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On the Stability of Non-Supersymmetric Quantum Attractors in String Theory

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Abstract

We study four dimensional non-supersymmetric attractors in type IIA string theory in the presence of sub-leading corrections to the prepotential. For a given Calabi-Yau manifold, the $D0 - D4$ system admits an attractor point in the moduli space which is uniquely specified by the black hole charges. The perturbative corrections to the prepotential do not change the number of massless directions in the black hole effective potential. We further study non-supersymmetric $D0 - D6$ black holes in the presence of sub-leading corrections. In this case the space of attractor points define a hypersurface in the moduli space.

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1 Introduction

It is well known that string theory provides a microscopic understanding of the origin of black hole entropy [1]. For a large class of intersecting brane configurations, it is possible to compute the leading order contribution for the degeneracy in the limit when the string coupling is weak. They agree with the entropy of the corresponding supergravity black holes, which exist in the appropriate weak curvature limits. Furthermore, there is a considerable progress in computing the sub-leading corrections to the microscopic counting of the degeneracy along with their macroscopic counterparts. For a recent review on these results see Ref. [2].

The microscopic counting gives the degeneracy of states as a function of the quantized charges of the intersecting brane configurations. From the supergravity side, the fact that the entropy should depend only on the black hole charges, at least in the case of single centered black holes, is evident from the attractor mechanism.

Soon after its revelation in the context of a simple supersymmetry preserving spherically symmetric black hole in $N = 2$ supergravity coupled to n vector multiplets [3], the attractor mechanism has been studied extensively [4–8]. The generalization of the attractor mechanism to supersymmetric, multi-centered black holes has been carried out [9–12]. It has also been subsequently used to compute the higher derivative corrections to the black hole entropy [13–16]. A systematic procedure for computing the black hole entropy using attractor mechanism in higher derivative gravity has been developed [17].

Though the attractor mechanism was originally perceived by explicitly solving the spinor conditions, it was soon realized that the mechanism is in fact a consequence of the extremality of the black hole [7,8]. This gave rise to the possibility of the existence of non-supersymmetric attractors along with their supersymmetric cousins. These non-supersymmetric attractors were explored in great detail in [18]. The existence of non-supersymmetric attractors in string theory was first shown in [19]. Various aspects of the non-supersymmetric attractors in string theory were subsequently studied [20–28]. Non-supersymmetric attractors were also shown to exist in a large class of gauged supergravity theories on symmetric spaces in four and five dimensions. New branches of solutions corresponding

to zero central charges were also explored in these models [29–31]. For some early results along these lines see Ref. [32–41]. Derivation of non-supersymmetric attractors in certain class of models in terms of the extremization of a fake superpotential has been proposed [42, 43]. This prescription has helped in finding the full flow in a number of examples [44–57]. The generalization of the attractor mechanism to black holes in anti-deSitter as well as deSitter spaces has also been carried out [58–64].

As it has been emphasized in [19,65], the non-supersymmetric attractors differ from the supersymmetric ones in a crucial way. The manifestation of the stability of the supersymmetric attractor is evident from the corresponding mass matrix of the effective black hole potential. On the other hand, for non-supersymmetric attractors $(n-1)$ of the $2n$ real scalar fields remain massless. Thus, one might expect that the sub-leading corrections will drastically change the criterion for stability of such attractors.

To understand the effect of the sub-leading corrections, $N = 2$ supergravity arising from the compactification of type IIA string theory on a simple two-parameter Calabi-Yau manifolds was considered [66]. It was noticed that, for non-supersymmetric $D0 - D4$ black holes, the perturbative corrections to the prepotential do not lift the massless mode. It was further suggested that in such cases, instanton corrections may lift the zero modes.

The goal of the present work is to extent the above result on the perturbative sub-leading correction to supergravity theories arising from string compactifications. We study both $D0 - D4$ as well as $D0 - D6$ configurations in presence of sub-leading corrections. In both these cases, the perturbative quantum corrections do not change the number of zero modes. Hence, the role of non-perturbative corrections become crucial for both the systems. The plan of the paper is as follows. In the next section we review some preliminaries on non-supersymmetric attractors in string theory [19]. In §3, we study the $D0 - D4$ configuration. §4 discusses the $D0 - D6$ configuration in detail. We first review the leading order results in §4.1. In §4.2 we solve the equation of motion and in §4.3 we derive the mass matrix and diagonalize. Finally, in §5 we summarize the results and in the appendix we workout some of the steps in detail.

2 Attractors in type IIA string theory

In this section we will briefly review some of the important results on static, spherically symmetric non-supersymmetric attractors in type IIA string theory. Such solutions were first explored in Ref. [19]. For the static, spherically symmetric black holes, the attractor point is obtained by extremizing the effective black hole potential [7]:

$$V = e^K \left[g^{a\bar{b}} \nabla_a W \left(\nabla_b W \right)^* + |W|^2 \right] \tag{2.1}$$

where $\nabla_a W = \partial_a W + \partial_a K W$. Here K is the Kähler potential in the moduli space. For $N = 2$ theories, it is determined in terms of the prepotential F as:

$$K = -\ln \Im \left(\sum_{a=0}^{N} \bar{X}^{\bar{a}} \partial_a F(X) \right) \tag{2.2}$$

Here $\Im(f)$ denotes imaginary part of f. The superpotential W for a configuration carrying q_a electrically charged and p^a magnetically charged branes is given by

$$W = \sum_{a=0}^{N} \left(q_a X^a - p^a \partial_a F \right) . \tag{2.3}$$

It is related to the central charge by $Z = e^{K/2} W$. Hence the supersymmetric attractor point is obtained by solving the algebraic equation $\nabla_a W = 0$. The non-supersymmetric attractor is obtained by extremizing the black hole effective potential $\partial_a V = 0$ and finding its critical points for which $\nabla_a W \neq 0$. For the potential (2.1), this condition becomes

$$\left(g^{b\bar{c}} \nabla_a \nabla_b W \overline{\nabla_c W} + 2 \nabla_a W \overline{W} + \partial_a g^{b\bar{c}} \nabla_b W \overline{\nabla_c W} \right) = 0 . \tag{2.4}$$

The non-supersymmetric attractor becomes stable, if the quadratic term in the effective potential about the corresponding critical point becomes positive definite. Or, in other words, the $2n \times 2n$ matrix of second derivatives of V must admit only positive Eigenvalues.

We will be interested in $N = 2$ supergravity theories obtained upon compactification of type IIA string theory on a Calabi-Yau three-fold $\mathcal{M}$. In this case, the leading term of the prepotential, in the large volume limit is given by

$$F = D_{abc} \frac{X^a X^b X^c}{X^0} , \tag{2.5}$$

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where $D_{abc} = (1/3!) \int_{\mathcal{M}} J_a \wedge J_b \wedge J_c$ are the triple intersection numbers associated with the Calabi-Yau manifold $\mathcal{M}$. Here the two-forms $\{J_a\}$ form a basis of the integral cohomology $H^2(\mathcal{M}, \mathbb{Z})$. The complex scalar fields $X^a, a = 1, \cdots, n$, parametrize the vector multiplet moduli space.

For a general $D0 - D4 - D6$ configuration, the supersymmetric attractor for this case has been obtained and it was shown that the macroscopic black hole entropy computed using attractor mechanism agrees with the microscopic counting [67]. In the case of non-supersymmetric attractor the solution was obtained in Ref. [19]. It was shown that the mass matrix at the non-supersymmetric critical point admits $(n + 1)$ positive and $(n - 1)$ zero Eigenvalues.

