DIFFRACTION OF ELECTROMAGNETIC WAVES BY A CLOUD OF PASSIVE VIBRATORS

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An approximate method is proposed for solving the problem of the diffraction of an electromagnetic plane wave by a cloud of passive vibrators in random motion. The field at a point in the far zone is found on the radiator side of the cloud. Expressions are found for the average power and for the spectrum.

Problems involving the diffraction of electromagnetic waves by a cloud of fine conductors (passive vibrators) are usually solved by reducing this cloud to a cloud of active, i.e., emitting, vibrators. It was shown in [1] that this approach does not yield perfectly accurate results. Below we propose a method for approximately solving for the diffraction field of the cloud at an observation point coincident with the radiator.

Formulation of the Problem

We consider a cloud consisting of a large number N of passive vibration of length l upon which an electromagnetic plane wave is incident along the z axis. We assume for simplicity that the cloud is a solid of revolution around the z axis. The origin of the cloud is at $z = 0$, and its end is at $z = a$. The vibrators fill the cloud at a uniform average density, are in random orientations, and are in random motion within the cloud. We assume the cloud to be nearly transparent for the incident wave, as is the case with either vibrators which are very small in comparison with the wavelength or which are separated by distances much greater than the wavelength. We are to find the field reflected from the cloud at an observation point $z = -R$ in the far zone ($R \gg a$). For definiteness we assume the observation point to be at the radiator.

Physical Picture of Diffraction by a Single Vibrator

Let us analyze diffraction by a single vibrator. Diffraction of an electromagnetic plane wave by a fine, straight conductor was described in [2]. When a wave is incident on the vibrator at some angle, a traveling-wave current is induced in the vibrator, with a phase velocity of $c/\sin\theta$ (where c is the velocity of the electromagnetic wave, and θ is the incidence angle, shown in Fig. 1); in addition, a standing wave is produced because of multiple reflections from the ends of the vibrator. The net current at the ends of the vibrator vanishes. The field reradiated by the vibrator is calculated as the resultant of the fields radiated by elementary dipoles dx (where x lies along the vibrator axis). If we assume that there is only a single current traveling wave, we find that the polarization vector also lies in the plane of the figure, and the amplitude of the reradiated field in the far zone, at a distance R from the vibrator, is

$$|E| = \frac{60\sigma E\cos\theta}{R}\,\frac{\cos\varphi\sin\left[\frac{\kappa l}{2}(\sin\theta-\sin\varphi)\right]}{\sin\theta-\sin\varphi}, \tag{1}$$

where $\sigma E\cos\theta$ is the amplitude of the current induced in the vibrator by the incident wave, $\kappa = 2\pi/\lambda$, and φ is the observation angle, shown in Fig. 1.

We see from Eq. (1) that the maximum reradiation occurs along the generatrix of a cone having a vertex angle of $\pi - 2\varphi$, approximately equal to 2θ in the case $\theta < \pi/2$ and $\kappa l \gg 1$. Figure 1 shows two lobes of

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the diffraction pattern, one along the reflection-law direction, and the other along the direction corresponding to the cophasal resultant of the fields from the elements dx; the field along these directions increases with increasing vibrator length. This picture shows that most of the radiation of a long vibrator results from the traveling wave induced in the vibrator. The standing wave, which emits radiation symmetric about the vibrator axis, must be quite weak. An exceptional case is that in which the wave is directly incident on the vibrator, and there is a pure standing wave. It follows from Eq. (1) that in the case of oblique incidence of the incident wave, the traveling wave no longer predominates over the standing wave in the case $\kappa l \ll 1$.

