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Surface tension and orthobaric densities for vibrating square well dumbbells. I

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Surface tensions and liquid-vapor orthobaric densities are calculated for a wide variety of vibrating square well dumbbells using discontinuous molecular dynamics simulations. The size of the vibration well, the elongation or bond distance of the two particles of the dumbbell, the asymmetry in size (and interaction range) of the two particles, and the depth of the interaction well are the variables whose effects are systematically evaluated in this work. Extensive molecular dynamics simulations were carried out and the orthobaric liquid-vapor densities are compared with those obtained previously by other authors using different methods of simulation for rigid and vibrating square well dumbbells. Surface tension values are reported for the first time for homonuclear and heteronuclear vibrating square well dumbbells as well as for all the simulated series. The molecular dynamics results of tangent homonuclear dumbbells are compared with those from Monte Carlo simulations also obtained in this work, as a way of checking the order of magnitude of the molecular dynamics results. The size of the vibration well is shown to have a small influence on the resulting properties. Decreasing elongation and the size of the second particle increase critical temperatures, liquid densities, and surface tensions. Moderate increases in the depth of the interaction well have the same effect. For larger asymmetries of the depth of the interaction well on the dumbbell particles, a strong association phenomenon is observed and the main effects are a maximum on the critical temperature for increasing well depth and a decrease in the surface tension. © 2010 American Institute of Physics. [doi:10.1063/1.3350531]

I. INTRODUCTION

The square well diatomic (SWD) is a simple model of molecular interactions having both repulsive and attractive potentials. Monte Carlo (MC) simulations represent this model as two spheres separated by a fixed distance and discontinuous molecular dynamics (DMD) treats it as a vibrating SWD (VSWD). Both particles are trapped in an intramolecular square well, which keeps them rattling or vibrating together. The main aim of this work is to present a systematic study of the behavior of VSWD to calculate orthobaric densities and surface tensions. Orthobaric densities for some systems have already been reported but there are no surface tension data published. The systematic study is based on some variables that account for the shape and the interaction of the molecules. The size of the vibrating well L_{vw} is the first variable selected. The elongation L (the distance between the centers of the particles) and the relative size of the second particle σ_2 are variables considered also in this study. The relative depth of the interacting square well of the second sphere ϵ_2 is used as the fourth variable taken into account. The diameter and the well depth of the first particle are kept constant for all series considered ($\sigma_1=1$ and $\epsilon_1=1$). Some of the series presented here show a strong association,

and using a combination of the last three variables mentioned allows us to cover the range from VSWD to associating fluids^{1,2} passing through a patchy model having just one patch.^{3,4} A systematic study of associating phenomena will be the subject of a separate paper that will be submitted shortly.

DMD has been used since the initial work for hard spheres by Alder *et al.*⁵ to study simple fluids that have complex physical behaviors. This procedure was extended to chains of spheres interacting with a square well potential by Rapaport.^{6,7} MC simulations of hard core chains were performed by Tildesley *et al.*,⁸ which obtained an equation of state for these series based on the calculated contact radial distribution functions. DMD simulations were applied to different forms of molecules by Chapela *et al.*^{9,10} Extensive DMD, MC, and Gibbs ensemble simulations were performed by Hall and co-workers^{11–13} for SWDs and its mixtures. They compared their results against the equation of state obtained by integral equations and perturbation theory. The liquid-vapor equilibrium of square well chains of different elongations and number of particles in the chain were studied by Elliot and co-workers.^{14,15} MC simulations by Escobedo *et al.*^{16,17} were performed for homonuclear SWD and homonuclear chains up to 100 tangent segments interacting with a SW potential. Kim *et al.*¹⁸ performed MC simulations for SWD at different attraction ranges of the SW. Lisal *et al.*¹⁹ studied homonuclear and heteronuclear SWDs. The data ob-

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tained with these computer simulations have been used to test perturbation^{11,13,20} and integral equation theories.^{13,21–24} The calculation of surface tension at the liquid-vapor interface of SWD has been reported using density functional theories^{14,25} but, to the best of our knowledge, this is the first time this property is calculated using simulation methods.

One of the main tasks of the molecular simulation community has been to compare and determine if the results obtained with the MD and MC algorithms for the same systems are equivalent, or if not, to explain the differences. As mentioned above, DMD and MC had been used to simulate molecular systems interacting with discontinuous potentials. In the case of the SWD, the results from these two methods (SWD via MC and VSWD via DMD) have been compared without knowing precisely how much do they differ. The existence of the vibrating well in the VSWD, compared to a delta function at a fixed distance in the SWD, makes the two molecular arrays different. This work sets out to test the hypothesis that they have equivalent orthobaric densities and surface tensions provided that the range of the vibrating well is kept small. The two methods use similar computing time in the simulations. MC has the advantage of been able to simulate both SWD and VSWD. DMD has the ability to calculate dynamical properties and has also, by virtue of the intramolecular potentials, the flexibility to represent many different shapes of molecules⁹ and to simulate them using the same code. The use of the virial route to obtain the surface tension is natural to this simulation method.²⁶ Displacing complex molecules by MC techniques requires procedures like the one put forward by Escobedo *et al.*¹⁷

This work is organized as follows: in Sec. II the molecular model and interaction potential are given, Sec. III includes a brief discussion of the DMD simulation method, Sec. IV gives an account of the thermodynamic properties calculated, Sec. V describes the simulated series in detail, Sec. VI presents the results, followed by a discussion of the main conclusions in Sec. VII. References and acknowledgments are included at the end.

II. MOLECULAR MODEL AND INTERACTION POTENTIAL

The VSWD is the model chosen in this work to represent a simple molecular system that shows a complex behavior. Figure 1 depicts a VSWD, where all the variables relevant to this work are pictorially defined. The only difference between SWD and VSWD is the intramolecular potential, a fixed distance for SWD and a SW potential with infinitely high walls for VSWD, to keep the particles in a molecule from drifting apart. This well is necessary to perform DMD on VSWD and one of the aims of this work is to assess the differences between these two models.

The interaction between two particles a and b is given by the following potential:

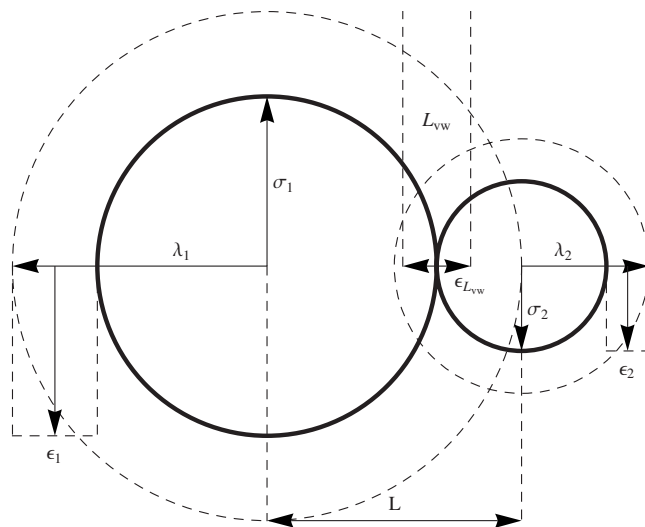


FIG. 1. Pictorial definition of the variables used. L_{vw} , L , σ_1 , σ_2 , ϵ_1 , ϵ_2 , λ_1 , λ_2 , and ϵ_{Lvw} . In this work, $\sigma_1=1$, $\epsilon_1=1$, $\lambda_1=\lambda\sigma_1$, $\lambda_2=\lambda\sigma_2$, with $\lambda=1.5$ and $\epsilon_{Lvw}=0$.

$$U(r_{ab}) = \begin{cases} \text{Intra} \\ \infty, & r_{ab} \leq L - L_{vw}/2 \\ \epsilon_{Lvw}, & L - L_{vw}/2 < r_{ab} \leq L + L_{vw}/2 \\ \infty, & r_{ab} > L + L_{vw}/2 \\ \text{Inter} \\ \infty, & r_{ab} \leq \sigma_{ab} \\ -\epsilon_{ab}, & \sigma_{ab} < r_{ab} \leq \lambda_{ab} \\ 0, & r_{ab} > \lambda_{ab}, \end{cases} \quad (1)$$

where r_{ab} is the distance between particles a and b , $\sigma_{ab} = (\sigma_a + \sigma_b)/2$ and $\epsilon_{ab} = \sqrt{\epsilon_a \epsilon_b}$ are the diameter and energy parameters obtained applying Lorentz–Berthelot rules, and $\lambda_{ab} = (\lambda_a + \lambda_b)/2$ is the range of the SW potential. The intramolecular interaction potential [top part of Eq. (1)] between two particles a and b in the same molecule (the vibrating well) is given by a SW of range L_{vw} centered at L , the elongation of the molecule. The depth of the well $\epsilon_{Lvw}=0$ for all cases considered in this work. The intermolecular interaction potential [bottom part of Eq. (1)] between two particles a and b in different molecules is given by a SW of size $\lambda_{ab}=\lambda\sigma_{ab}$ with a depth of the well ϵ_{ab} . In this work, a generic constant $\lambda=1.5$ is used and $\lambda_a=\lambda\sigma_a$ and $\lambda_b=\lambda\sigma_b$. In heteronuclear SWDs, when $\sigma_b \rightarrow 0 \Rightarrow \lambda_b \rightarrow 0$, the model becomes one of associating molecules.²⁷

III. DISCONTINUOUS MOLECULAR DYNAMICS

DMD is the chosen procedure to carry out the simulations of the model system described in Sec. IV. Rapaport⁶ gave a detailed account of the procedure to simulate molecular shapes, where the particles are kept within the molecule with strings attached to every pair. Chapela *et al.*⁹ used trapping wells as intramolecular potentials in order to achieve the same purpose. A brief account of the second approach is presented here.

