Secular changes of LOD associated with a growth of the inner core

C. Denis[1,⋆], **K.R. Rybicki**[2], and **P. Varga**[3]

[1] Musée National d'Histoire Naturelle de Luxembourg & European Centre for Geodynamics and Seismology, 19 rue J. Welter, 7256 Walferdange, Luxembourg
[2] Institute of Geophysics of the Polish Academy of Sciences, ul. K. Janusza 64, 01-452 Warszawa, Poland
[3] Seismological Observatory of the Geodetic and Geophysical Research Institute, Meredek 18, 1112 Budapest, Hungary

Received 2005 Apr 22, accepted 2005 Dec 5
Published online 2006 Apr 20

Key words Earth rotation – secular variation of the length of day – differential rotation

From recent estimates of the age of the inner core based on the theory of thermal evolution of the core, we estimate that nowadays the growth of the inner core may perhaps contribute to the observed overall secular increase of LOD caused mainly by tidal friction (i.e., 1.72 ms per century) by a relative decrease of 2 to 7 μs per century. Another, albeit much less plausible, hypothesis is that crystallization of the inner core does not produce any change of LOD, but makes the inner core rotate differentially with respect to the outer core and mantle.

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1 Introduction

There seems to be a consensus among Earth scientists that the inner core is a relatively recent structure inside the Earth, and that its formation is still going on at present. The energy released by inner core formation seems to heat up the outer core and maintain there the convective motions that generate the geomagnetic field. Another effect of inner core formation would be a slight secular decrease in time of the Earth's inertia moment, causing possibly a small decrease of the length of the day (LOD). The latter would, of course, be masked by the very much larger secular increase of LOD brought about by tidal friction. Another *a priori* possible hypothesis is that a small differential rotation between inner core, outer core, and mantle is produced as a result of the growth of the inner core, provided the couplings at the core-mantle boundary (CMB), or at the inner core boundary (ICB), are small enough to allow for differential rotations. Anyway, it seems interesting for the mind to get a quantitative idea of the possible contribution of inner core growth to the secular change of LOD or to the rotation characteristics of the core. For this purpose, we use a very simple mechanical model as a first approximation; in principle, it is possible to consider more sophisticated models but, owing to the fact that many of the important physical parameters are not known accurately, the final result does not seem worth the effort making the calculations more complicated.

2 Change of the inertia moment for a simple model

Let us consider a mechanical system consisting of a central sphere of radius r_i and uniform density ρ_i, surrounded by a spherical layer of inner radius r_i, outer radius r_o, and uniform density ρ_o. The total mass of this system is $M_\mathrm{c} = M_\mathrm{i} + M_\mathrm{o}$, where $M_\mathrm{i} = 4\pi \int_0^{r_\mathrm{i}} \rho_\mathrm{i}\, r^2 \mathrm{d}r$ is the mass of the inner sphere, and $M_\mathrm{o} = 4\pi \int_{r_\mathrm{i}}^{r_\mathrm{o}} \rho_\mathrm{o}\, r^2 \mathrm{d}r$ is the mass of the outer spherical layer. We assume that, by some process, the inner radius r_i is changing with time, in such a way that the density in the inner sphere keeps the same value ρ_i and that the total mass

$$M_\mathrm{c} = \frac{4\pi}{3}\left[\rho_\mathrm{i} r_\mathrm{i}^3 + \rho_\mathrm{o}\left(r_\mathrm{o}^3 - r_\mathrm{i}^3\right)\right] \tag{1}$$

of the system remains constant as well. Assuming, moreover, that the outer radius r_o does not get changed appreciably during this process, we find that only the density ρ_o of the outer layer changes with r_i, in such a way that ρ_o decreases when r_i increases, and vice-versa, according to the law

