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An MAGDM based on constrained FAHP and FTOPSIS and its application to supplier selection ☆

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ABSTRACT

The multiple attribute decision making (MADM) is widely used to rank alternatives with respect to multiple attributes. A new method for the multiple attribute group decision making (MAGDM) is proposed in this paper. In our method, linguistic terms are used during the whole evaluation process, the constrained fuzzy analytic hierarchy process is adopted to measure the relative importance of attributes, which is converted into the deterministic weight vector by using the extent analysis technique, the fuzzy TOPSIS is then used to rank the alternatives. With these improvements and other transformation skills, our new algorithm can better resolve the fuzzy information by decreasing its uncertain level, more scientific and accurate attribute weights can thus be obtained. More importantly, it can significantly reduce the computation amount and can provide more reasonable and robust ranking results. All these advantages are demonstrated by applying our new method to two supplier selection problems, typical complex MAGDM problems investigated extensively due to their practical importance. The sensitivity analysis and comparison with existing approaches sufficiently show the practicality, robustness and efficiency of our new algorithm, which can be applied to different kinds of complex MAGDM problems in reality.

1. Introduction

The multiple attribute decision making (MADM) is an important research area in modern decision making science. It has been widely applied in many fields such as economy, environment and management. Two typical MADM methods are the analytic hierarchy process (AHP) and the technique for order preference by similarity to an ideal solution (TOPSIS). The AHP is proposed by Saaty [1]. It has received more and more attention [2–5] since its appearance. This method is clear in structure and easy to understand. Nevertheless, it requires pairwise comparisons between attributes and alternatives in order to set up decision matrices, which result in huge computation and low accuracy. The TOPSIS is proposed by Hwang and Yoon [6]. They think that a good alternative should be the one that is nearest to the positive ideal alternative, and at the same time, is farthest from the negative ideal alternative. The TOPSIS has been studied and applied extensively [7–15]. In TOPSIS, it is necessary to assign attribute weights of alternatives to reflect their relative importance. At present, the equal weight is usually adopted for simplicity, or the attribute weights are subjectively determined, the shortcomings of these methods are obvious. In practice, different attributes have different importance, so it is not reasonable to assign them equal weights. While for the weights selected subjectively, one can hardly guarantee their objectivity and reasonability during the

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decision making process. To overcome disadvantages of these two methods, one can first determine the weights of attributes by AHP, then weight the decision matrix and rank alternatives by TOPSIS. In this way, the intensive computation required in determining decision matrices can be greatly reduced, while the weight selection difficulty in TOPSIS can be reasonably settled.

Actually, different ways to combine AHP and TOPSIS have been investigated in many papers. In these methods, the attribute weights are often determined by AHP, the alternatives are ranked by TOPSIS. For example, Lin solves the customer-driven product design problem by using AHP and TOPSIS in [16]. But in this method, the evaluations about attributes and alternatives are all supposed to be deterministic numbers. The combination of AHP and the fuzzy TOPSIS (FTOPSIS) method is applied in [17] to solve the solid waste transshipment site selection problem. In that paper, the attribute weights determined by AHP are real numbers, and the evaluations of alternatives with respect to different attributes are in linguistic terms. Similarly, the weapon selection problem is investigated in [18] by using AHP and FTOPSIS. Unlike the above papers, the fuzzy AHP (FAHP) and TOPSIS are used in [19] to evaluate Turkish cement firms, where linguistic terms are adopted to evaluate attributes and real numbers are used to evaluate alternatives. The FAHP and TOPSIS methodologies are used to evaluate hazardous waste transportation firms in [20]. Through combining above techniques, the Turkish bank sector evaluation problem is studied in [21]. Furthermore, the shopping center selection problem [22] and production system performance problem [23] are solved by combining FAHP and FTOPSIS, where both attribute and alternative evaluations are in linguistic terms.

Fuzziness and uncertainties are often encountered in practice. During the decision making process, people are often reluctant or unable to assign accurate values in the evaluation process due to the following typical reasons: the social environment is rather complex; the people thinking is usually uncertain, ambiguity and vague; the expert's knowledge might be limited. People prefer to provide their evaluations in linguistic terms. For example, one would like to use "good", "very good", "medium","bad" or "very bad" to express his/her preference in evaluating the product quality or performance. Fuzzy set theory is an effective tool to deal with the uncertainty and vagueness. It can incorporate uncertain information, incomplete information, unavailable information and partially ignorant facts in the decision model. When evaluating alternatives, the adopted linguistic terms are translated into trapezoidal fuzzy numbers or triangular fuzzy numbers. The ranking of alternatives relies on results from the fuzzy number operation. Unfortunately, usual mathematical operations might generate meaningless results when applied to fuzzy numbers, the direct unrevised application of fuzzy arithmetic could lead to questionable results, due to a deficient generalization from the real number arithmetic to the fuzzy intervals one [24]. Unique rules are got when arithmetic operations are performed on real numbers, but this principle is not valid in the fuzzy arithmetic. To overcome this problem, Klir [25] investigates fuzzy operations under constraints. By imposing requisite constraints, the uncertainty degree is reduced and relatively accurate results can be obtained. Enea and Piazza [24] apply this kind of constrained fuzzy operations to AHP. Tiryaki and Ahlatcioglu [26] study the fuzzy portfolio selection by using the constrained FAHP.

As a further improvement to existing researches, in this paper, we initially use the constrained fuzzy AHP (CFAHP) to determine the attribute weights, then rank alternatives by the FTOPSIS; meanwhile, evaluations of both attributes and alternatives are in linguistic terms. With these improvements and the simultaneous application of other skills such as the extent analysis technique, our new method can further resolve the fuzzy information by reducing its uncertain level, and can obtain more scientific and accurate attribute weights. Meanwhile, it can significantly reduce the computation complexity, and more importantly, it can provide more reasonable and robust ranking results. All these characteristics ensure that our new algorithm can efficiently solve different kinds of complex MAGDM problems in practice. We will demonstrate the above arguments one after another in the following sections.

With the development of information technology and increasing competition among companies, the supply chain management has become an important factor affecting the company competitiveness. The evaluation and selection of suppliers are critical for setting up an efficient, rapid response supply chain. There are usually many qualitative and quantitative attributes to be considered. The supplier selection problem is thus a MADM problem. On the other hand, with the business globalization, the supplier selection problem has become more and more complex. In order to choose an appropriate supplier, several decision makers from different departments in the company are often involved, so it is also a typical MAGDM problem.

Based on the fuzzy set theory, some methods have been proposed for the supplier selection problem. A hierarchy fuzzy MADM model is proposed by Chu and Lin [15], they adopt linguistic terms to assess the ratings and weights of supplier selection attributes and then use FTOPSIS to rank the suppliers. Chen et al. [27] solve the supplier selection problem by using the extended fuzzy AHP-based approach. In their paper, triangular fuzzy numbers are used to express decision makers' assessments to set up decision matrices, and the weight vectors are derived according to the fuzzy synthetic extent. Priority weights of suppliers are determined as the product of the weight of each supplier and the weight of the corresponding attribute, the suppliers are then ranked by their priority weights. Amir et al. [28] use linguistic terms to evaluate alternatives and attributes, then aggregate the evaluation values and defuzzify them, here VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) is used to rank the alternatives. Önüt et al. [29] investigate a telecommunication company selection problem in the fuzzy environment by using the analytic network process and TOPSIS.

Although many methods have been proposed to solve it, the general supplier selection problem has not been well solved due to its complexity. Until now, we have not found any research using CFAHP and FTOPSIS to solve the supplier selection problem. As an application, our new method will be applied to solve this kind of MAGDM problems.

The rest of this paper is organized as follows: fuzzy numbers and the corresponding distance are briefly introduced in Section 2. The CFAHP is presented in Section 3. The new method is presented in Section 4. Two supplier selection problems are then solved in Section 5 to illustrate the application and superiority of our method. The last section summarizes the paper.

