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Warranty and discrete preventive maintenance

C.S. Kim[a,*], I. Djamaludin[b], D.N.P. Murthy[b]

[a]*Department of Industrial Engineering, Sangji University, Wonju 220-702, South Korea*
[b]*Department of Mechanical Engineering, The University of Queensland, Australia*

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Abstract

Preventive maintenance actions over the warranty period have an impact on the warranty servicing cost to the manufacturer and the cost to the buyer of fixing failures over the life of the product after the warranty expires. However, preventive maintenance costs money and is worthwhile only when these costs exceed the reduction in other costs. The paper deals with a model to determine when preventive maintenance actions (which rejuvenate the unit) carried out at discrete time instants over the warranty period are worthwhile. The cost of preventive maintenance is borne by the buyer.

Keywords: Warranty; Discrete preventive maintenance; Virtual age; Age reduction

1. Introduction

All products are unreliable in the sense that they eventually fail. An item failure can occur early in its life due to manufacturing defects or late in its life due to degradation of the item. The degradation is dependent on age and usage. Most products are sold with a warranty that offers protection to buyers against early failures over the warranty period. The warranty period offered has been progressively getting longer. For example, the warranty period for cars was 3 months in the early thirties and this changed to one year in the sixties and currently it varies from three to five years. With extended warranties, an item is covered for a significant part of its useful life. This implies that failures due to degradation can occur within the warranty period. The degradation of item can be controlled by preventive maintenance and this reduces the likelihood of failures. This implies that preventive maintenance becomes important when warranty periods are long.

Offering warranty implies additional costs to the manufacturer. This is the cost of repairing item failures (through corrective maintenance) over the warranty period. Preventive maintenance during the warranty period can reduce this cost. Since the buyer pays nothing for repairs during the warranty period, there is no incentive for him/her to invest any effort into preventive maintenance. It is worthwhile for the manufacturer to carry out preventive maintenance only if the reduction in the warranty servicing cost is greater than the extra cost incurred with preventive maintenance.

However, from the buyer's perspective, investment in preventive maintenance during the warranty period and after the warranty has expired can have a significant impact on the maintenance cost after the warranty has expired which is borne by the buyer. As a result, buyer's preventive maintenance actions (during the warranty period and afterwards) need to be determined in the lifecycle context.

In this paper we develop a framework to study preventive maintenance for items sold with warranty from both buyer and manufacturer perspectives. Such a framework allows one to build alternate models to determine optimal preventive maintenance strategies. The outline of the paper is as follows. In Section 2, we give a brief overview of product warranty and maintenance so as to set the background for the main contribution of the paper. Following this, we develop a framework to study preventive maintenance for items sold with warranty in Section 3. In Section 4, we carry out a review of the literature dealing with warranty and maintenance against this framework. Section 5 deals with the new model formulation and its

* Corresponding author. Fax: +82-33-743-1115.
E-mail address: dowoo@mail.sangji.ac.kr (C.S. Kim).

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analysis. Finally, we conclude with some discussions of topics for future research.

2. Warranty and maintenance

2.1. Product warranty

A warranty is a contract between buyer and manufacturer that becomes effective on the sale of an item. The purpose of a warranty is basically to establish liability in the event of a premature failure of an item, where 'failure' is meant as the inability of the item to perform its intended function. The contract specifies the promised performance and if this is not met the means for the buyer to be compensated. The contract also specifies the buyer's responsibilities with regards to due care and operation of the purchased item.

There are many aspects of product warranty and these are covered in Blischke and Murthy (1996) [1]. Murthy and Djamaludin [2] reviewed the warranty literature over period 1990–2001. One aspect of particular interest to manufacturer is warranty cost-the cost associated with the servicing of claims occurring under warranty. This issue has received a lot of attention and further details can be found in Blischke and Murthy (1994) [3].

2.2. Maintenance

Maintenance can be defined as actions to (i) control the deterioration process leading to failure of a system and (ii) restore the system to its operational state through corrective actions after a failure. The former is called 'preventive' maintenance and the latter 'corrective' maintenance.

Corrective maintenance (CM) actions are unscheduled actions intended to restore a system from a failed state to a working state. This involves either repair or replacement of failed components. In contrast, preventive maintenance actions are scheduled actions carried out to either reduce the likelihood of a failure or prolong the life of the component.

In general, preventive maintenance is carried out at discrete time instants. In the case where they are done fairly frequently (the mean time between maintenance actions $\ll$ the life of the item), then they can be approximated as occurring continuously over time. This results in the modelling and analysis becoming simpler.

Several review papers on maintenance have appeared over the last 30 years. These include Pierskalla and Voelker (1976) [4], McCall (1965) [5], Sherif and Smith (1976) [6], Jardine and Buzzacot (1985) [7], Thomas (1986) [8], Gits (1992) [9], Pintelton and Gilders (1992) [10], Dekker (1996) [11] and Scarf (1997) [12], Valdez-Flores and Feldman (1989), [13]. Cho and Parlar (1991) [14] and Dekker et al (1997) [15] deal with the maintenance of multi-component systems. These contain references to the large number of papers and books dealing with maintenance.

