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On the rogue waves propagation in non-Maxwellian complex space plasmas

S. A. El-Tantawy,[1,a)] E. I. El-Awady,[1,b)] and M. Tribeche[2,c)]
[1]*Department of Physics, Faculty of Science, Port Said University, Port Said 42521, Egypt*
[2]*Plasma Physics Group, Theoretical Physics Laboratory, Faculty of Physics, University of Bab-Ezzouar, USTHB, BP 32, El Alia, Algiers 16111, Algeria*

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The implications of the non-Maxwellian electron distributions (nonthermal/or suprathermal/or nonextensive distributions) are examined on the dust-ion acoustic (DIA) rogue/freak waves in a dusty warm plasma. Using a reductive perturbation technique, the basic set of fluid equations is reduced to a nonlinear Schrödinger equation. The latter is used to study the nonlinear evolution of modulationally unstable DIA wavepackets and to describe the rogue waves (RWs) propagation. Rogue waves are large-amplitude short-lived wave groups, routinely observed in space plasmas. The possible region for the rogue waves to exist is defined precisely for typical parameters of space plasmas. It is shown that the RWs strengthen for decreasing plasma nonthermality and increasing superthermality. For nonextensive electrons, the RWs amplitude exhibits a bit more complex behavior, depending on the entropic index q. Moreover, our numerical results reveal that the RWs exist with all values of the ion-to-electron temperature ratio σ for nonthermal and superthermal distributions and there is no limitation for the freak waves to propagate in both two distributions in the present plasma system. But, for nonextensive electron distribution, the bright- and dark-type waves can propagate in this case, which means that there is a limitation for the existence of freak waves. Our systematic investigation should be useful in understanding the properties of DIA solitary waves that may occur in non-Maxwellian space plasmas. © *2015 AIP Publishing LLC*.
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I. INTRODUCTION

The observations in space plasma environments indicate the perturbation of the Maxwellian plasma[1] which may occur in the perimeter of a spacecraft because of interactions between the spacecraft and thermospheric gases, photoemission, or electron emissions from other devices on the spacecraft. The non-Maxwellian plasma distributions may also occur in nature as a mixture of ionospheric and magnetospheric plasmas or secondaries produced by photoionization in the thermosphere or auroral precipitation. Mostly, these nonthermal velocity distributions include a ring structure, and the simplest analytical way to model such effects is by the Cairns distribution,[2] where the influence of nonthermal electrons on the existence conditions of ion-acoustic solitary structures has been included. Such electron distribution with enhanced population of energetic electrons allows for the existence of both positive and negative density perturbations. On the other hand, such nonthermal electron populations may be distributed isotropically in velocities or possess a net streaming motion with respect to the background plasma, and their presence was confirmed by many observations of space plasmas.[3] Their characteristics have been observed in the upper ionosphere by Freja and Viking satellites.[3] They offer a considerable increase in richness of the wave motion that may exist in such plasmas and further significantly influence the conditions required for the formation of these waves.

An alternative approach to the Cairns distribution is the so called kappa distribution function. The latter mainly differs from the Maxwellian in the nature of the high energy tail. In this paper, we devote attention to the effect of non-Maxwellian electron distribution functions on the collective plasma behavior through a nonlinear analysis of perturbed fluid equations. Such energetic electrons populate the superthermal region of the velocity distribution and thus result in a long-tailed distribution function. The form of a phenomenological long-tailed distribution, termed the kappa-(κ) distribution, where κ defines the strength of superthermality, was first postulated by Vasyliunas,[4] who succeeded in reproducing the observed power-law dependence at high energies. The spectral index κ measures the slope of the energy spectrum of the superthermal particles at the tail of the distribution function such that smaller (larger) values of κ represent high (low) concentrations of superthermal particles in the tail of the distribution function and $\kappa > 2/3$ should hold for a physically meaningful solution. In the limit $\kappa \to \infty$, the κ-distribution function reduces to the well-known Maxwellian limit. The kappa distribution has been successfully employed to explain many astrophysical and space plasma situations, e.g., in the Earth's magnetosphere,[5] the auroral zone,[6] the solar wind,[7] and the interstellar medium.[8] Interestingly, non-Maxwellian energetic particles are also observed in laboratory plasmas.[9]