Diffraction by a Cloud of Fixed Vibrators

We consider a cloud of N rather long, fixed vibrators. We divide the cloud into n identical layers by planes normal to the z axis. If the thickness of each such layer is much less than the wavelength of the incident wave, we can assume, on the basis of the picture described above for diffraction by a single vibrator, that all the vibrators in a layer radiate in phase. We therefore assume that at z = –R the electric field of the wave reradiated by layer s is

$$E_s = h\,\Gamma_{s-1}\, e^{i\omega t - 2i\kappa(n-1)\Delta s}.$$

We call the positive real quantity Γ_{s-1} the diffraction coefficient of layer s; h is the complex attenuation factor of the field at the observation point; and Δ is the thickness of an elementary layer, i.e., a/n. The attenuation factor h is the same for all layers since the cloud is nearly transparent and because of the condition $R \gg a$.

The field at z = –R is the resultant of the fields from all the layers:

$$E = he^{i\omega t}\sum_{s=0}^{n-1}\frac{\Gamma_s}{\Delta}e^{2i\kappa\Delta s}\Delta.$$

In the limit as $n \to \infty$ we find

$$E = he^{i\omega t}\int_0^a \frac{\Gamma(z)}{dz}e^{-2i\kappa z}\,dz.$$

By definition, $\Gamma(z)$ is the diffraction coefficient of the elementary layer at coordinate z, so the ratio $\Gamma(z)/dz = \gamma(z)$ is the diffraction coefficient per unit length. We call this ratio the "diffraction function" of the cloud. This function is obviously related to the structure of the cloud along the z axis. By definition, $\gamma(z)$, a real function of z, vanishes everywhere on the z axis except on the interval $0 < z < a$, at whose ends it also vanishes. Accordingly, we have $E = \dot{E}e^{i\omega t}$, where $\dot{E}$ is the complex amplitude of the diffracted field at z = –R:

$$\dot{E} = h\int_0^a \gamma(z)\,e^{-2i\kappa z}\,dz. \qquad (2)$$

Exploiting the symmetry of the diffracted fields of infinitesimally thin layers, we can write the complex amplitude of the diffracted field at a distance R from the cloud on the other side of the cloud as

$$\dot{E}_R = h\int_0^a \gamma(z)\,dz. \qquad (3)$$

Since the cophasal fields are summed at any remote point on the left of the cloud, we have

$$\dot{E}_R = \beta N, \qquad (4)$$

where β is a complex coefficient which is independent of both the cloud dimensions and configuration.

Let us evaluate $|\beta|$ under the assumption that only traveling waves are excited in the vibrators. We use Eq. (1) for this purpose, bearing in mind that along the direction corresponding to the cophasal summation of the fields (i.e., along the propagation direction of the incident wave), this equation becomes

$$|\dot{E}_s| = \frac{30\,\sigma E}{R}\cos^2\theta_s\,\kappa l,$$

where $\theta_s < \pi/2$ is the angle formed by the axis of vibrator s and the plane of the wave front. Summing the

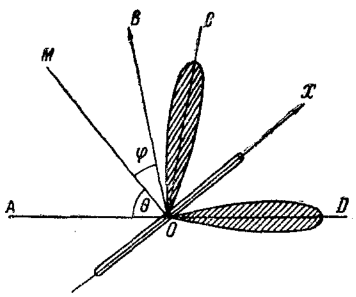

Fig. 1. AO) Direction of the incident wave; OB) direction to the observation point; OC) direction of the reflected field; OD) direction of the cophasal resultant of the elementary fields; OM) normal to x.

fields from all vibrators at z = R, and assuming all orientations of the vibrator to be equiprobable, we find

$$|\dot{E}_R| = \frac{30\,\pi\sigma E}{R}\,\frac{l}{\lambda}\,N$$

(we neglect the random part of the field, since the number N is large). Comparing this equation with Eq. (4), we find

$$|\beta| = g\frac{l}{\lambda}, \qquad g = \frac{30\,\sigma E}{R}. \tag{5}$$

In deriving an expression for g, we took into account the fact that the vibrators do not lie in the polarization plane of the incident wave. From Eqs. (2)-(4) we find

$$\dot{E} = \beta N \frac{\int\limits_0^a \gamma(z)\, e^{-2i\kappa z}\, dz}{\int\limits_0^a \gamma(z)\, dz}. \tag{6}$$

Since β depends on R, the vibrator length, and the wavelength R, we consider the properties of the field described by Eq. (6), assuming these parameters to be specified. Then we can extract information about the properties of field (6) the function

$$I(a) = \frac{\int\limits_0^a \gamma(z)\, e^{-2i\kappa z}\, dz}{\int\limits_0^a \gamma(z)\, dz}.$$

The problem is now one of studying the properties of the functions $\gamma(z)$ and $I(a)$.

