ELSEVIER

Cryogenics 39 (1999) 235–239

Cryogenics

Modelling and cooling behaviour of Peltier cascades

T. Metzger, R.P. Huebener *

Physikalisches Institut, Lehrstuhl Experimentalphysik II, Universität Tübingen, Morgenstelle 14, D-72076 Tübingen, Germany

Received 4 December 1998; received in revised form 19 January 1999; accepted 21 January 1999

Abstract

For evaluating the potential of Peltier cooling for the operation of superconducting electronics based on the cuprate superconductors, we discuss the model describing the cooling behaviour of Peltier cascades. The general conclusions derived from this model are presented, and good agreement with an experimental study is found. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Peltier cooling

1. Introduction

The increase of the superconducting transition temperature in some of the cuprate superconductors to values above 130 K [1,2] has generated new interest in the exploration of Peltier cooling for the operation of superconducting electronics. This requires the search for new Peltier materials with a sufficiently large figure of merit Z down to temperatures considerably lower than the temperature range, for which most of the Peltier systems at present commercially available, are optimized. For illustrating the situation, recently we have performed a Peltier cooling experiment using five stages of commercially available Peltier modules [3]. Fixing the temperature on the warm end of this system at 282 K using cooling water, the lowest temperature reached at the cold end of the cascade was 149 K.

In this paper we investigate the cooling behaviour of Peltier cascades using a theoretical model developed for describing a cascade. These theoretical discussions are supplemented by experiments using cascades built from single-stage Peltier modules commercially available. From this work we arrive at some general conclusions regarding the optimum cascade operation. A detailed discussion of the Peltier materials' properties is not given in this paper. A recent discussion of the specific materials aspects can be found elsewhere [4].

2. Modelling of a cascade

In the simplest case it is assumed that the Peltier legs of all N stages of a cascade have the same geometry and that only their number n_i ($i = 1,\dots, N$) varies in the different stages. Furthermore, the Seebeck coefficient S, the heat conductivity κ, and the electric resistivity ρ are assumed to be the same for the n-type and the p-type materials and independent of temperature. If the temperature of the warm side of the cascade is fixed to T_0 and if there is no heat load on the cold side ($Q_N = 0$), then the electric currents J_i applied to each stage are the only free parameters, determining all temperatures T_i and the heat flow Q_{i-1} from stage i to stage $i - 1$. These variables are shown schematically in Fig. 1.

With A and l denoting the cross section and the length

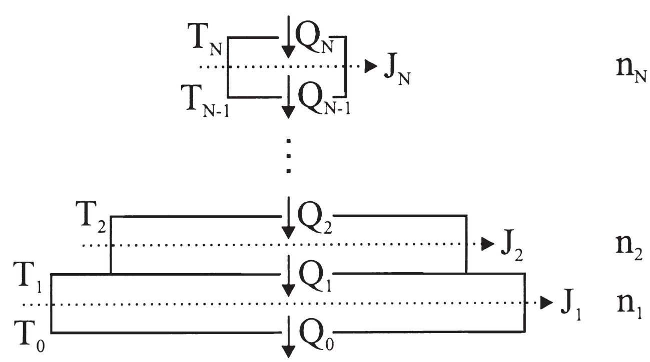

Fig. 1. Schematics of a Peltier cascade indicating the relevant variables.

* Corresponding author. Tel.: + 0049-7071-29-76315; fax: + 0049-7071-29-5406; e-mail: prof.huebener@uni-tuebingen.de

PII: S0011-2275(99)00019-3

of a Peltier leg, respectively, we introduce the thermal conductance $K = \kappa A/l$ and the electric resistance $R = \rho l/A$. The cascade can be modelled with $2N$ equations [5,6]:

$$Q_i = n_i\left(SJ_iT_i - \frac{1}{2}J_i^2R - K(T_{i-1} - T_i)\right) \tag{1a}$$

$$Q_{i-1} = n_i\left(SJ_iT_{i-1} + \frac{1}{2}J_i^2R - K(T_{i-1} - T_i)\right). \tag{1b}$$

These equations describe the heat flow into and out of the ith stage. The first term on the right side is due to the Peltier effect. The other two can be calculated from the differential equation describing the local temperature distribution within the Peltier leg arising from Joule heating and thermal conduction alone [7]. We emphasize that these two terms must not be regarded separately but as one result of the mathematical problem. For instance, one must not be misled into concluding that half of the Joule heat J_i^2R is transported to the cold end and half to the warm end of the stage. (In reality, the total heat J_i^2R is transported to the cold end.) From Eq. (1a) and (1b) we can calculate the temperature of the cold side T_N as a function of the electric currents J_i, and we can find the optimum current settings to minimize T_N.

