IL NUOVO CIMENTO Vol. XIX, N. 6 16 Marzo 1961

Towards a Two-Field Theory of Elementary Particles.

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(ricevuto il 23 Dicembre 1960)

Summary. — It is shown why the symmetry principle between the baryon triplet (Λnp) and lepton triplet (μ e ν) suggests a two-field theory of elementary particles. One massless spinor field is used to describe the nucleons and light leptons and a second spinor field with finite bare mass the «strange» particles Λ and μ. The two-field model resembles the theories of Heisenberg and Nambu in several respects but there are also important differences which are spelled out.

1. – Introduction.

The number of particles (π, K, $\mathcal{N}$, Λ, Σ, Ξ) which participate in the strong interactions is certainly much larger than the number of invariance principles which characterize these interactions. If we leave aside for the moment the discrete transformations P, C, T (which are associated with the improper Lorentz group), we find that the strong interactions are invariant under two independent gauge transformations (baryon and strangeness) and the isospin rotation group; this follows from the well-known fact that the charge number is related to the baryon and strangeness numbers and the third component of the isospin through the Gell-Mann–Nishijima equation:

$$Q = I_3 + \frac{S+B}{2}\,. \tag{1}$$

Hence, insofar as the integral degrees of freedom are concerned, all the strongly

(*) Guest Professor under the Ford Foundation and Guggenheim Fellow, on leave from the University of Rochester for the academic year 1960/61.

(**) Supported by National Science Foundation, U.S.A.

interacting particles can be represented by three fields: two fields comprising an isospinor and one field with unit baryon number (if the isospinor is K) or unit strangeness (if the isospinor is $\mathcal{N}$ or Ξ). While SAKATA's (1) choice of the baryon triplet (Λ, n, p) as the three fundamental fields is evidently not unique, it has several advantages as we shall see below.

Although a three-field theory such as Sakata's satisfies all the group-theoretic requirements (2) of the strong interactions, there still remain the particles which do not interact strongly at all, namely the leptons. At first sight, the number of additional fundamental fields required to describe the leptons is also three in number: in order to take account of the charge and lepton gauge transformations as well as the « strange » difference between the muon and the electron; it is natural to choose the observed lepton triplet (μ^-, e^-, ν) as the three fundamental lepton fields. If we introduce formally an isospin space and strangeness gauge transformation for the leptons, the analogue of the Gell-Mann–Nishijima equation becomes:

$$Q = I_3 + \frac{S-L}{2} \tag{1a}$$

where μ^- is an $S=-1$ isoscalar and (e^-, ν) is an $S=0$ isospinor. The correspondence between Sakata's baryon triplet (Λ, n, p) and the lepton triplet (μ^-, e^-, ν) is now apparent; indeed, eq. (1) and (1a) can be consolidated into the single relation (3,4):

$$Q = I_3 + \frac{S+B-L}{2}\,. \tag{1b}$$

Furthermore, the implications of the symmetry principle $\Lambda \leftrightarrow \mu^-$, $n \leftrightarrow e^-$, $p \leftrightarrow \nu$ (hereinafter called the BL symmetry principle) for the weak interactions have been discussed in detail (3). It is also possible to extend the BL symmetry principle to electromagnetic interactions (*e.g.* the absence of a *fast* $\Lambda \to n + \gamma$ decay implies the similar absence of a *fast* $\mu^- \to e^- + \gamma$ decay).

The Nagoya group (5) has attempted to construct a four-field theory for

(1) S. SAKATA: *Prog. Theor. Phys.*, **16**, 686 (1956); This type of theory is similar to that of E. FERMI and C. N. YANG: *Phys. Rev.*, **76**, 1739 (1949); cfr. also M. M. LÉVY and R. E. MARSHAK: *Suppl. Nuovo Cimento*, **11**, 366 (1954).

(2) W. E. THIRRING: *Nucl. Phys.*, **10**, 97 (1959); see also Y. YAMAGUCHI: *Prog. Theor. Phys. Suppl.*, **11**, 1, 37 (1959); and M. IKEDA, S. OGAWA and Y. OHNUKI: *Prog. Theor. Phys.*, **22**, 715 (1959); and **23**, 1073 (1960).

(3) A. GAMBA, R. E. MARSHAK and S. OKUBO: *Proc. Nat. Acad. Sci.*, **45**, 881 (1959).

(4) V. I. GOLDANSKI: *Nucl. Phys.*, **6**, 531 (1958); O. KLEIN: *Arkiv. Swed. Acad. Sci.*, **16**, 191 (1959); I. SAAVEDRA: *Nucl. Phys.*, **10**, 6 (1959).

(5) Z. MAKI, M. NAKAGAWA, Y. OHNUKI and S. SAKATA: *Prog. Theor. Phys.*, **23**, 1174 (1960); Y. KATAYAMA and M. TAKETANI: preprint (Kyoto); M. E. MAYER: *Nuovo Cimento*, **17**, 802 (1960).

baryons and leptons on the basis of the BL symmetry principle. They assume that the four fundamental fields are the lepton triplet (μ^-, e^-, ν) plus a positively charged boson B^+ which is supposed to be « strongly » coupled to the leptons. Each member of the baryon triplet (Λ, n, p) is hypothesized to be a composite of B^+ and the corresponding lepton, *i.e.* $(\Lambda, n, p) = B^+(\mu^-, e^-, \nu)$. Unfortunately, the Nagoya model cannot prevent a process like $n(= B^+e^-) + \bar{p}(= B^-\bar{\nu}) \to e^- + \bar{\nu}$ from being a « strong » reaction without postulating an additional selection rule. This is not surprising since this type of theory implicitly assumes the identity of the isospin space and the strangeness gauge transformation for the baryon and lepton triplets for *all* interactions.

It would thus appear that, within the framework of the standard perturbation-theoretic treatment of interacting fields, a minimum of six fields is required to explain the group properties of the strongly interacting particles and leptons despite the BL symmetry principle. The possibility of developing a theory of elementary particles with fewer than six fields is therefore intimately connected with the invalidity of the perturbation expansion and the breakdown of the « adiabatic theorem » resulting from the infinite number of degrees of freedom in quantum field theory. These points are briefly discussed in Section **2** as the rationale for theories of the type proposed by HEISENBERG (6) and NAMBU (7). In Section **3**, we summarize and compare the essential ideas and difficulties of Heisenberg's one-field theory and Nambu's « superconductivity » theory of elementary particles; the emphasis is on the significance of the invariance principles employed by the respective authors and not on the detailed quantitative predictions of baryon and meson masses. In Section **4**, we outline our own suggestions for a possible two-field theory of elementary particles; while we take over some concepts from both the Heisenberg and Nambu theories, we ground our theory firmly in the BL symmetry between the baryon and lepton triplets. Some of the problems faced by our theory are also spelled out.

2. – Non-perturbative solutions in quantum field theory.

In quantum field theory it is customary to decompose the total Hamiltonian into two parts; the free part H_0 and the interaction part H_1:

$$H = H_0 + H_1 . \tag{2}$$

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(6) H. P. DÜRR, W. HEISENBERG, H. MITTER, S. SCHLIEDER and K. YAMAZAKI: *Zeits. Naturfor.*, **14** a, 441 (1959); hereinafter we shall call this the Heisenberg theory.

