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A theory of the photomagnetoelectric effect with injection-level-dependent lifetime [a)]

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The theory of the photomagnetoelectric effect has been worked out for the case in which the lifetime is dependent on the injection level and the recombination mechanism is described by the Shockley-Read statistics. Under the small signal hypothesis, the nonlinear continuity equation has been solved by perturbative method. An example is presented which evidences the deviation of the theoretical behavior of the spectral distribution of the short-circuit current from the standard case in which the linear continuity equation is considered and the lifetime is constant.

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I. INTRODUCTION

In the analysis of carrier transport in semiconductors, the photomagnetoelectric (PME) effect has been widely used together with photoconductivity (PC) measurements.[1-3]

The theory of the PME effect has been developed by various authors under a variety of conditions including the independence of the lifetime of the injection level. Very few authors have introduce an empirical functional dependence of the lifetime τ on the excess carrier concentration Δn to explain the experimental behavior of the I_{PME} current versus the light intensity,[4] and few approaches have been made to explain the special distribution of the I_{PME} current and the photoconductance ΔG in the presence of trapping.[5,6]

In this work, we develop a theoretical approach which takes into account a dependence of the lifetime on the injection level assuming a recombination mechanism of the holes and electrons in the semiconductor described by the Shockley-Read statistics.[7]

A perturbative method is used in order to solve the (nonlinear) continuity equation; the solution is a series of functions convergent in a wide range of values of the perturbative parameter. The expressions of I_{PME} and ΔG are also calculated, considering only the first two terms of the perturbative series and their dependence on both wavelength and the intensity of the incident light is discussed.

II. THEORY

The continuity equations for the charge carriers through a semiconductor, under arbitrary carrier generation rate and in the steady-state condition, are the following:

$$-\frac{\Delta n}{\tau_n}+\tfrac{1}{q}\operatorname{div}\mathbf{J}_n+g(x,y,z,t)=0\,, \tag{2.1}$$

$$-\frac{\Delta p}{\tau_p}-\tfrac{1}{q}\operatorname{div}\mathbf{J}_p+g(x,y,z,t)=0\,, \tag{2.2}$$

where Δn and Δp are the excess densities of the electrons and holes τ_n and τ_p are the electron and hole lifetimes, J_n and J_p are the current densities of the electrons and holes, $g(x,y,z,t)$ is the generation rate, and finally, q is the absolute value of the electronic charge.

Let us assume: (i) local charge neutrality $\Delta n=\Delta p$, (ii) small Hall angles, (iii) incident light in the y direction, (iv) magnetic field in the z direction, (v) generation rate dependent on the y coordinate only, $g(x,y,z,t)\equiv g(y)$, and (vi) I_{PME} and I_{PC} currents in the x direction.

Under these hypotheses, the electron current density in the incident light direction, J_{ny}, is given by

$$J_{ny}=qD\frac{\partial}{\partial y}\Delta n\,, \tag{2.3}$$

where the diffusion coefficient D is

$$D=D_n\frac{1+c+2(\Delta n/n_0)}{b+c+(1+b)(\Delta n/n_0)}\,, \tag{2.4}$$

where $c=p_0/n_0$, $b=\mu_n/\mu_p$, D_n is the electron diffusion constant, n_0 and p_0 are the electron and hole concentrations in the dark, and μ_n and μ_p are the electron and hole mobilities.

We suppose that the recombination process occurs through a single flaw in the forbidden gap and obeys Shockley-Read statistics.

Assuming that the majority carrier density is greater than the density N_t of traps, it can be proved[7] that the lifetime is given by

$$\tau=\tau_{p_0}\frac{n_0+n_1+\Delta n}{n_0+p_0+\Delta n}+\tau_{n_0}\frac{p_0+p_1+\Delta n}{n_0+p_0+\Delta n}\,, \tag{2.5}$$

where τ_{p_0} (τ_{n_0}) represents the lifetime of holes (electrons) for capture by N_t centers which are completely filled (free) with (of) electrons; $n_1(p_1)$ is the number of electrons (holes) in the conduction (valence) band for the case in which the Fermi level falls at the energy level E_t of traps.

