RHEOLOGICAL MODELLING OF MEASURING SYSTEMS OF QUARTZ GRAVITY METERS

JIŘÍ TREŠL

*Geophysical Institute, Czechosl. Acad. Sci., Prague**)

Резюме: *С помощью модели вязкоупругого тела теоретически исследованы некоторые явления, регистрированые в течении испытаний давлением кварцевых гравиметров. Решением соответствующих реологических уравнений выведены выражения, описывающие реакцию измерительной системы в зависимости от временного хода возмущений давления.*

Summary: *Some phenomena, recorded during pressure tests of quartz gravity meters, are studied theoretically using a model of the visco-elastic continuum. Expressions, describing the response of the measuring system as a function of the time variation of pressure disturbances, are derived by solving the appropriate rheological equations.*

1. INTRODUCTION

In accurate gravity measurements, one has to consider the disturbing effect of various external factors (temperature, pressure, magnetic field, vibrations) and try to eliminate them. As a rule, a series of measurements is made at various values of the factor involved and the required dependence is derived from the results. Such experimental measurements have to be made individually for each gravity meter.

A number of authors [1—5] found that changes of atmospheric pressure affected the readings of quartz gravity meters. They observed the following effects:

i) a change in the reading on the gravity meter scale caused by a change in atmospheric pressure (the barometric effect proper);

ii) elastic after-effect observed for a certain period after the pressure disturbance had stabilized;

iii) visco-elastic hysteresis generated by cyclic pressure changes.

In describing these phenomena theoretically, it will be assumed that the gravity meter contains certain structural elements which are deformed by the pressure changes and cause the reading on the scale to change (e.g. membranes). The behaviour of these systems will be described by means of a rheological model.

2. BASIC RHEOLOGICAL MODELS

The general solution of relations between stresses, strains and their time variations is the subject of rheology [6, 7]. The properties of basic rheological models will be mentioned briefly.

Hooke's elastic body (H) is the basic model of the classical theory of elasticity and expresses the law of proportionality between stress $s(t)$ and strain $u(t)$:

*) Address: Boční II, 141 31 Praha 4 - Spořilov.

(1) $$u = (2\mu)^{-1}\, s\,, \quad \mu = \text{const.}$$

Here μ is the modulus of elasticity. If $\mu \to \infty$, Hooke's elastic body becomes Euclid's absolutely rigid body, which is the basic model of classical mechanics. The ideal spring is the mechanical analogy of Hooke's body.

Newton's viscous fluid (N) is the basic model of hydrodynamics. It characterizes substances whose rate of strain is proportional to the stress:

(2) $$\dot{u} = \mathrm{d}u/\mathrm{d}t = (2\eta)^{-1}\, s, \quad \eta = \text{const.}$$

Here η is the viscosity coefficient. The mechanical analogy is a piston with holes moving in a cylinder filled with fluid.

Kelvin's continuum ($\mathrm{K} = \mathrm{H} \mid \mathrm{N}$) is created by arranging elements H and N parallel (Fig. 1). The resultant stress is the sum of the stresses in both branches:

(3) $$2\eta\dot{u} + 2\mu u = s\,.$$

Assume that stress $s(t)$ is equal to the constant value s in the time interval $\langle 0, T\rangle$ and to zero outside this interval. We put

(4) $$s = s_0[h(t) - h(t - T)]\,,$$

where $h(t)$ is Heaviside's unit function. Laplace's transformation is used to solve Eq. (3):

(5) $$F(p) = \int_0^\infty f(t) \exp(-pt)\, \mathrm{d}t\,.$$

The image of Eq. (3) for $s(t)$ given by Eq. (4) under zero initial conditions will read

(6) $$2p(\mu + \eta p)\, U(p) = s_0[1 - \exp(-pT)]\,.$$

The inverse transformation yields the time variation of the strain:

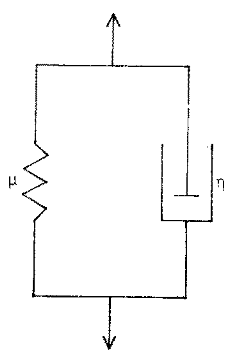

Fig. 1.

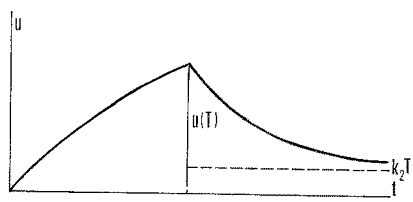

Fig. 2.

