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Free vibration and bending analysis of circular Mindlin plates using singular convolution method

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SUMMARY

Circular plates are important structural elements in modern engineering structures. In this paper a computationally efficient and accurate numerical model is presented for the study of free vibration and bending behavior of thick circular plates based on Mindlin plate theory. The approach developed is based on the discrete singular convolution method and the use of regularized Shannon's delta kernel. Frequency parameters, deflections and bending moments are obtained for different geometric parameters of the circular plate. Comparisons are made with existing numerical and analytical solutions in the literature. It is found that the DSC method yields accurate results for the free vibration and bending problems of thick circular plates. Copyright © 2008 John Wiley & Sons, Ltd.

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KEY WORDS: discrete singular convolution; circular plate; Mindlin plate theory; free vibration; bending

1. INTRODUCTION

The method of discrete singular convolution (DSC) [1] has been used extensively for the vibration analysis of structures. DSC method has emerged as a new approach for numerical solutions to differential equations. This new method has a potential approach for computer realization as a wavelet collocation scheme [2–4]. The use of the DSC method for vibration analysis of beams, plates and shells [5–19] has been proven to be quite satisfactory. Free vibration analysis of isotropic

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and laminated conical shells with or without rotating has been investigated by the present author [20–24].

Thin and thick circular plates have been widely used in many engineering applications, for example, civil and mechanical engineering, nuclear and aerospace structures. The bending and vibration analysis of circular plates is, therefore, of great importance in practical design. Static, vibration and buckling analysis of circular plates has been carried out by analytical [25–36] and numerical methods [37–46]. Recently, Liew *et al.* [47–49] and Han and Liew [39, 43, 50] applied differential quadrature method for vibration, bending and buckling analysis of circular plates. Yamada and co-workers [51] and Irie *et al.* [51–54] have also studied free vibration of annular and circular plates. Some exact results are presented by Xiang [35, 36]. Zhou *et al.* [55–57] developed Chebyshev–Ritz method for some circular plate problems. Liew and Han [58] proposed a four-node differential quadrature for Mindlin plates. Some important studies concerning the analysis of circular plates have been carried out by Wu *et al.* [59], Wu and Liu [60, 61], Leissa and Narita [26], Wang and Aung [62], Laura *et al.* [63], Rossi and Laura [64] and Wang and Lee [65]. Literature dealing with vibration and bending analysis of circular plates has been reported by many researchers; for instance, see References [40, 41, 55, 66, 67].

The aim of the present paper is to propose a relatively new numerical approach using the method of DSC for the solution to the free vibration and bending problem of circular plates. Frequency parameters, deflections and bending moments are obtained. This is the first instance in which the DSC method has been adopted for free vibration and bending analysis of Mindlin circular plates. A detailed literature related to circular and annular plates has also been given in references.

2. DISCRETE SINGULAR CONVOLUTION

The DSC method is an efficient and useful approach for the numerical solutions to differential equations. This method was introduced by Wei [1]. Similar to some other numerical methods, the DSC method discretizes the spatial derivatives and, therefore, reduces the given partial differential equations into an eigenvalue problem. The mathematical foundation of the DSC algorithm is the theory of distributions and wavelet analysis [2]. Wei and his co-workers first applied the DSC algorithm to solve mechanics problems [2–15]. Zhao *et al.* [10, 11] analyzed the high-frequency vibration of plates and plate vibration under irregular internal support using DSC algorithm. Wan *et al.* [17, 18] studied some fluid mechanics problem using DSC method. Civalek [20–24] gives numerical solution to free vibration problem of rotating and laminated conical shells and nonlinear analysis of plates on elastic foundation. Free vibration analysis of conical panels has been studied by Civalek [68]. Gu and Wei [69] proposed DSC method for shock capturing. These studies indicate that the DSC algorithm works very well for the vibration analysis of plates, especially for high-frequency analysis of rectangular plates. More recently, Lim *et al.* [13, 14] presented the DSC–Ritz method for the free vibration analysis of Mindlin plates and thick shallow shells.

