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AVAILABILITY ANALYSIS OF SYSTEMS WITH TWO TYPES OF REPAIR FACILITIES

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Abstract

This paper presents two mathematical models of repairable systems. Failed systems repair times are arbitrarily distributed. Two types of repair facilities are needed to repair a failed system. Laplace transforms of the state probability equations are developed.

INTRODUCTION

Most failed systems are repaired for their recycle usage. This paper presents two mathematical models I and II of repairable systems with two types of repair facilities. Other related models maybe found in references [2-3] .

Model I represents a repairable system with a degraded and a catastrophically failed states. In addittion two types of repair facilities (overhaul and minor repair) are required to repair the failed system. Therefore, it has two repair facility states. In other words the operational system may fail in its degraded (system failed partially) or catastrophic mode. A degraded system may also fail catastrophically. Furthermore, the degraded system is repaired. The failed system requires two types of repair (For example a major overhaul or a minor repair). Failed system repair times are arbitrarily distributed. Transition diagram of the system is shown in Figure 1.

Similarly, Model II represents a system which can fail in n mutually exclusive failure modes. In addittion system failed in anyone of the n failure modes may require a major overhaul or minor repair. In other words if the system fails in anyone of the failure modes it requires two types of repair facilities. System repair times are arbitrarily distributed. Typical examples of this system (at n=2) are:

a) An electrical transformer (open and shorted failure modes).

b) A motor (open and shorted failure modes).

c) A fluid flow valve (open and closed failure modes).

d) A complex system with n number of well defined mutually exclusive failure modes. System transition diagram is shown in Figure 2.

The supplementary variable method [1] is used to develop equations for both models I and II.

ASSUMPTIONS

The following were assumed to develop mathematical models, I and II:

Model I:

1. Failures are statistically independent.
2. Repaired system is as good as new.
3. System failure rates are constant. In addittion rates to carry out system overhaul or minor repair are constant.
4. Repair rate from system degraded mode is constant.
5. Failed system repair rates are arbitrarily distributed.
6. A failed system needs either overhaul or a minor repair, therefore, two types repair facilities are assumed.
7. System states are: good (0), degraded or partially failed (1), catastrophically failed (2), overhaul (3), minor repair (4).

Model II

1. Failures are statistically independent.
2. Repaired system is as good as new.
3. Operational system can fail in anyone of the n mutually exclusive failure modes.
4. System failed in any one of the failure modes may need anyone of the two types of repair facilities. For example one is for

the system overhaul and the other one is for the system minor repair.

5. System failure rates are constant. Rates to carry out overhaul or minor repair are also constant.
6. System repair times are arbitrarily distributed.
7. System is repaired from each failure mode after the type of repair (i.e. overhaul or minor) is detected.
8. System states are: good (0), failed (1,1;2,1;3,1;----n,1) overhaul (1,2;2,2;3,2;---n,2), minor repair (1,3;2,3;3,3;---n,3).

NOTATION

It is separately defined for models I & II.

Model I

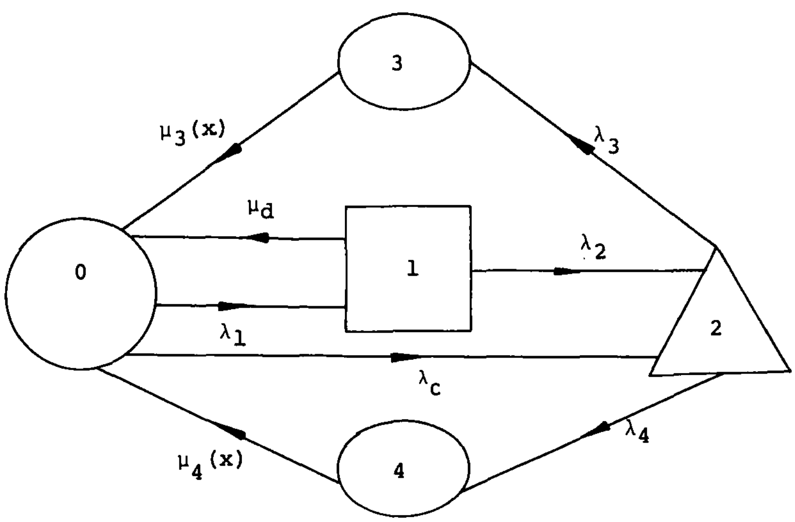

Figure 1. Model I transition diagram.

