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A Study of Decoupling with Skeleton Matrix in Two-Degree-of-Freedom Generalized Minimum Variance Control

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SUMMARY

There are many multi-input multi-output (MIMO) systems in chemical plants, and they have multiple time delays of different length in each input and output pair. This paper explains a two-degree-of-freedom (2DOF) control system based on generalized minimum variance control (GMVC) for MIMO systems. It can improve the tracking performance with respect to the reference signals and the response properties for the disturbance. The states between the sampling period can be expressed by using the modified z transform to take account of multiple time delays. Additionally, a tracking controller is designed to decouple the plant. © 2011 Wiley Periodicals, Inc. Electr Eng Jpn, 176(1): 28–36, 2011; Published online in Wiley Online Library (wileyonlinelibrary.com). DOI 10.1002/eej.21046

Key words: generalized minimum variance control; MIMO system; multiple time delays; modified z-transform; two-degree-of-freedom; decoupling; skeleton matrix.

1. Introduction

Chemical plants and other industrial processes often involve multiple-input multiple-output (MIMO) systems with different time delays. Such systems require advanced control, but precise prediction of different delay times is difficult. For example, self-tuning control (STC) [1] is a method of process control. With generalized minimum variance control (GMVC), typically used for STC design, a control system with good tracking performance can be designed for a dead-time system including unstable zero points. However, different time delays are difficult to handle with high accuracy, and a desirable response previously could not be obtained because time delays were approximated uniformly. The authors proposed a GMVC using a modified z-transform capable of accurately taking account of delay times [2]; the problem with the method was that adjustment of the target response characteristics inevitably resulted in a change of the disturbance response characteristics.

In this study, we propose a multivariable 2-DOF control system based on GMVC that can deal with different delay times for every input-output, while allowing independent specification of the target tracking characteristics and disturbance rejection performance. We verify the proposed method by applying it to the distillation column model of Wood and Berry [3].

2. Problem Formulation

Assume a stable controlled object with different input-output time delays:

$$G(s) = \begin{pmatrix} G_{11}(s)e^{-L_{11}s} & \cdots & G_{1n}(s)e^{-L_{1n}s} \\ \vdots & \ddots & \vdots \\ G_{n1}(s)e^{-L_{n1}s} & \cdots & G_{nn}(s)e^{-L_{nn}s} \end{pmatrix} \tag{1}$$

Below we consider the design of control system for the MIMO plant described by Eq. (1).

3. Modified z-Transform [4]

When there are fractional time delays $\overline{L}_{11} \ldots \overline{L}_{nn}$ (not multiples of the sampling period), multiple input terms occur within one sampling interval, which is reflected in the respective outputs. In this case, the modified z-transform has been proposed to represent the output values. The following can be derived by transforming the first line of Eq. (1) into a continuous-time system:

$$\dot{x}_1(t) = \Gamma_1 x_1(t) + \Phi_1 u(t - L_1) \tag{2}$$

$$y_1(t) = C_1 x_1(t) \tag{3}$$

Here,

$$x_1(t) := [x_{11}(t), \cdots, x_{1n}(t)]^T$$
$$u(t - L_1) := [u(t - L_{11}), \cdots, u(t - L_{1n})]^T$$
$$\Gamma_1 := diag[\gamma_{11}, \cdots, \gamma_{1n}]$$
$$\Phi_1 := diag[\phi_{11}, \cdots, \phi_{1n}]$$

When the modified z-transform is applied to every element of this system, the following is obtained:

$$x_1(k+1) = A_1 x_1(k) + B_1 u(k-j) + \bar{B}_1 u(k-j-1) \quad (4)$$

$$y_1(k) = C_1 x_1(k) \quad (5)$$

A_1, B_1, and $\overline{B}_1$ are as follows:

$$A_1 := diag[e^{\gamma_{11}T_s}, \cdots, e^{\gamma_{1n}T_s}]$$
$$B_1 := diag[B_{11}, \cdots, B_{1n}]$$
$$\bar{B}_1 := diag[\bar{B}_{11}, \cdots, \bar{B}_{1n}]$$
$$B_{1n} := \int_{\bar{L}_{1n}}^{T_s} e^{\gamma_{1n}(T_s - \tau)} \phi_{1n} d\tau$$
$$\bar{B}_{1n} := \int_{T_s}^{0} e^{\gamma_{1n}(T_s - \tau)} \phi_{1n} d\tau$$

4. Design of Multivariable 2-DOF Generalized Minimum Variance Control [5]

The configuration of the proposed control system is shown in Fig. 1. As can be seen from the diagram, complete 2-DOF control is configured with respect to the target tracking characteristic and the disturbance rejection performance. The controlled object is decoupled by the tracking controller, and the modified z-transform is implemented by the disturbance controller.

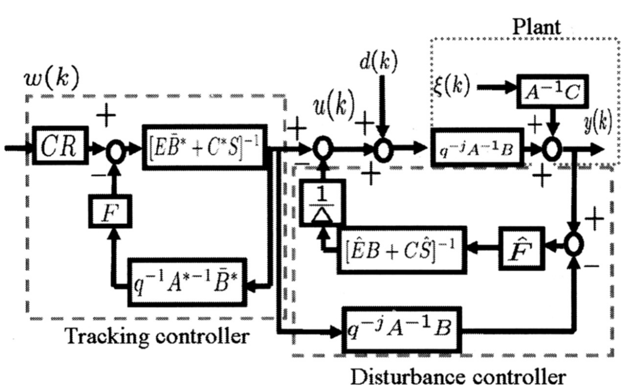

Fig. 1. Block diagram of proposed multivariable 2-DOF control.

