SIMILARITY SOLUTION FOR FREE AND FORCED CONVECTION HYDROMAGNETIC FLOW OVER A HORIZONTAL SEMI-INFINITE PLATE THROUGH A NON-HOMOGENEOUS POROUS MEDIUM

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Abstract. A similarity analysis for the free and forced convection hydromagnetic flow over a horizontal semi-infinite flat plate through a non-homogeneous porous medium is presented, taking into account the hydrostatic pressure variation normal to the flat plate. The similarity solution of the problem under consideration is obtained under certain valid simplifying assumptions when, (i) the plate temperature is inversely proportional to the square root of the distance from the leading edge, (ii) the intensity of the applied magnetic field, normal to the plate, changes with the inverse square root of the distance from the leading edge, and (iii) the permeability of the porous medium, occupying a semi-infinite region of the space bounded by the flat plate, is proportional to the distance measured in the direction of the flow. A numerical solution of the resulting system of ordinary differential equations of motion and energy is obtained, depending on the Prandtl number Pr, the magnetic parameter M_n, the buoyancy parameter λ, and the permeability parameter P_m. The variations of the fundamental quantities of the problem are shown graphically followed by a quantitative discussion.

1. Introduction

Flows through porous media are very much prevalent in nature and, therefore, the study of flow through porous media has become of principal interest in many scientific and engineering applications. This type of flow is of great importance in petroleum engineering and many studies of the movement of natural gas, oil, and water through oil reservoirs have been carried out. Further, to study underground water resources and seepage of water in river beds one also needs to investigate the flow of fluid through porous media.

The results of flow through porous media are also of great interest in astrophysics and geophysics especially in the interaction of the geomagnetic field with the fluid in the geothermal region. The structures of stars and planets are known to be greatly influenced by thermal convection in their interior (Weir, 1976) and magnetic fields have also been observed in many stars (Cramer and Pai, 1973). Water in the geothermal region is an electrically conducting liquid because of high temperature. Another potential geophysical application of the flow through porous media is in the exploration of geopressured reservoirs. In these reservoirs, water at elevated temperature exists at enormously high pressure because of the weight of the overlying rock and the geomagnetic field. Thus, there are numerous practical uses of fluid flow through porous media.

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The investigation of the flow streaming into a porous and permeable medium with arbitrary but smooth wall surface was obtained by Yamamoto and Iwamura (1976), on the basis of the Euler equation and a generalized Darcy's law in which the convective acceleration was taken into account (Yamamoto and Yoshida, 1974). Darcy's law may be used as a basic equation for the flow through porous media. This law states that the (seepage) velocity is proportional to the pressure gradient but it does not include the convective acceleration of the fluid. This law is, therefore, considered to be valid for low speed flows, whereas the speed in the porous medium is not always small and this convective term may be important. To analyze this kind of flow, Yamamoto and Yoshida (1974) employed a generalized form of Darcy's law in which the convection term was taken into account. A theoretical analysis of two-dimensional flow of a viscous, incompressible fluid through a porous medium bounded by an infinite porous plate was presented by Varshney (1979). Raptis (1984) studied the viscous boundary-layer flow through a very porous medium bounded by a semi-infinite plate when the free-stream velocity is constant. The same problem for the case where the free-stream velocity is not constant has been studied by Raptis and Takhar (1987).

In recent years, the subject of magnetohydrodynamics (MHD) has attracted many authors because of its applications to astrophysics, geophysics, and engineering (Cramer and Pai, 1973). Gulab and Mishra (1977) obtained an exact solution of the unsteady motion of an electrically-conducting, incompressible, and viscous fluid through a porous medium under the action of a transverse magnetic field. The free-convective MHD flow, through a porous medium, over an infinite vertical plate has been studied by many authors. Raptis and Kafousias (1982a, b) have studied the two-dimensional steady flow of an electrically-conducting fluid through a porous medium, occupying a semi-infinite region of space bounded by an infinite, vertical, and porous limiting surface under the action of a transverse magnetic field. The MHD free-convective and mass transfer flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux has been studied by Raptis and Kafousias (1982a, b). In all the above-mentioned studies, body forces, such as buoyancy forces were taken into account only with respect to their tangential component. Force components normal to the surface were neglected as higher-order terms, with the result of no pressure variation across the boundary layer.

