THEORY OF REGIMES WITH PEAKING IN COMPRESSIBLE MEDIA

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UDC 517.958.533.6

The basic ideas and results of investigations of the effect on a compressible medium of a special class of gas-dynamic and thermal regimes (regimes with peaking) are expounded. A mathematical description is given of new phenomena caused by such effects (the effect of localization, formation of structures, shockless compression of continuous media). The mathematical model studied was used in a number of applications, in particular, to problems of the physics of plasmas.

INTRODUCTION

Results of theoretical investigations of the effect on a continuous medium of boundary regimes with peaking are expounded in this work.

The mathematical model describing a continuous medium includes the time-dependent equations of gas dynamics [99, 126, 128] together with the corresponding equations of state and, in the more general case, additional equations taking into account various physical processes [15, 22, 129] (thermal conductivity, viscosity, heat sources, pressure, and diffusion of the magnetic field). The functions $F_i(\mathbf{r}, t)$ sought depend on the spatial coordinates r and the time t.

Formulation of the problem includes in correspondence with physical conditions the prescription of a number of functions sought on particular surfaces bounding the medium (boundary conditions) and also the prescription of the values of the desired functions at the initial time (inertial conditions).

A special feature of the present work is the consideration of a particular class of boundary conditions. Namely, we shall be interested in processes in a continuous medium when some of the desired functions $F_i(\mathbf{r}, t)$, for example, the density of the medium or the pressure in it, grow without bound ($F_i(\mathbf{r}, t)\to\infty$) during the course of the process as the time t approaches a finite time $t_f (t\to t_f<+\infty)$. It is hereby said that the given quantities F_i behave in a regime with peaking. One of the reasons for such behavior may be the prescription of a regime with peaking as a boundary condition: for a number of F_i on surfaces Γ bounding the medium there are given regimes with peaking $F_i(\mathbf{r}, t)|_{\mathbf{r}\in\Gamma}\to\infty$ as $t \to t_f$.

There are also other reasons for the occurrence of regimes with peaking: spatial inhomogeneity of the initial data and other geometric factors leading to the appearance of cumulative effects [12, 16, 62, 128, 134], intense heat sources (combustion of explosive character) [30, 34, 73-75, 77, 92, 93, 119, 120, 138-140]. Analysis of the development in a continuous medium of regimes with peaking under the effect of these reasons is beyond the framework of the present work.

In the work principal attention is devoted to the analysis of self-similar problems whose solutions are a special case of group-invariant solutions admitted by the system of equations of gas dynamics and by the equations of gas dynamics with consideration of dissipative processes [21, 55, 78, 109]. Group-invariant solutions are an important class of solutions of systems of hyperbolic and parabolic nonlinear equations and represent the asymptotics which the state of the medium approaches in the course of time [12, 34]. Analysis of group-invariant solutions makes it possible to find regimes in which development of processes in the medium occurs with peaking.

In self-similar regimes the dependence of all quantities on time has power form [62, 126, 128].* In this case the regime with peaking for the dependence $F_{i_0}(t)$ of some quantity

*Exponential dependence on time [128] is not considered here.

Translated from Itogi Nauki i Tekhniki, Seriya Sovremennye Problemy Matematiki, Noveishie Dostizheniya, Vol. 28, pp. 3-94, 1986.

0090-4104/88/4105-1163$12.50

$F_i(\mathbf{r}, t)$ on a surface $\mathbf{r} \in \Gamma$ is given by the expression

$$F_{0i}(t) = B_{0i}(t_f - t)^{n_i}. \tag{1}$$

Here B_{0i} = const, $n_i < 0$, $t < t_f$, and $t \to t_f < +\infty$. To avoid future confusion we note that since the equations considered in the present work do not change if t is replaced by $t' = t + \tau$, τ = const (they admit "translation in time"), the notation (1) is equivalent to the notation

$$F_{0i}(t) = B_{0i}(-t)^{n_i}. \tag{2}$$

In order to exclude additional dimensionless parameters contained in non-self-similar inertial data we assume in (2) that $t \in (-\infty, 0)$: $-\infty < t < 0$, $t \to 0$ (for more details see Sec. 2, Chap. 2). Similarly, in a self-similar problem without peaking [19-21, 23-28, 40, 41, 77, 86, 87, 107, 126, 128, 141] $t > 0$ and $t \in (0, +\infty)$, $t \to +\infty$, while the boundary regimes have the form

$$F_{0i}(t) = B_{0i} t^{n_i}. \tag{3}$$

Why has attention been focused on regimes with peaking in the analysis of boundary effects on a continuous medium?

First of all, the desire to consider compression of the medium within the framework of self-similarity of first kind in which the variation of quantities are determined by variation of the boundary conditions necessarily leads to the formulation of boundary value problems with a peaking regime on the boundary. Otherwise, as is known, a shock wave may arise in the solution which cumulates according to a self-similar law [134] which is in no way connected with changes of quantities on the boundary.

Secondly, the form of the dependence of all $F_i(\mathbf{r}, t)$ on time in self-similar problems with $t < 0$(2) and with $t > 0$(3) is the same. This makes it possible to use experience accumulated in solving self-similar problems with $t > 0$ [19-21, 23-28, 40, 107, 126, 128].

Thirdly, the "similarity" of (2) and (3) enables us to pose the question of the existence of "mirror" regimes in which the spatial distributions of quantities are the same but vary with time in opposite fashion (see Sec. 3, Chap. 2): for some $n < 0$ in (2), (3) we have $\lim_{t \to -0} (-t)^n \to +\infty$, while $\lim_{t \to +\infty} t^n \to 0$. It is essential that in "mirror" regimes the direction of the time is the same: it increases (both for $-\infty < t < 0$, $t \to 0^-$ and for $0 < t < +\infty$, $t \to +\infty$ the value of the time t increases). Therefore, such "reversal of process in time" does not contradict the second law of thermodynamics: the change of entropy in "mirror" regimes is the same [64, 72].

Finally, the study of regimes with peaking [6, 7, 45-50, 57, 66-69, 74, 77, 88-92, 117-120, 138-140] led to the discovery of the effect of localization of thermal, hydrodynamic, and magnetohydrodynamic processes in particular portions of the medium and the phenomenon connected with it of nonstationary structures in a continuous (in general, dissipative) medium. To the authors this reason is one of the most important. The self-similar formulation makes it possible to study analytically conditions for the occurrence of structures. Thus, on the one hand constructive laws are given for constructing organization of the medium [93], while on the other hand the role of regimes with peaking is clarified as the asymptotics of the behavior of the medium for a particular course of processes in it. The study of regimes with peaking was stimulated to a large extent by the capacity discovered of ensuring conditions for the occurrence of organization in a continuous medium. The effect of localization of heat was first discovered by Samarskii and Sobol' [122]. The corresponding exact solution of the nonlinear heat equation received the name of a "steady-state heat wave."

It is useful to trace historically the interrelation of the concepts of a regime with peaking, localization, and occurrence of structures in a medium.

