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On the interaction of numerical and size information in digit comparison: A behavioral and event-related potential study

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Abstract—To investigate effects of dimensional congruity in digit comparison processes we recorded response latencies, error rates and event-related potentials (ERPs) in a situation when the relevant and the irrelevant dimension of judgement, the digit's physical size or its numerical value, varied unpredictably from trial to trial. Congruity effects may arise at early processing stages, where both numerical and size information are mapped onto a common, integrated representation. Alternatively, numerical and size information may be processed in functionally independent channels and interact only at a later stage of response activation.

For both, numerical and size comparisons, reliable distance effects on ERPs (starting 224 ms post-stimulus at frontal electrode sites) and on behavioral data were observed. Congruity effects follow a timecourse that mirror-images the temporal pattern found for the distance effects: while distance effects on difference potentials start about 64 ms earlier for numerical comparisons, congruity effects due to the irrelevant attribute are seen about 88 ms earlier for size comparisons. While Lateralized Readiness Potentials (LRPs) for incongruent trials are shifted in time, there was no indication of an activation for the incorrect response in incongruent trials, as an account of congruity effects in terms of response competition would predict. The most parsimonious explanation of our results is that numerical and size information are extracted in parallel, are both converted into an integrated representation and potentially interact during these early parallel stages. © 1998 Elsevier Science Ltd. All rights reserved.

Key Words: ERPs; P300; LRPs; mental comparison; interference effects; early vs late locus of interaction.

Introduction

Humans are extremely efficient in processing and comparing symbolic information such as digits representing distinct numerical values. Judgements of numerical magnitude involve "symbolic" information because the relevant dimension—the abstract numerical value—cannot be deduced from the graphical appearance of the digits as such; rather, their numerical meaning relies on previously learned conventions and must be held in, and retrieved from, memory in order to successfully perform tasks such as digit comparison. Moyer and Landauer [19] first demonstrated that the time to determine the numerically larger out of two simultaneously presented digits systematically decreases with their numerical difference (the numerical distance effect; cf. [3–6, 18]). To account for this effect, Moyer and Landauer [19] proposed that the digits are automatically converted to percept-like analog quantities and that these analog representations are then in turn processed much like sensory representations of stimuli which differ along some extensive physical dimension.

One line of support for this hypothesis derives from Stroop-type interference experiments in which the digits are presented with varying physical (i.e. font) sizes. Given the hypothesis of an analog representation of numerical magnitude, one would expect facilitation or interference, depending on whether the relation of physical size and numerical value is congruent, or not. This size congruity effect predicted by the analog representation model was indeed first obtained by Besner and Coltheart [1]. In principle, their results are in line with a serial processing model in which the physical features of the digit (including its physical size) are first identified before the digit's numerical value is retrieved from memory and is then compared to the numerical value of the other digit (or to a fixed standard) to activate the appropriate response. According to the serial view, the output of the earlier

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stage of physical feature encoding may influence subsequent processes during later stages at which numerical information is retrieved and compared, but not vice versa. In particular, the digit's physical size may well influence processing of numerical information, but according to the serial model the numerical value of a digit should not in turn influence physical size comparisons.

Henik and Tzelgov [11] and Tzelgov *et al.* [23] first studied the "complementary" task of judging the digit's physical size, independent of its numerical value. Contrary to the expectations from the serial model, they found that the (irrelevant) numerical value of the digit induced systematic congruity effects on the latency of physical size judgements. These findings of bidirectional, mutual interference of size and numerical information strongly argue against any strictly serial model, which we will, therefore, not consider further in the sequel. Even so, two broad interpretations in terms of parallel processing remain, which differ in their assumptions about the locus of the observed congruity effects.

Under one interpretation (Fig. 1a), extending Moyer and Landauer's original proposal, both, the digit's physical size and its numerical value are first mapped onto an integrated internal analog representation, which is then processed further to activate the appropriate response. According to this view, congruity effects with both, numerical and size comparisons occur at an early processing stage where both, size and numerical information are converted into an integrated representation and potentially interact with each other. Under this model, the digit's attributes have no individual access to the stage of response activation; rather, they must first pass a processing stage at which the separate information from the two attributes is integrated into a single representation.

An alternative account (Fig. 1b) of size congruity effects, however, is that size and numerical information are first processed in parallel, functionally independent channels and that both of them can separately activate a specific "subresponse". Therefore, no congruity effects are predicted for early processing stages such as digit identification. Under this interpretation, both digit attributes have independent access to the stage of response activation and they potentially interact only during these later processing stages.

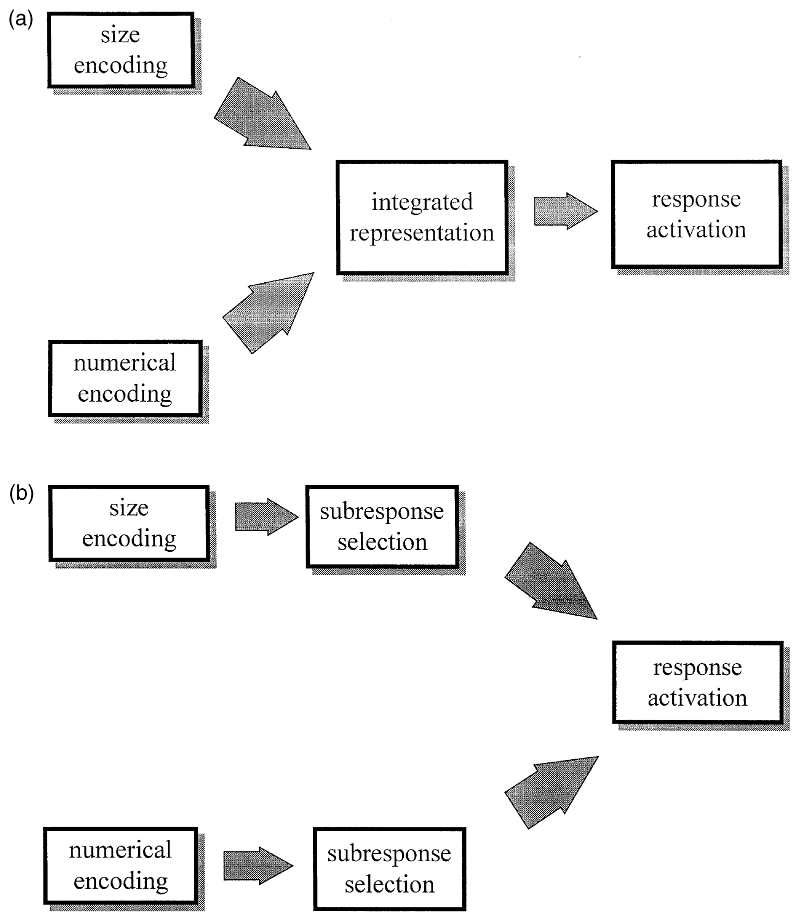

Fig. 1. A: Schematic model of early interaction: size and numerical information are first integrated into a joint analog representation, only then are responses prepared and executed. The channels processing size and numerical information have no direct access to the response module, but only through the integrated analog representation. B: Schematic model of late interaction: size and numerical information are separately processed up to a level at which separate subresponses for each attribute are selected, which may then induce facilitation or inhibition at the stage of overt response preparation and execution.

