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RESEARCH PAPER

Analysis of mode III crack perpendicular to the interface between two dissimilar strips

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Abstract The present work is concerned with the problem of mode III crack perpendicular to the interface of a bi-strip composite. One of these strips is made of a functionally graded material and the other of an isotropic material, which contains an edge crack perpendicular to and terminating at the interface. Fourier transforms and asymptotic analysis are employed to reduce the problem to a singular integral equation which is numerically solved using Gauss–Chebyshev quadrature formulae. Furthermore, a parametric study is carried out to investigate the effects of elastic and geometric characteristics of the composite on the values of stress intensity factor.

Keywords Composite · Interface · Perpendicular crack · Anti-plane shear stress · Fourier transform · Singular integral equation

1 Introduction

In recent years, layered structures, such as coating–substrate systems, are receiving graet attention in various engineering fields, and analysis of cracks along or perpendicular to interfaces within such structures is one of the most important subjects in fracture mechanics. As to earlier efforts on this subject, one can refer to Williams [1], Sih and Rice [2], and England [3] for the case of cracks along the interface between bonded dissimilar materials, while refer to Zak and Williams [4], Cook and Erdogan [5], and Gupta [6] for the case of cracks perpendicular to the bimaterial interface. These works are normally based on the material properties discontinuities across the interface [1–6]. Owing to such drawbacks in the interface modelling, the obtained results were characterized by a meaningless oscillatory behavior or a nonsquare-root singularity [1–6]. It is found that these unusual results can be improved when the model of functionally graded materials (FGM's) is employed [7].

Significant works have been accomplished for analysing interfacial crack problems involving FGM's. Delale and Erdogan [8] considered the problem of an interfacial crack located between two bonded half planes, one of which was homogeneous while the other was FGM so that the elastic properties were continuous at the interface. Similar works of an anti-plane shear counterpart were considered by Erdogan and Ozturk [9], Ozturk and Erdogan [10], and Shbeeb and Binienda [11]. The problem of a crack perpendicular to the interface was examined by Erdogan et al. [12], Choi [13,14], Guo et al. [15], and Yue et al. [16]. Further, the problem of a crack at an arbitrary angle to the graded interfacial zone, was considered by Shbeeb et al. [17] and Choi [18–20].

In spite of all these efforts, analysis of crack problems in composite structures is still of significant interests. The present work is concerned with a nonhomogeneous composite consisting of two bonded dissimilar strips, one of which is made of an isotropic material and the other of an FGM. The isotropic strip contains an edge crack perpendicular to and terminating at the interface. The crack faces are subjected to anti-plane shear traction, as shown in Fig. 1. This problem may be encountered in repairing and strengthening some of marine structures [21]. The difference between this work and the previous ones is that the crack terminates at the interface and both of the strips are of finite height. The method of solution is based on Fourier transforms and asymptotic analysis,

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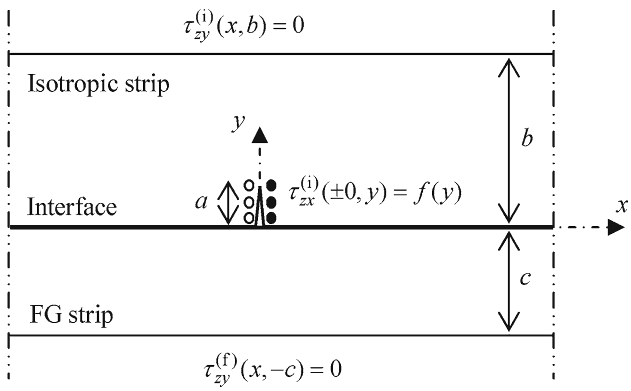

Fig. 1 Mode III crack perpendicular to the interface between two dissimilar strips

which are used to reduce the problem to a singular integral equation with a Cauchy kernel solvable numerically by using Gauss–Chebyshev integration formulae.

2 Formulation of the problem

Consider a non-homogeneous composite consisting of an isotropic strip bonded to another one of an FGM. The isotropic strip contains a sharp through edge crack perpendicular to the interfacial line and terminates at it. The crack faces are subjected to anti-plane shear traction as shown in Fig. 1. The shear modulus of FG strip is governed by:

$$\mu^{(\mathrm{f})} = \mu^{(\mathrm{i})} e^{\gamma y}, \quad -c \le y \le 0, \tag{1}$$

where the superscripts (f) and (i) denote FG and isotropic strips, respectively, $\mu^{(l)}$, $(l = \mathrm{f}, \mathrm{i})$, are the anti-plane shear modulus and γ is a constant characterizing the nonhomogeneity of the composite.

