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Terahertz generation from laser filaments in the presence of a static electric field in a plasma

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Two femtosecond laser pulses with frequencies ω_1 and ω_2 that undergo filamentation in a plasma couple nonlinearly in the presence of a transverse, static electric field to generate terahertz wave at the frequency $\omega_1 - \omega_2$. The coupling is enhanced in the presence of the static electric field. We develop a theoretical model and observe over 30 times increase in the magnitude of normalized terahertz amplitude by applying a dc electric field ~ 50 kV/cm. © *2011 American Institute of Physics*. [doi:10.1063/1.3671973]

I. INTRODUCTION

Since the early pioneering experiments on terahertz (THz) waves by Hamster *et al.*,[1] there is a surging interest in their generation owing to multiple applications in the field of medical imaging,[2] material characterization,[3] remote sensing,[4] and security screening.[5] Of the various techniques to generate terahertz waves from ultra short, femtosecond (fs) laser pulses employing semiconductors,[6] photoconductors,[7] dielectrics,[8] and metallic surfaces,[9] terahertz pulse energy obtained from laser filaments is quite high. Wang *et al.*,[10] observed two orders of magnitude enhancement of THz power at a distance of 10 m using two colour laser filaments.

Femto second filamentation occurs when incident power of a laser pulse exceeds the critical value $P_{cr} = 3.72\lambda^2/8\pi n_0 n_2$, where n_0 and n_2 are the refractive index and optical Kerr constant, respectively, at the laser frequency. Since the fs laser requires no nonlinear crystal in its path, effects such as medium breakdown and laser absorption are minimized. One may therefore employ higher laser intensity to obtain high terahertz output power. Another potential advantage is the feasibility of remote generation of THz waves by controlling the onset of laser filamentation in air when effects such as beam diffraction, absorption by water vapour and air group velocity dispersion are minimized. Recently Wang *et al.*,[10] experimentally investigated remotely generated THz radiation produced by two colour femtosecond filamentation in air. They observed 570 nJ THz pulse energy at frequency ≤ 5.5 THz, at a distance of $\sim$16 m.

Earlier, broadband THz radiation was generated within the plasma filaments formed by an ultrashort $\sim$100 *fs* laser pulse propagating in air by Ladouceur *et al.*[11] Plasma filaments were formed through multiphoton ionization. The measured electron density within the plasma filament was $\sim 10^{16}$ cm^{-3} corresponding to a plasma frequency of $\sim$0.9 THz. Typical power conversion efficiency was $\sim 10^{-9}$. The THz frequency is typically determined by the laser pulse duration. Due to its simplicity this remote generation THz source has attracted attention worldwide. Tzortzakis *et al.*[12] observed $\leq$THz radiation in a direction perpendicular to the filament. Sprangle *et al.*[13] carried out theory and simulation explanations for this radially directed terahertz waves. Amico *et al.*[14] observed conical forward THz emission from femtosecond laser beam filamentation in air. For a 150 fs pulse duration, yielding an energy of 4 mJ, a single filament was formed by focusing laser pulses in air. A strong, radially polarized, forward directed broadband THz radiation emitted by the plasma filament was detected. Wu *et al.*,[15] studied THz radiation mechanism from a plasma filament formed by an intense fs laser pulse using a 2D-PIC simulation. The nonuniform electron density of the plasma filament couples with the nonlinear velocity induced by the laser ponderomotive force to generate a nonlinear current at the THz frequency. Recently, Houard *et al.*[16] observed 3 orders of magnitude enhancement of THz energy due to filamentation of a fs laser pulse in air in the presence of a static, transverse electric field $\sim$10 KV/cm. The laser intensity was $\sim 9 \times 10^{13}$ W/cm^2. The emitted terahertz wave was found to be linearly polarized in the plane containing the static electric field.

In this paper, we develop an analytical formalism for terahertz generation from laser filaments in the presence of a static electric field in a plasma. The mechanism of THz generation is as follows: Consider two transversely amplitude modulated laser beams with electric fields $\vec{E}_1(\omega_1, \vec{k}_1)$ and $\vec{E}_2(\omega_2, \vec{k}_2)$ propagating along $\hat{z}$, polarized along $\hat{y}$, and amplitude modulated along $\hat{x}$. The plasma has a dc electric field along $\hat{x}$, that imparts a dc drift $\vec{v}_{dc}$ to electrons. The lasers exert a ponderomotive force $\vec{F}_{p\omega}$ as well as a static (space periodic) ponderomotive force $\vec{F}_{pq}$ on electrons. In the steady state, the static ponderomotive force is balanced by the pressure gradient force, producing a transverse density ripple, $n_{0,q}$ at zero frequency and wavevector $\vec{q}$. The beat frequency ponderomotive force is primarily along $\hat{z}$ and gives rise to velocity and density oscillations $\vec{v}_{\omega,\vec{k}}$ and $n_{\omega,\vec{k}}$. The density oscillation beats with the dc drift to produce a transverse current $\vec{J}_{\omega,\vec{k}}$ that is suitable for THz generation.

