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Pion Production in Reactions $p+p \to p+p+\pi^0$ and $p+p \to d+\pi^+$ near the Threshold

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Relativistic calculations for the S wave pion production cross sections of the reactions $p+p \to p+p+\pi^0$ and $p+p \to d+\pi^+$ are carried out. Importance of the small components of the deuteron relativistic wave functions in the description of the $p+p \to d+\pi^+$ cross section is studied. The problem of the relativistic description of a bound state is discussed.

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1. Introduction

Calculation of the cross sections for the pion production in pp collisions is one of the tests for the reliability of field theory models and also a necessary stage for understanding processes of the pion absorption in nuclei. Both dynamics of the πN interaction and the internucleonic correlations appear to be important in those reactions. Therefore it is interesting to ascertain mechanisms of the pion absorption on an interacting nucleon pair and to determine their relative contributions. Normally people distinguish the single-nucleon mechanism and the mechanism with rescattering. In the first case a pion is considered to be absorbed by one nucleon, and in the second case it is scattered by a nucleon, then it propagates being off-shell, and finally it is absorbed by another nucleon. In the latter case, it is necessary to take into account off-shell behaviour of the πN vertex amplitude. While calculating such kind of reactions it is preferable to make use of the covariant approach in order to take into account relativistic effects, in particular, small components of the deuteron relativistic wave function (for the reaction $pp \to d\pi^+$).

We shall consider the threshold production of pions in reactions

$$p+p \to p+p+\pi^0, \tag{1}$$

$$p+p \to d+\pi^+. \tag{1'}$$

For reaction (1) we shall consider the transition from the state ${}^{3,3}P_0(S=1,\ T=1,\ L=1,\ J=0)$ to the state ${}^{3,1}S_0$. Near the threshold (i.e. for small momenta of the pion) the contribution of this transition to the reaction cross section is predominant. In general, the total cross section of the S wave pion production may be presented for the reaction (1) as follows [1]

$$\sigma_{\mathrm{tot}} = \alpha \eta^2 + \beta \eta^6 \tag{2}$$

with constants α and β, η being the pion maximum momentum in units of the pion mass μ. Transition ${}^{3,3}P_0 \to {}^{3,1}S_0$ contributes only to the first term of (2).

To calculate the amplitudes of reactions (1, 1′) we shall use the πN interaction Lagrangian proposed in [2]. Let us note that according to calculation of this paper $\alpha = 65$ μbn which is essentially bigger than the experimental value [3]

$$\alpha = 27 \pm 10\ \mu\text{bn}. \tag{3}$$

In distinction from [2] we shall take into consideration the interaction of nucleons in the initial and

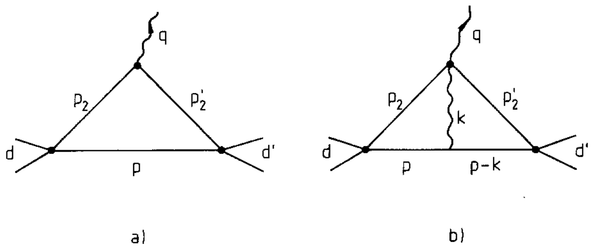

Fig. 1a and b. Main mechanisms of the s wave pion production in the reactions $p+p \to p+p+\pi^0$ and $p+p \to d+\pi^+$ near the threshold: **a** – single nucleon mechanism, **b** – two-nucleon mechanism with pion rescattering

final states and also the single-nucleon mechanism of the pion production.

Two diagrams contributing to the amplitudes of the reactions (1,1′) are presented by Fig. 1: diagram **a** corresponds to the single-nucleon mechanism, **b** – to the mechanism with rescattering, i.e. to the "two-nucleon" mechanism.

While calculating the reaction (1′) cross section we shall use not only the usual nonrelativistic wave function of the deutron (calculated for Reid potential [4]), but also the relativistic one parameterized according to [5]. In this connection it is expedient to discuss certain aspects of the relativistic description of a bound state.