2. Basic concepts

Uncertainties and vagueness exist extensively in the decision making process, which are often characterized by linguistic variables. Then linguistic variables are translated into fuzzy numbers. Two commonly used fuzzy numbers are the trapezoidal fuzzy number and the triangular fuzzy number.

A fuzzy number is a special fuzzy set $F = \{(x, \mu_F(x)), x \in R\}$, where x takes its value on the real line R: $-\infty \le x \le +\infty$, and $\mu_F(x)$ is a continuous mapping from R into the closed interval [0,1]. A trapezoidal fuzzy number can be denoted by $\tilde{A} = (a, b, c, d)$, where $a \le b \le c \le d$. The membership of $\tilde{A}$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \text{ or } x > d, \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ 1, & b \le x \le c, \\ \dfrac{d-x}{d-c}, & c \le x \le d. \end{cases} \tag{1}$$

If $b = c$, the trapezoidal fuzzy number reduces to the triangular fuzzy number. In the following, A triangular fuzzy number is represented as $\tilde{A} = (l, m, u)$. If $l = m = u$, the triangular fuzzy number degenerates to a real number.

In this paper, the triangular fuzzy number is adopted to represent the linguistic variables in the CFAHP. The main reason for using the triangular fuzzy number is that it is intuitively easy for decision makers to use and calculate. In addition, the triangular fuzzy number has been proven to be an effective way for modeling the subjective and/or imprecise information [4,5,9–12,17–24].

The distance between two triangular fuzzy numbers $\tilde{A_1} = (l_1, m_1, u_1)$ and $\tilde{A_2} = (l_2, m_2, u_2)$ can be defined as [27]

$$d(\tilde{A_1}, \tilde{A_2}) = \sqrt{\frac{1}{3}[(l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2]}. \tag{2}$$

The distance between two trapezoidal fuzzy numbers $\tilde{m} = (m_1, m_2, m_3, m_4)$ and $\tilde{n} = (n_1, n_2, n_3, n_4)$ can be calculated as [27]

$$d(\tilde{m}, \tilde{n}) = \sqrt{\frac{1}{4}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 + (m_4 - n_4)^2]}. \tag{3}$$

One can refer to [1] for more detailed discussion about fuzzy numbers.

3. Constrained fuzzy AHP

Different fuzzy AHPs have been proposed by various authors. However, as pointed out in the introduction, the direct application of usual fuzzy arithmetic operation could lead to questionable results due to a deficient generalization from the real number arithmetic to the fuzzy intervals one. Fuzzy arithmetic with requisite constraints is thus introduced by Klir [25], and is applied in the FAHP by Enea and Piazza [24] to avoid situations in which the traditional mathematical operations give meaningless results when applied to fuzzy numbers. For example, for two fuzzy numbers $\tilde{A} = [A_l, A_m, A_u]$ and $\tilde{B} = [B_l, B_m, B_u]$, we consider the fuzzy arithmetic expression $\tilde{A}/(\tilde{A}+\tilde{B})$. The fuzzy arithmetic operation under an equality constraint is defined as follows:

$$^{\alpha}[\tilde{A}/(\tilde{A} + \tilde{B})] = ^{\alpha}[A_l/(A_l + B_u), A_u/(A_u + B_l)].$$

The fuzzy arithmetic operation without the equality constraint is

$$^{\alpha}[\tilde{A}/(\tilde{A} + \tilde{B})] = ^{\alpha}[A_l/(A_u + B_u), A_u/(A_l + B_l)].$$

Therefore, the width of the corresponding interval of operation $\tilde{A}/(\tilde{A} + \tilde{B})$ under the above two methods, respectively, is reduced by

$$\frac{A_l}{A_l + B_u} - \frac{A_l}{A_u + B_u} + \frac{A_u}{A_l + B_l} - \frac{A_u}{A_l + B_u}.$$

For two triangular fuzzy numbers $A = [1, 2, 3]$, $B = [2, 3, 4]$, the result of $\tilde{A}/(\tilde{A} + \tilde{B})$ is $[1/(3+4), 2/(2+3), 3/(1+2)] = [1/7, 2/7, 1]$ if the equality constraint is not considered. But, if the equality constraint is considered, then the result is $[1/(1 + 4), 2/(2 + 3), 3/(3 + 2)] = [1/5, 2/5, 3/5]$. By making use of the equality constraint, we get a smaller interval which does not contain impossible values. Applying this requisite constraint fuzzy operation to the FAHP, a lower level of vagueness is achieved because a larger amount of information is used. Then we can get the results with a minor uncertainty since we exclude the ones impossible and reduce the uncertainty [24].

With the above observation, as a preparation for introducing our new method, we discuss in this section how to determine the fuzzy synthetic extent by using CFAHP.

Suppose that there are m alternatives and n attributes. The fuzzy pairwise comparison matrix of attributes is denoted by S, whose elements are $\tilde{s}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $1 \le i, j \le n$. To ensure the symmetry, we set

$$\tilde{s}_{ji} = \left(\frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}}\right), \quad \forall i \neq j, \tilde{s}_{ji} = (1, 1, 1), \forall i = j.$$

Denote the fuzzy synthetic extent by $\tilde{s}_i = (s_{li}, s_{mi}, s_{ui})$. Here, s_{mi} is determined by

$$s_{mi} = \sum_{j=1}^{n} m_{ij} \times \left(\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}\right)^{-1}. \tag{4}$$

To evaluate s_{li}, we need to construct a matrix $B_i = (b_{kj})$, $i, j, k = 1, \ldots, n$, which satisfies the following constraints:

$$b_{jj} = 1, \tag{5}$$

$$b_{jk} = 1/b_{kj}, \tag{6}$$

$$b_{ij} = l_{ij}, \quad \forall j \neq i, \tag{7}$$

$$b_{kj} = \left\{x \mid y = \max\left(x + \frac{1}{x}\right), \quad \forall x \in [l_{kj}, u_{kj}]\right\}, \forall k \neq i, j \neq i, j > k. \tag{8}$$

Once B_i, $i = 1, 2, \ldots, n$, are built, we can calculate s_{li}, $i = 1, 2, \ldots, n$, by

$$s_{li} = \sum_{j=1}^{n} b_{ij} \times \left(\sum_{k=1}^{n}\sum_{j=1}^{n} b_{kj}\right)^{-1}, \quad i = 1, 2, \ldots, n. \tag{9}$$

In the same way, in order to determine s_{ui}, $i = 1, 2, \ldots, n$, we need to construct a matrix $C_i = (c_{kj})$, $i, j, k = 1, \ldots, n$, which satisfies the following constraints:

$$c_{jj} = 1, \tag{10}$$

$$c_{jk} = 1/c_{kj}, \tag{11}$$

$$c_{ij} = u_{ij}, \quad \forall j \neq i, \tag{12}$$

$$c_{kj} = \left\{x \mid y = \min\left(x + \frac{1}{x}\right), \forall x \in [l_{kj}, u_{kj}]\right\}, \quad \forall k \neq i, j \neq i, j > k. \tag{13}$$

Once C_i, $i = 1, 2, \ldots, n$, are built, we can calculate s_{ui}, $i = 1, 2, \ldots, n$, by

$$s_{ui} = \sum_{j=1}^{n} c_{ij} \times \left(\sum_{k=1}^{n}\sum_{j=1}^{n} c_{kj}\right)^{-1}, \quad i = 1, 2, \ldots, n. \tag{14}$$

The advantage of the above method is that much information is utilized in calculating the fuzzy synthetic extent, which is useful for reducing uncertainty. Therefore, our method is more efficient than the traditional FAHP.

