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A New Coupling Term for Dual-Porosity Models

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Numerous models are available for simulating dual-porosity flow of groundwater, heat, or solute in a fractured porous medium. Each approach includes various approximations and results in models of varying complexity. One of the simplest models has been proposed by both *Barenblatt et al.* (1960) and *Warren and Root* (1963). It assumes that the flow exchange (of fluid, heat, or solute) between the two porosity systems can be written as a linear function of the difference between averaged driving potentials (pressure, temperature, or concentration). This has been called a quasi-static formulation because it can be shown to be correct only for long dimensionless time. A new formulation of this model is proposed here which retains the original simplicity but is not restricted to very slow transient flows. The integral method is used to derive the coupling term for the early parts of the transient regime.

INTRODUCTION

Dual-porosity models of varying complexity have been used to simulate the flow of fluid, heat, or solute in a fractured porous medium. Such models are required when one source of porosity provides the transmissive path for transport and the other provides most of the storage capacity. Generally, the fracture flow system provides the transport path of least resistance, and the pores in the matrix blocks provide the largest portion of the storage capacity. In this case, coupling between the two systems must be addressed. The coupling accounts for the transfer of fluid, heat, or solute between the two porosity systems.

Most models initially consider a system of parallel fractures that align with the flow direction (Figure 1). A tortuosity factor must be included in the fracture flow equations when the fractures are not parallel or aligned with the flow direction. Unequal spacing of the fractures needs to be considered, although it will be assumed here that a single representative fracture spacing is suitable. Finally, the existence of fractures perpendicular to the flow should be considered when considering unsaturated flow because these may offer a barrier to the flow in the matrix pore flow system. However, unsaturated flow will not be considered here.

The following discussion reviews many of the different types of models that have been previously considered. References are made to the originating papers of each model type. A more complete review of the previous work is given by *Berkowitz et al.* [1988].

One model for dual-porosity flow was proposed by *Martinez* [1988]. In this model, similarity solutions of the flow from the fracture to the matrix pore system were coupled to the fracture flow solution via a source term. This approach is similar to that used by *Bibby* [1981] to calculate solute transport in a saturated fractured medium. The similarity solution requires that the fracture spacing be sufficiently large so that the effect of neighboring fractures not be felt. This requirement limits the model to early times in transient flows. When solving the heat, solute, or saturated flow problem, the governing differential equations are linear. In this case, convolution integrals can be used to calculate a variety of different applications. The nonlinearity of the unsaturated pressure problem limits the application of this method to a finite set of applications. The Martinez/Bibby model requires that the transport in the matrix pore system parallel to the flow direction be negligible.

The *Neretnieks* [1980] and *Neretnieks and Rasmuson* [1984] model also couples a one-dimensional solution for the diffusion of solute in the matrix to a multidimensional fracture transport calculation. This coupling is similar to the Martinez/Bibby model. The matrix pore system is partitioned perpendicular to the fracture at different locations. The finite difference solution of a matrix flow equation in the direction perpendicular to the fracture yields the source/sink term for the fracture flow equation. Matrix transport is restricted to a direction perpendicular to the fracture. Whereas the Bibby model requires storage of the mathematical result at previous times to allow calculation of the convolution integral, the Neretnieks model requires storage of the mathematical solution at various matrix node points. This storage allows solution of a variety of applications and is not limited by the existence of similarity solutions.

The *Barenblatt et al.* [1960] model is similar except that only one pressure is used to represent the matrix pore system at each location. A similar model is proposed by *Warren and Root* [1963]. Both of these models apply only to slow transient applications because only the slowest term in a Fourier expansion is retained [*Dykhuizen*, 1987]. However, they are not limited to flows that are dominated by the fracture network. Therefore these models can be used for a domain that includes both saturated and unsaturated regimes, where fractures may be almost dry. The coupling term in this model is derived for various geometries by *van Genuchten and Dalton* [1986].

Peters and Klavetter [1988] proposed a model similar to that of Barenblatt except they assumed that the coupling of the two flow systems was instantaneous. This results in a single equivalent porous media. However, because it is not a dual-porosity model, it is not considered here.

