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Ion Permeability of a Membrane with Soft Polar Interfaces. 1. The Hydrophobic Layer as the Rate-Determining Step

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We have considered the ion transport across a membrane with extended soft permeable interfaces (polar zones), placed in an aqueous solution, under short-circuit conditions. We have taken into account the existence of fixed charges and dipoles in these membrane interphases. The membrane has been modeled as composed of three layers: an inner hydrophobic layer and two polar zones. Nernst–Planck's equation has been used for describing the ion transport, and Goldman's approximation has been assumed to be valid for the hydrophobic layer. In this paper the limiting case of the internal hydrophobic layer as the rate-controlling step for ion transport has been studied. The influence of the electrolyte concentration, the surface dipole density, and the thickness of the polar zone on the total ion flux and permselectivity has been analyzed. It has been shown that the permselectivity of such a system depends significantly on all these parameters. Particularly, the permselectivity of the same membrane changes with bulk electrolyte concentration. A general expression for cation selectivity has been obtained. The connection of the developed theory with experimental results has been discussed.

1. Introduction

There are two main reasons for a detailed consideration of the ion transport process in the membranes mentioned in the title. Firstly the soft interfaces are a distinctive feature of a large group of membranes, particularly of lipid and biological membranes.^{1–5} Such membranes have two extended loose boundary zones that separate the relatively compact inner hydrocarbon membrane core from the surrounding solutions. In the case of lipid membranes those are the zones of lipid polar heads, whereas in the case of biological membranes they are defined by the cytoskeleton, spectrin layer, glycocalyx, and/or other surface structures.^{2–4} Such a description can also be applied to nonbiological colloid systems, particularly to hydrophilic polymer membranes.^{5,6} The important feature of these zones is that they are filled with water molecules and ions from the solution in addition to the membrane surface molecular structures. Usually, the membrane fixed charges and dipoles are located inside this boundary zone, and in this case the membrane surface dipoles can exert some influence on the electric potential profile of the system.^{7,8}

As is known, the membrane charges and the electric potential profile in adjacent solutions change the ionic permeability of the membranes,^{1,9–16} irrespective of the specific nature of ionic transport. It has usually been assumed that all charges and surface dipoles are located at a single, infinitely thin, boundary surface. This assumption is not valid for many membrane systems, e.g., for lipid and biological membranes. As we have mentioned above, in reality for such membranes fixed charged groups are distributed throughout a region of finite thickness, e.g., in a soft polar interface.^{2,3}

There have been many studies in the last decade concerned with membranes with soft interfaces (see refs 5, 8, and 17 and references therein). However, they have dealt mainly with two problems, namely the influence of the spatial distribution of the surface charge on the electrophoretic behavior of cells, and the interaction between lipid bilayers. The influence of polar zone features on ion transport has not been analyzed yet. Apart from this, ionic and charge interactions in the polar region of a phospholipid layer have been shown to be important

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in determining ionic permeability.¹⁸ In this connection it is worth mentioning here the second reason for a detailed consideration of the ion transport process in membranes with soft polar interfaces. Different membranes having different kinds of lipids usually have similar inner hydrophobic parts which consist of short CH₂ chains, while at the same time their polar zones can be remarkably different.^{1,19} For hydrophilic ions, such as Ca²⁺, Na⁺, and K⁺ the inner hydrophobic layer of the membrane is the main barrier and, hence, the permeability of any hydrophilic ion in different lipid membranes should be expected to be the same. However, this is not true (see for example ref 20). It means that the polar zones of such membranes exert some influence on ionic transport. In addition, the experiments also demonstrate that in other cases it is the polar zones which determine the transport properties of the membrane system.^{21,22}

In the present study we shall analyze the influence of several polar zone parameters on the steady-state ionic transport through a membrane with soft interfaces. The ionic transport will be described in terms of a relatively simple model based on Nernst–Planck's equation for the steady state. Moreover, we shall consider separately the transport controlled by a concentration gradient with zero potential difference (short-circuit conditions) and the transport controlled by an external electric field when the electrolyte concentrations on both sides of the membrane are the same (open-circuit conditions). Our goal, in this study, is to examine the influence of the main parameters of the polar zone upon the permeability, permselectivity, and conductance of a membrane with soft polar interfaces. These parameters are the polar zone thickness L , the surface density of charges σ and dipoles ν , and the polar zone dielectric constant ϵ_p . We shall show that these parameters can influence sufficiently the ion transport across the membrane, and hence they have to be taken into account for a correct interpretation of any experimental data.

Generally, the transport of ions through the system under consideration includes a few steps: (a) passage of ions across the unstirred layers; (b) passage of ions across the dipole interfaces; (c) ion diffusion and migration in the polar zones; and (d) transport through the hydrophobic layer. It is necessary to note that the determination of the exact condition for the steady-state ion flux is not an easy task. Generally, the time-dependent transport problem needs to be solved and analyzed for $t \rightarrow \infty$. However, the numerical solution of the time-dependent equations is rather complicated.²³ The difficulties involved in that numerical solution could shield our main aim of analyzing the influence of the structure of the polar zone on ion transport. So, for the sake of simplicity, in the present study we shall consider only three limiting cases, namely (i) the inner hydrophobic layer as the rate-determining step, (ii) the polar zones as the rate-determining step, and (iii) the dipole interfaces as the rate-determining step. The mathematical criteria for them will also be obtained.

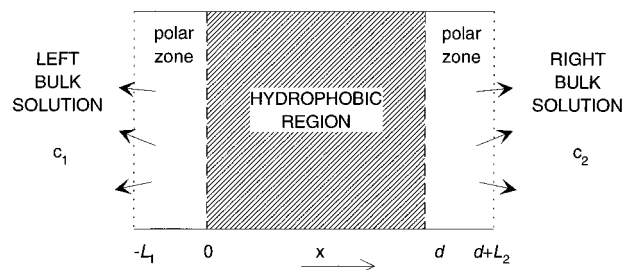


Figure 1. Schematic cross section of the membrane system. The hydrophobic region is shown cross-hatched and the charges and dipoles are assumed to be on the planes at $x = -L_1$ and $x = d + L_2$.

Within this study of the influence of the polar zone structure on ionic transport, we shall pay special attention to the surface dipoles. It is well-known that they create an electric barrier for ions. Apart from this, in the case of nonzero polar zone thickness they have some effect on the electric field distribution in solution^{7,8} and thereby on ion transport.

The real translocation of ions across this kind of membrane is of course a complicated process, too complex to model in a simple way. Nonetheless, it seems appropriate to exhaust the possibilities of a simple model before introducing additional complications. In order to clarify the main characteristics of these membranes, we deliberately have chosen an ideal simple model. However, further work is being carried out considering the nonlinear solution of Poisson–Boltzmann's equation and taking into account the decrease of the dielectric permittivity of water near the membrane surface.

2. Theory

2.1. General Description of the Model. Let us consider a planar, charged inhomogeneous membrane separating two aqueous solutions of a binary electrolyte with concentrations c_1 and c_2 on the left and right side, respectively, of the membrane. The membrane consists of three layers: a central hydrophobic one and two hydrophilic zones. The geometry of the system is illustrated in Figure 1. Let us choose a Cartesian spatial coordinate system such that the y and z axes lie in the membrane plane and the x axis is perpendicular to it. The origin of the coordinate system is on the left boundary of the hydrophobic layer, the thickness of the latter being d . The aqueous solution region extends from $x = -\infty$ to $x = 0$ and from $x = d$ to $x = d + \infty$. The hydrophilic zones extend from $x = -L_1$ to $x = 0$ and from $x = d$ to $x = d + L_2$. Generally, there are solution molecules (i.e., water molecules and ions) inside the membrane; however, their distribution is not homogeneous. Particularly, in the hydrophobic layer their fraction F_s is much smaller than the fraction F_m of membrane molecule groups (i.e., $F_m \gg F_s$). For simplicity we will assume that their contribution to the hydrophobic region electrostatic properties is negligible (i.e. $F_m = 1$ and $F_s = 0$).

Usually the fixed charges and dipoles of the membrane are located inside the hydrophilic zones and are strongly bound to the hydrophobic core. Apart from this, the hydrophilic zones have a number of water molecules and ions from the bath solution. In contrast to the hydrophobic layer, here $F_m \ll F_s$. We will assume that the main contribution to the electrostatic properties of the hydrophilic region is due to the solution molecules (i.e. $F_m = 0$ and $F_s = 1$). Although we shall denote these zones as *polar zones*, strictly speaking, only a few membranes (e.g., lipid membranes) have their polar groups only in these regions. So the hydrophilic zone thickness L is a phenomenological parameter that describes the extent of water and ion penetration into this loose region of the membrane. Bulk solution dielectric properties are assumed to be different from those of polar zones. Let us denote the dielectric permittivity of each region as ϵ for water, ϵ_p for polar zones, and ϵ_h for the hydrophobic part of the membrane.