The Maxwellian distribution arising from the Boltzmann–Gibbs (BG) statistics is believed valid universally for the macroscopic ergodic equilibrium systems. However, for systems endowed with long-range interactions, such as plasmas and gravitational systems, where non-equilibrium stationary

a)E-mail: samireltantawy@yahoo.com
b)E-mail: eielawady@hotmail.com
c)Electronic addresses: mouloudtribeche@yahoo.fr and mtribeche@usthb.dz

states may exist, Maxwellian distribution might be inadequate for the description of the systems. For example, the experimentally measured phase velocity of the ion acoustic waves was 70% higher than the theoretical value derived under the presupposition that the plasma is described by Maxwellian distribution.[10] In the experiment for measuring the ion acoustic waves, the energy distribution of electrons may actually be not the Maxwellian one, and hence, it is hard to determine the valid electron temperature.[11] In fact, the non-Maxwellian velocity distributions for electrons in plasma were already measured in the experiment where the temperature gradient was steep.[12] The non-Maxwellian velocity distributions for ions were also reported in the studies of the earth's plasma sheet, the solar wind, and the long-range interacting systems containing plentiful superthermal particles, i.e., the particles with the speeds faster than the thermal speed.[13]

Recent works have been focused on a new statistical approach, i.e., nonextensive statistics (or Tsallis statistics). The latter is described by a nonextensive parameter q. For $q \neq 1$, it gives power-law distribution functions, and only when the parameter $q \to 1$, the Maxwellian distribution is recovered.[14] It is thought to be a useful generalization of the BG statistics and to be appropriate for the statistical description of the long-range interaction systems, characterizing the nonequilibrium stationary states.[15] Lima *et al.*[16] have shown that, for electrostatic plane-wave propagation in plasmas, Tsallis formalism presents a good fit to the experimental data, while the standard Maxwellian distribution only provides a crude description. By restricting the value of the q-parameter from the experimental data, one can achieve a good agreement between the theory and the experiment.[17] Furthermore, the flexibility provided by the nonextensive parameter q enables one to obtain a good agreement between the theory and experiment.

During the last decades, many authors have paid attention to the nonlinear structures (solitary and shock excitations) in nonlinear medium and non-Maxellian plasmas (see for example, Refs. 18–25). In view of the crucial importance of the effects of the plasma applications with non-Maxwellian electron distributions, there is an urgent need for investigating the effects of non-Maxwellian electrons on the propagation of freak (rogue) waves in complex plasmas, which appear from nowhere and disappear without a trace. Inspired by the ubiquity of this challenging phenomenon, we have undertaken an investigation of the occurrence of dust-ion-acoustic (DIA) freak waves in a dusty plasma composed of warm positive ions, non-Maxwellian electrons, and stationary negative dust. Particular questions to be answered are (i) can the nonlinear Schrödinger (NLS) equation which is derived from the modified Korteweg-de Vries (mKdV) equation support the propagation of rogue waves or not? (ii) how rogue pulses are influenced by the non-Maxwellian electrons? and (iii) how can the ion-to-electron temperature ratio and non-Maxwellian parameters modify the trajectories of the rogue excitations? The results from this work are expected to contribute to the understanding of the nonlinear excitations that may appear in the space plasma.

II. MATHEMATICAL MODEL AND EVOLUTION EQUATIONS

Our theoretical plasma model for describing dust-ion-acoustic freak waves in collisionless, unmagnetized plasma consists of warm positive ions, non-Maxwellian electrons, and stationary negative dust. As is usual for ion acoustic structures, it is assumed that the wave phase speed (υ_{ph-i}) lies between the ion thermal speed (υ_{th-i}) and electron thermal speed (υ_{th-e}), i.e., $\upsilon_{th-i} \ll \upsilon_{ph-i} \ll \upsilon_{th-e}$, to avoid Landau damping.

