RESEARCH PROBLEMS

Synthetic data experiments using system dynamics models: a survey of results and a research agenda

Catherine M. Crawford, David F. Andersen, and George P. Richardson

In this section the *System Dynamics Review* presents problems having the potential to stimulate system dynamics research. Articles may address real-world dynamic problems that could be approached fruitfully from the system dynamics perspective, or methodological problems affecting the field. A paper submitted to the *Research Problems* section should concisely motivate and define a problem and start a process of conceptualization or formulation that can open the way for further studies. Manuscripts not exceeding 2,000 words should be sent to George P. Richardson, Milne 308, Rockefeller College, S.U.N.Y., Albany, NY 12222, U.S.A.

Catherine M. Crawford is an assistant professor in human service studies at Cornell University. She received her Ph.D. degree in public administration from the Rockefeller College, State University of New York at Albany. Her research interests include health policy issues and exploring alternative designs in program evaluation. *Address:* Human Services Studies, N 132 Martha van Rensselaer Hall, Cornell University, Ithaca, NY 14853, U.S.A.

Over the past decade, a number of published papers, working papers, and dissertations in the field of system dynamics have relied on synthetic data experiments as a research design (Sterman 1988; Sterman and Richardson 1985; Sterman, Richardson, and Davidsen 1988; Crawford 1988; Kelly 1984; McCaffrey et al. 1985; Mass and Senge 1980; Morecroft 1977; Richardson 1981; Senge 1974; 1975; 1977; Forrester 1979). In addition, a number of related papers, books, and dissertation projects have commented on the appropriate use of statistical analyses within continuous-time dynamic systems, often providing a theoretical basis for synthetic data experiments (Andersen 1983; Peterson 1975; 1980; Belsley, Kuh, and Welsch 1980; Bergstrom 1976; Johnson 1980; Luecke and McGinn 1975). The growing stream of theoretical and applied work points to the great potential that synthetic data experiments may have for investigating both technical and conceptual problems relating to system dynamics modeling.

Recognizing this potential, a group of researchers gathered at the Rockefeller Institute of Government, State University of New York at Albany, in June 1988 to discuss the results of synthetic data experiments to date and to consider how an organized program of research in synthetic data should proceed. The assumptions of the gathered researchers were several. First, while synthetic data experiments appear to have great potential, the experiments have often led to inconclusive results. The mixed record suggests that perhaps a more directed research agenda is necessary to realize the promising potential of synthetic data experiments in system dynamics. Second, the field as a whole has relatively limited research efforts that can be invested in these experiments. Thus, it would seem that efficient scientific inquiry is a necessity. Finally, this research would be most efficient if a community of involved researchers could agree upon a set of important questions, starting assumptions, and preferred approaches. The purpose of this article is to propose a research program in synthetic data experiments for discussion, debate, and ultimately implementation by interested researchers.

The synthetic data design

In general, we define synthetic data experiments as research projects that involve two computer-based models—a data-generating model and an estimation model. Typically, the data-generating model is a simulation model (for our purposes, think of a system dynamics model) that is designed to represent some aspects of a real-world system. Data are generated by this model under a variety of stochastic conditions (process error is fed into the model, output variables are observed with measurement

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David F. Andersen is associate professor at the Rockefeller College of Public Affairs and Policy, State University of New York at Albany, and Director of the Rockefeller Institute of Government. His teaching and research focuses on the application of system dynamics to public policy problems, decision making in the public sector, and public information resource management. He currently serves the System Dynamics Society as vice president for publications.

George P. Richardson is associate professor at the Rockefeller College of Public Affairs and Policy, State University of New York at Albany. He is coauthor of *Introduction to System Dynamics Modeling with DYNAMO* and executive editor of the *System Dynamics Review*.

error, and so on), and these data may be sampled in a variety of ways. For example, multiple simulations may be used to create a cross-sectional sample, or a single run may be sampled through simulated time (at various sampling intervals) to create a longitudinal synthetic data set.