Equation (2.5) can be written in the more compact form

$$\tau=\frac{\tau_0+[\tau_\infty/(n_0+p_0)]\Delta n}{1+[\Delta n/(n_0+p_0)]}\,, \tag{2.6}$$

where τ_0 and τ_∞ are the lifetimes at very small and very high injection levels, respectively, i.e.,

[a)]Work supported in part by Consiglio Nazionale delle Ricerche, Italy.

$$\tau_0 = \lim_{\Delta n \to 0} \tau = \tau_{p_0} \frac{n_0 + n_1}{n_0 + p_0} + \tau_{n_0} \frac{p_0 + p_1}{n_0 + p_0},$$

and

$$\tau_\infty = \lim_{\Delta n \to \infty} \tau = \tau_{n_0} + \tau_{p_0}.$$

After some manipulations, Eq. (2.6) becomes

$$\tau = \frac{\tau_0}{1 + [\epsilon/1 + \alpha(\Delta n/n_0)](\Delta n/n_0)} \tag{2.7}$$

with

$$\epsilon = \frac{1}{1+c} \frac{\tau_0 - \tau_\infty}{\tau_0}, \tag{2.8}$$

and

$$\alpha = \frac{\tau_\infty}{\tau_0(1+c)}. \tag{2.9}$$

From Eq. (2.8) one can easily verify that the value of ϵ ranges between 0 and 1, when τ_0 is greater than τ_∞, while it is negative for τ_∞ greater than τ_0. Notice that if $\alpha \Delta n/n_0 < 1$, Eq. (2.7) can be written as

$$\tau = \frac{\tau_0}{1 + \sum_{j=0}^{\infty} \epsilon(-1)^j \alpha^j (\Delta n/n_0)^{j+1}}. \tag{2.10}$$

We notice here that it seems very difficult to treat analytically the continuity equation which arises from Eqs. (2.4) and (2.7) via Eq. (2.3). In order to obtain an analytical solution of such an equation, we have introduced the following simplifying assumptions: (1) the lifetime is given by Eq. (2.10) for $j = 0$, namely,

$$\tau \simeq \frac{\tau_0}{1 + \epsilon(\Delta n/n_0)}, \tag{2.11}$$

(2) the small injection hypothesis ($\Delta n \ll n_0$) is valid.

In this case the diffusion coefficient D can be taken constant as Δn is changing. We observe that the lifetime also depends on the injection level,i.e., on $\Delta n/n_0$, but stronger than the diffusion coefficient, particularly for $\epsilon < 1$. In this case, it is straightforward to verify that the diffusion coefficient departs from the small-signal value, as Δn is changing, much less than the lifetime does.

Thus the assumptions (1) and (2) are quite reliable for $\epsilon < 1$.

Taking into account Eq. (2.3) and assumptions (i)–(vi) together with (1) and (2), the continuity Eq. (2.1) reads

$$D \frac{d^2 \Delta n}{dy^2} - \frac{1}{\tau_0}\left(1 + \epsilon \frac{\Delta n}{n_0}\right)\Delta n = -g(y), \tag{2.12}$$

to which one can associate the usual boundary conditions at the front and back surfaces of the sample:

$$D \left.\frac{d\Delta n}{dy}\right|_{y=0} = s_1 \Delta n(0), \tag{2.13a}$$

$$D \left.\frac{d\Delta n}{dy}\right|_{y=w} = -s_2 \Delta n(w), \tag{2.13b}$$

where s_1 and s_2 are, respectively, the surface recombination velocities on the front and on the back surfaces, and w is the sample thickness.

In the following, we shall deal with the simple case in which

$$g(y) = Ik \exp(-ky), \tag{2.14}$$

where k is the absorption coefficient of the incident monochromatic radiation and I is the photon flux density per second. Moreover, for simplicity's sake we shall use the following:

$$D \left.\frac{d\Delta n}{dy}\right|_{y=w} = 0, \tag{2.13c}$$

instead of the boundary condition (2.13b).

