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Relative impatience determines preference between contract bargaining and repeated bargaining

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Abstract. In this paper we consider the effect of the 'impatience ratio' I (of the worker discount factor to the firm discount factor) on the preferences of the players between two bargaining schemes in an asymmetric information wage bargaining context. The firm has private information about the worker's value and the worker makes wage demands. In the contact bargaining scheme, a wage demand which is accepted in one period is binding for all future periods (and hence the bargaining ends after acceptance of a wage demand). In the repeated bargaining scheme, the parties continue to bargain irrespective of whether the worker has been hired or not, and any accepted wage demand is only valid for the period in which it was accepted. We establish the following results under the assumption that the worker's value is uniformly distributed on an interval: When the firm is more patient than the worker ($I < 1$) both parties prefer contract bargaining, and when the worker is more patient than the firm ($I > 1$) both prefer repeated bargaining. For any value of I, the preferred type of bargaining gives the lower unemployment.

The work of Bae has already shown that when players are equally patient ($I = 1$) the players are indifferent between the two schemes, regardless of the distribution of the worker's value. This paper shows that Bae's indifference result (Bae, 1991) cannot be extended to unequally patient players.

Key words: Bargaining, incomplete information

1. Introduction

The impatience of the players, as quantified by their discount factors, is well known to have a significant effect on the equilibrium outcome of an otherwise fixed bargaining game. According to the equivalence result of Bae (1991), equally impatient players are indifferent between two alternative multiperiod bargaining schemes: *repeated bargaining* (RB) and *contract bargaining* (CB). In this paper we show that Bae's result does not extend to bargaining between parties with different discount factors. We show that, for uniform beliefs (about the value of the object of bargaining to the buyer), the relative impatience of the two players completely determines their preferences between RB and CB, and that the two players always have the same preference. For ease of exposition and for additional observations regarding unemployment, we use a labor market setting in which a worker and a firm bargain over the wage. Our models are however equally applicable to bargaining contracts for the repeated sale or rental of a good.

We consider wage bargaining models in which the firm (buyer) has a private information reservation price regarding the value of hiring the worker (seller). We make the assumption that this reservation price (worker's value v) is uniformly distributed over an interval, which we may normalize to the unit interval $[0, 1]$. In the repeated bargaining (RB) scheme, the worker makes a wage demand in each (of two) period, which may be accepted or rejected by the firm. If the wage demand is accepted, the worker is hired for that period; if it is rejected the worker is assumed to be unemployed. The worker's second period demand may depend on whether or not he was hired in the first period. The contract bargaining (CB) scheme is similar, except that a wage demand which is accepted in period one is binding for period 2 as well. The CB scheme is essentially equivalent to a single trade bargaining game (see Fudenberg, Levine, & Tirole, 1985). The dichotomy between the two schemes was first discussed by Hart and Tirole (1988).

Note that in both these bargaining schemes it is the uninformed party (the worker) who makes all the offers, and this results in inefficiencies (and unemployment, in the labor setting). Of course if the firm sets the wage then at the (subgame perfect) equilibrium the worker accepts a minimal wage in both periods. In realistic settings it is very common for the seller (uninformed party) to set prices which buyers with different valuations can accept or reject. Labor disputes and strikes can be explained in terms of workers making wage demands while uncertain about the firm's profitability. Of course in reality bargaining is a dynamic process in which offers and counter–offers are made. Since we want to focus on the possibility of extending Bae's equivalence result (players' indifference between RB and CB) we make the assumption that the worker makes all the offers.

The main findings of the paper can be simply expressed in terms of the 'impatience ratio' I of the worker discount factor to the firm discount factor. When the worker is more patient than the firm ($I > 1$) both parties prefer RB. When the firm is more patient than the worker ($I < 1$) both players prefer CB. The intuition for these results is that at the equilibrium of RB compared with that of CB the firm's profits are moved forward and wages backward. Hence an impatient firm is better off under RB. When $I = 1$, both players are indifferent (this was established by Bae under more general assumptions regarding the distribution of the worker's value to the firm). We find furthermore that for any value of the impatience ratio I, the preferred type of bargaining gives the lower unemployment (expected number of periods without an accepted wage offer).