There are, however, flow phenomena which cannot adequately be described by means of the classical boundary-layer equations. Examples are the bending of a hot, non-vertical jet in a gravity field, and the influence of buoyancy forces on the flow over a heated or cooled horizontal plate. In the latter problem the body forces normal to the surface may even result in separation of the boundary layer flow. Approximate solutions for the mixed convection flow, over horizontal flat plates, in the non-magnetic case and in the absence of a porous medium, have been obtained by many authors (Hieber, 1973; Leal, 1973; and Chen *et al.*, 1977).

Schneider (1979) has studied the effect of buoyancy forces on the steady, laminar, plane flow over a horizontal plate, taking into account the hydrostatic pressure variation normal to the plate. An exact similarity solution was given for the case of a wall temperature that was inversely proportional to the square root of the distance from the leading edge.

The aim of the present work is to investigate the free and forced convection hydromagnetic steady laminar flow, over a horizontal semi-infinite flat plate, through a non-homogeneous porous medium occupying a region bounded by the horizontal plate. The fluid is assumed to be viscous, incompressible, and electrically conducting and the flow field is affected by a transversely applied magnetic field. A similarity solution of the problem under consideration is obtained under the following assumptions:

(i) The plate temperature is inversely proportional to the square root of the distance from the leading edge;

(ii) the intensity of the applied magnetic field, normal to the plate, changes with the inverse square root of the distance from the leading edge;

(iii) the permeability of the porous medium is proportional to the distance measured in the direction of the flow.

In Section 2, the mathematical analysis of the problem is presented while the similarity solution is obtained in Section 3. Finally, in Section 4, a numerical solution of the resulting ordinary differential equations of motion and energy is obtained and the influence of the dimensionless parameters entering into the problem, especially the magnetic parameter M_n and the permeability parameter P_m, is discussed.

2. Mathematical Analysis

Let us consider the two-dimensional steady flow of an incompressible and electrically-conducting fluid of density ρ_∞, viscosity μ_∞, and electrical conductivity σ, through a porous medium of permeability K, occupying a semi-infinite region of the space bounded by a semi-infinite horizontal flat plate. The plate is aligned parallel to a uniform free stream with velocity U'_∞, pressure P_∞, and temperature T'_∞. A Cartesian coordinate system $Ox'y'$ is used with the origin at the leading edge of the plate. The velocity components of the fluid are u' and v' in the x' and y' directions, respectively, taken parallel and perpendicular to the plate. The plate is maintained at a certain temperature T'_w which may depend on the longitudinal coordinate x'. A magnetic field is applied transversely to the direction of the flow. This magnetic field is constant in the direction perpendicular to the plate, throughout the thickness of the boundary layer. The induced electric current does not distort appreciably the applied magnetic field. This is a valid assumption for small magnetic Reynolds numbers. The coefficient of electrical conductivity σ is a scalar and remains constant everywhere. Taking into account the hydrostatic pressure variation normal to the plate, assuming constant transport coefficients and applying the Boussinesq approximation, we obtain the following boundary-layer equations:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 , \tag{1}$$

