See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/235393827

# Electronegativity, Charge Transfer, Crystal Field Strength, and the Point Charge Model Revisited

ARTICLE in THE JOURNAL OF PHYSICAL CHEMISTRY A · JANUARY 2013

Impact Factor: 2.69 · DOI: 10.1021/jp3117566 · Source: PubMed

CITATIONS

CITATION

5

READS

72

# 2 AUTHORS:



Peter Anthony Tanner

The Hong Kong Institute of Education

355 PUBLICATIONS 4,367 CITATIONS

SEE PROFILE



**Lixin Ning** 

Anhui Normal University

63 PUBLICATIONS 425 CITATIONS

SEE PROFILE

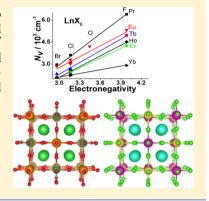
pubs.acs.org/JPCA

# Electronegativity, Charge Transfer, Crystal Field Strength, and the Point Charge Model Revisited

Peter A. Tanner\*,† and Lixin Ning‡

Supporting Information

**ABSTRACT:** Although the optical spectra of  $LnCl_6^{3-}$  systems are complex, only two crystal field parameters,  $B_{40}$  and  $B_{60}$ , are required to model the *J*-multiplet crystal field splittings in octahedral symmetry. It is found that these parameters exhibit  $R^{-5}$  and  $R^{-7}$  dependence, respectively, upon the ionic radius  $Ln^{3+}(VI)$ , but not upon the Ln-Cl distance. More generally, the crystal field strengths of  $LnX_6$  systems (X = Br, Cl, F, O) exhibit linear relationships with ligand electronegativity, charge transfer energy, and fractional ionic character of the Ln-X bond.



# 1. INTRODUCTION

Figure 1a shows the emission spectra of two europium (Eu³+) ion solid-state systems: Ba₂YNbO<sub>6</sub> (Figure 1b, doped with Eu³+) and neat Cs₂NaEuCl<sub>6</sub> (Figure 1c), which comprise EuX<sub>6</sub>, X = O, Cl moieties, respectively. The spectra are recorded at low temperature so that features are more clearly resolved. Both spectra have previously been interpreted in detail¹-³ as arising from intraconfigurational 4f<sup>6</sup> electronic and vibronic transitions from the  $^5D_J$  (J = 0, 1) states of Eu³+ to the levels of the  $^7F_J$  (J = 0-6) multiplet term (Figure 1d). The notation  $^{2S+1}L_J$  concisely gives the electron spin (S), orbital (L), and total (J) angular momentum of the Eu³+ multiplet. The spectra are complex and a chemist might ask, "What information can we get from such spectra, and how can we condense this information?" This study throws some light upon these questions.

Knowledge of the vibrations of these systems, together with the study of the absorption spectra, enables spectral assignments to be made for the 4f<sup>6</sup> energy levels. These energy levels correspond to just two multiplets:  $^5\mathrm{D}$  and  $^7\mathrm{F}$ , which are each split into 2L+1 states by spin—orbit coupling, and most of these states are further split by the crystal field (Figure 1d). The symmetry of these systems is octahedral for  $\mathrm{Cs_2NaEuCl_6}$ , or nearly so for  $\mathrm{Ba_2YNbO_6:Eu^{3+}}$ , and the crystal field may be modeled by two crystal field parameters (CFP),  $B_{kq^\prime}$  namely,  $B_{40}$  and  $B_{60}$ , as has previously been performed. These two parameters summarize concisely the crystal field splittings of multiplets. An attempt was made to provide a physical meaning for the crystal field parameters in a previous publication. In the present study, the relationships between crystal field parameters, crystal field strength, electronegativity and fractional ionic character are investigated.

