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On deficiencies of common ordering policies for multi-level inventory control

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Abstract. We consider two common types of ordering policies for multi-level inventory control: installation stock (R,Q)-policies and echelon stock (R,Q)-policies. The batch quantities are assumed to be given, but each policy is optimized with respect to its reorder point R. We demonstrate that there is no bound for the worst case performance ratio of these policies when applied to distribution inventory systems with a central warehouse and a number of retailers.

Zusammenfassung. Es werden zwei gebräuchliche Typen von Bestellpolitiken für mehrstufige Lagerhaltungsprobleme untersucht: (R,Q)-Politiken auf Basis lokaler Bestandsinformationen sowie (R,Q)-Politiken auf Basis systemweiter Bestände (Echelon-Bestände). Während die Losgrößen Q als gegeben angenommen werden, wird jede Politik bezüglich der Bestellpunkte R optimiert. Es wird für den Fall der Anwendung auf ein Distributionssystem mit einem Zentral- und mehreren Verkaufslägern gezeigt, daß unter Worst-Case-Bedingungen der relative Kostennachteil für jeden der beiden Politiktypen unbeschränkt groß werden kann.

Key words: Inventory, production, multi-echelon inventory systems, reorder point policies

Schlüsselwörter: Bestellpunktregeln, Lagerhaltung, Produktion, mehrstufige Lagerhaltungssysteme

1. Introduction

The most common way to control a multi-level inventory system is to use an installation stock reorder point policy. This means that ordering decisions at each installation are based exclusively on the inventory position at this installation. An alternative is to use an echelon stock policy, i.e. to base the ordering decisions on the echelon inventory position. The echelon inventory position is obtained by adding the installation inventory positions at the installation and all its downstream installations. The echelon stock concept was introduced by Clark and Scarf (1960).

In general, neither of these two types of policies are optimal but will provide good approximate solutions. Axsäter and Rosling (1993) have recently shown that echelon stock (R,Q)-policies dominate installation stock (R,Q)-policies for serial and assembly systems under reasonable assumptions. For distribution inventory systems with a central warehouse and a number of retailers it is known (Axsäter and Juntti, 1996), that installation stock (R,Q)-policies and echelon stock (R,Q)-policies may outperform each other under different assumptions. In the considered numerical cases the differences were very small, though.

This note demonstrates with the aid of two examples that for distribution inventory systems with given batch quantities, an installation stock (R,Q)-policy may fail when an echelon stock (R,Q)-policy provides an optimal solution with zero cost, and vice versa, an echelon stock (R,Q)-policy may fail when an installation stock (R,Q)-policy provides an optimal solution with zero cost. Clearly this means that neither policy has a finite bound for its worst case performance ratio.

2. Basic assumptions

We shall consider an inventory system with a central warehouse and two retailers. See Figure 1.

The retailers replenish their stock from the warehouse, and the warehouse replenishes its stock from an outside supplier. All facilities apply continuous review installation stock or echelon stock (R,Q)-policies (or (R,nQ)-policies since several batches may be ordered if needed). Stockouts at both echelons are backordered and delivered

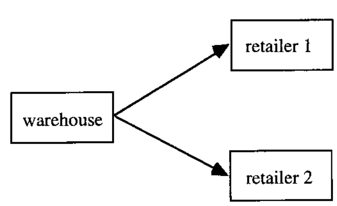

Fig. 1. Considered inventory system

on a first come–first served basis.The batch quantities are assumed to be given, but the reorder points are optimized for each policy. The leadtime for deliveries to the warehouse is constant. The transportation times for deliveries from the warehouse to the retailers are also constant. We introduce the following notations:

L_i = transportation time for an order to arrive at retailer i from the warehouse,
L_0 = leadtime for an order to arrive at the warehouse from the outside supplier,
R_i = reorder point at retailer i for an installation stock policy,
$\mathit{R_i}$ = reorder point at retailer i for an echelon stock policy,
R_0 = reorder point at the warehouse for an installation stock policy,
$\mathit{R_0}$ = reorder point at the warehouse for an echelon stock policy,
Q_i = batch size at retailer i,
Q_0 = batch size at the warehouse,
h_i = holding cost per unit and time unit at retailer i,
h_0 = holding cost per unit and time unit at the warehouse.

