Radiophysics and Quantum Electronics, Vol. 36, No. 8, 1993

EFFECT OF EXTERNAL-NOISE ARRIVAL-ANGLE FLUCTUATIONS ON STATISTICAL CHARACTERISTICS OF ADAPTIVE ARRAY

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UDC 621.396

It is well-known that adaptive antenna arrays (AAA) greatly improve the effectiveness of spatial signal processing under conditions of *a priori* uncertainty. At the same time, the presence of various types of unsteadiness in the propagation channel can considerably impair AAA performance. The effectiveness of processing in the presence of various kinds of distortion of both the desired signal and noise has been investigated [1, 2]. However, the studies have been mainly limited to fluctuations that were either ultrafast or ultraslow in comparison with the adaptation time of the algorithm. Here we shall examine the effect of totally correlated fluctuations of the wave front of external noise with an arbitrary correlation time on the statistical characteristics of an adaptive antenna array that maximizes the signal-to-noise ratio. Note that fluctuations of this type are typical in problems of hydroacoustics and can be produced, for example, by angular oscillations of the antenna as well as of the wave front of the noise. Frequency fluctuations of the noise signal have the same result.

We shall select strictly correlated (coherent) fluctuations ($\varphi_k(t) \equiv \varphi(t)$) as a model of the external-noise wave-front fluctuations. A situation such as this can arise, for example, when the external-noise arrival angle fluctuates $\Theta(t)$ about its mean value Θ_m. Assuming that the angle variation $\Delta\Theta(t) = \Theta(t) - \Theta_m$ is small ($\Delta\Theta << 1$), the random noise vector $\vec{S}_n(t)$ can be written as follows:

$$\vec{S}_n(t) = D(t)\vec{S}_n,\ D(t) = \mathrm{dia}\{1, \exp[j\varphi(t)], \ldots, \exp[j(N-1)\varphi(t)]\},$$

$$D(t) \approx 1 + j\varphi(t)E_1 - \frac{1}{2}\varphi^2(t)E_2,$$

where $\varphi(t) = (2\pi d/\lambda)\cos(\Theta_m)\Delta\Theta(t)$ is the random phase advance between adjacent elements of the AAA corresponding to angle variation $\Delta\Theta(t)$, $E_1 = \mathrm{dia}[0, 1, \ldots, N-1]$, and $E_2 = \mathrm{dia}[0, 1, \ldots, (N-1)^2]$. We shall consider $\varphi(t)$ a Gaussian process with zero mean value $\langle\varphi(t)\rangle = 0$. Assuming that the fluctuation variance σ_φ^2 is small ($\sigma_\varphi^2 << (N-1)^{-2}$), formulas for the mean value of vector $\vec{S}_n(t)$ and the correlation matrix $K_{ss}(\tau) = \langle S_n^*(t)S_n^T(t)\rangle$ can be written as follows:

$$\langle\vec{S}_n(t)\rangle = (1 - \frac{1}{2}\sigma_\varphi^2 E_2)\vec{S}_n,$$

$$\begin{aligned} K_{ss}(\tau) &= \vec{S}_n^*\vec{S}_n^T + [\exp(k(\tau)) - 1]E_1\vec{S}_n^*\vec{S}_n^T E_1 - \\ &-\frac{1}{2}[\exp(\sigma_\varphi^2) - 1](E_2\vec{S}_n^*\vec{S}_n^T + \vec{S}_n^*\vec{S}_n^T E_2). \end{aligned} \tag{1}$$

where $k(\tau)$ and σ_φ^2 are the autocorrelation function and variance of process $\varphi(t)$. Taking (1) into account, for the mean values of the stochastic input-signal matrix $M(t) = \vec{X}^*(t)\vec{X}^T(t)$ and the fluctuational component $\Phi(t)$ we can write

$$\langle M(t)\rangle = R_{xx} + \eta_0^2\nu_n\sigma_\varphi^2(E_1\vec{S}_n^*\vec{S}_n^T E_1 - \frac{1}{2}E_2\vec{S}_n^*\vec{S}_n^T - \frac{1}{2}\vec{S}_n^*\vec{S}_n^T E_2),$$

$$\langle\Phi(t)\rangle = \eta_0^2\nu_n\sigma_\varphi^2(E_1\vec{S}_n^*\vec{S}_n^T E_1 - \frac{1}{2}E_2\vec{S}_n^*\vec{S}_n^T - \frac{1}{2}\vec{S}_n^*\vec{S}_n^T E_2).$$

Scientific-Research Institute of Applied Mathematics and Cybernetics, Nizhny Novgorod State University. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Radiofizika, Vol. 36, No. 8, pp. 840-842, August, 1993. Original article submitted April 22, 1993.

0033-8443/93/3608-0576$12.50

Limiting ourselves to the case of a single acting powerful ($\nu_n >> 1$) noise and using a procedure described elsewhere [3] to find the stochastic vector of the weight factors $\vec{W}(t)$, it is not difficult to derive a formula for the mean value of the first approximation $\langle \vec{W}_1(t) \rangle$

$$\langle \vec{W}_1(t) \rangle = -\sigma_\varphi^2 \nu_n \frac{\mu_0}{1+\mu_0} A \vec{W}_0,$$

where $A = [E_1 - (n_1/N)I]\vec{S}_n^* \vec{S}_n^T E_1$ is a matrix that is a function of the external-noise arrival angle and $n_1 = \vec{S}_n^T E_1 \vec{S}_n^*$. Note that the approximation $\vec{S}_n \vec{W}_0 << 1$ remains valid for our computations.