Investigation by numerical methods of the nonlinear equations of magneto-gas dynamics for a thermally conducting and conducting (and in a number of cases radiating) medium provided the possibility of observing the complex picture of the interaction of thermal and magnetohydrodynamical processes [20, 24, 28, 88, 89, 91, 93, 130]. Conditions were found under which in the medium there arose self-sustaining temperature and current formations (T-layers) localized on particular portions of matter. Particular fluid particles of the medium in which electric currents are primarily concentrated are heated to temperatures much greater than the average temperature of the medium. These structures are sustained due to dissipation of the energy of the magnetic field as Joule heat during diffusion of the field in the medium. The mechanisms contributing to the transfer of energy from the region where it was emitted to a T-layer restrained and limited its development. These mechanisms include hydrodynamic motion (shock waves and the flow of plasma excited by the growth of the pressure of matter in the region of a T-layer due to explosive-type evolution of heat in it), processes of thermal conductivity, and radiation. A T-layer is a developed nonlinear stage of the evolution of a superheated instability in magnetohydrodynamics [20, 24, 28, 44, 61, 79, 116, 121, 123, 130].

The construction of special (self-similar) solutions of the original system of equations of magnetohydrodynamics contributed in a major way to the understanding of the mechanisms of formation and evolution of a T-layer. Self-similar solutions [20, 24, 28, 130] described the scattering of a plasma against the magnetic field in the so-called regular regime [106, 107].

A special feature of this regime is the following.

We consider one-dimensional motions of the medium: $F_i(\mathrm{r}, t) = F_i(r, t)$ depends only on two variables: the spatial coordinate r and the time t. In this case it is expedient to introduce Lagrangian coordinates x and t where each given coordinate x is connected with a particular fluid particle of the medium [99]. The boundary surface Γ now corresponds to some fixed value $x = x_0 : \mathrm{r} \in \Gamma \Rightarrow x = x_0$; a boundary condition of the form (2), (3) occurs for $x = x_0$: $B_{0i}(\pm t)^{n_i} = F_i(x_0, t)$. The total mass of matter in the problems considered is finite. This means that the parameter x_0 is finite: $x_0 : x_0 < +\infty$. In this case the system of equations of gas dynamics (including the case of consideration of dissipative processes) has a solution in separated variables:

$$F_i(x, t) = B_i(t) f_i(x). \tag{4}$$

It is called a regular regime.*

The form of the self-similar solution (4) found in [20, 24, 28, 130] indicated a remarkable feature of the regular regime: nonuniformly heated [if $f_i(x)$ is the temperature] or nonuniformly compressed [if $f_i(x)$ is the density] matter remained nonuniformly heated or nonuniformly compressed during the course of the entire process. The solutions (4) described the localization in the sense of absence of heat flow of other forms of energy from inhomogeneities in the surrounding medium. In the self-similar solutions containing a T-layer studied in [20, 24, 28, 130] the processes of evolution and propagation of heat were coordinated in time.

Problems with boundary conditions of the form (3) with $t \to +\infty$ were studied in [20, 24, 28, 130]. In this case localized solutions in the absence of evolution of heat do not seem unusual. For example, they describe the process of cooling of a finite mass of gas on the boundary of which the temperature drops with time. The isentropic regime of expansion of a mass of gas supported by a piston on which the pressure decreases with time is thus the more trivial. The density of matter in all solutions of the type (3) in [20, 24, 28, 130] drops with time. These are rarefaction regimes. Therefore, the absence of shock waves in them is not surprising (the position of the shock front cannot correspond to a fixed coordinate x: a flow of matter always exists across a strong discontinuity of this type [99]).

It is naturally desirable to investigate regimes of the form (4) with an increase of the density. In this case the solution (4) describes a process of shockless compression which in itself is not trivial. To obtain such a solution it is necessary to go over to the boundary conditions from the form (3) to the form (2), i.e., to introduce into the problem regimes with peaking. Regimes of shockless compression (4), (2) were constructed and studied for the system of equations of gas dynamics with consideration of a number of dissipative processes (and also in the case where the latter are absent) [65-70, 112]. The investigation was based on the formal method of introducing values $t < 0$ in formulas of the form (4), (3) (see Sec. 1, Chap. 1). The foundation for purposeful and regular study of the effect of regimes with peaking on continuous media was thus laid.

Properties characteristic for self-similar solutions with separated variables [20, 24, 28, 106, 107] in the case of regimes with peaking revealed all unusual features [65-70, 112].

A finite mass of matter was compressed but without the occurrence of shock waves. Through the boundaries of this mass it was possible to realize an input of thermal energy in a regime with peaking, and the heat did not propagate throughout the entire mass but

*In the case of boundary conditions (2), (3) of power form all $B_i(t)$ in (4) also have power form: $B_i(t) = B_{0i}(\pm t)^{n_i}$.

remained localized on a particular portion.

Conditions for the presence in the medium of thermal structures — portions of mass heated to temperatures considerably above the average temperatures — were formulated in analogy to the works [20, 24, 28, 130] but in application to another physical situation. The heat is not lost by these portions to the surrounding colder medium, while the temperature in them grows in a regime with peaking (2). Analysis of self-similar problems [66, 67, 69, 70] makes it possible to obtain altogether definite forms and dimensions of the thermal structures. Here, in contrast to the regimes described in [20, 24, 28], localization of heat is not accompanied by cooling due to expansion; thermal structures exist not as a result of a particular "balance" between the evolution of heat due to some dissipative processes and the loss of thermal energy expended on expansion but in the presence of only heating mechanisms (evolution of heat and work of compression). In a continuous medium in the presence of heat sources and sinks there may exist temperature inhomogeneities which are stationary or evolve for an infinitely long time [3, 11, 39, 56, 83, 84, 97, 102, 132]. Thermal structures in the case of the action in the medium of heat sources only are strongly nonstationary, they evolve in a regime with peaking, and they exist only a finite time: for the start of the process from any finite time t_0 ($t_0 < t < t_f$, $t \to t_f$) the solution exists only in an interval $[t_0, t_f)$ with bounded measure:

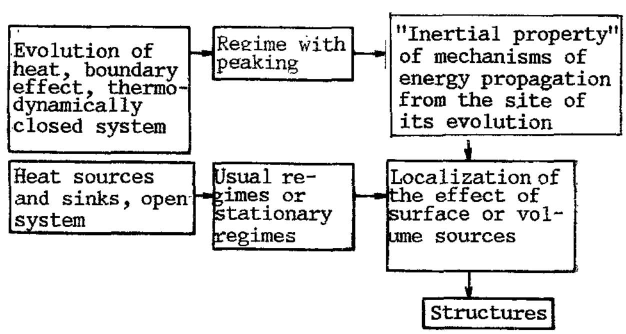
$$t_f - t_0 < +\infty.$$

The extraordinary nature of these facts stimulated in an essential way the investigations of the effect of regimes with peaking on a heat-conducting medium without consideration of gas-dynamical motion. It seemed natural that the simpler model (a boundary value problem for the heat equation or the Cauchy problem for the heat equation with a heat source) would make it possible to study effects of localization and occurrence of structures in the most complete and rigorous manner.

