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Impinging stagnation flows

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Jets of different fluid properties impinge head-on. The region near the stagnation point is investigated. Similarity solutions of the Navier–Stokes equations are matched at the interface. It is found that the flow field depends heavily on the ratios of viscosity and density.

Recently, Wang[1] studied the stagnation flow toward a heavier quiescent fluid surface. Since the pressure is not matched at the surface, the interface is kept approximately horizontal by gravity. In the present paper, two dynamic impinging stagnation flows are considered. This situation occurs in transpiration cooling, ablation cooling, and especially injection cooling problems.[2] All parameters will be matched at the interface. We assume the two fluids are almost immiscible with constant properties.

Let the fluid with the lower density be denoted by the subscript 1 and fluid with the higher density be denoted by the subscript 2 (see Fig. 1). The interface is at $y_i = 0$ $(i = 1,2)$. The flow far from the interface is assumed to be potential:

$$u_i = b_i x, \quad v_i = -m b_i y_i, \tag{1}$$

where u, v are velocities in the x, y directions, m equals 1 for two-dimensional flow, and m equals 2 for axisymmetric flow. The constant b_i denotes the strength of the stagnation flow. Using the similarity transformation

$$u_i = b_i x f_i'(\eta_i), \quad v_i = -m\sqrt{\nu_i b_i}\, f_i(\eta_i), \quad \eta_i \equiv y_i/\sqrt{\nu_i/b_i}, \tag{2}$$

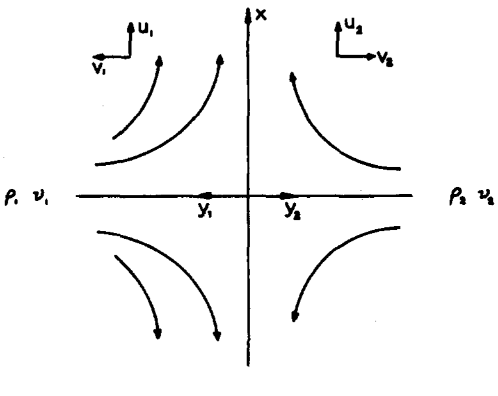

FIG. 1. The coordinate system.

the Navier–Stokes equations become

$$f_i''' + m f_i f_i'' - (f_i')^2 + 1 = 0, \tag{3}$$

$$p_i = p_{0i} - \rho_i (b_i^2 x^2/2 + v_i^2/2 - \nu_i v_{iy}). \tag{4}$$

Boundary conditions are

$$f_i(0) = 0, \quad f_i'(\infty) = 1 \tag{5}$$

and all velocities, stresses, and pressure are matched at every point across the interface. Setting $p_1 = p_2$ at $\eta_i = 0$ in Eq. (4) yields

$$\rho_1 b_1^2 = \rho_2 b_2^2, \tag{6}$$

$$p_{01} - m\rho_1\nu_1 b_1 f_1'(0) = p_{02} - m\rho_2\nu_2 b_2 f_2'(0). \tag{7}$$

Thus for given densities of two impinging jets the stagnation point adjusts to a position such that the local strengths b_i satisfy Eq. (6). Also, from Eq. (7), we see that the inviscid stagnation pressures p_{0i} are not equal; they are modified by

TABLE I. Solutions in terms of initial values.

β	$f''(0), m=1$	$f''(0), m=2$
0	1.232 588	1.311 938
0.1	1.146 6	1.229 1
0.2	1.051 1	1.133 7
0.3	0.946 8	1.026 7
0.4	0.834 1	0.908 7
0.5	0.713 3	0.780 3
0.6	0.584 7	0.642 2
0.7	0.449 0	0.494 7
0.8	0.306 1	0.338 2
0.9	0.156 3	0.173 2
1.0	0	0
1.1	−0.162 8	−0.181 2
1.2	−0.331 7	−0.370 0
1.3	−0.506 6	−0.566 2
1.4	−0.687 4	−0.769 7
1.5	−0.873 9	−0.980 2
2	−1.887 3	−2.131 1
3	−4.276 5	−4.871 6
5	−10.264 7	−11.802 2