Consider a distribution T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined as [16]

$$F(t)=(T*\eta)(t)=\int_{-\infty}^{\infty} T(t-x)\eta(x)\,\mathrm{d}x \tag{1}$$

where $T(t-x)$ is a singular kernel. Recently, the use of some new kernels and regularizer, such as delta regularizer [43, 44], was proposed to solve applied mechanics problem. Shannon's kernel

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is regularized as [13]

$$\delta_{\Delta,\sigma}(x-x_k)=\frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)}\exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right],\quad \sigma>0 \tag{2}$$

where Δ is the grid spacing. It is also known that the truncation error is very small due to the use of the Gaussian regularizer; the above formulation given in Equation (2) is practical and has an essentially compact support for numerical interpolation. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ given by

$$\left.\frac{\mathrm{d}^n f(x)}{\mathrm{d}x^n}\right|_{x=x_i}=f^{(n)}(x)\approx\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(n)}(x_i-x_k)f(x_k)\quad (n=0,1,2,\ldots) \tag{3}$$

where $\delta_{\Delta}(x-x_k)=\Delta\delta_{\alpha}(x-x_k)$ and superscript (n) denotes the nth-order derivative; $2M+1$ is the computational bandwidth that is centered around x and is usually smaller than the whole computational domain.

3. GOVERNING EQUATIONS

3.1. Free vibration analysis

Free vibration analysis of thick annular and circular plates has been reported in the literature by Liew *et al.* [70–75], Kitipornchai *et al.* [76], Xiang *et al.* [77], Liew [78], Kirkhope and Wilson [79], Chen [80], Reddy *et al.* [81] and Celep [82]. Some further results related to the analysis of circular and annular plates have also been found in [83–85]. In the present study, free vibration analysis of thick circular plates is analyzed by a relatively new numerical method. Figure 1 shows an isotropic circular plate with uniform thickness h and radius a. The governing equations for axisymmetric free vibration are given [38] by

$$H^2\left(R^2\frac{\partial^2\Psi}{\partial R^2}+R\frac{\partial\Psi}{\partial R}-\Psi\right)-6\kappa(1-\upsilon)R^2\left(\Psi+\frac{\partial W}{\partial R}\right)-H^2R^2\frac{\partial^2\Psi}{\partial T^2}=0 \tag{4}$$

$$\left(R\frac{\partial^2 W}{\partial R^2}+\frac{\partial W}{\partial R}+R\frac{\partial\Psi}{\partial R}+\Psi-\frac{2R}{(1-\upsilon)\kappa}\frac{\partial^2 W}{\partial T^2}\right)=0 \tag{5}$$

In the above equations, the following dimensionless parameters are used:

$$R=r/a,\quad W=w/a,\quad \Psi=\psi,\quad H=h/a,\quad T=t/t_0,\quad t_0=\sqrt{\frac{\rho a^2(1-\upsilon^2)}{E}} \tag{6}$$

where w is the transverse deflection, D is the flexural rigidity of the plate given by $D=Eh^3/12(1-\upsilon^2)$, κ is the shear correction factor, ψ is the bending rotations of the normal about the r-axis, ρ is the density of the plate material, t is the time, and E, G and υ are Young's modulus,

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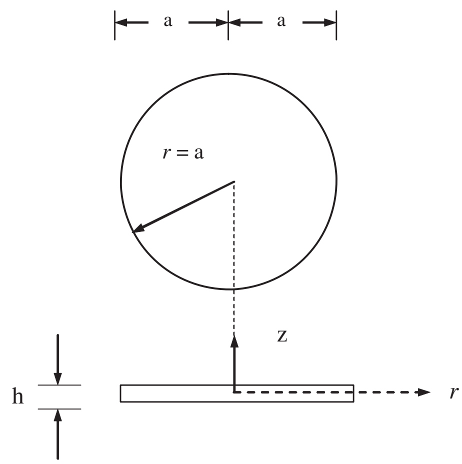

Figure 1. Coordinate system and a typical scheme of a circular plate.

shear modulus and Poisson's ratio, respectively. Displacement and rotations are expressed as

$$W(R,T)=W_j(R)e^{i\Omega_j T} \tag{7}$$

$$\Psi(R,T)=\Psi_j(R)e^{i\Omega_j T} \tag{8}$$

where Ω_j is the eigenvalue of the jth mode of vibration. Substituting Equations (7)–(8) into Equations (4)–(5) the governing equations of motions become [38]

$$H^2\left(R^2\frac{d^2\Psi}{dR^2}+R\frac{d\Psi}{dR}-\Psi\right)-6\kappa(1-v)R^2\left(\Psi+\frac{dW}{dR}\right)-H^2R^2\Omega^2\Psi=0 \tag{9}$$

$$R\frac{d^2W}{dR^2}+\frac{dW}{dR}+R\frac{d\Psi}{dR}+\Psi+\frac{2R}{(1-v)\kappa}W=0 \tag{10}$$

