AIP | Physics of Plasmas
#1 JOURNAL IN 2014 GOOGLE SCHOLAR METRICS FOR THE PLASMA & FUSION CATEGORY

Stimulated Raman scattering coupled to decay instability in a plasma channel

Ranjeet Singh and V. K. Tripathi

Citation: Physics of Plasmas **19**, 012109 (2012); doi: 10.1063/1.3675851
View online: http://dx.doi.org/10.1063/1.3675851
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/19/1?ver=pdfcov
Published by the AIP Publishing

Stimulated Raman scattering coupled to decay instability in a plasma channel

Ranjeet Singh[a] and V. K. Tripathi
Department of Physics, Indian Institute of Technology Delhi, New Delhi 110016, India

(Received 2 May 2011; accepted 21 November 2011; published online 20 January 2012)

A non local theory of Stimulated Raman scattering (SRS) coupled to decay instability in a plasma channel is developed. The primary Langmuir wave, produced in the Raman backscattering process, decays into a secondary Langmuir wave of longer wavelength and an ion acoustic wave. This diversion of energy, along with linear Landau damping of the primary Langmuir wave, slows down the Raman process. The nonlocal effects cause further reduction in the growth rate. © *2012 American Institute of Physics*. [doi:10.1063/1.3675851]

I. INTRODUCTION

Stimulated Raman scattering (SRS) is an important nonlinear process in high power laser plasma interaction.[1,2] In this process, the ponderomotive force by the pump of frequency ω_0 and its stockes sideband at $\omega_0-\omega$ drives a plasma wave at frequency ω. The latter enhances the scattering of the pump, reinforcing the side band. Many experiments[3–6] have shown that the daughter plasma wave further undergoes Langmuir decay instability (LDI), producing a secondary Langmuir wave and an ion acoustic wave (IAW). This diversion of energy from the plasma wave can play an important role in the nonlinear saturation of SRS.[7–10] Depierreux *et al.*[11] have simultaneously observed secondary IAW and Langmuir wave resonantly produced by LDI, confirming the process linked to SRS. They also observed the secondary decay of the LDI generated Langmuir wave and an ion acoustic wave. In LDI (Refs. 12 and 13) cascade, energy is transferred from the primary Langmuir wave to the secondary Langmuir wave which is not in resonance with SRS. Thus, LDI coupling can act as a sink of energy for the SRS process and can be an efficient nonlinear saturation mechanism for SRS.

Kolber *et al.*[14] have given a one dimensional saturation model for SRS (Ref. 15) via the parametric decay instability threshold and depending on the SRS convective gain factor. Salcedo *et al.*[16] have studied stimulated Raman backscattering and associated nonlinear coupling in space–time of a large number of modes using Eulerian Vlasov–Maxwell code. Sajal and Tripathi[17] have studied SRS-LDI coupling in a magnetized plasma. The Langmuir wave in the presence of magnetic field goes over to lower hybrid wave and the growth rate of stimulated Raman backscattering process is decreased.

In this paper we develop a non local theory of coupled SRS-LDI process in a plasma channel. The plasma channel[18] that could be created by a prepulse, supports electromagnetic, Langmuir and ion acoustic eigen modes. A large amplitude electromagnetic eigen mode of frequency and wave number (ω_0, k_0) excites an electromagnetic sideband wave (ω_1, k_1) and a primary electron plasma wave (EPW) (ω, k). The primary EPW decays into an ion acoustic wave (Ω, q) and a secondary EPW $(\Omega-\omega, q-k)$. The ponderomotive force due to the pump and electromagnetic sideband drives the primary EPW. The density perturbation associated with it couples with the pump to produce a nonlinear current driving the sideband. The oscillatory velocity of plasma electrons due to the primary EPW couples with the density perturbation due to the ion acoustic wave to produce a non linear current driving the secondary EPW. Primary and secondary EPW exerts a ponderomotive force on electrons driving the ion acoustic wave. We deduce coupled differential equations for the interacting waves in Sec. II and solve them numerically to study the temporal evolution of waves. The results are discussed in Sec. III.

II. INSTABILITY ANALYSIS

Consider a plasma channel with parabolic density profile

$$n_0^0 = n_{0m}^0\left(1 + x^2/a^2\right). \tag{1}$$

A laser propagates through it with electric field

$$\mathbf{E}_0 = \hat{y}A_0 e^{-i(\omega_0 t - k_{0z}z)}, \tag{2}$$

where A_0 is governed by the mode structure equation (deduced from the wave equation)

$$\begin{aligned} &\nabla^2\mathbf{E}_0 + \left(\omega^2\varepsilon(\omega_0)/c^2\right)\mathbf{E}_0 = 0,\\ &\frac{\partial^2 A_0}{\partial x^2} + \left[\frac{\omega_0^2-\omega_p^2}{c^2} - k_{0z}^2 - \frac{\omega_p^2}{c^2}\frac{x^2}{a^2}\right]A_0 = 0, \end{aligned} \tag{3}$$

where $\varepsilon(\omega_0) = 1 - \omega_p^2 n_0^0/\omega_0^2 n_{0m}^0$, $\omega_p = \left(n_{0m}^0 e^2/m\varepsilon_0\right)^{1/2}$, $-e$ and m are the electronic charge and mass. In the slab geometry, in which Eq. (2) is written, since $\mathbf{E}\,||\,\hat{y}$ and field has no y variation, E_z is identically zero. In the radial geometry, E_z indeed will be non-zero. However, for spot size much bigger than a parallel wavelength $E_z \sim (1/k_z r_0)E_\perp << E_\perp$, hence,

[a]Electronic mail: ranjeetsinghranjeet@yahoo.com.

axial electric field should not play a significant role in the non-linear coupling.

For the fundamental eigen mode

$$A_0 = A_{00} e^{-x^2/2r_0^2}, \tag{4}$$

$$r_0 = \left(\frac{ac}{\omega_p}\right)^{1/2}, \tag{5}$$

and the eigen value condition $\left(\omega_0^2 - \omega_p^2 - k_{0z}^2 c^2\right)\left(a/\omega_p c\right) = 1$, gives the dispersion relation

$$\omega_0^2 = \omega_p^2 + k_{0z}^2 c^2 + \omega_p\left(\frac{c}{a}\right). \tag{6}$$

The channel also supports Langmuir and ion acoustic eigen modes for which the mode structure equation could be deduced from the local linear dispersion relations. For the Langmuir eigen mode, having (t, z) variation of wave potential as $\phi = A_\omega e^{-i(\omega t - k_z z)}$, the mode structure equation from

$$\varepsilon\phi \equiv \left(\omega^2 - \omega_p^2 \frac{n_0^0}{n_{0m}^0} - 3k^2 v_{th}^2\right)\frac{1}{\omega^2}\phi = 0, \tag{7}$$

on replacing k^2 by $k_z^2 - \partial^2/\partial x^2$, turns out to be

$$\frac{\partial^2 A_\omega}{\partial x^2} + \left(\frac{\omega^2 - \omega_p^2}{3v_{th}^2} - k_z^2 - \frac{\omega_p^2}{3v_{th}^2}\frac{x^2}{a^2}\right)A_\omega = 0, \tag{8}$$

where $v_{th} = \sqrt{T_e/m}$ is the electron thermal speed and T_e is the electron temperature. For the '$\mathrm{n_{th}}$' eigen mode it gives

$$A_\omega = A_{\omega 0} H_n(x/r_\omega) e^{-x^2/2r_\omega^2}, \quad r_\omega = \left(\sqrt{3} a v_{th}/\omega_p\right)^{1/2},$$
$$\omega^2 = \omega_p^2 + 3k_z^2 v_{th}^2 + \omega_p\left(\frac{\sqrt{3} v_{th}}{a}\right), \tag{9}$$

where $H_n(x/r_w)$ is the Hermite polynomial and the third term on the right hand side of Eq. (9) is due to inhomogeneity in the density of plasma channel and finite size of the channel. In a cold plasma ($v_{th} = 0$), localized Langmuir eigen mode doesn't exist. One may note that $r_\omega << r_0$, i.e., the Langmuir mode is more strongly localized than the electromagnetic mode. The fundamental Langmuir mode is more strongly localized and that Langmuir mode will be driven which has the maximum cross-sectional overlap with the driving ponderomotive force of the laser

