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Control of Trajectory for Flying Round Obstacles by the Methods of Analytical Mechanics

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Abstract—Control of trajectory for flying round an obstacle is determined through optimization by the generalized work and a hierarchy of criteria. The methods of analytical mechanics are applied to form real-time controls under given constraints.

1. INTRODUCTION

We study the terminal positional trajectory control for flying round an obstacle in space. Solution by optimal control methods using classical quality criteria [1] yields an open-loop control. A positional control can be found on board in real time using the Krasovskii quality control and suitable prediction algorithms [1–4]. The solution of the first-order linear partial differential equation is determined from a Cauchy problem for the prediction model. According to [5, 6], the general solution for the equations of the model of space motion of a flying vehicle as a solid in connected axes (spiral prediction) [5] and a trajectory coordinate system can be found [6]. An algorithm for sequential optimization by two quality criteria with spiral prediction is designed in [7]. Since the variational principles and optimal control problems originated in classical mechanics as a result of mechanical harmony and stability in nature, the methods of classical mechanics [8] may prove helpful in optimal control [9–11 and others]. In particular, a terminal control for a flying vehicle moving in a longitudinal plane is found by the methods of analytical mechanics in [11].

This paper is the continuation the results of [11] on the optimal control of a flying vehicle in space for flying round obstacles by a hierarchy of two quality criteria with the use of the methods of analytical mechanics for solving the corresponding first-order partial differential equation.

2. FORMULATION OF THE PROBLEM

The motion of a flying vehicle as a material point in space can be described by the vector differential equation [3, 4]

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}, \mathbf{Y}, t), \quad \dot{\mathbf{Y}}_0 = \mathbf{U},\ \mathbf{x}(t_0) = \mathbf{x}_0, \tag{2.1}$$

where

$$\mathbf{X} = (v, \theta, \varphi, l, h, z)^{\mathrm{T}}, \quad \mathbf{Y} = (n_y, n_z)^{\mathrm{T}}, \quad \mathbf{Y}_0 = (\mathbf{Y}^{\mathrm{T}}, t_f)^{\mathrm{T}}, \quad \mathbf{x} = (\mathbf{X}^{\mathrm{T}}, \mathbf{Y}^{\mathrm{T}})^{\mathrm{T}},$$
$$f_1 = g(n_x - \sin\theta), \quad f_2 = \frac{g}{v}(n_y - \cos\theta), \quad f_3 = -\frac{g}{v}\frac{n_z}{\cos\theta}, \quad f_4 = v\cos\theta\cos\varphi,$$
$$f_5 = v\sin\theta, \quad f_6 = -v\cos\theta\sin\varphi, \quad f_7 = u_1, \quad f_8 = u_2, \quad f_9 = u_3,$$
$$n_x = -A_x/(mg), \quad A_x = c_x qS, \quad \alpha = n_y/n_y^{\alpha}, \quad \beta = n_z/n_z^{\beta},$$
$$n_y^{\alpha} = c_y^{\alpha} qS/(mg), \quad n_z^{\beta} = c_z^{\beta} qS/(mg), \quad q = \rho v^2/2;$$

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v, θ, φ, l, h, and z are the velocity, slope, and rotation of the trajectory, and coordinates of the center of mass of the vehicle in space, n_x, n_y, and n_z are the longitudinal, normal, and lateral components of acceleration, m is the mass, S is a given parameter, $c_x(v,h,\alpha,\beta)$, $c_y^{\alpha}(v,h)$, $c_z^{\beta}(v,h)$, $\rho(h)$, and $g(h)$ are given functions, and $\mathbf{U}$ is the control vector.

Our problem now is to move the vehicle to a given point by flying round an obstacle. The quality criterion is defined by the Krasovskii functional [1]

$$I = V_f(\mathbf{x}_f,\ t_f) + \int\limits_{t_0}^{t_f} \left(Q(\mathbf{x},t) + \frac{1}{2}\left(\mathbf{U}^{\mathrm{T}} k \mathbf{U} + \mathbf{U}_o^{\mathrm{T}} k \mathbf{U}_o\right)\right) dt, \tag{2.2}$$

where V_f and Q are given positive-definite functions, $k = \operatorname{diag}(k_1,k_2,k_3)$, $\mathbf{U} = (u_1,u_2,u_3)^{\mathrm{T}}$, $\mathbf{U}_o = (u_{o1},u_{o2},u_{o3})^{\mathrm{T}}$, and k_i, $i=\overline{1,3}$, are given coefficients.