Finally, the fracture and matrix systems can be modeled using a discrete mesh that explicitly includes fractures as found in the field or accounts for them by using repeating representative blocks [e.g., *Fogden et al.*, 1988; *Wang and Narasimhan*, 1985; *Nuttall and Ray*, 1980]. Using a discrete

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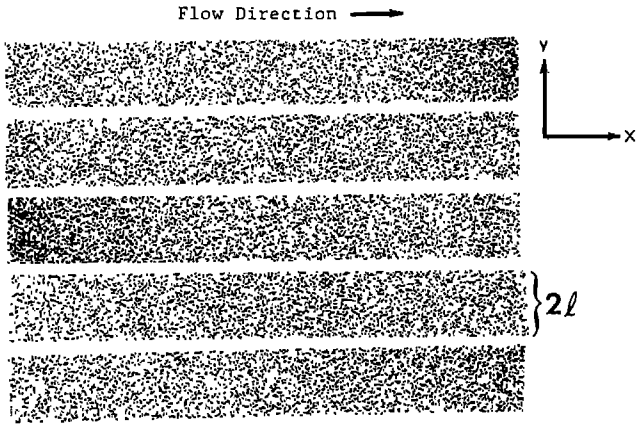

Fig. 1. Simplistic parallel fracture geometry.

mesh typically requires the greatest numerical effort, but it has no assumption that limits its accuracy for transient applications. Models of this sort typically allow modeling the fracture system in one less dimension than required for the matrix system [*Noorishad and Mehran*, 1982]. *Grisak and Pickens* [1980] developed a similar model but ignored transport in the matrix system parallel to the fractures, so their model is more like those described previously.

THEORY

For dual porosity flow the following equation set is proposed for the pressure field Ψ (similar to the Barenblatt model):

$$C_m \frac{\partial \overline{\Psi_m}}{\partial t} = \nabla[K_m \nabla \overline{\Psi_m}] + W_1 \tag{1}$$

$$C_f \frac{\partial \Psi_f}{\partial t} = \nabla[K_f \nabla \Psi_f] - W_1 \tag{2}$$

Each flow network (the matrix pore system and the fracture network) is assumed to constitute a continuum, and therefore all values represent averages over a representative elementary volume. This averaging should not be confused with the averaging required to obtain pressures for the matrix pore system where large gradients may appear as the fracture surface is approached. The overbars denote the averaging over the matrix pore system between fracture surfaces. No such distinction is made for the fracture flow system because the fractures are typically so narrow and diffusion so rapid that the local value is very close to the average value.

In equations (1) and (2) the capacitances C and conductivities K have been weighted by the area (or volume) fraction of each of the flow systems; W_1 represents the coupling term or cross flow from the fracture system to the matrix pore system. These equations can be rewritten to represent the transport of solute (where B represents the solute concentration [*Dykhuizen*, 1987]),

$$\hat{C}_m \frac{\partial \overline{B_m}}{\partial t} = \nabla[\hat{K}_m \partial \overline{B_m}] + W_1(B_x - \overline{B_m}) - V_m \frac{\partial \overline{B_m}}{\partial t} + W_2 \tag{3}$$

$$\hat{C}_f \frac{\partial B_f}{\partial t} = \nabla[\hat{K}_f \nabla B_f] - W_1(B_x - B_f) - V_f \frac{\partial B_f}{\partial t} - W_2 \tag{4}$$

or the transport of heat (where T represents temperature),

$$\bar{C}_m \frac{\partial \overline{T_m}}{\partial t} = \nabla[\bar{K}_m \nabla \overline{T_m}] + W_1(T_x - \overline{T_m}) - V_m \frac{\partial \overline{T_m}}{\partial t} + W_3 \tag{5}$$

$$\bar{C}_f \frac{\partial T_f}{\partial t} = \nabla[\bar{K}_f \nabla T_f] - W_1(T_x - T_f) - V_f \frac{\partial T_f}{\partial t} - W_3 \tag{6}$$

In the above four equations, submodels are required for the incorporation of appropriate capacitances and conductivities. The submodels will not be considered here. The solute and heat equations include two extra terms that represent convection of the conserved quantity, both within and between the two-flow systems. The subscript x denotes the temperature or the solute concentration of the cross flow between the flow systems [*Dykhuizen*, 1987]. The solute and heat equations are, otherwise, quite similar to the flow equation. If the coupling between the two systems is diffusive, it is expected that the coupling term (at the far right in each equation) will have an identical form in all three equation sets.