$P_i(t)$	Probability that system is in state i, at time t; i=0 (unfailed), i=1 (degraded or partially failed), i=2 (failed), i=3 (overhaul), i=4 (minor repair).
$p_i(x,t)$	Probability density (with respect to repair time) that the failed system is in state i and has an elapsed repair time x; for i=3 (overhaul), i=4 (minor repair).
$\mu_i(x), q_i(x)$	Repair rate (a hazard rate) and probability density

function of repair times when system is in state i and has an elapsed repair time of x; for i=3 (overhaul), i=4 (minor repair).

λ_i Constant ith mode failure rate, i=1 (degraded mode), i=2 (degradation to catastrophic mode), i=c (catastrophic mode).

λ_j Constant rate of overhauls (j=3) or minor repair (j=4).

μ_d Costant system repair rate from degradation state.

s Laplace transform variable.

Model II

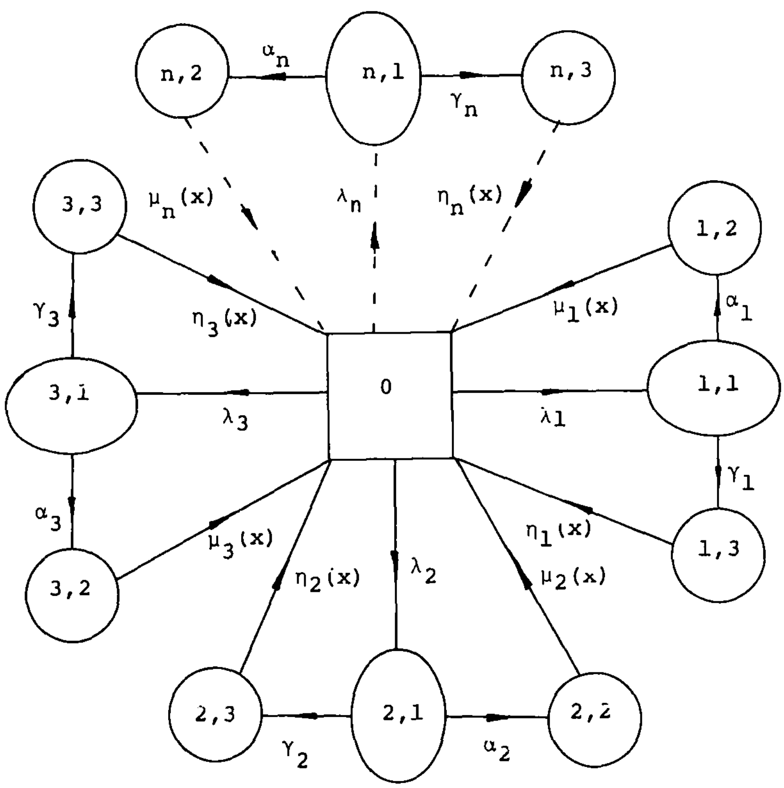

Figure 2. Model II transition diagram.

n Number of system failure modes.

$P_0(t)$ Probability that system is in unfailed state at time t.

$P_{i,1}(t)$	Probability that system is in ith(i,1) failure state at time t, for i=1,2,3,4---n.
$p_{i,j}(x,t)$	Probability density (with respect to repair time) that the failed system is in state (i,j) and has an elapsed repair time of x, for (i,j) = (1,2);(2,2);(3,2)----(n,2)(overhaul); (i,j) = (1,3);(2,3);(3,3);----(n,3)(minor repair).
$\mu_i(x)$ $q_{i,j}(x)$	Repair rate (a hazard rate) and probability density function of repair times when system is in state (i,j) and has an elapsed repair time of x; for (i,j)=(1,2);(2,2);(3,2);---(n,2) (overhaul); i=1,2,3---n.
$\eta_i(x)$ $q_{i,j}(x)$	Repair rate (a hazard rate) and probability density function of repair times when system is in state (i,j) and has an elapsed repair time of x; for (i,j)=(1,3);(2,3);(3,3);---(n,3); (minor repair);i=1,2,3----n.
λ_i	Constant ith failure mode system failure rate, for i=1,2,3----n.
α_i	Constant system ith mode overhaul rate, for i=1,2,3---n.
γ_i	Constant system ith mode minor repair rate, for i=1,2,3---n.
s	Laplace transform variable.