3. Framework for the study of warranty and maintenance

For the manufacturer the time interval of interest is the warranty period. In the case of non-renewing warranty, the warranty period is fixed. In the case of renewing warranty, the warranty period is a random variable. The variable of interest is the warranty cost incurred by the manufacturer over the warranty period.

For the buyer, the time interval of interest is the period from the instant an item is purchased to the instant when it is disposed or replaced. This includes the warranty period and the post-warranty period. The cost of rectification over the warranty period depends on the type of warranty. It can vary from no cost (in the case of free replacement warranty) to cost sharing (in the case of pro rata warranty). The cost of rectification during the post-warranty period is borne completely by the buyer. The variable of interest to the buyer is the cost of maintaining an item over its useful life.

Preventive maintenance actions are carried out either to reduce the likelihood of a failure and/or to prolong the life of an item. Preventive maintenance can be perfect (restoring the item to 'good-as-new') or imperfect (restoring the item to a condition that is between as 'good-as new' and as 'bad-as-old'). Corrective maintenance can be either minimal (repairing to back-as-old), imperfect or perfect. See Lindqvist [16] for more on modelling of repairable systems.

Preventive maintenance over the warranty period has an impact on the warranty servicing cost. It is worthwhile for the manufacturer to carry out this maintenance only if the reduction in the warranty cost exceeds the cost of preventive maintenance. From the buyer's perspective, a myopic buyer might decide not to invest in any preventive maintenance over the warranty period since the manufacturer rectifies all failures over the warranty period at no cost to the buyer. Investing in maintenance is viewed as an unnecessary cost and hard to justify. However, from a life cycle perspective the total life cycle cost to the buyer is influenced by maintenance actions during the warranty period and the post warranty period. This implies that the buyer needs to evaluate the cost under different scenarios for preventive maintenance actions. This raises several interesting questions and include the following:

1. Should preventive maintenance be used during the warranty period?
2. If so, what should be the optimal maintenance effort? Should the buyer or the manufacturer pay for this or should it be shared?
3. What level of maintenance should the buyer use during the post warranty period?

A proper evaluation of alternate options requires realistic models and analytical tools.

Another complicating factor is the information aspect. This relates to various issues such as the state of item, type of distribution function appropriate for modelling failures, parameters of the distribution function, etc. Two extreme situations are complete information and no information. Often, the information available to manufacturer and buyer is somewhere in between these two extremes and can differ. Quality variations (so that all items are not statistically similar) add yet another dimension to the complexity.

As such, effective study of preventive maintenance for products sold under warranty requires a framework that incorporates the factors discussed above. Many different models can be built depending on the number of factors included and the nature of their characterization.

3.1. A brief review of the literature linking warranty and maintenance

A brief chronological review of models dealing with warranty and preventive maintenance is as follows:

- Ritchken and Fuh (1986) [17]: PM (involving periodic replacement policy) after the expiry of warranty and infinite time horizon.
- Chun and Lee (1992) [18]: Periodic PM (which rejuvenate the unit) during and after the expiry of warranty; CM involves minimal repair and infinite time horizon.
- Chun (1992)[19]: Deals with a model similar to that in Ref. [18] but the focus is on the warranty cost to the manufacturer as opposed to the buyer.
- Jack and Dagpunar (1994) [20]: Show that the periodic PM policy in Ref. [19] is not optimal and derive the optimal policy.
- Dagpunar and Jack (1994) [21]: Deals with a model similar to that in Ref. [20].
- Sahin and Polatoglu (1996) [22]: PM actions after the expiry of warranty and CM actions involving minimal repair with the buyer paying the costs of CM actions.
- Sahin and Polatoglu (1998) [23]: Deals with PM actions and extended warranties.
- Monga and Zuo (1998) [24]: Deals with PM actions at the design stage taking into account product development and burn-in.
- Djamaludin, et al. (2001) [25]): Develop a framework to study preventive maintenance actions and propose a new model involving continuous PM actions.

Finally, extended warranties can be viewed as maintenance service contracts. Padmanabhan (1996) [26] and Murthy and Padmanabhan [27] deal with extended warranties and Murthy and Ashgarizadeh [28,29] deal with maintenance service contracts.

4. New model formulation

The following notation is used in this section.

L	life of product
W	warranty period
m	preventive maintenance level (decision variable) $[0 \le m \le M]$ $[m = 0$ implies no preventive maintenance$]$
M	upper limit on maintenance level
$F_0(t)$	failure distribution function with no preventive maintenance
$f_0(t)$	failure density function with no preventive maintenance
$r_0(t)$	ROCOF with no preventive maintenance
$r_m(t)$	ROCOF with preventive maintenance
C_R	cost of each repair
$C_m(t)$	maintenance cost with maintenance level m
β	shape parameter for Weibull distribution
θ_0	scale parameter for Weibull distribution
Δ	period for preventive maintenance actions
$\delta(m)$	age reduction (rejuvenation) with maintenance level m
$v(t)$	virtual age of the item at time t
C_{BX}	buyer's expected lifecycle cost under Option X
C_{MX}	expected warranty cost per unit to manufacturer under Option X
$N_X(W,L)$	expected number of failures over $[W,L)$ under Option X
$X :=$	A: no preventive maintenance, B: preventive maintenance over $[0,L)$, C: preventive maintenance only during $[W,L)$

4.1. Product warranty and item life

The product is repairable and sold with a non-renewing free replacement warranty policy with a warranty period W. All failures in the warranty period $[0,W)$ are rectified by the manufacturer at no cost to the buyer. The product has a useful life L and the cost of rectifying failures over the interval $[W,L)$ is borne by the buyer.

