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Microwave and infrared spectra of CO-(pH₂)₂, CO-(oD₂)₂, and mixed CO-pH₂-He trimers

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Abstract The microwave and infrared spectra of CO-(pH₂)₂, CO-(oD₂)₂, and CO-pH₂-He trimers are predicted by performing exact bound state calculations on the global potential energy surfaces defined as the sum of accurately known two-body pH₂-CO or oD₂-CO (in Li et al. J Chem Phys 139:164315, 2013), pH₂-pH₂ or oD₂-oD₂ (in Patkowski et al. J Chem Phys 129:094304, 2008), and pH₂-He pair potentials. A total of four transitions have been reported to date, three in the infrared region, and one in the microwave region, which are in good agreement with our theoretical predictions. Based on selection rules, new transitions for $J \leq 3$ have been predicted, and the corresponding transition intensities at different temperatures are also calculated. These predictions will serve as a guide for new experiments. The weak and tentatively assigned transitions are verified by our calculations. Three-body effects and the quality of the potential are discussed. A reduced-dimension

treatment of the pH₂ or oD₂ rotation has been employed by applying the hindered-rotor averaging technique of Li et al. (J Chem Phys 133:104305, 2010). A technique for displaying the three-dimensional pH₂ or oD₂ density in the body-fixed frame is used and shows that in the ground state, the two pH₂ or two oD₂ molecules are localized, while the He's are delocalized.

Keywords Microwave spectra · Infrared spectra · CO-(pH₂)₂ trimers · CO-(oD₂)₂ trimers · CO-pH₂-He trimers · Reduced-dimension treatment · Exact bound state calculations · Three-body effects

1 Introduction

At a very low temperature (<2 K), high-resolution infrared or microwave spectra of a single chromophore molecule doped in helium droplets or clusters have provided a unique opportunity to study quantum solvation and microscopic superfluidity [1–20]. Para-hydrogen (pH₂) or ortho-deuterium (oD₂), like ⁴He atoms, are bosons. Similarly, a chromophore molecule embedded in pH₂ or oD₂ clusters can also be considered as a possible route to investigate quantum solvation and the superfluidity of pH₂ or oD₂ [21–30]. Recently, combining experimental measurements and theoretical simulations, superfluid response for pH₂ to dopant rotation has been first elucidated in CO₂-(pH₂)_N clusters [26]. However, due to localization effects, the superfluid fraction of larger doped hydrogen clusters becomes suppressed. Carbon monoxide (CO) is a gentler probe molecule with much weaker and less anisotropic interaction with pH₂ and a relatively large rotational constant. These features lead to a delocalized distribution of pH₂ molecules with respect to CO. Persistent molecular

Dedicated to Professor Guosen Yan and published as part of the special collection of articles celebrating his 85th birthday.

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superfluid response to dopant rotation in $\text{CO-(pH}_2)_N$ clusters has been observed, and this conclusion is supported by both microwave spectroscopy and the theoretical analysis [31].

An understanding of the experimental spectra at the microscopic scale requires complementary computer simulation studies based on accurate intermolecular potential energy functions. Very recently, two reliable ab initio analytical potential energy surfaces (PESs) for $\text{H}_2\text{-CO}$ dimer have been reported, referred to hereafter V_{12} [32, 33] and V_{MLRQ} [34], respectively. Both PESs explicitly account for the intra-molecular C–O stretch coordinate and allow to describe the infrared spectrum in the region of the fundamental stretching vibration of C–O around $2,143.2712\text{ cm}^{-1}$. The predicted infrared spectra calculated from these two PESs have already been shown to agree extremely well with the experimental spectra of the $\text{pH}_2\text{-}$ and $\text{oH}_2\text{-CO}$ dimers [32–34]. Studies of $\text{CO-(pH}_2)_2$ trimer are important for testing the accuracy of pair-wise additive approach by sum of two-body potentials to simulate systems with tens of pH_2 molecules, looking for three-body effects, and for testing the validity of quantum Monte Carlo methods for molecular clusters, since comparisons with exact quantum results are possible.

Although a large number of infrared and microwave transitions were recorded and assigned for CO-He and CO-pH_2 dimers [33, 35–37], only a few transitions for $\text{CO-(pH}_2)_2$, $\text{CO-(oD}_2)_2$, and $\text{CO-pH}_2\text{-He}$ trimers were reported, and some weak or tentatively assigned transitions remain to be verified or reassigned [22, 31]. Therefore, reliable theoretical predictions based on exact quantum calculations will provide valuable guidance in confirming or reassigning the measured infrared spectra. Previous work on exact quantum calculations of infrared and microwave spectra for trimer complexes containing one CO, CO_2 , N_2O , and OCS molecule attached to two He atoms have been reported, by Wang and Carrington [38–41] and by Li et al. [42], which yield excellent agreement with experiment

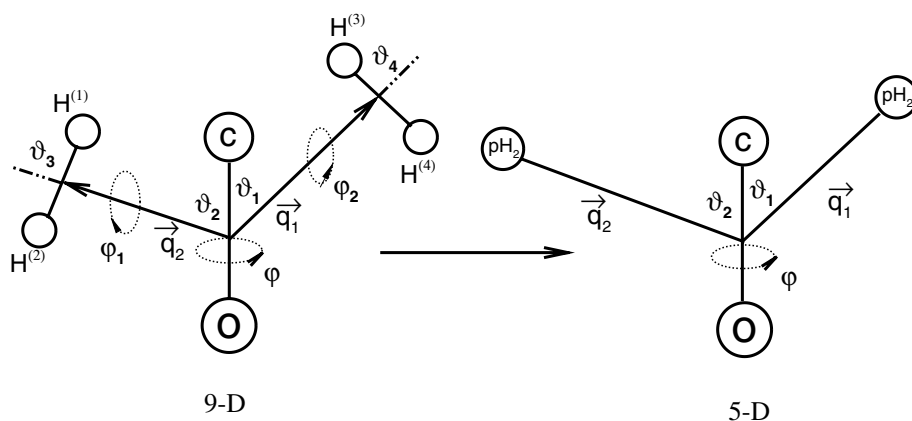
observations. Exact rovibrational spectra calculations for the trimer containing a single chromophore molecule doped with two pH_2 molecules, such as $\text{CO}_2\text{-(pH}_2)_2$, were first reported by Li et al. [27] by applying the hindered-rotor averaging technique to treat pH_2 as a point-like particle and consequently reduce the dimension of the potential energy surface. To date, however, exact quantum calculations for the trimer containing a chromophore molecule doped with two oD_2 or with mixed pH_2 and He have not been reported. Further more, exact quantum calculations for $\text{CO-(pH}_2)_N$ clusters have only been limited to CO-pH_2 dimer. In the present paper, we extend such exact quantum calculations to the cases of two pH_2 , or two oD_2 , or mixed pH_2 and He, attached to one CO dopant molecule. The results are used to help assign or confirm the experimental spectra of $\text{CO-(pH}_2)_2$, $\text{CO-(oD}_2)_2$, and $\text{CO-pH}_2\text{-He}$.

2 Computational methods

2.1 Geometry and reduced-dimension treatment

There are six atoms in $\text{CO-(pH}_2)_2$ system, and it takes a 12-dimensional (12D) PES to describe the interaction within the system. If we treat linear CO and the two H_2 as rigid molecules, taking only the intermolecular coordinates into consideration, there are still nine intermolecular degrees of freedom. The geometry of this complex in which CO is rigidly linear, and can be described naturally using the Jacobi coordinates ($q_1, q_2, \vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \varphi, \varphi_1, \varphi_2$) is shown on the left panel of Fig. 1, where q_1 and q_2 are the distances from the center of mass of CO to the centers of mass of the two H_2 molecules, ϑ_1 and ϑ_2 are the angles between \mathbf{q}_1 or \mathbf{q}_2 and a vector pointing from atom O to atom C, ϑ_3 and ϑ_4 are the angles between \mathbf{q}_1 and a vector pointing from atom H_2 to atom H_1 , and from \mathbf{q}_2 to a vector pointing from atom H_4 to atom H_3 , φ is the dihedral angle between two planes defined by vector \mathbf{q}_1 and \mathbf{q}_2 with the

Fig. 1 Jacobi coordinates for the $\text{CO-(H}_2)_2$ complex: left side, the geometry describe with nine dimensions of intermolecular coordinates; right side, the geometry described with five dimensions of intermolecular coordinates and with the two para-hydrogen treated as spherical particles



axis of the CO molecule, φ_1 and φ_2 are the azimuthal angles for rotation of $\text{H}_1\text{--H}_2$ and $\text{H}_3\text{--H}_4$ about vectors \mathbf{q}_1 or \mathbf{q}_2 , respectively.

It is still difficult to solve the nine-dimensional intermolecular Schrodinger equation. However, because the nuclear spins are anti-parallel for para-hydrogen, the rotational angular momentum, J , must be even. We know that the ground-state rotational wave function of an isolated ground-state ($J = 0$) H_2 molecule is precisely spherically symmetric, and the energy spacing to its first-excited ($J = 2$) level is relatively large (354.3735 cm^{-1}). It is therefore desirable to treat pH_2 as a spherical particle. Following this approximation, the nine-dimensional intermolecular coordinates for a $\text{CO}\text{--}(\text{pH}_2)_2$ complex can be reduced to five coordinates ($q_1, q_2, \vartheta_1, \vartheta_2, \varphi$) shown on the right panel of Fig. 1, greatly reducing the computational effort. However, it should be noted that pH_2 is not truly spherical when interacting with other species. A simple spherical average over the orientations of the H_2 moiety does not provide an accurate description [43].

