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SPECIAL ISSUE PAPER

Simulation of multiple fluids with solid–liquid phase transition

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ABSTRACT

Physically based multiphase fluid simulation has been a hot topic in computer graphics. Since there are complex changing interface topology and interactions among air, solid, and different fluids, few papers have devoted to simulate the multiple fluids phenomena with solid–liquid phase transition. In this paper, the thermal fluid model for phase transition combined with free surface tracking is used to describe the interaction between air and fluids. Then a new model based on hierarchical lattice is proposed to process the solid–liquid interaction and the phase transition in the solid–liquid interface. Further, with the use of hybrid interaction with multidistribution functions, different realistic multiple fluids phenomena are rendered with different lattice sizes. Copyright © 2012 John Wiley & Sons, Ltd.

KEYWORDS

multiple fluids; solid–liquid phase transition; free surface; hierarchical lattice

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1. INTRODUCTION

Multiple fluids phenomena involving both miscible and immiscible fluids are often observed in our life, such as dropping ink, cocktail drinks, slurry flows, aerosols, debris flows, and lava. In these examples, a pair of fluids is mixed smoothly while creating the sharp interface with air. Complex fluid phenomena involve more diverse combinations of miscible and immiscible fluids.

Especially, there is often phase transition including melting or freezing during the thermal conducting process. To model the interaction and phase transition between different components is indeed a highly challenging task. Because different components are changing along with the interface and interacting with different media, the free surfaces generation and solid–liquid coupling associated with separation, mixing, and melting or freezing are more difficult than simulating the single-component flow.

Similar to the single fluid model, there are two types of multicomponent models: discrete phase models (particle-based Largranigian formation [1,2]) and continuous phase models, which includes single fluid approach (volume of fluid [3], cavitation models [4]), and two-fluid approach (Eulerian–Eulerian models [5] and Eulerian–granular models [6]). Here, space lattice is often used for fluid volume, coupling them through appropriate kinematics and dynamic conditions. However, different attributes and shapes for multiple fluids cannot be ideally matched on one lattice [7]. The solid liquid coupling cannot be fully showed with solid objects as rigid body [8] when phase transition occurs. Few papers have noticed multiple fluids with solid–liquid interaction where phase transitions like melting and freezing occur.

In this paper, we focus on solid–liquid interactions with thermal phase transition for multiple fluids simulation using hierarchical lattices and multidistribution functions. Our main contributions are as follows:

Combination of free surface and thermal fluid model for phase transition. On the basis of the thermal lattice Boltzmann method model, heat distribution function and density distribution function are calculated considering the exchange at solid–gas and liquid–gas interfaces. With the use of interface lattices, a free surface tracking is proposed for gas–liquid coupling.

New solid–liquid coupling model compatible with phase transition. A new model based on hierarchical lattices is proposed to describe the solid–liquid interaction and phase transition along the solid–liquid interface. Exchanges of kinetic and thermal energy are processed on special cells named *MSolid* and *MLiquid*. And solid and liquid are tracked on different hierarchy by two-way interpolating.

Hybrid interaction for multiple fluids simulation with multidistribution functions. Multidistribution functions are presented to integrate the hierarchical lattices model and the existing two-fluid lattice Boltzmann method (LBM) to

realize multiple fluids simulation with phase transition and solid–liquid coupling.

The rest of this paper is organized as follows. After investigating the previous works in Section 2, the phase transition model and the method of tracking the free surface are introduced in Section 3. In Section 4, hierarchical lattices model is explained. Then we describe the hybrid model based on multidistribution function for multiple fluids in Section 5 and briefly explain our results in Section 6. Finally, the conclusion and future work are outlined in the last section.

2. RELATED WORKS

There are some works on the simulation of phase transition, solid–liquid and gas–liquid interactions, and the multiple fluids simulation.

The history of simulating the phase transition goes back to works by Miller *et al.* in 2001 [9] in which kinetic reaction equation is introduced for a model of phase transition. Then smoothed-particle hydrodynamics (SPH) [10,11] and LBM [12] are introduced for phase transition. But in LBM-based models, the solid is fixed on the lattices, and the solid–liquid coupling model is greatly simplified. There are also some numerical models for phase transition for multiphase flow, such as reaction–diffusion models [13] and triplet production models [14]. However, they are too complex to be directly used in fluid simulation. For ice melting, Losasso *et al.* [15] combined several modern methods like particle level set to model the melting and burning of solid materials. Fujisawa *et al.* [16] simulated the melting of ice on the basis of thermal dynamics. Recently, Iwasaki *et al.* [17] proposed a particle-based method for the simulation of melting and freezing of ice objects. Similar to melting phenomena, Wojtan *et al.* [18] proposed a method for animating corrosion and erosion.

