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Optimal implicit collusion in repeated procurement auctions

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Abstract We develop a model of implicit collusion in repeated procurement auctions in which suppliers can only observe past auction prices, but not all bids and the identities of winners. We focus on symmetric perfect public equilibria (SPPE) and use the dynamic programming techniques to characterize the optimal SPPE. We allow for a public randomization device and find that the implementation of the optimal collusive equilibrium can be simply obtained by using the bang-bang property, because suppliers just need to adopt the collusive bidding schedule and depending on the past winning bids, with a certain probability they stick to that strategy or move to Nash reversion. The optimal implicit collusion is characterized by a rigid-bidding scheme with punishments occurring on the equilibrium path. As a result, the optimal collusion can not achieve full efficiency.

Keywords Implicit collusion · Procurement auctions · Repeated games

JEL Classification D44 · D82 · D86

1 Introduction

Repeated auctions provide an ideal ground for bidder collusion, because in an repeated environment where the same set of bidders interact time and time again reputation can be built, implicit transfers can be made, and noncooperative players can be punished. A subject of major interest in repeated auctions is the nature and degree of implicit collusion that can be sustained amongst bidders, via strategies that make current period

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biddings depend upon aspects of the past history of individual bidder behavior. The scope of such strategies clearly is limited by the extent of the bidder's knowledge of that history. By far the most attention has been given to the case in which all bids and/or the identities of winners in each previous period are common knowledge (see the literature review in Sect. 1.1). While in our setting, we explore the consequences of invoking the opposite polar assumption about information that bidders cannot observe one another's past bids or the identities of past winners; thus the only information available to players is past winning bids in all previous auctions. Therefore, the public history of the game consists only of past winning bids.[1]

Formally, we consider an infinitely repeated procurement auction without communication, no transfer payment and suppliers (bidders) can only observe past auction prices. Each supplier is privately informed of its cost type in each period. There is a continuum of possible costs, and the cost realization is i.i.d. across time and suppliers, and so the repeated game has a recursive structure. We restrict attention to symmetric perfect public equilibria (SPPE), whereby following every public history, suppliers adopt symmetric bidding schedules.[2] Following the approach of Abreu et al. (1986), Abreu et al. (1990) (hereafter APS), we apply the tools of dynamic programming to this setting and transform the infinitely repeated game into an equivalent static game whose payoffs are the ones of the stage game augmented by a continuation payoff which represents the present value of future payoffs (Proposition 1). Following APS, any SPPE collusive scheme at a given point in time can be described by (i) a first-period bidding strategy for each cost type and (ii) an associated equilibrium continuation payoff for each vector of current auction price, where the continuation payoff is symmetric across suppliers. In an SPPE, therefore, colluding suppliers move symmetrically through any collusion or punishment (Nash-reversion) phases.

We observe that a collusive scheme must satisfy two kinds of incentive constraints. First, for every supplier and cost type, the short-term gain from deviation, i.e., with a bid that is not assigned to any cost type and that thus represents a clear deviation, must be unattractive. This corresponds to the admissibility constraint, which serves as a counterpart to the traditional participation constraint. Second, the proposed conduct must also be such that no supplier is ever attracted to misrepresent its private information and submit a bid intended for a different cost type. This corresponds to the standard incentive compatibility constraint.

[1] For example, in the Dutch clock auction, only the winning bid is announced publicly.

[2] In a related context, Athey et al. (2004), Skrzypacz and Hopenhayn (2004), and Blume and Heidhues (2008) also focus on symmetric equilibrium of this kind. Our focus on symmetric strategies is based on the following considerations: first, as argued by Athey, Bagwell and Sanchirico, "SPPE maybe the only available option if firms cannot observe individual firm behavior. This occurs, e.g., in procurement auctions with more than two bidders, if the winning bid–but not the name of the winner–is announced". Second, SPPE are appealingly simple and may be descriptive of less formal (and perhaps tacit) collusive ventures, while APPE (asymmetric PPE) are quite sophisticated and may be most plausible when a small number of firms interact frequently and communicate explicitly. Third, asymmetric schemes allow one firm to enjoy a more profitable continuation value than another. Such schemes thus facilitate transfers from one firm to another. However, optimal APPE collusion may require firms track and reward individual firm behavior over time, which mismatches the assumption that players can only observe past winning bids and not past actions in our model setting.

We turn next to explore whether suppliers can then support better-than-Nash profits. We find that there exist equilibria of surprising simplicity which are optimal among all pure strategy SPPE. In these equilibria, the supplier can coordinate their bids according to the admissible bidding rule and can do better than Nash competition, in which suppliers repeatedly play the Nash equilibrium of the static game. To compute which bid to submit in current period a supplier simply needs to remember the winning price in the previous period. Furthermore, credible punishments can be available to satisfy admissibility, which will assure the stability of implicit collusion (Lemma 1). The harshest punishment inflicted on the defector gives him the same payoffs as the single stage Nash equilibrium (Proposition 2). The optimal collusive mechanism must exhibit the bang-bang property which implies that the continuation payoffs of any SPPE (including the most collusive) can be sustained by an SPPE which after every public history of the first period only uses the two extreme continuation values (Proposition 3). In other words, the choice of optimal continuation values can be reduced to assign a probability to every possible public history after period 1. The extreme simplicity of implementing the optimal collusive scheme should be emphasized: with a positive probability suppliers revert to the stage game Nash equilibrium forever; with the remaining probability suppliers continue to play the current period strategy and receive the maximal continuation value (Proposition 4).

In an SPPE, the informational costs of collusion may be manifested in two ways. First, the bids of lower-cost suppliers are distorted downward so that higher-cost suppliers find lower prices less appealing and have no incentive to deviate. Second, following the selection of lower prices, the collusive scheme may sometimes call for a future equilibrium-path punishment. The current-period benefit of a lower price then may be of sufficient magnitude to compensate for the future punishment only if supplier truly has lower costs in the current period. In the optimal collusive scheme, suppliers may neutralize the informational costs of collusion altogether, by adopting a rigid-bidding scheme, in which bids are rigid over intervals of costs (i.e., the optimal bidding scheme is a weakly increasing step function) (Lemma 2). More generally, the optimal SPPE collusion is characterized by a rigid-bidding scheme, with punishments following some auction price realizations. The implication of the scheme is that it sacrifices efficiency benefits and thus the optimal SPPE are not efficient (Proposition 5). The scheme highlights the central tradeoff between efficiency benefits and informational costs that colluding suppliers must reconcile.

1.1 Relation to the literature

Recently there has been some progress made into understanding the behavior of rational bidders colluding in repeated auctions. One of the very first questions that comes to mind, is this: what is the scope of collusion in a repeated auction?

Theories of collusion in auctions highlight the role of communication among bidders. The seminar paper by McAfee and McMillan (1992) (thereafter, M&M) focuses on incentives in a static first price auction and show that a strong cartel (i.e., side transfers are allowed) with pre-stage communication can achieve full efficiency for a wide range of discount factors. They also prove that for a weak cartel (without transfers) a bid rotation scheme (BRS) can outperform the static Bayesian Nash equilibrium. The drawback of BRS is that, there is no truthtelling problem and the good rarely goes to the player with highest valuation (the winner of the auction is determined regardless of her valuation). Our information setup is more restrictive than M&M as we don't allow communication and all bids except winning bids are not publicly observed. Hence the folk theorem does not apply: there is not enough information in the public signal to provide sufficient incentives for the bidders. In addition, our paper models a repeated relationship in which deviation is deterred through future equilibrium punishment. However, in the work of M&M, there is no incentive to adhere to such a collusive agreement if there is no punishment for violating it openly. M&M also show that a fixed price (i.e., "identical bidding") is the optimal strategy for a weak cartel. In our analysis of the optimal SPPE, we generalize the weak-cartel model, since the static mechanism we analyzed is directly derived from a repeated game and allows for a proper choice of continuation payoffs (corresponding to symmetric punishments occur on the equilibrium path). The analysis here extends their results, by formally connecting the static results to the repeated game context and demonstrating that a rigid bidding scheme is optimal even when punishments are allowed. Our rigid-bidding finding thus provides additional theoretical support for the practice of identical bidding.

The static analysis of bidder collusion with communication is extended to a repeated framework firstly by Aoyagi (2003), who analyzes 2-bidder collusion in repeated auctions when no side transfer is possible. In contrast to M&M's static bid rotation, Aoyagi (2003) constructs a dynamic BRS in which bidders' coordination is based on communication history. In the scheme, play rotates among different phases that treat the bidders differently and collusion is sustained through the adjustment of continuation payoffs in a way that partially compensates for the lack of monetary transfer. The results show that when the stage auction is the IPV and first-price, the dynamic BRS is an equilibrium for sufficiently patient bidders and yields a strictly higher payoff than the optimal static scheme proposed by M&M. Aoyagi (2007) extended the above analysis to finite signal set. He studies collusion in repeated auctions when bidders communicate prior to each stage auction, and identifies conditions under which an equilibrium collusion scheme is fully efficient in the sense that the bidders' payoff is close to what they get when the object is allocated to the highest valuation bidder at the reserve price in every period. However, the stronger conclusion obtained in the paper derives from the self-decomposability techniques available for finite-action games. While it is more standard in the auction literature to use a continuous signal space, we know of no continuous counterpart to the above technique.

