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Fast Selective Elimination of Spiral Waves

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Elimination of spiral waves prevents transition to chaotic state in excitable media of different physicochemical nature. In cardiac muscle, to prevent cardiac death, the spiral waves are usually removed together with all propagating waves by a strong electric shock: 5 kV, 20 A ("defibrillation"). We have found an approach to extinguish spiral waves without destroying normally propagating waves. Chemical excitation waves in a spatial open reactor with the Belousov−Zhabotinsky reaction controlled by light were used as an experimental model. We found three different scenarios, depending on the rate of the light change. (i) When the light intensity was increased immediately, the spiral wave was eliminated. (ii) When it was increased slowly enough, the spiral wave survived, increasing its core and diminishing its rotation rate. (iii) When it was increased gradually, at an intermediate rate, the spiral wave survived, but multiple wave breaks appeared at the periphery. Computer modeling has shown that the results obtained are generic and relevant for excitable media of various nature.

Introduction

Chemical waves in the Belousov−Zhabotinsky (BZ) reaction are a very convenient model to study propagation of excitation in active media.[1−5] They show either a regular propagation, which can be well controlled by various means,[5−11] or an irregular "chaotic" one, which is very hard to control, resulting in turbulent-like patterns.[1,12−14] The complex patterns usually arise when propagating fronts lose stability and can be eventually broken. The wave breaks give birth to rotating spiral waves, or vortices.[2,15]

There are two ways to preserve an excitable medium against chaotic wave propagation: either by avoiding the conditions of wave front instability or eliminating quickly already existing vortices. Only a few mechanisms for the spontaneous wave breaking are known: the local inhomogeneties,[15] particular geometry and boundary conditions of the medium,[13,16] and external extrastimulus.[22] Even fewer choices exist for extinguishing spiral waves: (1) inducing the spiral wave drift and propelling it slowly out of the medium and (2) suppressing totally at the wave propagation in the system. The drift of the spiral wave, caused by various means (synchronization,[21] periodic modulation of the excitability,[6,9] imposed gradient of the excitability,[11] external electric field[7,8]), is a very slow process, usually several times less in magnitude than the speed of propagating waves. On the contrary, total suppression can be much faster if the appropriate parameter is chosen, but it destroys all wave propagation, so it is costly for biological systems.[22]

In the present work, we have found the possibility of eliminating spiral waves with a fast decrease of the excitability, but without suppression of the wave propagation. We also studied the stability of the wave pattern and its possible transitions to the complex state depending on the rate of transition between the two steady states, both situated far from the boundary of stability.

As an experimental model, we used the spatial open reactor[17,27] with the light-sensitive BZ reaction.[18,19] The intensity of light illumination was used to control the excitability of the medium. Light is known to inhibit the BZ reaction,[18−20] and the change of light illumination gives us the possibility to control the system with the only delay, defined by chemical kinetics, known to be no more than a few seconds.[20] The possibility of destabilizing the wave pattern with an increase of light intensity was already shown in ref 29.

A simple regular wave pattern consisting of a single rotating spiral wave was studied. We found three different scenarios for the development of the wave pattern, depending on the rate of light change. First is "shock" when the transition from one excitable state to another one was made instantaneously (much faster than the rate of spiral wave rotation), the initial spiral wave completely disappeared; the wave pattern was eliminated and, because the system remained excitable, after a short period of time developed anew. Second is "adaptation": when transition was made very slowly (in the characteristic time scale, about 100 periods of rotations of the spiral wave between initial and final states), the spiral wave survived, reshaping its core and changing its rotation rate; no new wave breaks were observed in the medium. Third is "broken wave-train": when the transition was made with moderate speed, but not slow enough, multiple wave breaks at the periphery of the spiral wave appeared; the transition induced new turbulent elements in the pattern. This means, for the same amplitude of the excitability change, one has a choice (1) to remove "turbulent" elements of the wave pattern (spiral waves); (2) not to change the wave pattern in general features (all already existing spiral waves survive, no new vortices appeared); or (3) to induce "turbulence" (multiple new spiral waves appear).