For the ion acoustic wave, taking the wave potential $\phi_\Omega = A_\Omega \exp[-i(\Omega t - qz)]$, the mode structure equation can be obtained from

$$\left(q^2 c_s^2 - \frac{\Omega^2}{\left(1 - \Omega^2 n_{0m}^0/\omega_{pi}^2 n_0^0\right)}\right)A_\Omega = 0, \tag{10}$$

by replacing q^2 by $q_z^2 - \partial^2/\partial x^2$ and assuming $\omega^2 \ll \omega_{pi}^2$, $x^2 < a^2$

$$\frac{\partial^2 A_\Omega}{\partial x^2} + \left(\frac{\omega^2/c_s^2}{1 - \omega^2/\omega_{pi}^2} - q_z^2 - \frac{\omega^2}{\omega_{pi}^2}\frac{\omega^2/c_s^2}{\left(1 - \omega^2/\omega_{pi}^2\right)^2}\frac{x^2}{a^2}\right)A_\Omega = 0, \tag{11}$$

where $\omega_{pi} = \left(n_{0m}^0 e^2/m_i\varepsilon_0\right)^{1/2}$, $c_s = (\gamma T_e/m_i)^{1/2}$ is the ion acoustic speed and $\gamma = 3$ is the adiabatic factor,[19] m_i is the ion mass and ions have been assumed to be singly ionized. For the fundamental mode Eq. (11) gives

$$A_\Omega = A_{\Omega 0} H_n(x/r_\Omega) e^{-x^2/2r_\Omega^2}, \; r_\Omega = \left[a\omega_{pi} c_s\left(1 - \Omega^2/\omega_{pi}^2\right)\right]^{1/2}\Big/\Omega,$$
$$\Omega^2 = \frac{\left(q^2 + 1/r_\Omega^2\right)c_s^2}{1 + \left(q^2 + 1/r_\Omega^2\right)c_s^2/\omega_{pi}^2}. \tag{12}$$

Now we study the nonlinear coupling between modes. The laser pump excites a primary EPW of electrostatic potential $\phi_\omega = A_\omega e^{-i(\omega t - kz)}$ and a backscattered sideband electromagnetic wave of electric field $\mathbf{E}_1 = \hat{y} A_1 e^{-i(\omega_1 t - k_1 z)}$, where $\omega_1 = \omega - \omega_0$ and $k_1 = k - k_0$. The primary EPW couples to an IAW of potential $\phi_\Omega = A_\Omega e^{-i(\Omega t - qz)}$ and a back scattered secondary EPW of electrostatic potential $\phi_{\Omega-\omega} = A_{\Omega-\omega} e^{-i((\Omega-\omega)t - (q-k)z)}$.

The pump and electromagnetic side band impart oscillatory velocities to electrons

$$\mathbf{v}_0 = e\mathbf{E}_0/mi\omega_0, \quad \mathbf{v}_1 = e\mathbf{E}_1/mi\omega_1 \tag{13}$$

and exert a ponderomotive force on them at (ω, k), $\mathbf{F}_{p\omega} = -\frac{m}{2}(\mathbf{v}_1 \cdot \nabla)\mathbf{v}_0 - \frac{e}{2}(\mathbf{v}_1 \times \mathbf{B}_0)$, where $\mathbf{B}_0 = \nabla \times \mathbf{E}_0/i\omega$. It simplifies to give $\mathbf{F}_{p\omega} = e\nabla\phi_{p\omega}$, where

$$\phi_{p\omega} = \frac{-m}{2e}(\mathbf{v}_0 \cdot \mathbf{v}_1) = \frac{eA_0 A_1}{2m\omega_0\omega_1} e^{-i(\omega t - kz)}. \tag{14}$$

The ponderomotive potential $\phi_{p\omega}$ and electrostatic potential ϕ_ω produce electron density perturbation, which on solving the equations of motion and continuity can be written as

$$n'_\omega \cong \frac{n_0^0 e}{m\omega^2}\nabla^2\left(1 - \frac{3v_{th}^2}{\omega^2}\nabla^2\right)(\phi_\omega + \phi_{p\omega}), \tag{15}$$

The density perturbation n_Ω associated with the ion acoustic wave also couples with the oscillatory velocity $\mathbf{v}_{\Omega-\omega}$ due to the secondary Langmuir wave to contribute to the density perturbation at ω, k. Using the equation of continuity this contribution comes out to be

$$n_\omega^{NL} = -\frac{1}{2}\frac{n_\Omega \mathbf{v}_{\Omega-\omega}^* \cdot \mathbf{k}}{\omega} \cong -\frac{k_z}{2\omega} n_\Omega v_{z\Omega-\omega}^*, \tag{16}$$

where $v_{z\Omega-\omega}^* = -e(q - k_z)\phi_{\Omega-\omega}/m(\Omega - \omega)$, $*$ refers to complex conjugate and n_Ω can be taken as $n_\Omega = n_0^0 e\phi_\Omega/T_e$.

The total density perturbation at (ω, k) is $n_\omega = n'_\omega + n_\omega^{NL}$. Using this in the Poisson's equation, $\nabla^2\phi_\omega = en_\omega/\varepsilon_0$ for the primary plasma wave, we obtain

$$\frac{\partial^2 A_\omega}{\partial x^2} + \left[\frac{\omega^2 - \omega_p^2}{3v_{th}^2} - k_z^2 - \frac{\omega_p^2}{3v_{th}^2}\frac{x^2}{a^2}\right]A_\omega$$
$$= \left[\frac{\omega_p^2}{3v_{th}^2}\left(1 + \frac{x^2}{a^2}\right) + k^2\right]\frac{eA_0A_1}{2m\omega_0\omega_1}$$
$$+ \frac{\omega^2}{3v_{th}^2}\frac{e(q-k)\omega_{pi}^2(1 + x^2/a^2)A_\Omega A_{\Omega-\omega}^*}{2m\omega k(\Omega - \omega)c_s^2}. \tag{17}$$

The last term accounts for energy diversion from the primary Langmuir wave to LDI. The density perturbation n_ω couples with $\mathbf{v}_0$ to produce nonlinear current density at $\omega_1(=\omega-\omega_0)$

$$\mathbf{J}_1^{NL} = -(1/2)n_\omega e\mathbf{v}_0^* = (1/2)\mathbf{v}_0^*\varepsilon_0 k^2\phi_\omega. \tag{18}$$

The linear current density at the side band is $\mathbf{J}_1^L = -n_0e\mathbf{v}_1 = -n_0e^2\mathbf{E}_1/mi\omega_1$.

The scattered sideband wave fields are governed by the Maxwell's equations, $\nabla\times\mathbf{E}_1 = -\partial\mathbf{B}_1/\partial t$ and $\nabla\times\mathbf{H}_1 = \left(\mathbf{J}_1^L+\mathbf{J}_1^{NL}\right)+\partial\varepsilon_0\mathbf{E}_1/\partial t$.

