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A THEOREM IN RELATIVISTIC ELECTRONICS

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Abstract

This paper presents a theorem that connects the dispersion relation of the Electron Cyclotron Maser[1] and the oscillation equation of the Gyromonotron. This theorem gives us a simple way of obtaining the osscillating characteristics of the Gyromonotron provided that dispersion relation of the ECRM is given. Though the theorem is proved only with the case of ECRM and Gyromonotron, it holds for other kinds of Electron Masers, FEL[4] etc. and corresponding osscillators.

I. Introduction

Consider an osscillator composed by an open uniform waveguide cavity and a helix electron beam as shown in figure 1. We want to decide the normal oscillating angular frequencies and gains of the exciting electromagnetic mode in the cavity.

Suppose that the normalized longitudinal field distributrion function of the cavity is $g(z)$ with $\int|g(z)|^2dz=1$ and its Fourier transform is given by

$$\tilde{g}(k)=\int_{-\infty}^{\infty}g(z)\exp(-jkz)\,dz$$

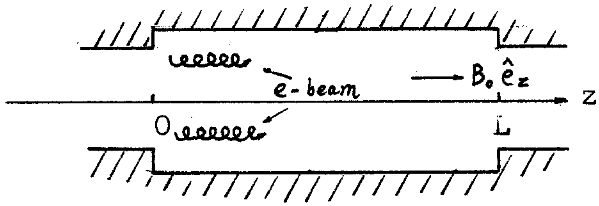

Fig.1 Configuration of Gyromonotron

And suppose that the dispersion relation for TEmn mode s-harmonic electron cyclotron maser is

$$D(k,jp)=0$$

in which

$$D(k,jp)=p^2+(k_c^2+k^2)c^2-\frac{\omega_p^2}{\gamma}\Big[\frac{p^2+k^2c^2}{(jp-kv_{\parallel}-s\omega_c)^2}\beta_\perp^2 Hms+\frac{jp-kv}{jp-kv_{\parallel}-s\omega_c}Qms+Ums\Big]$$

Theroem: The normal osscillating frequencies and gains of the stimulated electromagnetic mode of the gyromonotron is given by

$$\langle D(k,jp)\rangle=0$$

here $\langle(\ldots)\rangle$ is defined by

$$\langle\ldots\rangle=\frac{1}{2\pi}\int\tilde{g}(k)\tilde{g}^*(k)(\ldots)dk$$

II. Proof of the theorem

Considering a uniform cylindrical waveguide cavity which is being stimulated by tenuous perturbing current $\mathbf{J}(\mathbf{x},t)$, and assuming that only a TEmn mode be excited to be oscillating, we can express the radiation fields in the cavity as

$$
\begin{aligned}
&E_R(\mathbf{x},\omega)=-j\omega\mu_0\frac{jm}{R}J_m(k_cR)e^{jm\varphi}g_\alpha(z)D(\omega)\\
&E_\varphi(\mathbf{x},\omega)=j\omega\mu_0k_cJ_m'(k_cR)e^{jm\varphi}g_\alpha(z)D(\omega)\\
&E_z(\mathbf{x},\omega)=0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1)\\
&H_R(\mathbf{x},\omega)=k_cJ_m'(k_cR)e^{jm\varphi}g_\alpha'(z)D(\omega)\\
&H_\varphi(\mathbf{x},\omega)=\frac{jm}{R}J_m(k_cR)e^{jm\varphi}g_\alpha'(z)D(\omega)\\
&H_z(\mathbf{x},\omega)=k_c^2J_m(k_cR)e^{jm\varphi}g_\alpha(z)D(\omega)
\end{aligned}
$$

in which $k_c=\chi_{mn}'/a_w$, χ_{mn}' is the nth root of equation $J'(x)=0$, a_w is the radius of the waveguide, $g_\alpha(z)$ is the αth longitudinal mode distribution function of the cavity, and $D(\omega)$ is the Fourier transform of the amplitude of the oscillation.

Transforming (1) back into the version of time domain, we have that

$$
\begin{aligned}
&E_R(\mathbf{x},t)=-\mu_0\frac{jm}{R}J_m(k_cR)e^{jm\varphi}g_\alpha(z)\dot{D}(t)\\
&E_\varphi(x,t)=k_c\mu_0J_m'(k_cR)e^{jm\varphi}g_\alpha(z)\dot{D}(t)\\
&E_z(x,t)=0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2)\\
&H_R(x,t)=k_cJ_m'(k_cR)e^{jm\varphi}g_\alpha'(z)D(t)\\
&H_\varphi(x,t)=\frac{jm}{R}J_m(k_cR)e^{jm\varphi}g_\alpha'(z)D(t)\\
&H_z(x,t)=k_cJ_m(k_cR)e^{jm\varphi}g_\alpha(z)D(t)
\end{aligned}
$$

in which $\dot{D}(t)=dD(t)/dt$.

