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Numerical analysis for finite-range multitype stochastic contact financial market dynamic systems

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In an attempt to reproduce and study the dynamics of financial markets, a random agent-based financial price model is developed and investigated by the finite-range multitype contact dynamic system, in which the interaction and dispersal of different types of investment attitudes in a stock market are imitated by viruses spreading. With different parameters of birth rates and finite-range, the normalized return series are simulated by Monte Carlo simulation method and numerical studied by power-law distribution analysis and autocorrelation analysis. To better understand the nonlinear dynamics of the return series, a q-order autocorrelation function and a multi-autocorrelation function are also defined in this work. The comparisons of statistical behaviors of return series from the agent-based model and the daily historical market returns of Shanghai Composite Index and Shenzhen Component Index indicate that the proposed model is a reasonable qualitative explanation for the price formation process of stock market systems.
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The statistical analysis of financial market index and return has long been a focus of economic research for a clearer understanding about the statistical properties of fluctuations of stock prices in globalized securities markets. With the emerging of new statistical analyzing methods and computer-intensive analyzing methods in the last decade, the empirical results of stock returns required the invention of new financial models to describe price movements in the market. Hence, many statistical behaviors, the so-called "stylized facts," such as fat-tail phenomenon, power-law distributions, volatility clustering, and multifractality, are revealed from empirical research by previous studies. For the sake of reproducing and explaining these stylized facts, various market models have been introduced, some of those approaches in this field by applying the stochastic interacting particle systems. In the present paper, we present a random agent-based financial price model by the finite-range multitype contact (FRMC) process. The FRMC process is different from ordinary contact process for it has more than one type of particles and the number of neighbors of every site can be varied in the process. Then, we investigate the fat-tail phenomena of normalized returns of the price model by the power-law distribution analysis, comparing with the daily returns of Shanghai Stock Exchange (SSE) and the standard Gaussian distribution ones. Besides, the power-law exponent of simulation data and historical data including SSE and Shenzhen Stock Exchange (SZSE) are figured out at 3 ± 0.3 by fitting in different regions, which means the simulation data and historical data are all asymptotically according with "inverse cubic law." At last, we study the volatility clustering feature of absolute returns of simulation data from the FRMC process and SSE by the autocorrelation analysis. The empirical results exhibit that there exists evident volatility clustering feature in the absolute return series both of the price model and the real financial market. Moreover, we introduced and plotted other two types of nonlinear autocorrelation functions: the q-order autocorrelation function and the multi-autocorrelation function. From figures, we can see that the simulation data from the price model and the historical data from the real financial market have the same trends and tracks in these two functions. In general, from the above analysis and results, it implies that the financial price model applied by the FRMC process, to some extent, is similar to the real financial market, and we can apply it to do more work in the field of the statistical analysis of financial market index and return.

I. INTRODUCTION

Recently, there has been a considerable interest in simulation of financial market dynamics in the field of statistical physics systems. A variety of models have been proposed, based on the competition between supply and demand, to model the main observed features of price dynamics, or the so called "stylized facts," fat-tailed distribution of returns, volatility clustering, aggregational gaussianity, and multifractality.[1–11] Capturing these features of financial markets is crucial for many purposes, including derivatives pricing, hedging, forecasting volatility, portfolio management, regulatory issues, value-at-risk, and so on. Some of these approaches come from the field of statistical physics systems or interacting particle systems.[12–21] For example, Lux and Marchesi[12] introduced an agent-based model in which chartist agents compete with fundamentalists agents, leading to power-law distributed returns as observed in real markets

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which contradicts the popular efficient market hypothesis. Zhang and Wang[20] invented an interacting-agent model of speculative activity explaining price formation in a financial market that is based on the finite-range contact particle system, where the epidemic spreading of the contact model is considered as the dissemination of investing information in the stock market.

The current interest in this subject stems from a recent simulation of financial market dynamics.[22–27] The root of that study is a microscopic model based on the contact process, a model for epidemic spreading in a continuous time Markov process.[28–31] One interpretation of the contact process is often regarded as a crude model for the spread of a disease. Healthy individuals become infected at a rate proportional to the number of the infected neighbors, and infected individuals recover at a constant rate. The proposed financial model derived from the contact process produces some desirable characteristics of real financial markets well, including fat-tail phenomenon, the power-law distribution, and long range correlation. In the present work, the FRMC process is applied to develop a random agent-based financial time series model. The FRMC process, as its name implied, is different from ordinary contact process for it has more than one type of particles in the process. Specifically in this paper, there are two types of particles, which can be interpreted as a model for the spreading of two kinds of diseases, one is heavy and the other is slight. They spread at different rates, proportional to the number of heavily and slightly infected neighbors, respectively, and both types of infected individuals could be recovered at a constant rate. The interesting part of the rule is that, a heavily infected individual can affect its slightly infected neighbors, changing them into heavily infected ones, while the converse is not possible. Furthermore, on the basic of autocorrelation analysis, we defined other two types autocorrelation which we called q-order autocorrelation function and multi-autocorrelation function. And the return behaviors for the simulation data from FRMC model, SSE Composite Index and SZSE Component Index are studied in the following. The fat-tail phenomenon and volatility clustering feature of returns are investigated by power-law distribution analysis and autocorrelation analysis, respectively.

