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Tables and Applications of the Bonferroni t-Statistics: A Revision of Dunn's Simultaneous t-Tests

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Abstract

Tables for Bonferroni-adjusted significance levels of Student's t are provided for $r = 3(1)30(5)50$ etc. and alphas of 0.05, 0.01 and 0.001. The tables are suggested for various applications, for example in replacing analysis of variance of k samples by r simultaneous t tests. Use of the tables is shown by numerical examples.

The adjustment of alpha for improving the efficiency of testing is made by WILKINSON for orthogonal comparisons and by HOLM for nonorthogonal comparisons.

Key words: Bonferroni t-statistics, Dunn's simultaneous t-tests

In a foregoing paper, LIENERT et al. (1982) have tabulated the Bonferroni z-statistic (normal distribution) and given a variety of its applications, including HOLM's (1979) version of simultaneous z-testing. Z-testing is applicable to compare means of r random samples with a mean of a normal population only if its variance is also known numerically.

Usually the experimenter compares means of r random samples drawn from a normal population with an unknown variance under r different treatments. If the variances of the r samples are homogeneous, the population variance may be estimated by the analysis of variance (ANOVA). ANOVA is the most effective procedure for detecting differences in treatment means. For interpreting a significance between samples F-test exhaustively, so called aposteriori t-tests have been designed: These are (1) DUNNETT's t-test for comparing $k-1$ treatments with one control, (2) FISHER's (1949, p. 56–58) least significant differences (LSD) test, (3) TUKEY's (1953) honestly significant differences (HSD) test for pairwise comparing treatments and (4) SCHEFFE's (1953) S method for non-pairwise comparisons. They are all based on experimentwise error-rates while other tests such as NEWMAN-KEULS (1952) and DUNCAN's (1955) range test are based on the so called stairstep approach to multiple or simultaneous testing. A similar statement is true for RYAN's (1960) method of adjusted significance levels and GABRIEL's (1964) simultaneous test procedure (STP).

While the above tests are to be made only if the preceding ANOVA has shown to be significant, DUNN's (1961) multiple comparison procedure (MCP) does

not require any preceding ANOVA. Thus MCP is implying apriori t-tests which are based on a comparisonwise error rate adjusted for simultaneous testing by Bonferroni's inequality (see MILLER, 1966, p. 8, and 1977).

1. DUNN'S multiple comparison procedure (MCP)

DUNN'S (1961) multiple comparison procedure (MCP) is based on the critical limits of Student's t distribution, which are calculated for Bonferroni-adjusted alpha-risks,

$$\alpha^* = \alpha/r$$

where r is the number of simultaneous tests designed prior to, or at least logically independent of, sampling observations X from N individuals under k treatments.

DUNN'S multiple comparison procedure works as follows:

If $r=1$ comparison of 2 out of k sample means is designed, then the classical α-limits of the t-distribution are valid, where α is the predesigned level of significance. If $r=2$ comparisons are to be made, not necessary orthogonal ones, thent he α-limits are replaced by $(\alpha/2)$-limits of the t-distribution to arrive at DUNN'S critical t-limits for 2 comparisons. Analogously for $r=3$ comparisons, the factual level α is replaced by the Bonferroni-adjusted $\alpha/3$ in calculating the t-limit of DUNN'S MCP, etc.

For a numerical illustration of MCP assume that $r=5$ simultaneous apriori-tests have been designed for a nominal level $\alpha=0{,}05$; what then is the adjusted BONFERRONI t-limit if a $k=4$ sample of $n=6$ individuals has been observed? The non-adjusted 5 %-limit for $df=4\cdot(6-1)=20$ is read from a suitable table (see KIRK, 1968, table D.4) to be $t=2{,}086$. The adjusted 5 %-limit is equal to the nonadjusted $5\%/5=1\%$ limit which reads to be $t=2{,}845$ (in table D.4). The same limit ($t=2{,}85$) is read from DUNN'S table of Bonferroni-adjusted t-limits (see KIRK, 1968, table D.16) for $df=20$ and $r=5$ comparisons. Thus DUNN'S critical limits of the t_D statistic are nothing more than the Bonferroni-adjusted limits of STUDENT'S t-distribution, where adjustment is made for the number r of mean comparisons from k samples.

However DUNN'S t-limits, as well as BAILEY'S (1977) and HUITEMA'S (1980, Table A6), are given only for selected numbers of comparisons and restricted to likewise selected degrees of freedom. Beyond that, some limits for $r\geqq 25$ are only estimated graphically. Therefore a retabulation of the BONFERRONI-t-statistic seems to be useful for DUNN'S MCP, as well as for other aims discussed later on.

2. Percentage points of the BONFERRONI t distribution

The most extensive tabulation of percentage points for one-sided Student's t-tests is given in FEDERIGHI (1959) at 20 alpha-levels between 0,25 and 0,0000001 with $df=1$ (1) 30 (5) 60 (10) 100; 200; 500; 1000; 2000; 10000 and ∞. The table

Table 1a

ALPHA =0.1000 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 =0.0500 (1-SIDED)

N \ R	1	2	3	4	5	6	7	8	9	10
3	2.353	3.182	3.740	4.177	4.541	4.857	5.138	5.392	5.625	5.841
4	2.132	2.776	3.186	3.495	3.747	3.961	4.148	4.315	4.466	4.604
5	2.015	2.571	2.912	3.163	3.365	3.534	3.681	3.810	3.926	4.032
6	1.943	2.447	2.749	2.969	3.143	3.287	3.412	3.521	3.619	3.707
7	1.895	2.365	2.642	2.841	2.998	3.128	3.238	3.335	3.422	3.499
8	1.860	2.306	2.566	2.752	2.896	3.016	3.117	3.206	3.285	3.355
9	1.833	2.262	2.510	2.685	2.821	2.933	3.028	3.111	3.184	3.250
10	1.812	2.228	2.466	2.634	2.764	2.870	2.960	3.038	3.107	3.169
11	1.796	2.201	2.431	2.593	2.718	2.820	2.906	2.981	3.047	3.106
12	1.782	2.179	2.403	2.560	2.681	2.779	2.863	2.934	2.998	3.055
13	1.771	2.160	2.380	2.533	2.650	2.746	2.827	2.896	2.957	3.012
14	1.761	2.145	2.360	2.510	2.624	2.718	2.796	2.864	2.924	2.977
15	1.753	2.131	2.343	2.490	2.602	2.694	2.770	2.837	2.895	2.947
16	1.746	2.120	2.328	2.473	2.583	2.673	2.748	2.813	2.870	2.921
17	1.740	2.110	2.316	2.458	2.567	2.655	2.729	2.793	2.848	2.898
18	1.734	2.101	2.304	2.445	2.552	2.639	2.712	2.775	2.829	2.878
19	1.729	2.093	2.294	2.433	2.539	2.625	2.697	2.759	2.813	2.861
20	1.725	2.086	2.285	2.425	2.528	2.613	2.683	2.744	2.798	2.845
21	1.721	2.080	2.278	2.413	2.518	2.601	2.671	2.732	2.784	2.831
22	1.717	2.074	2.270	2.405	2.508	2.591	2.661	2.720	2.772	2.819
23	1.714	2.069	2.264	2.398	2.500	2.582	2.651	2.710	2.761	2.807
24	1.711	2.064	2.258	2.391	2.492	2.574	2.642	2.700	2.751	2.797
25	1.708	2.060	2.252	2.385	2.485	2.566	2.634	2.692	2.742	2.787
26	1.706	2.056	2.247	2.379	2.479	2.559	2.626	2.684	2.734	2.779
27	1.703	2.052	2.243	2.373	2.473	2.552	2.619	2.676	2.726	2.771
28	1.701	2.048	2.238	2.368	2.467	2.546	2.613	2.669	2.719	2.763
29	1.699	2.045	2.234	2.364	2.462	2.541	2.607	2.663	2.713	2.756
30	1.697	2.042	2.231	2.360	2.457	2.536	2.601	2.657	2.706	2.750
35	1.690	2.030	2.215	2.342	2.438	2.515	2.579	2.633	2.681	2.724
40	1.684	2.021	2.204	2.329	2.423	2.499	2.562	2.616	2.663	2.704
45	1.679	2.014	2.195	2.319	2.412	2.487	2.549	2.602	2.648	2.690
50	1.676	2.009	2.188	2.311	2.403	2.477	2.539	2.591	2.637	2.678
55	1.673	2.004	2.183	2.304	2.396	2.469	2.530	2.583	2.628	2.668
60	1.671	2.000	2.178	2.299	2.390	2.463	2.524	2.575	2.620	2.660
70	1.667	1.994	2.171	2.291	2.381	2.453	2.513	2.564	2.608	2.648
80	1.664	1.990	2.165	2.284	2.374	2.445	2.505	2.555	2.600	2.639
90	1.662	1.987	2.161	2.280	2.368	2.440	2.499	2.549	2.593	2.632
100	1.660	1.984	2.158	2.276	2.364	2.435	2.494	2.544	2.587	2.626
200	1.653	1.972	2.143	2.258	2.345	2.414	2.472	2.520	2.563	2.601
300	1.650	1.968	2.138	2.253	2.339	2.407	2.464	2.513	2.555	2.592
400	1.649	1.966	2.135	2.250	2.336	2.404	2.461	2.509	2.551	2.588
500	1.648	1.965	2.134	2.248	2.334	2.402	2.459	2.507	2.549	2.586
600	1.647	1.964	2.133	2.247	2.333	2.401	2.457	2.505	2.547	2.584
700	1.647	1.963	2.132	2.246	2.332	2.400	2.456	2.504	2.546	2.583
800	1.647	1.963	2.132	2.246	2.331	2.399	2.455	2.503	2.545	2.582
900	1.647	1.963	2.131	2.245	2.330	2.398	2.455	2.503	2.544	2.581
1000	1.646	1.962	2.131	2.245	2.330	2.398	2.454	2.502	2.544	2.581
∞	1.645	1.960	2.128	2.241	2.326	2.394	2.450	2.498	2.539	2.576

Continued Table 1a

ALPHA = 0.1000 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0500 (1-SIDED)

N \ R	11	12	13	14	15	16	17	18	19	20
3	6.042	6.232	6.410	6.580	6.741	6.895	7.043	7.185	7.322	7.453
4	4.732	4.851	4.963	5.068	5.167	5.261	5.351	5.437	5.519	5.598
5	4.129	4.219	4.303	4.382	4.456	4.526	4.592	4.655	4.716	4.773
6	3.788	3.863	3.932	3.997	4.058	4.115	4.169	4.221	4.270	4.317
7	3.570	3.636	3.696	3.753	3.806	3.855	3.902	3.947	3.989	4.029
8	3.420	3.479	3.534	3.584	3.632	3.677	3.719	3.759	3.796	3.833
9	3.310	3.364	3.415	3.462	3.505	3.547	3.585	3.622	3.657	3.690
10	3.225	3.277	3.324	3.368	3.409	3.448	3.484	3.518	3.551	3.581
11	3.159	3.208	3.253	3.295	3.334	3.370	3.404	3.437	3.467	3.497
12	3.106	3.153	3.196	3.236	3.273	3.308	3.341	3.371	3.401	3.428
13	3.062	3.107	3.149	3.187	3.223	3.256	3.288	3.318	3.346	3.372
14	3.025	3,069	3.109	3.146	3.181	3.214	3.244	3.273	3.300	3.326
15	2.994	3.026	3.076	3.112	3.146	3.177	3.207	3.235	3.261	3.286
16	2.967	3.008	3.047	3.082	3.115	3.146	3.175	3.202	3.228	3.252
17	2.943	2.984	3.022	3.056	3.089	3.119	3.147	3.173	3.199	3.222
18	2.923	2.963	3.000	3.034	3.065	3.095	3.123	3.149	3.173	3.197
19	2.904	2.944	2.980	3.014	3.045	3.074	3.101	3.127	3.151	3.174
20	2.888	2.927	2.963	2.996	3.026	3.055	3.082	3.107	3.131	3.153
21	2.874	2.912	2.947	2.980	3.010	3.038	3.065	3.090	3.113	3.135
22	2.861	2.899	2.933	2.965	2.995	3.023	3.049	3.074	3.097	3.119
23	2.849	2.886	2.921	2.952	2.982	3.009	3.035	3.059	3.082	3.104
24	2.838	2.875	2.909	2.941	2.970	2.997	3.022	3.046	3.069	3.091
25	2.828	2.865	2.899	2.930	2.959	2.986	3.011	3.035	3.057	3.078
26	2.819	2.856	2.889	2.920	2.949	2.975	3.000	3.024	3.046	3.067
27	2.811	2.847	2.880	2.911	2.939	2.966	2.990	3.014	3.036	3.057
28	2.803	2.839	2.872	2.902	2.930	2.957	2.981	3.004	3.026	3.047
29	2.796	2.832	2.864	2.894	2.922	2.949	2.973	2.996	3.018	3.038
30	2.789	2.825	2.857	2.887	2.915	2.941	2.965	2.988	3.009	3.030
35	2.762	2.797	2.828	2.857	2.885	2.910	2.933	2.955	2.976	2.996
40	2.742	2.776	2.807	2.836	2.862	2.887	2.910	2.931	2.952	2.971
45	2.726	2.760	2.791	2.819	2.845	2.869	2.892	2.913	2.933	2.952
50	2.714	2.747	2.777	2.805	2.831	2.855	2.877	2.898	2.918	2.937
55	2.704	2.737	2.767	2.794	2.820	2.844	2.866	2.887	2.906	2.925
60	2.696	2.729	2.758	2.785	2.811	2.834	2.856	2.877	2.896	2.915
70	2.683	2.715	2.745	2.771	2.796	2.820	2.841	2.862	2.881	2.899
80	2.674	2.705	2.734	2.761	2.786	2.809	2.830	2.850	2.869	2.887
90	2.666	2.698	2.727	2.753	2.777	2.800	2.821	2.841	2.860	2.878
100	2.660	2.692	2.720	2.747	2.771	2.793	2.815	2.834	2.853	2.871
200	2.634	2.665	2.693	2.718	2.742	2.764	2.784	2.803	2.821	2.839
300	2.626	2.656	2.683	2.709	2.732	2.754	2.774	2.793	2.811	2.828
400	2.621	2.651	2.679	2.704	2.727	2.749	2.769	2.788	2.806	2.823
500	2.619	2.649	2.676	2.701	2.724	2.746	2.766	2.785	2.803	2.820
600	2.617	2.647	2.674	2.699	2.723	2.744	2.764	2.783	2.801	2.817
700	2.616	2.646	2.673	2.698	2.721	2.743	2.763	2.782	2.799	2.816
800	2.615	2.645	2.672	2.697	2.720	2.742	2.762	2.780	2.798	2.815
900	2.614	2.644	2.671	2.696	2.719	2.741	2.761	2.780	2.797	2.814
1000	2.614	2.644	2.671	2.696	2.719	2.740	2.760	2.779	2.797	2.813
∞	2.609	2.638	2.665	2.690	2.713	2.734	2.754	2.773	2.790	2.807

ontinued Table 1a

LPHA =0.1000 (2-SIDED) T (ALPHA/R, N)
KPHA/2 =0.0500 (1-SIDED)