Substituting (2) into the Maxwell equation

$$\nabla\times\mathbf{H}=\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}+\mathbf{J}(\mathbf{x},t) \qquad (3)$$

and applying the normalization technique, we find the equation for $D(t)$:

$$\ddot{D}(t)g_\alpha(z)+c^2\left[k_c^2g_\alpha(z)-g_\alpha''(z)\right]D(t)=\frac{-1}{\varepsilon_0N_{mn}^2}\int_{S_W}\mathbf{J}_\perp(\mathbf{x},t)\cdot\mathbf{E}^*(R,\varphi)d\sigma \qquad (4)$$

in which

$$\mathbf{E}_{\perp}^{!}(R,\varphi)=-\mu_0 \frac{jm}{R}J_m(k_c R)e^{jm\varphi}\hat{e}_R+k_c\mu_0 J_m^{!}(k_c R)e^{jm\varphi}\hat{e}_\varphi$$

$$N_{mn}^{!}=\pi k_c^2\mu_0 a_w^2(1-\frac{m}{k_c^2 a_w^2})J_m^2(k_c a_w)$$

$$\mathbf{J}_{\perp}(\boldsymbol{x},t)=-|e|\iint_{S_W}\left[v_{\perp}\cos(\phi-\varphi)\hat{e}_R+v_{\perp}\sin(\phi-\varphi)\hat{e}_\varphi\right]f_1\,d\mathbf{P}$$

with $d\sigma$ denoting $RdRd\varphi$, s_w representing tne cross section of the waveguide,and $\mathbf{f}_1$ standing for the perturbed distribution function of the e-beam.(R, φ, z) are the cylindrical coordinates with axis z along the axis of the cavity.

Eq.(4) describes the action of the beam upon the radiation field in the cavity;the influence of the radiation field upon the beam is governed by the linearized Vlasov equation:

$$\frac{\partial f_1}{\partial t}+\mathbf{v}\cdot\nabla f_1-|e|\mathbf{v}\times B_0\hat{e}_z\cdot\nabla_p f_1=$$
$$|e|\left[\mathbf{E}(\boldsymbol{x},t)+\mathbf{v}\times\mathbf{B}(\boldsymbol{x},t)\right]\cdot\nabla_p f_0 \qquad (5)$$

For simplicity,we introduce a six-dimensional phase coordinates (R_g, φ_g, r_L, Θ, $p_{\parallel}$, z) to replace the general coordinates (x, y, z, p_x, p_y, p_z),which are defined by

$$\begin{cases} R_g=\left[(x-p_y/a)^2+(y+p_x/a)^2\right]^{1/2} \\ \varphi_g=\mathrm{arctg}(\frac{y+p_x/a}{x-p_y/a}) \\ r_L=(p_x^2+p_y^2)^{1/2}/a \\ \Theta=\mathrm{arctg}(p_y/p_x)-az/p_z-\pi/2 \\ p_{\parallel}=p_z \end{cases} \qquad (6)$$

where $a=|e|B_0$.

In terms of the introduced coordinates (R_g, φ_g, r_L, Θ, $p_{\parallel}$, z),Eq.(5) can be reduced to the following form

$$\frac{\partial f_1}{\partial t}+v_{\parallel}\frac{\partial f_1}{\partial z}=T_1\frac{\partial f_0}{\partial R_g}+T_2\frac{\partial f_0}{\partial \varphi_g}+T_3\frac{\partial f_0}{\partial r_L}+T_4\frac{\partial f_0}{\partial \Theta}+T_5\frac{\partial f_0}{\partial p_{\parallel}} \qquad (7)$$

in which

$$T_1=\frac{|e|}{a}\Big[(E_R-v_{\parallel}B_\varphi)\sin(\varphi_g-\varphi)-(E_\varphi+v_{\parallel}B_R)\cos(\varphi_g-\varphi)+ v_{\perp}B_z\sin(\varphi_g-\theta)\Big]$$