These investigations started from the solution mentioned above [122] in separted variables of the form (4). The study of the boundary effect on a semi-infinite medium ($0 < r < +\infty$) showed that the parameters in the regime with peaking on the boundary of the medium and the coefficients of the equation determined that portion of the mass to which the action of the boundary regime was propagated. Regimes with peaking of different form, including self-similar regimes but not coinciding with the separation of variables (4), were considered. A class of boundary regimes was distinguished in which the effect on the medium was localized, a class of regimes was indicated in which localization was absent [32, 73, 76, 77, 88, 90, 115, 118, 120], and a number of theorems were proved on localization for boundary regimes and coefficients of the equation of sufficiently general form and on localization in the Cauchy problem for a rather general form of nonlinear heat equation with a heat source [29-37, 57, 58, 74, 77, 92-98, 115, 117-120]. The physical meaning of the phenomenon of heat localization connected with the existence of spatial distributions of temperature possessing an inertial property [77] was clarified in the study of the Cauchy problem for a heat-conducting medium without a source.

As a result of the investigations conducted the concept of localization was sharpened and refined. It could now be applied in much more general situations than in the special examples of the works [20, 24, 28] and [66-70]. It is proposed to associate with localization only the effect of a surface or volume source which results in an effective change of the state of the medium only in a finite portion of it.

Study within the framework of the problems described of the phenomenon of the occurrence of ordered structures made it possible to propose the following scheme of their formation



The strongly nonstationary property (regime with peaking) plays in this scheme the same role as "openness" in the thermodynamical sense of a system with ordinary regimes (existing an arbitrarily large time).

The necessity arises of returning in a regular manner, equipped with the conceptual tools developed, to the system of equations of gas dynamics (including the consideration of dissipative processes) and displaying a class of regimes leading to localization by using the mathematical approaches developed for parabolic equations.

Such a program has been realized in application to problems of gas dynamics (Chap. 1) where, in particular, a class of localized boundary regimes is determined, and conditions for the formation of specific gas-dynamical structures are obtained.

The problem is beginning to be realized of investigating from this point of view compressible media with consideration of heat conduction and other physical effects for which regimes with peaking is separated variables (Chap. 2) have been studied in a comparatively complete manner. For example, rather broad classes of equations of state of the medium and of models of processes of heat conduction and heat evolution in it admitting solutions of the form (4) are indicated [45, 46].

Properties of regimes with peaking established in the study of various models of continuous media are important for a variety of applications which occasions the necessity of further detailed investigation and mathematical justification of them.

CHAPTER 1

BOUNDARY REGIMES WITH PEAKING IN GAS DYNAMICS

1. Effects of Localization and Formation of Structure on Compression of a Gas in a Regime with Peaking

1.1. Formulation of the Problem, Methodological Approach, and Terminology.

The natural mathematical formulation of the problem of boundary regimes with peaking in gas dynamics is the determination of gas flows arising under the action of a piston (the boundary of matter) with quantities on it which grow without bound in finite time. This contains the difference of the flows investigated from familiar regimes with peaking in gas dynamics generated, for example, by geometric factors (cumulation) [16, 26] and also from flows excited by a piston on which the quantities vary in an "ordinary" regime [124, 138, 40, 87, 86, 27].

A number of solutions of the problem of a piston compressing an isentropic, polytropic gas in a regime with peaking are known which have been discussed in connection with the search for means of supercompression of a gas for achieving extremel physical conditions [137, 133]. In these solutions the "gradient catastrophe" and formation of shock waves does not occur — the entropy of the gas does not increase for any degrees of compression. Because of this they describe an optimal process (with respect to energy expenditures for achieving a given density).

In [128] the trajectory is determined of a flat piston compressing an initially homogeneous and quiescent gas which ensures the intersection of all characteristics and of the piston simultaneously at a single point (the time of intersection is the peaking time $t = t_f$). The pressure on the piston varies according to the law

$$p(0, t) = p^0 t_f^{2\gamma/(\gamma+1)} (t_f - t)^{-2\gamma/(\gamma+1)}, \quad 0 \leqslant t < t_f \tag{1}$$

(where p^0 is the pressure of the quiescent gas and γ is the adiabatic exponent), so that $p(0, t) \to \infty$, $t \to t_f$. As $t \to t_f$ the pressure, density, and speed grow without bound: isentropic supercompression of a finite mass of gas occurs. This solution is essentially a Riemann wave with a piston adjoined to it on which the pressure is given in correspondence with (1).

Analogous centered waves of isentropic supercompression of gas in cylindrical and spherical geometries were constructed in [136, 80-82, 59, 60].

Isentropic supercompression has been studied also by means of constructing solutions of the equations of gas dynamics in separated (time and mass) variables [135]. For example, in [70] the explicit solution

$$p(x, t) = A (t_f - t)^{n_S} x^{2\gamma/(\gamma+1)}, \quad t < t_f, \tag{2}$$

was used where $x \geq 0$ is the mass coordinate, $n_S = -2\gamma(N + 1)/[2 + (N + 1)(\gamma - 1)]$, N = 0, 1, 2 is the symmetry exponent, and the constant A is determined from the boundary conditions.

The solutions described for $t \to t_f$, i.e., at the asymptotic stage, have the same regime with peaking for a pressure on the piston

$$p(0,\ t)=p_0(t_f-t)^{n_S},\quad t<t_f, \tag{3}$$

ensuring isentropic supercompression of a finite mass of gas. For a fixed type of symmetry solutions in characteristics and in separated variables differ only by details of the boundary conditions. In particular, the law for the pressure on the piston (1) coincides with formula (2) for N = 0 and $x = x_p$, where x_p is the coordinate of the piston.

We note that the approach using separation of variables carries over to the case of the presence in the medium of a large number of different physical processes (Chap. 2). Shock-free (optimal) regimes of supercompression were found on the basis of it [65-70] for various problems of plasma physics simultaneously and independently of the familiar project [137, 112].*

In Chap. 1 we study general properties of boundary regimes with peaking in gas dynamics. The following questions are posed: What is the classification of flows arising under the effect of the piston; in the processes considered do there exist "analogues" of the effects of heat localization, combustion, and formation of structures (which arise in the evolution of regimes with peaking in incompressible, heat-conducting media [119, 117]), and in what sense are these effects understood in application to gas dynamics; are more general (non-isentropic) regimes of shock-free supercompression of gas realized, and how does the character of compression depend on the distribution of entropy in matter?

Since questions of principle are concerned, our main attention is devoted to one-dimensional flows of an ideal, perfect, polytropic gas excited by a plane (in some cases a spherical or cylindrical) piston. The pressure (or speed) on the piston grows in a regime with peaking.

In contrast, for example, to parabolic equations, the equations of gas dynamics do not have a final mathematical foundation [126, 128, 114]. Therefore, we use a semiheuristic approach to clarify the questions posed.

The essence of the approach consists in the following. We construct and investigate classes of special, for example, invariant solutions with peaking. They are then used to determine space–time "boundaries" between classes of general flows. It is actually assumed that the flows studied depend in a continuous manner on the boundary conditions. This natural hypothesis can be verified and corroborated by direct numerical modeling of the corresponding problems according to the techniques of [111]. This approach makes it possible to obtain a rather complete picture of qualitative properties of boundary regimes with peaking in gas dynamics and their quantitative characteristics.