The same membrane can exhibit different polar zone thicknesses according to its phase state and external conditions.^{1,19}

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For example, it is known that Ca^{2+} and prolepherin²⁴ push out the water molecules from the lipid polar zone and thereby decrease its thickness. In addition, NMR and ESR data suggest that the presence of ions increases the mobility of the lipid head groups²⁶ and, hence, the polar zone thickness. It is obvious also that different membranes have different polar zone thicknesses. In this connection, the question about the influence of the polar zone thickness on the properties of the membrane arises. To our knowledge, direct measurement of L usually involves great experimental difficulties. In some cases it is possible to get the value of L from neutron scattering.²⁵ On the other hand, indirect estimations may also lead to this value. For example, for lipid membranes it was obtained from^{1,2,19} $L = 3\text{--}8$ to $12\text{--}14$ Å,^{25,26} for a thylakoid membrane³ it was obtained $L = 10\text{--}15$ Å, for a lipid membrane with brain gangliosides and cerebroside²⁷ it was obtained $L = 20$ Å, for erythrocytes⁴ it was obtained $L = 35\text{--}55$ Å, and for some cells²⁸ it was obtained $L = 40\text{--}70$ Å.

As we mentioned in the Introduction, our aim is to analyze the influence of the polar zone thickness L on ion transport through the membrane, along the x direction. For simplicity we assume that the membrane fixed charge and dipole densities depend only upon x and not upon z or y . Besides, we shall consider here only the effect of the normal x component of the dipoles. This is because the lateral component contributes much less than the x component of the electric dipole to the electric field.²⁹ Thus, by neglecting the finite size of the dipoles, any membrane dipole distribution within the polar zones can be represented as a set of infinitely thin planes, uniformly covered by point dipoles with only an x component.^{29b} To avoid unnecessary mathematical complexity, we shall analyze here the simplest case when the electric dipoles in each polar zone are located on one plane that is placed on the external boundary of the polar zone. We will consider two limiting cases for the membrane charge distribution, the first being when all charges inside the polar zone are located near a single plane, which coincides with the external boundary of the polar zone. We shall refer to this case as that of *constant charge*, and the total effective fixed charge density of each polar zone can be mathematically represented as

$$\rho_{\text{fix}}(x) = \sigma\delta(x-L) - \nu\delta'(x-L) \quad (1)$$

where σ is the surface charge density, ν is the surface dipole density of the normal dipole component, $\delta(x-L)$ is Dirac's δ function, and $\delta'(x-L)$ is its derivative.

The second kind of charge distribution, hereafter referred to as that of *constant charge density*, corresponds to the case of a uniform distribution of the membrane fixed charges in polar zones. In this case, the effective fixed charge density reads

$$\rho_{\text{fix}}(x) = \Omega - \nu\delta'(x-L) \quad (2)$$

where Ω is a constant volume charge density in the polar zone.

We will study this system under short-circuit conditions, which means equal electric potential on both bulk solutions, and will find the steady-state ion fluxes of the cations j^+ and anions j^- . Classically, the steady-state transport of ions across a medium

is described by the combination of Nernst–Planck's electrodiffusion equation^{9–12}

$$j^\pm = -D^\pm(x) \left[\frac{dc^\pm(x)}{dx} + z^\pm c^\pm(x) \frac{d\psi(x)}{dx} \right] \quad (3)$$

and Poisson's equation (see also the Appendix)

$$\frac{d}{dx} \left(\epsilon \frac{d\psi}{dx} \right) = -4\pi\beta\rho \quad (4a)$$

$$\rho \equiv \rho_{\text{fix}} + F(z^+c^+ + z^-c^-) \quad (4b)$$

Here $D^\pm(x)$, $c^\pm(x)$, and z^\pm denote the diffusion coefficient, local molar concentration, and valency of the positive and negative ions of the solution and F is Faraday's constant. The dimensionless electric potential $\psi(x)$ is connected with the real electric potential $\phi(x)$ in the usual way $\psi(x) = \beta\phi(x)$ and $\beta^{-1} = k_B T/e = 25.7$ mV (see also the Appendix). k_B is Boltzmann's constant, and e is the elementary charge. Generally, when ions pass through any complex inhomogeneous medium, diffusion coefficients and dielectric constants are spatial variables.

First of all let us discuss eq 3. With the help of the integration factor $\exp(z^\pm\psi)$, this eq can be rewritten as^{1,14,30}

$$j^\pm = z^\pm P^\pm \{ c^\pm(x_1) \exp[z^\pm\psi(x_1)] - c^\pm(x_2) \exp[z^\pm\psi(x_2)] \} \quad (5)$$

where $x = x_1$ and $x = x_2$ are the boundaries of the medium under consideration and the parameter P^\pm is determined by the expression

$$P^\pm = \left[\int_{x_1}^{x_2} \frac{\exp[z^\pm\psi(x)]}{D^\pm(x)} dx \right]^{-1} \quad (6)$$

It is worth stressing that eqs 5 and 6 do not solve the ion transport problem. They are only another form of Nernst–Planck's equation (eq 3). However, in certain cases it is more convenient to use them instead of the original equation. As already mentioned, we want to concentrate on analyzing the ion transport through a membrane that separates two solutions with different concentrations but with the same electric potential, namely $\psi(x_1) = \psi(x_2) = 0$. In this case eq 5 becomes

$$j^\pm = z^\pm P^\pm [c^\pm(x_1) - c^\pm(x_2)] \equiv z^\pm P^\pm \Delta c^\pm \quad (7)$$

Now the parameter P^\pm can be interpreted as an effective permeability of the system for each ionic species. This permeability is defined through the integral expression (eq 6). From one side, all transport properties of the system are summarized in P^\pm . From another side, this parameter can be measured in experiments. This is the reason why we use this parameter to show our results. If the membrane can be represented as a set of homogeneous layers, then eq 6 is valid for each part of it and can be written as

$$P_m^\pm = D_m^\pm / \int_{b_1}^{b_2} \exp[z^\pm\psi_m(x)] dx \quad (8)$$

where b_1 and b_2 are the limits of the m part of the membrane system according to the following notation: $m = h$, hydrophobic layer; $m = p_1$ and p_2 , polar zones; $m = d_1$ and d_2 , dipole interfaces. In addition, D_m^\pm and $\psi_m(x)$ are the ionic diffusion coefficients and electric potential, respectively, in part m . Thus, according to eq 6, the permeability P^\pm of the whole system is connected with the permeabilities of all parts in this way:

$$\frac{1}{P^\pm} = \sum_m \frac{1}{P_m^\pm} \quad (9)$$

We will assume that ionic transport across unstirred layers and through the boundaries of the hydrophobic layer is carried

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out rapidly enough so as not to have any effect on the total permeability of the system P^\pm . The permeabilities of all these parts can be easily obtained from expression 6. It is worth making a few general comments on the mathematical side of the problem. Strictly speaking, the problem cannot be solved with the help of only three equations (eqs 3 and 4), because five unknown functions have to be found: j^+ , j^- , $c^+(x)$, $c^-(x)$, and $\psi(x)$. Therefore, in order to obtain j^+ , j^- , the system of eqs 3 and 4 has to be supplemented with two equations and/or with the equations that can determine the connection between $c^\pm(x)$ and $\psi(x)$. Generally, there are two ways to do it. The first way (direct and general one) is to consider the corresponding time-dependent transport problem instead of using the steady-state equations (eqs 3 and 4) (see ref 23 and references therein) and analyze it for $t \rightarrow \infty$. In this case, the Poisson and Nernst–Planck equations have to be supplemented by two continuity equations. Then, the dependence of the concentration on the potential in the steady-state can be obtained as a result of the solution of the whole system of equations.

This approach seems not suitable here because the numerical solution of the time-dependent equations in our system is rather complicated.²³ The difficulties involved in obtaining a numerical solution could hide our main objective of analyzing the influence of the structure of the polar zone on ion transport. Besides, this approach also demands some *a priori* assumptions about boundary conditions that can influence significantly the final result. In many cases another approach is more convenient (and we shall follow it here). It consists in introducing *a priori* some determined dependence between $c^\pm(x)$ and $\psi(x)$. It is impossible to get this dependence generally. For each specific system, this dependence can be found from the preliminary analysis of its physicochemical characteristics. In fact, this approach is used in Goldman and Planck's approximation (see refs 9–11) and also when the double layer near the membrane is assumed to be in thermodynamic equilibrium (even with a nonzero ion flux across it^{13,14}).