4.2. Item failures

Since the product is repairable, any failure can be rectified through repair. We confine our attention to rectification through minimal repair. Under minimal repair the failure rate after repair is nearly the same as that just before failure. We assume that the time to rectify a failure is small in relation to the mean time between failures and that it can be ignored.

We first consider the case with preventive maintenance [Option A]. Let $F_0(t)[f_0(t)]$ denote the product failure distribution [density] function. From Barlow and Hunter

(1960) [30] the rate of occurrence of failures (ROCOF) is given by the failure rate function $r_0(t) = f_0(t)/[1 - F_0(t)]$. In other words, failures over time occur according to a non-homogeneous Poisson process with intensity function given by the failure rate function.

4.3. Discrete preventive maintenance actions

Preventive maintenance is carried out at discrete time instants $\tau_1, \tau_2, \cdots, \tau_j, \cdots$ and $\tau_0 = 0$. The effect of preventive maintenance is that it results in a rejuvenation of the item so that it effectively reduces the age of the item. The reduction in the age depends on the maintenance effort (m) used. The maintenance effort is constrained so that $0 \le m \le M$. $m = 0$ corresponds to no preventive maintenance and M is the upper limit of maintenance effort. Larger value of m corresponds to greater maintenance effort. Let ν_{j-1} denote the virtual age of the item after the $(j-1)^{\text{st}}$, $j \ge 2$, preventive maintenance action. The concept of virtual age was first proposed by Kijima [31] and has been used by Sahin and Polatoglu [22] in the context of preventive maintenance and extended warranty. The virtual age before the j^{th} preventive maintenance is given by $\nu_{j-1} + (\tau_j - \tau_{j-1})$, with $\nu_0 = 0$. If maintenance effort level m is used, then the virtual age after the j^{th} preventive maintenance is given by

$$v_j = v_{j-1} + \delta(m)(\tau_j - \tau_{j-1}) \tag{1}$$

with $\delta(m) \in [0,1]$. $\delta(m)$ is a decreasing function of m with $\delta(0) = 1$ and $\delta(M) = 0$. This implies that as the preventive maintenance effort is increased, the effect of aging is reduced. With $m = M$ at every preventive maintenance action, then the item is restored back to as good as new after each preventive maintenance action. If $m = 0$, then $\nu_j = \tau_j$, $j \ge 1$.

We assume that the level of preventive maintenance effort used is the same throughout the life. As a result, with preventive maintenance m, the item's virtual age at time t is given by

$$v(t) = v_{j-1} + t - \tau_{j-1},\ \tau_{j-1} \le t < \tau_j,\ \ j = 1, 2, \cdots \tag{2}$$

Since failures are repaired minimally and the repair times are negligible, the rate of occurrence of failures (ROCOF) is given by

$$r[v(t)] = r(v_{j-1} + t - \tau_{j-1}),\ \tau_{j-1} \le t < \tau_j,\ j = 1, 2, \cdots \tag{3}$$

In other words, failures over time occur according to a non-stationary Poisson process with intensity function given by the failure rate for the virtual age.

We consider the following three options:

- *Option A*: no preventive maintenance action over the life of the item
- *Option B*: periodic preventive maintenance over $[0, L)$ with $\tau_j = j\Delta, j \ge 1$.
- *Option C*: no preventive maintenance over the warranty period and discrete preventive maintenance over $[W, L)$. As a result, $\tau_j = W + j\Delta, j \ge 1$.

Let n_1, n_2 and n_3 denote the number of preventive maintenance actions over $[0, W)$, $[0, L)$ and $[W, L)$ respectively. Note that these are all zero for Option A and $n_1 = 0$ for Option C. Let $r_m(t)$ denote the ROCOF with preventive maintenance effort m. The characterization of this for the three options is as follows:

Option A: $m = 0$ and as a result the ROCOF is given by

$$r_m(t) = r_0(t) \tag{4}$$

Option B: with preventive maintenance level m, the ROCOF is given by

$$\begin{aligned} &r_m(t) = r_0(v_{j-1} + t - \tau_{j-1}),\ \tau_{j-1} \le t < \tau_j, \\ &j = 1, 2, \cdots, n_2\ r_m(t) = r_0(v_{n_2} + t - \tau_{n_2}),\ \tau_{n_2} \le t \le L \end{aligned} \tag{5}$$

with $\tau_j = j\Delta, j \ge 1$. Note that $[r_0(t) - r_m(t)]$ is an increasing function of function of t except at time instants when preventive maintenance is being carried out.

Option C: with preventive maintenance level m, the ROCOF is given by

$$r_m(t) = \begin{cases} r_0(t) & for 0 \le t < \tau_1 \\ [r_0(\tau_1) - r_m(\tau_1)] & \\ \quad + r_0(v_j + t - \tau_j) & for \tau_j \le t < \tau_{j+1}, j = 1, 2, \cdots, (n_3 - 1) \\ [r_0(\tau_1) - r_m(\tau_1)] & \\ \quad + r_0(v_{n_3} + t - \tau_{n_3})\, for \tau_{n_3} \le t \le L & \end{cases} \tag{6}$$

with $\tau_j = W + j\Delta, j \ge 1$.

