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Comparison of Theoretical with Experimental *p–n* Junction Recombination Effects*

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The current–voltage characteristics of an inhomogeneously doped *p–n* junction have been calculated using full Shockley–Read statistics. The equations solved include calculation of the minimal lifetimes from an inhomogeneous flaw distribution and the appropriate capture cross sections. When Poisson's equation and the hole and electron continuity equations are solved numerically, the injection parameter for the junction is found to be current dependent. When flaw densities and capture cross sections are adjusted to fit experimental measurements, the resulting minimal lifetimes are much shorter than those usually considered in the literature. This discrepancy is explained by the restriction of the recombination process to the depletion regions and the absence of recombination in the quasineutral regions of the device.

I. INTRODUCTION

The current–voltage characteristic of a silicon *p–n* junction has been calculated under vary minimal physical assumptions.[1,2] The impurity profile of the junction calculated is shown in Fig. 1. It has been found in junctions of this type that the generation–recombination current over much of the operating range of the junction is the dominant term. In order to compute lifetimes in such a junction, a flaw profile as shown in Fig. 2 is introduced.

II. LIST OF SYMBOLS

ψ	electrostatic potential
q	charge on the electron
K	relative dielectric constant
ϵ_0	permittivity of free space
n	electron density
p	hole density
N_a^-	density of ionized acceptors
N_d^+	density of ionized donors
E	electric field
J_p	hole current density
μ_p	hole mobility
D_p	hole diffusion constant
J_n	electron current density
μ_n	electron mobility
D_n	electron diffusion constant
U	total generation–recombination rate
$\mathfrak{F}_j$	Fermi–Dirac integral of order j
x	distance variable
L	device length
γ_p	reciprocal hole mobility
R_p	hole Fermi energy reduction factor
R_n	electron Fermi energy reduction factor
γ_n	reciprocal electron mobility
R	recombination rate
p_0	equilibrium hole density
p_e	excess hole density, $p_e=p-p_0$
p_1	hole density when $\phi=E_f$
n_0	equilibrium electron density
n_e	excess electron density, $n_e=n-n_0$
n_1	electron density when $\phi=E_f$
τ_{p0}	minimal electron lifetime
τ_{n0}	minimal hole lifetime
N_f	flaw density
$\bar{v}$	carrier thermal velocity
c_n	capture cross section for electrons
c_p	capture cross section for holes
m_e	mass of an electron
ϕ	equilibrium Fermi energy
E_c	energy of the conduction band
E_g	bandgap energy
E_v	energy of the valance band
E_f	flaw energy level

III. THEORY

Equation (1): Basic Equations

In order to get reasonable agreement between theory and experiment, the basic equations shown below must be solved[3]:

$$\nabla^2\psi=(q/K\epsilon_0)(n-p+N_a^- - N_d^+), \tag{1a}$$

$$\mathbf{E}=-\nabla\psi, \tag{1b}$$

$$\mathbf{J}_p=q\mu_p p\mathbf{E}-qD_p\nabla p, \tag{1c}$$

$$\mathbf{J}_n=q\mu_n n\mathbf{E}+qD_n\nabla n, \tag{1d}$$

$$\nabla^*\mathbf{J}_p=-qU, \tag{1e}$$

$$\nabla^*\mathbf{J}_n=qU, \tag{1f}$$

$$D_p=\mu_p\frac{kT}{q}\left[\mathfrak{F}_{1/2}\left(\frac{E_c-E_g-\phi}{kT}\right)\Big/\mathfrak{F}_{-1/2}\left(\frac{E_c-E_g-\phi}{kT}\right)\right], \tag{1g}$$

$$D_n=\mu_n\frac{kT}{q}\left[\mathfrak{F}_{1/2}\left(\frac{\phi-E_c}{kT}\right)\Big/\mathfrak{F}_{-1/2}\left(\frac{\phi-E_c}{kT}\right)\right], \tag{1h}$$

$$\mathfrak{F}_j(\eta)=[\Gamma(j+1)]^{-1}\int_0^\infty\frac{\epsilon^j d\epsilon}{1+\exp(\epsilon-\eta)}. \tag{1i}$$