N \ R	21	22	23	24	25	26	27	28	29	30
3	7.581	7.704	7.824	7.940	8.053	8.162	8.269	8.374	8.476	8.575
4	5.673	5.747	5.817	5.885	5.951	6.015	6.078	6.138	6.197	6.254
5	4.829	4.882	4.933	4.983	5.030	5.076	5.121	5.164	5.207	5.247
6	4.362	4.405	4.446	4.486	4.524	4.561	4.597	4.632	4.665	4.698
7	4.068	4.105	4.140	4.174	4.207	4.239	4.269	4.299	4.328	4.355
8	3.867	3.900	3.931	3.962	3.991	4.019	4.046	4.072	4.098	4.122
9	3.721	3.751	3.780	3.808	3.835	3.860	3.885	3.909	3.932	3.954
10	3.611	3.639	3.666	3.691	3.716	3.740	3.763	3.785	3.807	3.827
11	3.524	3.551	3.576	3.600	3.624	3.646	3.668	3.689	3.709	3.728
12	3.455	3.480	3.504	3.527	3.550	3.571	3.591	3.611	3.630	3.649
13	3.398	3.422	3.445	3.467	3.489	3.509	3.529	3.548	3.566	3.584
14	3.350	3.374	3.396	3.417	3.438	3.458	3.477	3.495	3.513	3.530
15	3.310	3.333	3.354	3.375	3.395	3.414	3.432	3.450	3.467	3.484
16	3.275	3.297	3.318	3.339	3.358	3.377	3.394	3.412	3.428	3.444
17	3.245	3.267	3.287	3.307	3.326	3.344	3.362	3.378	3.395	3.410
18	3.219	3.240	3.260	3.279	3.298	3.316	3.333	3.349	3.365	3.380
19	3.196	3.216	3.236	3.255	3.273	3.291	3.307	3.323	3.339	3.354
20	3.175	3.195	3.215	3.233	3.251	3.268	3.285	3.301	3.316	3.331
21	3.156	3.176	3.196	3.214	3.231	3.248	3.265	3.280	3.295	3.310
22	3.140	3.159	3.178	3.196	3.214	3.230	3.246	3.262	3.277	3.291
23	3.125	3.144	3.163	3.181	3.198	3.214	3.230	3.245	3.260	3.274
24	3.111	3.130	3.149	3.166	3.183	3.199	3.215	3.230	3.244	3.258
25	3.098	3.117	3.136	3.153	3.170	3.186	3.201	3.216	3.230	3.244
26	3.087	3.106	3.124	3.141	3.158	3.174	3.189	3.204	3.218	3.231
27	3.076	3.095	3.113	3.130	3.147	3.162	3.177	3.192	3.206	3.219
28	3.067	3.085	3.103	3.120	3.136	3.152	3.167	3.181	3.195	3.208
29	3.057	3.076	3.094	3.110	3.127	3.142	3.167	3.171	3.185	3.198
30	3.049	3.067	3.085	3.102	3.118	3.133	3.148	3.162	3.176	3.189
35	3.015	3.033	3.050	3.066	3.081	3.096	3.110	3.124	3.137	3.150
40	2.989	3.007	3.023	3.039	3.055	3.069	3.083	3.096	3.109	3.122
45	2.970	2.987	3.003	3.019	3.034	3.048	3.062	3.075	3.088	3.100
50	2.955	2.972	2.988	3.003	3.018	3.032	3.045	3.058	3.071	3.083
55	2.942	2.959	2.975	2.990	3.005	3.018	3.032	3.045	3.057	3.069
60	2.932	2.948	2.964	2.979	2.994	3.007	3.021	3.033	3.045	3.057
70	2.916	2.932	2.948	2.962	2.977	2.990	3.003	3.016	3.028	3.039
80	2.904	2.920	2.935	2.950	2.964	2.977	2.990	3.003	3.014	3.026
90	2.895	2.911	2.926	2.940	2.954	2.967	2.980	2.992	3.004	3.016
100	2.887	2.903	2.918	2.933	2.946	2.960	2.972	2.984	2.996	3.007
200	2.855	2.870	2.885	2.899	2.912	2.925	2.937	2.949	2.960	2.971
300	2.844	2.859	2.874	2.887	2.901	2.913	2.925	2.937	2.948	2.959
400	2.839	2.854	2.868	2.882	2.895	2.908	2.920	2.931	2.942	2.953
500	2.835	2.850	2.865	2.879	2.892	2.904	2.916	2.928	2.939	2.949
600	2.833	2.848	2.863	2.876	2.889	2.902	2.914	2.925	2.936	2.947
700	2.832	2.847	2.861	2.875	2.888	2.900	2.912	2.924	2.935	2.945
800	2.831	2.846	2.860	2.874	2.887	2.899	2.911	2.922	2.933	2.944
900	2.830	2.845	2.859	2.873	2.886	2.898	2.910	2.921	2.932	2.943
1000	2.829	2.844	2.858	2.872	2.885	2.897	2.909	2.921	2.932	2.942
∞	2.823	2.838	2.852	2.865	2.878	2.891	2.902	2.914	2.925	2.935

Continued Table 1a

ALPHA =0.1000 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 =0.0500 (1-SIDED)

N \ R	35	40	45	50	100	250
3	9.042	9.465	9.854	10.21	12.92	17.60
4	6.520	6.758	6.975	7.173	8.610	10.92
5	5.436	5.604	5.755	5.893	6.869	8.363
6	4.848	4.981	5.100	5.208	5.959	7.074
7	4.483	4.595	4.695	4.785	5.408	6.311
8	4.235	4.334	4.421	4.501	5.041	5.811
9	4.056	4.146	4.225	4.297	4.781	5.461
10	3.922	4.005	4.078	4.144	4.587	5.202
11	3.817	3.895	3.963	4.025	4.437	5.004
12	3.733	3.807	3.871	3.930	4.318	4.847
13	3.664	3.735	3.796	3.852	4.221	4.721
14	3.607	3.675	3.734	3.787	4.140	4.616
15	3.559	3.624	3.681	3.733	4.073	4.528
16	3.517	3.581	3.636	3.686	4.015	4.454
17	3.481	3.543	3.597	3.646	3.965	4.390
18	3.450	3.510	3.563	3.610	3.922	4.334
19	3.422	3.481	3.533	3.579	3.883	4.285
20	3.398	3.455	3.506	3.552	3.850	4.241
21	3.376	3.432	3.482	3.527	3.819	4.203
22	3.356	3.412	3.461	3.505	3.792	4.168
23	3.338	3.393	3.442	3.485	3.768	4.137
24	3.322	3.376	3.424	3.467	3.745	4.109
25	3.307	3.361	3.408	3.450	3.725	4.083
26	3.293	3.346	3.393	3.435	3.707	4.060
27	3.281	3.333	3.380	3.421	3.690	4.038
28	3.269	3.321	3.367	3.408	3.674	4.018
29	3.258	3.310	3.356	3.396	3.659	4.000
30	3.248	3.300	3.345	3.385	3.646	3.983
35	3.208	3.258	3.301	3.340	3.591	3.914
40	3.178	3.227	3.269	3.307	3.551	3.864
45	3.155	3.203	3.244	3.281	3.520	3.825
50	3.137	3.184	3.225	3.261	3.496	3.795
55	3.123	3.169	3.209	3.245	3.476	3.770
60	3.111	3.156	3.196	3.232	3.460	3.750
70	3.092	3.137	3.176	3.211	3.435	3.719
80	3.078	3.122	3.161	3.195	3.416	3.696
90	3.067	3.111	3.149	3.183	3.402	3.678
100	3.058	3.102	3.140	3.174	3.390	3.664
200	3.020	3.062	3.099	3.131	3.340	3.601
300	3.007	3.049	3.085	3.118	3.323	3.580
400	3.001	3.043	3.079	3.111	3.315	3.570
500	2.998	3.039	3.075	3.107	3.310	3.564
600	2.995	3.036	3.072	3.104	3.307	3.560
700	2.993	3.034	3.070	3.102	3.304	3.557
800	2.992	3.033	3.069	3.100	3.303	3.555
900	2.991	3.032	3.068	3.099	3.301	3.553
1000	2.990	3.031	3.067	3.098	3.300	3.552
∞	2.983	3.023	3.059	3.090	3.291	3.540

Table 1b

ALPHA = 0.0500 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0250 (1-SIDED)

N \ R	1	2	3	4	5	6	7	8	9	10
3	3.182	4.177	4.857	5.392	5.841	6.232	6.580	6.895	7.185	7.453
4	2.776	3.495	3.961	4.315	4.604	4.851	5.068	5.261	5.437	5.598
5	2.571	3.163	3.534	3.810	4.032	4.219	4.382	4.526	4.655	4.773
6	2.447	2.969	3.287	3.521	3.707	3.863	3.997	4.115	4.221	4.317
7	2.365	2.841	3.128	3.335	3.499	3.636	3.753	3.855	3.947	4.029
8	2.306	2.752	3.016	3.206	3.355	3.479	3.584	3.677	3.759	3.833
9	2.262	2.685	2.933	3.111	3.250	3.364	3.462	3.547	3.622	3.690
10	2.228	2.634	2.870	3.038	3.169	3.277	3.368	3.448	3.518	3.581
11	2.201	2.593	2.820	2.981	3.106	3.208	3.295	3.370	3.437	3.497
12	2.179	2.560	2.779	2.934	3.055	3.153	3.236	3.308	3.371	3.428
13	2.160	2.533	2.746	2.896	3.012	3.107	3.187	3.256	3.318	3.372
14	2.145	2.510	2.718	2.864	2.977	3.069	3.146	3.214	3.273	3.326
15	2.131	2.490	2.694	2.837	2.947	3.036	3.112	3.177	3.235	3.286
16	2.120	2.473	2.673	2.813	2.921	3.008	3.082	3.146	3.202	3.252
17	2.110	2.458	2.655	2.793	2.898	2.984	3.056	3.119	3.173	3.222
18	2.101	2.445	2.639	2.775	2.878	2.963	3.034	3.095	3.149	3.197
19	2.093	2.433	2.625	2.759	2.861	2.944	3.014	3.074	3.127	3.174
20	2.086	2.423	2.613	2.744	2.845	2.927	2.996	3.055	3.107	3.153
21	2.080	2.414	2.601	2.732	2.831	2.912	2.980	3.038	3.090	3.135
22	2.074	2.405	2.591	2.720	2.819	2.899	2.965	3.023	3.074	3.119
23	2.069	2.398	2.582	2.710	2.807	2.886	2.952	3.009	3.059	3.104
24	2.064	2.391	2.574	2.700	2.797	2.875	2.941	2.997	3.046	3.091
25	2.060	2.385	2.566	2.692	2.787	2.865	2.930	2.986	3.035	3.078
26	2.056	2.379	2.559	2.684	2.779	2.856	2.920	2.975	3.024	3.067
27	2.052	2.373	2.552	2.676	2.771	2.847	2.911	2.966	3.014	3.057
28	2.048	2.368	2.546	2.669	2.763	2.839	2.902	2.957	3.004	3.047
29	2.045	2.364	2.541	2.663	2.756	2.832	2.894	2.949	2.996	3.038
30	2.042	2.360	2.536	2.657	2.750	2.825	2.887	2.941	2.988	3.030
35	2.030	2.342	2.515	2.633	2.724	2.797	2.857	2.910	2.955	2.996
40	2.021	2.329	2.499	2.616	2.704	2.776	2.836	2.887	2.931	2.971
45	2.014	2.319	2.487	2.602	2.690	2.760	2.819	2.869	2.913	2.952
50	2.009	2.311	2.477	2.591	2.678	2.747	2.805	2.855	2.898	2.937
55	2.004	2.304	2.469	2.583	2.668	2.737	2.794	2.844	2.887	2.925
60	2.000	2.299	2.463	2.575	2.660	2.729	2.785	2.834	2.877	2.915
70	1.994	2.291	2.453	2.564	2.648	2.715	2.771	2.820	2.862	2.899
80	1.990	2.284	2.445	2.555	2.639	2.705	2.761	2.809	2.850	2.887
90	1.987	2.280	2.440	2.549	2.632	2.698	2.753	2.800	2.841	2.878
100	1.984	2.276	2.435	2.544	2.626	2.692	2.747	2.793	2.834	2.871
200	1.972	2.258	2.414	2.520	2.601	2.665	2.718	2.764	2.803	2.839
300	1.968	2.253	2.407	2.513	2.592	2.656	2.709	2.754	2.793	2.828
400	1.966	2.250	2.404	2.509	2.588	2.651	2.704	2.749	2.788	2.823
500	1.965	2.248	2.402	2.507	2.586	2.649	2.701	2.746	2.785	2.820
600	1.964	2.247	2.401	2.505	2.584	2.647	2.699	2.744	2.783	2.817
700	1.963	2.246	2.400	2.504	2.583	2.646	2.698	2.743	2.782	2.816
800	1.963	2.246	2.399	2.503	2.582	2.645	2.697	2.742	2.780	2.815
900	1.963	2.245	2.398	2.503	2.581	2.644	2.696	2.741	2.780	2.814
1000	1.962	2.245	2.398	2.502	2.581	2.644	2.696	2.740	2.779	2.813
∞	1.960	2.241	2.394	2.498	2.576	2.638	2.690	2.734	2.773	2.807

Continued Table 1b

ALPHA =0.0500 (2-SIDED) T (ALPHA/R, N)
ALPHA/2=0.0250 (1-SIDED)

N \ R	11	12	13	14	15	16	17	18	19	20
3	7.704	7.940	8.162	8.374	8.575	8.768	8.952	9.129	9.300	9.465
4	5.747	5.885	6.015	6.138	6.254	6.364	6.469	6.570	6.666	6.758
5	4.882	4.983	5.076	5.164	5.247	5.326	5.400	5.471	5.539	5.604
6	4.405	4.486	4.561	4.632	4.698	4.760	4.820	4.876	4.929	4.981
7	4.105	4.174	4.239	4.299	4.355	4.408	4.459	4.506	4.551	4.595
8	3.900	3.962	4.019	4.072	4.122	4.169	4.214	4.256	4.296	4.334
9	3.751	3.808	3.860	3.909	3.954	3.997	4.037	4.075	4.111	4.146
10	3.639	3.691	3.740	3.785	3.827	3.867	3.904	3.939	3.973	4.005
11	3.551	3.600	3.646	3.689	3.728	3.765	3.800	3.833	3.865	3.895
12	3.480	3.527	3.571	3.611	3.649	3.684	3.717	3.749	3.778	3.807
13	3.422	3.467	3.509	3.548	3.584	3.618	3.649	3.679	3.708	3.735
14	3.374	3.417	3.458	3.495	3.530	3.562	3.593	3.621	3.649	3.675
15	3.333	3.375	3.414	3.450	3.484	3.515	3.545	3.573	3.599	3.624
16	3.297	3.339	3.377	3.412	3.444	3.475	3.504	3.531	3.556	3.581
17	3.267	3.307	3.344	3.378	3.410	3.440	3.468	3.494	3.519	3.543
18	3.240	3.279	3.316	3.349	3.380	3.410	3.437	3.463	3.487	3.510
19	3.216	3.255	3.291	3.323	3.354	3.383	3.409	3.435	3.459	3.481
20	3.195	3.233	3.268	3.301	3.331	3.359	3.385	4.410	3.433	3.455
21	3.176	3.214	3.248	3.280	3.310	3.337	3.363	3.388	3.411	3.432
22	3.159	3.196	3.230	3.262	3.291	3.318	3.344	3.368	3.390	3.412
23	3.144	3.181	3.214	3.245	3.274	3.301	3.326	3.350	3.372	3.393
24	3.130	3.166	3.199	3.230	3.258	3.285	3.310	3.333	3.355	3.376
25	3.117	3.153	3.186	3.216	3.244	3.270	3.295	3.318	3.340	3.361
26	3.106	3.141	3.174	3.204	3.231	3.257	3.282	3.304	3.326	3.346
27	3.095	3.130	3.162	3.192	3.219	3.245	3.269	3.292	3.313	3.333
28	3.085	3.120	3.152	3.181	3.208	3.234	3.258	3.280	3.301	3.321
29	3.076	3.110	3.142	3.171	3.198	3.223	3.247	3.269	3.290	3.310
30	3.067	3.102	3.133	3.162	3.189	3.214	3.237	3.259	3.280	3.300
35	3.033	3.066	3.096	3.124	3.150	3.174	3.197	3.218	3.239	3.258
40	3.007	3.039	3.069	3.096	3.122	3.145	3.168	3.188	3.208	3.227
45	2.987	3.019	3.048	3.075	3.100	3.123	3.145	3.165	3.185	3.203
50	2.972	3.003	3.032	3.058	3.083	3.106	3.127	3.147	3.166	3.184
55	2.959	2.990	3.018	3.045	3.069	3.091	3.113	3.132	3.151	3.169
60	2.948	2.979	3.007	3.033	3.057	3.080	3.101	3.120	3.139	3.156
70	2.932	2.962	2.990	3.016	3.039	3.061	3.082	3.101	3.119	3.137
80	2.920	2.950	2.977	3.003	3.026	3.048	3.068	3.087	3.105	3.122
90	2.911	2.940	2.967	2.992	3.016	3.037	3.057	3.076	3.094	3.111
100	2.903	2.933	2.960	2.984	3.007	3.029	3.049	3.067	3.085	3.102
200	2.870	2.899	2.925	2.949	2.971	2.992	3.011	3.029	3.046	3.062
300	2.859	2.887	2.913	2.937	2.959	2.979	2.998	3.016	3.033	3.049
400	2.854	2.882	2.908	2.931	2.953	2.973	2.992	3.010	3.027	3.043
500	2.850	2.879	2.904	2.928	2.949	2.970	2.989	3.006	3.023	3.039
600	2.848	2.876	2.902	2.925	2.947	2.967	2.986	3.004	3.020	3.036
700	2.847	2.875	2.900	2.924	2.945	2.965	2.984	3.002	3.019	3.034
800	2.846	2.874	2.899	2.922	2.944	2.964	2.983	3.001	3.017	3.033
900	2.845	2.873	2.898	2.921	2.943	2.963	2.982	3.000	3.016	3.032
1000	2.844	2.872	2.897	2.921	2.942	2.962	2.981	2.999	3.015	3.031
∞	2.838	2.865	2.891	2.914	2.935	2.955	2.974	2.991	3.008	3.023

