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# Total Widths of Radially Excited V-Mesons in Relativistic Harmonic-Oscillator Models (*).

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**Summary.** — The purpose of this study is to make a quantitative estimate of the *total* widths of radial excitations of V-mesons through a systematic saturation of available channels, by means of certain current relativistic versions of the harmonic-oscillator model. In this connection, we test the effect of the usual radiation quantum (RQ) hypothesis *vs.* that of the (more recent) quark pair creation (QPC) assumption. While the former yields extremely large widths, increasing exponentially with the excitation quantum number, the latter yields interesting enough unusually small widths almost competing with the order of magnitudes found for the narrow resonances. Possible means of detection of these resonances, to throw light on the principle viability of the QPC hypothesis, are discussed.

## 1. – Introduction.

The unusually narrow widths of the $\psi|J$, $\psi'$, etc. resonances [1,2] immediately opened up several new lines of thought regarding hadronic structure and dynamics. Most of them have been (perhaps rightly) concerned with some

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new quantum number (charm, colour, etc.) ([3,4]) and concomitant selection rules for the suppression of their decays. Interestingly, more conventional mechanisms ([5,6]) have not been totally lacking, such as the coupling of the $\bar{\Omega}^+\Omega^-$ ($L=2$) structure for the $\psi$-meson to inhibit the decays through a centrifugal barrier. However, there is no evidence for any sustained interest in such mechanisms presumably because of very low estimates for $\Gamma(\psi \to e^+e^-)$ and $\Gamma(\psi' \to \to e^+e^-)$ ([7]). On the other hand, the mechanism of radial excitations of the conventional V-mesons has not received any attention in the literature. Indeed, there appears to be a tacit belief that typical hadronic widths in this region are of the order of several hundred MeV, though no generally accepted theory seems to be available to bear on this issue.

The purpose of this paper is *not* to suggest the mechanism of radial excitations of the regular V-mesons as a serious candidate for understanding the structure of the new mesons—indeed we consider such a possibility highly unlikely—but to offer a somewhat serious discussion on the issue of how big the *total widths* of such excitations are expected to be and how they vary with energy, for it appears to us that, independently of the question of any relevance (or otherwise) of the radial excitation mechanism in the context of the new resonances, the behaviour of these successive excited states of V-mesons is ,of *intrinsic* physical interest, since their existence is directly linked with the very quark model ([8]) which describes their unexcited or low-excited counter-

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(8) J. J. J. KOKEDEE: *The Quark Model* (New York, N. Y., 1969).

parts. If they have not been seriously looked for, or have been dismissed too quickly, it is perhaps because there is hardly any theoretical guideline for their detection, and not because of any convincing reason against their physical existence. In particular, the harmonic-oscillator [9] (h.o.) quark model, which works so well for the low-lying excitations, makes definite predictions on their locations and couplings without the introduction of any fresh parameters.

We should like to keep on record the results of our predictions on *total* widths and important branching ratios for these excitations. The motivation for these calculations came partly from the above considerations and partly from our recent involvement with a more general question of what should constitute an acceptable framework for the evaluation of hadronic couplings. Though the radiation quantum (RQ) [10] hypothesis has been the traditional tool for the evaluation of these couplings, the quark pair creation (QPC) [11] philosophy has, in more recent times, come in for increasing attention, with h.o. wave functions providing the necessary calculational handle on the various matrix elements. In the preceding paper [12] we have tried to examine the the relative merits of these two points of view in the context of the behaviour of *total widths* with the excitation quantum number $N$, using a suitable inclusiveness ansatz for their estimation. We have also examined two different options for a relativistic formulation of each of the RQ and QPC points of view, *viz.* a 4-dimensional covariant formulation, mainly due to KIM [13], and an earlier 3-dimensional *noncovariant* formalism due to LICHT and PAGNAMENTA [14]. We found, rather to our surprise, that, though the RQ hypothesis predicts widths exponentially growing with $N$, the QPC yields a very different pattern, at least with the LP formulation. This result encouraged us to look more closely into the predictions of QPC on the widths of radially excited V-mesons, which we record in the present paper, along with its RQ counterparts.