Given the initial positions and velocities of the system of N molecules, the collision times are calculated for each pair.

The smallest time is selected and the particles are moved at a constant velocity to the collision position. Once there the collision between two particles is solved and the new velocities are obtained. Collision times for the two particles that just collided are calculated again and the process is started again. Time saving schemes, such as neighbor list and maximum collision time to keep this list as short as possible, are employed.

Liquid-vapor interface simulations are performed using a parallelepiped box with periodic boundary conditions in three directions. The molecules initially fill the simulation cell at a density within the unstable region of the phase diagram. A fast phase separation is observed after several thousands of collision and a liquid slab surrounded by vapor is formed.

IV. CALCULATED PROPERTIES

Equations to calculate pressure, vapor pressure, surface tension, density profiles, orthobaric densities, and critical points are given. Expressions for pressure and surface tension are given for both methods, DMD and MC.

A. DMD pressure and surface tension

Pressure is obtained using the Clausius' virial theorem,^{1,28}

$$\langle P \rangle = \rho k_B T + \frac{1}{3Vt} \sum_{\text{col's}} m_i \Delta \mathbf{v}_{ij} \cdot \mathbf{r}_{ij}, \quad (2)$$

where $\langle P \rangle$ is the time average pressure, ρ is the atomic number density, k_B is the Boltzmann constant, and T is the temperature. The total elapsed simulation time t , which is the sum of individual times of each collision performed, m is the mass of the particles, $\Delta \mathbf{v}_{ij}$ is the velocity difference vector of the colliding pair of particles i and j before and after the collision, and \mathbf{r}_{ij} is the vector joining the centers of the two atoms. The $\Sigma_{i,j}$ run through the total number of collision times. $\langle \cdots \rangle$ means time average.

The components of the pressure tensor are obtained using the Clausius' virial theorem as given in

$$\langle P_{xx} \rangle = \rho k_B T + \frac{1}{Vt} \sum_{\text{col's}} m_i \Delta v_{xij} x_{ij}, \quad (3)$$

where P_{xx} is the instantaneous xx diagonal component of the pressure tensor, V is the system volume, Δv_{xij} is the x component of the velocity difference vector of the colliding pair of particles i and j before and after the collision, and x_{ij} is the x component of the vector joining the centers of the two atoms.

The surface tension of a planar interface was calculated using the mechanical definition of the atomic pressure tensor,^{28,33}

$$\gamma = \frac{L_x}{2} \left[\langle P_{xx} \rangle - \frac{1}{2} (\langle P_{yy} \rangle + \langle P_{zz} \rangle) \right], \quad (4)$$

where $P_{\alpha\alpha}$ are the diagonal components of the pressure tensor and L_x is the length of the simulation box in the x direction. The factor $1/2$ outside of the square brackets is to av-

erage the surface tension of the two interfaces in the system. Care must be taken to avoid finite size effects by choosing a box large enough to have a surface area (area of the box parallel to the interface) of at least 10 atomic diameters.²⁹⁻³² The vapor pressure is obtained as the time average of the pressure tensor component normal to the interface $P_v = \langle P_{xx} \rangle$.

B. MC pressure and surface tension

In MC the surface tension is calculated using Eq. (4) but the instantaneous component P_{xx} of the pressure tensor is calculated using the molecular version given by

$$P_{xx} = \rho_m k_B T + \frac{1}{V} \sum_i \sum_{j>i} \mathbf{F}_{ij} \cdot \mathbf{R}_{ij}, \quad (5)$$

where ρ_m is the molecular density and $\mathbf{R}_{ij} = \mathbf{R}_i - \mathbf{R}_j$ with \mathbf{R}_i as the center of mass position of molecule i . The force between particles i and j at a distance \mathbf{r}_{ij} in two different molecules is

$$\mathbf{F}_{ij} = - \frac{\partial u(r_{ij})}{\partial \mathbf{r}_{ij}} \frac{\mathbf{r}_{ij}}{r_{ij}}, \quad (6)$$

where for discontinuous potentials,^{33,34}

$$\frac{\partial u(r_{ij})}{\partial r_{ij}} = -k_B T \delta(r_{ij} - r_\sigma), \quad (7)$$

where r_σ stands for every discontinuity of the intermolecular potential. The delta function was evaluated numerically,

$$\delta(r_{ij} - r_\sigma) = \frac{\Theta(r - r_\sigma) - \Theta(r - [r_\sigma + \Delta r_\sigma])}{\Delta r_\sigma}, \quad (8)$$

where $\Theta(x)$ is the step function and Δr_σ is an arbitrary distance from r_σ . The approximation in Eq. (8) is valid when $\Delta r_\sigma \rightarrow 0$.

The components of the pressure tensor were calculated as $P_{xx} = P_{xx}(r_\sigma^+) + P_{xx}(\lambda r_\sigma^-) + P_{xx}(\lambda r_\sigma^+)$. The procedure to calculate $\delta(r_{ij} - r_\sigma)$ was the same as that used for the spherical case.³⁵ $P_{xx}(r_\sigma^+)$ is obtained by summing distances over pairs at $r_\sigma < r_{ij} < r_\sigma + \Delta r_\sigma$, $P_{xx}(\lambda r_\sigma^-)$ for distances $(\lambda r_\sigma - \Delta r_\sigma) \leq r_{ij} \leq \lambda r_\sigma$, and $P_{xx}(\lambda r_\sigma^+)$ for distances $\lambda r_\sigma \leq r_{ij} \leq (\lambda r_\sigma + \Delta r_\sigma)$. The components P_{yy} and P_{zz} were obtained in the same way as P_{xx} . Values of the arbitrary distance $\Delta r_\sigma / \sigma_1 = 0.005, 0.01, 0.02, 0.03$, and 0.04 were used to calculate $\delta(r_{ij} - r_\sigma)$. At the end of the simulation, the components of the pressure tensor were extrapolated to $\Delta r_\sigma \rightarrow 0$ by using a linear regression. There are alternative ways to obtain the surface tension using MC simulations²⁶ without calculating the components of the pressure tensor, as done here with this δ function procedure.

C. Density profile

The density profile is

$$\rho(x) = \frac{\langle N(x, x + \Delta x) \rangle}{\Delta V}, \quad (9)$$

where $\langle N(x, x + \Delta x) \rangle$ is the average number of particles with positions between x and $x + \Delta x$ and ΔV is the volume of a slab. The liquid and vapor coexisting densities were obtained

TABLE I. Description of the simulation series performed in this study. The first particle of the VSWD always has values of $\sigma_1=1$, $\epsilon_1=1$, $\lambda_1^*=\lambda\sigma_1$, and $\lambda_2^*=\lambda\sigma_1$. The last two columns give T_c^* and ρ_c^* obtained with Eqs. (11) and (12).

Series	σ_2^*	λ_2^*	ϵ_2^*	L^*	L_{vw}^*	T_c^*	ρ_c^*
1	1	1.5	1	1	0.05	1.55	0.299
2	1	1.5	1	1	0.1	1.54	0.300
3	1	1.5	1	1	0.2	1.56	0.298
4	1	1.5	1	0.5	0.05	2.40	0.409
5	1	1.5	1	0.5	0.1	2.37	0.414
6	1	1.5	1	0.5	0.2	2.37	0.421
7	1	1.5	1	0.25	0.05	3.48	0.526
8	1	1.5	1	0.25	0.1	3.42	0.521
9	1	1.5	1	0.25	0.2	3.37	0.525
10	0.5	0.75	1	0.75	0.1	1.62	0.538
11	1	1.5	2	1	0.1	2.29	0.290
12	1	1.5	4	1	0.1	3.40	0.285
13	1	1.5	8	1	0.1	5.30	0.284
14	0.5	0.75	2	0.75	0.1	1.98	0.517
15	0.5	0.75	4	0.75	0.1	2.49	0.500
16	0.5	0.75	8	0.75	0.1	2.18	0.530
17	0.25	0.375	1	0.625	0.1	1.53	0.585
18	0.25	0.375	2	0.625	0.1	1.71	0.568
19	0.25	0.375	4	0.625	0.1	1.96	0.579
20	0.25	0.375	8	0.625	0.1	2.04	0.588

by fitting the calculated density profile to a hyperbolic tangent function,

$$\rho(x) = \frac{1}{2}(\rho_L + \rho_V) - \frac{1}{2}(\rho_L - \rho_V)\tanh[2(x - x_0)/\delta], \quad (10)$$

where ρ_L and ρ_V are the liquid and vapor densities at the orthobaric curve, respectively, δ is the thickness of the interface, and x_0 is the Gibbs equimolar dividing surface of the interface.

