Characteristic features of evaporative cooling of droplets in high-temperature flows

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CHARACTERISTIC FEATURES OF EVAPORATIVE COOLING OF DROPLETS IN HIGH-TEMPERATURE FLOWS

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Numerical investigation of the evaporative cooling of water droplets in a high-temperature gas flow (temperature above 1000°C) has been carried out for two limiting regimes: a continuous medium and a free-molecular regime. The results of modeling have shown that with a small content of water vapor in the flow, due to evaporative cooling the droplet temperature attains a stationary value that is lower than the stream temperature by hundreds of degrees.

Keywords: *heat transfer coefficient, droplet temperature, flow reactor, thermophoresis.*

Introduction. Evaporation of liquid droplets in a gaseous medium has been investigated for a rather long time already [1], but as ever it attracts the attention of engineers and specialists from many areas of applied science. For example, injection of water droplets is widely used in the process of cooling hot gases, in suppression of fire areas, in cleaning flue gases in industrial furnaces, and in many other practically important processes. It should be noted specially that the evaporative cooling of water droplets is widely used in natural draft cooling towers of electric power stations and industrial enterprises, as well as in ventilation and conditioning systems [2].

The processes of heat and mass exchange of droplets with a gas flow are intensified considerably on decrease of the droplets' radii. Of particular interest is evaporative cooling of sufficiently small droplets, in particular, those with their radii lying in the submillimeter range, where such a cooling possesses a number of characteristic features of practical interest [3]. Evaporation of binary droplets has its specific features and was investigated in a diffusion approximation in [4]. The results of investigation of evaporation of multicomponent droplets of a fuel are presented in [5]. It should be noted that nonlinear processes of heat and mass transfer connected with vapor condensation of droplets in a laminar diffusion chamber were considered in detail in [6].

Evaporative cooling of droplets having radius of the order of microns at a lowered pressure can be described in a free-molecular approximation, which greatly simplifies their mathematical simulation. Experiments and calculations showed that in the free-molecular approximation the processes of heat and mass transfer of droplets are intensified substantially [7, 8].

An unusual phenomenon was discovered, in particular, in [3, 7, 8] residing in the fact that during evaporation the temperature of droplets rapidly attains a stationary value that can differ greatly from the temperature of the high-temperature stream by hundreds of degrees, which substantially determines the rate of droplet evaporation.

The aim of the present work was to construct mathematical models and to show the results of numerical investigation of evaporative cooling obtained on the basis of these models for droplets of submillimeter and micron ranges. In the first example, we consider evaporative cooling in the regime of a continuous medium, and in the second example, in a free-molecular regime. Each of the examples has its own independent practical importance.

1. Mathematical Model of Evaporative Cooling of Droplets in the Approximation of a Continuous Medium. We consider a droplet with the initial temperature T_d which is sprayed by a sprayer into a transverse hot gas stream with the initial temperature T_g and a low mass density of stream ρ in the stream. We assume that the size of droplets lies within the range 50–200 μm, so that we can use the continuous medium approximation.

The motion of a single droplet in the transverse gas steam is described by a system of differential equations:

$$\frac{dv_x}{dt} = \frac{3C\rho_g v_x}{8\rho_d R_d} v_{\text{long}}, \tag{1}$$

$$\frac{dv_z}{dt} = \frac{\rho_g - \rho_d}{\rho_d} g - \frac{3C\rho_g (v_z - u_g)}{8\rho_d R_d} v_{\text{long}}, \tag{2}$$

$$\frac{dx}{dt} = v_x, \tag{3}$$

$$\frac{dz}{dt} = v_z. \tag{4}$$

An analysis of these equations shows that several milliseconds after entry into the gas stream, the velocity of droplets of radius R_d virtually coincides with the vapor-gas flow velocity. An important consequence of this fact is that the processes of heat and mass exchange of a droplet with the vapor-gas flow proceed rather slowly, practically in a diffusion regime.

The processes of evaporative cooling of droplets in the case of their moderate concentration in a gas stream are described by a system of ordinary differential equations [2] that in our example account additionally for radiative heat transfer.

