Quart. J. R. Met. Soc. (1970), **96**, pp. 138-143

551.508.26 : 551.508.54 : 551.551.2 : 551.465.7

Spectra and cospectra of turbulence over water

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(Manuscript received 20 October 1969)

SUMMARY

Composite diagrams of spectra and cospectra were constructed from eight cases of velocity component and temperature measurements made over the water by an acoustic anemometer. Their universal form is discussed.

1. INTRODUCTION

With the improvement of sensors and data reduction schemes in recent years, the determination of atmospheric turbulence structure in the surface boundary layer is becoming both more complete and more reliable. An examination of the consistency between various measurements provides insight into the mechanisms of atmospheric turbulence.

During the spring and summer of 1968 an intensive observational programme was carried out by the Institute of Oceanography of the University of British Columbia at an experimental site on a tidal flat, about 400 m from the nearest beach. The results reported here were obtained using a Kaijo Denki three-dimensional acoustic anemometer-thermometer. This sensor is the product of the developmental effort of Mitsuta, Miyake and Kobori (1967) at Kyoto University in co-operation with the engineering staff of Kaijo Denki in Tokyo (1967). This particular acoustic anemometer is designed to obtain the advantages of both continuous-wave and pulse techniques. Vertical velocities are sensed by using two vertical paths closely separated, over which sound is transmitted in opposite directions; horizontal winds are sensed by 2 similar pairs of acoustical arrays oriented 120° apart in the horizontal plane. The transmitters send out bursts of 100 kHz sound 400 times per second, and the travel time to the third zero crossing within each received pulse is identified and converted into an analogue signal. The component of wind speed, in the direction of each array, is proportional to the difference in travel times between sound paths with opposite directions.

The average time of flight of the two vertical sound pulses gives a measure of the speed of sound and thus of the temperature.

Data from 8 runs were selected for discussion in terms of normalized spectra and cospectra. They were chosen to coincide with other results presented by Miyake *et al.* (1970).

2. EXPERIMENTS, METHODS OF DATA REDUCTION

All data reported here were obtained from the three-dimensional sonic anemometer, mounted from the bottom by a pole which had a cross arm on a carriage which could be raised or lowered to adjust for the tide level and the desired level of observation. Each run was about 50 minutes in duration; the statistics presented were computed for data collected during the time intervals given in Table 1.

All experimental data were recorded in analogue form, using a 14 channel IRIG FM scheme on an Ampex FR 1300 tape recorder.

Data analyses were carried out in analogue form. As 2 channels of data were played back at 60 inches per second, 8 times the speed of the original recording, each signal from the tape recorder was passed through a high pass filter with the low cut off frequency at 0·024 Hz. Thus the analysis contained data between 0·003 Hz and 2·5 kHz in the recorded time scale. Variances and covariances were computed by integration of signals multiplied on an analogue computer. The low frequency cut-off of the analogue filter was then shifted in logarithmic intervals of ¼ decade, and spectral and cospectral energy computed as the difference between successive energy estimates.

TABLE 1. SUMMARY OF THE DATA AND FLUCTUATION STATISTICS

Run No.	Time of Start	Duration	U	Ht. of sonic anemometer	Z/L	σ_u	σ_w	u_* from $-\overline{u'w'}$	r_{uw}	σ_T	$\overline{w'T'}$	r_{wT}	From $\phi_{11}(\kappa)$, u_*	σ_v	$\frac{\sigma_w}{u_*}$
		Minutes	cm s^{-1} at 5 m	cm	at 150 cm	cm s^{-1}	cm s^{-1}	cm s^{-1}		°C	cm s^{-1} °C		cm s^{-1}	cm s^{-1}	
1	29-8-68 11·05	40	432	170	− 0·13	31·5	16·6	12·4	− 0·293	0·183	1·18	0·388	13·0	29·3	1·34
2	30-8-68 11·17	33	728	140	− 0·04	70·4	32·4	26·1	− 0·299	0·180	3·41	0·585	25·6	47·1	1·24
3	30-8-68 16·25	35	737	150	0·01	71·8	36·7	26·9	− 0·274	0·111	− 0·45	− 0·107	29·0	52·3	1·36
4	3-9-68 14·12	40	444	370	− 0·04	38·2	22·1	15·7	− 0·290	0·117	0·67	0·258	16·8	35·5	1·41
5	4-9-68 17·17	48	398	180	− 0·03	44·4	22·8	11·8	− 0·138	0·110	0·23	0·092	15·7	45·6	1·92
6	17-9-68 15·12	48	385	220	− 0·02	47·0	21·4	13·0	− 0·169	0·071	0·24	0·157	14·5	45·0	1·64
7	18-9-68 11·05	63	860	450	− 0·02	74·5	42·3	27·9	− 0·246	0·209	2·11	0·239	35·1	63·7	1·51
8	18-9-68 13·36	26	508	160	− 0·08	40·2	21·5	15·8	− 0·288	0·201	1·49	0·344	16·6	40·8	1·36

