AIP | Journal of Applied Physics

Adiabatically implementing quantum gates

Jie Sun, Songfeng Lu, and Fang Liu

Citation: Journal of Applied Physics **115**, 224901 (2014); doi: 10.1063/1.4882018
View online: http://dx.doi.org/10.1063/1.4882018
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/115/22?ver=pdfcov
Published by the AIP Publishing

JOURNAL OF APPLIED PHYSICS **115**, 224901 (2014)

Adiabatically implementing quantum gates

Jie Sun, Songfeng Lu,[a)] and Fang Liu
School of Computer Science and Technology, Huazhong University of Science and Technology, Wuhan 430074, China

(Received 14 December 2013; accepted 26 May 2014; published online 9 June 2014)

We show that, through the approach of quantum adiabatic evolution, all of the usual quantum gates can be implemented efficiently, yielding running time of order $O(1)$. This may be considered as a useful alternative to the standard quantum computing approach, which involves quantum gates transforming quantum states during the computing process. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4882018]

I. INTRODUCTION

Adiabatic-evolution-based quantum computing has attracted much attention in recent years, since it was first proposed by Farhi *et al.*[1] Although there is considerable interest in adiabatic quantum algorithms that solve NP problems, other interesting problems have also been studied with this different approach to quantum computing, such as the Deutsch–Jozsa algorithm,[2,3] factorization of integers,[4] hidden subgroup problem,[5] and Grover's database search.[6]

The big break-through made in quantum adiabatic computation is in its computing power, being polynomially equivalent to the gate model quantum computation.[7,8] However, it is commonly believed that the former approach to quantum computing is advantageous over the latter in some other aspects, such as more robust against environmental noise,[9] quantum decoherence, and certain control errors.[10]

Let us recall the basic ideas involved in an adiabatic quantum computation paradigm. Consider a quantum system whose Hamiltonian varies sufficiently "slowly" from an initial Hamiltonian H_0 to a final Hamiltonian H_1, during which the corresponding ground state of the system evolves from an initial state to a final one. The quantum system evolves according to the Schrödinger equation

$$i\frac{d}{dt}|\varphi(t)\rangle = \tilde{H}(t)|\varphi(t)\rangle, \tag{1}$$

where $|\varphi(t)\rangle (0 \le t \le T)$ is the instantaneous state of the quantum system, and

$$\tilde{H}(t) = \hat{H}(s) = (1-s(t))H_0 + s(t)H_1, \tag{2}$$

with $s = s(t)$ a monotonic function satisfying $s(0) = 0$ and $s(T) = 1$. The running time T of this adiabatic evolution can be estimated in quantum mechanics given by the adiabatic theorem[11]

$$T \ge \frac{D_{max}}{\varepsilon g_{min}^2}, \quad 0 < \varepsilon \ll 1, \tag{3}$$

where

$$D_{max} = \max_{s\in[0,1]} \left| \left\langle E_1(s) \middle| \frac{d\hat{H}(s)}{ds} \middle| E_0(s) \right\rangle \right|, \tag{4}$$

$$g_{min} = \min_{s\in[0,1]} [E_1(s) - E_0(s)], \tag{5}$$

with $E_k(s)$ and $|E_k(s)\rangle$ the eigenvalues and corresponding eigenvectors of $\hat{H}(s)$ $(k = 0, 1)$, respectively. In an unstructured adiabatic quantum search, the ground state of the initial Hamiltonian H_0 is usually chosen as the equal superposition of all the elements in the database, and the final Hamiltonian H_1 is set as that for which the ground state encodes the solution. Generally, to ensure that there are also no degeneracies in the energy spectrum during the evolution, the initial and final Hamiltonians are usually chosen such that no symmetries exist in $\hat{H}(s)$ by ensuring that they are diagonal in different bases.

In this paper, we study adiabatically implementing the usual quantum gates, which are frequently used in circuit model of quantum computing. It is found that all these quantum gates can be implemented efficiently by adiabatic evolution, which means the running time is of $O(1)$. Note that here we only undertake a theoretical discussion of the quantum-gate implementations. A discussion of realistic experimental implementations of quantum logical gates for multilevel systems can be found in Ref. 12. The implementations were demonstrated through decoherence control under the adiabatic method using phase modulated laser pulses. The rest of the paper is organized as follows. In Sec. II, we first show a theorem that is the basis for time complexity estimation of the adiabatic evolutions for the quantum gates shown later. In the part followed, we discuss the adiabatically implementing all the usual quantum gates in detail, which are classified into three categories: one-qubit quantum gates implementations, two-qubit quantum gates implementations, and three-qubit quantum gates implementations. The last section is devoted to end the whole paper.