In view of the above, it is important to consider attractors in the presence of sub-leading corrections to the prepotential (2.5). For a large number of two and three parameter models, holomorphic, perturbative corrections to the prepotential in type IIA supergravity has been computed using mirror symmetry [68–70]. The prepotential has the general form:

$$F = D_{abc} \frac{X^a X^b X^c}{X^0} + \alpha_{0a} X^a X^0 + \beta (X^0)^2 + \text{nonperturbative terms} \qquad (2.6)$$

with $\alpha_{0a} = -(1/24) \int_{\mathcal{M}} c_2 \wedge J_a$ and $\beta = -i\zeta(3)\chi/(16\pi^3)$. Here c_2 and χ are respectively the second Chern class and the Euler number of the Calabi-Yau three fold $\mathcal{M}$. The supersymmetric black holes in presences of these sub-leading terms were studied and the correction to the black hole entropy was obtained [71]. In the following sections we will discuss the effect of such corrections to the prepotential on the extremal non-supersymmetric black holes.

3 The $D0 - D4$ system

In this section, we will consider the $D0 - D4$ system in the presence of perturbative sub-leading corrections. To find the non-supersymmetric attractors we need to explicitly solve eq.(2.4). Let us first compute the Kähler potential (2.2):

$$K = -\ln \Im \left(\bar{X}^0 \partial_0 F(X) + \sum_{a=1}^{N} \bar{X}^{\bar{a}} \partial_a F(X) \right) .$$

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It is straightforward to see that the additional contributions from the sub-leading terms $\alpha_{0a}X^aX^0 + \beta(X^0)^2$ to $\bar{X}^0\partial_0 F(X) + \sum_{a=1}^{N}\bar{X}^{\bar{a}}\partial_a F(X)$ is given by

$$\alpha_{0a}(X^a\bar{X}^0 + \bar{X}^aX^0) + 2\beta X^0\bar{X}^0 .$$

For Calabi-Yau manifolds which are obtained as complete intersections of product of projective spaces, the second Chern class is, $c_2 = c_2^{ab} J_a \wedge J_b$, where the coefficients c_2^{ab} are determined in terms of the degree and weights of various projective coordinates [69]. This implies that the α_{0a}s are all real and hence they do not appear in the expression for the Kähler potential, which involves only the imaginary part of the above quantity. On the other hand, β is pure imaginary and hence can appear in the expression for the Kähler potential. However, as it was shown in Ref. [71], it can be eliminated by a symplectic transformation. Hence from now on we will ignore this term in the prepotential (2.6).

Thus we find

$$K = -\ln\left(-iX^0\bar{X}^0 D_{abc}\left(\frac{X^a}{X^0} - \frac{\bar{X}^a}{\bar{X}^0}\right)\left(\frac{X^b}{X^0} - \frac{\bar{X}^b}{\bar{X}^0}\right)\left(\frac{X^c}{X^0} - \frac{\bar{X}^c}{\bar{X}^0}\right)\right) .$$

For the $D0 - D4$ system, the superpotential W is given by

$$W = q_0 X^0 - \sum_{a=1}^{N} p^a \partial_a F$$

Here q_0 is the charge of the $D0$ brane where as p^a are the charges of $D4$ branes wrapped on the 4-cycles of $\mathcal{M}$. After substituting for $\partial_a F$ from eq.(2.6), we find

$$W = q_0 X^0 - 3D_{ab}X^aX^b/X^0 - \alpha_0 X^0 .$$

In the above we have introduced $D_{ab} = D_{abc}p^c$ and $\alpha_0 = \alpha_{0a}p^a$. We also define $D_a = D_{ab}p^b$ and $D = D_ap^a$ for later use.

For convenience, from now on we will introduce the scalar fields $x^a = X^a/X^0$ and set the gauge $X^0 = 1$. The expression for the Kähler potential K and superpotential W in this gauge is given by:

$$
\begin{align}
K &= -\ln\left(-iD_{abc}(x^a - \bar{x}^a)(x^b - \bar{x}^b)(x^c - \bar{x}^c)\right) , \tag{3.1}\\
W &= (q_0 - \alpha_0) - 3D_{ab}x^ax^b . \tag{3.2}
\end{align}
$$

We notice that the only change in the superpotential is a shift of the $D0$ brane charge by α_0. Thus the computations will be identical to the one with the classical black hole with prepotential $F = D_{abc} X^a X^b X^c / X^0$. In particular, if we set the ansatz $x^a = p^a t$ we find that, for $t \neq 0$, the equations of motion (2.4) reduces to:

$$(q_0 - \alpha_0 - t^2 D)(q_0 - \alpha_0 + t^2 D) = 0 \ . \tag{3.3}$$

Clearly, the supersymmetric solution corresponds to $t = i\sqrt{(q_0 - \alpha_0)/D}$ where as the non-supersymmetric solution is given by $t = i\sqrt{(\alpha_0 - q_0)/D}$. The supersymmetric solution exists when $(q_0 - \alpha_0)D > 0$ where as the non-supersymmetric solution exists in the opposite domain $(\alpha_0 - q_0)D > 0$. The entropy of the black hole, in both the cases is given by $S = 2\pi \sqrt{|(q_0 - \alpha_0)D|}$.

To understand the stability of the non-supersymmetric attractor, we need to consider fluctuations of the field x^a around the attractor point:

$$x^a = i p^a \sqrt{\frac{(\alpha_0 - q_0)}{D}} + \delta\xi^a + i\delta y^a$$

and keep terms up to quadratic order in the black hole effective potential (2.1). We can read the mass matrix M from [19]:

$$M = 48 e^{K_0} \frac{(\alpha_0 - q_0)}{D} \left\{ (3 D_a D_b - D D_{ab}) \otimes I + D D_{ab} \otimes \sigma^3 \right\} \ . \tag{3.4}$$

Here I is the 2×2 identity matrix, σ^i are the Pauli matrices in the basis in which σ^3 is diagonal and K_0 is the Kähler potential evaluated at the non-supersymmetric extremum. The only change from the leading result is in the pre-factor $e^{K_0}(-q_0/D)$ where q_0 is replaced by $(q_0 - \alpha_0)$. Though there is a shift in mass for all the massive modes, the zero modes remain unchanged. Thus, for $D(\alpha_0 - q_0) > 0$, the mass matrix still has $(n+1)$ positive Eigenvalues and $(n-1)$ zero Eigenvalues.

In this section we have noticed that the non-supersymmetric attractor for $D0 - D4$ system in the presence of perturbative sub-leading correction can be obtained from the leading order solution quite trivially by a shift of the $D0$ brane charge. Hence all the results pertaining to the leading solution hold. Especially the condition for stability of the attractor does not change. In the next section we will study the $D0 - D6$ system, where we will see that the solution can no longer be derived in such a straightforward manner.

4 The $D0 - D6$ system

The $D0 - D6$ system is peculiarly different from other D-brane bound states in type IIA theory. Here we will first summarize the leading order result [65] and subsequently find the attractor solution in the presence of sub-leading corrections.

4.1 The leading order solution

Unlike the $D0 - D4$ bound state (along with $D0 - D4 - D6$ and $D0 - D2 - D4 - D6$ bound states), the $D0 - D6$ bound state does not admit supersymmetric solution in the absence of a B-field. One can see this in the following [72][1]:

For a supersymmetric $D0$ brane there must exist spinors ϵ^α and $\hat{\epsilon}_\beta$ satisfying $\hat{\epsilon}_\beta = \Gamma^0_{\beta\alpha}\epsilon^\alpha$, where as a $D6$ brane preserving supersymmetry need to satisfy the spinor condition $\hat{\epsilon}_\beta = (\Gamma^0\Gamma^1\cdots\Gamma^6)_{\beta\alpha}\epsilon^\alpha$. For the supersymmetric $D0 - D6$ bound state to exist both the conditions must be satisfied simultaneously and hence the spinor ϵ will necessarily have to obey $\Gamma^1\cdots\Gamma^6\epsilon = \epsilon$. Since $(\Gamma^1\cdots\Gamma^6)^2 = -1$, this equation can never admit a nonzero solution for the spinor ϵ.

