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Macroeconomic fluctuations and corporate financial fragility

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ABSTRACT

Using a large sample of accounting data for non-financial companies in France, this paper studies the interactions between macroeconomic shocks and companies' financial fragility. We consider links in both directions, namely whether firms' bankruptcies are affected by macroeconomic variables, and whether bankruptcies determine the business cycle. We estimate forecasting equations for firms' bankruptcy using Shumway's (2001) approach and study the joint dynamics of bankruptcies and macroeconomic variables within an exogenous VAR type model estimated at the sector level. We find evidence of reciprocal links between the bankruptcy rate and the output gap and highlight significant "second round effects" of shocks to the output gap on bankruptcies. We show how taking into account the dynamic transmission of macroeconomic shocks matters in stress testing exercises.

1. Introduction

The financial crisis that emerged in the summer of 2007, characterized by the most severe recession in the post-war period and a historical level of business bankruptcies in many countries, has highlighted the need to identify the link between bankruptcies and macroeconomic developments in a dynamic perspective. This is important for the proper implementation of "stress tests" of credit risk that are designed to assess the resilience of the financial sector, notably banks' loan portfolios, to exceptional but plausible macroeconomic shocks. In contrast to the way stress tests are usually carried out, with a one-way impact of the macroeconomic environment on the financial sector, we highlight the need to take account of "second round" effects, namely the reverse impact of bankruptcies on the macroeconomy. To this end, we estimate a 2-equation VAR type model linking the output gap and the bankruptcy rate, also using detailed information on the financial situation of individual firms.

We illustrate our methodology in the case of France and show that second round effects do matter. Fig. 1, where we report the number of corporate bankruptcies in France and a simple measure of the output gap (in inverted scale), provides *prima facie* evidence of the link between the two variables: in the wake of the crisis, in 2010, bankruptcies reached levels that were equivalent to those in the 1992–1993 period.[1] However, taking a longer perspective, one can observe different cases: either bankruptcies led the output gap as in the early 1990s and in 2001–2002, or the output gap led bankruptcies as in the late 1990s, or the two were independent as in 1993–1994, when the increase in the number of bankruptcies outpaced that of the output gap. This was also the case in 2003–2007: more bankruptcies occurred in spite of the upward phase of the business cycle. The Banque de France (2009) stressed in particular that the higher level of business creations during the 2003–2007 period – itself correlated with the business cycle – may explain

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[1] The output gap is computed as the residual of a regression of the logarithm of real GDP on an intercept and a time trend.

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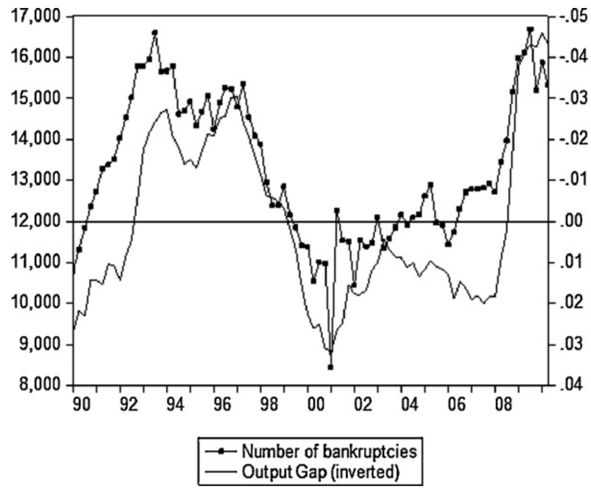

Fig. 1. Number of bankruptcies and the output gap in France (1990–2010). Source: Banque de France.

the increase in the number of bankruptcies. This calls for a more detailed analysis of the dynamic link between business bankruptcies and the business cycle.

There is agreement in the financial economics literature regarding the existence of a link between bankruptcies and the business cycle. This topic has already been extensively investigated and it is now acknowledged that some interaction exists. This led the Basel Committee on Banking Supervision to recommend in 2004 a regulatory framework (commonly known as Basel II) to, *inter alia*, take account of the adverse effect of the macro-economy on banks' loan portfolios, particularly in the implementation of stress tests. More recently, the Basel III framework introduced capital buffers that increase the cost of credit in the upturn, but reduce it during the downturn, highlighting the effect of capital losses on the supply of credit, hence on the business cycle. However, there is no agreement on the channels by which bankruptcies and the business cycle interact, nor on how to measure the link.

Regarding the channels of interaction, the business cycle affects the environment of firms, and hence may explain, with a lag, the changes in bankruptcies over time, in addition to firm-specific variables like financial ratios. On the other hand, bankruptcies may affect the business cycle, marginally through lost capacities of production, and more significantly through credit rationing as shocks to credit supply have often been shown to be leading indicators of the business cycle (Bernanke and Gertler, 1989; Lown and Morgan, 2006). In addition, banks may limit credit supply because they become more risk averse when they observe more bankruptcies or because larger losses constrain their ability to expand assets.

As far as measurement is concerned, the approaches followed in many studies are usually partial, as they focus on one-way interactions between bankruptcies and the business cycle. As mentioned above, our study contributes to this literature by focusing on the French case, providing evidence of two-way interactions, as well as showing how taking account of the dynamic transmission of macroeconomic shocks matters in stress testing exercises.

This paper attempts to merge two strands of the quantitative economic literature regarding how the macroeconomic environment affects financial fragility, and conversely how financial fragility affects the business cycle. We also consider evidence that points to two-way interactions between business bankruptcies and the macroeconomy.

The first strand in the literature is the growing number of quantitative papers focusing on the impact of macroeconomic conditions on the bankruptcy of firms. The different contributions can be distinguished either according to the types of data used, or the method implemented, with overlaps between the two types of papers. *First, regarding data*, macroeconomic indicators have been introduced into the estimation of credit risk models for portfolio management, but we should distinguish between models that use financial market data and models that use accounting data. In the first case, we are mainly looking at large quoted companies. See Allen and Saunders (2004) for a survey of these papers.[2] In the latter case, we consider a larger set of non-financial companies. *Second, regarding the methods used*, we should distinguish between (i) a large number of papers starting from Altman's (1968) seminal paper based on discriminant analysis that predict business failures but without introducing macroeconomic variables, (ii) papers introducing macroeconomic variables using the multi-period Logit model advocated by Allison (1982) and Shumway (2001), (iii) duration models and (iv) other econometric methods. Regarding the first group of papers, we should mention Altman and Saunders (1998), Benito et al. (2004), Bernhardsen (2001) and Bunn and Redwood (2003). Regarding the second group of papers, we refer to the methodology initiated by Allison (1982) and most notably applied by Shumway (2001), who use a particular Logit model in order to measure the dynamic relationship between macroeconomic variables and bankruptcies. In our case, we have access to a large sample of non-financial French firms (an average of 80,000 firms per year) which are observed over a sufficiently long period and make it possible to take into account the progressive deterioration of their financial conditions in predicting business failures, unlike the first generation Logit or Probit models, which only provide a static analysis, period by period, based on a cross-section of accounting ratios, hence without macroeconomic variables. Applications of this method include Chava and Jarrow (2004) and Campbell et al. (2008). In the latter paper, the macroeconomic environment is introduced through financial market variables. In addition, Beck et al. (1998) and Glennon and Nigro (2005) use dummy variables to capture the effects of the business cycle. Hillegeist et al. (2004) introduce the aggregate failure rate of US firms as a proxy for the growth rate of GDP. Nam et al. (2008) study defaults of Korean quoted companies and introduce exchange rate volatility as a macroeconomic variable. Jacobson et al. (2005) use Shumway's (2001) approach to model the default risk of Swedish companies. The third group of papers use duration models, for example Carling et al. (2007), Duffie et al. (2007), Bonfim (2009), Bhattacharjee et al. (2009) and Koopman et al. (2009). But despite the main advantages outlined above, the use of duration models remains limited due to left-censoring problems. Indeed, when the observation period is short, most firms in the dataset were created before the observation period, implying that firms' time at risk may be much greater than the observation period. The fourth group of papers, from the point of view of methodology, use a variety of econometric techniques to estimate bankruptcies, also taking into account macroeconomic variables: Hamerle et al. (2004) estimate a random effect Logit model of bankruptcies for German companies; Bonfim (2009) uses a random effect Probit model for Portuguese firms and Qu (2008) uses a fixed-effect LSDV model. Pederzoli and Torricelli (2005) estimate a state-dependent static Probit model of default, distinguishing between expansion

[2] Credit risk models based on financial market information and designed for the pricing of a portfolio of corporate bond fall into two categories: "structural" models derived from the financial literature (Merton, 1974; Black and Cox, 1976; Hull and White, 2004), "reduced form" models (Jarrow and Turnbull, 1992, 1995; Duffie et al., 1996). However, a limitation of these papers is that they concentrate on a subset of non-financial companies, namely quoted companies or those with access to financial markets.

and recession periods. Jéminez and Saurina (2006) measure the effects of the growth of bank lending on banks' defaults using a random effect Logit model.

A second strand of the literature looks at how the financial fragility of firms affects the business cycle. Such a question is particularly relevant to macroeconomic forecasting with a view to incorporating information at the microeconomic level. Several papers investigate how financial variables, and in particular the financial position of corporate firms, affect the business cycle. In particular, Lown and Morgan (2006) provide evidence that indicators of financial fragility, as measured by business failures, together with credit standards have explanatory power for the growth of bank loans and GDP, on top of standard measures of interest rates on loans.

As indicated above, our objective is to focus on the so-called "second round effects" by taking account of two-way interactions between macroeconomic developments and financial fragility. Very few papers take this approach. In the paper, in order to investigate these "second round effects", we examine how a given initial macroeconomic shock impacts the financial position of firms, which in turn affects macroeconomic variables. Many papers in the stress testing literature design sophisticated macroeconomic scenarios. However, only a small number of them really consider the two-way interactions between bankruptcies and the macroeconomy. Regarding the first type of stress tests – i.e. one-way interaction – one should mention single equation models such as those studied by Sorge and Virolainen (2006). Simons and Rolwes (2009) also use a single equation but focus on the dynamic (autoregressive) structure of Dutch companies' default rate, estimated in a separate Logit model. Multi-equation models can also be used in this first type of stress test. For example, VAR models are used by Alves (2005), highlighting the long-run common dynamics across sectors; Pesaran et al. (2005) as well as Castrén et al. (2010) use GVAR models for the design of the macroeconomic scenarios that affect default probabilities, but there is no feedback effect from bankruptcies to the macroeconomy. To our knowledge, only three papers really consider two-way interactions between aggregate bankruptcies and the macroeconomy. Jacobson et al. (2005) present a model in which observed bankruptcies are introduced as endogenous variables in a VAR with other macroeconomic variables. They also estimate a "micro-macro" model where estimated bankruptcies from a Logit equation with microeconomic variables are introduced as additional variables in an exogenous VAR (VARX) model. However, in the latter case, bankruptcies are no longer fully endogenous. Koopman and Lucas (2005) uncover cyclical comovements between GDP and business failures at long frequencies using a multivariate unobserved component model. Finally, Sommar and Shahnazarian (2009) use a structural credit risk model for calculating the empirical Expected Default Frequency (EDF) for listed companies, and investigate the long-run relationships between expected bankruptcies and macroeconomic developments using a Vector Error Correction Model (VECM). In comparison with the last two papers, we stress the need to use micro-data, as well as to consider a broader set of companies than only those with access to the financial market.

In this paper, we follow the "micro-macro" approach of Jacobson et al. (2005) that we apply to the French case, but consider business bankruptcies as fully endogenous variables, allowing for two-way interactions at the sector level. We provide evidence of second round effects based on the persistence of the shocks to the business cycle, but also explained by the statistically and economically significant effect of bankruptcies on the output gap in our sample.

