Empirical Tests of Valuation Models for Options on T-Note and T-Bond Futures

Nusret Cakici
Sris Chatterjee
Avner Wolf

INTRODUCTION

The purpose of this article is to test two valuation formulas for pricing American options on T-Note and T-Bond futures. One model is the European pricing formula of Black (1976) and the other model is that of Barone-Adesi and Whaley (1987) who derive a quadratic approximation for pricing American options. Options on these interest rate futures belong to the most actively traded contracts at the Chicago Board of Trade (CBOT). Options on U.S. Treasury-Bond futures were first introduced in October, 1982. Options on U.S. Treasury-Note futures began trading in May, 1985.

Upon exercise, a futures option holder acquires a long or short futures position (depending on whether the option holder is holding a call or put) with a futures price equal to the exercise price of the option. Because of the daily marking-to-market, the option holder's account is credited at the end of the day with an amount equal to the futures price less the exercise price, in the case of a call, and the exercise price less the futures price in the case of a put. Therefore, exercising the option is equivalent to receiving in cash the value of exercising the option.

Black (1976) was the first to provide a valuation formula for European options on forward contracts, but his formula is also valid for European options on futures contracts when the short-term interest rate is nonstochastic, because the price of a futures contract is equal to the price of a forward contract in this setting. Wolf (1984) discusses the reasons for the popularity of Black's model and provides extensive sensitivity analysis. Shastri and Tandon (1986) show that Black's European pricing model performs reasonably well for the futures options in S&P 500 Stock Index and West German Mark. Whaley (1986) compares Black's model with the Barone-Adesi–Whaley approach to valuing American futures options in the case of S&P 500 futures.

In Black's model the short-term riskless interest rate is constant through time. Since interest rate uncertainty is the driving force behind bond prices and the volatility of long-term bond futures, it may seem on the surface that options on T-Bond and T-Note futures

---

Nusret Cakici is an Assistant Professor of Finance at the School of Business, Rutgers University.

Sris Chatterjee is an Assistant Professor of Finance at Fordham University.

Avner Wolf is a Professor of Finance at Baruch College, City University of New York.

The Journal of Futures Markets, Vol. 13, No. 1, 1–13 (1993)

CCC 0270-7314/93/010001-13

may not be priced correctly by applying Black's model. However, Whaley (1986) points out that the driving force behind bond price and bond futures volatility is the uncertainty in the *long-term interest rate,* and that this uncertainty is, to some degree, separable from the assumptions on the behavior of the short-term interest rate. Whaley also argues that the assumption of a constant short-term interest rate is untenable *even* for stock option pricing, because the volatility of stock prices depends to some extent on the uncertainty of interest rates. Some researchers have tested more complex models that incorporate both interest rate uncertainty and volatility in the spot price of the underlying commodity. [See, for example, Brennan and Schwartz (1980) for an application to the valuation of convertible bonds, Ramaswamy and Sundaresan (1985) as well as Bailey (1987) for applications to valuation of futures options, and Courtadon (1983) for applications to valuation of bond options.] The major advantage of using models with stochastic interest rates is that they are more realistic. Of course, this is true only if the estimated interest rate process is reasonable. However, models with stochastic interest rates have to be solved by numerical methods, because there are no closed-form solutions. These numerical methods become prohibitively expensive when applied to transactions data. Therefore, the tests used here are restricted to Black's model and its modification by Barone-Adesi and Whaley (1987) for American options. This is a necessary first step in order to find out the accuracy of the model with constant short-term interest rate. The approach is similar to that of Whaley (1986) who examines the mispricing of S&P 500 futures options with the same model.

VALUATION MODELS

European Futures Options

Black (1976) provides the basic framework. The following assumptions are necessary to develop his formulas:

(A1) Capital markets are perfect and frictionless in that there are no transactions costs and no short sales constraints.

(A2) The short-term riskless interest rate is constant through time.

(A3) The instantaneous futures price relative is described by the stochastic differential equation:

$$dF/F = \mu\, dt + \sigma\, dz \tag{1}$$

where μ is the expected instantaneous price change relative of the futures contract, σ is the instantaneous standard deviation, and z is the standard Weiner process.