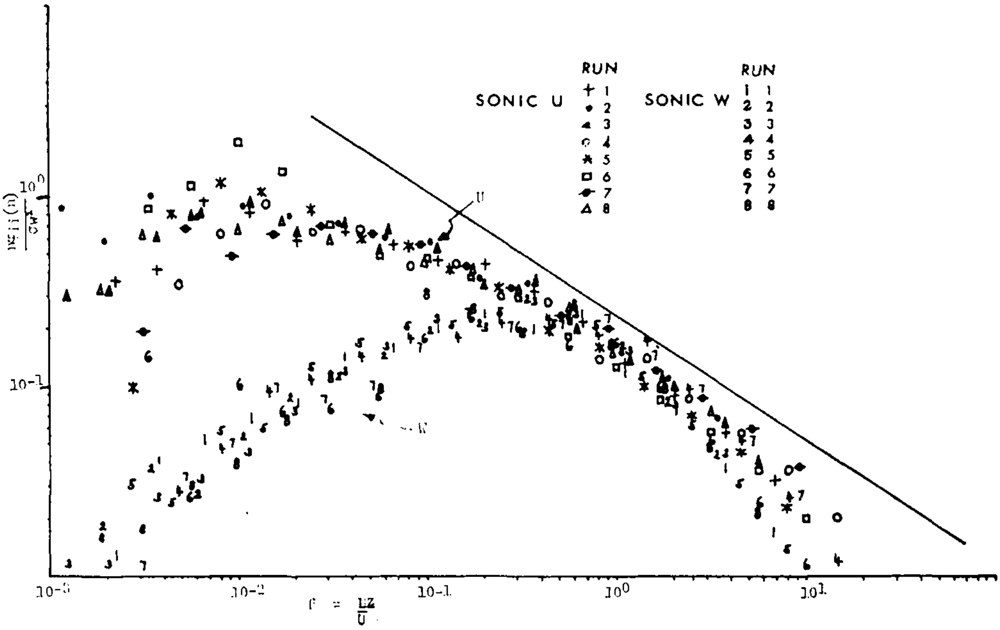

Figure 1. Normalized, logarithmic u and w spectra.

3. Spectra and cospectra

Spectra ϕ_{11} of the component u in the direction of the mean horizontal wind U, and ϕ_{33} of the vertical component w, are presented in Fig. 1, spectra ϕ_{22} of the transverse component v are presented in Fig. 2. The abscissa of these diagrams is the ' natural ' non-dimensional frequency f defined as $f = nz/U$, where n is frequency in hertz, z the elevation of the point of observation and U is the mean wind speed. The vertical co-ordinate is normalized by the standard deviation of vertical velocity w. It is customary, and in a way more defensible, to normalize the spectral values against $u_*{}^2$ rather than $\sigma_w{}^2$. However, it is our experience that measured values of $u_*{}^2$, taken as an