This paper builds on the wage determination model of Alpern and Snower (1988) where the worker learns about his productivity through making wage demands. In their model the firm does not behave strategically – it simply accepts demands below the worker's value. In Reyniers (1992) this model was extended to allow for a strategic firm which distorts the learning process of a boundedly rational worker. There the firm makes a first period wage demand which is below the worker's value, passing up a first period profit in order to disrupt the worker's learning process. In the current paper, both parties act strategically.

The paper is organized as follows. In sections 2 and 3 respectively, we give the explicit models for Repeated Bargaining and Contract Bargaining, and their unique equilibria. In section 4 we state our results (Lemma 3 and Theorem 4) on the comparison of these two schemes. Section 5 gives our conclusions. The proofs of Lemma 3 and Theorem 4 are given in the Appendix.

2. Repeated bargaining

In this section we analyze repeated bargaining between a worker and a firm as a Bayesian game and we determine the unique perfect Bayesian equilibrium (see for example Myerson 1991). A worker W (the seller) and a firm F (buyer) bargain over wage (rental price). The firm has private information on the worker's per period value v. Following the Bayesian game formulation, we call v the 'type' of the firm. We assume that v is uniformly distributed over the open unit interval $(0, 1)$. Both players are risk neutral and impatient, with discount factors δ_F for the firm and δ_W for the worker. Both discount factors are in $(0, 1)$. The distribution of v and the values of δ_F, δ_W are common knowledge. The part of the model given in this paragraph holds for contract bargaining as well.

In a Bayesian game it is necessary to specify an action space which is a strategy for each type of each player. The action space for the worker is $[0, 1]^3$, in which the worker picks a triple (w_1, w_a, w_r), where w_1 denotes the first period wage demand, and w_a and w_r are the second period demands in the respective cases where the first period demand was **a**ccepted or **r**ejected. The possible actions for each type of firm are the sets of wage demands which are accepted in each period. Perfectness implies that in the second period the firm hires the worker whenever his second period wage demand does not exceed his value. It can be shown that the only possible perfect equilibrium actions for the firm in the first period are of the cutoff form $f : (0, 1) \to (0, 1)$, where firms of type at least $f(w_1)$ accept the demand w_1, while those of smaller type reject this demand. The respective payoffs Π_F and Π_W to the firm and worker when the worker's value (firm's type) is v are given in the following table:

	Π_F	Π_W
Worker hired in both periods	$(v - w_1) + \delta_F(v - w_a)$	$w_1 + \delta_W w_a$
Worker hired in period 1 only	$(v - w_1)$	w_1
Worker hired in period 2 only	$\delta_F(v - w_r)$	$\delta_W w_r$
Worker hired in neither period	0	0

To determine the firm's optimal response to a worker strategy (w_1, w_a, w_r) we need to consider two options: Accepting the demand w_1 gives the firm a profit of $v - w_1 + \delta_F(v - w_a)^+$, while rejecting this demand gives a payoff of

