Confinement, Quartet Mechanism and Cluster Property.

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Confinement implies a violation of the cluster property (1). STROCCHI (2) has shown that non-Abelian gauge QFT are the only theories, among renormalizable QFT, in which a violation of the cluster property can arise. The violation of the cluster property typically arises in a theory where some two-point Green's functions in the momentum space has a double-pole singularity at $q^2 = 0$ (massless dipole propagators) (*).

From another point of view KUGO (4) proposed a confinement criterion based on the so-called quartet mechanism.

One can ask whether the two approaches are related, *i.e.* whether the Kugo mechanism leads to a violation of the cluster property.

Here we shall give a heuristic argument supporting an adfirmative answer. This argument is suggested by the superfield formulation of the gauge theories proposed in ref. (5).

Let us consider the $(4+2)$-dimensional superspace M obtained adding two Grassmann variables θ and $\bar{\theta}$ to the Minkowski space. In M we define the one-form $\hat{\alpha}$:

$$\hat{\alpha} = \hat{U}^+(\boldsymbol{A}_\mu \, \mathrm{d}x^\mu)\, \hat{U} + \hat{U}^+ \mathrm{d}\hat{U}\,, \tag{1}$$

where

$$\hat{U}(x, \theta, \bar{\theta}) = \exp\left[\theta\bar{\boldsymbol{c}}(x) + \bar{\theta}\boldsymbol{c}(x) + \theta\bar{\theta}(\boldsymbol{B}(x) + \tfrac{1}{2}(\boldsymbol{c}\bar{\boldsymbol{c}} + \bar{\boldsymbol{c}}\boldsymbol{c}))\right] \tag{2}$$

and

$$\boldsymbol{A}_\mu(x) = A_\mu^\alpha(x)\tau^\alpha\,, \qquad \boldsymbol{c} = c^\alpha\tau^\alpha\,, \qquad \bar{\boldsymbol{c}} = \bar{c}^\alpha\tau^\alpha\,, \qquad \boldsymbol{B} = B^\alpha\tau^\alpha;$$

A_μ^α, B^α are real, θ, c^α are Hermitian and $\bar{\theta}, \bar{c}^\alpha$ are anti-Hermitian.

(1) H. ARAKI, K. HEPP and D. RUELLE: *Helv. Phys. Acta*, **35**, 164 (1962).
(2) F. STROCCHI: *Phys. Lett. B*, **62**, 60 (1976); *Phys. Rev. D*, **17**, 2010 (1978).
(*) A confined model with dipole gluon fields has been studied in ref. (3).
(3) E. D'EMILIO and M. MINTCHEV: Pisa IFUP preprint (1980).
(4) T. KUGO: *Phys. Lett. B*, **83**, 93 (1979).
(5) L. BONORA and M. TONIN: *Phys. Lett. B*, **98**, 48 (1981).

τ^α are the anti-Hermitian representative matrices which generate the Lie algebra of the gauge group G. $A^\alpha_\mu(x)$ are the gauge fields, $c^\alpha(x)$ and $\bar{c}^\alpha(x)$ the anticommuting FP fields and $B^\alpha(x)$ the auxiliary fields.

From eq. (1) one can write

$$\hat{\alpha} = \boldsymbol{\Phi}_\mu(x, \theta, \bar{\theta})\, \mathrm{d}x^\mu + \boldsymbol{\eta}(x, \theta, \bar{\theta})\, \mathrm{d}\bar{\theta} + \bar{\boldsymbol{\eta}}(x, \theta, \bar{\theta})\, \mathrm{d}\theta \,, \tag{3}$$

where the superfields Φ^α_μ, η^α, $\bar{\eta}^\alpha$ are given by

$$\begin{cases} \Phi^\alpha_\mu(x, \theta, \bar{\theta}) = A^\alpha_\mu(x) + \theta D_\mu \bar{c}^\alpha(x) + \bar{\theta} D_\mu c^\alpha(x) + \theta\bar{\theta}\big(D_\mu B(x) + D_\mu c(x) \times \bar{c}(x)\big)^\alpha \,, \\ \eta^\alpha(x, \theta, \bar{\theta}) \;= c^\alpha(x) + \theta \bar{B}^\alpha(x) - \dfrac{\bar{\theta}}{2}\big(c(x) \times c(x)\big)^\alpha + \theta\bar{\theta}\big(\bar{B}(x) \times c(x)\big)^\alpha \,, \\ \bar{\eta}^\alpha(x, \theta, \bar{\theta}) \;= \bar{c}^\alpha(x) - \dfrac{\theta}{2}\big(\bar{c}(x) \times \bar{c}(x)\big)^\alpha + \bar{\theta} B^\alpha(x) + \theta\bar{\theta}\big(c(x) \times B(x)\big)^\alpha \end{cases} \tag{4}$$

with

$$B^\alpha(x) + \bar{B}^\alpha(x) + \big(c(x) \times \bar{c}(x)\big)^\alpha = 0 \,.$$