4.1 Design of tracking controller

The target tracking controller is designed using GMVC, while the controlled object is decoupled. If the inverse matrix of controlled object is used for decoupling, then the controlled object can be interpreted as a unit matrix in design. In addition, coupled terms can be eliminated in the models, even though cross-controllers are employed. However, in real plants, the controlled objects include parameter errors that strongly affect the system response. Thus, the inverse matrix is obtained from a skeleton matrix derived by decomposition of the object's diagonal matrix, and decoupling is performed to improve robustness to parameter errors [6]. First, let us consider the decoupling algorithm.

Let $G_0(s)$ denote the skeleton matrix that can be obtained by diagonal-matrix decomposition of the transfer function matrix $G(s)$. Now assume that $g_{0d}(s)$ is a common denominator polynomial for $G_0(s)$, and that $G_{0n}(s)$ is a polynomial matrix; that is,

$$G(s) = G_{diag2}(s) G_0(s) G_{diag1}(s) \quad (6)$$

$$G_0(s) = G_{0n}(s)/g_{0d}(s) \quad (7)$$

Then diagonal-matrix decomposition can be applied to the transfer function matrix $G(s)$ as follows:

$$G(s) = 1/g_{0d}(s)[G_{diag2}(s)G_{0ndiag2}(s)] \cdot G_{0n0}(s) \cdot [G_{0ndiag1}(s)G_{diag1}(s)] \quad (8)$$

Here $G_{0n0}(s)$ is a minimal-order polynomial matrix, and the other matrices are diagonal matrices.

Now consider the stabilized inverse matrix of the polynomial matrix. Let adj$Q(s)$ and det$Q(s)$ denote, respectively, the adjugate matrix and the determinant of the inverse matrix inv$Q(s)$ of polynomial matrix $Q(s)$. A mirror image with respect to the imaginary axis occurs when the determinant roots are unstable. With det$Q_{Plus}(s)$ denoting the resulting polynomial, the transfer function matrix [adj$Q(s)$/det$Q_{Plus}(s)$] is a stabilized matrix.

If the inverse matrix of $[G_{0ndiag1}(s)G_{diag1}(s)]$ is stable and has proper elements, then the decoupling compensator $G_{dep}(s)$ can be expressed as

$$G_{dcp}(s) = \text{inv}[G_{0ndiag1}(s)G_{diag1}(s)] \cdot \frac{\text{adj}G_{0n0}(s)}{\text{det}G_{0n0Plus}(s)} \quad (9)$$

and the following diagonalization is obtained when this compensator is implemented:

$$G_s G_{dcp}(s) = 1/g_{0d}(s)[G_{diag2}(s)G_{0ndiag2}(s)] \cdot \frac{\text{det}G_{0n0}(s)}{\text{det}G_{0n0Plus}(s)} \quad (10)$$

The controlled object is represented by the following model, with the decoupling controller obtained from the inverse matrix of the skeleton matrix of the real object:

$$A^*(q^{-1})y(k) = q^{-1}\bar{B}^*u(k) + C^*(q^{-1})\xi(k) \qquad (11)$$

$$A^*(q^{-1}) = I + A_1^*q^{-1} + \cdots + A_{n_A}^*q^{-n_A} \qquad (12)$$

$$B^*(q^{-1}) = B_0^* + B_1^*q^{-1} + \cdots + B_{n_B}^*q^{-n_B} \qquad (13)$$

$$C^*(q^{-1}) = I + C_1^*q^{-1} + \cdots + C_{n_C}^*q^{-n_C} \qquad (14)$$

Here $A^*(q^{-1})$, $B^*(q^{-1})$, $C^*(q^{-1})$ are $n \times n$ matrix polynomials of the time-delay operator q^{-1}, and $y(k)$, $u(k)$, and $\xi(k)$ represent the output, input, and noise terms, respectively. In addition, $\bar{B}^* = B^*(1)$. The steady-state value is used to improve robustness. Since the time-delay step is 1 in Eq. (11), the following control rules are formulated so as to minimize evaluation function (15):

$$J = \mathbf{E}\left[h^T(k+1)h(k+1)\right] \qquad (15)$$

$$h(k+1) = P(q^{-1})y(k+1) - R(q^{-1})w(k+1) + S(q^{-1})u(k) \qquad (16)$$

$$P(q^{-1}) = \begin{pmatrix} P_{11}(q^{-1}) & \cdots & P_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ P_{n1}(q^{-1}) & \cdots & P_{nn}(q^{-1}) \end{pmatrix} \qquad (17)$$

$$S(q^{-1}) = \begin{pmatrix} S_{11}(q^{-1}) & \cdots & S_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ S_{n1}(q^{-1}) & \cdots & S_{nn}(q^{-1}) \end{pmatrix} \qquad (18)$$

$$R(q^{-1}) = \begin{pmatrix} R_{11}(q^{-1}) & \cdots & R_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ R_{n1}(q^{-1}) & \cdots & R_{nn}(q^{-1}) \end{pmatrix} \qquad (19)$$

Here $P(q^{-1})$, $S(q^{-1})$, and $R(q^{-1})$ are weight matrix polynomials for, respectively, the controlled variable, the manipulated variable, and the target value, with orders n_P, n_S, and n_R. The target tracking characteristic is improved by tuning these weights. Furthermore, offset occurs if the weights are selected arbitrarily, and hence weight adjustment [3] is used to set the weights so that $R(1) = P(1) + S(1)B^{*-1}(1)A^*(1)$ (see appendix). In particular, when $S(q^{-1})$ is set low, the manipulated variable is heavily used, which contributes to the responsiveness of the closed-loop system but degrades robustness; conversely, when S is set high, the responsiveness of the closed-loop system drops but the robustness increases, as has been shown by experience.