The rotational constant for pH_2 (59.322 cm^{-1}) is more than 31 times larger than that of CO in its ground (1.9225288 cm^{-1}) or excited state (1.9050256 cm^{-1}) [44]. It therefore seems reasonable to perform a Born–Oppenheimer-type separation of the fast rotational motion of pH_2 from the relative slower CO rotation. Recently, Li, Roy, and Le Roy introduced a much better reduced-dimension description which they call an “adiabatic-hindered-rotor (AHR)” method. This approximation yields an effective $\text{pH}_2\text{--}\{\text{linear molecule}\}$ interactions that are an order of magnitude more accurate than those obtained using a simple spherical-average approximation. We expect that with such an AHR approximation, the $\text{CO}\text{--}(\text{pH}_2)_2$ trimer can be accurately described with the left over five degrees of freedom shown in the right panel of Fig. 1. Based on our previous tests for $\text{CO}\text{--oD}_2$ dimer, as shown in Table 2 of Ref. [43], we found the AHR treatment of oD_2 is still feasible and reliable, and the differences between $\{2\text{D--}4\text{D}\}$ are smaller than 0.092 cm^{-1} for vibrational levels, which are about six times better than those obtained by spherical treatment. Therefore, similar treatment has been applied for the other two trimers of $\text{CO}\text{--}(\text{oD}_2)_2$ and $\text{CO}\text{--pH}_2\text{--He}$.

The rovibrational energy levels of $\text{CO}\text{--}(\text{pH}_2)_2$, $\text{CO}\text{--}(\text{oD}_2)_2$, and $\text{CO}\text{--pH}_2\text{--He}$ were calculated using the radial discrete variable representation (DVR) and parity-adapted angular finite basis representation (FBR) methods described in Section II. The potential we used for each trimer is a sum of three pair potentials. The $\text{CO}\text{--pH}_2$, $\text{CO}\text{--oD}_2$, and $\text{oD}_2\text{--oD}_2$ potentials were generated by an “adiabatic-hindered-rotor” average treatment of the H_2 or D_2 rotation using dimer potentials taken from Refs. [32, 34, 45], while $\text{pH}_2\text{--pH}_2$ potentials were generated by a “spherical” average treatment of the H_2 rotation using dimer potential taken

from Ref. [45]. For $\text{CO}\text{--pH}_2\text{--He}$ and $\text{CO}\text{--}(\text{He})_2$ complexes, the $\text{CO}\text{--pH}_2$, $\text{CO}\text{--He}$, and $\text{He}\text{--He}$ potentials of Refs. [34, 46, 47] were used. The rotational constants B_{CO} required for these calculations were fixed at the experimental values of 1.9225288 and 1.9050256 cm^{-1} for CO in their ground ($v_3 = 0$) and excited ($v_3 = 1$) states [44], respectively. In our calculations, the masses were set at 1.00782503207 u for H, 2.0141017778 u for D, 4.00260324 u for ^4He , 15.994914635 u for ^{16}O , and 12 u for ^{12}C . An 40-point sine DVR grid range from 3.0 to 30.0 bohr was used for the radial r_1 or r_2 stretching coordinate. For the angular basis, we use $l_{\text{max}} = m_{\text{max}} = 25$; 30 Gauss–Legendre quadrature points were used for the integration over θ_1 or θ_2 , while 64 equally spaced points in the range $[0, 2\pi]$ were used for the integration over ϕ . It is useful to apply a potential ceiling value to reduce the spectral range and accelerate the convergence of the Lanczos calculation [48]. In our calculations, we use a ceiling of $1,000\text{ cm}^{-1}$ and have confirmed that low-lying energy levels change by less than 0.001 cm^{-1} when the ceiling is raised to $10,000\text{ cm}^{-1}$.

2.2 Hamiltonian and basis functions

Following our previous work for $\text{CO}_2\text{--}(\text{pH}_2)_2$ trimer [27], in Radau coordinate, we use the Hamiltonian and basis functions based mainly on the work of Wang et al. [39] and on fundamental ideas presented by Mladenovic [49]. In recent years, this approach have been successfully used to treat $\text{N}_2\text{O}\text{--}(\text{He})_2$, $\text{CO}\text{--}(\text{He})_2$, $\text{CO}_2\text{--}(\text{He})_2$, $\text{OCS}\text{--}(\text{He})_2$, and $\text{CO}_2\text{--}(\text{pH}_2)_2$ complexes. The method is briefly reviewed here, and the reader is referred to Refs. [39] and [27] for further details. With the AHR approximation, the rovibrational Hamiltonian of the $\text{CO}\text{--}(\text{pH}_2)_2$ complex in the CO molecule-fixed frame has the form (in a.u.) [39, 49–51],

$$\hat{H} = \hat{T}_{\text{str}} + \hat{T}_{\text{diag}} + \hat{T}_{\text{off}} + \hat{T}_{\text{Cor}} + \bar{V}(r_1, r_2, \theta_1, \theta_2, \phi) \quad (1)$$

in which

$$\hat{T}_{\text{str}} = -\frac{1}{2m_{\text{H}_2}} \frac{\partial^2}{\partial r_1^2} - \frac{1}{2m_{\text{H}_2}} \frac{\partial^2}{\partial r_2^2} \quad (2)$$

$$\begin{aligned} \hat{T}_{\text{diag}} = & -\left(\frac{1}{2m_{\text{H}_2}r_1^2} + B_{\text{CO}}\right) \left[\frac{\partial^2}{\partial \theta_1^2} + \cot \theta_1 \frac{\partial}{\partial \theta_1} - \frac{1}{\sin^2 \theta_1} (\hat{J}_z - \hat{l}_{2z})^2 \right] \\ & + \left(\frac{1}{2m_{\text{H}_2}r_2^2} + B_{\text{CO}}\right) \hat{l}_2^2 + B_{\text{CO}} [\hat{J}^2 - 2(\hat{J}_z - \hat{l}_{2z})^2 - 2\hat{J}_z \hat{l}_{2z}] \end{aligned} \quad (3)$$

$$\hat{T}_{\text{off}} = B_{\text{CO}} [\hat{l}_{2+} \hat{a}_1^- + \hat{l}_{2-} \hat{a}_1^+] \quad (4)$$

$$\hat{T}_{\text{Cor}} = -B_{\text{CO}} [\hat{J}_- \hat{a}_1^+ + \hat{J}_+ \hat{a}_1^- + \hat{J}_- \hat{l}_{2+} + \hat{J}_+ \hat{l}_{2-}] \quad (5)$$

where

$$\hat{J}_{\pm} = \hat{J}_x \pm i\hat{J}_y, \quad \hat{l}_{2\pm} = \hat{l}_{2x} \pm i\hat{l}_{2y} \quad (6)$$

$$\hat{a}_1^{\pm} = \pm \frac{\partial}{\partial \theta_1} - \cot \theta_1 (\hat{J}_z - \hat{l}_{2z}) \quad (7)$$

The details of the coordinate system are shown in Fig. 2. r_1 and r_2 are the lengths of Radau (or orthogonalized satellite) vectors \mathbf{r}_1 and \mathbf{r}_2 , which are linear combinations of the Jacobi vectors \mathbf{q}_1 and \mathbf{q}_2 running from the center of mass of CO to the centers of the pH_2 molecules [49]. The polyspherical angles $(\theta_1, \theta_2, \phi)$ are determined by the three vectors $(\mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2)$. θ_1 is the angle between the dopant axis \mathbf{r}_0 and \mathbf{r}_1 , θ_2 and ϕ are the polar angles that specify the orientation of \mathbf{r}_2 with respect to the CO molecule-fixed frame. m_{H_2} is the mass of the pH_2 molecule and B_{CO} is the rotation constant of CO. The operators \mathbf{J}_x , \mathbf{J}_y , and \mathbf{J}_z are the components of the total angular momentum operator \mathbf{J} in the body-fixed frame, the z axis of the body-fixed frame lies along the Jacobi radial vector \mathbf{r}_0 , and the x axis is in the plane that contains \mathbf{r}_0 and one pH_2 molecule. Here, $\bar{V}(r_1, r_2, \theta_1, \theta_2, \phi)$ is the total potential energy function which is represented as a sum of two two-dimensional AHR pH_2 -CO potentials [34, 43] plus the one-dimensional AHR pH_2 - pH_2 intermolecular potential [45]. The above Hamiltonian contains full vibration-rotation coupling.

A sine DVR grid [52] is used for the r_1 and r_2 degrees of freedom, while parity-adapted rovibrational basis functions are used for the angular part. A complete product basis function is

$$u_{l_1, l_2, m_2, K}^{J, M, P} = \frac{1}{\sqrt{2(1 + \delta_{m_2, 0} \delta_{K, 0})}} [|l_1, l_2, m_2, K; J, M\rangle + (-1)^{J+P} |l_1, l_2, -m_2, -K; J, M\rangle] \quad (8)$$

where $K > 0$, and $P = 0$ and 1 correspond to even and odd parities, respectively. If $K = 0$, then it is necessary to apply the constraint $m_2 \geq 0$ to get rid of the redundant basis. The combination $m_2 = K = 0$ and $(-1)^{J+P} = -1$ is not allowed. The parity-adapted basis makes it possible to calculate even and odd parity levels separately. Within each parity block, we use the symmetry-adapted Lanczos algorithm (SAL) [53, 54] to compute states that are symmetric (A) and antisymmetric (B) with respect to exchange of the two pH_2 or oD_2 in trimers of $\text{CO-(pH}_2)_2$ or $\text{CO-(oD}_2)_2$, while no exchange symmetry applied for CO-pH_2 -He Trimer.