To apply the above equations, a model is required for the coupling term. This is the essence of this paper and will be considered here. For simplicity, only the flow equation will be used in the following derivation of the coupling term. Pressure will represent temperature, concentration, or pressure.

The integral approximation method will be used to derive the coupling term. Classical works providing foundations for the integral approximation are given by *Pohlhausen* [1921] and *Goodman* [1964].

Later classic applications of this method include those by *Eckert and Drake* [1972] and *Schlichting* [1968]. In a very recent paper, *Zimmerman and Bodvarsson* [1989] use the integral method for unsaturated flow of fluid in a porous media. This work is similar to what is done in this paper, except it only applies to early time transients or infinite fracture spacings.

In implementing this method it is assumed that the fractures form a parallel set of planes in the direction of the flow and the existence of perpendicular fractures do not effect the results. It is further assumed that the flow conditions are such that the flow in the matrix pores is negligible compared to the flow in the fractures. Also, the flow of groundwater in the matrix pores is primarily perpendicular to the fractures.

A submodel for the matrix pores between the fractures is now considered. For the above assumptions the following equation applies in the matrix pores where the fractures are in planes of constant y and the flow is in the x direction:

$$C_m \frac{d}{dt} \int_0^l \Psi_m(x, y, t)\, dy = K_m \left.\frac{\partial \Psi_m}{\partial y}\right|_{y=l} \tag{7}$$

where $y = 0$ is midway between fractures, $2l$ is the fracture spacing, Ψ_m is the matrix pressure, C_m is the matrix capacitance, and K_m is the matrix conductivity.

To use the above equation, a pressure profile in the matrix pore system must be assumed. The accuracy of the solution is strongly affected by the accuracy of the assumed profiles.

In this paper, two separate profiles will be used: one will be used for early times and one for late times. This division is not arbitrary, and justifications will be offered for both the choice of the functional form and switchover time. The late time solution derivation will be shown first, because its choice determines many of the parameters for the early time solution.

First, assume that the fluid pressure in the fractures, which surround the matrix pore system, is a known function of time. (The coupling of the two systems will be presented later.) Because the fluid pressure will be continuous, the matrix pore fluid pressure at the fracture edge ($y = l$) will be the same as the fracture fluid pressure:

$$\Psi_m(x, l, t) = \Psi_f(x, t) \tag{8}$$

Assuming that the fracture fluid pressure varies smoothly and monotonically at large values of time, the fundamental mode of the matrix solution will be dominant [*Carslaw and Jaeger*, 1959]. The fundamental mode is the one with the largest time constant. Therefore the following functional form is proposed:

$$\Psi_m(x, y, t) = [\Psi_f(x, t) - \Psi_{m0}] \cdot \left[1 - f(x, t) \cos \frac{\pi y}{2l}\right] + \Psi_{m0} \tag{9}$$

where $f(x, t)$ is function of time and position, and Ψ_{m0} is an assumed constant initial matrix pressure level. It will be shown that the function $f(x, t)$ will not have to be determined, for it will be eliminated via the integral method.

Using (9), (7) becomes

$$\frac{d\overline{\Psi_m}}{dt} = \frac{K_m \pi}{C_m 2l^2} f(x, t)[\Psi_f(y, t) - \Psi_{m0}] \tag{10}$$

where

$$\overline{\Psi_m}(y, t) = \frac{1}{l} \int_0^l \Psi_m(x, y, t)\, dy \tag{11}$$

Using (9) and (11) to relate $f(x, t)$ to the mean matrix and fracture system fluid pressures, the following is obtained:

$$\overline{\Psi_m} = \Psi_f - [\Psi_f(y, t) - \Psi_{m0}] \frac{2f(x, t)}{\pi} \tag{12}$$

Equation (12) is used to eliminate the function $f(x, t)$ in (10). This yields the governing equation for large times

$$\frac{d\overline{\Psi_m}}{dt} = \frac{K_m}{C_m} \left(\frac{\pi}{2l}\right)^2 [\Psi_f(y, t) - \overline{\Psi_m}(y, t)] \tag{13}$$

Equation (13) was also derived by Laplace transform methods [*Wilson and Dudley*, 1987] or series expansion [*Dykhuizen*, 1987] for the quasi-steady approximation.