Similarly, force resultants and deformation variables are given in normalized form as

$$M_r=\frac{D}{a}\left(\frac{\partial\Psi}{\partial R}+\frac{v}{R}\Psi\right) \tag{11a}$$

$$M_\theta=\frac{D}{a}\left(\frac{\partial\Psi}{\partial R}+\frac{\Psi}{R}\right) \tag{11b}$$

$$Q_r=\kappa Gh\left(\Psi+\frac{\partial W}{\partial R}\right) \tag{11c}$$

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Employing the DSC method, the governing equations (9)–(10) become

$$H^2\left[R^2\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta R)\Psi_k+R\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k\right]$$

$$-6\kappa(1-v)R^2\left[\Psi_i+\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta R)W_k\right]-H^2R^2\Omega^2\Psi_i=0 \tag{12a}$$

$$R\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta R)W_k+\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta R)W_k+R\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k$$

$$+\Psi_i+\frac{2R}{(1-v)\kappa}W_i=0 \tag{12b}$$

3.2. Static analysis

For axisymmetric bending, the equilibrium equations of the plate are given by [39]

$$H^2\left(R^2\frac{\mathrm{d}^2\Psi}{\mathrm{d}R^2}+R\frac{\mathrm{d}\Psi}{\mathrm{d}R}-\Psi\right)-6\kappa(1-v)R^2\left(\Psi+\frac{\mathrm{d}W}{\mathrm{d}R}\right)=0 \tag{13a}$$

$$R\frac{\mathrm{d}^2W}{\mathrm{d}R^2}+\frac{\mathrm{d}W}{\mathrm{d}R}+R\frac{\mathrm{d}\Psi}{\mathrm{d}R}+\Psi+\frac{R}{\kappa GH}q=0 \tag{13b}$$

Similarly, the governing equations for bending are given in DSC form as below:

$$H^2\left[R^2\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta R)\Psi_k+R\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k\right]$$

$$-6\kappa(1-v)R^2\left[\Psi_i+\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta R)W_k\right]=0 \tag{14a}$$

$$R\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta R)W_k+\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta R)W_k+R\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k$$

$$+\Psi_i+\frac{R}{\kappa GH}q_i=0 \tag{14b}$$

Three different types of boundary conditions are considered. These are given as follows:

Simply supported edge (S):

$$W=0,\quad M_r=0 \tag{15}$$

Clamped edge (C):

$$W=0,\quad \Psi=0 \tag{16}$$

DOI: 10.1002/cnm

Free edge (F):

$$\Psi + \partial W/\partial R = 0 \quad \text{and} \quad v\Psi/R + \partial\Psi/\partial R = 0 \tag{17}$$

After normalization, these boundary conditions can be given in DSC form as

(a) *Simply supported edge* (S):

$$W_N = 0 \tag{18a}$$

$$\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k + v\Psi_N = 0 \tag{18b}$$

(b) *Clamped edge* (C):

$$W_N = 0 \tag{18c}$$

$$\Psi_N = 0 \tag{18d}$$

(c) *Free edge* (F):

$$\Psi_N + \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R) W_k = 0 \tag{18e}$$

$$v\Psi_N + \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R)\Psi_k = 0 \tag{18f}$$

For the plate central point, the regularity and central support conditions must be considered. These are given in discretized forms as follows:

(d) *Regularity conditions* (R):

$$\Psi_1 = 0 \tag{19a}$$

$$\sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta R) W_k = 0 \tag{19b}$$

(e) *Rigid central support* (D):

$$W_1 = 0 \tag{20a}$$

$$\Psi_1 = 0 \tag{20b}$$

The implementation of boundary conditions is a difficult task in the application of DSC method. In this study, we used the same procedure proposed by Wei *et al.* [8, 9, 12] and Zhao *et al.* [10, 11]. In this paper, details of the implementation of boundary conditions in the DSC method are not given; interested readers may refer to the works of [7–24]. The Poisson ratio was taken to be 0.3 in numerical calculations.