Once sampled, the synthetic data are then used as inputs to a statistical model that is used to estimate some aspect of the data-generating model, such as important parameters or elements of system structure. The key to these experiments is that the exact structure and parameters of the data-generating model are known in advance. The ability of the estimation model to recapture features of the data-generating model can be used as a more or less pure test of the ability of the statistical sampling and estimation techniques to recover accurately the known properties of a data-generating system.

The origins of the application of synthetic data experiments to the study of estimating the parameters of a social system can be traced back at least to the early 1970s. Brunner and Brewer (1971) performed a synthetic data experiment on a formal model of "modernization and mass politics" (pp. 16–22) in order to investigate the implications of public policies such as birth control, austerity programs, and taxes in Turkey and the Philippines. They found that ordinary least squares (OLS) regression can correctly retrieve perfectly specified parameters, but when feedback loops are slightly misspecified or simplified in the estimation model, the OLS estimates can be far off. They concluded that statistical estimation provides only weak clues about system structure:

> The behavior of a class of systems does not tell us everything we need to know about their common structure; conversely, a common structure produces different patterns of behavior in different contexts. The fitting of regression equations to data from different contexts produces coefficients that describe these contexts and that may contain clues about structure. These clues are not unambiguous and must be evaluated in the light of other evidence. To an overwhelming extent, however, structural information is imposed on the data rather than derived from it. (p. 168)

Recent experiments conducted in system dynamics have continued with this line of reasoning. They have also investigated relatively narrow and technical questions, such as how much measurement error might disrupt the ability of single-equation regression models to recover accurately parameters from a dynamic system (Senge 1975), as well as more conceptual questions that are more broadly associated with social science research. For example, Forrester (1979) evaluated the effects of common misspecifications in dynamic stock adjustment models, an important class of models in the study of household behavior and the business cycle. Also, Crawford (1988) investigated the effects of omitting feedback effects in the design of statistical cross-sectional evaluations of government programs.

Motivating questions and levels of research

One key question for synthetic data studies is how to estimate parameters in system dynamics models (Andersen 1981; Peterson 1980). Nonlinear, continuous, multiloop structures violate many of the hypotheses of commonly used econometric techniques. Often, this basic question evokes even more fundamental ones: Is it possible or even sensible to attempt to statistically estimate parameters in a system dynamics model? Under what conditions might it be impossible to estimate what types of parameters? (Richardson 1981; Senge 1974; 1975; 1977) Questions of parameter estimation cannot be neatly separated from questions of how to identify structural components of a system that are (statistically) significant in the overall dynamic performance of the system. The question can become, Under what conditions (if any) can statistical techniques help us to estimate or formulate portions of a model's structure? (Mass and Senge 1980; Morecroft 1977)

Recently, several researchers have suggested that synthetic data experiments might be useful in investigating chaotic dynamic systems (Mosekilde, Aracil, and Allen 1988; Reiner, Munz, and Weidlich 1988). Empirical investigations of the properties of chaotic systems in the social and managerial sciences are very difficult because the time series necessary to distinguish between chaos and other modes of behavior, such as stochastic processes or quasi-periodicity, may be on the order of thousands of years (Chen 1988; Reiner, Munz, and Weidlich 1988; Andersen 1988; Sterman 1989). Hence, generating data from a chaotic model and using an estimation model to recover parameters and structure may be the only way to conduct empirical investigations of chaotic systems and to investigate the ability of various statistical techniques to correctly diagnose those systems. This approach has been used successfully to test procedures to estimate Lyapunov exponents from time series data (Wolf et al. 1985).