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = - \frac{1}{\rho_\infty} \frac{\partial P'}{\partial x'} + \nu_\infty \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B^2}{\rho_\infty} u' - \frac{\nu_\infty}{K} u' , \tag{2}$$

$$\frac{1}{\rho_\infty}\frac{\partial P'}{\partial y'} = g\beta_\infty(T' - T'_\infty), \tag{3}$$

$$u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho_\infty C_p}\frac{\partial^2 T'}{\partial y'^2}, \tag{4}$$

where v_∞ is the kinematic viscosity; P, the pressure inside the boundary layer; T', the fluid temperature; g, the acceleration of gravity; β_∞, the thermal expansivity of the fluid in the undisturbed state; k, the thermal conductivity; B, the magnetic induction; and C_p, the specific heat at constant pressure. The last two terms on the right-hand side of the momentum equation (2) signify the electromagnetic body force and the additional resistance due to the porous medium with permeability K, respectively. Also, the viscous dissipation and Joule heating terms on the right-hand side of the energy equation (4) are assumed to be negligible. This is a valid assumption for low speed flows.

The boundary conditions of the problem are:

$$\left.\begin{array}{lllll} y' = 0: & u' = 0, & v' = 0, & T' = T'_w(x'), & x' > 0, \\ y' \to \infty: & u' = U_\infty, & v' = 0, & T' = T'_\infty, & P' = P_\infty, \end{array}\right\} \tag{5}$$

where P_∞ is the hydrostatic pressure in the undisturbed fluid.

We introduce now the following non-dimensional parameters (Schneider, 1979)

$$\begin{gathered} x = x'/L, \quad y = \frac{\sqrt{\mathrm{Re}}}{L}y', \quad u = u'/U_\infty, \quad v = \sqrt{\mathrm{Re}}\,v'/U_\infty, \quad \mathrm{Re} = \rho_\infty U_\infty L/\mu_\infty, \\ \theta = (T' - T'_\infty)/T^*, \quad P = (P' - P_\infty)/\rho_\infty U_\infty^2, \quad A_r = gL\beta_\infty T^*/U_\infty^2, \end{gathered} \tag{6}$$

where Re is the Reynolds number; A_r, the Archimedes number; L, a specific value of the x-coordinate where $T'_w - T'_\infty = T^*$ and T^* represents a characteristic temperature difference between the plate and the free stream.

In view of (6), Equations (1)–(4) reduce to the dimensionless form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \sigma\frac{B^2 L}{\rho_\infty U_\infty}u - \frac{v_\infty L}{U_\infty K}u, \tag{8}$$

$$\frac{\partial P}{\partial y} = \lambda\theta, \tag{9}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}, \tag{10}$$

where $\lambda = A_r/\sqrt{\mathrm{Re}}$ and Pr is the Prandtl number $(\rho_\infty v_\infty C_p/k)$. The boundary conditions

(5) become now:

$$\left.\begin{array}{lllll} y=0: & u=0\,, & v=0\,, & \theta=\theta_w(x)\,, & x>0\,, \\ y\to\infty: & u=1\,, & \theta=0\,, & P=0\,. & \end{array}\right\} \tag{11}$$

3. Similarity Solution

Introducing a stream function $\psi=\psi(x,y)$ such that

$$u=\frac{\partial\psi}{\partial y}\,,\qquad v=-\frac{\partial\psi}{\partial x}\,, \tag{12}$$

the continuity equation (7) is automatically satisfied, while the system of Equations (8)–(10) and the boundary conditions (11) become:

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\lambda\int_y^\infty\frac{\partial\theta}{\partial x}\,\mathrm{d}y+\frac{\partial^3\psi}{\partial y^3}-\left(\frac{\sigma B^2L}{\rho_\infty U_\infty}+\frac{\nu_\infty L}{U_\infty K}\right)\frac{\partial\psi}{\partial y}\,, \tag{13}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}\,, \tag{14}$$

$$\begin{array}{lllll} y=0: & \psi=0\,, & \dfrac{\partial\psi}{\partial y}=0\,, & \dfrac{\partial\psi}{\partial x}=0\,, & \theta=\theta_w(x)\,,\quad x>0\,, \\ y\to\infty: & \dfrac{\partial\psi}{\partial y}=1\,, & \theta=0,\ \text{respectively}\,. & & \end{array} \tag{15}$$

In Equations (13) and (14) the pressure P has been eliminated by integrating the normal momentum equation (9) with respect to y and using the boundary condition $P=0$ as $y\to\infty$.