D. Critical temperature and density

The estimation of the critical point was made using the procedure included by Singh *et al.*³⁶ using a least squares fit of the law of rectilinear diameter and scaling relation,

$$\rho_L - \rho_V = C_1 \left(1 - \frac{T}{T_c}\right)^{\beta_c} + C_2 \left(1 - \frac{T}{T_c}\right)^{\beta_c + \Delta}, \quad (11)$$

where C_1 and C_2 are adjusting constants. The critical exponent $\beta_c=0.325$ and $\Delta=0.51$. The critical temperature T_c from Eq. (11) is used in the rectilinear diameter relation to obtain the critical density ρ_c ,

$$\frac{\rho_L + \rho_V}{2} = \rho_c + C_3(T - T_c), \quad (12)$$

with C_3 as another constant.

V. DESCRIPTION OF THE SIMULATED SERIES

The reducing parameters are $\sigma_1=1$ for distance and $\epsilon_1=1$ for energy. The size of the vibration well $L_{vw}^*=L_{vw}/\sigma_1$, the elongation $L^*=L/\sigma_1$, the relative size of the two particles $\sigma_2^*=\sigma_2/\sigma_1$, and the relative depth of the interaction well $\epsilon_2^*=\epsilon_2/\epsilon_1$ are used as variables in this study. The ranges of the potential wells are always $\lambda_1^*=\lambda\sigma_1^*$ and

$\lambda_2^*=\lambda\sigma_2^*$ using the generic constant $\lambda=1.5$. Figure 1 gives a pictorial description of these variables. The reduced properties are as follows: $T^*=k_B T/\epsilon_1$ for temperature, $\rho^*=\rho\sigma_1^3$ for density, $P^*=P\sigma_1^3/\epsilon_1$, for pressure, and $\gamma^*=\gamma\sigma_1^2/\epsilon_1$ for surface tension.

The series simulated are defined in detail in Table I where the values of the selected variables σ_2^* , λ_2^* , ϵ_2^* , L^* , and L_{vw}^* are given for each series. The critical temperatures T_c^* and densities ρ_c^* obtained from the simulation data are included in the last two columns.

All runs were carried out at the UAM-Iztapalapa cluster. The box was a parallelepiped of 11.9 atomic diameters σ_1 in the y and z directions and six or eight times more in the x direction. The number of particles used in all cases are 5000 (2500 dumbbells) with mass $m=1$, initially positioned in a fcc crystalline array. Blocks of 100×10^6 collisions were performed, with partial results kept every 25×10^6 and accumulation of statistics every 100 000 collisions. An initial equilibration period of an average of 50 blocks were allowed to run. Once the equilibration period was done, the maximum number of blocks for one run was 131, the minimum was 18, and the average was 43 blocks. Surface tension calculation requires long runs as performed in this work because the mean value is obtained from differences of pressure tensor components. When there was need of continuing one run at another set of temperatures, the final configuration of the previous run was selected as the initial configuration.

VI. RESULTS

The results are presented in four main subsections and in Tables II–VI, where simulation results for γ^* , P^* , ρ_L^* , and ρ_V^* are given for the 20 series included in Table I. Table III includes the results for the MC runs performed. Section VI A

TABLE II. Simulation results for series 1 and 2. Values for surface tension γ^* , vapor pressure P_v^* , orthobaric liquid ρ_L^* , and vapor ρ_v^* densities are given for each temperature T^* , where simulation values are available. Subindices indicate the error in each data, for example, 0.627_{15} means that the value is 0.627 ± 0.015 . Errors are calculated as the mean square deviation of the data divided by the number of blocks performed in each run.

Series 1				
T^*	γ^*	P_v^*	ρ_L^*	ρ_v^*
1	0.627 ₁₅	0.0007 ₀	0.7561 ₁	0.001 77 ₃
1.05	0.545 ₁₈	0.0014 ₁	0.7408 ₁	0.002 85 ₄
1.1	0.488 ₁₁	0.0023 ₁	0.7251 ₁	0.004 60 ₅
1.15	0.423 ₁₃	0.0039 ₁	0.7080 ₁	0.007 52 ₇
1.175	0.404 ₁₀	0.0047 ₁	0.6993 ₁	0.008 87 ₈
1.2	0.375 ₁₃	0.0057 ₁	0.6895 ₁	0.010 47 ₈
1.225	0.336 ₁₃	0.0073 ₁	0.6798 ₁	0.014 00 ₉
1.25	0.303 ₁₅	0.0087 ₁	0.6707 ₁	0.016 54 ₉
1.275	0.290 ₁₁	0.0105 ₁	0.6590 ₁	0.020 11 ₁₁
1.3	0.253 ₁₁	0.0122 ₂	0.6478 ₁	0.023 38 ₁₂
1.325	0.244 ₁₃	0.0148 ₂	0.6357 ₁	0.027 91 ₁₂
1.35	0.183 ₁₁	0.0177 ₃	0.6235 ₁	0.035 34 ₁₆
1.375	0.162 ₈	0.0205 ₂	0.6086 ₁	0.040 88 ₁₅
1.425	0.108 ₁₀	0.0276 ₄	0.5762 ₁	0.058 06 ₂₀
1.475	0.082 ₈	0.0365 ₄	0.5348 ₁	0.084 11 ₂₅
Series 2				
1	0.615 ₉	0.0007 ₀	0.7566 ₂	0.001 49 ₃
1.05	0.551 ₈	0.0014 ₀	0.7419 ₁	0.002 73 ₆
1.1	0.485 ₇	0.0023 ₀	0.7261 ₁	0.004 58 ₈
1.15	0.428 ₇	0.0038 ₀	0.7114 ₂	0.008 03 ₁₄
1.175	0.410 ₁₁	0.0049 ₁	0.6992 ₁	0.009 64 ₇
1.2	0.356 ₆	0.0058 ₁	0.6898 ₁	0.010 14 ₁₇
1.225	0.346 ₁₁	0.0072 ₁	0.6808 ₁	0.013 14 ₉
1.25	0.307 ₆	0.0086 ₁	0.6710 ₁	0.016 57 ₃₁
1.275	0.281 ₁₅	0.0102 ₁	0.6595 ₁	0.019 19 ₁₁
1.3	0.267 ₇	0.0124 ₁	0.6478 ₁	0.024 58 ₃₉
1.325	0.213 ₉	0.0146 ₂	0.6375 ₁	0.027 42 ₁₃
1.35	0.200 ₅	0.0171 ₁	0.6299 ₁	0.030 43 ₅₄
1.375	0.164 ₁₁	0.0204 ₂	0.6090 ₁	0.040 48 ₁₅
1.425	0.112 ₇	0.0273 ₃	0.5777 ₁	0.053 89 ₁₈
1.475	0.064 ₈	0.0364 ₄	0.5311 ₁	0.088 64 ₂₆

gives the analysis of the effect of the size of the vibrating well L_{vw}^* for various elongations L^* of homonuclear dumbbells. Section VI B analyzes the effect of the elongation L^* for the case for overlapping homonuclear dumbbells. Section VI C discusses the effect of decreasing the size of one of the particles σ_2^* for tangent heteronuclear dumbbells. Section VI D includes the results of the effect of increasing the well depth ϵ_2^* for homonuclear and heteronuclear dumbbells.

A. Effect of L_{vw}^* : The size of the vibrating well

The effect of L_{vw}^* is quite small, and in order to give a much accurate picture of the comparison of the results for the orthobaric densities with previously reported data, a reduced portion of series 2 is given in Fig. 2, which presents data obtained for homonuclear VSWD with $L^*=1$ and $L_{vw}^*=0.05$, 0.1, and 0.2. These results are compared with MC results by Kim *et al.*,¹⁸ MC simulations by Escobedo and de Pablo¹⁶ and Yethiraj and Hall¹³ and DMD results by Hu *et al.*¹⁰ The agreement among the data is fair, and the scatter is around

TABLE III. Simulation results for series 3–6. See the caption of Table II. This table includes, at the bottom, the results from the MC runs performed.