For one-dimensional structures, the warm positive ions is described by the dimensionless fluid equations[26]

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)u + 3\sigma n\frac{\partial}{\partial x}n + \frac{\partial}{\partial x}\phi = 0. \tag{2}$$

These equations are closed by Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n + \alpha_d. \tag{3}$$

In Eqs. (1)–(3), $n(n_e)$ is the normalized ion (electron) number density, u is the normalized ion fluid velocity, $\sigma = T_i/T_e$ is the temperature ratio of the ions-to-electrons, $\alpha_d = Z_d n_d^{(0)}/n_i^{(0)}$ is the fraction of negative charge residing on the dust, where $n_d^{(0)}$ ($n_i^{(0)}$) and Z_d are the unperturbed dust (ions) number density and the dust charge number, respectively, ϕ is the normalized electric potential, and x and t are the normalized space and time variables, respectively. It should be mentioned here that we use the same normalized basic equations that have been used by El-Tantawy *et al.*[27] for studying nonlinear ion-acoustic structures in a dusty plasma with superthermal particles.

In a collisionless plasma, the most commonly used distribution is the Maxwellian velocity distribution, which is a distribution describing the thermal equilibrium. However, space, astrophysical, and laboratory plasma observations indicate clearly the presence of electron populations which are far away from their thermodynamic equilibrium.[28–31] Thus, for electrons obeying non-Maxwellian distributions, some cases will be considered in this paper for examining DIA freak waves in a dusty plasma with: (i) Nonthermal electron distribution,[26,32] (ii) Kappa electron distribution,[33,34] and (iii) Nonextensive electron distribution.

For Cairns distribution,[2] the normalized electron number density is given by

$$n_e = \mu[1 - \beta\phi + \beta\phi^2]\exp(\phi), \tag{4}$$

where $\beta = 4\delta/(1+3\delta)$. The parameter δ determines the number of nonthermal electrons present in our plasma model. In the thermal limiting case ($\delta = 0$), the density reduces to its well-known Boltzmann counterpart.[6] The electron number density n_e has been obtained by integration of the Cairns distribution over the velocity space. Here, $\mu = n_e^{(0)}/n_i^{(0)}$ is the electrons concentration, where $n_e^{(0)}$ is the unperturbed density of the electrons and is connected to the negative dust concentration α_d via the neutrality condition as, $\mu = 1 - \alpha_d$.

One major characteristic associated with collisionless space plasmas is the development of non-Maxwellian velocity distribution that in many circumstances can be represented by the so called κ–(Kappa) distribution.[33] As the fast particles are nearly collisionless in space plasmas and some laboratory experiments, they are easily accelerated and tend to produce non-equilibrium velocity distribution function with long high-energy tails decreasing as a power law in this energy range. The normalized electrons number density in this case can be expressed as[35]

$$n_e = \mu\left[1 - \frac{\phi}{\tilde{b}}\right]^{-\tilde{a}}, \tag{5}$$

where $\tilde{a} = (\kappa - 1/2)$, $\tilde{b} = (\kappa - 3/2)$, and the spectral index $\kappa > 3/2$ determines the slope of the superthermal energy distribution. Kappa distribution reduces to the Maxwellian distribution for $\kappa \to \infty$.

For Tsallis (nonextensive) distribution, the normalized electron number density is given by[36]

$$n_e = \mu[1 + \tilde{c}\phi]^{\tilde{d}}, \tag{6}$$

where $\tilde{c} = (q-1)$, $\tilde{d} = (q+1)/2(q-1)$ and the parameter q stands for the strength of electron nonextensivity. Before proceeding further, it may be useful to note that the nonextensive parameter q includes two cases, i.e., superextensivity for $q < 1$ and subextensivity for $q > 1$. Recently, Ait Gougam and Tribeche [see Ref. 37 for more details] demonstrated that the acceptable range for $q < 1$ is narrowed to $1/3 < q < 1$, in order that the physical requirement of energy equipartition is preserved. In the extensive limiting case ($q = 1$), distribution (6) reduces to the well-known Maxwell-Boltzmann velocity distribution.[38]

Finally, Eqs. (4)–(6) can be combined to obtain

$$n_e = \mu + \alpha_1\phi + \alpha_2\phi^2 + \alpha_3\phi^3 + \ldots, \tag{7}$$

where

$$\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{bmatrix}^T = \mu\begin{cases}\left[(1-\beta), \dfrac{1}{2}, \dfrac{1}{6}(1+3\beta)\right] & \text{for nonthermal case,}\\ \left[\dfrac{\tilde{a}}{\tilde{b}}, \dfrac{\alpha_1}{2\tilde{b}}(\tilde{a}+1), \dfrac{\alpha_2}{3\tilde{b}}(\tilde{a}+2)\right] & \text{for superthermal case,}\\ \left[\tilde{c}\tilde{d}, \dfrac{\alpha_1\tilde{c}}{2}(\tilde{d}-1), \dfrac{\alpha_2\tilde{c}}{3}(\tilde{d}-2)\right] & \text{for nonextensive case.}\end{cases} \tag{8}$$

This combination will be used hereafter.

