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MARTI ON DESCRIPTIONS IN CARNAP'S S2

ABSTRACT. This note is a friendly amendment to Marti's analysis of the failure of Føllesdal's argument that modal distinctions collapse in Carnap's logic S2. Føllesdal's argument turns on the treatment of descriptions. Marti considers how modal descriptions, which Carnap banned, might be handled; she adopts an approach which blocks Føllesdal's argument, but requires a separate treatment of non-modal descriptions. I point out that a more general treatment of descriptions in S2 is possible, and indeed is implicit in Marti's informal discussion, and that this treatment also blocks Føllesdal's argument. Further, I show by a semantic argument that no revised version of Føllesdal's argument could establish a collapse of modal distinctions.

This note is a friendly amendment to Marti's recent analysis[1] of the failure of Føllesdal's argument that modal distinctions collapse in Carnap's modal logic S2. Føllesdal's argument turns on the treatment of descriptions. Marti considers how descriptions containing modal operators, which Carnap banned from his system, might be handled; she adopts an approach to such descriptions which blocks Føllesdal's argument, but is restricted to modal descriptions and requires a separate treatment of non-modal descriptions. I point out that a more general treatment of descriptions in Carnap's logic is possible, and indeed is implicit in Marti's informal discussion, and that this treatment also blocks Føllesdal's argument for the collapse of modal distinctions. Further, I show by a semantic argument that no revised version of Føllesdal's argument could establish such a collapse of modal distinctions. I will assume familiarity with Marti's paper in the interest of brevity.

Carnap's contextual analysis of non-modal descriptions took the form:

$$\text{(C)} \qquad Q(\iota x Px) \quad =_{\text{df}} \exists y(\forall x(Px \leftrightarrow x = y) \,\&\, Qy) \vee (\sim \exists y \forall x(Px \leftrightarrow x = y) \,\&\, Qa^*).^2$$

As Marti points out, there is a difficulty in extending (C) to modal descriptions. For example, on (C) descriptions of the form $\iota x \Box Px$ will denote a^* in almost all cases (see below). The problem here can be brought out by considering the condition:

$$(*) \qquad \exists y \forall x(Px \leftrightarrow x = y)$$

which is required for $\iota x Px$ to count as proper. We expect $(*)$ to say that there is exactly one P. But if P contains modal operators, $(*)$ is much stronger, since the $\leftarrow$ half of $(*)$ says not only that Py but that Px for *any* individual concept x which *coincides* with y in the actual state-description. To get around this Marti proposes to strengthen $(*)$ to

(M*) $\quad \exists y \forall x (Px \leftrightarrow \Box x = y)$

and to modify (C) accordingly, when Px contains modal operators (call this modified analysis '(CM)').[3] Marti shows that Føllesdal's argument for the collapse of modal distinctions fails to go through, given (CM).

However, (CM) is not entirely satisfactory. First, as Marti points out, (CM) cannot be used for non-modal descriptions. For such descriptions the original analysis (C) must be retained; otherwise all non-modal descriptions will denote a^*. Thus we do not get a single treatment of all descriptions. Secondly, (CM) does not accord with Marti's own intuitions concerning the denotations of descriptions of the form $\iota x \Box Px$. She writes: "[s]uppose that there is an individual concept whose object-value in each state-description satisfies Px and whose object-value in the actual state-description uniquely satisfies Px. That object-value should be the descriptum." (p. 584) In her footnote 20, she adds that "[t]here may be more than one such individual concept. . . What is relevant here is that all concepts whose object-values satisfy Px at every state-description coincide in their object-value at the actual one." However, on (CM) in such a case the description will be improper and denote a^*. (CM) requires that there be exactly one individual concept making $\Box Px$ true for $\iota x \Box Px$ to be proper; and by Marti's own account this seems too strong.

These problems with (CM) can be easily repaired by a different modification of $(*)$ and (C), however. $(*)$ is the most compact among a family of sentences which are, in ordinary first-order logic, equivalent ways of saying that there is exactly one P. Another member of this family is:

$(**)$ $\quad \exists y (Py \,\&\, \forall x (Px \rightarrow x = y)).$

In ordinary first-order logic this is equivalent to $(*)$; but in Carnap's S2 with quantification over individual concepts and '$=$' representing coincidence, $(*)$ and $(**)$ are *not* equivalent when Px contains modal operators. In particular

(1) $\quad \exists y \forall x (\Box Px \leftrightarrow x = y)$ is not equivalent to

(2) $\quad \exists y (\Box Py \,\&\, \forall x (\Box Px \rightarrow x = y))$;

(1) can be true only if there is exactly one individual which has P in the actual state-description, and every individual has P in every other

state-description,[4] but (2) will be true in the case described by Marti on p. 584 and footnote 20 quoted above. This suggests that (2) should be the condition for treating $\iota x \Box Px$ as proper and that a better treatment of modal descriptions will result if we replace ($*$) not with (M*) but with ($**$), yielding:

$$(\mathrm{C}') \qquad Q(\iota x P x) \quad =_{\mathrm{df}} \exists y(Py \;\&\; \forall x(Px \rightarrow x = y) \;\&\; Qy) \vee \\ (\sim \exists y(Py \;\&\; \forall x(Px \rightarrow x = y)) \;\&\; Qa^*).$$

Moreover, since ($*$) and ($**$) *are* equivalent when Px contains no modal operators, (C) and (C$'$) are equivalent in that case and (C$'$) can be adopted as an analysis of *all* descriptions.