3 Results and discussion

3.1 Features of the five-dimensional potential energy surface

Figure 3 shows how the well depth of our vibrationally averaged five-dimensional V_{MLRQ} ground-state PES for

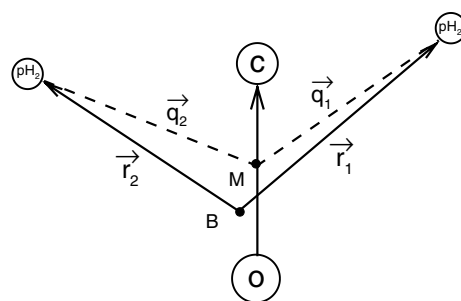


Fig. 2 M is the center of mass of CO, and B is the canonical point for the Radau vectors. \mathbf{q}_1 and \mathbf{q}_2 are Jacobi vectors. \mathbf{r}_1 and \mathbf{r}_2 are Radau vectors. ϕ is a dihedral angle between \mathbf{r}_1 and \mathbf{r}_2 around \mathbf{r}_0 . $\theta_1(\theta_2)$ are angles between \mathbf{r}_0 and $\mathbf{r}_1(\mathbf{r}_2)$

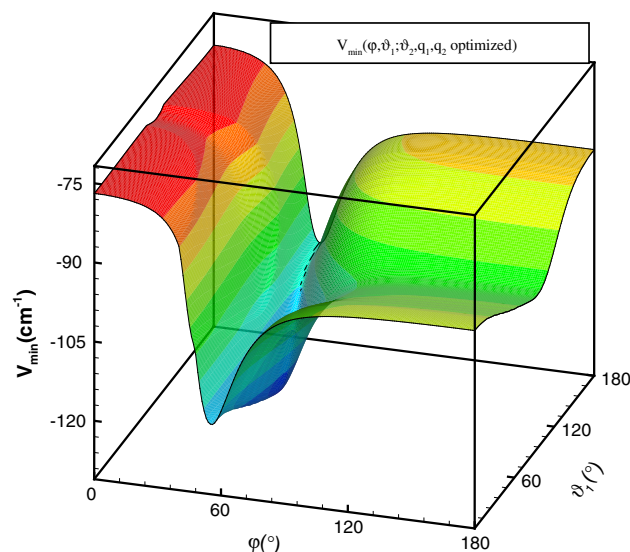


Fig. 3 Minimum energy on our vibrationally averaged five-dimensional PES for $\text{CO}(\nu = 0)\text{-(pH}_2)_2$ trimer as a function of angles ϕ and θ_1 , for optimized values of θ_2 , q_1 , and q_2

$\text{CO}(\nu = 0)\text{-(pH}_2)_2$ system depends on ϕ and θ_1 , when θ_2 , q_1 , q_2 are optimized to minimize the energy for each (ϕ, θ_1) . As seen there, the global minima with well depth of -126.06 cm^{-1} occur at the geometries $(\theta_1 = 95.6^\circ, \theta_2 = 95.6^\circ)$ with $q_1 = q_2 = 3.57 \text{ \AA}$ and $\phi = 58.0^\circ$ (or by symmetry of $\phi = 302.0^\circ$). Between these two identical global minima, there exists a saddle point with energy -101.90 cm^{-1} located at $\theta_1 = \theta_2 = 95.3^\circ$ with $q_1 = q_2 = 3.57 \text{ \AA}$ and $\phi = 180.0^\circ$, which corresponds to a coplanar geometry with the two pH_2 molecules on opposites of the CO molecule. In addition, Fig. 3 also shows local minimum with energy of -77.59 cm^{-1} occurs at the coplanar geometry $(\theta_1 = 94.7^\circ, \theta_2 = 180.0^\circ)$ with $q_1 = 3.56 \text{ \AA}$, $q_2 = 7.01 \text{ \AA}$, and $\phi = 0.0^\circ$. Parameters characterizing the various stationary configurations and energies for the $\text{CO-(pH}_2)_2$ trimer are summarized in Table 1, where

Table 1 Properties of stationary points of the $\text{CO}(\nu = 0)\text{-(pH}_2)_2$ potential energy surface, and comparisons with those results for $\text{CO}(\nu = 0)\text{-(oD}_2)_2$ and $\text{CO}(\nu = 0)\text{-(He)}_2$ surfaces

	Global minimum	Saddle point	Local minimum
$\text{CO-(pH}_2)_2(\text{V}_{\text{MLRQ}})$	{3.57, 3.57, 95.6, 95.6, 58.0, -126.06}	{3.57, 3.57, 95.3, 95.3, 180.0, -101.90}	{3.56, 7.01, 94.7, 180.0, 0.0, -77.59}
$\text{CO-(pH}_2)_2(\text{V}_{12})$	{3.57, 3.57, 94.7, 94.7, 58.0, -126.22}	{3.57, 3.57, 94.7, 94.7, 180.0, -102.07}	{3.56, 7.01, 94.0, 180.0, 0.0, -77.74}
$\text{CO-(oD}_2)_2(\text{V}_{\text{MLRQ}})$	{3.57, 3.57, 95.2, 95.2, 58.0, -126.91}	{3.56, 3.56, 94.6, 94.6, 180.0, -102.76}	{3.56, 7.01, 96.0, 180.0, 0.0, -78.07}
$\text{CO-(He)}_2(\text{V}_{\text{MLRQ}})$	{3.39, 3.39, 120.8, 120.8, 51.9, -53.47}	{3.39, 3.39, 120.8, 120.8, 180.0, -45.90}	{3.37, 6.31, 118.3, 0.0, 0.0, -32.22}

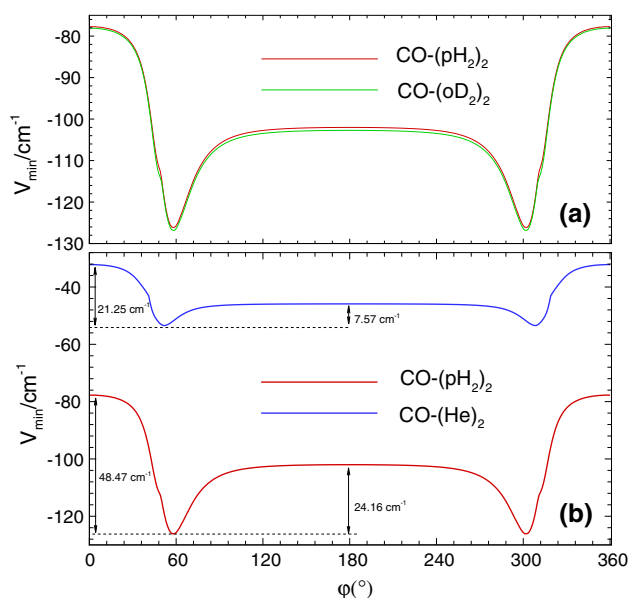
All entries are given as $\{q_1[\text{\AA}], q_2[\text{\AA}], \vartheta_1^\circ, \vartheta_2^\circ, \varphi^\circ, \Delta V[\text{cm}^{-1}]\}$

they are consistent with corresponding values obtained on V_{12} PES, and compared with properties of the analogous $\text{CO-(oD}_2)_2$ and CO-(He)_2 species. As shown in Table 1, all the stationary points (the global minimum, saddle point, and local minimum) for $\text{CO-(pH}_2)_2$ are more than twice as deep as those of CO-(He)_2 . This will be mostly due to the polarizability of pH_2 (5.414 au) [55] is four times larger than that for He (1.383 au) [56]. Comparing $\text{CO-(pH}_2)_2$ and $\text{CO-(oD}_2)_2$ yields the differences at the global minimum, saddle point, and local minimum are 0.85, 0.86, and 0.48 cm^{-1} , respectively. Those differences originate from the fact that $B_{\text{D}_2}/B_{\text{H}_2} \approx 0.5$, which means the solution of the two-dimensional angular rotational equation for oD_2 systems will have substantially larger contributions from basis functions corresponding to $L > 0$ [43], where L is the angular momentum quantum number of H_2 rotation. Figure 4 shows the minimum energy paths between equivalent global minima as a function of φ , for optimized values of $\vartheta_1, \vartheta_2, q_1$ and q_2 .

As clearly shown in Fig. 4(b), the comparison between $\text{CO-(pH}_2)_2$ and CO-(He)_2 shows that the anisotropy with respect to the dihedral torsional motion of the two pH_2 molecules in $\text{CO-(pH}_2)_2$ (48.47 cm^{-1}) is more than twice as strong as that for the analogous motion in the CO-(He)_2 (21.25 cm^{-1}) complex. It also shows that the barrier between two adjacent minima of $\text{CO-(pH}_2)_2$ (24.16 cm^{-1}) is more than three times higher than that for CO-(He)_2 (7.57 cm^{-1}). As would be expected, these barrier heights are approximately equal to the well depths for $\text{pH}_2\text{-pH}_2$ (24.71 cm^{-1}) [45] and He-He (7.65 cm^{-1}) [47] dimers, respectively. These differences in PESs will still be reflected in the different spectra of the Trimers.

3.2 Rovibrational energy levels and band origin shifts

The calculated intermolecular rovibrational energy levels for the ground ($\nu = 0$) states of $\text{CO-(pH}_2)_2$, $\text{CO-(oD}_2)_2$, and $\text{CO-pH}_2\text{-He}$ trimers with $J = 0, 1, 2$, and 3 are listed in Table 2, which are relative to the corresponding zero point energies of -42.907 , -59.326 , and -26.671 cm^{-1} , respectively. Those for excited ($\nu = 1$) (not shown here) are very

**Fig. 4** Minimum energy path on our vibrationally averaged five-dimensional PES for $\text{CO}(\nu = 0)\text{-(pH}_2)_2$ trimer as functions of angle φ for optimized values of ϑ_1, ϑ_2 , and q_1, q_2

similar and listed in Table S1 of the supplementary material. The rovibrational energy levels are labeled by quantum numbers ($J, P, n_{J,P}$). The only good quantum numbers are the overall angular momentum J and parity $P = \text{e or f}$. The label $n_{J,P}$ numbers consecutive states for each (J, P). As shown in Table 2, symmetry (A) and antisymmetry (B) are used to label the symmetries with respect to permutation of the two pH_2 or oD_2 molecules, (A, e) and (B, e) for even parity and (A, f) and (B, f) for odd parity. Because of the zero nuclear spin of the ^4He , pH_2 , oD_2 , therefore for $\text{CO-(pH}_2)_2$, CO-(He)_2 and $\text{CO-(oD}_2)_2$ trimers, only (A, e) and (A, f) states are physically allowed. However, for $\text{CO-pH}_2\text{-He}$ trimer, without exchange symmetry between pH_2 and He, only even and odd parity blocks are used to sort the rovibrational energy levels.