The one-step-ahead controlled variable $y(k+1)$ in Eq. (16) cannot be obtained at the current instant. Thus, the following Diophantine equation is introduced:

$$C^*(q^{-1})P(q^{-1}) = E(q^{-1})A^*(q^{-1}) + q^{-1}F(q^{-1}) \qquad (20)$$

Here n_E and n_F denote the orders of $E(q^{-1})$ and $F(q^{-1})$, respectively. By virtue of the above, the control rule to minimize evaluation function (15) is as follows:

$$u(k) = \left[E(q^{-1})\bar{B} + C^*(q^{-1})S(q^{-1})\right]^{-1} \cdot \left[C^*(q^{-1})R(q^{-1})w(k+1) - F(q^{-1})y(k)\right] \qquad (21)$$

In the tracking controller shown in Fig. 1, the feedback element described by apparent plant model (11) affects control rule (21). With this in mind, the final equations for the control rule and the closed-loop system are obtained as follows:

$$u(k) = \left[P(q^{-1})A^{*-1}(q^{-1})\bar{B}^* + S(q^{-1})\right]^{-1} \cdot R(q^{-1})w(k+1) \qquad (22)$$

$$y(k) = q^{-1}\left[T(q^{-1})\right]^{-1}R(q^{-1})w(k+1) + \left[T(q^{-1})\right]^{-1}\left[P(q^{-1})A^{*-1}(q^{-1})\bar{B}^* + S(q^{-1})\right] \cdot B^{*-1}(q^{-1})C^*(q^{-1})\xi(k) \qquad (23)$$

$$T(q^{-1}) := \left[P(q^{-1}) + S(q^{-1})B^{*-1}(q^{-1})A^*(q^{-1})\right] \qquad (24)$$

4.2 Design of disturbance controller

The disturbance controller is designed using a servo GMVC [7]. Consider the following model that represents the time delays accurately by means of modified z-transform:

$$A(q^{-1})y(k) = q^{-j}Bu(k) + C(q^{-1})\xi(k) \qquad (25)$$

$$A(q^{-1}) = I + A_1q^{-1} + \cdots + A_{n_A}q^{-n_A} \qquad (26)$$

$$B(q^{-1}) = B_0 + B_1q^{-1} + \cdots + B_{n_B}q^{-n_B} \qquad (27)$$

$$C(q^{-1}) = I + C_1q^{-1} + \cdots + C_{n_C}q^{-n_C} \qquad (28)$$

$$q^{-j} = \begin{pmatrix} q^{-j_{11}} & \cdots & q^{-j_{1n}} \\ \vdots & \ddots & \vdots \\ q^{-j_{n1}} & \cdots & q^{-j_{nn}} \end{pmatrix} \qquad (29)$$

Here $A(q^{-1})$, $B(q^{-1})$, and $C(q^{-1})$ are $n \times n$ matrix polynomials of time-delay operator q^{-1}, and $y(k)$, $u(k)$, and $\xi(k)$ represent the output, input, and noise terms, respectively. The controller is designed so that transfer characteristic from the disturbance $d(k)$ to the controlled variable $y(k)$ complies with servo GMVC. When the disturbance controller is configured as shown in Fig. 1, the control rule for the disturbance is as follows:

$$G_{cd}(k) = \frac{1}{\Lambda}\left[\hat{E}(q^{-1})B(q^{-1}) + \hat{S}(q^{-1})\right]^{-1}\hat{F}(q^{-1}) \qquad (30)$$

Here $\Delta = 1 - q^{-1}$. In addition, $\hat{E}(q^{-1})$ and $\hat{F}(q^{-1})$ can be determined uniquely by solving the following Diophantine equations, where q^{-j} is as in Eq. (29):

$$\hat{P}(q^{-1}) = \Delta\hat{E}(q^{-1})A(q^{-1}) + q^{-j}\hat{F}(q^{-1}) \qquad (31)$$

$$\hat{P}(q^{-1}) = \begin{pmatrix} \hat{P}_{11}(q^{-1}) & \cdots & \hat{P}_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ \hat{P}_{n1}(q^{-1}) & \cdots & \hat{P}_{nn}(q^{-1}) \end{pmatrix} \tag{32}$$

$$\hat{S}(q^{-1}) = \begin{pmatrix} \hat{S}_{11}(q^{-1}) & \cdots & \hat{S}_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ \hat{S}_{n1}(q^{-1}) & \cdots & \hat{S}_{nn}(q^{-1}) \end{pmatrix} \tag{33}$$

$$\hat{E}(q^{-1}) = \begin{pmatrix} \hat{E}_{11}(q^{-1}) & \cdots & \hat{E}_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ \hat{E}_{n1}(q^{-1}) & \cdots & \hat{E}_{nn}(q^{-1}) \end{pmatrix} \tag{34}$$

$$\hat{F}(q^{-1}) = \begin{pmatrix} \hat{F}_{11}(q^{-1}) & \cdots & \hat{F}_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ \hat{F}_{n1}(q^{-1}) & \cdots & \hat{F}_{nn}(q^{-1}) \end{pmatrix} \tag{35}$$