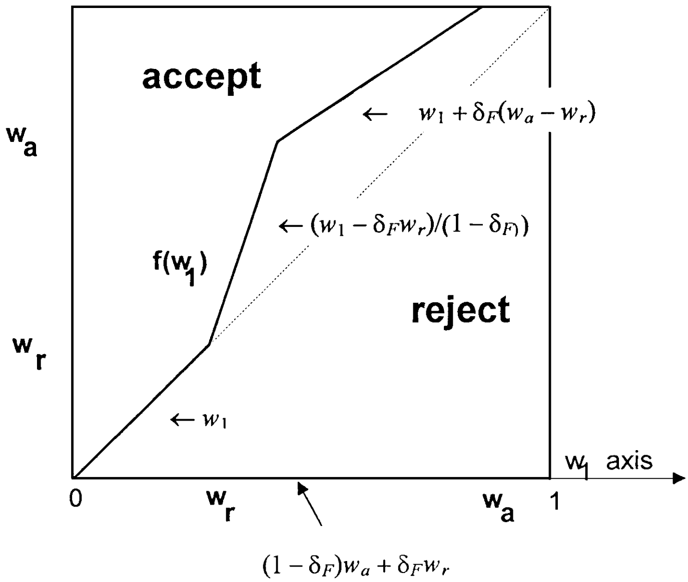

Fig. 1a. Firm cutoff f when $w_a \geq w_r$

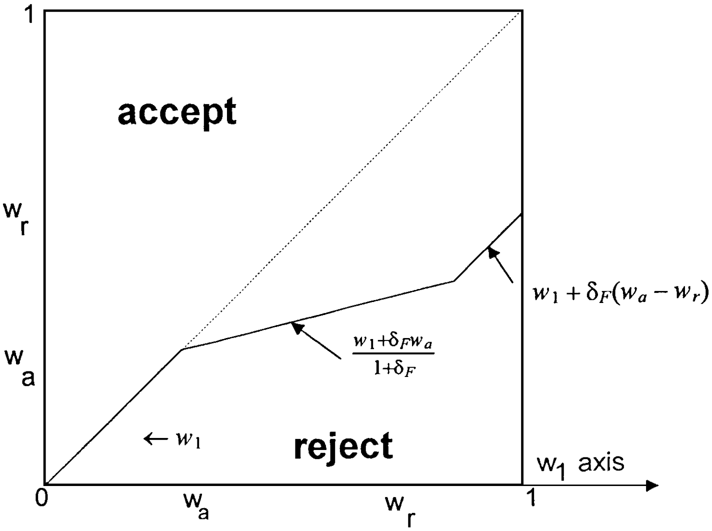

Fig. 1b. Firm cutoff f when $w_a < w_r$

$\delta_F(v - w_r)^+$, where $x^+ = \max(0, x)$. The firm types v for which each of these expressions is the greater form a partition of $(0, 1)$ into two intervals $(f(w_1), 1)$ and $(0, f(w_1))$. Hence only cutoff strategies can be optimal responses and consequently we need not consider other firm strategies. The optimal response to the worker strategy (w_1, w_a, w_r) is shown in Figures 1a and 1b and given explicitly below. The form depends on the sign of $w_r - w_a$. For $w_r \leq w_a$ we have

$$f_{w_a,w_r}(w_1) = \begin{cases} w_1, & \text{if } w_1 \le w_r \\ \dfrac{w_1 - \delta_F w_r}{1-\delta_F}, & \text{if } w_r < w_1 \le (1-\delta_F)w_a + \delta_F w_r \\ w_1 + \delta_F(w_a - w_r) & \text{if } (1-\delta_F)w_a + \delta_F w_r < w_1 \le 1 - \delta_F(w_a - w_r) \\ 1 & \text{if } 1 - \delta_F(w_a - w_r) < w_1. \end{cases}$$

For $w_a < w_r$, we have

$$f_{w_a,w_r}(w_1) = \begin{cases} w_1, & \text{if } w_1 \le w_a \\ \dfrac{w_1 + \delta_F w_a}{1+\delta_F}, & \text{if } w_a < w_1 \le (1+\delta_F)w_r - \delta_F w_a \\ w_1 + \delta_F(w_a - w_r) & \text{if } (1+\delta_F)w_r - \delta_F w_a < w_1. \end{cases}$$

Note that when $w_r \le w_a$ (as drawn in Figure 1a), the cutoff curve f lies above the 45° line, which means that a worker is hired only if his value is (significantly) larger than his wage demand. However if $w_a < w_r$ (see Figure 1b) then the curve lies on or below the 45° line, which indicates that a worker can be hired at wage demands which exceed his value, at a loss (in the first period) to the firm. (We will see later that at equilibrium the situation is as in Figure 1a, since we will have $\overline{w}_r = \overline{w}_a/2$.)

