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Coupling with the Jahn-Teller Mode for Triplet States of MF_6 ($M = Mn^{2+}$, Cr^{3+}) Complexes: Dependence on the M-F Distance and Influence on the Stokes Shift

By

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The microscopic origin of the $V_{\rm E}$ coupling constant with the Jahn-Teller mode ${\rm E_g}$ for orbital triplet states corresponding to several excited states of ${\rm MnF_6^{4-}}$ and the ${}^4{\rm T_{2g}}$ state of ${\rm CrF_6^{3-}}$ is explored by molecular orbital (MO) calculations. $V_{\rm E}$ is shown to be determined by the splittings $\Delta_{\rm e}$ and $\Delta_{\rm t}$ induced in the antibonding ${\rm c_g^*}$ and ${\rm t_{2g}^*}$ levels by a $Q_{\theta}(\sim 3z^2-r^2)$ distortion and so $V_{\rm E}<0$ for the ${}^4{\rm T_{2g}}$ state of ${\rm CrF_6^{3-}}$ leading to a compressed octahedron as equilibrium geometry. $V_{\rm E}$ values have been derived from self-consistent charge extended Hückel and MSX α calculations performed at different Q_{θ} values. The results for the ${}^4{\rm T_{1g}}({\rm G})$, ${}^4{\rm T_{2g}}({\rm G})$, and ${}^4{\rm T_{1g}}({\rm P})$ states of ${\rm MnF_6^{4-}}$ are reasonably close to the experimental figures. For the first excited state ${}^4{\rm T_{1g}}({\rm G})$ the value $V_{\rm E}\approx 60~{\rm cm^{-1}}/{\rm pm}$ found for a metal–ligand distance R equal to 213 pm leads to a Huang-Rhys factor $S_{\rm E}=1.5$ also close to experimental findings. On passing from this case to the ${}^4{\rm T_{2g}}$ state of ${\rm CrF_6^{3-}}$ $|V_{\rm E}|$ increases by a factor of about two but $S_{\rm E}=1.2$ in agreement with experimental data for Rb₂KGaF₆: ${\rm Cr^{3+}}$. The latter figure implies a Stokes shift $E_{\rm S}^0(E)$ due to the $E_{\rm g}$ mode equal to 1200 cm⁻¹ which is about 50% of the total Stokes shift. As a salient feature it is shown that for both ${\rm MnF_6^{4-}}$ and ${\rm CrF_6^{3-}}$ complexes $V_{\rm E}$ strongly depends upon R, as it also happens to the $V_{\rm A}$ coupling constant with the symmetric $A_{\rm 1g}$ mode. This reasonably explains the increase of the Stokes shift upon increasing R recently observed for ${\rm Mn^{2+}}$ -doped fluoroperovskites. Although pure crystal-field (CF) theory gives rise to $V_{\rm E}$ values much smaller than the experimental ones it is shown that the relation $V_{\rm A}/V_{\rm E}=\sqrt{2}$ derived for the first

1. Introduction

Photoluminescence is one of the most attractive phenomena associated with transition metal (TM) impurities in insulators. In this process an optical absorption band peaked at energy $E_{\rm a}$ gives rise to an emission band whose maximum is not at $E_{\rm a}$ but at $E_{\rm e} < E_{\rm a}$. Since the first explanation of this experimental fact (known as Stokes rule) through the concepts of photon and energy conservation [1] models have been proposed to explain the microscopic origin of the difference $E_{\rm a} - E_{\rm e}$ (usually called Stokes shift) in terms of a local lattice relaxation [2 to 6]. Because of this the equilibrium metal-ligand distance changes when we go from an electronic state to another one n, as the associate

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energy $E_n(\{Q_k\})$ is in fact a function of the normal coordinates $\{Q_k\}$ of the involved nuclei. Such coordinates are assumed to be the same for any electronic state. The values $\{Q_k\} \equiv 0$ correspond to the equilibrium situation for the electronic ground state.

If we consider in principle an orbitally singlet excited state, its energy $E_n(\{Q_k\})$ can be written as

$$E_n(\{Q_k\}) = E_{n+}^0 \sum_{k} \{V_{nk}Q_k + \frac{1}{2} \mu \omega_{nk}^2 Q_k^2 + \dots\},$$
 (1)

while for the ground state $E_0(\{Q_k\})$ has the form

$$E_0(\{Q_k\}) = E_0^0 + \sum_k \left\{ \frac{1}{2} \mu \omega_{0k}^2 Q_k^2 + \ldots \right\}.$$
 (2)

The dependence on $\{Q_k\}$ of the energies comes from the corresponding dependence of the adiabatic Hamiltonian $H(\mathbf{r}, \{Q_k\})$, which can simply be written as

$$H(\mathbf{r}, \{Q_k\}) = H(\mathbf{r}, 0) + \sum_{k} (\partial V/\partial Q_k) Q_k + \dots$$
 (3)

If only the linear term in $\{Q_k\}$ is retained in (3) we are working in the so-called *linear* electron-phonon coupling regime. In this regime $\omega_{nk} = \omega_{0k} \equiv \omega_k$ and the minimum value of $E_n(\{Q_k\})$ appears at the coordinates $\{Q_{nk}^0\}$ given by

$$Q_{nk}^0 = -\frac{V_{nk}}{\mu \omega_k^2} \,. \tag{4}$$

Thus, $E_n(\{Q_{nk}^0\})$ is equal to

$$E_n(\lbrace Q_{nk}^0\rbrace) = E_n^0 - \sum_k S_{nk} \hbar \omega_k \,,$$

$$S_{nk}\hbar\omega_k = \frac{V_{nk}^2}{2\mu\omega_k^2} \,, \tag{5}$$

where S_{nk} depicts the Huang-Rhys factors associated with the *n*-th electronic excited state and the *k*-th vibrational mode. Within this scheme, if the maximum of an absorption band appears at $E_n^0 - E_0^0$ (following the Franck-Condon principle), the corresponding emission peak corresponds to $E_n(\{Q_{nk}^0\}) - E_0(\{Q_{nk}^0\})$ and thus the Stokes shift is just given [5] by

$$E_{\rm S}^0 = 2\sum_k S_{nk}\hbar\omega_k \,. \tag{6}$$

If the electronic transition is not allowed but assisted [7, 8] by an odd phonon of frequency $\omega_u/2\pi$, then the final expression for the Stokes shift becomes

$$\begin{split} E_{\mathrm{S}} &= E_{\mathrm{S}}^{0} + \Delta E_{\mathrm{u}} ,\\ \Delta E_{\mathrm{u}} &= 2\hbar \omega_{\mathrm{u}} \tanh \left(\hbar \omega_{\mathrm{u}} / 2kT \right). \end{split} \tag{7}$$

Therefore, to gain a better insight into the experimental Stokes shift a microscopic understanding of the coupling coefficients V_{nk} is required. In the case of TM impurities in insulators it is known that the electronic properties can reasonably be described by means of the MX_p complex formed by the impurity with the p ligands. Moreover, in cases like Cr^{3+} -doped elpasolite lattices the low temperature optical spectra reveal that

coupling occurs with local modes of the MX_p complex [9 to 14]. In particular progressions involving the symmetric $\mathrm{A}_{1\mathrm{g}}$ and the Jahn-Teller E_{g} modes have been well observed in the ${}^4\mathrm{T}_{2\mathrm{g}} \to {}^4\mathrm{A}_{2\mathrm{g}}$ electronic transition involving the first excited state ${}^4\mathrm{T}_{2\mathrm{g}}$. Moreover, it has been pointed out [7, 10] that the room temperature Stokes shift can essentially be understood only through the coupling with the two stretching modes $\mathrm{A}_{1\mathrm{g}}$ and E_{g} . In other words it can simply be written

$$E_{\rm S}^0 = E_{\rm S}^0({\rm A}) + E_{\rm S}^0({\rm E}), \tag{8}$$

where

$$E_{\rm S}^{0}(A) = 2S_{\rm A}\hbar\omega_{\rm A} ,$$

$$E_{\rm S}^{0}(E) = 2S_{\rm E}\hbar\omega_{\rm E} ,$$
(9)

and so only two coupling constants V_A and V_E related to the first excited state would determine their local relaxation and consequently E_S .

