On the Growth of Proton-Proton Total Cross-Section in the ISR Region.

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(ricevuto il 6 Novembre 1974)

In this paper we would like to examine the energy dependence of the proton-proton total cross-section in the energy range 290 GeV to 2000 GeV, as observed (1) in the Intersecting Storage Ring (ISR) at CERN. The direct measurements of σ_{tot} made at ISR have large errors and therefore it is not possible to distinguish between the $\ln s$ and $\ln^2 s$ rise of σ_{tot}. The new ingredient of our present analysis consists in the experimental single-particle inclusive spectra of pions up to ISR energies, which give us information about $\langle n \rangle \sigma_{tot}$. We present arguments to whow that the shape of the rapidity distribution of π^- is compatible with the linear growth of σ_{tot} with $\ln s$, whereas it is incompatible with its growth as $\ln^2 s$.

The rapidity distribution $E\,\mathrm{d}^3\sigma/\mathrm{d}^3p$ *vs.* y of π^- in the projectile frame of ref. (2) has been compiled by ANTINUCCI *et al.* (3) for transverse momentum p_T of 0.4 GeV/c; recently ALPER *et al.* (4) have given this distribution for p_T in the range $(0.2 \div 1.0)$ GeV/c and for $\sqrt{s} = 30.6$ GeV and 52.8 GeV. We have parametrized these data assuming that the invariant cross-section $E\,\mathrm{d}^3\sigma/\mathrm{d}^3p$ factorizes into p_T and y as

$$E\,\mathrm{d}^3\sigma/\mathrm{d}^3p = g(y)\exp[-bp_T]\,, \tag{1}$$

where

$$b = 6.0\ (\mathrm{GeV/c})^{-1}.$$

(1) U. AMALDI, R. BIANCASTELLI, C. BOSIO, G. MATTHIAE, J. V. ALLABY, W. BARTEL, G. COCCONI A. N. DIDDENS, R. W. DOBINSON and A. M. WETHERELL: *Phys. Lett.*, **44** B, 112 (1973); S. R. AMENDOLIA, F. BELLETTINI, P. L. BRACCINI, C. BRADASCHIA, R. CASTALDI, V. CAVASINNI, C. CERRI, T. DEL PRETE, L. FOA, P. GIROMINI, P. LAURELLI, A. MENZIONE, L. RISTORI, G. SANGUINETTI and M. VALDATA: *Phys. Lett.*, **44** B, 119 (1973); G. BELLETTINI: *Fifth International Conference on High-Energy Collisions*, (Stony Brook, 1973).

(2) The rapidity is defined by $y = \frac{1}{2}\ln[(E+p_L)/(E-p_L)]$, where E is the energy of the particle and p_L is its longitudinal momentum. In the projectile frame of reference it is defined as $y = y_{beam} - y_{c.m.}$, where $y_{c.m.}$ si the c.m. rapidity of the particle and $y_{beam} \simeq \frac{1}{2}\ln(s/m_p^2)$ the c.m. rapidity of the incoming particle.

(3) M. ANTINUCCI, A. BERTIN, P. CAPILUPPI, M. D'AGOSTINO-BRUNO, A. M. ROSSI, G. VANNINI, G. GIACOMELLI and A. BUSSIERE: *Lett. Nuovo Cimento*, **6**, 121 (1973).

(4) B. ALPER, H. BOGGILD, P. BOOTH, F. BULOS, L. J. CARROLL, G. VON DARDEL, G. DAMGAARD, B. DUFF, F. HEYMANN, J. N. JACKSON, G. JARLSKOG, L. JONSSON, A. KLOWNING, L. LEISTAM, E. LILLETHUN, G. LYNCH, S. OLGAARD NIELSEN, M. PRENTICE, D. QUARRIE and J. M. WEISS: *Phys. Lett.*, **47** B, 75 (1973).