In sect. **2**, we summarize our essential assumptions and results on the different channels, referring for most of the calculational techniques and other details to the above paper. In sect. **3**, we discuss the significance of these results from the point of view of i) a choice between RQ and QPC and ii) a possible detection of some of these excitations through more recent experiments [15].

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## 2. – Inclusiveness ansatz and numerical results.

In this section we outline the essential assumptions for the estimation of total widths of successive radial excitations of V-mesons (ρ, ω, φ). We have considered PP, PV, VV, $B\overline{B}$, $\overline{B}B^*$ and $B^*\overline{B}^*$ channels and tabulated in table I

TABLE I. – *List of the spin matrix elements and partial widths $\Gamma'$ corresponding to the various decay modes.*

| Modes | Invariant coupling | $\Gamma'$ |
|---|---|---|
| $V \to PP$ | $2gM_V V^a_\mu \pi_b \pi_c \varepsilon_{abc}$ | $(g^2/6\pi)k^3$ |
| $V \to PV$ | $g\varepsilon_{\mu\nu\lambda\sigma} k_\mu V_\nu W_\lambda (P_\sigma + p_\sigma)$ | $(g^2/3\pi)k^3$ |
| $V \to VV$ | $gM_V U_\mu(p) W_\mu(P) V_\nu(k)(P_\nu + p_\nu)$ | $(g^2/24\pi\gamma)k^3(\alpha + 6\gamma)$ |
| $V \to \mathcal{N}\overline{\mathcal{N}}$ (charge) | $gM_V \bar{U}_B i\varrho_\mu \gamma_\mu U_{\overline{B}}$ | $(-g^2/6\pi)k\alpha$ |
| $V \to \mathcal{N}\overline{\mathcal{N}}$ (magnetic) | $g\bar{U}_B i\sigma_{\mu\nu} k_\mu \varrho_\nu U_{\overline{B}}$ | $(g^2/12\pi)k(\alpha + 6\gamma)$ |
| $V \to \overline{B}B^*$ (magnetic) | $g\bar{N}^{(k)}_{\underline{m}'} i\varepsilon_{\mu\nu\lambda\varrho} \gamma_\mu \varrho_\nu P_\lambda \Delta^{(p)}_{\varrho m}$ | $(g^2/18\pi\gamma)k[-\alpha\beta + \gamma(\delta + 4\beta)]$ |
| $V \to B^*\overline{B}^*$ (charge) | $gM_V \bar{U}_\mu i\gamma\varrho U_\mu$ | $(g^2/27\pi\gamma^2)k[2\alpha(2\gamma^2 + \beta^2) + 7\beta\gamma^2]$ |
| $V \to B^*\overline{B}^*$ (magnetic) | $g\bar{U}_{\mu'} i\sigma_{\mu\nu} \varrho_\nu k_\mu U_{\mu'}$ | unwieldy |

the coupling structures for various spin matrix elements ([16]) as well as the corresponding partial widths, where $m$, $m'$ are the masses of the decay products with $\underline{m}'$ corresponding to the *lower* spin product, and $\boldsymbol{k}$ is the c.m. 3-momentum. The coupling parameter $g$ incorporates the effects of the form factor and the $SU_6$ coefficients. The various symbols are defined as

$$\alpha = M^2 + 2m^2\,, \tag{2.1}$$

$$\beta = pk = (-M^2 + m^2 + m'^2)/2\,, \tag{2.2}$$

$$\gamma = m^2\,, \tag{2.3}$$

$$\delta = M^2 + mm'\,. \tag{2.4}$$

We have tried to saturate the contribution from different channels through the assumption of effective two-body decays into successively higher resonances up to the maximum excitation allowed by the mass kinematics of the h.o. model, *i.e.* a standard spacing $\Delta M^2 \approx 1\ (\text{GeV})^2$ for successive resonances. This practical device, which has been used in the absence of a formal (model independent) theory for the calculation of total widths, takes full account of the

([16]) A. N. MITRA: *Supermultiplet interactions among hadrons and resonances* (Department of Physics, University of Delhi, Delhi-7).

combinatorial effects involved in the counting process and conforms fairly well to many observed characteristics of multibody decays via quasi-two-body systems [17]. The $SU_6$ degrees of freedom of quarks which have little asymptotic effect on the variation of total widths have been ignored and all the decay products have been assumed to be in the ground state.