In the above computation, we need to find the maximum value point and minimum value point of $x + \frac{1}{x}$ on the interval $[l_{kj}, u_{kj}]$, respectively, so that we can determine b_{kj} and c_{kj}, $k, j = 1, \ldots, n$. Because

$$\left(x + \frac{1}{x}\right)' = 1 - \frac{1}{x^2}, \qquad \left(x + \frac{1}{x}\right)'' = \frac{2}{x^3},$$

$x + \frac{1}{x}$ is monotonically increasing on the interval $[1, +\infty)$, and monotonically decreasing on the interval $(0, 1)$. Thus its minimum value is attained at $x = 1$. If $u_{kj} < 1$, we have $b_{kj} = l_{kj}, c_{kj} = u_{kj}$. If $l_{kj} > 1$, we have $b_{kj} = u_{kj}, c_{kj} = l_{kj}$. If $l_{kj} < 1 < u_{kj}$ and $(l_{kj} + \frac{1}{l_{kj}}) > (u_{kj} + \frac{1}{u_{kj}})$, we have $b_{kj} = l_{kj}$. If $l_{kj} < 1 < u_{kj}$ and $(u_{kj} + \frac{1}{u_{kj}}) > (l_{kj} + \frac{1}{l_{kj}})$, we have $b_{kj} = u_{kj}$. If $(l_{kj} + \frac{1}{l_{kj}}) = (u_{kj} + \frac{1}{u_{kj}})$, either l_{kj} or u_{kj} can be selected, since $(l_{kj} + \frac{1}{l_{kj}})$ or $(u_{kj} + \frac{1}{u_{kj}})$ would appear in the denominator under this situation, it does not affect the final weight value. For this reason, we always choose l_{kj} in the following. If $l_{kj} < 1 < u_{kj}$, we have $c_{kj} = 2$.

The above determined synthetic extents $\tilde{s}_i = (s_{li}, s_{mi}, s_{ui})$, $i = 1, 2, \ldots, m$, can be used to measure the relative importance of attributes. What is better, different from existing papers, we further convert them into the deterministic weight vector by using the extent analysis technique in [30]. The concrete steps are as follows:

Step 1. Define the possibility of $\tilde{s}_2 = (s_{l2}, s_{m2}, s_{u2}) \ge \tilde{s}_1 = (s_{l1}, s_{m1}, s_{u1})$ by

$$v(\tilde{s}_2 \ge \tilde{s}_1) = \sup_{y \ge x}[\min(\mu_{\tilde{s}_1}(x), \mu_{\tilde{s}_2}(y))],$$

which can be equivalently expressed as

$$v(\tilde{s}_2 \geq \tilde{s}_1) = \mathrm{hgt}(\tilde{s}_2 \cap \tilde{s}_1) = \mu(d) = \begin{cases} 1, & s_{m2} \geq s_{m1}, \\ 0, & s_{l1} \geq s_{u2}, \\ \dfrac{s_{l1} - s_{u2}}{(s_{m2} - s_{u2}) - (s_{m1} - s_{l1})}, & \text{otherwise.} \end{cases} \tag{15}$$

In order to compare $\tilde{s}_1$ and $\tilde{s}_2$, we need to compute $v(\tilde{s}_2 \geq \tilde{s}_1)$ and $v(\tilde{s}_1 \geq \tilde{s}_2)$.

Step 2. The possibility degree of a triangular fuzzy number $\tilde{s}$ being greater than other m triangular fuzzy numbers $\tilde{s}_i, i = 1, 2, \ldots, m$, is defined by

$$v(\tilde{s} \geq \tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_m) = v[(\tilde{s} \geq \tilde{s}_1) \text{ and } (\tilde{s} \geq \tilde{s}_2) \text{ and } \cdots \text{ and } (\tilde{s} \geq \tilde{s}_m)] = \min_i v(\tilde{s} \geq \tilde{s}_i). \tag{16}$$

Let $d'(\tilde{s}_i) = \min_k v(\tilde{s}_i \geq \tilde{s}_k), k = 1, 2, \ldots, m, k \neq i$. The deterministic weight vector can then be determined by

$$W' = (d'(\tilde{s}_1), d'(\tilde{s}_2), \ldots, d'(\tilde{s}_m)). \tag{17}$$

Step 3. The normalized weight vector is defined as

$$W = (d(\tilde{s}_1), d(\tilde{s}_2), \ldots, d(\tilde{s}_m)), \tag{18}$$

which is a non-fuzzy vector.

By utilizing the fuzzy arithmetic with requisite constraints and the extent analysis technique, the above new method can not only exclude impossible values, reduce the vagueness level and use more information, but determine more accurate, deterministic (instead of fuzzy in current researches such as [19,20]) priority vector, which make it more efficient and practical for real applications. Nevertheless, in CFAHP, it is necessary to compare the attributes with respect to each of alternatives to set up decision matrices. While to determine the synthetic extent, we need to set up many matrices like B and C, so the computation amount is large. In order to avoid these shortcomings, we adopt the FTOPSIS to rank the alternatives to further improve our new method.

4. New group decision method based on CFAHP and FTOPSIS

With the above preparations, we can now present the whole process of our new method for group decision making, which are divided into three stages. At the first stage, we analyze the problem to select the attributes and alternatives of the decision making problem, and set up a hierarchy structure; at the second stage, we determine the attributes' weights by CFAHP; at the third stage, we rank the alternatives by FTOPSIS.

To avoid the subjective bias induced by a single decision maker, several decision makers are involved in our group decision making method, which is very important for ensuring reasonableness of evaluation results. By combining CFAHP and FTOPSIS, our new algorithm can then be developed.

Consider an MAGDM problem. Suppose that there are t experts, and there are m alternatives to be evaluated according to n attributes. The concrete steps are as follows.

Step 1. Analyze the problem to set up a hierarchical structure. In general, it should include goal, attribute, and alternative levels. If the problem is very complicated, several levels of sub-attributes can be included.

Step 2. Through pairwise comparisons of attributes, t experts set up their decision matrices $S_i, i = 1, 2, \ldots, t$. Apply the geometric average method to integrate all the opinions of experts and calculate the composite decision matrix as follows:

$$S = (S_1 \otimes S_2 \otimes \cdots \otimes S_t)^{\frac{1}{t}}.$$

After that, we examine the consistency of this composite decision matrix by using the algorithm in [29]. If the composite decision matrix is consistent, go to Step 3, otherwise, ask experts to modify their matrices.

Step 3. With S, calculate decision matrices $B_i, i = 1, 2, \ldots, n$, according to (5)–(8), and $C_i, i = 1, 2, \ldots, n$, according to (10)–(13), and then calculate the synthetic extents $\tilde{s}_i = (s_{li}, s_{mi}, s_{ui}), i = 1, 2, \ldots, n$, according to (4), (9) and (14).

Step 4. Determine criteria weights according to (15)–(17), and normalize them to get the weight vector:

$$W = (w_1, w_2, \ldots, w_n).$$

Step 5. Choose the linguistic terms to evaluate the alternatives. Based on the experts' opinions on alternatives, determine decision matrices $A^i_{m\times n}, i = 1, 2, \ldots, t$. Through the geometric average method, integrate all the normalized matrices to obtain the synthetic decision matrix

$$A_{m\times n} = (\tilde{a}_{ij})_{m\times n} = (A^1 \otimes A^2 \otimes \cdots \otimes A^t)^{\frac{1}{t}}.$$

Step 6. Calculate the weighted synthetic decision matrix $A'_{m\times n} = (\tilde{a}'_{ij})_{m\times n}$ with $\tilde{a}'_{ij} = w_j \tilde{a}_{ij}$.

Step 7. Determine the FPIS and FNIS by

$$\text{FPIS: } A^* = \{(\tilde{v}^*_j = \max_i \tilde{v}_{ij} \mid j \in J), (\tilde{v}^*_j = \min_i \tilde{v}_{ij} \mid j \in J'), j = 1, 2, \ldots, n\} \tag{19}$$

and

$$\text{FNIS: } A^- = \{(\tilde{v}_j^- = \min_i \tilde{v}_{ij} \mid j \in J), (\tilde{v}_j^- = \max_i \tilde{v}_{ij} \mid j \in J'), j = 1, 2, \ldots, n\}, \tag{20}$$

where $J = \{j = 1, 2, \ldots, n \mid j \in BCs\}$, $J' = \{j = 1, 2, \ldots, n \mid j \in CCs\}$. The corresponding FPIS and FNIS vectors are denoted by $A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*)$ and $A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-)$, respectively. *BCs* is the set of benefit attribute set and *CCs* is the set of cost attribute set.