For process (2.1), the optimal control by criteria (2.2) is $\mathbf{U} = -k\partial V/\partial \mathbf{Y}_0$, where V is the solution of the first-order linear partial differential equation (the Lyapunov equation) [1]

$$\frac{\partial V}{\partial t} + H_m(\mathbf{x},t) = -Q(\mathbf{x},t), \quad H_m(\mathbf{x},t) = \frac{\partial V}{\partial \mathbf{x}}\mathbf{f}(\mathbf{x},t) \tag{2.3}$$

under the boundary condition $V(\mathbf{x},t_f) = V_f(\mathbf{x},t_f)$. Here $H_m = H_{|\mathbf{U}=\mathbf{U}_o} = H_{|\mathbf{U}=0}$ and H is the Hamiltonian of the initial system.

Since Eq. (2.3) is linear, to optimize by criterion (2.2), we take $V = V_r + V_s$, $Q = Q_r + Q_s$, and

$$\frac{\partial V_r}{\partial t} + H_r(\mathbf{x},t) = -Q_r(\mathbf{x},t), \quad \frac{\partial V_s}{\partial t} + H_s(\mathbf{x},t) = -Q_s(\mathbf{x},t),$$
$$H_r(\mathbf{x},t) = \frac{\partial V_r}{\partial \mathbf{x}}\mathbf{f}(\mathbf{x},t), \quad H_s(\mathbf{x},t) = \frac{\partial V_s}{\partial \mathbf{x}}\mathbf{f}(\mathbf{x},t), \tag{2.4}$$

where Q_r is the integrand in the criterion defining the control quality and Q_s is a penalty function characterizing the constraints on the state vector. Then $\mathbf{U} = \mathbf{U}_r + \mathbf{U}_s$. Here there may be boundary conditions for the functions V_r and V_s.

Considerable computational resources are required for passing from the partial differential equation to the equations of the prediction model and subsequent numerical integration in every control cycle [1–3]. Equation (2.3) can be solved by the Jacobi method [8], which is helpful in finding the final equations of motion of the system with a given function H from the complete integral of Eq. (2.3). The complete integral of the Lyapunov equation can be found by the method of separation of variables [8]. Let us study canonical transformations of variables that preserve the equations of the prediction model for vectors $\mathbf{x}$ and conjugate variables $\mathbf{p}^{\mathrm{T}} = \partial V/\mathbf{x}$ in canonical form. To solve the problem, let us pass from the variables $\mathbf{x}$ and $\mathbf{p}$ to new variables $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ of the same dimension. Let us choose a transformation under which the coordinates ν_i, $i=\overline{1,n}$, are contained in the Hamiltonian of the system $H(\boldsymbol{\lambda},t)$. Then the canonical equations take the form

$$\dot{\boldsymbol{\lambda}}^{\mathrm{T}} = -\frac{\partial H}{\partial \boldsymbol{\nu}} = 0, \quad \dot{\boldsymbol{\nu}}^{\mathrm{T}} = \frac{\partial H}{\partial \boldsymbol{\lambda}} = \gamma, \quad \boldsymbol{\nu} = \boldsymbol{\gamma} t + \boldsymbol{\delta}, \quad \boldsymbol{\nu}^{\mathrm{T}} = \frac{\partial V}{\partial \boldsymbol{\lambda}},$$

where $\boldsymbol{\delta}$ is a vector of integration constants of appropriate dimension. Hence the function V on the optimal solution must satisfy Eq. (2.3) and relations $\mathbf{p}^{\mathrm{T}} = \partial V/\partial \mathbf{x}$ and $\boldsymbol{\nu}^{\mathrm{T}} = \partial V/\partial \boldsymbol{\lambda}$.