In a repeated model of oligopoly, Athey and Bagwell (2001) consider collusion with communication when each player's private signal is binary. They show that sophisticated collusion, in which firms track and reward individual firm behavior over time, communicate and allocate market shares, is characterized by market shares that are unstable over time. Athey and Bagwell (2001) considers a model with a continuum of types and studies optimal symmetric schemes, while allowing that firms may be impatient and that demand may be volatile. The paper provides a formal interpretation for the traditional view that standard collusion entails fixed prices and stable market shares over time. Our paper can also predict that the standard collusion will result in rigid bidding. More importantly, we have shown that the optimal SPPE can be achieved with more limited information structure and can be simply implemented by using the bang-bang property.

The information structure in our paper is very similar to Skrzypacz and Hopenhayn (2004), who consider collusion in repeated IPV auctions when no communication is available and the only information available to players is the identities of winners in all previous auctions. They show that in the 2-player case a cartel is not able to achieve full or even approximate efficiency, no matter how patient it is. However, large cartels can achieve almost efficient collusion. Related results have been obtained by Blume and Heidhues (2008) who also study tacit collusion without communication in which bidders can only observe past winners and not their bids. SH construct two schemes that deliver nonmarginal improvements conditional on every history of play: the exclusion scheme and the chips scheme. The two schemes are too complicated for analytical analysis and it is not known what are the improvements it delivers for distributions other than uniform,[3] and it is not known how the equilibrium bidding functions look like when there are more than two players (more detailed analysis about the differences between our paper and SH can be found in the end of Sects. 4.3 and 5). Johnson and Robert (1999) extend the static analysis of M&M to a series of auctions where bidders meet repeatedly and draw independent private values in each auction. They find that when cartels can use a randomization rule to overcome the incentive compatibility problems, optimal collusion can be implemented by identical bids and rotating bid schemes. however, in our paper, the incentive constraints are assured by pricing distortions and future punishments (Nash reversion).[4]

It follows from the analysis in Rachmilevitch (2013) that with two valuations the first-best can be achieved exactly, without public randomization, communication or transfers, in any repeated standard IPV auctions. Rachmilevitch (2014) demonstrates that in a repeated 2-bidder IPV first-price auction with three valuations, the first-best can be approximated as closely as one wishes, without using communication, transfers, or public randomization.[5] Obviously, in every stage the last period's winner and bids are all observable and thus the collusive strategy conditions on public histories including bids and the identities of winners. The first-best collusion results in a 2-

[3] SH compute the payoff for the special case of a uniform distribution.

[4] Other differences between our paper and that of Johnson and Robert are as follows:(1) in our imperfect monitoring context, the optimal punishment strategy will not involve deterministic Nash reversion. However, in the model of Johnson and Robert, when a bidder observes a high price, he will take this outcome as convincing evidence of deviation. As a result, the punishment strategy will involve deterministic Nash reversion; (2) we focus upon SPPE, while the latter use the equilibrium concept of subgame perfect equilibrium. Obviously, all discussed SPPE can not necessarily be implemented as SPNE. There exists other SPNE in which the bidders condition their strategies not only on the public history of the auction, but also on private histories. Moreover, if private strategies are adopted, the APS approach used in their paper cannot work.

[5] The idea behind the collusive strategy he proposes is simple: in each round, the last period's loser has more weight in the decision on the identity of the current winner. This privilege translates to an implicit transfer-losing today is accompanied by a larger continuation value (relatively to winning) because in the next period the current loser will have greater weight in choosing the winner. Under some restrictions on the distribution of valuations, this strategy is sustainable as a PPE.

bidders model by Rachmilevitch (2013, 2014) can not be extended to more general case when there are many bidders and the identities of winners are anonymous.

Our paper is also related to the literature on infinitely repeated games with imperfect public information. Under general conditions, Fudenberg et al. (1994) show that one can establish a Folk Theorem in PPE for finite-action-set games if there is a sufficient number of public outcomes which are observable. In a similar framework, but with finitely many types, Horner and Jamison (2007) show that an all-inclusive ring with finitely many (more than two) valuations can approximate first-best profits when the discount factor is close to one using review strategies.

The paper is organized as follows: Sect. 2 presents the model to be studied through a series of assumptions. In Sect. 3, we use the method developed by APS to transform the repeated game into an equivalent static game. In Sect. 4, we characterize the optimal collusive schemes and investigate the implementation of the optimal collusive equilibrium. We show the inefficient results of our optimal collusive schemes and explore its implications in Sect. 5. Section 6 contains a discussion of our main modeling assumptions. Some conclusions and policy implications are offered in Sect. 7. Many of the proofs can be found in the Appendix.

2 Model formulation

A set of $N = \{1, 2, \ldots, n\}$ of ex-ante identical and risk-neutral suppliers compete in an infinitely sequence of auctions, where a single indivisible object is procured in every period through a first price sealed bid auction.[6] All potential suppliers submit bids on the unit price of the objects. At the beginning of each period each supplier is privately informed of his own cost c_i. These costs are independently and identically drawn from a continuous and differentiable cumulative distribution F, with positive density f over a compact set $[\underline{c}, \overline{c}]$, which is common knowledge. Define $H(c) = \frac{F(c)}{f(c)}$ and assume H is an increasing function of c, i.e., $F(c)$ is log-concave. In addition, costs are independent across time. In particular, we assume that (i) suppliers cannot communicate before each auction; (ii) suppliers cannot resell the good nor use side payments; (iii) suppliers do not observe each others' bids and the identities of past winners, and they can only observe the winning bid after each period's auction.

The procurement auction can be modelled by an N-person infinitely repeated game with discounting. The first step in defining the game is to specify the single-period component game G.

The single-period game G After the suppliers learn their respective cost types, they simultaneously submit bids. Let $b_i \in \mathbb{R}_+$ denotes the bid submitted by supplier i, with $b \equiv (b_1, \ldots, b_n)$ then representing the associated bid profile. Attention is restricted to an equilibrium in which the stage bidding function $b_i(c)$ is non-decreasing in c. A bidding strategy for supplier i is a function $\beta_i(c_i)$ mapping from the set of cost types, $[\underline{c}, \overline{c}]$, to the set of possible bids, $\mathbb{R}_+$. A bidding strategy profile is thus a vector $\beta(c) \equiv (\beta_i(c_i), \beta_{-i}(c_{-i}))$, where $c \equiv (c_i, c_{-i})$ is the vector of cost types and $\beta_{-i}(c_{-i})$ is the profile of rival bidding strategies. Each supplier chooses its bidding strategy with

[6] If $n = 2$, the identity of the winner is always common knowledge, so we assume that $n > 2$.

the goal of maximizing its expected profit, given its cost type. To represent a supplier's expected profit, we require two further definitions. First, we define $\pi(b, c) \equiv b - c$ as the profit that a supplier receives when it submits the bid b and has cost type c and wins the auction. Second, we specify an allocation function, $p_i(b)$, that indicates supplier i's winning probability when the vector of realized bids is b.

We may now represent supplier i's interim profit, which is the expected profit for supplier i when it has cost type c_i, submits the bid b_i and anticipates that rival bids will be determined by the rival bidding strategy profile, $\beta_{-i}(c_{-i})$. With $\overline{p}(b_i; \beta_{-i}) \equiv E_{c_{-i}}[p_i(b_i, \beta_{-i}(c_{-i}))]$, supplier i's interim profit function may be written as

$$\overline{\pi}(b_i, c_i; \beta_{-i}) \equiv \pi(b_i, c_i)\overline{p}(b_i; \beta_{-i}).$$

The repeated game $G^{\infty}(\delta)$

$G^{\infty}(\delta)$ is the infinitely repeated auction game defined by the component game G and the common discount factor $\delta \in (0, 1)$. Formally, we describe the repeated game in the following terms. A full path of play is an infinite sequence $\{c^t, \beta^t\}$, with a given pair in the sequence representing a vector of types and bidding schedules at date t. The infinite sequence implies a public history of realized auction price vectors, $\{b_w^t\}$, and pathwise payoffs for supplier i may be thus defined as

$$u_i(\{c^t, \beta^t\}) = \sum_{t=1}^{\infty} \delta^{t-1}\pi(b_i^t, c_i^t)p_i(b_i^t, b_w^t)$$

At the close of period τ, supplier i possesses an information set, which may be written as $h_i = \{c_i^t, b_i^t, b_w^t\}_{t=1}^{\tau}$. (The null history is the supplier's information set at the beginning of the first period.) A (pure) strategy for supplier i, $s_i(h_i)(c_i)$, associates a bid schedule with each information set h_i. Each strategy profile $s = (s_1, \ldots, s_n)$ induces a probability distribution over play paths $\{c^t, \beta^t\}$ in the usual manner. The expected discounted payoff from s is thus the expectation $\overline{u}_i(s) = E[u_i(\{c^t, \beta^t\})]$ taken with respect to this measure on play paths.

