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Combinations of Circular Currents for Producing Uniform Magnetic Field Gradients

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The theory of magnetic field gradient production by a limited number of circular current loops with a common axis is briefly restated. The best positions for combinations of two loops and four loops, carrying currents of equal magnitude, are computed. The best ratio of breadth and depth of winding for coils of rectangular section, approximating most closely to these ideal solutions are also computed.

IN a previous paper[1] combinations of circular currents for producing uniform magnetic fields have been considered in some detail. The combinations here considered are so similar in most of their features that the notation there employed is also suitable for the present discussion.

We can, without loss of generality, require that the magnetic field at the origin vanish, i.e., $H_0=0$. It then follows that the number of loops is even and that the currents in the two hemispheres circulate in opposite directions about the axis. A representative pair out of a total of $t(=p_{\max})$ loops then has

$$r_{t-p+1}=r_p,\quad u_{t-p+1}=-u_p,\quad i_{t-p+1}=-i_p. \quad (1)$$

The potential function for a combination of loops is, as before, in Gaussian units,

$$V=\sum V_p=\frac{2\pi}{c}\Big\{\sum i_p+\sum i_p u_p + \sum_{n=1}^{n=\infty}\frac{1}{n}\sum_p\left[i_p(1-u_p{}^2)r_p{}^{-n}P_n'(u_p)\right]r^nP_n(u)\Big\}, \quad (2)$$

and, as before, we put

$$S_n=\frac{r_1{}^n}{n}\sum_p\left[i_p(1-u_p{}^2)r_p{}^{-n}P_n'(u_p)\right], \quad (3)$$

but now we have

$$I=\sum i_p=0. \quad (4)$$

The magnetic field gradient at the origin is axial and at any point it is obtained by finding the second differential coefficient of V with respect to the axial coordinate $z=ru$. We wish this gradient to be as nearly as possible independent of r, so it will be better to arrange the resulting series in ascending powers of r, rather than in ascending powers of z. This series then takes the form

$$\frac{\partial^2 V}{\partial z^2}=\frac{2\pi}{c}\sum_{n=2}^{n=\infty} S_n r_1{}^{-n}n(n-1)r^{n-2}P_{n-2}(u). \quad (5)$$

For the symmetrical pairs of loops here postulated only even values of n appear in the summation. The term for $n=2$ is already independent of r and contributes the constant gradient we wish to preserve by eliminating terms of higher order. This gradient depends upon the values of u_p, through its dependence upon S_2. The deviation from uniformity of gradient depends upon r^2 unless S_4 be made to vanish by proper choice of the u_p. The equation to be satisfied is

$$(1-u_p{}^2)P_4'(u_p)=0, \quad (6)$$

or

$$7u_p{}^4-10u_p{}^2+3=0. \quad (7)$$

Since the roots $u_1=1$, $u_2=-1$ correspond to an impracticable case we must take the other pair $u_1=(3/7)^{\frac{1}{2}}$, $u_2=-(3/7)^{\frac{1}{2}}$. This solution has already been used in a magnetometer[2] designed in this laboratory.

Still greater uniformity requires both $S_4=0$ and $S_6=0$, and at least two pairs of loops. Suppose we require the same current to flow in all four loops. We have three independent variables at our disposal, u_1, u_2, and r_2/r_1, and can therefore get rid of S_8 in addition to S_4 and S_6. The equations to be solved are

$$\begin{aligned}(1-u_1{}^2)r_1{}^{-4}P_4'(u_1)+(1-u_2{}^2)r_2{}^{-4}P_4'(u_2)&=0,\\ (1-u_1{}^2)r_1{}^{-6}P_6'(u_1)+(1-u_2{}^2)r_2{}^{-6}P_6'(u_2)&=0,\\ (1-u_1{}^2)r_1{}^{-8}P_8'(u_1)+(1-u_2{}^2)r_2{}^{-8}P_8'(u_2)&=0.\end{aligned} \quad (8)$$

[1] L. W. McKeehan, R. S. I. **7**, 150–153 (1936).

[2] L. W. McKeehan, R. S. I. **5**, 265–268 (1934).

TABLE I. *Combinations with identical currents.*

ORDER OF ERROR IN V m	NUMBER OF LOOPS $p$max	COORDINATES OF LOOPS LINEAR r_p/r_1	ANGULAR u_p	GRADIENT EFFICIENCY S_2/I *	NOTES
6	2	1	0.654654	0.210424	(a)
10	4	1	0.834267	0.226471	(b)
		1.024562	0.474688		

(a) $B_1^2/D_1^2=37/44$; $B_1/D_1=0.9170$
(b) $B_1/D_1=2.3965$; $B_2/D_2=1.6636$
* Since I or Σi_p is zero, the divisor here is taken as twice the current in the hemisphere for which u_p is positive. The maximum efficiency possible is only $3^{\frac{1}{2}}/6=0.288675$.

A solution of these equations by the usual method of successive approximations gives results included in Table I.

A coil of finite cross section introduces nonuniformity of gradient, and if the cross section is rectangular, of breadth, B_p, depth, D_p, the nonuniformity is least when

$$\frac{B_p^2}{D_p^2}=\frac{90-665u_p^2+1260u_p^2-693u_p^6}{105-735u_p^2+1323u_p^4-693u_p^6}. \tag{9}$$

As in the corresponding case for uniformity of field there are critical ranges of u_p, within which even such partial compensation is impossible, limited by nearly equal roots of the numerator ($u_p=0.9788$, 0.8076, 0.4559) and of the denominator ($u_p=1$, 0.8302, 0.4688). It will be observed that the four-loop solution has both its values of u_p near excluded ranges, but the indicated ratio of breadth to depth is not so far different from unity as to make construction awkward.

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Limitations of Tubular Ground Glass Joints

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A consideration of the stresses involved in ground glass joints when used in vacuum apparatus indicates that the weakest part of such a joint is the wall of the outer member. For dependable large joints all the dimensions should be increased proportionally. After grinding it is suggested that the surfaces be etched slightly to relieve strains.

THE usual type of ground glass joint does not seem to hold up well in diameters greater than 2 or 3 inches. It is of practical interest therefore to determine the reasons for such breakdown.

The accompanying figure shows a section through such a joint. When the joint is under vacuum conditions there will be a force pushing it together due to the air pressure on the ends of the tubing. The magnitude of this force in the direction of the axis is, approximately

$$F=P\pi a^2,$$

where P is the atmospheric pressure and a is the radius of the joint. Since we are interested only in the order of magnitude of the stresses involved, the average radius, a, will be used throughout.

The radial component of the force tending to split the joint open is

$$F_r=P\pi a^2 \cot\theta,$$

where θ is the angle of the joint as shown. The pressure outward in the radial direction is therefore

$$P_r=Pa\cot\theta/2l. \tag{1}$$

Consider now an element of the joint limited by planes perpendicular to the axis and of length dz and width $ad\phi$.

The sum of the components, perpendicular to a diameter, of all the radial forces acting over the elemental areas, $ad\phi dz$ is

$$\int_0^\pi P_r a\sin\phi d\phi dz=P_r adz.$$

If the thickness is i, the tensile stress is given by

$$T=P_r adz/idz=P_r a/i.$$