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RESEARCH ARTICLE

Fermions tunnelling from the rotating 5-D Myers–Perry black hole

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Abstract Recent researches on the Hawking radiation of black holes show that the Hawking temperature can be obtained by fermion tunnelling method. In this paper, we extend this method to a 5-D space-time and view the Hawking radiation of the Myers–Perry black hole with two independent angular momenta. As a result, the Hawking temperature is obtained, which is the same as that obtained by other methods.

Keywords Fermions · Hawking radiation · Tunnelling · Myers–Perry black hole

1 Introduction

Hawking made a remarkable discovery that a black hole can radiate thermally and the temperature is true in 1974 [1]. Since then, Hawking radiation has attracted many people's attention and many methods have been brought forward to derive it [2–12]. The most simply and available method is the semi-classical quantum approach modeling Hawking radiation as a tunnelling effect, which was proposed by Kraus and Wilczek [7] and was developed by a lot of researchers [13–22]. All of these researches can successfully recover the corrected Hawking temperature and play a great role on further research of black holes. As far as we know, the key point of tunnelling method

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is to calculate the imaginary part of the action for the emitted particles and we can get the tunnelling probability for the classically forbidden trajectory from inside to outside horizon by using the WKB approximation as $\Gamma \sim \exp(-2ImI)$. Up to now, there are mainly two approaches to obtain the imaginary part of action. First is the Null Geodesic method which was put forward by Parikh and Wilczek. Using Hamilton's equation on null geodesics and integrating the radial momentum p_r of the emitted particles, one can get the imaginary part of the action [23].Secondly, one can use Hamilton–Jacobi method to get the imaginary part of the action [24]. Following this work, people made a great deal of successes. However, almost all of the previous work limited to the scalar particle' tunnelling radiation. In fact, a black hole can radiate all kinds of particles. Recently, Kerner and Mann introduce the tunnelling method to study the Hawking radiation of spin 1/2 uncharged particles and recovered the Hawking temperature of the general non-rotating black hole and the Rindler spacetime [25]. Subsequently, this method was widely used in more general spacetime, such as BTZ black holes, rotating Einstein–Maxwell, Dilaton–Axion black holes, Kerr–Newman black holes, dynamical black holes and de-Sitter spaces [26–30]. All of these work support Kerner and Mann's opinion and indicate the universality of this method. At present, this method has been extended to the five space-time [31]. For the 5-D case, starting with the covariant Dirac equation and choosing five proper independent γ^μ matrices according to the feature of the horizon topology of the black holes, we can view the Hawking radiation of 5-D case. In this paper, we extend fermion tunnelling to the space-time of a 5-D rotating Myers–Perry black hole [32] with the horizon of topology S^3. The Myers–Perry metric is vacuum solution of Einstein's equation describing general rotating black hole spacetime as well as with two independent angular momenta.

The remainder of this paper is outlined as follows. In the next section, the metric of the Myers–Perry black hole is written and the Hawking radiation of the Myers–Perry black hole is investigated and the Hawking temperature is obtained by fermion tunnelling. Finally, we will give some conclusion and discussion.

2 Fermion tunnelling from 5-D rotating Myers–Perry black hole

According to Ref. [32], the metric of the 5-D rotating black hole can be written in Boyer–Lindquist coordinate as the most simple form as

$$\begin{aligned} ds^2 = -dt^2 + \Sigma\left(\frac{r^2}{\Delta}dr^2 + d\theta^2\right) + (r^2+a^2)\sin^2\theta d\varphi^2 + (r^2+b^2)\cos^2\theta d\phi^2 \\ +\frac{m}{\Sigma}\left(dt^2 - a\sin^2\theta d\varphi - b\cos^2\theta d\phi\right)^2, \end{aligned} \tag{1}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta + b^2\sin^2\theta, \quad \Delta = (r^2+a^2)(r^2+b^2) - mr^2. \tag{2}$$

Note that m is the physical mass of the black hole, while the parameters a and b are associated with its two independent angular momenta, respectively. The event (inner) horizons of the black hole are

$$r_{\pm}^2 = \frac{1}{2}[(m-a^2-b^2) \pm \sqrt{(m-a^2-b^2)^2 - 4a^2b^2}]. \tag{3}$$

The components of the inverse metric take the following forms as

$$\begin{aligned}
g^{tt} &= -\frac{\Delta m + \Delta\Sigma + m^2r^2}{\Delta\Sigma}, \quad g^{rr} = \frac{\Delta}{\Sigma r^2}, \quad g^{t\varphi} = -\frac{ma(r^2+b^2)}{\Sigma\Delta},\\
g^{t\phi} &= -\frac{mb(r^2+a^2)}{\Sigma\Delta}, \quad g^{\theta\theta} = \frac{1}{\Sigma}, \quad g^{\varphi\phi} = -\frac{mab}{\Sigma\Delta},\\
g^{\phi\phi} &= \frac{1}{\Sigma}\left[\frac{1}{\sin^2\theta} + \frac{(b^2-a^2)(r^2+b^2) - mb^2}{\Delta}\right],\\
g^{\varphi\varphi} &= \frac{1}{\Sigma}\left[\frac{1}{\cos^2\theta} + \frac{(a^2-b^2)(r^2+b^2) - ma^2}{\Delta}\right].
\end{aligned} \tag{4}$$