II. ADIABATIC EVOLUTION FOR IMPLEMENTING QUANTUM GATES

For convenience, we begin with a proof of the following theorem for use later.

[a)]Author to whom correspondence should be addressed. Electronic mail: lusongfeng@hotmail.com

Theorem: Suppose the initial state of an adiabatic quantum system is $|\alpha\rangle$, and the final state is $|\beta\rangle$; then the running time of a global adiabatic evolution is estimated as $O(|\langle\alpha|\beta\rangle|^{-2})$.

Proof: The proof is straight forward, so long as we note that the Hamiltonian for the adiabatic evolution can be written as

$$\hat{H}(s) = (1-s)H_0 + sH_1 = (1-s)(-|\alpha\rangle\langle\alpha|) + s(-|\beta\rangle\langle\beta|). \quad (6)$$

Suppose the eigenvalues and their corresponding eigenvectors of the Hamiltonian $\hat{H}(s)$ are given by $E_k(s)$ and $|E_k(s)\rangle (k=0,1)$, respectively. Then, we have

$$\hat{H}(s)|E_k(s)\rangle = E_k(s)|E_k(s)\rangle, \quad k=0,1. \quad (7)$$

Left multiplying Eq. (6) by $\langle\alpha|$ and $\langle\beta|$, respectively, and combining with Eq. (7), yields

$$\frac{s\langle\alpha|\beta\rangle}{s-1-E_k(s)} = \frac{s+E_k(s)}{(s-1)\langle\beta|\alpha\rangle}, \quad k=0,1, \quad (8)$$

from which we can obtain the eigenvalues for $\hat{H}(s)$,

$$E_0(s) = \frac{-1-\sqrt{1+4s(s-1)(1-|\langle\alpha|\beta\rangle|^2)}}{2}, \quad (9)$$

$$E_1(s) = \frac{-1+\sqrt{1+4s(s-1)(1-|\langle\alpha|\beta\rangle|^2)}}{2}. \quad (10)$$

Therefore, the minimum eigenvalue gap is

$$g_{min} = \min_{s\in[0,1]}[E_1(s)-E_0(s)] = |\langle\alpha|\beta\rangle|. \quad (11)$$

According to the adiabatic theorem, this implies that the running time for this type of adiabatic evolution is $O(|\langle\alpha|\beta\rangle|^{-2})$, completing the proof.

We now turn to the adiabatic implementation of the usual quantum gates frequently used in circuit models. These are classified into three categories: one-qubit-gate implementations, two-qubit-gate implementations, and three-qubit-gate implementations. The main idea behind the adiabatic implementation can be summarized as follows: the ground state of the initial Hamiltonian is select as the initial quantum state that needs to be transformed by the quantum gate, and a final Hamiltonian is designed such that its ground state corresponds to the final quantum state determined by the quantum gate. As we shall see next, in the adiabatic implementations of nearly all quantum gates, a simple linear evolution path is sufficient to guarantee a resultant efficiency for the algorithm of $O(1)$ running time.

A. One-qubit quantum gates

1. Hadamard gate

We know that the matrix form for the Hadamard gate is[13]

$$\mathbf{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

When this quantum gate is applied to the single quantum state $|0\rangle$ or $|1\rangle$, its effect is

$$\mathbf{H}|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad \mathbf{H}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \quad (12)$$

Therefore, suppose the adiabatic Hamiltonian is designed to be of form in Eq. (6), in which

$$|\alpha\rangle = \frac{E_0|0\rangle + E_1|1\rangle}{\sqrt{E_0^2+E_1^2}} \quad (13)$$

and

$$|\beta\rangle = \frac{\frac{1}{\sqrt{2}}(E_0+E_1)|0\rangle + \frac{1}{\sqrt{2}}(E_0-E_1)|1\rangle}{\sqrt{E_0^2+E_1^2}}. \quad (14)$$

By the above theorem, and setting $E_k = k+1 (k=0,\ 1)$, we know that the time complexity of the linear adiabatic evolution is

$$T = O(|\langle\alpha|\beta\rangle|^{-2}) = O(1), \quad (15)$$

which implies that the adiabatic implementation of the quantum gate is efficient.