So far, this model is not sufficiently realistic since the properties S, κ and ρ of the Peltier material do vary significantly with temperature. Also, we must allow for a different geometry (A_i and l_i) of the Peltier legs at each stage. To meet both requirements, we define for each stage

$$S_i = S(T_i),\ S_{i-1} = S(T_{i-1}), \tag{2}$$

$$K_i = \kappa\left(\frac{T_i + T_{i-1}}{2}\right)\frac{A_i}{l_i} \tag{3}$$

and

$$R_i = \rho\left(\frac{T_i + T_{i-1}}{2}\right)\frac{l_i}{A_i}. \tag{4}$$

The Seebeck coefficient, describing the Peltier heat by means of the Thomson relation, is taken at the temperature of the leg boundaries, whereas for κ and ρ we take the values at the mean temperature of the leg. In an optimally operated cascade the temperature difference over each stage is relatively small due to the heat load from the upper stages and the reduced operating temperatures. Typically it ranges between 50 K for the bottom stage and 10 K for the top stage (we note that a single stage can provide a temperature drop of up to 70 K from room temperature). Therefore, this treatment of the temperature dependence provides a reasonable approximation. The system of equations can be solved using an iteration loop for successively calculating the temperatures T_i with higher accuracy.

This model has been extended further by allowing a finite heat load to the cold side of the cascade. We also discuss the case where the transport properties of the n-type and the p-type materials are different.

3. Experimental confirmation

With this model we simulated a variety of cascades, built from single-stage modules and placed in a Peltier precooled copper box as schematically shown in Fig. 2. The cooling behaviour of the copper box with a heat load inside has been measured and implemented in the theoretical model. The temperature dependence of the material properties was given by the manufacturer [8]. The most promising three-, four-, and five-stage configurations were actually built and tested in the copper box. It could be shown that the maximum temperature drop of a cascade is reached with the calculated optimum current settings, that its value is as expected, and that it displays a flat maximum, i.e., small changes in the current settings affect the temperature of the cold side only very little.

The configuration which produced the best results is shown in Fig. 2. The heat sink consisted of two copper plates cooled by water. The copper box which contained the actual cascade was precooled by one Peltier stage at the top and at the bottom[1]. Inside the box a four-stage cascade was placed[2]. Its two bottom stages and its two top stages were operated electrically in series, respect-

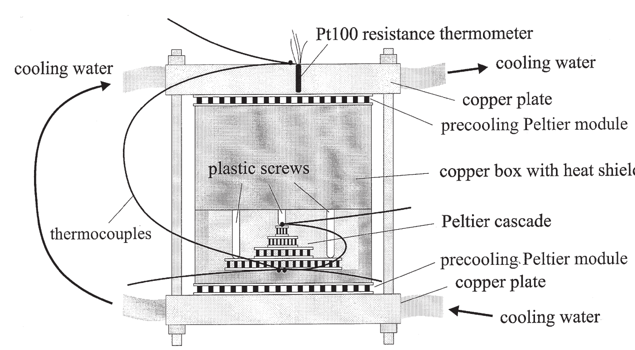

Fig. 2. Experimental set-up for investigating the cascade behaviour.

[1] The module at the top was model no. CP2-127-10L, operated at a current of 8.9A, the module at the bottom was model no. CP2-127-06L, operated at 13.8A, from Melcor Corp., Trenton, NJ, USA.

[2] The modules used were model nos. CP1.4-127-10L, CP1.4-31-10L, OT1.2-32-F0, and OT1.2-7-F1 from Melcor Corp., Trenton, NJ, USA.

ively, and the current leads were thermally coupled to the lower stages of the cascade. This is to reduce the heat flow to the cascade from outside the box. The calculated optimum current settings were $J_1 = 2.8$ A, $J_2 = 2.4$ A, $J_3 = 0.6$ A and $J_4 = 0.7$ A. All thermal contacts were made with thermally conducting paste and by applying mechanical pressure (using plastic screws inside the box). The temperature of the cooling water T_{water} was recorded by a Pt100 resistance thermometer. Two thermocouples (E type, ϕ 76 μm) measured the temperatures at the bottom and the top of the four-stage cascade. The whole set-up was placed in a vacuum chamber.