Step 8. Use the distance formula (2) between two triangular fuzzy numbers and (21)–(22) to calculate the distance D_i^*, $i = 1, 2, \ldots, m$, of each point to the FPIS and D_i^-, $i = 1, 2, \ldots, m$, of each point to the FNIS, respectively,

$$D_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1, 2, \ldots, m, \tag{21}$$

$$D_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1, 2, \ldots, m. \tag{22}$$

Step 9. Determine the relative closeness CC_i, $i = 1, 2, \ldots, m$, of each alternative as follows.

$$CC_i = \frac{D_i^-}{D_i^- + D_i^*}, \quad i = 1, 2, \ldots, m. \tag{23}$$

Step 10. Rank the alternatives in the descending order according to their relative closeness values. In this way, the larger the alternative's relative closeness value, the farther it is from the FNIS and the nearer it is to the FPIS.

While maintaining their advantages of current papers, the above new method overcomes the shortcomings of typical researches from the following perspectives: the proper and full description of fuzzy information [17–21], the computation efficiency [22,23], and the implementation [31]. Especially, except for advantages demonstrated at the end of Section 3, the other advantages of our new algorithm are: several experts are involved in our group decision making method; it sufficiently utilizes the clarity and simplicity of FAHP, while the obtained attribute weights are much accurate due to the introduction of CFAHP; the attribute weight vector determined through CFAHP is more reasonable, which overcomes the disadvantages of the usual equal-weight method and the subjectively weighting method; the alternatives are ranked by FTOPSIS, the huge computation required by the traditional FAHP is thus avoided; while ensuring its reasonableness, our algorithm significantly reduces the heavy computation because of the simultaneous application of CFAHP and FTOPSIS; what is more important, compared with existing methods, linguistic terms are used in the evaluation of both attributes and alternatives, which not only make it convenient for the application of our algorithm, but also result in accurate results with the help of CFAHP.

Before applying our new algorithm to concrete decision problems, we would like to further demonstrate its efficiency by examining its computation complexity. Actually, by calculating the number of basic operations at each step, it is easy to see that the worst-case time complexity of our algorithm is $O(n^3mt)$. Moreover, in our algorithm, the times of basic operations increases linearly with respect to the number of decision experts, while the times of basic operations is a cubical polynomial function in terms of the number of attributes. The computation complexity theory thus tells us that our new algorithm is a practical and efficient polynomial-time algorithm for solving complex MAGDMs.

5. Supplier selection by the new method

As an illustration to the practicality and efficiency of our new algorithm, we consider its application in two supplier selection problems.

Example 1. In architecture industry, the material choice is a key factor affecting the quality and cost of the building project. The corresponding supplier selection problem is rather important to improve the company's competitiveness. Suppose that an architecture company wants to select the most appropriate supplier for one important material such as cement. After the pre-evaluation, eight alternatives $A_1, A_2, \ldots, A_8$ are remained for further evaluation. Three experts DM_1, DM_2 and DM_3 are invited as decision makers. Four attributes: C_1 (the price of the product), C_2 (the quality of the product), C_3 (the delivery time) and C_4 (the risk) are taken into consideration. When our new method is applied to solve this problem, the concrete procedure would be as follows.

Step 1. Set up the hierarchy structure as in Fig. 1.

Step 2. Choose linguistic terms in Table 1 to compare attributes, translate the fuzzy linguistic terms into triangular fuzzy numbers and then set up decision matrices by using the scales in Table 1, which has been widely used in papers such as [32].

During the matrix construction, all the diagonal elements are set to $(1, 1, 1)$. It is only necessary to evaluate the elements above the diagonal: $\tilde{s}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $i < j$. Accordingly, the elements below the diagonal are set to $\tilde{s}_{ji} = \left(\frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}}\right)$, $i > j$.

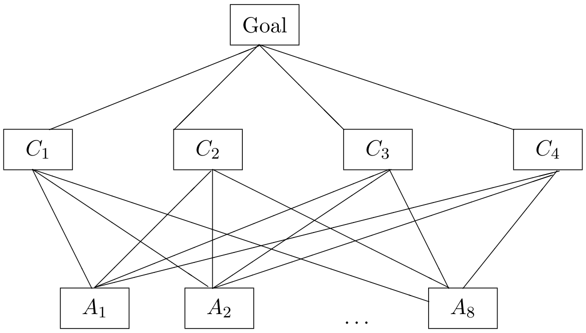

Fig. 1. The hierarchy structure of an architecture supplier problem.

Table 1
Linguistic terms to compare attributes.

Definition	Triangular fuzzy number
Just equal	(1, 1, 1)
Equally important	(1/2, 1, 3/2)
Weakly important	(1, 3/2, 2)
Moderately important	(3/2, 2, 5/2)
Strongly important	(2, 5/2, 3)
Absolutely important	(5/2, 3, 7/2)

Finally, the decision matrices given by D_1, D_2 and D_3 are the following S_1, S_2 and S_3, respectively.

$$S_1 = \begin{pmatrix} (1,1,1) & (3/2,2,5/2) & (1/2,1,3/2) & (1,3/2,2) \\ (2/5,1/2,2/3) & (1,1,1) & (5/2,3,7/2) & (2/5,1/2,2/3) \\ (2/3,1,2) & (2/7,1/3,2/5) & (1,1,1) & (3/2,2,5/2) \\ (1/2,2/3,1) & (3/2,2,5/2) & (2/5,1/2,2/3) & (1,1,1) \end{pmatrix},$$

$$S_2 = \begin{pmatrix} (1,1,1) & (3/2,2,5/2) & (2/5,1/2,2/3) & (1/2,1,3/2) \\ (2/5,1/2,2/3) & (1,1,1) & (3/2,2,5/2) & (2/7,1/3,2/5) \\ (3/2,2,5/2) & (2/5,1/2,2/3) & (1,1,1) & (5/2,3,7/2) \\ (2/3,1,2) & (5/2,3,7/2) & (2/7,1/3,2/5) & (1,1,1) \end{pmatrix},$$

$$S_3 = \begin{pmatrix} (1,1,1) & (1/2,1,3/2) & (3/2,2,5/2) & (2/5,1/2,2/3) \\ (2/3,1,2) & (1,1,1) & (2/5,1/2,2/3) & (3/2,2,5/2) \\ (2/5,1/2,2/3) & (3/2,2,5/2) & (1,1,1) & (3/2,2,5/2) \\ (3/2,2,5/2) & (2/5,1/2,2/3) & (2/5,1/2,2/3) & (1,1,1) \end{pmatrix}.$$

Step 3. To ensure the symmetry, aggregate the above decision matrices by the geometric average method and obtain:

$$S_4 = \begin{pmatrix} (1,1,1) & (1.0400, 1.5874, 2.1086) & (0.6694, 1, 1.3572) & (0.5848, 0.9086, 1.2599) \\ (0.4743, 0.6300, 0.9615) & (1,1,1) & (1.1447, 1.4422, 1.8001) & (0.5555, 0.6934, 0.8736) \\ (0.7368, 1, 1.4938) & (0.5555, 0.6934, 0.8736) & (1,1,1) & (1.7784, 2.2894, 2.7967) \\ (0.7937, 1.1006, 1.7100) & (1.1447, 1.3389, 1.8001) & (0.3576, 0.4368, 0.5623) & (1,1,1) \end{pmatrix}$$

we examine the consistency of the above composite decision matrix by using the algorithm in [33]. Since the consistency index value is 0.7602, so S_4 is consistent.