(7) Y. NAMBU and G. JONA-LASINIO: preprint (University of Chicago); this will be referred to as the Nambu theory. Cfr. also J. GOLDSTONE: CERN preprint.

The diagonalization of H_0 is straightforward and yields the continuous energy spectrum. However, the same is not true for H, where, in general, diagonalization is impossible except for a few rather trivial examples. Nevertheless, one expects a sort of correspondence between H_0 and H which is expressed by the so-called « adiabatic theorem ». This theorem asserts that, except for possible bound states, the eigenfunctions of H can be obtained from those of H_0 by adiabatically switching on the interaction. In other words, new types of states other than those corresponding to states of H_0 should not arise except for possible bound states. The adiabatic theorem obviously holds within the framework of perturbation theory.

The HEISENBERG (6) and NAMBU (7) programmes for constructing a theory of elementary particles depend for their success upon the existence of non-perturbative solutions of the field equations and the breakdown of the adiabatic theorem. It is therefore of great importance that HAAG has proved (8) (Haag's theorem) that the adiabatic theorem need not hold in quantum field theory. Haag's theorem essentially states that the Hilbert space constructed from the eigenfunctions of H (so that H is a proper operator in this space) is completely inequivalent to those of H_0, or, more precisely, that the domain of H is completely disjoint from that of H_0 in some wider space (except, of course, for a null element). This fact, related as it is to the unbounded character of the interaction Hamiltonian H_1, is sufficient to invalidate the perturbation expansion and the adiabatic theorem. If H_1 were a completely continuous transformation (*i.e.* if any element f_n of the Hilbert space satisfied the condition $f_n \to f$ weakly (f is also an element of the Hilbert space) then $H_1 f_n \to H_1 f$ strongly), then the limit (roughly speaking, continuous) spectrum of H would have to agree with that of H_0 (Weyl's theorem (9)). If this were the case, then it would be impossible to construct theories of the Fermi-Yang or Sakata type (1) since the continuous spectrum of the pion is lower than that of the nucleon. Thus the possibility of composite models probably depends upon the unbounded character of H_1 and the Haag theorem.

The above situation is connected with the appearance of the inequivalent representations (10) of the commutation rings when we have an infinite number of degrees of freedom. Suppose that we have N anti-commuting operators $a_1, ..., a_N$ and its adjoints satisfying the following commutation relations:

$$(3) \qquad \{a_i, a_j^*\}_+ = \delta_{ij}, \quad \{a_i, a_j\}_+ = 0, \quad \{a_i^*, a_j^*\}_+ = 0 \qquad (i, j = 1, ..., N).$$

(8) R. HAAG: *Kgl. Danske Videns. Selb. Mat. Fys. Medd.*, **29**, 12 (1955); P. W. HALL and A. S. WIGHTMAN: *Kgl. Danske Videns. Selb. Mat. Fys. Medd.*, **31**, 5 (1957).

(9) See *e.g.* F. RIESZ and B. SZ. NAGY: *Functional Analysis*, *2nd.* Edition (New York, 1953), pp. 367.

(10) R. HAAG: see ref. (8); L. GÅRDING and A. S. WIGHTMAN: *Proc. Nat. Acad. Sci.* **40**, 617, 622 (1954); A. S. WIGHTMAN and S. SCHWEBER; *Phys. Rev.*, **98**, 812 (1955).

When N is finite, there is only one essentially independent irreducible representation of this anticommutating ring. It is the usual representation, whose bases are the direct product $\Phi_1 \otimes \Phi_2 \otimes ... \otimes \Phi_N$, where Φ_i takes on only two possible values in conformity with the Pauli principle. However, when N becomes infinite, a new situation arises and we can have an infinite number of representations; *i.e.* we can have an infinite number of Hilbert spaces, which are orthogonal to each other ([11]). In some simple soluble examples ([11]), there is a one to one correspondence between the total Hamiltonian H and one of these inequivalent representations; more precisely, H is a proper operator in a subspace of only one of these Hilbert spaces. However, there is in principle no reason why H may not be a proper operator in several such Hilbert spaces which are orthogonal to each other. It is this possibility which permits one to entertain the hope that one field may be employed to describe several particles and, indeed, this is the foundation of our own theory (see Section 4). The important point is that we must find the Hilbert space, or spaces, in which H is a proper operator (strictly speaking, only in a dense subset of these Hilbert spaces, since H is an unbounded operator). Unfortunately, this is practically impossible to accomplish at the present time since we cannot solve the problem exactly (even if there were no difficulty with the divergences). The best that we can hope to achieve is to find in a self-consistent fashion an approximate Hilbert space or spaces.

We shall illustrate the above remarks by sketching a modified version of Nambu's ([7]) procedure. Suppose, we start with the following Lagrangian:

$$\left\{\begin{aligned} \mathscr{L}_0 &= -\bar{\psi}\left(\gamma\frac{\partial}{\partial x}\right)\psi\,, \\ \mathscr{L}_1 &= g\,(\bar{\psi}\psi)(\bar{\psi}\psi)\,, \end{aligned}\right. \tag{4}$$

where ψ is an ordinary Dirac four-component spinor. Eq. (4) is different from Nambu's, since it does not include the pseudoscalar term. Now let us suppose that the correct Hilbert space $\mathscr{H}_m$ of this theory is built up from the following asymptotic field φ_m with the mass m:

$$\left(\gamma\frac{\partial}{\partial x} + m\right)\varphi_m = 0\,. \tag{5}$$

In this way, we have an infinite number of Hilbert spaces $\mathscr{H}_m$ corresponding to different masses m. The problem is to find the correct (or approximately correct) Hilbert space corresponding to mass m. In the first approximation in the coupling constant g, $\mathscr{L}_1$ can be replaced by (we neglect the pairing

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([11]) R. HAAG: see ref. ([8]); L. VAN HOVE: *Physica*, 18, 145 (1952); Y. NAMBU and G. JONA-LASINIO: ref. ([7]).

of ψ and $\bar{\psi}$ belonging to other combinations, *e.g.* $\widehat{\bar{\psi}\psi\bar{\psi}\psi)}$:

$$\mathscr{L}_1 \simeq 2g\langle\bar{\psi}\psi\rangle_0(\bar{\psi}\psi)\,, \tag{6}$$

where $\langle\bar{\psi}\psi\rangle_0$ denotes the vacuum expectation value with respect to the Hilbert space $\mathscr{H}_m$. Hence, we can determine the mass m in a self-consistent fashion and, in first approximation, by the formula:

$$m = -\,2g\langle\bar{\psi}\psi\rangle_0\,. \tag{7}$$

Furthermore, in the low mass approximation, we have:

$$\langle\bar{\psi}\psi\rangle_0 \simeq -\,Z_2\,\mathrm{Tr}\,S_F(0,\,m)\,, \tag{8}$$

where Z_2 is a renormalization constant and consequently:

$$m = 2g'\,\mathrm{Tr}\,S_F(0,\,m)\,. \tag{9}$$

Eq. (9) is the equation given by NAMBU, where g' is the renormalized coupling constant.

