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# Galvanomagnetic Effects in III–V Compound Semiconductors

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# Galvanomagnetic Effects in III–V Compound Semiconductors*

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The influence of various structural characteristics in the III-V compounds on galvanomagnetic properties is discussed. Evidence for the scattering of charge carriers by polar optical modes is reviewed, and the behavior of Hall and magnetoresistance coefficients is examined in regard to the conduction band structure. Unique characteristics, imparted by light masses in certain bands, include high mobilities and large magnetoeffects associated either with transport in the band or with ionization energies of the impurity centers. The importance of avoiding inhomogeneities, either in specimen or in magnetic field, when measuring Hall coefficient or magnetoresistance in high-mobility materials is emphasized. Illustrations are given of the effects of nonuniformities in carrier concentration or in applied magnetic field on various galvanomagnetic phenomena.

THE influence on galvanomagnetic properties of charge-carrier scattering mechanisms and of band structure is exhibited in interesting ways in the different III-V compounds. In connection with the scattering, the fact that there exists two dissimilar atoms in the unit cell of the crystal, and that certain degrees of ionic bonding are present in the III-V semiconductors, renders it likely that polar optical modes might play a significant role in the interaction between the charge carriers and the lattice. That polar scattering may, in fact, predominate over acoustic phonon scattering at room temperature in such materials as InSb, InAs, InP, and GaAs is suggested by the theoretical calculations of mobility done by Ehrenreich.[1–4] Results on additional III-V compounds are available from calculations made by Hilsum.[5] Attempts to obtain further information on the scattering in many of these materials have caused measurements to be made of the transverse and longitudinal Nernst effects—the latter quantity being essentially the magnetothermoelectric power. Unfortunately, there is no unanimity in the interpretation of the results. For example, investigators such as Rodot[6] cited the behavior of the thermomagnetic effects at room temperature in indium antimonide as evidence for a predominance of scattering by polar optical modes, while Zhuse and Tsidil'kovskii[7] arrived at the conclusion that acoustic phonon scattering is the significant process. Nasledov and collaborators also favored the acoustic phonon process for scattering in indium arsenide[8] and gallium arsenide.[9–11] The interpretation of thermomagnetic phenomena often is complicated by ambipolar effects, by mixed scattering mechanisms, and by the fact that for polar optical scattering the relaxation time approximation is applicable only under limiting conditions.[12]

Unique features in the band structure of many of the III-V materials are reflected in the galvanomagnetic properties. Of particular interest has been the importance of the nonparabolic nature of the dependence of energy on wave number for states away from the conduction band edge,[13] especially in the small bandgap materials such as indium antimonide and indium arsenide. In connection with the heavy-mass valence band, the possible importance of linear terms in $k$, caused by the lack of inversion symmetry in these compounds, has been discussed for the case of indium arsenide.[14] In GaSb and GaAs, it is known that a set of subsidiary conduction band minima exist at approximately 0.08 and 0.36 ev, respectively[4,15] at room temperature, above the principal minimum which has [000] symmetry. In a certain temperature region, therefore, electrons will be thermally activated from the principal conduction band to the subsidiary band, where the mobility is lower. As a result, a maximum is observed in the behavior of the Hall coefficient with temperature.

A number of interesting galvanomagnetic phenomena result from the low effective masses in certain bands and the resulting high mobilities found in such materials as indium antimonide, indium arsenide, and to a lesser extent, gallium arsenide. These low effective masses are also responsible for some unique characteristics associated with the donor levels. In indium antimonide, for example, the low conduction band effective mass together with a relatively large dielectric constant yields

* Supported in part by the Air Force Office of Scientific Research.