# 2. THEORY

The simplest theoretical model, the point charge electrostatic model (PCEM)<sup>5,6</sup> envisages the electrostatic field experienced by the lanthanide ion to result from the external charge densities in the crystal, modeled as point charges. The expressions for the parameters for the case of octahedral symmetry of Ln<sup>3+</sup> are particularly simple:<sup>5</sup>

$$B_{40} = (-7/2)Z_{\text{eff}}e^2\langle r^4\rangle R^{-5}$$
 (1)

$$B_{60} = (-3/4)Z_{\text{eff}}e^2 \langle r^6 \rangle R^{-7}$$
 (2)

where r is the 4f radial coordinate, and  $\langle r^k \rangle$  denotes the mean value of  $r^k$  relative to the  $R_{4f}(r)$  function, R is the metal—ligand distance, and  $Z_{\text{eff}}e$  is the effective ligand charge.

The ratio of  $B_{40}/B_{60}$  according to the PCEM is

$$Ratio(B_{40}/B_{60}) = (14/3)[\langle r^4 \rangle / \langle r^6 \rangle]R^2$$
(3)

The expression for the scalar crystal field strength is

$$N_{\nu} = \left[ \sum_{k,q} \frac{4\pi}{2k+1} (B_{kq})^2 \right]^{1/2} \tag{4}$$

For  $LnX_6^{3-}$ , with  $O_h$  site symmetry of  $Ln^{3+}$ , only k = 4, 6 and  $q = 0, \pm 4$  are nonzero, and

Received: November 29, 2012 Revised: January 29, 2013 Published: January 30, 2013

<sup>&</sup>lt;sup>†</sup>Department of Science and Environmental Studies, The Hong Kong Institute of Education, 10 Lo Ping Street, Tai Po, Hong Kong S.A.R., P.R. China

<sup>&</sup>lt;sup>‡</sup>Department of Physics, Anhui Normal University, Wuhu, Anhui 241000. P.R. China

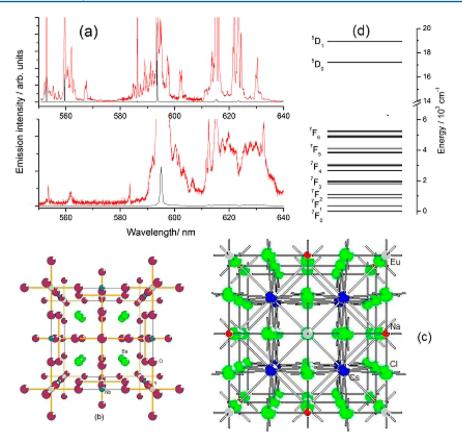


Figure 1. (a) 10 K emission spectra of  $Cs_2NaEuCl_6$  (top: 465 nm excitation) and  $Ba_2YNbO_6$ : $Eu^{3+}$  (bottom: 528.5 nm excitation) between 550 and 640 nm, with scale expansions of 100 times in red. The most intense band at 593–594 nm corresponds to the  $^5D_0 \rightarrow ^7F_1$  transition. (b), (c) Crystal structures of  $Ba_2YNbO_6$  and  $Cs_2NaEuCl_6$ . (d) Lower energy level scheme of  $Eu^{3+}$ .

$$B_{4\pm4} = \sqrt{\frac{5}{14}} B_{40} \text{ and } B_{6\pm4} = -\sqrt{\frac{7}{2}} B_{60}$$
 (5)

Besides, because  $B_{40} \gg B_{60}$  for LnX<sub>6</sub><sup>3-,4</sup> the sixth-rank  $B_{kq}$  can be omitted. Then,

$$N_{\rm v} = \left[ \sum_{k,q} \frac{4\pi}{2k+1} (B_{kq})^2 \right]^{1/2}$$

$$= \sqrt{\frac{4\pi}{9}} \left[ (B_{40})^2 + (B_{44})^2 + (B_{4-4})^2 \right]^{1/2}$$

$$= \frac{4}{3} \sqrt{\frac{3\pi}{7}} B_{40}$$
(6)

