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Electronic Pile Simulator

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A device has been made that follows the same set of kinetic equations as a chain-reacting pile. The voltage output varies as a function of time as the neutron flux would vary in a pile. A potentiometer varies the effective multiplication factor and can be considered as a control rod. Five delayed neutron periods are simulated. A high precision electronic integrator is used, and the instrument is capable of an accuracy much better than one percent.

THE neutron flux in a graphite-uranium chain-reacting pile decreases or increases when a control rod or other absorber is moved into or out of the pile. The way in which the flux increases or decreases is of some interest as one might wish to keep the pile output constant by an automatic controller or one might wish to know the change in flux with time produced by an absorber. It is possible to make an analog computer or simulator that responds exactly as a pile does to changes in absorption. Such an instrument serves admirably for the design of controllers or for demonstration or training.

If one assumes that the neutrons produced by the fission process are slowed down without absorption and are absorbed only as thermal neutrons, then the number of thermal neutrons per cm³ (n) is given by the relation

$$dn/dt=(\delta k_{\text{effective}}/kl^*)n-(\beta/l^*)n+e^{\tau\Delta}\sum_i \lambda_i C_i+S, \quad (1)$$

where

$l^*=l/ke^{\tau\Delta}=(1/ke^{\tau\Delta})\cdot(\lambda_a/v)$ = effective mean life of the neutrons from formation by fission to absorption or loss from pile by leakage.

$k=\nu\,\Sigma_f/\Sigma_{\text{total}}$ = number of neutrons per fission × (macroscopic cross section for fission/macroscopic total absorption cross section).

k = multiplication constant that pile lattice would have if it were of infinite extent.

τ = "Fermi age" of the neutrons, a measure of the net distance traveled by a neutron while being slowed down.

Δ = Laplacian representing the relative spatial distribution of the neutron flux. This is essentially a constant with time since the flux distribution is independent of the power level of the pile.

$\delta k_{\text{effective}}$ = reactivity of the pile $=(L^2\Delta-1+ke^{\tau\Delta})$.

L = diffusion length for thermal neutrons.

β = fraction of the fission neutrons coming from all delayed neutron emitters.

λ_i = decay constant of the ith kind of delayed neutron emitter.

C_i = concentration in atoms per cm³ of the ith kind of delayed neutron emitter.

S = source in neutrons per cm³ per second, other than those produced from neutron induced fission.

The first term on the right-hand side of Eq. (1) represents the effect of control rods or other absorbers upon the pile flux and is zero when the pile is in equilibrium at any value of neutron flux. The second term is inserted to correct for the fraction of the fission neutrons of each generation that does not appear instantaneously, and the third term represents the contribution of each of the delayed neutron emitters. The second and third terms must be equal when the pile is in a steady state at a constant flux. The fourth term represents the contribution of spontaneous fission, cosmic rays and other neutron sources that are not affected by adjusting the control rods of the pile.

The concentration of each delayed neutron emitter is a function of time depending on the past history of the pile. The rate of change of each delayed emitter is given by Eq. (2).

$$dC_i/dt=-\lambda_i C_i+(e^{-\tau\Delta}/l^*)\beta_i n. \quad (2)$$

Here β_i represents the fraction of the total fission neutrons formed from this particular delayed emitter. It is clear that the sum of these fractions β_i must be equal to the total fraction β.

The solution of differential equations of this type with variable coefficients can generally be accomplished by the use of a feed-back equation solver, which in principle is a very quick-operating trial and error method generally employing electrical circuits. In the electronic pile simulator the number of neutrons per cubic centimeter, n, is represented by a voltage and the other quantities involved in the equation are represented by other voltages or currents or by suitable circuit constants.

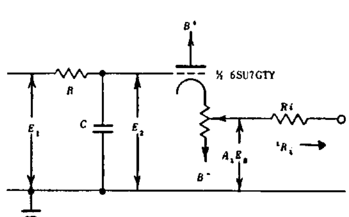

FIG. 1.