Though Eq. (2.13c) is certainly less general than Eq. (2.13b), the first is quite adequate in a large number of experimental situations. However, using a procedure similar to that employed for the simplest case, one can also treat analytically the nonlinear continuity Eq. (2.12) with the more general boundary conditions (2.13a) and (2.13b).

III. PERTURBATIVE SOLUTION OF THE NONLINEAR CONTINUITY EQUATION

Consider Eq. (2.12) with the boundary conditions (2.13a) and (2.13c). Let us introduce the adimensional quantities $W = w/L$, $Y = y/L$, $S_1 = s_1 w/D$, where $L^2 = \tau_0 D$.

By performing the change of variable $u = \Delta n/n_0$, Eq. (2.12) and the boundary conditions (2.13a) and (2.13c) become, respectively,

$$\frac{d^2 u}{dY^2} - (1 + \epsilon u)u = N \exp(-\mu Y), \tag{3.1}$$

and

$$\left.\frac{du}{dY}\right|_{Y=0} = au(0), \tag{3.2a}$$

$$\left.\frac{du}{dY}\right|_{Y=W} = 0, \tag{3.2b}$$

where

$$a = \frac{S_1}{W}, \quad N = -\frac{IkL^2}{Dn_0}, \quad \mu = kL = \frac{K}{W}, \tag{3.3}$$

with $K = kw$ and $Y \epsilon (0, W)$.

Equation (3.1), with the boundary conditions (3.2), is a nonlinear differential equation which is not solvable, as far as we know, in terms of known functions.

We observe that the solution of Eq. (3.1) [with the boundary conditions (3.2)] depends on the parameter ϵ, which is associated to the nonlinear term. Here we look for such a solution as a power series in the independent variable ϵ, that is

$$u(Y;\epsilon) = \sum_{n=0}^{\infty} \epsilon^n u_n(Y). \tag{3.4}$$

After substitution of Eq. (3.4) into Eq. (3.1), we obtain the following set of simultaneous equations:

$$\frac{d^2 u_0}{dY^2} - u_0 = N \exp(-\mu Y), \tag{3.5}$$

$$\frac{d^2 u_1}{dY^2} - u_1 = u_0^2; \tag{3.6}$$

$$\frac{d^2u_n}{dY^2} - u_n = \sum_{k=0}^{n-1} u_k u_{n-1-k}, \tag{3.7}$$

with the boundary conditions

$$\left.\frac{du_n}{dY}\right|_{Y=0} = au_n(0), \tag{3.8a}$$

$$\left.\frac{du_n}{dY}\right|_{Y=W} = 0, \tag{3.8b}$$

for $n \geqslant 0$.

Taking account of Eqs. (3.8), the general integral of Eq. (3.7) (for $n \geqslant 1$) is given by

$$u_n(Y) = \alpha_n \exp(Y) + \beta_n \exp(-Y) + F_n(Y), \tag{3.9}$$

where

$$F_n(Y) = \sum_{k=0}^{n-1} \int_0^Y u_k(t) u_{n-1-k}(t) \sinh(Y-t)dt, \tag{3.10}$$

and

$$\alpha_n = -\frac{(1+a)G_n(W)}{(1+a)\exp(W) - (1-a)\exp(-W)}, \tag{3.11}$$

$$\beta_n = -\frac{(1-a)G_n(W)}{(1+a)\exp(W) - (1-a)\exp(-W)}, \tag{3.12}$$

$$G_n(W) = \sum_{k=0}^{n-1} \int_0^W u_k(t) u_{n-1-k}(t) \cosh(W-t)dt. \tag{3.13}$$

The general integral of Eq. (3.5), which is the linearized form of Eq. (3.1), reads

$$u_0(Y) = \alpha_0 \exp(Y) + \beta_0 \exp(-Y) + N\int_0^Y \exp(-\mu t)\sinh(Y-t)dt, \tag{3.14}$$