II. AGENT-BASED FINANCIAL PRICE MODEL FROM FRMC PROCESS

As a metaphor, we can use the "tree, bush, and grass" to describe the FRMC model.[28,31,32] In the interpretation $2 =$ tree, $1 =$ bush, and $0 =$ grass, tree and bush die at the same rate, after death they become grasses, and they have different birth rates. If a new tree is born and sent to the site of a bush then at that site, a tree grows up and the bush dies. But if a new bush is born and sent to the site of a tree, then no action is taken at that site, the tree is still alive (at each site, only one plant is allowed to grow up). If a new tree (or bush) is born and sent to the site of a grass, then at that site, a new tree (or bush) will be grown up. A brief mathematical description of FRMC model is given in the follows, for details see Refs. 29–31. The FRMC process on $\mathbb{Z}^d$ is a continuous time Markov process $\{\eta_s, s > 0\}$ with infection parameters λ_1 and λ_2 (the λ_1 and λ_2 are the rates of a Poisson process) on the configuration space $\{0,1,2\}^{\mathbb{Z}^d}$, i.e., $\eta_s(x) \in \{0,1,2\}$ for $x \in \mathbb{Z}^d$, $\eta_s(x) = 0$ is interpreted as vacant at site x and $\eta_s(x) = i$ $(i = 1, 2)$ is interpreted as that there is an "i" type of particles at site x. In this model, the process evolves according to the following rules: (i) particles die at rate 1; (ii) particles of type i give birth at rate λ_i $(i = 1, 2)$; (iii) a new particle from x is sent to y chosen at random from $\{y : 0 < |y - x| \le L\}$ (where the finite-range L is a positive integer); and (iv) when $\eta_s(y) \ge \eta_s(x)$, then the birth is suppressed.

In the present paper, we assume that stock price fluctuation results from the traders' investment decisions towards the stock market and suppose that investment attitudes are represented by the particles of the FRMC model. We may classify attitudes into three categories according to levels of strength: strong attitudes, weak attitudes, and neutral attitudes, corresponding to 2 type particles "tree," 1 type particles "bush," and 0 type particles "grass," respectively. This is also reasonable in context of real world, for investment attitudes of different in real market are not always of same strength, that is, some investors (usually regarded as chartists investors) are more confident in their decisions while others (regarded as retail investors) are not so sure and relatively easy to alter. Let $\eta_s^{A_1,A_2}$ denote the state at time s with initial state $A_1 = \{x \in \mathbb{Z}^d : \eta_0(x) = 1\}$ and $A_2 = \{x \in \mathbb{Z}^d : \eta_0(x) = 2\}$. More generally, we consider the initial distribution as v_{θ_1} and v_{θ_2}, that is, each site is independently occupied by type 1 particles with probability θ_1 and by type 2 particles with probability θ_2 $(\theta_1 + \theta_2 \le 1)$, and let $\eta_s^{\theta_1,\theta_2}$ denote the corresponding FRMC model with initial distribution v_{θ_1} and v_{θ_2}. In this paper, we treat the initial distribution v_{θ_1} and v_{θ_2} as two constants labeled as the initial density θ_1 and θ_2 for short, where they represent the probability of each type particles, respectively, at time 0. The most significant feature of the FRMC process is that survival and extinction can both occur, which one occurs depends on the values of the rates λ_1 and λ_2 that represent the propagation rates of strong attitudes and weak attitudes in this agent-based financial price model.

Now, we consider a financial model of auctions for a stock index in a market. Suppose that the market for this stock consists of N (N is large enough) traders, who are located in a line $\{0, 1, \ldots, N\}$. At the beginning of each trading day $(t = 1, 2, \ldots, T)$, we select two fractions of traders (with initial density θ_1 and θ_2) randomly in the system who take weak attitudes and strong attitudes, respectively. Assume that each trader can spread his/her attitude several times at each day, and let l be the time length of trading time in each trading day, we denote the stock price at time s in the t-th trading day by $\mathscr{P}_t(s)$, where $s \in [0, l]$. For simplicity, we introduce the graphical representation of one-dimensional FRMC model with configuration space $\{0,1,2\}^{\mathbb{Z}}$ and different ranges $L = 2$ and $L = 3$, respectively, in Fig. 1, which is useful to illustrate and simulate the model. We consider, for each pair $x, y \in \{0, 1, \ldots, N\}$ with $0 < |x - y| \le L$, let $\{T_{1n}^{(x,y)}, n \ge 1\}$, $\{T_{2n}^{(x,y)}, n \ge 1\}$ and $\{U_n^x, n \ge 1\}$ be jump points of Poisson processes, with rates λ_1, λ_2, and 1, respectively. The x and y stand for different traders, while the

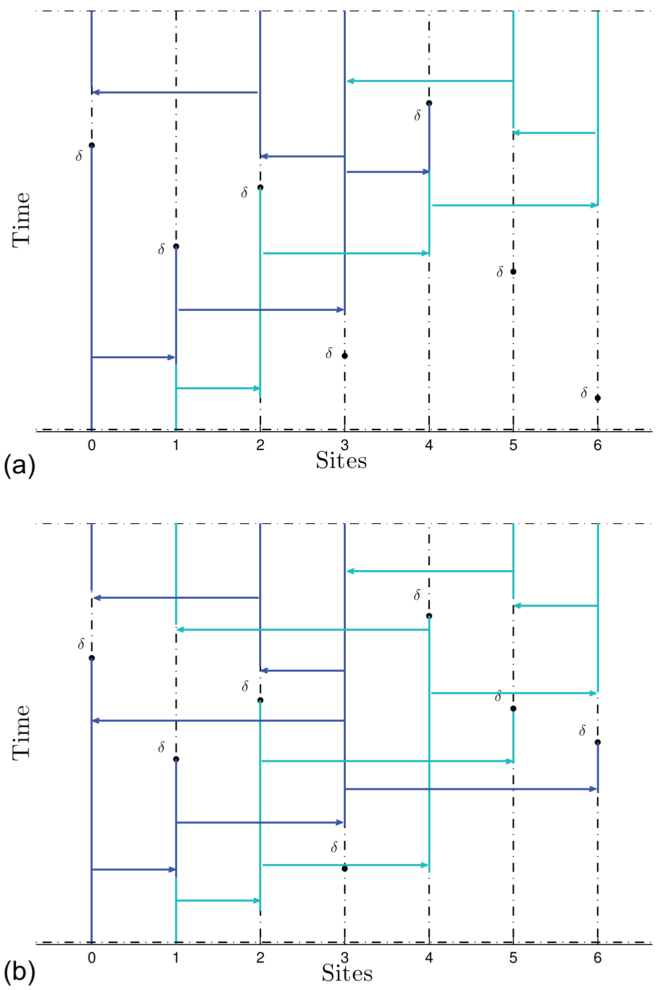

FIG. 1. Graphical representations of FRMC model with (a) $L = 2$ and (b) $L = 3$. The dark blue directed arrows represent the spread of strong attitudes, while the light blue directed arrows are for weak attitudes.