Under our assumptions, cost types are i.i.d. across time and suppliers, and so the repeated game has a recursive structure. It is therefore nature to follow Fudenberg et al. (1994) and employ sequential equilibrium as the solution concept. Specifically, we focus on sequential equilibrium in which each player's strategy only depend on the publicly available history. Such strategies are called public strategies and such public strategies form the sequential equilibrium after every public history which we referred as perfect public equilibria (PPE).[7] Deviations to nonpublic strategies are permitted but irrelevant. As long as the other players use public strategies, private history can be ignored when evaluating future payoffs. PPE are recursive and therefore dynamic programming techniques apply: The one-deviation principle holds, i.e., one can verify

[7] Any PPE can be made into a Perfect Bayesian Equilibrium (PBE) by choosing any beliefs that conform with Bayes'rule. Since behavior depends only on public history, these beliefs do not matter for evaluating future payoffs. Therefore the condition of consistency is immaterial. PPE also satisfy one of the two conditions for a sequential equilibrium, sequential rationality. Thus, we may think of PPE as sequential equilibria.

that a particular strategy profile is a PPE by checking that no player can gain after any history by deviating from his prescribed strategy once and conforming with it forever after. A public strategy may thus be abbreviated as a map from finite public histories $\{b_w^t\}_{t=1}^{\tau}$ to bidding schedules. We further restrict attention to SPPE, whereby following every public history, suppliers adopt symmetric bidding schedules: $s_i(b_w^1, \ldots, b_w^{\tau}) = s_j(b_w^1, \ldots, b_w^{\tau}), \forall i, j, \tau, b_w^1, \ldots, b_w^{\tau}$. Symmetry means that all suppliers suffer future punishments and rewards together on an industry-wide basis.

3 The dynamic programming

In this section, we apply the approach of Abreu et al. (1986, 1990) to present a "Factored Program" and establish a relationship between solutions to this program and optimal SPPE. APS were the first to use a generalization of the techniques of dynamic programming to analyze noncooperative games. This permits a very convenient representation of repeated games as much more tractable single stage games. They restrict attention to SSE's in which each player makes his current actions depend upon public (price) signal realizations and show that private quantity histories can be ignored.[8] We rule out private strategies by assumption and thus we can apply the APS approach.

Following APS, any symmetric public strategy profile s at a given point in time can be factored into (i) a first-period bidding strategy β for each cost type and (ii) an associated equilibrium continuation value u for each vector of current auction prices, where the continuation value is symmetric across suppliers. Using the approach of APS, we can transform the repeated game into an equivalent static game whose payoffs are the ones of the stage game augmented by a continuation payoff. Since the strategy profile is symmetric, we omit the subscript i thereafter for simplicity. Define $\overline{\pi}(b, c)$ as the stage game expected payoff of a supplier with cost type c who bids b given that the other suppliers following strategy β. Define $E_{b_w}[u(\beta(c), b_w)]$ as the expected continuation payoff for a supplier having bid according to the bidding strategy β, given the current winning bid is b_w. Furthermore we define $E_c\{\Pi(c; \beta, u)\} := E_c\{\overline{\pi}(\beta, c) + \delta E_{b_w}[u(\beta(c), b_w)]$ as each supplier's expected payoff from β.

In the following take V to be the set of perfect payoffs a supplier can receive in various SPPE of the repeated game. We now consider the Factored Program, in which we choose factorizations directly in order to maximize a supplier's expected payoff, subject to (i) the feasibility constraint that the continuation payoff rests always in the SPPE value set and (ii) the incentive constraint that a supplier cannot gain by deviating to an alternative bidding strategy (given the continuation payoff function and under the assumption that other suppliers follow the bidding strategy):

The factored program Suppliers choose bidding schedule β and continuation payoff function u to maximize

[8] In the model of APS, the problem of consistency does not arise, because every price observation is compatible with the beliefs and actions prior to period t do not affect payoffs from t onward. Any SSE is thus independent of private histories and beliefs about the past are irrelevant (Hence each player faces the same future environment regardless of his initial action). Otherwise, if players use private strategies in which they condition on their past private histories, the whole APS approach does not work.

$$E_c\{\overline{\pi}(\beta, c) + \delta E_{b_w}[u(\beta(c), b_w)]\}$$
$$subject\ \ to: \forall\, b_w \in \mathbb{R}^n_+,\ \ u(b_w) \in V,\ \ and$$
$$\forall\ b,\ \ E_c\{\overline{\pi}(\beta, c) + \delta E_{b_w}[u(\beta(c), b_w)]\} \geq E_c\{\overline{\pi}(b, c) + \delta E_{b_w}[u(b(c), b_w)]\}.$$

Proposition 1 *Consider any symmetric public strategy profile s with the corresponding factorization (β, u). Then s is an optimal SPPE if and only if (β, u) solves the Factored Program.*

Proposition 1 states that, when each supplier makes his actions depend only upon past signal realizations which are publicly observed (not on his own previous actions), we may characterize the set of optimal SPPE by solving the Factored Program. This is a preliminary result used in the following Sect. 4 when we analyze the optimal collusive schemes. Therefore, to characterize the optimal SPPE in Sect. 4, we just need to focus on the factorized pair (β, u) rather than the symmetric public strategy profile s.

Propositions 1 is now a corollary to the propositions on self-generation and factorization in APS. [9] APS develop an oligopoly theory where implicit collusion can be sustained amongst quantity-setting firms, via strategies that make output levels at time t depend upon past prices and firm's own past quantities. By employing the technology of self-generation and factorization, they characterize the optimal SSE of the discounted oligopolistic supergame and exhibit its extremely simple intertemporal structure. We adopt the analytical tools to a different model setting in which the colluding suppliers take past auction prices as coordination signals to reach implicit collusion in infinitely repeated procurement auctions. The main difference is that we allow for an endogenous support of the public signal and thus the support of the signal is itself determined in equilibrium. In the APS model, by contrast, the support of the publicly observed market price is independent of the private output selections made by suppliers. In particular, if suppliers employ a rigid-bidding scheme as shown by Lemma 2, then in equilibrium the support of the public signal is degenerate, as rival suppliers expect to observe the rigid auction price, no matter what cost realization the supplier experiences.

It is also possible to analyse the set of equilibrium. If suppliers can randomize over continuation equilibrium (e.g. using a public randomization device), then the set of SPPE values is convex and is thus fully characterized when the best and the worst SPPE are found, where the worst equilibrium value is attained by minimizing rather than maximizing the objective in the Factored Program. In an SPPE, therefore, colluding suppliers move symmetrically through any collusion or punishment phases. In Sect. 4.3, we prove that u may be chosen to have a "bang–bang" property that makes the intertemporal structure of the equilibrium entirely elementary.

[9] The technique was shown to be applied to many classes of supergames. In their textbook, Mailath and Samuelson (2006) present the classic folk theorem and reputation results for games of perfect and imperfect public monitoring, with the benefit of the modern analytical tools of decomposability and self-generation.

4 The optimal collusive schemes

The goal of this section is to characterize the optimal symmetric collusive schemes. Remark that under the information structure imposed, a cartel must and can be all encompassing. This is because a cartel cannot distinguish cheating by a member or non-member, and it can credibly menace any deviation with a punishment. The obvious question becomes what is the form of collusion which should be used to guarantee the cartel the highest discounted expected payoffs? Can some kind of defection be permitted? How should defectors be punished? In this section, we provide the main ideas behind our construction and relate them to these questions.

4.1 The detection of two kinds of deviations

We begin by observing that a SPPE in a collusive scheme must be immune to two kinds of deviations and correspondingly there are two types of constraints which need to be satisfied. One corresponds to the admissibility constraint which states that if β is the bidding strategy, the supplier with type c prefers bidding in β other than outside β, i.e., suppliers obey to the collusive bidding strategy and have no incentive to deviate. When a supplier chooses a bid not specified in the range of β for any cost realization, it has unambiguously deviated: the deviant price is “off the equilibrium path”. As is standard, such kind of deviations are most effectively deterred when suppliers use the worst available punishment as a threat. Another constraint will be referred to the usual incentive compatibility constraint: if β is the collusive bidding strategy, then the supplier with type c prefers bidding $\beta(c)$ to bidding $\beta(\widehat{c})$. A supplier may deviate by this way when it submits a bid that is assigned under β to some cost level, but not its own. For example, a supplier may be tempted to submit the lower bid assigned to a lower cost realization in order to increase its chances of winning the auction. Moreover, a rival can not be sure that the deviating supplier was not truly of the cost type that it is imitating: the deviant bid is “on the equilibrium path”. This kind of deviation can be prevented if the collusive scheme imposes a punishment when low bids are submitted. But such a punishment would be done at the cost of efficiency, since it would occur along the equilibrium path of play, whenever suppliers actually realize low costs.