According to the method of separation of variables, we assume that

$$V(\mathbf{x},\boldsymbol{\lambda},t) = \sum_{i=1}^{n} V_i(x_i,\boldsymbol{\lambda}) + V_0(t). \tag{2.5}$$

Then $p_i = \partial V/\partial x_i = \partial V_i/\partial x_i$. Solving Eq. (2.3) for every derivative $p_i = \partial V/\partial x_i$, we obtain $p_i = p_i(x_i, p_j)$, $j = \overline{1,n}$, but we must obtain $p_i = \partial V_i/\partial x_i$. This contradiction is eliminated by equating certain combinations of remaining variables to constants: $\psi_i(\mathbf{x}, \partial V/\partial \mathbf{x}) = \lambda_i$, $i = \overline{1,n}$. If the last equations can be solved for $\partial V/\partial x_i$, then we obtain $p_i = p_i(x_i, \boldsymbol{\lambda})$. Here λ_i are arbitrary constants obtained in the course of separation. Integrating the equations for $p_i = \partial V_i/\partial x_i$, by virtue of (2.5), we obtain $V = V(\mathbf{x}, \boldsymbol{\lambda}, t)$. Now differentiating with respect to $\boldsymbol{\lambda}$, we find $\boldsymbol{\nu}^{\mathrm{T}} = \partial V/\partial \boldsymbol{\lambda}$. Solving these equations for $\mathbf{x}$, we obtain $x_i = x_i(\boldsymbol{\lambda}, \boldsymbol{\nu}, t)$, which define the trajectory of the model. Knowing this trajectory, we can find $\mathbf{p}$ and then the control.

Applying this procedure, we shall find a control for the trajectory of motion in flying round an obstacle.

3. A FLIGHT CONTROL ALGORITHM BASED ON CANONICAL TRANSFORMATIONS

In analogy with the case of longitudinal motion [11], let us consider the part of the function H_m with which Eq. (2.3) is integrated by the method of separation of variables

$$H^{(1)} = p_\theta \frac{g}{v}(n_y - \cos\theta) - p_\varphi \frac{g n_z}{v\cos\theta} + p_l v\cos\theta\cos\varphi + p_h v\sin\theta - p_z v\cos\theta\sin\varphi + p_r Q_s(l,h,z)$$

with regard for the supplementary variable r satisfying the equation $\dot r = Q_s(l,h,z)$. Let us introduce new variables, assuming that

$$\begin{gathered}
V^{(1)} = V_2(\theta) + V_3(\varphi) + V_4(l) + V_5(h) + V_6(z) + V_7(t) + V_8(r),\\
\frac{\partial V_4}{\partial l} = \lambda_4, \quad \frac{\partial V_5}{\partial h} = \lambda_5, \quad \frac{\partial V_6}{\partial z} = \lambda_6, \quad \frac{\partial V_8}{\partial r} = \lambda_8,\\
-\frac{\partial V_3}{\partial \varphi}\frac{g n_z}{v\cos\theta} - \lambda_6 v\cos\theta\sin\varphi + \lambda_4 v\cos\theta\cos\varphi + 0.5\lambda_8 Q_s(l,h,z) = \lambda_3,\\
\frac{\partial V_2}{\partial \theta}\frac{g}{v}(n_y - \cos\theta) + \lambda_5 v\sin\theta + \lambda_3 + 0.5\lambda_8 Q_s(l,h,z) = \lambda_2, \quad \frac{\partial V_7}{\partial t} + \lambda_2 = \lambda_7^{(1)}.
\end{gathered} \tag{3.1}$$

Integrating the last system with respect to suitable coordinates, we obtain an expression for $V^{(1)}(\theta,\varphi,l,h,z,t,\boldsymbol{\lambda})$, which yields the coordinates $\nu_i = \partial V/\partial \lambda_i$:

$$\nu_4 = l - l_0 + \frac{v^2\cos^2\theta}{g n_z}\int\limits_{\varphi_0}^{\varphi}\cos\varphi\, d\varphi,$$

$$\nu_5 = h - h_0 - \int\limits_{\theta_0}^{\theta}\frac{v\sin\theta}{\frac{g}{v}(n_y - \cos\theta)}\, d\theta,$$