We shall follow the second approach; i.e., we shall introduce some additional assumptions that help us to find the connection between the concentration and the electric potential profiles. Moreover such assumptions have to be introduced for each separate part of the membrane system, namely for the hydrophobic layer, the polar zones, and the bulk electrolyte solutions. As regards the latter we will consider them to be always in thermodynamic equilibrium so that the concentration profiles $c_i^\pm(x)$ can be determined by Boltzmann's equation

$$c_i^\pm(x) = c_i \exp[-z^\pm \psi_i(x)] \quad (i = 1, 2) \quad (10)$$

where c_i denotes the bulk solution concentration on both sides. Because of the small dielectric constant of the hydrophobic layer ($\epsilon_h = 2-3^{31}$), the Debye length there is usually much bigger than its thickness²⁵ ($d \approx 30 \text{ \AA}^3$), so Goldman's approximation³² is valid here and hence the concentration profile is determined by the following expression¹²

$$c_h^\pm(x) = c_i \exp(z^\pm \Delta\psi x/d) + \frac{j^\pm d}{z^\pm D_h^\pm \Delta\psi} [1 - \exp(z^\pm \Delta\psi x/d)] \quad (11)$$

where $\Delta\psi = \psi(d) - \psi(0)$.

Unlike the case of the hydrophobic layer, a similar general hypothesis cannot be made for the polar zones. The reason is that we are interested in the cases where their thickness is of the same order as the Debye's length. However, in limiting cases a realistic assumption can be made to determine the concentration profiles in the polar zones. For the sake of simplicity we shall consider here only three limiting cases: (i) the inner hydrophobic layer as the rate-determining step; (ii) the polar zones as the rate-determining step; (iii) the dipole interfaces as the rate-determining step. It is obvious that the concentration profiles will be different for each one of these cases. Let us consider now what concentration profiles follow from the assumption that some specific part of the membrane system is the rate-determining step.

(i) *The Rate Determining Step Is the Inner Hydrophobic Layer.* In this case, the permeabilities of all parts of the membrane except the hydrophobic layer are assumed to be large enough. This can happen if $D_h^\pm \ll D_p^\pm$ and $D_h^\pm \ll D_d^\pm$. For polar zones it means that very small deviations from thermodynamic equilibrium inside them will be enough to give a finite value for the ion flux. Mathematically, it means that the ion flux has to be finite despite the fact that its diffusion coefficient tends to infinity $D_{pi}^\pm \rightarrow \infty$; $D_{di}^\pm \rightarrow \infty$. This can occur only if the expression in brackets within eqs 3 and 5 is equal to zero for polar zones and hence the concentration profiles inside them are determined by Boltzmann's law, which can be expressed in a general form as

$$c_{pi}^\pm(x) = c_{pi} \exp[-z^\pm(\psi_{pi}(x) - \psi_{pi}^*)] \quad (12)$$

where $\psi_{pi}(x)$ is the electric potential profile, ψ_{pi}^* is an "additional" potential ($\psi_{pi}^* = \text{constant}$), and c_{pi} is the bulk electrolyte concentration in the i polar zone. By introducing ψ_{pi}^* in the eq 12, we should like to stress that generally the electrolyte inside the polar zones can be charged even for big polar zones, i.e., the electroneutrality condition is not valid here as a rule.

Now, to find the concentration profiles $c_{pi}^\pm(x)$ it is necessary to determine ψ_{pi}^* and c_{pi} in this limiting case. This can be done by using the condition that the dipole interfaces are not the rate-controlling step either in this limiting case. Then, by repeating for dipole interfaces the above speculations made for the polar zones, the following expression for the left dipole interface (i.e., at $x = -L$) can be obtained from eq 5

$$c_1^\pm(-L-0) \exp[z^\pm \psi_1(-L-0)] = c_{p1}^\pm(-L+0) \exp[z^\pm \psi_{p1}(-L+0)] \quad (13)$$

Then from equations 10, 12, and 13 we finally get

$$c_1 = c_{p1} \exp(z^\pm \psi_{p1}^*) \quad (14)$$

Similar expressions can also be obtained for the right dipole interface. As these expressions are valid both for cations ($z^+ = 1$) and anions ($z^- = -1$), it means that in the considered case for both polar zones

$$\psi_{pi}^* = 0 \quad (15a)$$

$$c_i = c_{pi} \quad (15b)$$

Now we have expressions for concentration profiles for all parts of the membrane system, namely expression 10 for bulk electrolyte solutions, expression 11 for the hydrophobic layer, and expressions 12 and 15 for polar zones. These equations, together with eq 3, allow us to solve the ion transport problem when the inner hydrophobic layer is the rate-controlling step.

(ii) *The Rate-Determining Step Is the Polar Zone.* Now let us deal with the second limiting case, when the polar zones $c_{pi}^\pm(x)$ are the rate-controlling step. The ion concentration profile in polar zones can be found from the following three conditions. First, the ion flux through the left polar zone and the right polar zone should be the same in the steady state; i.e.,

$$J_{p1}^\pm = J_{p2}^\pm \quad (16)$$

The second condition follows, as in the previous case, from the assumption that the dipole interfaces are not the rate-controlling step either, and hence the solutions near the dipole interfaces are in thermodynamic equilibrium; i.e., eq 13 is also valid in this case. However, only the left side of this equation, corresponding to the bulk solution, can be combined now with Boltzmann's eq 10 because the expression (eq 12) is not valid in this case. So eq 13 becomes

$$c_1 = c_{p1}^\pm(-L+0) \exp[z^\pm \psi_{p1}(-L+0)] \quad (17)$$

A similar expression can also be obtained for the right dipole interface. The assumption that now the hydrophobic layer is not the rate-controlling step leads to the third condition. As eq

(31) Petihig, R. *Dielectric and Related Molecular Processes*; Special Periodic Report 3; Royal Chemical Society: London, 1977; p 219.

(32) Goldman, D. E. *J. Gen. Physiol.* **1943**, 37, 27.

5 is valid for the hydrophobic layer, from the same reasoning used above for the polar zones we can get the conditions for the ionic concentration on both boundaries of the hydrophobic layer

$$c_{p1}^{\pm}(0) \exp[z^{\pm}\psi_{p1}(0)] = c_{p2}^{\pm}(d) \exp[z^{\pm}\psi_{p2}(d)] \quad (18)$$

These expressions 16–18 together with eq 10 determine the concentration profile in the whole membrane system when the polar zones are the rate-controlling step. We shall discuss this case elsewhere.³³

(iii) *The Rate-Determining Step Is the Dipole Interface.* Similarly to the previous case the ion flux through the left dipole interface and that through the right dipole interface should be the same in the steady state; i.e.,

$$j_{d1}^{\pm} = j_{d2}^{\pm} \quad (19)$$

As the electrolyte in polar zones is now in thermodynamic equilibrium, then eq 12 is valid. However, in contrast to the case in eq 15 now $\psi_{pi}^* \neq 0$ and $c_i \neq c_{pi}$ and because eq 13 is not valid in this case. But eq 18 remains valid in the hydrophobic layer. Moreover, as eq 12 holds also, then from eqs 12 and 18 it follows that the concentrations and charges of the solutions in both polar zones are equal

$$\psi_{p1}^* = \psi_{p2}^* \neq 0 \quad (20a)$$

$$c_{p1} = c_{p2} \quad (20b)$$

These equations (eqs 16–18) together with eq 10 determine the concentration profile in the whole membrane system when the dipole interfaces are the rate-controlling step. We shall discuss them in detail elsewhere.³³

2.2. The Hydrophobic Layer as the Rate-Determining Step. Let us consider in detail the first limiting case, when the hydrophobic layer is the rate-determining step. Then, as follows from eqs 5 and 10 the steady-state ion flux is

$$j^{\pm} = z^{\pm}P_h^{\pm}(c_1 - c_2) = z^{\pm}P_h^{\pm}\Delta c \quad (21)$$

Then, by using Goldman's assumption that the electric field inside the hydrophobic layer is constant, the following expression for the permeability can be obtained from eq 8:

$$P_h^{\pm} = z^{\pm} \frac{D_h^{\pm}}{d} [\psi_{p1}(0) - \psi_{p2}(d)] / \{ \exp[z^{\pm}\psi_{p1}(0)] - \exp[z^{\pm}\psi_{p2}(d)] \} \quad (22)$$

which for small potentials can be written as

$$P_h^{\pm} \approx \frac{D_h^{\pm}}{d} \left\{ 1 - \frac{z^{\pm}}{2} [\psi_{p1}(0) + \psi_{p2}(d)] \right\} \quad (23)$$