(7) $$u(t) = u_1[1 - \exp(-t/\tau)] \qquad \text{for} \quad 0 \leqq t \leqq T\,,$$

$$u(t) = u(T) \exp\left[-(t - T)/\tau\right] \quad \text{for} \quad t > T,$$
$$u_1 = s_0/2\mu .$$

Here we introduced $\tau = \eta/\mu$. The time variation of the strain for Kelvin's continuum is shown in Fig. 2. After stressing, the strain increases exponentially and it would reach the limiting value of elastic strain u_1 for $t \to \infty$. Once the stress is released at time $t = T$, the strain decreases exponentially with time and reaches the value $u(T)\exp(-1)$ at $t = T + \tau$. The time interval τ represents the period of the elastic after-effect. For $\eta = 0$, $\tau = 0$, $K \to$ H, and for $\mu = 0$, $\tau \to \infty$, K $\to$ N.

Maxwell's continuum (M = H$-$N) is created by arranging elements H and N in series (Fig. 3). In this arrangement the rates of strain add up:

(8) $$\dot{u} = (2\mu)^{-1}\,\dot{s} + (2\eta)^{-1}\,s .$$

If the stress is again assumed to vary as in (4)), Laplace's image of Eq. (3) under zero initial conditions will read

(9) $$p^2\,U(p) = s_0[(\tfrac{1}{2}p/\mu) + (\tfrac{1}{2}/\eta)]\,[1 - \exp(-pT)] .$$

The inverse transformation then yields

(10) $$u(t) = u_1 + k_1 t \quad \text{for} \quad 0 < t \leqq T,$$
$$u(t) = k_1 T \qquad \text{for} \quad t > T, \quad k_1 = \tfrac{1}{2}s_0/\eta .$$

The time variation of the strain is shown in Fig. 4. After stressing, the strain increases linearly with time (constant-velocity flow). Once the stress is released at time $t = T$, the elastic component of the strain vanishes, but the constant component $k_1 T$ remains.

Fig. 3.

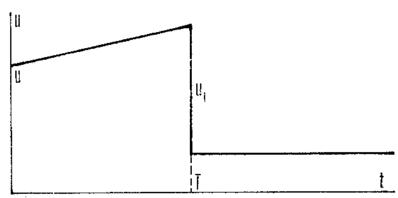

Fig. 4.

Maxwell's continuum thus behaves like a viscous fluid. For $\eta \to \infty$, M $\to$ H, and for $\mu \to \infty$, M $\to$ N.

3. JEFFREY'S CONTINUUM

The above deliberations indicate that the elastic after-effect can be described by the model of Kelvin's continuum and slow flow by the model of Maxwell's continuum.

Since both phenomena were observed simultaneously in the pressure tests of gravity meters, a combinations of elements N, M and K must be used for a complete description. In principle, the following immediately higher combinations can be created:

$$\text{(11)} \qquad \mathrm{J} = \mathrm{N} \mid \mathrm{M} = \mathrm{N} \mid (\mathrm{H} - \mathrm{N}) \quad \text{(Jeffrey's continuum)},$$

$$\text{(12)} \qquad \mathrm{L} = \mathrm{N} - \mathrm{K} = \mathrm{N} - (\mathrm{H} \mid \mathrm{N}) \quad \text{(Lethersich's continuum)}.$$

It can be proved that both models are equivalent with regard to mathematical description [6]. However, from the physical point of view it is more correct to use Jeffrey's continuum in which stresses are transferred from the solid to the liquid phase. The basic equation for Jeffrey's continuum reads

$$\text{(13)} \qquad \ddot{u} + (\tau)^{-1}\,\dot{u} = C_1\dot{s} + C_2 s\,.$$

The constants τ, C_1, C_2 are defined as follows:

$$\text{(14)} \qquad \tau = \eta_N\eta_M/[\mu(\eta_N + \eta_M)]\,, \quad C_1 = (2\eta_N)^{-1}\,, \quad C_2 = \mu(2\eta_N\eta_M)^{-1}\,.$$

The meaning of the individual symbols can be seen in Fig. 5.