4.2 The ability to punish

Once the defection has been detected, the punishment should be severe enough to deter it. The punishment should be specified so as to make any deviator sufficiently unhappy that any deviations in the collusion phase are unprofitable. Unless a further deviation is detected within the punishment phase, suppliers revert to the collusion phase. In other words, play begins with the collusion phase, and any deviation from the collusive scheme triggers the punishment phase where the one-shot Nash equilibrium of the stage auction is played.

Now consider a collusive mechanism $\{\beta(\cdot), u(\cdot)\}$, where β is the bidding strategy mapping types into bids and where $u(\cdot)$ is the future expected payoffs when the current winning bid is b_w. Let $Q(\beta, c)$ denote the probability that a supplier bidding $\beta(c)$ wins the auction given that all the other suppliers bid according to β. Since the buyer procures the item from the supplier with the lowest bid, and in the case of draws randomizes, $Q(\beta, c)$ can be written explicitly. If c is in an interval where β is monotonic increasing in c, then obviously $Q(\beta, c) = (1 - F(c))^{n-1}$. Otherwise if β is constant within the interval $[c_i, c_j]$ then for all $c \in [c_i, c_j]$ we have

$$Q(\beta, c) = \frac{(1 - F(c_i))^n - (1 - F(c_j))^n}{n(F(c_j) - F(c_i))} \equiv Q(c_i; c_j)$$

Incentive compatibility can thus be written as:

$$\begin{aligned} U(c; \beta, u) &:= [\beta(c) - c]Q(\beta, c) + \delta E_{b_w}[u(\beta(c), b_w)] \\ &\geq [\beta(\widehat{c}) - c]Q(\beta, \widehat{c}) + \delta E_{b_w}[u(\beta(\widehat{c}), b_w)] = U(\widehat{c}; \beta, u), \forall\ \widehat{c} \in [\underline{c}, \overline{c}] \end{aligned}$$

where

$$E_{b_w}[u(\beta(c), b_w)] = \int_{\underline{c}}^{c} u(\beta(x)) \cdot n[1 - F(x)]^{n-1} f(x)dx + u(\beta, b_w)[1 - F(c)]^{n-1}$$

For given β and u, the incentive constraint requires that a supplier with type c does not better by reporting that its type is c than by reporting some other type $\widehat{c}$, when other suppliers are presumed to report truthfully. According to the standard results in the mechanism literature (see Myerson (1981), we know that incentive compatibility is equivalent to the following two conditions:

$$\begin{gathered} \frac{d}{dc}U(c; \beta, u) = -Q(c; \beta) \\ Q(c; \beta)\ is\ nonincreasing\ in\ c \end{gathered}$$

We can thus obtain a more convenient characterization of incentive compatibility which lets us explicitly solve for the bidding strategy in terms of type conditional probabilities of winning:

$$\beta(c) = c + \left[\frac{\int_c^{\overline{c}} Q(s; \beta)dx + \delta \int_c^{\overline{c}} [u(\beta(x)) - u(\beta(c))]n[1 - F(x)]^{n-1} f(x)dx}{Q(c; \beta)} \right] \tag{1}$$

It is important to note that the bidding strategy will generally be composed of (positively) sloped and flat regions. McAfee and McMillan (1992) shows that, when cartel members are not allowed to make transfer payments to each other, the cartel can not do better than using a perfectly flat bidding scheme, i.e., optimal collusion has every bidder submitting the same bid (the reserve price). However, in their model, the information necessary to implement this optimal collusion concludes the identity and bid of the winning bidder, which are both assumed to be public.

To ensure that the cartel members comply with the collusion scheme, one enforcement avenue is to appeal to a grim trigger strategy in an infinitely repeated auction context, which requires us to impose another admissibility constraints. The admissibility constraints will be relevant whenever β contains flat regions. The following lemma furnished some preliminary results.

Lemma 1 *Let $\underline{u}$ denote the lowest expected continuation payoff credibly attainable. An incentive compatible collusive mechanism $(\beta(\cdot), u(\cdot))$ is admissible if and only if for all c^* at the lowest edge of a flat bidding range we have:*

$$\begin{aligned}&[\beta(c^*) - c^*]Q(c^*; \beta) + \delta u(\beta(c^*))[1 - F(c^*)]^{n-1}\\ &\quad\geq [\beta(c^*) - c^*][1 - F(c^*)]^{n-1} + \delta \underline{u}[1 - F(c^*)]^{n-1}\end{aligned}$$

The above admissibility constraint is similar to "the trigger strategy" constraint in repeated oligopoly models where all firms charge the monopoly price until a deviation is observed in which event all firms revert to competitive pricing for a specified number of periods. In the above inequality, we compare expected gains to deviation with expected gains to compliance. The proof consists primarily of showing the existence of discontinuities in the bidding strategy, showing that the imposition of the harshest possible punishment is always beneficial. Finally we can prove that if all c^* at the lowest edge of a flat bidding range respect admissibility then all types do. Using the bidding strategy given by Eq. (1) and the above Lemma 1, we can rewrite admissibility as:

$$\begin{aligned}&\delta[u(\beta(c^*)) - \underline{u}]\\ &\quad\geq \left[\int_{c^*}^{\overline{c}} Q(x; \beta)dx + \delta \int_{c^*}^{\overline{c}} [u(\beta(x)) - u(\beta(c^*))]n[1 - F(x)]^{n-1} f(x)dx\right]\\ &\qquad\times \frac{[1 - F(c^*)]^{n-1} - Q(c^*; \beta)}{[1 - F(c^*)]^{n-1} Q(c^*; \beta)}\end{aligned} \tag{2}$$

Note that when the bidding strategy is strictly increasing, we have $Q(c^*, \beta) = [1 - F(c^*)]^{n-1}$, and then admissibility is automatically satisfied.

Define $\overline{M}$ ($\underline{M}$) as an incentive compatible and admissible mechanism which yields the highest (lowest) level of expected continuation payoff $\overline{u}$ ($\underline{u}$). We can show that the set of perfect payoffs is compact, so we are assured that $\overline{M}$ and $\underline{M}$ are well defined. The remainder of this section aims to characterize $\overline{M}$ and $\underline{M}$.

To satisfy admissibility, credible punishments must be available. Proposition 2 shows that a defector can be threatened with noncooperative payoffs in all future auctions.

Proposition 2 *Under the admissibility constraint, the threat to a grim trigger strategy gives suppliers noncooperative payoffs associated with the stage game Nash equilibrium in the infinitely repeated auction context.*

Proposition 2 is an important step in endogenizing punishments as it characterizes the gains to optimal (in the sense of Abreu (1986)) punishments. The proof relies heavily on the fact that $H(c)$ is increasing in c along with the fact that the only information released is the winning bid. Inequality (2) leads us to an important result on collusion in repeated auctions which can be given by the following Proposition 3.

4.3 The implementation of the optimal collusive equilibrium

In an imperfect monitoring context, a supplier cannot observe the bid choices of rivals but that all suppliers observe a public signal (the auction price) that is influenced both by bid choices and an unobserved cost shock. A colluding supplier that witnesses a low auction price then faces an inference problem, as it is unclear whether the low-price outcome arose as a consequence of a favorable cost shock or a secret deviation by a rival. Consequently, in this imperfect monitoring context, the optimal punishment strategy will not involve deterministic Nash reversion. We thus allow for a public randomization device. In the stage game with public randomization, a realization of a public random variable is first drawn and observed by all players. Then each player chooses an action. This represents a situation in which active collusion is formally forbidden and thus pursued through tacit coordination on a public signal.

If suppliers can randomize over continuation equilibria (e.g. using a public randomization device), then the set of continuation payoffs is the convex hull of the set of values which can be supported by the pure strategies of the stage game. Thus the set of payoffs supported by SPPE is compact, non-empty and convex. Moreover, the set of SPPE continuation payoffs is fully characterized when the best and worst SPPE are found, where the worst equilibrium value is attained by minimizing rather than maximizing the objective in the program.

Formally, as defined in the last section, let V be the set of payoffs supported by SPPE. Proposition 3 shows that if the pair $(\beta(\cdot), u(\cdot))$ is admissible with respect to the compact set V, then the function u will take on only two extreme values, i.e., $\overline{u} \equiv \sup V$ and $\underline{u} \equiv \inf V$. Thus, we have $V = [\underline{u}, \overline{u}]$ and a SPPE that supports the ex-ante maximal continuation payoff $\overline{u}$ always exists. Consequently, in the optimal collusive scheme, the suppliers will gain the maximal continuation payoff $\overline{u}$. We focus on this optimal SPPE as it is natural to assume that suppliers insist on this equilibrium whenever possible, because it ensures the highest payoffs.