Continued Table 1b

ALPHA = 0.0500 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0250 (1-SIDED)

N \ R	21	22	23	24	25	26	27	28	29	30
3	9.624	9.778	9.928	10.07	10.21	10.35	10.49	10.62	10.74	10.87
4	6.847	6.933	7.016	7.096	7.173	7.248	7.321	7.392	7.461	7.529
5	5.666	5.726	5.784	5.840	5.893	5.945	5.996	6.045	6.092	6.138
6	5.030	5.077	5.122	5.166	5.208	5.248	5.288	5.326	5.362	5.398
7	4.636	4.676	4.714	4.750	4.785	4.819	4.852	4.884	4.915	4.944
8	4.370	4.405	4.438	4.470	4.501	4.531	4.559	4.587	4.614	4.640
9	4.179	4.210	4.240	4.269	4.297	4.324	4.349	4.374	4.398	4.422
10	4.035	4.064	4.092	4.118	4.144	4.168	4.192	4.215	4.237	4.259
11	3.923	3.950	3.976	4.001	4.025	4.048	4.070	4.091	4.112	4.132
12	3.833	3.859	3.884	3.907	3.930	3.951	3.972	3.992	4.012	4.031
13	3.760	3.785	3.808	3.830	3.852	3.873	3.893	3.912	3.930	3.948
14	3.699	3.723	3.745	3.767	3.787	3.807	3.826	3.845	3.863	3.880
15	3.648	3.670	3.692	3.713	3.733	3.752	3.770	3.788	3.805	3.822
16	3.604	3.626	3.647	3.667	3.686	3.705	3.723	3.740	3.756	3.773
17	3.565	3.587	3.607	3.627	3.646	3.664	3.681	3.698	3.714	3.730
18	3.532	3.553	3.573	3.592	3.610	3.628	3.645	3.661	3.677	3.692
19	3.503	3.523	3.543	3.561	3.579	3.597	3.613	3.629	3.645	3.660
20	3.477	3.497	3.516	3.534	3.552	3.569	3.585	3.601	3.616	3.630
21	3.453	3.473	3.492	3.510	3.527	3.544	3.560	3.575	3.590	3.604
22	3.432	3.452	3.470	3.488	3.505	3.521	3.537	3.552	3.567	3.581
23	3.413	3.432	3.451	3.468	3.485	3.501	3.517	3.531	3.546	3.560
24	3.396	3.415	3.433	3.450	3.467	3.483	3.498	3.513	3.527	3.540
25	3.380	3.399	3.417	3.434	3.450	3.466	3.481	3.495	3.509	3.523
26	3.366	3.384	3.402	3.419	3.435	3.450	3.465	3.480	3.494	3.507
27	3.353	3.371	3.388	3.405	3.421	3.436	3.451	3.465	3.479	3.492
28	3.340	3.359	3.376	3.392	3.408	3.423	3.438	3.452	3.466	3.479
29	3.329	3.347	3.364	3.381	3.396	3.411	3.426	3.440	3.453	3.466
30	3.319	3.336	3.353	3.370	3.385	3.400	3.414	3.428	3.442	3.454
35	3.276	3.293	3.309	3.325	3.340	3.354	3.368	3.382	3.394	3.407
40	3.244	3.261	3.277	3.292	3.307	3.321	3.334	3.347	3.360	3.372
45	3.220	3.237	3.252	3.267	3.281	3.295	3.308	3.321	3.333	3.345
50	3.201	3.217	3.233	3.247	3.261	3.275	3.288	3.300	3.312	3.324
55	3.186	3.202	3.217	3.231	3.245	3.258	3.271	3.284	3.295	3.307
60	3.173	3.189	3.204	3.218	3.232	3.245	3.258	3.270	3.281	3.293
70	3.153	3.168	3.183	3.197	3.211	3.224	3.236	3.248	3.260	3.271
80	3.138	3.153	3.168	3.182	3.195	3.208	3.220	3.232	3.243	3.254
90	3.127	3.142	3.156	3.170	3.183	3.196	3.208	3.220	3.231	3.242
100	3.118	3.133	3.147	3.161	3.174	3.186	3.198	3.210	3.221	3.232
200	3.077	3.092	3.106	3.119	3.131	3.144	3.155	3.166	3.177	3.187
300	3.064	3.079	3.092	3.105	3.118	3.130	3.141	3.152	3.163	3.173
400	3.058	3.072	3.085	3.098	3.111	3.123	3.134	3.145	3.155	3.166
500	3.054	3.068	3.081	3.094	3.107	3.118	3.130	3.141	3.151	3.161
600	3.051	3.065	3.079	3.092	3.104	3.116	3.127	3.138	3.148	3.158
700	3.049	3.063	3.077	3.090	3.102	3.114	3.125	3.136	3.146	3.156
800	3.048	3.062	3.075	3.088	3.100	3.112	3.123	3.134	3.145	3.155
900	3.047	3.061	3.074	3.087	3.099	3.111	3.122	3.133	3.134	3.154
1000	3.046	3.060	3.073	3.086	3.098	3.110	3.121	3.132	3.143	3.153
∞	3.038	3.052	3.065	3.078	3.090	3.102	3.113	3.124	3.134	3.144

Continued Table 1b

ALPHA = 0.0500 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0250 (1-SIDED)

N \ R	35	40	45	50	100	250
3	11.45	11.98	12.47	12.92	16.33	22.20
4	7.841	8.122	8.376	8.610	10.31	13.03
5	6.352	6.541	6.713	6.869	7.976	9.678
6	5.563	5.709	5.840	5.959	6.788	8.025
7	5.082	5.202	5.310	5.408	6.082	7.063
8	4.759	4.864	4.957	5.041	5.617	6.442
9	4.529	4.622	4.706	4.781	5.291	6.010
10	4.357	4.442	4.518	4.587	5.049	5.694
11	4.223	4.303	4.373	4.437	4.863	5.453
12	4.117	4.192	4.258	4.318	4.716	5.263
13	4.030	4.101	4.164	4.221	4.597	5.111
14	3.958	4.026	4.086	4.140	4.499	4.985
15	3.897	3.963	4.021	4.073	4.417	4.880
16	3.846	3.909	3.965	4.015	4.346	4.791
17	3.801	3.862	3.917	3.965	4.286	4.714
18	3.762	3.822	3.874	3.922	4.233	4.648
19	3.727	3.786	3.837	3.883	4.187	4.590
20	3.697	3.754	3.804	3.850	4.146	4.539
21	3.669	3.726	3.775	3.819	4.110	4.493
22	3.645	3.700	3.749	3.792	4.077	4.452
23	3.623	3.677	3.725	3.768	4.047	4.415
24	3.603	3.656	3.703	3.745	4.021	4.382
25	3.584	3.637	3.684	3.725	3.996	4.352
26	3.567	3.620	3.666	3.707	3.974	4.324
27	3.552	3.604	3.649	3.690	3.954	4.299
28	3.538	3.589	3.634	3.674	3.935	4.275
29	3.525	3.575	3.620	3.659	3.918	4.254
30	3.513	3.563	3.607	3.646	3.902	4.234
35	3.463	3.511	3.553	3.591	3.836	4.153
40	3.426	3.473	3.514	3.551	3.788	4.094
45	3.398	3.444	3.484	3.520	3.752	4.049
50	3.376	3.421	3.461	3.496	3.723	4.014
55	3.358	3.403	3.442	3.476	3.700	3.986
60	3.344	3.388	3.426	3.460	3.681	3.962
70	3.321	3.364	3.402	3.435	3.651	3.926
80	3.304	3.346	3.383	3.416	3.629	3.899
90	3.291	3.333	3.369	3.402	3.612	3.878
100	3.280	3.322	3.358	3.390	3.598	3.862
200	3.234	3.274	3.309	3.340	3.539	3.789
300	3.219	3.258	3.293	3.323	3.519	3.765
400	3.211	3.250	3.285	3.315	3.510	3.754
500	3.207	3.246	3.280	3.310	3.504	3.747
600	3.204	3.243	3.277	3.307	3.500	3.742
700	3.202	3.240	3.274	3.304	3.497	3.739
800	3.200	3.239	3.273	3.303	3.495	3.736
900	3.199	3.237	3.271	3.301	3.493	3.734
1000	3.198	3.236	3.270	3.300	3.492	3.733
∞	3.189	3.227	3.261	3.291	3.481	3.719

Table 2a

ALPHA = 0.0200 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0100 (1-SIDED)

N \ R	1	2	3	4	5	6	7	8	9	10
3	4.541	5.841	6.741	7.453	8.053	8.575	9.042	9.465	9.854	10.21
4	3.747	4.604	5.167	5.598	5.951	6.254	6.520	6.758	6.975	7.173
5	3.365	4.032	4.456	4.773	5.030	5.247	5.436	5.604	5.755	5.893
6	3.143	3.707	4.058	4.317	4.524	4.698	4.848	4.981	5.100	5.208
7	2.998	3.499	3.806	4.029	4.207	4.355	4.483	4.595	4.695	4.785
8	2.896	3.355	3.632	3.833	3.991	4.122	4.235	4.334	4.421	4.501
9	2.821	3.250	3.505	3.690	3.835	3.954	4.056	4.146	4.225	4.297
10	2.764	3.169	3.409	3.581	3.716	3.827	3.922	4.005	4.078	4.144
11	2.718	3.106	3.334	3.497	3.624	3.728	3.817	3.895	3.963	4.025
12	2.681	3.055	3.273	3.428	3.550	3.649	3.733	3.807	3.871	3.930
13	2.650	3.012	3.223	3.372	3.489	3.584	3.664	3.735	3.796	3.852
14	2.624	2.977	3.181	3.326	3.438	3.530	3.607	3.675	3.734	3.787
15	2.602	2.947	3.146	3.286	3.395	3.484	3.559	3.624	3.681	3.733
16	2.583	2.921	3.115	3.252	3.358	3.444	3.517	3.581	3.636	3.686
17	2.567	2.898	3.089	3.222	3.326	3.410	3.481	3.543	3.597	3.646
18	2.552	2.878	3.065	3.197	3.298	3.380	3.450	3.510	3.563	3.610
19	2.539	2.861	3.045	3.174	3.273	3.354	3.422	3.481	3.533	3.579
20	2.528	2.845	3.026	3.153	3.251	3.331	3.398	3.455	3.506	3.552
21	2.518	2.831	3.010	3.135	3.231	3.310	3.376	3.432	3.482	3.527
22	2.508	2.819	2.995	3.119	3.214	3.291	3.356	3.412	3.461	3.505
23	2.500	2.807	2.982	3.104	3.198	3.274	3.338	3.393	3.442	3.485
24	2.492	2.797	2.970	3.091	3.183	3.258	3.322	3.376	3.424	3.467
25	2.485	2.787	2.959	3.078	3.170	3.244	3.307	3.361	3.408	3.450
26	2.479	2.779	2.949	3.067	3.158	3.231	3.293	3.346	3.393	3.435
27	2.473	2.771	2.939	3.057	3.147	3.219	3.281	3.333	3.380	3.421
28	2.467	2.763	2.930	3.047	3.136	3.208	3.269	3.321	3.367	3.408
29	2.462	2.756	2.922	3.038	3.127	3.198	3.258	3.310	3.356	3.396
30	2.457	2.750	2.915	3.030	3.118	3.189	3.248	3.300	3.345	3.385
35	2.438	2.724	2.885	2.996	3.081	3.150	3.208	3.258	3.301	3.340
40	2.423	2.704	2.862	2.971	3.055	3.122	3.178	3.227	3.269	3.307
45	2.412	2.690	2.845	2.952	3.034	3.100	3.155	3.203	3.244	3.281
50	2.403	2.678	2.831	2.937	3.018	3.083	3.137	3.184	3.225	3.261
55	2.396	2.668	2.820	2.925	3.005	3.069	3.123	3.169	3.209	3.245
60	2.390	2.660	2.811	2.915	2.994	3.057	3.111	3.156	3.196	3.232
70	2.381	2.648	2.796	2.899	2.977	3.039	3.092	3.137	3.176	3.211
80	2.374	2.639	2.876	2.887	2.964	3.026	3.078	3.122	3.161	3.195
90	2.368	2.632	2.777	2.878	2.954	3.016	3.067	3.111	3.149	3.183
100	2.364	2.626	2.771	2.871	2.946	3.007	3.058	3.102	3.140	3.174
200	2.345	2.601	2.742	2.839	2.912	2.971	3.020	3.062	3.099	3.131
300	2.339	2.592	2.732	2.828	2.901	2.959	3.007	3.049	3.085	3.118
400	2.336	2.588	2.727	2.823	2.985	2.953	3.001	3.043	3.079	3.111
500	2.334	2.586	2.724	2.820	2.892	2.949	2.998	3.039	3.075	3.107
600	2.333	2.584	2.723	2.817	2.889	2.947	2.995	3.036	3.072	3.104
700	2.332	2.583	2.721	2.816	2.888	2.945	2.993	3.034	3.070	3.102
800	2.331	2.582	2.720	2.815	2.887	2.944	2.992	3.033	3.069	3.100
900	2.330	2.581	2.719	2.814	2.886	2.943	2.991	3.032	3.068	3.099
1000	2.330	2.581	2.719	2.813	2.885	2.942	2.990	3.031	3.067	3.098
∞	2.326	2.576	2.713	2.807	2.878	2.935	2.983	3.023	3.059	3.090

Continued Table 2a

ALPHA = 0.0200 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0100 (1-SIDED)