The governing equations for such stress disturbance problems can be written as [22]:

$$\nabla^2 W^{(\mathrm{i})} = 0, \quad (|x| < \infty, 0 < y < b), \tag{2a}$$

$$\nabla^2 W^{(\mathrm{f})} + \gamma \frac{\partial W^{(\mathrm{f})}}{\partial y} = 0, \quad (|x| < \infty, -c < y < 0), \tag{2b}$$

where ∇^2 is the two dimensional Laplacian operator, and $W^{(l)}$, $(l = \mathrm{f}, \mathrm{i})$ are the out of plane displacements. Furthermore, the following boundary conditions must be considered:

- Along the external boundaries:

$$\tau_{zy}^{(\mathrm{i})}(x, b) = \tau_{zy}^{(\mathrm{f})}(x, -c) = 0, \tag{3}$$

- Along the interface:

$$\lim_{y\to 0^+} W^{(\mathrm{i})}(x, y) = \lim_{y\to 0^-} W^{(\mathrm{f})}(x, y), \quad (x \ne 0), \tag{4a}$$

$$\lim_{y\to 0^+} \tau_{zy}^{(\mathrm{i})}(x, y) = \lim_{y\to 0^-} \tau_{zy}^{(\mathrm{f})}(x, y), \quad (x \ne 0). \tag{4b}$$

- Along the crack faces:

$$\lim_{x\to 0^\pm} \tau_{zx}^{(\mathrm{i})}(x, y) = f(y), \quad (0 \le y < a), \tag{5}$$

where $f(y)$ is a known traction applied along the crack faces as shown in Fig. 1 and $\tau_{zx}^{(l)}, \tau_{zy}^{(l)}, (l = \mathrm{i}, \mathrm{f})$, are the components of anti-plane shear stress:

$$\tau_{zx}^{(l)} = \mu^{(l)} \frac{\partial W^{(l)}}{\partial x}, \quad \tau_{zy}^{(l)} = \mu^{(l)} \frac{\partial W^{(l)}}{\partial y}, \quad (l = \mathrm{i}, \mathrm{f}). \tag{6}$$

3 Solution of the problem

For the cracked (isotropic) strip, let the displacement and stresses be expressed as the sum of two parts as follows [17]:

$$W^{(\mathrm{i})}(x, y) = W_{(1)}^{(\mathrm{i})}(x, y) + W_{(2)}^{(\mathrm{i})}(x, y), \tag{7a}$$

$$\tau_{zl}^{(\mathrm{i})}(x, y) = \tau_{zl(1)}^{(\mathrm{i})}(x, y) + \tau_{zl(2)}^{(\mathrm{i})}(x, y), \ (l = x, y), \tag{7b}$$

where the subscript (1) denotes the solution of the infinite plane containing a crack along $(x \to \pm 0, 0 < y < a)$, while the subscript (2) refers to the solution of the isotropic strip without the crack, as shown in Fig. 2.

On substitution of Eqs. (7) into Eqs. (2)–(5), the governing equations for the cracked isotropic strip may be written as:

$$\nabla^2 W_{(1)}^{(\mathrm{i})} = 0, \quad (|x| < \infty, |y| < \infty), \tag{8a}$$

$$\nabla^2 W_{(2)}^{(\mathrm{i})} = 0, \quad (|x| < \infty, 0 < y < b). \tag{8b}$$

The boundary conditions are:

$$\tau_{zy(1)}^{(\mathrm{i})}(x, b) + \tau_{zy(2)}^{(\mathrm{i})}(x, b) = \tau_{zy}^{(\mathrm{f})}(x, -c) = 0, \tag{9}$$

$$\lim_{y\to 0^+} [W_{(1)}^{(\mathrm{i})}(x, y) + W_{(2)}^{(\mathrm{i})}(x, y)] = \lim_{y\to 0^-} W^{(\mathrm{f})}(x, y), \quad (x \ne 0), \tag{10}$$

$$\lim_{y\to 0^+} [\tau_{zy(1)}^{(\mathrm{i})}(x, y) + \tau_{zy(2)}^{(\mathrm{i})}(x, y)] = \lim_{y\to 0^-} \tau_{zy}^{(\mathrm{f})}(x, y), \quad (x \ne 0), \tag{11}$$

$$\lim_{x\to 0^\pm} [\tau_{zx(1)}^{(\mathrm{i})}(x, y) + \tau_{zx(2)}^{(\mathrm{i})}(x, y)] = f(y), \quad (0 \le y < a). \tag{12}$$