Synthetic data experiments can also shed light on research design in the social, management, and policy sciences. For example, Do systematic biases arise from the common practice of specifying linear models? (Andersen 1982) How might the use of cross-sectional data to approximate dynamic phenomena bias research results? (Richardson 1981; Andersen and Fugleberg 1988) How might the inadvertent omission of feedback effects from the research design and the collection and analysis of data bias or otherwise affect final research conclusions? (Crawford 1988) How might the existence of nonlinearities in the real world facilitate or impede researchers' ability to estimate parameters or the effectiveness of programs in applied situations? (Leucke and McGinn 1975; Andersen 1981; 1983; McCaffrey et al. 1985)

This wide range of questions has been approached with research based on an equally wide range of data-generating and estimating models. While most synthetic data experiments have used nonlinear data-generating models, some researchers, such as Andersen and Fugleberg (1988), have chosen to focus on very simple linear structures in an attempt to settle some basic methodological questions. Most of these data-generating models are corrupted by both process error and measurement error; quite often the data are observed either longitudinally or with a discrete sampling interval. Estimation techniques have included ordinary least squares, two-stage least squares, generalized least squares, instrumental variables, and simultaneous equation techniques. While several researchers have suggested that additional estimation techniques, such as Kalman filtering or logistic regression, should be investigated, we are not aware of any cases where this has actually been done.

In addition to differing with regard to model type, errors, and estimation techniques, synthetic data experiments differ in their internal designs. Some experiments use longitudinally sampled data, some use cross-sectional data. Some experiments use a nonlinear data-generating model but specify the estimation model as linear. Some experiments deliberately omit variables in the specification, while other designs experiment with which variables to include or exclude from the analysis, often classifying endogenous system dynamics variables as exogenous econometric variables. Finally, some researchers have deliberately misspecified a dynamic system as a cross-sectional system or have used the synthetic data experiment to test for the effectiveness of forecasting techniques.

There are several problems associated with using such a wide variety of synthetic data designs. First, results often are not strictly comparable. That is, because each experiment uses a different model or design, it is difficult to discern whether the designs and results of two separate studies are in fact comparable or are testing for the same effect (such as the effect of measurement error). Second, the existing research contains few replications. That is, promising results are not being replicated under a wide variety of conditions to test their robustness. Third, results are often inconclusive: the ability or inability of the estimation model to recover a parameter could be due to one of several possible effects in the experiment, and additional work appears necessary to rule out alternative explanations. But since replications are not common, this type of cumulative research has not occurred. Finally, it seems that more attention to simpler, perhaps linear, data-generating models is needed, where hypotheses or results might be derived analytically. Such closed-form analyses of the possible effects of measurement or specification errors would create a stronger bridge to existing econometric literature and provide a more solid foundation for working with complicated nonlinear dynamic systems.

To correct for some of these problems in existing synthetic data experiments, we propose a more systematic program of research. First, we describe several general types of projects. Then, a number of caveats are suggested that we believe should guide all synthetic data research projects in the field of system dynamics. Finally, to add specificity to this description, the Appendix suggests a number of focused research questions that could fill in missing gaps in knowledge and launch the field into a more systematic mode of inquiry.

A proposed research agenda

The following are five general classes of synthetic data research projects, arranged approximately in order of complexity.

Replication

Existing studies need to be replicated in two ways. First, the same data sets need to be estimated using different estimation techniques. For example, a result estimated with ordinary least squares should be replicated using more complex and appropriate techniques, such as simultaneous equations or instrumental variable techniques. Second, the same estimation techniques should be applied to a variety of data-generating models. Where possible, the mathematics should be derived when the data-generating model is tractable (linear or with manageable nonlinearities). In any case, a researcher interested in entering this field should probably begin by replicating exactly a previously published piece of work.

Longitudinal misspecification

A very intriguing line of research has tried to determine what happens when a dynamic system is approximated with cross-sectional data. However, in general, these results have been inconclusive. We suggest that the existing cross-sectional experiments, such as those by McCaffrey et al. (1985) and Crawford (1988), be replicated with longitudinal data. Before sampling cross-sectionally from a nonlinear system, a researcher should sample from a purely linear and very simple system.

In general, we believe that all these experiments should begin with very simple models, absent all forms of stochastic inputs, and move systematically toward more complicated models and inputs, with the complications added one small step at a time.