Following Whaley (1986), assumption (A3) defines the dynamics of the futures price movement with no reference to the relationship between the futures price and the price of the underlying spot commodity.[1]

[1]If the cost-of-carry relationship holds, then (A3) is consistent with the assumption that the spot price of the underlying commodity also follows a stochastic differential equation given by:

$$dS/S = \alpha\, dt + \sigma\, dz \tag{2}$$

where $\mu = \alpha - (r - \delta)$, r = riskless interest rate, and δ represents the dividend yield (or its analog) on the spot commodity. In the case of T-Bond and T-Note futures, the basic cost-of-carry relationship is further complicated by the presence of delivery options which include quality and timing options. [See Boyle (1989) for details.] Therefore, the stochastic differential equation for the price of the spot commodity will be more complex than eq. (2) and cannot be specified easily.

Under the above assumptions, the value of a European call option (denoted by c) on a futures contract is:

$$c(F,T;x) = e^{-rT}[FN(d_1) - XN(d_2)] \tag{3}$$

where:

F = current futures price,

X = exercise price of futures option,

T = time to expiration of futures option,

N = the cumulative density function for univariate normal distribution,

$d_1 = \left[\ln(F/X) + 0.5\,\sigma^2 T\right]/(\sigma\sqrt{T})$, and

$d_2 = d_1 - \sigma\sqrt{T}$

The value of a European put option (denoted by p) is:

$$p(F,T;X) = e^{-rT}[XN(-d_2) - FN(-d_1)] \tag{4}$$

American Futures Options

Prices of American futures options are calculated by using the algorithm of Barone-Adesi and Whaley (1987). This algorithm is based on a quadratic approximation method and is computationally faster than the finite difference or the compound option approximation method. The value of an American call option on a futures contract is given by:

$$C(F,T;X) = c(F,T;X) + A_2(F/F^*)^q \qquad \text{when } F < F^* \tag{5a}$$

and

$$C(F,T;X) = F - X \qquad \text{when } F \geqslant F^* \tag{5b}$$

and where

$$A_2 = (F^*/q)\{1 - e^{-rT}N[d_1(F^*)]\}$$
$$d_1(F^*) = \left[\ln(F^*/X) + 0.5\,\sigma^2 T\right]\Big/\left(\sigma\sqrt{T}\right)$$
$$q = \left[1 + \sqrt{(1 + 4k)}\right]/2$$
$$k = 2r/[\sigma^2(1 - e^{-rT})]$$

F^* is the critical futures price above which the American futures option should be exercised immediately. The critical price is determined by numerical methods by solving

$$F^* - X = c(F^*,T;X) + \{1 - e^{-rT}N[d_1(F^*)]\}F^*/q \tag{6}$$

The quadratic approximation of the American put option on a futures contract is given by:

$$P(F,T;X) = p(F,T;X) + A_1(F/F^{**})^Q, \qquad \text{when } F > F^{**} \tag{7a}$$

and

$$P(F,T;X) = X - F \qquad \text{when } F \leqslant F^{**} \tag{7b}$$

and where

$$A_1 = -(F^{**}/Q)\{1 - e^{-rT}N[-d_1(F^{**})]\}$$
$$Q = \left[1 - \sqrt{(1 + 4k)}\right]/2$$

and where all other notations are the same as in the case of the American call. F^{**} is the critical futures price below which the American futures option should be exercised immediately and is determined by numerically solving the following equation:

$$X - F^{**} = p(F^{**}, T; X) - \{1 - e^{-rT}N[-d_1(F^{**})]\}F^{**}/Q \tag{8}$$

Data

Data for T-Note and T-Bond futures options traded on the Chicago Board of Trade (CBOT) are used to test the models. The data-set consists of all transaction prices (referred to as "time and sales data" by CBOT) on futures options from January 2, 1987 to December 31, 1987.[2] The volume of a transaction is not reported. The original sample from CBOT has 595,116 T-Bond futures prices, 138,145 call option, and 138,802 put option prices. For T-Notes, there are 87,677 futures prices, 8958 call option, and 12,575 put option prices. However, the actual sample used in this study is smaller than the original sample because all "nominal"[3] prices and all option prices which are less than the option's intrinsic value are eliminated. The futures option pricing model determines the futures and option prices simultaneously. To satisfy this requirement, the option price is matched to the futures price generated by the futures trade closest in trading time to the option trade.[4] The final sample consists of 3643 calls and 5106 puts for T-Notes, and 62,065 calls and 60,224 puts for T-Bonds. Table I provides more detailed information about the final sample and indicates that at-the-money options are more active. With respect to time to maturity, most of the trading activity is in the nearest contract month.