$$\nu_6 = z - z_0 - \frac{v^2\cos^2\theta}{g n_z}\int\limits_{\varphi_0}^{\varphi}\sin\varphi\, d\varphi,$$

$$\nu_3 = -\frac{v\cos\theta}{g n_z}(\varphi - \varphi_0) - \int\limits_{\theta_0}^{\theta}\frac{d\theta}{\frac{g}{v}(n_y - \cos\theta)},$$

$$\nu_8 = r - r_0 - \frac{1}{2}\int\limits_{\theta_0}^{\theta}\frac{Q_s d\theta}{\frac{g}{v}(n_y - \cos\theta)} + \frac{1}{2}\frac{v\cos\theta}{g n_z}\int\limits_{\varphi_0}^{\varphi} Q_s\, d\varphi,$$

$$\nu_2 = -\int\limits_{t_0}^{t} dt + \int\limits_{\theta_0}^{\theta}\frac{d\theta}{\frac{g}{v}(n_y - \cos\theta)}, \quad \nu_7 = \int\limits_{t_0}^{t} dt.$$

Hence, using tabulated integrals [12], we determine the variables l, h, z, φ, and θ at instant t (formulas for h and θ are given in [11]):

$$l = l_0 - \frac{v^2 \cos^2\theta}{gn_z}(\sin\varphi - \sin\varphi_0), \quad z = z_0 - \frac{v^2\cos^2\theta}{gn_z}(\cos\varphi - \cos\varphi_0), \tag{3.2}$$
$$\varphi = \varphi_0 - \frac{t - t_0}{v\cos\theta} gn_z.$$

From (3.1) we obtain

$$p_\varphi = \frac{\partial V_3}{\partial\varphi} = -[\lambda_3 - v\cos\theta\cos(\varphi - \mu) - 0.5\lambda_8 Q_s(l,h,z)]\frac{R_\varphi}{v\cos\theta},$$
$$p_\theta = \frac{\partial V_2}{\partial\theta} = [\lambda_2 - \lambda_3 - \lambda_5 v\sin\theta - 0.5\lambda_8 Q_s(l,h,z)]\frac{R_\theta}{v},$$
$$R_\theta = \frac{v^2}{g(n_y - \cos\theta)}, \quad R_\varphi = -\frac{v^2\cos^2\varphi}{gn_z}. \tag{3.3}$$

Let us take

$$V_{f_r}(\mathbf{x}, t_f) = V_f(\mathbf{x}, t_f) + r(t_f)$$

and

$$V_f = 1/2\rho_1(l(t_f) - l_f)^2 + 1/2\rho_2(h(t_f) - h_f)^2 + 1/2\rho_3(z(t_f) - z_f)^2 + 1/2\rho_4(\theta(t_f) - \theta_f)^2 + 1/2\rho_5(\varphi(t_f) - \varphi_f)^2.$$

We shall find λ_4, λ_5, and λ_6 under the boundary conditions

$$p_l(t_f) = \frac{\partial V_f}{\partial l_f} = \rho_1(l(t_f) - l_f) = \lambda_4,$$
$$p_h(t_f) = \frac{\partial V_f}{\partial h_f} = \rho_2(h(t_f) - h_f) = \lambda_5,$$
$$p_z(t_f) = \frac{\partial V_f}{\partial z_f} = \rho_3(z(t_f) - z_f) = -\sin\mu = \lambda_6,$$
$$\mu = -\arctan(\lambda_6/\lambda_4).$$

Hence, by (3.3), we obtain

$$\lambda_3 = v\cos\theta(\cos(\varphi_f - \mu) + p_\varphi(t_f)/R_\varphi(t_f) + 0.5\lambda_8 Q_s(l,h,z)),$$
$$\lambda_2 = \lambda_3 + \lambda_5 v\sin\theta_f + vp_\theta(t_f)/R_\theta(t_f) + 0.5\lambda_8 Q_s(l,h,z),$$
$$p_\theta(t_f) = \rho_4(\theta(t_f) - \theta_f),$$
$$p_\varphi(t_f) = \rho_5(\varphi(t_f) - \varphi_f).$$

Here $l(t_f)$, $h(t_f)$, $z(t_f)$, $\theta(t_f)$, and $\varphi(t_f)$ are computed by formulas (3.2), and ρ_i, $i = \overline{1,5}$, are given coefficients.