Equation (2): Hole and Electron Integral Equations

A list of symbols used here is given in Sec. II. These equations are solved by a numerical procedure. This numerical procedure consists first, of solving the Poisson equation using conventional, numerical, tridiagonalization techniques; and second, by rearranging the transport and continuity equations in the form shown below for hole and electron densities:

$$p(x)=e^{-\psi_p}\left(-\int_0^L \gamma_p J_p e^{\psi_p}dx'+p(L)\exp[\psi_p(L)]\right) \tag{2a}$$

$$\gamma_p=1/\mu_p(x) \tag{2b}$$

$$\psi_p=\psi(x)/R_p \tag{2c}$$

$$R_p=\left[\mathfrak{F}_{1/2}\left(\frac{E_c-E_g-\phi}{kT}\right)\Big/\mathfrak{F}_{-1/2}\left(\frac{E_c-E_g-\phi}{kT}\right)\right] \tag{2d}$$

$$n(x)=e^{\psi_n}\left(\int_0^L \gamma_n J_n e^{-\psi_n}dx'+n(L)\exp[-\psi_n(L)]\right) \tag{2e}$$

$$\gamma_n=1/\mu_n(x) \tag{2f}$$

$$\psi_n=\psi(x)/R_n \tag{2g}$$

$$R_n=\left[\mathfrak{F}_{1/2}\left(\frac{\phi-E_c}{kT}\right)\Big/\mathfrak{F}_{-1/2}\left(\frac{\phi-E_c}{kT}\right)\right], \tag{2h}$$

Equation (3): Generalized Current Integral Equations

Below for the appropriate hole and electron currents[4]:

$$J_p(x)=\int_0^x Udx+\left(p(L)\exp[\psi_p(L)]-p(0)\exp[\psi_p(0)]-\int_0^L\gamma_p\exp(\psi_p)\int_0^{x'}Udx''dx'\right)\Big/\int_0^L\gamma_p\exp(\psi_p)dx' \tag{3a}$$

$$J_n(x)=-\int_0^x Udx+\left(n(0)\exp[-\psi_n(0)]-n(L)\exp[-\psi_n(L)]+\int_0^L\gamma_n\exp(-\psi_n)\int_0^{x'}Udx''dx'\right)\Big/\int_0^L\gamma_n\exp(\psi_n)dx'. \tag{3b}$$

Equation (4)

The generation–recombination current can then be obtained by subtracting the change in either hole and electron current across the device, yielding that part of the current which is transported by the generation–recombination process. In the forward biased case, the generation–recombination term U is given entirely by the recombination rate R which is shown below; these equations are the full Shockley–Read equations[5]:

$$R=(p_0n_e+n_0p_e+n_ep_e/\tau_{p0}(n_1+n)+\tau_{n0}(p_1+p)) \tag{4a}$$

$$\tau_{p0}=1/N_f\bar{v}c_p \tag{4b}$$

$$\tau_{n0}=1/N_f\bar{v}c_n \tag{4c}$$

$$\bar{v}=(2m_ekT)^{1/2} \tag{4d}$$

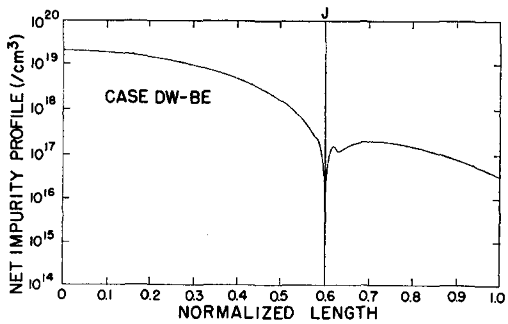

FIG. 1. Net impurity profile.

It should be noted that the minimal lifetimes τ_{p0} and τ_{n0} are computed using capture cross sections for holes and electrons, the mean thermal velocity of carriers, and the flaw density (shown in Fig. 2). This causes the effective lifetime to be a function of the bias level and of position throughout the device through the entire calculation.