$$T_2=\frac{|e|}{aR_g}\Big[(E_R-v_{\parallel}B_\varphi)\cos(\varphi_g-\varphi)+(E_\varphi+v_{\parallel}B_R)\sin(\varphi_g-\varphi)+$$

$$v_\perp B_z \cos(\varphi_g-\theta)$$
$$T_3=\frac{|e|}{a}\left[(E_R-v_{/\!/}B_\varphi)\sin(\varphi-\theta)+(E_\varphi+v_{/\!/}B_R)\cos(\varphi-\theta)\right]$$
$$T_4=\frac{|e|}{ar_\perp}\Big\{-(E_R-v_{/\!/}B_\varphi)\cos(\varphi-\theta)+(E_\varphi+v_{/\!/}B_R)\sin(\varphi-\theta)$$
$$-v_\perp B_z-z\frac{p_\perp^2}{p_{/\!/}^2}\omega_c\left[B_R\cos(\varphi-\theta)-B_\varphi\sin(\varphi-\theta)\right]\Big\}$$
$$T_5=|e|\left\{E_z-v_\perp\left[B_R\cos(\varphi-\theta)-B_\varphi\sin(\varphi-\theta)\right]\right\}$$

with $\theta=\Theta+az/p$.

It should be noted that, in deriving (7), we have utilized the geometry relations of (6) as shown in figure 2.

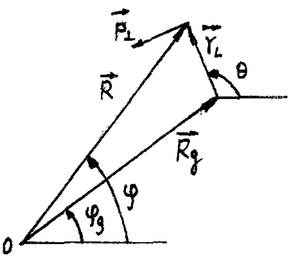

Fig.2 Geometry of coordinate transformation

Fourier-Laplace transforming Eqs.(4) and (7), assuming that the electron beam is axially symmetric yielding that $\partial f_0/\partial\varphi_g=\partial f_0/\partial\Theta=0$, and then combining the two resulting equations, we have that

$$(p^2+\omega_{mn}^2)\tilde{g}_\alpha(k)\tilde{D}(p)=D_I(k,p)+D_{II}(k,p) \qquad (8)$$

where

$$D_I(k,p)=\left[pD(t=0)+\dot{D}(t)\Big|_{t=0}\right]\tilde{g}_\alpha(k)+\frac{k_c\mu_0\,|e|\,a}{\varepsilon_0 N_{mn}^!}\sum_{s=-\infty}^{\infty}$$
$$\iiiint\!\!\int R_g r_\perp\, dR_g\, d\varphi_g\, dr_\perp\, d\Theta\, dp_{/\!/}\, v_\perp J_{m-s}(k_c R_g)\cdot$$
$$J'(k_c r_\perp)\frac{f_1(k+s\omega_c/v_{/\!/},\,t=0)}{p+jkv_{/\!/}+js\omega_c}e^{-j(m-s)\varphi_g-js\Theta}$$
$$-\frac{8\pi^3k_c^2\mu_0^2e^2}{\varepsilon_0 N_{mn}^!}D(t=0)\sum_s\iiint R_g r_\perp\, dR_g\, dr_\perp\, dp_{/\!/}\, v_\perp$$
$$J_{m-s}(k_c R_g)J_s'(k_c r_\perp)\tilde{g}_\alpha(k)\left[J_{m-s}'(k_c R_g)J_s(k_c r_\perp)\right.$$
$$\left.\cdot\frac{\partial f_0}{\partial R_g}+J_{m-s}(k_c R_g)J_s'(k_c r_\perp)\frac{\partial f_0}{\partial r_\perp}\right](p+jkv_{/\!/}+js\omega_c)^{-1}$$

$$D_{\mathrm{I}}(k,p)=-8\pi^3\frac{k_c^2\mu_0^2e^2}{\varepsilon_0N_{mn}^1}\tilde{D}(p)\tilde{g}_\alpha(k)\sum_s\iiint R_g r_L\, dR_g\, dr_L\, dp_{/\!/}\cdot$$
$$v_\perp J_{m-s}(k_cR_g)J_s'(k_cr_L)\left\{\left[-J_{m-s}'(k_cR_g)J_s(k_cr_L)\right.\right.$$
$$\left.+\frac{m-s}{R_g}\frac{jv_\perp}{p+jkv_{/\!/}+js\omega_c}J_{m-s}(k_cR_g)J_s'(k_cr_L)\right]\cdot$$
$$\frac{\partial f_0}{\partial R_g}+\left[(p+jkv_{/\!/})\frac{\partial f_0}{\partial r_L}-jav_\perp k\frac{\partial f_0}{\partial p_{/\!/}}\right]J_{m-s}(k_cR_g)\cdot$$
$$\left.J_s'(k_cr_L)\frac{1}{p+jkv_{/\!/}+js\omega_c}\right\}$$