We now note that two asymptotic fields $\varphi_{\pm m}$ can be associated with ψ by following Touschek's « ansatz » ([12]). Our equation of motion is:

$$\gamma\frac{\partial}{\partial x}\psi - g\,[(\bar{\psi}\psi)\psi + \psi(\bar{\psi}\psi)] = 0\,. \tag{10}$$

Let us put:

$$\begin{cases} \psi_1 = \psi\,, \\ \psi_2 = g[(\bar{\psi}\psi)\psi + \psi(\bar{\psi}\psi)]\,. \end{cases} \tag{11}$$

Then the equation of motion is:

$$\gamma\frac{\partial}{\partial x}\psi_1 - \psi_2 = 0\,. \tag{10a}$$

Now suppose that ψ_1 and ψ_2 go asymptotically to φ_1 and φ_2 respectively and

([12]) A. TOUSCHEK: *Nuovo Cimento*, **13**, 394 (1959); in this paper TOUSCHEK also shows that, under certain conditions, a doubling of states is necessary for a finite mass particle independent of the approximation when one starts with a two-component field.

that moreover:

$$\gamma \frac{\partial}{\partial x} \psi_1 \xrightarrow[\text{(weakly)}]{} \gamma \frac{\partial}{\partial x} \varphi_1 ,$$

then we have:

$$\gamma \frac{\partial}{\partial x} \varphi_1 - \varphi_2 = 0 , \tag{12}$$

where φ_1 and φ_2 are assumed to satisfy the free field equations:

$$(\Box - m^2)\varphi_1 = 0 , \qquad (\Box - m^2)\varphi_2 = 0 . \tag{13}$$

(We must modify this equation, when there exists a finite bare mass m_0; however, the final formula is similar to eq. (15)). If we operate with $\gamma(\partial/\partial x)$ on eq. (12), we get:

$$\gamma \frac{\partial}{\partial x} \varphi_2 - m^2\varphi_1 = 0 . \tag{14}$$

Then, if we define:

$$\begin{cases} \varphi_m \;\; = \varphi_2 - m\varphi_1 , \\ \varphi_{-m} = \varphi_2 + m\varphi_1 , \end{cases} \tag{15}$$

we obtain

$$\begin{cases} \left(\gamma \dfrac{\partial}{\partial x} + m\right) \varphi_m \;\; = 0 , \\ \left(\gamma \dfrac{\partial}{\partial x} - m\right) \varphi_{-m} = 0 . \end{cases} \tag{16}$$

Thus, we may interprete φ_m and φ_{-m} as the asymptotic fields corresponding to the two solutions m and $-m$ (except for a trivial numerical factor). Furthermore, we can show that the two Hilbert spaces $\mathscr{H}_m$ and $\mathscr{H}_{-m}$ are orthogonal [7] and that all matrix elements of $\varphi_{\pm m}$ with respect to $\mathscr{H}_{\mp m}$ must be zero. We can check this in the same approximation in which we derived the mass m, since:

$$\begin{aligned} (\Phi^*_{(\mp m)}, \varphi_{\pm m} \Phi'_{(\mp m)}) &\simeq (\Phi^*_{(\mp m)}, (\psi_2 \mp m\psi_1) \Phi'_{(\mp m)}) \simeq \\ &\simeq [-2g' \operatorname{Tr} S_F(0, \mp m) \mp m] (\Phi^*_{(\mp m)}, \psi \Phi'_{(\mp m)}) \equiv 0 , \end{aligned} \tag{17}$$

by virtue of eq. (9). In the above, $\Phi_{(\mp m)}$ and $\Phi'_{(\mp m)}$ are any arbitrary vectors belonging to $\mathscr{H}_{\mp m}$. Of course, these are not rigorous statements but illustrate the type of non-perturbative self-consistent result which is possible.

We have interpreted the two Nambu solutions $\pm m$ as corresponding to two independent fields whose asymptotic fields are given by φ_{+m} and φ_{-m}. However, this interpretation has several difficulties. One difficulty is that we cannot do the same for the $m=0$ solution of eq. (9). For $m=0$, the corresponding asymptotic field φ_0 is given by $\psi\to\varphi_0$ by the same reasoning; however, we already have

$$\psi=\frac{1}{2m}(\psi_2+m\psi_1)-\frac{1}{2m}(\psi_2-m\psi_1)\to\frac{1}{2m}(\varphi_m-\varphi_{-m})\,,$$

which cannot be reconciled with $\psi\to\varphi_0$. To avoid this dilemma we may suppose that a single ψ may give rise to more than one asymptotic field. Then, the consequence is essentially the same as doubling the Hilbert space and extending the meaning of ψ to the larger space. We assume this to be the case in the future, whenever a similar situation arises. The second difficulty is that instead of the Lagrangian (4) we may take:

$$\mathscr{L}_1=g[(\bar\psi\psi)(\bar\psi\psi)-(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)] \tag{18}$$

which is the one adopted by NAMBU [7]. If we proceed in the same way as before, the second term $\bar\psi\gamma_5\psi$ does not yield any contribution to eq. (9) since $\mathrm{Tr}\,\langle\bar\psi\gamma_5\psi\rangle_0=0$. Hence the conclusions should be the same as in the previous case. However, in contrast to (4), the Lagrangian (18) possesses the property of chirality invariance, namely it is invariant under the transformation:

$$\psi\to\exp[i\alpha\gamma_5]\cdot\psi\,. \tag{19}$$

Then, as was noted by NAMBU, this transformation is not a proper operator in a given Hilbert space $\mathscr{H}_m$. When we define $\varphi_m^{(\alpha)}$ by:

$$\left(\gamma\frac{\partial}{\partial x}+m\cdot\exp[2i\alpha\gamma_5]\right)\varphi_m^{(\alpha)}=0\,, \tag{20}$$

we can construct the Hilbert space $\mathscr{H}_m^{(\alpha)}$ from $\varphi_m^{(\alpha)}$. Then, the chirality transformation (19) brings $\mathscr{H}_m$ into $\mathscr{H}_m^{(\alpha)}$ so that now we have an infinite number of spaces $\mathscr{H}_m^{(\alpha)}$ instead of only two, $\mathscr{H}_m$ and $\mathscr{H}_{-m}$. The question now arises as to whether we should interpret this result as implying an infinite mass degeneracy or whether we should regard the total Hamiltonian as being a proper operator in only a small number of orthogonal Hilbert spaces, say $\mathscr{H}_0$, $\mathscr{H}_m$ and possibly $\mathscr{H}_{-m}$. We prefer the latter interpretation since the argument which led to the asymptotic fields $\varphi_{\pm m}$, cannot be extended to the present case. Thus, we take the view that also for eq. (18) only two independent fields $\varphi_{\pm m}$ exist and that the chirality transformation (19) is not defined in our space (since it can be shown that an operator U satisfying the relation $U^{-1}\psi U=\exp[i\alpha\gamma_5]\cdot\psi$ does not exist in our space).

3. – Comparison of the Heisenberg and Nambu theories.