The following definitions (Schneider, 1979) are now made

$$\eta=yx^{-1/2}\,,\quad \psi=x^{1/2}f(\eta)\,,\quad \theta_w(x)=x^{-1/2}\quad\text{and}\quad \theta=\theta_w\phi(\eta)=x^{-1/2}\phi(\eta)\,. \tag{16}$$

Then Equations (13) and (14) and the boundary conditions (15), become

$$2f'''(\eta)+f(\eta)f''(\eta)+\lambda\eta\phi(\eta)-\frac{2B^2L}{\rho_\infty U_\infty}\,xf'(x)-\frac{2\nu_\infty L}{U_\infty K}\,xf\,'(\eta)=0\,, \tag{17}$$

$$2\phi''(\eta)+\mathrm{Pr}\{f(\eta)\phi(\eta)\}'=0\,, \tag{18}$$

$$\left.\begin{array}{llll} \eta=0: & f'=0\,, & f=0\,, & \phi=1\,, \\ \eta\to\infty: & f'=1\,, & \phi=0\,, & \end{array}\right\} \tag{19}$$

respectively; and dashes mean differentiation with respect to η.

The last two terms on the right-hand side of the momentum equation (17), signifying the electromagnetic body force and the resistance due to the porous medium, respectively, are not independent of x in general and it is seen that a similarity solution may be obtained in the case where these terms do become independent of x. This can only be done (i) with a magnetic field varying with x as $B = B_0 x^{-1/2}$ and (ii) with a permeability K varying as $K = K_0 x$. Such a situation for the magnetic field in the case of MHD free convective flows has been considered by Lykoudis (1962). On the other hand, the porous material containing the fluid is in fact a non-homogeneous medium. Sometimes, for the sake of analysis, it is possible to replace it with a homogeneous fluid which has dynamical properties equal to the local averages of the original non-homogeneous continuum. Then, one can study the motion of the hypothetical homogeneous fluid under the action of the properly averaged external forces. Thus, the complicated problem of the motion of a viscous fluid in a porous solid reduces to the motion of a homogeneous fluid with some additional resistances. However, it is well known that porous media are often anisotropic, as reflected in different values of the permeability and the formation factor, depending on the direction of measurement. Various porous media, such as sandstones, have a layered structure and the permeability parallel to the layer is mostly greater than in the perpendicular direction. In many applications of hydrodynamics in porous media to the flow of fluids through underground strata, for instance, it is difficult to obtain a representative sample of the medium in question. It is, therefore, necessary to establish a hypothetical model for the porous medium under consideration. So, in our case, we assume that the permeability K, of the porous medium, is not constant but it is proportional to the coordinate x along the plate, e.g., $K = K_0 x$. It is common practice in fluid mechanics to start with simple models to investigate the various effects. The problem considered here, although idealized, retains the essential features of the investigation.

Under the above assumptions the system of Equations (17) and (18) becomes

$$2f'''(\eta) + f(\eta)f''(\eta) + \lambda\eta\phi(\eta) - 2\{M_n + P_m\}f'(\eta) = 0\,, \tag{20}$$

$$2\phi''(\eta) + P_r\{f(\eta)\phi(\eta)\}' = 0\,, \tag{21}$$

subjected to the same boundary conditions (19). The dimensionless parameters in Equation (20) are defined as $M_n = \sigma B_0^2 L/\rho_\infty U_\infty$ (magnetic parameter) and $P_m = \nu_\infty L/U_\infty K_0$ (permeability parameter), where B_0 and K_0 are a characteristic magnetic induction and permeability, respectively.

It is worth noting that, from the variation of the magnetic field with x ($B = B_0 x^{-1/2}$), one can see that there is a singular point at $x = 0$. However, it is well known that the boundary-layer equations are not valid there. The present situation is exactly the same as the problem of existence of similarity solutions in laminar flow with mass transfer at the wall where in the case of the flat plate it is demanded that the rate of mass injection be inversely proportional to the square root of x. Both types of singularity are integrable (Lykoudis, 1962).