$$\rho_\mathrm{o}(t) = \frac{M_\mathrm{c} - \frac{4}{3}\pi\rho_\mathrm{i} r_\mathrm{i}^3(t)}{\frac{4}{3}\pi\left[r_\mathrm{o}^3 - r_\mathrm{i}^3(t)\right]} = \frac{\alpha_\mathrm{c} - \rho_\mathrm{i} r_\mathrm{i}^3}{r_\mathrm{o}^3 - r_\mathrm{i}^3}, \quad \alpha_\mathrm{c} = \frac{3M_\mathrm{c}}{4\pi}, \tag{2}$$

where the variable t denotes the time. The rate of the density change with time in the outer layer is provided by

$$\frac{\mathrm{d}\rho_\mathrm{o}}{\mathrm{d}t} = -\frac{3r_\mathrm{i}^2\left[\rho_\mathrm{i}\left(r_\mathrm{o}^3 - r_\mathrm{i}^3\right) - \alpha_\mathrm{c} + \rho_\mathrm{i} r_\mathrm{i}^3\right]}{\left(r_\mathrm{o}^3 - r_\mathrm{i}^3\right)^2}\frac{\mathrm{d}r_\mathrm{i}}{\mathrm{d}t}. \tag{3}$$

Let $\Delta\rho(t)$ denote the changing density jump $\rho_\mathrm{i} - \rho_\mathrm{o}(t)$ at the changing inner boundary $r = r_\mathrm{i}(t)$ at each instant t. Then the formulae (1) and (2) can be recast into the form

$$M_\mathrm{c} = \frac{4\pi}{3}\left[\Delta\rho(t) r_\mathrm{i}^3(t) + \rho_\mathrm{o}(t) r_\mathrm{o}^3\right], \tag{4}$$

⋆ Corresponding author: denis@astro.ulg.ac.be

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$$\Delta\rho(t) = \rho_\mathrm{i} - \frac{\alpha_\mathrm{c} - \rho_\mathrm{i} r_\mathrm{i}^3(t)}{r_\mathrm{o}^3 - r_\mathrm{i}^3(t)}, \tag{5}$$

respectively. The time derivative of $\Delta\rho(t)$ is, obviously, the negative time derivative of ρ_o. After some straightforward algebraic operations, we obtain

$$\frac{\mathrm{d}\Delta\rho}{\mathrm{d}t} = -\frac{\mathrm{d}\rho_\mathrm{o}}{\mathrm{d}t} = \frac{3\rho_\mathrm{i}^2\Delta\rho}{r_\mathrm{o}^3 - r_\mathrm{i}^3}\frac{\mathrm{d}r_\mathrm{i}}{\mathrm{d}t}. \tag{6}$$

We wish to estimate the rate of change in time of the moment of inertia I_c about an axis passing through the centre of the sphere, resulting from a prescribed rate of change in time of the inner radius ρ_i. The total inertia moment $I_\mathrm{c}(t)$ at any instant of time t is the sum $I_\mathrm{c} = I_\mathrm{i} + I_\mathrm{o}$ of the inner inertia moment $I_\mathrm{i}(t) = \frac{8}{3}\pi\int_0^{r_\mathrm{i}(t)} \rho_\mathrm{i} r^4 \mathrm{d}r$ and of the outer inertia moment $I_\mathrm{o}(t) = \frac{8}{3}\pi\int_{r_\mathrm{i}(t)}^{r_\mathrm{o}} \rho_\mathrm{o} r^4 \mathrm{d}r$. Unlike the total mass M_c, the total inertia moment I_c of the system does not remain constant during the assumed process. For our simple model, we have

$$I_\mathrm{c}(t) = \frac{8\pi}{15}\left\{\rho_\mathrm{i} r_\mathrm{i}^5(t) + \rho_\mathrm{o}(t)\left[r_\mathrm{o}^5 - r_\mathrm{i}^5(t)\right]\right\} \tag{7}$$

or, alternatively,

$$I_\mathrm{c}(t) = \frac{8\pi}{15}\left[\Delta\rho(t) r_\mathrm{i}^5(t) + \rho_\mathrm{o}(t) r_\mathrm{o}^5\right]. \tag{8}$$