Step 4. Compute the fuzzy synthetic extents by using CFAHP. Concretely, in order to calculate s_{li}, $i = 1, 2, 3, 4$, we generate the following matrices B_1, B_2, B_3 and B_4. In order to calculate s_{ui}, $i = 1, 2, 3, 4$, we generate the following matrices C_1, C_2, C_3 and C_4.

$$B_1 = \begin{pmatrix} 1 & 1.0400 & 0.6694 & 0.5848 \\ 0.9615 & 1 & 1.8001 & 0.5555 \\ 1.4939 & 0.5555 & 1 & 2.7967 \\ 1.7100 & 1.8001 & 0.3576 & 1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 2.1084 & 0.6694 & 0.5848 \\ 0.4743 & 1 & 1.1447 & 0.5555 \\ 1.4939 & 0.8736 & 1 & 2.7967 \\ 1.7100 & 1.8002 & 0.3576 & 1 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 1 & 2.1086 & 1.3572 & 0.5848 \\ 0.4742 & 1 & 1.8002 & 0.5555 \\ 0.7368 & 0.5555 & 1 & 1.7784 \\ 1.7100 & 1.8002 & 0.5623 & 1 \end{pmatrix}, \quad B_4 = \begin{pmatrix} 1 & 2.1086 & 0.6694 & 1.2599 \\ 0.4743 & 1 & 1.8001 & 0.8736 \\ 1.4939 & 0.5555 & 1 & 2.7967 \\ 0.7937 & 1.1447 & 0.3576 & 1 \end{pmatrix},$$

Table 2
Linguistic terms for ranking the alternatives.

Very poor (VP)	(0, 0, 0.1)
Poor (P)	(0, 0.1, 0.3)
Medium poor (MP)	(0.1, 0.3, 0.5)
Fair (F)	(0.3, 0.5, 0.7)
Medium good (MG)	(0.5, 0.7, 0.9)
Good (G)	(0.7, 0.9, 1.0)
Very good (VG)	(0.9, 1.0, 1.0)

Table 3
The rating of the alternatives.

	C_1			C_2			C_3			C_4		
	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3
A_1	G	MG	F	VG	MG	G	G	F	G	F	G	G
A_2	MG	G	MG	G	G	MG	VG	MG	MG	MG	MG	VG
A_3	MG	VG	G	MP	F	F	G	F	MG	VG	VG	F
A_4	MG	MG	MG	VG	VG	VG	MG	MG	F	G	G	VG
A_5	VG	VG	VG	G	MG	G	F	MP	F	F	VG	F
A_6	F	G	G	MG	F	F	MG	G	VG	MG	MG	MG
A_7	F	MP	F	G	G	MG	G	VG	G	F	G	G
A_8	VG	G	G	F	MP	F	MG	G	MG	G	MG	MG

$$C_1=\begin{pmatrix}1 & 2.1086 & 1.3572 & 1.2599\\ 0.4742 & 1 & 1.1447 & 0.8736\\ 0.7368 & 0.8736 & 1 & 1.7784\\ 0.7937 & 1.1447 & 0.5623 & 1\end{pmatrix},\qquad C_2=\begin{pmatrix}1 & 1.0400 & 1 & 1\\ 0.9615 & 1 & 1.8001 & 0.8736\\ 1 & 0.5555 & 1 & 1.7784\\ 1 & 1.1447 & 0.5623 & 1\end{pmatrix},$$

$$C_3=\begin{pmatrix}1 & 1.0400 & 0.6694 & 1\\ 0.9615 & 1 & 1.1447 & 0.8736\\ 1.4938 & 0.8736 & 1 & 2.7967\\ 1 & 1.1447 & 0.3576 & 1\end{pmatrix},\qquad C_4=\begin{pmatrix}1 & 1.0400 & 1 & 0.5848\\ 0.9615 & 1 & 1.1447 & 0.5555\\ 1 & 0.8736 & 1 & 1.7784\\ 1.7100 & 1.8001 & 0.5623 & 1\end{pmatrix}.$$

Calculate $\tilde{s}_1, \tilde{s}_2, \tilde{s}_3$, and $\tilde{s}_4$ according to (4), (9) and (14), respectively, we have

$$\tilde{s}_1=(0.1798, 0.2626, 0.3347), \qquad \tilde{s}_2=(0.1649, 0.2199, 0.2773),$$
$$\tilde{s}_3=(0.2259, 0.2910, 0.3552), \qquad \tilde{s}_4=(0.1798, 0.2264, 0.2982).$$

Step 5. Determine the weight vector by using the extent analysis technique, that is, the steps given in Section 3. First calculate the possibility of $\tilde{s}_i \geq \tilde{s}_j$, $i \neq j$, then choose the smallest one as the weight for $\tilde{s}_i$. Concretely, we have

$$v(\tilde{s}_1 \geq \tilde{s}_2)=1, \qquad v(\tilde{s}_1 \geq \tilde{s}_3)=0.7930, \qquad v(\tilde{s}_1 \geq \tilde{s}_4)=1,$$
$$d'(\tilde{s}_1)=\min_i v(\tilde{s}_1 \geq \tilde{s}_i)=0.7930, \quad i=2,3,4.$$
$$v(\tilde{s}_2 \geq \tilde{s}_1)=0.6954, \qquad v(\tilde{s}_2 \geq \tilde{s}_3)=0.4196, \qquad v(\tilde{s}_2 \geq \tilde{s}_4)=0.9375,$$
$$d'(\tilde{s}_2)=\min_i v(\tilde{s}_2 \geq \tilde{s}_i)=0.4196, \quad i=1,3,4.$$
$$v(\tilde{s}_3 \geq \tilde{s}_1)=1, \qquad v(\tilde{s}_3 \geq \tilde{s}_2)=1, \qquad v(\tilde{s}_3 \geq \tilde{s}_4)=1,$$
$$d'(\tilde{s}_3)=\min_i v(\tilde{s}_3 \geq \tilde{s}_i)=1, \quad i=1,2,4.$$
$$v(\tilde{s}_4 \geq \tilde{s}_1)=0.7658, \qquad v(\tilde{s}_4 \geq \tilde{s}_2)=1, \qquad v(\tilde{s}_4 \geq \tilde{s}_3)=0.5281,$$
$$d'(\tilde{s}_4)=\min_i v(\tilde{s}_4 \geq \tilde{s}_i)=0.5281, \quad i=1,2,3.$$

The weight vector is thus $W'=(d'(\tilde{s}_1), d'(\tilde{s}_2), d'(\tilde{s}_3), d'(\tilde{s}_4))=(0.7930, 0.4196, 1, 0.5281)$. Normalize it to obtain the final weight vector

$$W=(d(\tilde{s}_1), d(\tilde{s}_2), d(\tilde{s}_3), d(\tilde{s}_4))=(0.2893, 0.1531, 0.3649, 0.1927).$$

Step 6. Given the linguistic terms for ranking the alternatives with respect to the attributes as those in Table 2, which is adopted from [16]. Build decision matrices shown in Table 3, and aggregate the attribute values by using the geometric average method to get the synthetic decision matrix presented in Table 4. With this matrix and the weight vector $W = (0.2893, 0.1531, 0.3649, 0.1927)$ determined at Step 5, we obtain the weighted decision matrix given in Table 5.

Step 7. Determine the FPIS and the FNIS as those in Table 6.

Table 4
The synthetic decision matrix.