Taking the curl of the former and using the latter, we obtain

$$\frac{\partial^2 A_1}{\partial x^2} + \left[\frac{\omega_1^2-\omega_p^2}{c^2} - k_{1z}^2 - \frac{\omega_p^2}{c^2}\frac{x^2}{a^2}\right]A_1 = \frac{\omega_1}{2c^2}k^2\frac{eA_0^*A_\omega}{m\omega_0}. \tag{19}$$

In a similar manner the density perturbation n_ω couples with $\mathbf{v}_1$ to produce nonlinear current density at $\omega_0(=\omega-\omega_1)$

$$\mathbf{J}_0^{NL} = -(1/2)n_\omega e\mathbf{v}_1^* = 1/2\mathbf{v}_1^*\varepsilon_0 k^2 A_\omega, \tag{20}$$

$$\mathbf{J}_0^L = -n_0e\mathbf{v}_0 = -n_0e^2\mathbf{E}_0/mi\omega_0. \tag{21}$$

Maxwell's equations for the pump, $\nabla\times\mathbf{E}_0 = -\partial\mathbf{B}_0/\partial t$ and $\nabla\times\mathbf{H}_0 = \left(\mathbf{J}_0^L+\mathbf{J}_0^{NL}\right)+\partial\varepsilon_0\mathbf{E}_0/\partial t$, yield,

$$\frac{\partial^2 A_0}{\partial x^2} + \left[\frac{\omega_0^2-\omega_p^2}{c^2} - k_{0z}^2 - \frac{\omega_p^2}{c^2}\frac{x^2}{a^2}\right]A_0 = \frac{\omega_0}{2c^2}k^2\frac{eA_1^*A_\omega}{m\omega_1}. \tag{22}$$

Primary and secondary plasma waves exert a ponderomotive force $\mathbf{F}_{p\Omega} = e\nabla\phi_{p\Omega}$ on electrons, at Ω, q_z with

$$\phi_{p\Omega} = \frac{-m}{2e}\left(\mathbf{v}_\omega\cdot\mathbf{v}_{\Omega-\omega}\right) \cong -\frac{m}{2e}v_{z\omega}v_{z\Omega-\omega}, \tag{23}$$

where $v_{z\omega} \cong -(ek_zA_\omega/m\omega)e^{-i(\omega t-kz)}$.

The self consistent and ponderomotive potentials ϕ_Ω, $\phi_{p\Omega}$ produce electron and ion density perturbations

$$n_\Omega \cong \frac{n_0^0 e}{T_e}\left(\phi_\Omega+\phi_{p\Omega}\right), \tag{24}$$

$$n_{i\Omega} \cong -\frac{n_0^0 e}{m_i\Omega^2}\nabla^2\phi_\Omega. \tag{25}$$

Using n_Ω and $n_{i\Omega}$ in the Poisson's equation, $\nabla^2\phi_\Omega = e(n_\Omega-n_{i\Omega})/\varepsilon_0$, we obtain

$$\frac{\partial^2 A_\Omega}{\partial x^2} + \left(\frac{\Omega^2/c_s^2}{1-\Omega^2/\omega_{pi}^2} - q_z^2 - \frac{\Omega^2}{\omega_{pi}^2}\frac{\Omega^2/c_s^2}{\left(1-\Omega^2/\omega_{pi}^2\right)^2}\frac{x^2}{a^2}\right)A_\Omega = \frac{\omega_{pi}^2}{c_s^2}\frac{(1+x^2/a^2)v_\omega(q-k)A_{\Omega-\omega}}{2(\Omega-\omega)}. \tag{26}$$

The electron density perturbation n_Ω associated with the ion acoustic wave beats with the oscillatory velocity $\mathbf{v}_\omega$ due to the primary Langmuir wave to produce nonlinear density at $(\Omega-\omega, q-k)$. From the equation of continuity we obtain

$$n_{\Omega-\omega}^{NL} = \frac{n_\Omega(q-k)v_\omega^*}{2(\Omega-\omega)}. \tag{27}$$

The linear density perturbation due to the secondary plasma wave is

$$n_{\Omega-\omega}^L \cong \frac{n_0^0 e}{m(\Omega-\omega)^2}\nabla^2\left(1-\frac{3v_{th}^2}{(\Omega-\omega)^2}\nabla^2\right)\phi_{\Omega-\omega}. \tag{28}$$

Using $n_{\Omega-\omega} = n_{\Omega-\omega}^L + n_{\Omega-\omega}^{NL}$ into the Poisson's equation, $\nabla^2\phi_{\Omega-\omega} = en_{\Omega-\omega}/\varepsilon_0$, we get

$$\frac{\partial^2 A_{\Omega-\omega}}{\partial x^2} + \left[\frac{(\Omega-\omega)^2-\omega_p^2}{3v_{th}^2} - (q_z-k_z)^2 - \frac{\omega_p^2}{3v_{th}^2}\frac{x^2}{a^2}\right]A_{\Omega-\omega} = -\frac{\omega_{pi}^2}{3v_{th}^2c_s^2}\frac{(1+x^2/a^2)(\Omega-\omega)}{2(q-k)}v_\omega^*A_\Omega. \tag{29}$$

If one ignores the nonlinear terms, Eqs. (19) and (29) offer the following mode structures,

$$A_1 = A_{10}e^{-x^2/2r_0^2}, \tag{30}$$

$$A_{\Omega-\omega} = A_{\Omega-\omega 0}H_n(x/r_\omega)e^{-x^2/2r_\omega^2}, \tag{31}$$

with dispersion relations

$$\omega_1^2 = \omega_p^2 + k_{1z}^2c^2 + \omega_p\left(\frac{c}{a}\right), \tag{32}$$

$$(\Omega-\omega)^2 = \omega_p^2 + 3(q_z-k_z)^2v_{th}^2 + \omega_p\left(\frac{\sqrt{3}v_{th}}{a}\right). \tag{33}$$

The Langmuir wave in the channel has effective sinusoidal character only in the inner region of the order of size of Langmuir eigenmode wave function, while beyond this outer region the Langmuir wave is evanescent. The coupling of the evanescent part of Langmuir wave with the ponderomotive force is weak. In the presence of finite RHS of Eqs. (17), (19), (22), (26), and (29), the eigen values are modified while the mode structure retains its character. Substituting for A_ω, A_1, A_0, A_Ω, and $A_{\Omega-\omega}$ in Eqs. (17), (19), (22), (26), and (29) and multiplying the resulting equations by A_j (where $j=\omega, 1, 0, \Omega$ or $\Omega-\omega$) and integrating on x, we obtain

$$\left[\left(\frac{\omega^2-\omega_p^2}{3v_{th}^2}-k_z^2\right)r_\omega - \frac{1}{r_\omega}\right]A_{\omega 0} = \frac{eA_{00}A_{10}}{2m\omega_0\omega_1}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\left(\frac{\omega_p^2}{3v_{th}^2}+k^2+\frac{r_0^2}{r_\omega^2\left(2r_\omega^2+r_0^2\right)}\right)$$
$$\times\left(\frac{2r_\omega^2r_0^2}{2r_\omega^2+r_0^2}\right)^{1/2} + \frac{\omega^2}{3v_{th}^2}\frac{e(q-k)\omega_{pi}^2}{2m\omega k(\Omega-\omega)c_s^2}A_{\Omega 0}A_{\Omega-\omega 0}^*\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}\left(\frac{2r_\omega^2r_0^2}{2r_\omega^2+r_0^2}\right)^{1/2}$$
$$\times\left(1+\frac{1}{a^2}\left(\frac{r_\omega^2r_0^2}{2r_\omega^2+r_0^2}\right)\right), \tag{34}$$

$$\left[\left(\frac{\omega_1^2-\omega_p^2}{c^2}-k_{1z}^2\right)r_0-\frac{1}{r_0}\right]A_{10} = \frac{\omega_1}{2c^2}\frac{k^2 e}{m\omega_0}\left(\frac{\sqrt{2}r_\omega r_0}{\sqrt{2r_\omega^2+r_0^2}}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}A_{00}^*A_{\omega 0}, \tag{35}$$

$$\left[\left(\frac{\omega_0^2-\omega_p^2}{c^2}-k_{0z}^2\right)r_0-\frac{1}{r_0}\right]A_{00} = \frac{\omega_0}{2c^2}\frac{k^2 e}{m\omega_1}\left(\frac{\sqrt{2}r_\omega r_0}{\sqrt{2r_\omega^2+r_0^2}}\right) \times\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}A_{10}^*A_{\omega 0}, \tag{36}$$

$$\left[\left(\frac{(\Omega-\omega)^2-\omega_p^2}{3v_{th}^2}-(q_z-k_z)^2\right)r_\omega-\frac{1}{r_\omega}\right]A_{\Omega-\omega 0} = \frac{\omega_{pi}^2}{3v_{th}^2c_s^2}\frac{ek(\Omega-\omega)}{2m\omega(q-k)}\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}\left(\frac{2r_\omega^2 r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)^{1/2} \times\left(1+\frac{1}{a^2}\left(\frac{r_\omega^2 r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\right)A_{\omega 0}^*A_{\Omega 0}, \tag{37}$$

$$\left[\left(\frac{\Omega^2/c_s^2}{1-\Omega^2/\omega_{pi}^2}-q_z^2\right)r_\Omega-\frac{1}{r_\Omega}\right]A_{\Omega 0} = -\frac{\omega_{pi}^2}{c_s^2}\frac{ek(q-k)}{2m\omega(\Omega-\omega)}\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}\left(\frac{1}{\pi r_\Omega^2}\right)^{-1/4} \times\left(\frac{2r_\Omega^2 r_\omega^2}{2r_\Omega^2+r_\omega^2}\right)^{1/2}\left(1+\frac{1}{a^2}\left(\frac{r_\Omega^2 r_\omega^2}{2r_\Omega^2+r_\omega^2}\right)\right)A_{\omega 0}A_{\Omega-\omega 0}. \tag{38}$$

