AIP Journal of Applied Physics

Derivation of a modified Fermi-Dirac distribution for quantum dot ensembles under nonthermal conditions

Huw D. Summers and Paul Rees

Citation: Journal of Applied Physics **101**, 073106 (2007); doi: 10.1063/1.2709614
View online: http://dx.doi.org/10.1063/1.2709614
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/101/7?ver=pdfcov
Published by the AIP Publishing

Derivation of a modified Fermi-Dirac distribution for quantum dot ensembles under nonthermal conditions

Huw D. Summers
School of Physics and Astronomy, Cardiff University, 5 The Parade, Cardiff CF24 3YB, United Kingdom

Paul Rees
Multidisciplinary Nanotechnology Centre, School of Engineering, University of Wales Swansea, Singleton Park, Swansea, SA2 8PP, United Kingdom

(Received 3 August 2006; accepted 9 January 2007; published online 9 April 2007)

Semiconductor quantum dot lasers offer significant advantages over traditional quantum well devices. However, the advantages due to the discrete density of states of a structure confined in all three spatial dimensions are usually not fully realized because of problems associated with the capture of carriers into the discrete states. In this paper we use a simple rate equation model to identify the processes that limit the performance of a quantum dot laser. This simplistic approach, while lacking the rigor of more complex models, allows us to develop a physical understanding of how the properties of the quantum dot electronic states effect the operation of a laser. The existence of a thermal, Fermi-Dirac distribution of carriers is shown to exist only when there are no recombination processes (either radiative or nonradiative). In a quantum well laser the rate of thermalization is much faster than the carrier loss processes and therefore the distribution appears to be close to Fermi-Dirac; however, in a quantum dot structure the slower capture/escape rates can cause nonthermal carrier distributions. The interplay of the radiative recombination and capture and escape rates in the dots is shown to define the mode of operation of the laser. An identity, derived simply in terms of the rates of carrier escape and spontaneous recombination and a confinement energy, predicts whether the carrier population is coupled across the dot ensemble. This will determine whether a semiconductor quantum dot laser exhibits single mode operation. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2709614]

I. INTRODUCTION

Quantum dot lasers are now well established and much effort has been channeled into understanding their operating characteristics. From a theoretical viewpoint the discrete density of states provided by zero-dimensional (0D) structures offers significant advantages for the performance of semiconductor laser diodes such as temperature insensitive operation, ultralow threshold current, and narrow linewidth.[1–3] Most of these advantages have been realized in practice and in many aspects quantum dot lasers have outperformed quantum well devices.[2,3] However, the link between theory and experiment has not been straightforward and the improved performance can seldom be linked wholly to simple arguments based on localization of electronic states. A particularly important aspect of their operation which has emerged is the degree of coupling between the isolated, localized electronic states of the dots and their interaction with the surrounding "wetting layer." In order to establish global populations within the laser charge carriers must be efficiently captured into the discrete dot states, and cross coupling across the dot ensemble maintained via continual escape to, and capture from, the wetting layer. The degree to which this can be achieved controls the fundamental operation of dot lasers as it determines whether thermal population distributions can be established[4] and affects the mode spectra through spatial and spectral hole burning.[5,6]

The dynamics of the intradot coupling have been modeled in a great deal of detail and are now fairly well understood.[7–10] Typical models are computationally complex, including calculations of the capture and escape rates of carriers into and out of the quantum dots, the involvement of the two-dimensional (2D) wetting layer, and solution of the spatial wave equation in the laser cavity.[11] Because of this complexity it is often difficult to gain a clear picture of dot laser operation in terms of basic physical processes. An extension of accepted laser models valid for higher-dimensional systems to these 0D systems also brings difficulties. While this may not be a problem within the specialist quantum dot community it does impose a barrier to understanding in the wider laser community, or indeed to the general physicist or engineer who wishes to gain some appreciation of how quantum dot lasers work.

In this paper we present a simpler analytic description of dot laser operation. This is an attempt to provide clear links to the underlying physics of the 0D system. We therefore sacrifice quantitative, predictive accuracy but obtain a model of the laser based on general physical principles. We employ a steady-state analysis of the time-dependent rate equations for charge carriers in the quantum dots and photons in the laser cavity. By consecutively introducing the terms for carrier capture/escape, spontaneous recombination, and stimulated recombination we identify the cumulative effect of these processes on the performance of semiconductor quantum dot lasers. By determining the relative strengths of these terms we are able to define criteria for the breakdown of Fermi-Dirac carrier distributions and derive an alternative

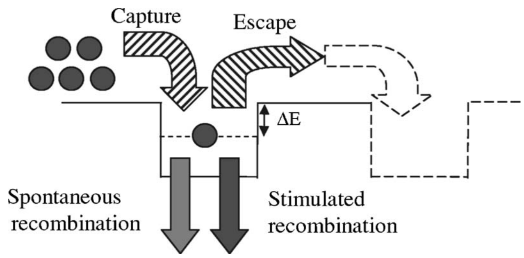

FIG. 1. Schematic showing the four processes that determine the occupation by electrons of an individual quantum dot in the semiconductor structure.

function valid in the nonthermal regime. This modified population distribution also leads to a simple identity, independent of dot composition and geometry, which determines the temperature of transition between thermal and nonthermal operations.