Thus, the transfer characteristic from the disturbance $d(k)$ to the controlled variable $y(k)$ can be derived as follows:

$$\begin{aligned} y(k) = {} & \Delta[\hat{P}(q^{-1})A^{-1}(q^{-1})B(q^{-1}) + \Delta\hat{S}(q^{-1})]^{-1} \\ & \cdot \left[\hat{E}(q^{-1})B(q^{-1}) + \hat{S}(q^{-1})\right] \\ & \cdot q^{-j}A^{-1}(q^{-1})B(q^{-1}) \cdot d(k) \end{aligned} \tag{36}$$

The disturbance response and robustness can be improved by tuning the weight polynomial matrices $\hat{P}(q^{-1})$ and $\hat{S}(q^{-1})$.

5. Numerical Examples

We demonstrate the effectiveness of the proposed design method by using the distillation column model proposed by Wood and Berry [3], which is shown in Fig. 2. The column is intended to extract methanol of specified purity from a crude methanol–water mixture.

Here $Y_1(s)$ is the concentration (%) of high-purity methanol produced by condensation of steam in the top portion of the column by cooling water.

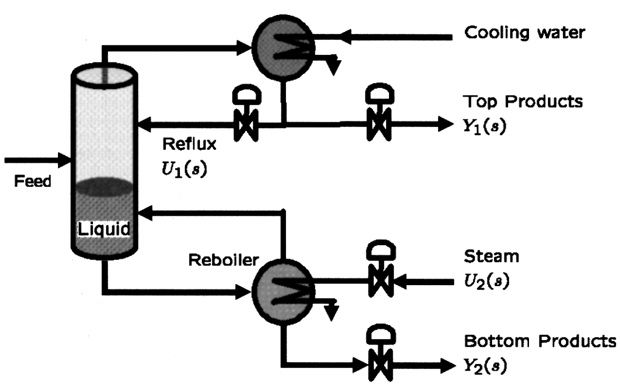

Fig. 2. Distillation column.

$Y_2(s)$ is the concentration (%) of low-purity methanol produced by heating the liquid in the bottom portion of column with a boiler.

$U_1(s)$ is the flow rate (lb/min) of the methanol input (the remaining methanol extracted from the top portion).

$U_2(s)$ is the flow rate (lb/min) of the steam input to the boiler.

This model of the distillation column can be expressed by the transfer function as follows:

$$G(s) = \begin{pmatrix} \dfrac{12.8}{16.7s+1}e^{-1s} & \dfrac{-18.9}{21.0s+1}e^{-3s} \\ \dfrac{6.6}{10.9s+1}e^{-7s} & \dfrac{-19.4}{14.4s+1}e^{-3s} \end{pmatrix} \tag{37}$$

In the simulations, we compared the following three cases in terms of control performance.

Case 1 Different time delays can be approximated by the maximum, minimum, or average value; approximation by the maximum value is known to offer the best control performance. In Case 1, the servo GMVC [7] is designed using approximation by the maximum value.

Case 2 According to Ref. 2, the time-delay steps are arranged so that all time delays fit into one sampling period, and the GMVC is designed using the modified z-transform.

Case 3 The sampling periods are set finely according to the 2-DOF GMVC with a modified z-transform as proposed in this study.

(*Target values*)

$w_1 \rightarrow$ change concentration to 80 (%) at instant $t = 150$ (min)

$w_2 \rightarrow$ change concentration to 30 (%) at instant $t = 20$ min

(*Disturbance*)

A disturbance $d = 4$ is applied to the manipulated variable $u_1(k)$ at instant $t = 400$ min.

(*Noise*)

$$\xi(k) = N(0, 0.01)$$

In addition, the time delays may fluctuate because of decreased motor output and other factors; thus, we applied a uniform variation by a factor of 1.5 in the simulations. On the other hand, the gain was not varied because the purpose was to estimate the robustness to time delays.

Case 1

The time delays were approximated uniformly as 7 min, and discretization by a sampling period of $T_s = 7$ min was applied; then the following representation by the CARMA model was obtained:

$$\begin{pmatrix} A_1(q^{-1}) & 0 \\ 0 & A_2(q^{-1}) \end{pmatrix} y(k)$$
$$= q^{-2} \begin{pmatrix} B_{11}(q^{-1}) & B_{12}(q^{-1}) \\ B_{21}(q^{-1}) & B_{22}(q^{-1}) \end{pmatrix} u(k) + I\xi(k) \quad (38)$$

where

$$A_1(q^{-1}) = 1 - 1.3741q^{-1} + 0.4712q^{-2}$$
$$A_2(q^{-1}) = 1 - 1.1411q^{-1} + 0.3236q^{-2}$$
$$B_{11}(q^{-1}) = 4.3827 - 3.1404q^{-1}$$
$$B_{12}(q^{-1}) = -5.3576 + 3.5231q^{-1}$$
$$B_{21}(q^{-1}) = 3.1275 - 1.9235q^{-1}$$
$$B_{22}(q^{-1}) = -7.4687 + 3.9295q^{-1}$$