IV. RESULTS

Using this technique, hole and electron currents and generation–recombination currents are calculated as shown in Figs. 3 and 4. These currents can be seen to agree in terms of the shape of the curve with experimental values of the current for a single junction of a diode connected transistor with a doping profile as given in Fig. 1. Experimental measurements on such a junction are given in Figs. 5 and 6. In order to

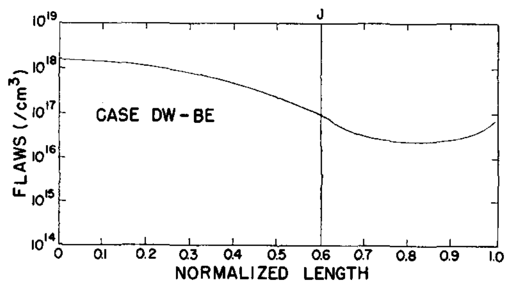

FIG. 2. Flaw profile.

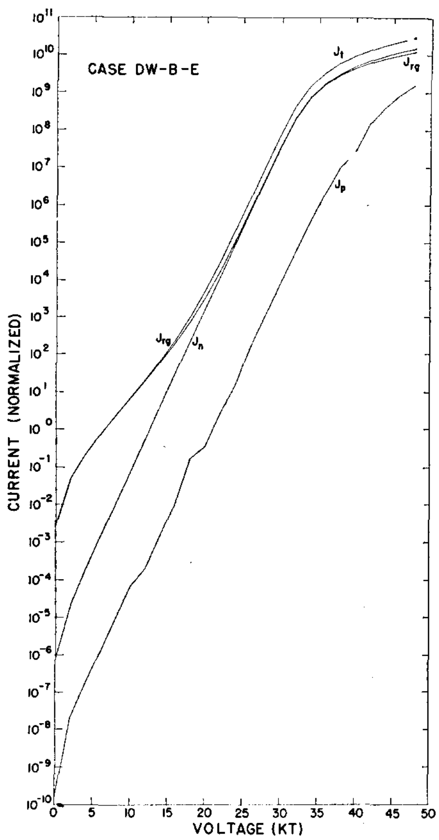

FIG. 3. Calculated forward current–voltage characteristic.

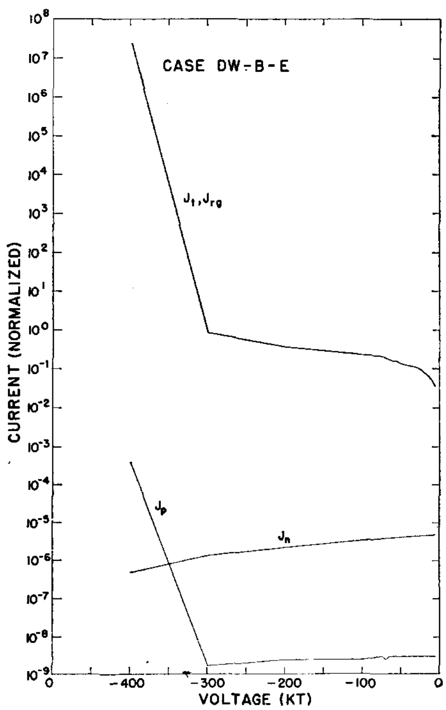

FIG. 4. Calculated reverse current–voltage characteristic.

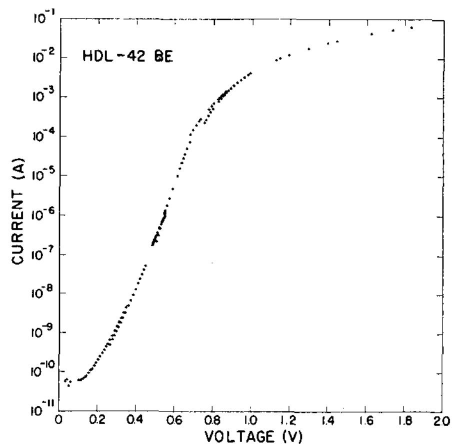

FIG. 5. Measured forward current–voltage characteristic.