The curvature two-form $\hat{\varrho}$

$$\hat{\varrho} = \mathrm{d}\hat{\alpha} + \tfrac{1}{2}[\hat{\alpha}, \hat{\alpha}]$$

is

$$\hat{\varrho} = \hat{U}^+(\boldsymbol{F}_{\mu\nu}\, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu)\, \hat{U} = \hat{\boldsymbol{F}}_{\mu\nu}(x, \theta, \bar{\theta})\, \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \,,$$

where

$$\boldsymbol{F}_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu] \,.$$

In this notation the Yang-Mills Lagrangian density equipped with a gauge-fixing term and the corresponding FP term is

$$\mathscr{L} = \mathrm{Tr}\left\{ -\frac{1}{4g^2}(\hat{\boldsymbol{F}}_{\mu\nu}\hat{\boldsymbol{F}}^{\mu\nu}) + \frac{\mathrm{d}}{\mathrm{d}\theta}\frac{\mathrm{d}}{\mathrm{d}\bar{\theta}}(\boldsymbol{\Phi}^\mu \boldsymbol{\Phi}_\mu) + \xi \left(\frac{\mathrm{d}\boldsymbol{\eta}}{\mathrm{d}\theta}\right)^2 \right\} . \tag{5}$$

This Lagrangian density is manifestely invariant under translations in $\bar{\theta}$ and θ. These nihilpotent translations are just the BRS [6] and the anti-BRS [7-9] transformations, generated by the nihilpotent charges Q_B and $\bar{Q}_\mathrm{B}$.

The Fock space $\mathscr{V}$ of the QFT defined by the Lagrangian density (5) has indefinite metric; the S-matrix invariant, semi-definite positive, physical subspace $\mathscr{V}_\mathrm{ph}$ is defined

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[6] C. BECCHI, A. ROUET and R. STORA: *Commun. Math. Phys.*, **42**, 127 (1975); *Ann. Phys. (N. Y.)*, **98**, 287 (1976).

[7] G. CURCI and R. FERRARI: *Nuovo Cimento A*, **32**, 151 (1976); *Phys. Lett. B*, **63**, 91 (1976); *Nuovo Cimento A*, **35**, 273 (1976).

[8] M. QUIROS and F. J. DE URRIES: preprint IEM (1980), Madrid; M. QUIROS, F. J. DE URRIES, J. HOYOS, M. L. MAZON and E. RODRIGUEZ: preprint IEM (1980), Madrid.

[9] I. OJIMA: *Prog. Theor. Phys.*, **64**, 625 (1980).

by the condition [5,9]

$$\mathscr{V}_{\mathrm{ph}} \equiv \{|\alpha\rangle \in \mathscr{V};\ Q_{\mathrm{B}}|\alpha\rangle = 0;\ \bar{Q}_{\mathrm{B}}|\alpha\rangle = 0\} \tag{6}$$

a strong version of the physical condition of ref. [7,10]. Since $\mathscr{V}$ is nondegenerate, to each $|\alpha, k\rangle \in \mathscr{V}$ with ghost number k, will correspond a *conjugate* state $|\alpha, -k\rangle$ with opposite ghost number such that

$$\langle \alpha, -k|\alpha, k\rangle \neq 0\,.$$

Let us recall briefly the quartet confinement mechanism purposed by Kugo [4].

If $\varrho(x)$ is the interpolating field of a particle of given mass and spin and $\varrho^{\mathrm{as}}(x)$ is the corresponding asymptotic field, we shall consider the BRS transform of $\varrho(x)$

$$[iQ_{\mathrm{B}}, \varrho(x)]_{\pm} = \sigma(x)\,. \tag{7}$$

Let us suppose that, in the channel of the (composite) operator $\sigma(x)$, a (bound) one-particle state is present with the same mass and spin of $\varrho(x)$. We shall call $\sigma^{\mathrm{as}}(x)$ the asymptotic field of this (bound) state. Then $\varrho^{\mathrm{as}}(x)$, $\sigma^{\mathrm{as}}(x)$, together with their conjugate fields, form a quartet and the corresponding particles appear in the physical space $\mathscr{V}_{\mathrm{ph}}$ only in zero-norm combinations, so that they are confined.