It is desired to match the above solution, which is appropriate for late times, to a solution that will yield accurate simulations for early times. Obviously, at early times the assumed profile (equation (9)) does not apply. In fact, a boundary layer is formed that travels from the fracture toward the center of the matrix block. Therefore the following profile will be assumed:

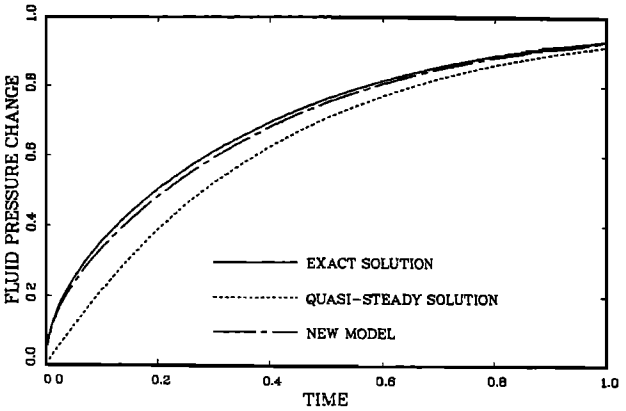

Fig. 2. Comparison of models for a step change in fracture fluid pressure. (Time nondimensionalized via equation (17).)

$$\Psi_m(x, y, t) = \Psi_{m0} + (\Psi_f(x, t) - \Psi_{m0}) \cdot \left[1 - \cos \frac{\pi(y - \delta(x, t))}{2(l - \delta(x, t))}\right] \quad \text{for } y > \delta(x, t) \tag{14a}$$

$$\Psi_m(x, y, t) = \Psi_{m0} \quad \text{for } y < \delta(x, t) \tag{14b}$$

where δ represents the distance to the boundary layer from the center of the matrix block. Initially, δ would be equal to l (the half fracture spacing).

Using (7) and (11) in a manner similar to that above, the governing equation for early times is obtained:

$$\frac{d\overline{\Psi_m}}{dt} = \frac{K_m}{C_m} \left[\frac{\Psi_f(x, t)}{l}\right]^2 \frac{(\pi/2 - 1)}{\overline{\Psi_m}(x, t)} \tag{15}$$

The choice of the switchover time now becomes clear. When the boundary layer is equal to the half fracture spacing, the two profiles will be identical ($f(x, t)$ will be unity at the transition and $\delta(x, t)$ will be zero). This condition makes the numerical value of the coupling term the same, since the flux at the boundary will be identical. This continuity will assure a smooth transition. The condition can be found by using the transition profile in (11) to obtain the following:

$$\overline{\Psi_m}(x, t) = [\Psi_f(x, t) - \Psi_{m0}](1 - 2/\pi) + \Psi_{m0} \tag{16}$$

APPLICATIONS

Before using this model to simulate a coupled transient flow in a fracture porous medium, some simple test problems were attempted to evaluate how this model compares to the quasi-steady model. To accomplish this, a nondimensional time is introduced:

$$\tau = K_m t / C_m l^2 \tag{17}$$

Two tests were run. The first shows the response of the average matrix fluid pressure to a step change in the fracture fluid pressure. Figure 2 shows a comparison of the exact solution, the quasi-steady solutions, and the new model presented here. As shown, the new model results in a significant improvement. The new model resulted in a switchover from the early (high) coupling term to the quasi-steady coupling term at a nondimensional time of 0.23. The

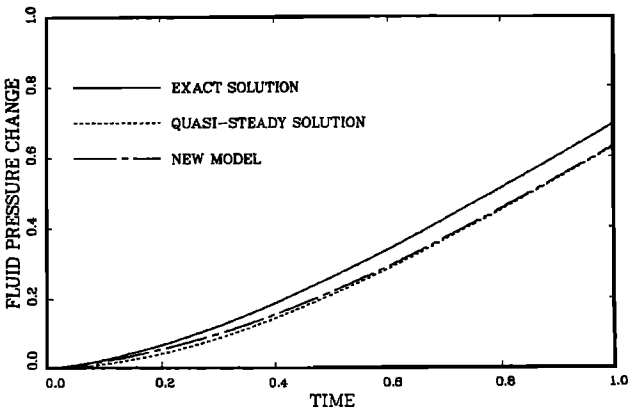

Fig. 3. Comparison of the models for a unit ramp in the fracture fluid pressure. (Time nondimensionalized via equation (17).)

switchover yields a smooth transition, and no discontinuity in either the value or the slope of the matrix pressure results.