Regarding the coupling coefficient V_A a recent work [8] has investigated its microscopic origin for the first excited states of O_h complexes of Cr^{3+} , Mn^{2+} , and V^{2+} involving halides as ligands. It has been shown [8] that: (i) V_A basically reflects the dependence of the cubic field splitting, 10Dq, upon the metal-ligand distance R. Such a dependence is usually written as

$$10Dq = CR^{-n} \tag{10}$$

in the vicinity $(\Delta R/R_0 \ll 1)$ of R_0 , where $R_0 = r(M) + r(X)$ involves the ionic radii of M and X. (ii) The R dependence of $E_S(A)$ in the vicinity of R_0 goes as $R^{6\gamma_A-2(n+1)}$, where γ_A is the Grüneisen coefficient for the A_{1g} mode. (iii) In the case of O_h complexes of divalent ions like Mn^{2+} or V^{2+} , $E_S(A)$ would increase when R increases.

As regards the latter statement recent experimental results [15] on Mn^{2+} -doped fluoroperovskites unambiguously demonstrate that the total *Stokes shift E*_S at room temperature *increases* as far as the Mn^{2+} -F⁻ distance increases.

Having in mind all these facts a main goal of the present work is not only to calculate $V_{\rm E}$ as a function of R for the first excited state of ${\rm CrF_6^{3-}}$ and ${\rm MnF_6^{4-}}$ complexes but specially to understand the microscopic origin of $V_{\rm E}$ in a direct way. After explaining the calculation procedure in Section 2 it is shown in Section 3 that the calculated values of $V_{\rm E}$ for several excited states of ${\rm MnF_6^{4-}}$ and ${\rm CrF_6^{3-}}$ are close to the experimental values [16, 17]. Moreover, it is possible to explain the observed increase experienced by the Stokes shift of ${\rm MnF_6^{4-}}$ (and also ${\rm VCl_6^{4-}}$ [8, 18]) upon increasing R.

2. Theoretical

2.1 Foundations

Linear coupling of electronic states $|T_{\mu}\rangle$ ($\mu=x,\,y,\,z$) (corresponding to an orbital triplet) with a r-degenerate Γ mode (characterized by the normal coordinates $Q_a,\,a=1\ldots r$) requires that not all the matrix elements $\langle T_{\mu}|\,\partial V/\partial Q_a\,|T_{\nu}\rangle$ are zero. Thus, as $\partial V/\partial Q_a$ belongs to the Γ representation it must be verified that $T\times T\supset \Gamma$ and so in the case of an octahedral complex A_{1g} and E_g are the only stretching modes which can be linearly coupled to an orbital triplet. The form of $Q_{\theta}(\sim 3z^2-r^2)$ and $Q_{\varepsilon}(\sim x^2-y^2)$ normal coordinates displaying the Jahn-Teller E_g mode are depicted in Fig. 1.

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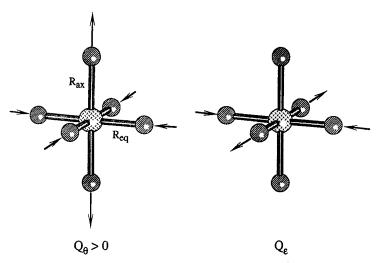


Fig. 1. Picture of the Q_{θ} and Q_{ε} normal coordinates corresponding to an octahedral MX₆ complex. $R_{\rm ax}$ and $R_{\rm eq}$ are the axial and equatorial metal-ligand distances, respectively, in D_{4h} symmetry appearing under Q_{θ} distortion

Under the action of $Q_{\theta} > 0$ ($Q_{\varepsilon} = 0$) an octahedron characterized by a metal-ligand distance R gives rise instantaneously to an elongated octahedron (D_{4h} symmetry) with two different metal-ligand distances $R_{\rm ax}$ and $R_{\rm eq}$ (defined in Fig. 1) such as $(R_{\rm ax} - R) = -2(R_{\rm eq} - R) = Q_{\theta}/\sqrt{3}$. To calculate the effects of the linear coupling with the E_g mode within the adiabatic approximation requires to evaluate the twelve matrix elements $\langle T_{\mu} | \partial V / \partial Q_{\alpha} | T_{\nu} \rangle$ ($\alpha = \theta, \varepsilon$). In other words the effects of the electron-phonon coupling term $(\partial V / \partial Q_{\alpha}) Q_{\alpha}$ (depicted in (3)) within the orbital triplet can be described by the Hamiltonian $H_{\rm cp}$ whose expression is given by

$$H_{ep} = \mathbf{V}_{\theta} Q_{\theta} + \mathbf{V}_{\varepsilon} Q_{\varepsilon} \,, \tag{11}$$

where \mathbf{V}_{θ} and \mathbf{V}_{ε} are symmetric 3×3 matrices involving $\langle \mathbf{T}_{\mu} | \partial V / \partial Q_{\theta} | \mathbf{T}_{\nu} \rangle$ and $\langle \mathbf{T}_{\mu} | \partial V / \partial Q_{\varepsilon} | \mathbf{T}_{\nu} \rangle$ matrix elements, respectively. As $H_{\rm ep}$ must be invariant under $O_{\rm h}$ group operations applied *simultaneously* on electronic \mathbf{V}_{θ} and \mathbf{V}_{ε} matrices and normal coordinates Q_{θ} and Q_{ε} , this determines all but one of twelve matrix elements. So the mentioned coupling can be described [19] by the simple *effective* Hamiltonian

$$H_{\text{eff}} = V_{\text{E}} \{ \mathbf{U}_{\theta} Q_{\theta} + \mathbf{U}_{\varepsilon} Q_{\varepsilon} \}, \tag{12}$$

where

$$\mathbf{U}_{\theta} = \begin{pmatrix} 1/2 & 0 & 0\\ 0 & 1/2 & 0\\ 0 & 0 & -1 \end{pmatrix}, \qquad \mathbf{U}_{\varepsilon} = \begin{pmatrix} -\sqrt{3}/2 & 0 & 0\\ 0 & \sqrt{3}/2 & 0\\ 0 & 0 & 0 \end{pmatrix}$$
(13)

operate upon the $|T_x\rangle$, $|T_y\rangle$, and $|T_z\rangle$ wavefunctions. The present problem is thus strongly connected with the changes of the polarizability tensor of an O_h molecule induced by E_g vibrations which appear in the Raman spectroscopy of octahedral complexes.

Adding the elastic energy term $(1/2)\mu\omega_{\rm E}^2(Q_\theta^2+Q_\varepsilon^2)$ to (12) the potential energy surfaces display three equivalent minima [19] at

$$\{Q_{\theta}^{0} = q_{\rm E}, \ Q_{\varepsilon}^{0} = 0\},$$

$$\{Q_{\theta}^{0} = -\frac{1}{2} q_{\rm E}, \ Q_{\varepsilon}^{0} = (\sqrt{3}/2) q_{\rm E}\},$$

$$\{Q_{\theta}^{0} = -\frac{1}{2} q_{\rm E}, \ Q_{\varepsilon}^{0} = -(\sqrt{3}/2) q_{\rm E}\};$$

$$q_{\rm E} = V_{\rm E}/\mu\omega_{\rm E}^{2},$$

$$(14)$$

stressing the full cubic symmetry of the problem. The involved decrement of energy is equal to

$$E_{\rm JT} = \frac{1}{2} \mu \omega_{\rm E}^2 q_{\rm E}^2 = \frac{1}{2} (V_{\rm E}^2 / \mu \omega_{\rm E}^2) = S_{\rm E} \hbar \omega_{\rm E}$$
 (15)

for each of the three equivalent surfaces.

It is worth noting that the point $\{Q_{\theta}=0;\ Q_{\varepsilon}=0\}$ is the intersection point of the three surfaces and not a branch point [20, 19]. Thus, the E_g mode in the present case behaves as a potentially active Jahn-Teller mode according to Toyozawa and Inoue [20]. It means that in the first electronic transition of CrF_6^{3-} or MnF_6^{4-} the resulting band does not exhibit any splitting coming from the Jahn-Teller coupling. Thus, in the present cases the existence of Jahn-Teller coupling in the excited state can be seen experimentally through the observation of progressions associated with the E_g mode in the low temperature optical spectra [9, 11]. Nevertheless, the shape of the optical bands at room temperature is the same as if only the coupling with the symmetric A_{1g} modes occurs. This explains the validity of (8) and (9) for the present case in spite of the fact that the excited state is not singlet but degenerate. At the same time (8) and (9) point out that an indirect evidence on the coupling to the E_g mode can also be extracted from the analysis of the experimental Stokes shift as it is discussed in [8].