Different interactions between gas, liquid, and solid phases are very important for multiphase fluid simulation regarded less of phase transition. As to the gas–liquid coupling, Kass *et al.* [19] first proposed the free surface model with shallow water equations. The height of the fluid is assumed to be the macrodensity of each lattice. Liu *et al.* [20] achieved shallow water equations on multiple lattices, which required more details in the delineation of the local human-intensive lattice. Thurey *et al.* have expanded LBM for free surface flow on 3D lattices [21]. For the solid–liquid coupling, typically fluids such as water (e.g., [22]) are simulated using Eulerian numerical methods, whereas solids such as cloth (e.g., [23,24]) are simulated with a Lagrangian numerical method. But it has proven to be difficult to couple these disparate simulation methods together. Thus, many researchers fully took Lagrangian approach [25,26]. Alternatively, Eulerian methods for both the fluid and solid are also studied, treating solids as in which solids are thought to be high viscosity or viscoelastic Eulerian fluids [27,28].

For multiple fluids simulation, multiple level sets are often used for interactions between multiple immiscible liquids [29]. Recently, Kang *et al.* [30] presented a hybrid approach for multiple fluids by combining distance functions (level set) with volume fractions. Another approach based on volume fraction was proposed [31] to simulate the miscible and immiscible flows simultaneously. SPH method is widely used too [32,33], and recently, Liu *et al.* [34] proposed an Liquid-Liquid Smoothed Particle Hydrodynamics (LLSPH) to simulate more than three fluids. Miscible fluids phenomena based on LBM are also presented [35–37]. Luo *et al.* [7] directly derived their two-fluid mixtures using a formal discretization procedure within kinetic theory. Thus, their two-fluid LBM, called TFLBM, inherited the sound physics and mathematical rigor incumbent in kinetic theory. And Zhu *et al.* [36] introduced a stable TFLBM by combining the TFLBM and the free surface LBM with subgrid model. However, the solid–liquid coupling and phase transition are avoided in the aforementioned models. Thus, the fluids are in the same phase: liquid.

In conclusion, most recent methods focus on either multiple fluids with liquid phase or the interaction between solid and liquid with single fluid. For the phenomena with phase transition and more interactions between different phases, there can be more than two types of fluids. Meanwhile, each fluid may have two phases: liquid and solid. To model this complex interaction and simulation, our approach needs to deal with

- Fluid model with free surface between gas and liquid;
- Heat transfer and phase transition of multiple fluids;
- Density transfer between multiple fluids;
- Interaction between solid and liquid phases.

3. THERMAL SOLID–LIQUID TRANSITION MODEL WITH FREE SURFACE TRACKED

Here, we focus on three different phases—solid, liquid, and gas—and the phase transition between liquid and solid. The single-phase equations for thermal fluid comprising melting and freezing are given by

$$\begin{aligned} &\nabla \cdot \mathbf{u} = 0 \\ &\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{p}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g} \\ &\frac{\partial E}{\partial t} + \nabla \cdot (\mathbf{u}E) = \nabla \cdot (k \nabla E) \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the velocity, p is the pressure, ρ is the density, ν is the kinematic viscosity, and $\mathbf{g}$ is the gravity. The thermal diffusivity is designated by $k = k(E)$, and E is the thermal energy.

To solve the macroscopic equation, the thermal LBM is introduced here [38], in which a multidistribution function method is applied. And there are two sets of distribution functions based on space lattice, one for the density and the other for the heat energy.

3.1. Heat Distribution Function

Denoting ξ as the liquid fraction in a cell, then in a simple approximation, it can be expressed as follows: [38]:

$$\xi(T) = \begin{cases} 1, & T > T_f \\ (T - T_i)/(T_f - T_i), & T_i < T < T_f \\ 0, & T < T_f \end{cases} \quad (2)$$

where T is the temperature. T_f and T_i represent the temperature of liquidus and solidus, respectively.

In thermal LBM, two sets of distribution functions, f_i and h_i, are defined. Here, f_i describes the mass and momentum transfer, and h_i represents the change of heat energy.

The collision and stream of the distributions are summarized by the motion equations.

$$h_i(\mathbf{x} + \mathbf{e}_i \Delta t, t + \Delta t) - h_i(\mathbf{x}, t) = -\frac{\Delta t}{\tau_h}(h_i(\mathbf{x}, t) - h_i^{eq}(\mathbf{x}, t)) \quad (3)$$

where $h_i(\mathbf{x}, t)$ represents the heat distribution function in i-direction, $\mathbf{e}_i$ is the discrete particle velocity vector in i-direction, and τ_h is the relaxation time.

3.2. Phase Transition

For phase transition, a method similar to enthalpy formulation is used here [39]. It is assumed that melting process occurs when the temperature ranges from T_i to T_f with $T_f - T_i$ to be a small value. The enthalpy method is to separate the sensible and latent heat components near the interface of solid and liquid. The latent heat component is expressed in term of the latent heat and liquid fraction $\xi(T)$ showed in Equation (2).