In order to assign the torsional motion states for $\text{CO-(pH}_2)_2$ trimer, it is very useful to yield a one-dimensional representation of the partial wave functions along the

Table 2 Calculated $J = 0, 1, 2, 3$ rovibrational energy levels (in cm^{-1}) for $\text{CO}(v=0)-(\text{pH}_2)_2$, $\text{CO}(v=0)-(\text{oD}_2)_2$, and $\text{CO}(v=0)-\text{pH}_2\text{-He}$ complexes, which are relative to the corresponding zero point energies of -42.907 , -59.326 , and -26.671 cm^{-1} , respectively

$\text{CO}(v=0)-(\text{pH}_2)_2$				$\text{CO}(v=0)-(\text{oD}_2)_2$				$\text{CO}(v=0)-\text{pH}_2\text{-He}$	
(A,e)	(B,e)	(A,f)	(B,f)	(A,e)	(B,e)	(A,f)	(B,f)	e	f
$J = 0$									
0.000 (0)	5.477	14.782	3.407 (1)	0.000 (0)	4.852	12.839	2.728 (1)	0.000 (0)	4.065 (1)
5.340 (2)	12.753	22.294	10.947 (3)	7.299 (2)	13.559	22.401	11.516 (3)	2.580 (2)	8.378 (3)
6.951	14.491	24.140	14.932	7.915	15.141	23.522	14.116	4.991	11.280
11.901 (4)	18.603	27.663	19.647 (5)	11.140 (4)	18.436	27.299	16.553 (5)	6.651	11.407
$J = 1$									
3.597	1.352	0.736	1.136	3.001	0.773	0.399	0.692	1.197	0.517
7.662	3.943	3.438	5.973	6.038	3.103	3.005	5.219	4.058	0.985
11.532	7.972	7.398	6.078	11.856	8.093	5.977	7.956	4.362	3.091
14.271	8.708	7.745	8.125	13.123	9.437	7.752	8.283	5.354	3.897
$J = 2$									
2.083	2.426	4.126	2.945	1.181	1.410	2.508	1.650	1.494	2.319
4.074	4.758	4.865	4.857	2.521	3.910	3.740	3.830	1.778	3.235
4.558	7.337	8.609	5.168	3.754	5.958	6.839	3.941	3.080	4.848
7.307	8.967	9.926	10.283	6.659	8.591	9.742	8.589	4.197	5.198
$J = 3$									
6.326	5.199	4.022	4.234	3.707	2.947	2.317	2.476	3.969	2.792
6.776	6.713	6.193	6.436	4.832	4.896	3.770	4.986	4.501	3.097
7.754	6.974	6.443	7.612	5.785	5.104	4.858	5.264	6.028	4.150
10.860	8.148	7.705	9.314	8.049	5.270	5.790	7.075	6.309	5.538

The torsional motion assignments are indicated in parentheses as ν_t

dihedral angle ϕ . In terms of the basis functions and Radau coordinate system, the wavefunction for the n 'th energy level for a given set of $\{J, M, P\}$ quantum numbers is,

$$\begin{aligned} \Psi_n^{J,M,P}(r_1, r_2, \theta_1, \theta_2, \phi; \alpha, \beta, \gamma) \\ = \sum_{\alpha_1, \alpha_2, l_1, l_2, m_2, K} \langle \theta_1, \theta_2, \phi; \alpha, \beta, \gamma | l_1, l_2, m_2; J, K, M, P \rangle \langle r_1 | \alpha_1 \rangle \langle r_2 | \alpha_2 \rangle \\ \times \langle \alpha_1, \alpha_2, l_1, l_2, m_2; J, K, M, P | \Psi_n^{J,M,P} \rangle, \end{aligned} \quad (9)$$

in which $\langle r_j | \alpha_j \rangle$ is a localized DVR basis function for particle j , and the coefficient $\langle \alpha_1, \alpha_2, l_1, l_2, m_2; J, K, M, P | \Psi_n^{J,M,P} \rangle$ is an eigenvector obtained via Lanczos diagonalization. The one-dimensional wave function along the dihedral angle ϕ , $\Psi_n^{J,M,P}$ is defined as

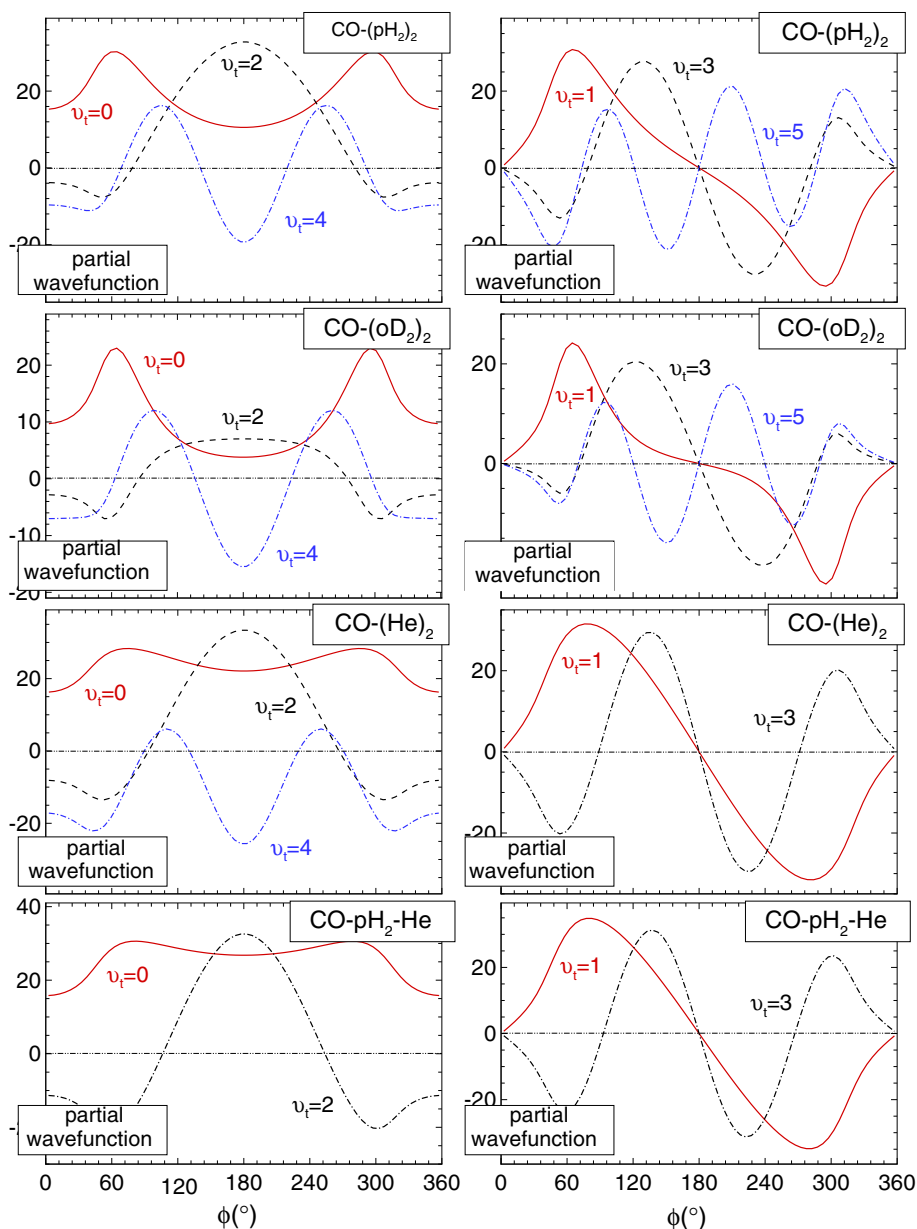
$$\begin{aligned} \psi^{J,M,P,n}(\phi) = \int dr_1 dr_2 \sin \theta_1 d\theta_1 \sin \theta_2 d\theta_2 \Psi_n^{J,M,P} \\ \times (r_1, r_2, \theta_1, \theta_2, \phi; \alpha, \beta, \gamma) \end{aligned} \quad (10)$$

The above integral can be evaluated on the same grid as the potential integral. The torsional motion states for $\text{CO}-(\text{pH}_2)_2$ trimer are assigned and indicated in parentheses as (ν_t) in Table 2, using the partial wave functions, which are yielded by integrating over all degrees of freedom except the dihedral angle ϕ . We make the assignment based on the nodal structure of the partial wave function shown in Fig. 5.

Table 3 presents the calculated zero point energies (ZPEs) and band origin shifts for CO containing dimer or trimer complexes formed from $\text{CO}(v)$ with one or two pH_2 , oD_2 , He, or mixed one pH_2 and one He. ZPEs are relative to the energies at the dissociation limits for corresponding dimers or trimers. All the band origin shifts are calculated by $\Delta\nu_0 = E_{v=1}^{\text{ZPE}} - E_{v=0}^{\text{ZPE}}$. As shown in Table 3, the band origin shift for $\text{CO}-(\text{pH}_2)_2$ trimer predicted by our five-dimensional reduced-dimension V_{MLRQ} PES is -0.369 cm^{-1} , which is almost twice the value of -0.179 cm^{-1} for $\text{CO}-\text{pH}_2$ dimer and slightly lower than the estimated experimental value of -0.355 cm^{-1} [31], which was indirectly determined by combining the microwave and the a-type infrared transitions, assuming that the rotational frequencies are the same in the ground and first-excited vibrational states. For $\text{CO}-(\text{oD}_2)_2$ or $\text{CO}-(\text{He})_2$ trimer, the predicted band origin shift of -0.420 or -0.049 cm^{-1} , is approximately equal to twice of -0.206 or -0.024 cm^{-1} for $\text{CO}-\text{oD}_2$ or $\text{CO}-\text{He}$ dimer.