The circuit of Fig. 1 is used for the solution of Eq. (2). In this circuit

$$dE_2/dt=i/C=(E_1-E_2)/RC. \quad (3)$$

If $1/RC=\lambda_1$ and E_1 represents n, then

$$dE_2/dt=-\lambda_i E_2+\lambda_i n. \quad (4)$$

This equation is like Eq. (2) with

$$(e^{-\tau\Delta}\beta_i/l^*\lambda_i)E_2=C_i.^1 \quad (5)$$

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** Contract W-35-058-eng. 71.

[1] The delayed neutron periods were taken from Table I, Bernstein, Preston, Wolfe, and Slattery, Phys. Rev. 71, 573 (1947).

The tube connected to E_2 is operated as a cathode follower. If the amplification factor (μ) of the tube is high and the load in the cathode circuit is a high resistance the gain (A) is nearly unity

$$A=1/[1+(1/g R_k)+1/\mu], \tag{6}$$

where R_k is the resistance of the load circuit measured from cathode to ground. In this instrument $A\approx 0.98$.

Potentiometers $R_{5,6,7,8,42}$ (Fig. 2) are located in the cathode circuits of the cathode followers and the gain at each potentiometer arm (A_i) includes the tube gain and the "gain" of the potentiometer. The internal resistance of the cathode follower and the potentiometer measured at the arm is so small that it can be neglected. To the arm of each potentiometer is attached the "adding" resistor R_i, and the far ends of these resistors are attached to point P which is essentially at a fixed potential. The current in each resistor is given by

$$i_{Ri}=(A_i/R_i)E_2=(A_i l^*/R_i\beta_i)e^{\tau\Delta}\lambda_i C_i. \tag{7}$$

Thus if the voltage $E_1=n$, then the current given by this circuit at P is one of the elements of the sum in the third term of Eq. (1) with a scale factor $A_i l^*/R_i\beta_i$. The first term in Eq. (1) can be represented very easily. Let us suppose a potentiometer is connected between a voltage representing n and a voltage representing $-n$. Then for the voltage E at the tap on the potentiometer we have

$$E=Dn, \tag{8}$$

where D is the linear scale of the potentiometer with zero at the center, $+1$ at the $+n$ end and -1 at the $-n$ end.

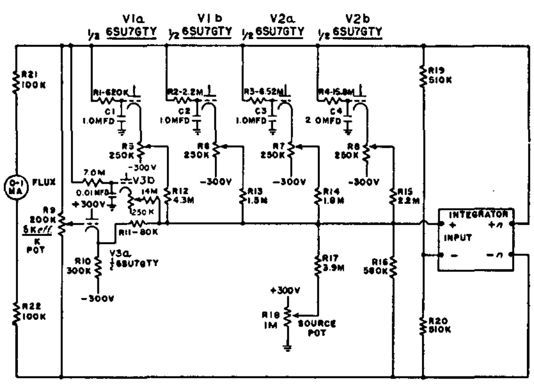

FIG. 2.

A cathode-follower circuit similar to that used in Fig. 1 is connected to the tap of the potentiometer and the current, i, in the output or adding resistor R_{11}, if the far end of this resistor is kept at a fixed potential, will be given by

$$i=(A_K D/R_{11})E$$
$$=[A_K/R_{11}(\delta k_{\text{eff. max.}}/kl^*)](\delta k_{\text{eff.}}/kl^*)n, \tag{9}$$

where A_K is the gain of the cathode follower and $(\delta k_{\text{eff.}}/kl^*)/(\delta k_{\text{eff. max.}}/kl^*)=D$. This current i represents the first term in Eq. (1) with a term in brackets as a scale factor. In this instrument $(\delta k_{\text{eff. max.}}/k)$ was arranged to be ± 0.05.

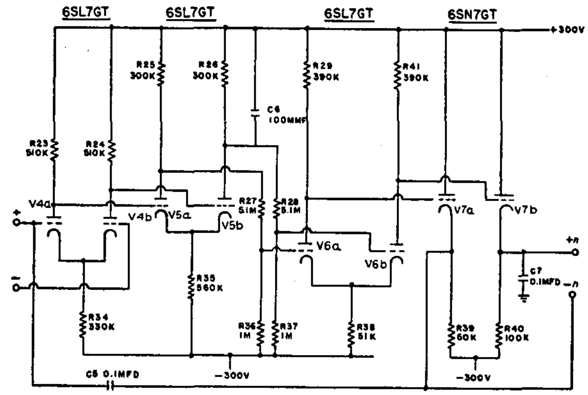

FIG. 3.