Both the Heisenberg and Nambu theories derive their significance from the possible existence of non-perturbative solutions of the postulated non-linear field equation. In each case, the author deduces a finite « baryon » mass from the self-interaction of a massless field. Within the framework of perturbation theory, the possibility of a finite mass would be excluded. NAMBU argues from the physical fact of superconductivity for the existence of a solutions of his field equation which is a non-analytic function of the coupling constant (13) but Heisenberg's theory is equally dependent on the existence of such solutions. Indeed, from the point of view of the baryon mass calculation, the Heisenberg equation (6):

$$\gamma \frac{\partial}{\partial x}\psi \pm g\gamma_5\gamma_\mu\psi\bar{\psi}\gamma_5\gamma_\mu\psi = 0\,, \tag{21}$$

differs in a non-essential way from Nambu's equation:

$$\gamma \frac{\partial}{\partial x}\psi - g\{\psi(\bar{\psi}\psi) + (\bar{\psi}\psi)\psi - \gamma_5\psi(\bar{\psi}\gamma_5\psi) - (\bar{\psi}\gamma_5\psi)\gamma_5\psi\} = 0\,. \tag{22}$$

Heisenberg's method of approximation for dealing with eq. (21) differs in certain details from Nambu's approximation for his eq. (22) but the resulting equation for the mass has the same structure:

$$m = mf(m^2l^2)\,, \tag{23}$$

where l is a length characteristic of the approximation (in Heisenberg's case $l = \sqrt{g}$ whereas in Nambu's case $l =$ cut-off length - see below). Eq. (23) possesses a solution $m = 0$—which is expected within the usual perturbation-theoretic framework and which is discarded (14) by both HEISENBERG and

(13) However, it must be continuous with respect to the coupling constant, in view of the theorem of Rellich (see: F. RIESZ and B. SZ. NAGY: loc. cit., pp. 169); this remark applies to the continuous spectrum of all theories.

(14) The reasons are different for the two authors; NAMBU discards the $m=0$ solution since he believes that it is unstable on two counts: 1) the analogy with superconductivity (where the superconducting state is stable and the normal state is not) and 2) on the basis of a calculation of the difference between the divergent zero-point energies. However, neither argument is conclusive: 1) the normal state becomes as stable as the superconducting state in the limit of infinite volume of the superconductor which corresponds more closely to the elementary particle case with its infinite number of degrees of freedom and hence there is no reason why two solutions should not be

NAMBU—and a solution $m \neq 0$ defined by:

$$f(m^2 l^2) = 1 \,. \tag{24}$$

If either eq. (21) or (22) were solved by means of perturbation theory, solution (24) would not be found.

It is of some interest to compare the approximation methods of Heisenberg and Nambu in order to assess more fully the role of l in eqs. (24). This comparison becomes more perspicuous if we employ a common notation; following HEISENBERG, we define the wave function:

$$\chi(x) = \langle 0 | \psi(x) | p \rangle \,, \tag{25}$$

where $|p\rangle$ is a state with momentum p, $\langle 0|$ is the vacuum state, and we assume that somehow a Hilbert space can be defined. Then, eq. (21) and (22) can be written in the form:

$$\gamma_{\alpha\beta} \frac{\partial}{\partial x} \chi_\beta(x) = -\, g \sum O_{\alpha\beta} \chi(x_\beta, x_\lambda | x'_\lambda) O_{\lambda\lambda'} \,, \tag{26}$$

where $O = \gamma_5 \gamma_\mu$ (eq. (21)) and $O = 1$, $i\gamma_5$ (eq. (22)) and

$$\chi(x_\alpha, y_\beta | z_\gamma) = \langle 0 | (\psi_\alpha(x) \psi_\beta(y) \overline{\psi}_\gamma(z))_+ | p \rangle \tag{27}$$

with $\chi(x, x|x)$ a suitable limit of $\chi(x, y|z)$. Furthermore, we define the τ-functions by

$$\tau(x) \equiv \chi(x) = \langle 0 | \psi(x) | p \rangle \,, \tag{28}$$

$$\tau(x, y|z) \equiv \chi(x, y|z) - F(y - z)\tau(x) + F(x - z)\tau(y) \,, \quad \text{etc.} \tag{29}$$

where

$$F(x - y) = \langle 0 | (\psi(x) \overline{\psi}(y))_+ | 0 \rangle \,. \tag{30}$$

It follows that the τ-functions are the connected parts of the $\chi(x, y|z)$ in the usual Feynman diagram technique.

HEISENBERG and, in essence, NAMBU now have recourse to the Tamm-

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equally stable in two orthogonal Hilbert spaces and 2) the meaning of the divergent zero-point energy is unclear at the present stage of quantum field theory. On the other hand, Heisenberg effectively discards the $m = 0$ solution since he wishes to describe the leptons by means of the « scale » transformation. Besides, in the Heisenberg theory, the propagator function is chosen in such a way that it is zero for $m = 0$ and therefore it is self-consistent to discard the $m = 0$ solution.

Dancoff approximation but at a different stage and with a different hypothesis concerning the propagator function $F(x-y)$. NAMBU simply sets $\tau(x, x|x) \equiv 0$ so that;

$$\chi(x_\alpha x_\beta | x_\gamma) \simeq F_{\gamma\beta}(0)\tau_\alpha(x) - F_{\alpha\gamma}(0)\tau_\beta(x)\,. \tag{31}$$

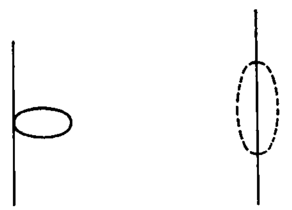
Thus, one obtains:

$$\gamma \frac{\partial}{\partial x}\tau(x) = -\,2g(\operatorname{Tr} F(0))\,\tau(x)\,. \tag{32}$$

If one now writes $\tau(x) = \exp[ipx]\cdot\tau(0)$ and $F(0) \simeq Z_2 S_F(0, m)$, eq. (32) becomes:

$$(i\gamma p)\,\tau(0) = -\,2g' \operatorname{Tr} S_F(0, m)\cdot\tau(0)\,. \tag{33}$$

Setting $(i\gamma p)\,\tau(0) = -\,m\,\tau(0)$, eq. (33) yields eq. (9). In the Heisenberg case, by assumption, the interaction term is taken as a «Wick product» and thus one must set $F(0) = 0$; therefore if one stops with $\tau(x, y|z) = 0$ one only obtains the $m = 0$ solution. Hence, Heisenberg goes to the next Tamm-Dancoff approximation, *i.e.* he sets $\tau(x_1 x_2 x_3 | x_4 x_5) \equiv 0$. In effect, NAMBU computes the mass on the basis of the bubble diagram (Fig. 1*a*), whereas HEISENBERG uses the more complicated diagram (Fig. 1*b*).



a) Nambu theory *b*) Heisenberg theory

Fig. 1. – Diagrams used to obtain finite mass solutions.

In addition, HEISENBERG and NAMBU choose different forms of the Green's function (30). NAMBU approximates $F(x-y)$ by

$$F(x-y) \simeq Z_2 \cdot \frac{i}{(2\pi)^4}\int \mathrm{d}^4 p\, \frac{i\gamma p - m}{p^2 + m^2}\, C(p^2, \Lambda^2) \exp[ip(x-y)]\,, \tag{34}$$

where $C(p^2, \Lambda^2)$ is a convergence factor (Λ^{-1} is the cut-off momentum) which must be inserted in order to produce a finite answer. This choice of propagator violates chirality invariance because of its dependence on m. The finite mass which follows from eq. (9) then turns out to be proportional to Λ^{-1}, *i.e.* $l = \Lambda$ in eq. (23). HEISENBERG attempts to maintain a chirality invariant propagator and convergence at the same time. This is achieved by writing down an expression for the propagator which implies an indefinite metric, namely:

$$F(x-y) \simeq \frac{+\,i}{(2\pi)^4}\int \mathrm{d}^4 p\,(i\gamma p)\left[\frac{1}{p^2+m^2} - \frac{1}{p^2} + \frac{m^2}{(p^2)^2}\right] \exp[ip(x-y)]\,. \tag{34a}$$

In this unorthodox fashion, HEISENBERG can produce a finite mass which is expressed in terms of the fundamental length of the theory, *i.e.* $l = \sqrt{g}$ in eq. (23). These different choices of F reflect the different assumptions which are made by HEISENBERG and by NAMBU concerning the underlying Hilbert space.