Laplace's transformation of Eq. (13) for the stress in the form of (4) and for zero initial conditions yields:

$$\text{(15)} \qquad p^2(p + \tau^{-1})\,U(p) = s_0(p\,C_1 + C_2)\,[1 - \exp(-pT)]\,.$$

After inverse transformation,

$$\text{(16)} \qquad \begin{aligned} u(t) &= u_1(t) + k_2 t && \text{for} \quad 0 \leqq t \leqq T\,, \\ u(t) &= u_1(T)\exp[-(t - T)/\tau] + k_2 T && \text{for} \quad t > T\,, \end{aligned}$$

$$\text{(17)} \qquad u_1(t) = \frac{s_0\eta_M^2}{2\mu(\eta_N + \eta_M)^2}\,[1 - \exp(-t/\tau)]\,, \quad k_2 = s_0/[2(\eta_N + \eta_M)\,.$$

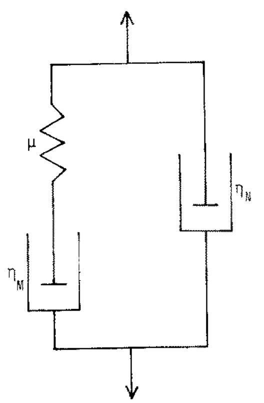

Fig. 5.

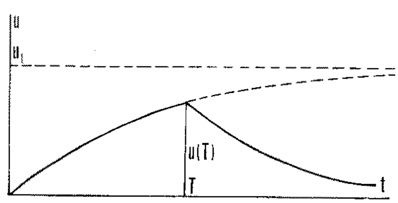

Fig. 6.

The time variation of the strain is shown in Fig. 6.

We shall now investigate two practically important cases of strain of Jeffrey's continuum due to cyclically acting strain. Let us assume that the stress varies with time as in Fig. 7. The appropriate Laplace's image is obtained by using the translation theorem in the following form:

$$S(p) = [s_0/(Tp^2)]\, Z(p)\,, \tag{18}$$

$$Z(p) = 1 - \exp(-Tp) - \exp[-(T + k\tau)\,p] + \exp[-(2T + k\tau)\,p] \pm$$
$$\pm \exp[-(2T + 2k\tau)\,p] \mp \exp[-(3T + 2k\tau)\,p] \mp \exp[-(3T + 3k\tau)\,p] \pm$$
$$\pm \exp[-(4T + 3k\tau)\,p]\,.$$

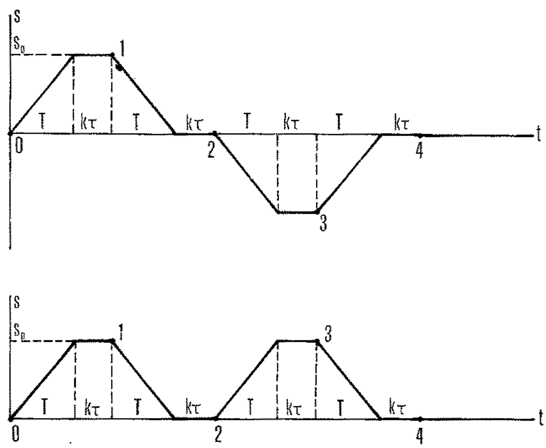

Fig. 7.

In the formula for $Z(p)$ the upper signs refers to the stress in Fig. 7a and the lower to the stress in Fig. 7b. Laplace's image of Eq. (13) will now read

$$p^3(p + \tau^{-1})\, U(p) = (s_0/T)(p\, C_1 + C_1)\, Z(p)\,. \tag{19}$$

Resolving into partial fractions and inverse transformation yield

$$u(t) = u_0(t)\, h(t) - u_c(t - T)\, h(t - T) - u_0(t - T - k\tau)\, h(t - T - k\tau) + \tag{20}$$
$$+ u_0(t - 2T - k\tau)\, h(t - 2T - k\tau) \pm u_0(t - 2T - 2k\tau)\, h(t - 2T - 2k\tau) \mp$$
$$\mp u_0(t - 3T - 2k\tau)\, h(t - 3T - 2k\tau) \mp u_0(t - 3T - 3k\tau)\, h(t - 3T - 3k\tau) \pm$$
$$\pm u_0(t - 4T - 3k\tau)\, h(t - 4T - 3k\tau)\,,$$

where $h(t)$ is Heaviside's unit function, and function $u_0(t)$ can be expressed as

$$u_0(t) = A' s_0(t/T) + B' s_0(t^2/T) - A' s_0(\tau/T)\,[1 - \exp(-t/\tau)]\,, \tag{21}$$

$$A' = \eta_M^2/[2\mu(\eta_N + \eta_M)^2]\,, \quad B' = [4(\eta_N + \eta_M)]^{-1}\,. \tag{22}$$