From the droplet mass conservation law we obtain an equation for the change in the droplet radius:

$$\frac{dR_d}{dt} = \frac{\gamma\,(\text{Re})\,(\rho - \rho_s\,(T_d))}{\rho_d}, \tag{5}$$

the energy conservation law yields an equation for the droplet temperature T_d [2]:

$$\frac{dT_d}{dt} = \frac{3\,[\alpha\,(\text{Re})\,(T_s - T_d) - \gamma\,(\text{Re})\,(c_w T_d - U)\,(\rho - \rho_s\,(T_d)) + \varepsilon\sigma\,(T_g^4 - T_d^4)]}{c_d \rho_d R_d}. \tag{6}$$

The first term of the sum on the right-hand side of Eq. (6) describes the convective heat flux to a droplet, the second term represents the heat flux connected with droplet evaporation, and the third term accounts for radiative heat transfer. The relative contribution of radiative heat transfer at temperatures of the order of 1300 K is equal to about 1%. In Eq. (6), the terms connected with the evaporation of water molecules are dominating due to the high value of U and to the heating of the droplet by convective heat transfer (the first term).

The characteristic time of temperature field equalization in the droplet is $\tau \sim R_d^2/(\pi^2 a^2)$, so $\tau \approx 10^{-4}$ s for a droplet of radius 100 µm. At times much in excess of τ, the approximation of the average temperature of a droplet T_d can be used. For droplets of smaller, size the applicability of the assumption on the average temperature of a droplet is even more justified.

The system of ordinary differential equations (1)–(6) entirely characterizes the process of droplet evaporation in a high-temperature gas stream; this system was solved numerically in the MathCAD 14 environment using the reliable and stable Runge–Kutta method of the 4th order.

The following initial conditions were taken for calculations: temperature of the vapor-gas stream T_g = 1273 K and its velocity u_g = 10 m/s, initial droplet velocity u_x = 20 m/s, initial temperature of the droplet T_{d0} = 293 K, density of water vapor in the sweeping gas stream ρ = 0.013 kg/m^3.

Results of calculations. The results of numerical simulation for the system of equations (1)–(6) are presented in Fig. 1. The dependence of the dimensionless radius of a droplet on the time of evaporation is given in Fig. 1a. The initial radius of the droplet R_{d0} served as the scale for the normalization of radii. It is seen that the duration of the evaporation process substantially exceeds the time of development of temperature fields inside the droplet, as well as the time of equalization of the droplet and stream velocities.

In the approximation of a continuous medium the rate of change of the droplet size depends on the radius due to the dependence of the mass transfer coefficient γ on the Reynolds number Re [2] at the initial time of the entry of
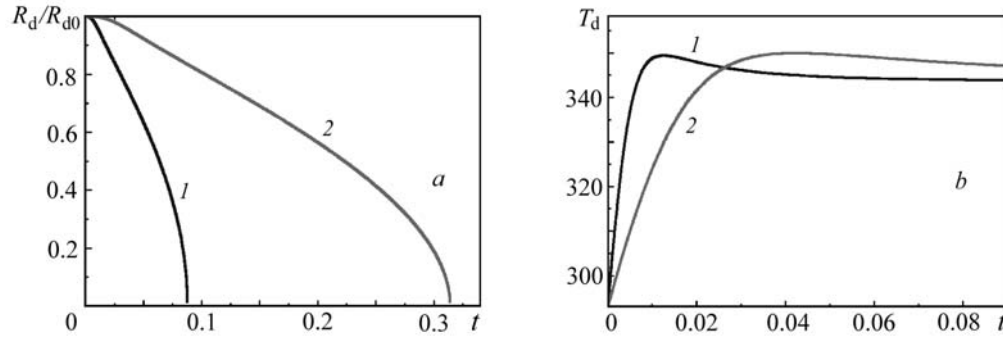
Fig. 1. Variation of the dimensionless droplet radius in a high-temperature stream (a) and the profile of its temperature (b): 1) $R_{d0} = 50$ μm; 1) 100 μm. t, s; T, K.

a droplet into a stream. In the diffusion regime the rate of variation of the droplet size is inversely proportional to the radius, which is seen from Fig. 1a.