There are some more fundamental differences between the Heisenberg and Nambu theories which are related to the interpretation of the field operator ψ. To make the point more clearly, consider the 2-component Weyl field φ:

$$\left(\sigma \frac{\partial}{\partial x}\right)\varphi = 0 . \tag{35}$$

It is well known that there is a one-to-one formal correspondence between the Weyl field φ and the Majorana field ψ defined by:

$$\psi = \begin{pmatrix} \varphi \\ \sigma_2 \varphi^* \end{pmatrix} . \tag{36}$$

This follows from the fact that if we use the representation for the Dirac matrices in which γ_5 is diagonal, then:

$$\gamma \frac{\partial}{\partial x} \psi = 0 , \qquad C^{-1}\overline{\psi} = \psi , \tag{37}$$

i.e. ψ is a Majorana field. This one-to-one correspondence between φ and ψ illustrates the importance of the physical interpretation of the field operator since a Weyl field does not admit the space-reflection operator whereas, for a Majorana field, particle and antiparticle are the same while space-reflection is an allowed operation. In this connection, it is convenient to distinguish between variational variables and canonical variables ([15]). Variational variables are the ones used in the variational principle for the Lagrangian with the proviso that one may adopt any arbitrary combinations as the variational variables. Thus, in the above simple example, φ or ψ can be a variational variable, but if we wish to interpret φ as a real Weyl field, then φ is canonical but not ψ. On the other hand, if we wish to interpret ψ as a real Majorana field, then ψ is canonical but not φ. The choice of canonical variable is not trivial; *e.g.* it has been shown in one simple soluble theory ([16]) that one

([15]) Such a distinction has already been made for different purposes by Y. TAKAHASHI: *Nuovo Cimento*, **1**, 414 (1955).

([16]) S. OKUBO: CERN preprint.

special choice of the canonical variable requires an indefinite metric and gives rise to divergences whereas another choice does not.

Let us now return to Heisenberg's equation:

$$\gamma \frac{\partial}{\partial x} \psi \pm g\gamma_5 \gamma_\mu \psi \overline{\psi} \gamma_5 \gamma_\mu \psi = 0 . \tag{38}$$

HEISENBERG uses the chirality transformation:

$$\psi \to \exp [i\alpha\gamma_5]\cdot\psi \tag{39}$$

as the generator for the fermion number N whereas NAMBU uses the usual gauge transformation:

$$\psi \to \exp [i\alpha]\cdot\psi \tag{40}$$

as the generator for N. In the Heisenberg theory, the transformation (40) is a generator for the third component I_3 of the isospin rotation. Indeed, (40) is a special case of the more general so-called « Pauli-Gürsey » transformation ([17])

$$\begin{gathered}\psi \to \alpha\psi + \beta\gamma_5 C^{-1}\psi , \\ |\alpha|^2 + |\beta|^2 = 1\end{gathered} \tag{41}$$

which is isomorphic to the isospin rotation group.

DÜRR ([18]) has shown that the transformations (39) and (41) can be written quite simply if we employ the 2-component spinors φ_1 and φ_2 (instead of the four-component ψ) defined as follows:

$$\begin{cases} \varphi_1 = \frac{1}{2}(1+\gamma_5)\psi , \\ \varphi_2 = \frac{1}{2}(1+\gamma_5)C^{-1}\overline{\psi} . \end{cases} \tag{42}$$

Then, eq. (38) reduces to (see Appendix):

$$\gamma \frac{\partial}{\partial x} \varphi = \pm g\sigma_\mu \varphi\varphi^* \sigma_\mu \varphi , \tag{43}$$

with $\varphi = \begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix}$. In the new notation, the chirality transformation (39) is con-

([17]) W. PAULI: *Nuovo Cimento*, **6**, 204 (1957); however, the possible existence of such a transformation was already noted by S. KAMEFUCHI and S. TANAKA: *Prog. Theor. Phys.*, **14**, 225 (1955).

([18]) H. P. DÜRR: preprint (Münich).

verted into the usual fermion gauge transformation:

$$\varphi_{1,2} \to \exp[i\alpha]\varphi_{1,2} \tag{44}$$

and the Pauli-Gürsey transformation (41) reduces to the usual isospin rotation:

$$\begin{cases} \varphi_1 \to \alpha\varphi_1 + \beta\varphi_2\,, \\ \varphi_2 \to -\beta^*\varphi_1 + \alpha^*\varphi_2\,. \end{cases} \tag{45}$$

Finally, the propagator (34a) takes on the simple form:

$$\langle 0|(\varphi(x)\varphi^*(y))_+|0\rangle = \tag{45a}$$
$$= \frac{-1}{(2\pi)^4}\int \mathrm{d}^4p(\sigma^+\cdot p)\left[\frac{1}{p^2+m^2} - \frac{1}{p^2} + \frac{m^2}{(p^2)^2}\right]\exp[ip(x-y)]\,.$$

The same decomposition defined by (42) does not lead to any simplification of propagator, etc., in Nambu's theory because of the different choice of canonical variable. That is to say, NAMBU takes the four-component « Dirac » field ψ as the canonical variable whereas HEISENBERG takes the two 2-component « Weyl » fields φ_1 and φ_2 as the canonical variables. This is the reason why in the Heisenberg theory the usual parity operation is no longer valid whereas in the Nambu theory, parity is still defined. The distinction between variational and canonical variables also explains another important point of difference between the Heisenberg and Nambu theories. On the basis of chirality invariance, NAMBU derives a conservation law for the axial vector current and thereby proves the existence of a zero mass pseudoscalar boson (identified as the pion) using some reasonable dispersion-theoretic arguments. However, the same deduction does not hold in Heisenberg's theory ([19]) since the chirality transformation is utilised to define the fermion number; in fact, the pion in Heisenberg's theory is of non-vanishing mass. This is not surprising, since the choice of the canonical variables is entirely different in the two theories.

The different choice of canonical variables—as well as of the fundamental non-linear equation—in the Heisenberg and Nambu theories flows from a different motivation in each case. HEISENBERG wishes to construct a theory of elementary particles based on *one* massless four-component field operator satisfying a non-linear equation whose invariance properties will automatically give rise to the known additive quantum numbers, isospins and multiplicative quantum numbers of all the elementary particles. (We shall not comment

([19]) Actually, in this case, the conservation of the current implies the separate C and P invariance of the matrix elements: W. E. THIRRING: *Nucl. Phys.*, **14**, 565 (1960).

on his hope to derive the properties and relative strengths of the strong, electromagnetic and weak interactions as well as the masses and properties of the particles). He has shown how to derive the fermion (additive quantum) number from chirality invariance and the isospin from « Pauli-Gürsey » invariance. However, the discrimination between the baryon and lepton numbers requires the introduction of the ill-defined « scale » transformation and the strangeness number necessitates the postulation of a vacuum with infinite isospin ([20]). HEISENBERG must introduce an additional hypothesis (he introduces the $l \rightarrow -l$ transformation) in order to cope with the multiplicative quantum numbers P and C. It is safe to say that Heisenberg's goal of a one-field theory is extremely ambitious and will be extraordinarily difficult to carry out.