To further investigate these alternative accounts of congruity effects we employed a variation of a paradigm introduced by Sudevan and Taylor [22] (see also [20]). In this paradigm, a cue serves to define the task-relevant stimulus attribute for each particular trial. In the present study, the cue indicated to the subjects to either judge the physical size, or the numerical value, of a single digit presented shortly thereafter. For numerical comparisons, subjects had to decide whether the numerical value of the presented digit was smaller or larger than a fixed numerical standard, which was equal to 5. The test digit could be 3, 4, 6 or 7, corresponding to a numerical distance from the standard of 1 (digits 4, 6) vs 2 (digits 3, 7). Likewise, for physical size comparisons, subjects had to decide whether the physical size of the presented digit was smaller or larger than a standard size (called size *c*) to which they were practiced before, during training blocks. The size of the test digit could be (in ascending order of size) *a*, *b*, *d*, or *e*, corresponding to a size distance from the size standard *c* of 1 (sizes *b*, *d*) vs 2 (sizes *a*, *e*). Thus, each digit was characterized by (i) its numerical value, and (ii) its physical size. A specific digit is called *congruent* if both, its physical size and its numerical value, would yield the same (correct) response under the two types of comparison, otherwise it is *incongruent*. For example, the digit 7 would be a congruent stimulus if presented with size *e*, but an incongruent stimulus, if presented with size *a*. Conversely, the digit 3 is congruent if presented with size *a*, but incongruent with size *e*. Note, that the congruity of a specific combination of numerical value and physical size is independent of the task-relevant attribute, so that estimates of congruity effects for the two types of comparisons are based on the same subsets of physical stimuli.

For the paradigm described, let us now reconsider the hypothesis that processing of physical size and numerical value interferes at an early stage, when both, size and numerical information are mapped onto an integrated representation (Fig. 1a). Suppose, processing of one of the two stimulus attributes, say, of numerical value, shows, on average, a speed advantage over judgements of the other attribute, physical size. If the onsets of the induced effects could be suitably monitored, we should then expect, first, earlier differential effects of numerical distance 1 vs numerical distance 2 for numerical comparisons, as compared to differential effects of size distance 1 vs size distance 2 for size comparisons. This pattern would simply reflect our hypothetical assumption that numerical information is, on average, processed faster than is size information. At the same time, this assumption would imply that for size comparisons the information about the irrelevant digit attribute (i.e. numerical magnitude) is, on average, available earlier than information about the relevant attribute, physical size. Thus, congruity effects should show up earlier for *size* comparisons as compared to numerical comparisons: for size comparisons, the irrelevant, but faster, attribute (numerical magnitude) could already exert its influence while the relevant attribute (physical size) is typically still being processed, whereas for numerical comparisons it is the relevant attribute which (over repeated trials) would usually have shorter finishing times. A similar reasoning would apply to the case that size information is, on average, processed faster than is numerical information: earlier distance effects for size judgements should then go together with earlier congruity effects for numerical judgements. In summary, if size and numerical information interact at an early stage we would expect that the order of onsets of congruity effects mirror-images the order of onsets found for the distance effects of the respective irrelevant dimension.

Consider now the alternative hypothesis that numerical and size information are represented and processed in functionally independent channels (Fig. 1b). Under this assumption, no differential congruity effect should occur before the stage of response activation; in particular, no systematic relation is predicted for the relative onset of distance and congruity effects during earlier stages preceding response activation. Furthermore, suppose that the onset and timecourse of response activation could be suitably monitored and that the "subresponse" suggested by, say, the numerical information, on average, accesses this stage earlier than the subresponse suggested by the size information. For size comparisons we would then expect an initial activation towards the incorrect response with incongruent stimuli, due to the earlier activation in response to the numerical information. At the same time, for congruent stimuli the individual subresponses to size and numerical information suggest the same overt response. Also, both effects should be absent or attenuated for numerical judgements, because, under our hypothetical assumption of faster numerical processing, the slower subreponse suggested by the size information will, on average, arrive too late to contribute an initial correct (congruent trials) or incorrect (incongruent trials) response activation.

The present study tries to distinguish between these contrasting predictions using event-related potentials (ERPs). ERPs provide a continuous record of synchronous neural activity which can aid in tracing the onset and timecourse of the processing stages that intervene between stimulus and response (cf. [10]). In particular, differences between ERPs can be used to monitor the onset and timecourse of electrophysiological effects attributable to variations of the independent variables, such as type of judgement, distance-to-the-standard and congruity in the present case. Furthermore, specific components of the ERP can be related to stimulus evaluation processes, while other components are thought to reflect response-related processes.

Distance effects are systematically related to a particular positive-going component of the ERP centered around the central parietal electrode site (Pz), the P300 [5, 8, 9, 16, 24]. The P300 usually shows a clearly identifiable peak between 300 and 700 ms after stimulus onset and its latency varies systematically with factors which influence the duration of stimulus evaluation processes such as stimulus discriminability or numerical distance in the present case. A recent literature review [25] indicates that the usefulness of P300 latency as an index of stimulus evaluation processes (cf. [13–15]) differs considerably across experiments, although for the present case of digit comparison previous work [5, 8, 9, 16, 24] has clearly established that distance effects are well reflected by characteristics of the P300.

Effects of selective response activation can be monitored by the so-called Lateralized Readiness Potential (LRP), which is related to the preparation for, and execution of, a motor movement. The LRP is derived from the difference of potentials measured at the left and right central brain sites $C3'$ and $C4'$ above the motor cortex; it is based on the fact that a maximum negativity occurs contralateral to the responding hand. By comparing the difference potentials $C3'$–$C4'$ under right-hand responses and left-hand responses main lateralization effects not related to response processes are subtracted out, so that the LRP monitors selective response activation (for detailed accounts of the LRP, see [2, 7, 17, 21]).