This can also be seen from the supergravity analysis by solving the condition $\nabla_a W = 0$. We will consider the known solution for supersymmetric $D0 - D4 - D6$ black hole [67] and show that the limit in which the $D4$ charges $p^a \to 0$ does not give a consistent solution. The supersymmetric $D0 - D4 - D6$ solution is given by:

$$x^a = p^a \frac{1}{2D}\left(p^0 q_0 \pm i\sqrt{q_0(4D - (p^0)^2 q_0)}\right) \tag{4.1}$$

We can see that, in the limit $p^a \to 0$ the imaginary part $\Im(x^a) = 0$ and hence the solution lies out side the Kähler cone.

Though the equation $\nabla_a W = 0$ can't be satisfied for $D0 - D6$ system, there is no obstruction for solving $\partial_a V = 0$. We will now show that this equation has a physical solution by taking the limit $p^a \to 0$ in the non-supersymmetric $D0 - D4 - D6$ solution. Let us consider the explicit expression for the non-supersymmetric $D0 - D4 - D6$ solution [19]. In this case, the solution is naturally given in terms of a parameter $s = \sqrt{(p^0)^2 - \frac{4D}{q_0}}$. There exists two branches in

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[1]The D-brane bound states in the presence of B field was discussed in [73]

the charge lattice $|s/p^0| > 1$ as well as $|s/p^0| < 1$ for which we have the solution, where as the line $|s/p^0| = 1$ corresponds to the singular limit $p^a \to 0$.

To obtain the leading order results for $D0 - D6$ solution, we will consider the $D0 - D4 - D6$ solution in the region $|s/p^0| > 1$ and take the limit $p^a \to 0$. It can be easily verified that the same result can be obtained by considering the limit $p^a \to 0$ in the region $|s/p^0| < 1$. The non-supersymmetric solution for $D0 - D4 - D6$ system valid in the region $|s/p^0| > 1$ is given by [19]

$$x^a = p^a(t_1 + it_2) \, ,$$

where t_1 and t_2 have the following form

$$t_1 = 2 \frac{(s + p^0)^{1/3} - (s - p^0)^{1/3}}{(s + p^0)^{4/3} + (s - p^0)^{4/3}} \tag{4.2}$$

$$t_2 = \frac{4s}{(s^2 - (p^0)^2)^{1/3}\left((s + p^0)^{4/3} + (s - p^0)^{4/3}\right)} \tag{4.3}$$

We see that t_1 remains finite and hence the real part of x^a vanishes in the limit $p^a \to 0$. On the other hand t_2 diverges in this limit and the resulting solution lies inside the Kähler cone. We find:

$$\lim_{p^a \to 0} x^a = \lim_{p^a \to 0} i \, \frac{p^a}{2} \left(-\frac{q_0}{Dp^0}\right)^{1/3} = i\hat{x}^a_{\mp} \left(\pm\frac{q_0}{p^0}\right)^{1/3} , \tag{4.4}$$

where

$$\hat{x}^a_{\pm} = \lim_{p^a \to 0} (\pm p^a / D^{1/3}) \, .$$

Here $\hat{x}^a_+$ and $\hat{x}^a_-$ are two real vector restricted to the hypersurfaces $D_{abc}\hat{x}^a_+\hat{x}^b_+\hat{x}^c_+ = 1$ and $D_{abc}\hat{x}^a_-\hat{x}^b_-\hat{x}^c_- = -1$ in the moduli space respectively. There are two branches of solutions for the $D0 - D6$ attractor:

$$x^a = \begin{cases} i\hat{x}^a_- \left(\frac{q_0}{p^0}\right)^{(1/3)} & \text{For } q_0p^0 > 0 \, , \\ i\hat{x}^a_+ \left(-\frac{q_0}{p^0}\right)^{(1/3)} & \text{For } q_0p^0 < 0 \, . \end{cases} \tag{4.5}$$

The entropy of the black hole, for both the cases, is given by $S = \pi|q_0p^0|$. Note that the solutions (4.5) obey

$$D_{abc}x^ax^bx^c = i\frac{q_0}{8p^0} \, . \tag{4.6}$$

This equation involves n real variables and hence defines a $(n-1)$ dimensional hypersurface in the $2n$ dimensional moduli space. Thus we find that the moduli are not completely fixed on the black hole horizon and the space of attractor points is specified by a $(n-1)$ dimensional hypersurface in the moduli space.

4.2 The sub-leading correction

We will now consider the $D0 - D6$ system in the presence of perturbative sub-leading corrections. The superpotential (2.3) for the system takes the form

$$W = q_0 - p^0 \alpha_{0a} x^a + p^0 D_{abc} x^a x^b x^c \ , \tag{4.7}$$

where as the Kähler potential is given by eq.(3.1). Unlike the $D0 - D4$ system, here we can't make a simple redefinition of charges to absorb the effect of subleading corrections and need to explicitly solve the equations of motion. We will substitute these expression for W and K in eq.(2.4)

$$\left(g^{b\bar{c}} \nabla_a \nabla_b W \overline{\nabla_c W} + 2 \nabla_a W \overline{W} + \partial_a g^{b\bar{c}} \nabla_b W \overline{\nabla_c W} \right) = 0 \ , \tag{4.8}$$

and solve it explicitly. Taking a clue from the previous subsection we consider the following ansatz for the scalar fields:

$$x^a = \hat{x}^a t \ , \tag{4.9}$$

for some real vector $\hat{x}^a$.

Before we proceed to solve eq.(4.8), we will try to solve the supersymmetry condition $\nabla_a W = 0$ and show that this equation does not admit any physically acceptable solution in the large volume limit. The leading order result was discussed in the previous sub-section and the world sheet analysis for nonexistence of the $D0 - D6$ bound state was presented in Ref. [72]. After a straightforward computation, we find

$$\nabla_a W = \hat{D}_a \left(3t^2 + \frac{3iW}{2p^0 \hat{D} t_2} \right) - \alpha_{0a} = 0 \ , \tag{4.10}$$

Here for convenience we introduce $\hat{D}_{ab} = D_{abc} \hat{x}^c$, $\hat{D}_a = \hat{D}_{ab} \hat{x}^b$ and $\hat{D} = \hat{D}_a \hat{x}^a$. The inverse of the matrix $\hat{D}_{ab}$ is denoted by $\hat{D}^{ab}$. We also define, $\hat{\alpha}_0 = \alpha_{0a} \hat{x}^a$

and set $t = t_1 + it_2$. Now, equating the real as well as the imaginary parts of the above equation to zero separately, we find

$$
\begin{aligned}
3\hat{D}(t_1{}^2 + t_2{}^2) - \hat{\alpha}_0 &= 0 \\
q_0 + p^0 \hat{D} t_1 (t_1{}^2 + t_2{}^2) - p^0 t_1 \hat{\alpha}_0 &= 0
\end{aligned}
\tag{4.11}
$$

Solving the above equations for t we get

$$
t = \frac{3q_0}{2p^0 \hat{\alpha}_0} + \frac{i}{2} \sqrt{\frac{4\hat{\alpha}_0}{3\hat{D}} - \frac{9q_0^2}{(\hat{\alpha}_0 p^0)^2}} \; .
\tag{4.12}
$$

We will focus on the imaginary part of t, since this is the quantity which appears in the expression for the Kähler potential and hence in the volume of the Calabi-Yau manifold. Apart from the charges q_0 and p^0 it depends on $\hat{\alpha}_0$ and $\hat{D}$, which involve the intersection numbers, weight of projective coordinates, degree of hypersurfaces, etc, and are $O(1)$ quantities. Since $\hat{x}^a$ is invariant under the scaling $\hat{x}^a \to \lambda \hat{x}^a$, we can't make its imaginary part arbitrarily large by choosing $\hat{x}^a$ of arbitrarily small size. Hence the imaginary part for this solution, if at all it exists for some Calabi-Yau manifold, can at best be of order $O(1)$. Thus the solution (4.11) is not valid in the large volume limit we are considering.

