Physics Today

Fonctions analytiques—Equations intégrales

Aristotle D. Michal and C. M. Ablow

Citation: Physics Today **12**(11), 50 (1959); doi: 10.1063/1.3060574
View online: http://dx.doi.org/10.1063/1.3060574
View Table of Contents:
http://scitation.aip.org/content/aip/magazine/physicstoday/12/11?ver=pdfcov
Published by the AIP Publishing

lems of stability: comparison of solutions, indirect methods using auxiliary functions, and more abstract topological methods. The first two of these were thoroughly investigated by Liapunov and in many cases enable explicit answers to be given to important questions. The third is mainly the work of Poincaré and is more general in scope though it is not easily applicable to other than second-order problems.

Malkin's work is devoted in the main to the Liapunov second method and centers around the construction of the appropriate Liapunov functions which are essentially semidefinite convex functions (with respect to the time variable) of the variables entering the equation. While there is no general method for constructing these functions, enough examples are given of the appropriate functions for fairly comprehensive classes of differential equations to enable the method described to be used in a variety of practical problems.

This is a mathematics book but designed to enable the reader to solve practical problems. A selected number of examples are worked out in an extended fashion and the general treatment goes to some pains to bring out the underlying concepts of the method. The exposition transcends questions of stability alone—in particular, full treatments are given of singular and degenerate cases which show how general methods must be modified to take care of the special circumstances. Among the features particularly worthy of note is the analysis of second-order systems using the methods of Poincaré and Liapunov and an extended discussion of equations with periodic coefficients indicating how their regions of stability are to be established.

Because the methods used are so powerful, the author has been able to produce a valuable book from the point of view of mathematical and practical interest with hardly any reference to non-Russian mathematicians. The serious reader will probably find it worthwhile at least to glance at Cesari's review, "Asymptotic Behavior and Stability Problems in Ordinary Differential Equations", in the *Ergebnisse der Mathematik* series in order to assure himself of the contribution of Western mathematicians to some of these problems and to recognize the existence of other topics of interest in this subject.

Fonctions analytiques—Equations intégrales. Vol. 1 of Le Calcul différentiel dans les Espaces de Banach. By Aristotle D. Michal. 150 pp. Gauthier-Villars, Paris, France, 1958. Paperbound $7.30. *Reviewed by C. M. Ablow, Stanford Research Institute.*

THE tendency toward abstraction and generalization characteristic of much of modern science is nowhere stronger than in mathematics. Established theorems are examined for excess hypotheses, and attempts are made to exhibit theorems as special cases of more general facts. For example, it is a classical theorem that the only continuous functions $f(x)$ satisfying $f(x+y) = f(x) + f(y)$ are constant multiples

of x. More generally the conclusion is valid if $f(x)$ is merely required to be bounded in the neighborhood of one value of x. More abstractly, in an inner product vector space with $\mathbf{x}$ a variable vector, for any such bounded linear functional there is a fixed vector $\mathbf{v}$ such that $f(\mathbf{x}) = \mathbf{v} \cdot \mathbf{x}$.

The present booklet pursues its abstractions with many concrete examples, stopping on almost every page to illustrate the general theory with familiar instances. The generalities themselves are clearly suggested by classical theorems. Thus after careful introductory paragraphs defining Banach space and linear normed rings, functions reminiscent of polynomials are described. Convergent infinite sums of such monomials are the analytic functions of this theory. Examples are the iterated and resolvent kernels of Fredholm integral equation theory.

The Fréchet differential is next introduced and its use in general perturbation theory explained. For example, the formulas for the variation in the solution of an integral equation due to a perturbation of its kernel are exhibited in very simple general forms. Similar readily apprehended formalities show the dependence of the solution of a differential equation on the coefficient functions in that equation.

The book is carefully and clearly written in simple, nonidiomatic French. With its subject matter in appliable mathematics and its numerous historical references, the book presents a good introduction to abstract functional analysis.

Dictionary of Guided Missiles and Space Flight. Edited by Grayson Merrill, C. W. Besserer, K. A. Ehricke, & B. B. Small. 688 pp. D. Van Nostrand Co., Inc., Princeton, N. J., 1959. $17.50. *Reviewed by Robert E. Street, University of Washington.*

ALTHOUGH this is the fifth volume in the series *Principles of Guided Missile Design,* it can by its very nature of being a dictionary be considered independent of the other volumes of the series. Rather it is intended, as the editor says, to be a sequel and companion to the *International Dictionary of Physics and Electronics* by the same publisher. The physicist who is familiar with the latter dictionary will find this one to be quite similar. The most common terms used in the field are defined both verbally and sometimes mathematically, when the latter is relevant. Quite a few terms are duplicated but on the whole the majority of mathematical and physical terms are not.

For example, terms in thermodynamics, kinetic theory, dynamics, etc., which were well defined in the earlier volume, are not repeated here. This does lead to some lack of balance, i.e., vector notation is used in some definitions but the definitions of the vector terms themselves are in the physics dictionary. Quite a number of physics and electronics words are given in expanded form, such as transistors, which now take up nine pages instead of two; many more fluid flow terms and a complete treatment of astronomical terms