Linear misspecification of known nonlinearities

Several researchers, like Mass and Senge (1980) and Richardson (1981), have deliberately misspecified a known nonlinearity as a linear function or substituted a nonlinear approximation that is linear in the parameters to be estimated. However, the results of these experiments have not been fully satisfying because it was not always clear whether it was the nonlinearity itself—how it interacted with process error—or something else that produced the reported estimation problem. We suggest that experiments of this type begin with an estimation model that correctly specifies the nonlinearity. Then one should carefully introduce measurement error, process error, and a discrete sampling interval one at a time *before* the linear misspecification is introduced. In this way, other possible and competing explanations of why linear specification did not work can be ruled out.

A special case of linear misspecification occurs when the estimation model has correctly specified causality but has grossly misspecified (perhaps a very simple linear form) functional forms. This type of experiment would be used to see if a known causal structure would appear to be statistically insignificant due to misspecification error. All of the preceding suggestions apply to this class of experiment. In addition, it would be best with this type of experiment to resimulate the estimated set of equations to determine the dynamics of the estimated system and the operational significance of the misspecification errors.

Nonlinear systems with complex (chaotic) dynamics

As noted, synthetic data experiments may be useful in helping to determine the empirical qualities of chaotic systems and which empirical techniques, if any, are capable in principle of correctly diagnosing the presence of chaos. The work done by Wolf et al. (1985) to approximate Lyapunov exponents from time series data is the only example we are aware of in this area. However, we believe that the caveats posited in the next section, representing lessons from over a decade of research, would probably apply to these experiments.

Dynamically oriented statistical approaches

Various researchers, like Peterson (1980), have suggested that more involved statistical tools, such as Kalman filtering, time series analysis, and combined cross-sectional and time series analysis, are the appropriate statistical tools to use in estimating dynamic systems. To date, relatively little empirical work using synthetic data experiments has employed any of these approaches (at least in the available system dynamics literature). These experiments are needed and should probably be guided by the following research caveats.

Caveats to guide synthetic data research

Based upon review and discussion of much of the synthetic data research over the past decade, several caveats emerge to guide future efforts.

Replicate and build

To the extent possible, new research should build directly upon previously published work. Rather than creating a new data-generating model or inventing a new experimental design, researchers should strive to use existing models and experimental designs. Whenever possible, an existing result should be replicated exactly before a new extension is proposed. This strategy would tend to produce research results that are truly cumulative.

Avoid sidetracks

For large synthetic data projects, a large number of runs, reruns, and estimations are often needed to answer even the simplest questions. These experiments can prove expensive in terms of computing time and researcher effort. It seems important to stress that, before undertaking a number of runs, the researcher should have clearly in mind exactly what she or he hopes to learn from those runs. The daily research program should be continually checked against a series of hypotheses concerning which effects are being ruled out, which conclusions included, and so on. In other words, always strive to increase the ratio of reflective thought to computer runs. Most researchers working in this area have had the experience of investing days or weeks in completing a set of runs and then losing sight of what was supposed to be accomplished. Always keep one of the big questions clearly in mind!

Use simple models

In general, researchers should use data-generating models of a low order (second or third order, preferably) and as structurally simple as possible (even linear, where appropriate). Too often, a research program has begun with higher-order systems of complicated structural form. The project then quickly bogs down under layers of formulation detail. Keep data-generating models simple, and add complications as needed, later on.

Walk before you run

This admonition is an extension of the "use simple models" rule and applies to all areas of the experiment. For instance, a data-generating model should be estimated with longitudinal data before the researcher moves to the more complicated cross-sectional case.

Always begin with an estimation model that is perfectly specified, with no measurement or process noise, and with the sampling interval set to the solution interval of the data-generating model. For this base case, the researcher should always expect to retrieve perfectly the parameters or the structure or whatever is of interest in the estimation model. Only after showing that these elements can be recovered for this base case (in effect, a controlled test of the experimental apparatus), should the researcher move on to other questions. Move on slowly. If a discrete sampling interval is to be used, investigate only the impact of sampling, increasing the interval in small steps. Then gradually introduce process error, then measurement error, and so on.