For instance, in the case of the gluons

$$[iQ_{\mathrm{B}}, A^{\alpha}_{\mu}] = \partial_{\mu} c^{\alpha} + (A_{\mu} x c)^{\alpha}\,. \tag{8}$$

If the operators $(A_{\mu} x c)^{\alpha}$ develop zero-mass bound states described by the asymptotic fields $C^{\mathrm{as}\,\alpha}_{\mu}$, the gluons are confined.

Let us consider eq. (8) together with

$$[i\bar{Q}_{\mathrm{B}}, A^{\alpha}_{\mu}] = (D_{\mu}\bar{c})^{\alpha}\,, \tag{9}$$

$$[i\bar{Q}_{\mathrm{B}}, D_{\mu} c^{\alpha}] = Y^{\alpha}_{\mu} = -[Q_{\mathrm{B}}, D_{\mu}\bar{c}^{\alpha}]\,, \tag{10}$$

where

$$Y^{\alpha}_{\mu} = D_{\mu} B^{\alpha} + (D_{\mu} c \times \bar{c})^{\alpha}\,.$$

We shall assume that $(D_{\mu}\bar{c})^{\alpha}$ and Y^{α}_{μ} too develop zero-mass bound states with asymptotic fields $\bar{C}^{\mathrm{as}\,\alpha}_{\mu}$ and $B^{\mathrm{as}\,\alpha}_{\mu}$. Then C^{as}_{μ}, $\bar{C}^{\mathrm{as}}_{\mu}$, B^{as}_{μ} and the gluons asymptotic fields A^{as}_{μ} form a quadruplet. In superfield notation [5] we have

$$\begin{aligned}\Phi^{\mathrm{as}}_{\mu}(x, \theta, \bar{\theta}) = A^{\mathrm{as}}_{\mu}(x) + \bar{\theta}\big(c^{\mathrm{as}}_{\mu}(x) + \partial_{\mu} c^{\mathrm{as}}_{\mu}(x)\big) + \theta\big(\bar{c}^{\mathrm{as}}_{\mu}(x) + \partial_{\mu}\bar{c}^{\mathrm{as}}_{\mu}(x)\big) + \\ + \theta\bar{\theta}\big(B^{\mathrm{as}}_{\mu}(x) + \partial_{\mu} B^{\mathrm{as}}_{\mu}(x)\big)\end{aligned} \tag{11}$$

in eq. (11) we have omitted the renormalization constants in front of the asymptotic fields, *i.e.* our asymptotic fields are not normalized.

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[10] T. Kugo and I. Ojima: *Phys. Lett. B*, **73**, 459 (1978); *Prog. Theor. Phys. Suppl.*, **66**, 31 (1979).

Let us write down the effective Lagrangian $\mathscr{L}_{\text{eff}}$ for these asymptotic fields. $\mathscr{L}_{\text{eff}}$ should satisfy the following requirements.

First of all it must be invariant under BRS and anti-BRS transformations.

Secondly it is quadratic in the asymptotic fields, since they are free fields. Moreover, the truncated effective Lagrangian $\hat{\mathscr{L}}_{\text{eff}}$,

$$\hat{\mathscr{L}}_{\text{eff}} = \mathscr{L}_{\text{eff}} - \mathscr{L}_{\text{g.f.}} ,$$

where $\mathscr{L}_{\text{g.f.}}$ is the gauge-fixing term, must be independent of the auxiliary fields B^α $(B^\alpha = B^{\alpha\,\text{as}})$.

Finally we remark that the original N-parameter gauge group G becomes the direct product of N Abelian U_1 groups.

The key point is that, if the bound states described previously exist, the gauge invariance of a Lagrangian term no longer ensures its BRS and anti-BRS invariance.