Experimental Method

The spatial open reactor was made as is described in ref 17. The 25 mm diameter, 0.4 mm thick disk of Vycor Corning porous glass was set between two CSTRs with the following chemical composition: tank A, H_2SO_4 0.3 M, $NaBrO_3$ 0.2 M, NaBr 0.05 M, SDS 0.2 mM, $CH_2(COOH)_2$ 0.1 M; tank B, H_2SO_4 0.3 M, $NaBrO_3$ 0.2 M, $Ru(bpy)_3$ 0.2 mM. The residence time was 40 min. The reaction takes place only in the porous glass, where chemicals are mixed by diffusion. Illumination and observation of the wave pattern were made through transparent windows at opposite sides of the reactor. For the

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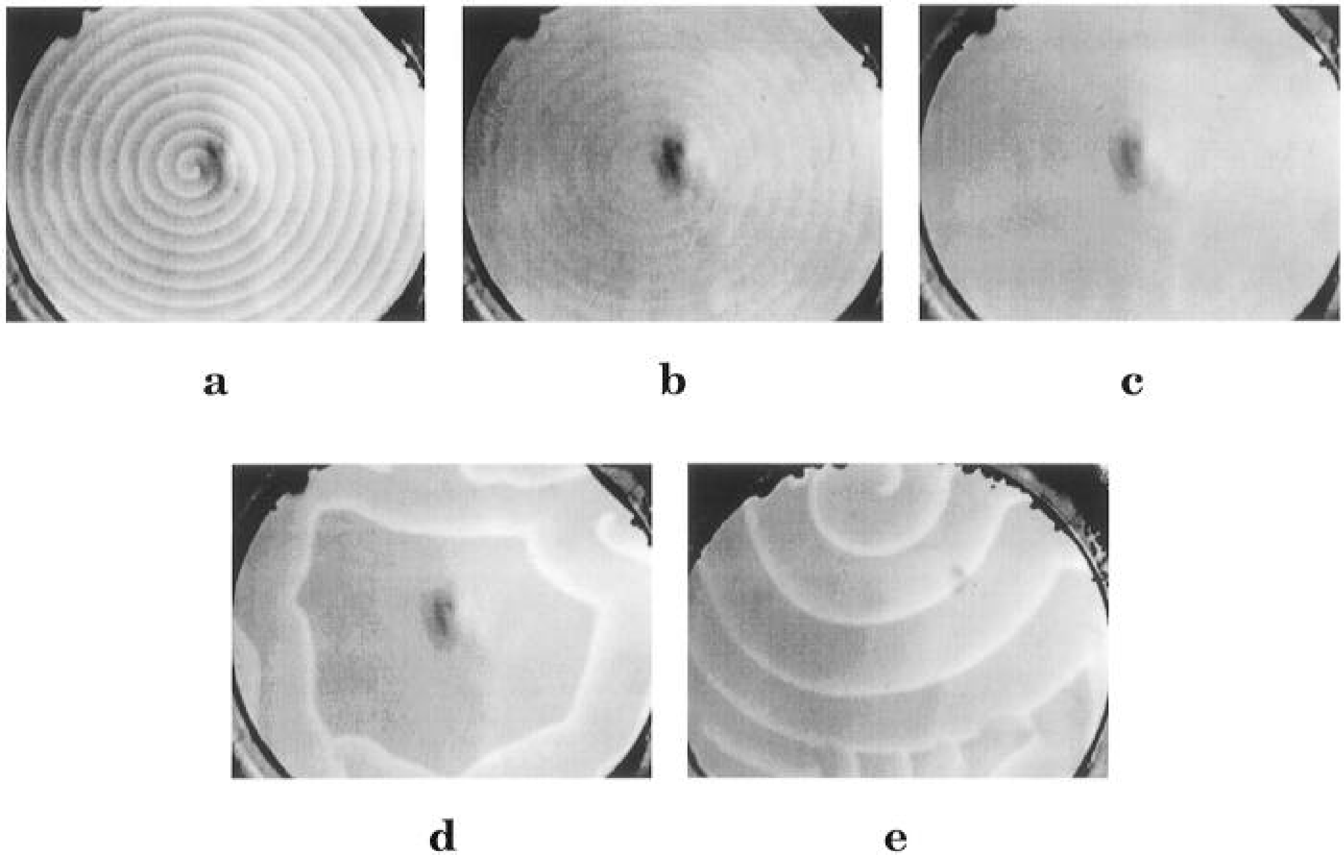

Figure 1. Death of the spiral wave in the BZ reaction under instantaneous increase of illumination (from 0.8 mW/cm^2 up to 12 mW/cm^2): (a) initial spiral wave (in almost dark conditions); (b) after the illumination was switched on, the waves stop propagation and gradually disappear (t = 30 s); (c) total disappearance of the wave pattern (t = 2 min); (d) waves spontaneously start propagation from the borders of the reactor under the same illumination as in parts b and d; (e) spiral wave was obtained under the same strong intensity of illumination.

illumination a slide projector with a 200 W tungsten filament lamp was used. The light beam was additionally collimated. The infrared part of the spectrum was cut off by optical filtering. A XC-77R CCD camera and "Sony" EV-C2000E VCR were used for the recording of the pattern. The illumination intensity was measured by electronic sensor, placed at the window of the reactor and calibrated by a powermeter ("Scientech" AC2500).