We will now turn into the exact solution for the non-supersymmetric attractor. Each of the terms in eq.(4.8) will be evaluated for the above ansatz in the appendix. Adding them together we find, after some simplification

$$
\begin{aligned}
0 = {} & \frac{3i}{\hat{D}t_2} \Big(2q_0{}^2 + 2(p^0)^2 \hat{D}^2 \bar{t}^4 t^2 + 4p^0 q_0 t_1 (\hat{D} t_1 \bar{t} - \hat{\alpha}_0) \\
& - 4(p^0)^2 \hat{D} t_1 \bar{t}^2 t \hat{\alpha}_0 + (2t_1{}^2 + t_2{}^2)(p^0)^2 \hat{\alpha}_0^2 \Big) \hat{D}_a \\
& + \Big(4p^0 t_1 \hat{\alpha}_0 - 4q_0 - 2ip^0 t_2 \hat{\alpha}_0 - 4\hat{D} p^0 t_1 \bar{t}^2 \Big) p^0 \hat{\alpha}_{0a} - \frac{i\hat{D}(p^0)^2 t_2}{3} \hat{T}_a \; ,
\end{aligned}
$$

where $\hat{T}_a = D_{apq} \hat{D}^{pb} \hat{D}^{qc} \alpha_{0b} \alpha_{0c}$. The real part of the above equation gives

$$
0 = 3t_1 (q_0 t_1 + p^0 |t|^2 (\hat{D}|t|^2 - \hat{\alpha}_0)) \hat{D}_a - \Big(q_0 + p^0 t_1 (\hat{D} t_1{}^2 - \hat{D} t_2{}^2 - \hat{\alpha}_0) \Big) \alpha_{0a}
\tag{4.13}
$$

Defining L to be

$$
L = \frac{3t_1 (q_0 t_1 + p^0 |t|^2 (\hat{D}|t|^2 - \hat{\alpha}_0))}{q_0 + p^0 t_1 (\hat{D} t_1{}^2 - \hat{D} t_2{}^2 - \hat{\alpha}_0)} \; ,
\tag{4.14}
$$

we find

$$
\alpha_{0a} = \hat{D}_a L
\tag{4.15}
$$

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The vectors $\hat{D}_a$ and α_{0a} depend on the property of the Calabi-Yau manifod. In addition, $\hat{D}_a$ depends on $\hat{x}^a$ as well. A priory there is no reason why both these vectors will be aligned in the same direction and hence this equation imposes restriction on $\hat{x}^a$. Multiplying $\hat{x}^a$ on both sides and summing over a we find $L = \hat{\alpha}_0/\hat{D}$ and $\hat{D}\alpha_{0a} = \hat{D}_a\hat{\alpha}_0$. The later of imposes restriction on $\hat{x}^a$ and hence defines the hypersurface of attractor points in presence of sub-leading terms.

The relation $L = \hat{\alpha}_0/\hat{D}$ gives

$$\frac{\hat{\alpha}_0}{\hat{D}} = \frac{3t_1(q_0t_1 + p^0|t|^2(\hat{D}|t|^2 - \hat{\alpha}_0))}{q_0 + p^0t_1(\hat{D}t_1{}^2 - \hat{D}t_2{}^2 - \hat{\alpha}_0)} \ , \tag{4.16}$$

This equation, along with the imaginary part of eq.(4.13) can be simultaneously solved to get t in terms of the charges q_0, p^0 and the geometric quantities $\hat{\alpha}_0, \hat{D}$. The algebraic equations, we get from eq.(4.13) in this process, are given as

$$\begin{aligned}
0 &= 36q_0{}^2 + 36(p^0\hat{D})^2(t_1{}^2 + t_2{}^2)^2(t_1{}^2 - t_2{}^2) + 6(p^0)^2(6t_1{}^2 + t_2{}^2)\hat{\alpha}_0^2 \\
&+ 72q_0p^0t_1(\hat{D}t_1{}^2 - \hat{\alpha}_0) - 2(p^0)^2\hat{D}(\hat{T}t_2{}^2 + 12t_1{}^2(3t_1{}^2 + t_2{}^2)\hat{\alpha}_0) \ , \tag{4.17}
\end{aligned}$$

and

$$3p^0\hat{D}^2t_1(t_1{}^2 + t_2{}^2)^2 + \hat{\alpha}_0(p^0t_1\hat{\alpha}_0 - q_0) + \hat{D}t_1(3q_0t_1 - 2(2t_1{}^2 + t_2{}^2)p^0\hat{\alpha}_0) = 0 \tag{4.18}$$

We can rescale the variables to write these two equations in a simple form. The resulting equations depend on one parameter only. The details are worked out in §A.1. Subsequently, we can eliminate t_1 and t_2 in sequence to get two cubic equations, which we can solve exactly. The resulting solution is

$$t_1 = \frac{q_0}{\hat{\alpha}_0p^0}\hat{F}_1\left(\frac{q_0^2\hat{D}}{\hat{\alpha}_0^3(p^0)^2}\right) \ , \tag{4.19}$$

$$t_2^2 = \frac{\hat{\alpha}_0}{\hat{D}}\hat{F}_2\left(4 - 27\frac{\hat{D}q_0^2}{\hat{\alpha}_0^3(p^0)^2}\right) \ , \tag{4.20}$$

where the functions $\hat{F}_1(x)$ and $\hat{F}_2(x)$ are defined to be

$$\hat{F}_1(x) = \frac{1}{6x(27x - 2)}\left(18x + \hat{f}_+(x)^{(1/3)} - \hat{f}_-(x)^{(1/3)}\right) \tag{4.21}$$

$$\hat{F}_2(x) = \frac{1}{6(x - 2)^2}\left(\hat{g}_+(x)^{(1/3)} - \hat{g}_-(x)^{(1/3)} - 6x\right) \ . \tag{4.22}$$

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The functions $\hat{f}_\pm(x)$ and $\hat{g}_\pm(x)$ in turn are given by

$$\hat{f}_\pm(x) = 12\left((2-27x)\sqrt{3x^3(27x-4)^3} \pm 9x^2(27x-4)^2\right) \qquad (4.23)$$

$$\hat{g}_\pm(x) = 4\left((x-2)^3\sqrt{x^7(x-4)} \pm (2x^3-40x^4-6x^5+8x^6-x^7)\right) \quad (4.24)$$

Substituting the solution for $t = t_1 + it_2$ in the entropy $S = \pi V(\hat{x}_0 t)$, we find, for the non-supersymmetric attractor

$$S = \frac{\pi p^0 \hat{\alpha}_0}{3}\sqrt{\frac{9q_0{}^2}{(p^0)^2\hat{\alpha}_0^2} - \frac{4\hat{\alpha}_0}{3\hat{D}}} \qquad (4.25)$$