The data on interest rate consist of the daily U.S. Treasury-Bill rate with one month to expiration.

Implied Parameters

Black's European option pricing formula as well as its American version has five parameters. These are F, E, T, r, and σ. All but σ are easily observable or can be estimated with little difficulty. The exercise price, E, the futures price, F, and the time to maturity, T, are written into the contract. The riskless rate is an easily observable market-determined value. The problem parameter is the volatility, σ.

There are several ways to estimate σ. For example, one can use historical data to calculate the standard deviation of futures price changes or one can use the implied volatilities. Implied volatilities reflect the market's best assessment of future volatility. Implied volatilities are used in this article.

Implied volatilities can also be estimated in several ways. The most common approaches are:

[2]If daily closing prices are used there may be a problem of nonsynchronous prices. Hopefully, using transactions data, we can eliminate this problem.

[3]The Chicago Board of Trade data manual defines a nominal price as the closing price of a contract which has not traded that day but has traded at some time since it was listed for trading.

[4]The time of a particular trade is first rounded to the nearest minute. For example, if the time for an option transaction is 10:04:51, then it is recorded as having occurred at 10:05. This rounding is done for both options and futures. If a futures transaction takes place at 10:05:15, then it is recorded as 10:05. Transactions which have the same rounded timing are then matched. The average difference in the true timing of an option and a futures trade is zero seconds, and the average absolute difference is 30 seconds. This procedure is different from the one used by Whaley (1986), and possibly biases the results toward finding that no trading profits can be earned. Thus, the tests are conservative in terms of detecting systematic pricing biases.

Table I
SUMMARY OF T-NOTE AND T-BOND FUTURES OPTIONS TRANSACTION DATA

F/E	# of Transactions Calls	# of Transactions Puts	Time-to-Expiration in Weeks	# of Transactions Calls	# of Transactions Puts
T-Notes					
$F/E < 0.90$	2	0	$T < 4$	912	956
$0.90 \leq F/E < 0.95$	123	55	$4 \leq T < 8$	1,193	1,878
$0.95 \leq F/E < 1.00$	2,411	2,388	$8 \leq T < 12$	992	1,422
$1.00 \leq F/E < 1.05$	1,090	2,544	$12 \leq T < 16$	376	527
$1.05 \leq F/E < 1.10$	16	118	$16 \leq T < 20$	144	287
$1.10 \leq F/E$	1	1	$20 \leq T$	26	36
All	3,643	5,106	All	3,643	5,106
T-Bonds					
$F/E < 0.90$	1,427	50	$T < 4$	14,445	14,377
$0.90 \leq F/E < 0.95$	13,325	1,758	$4 \leq T < 8$	15,909	16,201
$0.95 \leq F/E < 1.00$	38,863	20,490	$8 \leq T < 12$	16,579	15,922
$1.00 \leq F/E < 1.05$	8,052	30,945	$12 \leq T < 16$	8,765	7,932
$1.05 \leq F/E < 1.10$	358	5,563	$16 \leq T < 20$	4,530	4,423
$1.10 \leq F/E$	40	1,418	$20 \leq T$	1,837	1,369
All	62,065	60,224	All	62,065	60,224

1. Implied volatilities can be calculated by using an at-the-money option or the option closest to being at-the-money. The rationale for this approach is based on the evidence that market prices of at-the-money options have the least pricing bias vis-a-vis model prices [Feinstein (1989); Geske and Roll (1984)].
2. The volatility can be estimated by employing a weighting scheme. Latane and Rendleman (1976) used this approach. If there are n options on any particular day, then n implied standard deviations σ_j' can be calculated for $j = 1, \ldots, n$ by setting the option's market price equal to the model price, i.e., $c_j = c_j(\sigma_j)$ and solving for σ_j. If these estimates are weighted and averaged to compute σ',

$$\sigma' = \Sigma(w_j \sigma_j')/\Sigma w_j$$

   where w_j is the weight applied to the jth estimate, then a weighted implied standard deviation (σ') is obtained.