number of neighbors of each trader is $2L$. For each ordered pair of distinct time lines from $\{x\}$ to $\{y\}$ with $0 < |x - y| \leq L$, we place light blue directed arrows at time points $\{T_{1n}^{(x,y)}, n \geq 1\}$ from x to y, according to a Poisson process with rate λ_1, independently of other Poisson processes. This indicates that if the trader x takes weak attitude then the trader y will take weak attitude if he takes neutral attitude. Similarly, we place dark blue directed arrows at time points $\{T_{2n}^{(x,y)}, n \geq 1\}$ from x to y, according to a Poisson process with rate λ_2, independently of other Poisson processes, which indicates that if the trader x takes strong attitude then the trader y will take strong attitude whatever he takes. Along each trader $\{x\}$, we place a "δ" at x according to a Poisson process with intensity 1 and independently of other time lines at time points $\{U_n^x, n \geq 1\}$. The effect of a δ is to recover the attitudes of the trader x from other to neutral attitudes. In this dynamics, weak attitude investors acquire some information, which is represented by a random variable ξ_t of standard uniform distribution on $(-1, 1)$, similarly ζ_t (independent of ξ_t) is defined for strong attitude investors. Then they will make some decisions on trading positions accordingly, and these investors send bullish, bearish, or neutral signal to their finite-range neighbors. Since strong attitude investors are much confident in their investment information, as it comes from the latest technique or sources, their decisions on trading positions are more aggressive. Thus, we suppose their decisions are amplified by some exponent γ in response to their information received and are represented by $\gamma\zeta_t$, where we treat the γ as the proportion exponent between weak attitudes and strong attitudes. The remaining traders will hold neutral attitudes towards the market. According to FRMC system, investors can affect each other and the information can be spread, which is supposed as the main factor of price fluctuations for the market.

Next, the evolving stock dynamics of present work is given as follows. For $s \in [0, l]$ and $t = 1, 2, \ldots, T$, let

$$\begin{aligned}\mathscr{B}_t(\theta_1, \theta_2, s) = (\xi_t \cdot |\{x : \eta_s^{\theta_1,\theta_2}(x) = 1\}| \\ + \gamma\zeta_t \cdot |\{x : \eta_s^{\theta_1,\theta_2}(x) = 2\}|)/N, \quad (1)\end{aligned}$$

where $|A|$ denotes the cardinality of a set A. The θ_1 and θ_2 represent the initial density, N is the number of traders, and γ is the proportion exponent between weak attitudes and strong attitudes. We define the stock price at t-th trading day as[33–35]

$$\mathscr{P}_t = \mathscr{P}_{t-1} \exp\{\beta\mathscr{B}_t(\theta_1, \theta_2, s)\}, \quad (2)$$

where $\beta > 0$ represents the depth parameter of the market, which measures sensitivity of price fluctuation in response to change in excess demand. Then we have

$$\mathscr{P}_t = \mathscr{P}_0 \exp\left\{\beta \sum_{k=1}^{t} \mathscr{B}_k(\theta_1, \theta_2, s)\right\}, \quad (3)$$

where $\mathscr{P}_0$ is the initial stock price at time $t = 0$. The change of the logarithm of stock price which called "return" from time t to $t + 1$ is given as follows:

$$r(t) = r_t = \ln\mathscr{P}_{t+1}(s) - \ln\mathscr{P}_t(s). \quad (4)$$

Next, we perform numerical simulation for this proposed model. During the simulation, the number of traders is $N = 500$, the initial fractions of traders leaders are $\theta_1 = 0.02$ and $\theta_2 = 0.04$, respectively, the proportion exponent is $\gamma = 2$, and the propagation rates and the range can be varying. Fig. 2 displays the price series of $L = 2$, $L = 3$ from FRMC system and SSE in the left and shows the corresponding return series, respectively, in the right.

Moreover, the descriptive statistics and Jarque Bera test[36] of return series of SSE, SZSE, and the simulation data which are grouped to four sets by different parameters from the FRMC system as shown in Table I. In the Jarque Bera test, the significance level is 5%, and for the lengths of return series are both the same value 2000, the critical values of them also are same at 5.96. The test returns the logical value $H = 1$ if it rejects the null hypothesis that the return series comes from a normal distribution with unknown mean and variance at the 5% significance level, while $H = 0$ if it cannot. From Table I, we can note that the kurtosis values are all higher than 3 and the JB (Jarque Bera) statistic values are all higher than the critical values 5.96 for the simulation data as well as the historical data, and both of them increase as the