The second term in Eq. (1) is obtained from a resistor (R_{16}, Fig. 2) connected to the voltage representing $-n$. The current in this resistor (if the opposite end is fixed in voltage) is

$$i=-n/R_{16}=-(l^*/R_{16}\beta)(\beta/l^*)n. \tag{10}$$

Nothing has been said, so far, about the fact that although the output of a cathode follower changes at a rate A times its input there is also a constant difference of voltage. This constant voltage difference is the bias required by the cathode-follower tube when its input is at its "zero input" or reference voltage. The reference voltage is obtained when the $+n$ and $-n$ points have no voltage difference. There are enough disposable values to allow one to arrange things so that the tubes are operating at all times on a satisfactory part of their characteristics. A potentiometer (source potentiometer) is included to represent a source of neutrons and compensates the residual constant voltage difference of the cathode followers which would by itself represent a source of neutrons. The potentiometers $R_{5,6,7,8,42}$ in the cathode follower circuits are adjusted so that the residual constant voltage difference term which must be corrected by the source potentiometer is quite small.

When the ends of the adding resistors like R_1 in Fig. 1, R_{11} and R_{14} (Fig. 2) are all connected together to a common point P there will be (if the point P is kept at a fixed voltage) a total current

$$i_{\text{total}}=[A_K/R_{11}(\delta k_{\text{eff. max.}}/kl^*)](\delta k_{\text{eff.}}/kl^*)n$$
$$+e^{\tau\Delta}\sum_i (A_i l^*/R_i\lambda_i)\lambda_i C_i-(l^*/R_{16}\beta)(\beta n/l^*)$$
$$+[(i_{\text{source}}+i_{\text{const}})/S]S, \tag{11}$$

where i_{const} is the current due to the several constant voltage differences in the cathode followers and i_{source} is the current contributed by R_{17}, the sum of these last two terms represents the source term in Eq. (1).

All that remains now is to connect point P to a circuit that will produce, as its output voltage, the time integral of the total current at P multiplied by an appropriate

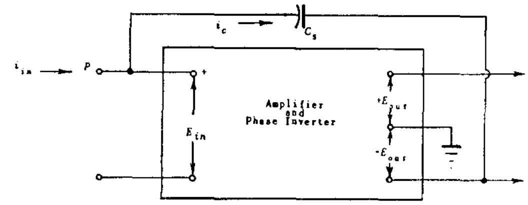

FIG. 4.

constant. This output voltage then would represent n. If a voltage changing equally but in the opposite direction is also produced, these voltages may then be connected to the parts of the circuit whose inputs are required to be n or $-n$. The whole instrument will then operate to give n as a function of time as required by the set of Eqs. (1) and (2). As long as the various elements of the circuit are operating in the range of voltages or currents in which their performance is satisfactory, the system approximates the relations we have assumed.

The integrator section of the instrument is the circuit whose output voltage is the time integral of its input current. The details of this section are shown in Fig. 3, and the operation may be more easily understood from the simplified diagram Fig. 4.

The point P is connected to a grid and one end of condenser C_5. This grid is in the negative grid region and the grid current is negligible so that the input current i_{in} at point P must be equal to the condenser current i_c but

$$\int i_c dt = C_5(E_{\text{out}} - E_{\text{in}}) = \int i_{\text{in}} dt. \tag{12}$$

If the voltage gain (A) of the amplifier in Fig. 4 is very great then E_{in} is very small compared with E_{out} and

$$\int i_{\text{total}} dt = \int i_{\text{in}} dt = C_5 E_{\text{out}} \tag{13}$$

is a good approximation.

The actual amplifier used (Fig. 3) is of the "differential" type in that it responds only to the difference in voltage between the two input grids. This effect is produced by the use of unbypassed cathode resistors.