II. THE MODEL

The schematic in Fig. 1 shows the four processes that determine the occupation, by electrons, of an individual quantum dot in the semiconductor structure. First, we have the capture of electrons from the surrounding wetting layer and also the possibility of electron escape from the quantum dot into the wetting layer. Electrons in the dot can recombine spontaneously (radiatively or nonradiatively), or if there is sufficient gain to overcome the cavity losses they can recombine via stimulated emission. The equilibrium occupancy of electrons in a specific dot confined state is defined by the rates of these four processes.

To determine the occupation of an ensemble of identical quantum dots and the subsequent operating performance of the semiconductor laser we use rate equations to describe the electrons in the quantum well wetting layer, electrons in a confined energy state in the quantum dots, and photons in the laser cavity,

$$\frac{d(f_{\mathrm{WL}}N_{\mathrm{WL}})}{dt}=\frac{I(1-f_{\mathrm{WL}})}{e}-\frac{f_{\mathrm{WL}}N_{\mathrm{WL}}(1-f_{\mathrm{QD}})}{\tau_d}+\frac{N_{\mathrm{QD}}f_{\mathrm{QD}}(1-f_{\mathrm{WL}})}{\tau_u},\tag{1}$$

$$\frac{d(f_{\mathrm{QD}}N_{\mathrm{QD}})}{dt}=\frac{f_{\mathrm{WL}}N_{\mathrm{WL}}(1-f_{\mathrm{QD}})}{\tau_d}-\frac{N_{\mathrm{QD}}f_{\mathrm{QD}}(1-f_{\mathrm{WL}})}{\tau_u}-\frac{N_{\mathrm{QD}}f_{\mathrm{QD}}}{\tau_{\mathrm{spon}}}-G(2f_{\mathrm{QD}}-1)S,\tag{2}$$

$$\frac{dS}{dt}=G(2f_{\mathrm{QD}}-1)S-\frac{S}{\tau_p}+\beta\frac{N_{\mathrm{QD}}f_{\mathrm{QD}}}{\tau_{\mathrm{spon}}},\tag{3}$$

where N_{QD} is the number of identical dots in the laser structure, i.e., the number of confined electron dot states; N_{WL} is the number of wetting layer (quantum well) electron states; S is the number of photons; τ_p is the photon lifetime defining the laser cavity loss; and β is the spontaneous emission coupling factor. To simplify our analysis we will only consider the lowest confined energy level in the quantum dots. As we are interested in the coupling between the dot and wetting layer and not the distribution in the wetting layer, we use the simplifying assumption that the wetting states N_{WL} are all at an energy ΔE above the dot state.

f_{QD} is the probability of occupancy of the dot states and f_{WL} is the probability of occupancy of the wetting layer states defined as

$$f_{\mathrm{QD}}=\frac{\text{number of electrons in the dots}}{N_{\mathrm{QD}}},\tag{4}$$

$$f_{\mathrm{WL}}=\frac{\text{number of electrons in the wetting layer}}{N_{\mathrm{WL}}}.\tag{5}$$

τ_d and τ_u are the lifetimes for electron capture into the quantum dots from the wetting layer and the electron escape from the dots to the wetting layer, respectively. These lifetimes are highly dependent on the activation energy ΔE, i.e., the energy difference between the confined electron energy state of the dot and the bottom of the wetting layer conduction band (see Fig. 1). Therefore these rate equations define the dynamics of electrons within dots with a specific confined energy level, i.e., dots of a particular geometry and composition, etc. The nature of quantum dot fabrication means that typical samples include dots with a wide distribution of size and composition, leading to an inhomogeneous spectrum of quantized energy levels. Using these equations, and simply varying the capture and escape lifetimes, we can determine the effect of the changing activation energy across an ensemble of dots on the laser operation.

τ_{spon}, the spontaneous recombination lifetime, and G, the optical gain parameter, are normally weakly dependent on the energy levels in the quantum dots; however, we will assume they are constant with respect to the quantum dot size and composition, which simplifies our analysis significantly. Also, we only consider the dynamics of electrons in the laser structure, which is typical of most laser models. This approximation assumes that the slower capture rates associated with electrons dominate the dynamics of the laser. In making this assumption we also note that the recombination must be monomolecular or excitonic (i.e., if an electron is present in the dot a hole is also assumed to be present) and charge neutrality is enforced within the dots. This allows us to neglect the dynamics of the holes in our analysis.

III. ANALYSIS

Rigorous analytical investigations of quantum dot population distributions have been published.[12] Here we specifically seek an analytic expression to describe the population statistics under conditions where thermal equilibrium breaks down. We use the steady-state solutions of the rate equation to determine the equilibrium occupancy of the quantum dots,

$$0=\frac{f_{\mathrm{WL}}N_{\mathrm{WL}}(1-f_{\mathrm{QD}})}{\tau_d}-\frac{N_{\mathrm{QD}}f_{\mathrm{QD}}(1-f_{\mathrm{WL}})}{\tau_u}-\frac{N_{\mathrm{QD}}f_{\mathrm{QD}}}{\tau_{\mathrm{spon}}}-G(2f_{\mathrm{QD}}-1)S.\tag{6}$$

In order to identify the relative impact of each of the electron capture/loss processes we introduce them one by one into the rate equations.