One may mention that the radial mode extent of the pump and scattered electromagnetic waves $\left(\sim(ac/\omega_p)^{1/2}\right)$ is much larger than that of the Langmuir waves $\left(\sim(\sqrt{3}av_{th}/\omega_p)^{1/2}\right)$. The ponderomotive force driver due to the two em waves certainly is finite across the cross-section of these beams. However, the response of the plasma to this force is large only in the narrow inner region $\left(\sim(\sqrt{3}av_{th}/\omega_p)^{1/2}\right)$. Beyond this region, the first x^2 term appearing in the Eq. (17) on RHS dominates, suppresses the contribution to the Langmuir mode.

These coupled mode equations can be converted into mode evolutions equations by expressing $\omega_j = \omega_j + i\partial/\partial t$. Neglecting the second order derivative and expressing in dimensionless form, for the fundamental mode we write

$$\frac{\partial F_{00}}{\partial\tau} = -\frac{i}{4}K^2W_0HX^2\left(\frac{\sqrt{2}r_\omega}{\sqrt{2r_\omega^2+r_0^2}}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}F_{10}^*F_{\omega 0}, \tag{39}$$

$$\frac{\partial F_{10}}{\partial\tau}+SF_{10} = -\frac{i}{4}K^2W_1HX^2\left(\frac{\sqrt{2}r_\omega}{\sqrt{2r_\omega^2+r_0^2}}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}F_{00}^*F_{\omega 0}, \tag{40}$$

$$\frac{\partial F_{\omega 0}}{\partial\tau}+T_\omega F_{\omega 0} = -\frac{i}{4}\left(W_p^2+K^2X^2W_0^2+\frac{r_0^2}{2r_\omega^2+r_0^2}\frac{r_\omega^2W_p^2}{a^2}\right) \times\frac{1}{HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\left(\frac{\sqrt{2}r_0}{\sqrt{2r_\omega^2+r_0^2}}\right)F_{00}F_{10} -\frac{i}{4}\frac{3v_{th}^2}{c_s^2}\frac{(Q-K)}{(\Omega_n-W)K}\left(\frac{\sqrt{2}r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right) \times\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}F_{\Omega 0}F_{\Omega-\omega 0}^*, \tag{41}$$

$$\frac{\partial F_{\Omega-\omega 0}}{\partial\tau}+T_{\omega-\Omega}F_{\omega-\Omega} = -\frac{i}{4}\frac{3v_{th}^2}{c_s^2}\frac{K}{W(Q-K)}\left(\frac{\sqrt{2}r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right) \times\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}F_{\omega 0}^*F_{\Omega 0}, \tag{42}$$

$$\frac{\partial F_{\Omega 0}}{\partial\tau} = \frac{i}{4}\frac{(Q-K)KW_0^2HX^2(1-\Omega_n^2)}{W\Omega_n(\Omega_n-W)}\left(\frac{\sqrt{2}r_\omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right) \times\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{-1/4}\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}F_{\omega 0}F_{\Omega-\omega 0}, \tag{43}$$

where T_ω and $T_{\Omega-\omega}$ are the damping rate of the primary and secondary EPW,[20] T_Ω is assumed to be zero and 'S' is the convection loss of the side band wave, $W=\omega/\omega_{pi}$, $F_{00}=eA_{00}/m\omega_0 v_{th}$, $F_{10}=eA_{10}/m\omega_1 v_{th}$, $(Q-K)=(q-k)/k_0$, $Q=q/k_0$, $F_{\omega 0}=eA_{\omega 0}/T_e$, $F_{\Omega 0}=eA_{\Omega 0}/T_e$, $F_{\Omega-\omega 0}=eA_{\Omega-\omega 0}/T_e$, $\tau=\omega_{pi}t$, $S=c/(8\pi L\omega_{pi})$, $W_0=\omega_0/\omega_{pi}$, $H=T_e/(mv_{th}^2)$, $X=k_0v_{th}/\omega_0$, $T_\omega=\left(\sqrt{\pi}W^2W_p^2/(W_0KX)^3\right)\exp\left(-W^2/(W_0KX)^2\right)$, $W_1=(\omega_1/\omega_{pi})$ $K=k/k_0$, $\Omega_n=\Omega/\omega_{pi}$, $K_1=k_1/k_0$, $(\Omega_n-W)=(\Omega-\omega)/\omega_{pi}$, and $T_{\Omega-\omega}=\left(\sqrt{\pi}W_{\Omega-\omega}^2W_p^2/(W_0KX)^3\right)\exp\left(-W_{\Omega-\omega}^2/(W_0KX)^2\right)$.

We now write the normalized field amplitudes for the fundamental modes $(n=0)$ in terms of their respective phases as

$$F_{jn} = f_{jn}(X)e^{i\theta_j(X)}, \tag{44}$$

where $j=0,1,\omega,\Omega$ or $\Omega-\omega$. On substituting this in Eqs. (39)–(43), and separating the real and imaginary parts we obtain

$$\frac{\partial f_{00}}{\partial\tau} = \frac{1}{4}K^2W_0HX^2\left(\frac{\sqrt{2}r_\omega}{\sqrt{2r_\omega^2+r_0^2}}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}f_{10}f_{\omega 0}\sin\Theta_1, \tag{45}$$

$$\frac{\partial f_{10}}{\partial\tau}+Sf_{10} = \frac{1}{4}K^2W_1HX^2\left(\frac{\sqrt{2}r_\omega}{\sqrt{2r_\omega^2+r_0^2}}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}f_{00}f_{\omega 0}\sin\Theta_1, \tag{46}$$

$$\frac{\partial f_{\omega0}}{\partial\tau}+T_\omega f_{\omega0}=-\frac{1}{4}\left(W_p^2+K^2X^2W_0^2+\frac{r_0^2}{2r_\omega^2+r_0^2}\frac{r_\omega^2W_p^2}{a^2}\right)\times\frac{1}{HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\left(\frac{\sqrt{2}r_0}{\sqrt{2r_\omega^2+r_0^2}}\right)f_{00}f_{10}\sin\Theta_1+\frac{1}{4}\frac{3v_{th}^2}{c_s^2}\frac{(Q-K)}{(\Omega_n-W)K}\left(\frac{\sqrt{2}r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\times\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}f_{\Omega0}f_{\Omega-\omega0}\sin\Theta_2, \tag{47}$$

$$\frac{\partial f_{\Omega-\omega0}}{\partial\tau}+T_{\omega-\Omega}f_{\omega-\Omega0}=\frac{1}{4}\frac{3v_{th}^2}{c_s^2}\frac{K}{W(Q-K)}\left(\frac{\sqrt{2}r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\times\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}f_{\omega0}f_{\Omega0}\sin\Theta_2, \tag{48}$$

$$\frac{\partial f_{\Omega0}}{\partial\tau}=\frac{1}{4}\frac{(Q-K)KW_0^2HX^2\left(1-\Omega_n^2\right)}{W\Omega_n(\Omega_n-W)}\left(\frac{\sqrt{2}r_\omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\times\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{-1/4}\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}f_{\omega0}f_{\Omega-\omega0}\sin\Theta_2, \tag{49}$$