This factorized form provides a qualitative description of the data and it is good enough for the arguments presented in this paper. The following expressions for $g(y)$ have been fitted to the data:

$$(1a)\quad g(y) = (4065.9 \pm 201.0)\exp\left[-(4.5 \pm 0.05)/y^{(0.32\pm0.01)}\right] + (10.0 \pm 0.1) \qquad \text{for } y < 2\,,$$

and

$$(1b)\quad g(y) = (66.3 \pm 0.9)\left[1 + (0.36 \pm 0.01)\,y\right] \qquad \text{for } y \geqslant 2\,.$$

In Fig. 1 we have shown the data for $p_T = 0.4$ GeV/c and the continuous curve shows the fit to the data according to eq. (1). The linear fit with $y \geqslant 2$ has $\chi^2/DF = 4$.

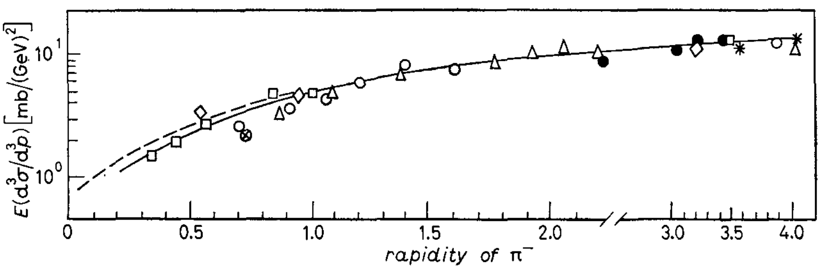

Fig. 1. – The invariant cross-section at $p_T = 0.4$ GeV/c of π^- in $p+p \to \pi^-+$anything is plotted against the laboratory rapidity of π^-. Data has been taken from the compilation of ANTINUCCI *et al.* (ref. (3)). The dashed line refers to 24 GeV data. The continuous line is our fit to the data.

Integration of eq. (1) gives

$$(2)\quad \langle n\rangle\,\sigma_{\text{tot}} = \frac{4\pi}{b^2}\int\limits_0^{y_{\max}} g(y)\,\mathrm{d}y\,,$$

where $y_{\max}$ is the c.m. rapidity of the incoming proton and can be written as

$$(3)\quad y_{\max} \simeq \tfrac{1}{2}\ln(s/m_p^2)\,.$$

Here $\langle n\rangle$ is the multiplicity (5) of produced π^-, σ_{tot} is the p-p total cross-section and m_p is the rest mass of the proton. For incident energies > 24 GeV, which correspond to $y_{\max} > 2$, the contribution of the parameterization (1*a*) to eq. (2) is a constant, whereas the contribution of (1*b*) to eq. (2) gives rise to terms in $\ln s$ and $\ln^2 s$. Thus we get

$$(2a)\quad \langle n\rangle\,\sigma_{\text{tot}} = -(20.5 \pm 3.0) + (11.8 \pm 0.2)\ln s + (1.0 \pm 0.03)\ln^2 s\,, \qquad \text{in mb.}$$

(5) The conventional definition of multiplicity is $\langle n\rangle = \sum n_i\sigma_i/\sigma_{\text{inel}}$, where σ_{inel} is the inelastic cross-section. For compatibility with our notation (see eq. (2) of the text) the conventional multiplicity should be multiplied by $\sigma_{\text{inel}}/\sigma_{\text{tot}}$ which is nearly a constant in the energy range considered in the text. To be precise $\sigma_{\text{inel}}/\sigma_{\text{tot}}$ varies from 0.79 to 0.83 in the energy range 20 GeV to 70 GeV, whereas it is 0.83 in the ISR energy range.

The presence of the $\ln^2 s$ term is due to the nonvanishing slope of the rapidity distribution for $y \geqslant 2$. We would like to stress that the signs of the coefficients cannot be reversed by making a different parametrization than what is used here.

