THE SOLUTION OF A SYSTEM OF r + 1 INCONSISTENT LINEAR EQUATIONS IN n UNKNOWNS BY THE METHOD OF LEAST SQUARES IN THE l_p NORM†

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Let there be given the inconsistent system of r + 1 complex linear equations in n unknowns of rank r:

$$h_s(z)=a_{s1}z_1+\ldots+a_{sn}z_n+b_s=0;\quad s=1,\ldots,r+1. \tag{1}$$

We consider the determinant

$$D=\begin{vmatrix} a_{11}\ldots a_{1r}b_1 \\ \vdots \\ a_{r+1,1}\ldots a_{r+1,r}b_{r+1}\end{vmatrix}. \tag{2}$$

We denote the cofactor of b_i in the determinant (2) by B_i, and the cofactor of a_{ij} by A_{ij}.

Then the solution of the system (1) by the method of least squares in the l_p norm is given by

$$|h_1(z^*)|^p+\ldots+|h_{r+1}(z^*)|^p=\frac{|D|^p}{(|B_1|^q+\cdots+|B_{r+1}|^q)^{p/q}}; \tag{3}$$

$$Z_j^*=\frac{A_{1j}|B_1|^{q-1}\dfrac{\overline{B}_1}{|B_1|}+\cdots+A_{r+1,j}|B_{r+1}|^{q-1}\dfrac{\overline{B}_{r+1}}{|B_{r+1}|}}{|B_1|^q+\cdots+|B_{r+1}|^q}+\frac{1}{D}\sum_{\nu=r+1}^{n}C_{j\nu}t_\nu, \tag{4}$$

where $j=1,\ldots,r$; $\nu=r+1,\ldots,n$; $C_{j\nu}=-\sum_{s=1}^{n}A_{sj}a_{s\nu}$; and t_ν is an arbitrary complex number.

Here, p and q are positive numbers such that

$$\frac{1}{p}+\frac{1}{q}=1.$$

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