	C_1	C_2	C_3	C_4
A_1	(0.4718, 0.6804, 0.8573)	(0.6804, 0.8573, 0.9655)	(0.5278, 0.7399, 0.8879)	(0.5278, 0.7399, 0.8879)
A_2	(0.5593, 0.7612, 0.9322)	(0.6257, 0.8277, 0.9655)	(0.6082, 0.7884, 0.9322)	(0.6082, 0.7884, 0.9322)
A_3	(0.6804, 0.8573, 0.9655)	(0.2080, 0.4217, 0.6257)	(0.4718, 0.6804, 0.8573)	(0.624, 0.7937, 0.8879)
A_4	(0.5000, 0.7000, 0.9000)	(0.9000, 1.0000, 1.0000)	(0.4217, 0.6257, 0.8277)	(0.7612, 0.9322, 1.0000)
A_5	(0.9000, 1.0000, 1.0000)	(0.6257, 0.8277, 0.9655)	(0.2080, 0.4217, 0.6257)	(0.4327, 0.6300, 0.7884)
A_6	(0.5278, 0.7399, 0.8879)	(0.3000, 0.5000, 0.7000)	(0.7612, 0.9322, 1.0000)	(0.5000, 0.7000, 0.9000)
A_7	(0.2080, 0.4217, 0.6257)	(0.6257, 0.8277, 0.9655)	(0.7612, 0.9322, 1.0000)	(0.5278, 0.7399, 0.8879)
A_8	(0.7612, 0.9322, 1.0000)	(0.2080, 0.4217,0.6257)	(0.5593, 0.7612, 0.9322)	(0.5593, 0.7612, 0.9322)

Table 5
The weighted decision matrix.

	C_1	C_2	C_3	C_4
A_1	(0.1365, 0.1968, 0.2480)	(0.1042, 0.1313, 0.1478)	(0.1926, 0.2700, 0.3240)	(0.1017, 0.1426, 0.1711)
A_2	(0.1618, 0.2202, 0.2697)	(0.0958, 0.1267, 0.1478)	(0.2219, 0.2877, 0.3402)	(0.1172, 0.1519, 0.1796)
A_3	(0.1968, 0.2480, 0.2793)	(0.0318, 0.0646, 0.0958)	(0.1722, 0.2483, 0.3128)	(0.1202, 0.1529, 0.1711)
A_4	(0.1447, 0.2025, 0.2604)	(0.1378, 0.1531, 0.1531)	(0.1539, 0.2283, 0.302)	(0.1467, 0.1796, 0.1927)
A_5	(0.2604, 0.2893, 0.2893)	(0.0958, 0.1267, 0.1478)	(0.0759, 0.1539, 0.2283)	(0.0834, 0.1214, 0.1519)
A_6	(0.1527, 0.2141, 0.2569)	(0.0545, 0.0856, 0.1165)	(0.2483, 0.3128, 0.3523)	(0.0964, 0.1349, 0.1734)
A_7	(0.0602, 0.1220, 0.1810)	(0.0958, 0.1267, 0.1478)	(0.2778, 0.3402, 0.3649)	(0.1017, 0.1426, 0.1711)
A_8	(0.2202, 0.2697, 0.2893)	(0.0318, 0.0646, 0.0958)	(0.2041, 0.2778, 0.3402)	(0.1078, 0.1467, 0.1796)

Table 6
The FPIS and the FNIS.

	C_1	C_2	C_3	C_4
FPIS	(0.2604, 0.2893, 0.2893)	(0.1378, 0.1531, 0.1531)	(0.2778, 0.3402, 0.3649)	(0.1467, 0.1796, 0.1927)
FNIS	(0.0602, 0.1220, 0.1810)	(0.0318, 0.0646, 0.0958)	(0.0759, 0.1539, 0.2283)	(0.0834, 0.1214, 0.1519)

Step 8. Calculate the distance of each alternative to the FPIS and FNIS, respectively. We then have $D_1^* = 0.1877, D_2^* = 0.1749, D_3^* = 0.2272, D_4^* = 0.1752, D_5^* = 0.2062, D_6^* = 0.1609, D_7^* = 0.2119, D_8^* = 0.2057, D_1^- = 0.2196, D_2^- = 0.2319, D_3^- = 0.1779, D_4^- = 0.2303, D_5^- = 0.1994, D_6^- = 0.2443, D_7^- = 0.1937, D_8^- = 0.1995$.

Step 9. Compute the relative closeness coefficients as follows. $CC_1 = 0.5544, CC_2 = 0.6472, CC_3 = 0.4661, CC_4 = 0.6581, CC_5 = 0.4815, CC_6 = 0.5303, CC_7 = 0.5181, CC_8 = 0.5406$.

Step 10. Rank the alternatives in the descending order of CC_i, $i = 1, 2, \dots, 8$, to obtain $A_2 \succ A_4 \succ A_8 \succ A_6 \succ A_1 \succ A_7 \succ A_3 \succ A_5$. Therefore, the best alternative is A_2.

To demonstrate the advantages of our new method (denoted as $M1$), we now consider the solution of the above example by using the method (denoted as $M2$) in [22], which integrates FAHP and FTOPSIS.

When the attribute weights are computed by FAHP, the fuzzy synthetic extents are calculated according to the following formula

$$\tilde{s}_i = \Big(\sum_{j=1}^{n} \tilde{s}_{ij}\Big) \Big/ \Big(\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{s}_{ij}\Big).$$

That is,

$$s_{li} = \Big(\sum_{j=1}^{n} l_{ij}\Big) \times \Big(\sum_{i=1}^{n}\sum_{j=1}^{n} u_{ij}\Big)^{-1},$$

$$s_{mi} = \Big(\sum_{j=1}^{n} m_{ij}\Big) \times \Big(\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}\Big)^{-1},$$

$$s_{ui} = \Big(\sum_{j=1}^{n} u_{ij}\Big) \times \Big(\sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}\Big)^{-1},$$

here $\tilde{s}_i = (s_{li}, s_{mi}, s_{ui})$, $i = 1, 2, \dots, n$, and $\tilde{s}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $i, j = 1, 2, \dots, n$.

The attribute weights are then determined by the extent analysis technique in [30]. With this attribute weight vector, we rank the alternatives by FTOPSIS, and the results under $M2$ are given in the right block of Table 7. The rank of alternatives is $A_4 \succ A_2 \succ A_1 \succ A_8 \succ A_6 \succ A_7 \succ A_5 \succ A_3$. The best alternative is thus A_4.

Table 7
The results of different methods.

	$M1$			$M2$		
	D_i^*	D_i^-	CC_j	D_i^*	D_i^-	CC_j
A_1	0.2196	0.2666	0.5483	0.2152	0.2677	0.5544
A_2	0.1703	0.3180	0.6512	0.1711	0.3139	0.6472
A_3	0.2417	0.2434	0.5018	0.2569	0.2243	0.4661
A_4	0.1882	0.2982	0.6131	0.1649	0.3137	0.6581
A_5	0.2609	0.2227	0.4605	0.2493	0.2315	0.4815
A_6	0.2083	0.2783	0.5719	0.2270	0.2563	0.5303
A_7	0.2278	0.2563	0.5294	0.2320	0.2494	0.5181
A_8	0.2000	0.2877	0.5899	0.2223	0.2616	0.5406

Table 8
The relative closeness values.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
CC_{12}	0.6198	0.6990	0.4031	0.6499	0.4298	0.5524	0.6676	0.4770
CC_{13}	0.5340	0.6384	0.5211	0.6166	0.5420	0.5464	0.4482	0.6057
CC_{14}	0.5496	0.6545	0.4715	0.6515	0.3684	0.5596	0.5819	0.5513
CC_{23}	0.5786	0.6391	0.3794	0.7443	0.6066	0.4351	0.4686	0.4315
CC_{24}	0.5612	0.6545	0.4781	0.6216	0.4810	0.5638	0.5403	0.5666
CC_{34}	0.5139	0.6247	0.5039	0.6908	0.4959	0.4867	0.4226	0.5589

Table 9
Ranking of the different methods.