N \ R	11	12	13	14	15	16	17	18	19	20
3	10.55	10.87	11.17	11.45	11.72	11.98	12.23	12.47	12.70	12.92
4	7.357	7.529	7.690	7.841	7.985	8.122	8.252	8.376	8.496	8.610
5	6.020	6.138	6.248	6.352	6.449	6.541	6.629	6.713	6.792	6.869
6	5.307	5.398	5.483	5.563	5.638	5.709	5.776	5.840	5.901	5.959
7	4.868	4.944	5.015	5.082	5.144	5.202	5.258	5.310	5.360	5.408
8	4.573	4.640	4.702	4.759	4.813	4.864	4.912	4.957	5.000	5.041
9	4.362	4.422	4.477	4.529	4.577	4.622	4.665	4.706	4.744	4.781
10	4.204	4.259	4.309	4.357	4.401	4.442	4.481	4.518	4.553	4.587
11	4.081	4.132	4.179	4.223	4.264	4.303	4.339	4.373	4.406	4.437
12	3.982	4.031	4.075	4.117	4.156	4.192	4.226	4.258	4.289	4.318
13	3.902	3.948	3.991	4.030	4.067	4.101	4.134	4.164	4.193	4.221
14	3.836	3.880	3.920	3.958	3.993	4.026	4.057	4.086	4.114	4.140
15	3.779	3.822	3.861	3.897	3.931	3.963	3.993	4.021	4.047	4.073
16	3.731	3.773	3.810	3.846	3.878	3.909	3.938	3.965	3.991	4.015
17	3.690	3.730	3.767	3.801	3.833	3.862	3.890	3.917	3.941	3.965
18	3.653	3.692	3.728	3.762	3.793	3.822	3.849	3.874	3.899	3.922
19	3.621	3.660	3.695	3.727	3.757	3.786	3.812	3.837	3.861	3.883
20	3.593	3.630	3.665	3.697	3.726	3.754	3.780	3.804	3.828	3.850
21	3.567	3.604	3.638	3.669	3.698	3.726	3.751	3.775	3.798	3.819
22	3.545	3.581	3.614	3.645	3.673	3.700	3.725	3.749	3.771	3.792
23	3.524	3.560	3.592	3.623	3.651	3.677	3.702	3.725	3.747	3.768
24	3.505	3.540	2.573	3.603	3.630	3.656	3.680	3.703	3.725	3.745
25	3.488	3.523	3.555	3.584	3.612	3.637	3.661	3.684	3.705	3.725
26	3.473	3.507	3.538	3.567	3.594	3.620	3.643	3.666	3.687	3.707
27	3.458	3.492	3.523	3.552	3.579	3.604	3.627	3.649	3.670	3.690
28	3.445	3.479	3.509	3.538	3.564	3.589	3.612	3.634	3.654	3.674
29	3.433	3.466	3.497	3.525	3.551	3.575	3.598	3.620	3.640	3.659
30	3.421	3.454	3.485	3.513	3.538	3.563	3.585	3.607	3.627	3.646
35	3.375	3.407	3.436	3.463	3.488	3,511	3.533	3.553	3.573	3.591
40	3.341	3.372	3.400	3.426	3.451	3.473	3.494	3.514	3.533	3.551
45	3.315	3.345	3.373	3.398	3.422	3.444	3.465	3.484	3.503	3.520
50	3.294	3.324	3.351	3.376	3.400	3.421	3.442	3.461	3.479	3.496
55	3.277	3.307	3.334	3.358	3.381	3.403	3.423	3.442	3.460	3.476
60	3.264	3.293	3.319	3.344	3.366	3.388	3.407	3.426	3.444	3.460
70	3.242	3.271	3.297	3.321	3.343	3.364	3.383	3.402	3.419	3.435
80	3.226	3.254	3.280	3.304	3.326	3.346	3.365	3.383	3.400	3.416
90	3.214	3.242	3.267	3.291	3.312	3.333	3.352	3.369	3.386	3.402
100	3.204	3.232	3.257	3.280	3.302	3.322	3.341	3.358	3.375	3.390
200	3.161	3.187	3.212	3.234	3.255	3.274	3.292	3.309	3.325	3.340
300	3.147	3.173	3.197	3.219	3.239	3.258	3.276	3.293	3.308	3.323
400	3.139	3.166	3.189	3.211	3.232	3.250	3.268	3.285	3.300	3.315
500	3.135	3.161	3.185	3.207	3.227	3.246	3.263	3.280	3.295	3.310
600	3.132	3.158	3.182	3.204	3.224	3.243	3.260	3.277	3.292	3.307
700	3.130	3.156	3.180	3.202	3.222	3.240	3.258	3.274	3.290	3.304
800	3.129	3.155	3.178	3.200	3.220	3.239	3.256	3.273	3.288	3.303
900	3.128	3.154	3.177	3.199	3.219	3.237	3.255	3.271	3.287	3.301
1000	3.127	3.153	3.176	3.198	3.218	3.236	3.254	3.270	3.286	3.300
∞	3.118	3.144	3.167	3.189	3.209	3.227	3.245	3.261	3.276	3.291

ontinued Table 2a

LPHA $=0.0200$ (2-SIDED) T (ALPHA/R, N)
LPHA/2 $=0.0100$ (1-SIDED)

N \ R	21	22	23	24	25	26	27	28	29	30
3	13.14	13.35	13.55	13.75	13.94	14.12	14.30	14.48	14.65	14.82
4	8.721	8.827	8.930	9.029	9.126	9.219	9.310	9.398	9.484	9.568
5	6.942	7.013	7.081	7.146	7.210	7.271	7.331	7.388	7.444	7.499
6	6.015	6.068	6.119	6.169	6.217	6.263	6.308	6.351	6.393	6.434
7	5.454	5.497	5.539	5.580	5.619	5.657	5.693	5.728	5.762	5.795
8	5.081	5.118	5.154	5.189	5.223	5.255	5.286	5.316	5.345	5.374
9	4.816	4.849	4.882	4.912	4.942	4.971	4.998	5.025	5.051	5.076
10	4.619	4.649	4.678	4.706	4.733	4.759	4.784	4.809	4.832	4.855
11	4.466	4.495	4.522	4.548	4.573	4.597	4.620	4.642	4.664	4.685
12	4.345	4.372	4.397	4.422	4.445	4.467	4.489	4.510	4.530	4.550
13	4.247	4.272	4.296	4.319	4.341	4.362	4.383	4.403	4.422	4.440
14	4.165	4.189	4.212	4.234	4.255	4.275	4.295	4.314	4.332	4.349
15	4.097	4.120	4.142	4.163	4.183	4.202	4.221	4.239	4.256	4.273
16	4.038	4.060	4.082	4.102	4.121	4.140	4.158	4.175	4.192	4.208
17	3.988	4.009	4.030	4.049	4.068	4.086	4.104	4.121	4.137	4.152
18	3.944	3.964	3.984	4.003	4.022	4.039	4.056	4.073	4.088	4.104
19	3.905	3.925	3.945	3.963	3.981	3.998	4.015	4.031	4.046	4.061
20	3.870	3.890	3.909	3.928	3.945	3.962	3.978	3.994	4.009	4.023
21	3.840	3.859	3.878	3.896	3.913	3.929	3.945	3.960	3.975	3.989
22	3.812	3.831	3.850	3.867	3.884	3.900	3.916	3.931	3.945	3.959
23	3.787	3.806	3.824	3.841	3.858	3.874	3.889	3.904	3.918	3.932
24	3.765	3.783	3.801	3.818	3.834	3.850	3.865	3.879	3.893	3.907
25	3.744	3.763	3.780	3.797	3.813	3.828	3.843	3.857	3.871	3.884
26	3.726	3.744	3.761	3.777	3.793	3.808	3.823	3.837	3.850	3.864
27	3.708	3.726	3.743	3.759	3.775	3.790	3.804	3.818	3.832	3.845
28	3.692	3.710	3.727	3.743	3.758	3.773	3.787	3.801	3.814	3.827
29	3.678	3.695	3.712	3.728	3.743	3.758	3.772	3.785	3.798	3.811
30	3.664	3.681	3.698	3.714	3.729	3.743	3.757	3.771	3.784	3.796
35	3.609	3.625	3.641	3.656	3.671	3.685	3.698	3.711	3.723	3.735
40	3.568	3.584	3.599	3.614	3.628	3.642	3.655	3.667	3.679	3.691
45	3.537	3.552	3.567	3.582	3.596	3.609	3.621	3.634	3.645	3.657
50	3.512	3.528	3.542	3.556	3.570	3.583	3.595	3.607	3.619	3.630
55	3.492	3.508	3.522	3.536	3.549	3.562	3.574	3.586	3.597	3.608
60	3.476	3.491	3.505	3.519	3.532	3.545	3.557	3.568	3.579	3.590
70	3.450	3.465	3.479	3.493	3.505	3.518	3.529	3.541	3.552	3.562
80	3.432	3.446	3.460	3.473	3.486	3.498	3.509	3.520	3.531	3.542
90	3.417	3.431	3.445	3.458	3.470	3.482	3.494	3.505	3.515	3.526
100	3.405	3.420	3.433	3.446	3.458	3.470	3.481	3.492	3.503	3.513
200	3.354	3.368	3.381	3.393	3.405	3.416	3.427	3.437	3.448	3.457
300	3.337	3.351	3.363	3.376	3.387	3.398	3.409	3.419	3.429	3.439
400	3.329	3.342	3.355	3.367	3.379	3.390	3.400	3.411	3.420	3.430
500	3.324	3.337	3.350	3.362	3.373	3.384	3.395	3.405	3.415	3.424
600	3.321	3.334	3.346	3.358	3.370	3.381	3.392	3.402	3.411	3.421
700	3.318	3.332	3.344	3.356	3.368	3.378	3.389	3.399	3.409	3.418
800	3.317	3.330	3.342	3.354	3.366	3.377	3.387	3.397	3.407	3.416
900	3.315	3.328	3.341	3.353	3.364	3.375	3.386	3.396	3.405	3.415
000	3.314	3.327	3.340	3.352	3.363	3.374	3.385	3.395	3.404	3.414
∞	3.304	3.317	3.330	3.341	3.353	3.364	3.374	3.384	3.394	3.403

Continued Table 2a

ALPHA = 0.0200 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0100 (1-SIDED)

N \ R	35	40	45	50	100	250
3	15.61	16.33	16.99	17.60	22.20	30.17
4	9.957	10.31	10.62	10.92	13.03	16.45
5	7.751	7.976	8.178	8.363	9.678	11.70
6	6.622	6.788	6.938	7.074	8.025	9.448
7	5.948	6.082	6.202	6.311	7.063	8.165
8	5.503	5.617	5.719	5.811	6.442	7.349
9	5.190	5.291	5.380	5.461	6.010	6.789
10	4.958	5.049	5.130	5.202	5.694	6.383
11	4.780	4.863	4.937	5.004	5.453	6.077
12	4.639	4.716	4.785	4.847	5.263	5.837
13	4.524	4.597	4.662	4.721	5.111	5.645
14	4.430	4.499	4.561	4.616	4.985	5.487
15	4.350	4.417	4.476	4.528	4.880	5.357
16	4.282	4.346	4.403	4.454	4.791	5.246
17	4.224	4.286	4.341	4.390	4.714	5.151
18	4.173	4.233	4.286	4.334	4.648	5.069
19	4.128	4.187	4.239	4.285	4.590	4.997
20	4.089	4.146	4.196	4.241	4.539	4.934
21	4.054	4.110	4.159	4.203	4.493	4.878
22	4.022	4.077	4.125	4.168	4.452	4.828
23	3.994	4.047	4.095	4.137	4.415	4.783
24	3.968	4.021	4.067	4.109	4.382	4.742
25	3.944	3.996	4.042	4.083	4.352	4.705
26	3.923	3.974	4.019	4.060	4.324	4.672
27	3.903	3.954	3.998	4.038	4.299	4.641
28	3.885	3.935	3.979	4.018	4.275	4.612
29	3.868	3.918	3.961	4.000	4.254	4.586
30	3.853	3.902	3.945	3.983	4.234	4.562
35	3.789	3.836	3.877	3.914	4.153	4.464
40	3.743	3.788	3.828	3.864	4.094	4.393
45	3.708	3.752	3.791	3.825	4.049	4.339
50	3.680	3.723	3.761	3.795	4.014	4.296
55	3.657	3.700	3.737	3.770	3.986	4.262
60	3.639	3.681	3.717	3.750	3.962	4.234
70	3.610	3.651	3.687	3.719	3.926	4.190
80	3.589	3.629	3.664	3.696	3.899	4.158
90	3.572	3.612	3.647	3.678	3.878	4.133
100	3.559	3.598	3.633	3.664	3.862	4.114
200	3.501	3.539	3.572	3.601	3.789	4.028
300	3.482	3.519	3.552	3.580	3.765	4.000
400	3.473	3.510	3.542	3.570	3.754	3.986
500	3.467	3.504	3.536	3.564	3.747	3.977
600	3.463	3.500	3.532	3.560	3.742	3.972
700	3.461	3.497	3.529	3.557	3.739	3.968
800	3.459	3.495	3.527	3.555	3.736	3.965
900	3.457	3.493	3.525	3.553	3.734	3.963
1000	3.456	3.492	3.524	3.552	3.733	3.961
∞	3.445	3.481	3.512	3.540	3.719	3.944

Table 2b

ALPHA =0.0100 (2-SIDED) T (ALPHA/R, N)
ALPHA/2=0.0050 (1-SIDED)

N \ R	1	2	3	4	5	6	7	8	9	10
3	5.841	7.453	8.575	9.465	10.21	10.87	11.45	11.98	12.47	12.92
4	4.604	5.598	6.254	6.758	7.173	7.529	7.841	8.122	8.376	8.610
5	4.032	4.773	5.247	5.604	5.893	6.138	6.352	6.541	6.713	6.869
6	3.707	4.317	4.698	4.981	5.208	5.398	5.563	5.709	5.840	5.959
7	3.499	4.029	4.355	4.595	4.785	4.944	5.082	5.202	5.310	5.408
8	3.355	3.833	4.122	4.334	4.501	4.640	4.759	4.864	4.957	5.041
9	3.250	3.690	3.954	4.146	4.297	4.422	4.529	4.622	4.706	4.781
10	3.169	3.581	3.827	4.005	4.144	4.259	4.357	4.442	4.518	4.587
11	3.106	3.497	3.728	3.895	4.025	4.132	4.223	4.303	4.373	4.437
12	3.055	3.428	3.649	3.807	3.930	4.031	4.117	4.192	4.258	4.318
13	3.012	3.372	3.584	3.735	3.852	3.948	4.030	4.101	4.164	4.221
14	2.977	3.326	3.530	3.675	3.787	3.880	3.958	4.026	4.086	4.140
15	2.947	3.286	3.484	3.624	3.733	3.822	3.897	3.963	4.021	4.073
16	2.921	3.252	3.444	3.581	3.686	3.773	3.846	3.909	3.965	4.015
17	2.898	3.222	3.410	3.543	3.646	3.730	3.801	3.862	3.917	3.965
18	2.878	3.197	3.380	3.510	3.610	3.692	3.762	3.822	3.874	3.922
19	2.861	3.174	3.354	3.481	3.579	3.660	3.727	3.786	3.837	3.883
20	2.845	3.153	3.331	3.455	3.552	3.630	3.697	3.754	3.804	3.850
21	2.831	3.135	3.310	3.432	3.527	3.604	3.669	3.726	3.775	3.819
22	2.819	3.119	3.291	3.412	3.505	3.581	3.645	3.700	3.749	3.792
23	2.807	3.104	3.274	3.393	3.485	3.560	3.623	3.677	3.725	3.768
24	2.797	3.091	3.258	3.376	3.467	3.540	3.603	3.656	3.703	3.745
25	2.787	3.078	3.244	3.361	3.450	3.523	3.584	3.637	3.684	3.725
26	2.779	3.067	3.231	3.346	3.435	3.507	3.567	3.620	3.666	3.707
27	2.771	3.057	3.219	3.333	3.421	3.492	3.552	3.604	3.649	3.690
28	2.763	3.047	3.208	3.321	3.408	3.479	3.538	3.589	3.634	3.674
29	2.756	3.038	3.198	3.310	3.396	3.466	3.525	3.575	3.620	3.659
30	2.750	3.030	3.189	3.300	3.385	3.454	3.513	3.563	3.607	3.646
35	2.724	2.996	3.150	3.258	3.340	3.407	3.463	3.511	3.553	3.591
40	2.704	2.971	3.122	3.227	3.307	3.372	3.426	3.473	3.514	3.551
45	2.690	2.952	3.100	3.203	3.281	3.345	3.398	3.444	3.484	3.520
50	2.678	2.937	3.083	3.184	3.261	3.324	3.376	3.421	3.461	3.496
55	2.668	2.925	3.069	3.169	3.245	3.307	3.358	3.403	3.442	3.476
60	2.660	2.915	3.057	3.156	3.232	3.293	3.344	3.388	3.426	3.460
70	2.648	2.899	3.039	3.137	3.211	3.271	3.321	3.364	3.402	3.435
80	2.639	2.887	3.026	3.122	3.195	3.254	3.304	3.346	3.383	3.416
90	2.632	2.878	3.016	3.111	3.183	3.242	3.291	3.333	3.369	3.402
100	2.626	2.871	3.007	3.102	3.174	3.232	3.280	3.322	3.358	3.390
200	2.601	2.839	2.971	3.062	3.131	3.187	3.234	3.274	3.309	3.340
300	2.592	2.828	2.959	3.049	3.118	3.173	3.219	3.258	3.293	3.323
400	2.588	2.823	2.953	3.043	3.111	3.166	3.211	3.250	3.285	3.315
500	2.586	2.820	2.949	3.039	3.107	3.161	3.207	3.246	3.280	3.310
600	2.584	2.817	2.947	3.036	3.104	3.158	3.204	3.243	3.277	3.307
700	2.583	2.816	2.945	3.034	3.102	3.156	3.202	3.240	3.274	3.304
800	2.582	2.815	2.944	3.033	3.100	3.155	3.200	3.239	3.273	3.303
900	2.581	2.814	2.943	3.032	3.099	3.154	3.199	3.237	3.271	3.301
1000	2.581	2.813	2.942	3.031	3.098	3.153	3.198	3.236	3.270	3.300
∞	2.576	2.807	2.935	3.023	3.090	3.144	3.189	3.227	3.261	3.291