In Sec. II, we determine ponderomotive force and obtain expressions for nonlinear velocity perturbation at the terahertz frequency $\omega = \omega_1 - \omega_2$ and wave vectors $\vec{k} = \vec{k}_1 - \vec{k}_2$, $\vec{k} + \vec{q}$ and nonlinear density perturbation at terahertz frequency and wave vectors $\vec{k},\ \vec{k}+\vec{q}$. By choosing the nonlinear coupling of the appropriate terms, an expression for nonlinear current density at frequency ω and wave vector $\vec{k}$ is obtained. In Sec. III, we determine terahertz wave equation and determine the normalized amplitude. A discussion of results is given in Sec. IV.

II. NONLINEAR CURRENT

Consider a plasma of electron density n_0^0 having a dc electric field $\vec{E}_{dc}||\hat{x}$. The dc field imparts a dc velocity to electrons,

$$\vec{v}_{dc} = -\frac{e\vec{E}_{dc}}{m\nu_e}, \tag{1}$$

where –e, m, and ν_e are the charge, mass, and collision frequency of electrons. We launch two transversely amplitude modulated (filamented) lasers into the plasma (c.f. Fig. 1) with electric fields,

$$\vec{E}_j = \hat{y}A_{j0}[1+\mu_j\cos qx]e^{-i(\omega_j t-k_j z)}; \quad j=1,2, \tag{2}$$

where μ_j is the index of modulation. The frequency difference of the lasers $\omega=\omega_1-\omega_2$ is in the terahertz range. The laser filaments impart oscillatory velocities to plasma electrons,

$$\vec{v}_j = \frac{e\vec{E}_j}{mi\omega_j}. \tag{3}$$

They also exert static ponderomotive force $\vec{F}_{pq}=e\nabla\phi_{pq}$ and beat frequency ponderomotive force $\vec{F}_{p\omega}=e\nabla\phi_{P\omega}$ on them, where

$$\phi_{pq} = \frac{e}{4m}\left[\frac{A_{10}^2\mu_1}{\omega_1^2}+\frac{A_{20}^2\mu_2}{\omega_2^2}\right]e^{iqx}, \tag{4}$$

$$\begin{aligned}\phi_{p\omega} &= -\left(\frac{m}{2e}\right)\vec{v}_1.\vec{v}_2^{\,*}\\ &= \frac{-eA_{10}A_{20}}{2m\omega_1\omega_2}\left[1+\frac{(\mu_1+\mu_2)}{2}e^{iqx}\right]e^{-i(\omega t-kz)}\end{aligned} \tag{5}$$

and $\vec{k}=\vec{k}_1-\vec{k}_2$. The static ponderomotive force causes ambipolar diffusion of plasma along $\hat{x}$. In the steady state, the static ponderomotive force is balanced by the pressure gradient force giving rise to zero frequency, transverse density ripple,

$$n_{0,q} = n_0^0\frac{e\phi_{pq}}{T_e} = \frac{n_0^0e^2}{4mT_e}\left[\frac{A_{10}^2\mu_1}{\omega_1^2}+\frac{A_{20}^2\mu_2}{\omega_2^2}\right]e^{iqx}, \tag{6}$$

where T_e is the equilibrium electron temperature. The beat frequency ponderomotive force imparts oscillatory velocity to electrons, governed by the equation of motion

$$m\frac{\partial\vec{v}_\omega^{NL}}{\partial t} = e\nabla\phi_{p\omega},$$

giving $\vec{v}_\omega^{NL} = \vec{v}_{\omega,\vec{k}}+\vec{v}_{\omega,\vec{k}+\vec{q}}$,

$$\vec{v}_{\omega,\vec{k}}^{NL} = \frac{e^2A_{10}A_{20}k}{2m^2\omega\omega_1\omega_2}e^{-i(\omega t-kz)}\,\hat{z} \tag{7}$$

$$\vec{v}_{\omega,\vec{k}+\vec{q}}^{NL} = \frac{e^2A_{10}A_{20}(\mu_1+\mu_2)}{4m^2\omega\omega_1\omega_2}[q\hat{x}+k\hat{z}]e^{-i[\omega t-kz-qx]}. \tag{8}$$