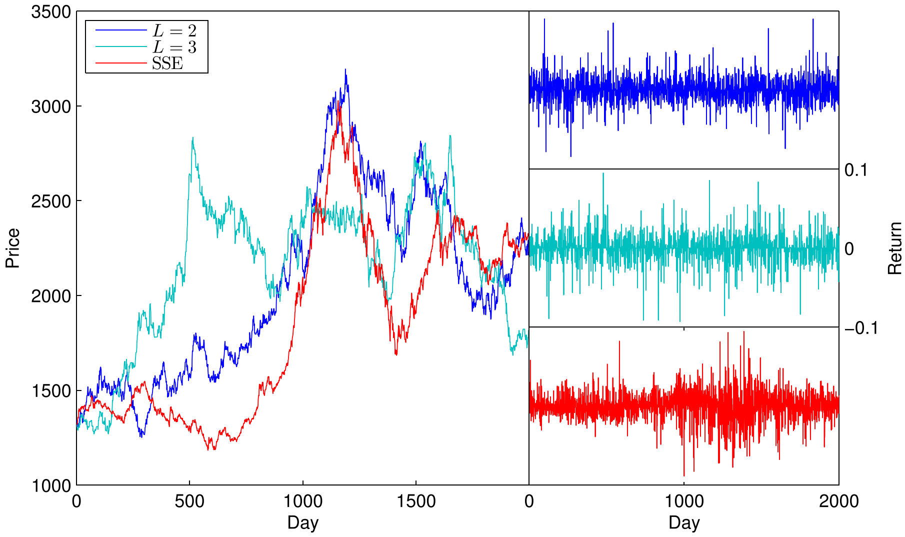

FIG. 2. The price series of $L = 2$, $L = 3$ from FRMC system and SSE in the left, and the corresponding return series, respectively, in the right.

value λ_1 (or λ_2) increases separately, which imply that with the value λ_1 (or λ_2) increasing separately, the fat-tail phenomena would become more significant since these return series are not matching a normal distribution, but more and more far away from a normal distribution. From the above, we would like to study the power-law distribution to obtain the more believable conclusion in Sec. III.

III. POWER-LAW DISTRIBUTIONS ANALYSIS

A power-law is the form taken by a remarkable number of regularities, or "laws," in economics and finance. It is a relation of the type $Y = kX^{\alpha}$, where Y and X are variables of interest, α is called the power-law exponent, and k is typically an unremarkable constant. Of course, the fit may be only approximate in practice and may hold only over a bounded range.[37] Many works have been indicated that stock market returns follow a power-law distribution at the tail part,[4,24,37–41] and moreover it seems that stock market crashes are not outliers to the power-law,[41] which means the tail distribution may give us both insights about the "normaltime" behavior of the market (inside the tails), and also the most extreme events. Hence, power-laws appear to describe histograms of relevant financial fluctuations, such as fluctuations in stock price, trading volume, and the number of trades, and a theory of power-law distributions in financial market fluctuations have been put forward by Gabaix *et al.*[4] In this section, we apply the power-law

TABLE I. Descriptive statistics and Jarque Bera test of simulation data and historical data.

Data			Descriptive statistics						J-B test	
L	λ_1	λ_2	Mean	Std	Max	Min	Kurtosis	Skewness	JB stat.	H
2	1.84	1.4	9.76×10^{-5}	0.015	0.095	−0.091	4.884	0.166	219.63	1
2	1.88	1.4	-2.29×10^{-4}	0.018	0.087	−0.097	5.018	−0.128	364.10	1
2	1.92	1.4	-4.43×10^{-4}	0.019	0.085	−0.089	5.367	−0.094	455.68	1
2	1.96	1.4	-8.99×10^{-4}	0.020	0.092	−0.098	6.126	−0.102	643.87	1
2	2.00	1.4	5.74×10^{-5}	0.021	0.091	−0.087	6.420	0.083	851.35	1
2	2.00	0.6	-4.39×10^{-4}	0.015	0.087	−0.097	4.896	−0.003	232.86	1
2	2.00	0.8	2.42×10^{-4}	0.017	0.098	−0.085	4.965	0.074	343.05	1
2	2.00	1.0	4.95×10^{-4}	0.018	0.095	−0.084	5.275	0.042	405.46	1
2	2.00	1.2	-6.41×10^{-4}	0.019	0.091	−0.095	6.116	−0.008	638.06	1
2	2.00	1.4	-5.74×10^{-5}	0.021	0.091	−0.087	6.420	0.083	851.35	1
3	1.50	1.2	2.80×10^{-5}	0.014	0.080	−0.096	4.700	−0.273	179.02	1
3	1.56	1.2	-1.55×10^{-4}	0.016	0.089	−0.092	5.268	−0.139	398.15	1
3	1.62	1.2	-1.05×10^{-4}	0.017	0.098	−0.086	6.205	0.082	699.81	1
3	1.68	1.2	-2.66×10^{-4}	0.019	0.094	−0.082	6.295	0.024	768.04	1
3	1.74	1.2	3.19×10^{-4}	0.022	0.094	−0.091	6.522	0.046	917.52	1
3	1.68	0.3	1.79×10^{-4}	0.016	0.095	−0.096	4.921	−0.035	261.70	1
3	1.68	0.6	-5.30×10^{-5}	0.017	0.086	−0.096	5.331	−0.166	429.59	1
3	1.68	0.9	-7.64×10^{-5}	0.018	0.096	−0.091	6.186	0.089	680.05	1
3	1.68	1.2	2.66×10^{-4}	0.019	0.094	−0.082	6.295	0.024	766.04	1
3	1.68	1.5	4.52×10^{-4}	0.020	0.093	−0.092	6.347	0.050	814.22	1
SSE			5.70×10^{-4}	0.018	0.095	−0.088	6.089	−0.144	602.32	1
SZSE			9.71×10^{-4}	0.020	0.096	−0.093	5.385	−0.193	486.44	1