For longitudinal motion, expression (3.1) contains $\lambda_3 = \lambda_4 v\cos\theta$ and we proceed as in [11]. The function $H^{(2)} = V_1(v) + V_7^{(2)}(t)$ is added to $H^{(1)}$ and variables are separated along the same lines as for the case of longitudinal motion [11].

Let us consider the function $H^{(3)} = (p_{n_y}, p_{n_z}, p_t)\mathbf{U} + 0.5\mathbf{U}^{\mathrm{T}}k\mathbf{U} + 0.5\mathbf{U}_o^{\mathrm{T}}k\mathbf{U}_o$, which upon addition to H_m yields the Hamiltonian of the initial system H. Minimizing H for $\mathbf{U}$, we obtain the control $\mathbf{U} = -k(p_{n_y}, p_{n_z}, p_t)^{\mathrm{T}}$. The values of p_{n_y}, p_{n_z}, and p_t are determined from the equations of the prediction model: $p_t = H_m(t_f)$,

$$p_{n_y} = \frac{\partial V}{\partial n_y} = \frac{v^2}{g} \int\limits_{\theta}^{\theta_f} \frac{\dfrac{\lambda_2 - \lambda_3}{v} - \lambda_5 \sin\theta - 0.5\lambda_8 Q_s}{(n_y - \cos\theta)^2} d\theta$$

$$= \frac{v^2}{g} \left[\frac{\lambda_2}{v} \int\limits_{\theta}^{\theta_f} \frac{d\theta}{(n_y - \cos\theta)^2} - \frac{1}{v} \int\limits_{\theta}^{\theta_f} \frac{\lambda_3 d\theta}{(n_y - \cos\theta)^2} - \lambda_5 \int\limits_{\theta}^{\theta_f} \frac{\sin\theta\, d\theta}{(n_y - \cos\theta)^2} - 0.5\lambda_8 \int\limits_{\theta}^{\theta_f} \frac{Q_s d\theta}{(n_y - \cos\theta)^2} \right],$$

$$p_{n_z} = \frac{\partial V}{\partial n_z} = \int\limits_{\varphi}^{\varphi_f} \frac{\dfrac{g}{v\cos\theta}[\lambda_3 + v\cos\theta\cos(\varphi - \mu) - 0.5\lambda_8 Q_s]}{\left(\dfrac{g}{v\cos\theta} n_z\right)^2} d\varphi$$

$$= -\frac{R\varphi}{n_z v \cos\theta} \left\{ \lambda_3(\varphi(t_f) - \varphi) - v\cos\theta[\sin(\varphi(t_f) - \mu) - \sin(\varphi - \mu)] - 0.5\lambda_8 \int\limits_{\varphi}^{\varphi_f} Q_s d\varphi \right\}.$$

For $\varphi = \text{const}$, we obtain $p_{n_z} = 0$. In this case, the problem degenerates, the projection of the trajectory of motion on the horizontal plane is transformed into a straight line $\lambda_3 = \lambda_4 v \cos\theta$, and p_{n_y} is defined by the expression

$$p_{n_y} = \frac{v^2}{g} \left\{ \frac{\lambda_2 - 0.5\lambda_8 Q_s}{v} \left[\left. \frac{-\sin\theta}{(1 - n_y^2)(n_y - \cos\theta)} \right|_{\theta}^{\theta_f} - \frac{n_y}{1 - n_y^2} \frac{g}{v}(t_f - t) \right] \right.$$

$$\left. -\lambda_4 \left[\left. \frac{-n_y \sin\theta}{(1 - n_y^2)(n_y - \cos\theta)} \right|_{\theta}^{\theta_f} - \frac{1}{1 - n_y^2} \frac{g}{v}(t_f - t) \right] + \lambda_5 \frac{1}{(n_y - \cos\theta)} \Big|_{\theta}^{\theta_f} \right\}.$$

Comparing this expression with that derived in [11], we can write it as the sum $p_{n_y} = p_{n_y}^r + p_{n_y}^s$, where $p_{n_y}^r$ defines the case with $Q_s = 0$, and $p_{n_y}^s$ is equal to the term differing from p_{n_y} by $p_{n_y}^r$. Then control is defined by $\mathbf{U} = \mathbf{U}_r + \mathbf{U}_s$, which agrees with (2.4).