   Previous researchers have employed various weighting schemes. Schmalensee and Trippi (1978) and Patell and Wolfson (1979) have used equal weights. Latane and Rendleman (1976) used the partial derivative of the call price with respect to σ as the weight. Chiras and Manaster (1978) used the elasticity of the call price with respect to σ as the weight.
3. Instead of using an explicit weighting scheme, one can use the market prices for the options to provide an implicit weighting scheme that yields an estimate which has as little prediction error as possible [see Whaley (1982)]. This third approach is used in this article.

In this estimation procedure, the market prices of the options on any day can be written as:

$$V_j = V_j(\sigma) + e_j \tag{9}$$

where $V_j(\sigma)$ is the model price. The estimate of σ is then obtained by minimizing the sum of squared residuals Σe_j^2, for $j = 1, \ldots, n$.

The estimation requires a nonlinear procedure. All futures option prices on a given day are used in the regression in eq. (9). Thus, all transaction prices with a given maturity are used in each regression. Table II presents the means and standard deviations for implied volatility. It can be seen from this table that the means of implied volatilities for T-Bonds are higher than those for T-Notes, as expected. On average, the volatility implied by call option transaction prices is lower than that implied by put option prices. This result is similar to Whaley (1986, p. 140), who speculates that the misspecification of the stochastic process governing the futures prices could be an explanation.

Table III compares the market prices with the prices obtained from Black's European futures option pricing formula and Barone-Adesi and Whaley's American futures option pricing formula. Table III indicates that deviation of market prices from the corresponding model prices are, in general, higher for the puts compared to the calls.

Tests of Biases in the Pricing Models

It has been demonstrated that Black's European model as well as its American version shows systematic mispricing when it is used to value foreign currency options or index

Table II
SUMMARY STATISTICS FOR ESTIMATES OF IMPLIED VOLATILITIES

Underlying Contract	Model	# of Observations	Mean	Standard Deviation
Calls				
T-Note	European (Black)	340	0.0842	0.0294
T-Note	American (Whaley)	340	0.0838	0.0294
T-Bond	European (Black)	529	0.1284	0.0304
T-Bond	American (Whaley)	529	0.1290	0.0336
Puts				
T-Note	European (Black)	340	0.0888	0.0304
T-Note	American (Whaley)	304	0.0883	0.0302
T-Bond	European (Black)	529	0.1366	0.0311
T-Bond	American (Whaley)	529	0.1365	0.0316

Black—European model prices calculated using Black European futures pricing formula. Whaley—American futures option prices are calculated using Barone-Adesi and Whaley algorithm.

Table III
COMPARISON OF MARKET AND MODEL PRICES

Underlying Contract	Model	# of Observations	MAE	MAPE
Calls				
T-Note	European (Black)	3,643	0.0431	8.3146
T-Note	American (Whaley)	3,643	0.0438	8.4208
T-Bond	European (Black)	62,065	0.0450	8.5499
T-Bond	American (Whaley)	62,065	0.0476	10.1356
Puts				
T-Note	European (Black)	5,106	0.0704	8.7984
T-Note	American (Whaley)	5,106	0.0720	9.1571
T-Bond	European (Black)	60,224	0.0887	10.3241
T-Bond	American (Whaley)	60,224	0.0921	11.2582

MAE is the mean value of the difference between market price and model price. MAPE is the mean value of the percent error. Black—European model prices calculated using Black's European futures pricing formula. Whaley—American futures option prices are calculated using Barone-Adesi and Whaley algorithm.

options [see Shastri and Tandon (1986) and Whaley (1986)]. To check whether these biases exist in the T-Note and T-Bond futures option market as well, this study tests for the existence of time-to-maturity bias and for bias related to the option being in- or out-of-the-money for every model.

Table IV presents the dollar mispricing of the American futures options pricing model. Results for the European pricing model are similar to the American model, and are not reported here. In reporting the systematic biases in Table IV, the total sample is first clustered by the degree the option is in-the-money and by the option's time to maturity. The average price deviation in each cluster is then reported in Table IV. There are three clusters in moneyness (F/E less than 0.98, F/E between 0.98 and 1.02, and F/E greater than 1.02). There are also three clusters in maturity (T less than 6 weeks, T between 6 and 12 weeks, and T greater than 12 weeks). Panel A of Table IV shows the mispricing for call and put options on T-Bond futures, and panel B shows the mispricing for T-Note futures options.

