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The polarized Debye sheath effect on Kadomtsev-Petviashvili electrostatic structures in strongly coupled dusty plasma

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We give a theoretical investigation on the dynamics of nonlinear electrostatic waves in a strongly coupled dusty plasma with strong electrostatic interaction between dust grains in the presence of the polarization force (i.e., the force due to the polarized Debye sheath). Adopting a reductive perturbation method, we derived a three-dimensional Kadomtsev-Petviashvili equation that describes the evolution of weakly nonlinear electrostatic localized waves. The energy integral equation is used to study the existence domains of the localized structures. The analysis provides the localized structure existence region, in terms of the effects of strong interaction between the dust particles and polarization force. © *2015 AIP Publishing LLC*.
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I. INTRODUCTION

The fundamental role of dust associated electrostatic excitations in the study of different environments (such as the cometary tails, the planetary rings, the gossamer ring of Jupiter, the asteroid zones, the interstellar clouds, the earth's mesosphere, ionosphere, and laboratory plasma, etc.[1]) has attracted a great deal of attention during the last few decades. Dust acoustic (DA) waves as one of the low-frequency dust associated waves were predicted theoretically by Rao *et al.*[2] and were verified experimentally by Barkan *et al.*[3] in laboratory. Due to the strength of interaction between dust grains, dusty plasma may go into a weakly ($\Gamma \ll 1$) or strongly ($\Gamma \gg 1$) coupled regimes, where Γ is a coupling parameter given by $\Gamma = q_d^2 \exp(-a_d/\lambda_d)/a_d k_B T_d$, in which q_d, a_d, T_d, and λ_d are, respectively, the dust grain charge, the interparticle distance, the dust kinetic temperature, and the Debye length.

A number of theoretical investigations have been carried out on the basic properties of DA waves in weakly and strongly coupled dusty plasma.[4–27] All of these works are valid in the absence of the effective dust temperature[20–27] (due to the strong electrostatic interactions among the highly negatively charged dust grains) and the polarization force[20,23,28–37] (i.e., the force due to the polarized Debye sheath). However, it has been shown that in the particular cases, for the low frequency DA waves these effects are quite important and need to be taken into account.

The notable aspect of the present investigation is that the effective electrostatic temperature arises due to the strong electrostatic interaction. In this case, the dust grains are highly negatively charged and thus the polarization force comes from the polarization of ion species around the dust particles.[20,28] Regardless of the sign of the dust charge, the polarization force is always in the direction of decreasing Debye length.[28] Furthermore, the polarization force is independent of the polarity of dust grains and for negative dust grains it is directed opposite to the electrostatic force. Khrapak *et al.*[30] showed that the polarization force has a decreasing effect on the phase velocity and its effect becomes more important when the size of dust particle increases. It has been shown that the polarization force affects drastically the basic features of electrostatic waves in space and laboratory dusty plasma.[20,23,28–37] Also, it is appropriate to add that the strong electrostatic interaction has a crucial role on the electrostatic structures in strongly coupled dusty plasma.[20–27] Mamun *et al.*[20] and Mamun and Cairns[21] employed the generalized hydrodynamic model to study the DA solitary[20] and shock[21] waves in a strongly coupled dusty plasma by taking account the effects of polarization force and effective dust temperature,[20] and arbitrarily charged strongly coupled dust.[21] Alinejad and Mamun[23] investigated the nonlinear propagation of DA solitons in a strongly coupled inhomogeneous dusty plasma in the presence of polarization force and effective temperature. Yaroshenko *et al.*[22] investigated the basic properties of the linear DA waves taking account the effect of strong electrostatic interactions between neighboring dust particles, near to the curve of liquid-crystal phase transition (in which $\Gamma \leq \Gamma_{cr}$, with Γ_{cr} being the threshold value of Γ for transition to the crystalline phase). They employed the fluid approach proposed by Gozadinos *et al.*[13] Pieper and Goree[12] experimentally studied the role of strong correlation among the highly charged dust on the propagation properties of the DA waves in a strongly coupled dusty plasma with $\Gamma \leq \Gamma_{cr}$, and they surprisingly observed the DA-like waves, which showed no resemblance to the dust lattice waves.[38] The experimental observation of Pieper and Goree[12] leads us to apply the fluid models for describing the wave dynamics in strongly coupled dusty plasmas with $\Gamma \leq \Gamma_{cr}$. The nonlinear features of the DA solitary waves have been discussed by Cousens *et al.*[24] in strongly coupled dusty plasmas. They used a fluid approach to describe the effect of strongly coupling on the characteristics of linear and nonlinear DA waves. Recently, the fluid approach employed to study the DA shock structure[25,27] and oblique solitary structure[26] in strongly coupled dusty plasma. Shahmansouri and Rezaei[25] and Cousens *et al*[27] discussed the formation and basic aspects of shock waves in strongly coupled dusty plasma, whereas Shahmansouri and Mamun[26] investigated the effect of obliqueness on the DA solitons in strongly coupled dusty plasmas.

We, in the present work, have taken into account the effects of the effective temperature and the polarization force on the features of dust acoustic solitary waves in a strongly coupled dusty plasma. To elucidate the physics behind the considered effects, we investigate the influence of the effective temperature and the polarization force on the existence domain of solitary structures in model at hand.

The layout of this paper is as follows: After Introduction, the basic equations governing our plasma model are presented in Sec. II. Then, the perturbation technique is employed to derive the Kadomtsev-Petviashvili (KP) equation in Sec. III. The energy integral equation is derived and a parametric investigation is presented to study the existence domain of the localized structures in Sec. IV. Finally, a discussion is provided in Sec. V.

II. THEORETICAL MODEL

Let us consider a three-dimensional strongly coupled dusty plasma consisting of inertialess electrons and ions, and inertial dust fluids of densities n_e, n_i, and n_d, respectively. Here, the plasma is supposed macroscopically homogeneous, quasi-neutral and at rest. The direction of wave propagation lies on the x-axis, and we assume a weak transverse perturbation perpendicular to the x-axis. At equilibrium, the charge neutrality reads as $n_{e0} + Z_d n_{d0} = n_{i0}$, where n_{e0}, n_{i0}, and n_{d0} are the equilibrium density of electrons, ions, and dust grains, respectively, and Z_d is the number of charge residing on the dust grain, from which we define $n_{e0}/Z_d n_{d0} = f$ and $n_{i0}/Z_d n_{d0} = 1 + f$. In order to discuss the low frequency waves where their characteristics speed is smaller than that of ions and electrons, the ions and electrons are considered to reside in local thermodynamics equilibrium. Therefore, we can suppose that the ions and electrons are Boltzmann distributed

$$n_e = n_{e0} \exp\left(\frac{e\phi}{k_B T_e}\right), \tag{1}$$

$$n_i = n_{i0} \exp\left(\frac{-e\phi}{k_B T_i}\right). \tag{2}$$

To study the electrostatic waves in three-dimensional strongly coupled dusty plasma, the fluid approach is employed, in which the strong electrostatic interaction is modeled by effective electrostatic temperature. Therefore, the nonlinear dynamics of the low frequency purely electrostatic perturbation mode is described by the momentum equation and the continuity equation. We note that in writing the momentum equation, there exist many forces acting on the dust grains. The polarization force associated with the electron-ion interactions and also the ion drag force is neglected in the present work. Of our interest here is the electrostatic force, the polarization force, and the gradient of effective electrostatic pressure. The electrostatic force between the dust grains is given by $\vec{F}_e = Z_d e \nabla\phi$.[1] The polarization force (i.e., the force due to the polarized Debye sheath) reads as[28]

$$\vec{F}_p = \frac{Z_d^2 e^2}{8\pi\varepsilon_\circ} \nabla \lambda_D^{-1}, \tag{3}$$

in which $1/\lambda_D = \sqrt{1/\lambda_{Di}^2 + 1/\lambda_{De}^2}$, $\lambda_{Di(e)} = \sqrt{\varepsilon_\circ k_B T_{i(e)}/e^2 n_{i(e)}}$, and $T_{i(e)}$ is the ion (electron) temperature. The polarization force in the normalized form can be obtained as