To illustrate the methodological approach and the terminology used, we consider the following problem. For the Hopf equation

$$\frac{\partial u}{\partial t}+A_0u^{\sigma}\frac{\partial u}{\partial x}=0,\quad A_0=\mathrm{Const}>0,\quad \sigma>0, \tag{4}$$

we study the first boundary value problem in $Q_T^+=(t_0,\ T)\times R_+^1$ with the conditions

$$u(t_0, x)=u_0(x)\geqslant 0,\ x\in R_+^1,\ \sup u_0<\infty, \tag{5}$$

$$u(t,\ 0)=u_1(t)\geqslant 0,\ t_0<t<T<\infty, \tag{6}$$

where the boundary function varies in a regime with peaking:

$$u_1(t)\to\infty,\ t\to T^-,\ t=T \text{ is the time of peaking.} \tag{7}$$

Equation (4), in particular, describes gas-dynamical flows of the type of simple waves (in this case here t is the time, x is the Lagrange mass coordinate, $\sigma = (\gamma + 1)/2\gamma$, u is the pressure, $A_0=(a_0^{-\gamma}\gamma)^{1/2}$, $(u/a_0)^{1/\gamma}$ is the density, and $(\gamma a_0^{-1/\gamma}u^{(\gamma+1)/\gamma})^{1/2}$ is the Lagrange sound speed) and is frequently used in modeling the equations of gas dynamics [we note that solutions corresponding to (1) and (2) are simple waves]. At the same time, for Eq. (4) (and for more general quasilinear equations of first order) it has been proved that the corresponding

*Without pretending to be complete, we mention a number of works devoted to this problem: [24, 20, 23, 9, 59, 112, 113, 22, 4, 129].

problems are well posed (existence and uniqueness of a generalized solution, theorems on comparing solutions with respect to the boundary conditions, etc.).

In the gas-dynamical interpretation problem (4)-(7) corresponds to the compression of gas by a piston on which the pressure grows without bound as $t \to T^-$.

In analogy with boundary regimes with peaking in problems of nonlinear heat conduction [118, 77] we introduce the following

Definition. We say that localization of perturbations occurs in problem (4)-(7) if there exists a constant $L < \infty$ such that

$$u(t, x) \leqslant M = \text{const} < \infty, \ x > L, \ t_0 \leqslant t < T.$$

The minimum possible value of L we call the depth of localization L^*.

From comparison theorems it follows that there can be only three possibilities:

1) $L^* > 0$. This means that $u(t, x) \to \infty$, $0 < x < L^*$, $t \to T^-$, i.e., the solution grows without bound in a region of finite dimensions while remaining bounded for $x > L^*$ (an S-regime; we use terminology similar to [118, 77]).
2) $L^* = 0$. The solution becomes infinite as $t \to T^-$ only at the boundary point $x = 0$ (an LS-regime); there exists a limit curve majorizing the solution for all $x > 0$, $t \leq T$.
3) As $t \to T^-$, $u(t, x) \to \infty$ for all $x > 0$. In this case we say that localization is absent (an HS-regime).

Problem (4)-(7) in the special case $u_1(t) = u_0(T - t)^n$, $n < 0$ and with corresponding initial data has self-similar solutions corresponding to S-, LS-, and HS-regimes for $n = -1/\sigma$, $n > -1/\sigma$, $n < -1/\sigma$, respectively (whereby for $n \geq -1/\sigma$ the solution is continuous, while for $n < -1/\sigma$ it contains a discontinuity). Analysis of their properties and comparison theorems give the following classification of unbounded, generalized solutions of the Hopf equation (details are contained in [8]).

1) Suppose in problem (4)-(7) $u_1(t) \leqslant u_0(T-t)^{-1/\sigma}$, $t < T$. Then localization occurs, and $L^* \leqslant A_0 u_0^\sigma$. If it is additionally known that $u_1(t) \geqslant \bar{u}_0(T-t)^{-1/\sigma}$, $t < T$, $\bar{u}_0 \leqslant u_0$, then $A_0 \bar{u}_0^\sigma \leqslant L^* \leqslant A_0 u_0^\sigma$ (an S-regime is realized).
2) If $u_1(t) \leqslant u_0(T-t)^n$, $n > -1/\sigma$, $t < T$, then $L^* = 0$, $u(T^-, x) \leqslant O[x^{n/(1+n\sigma)}]$, $x > 0$ (an LS-regime occurs, and a limit curve exists).
3) If $u_1(t) \geqslant u_0(T-t)^n$, $n < -1/\sigma$, $t < T$, then as $t \to T^-$, $u(t, x) \geqslant u_0(T-t)^n \to \infty$, i.e., localization is absent — an HS-regime (and a discontinuity with unbounded amplitude as $t \to T^-$ is necessarily formed in the solution).

Depending on the rate of growth of the boundary condition ["fast" and "slow" laws (7)] regimes of propagation of perturbations different in their properties are realized; the "boundaries" between them and detailed estimates for the solutions [8] are given by self-similar solutions (analogous conclusions were obtained for the process of heat conduction [118, 31, 32, 77]).

From dimension theory [126], group properties of differential equation [109, 78, 21], and other considerations [128, 41, 127] it is known that the equations of gas dynamics in mass coordinates in the case of a polytropic gas may admit the following special solutions:

$$1) \ u_i(x, t) = f_i(Dt - x),$$

$$2) \ u_i(x, t) = A_i t^{n_i} f_i(x/B_i t^m),$$

$$3) \ u_i(x, t) = A_i e^{n_i t} f_i(x/B_i e^{mt}),$$

$$4) \ u_i(x, t) = A_i e^{n_i x} f_i(B_i e^{mx}/t),$$

$$5) \ u_i(x, t) = \varphi_i(t) f_i(x).$$

The cases 1-5 exhaust all known special solutions of the equation of gas dynamics.

It is obvious that solutions 1 and 3 cannot describe boundary regimes with peaking (since it is necessary that at the point of the piston $x = 0$ the quantities grow without bound as $t \to t_f < \infty$). There remain the cases 2 — power self-similarity, 4 — exponential self-similarity, and solutions 5 in separated variables. Cases 2 and 4 are studied in detail in Sec. 1 (where, as it turns out, principal attention should be devoted to power self-similarity); case 5 is studied in Sec. 2. For solutions 2 and 4 in the region of continuous flow known beforehand (just as for solutions 1, 3) the form of the adiabatic integral has, respectively, power and exponential distribution in the x coordinate. For solutions 5 the entropy function is arbitrary.

After these preliminary remarks we pose the basic problem of Sec. 1.

We consider one-dimensional plane flows of an ideal gas described by the system of equations

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{\partial u}{\partial x},\quad \frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x},\quad \frac{\partial}{\partial t}(p\rho^{-\gamma})=0, \tag{8}$$

where p, ρ, and u are the pressure, density, and speed of matter; $x \geq 0$ is the mass coordinate.