Relation between the Diffraction Function and the Cloud Structure and the Number of Vibrators

We assume that an elementary layer of a specific cloud contains n(z) infinitesimally short vibrators. Since the fields from all vibrators within a layer are cophasal at the observation point, the quantity $\gamma(z)$ must be an additive function of the number of vibrators of a given layer. We can thus write

$$\gamma(z) = \varkappa n(z). \tag{7}$$

The quantity $\varkappa$, independent of z, is a proportionality factor. Hence, in particular, we find that with $n(z_1) = n(z_2)$ we have

$$\gamma(z_1) = \gamma(z_2). \tag{8}$$

In the case of a statistically uniform arrangement of vibrators within the cloud, we have

$$n(z) = \mu S(z),$$

where S(z) is the cross-sectional area of the cloud at coordinate z, and μ is a proportionality factor. From Eq. (7), we find, assuming the cloud to be a solid of revolution,

$$\gamma(z) = \delta r(z),$$

where r(z) is the radius of cross section S(z), and δ is a coefficient. Accordingly the function $\gamma(z)$ in the case of a cloud filled uniformly by vibrators is of the same form as function r(z). We have thus obtained a graphic representation of the function $\gamma(z)$ by using the particular case of a uniform distribution of vibrators in a cloud having the form of a solid of revolution. All the arguments above remain valid for a cloud of arbitrary shape, since it is unnecessary to specify a particular shape for the cloud.

Let us consider some particular examples of the behavior of $I(a)$ for various $\gamma(z)$.

1. Homogeneous Function:

$$\gamma(z) = \begin{cases} \dfrac{1}{a} & \text{for} \quad 0 < z < a \\ 0 & \text{for} \quad 0 \geqslant z \geqslant a, \end{cases}$$

$$I(a) = \frac{\lambda}{2\pi a} \sin \kappa a \; e^{-i\kappa a}.$$

The modulus of I(a), which decreases with increasing a, periodically passes through zero. Consequently, the field at the observation point gradually fades as the cloud expands, oscillating about a zero value.

2. Quadratic Function:

$$\gamma(z)=\begin{cases}\dfrac{1}{a^3}\left(z-\dfrac{a}{2}\right)^2 & \text{for } 0<z<a\\ 0 & \text{for } 0\geqslant z\geqslant a,\end{cases}$$

$$I(a)=\frac{3\lambda}{2\pi a}\sin\kappa a\; e^{-2i\kappa a}.$$

3. Quadratic Function:

$$\gamma(z)=\begin{cases}\dfrac{1}{4a}-\dfrac{1}{4a^3}\left(z-\dfrac{a}{2}\right)^2 & \text{for } 0<z<a\\ 0 & \text{for } 0\geqslant z\geqslant a,\end{cases}$$

$$I(a)=\frac{3\lambda^2}{16\pi^2 a^2}\cos\kappa a\, e^{-2i\kappa a}.$$

The modulus I(a) behaves as in the preceding cases, but is very small, since we have $\lambda \ll a$.

In these examples we have adopted definite functions $\gamma(z)$. Our analysis of these cases has a purely formal meaning, since the random spatial distribution of vibrators implies that the diffraction function must be a random function z. We write the actual diffraction function as

$$\gamma(z)=\langle\gamma(z)\rangle\,[1+\Phi(z)], \tag{9}$$

where $\langle\gamma(z)\rangle$ is the mathematical expectation of random function $\gamma(z)$, and $\Phi(z)$ is a random function having a vanishing mathematical expectation. Equation (9) does not simply describe a particular cloud: it describes a set of possible cases corresponding to the same average behavior of the diffraction functions.