A similar analysis for the worker's optimal response first finds w_a and w_r as functions of f and w_1. Then w_1 is optimized. The details are as follows.

If the worker's first period wage demand is accepted, which implies $f(w_1) < 1$, and he knows that the firm's cutoff strategy is given by the function f, his second period expected payoff maximimization problem is

$$\max_{f(w_1) \le w_a \le 1} w_a \frac{(1-w_a)}{(1-f(w_1))}. \tag{1}$$

If his first period wage demand is not accepted, which implies $f(w_1) > 0$, his second period expected payoff maximization problem is

$$\max_{0 \le w_r \le f(w_1)} w_r \frac{(f(w_1) - w_r)}{f(w_1)}. \tag{2}$$

The equilibrium second period wage demands w_a and w_r in terms of w_1 (which will be determined later) are therefore given by the fixed point equations

$$w_a = \max(f_{w_a,w_r}(w_1), 1/2) \quad \text{and} \tag{3}$$

$$w_r = f_{w_a,w_r}(w_1)/2. \tag{4}$$

Since in an optimal worker's response both (3) and (4) hold, and hence $w_a \ge w_r$, only the piecewise linear function $f_{w_a,w_r}(w_1)$ has to be considered. The first piece of $f_{w_a,w_r}(w_1)$ indicates $f_{w_a,w_r}(w_1) \le w_r$ and hence is not valid at equilibrium because of (4). Since $w_a \ge f_{w_a,w_r}(w_1)$ in an optimal worker's response, only the second piece of $f_{w_a,w_r}(w_1)$ is relevant at equilibrium.

To determine the optimal first period wage demand $\overline{w}_1$, the total expected payoff over the two periods should be considered:

$$\begin{aligned}\Pi_W &= w_1(1 - f_{w_a,w_r}(w_1)) \\ &\quad + \delta_W w_a(1 - f_{w_a,w_r}(w_1))\frac{(1 - w_a)}{(1 - f_{w_a,w_r}(w_1))} \\ &\quad + \delta_W w_r f_{w_a,w_r}(w_1)\frac{f_{w_a,w_r}(w_1) - w_r}{f_{w_a,w_r}(w_1)}. \end{aligned} \tag{5}$$

The first term in Π_W is the expected first period payoff. The second term is the discounted expected second period payoff if the worker's wage demand is accepted in the first period. The fraction $(1 - w_a)/(1 - f_{w_a,w_r}(w_1))$ is the conditional probability of being hired at w_a given that the worker was hired at w_1. The third term is the discounted expected second period payoff if the worker was not hired in period 1.

Substituting $f_{w_a,w_r}(w_1), w_a$ and w_r from (3) and (4) in terms of w_1 into the formula for Π_W above, and maximizing, gives the worker's strategy for $\overline{w}_1$.

The nature of the unique equilibrium in the repeated bargaining context is summarized in the following result. We call it a Lemma because it will be used, together with a corresponding result for the contract bargaining scheme, to prove our main comparative result, Theorem 4. For later comparisons, we will call the firm cutoff function $\bar{f}_{RB}$ to indicate the repeated bargaining scheme.

Lemma 1. *At the perfect Bayesian equilibrium of the repeated bargaining game:*

1. *The worker's strategy is given by*

$$\overline{w}_1 = \frac{(2 - \delta_F)(2 - \delta_F + 2\delta_W)}{4(2 - \delta_F + 2\delta_W)}, \tag{6}$$

$$\overline{w}_a = \frac{2 - \delta_F + 2\delta_W}{4 - 2\delta_F + 3\delta_W}, \tag{7}$$

$$\overline{w}_r = \frac{\overline{w}_a}{2}. \tag{8}$$

2. *The firm's strategy is given by*

$$\bar{f}_{RB}(w_1) = \begin{cases} w_1 & \textit{if } w_1 \le \overline{w}_r, \\ \dfrac{w_1 - \delta_F \overline{w}_r}{1 - \delta_F} & \textit{if } \overline{w}_r < w_1 \le (1 - \delta_F)\overline{w}_a + \delta_F \overline{w}_r, \\ w_1 + \delta_F(\overline{w}_a - \overline{w}_r) & \textit{if } (1 - \delta_F)\overline{w}_a + \delta_F \overline{w}_r < w_1 \le 1 - \delta_F(\overline{w}_a - \overline{w}_r), \\ 1 & \textit{if } 1 - \delta_F(\overline{w}_a - \overline{w}_r) < w_1. \end{cases}$$