The first term on the r.h.s. of Eq. (21) describes the elastic strain, the second term slow viscous flow and the third term the elastic after-effect. Constants A' and B' depend only on the mechanical properties of the gravity meter. The stress s_0 is caused by a change in barometric pressure, ΔP, and the corresponding strain u is reflected in the change of the reading on the gravity meters scale n. Assuming small disturbances, we may put $s_0 = k_1 \, \Delta P$, $u = k_2 n$, where k_1 and k_2 are constants. Assuming pressure variations as in Fig. 7a and $T > \tau$, it holds that

$$(23) \quad \begin{aligned} & n_0 = n(0) = 0\,, \quad n_1 = n(T + k\tau) = A\,\Delta P + B\,\Delta P(T + 2k\tau) - C\,, \\ & n_2 = n(2T + 2k\tau) = 2B\,\Delta P(T + k\tau) + C\,, \\ & n_3 = n(3T + 3k\tau) = -A\,\Delta P + B\,\Delta PT + C\,, \quad n_4 = n(4T + 4k\tau) = -C\,. \end{aligned}$$

$$(24) \quad \begin{gathered} C = A'k_1\,\Delta P(\tau/T)\exp(-k)\left[1 - \exp(-T/\tau)\right]/k_2\,, \\ A = A'k_1/k_2\,, \quad B = B'k_1/k_2\,. \end{gathered}$$

Constants A and B can be determined using these relations from experimental measurements in a pressure chamber. Constant C (the residual elastic after-effect value) will become practically negligible after a sufficiently long relaxation period ($k \geqq 3$).

As regards the stress shown in Fig. 7b, it holds that

$$(25) \quad \begin{aligned} & n_0 = n(0) = 0\,, \quad n_1 = n(T + k\tau) = A\,\Delta P + B\,\Delta P(T + 2k\tau) - C\,, \\ & n_2 = n(2T + 2k\tau) = 2B\,\Delta P(T + k\tau) + C\,, \\ & n_3 = n(3T + 3k\tau) = A\,\Delta P + B\,\Delta P(3T + 4k\tau) - C\,, \\ & n_4 = n(4T + 4k\tau) = 4B\,\Delta P(T + k\tau) + C\,. \end{aligned}$$

This system of equations describes the variation of the individual strain components in measuring the gravity difference between points P and Q using the pattern $PQPQP$. If the constants A and B have been determined in advance from pressure chamber measurements, the appropriate corrections may be introduced.

4. CONCLUSION

The purpose of this paper was the theoretical study of some of the phenomena recorded during pressure tests of gravity meters. It was proved that these effects can be described by the rheological model of Jeffrey's visco-elastic continuum.

The elastic after-effect decreases exponentially with time and can be eliminated practically during measurements if a particular procedure is used. The visco-elastic hysteresis is proportional to the integral of the pressure change with time. During field measurements, it is manifest by continuous time variations of the readings of the gravity meter scale and it can be included in the instrument drift. The barometric effect proper is contained in the results of the measurements in full and may become the source of systematic errors. It must be eliminated from precise gravity measurements by introducing a calculation correction which must be determined individually for each gravity meter by means of pressure chamber measurements.

Received 23. 5. 1984 *Reviewer: V. Tobyáš*

References

[1] C. Gantar, C. Morelli: New Experimental Data about Temperature and Pressure Effects on Worden Gravity-Meters. Boll. Geof. teor. ed appl., 19 (1963), 174.
[2] L. Gargalovič et al.: Influence of Atmospheric Pressure Variations on Measurements Made with Sharpe and Worden Gravimeters. Studia geoph. et geod., 22 (1978), 93.
[3] L. Kubáčková et al.: Application of the Restricted *D*-Optimal Design to Gravimetry. Studia geoph. et geod., 23 (1979), 10.
[4] G. Csapó: The Effect of Barometric Changes on Readings of Sharpe and Worden Gravimeters. Investigation of the Effects of External Factors upon Gravimeters, Edice VÚGTK, ř. 4, Praha 1979.
[5] K. Diviš, V. Höschl, S. Hrách: Some Pressure Test Phenomena of Worden and CG-2 (Sharpe) Gravimeters. Studia geoph. et geod., 25 (1981), 14.
[6] M. Рейнер: Реология. Наука, Москва 1965.
[7] П. Мельхиор: Физика и динамика планет (ч. 2). Мир, Москва 1976.