2. A. A. Kuskov, "The phonon model of rupture of a loaded atomic chain," Fiz. Tverd. Tela, 21, No. 10, 3095-3099 (1979).
3. A. A. Kuskov and V. I. Vettegren', "Calculation of the endurance of a loaded chain of atoms in anharmonic approximation," Fiz. Tverd. Tela, 22, No. 11, 3350-3358 (1980).
4. S. N. Zhurkov, "On the physical basis of strength," Fiz. Tverd. Tela, 22, No. 11, 3344-3349 (1980).
5. S. N. Zhurkov, "The dilatonic mechanism of the strength of solids," Fiz. Tverd. Tela, 25, No. 10, 3119-3123 (1983).
6. V. A. Petrov, "The dilatonic model of thermofluctuational crack nucleation," Fiz. Tverd. Tela, 25, No. 10, 3124-3127 (1983).
7. P. Resibois and M. de Lenert, The Classical Kinetic Theory of Liquids and Gases [Russian translation], Mir, Moscow (1980).
8. É. M. Kartashov, V. V. Shevelev, A. A. Valishin, and G. M. Bartenev, "Limit characteristics of brittle failure of polymers," Vysokomol. Soedin., (A) 28, No. 4, 805-809 (1986).
9. V. V. Shevelev and É. M. Kartashov, "Theory of brittle failure of high-strength materials," Fiz. Khim. Mekh. Mater., 24, No. 6, 11-14 (1988).
10. S. N. Zhurkov and V. S. Kuksenko, "The micromechanics of the failure of polymers," Mekh. Polim., No. 5, 792-801 (1974).
11. A. I. Slutsker and V. S. Kuksenko, "Nucleating submicroscopic cracks in loaded polymers," Mekh. Polim., No. 1, 84-94 (1975).
12. V. A. Petrov, "Thermodynamic approch to the micromechanism of failure of solids," Fiz. Tverd. Tela, 25, No. 10, 3110-3113 (1983).
13. V. V. Panasyuk, Limit Equilibrium of Brittle Bodies with Cracks [in Russian], Naukova Dumka, Kiev (1968).
14. E. Janke, F. Emde, and F. Lesch, Special Functions [Russian translation], Nauka, Moscow (1977).
15. G. M. Bartenev, Strength and the Mechanism of Failure of Polymers [in Russian], Khimiya, Moscow (1984).
16. É. M. Kartashov and G. M. Bartenev, "Processes of failure of polymers in brittle and quasibrittle states," Vysokomol. Soedin., (A) 23, No. 4, 904-912 (1981).

INTERACTION OF SURFACE CRACKS IN A UNIAXIALLY EXTENDED PLATE

A. E. Elagin

UDC 539.375

A method of experimentally evaluating the effects of interaction of normal dilatational surface cracks in a uniaxially extended plate with finite dimensions was suggested and approved. The effect of interaction was evaluated according to the ratio of the displacements of the lips of an interacting and of an isolated crack with identical dimensions. The obtained results were used for calculating the stress intensity factors K_I for systems of surface defects in plates with finite dimensions.

The present communication explains the results of the elaboration and approval of a method of the experimental evaluation of the effects of interaction of normal dilatational surface cracks in a uniaxially extended plate with finite dimensions. The concept of interaction of cracks denotes the existence of mutual influence of stress, strain, and displacement fields for cracks whose dimensions are comparable with the distance between them. The problem of collective interaction of cracks is complex, and as had been noted, "the solution

---

Institute of the Strength of Materials, Academy of Sciences of the Ukrainian SSR, Kiev.
Translated from Problemy Prochnosti, No. 3, pp. 14-17, March, 1990. Original article submitted August 12, 1989.

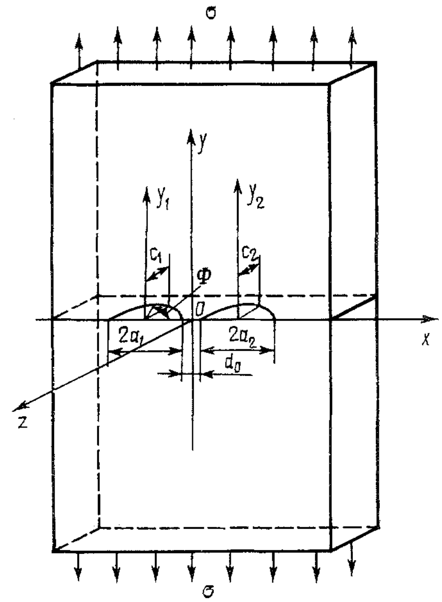

Fig. 1. Specimen PS(T) with two collinear surface cracks.

of even the simplest problems of this class has so far not even a qualitative description" [1].