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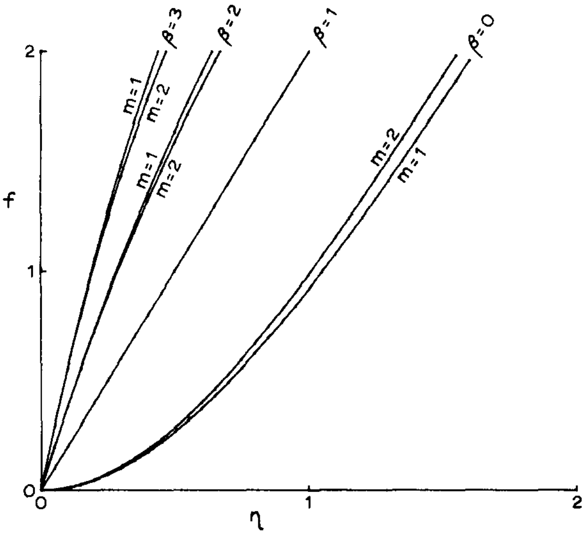

FIG. 2. The function $f(\eta)$ for various β.

viscous terms. Matching velocities and shear, we find

$$b_1 f_1'(0) = b_2 f_2'(0), \tag{8}$$

$$\rho_1 \nu_1 \frac{\partial u_1}{\partial y_1}(0) = -\rho_2 \nu_2 \frac{\partial u_2}{\partial y_2}(0), \tag{9}$$

$$\rho_1 \nu_1^{1/2} b_1^{3/2} f_1''(0) = -\rho_2 \nu_2^{1/2} b_2^{3/2} f_2''(0). \tag{10}$$

Equations (3), (5), (6), (8), and (10) are to be solved.

The problem can be simplified as follows. First, the generic problem

$$f''' + mff'' - (f')^2 + 1 = 0, \tag{11}$$

$$f(0) = 0, \quad f'(0) = \beta, \tag{12}$$

$$f'(\infty) = 1 \tag{13}$$

is solved for given m and β. When $\beta = 0$, the well-known stagnation flows on a solid surface are recovered. Some solutions for $\beta < 1$ are published in Ref. 1 but in the present case it is necessary to extend β to values larger than unity where

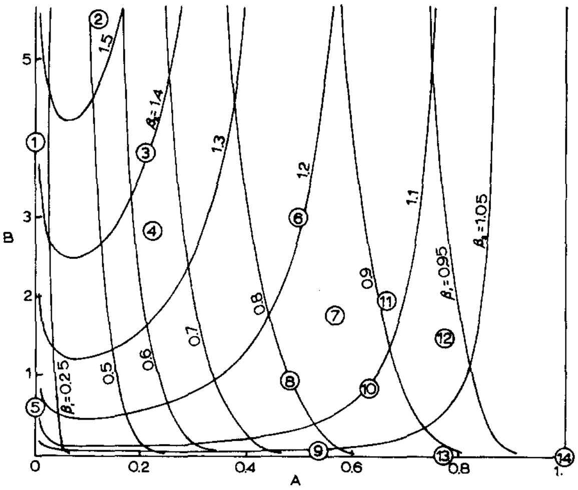

FIG. 3. Curves of constant β_1 or β_2 as functions of A,B ($m = 1$). Sample fluid pairs: (1) SO_2 to water, (2) CCl_4 to Hg, (3) He to Ne, (4) Br to Hg, (5) air to paraffin, (6) Ne to Ar, (7) CH_4 to C_2H_4, (8) H_2 to He, (9) water to H_2SO_4, (10) water to brine, (11) air to CO_2, (12) castor oil to glycerin, (13) water to glycerin, (14) identical fluids.

