International Journal of Infrared and Millimeter Waves, Vol. 18, No. 1, 1997

EQUIVALENT NETWORK PARAMETERS OF MILLIMETER WAVE E-PLANE SEPTUM DISCONTINUITIES WITH FINITE CONDUCTIVITY

Jin-Ping Xu

Department of Electronic Engineering
Nanjing University of Aeronautics and Astronautics
Nanjing, 210016, People's Republic of China

Received October 5, 1996

ABSTRACT

The equivalent T—network parameters of millimeter wave E—plane discontinuities of bilateral metal septa with finite conductivity are calculated by the method of lines. An efficient approach is presented to extend this numerical method to EM boundary value problems with imperfectly conducting metal boundaries parallel to the discretization lines. Numerical results of both equivalent reactances and resistances in the T—network are obtained. The insertion losses of a Ka—band filter are calculated based on the lossy equivalent circuit. Numerical results are in agreement with the published experimental results.

1. INTRODUCTION

E—plane integrated septum discontinuities in rectangular waveguide are widely applied to design millimeter—wave filters. A number of approaches, such as the variational method[1], the mode matching technique[2], the residue—calculus technique[3], the spectral—domain technique[4] and the method of lines[5] have been proposed in order to calculate the equivalent network parameters of var-

0195-9271/97/0100-0173$12.50/0 © 1997 Plenum Publishing Corporation

ious E—plane septa. However, the septa have to be assumed as perfectly conducting sheets in the formulation procedures by all the proposed models. Consequently, the obtained equivalent T—network parameters are purely inductive.

It is well known that metallic losses should not be neglected for the Q—factor of a resonator and the insertion loss in pass—band of a filter. The conductor losses become even more pronounced for multiple—resonator filters in millimeter—wave bands. The lossless equivalent circuit parameters of the septum discontinuities become insufficient to the accurate design of these components.

In this paper, the method of lines combined with collocation technique is proposed and applied to calculate the equivalent circuit parameters of E—plane discontinuities of bilateral metal septa with finite conductivity. The method of lines is extended to EM boundary value problems with imperfectly conducting metal boundaries. A hybrid homogeneous boundary condition, which includes the surface impedance of the imperfect metal, is introduced to derive a system of inhomogeneous differential equations of the discretized potential functions. The differential equations are solved by an efficient approximate procedure. A set of linear equations for reflection coefficients is established based on the principle of collocation. By solving the linear equations in the case of even and odd mode excitation, both equivalent reactances and resistances in the lossy equivalent T—network are obtained. The equivalent parameters have been used to compute the insertion losses of a five—resonator bilateral fin—line filter at Ka—band.

2. FORMULATIONS

The geometry of a E—plane bilateral septum discontinuity is sketched in Fig. 1 a. The septum width and thickness are denoted by w and t, respectively. The thickness and dielectric constant of the dielectric substrate are represented by d and ε_r, respectively. The metal septa have a finite conductivity σ. The equivaleut T—network of the septum discontinuity consists of both reactances and resistances, as shown in Fig. 1b. In the previous works, only the reactances jX_s and jX_p in the equivalent circuit can be calculated due to the assumption of perfectly conducting. In the following formulations, the equivalent resistances R_s and R_p together with the reactances will be determined simultaneously.

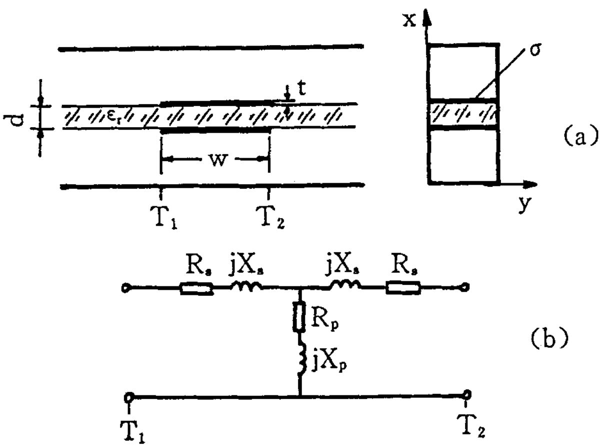


Fig. 1 Geometry and equivalent T—network of a bilateral E—plane septum discontinuity

When the septa are made of good conductor, we can make the following reasonable assumptions since the loss is slight. (i) Even and odd mode principle is applicable as the structure is symmetrical along z—axis. (ii) TE and TM modes can exist independently, which means that the coupling among the modes is neglected.