The paper is organized as follows. In Section 2 we explain our modelling choices. In Section 3 we present the data and the main results we obtain regarding the bilateral effects of macroeconomic conditions on bankruptcies. Variant scenarios and stress tests are considered in Section 4. Section 5 concludes, notably regarding the trade-offs that one may need to make when implementing such a model.

2. Modelling choices

In order to study the dynamic impact of financial fragility on the business cycle, Jacobson et al. (2005) use a VAR model in which the output gap and other macroeconomic variables are included together with indicators of financial fragility. We follow their approach, with a few differences in our modelling choices. First, the frequency of our data is annual – and not quarterly – and we focus on the sector level – not the aggregate level.[3] Second, the fragility indicator we introduce is observable and not estimated. In return, we dot not estimate *stricto sensu* a VAR model, but rather a two-equation system, the equations of which are estimated separately. However, we check *ex post* that the system can be inverted as a standard VAR model in order to provide impulse response functions that allow us to investigate the second round effects we focus on. It is worth emphasizing that an assumption of our modelling approach is the homogeneity across sectors regarding the impact of the business cycle on financial fragility and vice versa. Obviously, it would be interesting to investigate a richer model that makes it possible to introduce different specifications for sectoral dynamics. Such an investigation is left for further research.

The first equation of our system is obtained by aggregating at the sector level a multi-period Logit type equation, that we estimate at the firm level to predict bankruptcies along the lines of Shumway (2001).

More precisely, we estimate the logarithm of the individual odd ratio as a linear function of firm-specific indicators of financial fragility that are drawn from the Banque of France's FIBEN database (see Section 3) and also of macroeconomic variables, all observed at an annual frequency over 17 years. By assuming that all sectors are homogenous regarding the determinants of the default rate and are therefore associated with the same model except for sector fixed-effects, we easily obtain sector results by simply aggregating the firm-specific Logit type model.

Concerning the second equation describing the reverse impact of bankruptcies on the business cycle and more specifically on the output gap, we estimate the parameters of the related linear regression using a PANEL-GMM method.

2.1. *Impact of macroeconomic conditions on the fragility of firms: the multi-period Logit model along the lines of Shumway (2001)*

To explain default risk, one often refers to a latent variable which is the ability of the company to meet its financial debt obligations. If the latent variable is smaller than a critical value (which can be assumed to be equal to 0), the firm goes bankrupt ($y_i = 1$, and $y_i = 0$ otherwise). Such an approach, implemented in the standard way, provides a static analysis of business failure risk. In contrast, as shown by Shumway (2001), a multi-period Logit model can provide a dynamic description of these default events.

The advantage of the multi-period Logit model stems from the fact that it is specified and can be estimated as a standard Logit model provided that the observations are firm-dates (i, t) and not just firms i. Thus, the dependent variable $Y_{i,t}$ is equal to 1 if firm i goes bankrupt at time t and 0 otherwise.[4]

[3] Simons and Rolwes (2009) also focus on the sectoral level.

[4] However, it is worth noting that the individuals are assumed to be independent in such a procedure. It is of course not the case when one considers year-firms

Note that the multi-period Logit model can provide forecasts of default at different horizons. This can be useful if we want to define different indicators of financial fragility, for example if we look at default events over a period instead of defaults at a given point in time.

More precisely, we will focus on the estimation of p_{it}:

$$p_{it} = \frac{1}{1+\exp\left(c+\sum_{j=1}^{J} c_{1j}\mathbf{1}_{ij} + Z'_{i,t_i-H}\beta_1 + X'_{t_i-H'}\beta_2\right)},$$

which is the probability that firm i defaults at t, conditional on the fact that it is still alive at date $t-1$. Z denotes a vector of firm-specific variables, and X a vector of K macroeconomic or sectoral variables with K strictly smaller than T, the total number of observation dates in the sample. The lags H and H' introduced for the microeconomic and aggregate explanatory variables, respectively, account for the lag in the availability of the information used to predict bankruptcy.

This probability depends on lagged information at the micro and macro levels. More precisely, we introduce lags H and H' equal to a minimum of 2 (years). This choice is essentially justified by data quality concerns, since the reliability of accounting data for companies close to bankruptcy or the year of default may be questionable. Indeed, when a company is about to go bankrupt, either it does not provide any accounting information, or this information is not reliable, or the company has already been restructured and is a different entity to the original company.

Moreover, we introduce sectoral effects through the dummy variables 1_{ij} with j denoting a sector index, $1_{ij}=1$ if firm i belongs to sector j and =0 otherwise.

It is interesting to note that the multi-period Logit model can be interpreted as a particular duration model with an exponential hazard function, as proved by Shumway (2001), provided that the lifespan $t_i - \underline{t_i}$ (i.e. the number of years before bankruptcy) is introduced as an additional explanatory variable alongside the Z_i and X variables. In that case, the observations are the firms and not the firm-dates as above and the hazard function is equal to $1-F_{LOGIT}$, where F_{LOGIT} is the cumulated distribution function of the Logit model (See Appendix A for more details).

In our case, the duration variable (the firm's age) has not been introduced because we do not know the date of creation of the firm $\underline{t_i}$. We can introduce the "time at risk" (namely the number of years in the sample before bankruptcy) as a proxy of the firm's age, but at the cost of creating a bias due to the fact that the lifespan of the firms which default at the beginning of the sample period can be severely underestimated. When introducing our "time at risk" variable in our sample, we observed that the estimates of the coefficients β_1 and β_2 were not too different from those obtained when not introducing the "time at risk" variable. However, to avoid any bias effect, in the end we dropped the "time at risk" variable. The model we chose to use is therefore the multi-period Logit model.

Finally, we are able to deduce a theoretical indicator of financial fragility at the firm level as a linear function of firm-specific indicators Z_{it} and aggregate variables X_{jt}, at the sector level, for example the output gap for sector j, or at the macroeconomic level, for the interest rate, the inflation rate, etc.:

$$\tau^{(th)}_{def,it} = \mathrm{Log}\frac{p_{it}}{1-p_{it}} = c + \sum_{j=1}^{J} c_{1j}\mathbf{1}_{ij} + Z'_{i,t_i-H}\beta_1 + X'_{j,t_i-H'}\beta_2,$$

that we rewrite as:

$$\tau^{(th)}_{def,it} = c + \sum_{j=1}^{J} c_{1j}\mathbf{1}_{ij} + A_{12}(L)X_{jt} + d(L)Z_{it},$$

where $A_{12}(L)$ and $d(L)$ denote, as usual, polynomial functions of the lag operator L.[5]

The previous equation, written at the firm level, can then be aggregated at the sector level, by averaging, for each sector j, the individual equations over the firms belonging to this sector ($\widetilde{Z}_{jt}$ is the sector average of Z_{it} for all firms i in sector j), providing the corresponding (theoretical) indicator of financial fragility $\tau^{(th)}_{def,jt}$.

Finally, replacing the theoretical $\tau^{(th)}_{def,jt}$ by its empirical counterpart $\tau_{def,jt} = \mathrm{Log}(f_{jt}/(1-f_{jt}))$ derived from the observed default frequency f_{jt} of sector j at year t leads to the introduction of a residual ϵ_{1jt} so that:

$$\tau_{def,jt} = c_{1j} + A_{12}(L)X_{jt} + d(L)\widetilde{Z}_{jt} + \epsilon_{1jt}. \tag{1}$$

Note that, in their VAR model, Jacobson et al. (2005) measure financial fragility by the average $(1/n)\sum_{i=1}^{n}\widehat{p}_{it}$ of the estimated individual default probabilities derived from the Logit model. By contrast, we measure the financial fragility of sector j by the logarithm of the odd ratio $(\mathrm{Log}(f_{jt}/(1-f_{jt})))$, deduced from the observed default frequency. Accordingly, our fragility indicator is observed and not estimated.

Moreover, we approximate our sectoral fragility indicator $\mathrm{Log}(f_{jt}/(1-f_{jt}))$ by the average of the logarithm of the individual odd ratios $\mathrm{Log}(p_{it}/(1-p_{it}))$ as following[6]:

$$\frac{1}{n_j}\sum_{\text{firm } i\in \text{sector } j}\mathrm{Log}\frac{p_{it}}{1-p_{it}} \approx \mathrm{Log}\frac{f_{jt}}{1-f_{jt}},$$

which allows us to specify the fragility indicator as a linear function of the explanatory X and Z, as expected in a VAR type model.

In what follows, Eq. (1) is the first equation of the system we will refer to.

2.2. Impact of bankruptcies on the macroeconomy and dynamic system

Conversely, to investigate the impact of the financial fragility of firms on macroeconomic conditions, we specify the dynamics of the aggregate variables X_{jt} for sector j as:

$$X_{jt} = c_{2j} + A_{21}(L)\tau_{def,jt} + A_{22}(L)X_{jt} + \epsilon_{2jt}, \tag{2}$$

where $\tau_{def,jt}$ denotes the indicator of firms' bankruptcy risk introduced into Eq. (1). Putting together Eqs. (1) and (2) leads us to write the dynamics according to the following system that can be

as individuals. Indeed, observations related to the same firm at different dates are not independent. That is why Shumway (2001) suggests correcting the number of degrees of freedom for the chi-square distributions used in the standard tests: the right number of degrees of freedom is thus the number of firms (provided that the likelihood is written conditionally on the macroeconomic factors, which justifies the independence assumption for different firms). For our model, we make the same correction as Shumway (2001).

[5] Note that the estimates of the coefficients $A_{12}(L)$ and $d(L)$ derived from the estimated multi-period model implemented at the firm level account for the non-linear dependence of the fragility vis-à-vis the X and Z variables as modelled by a Logit model.

[6] We indeed checked that the approximation is valid in our case, because the probability of default is close to zero.

interpreted as a constrained exogenous VAR (VARX) type model of the form:

$$\begin{bmatrix} \tau_{def,jt} \\ X_{jt} \end{bmatrix} = \begin{bmatrix} c_{1j} \\ c_{2j} \end{bmatrix} + \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{bmatrix} \tau_{def,jt} \\ X_{jt} \end{bmatrix} + d(L) \begin{bmatrix} \widetilde{Z}_{jt} \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_{1jt} \\ \epsilon_{2jt} \end{bmatrix}. \qquad \text{(A)}$$

At the macroeconomic or sector level, the output gap measures the deviation of output from its trend. An increase in the output gap indicates more favourable prospects for companies, implying less risk of default.

According to the hypothesis of homogeneity within and across sectors, the coefficient of the lag operators $A(L)$ and $d(L)$ are the same for all sectors, which are only distinguished through the fixed effects $c_{.j}$.

In accordance with Eq. (1) which is estimated by aggregating the multi-period Logit model specified at the firm level, we assume that $\{A_{11}(L) = 0\}$, which means that the lagged values of $\tau_{def,jt}$ do not explain the current value of $\tau_{def,jt}$. In that sense, the VAR type specification we select is constrained, but this constraint allows us to introduce micro-information into our model, as Eq. (1) of our system is obtained by averaging equations specified at the firm level, as explained before. Moreover, we checked *ex post* that the corresponding estimated residuals $\widehat{\epsilon}_{1jt}$ do not display serial correlations, which indirectly confirms the validity of the constraint $\{A_{11}(L) = 0\}$.

Second, we assume that $\{A_{12}(0) = 0\}$. This excludes the contemporaneous value of X_{jt} in the first equation. However, we assume that $A_{21}(0) \neq 0$ to account for potential contemporaneous correlation between $\tau_{def,jt}$ and X_{jt}.[7]

Note that the constraint $\{A_{12}(0) = 0\}$ would correspond to a standard Choleski decomposition of the variance matrix of the residuals in a VAR specification, as used for example in Jacobson et al. (2005), but with a different ordering of the variables.[8] This constraint expresses the idea that the macroeconomic environment may explain the progressive deterioration of the financial position, hence leading to more bankruptcies a few periods later, while the reverse effect of bankruptcies on the business cycle is likely to occur more rapidly either directly through lost production capacities, or indirectly via procyclical credit rationing.[9] However, it is worth emphasizing that such an instantaneous causality scheme, as well as the others available in the literature, are identification assumptions that cannot be tested.