4. Numerical Solution and Discussion

In the system of Equations (20) and (21) the magnetic parameter M_n and the permeability parameter P_m appear only through their sum $M_n + P_m$. Thus we have to consider the variation of only three parameters: (i) the Prandtl number Pr; (ii) the buoyancy parameter λ; (iii) the sum $k(= M_n + P_m)$ of the magnetic and permeability parameter M_n and P_m.

For the solution of the system of Equations (20) and (21) under their boundary conditions (19) we considered the initial set of equations in the form

$$2f''' + ff'' - 2kf' = -\lambda\eta\phi(\eta), \tag{22}$$

$$2\phi'' + P_r(f\phi)' = 0. \tag{23}$$

Equation (23) can be integrated immediately to yield

$$\phi(\eta) = \exp\left[-\mathrm{Pr}/2 \int_0^{\eta} f(\eta)\,\mathrm{d}\eta\right]. \tag{24}$$

Equation (22) can be considered as a second-order ordinary differential equation in $f'(\eta)$ if $\phi(\eta)$ is known. This equation is solved numerically using an algorithm employing a tridiagonal scheme, enabling a new approximation for f' to be produced. Then a new value of ϕ is obtained by numerical integration in Equation (24). The iterative numerical scheme we applied was based on the above remarks and provided a stable, accurate, and rapidly convergent numerical method. Numerical computations were carried out for Pr $= 0.71$, for $\lambda = 0, 0.5, 1$ and for $k = 0, 0.1$, and 0.2. The case $k = 0$ ($M_n = 0, P_m = 0$) ($K_0 \to \infty$)) corresponds to a non-magnetic fluid flow in the absence of a porous medium while the general case $k \neq 0$ (0.1 or 0.2) includes two special limiting cases: (i) in the absence of a porous medium so that $P_m = 0$ and $k = M_n$, and (ii) when there is no applied magnetic field so that $M_n = 0$ and $k = P_m$.

Our results are illustrated in Figures 1 and 2 showing the variation of fluid velocity and temperature with the various parameters, while Tables I and II give the appropriate values of the dimensionless wall shear stress $f''(0)$, the Stanton number St* $(= \sqrt{\mathrm{Re}}\,\mathrm{St})$ and the displacement thickness $U_\infty(v_\infty x')^{-1/2}\,\delta^*$, defined below.

Figure 1 shows the variation of the tangential velocity profiles $f'(\eta)$ and fluid temperature $\phi(\eta)$ in the case of air (Pr $= 0.71$) for different values of the buoyancy parameter λ and the magnetic and permeability parameter k, while Figure 2 shows the variation of the same physical quantities in the case of water (Pr $= 7$). It is seen from these figures that the velocity profile f' increases with the buoyancy parameter λ but decreases as the parameter k increases. Since k involves the magnetic parameter M_n and the permeability parameter P_m, an increase in k implies an increase in M_n or in P_m. Hence, the application of a magnetic field helps in reducing the fluid velocity. The converse holds in the case of the permeability parameter K_0. An increase of P_m helps in reducing the fluid velocity, but as P_m is inversely proportional to K_0, an increase of P_m means