In the servo GMVC [7], $P(1) = R(1)$ must be satisfied in order to remove the offset with respect to the target value and disturbance; $S(q^{-1})$ is set arbitrarily. The following weight matrix polynomial was used for control system design:

$$P(q^{-1}) = (1 - 0.5q^{-1})I, \quad R(q^{-1}) = 0.5I$$
$$S(q^{-1}) = \begin{bmatrix} 0 & 50 \\ 5 & 0 \end{bmatrix} \quad (39)$$

The controller parameters $E(q^{-1})$ and $F(q^{-1})$ were

$$E(q^{-1}) = \begin{pmatrix} 1 + 1.8741q^{-1} & 0 \\ 0 & 1 + 1.6411q^{-1} \end{pmatrix} \quad (40)$$
$$F(q^{-1}) = \begin{pmatrix} F_1(q^{-1}) & 0 \\ 0 & F_2(q^{-1}) \end{pmatrix} \quad (41)$$

where

$$F_1(q^{-1}) = 2.6041 - 2.9872q^{-1} + 0.8831q^{-2}$$
$$F_2(q^{-1}) = 2.0492 - 2.0802q^{-1} + 0.5310q^{-2}$$

The responses of the closed-loop system are shown in Fig. 3; in addition, the responses of the control-loop system with time delays increased by a factor of 1.5 are shown in Fig. 4. Since the time delays were not estimated accurately, stabilization proved impossible unless the input weight S was set high. As a result, the responsiveness deteriorated. In addition, a considerable coupling effect occurred. On the other hand, approximation by the maximum value provided good robustness to delay time fluctuation.

Case 2

The model of the distillation column involved mixed time delays of 1, 3, and 7 min. Thus, the sampling period was set to $T_s = 8$ min so as to fit the internal states into one sampling period, and the modified z-transform was applied in order to deal with fractional time delays. The following representation by the CARMA model was obtained:

$$\begin{pmatrix} A_1(q^{-1}) & 0 \\ 0 & A_2(q^{-1}) \end{pmatrix} y(k)$$
$$= q^{-1} \begin{pmatrix} B_{11}(q^{-1}) & B_{12}(q^{-1}) \\ B_{21}(q^{-1}) & B_{22}(q^{-1}) \end{pmatrix} u(k) + I\xi(k) \quad (42)$$

where

$$A_1(q^{-1}) = 1 - 1.3026q^{-1} + 0.4232q^{-2}$$
$$A_2(q^{-1}) = 1 - 1.0538q^{-1} + 0.2754q^{-2}$$
$$B_{11}(q^{-1}) = 4.3827 - 2.5051q^{-1} - 0.3342q^{-2}$$
$$B_{12}(q^{-1}) = -4.0044 + 0.4973q^{-1} + 1.2282q^{-2}$$

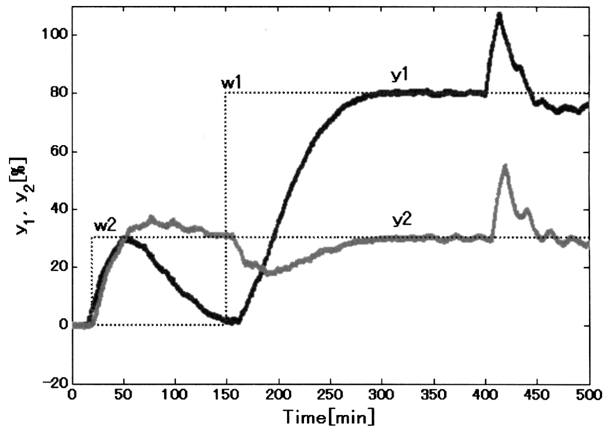

Fig. 3. Responses of closed-loop system of conventional GMVC.

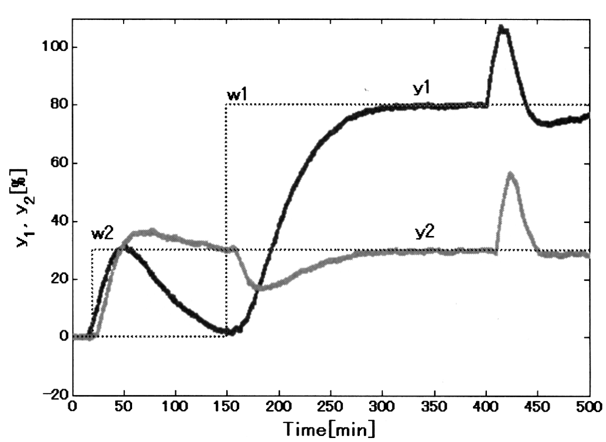

Fig. 4. Responses of closed-loop system of conventional GMVC with varied parameters.

$$B_{21}(q^{-1}) = 0.5786 + 2.5214q^{-1} - 1.6371q^{-2}$$
$$B_{22}(q^{-1}) = -5.6910 + 0.1536q^{-1} + 1.2375q^{-2}$$

In the servo GMVC [2] using the modified z-transform, $P(1) = R(1)$ must be satisfied in order to remove the offset with respect to the target value and disturbance; $S(q^{-1})$ is set arbitrarily. The following weight matrix polynomial was used for control system design:

$$P(q^{-1}) = (1 - 0.5q^{-1})I, \quad R(q^{-1}) = 0.5I$$
$$S(q^{-1}) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{43}$$