To summarize, including the constraints, the system we estimate is the following:

$$\begin{bmatrix} \tau_{def,jt} \\ X_{jt} \end{bmatrix} = \begin{bmatrix} c_{1j} \\ c_{2j} \end{bmatrix} + \begin{bmatrix} A_{12}(L)X_{jt-1} \\ A_{21}(L)\tau_{def,jt} + A_{22}(L)X_{jt-1} \end{bmatrix} + d(L) \begin{bmatrix} \widetilde{Z}_{jt} \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_{1jt} \\ \epsilon_{2jt} \end{bmatrix}. \qquad \text{(A}')$$

Finally, it is worth noting that the aggregate counterpart Z_{jt} of the firm-specific financial or accounting ratios Z_{it} are supposed to be exogenous in the model. The formal testing of such a property is reserved for future work, but we refer to the results obtained by Jacobson et al. (2005) to justify this assumption.

[7] We checked *ex post* that there is no correlation between the residuals of the two equations, so that an impulse response analysis on orthogonal shocks can be implemented.

[8] In Jacobson et al. (2005), the default probability is introduced in the last position in the VAR model, so that defaults are assumed to be impacted contemporaneously by all the macroeconomic series, while defaults affect the macroeconomy with a lag.

[9] Lown and Morgan (2006), who measure the impact of credit standards from the Senior Loan Officer survey, which includes banks' assessment of failure risk, as well as Dun & Bradstreet's indicator of bankruptcies, observe that the maximum impact on GDP is reached by the end of the first year (between 3 and 4 quarters) in the case of the US.

3. Data and estimation results

We now present the data that we use, and summarize successively the estimation results obtained for the two equations in our system.

3.1. Data sources

We use detailed accounting data on French companies at the individual level from the Banque de France FIBEN database for the period 1990–2006, from which we extract a sample.[10] This database is used to detect bankruptcies, which are defined as the opening of a judicial procedure for termination of business, independently of the actual legal outcome.[11] Although aggregate data are available for a more recent period, the analysis was carried on a shorter time period. There are two reasons for this. First, individual data are only available with a more substantial lag than for aggregate figures. Second, the 2006 Bankruptcy Act significantly changed the legal procedure, so that we may suspect a shift in the relationship.[12]

It should be borne in mind that the FIBEN database excludes very small companies and is therefore less complete than the set of income tax returns used, e.g., by Domens (2006). Nevertheless, the full set of FIBEN data is referred to as "FIBEN exhaustive" in Table 2.

Our sample of individual data from the FIBEN database initially included yearly accounting information on a sample of 259,890 non-financial companies in France over the 1990–2006 period (hence a total of 1,551,003 accounting statements) with a total of 35,875 bankruptcies.

As is typical with individual data, the database was filtered for outliers since there are a number of extreme values among the observations of the financial ratios constructed from raw data. To ensure that statistical results are not unduly influenced by outliers, the observations were "winsorized": we replaced all observations with a value above the 99th percentile of each variable by that

[10] Two types of data are used, namely financial information on firms and data on bankruptcies. Regarding individual financial data on non-financial companies in France, several sources are available, either from INSEE or from the Banque de France FIBEN database. There are also different sources on corporate bankruptcies, either from INSEE or the Banque de France. In our paper, we use an unbalanced sample of individual companies from the FIBEN database, for which we have information on the date and cause of exit from the sample. This allows us to measure precisely the occurrence of defaults. We check that our sample is representative of national developments using the comprehensive data published by the Banque de France on corporate defaults at the sector level, concentrating on the 1990–2006 period. The data from the Banque de France are quite consistent with those from INSEE at the sector level (see in particular Domens, 2006). In order to compute bankruptcy rates at the sector level, the aggregate number of failures by sector is divided by the number of companies per sector, using INSEE data from the Alisse database. We adjust the level of the computed default probabilities to ensure that they are identical with the ones published by Nahmias (2005) for the years 2002–2004.

[11] Three different judicial procedures are available to deal with corporate bankruptcies in France. When payments are suspended, the Court can open a procedure leading either to a restructuring ("*redressement judiciaire*") or a termination of business ("*liquidation judiciaire*"). Since 2006, companies which have not suspended payments can also make use of a third procedure of "safeguard". However, in all three cases, before the Court is called upon, there is a large number of out-of-court settlements, i.e. which are non-public, but increasingly binding, ranging from an "ad hoc mandate" to a "conciliatory procedure". They are based either on the 1985 Corporate Act, or on jurisprudence, part of which was confirmed by the 2006 Bankruptcy Act. They all offer several additional years to the debtor company before the suspension of payments is declared. The time lags between financial difficulties and the actual bankruptcy are therefore explained by the availability of many legal instruments designed to avoid bankruptcy. In addition, many of the stakeholders (including the entrepreneur) have an interest in not proceeding too fast.

[12] Assessing the consequences of the 2006 Bankruptcy Act is reserved for future work.

value. All values lower than the first percentile of each variable were corrected in the same manner.

Financial accounts are published continuously during the year. Most of them cover the period until 31 December and are typically published in the earlier part of the following year, but they may be made available with a time lag. In addition, some companies close their accounts during the year. We assume therefore that financial accounts published up to 30 June of year t actually cover operations for year $t-1$.[13]

We also have information on the sector to which individual companies belong. Based on the French classification of activities (NAF) we have a total of 10 sectors, namely Agri-Food, Consumer Goods, Capital Goods, Intermediate Goods, Construction, Commerce, Transport, Business Services and Personal Services. When comparing the bankruptcy data in our sample with those available at the sector level, it appears that it is only since 1994 that disaggregated data on bankruptcies are reliable, hence a total of 12 years. We have decided therefore to concentrate on the 1995–2006 period.

We also use data on payment incidents, that is defaults on commercial debt, collected by the Banque de France. Payment incidents are defined as the inability to meet payment obligations on commercial bills because of insufficient cash at hand, or liquidation. Such events do not automatically lead to bankruptcy (as measured by default on financial debt), but can be viewed as a harbinger of future bankruptcy.[14]

3.2. Evidence of the impact of the business cycle on bankruptcies

We now focus on the estimation of the first equation of system (A) at the individual firm level (and not sector level), but introduce sector fixed effects. The bankruptcy probability of a firm i in year t given that the company has not gone bankrupt until $t-1$ is specified as a function of (micro) financial information at $t-3$ and lagged macroeconomic information (hereafter denoted as case "$H-3$"). As explained before, we estimate a multi-period Logit model with the dependent variable $Y_{i,t}$ ($Y_{i,t}=1$ if firm i, still operating at date $t-1$, goes bankrupt at t; $Y_{i,t}=0$ otherwise) and with lagged macroeconomic variables and lagged financial ratios as independent variables.[15]

Thus, the probability of bankruptcy p_{it} underlying the indicator of financial fragility is the probability $P(Y_{i,t}=1)$.

3.2.1. Explanatory variables

In the multi-period Logit specification, as indicated in Section 2.1, we introduced variables with a lag of H periods. This is akin to projections of bankruptcies at horizon $t+H$ (conditional on the absence of bankruptcy at $t+H-1$), based on information at date t.[16] As already mentioned, in what follows, we concentrate on the case with H equal to 3 for the financial variables because of the poor quality of accounting data when the company is about to go bankrupt. We also consider shorter horizons, and therefore introduce macroeconomic variables with shorter lags.[17]

Several recent research papers on bankruptcies point to the robustness of the analysis to its extension to long horizons, such as Campbell et al. (2008) in the USA (horizons from 24 to 36 months) as well as Löffler and Maurer (2008), who consider a range of horizons from 2 to 5 years. The other studies that include lagged macroeconomic variables have already been discussed in Section 1.

Concerning the explanatory variables, we investigate several sets of such variables. In particular, the forecasting models incorporate Altman's (1968) explanatory variables, as well as some variables drawn from reports of the Companies Observatory of the Banque de France (Bardos et al., 2004).[18] But, unlike Shumway (2001), we do not introduce any financial market-driven explanatory variables because we include in the sample a large number of small and medium-sized companies, which do not have access to financial markets.

For each specification of the endogenous variable, we estimate three models: the first one with only financial ratios as independent variables; the second including information about default events and debt with two financial dummy variables:

- IP = 1 if the firm experiences at least one non-payment incident on its commercial debt during the year (=0, otherwise);
- Tax Arrears = 1 if the company is indebted vis-à-vis the government (Tax and Social Security) (=0 otherwise).

In the third specification, we add macroeconomic variables as explanatory variables, introducing an output gap variable (GAP), the long-term interest rate, the inflation rate and the dollar/euro nominal exchange rate (amount of euro per 1 USD).[19] These variables are included with lags, but impact bankruptcies at year t.

Such an approach allows us to measure the information content of the additional variables.

We now provide more details about the definition of the different financial variables that we introduce in the first step model. The ratios that we have retained are the following ones (see Appendix B for further details on the definition of variables and summary statistics):

- Profitability = Total Gross Income/Total assets;
- Leverage = Total Borrowing/Total Liabilities (including equity);
- Liquidity = Liquid Assets/Financial Debt;
- Business credit received (in days of purchases) = 360 × Accounts Payable/Purchases (VAT included);

[13] More precisely, data available between 1 January and 30 June of year t will appear in Year $t-1$, which will therefore cover dates from 1 July $t-1$ to 30 June t.

[14] In our sample, among the firms that go bankrupt, 11.9% experienced a payment incident, while the proportion is only 2.22% for other firms.

[15] The reliance on past information means that the sample at hand is limited to firms that were in operation two ($H=2$) or three years ($H=3$) before going bankrupt. While the model could in principle also be run on the sample of firms that were created the year before bankruptcy, the latter model might be unreliable given the poor quality of data for these firms. In practice, at a given point in time, the model is therefore used to forecast the probability of failure by companies with a minimum number of years in operation.

[16] We project bankruptcy at horizon t, conditional on the absence of bankruptcy until $t-1$, using information available until $t-H$ (actually $t-H$ and $t-H+1$). This is equivalent to projecting bankruptcy at horizon $t+H$, using macroeconomic information at date t and $t+1$, with $H>1$.

[17] As indicated in Section 3.1, financial information at $t-3$ corresponds to information published between 1 July of year $t-3$ and 30 June of year $t-2$. This is the choice usually made by practitioners in France (Bardos et al., 2004). As indicated earlier, taking financial information at $t-1$ is possible but this extended dataset would be less reliable, due to the non-publication of financial information by many companies in the year in which the bankruptcy procedure is launched, or the year before. We decided therefore to exclude these companies from the analysis. Appendix C presents results with financial information at $t-2$. Robustness checks based on $t-1$ information are available from the authors upon request.

[18] The variables suggested by Altman to predict companies' default are described extensively in Altman (1993). They include the following ratios: working capital to total assets (WC/TA); retained earnings to TA (RE/TA); earnings before interest and taxes to TA (EBIT/TA); market equity to total liabilities (ME/TL) and sales to TA (S/TA).

[19] We compute the output gap as the deviation from a linear trend. Although there are several methods available for computing the output gap, they do not have a decisive impact on the results, as shown in Appendix D.1.

• Int = Interest paid/(Interest paid + Total Gross Income).[20]

As mentioned above, we have 1,551,003 year-firm observations and 35,875 bankruptcies among them. As we are focusing on bankruptcies at the 3-year horizon, our sample is reduced to 863,005 year-firm observations, including 13,377 default events. The difference corresponds to the exclusion of companies that go bankrupt within the first two years of their existence.