Method	Ranking
CC_{12}	$A_2 \succ A_7 \succ A_4 \succ A_1 \succ A_6 \succ A_8 \succ A_5 \succ A_3$
CC_{13}	$A_2 \succ A_4 \succ A_8 \succ A_6 \succ A_5 \succ A_1 \succ A_3 \succ A_7$
CC_{14}	$A_2 \succ A_4 \succ A_7 \succ A_6 \succ A_8 \succ A_1 \succ A_3 \succ A_5$
CC_{23}	$A_4 \succ A_2 \succ A_5 \succ A_1 \succ A_7 \succ A_6 \succ A_8 \succ A_3$
CC_{24}	$A_2 \succ A_4 \succ A_8 \succ A_6 \succ A_1 \succ A_7 \succ A_5 \succ A_3$
CC_{34}	$A_4 \succ A_2 \succ A_8 \succ A_1 \succ A_3 \succ A_5 \succ A_6 \succ A_7$
M_1	$A_2 \succ A_4 \succ A_8 \succ A_6 \succ A_1 \succ A_7 \succ A_3 \succ A_5$
M_2	$A_4 \succ A_2 \succ A_1 \succ A_8 \succ A_6 \succ A_7 \succ A_5 \succ A_3$

Table 10
Linguistic terms for ranking the alternatives.

Very poor (VP)	(0, 0, 0.1, 0.2)
Poor (P)	(0.1, 0.2, 0.2, 0.3)
Medium poor (MP)	(0.2, 0.3, 0.4, 0.5)
Fair (F)	(0.4, 0.5, 0.5, 0.6)
Medium good (MG)	(0.5, 0.6, 0.7, 0.8)
Good (G)	(0.7, 0.8, 0.8, 0.9)
Very good (VG)	(0.8, 0.9, 1.0, 1.0)

Now we carry out the sensitivity analysis in terms of replacing one criterion weight with another criterion weight as that in [29]. Since there are four criteria, we have six different combinations. We calculate the relative closeness coefficients for each combination and denote it as CC_{ij} when the criterion i and criterion j are exchanged. For example, CC_{12} means the weights of criterion 1 and criterion 2 have been exchanged. For each combination, we calculate the weighted decision matrix and determine the corresponding FPIS and FNIS. Then the relative closeness to the ideal solution is calculated. The obtained results are shown in Table 8, the ranking results under our new method and $M2$ are also listed for comparison. From the results, we can see that A_2 is still the best alternative as that in our new algorithm, in four of six combinations. A_4 becomes the best alternative in two combinations CC_{23} and CC_{34}, the same results as that under M_2. As we can see from Step 5, the weight of criterion 3 is the largest among four criteria, therefore when we exchange the weights of criterion 2 and criterion 3, and those of criterion 3 and criterion 4, the alternative 4 would become the best alternative. Consequently, the alternative 2 is indeed the best alternative. These sensitivity analysis results obviously show the robustness of our new method.

From the above results, we can see that different methods result in different best alternatives. Compared with existing approaches like that in [22], our new method can help decision makers to find the truly best, robust alternative. On the other hand, as we can easily see from results in Tables 7–10 that, by virtue of the utilization of much more information and several transformation skills, our algorithm can generate more distinguishable and accurate results. This can not only enhance the stability of the ranking results of the alternatives, but also make it easy for decision makers to discriminate among alternatives. The decision makers are thus more confident about their selection decisions. These advantages are very important for the efficient application of our new method in practice.

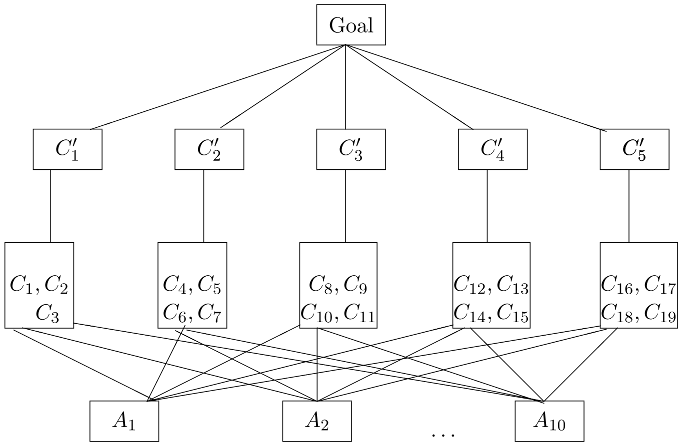

Fig. 2. The hierarchy structure of a global supplier problem.

To further demonstrate the feasibility and practicality of our new method for solving complex MAGDM problems, we consider another complex global supplier selection problem. Miller points out that most decision makers cannot simultaneously handle more than 7 factors when making decision [34]. Therefore, we should decompose the complex decision problem into some manageable sub-problems. In this example, we first consider those main factors which affect the supplier selection as criteria, and then decompose the criteria into attributes. A four-level hierarchy structure is set up for this problem: the objective level, the criteria level, the attribute level and the alternative level. Here, only one decision maker is considered due to the space limitation. If there are several decision makers, we can first aggregate the decision matrices into a collective decision matrix by the geometric average method. The concrete problem can then be described as follows.

Example 2. This global supplier selection problem is adapted from [35]. Since there are many factors affecting the supplier selection decision, we choose the following main factors as criteria: C'_1—the overall cost of the product, C'_2—the quality of the product, C'_3—the service performance of supplier, C'_4—the supplier's profile, C'_5—the risk factor. The criteria can be decomposed into various attributes. The factors affecting C'_1 can be stated as: C_1-the product price, C_2—the freight cost, C_3—the tariff and custom duties. The quality of the product (C'_2) is analyzed according to the following attributes: C_4—the rejection rate of the product, C_5—the increased lead time, C_6—the quality assessment, C_7—the remedy for quality problems. The service performance (C'_3) can be measured as follows: C_8—the delivery schedule, C_9—the technological and R&D support, C_{10}—the response to changes, C_{11}—the ease of communication. The characteristics of the suppliers (C'_4) can be stated as: C_{12}—the financial status, C_{13}—the customer base, C_{14}—the performance history, C_{15}—the production facility and capacity. The risk factor (C'_5) affecting the selection process of the global supplier can be analyzed as follows: C_{16}—the geographical location, C_{17}—the political stability, C_{18}—the economy, C_{19}—the terrorism. After the pre-evaluation, there are ten alternatives A_1–A_{10} left for further evaluation.

We use our new method to solve this problem. The hierarchy structure is constructed as in Fig. 2. The linguistic terms in Table 1 are used to compare attributes. In order to determine the attribute weight vector, we compare the criteria to set up a decision matrix as S_5 in the second level. By using the CFAHP, we can obtain the criteria weight vector $W' = (0.3271, 0.2935, 0.2280, 0.0126, 0.1389)$. In the third level, we compare the attributes under each criterion to set up decision matrices as $S_6, S_7, S_8, S_9, S_{10}$. By using the CFAHP, we can determine the attribute weight vectors as $W_1 = (0.1497, 0.1513, 0.0261)$, $W_2 = (0.0861, 0.0533, 0.1345, 0.0195)$, $W_3 = (0.0522, 0.0731, 0.0457, 0.0570)$, $W_4 = (0.0041, 0.0039, 0.0020, 0.0026)$ and $W_5 = (0.0296, 0.0436, 0.0250, 0.0407)$, respectively.

$$S_5 = \begin{pmatrix} (1,1,1) & (3/2,2,5/2) & (1,3/2,2) & (3/2,2,5/2) & (1/2,1,3/2) \\ (2/5,1/2,2/3) & (1,1,1) & (1/2,1,3/2) & (3/2,2,5/2) & (2,5/2,3) \\ (1/2,2/3,1) & (2/3,1,2) & (1,1,1) & (2,5/2,3) & (1/2,2/3,1) \\ (2/5,1/2,2/3) & (2/5,1/2,2/3) & (1/3,2/5,1/2) & (1,1,1) & (1,3/2,2) \\ (2/3,1,2) & (1/3,2/5,1/2) & (1,3/2,2) & (1/2,2/3,1) & (1,1,1) \end{pmatrix},$$

$$S_6 = \begin{pmatrix} (1,1,1) & (3/2,2,5/2) & (1/2,1,3/2) \\ (2/5,1/2,2/3) & (1,1,1) & (2,5/2,3) \\ (2/3,1,2) & (1/3,2/5,1/2) & (1,1,1) \end{pmatrix},$$

$$S_7 = \begin{pmatrix} (1,1,1) & (5/2,3,7/2) & (1/3,2/5,1/2) & (1/2,2/3,1) \\ (2/7,1/3,2/5) & (1,1,1) & (1/2,1,3/2) & (3/2,2,5/2) \\ (2,5/2,3) & (2/3,1,2) & (1,1,1) & (1,3/2,2) \\ (1,3/2,2) & (2/5,1/2,2/3) & (1/2,2/3,1) & (1,1,1) \end{pmatrix},$$