$$\vec{F}_p = \frac{Z_d e^2}{8\pi\varepsilon_\circ T_i \lambda_{D0}} \left[d_1 - \left(\frac{d_1^2}{d_0} - 3d_2\right)\phi\right] \nabla\phi, \tag{4}$$

in which $d_1 = -(1 + f - f\sigma^2)/2(1 + f + f\sigma)$, $d_2 = (1 + f + f\sigma^3)/6(1 + f + f\sigma)^2$, and $\sigma = T_i/T_e$. The normalization parameters used are given at the end of the section. For details of the derivation of Eq. (4), see the Appendix. Note that if $T_e \gg T_i$, we have $d_1 = -1/2$, $d_2 = n_{d0} Z_{d0}/6n_{i0}$, which yields $\lambda_D \cong \lambda_{Di}$. It appears appropriate here to add that, regardless of the sign of the dust charge, the polarization force is always in the direction of decreasing Debye length.[28] Furthermore, the polarization force is independent of the polarity of dust grains and for negative dust grains it is directed opposite to the electrostatic force. In dusty plasma when the dust grains are highly negatively charged, the polarization force comes from the polarization of plasma ions around the dust particle.[20,28] On the other hand, we employed the fluid approach to study the strongly coupled dusty plasma. For which we have adopted the concept of the effective electrostatic pressure[13] to describe the strong interaction between the dust grains, given by $P_{eff} = n_d k_B T_{eff}$, where T_{eff} is the effective temperature defined as[21–27]

$$T_{eff} = \frac{T_{eff}^{(0)}}{a_d} \frac{(1+\kappa)}{(1+\kappa_0)} e^{-(\kappa-\kappa_0)}, \tag{5}$$

in which $a_d = 1/\sqrt[3]{n_d}$ is the mean inter-particle distance, κ is the lattice parameter given by a_d/λ_D, N_{nn} is the number of nearest neighbors, and $T_{eff}^{(0)} = N_{nn} Z_d^2 e^2 (n_{d0})^{1/3} (1+\kappa_0)e^{-\kappa_0}/12\pi\varepsilon_\circ T_0$. The effective temperature has typically a few orders larger than the dust kinematic temperature[22] in the vicinity of the crystal-liquid phase transition. The effective electrostatic temperature is dependent on the various varying quantities and thus may be considered as a dynamically varying quantity.[24–27] Accordingly, the normalized momentum transfer equation in a 3-dimensional one-fluid model is given by

$$n_d \left[\frac{\partial}{\partial t} + (\mathbf{u}_d \cdot \nabla)\right] \mathbf{u}_d = n_d \nabla\phi + n_d R \left[d_1 - \left(\frac{d_1^2}{d_0} - 3d_2\right)\phi\right] \nabla\phi - \nabla(n_d T_{eff}), \tag{6}$$

where $R = Z_d e^2/8\pi\varepsilon_\circ k_B T_i \lambda_{D0}$ is a parameter determining the effect of polarization force, and $\mathbf{u}_d = (u, v, w)$ is the dust fluid velocity. The continuity equation, governs the evolution of the dust number density, is as follows:

$$\left[\frac{\partial}{\partial t} + (\mathbf{u}_d \cdot \nabla)\right] n_d = -n_d \nabla \cdot \mathbf{u}_d. \tag{7}$$

Then, the Poisson's equation completes the fluid equations

$$\nabla^2 \phi \cong n_d - 1 + \phi + b_1 \phi^2 + b_2 \phi^3, \tag{8}$$

in which $b_1 = d_1/(1 + f + f\sigma)$ and $b_2 = d_2/(1 + f + f\sigma)$. Equations (4)–(8) are the basic equations which describe the nonlinear evolution of the low frequency dust associated electrostatic waves. In the following, we introduce the normalization process which has been used in Eqs. (4)–(8): the number density n_j is normalized by its equilibrium value n_{j0}, the dust fluid speed u_d is normalized by $c_t = \sqrt{k_B T_0/m_d}$, the space variable r is normalized by $\lambda_{D0} = \sqrt{\varepsilon_\circ k_B T_0/n_{d0} Z_d^2 e^2}$, the time variable t is normalized by $1/\omega_{pd} = \sqrt{\varepsilon_\circ m_d/n_{d0} e^2 Z_d^2}$, the electrostatic potential ϕ is normalized by $k_B T_0/eZ_d$, and the effective temperature T_{eff} is normalized by $T_0 = Z_d^2 n_{d0} T_i T_e/(n_{i0} T_e + n_{e0} T_i)$.

A. Linear dispersion relation

To derive the linear dispersion relation for the dust-acoustic waves in a strongly coupled dusty plasma, we employ the normal mode analysis, i.e., we linearize Eqs. (1)–(8) to a first order approximation, and perform Fourier transformation of these linearized equations. Then, neglecting the zeroth-order fields and velocities, the linear dispersion relation can be written in the form

$$\omega^2 = \frac{k^2}{1 + k^2}(1 - c_2 + Rd_1) + k^2\left(c_1 + T_{eff}^{(0)}\right), \tag{9}$$

where $c_1 = \frac{T_{eff}^{(0)}}{3} \frac{1+\kappa_0+\kappa_0^2}{1+\kappa_0}$ and $c_2 = \frac{T_{eff}^{(0)}}{2} \frac{\kappa_0^2}{1+\kappa_0} \frac{1+f-f\sigma^2}{(1+f+f\sigma)^2}$. It must be added that in the present model, the effective electrostatic temperature, as a dynamical quantity, is a function of ϕ and n_d. Indeed, T_{eff} through the lattice parameter $\kappa(= 1/\sqrt[3]{n_d}\lambda_{\rm D}(\phi))$ is dependent on ϕ and n_d. By Taylor expanding the dynamic variables in T_{eff} and using Eq. (A2) in the Appendix, we can obtain the effective electrostatic temperature perturbations including the coefficients c_1 and c_2 for the dust number density and electrostatic potential, respectively.

The aforementioned equation represents the dispersion relation of the DA waves in strongly coupled dusty plasma in the presence of the polarization force. In the weakly coupled limit, i.e., $\{T_{eff}^{(0)}, c_1, c_2\} \to 0$, and in the absence of the polarization force, i.e., $R \to 0$, the dispersion relation (9) reduces to the usual DA dispersion relation.[2] Also, if we ignore the polarization force, Eq. (9) recovers the result of Cousens *et al.*[24] that is obtained for DA waves in strongly coupled dusty plasma. On the other hand, if we consider the effective temperature as a constant quantity, i.e., $T_{eff}^{(0)} \neq 0$ and $\{c_1, c_2\} \to 0$, then our dispersion relation reduces to $\omega^2 = \frac{k^2(1+Rd_1)}{1+k^2} + k^2 T_{eff}^{(0)}$, which can be considered as a dispersion relation of DA waves in a strongly coupled dusty plasma in the presence of the polarization force.[20] Restoring the dimensions to Eq. (9), the dispersion relation takes the following form:

$$\omega^2 = \frac{\omega_{pd}^2 \lambda_{D0}^2 k^2}{1 + \lambda_{D0}^2 k^2}(1 - c_2 + Rd_1) + \omega_{pd}^2 \lambda_{D0}^2 k^2\left(c_1 + T_{eff}^{(0)}\right). \tag{10}$$

The phase speed of DA waves in this system can be obtained from the long wavelength limit of Eqs. (9) and (10) in the normalized and dimensional form, respectively, as follows:

$$v_{ph} = \frac{\omega}{k} = \left(1 - c_2 + Rd_1 + c_1 + T_{eff}^{(0)}\right)^{1/2} \tag{11}$$

and

$$V_{ph} = \frac{\omega}{k} = \left[\frac{k_B}{m_d}\left(T_0 - c_2 T_0 + Rd_1 T_0 + c_1 T_0 + T_{eff}^{(0)} T_0\right)\right]^{1/2}. \tag{12}$$