According to the latter assumption, the incident TE_{10} mode will excite only TE_{m0} $(m=1,2,\cdots)$ modes on account of the structural uniformity along y—direction. The resultant fields can be expressed by means of a vector potential $\vec{\Pi}=\vec{u}_y\varphi(x,z)$ as follows[6]

$$\vec{E} = -j\omega\mu\vec{\Pi} \tag{1.a}$$
$$\vec{H} = \nabla\times\vec{\Pi} \tag{1.b}$$

where $\varphi(x,z)$ satisfies the Helmholtz equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}+\varepsilon k_0^2\varphi=0 \tag{2}$$

in which $\varepsilon=\varepsilon_r$ in the dielectric region and $\varepsilon=1$ elsewhere, k_0 is the wave number in free—space. Taking advantage of structural symmetry, only a quarter of the whole region is considered, as shown in Fig. 2. It is further divided into three regions, namely, region Ⅰ, Ⅱ and Ⅲ. The potentials in the three regions are denoted with φ^{I}, φ^{II}

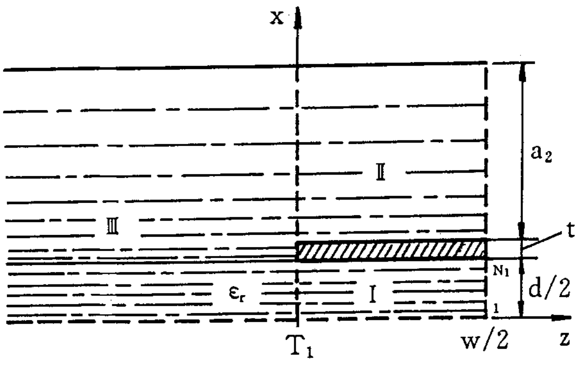


Fig. 2 A quarter of the structure and its discretization

and φ^{II}, respectively. The tangent electric and magnetic fields on the surface of the metal septum are related by

$$\vec{E}_t = Z_m \vec{u}_n \times \vec{H}_t \tag{3}$$

where $Z_m = (1+j)\sqrt{\omega\mu/2\sigma}$ is the surface impedance of the metal. Combining equation (1) and (3), a hybrid homogeneous boundary condition of the potentials on the surface of the septum can be derived as follows

$$(1 + p\frac{\partial}{\partial u_n})\varphi^i = 0, \qquad i = \mathrm{I}, \mathrm{II}, \mathrm{III} \tag{4}$$

in which $p = -Z_m/j\omega\mu$.

Region I, II and III are discretized in x —direction by a set of lines parallel to z —axis as depicted in Fig. 2. An equidistant discretization pattern and a nonequidistant one are arranged for the dielectric substrate region and the air region, respectively.

Let us first consider the potential φ^I in region I. It is now replaced by a set $\vec{\varphi}^I = (\varphi_1^I, \varphi_2^I, \cdots, \varphi_{N_1}^I)^t$ at the lines $x = (i - 0.5)h_1 (i = 1, 2, \cdots, N_1)$. The potential at the last line $x = (N_1 - 0.5)h_1$ can be expressed in terms of Taylor series

$$\varphi_{N_1}^I = \varphi_{N_1+1}^I - h_1 \frac{\partial \varphi_{N_1+1}^I}{\partial x} + h_1^2 \frac{\partial^2 \varphi_{N_1+1}^I}{\partial x^2} + \cdots \tag{5}$$

in which $\varphi_{N_1+1}^I = \varphi^I(x, z)|_{x=d/2}$. The terms higher than the first derivative in equation (5) are truncated. Substituting equation (4) into the truncated equation, we can deduce a relation for the potential at the last line and that on the surface of the septum as follows

$$\varphi_{N_1+1}^I = \varphi_{N_1}^I \frac{p}{h_1} / (\frac{p}{h_1} - 1) \tag{6}$$