3.2.2. Empirical results

Table 1 summarizes the main results obtained over the 1994–2006 period, using financial variables at $t-3$ and macroeconomic variables at $t-2$ and $t-3$ as explanatory variables (case denoted "$H=3''$").[21] The results obtained for case "$H=2''$ with financial information at $t-2$ and macroeconomic variables at $t-1$ and $t-2$ are given in Appendix C (Table 8). Note that they are very similar to the ones in Table 1.

First, the microeconomic or financial ratios have a significant information content, with the expected sign for the associated β parameters. The financial dummy variables also have, as expected, a significant (positive) impact on the bankruptcy rate. The macroeconomic variables have a significant coefficient with the expected sign. The GAP variable enters with a negative sign indicating that a lower GAP is associated with higher failures as displayed in Fig. 2. An increase in the interest rate results in higher borrowing costs, hence leading to an increase in the likelihood of bankruptcies. A depreciation of the nominal exchange rate vis-a-vis the US dollar, conditional on inflation and the business cycle, has two opposite effects: imports are more expensive (supply effects), while exports are less competitive (demand effects). In our case, the first effect appears to dominate with a lag of three years, given the presence of the output gap measuring the demand effect.[22] It is therefore likely to anticipate increasing bankruptcies for firms in the home country. Moreover, the discriminant power of the model, as measured by the AUC (Area Under Curve) criterion, increases when additional variables are included.[23]

It is usually observed that adding macroeconomic variables in the form of time series may introduce multicollinearity into the model (Beck and Rucker, 1997), and consequently biases on standard errors. We therefore assess the usefulness of the additional variables using a likelihood ratio (LR) test for nested models (under H_0, all coefficients on macroeconomic variables are equal to zero). The null hypothesis of absence of additional information is clearly rejected, as indicated in Table 1. Furthermore, the coefficients on microeconomic variables are stable across models (I, II and III). We also check, on the basis of a Spearman rank correlation test at the sector level, that there is no remaining cross-correlation among residuals, hence that the macroeconomic variables capture most of the common variations.

Second, based on these models it is therefore possible to compute the aggregate failure frequency for each year. This is the ratio of the number of firms going bankrupt divided by the number of firms. As shown in Fig. 2, the models perform quite well, and the estimated bankruptcy rate with model III (black long dashed line) is actually quite close to the observed one (black short dashed line).[24]

Fig. 2 also displays the output gap (GAP) with an inverted scale. We can conclude from the figure that bankruptcies estimated with model III, as well as actual bankruptcies, are indeed negatively correlated with the output gap. Negative output gaps are associated with higher bankruptcies. Note that such a comovement is not directly the consequence of the inclusion of macroeconomic variables in Eq. (1) of system (A), since these variables are introduced into the model with a lag equal to 2–3 periods. For example, expected bankruptcies for the year 2000 are based on financial information in 1997 and the macroeconomic situation in 1998–1997. However we show that at the aggregate level expected bankruptcies in 2000 are negatively correlated with the output gap in 2000. As indicated in Section 3.3, we take such a comovement into account when modelling the output gap.[25]

3.2.3. Robustness checks

We comment here on the results of several robustness checks. See Appendix D for details.

First we introduce alternative estimates of the output gap, using different degrees of smoothing: as shown in Table 9 of Appendix D.1, the results are broadly similar across indicators of the output gap. In particular, the effect of the output gap indicator is always statistically significant.

Second, we test for in- and out-of-sample performance of the optimal model (III). The estimation of model III for different subperiods provides evidence that the coefficients are stable and that the results are robust.[26] In-sample performance is satisfactory. Out-of-sample results are also quite satisfactory, but slightly better for "$H=2''$, i.e. when using lag 2 for the financial variables and lags 1 and 2 for the macroeconomic variables, than for "$H=3''$.

Finally, we investigate the effect of the transformation of the macroeconomic variables. The use of multi-period Logit models with observed time series may provide forecasts that are very sensitive to abrupt changes in macroeconomic variables and that therefore exhibit a jagged behaviour. To avoid this problem, several studies (Beck and Rucker, 1997; Beck et al., 1998) advocate the use of spline functions to smooth the macroeconomics series and get more realistic out-of-sample forecasts. Eubank (1988) suggests in particular the use of "natural cubic splines". We apply this method.

The estimation of the model with spline transformations of the variables for the case "$H=2''$ improves upon our previous results (available upon request from the authors) and offers better in and out-of-sample forecasts for model III. The improvement is mainly visible for the case "$H=3''$ (see Fig. 8 in Appendix D.2).

3.3. Impact of bankruptcies on the business cycle

We refer now to Eq. (2) of system (A) with just one aggregate indicator measuring the business cycle, namely the output gap. Such an equation is similar to the one introduced by Jacobson et al. (2005), where bankruptcies have an impact on macroeconomic variables but only in the following period.

We use data on value added at the sector level from INSEE National Accounts to construct an output gap indicator at the sector

[20] This indicator is also used by Jacobson et al. (2005). An alternative indicator is Interest Paid/Financial Debt, which yields very similar results.

[21] We use automatic procedures to select explanatory variables including lagged variables, namely stepwise, forward and backward. All these procedures select the same variables as the ones presented in Table 1.

[22] In the case of France, a change in the value of the US dollar directly affects the imported energy bill, which is paid in US dollars, while the dollar area is only a minor export market for the country, whose trading partners on the export side are rather located in the euro area (60%), the UK, Switzerland and Eastern Europe.

[23] That is, its ability to discriminate between firms going bankrupt and those that are not going bankrupt.

[24] The RMSE of Model III is 0.122%, while for Model II and Model I it is 0.672% and 0.692%, respectively.

[25] Note that in the VAR framework of system (A'), the comovement is taken into account through the other equation, namely the equation for the output gap.

[26] As in many other empirical studies, we split our sample into two parts corresponding to 2/3 (for estimation) and 1/3 (for out-of-sample analysis).

Table 1
Estimation of individual bankruptcy rates at date *t* using multi-period Logit models: case "*H* = 3″.

Variable	Model I		Model II		Model III	
	coef.	s.e.	coef.	s.e.	coef.	s.e.
Micro variables						
$Cst.$	-6.0702^{***}	0.0592	-6.3487^{***}	0.0690	-17.7689^{***}	0.3337
$Profit_{t-3}$	-2.7730^{***}	0.0934	-2.6064^{***}	0.0935	-2.7160^{***}	0.0942
$Leverage_{t-3}$	3.2956^{***}	0.0457	3.0779^{***}	0.0464	3.0446^{***}	0.0467
Liq_{t-3}	-0.0091^{***}	0.0009	-0.0082^{***}	0.0009	-0.0069^{***}	0.0009
$Bus.Cred._{t-3}$	0.00081^{**}	0.00019	0.00082^{**}	0.00019	0.0006	0.0002
Int_{t-3}	0.2464^{***}	0.0193	0.2230^{***}	0.0193	0.1658^{***}	0.0191
$Size(large)_{t-3}$	-1.8121^{***}	0.1000	-1.7036^{***}	0.1001	-1.5553^{***}	0.1003
$Size(medium)_{t-3}$	-0.3564^{***}	0.0404	-0.3182^{***}	0.0406	-0.1749^{**}	0.0410
IP_{t-3}			1.1403^{***}	0.0291	1.0774^{***}	0.0297
$TaxArrears_{t-3}$			0.3298^{***}	0.0418	0.3234^{***}	0.0419
Macro variables						
GAP_{t-3}					-0.2472^{***}	0.0208
INF_{t-3}					-0.4338^{***}	0.0287
IRL_{t-3}					0.5265^{***}	0.0327
$Ex.Rate_{t-3}$					0.0543^{***}	0.0026
GAP_{t-2}					-0.3954^{***}	0.0229
INF_{t-2}					-0.5933^{***}	0.0260
IRL_{t-2}					0.1211^{**}	0.0214
$Ex.Rate_{t-2}$					0.0397^{***}	0.0018
$\widetilde{R}^{2}$ [c]	0.0914		0.1015		0.131	
LR test [a]			235.86^{***}		998.21^{***}	
AUC [b]	0.764		0.773		0.803	
Nb.obs.	863,005		863,005		863,005	

This table provides the estimation results, using a multi-period Logit model, for the probability of bankruptcy of individual firms at date *t*, based on three sets of explanatory variables: with financial (or micro) variables only (Model I), with the addition of a few dummy variables (Model II), and when macroeconomic variables are also added (Model III). The definition of variables is in Appendix B.

[a] Likelihood Ratio test statistic for nested models $\sim\chi^2_{(df)}$ with $df = 2$ (resp. 8) for Model *I* vs *II* (resp. Model *II* vs *III*).

[b] Area Under the Receiver Operating Characteristics (ROC) Curve.

[c] Nagelkerke's $\widetilde{R}^2$ is obtained as: $R^2/R^2_{\max}$ with $R^2 = 1 - \{LnL_{\max}(\beta = 0)/LnL_{\max}(\widehat{\beta})\}^{2/N}$ and $R^2_{\max} = 1 - \{LnL_{\max}(\beta = 0)\}^{2/N}$. $\widetilde{R}^2$ measures the discrepancy between the Logit models with $(\widehat{\beta})$ or without $(\beta = 0)$ the dependent variables *X*. *N* is the sample size.

*Significance at 10% level.

** Significance at 5% level.

*** Significance at 1% level.

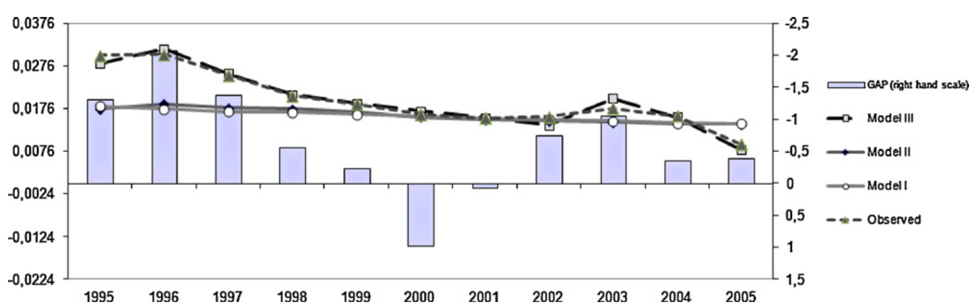

Fig. 2. Aggregate bankruptcy rate observed and estimated at t for the case "*H* = 3″. The Figure displays the negative relationship between the output gap (GAP, with inverted scale) and the bankruptcy rate estimated by model III (black long dashed line with box), which is closer to the observed rate (black short dashed line with triangle) than the one estimated by model II (grey solid line with diamond) or model I (grey solid line with box). See results in Table 1.

level.[27] We regress the output gap at the sector level (GAP_j) on its past values and the (contemporaneous) financial fragility indicator $\tau_{def,j}$. Parameters are estimated by GMM, using Blundell and Bond's (1998) System GMM method, since OLS is biased due to the correlation between the error term and the lagged endogenous variable.

The results are reported in Table 2, when the financial fragility indicator is computed for the three databases – the INSEE data, the exhaustive FIBEN database, and the "FIBEN sample" – that we used to estimate our multi-period Logit model.[28] The results are quite similar, indicating that the FIBEN sample is representative of macroeconomic developments. As indicated in Table 2, the sector bankruptcy ratio appears to contain statistically significant information, in addition to the AR(2) specification of the dynamics of the output gap: the related coefficient is equal to −0.2429 whereas the coefficients of GAP_{-1} and GAP_{-2} are 0.8874 and −0.3682. Note also that we cannot find any significant coefficient for lagged values of the bankruptcy variable, so that it only enters contemporaneously.

[27] As previously, the output gap at the sector level is computed as the residual of a regression of the logarithm of sectoral real value added on a linear trend and an intercept, using data from INSEE National Accounts (working day adjusted).

[28] The "FIBEN sample" is also the database used by the Banque de France for the calculation of scores for the financial assessment of companies.

Table 2
Dynamic Panel estimates of the output gap at the sector level (GAP_j).