The theoretical model predicted a total temperature drop of 128 K from $T_{\text{water}} = 284$ K for the current settings mentioned above. In the experiment the temperature of the cooling water was 286 K, and we could reach a cold side temperature of 158 K (i.e., a temperature drop of 128 K), indicating good confirmation of our model. The current settings of the inner four stages have been varied according to Table 1 in order to verify the position of the maximum temperature drop. From Table 1 we see that the maximum temperature drop is relatively insensitive to the current values, as expected from our model calculations.

For demonstrating the time scale, in a second experiment all Peltier currents were switched on at the same time. The temporal behaviour of the temperatures is shown in Fig. 3. The power dissipation for the maximum temperature drop reached a total of 350 W, with 24 W being produced inside the copper box.

Finally, we point out that the optimum Peltier cascade must not be constructed from individual modules, thermally coupled to each other. Instead, a strong improvement of the thermal coupling between two subsequent stages can be achieved by mounting the Peltier legs from both sides directly on a *single* ceramic plate separating the two stages (in this way eliminating two interfaces). In addition, in order to reduce the heat load on each stage from the attached current carrying wires, all stages must be connected in series electrically. This can be achieved by a proper geometric arrangement of the Peltier legs in each stage.

Table 1
Total temperature drop of the four-stage cascade inside the copper box and the precooling stage outside the copper box for various current settings

	$J_3 = J_4$		
	0.5 A	0.65 A	0.9 A
$J_1 = J_2$			
2.2 A	126.0 K	127.8 K	126.5 K
2.6 A	128.6 K	**128.6 K**	127.4 K
3.2 A	126.7 K	128.5 K	127.4 K

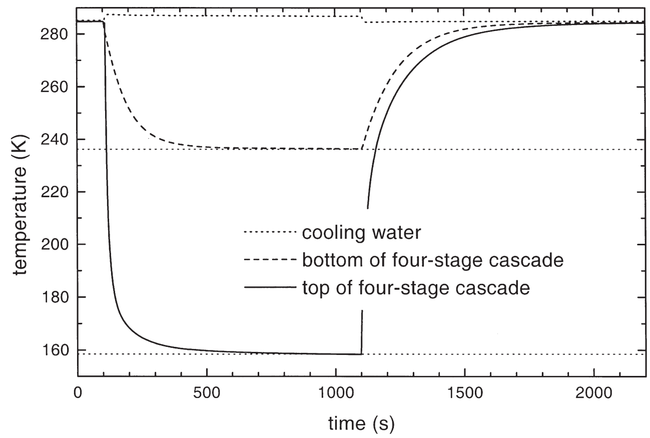

Fig. 3. Temperature at the three points as indicated for the Peltier system shown in Fig. 2 plotted vs time. All Peltier currents have been switched on (100 s) and switched off (1100 s) at the same time. Further details are given in the text.

4. Cascade behaviour

As expected from the linear equations describing the cascade behaviour, the figure of merit $Z = S^2/\kappa\rho$, used for evaluating single-stage Peltier devices, is still valid for cascades in the following sense: for any given cascade geometry, Z is the only material property which affects the maximum possible temperature drop $\Delta T_{\max}$, which always increases with increasing Z. Varying S, κ and ρ in such a way that Z remains constant only results in different optimum current settings and heat flow from stage to stage.

For a given material, the only geometric parameters determining $\Delta T_{\max}$ are the *stage sizes*

$$B_i \equiv \left(n_i \frac{A_i}{l_i}\right)$$

of all stages. Changes of the inner structure of these parameters only affect the optimum current settings.

We also considered cascades of *uniform* stage size ratio B_i/B_{i+1}. Concerning their cooling properties, such configurations appear favorable, if the size ratio of the bottom to the top stage is not too small. In order to exclude effects from the temperature dependence of the material properties, we chose temperature independent parameters $S = 169$ μV/K, $\kappa = 23.6$ mW/cm·K and $\rho = 0.629$ mΩ·cm, which are the values of the melcor material at 200 K [8]. As the warm side temperature of the cascade we took 284 K.