For call options on T-Bond futures, only the in-the-money calls ($F/E > 1.02$) seem to be mispriced in terms of their economic significance. The in-the-money calls are underpriced by the model. This underpricing is most pronounced for the intermediate maturity of $6 < T < 12$, where T is maturity in weeks. These two results are comparable to Whaley (1986). However, unlike Whaley (1986), no mispricing is detected for the out-of-the-money and at-the-money calls. As for the model's maturity bias, the underpricing of the short maturity in-the-money calls seems to be the only economically

Table IV

SUMMARY OF AVERAGE MISPRICING ERRORS OF AMERICAN FUTURES OPTION PRICING MODELS BY THE OPTION'S MONEYNESS (F/E) AND BY THE OPTION'S TIME TO EXPIRATION IN WEEKS (T) FOR CALL AND PUT OPTIONS ON T-BOND AND T-NOTE FUTURES (THE NUMBERS IN PARENTHESES ARE THE NUMBER OF OBSERVATIONS)

Panel A: T-Bond	$C - C(F,T;E)$				$P - P(F,T;E)$			
	$T < 6$	$6 < T < 12$	$T > 12$	All T	$T < 6$	$6 < T < 12$	$T > 12$	All T
$F/E < 0.98$	0.0083	−0.0031	−0.0064	−0.0010	−0.0315	−0.0863	−0.1987	−0.0875
	(10,056)	(16,026)	(11,176)	(37,258)	(3,286)	(3,297)	(1,692)	(8,375)
$0.98 \leq F/E < 1.02$	−0.0196	0.0179	0.0462	0.0395	−0.0150	−0.0258	−0.0359	−0.0228
	(11,254)	(8,110)	(3,583)	(22,947)	(14,003)	(12,397)	(6,017)	(32,399)
$F/E \geq 1.02$	0.0494	0.1444	0.1136	0.0983	0.0806	0.1352	0.1647	0.1308
	(781)	(706)	(373)	(1,860)	(4,814)	(8,621)	(6,015)	(19,450)
All F/E	−0.0045	0.0079	0.0091	0.0038	0.0034	0.0230	0.0320	0.0178
	(22,091)	(24,842)	(15,132)	(62,065)	(22,103)	(24,397)	(13,724)	(60,224)

Panel B: T-Note	$C - C(F,T;E)$				$P - P(F,T;E)$			
	$T < 6$	$6 < T < 12$	$T > 12$	All T	$T < 6$	$6 < T < 12$	$T > 12$	All T
$F/E < 0.98$	0.0173	−0.0098	−0.0181	−0.0041	−0.0230	−0.0743	−0.1757	−0.0666
	(295)	(515)	(241)	(1,051)	(297)	(392)	(91)	(780)
$0.98 \leq F/E < 1.02$	−0.0017	0.0096	0.0235	0.0061	0.0022	−0.0003	0.0002	0.0008
	(1,128)	(1,014)	(298)	(2,440)	(1,338)	(1,508)	(580)	(3,426)
$F/E \geq 1.02$	0.0197	0.0858	0.1870	0.0492	0.0727	0.1237	0.1662	0.1185
	(95)	(50)	(7)	(152)	(240)	(481)	(179)	(900)
All F/E	0.0034	0.0057	0.0072	0.0049	0.0073	0.0126	0.0163	0.0112
	(1,518)	(1,579)	(546)	(3,643)	(1,875)	(2,381)	(850)	(5,106)

significant mispricing. When all the calls are aggregated, the overall maturity bias is slight underpricing for longer maturities (0.0079 and 0.0091, respectively), and slight overpricing for the shortest maturity (−0.0045). But these mispricings are not economically significant. As for call options on T-Note futures, the maximum mispricing is also for the in-the-money options. The aggregate mispricing for in-the-money options is 0.0492, which is about half the mispricing for in-the-money call options on T-Bond futures. The only qualitative difference is that in the case of T-Note futures options, the longest maturity in-the-money calls seem to have the highest mispricing.