Let us now reconsider the models discussed previously in terms of difference potentials, P300 and LRP. Under the hypothesis of an early and common locus of size distance and numerical distance effects (Fig. 1a) we would expect that (a.i) numerical distance has a main effect on early difference potentials and P300 for numerical comparisons, (a.ii) size distance has a main effect on early difference potentials and P300 for size comparisons, (a.iii) congruity has a main effect on early difference potentials and P300 in both tasks, which should (a.iv) mirror-image the temporal order of effect onsets found in (a.i) and (a.ii); finally, (a.v) congruity effects may delay the LRP onset along the time axis, but should not lead to an initial deflection towards the incorrect response.

On the other hand, if numerical and size information is processed in functionally independent channels (Fig. 1b) and interact only via response competition, we expect that (b.i) congruity effects should not show up in early difference potentials and P300—even if size distance and numerical distance *individually* do induce main effects on difference potentials, or P300. Furthermore, (b.ii) the faster processed irrelevant attribute should induce an initial deflection of the LRP towards the incorrect response in incongruent trials. Conversely, (b.iii) the slower processed irrelevant attribute should not induce an initial deflection of the LRP towards the incorrect response in incongruent trials, although it may contribute incorrect response activation after the LRP onset for the correct response. Finally (b.iv), congruent trials should never induce an initial deflection of the LRP towards the incorrect response, but the LRP may onset earlier because both attributes co-activate the same subresponse. Clearly, because part of predictions (b.iii) and (b.iv) is shared by both models, special attention will be paid to predictions (b.i), and (b.ii). We also note that predictions (a.i) and (a.ii) of the early interaction model are also compatible with a response competition account and thus do not by themselves discriminate between the two models.

Method

Subjects

Fourteen young adults each participated in a single session lasting about 4 h. The data from two subjects were discarded because of too many eye-movement artifacts. All remaining 12 subjects (six male) were right-handed and had normal or corrected-to-normal vision; they were paid DM10 per hour ($\approx$\$6). Their mean age was 23 years ($r = 19$–26).

Stimuli and apparatus

Four different digits (3, 4, 6, 7) were used in four different font sizes (*a*, *b*, *d*, *e*), yielding 16 physically different stimuli used throughout the experiment. All digits were first generated from a. standard Courier font set and then enlarged with a vector graphic; from the viewing distance of 80 cm, the digit heights were seen at an angle of 1.6, 2.0, 3.0 and 3.8° with the font sizes *a*, *b*, *d*, *e*, respectively. The digits were presented on a 60 Hz video monitor, the display timing being synchronized with the video refresh cycle.

Procedure

Each trial started with the presentation of a small ($0.4° \times 0.4°$) fixation square at the center of the screen. This square was either red or green, indicating to the subjects to judge either the physical size, or the numerical value of the forthcoming digit. The color-to-task mapping was counterbalanced across subjects. After an onset asynchrony uniformly distributed between 1000 ms and 1200 ms, the digit replaced the fixation square for 150 ms. For numerical comparisons, subjects had to indicate whether the numerical value of the digit was smaller or larger than the numerical standard, which was equal to 5. For size comparisons subjects had to indicate whether the physical size of the digit was smaller or larger than the physical standard, which had the intermediate font size, *c* (height of 2.5°). The next trial then started 2500 ms after the presentation of the digit. In both tasks and for all subjects, "smaller" responses required a left-hand button press and "larger" responses a right-hand button press with the index finger. Instructions encouraged fast responses, but urged the subjects to keep the overall error rate below 10%.

At the start of the experiment, subjects were first repeatedly shown all 16 different stimuli (the four digits in the four sizes) on screen, and then also all digits in the intermediate font size, *c*, which served as the size standard. To adapt to the task and to the physical size relations, all subjects were given two practice blocks. Each block consisted of four presentations of the 16 different stimuli under both tasks; thus, a block comprised a total of 128 trials. Each block lasted for about 10 min and was followed by a subject-paced break. Following the two practice blocks, data were collected from eight blocks of 128 trials (plus two warm-up trials) each. Feedback (mean RT and error rate for each task) was provided after each block.

ERP Recording and Data Analysis

The EEG was recorded from the scalp using tin electrodes embedded in an elasticated cap (Electro-Cap International) and located at the standard sites Fz, Cz, Pz, Oz and from sites 4 cm to the left ($C3'$) and right ($C4'$) of Cz. All these electrodes were referenced to the left ear lobe. Eye movements were monitored with electrodes at both outer canthi (horizontal electro-oculogram, EOG) and at the infra-orbital ridge of the right eye (vertical EOG). EEG signals were filtered with bandpass of 0.016–64 Hz and sampled on-line at a rate of 4 ms per point for an epoch of 1024 ms, starting 100 ms prior to stimulus presentation. Trials on which base-to-peak EOG amplitude exceeded 75 μV, drift of the EEG from baseline exceeded 100 μV, or on which amplifier saturation occurred were excluded from averaging. On average, 12% of trials were rejected as artifacts; the data of two subjects with rejection rates exceeding 25% were discarded from further analysis.

Following Coles [2, 20], the LRP was defined as follows:

$$\mathrm{LRP} = \tfrac{1}{2}\cdot[\mathrm{Mean}\,(C4'\text{–}C3')_{\text{left-hand response}} + \mathrm{Mean}\,(C3'\text{–}C4')_{\text{right-hand response}}],$$

and was calculated separately for consecutive epochs of 8 ms for each subject, each of the four combinations of congruity (congruent vs incongruent) and type of comparison (size vs numerical). With this convention, negative (positive) values of LRP indicate a selective activation of the correct (incorrect) response.

Results

Behavioral results

Across all subjects, 12 (subjects) $\times$ 32 (replications) = 384 trials were collected for each of the 32 basic conditions (2 tasks $\times$ 4 digits $\times$ 4 sizes) of the experiment. Mean RT and mean error rate were subjected to an ANOVA with factors task (size vs numerical comparisons), distance-to-the-standard (small vs large, or 1 vs 2), and congruity (congruent vs incongruent).

Mean correct RT

Mean RTs are shown in Table 1. The factor task had only a marginally significant main effect on mean RT (size comparisons: 496 ms, numerical comparisons: 475 ms; $F(1,11) = 3.33$, $P = 0.095$). Within each task, reliable distance-to-the-standard main effects were observed: for size comparisons, the small distance (i.e. digit sizes b, d) yielded a mean RT of 516 ms, while for the large distance (i.e. digit sizes a, e) mean RT was 475 ms, $F(1,11) = 138.73$, $P < 0.001$. Similarly, for numerical comparisons, distance = 1 (i.e. digits 4, 6) yielded a mean RT of 485 ms, while for numerical distance = 2 (i.e. digits 3, 7) mean RT was 464 ms, $F(1,11) = 56.76$, $P < 0.001$.