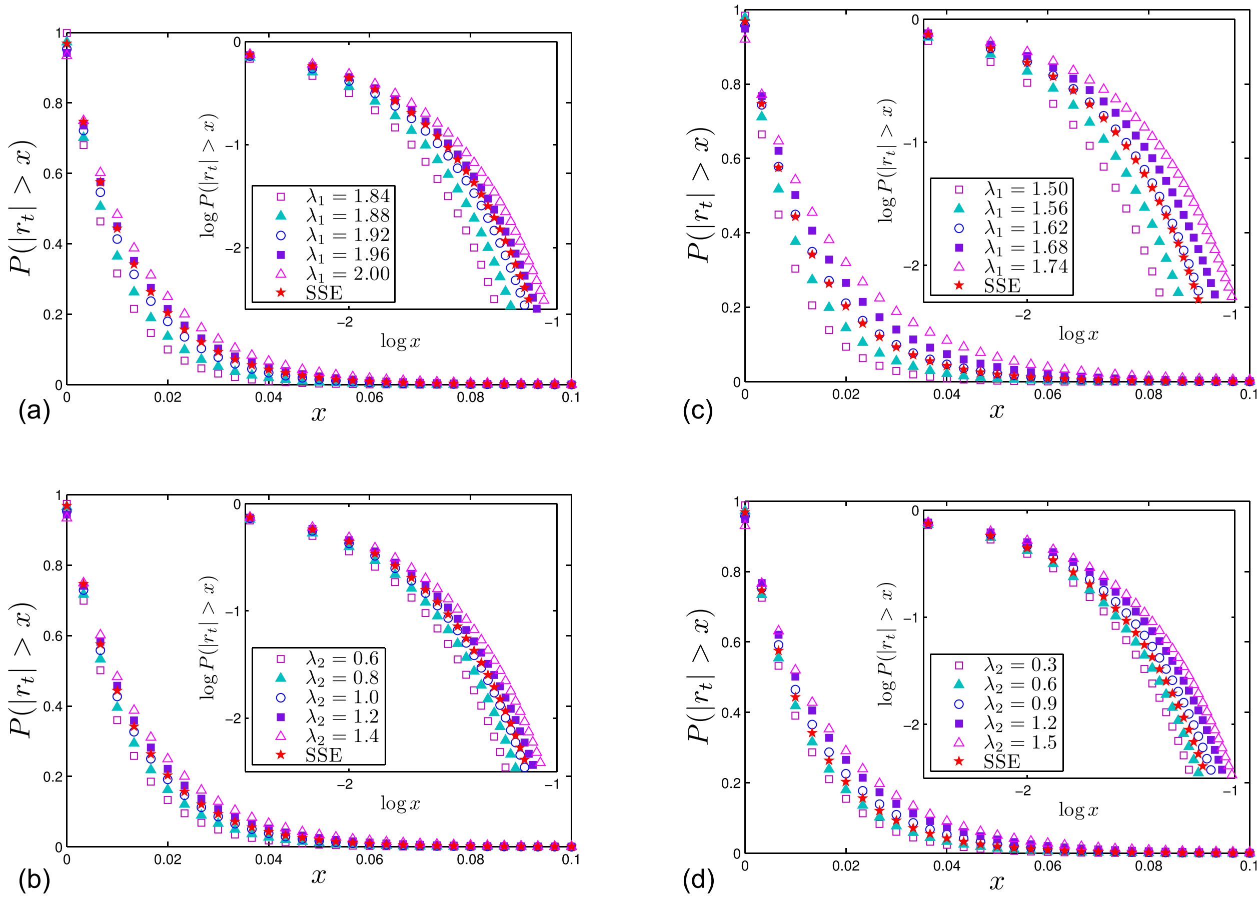

FIG. 3. Plots and log-log plots of cumulative distributions of return series for the simulation data and the historical data.

distribution analysis to compare the fat-tail phenomenon of return series of SSE, SZSE, and the simulation data form FRMC system. According to Sec. II, r_t is the return of a given stock or the proposed financial model, and from Ref. 4, the probability that a return has an absolute value larger than x is found empirically to be

$$P(|r_t| > x) \sim x^{-\alpha}, \tag{5}$$

with the exponent $\alpha \approx 3$. Hence, it is also called the "inverse cubic law."

Fig. 3 exhibits the power-law distributions and the log-log plot of cumulative distributions of normalized return series of SSE, the simulation data from the FRMC system, respectively. The simulation data from the FRMC system are classified to four sets by different parameters: (i) The propagation rate of weak attitudes λ_1 is changed with the same range $L = 2$ and propagation rate of strong attitudes $\lambda_2 = 1.4$. (ii) The propagation rate of strong attitudes λ_2 is changed with the same values $L = 2$ and $\lambda_1 = 2.0$. (iii) The propagation rate of weak attitudes λ_1 is changed with the same values $L = 3$ and $\lambda_2 = 1.2$. (iv) The propagation rate of strong attitudes λ_2 is changed with the same values $L = 3$ and $\lambda_1 = 1.68$. These four sets are displayed in Figs. 3(a)–3(d), respectively.

From these plots of cumulative distributions of returns, we note that for all the simulation data, the corresponding tracks are similar with the track of return series of SSE (the red stars in the plots), which suggests that the simulation data and the historical data both have the fat-tail phenomena. Also, these log-log plots of cumulative distributions show that the track with larger value λ_1 (or λ_2) lies above the one with smaller values. It indicates that when the value λ_1 (or λ_2) increases separately, the power-law exponent α increases, which means that the fat-tail phenomena becomes more significant as the value λ_1 (or λ_2) increases separately. Because the propagation rate of weak attitudes λ_1 and the propagation rate of strong attitudes λ_2 of the FRMC system relate to the number of weak attitude investors and the number of strong attitude investors in the market, as λ_1 (or λ_2) increases separately, the density of investors in the market will increases, and then the market will become more swarming that the investors are more likely to group together, following the large fluctuations of the stock prices as a result. Moreover, for the better simulation, the values of λ_1 and λ_2 with range $L = 2$ are larger than those with the range $L = 3$. Since the range L of the FRMC system relates to the number of neighbors in the market, as L increases, the propagation of investment information becomes larger. In order to closer to the real market by simulation, when we choose larger range L, we should decrease the values of two propagation rates.