In the case of put options on T-Bond futures, the results are similar to those reported by Whaley (1986). Out-of-the-money puts are overpriced by the model and in-the-money puts are underpriced. As in Whaley (1986), the moneyness-bias and the maturity-bias seem to be monotonically related for both in-the-money puts and out-of-the-money puts. Unlike the call options on T-Bond futures, the maturity bias of puts is economically more significant. This result, too, is similar to Whaley (1986). For put options on T-Note futures, significant maturity and moneyness bias is found and, also, a monotonic relation exists between these two biases for both in-the-money and out-of-the-money options. This is the only qualitative difference between the put options on T-Note futures and the put options on the T-Bond futures.

Market Efficiency Tests

The systematic biases reported in Table IV may result, because the futures options pricing models are misspecified, or because the market for T-Note and T-Bond futures options is inefficient, or because of both inefficiency and misspecification. One way to determine the cause of the bias is to test whether abnormal rates of return after transaction costs can be earned by trading futures options on the basis of model prices. If abnormal profits after transaction costs can be earned, then it is likely that the market is inefficient. The price deviations signal profit-making opportunities. If abnormal profits cannot be earned, then one cannot reject the joint null hypothesis that the model is correctly specified and the market is efficient.

A "buy and hold" hedging strategy is used which is the same as in Whaley (1986). Each day, options are priced by using the American futures option model and an implied volatility computed from the previous day's transaction prices. The implied volatility is estimated separately for calls and puts. One estimate is obtained for call options by using all the transaction prices for the day's call options, and another estimate is obtained from the put option transaction prices. [This procedure is consistent with the demonstration in Whaley (1986) that the two σ estimates may be different.] A nonlinear procedure is used to estimate the implied volatility.

The hedge formed at an instant of time depends on the transaction price and the nature of the mispricing. Let C_F (P_F) be the partial derivative of the call (put) price with respect to the futures price. Then the strategy is as described below:

Nature of Mispricing	Futures Options Position	Futures Position
Undervalued call	Long 1 contract	Short C_F contract
Overvalued call	Short 1 contract	Long C_F contract
Undervalued put	Long 1 contract	Short P_F contract
Overvalued put	Short 1 contract	Long P_F contract

The "buy and hold" hedge portfolio is formed according to the weights described above and is held until the futures option/futures expiration, or until the end of the sampling period, whichever comes first. At the end of the holding period, the portfolio is liquidated and profits are calculated.[5] To allow for transactions costs, two sets of hedge portfolios are formed: one set of hedge portfolios is formed only when the price deviation between actual and model prices is equal to or greater than 5 cents; and the other set of hedge portfolios is formed when this difference is equal to or greater than 10 cents. For a retail customer, a signaling threshold of less than 10 cents per contract may not be a worthwhile signal if transaction costs are included. Since transaction costs are less for traders, however, the 5 cent threshold is also included.

Tables V and VI illustrate the results of these hedging tests. The results indicate that, in terms of hedging profits, there is no significant difference between Black's (1976) model for European option pricing and the competing American pricing model. The tables also show that statistically significant profits can be earned by these model-predictions only with (i) overvalued calls and puts on T-Bond futures when the signaling price difference is at least \$0.10; and with (ii) undervalued puts on T-Note puts when the signaling price difference is at least \$0.05. In all other cases only negative profits are earned. The economic significance of the profit has to be judged by considering the transaction costs. The transaction cost for a floor trader is usually quite low. But even for floor traders, the overvalued puts do not offer any opportunity for making a profit net of transaction costs.

Comparison to Other Empirical Studies on Futures-Options

Shastri and Tandon (1986) compared Black's European pricing model with a modified version of the Geske–Johnson (1984) American pricing model for S&P 500 and West German Mark futures. They found that the simple European model performs as well as the American model, especially for options near maturity and not deep in the money. Whaley (1986) compared Black's model with the Barone-Adesi–Whaley (1986) approach to valuing American futures options for S&P 500 futures. Since this study is more closely related to Whaley's study, a detailed comparison of Whaley's results with the results of this study follows.