NAMBU seems to be more modest in his objectives. He appears to be willing to introduce the baryon gauge transformation, isospin rotation and strangeness gauge transformation *ad hoc* (by working with three Dirac fields) and he makes no attempt at the present time to deal with the leptons. His chief interest is in pushing the analogy between the chirality invariance of his non-linear field equation, the finite mass solution and the zero-mass pseudoscalar boson on the one hand and (ordinary) gauge invariance, the energy gap and the collective oscillations in superconductivity on the other. NAMBU has suggested introducing a non-zero bare mass ($m_0 \neq 0$) into his equation in order to obtain a finite mass for the pion (but as we shall see, we propose to introduce $m_0 \neq 0$ for a completely different purpose).

4. – Two-field theory.

We believe that the BL symmetry principle stated in the introduction suggests a two-field theory which, while utilizing several key concepts of the Heisenberg and Nambu theories, may possess certain advantages over both. There are also serious problems which will be mentioned at the end.

Let us first consider the isodoublets (n, p) and (e^-, ν); the isosinglets Λ and μ^- will be treated later. It is reasonable to assume that the mass differences between (n, p) and (e^-, ν) are due to electromagnetic effects and hence, if the electromagnetic interaction is neglected, that the nucleons possess equal mass M and the light leptons zero mass. Suppose we start with the Heisenberg

([20]) Recently, HEISENBERG (private communication) has given an argument justifying this approach by using the analogy with superconductivity and ferromagnetism in still another fashion; he argues that the non-zero isospin of the vacuum is only true locally (the over-all vacuum benig symmetrical) just as the breakdown of the usual gauge invariance is only true locally (the gauge invariance returning for large distances).

Lagrangian:

$$
(46)\qquad \left\{\begin{aligned}
&\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1\,,\\
&\mathscr{L}_0 = -\frac{1}{4}\left[\bar{\psi}\gamma\frac{\partial}{\partial x}\psi - \frac{\partial\bar{\psi}}{\partial x}\gamma\psi + \psi\gamma^T\frac{\partial\bar{\psi}}{\partial x} - \frac{\partial\psi}{\partial x}\gamma^T\bar{\psi}\right],\\
&\mathscr{L}_1 = g\colon(\bar{\psi}\gamma_5\gamma_\mu\psi)(\bar{\psi}\gamma_5\gamma_\mu\psi)\colon
\end{aligned}\right.
$$

We note that the four-fermion interaction:

$$
(47)\qquad (\bar{\psi}Q\psi - \psi Q^T\bar{\psi})(\bar{\psi}Q\psi - \psi Q^T\bar{\psi})
$$

is invariant under both the chirality transformation (39) and Pauli-Gürsey transformation (41) if (see Appendix):

$Q = A$ or T for Pauli-Gürsey transformation

if

$Q = A$ or V for chirality transformation.

Thus only $Q =$ axial vector satisfies the invariance under both transformations, which is, of course, the basis of Heisenberg's claim that his non-linear equation is unique (as long as one limits oneself to the simplest type of non-linear spinor equation). It should be emphasized that zero bare mass ($m_0 = 0$) is essential for the validity of both chirality and Pauli-Gürsey invariance.

We now adopt Heisenberg's viewpoint, *i.e.* we think of the two 2-component Weyl fields φ_1 and φ_2 defined by (42) as the canonical variables. Then as has been noted, the field φ describes an isospinor. Thus, as in the Heisenberg theory, we identify the non-perturbative finite mass solution with the nucleon (n, p). Furthermore, to define the parity, we propose to combine the $m = \pm M$ solutions into a four-component Dirac particle with mass M (this bears only a formal resemblance to the $l \to -l$ transformation). In contrast to the Heisenberg theory, we do not discard the $m = 0$ solution but instead identify the associated isospinor with the light lepton (e^-, ν). It follows that the light lepton must be described by a two-component spinor. While this appears satisfactory for the neutrino, it is certainly not true for the non-neutral member of the light lepton doublet, namely the electron. To explain this deficiency, we hypothesize that the other two components of the four-component Dirac spinor actually describing the electron, are generated by the electromagnetic interaction; *i.e.*, we may expect that the non-perturbative finite mass solution for the electron (due to the chirality-invariant electromagnetic interaction) also involves a doubling [12] of the number of components in the same sense as the baryon case above. In this context the electromagnetic field could be considered as a separate fundamental field.

The next step is to assume the existence of an equation like (23) and to argue, with NAMBU, that the $m = 0$ and $m \neq 0$ solutions refer to inequivalent representations of the anti-commuting ring and that therefore the light lepton and nucleon Hilbert spaces are orthogonal to each other. Consequently, the field operator φ or (φ_1, φ_2) possesses no non-zero transition matrix elements from one Hilbert space to the other. Thus we can double the space and extend the meaning of the field operator φ to the product space of the light lepton and nucleon Hilbert spaces instead of the direct sum of the spaces.

The orthogonality of the two spaces implies that nucleons never transform into the light leptons, *i.e.* a kind of the super-selection rule operates, as has been already pointed out by NAMBU. We also note that in this approximation, there is no β-decay interaction since $n + \bar{p} \to e + \bar{\nu}$ is forbidden. It is even tempting to attribute the weak β-decay to a breakdown of the orthogonality of the lepton and baryon Hilbert spaces resulting from the non-zero mass of the electron (by breakdown is meant that the field operators possess non-vanishing matrix elements between the two spaces). Such an explanation would have the virtue that it would at the same time provide a natural basis for the apparent dominance of the charged currents over the neutral currents in weak interactions. The two members of the current contributing to the weak interaction differ in charge because they must differ in mass in order to break the orthogonality of the Hilbert spaces. The electromagnetic interaction is the mechanism whereby the weak interactions become possible. From this viewpoint intermediate charged bosons are unnecessary.

Thus, in the proposed picture, we can readily explain the isospinor character of (n, p) and (e^-, ν) and also we have a super-selection rule which distinguishes between the baryon and lepton numbers. There is, however, a fundamental difficulty: there is no guarantee that the four-lepton interaction is appreciably weaker than the four-baryon interaction. The tremendous difference in observed strengths between the four-lepton and four-baryon interactions must be ascribed, from our viewpoint, to the strikingly different character of the $m = 0$ and $m = \pm M$ solutions ([21]). Since the non-perturbative finite mass solution is a reflection of the non-analytic dependence on the coupling constant, we argue that the effective coupling is an extremely sensitive function of the physical fermion mass and is responsible for the relative strengths of the four-baryon and four-lepton interactions; for example, we

([21]) L. VAN HOVE'S work *Physica*, **25**, 365 (1959) in this connection is unfortunately not relevant since he could only show that in a certain model a strong interaction not satisfying a prescribed invariance principle can be converted into a strong interaction obeying the invariance principle *plus* a weak interaction.

may conjecture that, in analogy with superconductivity, the effective coupling constant is of the form $g \exp[-1/m^2 g]$ or $mf(m^2, g)$. (In superconductivity, the energy gap E is related to the phonon-electron coupling constant g by means of $E \approx \exp[-(W/g^2)]$, where W is the density of states per unit energy at the Fermi surface.) We had hoped to demonstrate the possibility of this behaviour using the Thirring « one-dimensoinal » four-fermion model ([22]); unfortunately, the coupling constant in this model must be dimensionless and the possibility of a non-perturbative finite mass solution is necessarily excluded. A crucial test of our type of theory will be to exhibit the possibility of a non-analytic dependence of the effective strength of the coupling on the physical masses of the four-fermions participating in the interaction.