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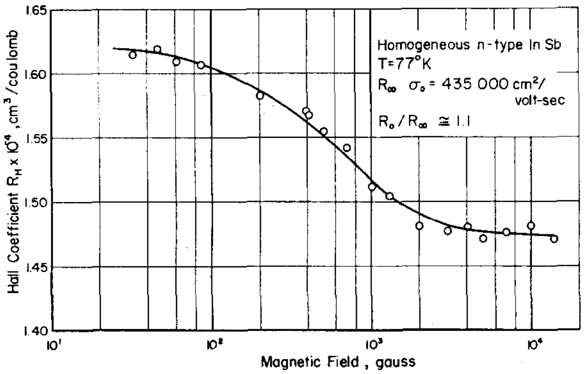


FIG. 1. Field dependence of Hall coefficient in high-purity indium antimonide.

a value of the order of $7\times10^{-4}$ ev for the ionization energy of a hydrogenic level.[16] This result is in agreement with the value of $6\times10^{-4}$ ev reported by Nasledov for ultrapure specimens.[10] The conditions mentioned above, however, are responsible for a relatively large effective Bohr radius, i.e., for a substantial spreading of the electronic wave functions centered on different impurity atoms. Therefore, not many donor atoms can be accommodated in the crystal before one notices results of overlap of these wave functions. In the case of indium antimonide it appears from measurements by both Sladek and Putley that, for *n*-type impurity concentrations even as low as $10^{14}$ atoms $cm^{-3}$, the donor levels are effectively merged with the conduction band, and no carrier "freeze-out" occurs at low temperatures.[17,18] The effect of a magnetic field is to reduce the overlap of the donor wave functions. Because of the small effective mass in InSb, a significant carrier freeze-out can be achieved at moderate magnetic field strengths in material of donor concentrations in the $10^{14}$ $cm^{-3}$ range.[17,19,20] In ultrapure specimens containing around $4\times10^{13}$ extrinsic electrons per $cm^3$, Putley has measured "effective" ionization energies of around $2\times10^{-4}$ ev in fields of 4000 gauss.[21] When cooled to 1.5°K, such a specimen exhibited sensitivity as a photoconductor for electromagnetic radiation having wavelengths of a few millimeters.

A high charge-carrier mobility in a semiconductor is useful not only because of the large augmentation of many galvanomagnetic phenomena, but also because the investigator can study the effects over the range from the weak to the strong magnetic field regions with the use of ordinary laboratory magnets. An example of such measurements is illustrated in Fig. 1, which shows the Hall coefficient at 77°K in an InSb sample cut from a region in a single crystal which exhibited a high degree of homogeneity. The magnetic field in the region 400–14 000 gauss was monitored by an NMR gaussmeter. Below 90 gauss, the field was obtained by means of Helmholtz coils. It is apparent that in high-mobility materials galvanomagnetic measurements must be taken at very low field strengths to determine weak-field coefficients. We note in the figure that the weak-field plateau for the Hall coefficient is not achieved until the magnetic field is reduced to below 50 gauss. Data of the type shown are useful in that the strong-field Hall coefficient is directly related to the charge-carrier density—independent of the scattering mechanism. Thus, the measurements provide convenient means for determining the conductivity mobility. The weak-field Hall coefficient, on the other hand, is influenced by the charge-carrier scattering and the band structure. It, of course, yields the Hall mobility. Further information on scattering and band structure can be obtained from the magnetoresistance mobility and the directional magnetoresistance coefficients.

Magnetoresistance measurements in various III-V compounds have been carried out by a number of investigators. With no attempt to be complete, we shall cite in the references examples for indium antimonide,[22,23] indium arsenide,[24,25] indium phosphide,[26] gallium arsenide,[26,27] and gallium antimonide.[28] By making allowances for rather severe measurement problems, one may conclude with reasonable safety that the results are consistent with approximately spherical energy surfaces for the principal minima of the conduction bands.