Fig. 1 shows a plot of $r_m(t)$ for the three different options for a fixed m. For a given t, the failure rate under Option B is less than under Option C which in turn is less than under Option A.

5. Model analysis

Since failures are rectified through minimal repair and the repair times are negligible, the expected number of failures over any interval is given by the integral of

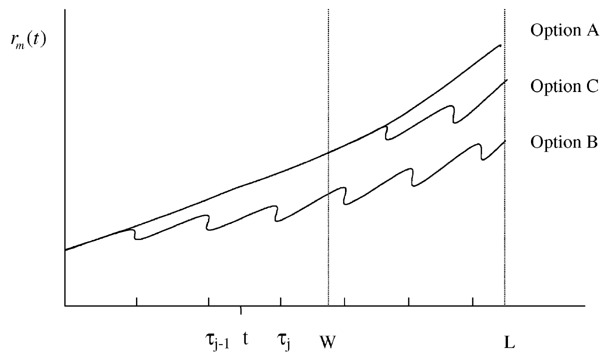

Fig. 1. ROCOF for Options A, B and C.

the ROCOF over the interval. As a result, the expected number of failures over warranty period is given by $\int_0^W r_m(t)\mathrm{d}t$ and over the post warranty period by $\int_W^L r_m(t)\mathrm{d}t$. $r_m(t)$ is given by (4)–(6) for Options A–C, respectively.

5.1. Buyer's expected lifecycle cost

Let C_m denote the cost for a preventive maintenance action with maintenance level m and this increases as m increases. Let C_R denote the cost of minimal repair to fix a failure. The expected costs to the buyer for the three options are as follows.

OPTION A: Let $N_A(W,L)$ denote the expected number of failures during (W,L). Then

$$N_A(W,L) = \int_W^L r_0(t)\mathrm{d}t = R_0(L) - R_0(W) \tag{7}$$

where $R_0(t) = \int_0^t r_0(x)\mathrm{d}x$. The expected cost to the buyer is given by

$$C_\mathrm{BA} = C_R N_A(W,L). \tag{8}$$

OPTION B: Let $N_B(W,L)$ denote the expected number of failures during (W,L). Then

$$N_B(W,L) = \int_W^{\tau_{n_1+1}} r_0(v_{n_1} + t - \tau_{n_1})\mathrm{d}t + \sum_{j=n_1+1}^{n_2-1} \int_{\tau_j}^{\tau_{j+1}} r_0(v_j + t - \tau_j)\mathrm{d}t + \int_{\tau_{n_2}}^{L} r_0(v_{n_2} + t - \tau_{n_2})\mathrm{d}t \tag{9}$$

The expected cost to the buyer is given by

$$C_\mathrm{BB} = C_m n_2 + C_\mathrm{R} N_B(W,L) \tag{10}$$

where n_2 is the largest integer less than (L/Δ).

OPTION C: Let $N_\mathrm{C}(W,L)$ denote the expected number of failures during (W,L). Then

$$N_C(W,L) = \int_W^{\tau_1} r_0(t)\mathrm{d}t + \sum_{j=1}^{n_3-1} \int_{\tau_j}^{\tau_{j+1}} [r_0(\tau_1) - r_\mathrm{m}(\tau_1) + r_0(v_\mathrm{j} + t - \tau_j)]\mathrm{d}t + \int_{\tau_{n_3}}^{L} [r_0(\tau_1) - r_\mathrm{m}(\tau_1) + r_0(v_{\mathrm{n}_3} + t - \tau_{n_3})]\mathrm{d}t \tag{11}$$

where $\tau_j = W + j\Delta$, $j = 1,2,\cdots n_3$ and n_3 is the largest integer less than $(L-W)/\Delta$. The expected cost to the buyer is given by

$$C_\mathrm{BC} = C_\mathrm{m} n_3 + C_\mathrm{R} N_\mathrm{C}(W,L). \tag{12}$$

The optimal preventive maintenance level m^* for Options B and C can be obtained by minimizing the expected cost. When m is modeled as a discrete variable, then m^* is obtained by evaluating the costs for the different values of m and then carrying out a relative comparison.

5.2. Manufacturer's expected warranty serving cost

The cost to the manufacturer is the warranty serving cost (the cost of rectifying failures under warranty). For Options A and C, the expected warranty servicing cost is given by

$$C_\mathrm{MA} = C_\mathrm{MC} = C_\mathrm{R} \int_0^W r_0(t)\mathrm{d}t. \tag{13}$$

For Option B, the expected warranty serving cost is given by

$$C_\mathrm{MB} = C_\mathrm{R} \int_0^{\tau_1} r_0(t)\mathrm{d}t + C_\mathrm{R} \sum_{j=1}^{n_1-1} \int_{\tau_j}^{\tau_{j+1}} r_0(v_j + t - \tau_j)\mathrm{d}t + C_R \int_{\tau_{n_1}}^{W} r_0(v_{n_1} + t - \tau_{n_1})\mathrm{d}t. \tag{14}$$

Since $r_\mathrm{m}(t) \le r_0(t)$, the expected warranty serving cost to the manufacturer under Option B is less than that under Options A or C. Hence, the manufacturer would always prefer to have the buyer carry out preventive maintenance during the warranty period.