A further insight into the Jahn-Teller coupling in an orbital triplet thus requires a microscopic interpretation of the coupling constant $V_{\rm E}$ appearing in the effective Hamiltonian (12). A simple way to do that is exploring the dependence of energies $E_{\mu}(Q_{\theta}, Q_{\varepsilon})$ on Q_{θ} associated with the $|T_{\mu}\rangle$ ($\mu=x,y,z$) states of the triplet. Thus, the comparison between (11) and (12) leads to

$$V_{\rm E} = -[\partial E_z/\partial Q_{\theta}]_{Q_{\epsilon}=0} = -[\partial (E_z - E_0)/\partial Q_{\theta}]_{Q_{\epsilon}=0} = \frac{2}{3} \left[\partial (E_x - E_z)/\partial Q_{\theta}\right]_{Q_{\epsilon}=0}.$$
(16)

So the coupling constant $V_{\rm E}$ can be determined through the energy variations induced by *static* tetragonal deformations. A deformation is characterized by $R_{\rm ax}$ and $R_{\rm eq}$ (Fig. 1) verifying $(R_{\rm ax}-R)+2(R_{\rm eq}-R)=0$.

2.2 Approximations

In the molecular orbital (MO) scheme the electronic state energies of a complex are written in terms of one-electron energies ε_n as well as of matrix elements of the two-electron operator e^2/r_{ij} . In the case of a cubic complex involving a d-cation there are ten independent matrix elements [21]. Usually, however, such matrix elements are written in terms of two effective Racah parameteres B and C. A careful study on MnF_6^{4-} has shown that this approximation is indeed good [22].

Let us now focus upon the dependence of ε_n and Racah parameteres upon the symmetric Q_A coordinate or equivalently upon R. Taking as a guide the case of the MnF₆⁴⁻ complex in fluoroperovskites it has been shown experimentally [23, 24] and theoretically [22, 25] that: (i) 10Dq, which is equal to $\varepsilon(e_g^*) - \varepsilon(t_{2g}^*)$ (where e_g^* and t_{2g}^* are the mainly 3d antibonding levels), is strongly sensitive to R changes. (ii) By contrast, the effective Racah parameteres B and C are nearly independent of R.

In view of these results we have assumed that $V_{\rm E}$ in (12) is essentially determined by the changes undergone by one-electron energies when the Q_{θ} deformation is switched on. Therefore, if we designate by simplicity as $\{x^2 - y^2, 3z^2 - r^2\}$ and $\{xy, xz, yz\}$ the components of the $e_{\rm g}^*$ and $t_{2\rm g}^*$ antibonding orbitals, respectively, we need *only* evaluate the changes $\delta \varepsilon(x^2 - y^2)$, $\delta \varepsilon(x, y)$, etc. induced by a Q_{θ} distortion. Moreover, as a closed shell (displaying $A_{1\rm g}$ symmetry) is not affected by a Q_{θ} distortion the centre of gravity theorem holds and so

$$\delta\varepsilon(x^2 - y^2) + \delta\varepsilon(3z^2 - r^2) = 0, \qquad \delta\varepsilon(xy) + 2\,\delta\varepsilon(xz) = 0. \tag{17}$$

2.3 Expression of V_E for the first excited state ${}^4T_{2g}$ of CrX_6^{3-}

The ground state $^4A_{2g}$ of the CrX_6^{3-} complex arises from the $3d^3$ configuration of isolated Cr^{3+} . Neglecting the mixing with configurations such as $3d^24s$, $3d^24p$, etc. the M=3/2 component of the $|^4A_{2g}\rangle$ wavefunction is just given [21] by the Slater determinant

$$|^{4}\mathbf{A}_{2g}\rangle = |\overset{+}{xy}, \overset{+}{xz}, \overset{+}{yz}|. \tag{18}$$

The M=3/2 component of the ${}^4\mathrm{T}_{2\mathrm{g}}$ state also involves one Slater determinant, and so the $|{}^4\mathrm{T}_{2\mathrm{g}}, z\rangle$ state is described [21] even in a D_{4h} symmetry by the wavefunction

$$|^{4}T_{2g}, z\rangle = |x_{z}^{+}, y_{z}^{+}, x^{2} + y^{2}|.$$
 (19)

Thus, by comparison with (18) it involves a jump of an electron placed in the $xy\pi$ -antibonding orbital to the $x^2-y^2\sigma$ -antibonding orbital.

Therefore, applying (16) to the calculation of $V_{\rm E}$ and assuming that only the changes experienced by one-electron energies are relevant, it is found

$$V_{\rm E} = \left[\partial \{\varepsilon(xy) - \varepsilon(x^2 - y^2)\}/\partial Q_{\theta}\right]_{Q_{\varepsilon} = 0}$$
(20)

which taking into account (17) becomes

$$V_{\rm E} = -[\partial \{ (1/2) \, \Delta_{\rm e} - (2/3) \, \Delta_{\rm t} \} / \partial Q_{\theta}]_{Q_{\rm e} = 0} \,, \tag{21}$$

where $\Delta_{\rm e}=\varepsilon(x^2-y^2)-\varepsilon(3z^2-r^2)$ and $\Delta_{\rm t}=\varepsilon(xy)-\varepsilon(xz;\,yz)$ are the splittings induced upon ${\rm e_g^*}$ and ${\rm t_{2g}^*}$ orbitals by the Q_{θ} deformation along $(Q_{\varepsilon}=0)$.

Therefore, as the splitting is much bigger for the e_g^* levels (involving σ bonding) than for the t_{2g}^* levels (where π bonding is present) and Δ_e is positive when $R_{ax} > R_{eq}$ (and thus $Q_{\theta} > 0$), it is expected to compute a negative value for V_E corresponding to the ${}^4T_{2g}$ excited state of CrX_6^{3-} complexes. In other words taking into account (14) the equilibrium geometry at the three equivalent minima would correspond to a compressed octahedron.

2.4 Expression of V_E for the first excited state ${}^4T_{1g}(G)$ of MnF_6^{4-}

The situation for this state is a little more complex than for the ${}^4\mathrm{T}_{2g}$ state of CrX_6^{3-} as it involves a mixing of three different configurations: $\mathrm{t}_{2g}^4\mathrm{e}_\mathrm{g}$, $\mathrm{t}_{2g}^3\mathrm{e}_\mathrm{g}^2$, and $\mathrm{t}_{2g}^2\mathrm{e}_\mathrm{g}^3$. Fortunately the amount of $\mathrm{t}_{2g}^4\mathrm{e}_\mathrm{g}^1$ configuration is 95% [17] and so we can discard the configuration interaction in a first approximation. If the M=5/2 component of the $|{}^6\mathrm{A}_{1g}\rangle$ ground state wavefunction is described by the Slater determinant

$$|^{6}A_{1g}\rangle = |xy, xz, yz, 3z^{2} - r^{2}, x^{2} - y^{2}|,$$
 (22)

then the M=3/2 component of the $|^4T_{1g}(G), z\rangle$ wavefunction is given [17] even in D_{4h} geometry by

$$|^{4}T_{1g}(G), z\rangle = |t_{2g}^{4}e_{g}; {}^{4}T_{1g}, z\rangle = |\dot{x}_{y}^{+}, \bar{x}_{y}, \dot{x}_{z}^{+}, \dot{y}_{z}^{+}, 3z^{2} + r^{2}|.$$
(23)

Thus, comparing (22) and (23) we see that there is a $x^2 - y^2 \to xy$ jump on passing from the ground to the excited state. This is the opposite situation to that found in Section 2.3 and thus the expression for $V_{\rm E}$ is just given by

$$V_{\mathrm{E}}(^{4}\mathrm{T}_{1\mathrm{g}}(\mathrm{G})) = -[\partial\{\varepsilon(xy) - \varepsilon(x^{2} - y^{2})\}/\partial Q_{\theta}]_{Q_{\varepsilon} = 0}$$
$$= [\partial\{(1/2)\Delta_{\mathrm{e}} - (2/3)\Delta_{\mathrm{t}}\}/\partial Q_{\theta}]_{Q_{\varepsilon} = 0}. \tag{24}$$

Therefore $V_{\rm E}$ would be positive for this case as it has been measured by Solomon and McClure [17].