To determine the dispersion of $\Phi(z)$ we make use of the fact that $\gamma(z)$ is an additive function of the number of vibrators intersected by an elementary layer. If a system consists of a certain number of independent parts (in this case of infinitesimally short vibrators), the relative fluctuation of any additive function of some state of the system [in this case the diffraction function $\gamma(z)$] is inversely proportional to the square root of the average number of parts [3]. This means that we have

$$\frac{\sqrt{\overline{(\Delta\gamma)^2}}}{\langle\gamma(z)\rangle}=\frac{1}{\sqrt{\langle n(z)\rangle}}. \tag{10}$$

From Eq. (9) we find

$$\frac{\sqrt{\overline{(\Delta\gamma)^2}}}{\langle\gamma(z)\rangle}=\sqrt{\overline{[\Delta\Phi(z)]^2}}. \tag{11}$$

Comparing Eqs. (10) and (11), we find the dispersion D(z) of random function $\Phi(z)$:

$$D(z)=\frac{1}{\langle n(z)\rangle}. \tag{12}$$

To find $\langle n(z)\rangle$ for a specified function $\langle\gamma(z)\rangle$, we assume that $\langle m(z)\rangle$ is the average number of vibrators at cross section z per unit length. Then according to Eq. (7) we can write

$$\langle\gamma(z)\rangle=\varkappa\,\langle m(z)\rangle. \tag{13}$$

Integrating this equation from 0 to a, and taking account of the fact that the cloud contains N vibrators, we find

$$\varkappa=\frac{1}{N}\int\limits_0^a\langle\gamma(z)\rangle\,dz.$$

From Eq. (13) we find

$$\langle m(z) \rangle = \frac{N \langle \gamma(z) \rangle}{\int_0^a \langle \gamma(z) \rangle dz}. \tag{14}$$

We can show that the average number of vibrators $\langle n(z) \rangle$ intersected by an elementary layer is smaller by a factor of π than the average number of vibrators in a layer of thickness $2l$; using this result, we can write

$$n(z) = \frac{2lN \langle \gamma(z) \rangle}{\pi \int_0^a \langle \gamma(z) \rangle dz}.$$

For a homogeneous function $\langle \gamma(z) \rangle$ we have

$$\langle n(z) \rangle = \frac{2Nl}{\pi a}. \tag{15}$$

Below we restrict the discussion to the case of a cloud having a homogeneous function $\langle \gamma(z) \rangle$. We find the dispersion from Eqs. (13) and (15):

$$D = \frac{\pi a}{2Nl}. \tag{16}$$

Let us determine the correlation function for random function $\Phi(z)$. The correlation radius of $\Phi(z)$ is equal to the vibrator length. According to Eq. (8), the function $\Phi(z)$ is completely correlated at the nearest coordinates, since the values of $\gamma(z)$ are due to segments of the same vibrators. For coordinates separated by a distance greater than the vibrator length, the correlation disappears completely because there are no physical bonds between the vibrators.

To determine the correlation function we also assume that for a homogeneous function $\gamma(z)$ the statistical properties of $\Phi(z)$ do not depend on the coordinate and that the correlation function is unambiguously governed by the coordinate difference $z - z' = \zeta$. Accordingly, the correlation function $R(\zeta)$ must satisfy the conditions:

1) $R(\zeta)$ is an even function;

2) $R(\zeta) = D$ for $\zeta = 0$;

3) $R(\zeta) = 0$ for $-l \geq \zeta \geq l$;

4) the spectrum determined by correlation function $R(\zeta)$ must be a nonnegative function of the linear density.