Matter is compressed by a piston located at the point $x = 0$ on which the pressure varies in a regime with peaking:

$$p(0,t)\to\infty,\ t\to t_f,\ t_0\leqslant t<t_f<\infty. \tag{9}$$

The initial parameters of the gas are bounded functions:

$$|u(x,t_0)|\leqslant M_1,\ p(x,t_0)\leqslant M_2,\ 0<M_3\leqslant\rho(x,t_0)\leqslant M_4,\ x\geqslant 0. \tag{10}$$

In analogy to the foregoing we introduce the

Definition. Localization occurs in problem (8)-(10) if as $t \to t_f$ the quantities become infinite only in a bound region (the region of localization), i.e., either in a region of finite size an S-regime occurs or at the point $x = 0$ an LS-regime occurs. If the parameters of the gas as $t \to t_f$ grow without bound for all $x > 0$, then localization is absent (an HS-regime; with time the effect of the piston is felt at any point of the medium).

We remark that since in S⁻ and LS-regimes outside the region of localization the parameters are finite for all $t_0\leqslant t\leqslant t_f$, it follows that the region of effect of the boundary condition, in contrast to HS-regimes, is uniformly bounded in time right up to time $t = t_f$.

Localization means that the possibility exists of unboundedly concentrating energy in finite portions of matter without touching its remaining part. In parts 2, 3 conditions are determined for the existence or absence of localization of the flows described by problems (8)-(10).

1.2. Self-Similar Regimes of Compression with Peaking. The first step in the study of problem (8)-(10) is the construction and analysis of a broad class of self-similar solutions of it [6, 7, 103, 52, 53] which may exist for special boundary conditions.

For power self-similarity the pressure on the piston is a power function of time:

$$p(0,\ t)=p_0(t_f-t)^n,\ n<0,\ t_0\leqslant t\leqslant t_f. \tag{11}$$

The gas in the initial state is cold and quiescent:

$$p(x,\ t_0)=u(x,\ t_0)=0,\ x\geqslant 0. \tag{12}$$

All energy contained in matter for $t > t_0$ is communicated to it by the piston which is convenient for purposes of this consideration.

In the region of continuity of the flow the adiabatic integral has the form

$$p(x,t)=a_0x^\delta\rho^\gamma(x,t),\quad a_0>0, \tag{13}$$

where a_0, δ are parameters characterizing the distribution of entropy.

By dimension theory [126] the parameters p_0, a_0 completely determine the self-similarity. Therefore, it is necessary to eliminate "superfluous" dimensional parameters from conditions (11), (12), i.e., to set the "start" of compression at $t_0 = -\infty$ and to consider the process for $-\infty\leqslant t<t_f$.

The first boundary of the gas brought into motion (the wave front) is the boundary with unperturbed matter [see (12)] on which we have

$$p(x_\Phi^+(t), t) = u(x_\Phi^+(t), t) = 0, \quad t < t_f. \tag{14}$$

Here $x_\Phi(t) \leqslant \infty$ is the coordinate of the front, and $p(x^-(t), t) = p(x^+(t), t)$, $u(x^-(t), t) = u(x^+(t), t)$ for continuous flows. If the flow is discontinuous and at the point $x_\Phi(t)$ there is a shock wave, then the quantities to the left and right are subject to the Hugoniot conditions (in this case to the right of the discontinuity of the density of the gas is assumed constant for simplicity: $\rho(x, t) = \rho^0$, $x > x_\Phi(t)$, $t < t_f$).

The solution of problem (11)-(14) can be represented in the form

$$\begin{gathered} p(x, t) = p_0 (t_f - t)^n \pi(\xi), \quad \rho(x, t) = \rho_0 (t_f - t)^k g(\xi), \\ u(x, t) = u_0 (t_f - t)^l v(\xi), \quad k = ((2-\delta) n - 2\delta)/(2\gamma + \delta), \\ l = ((\gamma - 1 + \delta) n + \delta)/(2\gamma + \delta), \quad u_0 = (p_0^{\gamma - 1 + \delta} a_0)^{1/(2\gamma + \delta)}, \\ \rho_0 = (p_0^{2-\delta} a_0^{-2})^{1/(2\gamma + \delta)}. \end{gathered} \tag{15}$$

Here $\pi(\xi)$, $g(\xi)$, $v(\xi)$ are dimensionless functions of the self-similar coordinate

$$\xi = x / [x_0 (t_f - t)^m], \quad m = \frac{(\gamma + 1) n + 2\gamma}{2\gamma + \delta}, \quad x_0 = (p_0^{\gamma+1} a_0^{-1})^{1/(2\gamma + \delta)}. \tag{16}$$

Problem (11)-(14) goes over into the following problem for self-similar functions:

$$\begin{gathered} m\xi g' + g^2 v' = kg, \quad \delta \xi^\delta g^{\gamma - 1} g' + m\xi v' = lv - \delta g^\gamma \xi^{\delta - 1}, \quad \pi = \xi^\delta g^\gamma, \\ \pi(0) = 1, \quad \pi(\xi_\Phi) = v(\xi_\Phi) = 0, \quad \xi_\Phi \leqslant \infty, \end{gathered} \tag{17}$$

where $v(0) < \infty$ which ensures boundedness of the speed of the piston for all $-\infty \leqslant t < t_f$.

The system (17) is invariant relative to the stretching transformation and change of variables:

$$\eta = \ln \xi, \quad g(\xi) = \xi^{\frac{2-\delta}{\gamma+1}} G(\eta), \quad v(\xi) = \xi^{\frac{\gamma - 1 + \delta}{\gamma+1}} V(\eta), \quad \pi(\xi) = \xi^{\frac{2\gamma + \delta}{\gamma+1}} P(\eta)$$

and reduces to an equation of first order:

$$\begin{gathered} dV/dG = \Delta_V / \Delta_G, \\ P = G^\gamma, \quad dV/d\eta = \Delta_V / \Delta, \quad dG/d\eta = \Delta_G, \quad \Delta = m^2 - \gamma G^{\gamma+1}, \\ \Delta_V = \frac{\gamma - 1 + \delta}{\gamma + 1} \gamma V G^{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} mV, \\ \Delta_G = \frac{2\gamma + \delta}{\gamma + 1} G^{\gamma + 2} - \frac{2m}{\gamma + 1} G - lVG^2. \end{gathered} \tag{18}$$

Equation (18) with corresponding boundary conditions can be analyzed by standard methods [131] with the help of which existence of solutions and their uniqueness are established.

The properties of the flows depend on the relation between the adiabatic exponent and the exponent of the rate of growth of pressure on the piston (the parameter n).

1) An S-regime [$n = n_S = -2\gamma/(\gamma + 1)$, $m = 0$] which corresponds to separation of the variables x and t [see (16)] [6, 52]. A solution exists for $\delta > -\gamma$ (for $\delta \leq -\gamma$ due to singularity of the entropy the conditions on the piston are not satisfied); it is unique, continuous, and has a fixed finite front $x_\Phi = x_\Phi(\delta, \gamma, a_0, p_0)$. The effective dimension of the compression wave (the half width), i.e., the coordinate of the point at which $p(x_{\text{ef.}}(t), t) = \frac{1}{2} p(0, t)$, also does not change with time.