(CM), however, had the virtue of blocking Føllesdal's argument for the collapse of modal distinctions. Fortunately, it is easy to build on Marti's analysis of the argument to show that (C$'$) equally blocks it. There are three versions of the argument to consider:

(a) The original argument (pp. 577–579) employing the non-modal description $\iota x(x = u \,\&\, p)$, and arguing to $\Box \iota x(x = u \,\&\, p) = u$ via the intermediate step

$$(16) \qquad \iota x(x = u \,\&\, p) = u$$

and Necessity of Identity can be analyzed exactly as Marti did. Since (C) and (C$'$) are equivalent in this case, (16) can be established, but as Necessity of Identity does not hold in S2, nothing special follows from this.

(b) The purely modal argument (pp. 585–587) employing the description $\iota x(x \equiv u \,\&\, p)$ and attempting to prove

$$(16) \qquad \iota x(x \equiv u \,\&\, p) \equiv u$$

commits the mistake that Marti points out on p. 586, of ignoring Carnap's convention that descriptions are to be assigned narrowest possible scope. From the assumption $p \,\&\, \Box(u \neq a^*)$ we readily deduce

$$(14) \qquad \exists y((y \equiv u \,\&\, p)) \,\&\, \forall x((x \equiv u \,\&\, p) \rightarrow x = y) \,\&\, y \equiv u),$$

but it is a fallacy to use (C$'$) to infer from this to

$$(16) \qquad \iota x(x \equiv u \,\&\, p) \equiv u, \quad \text{that is:}$$

$$\Box(\iota x(x \equiv u \,\&\, p) = u).$$

To derive (16) from (C$'$), given Carnap's conventions on the scope of descriptions, we need

$$(14') \qquad \Box(\exists y((y \equiv u \,\&\, p) \,\&\, \forall x((x \equiv u \,\&\, p) \rightarrow x = y) \,\&\, y = u)),$$

which is not implied by (14). This analysis of course exactly parallels Marti's (p. 586).

(c) Finally the "mixed" argument considered in Marti's Appendix 2 (pp. 588–591) again commits the same error, detected by Marti on p. 590, of ignoring Carnap's convention governing the scope of descriptions.

To sum up, Marti's claim that "[i]f we were to introduce definite descriptions with modal operators in S2, we would have two alternatives," (C) and (CM), can be strengthened to include a third, most plausible account of descriptions, (C$'$), without impugning her conclusion that "[i]n any case, Føllesdal's contention that Carnap's system is saved by the restriction on the internal structure of definite descriptions is not justified." (pp. 586–587)

Marti's analysis thus shows that Føllesdal's specific argument for the collapse of modal distinctions in S2 is fallacious; however it remains *a priori* conceivable that a revised version of the argument might yet establish such a collapse. I conclude with a sketch of a semantic proof that modal distinctions do not collapse in S2 with modal descriptions analyzed contextually by any of (C), (CM), (C$'$). In *Meaning and Necessity*, Carnap presents S2 semantically, not proof-theoretically, yet offers a contextual analysis of definite descriptions, no doubt inspired by Russell. I present Carnap's semantics in the now-standard possible-worlds framework, rather than using his own terminology of state-descriptions. Work with a language for quantified modal logic with identity, with connectives $\rightarrow$, $\sim$, and $\Box$, quantifier $\forall$, individual constants, n-ary predicates for each n, and a distinguished individual constant a^*, designating the referent of "bad" descriptions. For Carnap's original S2, include a description operator ι which forms a term $\iota x A$ whenever A is a formula containing *no* modal operators.

A model is a quintuple $\langle W, @, D, d^*, I\rangle$ where W ("worlds") and D ("domain of objects") are non-empty, $@ \in W$ ("actual world"), $d^* \in D$ (denotation for "bad" descriptions) and I is an interpretation function; if c is an individual constant, $I(c) \in D$; $I(a^*) = d^*$; if P is an n-ary predicate and $w \in W$, $I(P, w) \subseteq D^n$.

Quantifiers range not over D, but over *individual concepts*, i.e. functions from W to D. Let an assignment a be a function mapping each variable x to an individual concept; let $a[i/x]$ be that assignment differing from a at most in mapping x to i. Given a model $M = \langle W, @, D, d^*, I\rangle$, define a function V_M assigning members of D to terms and truth-values to formulas, relative to world w and assignment a: $V_M(x, w, a) = (a(x))(w)$ for each variable x. $V_M(c, w, a) = I(c)$ for each constant c. $V_M(Pt_1 \ldots t_n, w, a) = T$ iff $\langle V_M(t_1, w, a), \ldots, V_M(t_n, w, a)\rangle \in I(P, w)$.