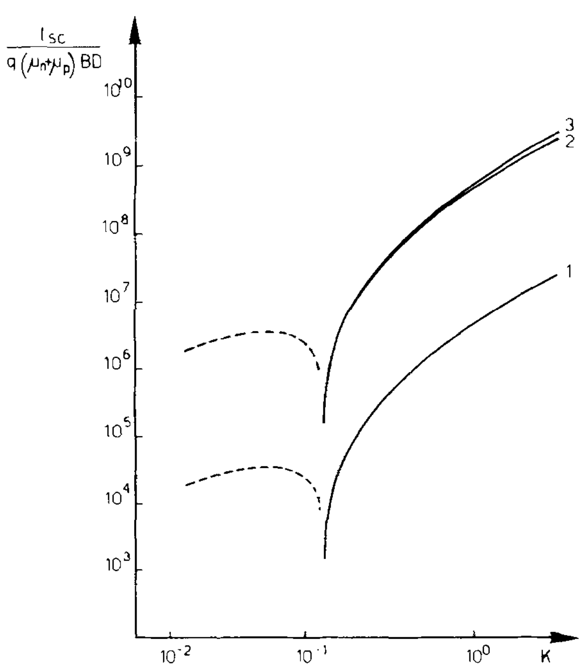

FIG. 1. Spectral distribution of the quantitiy $I_{sc}/q(\mu_n+\mu_p)BD$ (cm^{-3}) as a function of K. Dotted (solid) lines represent negative (positive) currents. Curve 1: $N_0 = 10^{-4}$ (the spectral distribution is unaffected by the value of ϵ, for ϵ ranging from 1 to -10). Curve 2: $N_0 = 10^{-2}$, $\epsilon = 0$. Curve 3: $N_0 = 10^{-2}$, $\epsilon = -10$.

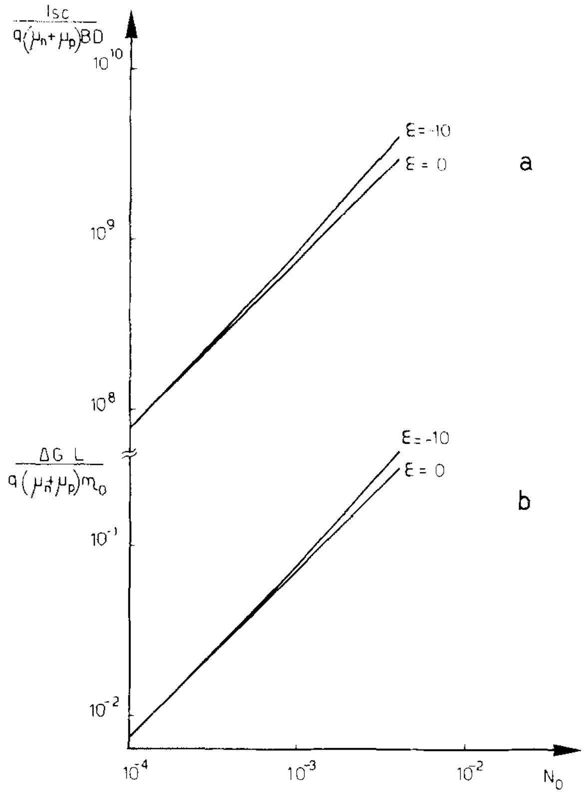

FIG. 2. Theoretical behavior of the quantities $I_{sc}/q(\mu_n+\mu_p)BD$ (cm^{-3}) and (a) $\Delta GL/q(\mu_n+\mu_p)n_0$ (cm^2), (b) as functions of the dimensionless quantity N_0 for two values of the perturbative parameter ϵ.

where

$$\alpha_0 = -\frac{(1+a)G(W)}{(1+a)\exp(W) - (1-a)\exp(-W)}, \tag{3.15}$$

$$\beta_0 = -\frac{(1-a)G(W)}{(1+a)\exp(W) - (1-a)\exp(-W)}, \tag{3.16}$$

$$G(W) = N\int_0^W \exp(-\mu t)\cosh(W-t)dt. \tag{3.17}$$

The series on the right of Eq. (3.4), where $u_0(Y)$ and $u_n(Y)$ ($n \geqslant 1$) are given, respectively, by Eqs. (3.14) and (3.9), is absolutely and uniformly convergent for

$$|\epsilon| \leqslant \delta < \frac{1}{4\max|u_0(Y)|}, \tag{3.18}$$

where Y is such that $0 \leqslant Y \leqslant W$ (see Appendix).