4.3   The zero modes

We saw that the leading $D0 - D6$ solution has $(n-1)$ zero modes. We are interested in studying the effect of sub-leading corrections to these zero modes. We will address this issue in this section. We need to consider the fluctuations around $\hat{x}^a t$:

$$x^a = \hat{x}^a t + \delta\xi^a + i\delta y^a$$

and keep term quadratic in the fields $\delta\xi^a$, δy^a in the effective black hole potential:

$$\begin{aligned}
V_{quad} &= \partial_a\partial_{\bar{d}}V(\delta\xi^a\delta\xi^d + \delta y^a\delta y^d) + \Re(\partial_a\partial_d V)(\delta\xi^a\delta\xi^d - \delta y^a\delta y^d)\\
&\quad -2\Im(\partial_a\partial_d V)\delta\xi^a\delta y^d.
\end{aligned} \qquad (4.26)$$

Here $\Re(\partial_a\partial_d V)$ and $\Im(\partial_a\partial_d V)$ are the real and imaginary parts of $(\partial_a\partial_d V)$ respectively. Each of these terms are computed explicitly in §A.2. Following [19], we express the mass matrix M in the form:

$$M = E\left(3\frac{\hat{D}_a\hat{D}_d}{\hat{D}} - \hat{D}_{ad}\right)\otimes\mathbf{I} + \hat{D}_{ab}\otimes(A\sigma^3 - B\sigma^1), \qquad (4.27)$$

The coefficients E, A and B appearing in the above formula are computed in §A.2 and are given by

$$\begin{aligned}
E &= e^{K_0}\left(\frac{4\hat{\alpha}_0^2(p^0)^2(3-u)(u+3)^2}{3\hat{D}(u-1)^2}\right)\\
A &= e^{K_0}\left(\frac{2\hat{\alpha}_0^2(p^0)^2(3-u)(u+3)^2}{3\hat{D}(u-1)^2}(u^2-2u-1)\right)
\end{aligned}$$

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$$B = e^{K_0} \left(\frac{2\hat{\alpha}_0^2(p^0)^2(3-u)(u+3)^2}{3\hat{D}(u-1)} \sqrt{(u+1)(3-u))} \right) \tag{4.28}$$

Here K_0 is the Kähler potential evaluated at the attractor point and the parameter u is given by $u = \hat{F}_3(\hat{D}q_0^2/(\hat{\alpha}_0^3(p^0)^2))$. The function $\hat{F}_3(x)$ is defined as

$$\hat{F}_3(x) = \sqrt{1 - 24x(\hat{F}_1(x))^2} \ . \tag{4.29}$$

Substituting the explicit expression for A and B, we can see that the 2×2 matrix $(A\sigma^3 - B\sigma^1)$ has the Eigenvalues $\pm E$. Thus the matrix M can be brought into block diagonal form, where one block contains a positive coefficient times the moduli space metric $g_{a\bar{b}}$ at the extremum and the other block contains a positive coefficient times the matrix $D_a D_b$. Since the moduli space metric is positive definite and any matrix of the form $D_a D_b$ has one positive and $(n-1)$ zero eigenvalues, the matrix M has $(n+1)$ positive and $(n-1)$ zero eigenvalues.

5   Conclusion

In this paper we have studied non-supersymmetric attractors in type IIA string theory compactified on a Calabi-Yau manifold, in the presence of perturbative sub-leading corrections to the perpotential. We discussed both $D0 - D4$ as well as $D0 - D6$ configurations. In both the cases there are $(n-1)$ massless modes. Because of the presence of non-trivial quartic terms, in the case of $D0 - D4$ black holes, all the vector multiplet moduli are attracted to a fixed point. On the other hand, for the $D0 - D6$ system there exists a $(n-1)$ dimensional hypersurface of attractor points. Thus, in this case the $(n-1)$ massless modes are exactly flat and the attractor mechanism occurs only in an $(n+1)$ dimensional subspace of the $2n$ dimensional moduli space for the vector multiples moduli. The quantum correction deforms this $(n-1)$ dimensional hypersurface but does not change its dimensionality. Since the perturbative corrections are not sufficient to lift the massless modes, they can only be lifted by non-perturbative terms in the potential. It would be interesting to consider explicit examples where these modes are stabilized by non-perturbative corrections. It would also be interesting to find the full flow and understanding it in terms the first order formalism. We hope to report on some of these issues in future.

6 Acknowledgments

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7 Note Added

The effect of sub-leading corrections in st^2 and t^3 models has been studies in [74, 75]. The effect of perturbative quantum corrections on mass less moduli and on flat directions had already been been obtained in [76] using symplectic transformations. We are grateful to Alessio Marrani for pointing out the above references to us.

A Appendix

In this appendix we will carry out some of the computations in detail. In §A.1 we will derive the attractor solution for $D0 - D6$ system and in §A.2 we will outline some of the steps involved in obtaining mass matrix for this system.

A.1 Nonsupersymmetric solution for $D0 - D6$ system

The black hole effective potential for $N = 2$ supergravity in four dimensions is given by

$$V = e^K \left[g^{a\bar{b}} \nabla_a W \left(\nabla_b W \right)^* + |W|^2 \right] \tag{A.1}$$

where $\nabla_a W = \partial_a W + \partial_a K W$. In terms of the $N = 2$ prepotential F, the Kähler potential K and superpotential W are given by

$$K = -\ln \Im \left(\sum_{a=0}^{N} \bar{X}^{\bar{a}} \partial_a F(X) \right)$$

$$W = \sum_{a=0}^{N} (q_a X^a - p^a \partial_a F) . \tag{A.2}$$

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We are interested in finding the non-supersymmetric attractors, which correspond to extremising this potential. The stable non-supersymmetric attractors correspond to the minima of V for which $\nabla_a W \neq 0$. The equation of motion is

$$\left(g^{b\bar{c}} \nabla_a \nabla_b W \overline{\nabla_c W} + 2\nabla_a W \overline{W} + \partial_a g^{b\bar{c}} \nabla_b W \overline{\nabla_c W} \right) = 0 . \tag{A.3}$$

This section deals with the attractor solution for the $D0 - D6$ system. We will first evaluate each term in the equation of motion separately. Subsequently we will add them up and simplify and eventually find the exact solution corresponding to the equation of motion.

We will first introduce some of the standard notations and express the Kähler potential, moduli space metric and its inverse in terms of them [19]:

$$\begin{aligned}
M_{ab} &= D_{abc}(x^c - \bar{x}^c) \\
M_a &= D_{abc}(x^b - \bar{x}^b)(x^c - \bar{x}^c) \\
M &= D_{abc}(x^a - \bar{x}^a)(x^b - \bar{x}^b)(x^c - \bar{x}^c) .
\end{aligned} \tag{A.4}$$

The Kähler potential is

$$K = -\ln(-iM) \tag{A.5}$$

The metric $g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K$ and its inverse are

$$\begin{aligned}
g_{a\bar{b}} &= \frac{3}{M}\left(2M_{ab} - \frac{3}{M}M_a M_b \right) , \\
g^{a\bar{b}} &= \frac{M}{6}\left(M^{ab} - \frac{3}{M}(x^a - \bar{x}^a)(x^b - \bar{x}^b) \right) ,
\end{aligned} \tag{A.6}$$