Also,

$$\lim_{x\to 0^+} \tau_{zx(1)}^{(\mathrm{i})}(x, y) = \lim_{x\to 0^-} \tau_{zx(1)}^{(\mathrm{i})}(x, y), \tag{13}$$

Furthermore, to ensure the regularity of the displacement and stresses (through the infinite plate), the following conditions must be satisfied:

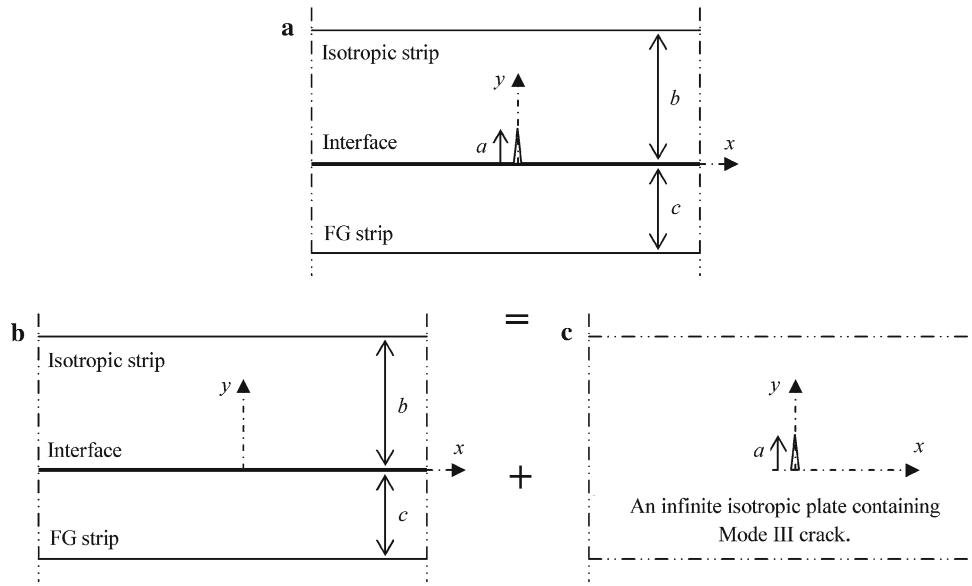

Fig. 2 Methodology of the solution of stress disturbance problems. **a** Original problem; **b** Problem of isotropic strip without crack; **c** Problem of an infinite isotropic plate containing mode III crack

$$\lim_{r\to\infty} W_{(1)}^{(\mathrm{i})}(x,y) = \lim_{r\to\infty} \tau_{zl(1)}^{(\mathrm{i})}(x,y) = 0, \quad (l = x, y), \tag{14}$$

where $r = \sqrt{x^2+y^2}$.

Applying Fourier transform (with respect to y) to Eq. (8a), one can find the regular solution satisfying Eq. (14) as follows:

$$W_{(1)}^{(\mathrm{i})}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} D_1 \mathrm{e}^{-|s|x}\mathrm{e}^{\mathrm{j}sy}\mathrm{d}s, \quad x > 0, \tag{15a}$$

$$W_{(1)}^{(\mathrm{i})}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} D_2 \mathrm{e}^{|s|x}\mathrm{e}^{\mathrm{j}sy}\mathrm{d}s, \quad x < 0, \tag{15b}$$

where D_1 and D_2 are unknown coefficients, and s is Fourier transform variable, $\mathrm{j} = \sqrt{-1}$.

By substituting Eq. (15) into Eqs. (6) and (13), one finds:

$$D_2 = -D_1. \tag{16}$$

Let

$$\lim_{x\to 0^+}\frac{\partial}{\partial y}W_{(1)}^{(\mathrm{i})}(x,y) - \lim_{x\to 0^-}\frac{\partial}{\partial y}W_{(1)}^{(\mathrm{i})}(x,y) = [1 - H(y-a)]\phi(y), \tag{17}$$

where $\phi(y)$ is unknown function, and $H(y-a)$ is a unit step function [23].