The measurement problems alluded to above refer to the necessity for avoiding any perturbation of the Hall field in the specimen. In high-mobility materials, even a slight shorting of the Hall voltage can produce large magnetoresistive effects. Disturbances of the Hall field can occur through external contacts or by means of inhomogeneities in the measurement sample. The first problem, the so-called geometrical effects, has been discussed by a number of authors quite generally,[29] as well as in the special case for measurements on indium antimonide[23] and indium arsenide.[24] The question of inhomogeneities must be given special consideration in

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the case of the III-V compounds because of the possibility of anisotropic segregation of impurities. This phenomenon has been demonstrated by several investigators in the case of indium antimonide,[30,31] and may be expected to be important in a number of other compounds. That impurity concentration gradients can seriously affect galvanomagnetic voltages was pointed out sometime ago by Herring,[32] who has recently published a theoretical treatment of the effect of random inhomogeneities on electrical and galvanomagnetic measurements.[33] He also discusses the effect of macroscopic inclusions and the behavior of the laminar model. The problem of gross inhomogeneities was examined by Bate and Beer, and explicit solutions to the boundary value problem were obtained for the case where the carrier concentration varied exponentially along the specimen,[34] and where the variations in conductivity and Hall coefficient were approximated by step functions.[35]

When inhomogeneities exist, the current distribution in the sample is, in general, modified by the magnetic field. Thus, the galvanomagnetic voltages become dependent on probe position. In addition, the voltage changes due to the inhomogeneities may predominate over the effects which are characteristic of the material. This latter point is illustrated by the directional magnetoresistance measurements of Rupprecht *et al.* on tellurium-doped indium antimonide.[36] These investigators found that the important consideration was not the relation of the current to a general crystallographic direction, but rather the relation of the current to the specific direction [111] in which the crystal had been pulled. When these directions coincided, the transverse magnetoresistance was much larger. Further measurements by Rupprecht[37] on crystals pulled also in [100] and [113] directions revealed a similar anisotropy, namely, that the largest transverse magnetoresistance occurred when the current was in the direction of pull. The magnitude of the effect, which was largest for [111] directions of pull, became progressively less for the [100] and the [113] directions. Another interesting observation was the importance of the pulling speed in accenting the anisotropy. For the [111] pull, the $\Delta\rho/\rho_0$ dropped from a largest value of 0.7 for a speed of 0.25 mm/min, to a largest value of 0.02 for a pulling speed of 4.2 mm/min.

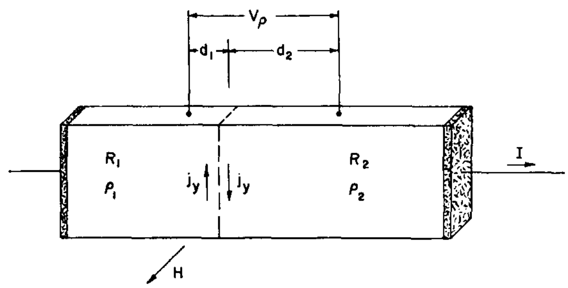


FIG. 2. Arrangement of resistivity probes in step-function model of inhomogeneous semiconductor. Length of specimen in the direction of current $I$ is $l$, width in the $y$ direction is $w$, and thickness in the $H$ direction is $b$.

An inhomogeneous medium will usually produce an intermixing of Hall and magnetoresistance effects. This is illustrated by the expressions for the measured transverse magnetoresistance and Hall effects in the step function model when the current is normal to the plane separating regions of resistivities, $\rho_1$ and $\rho_2$, and respective Hall coefficients, $R_1$ and $R_2$[35]:

$$\left[\frac{\Delta\rho}{\rho_0}\right]_m=\left[\frac{\Delta\rho}{\rho_0}\right]+\frac{d_2R_2-d_1R_1}{d_2\rho_2{}^0-d_1\rho_1{}^0}\,\frac{(R_2-R_1)H^2}{\rho_1+\rho_2},\qquad d_1,\, d_2\ll w\ll l, \tag{1}$$

$$R_m=\frac{R_1\rho_2+R_2\rho_1}{\rho_1+\rho_2},\qquad R\equiv R(H),\quad \rho\equiv\rho(H). \tag{2}$$