The wave pattern spontaneously developed in the reactor 20−30 min after the mixing of the reagents. Usually waves started from the borders of the porous glass, where it is connected with the acrylic walls of the reactor. The spiral wave was created from wave breaks obtained by illumination of one part of the reactor with an intensity sufficient to suppress all the wave propagation and shadowing the other part of the reactor. After reducing the light intensity to the minimal level appropriate for the recording and observation, the wave breaks evolved into rotating spiral waves. One of them was selected and shadowed by a piece of black paper. In the shadowed area the rate of spiral wave rotation was faster than in the illuminated part of the reactor, so with time all the spiral waves were pushed out from the medium, except for the shadowed one.[21] The location of the chosen spiral could also be adjusted using the method of periodic illumination, described in refs 6 and 9.

Experimental Results

Figure 1 shows the result of the instantaneous increase of the illumination intensity of the reaction area. The initial spiral wave rotated at low intensity of illumination (0.8 mW/cm^2), Figure 1a. After an increase of the illumination intensity up to 12 mW/cm^2 the spiral wave was destroyed. The waves emitted by the spiral wave stopped propagating and gradually disappeared, Figure 1b,c. Destroying the spiral wave by instantaneous increase of illumination is a threshold phenomenon. Until the light intensity was lower than 10.4 mW/cm^2, the spiral wave reshaped significantly, but no wave breaks and no topological changes of the wave pattern appeared. When the intensity of the illumination exceeded 11.2 mW/cm^2, the spiral wave was eliminated. In several minutes after terminating the spiral, under the same intensity of the illumination (12 mW/cm^2), new waves spontaneously arose from the borders of the reactor, and a new wave pattern developed, Figure 1d. This means that the system remained excitable. Also, at the same intensity of the illumination, the system could maintain the spiral waves as well: new spiral waves were created by breaking the propagating fronts with local strong illumination, Figure 1e. Thus, rapid increase of the illumination intensity (decrease of excitability) resulted in complete elimination of the spiral wave, while the system remained excitable.

When the intensity of the illumination increased slowly (0.8 mW/cm^2 per 3 min), we observed the gradual transformation of the spiral wave. With the increase of the illumination intensity the wave speed decreased, as well as the rotation rate of the spiral wave. The core of the spiral grew in size and the wavelength of the spiral wave increased, Figure 2b,c. Also, with the increase of the illumination power, we observed the transition from stationary circular rotation of the spiral wave to complex rotation (so-called "meandering"[4]). The amplitude of the meandering grew with the power of the light, resulting in a specific nonsymmetric shape of the spiral, Figure 2b,c. Despite the significant reshaping of the spiral wave, no visible indication for the instability of the propagating waves was observed. Thus,

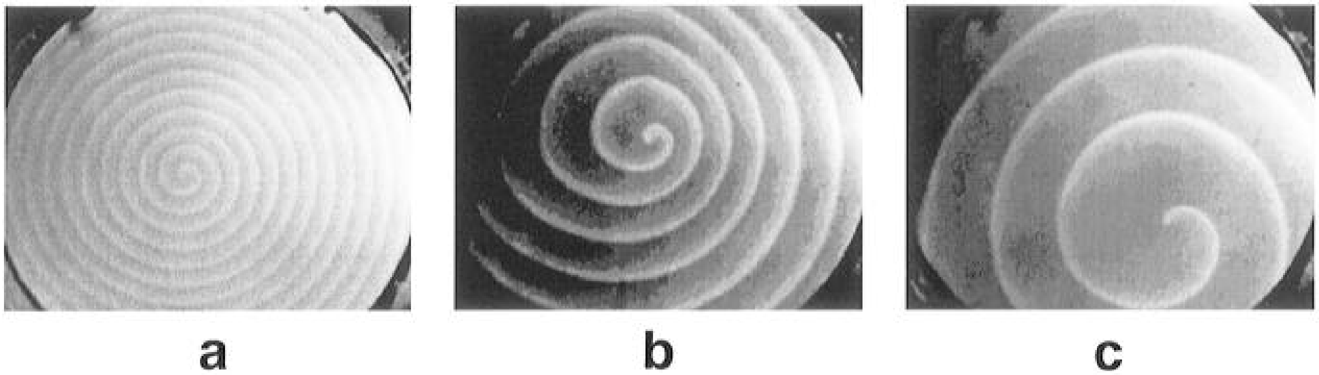

Figure 2. Adaptation of the spiral wave to the slow increase of illumination. The illumination I was increased slowly, 0.8 mW/cm^2 per 3 min: (a) spiral wave in the beginning of the experiment, $I = 0.8$ mW/cm^2; (b) $I = 9.6$ mW/cm^2; (c) $I = 12.8$ mW/cm^2.