It is to be noted that a careful analysis of ALPER *et al.* [4] has shown that the slope [6] of the rapidity distribution decreases from 0.18 at $\sqrt{s} = 30.6$ GeV to 0.13 at $\sqrt{s} = 52.8$ GeV. This essentially gives rise to a small admixture of a $\ln^3 s$ term with a *negative* coefficient in eq. (2*a*). Addition of this term which is proportional to y^2 in eq. (1*b*) does not improve the quality of the fit and hence it is not used in the fit of eq. (1*b*). But it is important to note that the $\ln^3 s$ term of $\langle n \rangle \sigma_{tot}$ has a negative coefficient and this point will prove to be an important clue to our arguments presented later. If there would have been an increase in the slope of the rapidity distribution with the increasing s, it could then be possible for $\langle n \rangle \sigma_{tot}$ to rise faster than $\ln^2 s$. The analysis of ALPER *et al.* therefore suggests [7] that with the increasing energy the $\ln s$ term in (2*a*) becomes more dominant than the $\ln^2 s$ term, thus leading gradually towards a flat plateau in the central region.

We now discuss the growth of the charged multiplicity and σ_{tot} with energy.

The energy dependence of the charged multiplicity, where $\pi^{\pm}$ constitute the major portion of the total multiplicity, as well as the multiplicity of single particles have been studied by many authors [3,8]. From these studies it is concluded that at lower energies, *i.e.* $E_{lab} \leqslant 100$ GeV, the multiplicity grows like a power law

$$\langle n \rangle \sim s^{\frac{1}{4}}\,, \tag{4}$$

whereas at higher energies in the range of $E_{lab} > 100$ GeV to 10^4 GeV the growth is slower and is given by

$$\langle n \rangle = a_1 + a_2 \ln s\,. \tag{5}$$

It is important to note that a_2 is positive whereas a_1 is negative.

The measurements of σ_{tot} exist up to about 2000 GeV. In the energy region 20 GeV to 60 GeV, where accurate measurements of σ_{tot} exist, it is seen that σ_{tot} remains practically constant (it varies only from 39 mb to 38.4 mb), whereas in the ISR region from 290 GeV to 2000 GeV there is a definite rise in σ_{tot}. There exist three different empirical fits to explain the rise in σ_{tot} in the ISR region:

$$\sigma_{tot} = 38.4 + 0.9 \ln^{\nu}(s/200) \qquad \text{with } \nu = 1.8 \pm 0.4 \quad (\text{ref. } (^1)), \tag{6}$$

$$\sigma_{tot} = 38.4 + 0.5 \ln^2(s/122) \qquad (\text{ref. } (^9)), \tag{7}$$

$$\sigma_{tot} = 24.9 + 39.9\, s^{-\frac{1}{4}} + 2.1 \ln s \qquad (\text{ref. } (^{10})). \tag{8}$$

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[6] ALPER *et al.* (ref. [4]), have parametrized the function $g(y)$ as proportional to $1 + \alpha y_{c.m.}$, where α is the slope. Converting the above dependence into laboratory rapidity y, as used in eq. (1*b*) of the text, the slopes of ALPER *et al.* becomes 0.4 and 0.3 at $\sqrt{s} = 30.6$ GeV and $\sqrt{s} = 52.8$ GeV, respectively, which are to be compared with our fitted slope of 0.36.

[7] A possible way out to take care of the slow decrease of the slope with energy for $y \geqslant 2$ is to parametrize $g(y)$ as $g(y) = (a_1 - a_2 s^{-\frac{1}{4}})(1 + a_3 s^{-\frac{1}{4}} y)$. This has a nice feature of its approach to scaling as $s^{-\frac{1}{4}}$. We need to have more measurements at different s values to test the above parametrization.

[8] S. N. GANGULI and P. K. MALHOTRA: *Phys. Lett.*, **42** B, 83 (1972); D. R. O. MORRISON: *Proceedings of the IV International Conference on High-Energy Collisions* (Oxford, 1972); A. GURTU: preprint TIFR-BC-74-9.

[9] E. LEADER and U. MAOR: *Phys. Lett.*, **43** B, 505 (1973); U. AMALDI: CERN-NP 73-5 (1973).

[10] S. N. GANGULI and A. SUBRAMANIAN: *Lett. Nuovo Cimento*, **10**, 235 (1974).