To investigate the propagation of nonlinear electrostatic structures in the present model, the reductive perturbation technique is employed.[39] According to this method, the independent variables Ψ ($\Psi \equiv n, u, \phi$) at a given position x and time t can be stretched as $\Psi = (1,0,0) + \sum_{m=1}^{\infty}\varepsilon^m\Psi^{(m)}$, where ε is a small parameter. Going parallel to that done in standard nonlinear analysis, one can obtain the following Korteweg-de Vries (KdV) like equation

$$\frac{\partial\phi}{\partial\tau} + B\phi\frac{\partial\phi}{\partial\zeta} + A\frac{\partial^3\phi}{\partial\zeta^3} = 0, \tag{9}$$

where $B = A[3(\lambda^2+\sigma)/(\lambda^2-3\sigma)^3 - 2\alpha_2]$, $A = (\lambda^2-3\sigma)^2/(2\lambda)$, and $\lambda = \sqrt{1/\alpha_1 + 3\sigma}$ represent, respectively, the coefficient of the nonlinear term, the coefficient of the dispersive term, and the linear wave phase velocity. When the negative dust concentration α_d is equal to its critical value α_{dc}, the coefficient of the quadratic nonlinear term in the KdV Equation (9) equals to zero, i.e., $B = 0$. For $B = 0$, one can get two roots for α_{dc} as

$$\alpha_{dc} = \frac{1}{24S_1^3\sigma}\left[3S_1^2(1+8S_1\sigma) \mp \sqrt{3S_1^3(2S_1 + 32S_2\sigma)}\right], \tag{10}$$

where $S_{1,2} = \alpha_{1,2}/\mu$. It is clear that the first root, with "−," satisfies the neutrality condition. However, the other root, with "+," has to be rejected because $\alpha_{dc} > 1$ and the neutrality condition is therefore not fulfilled.

As a consequence of the existence of this critical value α_{dc}, the higher-orders have to be taken into account. Therefore, if we use the independent variables $\zeta = \varepsilon(x - \lambda t)$ and $\tau = \varepsilon^3 t$ rather than $\zeta = \varepsilon^{1/2}(x - \lambda t)$ and $\tau = \varepsilon^{3/2}t$, the following mKdV equation is obtained[40]

$$\frac{\partial\phi}{\partial\tau} + C\phi^2\frac{\partial\phi}{\partial\zeta} + A\frac{\partial^3\phi}{\partial\zeta^3} = 0, \tag{11}$$

where $C = 3A[(5\lambda^4 + 30\lambda^2\sigma + 9\sigma^2)/(\lambda^2-3\sigma)^5 - 2\alpha_3]/2$.

The nonlinear Schrödinger equation (NLSE) is used to study the modulational instability (MI) of weakly nonlinear wavepackets, which can be derived by the asymptotic reduction from the mKdV Eq. (11).[41,42] To examine the MI of a weakly nonlinear DIA wavepackets at the critical value of the negative dust concentration α_{dc}, the solution of Eq. (11) can be expressed in terms of weakly modulated sinusoidal wave potential ϕ, as

$$\phi = \sum_{m=1}^{\infty}\varepsilon^m\sum_{l=-m}^{m}\phi_m^{(l)}(X,T)\exp[il(k\zeta - \omega\tau)], \tag{12}$$

where k (ω) is the wavenumber (wave frequency) of the carrier DIA waves. The slow space and time variables X and T

can be stretched as: $\zeta = \varepsilon(X - v_g T)$ and $\tau = \varepsilon^2 T$, where v_g is the group velocity of the envelope wave. It is convenient to note that all the perturbed states depend on the fast scales through the phase $(k\zeta - \omega\tau)$, whereas the slow scales (X, T) enter the arguments of the *m-th* harmonic amplitude $\phi_m^{(l)}$ which must be real. The coefficients in Eq. (12) have to satisfy the condition $\phi_{-m}^{(l)} = \phi_m^{(l)*}$, where the asterisk notation (*) stands for the complex conjugate.