3.3. Density Distribution Function

The phase transition zone is treated like a porous medium. The fluid penetration into the medium depends on its permeability. In the standard LBM model, it is assumed that the densities are uniformly distributed throughout the volume of each cell. However, if the cell is totally full of solid, the stream step should not be taken at all, so we just treat it as the no-slip boundary condition. Thus, the $f_i(\mathbf{x}, t)$ will be totally reflected. We note $f_i^*(x, t+\Delta t)$ as the result of collision step. Then the stream step of density functions in the porous media can be written as

$$f_i(\mathbf{x}, t + \Delta t) = f_i^*(\mathbf{x}, t + \Delta t) + (1 - \xi(\mathbf{x})) \times \left(f_{\tilde{i}}^*(\mathbf{x} + \mathbf{e}_i \Delta t, t + \Delta t) - f_i^*(\mathbf{x}, t + \Delta t) \right) \quad (4)$$

where $\xi(\mathbf{x})$ is the liquid fraction at the position $\mathbf{x}$ and $\tilde{i}$ is the opposite direction of i.

Equation (4), at the macro level, means that for one cell, the higher the percent of solid is, the less the density exchanged with neighbors. When it is full of solid, there will be no density exchanged with adjacent cells.

3.3.1. Free Surface Tracking with Interface lattices.

Gas–liquid coupling is encountered in fluid simulation with free surface. As the space discretized into lattice, there exist fluid region and nonfluid region. In thermal LBM, two distribution functions, one for density and one for heat energy, are evolving on space lattices. To track the free surface, here three types cells are used as in [40]: *gas*, *liquid*, and *interface* cell. *Interface* cell is the interspace of other two types of cells and contains both liquid and gas, just as shown in Figure 1. The states of these cells will be changed along with fluid movement.

In [40], Thurey maintained the mass of liquid in every cell and update the liquid fraction $\varepsilon = m/\rho$ every step to determine the conversion of the cell types. After the initialization of the cell types, the mass is transferred with the density distribution function again.

When coupling the thermal LBM and the free surface tracking, three physical quantities are transferred: mass m, density ρ, and heat energy E. So the transfers of the mass and density need to be modified. Because the density is now transferred by Equation (4), the difference of inpouring and outpouring mass should be calculated by

$$\Delta m_i(\mathbf{x}, t + \Delta t) = \xi(\mathbf{x}) \cdot (f_i(\mathbf{x} + \mathbf{e}_i \Delta t, t) - f_i(\mathbf{x}, t)) \cdot \frac{(\varepsilon(\mathbf{x} + \mathbf{e}_i \Delta t, t) + \varepsilon(\mathbf{x}, t))}{2} \quad (5)$$

The change of mass in each cell can be directly obtained by accumulating Δm_i in all directions. *Gas* cells are cells with no mass with no participation in the mass exchange. But because *gas* cells have heat energy and density, they exchange them with *interface* cells, so the heat energy distribution function and density distribution function in *interface* cells both need to be reconstruction.

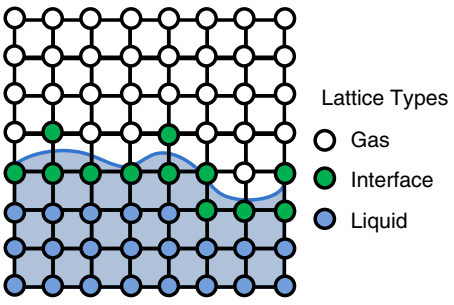

Figure 1. Cell types in the gas–liquid coupling lattice.

Here, the reconstruction of density distribution function changes into

$$f_i(\mathbf{x}, t+1) = \left(f_i^{eq}(A) + f_{\tilde{i}}^{eq}(A)\right) \cdot \xi(\mathbf{x}) + f_{\tilde{i}}(\mathbf{x}, t) - 2\xi(\mathbf{x}) \cdot f_{\tilde{i}}(\mathbf{x}, t) \quad (6)$$

And for the reconstruction of the heat energy distribution function, to be simplified, we assume that the heat energy exchange at surface is proportional to the difference of the heat energies of two sides. And the heat energy of the gas E_g is treated as constant, because the *gas* cells keep no physical quantities in them. So the heat exchange between *liquid–gas* cells can be written as

$$h_i(\mathbf{x}, t+\Delta t) - h_{\tilde{i}}(\mathbf{x}, t) = \kappa(E(\mathbf{x}, t) - E_g) \quad (7)$$

where κ is the factor of the proportionality and $E(\mathbf{x}, t)$ is the heat energy at the position $\mathbf{x}$.

4. HIERARCHICAL LATTICES FOR SOLID–LIQUID COUPLING

When considering phase transition, the moving objects may melt when its temperature reaches a certain threshold. If only by using one same lattice for distribution functions of solid and fluid, the interaction between them depends on the resolution of lattice, and the movement of melting solid is hard to track. So we proposed a new method with hierarchical lattices to describe them.