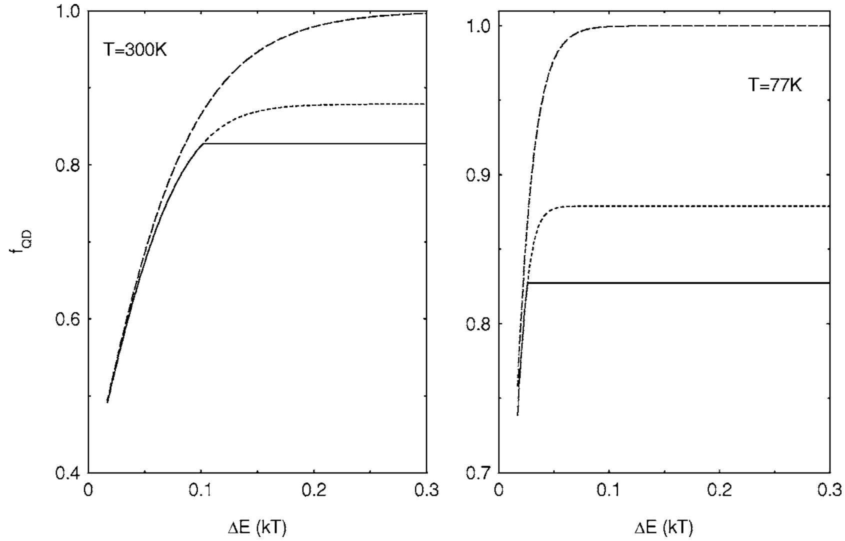

FIG. 2. The probably of occupancy, f_{QD}, of the quantum dot states vs the activation energy, $\Delta E/kT$, for temperatures of 300 and 77 K. (Dashed line) no electron recombination; (dotted line) with spontaneous recombination; (solid line) with spontaneous and stimulated recombination.

A. No spontaneous or stimulated emission

We start our analysis by considering the case where the spontaneous and stimulated recombination terms are ignored. Although this does not correspond to any physically meaningful situation it highlights the role of carrier capture and escape in establishing a thermal population distribution across a dot ensemble. In this case the only rates included in the carrier equation are those describing the downward transition (or capture) rate and the upward (escape) rate of carriers between the wetting layer and the quantum dot electronic states, i.e.,

$$0 = \frac{f_{\mathrm{WL}} N_{\mathrm{WL}} (1 - f_{\mathrm{QD}})}{\tau_d} - \frac{N_{\mathrm{QD}} f_{\mathrm{QD}} (1 - f_{\mathrm{WL}})}{\tau_u}. \tag{7}$$

This gives an expression for the carrier occupancy, f_{QD} of the quantum dot in terms of the carrier capture and escape rates:

$$f_{\mathrm{QD}} = \frac{1}{1 + \dfrac{N_{\mathrm{QD}}(1 - f_{\mathrm{WL}})}{f_{\mathrm{WL}} N_{\mathrm{WL}}} \dfrac{\tau_d}{\tau_u}}. \tag{8}$$

At this point we note that the capture and escape rates for electrons interacting with a phonon (boson) bath at a temperature T can be described by[7]

$$\tau_u = \tau_0 \{ e^{\Delta E/kT} - 1 \} \tag{9}$$

and

$$\frac{\tau_d}{\tau_u} = \exp\left(-\frac{\Delta E}{kT} \right), \tag{10}$$

where τ_0 represents the electron thermalization lifetime due to spontaneous emission of phonons, and where the activation energy, ΔE, is the confining potential in the dot. Combining Eqs. (8) and (10) gives an occupancy factor of

$$f_{\mathrm{QD}} = \frac{1}{1 + \gamma \exp\left(-\dfrac{\Delta E}{kT} \right)}, \tag{11}$$

where

$$\gamma = \frac{N_{\mathrm{QD}}(1 - f_{\mathrm{WL}})}{f_{\mathrm{WL}} N_{\mathrm{WL}}}. \tag{12}$$

Therefore we obtain a Fermi-Dirac distribution for the equilibrium occupancy. This is as expected since the model considers a closed system (no carrier loss) thermally coupled to the wetting layer via interaction with a phonon bath. The assumptions made so far only allow two processes: the escape of electrons from the quantum dots to the wetting layer and the capture of electrons from the wetting layer to the quantum dots. Electrons spend an infinite time in the wetting layer/dot system and an electron in any dot energy state can scatter into any other dot state via the wetting layer. In this way we have total coupling between all of the dot states and the wetting layer leading to a global electron distribution.

If we assume a constant occupancy for the wetting layer, f_{WL}, we can vary the confining potential in order to assess the state occupancy across a size-broadened dot ensemble. This is plotted in Fig. 2, at temperatures of 300 and 77 K (dashed line) for a wetting layer occupancy of 0.01. For this calculation, we have assumed that the number of dots in the laser structure is 10^6 and we have only one confined electron state per dot. The number of electronic states in the wetting layer is taken to be 10^8, giving a typical ratio of densities of states of 1:100 (Ref. 13). The values of τ_0 and τ_{spon} are taken to be 3 ps and 0.3 ns, respectively. This electron occupancy is described by a Fermi-Dirac distribution since the dots are in thermal equilibrium with each other via their interaction with the wetting layer. The occupancy approaches unity as the confining potential is increased because of the concentration of electrons in the dots rather than the wetting layer.

In this simple case we have ignored all loss terms. This is clearly at odds with the first equation describing the carrier occupancy of the wetting layer where the sum of the escape and capture terms must equal the pumping rate. However, by defining a wetting layer occupancy we circumvent the need to use this equation to define a wetting layer occupancy (and therefore quantum dot occupancy) in terms of an injection current I. Physically, this is equivalent to instantaneously injecting electrons and allowing them to equilibrate with no further injection or loss.