For $\text{CO}-\text{pH}_2\text{-He}$ trimer, two $\text{CO}-\text{He}$ potentials of CBS + Corr [57] and V333 [46] were used for comparison. The CBS + Corr surface by Peterson and McBane was a three-dimensional ab initio PES that explicitly incorporates dependence on the C–O stretching, obtained with state-of-the-art ab initio and extrapolation methods. The V333 surface [46] by Chuaqui et al. [57] was a two-dimensional PES with CO fixed at its equilibrium geometry, obtained by fitting

Fig. 5 Partial wave functions for the torsional motion ν_t of CO-(pH₂)₂ complex as a function of the azimuthal angle ϕ , compared with those of CO-(oD₂)₂, CO-(He)₂, CO-pH₂-He complexes



a potential to observed transitions near the fundamental band of CO. As we expected, the calculated band origin shift for CO-pH₂-He trimer is -0.205 cm^{-1} on CBS + Corr potential for CO-He, which is approximately equal to the sum of the band origin shifts of -0.179 and -0.024 cm^{-1} for CO-pH₂ and CO-He dimers, respectively. This indicates that the density for pH₂ or He in CO-pH₂-He trimer has very similar distribution as that in their dimers. Due to V333 potential for CO-He without taking into account the CO vibration, the predicted band origin shift on this surface for CO-pH₂-He trimer is -0.019 cm^{-1} different from that on CBS + Corr potential, which is very close to the calculated shift value of -0.024 cm^{-1} for CO-He dimer.

3.3 Predicted microwave and infrared transitions

For CO-(pH₂)₂, the predicted microwave (MW) and infrared (IR) transition frequencies (cm^{-1}) from our AHR 5D V_{MLRQ} potential are listed in Tables 4 and 5, respectively, and compared with those calculated on V_{12} potential and experimental results. The lower and upper states involved in the transitions are numbered with the set of parameters ($J, P, n_{J,P}$). For infrared transitions, to compare our computed energy transitions directly with experimental results, we calculate IR rovibrational transitions from

$$\nu = \nu_0(\text{CO}) + E_{\nu=1}^{\text{upper}} - E_{\nu=0}^{\text{lower}} \quad (11)$$

Table 3 The zero point energies (ZPEs) and band origin shifts for the complexes formed from stretch CO(*v*) with one or two pH₂, oD₂, or mixed one pH₂ and one He

Complexes	$E_{v=0}^{\text{ZPE}}$	$E_{v=1}^{\text{ZPE}}$	$\Delta\nu_0(\text{calc.})$	$\Delta\nu_0(\text{obs.})$
CO(<i>v</i>)–pH ₂	–19.371	–19.550	–0.179	–0.179 ^a
CO(<i>v</i>)–(pH ₂) ₂	–42.907	–43.276	–0.369	–0.355 ^b
CO(<i>v</i>)–oD ₂	–25.630	–25.836	–0.206	–0.202 ^c
CO(<i>v</i>)–(oD ₂) ₂	–59.326	–59.746	–0.420	
CO(<i>v</i>)–He ^f	–6.431	–6.455	–0.024	–0.025 ^d
CO(<i>v</i>)–(He) ₂ ^f	–13.130	–13.179	–0.049	–0.049 ^e
CO(<i>v</i>)–pH ₂ –He ^g	–26.671	–26.857	–0.186	
CO(<i>v</i>)–pH ₂ –He ^f	–26.459	–26.664	–0.205	

ZPEs are relative to the energies at the dissociation limits for corresponding dimers or trimers. $\Delta\nu_0 = E_{v=1}^{\text{ZPE}} - E_{v=0}^{\text{ZPE}}$ is the resulting band origin shifts. All energies are in cm^{–1}

^a Reference [35]. ^b Reference [31]. ^c Reference [58]

^d Reference [36]. ^e Reference [38]

^f This calculation is with the CBS + Corr potential [57]

^g This calculation is with the V333 potential [46]

where, $\nu_0(\text{CO}) = 2,143.2712 \text{ cm}^{-1}$ is the experimental fundamental vibrational transition frequency of a free CO molecule.

As shown in column 2 of Tables 4 or 5, the transition energies yielded by our V_{MLRQ} PES are seen to agree very well with the experimental values shown in column 3 of Tables 4 or 5. The differences seen in column 5 are very small both for microwave and infrared transitions, yielding a root-mean-square (rms) discrepancy of only 0.010 cm^{-1} for all 4 observed transitions (1 for MW and 3 for IR), and also consistent with those obtained on the V_{12} potential

as shown in column 5 with the same rms discrepancy for those 4 experimental transitions. To predict more possible microwave and infrared transitions for CO–(pH₂)₂ trimer, the line strength and relative intensity at temperatures 0.6 K (effective experimental rotational temperature) are also computed from the wave functions. Note that the line strength does not depend on the temperature. Subject to the selection rules $\Delta J = 0, \pm 1$, $\Delta P = e \leftrightarrow f$, and with line strength larger than $10^{-3}(\mu_{\text{CO}}^2)$, 64 possible allowed transitions for MW and 129 for IR with $J \leq 3$ and $n \leq 4$ are predicted. To select most possible microwave and infrared transitions for CO–(pH₂)₂ trimer, all the relative intensities at temperature 0.6 K are calculated, and expressed relative to (1, f, 2)–(0, e, 1) transition, whose intensity is set to 1. With the restriction of the relative intensity larger than 10^{-3} , only 15 strongest transitions for MW and 18 for IR are kept and listed in Tables 4 and 5, respectively. The predicted 129 line strengths and 18 relative intensities for CO–(pH₂)₂ trimer at temperature of 0.6 K are clearly illustrated in the uppermost segment of Fig. 6.

For CO–(oD₂)₂ and CO–pH₂–He, the predicted infrared transition frequencies expressed relative to the band origin $2,143.2712 \text{ cm}^{-1}$ of a free CO from our AHR 5D V_{MLRQ} potential are given in Tables 6 and 7, respectively. The lower and upper states involved in the transitions are also numbered with the set of parameters ($J, P, n_{J,P}$). Only the transitions between A+ and A– states are physically allowed for CO–(oD₂)₂, but this restriction does not apply to CO–pH₂–He. Subject to overall selection rules $\Delta J = 0, \pm 1$, $\Delta P = e \leftrightarrow f$, with line strength larger than $10^{-3}(\mu_{\text{CO}}^2)$, $J \leq 3$ and $n \leq 4$, 149 line strengths and 32 most possible allowed IR transitions at temperature of 0.6 K for

Table 4 Predicted microwave frequencies (cm^{–1}), line strengths, and relative intensities at temperature of 0.6 K for CO–(pH₂)₂ complex from vibrationally averaged five-dimensional V_{12} or V_{MLRQ} PES, comparison with experimental results

	Transitions			Diff. (Calc. – Obs.)		Line strength	Intensity (0.6 K)
	Upper–lower	V_{MLRQ}	Obs.	V_{12}	V_{MLRQ}		
The lower and upper states involved in the transitions are numbered with the set of parameters ($J, P, n_{J,P}$). The line strength values are multiplied by 100	(1, f, 1)–(0, e, 1)	0.736	0.741	0.002	–0.005	0.061	0.047
	(2, f, 2)–(2, e, 1)	2.783				1.853	0.010
	(1, e, 1)–(1, f, 1)	2.862				0.957	0.126
	(1, f, 2)–(0, e, 1)	3.438				1.294	1.000
	(2, e, 3)–(1, f, 1)	3.822				0.679	0.090
	(3, f, 2)–(2, e, 1)	4.110				0.521	0.003
	(1, f, 3)–(2, e, 1)	5.315				0.225	0.001
	(3, f, 4)–(2, e, 1)	5.622				0.162	0.001
	(0, e, 3)–(1, f, 1)	6.951				0.205	0.158
	(2, f, 3)–(2, e, 1)	6.526				0.312	0.002
	(1, e, 2)–(1, f, 1)	6.926				0.079	0.011
	(1, f, 3)–(0, e, 1)	7.236				0.105	0.082
	(1, f, 4)–(0, e, 1)	7.398				0.326	0.252
	(2, f, 4)–(2, e, 1)	7.844				0.489	0.003
	(1, e, 3)–(1, f, 1)	10.796				0.141	0.019

Table 5 Predicted infrared transition frequencies (cm^{-1}), line strengths, and relative intensities at temperature of 0.6 K for $\text{CO-(pH}_2)_2$ complex from vibrationally averaged five-dimensional V_{12} or V_{MLRQ} PES, comparison with experimental results

Transitions	Diff. (calc. – obs.)		Line Strength	Intensity (0.6 K)
	Upper-lower	V_{MLRQ}	V_{12}	V_{MLRQ}
(0, e, 1)–(1, f, 1)	2,142.167			
(1, f, 1)–(0, e, 1)	2,143.638	2,143.657	0.007	0.019
(2, e, 1)–(1, f, 1)	2,144.250			
(2, f, 1)–(2, e, 1)	2,144.945			
(2, f, 2)–(2, e, 1)	2,145.671			
(1, e, 1)–(1, f, 1)	2,145.747			
(1, f, 2)–(0, e, 1)	2,146.324	2,146.330	0.015	0.006
(2, e, 3)–(1, f, 1)	2,146.713	2,146.712	0.011	–0.001
(3, f, 2)–(2, e, 1)	2,147.586			
(1, f, 3)–(2, e, 1)	2,148.201			
(3, f, 4)–(2, e, 1)	2,148.893			
(0, e, 3)–(1, f, 1)	2,149.108			
(2, f, 3)–(2, e, 1)	2,149.414			
(1, e, 2)–(1, f, 1)	2,149.809			
(1, f, 3)–(0, e, 1)	2,150.283			
(1, f, 4)–(0, e, 1)	2,150.637			
(2, f, 4)–(2, e, 1)	2,150.733			
(1, e, 3)–(1, f, 1)	2,153.693			