At the event horizons of the Myers–Perry black hole, the angular velocities are, respectively,

$$\Omega_{h1} = \frac{a}{a^2+r_+^2}, \quad \Omega_{h2} = \frac{b}{b^2+r_+^2}, \tag{5}$$

Now we proceed to apply fermions tunnelling method to the Myers–Perry black hole. In curve spacetime, the motion equation of the fermion satisfies the following Dirac equation

$$i\gamma^\mu D_u\psi + \frac{m}{\hbar}\psi = 0. \tag{6}$$

where $D_u = \partial_u + \Omega_u$, $\Omega_\mu = \frac{i}{2}\Gamma_\mu^{\alpha\beta}\sum_{\alpha\beta}$, $\sum_{\alpha\beta} = \frac{i}{4}[\gamma^\alpha, \gamma^\beta]$. There are many different ways to choose γ^μ matrices. In this paper, considering the γ^μ matrices satisfy $\{\gamma^\mu, \gamma^\upsilon\} = 2g^{\mu\upsilon}I$ as well as the principle of convenience for calculation, we choose the 5-D space-time coordinates as $x^u = (t, r, \theta, \varphi, \phi)$ and matrices $\gamma^u = (\gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^4)$ according to the feature of this black hole, namely

$$\begin{aligned}
&\gamma^0 = M\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \gamma^1 = \sqrt{g^{rr}}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \quad \gamma^2 = \sqrt{g^{\theta\theta}}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix},\\
&\gamma^3 = D\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + E\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix},\\
&\gamma^4 = G\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} + H\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix} + K\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}.
\end{aligned} \tag{7}$$

in which the σ^i are the Pauli matrices expressed as

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{8}$$

where

$$M=\sqrt{g^{tt}},\quad D=\frac{g^{t\varphi}}{\sqrt{g^{tt}}},\quad G=\frac{g^{t\phi}}{\sqrt{g^{tt}}},\quad E=\sqrt{g^{\varphi\varphi}-\frac{(g^{t\varphi})^2}{g^{tt}}},$$
$$H=\frac{g^{\phi\varphi}g^{tt}-g^{t\varphi}g^{t\phi}}{\sqrt{g^{\varphi\varphi}(g^{tt})^2-(g^{t\varphi})^2g^{tt}}},\tag{9}$$
$$K=\sqrt{-g^{\phi\phi}g^{tt}\left[g^{\varphi\varphi}-\frac{(g^{t\varphi})^2}{g^{tt}}\right]+(g^{t\phi})^2\left[g^{\varphi\varphi}-\frac{(g^{t\varphi})^2}{g^{tt}}\right]+\left(g^{\phi\varphi}-\frac{g^{t\phi}g^{t\varphi}}{g^{tt}}\right)^2g^{tt}}.$$

We only discuss the spin up state of the fermion because that the spin down and the spin up resemble to each other. Adopting the following wave function with spin up

$$\psi_{\uparrow}(t,r,\theta,\varphi,\phi)=\begin{pmatrix}A\,(t,r,\theta,\varphi,\phi)\\0\\B\,(t,r,\theta,\varphi,\phi)\\0\end{pmatrix}\exp\left[\frac{i}{\hbar}I_{\uparrow}\,(t,r,\theta,\varphi,\phi)\right],\tag{10}$$

and inserting it into the Dirac equation yields

$$-B\Big(iM\partial_tI_{\uparrow}+\sqrt{g^{rr}}\partial_rI_{\uparrow}+iD\partial_{\varphi}I_{\uparrow}+iG\partial_{\phi}I_{\uparrow}\Big)+A\left(m+iK\partial_{\phi}I_{\uparrow}\right)=0,\tag{11}$$

$$-B\left(i\sqrt{g^{\theta\theta}}\partial_{\theta}I_{\uparrow}+E\partial_{\varphi}I_{\uparrow}+H\partial_{\phi}I_{\uparrow}\right)=0,\tag{12}$$

$$A\Big(iM\partial_tI_{\uparrow}-\sqrt{g^{rr}}\partial_rI_{\uparrow}+iD\partial_{\varphi}I_{\uparrow}+iG\partial_{\phi}I_{\uparrow}\Big)+B\big(m-iK\partial_{\phi}I_{\uparrow}\big)=0,\tag{13}$$

$$-A\left(i\sqrt{g^{\theta\theta}}\partial_{\theta}I_{\uparrow}+E\partial_{\varphi}I_{\uparrow}+H\partial_{\phi}I_{\uparrow}\right)=0.\tag{14}$$