Series 3				
T^*	γ^*	P_v^*	ρ_L^*	ρ_v^*
1	0.621 ₇	0.0007 ₀	0.7600 ₁	0.001 48 ₄
1.05	0.556 ₇	0.0014 ₀	0.7440 ₁	0.003 27 ₈
1.1	0.494 ₇	0.0023 ₀	0.7296 ₁	0.005 15 ₁₁
1.15	0.435 ₈	0.0037 ₁	0.7101 ₁	0.006 94 ₁₅
1.175	0.401 ₁₂	0.0047 ₁	0.7014 ₁	0.008 54 ₇
1.2	0.357 ₇	0.0057 ₁	0.6913 ₁	0.010 11 ₁₅
1.225	0.341 ₁₂	0.0072 ₁	0.6820 ₁	0.013 14 ₉
1.25	0.318 ₆	0.0086 ₁	0.6719 ₁	0.014 61 ₂₅
1.275	0.301 ₈	0.0106 ₂	0.6614 ₁	0.020 19 ₁₁
1.3	0.253 ₆	0.0121 ₁	0.6484 ₁	0.025 65 ₄₂
1.325	0.232 ₉	0.0149 ₁	0.6371 ₁	0.028 04 ₁₃
1.35	0.186 ₆	0.0173 ₁	0.6229 ₁	0.034 43 ₆₇
1.375	0.173 ₈	0.0207 ₂	0.6095 ₁	0.041 65 ₁₅
1.425	0.108 ₉	0.0273 ₃	0.5778 ₁	0.056 52 ₁₇
1.475	0.059 ₇	0.0368 ₃	0.5367 ₁	0.082 33 ₂₀
Series 4				
1.4	1.354 ₂₃	0.0006 ₁	1.0984 ₃	0.000 78 ₂
1.6	1.051 ₁₈	0.0037 ₁	1.0379 ₂	0.004 12 ₉
1.8	0.744 ₂₀	0.0123 ₂	0.9708 ₂	0.014 06 ₂₈
2	0.416 ₁₃	0.0317 ₃	0.8880 ₂	0.041 23 ₇₀
2.2	0.177 ₁₂	0.0689 ₅	0.7919 ₁	0.083 63 ₁₃₉
Series 5				
1.4	1.351 ₂₃	0.0006 ₀	1.1094 ₂	0.002 27 ₂
1.6	1.050 ₂₂	0.0035 ₁	1.0442 ₃	0.004 87 ₉
1.8	0.751 ₁₈	0.0123 ₁	0.9763 ₂	0.015 19 ₂₇
2	0.439 ₁₇	0.0317 ₂	0.8957 ₂	0.035 50 ₆₁
2.2	0.180 ₁₀	0.0698 ₅	0.7856 ₂	0.101 20 ₁₆₂
Series 6				
1.4	1.523 ₃₂	0.0004 ₁	1.1478 ₈	0.000 83 ₁
1.6	1.093 ₂₂	0.0031 ₁	1.0794 ₅	0.004 28 ₄
1.3	0.405 ₇	0.0199 ₂	1.2069 ₁	0.033 40 ₂₉
1.325	0.400 ₇	0.0234 ₂	1.1915 ₁	0.044 13 ₄₃
1.8	0.796 ₁₅	0.0117 ₂	1.0062 ₅	0.013 92 ₆
2	0.484 ₁₃	0.0309 ₃	0.9198 ₄	0.038 23 ₆
2.2	0.183 ₁₁	0.0686 ₅	0.7996 ₄	0.098 49 ₉
MC				
1.20	0.35 ₂	0.0100	0.699	0.0112
1.25	0.27 ₂	0.0074	0.668	0.0106
1.30	0.26 ₂	0.0083	0.644	0.0222
1.35	0.20 ₂	0.0157	0.620	0.0350
1.40	0.13 ₃	0.0207	0.588	0.0492
1.45	0.08 ₃	0.0309	0.550	0.0730
1.50	0.03 ₃	0.0403	0.480	0.1210

the points obtained in this work. However, the best agreement is with the MC data of Kim *et al.*¹⁸ The DMD results by Hu *et al.*¹⁴ are high with respect to the rest of the data, probably due to the fact that the reduced elongation of their dumbbells L^* is not 1 but 0.985, producing a smaller molecule and hence a higher critical point. In general, this variable has a small impact on the properties selected in comparison to the effects of the rest of the variables analyzed. Since the data are at the same temperature, a direct comparison of the results was performed. Data with $L_{vw}^*=0.1$ were

TABLE IV. Simulation results for series 7–11. See the caption of Table II.

Series 7				
T^*	γ^*	P_V^*	ρ_L^*	ρ_V^*
2.5	1.366 ₂₂	0.0166 ₂	1.2036 ₁₉	0.014 65 ₁₁
2.61	1.202 ₂₂	0.0240 ₂	1.1772 ₁₈	0.017 75 ₁₄
2.72	0.984 ₁₉	0.0352 ₂	1.1536 ₁₃	0.022 77 ₉
2.83	0.848 ₁₈	0.0484 ₃	1.1082 ₁₉	0.042 08 ₁₆
2.94	0.645 ₁₇	0.0666 ₄	1.0924 ₁₀	0.069 08 ₁₅
3.05	0.475 ₁₄	0.0879 ₄	1.0370 ₁₁	0.075 12 ₂₇
3.16	0.337 ₁₄	0.1144 ₅	0.9985 ₁₀	0.108 11 ₃₅
3.27	0.205 ₁₄	0.1480 ₆	0.9070 ₁₁	0.171 00 ₃₇
Series 8				
2.5	1.395 ₂₂	0.0172 ₂	1.2024 ₁₆	0.017 63 ₁₁
2.61	1.193 ₂₀	0.0253 ₂	1.1757 ₁₅	0.020 45 ₁₀
2.72	0.967 ₁₄	0.0355 ₂	1.1422 ₁₃	0.028 24 ₁₈
2.83	0.807 ₁₅	0.0501 ₃	1.1120 ₁₂	0.044 22 ₁₆
2.94	0.618 ₁₆	0.0679 ₄	1.0712 ₁₂	0.057 88 ₁₅
3.05	0.461 ₁₅	0.0904 ₄	1.0319 ₁₂	0.074 60 ₁₉
3.16	0.348 ₁₂	0.1188 ₄	0.9876 ₁₂	0.102 19 ₃₄
3.27	0.184 ₉	0.1538 ₆	0.9083 ₁₀	0.173 04 ₂₄
Series 9				
2.5	1.334 ₁₈	0.0188 ₁	1.2083 ₁₃	0.018 62 ₁₄
2.61	1.151 ₂₁	0.0275 ₂	1.1786 ₁₈	0.023 28 ₁₁
2.72	0.979 ₁₅	0.0394 ₃	1.1453 ₁₁	0.033 81 ₁₅
2.83	0.747 ₁₂	0.0552 ₄	1.1087 ₁₂	0.048 33 ₁₅
2.94	0.581 ₁₄	0.0754 ₃	1.0754 ₁₂	0.059 46 ₁₅
3.05	0.422 ₁₃	0.1000 ₅	1.0246 ₁₄	0.101 00 ₃₁
3.16	0.275 ₁₁	0.1312 ₅	0.9645 ₁₁	0.117 82 ₃₅
3.27	0.135 ₈	0.1692 ₅	0.8886 ₆	0.185 18 ₃₉
Series 10				
1	0.907 ₉	0.0014 ₀	1.3995 ₂	0.003 15 ₄
1.05	0.817 ₁₁	0.0025 ₁	1.3706 ₁	0.004 00 ₆
1.1	0.744 ₉	0.0041 ₁	1.3421 ₂	0.006 12 ₇
1.15	0.668 ₁₀	0.0064 ₁	1.3130 ₂	0.014 52 ₁₆
1.2	0.584 ₈	0.0097 ₁	1.2797 ₂	0.017 53 ₁₇
1.225	0.543 ₈	0.0118 ₁	1.2639 ₁	0.022 04 ₂₀
1.25	0.492 ₇	0.0141 ₁	1.2459 ₁	0.022 97 ₂₃
1.275	0.458 ₇	0.0168 ₁	1.2304 ₁	0.024 93 ₂₁
1.3	0.405 ₇	0.0199 ₂	1.2069 ₁	0.033 40 ₂₉
1.325	0.400 ₇	0.0234 ₂	1.1915 ₁	0.044 13 ₄₃
1.35	0.337 ₈	0.0273 ₁	1.1697 ₁	0.044 59 ₄₁
1.375	0.303 ₆	0.0318 ₂	1.1495 ₂	0.053 73 ₅₀
1.425	0.224 ₇	0.0426 ₃	1.1101 ₁	0.076 37 ₆₂
1.475	0.169 ₇	0.0557 ₃	1.0456 ₁	0.099 42 ₈₅
1.525	0.106 ₁₁	0.0722 ₃	0.9850 ₁	0.151 37 ₁₂₈
1.575	0.033 ₃	0.0933 ₃	0.8745 ₂	0.212 17 ₁₇₉
Series 11				
1.4	0.880 ₁₅	0.0006 ₀	0.7658 ₁	0.001 44 ₃
1.5	0.788 ₁₆	0.0015 ₁	0.7478 ₁	0.002 96 ₅
1.6	0.655 ₁₂	0.0032 ₁	0.7227 ₁	0.003 71 ₆
1.7	0.537 ₁₄	0.0064 ₁	0.6988 ₁	0.008 73 ₁₀
1.8	0.438 ₁₃	0.0110 ₁	0.6700 ₁	0.013 96 ₁₁
1.9	0.324 ₁₁	0.0176 ₁	0.6456 ₁	0.021 30 ₂₂
2	0.223 ₉	0.0278 ₂	0.6093 ₁	0.035 21 ₃₃
2.1	0.133 ₉	0.0430 ₂	0.5568 ₁	0.051 35 ₄₃

selected as the base results and the other two were compared to these. Table VII gives the average percentage deviations of these comparisons. For $L^*=1$, averages are within simu-

TABLE V. Simulation results for series 12–15. See the caption of Table II.