Proposition 3 *Let $(\beta(\cdot), u(\cdot))$ be an admissible pair with respect to a compact set $V \subseteq R$. Let $\overline{u} \equiv \sup V$ and $\underline{u} \equiv \inf V$. Then the function u must exhibit the bang-bang property such that $u : \mathbb{R}^n_+ \longrightarrow \{\underline{u}, \overline{u}\}$ and for the optimal collusive scheme, $\overline{M} = (\beta(\cdot), u(\cdot))$, we have*

$$u(b) = \overline{u} \quad for\ almost\ every\ b \in \beta$$

According to the bang-bang property, in the admissible collusive scheme, the expected continuation payoffs can take on only two extremal values. The maximal value can be achieved when the admissibility constraint can be satisfied and all cartel members comply with the collusive bidding strategy, while the minimal value corresponds to the case when some certain defection is detected and the game reverts to

Nash competition. The relevant collusive equilibrium has the following features: in the collusive phase, suppliers bid according to the collusive bidding scheme β and enjoy the maximal continuation payoffs $\overline{u}$; and any bid not in β will be treated as defection and all suppliers obtain non-cooperative Nash payoffs $\underline{u}$. Suppliers coordinate on a randomization device and either jointly stay in the collusive phase or enter the punishment phase. As a result, the implementation of the optimal collusive equilibrium is obtained by using the so-called bang-bang property given by the above Proposition 3, which ensures that it is possible to implement the optimal SPPE randomizing only between the two extremal points of the set V.

Formally, the implementation of the optimal collusive equilibrium is equivalent to solve the following problem:

$$\overline{u} = \max_{\beta,\lambda} u$$

$$s.t. \quad u = \overline{\pi}(\beta(c_i), c_i; \beta) + \delta[(1-\lambda)u + \lambda \underline{u}] \tag{3}$$

$$\forall\, \mathbf{b}_w \in \mathbb{R}^n_+, \;\; u(\mathbf{b}_w) \in V \tag{4}$$

$$\forall\, \widetilde{\beta}, \;\; u(\beta) \geq u(\widetilde{\beta}) \tag{5}$$

where λ is the conditional probability of entering into a perpetual Nash reversion given the state of nature (the history of the winning bids). Condition 1 (Eq. (3)) is the stage game payoff when all suppliers play β augmented by the expected continuation payoff defined by the bang-bang implementation. Condition 2 (Eq. (4)) is the feasibility constraint that the continuation payoff rests always in the SPPE value set. Condition 3 (Eq. (5)) is the incentive constraint that a supplier cannot gain by deviating to an alternative bidding schedule (given the continuation payoff function and under the assumption that other firms follow the bidding schedule). The maximal payoff $\overline{u}$ is obtained with the following equilibrium strategies:

Proposition 4 *In the imperfect monitoring context, the implementation of the optimal collusive equilibrium is obtained by using the bang-bang property. This implies that, suppliers just need to comply with the collusion scheme and depending on the past winning bids, they stick to that collusive scheme with probability* $1-\lambda$ *or switch to Nash reversion with probability* $\lambda > 0$.

Proposition 4 describes supplier's optimal strategy. The implementation of the optimal collusive scheme is very simply obtained by using the bang-bang property, because suppliers just need to start playing the collusive bidding scheme β and depending on the public information (the past winning bids), with a certain probability they stick to that strategy or move to Nash reversion. Suppliers never find it optimal to punish when they observe low auction price. On the contrary, when they observe low auction price, they optimally punish with a certain probability $\lambda > 0$, to give the right incentives to sustain the collusive equilibrium.[10]

[10] This implies that, under no circumstances will suppliers be able to infer from the public information that with probability 1, someone has deviated from collusive equilibrium behavior.

It is worthwhile to note that the bang-bang property does not arise in the work of Skrzypacz and Hopenhayn (2004), which has a much closer information structure. The public history of the game in SH consists only of the identities of past winners and thus the tacit collusive schemes involving history-dependent temporary exclusion of players. The expected continuation payoffs of one player after winning and losing are different. Losing is rewarded with higher continuation payoff than winning. The asymmetric continuation payoffs conditional on winning or losing thus play the role of implicit transfers and thus facilitate transfers from winners to losers, although explicit transfer payment is not allowed among players. Thereby the implementation of the collusive scheme involves no punishments (Nash reversion). As a result, there is no punishment on the equilibrium path and the bang-bang property does not arise. While in our paper the symmetric continuation payoffs prevent such cross-supplier transfers and there are punishments on the equilibrium path, which lead to the continuation payoffs taking on two extremal values (one corresponds to the collusion phase and another corresponds to the punishment phase). Consequently, the bang-bang property arises in our model while not in the work of SH.

5 The inefficiency of the optimal collusive schemes

The full characterization of bidding strategies in any SPPE allows us to provide a bound on the scope of collusion. A collusive scheme is efficient in a given auction if with probability 1 the winner of the object is the supplier with the lowest realized cost. The next lemma narrows down the class of optimal collusive bidding functions.

It follows directly from Proposition 3 that the bidding function takes on the familiar form:

$$\beta(c) = c + \frac{\int_c^{\overline{c}} Q(x, \beta) dx}{Q(c, \beta)}$$

It is easy to verify that any flat section in the bidding function must be preceded and followed by discontinuities. Suppose that a flat section somewhere in the bidding function starts at cost parameter $c^* > \underline{c}$ and ends at $c^{**} < \overline{c}$. Since the probability of a bidder with production cost c^* winning is discretely greater than the probability of a bidder with parameter $c^* + \varepsilon$ for any $\varepsilon > 0$, the denominator of the above equation increases discretely at c^*. Therefore $\beta(c^*)$ must increase discretely as well. The argument that a flat section must be followed by a discontinuity is precisely the same.

Lemma 2 *If $\overline{u} > \underline{u}$, then the optimal collusive SPPE entails bidding strategies that are weakly increasing step functions.*

The proof of Lemma 2 can be found in the appendix. The implications of this lemma are strong as it implies that an optimally colluding cartel will use a bidding function which has a finite range. To give an example, suppose $c_2 > c_1 > c_0$ and $b_1 > b_0$, all bidders with type in $[c_0, c_1]$ bid b_0, all bidders with type in $[c_1, c_2]$ bid b_1 etc. This rigid-bidding scheme is supported by the threat that if any other auction price is observed, the suppliers will revert to the worst SPPE, which delivers continuation payoff $\underline{u}$. Therefore, the optimal implicit collusion is characterized by a rigid-bidding scheme in which suppliers' bids are rigid over intervals of costs (i.e., the optimal bidding scheme is a weakly increasing step function), with punishments occurring on the equilibrium path.

The rigid-bidding scheme has benefits and costs. An important benefit of this scheme is that it satisfies the incentive constraint without reversion to Nash retaliation. However, an evident cost of the rigid-bidding scheme is that it sacrifices efficiency benefits: it may be that one supplier has a low cost while another supplier has a high cost, but under the rigid-bidding scheme, the later has the opportunity to serve the market. The content of Lemma 2 is that the benefits of the rigid-bidding scheme exceed the costs, provided that the best (optimal) SPPE value exceeds the worst SPPE value. The results of Lemma 2 also suggest to policymakers that evidence of a rigid bidding scheme in repeated procurement may indicate the possibility of implicit collusion and thus be considered seriously.

We proceed to give some explanation about why the bidding function in our paper is so different from Skrzypacz and Hopenhayn (2004). SH characterize the bidding function in second price auction with 2 players and thus the identity of the winner in any subgame of the game is common knowledge for both players. The expected continuation payoffs of one player are conditioned on him winning or losing the current auction. Moreover, losing is rewarded with higher expected continuation payoffs than winning (i.e., $c > 0$.[11] See detailed analysis in Sect. 2.1 of their work), and thus "the first stage of the dynamic game is equivalent to a one shot game in which the support of players' values is shifted to the left by c." The class of bidding functions is limited by the imposed information structure. There is no punishment (Nash reversion) and the implementation of the collusive scheme is through the asymmetric expected continuation payoffs conditioning on winning or losing the current auction, which act as a role of implicit transfers between winners and losers. More importantly, as they detailed in the end of Sect. 2.1, when there are $N > 2$ players and when the continuation payoff of a loser depends on the identity of the winner, the characterization of the bidding functions becomes much more complicated.

In our model, the SPPE must be immune to two kinds of current-period deviations and correspondingly there are two types of constraints, i.e., the admissibility constraint and the usual incentive compatibility constraint. By incorporating these two types of constraints, we characterize the bidding function in a first price procurement auction with $N > 2$ players.