The lower and upper states involved in the transitions are numbered with the set of parameters ($J, P, n_{J,P}$). The line strength values are multiplied by 10

$\text{CO-(oD}_2)_2$, and 118 line strengths and 37 most possible IR transitions for $\text{CO-pH}_2\text{-He}$ are predicted, and shown in the middle and lowermost segments of Fig. 6, respectively. Although MW or IR transitions for $\text{CO-(oD}_2)_2$ have not been reported yet, one transition marked “m” in Fig. 7 of Ref. [22] was tentatively assigned as b-type $R(0)$ of the mixed ternary cluster $\text{CO-pH}_2\text{-He}$. Based on the calculated line position and intensity, this line may correspond to the (1, f, 4)–(0, e, 1) transition. We are confident for this assignment, not only because the calculated transition at $2,146.958\text{ cm}^{-1}$ are in good agreement with the experimental position at $2,146.93\text{ cm}^{-1}$, but also the relative intensity for (1, f, 4)–(0, e, 1) at temperature 0.6 K is the strongest among all predicted transitions for $\text{CO-pH}_2\text{-He}$ complex.

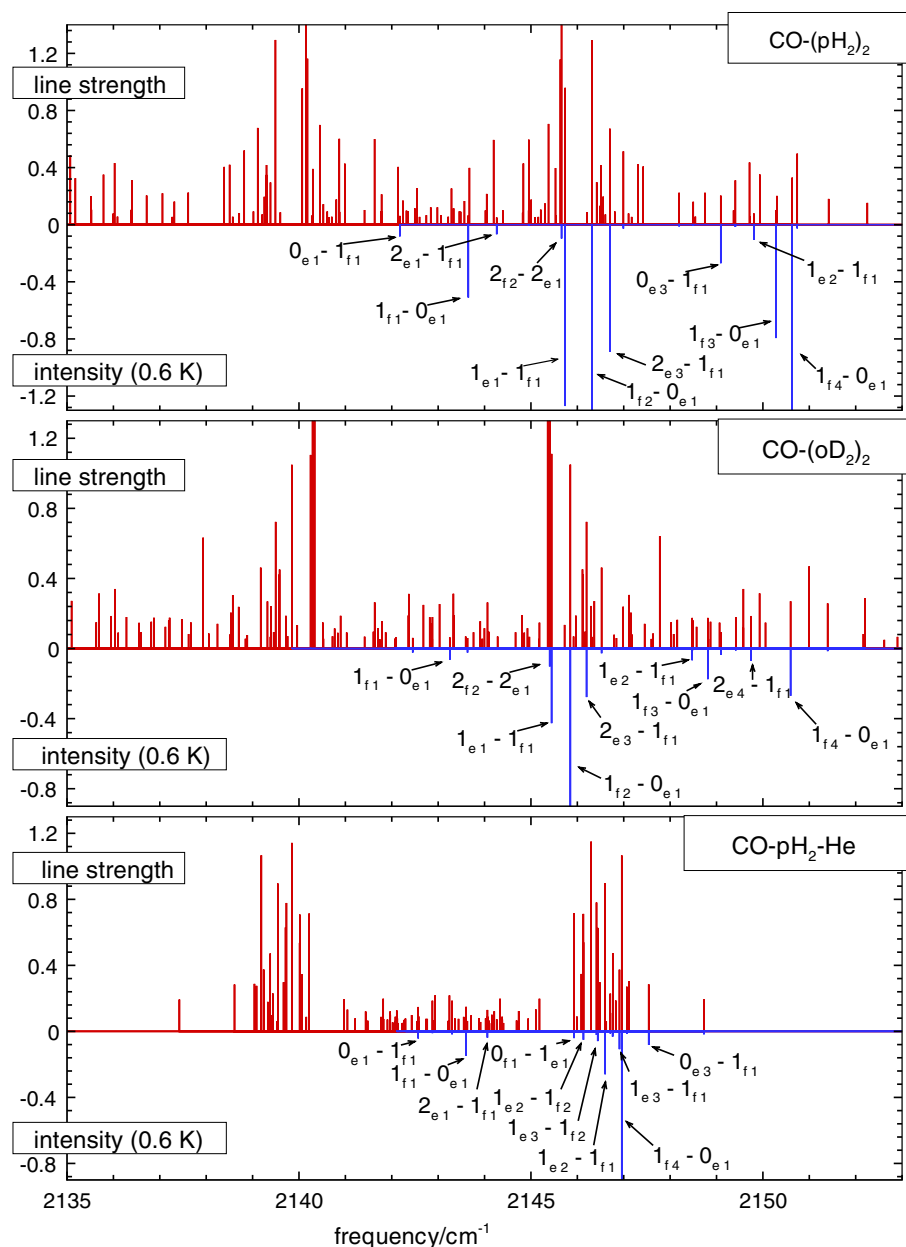
3.4 The wave functions and “solvent” density distributions

Examining the wave function of some of the vibrational states with $v = 0$ and $J = 0$ helps us to understand the nature about the motions of the pH_2 , oD_2 , and He in their complexes. We are most interested in the ϕ dependence of the wave function because this coordinate plays an important role in assigning and understanding many vibrational states. Figure 5 shows the reduced wave functions with (A,e) symmetry and (B,f) symmetry for $v_t = 0, 1, 2, 3, 4$, and 5 levels of $\text{CO-(pH}_2)_2$, obtained by integrating the wave function over the coordinates of r_1, r_2, θ_1 and θ_2 at each value of ϕ , and compared with those of $\text{CO-(oD}_2)_2$, CO-(He)_2 , and $\text{CO-pH}_2\text{-He}$ trimers. As shown in Fig. 5,

the wave functions for the $v_t = 1, 3$, and 5 levels shown on the right side have strikingly regular nodal structures along the ϕ coordinate with exact zero at $\phi = 0$ or $\phi = 180^\circ$, which implies that there is little coupling between ϕ with the other coordinates. While the wave functions for $v_t = 0, 2$, and 4 levels shown on the left side have no strict characteristics as torsional states, which is probably due to the fact that there is strong coupling between ϕ with the other coordinates that distort their character. For CO-(He)_2 and $\text{CO-pH}_2\text{-He}$ trimers, there exist strong coupling between many (A,e) or (A,f) states, and it is hard to assign those states as torsional motions; thus, we only presented the lowest possible torsion states with $v_t < 4$. In Fig. 5, it can be clearly seen that the wave functions for the $v_t = 0, 2$, and 4 levels are quite different between the $\text{CO-(pH}_2)_2$ and CO-(He)_2 complexes, but very similar for $\text{CO-(pH}_2)_2$ and $\text{CO-(oD}_2)_2$, and for CO-(He)_2 and $\text{CO-pH}_2\text{-He}$ complexes. In particular, for the $v_t = 0$ and 1 levels, the highly localized distribution for pH_2 or oD_2 in $\text{CO-(pH}_2)_2$ or $\text{CO-(oD}_2)_2$ complexes contrasts sharply with that for He in CO-(He)_2 or $\text{CO-pH}_2\text{-He}$ complexes. In addition to the expected excluded volumes near $\phi = 0$ and $\phi = 360^\circ$, for the ground state of $\text{CO-(pH}_2)_2$ or $\text{CO-(oD}_2)_2$, there are local maxima at $\phi = 58^\circ$ and $\phi = 302^\circ$ corresponding to the pH_2 or oD_2 particles lying at global minimum positions, while the He distribution in CO-(He)_2 or $\text{CO-pH}_2\text{-He}$ is delocalized, even with a lot of distribution at $\phi = 180^\circ$.

Figure 7 shows the three-dimensional density for the two pH_2 molecules (upper), two He atoms (middle), or mixed

Fig. 6 Calculated infrared spectra of CO-(pH₂)₂ (upper panel), CO-(oD₂)₂ (middle panel), and CO-pH₂-He (lower panel) for line strength (upward pointing lines) and relative intensity at 0.6 K (downward pointing lines), respectively. The intensities are expressed relative to (1, f, 2)-(0, e, 1) transition of CO-(pH₂)₂ or CO-(oD₂)₂, and (1, f, 4)-(0, e, 1) transition of CO-pH₂-He complexes, respectively, whose intensity is equal 1. All the line strength values are multiplied by 10. The relative intensity of CO-(pH₂)₂ are also multiplied by 100



one pH₂ and one He (bottom) in the body-fixed frame, as calculated for the ground state of CO-(pH₂)₂, CO-(He)₂, or CO-pH₂-He using the procedure described in Ref. [27]. It is very useful to develop some intuitive feeling regarding the nature of these species. Iso-surfaces are used to represent the three-dimensional densities, and the z axis is defined to lie on the CO molecular axis. One significant feature of these representations is the fact that the two pH₂ molecules or two He molecules appear to have very different density distributions. It is clear that the density distribution of the first pH₂ or He particle in CO-(pH₂)₂ or CO-(He)₂ trimer is represented as a disk and somewhat delocalized in the yz plane, while that of the second pH₂ in CO-(pH₂)₂ trimer is highly localized, with two density maxima at regions

associated with the pH₂-pH₂ potential minimum. In contrast, the density distribution of the second helium atom in CO-(He)₂ trimer (middle panel) is highly delocalized relative to the first, being distributed on an incomplete ring wrapped around the CO molecule. For CO-pH₂-He trimer, the density for pH₂ as particle-1 is located in the yz plane as reference, while the density for He as particle-2 is relative to pH₂. As shown in the lowest panel of Fig. 7, the more localized He distribution in CO-pH₂-He than in CO-(He)₂ is related to the stronger interaction (deeper well) between the pH₂-He interaction than the He-He one. For all three cases, the density for particle-2 is excluded from the region near particle-1 by the short-range repulsive wall of the pH₂-pH₂, He-He, or pH₂-He potential.