If we differentiate the latter with respect to time, substitute Eq. (6) into the result, and put

$$\eta(t) = \frac{r_\mathrm{i}(t)}{r_\mathrm{o}}, \tag{9}$$

then we find after some straightforward mathematical operations that

$$\frac{\mathrm{d}I_\mathrm{c}}{\mathrm{d}t} = \frac{8\pi}{15} r_\mathrm{o}^5 \Delta\rho\,\eta^2\left(5\eta^2 - 3\frac{1-\eta^5}{1-\eta^3}\right)\frac{\mathrm{d}\eta}{\mathrm{d}t}. \tag{10}$$

The latter formula is quite elegantly concise and relates the changes of I_c to the variable quantities η, $\mathrm{d}\eta/\mathrm{d}t$ and $\Delta\rho$ that define the variable inner core boundary $r = r_\mathrm{i}$. Knowing η, or equivalently $r_\mathrm{i} = \eta r_\mathrm{o}$, as a function of time, we can use Eq. (5) to evaluate $\Delta\rho(t)$ and then Eq. (10) to compute $\mathrm{d}I_\mathrm{c}/\mathrm{d}t$. Nevertheless, even though we must perhaps accept the loss of some conciseness, it seems useful to provide a formula that relates $\mathrm{d}I_\mathrm{c}/\mathrm{d}t$ only to the inner radius r_i and its time rate of change $\mathrm{d}r_\mathrm{i}/\mathrm{d}t$, and not to $\Delta\rho$ or ρ_o as well. To achieve this goal, we eliminate ρ_o in Eq. (7) by making use of Eq. (2) *before* we differentiate Eq. (7) with respect to time. We find

$$\begin{aligned}\frac{\mathrm{d}I_\mathrm{c}}{\mathrm{d}t} &= \frac{8\pi}{15} r_\mathrm{o}^5 \rho_\mathrm{i}\,\eta^2 \\ &\times \left[5\eta^2 - 3F_1(\eta) + \left(\beta_\mathrm{c} - \eta^3\right)F_2(\eta)\right]\frac{\mathrm{d}\eta}{\mathrm{d}t},\end{aligned} \tag{11}$$

with

$$F_1(\eta) = \frac{1-\eta^5}{1-\eta^3}, \quad F_2(\eta) = \frac{2\eta^5 - 5\eta^2 + 3}{(1-\eta^3)^2}, \tag{12}$$

and

$$\beta_\mathrm{c} = \frac{\alpha_\mathrm{c}}{\rho_\mathrm{i} r_\mathrm{o}^3} = \frac{M_\mathrm{c}}{\frac{4}{3}\pi\rho_\mathrm{i} r_\mathrm{o}^3}. \tag{13}$$

It is easy to check that we obtain Eq. (11) if we substitute into Eq. (10) the relation (5) expressed in non-dimensional variables, i.e., $\Delta\rho = \rho_\mathrm{i}\left[1 - (\beta_\mathrm{c} - \eta^3)/(1-\eta^3)\right]$. Eq. (11) can be put to the following somewhat simpler form:

$$\frac{\mathrm{d}I_\mathrm{c}}{\mathrm{d}t} = \frac{8\pi}{15} r_\mathrm{o}^5 \rho_\mathrm{i}\frac{(\beta_\mathrm{c}-1)\eta^2(3+6\eta+4\eta^2+2\eta^3)}{(1+\eta+\eta^2)^2}\frac{\mathrm{d}\eta}{\mathrm{d}t}. \tag{14}$$

For $\rho_\mathrm{i} > \rho_\mathrm{o}$ we have $\beta_\mathrm{c} < 1$, and Eq. (13) shows that under these circumstances the inertia moment decreases whenever the inner radius increases, and vice-versa.