The velocity perturbations give rise to density perturbations $n_{\omega,\vec{k}}^{NL},\ n_{\omega,\vec{k}+\vec{q}}^{NL}$ in compliance with the equation of continuity

$$\frac{\partial n_{\omega,\vec{k}}^{NL}}{\partial t}+\nabla.(n_0^0\vec{v}_{\omega,\vec{k}}^{NL})=0, \quad \frac{\partial n_{\omega,\vec{k}+\vec{q}}^{NL}}{\partial t}+\nabla.(n_0^0\vec{v}_{\omega,\vec{k}+\vec{q}}^{NL})=0 \tag{9}$$

$$\begin{aligned}n_{\omega,k}^{NL} &= \frac{n_0^0e^2A_{10}A_{20}k^2}{2m^2\omega^2\omega_1\omega_2}e^{-i(\omega t-kz)},\\ n_{\omega,\vec{k}+\vec{q}}^{NL} &= \frac{n_0^0e^2A_{10}A_{20}(\mu_1+\mu_2)(q^2+k^2)}{4m^2\omega^2\omega_1\omega_2}.\end{aligned} \tag{10}$$

We look for nonlinear current density at $\omega,\vec{k}$. It arises due to the coupling between (i) equilibrium plasma density n_0^0 and nonlinear velocity $\vec{v}_{\omega,\vec{k}}^{NL}$, (ii) nonlinear density perturbation $n_{\omega,\vec{k}}^{NL}$ and dc electron velocity $\vec{v}_{dc}$, and (iii) zero frequency transverse density ripple $n_{0,q_\perp}^*$ and nonlinear velocity $\vec{v}_{\omega,\vec{k}+\vec{q}}^{NL}$. One may write the nonlinear current density as,

$$\vec{J}_{\omega,\vec{k}}^{NL} = -n_{0,q}^*e\vec{v}_{\omega,\vec{k}+\vec{q}}^{NL}-n_0^0e\vec{v}_{\omega,\vec{k}}^{NL}-n_{\omega,\vec{k}}^{NL}e\vec{v}_{dc} \tag{11}$$

$$J_{x\omega,\vec{k}}^{NL} = \frac{-n_0^0e^4A_{10}A_{20}}{2m^3\omega\omega_1\omega_2}\left[\frac{k^2E_{dc}}{\nu_e\omega}+\frac{eq}{8T_e}\left(\frac{\mu_1A_{10}^2}{\omega_1^2}+\frac{\mu_2A_{20}^2}{\omega_2^2}\right)\right]e^{-i(\omega t-kz)} \tag{12}$$

$$\begin{aligned}J_{z\omega,\vec{k}}^{NL} = \Bigg\{&\frac{-n_0^0e^5A_{10}A_{20}(\mu_1+\mu_2)k}{16m^3T_e\omega\omega_1\omega_2}\left[\frac{\mu_1A_{10}^2}{\omega_1^2}+\frac{\mu_2A_{20}^2}{\omega_2^2}\right]\\ &-\frac{n_0^0e^3A_{10}A_{20}k}{2m^2\omega\omega_1\omega_2}\Bigg\}e^{-i(\omega t-kz)}.\end{aligned} \tag{13}$$

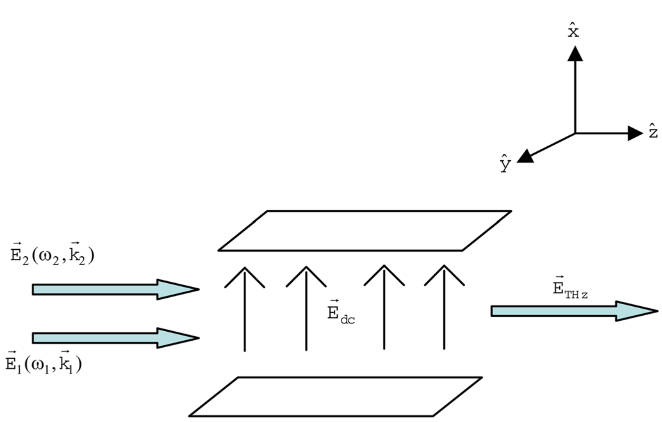

FIG. 1. (Color online) Schematic diagram.