The second derivative of $\vec{\varphi}^I$ with respect to x can be expressed as

$$\frac{\partial^2 \vec{\varphi}^I}{\partial x^2} = -\frac{1}{h_1^2}[P]\vec{\varphi}^I - (0,0,\cdots,q\varphi_{N_1}^I)^t \tag{7}$$

where $q = -(p/h_1)/(p/h_1 - 1)/h_1^2$, $[P]$ is a difference matrix of the second order. Inserting equation (7) into (2) leads to an inhomogeneous differential equation

$$\frac{d^2 \vec{\varphi}^I}{d z^2} - (\frac{1}{h_1^2}[P] - k_0^2[I])\vec{\varphi}^I = (0,0,\cdots,q\varphi_{N_1}^I)^t \tag{8}$$

The orthogonal matrix $[T]$, that has the property $[T]^t[P][T] = diag(\lambda_i)$, is employed so that equation (8) can be transformed into the following form

$$\frac{d^2 \vec{u}^I}{d z^2} - diag(\frac{\alpha_i^2}{h_1^2})\vec{u}^I = q[T]^t(0,0,\cdots,\varphi_{N_1}^I)^t \tag{9}$$

where $\vec{u}^I = [T]^t \vec{\varphi}^I$ is the transformed potential, $\alpha_i^2 = \lambda_i - h_1^2 \varepsilon_r k_0^2$. As the inhomogeneous term in equation (9) includes the unknown function $\varphi_{N_1}^I$, this equation is still a coupled one. It is difficult to solve the equation exactly. Inspecting this equation we can find that, when $q=0$ the solution is excatly the one for lossless case, and it can be expressed in terms of simple hyperbolic functions. Since the septum is made of good conductor, the field differs little from that of lossless case. Therefore, the solution for the lossless case at the line $x = (N_1 - 0.5)h_1$ is a very good approximate function that can be employed as a substitute for $\varphi_{N_1}^I$ in the inhomogeneous term in equation (9). By this approximate procedure, the solution for the lossy case can be derived analytically. For even mode excitation, for example, the solution of the discrete potentials in the transformed domain is

$$u_i^I = \sum_{j=1}^{N_1} S_{ij}^I(z) A_j \qquad i = 1,2,\cdots,N_1 \tag{10}$$

in which

$$S_{ij}^{I}(z)=\begin{cases} h_1^2 q\dfrac{T_{N_1 i}T_{N_1 j}}{\lambda_j-\lambda_i}\cosh\dfrac{\alpha_j}{h_1}(\dfrac{w}{2}-z) & j\neq i \\ \cosh\dfrac{\alpha_j}{h_1}(\dfrac{w}{2}-z)-\dfrac{1}{2\alpha_i/h_1}qT_{N_1 i}^2 z\sinh\dfrac{\alpha_j}{h_1}(\dfrac{w}{2}-z) & j=i \end{cases} \tag{11}$$

and $A_j(j=1,2,\cdots,N_1)$ are expansion coefficients.

By a reverse transformation to $\vec{u}^I$, the potential $\vec{\varphi}^I$ in original domain can readily be derived.

In regin II, the solution for $\vec{\varphi}^{II}$ is obtained in a similar way by introducing normalized potential functions[7] with respect to the sizes of the intervals of the discretization lines.

Region III is regarded as half a uniform waveguide partially filled with dielectric. By applying the methed of lines to the Helmholtz equation of the potential φ^{III}, an eigenvalue equation of the normalized potential can be established. Solving this equation by the QL—algorithm leads to the propagation constants of the eigenmodes TE_{m0} as well as the potential functions at the discretization lines.