Finally, we may wish to write Eq. (14) in an entirely non-dimensional form by using the non-dimensional inertia coefficient

$$y_\mathrm{c} = \frac{I_\mathrm{c}}{M_\mathrm{c} r_\mathrm{o}^2} \tag{15}$$

instead of the inertia moment I_c itself. Then Eq. (14) becomes

$$\frac{\mathrm{d}y_\mathrm{c}}{\mathrm{d}t} = \mu_\mathrm{c}\frac{(\beta_\mathrm{c}-1)\eta^2(3+6\eta+4\eta^2+2\eta^3)}{(1+\eta+\eta^2)^2}\frac{\mathrm{d}\eta}{\mathrm{d}t}, \tag{16}$$

where the constant μ_c represents two fifth of the ratio of the inner density ρ_i to the average density $\bar{\rho}_\mathrm{c} = M_\mathrm{c}/(\frac{4}{3}\pi r_\mathrm{o}^3)$ of the model, i.e., $\mu_\mathrm{c} = \frac{2}{5}(\rho_\mathrm{i}/\bar{\rho}_\mathrm{c})$.

3 Rate of the inner core growth according to the theory of thermal evolution of the core

Let us apply the foregoing formulae to the growth of the inner core in order to get a quantitative idea about the secular rate of change of the inertia moment that this process may cause. Thermodynamic considerations dealing with the heat balance associated with the cooling and solidification of the inner core on one hand, thermal and compositional convection in the outer core on the other hand, led Buffett et al. (1996) to the equation

$$f(\eta)d\eta = H(t)\mathrm{d}t. \tag{17}$$

The variable η denotes the ratio of the (growing) inner core radius $r_\mathrm{i} = r_\mathrm{i}(t)$ with respect to the outer core radius r_o that we consider in this context constant at the value $r_\mathrm{o} = 3480$ km. The forcing term $H(t)$ on the r.h.s. of Eq. (17) represents the heat flux across the core-mantle boundary (CMB) associated with the cooling and solidification processes. The motion of the inner core boundary (ICB) upwards results from the fact that in the course of time, the temperature drops below the freezing point of iron in the iron-rich mixture of elements at the base of the liquid outer core. Then, in this layer, iron and siderophile elements crystallize, but the lighter, more volatile minerals, which do not fit into the crystal lattice of iron and iron-like elements, move upwards by the mere effect of buoyancy. The newly formed solid iron layer is now also a part of the inner core, and as a net result the ICB has been shifted upwards as well. We notice that in this scenario, the inner core started to form when the Earth had sufficiently cooled down at the Earth's centre for iron to solidify. That might have been a very long time after the core itself had formed; it is obvious that the very epoch

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when inner core formation set in is determined by the rate of cooling of the outer core and, thus, by the heat flux $H(t)$ across the CMB.

Buffett et al. (1996) do not provide a general expression for the function $f(\eta)$. However, as long as η remains small enough, we may approximate $f(\eta)$ by

$$f(\eta) \approx (2\eta + 3\mathrm{A}\eta^2)\mathcal{M}\,, \tag{18}$$

neglecting terms that contain powers of η larger than 2. The non-dimensional parameter A is the algebraic sum of three terms which are not known very accurately. They represent the effect of composition on the liquidus temperature, the effect of latent heat release, and the effects of gravitational energy release and ohmic dissipation due to compositional rearrangement. Thus, the quantity A describes the relative importance of the various physical processes intervening in the evolution of the core. For the present epoch, the values of the parameters provided by Buffett et al. (1996) yield $\mathrm{A} = 1.57$. The multiplication constant $\mathcal{M}$ represents the heat that must be extracted to cool the entire core to its solidification temperature. Buffett et al. (1996) furnish $\mathcal{M} = 1.88 \times 10^{30}$ J. It should be clear, however, that there exists an overall uncertainty in all parameter values that reaches at least several percent, but could be much larger. Owing to this uncertainty, it seems justified to neglect terms of $\mathcal{O}(\eta^3)$ in the function $f(\eta)$[1].