Therefore, we conclude that the function (3.4), for any ϵ verifying the inequality (3.18), is *defined* on the whole closed interval $(0, W)$ and is the *solution* of Eq. (3.1) with the boundary conditions (3.2).

IV. DISCUSSION

In Sec. III we have obtained the general form of the function $u(Y)$, from which the excess carrier distribution in the sample can be deduced.

The PME short-circuit current per unit width I_{sc} can be calculated by the usual expression[2]

$$I_{sc} = -qD(\mu_n + \mu_p)B\,[\Delta n(w) - \Delta n(0)], \tag{4.1}$$

while the photoconductance per unit width to length ratio ΔG can be evaluated from the following relationship:

$$\Delta G = q(\mu_n + \mu_p)\int_0^w \Delta n(y)dy . \quad (4.2)$$

The explicit computation of I_{sc} and ΔG from Eqs. (4.1) and (4.2) is straightforward, but quite tedius. Thus their analytical expressions will be omitted here.

In order to discuss the consequence of the lifetime behavior on the PME short-circuit current and the photoconductance, we have computed these quantities by using Eqs. (4.1) and (4.2) and the excess carrier density relation truncated at the first order.

In Fig. 1 the quantity $I_{sc}/q(\mu_n + \mu_p) BD$ is reported as a function of the adimensional quantity K, for two different values of the light intensity, and for different values of the parameter ϵ. The computation refers to a thick sample ($W = 75$) with $S_1 = 10$ and $n_0 = 10^{11}$ cm^{-3}.

As one can see from Fig. 1, at low-intensity level the spectral distribution of the PME short-circuit current is unaffected by the value of ϵ. At higher intensity, but always in the small signal case, a variation of spectral distribution as a function of the parameter ϵ is evidenced in the region of high values of K. We can conclude that the correction due to the lifetime dependence on the injection level becomes important when the light is strongly absorbed in the sample. Similar considerations can also be made for the photoconductance.

In Fig. 2(a) the PME short-circuit current is shown as a function of the adimensional quantity $N_0 = Iw/DW^2n_0$, which is proportional to the light intensity. The computation has been carried out with light strongly absorbed ($K = 10$), for different values of the parameter ϵ. When $\epsilon = 0$, I_{sc} is a linear function of the light intensity, as provided by the unperturbed small-signal theory.[2]

On the contrary, when $\epsilon = -10$ a clear shift from the linear behavior becomes evident. In a log-log plot, I_{sc} will appear as a superlinear function of the light intensity. A similar behavior is also shown by the photoconductance [see Fig. 2(b)].

In conclusion, we note that in Fig. 1, 2(a) and 2(b), respectively, we have limited ourselves to consider the behavior of I_{sc} and ΔG versus the light intensity, for K and N_0 such that $10^{-2} \leqslant K \leqslant 5$ and $10^{-4} \leqslant N_0 \leqslant 2.5\times 10^{-3}$.

This choice is motivated by the need to remain within the limitation (3.18) which assures the convergence of the perturbative series (3.4). However, even if we limit ourselves to consider the above mentioned example, the analysis of Figs. 1 and 2 shows that the model of the PME effect proposed in this paper gives rise to appreciable deviations (up to about the 25%) with respect to the model with constant lifetime.

Many thanks are due to Prof. C. Manfredotti for valuable discussions.

APPENDIX

Here we shall show that the series on the right of Eq. (3.4) is absolutely and uniformly convergent whenever $|\epsilon| \leqslant \delta$, where

$$\delta < \frac{1}{4\max|u_0(Y)|},$$

for any $Y \epsilon (0, W)$.

In doing so, let us begin by noticing that all the functions $u_n(Y)$ ($n \geqslant 0$), as given by Eqs. (3.14) and (3.9), are continuous on the closed interval $(0, W)$. Consequently, in virture of the extreme-value theorem,[8] *any* function $|u_n(Y)|$ has an absolute maximum somewhere in $(0, W)$.

