Equivalent Temperature of an Electron Beam

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Letters to the Editor

Equivalent Temperature of an Electron Beam

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1. Introduction.—Parzen and Goldstein[1] in a recent article have shown the effects of a distribution of electron velocities on the gain of a traveling wave tube. For their assumption that the longitudinal velocity distribution has an equivalent temperature equal to that of the cathode, substantial reduction in gain is predicted for some types of tubes.

The purpose of this letter is to discuss the equivalent temperature to be expected in the beam of a practical traveling wave tube. It is shown that the effect of beam acceleration is to reduce the spread in longitudinal velocities by a large factor for cases of non-converging electron guns. The effects of long drift times and converging guns are discussed in a qualitative manner. These effects can result in a high beam temperature from other considerations.

2. Temperature Calculation.—For this derivation it is assumed that the beam is accelerated from the cathode in substantially rectilinear flow with the applied fields acting in the longitudinal direction only. It is also assumed that the electrons do not interact with each other on a microscopic scale.

The distribution function of the longitudinal velocities at the potential minimum just in front of the cathode is

$$dn=(e\bar{n}/kT_c)\exp(-eV^*/kT_c)dV^*,$$

where $\bar{n}$ is number passing across per unit time and

$$V^*=u_{zm}^2/(2e/m).$$

Here u_{zm} is the electron velocity at the potential minimum. After acceleration to a potential V_0 above cathode potential, an electron with an initial kinetic energy eV^* at the minimum will have a final velocity given by

$$u_z=(2(e/m)(V_0+V_m+V^*))^{\frac{1}{2}},$$

where $-V_m$ is the potential at the minimum. If V^* is very small in comparison to V_0, the velocity may be closely approximated by

$$u_z=(2(e/m)(V_0+V_m))^{\frac{1}{2}}[1+(V^*/2(V_0+V_m))].$$

The mean velocity of the stream is given by

$$\bar{u}_z=\frac{1}{n}\int_n u dn \quad \text{(see footnote *)}$$

$$=\frac{e}{kT_c}\int_0^\infty\left(2\frac{e}{m}(V_0+V_m)\right)^{\frac{1}{2}}\left[1+\frac{V^*}{2(V_0+V_m)}\right]\exp\left(-\frac{eV^*}{kT_c}\right)dV^*$$

$$=(2(e/m)(V_0+V_m))^{\frac{1}{2}}[1+(kT_c/2e(V_0+V_m))].$$

The equivalent beam temperature is given by

$$T_b=(1/kn)\int m(u_z-\bar{u}_z)^2dn$$

$$=\frac{e^2}{2k^2T_c(V_0+V_m)}\int_0^\infty\left[V^*-\frac{kT_c}{e}\right]^2\exp\left(-\frac{eV^*}{kT_c}\right)dV^*.$$

$$T_b=\left[\frac{kT_c}{2e(V_0+V_m)}\right]T_c.$$

3. Effects of Long Drift Times.—For $V_0=1000V$ and $T_c=1000°K$, the above formula predicts an effective beam temperature of about 1/20 of a degree absolute, rather than 1000°. This is the temperature corresponding to the longitudinal velocity spread only. The transverse velocity spread remains the same as it was at the time of emission, which corresponds to the temperature of the cathode.

If the beam is allowed to drift for a sufficiently long time, these transverse velocities will have their energies redistributed so that each degree of freedom has an equal thermal velocity distribution. This would imply that the final beam temperature should approach approximately $\frac{2}{3}$ of the cathode temperature if allowed to drift for an indefinite distance. There is, however, some evidence that the microscopic interactions between the electrons are sufficiently infrequent or ineffective that it may take a very long drift time to re-establish equilibrium in practical tubes, depending upon the charge density in the beam.