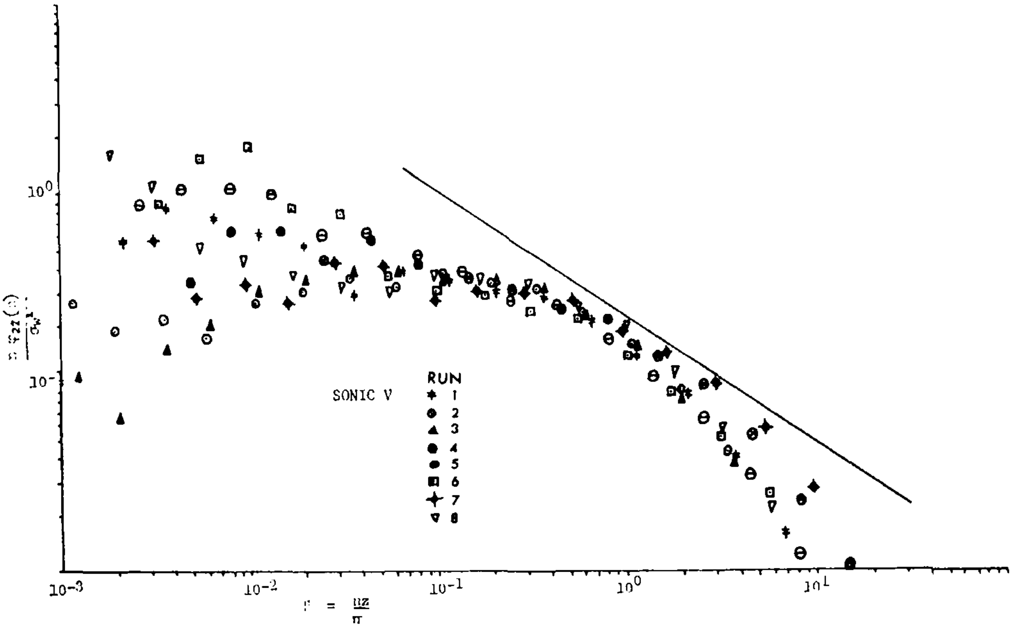

Figure 2. Normalized, logarithmic v spectra.

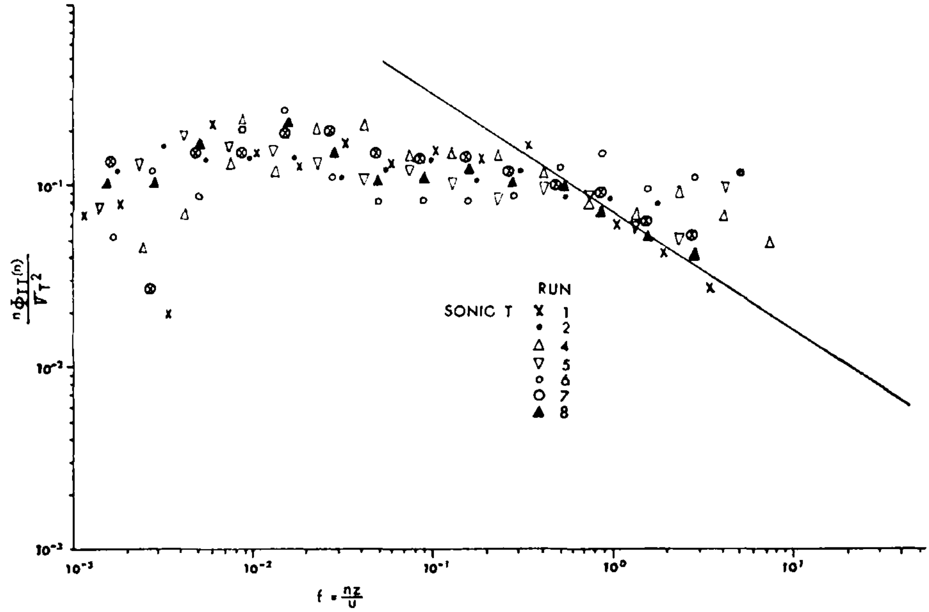

Figure 3. Normalized, logarithmic T spectra.

integral of the uw cospectrum, are subject to much larger statistical fluctuations than are values of $\sigma_w{}^2$. This is perhaps to be expected since spectral values of uw can in principle have either sign, while those of w^2 are positive-definite. Further, the uw spectrum has greater relative contributions from low frequencies than has the w'^2 spectrum and so is subject to larger statistical uncertainties for this reason. The straight line drawn on each log-log plot has a slope of $-2/3$, (corresponding to the Kolmogoroff $-5/3$ law).