Both tasks showed reliable congruity effects: for size comparisons, mean RT for congruent digits (471 ms) differed from mean RT for incongruent digits (520 ms), $F(1,11) = 65.40$, $P < 0.001$. For numerical comparisons, congruent digits yielded a mean RT of 462 ms, as compared to 488 ms for incongruent digits, $F(1,11) = 23.70$, $P < 0.001$. The task $\times$ congruity interaction was significant, indicating that the congruity effect was larger with size comparisons than with numerical comparisons (49 ms vs 26 ms, $F(1,11) = 8.03$, $P = 0.016$). For numerical comparisons, the effects of distance and congruity were found to be additive (for their interaction, $F(1,11) < 1$), but for size comparisons, the interaction of distance and congruity was significant ($F(1,11) = 4.93$, $P = 0.048$): for the small distance the effect of congruity was 56 ms, whereas for the large distance it was 41 ms.

In summary, for both types of comparison reliable effects of distance and congruity were observed. Interference effects were found to be mutual, but their magnitudes are asymmetric, being more pronounced for size comparisons. Also, distance and congruity interact for size comparisons, but not for numerical comparisons. Given the (marginally significant) overall speed advantage for numerical comparisons, these results are consistent with a view that numerical information is, on average, available earlier than size information, at a point of time when size information is typically still processed. Thus, irrelevant numerical information will induce larger congruity effects on the typically still ongoing processing of relevant size information than vice versa. On the other hand, the very existence of congruity effects (although smaller) for numerical comparisons shows that at least in some (smaller) proportion of all trials, irrelevant size information is available early enough to influence in turn the ongoing numerical comparison. Taken together this pattern is consistent with the view that (over repeated trials) the distribution of latencies after which information about physical size and numerical value is available overlap, but with the mean of the latter distribution being smaller.

Table 1. Effects of distance and congruity on mean RT and error rate

	Numerical comparison				Size comparison			
	Congruent		Incongruent		Congruent		Incongruent	
	$D = 1$	$D = 2$	$D = 1$	$D = 2$	$D = 1$	$D = 2$	$D = 1$	$D = 2$
Mean RT in *ms*	474	450	496	479	488	455	544	496
Mean error rate in %	3.2	1.9	7.9	5.4	12.5	3.2	24.5	9.8

According to this view, we should expect that decreasing the distance-to-the-standard of the relevant attribute yields an increase of the associated congruity effect, because the irrelevant information will then, on average, have more opportunity (i.e. time) to influence the comparison process. For example, for size comparisons decreasing size distance-to-the-standard from large to small should yield an increase of the associated congruity effect, because the irrelevant numerical information could then influence the size comparison during a longer time interval. This prediction is in fact confirmed by the significant distance × congruity interaction for size comparisons, which derives from a larger congruity effect for the close distance. A similar effect is predicted for numerical comparisons, when numerical distance is decreased from 2 to 1; however, this effect was not found to be significant. An obvious explanation for this negative finding is that distance effects are significantly smaller for numerical than for size comparisons (21 ms vs 41 ms; $F(1,11) = 21.80$, $P < 0.001$, for the task × distance interaction). As a consequence, the predicted interaction effect of decreasing numerical distance from 2 to 1 must clearly be considerably smaller than the corresponding effect for size comparisons.

Another related prediction from this interpretation is that congruity effects should increase with increasing distance-to-the-standard of the *irrelevant* attribute. The reason is that with increasing distance, the information from the irrelevant attribute is, on average, available earlier and should, thus, be better able to interfere with the processing of the relevant attribute. For numerical comparisons, this expectation is in fact borne out: for the small size distance (i.e. sizes *b*, *d*), the congruity effect is 17 ms and for the large size distance (i.e. sizes *a*, *e*), it is 35 ms ($F(1,11) = 6.74$, $P = 0.025$). For size comparisons, the corresponding means are similarly ordered: 47 ms for a numerical distance of 1 (i.e. digits 4, 6), and 50 ms for a numerical distance of 2 (i.e. digits 3, 7), but their difference is not significant ($F(1,11) < 1$). Again, an obvious explanation for this latter negative finding is that distance effects are only half as large for numerical comparisons than for size comparisons; thus, for size comparisons, increasing numerical distance does not seem to speed up processing of the irrelevant numerical information to a degree that could induce significant changes of the observed congruity effects. An alternative interpretation is that, for size comparisons, the irrelevant numerical information may be only roughly categorized as being smaller or larger than 5, so that more fine grained numerical distance information does not in addition influence the congruency effect for size judgements (cf. [23]).

Error data

Mean error rates are shown in Table 1. Overall, the error rate equated 8.5%. In general, error rates confirmed the significance pattern found for mean RT, with the exception that, in addition, significantly more errors occurred with size comparisons (12.5%) than with numerical comparisons (4.6%; $F(1,11) = 27.66$, $P < 0.001$). Within each task, distance-to-the-standard effects were significant: for size comparisons, error rates were 18.5% (small distance) vs 6.5% (large distance), $F(1,11) = 68.63$, $P < 0.001$. For numerical comparisons, error rates were 5.5% (distance = 1) vs 3.6% (distance = 2), $F(1,11) = 7.40$, $P = 0.020$. Again, under both tasks reliable congruity effects were observed: for size comparisons, error rate for congruent digits (7.8%) differed from the error rate for incongruent digits (17.1%), $F(1,11) = 55.36$, $P < 0.001$. For numerical comparisons, the corresponding error rates are 2.5% and 6.7%, respectively, $F(1,11) = 26.98$, $P < 0.001$. As with mean RT, the task × congruity interaction was significant, indicating that congruity had a larger effect on error rates for size comparisons as compared to numerical comparisons (9.3% vs 4.1%, $F(1,11) = 13.46$, $P = 0.004$). Also, for size comparisons, the interaction of distance and congruity was significant ($F(1,11) = 7.86$, $P = 0.017$), with a larger congruity effect for the small than the large distance. For numerical comparisons, the corresponding interaction of distance and congruity was again not significant ($F(1,11) = 2.22$; $P = 0.166$).

Overall, error rates complement the pattern of mean RT findings and rule out simple explanations of mean RT effects on the basis of speed-accuracy trade-offs. Both tasks show reliable distance and congruity error rate effects and mutual interference, which, however, is more pronounced for size comparisons. The large main task effect on error rate suggests that the effect of the type of comparison on mean RT, which is only marginally significant, would be larger for comparable error rates under the two tasks.