One such piece of evidence can be deduced from the measurements of the noise currents in an electron beam reported by Cutler and Quate.[2] In the theories of Pierce,[3] Rack,[4] and Peterson,[5] the velocity fluctuations at the cathode are assumed to be the major source of noise in vacuum tubes. These velocity fluctuations cause fluctuations in electron transit times during acceleration and drift which result in convection current and velocity fluctuations which are distinctly correlated with one another. In a drifting beam, such correlated fluctuations will cause a repeating pattern of interference between the space-charge waves so established. Cutler and Quate[2] observed these interference effects and they show that they are readily explainable in terms of the theories of Rack, Pierce, and Peterson. The waves they observe are of such a magnitude that the velocity fluctuations at the maximum are rather close to that which Rack gives for the fluctuations at the cathode, namely

$$\bar{v}^2=\frac{(4-\pi)(e/m)kT_cB}{I_0}.$$

The fact that such maximum fluctuations were observed does not imply that the thermal velocity distribution in the final beam corresponds to the cathode temperature as these fluctuations are produced by a wave propagation phenomenon which involves the beam as a whole. Rather, one would expect a considerably greater magnitude of noise if the beam had acquired an equilibrium maxwellian velocity distribution by interaction with the transverse thermal velocities. Rack's formula applies to the unsymmetrical velocity distribution at the potential minimum next to the cathode surface. The appropriate formula for the velocity fluctuations in a symmetrical random maxwellian velocity distribution is

$$\bar{v}^2=\frac{4(e/m)kTB}{I_0},$$

which is about 5 db larger for an assumed equilibrium temperature $\frac{2}{3}$ that of the cathode. If thermal equilibrium had been established then it might be expected that new velocity fluctuations of this larger magnitude would have appeared in the process, generating space charge waves in the beam with associated noise currents of larger magnitude. Cutler and Quate did not observe such effects so that it might be deduced that thermal equilibrium had not been established in their beam. This would imply that the effective beam temperature, considering the longitudinal velocity distribution only, was much lower than the cathode temperature in their experiment.

4. Effects of Beam Focusing.—A sharply converging electron gun can result in a rather wide distribution of longitudinal velocities. This will occur because of the transverse electron velocity distribution in the accelerating and focusing region. The beam contains many electrons which have not followed the ideal trajectories and consequently have been accelerated in an abnormal manner so that they have rather widely dispersed transverse velocities. These electrons must also have a dispersion in their longitudinal velocities because of energy considerations. This can increase the effective beam temperature markedly for guns with a high degree of beam convergence.

Methods of beam focusing which cause a variation in dc velocity over the beam cross section will cause effects similar to those of high beam temperature. If the beam is focused in the ideal "Brillouin flow" condition,[6] no difficulty of this sort is to be expected as the longitudinal velocity of the electrons is uniform throughout the beam.

5. *Summary.*—The equivalent temperature of the longitudinal velocity distribution in an electron beam may be very low in cases where the electron gun is not of the converging type and where the beam is focused in the "Brillouin flow" condition. If a beam is allowed to drift for an indefinite time, then the transverse velocities may re-establish an equilibrium at about $\frac{2}{3}$ the cathode temperature, but there is evidence that this may take a rather long drift time. Converging electron guns and improper focusing conditions will increase the effective beam temperature.

[1] Philip Parzen and Ladislas Goldstein, J. Appl. Phys. **22**, 398–401 (1951).

* Strictly speaking, dn in this formula specifies the distribution function per unit volume, rather than the number passing per unit time. However, because of the small velocity spread, the difference between the two forms is negligible in this case.

[2] C. C. Cutler and C. F. Quate, Phys. Rev. **80**, 875 (1950).
[3] J. R. Pierce, *Traveling Wave Tubes* (D. Van Nostrand Company, Inc., New York, 1950), pp. 148–155.
[4] A. J. Rack, Bell System Tech. J. **17**, 592 (1938).
[5] L. C. Peterson, Proc. Inst. Radio Engrs. **35**, 1264 (1947).
[6] J. R. Pierce, *Theory and Design of Electron Beams* (D. Van Nostrand Company, Inc., New York, 1950), pp. 152–155.

Comments on "Equivalent Temperature of an Electron Beam"

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MR. HINES'S derivation of the variation of temperature in a rectilinear beam is correct if the electron-electron interaction is neglected. A similar derivation may be found on page 126 of the book, *Klystrons and Microwaves Triodes*, by Hamilton, Knipp, and Kuper of the M.I.T. Series (McGraw-Hill Book Company, Inc., New York, 1948). Mr. Hines's result is most easily obtained from Eq. (40) of the mentioned paper by Parzen and Goldstein. Thus, assuming no interaction, $q=0$ and hence $Tv^2=$constant. However, the electron-electron interaction may not be neglected. The experiments of Culter and Quate have not as yet been sufficiently interpreted to indicate the degree of interaction. There are effects due to the finite lateral extension of the electron beam which remain to be explained. Accurate measurements of the period of the space charge waves in the drift tube may cast some light on the actual electron temperature in the drift tube.

