AIP | Physics of Fluids

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Citation: Physics of Fluids **21**, 2115 (1978); doi: 10.1063/1.862121
View online: http://dx.doi.org/10.1063/1.862121
View Table of Contents: http://scitation.aip.org/content/aip/journal/pof1/21/11?ver=pdfcov
Published by the AIP Publishing

Slip of liquid inside a channel exit

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(Received 28 November 1977; final manuscript received 7 August 1978)

Navier's slip coefficient boundary condition at the channel wall reduces die swell of Newtonian liquid exiting at low Reynolds number, and appears to relieve the stress singularity associated with Stokes' no-slip hypothesis, according to a finite element analysis.

Experimental evidence mounts that Stokes' no-slip hypothesis[1] can be violated by viscous liquids upstream of channel exits.[2,3] Although theoretical evidence exists that Stokes' hypothesis would require unbounded shear stress at a channel exit,[4] it has been adopted in finite element analyses of die swell and related flows with static meniscus separation lines.[5-7] We employ Navier's slip hypothesis[8] to alleviate the separation line singularity as simply as possible, and to investigate the effects of wall slip on the die-swell flow field at low Reynolds number.

The Navier–Stokes equations for two-dimensional, incompressible, steady flow of Newtonian liquid as shown in Fig. 1 are solved by the Galerkin finite element method based on the divergence form of the equations.[7] A mixed interpolation consisting of quadratic serendipity elements on rectangles for Cartesian velocity components and bilinear elements on rectangles for pressure is chosen.[7,9] Conversion from channel flow to a jet of constant width is taken to occur in a distance $4b$, where b is the channel half-width. Additional calculations revealed that increasing this domain length did not change the low Reynolds number results presented here; a length of $4b$ was also arrived at in another analysis of channel exit flow.[7] The upstream and downstream, symmetry plane, and free-surface boundary conditions are handled in the usual way as essential or natural conditions. Navier's slip condition at the wall is a boundary condition of the third kind: $\beta \mathbf{t}\cdot\mathbf{T}\cdot\mathbf{n}+\mathbf{t}\cdot\mathbf{u}=0$, where $\mathbf{T}=-p\mathbf{1}+\boldsymbol{\tau}$ is the stress tensor, p is the pressure, $\mathbf{1}$ is the unit tensor, $\boldsymbol{\tau}$ is the viscous stress tensor, $\mathbf{u}$ is the slip velocity, $\mathbf{n}$ is the unit normal, and $\mathbf{t}$ is the unit tangent at the wall. β is the slip coefficient ($1/\beta$ is the coefficient of momentum transfer down the velocity discontinuity, much as viscosity μ is the coefficient of momentum transfer down a velocity gradient). The limit $\beta \to 0$ gives no slip and the limit $\beta \to \infty$ gives perfect slip, i.e., vanishing wall shear stress. β may well depend on the stress tensor $\mathbf{T}$ and otherwise on location along the wall. Here, we present results for constant β only.

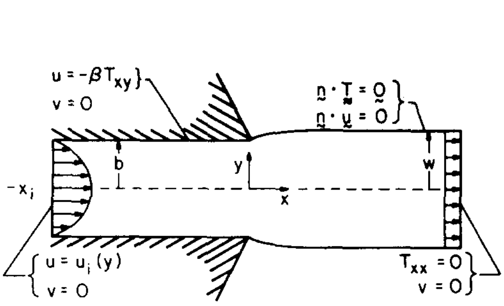

FIG. 1. Channel exit and issuing sheet, with boundary conditions.

The upstream velocity boundary condition must accord with the wall-slip boundary condition. The downstream velocity boundary condition is uniform, parallel flow. These conditions are applied at a distance $2b$ upstream and downstream, respectively, from the exit.

The free surface shape is calculated by successively adopting a shape, setting aside the kinematic boundary condition, solving for the flow field, and then generating a new shape by means of the integral analog of the kinematic condition

$$\int_0^{h(x)} u\,dy = Q, \tag{1}$$

where $h(x)$ is the free surface location sought, and Q is the volumetric rate per unit length. At times the u field must be extrapolated outside the current $h(x)$, as is true of the oft-used kinematic condition.[5-7] The latter suffers difficulty at the channel exit which is avoided by this integral analog, however.