Continued Table 2b

ALPHA =0.0100 (2-SIDED) T (ALPHA/R, N)
ALPHA/2=0.0050 (1-SIDED)

N \ R	11	12	13	14	15	16	17	18	19	20
3	13.35	13.75	14.12	14.48	14.82	15.15	15.46	15.76	16.05	16.33
4	8.827	9.029	9.219	9.398	9.568	9.729	9.883	10.03	10.17	10.31
5	7.013	7.146	7.271	7.388	7.499	7.604	7.703	7.798	7.889	7.976
6	6.068	6.169	6.263	6.351	6.434	6.512	6.586	6.657	6.724	6.788
7	5.497	5.580	5.657	5.728	5.795	5.859	5.919	5.976	6.030	6.082
8	5.118	5.189	5.255	5.316	5.374	5.428	5.479	5.527	5.573	5.617
9	4.849	4.912	4.971	5.025	5.076	5.124	5.169	5.211	5.252	5.291
10	4.649	4.706	4.759	4.809	4.855	4.898	4.939	4.977	5.014	5.049
11	4.495	4.548	4.597	4.642	4.685	4.724	4.762	4.798	4.831	4.863
12	4.372	4.422	4.467	4.510	4.550	4.587	4.622	4.655	4.687	4.716
13	4.272	4.319	4.362	4.403	4.440	4.475	4.508	4.540	4.569	4.597
14	4.189	4.234	4.275	4.314	4.349	4.383	4.414	4.444	4.472	4.499
15	4.120	4.163	4.202	4.239	4.273	4.305	4.336	4.364	4.391	4.417
16	4.060	4.102	4.140	4.175	4.208	4.239	4.268	4.296	4.322	4.346
17	4.009	4.049	4.086	4.121	4.152	4.182	4.210	4.237	4.262	4.286
18	3.964	4.003	4.039	4.073	4.104	4.133	4.160	4.186	4.210	4.233
19	3.925	3.963	3.998	4.031	4.061	4.089	4.116	4.141	4.164	4.187
20	3.890	3.928	3.962	3.994	4.023	4.051	4.077	4.101	4.124	4.146
21	3.859	3.896	3.929	3.960	3.989	4.016	4.042	4.066	4.088	4.110
22	3.831	3.867	3.900	3.931	3.959	3.985	4.010	4.034	4.056	4.077
23	3.806	3.841	3.874	3.904	3.932	3.958	3.982	4.005	4.027	4.047
24	3.783	3.818	3.850	3.879	3.907	3.932	3.956	3.979	4.000	4.021
25	3.763	3.797	3.828	3.857	3.884	3.909	3.933	3.955	3.976	3.996
26	3.744	3.777	3.808	3.837	3.864	3.888	3.912	3.934	3.955	3.975
27	3.726	3.759	3.790	3.818	3.845	3.869	3.892	3.914	3.934	3.954
28	3.710	3.743	3.773	3.801	3.827	3.851	3.874	3.896	3.916	3.935
29	3.695	3.728	3.758	3.785	3.811	3.835	3.858	3.879	3.899	3.918
30	3.681	3.714	3.743	3.771	3.796	3.820	3.842	3.863	3.883	3.902
35	3.625	3.656	3.685	3.711	3.735	3.758	3.779	3.799	3.818	3.836
40	3.584	3.614	3.642	3.667	3.691	3.713	3.733	3.753	3.771	3.788
45	3.552	3.582	3.609	3.634	3.657	3.678	3.698	3.717	3.735	3.752
50	3.528	3.556	3.583	3.607	3.630	3.651	3.670	3.689	3.707	3.723
55	3.508	3.536	3.562	3.586	3.608	3.629	3.648	3.666	3.684	3.700
60	3.491	3.519	3.545	3.568	3.590	3.611	3.630	3.648	3.665	3.681
70	3.465	3.493	3.518	3.541	3.562	3.582	3.601	3.619	3.635	3.651
80	3.446	3.473	3.498	3.520	3.542	3.561	3.580	3.597	3.613	3.629
90	3.431	3.458	3.482	3.505	3.526	3.545	3.563	3.580	3.597	3.612
100	3.420	3.446	3.470	3.492	3.513	3.532	3.550	3.567	3.583	3.598
200	3.368	3.393	3.416	3.437	3.457	3.476	3.493	3.509	3.524	3.539
300	3.351	3.376	3.398	3.419	3.439	3.457	3.474	3.490	3.505	3.519
400	3.342	3.367	3.390	3.411	3.430	3.448	3.465	3.481	3.495	3.510
500	3.337	3.362	3.384	3.405	3.424	3.442	3.459	3.475	3.490	3.504
600	3.334	3.358	3.381	3.402	3.421	3.439	3.455	3.471	3.486	3.500
700	3.332	3.356	3.378	3.399	3.418	3.436	3.453	3.468	3.383	3.497
800	3.330	3.354	3.377	3.397	3.416	3.434	3.451	3.466	3.481	3.495
900	3.328	3.353	3.375	3.396	3.415	3.433	3.449	3.465	3.480	3.493
1000	3.327	3.352	3.374	3.395	3.414	3.431	3.448	3.464	3.478	3.492
∞	3.317	3.341	3.364	3.384	3.403	3.421	3.437	3.452	3.467	3.481

Continued Table 2b

ALPHA = 0.0100 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0050 (1-SIDED)

N \ R	21	22	23	24	25	26	27	28	29	30
3	16.60	16.86	17.11	17.36	17.60	17.83	18.06	18.28	18.50	18.71
4	10.44	10.56	10.68	10.80	10.92	11.03	11.13	11.24	11.34	11.44
5	8.059	8.139	8.217	8.291	8.363	8.433	8.501	8.567	8.630	8.693
6	6.850	6.909	6.966	7.021	7.074	7.125	7.175	7.223	7.269	7.314
7	6.131	6.179	6.224	6.268	6.311	6.352	6.391	6.429	6.467	6.503
8	5.659	5.700	5.738	5.775	5.811	5.846	5.879	5.911	5.943	5.973
9	5.328	5.363	5.397	5.429	5.461	5.491	5.520	5.548	5.576	5.602
10	5.082	5.114	5.145	5.174	5.202	5.229	5.256	5.281	5.305	5.329
11	4.894	4.923	4.951	4.978	5.004	5.029	5.053	5.076	5.099	5.120
12	4.745	4.772	4.798	4.823	4.847	4.871	4.893	4.914	4.935	4.955
13	4.624	4.650	4.675	4.698	4.721	4.742	4.763	4.784	4.803	4.822
14	4.525	4.549	4.572	4.595	4.616	4.637	4.657	4.676	4.694	4.712
15	4.441	4.464	4.487	4.508	4.528	4.548	4.567	4.585	4.603	4.620
16	4.370	4.392	4.414	4.434	4.454	4.473	4.491	4.509	4.526	4.542
17	4.308	4.330	4.351	4.371	4.390	4.408	4.425	4.442	4.459	4.475
18	4.255	4.276	4.296	4.315	4.334	4.351	4.369	4.385	4.401	4.416
19	4.208	4.229	4.248	4.267	4.285	4.302	4.319	4.335	4.350	4.365
20	4.167	4.187	4.206	4.224	4.241	4.258	4.274	4.290	4.305	4.319
21	4.130	4.149	4.168	4.186	4.203	4.219	4.235	4.250	4.265	4.279
22	4.097	4.116	4.134	4.152	4.168	4.184	4.200	4.215	4.229	4.243
23	4.067	4.086	4.104	4.121	4.137	4.153	4.168	4.183	4.197	4.210
24	4.040	4.058	4.076	4.093	4.109	4.124	4.139	4.154	4.167	4.181
25	4.015	4.034	4.051	4.067	4.083	4.098	4.113	4.127	4.141	4.154
26	3.993	4.011	4.028	4.044	4.060	4.075	4.089	4.103	4.116	4.129
27	3.972	3.990	4.007	4.023	4.038	4.053	4.067	4.081	4.094	4.107
28	3.953	3.971	3.987	4.003	4.018	4.033	4.047	4.061	4.074	4.086
29	3.936	3.953	3.969	3.985	4.000	4.014	4.028	4.042	4.055	4.067
30	3.919	3.936	3.953	3.968	3.983	3.997	4.011	4.024	4.037	4.049
35	3.853	3.870	3.885	3.900	3.914	3.928	3.941	3.953	3.966	3.977
40	3,805	3.821	3.836	3.850	3.864	3.877	3.889	3.902	3.913	3.925
45	3.768	3.783	3.798	3.812	3.825	3.838	3.850	3.862	3.874	3.885
50	3.739	3.754	3.768	3.782	3.795	3.807	3.819	3.831	3.842	3.853
55	3.715	3.730	3.744	3.758	3.770	3.783	3.795	3.806	3.817	3.828
60	3.696	3.710	3.724	3.738	3.750	3.762	3.774	3.785	3.796	3.807
70	3.666	3.680	3.694	3.706	3.719	3.731	3.742	3.753	3.764	3.774
80	3.644	3.657	3.671	3.683	3.696	3.707	3.719	3.729	3.740	3.750
90	3.626	3.640	3.653	3.666	3.678	3.689	3.700	3.711	3.721	3.731
100	3.613	3.626	3.639	3.652	3.664	3.675	3.686	3.696	3.707	3.716
200	3.552	3.565	3.578	3.590	3.601	3.612	3.622	3.632	3.642	3.651
300	3.533	3.545	3.558	3.569	3.580	3.591	3.601	3.611	3.621	3.630
400	3.523	3.536	3.548	3.559	3.570	3.581	3.591	3.601	3.610	3.619
500	3.517	3.530	3.542	3.553	3.564	3.575	3.585	3.595	3.604	3.613
600	3.513	3.526	3.538	3.549	3.560	3.571	3.581	3.590	3.600	3.609
700	3.510	3.523	3.535	3.546	3.557	3.568	3.578	3.587	3.597	3.606
800	3.508	3.521	3.533	3.544	3.555	3.566	3.576	3.585	3.595	3.604
900	3.507	3.519	3.531	3.543	3.553	3.564	3.574	3.584	3.593	3.602
000	3.505	3.518	3.530	3.541	3.552	3.563	3.573	3.582	3.591	3.600
∞	3.494	3.506	3.518	3.529	3.540	3.550	3.560	3.570	3.579	3.588

Continued Table 2b

ALPHA = 0.0100 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0050 (1-SIDED)

N \ R	35	40	45	50	100	250
3	19.70	20.60	21.43	22.20	28.00	38.03
4	11.90	12.31	12.69	13.03	15.54	19.60
5	8.980	9.235	9.466	9.678	11.18	13.50
6	7.523	7.708	7.874	8.025	9.082	10.67
7	6.668	6.814	6.945	7.063	7.885	9.089
8	6.112	6.234	6.343	6.442	7.120	8.098
9	5.723	5.830	5.925	6.010	6.594	7.424
10	5.438	5.533	5.618	5.694	6.211	6.939
11	5.220	5.306	5.383	5.453	5.921	6.574
12	5.047	5.128	5.199	5.263	5.694	6.291
13	4.909	4.984	5.051	5.111	5.513	6.065
14	4.794	4.865	4.928	4.985	5.363	5.881
15	4.698	4.766	4.826	4.880	5.239	5.727
16	4.617	4.682	4.739	4.791	5.134	5.598
17	4.547	4.609	4.665	4.714	5.044	5.488
18	4.486	4.547	4.600	4.648	4.966	5.393
19	4.433	4.491	4.543	4.590	4.897	5.310
20	4.386	4.443	4.493	4.539	4.837	5.237
21	4.344	4.399	4.449	4.493	4.784	5.172
22	4.306	4.361	4.409	4.452	4.736	5.114
23	4.272	4.326	4.373	4.415	4.693	5.062
24	4.242	4.294	4.340	4.382	4.654	5.015
25	4.214	4.265	4.311	4.352	4.619	4.973
26	4.188	4.239	4.284	4.324	4.587	4.934
27	4.165	4.215	4.259	4.299	4.558	4.899
28	4.143	4.193	4.236	4.275	4.530	4.866
29	4.124	4.172	4.215	4.254	4.506	4.836
30	4.105	4.154	4.196	4.234	4.482	4.808
35	4.031	4.077	4.117	4.153	4.389	4.696
40	3.976	4.020	4.059	4.094	4.321	4.615
45	3.935	3.978	4.016	4.049	4.269	4.554
50	3.902	3.944	3.981	4.014	4.228	4.505
55	3.876	3.917	3.953	3.986	4.196	4.466
60	3.854	3.895	3.930	3.962	4.169	4.435
70	3.820	3.860	3.895	3.926	4.127	4.385
80	3.795	3.834	3.868	3.899	4.096	4.349
90	3.776	3.814	3.848	3.878	4.072	4.321
100	3.761	3.799	3.832	3.862	4.053	4.298
200	3.693	3.729	3.761	3.789	3.970	4.201
300	3.671	3.707	3.738	3.765	3.944	4.169
400	3.660	3.696	3.726	3.754	3.930	4.154
500	3.654	3.689	3.720	3.747	3.922	4.144
600	3.649	3.684	3.715	3.742	3.917	4.138
700	3.646	3.681	3.712	3.739	3.913	4.134
800	3.644	3.679	3.709	3.736	3.910	4.131
900	3.642	3.677	3.707	3.734	3.908	4.128
1000	3.641	3.675	3.706	3.733	3.906	4.126
∞	3.628	3.662	3.692	3.719	3.891	4.107

Table 3a

ALPHA = 0.0020 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0010 (1-SIDED)