4. NUMERICAL RESULTS

First, convergence study is carried out for two plates: clamped and completely free circular Mindlin plate. To determine the accuracy and convergence of the present method for vibration and bending

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analysis of circular Mindlin plates, numerical experimentation was carried out by varying the number of grid points N. For brevity, the letters C, S and F are used to denote a clamped edge, a simply supported edge and a free edge, respectively. Results related to free vibration computed for different number of terms in N are shown in Table I and Figures 2 and 3. Table I presents the comparison of frequency parameters for a clamped circular Mindlin plate for different thickness-to-radius ratios. The finite-element-based results given by Lin and Tseng [67] and the results given by Irie *et al.* [54] are presented for comparison. It is found from Table I that $N \geqslant 15$ is required for reasonable numerical solutions. Good agreement of present results with the analytical solutions is observed. In general, it is obvious that excellent agreement between the present results and the results of Irie *et al.* [54] is achieved. Figures 2 and 3 show the convergence of the first three frequencies for different grid numbers. Simply supported and the free boundary conditions are considered for the results given in these figures. It is seen from these figures that when the grid point numbers reach $N = 11$, the present method gives accurate predictions for the first vibration modes. For second and third modes of vibration, however, the accurate results are obtained

Table I. Comparison of natural frequencies ($\Omega = (\omega a^2)\sqrt{\rho h/D}$) of clamped circular plates.

	h/a			
Sources	0.05	0.1	0.15	0.20
FEM [67]	10.0047	9.812	9.453	9.016
Rayleigh–Ritz [54]	10.145	9.941	9.629	8.807
Present DSC results $N = 13$	10.156	9.984	9.647	8.814
Present DSC results $N = 15$	10.147	9.981	9.633	8.808
Present DSC results $N = 18$	10.147	9.981	9.632	8.808

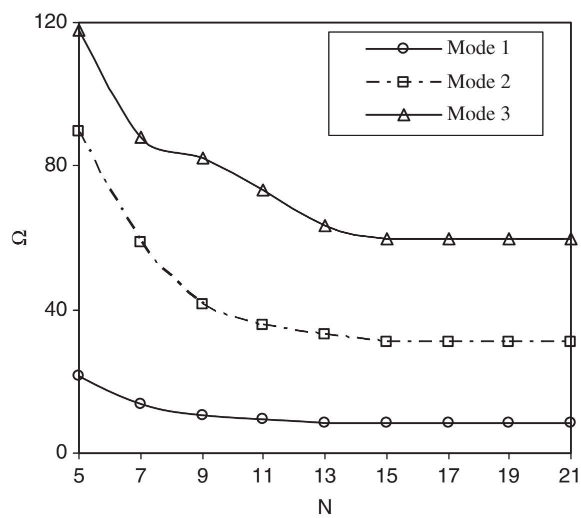

Figure 2. Convergence and accuracy of a frequency parameter for the first three modes with grid numbers for a completely free circular plate ($h/a = 0.2$).

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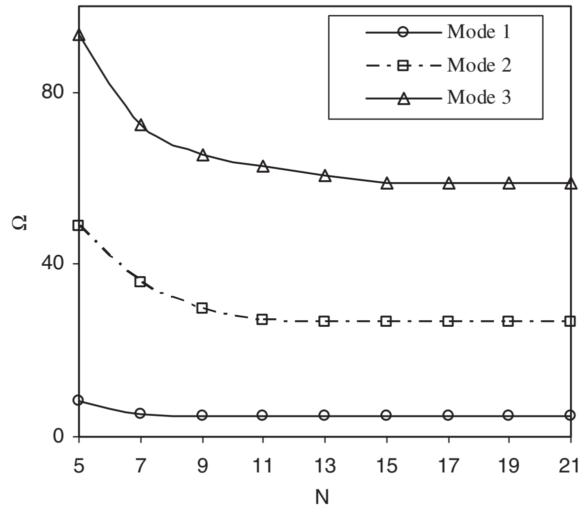

Figure 3. Convergence and accuracy of a frequency parameter for the first three modes with grid numbers for a simply supported circular plate ($h/a=0.15$).