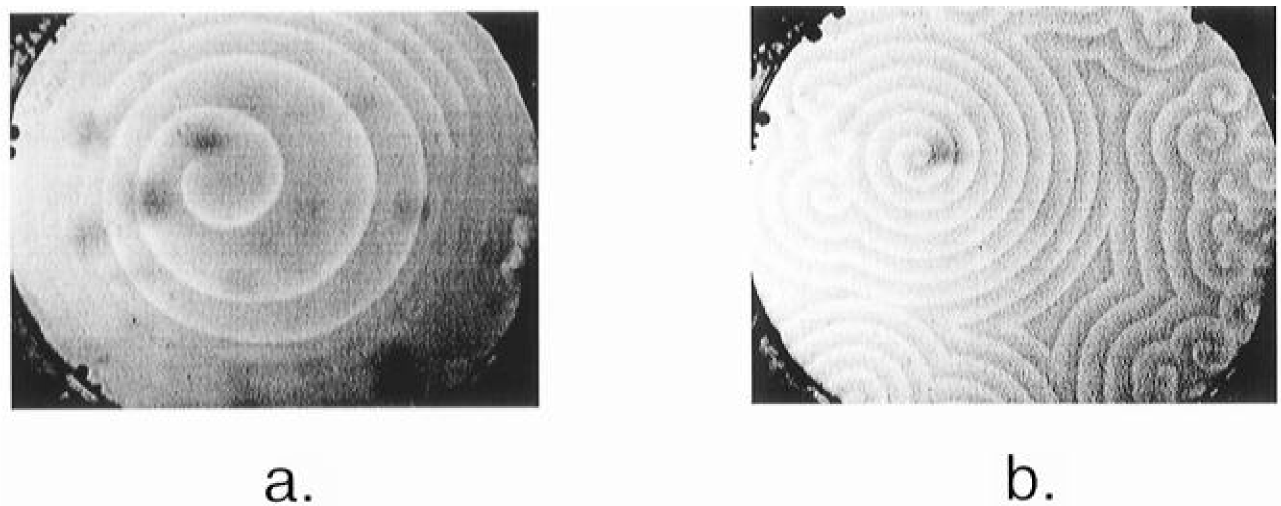

Figure 3. Wave breaks appearing under fast gradual increase of illumination. The illumination I was increased with the speed 0.8 mW/cm^2 per min: (a) waves broken at $I = 11.2$ mW/cm^2; (b) new spiral waves appeared from the wave breaks when the intensity of the illumination I was decreased to 1.6 mW/cm^2.

a slow increase of illumination resulted in adaptation of the spiral wave to the same light intensity at which the spiral was eliminated (Figure 1).

We also found that in between the elimination and the adaptation of the spiral wave the wave breaks can appear, resulting in irregular behavior of the system. Figure 3 shows the experiment with the intensity of illumination increasing gradually (0.8 mW/cm^2 per minute), which is not slow enough for the adaptation process. The wave breaks appeared at the periphery of the spiral, Figure 3b. They served as the origins for the new spiral waves, especially when the intensity of the illumination was decreased, Figure 3c. It is interesting to note that the wave breaks appeared at the periphery of the spiral wave only: the waves situated closer to the core of the spiral wave were not broken. This is very unusual behavior, because in most excitable media, instabilities of propagating waves arise first close to the core of spiral wave.

Computer Simulations

The light affects many characteristics of the chemical excitable medium. In a numerical simulation it is easy to change only selected parameters of the medium. The modified Barkley model[24] was used:

$$U_t = U(1 - U)[U - U_{th}(V)] + \Delta U + \zeta$$

$$V_t = (U - V)/\tau(U,V)$$

Here, U is an activator variable, describing the excitation, and V is an inhibitor variable; its slow dynamics is responsible for the returning of the system back to the resting point. $U_{th}(V) = (V + a)/b$; a and b are parameters. a is the amplitude of variable V, and b is the excitation threshold. $\tau(U,V) = \tau_1$ for $U > U_{th}$ or $\tau(U,V) = \tau_2$ for $U < U_{th}$. τ is the characteristic time scale: τ_1 is the pulse duration, and τ_2 is the restoration time (refractoriness). ζ is a small additive noise, which was added at every time step of integration as $U_{ij} = U_{ij} + 0.01\eta$, where η is a random uniformly distributed number, $-1 < \eta < 1$.