Variable	INSEE		FIBEN (exhaustive)		FIBEN (sample)	
	coef.	s.e.	coef.	s.e.	coef.	s.e.
$GAP_{j,-1}$	1.0804***	0.0670	1.0569***	0.0649	0.8874***	0.0721
$GAP_{j,-2}$	−0.5198***	0.0521	−0.5032***	0.0548	−0.3682***	0.0567
$\tau_{def,jt}$	−0.0841**	0.0387	−0.0997*	0.0535	−0.2429*	0.1324
m_1	0.02		0.02		0.02	
m_2	0.11		0.12		0.26	
Sargan[a]	0.40		0.39		0.27	
$Nb.obs.$	120		120		110	

We regress the output gap at the sector level on its past values and on the financial fragility indicator, comparing the results from the same model applied to various datasets on bankruptcies, either the ones published by INSEE, or the full FIBEN database or the "FIBEN sample" used for Eq. (1). The estimation technique used is System Dynamic Panel GMM with the Blundell and Bond's (1998) 1-Step method. The estimation period is 1995–2006 and the number of sectors is 10. m_1 and m_2 are p-values of Arellano–Bond tests of $AR(1)$ and $AR(2)$ serial correlation, respectively. $\tau_{def,jt} = \text{Log}(f_{jt}/(1-f_{jt}))$.

[a] p-value of Sargan's over identification test of instruments.
* Significance at 10% level.
** Significance at 5% level.
*** Significance at 1% level.

We find that the values of the estimates are relatively close for the different cases. We do not find any serial correlations of order 2 (Arellano and Bond test). Moreover, the instruments are validated by the Sargan/Hansen test.

Let us mention here that we also estimated a more comprehensive model of GAP, with additional independent variables (long-term interest rate and inflation); the coefficient of the bankruptcy rate remains significant with a quite similar value (results are available upon request).

4. Variant scenarios and stress testing

Until now, we have investigated separately the one-way impact of (i) macroeconomic conditions on bankruptcies, (ii) bankruptcies on the business cycle. In particular, (i) can be illustrated through traditional stress test exercises, using the multi-period Logit model to investigate how shocks to the output gap affect the distribution of bankruptcies.

But, as stated above, we aim to take into account the dynamic transmission of macroeconomic shocks in stress testing exercises and for this purpose we also investigate feedback effects, using the system of two equations introduced in Section 2 with the output gap and bankruptcies. Within such a system, we implement a traditional impulse response analysis in order to quantify the impact on bankruptcies of an output gap shock.

Note that the estimates of the parameters of the system are reported in Tables 1 and 2, as we estimate the two equations separately.

The constraints imposed on the first equation by the aggregation method we have adopted have been *ex post* justified by checking that the corresponding estimated residuals do not present any serial correlations. Concerning the second equation, the endogeneity of the indicator of financial fragility has been taken into account by using relevant instruments. Thus, the results of the AR(2) and Sargent tests proposed by Arellano and Bond (1991), allow us to claim that the parameters of the second equation are correctly estimated.

4.1. Single equation stress test exercise: impact of an adverse shock on the output gap on the distribution of bankruptcy probabilities

We consider here a simple scenario of a severe recession, with an adverse shock to the output gap. We distinguish whether or not we are using the most comprehensive model on macroeconomic variables described in Appendix E.2, where we take into account the fact that a shock to the output gap is associated with a change in interest rates and inflation.

The scenario is characterized by a drop in the output gap of 2 standard deviations in 2002 (this corresponds to a more negative output gap, which in absolute value terms reaches the maximum amplitude observed over the 1990–2006 period) instead of the small decline observed in the data. For each scenario, we provide $\overline{\tau}_{def,t}$, which is the average across all firms of the bankruptcy indicator $\tau_{def,it}$, as well as the average of the distribution of bankruptcy probabilities p_{it}, and its various quantiles.

As indicated in Table 3, which provides indicators of the distribution of bankruptcies for all years, the shock shifts the bankruptcy probability distribution to the right, indicating that a higher fraction of firms experience a higher bankruptcy rate. When taking into account the change in the output gap alone, the mean of the distribution shifts from 1.5% to 1.7% and the median shifts from 0.9% to 1%. When taking into account the effect on the other macroeconomic variables (inflation and interest rates), there is hardly any further change in the overall distribution compared with the first stress scenario.

However, the shift in the distribution of bankruptcy probabilities year-by-year is more significant than for the overall distribution. Given that the shock affects the bankruptcy probability with a lag, the distribution is mainly affected in 2004 and 2005. We can see from Table 4 (2004) and Table 5 (2005) that the upward shift in mean for 2004 (from 1.55% to 3.28%) is greater than for 2005 (from 0.79% to 1.28%). This can be explained by the fact that, as indicated in Table 1, the coefficient of lag 2 of GAP is larger in absolute value terms than that of lag 3 in the bankruptcy equation.

We can also observe an increase in $\overline{\tau}_{def}$, which is a monotonic function of the default probability. The indicator $\overline{\tau}_{def}$ increases from −4.56 to −3.77 in 2004 (+0.80) and from −5.27 to −4.78 in 2005 (+0.49). When taking into account the whole macroeconomic environment (line "after stress-with full macro"), $\overline{\tau}_{def}$ increases further in 2005 (from −4.78 to −4.50), while it decreases in 2004 (from −3.77 to −4.10).

It appears therefore that the full set of conditioning variables matters in assessing the impact of a scenario on the bankruptcy rate. Indeed, as Tables 3–5 indicate, the bankruptcy profile is slightly different when the sensitivity of the bankruptcy rate to inflation and interest rates is accounted for. With the output gap only, the bankruptcy rate increases more in 2004 (+1.72 pts, from 1.55% to 3.28%) than in 2005 (+0.49 pt from 0.79% to 1.27%). When all the macroeconomic variables are introduced, the impact is also

Table 3
Distribution of bankruptcy probabilities before and after stress.

(Prob. as a %)	Mean	S.e.	P05	P25	P50	P75	P95	P99	P99.5	$\bar{\tau}_{def}$
Before stress	1.550	2.358	0.053	0.3524	0.914	1.862	4.946	11.07	15.12	−4.892
After stress without macro	1.732	2.556	0.054	0.4019	1.029	2.090	5.559	12.18	16.45	−4.785
After stress with full macro	1.694	2.478	0.050	0.4124	1.036	2.054	5.320	11.75	15.95	−4.795

The table displays the distribution of bankruptcy probabilities before and after a scenario of stress characterized by a drop in the output gap of 2 standard deviations in 2002. The line "After stress – without macro" indicates that the macroeconomic variables (INF and IRL) do not react to the shock on GAP. "After stress – with full macro" indicates that macroeconomic variables (INF and IRL) react to the shock on GAP (see Appendix E). We report the mean, standard deviation and various quantiles of the distribution of bankruptcy probabilities for the whole sample. We also report the average of the bankruptcy indicator.

Table 4
Distribution of bankruptcy probabilities in 2004 before and after stress.

(Prob. as a %)	Mean	s.e.	P05	P25	P50	P75	P95	P99	P99.5	$\bar{\tau}_{def}^{2004}$
Before stress	1.554	1.844	0.214	0.593	1.061	1.858	4.346	8.965	11.88	−4.557
After stress without macro	3.281	3.539	0.469	1.297	2.307	4.000	9.090	17.81	22.89	−3.768
After stress with full macro	2.2402	2.709	0.337	0.933	1.664	2.901	6.689	13.45	17.55	−4.101

The table displays the distribution of bankruptcy probabilities in 2004, before and after a scenario of stress characterized by a drop in the output gap of 2 standard deviations in 2002. The line "After stress – without macro" indicates that the macroeconomic variables (INF and IRL) do not react to the shock on GAP. "After stress – with full macro" indicates that macroeconomic variables (INF and IRL) react to the shock on GAP (see Appendix E). We report the mean, standard deviation and various quantiles of the distribution of bankruptcy probabilities in 2004. We also report the average of the bankruptcy indicator.

more pronounced in 2004 (+0.68 pt, from 1.55% to 2.24%) than in 2005 (+0.38 pt, from 0.79% to 1.17%), but the difference is less significant than in the previous case.

4.2. Dynamic stress test exercise using an impulse response analysis

We now consider system (A) of two equations with just two components: the default ratio τ_{def} and the output gap.

The first equation, derived from the multi-period Logit model measures the effect of activity on future bankruptcies. The second equation measures the impact of bankruptcies on the output gap in addition to the autoregressive effects. Linking the two equations allows us to implement an impulse response analysis to examine how shocks to activity are propagated and affect the default ratio as well as the output gap over time. This is similar to the approach of Jacobson et al. (2005) and De Graeve et al. (2008) but here we introduce the observed default ratio – and not the estimated one – as an endogenous component. Moreover, we work at the sector level and not at the aggregate level like the aforementioned authors.

A further refinement of the model for Eq. (1) is to introduce inflation, long-term interest and exchange rates (denoted M below), in accordance with multi-period Logit model III. However, we do not regard these variables as additional components of the VARX model, because they are not sector-specific. Nevertheless, we take into account their response to an output gap shock through a satellite VAR model estimated on quarterly data (see Appendix E.2).

We consider shocks to the output gap and bankruptcies that are identical for all sectors.

Our two-equation system at the sector level (j) is therefore:

$$\begin{cases} \tau_{def,jt} = a_1 GAP_{jt-2} + b_1 GAP_{jt-3} + c_1 M_{t-2} + d_1 M_{t-3} + e_1 Z_{j,t-3} + \epsilon_{1jt} \\ GAP_{jt} = b_2 GAP_{jt-1} + c_2 GAP_{jt-2} + a_2 \tau_{def,jt} + \epsilon_{2jt} \end{cases}, \tag{B}$$

where $M_t = (INF, IRL, Ex.Rate)$ and Z_t are firm-specific variables. Its parameters are estimated as explained before.

We consider shocks to the output gap and bankruptcies that are identical for all sectors and we invert the previous system like a VAR model (see Appendix E for the complete system) in order to get the standard impluse response functions, according to:

$$\begin{pmatrix} \tau_{def,jt} \\ \chi_{jt} \end{pmatrix} = F(L) \begin{pmatrix} \epsilon_{1jt} \\ \epsilon_{2jt} \end{pmatrix}, \tag{3}$$

where $\chi_{jt} = GAP_{jt} - a_2 \tau_{def,jt}$.

IRFs to a positive one standard deviation shock to the output gap can be computed using the standard method on our system. They indicate that the default ratio responds negatively to an output gap shock and the output gap responds negatively to a bankruptcy shock. We provide the standard error around the IRFs using a Monte Carlo simulation (with random draws on the a's, b's as well as the c and d coefficients).

In Fig. 3, notice that the impacts of the shocks are transitory and return to zero after a few years. However, they are more persistent and larger than the ones highlighted in the previous stress test exercises, based on the first equation only. One should acknowledge that these results are partly a consequence of the more persistent nature of the output gap shock: arguably in our system of equations, since the output gap is an autoregressive process, a shock to the output gap at date t is propagated at t+1. Nevertheless, in the output gap equation, the default ratio τ_{def} also enters with a significant coefficient.

Table 5
Distribution of bankruptcy probabilities in 2005 before and after stress.

(Prob. as a %)	Mean	s.e.	P05	P25	P50	P75	P95	P99	P99.5	$\bar{\tau}_{def}^{2005}$
Before stress	0.789	1.048	0.103	0.288	0.521	0.926	2.213	4.847	6.659	−5.272
After stress without macro	1.275	1.625	0.169	0.472	0.850	1.508	3.572	7.699	10.46	−4.779
After stress with full macro	1.166	2.054	0.223	0.619	1.114	1.972	4.642	9.878	13.31	−4.506

The table displays the distribution of bankruptcy probabilities in 2005, before and after a scenario of stress characterized by a drop in the output gap of 2 standard deviations in 2002. The line "After stress – without macro" indicates that the macroeconomic variables (INF and IRL) do not react to the shock on GAP. "After stress – with full macro" indicates that macroeconomic variables (INF and IRL) react to the shock on GAP (see Appendix E). We report the mean, standard deviation and various quantiles of the distribution of bankruptcy probabilities in 2005. We also report the average of the bankruptcy indicator.