Fig. 4 shows the cold side temperatures reached theoretically for different stage numbers (solid lines) plotted versus the stage ratio B_i/B_{i+1}. We have also displayed the asymptotes (dotted lines) for each number of stages which represent the limit when B_i/B_{i+1} goes to infinity, i.e., when each stage has virtually no heat load from the stages above. We see that a cascade with more stages

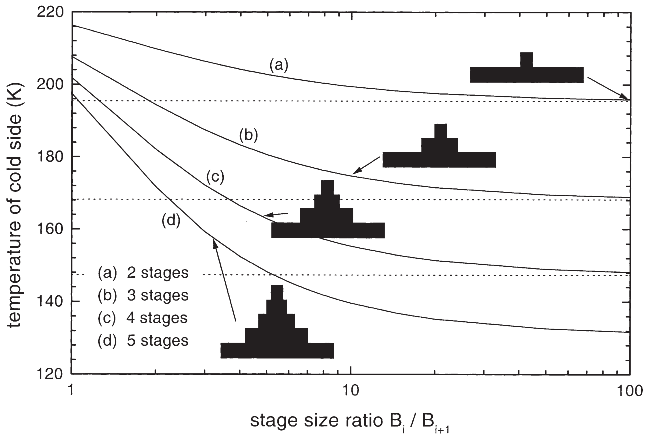

Fig. 4. Calculated cold side temperature vs the stage size ratio B_i/B_{i+1} for different stage numbers as indicated and for the following parameter values: $S = 169$ μV/K, $\kappa = 23.6$ mW/cm·K, $\rho = 0.629$ mΩ·cm, and $T_0 = 284$ K.

need not be better, if its stage size ratio is chosen too small. Also shown schematically are four cascades with increasing number of stages but constant ratio B_1/B_N. With increasing number of stages, the additional intermediate stages yield less additional cooling. From an experimental point of view, for cascades with too many stages or too large values of the stage size ratio B_i/B_{i+1}, the vertical or lateral heat flow from stage to stage becomes a problem, and the calculated temperature drop is difficult to realize. From these arguments, cascades with no more than 6 stages and with a stage size ratio B_i/B_{i+1} in the range 4–10 appear favorable.

Turning our attention to cascades carrying a heat load, we note that the temperature drop of a cascade decreases linearly with increasing heat load similar to the case of a single stage. In our calculations we have found that further optimization of the electric currents according to the heat load has only negligible effects on the cooling behaviour. Therefore, all calculations have been done with the optimum current settings for the case of zero heat load.

To see what happens within the cascade when a heat load is applied, we calculated all temperatures of a three-stage cascade with the following parameters: $n_1 = 14$, $n_2 = 62$, $n_3 = 142$, $A_i/l_i = 0.0768$ cm, $T_0 = 284$ K and, again, temperature independent material properties $S = 169$ μV/K, $\kappa = 23.6$ mW/cm·K and $\rho = 0.629$ mΩ·cm. Fig. 5 shows the results: here we have plotted the temperature of the three stages versus the heat load on the cold side of the cascade. We see that the top temperature T_3 is most strongly affected by the heat load, whereas the lower stages are affected much less.

To get a more complete view of the cascade behaviour under heat load, we simulated all meaningful configurations of the three modules discussed in conjunction with Fig. 5, with all parameters as given there. The total temperature drop of these configurations as a function of the

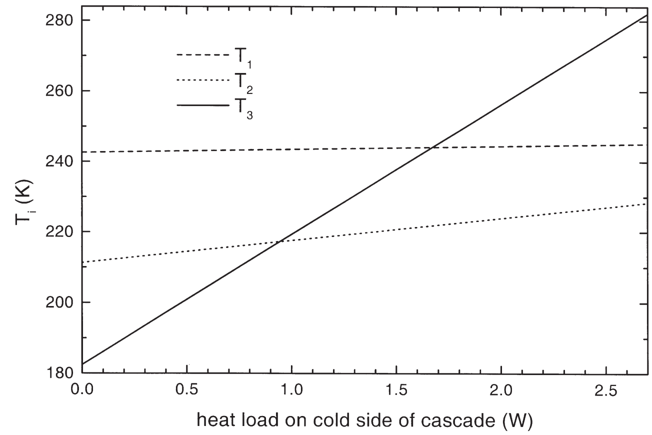

Fig. 5. Calculated temperatures T_1, T_2, and T_3 of a three-stage cascade plotted vs the heat load on the cold side for the same parameters as in Fig. 4 and $n_1 = 14$, $n_2 = 62$, $n_3 = 142$, and $A_i/l_i = 0.0768$ cm.