$$\frac{\partial\Theta_1}{\partial\tau}=\frac{1}{4}K^2HX^2\left(\frac{\sqrt{2}r_\omega}{\sqrt{2r_\omega^2+r_0^2}}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}\left(W_0\frac{f_{10}f_{\omega0}}{f_{00}}\cos\Theta_1+W_1\frac{f_{00}f_{\omega0}}{f_{10}}\cos\Theta_1\right)-\frac{1}{4}\left(W_p^2+K^2X^2W_0^2+\frac{r_0^2}{2r_\omega^2+r_0^2}\frac{r_\omega^2W_p^2}{a^2}\right)\frac{1}{HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\left(\frac{\sqrt{2}r_0}{\sqrt{2r_\omega^2+r_0^2}}\right)\times\frac{f_{00}f_{10}}{f_{\omega0}}\cos\Theta_1-\frac{1}{4}\frac{3v_{th}^2}{c_s^2}\frac{(Q-K)}{(\Omega_n-W)K}\left(\frac{\sqrt{2}r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}\frac{f_{\Omega0}f_{\Omega-\omega0}}{f_{\omega0}}\cos\Theta_2, \tag{50}$$

$$\frac{\partial\Theta_2}{\partial\tau}=\frac{1}{4}\frac{(Q-K)KW_0^2HX^2\left(1-\Omega_n^2\right)}{W\Omega_n(\Omega_n-W)}\left(\frac{\sqrt{2}r_\omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{-1/4}\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}\times\frac{f_{\omega0}f_{\Omega-\omega0}}{f_{\Omega0}}\cos\Theta_2+\frac{1}{4}\left(W_p^2+K^2X^2W_0^2+\frac{r_0^2}{2r_\omega^2+r_0^2}\frac{r_\omega^2W_p^2}{a^2}\right)\frac{1}{HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\left(\frac{\sqrt{2}r_0}{\sqrt{2r_\omega^2+r_0^2}}\right)\times\frac{f_{00}f_{10}}{f_{\omega0}}\cos\Theta_1+\frac{1}{4}\frac{3v_{th}^2}{c_s^2}\left(\frac{(Q-K)}{(\Omega_n-W)K}\frac{f_{\Omega0}f_{\Omega-\omega0}}{f_{\omega0}}\cos\Theta_2+\frac{K}{W(Q-K)}\frac{f_{\omega0}f_{\Omega0}}{f_{\Omega-\omega0}}\cos\Theta_2\right)\times\left(\frac{\sqrt{2}r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\left(1+\frac{r_\omega^2}{a^2}\frac{r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}, \tag{51}$$

where $\Theta_1=\theta_\omega-\theta_1-\theta_0$ and $\Theta_2=\theta_\Omega-\theta_\omega-\theta_{\Omega-\omega}$. These coupled Eqs. (45)–(51) determine the behavior of the different waves involved in coupled SRS and decay instability processes. We have solved them numerically for the normalized amplitudes of pump wave (f_{00}), sideband wave (f_{10}), primary EPW ($f_{\omega0}$), secondary EPW ($f_{\Omega-\omega0}$) and IAW ($f_{\Omega0}$) as a function of normalized time $(\tau=\omega_{pi}t)$. Initial conditions for solving the coupled differential equations are $f_{10}(0)=0.005$, $f_{\omega0}(0)=0.001$, $f_{\Omega-\omega0}(0)=0.001$, $f_{\Omega0}(0)=0.005$, $\Theta_1(0)=0.1$, and $\Theta_2(0)=1.95$. The length of the plasma L $\cong$ 400 μm, $\lambda_0=1\mu$m, $a=100/(2\pi/\lambda_0)$, $\omega_p^2/\omega_0^2=0.04, f_{00}(0)=0.2$, and $T_e=$400 eV.

Fig. 1 shows the evolution of pump and scattered sideband wave amplitudes with normalized time and Fig. 2 shows the amplitude variation of Primary and secondary EPW with normalized time τ for different radial mode number. The pump amplitude decreases with τ while the scattered sideband wave amplitude increases. The sideband wave amplitude attains saturation and then falls, giving back and forth transfer of energy between the pump electromagnetic wave and the Raman scattered sideband wave. It reveals that pump depletion is one of the saturation mechanisms of Raman instability. The behaviour is similar when the interaction of Langmuir eigen mode of different radial mode number is considered. In Fig. 2, as the primary plasma wave amplitude increases it couples with the ion acoustic wave and enhances the amplitude of the secondary plasma wave which is non resonant with the SRS process and saturates the primary plasma wave amplitude. The limitation of the model is that, if the pulse is short enough, the ion acoustic wave will not be able to stop the Raman scattering within the pulse, typical decay processes are examined to limit the

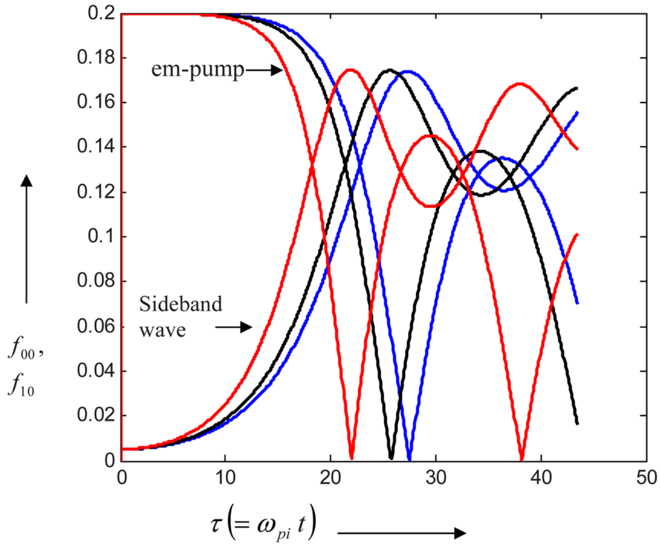

FIG. 1. (Color online) Variation of pump amplitude f_{00} (blue for $n=0$, black for $n=1$, red for $n=2$) and scattered sideband wave amplitude f_{10} (blue for $n=0$, black for $n=1$, red for $n=2$) with normalized time τ for the parameters $\lambda_0 = 1\ \mu$m, $L = 400\ \mu$m, $a = 100/(2\pi/\lambda_0)$, $\omega_{p0}^2/\omega_0^2 = 0.04$, $f_{00}(0) = 0.2$, and $T_e = 400\,$eV.

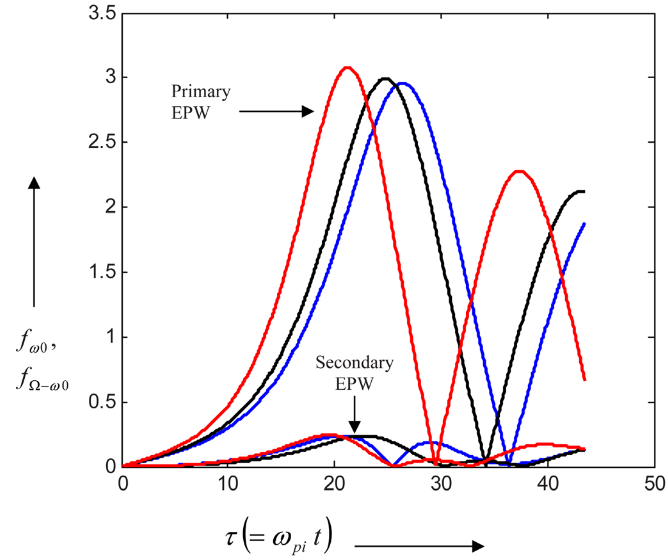

FIG. 2. (Color online) Variation of amplitude of Primary EPW $f_{\omega 0}$ (blue for $n=0$, black for $n=1$, red for $n=2$) and secondary EPW $f_{\Omega-\omega 0}$ (blue for $n=0$, black for $n=1$, red for $n=2$) with normalized time τ for the parameters as in Fig. 1.

amount of Raman scattering with in the laser pulse. For this to happen, at least one ion acoustic period must fit inside the pulse, i.e., if the ion acoustic period is a fraction of a picosecond, than the laser pulse must be at least of that order or longer.