The two nearly equal resistors, R_{19}, R_{20}, feed one of the input grids ($-$input) with one-half of the algebraic sum of the $+n$ and $-n$ output voltages, and only the difference between the voltage of this grid and the voltage at point P ($+$input) is amplified. This arrangement converts the circuit into a bridge and greatly reduces the effect of any errors in the phase-inverting action which produces $+n$ and $-n$; in addition if R_{19} is the right amount larger than R_{20} the effective gain of the integrator may be made infinite. A condenser C_7 is connected from the $+n$ output to ground to make the loading symmetrical.

It can be seen that the action of the integrator-amplifier opposes any change in the voltage difference between the $+$input and $-$input by supplying a current through the integrating condenser C_5. This keeps point P at a constant voltage relative to the algebraic sum of $+n$ and $-n$, that is, approximately constant with respect to ground as was assumed when considering the currents in the adding resistors.

Using Eqs. (13) and (11) we find

$$(C_5)dn/dt = [A_K/R_{11}(\delta k_{\text{eff. max.}}/kl^*)](\delta k_{\text{eff.}}/kl^*)n + e^{\tau\Delta}\sum_i [A_i l^*/R_i\beta_i]\lambda_i C_i - (l^*/R_{16}\beta)(\beta n/l^*) + [(i_{\text{source}} + i_{\text{const}})/S]S. \tag{14}$$

The quantities in the brackets are scale factors for each term. The condenser C_5 is 0.1 microfarad, which is the largest value for which the maximum integration current required can be supplied by tubes $V3_a$ and $V7_a$. The choice of C_5 fixes the value to which the other scale factors must be adjusted.

The errors in this instrument are mostly in the integrating circuit and these errors arise as follows:

Let us suppose that the amplifier of the integrator has an initial gain of 1000 and the resistor pair R_{19}–R_{20} is adjusted to supply just the amount of positive feedback required to make the effective gain infinite. If the differential gain of the amplifier falls to 500 at full output

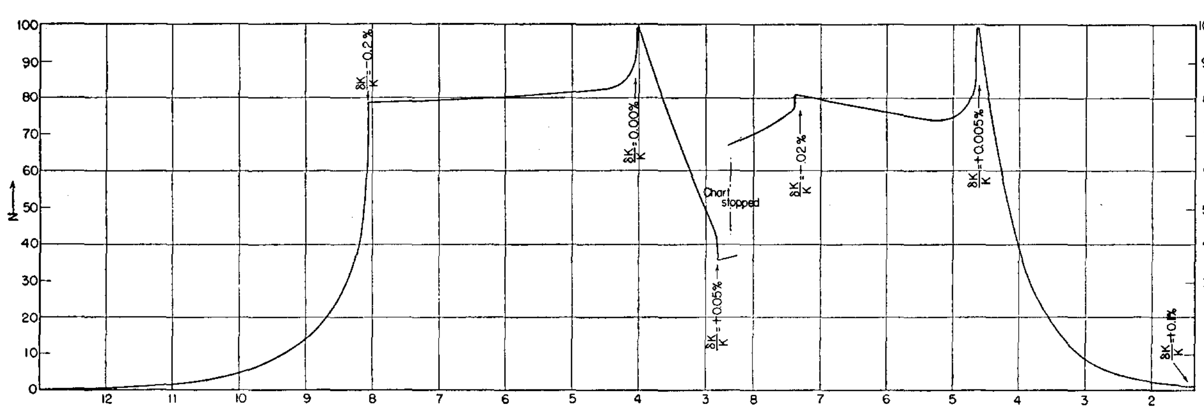

FIG. 5.

(100 volts), the signal required at the grids would be 0.2 volt, and the positive feedback would supply only 0.1 volt thus requiring the input signal to supply the remaining 0.1 volt. If the simulator output was rising initially with a 30 minute period due to a signal from the $\delta k_{\text{effective}}$ potentiometer of $\delta k/k_{\text{effective}}=0.005$ percent, the output would just cease rising when this amount of non-linearity had set in. Another kind of error due to the integrator circuit is drift or fluctuation of the output due to changes in contact potential in the first double triode. These fluctuations limit the range of n to about 5×10^3.