Figure 1b shows the change in the temperature of droplets when they occur in a transverse high-temperature vapor-gas stream. According to Eq. (6), the rate of heating is higher in finer droplets.

At high temperatures the stationary temperature of a droplet is given by the vanishing of the right-hand side of Eq. (6). In some evaporation regimes, when there is a sufficient amount of water vapor ρ in the vapor-gas stream, the attainment of a stationary value is also possible.

Neglecting the radiative heat transfer, from Eq. (6), after transformations, we obtain an approximate equation for the stationary temperature T_d:

$$T_g \approx T_d + \frac{D}{\lambda}\left(c_w T_d - U\right)\left(\rho - \rho_s\left(T_d\right)\right).\tag{7}$$

Equation (7) determines the stationary temperature of a droplet, which is independent of the radius of the latter. The main characteristic feature of the right-hand side of Eq. (7) is the exponential dependence of the density of saturated vapors on temperature and the high magnitude of the factor due to the latent heat of phase transition U, with the factor $D/\lambda \sim 10^{-2}$. It is important to emphasize that the effect of evaporative cooling occurs when the density of water vapors in the stream is rather small and the second term in Eq. (7) is positive. The preliminary results of this part of the work were published in [3].

2. Mathematical Model of Evaporative Cooling of a Droplet in a Free-Molecular Regime. The second example of the evaporative cooling of a water droplet is associated with the problem of heating and motion of a micron droplet that with a vapor-air stream enters a flow reactor with a high wall temperature. At the initial moment, the content of water vapor in the stream is such that the droplet is in equilibrium with the vapor, and it does not evaporate. Such conditions in the stream appear when nanoparticles are produced by the method of low pressure spray pyrolysis in an aerosol reactor [7 and references therein].

According to [7], the flow of a gas stream in a flow reactor is laminar, so that the Reynolds number Re = $u_0 R_r \rho_g / \eta \sim 100$. The gas velocity profile in the reactor is approximately described by the Poiseuille equation that accounts for the change in the average gas stream velocity as it is heated in the reactor:

$$u\left(r\right) = 2u_{g0}\left(T_g\right)\left|1 - \left(\frac{r}{R_r}\right)^2\right|,\tag{8}$$

where $u(r)$ is the velocity profile, R_r is the reactor radius, and u_{g0} is the initial velocity of the gas-carrier flow.

The change in the gas temperature is described by the equation of convective heat conduction:

$$u\left(r, z\right)\frac{\partial T_g}{\partial z} = \frac{1}{\rho_g c_g}\left|\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda\left(r, z\right) r \frac{\partial T_g}{\partial r}\right)\right|,\tag{9}$$

where ρ_g, c_g, $\lambda(r, z)$ are the density, heat capacity, and thermal conductivity of the gas-carrier, respectively. Moreover, for calculation the following continuity equation in the integral form was used:

$$u_0 \rho (T_{g0}) = \bar{u} (T) \, \rho (T) , \tag{10}$$

where $\bar{u}(T)$ is the average velocity of the gas-carrier flow over the reactor section.

On the axis of the flow reactor the condition of the cylindrical symmetry of the temperature field is set:

$$\frac{\partial T_g (0, z)}{\partial r} = 0 . \tag{11}$$

The gas near the flow reactor wall is heated faster than in the central portion. This leads to the appearance of the radial gradient of the gas-carrier temperature, and the droplet is exposed to the thermophoretic force. Thus, this force changes the radial position of the droplet, which is described by the equation

$$\frac{dR_{rad}}{dz} = \frac{u_{tr}}{u (r)} , \tag{12}$$

where R_{rad} is the radial position of the droplet; u_{tr} is the rate of thermophoresis [9]:

$$u_{tr} = -\frac{3}{4} \frac{\eta \nabla T}{\rho_g T_{g0}} . \tag{13}$$

Since a lowered pressure is maintained in the reactor, for a droplet of radius of the order of a micron the thermophoresis rate is calculated in the free-molecular approximation; in this regime the thermophoresis rate is independent of the droplet radius.