The controller parameters $E(q^{-1})$ and $F(q^{-1})$ were

$$E(q^{-1}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{44}$$
$$F(q^{-1}) = \begin{pmatrix} F_1(q^{-1}) & 0 \\ 0 & F_2(q^{-1}) \end{pmatrix} \tag{45}$$

where

$$F_1(q^{-1}) = 1.8026 - 1.7258q^{-1} + 0.4232q^{-2}$$
$$F_2(q^{-1}) = 1.5538 - 1.3292q^{-1} + 0.2754q^{-2}$$

The responses of the closed-loop system are shown in Fig. 5; in addition, the responses of the control-loop system with time delays increased by a factor of 1.5 are shown in Fig. 6. A rather good target response was achieved, but the disturbance could not be rejected sufficiently. In addition, when the time delays changed, the response became oscillatory.

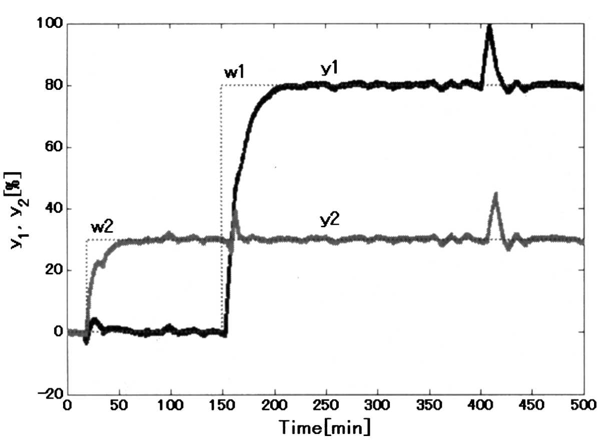

Fig. 5. Responses of closed-loop system of GMVC using modified z-transform.

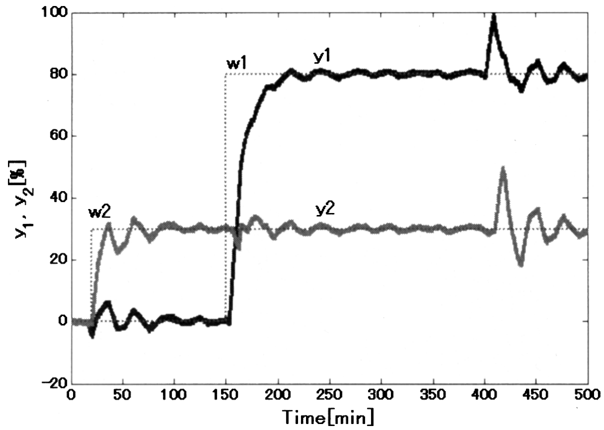

Fig. 6. Responses of closed-loop system of GMVC using modified z-transform with varied parameters.

Case 3

The sampling period was set to $T_s = 2$ min, and the fractional time delays were estimated accurately using the modified z-transform. In addition, the 2-DOF control system was configured so that the target response and disturbance rejection could be controlled independently. The following decoupling was used in the control rules. First, diagonal-matrix decomposition was applied to $G(s)$:

$$G(s) = \frac{1}{g_{0d}} \begin{pmatrix} e^{-1s} & 0 \\ 0 & e^{-3s} \end{pmatrix} \cdot \begin{pmatrix} -18.9cd(2s+1) & 0 \\ 0 & 6.6ab(s+1) \end{pmatrix} \cdot \begin{pmatrix} \frac{-12.8}{18.9} b(s+1) & a(-s+1) \\ d(-2s+1) & \frac{-19.4}{6.6} c(2s+1) \end{pmatrix} \tag{46}$$

Here,

$$a = 16.7s + 1, \quad b = 21.0s + 1$$
$$c = 10.9s + 1, \quad d = 14.4s + 1$$
$$g_{0d}(s) = abcd(s+1)(2s+1)$$

The time delays remaining in the skeleton matrix are approximated by Padé approximants, and the decoupled model can be expressed as follows:

$$G(s)G_{dcp}(s) = \begin{pmatrix} \frac{-18.9}{ab(s+1)} e^{-1s} & 0 \\ 0 & \frac{6.6}{cd(2s+1)} e^{-3s} \end{pmatrix} \tag{47}$$

In terms of the CARMA model, the following is obtained:

$$\begin{pmatrix} A_1^*(q^{-1}) & 0 \\ 0 & A_2^*(q^{-1}) \end{pmatrix} y(k)$$
$$= q^{-1} \begin{pmatrix} B_1^*(q^{-1}) & 0 \\ 0 & B_2^*(q^{-1}) \end{pmatrix} u(k) + I\xi(k) \quad (48)$$

where

$$A_1^*(q^{-1}) = 1 - 1.9316q^{-1} + 1.0496q^{-2} - 0.1092q^{-3}$$
$$A_2^*(q^{-1}) = 1 - 2.0706q^{-1} + 1.3508q^{-2} - 0.2665q^{-3}$$
$$B_1^*(q^{-1}) = -0.04397 - 0.1088q^{-1} - 0.01474q^{-2}$$
$$B_2^*(q^{-1}) = 0.02044 + 1.3508q^{-1} - 0.2665q^{-2}$$