The second test (Figure 3) shows the response of the average matrix fluid pressure to a unit ramp of the fracture fluid pressure. Again, the new model exhibits some improvement.

The final test is to calculate an example problem where the two solute concentrations are actually coupled. The following equation set is postulated here to simulate solute transport in a dual porosity medium:

$$\frac{\partial B_f}{\partial t} + \frac{V_f}{\theta_f}\frac{\partial B_f}{\partial x} - D_f\frac{\partial^2 B_f}{\partial x^2} = \frac{-W_2}{\theta_f}$$
$$\frac{\partial \overline{B_m}}{\partial t} = \frac{W_2}{\theta_m} \tag{18}$$

where θ represents porosities, D represents the diffusion/dispersion coefficient, and W_2 is the coupling term. For comparison with (3) and (4) the solute conductivities $\hat{K}$ are assumed to be equal to θD, and the capacitances $\hat{C}$ are equal to θ.

The model for the coupling term is obtained from (15) for early times and (13) for late times. Therefore, using the above definitions for conductivities and capacitances, the following is obtained:

$$W_2 = \frac{\theta_m D_m}{l^2}(\pi/2 - 1)\frac{B_f^2}{\overline{B_m}} \quad \text{if } \overline{B_m} < B_f(1 - 2/\pi)$$
$$W_2 = \frac{\theta_m D_m \pi^2}{2l^2}(B_f - \overline{B_m}) \quad \text{if } \overline{B_m} > B_f(1 - 2/\pi) \tag{19}$$

This equation set ((18) and (19)) will yield averaged fracture and matrix solute concentration as a function of distance and time. Note that convection and diffusion along the flow direction are assumed to be negligible in the matrix pores. For this problem the fracture concentration is of primary interest because transport in the fractures is dominant. Therefore the fracture concentration results will be presented.

The quasi-steady model uses the late time formulation for the coupling term at all times. At early times this approximation underestimates the extent of the coupling. In fact, in the limit of infinite fracture spacing, the coupling term goes to zero in both the fracture and the matrix governing equations. However, if the early time choice is used, the coupling remains finite in the fracture system equation. This term remains finite because the matrix concentration and the fracture porosity will both scale inversely with the fracture spacing.

TABLE 1. Parameters for Calculating Example Solute Transport Problem

	Value	Unit
V_f	1. E-10	m/sec
θ_f	1.E-4	—
θ_m	.1	—
D_f	1.E-11	m^2/sec
D_m	1.E-11	m^2/sec
l	0.075	m

Read 1.E-10 as $1. \times 10^{-10}$.

The governing equations were solved numerically by the method of lines. The mesh contained 35 nodes 0.3333 cm apart. Table 1 presents the parameters used. The results from both the quasi-steady model and the new model are presented in Figure 4, as is an analytical solution to a similar problem for comparison. The analytical solution corresponds to the same equation set except that there is no axial diffusion in the fracture flow system and it uses an infinite fracture spacing [*Carslaw and Jaeger*, 1959]. The time scale in Figure 4 is scaled by the fracture travel time and thus is nondimensional. The following equation defines the time scaling for Figure 4:

$$\tau = V_f t / X\theta_f \tag{20}$$

where X is 10 cm, the output position.

This scaling results in the solute front arriving at the output location at a dimensionless time of unity, if the coupling is deleted from the model.

As can be seen, the quasi-steady model indicates little delay of the solute transport for this problem. This is because the coupling term is almost zero in the quasi-steady model of this problem. The new model provides a much better estimate of the solute transport.