2. Relativistic Description of a Bound State in Field Theory

It is well-known that relativistic equation for two-particle system has been proposed by H. Bethe and E. Salpeter [6–8] (see Figs. 2 and 3).

There are difficulties connected with the Bethe-Salpeter equation. Namely: since it is a four-dimensional integral equation we can reduce it to the two-dimensional one, but not to the one-dimensional (as it is in the case of the Lippman-Schwinger equation) using the partial wave expansion. Also a serious difficulty arises because of the fact that the Bethe-Salpeter wave function depends on two time variables and does not allow its usual probability interpretation. At the same time the problem of boundary conditions becomes much more complicated. There are also difficulties connected with the necessity to deal with an equation in Minkowski space except for some approaches like Wick-Cutcosky model [9] in which an analytic continuation to Euclidean variables is used.

To overcome those difficulties A. Logunov and A. Tavkhelidze have proposed three-dimensional quasipotential equation [10]. Equations of this kind have been considered by R. Blankenbeckler and R.

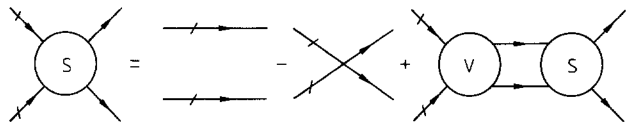

Fig. 2. Grafic representation of the Bethe-Salpeter equation

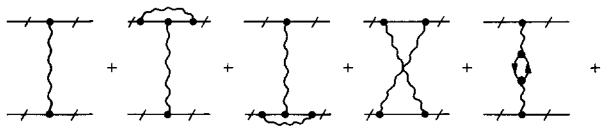

Fig. 3. Irreducible four-fermion diagrams constributing to the kernel V

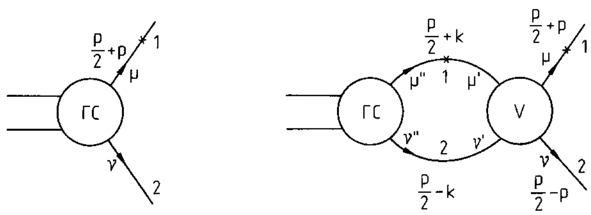

Fig. 4. Grafic representation of the vertex $\Gamma(p)$ of the deutron-two-nucleons interaction nucleon 1 being on-shell

Sugar [11], and V. Alessandrini and R. Omnes [12].

In Ref. 13 it is shown that the equation of the same type can be derived from the Bethe-Salpeter equation in the approximation of instantaneous interaction.

Alternative three-dimensional quasipotential equations were proposed by V. Kadyshevsky [14] and F. Gross [15] (see also [16]).

To describe the deuteron wave function the approach [15] seems to be more convenient. We shall consider the deutron-two-nucleon vertex $\Gamma(p)$ with one on-shell nucleon (Fig. 4, particle 1). If the second nucleon is also on-shell such a vertex depends on two invariant functions $F(p_2^2)$ and $G(p_2^2)$

$$\Gamma_{0n} = F(p_2^2)\,\gamma \cdot \xi + \frac{G(p_2^2)}{m}\, p \cdot \xi, \tag{4}$$

ξ being the deutron spin operator. This follows from the consideration of two independent amplitudes (two possible helicities) for the transition $1 \to \frac{1}{2} + \frac{1}{2}$. While nucleon 2 is off-shell the number of independent amplitudes is doubled due to the additional transition $\frac{1}{2} \to 0 + \frac{1}{2}$ in which one nucleon radiates a pion. Hence

$$\Gamma(p) = F(p_2^2)\,\gamma \cdot \xi + \frac{G(p_2^2)}{m}\, p \cdot \xi - \frac{m - \not{p}_2}{m} \cdot \left[H(p_2^2)\,\gamma \cdot \xi + \frac{I(p_2^2)}{m}\, p \cdot \xi \right]. \tag{5}$$