2.5 The crystal-field approach

Electronic levels of a MX_6 complex, like t_{2g}^* and e_g^* , are molecular orbitals and so for instance the wavefunction corresponding to t_{2g}^* can be briefly written as

$$|\mathbf{t}_{2g}^{*}\rangle = N\{|d\rangle - \lambda_{\pi}|\chi_{\pi}\rangle\}, \tag{25}$$

where $|d\rangle$ belongs to the central ion and $|\chi_{\pi}\rangle$ is a suitable LCAO of valence π ligand orbitals.

Despite this fact it is also true that the crystal field approach is usually taken as a reference model [21]. Thus, we report here the values of $V_{\rm E}$ obtained in the crystal-field approach for the cases discussed in Sections 2.3 and 2.4. The expressions reported in [17] contain some mistakes with respect to the present ones.

If an octahedral complex whose metal-ligand distance is R is slightly distorted by Q_{θ} deformation leading to $(R_{\rm ax} - R) = -2(R_{\rm eq} - R) = Q_{\theta}/\sqrt{3}$ then the shift energy experienced by $x^2 - y^2$ and xy levels is just given by

$$\delta\varepsilon(x^{2} - y^{2}) = Z_{L}e^{2}[\{(25/7)\langle r^{4}\rangle_{d}/R^{6}\} - \{(36/7)\langle r^{2}\rangle_{d}/R^{4}\}](R_{eq} - R),$$

$$\delta\varepsilon(xy) = Z_{L}e^{2}[\{(100/21)\langle r^{4}\rangle_{d}/R^{6}\} - \{(36/7)\langle r^{2}\rangle_{d}/R^{4}\}](R_{eq} - R),$$
(26)

where $Z_{L}e$ is the ligand charge. Shift energies like $\delta\varepsilon(3z^2-r^2)$ or $\delta\varepsilon(xz)$ are obtained from (17) and (26).

Therefore, for the ${}^4T_{2g}$ state of CrF_6^{3-} V_E becomes equal to

$$V_{\rm E} = -(25\sqrt{3}/18) \ Z_{\rm L} e^2 \langle r^4 \rangle_{\rm d} / R^6 \tag{27}$$

and thus it is negative indeed. It is worth noting that $V_{\rm E}$ is independent of $\langle r^2 \rangle_{\rm d}$ and then connected to the coupling coefficient $V_{\rm A}$ for the symmetric $A_{\rm 1g}$ mode. Calling $E_{n0} = E_n^0 - E_0^0$, $V_{\rm A}$ for the present case is just given [8] by

$$V_{\rm A} = \partial E_{n0} / \partial Q_{\rm A} = (1/\sqrt{6}) \left(\partial 10 Dq / \partial R \right) \tag{28}$$

and so for the ${}^4\mathrm{T}_{2\mathrm{g}}$ state of CrX_6^{3-} in the crystal field model V_A becomes

$$V_{\rm A} = -(25/3\sqrt{6}) Z_{\rm L} e^2 \langle r^4 \rangle_{\rm d} / R^6 = -(5/\sqrt{6}) (10Dq/R),$$
 (29)

where

$$10Dq = (5/3) Z_{L}e^{2}\langle r^{4}\rangle_{d}/R^{5}.$$
(30)

Therefore, comparing (27) and (29) the following relation is found:

$$V_{\rm A} = \sqrt{2} V_{\rm E} \,. \tag{31}$$

In the case of MnX₆⁴⁻ complexes, as the first excited state $^4T_{1g}$ is essentially described by a $t_{2g}^4e_g$ configuration, it involves a $e_g \to t_{2g}$ jump and so V_A is given by

$$V_{\rm A} = -(1/\sqrt{6}) \ (\partial 10Dq/\partial R) \tag{32}$$

and thus relation (31) also holds. Applying (31) and (32) to $\mathrm{MnF_{6}^{4-}}$, putting $\langle r^4 \rangle_{\mathrm{d}} = 5.286$ a.u. [26] and $R = 213 \,\mathrm{pm} \ V_{\mathrm{E}} = 12 \,\mathrm{cm^{-1}/pm}$ and $10Dq = 1826 \,\mathrm{cm^{-1}}$ is found. The comparison with experimental values for RbMnF₃ [17, 24], $V_{\mathrm{E}} = 65 \,\mathrm{cm^{-1}/pm}$ and $10Dq = 7500 \,\mathrm{cm^{-1}}$, underlines the inadequacy of the crystal-field theory for quantitative purposes. In fact a quantity like 10Dq mainly reflects the different chemical bonding displayed by t_{2g}^* and e_{g}^* orbitals, which is not taken into account in the crystal-field framework.

2.6 Higher excited states of MnF_6^{4-} : ${}^4T_{2g}(G)$, ${}^4T_{2g}(D)$, and ${}^4T_{1g}(P)$

Let us start discussing the second excited state ${}^4T_{2g}(G)$. The wavefunction for this state is certainly more complex than that for ${}^4T_{1g}(G)$ as it involves an important mixing of $t_{2g}^4e_g$, $t_{2g}^3e_g^2$, and $t_{2g}^2e_g^3$ configurations though the first one is the dominant. Therefore, the $[{}^4T_{2g}(G),\,z\rangle$ wavefunction for this case can be briefly written [17] as

$$|^{4}T_{2g}(G), z\rangle = a |t_{2g}^{4}e_{g}; {}^{4}T_{2g}, z\rangle + b |t_{2g}^{3}e_{g}^{2}; {}^{4}T_{2g}, z\rangle + c |t_{2g}^{2}e_{g}^{3}; {}^{4}T_{2g}, z\rangle,$$
(33)

where the M = 3/2 components are given by

$$|\mathbf{t}_{2x}^{4}\mathbf{e}_{g}; {}^{4}\mathbf{T}_{2g}, z\rangle = |\dot{y}_{z}^{+}, \dot{x}_{z}^{+}, \dot{x}_{y}^{+}, \dot{x}_{y}^{-}, \dot{x}_{z}^{2} + y^{2}|,$$
 (34)

$$|\mathbf{t}_{2g}^2 \mathbf{e}_{\mathbf{g}}^3; {}^4\mathbf{T}_{2g}, \ z\rangle = |x^+z, y^+z, 3z^2 - r^2, 3z^2 - r^2, x^2 - y^2|,$$
 (35)

and the contribution of $|\mathbf{t}_{2g}^3 \mathbf{e}_{\mathrm{g}}^2$; $^4\mathrm{T}_{2g}$, $z\rangle$ to (33) is unimportant. So by comparison with the ground state, (34) involves a $3z^2-r^2\to xy$ jump, while (35) involves the opposite $xy\to 3z^2-r^2$ jump. Therefore, for the $^4\mathrm{T}_{2g}(\mathrm{G})$ state of MnF_6^{4-} the Jahn-Teller coupling coefficient V_{E} can be written as follows:

$$V_{\rm E}(^4\mathrm{T}_{2\mathrm{g}}(\mathrm{G})) = (a^2 - c^2) \left[\partial \{\varepsilon(3z^2 - r^2) - \varepsilon(xy)\} / \partial Q_{\theta} \right]_{\Omega = 0}$$
(36)

or, in terms of Δ_e and Δ_t ,

$$V_{\rm E}(^{4}\mathrm{T}_{2g}(\mathrm{G})) = -(a^{2} - c^{2}) \left[\partial \{(1/2) \Delta_{\rm e} + (2/3) \Delta_{\rm t} \} / \partial Q_{\theta} \right]_{Q_{\epsilon} = 0}.$$
 (37)

This expression points out that the admixture of the $\mathrm{t}_{2\mathrm{g}}^2\mathrm{e}_{\mathrm{g}}^3$ configuration (and also $\mathrm{t}_{2\mathrm{g}}^3\mathrm{e}_{\mathrm{g}}^2$) in the $\mathrm{t}_{2\mathrm{g}}^4\mathrm{e}_{\mathrm{g}}$ one tends to decrease the V_{E} value expected for the case where $|^4\mathrm{T}_{2\mathrm{g}}(\mathrm{G}),z\rangle$ is just described by $|\mathrm{t}_{2\mathrm{g}}^4\mathrm{e}_{\mathrm{g}};^4\mathrm{T}_{2\mathrm{g}},z\rangle$ alone (a=1). Moreover, as $\mathrm{t}_{2\mathrm{g}}^4\mathrm{e}_{\mathrm{g}}$ is the dominant contribution in the $^4\mathrm{T}_{2\mathrm{g}}(\mathrm{G})$ state, equation (37) implies that V_{E} is negative in qualitative agreement with the measurements reported by Solomon and McClure [17].