These expressions (eqs 21–23) determine the separate ion fluxes j^{\pm} and the total flux $j = j^+ + j^-$

$$j = (P_h^+ - P_h^-)\Delta c = \frac{1}{d} \frac{[\psi_{p1}(0) - \psi_{p2}(d)][D_h^+ - D_h^- \exp(\psi_{p1}(0) + \psi_{p2}(d))]}{\exp(\psi_{p1}(0)) - \exp(\psi_{p2}(d))} \Delta c \quad (24)$$

The total flux j is the main characteristic of the membrane system that is usually measured in experiments. So, the theoretical approach here developed can be tested on the basis of this expression (eq 24). Moreover, to get qualitative estima-

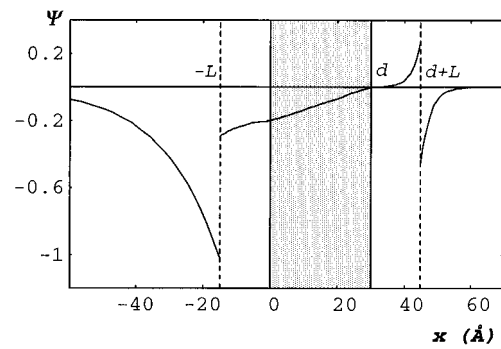


Figure 2. Example of the typical electric potential profile in the considered membrane system.

tions, the following approximate expression can be used instead of eq 24:

$$j \approx \frac{1}{d} \{ D_h^+ - D_h^- [\psi_{p1}(0) + \psi_{p2}(d)] \} \Delta c \quad (25)$$

To study the characteristic difference in membrane permeability for cations and anions, we shall consider also the permselectivity¹⁰ η defined as

$$\eta = -\frac{j^-}{j^+} = \frac{P_h^+}{P_h^-} = \frac{D_h^+}{D_h^-} \exp(\psi_{p1}(0) + \psi_{p2}(d)) \quad (26)$$

2.3. The Electric Potential Profile. Finally, to solve our original problem, we have to determine the potential profile $\psi(x)$ in the whole system when the hydrophobic layer is the rate-controlling step. For this limiting case, the potential distributions are governed by Poisson–Boltzmann's equation in the polar zones and bulk solutions and by Laplace's equation in the hydrophobic layer. For small potentials these equations can be summarized in this form (see the Appendix) ($i = 1, 2$):

$$\frac{d^2\psi}{dx^2} = \begin{cases} 0; & 0 < x < d \\ \frac{\epsilon}{\epsilon_p} [k_i^2\psi(x) - \alpha\rho_i(x)]; & 0 \leq |x| < L_i \\ \frac{\epsilon}{\epsilon_p} [k_i^2\psi(x) + \alpha\nu\delta'(|x| - L_i)]; & L_i \leq |x| < \infty \end{cases} \quad (27a)$$

$$\bar{x} = \begin{cases} x & i = 1 \\ x - d & i = 2 \end{cases} \quad (27b)$$

where $\rho_i(x)$ denotes the volume density of the fixed charges in polar zone i and is determined by eq 1 or 2, ν_i is the surface dipole density of the normal dipole component in polar zone i , k_i is the inverse Debye length of the i solution, ϵ is the dielectric constant of the solution ($\epsilon = 78$ for water), ϵ_p is the dielectric constant of the polar zone ($\epsilon_p = 10$ – 40),³⁵ and $\alpha = 4\pi\beta/\epsilon$.

The general solution of eq 27 can be found in the Appendix, and it allows us to analyze membranes with any charge distributions and dipole densities. However, to clarify the main properties of our membrane system, we shall concentrate on the simplest membrane system, namely a symmetrical membrane in the case denoted as *constant charge* and neglecting the difference between ϵ_p and ϵ , i.e.,

$$\begin{aligned} L_1 &= L_2 \equiv L \\ \rho_1 &= \rho_2 \equiv \rho \\ \nu_1 &= -\nu_2 \equiv -\nu \\ \epsilon_p &= \epsilon = 78 \end{aligned} \quad (28)$$

The potential profile in such system is shown in Figure 2. Note that for qualitative estimations of j and η it is possible to neglect the influence of the surface potential of one side of the membrane

(33) Aguilera, V.; Belaya, M.; Levadny, V. In preparation.
(34) (a) Barlow, C. A. In *Physical Chemistry: IXA, Electrochemistry*; Eyring, H., Ed.; Academic Press: New York, 1970; p 167. (b) Raudino, A.; Mauzerall, D. *Biophys. J.* **1986**, *50*, 441. (c) Coster, H. G. L.; Smith, J. R. *Biochim. Biophys. Acta* **1974**, *373*, 151.

(35) (a) Gross, E.; Bedlack, R. S.; Loew, L. M. *Biophys. J.* **1994**, *67*, 208. (b) Franklin, J. C.; Cafiso, D. S. *Biophys. J.* **1993**, *65*, 289.

on the other (i.e., to assume $\gamma = 0$ in eq A-9 from the Appendix). Then, $\psi_{p1}(0)$ and $\psi_{p2}(d)$ in expressions 25 and 26 become

$$\begin{aligned}\psi_{p1}(0) &\cong \alpha \left(\frac{\sigma}{k_1} - \nu \right) \exp(-k_1 L) \\ \psi_{p2}(d) &\cong \alpha \left(\frac{\sigma}{k_2} - \nu \right) \exp(-k_2 L)\end{aligned}\quad (29)$$

In short, expressions 22, 24, 26, and 29 determine all the transport characteristics of a membrane with polar zones and surface dipoles when the hydrophobic layer is the rate-controlling step.

3. Discussion

It has been common to neglect the existence of the extended polar zones and dipoles on the membrane surface when working with experimental data on ion transport. In order to demonstrate the importance of these characteristics of the membrane structure for the correct interpretation of ionic transport across membranes with soft interfaces, we shall compare some calculations done on the basis of our approach with others based on the commonly accepted approach.

We start by analyzing the influence of the surface dipoles on the ion transport across the membranes. Currently, the widespread hypothesis is that the negative surface dipoles cause the anion selectivity of the membrane (see for example ref 35). In particular, Flewelling and Hubbell³⁶ have developed a theory of the selectivity of lipid membranes for hydrophobic anions and cations. According to their theory, the ion permeability of these membranes is determined by the Born-image energy, the hydrophobic energy, and the dipole electric energy. Moreover, the positive dipole potential (i.e. $\nu_2 = \nu < 0$ in our notation) of the ester groups of lipid carbonyls is believed to be the main contribution to the dipole electric energy. It results in a low energy barrier for anions as compared to cations and anion selectivity of lipid membranes. This theory has been corroborated by an experimental investigation of organic ion permeation across lipid bilayers.³⁶ However, the generalization of this theory to other membrane systems is not supported by the experimental results. In particular, planar lipid bilayers formed from thylakoid lipids showed a selective permeability to K^+ over Cl^- (see ref 20 and references therein) but their membrane dipole potential is known to be positive inside ($\nu < 0$ in our notation).³⁷ So it appears that the dipole theory of membrane permselectivity is in contradiction of the observed cation selectivity (see discussion in ref 20). Our theoretical results show that the sign of the surface dipole by itself is not enough to explain the cation or anion permselectivity of the membrane system. To demonstrate this, we consider the ion flux calculated according to eqs 21, 22, and 29. To avoid overcomplicating the expression, we shall omit now the subindex "h".

Let the concentration of the left side solution c_1 be fixed and c_2 be variable. Let us recall that we are studying here the case of short-circuit conditions, i.e., when, $\psi(-\infty) = \psi(\infty) = 0$, and hence there is no voltage drop across the whole membrane system. However, the total ion flux is not zero ($j = j^+ + j^- \neq 0$). The dipoles usually play a twofold role: firstly they charge the interior of the membrane;^{35,36} secondly, they influence the outside double electrical layer.^{7,8} Therefore, to rule out the second effect