Once factors like sampling interval, process error, and measurement error have been systematically examined, more complicated questions, such as what happens when a nonlinearity is misspecified as a linear function, can be investigated.

Knit together and build

Researchers working in various synthetic data areas should keep in touch with what is going on in other fields. System dynamicists should strive to knit their contributions together, linking to and building upon the work of others. Solid bodies of literature relating to estimation of dynamic systems already exist in the engineering and econometric literature (for example: Belsley, Kuh, and Welsch 1980; Bergstrom 1976; Johnson 1980; Schweppe 1973). We must link and build.

Appendix: Five specific research projects[1]

1. Solve the sampling interval problem

Richardson (1981) and Andersen and Fugleberg (1988) have quite clearly noted that a discrete sampling interval greater than the solution interval of a dynamic system can create considerable bias when estimating parameters within a dynamic system. This result appears to hold for both nonlinear and simple linear systems. Sampling every DT is cumbersome in synthetic data experiments and is meaningless in real-world situations. Someone needs to figure out what to do about the sampling interval issue. Several things are waiting to be done. First, the Richardson and Andersen/Fugleberg results could be derived in closed-form mathematics to see more precisely how this bias occurs. Second, Richardson has proposed that averaging might be used to eliminate the sampling interval bias. This hypothesis should be systematically investigated. Third, simple heuristics might be developed to let researchers know what sampling interval is small enough (e.g., choose sampling interval less than one-quarter of the shortest time constant of the system being studied). In all of these investigations, the econometric and engineering literature should be thoroughly searched for possible solutions (e.g., Bergstrom 1976).

2. Test estimation problems using a simple chaotic model

The suggestion that synthetic data experiments could be used to estimate structure within a chaotic system is an interesting one. This exploratory program should begin with a low-order chaotic model of relatively simple functional form. The Roessler and Lorenz models are particularly suitable because they are linear in the parameters to be estimated. Richardson, for example, demonstrated at the Albany meeting that the parameters in the Roessler model can be easily retrieved from chaotic data using naive OLS regression (as long as sampling interval is the same as DT). More complex low-order structures, such as the migration models discussed by Mosekilde, Aracil, and Allen (1988) and Reiner, Munz, and Weidlich (1988), could then be explored. Heeding the caveats outlined previously, one should first ascertain that the model can be estimated when the estimation model is perfectly specified and no noise exists. Subsequent experiments would investigate the effects of introducing measurement error, process error, or a discrete sampling interval into the system.

3. Test for the effects of cross-sectional misspecification of a dynamic system

Crawford (1988) conducted an experiment that entailed cross-sectional misspecification. Her experiment should be replicated using a linear first-order system. The system dynamics literature contains no clear explanations of what happens when cross-sectional data are used to estimate a dynamic system. This line of research needs to be more systematically explored using careful controls.

4. Compare Kalman filtering estimation and regression-based estimation on progressively complex systems

Peterson (1980) and Andersen (1981) have both suggested that this experiment would be an interesting one. This synthetic data experiment should be carefully performed and reported in the system dynamics literature.

5. Misspecify nonlinear models in linear form

Here one would correctly include all the causal links in a system but functionally misspecify all links as linear functions. The goal of this experiment would be to see which of the known links prove to be statistically significant or not significant and to determine why. Given the sampling interval difficulties Richardson (1981) encountered exploring this area, experiments in linear misspecification should be carefully designed to follow the "walk before you run" caveat.

Note

1. For a more detailed description of the range of questions to which synthetic data techniques have been applied, interested readers are encouraged to browse the complete references in an annotated bibliography available from Professor George P. Richardson, Milne 308, Rockefeller College of Public Affairs and Policy, 135 Western Avenue, Albany, NY 12222.

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