EQUATIONS

Models I & II equations are developed in the following sections:

Model I

The associated system of differential equations with Figure 1 are:

$$\frac{dP_0(t)}{dt} + P_0(t)(\lambda_1+\lambda_c) = P_1(t)\mu_d + \int_0^\infty p_4(x,t)\mu_4(x)dx + \int_0^\infty p_3(x,t)\mu_3(x)dx \tag{1}$$

$$\frac{dP_1(t)}{dt} + P_1(t)(\lambda_2+\mu_d) = P_0(t)\lambda_1 \tag{2}$$

$$\frac{dP_2(t)}{dt} + P_2(t)(\lambda_3 + \lambda_4) = P_0(t)\lambda_c + P_1(t)\lambda_2 \tag{3}$$

$$\frac{\partial p_j(x,t)}{\partial t} + \frac{\partial p_j(x,t)}{\partial x} + p_j(x,t)\mu_j(x) = 0 \tag{4}$$

for j = 3 or 4.

$$p_j(0,t) = P_2(t)\lambda_j \tag{5}$$

for j = 3 or 4.

$P_i(0) = 1$, for $i=0$, otherwise $P_i(0) = 0$, for $i=1,2$; $p_j(x,0)$ for all j.

The Laplace transforms of the solution are (6)-(9):

$$P_0(s) = \left[A_1 - \frac{\mu_d\lambda_1}{A_2} - \{G_4(s)\lambda_4+\lambda_3 G_3(s)\}\{\lambda_c+\frac{\lambda_1\lambda_2}{A_2}\}\frac{1}{A_3}\right]^{-1} \tag{6}$$

$A_1 \equiv s + \lambda_1 + \lambda_c$

$A_2 \equiv s+\lambda_2+\mu_d$

$A_3 \equiv s+\lambda_3+\lambda_4$

$G_i(s) \equiv \int_0^\infty \exp(-sx)\ q_i(x)\ dx$

for i = 3 or 4

$$P_1(s) = \lambda_1 P_0(s)/ A_2 \tag{7}$$

$$P_2(s) = \{\lambda_c + \lambda_1\ \lambda_2/ A_2\}P_0(s)/ A_3 \tag{8}$$

$$P_i(s) = \{1-G_i(s)\}P_2(s)\lambda_i/s \tag{9}$$

for i= 3 or 4

Model II

The associated system of differential equations with Figure 2 are:

$$\frac{dP_0(t)}{dt} + P_0(t) \left(\sum_{i=1}^{n} \lambda_i\right) = \int_0^\infty \left(\sum_{i=1}^{n} P_{i,2}(x,t)\mu_i(x)\right)dx + \int_0^\infty \left(\sum_{i=1}^{n} P_{i,3}(x,t)\eta_i(x)\right)dx \quad (10)$$

$$\frac{dP_{i,1}(t)}{dt} + (\alpha_i + \gamma_i)P_{i,1}(t) = P_0(t)\lambda_i \quad (11)$$

$$\frac{\partial p_{i,2}(x,t)}{\partial x} + \frac{\partial p_{i,2}(x,t)}{\partial t} + \mu_i(x)\, p_{i,2}(x,t) = 0 \quad (12)$$

$$\frac{\partial p_{i,3}(x,t)}{\partial x} + \frac{\partial p_{i,3}(x,t)}{\partial t} + \eta_i(x)p_{i,3}(x,t) = 0 \quad (13)$$

$$p_{i,2}(0,t) = \alpha_i P_{i,1}(t) \quad (14)$$

$$p_{i,3}(0,t) = \gamma_i P_{i,1}(t) \quad (15)$$

In the case of equations (11)-(15):

$i = 1,2,3,4 --- n$

$P_i(0) = 1$, for $i=0$, otherwise $P_{i,1}(0) = 0$, for $i=1,2,3,---n$; $p_{i,j}(x,0)=0$ for $j=2$ or 3; $i=1,2,3,---n$.

The Laplace transforms of the solution are (15)-(19):

$$P_0(s) = \left[s + \sum_{i=1}^{n}\lambda_i - \sum_{i=1}^{n}\alpha_i A_i G_{i,2}(s) - \sum_{i=1}^{n}\gamma_i A_i G_{i,3}(s)\right]^{-1} \quad (16)$$

$A_i \equiv \lambda_i / s+\alpha_i+\gamma_i$, for $i=1,2,3,---n$

$$G_{i,j}(s) \equiv \int_0^\infty \exp(-sx)\, q_{i,j}(x)\, dx$$

for j=2 or 3, i= 1,2,3,4----n.

$$P_{i,1}(s) = P_0(s)A_i \tag{17}$$

$$P_{i,2}(s) = \{1-G_{i,2}(s)\}\ \alpha_i A_i/s \tag{18}$$

$$P_{i,3}(s) = \{1-G_{i,3}(s)\}\gamma_i A_i/s \tag{19}$$

For equations (16)-(19): i=1,2,3,----n

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