Such a form of $\Gamma(p)$ reduces to (4) if the nucleon 2 is on-shell. Using the dispersion method it was shown [17] that (5) is the most general expression for deutron-two-nucleon vertex if the nucleon 2 is off-shell. This vertex is connected with the deutron (relativistic) wave function

$$\langle p|D\rangle = \int \mathrm{d}x\, e^{ip_1x} \langle O|T(\psi(x)\psi(0))|D\rangle \tag{6}$$

by the following relation

$$[S(p_2)\Gamma(p)S(-p_1)C]_{\alpha_2\alpha_1} = i(2\pi)^{-3/2}(2m_d)^{-1/2}[\langle p|D\rangle]_{\alpha_1\alpha_2}, \tag{7}$$

where $S(p)=(\not p-m)^{-1}$, C is the charge conjugation matrix, and according to the definition

$$\langle p_1|(m-\not p_2)\psi(0)|D\rangle = \Gamma(p)\,C\bar u^T(p_1). \tag{8}$$

It can be shown that for the vertex $\Gamma(p)$ one can obtain [5] an equation similar to the equation for a bounded state [16]:

$$(\tilde\Gamma C)_{\mu\nu}(p) = -\int \frac{d^3k}{(2\pi)^3} V_{\mu\mu',\nu\nu'}(p,k,P) \cdot G_{\mu'\mu'',\nu'\nu''}(k,P)(\tilde\Gamma C)_{\mu''\nu''}(k) \tag{9}$$

where $(\Gamma C)_{\nu\mu}=(\Gamma C)_{\mu\nu}$, $G_{\mu'\mu'',\nu'\nu''}$ is a two-particle Green function

$$G_{12}(k,P) = \frac{[m+\gamma(k+\frac{1}{2}P)]_1\,[m+\gamma(-k+\frac{1}{2}P)]_2}{2E_k m_d(2E_k-m_d)}. \tag{10}$$

The operator (γp_2+m) can be divided in two parts: with positive and negative energies. Therefore we have two normalized wave functions of the deutron

$$\psi^+_{rs}(\mathbf{p}) = \frac{m}{[2m_d(2\pi)^3]^{1/}} \frac{\bar u^{(s)}(-p)\Gamma C\bar u^{(r)T}(\mathbf{p})}{Ep(2E_p-m_d)},$$
$$\psi^-_{rs}(\mathbf{p}) = -\frac{m}{[2m_d(2\pi)^3]^{1/2}} \frac{\bar v^{(s)}(p)\Gamma C\bar u^{(r)T}(\mathbf{p})}{E_p m_d}. \tag{11}$$

Using these expressions one can connect deutron wave functions u, w, v_t and v_s [5] with invariants F, G, H and I.

In paper [5] deutron relativistic wave functions were obtained solving wave equation (9) numerically. In this approach one-boson exchange by π, ρ, ω and σ mesons was taken into account and πNN coupling was considered as the sum of pseudovector and pseudoscalar interactions

$$g_{\pi N}\left[\lambda\gamma_5+(1-\lambda)\frac{(p_f-p_i)\cdot\gamma\gamma_5}{2m}\right]. \tag{12}$$

In [5] a set of solutions for the deutron wave function was obtained depending on the parameter λ values. In the present paper we use these results.