Taken together, our behavioral results confirm previous principal findings concerning numerical and size distance effects and the mutual interference of physical size and numerical value in digit comparison [1, 11, 23]. They are consistent with a view that size and numerical information are initially processed in parallel channels, with finishing time distributions which overlap, yielding mutual interference, but which (at least under the conditions of the present experiment with its relatively difficult size discrimination) show a shorter mean for numerical information, yielding interference effects which differ significantly in magnitude. In line with this interpretation, for size comparisons, prolonging the stage of size information processing by decreasing size distance from large to small, or speeding up the stage of numerical information processing by increasing numerical distance from 1 to 2, increases the congruity effects. The absence of the corresponding effects for numerical comparisons may be due to the considerably smaller distance main effect for numerical judgements.

So far, our interpretation of the behavioral results in terms of finishing time distributions and their relative overlap is, in principle, vague with respect to the locus of interaction. In particular, it is not clear whether the output of the channels for size information and numerical information are mapped directly onto a joint, integrated representation and potentially interact at a level before the overt response is activated, or whether both channels directly and independently activate the response suggested by the individual partial information conveyed by them. To further distinguish these possibilities we next turn to the ERP results.

Event-related potentials

Mean amplitudes across subjects were calculated for consecutive epochs of 8 ms and then selectively averaged according to task (numerical vs size comparison) and to distance (small vs large, 1 vs 2) and congruity (congruent vs incongruent) within each task. The statistical significance of these factors were evaluated by repeated measures ANOVAs in which each factor was tested against its interaction with the subjects factor. No correction for violations of sphericity needed to be applied because only effects with a single degree of freedom in the numerator were evaluated.

Frontal difference potentials

The earliest reliable electrophysiological effects due to distance and congruity were found at the frontal electrode site Fz.

Figure 2 shows the effects of distance (upper panel) and congruity (lower panel) for numerical (left side) and size (right panel) comparisons at the frontal electrode site Fz. Differences which were reliable at the $P < 0.05$ level are indicated by lines. For numerical comparisons, first reliable effects of numerical distance 1 vs 2 (upper left panel) occur in the frontal N2 latency range after 224 ms and last up to 400 ms. They consist of a larger negativity induced by the numerical distance 1 (digits 4, 6) as compared to the numerical distance of 2 (digits 3, 7). For size comparisons, reliable effects of size distance (upper right panel) start only after 288 ms, i.e. more than 60 ms later than the effects of numerical distance for numerical comparisons. The two lower panels show that this temporal pattern of onset latencies is reversed for the congruity effects. For size comparisons (lower right panel), the first reliable effects of congruity occur after 280 ms and consist of a larger positivity for congruent as compared to incongruent digits. For numerical comparisons, it is only after 368 ms that effects of congruity become significant. A similar pattern of results, although temporally slightly delayed, is found at the central electrode site Cz.

In summary, we find reliable electrophysiological effects of distance and congruity at the frontal and central midline electrodes Fz and Cz, which differ in their onset latencies. Two results are of greatest importance in the present context. First, while the individual main effects of size distance and numerical distance *per se* are easily reconciled with a response competition account of congruity effects, we note that, in addition, the congruency of numerical and size information induces reliable effects at the midline electrodes after 280 ms (size comparisons) and 368 ms (numerical comparisons). As congruity effects cannot be related to the information of just a single digit attribute, but are defined by comparisons of specific combinations of both digits attributes, there must be some kind of functional interaction in the processing of numerical and size information at this point in time. Second, the order of effect onsets across the two tasks is reversed for distance and congruity effects: while distance effects are seen earlier with numerical comparisons, congruity effects arise earlier with size comparisons. In general, congruity effects for one task show onsets of about 60 ms after distance effects arise with the irrelevant attribute. This pattern is consistent with a view that under the conditions of the present experiment numerical information is, on average, obtained earlier than size information. It thus shows earlier distance effects and is at the same time, on average, earlier able to influence in turn size information processing.

P300

A clear positive peak could be identified under all conditions in the mean ERPs of each individual subject that was maximal at the Pz electrode site and rapidly decreased towards the frontal sites. For each combination of task, distance, and congruity, and for each subject, P300 peak amplitude was defined as the maximum positivity at the Pz electrode between 300 and 700 ms after stimulus onset, and P300 latency was defined as the point of time when this peak occurred. The results of the P300 analysis are summarized in Table 2 and the grand average waveforms are shown in Fig. 3.

For numerical comparisons, numerical distance has a significant effect on P300 latency, which decreases with increasing numerical distance. On the other hand, for size comparisons, no reliable effects of size distance on P300 latency were observed. Conversely, congruity has significant effects on P300 latency for size comparisons, but not for numerical comparisons.

In summary, reliable P300 latency effects of numerical distance, but not of size distance were found. Thus, for numerical comparisons the relevant numerical distance induces reliable P300 latency effects, but the irrelevant size information does not affect P300 latency, i.e. there are no effects of congruity. On the other hand, for size comparisons the relevant size distance has no effects on P300 latency, whereas the irrelevant numerical information does affect P300 latency, i.e. there are reliable effects of congruity. Taken together these results are consistent with a view that numerical distance generally