In the light-sensitive BZ reaction, light illumination increases the rate of bromide production, thus decreasing excitability by increasing the threshold and refractoriness.[18−20] So, we have modeled the effects of light on the dynamics of spiral waves by increasing the parameters b and τ_2.

Figure 4 demonstrates elimination of the spiral wave under instantaneous increase of threshold b. Waves stopped propagating, became thinner, Figure 4b, and finally disappeared, Figure 4c (small excited spots at the boundaries also disappeared). After that, to prove that normal wave propagation is possible with the same set of parameters, circular waves (Figure 4d) and a spiral wave (Figure 4e) were initiated. The results compare favorably with the experiments displayed in Figure 1.

Figure 5 (cf. Figure 2) shows adaptation of the spiral wave to the slow increase of the threshold. From the same initial conditions as in Figure 4a, parameter b was increased gradually, 0.025 per 4 time units, up to the same value, 0.225, at which the spiral wave was eliminated in Figure 4. In Figure 5b, contrary to Figure 4b, waves did not become thinner and did not stop propagating. Wavelength increased, and the spiral wave

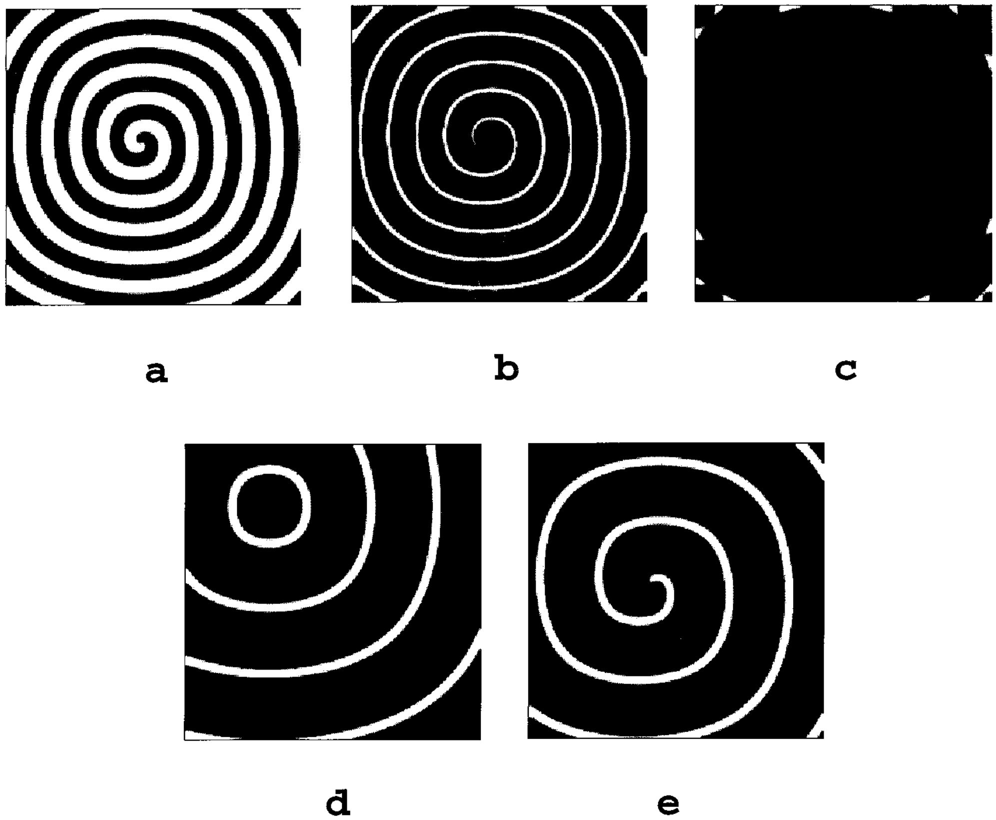

Figure 4. Death of the spiral wave in numerical simulation under stantaneous increase of the threshold. Parameter b was increased from 0.025 up to 0.225 at $t = 0$: (a) initial spiral wave ($b = 0.025$); (b) waves stop propagation, become thinner, and gradually disappear ($t = 53$, $b = 0.225$); (c) wave pattern disappears ($t = 66$); (d) concentric waves in the same medium as in part b ($b = 0.225$); (e) spiral wave in the same medium ($b = 0.225$). Modified Barkley's model: $a = 1$, $\tau_1 = \tau_2 = 100$, $\zeta = 0$, box size $L = 800$, 256×256 grid points, d$t = 0.992$. Excited state ($U > 0.5$) is shown in white.