Hence, the plot of  $N_{\rm v}$  vs  $B_{40}$  is expected to have the slope 1.55. The validity of this relationship is demonstrated from experimental values of  $B_{40}$  and  $B_{60}$  from the  ${\rm LnCl_6}^{3-}$  series in Figure S1, Supporting Information, where the slope is 1.62  $\pm$  0.02. Because, with the PCEM model for the crystal-field interaction, the  $B_{40}$  parameter may be expressed as in eq 1, when substituting eq 6 into eq 1, we obtain

$$N_{\rm v} = \sqrt{\frac{28\pi}{3}} \left( -Z_{\rm eff} e^2 \right) \frac{\langle r^4 \rangle}{R^5} \tag{7}$$

In eq 6, the effective ligand charge  $Z_{\rm eff}$  may be expressed as

$$Z_{\text{eff}} = f \cdot Z \tag{8}$$

where Z is the formal ionic charge of the ligands and f is the fractional ionic character of the chemical bonds. To estimate the value of f, Pauling's empirical formula may be employed:

$$f = 1 - \exp[-0.25(\chi_{ln} - \chi_{x})^{2}]$$
 (9)

where  $\chi_{Ln}$  and  $\chi_{X}$  are the electronegativities of the lanthanide and the ligand, respectively. Substituting eqs 8 and 9 into eq 7, we have

$$N_{\rm v} = \sqrt{\frac{28\pi}{3}} \left[ 1 - e^{-0.25(\chi_{\rm Ln} - \chi_{\rm L})^2} \right] (-Ze^2) \frac{\langle r^4 \rangle}{R^5}$$
 (10)

If we repace the Ln–X distance R by the Shannon ionic radius  $r_{\rm rad}$ ,  $r[{\rm Ln}^{3+}({\rm VI})]$ , according to the following discussions in the text, the above expression turns into

$$N_{\rm v} = \sqrt{\frac{28\pi}{3}} \left[ 1 - e^{-0.25(\chi_{\rm Ln} - \chi_{\rm L})^2} \right] (-Ze^2) \frac{\langle r^4 \rangle}{r_{\rm rad}^5}$$
(11)

#### 3. RESULTS AND DISCUSSION

Using the Ln–Cl distances, R, from the lattice parameters of  $Cs_2NaLnCl_6$  systems<sup>7</sup> and the values of  $\langle r^k \rangle$  and  $B_{kq}$  in the Supporting Information, Figure 2a uncovers that the slope for the plot of  $\log[B_{40}/\langle r^4 \rangle]$  versus  $\log R$  (i.e., representing the power of R), is  $-33.5 \pm 1.8$ . Alternatively, using values of R from ab initio calculations, the slope is  $-(15.8 \pm 1.2)$ . Both values would appear to show that the formula from the PCEM model is invalid. However, when the  $Ln^{3+}(VI)$  ionic radius is employed, instead of R, as in Figure 2b, the slope of  $-5.5 \pm 0.4$  is found, in reasonable agreement with eq. 1. The analogous plots for  $B_{60}$  employing

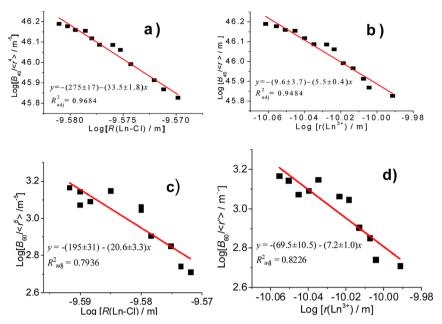


Figure 2. (a), (b) Plot of  $\log[B_{40}/\langle r^6 \rangle]$  and (c), (d)  $\log[B_{60}/\langle r^6 \rangle]$  against  $\log[R(\text{Ln-Cl})]$  from lattice parameter data (a), (c) and against  $\log[r(\text{Ln}^{3+}(\text{VI})]]$  from Shannon ionic radii (b), (d).