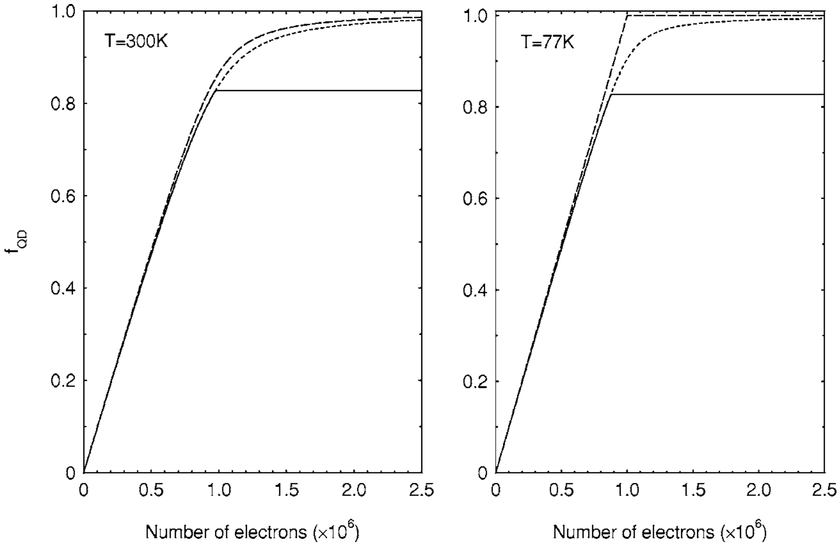

FIG. 3. The probably of occupancy, $f_{\rm QD}$, of the quantum dot states vs the number of injected electrons for temperatures of 300 and 77 K. (Dashed line) no electron recombination; (dotted line) with spontaneous recombination; (solid line) with spontaneous and stimulated recombination.

The electrons in the quantum dot laser are distributed between the dot states and the wetting layer, however, only the electrons in the dot states contribute to gain to obtain lasing. The total number of carriers injected into the system is given by

$$N_{\rm total} = f_{\rm QD} N_{\rm QD} + f_{\rm WL} N_{\rm WL}, \tag{13}$$

and it useful to plot the carrier occupancy of the dot states as a function of the total number of injected carriers. This is shown in Fig. 3 for a typical activation energy of 0.1 eV (dashed line) for temperatures of 300 and 77 K. At low numbers of injected electrons, the majority thermalize into the quantum dot states, however, as the number of electrons injected increases they successively populate the wetting layer especially at high temperature.

B. Including spontaneous emission

If we now include the spontaneous recombination term, the steady-state carrier rate equation becomes

$$0 = \frac{N_{\rm WL} f_{\rm WL}(1 - f_{\rm QD})}{\tau_d} - \frac{N_{\rm QD} f_{\rm QD}(1 - f_{WL})}{\tau_u} - \frac{N_{\rm QD} f_{\rm QD}}{\tau_{\rm spon}}, \tag{14}$$

and we can introduce a reduced lifetime

$$\frac{1}{\tau_{\rm eff}} = \frac{1 - f_{\rm WL}}{\tau_u} + \frac{1}{\tau_{\rm spon}}, \tag{15}$$

which describes the rate of carrier loss from the quantum dot, allowing us to write

$$f_{\rm QD} = \frac{1}{1 + \gamma\left\{\exp\left(-\dfrac{\Delta E}{kT}\right) + \dfrac{\tau_d}{\tau_{\rm spon}}\dfrac{1}{(1 - f_{\rm WL})}\right\}}, \tag{16}$$

using Eq. (10),

$$f_{\rm QD} = \frac{1}{1 + \gamma \exp\left(-\dfrac{\Delta E}{kT}\right)\left\{1 + \dfrac{\tau_u}{\tau_{\rm spon}}\dfrac{1}{(1 - f_{\rm WL})}\right\}}. \tag{17}$$

$f_{\rm QD}$ in this case is plotted versus the activation energy (the dotted line) in Fig. 2 and also versus the number of injected carriers in Fig. 3. The presence of spontaneous recombination reduces the state occupancy due to the factor

$$\frac{\tau_u}{\tau_{\rm spon}}\frac{1}{(1 - f_{\rm WL})} \tag{18}$$

in the denominator of Eq. (17). Equation (17) is analogous to the modified Fermi function first derived by Jiang and Singh.[14] The physical interpretation of this is straightforward: the loss of carriers from the system via recombination will reduce the occupancy. The degree of this reduction depends on the rate of loss due to spontaneous recombination relative to the rate of escape from the dot to the wetting layer. The term $1 - f_{\rm WL}$ in the denominator of Eq. (17) represents a reduction in the electron escape rate due to the filling of the wetting layer, which limits the number of available states for the escape process. It is interesting to note that the strict interpretation of Eq. (17) is that a thermal carrier distribution can never be achieved in a quantum dot system. From a theoretical point of view this must be the case. The Fermi-Dirac distribution is a specific example of a grand canonical distribution and is derived under the assumption of an equal *a priori* probability of occupancy of each of the system microstates. In the context of an ensemble of quantum dots, all the available quantum dot states act as a thermal and particle reservoir for any one specific dot state. For the electrons in dot states that recombine, the interdot coupling is reduced and so a global thermal equilibrium cannot be established.

In practice it is the relative rate of the carrier loss ($\tau_{\rm spon} \sim$ ns) and carrier redistribution ($\tau_u \sim$ ps) that is important, as defined in Eq. (17) and shown in Figs. 2 and 3. Provided that the confinement potential is $< 2 - 3$ kT the escape of carriers is very rapid and a pseudothermal equilibrium can be established before recombination takes place.