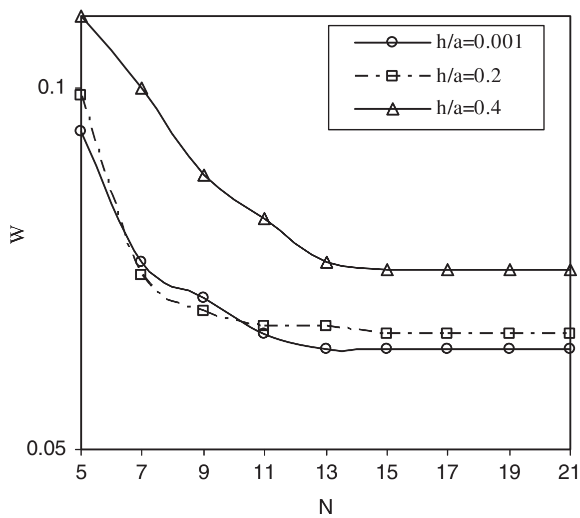

Figure 4. Convergence and accuracy of deflection of a simply supported circular plate with grid numbers.

for $N=15$. For further validation the capability of the present method in static analysis of circular Mindlin plates, we consider a simply supported circular plate. The variation of deflection with grid numbers for different thicknesses is depicted in Figure 4. The reasonably acceptable results for deflections are obtained when the grid point numbers reach $N\geqslant 13$. On the basis of convergence study, unless otherwise specified, we set the grid number $N=15$.

In Table II, the first three frequencies for circular plates with different boundary conditions are listed for $h/a=0.3$. The values presented by Liew and Yang [48] are used for comparison. The discrepancy between Liew and Yang's results [48] and present results is less than 1%. The fundamental frequencies of clamped circular plates are obtained and given in Table III with the results of Liew and Yang [25] and Zhou *et al.* [55]. As seen in Table III, there is good agreement between the results of Liew and Yang [25] and Zhou *et al.* [55] and the present results. It is also concluded from Tables II and III that the frequency parameters decrease rapidly as the thickness of the plate increases. Also, clamped plate has the highest frequency parameter Ω. Table IV shows the comparison between the present results of central deflections (wD/qa^4) and the analytical results of Reismann [86] and differential quadrature (DQ) results of Han and Liew [43] for various thickness-to-radius ratios. It should be noted that the results are in good agreement. Tables V and VI show the bending moments and central deflections of Mindlin circular plate with clamped and simply supported edge conditions, respectively. An increase in the thickness-to-radius ratio (h/a) from 0.001 to 0.2 leads to an increase in the bending and deflection parameter. Figure 4 displays the values of the frequencies as a function of thickness ratio for different mode numbers. It is shown

Table II. Comparison of the first three frequencies for circular plates with different boundary conditions ($h/a=0.3$).

	Completely free		Clamped	
Mode	Present study	Liew and Yang [48]	Present study	Liew and Yang [48]
1	4.9006	4.9005	8.4681	8.4676
2	8.0351	8.0344	15.4594	15.453
3	10.4400	10.439	22.6698	22.667

Table III. Comparison of the fundamental frequencies of clamped circular plates.

	Sources		
h/a	Present study	Liew and Yang [25]	Zhou *et al.* [55]
0.1	9.981	9.9909	9.9755
0.3	8.468	8.4676	8.4606
0.5	6.810	6.8068	6.8027

Table IV. Comparison of central deflections (wD/qa^4) of a Mindlin circular plate with clamped edge.

	Sources		
h/a	Present study	Han and Liew [43]	Exact [86]
0.001	0.01563	—	0.01562
0.01	0.01561	0.01561	0.01563
0.02	0.01564	0.01563	0.01565
0.05	0.01578	0.01578	0.01580
0.1	0.01633	0.01631	0.01634
0.2	0.01846	0.01846	0.01848

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Table V. Bending moments of clamped circular plates for different thicknesses.

h/a	M_x/qa^2	M_y/qa^2
0.001	0.08116	0.08116
0.01	0.08120	0.08120
0.02	0.08119	0.08119
0.05	0.08120	0.08120
0.1	0.08121	0.08121
0.2	0.08121	0.08121

Table VI. Central deflections (wD/qa^4) of a Mindlin circular plate with a simply supported edge.

	wD/qa^4		
h/a	$N=11$	$N=13$	$N=15$
0.001	0.06373	0.06371	0.06371
0.025	0.06390	0.06388	0.06388
0.15	0.06521	0.06652	0.06520
0.25	0.06658	0.06656	0.06656
0.40	0.73122	0.73119	0.73118