TABLE I. – *Summary of fits to proton-proton total cross-section.*

Type of fits	χ^2	Data points	Energy range
$\sigma_{tot} = 23.0 + 44.0\, s^{-\frac{1}{2}} + 2.4 \ln s$	39	37	12 GeV to 2067 GeV
$\sigma_{tot} = 26.3 + 2.1 \ln s$	13	16	290 GeV to 2067 GeV
$\sigma_{tot} = 38.7 + 1.2[\ln(s/200)]^{1.4 \pm 0.3}$	13	16	290 GeV to 2067 GeV
$\sigma_{tot} = 38.5 + 0.5 \ln^2(s/105)$	38	37	12 GeV to 2067 GeV
$\sigma_{tot} = 38.5 + 0.4 \ln^2(s/80)$	13	16	290 GeV to 2067 GeV

We have refitted these expressions with all the available data and the results are summarized in Table I. All the three fits are equally good and it is not possible to decide the correct energy dependence from σ_{tot} measurements alone. The energy dependence of σ_{tot} in the ISR region can be obtained if we combine the energy dependence of $\langle n \rangle \sigma_{tot}$, as obtained from π inclusive spectra, and the energy dependence of $\langle n \rangle$; this is discussed below.

The energy dependence of $\langle n \rangle\, \sigma_{tot}$, as discussed in the beginning and given by eq. (2*a*), shows $\ln s$ and $\ln^2 s$ terms. In the energy range $(20 \div 60)$ GeV, σ_{tot} remains practically constant and therefore it is expected from the energy dependence of $\langle n \rangle \sigma_{tot}$ that $\langle n \rangle$ should have terms in $\ln s$ and $\ln^2 s$. This depencence is fitted to the available π^- data in the range 13 GeV to 100 GeV. The result of the fit is given by

$$\langle n \rangle = -0.9 + 0.4 \ln s + 0.04 \ln^2 s \tag{9a}$$

with $\chi^2 = 0.8$ for 9 data points. The expected energy dependence of the π^- multiplicity in the above energy range can also be obtained from eq. (2*a*) by taking the average σ_{tot}, which we take as 38.6 mb, and the average value of $\sigma_{inel}/\sigma_{tot}$ which we take as 0.81. Thus the energy dependence of the conventional multiplicity [5] is obtained from eq. (2*a*) by dividing it by 31.3 and the result is

$$\langle n \rangle = -(0.7 \pm 0.1) + (0.4 \pm 0.01) \ln s + (0.03 \pm 0.001) \ln^2 s\,. \tag{9b}$$

Comparison of (9*b*) with (9*a*) brings out the internal consistency of eq. (2*a*) with the observed multiplicity and σ_{tot}.

In the ISR energy range, *i.e.* 290 GeV to 2000 GeV, the energy dependence of σ_{tot} is deduced by examining the following four cases:

i) The maximum possible growth of σ_{tot} with energy can be obtained from eq. (2*a*) by assuming that the multiplicity has become constant from 290 GeV onwards. By extrapolating the fitted relation (9*a*) to 290 GeV we get the conventional multiplicity as 3.0; by multiplying it by $\sigma_{inel}/\sigma_{tot}$ we get the value of $\langle n \rangle$ to be used in eq. (2*a*) as 2.5. The values of σ_{tot} thus obtained for various s values are listed in Table II along with the measurements [1] at the ISR. From Table II we see that the deduced growth of σ_{tot}, with terms in $\ln s$ and $\ln^2 s$, is much faster than the experimental observation. This is quite expected because we know that the observed multiplicity [3] in the range $(290 \div 1500)$ GeV is not constant; it increases from 3 to 3.9 after taking into account the increase in σ_{inel}.