The known theoretical solutions, e.g., [2, 3], confine themselves to cases of fairly large distances between the cracks although the disposition of interacting cracks close to each other is of the greatest interest [4-7]. From among the experimental works, [8] might be mentioned, where the method of determining K_I by the speed of a slowly growing crack in models of organic glass is used for investigating the interaction of systems of through cracks with different configuration. However, this method is not applicable to systems of surface cracks because the correlation between the crack growth rate and the magnitude of K_I is ambiguous [9]; this ambiguity is apparently due to the unbalanced structure of the end zone of the surface crack [8].

It follows from the obtained results [4-8] that of fundamental interest are three (basic) configurations of systems of interacting cracks: collinear, parallel cracks, and also their combinations, i.e., a system of collinear cracks with parallel cracks situated above the neck between them. The first and the third arrangement were realized in the present work.

Static tensile tests were carried out with flat specimens PS(T), 100 × 295 mm in size, made of sheet polymethyl metacrylate (PMMA) marque TOSP (modulus of elasticity E = 3.03 GPa, Poisson ratio $\nu = 0.37$), 8 mm thick. The method of impact wedging of the initiating notch was used to provide surface cracks with specified dimensions and orientation in the specimens (Fig. 1).

It should be noted that the method of providing the initial cracks can have a substantial influence on the state of stress and strain near the crack front. Closest to the theoretical model of a crack in the form of a mathematical slit are fatigue and creep cracks grown at comparatively low load levels. However, when the initial cracks are initiated by such methods, the load is applied over the entire section of the specimen, and it is practically impossible to obtain an ordered system of cracks. The method of local load application was therefore used, i.e., the method of impact wedging of the initiating notch, which, judging by the results of previous work [10], yields measured values of $v(x, 0)$ that are lower than the theoretical values.

On the front face of the specimen we provided a network of base points consisting of lines symmetrically arranged about the Ox- and Oy-axes and parallel to them, equally spaced in the direction of the Ox-axis, and also situated at distances y equal to 0.5, 1, 2, 4 mm from the plane of the cracks. For each of the base points with the coordinates (x; ±y) thus obtained we recorded by a tensometric shackle with adjustable distance between the apexes of the reference indentors at least three elastic sections of the diagram load P vs.

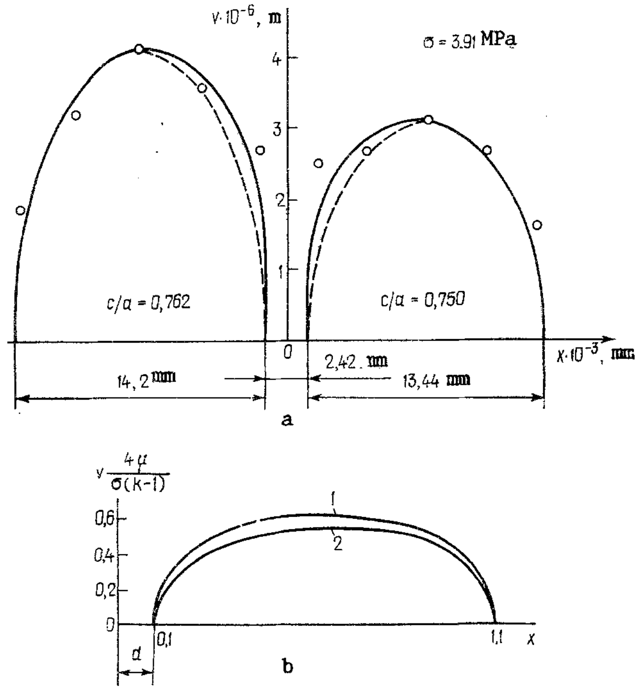

Fig. 2. Experimental profiles of interacting collinear surface cracks (a) and theoretical profiles of interacting through cracks (b) under tension

$$\left(\mu = E/2(1+\nu);\ k = \begin{cases} (3-\nu)/(1+\nu) & \text{—plane-parallel state,} \\ (3-4\nu) & \text{— plane strain} \end{cases}\right):$$

1, 2 —two cracks and one crack, respectively.

displacement 2v. By using in analogy to [11] the linear interpolation of the measured displacements to the axis of ordinates, the sought displacement of the crack lips v was found.