We now turn to the μ and Λ. The muon possesses a huge mass (on the scale of the electron mass) and it is extremely unlikely that this mass has an electromagnetic origin ([23]). We postulate that the « strange » mass of the muon is due to the existence of a bare mass $m_0 \neq 0$ in the Lagrangian from the beginning and we add a second field ψ' with $m_0 \neq 0$ to the Lagrangian, namely:

$$\mathscr{L}_{m_0} = -\frac{m_0}{2}(\bar{\psi}'\psi' - \psi'\bar{\psi}') . \tag{48}$$

We now suppose that there is a non-linear interaction between the ψ and ψ' fields and that the μ and Λ masses arise from a displacement of the $m=0$ and $m=M$ solutions for ψ alone ([24]). However, in order not to affect the existence of the $m=0$ solution (see eq. (23)) to describe the light leptons, it is sufficient for the total Lagrangian for the two fields ψ and ψ' to be invariant under the restricted chirality transformation:

$$\psi \to \gamma_5 \psi , \qquad \psi' \to \psi' . \tag{49}$$

In a sense, the (finite) m_0 plays the role of a strangeness quantum number. This is to be compared with Nambu's proposal of a finite m_0 to enable him to obtain a finite pion mass. From our viewpoint, we interpret the chirality

([22]) W. E. THIRRING: *Ann. Phys.*, **3**, 91 (1958); *Nuovo Cimento*, **9**, 1007 (1958); V. GLASER: *Nuovo Cimento*, **9**, 990 (1958).

([23]) See, however, the work by J. LEAL FERREIRA and Y. KATAYAMA: *Prog. Theor. Phys.*, **23**, 776 (1960).

([24]) H. KITA and E. PREDAZZI: *Nuovo Cimento*, **17**, 908 (1960); have used a similar method for explaining the mass difference between the muon and electron. They utilize a strong cut-off dependence to obtain the large muon mass from zero bare mass in a way which we consider unreasonable.

transformation in the same way as HEISENBERG and this problem does not arise. Moreover, if Nambu's $m = 0$ solution is identified with the light lepton, the introduction of a finite bare mass sufficient to explain the pion mass would be expected to lead to an excessive light lepton mass, unless one is lucky.

Because of the finite bare mass, the $\mathscr{L}_{m_0}$ part of the Lagrangian is not invariant under the Pauli-Gürsey transformation so that the four-component ψ is maintained as the canonical variable and remains, as experiment requires, an isoscalar; *i.e.* the isospin transformation for ψ' is $\psi' \to \psi'$, in contrast to eq. (41). This is to be compared to the four-component ψ which is interpreted as two canonical two-component Weyl spinors φ_1 and φ_2. In this fashion, two four-component spinors ψ and ψ', the first with zero bare mass and the second with finite bare mass, may reproduce simultaneously the baryon triplet (Λ, n, p) and the lepton triplet (μ^-, e^-, ν). The BL symmetry principle has led in a natural way to a *two-field* theory of elementary particles.

We recognize that our two-field theory is more of a programme than a theory. It is less ambitious than Heisenberg's one-field theory in that the strangeness is introduced from the start by means of a second field with $m_0 \neq 0$. It differs from HEISENBERG in exploiting the inequivalent representations to differentiate between the baryons and the leptons rather than employ the « scale transformation » with its attendant difficulties. It shares Heisenberg's dilemma that separate P and C invariance of the strong four-baryon interaction can only be achieved by means of an additional assumption (unless Thirring's programme [25] of deriving separate P and C invariance from the CP invariance of this type of theory is successful). The two-field theory is more ambitious than Nambu's theory because the isospin, strangeness and lepton quantum numbers are supposed to follow from the group-theoretic properties of only two fields. It differs from NAMBU in seeking the analogy with superconductivity not through the chirality invariance, but rather through the sensitive dependence of the four-fermion interaction upon the physical fermion mass. The common difficulty of all of these theories is that serious calculations are almost impossible. One may only hope that the judicious choice of model calculations may lend support to the qualitative features of one of these theories and suggest certain types of experiments which may be particularly incisive.

* * *

We are indebted to Professors V. GLASER, W. HEISENBERG, L. VAN HOVE and Y. YAMAGUCHI for valuable conversations and to Professor Y. NAMBU for helpful correspondence. We also wish to thank CERN for its hospitality.

(25) W. E. THIRRING: *Nucl. Phys.*, **10**, 97 (1959); **14**, 565 (1960).

APPENDIX

The group properties of n four-component spinors.

For the purposes of reference, it is useful to record in this Appendix an extension of the work of DÜRR [18] and THIRRING [25] on the group properties of various types of four-fermion interactions among n four-component spinors. Let us consider n four-component spinors ψ_i $(i=1, \dots, n)$, and assume invariance under the unitary group U_n among them. We take zero bare mass so that the Lagrangian is:

$$\text{(A.1)} \qquad \begin{cases} \mathscr{L}_0 = -\dfrac{1}{4}\left[\overline{\psi}\gamma \dfrac{\partial}{\partial x}\psi - \dfrac{\partial\overline{\psi}}{\partial x}\gamma\psi + \psi\gamma^T\dfrac{\partial\overline{\psi}}{\partial x} - \dfrac{\partial\psi}{\partial x}\gamma^T\overline{\psi}\right], \\ \mathscr{L}_1 = \dfrac{1}{4}g(\overline{\psi}Q\psi - \psi Q^T\overline{\psi})(\overline{\psi}Q\psi - \psi Q^T\overline{\psi}), \end{cases}$$

where $\psi = (\psi_1, \dots, \psi_n)$ is a $4n$-component spinor, Q denotes appropriate γ matrices, and

$$\overline{\psi}Q\psi = \sum_{i=1}^{n}\overline{\psi}_i Q\psi_i .$$

Eq. (A.1) is invariant under the unitary **group** U_n:

$$\text{(A.2)} \qquad \psi_i \to \sum_{j=1}^{n} a_{ij}\psi_j , \qquad \sum_{j=1}^{n} a^*_{ij}a_{kj} = \delta_{ik} .$$

We shall show that for a special choice of Q, eq. (A.1) is invariant under a much larger group.