We have examined the nonlinear coupling of the pump and scattered wave with Langmuir eigen mode of different radial mode numbers. We find that the mode evolution pattern is similar to one, when Langmuir wave propagates in the fundamental mode. Nonlocal theory here implies that the eigen mode is fed energy in certain region but the mode amplitude is enhanced over the entire radial extent of the mode. The mode evolution equation for the higher order radial mode number ($n = 1$, 2) can be written as

$$\frac{\partial f_{01}}{\partial \tau} = \frac{1}{4}K^2 W_0 H X^2\left(\frac{2\sqrt{2}r_0 r_\omega}{\sqrt{\pi}\left(2r_\omega^2+r_0^2\right)}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/4} f_{11} f_{\omega 1}\sin\Theta_1, \tag{52}$$

$$\frac{\partial f_{11}}{\partial \tau} + S f_{11} = \frac{1}{4}K^2 W_1 H X^2\left(\frac{2\sqrt{2}r_0 r_\omega}{\sqrt{\pi}\left(2r_\omega^2+r_0^2\right)}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/4} \times f_{01} f_{\omega 1}\sin\Theta_1, \tag{53}$$

$$\frac{\partial f_{\omega 1}}{\partial \tau} + T_\omega f_{\omega 1} = -\frac{1}{4}\left(W_p^2 + K^2X^2W_0^2 + \frac{2r_0^2}{2r_\omega^2+r_0^2}\frac{r_\omega^2 W_p^2}{a^2}\right) \times \frac{1}{HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\left(\frac{2\sqrt{2}r_0^2}{\sqrt{\pi}\left(2r_\omega^2+r_0^2\right)}\right) f_{01} f_{11}\sin\Theta_1 + \frac{1}{4}\frac{3v_{th}^2}{c_s^2}\frac{(Q-K)}{(\Omega_n - W)K}\left(\frac{8\sqrt{2}r_\Omega^3 r_\omega}{\sqrt{\pi}\left(2r_\Omega^2+r_\omega^2\right)^2}\right)\left(1+\frac{2}{a^2}\left(\frac{2r_\omega^2 r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\right) \times \left(\frac{1}{\pi r_\Omega^2}\right)^{1/4} f_{\Omega 1} f_{\Omega-\omega 1}\sin\Theta_2, \tag{54}$$

$$\frac{\partial f_{\Omega-\omega 1}}{\partial \tau} + T_{\omega-\Omega} f_{\omega-\Omega 1} = \frac{1}{4}\frac{3v_{th}^2}{c_s^2}\frac{K}{W(Q-K)}\left(\frac{8\sqrt{2}r_\Omega^3 r_\omega}{\sqrt{\pi}\left(2r_\Omega^2+r_\omega^2\right)^2}\right) \times \left(1+\frac{2}{a^2}\frac{2r_\omega^2 r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4} f_{\omega 1} f_{\Omega 1}\sin\Theta_2, \tag{55}$$

$$\frac{\partial f_{\Omega 1}}{\partial \tau} = \frac{1}{4}\frac{(Q-K)KW_0^2 H X^2\left(1-\Omega_n^2\right)}{W\Omega_n(\Omega_n - W)}\left(\frac{8\sqrt{2}r_\omega^2 r_\Omega^2}{\sqrt{\pi}\left(2r_\Omega^2+r_\omega^2\right)^2}\right) \times \left(1+\frac{2}{a^2}\frac{2r_\omega^2 r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{-1/4}\left(\frac{1}{\pi r_\omega^2}\right)^{1/2} f_{\omega 1} f_{\Omega-\omega 1}\sin\Theta_2, \tag{56}$$

$$\frac{\partial \Theta_1}{\partial \tau} = \frac{1}{4}K^2 H X^2\left(\frac{2\sqrt{2}r_0 r_\omega}{\sqrt{\pi}\left(2r_\omega^2+r_0^2\right)}\right)\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}\left(W_0\frac{f_{11}f_{\omega 1}}{f_{01}}\cos\Theta_1 + W_1\frac{f_{01}f_{\omega 1}}{f_{11}}\cos\Theta_1\right) - \frac{1}{4}\left(W_p^2 + K^2X^2W_0^2 + \frac{2r_0^2}{2r_\omega^2+r_0^2}\frac{r_\omega^2 W_p^2}{a^2}\right)\frac{1}{HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\left(\frac{2\sqrt{2}r_0^2}{\sqrt{\pi}\left(2r_\omega^2+r_0^2\right)}\right)\frac{f_{01}f_{11}}{f_{\omega 1}} \times \cos\Theta_1 - \frac{1}{4}\frac{3v_{th}^2}{c_s^2}\frac{(Q-K)}{(\Omega_n - W)K}\left(\frac{8\sqrt{2}r_\Omega^3 r_\omega}{\sqrt{\pi}\left(2r_\Omega^2+r_\omega^2\right)^2}\right)\left(1+\frac{2}{a^2}\left(\frac{2r_\omega^2 r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}\frac{f_{\Omega 1}f_{\Omega-\omega 1}}{f_{\omega 1}}\cos\Theta_2, \tag{57}$$

$$\frac{\partial\Theta_2}{\partial\tau} = \frac{1}{4}\frac{(Q-K)KW_0^2HX^2\left(1-\Omega_n^2\right)}{W\Omega_n(\Omega_n-W)}\left(\frac{8\sqrt{2}r_\omega^2r_\Omega^2}{\sqrt{\pi}\left(2r_\Omega^2+r_\omega^2\right)^2}\right)\left(1+\frac{2}{a^2}\frac{2r_\omega^2r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{-1/4}\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}$$
$$\times\frac{f_{\omega1}f_{\Omega-\omega1}}{f_{\Omega1}}\cos\Theta_2+\frac{1}{4}\left(W_p^2+K^2X^2W_0^2+\frac{2r_0^2}{2r_\omega^2+r_0^2}\frac{r_\omega^2W_p^2}{a^2}\right)\frac{1}{HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\left(\frac{2\sqrt{2}r_0^2}{\sqrt{\pi}\left(2r_\omega^2+r_0^2\right)}\right)$$
$$\times\frac{f_{01}f_{11}}{f_{\omega1}}\cos\Theta_1+\frac{1}{4}\frac{3v_{th}^2}{c_s^2}\left(\frac{(Q-K)}{(\Omega_n-W)K}\frac{f_{\Omega1}f_{\Omega-\omega1}}{f_{\omega1}}\cos\Theta_2+\frac{K}{W(Q-K)}\frac{f_{\omega1}f_{\Omega1}}{f_{\Omega-\omega1}}\cos\Theta_2\right)$$
$$\times\left(\frac{8\sqrt{2}r_\Omega^3r_\omega}{\sqrt{\pi}\left(2r_\Omega^2+r_\omega^2\right)^2}\right)\left(1+\frac{2}{a^2}\left(\frac{2r_\omega^2r_\Omega^2}{2r_\Omega^2+r_\omega^2}\right)\right)\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}, \tag{58}$$

$$\frac{\partial f_{02}}{\partial\tau} = \frac{1}{4}K^2W_0HX^2\left(\frac{2r_0^2}{\left(2r_\omega^2+r_0^2\right)}-1\right)\frac{r_\omega}{\sqrt{2r_\omega^2+r_0^2}}\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}f_{12}f_{\omega2}\sin\Theta_1, \tag{59}$$

$$\frac{\partial f_{12}}{\partial\tau}+Sf_{12} = \frac{1}{4}K^2W_1HX^2\left(\frac{2r_0^2}{\left(2r_\omega^2+r_0^2\right)}-1\right)\frac{r_\omega}{\sqrt{2r_\omega^2+r_0^2}}\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}f_{02}f_{\omega2}\sin\Theta_1, \tag{60}$$