Equation (12) is very important for investigation of linear and nonlinear features of DA waves in the plasma system. Since the velocity of solitary waves has a minimum given by the sound speed in plasma, Eq. (12) determines the lower bound of the solitary wave velocity. To investigate the influence of relevant physical parameters, such as temperature of electrons and ions, and also their number density, on the linear properties of DA waves, we have numerically investigated the dispersion relation (9). The behavior of the phase velocity with wave number is investigated in four different cases, in the presence and absence of the effects of polarization force and effective electrostatic temperature, in Fig. 1. We observe that the effective electrostatic temperature leads to increase of the phase velocity, while the polarization force shows an inverse effect, between them the influence of the effective electrostatic temperature is more pronounced. Figure 2(a) represents that the ion temperature has a decreasing effect on the phase velocity. Furthermore, an increase in the ion temperature decreases the effect of the polarization

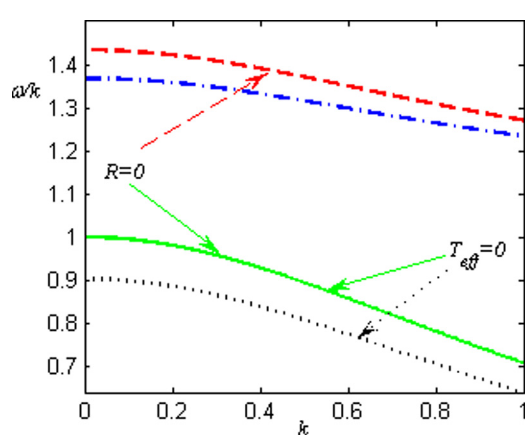

FIG. 1. The phase velocity with respect to the wave number, for parameters: $k_B T_i = 0.03\,\text{eV}$, $k_B T_e = 3\,\text{eV}$, $n_{i0} = 5 \times 10^{13}\,\text{m}^{-3}$, $n_{e0} = 4 \times 10^{13}\,\text{m}^{-3}$, and $r_d = 3 \times 10^{-7}\text{m}$. The solid line refers to $T_{eff} = 0$ and $R = 0$, the dotted line refers to $T_{eff} = 0$ and $R \neq 0$, the dashed line refers to $T_{eff} \neq 0$ and $R = 0$, the dotted-dashed line refers to $T_{eff} \neq 0$ and $R \neq 0$.

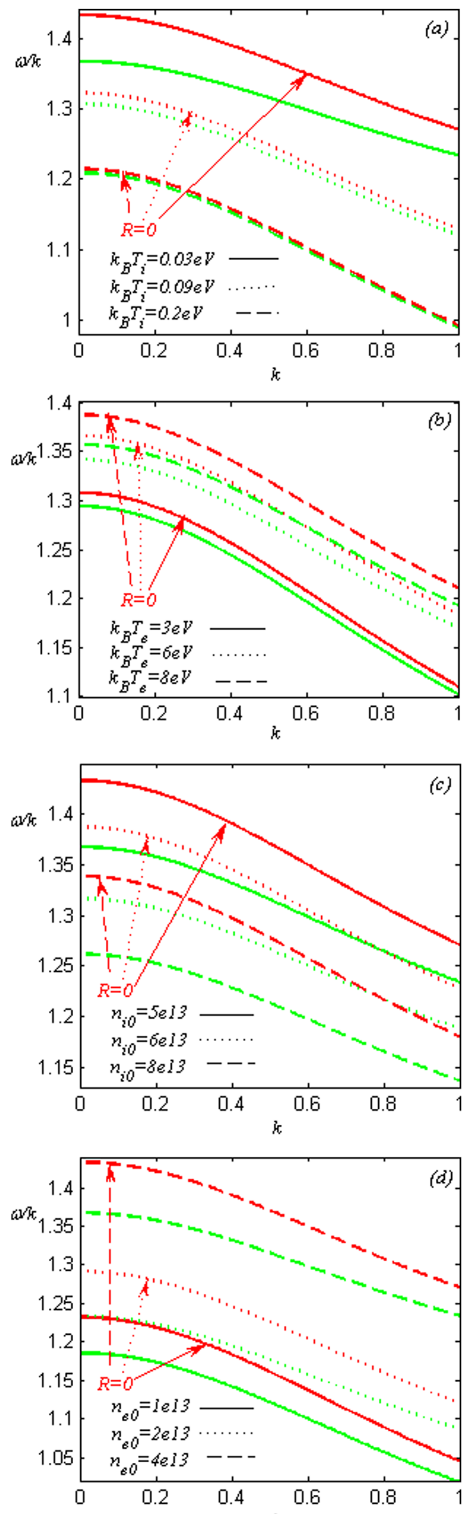

FIG. 2. The phase velocity with respect to the wave number (a) for $k_BT_i = 0.03\,\mathrm{eV}$, $0.09\,\mathrm{eV}$, $0.2\,\mathrm{eV}$ with parameters: $k_BT_e = 3\,\mathrm{eV}$, $n_{i0} = 5\times10^{13}\,\mathrm{m}^{-3}$, $n_{e0} = 4\times10^{13}\,\mathrm{m}^{-3}$; (b) for $k_BT_e = 3\,\mathrm{eV}$, $6\,\mathrm{eV}$, $8\,\mathrm{eV}$ with parameters: $k_BT_i = 0.1\,\mathrm{eV}$, $n_{i0} = 5\times10^{13}\,\mathrm{m}^{-3}$, $n_{e0} = 4\times10^{13}\,\mathrm{m}^{-3}$; (c) for $n_{i0} = 5\times10^{13}$, 6×10^{13}, 8×10^{13} with parameters: $k_BT_e = 3\,\mathrm{eV}$, $k_BT_i = 0.03\,\mathrm{eV}$, $n_{e0} = 4\times10^{13}\,\mathrm{m}^{-3}$; (d) for $n_{e0} = 1\times10^{13}$, 2×10^{13}, 4×10^{13} with parameters: $k_BT_e = 3\,\mathrm{eV}$, $k_BT_i = 0.03\,\mathrm{eV}$, $n_{i0} = 5\times10^{13}\,\mathrm{m}^{-3}$.

force. But this behavior with the electron temperature is completely inverse (Fig. 2(b)). On the other hand, the ion number density leads also to decrease of the phase velocity (Fig. 2(c)), and it is seen to vary about 10% when the ion number density changes from 5×10^{13} to $8\times10^{13}\,\mathrm{m}^{-3}$. Figure 2(d) shows that the electron number density have a stronger and inverse effect on the phase velocity. It can be seen from Fig. 2 that the influence of number densities on the polarization force is small in comparison with the effect of relevant temperatures.

III. DERIVATION OF KADOMTSEV-PETVIASHVILI EQUATION

We adopt the standard reductive perturbation method to investigate the nonlinear propagation of DA waves in a strongly coupled dusty plasmas under transverse perturbation to obtain the KP equation. The dependent variables are expanded as

$$\begin{pmatrix} n_d \\ u \\ v \\ w \\ \phi \\ T_{eff} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ T_{eff}^{(0)} \end{pmatrix} + \varepsilon \begin{pmatrix} n^{(1)} \\ u^{(1)} \\ \varepsilon^{1/2} v^{(1)} \\ \varepsilon^{1/2} w^{(1)} \\ \phi^{(1)} \\ T_{eff}^{(1)} \end{pmatrix} + \varepsilon^2 \begin{pmatrix} n^{(2)} \\ u^{(2)} \\ \varepsilon^{1/2} v^{(2)} \\ \varepsilon^{1/2} w^{(2)} \\ \phi^{(2)} \\ T_{eff}^{(2)} \end{pmatrix} + \cdots. \tag{13}$$

We note that the appearance of transverse velocity components at a higher order of ε (relative to the parallel component u) comes from an anisotropy induced by the influence of transverse perturbation. To derive the KP equation, we first stretch the time and space coordinates in Eqs. (6)–(8) regarding the style of Kadomtsev and Petviashvili[39] as follows:

$$\xi = \varepsilon^{1/2}(x - V_p t),\ Y = \varepsilon y,\ Z = \varepsilon z,\ \tau = \varepsilon^{3/2} t, \tag{14}$$

where ε refers to a real and small parameter, which measures the weakness of the amplitude or dispersion, and V_p is the phase speed. Substituting Eqs. (13) and (14) into Eqs. (6)–(8), and collecting the terms in the different powers of ε, the lowest order of ε leads to

$$\begin{pmatrix} u^{(1)} \\ n^{(1)} \\ \phi^{(1)} \\ T_{eff}^{(1)} \end{pmatrix} = \begin{pmatrix} 0 & V_p & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \dfrac{-V_p}{1+Rd_1} & \dfrac{T_{eff}^{(0)}}{1+Rd_1} & 0 & \dfrac{1}{1+Rd_1} \\ 0 & c_1 & c_2 & 0 \end{pmatrix} \begin{pmatrix} u^{(1)} \\ n^{(1)} \\ \phi^{(1)} \\ T_{eff}^{(1)} \end{pmatrix}. \tag{15}$$

