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Self-Induced Transparency of Excitons[2])

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As is well known, the action of ultrashort resonant laser radiation pulses is able to induce coherent states of excitons /1/. The phenomena of self-induced transparency (SIT) can then be observed in the exciton part of the spectrum. The possibility of SIT due to excitons has been object of intensive study in the last years /1 to 4/. In a very interesting paper Moskalenko et al. /5/ studied the possibility of SIT due to excitons in semiconductors and showed that the existence of bleaching and darkening waves is determined by the sign of the exciton-exciton interaction constant g and concluded that g must be positive, i.e., a repulsive interaction constant. In solving the coupled equations that describe the exciton-photon system Moskalenko et al. /5/ ignore the second derivatives with respect to x of the amplitudes of the exciton and the photon fields. With this approximation they find erroneously that the exciton-exciton interaction constant g must be positive. In this note we show that solving the equations that describe the inhomogeneous states of excitons and photons in a rigorous way, the condition for SIT is $g < 0$ or the exciton-exciton interaction must be attractive. We start from the coupled equations for excitons and photons and assume that the amplitudes of the excitons a and of the positive frequency part of a coherent electromagnetic field E^+ depend only on the coordinate x and time t /5/,

$$i\hbar \frac{\partial a}{\partial t} = \hbar\Omega_1 a - \frac{\hbar^2}{2m}\frac{\partial^2 a}{\partial x^2} + g|a|^2 a - dE^+ \quad , \tag{1}$$

$$\frac{\partial^2 E^+}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E^+}{\partial t^2} = \alpha \frac{\partial^2 a}{\partial t^2} \quad , \tag{2}$$

where Ω_1 is the limiting frequency of transverse photons, m is the translational mass of the exciton, g is the exciton-exciton interaction constant, d is the dipole moment of the transition from the ground state to the exciton state of the crystal, c is the velocity of light in the crystal, and $\alpha = 4\pi d/v_0 c^2$ with v_0 being the volume of the unit cell. Now to solve the system (1), (2) we take the solutions of such equations in the form

$$a = \phi(x, t)\exp(i(Kx - \omega t)) \quad , \tag{3}$$

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$$E^{+} = \rho(x, t)\exp(i(Kx - \omega t)) , \tag{4}$$

where $\phi(x, t)$ and $\rho(x, t)$ are real functions and ω and K are the frequency and wave number of the carrier, respectively. We now substitute (3) and (4) in (1) and (2) and separate the real and imaginary parts

$$\hbar\omega\phi = \hbar\Omega_1\phi - \frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + \frac{\hbar^2}{2m}K^2\phi + g\phi^3 - d\rho , \tag{5}$$

$$\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{m}K\frac{\partial\phi}{\partial x} \tag{6}$$

for (1) and

$$\frac{\partial^2\rho}{\partial x^2} - K^2\rho - \frac{1}{c^2}\frac{\partial^2\rho}{\partial t^2}\frac{\omega^2}{c^2}\rho = \alpha\frac{\partial^2\phi}{\partial t^2} - \alpha\omega^2\phi , \tag{7}$$

$$2K\frac{\partial\rho}{\partial x} + \frac{2\omega}{c^2}\frac{\partial\rho}{\partial t} = -2\alpha\omega\frac{\partial\phi}{\partial t} \tag{8}$$

for (2).

Introducing the traveling variable $\xi = x - vt$, where v is the velocity of the wave packet in (5) to (8) and imposing the usual boundary conditions on $\phi(x, t)$ and $\rho(x, t)$ for a localized pulse-like solution, i.e., $\phi(\xi = \pm\infty) = \rho(\xi = \pm\infty) = \phi_\xi(\pm\infty) = \rho_\xi(\pm\infty) = 0$ (the subscript ξ in ϕ and ρ means partial derivative relative to the variable ξ) we have

$$\rho(\xi) = \frac{\alpha v^2}{1-(v/c)^2}\phi(\xi) , \tag{9}$$

$K = mv/\hbar$ is self-consistently satisfied and we allowed for the fact that $v = d\omega/dK$. Now we substitute (9) in (5) and have

$$\frac{\partial^2\phi}{\partial\xi^2} + (K^2 - K_1^2)\phi - \frac{2mg}{\hbar^2}\phi^3 = 0 , \tag{10}$$

where $K_1^2 = (2m/\hbar)\bar{\Omega}_1$ with

$$\bar{\Omega}_1 = \Omega_1 - \frac{\Omega_o(v/c)^2}{1-(v/c)^2} \tag{11}$$

and $\Omega_o = 4\pi d^2/\hbar$ ($v_o = 1$). The solution of (10) is easily found by direct integration giving the following normalized expression for $\phi(\xi)$:

$$\phi(\xi) = \frac{1}{\sqrt{2L}}\,\mathrm{sech}\left(\frac{\xi-\xi_o}{L}\right), \tag{12}$$

where $L = 1/\bar{K}$ is the width of the pulse soliton and $\bar{K}^2 = K_1^2 - K^2$. The normalization condition sets a value for the propagation velocity v of such a pulse found by the relation $L = -2\hbar^2/mg$ that is a function of the physical parameters of the crystal under consideration and the velocity of light in such a crystal. Finally we substitute (12) in (9) and find the amplitude of the electromagnetic field that propagates in the crystal without loss. This solution shows that the excitonic medium is transparent only for g attractive.

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