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Circularly polarized wave propagation in magnetofluid dynamics for relativistic electron-positron plasmas

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The dispersion relation for circularly polarized electromagnetic waves propagating in the direction of an external magnetic field in a relativistic electron-positron plasma with arbitrary constant drift velocities is obtained for constant temperature in the homentropic regime. This result is an exact solution of the nonlinear magnetofluid unification field formalism introduced by S. Mahajan [Phys. Rev. Lett. **90**, 035001 (2003)], where the electromagnetic and fluid fields are coupled through the relativistic enthalpy density. The behavior of electromagnetic and Alfvén branches of the dispersion relation are discussed for different temperatures. © *2009 American Institute of Physics*. [doi:10.1063/1.3272667]

I. INTRODUCTION

Relativistic electron-positron plasmas have received much attention because they are relevant in several environments, either of astrophysical or laboratory nature. Examples of this are accretion disks,[1–3] models of early universe,[4,5] ultraintense lasers,[6] laboratory and tokamak plasmas,[7,8] pulsar magnetospheres,[9,10] or hypothetical quark stars.[11] Several effects in these plasmas relate to wave propagation, such as the proposed pulsar radio emission processes,[12] bulk acceleration of relativistic jets,[13] quasar relativistic jets,[14] or electron-positron pair annihilation into one-photon in the presence of a strong magnetic field.[15]

In several of the environments mentioned above, relativistic effects and temperature play an important role; thus it is fundamental to understand wave propagation modes in relativistic plasmas with temperature. In this paper we will focus in the particular case of circularly polarized electromagnetic waves, which, although simple, allows us to study in detail the effect of relativistic temperatures on wave propagation in relativistic hot plasmas.

Exact solutions for the plasma equations can be found for cold nonrelativistic plasmas. For instance, circularly polarized Alfvén electromagnetic waves propagating parallel to an external magnetic field are an exact solution of the magnetohydrodynamic equations even when the amplitude is large.[16] Also, a circularly polarized wave in a multiple ion species plasma with drifts is a finite amplitude solution of the cold plasma model.[17] The nonlinear propagation of circularly polarized electromagnetic waves in unmagnetized electron-positron-ion plasmas has also been studied in the cold[18] and relativistically hot[19] cases, showing the existence of stable localized structures.

Here we propose an approach which permits to find an exact solution for the propagation modes in a relativistic electron-positron plasma with constant, arbitrary temperature, within the context of a fluid theory. This can be done by basing our approach on the magnetofluid field unification formalism in Ref. 20.

In this unification approach, the whole plasma is treated as a unique field where the electromagnetic field is coupled with the charged fluid field through a function that carries statistical information of the system. This leads to a simple and elegant way to describe relativistic plasmas. Using this approach, we derive the dispersion relation for circularly polarized waves of arbitrary amplitude propagating along a constant magnetic field, for arbitrary temperatures.

The paper is organized as follows. In Sec. II, a brief summary of the magnetofluid unification approach is presented. Then, in Sec. III, the dispersion relation for circularly polarized waves is derived. In Sec. IV, it is solved numerically and several features are discussed. Finally, in Sec. V, results are summarized and conclusions are outlined.

II. MAGNETOFLUID UNIFICATION

Usually, the interaction of particles with electromagnetic fields is described by introducing a "minimal coupling" in the momentum, $\mathbf{p}\rightarrow\mathbf{p}-e\mathbf{A}/c$, where $\mathbf{A}$ is the vector potential. This leads to an energy-momentum conservation equation. This equation and Maxwell equations, describe the basic dynamics of charged relativistic particles in plasmas.

In Ref. 20 it was suggested that the coupling of a relativistic charged fluid, at a given temperature, with the electromagnetic field can be described by an antisymmetric field tensor that contains the statistical information of the system. Thus, the set of equations to describe the plasma dynamics in the homentropic regime, for species j, is the Maxwell equations, the continuity equation, and the equation

$$q_j U_{j_\mu} M_j^{\nu\mu}=0. \tag{1}$$

The field $M_j^{\mu\nu}$ is the tensor that couples the electromagnetic and the fluid fields. This tensor is defined as $M_j^{\mu\nu}=F^{\mu\nu}+(m_jc^2/q_j)S_j^{\mu\nu}$, with $F^{\mu\nu}$ as the electromagnetic tensor, q_j is the charge, and where it is introduced the new antisymmetric tensor $S_j^{\mu\nu}=\partial^\mu(f_jU_j^\nu)-\partial^\nu(f_jU_j^\mu)$ representing the relativistic

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thermal fluid. Here, $U_j^{\mu} \to (\gamma_j, \gamma_j \mathbf{v}_j / c)$ is the four-velocity of the fluid, $\mathbf{v}_j$ is the species velocity, $\gamma_j = (1 - v_j^2/c^2)^{-1/2}$, $v_j^2 = \mathbf{v}_j \cdot \mathbf{v}_j$, and c is the speed of light.