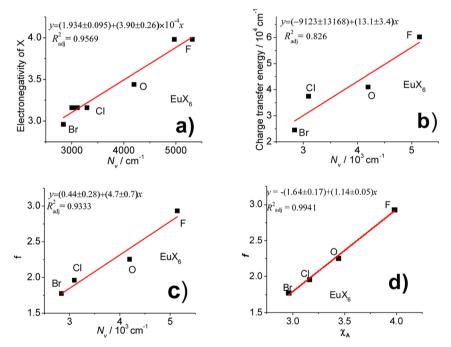


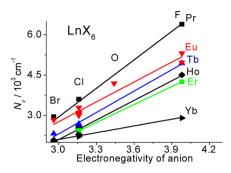
Figure 3. Plots of crystal field strength,  $N_{\rm v}$  (abscissa) against (a) electronegativity, (b) charge transfer band maximum, and (c) fractional ionic character, for EuX<sub>6</sub> systems. (d) shows fractional ionic character vs electronegativity of X in EuX<sub>6</sub>. Refer to the text for explanations. The  $N_{\rm v}$  values in (a) are calculated from the crystal field parameters in refs 4 and 12 (Cs<sub>2</sub>NaEuCl<sub>6</sub> and Cs<sub>2</sub>NaYCl<sub>6</sub>: Eu<sup>3+</sup>); refs 13 and 14 (Cs<sub>2</sub>NaYF<sub>6</sub>:Eu<sup>3+</sup>); ref 3 (Ba<sub>2</sub>YNbO<sub>6</sub>:Eu<sup>3+</sup>); and the fitting of the energy level data in ref 15 (Cs<sub>2</sub>NaLuBr<sub>6</sub>: Eu<sup>3+</sup>). In Figure 2b, the CT maxima are taken from the values given in ref 16 (Cs<sub>2</sub>NaEuCl<sub>6</sub>), ref 17 (Cs<sub>2</sub>NaYF<sub>6</sub>:Eu<sup>3+</sup>), refs 16, 18, and 19 (Y<sub>2</sub>O<sub>3</sub>:Eu<sup>3+</sup> and EuO<sub>6</sub><sup>9-</sup> impurities), and ref 20 (EuBr<sub>6</sub><sup>3-</sup> in tetraethylammonium bromide saturated anhydrous acetonitrile solution).

R(Ln-Cl) from the lattice parameter and  $r(\text{Ln}^{3+}(\text{VI}))$  yield slopes of  $-(20.6 \pm 3.3 \text{ and } -7.2 \pm 1.0, \text{respectively (Figure 2c,d)}, \text{compared}$  with the value 7.0 from eq 2. These results are taken to mean that the  $\text{Ln}^{3+}$ -ligand interaction is ionic and that the point charges are not located at the distance R(Ln-Cl) but at the distance  $r(\text{Ln}^{3+})$ .

The values of the Ratio( $B_{40}/B_{60}$ ) (eq 3) for Pr<sup>3+</sup> and Tm<sup>3+</sup> in Cs<sub>2</sub>NaLnCl<sub>6</sub> are, respectively, 8.6 and 9.8 from the experimental data fitting given in ref 4, 3.0 and 3.4 by using ionic radii  $r[\text{Ln}^{3+}(\text{VI})]$  in eq 3, and 22.3 and 30.7 by using R(Ln-Cl) from

the lattice parameters in eq 3. The agreement is better when ionic radii are used instead of ligand—metal distances.

Although this modified PCEM model underestimates the ratio of  $B_{40}/B_{60}$ , it provides a reasonable guide to the power variations of crystal field parameters with distance so that we now pursue it further. Note that the slower variation  $(r^{-5})$  of  $B_{40}$  with distance leads to its dominance over  $B_{60}$  for the  $\mathrm{LnCl_6}^{3-}$  elpasolite systems, and it is also responsible for the dominance of the  $\mathrm{Cl}^-$  nearest neighbor contribution to the crystal field.



**Figure 4.** Plot of  $N_v$  against electronegativity of anion for LnX<sub>6</sub> moieties. Data from refs 3, 4, 12, 14, 15, and 22–28.