As regards now the ${}^{4}T_{2g}(D)$ state it is important to remark that the wavefunction $|{}^{4}T_{2g}(D), z\rangle$ is also described by (33) though with different mixing coefficients a, b, and c from those found for ${}^{4}T_{2g}(G)$. Therefore equation (37) can also be applied to calculate $V_{\rm E}$ for ${}^{4}T_{2g}(D)$.

Different from what is found for the first excited state ${}^4T_{1g}(G)$, in ${}^4T_{1g}(P)$ there is a significant configuration mixing where $t_{2g}^2e_g^3$ is the dominant one [17]. As

$$|\mathbf{t}_{2g}^2 \mathbf{e}_{\mathbf{g}}^3; {}^4 \mathbf{T}_{1g}, \ z\rangle = |\dot{y}_z^+, \dot{x}_z^+, 3z^2 + r^2, \ x^2 + y^2, \ x^2 - y^2|$$
 (38)

it involves (when compared to the wavefunction (22) of the ground state) a $xy \to x^2 - y^2$ jump. Thus, if $|^4T_{1g}(G), z\rangle$ is written as

$$|^{4}T_{1g}(G), z\rangle = a |t_{2g}^{4}e_{g}; {}^{4}T_{1g}, z\rangle + b |t_{2g}^{3}e_{g}^{2}; {}^{4}T_{1g}, z\rangle + c |t_{2g}^{2}e_{g}^{3}; {}^{4}T_{1g}, z\rangle,$$
(39)

following a similar procedure to that developed for ${}^4T_{2g}$ states, it is finally found

$$V_{\rm E}(^{4}{\rm T}_{1g}({\rm P})) = (a^{2} - c^{2}) \left[\partial \{ (1/2) \, \varDelta_{\rm e} - (2/3) \, \varDelta_{\rm t} \} / \partial Q_{\theta} \right]_{Q_{\rm e} = 0} \tag{40}$$

which coincides with (24) when $a^2 = 1$. As for ${}^4\mathrm{T}_{1g}(\mathrm{P})$ $c^2 > a^2$ [17], V_{E} would be negative in qualitative agreement with experimental measurements [17].

3. Results and Discussion

3.1 Calculation methods

Calculation of one-electron energies has been carried out for isolated MnF₆⁴⁻ and CrF₆⁵⁻ octahedral complexes for three values of the metal-ligand distance R=206, 213, and 220 pm for MnF₆⁴⁻ and R=185, 191, and 195 pm for CrF₆³⁻. In order to simulate the Q_{θ} distortion ($Q_{\epsilon}=0$) of the Jahn-Teller E_g mode coupled to orbital triplet states T we have performed calculations with tetragonal D_{4h} symmetry and so there are two different metal-ligand distances, $R_{\rm eq}$ and $R_{\rm ax}$. For each value of R four calculations have been made, corresponding to $R_{\rm eq}-R=+2$, +1, -1, -2 pm with $R_{\rm ax}-R=-2(R_{\rm eq}-R)$.

To be sure of the predicted trends, two different MO methods have been used simultaneously: the multiple scattered Xa(MSXa) and the self-consistent charge extended Hückel (SCCEH) methods. The results obtained through both methods are rather similar.

SCCEH calculations have been carried out following the procedure developed by Ammeter et al. [27] and Bash et al. [28], using accurate Clementi-Roetti [29] wavefunctions. We have verified that the use of poorer-quality wavefunctions significantly affects the dependence of the one-electron splittings Δ_e and Δ_t upon R and Q_θ . In the calculations performed at fixed R but for different Q_θ values only the overlap integrals are varied

and so the relations (17) are reasonably verified. A standard version of the MSX α method [30] has been used for spin-unrestricted calculations. The α parameter for the different atoms was taken from Schwarz's compilation [31]. Sphere radii were chosen according to the Norman procedure [32].

Before accomplishing the present calculations on $\mathrm{MnF_6^{4^-}}$ and $\mathrm{CrF_6^{3^-}}$ at different values of the Q_θ coordinate we have verified that both methods lead to reasonable results concerning the parameters of the undistorted octahedral complexes. For instance the 10Dq values calculated for $\mathrm{MnF_6^{4^-}}$ (at $R=213\,\mathrm{pm}$) are equal to $7940\,\mathrm{cm^{-1}}$ (MSX α) and $7300\,\mathrm{cm^{-1}}$ (SCCEH). This value has to be compared with $10Dq=7260\,\mathrm{cm^{-1}}$ measured for RbCdF₃: $\mathrm{Mn^{2+}}$ [24], where the $\mathrm{Mn^{2+}}$ -F⁻ distance has been determined to be equal to $213\,\mathrm{pm}$ [24, 33]. In the case of $\mathrm{CrF_6^{3^-}}$ the 10Dq values obtained at $R=191\,\mathrm{pm}$ are equal to $15480\,\mathrm{cm^{-1}}$ (SCCEH) and $16250\,\mathrm{cm^{-1}}$ (MSX α) and thus not far from $10Dq=16000\,\mathrm{cm^{-1}}$ for $\mathrm{K_2NaGaF_6}$: $\mathrm{Cr^{3+}}$ [13]. We have also verified that the SCCEH and MSX α methods lead to unpaired spin densities not far from experimental ones. A similar situation was found for complexes involving $\mathrm{Cu^{2+}}$, $\mathrm{Ag^{2+}}$, or $\mathrm{Ni^+}$ ions [34, 35] where more details on the employed SCCEH and MSX α methods can be found.

3.2 Results for MnF_6^{4-} at R=213 pm

In Table 1 are collected $\Delta_{\rm e}$ and $\Delta_{\rm t}$ values calculated for different Q_{θ} values but keeping the same average value of the Mn²⁺-F⁻ distance at R=213 pm. It can be seen that both methods give rise to similar results. In particular as expected $|\Delta_{\rm e}|$ is clearly higher than $|\Delta_{\rm t}|$.

3.2.1 The first excited state ${}^4T_{1g}(G)$

Using the data gathered in Table 1 together with (24) the coupling constant $V_{\rm E}$ can easily be calculated. The results together with the calculated value of the coupling constant $V_{\rm A}$ for the symmetric $A_{\rm 1g}$ mode are reported in Table 2.

It can be noticed that the SCCEH value ($V_{\rm E}=63~{\rm cm^{-1}/pm}$) and the MSX α one ($V_{\rm E}=58~{\rm cm^{-1}/pm}$) for $R=213~{\rm pm}$ are very close. Moreover, both values lie not far from the experimental one $V_{\rm E}=65~{\rm cm^{-1}/pm}$ reported by Solomon and McClure [17] for RbMnF₃. Taking now $\hbar\omega=287~{\rm cm^{-1}}$ [17, 36] and $V_{\rm E}=60~{\rm cm^{-1}/pm}$, equation (15) gives $E_{\rm JT}=420~{\rm cm^{-1}}$ and $S_{\rm E}=1.5$. Such values are consistent with those derived

Table 1 Calculated values of $\Delta_{\rm e}=\varepsilon(x^2-y^2)-\varepsilon(3z^2-r^2)$ and $\Delta_{\rm t}=\varepsilon(xy)-\varepsilon(xz;\,yz)$ one-electron splittings for MnF $_6^4$ with D_{4h} geometry keeping the same average value R=213 pm. Results are shown for different values of the $Q_\theta=-2\sqrt{3}\;(R_{\rm eq}-R)$ coordinate, where $R_{\rm ax}+R_{\rm eq}=3R$. $\Delta_{\rm e}$ and $\Delta_{\rm t}$ are given in cm $^{-1}$ while $R_{\rm eq}-R$ and Q_θ are in pm

	$ ext{MSX}lpha$			SCCEH	
$R_{ m eq}$ – R	$Q_{m{ heta}}$	$\Delta_{ m e}$	Δ_{t}	Δ_{e}	⊿t
2	-6.93	-1190	-310	-1197	-271
1	-3.46	- 588	-146	- 604	-133
-1	3.46	590	143	611	128
$ \begin{array}{l} -1 \\ -2 \end{array} $	6.93	1190	270	1228	252