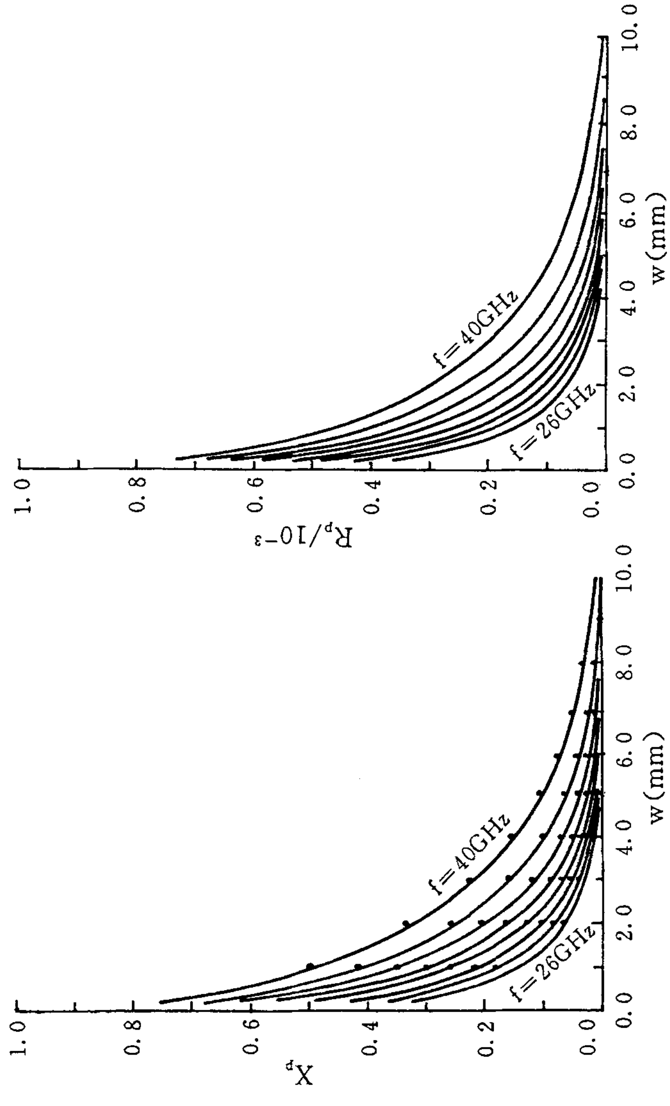
Finally, the continuous conditions of tangent eletric and magnetic fields at the interface of region III and I, region III and II as well as the hybrid boundary condition on the edge of the septum are utilized to establish a set of linear equations of reflection coefficients of all the modes. Solving these equations in the case of both even and odd mode excitation, the reflection coefficients b_1^e (for even excitation) and b_1^o (for odd excitation) of the dominant TE_{10} mode are obtained. Then the normalized input impedances at reference plane T_1 are determined by the relation $Z_{e,o}=(1+b_1^{e,o})/(1-b_1^{e,o})$. The equivalent lossy T — network parameters in Fig. 1b are calculated from the following relations

$$R_s+jX_s=Z_o \tag{12.a}$$

$$R_p+jX_p=\frac{1}{2}(Z_e-Z_o) \tag{12.b}$$

3. NUMERICAL RESULTS

A computer program has been implemented to calculate the lossy equivalent T—network parameters. Fig. 3 shows the equivalent