4.1. Hierarchical Lattices for Liquid and Solid

Similar with *solid level set* method to update the local lattices of solid [18], in our hierarchical lattices, the liquid lattice is fixed on global coordinate system, and another lattice for solid is constructed in local coordinate. Keep a bounding box for every object and discretize it into a solid lattice with heat distribution functions and fraction of solid, as shown in Figure 2. The solid lattice is changing along with the movement of the object. A transformation matrix $\mathbf{R}$ from local to global coordinate is maintained. So the solid lattice is on local coordinate system; a point at the position $\mathbf{x}$ in solid lattice has the coordinate $\mathbf{R} \cdot \mathbf{x}$ in liquid lattice.

4.2. Two-Way Interpolating for Two Lattices

On the basis of the aforementioned hierarchical lattices, the total evolution process of solid–liquid coupling is shown in Figure 3.

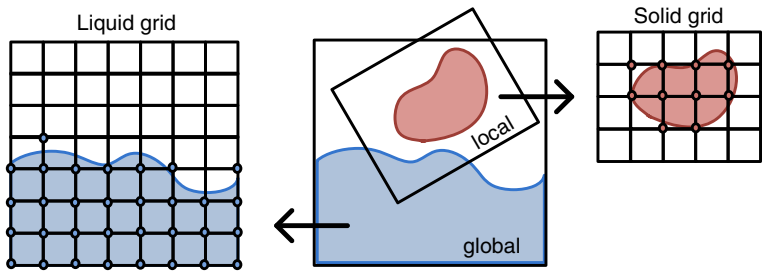

Figure 2. The hierarchical lattices for liquid and solid.

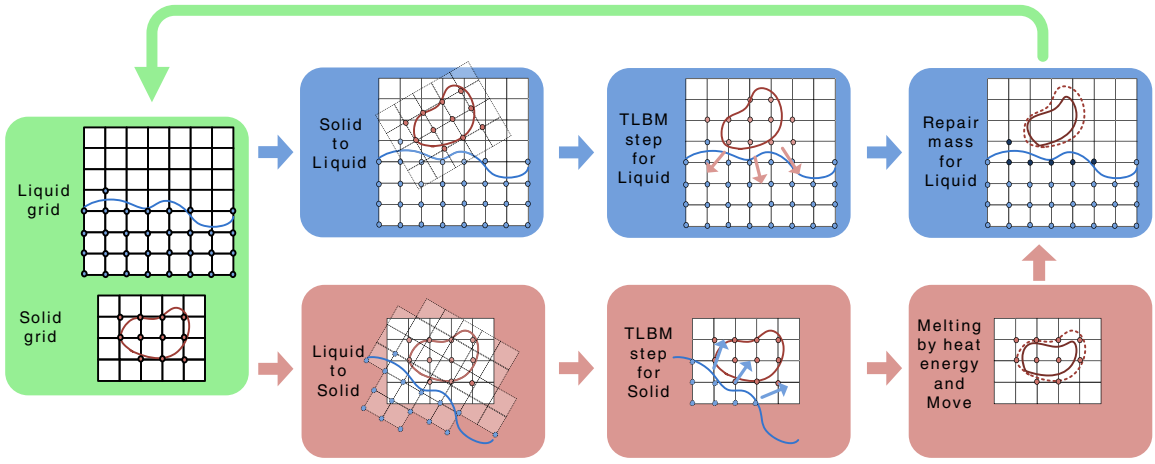

Figure 3. Overview of the solid–liquid coupling process, and the evolutions of solid and liquid are separated and take place on hybrid lattices.

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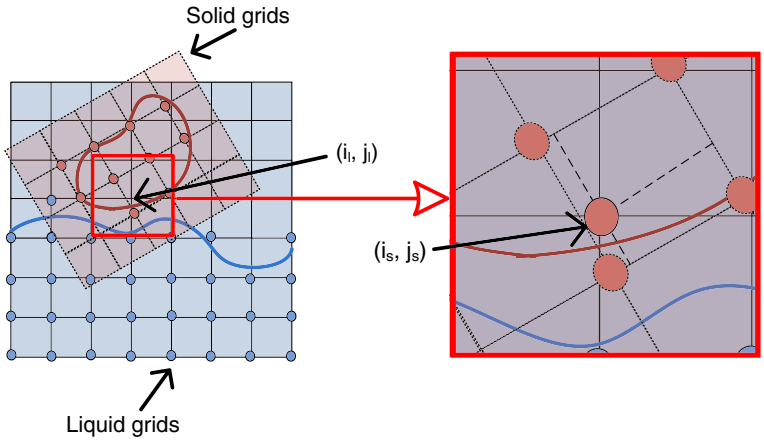

Figure 4. The full-line circle on liquid lattice representing a *MSolid* cell is interpolated from the four dotted-line circles on the solid lattice.