We substitute formula (18) into Eq. (17), assume that the parameter A is approximately time-independent, and integrate with respect to time t from the instant $t = 0$ at which the temperature first fell below the liquidus of iron at the centre of the Earth, up to an arbitrary instant $t = \tau$:

$$\eta^2 + \mathrm{A}\eta^3 = \mathcal{M}^{-1}\int_0^{\tau} H(t)\mathrm{d}t. \tag{19}$$

Thus, specific predictions for the growth of the inner core necessitate an estimate of the heat flux function $H(t)$. According to Stacey (1992, p. 301), the net cooling rate of the Earth is at present about 10 TW, i.e., approximately one fourth of the heat flux measured at the Earth's surface (Sclater, Jaupart & Galson 1980) [Note: $1\,\mathrm{TW} = 10^{12}$ W]. Stacey attributes a large fraction of the 10 TW to the cooling of the mantle, the rest stemming from the cooling of the crust and core. His preferred estimate of the present-day heat loss from the core is 3.0 TW, in good agreement with an estimate derived by Sleep (1990) on the basis of the heat transported by mantle plumes. Stevenson, Spohn & Schubert (1983) and Mollett (1984) find that the heat flux from the core may have slowly decreased in time, the amount depending on poorly known initial conditions. Accordingly, Buffett et al. (1996) consider $\overline{\mathrm{H}} = 4.0$ TW as a typical time-averaged value of H, but they also consider $\overline{\mathrm{H}} = 6.0$ TW and $\overline{\mathrm{H}} = 2.5$ TW as high and low values to span a plausible range of solutions. It is obvious that a variation of H with time could easily be included in the solution of Eq. (19), but such a level of detail seems unnecessary given the already large uncertainty of the time-averaged value.

In terms of such a time-averaged value $\overline{\mathrm{H}}$ of H, Eq. (19) can be written as

$$\eta^2 + \mathrm{A}\eta^3 = \mathcal{M}^{-1}\overline{\mathrm{H}}\,\tau. \tag{20}$$

The general solution of this cubic equation consists of two complex conjugate roots, and one real root. Their symbolic expressions, even that of the physically acceptable real root, are quite involved; there is no need to reproduce them here.

The present age τ_o of the inner core can be estimated by putting the present radius, corresponding to $\eta_\mathrm{o} = 0.351$, into Eq. (20). If we express $\overline{\mathrm{H}}$ in TW [$1\,\mathrm{TW} = 10^{12}\,\mathrm{J\,s^{-1}}$] and τ in Ga [$1\,\mathrm{Ga} \approx 3.1557 \times 10^7$ s], we obtain

$$\tau_\mathrm{o} \approx 11.3\,\overline{\mathrm{H}}^{-1}. \tag{21}$$

Considering the range of values of $\overline{\mathrm{H}}$ stated above, the age of the Earth's solid inner core should be comprised between 1.88 Ga (for $\overline{\mathrm{H}} = 6.0$ TW) and 4.52 Ga (for $\overline{\mathrm{H}} = 2.5$ TW). The latter value, corresponding to an upper bound for the age of the inner core, is hardly realistic since it implies that cooling of the Earth, once the liquid core was formed, should have been extremely fast. Stacey's (1992) preferred value, $\overline{\mathrm{H}} = 3.0$ TW, leads to a cooling rate of the core that seems plausible, and to an age of ca. 3.8 Ga for the birth of the inner core.

However, almost everything known or inferred about the inner core, from seismology or indirect inference, is controversial (Anderson 2002). Thus, according to recent estimates (Labrosse, Poirier & Le Mouël 1997, 2001; Labrosse 2002), Stacey's value $\overline{\mathrm{H}} = 3.0$ TW seems to underestimate the actual rate of heat flux across the CMB by at least a factor 2, and perhaps by a factor 3 or more. Consequently, the inner core seems to be younger than 1.9 Ga; in fact, a plausible age is about 1 Ga. Labrosse & Macouin (2003) argue that prior to inner core crystallization (i.e., more than 1 Ga ago), the geomagnetic field was maintained by thermal convection driven by cooling of the core material, not by solidification of iron in the core. More information concerning constraints on the heat flux imposed by the geodynamo is provided by Gubbins et al. (2003).