TABLE II. – *Comparison of σ_{tot} as measured at* ISR *with the expectation from the rapidity distribution under the assumption of constant multiplicity beyond* 290 GeV.

p_{lab} (GeV/c)	σ_{tot} (mb)	
	Measured values	Expected values
290	39.1 ± 0.4	38.1 ± 1.4
500	40.5 ± 0.5	43.5 ± 1.4
1070	42.5 ± 0.5	51.7 ± 1.5
1480	43.2 ± 0.6	55.5 ± 1.5
2067	44.0 ± 0.8	59.2 ± 1.6

ii) We now consider the case with multiplicity growing as $\ln s$, $\langle n\rangle = a_1 + a_2 \ln s$, where a_1 has always a negative value [3,8,11], and σ_{tot} growing as $\ln^2 s$, $\sigma_{tot} = b_1 + b_2 \ln^2 s$, where both b_1 and b_2 have positive values [9]. Now we multiply $\langle n\rangle$ and σ_{tot} and compare the product with the observed dependence of $\langle n\rangle\, \sigma_{tot}$ as given by eq. (2*a*). We find that the coefficient of $\ln^2 s$ as obtained here, $a_1 b_2$, is a negative quantity, whereas the observed dependence of eq. (2*a*) wants it to be a positive quantity. Also the coefficient of $\ln^3 s$, which is $a_2 b_2$, is positive here, whereas the experimental observation of ALPER *et al.* [4] demands the coefficient to be negative. Hence the form of $\sigma_{tot} = b_1 + b_2 \ln^2 s$ is incompatible with eq. (2*a*).

iii) We now consider the dependence of σ_{tot} as $\sigma_{tot} = b_1 + b_2 \ln s + b_3 \ln^2 s$, with b_3 having a positive value and the multiplicity growing as in ii). We again multiply $\langle n\rangle$ and σ_{tot}. We see that the coefficient of $\ln^3 s$ here is $a_2 b_3$, which is a positive quantity. But the experimental observation of ALPER *et al* [4], as mentioned earlier, shows that the coefficient of the $\ln^3 s$ term must be negative and hence the form of σ_{tot} discussed here is also not compatible with the observed dependence of $\langle n\rangle\, \sigma_{tot}$.

iv) Here we consider the energy dependence of σ_{tot} as $\sigma_{tot} = b_1 + b_2 \ln s$, where both b_1 and b_2 have positive values (see Table I) and the multiplicity [11] grows as in ii). We now multiply $\langle n\rangle$ and σ_{tot} and it comes out to be

$$\langle n\rangle\, \sigma_{tot} \approx -26 + 11 \ln s + 1.0 \ln^2 s\,. \tag{10}$$

This is to be compared with the observed $\langle n\rangle \sigma_{tot}$ as given by eq. (2*a*). We find that the two expression eqs. (10) and (2*a*) are in good agreement with each other and there is no wrong sign for the coefficients as found for the cases ii) and iii).

The above arguments are based on the assumption that the charged-particle multiplicity grows as $\ln s$ which is supported by experimental data [3,8] up to 10^4 GeV. If the σ_{tot} has a $\ln^2 s$ term, the dependence of the multiplicity on energy has to be a complicated function of $\ln s$ such that $\langle n\rangle\, \sigma_{tot}$ satisfies the functional form of eq. (2*a*). It also needs eventually that the slope of the rapidity distribution should increase with energy. But the present observation at ISR [4] shows that the slope [7] of the rapidity distribution

[11] The multiplicity of π^- in the energy range $(200 \div 1500)$ GeV has large statistical errors. The approximate energy dependence can be given by $\langle n\rangle \approx -1.0 + 0.5 \ln s$.

decreases with increasing energy, thereby implying that the $\ln^2 s$ term in eq. (2a) becomes weaker with energy and therefore the rise of σ_{tot} as $\ln^2 s$ is not supported by the data.

We thus conclude from the rapidity distribution of the single-particle inclusive spectrum of π^- from p-p collisions up to ISR energies and the observed energy dependence of the multiplicity (like $\ln s$) that the rise in p-p total cross-section as observed in the ISR energy range is consistent with a $\ln s$ dependence on energy and is inconsistent with the growth of σ_{tot} as $\ln^2 s$.

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The author is grateful to Drs. A. SUBRAMANIAN, S.M. ROY and PROBIR ROY for helpful discussions.

Direttore responsabile: CARLO CASTAGNOLI

Stampato in Bologna dalla Tipografia Compositori coi tipi della Tipografia Monograf
Questo fascicolo è stato licenziato dai torchi il 31-I-1975