To obtain Eq. (15), we keep in mind that the perturbations tend to zero at $\xi \to \pm\infty$. The solution to Eq. (15) yields the following linear dispersion relation:

$$V_p = (1 - c_2 + T_{eff}^{(0)} + c_1 + Rd_1)^{1/2}. \tag{16}$$

This is similar to that obtained in Eq. (11) for the phase velocity. On the other hand, we can write the lowest order x and y components of the momentum equation as

$$\frac{\partial v^{(1)}}{\partial \xi} = -\left[\frac{1 + Rd_1 - c_2 + T_{eff}^{(0)} + c_1}{V_p}\right]\frac{\partial \phi^{(1)}}{\partial Y}, \qquad (17)$$

$$\frac{\partial w^{(1)}}{\partial \xi} = -\left[\frac{1 + Rd_1 - c_2 + T_{eff}^{(0)} + c_1}{V_p}\right]\frac{\partial \phi^{(1)}}{\partial Z}. \qquad (18)$$

The next order in ε gives a set of equations in the second order perturbed quantities, as follows:

$$-V_p\frac{\partial n^{(2)}}{\partial \xi} + \frac{\partial n^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi}\left(n^{(1)}u^{(1)}\right) + \frac{\partial u^{(2)}}{\partial \xi} + \frac{\partial v^{(1)}}{\partial Y} + \frac{\partial w^{(1)}}{\partial Z} = 0, \qquad (19)$$

$$\begin{aligned} -V_p\frac{\partial u^{(2)}}{\partial \xi} &+ \frac{\partial u^{(1)}}{\partial \tau} + u^{(1)}\frac{\partial u^{(1)}}{\partial \xi} - V_p n^{(1)}\frac{\partial u^{(1)}}{\partial \xi} \\ &= \frac{\partial \phi^{(2)}}{\partial \xi} + n^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + Rd_1\frac{\partial \phi^{(2)}}{\partial \xi} + Rd_1 n^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} \\ &+ R\left(-\frac{d_1^2}{d_0} + 3d_2\right)\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} - T_{eff}^{(0)}\frac{\partial n^{(2)}}{\partial \xi} \\ &- \frac{\partial}{\partial \xi}\left(T_{eff}^{(1)}n^{(1)}\right) - \frac{\partial T_{eff}^{(2)}}{\partial \xi}, \end{aligned} \qquad (20)$$

$$T_{eff}^{(2)} = c_1 n^{(2)} + c_2\phi^{(2)} + c_3(n^{(1)})^2 + c_4 n^{(1)}\phi^{(1)} + c_5(\phi^{(1)})^2, \qquad (21)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = n^{(2)} + \phi^{(2)} + b_1\left(\phi^{(1)}\right)^2, \qquad (22)$$

where $c_3 = \frac{T_{eff}^{(0)}}{18}\frac{\kappa_0^3 - 3\kappa_0^2 - 2\kappa_0 - 2}{1+\kappa_0}$, $c_4 = -\frac{T_{eff}^{(0)}b_1}{3}\frac{\kappa_0^2(\kappa_0 - 1)}{(1+\kappa_0)}$, $c_5 = \frac{T_{eff}^{(0)}(\kappa_0 b_1^2 - 3b_2)}{2}\frac{\kappa_0^2}{1+\kappa_0}$, for derivation of coefficients c_1–c_5 see the explanation after Eq. (9). Then, taking the second derivative of Eq. (22) with respect to ξ, and using Eqs. (15)–(21), we eliminate the second order perturbed quantities to obtain the following KP equation:

$$\begin{aligned} \frac{\partial}{\partial \xi}\left[\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3}\right] \\ + C\left[\frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}\right]\phi^{(1)} = 0, \end{aligned} \qquad (23)$$

where

$$A = -\frac{1 + Rd_1 - R\left(-\frac{d_1^2}{d_0} + 3d_2\right) + 2\left(1 - c_2 + Rd_1 + T_{eff}^{(0)} + c_1\right) + 2(c_1 - c_2 + c_3 - c_4 + c_5) + 2b_1(1 - c_2 + Rd_1)}{2\sqrt{1 - c_2 + Rd_1 + T_{eff}^{(0)} + c_1}}, \qquad (24)$$

$$B = \frac{1 - c_2 + Rd_1}{2\sqrt{1 - c_2 + Rd_1 + T_{eff}^{(0)} + c_1}}, \qquad (25)$$

$$C = \frac{1}{2}\sqrt{1 - c_2 + Rd_1 + T_{eff}^{(0)} + c_1}. \qquad (26)$$

The preceding equation is the well known KP equation, and governs the evolution of the first order approximation of the electrostatic potential corresponding to the DA waves in a strongly coupled dusty plasma in the presence of polarization force. Indeed, the last term in Eq. (23) comes from the effect of transverse perturbation, as in the absence of this effect the KP equation (23) reduces to the usual KdV equation. Thus, we expect that the transverse perturbation, through the last term in Eq. (23), affects the properties and existence domain of the solitary structures. In the following, we investigate the effects of transverse perturbation, strong interaction between dust particle and polarization force on the basic properties as well as the existence domain of the solitary structures.

A. Exact solution of KP equation

An exact solution in the form of a solitary wave can be obtained via the generalized expansion method into Eq. (23). For this purpose, we transform the independent variables X, Y, and ξ into a new coordinate[40]

$$\begin{aligned} \zeta &= l_\xi \xi + l_Y Y + l_Z Z - U\tau \\ &= \varepsilon^{1/2} l_\xi (x - V_p t) + \varepsilon(l_Y y + l_Z z) - \varepsilon^{3/2} Ut, \end{aligned} \qquad (27)$$

where U is an arbitrary constant speed, and l_ξ, l_Y, and l_Z are, respectively, the directional cosines of the wave vector $\underline{k}$ along the x, y, and z axes, so that $l_\xi^2 + l_Y^2 + l_Z^2 = 1$. Thus, a plane KdV soliton in the steady state condition can be obtained as follows:

$$\phi^{(1)} = \phi_{\max} \sec h^2\left(\frac{\zeta}{\Delta}\right), \qquad (28)$$

where

$$\phi_{\max} = 3(Ul_\xi - C(l_Y^2 + l_Z^2))/Al_\xi^2 \qquad (29)$$

and

$$\Delta = \sqrt{8Bl_\xi^4/(Ul_\xi - C(l_Y^2 + l_Z^2))} \qquad (30)$$

give the normalized amplitude and width of solitary wave. The localized solitary wave solution (28) describes a pulse excitation in which the nonlinearity and dispersion are balanced. These soliton solutions are all of infinite extent in one-direction and never propagate along the y and z axis.[41] The existence of such soliton solution needs to satisfy some essential conditions, which will be discussed in Sec. IV. It is easy to show that such electrostatic structures have bipolar electric fields. One can obtain an exact expression for the electric field vector from Eq. (28) via $\vec{E} = -\nabla\phi^{(1)}$ as follows:

$$\vec{E} = \frac{3}{A\sqrt{B}}\left(Ul_\xi - C\left(l_Y^2 + l_Z^2\right)\right)^{3/2} \sec h^2\left(\frac{\zeta}{\Delta}\right)\tanh h\left(\frac{\zeta}{\Delta}\right)\frac{\vec{\zeta}}{|\zeta|}. \tag{31}$$

This is similar to that discussed in Refs. 42 and 43. Further discussion on the bipolar electric field structures are discussed in Sec. IV.