The admissibility constraint can be assured by the credible punishments imposed by Nash reversion (Proposition 2). To satisfy the incentive compatibility constraint, the scheme must be constructed so that a higher-cost supplier has no incentive to misrepresent its cost as lower, by selecting a lower bid and thereby securing for itself the probability of winning. The bids of lower-cost suppliers may be distorted downward. This is a potentially effective means of eliciting truthful cost information, since higher-

[11] Here $c = \delta(w^2 - w^1)$, where w^2 and w^1 denote the expected continuation payoffs of one player conditional on him losing and winning the current auction respectively, δ denotes the common discount factor.

cost suppliers find lower bids less appealing. Furthermore, following the selection of lower bids, the collusive scheme may sometimes call for a future equilibrium-path punishment. A collusive bidding function is one in which a supplier maximizes its payoff taking into account that any deviation increases current expected payoff but lowers future expected payoff by making a punishment more likely. Once the defection has been detected, the punishment should be severe enough to deter it. In an SPPE, the informational costs may be manifested in pricing distortions and future punishments (Nash reversion) on the equilibrium path that are required to dissuade suppliers from deviation. Suppliers may neutralize the informational costs of collusion altogether, by adopting a rigid-bidding scheme, in which bids fluctuate less in response to cost change that they would otherwise in order not to disturb existing collusive discipline.

More generally, collusive bidding schemes may be strictly increasing over some intervals of costs and rigid over other regions, with punishments following some auction price realizations. This implies that colluding suppliers adjust bid occasionally in response to change in cost conditions. Rigid bidding captures the extent to which information cost limits colluding suppliers' ability to respond to their respective cost positions. Bid rigidity is appealing to collusive suppliers, because a rigid bidding collusive scheme prevents deviation and reduces the risk of reversion to Nash competition. As a result, it is not possible for suppliers to achieve full efficiency benefits, if they use rigid bidding schemes.

Proposition 5 *The optimal SPPE are not efficient in our collusive scheme (with no transfer or communication, in which suppliers condition only on the past auction prices).*

There are some observations we want to make about this proposition.[12]

First, it shows that, no matter whether the cartel is patient or impatient the cartel cannot extract all surplus. That corresponds to our original motivation for the information structure: the benefits from explicit communication before each auction as well as from creating more complicated schemes (that keep track of all past bids and the identities of the winners) are very limited for the cartels in the repeated procurement auction market especially when the cartel is all-inclusive. As communication is illegal in practice and complicated schemes can be difficult to monitor and coordinate on, suppliers have a tendency to use simple collusive schemes, like the ones presented in our model setting.

Second, the important insight from Proposition 5 is that our information setup is more restrictive than in any of the existing research, as we do not allow communication and not all bids are publicly observed. The suppliers can publicly observe only the past winning bids and there is no pre-stage communication, and thus efficient collusion is not feasible. Moreover, from the comparison of our paper with that of SH, the important recommendation for the auction designer is that, in order to counteract bidder collusion, he can limit the information released to set up barrier for bidders to achieve desired coordination (by releasing the information of past winning bids rather than the identities of the winners). Our paper provides a formal theory in which buyer's

[12] The proof of this proposition is immediate given the above characterization of the rigid bidding scheme, so we omit it here.

information revelation policy affect the ability of sellers to collude in an infinitely repeated procurement auction.

6 Discussion of modeling assumptions

The distinguishing feature of our theoretical analysis is that we assume suppliers can observe only the auction price history rather than the levels of the bids and the identities of the winners after each auction. This approach is in contrast with that of the existing literature, which attempts to characterize the collusive equilibrium by adopting the information history of the past bids and (or) the identities of the winners. The assumption in our modelling choice is based on the following considerations: first, as analyzed by some literature, when bidder can observe its rival's bids (Rachmilevitch 2013, 2014) or the identities of past winners (Skrzypacz and Hopenhayn 2004), the first-best collusive outcome can be achieved (or at least approximated) even without using communication, transfers, or public randomization. Thus it is an open question whether the first-best can be achieved in repeated auctions when only the past auction prices are publicly known. With information so limited in our model setting, we find that efficient collusion is impossible . Combined with the existing efficient collusion results in which bids submitted by all bidders and (or) the identities of winners in each previous period are common knowledge, the present analysis would make it possible to discuss the effectiveness of information release policy during auction process for the buyer when collusion is a concern. For instance, in public procurement frequently the law forces the public administration to publicly reveal both the winner's identity and the winning bid. Given the results of this paper, the present policy maybe suboptimal when tacit collusion is a real threat.[13]

Second, to provide some perspective on this paper's assumption, other research has examined issues involving information revelation and focus on how revealing bids and winners' identities can reveal private information that affects future competition, but under the presumption that collusion is not an issue. Thomas (2010) investigate how procurement costs are affected by information revelation policies available to buyers sequentially offering procurement contracts. Expected prices are lowest when sellers learn nothing until all contracts are allocated, are higher when they learn all sellers price offers as contracts are allocated, and typically are even higher when they learn only the winner's identity, or the winner's identity and price offer. The results suggest that buyers engaged in repeated procurement may pay less by not revealing the information of the winner's identity, and under some conditions the buyer may prefer to reveal only the winner's price offer.[14]

[13] The author thanks to one anonymous reviewer to point out this.

[14] In addition, there maybe good economic reasons for greater information disclosure in certain applications. A common theme in the auction literature assuming affiliated values is that revenue will be enhanced by designing auction that reveals as much information as possible (McAfee and McMillan 1987; Milgrom 2004). "Transparency in bidding" has been touted by the federal government to decrease the scope for corruption by the auctioneer and increase efficiency in environments with externalities. However, these arguments for revealing information presumes that collusion is not an issue.

Finally, as shown by Marshall and Marx (2009), information disclosure about the identities of the active bidders and the identity of the current high bidder, increase the susceptibility to collusion. Often a bidding cartel in repeated auctions will organize itself so that the continuation payoff acting as the role of implicit transfer payment depends on who wins the object in the current period auction (Aoyagi 2003, 2007; Skrzypacz and Hopenhayn 2004; Blume and Heidhues 2008). This type of collusive organization is facilitated when auctioneers release detailed information about the identities of the winners. By withholding information about the identities of the winners, the auction designer potentially can create opportunities for the winner to circumvent payments to its co-conspirators. Thus, making the winner's identity anonymous can make collusion more difficult and less palatable to a typical bidder collusion. The above considerations motivate us to investigate if and when there exists a fully efficient collusion scheme when bidders can only observe past auction prices, but not all bids and the identities of winners.

Throughout this paper, we focus upon SPPE in which each supplier's strategy conditions only upon the publicly observed history of auction prices. If suppliers are allowed to use private strategies in which they condition their behavior on private histories, then the whole APS approach does not work. This is because APS restrict attention to SSE's in which each player makes his actions depend only upon publicly observed history of signal realizations (not on his own previous actions). We show that if suppliers condition their behavior only on public history, then their average payoffs are bounded away from collusive efficiency. However, it remains an open question whether all SPNE payoffs are bounded away from collusive efficiency when private strategies are permitted.

7 Conclusions and policy implications

This paper considers implicit collusion in an infinitely repeated procurement auction when no communication is available and the only public information available to suppliers is the winning bid in all previous auctions, without knowing anything with regard to the identities of the winners or the bid (action) choices of rivals. The cost types are distributed i.i.d. across time and across suppliers. In this model setting, when a supplier observes a low price, it is unclear whether the low-price outcome arose as a consequence of a favorable cost shock or a secret deviation by a rival. In this imperfect monitoring context, the optimal punishment strategy will not involve deterministic Nash reversion. Thus, we allow for a public randomization device and suppliers pursue collusion through tacit coordination on a public signal.

By adopting the technology of dynamic programming, we succeed in characterizing the optimal equilibrium for the cartel of bidders. We show that the optimal implicit collusion can be implemented by the bang-bang property: suppliers just need to start playing the collusive bidding schedule and enjoy the maximal continuation payoffs, and depending on the public signals (past winning bids), with a certain probability they stick to that collusive strategy or move to Nash reversion. The optimal implicit collusion is characterized by a rigid-bidding scheme with punishments occurring on the equilibrium path. The information cost is formalized in terms of pricing distortions and future punishments (Nash reversion) that are required to dissuade suppliers from deviation. As a result, it is not possible for suppliers to achieve full efficiency benefits.

The results of this paper have important policy implications. Given the existing results that the optimal collusive scheme is asymptotically efficient when players can observe past bids and /or the identities of winners, one important recommendation for the public administration is that, in order to counteract bidder collusion, he can limit the information released to set up barrier for bidders to achieve desired coordination (by releasing the information of past winning bids rather than all bids and the identities of the winners). The results of this paper also suggest to policymakers that evidence of a rigid-bidding scheme in repeated procurement may indicate the possibility of implicit collusion and thus be considered seriously.