On differentiating Eq. (20), we obtain

$$\frac{\mathrm{d}\eta}{\mathrm{d}\tau} = \frac{\mathcal{M}^{-1}\overline{\mathrm{H}}}{2\eta + 3\mathrm{A}\eta^2} \tag{22}$$

for any epoch $t = \tau$. In particular, for the present epoch $t = \tau_\mathrm{o}$, the growth rate of the inner core is given by the formula

$$\left[\frac{\mathrm{d}\eta}{\mathrm{d}\tau}\right]_{\tau_\mathrm{o}} \approx 0.013\,\overline{\mathrm{H}}. \tag{23}$$

4 Effect of the inner core growth on the inertia moment and length of the day

The fourth column of Table 1 shows the growth rates of the inner core radius for the present epoch according to various

[1] Note that $\mathcal{O}(\eta^3) = \mathcal{O}(0.351^3) \approx 0.04$ for the present size of the inner core.

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Table 1 Present growth rates of the inner core radius r_i and the associated rates of change of the inertia moment I_c of the core, according to different hyptheses concerning its present age τ_o.

τ_o (Ga)	$\overline{\mathrm{H}}$ (TW)	$[\mathrm{d}\eta/\mathrm{d}t]_{\tau_\mathrm{o}}$ (Ga^{-1})	$[\mathrm{d}r_\mathrm{i}/\mathrm{d}t]_{\tau_\mathrm{o}}$ ($\mathrm{km\,Ga}^{-1}$)	$[\mathrm{d}I_\mathrm{c}/\mathrm{d}t]_{\tau_\mathrm{o}}$ ($10^{35}\,\mathrm{kg\,m^2\,Ga^{-1}}$)
0.5	22.6	0.2938	1022.4	−1.316
1.0	11.3	0.1469	511.2	−0.658
1.5	7.53	0.0979	340.8	−0.438
2.0	5.65	0.0734	255.6	−0.329
2.5	4.52	0.0588	204.5	−0.263
3.0	3.77	0.0490	170.4	−0.219
3.5	3.23	0.0420	146.1	−0.188
4.0	2.82	0.0367	127.8	−0.164
4.5	2.51	0.0326	113.6	−0.146

assumptions concerning the present age of the inner core or, equivalently, according to various assumptions concerning the average heat flux rate across the CMB. A plausible lower limit of the present age of the inner core is 1 Ga, a plausible upper limit is 3.5 Ga. The corresponding growth rates are 511.2 and 146.1 $\mathrm{km\,Ga^{-1}}$, respectively.

Considering the values given in Table 2, which provides some characteristic parameters pertaining to the Earth models PREM (Dziewonski & Anderson 1981), PREMM and CGGM (Denis et al. 1997), we shall assume that $\beta_\mathrm{c} \approx 0.87$, $\rho_\mathrm{i} \approx 12.5 \times 10^3\,\mathrm{kg\,m^{-3}}$. For these values, Eq. (14) yields $\mathrm{d}I_c/\mathrm{d}t \approx -6.58 \times 10^{34}\,\mathrm{kg\,m^2\,Ga^{-1}}$ if $\tau_\mathrm{c} = 1\,\mathrm{Ga}$, and $\mathrm{d}I_c/\mathrm{d}t \approx -1.88 \times 10^{34}\,\mathrm{kg\,m^2\,Ga^{-1}}$ if $\tau_\mathrm{c} = 3.5\,\mathrm{Ga}$ (see the last column of Table 1).