On the other hand, one can obtain an expression for the soliton energy in terms of its amplitude and width[44,45] by integrating from $(\phi^{(1)}(\zeta))^2$, as follows:

$$K = \int_{-\infty}^{\infty} (\phi^{(1)}(\zeta))^2 d\zeta, \tag{32}$$

which leads to

$$K = \frac{4}{3}\phi_m^2\Delta. \tag{33}$$

The KP equation (23) has been studied extensively due to its application in different physical situations. Analytical[46–48] and numerical[49,50] investigations show that under a periodic transverse perturbation, a plane KdV soliton may be transformed into a chain of two-dimensional KP solitons. The solitary wave solution such as Eq. (28) was shown to be unstable to the long-wavelength transverse perturbations for $C < 0$ (which is well known as the positive dispersion case), whereas it is stable for $C > 0$ (the negative dispersion case).[51,52] It is clear that in the present plasma model $C > 0$ and it regards as a negative dispersion case (due to $C > 0$). This reflects the fact that the soliton solution (28) is stable with respect to the long-wavelength transverse perturbations.

IV. ENERGY INTEGRAL AND PARAMETRIC INVESTIGATION OF SOLITARY WAVE EXISTENCE

It is shown by Kadomtsev and Petviashvili[39] that in the small amplitude asymptotic limit, a solitary wave under the transverse perturbation is described by KP equation. The solitons remain stable against such perturbations.[39,53,54] In contrast to the KdV equation, the soliton solution of KP equation is dependent on the sign of dispersion coefficient. Kadomtsev and Petviashvili[39] have shown that in a one-dimensional system with negative dispersion, the perturbations can be transformed from soliton to the medium and thus in this case the solitons are stable with respect to the weak transverse perturbations. The effect of different types of perturbations on the two dimensional solitons has been considered by Kako and Rowlands.[53] They investigated the stability of solitary structures in their model. In the model at hand, we discuss the stability properties of DA solitons in strongly coupled dusty plasma under the transverse perturbation. This can be demonstrated by a method based on energy consideration, which it needs to obtain the potential energy, namely, the Sagdeev potential. Using the transformation (27) into Eq. (23), we obtain the following differential equation:

$$l_\xi \frac{\partial^2}{\partial\zeta^2}\left[-U\phi^{(1)} + \frac{A}{2}l_\xi\left(\phi^{(1)}\right)^2 + Bl_\xi^3\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}\right] + C\left[l_Y^2 + l_Z^2\right]\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} = 0, \tag{34}$$

and after integrating twice, Eq. (34) takes the following form:

$$\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} = \frac{U}{Bl_\xi^3}\phi^{(1)} - \frac{A}{2Bl_\xi^2}\left(\phi^{(1)}\right)^2 - \frac{C}{Bl_\xi^4}\left[l_Y^2 + l_Z^2\right]\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} + h_1\zeta + h_2, \tag{35}$$

in which h_1 and h_2 are the integrating constants. Then, multiplying both sides of Eq. (35) with $d\phi^{(1)}/d\zeta$ and integrating once we can obtain the following energy-like equation:

$$\frac{1}{2}\left(\frac{d\phi^{(1)}}{d\zeta}\right)^2 + \psi\left(\phi^{(1)}\right) = 0, \tag{36}$$

where $\psi(\phi^{(1)})$ is the pseudopotential or Sagdeev potential and is given by

$$\psi\left(\phi^{(1)}\right) = \frac{-Ul_\xi + C\left(l_Y^2 + l_Z^2\right)}{2Bl_\xi^4}\left(\phi^{(1)}\right)^2 + \frac{A}{6Bl_\xi^2}\left(\phi^{(1)}\right)^3. \tag{37}$$

It should be noted that to obtain Eq. (37) the appropriate boundary conditions have been employed, namely, $\{\phi^{(1)}; d\phi^{(1)}/d\zeta; d^2\phi^{(1)}/d\zeta^2\}|_{\zeta\to\pm\infty} \to 0$, which lead to $h_1 = h_2 = 0$. Equation (36) is the well-known equation in the form of the "energy integral" of an oscillating particle of unit mass, with velocity $\partial\phi^{(1)}/\partial\zeta$ and position $\phi^{(1)}$ in a potential well $\psi(\phi^{(1)})$. The first term in Eq. (36) can be considered as the kinetic energy of the unit mass, and $\psi(\phi^{(1)})$ is the potential energy. Since the kinetic energy is a positive quantity, it requires that $\psi(\phi^{(1)}) \le 0$ for the entire of motion, i.e., in the interval $0 < \phi < \phi_{\max}(\phi_{\min} < \phi < 0)$ for the compressive (rarefactive) solitary waves, where $\phi_{\max}(\phi_{\min})$ is the maximum (minimum) value of ϕ for which $\psi(\phi) = 0$.

Furthermore, Eq. (37) describes the Sagdeev potential for DA solitons in a strongly coupled dusty plasma. The existence condition of solitary wave solution (28) requires that $d^2\psi/d\phi_1^2|_{\phi_1=0} < 0$, which implies that

$$\frac{d^2\psi}{d\phi_1^2}|_{\phi_1=0} = -\frac{Ul_\xi - C\left(l_X^2 + l_Y^2\right)}{2Bl_\zeta^4} = \frac{-S}{l_\zeta^4} < 0. \tag{38}$$

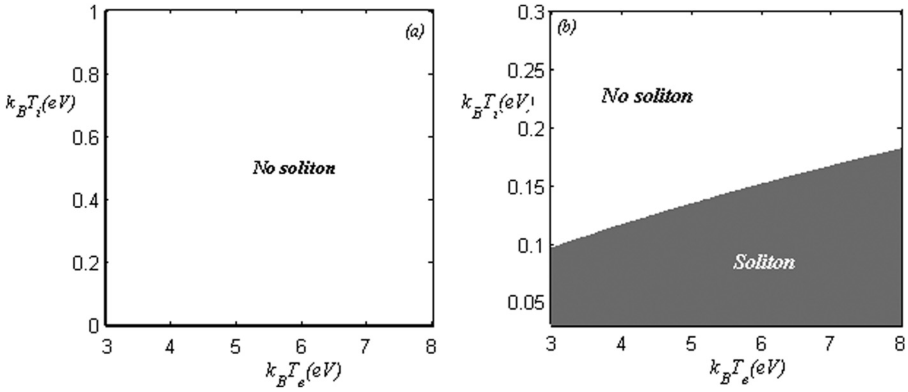

FIG. 3. Plot of $S=0$ in space (k_BT_e, k_BT_i), for (a) $R=0$ and (b) $R\neq 0$, where $T_{eff}=0$ and we have taken the parameters $n_{i0}=5\times 10^{13}$ m^{-3}, $n_{e0}=4\times 10^{13}\,\mathrm{m}^{-3}$, $r_d=2\times 10^{-7}$ m, $U=1$, and $l_\xi=0.805$.

The above expression shows that the solitary wave solution (28) exists whenever the condition

$$S=\left[Ul_\xi - C\left(l_X^2+l_Y^2\right)\right]/2B>0, \tag{39}$$

is satisfied. The existence domain of solitary wave solution can be explored by plotting the curve $S=0$ that separates the parameter space into two regions, one in which $S>0$ and solitary waves exist and another one where $S<0$ and solitary waves do not exist. We note that the sign of S mainly is determined by l_ξ rather than other parameters and in turn, the values of l_ξ that satisfy the existence condition are strongly controlled by arbitrary parameter U. This statement is justified in the following manner. To examine the effect of relevant physical parameters, such as electrostatic interaction between dust grains, concentrations and temperatures of plasma species, on the stability properties of IA waves, we have numerically investigated the condition (39). Figure 3 shows the plot of $S=0$ in space (k_BT_e, k_BT_i), in weakly coupled regime (i.e., for $T_{eff}\cong 0$), in the absence (Fig. 3(a)) and in the presence (Fig. 3(b)) of polarization force. It is shown that for the parameters $n_{i0}=5\times 10^{13}\,\mathrm{m}^{-3}$, $n_{e0}=4\times 10^{13}\,\mathrm{m}^{-3}$, $r_d=2\times 10^{-7}\mathrm{m}$, $U=1$, and $l_\xi=0.805$, we have $S<0$ and thus the weakly coupled dusty plasma system in the absence of polarization force does not support the essential conditions for the existence of solitary structures. But in the presence of polarization force,