Table 2 Calculated values of the coupling coefficients $V_{\rm A}$ and $V_{\rm E}$ at different metal-ligand distances R corresponding to the first excited state of ${\rm MnF_6^{4-}}$ (${}^4{\rm T_{1g}}({\rm G})$) and ${\rm CrF_6^{3-}}$ (${}^4{\rm T_{2g}}$). $V_{\rm E}$ and $V_{\rm A}$ are given in cm $^{-1}/{\rm pm}$ while R is in pm

		$MSX\alpha$			SCCEH		
system	R	$V_{\rm A}$	$V_{ m E}$	$V_{ m A}/V_{ m E}$	V_{A}	$V_{ m E}$	$V_{ m A}/V_{ m E}$
MnF ₆ ⁴⁻	206	82	71	1.16	108	77	1.40
U	213	66	58	1.14	85	63	1.35
	220	58	52	1.12	68	51	1.33
CrF_6^{3-}	185	-170	-141	1.21	-225	-165	1.36
Ū	191	-145	-123	1.18	-182	-130	1.40
	195	-130	-111	1.17	-159	-113	1.41

from the analysis of experimental splitting in the ${}^4T_{1g}(G)$ state due to spin-orbit coupling, which is significantly reduced by the Ham effect [37]. As the spin-orbit constant λ for the ${}^4T_{1g}(G)$ state is about 30 cm⁻¹ [36], the value $E_{JT} = 420$ cm⁻¹ explains a posteriori that in this case the Jahn-Teller effect dominates over the spin-orbit interaction.

The values $S_{\rm E}=1.5$ and $\hbar\omega_{\rm E}=287\,{\rm cm}^{-1}$ lead to $E_{\rm S}^0({\rm E})=860\,{\rm cm}^{-1}$. This value can be compared with the room temperature experimental value $E_{\rm S}=1550\,{\rm cm}^{-1}$ for RbCdF₃: Mn²⁺ [15] where the metal-ligand distance R is equal to 213 pm [15]. As the quantity $\Delta E_{\rm u}$ is equal to $\approx 300\,{\rm cm}^{-1}$ at room temperature for RbCdF₃: Mn²⁺ [15] the present analysis underlines that $E_{\rm S}^0({\rm E})$ can be the dominant contribution to the experimental Stokes shift of MnF₆⁴⁻ in accord with previous conclusions [8].

3.2.2 The excited states
$${}^4T_{2g}(G)$$
, ${}^4T_{2g}(D)$, and ${}^4T_{1g}(P)$ of MnF_6^{4-}

Using the figures given in Table 1 together with expressions (37) and (40) and the values of the coefficients a, b, and c (describing the configuration mixing), the $V_{\rm E}$ values corresponding to the three states ${}^4{\rm T}_{2\rm g}({\rm G})$, ${}^4{\rm T}_{2\rm g}({\rm D})$, and ${}^4{\rm T}_{1\rm g}({\rm P})$ can be derived. The results are collected in Table 3. The employed coefficients a, b, and c are those given in [17]. Similar values were obtained through the analysis reported in [24]. It can be seen that both the SCCEH and the MSX α methods lead to very similar results as for the

Table 3 Calculated values of $V_{\rm E}$ for three T excited states of MnF₆⁴⁻ at $R=213\,{\rm pm}$ compared to the experimental ones reported by Solomon and McClure [17]. The values of the coefficient of configuration mixing a^2-c^2 are also taken from [17]. $V_{\rm E}$ is given in cm⁻¹/pm

state		$V_{ m E}$			
	a^2-c^2	SCCEH	MSXα	experimental	
$^{4}\mathrm{T}_{2\mathfrak{g}}(\mathrm{G})$	0.53	-59	-59	-36	
$^{4}\mathrm{T}_{2g}(\mathrm{D})$	0.25	-2 8	-28	-23	
$^{4}\mathrm{T}_{2g}(G) \\ ^{4}\mathrm{T}_{2g}(D) \\ ^{4}\mathrm{T}_{1g}(P)$	-0.66	-42	-38	-37	

⁴T_{1g}(G) state.

Despite the approximations involved in the present calculations and the experimental uncertainties the comparison between theoretical and experimental values for the three states can be considered as reasonably good. Moreover, the closeness to experimental values is better in the present case than in the first work reported by Nikiforov et al. [38]. The results collected in Tables 2 and 3, and in Section 3.2.1 support indirectly the validity of the approximations discussed in Section 2.2.

3.3 Results for the $^4T_{2g}$ state of CrF_6^{3-} at $R=191\;pm$

Table 2 shows that the $|V_E|$ value of CrF_6^{3-} at R=191 is about twice that found for MnF_6^{4-} at R=213 pm. This situation which also holds for V_A can be compared with that experienced by 10Dq. This quantity also increases by a factor of ≈ 2 on passing from MnF_6^{4-} to CrF_6^{3-} [8].

As no experimental measurements of $V_{\rm E}$ for ${\rm CrF_6^{3-}}$ have been reported to our knowledge we can check the validity of the values in Table 2 for ${\rm CrF_6^{3-}}$ indirectly analysing the obtained values for $S_{\rm E}$, $E_{\rm JT}$, and $E_{\rm S}^0({\rm E})$. The values collected in Table 4 have been derived using $\hbar\omega_{\rm E}=481~{\rm cm^{-1}}$ measured [13, 14] for ${\rm K_2NaGaF_6:Cr^{3+}}$.

It is worth noting that though $V_{\rm E}$ for the present case is about twice the value found for the first excited state ${}^4{\rm T}_{1\rm g}({\rm G})$ of ${\rm MnF_6^{4-}}$, the calculated value of the Huang-Rhys factor is, however, similar for both cases. This comes from the increase usually experienced by the vibrational frequencies on passing from divalent to trivalent complexes [39].

As regards the value of $S_{\rm E}$ itself it has recently been estimated [40] in Rb₂KGaF₆: Cr³⁺ through the analysis of the intensities displayed by the peaks corresponding to the E_g vibrational progression. The value given in Table 4 is close to the experimental estimation [40] $S_{\rm E} = 1.2$. As regards the value of $E_{\rm JT}$ the values collected in Table 4 are not far but a little higher than $E_{\rm JT} = 391~{\rm cm}^{-1}$ calculated by Woods et al. [13].

Compared to the room temperature experimental value $E_{\rm S}=2800~{\rm cm}^{-1}$ reported [13] for K₂NaGaF₆: Cr³⁺, the value $E_{\rm S}^0({\rm E})\approx 1200~{\rm cm}^{-1}$ derived from Table 4 points out that in the present case the contribution of $E_{\rm S}^0({\rm E})$ to the total Stokes shift would not exceed 50%. This conclusion is thus in agreement with the analysis carried out in [8] and is supported by the $V_{\rm A}$ value given in Table 2. In fact taking $V_{\rm A}=165~{\rm cm}^{-1}/{\rm pm}$ and using $\hbar\omega_{\rm A}=568~{\rm cm}^{-1}$ observed experimentally [13, 14] for K₂NaGaF₆: Cr³⁺ it is found $S_{\rm A}=1.3$ and $E_{\rm S}^0({\rm A})=1480~{\rm cm}^{-1}$.

The present analysis thus supports that the experimental Stokes shift at high temperatures can essentially be understood in terms of coupling only with E_g and A_{1g} modes of the CrF_6^{3-} complex. A similar conclusion emerges from the first calculations on $CrCl_6^{3-}$ using the Amsterdam density functional (ADF) code [41].