Series 12				
T^*	γ^*	P_V^*	ρ_L^*	ρ_V^*
2.2	1.007 ₂₈	0.0011 ₁	0.7544 ₂	0.000 91 ₂
2.3	0.919 ₁₈	0.0018 ₁	0.7388 ₁	0.002 84 ₂
2.4	0.843 ₃₄	0.0031 ₁	0.7266 ₁	0.002 21 ₃
2.5	0.752 ₂₄	0.0051 ₁	0.7107 ₁	0.004 89 ₄
2.6	0.660 ₂₄	0.0075 ₁	0.6929 ₁	0.006 11 ₆
2.7	0.578 ₁₇	0.0108 ₂	0.6729 ₁	0.009 20 ₅
2.8	0.501 ₂₆	0.0159 ₂	0.6571 ₁	0.014 93 ₁₀
2.9	0.383 ₂₃	0.0221 ₃	0.6321 ₁	0.016 96 ₇
3	0.300 ₁₅	0.0301 ₂	0.6077 ₂	0.025 78 ₅
3.1	0.235 ₁₅	0.0397 ₃	0.5757 ₂	0.035 39 ₄
3.2	0.174 ₁₃	0.0530 ₄	0.5383 ₁	0.052 93 ₄
Series 13				
2.9	1.163 ₈₄	−0.0001 ₁	0.7972 ₃	0.000 00 ₀
3.1	0.953 ₆₆	−0.0001 ₁	0.7921 ₄	0.000 02 ₀
3.3	0.987 ₄₇	0.0000 ₁	0.7730 ₃	0.000 10 ₀
3.5	0.838 ₄₀	0.0002 ₁	0.7555 ₂	0.000 24 ₀
3.7	0.872 ₄₀	0.0008 ₁	0.7382 ₂	0.000 53 ₀
3.9	0.724 ₃₂	0.0018 ₁	0.7176 ₂	0.001 00 ₁
4.1	0.684 ₃₃	0.0036 ₁	0.6985 ₂	0.001 99 ₁
4.3	0.596 ₃₀	0.0067 ₂	0.6762 ₂	0.003 94 ₁
4.5	0.470 ₂₇	0.0110 ₂	0.6503 ₂	0.005 56 ₂
Series 14				
1.4	0.685 ₁₃	0.0062 ₁	1.3213 ₂	0.010 89 ₁₁
1.5	0.561 ₁₃	0.0126 ₁	1.2623 ₂	0.016 13 ₁₂
1.6	0.409 ₈	0.0228 ₂	1.2069 ₁	0.031 82 ₂₆
1.7	0.279 ₈	0.0387 ₂	1.1261 ₁	0.046 88 ₄₃
1.8	0.152 ₇	0.0630 ₃	1.0144 ₃	0.124 33 ₁₀₀
Series 15				
1.6	0.537 ₂₃	0.0008 ₁	1.3383 ₇	0.000 58 ₁
1.7	0.469 ₂₁	0.0019 ₁	1.3072 ₇	0.001 26 ₂
1.8	0.400 ₂₁	0.0041 ₁	1.2669 ₅	0.005 74 ₆
1.9	0.369 ₁₉	0.0080 ₁	1.2286 ₃	0.009 22 ₁₂
2	0.295 ₁₃	0.0139 ₁	1.1814 ₄	0.016 59 ₁₂
2.1	0.254 ₁₀	0.0235 ₂	1.1205 ₃	0.028 30 ₂₇
2.2	0.171 ₁₁	0.0380 ₃	1.0600 ₈	0.045 00 ₉
2.3	0.078 ₁₁	0.0575 ₃	0.9411 ₈	0.091 44 ₁₆

lation errors for all the properties ($<0.6\%$) except for ρ_V^* , which is ($\sim 3\%$). As the particles in the molecule get closer together the average percentage deviations increase for all the properties, as can be seen in Fig. 2 and Table VII. These might be due to the fact that increments in percentage variations in molecular volume follow a similar tendency as the increments for the relative percentage deviations, as shown in the last three columns of Table VII. Careful analysis of the distribution of the percentage deviations indicate that there is not a tendency to any particular bias due to temperature.

Another way to analyze these results is through the critical properties. The effect of L_{vw}^* on T_c^* and ρ_c^* is quite minor as well. For values of L_{vw}^* going from 0.05, to 0.1, and to 0.2, T_c^* increases from $1.55 \pm 0.4\%$, to $2.38 \pm 0.7\%$, and to $3.42 \pm 1.6\%$ for $L^*=1$, 0.5, and 0.25, respectively. For the same values of L_{vw}^* and L^* , the critical density ρ_c^* increases from $0.3 \pm 0.3\%$, to $0.41 \pm 1.5\%$, and to $0.5 \pm 0.5\%$. The variations in these calculated values are, in all cases, less

TABLE VI. Simulation results for series 16–20. See the caption of Table II.

Series 16				
T^*	γ^*	P_v^*	ρ_L^*	ρ_v^*
1.9	0.297 ₂₆	0.0001 ₀	1.2455 ₁₀	0.003 16 ₄
2	0.104 ₂₂	0.0007 ₁	1.2025 ₇	0.032 25 ₂₉
2.1	0.039 ₁₂	0.0020 ₁	1.0495 ₈	0.049 39 ₅₁
Series 17				
1	0.790 ₁₀	0.0030 ₁	1.5456 ₆	0.006 28 ₃
1.06	0.700 ₁₁	0.0056 ₁	1.5032 ₅	0.011 14 ₄
1.12	0.590 ₁₁	0.0093 ₁	1.4602 ₅	0.018 00 ₇
1.18	0.512 ₁₀	0.0149 ₁	1.4142 ₄	0.027 84 ₇
1.24	0.417 ₇	0.0226 ₂	1.3629 ₅	0.042 93 ₈
1.3	0.326 ₆	0.0326 ₂	1.3106 ₅	0.060 12 ₁₀
1.36	0.237 ₆	0.0466 ₂	1.2454 ₅	0.091 68 ₇
1.42	0.146 ₄	0.0636 ₂	1.1663 ₅	0.128 66 ₁₆
1.46	0.096 ₃	0.0777 ₂	1.0135 ₁₁₂	0.166 31 ₉
1.5	0.046 ₃	0.0944 ₂	0.9241 ₃₆	0.226 99 ₁₃
Series 18				
1	0.877 ₁₅	0.0004 ₀	1.6573 ₁₀₁	0.001 42 ₂
1.1	0.775 ₁₀	0.0018 ₀	1.6060 ₂₆	0.003 75 ₃
1.2	0.662 ₉	0.0050 ₁	1.5384 ₂₈	0.007 61 ₃
1.3	0.549 ₇	0.0116 ₁	1.4649 ₁₇	0.019 46 ₆
1.4	0.409 ₆	0.0228 ₁	1.3801 ₆	0.042 90 ₇
1.5	0.281 ₅	0.0410 ₂	1.2886 ₈	0.069 62 ₁₀
1.6	0.143 ₄	0.0678 ₂	1.1786 ₆	0.124 04 ₁₅
1.7	0.019 ₃	0.1082 ₂	0.7145 ₄	0.286 78 ₂₁
Series 19				
1.24	0.611 ₂₅	0.0000 ₀	1.5175 ₅₃	0.001 67 ₂
1.36	0.485 ₂₃	0.0006 ₀	1.4536 ₄₅	0.005 41 ₄
1.48	0.341 ₁₆	0.0026 ₁	1.3614 ₃₇	0.019 59 ₂₀
1.6	0.280 ₁₂	0.0083 ₁	1.2572 ₁₇	0.046 85 ₁₈
1.72	0.180 ₈	0.0207 ₁	1.1381 ₁₂	0.105 05 ₂₉
1.84	0.083 ₆	0.0445 ₂	1.0000 ₀	0.180 00 ₀
Series 20				
1.3	0.480 ₄₆	-0.0001 ₀	1.7243 ₁₅₄	0.000 00 ₀
1.45	0.386 ₂₈	0.0002 ₀	1.5783 ₆₂	0.000 51 ₁
1.6	0.290 ₂₁	0.0014 ₁	1.4486 ₄₇	0.014 37 ₉
1.75	0.161 ₁₇	0.0047 ₁	1.3277 ₂₆	0.040 18 ₁₈
1.9	0.061 ₈	0.0150 ₂	1.1500 ₁₉	0.150 00 ₅₅

than 1.6%. This could be important because wider vibrating wells promote much faster simulations since tightly constrained vibrating wells are rapidly colliding. Faster simulations can probe longer time scales and mesoscale behavior. There may be theoretical implications also since such behavior would indicate the feasibility of simplifying assumptions.