6. Numerical example

We assume that the failure distribution $F_0(t)$ is given by the Weibull distribution with shape parameter β and scale parameter θ_0. As a result, the ROCOF with no preventive maintenance action is given by a Weibull intensity function [32]. Let $\delta(m) = (1+m)\mathrm{e}^{-m}$, $m \ge 0$ and integer, then $\delta(m)$ is a decreasing function of m. We assume five discrete levels of preventive maintenance and Table 1 shows $\delta(m)$ and C_m for the five levels.

Let $W = 2$ (years), $L = 8$ (years),$\beta = 2$ and $\theta_0 = 2$. This implies that the mean time to first failure is 1.8 years. We assume that maintenance is carried out at discrete time τ_j, with $\Delta = 0.33$ year between preventive maintenance actions so that maintenance is carried out every 4 months.

6.1. Buyer's expected lifecycle cost

Table 2 shows the expected lifecycle costs to the buyer under Options A, B and C as C_R varies from \$20 to \$500 in increments of \$20.

Table 1
Maintenance level m, improvement $\delta(m)$ and cost C_m

Maintenance level m	$\delta(m)$	C_m (\$)
0	1.00	0
1	0.74	10
2	0.41	30
3	0.20	60
4	0.09	100
5	0.04	160

Table 2
Buyer's expected lifecycle costs for Options A, B and C ($\beta = 2$ and C_R varying)

C_R	Option A	Option B					Option C				
	$M = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 1$	$m = 2$	$M = 3$	$m = 4$	$m = 5$
20	300.00	463.37	847.74	1507.75	2436.56	3861.72	410.12	682.92	1168.22	1859.77	2926.25
40	600.00	686.74	975.48	1575.51	2473.12	3883.45	640.25	825.84	1256.44	1919.55	2972.49
60	900.00	910.11	1103.23	1643.26	2509.67	3905.17	870.37	968.76	1344.66	1979.32	3018.74
80	1200.00	1133.48	1230.97	1711.01	2546.23	3926.90	1100.49	1111.69	1432.88	2039.09	3064.99
100	1500.00	1356.85	1358.71	1786.70	2582.79	3948.62	1330.61	1254.61	1521.10	2098.86	3111.23
120	1800.00	1580.22	1486.45	1846.52	2619.35	3970.34	1560.74	1397.53	1609.32	2158.64	3157.48
140	2100.00	1803.59	1614.19	1925.38	2655.90	3992.07	1790.86	1540.45	1697.53	2218.41	3203.73
160	2400.00	2026.96	1741.93	1994.72	2692.46	4013.79	2020.98	1683.37	1785.75	2278.18	3249.97
180	2700.00	2250.33	1869.68	2064.05	2729.02	4035.52	2251.11	1826.29	1873.97	2337.96	3296.22
200	3000.00	2473.70	1997.42	2133.39	2765.58	4057.24	2481.23	1969.22	1962.19	2397.73	3342.46
220	3300.00	2697.07	2125.16	2202.73	2802.13	4078.96	2711.35	2112.14	2050.41	2457.50	3388.71
240	3600.00	2920.44	2252.90	2272.07	2838.69	4100.69	2941.47	2255.06	2138.63	2517.27	3434.96
260	3900.00	3143.81	2380.64	2341.41	2875.25	4122.41	3171.60	2397.98	2226.85	2577.05	3481.20
280	4200.00	3367.18	2508.38	2410.75	2911.81	4144.14	3401.72	2540.90	2315.07	2636.82	3527.45
300	4500.00	3590.55	2636.13	2480.09	2948.37	4165.86	3631.84	2683.82	2403.29	2696.59	3573.70
320	4800.00	3813.92	2763.87	2549.43	2984.92	4187.58	3861.97	2826.74	2491.51	2756.37	3619.94
340	5100.00	4037.29	2891.61	2618.77	3021.48	4209.31	4092.09	2969.67	2579.73	2816.14	3666.19
360	5400.00	4260.66	3019.35	2688.11	3058.04	4231.03	4322.21	3112.59	2667.95	2875.91	3712.44
380	5700.00	4484.03	3147.09	2757.45	3094.60	4252.76	4552.34	3255.51	2756.16	2935.69	3758.68
400	6000.00	4707.40	3274.83	2826.79	3131.15	4274.48	4782.46	3398.43	2844.38	2995.46	3804.93
420	6300.00	4930.77	3402.58	2896.13	3167.71	4296.20	5012.58	3541.35	2932.60	3055.23	3851.18
440	6600.00	5154.14	3530.32	2965.47	3204.27	4317.93	5242.70	3684.27	3020.82	3115.00	3897.42
460	6900.00	5377.51	3658.06	3034.81	3240.83	4339.65	5472.83	3827.20	3109.04	3174.78	3943.67
480	7200.00	5600.88	3785.80	3104.15	3277.38	4361.38	5702.95	3970.12	3197.26	3234.55	3989.91
500	7500.00	5824.25	3913.54	3173.49	3313.94	4383.10	5933.07	4113.04	3285.48	3294.32	4036.16