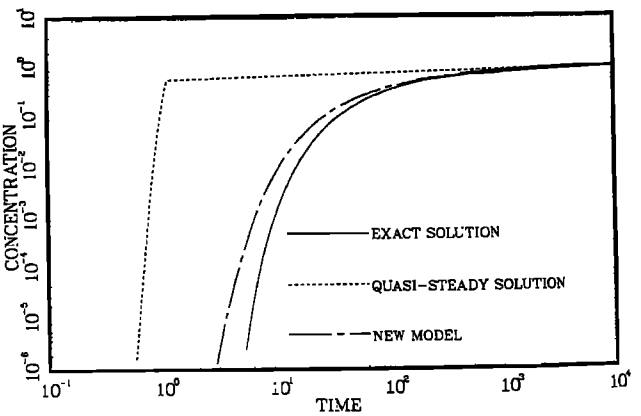

Fig. 4. Comparison of breakthrough curves for 10-cm depth (fracture concentration). (Time nondimensionalized via equation (20).)

DISCUSSION

The parameters used in these calculations were chosen to illustrate the large differences between the two models. These differences arise when the quasi-steady model time constant (see equation (17)) for diffusion into the matrix material is large (3 orders of magnitude larger in this case) compared to the fracture travel time. This resulted in the new model primarily using the early time coupling term. The early time coupling is much larger and resulted in an order-of-magnitude increase in the amount of solute diffusing into the matrix pores (although the average matrix concentration still remains 2 orders of magnitude below the fracture concentration). Therefore the arrival of the solute is shown to be significantly delayed in the new model.

The difference between the new model and the analytical result can be attributed to four causes. The largest source of error is numerical diffusion present in the solution of the finite-differenced equations. The slope of the breakthrough curve of the new model at low concentrations is very sensitive to numerical diffusion. The run presented was performed with very small node spacings and central differences to minimize the effect of numerical diffusion. Other runs with larger node spacings and upwind differences resulted in a more gradual rise in the breakthrough curve. However, the region between dimensionless concentrations of 0.1 and 1.0 were hardly effected. The second cause of a difference is the treatment of the fracture spacing. The analytical solution corresponds to a fluid flow of the same velocity as the new model but with infinite fracture spacing. Therefore the analytic model did not include the effect of solute from a neighboring fracture. Because this does happen in the new model solution near the top of the simulation region, the new model resulted in a somewhat earlier arrival of the solute downstream.

The last two causes for the differences result in relatively minor errors. The new model includes axial diffusion, which was determined to be very small in this case but was not included in the analytical solution. Finally, the approximations in the integral method result in some disagreement with the analytical solution.

It is interesting to note that this coupling term has been investigated for many years in the chemical engineering literature. *Gluekaug* [1955] reviews a series of approximate forms for the coupling term in the analysis of spherical ion exchange column particles. These proposed forms include one that is identical to the late time form presented here. Another one of the proposed forms for the coupling term was very similar to the early time form presented here; however, it included a second term to prevent overshoot of the equilibrium condition. It is felt that in this paper presents the first attempt at using two different forms of the coupling term to enable accurate simulation of both the early and late behavior.

CONCLUSIONS

A new model has been derived for the coupling of two flow systems in a dual porosity model. The new formulation eliminates quasi-steady approximation in deriving the coupling term. This approximation has been described by *Berkowitz et al.* [1988] as a major shortcoming. The only extra calculation required is a choice in the coupling term based upon a switchover relation. This model can be used for the transport of fluid, heat, or solute by simply switching the potential used in the formulation.

The model is best applied to the flow of heat or solute where the governing differential equation for the matrix submodel is linear. For the transport of heat or solute in the matrix subsystem a linear governing differential equation is typically assumed, and therefore the assumed profiles will be very good. The flow of groundwater in a saturated or nearly saturated medium will also be linear. However, application of this method to the nonlinear flow of a highly unsaturated medium may require different functional forms for the matrix pore system profiles [*Zimmerman and Bodvarsson*, 1989]. Also, the model relies on functional shapes that are appropriate only for monotonically varying transient pressures. Use of the model for oscillatory boundary conditions will not yield good results.

The new coupling term is nonlinear. This makes the solution somewhat more difficult than the quasi-steady model (which is linear), but numerical solutions are typically always used. Because of the decreased stiffness of the model equations the new model can often take orders-of-magnitude less computer time for completion. Example problems are presented which show improved predictions of solute transport with the new model.

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