B. Effect of L^* : Elongation

The elongation has a much larger effect on the properties observed. In terms of the critical temperature, the average values goes from $T_c=3.42$, to 2.38, and to 1.55 for $L^*=0.25$, 0.5, and 1, respectively. This means that the T_c 's have a maximum decrease of 45%, while the critical densities present average values that go from $\rho_c^*=0.52$, to 0.41, and to 0.3 with a maximum decrease of 57%. Figure 3 includes series 1–9 and gives orthobaric densities ρ_L^* and ρ_v^* (top),

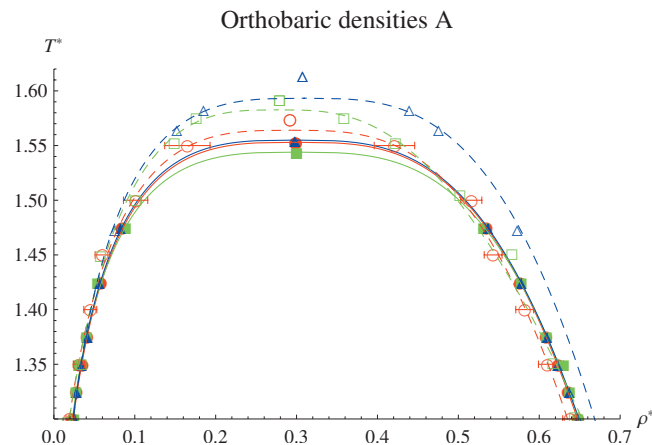


FIG. 2. Effect of L_{vw}^* on orthobaric densities. The figure plots the orthobaric densities of homonuclear vibrating dumbbells (series 1–3). Full circles, squares, and triangles are the results for VSWD with L_{vw}^* values of 0.05, 0.1, and 0.2, respectively. Open circles are SWD MC values by Kim *et al.* (Ref. 18), open squares are SWD MC data by Escobedo *et al.* (Ref. 16), and Yethiraj *et al.* (Ref. 13), and open triangles are VSWD DMD results by Hu *et al.* (Ref. 14). VSWD and SWD are different systems. The main discrepancy of these two systems is the intramolecular potential. While the SWD is bonded at a fix distance, the VSWD is bonded by a SW potential of range L_{vw}^* with infinite walls.

surface tension γ^* (middle), and vapor pressure P_v (bottom) for elongations $L^*=1$, 0.5, and 0.25 and vibrating well lengths $L_{vw}^*=0.05$, 0.1, and 0.2. Elongation L^* has a large effect on these properties, increasing considerably the critical temperature and producing a small increase in critical density with decreasing elongation. Orthobaric densities are compared with the values reported by Hu *et al.*¹⁴ for $L^*=0.97$, 0.6, and 0.4. Even though elongations do not match, the general trend of the curves and their position are correct. Surface tension is included in the middle panel of the plot. The slope of γ^* versus T^* increases slightly with decreasing L^* and the value of $L_{vw}^*=0.2$ has only a limited effect for values of $L^*=0.5$ and 0.25. In the first case, γ^* increases at low temperatures and decreases at high temperatures for $L^*=0.25$. This same middle figure reports the values for the surface tension γ^* obtained with MC (included at the bottom part of Table VII) using equations given in Sec. IV B. At the bottom panel, the results for vapor pressure P_v^* are given for the same three series and compared with the reported DMD values by Hu *et al.*¹⁴ for $L^*=1$. The agreement of these data is fair.

C. Effect of σ_2^* : Size of the second particle

The particle size has a large effect on both critical temperature and density. Care must be exercised in the analysis of these data because the variation in σ_2^* is accompanied by a change in the range of the potential well given by $\lambda_2^*=\lambda\sigma_2^*$, with $\lambda=1.5$. Tangent VSWDs with second particle sizes $\sigma_2^*=0.25$, 0.5, and 1 were simulated to evaluate this effect and four different values of $\epsilon_2^*=1$, 2, 4, and 8 are included. Average values of critical density decrease monotonically for the σ_2^* 's considered as: $\rho_c^*=0.58 \pm 1.5\%$, $0.52 \pm 3.2\%$, and $0.29 \pm 2.5\%$, respectively. Each one of the ρ_c^* curves for the ϵ_2^* 's mentioned above behave in the same manner. The maximum deviation of the critical density for

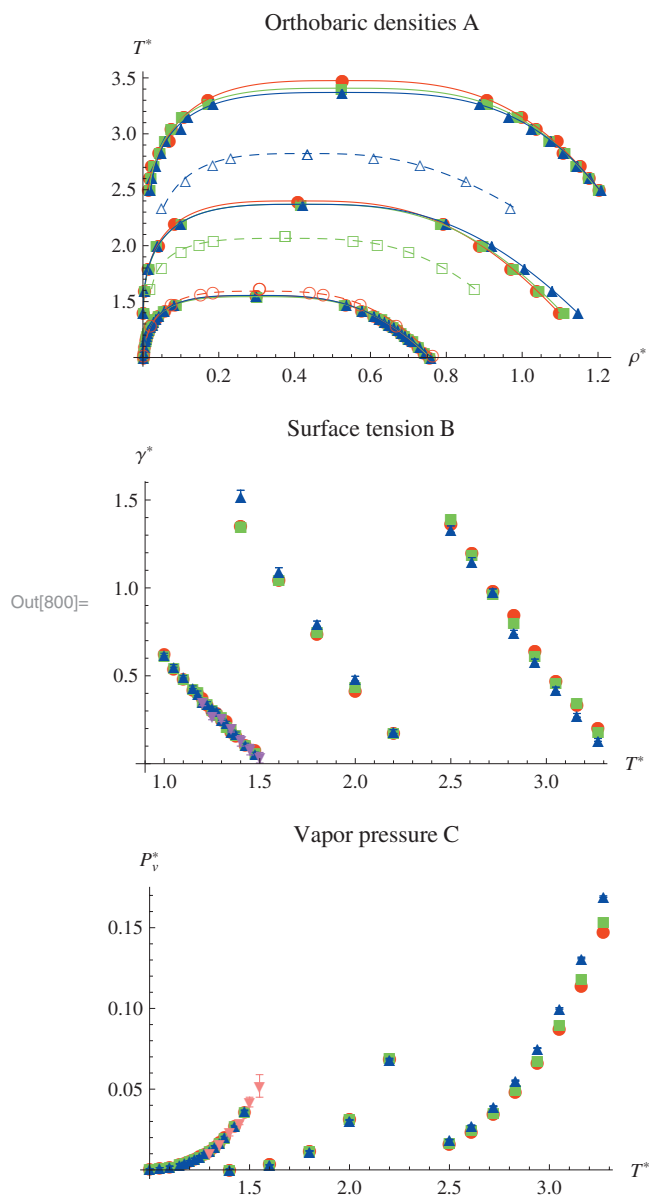


FIG. 3. Effect of L_{vw}^* on orthobaric densities (top), surface tension (middle), and vapor pressure (bottom) which correspond to tangent and overlapping homonuclear VSWD having $L^*=1, 0.5, 0.25$ (series 1–9). The top panel includes the orthobaric densities with full circles, squares, and triangles from results of this work for VSWD with $L_{vw}^*=0.05, 0.1, 0.2$, respectively. From the bottom up curves represent series with elongation $L^*=1, 0.5, 0.25$. Open circles, squares, and triangles are the values reported by Kim *et al.* (Ref. 18) for $L^*=0.97, 0.6, 0.4$, and $L_{vw}^*=0.03, 0.1, 0.1$. Surface tension data are depicted at the bottom panel with decreasing values of $L^*=1, 0.5, 0.25$ going from left to right.