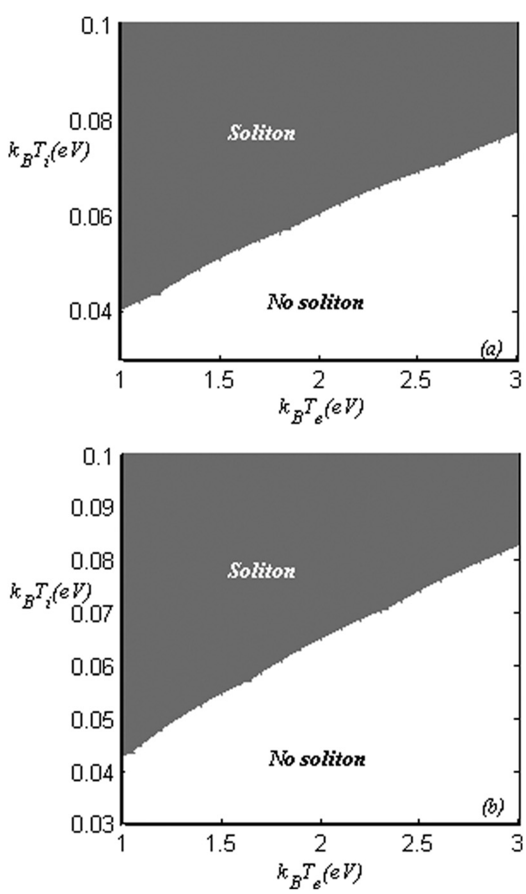

FIG. 4. Plot of $S=0$ in space (k_BT_e, k_BT_i), for (a) $R=0$ and (b) $R\neq 0$, where $T_{eff}\neq 0$ and we have taken the parameters $n_{i0}=8\times 10^{13}\,\mathrm{m}^{-3}$, $n_{e0}=1\times 10^{13}\,\mathrm{m}^{-3}$, $r_d=2\times 10^{-7}\mathrm{m}$, $U=1$, and $l_\xi=0.9$.

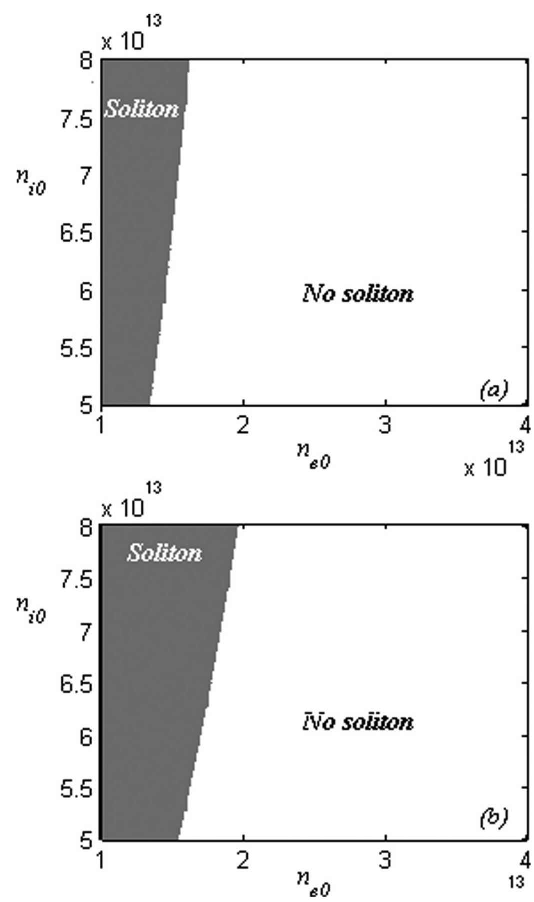

FIG. 5. Plot of $S=0$ in space (n_{e0}, n_{i0}), for (a) $R=0$ and (b) $R\neq 0$, where $T_{eff}\neq 0$ and we have taken the parameters $k_BT_i=0.03\,\mathrm{eV}$, $k_BT_e=3\,\mathrm{eV}$, $r_d=2\times 10^{-7}\mathrm{m}$, $U=1$, and $l_\xi=0.95$.

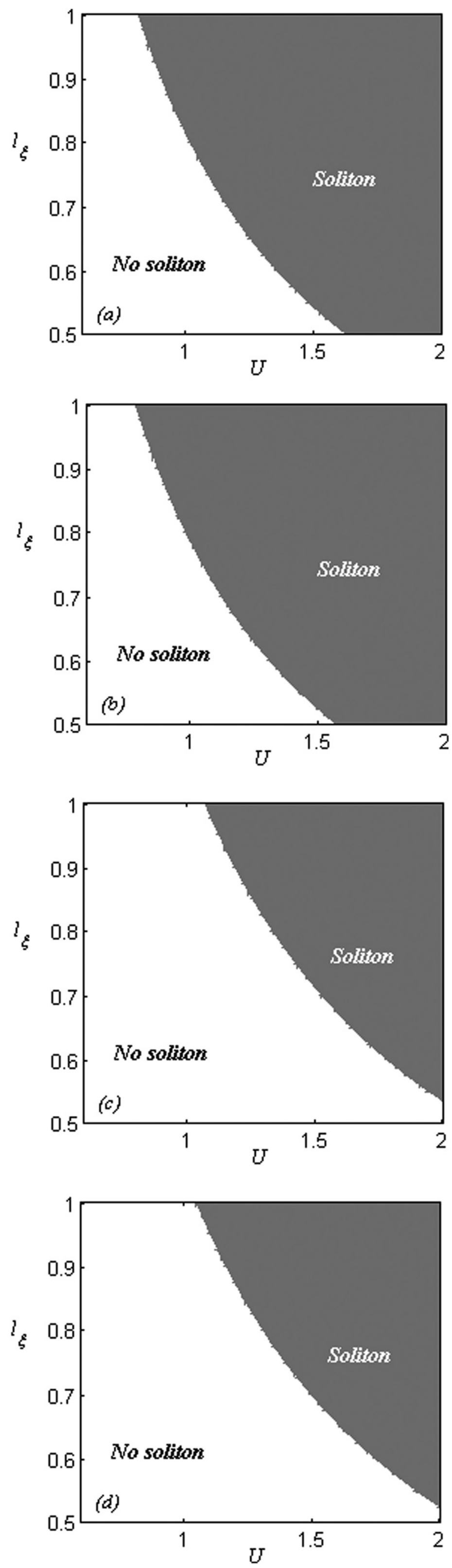

FIG. 6. Plot of $S=0$ in space (l_ξ, U), for (a) $R=0, T_{eff}=0$ and (b) $R\neq 0, T_{eff}=0$, (c) $R=0, T_{eff}\neq 0$, and (d) $R\neq 0, T_{eff}\neq 0$, for parameters $k_BT_i=0.03\,\text{eV}$, $k_BT_e=3\,\text{eV}$, $n_{i0}=8\times 10^{13}\,\text{m}^{-3}$, $n_{e0}=4\times 10^{13}\,\text{m}^{-3}$, and $r_d=2\times 10^{-7}\,\text{m}$.

this system predicts the occurrence of solitons for the lower values of ion temperature. This might be one of the reasons that why we have considered this interaction into account in the present study.