Table II shows the power-law exponents α (see Eq. (5)) of return series of SSE, SZSE, and the simulation data from the FRMC system by fitting in two regions (α_1 is fitted in the last 10% data of each series, and α_2 is fitted in the last 7.5% data of each series). We can see that as the value λ_1 (or λ_2)

TABLE II. Power-law exponents of simulation data and historical data.

$L=2$ and $\lambda_2=1.4$			$L=2$ and $\lambda_1=2.0$		
λ_1	α_1	α_2	λ_2	α_1	α_2
1.84	2.7469	2.9187	0.6	2.8138	3.0463
1.88	2.8557	2.9827	0.8	2.9000	3.0744
1.92	2.9377	3.0887	1.0	2.9688	3.1109
1.96	3.0837	3.2146	1.2	3.0776	3.1958
2.00	3.1229	3.2453	1.4	3.1229	3.2453
$L=3$ and $\lambda_2=1.2$			$L=3$ and $\lambda_1=1.68$		
λ_1	α_1	α_2	Λ_2	α_1	α_2
1.50	2.7279	2.9386	0.3	2.8780	3.0146
1.56	2.8805	3.0623	0.6	2.9740	3.1298
1.62	3.0620	3.2017	0.9	3.0911	3.2258
1.68	3.1325	3.2493	1.2	3.1325	3.2493
1.74	3.2165	3.2822	1.5	3.1948	3.2622
	α_1	α_2		α_1	α_2
SSE	3.0389	3.1511	SZSE	3.0137	3.1601

increases separately, the power-law exponent increases, which can also obtained from Fig. 3. Additionally, for both the simulation data, SSE, and SZSE, the corresponding power-law exponents are nearly to 3 ± 0.3, which means that the simulation data and the historical data are all asymptotically according with "inverse cubic law."

IV. AUTOCORRELATION ANALYSIS

Volatility clustering feature is noted by Mandelbrot in 1963 that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes."[42] Many financial market time series show the volatility clustering feature.[43–46] In this section, we apply the autocorrelation analysis to study the volatility clustering feature of returns for SSE and the simulation data from the FRMC system. The autocorrelation of a random process describes the correlation between the values of the process at different times, as a function of the two times or of the time lag. Since the autocorrelation is also the cross-correlation of a signal with itself, we transform the definition of cross-correlation function given by Ref. 47 to the definition of autocorrelation function in this section as follows:

$$A(r_t,k)=\frac{\sum_{i=k+1}^{T} r_i r_{i-k}}{\sum_{i=1}^{T} r_i^2},\quad k=0,1,\ldots,T-1,\qquad(6)$$

where r_t is the return series of simulation data or historical data and its length is T, while k represents the time lag of autocorrelation function.

To study the volatility clustering feature of returns, at first, we consider the absolute return series with different exponents q, labeled as $|r_t|^q$, that is, we replace the return series r_t with $|r_t|^q$ in Eq. (6). We compare the autocorrelation function of return series with those of absolute return

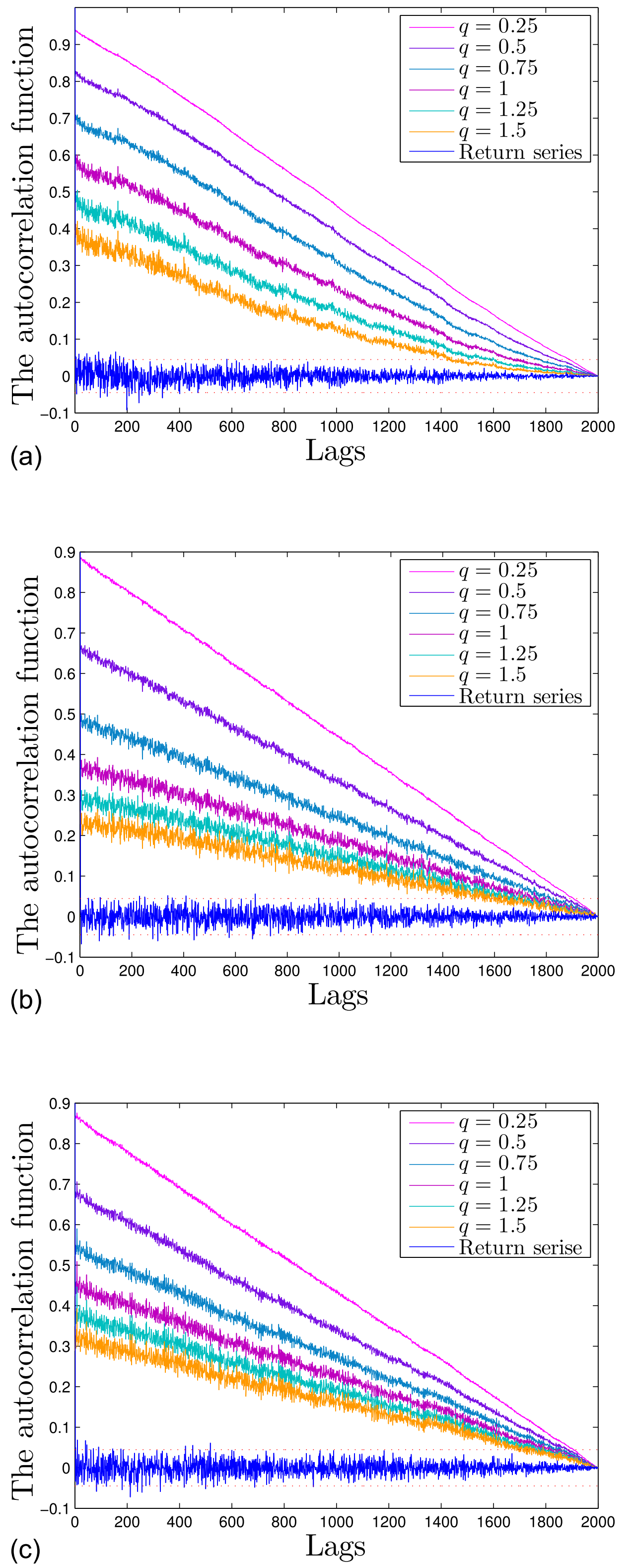

FIG. 4. Autocorrelation functions of return series and corresponding absolute returns with different exponents $|r_t|^q$ for SSE (a), the simulation data of the proposed model with range $L=2$ (b) and $L=3$ (c).