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Appendix A: Proof of Proposition 1

For each $W \subseteq \mathbb{R}$, define $B(W) = \{E\{\Pi(c;\beta,u)\}|(\beta,u)$ *is admissible with respect to* $W\}$. $B(W) \subseteq R$ represent the collusive suppliers' total expected payoffs in pure strategy equilibrium in which each first-period stage game gain is followed by some symmetric expected future payoff drawn from W. At the end of the single period of the truncated game, suppliers receive their conventional one-period profits, plus some element of W (the same for each supplier), depending on the past winning bids that arises; the expected value of this sum, discounted to the beginning of the period, will be the one element of $B(W)$.

For any SPPE s, the value to supplier i of the successor SPPE specified following a given first-period winning price b_w must be independent of the bid that i submitted in period 1. This is because no one else has observed that bid, and hence i faces the same future environment regardless of his initial bid. Thus the value of s for each supplier can be factorized into two terms: the gain from first-period game, and the discounted expected value of a reward function $E_{b_w}(u(b_w)|\beta) = v(s|_{h(1)=(b_w^1,s^1)})$. Since $s|_{h(1)=(b_w^1,s^1)}$ is an SPPE, this reward function is drawn from V. This together with the constraints that the suppliers are willing to adopt strategy $s^1 = (\beta_1, \cdots, \beta_n)$ in period 1, means precisely that (β, u) is admissible with respect to V. Thus the requirements for s to be an SPPE are exactly those needed for $v(s)$ to be in $B(V)$, and therefore $V = B(V)$.

We have shown that V can be recovered from the function B. In addition, V is a fixed point of B and V is also the largest bounded fixed point of B. Moreover, since V is self-generating, any element w of V is the payoff of $\hat{s}(w)$, the SPPE constructed in Proposition 1.[15] This supergame equilibrium is described entirely by two functions β and u, and the number w.

[15] For each $w \in B(W)$, $\hat{s}(w)$ is an SPPE with $v_i(\hat{s}(w)) = w$ for all $i = 1, \ldots, n$.

Appendix B: Proof of Proposition 2

Gains to a player with cost parameter c from the collusive mechanism (β, u) are:

$$\int_{c}^{\overline{c}} Q(x; \beta)dx + \delta \int_{\underline{c}}^{\overline{c}} u(\beta(c))(1 - F(c))^{n-1} dF(c)$$

which can be used to calculate ex-ante rents:

$$\int_{\underline{c}}^{\overline{c}} H(c)Q(c; \beta)dF(c) + \delta \int_{\underline{c}}^{\overline{c}} u(\beta(c))(1 - F(c))^{n-1} dF(c)$$

Consider infinite, or unrelenting punishments. Let (β_1, U_1) be an incentive compatible collusive mechanism which minimizes the above equation:

$$\min_{\beta_1, U_1} : \int_{\underline{c}}^{\overline{c}} H(c)Q(c; \beta)dF(c) + \delta \int_{\underline{c}}^{\overline{c}} u(\beta(c))(1 - F(c))^{n-1} dF(c)$$

Since payoffs are constrained to be in V, the perfect equilibrium set, and we have proved in Proposition 2 that $V = B(V)$, there exists another collusive couple, (β_2, u_2) giving the same payoffs as $\int_{\underline{c}}^{\overline{c}} \left[E_b(u_1(b)|\beta_1(c)) \right] dF(c)$. Working iteratively in this manner one obtains that the payoff to this punishment is

$$\sum_{i=1}^{\infty} \delta^{i-1} \int_{\underline{c}}^{\overline{c}} H(v)Q(c; \beta_i)dF(v)$$

Incentive compatibility requires $Q(\cdot; \beta)$ to be non decreasing. The above expression is minimized when we rule out constant bidding regions, i.e., when the bidding rules are constrained to induce efficiency so that $Q(c; \beta) = (1 - F(c))^{n-1}$. The single stage Nash strategy accomplishes this task while satisfying incentive compatibility. Note that when bidding rules are constrained to induce efficiency payoffs are necessarily equal to the Nash equilibrium stage game payoffs.

The next step is to show that participants always bid with probability one. Suppose a mechanism which randomly selects agents not to bid. The identity of the winning bidder is unobservable, so future gains can only be a function of the winning bid. Therefore for a certain agent with a given type to be indifferent between bidding and not bidding, all agents with lower cost must strictly prefer bidding and all agents with higher cost must strictly prefer not bidding. Consider a mechanism that in the first round outlaws bidding above $r > 0$, and let μ_0 be the future expected gains in nobody bids. Admissibility of the mechanism implies:

$$\delta\mu_0(1 - F(r))^{n-1} \geq r(1 - F(r))^{n-1} + \delta\underline{u}(1 - F(r))^{n-1}$$

And since $(1 - F(r))^{n-1} > F(x)(1 - F(x))^{n-1}$ for all x in $[0, r]$ we have that:

$$r(1-F(r))^{n-1}+\delta\underline{u}(1 - F(r))^{n-1} > \int_0^r F(x)(1 - F(x))^{n-1}dx+\delta\underline{u}(1 - F(r))^{n-1}$$

This previous expression represents using the Nash bidding strategy in the interval $[0, r]$. Therefore we have contradicted the supposition that outlawing bidding in a certain region dominated the Nash bidding strategy.

Appendix C: Proof of Proposition 3

Let $\{Q(\cdot; \beta), u(\cdot)\}$ be an optimal collusive mechanism, generating $\overline{u}$, for which the bang-bang property does not hold. Therefore, there exists an interval, $[c_1, c_2]$, such that $u(\beta(c)) < \overline{u}$ for all $c \in [c_1, c_2]$. In this case we show that there exists an alternative incentive compatible, admissible mechanism which generates higher rents for all types. This mechanism consists of raising $u(\beta(c))$ by some small amount Δu whenever $c \in [c_1, c_2]$. Denote this new continuation function by u^*. All flating bidding regions above c_2 are cut into two regions otherwise, the probability of winning is held constant for all other types. From equation (1), if we increase continuation payoffs in $[c_1, c_2]$ and wish to preserve type conditional probabilities of winning an auction $(Q(\cdot, \beta))$, it is necessary to change bids in order to retain incentive compatibility. Bids of types lower than c_1 are unaffected by such a change. Studying inequality (2) immediately shows that types in $[c_1, c_2]$ will satisfy admissibility. The variation in the rents to $c < c_2$ are given by:

$$\Delta U(c_2) = [(1 - F(c_1))^{n-1} - (1 - F(c_2))^{n-1}]\Delta u$$

which is strictly positive for all $\Delta u > 0$.

Consider the interval $[\underline{c}_i, \overline{c}_i]$ where $\underline{c}_i \geq c_2$ and for which the bidding function is flat. Find recursively a $\hat{c}_i$ where the following holds:

$$(\hat{c}_i - \underline{c}_i)[Q(\underline{c}_i, \overline{c}_i) - Q(\underline{c}_i, \hat{c}_i)] = \Delta U(\underline{c}_i)$$

Create a new incentive compatible bidding function using the new continuation function u^*, β^* different from β only in that $Q(\cdot; \beta^*)$ exhibits discontinuities at all $\hat{c}_i$ but constant on all the $[\underline{c}_i, \hat{c}_i]$ and $[\hat{c}_i, \overline{c}_i]$.

We now have $\Delta U(c) = (c - \hat{c}_i)[Q(\hat{c}_i, \overline{c}_i - Q(\underline{c}_i, \overline{c}_i)] \geq 0$ for all $c \in [\hat{c}_i, \overline{c}_i]$. Additionally for all $c \in [\overline{c}_i, \hat{c}_i]$, we have $\Delta U(c) = [c - \underline{c}_i][Q(\underline{c}_i, \hat{c}_i) - Q(\underline{c}_i, \overline{c}_i)] + \Delta U(\underline{c}_i) \geq 0$. Hence the variation in rents for all types is non negative by construction, which implies that this new constructed mechanism can generate higher rents for all types. It remains to verify that the new mechanism is admissible for all $c > c_2$ if the original mechanism was assumed to be admissible.