Upon substituting into Eq. (11), and then isolating various orders in m and respective l-th harmonic contributions, one finally obtains a system for the harmonic contributions to each order. The first-order $(m=1)$ with the first harmonic $(l=1)$ gives the linear dispersion relation $\omega = -Bk^3$. The second-order $(m=2)$ with the first-harmonic $(l=1)$ yields the group velocity $v_g = -3Bk^2$. For the third-order $(m=3)$ with the first-harmonic $(l=1)$, the compatibility condition will be found, from which one can obtain the following NLSE:

$$i\frac{\partial\psi}{\partial T} + \frac{1}{2}P\frac{\partial^2\psi}{\partial X^2} + Q|\psi|^2\psi = 0. \tag{13}$$

Here, $\phi_1^{(1)} \equiv \psi$ for brevity. The coefficients of the dispersion and nonlinear terms are, respectively, given by

$$P = \frac{-3k}{\lambda}(\lambda^2 - 3\sigma)^2,$$

and

$$Q = -\frac{3k}{4\lambda}\left[\frac{(5\lambda^4 + 30\lambda^2\sigma + 9\sigma^2)}{(\lambda^2 - 3\sigma)^3} - 2\alpha_3(\lambda^2 - 3\sigma)^2\right].$$

III. FREAK WAVES AND PARAMETRIC INVESTIGATIONS

The NLSE has solution in the form of a monochromatic wave.[43] This wave solution becomes bright (unstable) or dark (stable) waves. Bright-type waves (unstable envelope pulses) occur when $PQ > 0$; whereas dark-type waves (stable envelope pulses) arise for $P/Q < 0$. Since P is always negative, the stability of monochromatic dust-ion-acoustic waves is dependent on the sign of Q. Therefore, the plane waves become unstable if $Q < 0$ and stable when $Q > 0$ for any value of the carrier wavenumber.[43,44]

One of the interesting results here is the modulational instability of the wavepackets leading to the generation of the freak waves. The latter appear suddenly and then disappear without any trace, in the region $PQ > 0$. As a result, the NLSE (13) in the region $PQ > 0$ has a rational solution that is located on a nonzero background and localized both in T and X directions as[44–46]

$$\psi(\zeta,\tau) = \sqrt{\frac{P}{Q}}\left[-1 + \frac{4(1+2iPT)}{1+4X^2+4P^2T^2}\right]\exp(iPT). \tag{14}$$

In order to gain some insight, we have investigated the effect of non-Maxwellian electron distributions on the existence of rogue wave profile as shown in Fig. 1. Figure 2 shows the way to propagate the rogue wave where the blue (white) color corresponds to the region of modulational unstable (stable) pulses, i.e., $PQ > 0$ $(PQ < 0)$. It is clear from this figure that the freak waves may propagate for various plasma parameters within the blue region. Our numerical results reveal that the freak waves exist with all values of the ion-to-electron temperature ratio σ for nonthermal and kappa distributions and there is no limitation for the freak waves propagation in both two distributions in the present plasma system (the figure not included here). However, it is worth to note that the Cairns distribution develops wings and may become unstable for $\delta > 0.25$ as noticed by Verheest and Pillay.[47] But, for q-distribution, the bright- and dark-type