For $p_{n_z} \neq 0$, the integral in p_{n_y} with λ_3 takes the form

$$\int\limits_{\theta}^{\theta_f} \frac{\lambda_3\, d\theta}{(n_y - \cos\theta)^2} = \int\limits_{\theta}^{\theta_f} \frac{v\cos\theta \left[\cos(\varphi_f - \mu) + \dfrac{p_\varphi(t_f)}{v^2 \cos\theta^2} g n_z \right]}{(n_y - \cos\theta)^2} d\theta$$

$$= v\cos(\varphi_f - \mu) \int\limits_{\theta}^{\theta_f} \frac{\cos\theta\, d\theta}{(n_y - \cos\theta)^2} - \frac{p_\varphi(t_f) g n_z}{v} \int\limits_{\theta}^{\theta_f} \frac{d\theta}{\cos\theta(n_y - \cos\theta)^2}$$

$$= v\cos(\varphi_f - \mu) \left[\left. \frac{n_y \sin\theta}{(n_y^2 - 1)(n_y - \cos\theta)} \right|_{\theta}^{\theta_f} + \frac{1}{n_y^2 - 1} \frac{g}{v}(t_f - t) \right] - \frac{p_\varphi(t_f) g n_z}{v} J,$$

where

$$J = \int\limits_{\theta}^{\theta_f} \frac{d\theta}{\cos\theta(n_y - \cos\theta)^2}.$$

Applying the substitution $e = \tan\dfrac{\theta}{2}$, we can reduce the integral J to the form

$$J = 2 \int \frac{(1 + e^2)^2\, de}{(1 + e^2)[n_y - 1 + (n_y + 1)e^2]^2}.$$

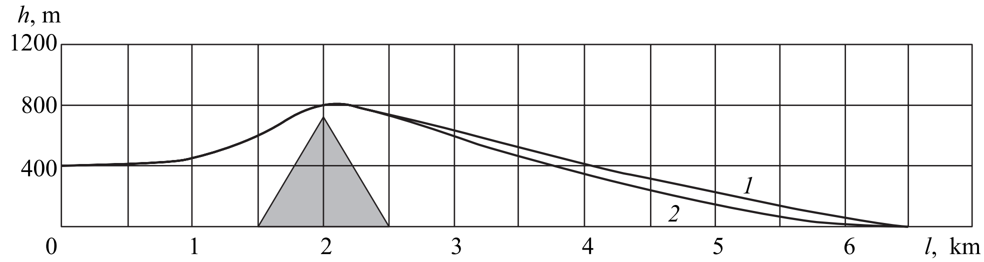

Figure.

Using tabulated integrals [12], for J we obtain

$$n_y = 1: \quad J = \frac{1}{2}\int \frac{(1+e^2)^2\, de}{e^4(1-e^2)} = -\frac{1}{6e^3} - \frac{3}{2e} + \ln\left|\frac{1+e}{1-e}\right|;$$

$$n_y = -1: \quad J = \frac{1}{2}\int \frac{(1+e^2)^2\, de}{1-e^2} = -\frac{e^3}{6} - \frac{3e}{2} + \ln\left|\frac{1+e}{1-e}\right|;$$

$$|n_y| < 1: \quad J = \frac{2}{(n_y+1)^2}\int \frac{(1+e^2)^2\, de}{(1-e^2)(e^2-b^2)^2}$$

$$= \frac{2}{(n_y+1)^2}\left[A_1 \ln\left|\frac{e+1}{e-1}\right| + B_1 \ln\left|\frac{e-b}{e+b}\right| - 2C_1\frac{e}{e^2-b^2}\right],$$