In the free-molecular approximation the equation describing a change in the radius is written in the form

$$\frac{dR_d}{dz} = \frac{m_w}{u (r) \, \rho_w} \left[\frac{p_p (T_g)}{\sqrt{T_g}} - \frac{p_s (T_d)}{\sqrt{T_d}} \right], \tag{14}$$

where m_w is the mass of a water molecule; ρ_w is the water density; p_p and p_s are the partial and saturated pressure of water vapors, respectively.

The equation for temperature variation is

$$\frac{dT_d}{dz} = \frac{3\sigma (T_{wall}^4 - T_d^4)}{u (r) \, c_w \rho_w R} - \frac{3T_d}{R_d} \frac{dR_d}{dz} + \frac{3}{u (r) \, c_w \rho_w R_d} A , \tag{15}$$

$$A = \rho_g \left(\frac{c_g T_g + 0.5kT_g}{\sqrt{2\pi k m_g T_g}} - \frac{c_g T_d + 0.5kT_d}{\sqrt{2\pi k m_g T_d}} \right) + p_p (T_g) \frac{c_w T_g + 0.5kT_g}{\sqrt{2\pi k m_w T_g}} - p_s (T_d) \frac{c_w T_d + 0.5kT_d}{\sqrt{2\pi k m_w T_d}} ,$$

where T_{wall} is the temperature of the reactor wall; c_w and c_g are the heat capacity per molecule of water and of the gas-carrier, respectively.

Results of calculations. The partial differential equation (9) was solved by the method of straight lines [10] together with a system of ordinary differential equations ((8), (10)–(15)) in the MathCAD 14 environment, with the use of the Runge–Kutta method of fourth order.

The initial conditions were: air flow temperature 273 K, temperature of the reactor walls 1600 K, and $u_0 =$ 5.5 m/s; numerical density of water molecules in the stream was equal to the numerical density of saturated water vapor at the initial temperature of the droplets entering the furnace.

Figure 2 demonstrates the variation in the carrier gas temperature as it moves in the reactor. It is seen that under such conditions the temperature virtually does not change at the flow center. Indeed, the ratio of the charac-
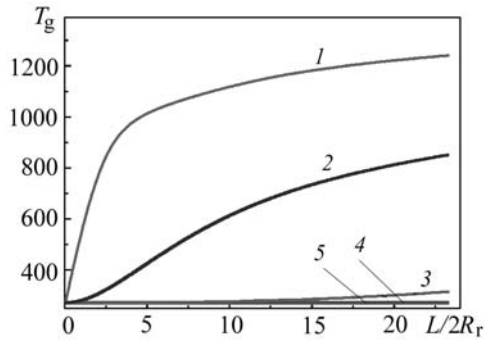
Fig. 2. Variation of the gas-carrier temperature in the reactor at a wall temperature of 1600 K: 1) at a distance of $0.9R_r$ from the reactor axis; 2) $0.8R_r$; 3) $0.6R_r$; 4) $0.3R_r$; 5) $0.1R_r$. T_g, K.
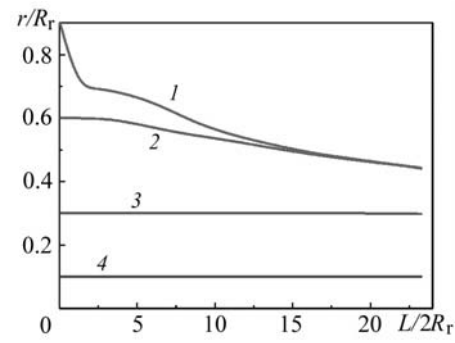
Fig. 3. Variation of the radial position of a droplet: 1) at a distance of $0.9R_r$ from the reactor axis; 2) $0.6R_r$; 3) $0.3R_r$; 4) $0.1R_r$.
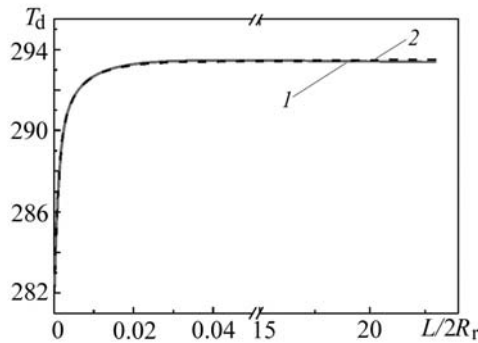