3. Pion Production in the Reaction $p+p \to p+p+\pi^0$ near the Threshold

a) Single-Nucleon Mechanism

Matrix element corresponding to the diagram of Fig. 1a can be written as follows (pseudoscalar coupling is used)

$$S_1 = -i\frac{\delta^4(d'+q-d)}{4\sqrt{\pi\mu d_0 d_0'}}\frac{g_{\pi N}}{2m}\int\frac{d^4p}{(2\pi)^4} \cdot \mathrm{Tr}[S_F^c(p)\bar\Gamma_{d'}(p,d'-p) \cdot S_F(p_2')\not q\gamma^5 S_F(p_2)\Gamma_d(d-p,p)]. \tag{13}$$

Notation corresponds to [19]. While calculating this matrix element we take into account only the positive pole of the nucleon-spectator propagator and neglect antinucleon degrees of freedom in the initial and final states, i.e. we substitute

$$S_F(p_2) = \frac{\not p_2+m}{m^2-p_2^2-i\varepsilon} \to \frac{\Lambda^+(p_2)}{2E_{p_2}-d_0}$$
$$S_F(p_2') = \frac{\not p_2'+m}{m^2-p_2'^2-i\varepsilon} \to \frac{\Lambda^+(p_2')}{2E_{p_2'}-d_0'}, \tag{14}$$

where Λ^+ is an appropriate projection operator. In the c.m. system

$$d=(2m+\mu,0), \quad d'=(2m,0), \quad q=(\mu,0). \tag{15}$$

The vertex function of the initial state $^{3,3}P_0$ is as follows

$$\Gamma(d-p,p) = \pi\sqrt{2d_0}(2E_p-d_0)\frac{u_1(p)}{p}E_p,$$
$$E_p=\sqrt{p^2+m^2}, \tag{16}$$

and for the final state $^{3,1}S_0$

$$\bar\Gamma(p,d'-p)=\bar\Gamma(p)\gamma_5=\pi u_0(p)\sqrt{2d_0}(2E_p-d_0') \tag{16'}$$

$u_0(p)$ and $u_1(p)$ being wave functions of the initial and final states normalized as in [19].

Using (14–16') we obtain from (13)

$$S_1 = -\frac{\delta^4(d'+q-d)}{\sqrt{4\pi\mu\mu m}}\frac{g_{\pi N}}{2m}\mu^2\int_0^\infty p^3u_0(p)\frac{m}{E_p}u_1(p)\,\mathrm{d}p.$$

The integral in this expression can be rewritten in the coordinate space:

$$I_1 \equiv -\mu^2 \int_0^\infty p^3 u_0(p) \frac{m}{E_p} u_1(p)\,\mathrm{d}p$$
$$\approx -\frac{\mu^2}{1+\mu/(2m)} \int_0^\infty \mathrm{d}r\, r^2 \frac{u_0(r)}{r} \left(\frac{d}{dr}+\frac{2}{r}\right) \frac{u_1(r)}{r}. \tag{17}$$

Here we have neglected the p^2-dependence of E_p putting $E_p = m+\mu/2$. As the calculation shows the accuracy of such an approximation for the reaction under consideration is $\sim 10\%$. The assymptotic behaviour of wave functions in (17) is usual:

$$\frac{u_l(r)}{r} \xrightarrow[r\to\infty]{} \sqrt{\frac{2}{\pi}} \frac{\sin(pr - l\pi/2+\delta_l)}{pr}. \tag{18}$$

The integral similar to (17) was obtained in the nonrelativistic approach [20].

b) Two-Nucleon Mechanism

The matrix element corresponding to the diagram of Fig. 1b can be written as follows

$$S_2 = \frac{\delta^4(d'+p-d)}{4\sqrt{\pi\mu d_0 d_0'}} \frac{g_{\pi N}}{2m} \int \frac{\mathrm{d}^4 p\, \mathrm{d}^4 k}{(2\pi)^8}$$
$$\cdot \mathrm{Tr}\,[S_F^c(p) \not{k} \gamma_5$$
$$\cdot \Delta_F(k) S_F^c(p-k) \bar{\Gamma}(p, d'-p) S_F(p_2')$$
$$\cdot T_{\mathrm{tr}}(s,t,u) S_F(p_2) \Gamma(d-p,p)], \tag{19}$$

where $\Delta_F(k) = (\mu^2 - k^2 - i\varepsilon)^{-1}$ is the rescattered pion propagator, with $k_0 = E_p - E_{p-k}$, and