The flow is realized as a steady-state compression wave localized in the region $x \leqslant x_\Phi$, where the parameters of the gas grow without bound as $t \to t_f$ (Fig. 1). For $x > x_\Phi$ there is motionless, cold gas [for $x > x_\Phi$ the solution is continued by the stationary solution of the system (8): $p(x, t) = u(x, t) = 0$, $\rho(x, t) = \rho(x)$; $x > x_\Phi$, $t < t_f$], the effect of the piston on which is not felt during the entire compression process. An example is the explicit solution for the isentropic case ($\delta = 0$, a simple wave):

$$p(x, t) = \begin{cases} p_0 (t_f - t)^{-\frac{2\gamma}{\gamma+1}} (1 - x/x_\Phi)^{\frac{2\gamma}{\gamma+1}}, & 0 \leqslant x \leqslant x_\Phi \\ 0, & x > x_\Phi = (\gamma p_0^{\gamma+1} a_0^{-1})^{1/2\gamma}. \end{cases} \tag{19}$$

2) An LS-regime ($n > n_S$, $m > 0$ [6, 103, 104, 53]). A solution exists for $\delta > -\gamma$, is unique, and is continuous for all $x \geq 0$, $t < t_f$. The front of the compression is located at

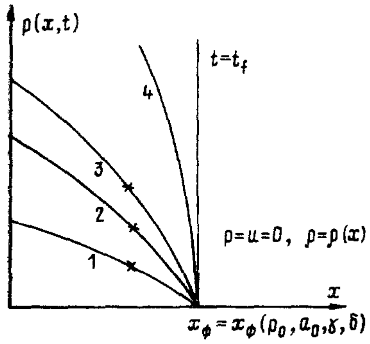


Fig. 1

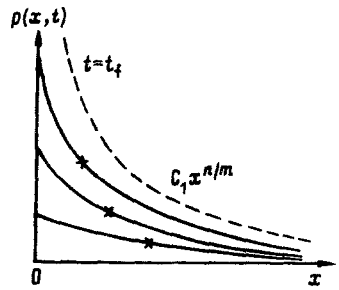


Fig. 2

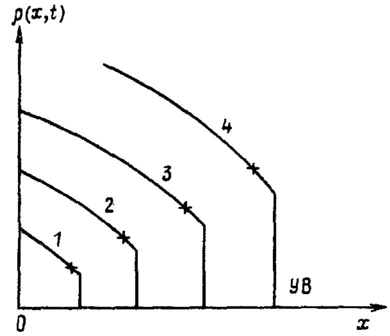


Fig. 3

Fig. 1. Steady-state compression wave. The crosses denote the half-width, $t_1 < t_2 < t_3 < t_4 < t_f$.

Fig. 2. Compression wave with contracted effective dimensions. The dashed curve indicates the limit curve, $t_1 < t_2 < t_3 < t_f$.

Fig. 3. Dynamics of the pressure in a self-similar HS-regime, $t_1 < t_2 < t_3 < t_4 < t_f$.

the point $x_\Phi(t) = \infty$, $t < t_f$ which agrees with (16) [for $x_\Phi(t) < \infty$ the perturbed region would contract in the course of time].

A compression wave with contracted effective dimensions is realized:

$$x_{\text{ef.}}(t) = \xi_{\text{ef.}} \cdot x_0 (t_f - t)^m \to 0, \quad t \to t_f, \tag{20}$$

where $\xi_{\text{ef.}} = \xi_{\text{ef.}}(n, \gamma, \delta)$ is such that $\pi(\xi_{\text{ef.}}) = 1/2$. The coordinate of any distinguished point, in particular, the "sound point" at which the mass speed of sound is equal to the propagation speed of a fixed self-similar state, is contracted according to a similar law. Between the piston (front) and the "sound point" the flow is "subsonic" ("supersonic"), respectively (in the previous case the "sound point" is situated on the wave front). The energy communicated to the gas is concentrated in a decreasing region near the boundary. Localization in the LS-regime means (see the definition on p. 1170) that there exist limit curves uniformly limiting for $t \to t_f$ the growth of gas-dynamical functions for all $x > 0$ (Fig. 2). For example, for the pressure we have

$$p(x, t) = C_1 x^{\frac{n}{m}} + C_2 x^{\frac{n-1}{m}} (t_f - t) + \ldots, \quad t \to t_f, \quad x > 0, \tag{21}$$
$$C_1 = C_1(n, \gamma, \delta) > 0, \quad C_2 = C_2(n, \gamma, \delta) < 0.$$

Any state with fixed parameters does not penetrate further than a fixed mass of gas. The pressure of matter and other quantities as $t \to t_f$ become infinite only on the piston.

At a time $t_1 \in (-\infty, t_f)$ the gas contains for $n < n_* = -2(\gamma+\delta)/(3\gamma-1+2\delta)$ finite energy and for $n \geq n_*$ infinite energy which is concentrated in a neighborhood of the wave front. After time t_1 as $t \to t_f$ either finite ($n > n_*$) or infinite ($n \leq n_*$) energy enters the gas and is concentrated near the boundary. In the isentropic case ($\delta = 0$, a simple wave) there is the analytic solution

$$\xi = \pi^{\sigma+1/n} - \pi^{\sigma}, \quad \sigma = (\gamma+1)/2\gamma.$$

<u>Remark.</u> In cases 1), 2) the solutions are continuous for all $x \leqslant x_\Phi$, $t < t_f$. From this and from (12), (13) it follows that the density of a compressible gas $\rho(x, t) \to 0$, $t \to t_0 = -\infty$, i.e., the radius of the piston $r(0, t) \to -\infty$, $t \to t_0$. This is one of the methodological differences of the given formulation from known self-similar problems of gas dynamics which contain the symbol of the density in the initial data [126, 62]. We note also that the regimes 1), 2) are "reversible" in time. With a change of the sign of the speed they describe self-similar expansion (finite or infinite) of a mass of gas pushing the piston. The spatial profiles of quantities are hereby reproduced in "reverse" order as compared with compression processes. "Time reversal" in self-similar regimes of compression with consideration of dissipative processes is considered in Chap. 2.

3) An HS-regime ($n < n_S$, $m < 0$) [105]. A solution exists for $n < -2$, $\delta = 2/(n+2)$, is unique, and has a finite front on which there is a shock wave. The coordinate of the shock wave and the half width grow without bound:

$$x_{\Phi}(t)=\xi_{\Phi}\cdot x_0\cdot(t_f-t)^m\to\infty,\quad t\to t_f;$$
$$x_{\text{ef}}(t)=\xi_{\text{ef}}\cdot x_0\cdot(t_f-t)^m\to\infty,\quad t\to t_f.$$

The motion encompasses the entire mass of gas lying in front of the piston (Fig. 3). At any point $x^* < \infty$ the parameters tend to infinity as $t \to t_f$. For example [see (15), (16)],

$$p(x^*,\ t)=p_0(t_f-t)^n\pi(\xi(x^*,\ t))\to p_0(t_f-t)^n\to\infty,\ t\to t_f.$$

Thus, for $n < n_S$ there is no localization. In this sense the flow is analogous to regimes of compression of a gas by a piston without peaking [124, 138, 40, 87, 85]. However, in the HS-regime the motion encompasses the entire medium not as $t \to \infty$ but after finite time. The flow for $n < n_S$ is a regime of superfast compression of matter.