When updating the liquid lattice, special lattices whose corresponding locations in the solid lattices are not empty are noted *MSolid* cells. The attributes of the *MSolid* cell including velocity, heat energy, and fraction of solid are obtained by trilinear/bilinear interpolation in 3D/2D space.

For the cell (i_l, j_l) on liquid lattice, its corresponding coordinate in the solid lattice is $(i_s, j_s) = \mathbf{R}^{-1}(i_l, j_l)$, so the attributes can be interpolated from this four neighbors, as shown in Figure 4.

When updating the solid lattice, special cells *MLiquid* are dealt as *MSolid*. But for the attributes of *MLiquid* cells, the density is neglected, because there is no density exchange between solid and liquid.

4.3. Kinetic and Thermal Energy Exchange

The coupling of solid and liquid takes the kinetic and thermal energies into consideration, which makes phase transition possible.

(1) *Exchange of kinetic energy*

With the use of *MSolid* and *MLiquid* cells, the momentum transferred from solid to liquid is dealt by [40]

$$f_i(\mathbf{x}, t + \Delta t)' = f_{\tilde{i}}(\mathbf{x}, t) + 6\omega_i \rho_f \mathbf{e}_i \cdot \mathbf{u}_o \qquad (8)$$

where ρ_f is the liquid density and $\mathbf{u}_o$ is the velocity at solid boundary. Equation (8) modifies the stream step when it is from an *MSolid* cell to a *liquid* or *interface* cell,

Resultant force $\mathbf{F}_o$ on solid by the liquid is calculated by

$$\mathbf{F}_o = \frac{\Delta x}{\Delta t} \sum_{\forall \mathbf{x}_b} \sum_{i=1}^{Q} \mathbf{e}_i \omega_o(\mathbf{x}_b + \mathbf{e}_i \Delta t, t) \times \left(f_{\tilde{i}}(\mathbf{x}_b + \mathbf{e}_i \Delta t, t) + f_i(\mathbf{x}_b + \mathbf{e}_i \Delta t, t + \Delta t) \right) \qquad (9)$$

where $\mathbf{x}_b$ is an indicator function that is equal to one when the cell at the position x is a fluid cell, and zero otherwise. Q is the number of discrete directions in an LBM lattice. ω_0 is equal to one when the cell is a fluid cell, and zero otherwise. And the torque on each cell is calculated in the similar way. With the resultant force and torque, the movements of solids are dealt by the mechanics of rigid body. The only difference is that the barycenter, moment of inertia, and mass of the solid need to be recalculated in each step.

(2) *Exchange of thermal energy*

The thermal LBM steps are taken on the solid and liquid lattices, respectively. The heat energy can be transferred between solid and liquid, which is different from the density, so the boundary condition calculated by Equation (7) is only used between air and solid or between air and liquid. After the heat energy is transferred, the phase transition may occur. According to Equation (2), we can obtain the $\xi(\mathbf{x})$ on the solid girds and then calculate the liquid fraction and update the mass on every cell. The melted part $\Delta m(\mathbf{x})$ of a cell that transfers to liquid is $m(\mathbf{x}) \cdot \xi(\mathbf{x})$, where $m(\mathbf{x})$ is the mass of the cell at position $\mathbf{x}$ before melting. Distribute this part of mass to the adjacent *liquid* and *interface* cells according the weights of the valid neighbors on the basis of their positions. If there is no *liquid* or *interface* cells, we change the adjacent cells into *interface* cells.

After updating of the mass, the temperature of the cell is set to T_i referred in Section 3.2 that represents

the solidus temperature, where the liquid in melting cells has been removed from the solid cells.

4.4. Mass Conservation and Solid Lattice Separation

There maybe is some mass generated and disappeared in the following situations:

First, in Equation (8), the momentum of the movement of solid is transferred to the fluid by adding or subtracting extra densities.

Second, in Section 4.2, when interpolating the local lattice to the global lattice, once the original cell is a *fluid* or *interface* cell, it will be changed into *MSolid*, and so the mass on that cell just disappears, denoted as M_{cover}.

So the total lost liquid mass is

$$\Delta M = -\sum_{\forall \mathbf{x}_b} 6\omega_i \rho_f \mathbf{e}_i \cdot \mathbf{u}_o + M_{\text{cover}} \tag{10}$$

Distributing it to adjacent *interface* cells averagely at each step, the mass conservation can be kept.

During the melting process, occasionally the solid may split into some pieces. We use the flood-fill method to detect the situation and add the new local coordinate system for each new piece. Then re-initialize new lattices with new barycenters, masses, energy distribution functions, and velocities.

