momentum to the rest frame with an inverse Lorentz boost. Because of this identification, the relativistic and non-relativistic two-body scattering matrices are identical functions of k.

Differences occur in how the two-body interactions appear in the three-body mass (rest energy) operator. In the relativistic case the non-linear relation between mass and energy, which must be respected for S-matrix cluster properties, implies that the two-body interaction in the three-body problem has the form

$$V_{12} = \sqrt{4k^2 + 4m^2 + 4mv_{NN} + q^2} - \sqrt{4k^2 + 4m^2 + q^2},$$

where q is the momentum of the spectator boosted to the rest frame of the non-interacting three-body system. The two-body interaction in the square root appears in the kernel of the Faddeev equation for the mass operator.

The relation between M^2 and the non-relativistic Hamiltonian leads to the following identification for the non-relativistic two-body and relativistic $2+1$-body right-half-shell transition matrix elements

$$\langle q', k'|T_{12}(z_r)|q, k\rangle = \frac{4m\langle k'|t_{12nr}(z_{nr})|k\rangle\delta(q' - q)}{\sqrt{4k^2 + 4m^2 + q^2} + \sqrt{4k'^2 + 4m^2 + q^2}},$$

where z_r and z_{nr} are the right-half-shell invariant mass squared and energy, respectively. The off-shell $T_{12}(z)$ is needed in the Faddeev kernel. We calculate this using the first resolvent equation which gives an integral equation for the off-shell $T_{12}(z)$ using the half-shell $T_{12}(z_r)$ as input [2].

This method is used to solve the Faddeev equations for the Poincaré-invariant mass operator. In addition, because we are interested in energies near the GeV scale, we have chosen to solve the equations by direct integration, rather than dealing with the large number of partial waves required for convergence [3, 4].

Our test calculations, using a Malfliet-Tjon type of two-body interaction, are converged for laboratory energies up to 2 GeV.

While the model interaction does not have the spin complexities of a realistic interaction, the success of these calculations suggests that the method outlined above is suitable for modeling reactions in the few GeV energy range. Note that similar calculations using partial wave methods with realistic interactions have been solved at lower energies [5].

2 Results

Comparison of the relativistic and corresponding non-relativistic calculations lead to observations that should also be relevant for realistic interactions. Note that this comparison does not involve a non-relativistic limit, instead relativistic and non-relativistic three-body calculations with interactions that are fit to the same two-body data are compared. All of the differences are due to the different ways in which the two-body dynamics appears in the three-body problem.

One important observation is that the effect of relativistic kinematics, which can be large and grows with energy, is largely canceled by dynamical effects.

For elastic scattering the largest difference between the relativistic and non-relativistic calculations is at large angles, above 60 degrees at 1 GeV; the difference increases as a function of beam energy.

The convergence of the multiple scattering series is not uniform. For elastic scattering the first order term is not adequate even at 2 GeV for large angles.

Similar behavior is observed for breakup calculations away from the quasi-elastic peaks. In addition, both inclusive and exclusive breakup calculations show a shift in the position of the quasielastic peaks relative to the peaks in the non-relativistic theory.

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