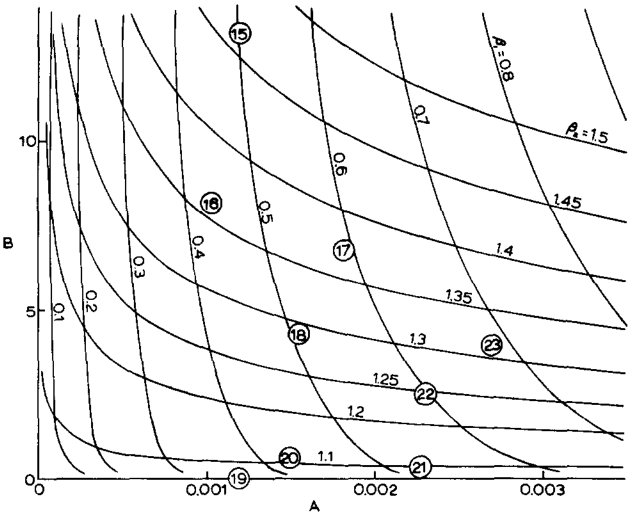

FIG. 4. Expanded regions of Fig. 3 for small A. (15) air to water, (16) air to brine, (17) CO_2 to water, (18) CO_2 to brine, (19) air to glycerin, (20) air to paraffin, (21) CO_2 to paraffin, (22) SO_2 to brine, (23) SO_2 to water ($m = 1$, two dimensional).

$f''(0)$ is negative. The results are shown in Table I. Figure 2 shows the universal profiles $f(\eta)$ for some values of β.

However, both β and $f''(0)$ are not known *a priori*. They are derived from matching conditions (9) and (10). Introduce two nondimensional ratios:

$$A \equiv \rho_1/\rho_2 \leqslant 1, \quad B \equiv \nu_1/\nu_2. \tag{14}$$

Eliminating b_i from Eqs. (6), (8), and (10) yields

$$\beta_1 = A^{1/2} \beta_2, \tag{15}$$

$$f_1''(0) = -A^{1/4} B^{1/2} f_2''(0). \tag{16}$$

Figure 3 shows the values of β_1 and β_2 for $m = 1$ and given A, B. The curves for $m = 2$ differ by a few percent and are not reproduced here. Figure 3 also shows the ratio A, B for some combinations of possible impinging fluids. For each given pair of fluids, there is only one strength ratio and only one set of β_1, β_2 for which steady impingement is possible. The flow field is then completely determined by b_i and β_i. For example, for the impingement of water and brine

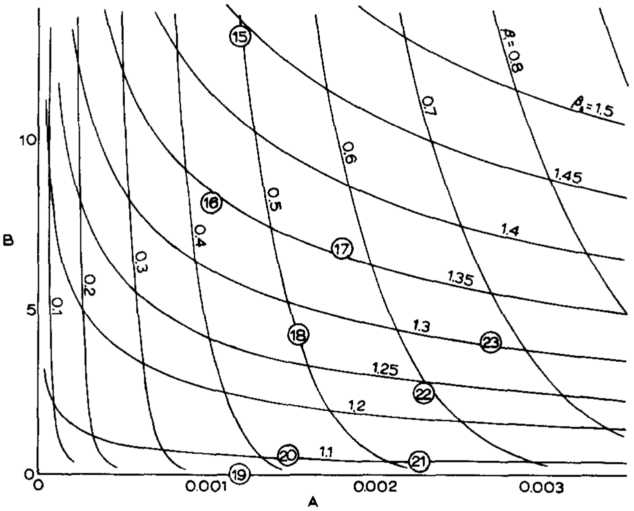

FIG. 5. Same legend as Fig. 4 except $m = 2$ (axisymmetric).

(22.8% NaCl) the densities and viscosities give $A = 0.63$ and $B = 0.86$. From Eq. (6) the strength ratio of the jets near the stagnation point is $b_1/b_2 = 1.26$. From Fig. 4 we find $\beta_1 = 0.87$, $\beta_2 = 1.1$. This means that near the interface brine is speeded up and water is slowed down. The velocity profiles are given by $f(\eta)$ and the pressure distribution is obtained from Eqs. (4) and (7).