All these conditions are satisfied by the approximating function

$$R(\zeta) = \begin{cases} De^{-\alpha|\zeta|} \cos \frac{\pi}{2l} \zeta & \text{for} \quad -l < \zeta < l \\ 0 & \text{for} \quad -l \geqslant \zeta \geqslant l. \end{cases} \tag{17}$$

The quantity α is determined below. Dispersion D is given by Eq. (16). Within a layer of thickness $2l$, diffraction function $\gamma(z)$ is completely or partially correlated, so we call such a layer a "correlation layer."

Cloud of Vibrators in Random Motion

Up to this point we have dealt with diffraction by clouds of fixed vibrators, treating the function $\gamma(z)$ as a function varying randomly from one case to the next. We have not discussed the mechanism for the transition between possible cases. This mechanism could be random motion of the vibrators, in which case the diffraction functions would depend on the time. If we assume that the shape of the cloud remains unchanged as the vibrators move or, equivalently, that $\langle \gamma(z) \rangle$ does not change, we can write the diffraction function as

$$\gamma(z, t) = \langle \gamma(z) \rangle [1 + \Phi(z, t)], \tag{18}$$

where $\Phi(z, t)$ is a random function of the coordinate and time and has a vanishing mathematical expectation.

Let us analyze the relationship between the functions $\Phi(z, t)$ and $\Phi(z)$. In the case of random vibrator motion, the function $\Phi(z, t)$ is a random function of the time because of the random exchange of vibrators between adjacent correlation layers. The rate at which this function changes depends on the relative velocity of the vibrators. Since only the motion along the z axis is significant here, the only important velocity components are those along the z axis. If the average velocity of the vibrators in their random motion along this axis is v, the vibrators do not manage to leave the correlation layers during time intervals $\tau < l/v$, so the function $\Phi(z)$ remains essentially constant. After time intervals $\tau > l/v$, the vibrators leave their correlation layers, and the function $\Phi(z)$ has become completely different. Accordingly, we can introduce a temporal correlation function $R(\tau)$ of the random function $\Phi(z, t)$. The steady-state motion, the correlation function $R(\tau)$ must have the same form as $R(\zeta)$, by virtue of the ergodicityof the ensemble. At $\tau = 0$ this function must be equal to unity, for when the random motion stops, the function $\Phi(z, t)$ converts into $\Phi(z)$. We can thus write

$$R(\tau) = \begin{cases} e^{-\varepsilon|\tau|} \cos \dfrac{\pi v}{2l} \tau & \text{for} \quad -\dfrac{l}{v} < \tau < \dfrac{l}{v} \\ 0 & \text{for} \quad -\dfrac{l}{v} \geqslant \tau \geqslant \dfrac{l}{v} \end{cases}, \tag{19}$$

where ε is to be determined.

We can show that random function $\Phi(z, t)$ is the product of two random functions of independent variables z and t, which can be separated:

$$\Phi(z, t) = \Phi(z) \cdot \varphi(t). \tag{20}$$

Since the number of vibrators in the cloud is constant, so is the integral $\int_0^a \gamma(z, t)dz$. Accordingly, Eq. (18) shows that $\int_0^a \Phi(z, t)dz$ is also independent of the time. Since the mathematical expectation of random function $\Phi(z, t)$ vanishes by assumption, we have $\int_0^a \Phi(z, t)dz = 0$ at any time. The integral on the left side of this equation is a product of the form $f(a)\varphi(t)$; since it vanishes at any time, we have $f(a) \equiv 0$, so the integrand must be only a product of the form $\Phi(z)\varphi(t)$, where $\int_0^a \Phi(z)dz = 0$. This latter equality holds in the case $\varphi(t) = \text{const}$, i.e., if the vibrators are fixed.

It follows that the mathematical expection of $\varphi(t)$ vanishes, there is unit dispersion, and the correlation function is given by Eq. (19). The mathematical expectation of $\Phi(z)$ vanishes, the dispersion is given by Eq. (16), and the correlation function is given by function (17).