The crystal field data can be summarized more succinctly, through the combination of the crystal field parameters to give the scalar crystal field strength,  $N_v$ , defined in eq 4.9,10 This strength, which is an invariant (i.e., it does not change under a transformation from one coordinate system to another) is related to the maximum splitting of a given *J* state of a rare earth ion in a crystal. It is a quantity that can characterize the crystal field strength in any kind of symmetry. To a chemist, interactions between ions lie in the realm of bonding, and ionic bonds concern those between atoms of widely different electronegativity. Sen has stated that electronegativity not only is an intuitive concept but also can be treated as a quantum chemical parameter. 11 The crystal field strengths derived for EuX<sub>6</sub> systems (X = Br, Cl, F, O) are plotted against the corresponding Pauling electronegativities of X,  $\chi_X$  in Figure 3a and a linear relation is found. The scatter in the values of crystal field strength for an individual EuX<sub>6</sub> system shows the typical errors in determination of N<sub>v</sub>. These errors mainly arise from (i) different numbers of parameters employed in the energy level fitting, (ii) different numbers of energy levels employed in the fits, (iii) incorrect spectral assignments, and (iv) errors in the calculation. The most electronegative ligand, F, representing the closest ligand and most ionic bonding, exhibits the largest value of  $N_{v}$ , in LnF<sub>6</sub><sup>3-</sup>.

Jørgensen gave a relationship between optical electronegativity and the position of the charge transfer (CT) band of a lanthanide ion system. In fact, the optical electronegativities are directly proportional to electronegativity values, so that a correlation is then expected between the CT band maximum and  $N_{\nu}$ . Estimations of CT band maxima for Eu<sup>3+</sup> are taken from the available literature and it is noted that the band positions are approximate because they vary according to the synthesis method and history of the sample, as well exhibiting marked differences in solution compared with the solid state. For example, the CT band of Eu–O in six coordination in the solid

state is estimated to be between 39200 and 43103 cm $^{-1}$ .  $^{16,19,22}$  Thus, Figure 3b shows the approximate linear relationship between CT band energy and  $N_{\rm v}$  for the EuX $_6$  systems. The CT energy relates to the two-center process in a crystal:

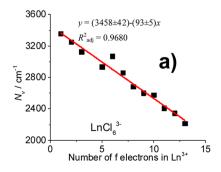
$$Ln^{3+}$$
 (GS) + VB  $\rightarrow Ln^{2+}$  (GS: vibrationally excited)  
+ VB (hole)

(12)where GS refers to ground electronic state, and the valence band (VB) is mainly composed of ligand p-electrons. The electron transfer is easier if the Ln–X bond is weak, i.e., small  $N_{\rm v}$ . From the linear relations in Figure 2a,b, an estimation of  $N_{\rm v}$  for the EuI $_6$ <sup>3-</sup> complex is  $1640 \pm 190 \, {\rm cm}^{-1}$ , taking the Pauling electronegativity of I as 2.5 and the CT band wavelength as 675 nm. This value would enable a simulation of crystal field splittings to be made for EuI $_6$ <sup>3-</sup>. Figure 3c shows that the relationship between  $N_{\rm v}$  and fractional ionic character, f, of the bond (eq 9) is linear, because that between f and electronegativity is also linear for Eu–X (Figure 3d).

The linearity of the plot of  $N_v$  and electronegativity is general for LnX<sub>6</sub> moieties (Figure 4). The ordinate intercept of the plot becomes less negative (Figure S1a, Supporting Information), and the slope becomes less positive (Figure S1b, Supporting Information), as the number of electrons in 4f<sup>N</sup> increases. This indicates a smaller discrimination in  $N_{\rm v}$  values for the different ligands when the lanthanide cation is smaller and more polarizing, which is a consequence of the smaller  $N_v$  values for the smaller cations, as shown for the  $LnCl_6^{3-}$  series in Figure 5a. The corresponding relation is also found for  $LnO_6$  systems (Figure 5b) but the  $N_v$  values were calculated from very incomplete data sets and are not as accurate. A similar linear plot has previously been given for  $B_{40}$  and the number of 4felectrons. The relations involving  $N_v$  (Figures 5a,b) ensue because  $B_{40}$  dominates over  $B_{60}$  for the LnX<sub>6</sub> systems so that it is linearly proportional to  $N_v$  (Figure S1, Supporting Information, and eq 6). This dominance of  $B_{40}$  results from its distance dependence, the predominantly ionic character and relatively long bond distances in  $LnX_6$  systems. This dominance of  $B_{40}$ , and the presence of such simple linear relations as presented herein, are unlikely to exist generally for other Ln<sup>3+</sup> systems where more than two crystal field parameters are involved, or even for the  $T_h$  symmetry systems  $Ln(NO_2)_6^{3-.29}$ 