If we assume that the couplings at the ICB and at the CMB are strong enough to preclude any differential rotation between inner core, outer core, and mantle, any change ΔI of the Earth's inertia moment I will give rise to a concomitant change $\Delta\Omega$ of the Earth's spin Ω according to the formula

$$\frac{\Delta\Omega}{\Omega} = -\frac{\Delta I}{I}, \tag{24}$$

i.e., neglecting the small difference between sidereal and solar days,

$$\frac{\Delta\mathrm{LOD}}{\mathrm{LOD}} \approx \frac{\Delta I}{I}. \tag{25}$$

Using the above estimates for $[\mathrm{d}I_\mathrm{c}/\mathrm{d}t]_{\tau_\mathrm{o}}$, namely -6.58×10^{34} respectively -1.88×10^{34} $\mathrm{kg\,m^2}$ per 10^9 years, we find that for a solid body rotation, the growth of the inner core contributes at present to the change of LOD by a decrease of 7.1 μs per century respectively 2.0 μs per century. Such a slight secular decrease of LOD of a few microseconds per century is, of course, entirely masked by the overall secular increase of about 1.72 ms per century at the present epoch. The latter value results from the combination of a secular increase of LOD amounting to (2.4 ± 0.1) ms/cy produced by tidal friction, and of a secular decrease of LOD of (0.6 ± 0.1) ms/cy caused by factors of non-tidal origin, such as post-glacial crustal uplift (e.g., Varga, Denis & Varga 1998).

Table 2 Some characteristic parameters for the Earth models PREM (Dziewonski & Anderson 1981), PREMM, and CGGM (Denis et al. 1997).

Parameter	PREM	PREMM	CGGM
r_i (10^6 m)	1.2215	1.2215	1.2215
r_o (10^6 m)	3.4800	3.4800	3.4800
R (10^6 m)	6.3710	6.3710	6.3710
$\eta = r_\mathrm{i}/r_\mathrm{o}$	0.3510	0.3510	0.3510
M_i (10^{24} kg)	0.0984	0.0946	0.0924
M_o (10^{24} kg)	1.8411	1.8150	1.8018
M_c (10^{24} kg)	1.9395	1.9096	1.8942
M (10^{24} kg)	5.9737	5.9737	5.9737
ρ_i ($10^3\,\mathrm{kg\,m^{-3}}$)	12.894	12.394	12.099
ρ_o ($10^3\,\mathrm{kg\,m^{-3}}$)	10.901	10.746	10.668
$\bar\rho_\mathrm{c}$ ($10^3\,\mathrm{kg\,m^{-3}}$)	10.987	10.817	10.730
$\bar\rho$ ($10^3\,\mathrm{kg\,m^{-3}}$)	5.515	5.515	5.515
I_i ($10^{37}\,\mathrm{kg\,m^2}$)	0.0059	0.0056	0.0055
I_o ($10^{37}\,\mathrm{kg\,m^2}$)	0.9066	0.8956	0.8879
I_c ($10^{37}\,\mathrm{kg\,m^2}$)	0.9125	0.9012	0.8934
I ($10^{37}\,\mathrm{kg\,m^2}$)	8.0185	8.0549	8.0549
$y = I/MR^2$	0.3307	0.3322	0.3322
α_c (10^{23} kg)	4.6302	4.5588	4.5221
β_c	0.8521	0.8728	0.8869
μ_c	0.4694	0.4583	0.4510

5 Some words about differential rotation

It is commonly assumed, either explicitly or implicitly, that the Earth as a whole undergoes solid body rotation, unlike gaseous or liquid cosmic bodies such as the sun or the giant planets. On the other hand, the boundary conditions imposed at the ICB and the CMB for studying global deformations of the Earth, such as the gravito-elastic free oscillations, the bodily tides, the Chandler wobble, the nearly-diurnal nutations, etc., are slip conditions allowing differential motion between inner core and outer core, or between outer core and mantle. In this respect, Denis (1991, 1993) has shown that the use of slip-free boundary conditions at CMB would change the eigenperiods for the Earth model PREM (Dziewonski & Anderson 1981) by quite large amounts, incompatible with the observed values. Similarly, the change from slip to slip-free conditions at the CMB would drastically change the values of the characteristic tidal numbers (Table 3) in a way incompatible with observation. The boundary conditions at the ICB have an effect as well, but the latter is much smaller and does not fundamentally change the values of the eigenperiods and tidal numbers.