Table 3
Buyer's expected lifecycle costs for Options A, B and C ($\beta = 3$ and C_R varying)

C_R	Option A	Option B					Option C				
	$M = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
20	1260.00	936.95	946.58	1503.14	2418.25	3846.54	899.12	813.52	1203.99	1885.84	2957.21
40	2520.00	1633.90	1173.15	1566.27	2436.50	3853.07	1618.23	1087.04	1327.98	1971.68	3034.42
60	3780.00	2330.85	1399.73	1629.41	2454.75	3859.61	2337.35	1360.56	1451.97	2057.52	3111.63
80	5040.00	3027.80	1626.30	1692.54	2473.00	3866.14	3056.46	1634.08	1575.96	2143.36	3188.84
100	6300.00	3724.75	1852.88	1757.26	2491.26	3872.68	3775.58	1907.60	1699.95	2229.21	3266.05
120	7560.00	4421.70	2079.45	1818.81	2509.51	3879.21	4494.70	2181.12	1823.94	2315.05	3343.25
140	8820.00	5118.65	2306.03	1884.16	2527.76	3885.75	5213.81	2454.65	1947.93	2400.89	3420.46
160	10080.00	5815.60	2532.61	1947.61	2546.01	3892.29	5932.93	2728.17	2071.92	2486.73	3497.67
180	11340.00	6512.55	2759.18	2011.06	2564.26	3898.82	6652.04	3001.69	2195.91	2572.57	3574.88
200	12600.00	7209.50	2985.76	2074.51	2582.51	3905.36	7371.16	3275.21	2319.90	2658.41	3652.09
220	13860.00	7906.45	3212.33	2137.96	2600.76	3911.89	8090.28	3548.73	2443.89	2744.25	3729.30
240	15120.00	8603.39	3438.91	2201.41	2619.01	3918.43	8809.39	3822.25	2567.88	2830.09	3806.51
260	16380.00	9300.34	3665.48	2264.86	2637.27	3924.97	9528.51	4095.77	2691.87	2915.94	3883.72
280	17640.00	9997.29	3892.06	2328.31	2655.52	3931.50	10247.63	4369.29	2815.86	3001.78	3960.93
300	18900.00	10694.24	4118.64	2391.77	2673.77	3938.04	10966.74	4642.81	2939.85	3087.62	4038.14
320	20160.00	11391.19	4345.21	2455.22	2692.02	3944.57	11685.86	4916.33	3063.84	3173.46	4115.34
340	21420.00	12088.14	4571.79	2518.67	2710.27	3951.11	12404.97	5189.85	3187.83	3259.30	4192.55
360	22680.00	12785.09	4798.36	2582.12	2728.52	3957.64	13124.09	5463.37	3311.82	3345.14	4269.76
380	23940.00	13482.04	5024.94	2645.57	2746.77	3964.18	13843.21	5736.90	3435.81	3430.98	4346.97
400	25200.00	14178.99	5251.51	2709.02	2765.02	3970.72	14562.32	6010.42	3559.80	3516.82	4424.18
420	26460.00	14875.94	5478.09	2772.47	2783.27	3977.25	15281.44	6283.94	3683.79	3602.66	4501.39
440	27720.00	15572.89	5704.67	2835.92	2801.53	3983.79	16000.55	6557.46	3807.78	3688.51	4578.60
460	28980.00	16269.84	5931.24	2899.37	2819.78	3990.32	16719.67	6830.98	3931.77	3774.35	4655.81
480	30240.00	16966.79	6157.82	2962.82	2838.03	3996.86	17438.79	7104.50	4055.76	3860.19	4733.02
500	31500.00	17663.74	6384.39	3026.28	2856.28	4003.39	18157.90	7378.02	4179.74	3946.03	4810.23