series with different exponents 0.25, 0.5, 0.75, 1.00, 1.25, and 1.50 in Fig. 4. The empirical results for SSE and the simulation data of the proposed model with range $L=2$ and $L=3$ are exhibited in (a), (b), and (c) of Fig. 4, respectively. We can note that the trends of autocorrelation functions of real market returns (Fig. 4(a)) are quite similar to those of returns of the proposed model (Figs. 4(b) and 4(c)). For the absolute return series with different exponents, the autocorrelation functions display a positive, significant, and slowly decaying with the time lag ranging from zero to maximum. This suggests that the dependence property of absolute return series is significantly strong for the small time lags, and the dependence declines when the time lag increases. Also, for different exponent values of q, the autocorrelation function shows the distinct decaying characteristic. It decreases prominently when the exponent q increases, where the largest value corresponds to $|r_t|^{0.25}$ and the smallest one corresponds to $|r_t|^{1.5}$. Besides, the series between the red dashed lines is the return series and the red dashed lines stand for the critical value of 95% confidence interval. We find that the autocorrelation functions of return series have not shown the above similar behavior, which exhibits that the dependence property is not obvious for the return series. From the above autocorrelation analysis, the empirical results display that the volatilities of absolute returns have the significant autocorrelation, and there exists evident volatility clustering feature in the absolute return series for the simulation data and the SSE.

From the above empirical research, the absolute return series exhibit the significant autocorrelation. Next, we define other two type autocorrelation functions. One is named after the q-order autocorrelation function as follows:

$$A^q(|r_t|,k) = \left(\frac{\sum_{i=k+1}^{T}|r_i||r_{i-k}|}{\sum_{i=1}^{T}|r_i|^2}\right)^q, \quad k=0,1,\ldots,T-1, \tag{7}$$

and the other one called the multi-autocorrelation function is defined as

$$A_q(|r_t|,k) = \left(\frac{\sum_{i=k+1}^{T}|r_i|^q|r_{i-k}|^q}{\sum_{i=1}^{T}|r_i|^{2q}}\right)^{\frac{1}{q}}, \quad k=0,1,\ldots,T-1, \tag{8}$$

where $|r_t|$ is the absolute return series of simulation data or historical data.

We compare the q-order autocorrelation functions for different exponent values of 1/4, 1/3, 1/2, 1, 2, 3, and 4 in Fig. 5, and the multi-autocorrelation functions for different exponent values of 0.25 0.5, 0.75, 1.00, 1.25, and 1.50 in Fig. 6. The corresponding empirical results for the SSE and the simulation data with range $L=2$ and $L=3$ are exhibited in (a), (b), and (c) of Figs. 5 and 6, respectively. These figures show that the trends of these two type autocorrelation functions of real market returns (Figs. 5(a) and 6(a)) are also quite similar to those of returns of the financial model (Figs. 5(b), 5(c), 6(b), and 6(c)). In Fig. 5, when exponent $q>1$, the q-order autocorrelation functions display a exponential decaying with the time lag ranging from zero to maximum, when $q<1$ they exhibit a logarithmic decaying, and when $q=1$ they show a linear decaying. In fact, q-order autocorrelation function is equal to the autocorrelation function of absolute return series when $q=1$. Furthermore, the curve of q-order autocorrelation function with the smaller exponent q lies above the one with the larger q, where the largest one corresponds to $q=1/4$ and the smallest one corresponds to $q=4$. While in Fig. 6, it is found that, the trends of multi-autocorrelation function can clearly recognize two phases as the time lag increases. Within the first phase, compared with $q>1$, the multi-autocorrelation functions of $q<1$ are relatively close to that of $q=1$, which means the intervals among them are relatively small. In the second phase,