5. HYBRID INTERACTION FOR MULTIPLE FLUIDS SIMULATION

The aforementioned coupling between gas–liquid and solid–liquid, respectively, is for only one type of fluid with solid–liquid phase transition. However, there maybe are more than two types of fluids; for example, some phenomena may have original liquid and solid phases of one type of fluid A and then mix it with another fluid B with liquid phase. Then melting and freezing of fluids A and B both may take place and mixed together.

5.1. Modeling Multiple Fluids

For miscible mixture simulation, two sets of density distribution function for each fluid are used, and the two-fluid equation is as follows [35]:

$$f_i^{\sigma}(\mathbf{x} + \mathbf{e}_i \Delta t, t + \Delta t) = f_i^{\sigma}(\mathbf{x}, t) + J_i^{\sigma\sigma} + J_i^{\sigma\varsigma} + F_i^{\sigma} \tag{11}$$

where σ, ς denote the two fluids and $J_i^{\sigma\sigma}$ and $J_i^{\sigma\varsigma}$ are the self-collision term and the cross-collision term exerting to fluid σ, respectively, which represents the interplay between the two components. F_i^{σ} is the external force.

The cross-collision term determines how strong the diffusion effect in the miscible mixture, so the miscibility of mixtures can be easily adjusted through the cross-collision coefficient, which should be more than 0.5 for simulating miscible mixtures.

5.2. Mixtures of Multiple Fluids with Multidistribution Functions

To achieve the solid–liquid coupling and phase transition in multiple fluids simulation, multidistribution functions are used to describe the new attributes on the lattices of both liquid and solid. So the distribution functions here includes $f_i^{\sigma}, f_i^{\varsigma}, h_i$.

For the exchange of the heat energy, the phases of fluid can be neglected because the heat transfer between solid and liquid in one cell, that is, the temperature, is unique. So h_i is transferred by Equations (3) and (7) in liquid and solid lattices.

Whereas for transfer of the densities of two fluids, the influence of heat energy is necessary, so Equation (11) is improved into

$$f_i^{\sigma}(\mathbf{x} + \mathbf{e}_i, t + \Delta t) = f_{\tilde{i}}^{\sigma}(\mathbf{x}, t) + \xi(\mathbf{x}) \left(f_i^{\sigma}(\mathbf{x}, t) - f_{\tilde{i}}^{\sigma}(\mathbf{x}, t) + J_i^{\sigma\sigma} + J_i^{\sigma\varsigma} + F_i^{\sigma} \right) \tag{12}$$

The states involved in this hybrid model are displayed in Figure 5. Here, no coupling between the two fluids is for solid phase, whereas in the liquid phase, the two fluids that interact with each other is described.

For every global (liquid) lattice in hybrid model, every cell has only one value of temperature, so a set of heat distribution functions is enough to model the heat transfer whereas two sets of density distribution functions are needed for two fluids.

Different from the liquid lattice, the density of every local (solid) lattice is not transferred on the solid lattice, so we keep just the mass and fractions of two fluids in the cells instead of two sets of density distribution functions and update them dynamically.

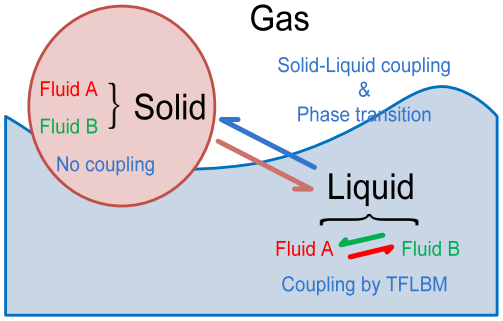

Figure 5. The states and the interactions between them.

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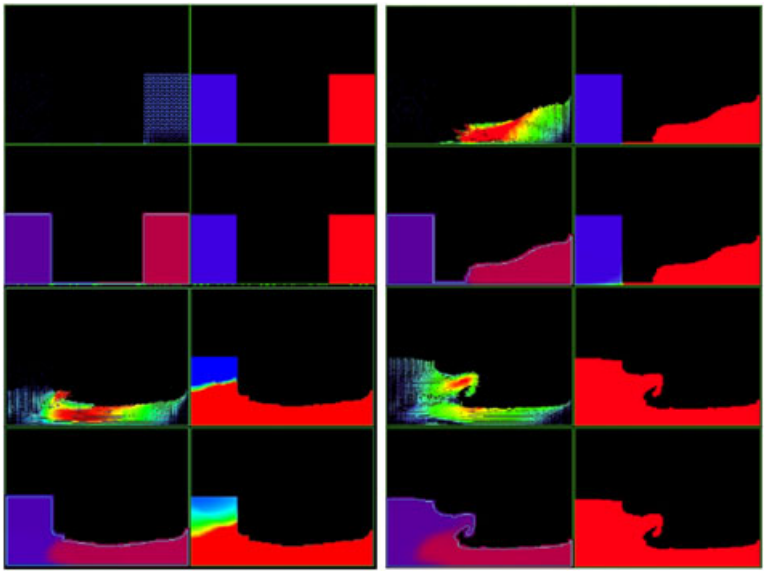

Figure 6. Four shortcuts of the 2D two fluids simulation with phase transition.