2. Pauli-X gate

The matrix form for Pauli-X gate is written as[13]

$$\mathbf{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Therefore, the quantum gate applied to the single quantum state produces

$$\mathbf{X}|0\rangle = |1\rangle, \quad \mathbf{X}|1\rangle = |0\rangle. \quad (16)$$

Suppose the initial state is same as that for Hadamard gate, and the final state is specified as

$$|\beta\rangle = \frac{E_0|1\rangle + E_1|0\rangle}{\sqrt{E_0^2+E_1^2}}. \quad (17)$$

Assume the instantaneous Hamiltonian has the usual linear form; because $|\langle\alpha|\beta\rangle| \neq 0$, setting $E_k = k+1$ $(k=0,\ 1)$ can thus yield a time complexity of order $O(1)$ for the corresponding adiabatic evolution.

3. Pauli-Y gate

The matrix form for this particular quantum gate takes the well-known form[13]

$$\mathbf{Y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$

and the effect that this gate has on a single quantum state is

$$\mathbf{Y}|0\rangle = i|1\rangle, \quad \mathbf{Y}|1\rangle = -i|0\rangle. \tag{18}$$

The initial Hamiltonian can be chosen as that of the Pauli-X gate; the final state is thus specified as

$$|\beta\rangle = \frac{E_0|1\rangle - E_1|0\rangle}{\sqrt{E_0^2 + E_1^2}} i. \tag{19}$$

However, if the system Hamiltonian is to retain its widely studied designed form $\hat{H}(s) = (1-s)H_0 + sH_1$, the running time tends to infinity by the above theorem as the two quantum states $|\alpha\rangle$ and $|\beta\rangle$ are orthogonal. To overcome this difficulty, we can use a nonlinear adiabatic evolution[14]

$$\hat{H}(s) = (1-s)H_0 + sH_1 + s(1-s)(|\alpha\rangle\langle\beta| + |\beta\rangle\langle\alpha|). \tag{20}$$

Left multiplying the above equation by $\langle\alpha|$ and $\langle\beta|$ and combining with Eq. (7), we obtain

$$\frac{s(s-1)}{s-1-E_k(s)} = \frac{E_k(s)+s}{s(1-s)}. \tag{21}$$

Similar to the above, the eigenvalues and minimum gap are

$$E_0(s) = -1, \quad E_1(s) = 0; \quad g(s) = E_1(s) - E_0(s) = 1, \tag{22}$$

which again imply $O(1)$ running time for the adiabatic evolution.

4. Pauli-Z gate

The matrix form for Pauli-Z gate can be written as[13]

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

When the gate is applied to the single quantum state, the result is

$$\mathbf{Z}|0\rangle = |0\rangle, \quad \mathbf{Z}|1\rangle = -|1\rangle. \tag{23}$$

If the initial state is $|\alpha\rangle$, the final state after adiabatic evolution corresponds to

$$|\beta\rangle = \frac{E_0|0\rangle - E_1|1\rangle}{\sqrt{E_0^2 + E_1^2}}. \tag{24}$$

As we can see, the Pauli-Z gate has changed the phase of the state $|1\rangle$. By the above theorem, and selecting $E_k = k+1$ $(k = 0, 1)$, we have

$$|\langle\alpha|\beta\rangle| = \frac{3}{5}. \tag{25}$$

Hence, the adiabatic evolution can be completed in time $O(1)$.

5. Phase gate

The phase gate has matrix form[13]

$$\mathbf{S} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}.$$

Given an initial state $|\alpha\rangle$ of the quantum system, then according to this representation of the phase gate, it evolves into the final state

$$|\beta\rangle = \frac{E_0|0\rangle + iE_1|1\rangle}{\sqrt{E_0^2 + E_1^2}}. \tag{26}$$

Hence,

$$|\langle\alpha|\beta\rangle| = \frac{\sqrt{E_0^4 + E_1^4}}{E_0^2 + E_1^2}. \tag{27}$$

Setting $E_k = k+1$ $(k = 0, 1)$, the adiabatic evolution procedure can be finished in running time of $O(1)$.