in which $\tilde{D}(p)=\int_0^\infty D(t)\exp(-pt)dt$, $\tilde{g}_\alpha(k)=\int_{-\infty}^{\infty}g_\alpha(z)\exp(-jkz)dz$, $f_1(k,t)=\int_{-\infty}^{\infty}f(z,t)\exp(-jkz)dz$, $\omega_{mn}^2=(k^2+k_c^2)c^2$.

Choosing such a $f_1(k,t=0)$ that $D_{\mathrm{I}}(k,p)=[pD(t=0)+\dot{D}(t)|_{t=0}]\tilde{g}_\alpha(k)$ and taking the equilibrium distribution function as

$$f_0=\frac{N_e}{4\pi R_g r_L a}\delta(R_g-R_{g0})\delta(r_L-r_{L0})\delta(p_{/\!/}-p_{/\!/0}),$$

we can simplify (8) to the following form

$$(p^2+\omega_{mn}^2)\tilde{g}_\alpha(k)\tilde{D}(p)=\left[pD(t=0)+\dot{D}(t)|_{t=0}\right]\tilde{g}_\alpha(k)+$$
$$\frac{k_c^2\mu_0^2e^2N_e}{\varepsilon_0N_{mn}^1m_0\gamma_0}\sum_s\left[\frac{p^2+k^2c^2}{(jp-kv_{/\!/0}-s\omega_{c0})^2}\beta_{\perp 0}^2H_{ms}+\right.$$
$$\left.\frac{jp-kv_{/\!/0}}{jp-kv_{/\!/0}-s\omega_{c0}}Q_{ms}+U_{ms}\right]\tilde{g}_\alpha(k)\tilde{D}(p) \qquad (9)$$

in which

$$H_{ms}=J_{m-s}^2(k_cR_{g0})J_s'^2(k_cr_{L0})$$
$$Q_{ms}=2H_{ms}+2k_cr_{L0}J_{m-s}(k_cR_{g0})J_s''(k_cr_{L0})\cdot$$
$$\left[J_{m-s}(k_cR_{g0})J_s''(k_cr_{L0})+\frac{m-s}{s}\frac{r_L}{R_g}J_{m-s}'(k_cR_{g0})J_s'(k_cr_{L0})\right]$$
$$U_{ms}=k_cr_{L0}J_s'(k_cr_{L0})\left\{J_s(k_cr_{L0})\left[\left(1-\left(\frac{m-s}{k_cR_{g0}}\right)^2\right)J_{m-s}^2(k_cR_{g0})-\right.\right.$$
$$\left.\left.J_{m-s}'^2(k_cR_{g0})\right]-2\frac{m-s}{s}J_s'(k_cr_{L0})J_{m-s}(k_cR_{g0})J_{m-s}'(k_cR_{g0})\right\}$$

and N_e is the electron beam density.

Multiplying (9) by $\tilde{g}^*(k)$, and then integrating it with respect to k, we can arrive at that

$$\tilde{D}(p)=\frac{pD(t=0)+\dot{D}(t)|_{t=0}}{\langle D(k,jp)\rangle} \qquad (10)$$

in which

$$D(k,jp)=p^2+\omega_{mn}^2-\frac{\omega_p^2}{\gamma_0}\sum_s \left[\frac{p^2+k^2c^2}{(jp-kv_{//0}-s\omega_{c0})^2}\beta_{\perp 0}^2 H_{ms}+\frac{jp-kv_{//0}}{jp-kv_{//0}-s\omega_{c0}}Q_{ms}+U_{ms}\right],$$

$\langle(\ldots)\rangle$ is defined by

$$\langle(\ldots)\rangle=\frac{1}{2\pi}\int \tilde{g}_\alpha(k)g_\alpha^*(k)\,(\ldots)dk$$

and $\omega_p^2=k_c^2\mu_0^2e^2N_e/(\varepsilon_0 m_0 N'_{mn})$.

From Eq.(10), we know that the long term characteristic of oscillation is determined by solving equation

$$\langle D(k,jp)\rangle=0 \qquad (11)$$

for jp or ω=jp. Therfore Eq.(11) can be referred as the characteristic equation of the gyromonotron based on ECRM interaction.