Fig. 3 gives normalized temperature spectra. In a similar manner cospectra of momentum flux ϕ_{13} and of heat-flux ϕ_{Tw} are presented in Fig. 4 and Fig. 5. In these cases graphs are normalized by $u_*{}^2$ and $\overline{Tw}$.

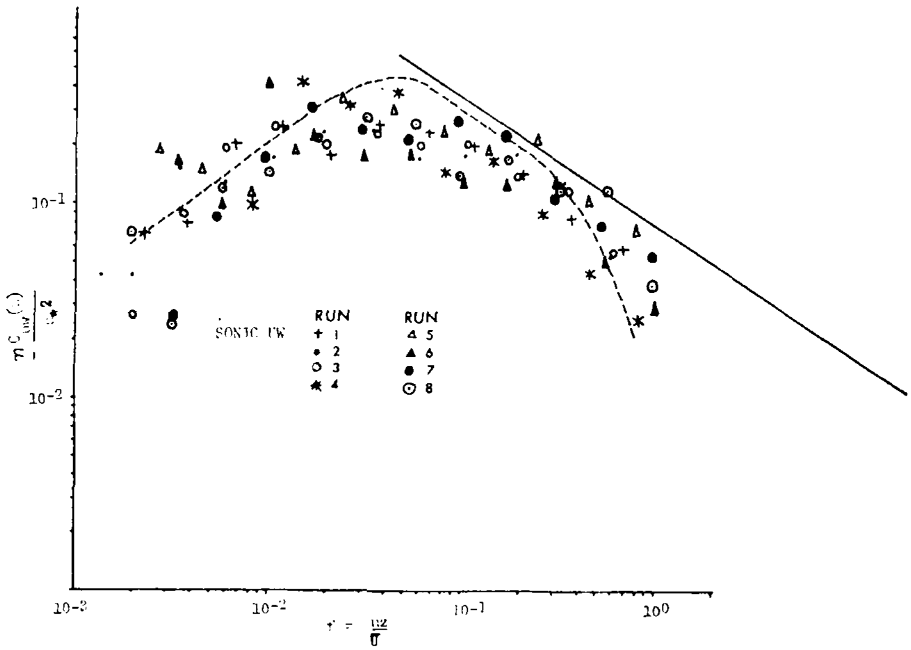

Figure 4. Normalized, logarithmic uw cospectra.

4. Discussion

(i) *Universal shape of w spectra*

From Fig. 1 there seems to be no question but that there is a similarity law which is applicable to the vertical velocity spectra.

The peak of the w spectra appears at $f = 2{\cdot}5 \times 10^{-1}$. Below this region one might fit a spectral power law with slope about $f^{0{\cdot}8}$. Above this peak one might fit a $-2/3$ power, but w spectra seem to be steeper than $-2/3$ slope in the region above about $f = 2$.

Since sensor response limitations become significant by $f = 10$, one must be cautious in drawing conclusions. Nevertheless above $f = 1$ the tendency seems to be for the w spectra to have less energy than those of the u component. Since, for isotropic turbulence,

$$\phi_{33} = \phi_{22} = -\frac{f^2}{2}\frac{d}{df}\left(\frac{\phi_{11}}{f}\right) \qquad (1)$$

this means that ϕ_{33} is significantly lower than the values required for isotropy. This agrees with the conclusion of Weiler and Burling (1967) and Smith (1967), but differs from that of Busch and Panofsky (1968).

(ii) *Spectra of u, and v*

The horizontal velocity component spectra both seem to have a similar form for most of the frequency range. However, it appears that u components follow a $-5/3$ power relation for about half a decade farther in the high frequency region than do v and w. The question as to how much of this may be caused by instrumental limitations will have to await close study of data obtained with instruments having better spacial resolution. (Such a study, using hot wire anemometers, is under way at this laboratory).

At low frequencies ϕ_{22} drops below the $-5/3$ law at a larger value of f than does ϕ_{11} – as might be expected from the nature of Eq. (1).

As has been pointed out before (Lumley and Panofsky 1964) the low frequency part of ϕ_{22} does not follow a similarity law. The ϕ_{11} spectrum is also scattered in the low frequency region, and deviates from the $-5/3$ law below $f = 3 \times 10^{-2}$.