$$A_1 = \frac{2}{(1-b^2)^2}, \quad B_1 = \frac{4b^2-(1-b^2)^2}{4b^3(1-b^2)^2}(1+b^2), \quad C_1 = \frac{(1+b^2)^2}{4b^2(1-b^2)}, \quad b^2 = \left|\frac{n_y-1}{n_y+1}\right|;$$

$$|n_y| > 1: \quad J = \frac{2}{(n_y+1)^2}\int \frac{(1+e^2)^2\, de}{(1-e^2)(e^2+b^2)^2}$$

$$= \frac{2}{(n_y+1)^2}\left[A_2 \ln\left|\frac{e+1}{e-1}\right| + B_2\frac{1}{b}\tan^{-1}\frac{e}{b} + C_2\left(\frac{e}{2b^2(e^2+b^2)} + \frac{1}{2b^3}\tan^{-1}\frac{e}{b}\right)\right],$$

$$A_2 = \frac{2}{(1+b^2)^2}, \quad B_2 = \frac{4}{(1+b^2)^2} - 1, \quad C_2 = (1-b^2)(1+b^2), \quad b^2 = \left|\frac{n_y-1}{n_y+1}\right|.$$

The intersection points of the predicted trajectory with the obstacle are iteratively determined by the analytical expressions given above. The number of iterations required to determine these points depends on the choice of the first approximation, and varied from 2 to 14 for our problem.

Figure (curve *1*) shows the numerical solution of the landing control problem in flying round a typical obstacle in a longitudinal plane [3] for $\rho_1 = \rho_2 = 0.002$, $\rho_3 = \rho_4 = \rho_5 = 0$, $k_1 = 0.2$, and $k_3 = 0.0005$. Landing accuracy worsens if boundary conditions for θ and φ are taken into account.

For the angular characteristics of landing accuracy to be satisfied, it is better to apply the sequential optimization algorithm [4] using a hierarchy of criteria

$$I_1 = V_{f_1}(\mathbf{x}_f, t_f), \quad V_{f_1} = 1/2\rho_4(\theta(t_f)-\theta_f)^2 + 1/2\rho_5(\varphi(t_f)-\varphi_f)^2,$$

$$I_2 = V_{f_2}(\mathbf{x}_f, t_f) + 1/2\int_{t_0}^{t_f}(\mathbf{U}^{\mathrm{T}}k\mathbf{U} + \mathbf{U}_o^{\mathrm{T}}k\mathbf{U}_o)dt,$$

$$V_{f_2} = 1/2\rho_1(l(t_f)-l_f)^2 + 1/2\rho_2(h(t_f)-h_f)^2 + 1/2\rho_3(z(t_f)-z_f)^2.$$

In this case, control is formed in the form $\mathbf{U} = \mathbf{U}_1 + \mathbf{U}_2$, where $\mathbf{U}_1 = (\Delta Y_1, \Delta Y_2, 0)^{\mathrm{T}}\delta(t)$ and $\delta(t)$ is a delta function. The values of ΔY_1 and ΔY_2 are chosen through iterations using the condition $V_{f_1} = 0$. The second-level control $\mathbf{U}_2 = (u_1, u_2, u_3)^{\mathrm{T}}$ is determined for the values of Y_1 and Y_2

thus chosen. Since these values of parameters of the model ensure first-level terminal conditions, this model does not match the trajectory forecast with the current load. Therefore, this algorithm must be applied on segments with $Q_s = 0$. To solve the landing problem on the segment in flying round the obstacle, we must first apply control, using the generalized work criterion (2.2) and then apply control, using a hierarchy of two criteria. Computation results for $\rho_1 = \rho_2 = 0.005$, $\rho_3 = 0$, $k_1 = 0.2$, and $k_3 = 0.0005$ are shown in the figure (curve *2*). Clearly, this control algorithm ensures flight round an obstacle and high landing accuracy both in linear and angular coordinates.

4. CONCLUSIONS

Trajectory control for flying round an obstacle is studied. An algorithm is designed to form the desired trajectory from analytical expressions for the predicted linear and angular coordinates of the flying vehicle and control. Computations corroborate the high effectiveness of joint optimization by functional generalized work and classical methods of analytical mechanics.

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