The last noticeable source of error is the variation in gain of the delayed neutron cathode followers. The current in these cathode followers varies about 30 percent from no output to full output. This change of current gives a variation of g of about 10 percent and results in a change of gain of about 0.03 percent. The effect due to the change of μ is still smaller. Since the total contribution of all the delayed neutron emitters is roughly 0.7 percent of the total neutrons, this small change in gain produces an error that has the same effect as a change in $\delta k_{\text{effective}}/k$ of 0.0002 percent.

The complete instrument must be adjusted experimentally to determine the exact setting of the source potentiometer, R_{18}, that represents zero source strength. It must be noted that this instrument, unlike the pile, is completely symmetrical with respect to the zero output point and may be caused to build up on either "positive" or "negative" neutrons depending on the source setting. In addition to the source zero the position of $\delta k_{\text{critical}}$ must be determined on the δk potentiometer. A satisfactory procedure is to set the δk potentiometer slightly more negative than the apparent position of $\delta k_{\text{critical}}$ and then slowly adjust the source potentiometer, allowing several minutes to elapse between each adjustment, until the n meter gives only an extremely small positive reading. The δk potentiometer should then be adjusted to cause the output to rise slowly (say 10–20 min. to reach full output) and the output plotted against time. If the δk potentiometer is at $\delta k_{\text{critical}}$ the output will rise linearly due to the small source; if the reactivity is positive the curve will bend upward; if the reactivity is negative the curve will bend downward. Slight readjustments can now be made until a very small change in the source potentiometer setting causes the output to rise or fall linearly with time. The setting on the source potentiometer is then zero source, and the setting on the δk potentiometer is $\delta k_{\text{critical}}$.

Figure 5 is a tracing made from a Brown recorder record of the output voltage n against time as the δk potentiometer, which represents the control rod of the pile, is moved as shown on the tracing. Note the transient effects whenever the control rod is moved. The output approaches a pure exponential curve as the transient dies away.

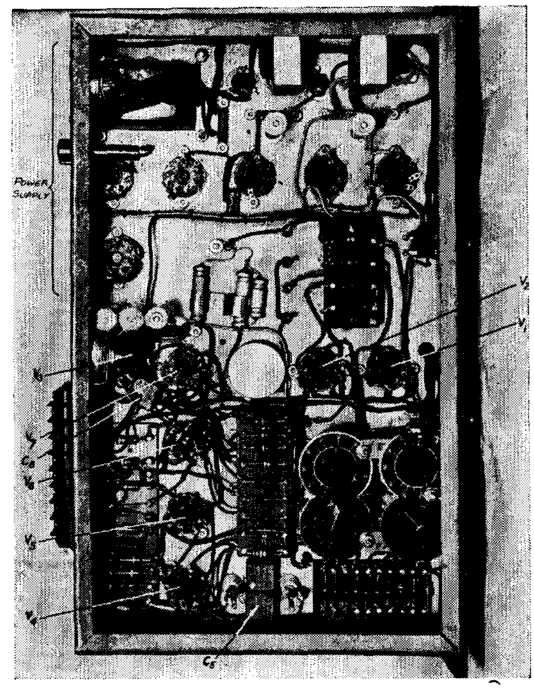

Fig. 6.

Figure 6 is a photograph of the bottom of the chassis of the simulator. The row of four tubes is the integrator section. All of the delayed neutron storage condensers except C_8 are invisible because they are above chassis. These condensers are Western Electric type D 161270 1 μf units chosen for their extremely low leakage and "soaking up" effect.

Anyone attempting the construction of an instrument of this type should place the leads with care, especially in the integrator section, and employ an oscilloscope to check the absence of high frequency parasitic oscillations. Condenser C_6 was added to suppress such an oscillation. When the δk potentiometer is mounted in a remote location it is necessary to shield separately the lead from the arm to prevent coupling between this lead and the $+n$ lead. Such coupling will produce oscillations at a high frequency.

We wish to acknowledge the valuable suggestions of F. H. Murray during the planning of this instrument.