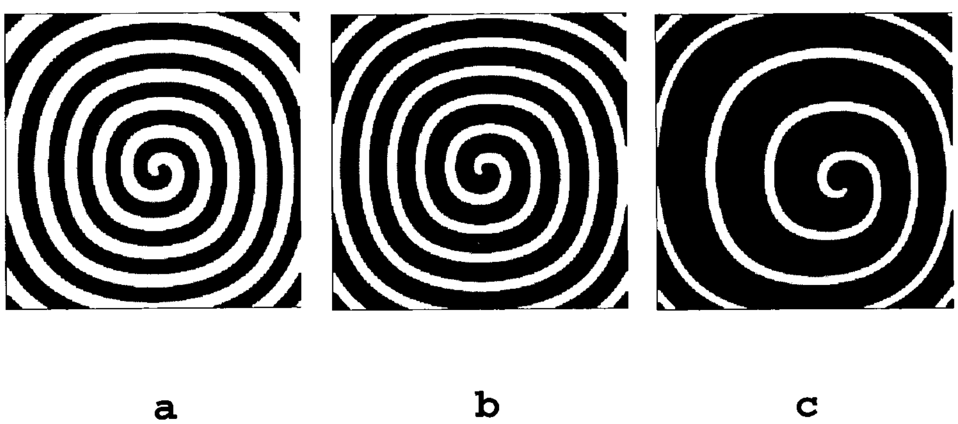

Figure 5. Adaptation of the spiral wave to the slow increase of the threshold. Parameter b was increasing slowly ($\delta b = 0.025$ per 400 time units, until it reaches a value $b = 0.225$): (a) initial spiral wave ($b = 0.025$); (b) $t = 2000$, $b = 0.15$; (c) $t = 4300$, $b = 0.225$. Finally the spiral relaxed to the same spiral as in Figure 4e. Other parameters are the same as in Figure 4.

survived, Figure 5c. A displacement of the spiral wave core from the center of the medium was observed, which also was usually observed in experiments with the BZ reaction (Figure 2).

Wave breaks appeared under the gradual, but relatively fast, decrease of the excitability (Figure 6). The core of the spiral wave grew in size (Figure 6a−c), adapting to the new set of parameters, while propagating waves became thinner and broke at the periphery of the spiral (Figure 5b,c).

Our results demonstrate that the core of the spiral is more flexible than the spiral wave periphery. Under the slow increase of light intensity the rotation speed of the tip slows down. The size of the core (radius of the rotation) grows, following the new set of concentrations, created by photochemistry, and results in increasing the wavelength $\lambda_@$ of the spiral. On the contrary, the propagating wave train is more rigid, because it was already created by the old-shape spiral wave, with definite frequency and wavelength. The only possible response is decrease in speed, not in wavelength.

Analytical Estimations

From the experiments and computer simulations one can see three different responses of the spiral wave to the decrease of excitability: adaptation; multiple wave breaks at the periphery of the spiral; elimination of the spiral wave. Below we estimate the conditions for transitions between these types of behavior.