First consider checking admissibility in $[\hat{c}_i, \overline{c}_i]$. With the original mechanism we have

$$\beta(\overline{c}_i) - \overline{c}_i = (\overline{c}_i - \hat{c}_i) + \frac{U(\hat{c}_i; \beta, u)}{Q(\underline{c}_i, \overline{c}_i)}$$

Under the new mechanism we have that:

$$\beta^*(\overline{c}_i) - \overline{c}_i = (\overline{c}_i - \hat{c}_i) + \frac{U(\hat{c}_i; \beta^*, u^*)}{Q(\hat{c}_i, \overline{c}_i)}$$

Since $U(\hat{c}_i; \beta, u) = U(\hat{c}_i; \beta^*, u^*)$, and $Q(\underline{c}_i, \overline{c}_i) < Q(\hat{c}_i, \overline{c}_i)$ we have that:

$$\beta^*(\overline{c}_i) - \overline{c}_i < \beta(\overline{c}_i) - \overline{c}_i$$

Since the original mechanism was assumed to be admissible, i.e.,

$$\delta[\overline{u} - \underline{u}][1 - F(\overline{c}_i)]^{n-1} \geq [\beta(\overline{c}_i) - \overline{c}_i][(1 - F(\overline{c}_i))^{n-1} - Q(\overline{c}_i; \beta, u)]$$

Also we know that $Q(\overline{c}_i; \beta^*) > Q(\overline{c}_i; \beta, u)$ and $\beta^*(\overline{c}_i) - \overline{c}_i < \beta(\overline{c}_i) - \overline{c}_i$, and thus

$$\begin{aligned}&[\beta(\overline{c}_i) - \overline{c}_i][(1 - F(\overline{c}_i))^{n-1} - Q(\overline{c}_i; \beta, u)] \\ &\quad > [\beta^*(\overline{c}_i) - \overline{c}_i][(1 - F(\overline{c}_i))^{n-1} - Q(\overline{c}_i; \beta^*, u^*)]\end{aligned}$$

So the admissibility constraint of the new mechanism is sure to be satisfied. Finally, check admissibility in $[\underline{c}_i, \hat{c}_i]$. Note that as Δu approaches zero, $\hat{c}_i$ must approach $\underline{c}_i$. It follows that for a Δu sufficiently close to zero, the admissibility constraint will be respected for $\hat{c}_i$.

Appendix D: Proof of Proposition 4

Proof To find the most collusive SPPE, I have recured to the recursive dynamic programming technique developed by Abreu et al. (1986, 1990). Any SPPE of the repeated game can be decomposed into a pair of first period price schedules $\beta(\cdot)$ and a continuation value function $u(\cdot)$. The continuation payoff $u(\cdot)$ depends on the public history of the first period, i.e. winning bid. In order for a decomposition pair $(\beta(\cdot), u(\cdot))$ to qualify as a SPPE, two necessary and sufficient conditions have to hold. First, an individual supplier should have no incentive to deviate from the current period price schedule given all other firms choose $\beta(\cdot)$. And second, all continuation values are drawn from the set of SPPE payoff values V. These two conditions are given by Eq.(4) and (5). Since the perpetual repetition of the Nash equilibrium of the stage game is always a SPPE, the set V is non-empty and its lowest possible value is $\underline{u}$. Convexify the set V by assuming that suppliers have access to a public randomization device at the end of each period. Then, the bang-bang property of optimal continuation values in an equilibrium implies that the value of any SPPE (including the most collusive) can be sustained by a SPPE which after every public history of the first period only uses the two extreme continuation values $\overline{u}$ and $\underline{u}$. In other words, the choice of optimal continuation values can be reduced to assigning a probability $\lambda \in (0, 1)$ to every possible history after period 1. With probability λ suppliers revert to the stage game Nash equilibrium forever. With the remaining probability suppliers continue to play the current period strategy and receive a continuation value of $\overline{u}$. □

Appendix E: Proof of Lemma 1

Suppose β to be constant on $[c_i, c_j]$. First remark that $Q(c, \beta)$ is discontinuous at c_i, and that in particular $Q(c_i, \beta) < [1 - F(c_i)]^{n-1}$. Suppose that the condition in the statement of the lemma is violated and that β is continous at c_i. Then since $u(\beta(c_j - \varepsilon)) \geq \underline{u}$ for all $\varepsilon > 0$, there exists a small ε such that bidders of type c_i prefer to bid $\beta(c_j - \varepsilon)$, violating the incentive compatibility constraints. Now suppose that β is discontinuous at c_i. From equation (1), one can verify that β is also discontinuous at c_j. Therefore there exists a $b \notin \beta$ which is arbitrarily close to $\beta([c_i, c_j])$ yet which discretely increases the probability of winning. A (credible) punishment must be available to dissuade such a deviation. This punishment can be made as severe as possible without harming cartel gains, in so far as it is only used off the equilibrium path, i.e., $b_w \notin \beta$.

To prove the only if part, notice that admissibility implies that all types respect the above inequality, and in particular c_i respect the above inequality.

To prove the if part, define $\beta^- = \lim_{c \to c_i^-} \beta(c)$ and $\beta^+ = \lim_{c \to c_i^+} \beta(c)$. Since β is discontinuous at c_i, we have $\beta^+ > \beta^-$. Any bid in $(\beta^-, \beta^+]$ will win the auction with probability $(1 - F(c_i))^{n-1}$. The inequality in the statement of the lemma implies that c_i prefers following his equilibrium strategy rather than deviating. Fix $c > c_i$. By monotonicity of preferences, c also prefers bidding β^- than bidding in $(\beta^-, \beta^+]$. A similar argument shows that all $c < c_i$ prefer bidding β^- than bidding in $(\beta^-, \beta^+]$. □

Appendix F: Proof of Lemma 2

Suppose an optimal collusive bidding function which is continuously increasing for some interval $[c_0, c_1] \subset [\underline{c}, \overline{c}]$. To show a contradiction, modify the bidding rule so that it awards the object to types in $[c_0, c_1]$ with equal probability and respects incentive compatibility. There are two cases which will be considered separately: (1) type c_0 weakly prefers the old scheme. (2) Type c_0 strictly prefers the new scheme. In each treatment incentive compatibility is verified while using the original value $\overline{u}$, then it is shown that the new bidding function generates a higher $\overline{u}$.

Case 1 Suppose that after invoking b_0', type c_0 is just as well off as under the old bidding function. Therefore bidding levels b_1 and b_2 can be maintained and all incentive compatibility constraints are satisfied. Now apply lemma 2 to see that the expected value to collusion has been increased. Suppose that type c_0 strictly prefers the old scheme. Then if b_1 and b_2 are held constant, the type indifferent between bidding b_0' and b_1 will be located to the right of c_0. Now raise b_1 and b_2 such that the indifferent type between bidding b_0' and b_1 remains c_2. Notice that this change in the bidding function has not changed the probabilities that any type lower than c_1 wins the auction and all incentive compatibility and admissibility constraints are respected. Now apply Lemma 2.

Case 2 Suppose that type c_0 strictly prefers the new scheme. Then the type indifferent between bidding b_0' and b_1 moves to the left of c_1. Call this new indifferent type c_1'. Incite c_1' to move back to c_1 by decreasing the continuation payoff if the winning bid is b_0'. Such a change produces a more profitable bidding function be Lemma 2, but may not produce higher rents. Invoke Proposition 4 to be assured of the existence of a collusive mechanism with a bidding function which is never continuously increasing and having the bang-bang property. This bang-bang mechanism uses a bidding function which is even more profitable. Furthermore, such a bang-bang mechanism generates $\overline{u} = \int_{\underline{c}}^{\overline{c}} H(c)Q(c;\beta^*)dF(c)$, where β^* is the newest bidding function. So since the bang-bang mechanism produces rents which are generated by a bidding function which dominates the original scheme, the proposition is proved. □

Appendix G: Proof of Lemma 3

The proof of Lemma 2 makes reference to the following lemma which we state and prove before the proof of Lemma 2.

Lemma 3 *For any $\underline{c} < c_0 < c_1 < \overline{c}$, it is the case that:*

$$\frac{\int_{c_0}^{c_1} H(c)dF(c)}{F(c_1) - F(c_0)} \geq \frac{\int_{c_0}^{c_1} H(c)dF(c)^n}{(1 - F(c_0))^n - (1 - F(c_1))^n}$$

Proof of Lemma 3. After an integration by parts the above inequality can be written as:

$$\int_{c_0}^{c_1} \frac{F(c_1) - F(c)}{F(c_1) - F(c_0)} H'(c)dc \geq \int_{c_0}^{c_1} \frac{(1 - F(c))^n - (1 - F(c_1))^n}{(1 - F(c_0))^n - (1 - F(c_1))^n} H'(c)dc$$

According to Assumption (A2), we know that $H'(c) > 0$, it suffices to show that:

$$\frac{(1 - F(c)) - (1 - F(c_1))}{(1 - -F(c_0)) - (1 - F(c_1))} \geq \frac{(1 - F(c))^n - (1 - F(c_1))^n}{(1 - F(c_0))^n - (1 - F(c_1))^n}, \ \forall c \in (c_0, c_1)$$

Since it is the case that$F(c_0) < F(c) < F(c_1)$, $1 - F(c)$ can be written as:

$$1 - F(c) = \lambda(1 - F(c_0)) + (1 - \lambda)(1 - F(c_1)) \ \ for\ some\ \lambda \in (0, 1)$$

Therefore after substitution and rearrangement it is sufficient to show that:

$$\Big[\lambda(1 - F(c_0)) + (1 - \lambda)(1 - F(c_1))\Big]^n \leq \lambda(1 - F(c_0))^n + (1 - \lambda)(1 - F(c_1))^n$$

which is always satisfied since $(1 - F(c))^n$ is a convex function of $(1 - F(c))$. $\square$

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