Substituting Eqs. (15) and (16) into Eq. (17), one finds:

$$D_1 = \int_0^a \frac{\mathrm{e}^{-\mathrm{j}st}}{\mathrm{j}2s}\phi(t)\mathrm{d}t. \tag{18}$$

After substitution of Eq. (18) into Eq. (15a), $W_{(1)}^{(\mathrm{i})}$ can be obtained in terms of $\phi(y)$ as follows:

$$\begin{aligned} W_{(1)}^{(\mathrm{i})}(x,y) &= \frac{1}{2\pi}\int_0^a \left(\int_{-\infty}^{\infty}\frac{\mathrm{e}^{\mathrm{j}s(y-t)}}{\mathrm{j}2s}\mathrm{e}^{-|s|x}\mathrm{d}s\right)\phi(t)\mathrm{d}t \\ &= \frac{-1}{2\pi}\int_0^a \left(\int_0^{\infty}\frac{\sin(t-y)s}{s}\mathrm{e}^{-sx}\mathrm{d}s\right)\phi(t)\mathrm{d}t \\ &= -\frac{1}{2\pi}\int_0^a \tan^{-1}\left(\frac{t-y}{x}\right)\phi(t)\mathrm{d}t, \end{aligned} \tag{19a}$$

from which and Eqs. (6), $\tau_{zl(1)}^{(\mathrm{i})}$, $(l = x, y)$ can be obtained as:

$$\tau_{zx(1)}^{(\mathrm{i})}(x,y) = \frac{\mu^{(\mathrm{i})}}{2\pi}\int_0^a \frac{t-y}{x^2+(t-y)^2}\phi(t)\mathrm{d}t, \tag{19b}$$

$$\tau_{zy(1)}^{(\mathrm{i})}(x,y) = \frac{\mu^{(\mathrm{i})}}{2\pi}\int_0^a \frac{x}{x^2+(t-y)^2}\phi(t)\mathrm{d}t. \tag{19c}$$

Similarly, using Fourier transform, the second part of the solution can be obtained as:

$$W_{(2)}^{(\mathrm{i})}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}(A_1\mathrm{e}^{sy} + A_2\mathrm{e}^{-sy})\mathrm{e}^{\mathrm{j}sx}\mathrm{d}s, \tag{20a}$$

$$\tau_{zx(2)}^{(\mathrm{i})}(x,y) = \frac{\mathrm{j}\mu^{(\mathrm{i})}}{2\pi}\int_{-\infty}^{\infty} s(A_1\mathrm{e}^{sy} + A_2\mathrm{e}^{-sy})\mathrm{e}^{\mathrm{j}sx}\mathrm{d}s, \tag{20b}$$

$$\tau_{zy(2)}^{(\mathrm{i})}(x,y) = \frac{\mu^{(\mathrm{i})}}{2\pi}\int_{-\infty}^{\infty} s(A_1\mathrm{e}^{sy} - A_2\mathrm{e}^{-sy})\mathrm{e}^{\mathrm{j}sx}\mathrm{d}s. \tag{20c}$$

The displacement and stresses of the FG strip can also be obtained in terms of Fourier integrals as follows:

$$W^{(\mathrm{f})}(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(B_1\mathrm{e}^{r_1y}+B_2\mathrm{e}^{r_2y})\mathrm{e}^{\mathrm{j}sx}\mathrm{d}s, \tag{21a}$$

$$\tau_{zx}^{(\mathrm{f})}(x,y)=\frac{\mathrm{j}\mu^{(\mathrm{i})}\mathrm{e}^{\gamma y}}{2\pi}\int_{-\infty}^{\infty}s(B_1\mathrm{e}^{r_1y}+B_2\mathrm{e}^{r_2y})\mathrm{e}^{\mathrm{j}sx}\mathrm{d}s, \tag{21b}$$

$$\tau_{zy}^{(\mathrm{f})}(x,y)=\frac{\mu^{(\mathrm{i})}\mathrm{e}^{\gamma y}}{2\pi}\int_{-\infty}^{\infty}(r_1B_1\mathrm{e}^{r_1y}+r_2B_2\mathrm{e}^{r_2y})\mathrm{e}^{\mathrm{j}sx}\mathrm{d}s, \tag{21c}$$

where $A_{(l)}$ and $B_{(l)}$, $(l=1,2)$, are unknown coefficients.