The arrangement of the resistivity probes is shown in Fig. 2; the Hall probes are located on the discontinuity at $y=\pm w/2$. In Eq. (1), the first term on the right is that due to the material alone, while the second term is that resulting from the inhomogeneity. We note that if $R$ and $\rho$ saturate, this term varies as $H^2$ and that it may be positive or negative. Thus, in strong magnetic fields this effect may predominate. In weak magnetic fields, it can also predominate if the material magnetoresistance coefficient $\Delta\rho/[\rho_0\mu_0{}^2H^2]$ is small, as it true of many $n$-type III-V materials. In the case of the Hall effect, the nonuniformities may introduce a significant magnetic field dependence. An example is shown by the data taken on a specimen in which a change in carrier density by a factor of 10 occurred in the current direction, as compared with the data measured on a uniform sample, shown in Fig. 3.

Thus, we see that measurement of magnetoresistance in strong magnetic fields is greatly complicated by the demands placed on the uniformity of the specimen. Actually, the situation is even more difficult when we are interested in extremely strong field strengths. For, as pointed out by Bate and Beer,[34] gradients in the magnetic field strength are in certain cases equivalent to gradients in the components of conductivity. This is evident from a comparison of the differential equations for the electric potential for the cases in question.

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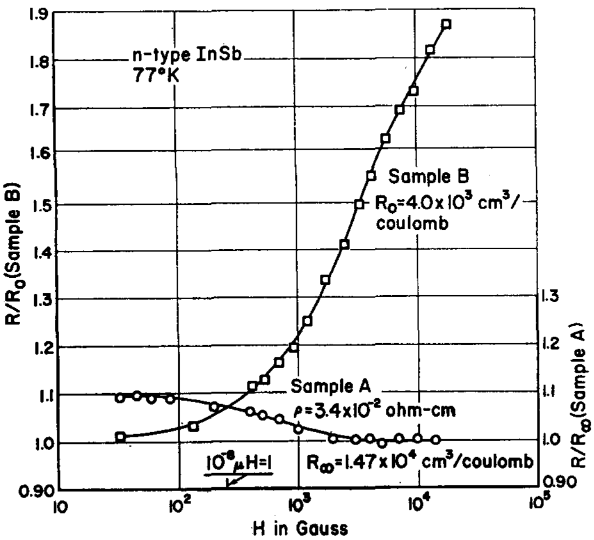


FIG. 3. Magnetic-field dependence of the normalized Hall coefficient of two high-purity $n$-type InSb samples. Carrier concentrations are: sample A: $4.25\times10^{14}$ cm$^{-3}$; sample B: $5\times10^{14}$ to $5\times10^{15}$ cm$^{-13}$ (after Bate *et al.*[35]).

(a) Nonuniformity in $\sigma_t$:

$$\sigma_t^{-1}d\sigma_t/dx\equiv K\neq 0, \quad d\beta/dx=0,$$

$$\nabla^2 V+K\frac{\partial V}{\partial x}+K\beta\frac{\partial V}{\partial y}=0.$$

(b) Nonuniformity in $H$:

$$d\beta/dx\neq 0, \quad d\sigma_t/dx=(\partial\sigma_t/\partial H)(dH/dx)$$

$$\nabla^2 V+\frac{1}{\sigma_t}\frac{\partial\sigma_t}{\partial H}\frac{dH}{dx}\frac{\partial V}{\partial x}+\left(\frac{\beta}{\sigma_t}\frac{\partial\sigma_t}{\partial H}+\frac{\partial\beta}{\partial H}\right)\frac{dH}{dx}\frac{\partial V}{\partial y}=0,$$

where $\beta[\equiv RH/\rho_H]$ is the tangent of the Hall angle, $\sigma_t$ are the diagonal components of the conductivity tensor, and $\pm\beta\sigma_t$ are the off-diagonal components. The forms of (b) at strong fields, $\beta>1$, are as follows:

(b-1) $\nabla^2 V+2K\frac{\partial V}{\partial x}+K\beta(x)\frac{\partial V}{\partial y}=0$, if $\Delta\rho/\rho_0$ saturates,

(b-2) $\nabla^2 V+K\frac{\partial V}{\partial x}+K\beta\frac{\partial V}{\partial y}=0$, if $\Delta\rho/\rho_0\sim H$,

where $K\equiv -H^{-1}dH/dx$.