Opening the moment bracket $\langle \Phi(t-\tau) Q L(\tau') Q^+ \Phi(t-\tau-\tau') \rangle$

$$\langle \Phi(t-\tau) Q L(\tau') Q^+ \Phi(t-\tau-\tau') \rangle = (\eta_0^2 \nu_n)^2 k(\tau') \times$$

$$\times \{\exp(-\alpha_0 \tau')(n_2 - \tfrac{n_1^2}{N}) \vec{S}_n^* \vec{S}_n^T + \exp(-\alpha_1 \tau')[(\tfrac{n_1^2}{N}) \vec{S}_n^* \vec{S}_n^T + N E_1 \vec{S}_n^* \vec{S}_n^T E_1]\},$$

where $n_2 = \vec{S}_n^T E_2 \vec{S}_n^*$, we obtain a formula for the mean value of the second approximation $\langle \vec{W}_2(t) \rangle$

$$\langle \vec{W}_2(t) \rangle = \sigma_\varphi \nu_n \frac{\mu_0}{1+\mu_0} \frac{\tau_{cor}}{\tau_{cor} + \tau_{ad}} A \vec{W}_0.$$

To determine the power characteristics of the AAA in the presence of noise-angle fluctuations, we need formulas for the vector $æ_W = \langle \Phi(t) \vec{W}_1(t) \rangle$ and correlation matrix of the correction vector $K_W(t, t) = \langle \vec{W}_1^*(t) \vec{W}_1^T(t) \rangle$. It can be shown that

$$æ_W = \langle \Phi(t) \vec{W}_1(t) \rangle = -\sigma_\varphi^2 \eta_0^2 \nu_n \frac{\mu_0}{1+\mu_0} \frac{\tau_{cor}}{\tau_{cor} + \tau_{ad}} B \vec{W}_0,$$

$$K_W(t,t) = \langle \vec{W}_1^*(t) \vec{W}_1^T(t) \rangle = \sigma_\varphi^2 N^{-2} \times \left[\frac{\mu_0}{1+\mu_0} (\vec{W}_0^T B^* \vec{W}_0^*) \frac{\tau_{cor}}{\tau_{cor} + \tau_{ad}} \vec{S}_n \vec{S}_n^+ + |\vec{S}_n^T \vec{W}_q| \frac{\tau_{cor}}{\tau_{cor} + \tau_i} (1 - N^{-1} \vec{S}_n \vec{S}_n^+) \right],$$

where $B = E_1 \vec{S}_n^* \vec{S}_n^T E_1$. Then for the total external- and internal-noise power and for the desired-signal power at the system output we can write

$$P_{nfl} = \left(1 - \sigma_\varphi^2 \nu_n \frac{1-\mu_0}{1+\mu_0} \frac{\vec{W}_0^T B^* \vec{W}_0}{N(1 - f_0^2/N^2)} \frac{\tau_{ad}}{\tau_{cor} + \tau_{ad}}\right) P_n, \tag{2}$$

$$P_{sfl} = \left(1 - 2\sigma_\varphi^2 \nu_n \frac{\mu_0}{1+\mu_0} \frac{\vec{W}_0^T B^* \vec{W}_0^*}{N(1 - f_0^2/N^2)} \frac{\tau_{ad}}{\tau_{cor} + \tau_{ad}}\right) P_s, \tag{3}$$

where $f_0 = \vec{S}_n^T \vec{S}_{ctl}$, where $\vec{S}_{ctl}$ is the control vector of the AAA directivity pattern in the absence of noise. Dividing (3) by (2), with allowance for the smallness of σ_φ^2, and considering that $\vec{W}_0^T B^* \vec{W}_0^* = |f_1 - (n_1/N) f_0|^2$, where $f_1 = \vec{S}_n^T E_1 \vec{S}_{ctl}$, we obtain a formula for the output signal-to-noise ratio in the presence of noise-angle fluctuations ρ_{fl}

$$\rho_{fl} = \left(1 - \psi(\Theta_m) \sigma_\varphi^2 \nu_n \frac{\tau_{ad}}{\tau_{cor} + \tau_{ad}}\right) \rho_0,$$

where $\psi(\Theta_m) = |f_1 - (n_1/N) f_0|^2 / N(1 - f_0^2/N^2)$. Note that in the case of independent external-noise wave-front fluctuations, ρ_{fl} is determined by a similar formula for $\psi(\Theta_m) = \psi = 1$ [3]. As is apparent from analysis of the formula, $\psi(\Theta_m)$ is maximal value when the noise is at the null of the AAA directivity pattern and minimal when the noise arrives at the maximum of a side lobe. The dependence of ρ_{fl} on ν_n, σ_φ^2, and τ_{cor} remains the same as in the case of independent external-noise arrival-angle fluctuations; noise arriving at the null of the initial directivity pattern of the AAA has a greater effect on the signal-to-noise ratio.

REFERENCES

1. A. A. Mal'tsev and G. V. Serebryakov, Izv. Vyssh. Uchebn. Zaved., Radiofiz., **34**, No. 1, 52 (1991).
2. A. A. Mal'tsev, Izv. Vyssh. Uchebn. Zaved., Radiofiz., **30**, No. 8, 1101 (1987).
3. I. V. Savinov and G. V. Serebryakov, Radiotekh. Élektron. (in press).