III. TERAHERTZ RADIATION GENERATION

The wave equation governing the propagation of terahertz wave can be written as,

$$\nabla^2\vec{E}+\frac{\omega^2}{c^2}\in\vec{E}-\nabla(\nabla.\vec{E}) = \frac{-4\pi i\omega}{c^2}\vec{J}^{NL}, \tag{14}$$

where $\in(\omega) = 1 - \omega_{P0}^2/\omega^2$ is the plasma permittivity at the terahertz frequency and $\omega_{P0} = (4\pi n_0^0 e^2/m)^{1/2}$. For the transverse (x) component of the field, Eq. (14) can be written as

$$\frac{\partial^2 E_x}{\partial z^2} + \alpha^2 E_x = R, \tag{15}$$

where $\alpha^2 = \omega^2 \in /c^2$ and $R = \frac{-4\pi i\omega}{c^2} J_x^{NL}$ can be written as,

$$R = \frac{\omega_{P0}^2 e^3 A_{10} A_{20} (\mu_1 + \mu_2) iq}{16 m^2 c^2 T_e \omega_1 \omega_2} \left[\frac{\mu_1 A_{10}^2}{\omega_1^2} + \frac{\mu_2 A_{20}^2}{\omega_2^2} \right] + \frac{\omega_{P0}^2 e^2 A_{10} A_{20} i k^2 E_{dc}}{2 m^2 c^2 \nu_e \omega \omega_1 \omega_2}.$$

The solution of the above equation over length L of the plasma column may be written as

$$E_x = \frac{R}{2\alpha} \frac{e^{i\alpha L}}{(\alpha - k)} [e^{-i(\alpha - k)L} - 1]. \tag{16}$$

The normalized terahertz amplitude can be written as

$$\left| \frac{E_x}{A_{10}} \right| \approx \frac{v'_{20} q'}{32} [\mu_1 v'^2_{10} + \mu_2 v'^2_{20}] \frac{(\mu_1 + \mu_2)}{\sqrt{\in}(\sqrt{\in} - 1)\omega'_1 k'^2} + \frac{v'_{20} v'_{dc}}{4\omega'_1 \omega' \sqrt{\in}(\sqrt{\in} - 1)}, \tag{17}$$

where $v'_{10} = eA_{10}/m\omega_1 c$, $v'_{20} = eA_{20}/m\omega_2 c$, $v'_{dc} = eE_{dc}/m\nu_e c$, $q' = qc/\omega_{P0}$, $k' = kv_{th}/\omega_{P0}$, $\omega'_1 = \omega_1/\omega_{P0}$, and $\omega' = \omega/\omega_{P0}$. One notices from Eq. (17) that the normalized terahertz amplitude is directly proportional to the magnitudes of static electric field, laser power and filament cross-section and inversely proportional to electron temperature, laser and terahertz frequencies, and plasma permittivity at terahertz frequency. We also observe that the contribution of second term (due to dc electric field) is much smaller than the first term (due to ponderomotive force at the beat frequency) at high filament intensities. Our theory is thus valid at moderate to low laser intensities $\sim 10^{14}$ W/cm^2. In Fig. 2, we plot normalized terahertz amplitude as a function of normalized terahertz frequency ω' for the following set of parameters: $v'_{10} \sim v'_{20} \sim 0.005$, $v'_{dc} = 0.053$, $q' = 0.3$, $k' \approx 0.01$, $\mu_1 \sim \mu_2 \sim 0.3$, and $\omega'_1 = 50$. It is observed that as one increases ω' in the range 2.0–5.0, normalized terahertz amplitude increases continuously from 0.77×10^{-5} to 1.9×10^{-5}. We also note that there is an order of magnitude enhancement in the normalized terahertz amplitude due to static electric field contribution. In Fig. 3, we plot normalized terahertz amplitude as a function of normalized terahertz frequency ω' at $v'_{10} \sim v'_{20} \sim 0.01$. We observe that the contribution of first and second term in Eq. (17) is almost same. As one increases ω' in the range 2.5–5.0, normalized terahertz amplitude increases from 2.46×10^{-5} to 5.9×10^{-5}. In Fig. 4, we plot normalized terahertz amplitude as a function of normalized dc velocity v'_{dc} for $\omega' = 3.0$. As one increases v'_{dc} from 0.053 to 0.27 corresponding to an increase in static electric field from 10 KV/cm to 50 KV/cm, the normalized terahertz amplitude increased over 4.5 times from 0.96×10^{-5} to 4.28×10^{-5}. Further