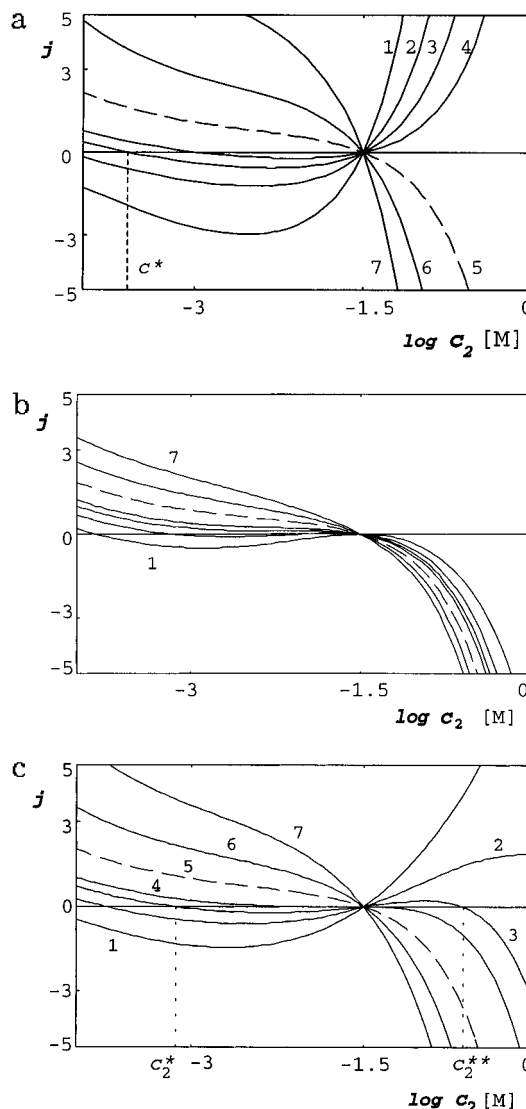


Figure 3. Total flux j through the hydrophobic layer in dimensionless units (D^+c_1/d) as a function of right solution concentration c_2 for a symmetrical membrane $\sigma_1 = \sigma_2 = \sigma = -10^{-4} \text{ e}/\text{\AA}^2$; $\nu_1 = -\nu_2$ in the case $c_1 = 10^{-1.5} \text{ M}$, for different dipole densities: (1) $\nu_2 = -2.5 \times 10^{-2} \text{ e}/\text{\AA}$; (2) $\nu_2 = -1.7 \times 10^{-2} \text{ e}/\text{\AA}$; (3) $\nu_2 = -1.17 \times 10^{-2} \text{ e}/\text{\AA}$; (4) $\nu_2 = -8.5 \times 10^{-3} \text{ e}/\text{\AA}$; (5) $\nu_2 = 0$; (6) $\nu_2 = 8.5 \times 10^{-3} \text{ e}/\text{\AA}$; (7) $\nu_2 = 1.7 \times 10^{-2} \text{ e}/\text{\AA}$. The thickness of the hydrophobic layer $d = 30 \text{ \AA}$ and the diffusion coefficients are related through $D^+/D^- = 2$. Here and everywhere e is the elementary charge. The thickness of the polar zone is (a) $L = 0$; (b) $L = 30 \text{ \AA}$; (c) $L = 10 \text{ \AA}$.

from the beginning, we will consider limiting cases, when the polar zones are very small ($L \approx 0$). Figure 3 shows the total ion flux in dimensionless units (D^+c_1/d units) calculated from eq 24 for $L = 0$ for different values of the surface dipole density ν . The dashed line has been obtained for $\nu = 0$; i.e., it corresponds to the classical study of the influence of the surface charge on ion transport (see for example refs 12 and 15). As it seems from the figure, the surface dipoles have a great influence on the total ion flux and the permselectivity. Moreover, in this case this influence is connected only with the charging of the membrane interior because the polar zone thickness is assumed to be zero and hence the dipoles have no effect on the outside double electrical layer.⁸ However, the total permselectivity is determined not only by the sign of ν but also by the electrolyte concentration c_1 and c_2 , by the surface charge density σ , and by the ratio D^+/D^- . Furthermore, in some cases the membrane is anion selective for some concentration range and cation selective for

(36) Flewelling, R. F.; Hubbell, W. L. *Biophys. J.* **1986**, *49*, 541.

(37) Oldani, D.; Hanser, H.; Nichols, B. W.; Phillips, M. C. *Biochim. Biophys. Acta* **1975**, *382*, 1.

another (see curves 3 and 4). Generally, for $L \cong 0$ the membrane is cation selective if

$$\nu > \frac{1}{2} \left[\sigma \left(\frac{1}{k_1} + \frac{1}{k_2} \right) - \frac{1}{\alpha} \ln \frac{D^+}{D^-} \right] \quad (30)$$

and hence for certain conditions in experiments with constant c_1 and variable c_2 the following *blocking concentration* c^* (see curve 3 in Figure 3a) can be obtained (for zero ion flux across the membrane):

$$c_2 = c^* = \theta \left[\frac{2\nu}{\sigma} - \frac{1}{k_1} + \frac{1}{\sigma \alpha} \ln \frac{D^+}{D^-} \right]^{-2} \quad (31)$$

where

$$\theta = \frac{\epsilon k_B T}{8\pi e^2} \quad (32)$$

The results for the other limiting case of very large polar zones ($L \gg k_1^{-1}$) are shown in Figure 3b. As follows from comparison of this figure with Figure 3a the difference between these cases lies in the fact that the permselectivity of the membranes with large polar zones for concentrated solutions ($c_2 > c_1$) does not depend on the sign of the surface dipoles and is determined only by the diffusion coefficients of the cations D^+ and anions D^- . The functional dependence $j(c_2)$ becomes more complicated for the intermediate cases of moderate polar zones (see Figure 3c). In particular, for certain conditions, there is a second *blocking concentration* in the range of large concentrations (curves 2 and 3). In this connection it is worth noting that the existence of these *blocking concentrations* is a general characteristic of the membrane systems under consideration. These concentrations are determined by the following condition that comes directly from eq 24:

$$\psi_{p1}(o) + \psi_{p2}(d) = \ln(D^+/D^-) \quad (33)$$

For the present case of constant c_1 and variable c_2 , this condition becomes valid if

$$4\sigma L < \nu B^2 \quad (34)$$

where

$$B = 1 + \frac{\sigma L}{\nu} + \left(\frac{\sigma}{k_1 \nu} - 1 \right) \exp(-k_1 L) + \frac{1}{\alpha \nu} \ln(D^+/D^-) \quad (35)$$

Then, for the following concentrations

$$c_2 = c^* = \theta \left(\frac{\sigma}{\nu B} \right)^2 \quad (36a)$$

$$c_2 = c^{**} = \theta \left(\frac{B}{L} \right)^2 \quad (36b)$$

no ionic flux crosses the membrane. Note that these *blocking concentrations* may serve as a test of our theory because they can be obtained experimentally.

Our theoretical results show two important peculiarities of the membrane systems with polar zones. First, the permselectivity of such systems is determined not only by

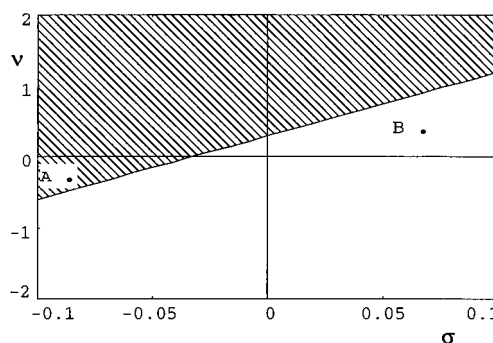


Figure 4. Plot of permselectivity η as a function of σ and ν . The cross-hatched region indicates cation selective membranes ($\eta > 1$). By changing L and c_2 , the boundary between the membranes with $\eta > 1$ and $\eta < 1$ can be rotated, thus changing the permselectivity of the membranes represented by points A and B.

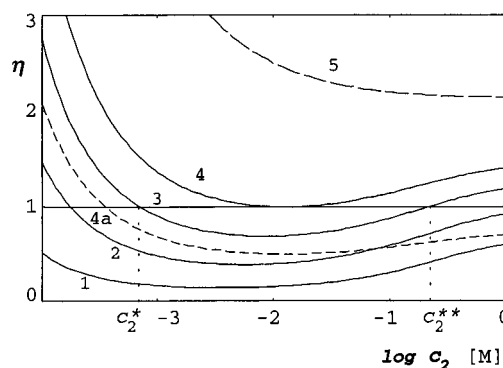


Figure 5. Permselectivity η as a function of right solution concentration c_2 for different values of the surface dipole density ν : (1) $\nu_2 = -2.5 \times 10^{-2} \text{ e/\AA}$; (2) $\nu_2 = -1.7 \times 10^{-2} \text{ e/\AA}$; (3) $\nu_2 = -1.17 \times 10^{-2} \text{ e/\AA}$; (4) $\nu_2 = -8.5 \times 10^{-3} \text{ e/\AA}$; (5) $\nu_2 = 0$. The polar zone thickness is $L = 10 \text{ \AA}$. All other parameters are the same as in Figure 3 except for the dashed line, which reproduces curve 4 for the case of $D^+/D^- = 1$.

the surface dipoles but also by the surface charges and by the thickness of the polar zone. From eqs 26 and 29 it follows that, generally speaking, a membrane is cation selective ($\eta > 1$) if

$$\nu > b_1 \sigma - b_2 \quad (37)$$

where

$$b_1 = \frac{k_1 \exp(k_1 L) + k_2 \exp(k_2 L)}{k_1 k_2 [\exp(k_1 L) + \exp(k_2 L)]} \quad (38a)$$

$$b_2 = \frac{1}{\alpha} \frac{\ln(D^+/D^-)}{\exp(-k_1 L) + \exp(-k_2 L)} \quad (38b)$$

Figure 4 shows a region in the $\{\sigma, \nu\}$ space which corresponds to a cation selective membrane (cross-hatched region). For example, point A in Figure 4 corresponds to a cation selective membrane although it has negative surface dipoles ($\nu < 0$), and in contrast, point B corresponds to an anion selective membrane although it has positive surface dipoles ($\nu > 0$).