Problems do arise for deep dot states from which the carrier escape time can be tens of picoseconds. The fundamental issue here is not unique to quantum dot systems; Eq. (17) can also be applied to a quantum well structure. However, in the 2D case interstate coupling takes place via scattering mechanisms with a time constant <100 fs. Thus, spontaneous recombination occurring over ns time scales can be neglected and a Fermi-Dirac distribution assumed.

We note that the inclusion of the spontaneous recombination term makes the rate equation for the number of electrons in the quantum dots and the wetting layer self-consistent. In the steady state we obtain the familiar balance between the electron injection and the loss of electrons due to spontaneous recombination.

C. Including stimulated emission

In terms of the operation of a laser device the analysis so far describes the subthreshold behavior. To describe the lasing regime we add stimulated recombination to the rate equation for the occupancy of the quantum dot states. This requires knowledge of the steady-state photon number S derived from Eq. (3), i.e.,

$$0 = G(2f_{\mathrm{QD}} - 1)S - \frac{S}{\tau_p} + \beta \frac{N_{\mathrm{QD}} f_{\mathrm{QD}}}{\tau_{\mathrm{spon}}}, \tag{19}$$

giving

$$S = \frac{\beta (N_{\mathrm{QD}} f_{\mathrm{QD}} / \tau_{\mathrm{spon}})}{(1/\tau_p) - G(2f_{\mathrm{QD}} - 1)}, \tag{20}$$

which is solved simultaneously with Eq. (2) to obtain the occupancy factor. This factor is plotted versus the activation energy (the solid line) for a fixed wetting layer occupancy factor of 0.01 in Fig. 2 and for a fixed activation energy of 0.1 eV as a function of the number of injected carriers in Fig. 3. As the carrier number increases the dot occupancy at a specific energy increases until the point at which there is sufficient gain to achieve lasing, i.e., the rate of stimulated recombination equals the rate of photon loss:

$$G(2f_{\mathrm{QD}} - 1)S = \frac{S}{\tau_p}. \tag{21}$$

At this point stimulated emission becomes the dominant process since this rate is much faster than the other three processes. Therefore any further carriers captured by the quantum dot are immediately lost through stimulated emission and the carrier occupancy is pinned at the threshold value. The threshold value of f_{QD} determined from Eq. (21) is

$$f_{\mathrm{QD}} = \frac{1 + G\tau_p}{2G\tau_p}, \tag{22}$$

and is dependent on the photon losses and the gain in the system.

It is clear from the foregoing analysis and from experimental observations that the relative populations of carriers in the wetting layer and the quantum dots are key to understanding the system. Figure 4 shows the electron number in the quantum dot structure and the quantum well wetting layer plotted versus the total number of electrons injected

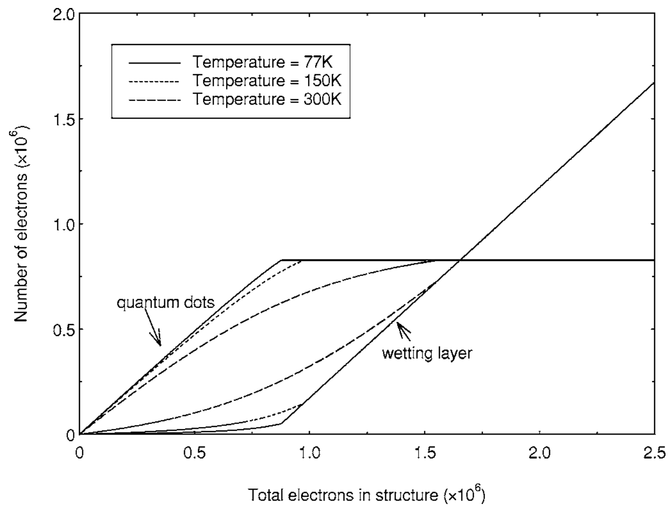

FIG. 4. The number of electrons in the quantum dot states and the quantum well wetting layer as a function of the total number of injected electrons for three different temperatures.

into the laser. As the injected electron number is increased the electron population in the quantum dot structure increases until the electron occupation pins, due to stimulated emission (if the gain overcomes the losses), or all the dot states, are occupied. When the activation energy is large or the temperature is low then injected electrons are effectively captured into the quantum dots; however, if the temperature is high or the activation energy low then the escape rate of carriers from the dots means a significant proportion of carriers occupy the wetting layer.[13] At low temperatures the number of carriers in the dots increases linearly as all electrons added to the system occupy the dot states until saturation, when all further electrons occupy the wetting layer. This abrupt change in the number of electrons in the dots naturally results in abrupt changes in the dot occupancy f_{QD} as observed in Fig. 3(b). Figure 4 also highlights the effect of the wetting layer on the occupancy of the quantum dots. At high temperatures, or if the activation energy is low (i.e., shallow energy levels), then the wetting layer starts to become increasingly occupied taking electrons that would have occupied the quantum dots.

At this point, an important conclusion can be drawn from Fig. 3, where the quantum dot occupancy is plotted versus the activation energy for two temperatures and for a fixed wetting layer occupancy. When the activation energy is small the quantum dot occupancy is too low to achieve lasing; the proximity of the quantum well wetting layer to the dot energy level means the electrons occupy the wetting layer rather than the dots. A typical quantum dot structure would consist of an ensemble of different size dots resulting in different activation energies. Therefore, according to Fig. 2(a) all quantum dots with activation energies below approximately 0.1 eV would not lase at 300 K, while only those below 0.03 eV would not lase at 77 K.