N \ R	1	2	3	4	5	6	7	8	9	10
3	10.21	12.92	14.82	16.33	17.60	18.71	19.70	20.60	21.43	22.20
4	7.173	8.610	9.568	10.31	10.92	11.44	11.90	12.31	12.69	13.03
5	5.893	6.869	7.499	7.976	8.363	8.693	8.980	9.235	9.466	9.678
6	5.208	5.959	6.434	6.788	7.074	7.314	7.523	7.708	7.874	8.025
7	4.785	5.408	5.795	6.082	6.311	6.503	6.668	6.814	6.945	7.063
8	4.501	5.041	5.374	5.617	5.811	5.973	6.112	6.234	6.343	6.442
9	4.297	4.781	5.076	5.291	5.461	5.602	5.723	5.830	5.925	6.010
10	4.144	4.587	4.855	5.049	5.202	5.329	5.438	5.533	5.618	5.694
11	4.025	4.437	4.685	4.863	5.004	5.120	5.220	5.306	5.383	5.453
12	3.930	4.318	4.550	4.716	4.847	4.955	5.047	5.128	5.199	5.263
13	3.852	4.221	4.440	4.597	4.721	4.822	4.909	4.984	5.051	5.111
14	3.787	4.140	4.349	4.499	4.616	4.712	4.794	4.865	4.928	4.985
15	3.733	4.073	4.273	4.417	4.528	4.620	4.698	4.766	4.826	4.880
16	3.686	4.015	4.208	4.346	4.454	4.542	4.617	4.682	4.739	4.791
17	3.646	3.965	4.152	4.286	4.390	4.475	4.547	4.609	4.665	4.714
18	3.610	3.922	4.104	4.233	4.334	4.416	4.486	4.547	4.600	4.648
19	3.579	3.883	4.061	4.187	4.285	4.365	4.433	4.491	4.543	4.590
20	3.552	3.850	4.023	4.146	4.241	4.319	4.386	4.443	4.493	4.539
21	3.527	3.819	3.989	4.110	4.203	4.279	4.344	4.399	4.449	4.493
22	3.505	3.792	3.959	4.077	4.168	4.243	4.306	4.361	4.409	4.452
23	3.485	3.768	3.932	4.047	4.137	4.210	4.272	4.326	4.373	4.415
24	3.467	3.745	3.097	4.021	4.109	4.181	4.242	4.294	4.340	4.382
25	3.450	3.725	3.884	3.996	4.083	4.154	4.214	4.265	4.311	4.352
26	3.435	3.707	3.864	3.974	4.060	4.129	4.188	4.239	4.284	4.324
27	3.421	3.690	3.845	3.954	4.038	4.107	4.165	4.215	4.259	4.299
28	3.408	3.674	3.827	3.935	4.018	4.086	4.143	4.193	4.236	4.275
29	3.396	3.659	3.811	3.918	4.000	4.067	4.124	4.172	4.215	4.254
30	3.385	3.646	3.796	3.902	3.983	4.049	4.105	4.154	4.196	4.234
35	3.340	3.591	3.735	3.836	3.914	3.977	4.031	4.077	4.117	4.153
40	3.307	3.551	3.691	3.788	3.864	3.925	3.976	4.020	4.059	4.094
45	3.281	3.520	3.657	3.752	3.825	3.885	3.935	3.978	4.016	4.049
50	3.261	3.496	3.630	3.723	3.795	3.853	3.902	3.944	3.981	4.014
55	3.245	3.476	3.608	3.700	3.770	3.828	3.876	3.917	3.953	3.986
60	3.232	3.460	3.590	3.681	3.750	3.807	3.854	3.895	3.930	3.962
70	3.211	3.435	3.562	3.651	3.719	3.774	3.820	3.860	3.895	3.926
80	3.195	3.416	3.542	3.629	3.696	3.750	3.795	3.834	3.868	3.899
90	3.183	3.402	3.526	3.612	3.678	3.731	3.776	3.814	3.848	3.878
100	3.174	3.390	3.513	3.598	3.664	3.716	3.761	3.799	3.832	3.862
200	3.131	3.340	3.457	3.539	3.601	3.651	3.693	3.729	3.761	3.789
300	3.118	3.323	3.439	3.519	3.580	3.630	3.671	3.707	3.738	3.765
400	3.111	3.315	3.430	3.510	3.570	3.619	3.660	3.696	3.726	3.754
500	3.107	3.310	3.424	3.504	3.564	3.613	3.654	3.689	3.720	3.747
600	3.104	3.307	3.421	3.500	3.560	3.609	3.649	3.684	3.715	3.742
700	3.102	3.303	3.418	3.497	3.557	3.606	3.646	3.681	3.712	3.739
800	3.100	3.303	3.416	3.495	3.555	3.604	3.644	3.679	3.709	3.736
900	3.099	3.301	3.415	3.493	3.553	3.602	3.642	3.677	3.707	3.734
1000	3.098	3.300	3.414	3.492	3.552	3.600	3.641	3.675	3.706	3.733
∞	3.090	3.291	3.403	3.481	3.540	3.588	3.628	3.662	3.692	3.719

Continued Table 3a

ALPHA =0.0020 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 =0.0010 (1-SIDED)

N \ R	11	12	13	14	15	16	17	18	19	20
3	22.92	23.60	24.24	24.85	25.43	25.99	26.52	27.03	27.52	28.00
4	13.35	13.65	13.93	14.20	14.45	14.69	14.92	15.13	15.34	15.54
5	9.872	10.05	10.22	10.38	10.53	10.67	10.81	10.94	11.06	11.18
6	8.164	8.292	8.412	8.524	8.630	8.729	8.824	8.914	9.000	9.082
7	7.172	7.272	7.366	7.453	7.535	7.612	7.685	7.755	7.821	7.885
8	6.532	6.616	6.693	6.765	6.833	6.896	6.957	7.014	7.068	7.120
9	6.088	6.160	6.227	6.289	6.347	6.402	6.454	6.503	6.549	6.594
10	5.763	5.827	5.886	5.942	5.993	6.042	6.087	6.131	6.172	6.211
11	5.516	5.574	5.628	5.678	5.724	5.768	5.809	5.849	5.886	5.921
12	5.322	5.375	5.425	5.471	5.514	5.554	5.592	5.628	5.662	5.694
13	5.165	5.215	5.261	5.304	5.344	5.382	5.417	5.451	5.482	5.513
14	5.036	5.084	5.127	5.167	5.205	5.240	5.274	5.305	5.335	5.363
15	4.929	4.974	5.015	5.053	5.089	5.123	5.154	5.184	5.212	5.239
16	4.838	4.881	4.920	4.957	4.991	5.023	5.023	5.081	5.108	5.134
17	4.759	4.801	4.838	4.874	4.906	4.937	4.966	4.993	5.019	5.044
18	4.691	4.731	4.768	4.802	4.833	4.863	4.891.	4.917	4.942	4.966
19	4.632	4.670	4.706	4.739	4.769	4.798	4.825	4.851	4.875	4.897
20	4.579	4.617	4.651	4.683	4.713	4.741	4.767	4.792	4.815	4.837
21	4.533	4.569	4.603	4.634	4.663	4.690	4.715	4.740	4.762	4.784
22	4.491	4.527	4.559	4.590	4.618	4.645.	4.669	4.693	4.715	4.736
23	4.453	4.488	4.520	4.550	4.578	4.604	4.628	4.651	4.673	4.693
24	4.419	4.454	4.485	4.514	4.541	4.567	4.590	4.613	4.634	4.654
25	4.388	4.422	4.453	4.482	4.508	4.533	4.556	4.579	4.599	4.619
26	4.360	4.393	4.424	4.452	4.478	4.502	4.525	4.547	4.568	4.587
27	4.334	4.367	4.397	4.424	4.450	4.474	4.497	4.518	4.538	4.558
28	4.311	4.343	4.372	4.399	4.425	4.449	4.471	4.492	3.512	4.530
29	4.289	4.320	4.349	4.376	4.401	4.425	4.447	4.467	4.487	4.506
30	4.268	4.300	4.328	4.355	4.380	4.403	4.424	4.445	4.464	4.482
35	4.186	4.215	4.243	4.268	4.291	4.313	4.334	4.353	4.371	4.389
40	4.126	4.154	4.180	4.205	4.227	4.248	4.268	4.287	4.304	4.321
45	4.080	4.108	4.133	4.156	4.178	4.199	4.218	4.236	4.253	4.269
50	4.044	4.071	4.096	4.119	4.140	4.160	4.178	4.196	4.213	4.228
55	4.015	4.041	4.066	4.088	4.109	4.128	4.147	4.164	4.180	4.196
60	3.991	4.017	4.041	4.063	4.083	4.103	4.121	4.137	4.153	4.169
70	3.954	3.979	4.002	4.024	4.044	4.063	4.080	4.097	4.112	4.127
80	3.926	3.951	3.974	3.995	4.015	4.033	4.050	4.066	4.081	4.096
90	3.905	3.930	3.952	3.973	3.992	4.010	4.027	4.043	4.058	4.072
100	3.888	3.913	3.935	3.955	3.974	3.992	4.009	4.025	4.039	4.053
200	3.814	3.837	3.859	3.878	3.896	3.913	3.929	3.943	3.957	3.970
300	3.790	3.813	3.834	3.853	3.870	3.887	3.902	3.917	3.931	3.944
400	3.778	3.801	3.821	3.840	3.858	3.874	3.889	3.904	3.917	3.930
500	3.771	3.794	3.814	3.833	3.850	3.866	3.882	3.896	3.909	3.922
600	3.767	3.789	3.809	3.828	3.845	3.861	3.877	3.891	3.904	3.917
700	3.763	3.785	3.806	3.824	3.842	3.858	3.873	3.887	3.900	3.913
800	3.761	3.783	3.803	3.822	3.839	3.855	3.870	3.884	3.898	3.910
900	3.759	3.781	3.801	3.820	3.837	3.853	3.868	3.882	3.895	3.908
1000	3.757	3.779	3.799	3.818	3.835	3.851	3.866	3.880	3.894	3.906
∞	3.743	3.765	3.785	3.803	3.820	3.836	3.851	3.865	3.878	3.891

ontinued Table 3a

ALPHA =0.0020 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 =0.0010 (1-SIDED)

N \ R	21	22	23	24	25	26	27	28	29	30
3	28.46	28.91	29.34	29.76	30.17	30.57	30.95	31.33	31.70	32.06
4	15.74	15.92	16.10	16.28	16.45	16.61	16.77	16.93	17.08	17.22
5	11.29	11.40	11.51	11.61	11.70	11.80	11.89	11.98	12.07	12.15
6	9.161	9.237	9.310	9.380	9.448	9.513	9.577	9.638	9.698	9.756
7	7.945	8.003	8.059	8.113	8.165	8.215	8.263	8.310	8.356	8.400
8	7.170	7.217	7.263	7.307	7.349	7.390	7.430	7.468	7.505	7.541
9	6.636	6.677	6.716	6.753	6.789	6.824	6.858	6.890	6.922	6.952
10	6.248	6.284	6.319	6.352	6.383	6.414	6.444	6.472	6.500	6.527
11	5.955	5.987	6.018	6.048	6.077	6.104	6.131	6.156	6.181	6.205
12	5.725	5.755	5.783	5.811	5.837	5.862	5.886	5.910	5.933	5.955
13	5.541	5.569	5.595	5.620	5.645	5.668	5.691	5.712	5.734	5.754
14	5.390	5.416	5.441	5.465	5.487	5.509	5.531	5.551	5.571	5.590
15	5.265	5.289	5.313	5.335	5.357	5.377	5.397	5.416	5.435	5.453
16	5.158	5.182	5.204	5.225	5.246	5.266	5.285	5.303	5.321	5.338
17	5.067	5.089	5.111	5.131	5.151	5.170	5.188	5.206	5.223	5.239
18	4.988	5.010	5.030	5.050	5.069	5.087	5.105	5.121	5.138	5.154
19	4.919	4.940	4.960	4.979	4.997	5.015	5.032	5.048	5.064	5.079
20	4.858	4.879	4.898	4.916	4.934	4.951	4.967	4.983	4.999	5.013
21	4.804	4.824	4.843	4.861	4.878	4.894	4.910	4.926	4.941	4.955
22	4.756	4.775	4.794	4.811	4.828	4.844	4.860	4.874	4.889	4.903
23	4.713	4.731	4.749	4.766	4.783	4.799	4.814	4.828	4.842	4.856
24	4.674	4.692	4.709	4.726	4.742	4.758	4.772	4.787	4.801	4.814
25	4.638	4.656	4.673	4.689	4.705	4.720	4.735	4.749	4.762	4.776
26	4.605	4.623	4.640	4.656	4.672	4.686	4.701	4.714	4.728	4.741
27	4.576	4.593	4.610	4.625	4.641	4.655	4.669	4.683	4.696	4.709
28	4.548	4.565	4.582	4.597	4.612	4.627	4.641	4.654	4.667	4.679
29	4.523	4.540	4.556	4.571	4.586	4.600	4.614	4.627	4.640	4.652
30	4.500	4.516	4.532	4.547	4.562	4.576	4.589	4.602	4.615	4.627
35	4.405	4.421	4.436	4.450	4.464	4.477	4.490	4.502	4.514	4.525
40	4.337	4.352	4.366	4.380	4.393	4.406	4.418	4.429	4.441	4.452
45	4.284	4.299	4.313	4.326	4.339	4.351	4.363	4.374	4.385	4.396
50	4.243	4.257	4.271	4.284	4.296	4.308	4.320	4.331	4.341	4.352
55	4.210	4.224	4.237	4.250	4.262	4.274	4.285	4.296	4.306	4.316
60	4.183	4.197	4.210	4.222	4.234	4.245	4.256	4.267	4.277	4.287
70	4.141	4.154	4.167	4.179	4.190	4.202	4.212	4.223	4.232	4.242
80	4.110	4.123	4.135	4.147	4.158	4.169	4.180	4.190	4.199	4.209
90	4.086	4.098	4.111	4.122	4.133	4.144	4.154	4.164	4.174	4.183
100	4.067	4.079	4.091	4.103	4.114	4.124	4.135	4.144	4.154	4.163
200	3.983	3.995	4.006	4.017	4.028	4.038	4.047	4.056	4.065	4.074
300	3.956	3.968	3.979	3.989	4.000	4.009	4.019	4.028	4.036	4.045
400	3.942	3.954	3.965	3.976	3.986	3.995	4.005	4.013	4.022	4.030
500	3.934	3.946	3.957	3.967	3.977	3.987	3.996	4.005	4.014	4.022
600	3.929	3.940	3.951	3.962	3.972	3.981	3.991	3.999	4.008	4.016
700	3.925	3.937	3.947	3.958	3.968	3.977	3.987	3.995	4.004	4.012
800	3.922	3.934	3.945	3.955	3.965	3.974	3.984	3.992	4.001	4.009
900	3.920	3.931	3.942	3.953	3.963	3.972	3.981	3.990	3.999	4.007
1000	3.918	3.930	3.941	3.951	3.961	3.970	3.979	3.988	3.997	4.005
∞	3.902	3.914	3.924	3.935	3.944	3.954	3.963	3.971	3.980	3.988

Continued Table 3a

ALPHA =0.0020 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 =0.0010 (1-SIDED)