$$\frac{\partial f_{\omega2}}{\partial\tau}+T_\omega f_{\omega2} = -\frac{1}{4r_\omega}\left(\left(W_p^2+K^2X^2W_0^2\right)\left(\frac{4\sqrt{2}r_\omega r_o^3}{\sqrt{\left(2r_\omega^2+r_0^2\right)^3}}-\frac{2\sqrt{2}r_\omega r_0}{\sqrt{2r_\omega^2+r_o^2}}\right)+\frac{W_p^2}{a^2}\left(\frac{3}{r_\omega^2}\frac{\left(2r_\omega^2r_0^2\right)^{5/2}}{\sqrt{\left(2r_\omega^2+r_0^2\right)^5}}\right.\right.$$
$$\left.\left.-\frac{\left(2r_\omega^2r_0^2\right)^{3/2}}{\sqrt{\left(2r_\omega^2+r_0^2\right)^3}}\right)\right)\frac{1}{2\sqrt{2}HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}f_{02}f_{12}\sin\Theta_1+\frac{1}{4r_\omega}\frac{3v_{th}^2}{c_s^2}\frac{(Q-K)}{(\Omega_n-W)K}\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}\frac{1}{2\sqrt{2}}$$
$$\times\left(\left(\frac{15}{r_\omega^4r_\Omega^2}\frac{\left(2r_\omega^2r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}-\left(\frac{6}{r_\omega^2r_\Omega^2}+\frac{3}{r_\omega^4}\right)\frac{\left(2r_\omega^2r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}+\left(\frac{2}{r_\omega^2}+\frac{1}{r_\Omega^2}\right)\frac{\left(2r_\omega^2r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}-\frac{\sqrt{2}r_\omega r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\right.$$
$$+\frac{1}{a^2}\left(\frac{105}{2r_\omega^4r_\Omega^2}\frac{\left(2r_\omega^2r_\Omega^2\right)^{9/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^9}}-\left(\frac{15}{r_\omega^2r_\Omega^2}+\frac{15}{2r_\omega^4}\right)\frac{\left(2r_\omega^2r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}+\left(\frac{3}{r_\omega^2}+\frac{3}{2r_\Omega^2}\right)\frac{\left(2r_\omega^2r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}\right.$$
$$\left.\left.-\frac{1}{2}\frac{\left(2r_\omega^2r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}\right)\right)f_{\Omega2}f_{\Omega-\omega2}\sin\Theta_2, \tag{61}$$

$$\frac{\partial f_{\Omega-\omega2}}{\partial\tau}+T_{\omega-\Omega}f_{\omega-\Omega2} = \frac{1}{4r_\omega}\frac{3v_{th}^2}{c_s^2}\frac{K}{W(Q-K)}\frac{1}{2\sqrt{2}}\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}\left(\left(\frac{15}{r_\omega^4r_\Omega^2}\frac{\left(2r_\omega^2r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}\right.\right.$$
$$\left.-\left(\frac{6}{r_\omega^2r_\Omega^2}+\frac{3}{r_\omega^4}\right)\frac{\left(2r_\omega^2r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}+\left(\frac{2}{r_\omega^2}+\frac{1}{r_\Omega^2}\right)\frac{\left(2r_\omega^2r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}-\frac{\sqrt{2}r_\omega r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)$$
$$+\frac{1}{a^2}\left(\frac{105}{2r_\omega^4r_\Omega^2}\frac{\left(2r_\omega^2r_\Omega^2\right)^{9/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^9}}-\left(\frac{15}{r_\omega^2r_\Omega^2}+\frac{15}{2r_\omega^4}\right)\frac{\left(2r_\omega^2r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}+\left(\frac{3}{r_\omega^2}+\frac{3}{2r_\Omega^2}\right)\frac{\left(2r_\omega^2r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}\right.$$
$$\left.\left.-\frac{1}{2}\frac{\left(2r_\omega^2r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}\right)\right)f_{\omega2}f_{\Omega2}\sin\Theta_2, \tag{62}$$

$$\frac{\partial f_{\Omega 2}}{\partial \tau}=\frac{1}{4r_\Omega}\frac{(Q-K)KW_0^2HX^2\left(1-\Omega_n^2\right)}{W\Omega_n(\Omega_n-W)}\frac{1}{2\sqrt{2}}\left(\frac{1}{\pi r_\Omega^2}\right)^{-1/4}\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}\left(\left(\frac{15}{r_\omega^4 r_\Omega^2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}\right.\right.$$
$$\left.-\left(\frac{6}{r_\omega^2 r_\Omega^2}+\frac{3}{r_\omega^4}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}+\left(\frac{2}{r_\omega^2}+\frac{1}{r_\Omega^2}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}-\frac{\sqrt{2}r_\omega r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)$$
$$+\frac{1}{a^2}\left(\frac{105}{2r_\omega^4 r_\Omega^2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{9/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^9}}-\left(\frac{15}{r_\omega^2 r_\Omega^2}+\frac{15}{2r_\omega^4}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}+\left(\frac{3}{r_\omega^2}+\frac{3}{2r_\Omega^2}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}\right.$$
$$\left.\left.-\frac{1}{2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}\right)\right)f_{\Omega-\omega 2}f_{\omega 2}\sin\Theta_2, \tag{63}$$

$$\frac{\partial \Theta_1}{\partial \tau}=\frac{1}{4}K^2HX^2\left(\frac{2r_0^2}{\left(2r_\omega^2+r_0^2\right)}-1\right)\frac{r_\omega}{\sqrt{2r_\omega^2+r_0^2}}\left(\frac{1}{\pi r_\omega^2}\right)^{1/4}\left(W_0\frac{f_{12}f_{\omega 2}}{f_{02}}\cos\Theta_1+W_1\frac{f_{02}f_{\omega 2}}{f_{12}}\cos\Theta_1\right)$$
$$-\frac{1}{4r_\omega}\left(\left(W_p^2+K^2X^2W_0^2\right)\left(\frac{4\sqrt{2}r_\omega r_o^3}{\sqrt{\left(2r_\omega^2+r_0^2\right)^3}}-\frac{2\sqrt{2}r_\omega r_0}{\sqrt{2r_\omega^2+r_o^2}}\right)+\frac{W_p^2}{a^2}\left(\frac{3}{r_\omega^2}\frac{\left(2r_\omega^2 r_0^2\right)^{5/2}}{\sqrt{\left(2r_\omega^2+r_0^2\right)^5}}\right.\right.$$
$$\left.\left.-\frac{\left(2r_\omega^2 r_0^2\right)^{3/2}}{\sqrt{\left(2r_\omega^2+r_0^2\right)^3}}\right)\right)\frac{1}{2\sqrt{2}HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\frac{f_{02}f_{12}}{f_{\omega 2}}\cos\Theta_1-\frac{1}{4r_\omega}\frac{3v_{th}^2}{c_s^2}\frac{(Q-K)}{(\Omega_n-W)K}\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}\frac{1}{2\sqrt{2}}$$
$$\times\left(\left(\frac{15}{r_\omega^4 r_\Omega^2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}-\left(\frac{6}{r_\omega^2 r_\Omega^2}+\frac{3}{r_\omega^4}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}+\left(\frac{2}{r_\omega^2}+\frac{1}{r_\Omega^2}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}-\frac{\sqrt{2}r_\omega r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)\right.$$
$$+\frac{1}{a^2}\left(\frac{105}{2r_\omega^4 r_\Omega^2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{9/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^9}}-\left(\frac{15}{r_\omega^2 r_\Omega^2}+\frac{15}{2r_\omega^4}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}+\left(\frac{3}{r_\omega^2}+\frac{3}{2r_\Omega^2}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}\right.$$
$$\left.\left.-\frac{1}{2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}\right)\right)\frac{f_{\Omega 2}f_{\Omega-\omega 2}}{f_{\omega 2}}\cos\Theta_2, \tag{64}$$