Weight adjustment [3] was used to remove the offset of the target value, and the steady-state value $R(1)$ of $R(q^{-1})$ was given by Eq. (49). The following weight matrix polynomial was used for control system design:

$$P(q^{-1}) = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$
$$S(q^{-1}) = \begin{bmatrix} 28 & 0 \\ 0 & 7 \end{bmatrix}$$
$$R(q^{-1}) = P(1) + S(1)B^{*-1}(1)A^*(1) \quad (49)$$
$$= \begin{bmatrix} -1.97 & 0 \\ 0 & 0.587 \end{bmatrix}$$

The controller parameters $E(q^{-1})$ and $F(q^{-1})$ were

$$E(q^{-1}) = \begin{pmatrix} E_1(q^{-1}) & 0 \\ 0 & E_2(q^{-1}) \end{pmatrix} \quad (50)$$
$$E_1(q^{-1}) = -0.5000$$
$$E_2(q^{-1}) = 0.5000$$
$$F(q^{-1}) = \begin{pmatrix} F_1(q^{-1}) & 0 \\ 0 & F_2(q^{-1}) \end{pmatrix} \quad (51)$$

where

$$F_1(q^{-1}) = -0.9658 + 0.5248q^{-1} - 0.05458q^{-2}$$
$$F_2(q^{-1}) = 1.0353 - 0.6754q^{-1} + 0.1333q^{-2}$$

The following CARMA model with modified z-transform was applied to the disturbance:

$$\begin{pmatrix} A_1(q^{-1}) & 0 \\ 0 & A_2(q^{-1}) \end{pmatrix} y(k)$$
$$= q^{-j} \begin{pmatrix} B_{11}(q^{-1}) & B_{12}(q^{-1}) \\ B_{21}(q^{-1}) & B_{22}(q^{-1}) \end{pmatrix} u(k) + I\xi(k) \quad (52)$$

where

$$A_1(q^{-1}) = 1 - 1.7963q^{-1} + 0.8065q^{-2}$$

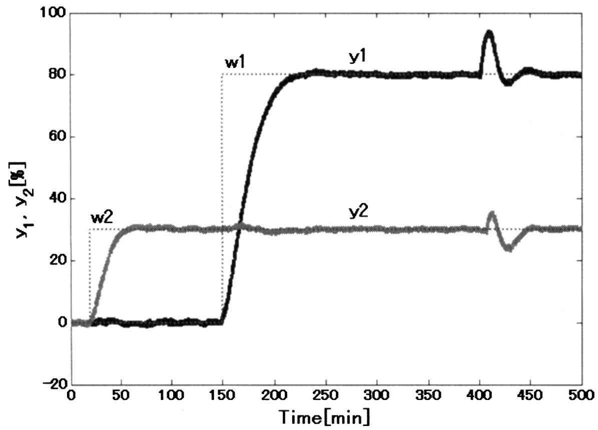

Fig. 7. Responses of closed-loop system of GMVC using modified z-transform.

$$A_2(q^{-1}) = 1 - 1.7027q^{-1} + 0.7244q^{-2}$$
$$B_{11}(q^{-1}) = 0.7440 + 0.02434q^{-1} - 0.6371q^{-2}$$
$$B_{12}(q^{-1}) = -0.8789 - 0.05833q^{-1} + 0.7434q^{-2}$$
$$B_{21}(q^{-1}) = 0.5786 + 0.02431q^{-1} - 0.4594q^{-2}$$
$$B_{22}(q^{-1}) = -1.3015 - 0.1309q^{-1} + 1.0106q^{-2}$$
$$q^{-j} = \begin{pmatrix} q^0 & q^{-1} \\ q^{-3} & q^{-1} \end{pmatrix}$$

There are no particular conditions to be satisfied by parameters $\hat{P}(q^{-1})$ and $\hat{S}(q^{-1})$ used in the disturbance controller. The following weight matrix polynomial was set by trial and error:

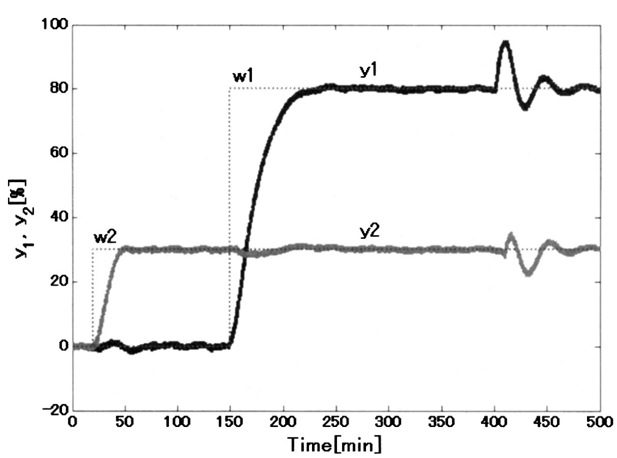

Fig. 8. Responses of closed-loop system of GMVC using modified z-transform with varied parameters.