Clearly the occupancy of the quantum dots and, therefore, the performance of the laser diode, are defined by the rates of carrier loss and generation at the operating point of the laser. In the steady state the four rates in the carrier rate

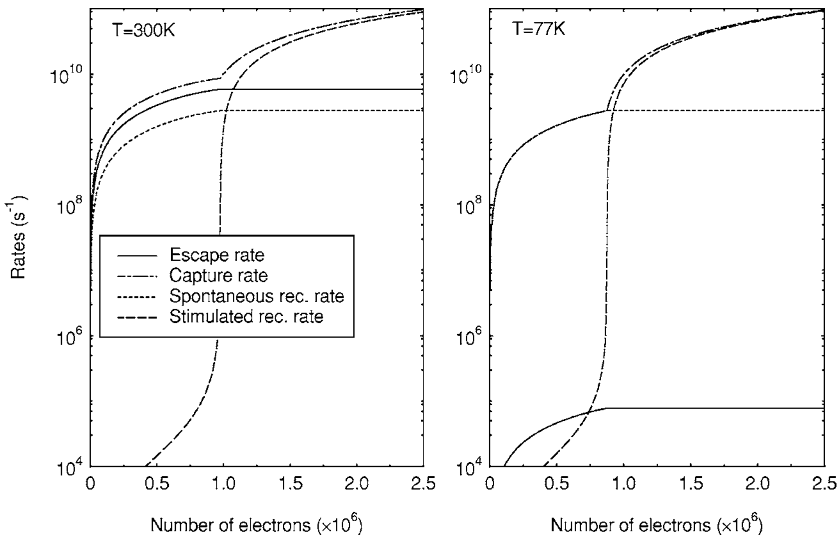

FIG. 5. The capture, escape, and recombination rates as a function of the total number of electrons injected into the dot laser for temperatures of 300 and 77 K.

equation must sum to zero and it is the balance of these rates which defines the occupancy of the quantum dot states:

$$0 = R_{\text{down}} - R_{\text{up}} - R_{\text{spon}} - R_{\text{stim}}. \tag{23}$$

Figure 5 shows the four loss/generation rates as a function of the number of injected carriers at two different temperatures of (a) 300 and (b) 77 K. It is clear that the stimulated emission rate determines the dependence of the other three rates on the injected carrier number. Below the lasing threshold the stimulated emission rate is negligible, however, above threshold the stimulated emission rate increases dramatically as the laser turns on. When lasing, the very fast stimulated emission rate causes the carrier occupancy of the quantum dots to pin at the threshold value. Consequently, the spontaneous emission rate and the carrier escape rate also become fixed. The main difference between the rates at 300 and 77 K is the carrier escape rate, which is reduced by four orders of magnitude at the lower temperature, due to the coupling with the phonon bath [Eq. (10)].

The performance of quantum dot lasers is strongly related to the relative strengths of the carrier escape process, which drives interdot coupling, and the recombination processes, which inhibit it. Below laser threshold the establishment of a thermal equilibrium carrier distribution requires $\tau_u \ll \tau_{\text{spon}}$. In the lasing regime if $\tau_{\text{stim}} < \tau_u$ then again there will be limited coupling between dots and this leads to spatial and spectral hole burning[5,6] due to the localized depletion of electrons within the dots. This produces associated power saturation and multimode operation[15] as the dot ensemble behavior changes from that of a homogeneous to an inhomogeneous system. The limitation both below and above laser threshold is the loss of electrons via recombination before they can redistribute within the ensemble. The drastic reduction in recombination lifetime at laser threshold has a particularly severe effect in quantum dot lasers compared to quantum well devices because of the relatively slow escape times. To elucidate these effects we have calculated the ratio of the spontaneous emission rate and the escape rate, and the ratio of the stimulated emission and the carrier escape rate. If these ratios are greater than 1, then the probability that carriers will be lost from the system is greater than the probability of cross coupling and thermalization between the dots,[15] and so these ratios are useful indicators of when thermal population distributions can no longer be maintained.

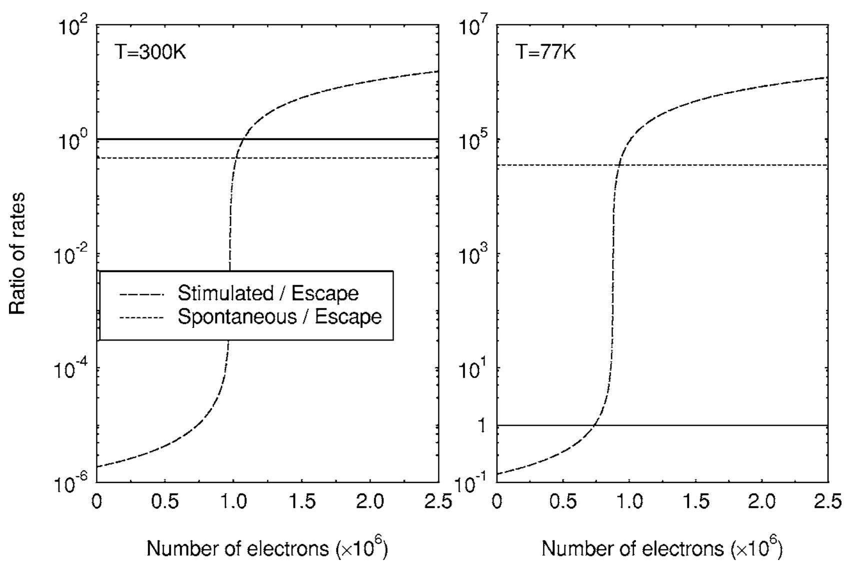

FIG. 6. The ratio of stimulated recombination and electron escape rate and the ratio of spontaneous recombination and electron escape rate as a function of the total number of electrons injected into the dot laser for temperatures of 300 and 77 K.