We will also need

$$\partial_a g^{b\bar{c}} = \frac{1}{6}\left(3M_a M^{bc} - MM^{pb}M^{qc}D_{apq} \right) - \frac{1}{2}\left(\delta_a{}^b(x^c - \bar{x}^c) + \delta_a{}^c(x^b - \bar{x}^b) \right) \tag{A.7}$$

For $D0 - D6$ system, the superpotential W and its covariant derivatives $\nabla_a W, \nabla_a \nabla_b W$ are given by

$$\begin{aligned}
W &= q_0 - p^0 \alpha_{0a} x^a + p^0 D_{abc} x^a x^b x^c \\
\nabla_a W &= -p^0 \alpha_{0a} + 3p^0 D_{abc} x^b x^c - \frac{3M_a W}{M} \\
\nabla_a \nabla_b W &= 6p^0 D_{abc} x^c + \frac{6W}{M}\left(\frac{3M_a M_b}{M} - M_{ab} \right) \\
&\quad + \frac{3p^0}{M}\left(M_a \alpha_{0b} + M_b \alpha_{0a} \right) - \frac{9p^0 x^p x^q}{M}\left(M_a D_{bpq} + M_b D_{apq} \right) \tag{A.8}
\end{aligned}$$

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We will use the ansatz $x^a = \hat{x}^a t = \hat{x}^a(t_1 + it_2)$. The inverse metric and its derivative in this ansatz has the form

$$
\begin{aligned}
g^{b\bar{c}} &= \frac{2t_2^2}{3}\hat{D}\left(\frac{3}{\hat{D}}\hat{x}^b\hat{x}^c - \hat{D}^{bc}\right) , \\
\partial_a g^{b\bar{c}} &= -\frac{it_2}{3}\hat{D}\left(\frac{3}{\hat{D}}(\hat{x}^c\delta_a^b + \hat{x}^b\delta_a^c - \hat{D}^{bc}\hat{D}_a) + \hat{D}^{ec}\hat{D}^{bf}D_{aef}\right) , \quad (A.9)
\end{aligned}
$$

where as the superpotential W and its covariant derivatives are given by

$$
\begin{aligned}
W &= q_0 + p^0\hat{D}t^3 - p^0\hat{\alpha}_0 t \\
\nabla_a W &= \hat{D}_a p^0\left(3t^2 + \frac{3iW}{2p^0\hat{D}t_2}\right) - p^0\hat{\alpha}_{0a} \\
\nabla_a \nabla_b W &= \hat{D}_{ab}\left(6p^0 t + \frac{3W}{2\hat{D}t_2^2}\right) \\
&\quad + \hat{D}_a\hat{D}_b\left(\frac{-9W}{2\hat{D}^2 t_2^2} + \frac{9ip^0 t^2}{\hat{D}t_2}\right) + \frac{3p^0}{2i\hat{D}t_2}\left(\hat{D}_a\alpha_{0b} + \hat{D}_b\alpha_{0a}\right) \quad (A.10)
\end{aligned}
$$

Let us evaluate each term in eq.(A.3).

$$
\begin{aligned}
g^{b\bar{c}}\nabla_a\nabla_b W\overline{\nabla_c W} &= \hat{D}_a\left(m_1\left(6p^0 t - \frac{3W}{\hat{D}t_2^2} + \frac{9ip^0 t^2}{t_2} + \frac{3p^0\hat{\alpha}_0}{2i\hat{D}t_2}\right)\right. \\
&\quad + \left. m_2\left(\frac{-9W\alpha}{2\hat{D}^2 t_2^2} + \frac{9ip^0 t^2\hat{\alpha}_0}{\hat{D}t_2} + \frac{3p^0\hat{T}}{2i\hat{D}t_2}\right)\right) \\
&\quad + \alpha_{0a}\left(\frac{3p^0 m_1}{2it_2} + m_2\left(6p^0 t + \frac{3W}{2\hat{D}t_2^2} + \frac{3p^0\alpha_0}{2i\hat{D}t_2}\right)\right) (A.11)
\end{aligned}
$$

Where $m_1 = \frac{2p^0\hat{D}t_2^2}{3}\left(2\bar{G} - \frac{3\hat{\alpha}_0}{\hat{D}}\right)$, $m_2 = \frac{2p^0\hat{D}t_2^2}{3}$, $G = 3t^2 + \frac{3iW}{2p^0\hat{D}t_2}$
and $\hat{T} = \hat{D}^{bc}\alpha_{0b}\alpha_{0c}$

$$
\begin{aligned}
\partial_a g^{b\bar{c}}\nabla_b W\overline{\nabla_c W} &= \hat{D}_a\left(it_2\hat{T} - \frac{4}{3}it_2\hat{D}G\bar{G}\right)(p^0)^2 - \frac{it_2\hat{D}(p^0)^2}{3}\hat{T}_a \\
&\quad + \alpha_{0a}\left(\frac{4}{3}it_2\hat{D}(G + \bar{G}) - 2it_2\hat{\alpha}_0\right)(p^0)^2 \quad (A.12)
\end{aligned}
$$

Where $\hat{T}_a = D_{apq}\hat{D}^{pb}\hat{D}^{qc}\alpha_{0b}\alpha_{0c}$

$$
2\nabla_a W\overline{W} = 2p^0\left(\hat{D}_a G - \alpha_{0a}\right)\overline{W} \quad (A.13)
$$

Adding up terms eq. (A.11), eq. (A.12), eq. (A.13) and using eq.(A.3) we get,

$$
\begin{aligned}
0 &= \frac{3i}{\hat{D}t_2}\left(2q_0^2 + 2(p^0)^2\hat{D}^2\bar{t}^4 t^2 + 4p^0 q_0 t_1(\hat{D}t_1\bar{t} - \hat{\alpha}_0)\right. \\
&\quad - \left. 4(p^0)^2\hat{D}t_1\bar{t}^2 t\hat{\alpha}_0 + (2t_1^2 + t_2^2)(p^0)^2\hat{\alpha}_0^2\right)\hat{D}_a
\end{aligned}
$$

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$$+ \left(4p^0 t_1 \hat{\alpha}_0 - 4q_0 - 2ip^0 t_2 \hat{\alpha}_0 - 4\hat{D}p^0 t_1 \bar{t}^2\right) p^0 \hat{\alpha}_{0a} - \frac{i\hat{D}(p^0)^2 t_2}{3} T_a \,,$$

where $\hat{T}_a = D_{apq}\hat{D}^{pb}\hat{D}^{qc}\alpha_{0b}\alpha_{0c}$. Taking the real part of the above equation we get.

$$\begin{aligned}
\Re(e^K \partial_a V_{eff}) &= \left(3t_1(q_0 t_1 + p^0|t|^2(\hat{D}|t|^2 - \hat{\alpha}_0))\right) p^0 \hat{D}_a \\
&+ \left(-q_0 + p^0 t_1(-\hat{D}t_1^2 + \hat{D}t_2^2 + \hat{\alpha}_0)\right) p^0 \alpha_{0a} = 0 \quad \text{(A.14)}
\end{aligned}$$

This gives

$$\alpha_{0a} = \hat{D}_a L \quad \text{(A.15)}$$

Where

$$L = \frac{3t_1(q_0 t_1 + p^0|t|^2(\hat{D}|t|^2 - \hat{\alpha}_0))}{q_0 + p^0 t_1(\hat{D}t_1^2 - \hat{D}t_2^2 - \hat{\alpha}_0)}. \quad \text{(A.16)}$$

Multiplying eq.(A.14) with $\hat{x}^a$ and separate its real and imaginary parts we get the following two equations.