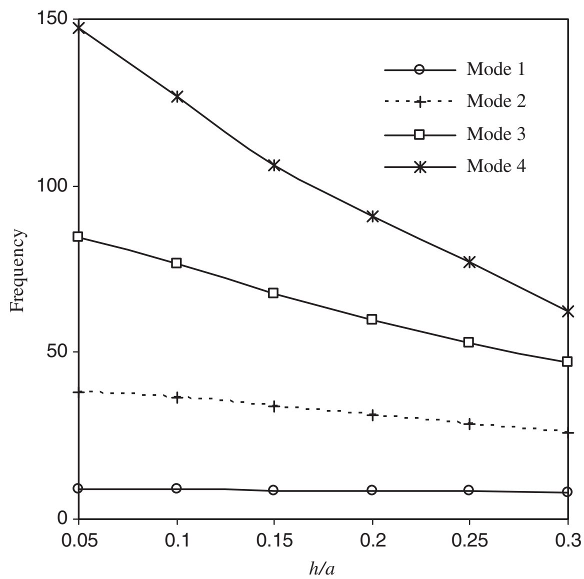

Figure 5. First four frequency parameters with different thicknesses of circular plates with a free edge.

from this figure that the increasing value of thickness always decreases the frequency parameter Ω for the third and fourth modes. However, the effect of thickness ratio on the frequency parameter is insignificant for the first mode. In other words, the results show that the thickness becomes more influenced as the mode numbers are bigger (Figure 5). Figure 6 describes the variation of frequency parameters with the mode numbers for different thicknesses. As can be observed from this figure, the frequency parameters generally increase with increasing mode number. The effects of boundary conditions on the frequencies of the circular Mindlin plates are depicted in Figure 7.

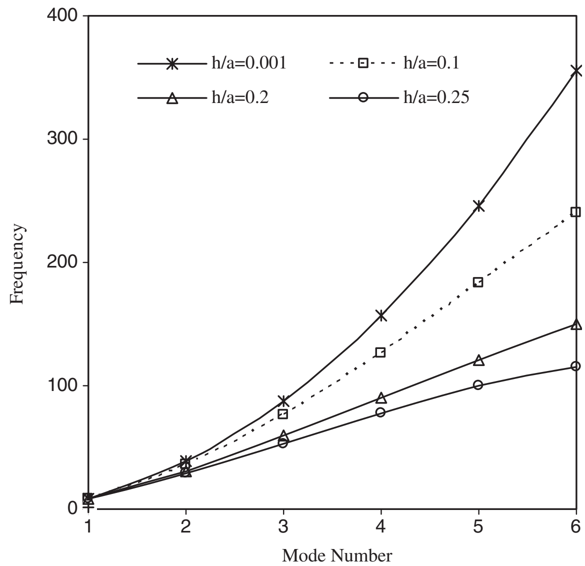

Figure 6. Variation of frequency parameters of free circular plates with mode numbers for different thicknesses.

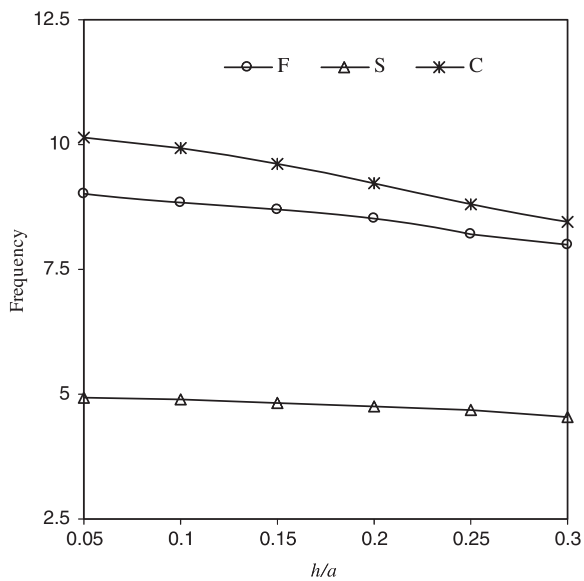

Figure 7. Variation of frequency parameters with thickness-to-radius ratio for different boundary conditions.