3. *Any worker hired in period 1 is also hired in period 2, since* $\overline{w}_a = \bar{f}(\overline{w}_1)$. *(No unemployment among insiders.)*
4. *The worker's first period wage demand* $\overline{w}_1 = (1-\delta_F)\ \overline{w}_a + \delta_F \overline{w}_r$ *maximizes the difference* $\bar{f}(w_1) - w_1$. *(See Figure 1a.)*
5. *The firm type which is indifferent between accepting and rejecting a first period wage demand makes a positive profit in period 1 only.*

We conclude this section with a few observations on the equilibrium solution. Note that the worker's equilibrium wage demand $\overline{w}_1$ is increasing in his discount factor δ_W. The intuition is that as he becomes more patient he will be more interested in securing a higher lower bound for his second period wage if he is hired, and this lower bound $\bar{f}_{RB}(w_1)$ is increasing in w_1. Note that if the worker is bargaining with a more patient firm then he demands a lower w_1 and a higher w_a. Equilibrium expected payoffs can be calculated for both parties. Note that the expected payoff of the firm is the firm's equilibrium payoff averaged over types. From these expected payoffs it is clear that both parties would prefer to bargain with an impatient opponent.

3. Contract bargaining

In this section we analyze the equilibrium behavior in the contract bargaining scheme. In contract bargaining the worker makes a demand w_{12}, which if accepted will be his wage for both periods 1 and 2. If it is not accepted he makes another demand w_2 for his second period wage. The same type of analysis as for repeated bargaining in the previous section can be used to determine the PBE. In particular, for a given first wage demand w_{12} and a given firm cutoff function $f(w_{12})$, the worker optimizes expected second period wages conditioned on $v < f(w_{12})$. This gives w_2 as a function of w_{12} and $f(w_{12})$. Next the firm optimizes $f(w_{12})$ given w_2. Finally w_{12} is optimized by the worker to maximize total expected discounted wages. The results of this process are given in the following lemma, together with some qualitative observations on the equilibrium behavior. We use the notation $\bar{f}_{CB}$ for the firm cutoff function to indicate the contract bargaining scheme.

Lemma 2. *At the perfect Bayesian equilibrium of the contract bargaining game:*

1. *The worker's strategy is given by*

$$\overline{w}_{12} = \frac{(1+\delta_W)(2+\delta_F)^2}{(1+\delta_F)(4(1+\delta_W)(2+\delta_F) - 2\delta_W(1+\delta_F))}, \tag{9}$$

$$\overline{w}_2 = \frac{(1+\delta_W)(2+\delta_F)}{4(1+\delta_W)(2+\delta_F) - 2\delta_W(1+\delta_F)}. \tag{10}$$

2. *The firm's strategy is given by*

$$\bar{f}(w_{12}) = \begin{cases} w_{12} & \textit{if } w_{12} \le \overline{w}_2, \\ w_{12} + \delta_F(w_{12} - \overline{w}_2) & \textit{if } \overline{w}_2 < w_{12} \le (1+\delta_F\overline{w}_2)/(1+\delta_F), \\ 1 & \textit{if } (1+\delta_F\overline{w}_2)/(1+\delta_F) < w_{12}. \end{cases}$$

3. *At equilibrium, with the worker demanding* $\overline{w}_{12}, \overline{w}_2$, *only the middle part of the firm's strategy is employed, so that a worker is only hired if his value exceeds his wage demand (by the stated amount).*
4. *The firm type which is indifferent between accepting and rejecting a first period wage demand makes a positive payoff (profit) in both periods.*

4. Players' preferences between RB and CB

In this section we will make a number of comparisons between equilibrium behavior in RB and in CB. The most important comparison is that of the preferences of the players between the two schemes, where we will show that Bae's result of indifference does not hold in general when the players have different discount factors. However we will show in general (for arbitrary discount factors, but assuming the uniform distribution of values) that the two players always have the same preference between the two schemes, and give a simple condition on relative impatience, which determines their common preference. The two results stated in this section, Lemma 3 and Theorem 4, have somewhat lengthy proofs which we relegate to the Appendix.