To illustrate the effect of a magnetic field gradient on the current distribution in the specimen, we consider case (b-1). The differential equation is then identical with that for the case of nonuniform conductivity and the expression given by Bate and Beer for the current density is[34]

$$j_x=\frac{I}{wb}\frac{\gamma/2}{\sinh\gamma/2}e^{-\gamma y/w}, \quad \gamma\equiv Kw\beta.$$

An important factor at strong fields is obviously $\beta$, the tangent of the Hall angle. Some recent measurements by Hieronymus and Weiss[38] on intrinsic InSb, which therefore possessed good charge-carrier homogeneity, suggest that values of $\beta$ of 30 are readily possible. This result yields a value for $\gamma$ of 0.75 for a specimen 1-cm wide in a field gradient such that $H^{-1}dH/dx$ is $2\frac{1}{2}\%$ per cm. The ratio of the current densities at the two edges of the sample is then $j_x(-w/2)/j_x(w/2)=2.1$. Thus, we see that when large Hall angles are involved even a relatively small degree of nonuniformity can have a significant effect.

Another item of interest at strong magnetic fields in high-mobility bands is the possible appearance of quantum effects in the transport properties. For example, in $n$-type InSb at 77°K, Landau splitting energies, $\hbar\omega$, of around $3kT$ can be obtained at 20 kgauss. This has led numerous investigators[39–42] to measure magnetoresistance in the region where $\hbar\omega\gg kT$ (or $\hbar\omega\gg\zeta$, where $\zeta$ is the Fermi level in degenerate material) to obtain information on electron orbit quantization effects. The additional condition $\omega\tau\gg1$—where $\tau$ is the carrier relaxation time, and the cyclotron frequency $\omega$ is given by $eH/m^*c$—is of course readily met in indium antimonide. Quantum effects can be responsible for a nonzero longitudinal magnetoresistance in isotropic solids, which does not saturate, and for failure of the transverse magnetoresistance to saturate.[43,44]

Most of the experimental results provide qualitatively a reasonably convincing argument for the observance of quantum effects in InSb. We know, of course, that any disturbance of the current distribution in the sample—whether by inhomogeneities in specimen or magnetic field, by contacting leads, or by surface conduction—will affect the saturation of the magnetoresistance and may produce significant results at large magnetic fields. Therefore, to secure data which can be used in quantitative studies of quantum effects imposes an extremely demanding task on the experimenter.[45]

We have indicated how inhomogeneities can dominate the galvanomagnetic behavior in high-mobility materials. That this may have been the state of affairs in the case of indium antimonide is suggested by the recent experiments of Weiss, who finds that a negligible magnetoresistance effect can be attributed to the elec-

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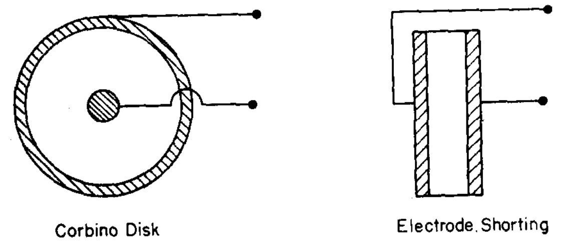


FIG. 4. Mechanisms to produce shorting of the Hall voltage.