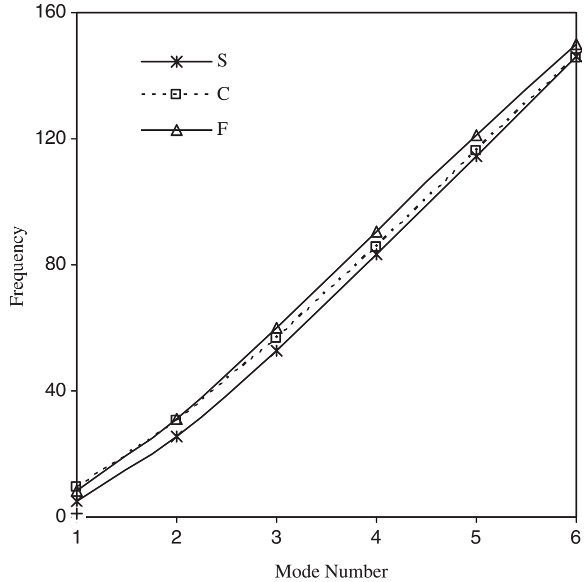

Figure 8. Variation of frequency parameters with mode sequence for different boundary conditions ($h/a=0.2$).

It is seen that the frequency parameter for all the three boundary conditions decreases as thickness increases. It is also shown that the effect of thickness on frequencies is marginal for circular plate with simply supported, namely, except for S boundary condition, there is considerable variation in the frequency parameters. In Figure 8, the variation of frequency parameters with mode sequence for different boundary conditions is presented for $h/a=0.2$. The frequency parameters rapidly increase with increasing mode number. It is also shown from the figure that the boundary conditions have little effect on the frequency when the mode number is increased for constant thickness. Figure 9 also shows a comparison study of the present DSC results with the finite element solutions obtained using the IDEAS software package. The first four axisymmetric frequencies of clamped circular Mindlin plates are presented for $h/a=0.05$ in this figure with the results given by Liew *et al.* [45] using the DQ method. Good agreement is achieved among the present results, the results given by Liew *et al.* [45] and the results produced by IDEAS using the Mindlin shell element. The results given by IDEAS include both symmetric and axisymmetric modes. We choose only axisymmetric results for comparison.

5. CONCLUSIONS

The DSC method has been applied to study the free vibration and static analysis of circular Mindlin plates. The equations of motion and bending for circular Mindlin plates are presented and the DSC method is used to develop the numerical solution used in this paper. The influences of boundary conditions, thickness-to-radius ratios and mode numbers on the frequency parameters of plates are discussed. The effect of grid numbers on the convergence of the results is also studied.

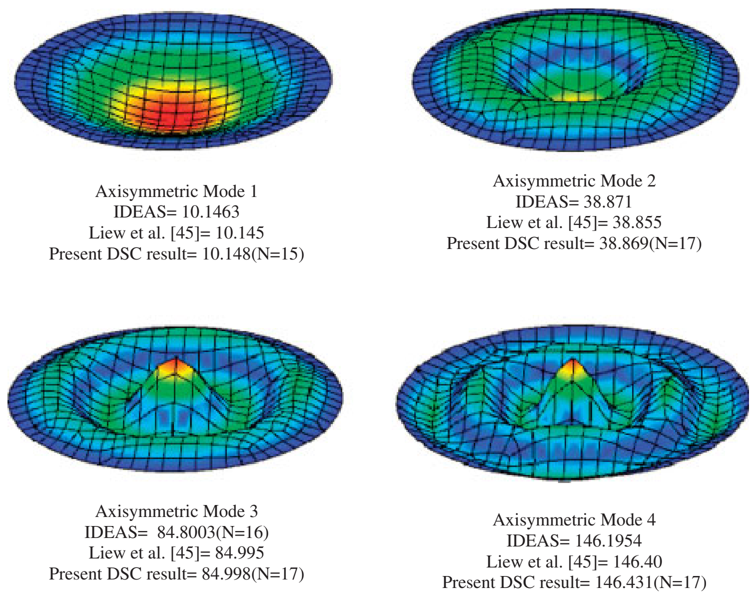

Figure 9. First four axisymmetric non-dimensional frequencies of clamped circular Mindlin plates.

Frequencies, deflections and bending moments are obtained. By comparison with the previously published results, it was shown that the natural frequencies and deflections can be determined with adequate accuracy. Therefore, we conclude that the method can be used as an alternative to existing numerical methods for solutions to vibration, buckling and bending problem of thick or thin plate.

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