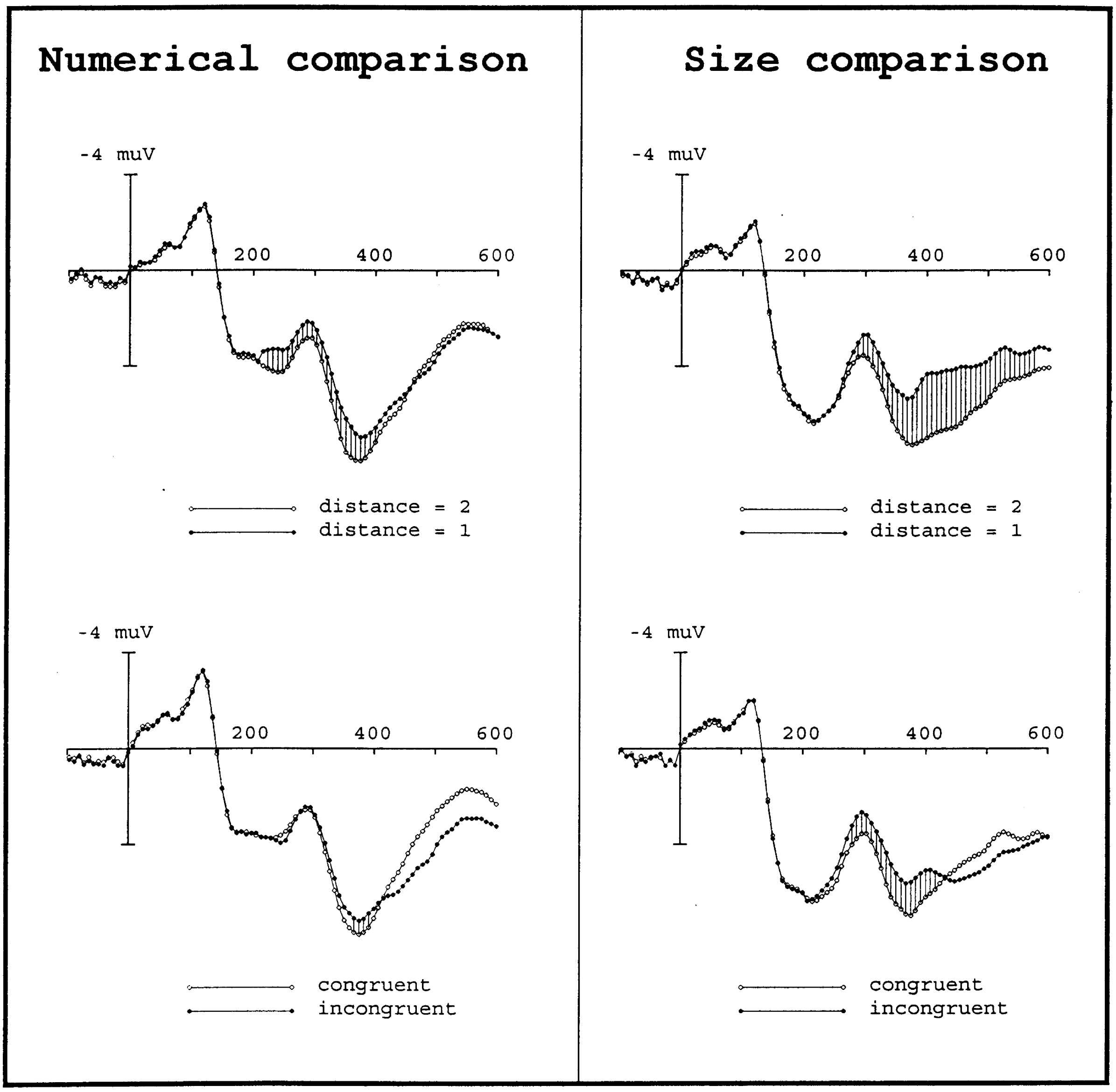

Fig. 2. Grand average (epoch length 8 ms) ERPs at frontal electrode site Fz. Left panels refer to numerical comparisons, right panels to size comparisons. Upper panels: distance effect, filled circles refer to distance 1, open circles to distance 2. Lower panels: congruity effect, filled circles refer to incongruent trials, open circles to congruent trials. Within each diagram, significantly different data points are joined by lines ($P < 0.05$).

Table 2. Effects of distance and congruity on P300 latency for size and numerical comparison

	Numerical comparison				Size comparison			
	No. distance		Congruity		Size distance		Congruity	
	$D = 1$	$D = 2$	Cong.	Inc.	Close	Far	Cong.	Inc.
Latency in *ms*	420	405	411	415	425	414	407	432
$F(1,11)$	8.25		0.29		1.04		4.96	
P	0.015		n.s.		n.s.		0.048	

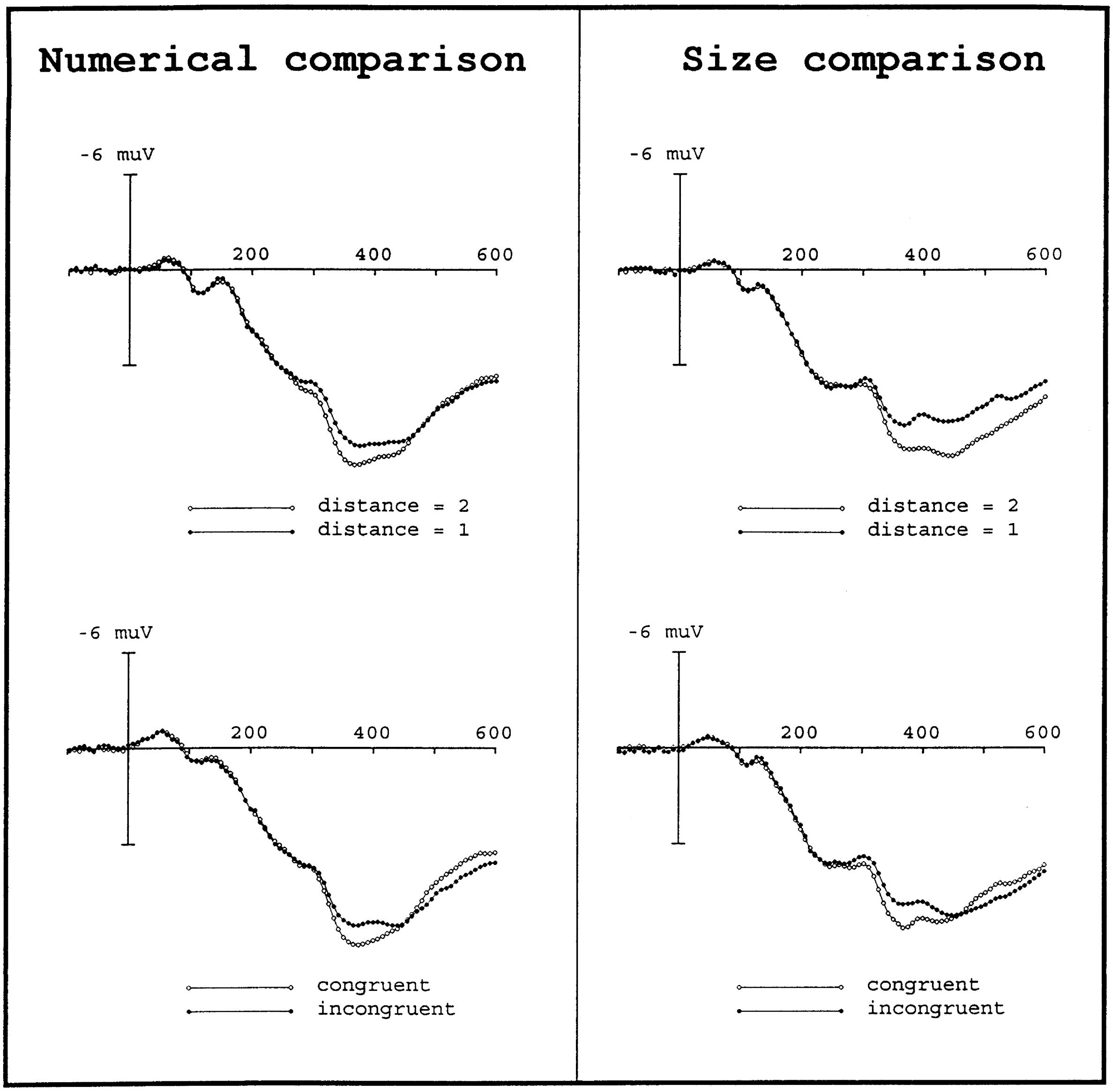

Fig. 3. Same as Figure 2, but for parietal electrode site Pz.