(iii) *Spectra of T*

As is shown in Table 1, the largest temperature fluctuation was of the order of 0·2°C. With this acoustic thermometer it is not to be expected that there will be a wide range of frequency where the signal is above noise level. Unlike the component of velocity, the observed temperature

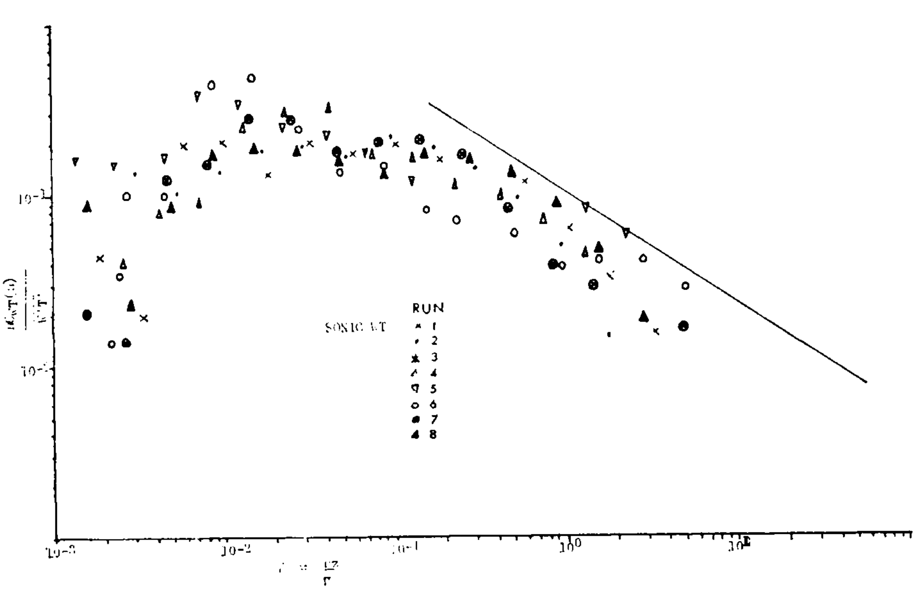

Figure 5. Normalized, logarithmic wT cospectra.

spectra do not follow the $-5/3$ power law below $f = 1$; rather they appear to have the characteristics of white noise in this range. At low frequencies there is an indication that the temperature spectra drop rather abruptly at about the frequency at which the w spectra becomes insignificant, near $f = 3 \times 10^{-3}$.

(*iv*) *Cospectra of u and w*

Cospectra normalized by u_*^2 are plotted in Fig. 4. The curve is that of Panofsky and Mares 1968). One can see some universal form of cospectra emerging, although our data indicates a rather flatter peak and higher values at low frequencies. Our peak is near 3×10^{-2} and at high frequency the spectra seem roughly to follow a $-5/3$ law. The values at low frequency show much more scatter than do those at high frequency but one might be able to fit a line with slope of $+0{\cdot}8$. The most important fact in regard to this diagram is that meaningful integrals of cospectra, i.e. the stress, can and should be computed from fluctuations of u and w which cover the range from 2×10^{-3} to 1 in terms of natural frequency f.

(*v*) *Cospectra w and T*

Cospectra of w and T, which describe heat transport, are plotted in Fig. 5. Those are quite similar to the cospectra of u and w. (The noise problem, which distorts the temperature spectrum of Fig. 3, is unimportant in the cospectrum since the noise is incoherent.) If there is any systematic difference, this study does not show it.

While this composite diagram does not make it very clear, the variability of cospectral statistics is much more pronounced for heat-flux in the low frequency range than is that of the momentum flux. This means that integrated values of heat-flux tend to be much less reliable than those for momentum flux, as was pointed out by Businger, Miyake, Dyer and Bradley (1967).

Acknowledgments

We would like to acknowledge the effort of all students in the Air-Sea Interaction programme in the gathering and processing of field data. The contributions of E. Jerome, M. Donelan, G. McBean deserve special mention. This research is supported by Office of Naval Research contract NR 083-207, Meteorological Branch of the Canadian Department of Transport, and the National Research Council of Canada.

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