The parameter f is a function of the temperature T. An explicit form for $f(T_j)$ can be obtained by assuming a given statistical behavior for the gas. For instance, if the system follows a Maxwell–Jüttner equilibrium distribution,[19,20]

$$f(T_j) = \frac{K_3\left(\dfrac{m_j c^2}{k_B T_j}\right)}{K_2\left(\dfrac{m_j c^2}{k_B T_j}\right)} \equiv f_j, \tag{2}$$

where K_2 and K_3 are the modified Bessel functions of order two and three, respectively, and k_B is the Boltzmann constant. However, within the treatment carried out in this paper, no explicit description for the function f is needed, and all the analytical and numerical results that follow [except for Eq. (13)] are independent of the underlying particle distribution function, as it should be for any fluid theory.

The spacelike components of Eq. (1) yield the plasma motion equation,

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_j \cdot \nabla\right)(f_j \gamma_j \mathbf{v}_j) = \frac{q_j}{m_j}\left(\mathbf{E} + \frac{\mathbf{v}_j}{c} \times \mathbf{B}\right) - \frac{1}{\bar{n}_j m_j \gamma_j} \nabla p_j. \tag{3}$$

An alternative form of this equation, in terms of the enthalpy density h, can be found in Ref. 21.

The formalism of magnetofluid unification in Ref. 20 is very general and allows us to study the dynamics of a charged fluid (plasma) in an electromagnetic field in a unified way, treating them as a single field. It is a basis for further theoretical developments, like the study of the interaction of plasmas with non-Abelian fields.[22] On the other hand, it also provides a general framework to study various plasma physics phenomena, taking into account relativistic temperature effects in a consistent way. In particular, the effect of relativistic temperatures on wave propagation in plasmas can be studied systematically. For the sake of simplicity, in this paper we will consider an equal mass electron-positron plasma, where a circularly polarized electromagnetic wave propagates along a constant background magnetic field.

III. CIRCULARLY POLARIZED WAVES

We consider a circularly polarized wave that propagates along the z axis, whose electric and magnetic fields are given by

$$\mathbf{E}(z,t) = E[\sin(kz - \omega t)\hat{x} - \cos(kz - \omega t)\hat{y}], \tag{4}$$

$$\mathbf{B}(z,t) = B[\cos(kz - \omega t)\hat{x} + \sin(kz - \omega t)\hat{y}], \tag{5}$$

respectively. In addition, there is a background uniform magnetic field $\mathbf{B}_0 = B_0 \hat{z}$. Electrons and positrons have equal constant drift velocities $\mathbf{v}_0 = v_0 \hat{z}$ and equal constant densities $n_p = n_e = n$. We will also denote their mass by $m_p = m_e = m$ and their charge by $q_p = -q_e = e$.

In principle, we assume that electrons and positrons have constant but different temperatures, so that $f_e \neq f_p$. It turns out, as we will see below, that the purely transverse circularly polarized wave is an exact solution of the field equations, which is consistent with the assumption of no pressure fluctuations, that is, no fluctuations in f.

The particle velocities induced by the wave field are purely transverse, hence, the amplitude of the circularly polarized velocity is proportional to the constant amplitude of the circularly polarized vector potential field (which can be easily shown in the Lorentz gauge, for instance), and the relativistic factor,

$$\gamma_j = (1 - \mathbf{v}_j^2/c^2 - \mathbf{v}_0^2/c^2)^{-1/2}, \tag{6}$$

is constant for both species.