In Fig. 2 we report die swell w/b and meniscus separation inclination i for slip coefficients from $\beta = 0$ (no slip) to $\beta = b/\mu$ (macroscopically obvious slip). To isolate the effect of slip, surface tension and Reynolds number are here set equal to zero (results where they are not will be given elsewhere). The meshes employed range from 9×3 elements to 12×4 elements, with elements diminishing in size toward the separation line. The code reproduced independent calculations of no-slip, creeping flow cases.[7] Moreover, mesh refinement in this range never changed the swell ratio more than 1%. That moderate slip ($\beta\mu/b \sim 0.01$) causes a de-

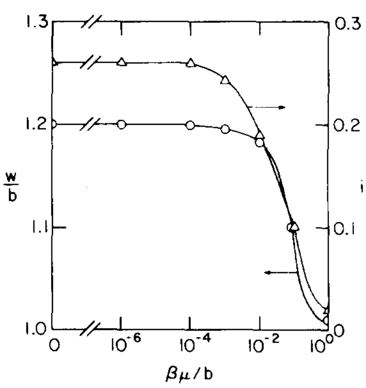

FIG. 2. Swell ratio (○) and meniscus separation inclination (△) vs slip coefficient. w is measured at $x = 2b$; i is chord slope from $x = 0$ to $x = 0.18b$ (a span of two elements).

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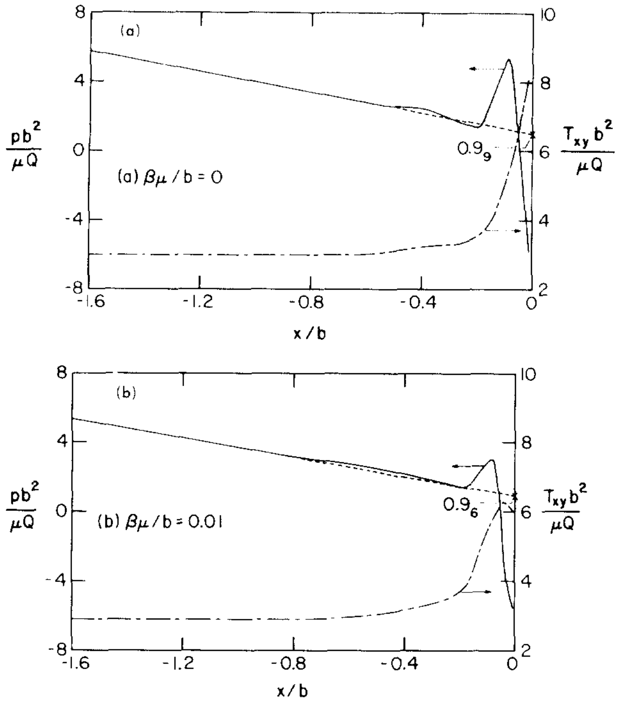

FIG. 3. Stress field components along channel wall: —, pressure; — — —, shear stress; - - - - - -x, extrapolated upstream pressure. (a) No slip. (b) Perceptible slip.

tectable decrease in swell ratio and meniscus separation inclination is plain from Fig. 2. This effect is observed in the laboratory with a very viscous, shear thickening, elastic liquid,[3] but our result shows that non-Newtonian normal stress response need not be present. The effect should be sought experimentally with highly viscous, Newtonian liquid.

Wall shear stress is plotted in Fig. 3 [the slip at the wall, $u(b) = -\beta T_{xy}$, is proportional]. The no-slip results, particularly of mesh refinement, are consonant with an integrable singularity $T_{xy} \to \alpha/\sqrt{x}$. Slip evidently removes the singularity, and when $\beta < 0.001\ b/\mu$, it does so without perceptibly altering the velocity and pressure fields, meniscus separation inclination, or swell ratio. Thus, the singularity may be tolerable for many purposes of theory and mathematical simulation. The extrapolation of the upstream pressure all the way to the channel exit is shown in Fig. 3. Even in the no-slip case it plainly differs from zero, assertions to the contrary notwithstanding.[10] Upstream influence from the exit should be greatest in the creeping flow limit we report here.

This research was supported by the National Science Foundation and University of Minnesota Computing Center. One of us (W. J. S.) holds a Bush Foundation Fellowship.

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Fermi–Pasta–Ulam recurrence in the two-space dimensional nonlinear Schrödinger equation

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(Received 24 July 1978)

Numerical solutions of the two-space dimensional nonlinear Schrödinger equation with spatially periodic boundary conditions have been obtained. It has been found that the long-time evolution exhibits the Fermi–Pasta–Ulam recurrence phenomenon for a wide range of initial conditions.

It has been shown by Zakharov[1] that the evolution of a weakly nonlinear, deep-water, gravity wave train subjected to two-space dimensional modulation is governed by the following equation for the complex envelope A:

$$i\left(\frac{\partial A}{\partial t} + \frac{\omega_0}{2k_0}\frac{\partial A}{\partial x}\right) - \frac{\omega_0}{8k_0^2}\frac{\partial^2 A}{\partial x^2} + \frac{\omega_0}{4k_0^2}\frac{\partial^2 A}{\partial y^2} - \frac{1}{2}\omega_0 k_0^2 |A|^2 A = 0\ , \tag{1}$$

where ω_0 and k_0 are the frequency and wavenumber of

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