Remark. For $n < n_S$ flows having closely related properties are realized also in the case where the density of the gas in front of the wave is variable: $\rho(x,t)=\rho^0x^{\alpha}$, $x>x_{\Phi}(t)$, $t<t_f$ [105].

Whether self-similar flows belong to an S-, LS-, or HS-regime is determined by the parameters n, γ and does not depend on the entropy distribution in the gas (the parameter δ) which influences in an essential manner the spatial behavior of such important quantities as the density and temperature.

Compression (heating) of some portion of matter depends [see (13)] on its entropy and the pressure in it. Therefore, even for monotone variation of the pressure (in regimes 1-3 the pressure and speed decrease monotonically from the piston to the front) individual portions of matter may be compressed (heated) more strongly than neighboring portions. For $n \geq n_S$ in compression waves there exist (unique) extrema of the density ($\delta < 0$) or the temperature ($\delta > 0$). In the isentropic case all quantities in a compression wave are monotonic.

Because of the properties of S- and LS-regimes the extrema are localized – in an S-regime they are connected with fixed particles of the gas (fixed with respect to mass) and have constant characteristic dimensions; in an LS-regime the coordinate and half width are contracted according to a law analogous to (20) (Fig. 4).

Local extrema of quantities are structures (in the present case gas-dynamical structures) – stable inhomogeneities having space–time order and contributing to complication of the organization of the process [93, 97, 11, 132, 39] (in the present case the process of compression). The properties of gas-dynamical structures (constancy or contraction of dimensions) and the reason for their formation (the effect of localization) unite them with nonstationary heat structures [119, 117, 58]. Complex gas-dynamical structures are considered in Sec. 2.

The properties established of S- and LS-regimes (shockless compression, localization, formation of structures, "reversibility") distinguish them in principle from regimes of gas compression without peaking [87, 86, 40, 41, 126].

Numerical calculation of problem (11)-(14) shows that self-similar solutions are stably reproduced with growth of the pressure by several orders of magnitude in spite of some distortions of the boundary condition (in numerical calculation it is not possible to exactly reproduce the singularities of the density and temperature present at the point $x = 0$).

Remark. If in place of (11) on the piston there is given a variation of speed in a regime with peaking $u(0,t)=u_0(t_f-t)^l$, $l<0$, then both problems are equivalent (their solutions are obtained from one another by simple renormalization [104]). For $l=l_S=-\frac{\gamma-1}{\gamma+1}$, $l>l_S$, $l<l_S$ S-, LS-, and HS-regimes, respectively, are realized. We note that analysis of exponential self-similarity (solution 4 in the table of part 1) gives no essentially new information regarding properties of boundary regimes with peaking.

1.3. Classification of Boundary Regimes with Peaking and Methods of Exciting Structures.

On the basis of the results of self-similar analysis and numerical modeling we give a classification of regimes of compression in problem (8)-(10) in the general (non-self-similar) case [50]. We first consider the most typical class of initial data – a homogeneous, quiescent gas ($t_0 = 0$ with no loss of generality):

$$p(x,0)=p^0>0,\ \rho(x,0)=\rho^0=(p^0a_0^{-1})^{1/\gamma},\ u(x,0)=0,\quad x>0. \tag{22}$$

1) The effect of boundary laws of the form

$$p(0,t)\leq p_0(t_f-t)^{n_S},\quad n_S=-2\gamma/(\gamma+1),\quad t<t_f, \tag{23}$$

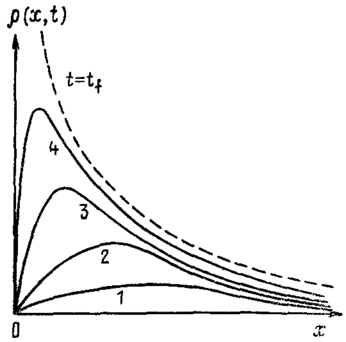


Fig. 4

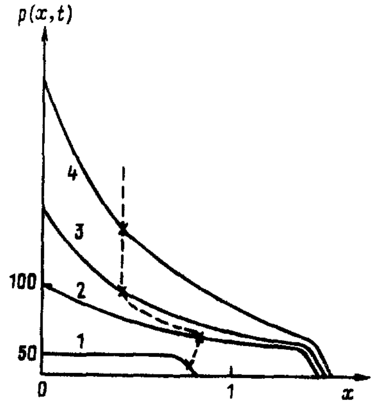


Fig. 5

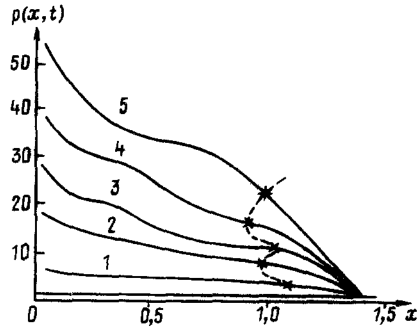


Fig. 6

Fig. 4. Self-similar structure of the density of an LS-regime, $t_1 < t_2 < t_3 < t_4 < t_f$.

Fig. 5. "Passage" to a self-similar S-regime in the presence of a shock wave. The dashed line indicates the trajectory of the half width. Parameters: $\gamma = 5/3$, $p^0 = a_0 = 1$, $p_0/p^0 = 50$, $t_0 = 0$, $t_f = 1$, $t_1 = 0.63$, $t_2 = 0.92$, $t_3 = 0.991$, $t_4 = 0.998$.

Fig. 6. Influence of deviations of the boundary law on establishing a self-similar S-regime. Parameters: $\gamma = 5/3$, $p(0,\ t) = (1 + 0.2\sin(1/(t_f - t)))(t_f - t)^{-2\gamma/(\gamma+1)}$, $a_0 = 1$, $t_f = 1$, $t_0 = 0$, $t_1 = 0.8$, $t_2 = 0.87$, $t_3 = 0.91$, $t_4 = 0.98$, $t_5 = 0.991$.

i.e., those majorized by a law of a self-similar S-regime, leads to localization of gas-dynamical motion on a finite mass of gas. The dimension of the region of localization x_Φ and the pressure in it can be bounded above by means of the solution (19) for the isentropic case in which the constant a_0 is determined either from the initial data (continuous flows) or can be estimated in terms of the parameters of the shock wave passing through the gas (see part 4).

If the boundary law is self-similar [equality sign in (23)], then "passage" to a self-similar S-regime of compression occurs; this happens as follows.

Suppose $p(0,\ 0) = p(x,\ 0) = p^0$, i.e., the initial pressure on the piston is equal to the pressure of the gas (in this case no shock wave occurs). The half width of the compression wave first increases. As the front of the compression wave approaches the boundary of the region of localization $x = x_\Phi$ [see (19)], the half wide then stabilizes. From this time on the flow is well described by the solution (19); for all $t < t_f$ the front of the wave does not penetrate beyond the depth of localization.