$$\frac{\partial \Theta_2}{\partial \tau}=\frac{1}{4r_\Omega}\frac{(Q-K)KW_0^2HX^2\left(1-\Omega_n^2\right)}{W\Omega_n(\Omega_n-W)}\frac{1}{2\sqrt{2}}\left(\frac{1}{\pi r_\Omega^2}\right)^{-1/4}\left(\frac{1}{\pi r_\omega^2}\right)^{1/2}\left(\left(\frac{15}{r_\omega^4 r_\Omega^2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}\right.\right.$$
$$\left.-\left(\frac{6}{r_\omega^2 r_\Omega^2}+\frac{3}{r_\omega^4}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}+\left(\frac{2}{r_\omega^2}+\frac{1}{r_\Omega^2}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}-\frac{\sqrt{2}r_\omega r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\right)$$
$$+\frac{1}{a^2}\left(\frac{105}{2r_\omega^4 r_\Omega^2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{9/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^9}}-\left(\frac{15}{r_\omega^2 r_\Omega^2}+\frac{15}{2r_\omega^4}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}+\left(\frac{3}{r_\omega^2}+\frac{3}{2r_\Omega^2}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}\right.$$
$$\left.\left.-\frac{1}{2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}\right)\right)\frac{f_{\Omega-\omega 2}f_{\omega 2}}{f_{\Omega 2}}\cos\Theta_2+\frac{1}{4r_\omega}\left(\left(W_p^2+K^2X^2W_0^2\right)\left(\frac{4\sqrt{2}r_\omega r_o^3}{\sqrt{\left(2r_\omega^2+r_0^2\right)^3}}-\frac{2\sqrt{2}r_\omega r_0}{\sqrt{2r_\omega^2+r_o^2}}\right)\right.$$

$$
+\frac{W_p^2}{a^2}\left(\frac{3}{r_\omega^2}\frac{\left(2r_\omega^2 r_0^2\right)^{5/2}}{\sqrt{\left(2r_\omega^2+r_0^2\right)^5}}-\frac{\left(2r_\omega^2 r_0^2\right)^{3/2}}{\sqrt{\left(2r_\omega^2+r_0^2\right)^3}}\right)\Bigg)\frac{1}{2\sqrt{2}HW}\left(\frac{1}{\pi r_\omega^2}\right)^{-1/4}\frac{f_{02}f_{12}}{f_{\omega 2}}\cos\Theta_1
$$

$$
+\frac{1}{4r_\omega}\frac{3v_{th}^2}{c_s^2}\left(\frac{(Q-K)}{(\Omega_n-W)K}\frac{f_{\Omega 2}f_{\Omega-\omega 2}}{f_{\omega 2}}\cos\Theta_2+\frac{K}{W(Q-K)}\frac{f_{\omega 2}f_{\Omega 2}}{f_{\Omega-\omega 2}}\cos\Theta_2\right)\frac{1}{2\sqrt{2}}\left(\frac{1}{\pi r_\Omega^2}\right)^{1/4}
$$

$$
\times\Bigg(\Bigg(\frac{15}{r_\omega^4 r_\Omega^2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}-\left(\frac{6}{r_\omega^2 r_\Omega^2}+\frac{3}{r_\omega^4}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}+\left(\frac{2}{r_\omega^2}+\frac{1}{r_\Omega^2}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}-\frac{\sqrt{2}r_\omega r_\Omega}{\sqrt{2r_\Omega^2+r_\omega^2}}\Bigg)
$$

$$
+\frac{1}{a^2}\Bigg(\frac{105}{2r_\omega^4 r_\Omega^2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{9/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^9}}-\left(\frac{15}{r_\omega^2 r_\Omega^2}+\frac{15}{2r_\omega^4}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{7/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^7}}+\left(\frac{3}{r_\omega^2}+\frac{3}{2r_\Omega^2}\right)\frac{\left(2r_\omega^2 r_\Omega^2\right)^{5/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^5}}
$$

$$
-\frac{1}{2}\frac{\left(2r_\omega^2 r_\Omega^2\right)^{3/2}}{\sqrt{\left(2r_\Omega^2+r_\omega^2\right)^3}}\Bigg)\Bigg). \tag{65}
$$

Similar analysis can be done for any further higher order mode to obtain the mode evolution equations of the pump and the scattered sideband waves.

III. CONCLUSIONS

The nonlocal theory of SRS coupled with decay instability reveals that SRS reflectivity is reduced significantly due to nonlocal effects and the latter has significant effect on the saturation of SRS since the secondary EPW and ion acoustic wave are non resonant for the SRS. The primary EPW damp fast and undergoes saturation. LDI acts as energy sink for the primary EPW. The maximum amplitude of the primary EPW is around eight times of the amplitude of the secondary EPW. The amplitude of primary EPW increases with time due to the SRS growth but then its amplitude falls due to diversion of energy into the LDI. The successive amplitude maxima of the wave exhibit a gradual decline due to the overall energy transfer from the wave to the plasma electrons via Landau damping.

ACKNOWLEDGMENTS

One of the authors (R.S.) is thankful to Manoj Kumar, K. K. Magesh Kumar, and Plasma Physics Group, IIT Delhi, India for their stimulating discussion and University of Delhi, Acharya Narendra Dev College for financial support.

[1] C. S. Liu and V. K. Tripathi, *Interaction of Electromagnetic Waves With Electron Beams and Plasmas* (World Scientific, Singapore, 1994), p. 88.
[2] W. L. Kruer, *The Physics of Laser Plasma Interaction* (Addison-Wesley, Redwood City, CA, 1988), p. 135.
[3] K. L. Baker, R. P. Drake, B. S. Bauer, K. G. Estabrook, A. M. Rubenchik, C. Labaune, H. A. Baldis, N. Renard, S. D. Baton, E. Schifano, A. Michard, W. Seka, and R. E. Behr, Phys. Rev. Lett. **77**, 67 (1996).
[4] C. Labaune, H. A. Baldis, B. S. Bauer, V. T. Tikhonchuk, and G. Laval, Phys. Plasmas **5**, 234 (1998).
[5] S. Depierreux, C. Labaune, J. Fuchs, D. Pesme, V. T. Tikhonchuk, and H. A. Baldis, Phys. Rev. Lett. **89**, 045001 (2002).
[6] B. Bezzerides, D. F. DuBois, and H. A. Rose, Phys. Rev. Lett. **70**, 2569 (1993).
[7] R. L. Berger, C. H. Still, E. A. Williams, and A. B. Langdon, Phys. Plasmas **5**, 4337 (1998).
[8] D. A. Russel, D. F. Dubois, and H. A. Rose, Phys. Plasmas **6**, 1294 (1999).
[9] D. S. Montgomery, B. B. Afeyan, J. A. Cobble, J. C. Ferna'ndez, M. D. Wilke, S. H. Glenzer, R. K. Kirkwood, B. J. MacGowan, J. D. Moody, E. L. Lindman, D. H. Munro, B. H. Wilde, H. A. Rose, D. F. Dubois, B. Bezzerides, and H. X. Vu, Phys. Plasmas **5**, 1973 (1998).
[10] D. S. Montgomery, J. A. Cobble, J. C. Fernandez, R. J. Focia, R. P. Johnson, N. Renard-LeGalloudec, H. A. Rose and D. A. Russel, Phys. Plasmas **9**, 2311 (2002).
[11] S. Depierreux, J. Fuchs, C. Labaune, A. Michard, H. A. Baldis, D. Pesme, S. Huller, and G. Laval, Phys. Rev. Lett. **84**, 2869 (2000).
[12] J. P. Palastro, E. A. Williams, D. E. Hinkel, L. Divol, and D. J. Strozzi, Phys. Plasmas **16**, 092304 (2009).
[13] T. Umeda, and T. Ito, Phys. Plasmas **15**, 084503 (2008).
[14] T. Kolber, W. Rozmus, and V. T. Tikhonchuk, Phys. Fluids B **5**, 138 (1993).
[15] C. S. Liu and V. K. Tripathi, Phys. Plasmas **3**, 3410 (1996).
[16] A. Salcedo, R. J. Focia, A. K. Ram, and A. Bers, Nucl. Fusion **43**, 1759 (2003).
[17] V. Sajal and V. K. Tripathi, Phys. Plasmas **11**, 4206 (2004).
[18] C. Durfee and H. Milchberg, Phys. Rev. Lett. **71**, 2409 (1993).
[19] F. F. Chen, *Introduction to Plasma Physics and Controlled Fusion*, 2nd ed. (Plenum, New York, 1984), Vol. 1.
[20] V. Sajal, and V. K. Tripathi, J. Phys. D: Appl. Phys. **40**, 453 (2007).