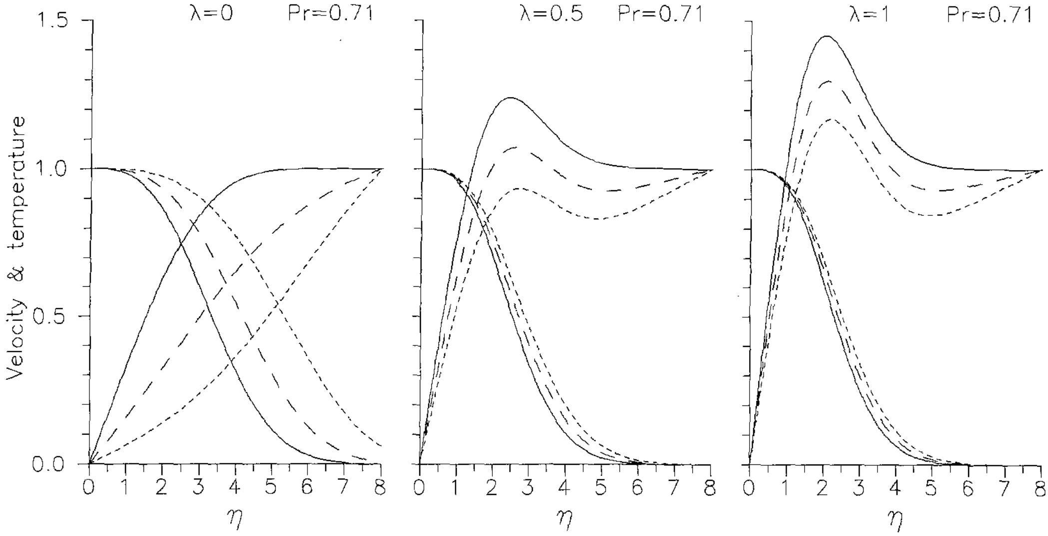

Fig. 1. Air flow velocity $f'(\eta)$ and temperature $\phi(\eta)$ profiles for various values of the parameters k and λ. Solid curves: $k = 0$; long-dashed curves: $k = 0.1$; short-dashed curves: $k = 0.2$.

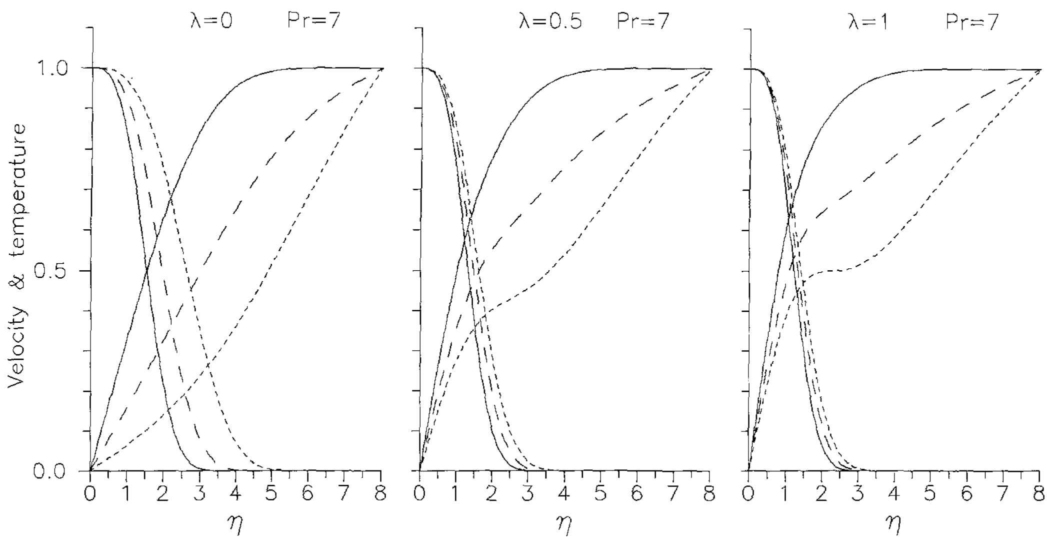

Fig. 2. Water flow velocity $f'(\eta)$ and temperature $\phi(\eta)$ profiels for various values of the parameters k and λ. Solid curves: $k = 0$; long-dashed curves: $k = 0$; short-dashed curves: $k = 0.2$.

a decrease of the permeability of the porous medium. On the other hand, fluid temperature decreases as the buoyancy parameter λ increases and the opposite is true when the parameter k increases. This increase of fluid temperature with k is more evident in the absence of buoyancy forces ($\lambda = 0$).