Table 4 Values of $V_{\rm E}$, $S_{\rm E}$, $E_{\rm JT}$, and $E_{\rm S}^0({\rm E})$ derived for ${\rm CrF_6^{3-}}$ at $R=191\,{\rm pm}$ and using $\hbar\omega_{\rm E}=481\,{\rm cm^{-1}}$. $E_{\rm JT}$ and $E_{\rm S}^0({\rm E})$ are given in ${\rm cm^{-1}}$ and $V_{\rm E}$ in ${\rm cm^{-1}/pm}$

method	$V_{ m E}$	$S_{ m E}$	$E_{ m JT}$	$E_{ m S}^0({ m E})$
SCCEH	-130	1.3	625	1250
MSXa	-120	1.2	575	1150

3.4 V_E for the first excited state of MnF_6^4 and CrF_6^3 : Dependence on R

The calculated values of $V_{\rm E}$ (and also of $V_{\rm A}$) at different Mn-F and Cr-F distances are also displayed in Table 2. It can be seen that $V_{\rm E}$ and $V_{\rm A}$ are both very sensitive to changes of the metal-ligand distance R. Following (10) let us write the dependence of $V_{\rm E}$ and $V_{\rm A}$ in the vicinity of R_0 as

$$V_{\rm A} = C_{\rm A} R^{-n_{\rm A}} ,$$

$$V_{\rm E} = C_{\rm E} R^{-n_{\rm E}} .$$
(41)

The calculated exponents n_A and n_E are given in Table 5 for both MnF_6^{4-} and CrF_6^{3-} systems. It is worth noting that

$$n_{\mathbf{A}} = n + 1, \tag{42}$$

where the exponent n was defined in (10).

Aside from telling us that the ratio V_A/V_E is nearly independent of R Tables 2 and 5 reveal that n_A is very close to n_E for both systems and for both employed methods.

Before stressing the importance of this fact concerning the experimental dependence of the Stokes shift upon R for $\mathrm{MnF_6^{4^-}}$ let us start analysing the variation of the Huang-Rhys factor S_{E} on going from $\mathrm{KMgF_3}:\mathrm{Mn^{2^+}}$ to RbMnF₃ [36]. In the first case the experimental value is $S_{\mathrm{E}}=1.35$, while in the second one $S_{\mathrm{E}}=1.6$ [36]. The equilibrium Mn–F distance for the ground state of $\mathrm{KMgF_3}:\mathrm{Mn^{2^+}}$ has been determined to be equal to (207 ± 1) pm while R=212 pm for RbMnF₃ [23, 24, 33]. Following a previous insight [8] into this kind of problems the increase of S_{E} upon increasing R can reasonably be explained taking into account (41) and the Grüneisen law for a local mode of the complex written as

$$\frac{\partial(\ln \omega_{\rm E})}{\partial(\ln R)} = -3\gamma_{\rm E} \,. \tag{43}$$

Therefore taking into account (15), (41), and (43) it is a simple matter to find the R dependence of $S_{\rm E}$ as follows:

$$S_{\rm E} \propto R^{9\gamma_{\rm E} - 2n_{\rm E}} \,. \tag{44}$$

Although $\gamma_{\rm E}$ has not been determined for complexes involving divalent cations, $\gamma_{\rm A}$ has been calculated for MnF₆⁴⁻ and VF₆⁴⁻ [25, 42]. The obtained values ($\gamma_{\rm A}=2.3$ and 2.9, respectively) indicate that in these cases $\gamma_{\rm A}$ lies between 2 and 3. Thus, assuming now that the Grüneisen coefficient $\gamma_{\rm E}$, corresponding also to a stretching mode, lies in the

Tabel 5 Values of the exponents n_A and n_E (defined in (41)) corresponding to the first excited state of MnF₆⁴⁻ and CrF₆³⁻ complexes calculated by means of SCCEH and MSX α methods

	SCCEH		$MSX\alpha$	
	$\overline{n_{A}}$	$n_{ m E}$	$\overline{n_{A}}$	$n_{ m E}$
MnF_e^{4-}	7.03	6.26	5.27	4.74
MnF ₆ ⁴⁻ CrF ₆ ³⁻	6.60	6.52	5.09	4.52

same range as $\gamma_{\rm A}$ it becomes possible to understand the increase of $S_{\rm E}$ when R increases, observed experimentally for MnF₆⁴⁻ [36]. In fact, taking $n_{\rm E}=6.3$ the exponent $9\gamma_{\rm E}-2n_{\rm E}$ which appears in (44) is positive provided $\gamma_{\rm E}>1.4$. Assuming for instance $\gamma_{\rm E}\approx 2.5$, the $S_{\rm E}$ is proportional to R^{10} and so if $S_{\rm E}=1.35$ for R=207 pm, $S_{\rm E}$ should be close to 1.7 when R=212 pm.

As pointed out in Section 1 recent studies on the room temperature Stokes shift of $\mathrm{Mn^{2+}}$ -doped fluoroperovskites have shown [15] that the experimental E_{S} value also increases significantly as far as R increases following a law $E_{\mathrm{S}} \propto R^{5.3}$.

It was previously pointed out that as

$$E_{\rm S}^0({\rm A}) \propto R^{6\gamma_{\rm A} - 2n_{\rm A}} \tag{45}$$

the exponent can be positive if $\gamma_{\rm A} > 2$, provided we accept $n_{\rm A} \approx 6$. Nevertheless, in order to well understand the microscopic origin of the observed [15] $E_{\rm S} \propto R^{5.3}$ dependence for MnF₆⁴⁻ it is necessary to be sure that $E_{\rm S}^0({\rm E})$ can exhibit a similar R dependence to that displayed by $E_{\rm S}^0({\rm A})$. This idea is thus certainly reinforced by the results shown in Table 5 which indicate that $n_{\rm A}$ and $n_{\rm E}$ are very close indeed. Although accurate values of $\gamma_{\rm A}$ and $\gamma_{\rm E}$ are required for further proceeding, the experimental dependence $E_{\rm S} \propto R^{5.3}$ can be consistent with $\gamma_{\rm A} = \gamma_{\rm E} = 2.8$ and $n_{\rm A} = n_{\rm E} = 5.7$ as a first approximation. The latter $n_{\rm A}$ values has been derived from (42) and the value n = 4.7 corresponding to the experimental dependence of 10Dq upon R in the case of MnF₆⁴⁻ complexes [23, 24].

It is worth stressing that within the present framework the R dependence of the Stokes shift $E_{\rm S}$ is strongly sensitive to the actual values of Grüneisen constants $\gamma_{\rm A}$ and $\gamma_{\rm E}$. In particular, assuming $\gamma_{\rm A}=\gamma_{\rm E}\equiv\gamma$ and $n_{\rm A}=n_{\rm E}=6$, $E_{\rm S}$ could decrease upon increasing R provided $\gamma<2$. Though additional experimental data are required first analysis [15] reveals that this is likely the behavior displayed by ${\rm CrF}_6^{3-}$ complexes in several elpasolite lattices. This is consistent with recent theoretical calculations for ${\rm CrF}_6^{3-}$ giving values of $\gamma_{\rm A}$ lying between 1 and 1.8 [13, 43].

4. Final Remarks

The present work offers a simple way for achieving a better insight into the microscopic origin of signs and absolute values of the coupling constant $V_{\rm E}$. The analysis displayed here for several excited states of ${\rm MnF_6^{4-}}$ and ${\rm CrF_6^{3-}}$ supports a posteriori that $V_{\rm E}$ is essentially determined by the energy changes undergone by antibonding one-electron orbitals.

Despite $V_{\rm E}$ and $V_{\rm A}$ values calculated within a pure crystal-field approach are substantially smaller than experimental ones it is also true that the relation $V_{\rm A}/V_{\rm E}=\sqrt{2}$ derived for the first excited states of ${\rm MnX_6^{4-}}$ and ${\rm CrX_6^{3-}}$ complexes is not far from the present MSX α and SCCEH results as is shown in Table 2. This point deserves a further investigation as a possible general relation between $V_{\rm A}$ and $V_{\rm E}$ is certainly of interest. In fact $V_{\rm A}$ can in principle be measured and calculated more easily than $V_{\rm E}$.

Aside from obtaining $S_{\rm E}$, $E_{\rm S}^0({\rm E})$, and $E_{\rm S}^0({\rm A})$ values which are compatible with experimental findings for ${\rm Mn}^{2+}$ in fluoroperovskites and ${\rm Cr}^{3+}$ in fluoroelpasolites it is stressed here that the dependence of $E_{\rm S}^0({\rm E})$ upon R can be very similar to that displayed by $E_{\rm S}^0({\rm A})$. Thus, this fact provides us with a reasonable explanation of the increase experienced by the Stokes shift of ${\rm Mn}^{2+}$ -doped fluoroperovskites when R increases such as it

has recently been reported in [15]. Having in mind the strong influence of Grüneisen constant γ_A and γ_E on the R dependence of the Stokes shift efforts for a more precise knowledge of them are however required.