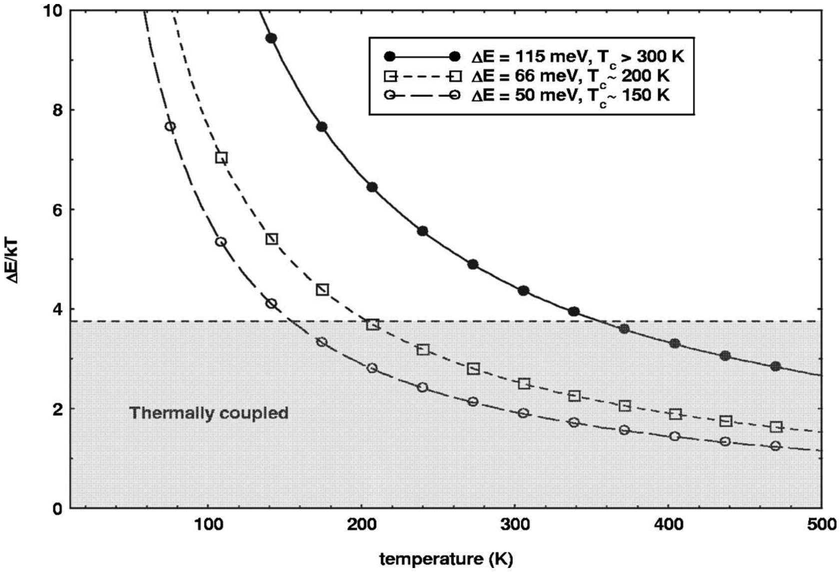

FIG. 7. $\Delta E/kT$ as a function of temperature and activation energy.

The ratios are plotted in Fig. 6 for temperatures of (a) 300 and (b) 77 K, and a confinement energy of 0.1 eV. The spontaneous recombination and escape rate are both linearly dependent upon the number of electrons and so their ratio is a constant. At 300 K the ratio is below 1, indicating that in this temperature range the thermal energy available to the carriers is sufficient to maintain a rapid carrier escape from the dot states. At 77 K the ratio is $\sim 10^4$ and so carrier recombination dominates and there is negligible cross coupling in the dot ensemble via electron escape and recapture in other dots. Measurements on quantum dot laser systems clearly show this behavior with nonthermal carrier distributions present at low temperature.[4]

Obviously the ratio of stimulated recombination and escape is highly nonlinear showing an abrupt increase at laser threshold. At 300 K the ratio is well below 1 at threshold and so the system can initially maintain thermal distributions in the lasing regime. Eventually, at higher carrier injection, the stimulated recombination dominates and will produce nonthermal distributions via hole-burning effects. At 77 K we see that the ratio is above 1 before threshold is reached, indicating that fully coupled dot lasing can never be achieved. This balance manifests itself in the mode distributions seen in quantum dot lasers. At room temperature, close to threshold, lasing can be achieved in a single mode as cross coupling in the dots feeds the excitation energy into one lasing transition. However, at higher injection multimode lasing is always seen. At low temperature multimode behavior is present from threshold.[15]

It is well known (e.g., Ref. 12) that for homogeneous operation at room temperature the minimum requirement is that the escape (coupling) rate must be greater than the spontaneous recombination (loss) rate,

$$R_{\mathrm{up}} > R_{\mathrm{spon}} \quad \text{or} \quad \frac{N_{\mathrm{QD}} f_{\mathrm{QD}}}{\tau_u} > \frac{N_{\mathrm{QD}} f_{\mathrm{QD}}}{\tau_{\mathrm{spon}}},$$

giving

$$\tau_{\mathrm{spon}} > \tau_u. \tag{24}$$

The relevance of this inequality is illustrated in our expression for the nonequilibrium Fermi function [Eq. (17)], where it controls the extent of the modification. Substituting for

$$\tau_u = \tau_0 \{ e^{\Delta E/kT} - 1 \}$$

[Eq. (9) as shown earlier in this article], which comes from the assumption of a thermal bath of bosons with τ_0 representing the carrier thermalization lifetime due to spontaneous emission of phonons,[7] we obtain (using $\tau_{\mathrm{spon}}/\tau_0 \gg 1$)

$$\frac{\Delta E}{kT} < \ln\left(\frac{\tau_{\mathrm{spon}}}{\tau_0}\right), \tag{25}$$

the minimum ratio of dot activation energy to thermal energy required to preserve homogeneous operation at a given temperature. It is interesting to note that we have lost the restraint of the knowledge of the carrier density and therefore the operating parameters of the laser. Therefore this provides a fundamental parameter with global applicability (independent of dot structure and composition) which allows us to assess whether the dot system will have a thermal distribution of electrons with knowledge only of τ_{spon} and τ_0. The relative rates of spontaneous emission and electron escape produce the breakdown of coupling. This leads to a nonthermal electron distribution and changes the occupation probability from a Fermi-Dirac distribution as described by Eq. (16). We note that this analysis considers only one electronic level in each quantum dot and so is valid for the loss of coupling between ground and excited states or between dot states and wetting layer. Despite being derived from a simplified rate analysis Eq. (25) provides a robust indicator of when interdot coupling breaks down and has real quantitative value.