#### 4. CONCLUSIONS

In summary, the small number of crystal field parameters for the  $O_h$  symmetry systems  $LnX_6$  has enabled a simple rationalization of the different concepts: crystal field strength, electronegativity, and charge transfer. The point charge electrostatic model has been shown to provide an approximate basis for these



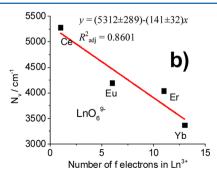


Figure 5. Plot of  $N_v$  against number of 4f electrons of  $Ln^{3+}$  in (a)  $LnCl_6^{3-}$  (data from ref 4) and (b)  $LnO_6^{9-}$  (data from refs 3, 31, and 30. In ref 30, the multiplicative factors arising from the reduced matrix elements were omitted from the values of crystal field parameters.).

relationships. It has been demonstrated that the PCEM model, which has previously been thought to give only the correct signs of the crystal field parameters, can be used to rationalize the distance dependence of the crystal field strengths, (or crystal field parameters) of the elpasolite and double perovskite systems. This rationalization using the ionic radius of Ln³+(VI) points to (or confirms) the important fact that the localization of Ln³+ 4f electrons remains in an inorganic compound with negligible contribution to bonding.

#### ASSOCIATED CONTENT

## **S** Supporting Information

Plot of  $N_{\rm v}$  against fourth rank crystal field parameter for LnCl<sub>6</sub><sup>3–</sup> systems (Figure S1); plots of (a) intercept and (b) slope from Figure 4 (Figure S2); and table of crystal field parameters and radial integrals. This material is available free of charge via the Internet at http://pubs.acs.org.

# AUTHOR INFORMATION

#### **Corresponding Author**

\*E-mail: peter.a.tanner@gmail.com.

#### **Notes**

The authors declare no competing financial interest.

#### ACKNOWLEDGMENTS

L.N. acknowledges support from the National Natural Science Foundation of China (Grant No. 11174005).