1. For the S&P 500 futures options, Whaley (1986) finds both a moneyness and a maturity bias. This study finds the same biases for T-Bond and T-Note futures options (see Table IV). However, the direction of bias is not the same as in Whaley (1986). It is found here that out-of-the-money call options are overvalued, whereas Whaley found that out-of-the-money call options are underpriced. For puts, the reverse is true. Regarding the maturity bias, Whaley reports that the bias is the same for both puts and calls, although the bias appears to be greater for put options. This study finds that the maturity bias works in opposite directions for calls and puts in the sample of T-Bond and T-Note futures options. The maturity bias does not seem to be more pronounced for puts in the sample used here.
2. Whaley (1986) reports that the standard deviation implied by call option transaction prices is lower, on average, than that implied by put option prices. The same result is found to be true for T-Bond and T-Note futures options in this study. This is documented in Table II.
3. Whaley (1986) reports that the lower transaction costs incurred by floor traders or institutional investors would allow opportunities for riskless arbitrage profits,

[5]As pointed out in Whaley (1986), by holding the portfolio open until expiration, at which time market price converges to intrinsic value, there is some assurance that observed profit opportunities will be captured.

Table V
AVERAGE PROFITS FROM HEDGE PORTFOLIOS FOR CALLS

Underlying Contract	Model	Minimum Abs. Pr. Dev.	# of Observations	Average # of Contracts	Average Profits	T Values
Undervalued calls						
T-Note	European (Black)	$\|DIF\| \geq 0.05$	872	0.3902	0.0147	0.34
T-Note	American (Whaley)	$\|DIF\| \geq 0.05$	813	0.3918	−0.0291	−0.67
T-Bond	European (Black)	$\|DIF\| \geq 0.05$	15,318	0.3295	−0.1242	−10.80
T-Bond	American (Whaley)	$\|DIF\| \geq 0.05$	14,179	0.3375	−0.1654	−14.49
T-Note	European (Black)	$\|DIF\| \geq 0.10$	383	0.3879	−0.2064	−3.43
T-Note	American (Whaley)	$\|DIF\| \geq 0.10$	363	0.3973	−0.2141	−3.56
T-Bond	European (Black)	$\|DIF\| \geq 0.10$	6,610	0.3419	−0.4094	−24.96
T-Bond	American (Whaley)	$\|DIF\| \geq 0.10$	6,399	0.3509	−0.4039	−26.24
Overvalued calls						
T-Note	European (Black)	$\|DIF\| \geq 0.05$	946	0.4432	−0.2806	−5.39
T-Note	American (Whaley)	$\|DIF\| \geq 0.05$	1,003	0.4388	−0.2938	−5.75
T-Bond	European (Black)	$\|DIF\| \geq 0.05$	13,883	0.3488	−0.0698	−4.47
T-Bond	American (Whaley)	$\|DIF\| \geq 0.05$	14,824	0.3395	−0.0706	−4.66
T-Note	European (Black)	$\|DIF\| \geq 0.10$	515	0.4240	−0.2184	−3.11
T-Note	American (Whaley)	$\|DIF\| \geq 0.10$	541	0.4209	−0.1993	−2.94
T-Bond	European (Black)	$\|DIF\| \geq 0.10$	6,887	0.3922	0.1046	4.47
T-Bond	American (Whaley)	$\|DIF\| \geq 0.10$	7,399	0.3847	0.1128	4.99

regardless of the choice of valuation models. This study finds that there is no significant difference between the valuation models, but finds that abnormal profit opportunity exists in only a few cases. Only overvalued calls and puts on T-Bond futures, and undervalued puts on T-Note futures seem to generate profit opportunities. Even these profits are economically insignificant when one considers transaction costs.

Table VI
AVERAGE PROFITS FROM HEDGE PORTFOLIOS FOR PUTS

Underlying Contract	Model	Minimum Abs. Pr. Dev.	# of Observations	Average # of Contracts	Average Profits	T Values
Undervalued puts						
T-Note	European (Black)	|DIF| ≥ 0.05	1,413	−0.5829	0.1340	3.74
T-Note	American (Whaley)	|DIF| ≥ 0.05	1,332	−0.5916	0.1220	3.39
T-Bond	European (Black)	|DIF| ≥ 0.05	17,983	−0.5363	−0.0678	−5.18
T-Bond	American (Whaley)	|DIF| ≥ 0.05	17,005	−0.5430	−0.1104	−8.44
T-Note	European (Black)	|DIF| ≥ 0.10	796	−0.5865	−0.1193	−2.82
T-Note	American (Whaley)	|DIF| ≥ 0.10	766	−0.5926	−0.1138	−2.69
T-Bond	European (Black)	|DIF| ≥ 0.10	10,604	−0.5421	−0.3334	−20.97
T-Bond	American (Whaley)	|DIF| ≥ 0.10	10,043	−0.5480	−0.3445	−21.85
Overvalued puts						
T-Note	European (Black)	|DIF| ≥ 0.05	1,916	−0.3904	−0.3883	−10.76
T-Note	American (Whaley)	|DIF| ≥ 0.05	1,985	−0.3860	−0.4085	−11.41
T-Bond	European (Black)	|DIF| ≥ 0.05	22,718	−0.3127	−0.0877	−7.49
T-Bond	American (Whaley)	|DIF| ≥ 0.05	23,829	−0.3136	−0.1037	−8.93
T-Note	European (Black)	|DIF| ≥ 0.10	1,243	−0.3656	−0.2326	−5.62
T-Note	American (Whaley)	|DIF| ≥ 0.10	1,330	−0.3654	−0.2406	−5.96
T-Bond	European (Black)	|DIF| ≥ 0.10	15,102	−0.2926	0.0731	5.58
T-Bond	American (Whaley)	|DIF| ≥ 0.10	16,077	−0.2939	0.0506	3.92