affects P300 latency. If these effects, in fact, index the evaluation of the numerical stimulus information, then an early interaction model (Fig. 1a) would predict that these same effects should also re-occur as congruity effects for size comparisons, because for size comparisons, congruity effects depend on whether the irrelevant numerical information is, or is not, congruent with the relevant size information. The data conform to this prediction in that for size comparisons, congruity reliably affects P300 latency. By the same token, because size distance did not reliably affect P300 latency for size comparisons, an early interaction model would predict that there should also be no effect of the irrelevant size information on numerical comparisons, which is also in line with our results. As noted in the Introduction, the fact that distance-to-the-standard of the relevant digit attribute has main effects on the P300 is easily reconciled with both, early and late interaction models. In particular, distance main effects do not in itself argue against models of late dimensional interaction. However, for size comparisons, we also find reliable effects of congruity on P300 latency. Because congruity effects require some kind of interaction in the processing of the separate attributes, and because the P300 is thought to reflect processes of numerical distance evaluation [8, 9, 20], these findings do not conform to predictions of a response competition account of the observed effects.

Lateralized readiness potentials (LRPs)

Grand average LRP waveforms across subjects are shown in Fig. 4, separately for numerical comparisons (left panel) and size comparisons (right panel). LRPs were

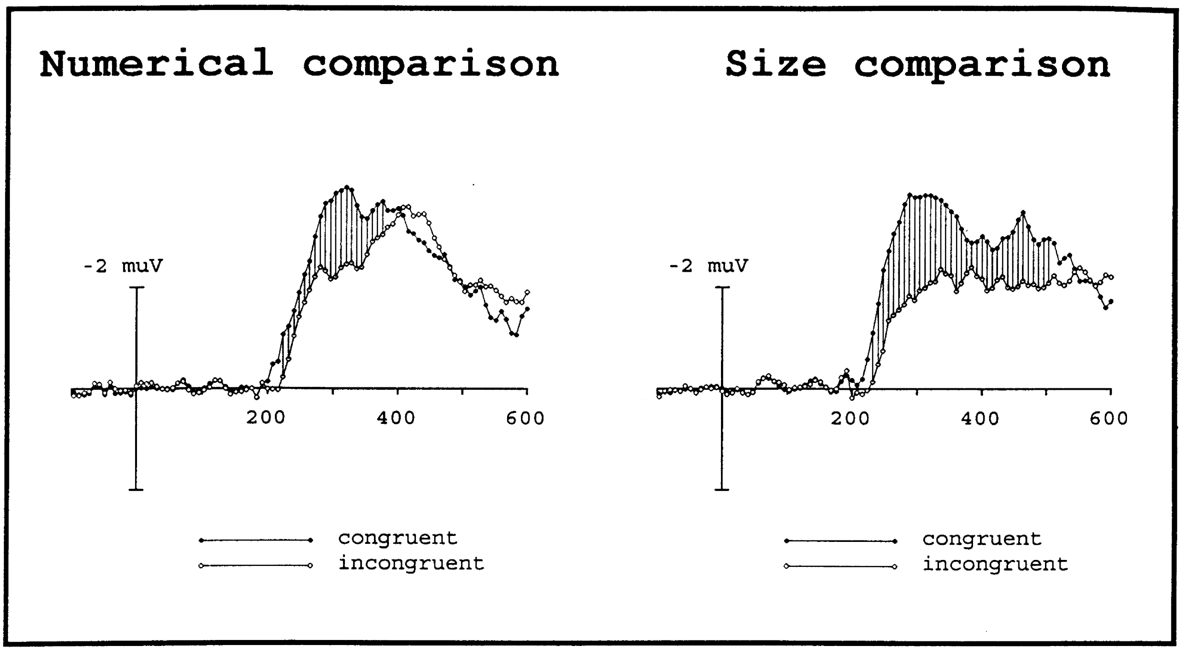

Fig. 4. Grand average LRP for numerical (left side) and size (right side) comparisons. Congruent trials: filled circles, incongruent trials: open circles. Within each diagram, significantly different data points are joined by lines ($P < 0.05$).

calculated for each subject, each 8 ms epoch and each type-of-comparison × congruity combination separately and then evaluated statistically by one-sample Wilcoxon tests (against the null hypothesis of zero LRP) and by two-sample Wilcoxon tests (against the null hypothesis of no effect of congruity on the LRP) using a significance level of $\alpha = 0.05\%$. We theoretically required five consecutively significant differences of the same polarity to protect our procedure against α-errors, because the successive Wilcoxon tests cannot be independent and because of the large number of tests performed (cf. [20]). Practically, however, Fig. 4 indicates that no isolated significant results occurred.

First, no significant positive deflection of the LRP, indicating an initial incorrect response activation, was observed under incongruent trials for either numerical comparisons or size comparisons. In fact, at no epoch came any LRP even close to being significantly positive. Second, for numerical comparisons (left panel of Fig. 4), a reliable negative deflection of the LRP, indicating activation of the correct response, starts 224 ms after congruent stimuli, but only 240 ms after incongruent stimuli. Compared to each other, LRPs for congruent and incongruent stimuli start to differ significantly 224 ms post-stimulus. Third, for size comparisons (right panel of Fig. 4), a reliable negative deflection of the LRP starts 232 ms after congruent stimuli, and 256 ms after incongruent stimuli. Compared to each other, LRPs for congruent and incongruent stimuli start to differ significantly 232 ms poststimulus.

Clearly, these LRP onset results do not conform to the predictions of a response competition account of the congruity effects. In particular, we find no evidence that irrelevant partial numerical information, or irrelevant partial size information, separately activates the response suggested by the irrelevant attribute when this information is incongruent with the information obtained about the relevant attribute. Main effects of congruity show up in the form of temporal shifts of the LRP onset; however, LRPs do not show any deflection towards incorrect responses for incongruent stimuli and thus, may simply reflect effects originally induced during earlier stages of processing.

General Discussion

The present study investigates comparison processes involving different attributes of digits in a situation when the relevant and the irrelevant attribute (their physical size, or their numerical value) changes unpredictably from trial to trial. Previous behavioral findings [1, 11, 23] suggest that size and numerical information are processed in parallel, or at least partially overlapping stages, and show a pattern of mutual interference (which may differ in magnitude) of the task-relevant with the task-irrelevant attribute. As outlined in the Introduction, the locus of this interference effect may be related to early processing stages which carry out processes of stimulus identification and evaluation (Fig. 1a). Alternatively, interference may be related to later processes, such as the activation of the appropriate response (Fig. 1b).