obtain this agreement of curve shapes, however, the flaw density and capture cross section for holes and electrons had to be approximately 10 times higher so as to yield minimal lifetimes approximately 10 times lower than those normally given in the literature[5] with $c_p = 1.0 \times 10^{-14}$ cm^2 and $c_n = 3.5 \times 10^{-14}$ cm^2. This is attributed to the fact that, in the relatively exact calculation carried out here, the generation–recombination process was restricted almost entirely to the depletion layer. The extent of the depletion layer and the spacial dependence of the generation–recombination process are shown in Figs. 7 and 8, respectively. From these considerations one can conclude that in order to accurately compute the current–voltage characteristics of

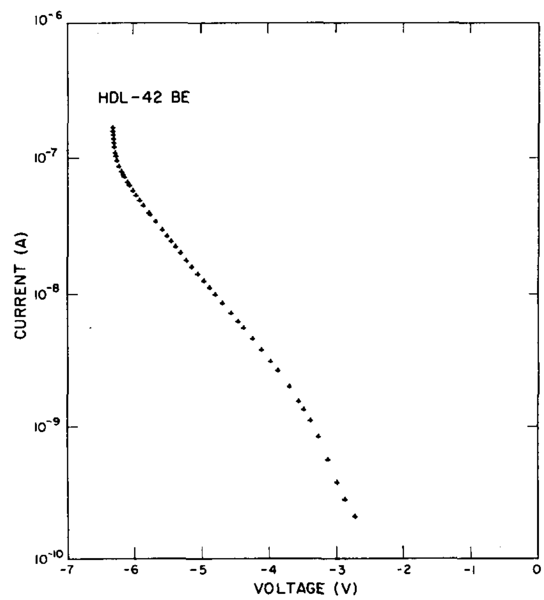

FIG. 6. Measured reverse current–voltage characteristic.

p–n junctions so that there is good agreement between theory and experiment, it is necessary to accurately know both the extent and the shape of the depletion layer and its effect upon the minimal lifetimes. This, in turn, requires that a specific model of flaw density and a specific set of recombination equations, such as the Shockley–Read[6] equations used here, be used. In addition, it requires that the capture cross sections normally available in the literature be adjusted and/or that the flaw densities usually considered be readjusted so that the calculation will be in agreement with experiments.

V. CONCLUSIONS

In order to make accurate measurements of the capture cross sections for holes and electrons through monovalent flaws, it is necessary to have a good idea of the extent of the depletion layer and, therefore, the doping profile in the specific p–n junction under consideration. This, in turn, tends to invalidate many of the assumptions used to reduce various sorts of experimental measurements to capture cross sections and would indicate that complete solution for the p–n junction using a complex computer program, such as the TACS1 program,[2] is necessary in most cases in order to obtain accurate information about the recombination processes which take place in silicon p–n junctions.

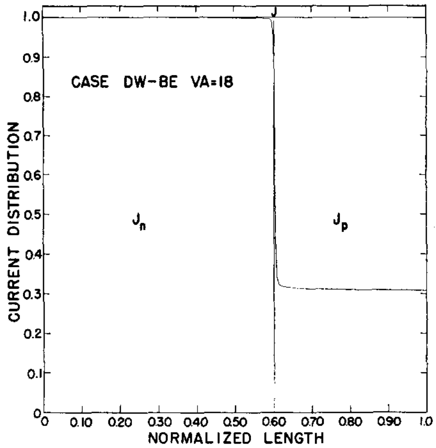

FIG. 8. Charge in current due to recombination.

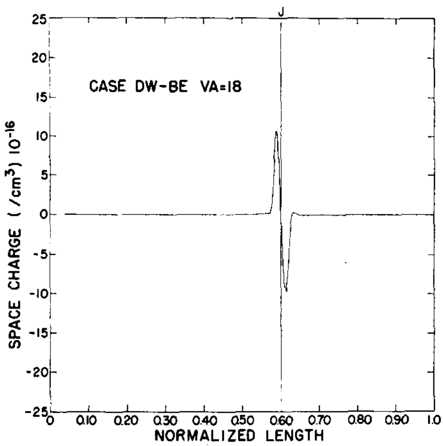

FIG. 7. Space-charge dipole under forward biased conditions.

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