SUMMARY AND CONCLUSION

This article tests Black's European futures option model and its American version in Barone-Adesi–Whaley's algorithm, using transactions data on T-Note and T-Bond futures options. The results presented here indicate that Black's (1976) European pricing model is indistinguishable from the American pricing model of Barone-Adesi and Whaley (1987) for pricing options on T-Bond and T-Note futures. The practical implication of these results is that Black's (1976) model is a fairly good approximation for pricing these contracts. Buy-and-hold hedge strategies reveal that statistically significant profits may be earned in only a few cases if the model prices are used as signals to construct hedge portfolios. But these profits are not economically significant.

Bibliography

Bailey, W. (1987): "An Empirical Investigation of the Market for Comex Gold Futures Options," *Journal of Finance,* 42:1187–1194.

Barone-Adesi, G., and Whaley, R. E. (1987): "Efficient Analytic Approximation of American Option Values," *Journal of Finance,* 42:301–320.

Black, F. (1976): "The Pricing of Commodity Contracts," *Journal of Financial Economics,* 3:167–179.

Boyle, Phelim (1989): "The Quality Option and Timing Option in Futures Contracts," *Journal of Finance,* 44:101–113.

Brennan, M. J., and Schwartz, E. S. (1980): "Analyzing Convertible Bonds," *Journal of Financial and Quantitative Analysis,* 15.

Chiras, D. P., and Manaster, S. (1978): "The Informational Content of Option Prices and a Test of Market Efficiency," *Journal of Financial Economics,* 6:213–234.

Courtadon, G. (1978, October): "Options on Default Free Bonds and Options on Default Free Bond Futures: A Comparison," NYU working paper.

Feinstein, S. P. (1989, May-June): "Forecasting Stock Market Volatility Using Options on Index Futures," *Economic Review,* Federal Reserve Bank of Atlanta, 12–30.

Geske, R., and Roll, R. (1984): "On Valuing American Call Options with the Black–Scholes European Formula," *Journal of Finance,* 39:443–455.

Latane, H., and Rendleman, R. J. (1976): "Standard Deviation of Stock Price Ratios Implied by Option Premia," *Journal of Finance,* 31:369–382.

Patell, J. M., and Wolfson, M. A. (1979): "Anticipated Information Releases Reflected in Call Option Prices," *Journal of Accounting and Economics,* 1:117–140.

Ramaswamy, K., and Sundaresan, S. (1985): "The Valuation of Options on Futures Contracts," *Journal of Finance,* 40:1319–1340.

Schmalensee, R., and Trippi, R. R. (1978): "Common Stock Volatility Expectations Implied by Option Premia," *Journal of Finance,* 33:129–147.

Shastri, K., and Tandon, K. (1986): "Options on Futures Contracts: A Comparison of European and American Pricing Models," *The Journal of Futures Markets,* 6:593–618.

Whaley, R. E. (1986): "Valuation of American Futures Options: Theory and Empirical Tests," *Journal of Finance,* 41:127–150.

Whaley, R E. (1982): "Valuation of American Call Options on Dividend-Paying Stocks," *Journal of Financial Economics,* 10:29–58.

Wolf, A. (1984): "Options on Futures: Pricing and the Effect of an Anticipated Price Change," *The Journal of Futures Markets,* 4:491–512.