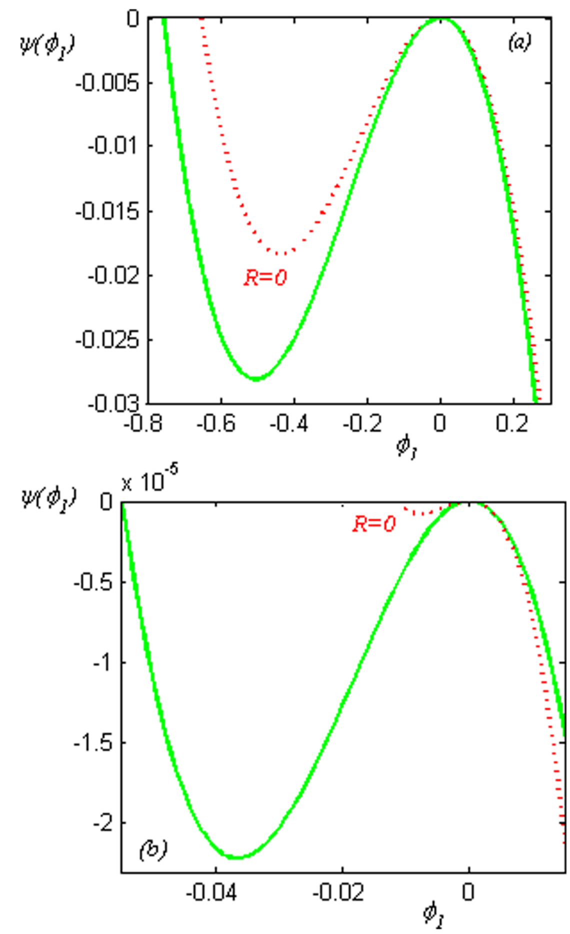

FIG. 7. The Sagdeev potential with respect to the electrostatic potential, for (a) $T_{eff}=0$ and (b) $T_{eff}\neq 0$. The dotted line refers to $R=0$ and the solid line refers to $R\neq 0$. For better clarity, the dotted line in panel (b) is magnified with factor 4. We have taken $k_BT_i=0.03\,\text{eV}$, $k_BT_e=3\,\text{eV}$, $n_{i0}=8\times 10^{13}\,\text{m}^{-3}$, $n_{e0}=4\times 10^{13}\,\text{m}^{-3}$, $r_d=2\times 10^{-7}\text{m}$, $U=1.1$, and $l_\xi=0.97$.

Figure 3 revisited in Fig. 4 considering the strong electrostatic interaction between dust grains (i.e., for $T_{eff}\neq 0$). However, in this case the present dusty plasma system for $T_{eff}=0$ supports the occurrence of solitary structures for all values of the ion temperature, but in the presence of strong interaction, there is a threshold which separates the space (k_BT_e, k_BT_i) into two parts, one in which $S<0$ (gray region) and another one where $S>0$(white region). In the presence of the polarization force, this threshold satisfies at the higher values of the ion temperature. This turns out that the polarization force decreases the primitive region (i.e., region of $S>0$) for the formation of solitary structures. We find that the effect of polarization force on the primitive region may be more pronounced in the region of low ion temperature, because of the higher values of parameter R in this region.

To see how the electron and ion number density affect the existence domain of the solitary structures, we have plotted $S=0$ in space (n_{e0}, n_{i0}) in Figs. 5(a) and 5(b) (in the absence and presence of polarization force, respectively). It can be seen that the existence domain is restricted to the lower values of electron number density. On the other hand,

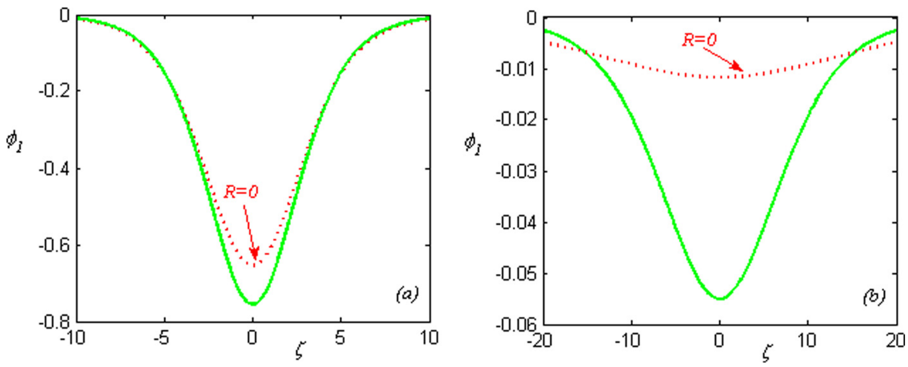

FIG. 8. The electrostatic potential as a function of ζ, for (a) the weakly and (b) strongly coupled regime in the presence and absence of polarization force. We have taken the parameters: $k_BT_i = 0.03\,\text{eV}$, $k_BT_e = 3\,\text{eV}$, $n_{i0} = 8 \times 10^{13}\,\text{m}^{-3}$, $n_{e0} = 4 \times 10^{13}\,\text{m}^{-3}$, $r_d = 2 \times 10^{-7}\text{m}$, $U = 1.1$, and $l_\xi = 0.97$. The dashed line refers to $R = 0$ and the solid line refers to $R \neq 0$.

the polarization force shifts the threshold to the higher values of electron number density (Fig. 5(b)).

The contour of parameter S in space (U, l_ξ) is plotted in Fig. 6, for weakly and strongly coupled dusty plasma in the presence and absence of the polarization force. It shows that the solitary wave solution (28) exists at the upper region of the threshold, while it cannot exist at the lower region. This figures show that the existence range of solitons increases with increasing l_ξ and U. The strong interaction between the dust grains decreases the existence domain of the solitary structures in space (U, l_ξ) (Figs. 6(c) and 6(d)), while the polarization force shows a reverse and smaller influence on the existence domain (Figs. 6(b) and 6(d)).

We have also examined the variation of the Sagdeev potential with the electrostatic potential ϕ in Fig. 7 for weakly (panel (a)) and strongly (panel (b)) coupled regimes, in which the solid line includes the polarization force. It can be seen that the depth of Sagdeev potential increases due to the presence of polarization force, and experiences a decrease when we consider the strong interaction between the dust grains. The corresponding electrostatic potential for weakly and strongly coupled regimes is depicted in Figs. 8(a) and 8(b), respectively. This implies that the larger solitons may be excited in the presence of polarization force (solid lines) in the weakly coupled regime (Fig. 8(a)).

The bipolar electric field structures associated with the rarefactive potentials are shown in Fig. 9, as a function of ζ for weakly and strongly coupled regimes in the presence and absence of polarization force. Panels (a) and (b) refer to the weakly and strongly coupled regimes, respectively. It can be seen that the strong interaction between the dust grains leads to the lower localized electric field with smaller maximum amplitude. The narrower solitons which come from the weakly coupled regime having steeper slopes lead to the higher values of the electric field amplitude.

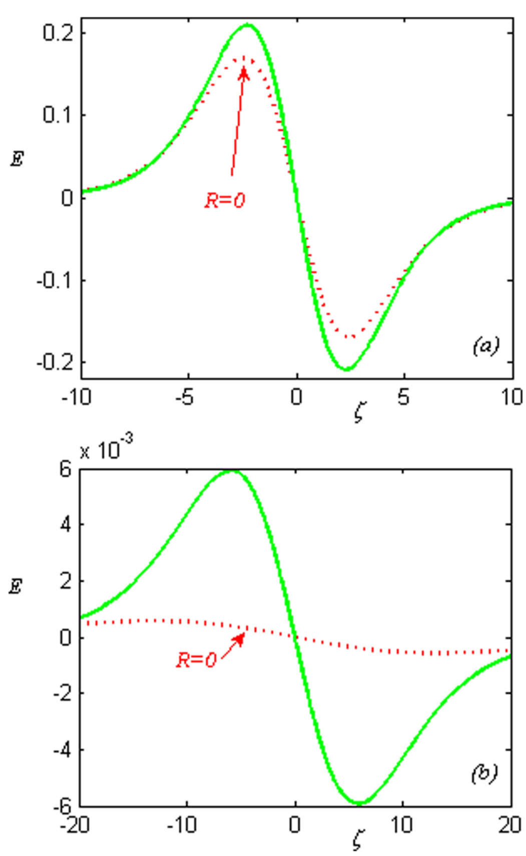

FIG. 9. The bipolar electric field structure as a function of ζ for (a) the weakly and (b) strongly coupled regime in the presence and absence of polarization force. We have taken the parameters: $k_BT_i = 0.03\,\text{eV}$, $k_BT_e = 3\,\text{eV}$, $n_{i0} = 8 \times 10^{13}\,\text{m}^{-3}$, $n_{e0} = 4 \times 10^{13}\,\text{m}^{-3}$, $r_d = 2 \times 10^{-7}\text{m}$, $U = 1.15$, and $l_\xi = 0.97$. The dashed line refers to $R = 0$ and the solid line refers to $R \neq 0$.