6. $\frac{\pi}{8}$ phase gate

This gate has a similar matrix form to the previous[13]

$$\mathbf{S}^{\frac{\pi}{8}} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix},$$

but this time the initial state $|\alpha\rangle$ evolves into the final state

$$|\beta\rangle = \frac{E_0|0\rangle + e^{i\pi/4}E_1|1\rangle}{\sqrt{E_0^2 + E_1^2}}. \tag{28}$$

Clearly,

$$|\langle\alpha|\beta\rangle| = \frac{\sqrt{E_0^4 + E_1^4 + \sqrt{2}E_0^2E_1^2}}{E_0^2 + E_1^2}. \tag{29}$$

Choosing $E_k = k+1$ $(k = 0, 1)$, it is easy to show that the running time for the adiabatic evolution is in $O(1)$ by the above theorem.

B. Two-qubit quantum gates

1. Controlled-NOT gate

The effect that the controlled-NOT gate has on a two-qubit quantum state can be expressed as[13]

$$|a\rangle|b\rangle \rightarrow |a\rangle|a \oplus b\rangle, \tag{30}$$

in which $\oplus$ represents addition modulo 2, with $|a\rangle$ and $|b\rangle$ the controlling qubit and target qubit, respectively. Therefore, if the controlling qubit is 1, the target qubit switches to its complementary value, otherwise remains unchanged. The equivalent matrix form of the quantum gate is given by

$$\textbf{Controlled}-\textbf{NOT Gate} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Suppose the system Hamiltonian, $\hat{H}(s)$ of Eq. (6), that adiabatically implements this quantum gate is given with

$$|\alpha\rangle = \frac{\sum_{k=0}^{3} E_k|k\rangle}{\sqrt{\sum_{k=0}^{3} E_k^2}} \tag{31}$$

and

$$|\beta\rangle = \frac{E_0|00\rangle + E_1|01\rangle + E_2|11\rangle + E_3|10\rangle}{\sqrt{\sum_{k=0}^{3} E_k^2}}. \tag{32}$$

For convenience, we set $E_k = k$ $(k = 0, 1, 2, 3)$. By the above theorem, the running time for the corresponding adiabatic evolution is shown to be of order $O(1)$.

2. Swap gate

Note that the matrix form for swap gate can be described as[13]

$$\textbf{Swap Gate} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Similar to the argument given for the controlled-NOT quantum gate, it is easy to demonstrate that the running time for the adiabatically implementing swap gate is of order $O(1)$.

3. Controlled-Z gate

From Ref. 13, we know that controlled-Z gate has an equivalent matrix representation

$$\textbf{Controlled} - \textbf{Z Gate} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Assuming the instantaneous quantum Hamiltonian has a linear form, in which the initial state is

$$|\alpha\rangle = \frac{\sum_{k=0}^{3} E_k|k\rangle}{\sqrt{\sum_{k=0}^{3} E_k^2}}, \tag{33}$$

then, the final state is

$$|\beta\rangle = \frac{E_0|00\rangle + E_1|01\rangle + E_2|10\rangle - E_3|11\rangle}{\sqrt{\sum_{k=0}^{3} E_k^2}}. \tag{34}$$

Setting $E_k = k + 1 (k = 0, 1, 2, 3)$, and by the above theorem, the time complexity for the adiabatic evolution is of order $O(|\langle\alpha|\beta\rangle|^{-2}) = O(1)$, where

$$|\langle\alpha|\beta\rangle| = \left|\frac{E_0^2 + E_1^2 + E_2^2 - E_3^2}{\sum_{k=0}^{3} E_k^2}\right|. \tag{35}$$

4. Controlled-phase gate

The equivalent matrix representation for this quantum gate is given by[13]

$$\textbf{Controlled} - \textbf{phase Gate} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix}.$$

Therefore, if the initial state is the same as Eq. (33), after adiabatic evolution, the target correspondingly is given by

$$|\beta\rangle = \frac{E_0|00\rangle + E_1|01\rangle + E_2|10\rangle + iE_3|11\rangle}{\sqrt{\sum_{k=0}^{3} E_k^2}}. \tag{36}$$

Clearly,

$$|\langle\alpha|\beta\rangle| = \frac{\sqrt{(E_0^2 + E_1^2 + E_2^2)^2 + E_3^4}}{\sum_{k=0}^{3} E_k^2}. \tag{37}$$

For simplicity, selecting $E_k = k + 1$ $(k = 0, 1, 2, 3)$ and after running time of $O(1)$ for the adiabatic evolution, the quantum state $|\alpha\rangle$ evolves into $|\beta\rangle$, which has the same effect as the controlled-phase gate applied to the initial state.