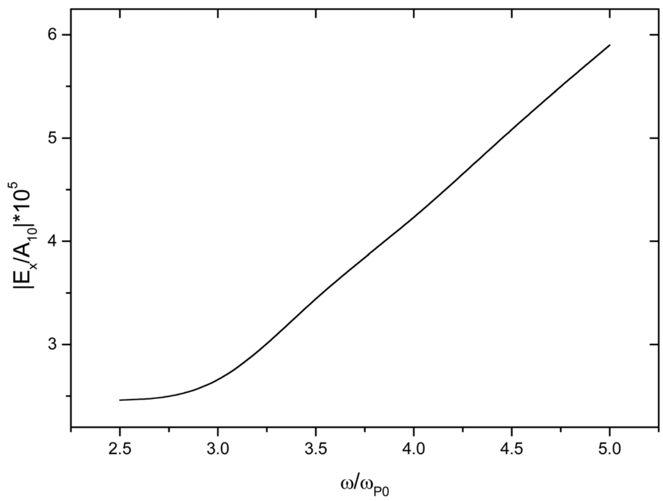

FIG. 3. Variation of normalized terahertz amplitude as a function of normalized terahertz frequency ω' at $v'_{10} \sim v'_{20} \sim 0.01$, $v'_{dc} = 0.053$, $q' = 0.3$, $k' \approx 0.01$, and $\omega'_1 = 50$.

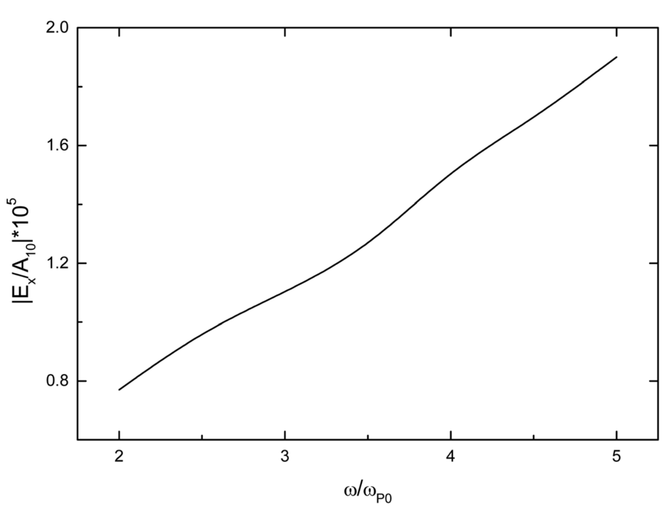

FIG. 2. Variation of normalized terahertz amplitude as a function of normalized terahertz frequency ω' for $v'_{10} \sim v'_{20} \sim 0.005$, $v'_{dc} = 0.053$, $q' = 0.3$, $k' \approx 0.01$, and $\omega'_1 = 50$.

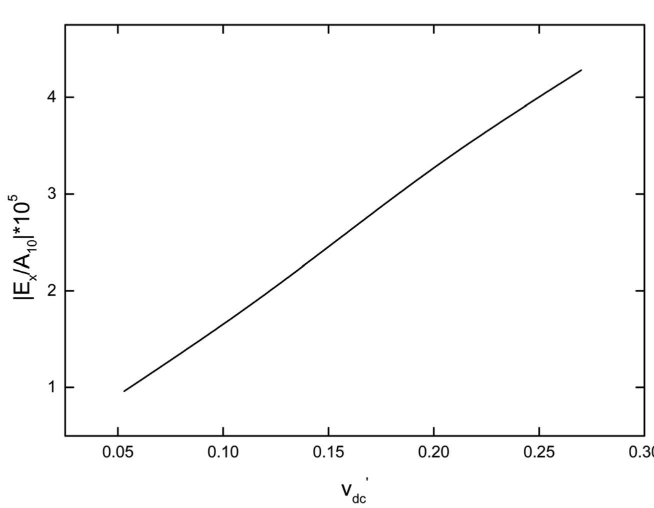

FIG. 4. Variation of normalized terahertz amplitude as a function of normalized dc velocity v'_{dc} for $\omega' = 3.0$, $v'_{10} \sim v'_{20} \sim 0.005$, $q' = 0.3$, $k' \approx 0.01$, and $\omega'_1 = 50$.

there is over 30 times increase in normalized terahertz amplitude by applying $E_{dc} \sim 50$ kV/cm.

IV. DISCUSSION

Two filamented laser beams in the presence of a static electric field generate terahertz wave at the beat frequency. The ratio of terahertz amplitude to that of the filament amplitude is $\sim 10^{-5}$ at laser intensities $\sim 10^{14}$ W/cm^2. The coupling is enhanced due to dc electric field whose effect is more pronounced at moderate to low laser intensities when relativistic effects are not important. We observe over 30 times increase in normalized terahertz amplitude by applying electric field $\sim$50 kV/cm. The terahertz amplitude increases with THz frequency.

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