This study has been undertaken under four alternative assumptions (RQ *vs.* QPC hypothesis and 4-dimensional covariant *vs.* 3-dimensional noncovariant relativistic formulations) to simulate the relativistic structure of hadron couplings. The noncovariant LP formulation, which was originally given for the Breit frame has also been applied in the c.m. frame of the decaying particle, thus motivating two distinct types of Lorentz-contraction factors. The overlap integrals, for $M \to MM$ and $M \to B\bar{B}$ transitions, in the covariant and noncovariant (c.m. frame) models are given in I and the corresponding matrix elements in the noncovariant Breit frame are listed in the appendix. To calculate the reduced coupling constant, the $\rho \to \pi\pi$ width has been used as input. Because of the low mass of the pions, $SU_3$ is badly violated. To overcome the effects of $SU_3$ violation, due to large mass differences arising from the Lorentz-contraction factors, the reduced coupling constants are redefined by extracting a suitable factor of the dimension of $(\text{mass})^2$ in case of covariant and noncovariant c.m. (only under the RQ hypothesis) formulations. For the remaining noncovariant matrix elements, the corresponding factor has the dimension of mass. The Lorentz-contraction factors so modified are shown in table II for the different cases, in which

$$w = \frac{3}{2} + \frac{k^2}{m_b^2} + \frac{k^2}{m_c^2}\,, \tag{2.5}$$

$$v = 2\boldsymbol{k}\left(\frac{E_{ck}}{m_c^2} - \frac{E_{bk}}{m_b^2}\right), \tag{2.6}$$

$$z = \frac{1}{2}\left(\frac{1}{m_b^2} + \frac{1}{m_c^2}\right), \tag{2.7}$$

$$x = 1 + \frac{k^2}{m_b^2} + \frac{k^2}{m_c^2}\,, \tag{2.8}$$

$$u = (4w^2 - v^2)^{\frac{1}{2}}\,, \tag{2.9}$$

$$y = E_{bk}E_{ck}\,, \tag{2.10}$$

$$y' = E_{ap}E_{bp} \tag{2.11}$$

($E_{bk}$, $E_{ck}$ and $E_{ap}$, $E_{bp}$ are the energies in the c.m. frame and in the Breit frame, respectively).

The total widths have been evaluated by using the Ramanujan formula for

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[17] PARTICLE DATA GROUP: *Phys. Rev. Lett.*, **50** B, 1 (1974).

TABLE II. – *Resultant contraction factors multiplying the overall matrix elements.*

| | 4-dimensional covariant | | 3-dimensional noncovariant | | | |
|---|---|---|---|---|---|---|
| | RQ | QPC | C.m. frame | | Breit frame | |
| | | | RQ | QPC | RQ | QPC |
| $M \to MM$ | $Z/x$ | $Z/u$ | $1/y$ | $(1/y)^{\frac{1}{2}}$ | $1/E_{bp}$ | $\sqrt{m_a} m_b/y'$ |
| $M \to B\bar{B}$ | do | do | $m_b m_c/y^2$ | $\sqrt{m_b m_c}/y$ | $m_b/E_{bp}^2$ | $m_a^2 m_b/y'^2$ |

equipartition ([18]) to simulate multibody decays. The latter is given as

$$p(N) \approx (4N\sqrt{3})^{-1} \exp\left[\pi\sqrt{2N/3}\right], \tag{2.12}$$

where the integer $N$, in the h.o. model, corresponds to the (mass)$^2$ of the decaying particle in units of 1 (GeV)$^2$. The variations of total widths with $N$ corresponding to various coupling structures are displayed in tables III,