Figure 6 is the simulated results with the updating attributes of two fluids with phase transition. At first, a box of cold blue fluid in solid phase is located on the left, and a box with hot red fluid in liquid phase is located on the right. In each picture, the left top is the velocity field (red for larger, blue for smaller); the right top is the phase of two fluids (red for liquid, blue for solid); the left bottom is the density of two fluids (red for one, blue for the other one); the right bottom is the heat energy (red for higher, blue for lower).

According to multidistribution functions for hybrid interaction, the solid–liquid coupling is also extended in different parts as following in state updating process.

Solid update. After the transfer of thermal energy on solid lattice, the phases of the cells are updated according to the temperature, and the melted mass of the two components is recorded to maintain the mass conservation.

Liquid update. On the liquid lattice, two fluids in one cell interact according to Section 5.1, and when a *liquid* or *interface* cell interacts with *gas* cell, the reconstruction step in [36] is taken.

Solid–liquid coupling. Before the *liquid update*, the extra mass from the melted solid is distributed for two fluids. And the force on the cell containing two fluids acts on both fluids, so Equation (8) should be modified into

$$f_i^{\sigma}(\mathbf{x}, t+\Delta t)' = f_{\tilde{i}}^{\sigma}(\mathbf{x}, t) + fraction_{\sigma} \cdot 6\omega_i \rho_f \mathbf{e}_i \cdot \mathbf{u}_o \tag{13}$$

where $fraction_{\sigma}$ is the fraction of fluid σ in the lattice. The equation for new torque and resultant force on solid is easy to deal with by replacing single f_i with the mixed $\sum_{\sigma} f_i^{\sigma}$.

6. RESULTS AND ANALYSIS

On the basis of the aforementioned method, we implement interactive rendering of different multiple fluids simulation with solid–liquid phase transition including burning candle, dropping solids into water, and ice sculpture melted by hot liquid, and so on. Our rendering machine is PC with Intel (R) Pentium (R) 3.4 GHz CPU, 3 GB memory, NVIDIA GeForce GTX260 Graphics card.

We test the combination of different models for different multiple fluids phenomena. The surface of the fluid is extracted by Marching Cube algorithm with the volume fractions on every cell and rendered by using Pov-ray.

The phase transition is simulated at first. Figure 7 shows two scenes with burning candle. As being heated by the flame, the heat energy makes solid melting, whereas the liquid that is far from the flame is cooled down by the air. Then multiphase fluid including phase transition and solid–liquid coupling with the scene of dropping cube and sphere in the water is rendered. In Figure 8, the solids are fusible; the phase transition and interaction are shown.

Later, a phenomenon with hot red fluid pouring on an ice sculpture is rendered here, just as shown in upper pictures in Figure 9. Here, the solid part is fixed on the space because of the lack of solid–liquid coupling. The water generated by the melted ice mixes with the red fluid.

Finally, hybrid framework of completed multiphase and two-component model is tested with a scene similar to Figure 8. The solids in this sample are solid phase of a red fluid originally and melt when touching the hot water, just as shown in Figure 10. From these simulating results, our method for multiple fluid simulation is effective and stable.

The integrated models and the lattice sizes in the models of every simulated scene are shown in Table I.

Figure 7. Two scenes of burning candle in different viewpoints, in which the free surface for gas–liquid coupling and the thermal lattice Boltzmann method for phase transition are used.

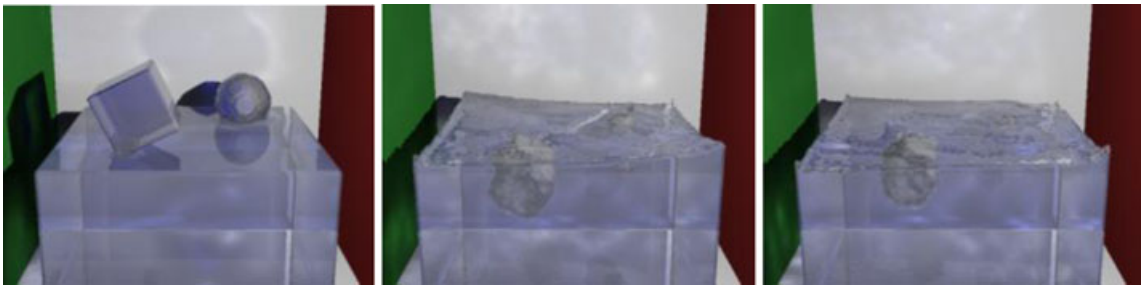

Figure 8. Dropping solids in water. The fusible solids are melting in the water.