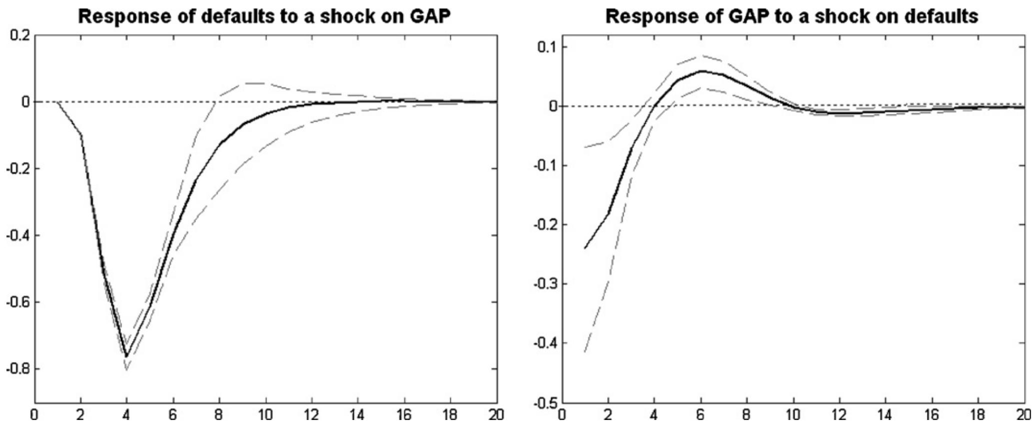

Fig. 3. Impulse Response Function from Panel VAR (%)-black solid line, with Monte Carlo confidence intervals at 90%, 1000 Trials (dashed grey lines). The Figure displays the response of the default ratio and the output gap to a positive one standard deviation shock to the GAP, including the effect of the initial shock to the GAP on the additional macroeconomic variables (INF, IRL). The simulation of our system (B) of two equations allows us to consider second round effects.

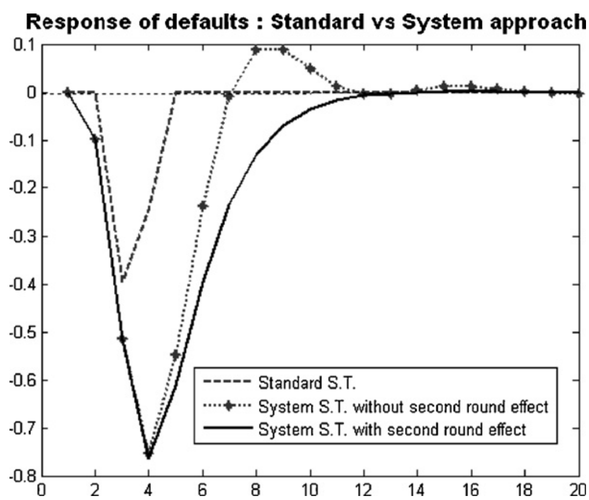

Fig. 4. Comparison of stress testing (S.T.) exercises. This Figure displays different impulse responses of defaults to a positive one standard deviation shock on the output gap with different stress test (S.T.) approaches namely, "Standard S.T" (single equation: first equation of system (B)), "System S.T. with second round effect" (running dynamically the whole system (B), as in Fig. 3) and "System S.T. without second round effect" (running dynamically the constrained system (B) with $a_2 = 0$).

In order to highlight the differences between the approaches, we show in Fig. 4 the response of bankruptcies to an output gap shock in the following different cases: (i) only considering the equation for bankruptcies (as in Section 4.1); (ii) using the bivariate system of equations; (iii) using the bivariate system of equations but constraining the coefficient of τ_{def} to be equal to zero in the output gap equation without re-estimating the whole system (this is labelled as the scenario "without second round effects").

With the default equation only (case i), a positive shock to the output gap at year t has a negative effect on the default ratio (hence on default frequency) at $t+2$ and $t+3$ (dotted line, at year 3 and 4 in Fig. 4). A one standard deviation shock to the output gap has an effect of −0.4 on the default ratio, a result which is consistent with the scenario presented in Section 4.1.[29] Within the VAR system, second round effects are taken into account, coming from the autoregressive feature of the GAP dynamics and from the impact of bankruptcies on the GAP (case ii). Bankruptcies return to baseline only after year 6 (solid line in Fig. 4). Moreover the effects are also greater: a positive shock to the output gap implies lower bankruptcies (Eq. (1)), which in turn is expected to improve the output gap (Eq. (2)), hence decreasing further the default ratio τ_{def} (Eq. (1)).

The maximum response of the default ratio to a one standard deviation shock to the output gap is almost −0.8, i.e. twice as large as in case i. However, comparing the solid line, on the one hand, and the dotted-dashed line (case iii), on the other hand, it turns out that the effect of the output gap on the probability of bankruptcy is not limited to the autoregressive part of the output gap, given the significant contribution of the default ratio to the output gap. Indeed, without the feedback effect of bankruptcies on the output gap, the IRF of bankruptcies is hump-shaped and less persistent (see dotted-dashed line in Fig. 4).

There is therefore evidence of second round effects from a shock to the output gap. This effect is both statistically and economically significant. This is therefore an important feature that needs to be taken into account when designing and implementing stress tests.

5. Conclusion

This paper reports empirical evidence of the links between macroeconomic cycles and changes in financial fragility measured at the microeconomic level by focusing on corporate firms in France.

We estimate a VAR type system of two equations at the sector level: the first describes an indicator of firms' bankruptcies – measured as the Log of the odd ratio – as a linear function of financial ratios and macroeconomic variables; the second specifies the dynamics of the output gap as a linear function of firms' bankruptcies.

The first equation is estimated at the firm level and then aggregated at the sector level. This equation is derived from a multi-period Logit model in order to take account of the progressive deterioration of the financial conditions of firms in predicting their potential default. This multi-period Logit model is easily implemented through a standard Logit estimation along the lines of Shumway (2001). We show that macroeconomic conditions do have an effect on the bankruptcy rate of corporate firms, as proved by the significant impact of lagged macroeconomic variables as well as financial ratios in the multi-period Logit model estimated for each firm. Indeed, we find that the output gap included in this model with lags of two and three years has a significant negative effect on the default probability estimated at the firm level.

[29] In Section 4.1, a two standard deviation shock to the output gap had an effect of 0.8 on τ_{def}. Since the model is linear in τ_{def} and GAP, a one standard deviation shock to GAP now has an effect half as large as the previous one on τ_{def}, namely of 0.4.

Second, by regressing the output gap on its lagged values and the observed bankruptcy indicator at the sector level, and by using a panel GMM estimation method, we find a negative coefficient for the bankruptcy rate, which is only significant when introduced contemporaneously into the equation. It provides evidence that the financial fragility of firms does indeed have an impact on the business cycle.

Third, using the system of the two previous equations, which assess the joint dynamics of the output gap and financial fragility, we highlight the importance of taking into account in stress testing exercises both the persistence of the output gap and the feedback effects of bankruptcies on the output gap. Indeed, given the statistically and economically significant effect of bankruptcies on the output gap, second round effects appear quite clearly. The fact that dynamic effects matter also implies that, following an output gap shock, the distribution of bankruptcies is expected to be more permanently affected than in standard stress testing exercises. The proper design of the macroeconomic environment of stress tests therefore requires taking account of the feedback effects that can be captured through financial fragility indicators, such as bankruptcy rates.

In the course of the paper we have identified a few trade-offs associated with our approach. First, endogenising defaults through the use of accounting data, which are annual, restricts the analysis to lower frequencies. But accounting data at the micro level allow a more robust estimation of the model parameters. They also cover, in our case, a larger set of companies than only quoted firms, given the existence of many small and medium-sized firms. While quoted firms offer a larger set of indicators, notably asset prices, financial markets data may sometimes provide inadequate signals during crisis periods. Second, using a bivariate system, together with exogenous variables, instead of a larger VAR, increases the robustness of the results as well as reduces the need for additional identification assumptions.

In this study, we assumed that all sectors are homogenous regarding the impact of the business cycle on the bankruptcy rate. Future research could consider estimating separate default models for the different sectors. In that case, our methodology to assess second round effects could be extended relatively easily. It would also be useful to compare our results with estimates of the impact of bankruptcies on the business cycle in other countries.

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Appendix A. Shumway's (2001) multi-period Logit as a particular duration model

We discuss here the link between Shumway's model and a duration model.

Let us suppose that we observe n firms over the period $[0, T]$ and for each firm i we define the lifespan d_i, that is, the duration before default. If t_i denotes the default date and $\underline{t_i}$ the date of creation of firm i,

$$t_i = \underline{t_i} + d_i$$

d_i can be interpreted as a realization of a random variable D_i.

Let us assume that the population is homogeneous and that the default events are independent across different firms. Such an assumption is at first sight questionable but becomes acceptable provided that the default events are investigated "conditionally on macroeconomic information", which corresponds to our specification choice.

Let f denote the density distribution function of D_i; it is identical for all firms, according to the homogeneity hypothesis. A firm i contributes to the likelihood by $f(d_i)^{y_i}$ (i.e. $y_i = 1$) if it goes bankrupt at $t_i = \underline{t_i} + d_i \le T$. Otherwise, it contributes by $S(c_i - \underline{t_i})^{1-y_i}$, where S denotes the survival function.[30]

The likelihood can be written as:

$$\begin{aligned} L(u_1 \dots, u_n) &= \prod_{i=1}^{n} f(u_i - \underline{t_i})^{y_i} S(u_i - \underline{t_i})^{1-y_i} \\ &= \prod_{i=1}^{n} h(u_i - \underline{t_i})^{y_i} \prod_{d^{(k)} < u_i - \underline{t_i}} (1 - h(d^{(k)})). \end{aligned}$$

with $u_i = \min(t_i, c_i)$ where t_i is the date of bankruptcy of firm i if it defaults ($y_i = 1$) and c_i the date at which firm i leaves the sample: $c_i = T$ if the firm is observed over the whole sample period or $c_i < T$ if firm i leaves the sample for a reason other than bankruptcy (in the case of a merger, for example); in both of the latter cases, $y_i = 0$. The duration observed for firm i is thus $u_i - \underline{t_i}$, by denoting $\underline{t_i}$ the date of the business start for firm i.

By introducing explanatory variables according to the specification of the hazard function proposed by Shumway (2001):

$$h\left(u_i / Z_{i,u_i-H}, X_{u_i-H'}\right) = \frac{1}{1 + \exp(c + Z'_{i,u_i-H}\beta_1 + X'_{u_i-H'}\beta_2 + \beta_3(u_i - \underline{t_i}))}$$

with Z_i and X denoting firm-specific and macroeconomic variables, respectively, the previous likelihood can be rewritten as:

$$\begin{aligned} L\left(u_1, \dots, u_n / Z_i; X\right) = &\prod_{i=1}^{n} \left(\frac{1}{1 + \exp(f(Z_{i,u_i}, X_{u_i}, u_i)} \right)^{y_i} \\ &\times \prod_{\underline{t_i} < t < u_i} \left(\frac{1}{1 + \exp(-f(Z_{i,t}, X_t, t - \underline{t_i}))} \right) \end{aligned} \quad (A.1)$$

by noting $f(Z_{it}, X_t, t - \underline{t_i}) = c + Z'_{i,t-H}\beta_1 + X'_{t-H'}\beta_2 + \beta_3(t - \underline{t_i})$.

This likelihood is formally the same as that of a Logit model provided the "statistical individual" (i, t) becomes a "firm-date".