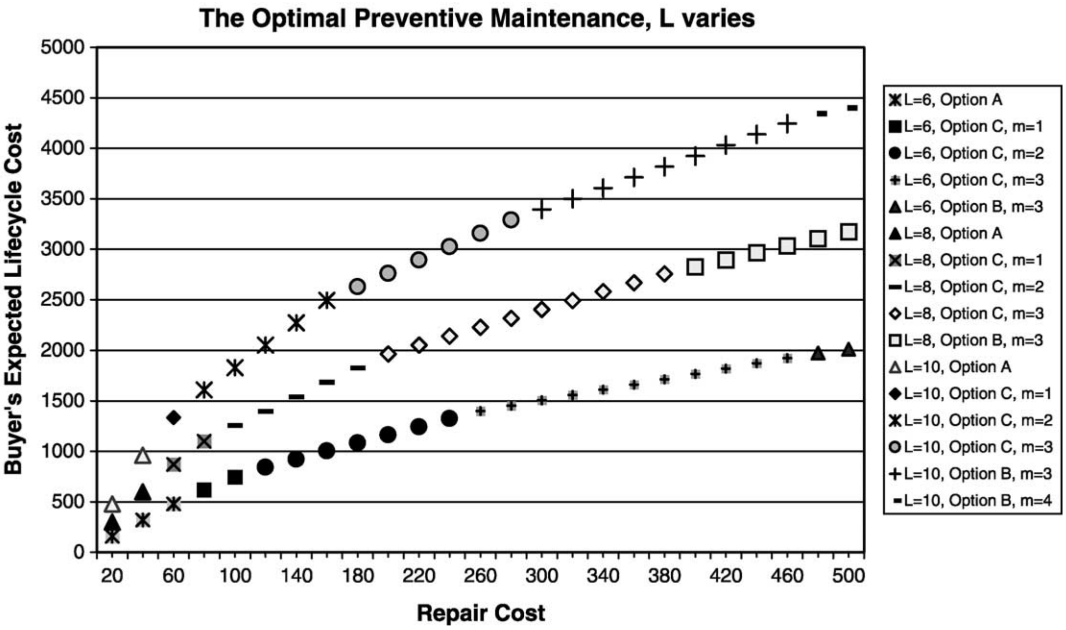

Fig. 2. Effect of L on the optimal preventive maintenance strategy.

The result shows that if the corrective maintenance cost is low ($\leq$ \$40), the optimal strategy is Option A (no preventive maintenance during the life of the item). As this cost increases, the optimal strategy switches to Option C for $60 \leq C_R \leq 380$, that is to carry out preventive maintenance after the warranty period has expired. The optimum preventive maintenance level increases as C_R increases. It starts at level 1 and reaches level 3. For still higher costs ($C_R >$ \$380), the optimal strategy is Option B (carry out preventive maintenance over the whole interval) with maintenance level 3.

The corresponding results for the case when $\beta = 3$ are shown in Table 3.

The results are similar to the previous case. However, Option A is never the optimal strategy. For $C_R \leq 100$ the optimal strategy is Option B and for $C_R \geq 120$ Option C is the optimal strategy. Also to note is that the maintenance levels used are higher.

We now study the effect of the model parameters changing from their nominal values indicated in the beginning of the section. We vary one parameter at a time.

6.1.1. *L varying*

We look at the effect of L (the useful life of the item) on the optimal preventive maintenance strategies. We consider two other values—$L = 6$ and $L = 10$ (years)—and Fig. 2 shows the corresponding results.

The results are similar. The effect of L on the optimal strategy is summarised in Table 4. As can be seen, the values of C_R where the optimal strategy switches from Option A to Option C and, from Option C to Option B decreases as L increases.

6.1.2. *Δ varying*

We vary Δ the time between preventive maintenance actions and consider the following two other values: $\Delta = 0.25$ year (preventive maintenance actions done every 4 months) and $\Delta = 0.5$ year (preventive maintenance actions carried out every 6 months). The results are presented in Fig. 3.

The results indicate that as Δ decreases from 0.33 to 0.25, Option B is never the optimal strategy. For the reverse situation (as Δ increases from 0.33 to 0.5) Option C becomes the optimal strategy for $C_R \geq 120$. Table 5 summarises the results.

6.2. *Manufacturer's expected warranty servicing cost per unit*

For the nominal parameter values for the model indicated in the beginning of Section 7, the expected warranty cost per unit to the manufacturer under the three Options A (or C) and B are given in Table 6 for C_R varying from \$20 to 500.

Table 4
Optimal preventive maintenance strategy as a function of L

L (years)	C_R (\$)	Optimal decision
6	$C_R \leq 60$	Option A
	$80 \leq C_R \leq 460$	Option C
	$C_R \geq 480$	Option B
8	$C_R \leq 40$	Option A
	$60 \leq C_R \leq 380$	Option C
	$C_R \geq 400$	Option B
10	$C_R \leq 40$	Option A
	$60 \leq C_R \leq 280$	Option C
	$C_R \geq 400$	Option B

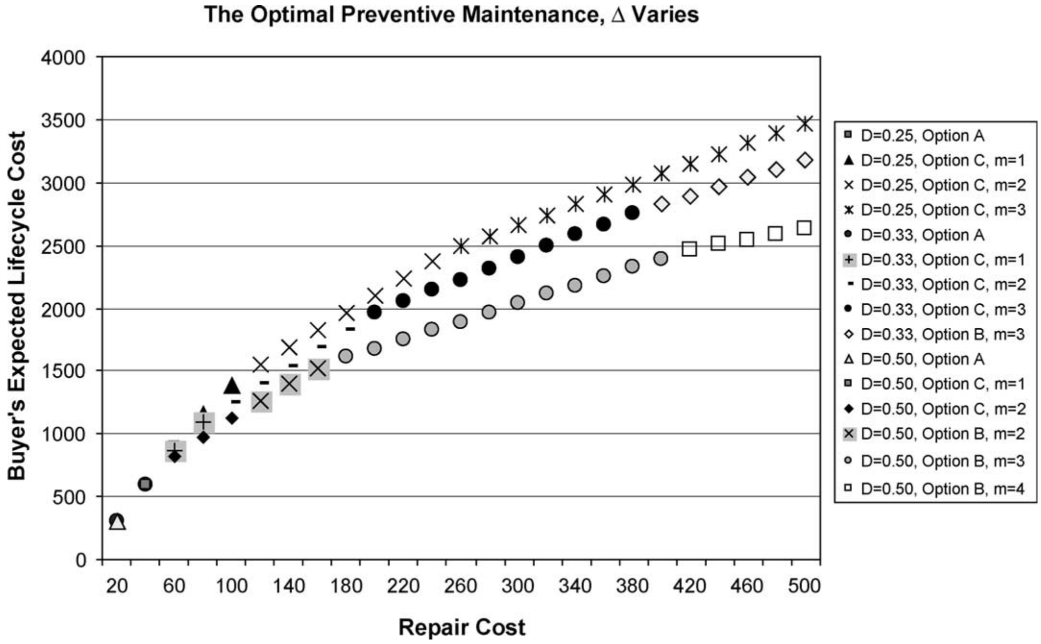

Fig. 3. Effect of Δ on the optimal preventive maintenance strategy. (Note: D in the legend is Δ).