To illustrate this, the ratio $\Delta E/kT$ is plotted in Fig. 7 as a function of temperature for three activation energies which correspond to measured values quoted in the literature [50 meV (Ref. 4), 66 meV (Ref. 16), and 115 meV (Ref. 17)]. The gray shaded area indicates the operating regime within which homogeneous operation can be maintained, i.e., $<\ln(\tau_{spon}/\tau_0)$, where typical values of 7 and 300 ps have been used for τ_0 and τ_{spon}, respectively. The temperatures required to maintain thermal carrier distributions are reported as 150 K in Ref. 4, 200 K in Ref. 15, and >300 K in Ref. 17. These are in good agreement with the analytical prediction depicted in Fig. 7. This is remarkable given that the referenced data is from different dot systems with differing emission wavelengths.

We note that having derived expressions for the population distribution under both thermal and nonthermal conditions [Eqs. (8) and (17)] we are able to go further than just stating a criteria for nonequilibrium operation. Equation (18) gives a quantitative measure of the modification of the Fermi-Dirac function and therefore the equilibrium statistics. Minimization of Eq. (18) naturally leads to the inequality $\tau_{spon} > \tau_u$, the criteria for thermal equilibrium operation.

IV. SUMMARY

In conclusion, we have investigated the effect of carrier recombination and escape and capture rates on the occupancy of electrons in the electronic states of the dots using rate equations describing electrons in the quantum dots, the quantum well wetting layer, and photons in the laser cavity. Under steady-state conditions the rates of carrier loss and capture in the quantum dots lead to a Fermi-Dirac energy distribution in the ensemble. However, we show that the Fermi-Dirac distribution is modified in the presence of any electron loss process; for example, spontaneous emission. This occurs in all laser structures but is more evident in quantum dot structures as the mismatch between escape and capture into the dot electronic states becomes comparable with the loss rate under certain circumstances.

The rate of carrier escape from the quantum dot compared with the spontaneous recombination rate at turn on is shown to determine whether the electron states couple and the laser operates in single mode. Our analysis provides a generic identity independent of dot structure and composition which allows us to assess whether the dot system will be in thermal equilibrium.

ACKNOWLEDGMENT

The authors would like to thank Professor Peter Blood for useful discussions and comments on the manuscript.

[1] M. Grundmann, Physica E (Amsterdam) **5**, 167 (2000).
[2] D. G. Deppe, D. L. Huffaker, S. Csutak, Z. Zhou, G. Park, and O. E. Shchekin, IEEE J. Quantum Electron. **35**, 1238 (1999).
[3] D. Bimberg, N. Kirkstaedter, N. N. Ledentsov, Zh. I. Alferov, P. S. Kop'ev, and V. M. Ustinov, IEEE J. Sel. Top. Quantum Electron. **3**, 196 (1997).
[4] H. D. Summers, J. D. Thomson, P. M. Smowton, P. Blood, and M. Hopkinson, Semicond. Sci. Technol. **16**, 140 (2001).
[5] R. Alizon, A. Bilenca, H. Dery, V. Mikhelashvili, G. Eisenstein, R. Schwertberger, D. Gold, J. P. Reithmaier, and A. Forchel, Appl. Phys. Lett. **82**, 4660 (2003).
[6] Y. Nambu, A. Tomita, H. Saito, and K. Nishi, Jpn. J. Appl. Phys., Part 1 **38**, 5087 (1999).
[7] H. Huang and D. G. Deppe, IEEE J. Quantum Electron. **37**, 691 (2001).
[8] M. Grundmann and D. Bimberg, Phys. Rev. B **55**, 9740 (1997).
[9] L. V. Asryan and R. A. Suris, IEEE J. Quantum Electron. **34**, 841 (1998).
[10] T. W. Berg, S. Bischoff, I. Magnusdottir, and J. Mork, IEEE Photon. Technol. Lett. **13**, 541 (2001).
[11] E. Gehrig and O. Hess, Phys. Rev. A **65**, 033804 (2002).
[12] L. V. Asryan and R. A. Suris, Semicond. Sci. Technol. **11**, 554 (1996).
[13] D. R. Matthews, H. D. Summers, P. M. Smowton, and M. Hopkinson, Appl. Phys. Lett. **81**, 4904 (2002).
[14] C. Q. Jiang and M. G. Singh, "Modeling and control of discrete event dynamical systems," presented at the Seventh International Conference on Mathematics and Computer Modeling.
[15] Z. Zou, O. B. Shchekin, G. Park, D. L. Huffaker, and D. G. Deppe, IEEE Photon. Technol. Lett. **10**, 1673 (1998).
[16] G. Park, O. B. Shchekin, and D. G. Deppe, IEEE J. Quantum Electron. **36**, 1065 (2000).
[17] N. V. Kryzhanovskaya, A. G. Gladyshev, S. A. Blokhin, M. V. Maksimov, E. S. Semenova, A. P. Vasil'ev, A. E. Zhukov, N. N. Ledentsov, V. M. Ustinov, and D. Bimberg, Semiconductors **39**, 1188 (2005).