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Some remarks on saturation of amplified spontaneous emission

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Analytical (approximate) solution of rate equations for amplified spontaneous emission in a two-level system, in the case of "fast" pumping, shows the mutual dependence of some parameters characterizing amplification and saturation, such as critical time (or critical length), saturation (or peak) time, and peak power. The spontaneous emission can be accounted for by an equivalent number of initial photons. These results are compared with numerical computer solutions for the N_2 laser.

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In a recent paper[1] the rate equations for amplification of spontaneous emission have been solved numerically for different pumping energies and excitation functions in the case of the 3371-Å N_2 laser, showing the dependence of peak power and saturation time on pumping energy, for "fast" and "slow" pumping. In this paper we show that some of these results can be obtained analytically, and that the relationships linking critical length, saturation time, pumping energy, etc., are very simple (at least if the pumping energy is not too low), and generally in agreement with computer calculations.

The rate equations for a two-level laser system pumped by a short (a Dirac δ function of time) pulse, at a given point of the active medium, are

$$\frac{dN}{dt} = -A(N_0+N) - 2c\sigma Nn, \tag{1a}$$

$$\frac{dn}{dt} = \frac{\Omega}{4\pi}\frac{A}{2}(N_0+N) + c\sigma Nn \tag{1b}$$

with the initial conditions, at $t=0$,

$$N = N_0 \quad \text{and} \quad n=0.$$

N is the inversion density; n is the photon density; σ is the cross section for stimulated emission; c is the velocity of light; A is the spontaneous emission probability from upper to lower level (for simplicity we suppose the branching ratio is unity); Ω is the solid angle in which spontaneously emitted photons can produce stimulated emission. As will be seen later, $\Omega(t)$ is appreciably involved only at the beginning of the process, so that we can take for $\Omega(t)$ the value of the exit solid angle seen from the beginning of the laser tube.

When the amplification process begins, and as long as

$$\tfrac{1}{2}(N_0-N) \ll N_0 \quad \text{or} \quad (4\pi/\Omega)\,n \ll N_0, \tag{2}$$

Eqs. (1) can be approximated by

$$\frac{dN}{dt} = -2AN_0 - 2c\sigma N_0 n, \tag{3a}$$

$$\frac{dn}{dt} = \frac{\Omega}{4\pi}AN_0 + c\sigma N_0 n. \tag{3b}$$

While later, when stimulated emission becomes predominant,

$$(\Omega/4\pi)AN_0 \ll c\sigma Nn, \tag{4}$$

we can write

$$\frac{dN}{dt} = -2c\sigma Nn, \tag{5a}$$

$$\frac{dn}{dt} = c\sigma Nn. \tag{5b}$$

In the case

$$N_0 \gg 2A/c\sigma \tag{6}$$

Eqs. (2) and (4) can be satisfied simultaneously and the two systems [Eqs. (3) and (5)] have a region of validity in common and can be connected.

Equations (5) can be solved by putting $N+2n=u$ and $N-2n=w$; then the variables can be separated, and the solution is

$$N(t) \approx \frac{N_0}{1+(2n_0/N_0)\exp(c\sigma N_0 t)}, \tag{7a}$$

$$n(t) \approx \frac{\frac{1}{2}N_0}{1+(N_0/2n_0)\exp(-c\sigma N_0 t)}, \tag{7b}$$

where n_0 (that we have considered $\ll N_0$) is the "effective initial n", that is, $n(t=0)$ if this approximation (without spontaneous emission) were valid, and can be obtained by connecting Eq. (7b) to the solution of system (3), which is

$$n(t) = \frac{\Omega}{4\pi}\frac{A}{c\sigma}[\exp(c\sigma N_0 t) - 1]. \tag{8}$$

TABLE I. Comparison of computer and analytical results for different values of initial density inversion of an N_2 laser excited by a "fast" pulse of relativistic electrons. N_0 = initial inversion density, t_s = saturation time, $n(t_s)$ = saturation photon density, and $P(t_s)$ = saturation power.