trons in the conduction band of indium antimonide.[46] Since the large magnitudes of the inhomogeneity effects result from perturbations of the Hall voltage, it is of interest to examine galvanomagnetic measurements under conditions such that the Hall field is absent. A common example is the Corbino disk[47] or the large width-to-length ratio specimen where the transverse field is shorted by the end electrodes[48] (see Fig. 4). The latter arrangement has been used by Goldberg to measure directly the magnetoconductivity in germanium.[49] Some unpublished data[50] on the Corbino magnetoresistance in InSb carried out at the Battelle Memorial Institute are shown in Fig. 5. It is readily shown that deviations from an $H^2$ dependence at strong fields result from a nonsaturation of the ordinary magnetoresistance. For the room temperature data on intrinsic InSb, this is attributed to the effect of the holes,[51] and the data shown are fitted using an appropriate electron-hole mobility ratio. In this case, spherical energy surfaces and acoustic phonon scattering were assumed. At 80°K, in the extrinsic range, the hole concentration is negligible and it appears likely that quantum effects may be responsible for the break away from an $H^2$ line. This point has been made by Frederikse and Hosler,[52] who found that the slope in a plot of their data at high magnetic fields approached a linear dependence in accordance with Argyres' predictions.[53]

Data at 77°K on a more pure sample of InSb are available from some recent measurements by Bate,[54] which also provided a check on the theoretical relation connecting the Corbino effect, transverse magnetoresistance, and conductivity mobility. The procedure

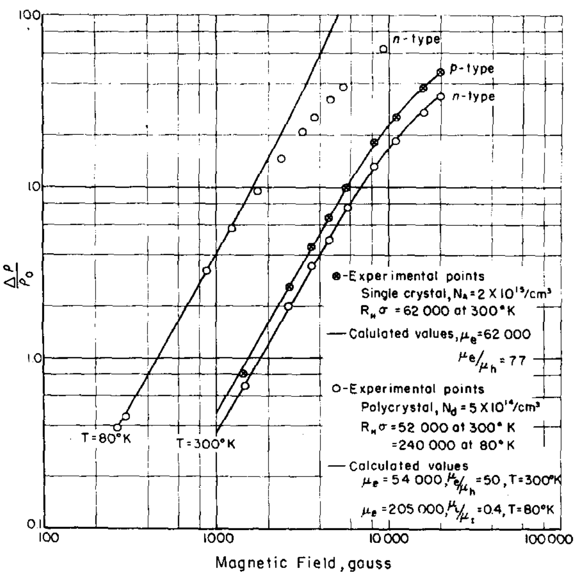


FIG. 5. Corbino magnetoresistance in indium antimonide (after Beer *et al.*[50]).

was to measure first the Corbino effect at 77°K as a function of magnetic field on a disk with soldered annular electrodes. Since the disk was 12.9 mm in diameter and 2.28 mm thick, with a hole 3.35 mm in diameter at the center, it was subsequently possible to cut out a sample in the shape of a rectangular parallelepiped, and then to measure directly the Hall effect, resistivity, and transverse magnetoresistance as functions of field at 77°K. A determination of the resistance of the contacts and connecting leads of the original Corbino disk was then made by subtracting from its measured resistance that resistance calculated, using its dimensions and the resistivity measured on the parallelepiped. The total resistance of the contacts and leads was about 35% of the disk resistance at zero field. The Corbino data were then corrected for contact effects by subtracting the contact resistance (assumed independent of magnetic field) from the resistance of the disk in the field. The resulting data, along with those of the transverse magnetoresistance, are shown in Fig. 6. When measurements are taken in the region where the Hall coefficient has saturated, it can be shown that a relation exists between the Corbino effect, the transverse magnetoresistance, and the conductivity mobility, as follows[50]:

$$\frac{\rho}{\rho_0}\left[\left(\frac{\rho}{\rho_0}\right)_c-\left(\frac{\rho}{\rho_0}\right)\right]\equiv F=\mu_0{}^2H^2,\quad R_H\rightarrow R_\infty.$$

The quantity $(\mu_0H)^2$ is shown as the "theoretical line" in Fig. 6, and values of $F$ calculated from the transverse and Corbino magnetoresistance data are plotted for comparison. The agreement is seen to be excellent.