#### REFERENCES

- (1) Morley, J. P.; Faulkner, T. R.; Richardson, F. S. J. Chem. Phys. 1982, 77, 1710–1723.
- (2) Thorne, J. R. G.; Jones, M.; McCaw, C. S.; Murdoch, K. M.; Denning, R. G.; Khaidukov, N. M. J. Phys.: Condens. Matter 1999, 11, 7851–7866.
- (3) Li, W.; Ning, L.; Tanner, P. A. J. Phys. Chem. A 2012, 116, 7337—7344.
- (4) Duan, C.-K.; Tanner, P. A. J. Phys. Chem. A 2010, 114, 6055-6062.
- (5) Görller-Walrand, C.; Binnemans, K. In *Handbook on the Physics and Chemistry of Rare Earths*; Gschneidner, K. A., Jr., Eyring, L., Eds.; Elsevier Science B.V.: Amsterdam, 1996; Vol. 23, Chapter 155, pp 121–283.
- (6) Garcia, D.; Faucher, M. In *Handbook on the Physics and Chemistry of Rare Earths*; Gschneidner, K. A., Jr., Eyring, L., Eds.; Elsevier Science B.V.: Amsterdam, 1995; Vol. 21, Chapter 144, pp 263–304.
- (7) Meyer, G. Prog. Solid. State Chem. 1982, 14, 141-219.
- (8) Ordejon, B.; Seijo, L.; Barandiaran, Z. J. Chem. Phys. 2003, 119, 6143-6149.
- (9) Auzel, F. Opt. Mater. 2002, 19, 89-94.
- (10) Auzel, F.; Malta, O. J. Phys. (Paris) 1983, 44, 201-206.
- (11) Sen, K. D. In Structure and Bonding (Berlin); Sen, K. D.; Jørgensen, C. K., Eds.; Springer-Verlag: Berlin, 1987; Vol. 36, front page No. 4.
- (12) Tanner, P. A.; Ravi Kanth Kumar, V. V.; Jayasankar, C. K.; Reid, M. F. J. Alloys Compd. 1994, 215, 349–370.
- (13) Thorne, J. R. G.; Jones, M.; McCaw, C. S.; Murdoch, K. M.; Denning, R. G.; Khaidukov, N. M. J. Phys.: Condens. Matter 1999, 11, 7851–7866.
- (14) Tanner, P. A.; Liu, Y.-L.; Edelstein, N.; Murdoch, K.; Khaidukov, N. M. J. Phys.: Condens. Matter 1997, 9, 7817–7836.
- (15) Dushin, R. B.; Barbanel, Y. A.; Kolin, V. V.; Kotlin, V. P.; Nekhoroshkov, S. N.; Chudnovskaya, G. P. *Radiochemistry* **2000**, 42, 53–60.
- (16) Duan, C.-K.; Tanner, P. A.; Meijerink, A.; Babin, V. J. Phys.: Condens. Matter 2009, 21, 395501 (9 pp.)..
- (17) Tanner, P. A.; Duan, C.-K.; Makhov, V. N.; Kirm, M.; Khaidukov, N. M. J. Phys.: Condens. Matter **2009**, 21, 395504 (10 pp).
- (18) Dorenbos, P. J. Phys.: Condens. Matter 2003, 15, 8417-8434.

- (19) Tanner, P. A.; Fu, L.; Cheng, B.-M. J. Phys. Chem. C 2009, 113, 10773–10779.
- (20) Nugent, L. J.; Baybarz, R. D.; Burnett, J. L.; Ryan, J. L. J. Phys. Chem. 1973, 77, 1528–1539.
- (21) Jørgensen, C. K. Mol. Phys. 1963, 6, 43-47.
- (22) Duan, C.-K.; Tanner, P. A.; Babin, V.; Meijerink, A. J. Phys. Chem. C **2009**, *113*, 12580–12585.
- (23) Ryan, J. L. Inorg. Chem. 1969, 8, 2053-2058.
- (24) Zhou, X.; Reid, M. F.; Faucher, M. D.; Tanner, P. A. J. Phys. Chem. B **2006**, 110, 14939–14942.
- (25) Zhou, X.; Tanner, P. A.; Faucher, M. D. J. Phys. Chem. C 2007, 111, 683-687.
- (26) Tanner, P. A.; Faucher, M. D.; Zhou, X. J. Phys. Chem. A 2011, 115, 2557–2567.
- (27) McCaw, C. S.; Murdoch, K. M.; Denning, R. G. Mol. Phys. 2003, 101, 427–438.
- (28) Berry, A. J.; Morrison, I. D.; Denning, R. G. Mol. Phys. **1998**, 93, 1–14
- (29) Tanner, P. A.; Li, W.; Ning, L. Inorg. Chem. 2012, 51, 2997-3006.
- (30) Calder, S.; Fennell, T.; Kockelmann, W.; Lau, G. C.; Cava, R. J.; Bramwell, S. T. *J. Phys.: Condens. Matter* **2010**, 22, 116007 (7 pp.)..
- (31) Zhou, W.-L.; Zhang, Q.-L.; Gao, J.-Y.; Liu, W.-P.; Ding, L.-H.; Yin, S.-T. Chin. Phys. B **2011**, 20, 016101 (8 pp).