With all these considerations, Eq. (3) yields

$$f_j \gamma_j \frac{\partial \mathbf{v}_j}{\partial t} = - f_j \gamma_j (\mathbf{v}_j + \mathbf{v}_0) \cdot \nabla \mathbf{v}_j + \frac{q_j}{m}\left(\mathbf{E} + \frac{(\mathbf{v}_j + \mathbf{v}_0)}{c} \times (\mathbf{B} + \mathbf{B}_0)\right). \tag{7}$$

All the circularly polarized quantities have a space-time dependence of the form $e^{ikz - i\omega t}$. We can write the velocity in the polarization representation as $v_{xj} + i v_{yj} = v_{j\perp} e^{i(kz - \omega t)}$, the electric field as $E_x + iE_y = E_{\perp} e^{i(kz - \omega t)}$, and the magnetic field as $B_x + iB_y = B_{\perp} e^{i(kz - \omega t)}$. Thus, Eq. (7) yields

$$v_{j\perp} = i \frac{q_j}{m}\left(\frac{E_{\perp} + iB_{\perp} v_0/c}{f_j \gamma_j \omega' - \Omega_{cj}}\right), \tag{8}$$

where $\Omega_{cj} = q_j B_0 / mc$ and $\omega' = \omega - k v_0$ are the gyrofrequency for both species and the Doppler shifted frequency, respectively.

From Maxwell equations it can be shown that

$$E_{\perp} = - i \frac{\omega}{ck} B_{\perp}, \quad \left(-k^2 + \frac{\omega^2}{c^2}\right) E_{\perp} = \frac{4\pi}{c^2} \frac{\partial J_{\perp}}{\partial t}, \tag{9}$$

where $J_{\perp} = J_x + iJ_y = \Sigma_j n q_j v_{j\perp}$ is the transverse current, and n is the density measured in the laboratory frame.

Using Eq. (9), Eq. (8) can be rewritten as

$$v_{j\perp} = \frac{\omega'}{f_j \gamma_j \omega' - \Omega_{cj}} \frac{q_j B_{\perp}}{mck}. \tag{10}$$

Finally, this leads to the following dispersion relation for an electron-positron plasma

$$\omega^2 - c^2 k^2 = \sum_{j=e,p} \omega_p^2 \left(\frac{\omega'}{f_j \gamma_j \omega' - \Omega_{cj}}\right), \tag{11}$$

with $\omega_p = \sqrt{4\pi n e^2 / m}$ as the electron plasma frequency. This dispersion relation is an exact solution of the plasma equations, for finite amplitude circularly polarized propagating waves along the magnetic field in a relativistic plasma with temperature.

It is interesting to note that Eq. (11) differs from the cold plasma case[23] only through a factor f_j. We can understand this as follows. For single particle motion, it is possible to convert a nonrelativistic result into a relativistic one by changing $m_j \to \gamma_j m_j$. This is possible because mass is in-

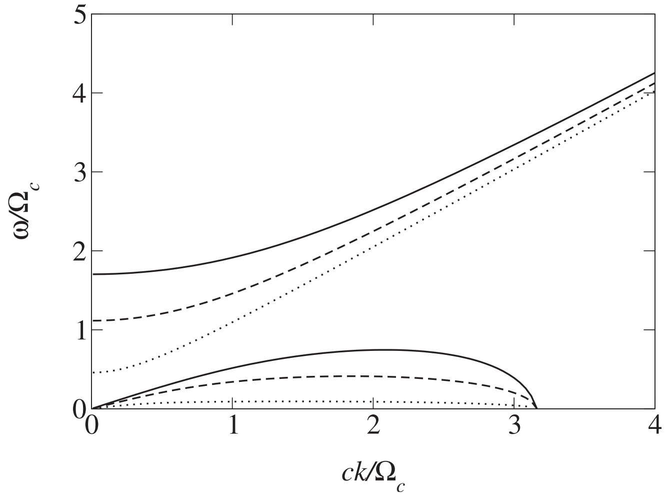

FIG. 1. Electromagnetic and Alfvén branches for the general dispersion relation, Eq. (12), for $a=1$, $\alpha=0.1$. Solid line: cold plasma case ($f=1$); dashed line: $f=2$; and dotted line: $f=10$.