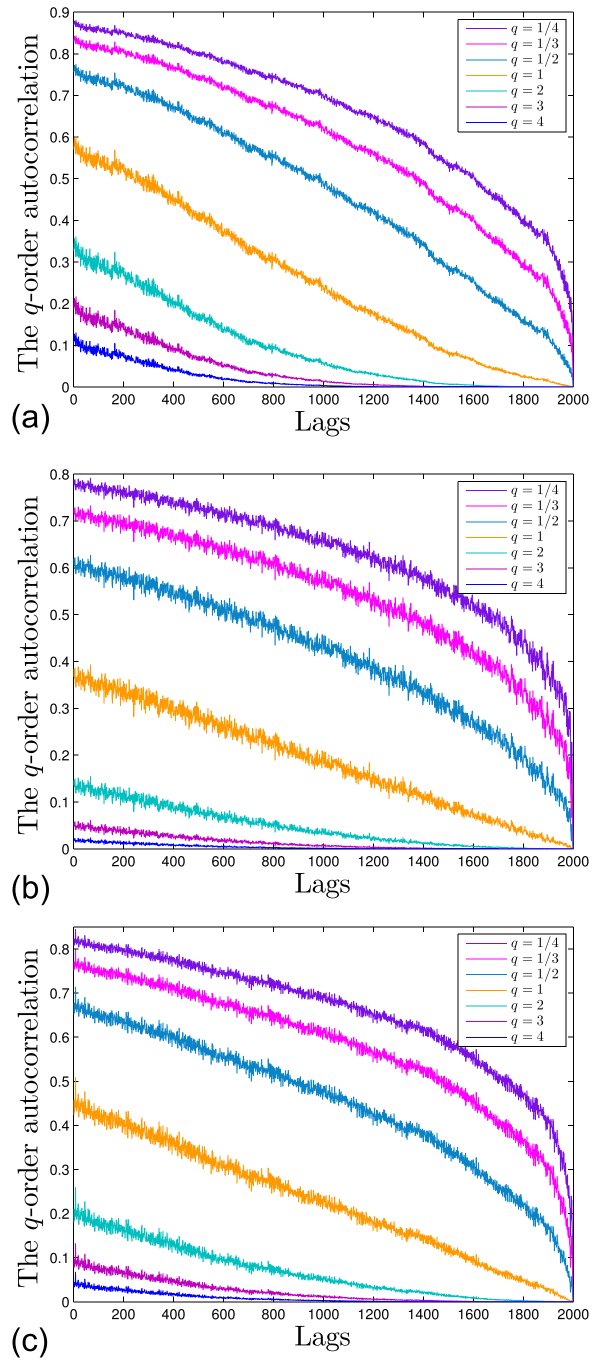

FIG. 5. q-order autocorrelation functions $A^q(|r_t|,k)$ for SSE (a), the simulation data of the proposed model with range $L=2$ (b) and $L=3$ (c).

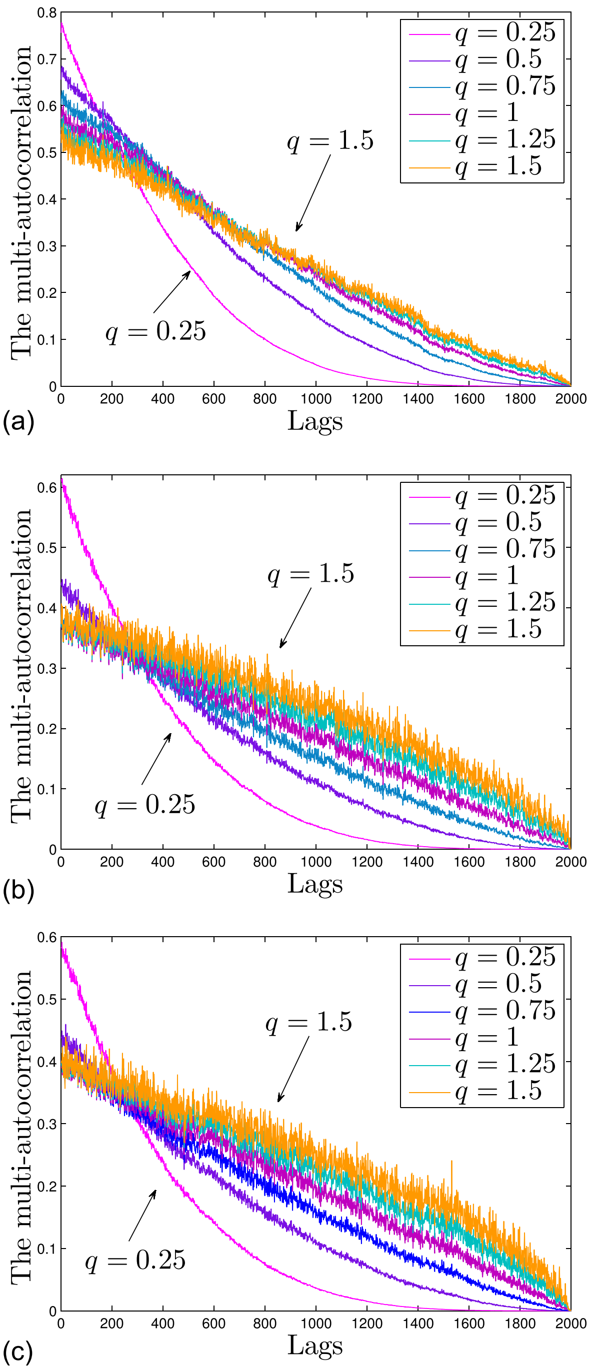

FIG. 6. Multi-autocorrelation functions $A_q(|r_t|, k)$ for SSE (a), the simulation data of the proposed model with range $L = 2$ (b) and $L = 3$ (c).

however, there are clear separation among these curves, and as the q increases, the multi-autocorrelation function becomes more smaller. Moreover, we can observe that the bounds of these two phases for the simulation data and the historical data are almost between 200 and 400 lags. In fact, the autocorrelation function of absolute return series is not only equal to q-order autocorrelation function but also to multi-autocorrelation function when $q = 1$. From the above modified autocorrelation analysis, they indicate that there exists the similar volatility clustering feature in q-order autocorrelation function and multi-autocorrelation for the simulation data of the proposed model and the SSE.

V. CONCLUSION

In the present work, we present a stock price model based on the finite-range multitype contact dynamic system and investigate the nonlinear behavior of the proposed price dynamics. The price evolution of the financial model possess the following features, investment attitudes of various levels of strength, and dispersal of two types of attitudes at different rates with different interaction mechanism. In the details, we suppose that investment attitudes are of different strength levels, and the propagation of different attitudes follows a finite-range multitype contact process. The dispersal of different attitudes is through the interaction between finite-range neighbors at different rates, which is measured by the propagation parameters λ_1 and λ_2. Then, we investigate and analyze the statistical behaviors of normalized returns of the price model by some analysis methods, including power-law distribution analysis and autocorrelation analysis. In autocorrelation analysis, we develop two types of autocorrelation which we called q-order autocorrelation function and multi-autocorrelation function. Moreover, we choose the daily returns of SSE Composite Index and SZSE Component Index as the historical data, and the simulation data which are derived from the finite-range multitype contact process with different parameters λ_1 and λ_2 are classified by $L = 2$ and $L = 3$. The comparisons of return behaviors between the historical data and the simulation data are similar.

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