$$\begin{aligned}
0 &= 36q_0^2 + 36(p^0\hat{D})^2(t_1^2 + t_2^2)^2(t_1^2 - t_2^2) + 6(p^0)^2(6t_1^2 + t_2^2)\hat{\alpha}_0^2 \\
&+ 72q_0 p^0 t_1(\hat{D}t_1^2 - \hat{\alpha}_0) - 2(p^0)^2\hat{D}(\hat{T}t_2^2 + 12t_1^2(3t_1^2 + t_2^2)\hat{\alpha}_0) \,, \quad \text{(A.17)}
\end{aligned}$$

and

$$3p^0\hat{D}^2 t_1(t_1^2 + t_2^2)^2 + \hat{\alpha}_0(p^0 t_1 \hat{\alpha}_0 - q_0) + \hat{D}t_1(3q_0 t_1 - 2(2t_1^2 + t_2^2)p^0\hat{\alpha}_0) = 0 \quad \text{(A.18)}$$

We need to solve these two equations for t_1 and t_2 in terms of $q_0, p^0, \hat{D}$ and $\hat{\alpha}_0$. We will first do some scaling of variables and parameters to simplify these two equations. Introducing the variables $\tilde{t}_1 = t_1(p^0\hat{\alpha}_0/q_0)$ and $\tilde{t}_2 = t_2\sqrt{\hat{D}/\hat{\alpha}_0}$, we find

$$\begin{aligned}
&9\hat{D}^2(p^0)^2 q_0^4 \hat{\alpha}_0^3 \tilde{t}_1^3(2 + \tilde{t}_1(\tilde{t}_2^2 - 2)) + (p^0)^6\hat{\alpha}^9 \tilde{t}_2^2(1 - 9\tilde{t}_2^4) \\
&+9\hat{D}^3 q_0^6 \tilde{t}_1^6 + 3\hat{D}(p^0)^4 q_0^2 \hat{\alpha}_0^6(3 - 6\tilde{t}_1 + \tilde{t}_1^2(3 - 2\tilde{t}_2^2 - 3\tilde{t}_2^4)) = 0 \quad \text{(A.19)}
\end{aligned}$$

and

$$3\hat{D}^2 q_0^4 \tilde{t}_1^5 + \hat{D}(p^0)^2 q_0^2 \hat{\alpha}_0^3 \tilde{t}_1^2(3 + \tilde{t}_1(6\tilde{t}_2^2 - 4)) - (p^0)^4 \hat{\alpha}_0^6(1 - \tilde{t}_1(1 - 2\tilde{t}_2^2 + 3\tilde{t}_2^4)) = 0 \quad \text{(A.20)}$$

We will now introduce the parameter $\tilde{D} = \hat{D}q_0^2/((p^0)^2\hat{\alpha}_0^3)$. The above equations take particularly simple form when expressed in terms of $\tilde{D}$:

$$9\tilde{D}(1 - \tilde{t}_1 + \tilde{D}\tilde{t}_1^3)^2 + (1 - 3\tilde{D}\tilde{t}_1^2)^2\tilde{t}_2^2 - 9\tilde{D}\tilde{t}_1^2\tilde{t}_2^4 - p\tilde{t}_2^6 = 0 \quad \text{(A.21)}$$

$$1 - \tilde{t}_1(1 + 3\tilde{D}^2\tilde{t}_1^4 - 2\tilde{t}_2^2 + 3\tilde{t}_2^4 + \tilde{D}\tilde{t}_1(3 + \tilde{t}_1(6\tilde{t}_2^2 - 4))) = 0 \quad \text{(A.22)}$$

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Eliminating $\tilde{t}_1$ and $\tilde{t}_2$ in succession, and after a bit simplification, we find

$$\tilde{D}(27\tilde{D} - 2)\tilde{t}_1^3 - 9\tilde{D}\tilde{t}_1^2 + 2\tilde{t}_1 - 1 = 0 \tag{A.23}$$

$$27(\tilde{d} - 2)^2\tilde{t}_2^6 + 81\tilde{d}\tilde{t}_2^4 + 18\tilde{d}^2\tilde{t}_2^2 + \tilde{d}^3 = 0 \tag{A.24}$$

Here, for easy reading, we have introduced $\tilde{d} = 4 - 27\tilde{D}$ in the second line. We finally get two cubic equations in terms of variables $\tilde{t}_1$ and $\tilde{t}_2^2$ which we can solve easily. The exact solution for the original variables t_1, t_2 in terms of the charges q_0, p^0 and parameters $\hat{D}, \hat{\alpha}_0$ is given in §4.2.

We will now find the black hole entropy. The Entropy of the non-supersymmetric solution determined by the value of the black hole effective potential at the critical point, $S = \pi V(\hat{x}_0 t)$. In terms of t_1 and t_2 we can write the effective potential has the following form:

$$V = \frac{3q_0^2 + 3\hat{D}^2|t|^6 - 6\hat{D}t_1^2|t|^2\hat{\alpha}_0 + (3t_1^2 + t_2^2)\hat{\alpha}_0^2 + 6q_0(\hat{D}t_1^3 - t_1\hat{\alpha}_0)}{6\hat{D}t_2^3} \tag{A.25}$$

Substituting the solution for t_1 and t_2 in the above expression, we get the entropy of the non-supersymmetric solution:

$$S = \frac{\pi p^0 \hat{\alpha}_0}{3}\sqrt{\frac{9q_0^2}{(p^0)^2\hat{\alpha}_0^2} - \frac{4\hat{\alpha}_0}{3\hat{D}}} \tag{A.26}$$

A.2 The mass matrix

In this section we will compute the mass matrix for the $D0 - D6$ system. We need the coefficients of the quadratic terms in the effective potential. It is straightforward to express them in terms of W and its covariant derivatives [19]:

$$\begin{aligned}
e^{-K_0}\partial_a\partial_d V &= \left\{ g^{b\bar{c}}\nabla_a\nabla_b\nabla_d W + \partial_a g^{b\bar{c}}\nabla_b\nabla_d W + \partial_d g^{b\bar{c}}\nabla_b\nabla_a W \right\} \overline{\nabla_c W} \\
&+ 3\nabla_a\nabla_d W\overline{W} + \partial_a\partial_d g^{b\bar{c}}\nabla_b W\overline{\nabla_c W} - g^{b\bar{c}}\partial_a g_{d\bar{c}}\nabla_b W\overline{W} \\
e^{-K_0}\partial_a\partial_{\bar{d}} V &= g^{b\bar{c}}\nabla_a\nabla_b W\overline{\nabla_c\nabla_d W} + \left\{ 2|W|^2 + g^{b\bar{c}}\nabla_b W\overline{\nabla_c W} \right\} g_{a\bar{d}} \\
&+ \partial_a g^{b\bar{c}}\nabla_b W\overline{\nabla_c\nabla_d W} + \partial_{\bar{d}} g^{b\bar{c}}\nabla_a\nabla_b W\overline{\nabla_c W} + 3\nabla_a W\overline{\nabla_d W} \\
&+ \partial_a\partial_{\bar{d}} g^{b\bar{c}}\nabla_b W\overline{\nabla_c W} \tag{A.27}
\end{aligned}$$

We need to explicitly evaluate these terms at the attractor point. Since the expressions are particularly lengthy, we will introduce some further notations and express various terms in eq.(A.27) in terms of them in a compact way. Define:

$$g_1 \;=\; 6t + \frac{3W}{2p^0 \hat{D} t_2{}^2}, \quad g_2 = \frac{-9W}{2p^0 \hat{D} t_2{}^2} + \frac{9it^2}{t_2} + \frac{3L}{it_2},$$

$$g_3 \;=\; \frac{2t_2{}^2}{3}\left(3g_1 + 2g_2\right), \quad g_4 = \frac{-2t_2{}^2 g_1}{3}, \quad g_5 = G - L, \quad g_6 = 3t^2 - L.$$