(b) Shunt resistances

(a) Shunt reactances

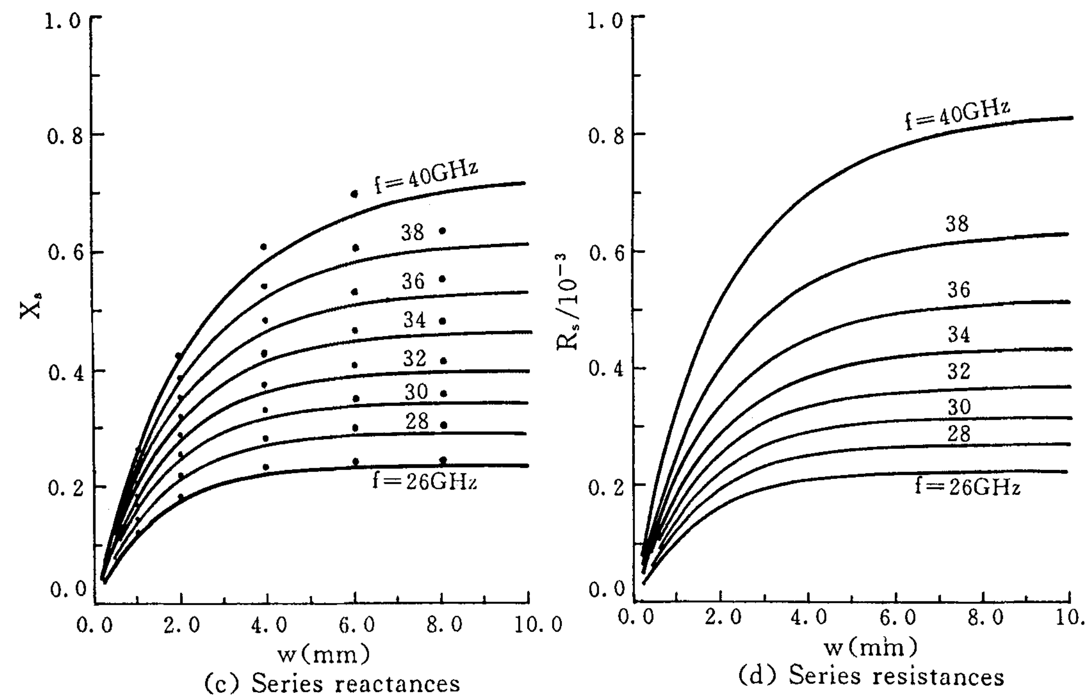


Fig. 3 Equivalent reactances and resistances of E-plane discontinuities of bilateral septa.

reactances and resistances of the E—plane discontinuities of bilateral septa in a Ka—band waveguide (WR28, $a \times b = 7.112 \times 3.556 \mathrm{mm}^2$). The other parameters used in the computation are $d = 0.254$mm, $t = 0.017$mm, $\varepsilon_r = 2.22$ and $\sigma = 5.80 \times 10^7$S/m. The dots in the shunt and series reactance diagrams are the results for lossless structure by the residue — calculus technique reported in reference [2]. It can be found that the equivalent reactances of the imperfectly conducting metallic septa differ very little from those of perfectly conducting ones. Let us look at the diagrams of the equivalent resistances. We can find that both shunt and series resistances vary in a similar pattern as their corresponding reactances. The wider the septum in size is, the greater the value of series resistance is, and the smaller the value of shunt resistance is. As frequency becomes higher, the value of both R_s and R_p increases more and more rapidly.

The equivalent resistance as a function of the septum thickness t has also be computed. Numerical results show that neither R_s nor R_p changes substantially with the thickness. When the septum width w has a smaller value, the variation in the value of R_p is a little more obvious than that of R_s.

The characteristics of the equivalent resistance in different frequency bands have also been investigated. It is found that in shorter millimeter wave bands, the values of the resistances are much greater than in lower microwave bands. Therefore, the conductor losses are much more pronounced in millimeter wave bands, as has been experienced by many researchers.

4. APPLICATION TO A E—PLANE FILTER

The lossy equivalent T—network parameters have been applied to investigate the insertion losses of a five—resonator E—plane bilateral fin—line filter in Ka—band. The structural dimensions used in calculation are the same as in [3]. Calculated insertion losses based on the lossy equivalent circuit are depicted in Fig. 4, where the measured results and the calculated results based on a lossless equivalent circuit are also plotted for comparison. The calculated insertion losses are less than 0.2 dB in the passband when the conductor loss is neglected. The computed results are 0.7~0.9 dB when the conductor loss is considered. The measured results are 1.1~1.3 dB including 0.3 dB associated with the test fixture[3]. It can be seen that the calculated insertion losses based on the lossy equivalent circuit are in better agreement with the measured results than the calculated results based on the lossless equivalent circuit.