The second peculiarity is that, for this kind of membrane system, permselectivity is no longer a constant parameter but depends on the electrolyte concentration. Figure 5 shows the dependence $\eta(c_2)$ for different values of the surface dipole density ν . It seems also that permselectivity is determined not only by ν but also by σ and c_2 . It is seen that permselectivity changes with c_2 . Moreover, in certain

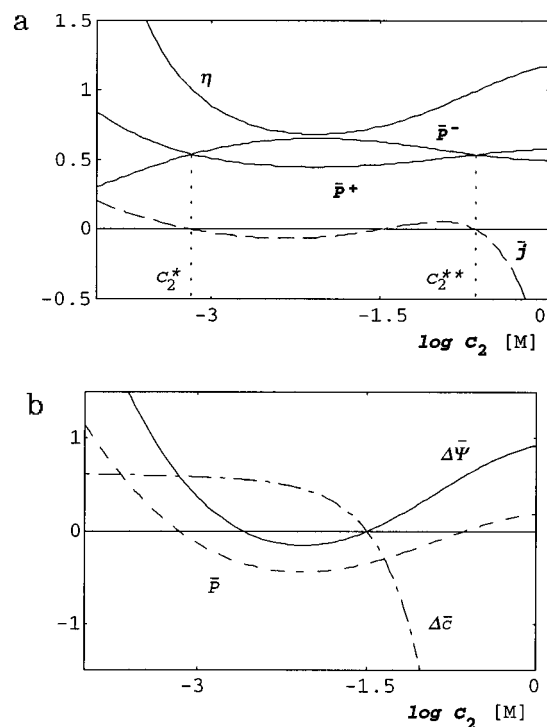


Figure 6. Dependence of the main transport parameters on the right solution concentration c_2 for the same membrane system represented by curve 3 in Figure 3c: (a) Permeability $\eta(c_2)$; normalized total flux $\bar{j} = j(c_2)/j(c_2 = 10^{-4})$; and normalized cation and anion permeability $\bar{P}^{\pm} = P^{\pm}(c_2)/P^{\pm}(c_2 = c_1)$ for this membrane system. (b) Transhydrophobic normalized potential $\Delta\bar{\psi} = \Delta\psi/\psi_{p1}(0)$ (—); total normalized permeability $\bar{P} = P/P^+$ ($c_2 = c_1$) (---); and normalized cation concentration difference $\Delta\bar{c} = \Delta c/c_1$ (— · —).

cases this dependence is nonmonotonic. It is also seen that the membrane is cation selective in one region (e.g. curve 3 in Figure 5 for $c_2 > c_2^*$ and $c_2 < c_2^*$) and anion selective in another one (e.g. curve 3 in Figure 5 for $c_2^* < c_2 < c_2^{**}$). It should be stressed that the variation of the permselectivity in our model is connected with the redistribution of the electric field in the system. The selectivity determined by diffusion coefficients—described here by the ratio D^+/D^- —remains unchanged. In particular, $D^+/D^- = 2$ for all curves in Figure 5 except curve 4a, for which $D^+/D^- = 1$.

What determines this behavior of the permselectivity $\eta(c_2)$? To clarify this problem, we consider now in detail a particular membrane system, e.g., that represented by curve 3 in Figure 3c. The dependence of permselectivity, flux, and cation and anion permeability upon the right bulk concentration is shown in Figure 6a for this membrane system. As is seen, both cation and anion permeability change with the concentration c_2 . Moreover, in the region where $c_2^* < c_2 < c_2^{**}$ the former is bigger than the latter ($P^+ > P^-$), but for the regions where $c_2 > c_2^*$ and the opposite happens ($P^+ < P^-$). The variation of P^{\pm} is determined mainly by the change of $\Delta\psi$ (strictly speaking, by the change in the potential $\psi_{p2}(d)$). For a negatively charged membrane ($\sigma < 0$) any decrease in the right electrolyte concentration leads to a decrease in the potential $\psi_{p2}(d)$. Generally, it has two sequences: (i) transhydrophobic potential increases $\Delta\psi$ (see Figure 6b) and (ii) cation concentration increases and anion concentration decreases at $x = d$. As for cation permeability, P^+ , the first effect accelerates the cation flux and hence increases the cation permeability of the hydrophobic zone. The second one decreases the difference of concentrations at the hydrophobic zone boundary and hence decreases the cation permeability.¹² However for our membrane

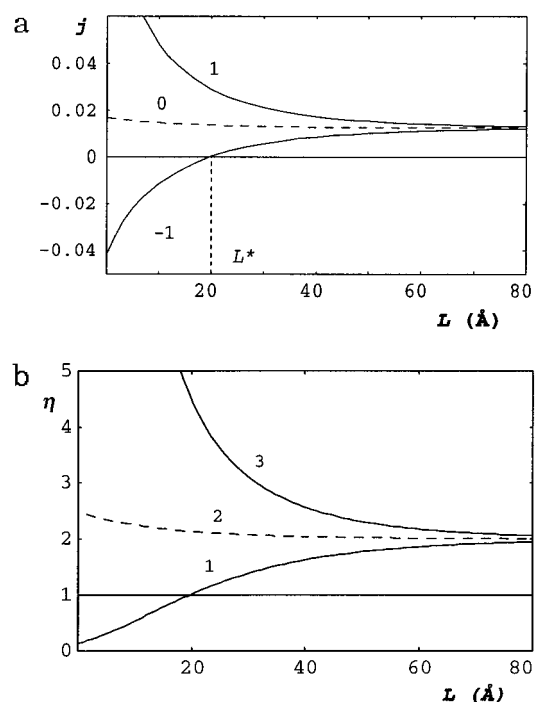


Figure 7. Influence of the polar zone thickness L on the main transport parameters of the considered membrane system in the case $c_1 = 0.1$ M, $c_2 = 0.225$ M, $\sigma_1 = \sigma_2 = \sigma = -10^{-4}$ e/Å²; for different dipole densities ($\nu_1 = -\nu_2$): (1) $\nu_2 = -1.7 \times 10^{-2}$ e/Å; (2) $\nu_2 = 0$; (3) $\nu_2 = 1.7 \times 10^{-2}$ e/Å. The thickness of the hydrophobic layer is $d = 30$ Å and $D^+/D^- = 2$. (a) Total flux j through the hydrophobic layer in dimensionless units (D^+c_1/d) as a function of L . (b) Permselectivity η as a function of L .

system the second effect, connected with the variations Δc^{\pm} , is negligible. Actually, as is seen in Figure 6b, the concentration difference Δc^{\pm} in the region $c_2 < 10^{-2}$ M is changing weakly but P^+ is growing. As follows from eq 29, $\Delta\psi$ consists of two terms, namely, the surface charge term and the dipole one; the first one is falling but the second one is growing upon the increase of the concentration. The balance between these terms determines the potential difference $\Delta\psi$ and hence the permeability P^{\pm} of the membrane system.

Let us consider now the influence of the polar zone thickness L on the ionic transport. As we have mentioned in the Introduction, this problem has to be studied because, firstly, it is known that lipid membranes of similar chemical composition can have the same inner hydrophobic layer but a quite different structure and size of their polar zones, as happens, e.g., in phosphatidylcholine and phosphatidylethanolamine bilayers.^{1,17} Secondly, the same membrane, under different conditions, may exhibit different polar zone sizes.²⁵ In Figure 7 the dependence of permselectivity η and total ion flux j on L is shown. Calculations have been done on the basis of eqs 22–24, 26, and 29. From Figure 7b it is seen that the membrane permselectivity η is changing with the polar zone thickness L . Moreover, in the region of the small L , permselectivity is determined mainly by the sign of the surface dipoles and the ratio between D^+ and D^- . The permselectivity changes with the increase of the polar zone thickness. Moreover, as the polar zone thickness is growing, an anion selective membrane (in the region of small L , see e.g. curve 1 in Figure 7b) can become cation selective in the region of large L (provided that $D^+ > D^-$).