all the ϵ_2^* 's is 50%. Variations in the critical temperature T_c^* have different behaviors for each ϵ_2^* , when the series of σ_2^* 's are considered. For $\epsilon_2^*=1$, the values of $T_c^*=1.53, 1.62$, and 1.54 present a maximum at $\sigma_2^*=0.5$. For $\epsilon_2^*=2$ and 4 , the values of T_c^* increase monotonically. For $\epsilon_2^*=8$, the values of $T_c^*=2.04, 2.18$, and 5.3 are flat at the beginning and then exhibit a large increase at $\sigma_2^*=1$. Figure 4 presents the results obtained for series 2, 10, and 17 and the data are given in Tables II, IV, and VI. This group of series presents a maximum in T_c^* at $\sigma_2^*=0.5$ for $\epsilon_2^*=1$. The upper panel presents the orthobaric densities and the lower one gives the surface ten-

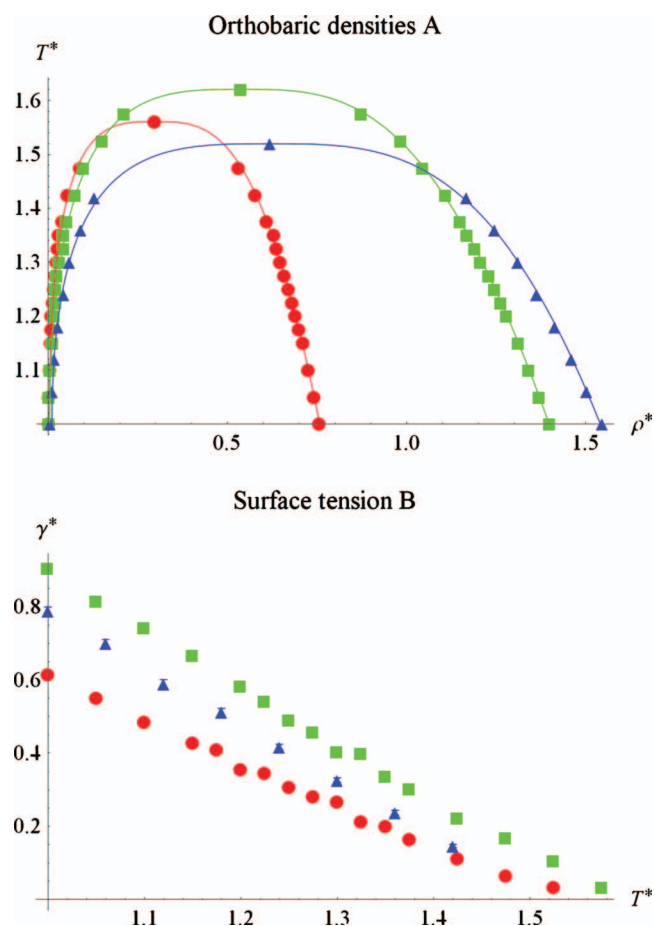


FIG. 4. Orthobaric densities (top) and surface tension (bottom) for series 2, 10, and 17 corresponding to tangent homonuclear and heteronuclear VSWDs with sizes of the second particle $\sigma_2^*=1$ (circles), 0.5 (squares), and 0.25 (triangles).

sion. In this figure the most striking effect is that the critical temperature increases at first from $\sigma_2^*=1$ to $\sigma_2^*=0.5$ and then it decreases for $\sigma_2^*=0.25$. This initial increase might be due to the fact that two opposing effects are acting in this case. It is known that a decrease in λ produces a decrease in the critical temperature^{37,38} and that a decrease in molecular volume σ_2^* produces an increase in critical temperature for the same well size and depth (see Fig. 3). It must be remembered that a change in σ_2^* means that λ_2^* also changes. Surface tensions show the typical shape with increasing values when going from $\sigma_2^*=1$ to $\sigma_2^*=0.5$ and decreasing later for $\sigma_2^*=0.25$.

D. Effect of ϵ_2^* : SW depth of the second particle

Series (2, 11, 12, 13), (10, 14, 15, 16), and (17, 18, 19, 20) are used to study the effect of the SW depth of the second particle ϵ_2^* . These three groups are for tangent VSWD with ϵ_2^* 's $1, 2, 4$, and 8 . Each group corresponds to values of $\sigma_2^*=1, 0.5$, and 0.25 , respectively ($\lambda_2^*=1.5\sigma_2^*$). The effect of ϵ_2^* on the critical density is very small. Average values of critical density for each σ_2^* considered are $\rho_c^*=0.58 \pm 1.5\%$, $0.52 \pm 3.2\%$, and $0.29 \pm 2.5\%$, respectively. Variations in the critical temperature T_c^* are larger and with different behaviors for each σ_2^* when the series of ϵ_2^* are considered. For $\sigma_2^*=1$, the values of $T_c^*=1.54, 2.29, 3.4$, and 5.3 increase

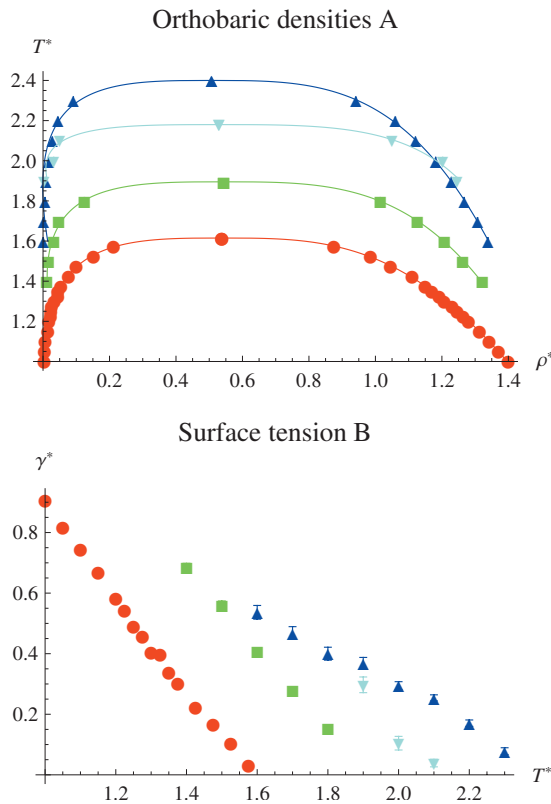


FIG. 5. Orthobaric densities (top) and surface tension (bottom) for series 10, 14, 15, and 16 corresponding to heteronuclear tangent VSWD with $\sigma_2^*=0.5$. Circles, squares, triangles, and inverted triangles are for $\epsilon_2^*=1, 2, 4$, and 8, respectively.

monotonically with a total increase of 3.4 times the initial value. For $\sigma_2^*=0.5$, the values of $T_c^*=1.62, 1.98, 2.49$, and 2.18 present a maximum at $\epsilon_2^*=4$ with a maximum increase, up to the maximum, of 1.54 times the initial value. For $\sigma_2^*=0.25$, the values of $T_c^*=1.53, 1.71, 1.96$, and 2.04 increase monotonically but it flattens at $\epsilon_2^*=4$, with a total increase of 1.3 times the initial value, hinting that for larger values of $\epsilon_2^*=8$, it might show a maximum value as in the case of $\sigma_2^*=0.5$. The upper panel of Fig. 5 gives orthobaric density results for heteronuclear tangent VSWD with $\sigma_2^*=0.5$, which is the most interesting case due to the existence of a maximum of the critical temperature. The values of the critical density $\rho_c^*=0.52\%-1.5\%$ are very similar, but the values of the critical temperature present a maximum, increasing systematically, at first, going from $T_c^*=1.62, 1.98$, and 2.49 for $\epsilon_2^*=1, 2$, and 4, respectively, and then decreases to $T_c^*=2.18$ for $\epsilon_2^*=8$.

TABLE VII. Analysis of the effect of L_{vw}^* for homonuclear VSWD with different elongations L^* , expressed as the absolute value of the average percentage deviations from the base case of $L_{vw}^*=0.1$. Values at the extreme right of the table are the percentage variations of the volume of the VSWD for $L_{vw}^*=0.05, 0.1$, and 0.2 for elongations $L^*=1, 0.5$ and 0.25. The increase in percentage variations with both L^* and L_{vw}^* follow similar percentage increases in the volume of the molecule.