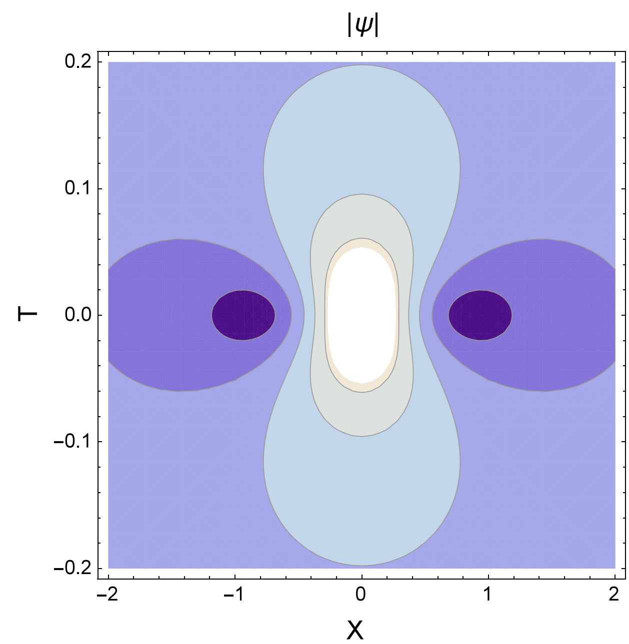

FIG. 1. The rogue wave profile $|\psi|$ is depicted against X and T directions.

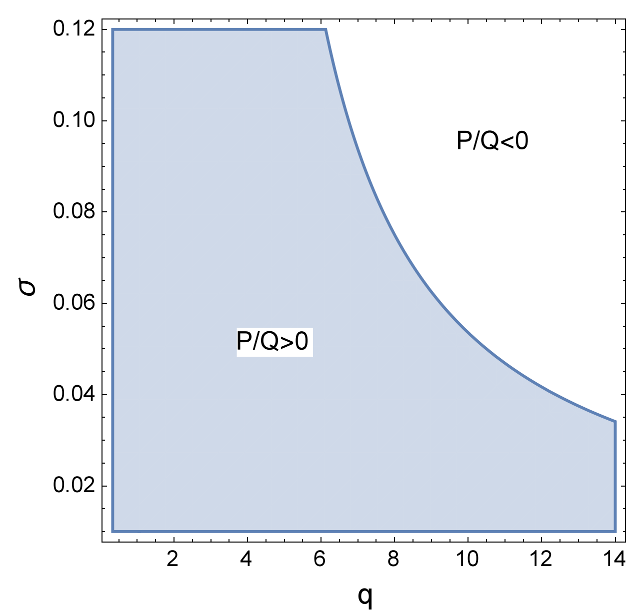

FIG. 2. The ratio P/Q is depicted against the ion-to-electron temperature ratio σ with the nonextensive parameter q. The blue (white) color represents the region where the unstable (stable) pulses set in.