We decompose ψ_i into two 2-component spinors φ_i and χ_i by means of:

$$\text{(A.3)} \qquad \varphi_i = \tfrac{1}{2}(1+\gamma_5)\psi_i , \qquad \chi_i = \tfrac{1}{2}(1+\gamma_5)C^{-1}\overline{\psi}_i .$$

Then defining:

$$\text{(A.4)} \qquad \varphi = (\varphi_1, \dots, \varphi_n) , \qquad \chi = (\chi_1, \dots, \chi_n) .$$

Eq. (41) becomes:

$$\text{(A.5)} \qquad \mathscr{L}_0 = -\frac{1}{4}\left[\overline{\varphi}\gamma\frac{\partial}{\partial x}\varphi - \frac{\partial\overline{\varphi}}{\partial x}\gamma\partial + \varphi\gamma^T\frac{\partial\overline{\varphi}}{\partial x} - \frac{\partial\varphi}{\partial x}\gamma^T\overline{\varphi}\right] - \\ -\frac{1}{4}\left[\overline{\chi}\gamma\frac{\partial}{\partial x}\chi - \frac{\partial\overline{\chi}}{\partial x}\gamma\chi + \chi\gamma^T\frac{\partial\overline{\chi}}{\partial x} - \frac{\partial\chi}{\partial x}\gamma^T\overline{\chi}\right],$$

and

$$\bar{\psi}Q\psi - \psi Q^T\bar{\psi} = \bar{\varphi}Q\varphi - \varphi Q^T\bar{\varphi} + \bar{\chi}(C^{-1}Q^TC)\chi - \\ - \chi(CQC^{-1})\bar{\chi} - \chi(CQ)\varphi + \bar{\varphi}(QC^{-1})\bar{\chi} - \varphi(Q^TC)\chi + \bar{\chi}(C^{-1}Q^T)\bar{\varphi}\,. \tag{A.6}$$

Note, furthermore, that:

$$\gamma_5\varphi = \varphi\,, \qquad \gamma_5\chi = \chi\,. \tag{A.7}$$

We now specialize Q.

i) $Q = V, A$, then:

$$\gamma_5^T(CQ)\gamma_5 = -CQ\,,$$

so that:

$$\chi(CQ)\varphi = \chi\gamma_5^T(CQ)\gamma_5\varphi = -\chi(CQ)\varphi = 0\,.$$

Thus:

$$\bar{\psi}Q\psi - \psi Q^T\bar{\psi} = \bar{\varphi}Q\varphi - \varphi Q^T\bar{\varphi} + \bar{\chi}(C^{-1}Q^TC)\chi - \chi(CQC^{-1})\bar{\chi}\,. \tag{A.8}$$

ii) $Q = S, T, P$, then:

$$\gamma_5 Q\gamma_5 = Q\,,$$

so that:

$$\bar{\varphi}Q\varphi = -\bar{\varphi}\gamma_5 Q\gamma_5\varphi = -\bar{\varphi}Q\varphi = 0\,.$$

Similarly:

$$\bar{\chi}Q\chi = 0\,,$$

so that:

$$\bar{\psi}Q\psi - \psi Q^T\bar{\psi} = -\chi(CQ)\varphi + \bar{\varphi}(QC^{-1})\bar{\chi} - \varphi(Q^TC)\chi + \bar{\chi}(C^{-1}Q^T)\bar{\varphi}\,. \tag{A.9}$$

On the basis of eq. (A.8) and (A.9), we have:

i) $Q = A$

$$\bar{\psi}Q\psi - \psi Q^T\bar{\psi} = \bar{\varphi}Q\varphi + \bar{\chi}Q\chi - \varphi Q^T\bar{\varphi} - \chi Q^T\bar{\chi}\,. \tag{A.10}$$

ii) $Q = V$

$$\bar{\psi}Q\psi - \psi Q^T\bar{\psi} = \bar{\varphi}Q\varphi - \bar{\chi}Q\chi - \varphi Q^T\bar{\varphi} + \chi Q^T\bar{\chi}\,. \tag{A.11}$$

iii) $Q = S$ or P

$$\bar{\psi}Q\psi - \psi Q^T\bar{\psi} = -\chi(CQ)\varphi - \varphi(CQ)\chi + \bar{\varphi}(QC^{-1})\bar{\chi} + \bar{\chi}(QC^{-1})\bar{\varphi}\,. \tag{A.12}$$

iv) $Q = T$

$$\bar{\psi}Q\psi - \psi Q^T\bar{\psi} = \bar{\varphi}(QC^{-1})\bar{\chi} - \bar{\chi}(QC^{-1})\bar{\varphi} - \chi(CQ)\varphi + \varphi(CQ)\chi \,. \tag{A.13}$$

Thus for $Q = A$, $\mathscr{L}_1$ is invariant under the $2n$-dimensional unitary group U_{2n}; this is clear if we define:

$$\Phi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} . \tag{A.14}$$

For $Q = V$, $\mathscr{L}_1$ is invariant under separate n-dimensional groups U_n among the χ and among the φ, *i.e.* it is invariant under $U_n \otimes U_n$. For $Q = S$ or P, let us introduce new variables:

$$\varphi' = \frac{1}{\sqrt{2}}(\varphi + \chi) \,, \qquad \chi' = \frac{-i}{\sqrt{2}}(\varphi - \chi) \,, \tag{A.15}$$

then:

$$\bar{\psi}Q\psi - \psi Q^T\bar{\psi} = -\varphi'(CQ)\varphi' - \chi'(CQ)\chi' + \bar{\varphi}'(CQ)\bar{\varphi}' + \bar{\chi}'(CQ)\bar{\chi}' . \tag{A.16}$$

Therefore, $\mathscr{L}_1$ is invariant under the $2n$-dimensional orthogonal group O_{2n}. Finally, for $Q = T$, if we define:

$$\Phi = \begin{pmatrix} \varphi_1 \\ \chi_1 \\ \varphi_2 \\ \chi_2 \\ \vdots \\ \varphi_n \\ \chi_n \end{pmatrix} , \qquad I = \begin{pmatrix} i\tau_2 & & 0 \\ & i\tau_2 & \\ 0 & & i\tau_2 \end{pmatrix} , \tag{A.17}$$

then $\mathscr{L}_1$ is invariant under:

$$\Phi \to U\Phi \qquad \text{where} \qquad U^T I U = I \,. \tag{A.18}$$

This means that $\mathscr{L}_1$ is invariant under the $2n$-dimensional symplectic group $S_p(2n)$. Note that $\mathscr{L}_0$ is invariant under U_{2n}.

We may summarize the results:

$$\begin{cases} Q = A; & U_{2n} \,, \\ Q = V; & U_n \otimes U_n \,, \\ Q = S \text{ or } P; & U_{2n} \cap O_{2n} \,, \\ Q = T; & U_{2n} \cap S_p(2n) \,. \end{cases} \tag{A.19}$$

The Heisenberg case corresponds to $n = 1$ and $Q = A$ and hence (A.19) the result is invariant under U_2. Also for $n = 1$ and $Q = T$, we have the two-dimensional unitary symplectic group which is equivalent to the two-dimensional unitary unimodular group; this is why $Q = T$ (together with $Q = A$) leads to invariance under the Pauli-Gürsey transformation but not to invariance under the chirality (gauge) transformation. In the case of $n = 2$, our choice, we have various possibilities depending upon the interaction. For the study of the « symmetrical » Sakata model, $n = 3$ and it appears that we have too many invariant quantities if we adopt a pure Q (*i.e.* not a mixture of various types of interactions). This would then be a problem with the « symmetrical » Sakata model [2,25].

RIASSUNTO (*)

Si mostra come il principio di simmetria tra il tripletto di barioni (Λnp) e il tripletto di leptoni (μeν) suggerisca per le particelle elementari una teoria a due campi. Si usa un campo spinoriale per descrivere i nucleoni e i leptoni leggeri e un secondo campo spinoriale con massa nuda finita per descrivere le particelle « strane » Λ e μ. Sotto molti aspetti il modello a due campi è simile alla teoria di Heisenberg e di Nambu ma esistono anche notevoli differenze che si mettono in evidenza.

(*) *Traduzione a cura della Redazione.*