Indeed, if one considers a standard Logit modelling with such individuals:

$$Y_{(i,t)} = 1 \text{ if firm } i \text{ defaults at date } t = t_i \Leftrightarrow W_{(i,t_i)} < 0$$

where $W_{(i,t)} = f(Z_{i,t}, X_t, t - \underline{t_i}) + \varepsilon_{(i,t)}$ denotes a latent variable characterizing the repayment capacity of firm i at date t, with residual $\varepsilon_{(i,t)}$ distributed as a Logit variable with a cumulated distribution function $F_{Logit}(x) = 1/(1 + \exp(-x))$, we can write:

$$\begin{aligned} P\left(Y_{(i,t)} = 1 / Z_{i,t_i-H}, X_{t_i-H'}, \underline{t_i}\right) &= F_{Logit}(-f(Z_{it_i}, X_t, t - \underline{t_i})) \\ &= h(t_i - \underline{t_i}) / Z_{i,t_i-H}, X_{t_i-H'}, t - \underline{t_i} \end{aligned}$$

[30] The survival function is defined as $S(d) = P(D > d) = 1 - F(d)$, where F is the cumulated distribution function of the D variable.

and the likelihood of the duration model $L(u_1, \ldots, u_n/\{Z_i\,; X\})$ in (A.1) can be rewritten as:

$$\prod_{i=1}^{n}[F_{Logit}(-f(Z_{i,u_i}, X_{u_i}, u_i))]^{y_i} \prod_{\underline{t_i}<t<u_i}[1-F_{Logit}(-f(Z_{it}, X_t, t-\underline{t_i}))] \tag{A.2}$$

or, equivalently, as a Logit-type likelihood for (i, t) individuals:

$$\prod_{1\le i\le n}\prod_{\underline{t_i}\le t\le u_i}[F_{Logit}(-f(Z_{i,t}, X_t, t-\underline{t_i}))]^{y_{(i,t)}} (1-F_{Logit}(-f(Z_{i,t}, X_t, t-\underline{t_i}))^{1-y_{(i,t)}} \tag{A.3}$$

with t always smaller than u_i, and greater than $\underline{t_i}$ for any i, because the observed firm-dates (i, t) are necessarily such that $\underline{t_i} < t \le u_i$.

The identity between (A.2) and (A.3) is easy to check by comparing the contributions of each firm i to both likelihoods.

For example, a firm i which defaults is such that $u_i = t_i$ and $y_i = 1$ and has a contribution to the duration-type likelihood (A.2) equal to:

$$F_{Logit}(-f(Z_{i,t_i}, X_{t_i}, t_i-\underline{t_i}))\prod_{\underline{t_i}<t<t_i}[1-F_{Logit}(-f(Z_{it}, X_t, t-\underline{t_i}))]$$

which is also its contribution to the Logit-type likelihood (A.3), because $y_{(i,t_i)} = 1$ and $y_{(i,t)} = 0$ for any t such that $\underline{t_i} < t < u_i (= t_i)$.

Appendix B. Definition of variables and summary statistics

See Tables 6 and 7.

Appendix C. Estimation results: bankruptcy rate at *t* with information at *t-2*

See Figs. 5 and 6.

Appendix D. Robustness checks

D.1. Alternative measures of the output gap

See Table 9.

D.2. In- and out-of-sample results

See Figs. 7 and 8.

Appendix E. Impulse responses from VARX type model

We describe here how to implement an impulse response analysis based on our VAR model, distinguishing between the standard case, where we only consider the output gap and defaults, and the more comprehensive model, with additional macroeconomic variables.

E.1. Standard case

Our system (B) of two equations can be expressed in state-space form as a "VARX(1) type" model, using $\tau_{def,jt}$ and χ_{jt} with

$\chi_{jt} = GAP_{jt} - a_2\tau_{def,jt}$

$$\begin{pmatrix}\tau_{def,jt}\\ \tau_{def,jt-1}\\ \tau_{def,jt-2}\\ \chi_{jt}\\ \chi_{jt-1}\\ \chi_{jt-2}\end{pmatrix} = \begin{pmatrix}0 & a_1a_2 & b_1a_2 & 0 & a_1 & b_1\\ 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ a_2b_2 & a_2c_2 & 0 & b_2 & c_2 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\tau_{def,jt-1}\\ \tau_{def,jt-2}\\ \tau_{def,jt-3}\\ \chi_{jt-1}\\ \chi_{jt-2}\\ \chi_{jt-3}\end{pmatrix} + c_1\begin{pmatrix}M_{t-2}\\0\\0\\0\\0\\0\end{pmatrix} + d_1\begin{pmatrix}M_{t-3}\\0\\0\\0\\0\\0\end{pmatrix} + e_1\begin{pmatrix}Z_{jt-3}\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix}\epsilon_{1jt}\\0\\0\\\epsilon_{2jt}\\0\\0\end{pmatrix}. \tag{E.1}$$

Or in a more compact form:

$$X_t = AX_{t-1} + c_1M_{t-2} + d_1M_{t-3} + e_1Z_{t-3} + E_t, \tag{E.2}$$

where $M_t = (INF, IRL, Ex\,.\,Rate)$ and Z_t are firm-specific variables.

We consider shocks to GAP_{jt} that are similar across all sectors, e.g. a recession affecting all sectors simultaneously, i.e. $\epsilon_{1jt} = \epsilon_t$, for all j so that we can omit the superscript j. Running Eq. (E.2) recursively yields at date $t + l$:

$$X_{t+l} = A^{l+1}X_{t-1} + \Sigma_{k=0}^{l}A^{l-k}(c_1M_{t+k-2} + d_1M_{t+k-3} + e_1Z_{t+k-3} + E_{t+k}). \tag{E.3}$$

Here we omit the constant terms, which do not contribute to the responses of the shocks ϵ. The IRFs are simply the appropriate cell of A^l for $l = 0, 1, 2,$ etc.

E.2. Impulse response analysis including a more comprehensive macroeconomic environment

A more comprehensive exercise requires introducing additional exogenous variables. This is the component $\Sigma_{k=0}^{l}A^{l-k}(c_1M_{t+k-2} + d_1M_{t+k-3} + e_1Z_{t+k-3})$ in (E.3), i.e. the impact of the Z_t's and the M_t's. In this respect we should distinguish between the Z_t's and the M_t's. The Z_t's may be assumed to be exogenous, consistently with the findings of Jacobson et al. (2005): macroeconomic variables do not affect individual (and sector-level) financial ratios directly.

Concerning the M_t's, however, a shock to ϵ_{2t} has an effect on inflation (*INF*), the long-term interest rate (*IRL*) and the exchange rate (*Ex . Rate*). In order to take these effects into account, we estimate a quarterly macroeconomic VAR of dimension 4, namely [*INF*, *IRL*, *Ex . Rate*, *GAP*]'.[31] The number of lags is determined by the Akaike criterion. It turns out that the output gap has no significant effect on the nominal exchange rate (IRFs are available upon request). We decided therefore to concentrate on a VAR of dimension 3, excluding the exchange rate for that particular stress test exercise: we focus on the response of a shock to the *GAP* on inflation and the interest rate. We aggregate the response at the annual frequency in order to derive a new path for M_t'**s**. We assume that the following quarterly inverted VAR yields:

$$\begin{pmatrix}INF_t\\ IRL_t\\ GAP_t\end{pmatrix} = H(L)Q^{-1}Q\begin{pmatrix}\eta_{1t}\\ \eta_{2t}\\ \eta_{3t}\end{pmatrix} = H(L)Q^{-1}\begin{pmatrix}\eta'_{1t}\\ \eta'_{2t}\\ \eta_{3t}\end{pmatrix}. \tag{E.4}$$

[31] The long-term interest rate is included in deviation to a quadratic trend in order to take into account the convergence to EMU until 1999. Very similar results are found assuming a linear trend until 1999 and no trend afterwards.

Table 6
Definition of variables.

Micro variables	
Profit	Profitability = Total Gross Income ("Résultat Brut Global")/Total Assets
Leverage	Leverage = Total Borrowing/Total Liabilities (including equity)
Liq	Liquidity = (Liquid Assets/Financial Debt), with Liquid Assets = Cash Assets + Short Term investments (including own stocks) ("Disponibilité (Brut) + Valeurs mobilières de placement (brut) (dont actions propres)") Financial Debt = bonds + bank credit + other funding liabilities ("Obligations + dettes bancaires + autres emprunts")
Bus . Cred .	Business credit received (in days of purchases) = 360 × Accounts Payable/Purchases (VAT included) (Accounts Payable = "dettes fournisseurs et comptes rattachés")
Int	Interest = Interest Paid and other Borrowing Fees/(Interest Paid and other Borrowing Fees + TGI), with TGI = Total Gross income
Size	Firm size = 1 (small), if Net Turnover <700,000 EUR; 2 (medium), if 700,000 EUR ≤ Net Turnover ≤ 40,000,000 EUR; 3 (large), if Net Turnover > 40,000,000 EUR
IP	Non-payment incident = 1 if the firm experiences at least one non-payment incident on its commercial debt during the year, and =0 otherwise.
Tax Arrears	Tax payable = 1 if the company is in debted to the government (Tax and Social Security), and =0 otherwise
Macro variables	
GAP	Output gap = deviation from a linear trend
INF	Inflation rate = calculated from GDP deflator
IRL	Long-term interest rate = IRL on government bonds
Ex . Rate	Nominal exchange rate = dollar/euro (amount of euro per 1 USD)

In this table we describe the variables used in our model. We also provide the corresponding item in the French corporate tax form ("liasse fiscale". We also introduce dummy variables to capture sectoral effects.

Table 7
Descriptive statistics for 3-year horizon data by firm type (Total number of firm-years $N = 863{,}005$).

Variable	Mean	s.d	Min	25%	50%	75%	Max
Non-bankrupted firms ($N = 849{,}628$)							
Profitability	0.1187	0.0996	−0.1975	0.0601	0.1048	0.164	0.532
Leverage	0.5528	0.2088	0.00000	0.4017	0.5498	0.699	1.195
Liquidity	4.9986	21.030	0.00000	0.0440	0.3211	1.563	172.9
Bus.Cred.	70.654	41.548	0.00000	42.763	65.634	89.33	300.0
Interest	0.1178	0.2972	−1.8372	0.0233	0.0769	0.175	2.250
Bankrupted firms ($N = 13{,}377$)							
Profitability	0.0716	0.1098	−0.1975	0.0204	0.0689	0.122	0.532
Leverage	0.7195	0.2015	0.06664	0.5906	0.7374	0.855	1.195
Liquidity	1.8660	11.879	0.00000	0.0093	0.0738	0.376	172.9
Bus. Cred.	84.059	46.481	0.00000	54.038	77.304	103.2	300.0
Interest	0.1796	0.5115	−1.8372	0.0460	0.1724	0.313	2.250

We report the mean, standard deviation and quartiles of financial variables on bankrupted and non-bankrupted firms. We are considering here the sample of firms that have been present in the FIBEN sample for at least 3 years. They either go bankrupt in the third year or later, or remain active during the whole sample. Firms that go bankrupt before year 3 are excluded, as the quality of accounting data may deteriorate when the firm gets close to bankruptcy. When looking at $H = 2$ projections, we consider the set of firms that have been present for at least 2 years (hence a total of 876,995 year-firm observations). The statistics are computed on winsorized data.

Here again, Q is an upper triangular matrix (with zeroes on the lower left part) indicating that shocks to the interest rate and inflation have no contemporaneous effect on the *GAP* at the quarterly frequency, while *GAP* affects *INF* and *IRL* contemporaneously.