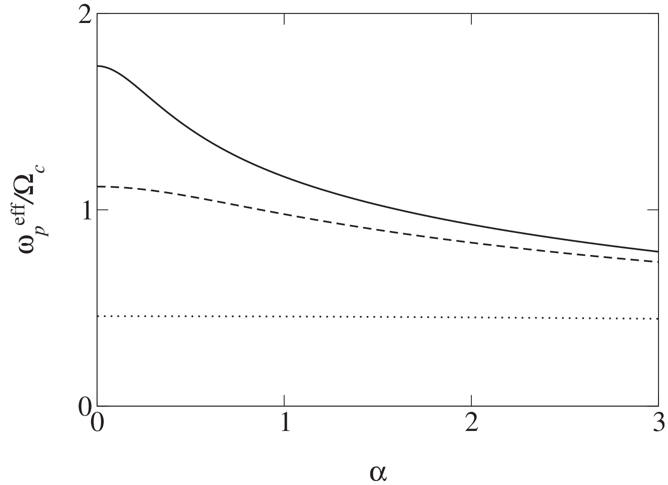

FIG. 2. Effective plasma frequency ω_p^{eff} as a function of wave amplitude α, for various temperatures. Solid line: cold plasma case ($f=1$); dashed line: $f=2$; and dotted line: $f=10$.

volved in the momentum equation. However, this simple replacement is not possible for a fluid, since velocity is related nonlinearly to momentum through γ_j. Therefore, the average on momentum $\langle \mathbf{p}_j \rangle$ is not equivalent to $\langle \gamma_j \rangle \langle m_j \mathbf{v}_j \rangle$. We can think, though, that there is some proportionality factor between both quantities, say f_j, such that $\langle \mathbf{p}_j \rangle = f_j \langle \gamma_j \rangle \langle m_j \mathbf{v}_j \rangle$. Thus, it would be possible to convert a nonrelativistic fluid force equation into a relativistic one with the same prescription as for single particles, $\langle m_j \mathbf{v}_j \rangle \rightarrow \langle \mathbf{p}_j \rangle$, but, in order to take into account the statistics, $m_j \rightarrow f_j \gamma_j m_j$. This is exactly what is needed to go from the cold fluid dispersion relation [$f_j=\gamma_j=1$ in Eq. (11)], to Eq. (11).

As an example, let us assume that the temperature of electrons and positrons is the same and that there is no drift. In this case, $f_p = f_e \equiv f$, and Eq. (11) becomes

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega}\left(\frac{1}{f\gamma_p\omega - \Omega_c} + \frac{1}{f\gamma_e\omega + \Omega_c}\right), \tag{12}$$

where $\Omega_c = eB_0/mc$ is the positron gyrofrequency.

For $f=1$, the dispersion relation for a cold relativistic plasma is recovered.[21,23] The same result as in Ref. 23 can be obtained from a kinetic approach,[24] in the limit where the velocity distribution is a Dirac delta [see, e.g., Eq. (23) in Ref. 24].

The opposite limit corresponds to $k_BT \gg mc^2$. If f_j is given by Eq. (2), then $f(T) \approx 4k_BT/mc^2 \gg 1$. Thus, Eq. (12) yields the dispersion relation for a very hot relativistic magnetized electron-positron plasma,

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega}\left(\frac{1}{\gamma_p\dfrac{4k_BT}{mc^2}\omega - \Omega_c} + \frac{1}{\gamma_e\dfrac{4k_BT}{mc^2}\omega + \Omega_c}\right). \tag{13}$$

In particular, we notice that when the temperature is very high, the transverse mode becomes a light mode $c^2k^2=\omega^2$, consistent with the relativistic decrease in the effective plasma frequency. Also, light wave modes are the solution of Eq. (13) in the ultrarelativistic limit $1/\gamma_j \rightarrow 0$. Notice that, although Eq. (13) was obtained for a specific functional form of f with temperature, as given by Eq. (2), the asymptotic behavior of transverse modes described here is expected to be independent of the particle distribution function, consistent with the fact that we are treating the plasma as a fluid.

IV. ANALYSIS OF THE DISPERSION RELATION

In order to study the dispersion relation [Eq. (12)] we will normalize all frequencies to the positron gyrofrequency Ω_c, and velocities to the speed of light c. It is convenient to define two adimensional parameters, related to the plasma frequency and to the wave amplitude,

$$a = \frac{\omega_p^2}{\Omega_c^2}, \quad \alpha = \frac{e|A|}{mc^2} = \frac{e|E_\perp|}{mc\omega} = \frac{e|B_\perp|}{mc^2k}, \tag{14}$$

where $\mathbf{A}$ is the vector potential of the wave. Notice that α corresponds to the particle transverse momentum due to the wave.