In order to make comparisons between the preferences of the players regarding the two schemes, we must first determine the ordering of the equilibrium wage demands. In turns out that the ordering of some of these demands is independent of the discount factors.

Lemma 3. *The equilibrium two-period wage demand in the contract bargaining (CB) scheme lies between the first period wage demand and the hired worker's second period wage demand in the repeated bargaining (RB) scheme:*

$$\overline{w}_1 \leq \overline{w}_{12} \leq \overline{w}_a.$$

The main result of this paper is that, regardless of the discount factors, the worker and firm (uninformed and informed players) have a common preference between RB and CB. To determine this common preference, we define the *impatience ratio* by

$$I = \delta_W / \delta_F.$$

The last result in Theorem 4 involves the notion of 'unemployment', which requires a few preliminary words of explanation. At equilibrium, each type v will be employed (hired) for $e(v)$ periods, where $e(v)$ is 0, 1, or 2. With our assumption that v is uniformly distributed, we see that total employment E is given by $\int_0^1 e(v)\, dv$, which would equal 2 if everyone was hired in both periods. Unemployment is consequently defined by $U = 2 - E$. For RB we have that (see Appendix for details) $U_{RB} = \bar{f}_{RB}(\overline{w}_1) + \bar{f}_{RB}(\overline{w}_1)/2$, and for CB we have that $U_{CB} = \bar{f}_{CB}(\overline{w}_{12}) + \bar{f}_{CB}(\overline{w}_{12})/2$, where in each case the sum is made up of first and second period unemployment. These formulae are explained in the Appendix. We can now state our main result.

Theorem 4. *At the unique PBE of the RB and CB games, the following hold:*

(a) *If* $I > 1$, *then both players prefer RB.*
(b) *If* $I < 1$, *then both players prefer CB.*

(c) *If* $I = 1$, *then both players are indifferent between CB and RB.*
(d) *In all cases, unemployment is least in the game preferred by both players.*

It should be observed that part (c) is a special case of Bae's result, special in the sense that we consider only the case where the value v is uniformly distributed. Cases (a) and (b), on the other hand, constitute a counterexample to an extension of Bae's result (indifference) beyond the common discount factor assumption, $\delta_F = \delta_W$ (or $I = 1$).

We devote the remainder of this section to an intuitive, nonrigorous, 'explanation' of why some of the preferences are as stated. To do this we consider two cases in which one of the players is myopic (cares only about the current period). Note that some of the arguments apply to arbitrary distributions of v, while others invoke Lemma 3 which is proved for the uniform distribution. The stated preferences of the players are not strict, they include the possibility of indifference.

First suppose that the firm is myopic. Then in RB the firm will hire in the first period iff $v \geq w_1$ and in CB iff $v \geq w_{12}$. Since the firm behaves the same in either scheme, the hired worker is better off having the option of making a second period wage demand higher than his first. So the worker prefers RB. The firm only cares about the first period wage demand, the lower the better. Since it is lower in RB (that is, $\overline{w}_1 < \overline{w}_{12}$, as in Lemma 3), this is the preferred scheme. Next suppose that the worker is myopic. Since the equilibrium firm cutoff function is lower in the CB scheme than in the RB scheme, the workers will prefer CB to RB.