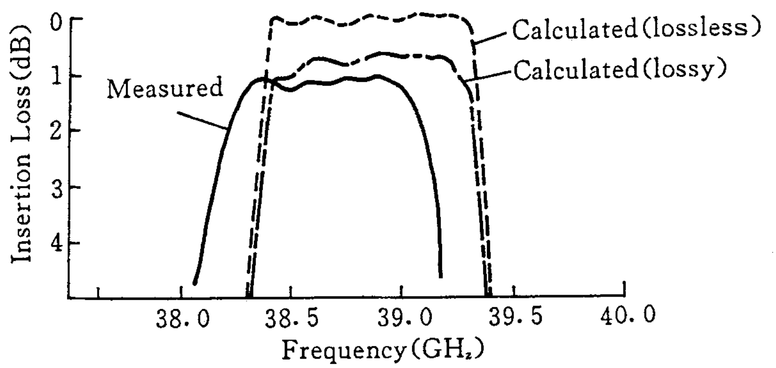


Fig. 4 Insertion loss characteristics of a five—resonator E—plane bilateral fin—line bandpass filter in Ka—band

5. CONCLUDLNG REMARKS

In this paper, an efficient approach is presented to extend the method of lines to EM boundary value problems with imperfectly conducting metal boundaries. The equivalent T—network parameters of millimeter wave E—plane septum discontinuities with finite conductivity have been calculated and applied to investigate the insertion loss charateristic of an E—plane filter in Ka—band. The calculated insertion losses are in agreement with measured results. Dielectric substrate loss is neglected in the formulation. In fact, the metallic loss is predominant when low loss dielectric is used. If the dielecric loss is required to be considered to obtain more accurate results, the theoretical model presented in this paper is directly applicable. The metallic loss has a considerable influence on the performances of millimeter wave components such as filters and resonators. Therefore, the lossy equivalent circuit parameters of E—plane septum discontinuities are very useful to calculate accurately the insertion loss of the E—plane circuits, which is particularly important when very low insertion loss in passband is specified.

ACKNOWLEDGMENT

The author wishes to thank Prof. C. H. Liang for his encouragement and support to this subject.

REFERENCES

[1] Y. Konish and K. Uenakada, "The design of a bandpass filter with inductive strip — planar circuit mounted in waveguide", IEEE Trans. on Microwave Theory Tech., vol. MTT — 22, no. 10, pp. 869—873, Oct. 1974.
[2] F. Arndt, et al., "Theory and design of low—insertion loss fin—line filters", IEEE Trans. on Microwave Theory Tech., vol. MTT—30, pp. 155—163, Feb. 1982.
[3] Y. C. Shih, T. Itoh and L. Q. Bui, "Computer—aided design of millimeter—wave E—plane filter", IEEE Trans. on Microwave Theory Tech., vol. MTT—31, no. 2, pp. 135—142, Feb. 1983.
[4] A. Rong and S. Li. "Generalized analysis of E—plane septa discontinuities", IEEE MTT—S, Int. Microw. Symp. Dig., 1987, pp. 721—724.
[5] A. Rong and S. Li, "Equivalent circuit parameters of millimeter wave E—plane strip discontinuities", Journ. of Communications (in Chinese), vol. 4, pp. 10—17, 1988.
[6] R. E. Collin, Field theory of guided waves, McGraw—Hill, New York, 1960.
[7] H. Diestel and S. B. Worm, "Analysis of hybrid field problems by the method of lines with nonequidistant discretization", IEEE Trans. on Microwave Theory Tech., vol, MTT — 32, no. 6, pp. 633—638, June 1984.