Therefore, at first sight, it would seem that inner core, outer core, and mantle might – and, in fact, should – rotate differentially if the inner core is growing slowly in time. Song & Richards (1996) as well as Su, Dziewonski & Jeanloz (1996) claimed that they had found seismological evidence for a differential rotation of the Earth's inner core. The inferred rotation rate is about 1° per year faster than the daily rotation of the mantle and the crust. Although very

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Table 3 Effect of a change of the boundary conditions at the ICB and at the CMB, respectively, on the characteristic tidal numbers of the anelastic PREM for a tidal forcing period of 12 hours and a harmonic degree $n = 2$. The numbers h and k are Love's numbers, l is Shida's number, and γ, δ, and Λ are, respectively, the clinometric, gravimetric, and astrometric tidal numbers.

At the ICB: At the CMB:	slip slip	slip-free slip	slip slip-free	slip-free slip-free
h	0.6141	0.6141	1.2526	1.2528
k	0.3038	0.3038	0.6214	0.6215
l	0.0860	0.0860	0.1150	0.1151
γ	0.6898	0.6898	0.3689	0.3688
δ	1.1583	1.1583	1.3204	1.3205
Λ	1.2178	1.2178	1.5064	1.5065

small, such a differential rotation would, strictly speaking, render the concept of a gravity potential and level surfaces meaningless for the Earth (Denis et al. 1998). However, the question of the superrotation of the inner core remains controversial at present, since the most recent seismological studies point to differential rotation rates less than 0.2° per year, and possibly to no differential rotation at all (e.g., Souriau, Garcia & Poupinet 2003). Actually, a majority of researchers dealing with the empirical evidence seem now to be convinced that there is no differential rotation at all, although differential convective motions obviously should occur.

An anonymous reviewer, whose comments we accept with pleasure and thanks, made a crude estimate that if the inner core was uncoupled from the outer core and the change of LOD was at the level of 1 microsecond per century, one revolution of the inner core should nowadays be about 10 seconds different from one revolution of the rest of the Earth. If this happened to be the case, any pattern in the inner core detectable by seismological means should be shifted with respect to a frame anchored in the mantle by about 15° in one year, an effect which definitely seems to be ruled out by seismological evidence. Therefore, the reviewer concludes that the inner core must be sufficiently coupled to the rest of the Earth that our assumption of a uniform rotation rate for the whole Earth is basically correct.

Actually, the question of whether there can exist a differential rotation of different layers within the Earth is quite complicated, even if we do not take into account the effect of tidal friction which amounts to an increase of LOD of several hours in 10^9 years.

On one hand, in the study of small periodic deformations, it is generally argued that viscous, topographic, gravitational and electromagnetic coupling at the CMB and ICB is too small to prevent slip from occurring. On the other hand, in the study of the Earth's rotation, it is generally assumed that large stretching and distorsion of geomagnetic field lines, and perhaps the irregular topographies of CMB and ICB, prevent differential rotations between inner core, outer core, and mantle, and make the Earth as a whole rotate as a solid body. Over the last few decades, many authors have tried to quantify the geodynamic effects of the different kinds of coupling without reaching a consensus. Two relatively recent papers dealing to some extent with this topic are those of Hollerbach (1998), Aurnou & Olson (2000), and Gubbins et al. (2003).

Despite the fact that some numerical simulations (Glatzmaier & Roberts 1996) seem to suggest that convective flow in the liquid core may sustain a differential rotation of the inner core, we should be aware that, when dealing with a growing inner core radius, the ICB ceases to be a *simple* interface between two continua. The fact that particles, belonging at a given instant t_1 to the ICB, belong at a later instant t_2 to the inner core, and that other particles, belonging at the instant t_1 to the ICB as well, but belong at the instant t_2 to the outer core, must obviously create a strong coupling between inner and outer cores, preventing differential rotation to occur as a result of the inner core growth.

Acknowledgements. Carlo Denis expresses his sincere thanks to the *Musée National d'Histoire Naturelle de Luxembourg* for funding partially this research. Péter Varga acknowledges with thanks sponsorship from the Hungarian Scientific Research Fund (Project T038123).

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