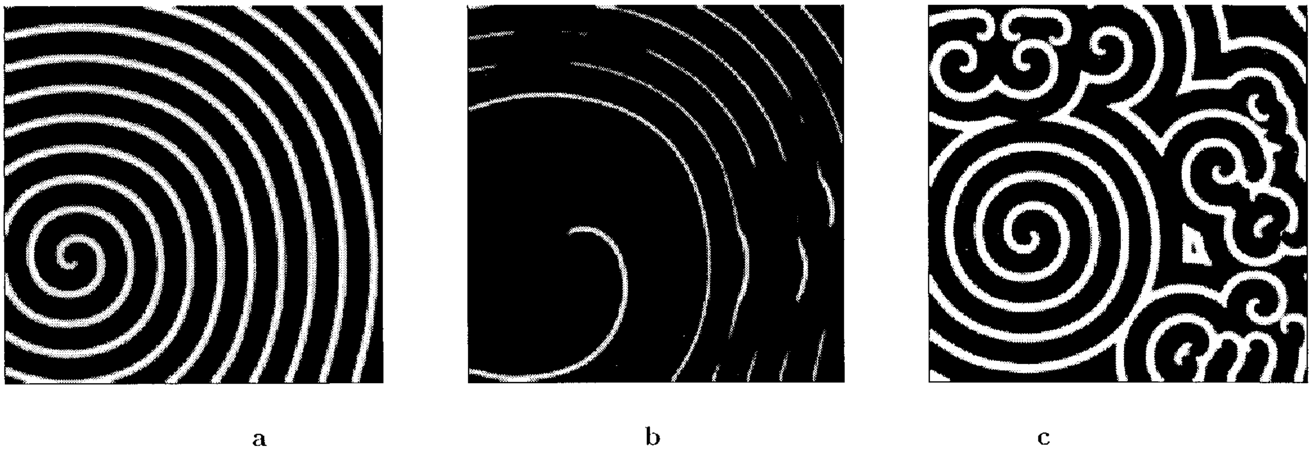

Figure 6. Wave breaks appearing under fast gradual increase of excitability: (a) Initial spiral wave, $b = 0.01$, $\tau_2 = 20$; (b) waves broken at $b = 0.065$, $\tau_2 = 65$ ($t = 1150$); (c) $t = 1950$, new spiral waves appeared from the wave breaks after the parameters were returned to initial values (at $t = 1200$). Parameters: $a = 0.85$, $\tau_1 = 20$. At times $t = 0$; 200; 400; 600; 800 parameters b and τ_2 were increased as follows: (b, τ_2) = (0.035, 30); (0.045, 40); (0.055, 50); (0.065, 60); (0.065, 65). $L = 800$, 256×256 grid points, d$t \cong 0.98$.

1. When the light intensity increases with time, wave breaks will first appear at the periphery of the wave pattern and not near the core of the spiral.

This is an unexpected behavior, because in most known examples (except Eckhaus instability[26,27]), instabilities of propagating waves arise first close to the core of the spiral wave.[28] Curvature of the wave is larger closer to the core, and the excitability there is smaller (e.g., velocity of wave propagation $C = C_0 - Dk$, where k is the curvature). This implies that the wave breaks would arrive first close to the core.

Illumination increases both the minimal possible wavelength λ_R for the propagating waves and the wavelength of the spiral $\lambda_@$. The condition for initiating a wave break is

$$\lambda_@ < \lambda_R \quad \text{or} \quad T < R \tag{1.1}$$

where T is an interval between waves, R is the refractoriness of excitable medium (given by the minimal period of propagating waves), and $\lambda = CT$. But λ_R increases globally (everywhere in the medium), and $\lambda_@$ increases only locally, near the spiral core (see Figures 2b and 6b). This implies that the wave breaks appear first not at the core but at ther periphery of the spiral.

In the experiment, light intensity I was increased linearly with time,

$$I(t) = I_0 + \alpha t \tag{1.2}$$

Refractoriness R and period $T_@$ of rotation of the spiral wave increase with light intensity I, while velocity C of wave propagation diminishes with I:

$$R = R(I), \quad T_@ = T(I), \quad C = C(I) \tag{1.3}$$

$R(I)$, $T(I)$, and $C = C(I)$ are functions, determined in the experiment.

Let us find when a wave situated at the periphery of the reactor can be broken. Let L be the distance between this wave and the core of the spiral (characteristic size of the reactor). The wave was sent $\delta t = L/C$ seconds ago. We will take this time as zero and denote light intensity at that time as I^0. Refractoriness at $t = 0$ was $R^0 = R(I^0)$, and the period of the spiral wave was $T_@{}^0 = T(I^0)$. At time δt, refractoriness $R(\delta t) = R(I^0 + \alpha\delta t)$. Condition 1.1 is equivalent to

$$T_@{}^0 < R(\delta t) \tag{1.4}$$

Or, $T_@(I^0) < R(I^0 + \alpha\delta t)$. From here, the restriction for α inducing a wave break is easily found: $R^{-1}[T_@(I^0)] < I^0 + \alpha\,\delta t$, or

$$\alpha\delta t > R^{-1}[T_@(I^0)] - I^0 \tag{1.5}$$

Formula 1.5 predicts than the wave breaks are easier to induce when (1) the reactor size is big (the critical value of α in (1.5) linearly decreases with size of the reactor, L, since $\delta t = L/C$) and (2) initial light intensity I^0 is smaller.

An intuitively clear interpretation of (1.5) is that the wave-train with the smallest wavelength should have enough time to pass all over the medium. Close to this situation there is the high-frequency instability of the wave-train, which can result in a preturbulent state of the medium.[13]

2. The spiral wave disappears when the light intensity increases.

Increase in light intensity results in a decrease of both the wave front velocity C and the wave tail velocity C_- and in an increase of both the recovery time R and the wavelength $\lambda_R = RC$ ("refractory tail"). In a stationary propagating wave, $C_- = C$. But when the refractory tail grows, velocity C_- decreases more than C:

$$C - C_- = \partial/\partial t\, \lambda_R = \alpha(RC' + R'C) \tag{2.1}$$

where $C' = \partial C/\partial I$, and $R' = \partial R/\partial I$ are characteristics of the medium, and the light intensity increases according to (1.2).