The present analysis about the optical properties of $\mathrm{Mn^{2+}}$ in fluoroperovskites and $\mathrm{Cr^{3+}}$ in fluoroelpasolites is based on theoretical results performed on isolated $\mathrm{MF_6}$ units $\mathrm{(M=Mn^{3+},\,Cr^{3+})}$. This procedure is reasonable provided the electrostatic potential exerted by the rest of the lattice upon the electrons in the complex is perfectly flat. This situation does not always happen as the flatness of such a potential depends on the type of crystalline lattice where the complex is embedded. It has been shown [44] that the addition of a non-flat potential on a $\mathrm{MF_6}$ complex modifies the 10Dq value and thus it is expected to modify also the constants $V_{\rm A}$ and $V_{\rm E}$. Thus, care has to be taken when comparing $V_{\rm E}$, $S_{\rm E}$, or $E_{\rm S}^0(\rm E)$ of a given complex but embedded in two different kinds of host lattices. This point is discussed in [45].

Through the present work it has been pointed out that the R dependence of $V_{\rm E}$ plays a significant role for understanding that associated with $S_{\rm E}$ and $E_{\rm S}^0({\rm E})$. Though the calculated values of $n_{\rm E}$ appear to give reasonable results it is also necessary to understand why $V_{\rm E}$ is so strongly dependent upon R. This is especially important if we take into account that the pure crystal-field model produces $V_{\rm E}$ and $V_{\rm A}$ values much smaller than the experimental ones. Therefore, to try to justify exponents $n_{\rm E}$ and $n_{\rm A}$ within a pure crystal-field picture is simply meaningless while the answer to this question has to be sought within the MO description of ${\rm MnF_6^{4-}}$ and ${\rm CrF_6^{3-}}$ complexes employed throughout the present paper. First analysis on the R dependence of 10Dq within the MO framework reveals [46] that in cases like ${\rm MnF_6^{4-}}$ or ${\rm CrF_6^{3-}}$ the very small amount of 2s orbitals of ${\rm F^-}$ in antibonding ${\rm e_g}^*$ orbitals plays a crucial role. Further work along this line is now in progress.

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References

- [1] A. Einstein, Ann. Phys. 17, 132 (1905).
- [2] N. F. Mott and R. W. Gurney, Electronic Processes in Ionic Crystals, Chap. 6, Dover Publ. Inc., New York 1940.
- [3] K. Huang and A. Rhys, Proc. Roy. Soc. A 204, 406 (1950).
- [4] M. Lax, J. chem. Phys. **20**, 1752 (1952).
- [5] J. Bourgoin and M. Lannoo, Point Defects in Semiconductors II, Springer-Verlag, Berlin 1983.
- [6] B. Henderson and G. I. Imbusch, Optical Spectroscopy of Inorganic Solids, Chap. 5, 6, Clarendon Press, Oxford 1989.
- [7] O. G. Holmes and D. S. McClure, J. chem. Phys. 26, 1686 (1957).
- [8] M. Moreno, M. T. Barriuso, and J. A. Aramburu, J. Phys.: Condensed Matter 4, 948 (1992).
- [9] J. FERGUSON, H. J. GUGGENHEIM, and D. L. WOOD, J. chem. Phys. 54, 504 (1971).
- [10] H. U. Gudel and T. R. Snellgrove, Inorg. Chem. 17, 1617 (1978).
- [11] R. KNOCHENMUSS, C. REBER, M. U. RAJASEKHARAN, and H. U. GUDEL, J. chem. Phys. 85, 4280 (1986).
- [12] M. C. MARCO DE LUCAS, F. RODRÍGUEZ, J. M. DANCE, M. MORENO, and A. TRESSAUD, J. Lum. 48/49, 553 (1991).
- [13] A. M. WOODS, R. S. SINKOVITS, J. C. CHARPIE, W. L. HUANG, R. H. BARTRAM, and A. R. ROSSI, J. Phys. Chem. Solids 54, 543 (1993).
- [14] J. F. DOLAN, A. G. RINZLER, L. A. KAPPERS, and R. H. BARTRAM, J. Phys. Chem. Solids 53, 905 (1992).

- [15] M. C. MARCO DE LUCAS, F. RODRÍGUEZ, and M. MORENO, Phys. Rev. B 50, 2760 (1994).
- [16] M. Y. CHEN, D. S. McClure, and E. I. SOLOMON, Phys. Rev., B 6, 1690 (1972).
- [17] E. I. SOLOMON and D. S. McClure, Phys. Rev. B 6 1697 (1972); B 9, 4690 (1974).
- [18] B. Galli, A. Hauser, and H. U. Gudel, Inorg. Chem. 24, 1697 (1985).
- [19] YU. E. PERLIN and B. S. TSUKERBLAT, The Dynamical Jahn-Teller Effect in Localized Systems, Ed. YU. E. PERLIN and W. WAGNER; North-Holland Publ. Co., Amsterdam 1984 and references cited therein.
- [20] Y. Toyozawa and M. Inoue, J. Phys. Soc. Japan 21, 1663 (1966).
- [21] S. SUGANO, Y. TANABE, and H. KAMIMURA, Multiplets of Transition-Metal Ions in Crystals, Academic Press New York 1970.
- [22] M. FLOREZ, L. SEIJO and L. PUEYO, Phys. Rev. B 34, 1200 (1986).
- [23] F. Rodríguez and M. Moreno, J. chem. Phys. 84, 692 (1986).
- [24] M. C. MARCO DE LUCAS, F. RODRÍGUEZ, and M. MORENO, J. Phys.: Condensed Matter 5, 1437 (1993).
- [25] V. Luaña, M. Bermejo, M. Florez, J. M. Recio, and L. Pueyo, J. chem. Phys. 90, 6409 (1989).
- [26] S. Fraga, J. Karwowski, and K. Saxena, Handbook of Atomic Data, Elsevier Publ. Co., Amsterdam 1976.
- [27] J. H. AMMETER, A. B. BURGI, J. C. THIBEAULT, and R. HOFFMANN, Amer. Chem. Soc. 100, 3686 (1978).
- [28] H. Bash, A. Viste, and H. B. Gray, J. chem. Phys. 44, 10 (1966).
- [29] E. CLEMENTI and C. ROETTI, Atomic Data nuclear Data Tables 14, 177 (1974).
- [30] K. H. Johnson, Adv. Quantum Chem. 7, 143 (1973).
- [31] K. Schwarz, Phys. Rev. B 5, 2466 (1972).
- [32] J. G. NORMAN, Mol. Phys. **31**, 1191 (1976).
- [33] M. T. Barriuso and M. Moreno, Phys. Rev. B 29, 3623 (1984).
- [34] J. A. ARAMBURU, M. MORENO, and M. T. BARRIUSO, J. Phys.: Condensed Matter 4, 9089 (1992).
- [35] R. VALIENTE, J. A. ARAMBURU, M. T. BARRIUSO, and M. MORENO, J. Phys.: Condensed Matter 6, 4515 (1994).
- [36] F. Rodríguez, H. Riesen, and H. U. Güdel, J. Lum. 50, 101 (1991).
- [37] F. S. Ham, Phys. Rev. 138, 1727 (1965).
- [38] A. E. Nikiforov, S. Y. U. Shashkin, and A. I. Krotkii, phys. stat. sol. (b) 98, 289 (1980).
- [39] K. NAKAMOTO, Infrared and Raman Spectra of Inorganic and Coordination Compounds, Wiley, New York 1986.
- [40] M. C. Marco de Lucas, Thesis, Universidad de Cantabria 1992.
- [41] K. Bellafrouh, C. Daul, H. U. Güdel, F. Gilardoni, and J. Weber, Theor. Chim. Acta 91, 215 (1995).
- [42] N. W. WINTER and R. M. PITZER, J. chem. Phys. 89, 446 (1988).
- [43] L. Seijo, Z. Barandiaran, and L. Pettersson, J. chem. Phys. 98, 4041 (1993).
- [44] K. Pierloot, E. van Praet, and L. G. Vanquickenborne, J. chem. Phys. 96, 4163 (1992).
- [45] M. C. Marco de Lucas, F. Rodríguez, and M. Moreno, phys. stat. sol. (b) 184, 247 (1994).
- [46] M. Moreno, M. T. Barriuso, and J. A. Aramburu, Internat. J. Quantum Chem. 52, 829 (1994).