Fluctuations of the Field of a Cloud of Moving Vibrators

Using Eq. (20), we write diffraction function (18) for a homogeneous cloud: $\gamma(z, t) = 1/a + (\varphi(t)/a)\Phi(z)$. Then we have

$$I(a, t) = \frac{\lambda e^{-i\kappa a}}{2\pi a} \sin \kappa a + \frac{\varphi(t)}{a} \int_0^a \Phi(z) e^{-2i\kappa z} dz. \tag{21}$$

The complex field amplitude at the observation point can be written, according to Eqs. (6) and (21), as

$$\dot{E} = \beta N I(a, t) = E_d + E_r.$$

The field at the observation point is thus the sum of two parts: a determinate part, having a complex amplitude

$$E_d = \frac{\beta N \lambda e^{-i\kappa a}}{2\pi a} \sin \kappa a,$$

and a random part,

$$E_r = \beta N \frac{\varphi(t)}{a} \int_0^a \Phi(z) e^{-2i\kappa z} dz.$$

We can then find the power of the determinate part of the field:

$$P_d = \frac{|\beta|^2 N^2 \lambda^2 S}{4\pi^2 a^2} \sin^2 \kappa a, \tag{22}$$

where S is the aperture of the receiving antenna.

The average power of the random part is expressed in terms of the field dispersion:

$$P_r = D\{E_r\}S = \frac{|\beta|^2 N^2}{a^2} D\{\varphi(t)\} D\{I\} S,$$

where

$$I = \int_0^a \Phi(z) e^{-2i\kappa z} dz.$$

Since we have $D\{\varphi(t)\} = 1$, we have

$$P_r = \frac{|\beta|^2 N^2}{a^2} D\{I\} S. \tag{23}$$

To find $D\{I\}$ we use the fact that correlation function (17) for random function $\Phi(z)$ is known. For this purpose we must evaluate the integral

$$D\{I\} = a \int_{-l}^{l} R(\zeta)\left(1 - \frac{|\zeta|}{a}\right) e^{-2i\kappa\zeta} d\zeta.$$

Evaluation for $a \gg l$ yields

$$D\{I\} = D \frac{2\alpha a[\alpha^2 + \rho^2 + 4\kappa^2]}{(\alpha^2 + \rho^2 + 4\kappa^2)^2 + 16\rho^2\kappa^2}\left[1 - \frac{\rho}{\alpha} e^{-\alpha l} \frac{\sqrt{(\alpha^2 + \rho^2 + 4\kappa^2)^2 - 16\rho^2\kappa^2}}{\alpha^2 + \rho^2 + 4\kappa^2} \cos(2\kappa l + \psi)\right],$$
$$\rho = \frac{\pi}{2l}, \quad \tan\psi = \frac{\alpha^2 + \rho^2 - 4\kappa^2}{4\alpha\kappa}. \tag{24}$$

In the case $l \ll \lambda$, $\kappa^2 \ll \alpha^2$, we have

$$D\{I\} \approx D \frac{2\alpha a}{\alpha^2 + \rho^2}. \tag{25}$$

From Eqs. (22) and (23) with $\sin \kappa a = 1$ we find

$$\frac{P_r}{P_d} = \frac{4\pi^2}{\lambda^2} D\{I\}. \tag{26}$$

It was shown in [1] that with $l \ll \lambda$

$$\frac{P_r}{P_d} = \frac{4\pi^2 a^2}{N\lambda^2}. \tag{27}$$

From Eqs. (26) and (27) we see that dispersion $D\{I\}$ must satisfy

$$D\{I\} = \frac{a^2}{N}. \tag{28}$$

Equating Eqs. (25) and (28), and using (16), we find

$$\alpha = \frac{\pi}{2l} \tag{29}$$

The inequality $\kappa^2 \ll \alpha^2$ holds here, because of the discussion above.