In the present work we carried out comparative measurements of the displacement of the lips of a surface crack produced by the method described above, and of a crack with a previously made initiating notch with the required shape on its front face. The results of the measurements, compared with the results calculated by known formulas [10, 12-15], are presented in Table 1. It can be seen that the method of growing the cracks influences the magnitude of the displacement of the lips of the surface crack: The experimental values are much smaller than the calculated ones. An analogous result was also obtained earlier on [10] where one of the possible variants of explaining the observed difference between the measured and the calculated values was suggested; the cause of this difference is apparently the discrepancy between the theoretical models and the conditions obtaining in the given experiment.

For the quantitative evaluation of the effect of interaction of dilatational cracks it was decided to use the ratio of the stress intensity factor K_I for the crack that is part of a system of interacting cracks, and an isolated crack with identical dimensions and analogous-

TABLE 1. Comparison of the Calculated and Experimental Values of the Maximal Displacement of the Lips of a Surface Crack under Unaxial Extension Ev'/σ_c

Method of inducing crack	Dimensions of crack	Experimental value	Calculated value obtained by formulas				
			[12, 13]	(7)—(9)—(11) [10]	(7), (8) [10]*	(19) [14]	(15), (16) [15]
Impact	$c/a=0{,}569$; $c/B=0{,}490$	0,936	1,143	2,421	2,361	2,331	3,113
Bending	$c/a=0{,}546$; $c/B=0{,}370$	1,175	1,504	2,312	2,298	2,224	2,917

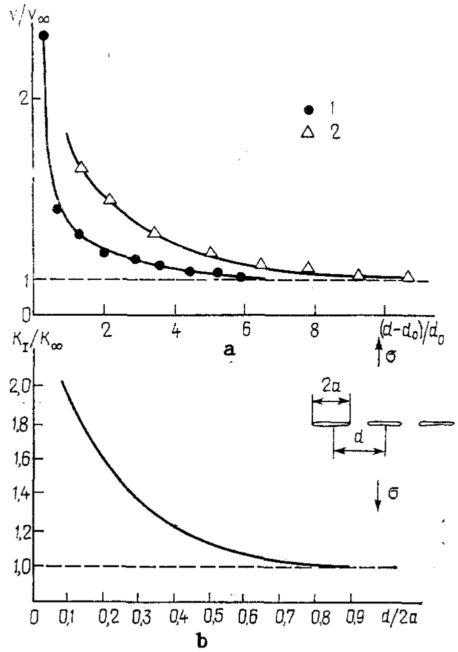

Fig. 3. Effect of interaction between collinear surface (a) and through cracks (b) in dependence on the distance between them: 1) $d_0 = 2.42 \cdot 10^{-3}$ m; 2) $d_0 = 0.8 \cdot 10^{-3}$ m. (The dashed lines correspond to the case of an isolated crack.)

ly loaded. However, as already mentioned above, at present there does not exist any acceptable method of calculating K_I for systems of interacting cracks lying close to each other. There are various indirect methods of determining K_I where, within the framework of some model, an arbitrary experimentally measured magnitude is converted into K_I [6, 8], but all these methods are subject to fairly stringent limitations which preclude their application to a broad range of problems.

In the present work the effect of interaction of cracks is evaluated according to the ratio of the experimentally measured displacements of the lips of cracks of the system and of an identical isolated crack in the body under consideration. Figure 2a shows the deformed profiles of two collinear interacting surface cracks under tension on the front face of the specimen ($z = 0$). The dots represent displacements measured by the above-described method, the solid lines show the profile of the crack plotted according to the experimental data, the dashed lines denote calculation by the formula

$$v = v^* \sqrt{1 - \left(\frac{x}{a}\right)^2}, \tag{1}$$

where v^* is the experimentally measured displacement of the crack lips on its base axis ($x = 0$). It can be seen from Fig. 2a that in interaction with a neighboring crack the shape of the deformed profile on the front face of the specimen deviates increasingly from elliptical shape. The same pattern is found in the case of interaction of collinear through cracks (Fig. 2b) [16].