TABLE I

Air flow: values of $f''(0)$, St*, and $(U_\infty/\nu_\infty x')^{1/2}\,\delta^*$ for various values of the parameters λ and k

	$f''(0)$			St*			$(U_\infty/\nu_\infty x')^{1/2}\,\delta^*$		
	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1.0$
$k = 0$	0.3325	0.8297	1.1403	1.8460	2.1941	2.3619	1.7177	0.1522	−0.4543
0.1	0.1562	0.6762	0.9829	1.5990	2.0448	2.2333	3.2456	0.9352	0.2364
0.2	0.0618	0.5606	0.8569	1.4175	1.9134	2.1179	4.6452	1.6344	0.8639

TABLE II

Water flow: values of $f''(0)$, St*, and $(U_\infty/\nu_\infty x')^{1/2}\,\delta^*$ for various values of the parameters λ and k

	$f''(0)$			St*			$(U_\infty/\nu_\infty x')^{1/2}\,\delta^*$		
	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1.0$
$k = 0$	0.3325	0.5363	0.6822	0.4411	0.4887	0.5177	1.7177	1.2757	1.0383
0.1	0.1562	0.3933	0.5443	0.3537	0.4301	0.4675	3.2456	2.4318	2.1072
0.2	0.0618	0.3061	0.4540	0.2798	0.3820	0.4260	4.6452	3.4646	3.0840

In each case of air (Pr = 0.71), and water (Pr = 7) the dimensionless wall shear $f''(0)$ increases with λ and decreases with k (Tables I and II).

The total heat transfer Q_w is determined with the help of the heat flux equation in the form

$$Q_w = \rho_\infty C_p \int_0^\infty \{(T' - T'_\infty)u'\}_{x=l}\,\mathrm{d}y\,, \tag{25}$$

where l is the length of the flat plate. Introducing the dimensionless parameters (6) and applying the similarity transformation (16) we obtain

$$\mathrm{St}^* = \sqrt{\mathrm{Re}}\;\mathrm{St} = \int_0^\infty \phi(\eta) f'(\eta)\,\mathrm{d}\eta = \text{constant}\,, \tag{26}$$

where the Stanton number is defined as $\mathrm{St} = Q_w/\rho_\infty U_\infty C_p T^* L$. It is interesting to note that $\phi'(0) = 0$ for all values of λ and k. This result, which follows directly from Equation (21) together with the boundary conditions (19), indicates that there is no local heat transfer at the plate surface for all $x > 0$. Nevertheless, although dissipation has been neglected, the temperature of fluid changes during the process. The paradox is resolved by recalling that the similarity solution requires a singular behaviour of the plate temperature at $x = 0$, and all the heat necessary to change the fluid temperature must be transferred at the singular point $x = 0$, the leading edge of the plate (Schneider, 1979). According to Equation (26) the Stanton number is independent of the plate length l, thereby confirming the statement that the total heat transfer takes place at the leading edge.

The values of St* are given in Tables I and II for the case of air and water, respectively, from which we conclude that St* increases with λ but decreases when k increases.

Finally, Tables I and II give the values of the displacement thickness $(U_\infty/\nu_\infty x')^{1/2}\,\delta^*$, calculated from the expression

$$\delta^* = \int_0^\infty \left\{1 - \frac{u'}{U_\infty}\right\} \mathrm{d}y' = \sqrt{\frac{\nu_\infty x'}{U_\infty}} \int_0^\infty \{1 - f'(\eta)\}\,\mathrm{d}\eta$$

or

$$\sqrt{\frac{U_\infty}{\nu_\infty x'}}\,\delta^* = \int_0^\infty \{1 - f'(\eta)\}\,\mathrm{d}\eta\,. \tag{27}$$

The variation of the displacement thickness with λ and k is self-evident and further discussion is redundant.

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