As it follows from Figure 7, actually, the permselectivity η and total ion flux j of membranes with large polar zones ($L \gg k_1^{-1}$) do not depend either on dipole density or on charge density and are determined only by anion and

cation diffusion coefficients. Note that there is a *blocking thickness* L^*

$$L^* \approx \left\{ \left(\frac{1}{k_1} + \frac{1}{k_2} \right) - \frac{1}{\alpha\sigma} (2\alpha\nu + \ln(D^+/D^-)) \right\} / \left[2 - \frac{\nu}{\sigma} (k_1 + k_2) \right] \quad (39)$$

for which the total ionic flux is zero.

One more interesting property of the function $j(L)$ is that, under certain conditions, this function becomes nonmonotonic. Particularly, in the case of $c_2 > c_1$, for a membrane system with surface charge density σ and dipole density ν which satisfy the following condition

$$k_1 > \frac{\sigma}{\nu} > k_2 \quad (40)$$

the total ion flux will be maximum if the polar zone thickness is

$$L^{**} \cong \frac{1}{k_1 - k_2} \ln \left(\frac{\nu k_2 - \sigma}{\sigma - \nu k_1} \right) \quad (41)$$

Summarizing the above considerations, a qualitative conclusion can be drawn: in the case of negative surface dipole densities $\nu < 0$, membranes with bigger polar zones should exhibit higher cation selectivity (curve 1 in Figure 7b) and hence higher cation permeability. This can explain the experimental fact that thylakoid membranes are more cation permeable than phospholipid membranes²⁰ because the polar zone thickness of the former is^{1,2,19} $L = 3-8$ Å while for the latter L is³ $11-15$ Å. However, it is necessary to note that polar zone thickness variation alone cannot explain the 5- or even 10-fold increase of the permeability of thylakoid membranes experimentally found.²⁰ In this connection, the assumptions made during the calculation of the electric profile $\psi(x)$ (namely we have assumed that the dielectric constant in the polar zone is the same as that in the aqueous solution) should be reconsidered. As we have mentioned above, it is not generally true; actually, for lipid membranes the dielectric constant of the polar zone is^{3,34} $\epsilon_p = 20-40$. We shall consider in detail the influence of the dielectric constant of the polar zone ϵ_p on the ion transport in a separate paper.³³ However, a preliminary analysis shows that a 5-10-fold increase of the permeability of the thylakoid membrane in comparison to other lipid membranes can be explained in the framework of our model by taking into account that ϵ_p is smaller than ϵ .

How can the model developed here be connected with the experiments? One example is the following. It is known that the addition of some small molecules to the solution changes the bilayer permeability, e.g., salicylamide and phloretin. Phloretin has been reported to increase dramatically the cation flux and decrease the anion flux across artificial and natural membranes.^{22,24,38,39} These experimental data have been interpreted as a result of conformational changes of the lipid head in such a manner that the surface density of the normal component of the dipoles changes,^{24,38,40} but this is not enough to account for the observed changes.²⁴ Besides, these NMR studies have shown that phloretin also pushes out the boundary water from the polar zone regions.²⁴ The latter in our model means that the effective thickness of the

polar zone L is changed. Hence the phloretin has a double effect on the bilayer: (1) it changes the normal component of surface dipoles, and (2) it changes the effective thickness of the polar zones. Taking into account both of these effects enables one to find a better interpretation of the experimental results.^{24,38}

4. Conclusions

The steady-state transport characteristics of membranes with two extended polar zones in an aqueous solution have been considered under short-circuit conditions. We have taken into account the existence of charges and dipoles in these zones. The total thickness of the membrane L is assumed to be on the same scale as the Debye's length of the solution, and consequently it is necessary to take into account the influence of the double electric layers. Ion flux is described with Nernst-Planck's equation. In this paper the limiting case of the rate-control step being the passage of ions through the internal hydrophobic layer in Goldmann's approach is presented. The influence of the electrolyte concentration and the thickness of the polar zone on the total ion flux and permselectivity is analyzed. It has been shown that the permselectivity of such a system depends significantly on the surface charges, the surface dipoles, the thickness of the polar zone, and the electrolyte concentration.

Permselectivity η changes with bulk electrolyte concentration. The same membrane can be either cation selective (in one region of concentrations) or anion selective (in the other one). In some cases the dependence $\eta(c_2)$ becomes nonmonotonic. A general expression for cation selectivity has been obtained (see eq 37 and Figure 4). The membrane system can exhibit anion selectivity in spite of the positive sign of the surface dipoles. We have obtained that two concentrations exist for which the total ion flux through the system vanishes, even with a gradient of concentration in the system. The existence of these *blocking concentrations* gives us the possibility to test experimentally the model presented here.

Concerning the polar zone thickness, we have shown that in the case of extended polar zones ($L \gg k^{-1}$) the influence of the surface charges and dipoles on permselectivity vanishes. However, this influence is significant for moderate values of L . Apart from it, in this case, for the same values of surface charge density and surface dipole density the permeability and permselectivity depend on L significantly. In particular, if $L > L^*$ (see the expression for L^* in eq 40), any membrane with larger polar zones will be more cation selective. We have also shown that the influence of the thickness of the polar zone on the permeability increases significantly when taking into account that the dielectric constant of the polar zones is much smaller than that of bulk solutions.

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Appendix: Electric Field Distribution in the System

In view of the electric characteristics of the membrane under consideration, we take into account the existence of the two semispaces occupied by solution and one hydrophobic layer. Note that, generally, the dielectric constant of the polar zones ϵ_p differs from those for an aqueous solution ($\epsilon = 81$) and for the hydrophobic region

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($\epsilon_h = 2-4$). The distribution of the electric potential $\phi(x)$ in the system is determined by Poisson's equation

$$\frac{d}{dx} \left[\epsilon(x) \frac{d\phi(x)}{dx} \right] = -4\pi\rho(x)$$

The electric charge density $\rho(x)$ describes here the volume charge density after taking into account all the sources of electric field in the system, which are in our case the mobile ions of the solution and the membrane fixed charges of the polar zone (represented by the volume density $\rho_{\text{fix}}(x)$, which includes the dipoles):

$$\rho(x) = \rho_{\text{fix}} + F(z^+ c^+ + z^- c^-)$$

The expression for $\rho_{\text{fix}}(x)$ is found in eqs 1 and 2 of the main text. For the sake of simplicity, we will consider here the case of small potentials to ensure the validity of Debye-Hückel's approximation. Then, the electric potential profile $\phi(x)$ in the system is governed by Poisson-Boltzmann's equation:

$$\frac{d^2\phi}{dx^2} = \begin{cases} k_1^2\phi(x) + \frac{4\pi\nu_1}{\epsilon}\delta'(x+L_1); & -\infty < x \leq -L_1 \\ \frac{\epsilon}{\epsilon_p} \left[k_1^2\phi(x) - \frac{4\pi}{\epsilon}\rho_1(x) \right]; & -L_1 < x \leq 0 \\ 0 & 0 < x < d \\ \frac{\epsilon}{\epsilon_p} \left[k_2^2\phi(x) - \frac{4\pi}{\epsilon}\rho_2(x) \right]; & d \leq x < d+L_2 \\ k_2^2\phi(x) + \frac{4\pi\nu_2}{\epsilon}\delta'(x-d-L_2); & d+L_2 \leq x < +\infty \end{cases} \quad (\text{A-1})$$

where k_i is the inverse Debye length of the solution in i semispace, which for a binary electrolyte ($|z^+| = |z^-| = z$) reads

$$k_i^2 = \frac{8\pi c_i e^2 z}{\epsilon K_B T} = \frac{8\pi c_i e z \beta}{\epsilon} \quad (i = 1, 2)$$

c_i is the bulk concentration in solution i . Here and everywhere subindex 1 denotes the left side of the membrane and subindex 2 denotes the right side. In dimensionless form, expression A-1 becomes after some modification

$$\frac{d^2\psi}{dx^2} = \begin{cases} 0; & 0 < x < d \\ \omega^2 [k_i^2 \psi(x) - \alpha \rho_i(x)]; & 0 \leq |\bar{x}| < L_i \\ \omega^2 [k_i^2 \psi(x) + \alpha \delta'(\bar{x} - L_i)]; & L_i \leq |\bar{x}| < \infty \end{cases} \quad i = 1, 2 \quad (\text{A-2})$$

$$\bar{x} = \begin{cases} x & i = 1 \\ x - d & i = 2 \end{cases} \quad \omega = \left(\frac{\epsilon}{\epsilon_p} \right)^{1/2}$$