A Note on the "Attenuation of Radio Signals Caused by Scattering"

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(Received July 2, 1951)

IN a recent paper[1] to the Journal of Applied Physics, A. H. LaGrone, W. H. Benson, Jr., and A. W. Straiton have integrated the Booker Gordon equation to obtain an expression for the attenuation of a plane wave due to scattering. Referring to Eq. (14) of their paper, it is seen that the shorter wavelengths are more rapidly attenuated, and indeed if we consider a limiting case, light should scarcely be propagated at all! A glance at Figs. 2 and 3 of the paper[1] indicates an error, for the curves of attenuation have no maxima and increase indefinitely with increasing l/λ. In deriving the expression for attenuation, the authors[1] have subtracted the power scattered in all directions from the incident power density, including the power scattered in the forward direction. In considering a plane wave, however, power scattered in a direction having a forward component is a part of the propagated wave. It is suggested, therefore, that the integration of the Booker Gordon equation be performed, not over the entire surface S, but

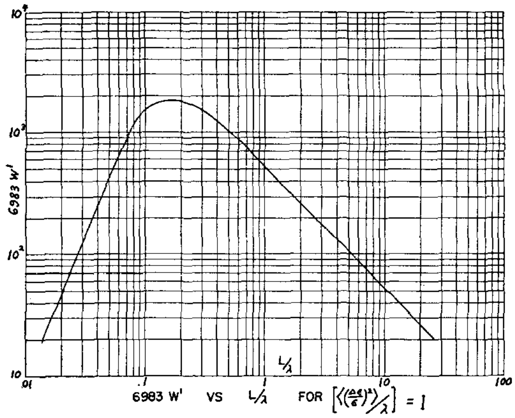

FIG. 1.

only over the hemisphere backward to the direction of propagation Denoting by W' the power scattered by unit scattering volume per unit incident power density in all directions making an angle equal to or greater than 90° with the direction of propagation, we have

$$W'=\int_{\theta=\pi/2}^{\pi}\int_{\delta=0}^{2\pi}\sigma(\theta,x)\sin\theta d\theta d\delta.$$

This integration yields

$$W'=\frac{\langle(\Delta\epsilon/\epsilon)^2\rangle\pi}{\lambda(2\pi l/\lambda)}\left\{\frac{1+5(2\pi l/\lambda)^2+8(2\pi l/\lambda)^4}{2+12(2\pi l/\lambda)^2+16(2\pi l/\lambda)^4}-\frac{1+2(2\pi l/\lambda)^2}{4(2\pi l/\lambda)^2}\log_e\frac{1+4(2\pi l/\lambda)^2}{1+2(2\pi l/\lambda)^2}\right\}.$$

Using W' as given by this expression, instead of the W given by Eq. (9), we arrive at more satisfactory attenuation curves. We submit, as a substitute for Fig. 2, a plot of the value of $6983W'$ *vs* the ratio of l/λ for the special case of $[\langle(\Delta\epsilon/\epsilon)^2\rangle/\lambda]=1$ (see Fig. 1). Unfortunately, neither the original approach[1] nor the one suggested in this note can be used to solve the problem indicated by the title of the paper.

[1] LaGrone, Benson, Jr., and Straiton, J. Appl. Phys. **22**, 672 (1951).

Reply to J. B. Smyth and C. P. Hubbard

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(Received July 25, 1951)

DR. J. B. Smyth and Mr. C. P. Hubbard of the Navy Electronics Laboratory, San Diego, California, have proposed that the paper "Attenuation of radio signals caused by scattering"[1] be modified to the extent that only backscattering be considered as the cause of attenuation. They point out that all scattering at $\theta<90°$ has a forward component and should be considered as part of the propagated wave.

It is difficult for the authors to see where such a modification would be an improvement to the original paper, if indeed, the proposed modification has any merit at all. The original paper reported the power scattered per unit macroscopic element of volume relative to the power incident on the volume. It was recognized that a fraction of this power would be propagated in the same direction as that of the incident wave, and that for large values of (l/λ), a receiver along the path and in the immediate vicinity of the scattering would receive an appreciable part of the