Thus we have proved the theorem in the case of gyrotron. In view of above proof procedure, it is evident that the theorem holds for other kinds of HF oscillators based upon corresponding maser and FEL interactions.

According to the theorem, to determine the characteristic frequencies of an oscillator based upon some electron beam instability, one just need to

1) find the dispersion relation of the beam instability $D(k,\omega)=0$,

2) write down the normalized longitudinal cavity field distribution function $g(z)$ and its Fourier transformation $\tilde{g}(k)$,

3) solve $\langle D(k,\omega)\rangle=0$ for ω.

It should be pointed out that, other than Eq.(11), the theorem can take different form. Let us come back to expression (9). Multiplying (9) by $\prod_s(jp-kv_{\parallel 0}-s\omega_{c0})^2$ before performing the operator $\langle\ldots\rangle$ on it, we have that

$$D(p)=\frac{\langle\prod_s(jp-kv_{\parallel 0}-s\omega_{c0})^2\rangle[pD(t=0)+\dot{D}(t)|_{t=0}]}{\langle\prod_s(jp-kv_{\parallel 0}-s\omega_{c0})^2\,D(k,jp)\rangle} \qquad (12)$$

From (12), we know that the normal oscillation characteristic is governed by

$$\langle\prod_s(jp-kv_{\parallel 0}-s\omega_{c0})^2 D(k,jp)\rangle = 0 \qquad (13)$$

We shall see in next section that Eq.(13) is more convenient and easier to solve than (11).

In practical situation, the cold cavity resonance frequency $jp^{(0)}$ is chosen to be close to one of the cyclotron frequency harmonic, i.e., $jp^{(0)} \approx l\omega_{c0}$; and because the electron beam density is sufficiently low, the solution to (13) is in fact near $jp^{(0)}$. So we can drop non-resonance terms, i.e., terms with s=l, from D(k,jp), and then Eq.(13) can be approximated by

$$\langle(jp-kv_{\parallel 0}-l\omega_{c0})^2 D_0(k,jp)\rangle = 0 \qquad (13')$$

in which $D_0(k,jp)$ is the simplified D(k,jp).

III. Application of theorem

To illustrate how to use the theorem, let us consider an example of open cavity gyrotron with its axial field distribution function given by

$$g(z)=\sqrt{\frac{2}{L}}\sin(k_\parallel z)\,\mathrm{rect}\left(\frac{z-L/2}{L}\right) \qquad (14)$$

here $k_\parallel=\pi/L$, L is the cavity length.

It is easy to obtain the Fourier transform of (14):

$$\tilde{g}(k)=\sqrt{\frac{2}{L}}\frac{2k_\parallel}{k_\parallel^2-k^2}\exp(-jkL/2) \qquad (15)$$

Substituting (15) and its complex conjugate into Eq.(11), we get that

$$\omega^2-\Omega^2-\frac{\omega_p^2}{\gamma_0}\sum_s\Big[\beta_{\perp 0}^2 H_{ms}\ (\omega^2-k_{/\!/}^2c^2)I_s^{(1)}+$$
$$(1-js\omega_{c0}I_s^{(2)})Q_{ms}\ +U_{ms}\Big]=0 \qquad (16)$$

in which $\Omega^2=(k_{/\!/}^2+k_c^2)c^2$,

$$I_s^{(1)}=\frac{2k_{/\!/}}{v_{/\!/0}^2}\Bigg\{\Big(L+\frac{1}{jk_{/\!/}}\Big)\Big[-\frac{v_{/\!/0}}{j\Omega_-}Le^{-j\Omega_-L/v_{/\!/0}}+\frac{v_{/\!/0}^2}{\Omega_-^2}(e^{-j\Omega_-L/v_{/\!/0}}-1)\Big]-\Big\{\frac{jv_{/\!/0}}{\Omega_-}L^2e^{-j\Omega_-L/v_{/\!/0}}-j\frac{2v_{/\!/0}}{\Omega_-}\Big(j\frac{v_{/\!/0}L}{\Omega_-}\exp(-j\Omega_-L/v_{/\!/0})+\frac{v_{/\!/0}^2}{\Omega_-^2}(e^{-j\Omega_-L/v_{/\!/0}}-1)\Big)\Big\}+\Big(L+\frac{j}{k_{/\!/}}\Big)\Big[\frac{jv_{/\!/0}L}{\Omega_+}e^{-j\Omega_+L/v_{/\!/0}}+\frac{v_{/\!/0}^2}{\Omega_+^2}(e^{j\Omega_+L/v_{/\!/0}}-1)\Big]-\Big\{\frac{jv_{/\!/0}L^2}{\Omega_+}e^{-j\Omega_+L/v_{/\!/0}}-\frac{j2v_{/\!/0}}{\Omega_+}\Big[\frac{jv_{/\!/0}L}{\Omega_+}e^{-j\Omega_+L/v_{/\!/0}}+\frac{v_{/\!/0}^2}{\Omega_+}(e^{-j\Omega_+L/v_{/\!/0}}-1)\Big]\Big\}\Bigg\}$$