5. Conclusions

In this paper we analyzed the preference of bargaining parties between contract bargaining and repeated bargaining. (Their preferences are determined by their payoffs at the unique perfect Bayesian equilibria of these two schemes.) Our results should be considered in relation to that of Bae (1991) who showed that when the parties have the same discount factor, they are indifferent between the two schemes. We considered models with an arbitrary pair of discount factors for the players, but to make our comparisons we made the more restrictive assumption that the buyer's reservation price (type) is uniformly distributed. This assumption enabled us to obtain explicit formulae for the parties' payoffs in the two schemes, as functions of the pair of discount factors. We find that indifference between the schemes holds only when the discount factors are identical, which shows that Bae's result is not robust. As such our main result (Theorem 4) can be viewed as a counterexample to an extension of Bae's result to arbitrary discount factors. We found moreover that both parties always have the same preferences between the schemes, regardless of their discount factors, and gave a simple determinant of this preference (the 'impatience ratio'). We do not know to what extent our results can be extended to other distributions of reservation price, and this would seem to be an area for future research. This problem could be attacked for explicit distributions by the method given here, but for arbitrary distributions methods closer to those of Bae will probably be required.

Our models have been presented in the context of wage bargaining, where the firm's type (reservation price) is the per period value of the worker's labor.

In this context we obtained an additional result (also for the uniform distribution and arbitrary discount factors), namely that unemployment is always less in the bargaining scheme (repeated or contract) that is preferred by the players. Further work is needed to see to what extent the players prefer the 'more efficient' scheme when more general distributions of reserve price are allowed.

Appendix (Proofs of Lemma 3 and Theorem 4)

Proof of Lemma 3: By (1), (4), and (2), we get $\overline{w}_1 - \overline{w}_{12}$ equal to

$$-\frac{\delta_F}{2(1+\delta_F)(4-2\delta_F+3\delta_W)(4+2\delta_F+3\delta_W+\delta_F\delta_W)}$$
$$\times ((8\delta_F - 2\delta_F^3) + 12\delta_F\delta_W + (2\delta_F^2\delta_W - \delta_F^3\delta_W)$$
$$+ 2\delta_W^2 + 7\delta_F\delta_W^2 + 2\delta_F^3\delta_W^2) \tag{11}$$

and

$$\overline{w}_{12} - \overline{w}_a = -\frac{\delta_F((8-2\delta_F^2) + 14\delta_W + 5\delta_F\delta_W + 4\delta_W^2 + \delta_F\delta_W^2)}{(1+\delta_F)(4-2\delta_F+3\delta_W)(8+4\delta_F+6\delta_W+2\delta_F\delta_W)}. \tag{12}$$

Since all the bracketed expressions in (11) and (12) are positive, the claimed ordering of the equilibrium wage demands is proved.

Proof of Theorem 4: If an agreement is not reached in the first period, the difference between the second period wage demands $\overline{w}_r$ in (8) for RB and $\overline{w}_2$ in (10) for CB is given by

$$\overline{w}_r - \overline{w}_2 = \frac{\delta_W\delta_F(\delta_F - \delta_W)}{2(4-2\delta_F+3\delta_W)(4+2\delta_F+3\delta_W+\delta_F\delta_W)}. \tag{13}$$

It follows that $\overline{w}_r$ and $\overline{w}_2$ are equal when $\delta_F = \delta_W$, when $\delta_F = 0$ and when $\delta_W = 0$. If $\delta_F > \delta_W$, then $\overline{w}_r > \overline{w}_2$ and if $\delta_F < \delta_W$, then $\overline{w}_r < \overline{w}_2$.