Fig. 4. Variation of the droplet temperature: 1) at a distance of $0.9R_r$ from the reactor axis; 2) $0.1R_r - 0.6R_r$. T_d, K.

teristic hydrodynamic time $\tau = L/u_0$ of the gas flow to the characteristic time of heating $\tau/\tau_{tr} = \pi^2 L a^2/(R_r^2 u_{g0}) \sim 0.1$; thus, the gas flow has no time for being heated while passing through the reactor.

The change in the radical position of a droplet at different initial positions in the flow reactor is shown in Fig. 3. It is of interest to note that at the distance $L/2R_r \approx 25$ there occurs concentration of droplets in the region situated at a distance $0.42\,R_r$ from the reactor axis.

Figure 4 demonstrates changes in the temperature of a droplet at its different initial positions in the flow reactor. It is seen that the temperature of the droplets entering the reactor at different initial positions rapidly attains its stationary value, and a droplet in the reactor is heated only by 11 K. At this slightly elevated temperature of the droplets and due to the condition that the entering vapor-gas stream is in material equilibrium with the droplets, in the considered high-temperature reactor the droplet radius virtually does not change. It is important to take into account this factor in connection with the development of the physical foundation for production of nanoparticles on pyrolysis of droplets [7, 8].

Conclusions. Mathematical models and results of numerical investigation of evaporative cooling of water droplets in a high-temperature gas stream are presented. Two problems have been solved: in the first one, to describe the processes of transfer between a droplet and the vapor-gas stream the approximation of a continuous medium was used, whereas in the second problem the approximation of a free-molecular regime is used. The very gas stream is described in the approximation of a continuous medium. The experimental evidence of the predictions made on the basis of the given mathematical models is given in [7, 11].

The main result obtained is that the evaporative cooling of submillimeter (and smaller) droplets at a small content of water vapors in a high-temperature flow prevents droplets from strong heating. Heat losses in evaporation of droplets are so appreciable that they evaporate practically at a constant temperature. It is important to take into ac-

count this fact in analyzing chemical reactions proceeding in multicomponent droplets [7 and the references therein], as well as in designing heat transfer equipment that employ droplet flows.

In particular, it was shown that a droplet with an initial temperature of about 300 K while entering into a direct flow reactor 1 m in length with a wall temperature of 1600 K at a pressure of 20 torr is heated only by 13 K; other details of the calculation are given in [12]. A larger droplet in the approximation of a continuous medium, when the evaporation rate is not so high as in a free-molecular regime, is heated by 80 K in a flow with a temperature of about 1300 K.

NOTATION

a, thermal diffusivity of a gas mixture, m^2/s; c, heat capacity per molecule, J/K; C, coefficient of aerodynamic resistance of air; D, diffusion coefficient, m^2/s; g, free fall acceleration, m/s^2; k, Boltzmann constant, J/K; Kn, Knudsen number; L, length of the reactor; m, mass of a molecule, kg; p, water pressure, Pa; P, carrier gas pressure, Pa; Q, specific mass flow rate, $kg/(m^2 \cdot s)$; R, radius, m; Re, Reynolds number; T, temperature, K; t, time, s; u, gas stream velocity, m/s; U, latent heat of phase transition per water molecule, J; v, velocity, m/s; x, z, coordinates, m; α, heat transfer coefficients, $W/(m^2 \cdot K)$; γ, mass transfer coefficient; ε, absorptivity; η, dynamic viscosity of air, m^2/s; λ, gas-carrier free path, m; $\lambda(r, z)$, thermal conductivity of air, $W/(K \cdot m)$; ρ, density, kg/m^3; σ, Stefan–Boltzmann constant, $W/(m^2 \cdot K^4)$; τ, hydrodynamic time, s. Subscripts: 0, initial value; d, refers to a droplet; f, finite value; g, gas-carrier; long, absolute length; p, partial; rad, radial position; r, reactor; s, saturated; tr, thermophoresis; w, water; wall, wall.

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