$$r_1,\ r_2=-\frac{\gamma}{2}\pm\sqrt{\frac{\gamma^2}{4}+s^2}.$$

Substituting Eqs. (19)–(21) into Eqs. (9)–(11), one can find after algebraic manipulations:

$$(s\mathrm{e}^{-2bs}-s)A_2-(r_1-r_1\mathrm{e}^{-(r_1-r_2)c})B_1=\mathrm{j}\frac{1-2\pi}{2}\int_0^a\mathrm{e}^{-t|s|}\phi(t)\mathrm{d}t, \tag{22a}$$

$$(s\mathrm{e}^{-2bs}+s)A_2-\left(s-\frac{sr_1}{r_2}\mathrm{e}^{-(r_1-r_2)c}\right)B_1=\mathrm{j}\frac{1-2\pi}{2}\int_0^a\mathrm{e}^{-t|s|}\phi(t)\mathrm{d}t, \tag{22b}$$

$$A_1-\mathrm{e}^{-2bs}A_2=\mathrm{j}\pi\int_0^a\frac{\mathrm{e}^{-t|s|}}{s}\phi(t)\mathrm{d}t, \tag{22c}$$

$$\frac{r_1}{r_2}\mathrm{e}^{-(r_1-r_2)c}B_1+B_2=0, \tag{22d}$$

from which, one can obtain:

$$A_2=\mathrm{j}\frac{1-2\pi}{2}\frac{(sr_1-r_1r_2)\mathrm{e}^{-(r_1-r_2)c}+(r_1r_2-sr_2)}{(s-s\mathrm{e}^{-2bs})(sr_2-sr_1\mathrm{e}^{-(r_1-r_2)c})+(s+s\mathrm{e}^{-2bs})(r_1r_2-r_1r_2\mathrm{e}^{-(r_1-r_2)c})}\int_0^a\mathrm{e}^{-t|s|}\phi(t)\mathrm{d}t. \tag{23}$$

Substituting Eqs. (19b), (20b), (22c) and (23) into Eq. (12), one can reduce the problem to the following singular integral equation:

$$\int_0^a\frac{1}{t-y}\phi(t)\mathrm{d}t+\int_0^a k(t,y)\phi(t)\mathrm{d}t=\frac{2\pi}{\mu^{(\mathrm{i})}}f(y),\quad (0<t,\ y<a), \tag{24}$$

where

$$k(t,y)=-\pi+\frac{2\pi-1}{2}\int_{-\infty}^{\infty}\frac{[r_1r_2-sr_2+(sr_1-r_1r_2)\mathrm{e}^{-(r_1-r_2)c}][\mathrm{e}^{2bs-(t+y)|s|}+\mathrm{e}^{-(t-y)|s|}]}{(\mathrm{e}^{2bs}-1)(sr_2-sr_1\mathrm{e}^{-(r_1-r_2)c})+(\mathrm{e}^{2bs}+1)(r_1r_2-r_1r_2\mathrm{e}^{-(r_1-r_2)c})}\mathrm{d}s, \tag{25}$$

which is not a bounded kernel because the singularity is expected when $t\to y$ and $|s|\to\infty$. To clarify the type of this singularity, one considers the following asymptotic expressions, (as $|s|\to\infty$):

$$sr_1\simeq s^2-\frac{\gamma}{2}s+\frac{\gamma^2}{8},$$

$$sr_2\simeq -s^2-\frac{\gamma}{2}s-\frac{\gamma^2}{8},$$

$$r_1r_2=-s^2,$$

$$k(t,y)=-\pi+\frac{2\pi-1}{2}\times\int_{-\infty}^{\infty}\langle I(s,t,y)-[I^{-\infty}(s,t,y)+I^{\infty}(s,t,y)],+[I^{-\infty}(s,t,y)+I^{\infty}(s,t,y)]\rangle\mathrm{d}s,$$

where $I^{\pm\infty}(s,t,y)$ represent the asymptotic expressions of the integrand, $I(s,t,y)$ as $s\to\pm\infty$.