V. CONCLUSIONS

A three dimensional strongly coupled dusty plasmas have been considered to examine the effects of effective electrostatic temperature and polarization force (i.e., the force due to the polarized Debye sheath) on the existence, formation, and profile of DA solitary waves. The strongly correlated negatively charged dust grains and weakly correlated electrons and ions are assumed in our plasma model. By using the reductive perturbation technique, we derived a KP equation describing the DA solitary structures. We employed the energy integral equation to study the existence domains of the solitary structures. The main points of this investigation are summarized as follows:

(i) In the linear regime, analysis of the linear dispersion properties shows that the strong interaction between the dust particles makes the solitary structures faster than that of in a weakly coupled dusty plasma. However, the polarization force shows an inverse effect on the phase velocity, and leads also to the slower solitons both in the strongly and weakly coupled regimes.

(ii) We have investigated the existence domain of the solitary solution through a parametric analysis and found that this domain is dependent on the concentration as well as the temperature of electrons and ions. The ion (electron) number density has an increasing (a decreasing) effect on the existence domain of solitary structures. The ion and electron temperatures also show a similar influence on the existence domain of solitary structures. On the other hand, the existence region of solitons experiences a wider range with increasing l_ξ and U.

(iii) The strong interaction between the dust particles restricts the existence domain of solitons to the smaller range, but the polarization force has a reverse effect. We have found that the effect of strong interaction is more pronounced than that of the polarization force.

(iv) The profile of solitary structures depends significantly on the effective electrostatic temperature and the polarization force. We observe that the solitary structures experience a reduction in their amplitude in the presence of strong interaction between the dust grains, whereas the polarization force increases the amplitude of solitons. Among the four possible cases, the solitary structures with the smallest amplitude refer to the case of $T_{eff} \neq 0$ and $R = 0$, while for $T_{eff} = 0$ and $R \neq 0$ the solitons amplitude reaches to their maximum value. Also, the corresponding electric fields in the weakly coupled regime are more localized.

Our results address the physical setting of localized structures in strongly coupled dusty plasma, adopting the multiple scale reductive perturbation approach. We expect that the present investigation should be useful for understanding the localized electrostatic disturbances in space, astrophysical, and laboratory dusty plasma.

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APPENDIX: DERIVATION OF THE POLARIZATION FORCE

In this Appendix, we give derivation of Eq. (4) in more details. The normalized Debye length $\bar{\lambda}_D$ (normalized by $\lambda_{D0} = \sqrt{\varepsilon_\circ k_B T_0 / n_{d0} Z_d^2 e^2}$) is given by

$$\bar{\lambda}_D = \lambda_D/\lambda_{D0} = 1/\lambda_{D0}\sqrt{1/\lambda_{Di}^2 + 1/\lambda_{De}^2} = \sqrt{(n_{i0}T_e + n_{e0}T_i)\Big/\left[n_{i0}\exp\left(\frac{-T_0\phi}{Z_dT_i}\right)T_e + n_{e0}\exp\left(\frac{T_0\phi}{Z_dT_e}\right)T_i\right]}. \tag{A1}$$

By Taylor expanding of the exponential functions, for small electrostatic potential ϕ, we obtain

$$\begin{aligned}\bar{\lambda}_D &= \sqrt{\frac{T_eZ_dn_{d0}\left(\dfrac{n_{i0}}{Z_dn_{d0}} + \dfrac{n_{e0}}{Z_dn_{d0}}\dfrac{T_i}{T_e}\right)}{n_{i0}\left[1 - \dfrac{T_0\phi}{Z_dT_i} + \dfrac{1}{2}\left(\dfrac{T_0}{Z_dT_i}\right)^2\phi^2 + \dots\right]T_e + n_{e0}\left[1 + \dfrac{T_0\phi}{Z_dT_e} + \dfrac{1}{2}\left(\dfrac{T_0}{Z_dT_e}\right)^2\phi^2 + \dots\right]T_i}} \\ &= \sqrt{\frac{1+f+f\sigma}{1+f+f\sigma + \left[-\dfrac{1+f-f\sigma^2}{1+f+f\sigma}\right]\phi + \left[\dfrac{1+f+f\sigma^3}{2(1+f+f\sigma)^2}\right]\phi^2 + \dots}}, \\ &= \sqrt{\frac{1+f+f\sigma}{\sum_{l=0}^{\infty}(l+1)d_l\phi^l}}\end{aligned} \tag{A2}$$

in which $n_{e0}/Z_dn_{d0} = f$, $n_{i0}/Z_dn_{d0} = 1+f$, $T_i/T_e = \sigma$, $d_0 = 1+f+f\sigma$, $d_1 = -\frac{1+f-f\sigma^2}{2(1+f+f\sigma)}$, and $d_2 = \frac{1+f+f\sigma^3}{6(1+f+f\sigma)^2}$. Then, $\nabla\lambda_D/\lambda_D^2$ can be approximated as follows:

$$\frac{\nabla\lambda_D}{\lambda_D^2} = \frac{1}{\lambda_{D0}^2}\frac{\nabla\bar{\lambda}_D}{\bar{\lambda}_D^2} = \frac{1}{\lambda_{D0}^2}\nabla\left[\sqrt{\frac{1+f+f\sigma}{\sum_{l=0}^{\infty}(l+1)d_l\phi^l}}\right] \Bigg/ \left[\sqrt{\frac{1+f+f\sigma}{\sum_{l=0}^{\infty}(l+1)d_l\phi^l}}\right]^2$$

$$= \frac{1}{\lambda_{D0}^2}\left[-\frac{1}{2}\frac{\sqrt{1+f+f\sigma}\sum_{l=0}^{\infty}l(l+1)d_l\phi^{l-1}\nabla\phi}{\left(\sum_{l=0}^{\infty}(l+1)d_l\phi^l\right)^{3/2}}\right] \Bigg/ \left[\frac{1+f+f\sigma}{\sum_{l=0}^{\infty}(l+1)d_l\phi^l}\right]$$

$$= \frac{-1}{2\lambda_{D0}^2}\frac{\sum_{l=0}^{\infty}l(l+1)d_l\phi^{l-1}\nabla\phi}{\sqrt{1+f+f\sigma}\left(\sum_{l=0}^{\infty}(l+1)d_l\phi^l\right)^{1/2}} \approx \frac{-1}{2\lambda_{D0}^2\sqrt{1+f+f\sigma}}\frac{(2d_1+6d_2\phi)\nabla\phi}{(d_0+2d_1\phi)^{1/2}}$$

$$= \frac{-1}{2\lambda_{D0}^2(1+f+f\sigma)}\frac{(2d_1+6d_2\phi)\nabla\phi}{\left(1+2\frac{d_1}{d_0}\phi\right)^{1/2}} = \frac{-\nabla\phi}{2\lambda_{D0}^2(1+f+f\sigma)}(2d_1+6d_2\phi)\left(1-\frac{d_1}{d_0}\phi\right)$$

$$\approx \frac{-\nabla\phi}{2\lambda_{D0}^2(1+f+f\sigma)}\left(2d_1+6d_2\phi-2\frac{d_1^2}{d_0}\phi\right) = \frac{-\nabla\phi}{\lambda_{D0}^2(1+f+f\sigma)}\left[d_1+\left(3d_2-\frac{d_1^2}{d_0}\right)\phi\right].$$

By using $(1+f+f\sigma) = Z_dT_i/T_0$, the above expression takes the following form:

$$\Rightarrow \frac{\nabla\lambda_D}{\lambda_D^2} \cong -\frac{T_0}{Z_dT_i\lambda_{D0}^2}\nabla\phi\left\{d_1+\left(3d_2-\frac{d_1^2}{d_0}\right)\phi\right\}. \quad (A3)$$

Therefore, the normalized polarization force can be written as

$$\tilde{F}_p = \frac{F_p}{m_dc_t\omega_{pd}} = \frac{\lambda_{D0}}{k_BT_0}F_p = -\frac{\lambda_{D0}}{k_BT_0}\frac{Z_d^2e^2}{8\pi\varepsilon_\circ}\frac{\nabla\lambda_D}{\lambda_D^2}$$
$$\cong \frac{Z_de^2}{8\pi\varepsilon_\circ T_i\lambda_{D0}}\nabla\phi\left\{d_1+\left(3d_2-\frac{d_1^2}{d_0}\right)\phi\right\}. \quad (A4)$$

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