An important issue is also to calibrate the relative size of the shocks affecting the output gap (*GAP*), interest rates and inflation. We proceed using the following steps:

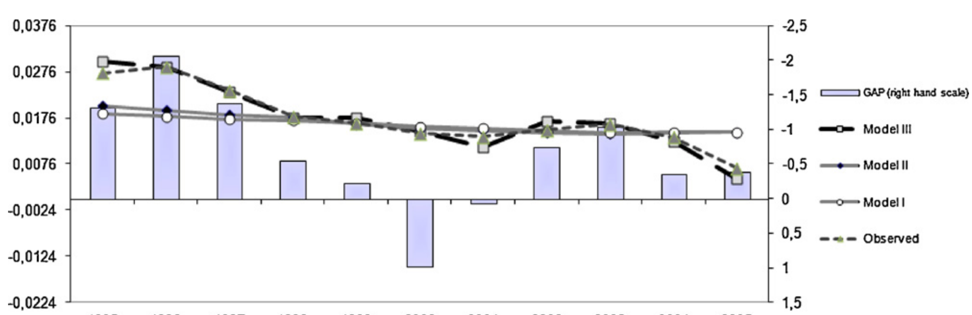

Fig. 5. Aggregate bankruptcy rate observed and estimated at t for the case "$H = 2$". (The RMSE of Model III is 0.136%, while for Model II and Model I it is 0.442% and 0.502%, respectively.) This figure displays the negative relationship between the output gap (GAP, with inverted scale) and the bankruptcy rate estimated by model III (black long dashed line with box), which is closer to the observed rate (black short dashed line with triangle) than the one estimated by model II (grey solid line with diamond) or model I (grey solid line with box). See results in Table 8.

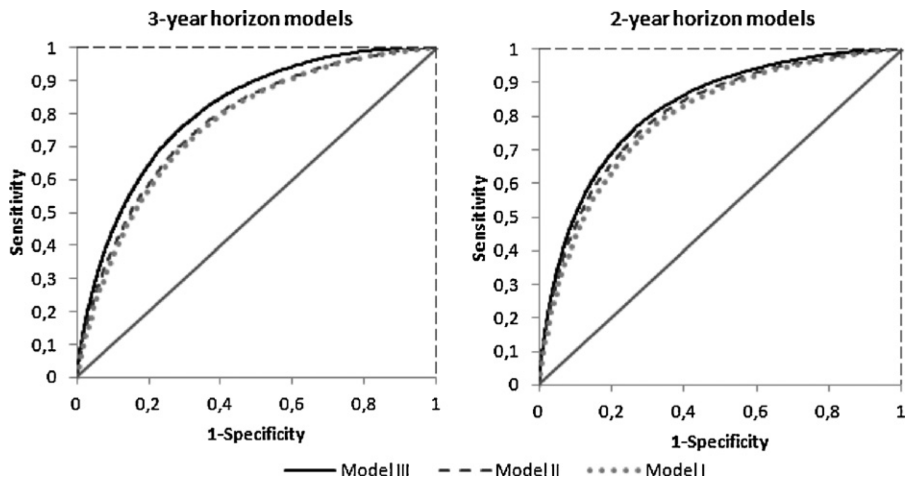

Fig. 6. ROC curves. This Figure displays ROC (Receiver Operating Characteristic) curves of different specifications of Logit model: model I, II and III and for the case "$H = 3$" and "$H = 2$". The area under the ROC curve (AUC) is a measure of how well a model can distinguish between two groups: bankrupted/non- bankrupted (see Tables 1 and 8).

Table 8
Estimation of individual bankruptcy rates at date t from multi-period Logit models: case "$H = 2$".

Variable	Model I		Model II		Model III	
	coef.	s.e.	coef.	s.e.	coef.	s.e.
Micro variables						
$Cst.$	−5.2646***	0.0534	−5.6045***	0.0641	−13.0380***	0.2570
$Profit_{t-2}$	−4.7857***	0.0914	−4.4819***	0.0917	−4.5351***	0.0922
$Leverage_{t-2}$	2.1766***	0.0344	2.0079***	0.0347	2.0041***	0.0350
Liq_{t-2}	−0.0215***	0.0016	−0.0186***	0.0015	−0.0167***	0.0014
$Bus.Cred._{t-2}$	0.0018***	0.00018	0.0017***	0.00019	0.00153	0.0002
Int_{t-2}	0.2806***	0.0173	0.2490***	0.0172	0.2082***	0.0171
$Size(large)_{t-2}$	−1.7848***	0.0975	−1.6306***	0.0976	−1.4946***	0.0979
$Size(medium)_{t-2}$	−0.3396***	0.0397	−0.2784***	0.0399	−0.1518*	0.0402
IP_{t-2}		1.3990***	0.0260	1.3279***	0.0265	
$Tax\,Arrears_{t-2}$		0.2919***	0.0407	0.2857***	0.0408	
Macro variables						
GAP_{t-2}					−0.1575***	0.0194
INF_{t-2}					−0.3501***	0.0276
IRL_{t-2}					0.3683***	0.0287
$Ex.Rate_{t-2}$					0.0488***	0.0023
GAP_{t-1}					−0.3302***	0.0222
INF_{t-1}					−0.3598***	0.0243
IRL_{t-1}					0.0557	0.0206
$Ex.Rate_{t-1}$					0.0120***	0.0018
$\widetilde{R}^2$[c]	0.1072		0.1245		0.1446	
LR test[a]			443.67***		515.15***	
AUC[b]	0.794		0.807		0.823	
$Nb.obs.$	876,995		876,995		876,995	

The table provides the estimation results, using a multi-period Logit model, for the probability of bankruptcy of individual firms at date t, based on three sets of explanatory variables: with financial (or micro) variables only (Model I), with the addition of a few dummy variables (Model II), and when macroeconomic variables are also added (Model III). The definition of variables is in Appendix B.

[a] Likelihood Ratio test statistic for nested models $\sim\chi^2_{(df)}$ with $df = 2$ (resp. 8) for Model *I* vs *II* (resp. Model *II* vs III).

[b] Area Under the Receiver Operating Characteristics (ROC) Curve.

[c] Nagelkerke's $\widetilde{R}^2$ is obtained as: $R^2/R^2_{\max}$ with $R^2 = 1 - \{LnL_{\max}(\beta = 0)/LnL_{\max}(\widehat{\beta})\}^{2/N}$ and $R^2_{\max} = 1 - \{LnL_{\max}(\beta = 0)\}^{2/N}$. $\widetilde{R}^2$ measures the discrepancy between the LOGIT models with $(\widehat{\beta})$ or without $(\beta = 0)$ the dependent variables X. N is the sample size.

* Significance at 10% level.

**Significance at 5% level.

*** Significance at 1% level.

Table 9
Coefficient associated with the output gap (*GAP*) according to different methods.

	GAP	*GAPHP*(λ = 20)	*GAPHP*(λ = 15)	*GAPHP*(λ = 8)
GAP_{-3}	−0.247 (0.021)	−0.008 (0.021)	−0.019 (0.022)	−0.035 (0.023)
GAP_{-2}	−0.395 (0.023)	−0.599 (0.024)	−0.633 (0.025)	−0.731 (0.022)
$GAP_{-3} + GAP_{-2}$	−0.643 (0.0190)	−0.607 (0.0196)	−0.652 (0.0202)	−0.766 (0.0243)

The table shows the impact of alternative measures of the output gap on the results of multi-period Logit estimations.
GAP is the deviation from (deterministic) trend; *HP*(λ) is the Hodrick Prescott filter with smoothing parameter λ; Standard deviations are in ().

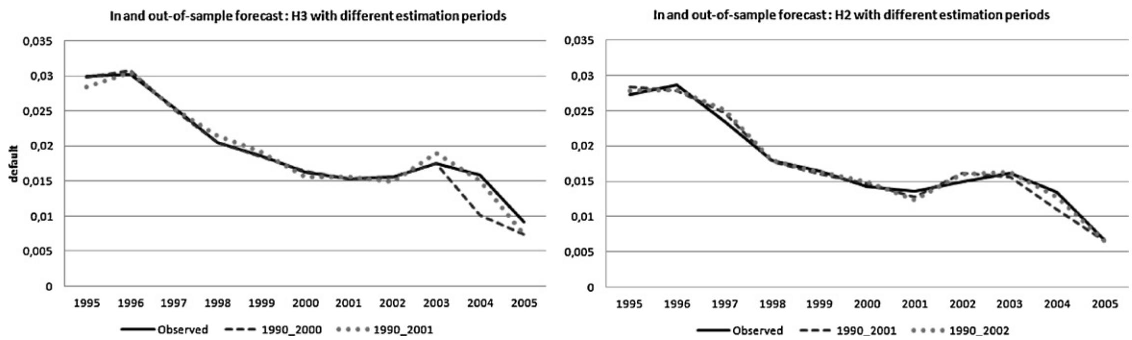

Fig. 7. In- and out-of-sample performance of Model III (H3 and H2) without spline method. This figure displays in- and out-of-sample forecast of the aggregate bankruptcy rate. For each case (H = 3 or H = 2), the observed rate (black solid line) is compared with the performance of two multi-period Logit models. In the case H = 3 (left-hand sub-figure), the models are estimated on the period 1990–2000 (black dashed line) and 1990–2001 (grey dotted line). In the case H = 2 (right hand subfigure), the models are estimated on the period 1990–2001 (black dashed line) and 1990–2002 (grey dotted line).

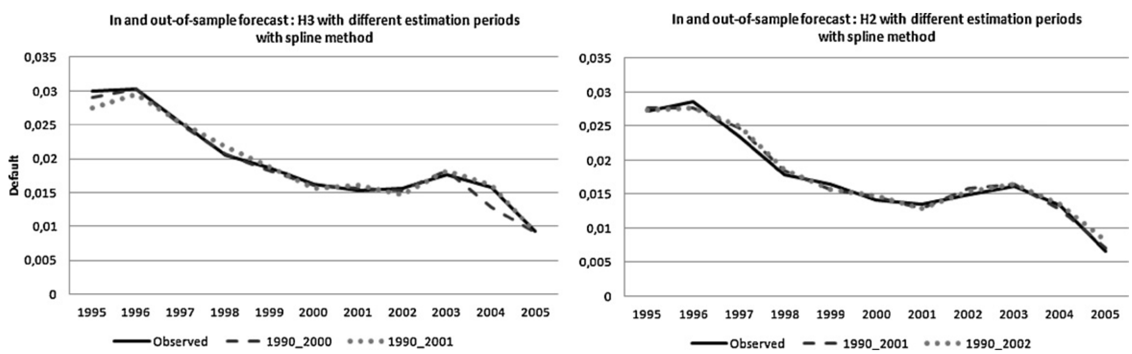

Fig. 8. In- and out-of-sample performance of Model III ($H3$ and $H2$) with spline method. This figure displays in- and out-of-sample forecast of the aggregate bankruptcy rate. For each case (H = 3 or H = 2), the observed rate (black solid line) is compared with the performance of two multi-period Logit models, which include spline transformations of macroeconomic variables. In the case H = 3 (left-hand sub-figure), the models are estimated on the period 1990–2000 (black dashed line) and 1990–2001 (grey dotted line). In the case H = 2 (right hand subfigure), the models are estimated on the period 1990–2001 (black dashed line) and 1990–2002 (grey dotted line).

- we start from the quarterly VAR, and compute the annualized value of the IRFs of inflation and interest rate to a unit shock to the output gap;
- we compute the response of a shock of one standard deviation to the innovation of the output gap in the annual/sectoral model, by simply multiplying the previous IRF by one standard deviation of the innovation in the annual/sectoral VAR
- we derive the annual path for INF_t and IRL_t which is the new path for $M_t's$ that we note $\widehat{M_t's}$
- we run recursively the equation, as in the previous subsection:

$$\widehat{X}_t = A^t X_0 + \Sigma_{k=1}^t A^{t-k}(c_1 \widehat{M}_{k-2} + d_1 \widehat{M}_{k-3} + e_1 Z_{k-3} + E_k). \tag{E.5}$$

The IRFs for GAP_{jt} and $\tau_{def,jt}$ are shown in Fig. 3.

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