After some lengthy but straightforward manipulations, Eq. (24) can be reduced to the following singular integral equation with Cauchy type singularity:

$$\int_0^a\frac{1}{t-y}\phi(t)\mathrm{d}t+\int_0^a\bar{k}(t,y)\phi(t)\mathrm{d}t=\frac{f(y)}{\mu^{(\mathrm{i})}},\quad (0<t,y<a), \tag{26}$$

where $\bar{k}(t,y)$ is a bounded kernel defined as:

$$\bar{k}(t,y)=\frac{-1}{2}+\frac{2\pi-1}{4\pi}\int_{-\infty}^{\infty}\left\langle\frac{[r_1r_2-sr_2+(sr_1-r_1r_2)\mathrm{e}^{-(r_1-r_2)c}][\mathrm{e}^{2bs-(t+y)|s|}+\mathrm{e}^{-(t-y)|s|}]}{(\mathrm{e}^{2bs}-1)(sr_2-sr_1\mathrm{e}^{-(r_1-r_2)c})+(\mathrm{e}^{2bs}+1)(r_1r_2-r_1r_2\mathrm{e}^{-(r_1-r_2)c})}-\mathrm{e}^{-(t-y)|s|}\right\rangle\mathrm{d}s. \tag{27}$$

Equation (26) can be solved numerically using Gauss–Chebyshev integration formulae [24–26] as follows:

Assume

$$\tau = \frac{2t}{a} - 1, \quad \zeta = \frac{2y}{a} C1. \tag{28}$$

Therefore, Eq. (26) is reduced to:

$$\int_{-1}^{1} \frac{1}{\tau - \zeta}\Phi(\tau)\mathrm{d}\tau + \int_{-1}^{1} K(\tau,\zeta)\Phi(\tau)\mathrm{d}\tau = \frac{F(\zeta)}{\mu^{(\mathrm{i})}}, \quad (-1 < (\tau,\zeta) < 1), \tag{29}$$

where $\Phi(\tau)$ and $F(\zeta)$ are equivalent to $\phi(t)$ and $f(y)$, respectively.

where $R(1)$ can be extrapolated using Krenk's formula [24] as follows:

$$R(1) = \frac{2}{2N+1}\sum_{l=1}^{N} \cot\left(\frac{\pi(2l-1)}{4N+2}\right) \sin\left(\frac{\pi N(2l-1)}{2N+1}\right) R(\tau_l). \tag{35}$$

4 Numerical results

The improper integral in Eq. (30) is evaluated numerically, with an error $\leq 10^{-9}$, using trapezoidal rule. Equations (32), (33) and (35) are solved with $N = 20$. This numerical scheme leads to solutions with sufficient accuracy for the considered elastic and geometric configurations. For simplicity, the anti-plane shear traction, (applied along the crack surfaces), are assumed to be of the form: $f(y) = \tau_0$, τ_0 is a constant. Further, for practical purposes, the values of the stress intensity factor (SIF) are normalized as follows:

$$K(\tau,\zeta) = -\frac{a}{4} + \frac{(2\pi-1)a}{8\pi}\int_{-\infty}^{\infty}\left\langle \frac{[r_1r_2 - sr_2 + (sr_1 - r_1r_2)\mathrm{e}^{-(r_1-r_2)c}][\mathrm{e}^{2bs-a-\frac{a}{2}(\tau+\zeta)|s|} + \mathrm{e}^{-\frac{a}{2}(\tau-\zeta)|s|}]}{(\mathrm{e}^{2bs}-1)(sr_2 - sr_1\mathrm{e}^{-(r_1-r_2)c}) + (\mathrm{e}^{2bs}+1)(r_1r_2 - r_1r_2\mathrm{e}^{-(r_1-r_2)c})} - \mathrm{e}^{-\frac{a}{2}(\tau-\zeta)|s|}\right\rangle \mathrm{d}s. \tag{30}$$

Let

$$\Phi(\tau) = R(\tau)\sqrt{\frac{1+\tau}{1-\tau}}, \tag{31}$$

where $R(\tau)$ is an unknown bounded continuous function over $[-1, 1]$.