$$I_s^{(2)}=-j\frac{2k_{/\!/}}{v_{/\!/0}}\Big\{\Big(L-\frac{j}{k_{/\!/}}\Big)\frac{jv_{/\!/0}}{\Omega_-}(e^{-j\Omega_-L/v_{/\!/0}}-1)-\Big[\frac{jv_{/\!/0}L}{\Omega_-}e^{-j\Omega_-L/v_{/\!/0}}+\frac{v_{/\!/0}^2}{\Omega_-^2}(e^{-j\Omega_-L/v_{/\!/0}}-1)\Big]+\Big(L+\frac{j}{k_{/\!/}}\Big)\frac{jv_{/\!/0}}{\Omega_+}(e^{-j\Omega_+L/v_{/\!/0}}-1)-\Big[\frac{jv_{/\!/0}L}{\Omega_+}e^{-j\Omega_+L/v_{/\!/0}}+\frac{v_{/\!/0}}{\Omega_+^2}(e^{-j\Omega_+L/v_{/\!/0}}-1)\Big]\Big\}$$

here $\Omega_\pm=\omega\pm k_{/\!/}v_{/\!/0}-s\omega_{c0}$, and $\omega=jp$.

Solution to (16) of desired order of approximation can be obtained by either numerical calculation method or iteration method. With iteration method, the zero order approximation solution of (16) can be chosen reasonably as the 'cold' resonance frequency of the cavity, i.e., $\Omega=(k_{/\!/}^2+k_c^2)^{1/2}c$.

If substitute (15) and its conjugate complex into (13'), we obtain that

$$\omega^4-2\langle B\rangle\omega^3-\Big[\langle B^2\rangle-\langle A\rangle c^2-\frac{\omega_p^2}{\gamma_0}(Q_{ml}-\beta_{\perp 0}^2H_{ml})\Big]\omega^2+\Big[2\langle AB\rangle c^2+\frac{\omega_p^2}{\gamma_0}Q_{ml}\ (l\,\omega_{c0})\Big]\omega-\langle AB^2\rangle c^2-\frac{\omega_p^2}{\gamma_0}\Big[Q_{ml}\ k_{/\!/}^2v_{/\!/0}^2+\beta_{\perp 0}^2H_{ml}\ k_{/\!/}^2c^2\Big]=0 \qquad (17)$$

where $\omega=jp$,

$\langle A\rangle=k_c^2+k_{/\!/}^2$,
$\langle B\rangle=l\,\omega_{c0}$,
$\langle B^2\rangle=k_{/\!/}^2v_{/\!/0}^2+(l\,\omega_{c0})^2$,
$\langle AB\rangle=(k_c^2+k_{/\!/}^2)l\,\omega_{c0}$,
$\langle AB^2\rangle=k_c^2k_{/\!/}^2c^2+(k_c^2+k_{/\!/}^2)(l\,\omega_{c0})^2+k_{/\!/}^4v_{/\!/0}^2$.

Since Eq.(17) is an algebrical equation of the fourth order,its roots are much easier to find than that of Eq.(16),making (17) of more practical value in engineering design than (16).

Eqs.(16) and (17) are two forms of linear theory for gyromonotron.Owing to the theorem,not only is their derivation simple but also their formulae are compact and easy to solve in contrast to the comlicated derivation procedure and the overelaborate formula of the general gyrotron theory [2,3].

IV. Conclusions

A useful theorem in the field of relativistic electronics has been proposed and proved in this paper.According to the theorem,the oscillating characteristic of any HF oscillator,under the condition of small signal,is connected directly with the dispersion relation of the electron beam instability on which the oscillator is based.Besides, the theorem provides a simple way of analysing the linear oscillating characteristic of the oscillator,making the theorem valuable in engineering.

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