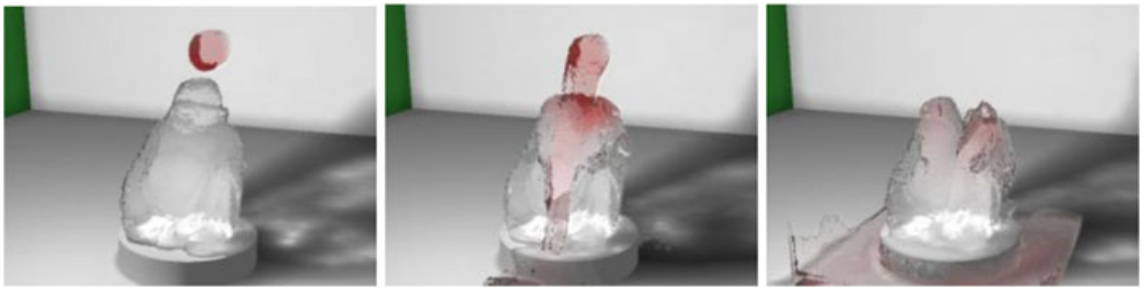

Figure 9. An ice sculpture watered by red warm liquid. Part of the sculpture melts into water, whereas part of the red water freezes on the sculpture.

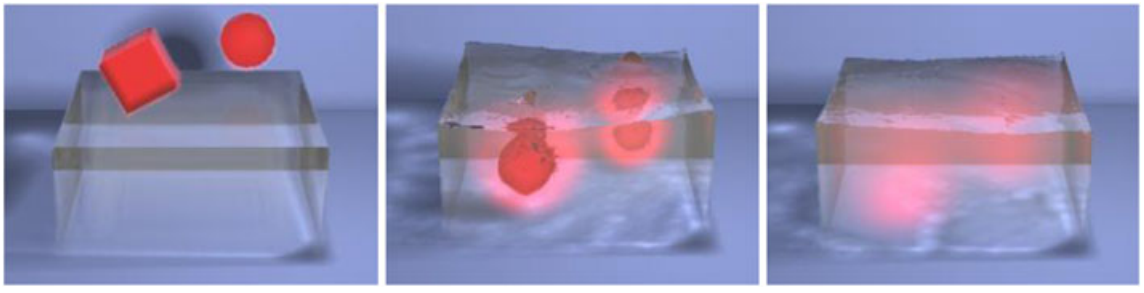

Figure 10. Dropping red solids in hot water. The red solids melt into red liquid and interact with water, where there are two types of liquids and one solid object mixed.

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Table I. Our simulating results by coupling different models.

Simulated models	Second per frame	Lattice size	Two component	Multiphase		
				Solid–liquid coupling	Gas–liquid coupling	Phase transition
Burning candle (Figure 7)	0.85	100^3	×	×	√	√
Fusible solids (Figure 8)	1.16	140^3	×	√	√	√
Ice sculpture (Figure 9)	1.21	140^3	√	×	√	√
Dropping solids (Figure 10)	1.38	140^3	√	√	√	√

Table II. The comparison of the capabilities between recent works and our method.

Recent works	Model	Number of fluids	Number of phases	Interaction		Phase transition
				Solid–liquid	Gas–liquid	
Liu *et al.* [34]	SPH	3	2	×	√	×
Iwasaki *et al.* [17]	SPH	1	3	√	√	√
Bao *et al.* [31]	VOF	3	2	×	√	×
Our method	LBM	2	3	√	√	√

SPH, smoothed-particle hydrodynamics; VOF, volume of fluid; LBM, lattice Boltzmann method.

7. CONCLUSION AND FUTURE WORKS

A new hybrid modeling method based on hierarchical lattices is presented here to describe the interaction, which can effectively process the solid–liquid interaction, phase transition in the solid–liquid interface, and free surface tracking for multiple fluids simulation. Here, the solid–liquid coupling model is competent to extend the phase transition model and two-fluid model. Our hybrid model is able to simulate multiple types of fluids with different phases. And the given method is lattice based on a framework of LBM whose main advantage is easy to be parallelized and accelerated by GPU. The solid–liquid coupling model is also easy to be combined with other lattices-based multiphase fluid solver. And from the test cases we list, the capability of our method covers multiple fluids, multiphase, gas–liquid and solid–liquid coupling, and phase transition. A comparison with other recent works is presented in Table II.

Future work includes the use adaptive model to show more details in multiple fluids and implement the algorithm on GPU to achieve considerable acceleration and apply it to simulate large-scale disaster phenomena with complex fluid system.

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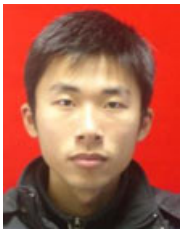

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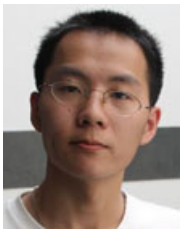

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