$V_M(t_1 = t_2, w, a) = T$ iff $V_M(t_1, w, a) = V_M(t_2, w, a)$. $V_M(\sim A, w, a) = T$ iff $V_M(A, w, a) = F$. $V_M(A \to B, w, a) = T$ iff $V_M(A, w, a) = F$ or $V_M(B, w, a) = T$. $V_M(\forall x A, w, a) = T$ iff $V_M(A, w, a[i/x]) = T$ for every i. $V_M(\Box A, w, a) = T$ iff $V_M(A, w', a) = T$ for every $w' \in W$.

If A is closed then for any assignments a and a', $V_M(A, w, a) = V_M(A, w, a')$ if a is any assignments, set $V_M(A, w) = V_M(A, a, w)$. Furthermore set $V_M(A) = V_M(A, @)$.

Given the convention that descriptions have narrowest scope, Carnap's contextual definition (C) of $\iota x A$ corresponds to a semantic treatment of descriptions as genuine terms, whose denotation is specified by the clause: (CS) $V_M(\iota x A, w, a) = d$ if for some i: (1) $i(w) = d$; *and* (2) for every i', if $i'(w) = d$, then $V_M(A, w, a[i'/x]) = T$; *and* (3) for every i', if $i'(w) \neq d$, then $V_M(A, w, a[i'/x]) = F$; $V_M(\iota x A, w, a) = d^*$ otherwise. We can take S2 to be the set of all sentences true in all models, given this additional semantic clause.

Since A contains no modal operators, (2) in (CS) can be replaced by: (2′) $V_M(A, w, a[i/x]) = T$. Thus we get the expected denotations for descriptions containing no modal operators; but if we enrich the language with modal descriptions, we can no longer replace (2) with (2′) in (CS), and (CS) implies, for example, that $V_M(\iota x \Box P x, w, a) = d^*$, *unless* (a) $I(P, w) = \{d\}$ $(d \neq d^*)$ *and* (b) $I(P, w') = D$ for all $w' \neq w$. This motivates the adoption of one of the following alternative semantics, corresponding to the contextual definitions (CM) and (C′) discussed above: (CMS) When A contains no modal operators, $V_M(\iota x A, w, a) = d$ if for some i: (1) $i(w) = d$; *and* (2) $V_M(A, w, a[i/x]) = T$; *and* (3) for every i', if $i'(w) \neq d$, then $V_M(A, w, a[i'/x]) = F$; $V_M(\iota x A, w, a) = d^*$ otherwise. When A contains modal operators, $V_M(\iota x A, w, a) = d$ if for some i: (1) $i(w) = d$; *and* (2) $V_M(A, w, a[i/x]) = T$; *and* (3) for every i', if $i' \neq i$, then $V_M(A, w, a[i'/x]) = F$; $V_M(\iota x A, w, a) = d^*$ otherwise. (C′S) $V_M(\iota x A, w, a) = d$ if (1) for some i, $V_M(A, w, a[i/x]) = T$; *and* (2) for every i, if $V_M(A, w, a[i/x]) = T$, then $i(w) = d$; $V_M(\iota x A, w, a) = d^*$ otherwise.

It is easy to check that under (CS), (CMS), (C′S), respectively all instances of (C), (CM), (C′) are valid. All that needs to be borne in mind is that descriptions are always given narrowest possible scope! Yet there are clearly models in which modal distinctions do not collapse; every model of the language without modal descriptions can be extended to a model of the language with modal descriptions under any of (CS), (CMS) or (C′S), and so any of (C), (CM), (C′) yields a conservative extension of S2. For example, if we consider a model M in which $D = \{d^*\}$, $W = \{@, w\}$, $I(P, @) = \{d^*\}$, $I(P, w) = \varnothing$, then Pa^* will be true in

M, $\Box Pa^*$ false in M, and the presence of modal descriptions will in no way change this, no matter how they are interpreted. Consequently, not only does Føllesdal's argument fail to show the collapse of modal distinctions in S2, but no revised version of the argument is waiting in the wings to take its place.

NOTES

[1] "Do Modal Distinctions Collapse in Carnap's System?' *Journal of Philosophical Logic* 23 (1994), 575–593. All references are to this article.

[2] Here a^* is the stipulated value for "improper" descriptions; as pointed out by Marti, Carnap applied this analysis under the convention that descriptions always have narrowest possible scope.

[3] She explicitly endorses such an analysis only for descriptions of the form $\iota x \Box Px$; but it seems implicit in her discussion that the analysis should extend to all descriptions containing modal operators.

[4] An anonymous referee pointed out an error in a previous version both here and at the corresponding point in the discussion of the semantic model below.

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