Holding costs under transportation are disregarded. Furthermore there are either shortage costs or given service requirements at the retailers. The considered demand processes are defined below.

3. Failure of installation stock policy

Let $L_0 = 0.5$ and $L_1 = L_2 = 0$. The batch quantities are $Q_0 = Q_1 = Q_2 = 1$. Assume demand processes at the retailers that are completely dependent in the following way. For any integer i, the demand in the time interval [i, i + 1) will either exclusively occur at retailer 1 or exclusively at retailer 2 with probability 0.5 for each alternative. The demand in the interval is deterministic and consists of a demand of size $1-\varepsilon$ at time t = i and a demand of size ε at time t = i + 0.5, where ε is a small number. Let us assume that no shortages are allowed.

Consider now the echelon stock policy $\mathit{R_0} = 0$ and $\mathit{R_1} = \mathit{R_2} = -(1-\varepsilon)$. Assume that the process starts at time t = 0 with one unit in stock at the warehouse and no stock at the retailers. The warehouse will then order at times 0.5, 1.5, 2.5, ... and one of the retailers at times 0, 1, 2, ... Evidently no shortages will occur since the deliveries from the warehouse are instantaneous. The average holding costs per time unit at the retailers are $\varepsilon(h_1 + h_2)/4$. There are no holding costs at the warehouse. Consequently the total costs will approach zero as $\varepsilon \to 0$.

An installation stock policy is nested, i.e. the warehouse can only order when one of the retailers orders. Since $Q_1 = Q_2 = 1$ we only need to consider integral values of R_0. If $R_0 = 0$ the retailers can use the same policy and will have the same costs as with the echelon stock policy, but the warehouse holding costs become $h_0/2$ per time unit. Another possibility is to use $R_0 = -1$. But this means that orders from the retailers will have to wait the time 0.5 at the warehouse. To avoid shortages we then need to have $R_1 = R_2 = 0$. In this way we avoid the holding costs at the warehouse but will face additional holding costs at the retailers that are $(h_1 + h_2)/2$ since the retailer without demand in the interval [i, i + 1) will have one unit in stock during the interval. In any way, for positive holding costs the total costs will not approach zero with ε.

4. Failure of echelon stock policy

Let $L_0 = L_1 = L_2 = 0$. Furthermore $Q_0 = Q_1 = 1$ and $Q_2 = 2$. The holding costs h_0 and h_1 are positive while $h_2 = 0$. Let the demand processes at the retailers be e.g. independent Poisson processes. Finally we assume that delays are not allowed at retailer 1, while there are no costs or constraints associated with delays at retailer 2.

Note that it may very well be optimal in a distribution system to use a retailer batch that is larger than the warehouse batch. This is the case both when the rationing policy at the warehouse is to deliver all or nothing of an order and when partial deliveries are allowed. Our example works in the same way for both these rationing policies.

It is obvious that the installation stock policy $R_0 = R_1 = -1$ is optimal and gives no costs at all. Note that R_2 does not affect the costs.

It is also easy to see that an echelon stock policy must necessarily give positive total costs. If $\mathit{R_1} > -1$ we will get holding costs at retailer 1. On the other hand, if $\mathit{R_1} = -1$ we must have $\mathit{R_0} \geq \mathit{R_2} + 1$ to avoid delays at the warehouse and consequently also at retailer 1. This is because the inventory position at retailer 2 may be $\mathit{R_2} + 2$. But when the inventory position at retailer 2 is $\mathit{R_2} + 2$ a demand at retailer 2 will then generate an unnecessary order for one unit at the warehouse. This means positive holding costs at the warehouse.

5. Conclusions

We have shown that for distribution inventory systems there exist situations when an installation stock (R,Q)-policy is optimal with zero cost while an echelon stock (R,Q)-policy fails. But also the opposite case is possible, i.e. an echelon stock policy may be optimal with zero cost while an installation stock policy fails. Since there are no bounds for the cost ratios, an immediate conclusion is that the choice of control policy can be more important than what previous numerical results have indicated. Our worst case results have been derived by considering rather extreme and artificial examples. These examples can be seen as limits of more "normal" systems. Still, an interesting topic for future research would be to analyse how our results will change under more realistic and restrictive assumptions concerning demand processes and cost structures.

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