Table 6 Predicted infrared transition frequencies (cm^{-1}), line strengths, and relative intensities at temperature of 0.6 K for $\text{CO}-(\text{oD}_2)_2$ complex from vibrationally averaged five-dimensional V_{MLRQ} PES

Transitions	Line	Intensity
Upper-lower	Calc.	Strength (0.6 K)
(1, f, 2)–(0, e, 1)	2,145.849	1.050 1.000
(1, e, 1)–(1, f, 1)	2,145.445	1.111 0.406
(2, e, 3)–(1, f, 1)	2,146.199	0.723 0.264
(1, f, 4)–(0, e, 1)	2,150.597	0.270 0.257
(1, f, 3)–(0, e, 1)	2,148.813	0.175 0.167
(2, f, 2)–(2, e, 1)	2,145.404	1.726 0.097
(1, e, 3)–(1, f, 1)	2,154.309	0.241 0.088
(0, e, 2)–(1, f, 1)	2,149.747	0.186 0.068
(1, e, 2)–(1, f, 1)	2,148.475	0.175 0.064
(1, f, 1)–(0, e, 1)	2,143.251	0.064 0.061
(2, e, 4)–(1, f, 1)	2,149.096	0.093 0.034
(3, f, 3)–(2, e, 1)	2,146.523	0.462 0.026
(0, e, 1)–(1, f, 1)	2,142.452	0.061 0.022
(2, e, 1)–(1, f, 1)	2,143.633	0.060 0.022
(2, f, 4)–(2, e, 1)	2,151.396	0.258 0.014
(1, f, 4)–(2, e, 1)	2,149.416	0.179 0.010
(2, f, 3)–(2, e, 1)	2,148.495	0.160 0.009
(3, f, 4)–(2, e, 1)	2,147.451	0.141 0.008
(3, e, 2)–(3, f, 1)	2,145.361	1.616 0.006
(1, f, 2)–(2, e, 1)	2,144.668	0.093 0.005
(1, f, 3)–(2, e, 1)	2,147.632	0.086 0.005
(1, f, 1)–(2, e, 1)	2,142.070	0.057 0.003
(3, f, 4)–(2, e, 2)	2,146.111	0.452 0.001
(3, e, 3)–(2, f, 1)	2,146.119	0.423 0.001
(1, f, 1)–(1, e, 1)	2,140.250	1.105 0.001
(0, e, 1)–(1, f, 2)	2,139.846	1.049 0.001
(1, f, 2)–(2, e, 2)	2,143.328	0.313 0.001
(1, e, 3)–(2, f, 1)	2,152.200	0.287 0.001
(1, e, 2)–(2, f, 1)	2,146.366	0.271 0.001
(2, f, 2)–(2, e, 2)	2,144.063	0.263 0.001
(2, e, 4)–(2, f, 1)	2,146.988	0.239 0.001
(1, f, 3)–(2, e, 2)	2,146.292	0.243 0.001

The lower and upper states involved in the transitions are numbered with the set of parameters ($J, P, n_{J,P}$). The line strength values are multiplied by 10

Figure 8 shows the “Solvent” pH_2 (upper panels), He (middle panels), and pH_2 -He (lower panels) densities in the first three torsional levels of $\text{CO}-(\text{pH}_2)_2$, $\text{CO}-(\text{He})_2$, and $\text{CO}-\text{pH}_2$ -He, respectively. For the $v_t = 0$ ground states, the density distribution patterns of particle-2 across three complexes are quite different, while for $v_t = 1, 2$, and 3 excited states, the density distribution patterns are similar among the three species, because at these levels energy patterns are determined by the nodal structures instead of

Table 7 Predicted infrared transition frequencies (cm^{-1}), line strengths, and relative intensities at temperature of 0.6 K for $\text{CO}-\text{pH}_2$ -He complex from vibrationally averaged five-dimensional V_{MLRQ} PES

Transitions	Line	Intensity
Upper-lower	Calc.	Strength (0.6 K)
(1, f, 4)–(0, e, 1)	2,146.958 ^a	1.066 1.000
(1, e, 2)–(1, f, 1)	2,146.599	0.898 0.244
(1, f, 1)–(0, e, 1)	2,143.601	0.149 0.140
(1, e, 3)–(1, f, 1)	2,146.908	0.374 0.102
(0, e, 3)–(1, f, 1)	2,147.539	0.285 0.077
(1, e, 3)–(1, f, 2)	2,146.440	0.627 0.055
(1, e, 2)–(1, f, 2)	2,146.131	0.540 0.048
(0, e, 1)–(1, f, 1)	2,142.569	0.148 0.040
(0, f, 1)–(1, e, 1)	2,145.929	0.715 0.038
(2, e, 1)–(1, f, 1)	2,144.061	0.130 0.035
(2, e, 4)–(1, f, 1)	2,146.761	0.112 0.030
(2, f, 3)–(2, e, 1)	2,146.413	0.781 0.020
(0, e, 4)–(1, f, 2)	2,148.729	0.194 0.017
(1, e, 1)–(1, f, 2)	2,143.299	0.186 0.016
(2, f, 4)–(1, e, 1)	2,147.067	0.271 0.014
(2, f, 4)–(2, e, 1)	2,146.770	0.474 0.012
(2, f, 3)–(1, e, 1)	2,146.710	0.229 0.012
(1, f, 2)–(1, e, 1)	2,142.876	0.186 0.010
(2, f, 3)–(2, e, 2)	2,146.129	0.712 0.009
(3, f, 4)–(2, e, 1)	2,147.113	0.303 0.008
(2, e, 2)–(1, f, 2)	2,143.881	0.079 0.007
(2, f, 1)–(1, e, 1)	2,144.209	0.122 0.006
(2, e, 1)–(1, f, 2)	2,143.593	0.060 0.005
(2, f, 4)–(2, e, 2)	2,146.486	0.297 0.004
(1, f, 1)–(2, e, 1)	2,142.108	0.127 0.003
(0, f, 3)–(1, e, 1)	2,153.142	0.062 0.003
(3, f, 4)–(2, e, 2)	2,146.829	0.187 0.002
(3, f, 1)–(2, e, 1)	2,144.382	0.072 0.002
(3, f, 2)–(2, e, 1)	2,144.691	0.066 0.002
(1, f, 2)–(2, e, 1)	2,142.579	0.060 0.002
(2, f, 1)–(2, e, 1)	2,143.912	0.057 0.001
(3, e, 3)–(3, f, 1)	2,146.297	1.150 0.001
(1, f, 2)–(2, e, 2)	2,142.295	0.078 0.001
(3, e, 3)–(2, f, 1)	2,146.770	0.268 0.001
(3, f, 1)–(2, e, 2)	2,144.098	0.065 0.001
(2, f, 1)–(2, e, 2)	2,143.628	0.063 0.001
(3, f, 2)–(2, e, 2)	2,144.407	0.051 0.001

The lower and upper states involved in the transitions are numbered with the set of parameters ($J, P, n_{J,P}$). The line strength values are multiplied by 10

^a The observed value is $2,146.93 \text{ cm}^{-1}$

inter-particle interactions and can be explained by a simple one-dimensional particle-in-a-box model along the ϕ coordinate [27, 39].

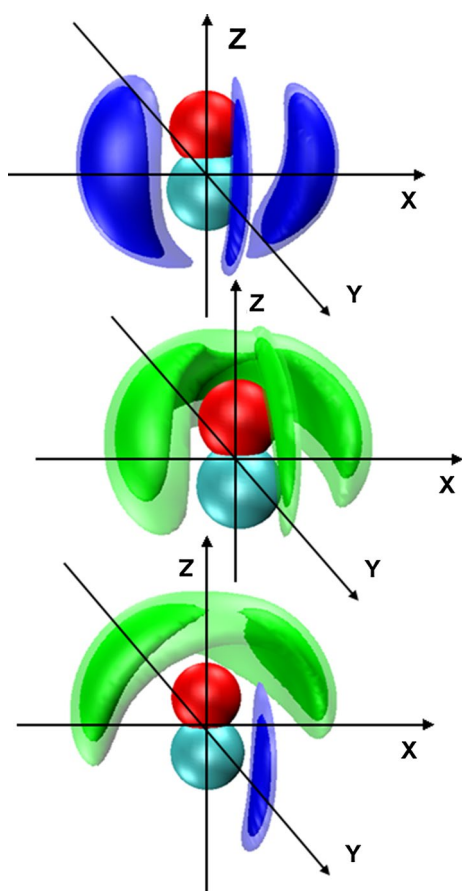


Fig. 7 Three-dimensional representations of the pH_2 density for the ground state $\text{CO}-(\text{pH}_2)_2$ trimer (upper panel) in the body-fixed frame, compared with the helium atom density for a ground-state $\text{CO}-(\text{He})_2$ trimer (middle panel) and the pH_2 -He density of a ground-state $\text{CO}-\text{pH}_2\text{-He}$ trimer (lower panel). The position of the first pH_2 or He defines the location of the yz plane, while the density of the second pH_2 or He particle is shown in the xy plane. These results were obtained with the values of the Gaussian standard deviations defining the densities set at $\sigma_x = \sigma_z = 0.25 \text{ \AA}$ and $\sigma_y = 0.1 \text{ \AA}$ for particle-1 and at $\sigma_x = \sigma_y = \sigma_z = 0.25 \text{ \AA}$ for particle-2