A wave disappears when the wave front reaches the tail. The distance between the front and the tail is

$$d = \lambda_@ - \lambda_R \tag{2.2}$$

where $\lambda_@$ is the wavelength (pitch) of the spiral. The time necessary to eliminate the spiral is

$$t_n = d/(C - C_-) \tag{2.3}$$

and the necessary light intensity is

$$I_n = I_0 + \alpha t_n = I_0 + d/(RC' + R'C) \tag{2.4}$$

In the the experiment, the spiral wave is destroyed only when the light changes quickly (Figure 1). Surprisingly, estimation 2.4 does not depend on α. What is the reason for this contradiction? The answer is the following. We have permitted an arbitrarily large time (2.3) for eliminating the spiral. The estimation is correct for eliminating 1D rotating waves (the pulse, rotating in a thin circular fiber) or 2D spiral waves rotating around a big obstacle. For a free spiral wave, there are two competing processes. One is described by (2.4). The second one is an increase of the core resulting in increasing $\lambda_@$ and increasing d in (2.2). Which of the processes prevails depends on the relation of the characteristic time scales. The spiral can be eliminated when

$$t_n < T_@ \quad \text{or} \quad \alpha > d/T_@(RC' + R'C) \qquad (2.5)$$

Our estimations show that there are two control parameters: the size of the reactor and the time scale of the light change. For sufficiently large reactors, both phenomena, 1 and 2, can be observed. In a small reactor, only phenomenon 2 can be observed because condition 1.5 becomes more restrictive than 2.5, and the spiral wave disappears earlier than it can induce wave breaks.

Discussion

We have found that a spiral wave can be eliminated by a transition in the parameter space, inside the region where normal wave propagation is not destroyed. The mechanism of elimination is quite different from what is already known from experiments and numerical simulations: the annihilation of two counterrotating spirals;[8] collision of the spiral with the boundary;[5] the global suppression of wave propagation. It is also different from aperiodic modulation of wave fronts resulting in wave breaks[29] (where there are some mistakes because light intensity of 300 mW/m^2 reported in ref 29 is definitely not enough to induce wave destabilization in the Ru-catalyzed BZ reaction).

There exists a region in the parameter space where spiral waves do not exist but plane and circular waves can propagate.[24] It is not surprising to destroy the spirals by coming to this peculiar region. But what we have found is different. All our parameters (initial and final values) belong to the region where spirals exist.

The light was chosen here for elimination of the rotating wave, but this is not the unique choice. It is also possible to change the temperature in the reactor (cool it down to decrease excitability) or reduce the acidity or bromate concentration.[25] What is required is that the excitability should be changed rapidly. For changing the chemical composition, the limiting factor is the residence time of the reactor, which is a few minutes at best, and for the temperature it is the heat capacity of the reactor, which also prevents the fast transition.

The validity of the results obtained is not limited by chemical excitable media. Our analytical estimations of light effects on the chemical system really do not exploit any chemical specificity. We manipulate the wavelength of the spiral wave, velocity of excitation propagation, and recovery time, which are general properties of any excitable system. The results were numerically reproduced with such a nonspecific model as FitzHugh−Nagumo equations, which rely only on generic (topological) properties of excitable media.

With suppression of the spiral wave there are two different problems: elimination of a single spiral and elimination of many spirals simultaneously. Resonant drift of the spiral wave,[6] termination of the spiral wave by application of excitation to the core region of the spiral, and antitachycardia pacing in cardiac muscle[22] are all methods that permit effective destroying of a single spiral wave but are absolutely not effective for elimination of many spirals. For resonant drift, for example, the direction of the drift is determined by the phase of rotation of the spiral wave, which is different for different spirals. Induced drift[21] of the spiral, in principle, permits elimination of many spiral waves, but the method is extremely slow; the speed of drift is 1 order of magnitude less than the speed of wave propagation.

The only known fast method for destroying many rotating spirals simultaneously is defibrillation of cardiac muscle with a very strong electric shock (5 kV, 20 A), which destroys all propagating waves, together with rotating spirals. Quite naturally, it damages excitable cells also. Our method is another approach that allows the elimination of spiral waves only, without suppression of normally propagating waves, and it requires less (and, hopefully, less damaging) intervention.

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