$$T_{\mathrm{tr}}(s,t,u) = -A_{\mathrm{tr}}(s,t,u) + \frac{\not{q}+\not{k}}{2} B_{\mathrm{tr}}(s,t,u)$$

is the matrix of the pion rescattering without the nucleon pole. The invariant amplitudes A_{tr} and B_{tr} depend on the dynamics of πN interaction. We use the πN Lagrangian proposed in [3] and take into account contributions of σ meson and Δ_{33} isobar (ρ meson does not contribute due to the isospin conservation). To calculate (19) we make substitution (14) and also

$$S_F^c(p) \to \frac{\pi i}{E_p}(m - \not{p}), \quad S_F^c(p-k) \to \frac{\pi i}{E_{p-k}}(m - \not{p} - \not{k}).$$

Finally we obtain

$$S_2 = -\frac{\delta^4(d'+q-d)}{4\sqrt{\pi\mu d_0 d_0'}} \frac{g_{\pi N}}{2m} \frac{1}{4\pi} \int \frac{\mathrm{d}^3 p\, \mathrm{d}^3 k}{(2\pi)^3}$$
$$\cdot u_0(|\mathbf{p}-\mathbf{k}|)\, \hat{T}(\mathbf{p},\mathbf{k})\, u_1(p),$$
$$\hat{T}(\mathbf{p},\mathbf{k}) = \frac{m}{E_p} \frac{m}{E_{\mathbf{p}-\mathbf{k}}} \Delta_F(k) \left[\left(\frac{\mu+2k_0}{2}\right.\right.$$
$$\left.\left.\cdot B_{\mathrm{tr}} - \frac{E_p}{m}\right) \hat{\mathbf{p}}\cdot\mathbf{k} + A_{\mathrm{tr}} \frac{k_0}{m} \hat{\mathbf{p}}\cdot\mathbf{p}\right], \tag{20}$$

Table 1. Cross section of the reaction $p+p \to p+p+\pi^0$ near the threshold

Λ_π, GeV/c	$\alpha = \sigma/\eta^2$, μbn	I_1	$I_2(\sigma)$	$I_2(\Delta_{33})$
0.938	11	−0.526	0.857	0.531
1.200	18	−0.526	0.928	0.697
1.400	11	−0.526	0.957	0.790

where $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$. To evaluate the integral in (20) we make the substitution

$$T(\mathbf{p},\mathbf{k}) \to \frac{1}{1+\mu/(2m)} \frac{1}{3/4\,\mu^2 + \mathbf{k}^2} (\mu B_{\mathrm{tr}} - A_{\mathrm{tr}})\, \hat{\mathbf{p}}\,\mathbf{k}. \tag{21}$$

Such a substitution is justified if we neglect the interaction of the initial and final states, i.e. if we put

$$u_1(p) = \frac{\delta(p-p_i)}{\mathbf{p}^2}, \quad u_0(p) = \frac{\delta(|\mathbf{p}-\mathbf{k}|)}{(\mathbf{p}-\mathbf{k})^2},$$
$$E_i = \sqrt{m^2 + \mathbf{p}_i^2} \approx m + \frac{\mu}{2}.$$

After all that the matrix element of the two-nucleon mechanism can be written as follows

$$S_2 \cong \frac{\delta^4(d'+q-d)}{\sqrt{4\pi\mu}} \frac{g_{\pi N}}{2m} \frac{I_2}{1+\mu/(2m)} \frac{1}{\mu},$$
$$I_2 = \frac{m\mu}{4\pi} \int \mathrm{d}r\, r^2 \frac{u_0(r)}{r} \frac{\partial f}{\partial r} \frac{u_1(r)}{r}. \tag{22}$$

To calculate the rescattering function $f(r)$ we use the form factor $g(k^2) = (\Lambda_\pi^2 - \mu^2)/(\Lambda_\pi^2 - k^2)$ and take into account k^2-dependence of A_{tr} and B_{tr} near the point $s = (m+\mu)^2$, $u = m(m-2\mu)$.