	γ^*	P_V^*	ρ_L^*	ρ_V^*	γ^*	P_V^*	ρ_L^*	ρ_V^*	L_{vw}^*	L_{vw}^*	L_{vw}^*
L^*/L_{vw}^*	0.05				0.2				0.05	0.1	0.2
1	0.1	-0.5	0.2	-3.2	0.6	-0.2	0.1	1.1	0.01	0.05	0.18
0.5	1.5	-0.9	0.4	6.0	5.1	-5.6	2.7	-4.6	1.7	3.3	6.7
0.25	-2.6	3.1	-1.2	6.2	-11.7	9.2	-0.5	11.4	2.8	5.6	11.1

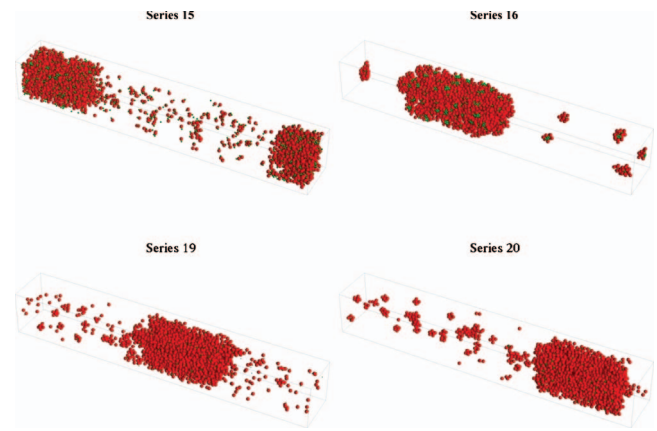


FIG. 6. The top panels show a typical snapshot configuration of series 15 and 16. Both have $\sigma_2^*=0.5$ and $\epsilon_2^*=4$ and 8, respectively, with T^* as near the critical point as possible. The bottom panels show a typical snapshot configuration of series 19 and 20. Both have $\sigma_2^*=0.25$ and $\epsilon_2^*=4$ and 8, respectively, with T^* as near the critical point as possible. Aggregates of molecules are seen in both phases of all pictures. The nature (size and shape) of them varies substantially for the top figures, in contrast to the bottom ones, which have very similar aggregates, only fewer in the left panel.

It is striking that the critical temperature is lower than that for the case of $\epsilon_2^*=4$ and the solid phase presents itself at a much higher temperature. This might be due to the strong association presented by this system. Figure 6 shows four configuration snapshots corresponding to series 15, 16, 19, and 20 as close to the critical point as possible. Series 15 and 16 are for heteronuclear VSWD with $\sigma_2^*=0.5$ and $\epsilon_2^*=4$ and 8, respectively. Series 19 and 20 are for heteronuclear VSWD with $\sigma_2^*=0.25$ and $\epsilon_2^*=4$ and 8, respectively. The nature of the association is qualitatively different. While series 15 ($\epsilon_2^*=4$) presents practically no association, series 16 ($\epsilon_2^*=4$) shows molecular aggregates that are evident even in the vapor phase, with the smaller more attractive particles, gathered in the inner part of the aggregate. Within the liquid phase one can appreciate the same aggregates but they seem to be larger and interconnected. In the case of Series 19 and 20 the nature of the association is very similar for both values of ϵ_2^* s, with a larger concentration of aggregates for the larger value of $\epsilon_2^*=8$. By direct inspection of the configuration snapshots it seems that, in this case of smaller molecules ($\sigma_2^*=0.25$), the attraction saturates and the aggregates are not able to grow larger than 5 or 6 dumbbells, while the larger molecules ($\sigma_2^*=0.5$) have sizes that go from 8 up to around 30 dumbbells. Series for $\sigma_2^*=1$ and 0.25 present orthobaric

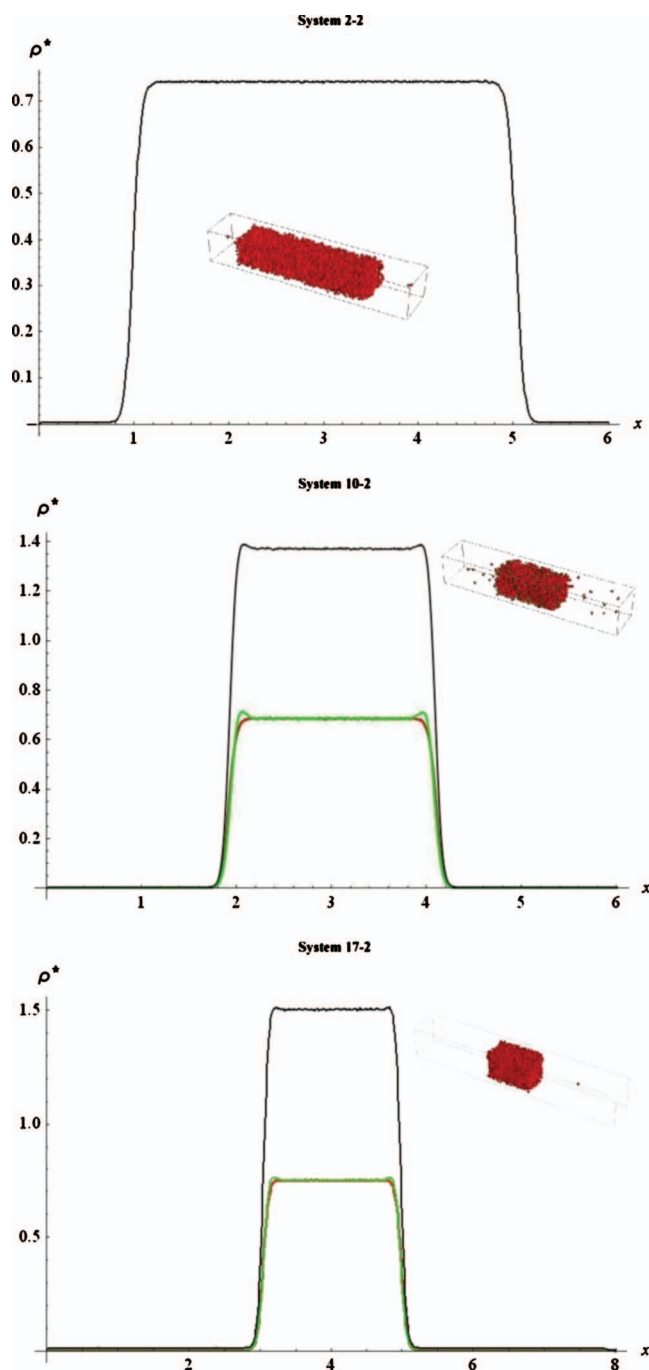


FIG. 7. Density profiles for homonuclear dumbbells series 2 with $\sigma_2^*=1$ at $T^*=1.05$, heteronuclear dumbbells of series 10 with $\sigma_2^*=0.5$ at $T^*=1.05$, and heteronuclear dumbbells of series 17 with $\sigma_2^*=0.25$ at $T^*=1.06$, from top to bottom. The three series have $\epsilon_2^*=1$. As an inset, every profile has a snapshot of one configuration taken during the run.

curves very similar to the ones shown in Fig. 5, except that all of the critical temperatures grow monotonically with growing ϵ_2^* .

For homonuclear tangent VSWD, the results for surface tension γ^* present increasing values of γ^* with similar slopes for $\epsilon_2^*=1, 2$, and 4. For $\epsilon_2^*=8$, γ^* increases also, but the slope with respect to temperature diminishes. Heteronuclear tangent VSWDs with $\sigma_2^*=0.5$, shown of the bottom panel of Fig. 5, present the same general behavior just described for the homonuclear tangent case for $\epsilon_2^*=1$ and 2.

For $\epsilon_2^*=4$, γ^* values are not as high than the homonuclear case and at lower temperatures, they have similar values with a diminishing slope. For $\epsilon_2^*=8$, γ^* values are smaller than those of the previous case described ($\epsilon_2^*=4$) and the curve of γ^* versus temperature presents a nonlinear tendency with larger error bars even for longer runs. Heteronuclear tangent VSWD with $\sigma_2^*=0.25$ shows very similar results as for the case of $\sigma_2^*=0.5$ (Fig. 5) except that the last two curves, corresponding to $\epsilon_2^*=4$ and 8, lie on top of each other.

In terms of the density profiles, Fig. 7 presents some typical examples for series 2, 10, and 17 where a well developed planar interface is formed. The homonuclear dumbbell (series 2) presents a usual profile without any specific features. Series 10 and 17 have a little absorption of type 2 atoms in the interphase, larger for $\sigma_2^*=0.5$ than for 0.25. Other profiles present similar features at low temperatures. The aggregates mention above has some effect on the profile and perhaps are responsible for the adsorption. As an inset, every profile has a snapshot of one configuration taken during the run.

VII. CONCLUDING REMARKS

A comprehensive study of the influence of four variables on surface tension γ^* and orthobaric densities ρ_L^* and ρ_V^* is presented. The four variables are the size of the vibration well L_{vw}^* , the elongation L^* (also known as the separation of the centers of the particles composing the VSWD) the relative size of the second particle σ_2^* , and the relative depth ϵ_2^* of the interaction well of the second particle. The size of the vibration well L_{vw}^* have a very small influence on the resulting properties. Decreasing elongation L^* increases the critical temperatures and densities. A decreasing size of the second particle σ_2^* in the tangent VSWD produces a large increase in the critical temperature and critical density. An increase in the depth of the interaction well ϵ_2^* produces a large increase in both critical temperature and critical density. Values of $\epsilon_2^*=4$ and 8 show a substantial association phenomena, which produce a decrease in the surface tension γ^* . All series analyzed produce a decrease in the surface tension with respect to the SW and to the tangent homonuclear VSWD in terms of the corresponding state construction. As an ongoing future work, the nature of the association phenomenon found is being studied in depth.

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