For both tasks, we observed reliable distance effects at several levels of analysis, from early electrophysiological effects in the frontal N2 latency range to overt RT and error rate effects. As noted in the Introduction, the finding of early ERP main effects due to size distance and numerical distance does not, in itself, distinguish models of early and late dimensional interaction; in particular, even under a model of late interaction, there may well be early main effects for each attribute separately. For numerical comparisons, reliable electrophysiological effects of numerical distance start about 60 ms earlier than size distance effects for size comparisons, in keeping with the shorter RTs (although only marginally significant) and lower error rates for numerical judgements. If size information and numerical information are mapped onto a common internal analog representation at an early stage of processing, then the different onsets of numerical distance and size distance effects lead to the prediction that the congruity effects observed under the two tasks should show an asymmetric timecourse that mirror-images the temporal pattern found for the onsets of distance effects. This expectation is in fact borne out in that early frontal congruity effects induced by the irrelevant attribute of the digit are seen about 60 ms earlier with *size* comparisons than with numerical comparisons (Fig. 3). In a similar way, the patterns of reliable P300 latency distance and congruity effects mirror-image each other across the two tasks: for numerical comparisons, numerical distance affects P300 latency, as does congruity for size comparisons. On the other hand, for size comparisons, size distance has no effect on P300 latency and there is also no P300 latency congruity effect due to the irrelevant size attribute for numerical comparisons. Finally, while we do find LRP onsets for incongruent trials to be shifted in time, there is no indication of an activation of incorrect responses, as an account of congruity effects in terms of response competition would predict. Taken together, then, our results clearly conform to the predictions of models which posit an early locus of dimensional interaction, and do not support an account of the observed congruity effects in terms of response competition.

The present results confirm and extend previous principal ERP findings on numerical distance [5, 8, 9, 16, 24] and size congruity effects [1, 11, 23]. The latter authors present evidence for a model of automatic processing of numerical information according to which for size comparisons RT effects of congruity, but no additional effect of the irrelevant numerical distance is predicted. Tzelgov *et al.* [23] assume that for size comparisons only a rough, automatic and fast numerical classification (“smaller vs larger than 5”) takes place; thus, a size congruity effect is predicted, but the finer graded numerical distance information is lost, so that no additional effect of numerical distance is predicted. Clearly, our results fit into this picture in that we did not find an RT effect of the irrelevant numerical distance for size comparisons either. In contrast to the study of Tzelgov *et al.*, which used two digit sizes, we used four digit sizes and thus were able to vary size distance also within a design that was completely orthogonal with respect to numerical value and physical size. Under these conditions, we do find a size distance effect on RT for numerical comparisons, i.e. when size is irrelevant. In terms of the Tzelgov *et al.* model, this could mean that size information, in contrast to numerical information, is automatically processed in more fine detail than is numerical information, even if it is irrelevant. Alternatively, our results could be interpreted to mean that the numerical distance main effects were simply too small to yield significant distance effects when size is relevant. If this latter interpretation is correct, we should expect to find numerical distance effects for size judgements when larger numerical distance main effects are induced, e.g. by using digits which are farther away from the standard, as in [5]. Consistent with this expectation, we may note that Dehaene [5], using the digits 1, 4, 6, 9 together with a standard of 5, reports earlier ERP effects than ours (starting about 174 ms post-stimulus) induced by his wider spaced numerical distances (1 vs 4 as compared to 1 vs 2 in the present study).

Our pattern of results may also be compared to the findings of Otten* *et al.* [20], who studied the joint processing of a digit’s numerical value, and its parity. In contrast to the present results, Otten *et al.* [20] found no P300 effects of numerical distance and congruity, but their LRPs show a clear activation of the incorrect response when the faster processed stimulus dimension (the digit’s numerical value) was irrelevant, i.e. for parity judgements. Also, consistent with earlier behavioral results of Sudevan and Taylor ([22]), RT interference effects were strictly unidirectional, in that congruity had no effect on numerical judgements, but only on parity judgements. Clearly, this pattern of results conforms to the predictions of a response competition account of congruity effects. In principle, this pattern of unidirectional interference effects is also compatible with a serial processing model, in which parity information can be accessed only after detailed numerical information (which independently activates the response it suggests) is available. Several important differences between the study of Otten *et al.* [20] and the present study may be noted.

First, there is, of course, no *a priori* reason to expect that a digit’s physical size and its parity influence processing of its numerical information in any comparable way. In fact, while the unidirectional interference results of Otten *et al.* [20] (and those in [22]) are compatible with the view that information about the parity of a digit is obtained only after its numerical information has been fully processed, the present bidirectional interference results (and those in [1, 11, 23]) point to a parallel activation of both, numerical and size information. These results might be expected if numerical magnitude has, in fact, an internal analog representation, because parity is a

* Here, and in the remainder of this article, we consider the “low-left, high-right” response condition of the Otten *et al.* [20] study only, which corresponds to the response mapping used in the present study.

strictly dichotomous, categorical property of which there cannot be a "more" or a "less". Thus, even if numerical magnitude does have an analog representation, parity is probably not a dimension it could easily interact with. In contrast, in the present study, to access numerical information, the subject must necessarily, at least to some degree, process physical features of the digit in order to identify the digit. Also, both, size and numerical information are fine-graded and admit a "directional" representation in terms of "more" or "less", which may be a requirement to integrate information about both attributes into a single analog representation and, thus, for early bidirectional interference. Furthermore, there are obvious procedural differences. For example, in the present study, the relevant stimulus attribute was indicated well before stimulus onset, whereas in the Otten *et al.* [20] study cue and digit were presented simultaneously. Sudevan and Taylor [22] showed that with increasing time between cue and digit performance improves markedly. Clearly, at least some open questions remain as a target for future research, such as Otten *et al.*'s finding of no distance effect on the P300 for numerical judgements which is not in line with earlier related studies [5, 8, 9, 16, 24] and the present results. In particular, it would be interesting to know whether the simultaneous presentation of the cue (which defines the relevant attribute) and the digit suppresses early electrophysiological distance effects (cf. [9]).

In conclusion, the most parsimonious explanation of our results is that numerical and size information are extracted in parallel, are both converted into an analog representation, which integrates information about both digit attributes, and potentially interact during these early stages of processing. Processes of selective response activation start only after an integrated representation of both attributes is obtained; their timecourse reflects main effects induced during earlier processing stages in the form temporal shifts, but does not show partial activations from each stimulus attribute separately.

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