THE SOLUTION OF A SYSTEM OF r+1 INCONSISTENT LINEAR EQUATIONS IN n UNKNOWNS BY THE METHOD OF LEAST SQUARES IN THE $l_{\,p}$ NORM \dagger

UDC 512.25

Let there be given the inconsistent system of r + 1 complex linear equations in n unknowns of rank r:

$$h_s(z) = a_{s_1}z_1 + \dots + a_{s_n}z_n + b_s = 0; \quad s = 1, \dots, r+1.$$
 (1)

We consider the determinant

$$D = \begin{vmatrix} a_{11} \dots a_{1r}b_1 \\ \vdots \\ a_{r+1,1} \dots a_{r+1,r}b_{r+1} \end{vmatrix}. \tag{2}$$

We denote the cofactor of b_i in the determinant (2) by B_i , and the cofactor of a_{ij} by A_{ij} .

Then the solution of the system (1) by the method of least squares in the $l_{\,\mathrm{p}}$ norm is given by

$$|h_1(z^*)|^p + \dots + |h_{r+1}(z^*)|^p = \frac{|D|^p}{(|B_1|^q + \dots + |B_{r+1}|^q)^{p/q}};$$
(3)

$$Z_{j}^{*} = \frac{A_{1j} |B_{1}|^{q-1} \frac{\overline{B}_{1}}{|B_{1}|} + \dots + A_{r+1, j} |B_{r+1}|^{q-1} \frac{\overline{B}_{r+1}}{|B_{r+1}|}}{|B_{r+1}|^{q}} + \frac{1}{D} \sum_{v=r+1}^{n} C_{jv} t_{v}, \tag{4}$$

where $j = 1, \ldots, r; \nu = r + 1, \ldots, n; C_{j\nu} = -\sum_{s=1}^{n} A_{Sj} a_{S\nu};$ and t_{ν} is an arbitrary complex number.

Here, p and q are positive numbers such that

$$\frac{1}{p} + \frac{1}{q} = 1.$$

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