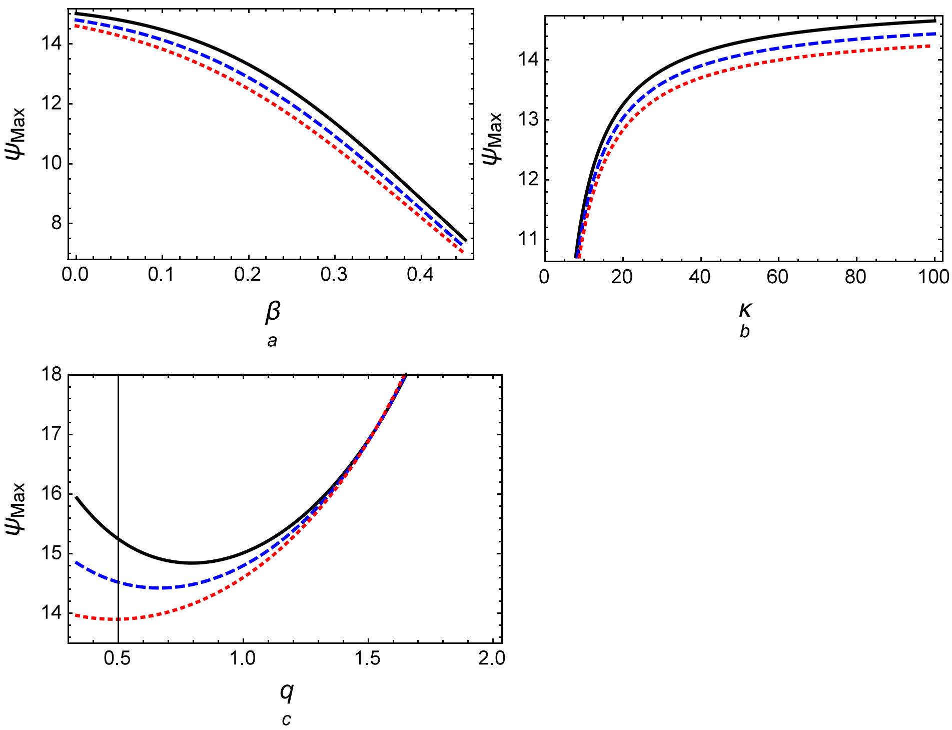

FIG. 3. Maximum amplitude of the rogue wave $\psi_{\rm Max}$ is plotted against (a) the nonthermal parameter β, (b) the superthermal index κ, and the nonextensive parameter q, with different values of the ion-to-electron temperature ratio σ, where $\sigma = 0.05$ (solid line), $\sigma = 0.08$ (dashed line), and $\sigma = 0.12$ (dotted line).

waves can propagate in this case, which means that there is a limitation for the existence of freak waves (see Fig. 2).

In addition, we numerically analyze the maximum rogue wave envelope amplitude $\psi_{\rm Max}(\psi(0,0) = \sqrt{9P/Q})$ and investigate how the non-Maxwellian parameters (β, κ) and the ion-to-electron temperature ratio σ change its profile. Figure 3(a) shows that an increase of σ and β would lead to the reduction of the rogue wave amplitude. It is noticed that for decreasing σ and β, the rogue wave amplitude becomes high and the DIA rogue wave concentrates an amount of energy that makes the pulses taller. However, if σ and β are increased, the energy decreases, reducing therefore the nonlinearity, and the pulses become shorter. The effects of superthermality on the amplitude of the rogue waves are highlighted in Fig. 3(b). In plasmas with higher proportions of superthermal electrons (that is, plasmas with a lower κ value), the nonlinearity parameter is affected, producing rogue waves with decreased amplitude. On the other hand, increasing the superthermal parameter κ (quasi-Maxwellian electrons for $\kappa = 100$) leads to an enhancement of the nonlinearity and then to a significant concentration of energy, which makes the pulses taller. It is interesting to mention that for Cairns distribution, the increase of β leads to an increase of non-Maxwellian particles due to the strong deviation from Maxwellian state. In contrast, for Kappa distribution, increasing κ causes the system to approach the Maxwellian case. Therefore, we can deduce that the freak wave amplitude has maximum values at Maxwellian state, i.e., at $\kappa \to \infty$ and $\beta \to 0$. Next, we turn to an analogous study of the effect of the nonextensive parameter q on the rogue waves profile. Before going to examine the effect of q on the rogue wave profile, it must be noted that the value of q must be lower than 3 to satisfy the neutrality condition ($\mu = 1 - \alpha_d$) and Eq. (10) with together. Figure 3(c) demonstrates that the rogue wave amplitude has complex behavior. One notes that, for $0.3 < q \lesssim 0.8$ and $\sigma = 0.05$, the amplitude of the DIA rogue pulses decays with the enhancement of q while the amplitude augments with an increase of q in the range $q > 0.8$.

IV. SUMMARY

The characteristics of DIA rogue waves have been investigated in a complex plasma composed of warm positive ions, non-Maxwellian electrons, and stationary negative dust. The main emphasis of this paper is to transform the mKdV equation to the NLSE for studying the properties of the rogue waves with electrons obeying non-Maxwellian distributions. For simplicity, we have adopted the quasineutrality hypothesis for our analytical study in the nonlinear part. We have determined and analyzed the $P/Q > 0$ domain with the different values of non-Maxwellian electron distributions where rogue waves may occur, while stable envelope pulses are found for $P/Q < 0$. The relevant plasma configurational parameters (e.g., the nonthermal parameter β, the superthermal index κ, the nonextensive parameter q, and the ion-to-electron temperature ratio σ) are seen to significantly modify the associated characteristics of DIA freak waves in our system. We have shown that the rogue wave amplitude of dust-ion-acoustic waves in a plasma decreases with stronger plasma nonthermality, increasing plasma superthermality, and increasing ion-to-electron temperature ratio. For the nonextensive case, the rogue wave amplitude exhibits complex behavior. Finally, the results of this investigation should be useful in understanding the properties of dust-ion-acoustic solitary waves that may appear in non-Maxwellian space plasmas.[3,48]

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