So, the term

$$\mathscr{L}_0 = -\tfrac{1}{4}\,\mathrm{Tr}(\boldsymbol{F}^{\text{as}}_{\mu\nu}\boldsymbol{F}^{\mu\nu\,\text{as}}) ,$$

where

$$\boldsymbol{F}^{\text{as}}_{\mu\nu} = \partial_\mu \boldsymbol{A}^{\text{as}}_\nu - \partial_\nu \boldsymbol{A}^{\text{as}}_\mu$$

is gauge invariant, but breaks the BRS and anti-BRS symmetry and therefore cannot appear in $\mathscr{L}_{\text{eff}}$.

The simplest quadratic, gauge-invariant as well as BRS and anti-BRS invariant effective Lagrangian is

$$\mathscr{L}_{\text{eff}} = -\,\mathrm{Tr}\left(\frac{\mathrm{d}}{\mathrm{d}\theta}\frac{\mathrm{d}}{\mathrm{d}\bar\theta}\left[\frac{1}{4}\,\boldsymbol{\Phi}^{\text{as}}_{\mu\nu}\boldsymbol{\Phi}^{\mu\nu\,\text{as}} + \zeta\boldsymbol{\Phi}^{\text{as}}_{\theta\mu}\boldsymbol{\Phi}^{\bar\theta\mu\,\text{as}}\right]\right), \tag{12}$$

where

$$\left\{\begin{aligned} \boldsymbol{\Phi}^{\text{as}}_{\mu\nu} &= \partial_\mu \boldsymbol{\Phi}^{\text{as}}_\nu - \partial_\nu \boldsymbol{\Phi}^{\text{as}}_\mu ,\\ \boldsymbol{\Phi}^{\text{as}}_{\theta\mu} &= \frac{\partial}{\partial\theta}\,\boldsymbol{\Phi}^{\text{as}}_\mu - \partial_\mu \bar{\boldsymbol{\eta}}^{\text{as}} ,\\ \boldsymbol{\Phi}^{\text{as}}_{\bar\theta\mu} &= \frac{\partial}{\partial\bar\theta}\,\boldsymbol{\Phi}^{\text{as}}_\mu - \partial_\mu \boldsymbol{\eta}^{\text{as}} , \end{aligned}\right. \tag{13}$$

and

$$\boldsymbol{\eta}^{\text{as}} = \boldsymbol{c}^{\text{as}} - \theta\boldsymbol{B}^{\text{as}} ,$$

$$\bar{\boldsymbol{\eta}}^{\text{as}} = \bar{\boldsymbol{c}}^{\text{as}} + \bar\theta\boldsymbol{B}^{\text{as}} .$$

In terms of the component fields, eq. (12) becomes

$$\mathscr{L}_{\text{eff}} = \mathrm{Tr}\,(\boldsymbol{F}^{\text{as}}_{\mu\nu}\partial^\mu \boldsymbol{B}^{\nu\,\text{as}} + \zeta\boldsymbol{B}^{\text{as}}_\mu \boldsymbol{B}^{\mu\,\text{as}} + \tfrac{1}{2}(\partial_\mu \bar{\boldsymbol{C}}^{\text{a}}_\nu - \partial_\nu \bar{\boldsymbol{C}}^{\text{s}}_\mu)(\partial_\mu \boldsymbol{C}^{\text{as}}_\nu - \partial_\nu \boldsymbol{C}^{\text{as}}_\mu)) . \tag{14}$$

We shall not consider a term proportional to $\mathrm{Tr}\left((\partial^2/\partial\theta\,\partial\bar\theta)\,\boldsymbol{\Phi}^{\text{as}}_\mu\boldsymbol{\Phi}^{\mu\,\text{as}}\right)$ in $\mathscr{L}_{\text{eff}}$, which would give a mass to the vector particles.

With a Gaussian integration over B_μ^{as} the effective Lagrangian (14) becomes

$$\mathscr{L}_{\text{eff}} \Rightarrow -\operatorname{Tr}\left(\frac{1}{4\zeta}(\partial_\mu \boldsymbol{F}_{\mu\nu}^{as})^2 + \frac{1}{2}(\partial_\mu \overline{\boldsymbol{C}}_\nu^{as} - \partial_\nu \overline{\boldsymbol{C}}_\mu^{as})(\partial_\mu \boldsymbol{C}_\nu^{as} - \partial_\nu \boldsymbol{C}_\mu^{as})\right). \tag{15}$$

The Lagrangian (15) implies that the propagator of the gluon fields in the momentum space is

$$\Delta_{\mu\nu}(q) = \frac{1}{q^4}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \dots . \tag{16}$$

This dipole propagator leads of course to a violation of the cluster property.

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