The result of calculation for the coefficient α (see (2)) and for the integrals I_1 and I_2 presented in the Table 1 as functions of Λ_π. The calculation is done using the Reid potential. Values of I_1 presented in Table 1 were calculated putting $E_p = m + \mathbf{p}^2/(2m)$; values of I_2 are presented as separate contributions due to the σ meson exchange and Δ_{33} isobar (values of I_1 and I_2 in the Table 1 were obtained for the wave function normalization differing from (41) by factor $\sqrt{2/\pi}$). Let us note that the model of [19] using Reid wave functions gave $\alpha = 10\,\mu$bn.

Table 1 shows that the calculation results fit the experimental value of α [3] for $\Lambda_\pi = 1{,}2$ GeV/c. Hence, the agreement with the experiment for the threshold cross section of the reaction (1) can be obtained in the framework of the Lagrangian (2) if one takes into account *(i)* interaction in the initial and final states, and *(ii)* single-nucleon mechanism of the pion production.

Table 2. Cross section of the reaction $p+p \to d+\pi^+$ near the threshold for $\Lambda_\pi = 1.2$ GeV/c

	Deutron non-relativistic wave function (Reid)	Deutron relativistic wave function					
		$\lambda=0$	$\lambda=0.2$	$\lambda=0.4$	$\lambda=0.6$	$\lambda=0.8$	$\lambda=1.0$
σ/η, μbn	153	153	216	248	193	165	166

4. Pion Production in the Reaction $p+p \to d+\pi^+$ near the Threshold

In our previous report [19], to describe the threshold production of π^+ in the reaction (1′) we used $\Lambda_\pi = 0.938$ GeV/c. Practical calculations were performed for the deutron nonrelativistic wave function (Reid potential) and for the relativistic one which according to (5) depends of the parameter λ reflecting the relative role of the pseudovector and pseudoscalar interactions in the relativistic case. The cross section of the s wave pion production in this reaction is proportional to η near the threshold. Table 2 presents this cross section for $\Lambda_\pi = 1.2$ GeV/c and for different types of the deutron wave function. Calculation was done in the framework of the model [19] (taking into account the $\mathbf{k}^2$-dependence of the invariant functions near the rescattered pion absorption point.

Experimental value of σ/η for the reaction (1′) lies between 200 and 300 μbn [21]. Table 2 shows that the calculated value fits the experimental one for the parameter λ of the deutron wave function lying between 0.2 and 0.6. Let us note that single-particle mechanism contribution to the cross section of the reaction (1′) is mainly due to the relativistic components of the deutron wave function and changes from 5% ($\lambda=0.2$) up to 30% (for $\lambda=0.6$). The single-particle mechanism contribution for the reaction (1) is of the same order of magnitude.

5. Conclusion

This paper presents an attempt to calculate the pion production cross section in reactions $p+p \to p+p+\pi^0$ and $p+p \to d+\pi^+$ near the threshold in the framework of a relativistic approach. Of course, there are some uncertainties in calculation of the covariantly defined functions, particularly while performing the noncovariant three-dimension integration. Nevertheless first results testify in favor of such an approach. The results give an indication that the pseudovector πNN coupling is predominant. It is shown that both, single-particle and two-particle, mechanisms contribute to the reactions under consideration and the contribution of the latter is of $\gtrsim 70\%$. We utilized here parameters of the paper [2] considering a certain success of the approach [2] in description of πN interaction, particularly, the ratio R_s of the velocities of the negative pion absorption on the pairs proton-neutron and proton-proton of a nucleus. Certainly, the final choice of the model parameters can be done only after detailed study of different channels of the pion production in nucleon-nucleon collisions near the threshold. Such a study is in progress now.

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