Equation (24) can be rewritten as

$$D\{I\} = D \frac{\alpha a(\alpha^2 + 2\kappa^2)}{\alpha^2 + 4\kappa^2}\left[1 - e^{-\frac{\pi}{2}} \frac{\sqrt{\alpha^4 + 4\kappa^4}}{\alpha^2 + 2\kappa^2} \cos(2\kappa l + \psi)\right],$$
$$\psi = \frac{\alpha^2 - 2\kappa^2}{2\alpha\kappa}. \tag{30}$$

The power spectrum of the random part of the field can be written on the basis of Eqs. (19) and (23) as

$$F(\Omega) = \frac{|\beta|^2 N^2 S}{a^2} D\{I\} \int_{-v/l}^{v/l} R(\tau) e^{-i\Omega\tau} d\tau.$$

From Eq. (29) for α we see that we have $\varepsilon = \pi v/2l$ in Eq. (19) and thus

$$F(\Omega) = \frac{|\beta|^2 N^2 S}{a^2} D\{I\} \frac{\varepsilon}{\pi} \frac{2\varepsilon^2 + \Omega^2}{4\varepsilon^4 + \Omega^4} \left[1 - e^{-\frac{\pi}{2}} \frac{\sqrt{4\varepsilon^4 + \Omega^4}}{2\varepsilon^2 + \Omega^2} \cos\left(\Omega \frac{l}{v} + \chi\right)\right],$$
$$\tan\chi = \frac{2\varepsilon^2 - \Omega^2}{2\varepsilon\Omega}. \tag{31}$$

RESULTS AND DISCUSSION

According to these arguments, the total power received by the receiving antenna is the sum of the power corresponding to the determinate part of the field and that corresponding to the random part:

$$P = P_d + P_r.$$

Components P_d and P_r are given by Eqs. (22) and (23). Using Eq. (5), we rewrite these expressions as

$$P_d = \frac{g^2 N^2 l^2 S}{4\pi^2 a^2} \sin^2 \kappa a, \tag{32}$$

$$P_r = \frac{g^2 N^2 l^2 S}{a^2 \lambda^2} D\{I\}. \tag{33}$$

These equations hold for $\kappa l \gg 1$, in which case we can neglect the reradiation of vibrators due to the standing waves. For this case we rewrite Eq. (30) in the simpler form

$$D\{I\} = \frac{a^2}{2N} \frac{\lambda^2}{l^2} [1 + 0.2 \sin 2\kappa l].$$

Then we have

$$P_r = \frac{g^2 N S}{2} [1 + 0.2 \sin 2\kappa l]. \tag{34}$$

From Eq. (34) we see that the average power of the random part of the reflected field is proportional to the number of vibrators and nearly independent of the vibrator length. On the other hand, the power of the determinate part of the field is proportional to the square of the number of vibrators and is a strong function of the vibrator length. If the cloud consists of a large number of long vibrators, the power of the determinate part of the field is thus predominant.

In the case $\kappa l \ll 1$, Eqs. (32) and (33) give results which are too low, since in their derivation we neglected the relatively strong radiation due to the standing waves. An inspection of the procedure used to derive Eqs. (32) and (33) shows that in the case $\kappa l \ll 1$ an account of this radiation affects components P_r and P_d in the same way. Accordingly, the ratio P_r/P_d in the case $\kappa l \ll 1$ does not depend on whether only the traveling waves, only the standing waves, or both are taken into account. This circumstance was used to determine α from Eq. (29).

The power spectrum of the random part of the field is given by

$$F(\Omega) = \frac{g^2 N}{\pi^2} t \frac{2 + (4ft)^2}{4 + (4ft)^4} \left[1 - 0.2 \frac{\sqrt{4 + (4ft)^4}}{2 + (4ft)^2} \cos(2\pi f t + \chi)\right], \tag{35}$$

where f is the frequency (in hertz) and $t = l/v$ is the correlation time.

We see from Eq. (35) that the high-frequency part of the spectrum becomes more intense as the vibrators are shortened. The spectral density reaches a maximum at $0.23 v/l$ Hz and tends toward zero as the frequency is increased, oscillating slightly about an average value.

We should point out that we have neglected the Doppler component of the spectrum, due to the random motion of the vibrators. It can be shown, however, that this component is relatively small.

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