TABLE III. – *Total widths in the covariant framework.*

| $n$ | QPC | | | RQ | | |
|---|---|---|---|---|---|---|
| | $\tilde{\Gamma}_\omega^{(B)}$ | $\tilde{\Gamma}_\varphi^{(B)}$ | $\tilde{\Gamma}_\rho^{(B)}$ | $\tilde{\Gamma}_\omega^{(A)}$ | $\tilde{\Gamma}_\varphi^{(A)}$ | $\tilde{\Gamma}_\rho^{(A)}$ |
| 0 | | 2.0 | 101.3 | | 2.3 | 128.6 |
| 1 | 447.1 | 21.1 | 182.1 | 553.6 | 140.8 | 374.4 |
| 2 | 515.4 | 45.0 | 233.0 | 1135.4 | 490.1 | 977.8 |
| 3 | 754.7 | 84.3 | 365.7 | 2605.2 | 1331.7 | 2510.0 |
| 4 | 954.7 | 123.0 | 486.6 | 4719.6 | 2638.7 | 4915.0 |
| 5 | 1239.7 | 175.8 | 658.9 | 8301.0 | 4907.8 | 9192.1 |
| 6 | 1633.8 | 248.3 | 901.1 | $1.4\cdot10^4$ | 8740.4 | $1.7\cdot10^4$ |
| 7 | 2172.7 | 447.6 | 1239.0 | $2.4\cdot10^4$ | $1.5\cdot10^4$ | $2.9\cdot10^4$ |
| 8 | 2905.3 | 483.2 | 1708.1 | $4.0\cdot10^4$ | $2.5\cdot10^4$ | $5.0\cdot10^4$ |
| 9 | 3898.1 | 667.2 | 2356.6 | $6.5\cdot10^4$ | $4.1\cdot10^4$ | $8.4\cdot10^4$ |
| 10 | 5240.4 | 915.5 | 3249.3 | $1.1\cdot10^5$ | $6.7\cdot10^4$ | $1.4\cdot10^5$ |

IV and V. On the other hand, it is found that the corresponding results with the actual combinatorial mechanism based on the inclusiveness ansatz defined in I exhibit a variation slower than the equipartition assumption, though the essential qualitative features (regarding exponential rise or fall) remain the same. Thus the results based on the relatively simpler assumption of equi-

([18]) G. H. HARDY: *Ramanujan* (Cambridge, 1940).

TABLE IV. – *Total widths obtained under the noncovariant* c.m. *formulation.*

| $n$ | QPC | | | RQ | | |
|---|---|---|---|---|---|---|
| | $\bar{\Gamma}^{(B)}_{\omega}$ | $\bar{\Gamma}^{(B)}_{\varphi}$ | $\bar{\Gamma}^{(B)}_{\rho}$ | $\bar{\Gamma}^{(A)}_{\omega}$ | $\bar{\Gamma}^{(A)}_{\varphi}$ | $\bar{\Gamma}^{(A)}_{\rho}$ |
| 0 | | 4.2 | 150.5 | | 2.1 | 153.1 |
| 1 | 275.0 | 114.5 | 272.1 | 580.7 | 286.1 | 490.0 |
| 2 | 607.1 | 149.7 | 284.1 | 5236.3 | 1967.0 | 3232.5 |
| 3 | 1382.8 | 442.6 | 944.8 | $5.5\cdot10^4$ | $1.7\cdot10^4$ | $4.3\cdot10^4$ |
| 4 | 49.4 | 125.7 | 51.6 | $1.5\cdot10^5$ | $5.9\cdot10^4$ | $1.1\cdot10^5$ |
| 5 | 87.5 | 22.3 | 58.8 | $3.0\cdot10^5$ | $1.5\cdot10^5$ | $2.2\cdot10^5$ |
| 6 | 39.2 | 14.8 | 26.5 | $5.5\cdot10^5$ | $2.6\cdot10^5$ | $3.9\cdot10^5$ |
| 7 | 5.0 | 1.3 | 3.5 | $9.5\cdot10^5$ | $4.5\cdot10^5$ | $6.7\cdot10^5$ |
| 8 | 1.0 | 0.06 | 0.11 | $1.6\cdot10^6$ | $7.6\cdot10^5$ | $1.1\cdot10^6$ |
| 9 | 0.2 | 0.2 | 0.14 | $2.6\cdot10^6$ | $1.2\cdot10^6$ | $1.8\cdot10^6$ |
| 10 | 0.18 | 0.08 | 0.11 | $4.1\cdot10^6$ | $2.0\cdot10^6$ | $2.9\cdot10^6$ |