In the case of the Corbino disk, one can establish for a radial inhomogeneity in conductivity or in magnetic

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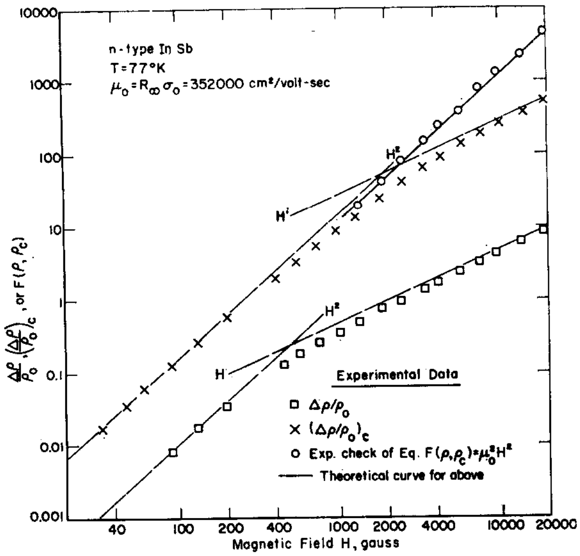


FIG. 6. Magnetic-field dependence of transverse magnetoresistance and Corbino magnetoresistance in InSb specimen. Data are also plotted to provide an experimental check of the theoretical relationship, valid when $R_H$ saturates:

$$\frac{\rho}{\rho_0}\left[\left(\frac{\rho}{\rho_0}\right)_c-\left(\frac{\rho}{\rho_0}\right)\right]\equiv F(\rho,\rho_c)=(\mu_0 H)^2.$$

field that the potential does not contain the Hall angle, and therefore the Corbino magnetoresistance involves only $\sigma_t(r)$, where $\sigma_t$ are the diagonal elements of the conductivity tensor, suitably averaged over the distance between the points where the potential is measured. There is no Hall field to produce the augmented effects at strong fields, and the potential assumes the simple form shown below.

$$V(r)-V(a)=-\frac{I}{2\pi b}\int_a^r \frac{dr'}{r'\sigma_t(r')},$$

where $b$ is the thickness of the disk, and $I$ is the current. The relationship is valid for any variation of $\sigma_t$ with $r$—so long as the integral exists—and $\sigma_t$ may depend explicitly on $r$ or implicitly through $H(r)$.

A similar situation exists for a magnetoconductivity measurement when the impurity or magnetic field gradient is in the direction of the electric field. In this case, the expression for the potential is

$$V(x)-V(0)=-\frac{I}{wb}\int_0^x \frac{dx'}{\sigma_t(x')},$$

where $wb$ is the cross-sectional area of the specimen normal to the $x$ direction, and $\sigma_t$ may depend on $x$ either explicitly or implicitly through $H(x)$. On the other hand, if the gradients in the above example are in the $y$ direction, one should measure an ordinary magnetoresistance effect with the current in the $x$ direction to avoid distortions from the inhomogeneity effect.[34] In such a case, when $\beta$ is independent of position, the expression for the potential can be put in the simple form

$$V(x,y)=-\left[I\Big/\left\{b\int_{-w/2}^{w/2} dy/\rho(y)\right\}\right][x+\beta y],$$

where the resistivity $\rho(y)$ is given by $\rho(y)\equiv\{\sigma_t(y)[1+\beta^2]\}^{-1}$. Hence, $V(x_1,y)-V(x_2,y)$ depends in this case only on an average of $\rho^{-1}(y)$.

Thus, we see that Corbino disk or magnetoconductivity measurements can be helpful in some cases in separating the contributions from nonuniformities in specimen or in magnetic field from the magnetoresistive behavior of the material proper. An experimental problem is to secure negligible resistance—or perhaps more important, uniform resistance—at the shorting contacts. Nevertheless, sensible results have been obtained in the case of indium antimonide, and it appears that the techniques may be useful in additional applications—especially where it is desirable to secure further support for data resulting from standard measurements.

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