C. Three-qubit quantum gates

1. Toffoli gate

The Toffoli gate takes as input three-qubit quantum states, for which two are controlling qubits and the third is a target qubit. Formally, it has the effect

$$|a\rangle|b\rangle|c\rangle \mapsto |a\rangle|b\rangle|ab \oplus c\rangle. \tag{38}$$

The matrix form for this quantum gate according to Ref. 13 is given by

$$\textbf{Toffoli} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

As before, the system Hamiltonian that drives the adiabatic evolution is set as that in Eq. (6) with

$$|\alpha\rangle = \frac{\sum_{k=0}^{7} E_k|k\rangle}{\sqrt{\sum_{k=0}^{7} E_k^2}} \tag{39}$$

and

$$|\beta\rangle = \frac{E_0|000\rangle + E_1|001\rangle + E_2|010\rangle + E_3|011\rangle}{\sqrt{\sum_{k=0}^{7} E_k^2}} + \frac{E_4|100\rangle + E_5|101\rangle + E_6|111\rangle + E_7|110\rangle}{\sqrt{\sum_{k=0}^{7} E_k^2}}. \quad (40)$$

Setting $E_k = k(k = 0, 1, 2, 3, 4, 5, 6, 7)$ and by the above theorem, it is easy to demonstrate that the running time for the adiabatic evolution is of order $O(1)$.

2. Fredkin gate

The adiabatic implementation of the Fredkin gate is essentially the same as that for the Toffoli gate. By Ref. 13, we have for this quantum gate the matrix representation

$$\textbf{Fredkin Gate} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Following the same prescription as for the previous, one can deduce the effect of the gate applied to the initial quantum state. The corresponding adiabatic evolution running time is, of course, of order $O(1)$.

III. DISCUSSION AND CONCLUSION

In the framework of adiabatic evolution model, all the usual quantum gates have been shown to be implemented efficiently, which means an order of $O(1)$ running time. Although the prior work of Andrecut *et al*. provides evidence that the Hadamard gate, controlled-NOT gate, and Toffoli gate can be implemented easily using adiabatic evolution,[15] here we give an alternative construction and extend their work to include all other frequently used quantum gates. It is not difficult to observe that, in adiabatically implementing the controlled-NOT and Toffoli gates, nonlinear evolution paths have been used in Ref. 15. Furthermore, the resultant system Hamiltonians add extra complexity in implementing the adiabatic evolution.[16] In contrast, in our approach, simple linear evolution paths are sufficient to attain the same goal. The only exception is the implementation of the Pauli-Y gate, in which a nonlinear evolution path has also been applied to avoid inefficiencies in the algorithm. And, some of the merits of the nonlinear approaches actually lies more on the fact that they represent the more practical implementable scenarios (see for, e.g., again Ref. 12 on Adiabatic Quantum Computing). However, here we do not mean to delivery the readers of the information that this approach is going to supersede all other existing modes of quantum computing. We only mean that, when executing transformations on quantum states of circuit model, adiabatic evolution may be considered as an alternative for quantum gates if their implementations are restricted by certain experimental circumstances. In the framework of the adiabatic quantum framework, logical implementation of quantum gates uses the language of ground states, spectral gaps and Hamiltonians instead of the standard unitary transformation language. It is noted that in each adiabatic implementation of the usual quantum gates discussed in this paper, the computational procedure is described by the continuous time evolution of a time-dependent Hamiltonian with limited energetic resources, which is often neglected in the unitary gate language.[13] In contrast to this, in Ref. 17, it is only possible to do the calculation of a quantum search in $O(1)$ time complexity by adding large energy to the system. Adiabatic quantum computing, which has taken a lot of realistic beating in the past few years, for referrals to the present status and the implementation issues of it, see for, e.g., Refs. 4 and 18–23.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant Nos. 61173050 and U1233119. The first author also gratefully acknowledges the support from the China Postdoctoral Science Foundation under Grant No. 2014M552041.