The expected payoffs to both players in the two models can be calculated and we can derive whether players prefer CB or RB. The difference between the worker's expected payoff in RB (Π_W^r) and CB (Π_W^c) is given by

$$\Pi_W^r - \Pi_W^c = \frac{\delta_F(\delta_W - \delta_F)(2+\delta_W)(4-\delta_F^2+4\delta_W+\delta_F\delta_W)}{4(1+\delta_F)(4-2\delta_F+3\delta_W)(4+2\delta_F+3\delta_W+\delta_F\delta_W)}. \tag{14}$$

For the firm, the expected payoff is interpreted as the payoff the firm expects before it learns the worker's value. The difference between the firm's expected payoff in RB (Π_F^r) and CB (Π_F^c) is given by

$$\Pi_F^r - \Pi_F^c = \frac{\delta_F \delta_W (\delta_W - \delta_F)}{8(-4 + 2\delta_F - 3\delta_W)^2 (4 + 2\delta_F + 3\delta_W + \delta_W \delta_F)^2}$$
$$\times ((64 - 16\delta_F^2 - 12\delta_F^3 - 5\delta_F^3 \delta_W) + 48\delta_F + 80\delta_W + 76\delta_F \delta_W$$
$$+ 6\delta_F^2 \delta_W + 24\delta_W^2 + 28\delta_F \delta_W^2 + 7\delta_F^2 \delta_W^2). \tag{15}$$

If follows that when the discount factors are equal $\delta_F = \delta_W$ the expected payoffs to both players are equal in both models and hence players are indifferent between RB and CB. If $\delta_F < \delta_W$ both players prefer RB whereas if $\delta_F > \delta_W$ both players are better off under CB. This proves parts (a) and (b). Part (c) is similar.

To establish part (d) regarding unemployment, assume a large population of workers all bargaining independently with the firm, half of whom are in their first period and half in their second. Workers with values below the optimal strategy for the firm evaluated at the optimal first period wage demand (i.e. $\bar{f}_{RB}(\overline{w}_1)$ for RB and $\bar{f}_{CB}(\overline{w}_{12})$ for CB) are not hired in the first period. First period unemployment (or the probability of disagreement for an individual worker) is thus $\bar{f}_{RB}(\overline{w}_1)$ or $\bar{f}_{CB}(\overline{w}_{12})$. Using the Bayesian equilibrium solutions $\bar{f}_{RB}(\overline{w}_1)$ and $\bar{f}_{CB}(\overline{w}_{12})$ for RB and CB respectively, we find that

$$\bar{f}_{RB}(\overline{w}_1) - \bar{f}_{CB}(\overline{w}_{12}) = \frac{\delta_F \delta_W (\delta_F - \delta_W)}{(4 - 2\delta_F + 3\delta_W)(4 + 2\delta_F + 3\delta_W + \delta_F \delta_W)}, \tag{16}$$

and hence first period unemployment is higher in RB if $\delta_F < \delta_W$. If $\delta_F > \delta_W$ first period unemployment is higher in CB. There is no difference with respect to first period unemployment if $\delta_F = \delta_W$. Second period unemployment is nonexistent for insiders (workers employed in the first period) in both models, by definition for CB and only at equilibrium for RB. In the second period, the outsiders (those not hired in the first period) demand a wage $\overline{w}_r$ in RB and $\overline{w}_2$ in CB. Only outsiders with values above their wage demand are hired and hence for outsiders, second period unemployment is $\overline{w}_r/\bar{f}_{RB}(\overline{w}_1) = 1/2$ for RB and $\overline{w}_2/\bar{f}_{CB}(\overline{w}_{12}) = 1/2$ for CB. The fraction of outsiders is however different in the two models: $\bar{f}_{RB}(\overline{w}_1)$ in RB and $\bar{f}_{CB}(\overline{w}_{12})$ in CB. The unemployment rate is thus $\bar{f}_{RB}(\overline{w}_1)$ or $\bar{f}_{CB}(\overline{w}_{12})$ for first period workers and $\bar{f}_{RB}(\overline{w}_1)/2$ or $\bar{f}_{CB}(\overline{w}_{12})/2$ for second period workers and hence total unemployment is $3/4\bar{f}_{RB}(\overline{w}_1)$ or $3/4\bar{f}_{CB}(\overline{w}_{12})$. From (16) we conclude that unemployment is higher in CB if $\delta_F < \delta_W$ and higher in RB if $\delta_F > \delta_W$. Note that both the worker and the firm prefer the bargaining protocol that leads to least unemployment, as claimed in part (d).

6. References

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