Then from the boundary conditions

$$\left.\frac{du_n(Y)}{dY}\right|_{Y=0} = au_n(0)$$

and

$$\left.\frac{du_n(Y)}{dY}\right|_{Y=W} = 0,$$

where $a > 0$, we deduce that any function $|u_n(Y)|$ takes its absolute maximum at a point belonging to the interval $(0, W)$, say $\bar{Y}_n$, such that $du_n/dY|_{Y=\bar{Y}_n} = 0$.

Using this relation, from Eqs. (3.9) and (3.10) and with the help of Eqs. (3.11) and (3.12) we obtain

$$u_n(\bar{Y}_n) = \int_0^{\bar{Y}_n} dt \sum_{k=0}^{n-1} u_k(t) u_{n-1-k}(t) \times \left(\sinh(\bar{Y}_n - t) - \frac{1+\lambda\exp(-2\bar{Y}_n)}{1-\lambda\exp(-2\bar{Y}_n)}\cosh(\bar{Y}_n - t)\right), \quad (A1)$$

where $\lambda = (1-a)/(1+a)$.

Using Eq. (A1) we can write

$$|u_n(Y)| \leqslant \int_0^{\bar{Y}_n} dt \sum_{k=0}^{n-1} |u_k(t)||u_{n-1-k}(t)| \times \left(\frac{1+\lambda\exp(-2\bar{Y}_n)}{1-\lambda\exp(-2\bar{Y}_n)}\cosh(\bar{Y}_n - 1) - \sinh(\bar{Y}_n - t)\right), \quad (A2)$$

for $n > 0$ and any $y \epsilon (0, W)$.

Since for any $n > 0$

$$\int_0^{\bar{Y}_n} dt \left(\frac{1+\lambda\exp(-2\bar{Y}_n)}{1-\lambda\exp(-2\bar{Y}_n)}\cosh(\bar{Y}_n - t) - \sinh(\bar{Y}_n - t)\right) = 1 - \frac{(1-\lambda)\exp(-\bar{Y}_n)}{1-\lambda\exp(-2\bar{Y}_n)} < 1, \quad (A3)$$

from Eq. (A2) we deduce that

$$|u_n(Y)| \leqslant c_n M_0^{n+1}, \quad (A4)$$

with $M_0 = \max|u_0(Y)|$ and

$$c_n = \sum_{k=0}^{n-1} c_k c_{n-1-k}, \quad (A5)$$

where $n = 1,2,3,\ldots$ and $c_0 \equiv 1$.

The coefficient (A5) can be explicitly evaluated in terms of n as follows.

Let us put

$$f(z) = \sum_{n=0}^{\infty} c_n z^n . \quad (A6)$$

In order that c_n obeys the recurrence relation (A5), it is necessary that

$$\left(\sum_{n=0}^{\infty} c_n z^n\right)\left(\sum_{m=0}^{\infty} c_m z^m\right) = \sum_{n=0}^{\infty} c_{n+1} z^n . \quad (A7)$$

Consequently, from Eqs. (A6) and (A7) we obtain

$$z f^2(z) - f(z) + 1 = 0\,, \tag{A8}$$

which gives

$$f(z) = \frac{1 - (1 - 4z)^{1/2}}{2z}\,, \tag{A9}$$

where we have taken into account the fact that $f(0)$ must be equal to unity. We have, therefore

$$\sum_{n=0}^{\infty} c_n z^n = \frac{1 - (1 - 4z)^{1/2}}{2z}\,, \tag{A10}$$

from which one gets

$$c_n = \frac{2^n (2n-1)!!}{(n+1)!}\,. \tag{A11}$$

In view of Eqs. (A4) and (A10), we conclude that

$$\sum_{n=0}^{\infty} |\epsilon^n u_n(Y)| \leqslant \frac{1 - (1 - 4\epsilon M_0)^{1/2}}{2\epsilon}\,,$$

where $|\epsilon| < 1/4M_0$.

Thus the assertion is proved.

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