By substituting Eq. (31) into Eq. (29), then employing Gauss–Chebyshev formulae [24–26], the problem can be reduced to the following linear algebraic equations:

$$\sum_{l=1}^{N} w_l\left[\frac{1}{\tau_l - \zeta_m} + K(\tau_l,\zeta_m)\right]R(\tau_l) = F(\zeta_m)/\mu^{(\mathrm{i})}, \quad (l, m = 1, N), \tag{32}$$

where N is the number of collocation points over $[-1, 1]$.

$$w_l = 2\frac{1+\tau_l}{2N+1}, \quad (l = 1, N), \tag{33a}$$

$$\tau_l = \cos\left(\frac{\pi(2l-1)}{2N+1}\right), \quad (l = 1, N), \tag{33b}$$

$$\zeta_m = \cos\left(\frac{2\pi m}{2N+1}\right), \quad (m = 1, N). \tag{33c}$$

Consequently, the mode III stress intensity factor can be obtained as [27]:

$$K_{\mathrm{III}} = \lim_{\substack{x\to\pm 0\\ y\to +a}} \sqrt{2\pi(y-a)}\,\tau_{zx}^{(\mathrm{i})}(x,y) = -\mu^{(\mathrm{i})}\sqrt{\frac{\pi a}{2}}R(1), \tag{34}$$

Normalized SIF= $\frac{K_{\mathrm{III}}}{\tau_0\sqrt{a/2}}$, τ_0 is the applied traction along the crack faces.

Figures 3 and 4 show that the values of the normalized SIF reach a minimum at $a/b \simeq 0.3$. For a relatively long cracks, $(a/b > 0.55)$, the values of the normalized SIF decrease with increasing shear modulus of the FG coating strip, whereas for short cracks $(a/b \leq 0.55)$, these values decrease with decreasing shear modulus, as shown in Fig. 3. Figure 4 shows

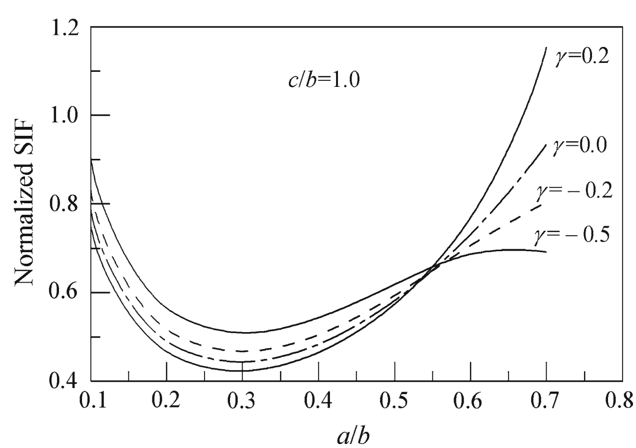

Fig. 3 Variation of the normalized SIF with the non-homogeneity constant γ and crack length

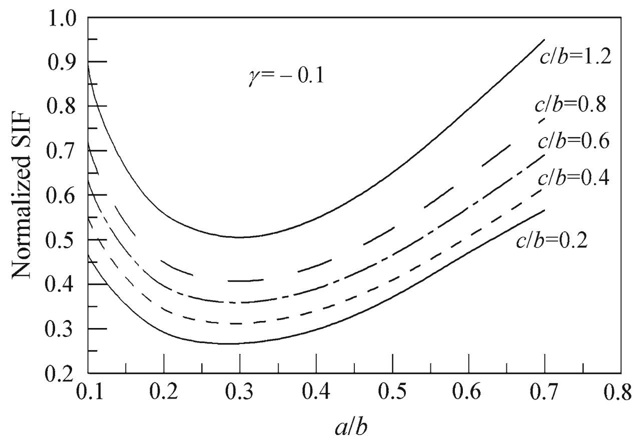

Fig. 4 Variation of the normalized SIF with the FG strip height and crack length

that the values of normalized SIF decrease with decreasing height of the FG coating strip.

5 Conclusion

The problem of mode III crack perpendicular to the interface of a bi-material composite, is analytically solved. The method of solution is based on Fourier transforms and asymptotic analysis, which are used to reduce the problem to a singular integral equation with a kernel of Cauchy type solvable numerically by using Gauss–Chebyshev integration formulae. This solution can be considered as an extension of analytical solutions of interfacial crack problems. The results may be applied to different composites involved in strengthening and coating with functionally graded materials.

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