[1]E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, "A quantum adiabatic evolution algorithm applied to random instances of an NP-complete problem," Science **292**, 472–476 (2001).
[2]S. Das, R. Kobes, and G. Kunstatter, "Adiabatic quantum computation and Deutsch's algorithm," Phys. Rev. A **65**, 062310 (2002).
[3]Z. Wei and M. Ying, "A modified quantum adiabatic evolution for the Deutsch-Jozsa problem," Phys. Lett. A **354**, 271–273 (2006).
[4]X. H. Peng, Z. Y. Liao, N. Y. Xu, G. Qin, X. Y. Zhou, D. Suter, and J. F. Du, "Quantum adiabatic algorithm for factorization and its experimental implementation," Phys. Rev. Lett. **101**, 220405 (2008).
[5]M. V. Panduranga Rao, "Solving a hidden subgroup problem using the adiabatic quantum-computing paradigm," Phys. Rev. A **67**, 052306 (2003).
[6]L. K. Grover, "Quantum mechanics helps in searching for a needle in a haystack," Phys. Rev. Lett. **79**, 325 (1997).
[7]A. Mizel, D. A. Lidar, and M. Mitchell, "Simple proof of equivalence between adiabatic quantum computation and the circuit model," Phys. Rev. Lett. **99**, 070502 (2007).
[8]D. Aharonov, W. v. Dam, J. Kempe, Z. Landau, S. Lloyd, and O. Regev, "Adiabatic quantum computation is equivalent to standard quantum computation," SIAM J. Comput. **37**, 166–194 (2007).
[9]A. M. Childs, E. Farhi, and J. Preskill, "Robustness of adiabatic quantum computation," Phys. Rev. A **65**, 012322 (2001).
[10]D. A. Lidar, "Towards fault tolerant adiabatic quantum computation," Phys. Rev. Lett. **100**, 160506 (2008).
[11]A. Messiah, *Quantum Mechanics*, 1st ed. (Dover, New York, 1999), pp. 744–763.
[12]D. Goswami, "Adiabatic quantum computing with phase modulated laser pulses," J. Phys. A **38**, L615–L626 (2005).
[13]M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
[14]E. Farhi, J. Goldstone, and S. Gutmann, "Quantum adiabatic evolution algorithms with different paths," e-print arXiv:quant-ph/0208135 (2002).
[15]M. Andrecut and M. K. Ali, "Adiabatic quantum gates and Boolean functions," J. Phys. A: Math. Gen. **37**, L267–L273 (2004).
[16]M. Andrecut and M. K. Ali, "Unstructured adiabatic quantum search," Int. J. Theor. Phys. **43**, 925–931 (2004).
[17]S. Das, R. Kobes, and G. Kunstatter, "Energy and efficiency of adiabatic quantum search algorithms," J. Phys. A: Math. Gen. **36**, 2839–2845 (2003).
[18]M. Grajcar, A. Izmalkov, and E. Il'ichev, "Possible implementation of adiabatic quantum algorithm with superconducting flux qubits," Phys. Rev. B **71**, 144501 (2005).
[19]N. Xu, J. Zhu, D. Lu, X. Zhou, X.-H. Peng, and J. Du, "Quantum factorization of 143 on a dipolar-coupling nuclear magnetic resonance system," Phys. Rev. Lett. **108**, 130501 (2012).

[20] V. E. Zobov and A. S. Ermilov, "Implementation of a quantum adiabatic algorithm for factorization on two qudits," J. Exp. Theor. Phys. **114**, 923–932 (2012).
[21] Z. Bian, F. Chudak, W. G. Macready, L. Clark, and F. Gaitan, "Experimental determination of Ramsey numbers," Phys. Rev. Lett. **111**, 130505 (2013).
[22] Y. Long, G. Feng, Y. Tang, W. Qin, and G. Long, "NMR realization of adiabatic quantum algorithms for the modified Simon problem," Phys. Rev. A **88**, 012306 (2013).
[23] T. Keating, K. Goyal, Y.-Y. Jau, G. W. Biedermann, A. J. Landahl, and I. H. Deutsch, "Adiabatic quantum computation with Rydberg-dressed atoms," Phys. Rev. A **87**, 052314 (2013).