cold side heat load is shown in Fig. 6. Again we see that the top module determines the range in which the temperature drop reaches zero. Independent of the number of stages, all cascades, where the smallest of the three modules is used as the top stage, provide no cooling at all when the heat load reaches 2–3 W. The optimum design of a cascade depends on the cooling requirements. If the expected heat load is small and a high temperature drop is desired, the top module should be small. On the other hand, if the heat load is larger, a cascade with a smaller number of stages and a relatively large top stage appears more favorable.

5. Summary and conclusions

Peltier cascades consisting of three, four, and five stages, respectively, were built from single-stage mod-

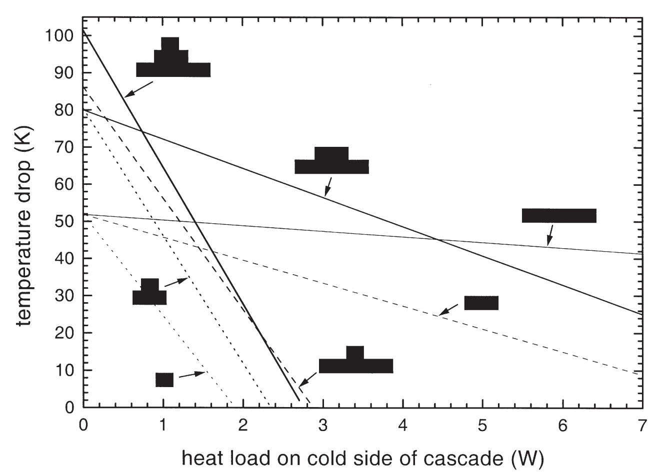

Fig. 6. Calculated temperature drop of the cold side plotted vs the heat load on the cold side for the different Peltier configurations indicated schematically by the black symbols. We show all meaningful configurations of the three modules discussed in conjunction with Fig. 5.

ules commercially available, and the temperature on the cold side was measured as a function of the operating currents. An additional Peltier stage served for cooling a copper box acting as an effective radiation shield for the higher stages placed inside the box. In addition, the cooling behaviour of the cascades was simulated using a theoretical model. Fixing the temperature on the warm side at 286 K using cooling water, on the cold side a minimum temperature of 158 K (temperature drop = 128 K) was reached with a five-stage cascade. This minimum temperature and the optimum values of the operating currents were exactly reproduced by our model. We have found both experimentally and theoretically, that the maximum temperature drop is relatively insensitive to the values of the operating currents. Optimum thermal coupling between all stages is essential for reaching the lowest possible temperature in a cascade. As shown by our model calculations, the expected heat load has important implications for the optimum design of a cascade.

Acknowledgements

This work was supported by the Bundesminister für Bildung, Wissenschaft, Forschung und Technologie (Project No. 13N6937/7).

References

[1] Putilin SN, Antipov EV, Chmaissem O, Marezzio M. Nature 1993;362:226.
[2] Schilling A, Cantoni M, Guo JD, Ott HR. Nature 1993;363:56.
[3] Huebener RP, Tsuei CC. Cryogenics 1998;38:325.
[4] Strähle JW, Rath S, Hans T, Metzger T, Nissel T, Huebener RP, Kemmler-Sack S. In: Proceedings of the XVII Int. Conf. on Thermoelectrics (ICT'98). Nagoya, Japan, 1998. p. 491.
[5] Hyun D-B, Kolomoets NV, Oh T-S, Hwang J-S, Shim J-D. In: Proceedings of the XVI Int. Conf. on Thermoelectrics (ICT'97). Dresden, Germany, 1997. p. 679.
[6] Drabkin I, Jouranlev O. In: Proceedings of the XVI Int. Conf. on Thermoelectrics (ICT'97). Dresden, Germany, 1997. p. 676.
[7] Jungnickel H, Agsten R, Kraus WE. Grundlagen der Kältetechnick. Berlin: Verlag Technik, 1990.
[8] Melcor Corp., Trenton, NJ, USA.