$$\hat{P}(q^{-1}) = \begin{bmatrix} 0.15 & -0.1 \\ 0.1 & -0.1 \end{bmatrix}$$

$$\hat{S}(q^{-1}) = \begin{bmatrix} 7 & 0 \\ 1 & 9 \end{bmatrix}$$

The control parameters $\hat{E}(q^{-1})$ and $\hat{F}(q^{-1})$ were as follows:

$$\hat{E}(q^{-1}) = \begin{pmatrix} \hat{E}_{11}(q^{-1}) & \hat{E}_{12}(q^{-1}) \\ \hat{E}_{21}(q^{-1}) & \hat{E}_{22}(q^{-1}) \end{pmatrix} \tag{53}$$

$$\hat{E}_{11}(q^{-1}) = 0.1500$$
$$\hat{E}_{12}(q^{-1}) = -0.1000 - 0.2703q^{-1}$$
$$\hat{E}_{21}(q^{-1}) = 0.1000 + 2796q^{-1} + 0.5216q^{-2} + 0.8115q^{-3}$$
$$\hat{E}_{22}(q^{-1}) = -0.1000 - 0.2703q^{-1}$$

$$\hat{F}(q^{-1}) = \begin{pmatrix} \hat{F}_{11}(q^{-1}) & \hat{F}_{12}(q^{-1}) \\ \hat{F}_{21}(q^{-1}) & \hat{F}_{22}(q^{-1}) \end{pmatrix} \tag{54}$$

$$\hat{F}_{11}(q^{-1}) = 0.4194 - 0.3904q^{-1} + 0.1210q^{-2}$$
$$\hat{F}_{12}(q^{-1}) = -0.4877 + 0.5835q^{-1} - 0.1958q^{-2}$$
$$\hat{F}_{21}(q^{-1}) = 1.1369 - 1.6914q^{-1} + 0.6545q^{-2}$$
$$\hat{F}_{22}(q^{-1}) = -0.4877 + 0.5835q^{-1} - 0.1958q^{-2}$$

The responses of the closed-loop system are shown in Fig. 5; in addition, the responses of the control-loop system with time delays increased by a factor of 1.5 are shown in Fig. 6. Since the target response and disturbance rejection can be controlled independently, a good target response is obtained, and the disturbance is rejected effectively. In addition, the delay time fluctuation does not produce substantial oscillation, and the coupling effects are weak.

Weight adjustment was used to set R for offset removal. Assuming that offset removal by weight adjustment is sufficient even though the weight P of the controlled variable is partly negative, we tuned P and S by trial and error. As a result, P is partly negative but the closed-loop system produces desirable control characteristics.

6. Conclusions

We configured a 2-DOF control system to deal with different time delays, and verified its robustness to time delay fluctuation. In particular, the target values were decoupled using a skeleton matrix, and the robustness to time delay fluctuation proved better than that of the widely used cross-controllers. In addition, accurate estimation of time delays was made possible by GMVC control using disturbance feedforward and a modified z-transform. In our simulations, negative concentrations occurred because only steady-state operation was modeled. As a result, the plots in Figs. 3 to 8 go beyond the 100% range; in real distillation columns, however, negative concentrations do not exist.

Thus, independent control of the target response and disturbance rejection performance was implemented by 2-DOF control; in addition, the control system showed robustness to time delay fluctuations. In the future, we plan to investigate robustness not only to time delay fluctuations, but also to gain fluctuations.

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APPENDIX

Weight Adjustment [3]

Let us calculate the target value offset. The error $e(k)$ between the controlled variable $y(k)$ and the target value $w(k)$ is defined as follows:

$$e(k) = w(k) - y(k) \tag{A.1}$$

The following is obtained by substituting it into Eq. (23):

$$\begin{aligned} e(k) = &\left[T(q^{-1})\right]^{-1} \left[\{T(q^{-1}) - R(q^{-1})\}w(k)\right. \\ &- \{P(q^{-1})A^{*-1}(q^{-1})\bar{B}^* + S(q^{-1})\} \\ &\left.\cdot B^{*-1}(q^{-1})C^*(q^{-1})\xi(k)\right] \end{aligned} \tag{A.2}$$

$$\begin{aligned} T(q^{-1}) = &\left[P(q^{-1})A^{*-1}(q^{-1})\bar{B}^* B^{*-1}(q^{-1})A^*(q^{-1})\right. \\ &\left.+ S(q^{-1})B^{*-1}(q^{-1})A^*(q^{-1})\right] \end{aligned} \tag{A.3}$$

The expectation is obtained as

$$\mathbf{E}\left[e(k)\right] = \left[T(q^{-1})\right]^{-1} \left[T(q^{-1}) - R(q^{-1})\right] w(k) \tag{A.4}$$

Using the final value theorem, let us calculate the offset for step variation of the target value:

$$\begin{aligned}\mathbf{E}\left[e(\infty)\right] &= \lim_{q\to 1}\left[I-q^{-1}\right]\left[T(q^{-1})\right]^{-1} \\ &\quad \left[T(q^{-1})-R(q^{-1})\right]\left[q-I\right]^{-1}q \\ &= \left[T(1)\right]^{-1}\left[T(1)-R(1)\right]\end{aligned} \tag{A.5}$$

In the above equation, the offset is removed when $T(1) - R(1) = 0$.

$$\begin{aligned}T(1)-R(1) &= P(1)A^{*-1}(1)B^{*}(1)B^{*-1}(1)A^{*}(1) \\ &\quad + S(1)B^{*-1}(1)A^{*}(1) - R(1) \\ &= P(1) + S(1)B^{*-1}(1)A^{*}(1) - R(1)\end{aligned} \tag{A.6}$$

Therefore, after arbitrary selection of $P(q^{-1})$ and $S(q^{-1})$, the offset can be removed by the following setting:

$$R(1) = P(1) + S(1)B^{*-1}(1)A^{*}(1) \tag{A.7}$$

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