	Computer			Analytical		
N_0 (m^{-3})	t_s (nsec)	$n(t_s)$ (m^{-3})	$P(t_s)$ (MW cm^{-3})	t_s (nsec)	$n(t_s)$ (m^{-3})	$P(t_s)$ (MW cm^{-3})
12.0×10^{17}	1.00	3.42×10^{17}	1230.0	0.96	3.00×10^{17}	1368.0
6.0×10^{17}	1.90	1.45×10^{17}	295.0	1.83	1.50×10^{17}	342.0
1.8×10^{17}	6.30	0.30×10^{17}	15.6	5.58	0.45×10^{17}	30.8
1.2×10^{17}	9.35	0.19×10^{17}	6.2	8.12	0.30×10^{17}	13.7

TABLE II. Comparison of computer and analytical results for an N_2 laser excited by "slow" current pulses [$f(t)$ rectangular functions of duration $> t_s$], $n(t_s)$ = saturation photon density and $P(t_s)$ = saturation power.

$f(t)$ (kA)	Computer $n(t_s)$ (m^{-3})	Computer $P(t_s)$ ($MW\,m^{-3}$)	Analytical $n(t_s)$ (m^{-3})	Analytical $P(t_s)$ ($MW\,m^{-3}$)
20	1.46×10^{17}	276	1.48×10^{17}	231
30	1.64×10^{17}	408	1.76×10^{17}	365
40	1.88×10^{17}	534	1.80×10^{17}	496
50	2.11×10^{17}	674	2.22×10^{17}	639
60	2.33×10^{17}	821	2.49×10^{17}	777
70	2.52×10^{17}	961	2.72×10^{17}	917
80	2.69×10^{17}	1097	2.85×10^{17}	1059
90	2.86×10^{17}	1239	3.05×10^{17}	1201
100	3.03×10^{17}	1390	3.28×10^{17}	1343

In the common validity region of the two systems [Eqs. (3) and (5), Eq. (8) can be approximated by

$$n(t)=\frac{\Omega}{4\pi}\frac{A}{c\sigma}\exp(c\sigma N_0 t) \tag{9}$$

and n_0 is obtained as

$$n_0=\lim_{t\to 0}\frac{\Omega}{4\pi}\frac{A}{c\sigma}\exp(c\sigma N_0 t)=\frac{\Omega}{4\pi}\frac{A}{\sigma c}\,. \tag{10}$$

This quantity plays the role of an initial photon density equivalent to spontaneous emission.

From Eq. (7b) we find that the saturation time t_s, or peaktime (that is, the time when the power reaches its maximum value, and then saturation begins to be predominant), is connected (for a given transition) to the initial inversion density by

$$t_s=\frac{1}{c\sigma N_0}\log\left(\frac{N_0}{2n_0}\right)\,. \tag{11}$$

In lasers with amplification of spontaneous emission, it is useful to introduce the concept of a critical length[2] L_{cr}, which can be expressed as

$$L_{cr}=1/\sigma N \tag{12}$$

to emphasize its meaning as mean free path for stimulated emission; or equivalently we can speak of a "critical time" $t_{cr}=L_{cr}/c$.

From Eqs. (11) and (12) we see that

$$t_s=t_{cr}\log(N_0/2n_0) \tag{13}$$

From Eq. (7) we see also that

$$n(t_s)=\tfrac{1}{4}N_0 \tag{14}$$

and the power per unit volume

$$P(t_s)=h\nu\mathring{n}(t_s)=\tfrac{1}{8}h\nu c\sigma N_2^0. \tag{15}$$

The values of t_s, $n(t_s)$, and $P(t_s)$ obtained by Eqs. (11), (14), and (15) are in agreement [except the two lowest values of $P(t_s)$] with those obtained by the numerical solution of system (1) for an N_2 laser excited by a pulse of relativistic electrons,[1] as can be seen in Table I.

The rate equations have also been solved numerically[1] (for N_2) for the case of "slow" pumping, that is, with an excitation which is a function of time $f(t)$ of duration comparable with, or greater than t_s. From our analytical considerations we can guess that, within a factor (of order unity) depending on the form of $f(t)$, the results for $n(t_s)$ and $P(t_s)$ should be the same, with an effective N_0 equal to the number of atoms excited to the upper level up to $t=t_s$:

$$N_0=(\Sigma/S)\,N_f\int_0^{t_s}f(t)\,dt$$

(where Σ is the excitation cross section of the upper laser level, S is the laser tube section, and N_f is the fundamental state population density).

Table II shows a comparison of computer and analytical results for N_2, with an excitation current $f(t)$ which is a rectangular function of time, beginning at $t=0$ and of duration $> t_s$. The results are in good agreement.

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[2]G.I. Peters and L. Allen, J. Phys. A 4, 238 (1971).