TABLE V. – *Comparison of total widths in the noncovariant Breit frame.*

| $n$ | QPC | | | RQ | | |
|---|---|---|---|---|---|---|
| | $\bar{\Gamma}^{(B)\prime}_{\omega}$ | $\bar{\Gamma}^{(B)\prime}_{\varphi}$ | $\bar{\Gamma}^{(B)\prime}_{\rho}$ | $\bar{\Gamma}^{(A)\prime}_{\omega}$ | $\bar{\Gamma}^{(A)\prime}_{\varphi}$ | $\bar{\Gamma}^{(A)\prime}_{\rho}$ |
| 0 | | 4.6 | 101.3 | | 3.4 | 114.1 |
| 1 | 43.6 | 38.4 | 36.6 | 1918.8 | 1117.9 | 1388.4 |
| 2 | 175.3 | 75.1 | 79.6 | $2.1\cdot10^4$ | $1.0\cdot10^4$ | $1.3\cdot10^4$ |
| 3 | 458.9 | 155.3 | 286.6 | $1.9\cdot10^5$ | $7.3\cdot10^4$ | $1.5\cdot10^5$ |
| 4 | 69.5 | 59.5 | 61.1 | $8.5\cdot10^5$ | $3.2\cdot10^5$ | $6.3\cdot10^5$ |
| 5 | 36.8 | 15.6 | 23.8 | $2.8\cdot10^6$ | $1.1\cdot10^6$ | $2.0\cdot10^6$ |
| 6 | 7.7 | 2.7 | 5.7 | $7.6\cdot10^6$ | $2.8\cdot10^6$ | $5.3\cdot10^6$ |
| 7 | 5.1 | 1.9 | 3.2 | $1.9\cdot10^7$ | $6.6\cdot10^6$ | $1.2\cdot10^7$ |
| 8 | 0.6 | 0.3 | 0.5 | $4.4\cdot10^7$ | $1.5\cdot10^7$ | $2.9\cdot10^7$ |
| 9 | 0.7 | 0.2 | 0.4 | $9.3\cdot10^7$ | $3.1\cdot10^7$ | $6.2\cdot10^7$ |
| 10 | 0.03 | 0.03 | 0.02 | $2.0\cdot10^8$ | $6.1\cdot10^7$ | $1.2\cdot10^8$ |

partition suffice, in principle, for a possible discrimination among the four available candidates (RQ *vs.* QPC and covariant *vs.* noncovariant). However, to have an idea of the variation of the widths with the combinatorial mechanism, sample figures in the noncovariant c.m. frame are given in table VI.

TABLE VI. – *Results of the combinatorial mechanism in the noncovariant* c.m. *frame.*