The effect of interaction of cracks depends on the distance between them. In distinction to through cracks, where d, the distance between them, is a constant magnitude, the distance between surface cracks in consequence of the curvature of their front is a function of the angle Φ (Fig. 1). On the front face of the specimen, i.e., with $\Phi = 0°$, $d(\Phi) = d_0$, at $\Phi = \frac{\pi}{2}$ $d(\Phi) = a_1 + a_2 + d_0$. If we denote by v the displacement of the crack lips measured on the front face of the specimen for points of the crack front determined by the angle Φ, and if we denote by v_∞ the displacements calculated by formula (1) for the corresponding values of x, we can evaluate the effect of interaction of the cracks in dependence on the distance between them $(d - d_0)/d_0$ by the ratio v/v_∞ (Fig. 3a). Figure 3b shows the results analogous-

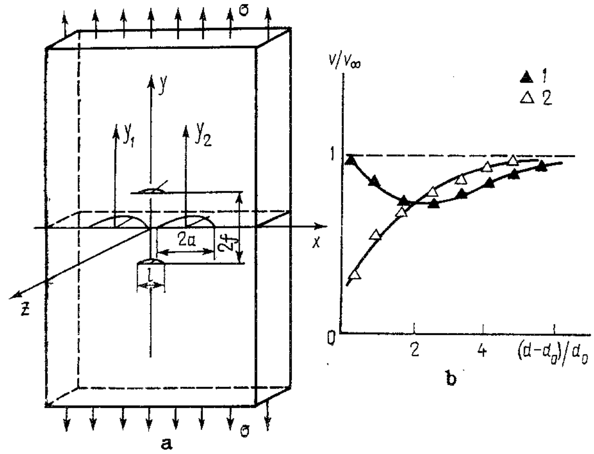

Fig. 4. Specimen PS (T) with a system of four interacting surface cracks (a) and the influence of the distance between cracks on the effect of interaction between the collinear cracks contained in the given system (b), for $f/\ell = 1.25$; $2a/\ell = 1.563$.

ly expressing the dependence of the effect of interaction under tension of collinear through cracks on the distance between them.

It is known from the theoretical solutions provided in the literature that when cracks interact, the rupture stresses become lower, i.e., the stress intensity factors increase, which applies to collinear cracks, but in some cases, conversely, the level of the rupture stresses becomes higher than the rupture stress in a part with an isolated defect with analogous dimension, in other words, the stress intensity factors become lower. For instance, we investigated the interaction of a macrocrack with a "bundle" of microcracks situated one above the other at its tip [17, 18]. It follows from the results of this work that such a disposition of the microcracks has a "protective effect," and its maximal value may attain 25% [18] and 77% [17]. According to the presented data [19, 20], the disposition of cracks above each other leads to a decrease of K_I.

With the object of experimentally studying these phenomena, and also to demonstrate the possibilities of the suggested experimental approach to the evaluation of the effects of interaction of cracks, we tested a specimen with a system of surface cracks consisting of two unequal collinear cracks (one larger 1, $c/a = 0.615$; $c/B = 0.494$, the other smaller 2, $c/a = 0.623$; $c/B = 0.520$) and two parallel small cracks ($c/a = 0.666$; $c/B = 0.390$) situated above the neck between the other cracks.

The results of the measurements (Fig. 4b) showed that in this case there is a protective effect ($v/v_\infty < 1$) and that the nature of the behavior of cracks 1 (dark dots) is different from that of cracks 2 (light dots). The opening of crack 1 is close to the case of isolation ($v/v_\infty \to 1$), whereas the opening of crack 2 upon approach to the neighboring crack 1 tends to zero.