4 Conclusions

Predicted rovibrational energy levels, microwave, and infrared spectra for $\text{CO}-(\text{pH}_2)_2$, $\text{CO}-(\text{oD}_2)_2$, and $\text{CO}-\text{pH}_2\text{-He}$ complexes have been obtained from the vibrationally averaged five-dimensional PESs as a sum of accurate $\text{CO}-\text{pH}_2$, $\text{CO}-\text{oD}_2$, or $\text{CO}-\text{He}$ and $\text{pH}_2\text{-pH}_2$, $\text{oD}_2\text{-oD}_2$, or $\text{pH}_2\text{-He}$ pair potentials. For $\text{CO}-(\text{pH}_2)_2$ complex, it is remarkable that the predictions are in good agreement with the experimental measurements of the microwave and infrared transitions, showing that the spectrum can be reliably predicted by this additive approach. Small remaining differences between experiment and theory may in part be due to our neglect of three-body contributions to the interaction energy. For $\text{CO}-(\text{oD}_2)_2$ trimer, the infrared spectrum

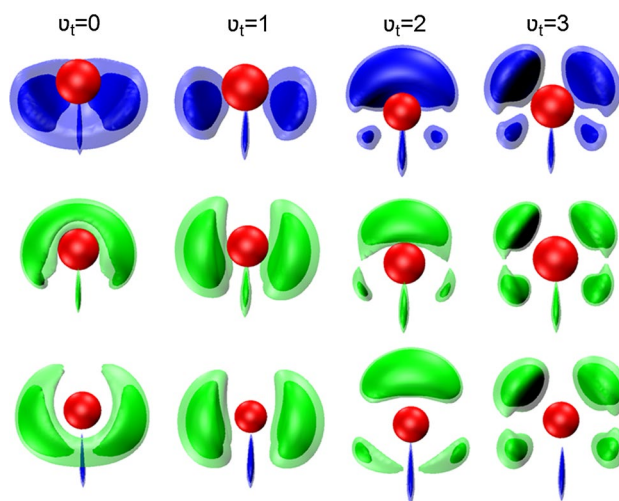


Fig. 8 Three-dimensional representations of the “solvent” (pH_2 , He and $\text{pH}_2\text{-He}$) density for $\text{CO}-(\text{pH}_2)_2$ (upper panels), $\text{CO}-(\text{He})_2$ (middle panels), and $\text{CO}-\text{pH}_2\text{-He}$ (lower panels) trimers in their four lowest torsion vibrational levels

has been predicted for the first time. For $\text{CO}-\text{pH}_2\text{-He}$, one observed IR transition tentatively assigned as b-type $R(0)$ was confirmed based on the calculated line position and intensity. Although many of the transitions for those three complexes have not been observed, our predictions will be a good guidance for future experimental research. The calculated band origin shift from vibrationally averaged five-dimensional PESs associated with the fundamental transition of CO is -0.369 cm^{-1} for $\text{CO}-(\text{pH}_2)_2$, which is also in good agreement with the experimental value of -0.355 cm^{-1} . The band origin shifts for $\text{CO}-(\text{oD}_2)_2$ and $\text{CO}-\text{pH}_2\text{-He}$ are also predicted for the first time. In any case, the result indicates that a reduced-dimension treatment of pH_2 or oD_2 rotation based on a “adiabatic-hindered-rotor” average over its relative orientations can yield accurate spectra predictions. Three-dimensional representation of the pH_2 , oD_2 , or He density in the body-fixed frame is very useful to develop some intuitive feeling regarding the nature of these species, and it clearly shows the density distribution of the second pH_2 , oD_2 , or He relative to the first one. The pH_2 and oD_2 distributions in the vibrational ground state of $\text{CO}-(\text{pH}_2)_2$ and $\text{CO}-(\text{oD}_2)_2$ are highly localized, with two maxima, while the He distribution in $\text{CO}-(\text{He})_2$ or $\text{CO}-\text{pH}_2\text{-He}$ is delocalized, with one maximum lying in an incomplete ring wrapped around the CO molecule axis at the O end.

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References

1. Grebenev S, Toennies JP, Vilesov AF (1998) *Science* 279:2083
2. Nauta K, Miller RE (2001) *J Chem Phys* 115:10254
3. Tang J, Xu Y, McKellar ARW, Jäger W (2002) *Science* 297:2030
4. Lehnig R, Jäger W (2006) *Chem Phys Lett* 424:146
5. Tang J, McKellar ARW (2003) *J Chem Phys* 119:754
6. Surin LA, Potapov AV, Dumesh BS, Schlemmer S, Xu Y, Raston PL, Jäger W (2008) *Phys Rev Lett* 101:233401
7. Tang J, McKellar ARW (2004) *J Chem Phys* 121:181
8. Tang J, McKellar ARW, Mezzacapo F, Moroni S (2004) *Phys Rev Lett* 92:145503
9. McKellar ARW (2008) *J Chem Phys* 128:044308
10. Xu Y, Jäger W (2003) *J Chem Phys* 119:5457
11. McKellar ARW, Xu Y, Jäger W (2006) *Phys Rev Lett* 97:183401
12. Tang J, McKellar ARW (2003) *J Chem Phys* 119:5467
13. McKellar ARW, Xu Y, Jäger W (2007) *J Phys Chem A* 111:7329
14. Li H, Ma YT (2012) *J Chem Phys* 137:234310
15. Wang L, Xie DQ, Le Roy RJ, Roy P-N (2012) *J Chem Phys* 137:104311
16. McKellar ARW (2007) *J Chem Phys* 127:044315
17. Knaap CJ, Xu Y, Jäger W (2011) *Mol Spectrosc* 268:130
18. Xu Y, Blinov N, Jäger W, Roy P-N (2006) *J Chem Phys* 124:081101
19. Toennies JP (2013) *Mol Phys* 111:1879
20. Zeng T, Roy P-N (2014) *Rep Prog Phys* 77:046601
21. Grebenev S, Sartakov B, Toennies JP, Vilesov AF (2000) *Science* 289:1532
22. Moroni S, Botti M, De Palo S, McKellar ARW (2005) *J Chem Phys* 122:094314
23. Tang J, McKellar ARW (2004) *J Chem Phys* 121:3087
24. Tang J, McKellar ARW (2005) *J Chem Phys* 123:114314
25. Michaud J, Xu Y, Jäger W (2008) *J Chem Phys* 129:144311
26. Li H, Le Roy RJ, Roy P-N, McKellar ARW (2010) *Phys Rev Lett* 105:133401
27. Li H, McKellar ARW, Le Roy RJ, Roy P-N (2011) *J Phys Chem A* 115:7327
28. Li H, Roy P-N, Le Roy RJ (2010) *J Chem Phys* 132:214309
29. Zeng T, Guillon G, Cantin JT, Roy P-N (2013) *J Phys Chem Lett* 4:239
30. Wang L, Xie DQ, Le Roy RJ, Roy P-N (2013) *J Chem Phys* 139:034312
31. Raston PL, Jäger W, Li H, Le Roy RJ, Roy P-N (2012) *Phys Rev Lett* 108:253402
32. Jankowski P, McKellar ARW, Szalewicz K (2012) *Science* 336:1147
33. Jankowski P, Surin LA, Potapov A, Schlemmer S, McKellar ARW, Szalewicz K (2013) *J Chem Phys* 138:084307
34. Li H, Zhang XL, Le Roy RJ, Roy P-N (2013) *J Chem Phys* 139:164315
35. McKellar ARW (1998) *J Chem Phys* 108:1811
36. McKellar ARW, Xu Y, Jäger W, Bissonnette C (1999) *J Chem Phys* 110:10766
37. Surin LA, Roth DA, Pak I, Dumesh BS, Lewen F, Winnewisser G (2000) *J Chem Phys* 112:4064
38. Wang XG, Carrington T Jr, McKellar ARW (2009) *J Phys Chem A* 113:13331
39. Wang XG, Carrington T Jr, Tang J, McKellar ARW (2005) *J Chem Phys* 123:34301
40. Wang XG, Carrington T Jr (2010) *Can J Phys* 88:779
41. Tang J, McKellar ARW, Wang XG, Carrington T Jr (2009) *Can J Phys* 87:417
42. Li H, Liu YD, Jäger W, Le Roy RJ, Roy P-N (2010) *Can J Phys* 88:1146
43. Li H, Roy P-N, Le Roy RJ (2010) *J Chem Phys* 133:104305
44. Telle H, Telle U (1981) *J Mol Spectrosc* 85:248
45. Patkowski K, Cencek W, Jankowski P, Szalewicz K, Mehl JB, Garberoglio G, Harvey AH (2008) *J Chem Phys* 129:094304
46. Chuaqui CE, Le Roy RJ, McKellar ARW (1994) *J Chem Phys* 101:39
47. Jeziorska M, Cencek W, Patkowski K, Jeziorski B, Szalewicz K (2007) *J Chem Phys* 127:124303
48. Bramley MJ, Tromp JW, Carrington T Jr, Corey GC (1994) *J Chem Phys* 100:6175
49. Mladeovic M (2000) *J Chem Phys* 112:1070
50. Gatti F, Lung C, Menou M, Justum Y, Nauts A, Chapuisat X (1998) *J Chem Phys* 108:8804
51. Yu HG (2002) *Chem Phys Lett* 365:189
52. Light JC, Hamilton IP, Lill JV (1985) *J Chem Phys* 82:1400
53. Chen R, Guo H (2001) *J Chem Phys* 114:1467
54. Wang XG, Carrington T Jr (2001) *J Chem Phys* 114:1473
55. Bishop DM, Cheung LM (1980) *J Chem Phys* 72:5125
56. Parker GA, Snow RL, Pack RT (1976) *J Chem Phys* 64:1668
57. Peterson KA, McBane GC (2005) *J Chem Phys* 123:084314
58. McKellar ARW (1991) *Chem Phys Lett* 186:58