N \ R	35	40	45	50	100	250
3	33.76	35.30	36.71	38.03	47.93	65.06
4	17.91	18.52	19.08	19.60	23.33	29.37
5	12.54	12.89	13.21	13.50	15.55	18.72
6	10.02	10.26	10.47	10.67	12.03	14.08
7	8.603	8.783	8.943	9.089	10.10	11.60
8	7.706	7.851	7.980	8.098	8.907	10.08
9	7.092	7.215	7.325	7.424	8.102	9.072
10	6.649	6.757	6.853	6.939	7.527	8.359
11	6.315	6.412	6.497	6.574	7.097	7.831
12	6.055	6.143	6.221	6.291	6.765	7.425
13	5.847	5.928	6.000	6.065	6.501	7.104
14	5.677	5.753	5.820	5.881	6.287	6.845
15	5.535	5.607	5.670	5.727	6.019	6.631
16	5.416	5.484	5.544	5.598	5.959	6.452
17	5.314	5.379	5.436	5.488	5.832	6.299
18	5.225	5.288	5.343	5.393	5.722	6.168
19	5.148	5.209	5.262	5.310	5.627	6.054
20	5.080	5.139	5.190	5.237	5.543	5.955
21	5.020	5.077	5.127	5.172	5.469	5.867
22	4.966	5.022	5.070	5.114	5.402	5.788
23	4.918	4.972	5.019	5.062	5.343	5.718
24	4.875	4.927	4.974	5.015	5.290	5.655
25	4.835	4.887	4.932	4.973	5.241	5.598
26	4.799	4.849	4.894	4.934	5.197	5.546
27	4.766	4.816	4.859	4.899	5.157	5.499
28	4.736	4.784	4.828	4.866	5.120	5.455
29	4.708	4.756	4.798	4.836	5.086	5.415
30	4.682	4.729	4.771	4.808	5.054	5.378
35	4.577	4.622	4.661	4.696	4.927	5.230
40	4.501	4.544	4.582	4.615	4.835	5.123
45	4.443	4.485	4.521	4.554	4.765	5.042
50	4.398	4.438	4.474	4.505	4.711	4.979
55	4.362	4.401	4.436	4.466	4.667	4.928
60	4.332	4.370	4.404	4.435	4.631	4.886
70	4.285	4.323	4.356	4.385	4.576	4.822
80	4.251	4.288	4.320	4.349	4.335	4.775
90	4.225	4.261	4.293	4.321	4.503	4.739
100	4.204	4.240	4.271	4.298	4.478	4.710
200	4.112	4.146	4.175	4.201	4.369	4.585
300	4.083	4.115	4.144	4.169	4.334	4.544
400	4.068	4.100	4.129	4.154	4.317	4.524
500	4.059	4.091	4.119	4.144	4.306	4.512
600	4.053	4.085	4.113	4.138	4.299	4.504
700	4.049	4.081	4.109	4.134	4.294	4.499
800	4.046	4.078	4.106	4.131	4.291	4.495
900	4.044	4.075	4.103	4.128	4.288	4.491
1000	4.042	4.073	4.101	4.126	4.285	4.489
∞	4.024	4.056	4.083	4.107	4.265	4.465

Table 3b

ALPHA =0.0010 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 =0.0005 (1-SIDED)

N \ R	1	2	3	4	5	6	7	8	9	10
3	12.92	16.33	18.71	20.60	22.20	23.60	24.85	25.99	27.03	28.00
4	8.610	10.31	11.44	12.31	13.03	13.65	14.20	14.69	15.13	15.54
5	6.869	7.976	7.693	9.235	9.678	10.05	10.38	10.67	10.94	11.18
6	5.959	6.788	7.314	7.708	8.025	8.292	8.524	8.729	8.914	9.082
7	5.408	6.082	6.503	6.814	7.063	7.272	7.453	7.612	7.755	7.885
8	5.041	5.617	5.973	6.234	6.442	6.616	6.765	6.896	7.014	7.120
9	4.781	5.291	5.602	5.830	6.010	6.160	6.289	6.402	6.503	6.594
10	4.587	5.049	5.329	5.533	5.694	5.827	5.942	6.042	6.131	6.211
11	4.437	4.863	5.120	5.306	5.453	5.574	5.678	5.768	5.849	5.921
12	4.318	4.716	4.955	5.128	5.263	5.375	5.471	5.554	5.628	5.694
13	4.221	4.597	4.822	4.984	5.111	5.215	5.304	5.382	5.451	5.513
14	4.140	4.499	4.712	4.865	4.985	5.084	5.167	5.240	5.305	5.363
15	4.073	4.417	4.620	4.766	4.880	4.974	5.053	5.123	5.184	5.239
16	4.015	4.346	4.542	4.682	4.791	4.881	4.957	5.023	5.081	5.134
17	3.965	4.286	4.475	4.609	4.714	4.801	4.874	4.937	4.993	5.044
18	3.922	4.233	4.416	4.547	4.648	4.731	4.802	4.863	4.917	4.966
19	3.883	4.187	4.365	4.491	4.590	4.670	4.739	4.798	4.851	4.897
20	3.850	4.146	4.319	4.443	4.539	4.617	4.683	4.741	4.792	4.837
21	3.819	4.110	4.279	4.399	4.493	4.569	4.634	4.690	4.740	4.784
22	3.792	4.077	4.243	4.361	4.452	4.527	4.590	4.645	4.693	4.736
23	3.768	4.047	4.210	4.326	4.415	4.488	4.550	4.604	4.651	4.693
24	3.745	4.021	4.181	4.294	4.382	4.454	4.514	4.567	4.613	4.654
25	3.725	3.996	4.154	4.265	4.352	4.422	4.482	4.533	4.579	4.619
26	3.707	3.974	4.129	4.239	4.324	4.393	4.452	4.502	4.547	4.587
27	3.690	3.954	4.107	4.215	4.299	4.367	4.424	4.474	4.518	4.558
28	3.674	3.935	4.086	4.193	4.275	4.343	4.399	4.449	4.492	4.530
29	3.659	3.918	4.067	4.172	4.254	4.320	4.376	4.425	4.467	4.506
30	3.646	3.902	4.049	4.154	4.234	4.300	4.355	4.403	4.445	4.482
35	3.591	3.836	3.977	4.077	4.153	4.215	4.268	4.313	4.353	4.389
40	3.551	3.788	3.925	4.020	4.094	4.154	4.205	4.248	4.287	4.321
45	3.520	3.752	3.885	3.978	4.049	4.108	4.156	4.199	4.236	4.269
50	3.496	3.723	3.853	3.944	4.014	4.071	4.119	4.160	4.196	4.228
55	3.476	3.700	3.828	3.917	3.986	4.041	4.088	4.128	4.164	4.196
60	3.460	3.681	3.807	3.895	3.962	4.017	4.063	4.103	4.137	4.169
70	3.435	3.651	3.774	3.860	3.926	3.979	4.024	4.063	4.097	4.127
80	3.416	3.629	3.750	3.834	3.899	3.951	3.995	4.033	4.066	4.096
90	3.402	3.612	3.731	3.814	3.878	3.930	3.973	4.010	4.043	4.072
100	3.390	3.598	3.716	3.799	3.862	3.913	3.955	3.992	4.025	4.053
200	3.340	3.539	3.651	3.729	3.789	3.837	3.878	3.913	3.943	3.970
300	3.323	3.519	3.630	3.707	3.765	3.813	3.853	3.887	3.917	3.944
400	3.315	3.510	4.619	3.696	3.754	3.801	3.840	3.874	3.904	3.930
500	3.310	3.504	3.613	3.689	3.747	3.794	3.833	3.866	3.896	3.922
600	3.307	3.500	3.609	3.684	3.742	3.789	3.828	3.861	3.891	3.917
700	3.304	3.497	3.606	3.681	3.739	3.785	3.824	3.858	3.887	3.913
800	3.303	3.495	3.604	3.679	3.736	3.783	3.822	3.855	3.884	3.910
900	3.301	3.493	3.602	3.677	3.734	3.781	3.820	3.853	3.882	3.908
000	3.300	3.492	3.600	3.675	3.733	3.779	3.818	3.581	3.880	3.906
∞	3.291	3.481	3.588	3.662	3.719	3.765	3.803	3.836	3.865	3.891

Continued Table 3b

ALPHA =0.0010 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 =0.0005 (1-SIDED)

N \ R	11	12	13	14	15	16	17	18	19	20
3	28.91	29.76	30.57	31.33	32.06	32.76	33.43	34.08	34.70	35.30
4	15.92	16.28	16.61	16.93	17.22	17.51	17.78	18.04	18.28	18.52
5	11.40	11.61	11.80	11.98	12.15	12.32	12.47	12.62	12.76	12.89
6	9.237	9.380	9.513	9.638	9.756	9.867	9.973	10.07	10.17	10.26
7	8.003	8.113	8.215	8.310	8.400	8.485	8.565	8.641	8.713	8.783
8	7.217	7.307	7.390	7.468	7.541	7.609	7.674	7.736	7.795	7.851
9	6.677	6.753	6.824	6.890	6.952	7.011	7.066	7.118	7.168	7.215
10	6.284	6.352	6.414	6.472	6.527	6.578	6.626	6.672	6.715	6.757
11	5.987	6.048	6.104	6.156	6.205	6.251	6.295	6.336	6.374	6.412
12	5.755	5.811	5.862	5.910	5.955	5.997	6.036	6.074	6.109	6.143
13	5.569	5.620	5.668	5.712	5.754	5.793	5.829	5.864	5.897	5.928
14	5.416	5.465	5.509	5.551	5.590	5.626	5.660	5.693	5.724	5.753
15	5.289	5.335	5.377	5.416	5.453	5.488	5.520	5.550	5.579	5.607
16	5.182	5.225	5.266	5.303	5.338	5.370	5.401	5.430	5.458	5.484
17	5.089	5.131	5.170	5.206	5.239	5.270	5.300	5.327	5.354	5.379
18	5.010	5.050	5.087	5.121	5.154	5.184	5.212	5.239	5.264	5.288
19	4.940	4.979	5.015	5.048	5.079	5.108	5.135	5.161	5.185	5.209
20	4.879	4.916	4.951	4.983	5.013	5.041	5.068	5.093	5.116	5.139
21	4.824	4.861	4.894	4.926	4.955	4.982	5.008	5.032	5.055	5.077
22	4.775	4.811	4.844	4.874	4.903	4.930	4.955	4.978	5.000	5.022
23	4.731	4.766	4.799	4.828	4.856	4.882	4.906	4.930	4.951	4.972
24	4.692	4.726	4.758	4.787	4.814	4.839	4.863	4.886	4.907	4.927
25	4.656	4.689	4.720	4.749	4.776	4.800	4.824	4.846	4.867	4.887
26	4.623	4.656	4.686	4.714	4.741	4.765	4.788	4.810	4.830	4.849
27	4.593	4.625	4.655	4.683	4.709	4.733	4.755	4.776	4.796	4.816
28	4.565	4.597	4.627	4.654	4.679	4.703	4.725	4.746	4.766	4.784
29	4.540	4.571	4.600	4.627	4.652	4.675	4.697	4.718	4.737	4.756
30	4.516	4.547	4.576	4.602	4.627	4.650	4.671	4.692	4.711	4.729
35	4.421	4.450	4.477	4.502	4.525	4.547	4.567	4.587	4.605	4.622
40	4.352	4.380	4.406	4.429	4.452	4.472	4.492	4.510	4.527	4.544
45	4.299	4.326	4.351	4.374	4.396	4.416	4.435	4.452	4.469	4.485
50	4.257	4.284	4.308	4.331	4.352	4.371	4.390	4.407	4.423	4.438
55	4.224	4.250	4.274	4.296	4.316	4.335	4.353	4.370	4.386	4.401
60	4.197	4.222	4.245	4.267	4.287	4.306	4.323	4.340	4.356	4.370
70	4.154	4.179	4.202	4.223	4.242	4.260	4.277	4.293	4.309	4.323
80	4.123	4.147	4.169	4.190	4.209	4.227	4.243	4.259	4.274	4.288
90	4.098	4.122	4.144	4.164	4.183	4.201	4.217	4.233	4.247	4.261
100	4.079	4.103	4.124	4.144	4.163	4.180	4.196	4.212	4.226	4.240
200	3.995	4.017	4.038	4.056	4.074	4.090	4.105	4.120	4.133	4.146
300	3.968	3.989	4.009	4.028	4.045	4.061	4.076	4.090	4.103	4.115
400	3.954	3.976	3.995	4.013	4.030	4.046	4.061	4.075	4.088	4.100
500	3.946	3.967	3.987	4.005	4.022	4.038	4.052	4.066	4.079	4.091
600	3.940	3.962	3.981	3.999	4.016	4.032	4.046	4.060	4.073	4.085
700	3.937	3.958	3.977	3.995	4.012	4.028	4.042	4.056	4.069	4.081
800	3.934	3.955	3.974	3.992	4.009	4.025	4.039	4.053	4.066	4.078
900	3.931	3.953	3.972	3.990	4.007	4.022	4.037	4.050	4.063	4.075
1000	3.930	3.951	3.970	3.988	4.005	4.020	4.035	4.048	4.061	4.073
∞	3.914	3.935	3.954	3.971	3.988	4.003	4.017	4.031	4.044	4.056

Continued Table 3b

ALPHA =0.0010 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 =0.0005 (1-SIDED)

N \ R	21	22	23	24	25	26	27	28	29	30
3	35.88	36.44	36.98	37.51	38.03	38.53	39.02	39.50	39.96	40.42
4	18.75	18.97	19.19	19.39	19.60	19.79	19.98	20.16	20.34	20.52
5	13.02	13.15	13.27	13.38	13.50	13.60	13.71	13.81	13.91	14.01
6	10.35	10.43	10.51	10.59	10.67	10.74	10.81	10.88	10.95	11.01
7	8.849	8.912	8.974	9.033	9.089	9.144	9.197	9.249	9.299	9.347
8	7.904	7.956	8.005	8.052	8.098	8.142	8.185	8.226	8.266	8.305
9	7.261	7.304	7.346	7.386	7.424	7.461	7.497	7.532	7.565	7.598
10	6.796	6.834	6.871	6.905	6.939	6.971	7.003	7.033	7.062	7.090
11	6.447	6.481	6.513	6.544	6.574	6.603	6.631	6.658	6.684	6.710
12	6.175	6.206	6.236	6.264	6.291	6.317	6.343	6.367	6.391	6.414
13	5.958	5.986	6.014	6.040	6.065	6.089	6.113	6.135	6.157	6.178
14	5.781	5.807	5.833	5.857	5.881	5.903	5.925	5.946	5.966	5.986
15	5.633	5.658	5.682	5.705	5.727	5.749	5.769	5.789	5.808	5.827
16	5.509	5.533	5.556	5.577	5.598	5.619	5.638	5.657	5.675	5.692
17	5.403	5.425	5.447	5.468	5.488	5.507	5.526	5.544	5.561	5.578
18	5.311	5.333	5.354	5.374	5.393	5.411	5.429	5.446	5.463	5.479
19	5.231	5.252	5.272	5.291	5.310	5.327	5.345	5.361	5.377	5.393
20	5.160	5.180	5.200	5.219	5.237	5.254	5.270	5.286	5.302	5.317
21	5.098	5.117	5.136	5.154	5.172	5.188	5.205	5.220	5.235	5.249
22	5.042	5.061	5.079	5.097	5.114	5.130	5.146	5.161	5.175	5.190
23	4.992	5.010	5.028	5.046	5.062	5.078	5.093	5.108	5.122	5.136
24	4.946	4.965	4.982	4.999	5.015	5.031	5.046	5.060	5.074	5.087
25	4.905	4.923	4.940	4.957	4.973	4.988	5.002	5.016	5.030	5.043
26	4.868	4.886	4.902	4.919	4.934	4.949	4.963	4.977	4.990	5.003
27	4.834	4.851	4.868	4.883	4.899	4.913	4.927	4.941	4.954	4.966
28	4.802	4.819	4.836	4.851	4.866	4.880	4.894	4.908	4.920	4.933
29	4.773	4.790	4.806	4.821	4.836	4.850	4.864	4.877	4.890	4.902
30	4.747	4.763	4.779	4.794	4.808	4.822	4.836	4.849	4.861	4.873
35	4.638	4.654	4.669	4.683	4.696	4.709	4.722	4.734	4.746	4.757
40	4.560	4.574	4.589	4.602	4.615	4.628	4.640	4.651	4.662	4.673
45	4.500	4.514	4.528	4.541	4.554	4.566	4.577	4.588	4.599	4.610
50	4.453	4.467	4.480	4.493	4.505	4.517	4.528	4.539	4.550	4.560
55	4.415	4.429	4.442	4.455	4.466	4.478	4.489	4.500	4.510	4.520
60	4.384	4.398	4.411	4.423	4.435	4.446	4.457	4.467	4.477	4.487
70	4.337	4.350	4.362	4.374	4.385	4.396	4.406	4.417	4.426	4.436
80	4.301	4.314	4.326	4.338	4.349	4.359	4.370	4.379	4.389	4.398
90	4.274	4.286	4.298	4.310	4.321	4.331	4.341	4.351	4.360	4.369
100	4.252	4.265	4.277	4.288	4.298	4.309	4.319	4.328	4.337	4.346
200	4.158	4.169	4.180	4.191	4.201	4.211	4.220	4.229	4.238	4.246
300	4.127	4.139	4.149	4.160	4.169	4.179	4.188	4.197	4.205	4.213
400	4.112	4.123	4.134	4.144	4.154	4.163	4.172	4.181	4.189	4.197
500	4.103	4.114	4.125	4.135	4.144	4.154	4.163	4.171	4.180	4.188
600	4.097	4.108	4.119	4.129	4.138	4.148	4.156	4.165	4.173	4.181
700	4.093	4.104	4.114	4.124	4.134	4.143	4.152	4.160	4.169	4.177
800	4.089	4.100	4.111	4.121	4.131	4.140	4.149	4.157	4.165	4.173
900	4.087	4.098	4.108	4.118	4.128	4.137	4.146	4.154	4.163	4.171
000	4.085	4.096	4.106	4.116	4.126	4.135	4.144	4.152	4.161	4.168
∞	4.067	4.078	4.088	4.098	4.107	4.117	4.125	4.134	4.142	4.149