The result shows that as the corrective maintenance cost increases, the expected warranty cost per unit to the manufacturer also increases. On the other hand, maintenance reduces the manufacturer expected warranty cost, and greater preventive maintenance effort on the part of the buyer implies smaller warranty servicing cost.

6.2.1. Comment

The results of Sections 6.1 and 6.2 show the effect of preventive maintenance on the buyer's life-cycle cost and manufacturer's warranty servicing cost. As the buyer's effort on preventive maintenance increases, the manufacturer warranty servicing cost decreases.

For the case $C_R = \$500$ and $\beta = 2$, from Table 2 it is seen that optimal preventive maintenance ($m = 3$) results in lower costs to both the manufacturer and buyer. This is a win–win situation.

If the buyer is myopic and does not invest in preventive maintenance during the warranty period then the manufacturer warranty servicing cost is higher. In this case, the manufacturer might get the buyer to invest in maintenance effort during the warranty period by offering monetary incentive as long as it is less than \$333.69. This is the reduction in the cost to the manufacturer resulting from buyer doing preventive maintenance at level 3 as opposed to no preventive maintenance.

Table 5
Optimal preventive maintenance strategy as a function of δ

δ (year)	C_R (\$)	Optimal Decision
0.25	$C_R \leq \$60$	Option A
	$C_R \geq \$80$	Option C
0.33	$C_R \leq 40$	Option A
	$60 \leq C_R \leq 380$	Option C
	$C_R \geq 400$	Option B
0.5	$C_R \leq 20$	Option A
	$40 \leq C_R \leq 100$	Option C
	$C_R \geq 120$	Option B

Table 6
Manufacturer's expected warranty servicing cost per unit for Options A (or C), and B, ($\beta = 2$ and C_R varying from \$20 to \$500)

C_R	Option A (or C)	Option B				
	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
20	20.00	15.60	10.10	6.65	4.86	4.01
40	40.00	31.19	20.20	13.30	9.72	8.01
60	60.00	46.79	30.30	19.96	14.58	12.02
80	80.00	62.38	40.40	26.61	19.44	16.03
100	100.00	77.98	50.50	33.26	24.30	20.04
120	120.00	93.58	60.60	39.91	29.16	24.04
140	140.00	109.17	70.70	46.57	34.02	28.05
160	160.00	124.77	80.80	53.22	38.88	32.06
180	180.00	140.36	90.90	59.87	43.74	36.06
200	200.00	155.96	101.00	66.52	48.60	40.07
220	220.00	171.56	111.10	73.18	53.46	44.08
240	240.00	187.15	121.20	79.83	58.32	48.09
260	260.00	202.75	131.30	86.48	63.18	52.09
280	280.00	218.34	141.40	93.13	68.03	56.10
300	300.00	233.94	151.50	99.79	72.89	60.11
320	320.00	249.54	161.60	106.44	77.75	64.11
340	340.00	265.13	171.70	113.09	82.61	68.12
360	360.00	280.73	181.80	119.74	87.47	72.13
380	380.00	296.32	191.90	126.40	92.33	76.14
400	400.00	311.92	202.00	133.05	97.19	80.14
420	420.00	327.52	212.10	139.70	102.05	84.15
440	440.00	343.11	222.20	146.35	106.91	88.16
460	460.00	358.71	232.30	153.01	111.77	92.16
480	480.00	374.30	242.40	159.66	116.63	96.17
500	500.00	389.90	252.50	166.31	121.49	100.18

7. Conclusion

In this paper we discussed preventive maintenance which is carried out by the buyer for item sold under warranty and formulated a simple model involving discrete preventive maintenance. We have also discussed for different maintenance level that is carried out during warranty period and after warranty expired as often is the case in real life. The result shows that for large failure costs the optimal result is to carry out different maintenance levels during warranty and after warranty expired.

The model can be extended in several ways and we indicate a few

1. The effect of maintenance often leads to the life of the item being extended. This implies that L increases with m.
2. We have confined our analysis to the free replacement policy and failed item being always repaired minimally. The analysis of other types of warranty policies for example, pro-rata, combination is yet to be carried out.
3. We have not studied the different incentive schemes and the related moral hazard issues. This is a topic for considerable new research.
4. We have assumed that the maintenance level is constant. Often, this is not realistic and the maintenance effort changes with the age of the item—less when it is new and more as it ages.
5. We have confined our attention to discrete preventive maintenance effort. Often, one employs both continuous (see Refs. [33,34]) and discrete (overhaul) preventive maintenance actions. This makes the problem more difficult and also interesting.

Some of these problems are currently under investigation by the authors.

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