Table 11
The rating of the alternatives.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
C_1	G	F	P	F	VG	G	MG	F	F	MP
C_2	F	G	G	F	MG	P	F	P	VG	G
C_3	F	F	MP	MG	G	F	G	VG	F	F
C_4	VG	G	F	F	G	P	F	F	MP	G
C_5	P	MG	G	MG	F	F	MP	VG	F	MG
C_6	MG	F	F	G	F	MG	F	G	MP	F
C_7	G	G	MP	VP	VG	G	F	F	G	MG
C_8	MP	MG	G	F	F	MG	VG	MG	G	MP
C_9	F	MG	P	G	G	P	VG	F	F	G
C_{10}	G	F	G	F	MG	MP	F	MG	G	P
C_{11}	MG	MG	G	MP	F	G	G	F	F	VG
C_{12}	F	MG	F	VP	VG	G	MG	P	F	F
C_{13}	MG	F	MP	VG	G	F	G	F	G	MP
C_{14}	G	MG	F	MP	F	VP	G	VG	P	VG
C_{15}	P	MG	F	G	VG	F	G	MG	VG	F
C_{16}	VG	F	MP	F	G	G	MP	MG	MG	MG
C_{17}	F	MG	MG	F	MP	VP	VG	MG	F	F
C_{18}	MG	MP	MG	F	VG	MP	F	G	G	F
C_{19}	VP	G	MG	G	F	P	F	MG	MP	VG

Table 12
The results.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
D_i^*	0.3006	0.2774	0.3602	0.3349	0.2314	0.4359	0.2946	0.3367	0.3308	0.2993
D_i^-	0.3704	0.3953	0.3116	0.3360	0.4417	0.2363	0.3772	0.3339	0.3405	0.3740
CC_i	0.5520	0.5876	0.4639	0.5008	0.6562	0.3515	0.5614	0.4979	0.5072	0.5554

$$S_8 = \begin{pmatrix} (1,1,1) & (2,5/2,3) & (1/2,2/3,1) & (2/5,1/2,2/3) \\ (1/3,2/5,1/2) & (1,1,1) & (5/2,3,7/2) & (1/2,1,3/2) \\ (1,3/2,2) & (2/7,1/2,2/5) & (1,1,1) & (1,3/2,2) \\ (3/2,2,5/2) & (2/3,1,2) & (1/2,2/3,1) & (1,1,1) \end{pmatrix},$$

$$S_9 = \begin{pmatrix} (1,1,1) & (1/2,1,3/2) & (3/2,2,5/2) & (1,3/2,2) \\ (2/3,1,2) & (1,1,1) & (5/2,3,7/2) & (1/3,2/5,1/2) \\ (2/5,1/2,2/3) & (2/7,1/3,2/5) & (1,1,1) & (2,5/2,3) \\ (1/2,2/3,1) & (2,5/2,3) & (1/3,2/5,1/2) & (1,1,1) \end{pmatrix},$$

$$S_{10} = \begin{pmatrix} (1,1,1) & (2/5,1/2,2/3) & (1/2,1,3/2) & (3/2,2,5/2) \\ (3/2,2,5/2) & (1,1,1) & (3/2,2,5/2) & (2/7,1/3,2/5) \\ (2/3,1,2) & (2/5,1/2,2/3) & (1,1,1) & (1,3/2,2) \\ (2/5,1/2,2/3) & (5/2,3,7/2) & (1/2,2/3,1) & (1,1,1) \end{pmatrix}.$$

Then the attribute weight of the alternatives can be determined by multiplying the weight calculated in the third level with the weight of the corresponding criterion. The resulting weight vector of attributes is

$$W = (0.1497, 0.1513, 0.0261, 0.0861, 0.0533, 0.1345, 0.0195, 0.0522, 0.0731, 0.0457, 0.0570, 0.0041, 0.0039, 0.0020, 0.0026, 0.0296, 0.0436, 0.0250, 0.0407).$$

If CFAHP is used to evaluate the ten alternatives, we have to construct evaluation matrices through the pairwise comparison of different alternatives under each attribute. There are many comparisons to make and we have to construct 19 pairwise comparison matrices, which leads to a huge amount of work and poor accuracy. Thus we use the FTOPSIS method to rank the alternatives.

Both the triangular fuzzy number and the trapezoidal fuzzy number are used in the literature to translate the linguistic arguments in FTOPSIS. Since the triangular fuzzy number is a special case of the trapezoidal fuzzy number, we translate the linguistic arguments into trapezoidal fuzzy numbers. The linguistic terms for ranking the alternatives are given as those in Table 2, which is also used in [36].

The decision maker evaluates the alternatives with respect to each attribute by using the linguistic arguments in Table 10 and sets up the decision matrix in Table 11. Then he computes the weighted decision matrix and determines the FPIS and FNIS. The distance of each alternative to the FPIS and FNIS, respectively, is calculated and the relative closeness coefficients are determined, given in Table 12. The alternatives are ranked accordingly, and we get $A_5 \succ A_2 \succ A_7 \succ A_{10} \succ A_1 \succ A_9 \succ A_4 \succ A_8 \succ A_3 \succ A_6$. The best alternative is A_5. Because of the space limitation, we do not show the step by step computation results here.

Example 2 illustrates that our new method can be equally used to deal with complex decision making problems.

6. Conclusions

In order to efficiently solve complex MAGDM problems and to find more reasonable decisions, a new fuzzy decision making method is proposed in this paper by comprehensively utilizing the CFAHP, FTOPSIS, the extent analysis technique and other transformation skills. The current researches are improved from perspectives such as: the description of the uncertain environment, the objectivity and accuracy of the attribute weights, the robustness of the ranking results and the overall computation efficiency. The new algorithm is applied to solve two supplier selection problems. Many factors have to be considered during the supplier selection process. Meanwhile, due to its fuzziness and uncertainty, decision makers usually evaluate different suppliers with the linguistic arguments. The supplier selection problem has not been well solved because of its complexity. We have not seen any research to use CFAHP, needless to say the combination of CFAHP and FTOPSIS, to solve the supplier selection problem. The main contributions of this paper can be summarized as follows.

The use of linguistic terms through the whole evaluation process can resolve the fuzzy and uncertain problem, and their proper transformations ensure that our new method has much wide application fields. Meanwhile, the fuzzy arithmetic with requisite constraints is used in FAHP to avoid situations in which the traditional mathematical operators give meaningless results when applied to fuzzy numbers. More reasonable and accurate attribute weights are obtained by CFAHP than the weights given by the decision maker subjectively or given by the traditional FAHP. In order to further reduce the large computation amount caused by comparing the criteria with respect to each of alternatives, FTOPSIS is used to rank the alternatives. All the above improvements ensure that our method is easy to apply and it can provide more scientific and robust results. The proposed method is used to solve a architecture material supplier selection problem and a global supplier selection problem, empirical results show that our new algorithm can provide more robust, reasonable and easy to apply ranking results when compared with the method using FAHP and FTOPSIS.

Further improvements of our algorithm might include the use of interval numbers or uncertain linguistic arguments to describe the uncertainties, the utilization of the multiple period information in decision making and the influence of correlation among attributes on the decision making process. Of course, the application of our new method to more complex MAGDM problems in reality, such as the personnel selection, the product selection, and the environment evaluation is also worthy of investigation. These are left for future study.

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