| $n$ | QPC | | | RQ | | |
|---|---|---|---|---|---|---|
| | $\Gamma_\omega^{(B)}$ | $\Gamma_\varphi^{(B)}$ | $\Gamma_\rho^{(B)}$ | $\Gamma_\omega^{(A)}$ | $\Gamma_\varphi^{(A)}$ | $\Gamma_\rho^{(A)}$ |
| 0 | | 4.2 | 150.2 | | 2.1 | 153.1 |
| 1 | 197.4 | 70.3 | 198.2 | 421.9 | 166.9 | 381.1 |
| 2 | 176.3 | 74.0 | 102.7 | 2344.5 | 1272.5 | 1739.0 |
| 3 | 714.9 | 235.3 | 553.4 | $3.2\cdot10^4$ | $1.1\cdot10^4$ | $2.4\cdot10^4$ |
| 4 | 291.1 | 139.6 | 294.4 | $1.9\cdot10^5$ | $6.5\cdot10^4$ | $1.4\cdot10^5$ |
| 5 | 180.5 | 75.3 | 276.2 | $9.4\cdot10^5$ | $3.2\cdot10^5$ | $8.0\cdot10^5$ |
| 6 | 260.4 | 70.5 | 226.6 | $4.4\cdot10^6$ | $1.4\cdot10^6$ | $3.6\cdot10^6$ |
| 7 | 83.0 | 48.8 | 110.1 | $1.6\cdot10^7$ | $6.4\cdot10^6$ | $1.5\cdot10^7$ |

## 3. – Discussion and conclusions.

Our emphasis in this paper has been to scrutinize the predictions of the h.o. model with the hypothesis of RQ and QPC and in terms of covariant and noncovariant relativistic treatments, regarding the variation of total widths in the so far untouched domain of radial excitations. An inclusiveness mechanism for the estimation of total widths from individual partial modes, as well as their mode of evaluation have been taken from I. We have listed the different figures in tables III-V to get a better idea of the actual range of numbers predicted under the different assumptions considered in this paper. The results of these different assumptions do not have much variation at lower excitations, but the differences show up sharply at higher excitations. The tables show that the RQ hypothesis is rather unattractive in view of the astronomical figures it yields even in the 3 GeV mass region. On the other hand, the QPC assumption (which also has certain intrinsic virtues ([11]) over the RQ hypothesis) shows an entirely different trend.

Indeed table III shows that the magnitudes obtained from the *covariant* QPC matrix elements are appreciably less than their RQ counterparts and the figures even appear to be generally reasonable. However, as the general analysis of I shows, the covariant QPC model predicts an exponential rise of $\Gamma_{\text{tot}}$. On the other hand, the noncovariant QPC model exhibits an altogether different and rather interesting trend. Thus tables IV-V reveal that the width first increases and then comes down abruptly (after $n \geqslant 3$). Indeed the very low magnitudes of the total widths tempt one to identify these with the observed narrow resonances, but for the fact that there still appears an order-of-magnitude difference ($\sim 10$).

It is of course still an open question as to what is the best form of relativistic formulation, covariant and noncovariant. Our general analysis seems to favour the latter, but, at a more observational level, it is interesting to speculate on some consequences of the LP formalism, should this have some element of truth in it. For example, one should expect that, if the widths of these radial excitations are indeed *that* narrow (albeit much bigger than the narrow resonances), they should in principle be observable in $e^+e^-$ collisions. On the other hand, the ones so far observed in these collisions do not quite seem to harmonize with the predictions of the QPC (LP) model for the radial excitations of V-mesons. To further test these ideas, one should perhaps look for these in $p\bar{p}$ and $\bar{n}p$ experiments. Of these, $p\bar{p}$ experiments are constrained by some selection rules which inhibit the production of even-$L$ (in particular, $L = 0$) $q\bar{q}$ states ([19]). This constraint is fortunately not operative on $\bar{n}p$ experiments, which, though more difficult to perform, seem to be within the realm of reality. It would therefore be of sufficient physical interest to check the specific predictions of these narrow widths of radially excited V-mesons in $\bar{n}p$ experiments.