The covariant derivatives of the superpotential, in terms of these quantities are

$$\nabla_a W \;=\; g_5 \hat{D}_a p^0$$

$$\nabla_a \nabla_b W \;=\; p^0 \left(g_1 \hat{D}_{ab} + g_2 \frac{\hat{D}_a \hat{D}_b}{\hat{D}} \right)$$

$$\nabla_a \nabla_b W g^{b\bar{c}} \;=\; p^0 \left(g_3 \hat{D}_a \hat{x}^c + g_4 \hat{D} \delta_a{}^c \right) \tag{A.28}$$

We are now in a position to compute the mass matrix. Let us first consider individual terms in $\partial_a \partial_{\bar{d}} V$ and simplify them. We find

$$g^{b\bar{c}} \nabla_a \nabla_b W \overline{\nabla_c \nabla_d W} \;=\; (p^0)^2 \left(\hat{D}_a \hat{D}_d \left(\bar{g}_1 g_3 + \bar{g}_2 g_3 + \bar{g}_2 g_4 \right) + \bar{g}_1 g_4 \hat{D} \hat{D}_{ad} \right)$$

$$3 \nabla_a W \overline{\nabla_d W} \;=\; 3(p^0)^2 g_5 \bar{g}_5 \hat{D}_a \hat{D}_d$$

$$2 g_{a\bar{d}} |W|^2 \;=\; \frac{3}{\hat{D} t_2{}^2} \left(\frac{3 \hat{D}_a \hat{D}_d}{2 \hat{D}} - \hat{D}_{ad} \right) W \overline{W}$$

$$g_{a\bar{d}} g^{b\bar{c}} \nabla_b W \overline{\nabla_c W} \;=\; (p^0)^2 g_5 \bar{g}_5 \left(3 \hat{D}_a \hat{D}_d - 2 \hat{D} \hat{D}_{ad} \right)$$

$$\partial_a g^{b\bar{c}} \nabla_b W \overline{\nabla_c \nabla_d W} \;=\; \frac{-4it_2 (p^0)^2 \hat{D} g_5}{3} \left(\bar{g}_1 \hat{D}_{ad} + \bar{g}_2 \frac{\hat{D}_a \hat{D}_d}{\hat{D}} \right)$$

$$\partial_{\bar{d}} g^{b\bar{c}} \overline{\nabla_c W} \nabla_a \nabla_d W \;=\; \frac{4it_2 (p^0)^2 \hat{D} \bar{g}_5}{3} \left(g_1 \hat{D}_{ad} + g_2 \frac{\hat{D}_a \hat{D}_d}{\hat{D}} \right)$$

$$\partial_a \partial_{\bar{d}} g^{b\bar{c}} \nabla_b W \overline{\nabla_c W} \;=\; \frac{2 (p^0)^2 g_5 \bar{g}_5}{3} \left(3 \hat{D}_a \hat{D}_d - 2 \hat{D} \hat{D}_{ad} \right)$$

Similarly, after simplification, the individual terms in $\partial_a \partial_d V$ are given by

$$\begin{aligned}
g^{b\bar{c}} \nabla_a \nabla_b \nabla_d W \overline{\nabla_c W} \;=\;& \frac{4 t_2{}^2 (p^0)^2 \bar{g}_5 \hat{D}}{3} \left[\hat{D}_{ad} \left(6 + \frac{15iW}{4 p^0 \hat{D} t_2{}^3} + \frac{9it}{t_2} + \frac{9t^2}{2 t_2{}^2} - \frac{3L}{2 t_2{}^2} \right) \right. \\
&+ \frac{\hat{D}_a \hat{D}_d}{\hat{D}} \left(\frac{3i(g_1 + g_2)}{2 t_2} - \frac{27iW}{4 p^0 \hat{D} t_2{}^3} - \frac{3 g_6}{t_2{}^2} - \frac{9 t^2}{t_2{}^2} \right. \\
&\left. \left. + \frac{9it}{t_2} + \frac{3L}{t_2{}^2} \right) \right]
\end{aligned}$$

$$\partial_a g^{b\bar{c}} \nabla_b \nabla_d W \overline{\nabla_c W} = \frac{-4it_2(p^0)^2 \hat{D} \bar{g}_5}{3} \left(g_1 \hat{D}_{ad} + g_2 \frac{\hat{D}_a \hat{D}_d}{\hat{D}} \right)$$

$$3\nabla_a \nabla_d W \overline{W} = 3\overline{W} p^0 \left(g_1 \hat{D}_{ad} + g_2 \frac{\hat{D}_a \hat{D}_d}{\hat{D}} \right)$$

$$-g^{b\bar{c}} \partial_a g_{d\bar{c}} \nabla_b W \overline{W} = \frac{-ig_5 p^0 \overline{W}}{t_2} \left(\frac{3\hat{D}_a \hat{D}_d}{\hat{D}} - 2\hat{D}_{ad} \right)$$

$$\partial_a \partial_d g^{b\bar{c}} \nabla_b W \overline{\nabla_c W} = \frac{2(p^0)^2 |g_5|^2}{3} \left(2\hat{D}\hat{D}_{ad} - 3\hat{D}_a \hat{D}_d \right)$$

We will now substitute the above expressions in eq.(A.27) for the two derivative terms of the potential, add them up and simplify. For easy reading, we will define the function $\hat{F}_3(x)$ as

$$\hat{F}_3(x) = \sqrt{1 - 24x(\hat{F}_1(x))^2} \ , \tag{A.29}$$

and introduce the parameter $u = \hat{F}_3(\hat{D} q_0^2 / (\hat{\alpha}_0^3 (p^0)^2))$. We find

$$e^{-K_0} \partial_a \partial_d V = (p^0 \hat{\alpha}_0)^2 \hat{D}_{ad} (3 - u)(3 + u)^2 \frac{u^2 - 2u - 1 + (u - 1)\sqrt{u^2 - 2u - 3}}{3\hat{D}(u-1)^2}$$

$$e^{-K_0} \partial_a \partial_{\bar{d}} V = \frac{2(p^0)^2 (u-3)(u+3)^2 \alpha^2}{3\hat{D}^2 (u-1)^2} \left(\hat{D}\hat{D}_{ad} - 3\hat{D}_a \hat{D}_d \right) \tag{A.30}$$

Here K_0 is the value of the Kähler potential at the critical point.

We can express the mass matrix in the form

$$M = E \left(3\frac{\hat{D}_a \hat{D}_d}{\hat{D}} - \hat{D}_{ad} \right) \otimes \mathbf{I} + \hat{D}_{ab} \otimes (A\sigma^3 - B\sigma^1), \tag{A.31}$$

where the coefficients E, A and B are given by

$$E = e^{K_0} \left(\frac{4\hat{\alpha}_0^2 (p^0)^2 (3 - u)(u + 3)^2}{3\hat{D}(u-1)^2} \right)$$

$$A = e^{K_0} \left(\frac{2\hat{\alpha}_0^2 (p^0)^2 (3 - u)(u + 3)^2}{3\hat{D}(u-1)^2} (u^2 - 2u - 1) \right)$$

$$B = e^{K_0} \left(\frac{2\hat{\alpha}_0^2 (p^0)^2 (3 - u)(u + 3)^2}{3\hat{D}(u-1)} \sqrt{(u+1)(3-u)} \right) \ . \tag{A.32}$$

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