In order to plot the dispersion relation, Eqs. (6), (10), and (11) are solved simultaneously for γ_j, $v_{j\perp}$, and ω, for a given k, as outlined in Ref. 24. In Fig. 1 the dispersion relation (12) is plotted for various values of f, and for $a=1$, $\alpha=0.1$. There are two branches: an electromagnetic branch, with a lower cutoff at the effective plasma frequency, and an Alfvén branch, which has an upper cutoff in wave number, and an upper cutoff in frequency.

The black dotted line corresponds to the relativistic cold plasma, $f=1$. It can be seen that, as mentioned in Sec. III, when the temperature increases the effective plasma frequency decreases, and the electromagnetic wave becomes a light wave. This effect can be better appreciated in Fig. 2, where the effective plasma frequency ω_p^{eff} (given by the lower frequency cutoff for the electromagnetic branch in Fig. 1) is plotted for the same values of f used in Fig. 1. ω_p^{eff} also decreases due to the relativistic effect on the mass, and thus we plot it as a function of α as well. However, for large enough temperatures, the variation of the plasma frequency with wave amplitude is negligible.

Regarding the Alfvén branch, Fig. 1 shows that it starts at the origin following the usual linear dispersion relation, $\omega = v_A k$. In this region, γ_p, $\gamma_e \simeq 1$. Then, the Alfvén velocity can be obtained from Eq. (12) by first rewriting it in the form,

$$c^2k^2 = \omega^2 - 2f\omega_p^2 \frac{\omega^2}{(f\omega)^2 - \Omega_c^2}.$$

If $\omega f \ll \Omega_c$, which is always satisfied for low enough frequencies, we find that the Alfvén velocity is given by

$$v_A = \frac{c}{\sqrt{1 + 2f\omega_p^2/\Omega_c^2}}. \tag{15}$$

As wave number increases along the Alfvén branch, there is an upper frequency cutoff ω_{crit} given by

$$\frac{\omega_{\mathrm{crit}}}{\Omega_c} = \frac{1}{f}\left[1 + \left(\frac{\alpha}{f}\right)^{2/3}\right]^{-3/2}. \tag{16}$$

Thus, when temperature increases, Alfvén waves are eventually confined to a very narrow frequency band. This can also be seen in Fig. 1.

Another interesting feature of the Alfvén branch is the existence of a maximum value for the wave number, $k_{\max}$. This upper cutoff occurs because one species (positrons, in this case) becomes ultrarelativistic, essentially moving with the speed of light in the wave field, thus, $\gamma_p = \infty$. Electrons do not resonate with the wave, so γ_e is finite. In order to obtain an analytic expression for the wave number cutoff for the Alfvén branch, let us first notice that, from Eq. (10), it follows that

$$\frac{\omega}{f\gamma_j\omega - \Omega_{cj}} = \pm \frac{1}{\alpha}\frac{v_{\perp j}}{c}, \tag{17}$$

where the plus (minus) sign corresponds to positrons (electrons), so the dispersion relation [Eq. (12)] can be written,

$$c^2k^2 = \omega^2 - \frac{\omega_p^2}{\alpha c}(v_{\perp p} - v_{\perp e}). \tag{18}$$

The cutoff occurs for $\omega = 0$, and in this limit $v_{\perp p} \simeq -c$ [notice that the left-hand term in Eq. (17) is negative along the Alfvén branch for all values of k], whereas $v_{\perp e}$ becomes negligible. Thus, it follows that the maximum wave number is given by

$$k_{\max} = \frac{\omega_p}{c\sqrt{\alpha}}. \tag{19}$$

This is consistent with the numerical result in Fig. 1. Notice, in particular, that $k_{\max}$ depends only on wave amplitude, not on temperature. One could argue that, if the transverse particle velocity is ultrarelativistic, then thermal velocities are not relevant.