Continued Table 3b

ALPHA =0.0010 (2-SIDED) T (ALPHA/R, N)
ALPHA/2 = 0.0005 (1-SIDED)

N \ R	35	40	45	50	100	250
3	42.55	44.49	46.27	47.93	60.40	81.98
4	21.33	22.06	22.72	23.33	27.77	34.95
5	14.46	14.86	15.22	15.55	17.90	21.54
6	11.31	11.58	11.81	12.03	13.56	15.85
7	9.570	9.766	9.943	10.10	11.21	12.86
8	8.483	8.640	8.780	8.907	9.783	11.05
9	7.748	7.879	7.996	8.102	8.827	9.867
10	7.220	7.334	7.435	7.527	8.150	9.034
11	6.825	6.926	7.016	7.097	7.647	8.421
12	6.519	6.610	6.692	6.765	7.261	7.952
13	6.275	6.359	6.434	6.501	6.954	7.583
14	6.076	6.154	6.224	6.287	6.706	7.285
15	5.911	5.985	6.050	6.109	6.502	7.041
16	5.773	5.842	5.904	5.959	6.330	6.836
17	5.654	5.721	5.779	5.832	6.184	6.663
18	5.552	5.616	5.672	5.722	6.058	6.514
19	5.463	5.524	5.578	5.627	5.949	6.385
20	5.385	5.444	5.496	5.543	5.854	6.273
21	5.315	5.373	5.423	5.469	5.769	6.173
22	5.254	5.309	5.358	5.402	5.694	6.085
23	5.198	5.252	5.300	5.343	5.626	6.006
24	5.148	5.201	5.248	5.290	5.566	5.935
25	5.103	5.155	5.200	5.241	5.511	5.871
26	5.062	5.112	5.157	5.197	5.461	5.812
27	5.024	5.073	5.117	5.157	5.415	5.759
28	4.989	5.038	5.081	5.120	5.373	5.710
29	4.957	5.005	5.048	5.086	5.335	5.666
30	4.928	4.975	5.017	5.054	5.299	5.624
35	4.808	4.853	4.892	4.927	5.156	5.458
40	4.722	4.764	4.802	4.835	5.053	5.339
45	4.657	4.698	4.733	4.765	4.975	5.249
50	4.606	4.645	4.680	4.711	4.914	5.149
55	4.564	4.603	4.637	4.667	4.865	5.122
60	4.530	4.568	4.602	4.631	4.825	5.076
70	4.478	4.515	4.547	4.576	4.763	5.005
80	4.439	4.475	4.507	4.535	4.717	4.953
90	4.410	4.445	4.476	4.503	4.682	4.913
100	4.386	4.421	4.451	4.478	4.654	4.881
200	4.283	4.316	4.344	4.369	4.533	4.743
300	4.250	4.282	4.309	4.334	4.494	4.698
400	4.233	4.265	4.292	4.317	4.474	4.676
500	4.224	4.255	4.282	4.306	4.463	4.663
600	4.217	4.248	4.275	4.299	4.455	4.655
700	4.212	4.243	4.270	4.294	4.450	4.648
800	4.209	4.240	4.267	4.291	4.446	4.644
900	4.206	4.237	4.264	4.288	4.442	4.640
1000	4.204	4.235	4.262	4.285	4.440	4.637
∞	4.185	4.215	4.241	4.265	4.417	4.611

may be used for obtaining BONFERRONI percentage points by interpolation for nominal levels α/r with $r=1\,(1)\,100$, say, for one-sided BONFERRONI t tests. For two-sided t tests, the most extensive tabulation has been made by SMIRNOV (1961) in his distribution function for $t=0(0.01)\,2{,}50(0.02)\,3{,}50(0.05)\,6{,}50$ with $df=1(1)\,35$, including auxiliary tables for larger t's and df's. SMIRNOV's table may likewise be used for getting two-sided Bonferroni levels by interpolation.

The most extensive tables which give the percentage points of t directly for Bonferroni-adjusted α-levels are BAILEY's tables (1977) and the tables of Huitema (1980). But both tables are restricted to two-sided testing at levels $\alpha=0.05$ and $\alpha=0.01$.

Below are the BONFERRONI-percentage points of t, calculated for r simultaneous t tests at all conventional levels of α (0.05, 0.01 and 0.001) in one-sided as well as in two-sided testing, (Tables 1–3). The method used in tabulation was via the inverse of the incomplete beta function (see MAJUMDER & BHATTACHARJEE, 1973, and CRAN et al. 1977).

Table 1ab gives the one-sided (a) and the two-sided (b) critical limits of BONFERRONI t at level $\alpha=0.05$ for $df=3\,(1)\,30\,(5)\,60\,(10)\,100\,(100)\,1000$ and ∞ according to Table A.4 in SNEDECOR and COCHRAN (1967), and for $r=1\,(1)\,30\,(5)\,50;\,100;\,250$ simultaneous tests. Table 2ab shows the same limits at level $\alpha=0.01$ and Table 3ab at level 0.001.

Note that the t distribution with $df=\infty$ is identical to the standard normal distribution. Thus, for large samples the Bonferroni t-test may be approximated by the Bonferroni z-test tabulated in LIENERT et al. (1982). Note furthermore that the t^2-distribution with any df's is identical to the F-distribution with $df_1=1$ and $df_2=$ df. Thus the ANOVA F-test for 2 independent samples of size n_1 and n_2 is identical to a t-test with $df=n_1+n_2-2$.

3. Two-sample Applications

The Bonferroni-adjusted t test called BONFERRONI-DUNN test (BD-test) below, is an alternative to ANOVA of unifactorial randomized group designs if the research worker is interested only in apriori justified comparisons between pairs of group means.

1. If the error variance has been identified by ANOVA from homoscedastically distributed k samples of size n_i, $i=1(1)\,k$, the test statistic given by

$$t_{ij}=(M_i-M_j)/s_{ij} \tag{1}$$

is to be evaluated for $r=k\,(k-1)/2$ and $df=\sum(n_i-1)$ in Table 1–3 (if all possible pairwise comparisons are to be made for an exhaustive interpretation). The error term s_{ij} in (1) is given by

$$s_{ij}^2=s^2\,(1/n_i+1/n_j) \tag{2}$$

where s^2 is the within-group variance of the randomized group ANOVA. If $k=5$

samples of size $n_i = N/k = 10$ and if their within-group variance $s^2 = 28.8$, as in Table 3.2–2 of KIRK's (1968) text book, then $s_{ij}^2 = 28.8 \cdot (1/10 + 1/10) = 5.76$ and $s_{ij} = 2.40$. If $M_1 - M_4 = -10.5$, then $t_{14} = -10.5/2.40 = -4.375$ exceeds the two-sided 1 %-limit of 3.52 for $df = 5\ (10-1) = 45$ and $r = 5.4/2 = 10$ pairwise comparisons in Table 2b.

The critical difference ${}_aD_{ij}$ between pairs of means from k independent samples is given by

$$(3) \qquad {}_aD_{ij} = {}_at_{ij}(s_{ij})$$

which is $3.52\ (2.40) = 8.47$ for the example above. Any of the 10 differences of pairs of means which exceeds the critical Bonferroni difference is significant at $\alpha = 1$ % level of significance.

2. The BD-test may also be applied to k independent samples of differences (increments) from pre-to-post-observations. If the increments are larger in $k-1$ experimental samples and 1 control sample, then $r = k-1$ BD-tests, called Paardifferenzen-t-Tests by BUCK (1975), may replace an ANOVA of increments.

3. Another problem to be attacked by BD-tests rather than by ANOVA is the comparison of α regression coefficients b with k independent bivariate samples; e.g., blood pressure regression onto age in k professional groups. Replacing M_i by b_i and M_j by b_j in (1), the term s^2 is defined as the residual variance (deviation from regression line variance) pooled over the k samples in the ANOVA evaluation of regressions (see SNEDECOR and COCHRAN, 1967, Ch. 14.6).

Note that the BONFERRONI-DUNN tests imply homogeneous variances of the k samples to be compared pairwise. Violation of the homogeneity requirement is least disturbing the k samples are of about equal if size, $n_i = N/k$ (see AHRENS, 1967, Ch. 2.1.2). If the variances are heterogeneous, from data inspection, the error variance s^2 in (1) could be better estimated from $s^2 = s_i^2 + s_j^2$ in like-sized samples, where s_i^2 and s_j^2 are the variance estimates of the two samples to be compared.

4. One sample t test application

1. The classical application of the one-sample t test is to identify whether one sample of N normally distributed observations, X, deviates from a given population mean (or theoretical standard). If k independent samples are to be evaluated this way, the one-sample t test has to be replaced by the BONFERRONI-DUNN test with the critical limits tabulated in Tables 1–3.

2. The most frequent application of the one-sample t test is the traditional evaluation of two paired samples of their differences, $d = X_2 - X_1$ which are, in bivariate normal populations, distributed univariately normal. The variance is estimated from the sample of N differences by $s_d^2 = \sum d^2/(N-1) - \bar{d}^2$. In the case of k paired samples of size N, obtained by repeated measurements from each of N individuals, the one-sample t test has to be replaced by the $k\ (k-1)/2$ BON-

FERRONI-DUNN tests

(4) $$t_{ij} = \bar{d}_{ij}/(s\sqrt{2/N})$$

where s^2 is the estimated error variance of a randomized block design with k repeated measurements per individual (see SNEDECOR and COCHRAN, 1967, Ch. 11.3). The t_{ij} statistic is to be evaluated for $df = (k-1)(N-1)$. The critical difference for the means of 2 out of k paired samples is, in BONFERRONI-DUNN one-sample testing, given by

(5) $$\bar{d}_{ij} = {}_{*}t_{ij} s\sqrt{2/N}$$

where ${}_{*}t_{ij}$ is to be read from Tables 1–3. Reading may be one-sided if there is a general location trend in the k paired samples as in case of response curves with k increasing dosages of an agent. Otherwise, ${}_{*}t_{ij}$ must be read as a two-sided statistic in tables 1–3, for $r = k(k-1)/2$ and $df = (k-1)(N-1)$ if no prediction is made as to the specified paired comparisons.

In the case of $k = 5$ treatments of soybean seeds (SNEDECOR and COCHRAN, 1967, Ch. 11.2) in $N = 5$ plots (blocks) the error variance was $s^2 = 5{,}41$ implying $s = 2{,}326$. At $\alpha = 0.05$ the BONFERRONI-DUNN t is read to be $t_{ij} = 2{,}473$ for $r = (5-1) = 4$ and $df = (5-1)(5-1) = 16$ from Table 1a of one-sided testing of each of 4 treatments against 1 control (check). Thus the critical difference is given by $\bar{d}_{ij} = 2{,}473\,(2{,}326)\sqrt{2/5} = 3{,}638$ from one-sided BONFERRONI-DUNN testing in comparing the above $k = 5$ paired samples. Since the mean failure rate 5,8 of Fermate-treatment sample lies beyond the mean failure rate of 10,8 in check sample, the difference $10{,}8 - 5{,}8 = 5{,}0$ exceeds the critical difference, thus being significant at level $\alpha = 0{,}05$.

BONFERRONI-DUNN testing via s^2 in randomized block designs requires the variance-covariance matrix to be symmetrical (see KIRK, 1972, p. 139). If this requirement is not met, then $k(k-1)/2$ paired differences BONFERRONI-DUNN tests may replace the ANOVA evaluation. The same statement is true if block and treatment effects are not additive, as required by ANOVA evaluation (see KIRK, 1968, p. 137) in randomized block designs.

5. BONFERRONI replaced by HOLM-adjustment

Simultaneous t testing via BONFERRONI is sometimes more effective than is global testing via ANOVA, especially in the case of one outlying sample. Generally speaking, BONFERRONI testing is less conservative than it is generally supposed (see FUCHS and KENNETT, p. 395). Nevertheless, some further improvements of BONFERRONI testing are available.

1. It is well known that some efficiency is gained if only r out of $k-1$ orthogonal comparisons are planned. In that case WILKINSON'S (1951) alpha-adjustment

(6) $$\alpha^* = 1 - (1-\alpha)^r$$

is a little more effective than is the BONFERRONI-adjustment, especially, if $\alpha = 0.05$ or larger (see WILKINSON, 1951).

A t-table for simultaneous control in that case is given by GAMES (1977). The control is a little pro-conservative, since SIDAK (1967) has shown that the Wilkinson adjustment is applicable also for selected nonorthogonal comparisons.

2. Though yet little known, HOLM'S (1979) alpha-adjustment is suitable in orthogonal as well as in nonorthogonal comparisons of any number from 2 to $k\,(k-1)/2$.

For k equal size samples, HOLM'S procedure requires to calculate t_i-values' $i = 1(1)\,r$, for all r planned comparisons and order them from high to low. Then the largest t_r is evaluated for α/r, the next-to largest t_i for $\alpha/(r-1)$ etc. down to the smallest $t_i = t_1$ which is to be evaluated for $\alpha/1$. Tables 1–3 may also be used for evaluating simultaneous t tests by HOLM'S adjustment. The gain in power relative to the Bonferroni adjustment is substantial at small numbers of r.

For non-equally sized samples, HOLM'S adjustment requires to read transgression probabilities P_i for t_i-values with now unlike df's, from extensive regular t tables (such as SMIRNOV'S, 1961) and order them from low to high. The comparison associated with the lowest $P_i = P_r$ is significant if P_r does not exceed α/r etc. and the comparison with the largest $P_i = P_1$ is still significant if $P_1 \leqq \alpha/1$. The downward procedure may be stopped if the first non-significant comparison has been identified.

3. A thoroughly compiled synopsis of simultaneous tests at multiple level α has been made available by SONNEMANN (1981) in a paper read at the 1981 Seminary of the Austrian-Switzerland Region of the International Biometric Society in Bad Ischl (Austria). A printed report is given by U. FERNER (1981).

Note that the HOLM and BONFERRONI t tests may be evaluated by tables 1–3, while WILKINSON t tests may not. Note furthermore that simultaneous tests must be two-sided and $k\,(k-1)/2$ in number if performed aposteriori, while apriori tests may be one-sided if the sign of a mean difference has been predicted under H_1 or if the alternative to H_0 is a trend in location from sample 1 to sample k.

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