We should note that A and B are not constant, but the components Ω_{μ} can be neglected to the lowest order in WKB because all of them include the factor $\hbar$ [25] .To solve above equations, we carry out variables of separation according to the symmetries of the Myers–Perry space-time as

$$I_{\uparrow}=-\omega t+J\varphi+L\phi+W(r,\theta)\tag{15}$$

where ω is the energy of the emitted fermion, and J and L are two independent angular momentum corresponding to the angle φ and ϕ, respectively. Then substitute Eq. (15) into Eqs. (11) and (13), we can get the following equations

$$-B\Big(-iM\omega+\sqrt{g^{rr}}\partial_rW(r,\theta)+iDJ+iGL\Big)+A(m+iKL)=0,\tag{16}$$

$$A\Big(-iM\omega-\sqrt{g^{rr}}\partial_rW(r,\theta)+iDJ+iGL\Big)+B(m-iKL)=0.\tag{17}$$

We know that the two equations have a non-trivial solution for A and B only if the determinant of the coefficient matrix vanishes, so we have

$$\begin{aligned}\partial_r W(r,\theta) &= \pm\sqrt{\frac{(\omega-\Omega_{h1}J-\Omega_{h2}L)^2-M^{-2}[m^2+K^2L^2]}{-g^{rr}M^{-2}}}\\ &= \pm\sqrt{\frac{(\omega-\Omega_{h1}J-\Omega_{h2}L)^2-M^{-2}[m^2+K^2L^2]}{-g^{rr}_{,r}(r,\theta)M^{-2}{}_{,r}(r,\theta)\,(r-r_+)^2}}\end{aligned} \tag{18}$$

Equation (18) indicates that $W(r,\theta)$ is a complex function and it have no explicit r dependence. But near the horizon of Myers–Perry black hole, the product of $M^{-2}_{,r}(r,\theta)g^{rr}_{,r}(r,\theta)(r-r_+)^2$ is independent of θ. Therefore, near the horizon $\partial_r W(r,\theta)$ is independent of θ. And the function $W(r,\theta)$ can be separated as

$$W(r,\theta) = W(r) + \Theta(\theta). \tag{19}$$

Now, we only concentrate on radial function $W(r)$. Integrating the pole at the horizon of the Myers–Perry black hole [32], and obtain the imaginary part of the emitted particle's action across the event horizon

$$\mathrm{Im}\,W_\pm = \pm\pi\frac{\omega-J\Omega_{h1}-L\Omega_{h2}}{\sqrt{g^{rr}(r,\theta)_{,r}M^{-2}(r,\theta)_{,r}}}\bigg|_{r=r_+}, \tag{20}$$

where + (−) correspond to the outgoing (ingoing) solutions. So the tunnelling probability of emitted fermion can be given as

$$\begin{aligned}\Gamma = &\frac{P(emission)}{P(absorption)} = \frac{\exp(-2\,\mathrm{Im}\,I_+)}{\exp(-2\,\mathrm{Im}\,I_-)} = \frac{\exp(-2\,\mathrm{Im}\,W_+)}{\exp(-2\,\mathrm{Im}\,W_-)}\\ &= \exp\left[-4\pi\frac{\omega-J\Omega_{h1}-L\Omega_{h2}}{\sqrt{g^{rr}_{,r}(r,\theta)M^{-2}_{,r}(r,\theta)}}\right]\Bigg|_{r=r_+}\\ &= \exp\left[-4\pi\frac{\omega-J\Omega_{h1}-L\Omega_{h2}}{2\left(r_+^2-r_-^2\right)/mr_+}\right]\end{aligned} \tag{21}$$

From the tunnelling probability of emitted fermion, the Hawking temperature can be derived as

$$T = \frac{\kappa}{2\pi} = \frac{r_+^2-r_-^2}{2\pi m r_+} \tag{22}$$

In this section, we calculated the Dirac particles' Hawking radiation from 5-D Myers–Perry black holes using the tunnelling approach and the result is coincided with that got via other method. Obviously, the result also proves the the robust of the tunnelling method.

3 Conclusion

In this paper, we have extended the fermion tunnelling method to a more general case and viewed 5-D rotating Myers–Perry black hole with two independent angular momenta. In our result, the Hawking temperature is corrected recovered and is full consistence with that obtained by other methods. But it is important to note that the angular momentum, energy and charge of the black hole are fixed with the particles emission from the black hole and the derived radiation spectrum only is leading term. When unfixed background space-time and self-gravitational interaction are taken into account, the derived result should be modified slightly. Also the Hawking can be derived from the spin down case.