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## APPENDIX

The overlap integrals in the noncovariant Breit frame for the process $M_a \to M_b + M_c$ under the RQ (taking $M_a$ as the radiation quantum) and QPC assumptions have been elaborated in paper I (sect. **2**). Substitution of the h.o. wave functions leads to

$$\bar{V}_A(n \to 00) = G_A(m_b/E_{bp}) \exp[-\boldsymbol{p}_a'^2/2] \qquad \left(\boldsymbol{p}_a' = \frac{m_b}{E_{bp}} \boldsymbol{p}_a\right) \tag{A.1}$$

and

$$\bar{V}_B(n \to 00) = (\tfrac{2}{3})^{\frac{3}{2}} G_B N_n (m_a m_b / E_{ap} E_{bp}) 3^{-n} L_n^{\frac{1}{2}}(\bar{X}) \exp[-\bar{Y}]\,, \tag{A.2}$$

where

$$\begin{cases} \bar{X} = \tfrac{2}{3}(2\boldsymbol{p}_b' - \boldsymbol{p}_a')^2\,, \qquad \bar{Y} = \tfrac{2}{3}(\boldsymbol{p}_a'^2 + \boldsymbol{p}_b'^2 - \boldsymbol{p}_a'\boldsymbol{p}_b')\,, \\ \boldsymbol{p}_b' = \dfrac{m_a}{E_{ap}} \boldsymbol{p}_b\,, \qquad N_n^{-2} = 2\pi\Gamma^3(n + 3/2)/n!\,. \end{cases} \tag{A.3}$$

The corresponding matrix elements for the transition $M_a \to B_b + \bar{B}_c$ are respectively given by

$$\bar{U}_A(n \to 00) = (3\sqrt{2}/4)^3 G_A (m_b/E_{bp})^2 \exp[-\boldsymbol{p}_a'^2] \tag{A.4}$$

(19) S. DEVONS, T. KOZLOWSKI, P. NEMETHY and S. SHAPIRO: *Phys. Rev. Lett.*, **27**, 1614 (1971).

and

$$(\text{A.5})\qquad \overline{U}_B(n \to 00) = (8/13)^{\frac{3}{2}}\, G_B N_n (m_a m_b / E_{ap} E_{bp})^2 (5/13)^n L_n^{\frac{1}{2}}(X) \exp[-Y]\,,$$

where

$$(\text{A.6})\qquad X = \frac{27}{65}\,\overline{X}\,, \qquad Y = \frac{4}{13}\left(\boldsymbol{p}_b'^2 + \frac{7}{2}\,p_b'^2 - \boldsymbol{p}_a'\boldsymbol{p}_b'\right).$$

For the calculation of total widths in the c.m. frame, one has $\boldsymbol{p}_a = 0$.

## ● RIASSUNTO (*)

Lo scopo di questo studio è di fare stime quantitative delle ampiezze totali delle eccitazioni radiali dei mesoni V attraverso una saturazione sistematica di canali disponibili per mezzo di certe versioni relativistiche correnti del modello dell'oscillatore armonico. A questo proposito si sperimenta l'effetto dell'usuale ipotesi del quanto di radiazione (RQ) nei confronti di quello della (più recente) ipotesi di creazione della coppia di quark (QPC). Mentre la prima fornisce ampiezze estremamente grandi che aumentano esponenzialmente con il numero quantico di eccitazione, la seconda fornisce, fatto abbastanza interessante, ampiezze insolitamente piccole quasi competitive con l'ordine di grandezza trovato per le risonanze strette. Si discutono possibili mezzi di rivelazione di queste particelle per chiarire la validità di principio dell'ipotesi del QPC.

(*) *Traduzione a cura della Redazione.*

**Полные ширины радиально возбужденных V-мезонов в моделях релятивистского гармонического осциллятора.**

**Резюме** (*). — Цель этой работы — произвести количественную оценку полных ширин радиальных возбуждений V-мезонов посредством систематического насыщения имеющихся каналов, используя токовые релятивистские варианты модели гармонического осциллятора. В связи с этим мы сопоставляем эффект гипотезы излучения кванта с результатами предположения рождения пары кварков. Первая гипотеза дает чрезвычайно большие ширины, возрастающие экспоненциально в зависимости от квантового числа возбуждения; вторая гипотеза приводит к необычно малым ширинам, которые почти сопоставимы с порядком величин, полученных для узких резонансов. Обсуждаются возможные способы детектирования этих резонансов, чтобы установить жизненность гипотезы рождения пары кварков.

(*) *Переведено редакцией.*