For very small density ratios, such as air and water ($A = 0.0012$, $B = 13.16$), Fig. 3 is inconvenient. The expanded region $0<A<0.0035$ is shown in Fig. 4 for $m = 1$. We find in this particular case $\beta_1 = 0.05$, $\beta_2 = 1.46$. Since β_1 is close to zero, the air flow is almost like the Hiemenz stagnation flow towards a solid plate. It is interesting to note that β_2 is not close to one, showing that the water flow cannot be considered inviscid. Because of the small density ratio, the strength of the water jet is about 3.5% of the air jet near the stagnation point. Figure 5 shows the expanded region for $m = 2$. The curves are similar to Fig. 4 except that the axisymmetrical cases yield slightly lower β_1 and β_2 values.

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[2]C. Y. Wang, AIAA J. 2, 178 (1964).

The energy dissipation function for turbulence closure theory

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Approximations for the substantial derivative of the eddy kinetic energy dissipation rate are discussed and compared to those previously proposed, particularly with respect to equilibrium solutions.

Approximations for the substantial derivative $d\epsilon/dt$ of the dissipation rate for eddy kinetic energy have been proposed, e.g., by Launder, Reece, and Rodi.[1] As another approximation, each of the terms T_ϵ, P_ϵ, and ϵ_ϵ appearing on the right-hand side of the rate equation for ϵ can be taken as proportional to the corresponding one for T, P, and ϵ appearing on the right-hand side of the rate equation for kinetic energy u^2:

$$\frac{du^2}{dt} = T + P - \epsilon, \tag{1}$$

$$\frac{d\epsilon}{dt} = T_\epsilon + P_\epsilon - \epsilon_\epsilon, \tag{2}$$

where T and T_ϵ stand for diffusive transport, P and P_ϵ stand for shear production, and ϵ and ϵ_ϵ for the viscous destruction rate. In Eqs. (1) and (2) $u^2 = \langle u_k u_k \rangle$ is twice the eddy kinetic energy, $P = 2\langle u_i u_k \rangle \partial U_i/\partial x_k$ is the turbulence production from mean shear $\partial U_i/\partial x_k$, and $\epsilon = 2\nu\langle \partial u_i/\partial x_k \times \partial u_i/\partial x_k \rangle$ is the molecular dissipation rate. One can indeed take

$$T_\epsilon = C_\epsilon (\epsilon/u^2) T, \tag{3a}$$

$$P_\epsilon = C_{\epsilon 1} (\epsilon/u^2) P, \tag{3b}$$

$$\epsilon_\epsilon = C_{\epsilon 2} (\epsilon/u^2) \epsilon, \tag{3c}$$

where C_ϵ, $C_{\epsilon 1}$, and $C_{\epsilon 2}$ are numerical constants.

Equations (1)–(3) bear a close resemblance to the ones proposed by other authors, e.g., Launder, Reece, and Rodi,[1] who take

$$C_\epsilon = 0.15, \quad C_{\epsilon 1} = 1.44, \quad C_{\epsilon 2} = 1.90. \tag{3d}$$

Primarily this is because the methods used to derive such equations are very similar, being, for the most part, based on dimensional arguments.

Equations (1)–(3) can lead to different kinds of stationary solutions, depending on the type of turbulent flow under consideration and the values for the numerical constants.

Let us first consider the case of a homogeneous equilibrium shear flow (homogeneous shear turbulence), where $d/dt = 0$ and $T = T_\epsilon = 0$. In such a case Eqs. (1)–(3) reduce to

$$P - \epsilon = 0 \tag{4a}$$

and

$$\frac{\epsilon}{u^2}(C_{\epsilon 1} P - C_{\epsilon 2}\epsilon) = 0, \tag{4b}$$

which imply

$$C_{\epsilon 1} = C_{\epsilon 2}. \tag{5}$$

The values chosen for $C_{\epsilon 1}$ and $C_{\epsilon 2}$ by Launder *et al.*[1] [see Eq. (3d)] differ from each other, which in principle would make it impossible for their model to describe homogeneous shear turbulence. This can nevertheless be rendered less crucial if one takes into account that it may almost be impossible to achieve homogeneous shear flow in practice, as there is some evidence from laboratory experiments (see, e.g., Champagne *et al.*[2]) that the turbulence length scale continues to increase downstream and consequently that ϵ simultaneously decreases.[3]

Let us now consider an inhomogeneous equilibrium shear flow, such as shear on an "infinite" medium. This, for