The dimension coefficient α is defined as

$$\alpha = \frac{4\pi}{\epsilon} \frac{e}{K_B T} = 4\pi\beta/\epsilon$$

Note that the electric potential and the electric field must satisfy the following matching conditions at the polar zone boundaries ($x = L_i$)

$$\psi(L_i+0) - \psi(L_i-0) = \alpha\nu_i$$

$$\oint_s E ds = \int_V \rho dV$$

The last equation in the case of *constant charge* (eq 1) becomes the following

$$\frac{d\psi}{dx}(L_i+0) - \frac{d\psi}{dx}(L_i-0) = \alpha\sigma$$

while in the case of *constant charge density* (eq 2) leads to

$$\frac{d\psi}{dx}(L_i+0) - \frac{d\psi}{dx}(L_i-0) = 0$$

If eq A-2 is combined with the usual boundary conditions

$$\psi(-\infty) = 0 \quad \psi(+\infty) = \psi_{\text{ex}}$$

where ψ_{ex} is the externally applied electric potential, then the following dimensionless electric potential distribution $\psi(x)$ can be obtained ($i = 1, 2$):

(a) for outside solutions ($-\infty < x < -L_1$ and $d+L_2 < x < +\infty$)

$$\psi_i(x) = \alpha\omega^2 \left\{ \left[\frac{\sigma_{oi}(L_i)}{k_i} - F_i \right] \times \exp[-(-1)^i k_i(\bar{x} + L_i)] - \psi_{\text{ex}} \delta_i^2 \right\} \quad (\text{A-3})$$

(b) for polar zones ($-L_1 < x < 0$ and $d < x < d+L_2$)

$$\psi_{pi}(x) = \alpha\omega^2 \left\{ \frac{\sigma_{pi}(\bar{x})}{k_i} \cosh(k_i\omega\bar{x}) + \frac{\sigma_{oi}(\bar{x})}{k_i} \times \exp[-(-1)^i k_i(\bar{x} + L_i)] - G_i(x) - \psi_{\text{ex}} \delta_i^2 \right\} \quad (\text{A-4})$$

(c) for the hydrophobic layer ($0 < x < d$)

$$\psi_h(x) = \left[\frac{x}{d} + A_1\omega^2 \right] \Psi + \alpha\omega^2 \frac{\sigma_{p1}(0)}{k_1} \quad (\text{A-5})$$

where the parameters F_i and G_i describe the dependence of the electric field distribution in the i part of the membrane system on the potential profile in the other part.

$$F_i = (-1)^i A_i \Psi / (\alpha\omega) \\ G_i(x) = (-1)^i \frac{A_i \Psi}{\alpha\omega} [\omega \cosh(k_i\omega\bar{x}) - (-1)^i \sinh(k_i\omega\bar{x})] \times [\omega \cosh(k_i\omega L_i) - (-1)^i \sinh(k_i\omega L_i)]$$

Other parameters are

$$\Psi = \frac{\omega^2}{\gamma} \left\{ \alpha \left[\frac{\sigma_{p2}(0)}{k_2} - \frac{\sigma_{p1}(0)}{k_1} \right] - \psi_{\text{ex}} \right\} \quad (\text{A-6})$$

$$\sigma_{oi}(\bar{x}) = \int_0^{\bar{x}} \frac{\rho_i(\bar{\tau}) \cosh(k_i\omega\bar{\tau}) + (-1)^i k_i \nu_i \sinh(k_i\omega\bar{\tau}) \delta(\bar{\tau} - L_i)}{\cosh(k_i\omega\bar{\tau}) + \omega \sinh(k_i\omega\bar{\tau})} d\tau \quad (\text{A-7})$$

$$\sigma_{pi}(\bar{x}) = \int_{|\bar{x}|}^{L_i} \frac{\rho_i(|\tau|) - (-1)^i k_i \nu_i \delta(|\tau| - L_i)}{\cosh(k_i \omega |\tau|) + \omega \sinh(k_i \omega |\tau|)} d\tau \quad (\text{A-8})$$

$$\gamma = 1 + \omega^2 \sum_{i=1}^2 \frac{\omega \cosh(k_i \omega L_i) + \sinh(k_i \omega L_i)}{\cosh(k_i \omega L_i) + \omega \sinh(k_i \omega L_i)} \gamma_i \quad (\text{A-9})$$

$$\gamma_i = \frac{\epsilon_h \omega}{\epsilon k_i d} \quad (\text{A-10})$$

$$A_i = \gamma_i / [\cosh(k_i \omega L_i) + \omega \sinh(k_i \omega L_i)] \quad (\text{A-11})$$

Kronecker's symbol δ_j^2 in eqs A-3 and A-4 allows the introduction of ψ_{ex} only in the right solution. Equations A-3–A-5 can be used for analyzing the influence of the polar zone thickness on the ion transport behavior in a general case. For the case mentioned above in the main text (specific charge distributions) they can be simplified. In particular, neglecting the differences between the polar zone dielectric constant ϵ_p and the aqueous solution one ϵ (i.e. assuming $\epsilon = 78$), expressions A-9–A-11 become

$$\omega = 1; \quad A_i = \gamma_i \exp(-k_i L_i); \quad \gamma = 1 + \gamma_1 + \gamma_2$$

Then, when all charges inside the polar zone are located near a single plane, i.e., in the case of *constant charge* (see eq 1 in the main text), eqs A-7 and A-8 become

$$\left. \begin{aligned} \sigma_{oi} &= \sigma_{oi}(L_i) = \sigma_j \cosh(k_i L_i) + (-1)^i k_i \nu_i \sinh(k_i L_i) \\ \sigma_{pi} &= \sigma_{pi}(z) = [\sigma_i - (-1)^i k_i \nu_i] \exp(-k_i L_i) \end{aligned} \right\} \quad (\text{A-12})$$

(Note that in this case $\sigma_{oi}(x) = 0$ everywhere except at $x = L_i$).

For the second kind of charge distribution denoted as *constant charge density* (see eq 2 in the main text), eqs A-7 and A-8 become

$$\left. \begin{aligned} \sigma_{oi}(L_i) &= \frac{\Omega_i}{k_i} \sinh(k_i L_i) + (-1)^i k_i \nu_i \sinh(k_i L_i) \\ \sigma_{oi}(x) &= \frac{\Omega_i}{k_i} \sinh(k_i x) \\ \sigma_{pi}(x) &= - \left\{ \frac{\Omega_i}{k_i} [1 - \exp(-k_i[|\bar{x}| - L_i])] + (-1)^i k_i \nu_i \right\} \times \\ &\quad \exp(-k_i L_i) \end{aligned} \right\} \quad (\text{A-13})$$

List of Symbols

Upper subindexes indicate the type of ion, and lower subindexes indicate the compartment of the membrane

system. All the values with subindex 1 deal with the left part of the considered system; those with subindex 2 deal with the right part. The subindexes “+” and “−” indicate cations and anions, respectively. The subindex “±” indicates both possibilities. The subindex d indicates dipole layers, the subindex p indicates polar zones, and the subindex h indicates the hydrophobic layer. α , β , γ , γ_i , θ , ω , Ω , σ_{oi} , σ_{pi} , F , G , A , B , b_1 , and b_2 are the auxiliary parameters defined in the text. In particular, $\beta = e/k_B T = 1/25.7 \text{ mV}^{-1}$ and $\alpha = 4\pi\beta/\epsilon$.

| | |
|-------------------|---|
| c_1, c_2 | left and right bulk solution concentration, respectively |
| $c^\pm(x)$ | cation and anion concentration, respectively |
| c_b^*, c_a^{**} | blocking concentrations |
| d | thickness of the hydrophobic layer of the membrane |
| D_m^\pm | diffusion coefficient of the cation and anion, respectively, of the m part of the membrane system |
| F | Faraday's constant |
| j^\pm | ionic flux of the cation and anion, respectively |
| j | total ionic flux |
| k_i | the inverse Debye length of the i solution |
| k_B | Boltzmann's constant |
| L_i | thickness of polar zone i |
| L^* | blocking polar zone thickness |
| P_m^\pm | permeability of the cation and anion, respectively, of the m part of the membrane system |
| P | total permeability of the membrane system |
| T | absolute temperature |
| x | spatial coordinate |
| z^\pm | valency of the cation and anion, respectively |
| ϵ | dielectric constant of the aqueous solution |
| ϵ_h | dielectric constant of the hydrophobic zone |
| ϵ_p | dielectric constant of the polar zone |
| η | permselectivity |
| $\phi(x)$ | electric potential |
| $\psi(x)$ | dimensionless electric potential: $\psi(x) = \beta\phi(x)$ |
| $\psi_{pi}(x)$ | electric potential profile in polar zone i |
| $\rho_i(x)$ | volume charge density in polar zone i |
| σ_i | surface charge density in polar zone i (for a symmetrical system $\sigma_1 = \sigma_2 = \sigma$) |
| ν_i | surface dipole density in polar zone i (for a symmetrical system $\nu_1 = \nu_2 = \nu$) |

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