Thus, on the basis of previous methodological work [9-11], a fairly effective and lucid method of quantitative evaluation of the interaction of cracks was worked out and approved. It was apparently the first time that the deformed profiles of surface cracks were experimentally determined, these cracks belonging to two characteristic and practically important arrangements of their relative disposition in a uniaxially extended plate with finite dimensions. Subsequently the results of the present investigation will be used within the framework of the method of weight functions in the calculation of the values of the parameter K_I.

The author expresses his appreciation to V. P. Naumenko, Dr. Sci., for stating the problem and participating in the analysis of the results of the investigations.

LITERATURE CITED

1. G. P. Cherepanov, "Present-day problems of fracture mechanics," Probl. Prochn., No. 8, 3-13 (1987).

2. V. V. Panasyuk, M. P. Savruk, and A. P. Datsyshin, Stress Distribution Near Cracks in Plates and Shells [in Russian], Naukova Dumka, Kiev (1976).
3. M. P. Savruk, Two-dimensional Problems of Bodies with Cracks [in Russian], Naukova Dumka, Kiev (1981).
4. M. V. Khai, I. P. Laushnik, and I. V. Kalynyak, "Interaction of closely spaced disk-shaped complanar cracks," Fiz.-Khim. Mekh. Mater., 21, No. 6, 87-88 (1985).
5. M. Isida, K. Hirota, H. Nogushi, and T. Yoshida, "Two parallel elliptical cracks in an infinite solid subjected to tension," Int. J. Fract., 27, No. 6, 31-48 (1985).
6. B. Farahmand and D. D. Raftopoulos, "A two-dimensional investigation on plates with oblique equal and unequal edge cracks subjected to tension," Int. J. Fract., 19, No. 3, 203-220 (1982).
7. Y. Muton, I. Sakamoto, M. Inoue, and T. Iwamoto, "Study on an assessment of defects for neighboring embedded cracks," J. Soc. Mater. Sci., 35, No. 393, 654-658 (1986).
8. N. A. Dolotova, V. I. Aleshin, V. A. Zeiliger, and M. I. Bessonov, "Experimental study of interaction and merger of collinear cracks," Probl. Prochn., No. 7, 16-23 (1987).
9. G. S. Pisarenko, V. P. Naumenko, and N. I. Nedelchev, "Characteristic of crack resistance of brittle material. What is that?" Probl. Prochn., No. 11, 17-24 (1985).
10. N. I. Nedelchev, "Comparison of the theoretical and experimental opening of a normal dilatational surface crack," Probl. Prochn., No. 12, 46-52 (1985).
11. V. P. Naumenko and N. I. Nedelchev, "Method and results of the experimental determination of the profile of a surface crack," Probl. Prochn., No. 12, 25-30 (1985).
12. A. E. Green and I. N. Sneddon, "The distribution of stress in the neighborhood of a flat elliptical crack in an elastic solid," Proc. Cambridge Phil. Soc., 46, 159-164 (1950).
13. J. C. Newman and I. S. Raju, "An empirical stress-intensity factor equation for the surface crack," Eng. Fract. Mech., 15, No. 1-2, 185-192 (1981).
14. V. A. Vainshtok, "Calculation of crack lip opening by the stress intensity factors," Probl. Prochn., No. 10, 35-38 (1988).
15. C. Mattheck, P. Morawietz, and D. Munz, "Stress intensity factor at the surface and at the deepest point of a semi-elliptical surface crack in plates under stress gradients," Int. J. Fract., 23, No. 3, 201-212 (1983).
16. J. Eftis, "Influence of load biaxiality on the fracture characteristics of two collinear cracks," Int. J. Fract., 24, No. 1, 59-80 (1984).
17. A. A. Rubinstein and H. C. Choi, "Macrocrack interaction with transverse array of microcracks," Int. J. Fract., 36, No. 1, 15-26 (1988).
18. M. Kachanov, "On crack—microcrack interaction," Int. J. Fract., 30, No. 4, R65-R72 (1986).
19. I. V. Kalynyak, I. P. Laushnik, and M. V. Khai, "Limit equilibrium of an infinite body containing closely spaced gas-filled disk-shaped cracks," Probl. Prochn., No. 8, 110-113 (1986).
20. F. F. Lange, "Interaction between overlapping parallel cracks; a photoelastic study," Int. J. Fract. Mech., 4, No. 3, 287-294 (1968).