Finally, it is interesting to calculate the group velocity of the waves. This is shown in Fig. 3, where the group velocity v_g is plotted for both branches of the dispersion relation, taking the same f values as in Fig. 1. The electromagnetic wave branch, as expected, starts with zero group velocity for small wave numbers, and approaches c as k increases. On the other hand, the Alfvén branch starts with v_g equal to the Alfvén velocity (15), and as the wave number increases, the group velocity tends to zero as $\omega \to \omega_{\mathrm{crit}}$, Eq. (16). Then it becomes negative, tending toward minus infinity when $k \to k_{\max}$, where the corresponding transverse velocity of the positrons equals the speed of light.

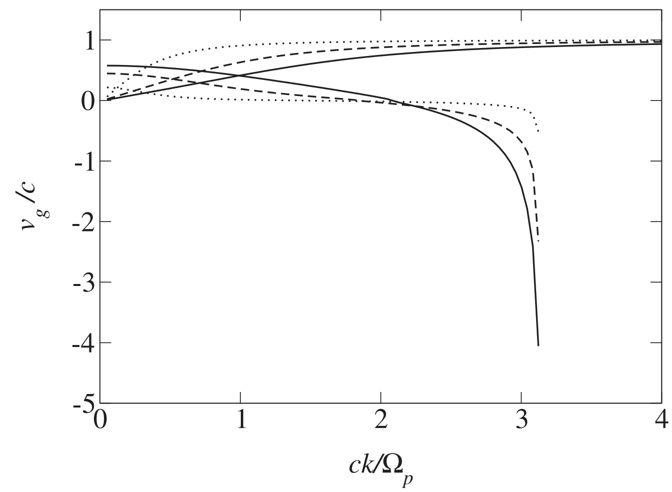

FIG. 3. Group velocity v_g for the waves in Fig. 1 vs wave number, for different temperatures, and for $a=1$, $\alpha=0.1$. Solid line: cold plasma case ($f=1$); dashed line: $f=2$; and dotted line: $f=10$. Curves tending asymptotically to $v_g/c=1$ correspond to the electromagnetic branch in Fig. 1, and curves diverging to $v_g/c=-\infty$ correspond to the Alfvén branch.

V. SUMMARY

In this paper we have used the magnetofluid unification formalism proposed in Ref. 20 to derive the dispersion relation of waves including relativistic temperature effects in a consistent way, within the context of a fluid theory. Thermal information is enclosed in a single function f_j, related to the enthalpy density for species j, which depends only on temperature and particle mass. In particular, we have studied the propagation of circularly polarized waves along a constant background magnetic field, improving on previous results for relativistic cold plasmas.[23,24]

Two branches are present: an electromagnetic and an Alfvén branch. The electromagnetic branch has a lower cutoff at an effective plasma frequency ω_p^{eff}, which decreases with temperature and wave amplitude due to the relativistic increase in effective mass of the particles. As temperature increases, though, the effect of wave amplitude on ω_p^{eff} becomes negligible. When relativistic effects are larger, either due to the Lorentz factors γ_j or the thermal function f_j, the electromagnetic wave becomes a nondispersive light wave.

The Alfvén branch, on the other hand, shows several interesting features. For very small wave numbers it is nondispersive, with an Alfvén velocity v_A which depends only on temperature, not wave amplitude. As wave number increases, a maximum frequency ω_{crit}, corresponding to a critical wave number k_{crit}, is reached. This upper cutoff depends both on wave amplitude and temperature. Thus, as temperature increases, Alfvén waves are confined to an ever narrower frequency bandwidth. At this critical wave number, the group velocity for Alfvén waves becomes zero. For $k > k_{\mathrm{crit}}$, the group velocity is negative, eventually becoming infinite at a certain maximum wave number $k=k_{max}$. This upper frequency cutoff for Alfvén wave numbers depends only on wave amplitude, not temperature. The anomalous behavior of the waves in this regime deserves further attention, and will be studied elsewhere.

We should stress here that we have found an exact solution to the relativistic fluid equations. A critical assumption is the fact that particle velocities are purely transverse with respect to the background magnetic field, which is true for circularly polarized electromagnetic waves of arbitrary amplitude. In particular, this means that no pressure or density fluctuations appear, and that f is constant. We plan to explore the possibility of extending the above analysis to include a pressure tensor,[25] a case in which we do not expect to find exact solutions, but which can certainly be studied numerically.

Other subjects currently under consideration are the study by means of computer simulations, which opens the possibility to consider more complicated propagation modes, the study of instabilities due to species drifts, which have been included in Eq. (11), and parametric decays.[26]

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