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References

1. Hawking, S.W.: Nature **248**, 30 (1974)
2. Damour, T., Ruffini, R.: Phys. Rev. D Part Fields **14**, 332 (1976). doi:10.1103/PhysRevD.14.332
3. Strominger, A., Vafa, C.: Phys. Lett. B **379**, 99 (1996)
4. Jing, J.L.: Chin. Phys. **10**, 234 (2001). doi:10.1088/1009-1963/10/3/312
5. Liu, L., Xu, D.Y.: Acta. Phys. Sin. **29**, 1624 (1980) (in Chinese)
6. Wu, S.Q., Cai, X.: Nuovo Cimento B **115**, 143 (2000)
7. Parikh, M.K., Wiltzek, F.: Phys. Rev. Lett. **85**, 5042 (2000). doi:10.1103/PhysRevLett.85.5042
8. Srinivasan, K., Padmanabhan, T.: Phys. Rev. D Part Fields **60**, 024007 (1999). doi:10.1103/PhysRevD.60.024007
9. Robinson, S.P., Wilcze, F.: Phys. Rev. Lett. **95**, 011303 (2005). doi:10.1103/PhysRevLett.95.011303
10. Iso, S.: metsu, H.U., Wilczek, F.: Phys. Rev. Lett. **96**, 151302 (2006). doi:10.1103/PhysRevLett.96.151302
11. Lin, K., Zeng, X.X., Yang, S.Z.: Chin. Phys. Lett. **25**, 390 (2008)
12. Jiang, Q.Q., Wu, S.Q.: Phys. Lett. B **647**, 200–206 (2007). doi:10.1016/j.physletb.2007.01.058
13. Zhang, J.Y., Zhao, Z.: Nucl. Phys. B **725**, 173 (2005). doi:10.1016/j.nuclphysb.2005.07.024
14. Zhang, J.Y., Zhao, Z.: JHEP **0510**, 055 (2005)
15. Yang, S.Z., Jiang, Q.Q., Li, H.L.: Chin. Phys. **14**, 24112005 (2005). doi:10.1088/1009-1963/14/11/016
16. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Rev. D Part Fields Gravit. Cosmol. **73**, 064003 (2006). doi:10.1103/PhysRevD.73.064003; Phys. Lett. B **651**, 58 (2007); Phys. Lett. B **651**, 65 (2007); Jiang, Q.Q., Wu, S.Q.: Phys. Lett. B **647**, 200 (2007)
17. Jiang, Q.Q.: Class. Quantum Gravity **24**, 4391 (2007). doi:10.1088/0264-9381/24/17/008
18. Yang, S.Z.: Chin. Phys. Lett. **22**, 2492 (2005)
19. Kerner, R., Mann, R.B.: Phys. Rev. D Part Fields Gravit. Cosmol. **75**, 084022 (2007). doi:10.1103/PhysRevD.75.084022
20. Angheben, M., Nadalini, M., Vanzo, L., Zerbini, S.: JHEP 05, 014 (2005)
21. Medved, A.J.M.: Phys. Rev. D Part Fields **66**, 124009 (2002). doi:10.1103/PhysRevD.66.124009
22. Medved, A.J.M., Vagenas, E.C.: Mod. Phys. Lett. A **20**, 1723 (2005). doi:10.1142/S0217732305018025
23. Parikh, M., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000)
24. Bekenstein, J.D.: Phys. Rev. D Part Fields **7**, 2333 (1973). doi:10.1103/PhysRevD.7.2333
25. Kerner, R., Mann, R.B.: Class. Quantum Gravity **25**, 095014 (2008) arXiv: 0710.0612 [hep-th]
26. Li, R., Ren, J.R.: Phys. Lett. B **661**, 370 (2008) arXiv: 0802.3954 [hep-th]
27. Kerner, R., Mann, R.B.: Phys. Lett. B **665**, 277 (2008) arXiv: 0803.2246 [hep-th]
28. Roberto, D.C., Luciano, V.: Europhys. Lett. **82**, 60001 (2008) arXiv: 0803.0435 [hep-th]
29. Chen, D.Y., Yang, H.T., Zu, X.T.: Eur. Phys. J.C. **56**, 119 (2008). doi:10.1140/epjc/s10052-008-0638-y

30. Ahmed, M.: 1991. Phys. Lett. B **258**, 318 (1991). doi:10.1016/0370-2693(91)91092-A
31. Jiang, Q.Q.: Phys. Rev. D Part Fields Gravit. Cosmol. **78**, 044009 (2008). doi:10.1103/PhysRevD.78.044009
32. Myers, R.C.: Perry Ann, M.J. Phys. N.Y. **172**, 304 (1986). doi:10.1016/0003-4916(86)90186-7