If $p(0,\ 0) > p(x,\ 0) = p^0$, i.e., the initial pressure on the piston is greater than the pressure of the medium, then decomposition of the discontinuity occurs at the piston, and a chock wave passes through the gas, changing its parameters. Nevertheless, for sufficient growth of the pressure on the piston the basic properties of a self-similar S-regime are reproduced. The shock wave during the course of the entire process moves with a slowly changing speed (due to the effect of localization the contact between the piston and the discontinuity is lost as the shock wave moves away from the piston). Behind the front of the shock wave compression occurs adiabatically (without the occurrence of new discontinuities) and "passes" to the asymptotics of a self-similar S-regime.

As $t \to t_f$ the shock wave actually "degenerates" into a weak discontinuity, since the (almost constant) pressure behind it steadily decreases as compared with the pressure created by the piston. Its presence changes only the depth of localization (see part 4). An example of numerical calculation of the S-regime in the presence of a jump of the pressure on the piston is presented in Fig. 5.

In the case when the law (23) is not exactly self-similar but is nearly so (for example, $p_{0_1}(t_f-t)^{n_S} \leqslant p(0,\ t) \leqslant p_{0_2}(t_f-t)^{n_S}$), an S-regime of compression with all its properties is realized in mean. The solution is bounded above and below by the corresponding self-similar solutions, while for the depth of localization there is the estimate $x_{\Phi_1} \leqslant x_\Phi \leqslant x_{\Phi_2}$ (a numerical example is given in Fig. 6).

2) If for (23) we have

$$p(0,\ t) \leqslant p_0(t_f-t)^n, \quad n_S < n < 0, \quad t < t_f, \tag{24}$$

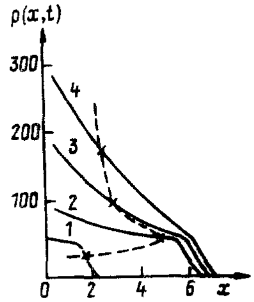


Fig. 7

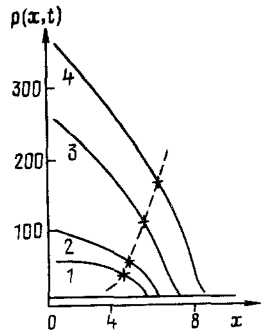


Fig. 8

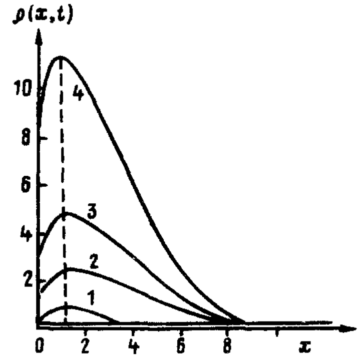


Fig. 9

Fig. 7. Establishing a self-similar LS-regime. Parameters: $\gamma = 5/3$, $n = -0.5$, $p^0 = a_0 = 1$, $p_0 = 50$, $t_f = 1$, $t_0 = 0$, $t_1 = 0.125$, $t_2 = 0.75$, $t_3 = 0.95$, $t_4 = 0.975$.

Fig. 8. An HS-regime of compression. Parameters: $\gamma = 5/3$, $p(0, t) = p_0 \exp(1/(t_f - 1) - 1)$, $p_0 = p^0 = a_0 = 1$, $t_f = 1$, $t_0 = 0$, $t_1 = 0.792$, $t_2 = 0.828$, $t_3 = 0.846$, $t_4 = 0.858$.

Fig. 9. Excitation of a density structure in an S-regime. Parameters: $\gamma = 5/3$, $a_0 = p^0 = 1$, $p_0/p^0 = 50$, $t_f = 1$, $t_0 = 0$, $t_1 = 0.61$, $t_2 = 0.915$, $t_3 = 0.974$, $t_4 = 0.992$.

i.e., the pressure on the piston grows no faster than in the case of a self-similar LS-regime, then the foregoing conclusions are considerably sharper — an SL-regime of compression is realized.

At the initial stage the half width of the compression wave increases. As its front approaches the depth of localization the half width halts and then begins to contract (during the course of the entire process the motion does not penetrate beyond the depth of localization).

The depth of localization in case (24) is determined from the fact that for any law (24) there is a boundary law majorizing it corresponding to a self-similar S-regime, i.e., $p(0, t) \leqslant p_{0S} \cdot (t_f - t)^{n_S}$, $t < t_f$. The minimal constant p_{0S} can obviously be found from the condition

$$p(0, t=0) \leqslant p_0 t_f^n = p_{0S} t_f^{n_S}.$$

The solution in case (24) is majorized by the solution (19) in which the constant $p_0 = p_{0S}$ and for the depth of localization there is the estimate (with consideration of the remarks made above concerning the constant a_0)

$$x_\Phi \leqslant (\gamma p_0^{\gamma+1} a_0^{-1})^{1/2\gamma} t_f^{\frac{2\gamma}{\gamma+1}+n}.$$

In case 2) the growth of gas-dynamical parameters in the medium is bounded by limit curves of a self-similar LS-regime. We note that the "infinite" wave front characteristic for the self-similar solution is not realized, since the process is begun at time $t_0 > -\infty$.

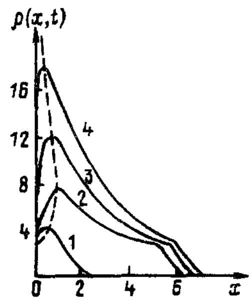
If the law (24) is a lower law, then at an advanced stage of the process the flow "passes" to self-similar asymptotics (with the exception of a neighborhood of the wave front). An example of a calculation is presented in Fig. 7 (as in the preceding case, the presence of a shock wave does not effect in principle the properties of the flow).

3) In the case where the speed of growth of pressure on the piston is no less than for some self-similar HS-regime,

$$p(0, t) \geqslant p_0 (t_f - t)^n, \quad n < n_s, \quad t < t_f, \tag{25}$$

compression takes place in an HS-regime (localization is absent). The motion extends to the entire mass of gas lying in front of the piston. All parameters as $t \to t_f$ grow without bound at any point of the medium. The half width and the wave front pass out to infinity (a shock wave with unbounded amplitude as $t \to t_f$ is necessarily formed on the front).

An example of a computation for an exponential (non-self-similar) law of peaking on the piston is given in Fig. 8. If the boundary law is a power law, then self-similar regularities



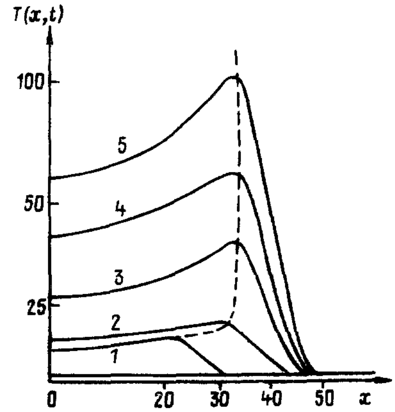


Fig. 10 Fig. 11

Fig. 10. Excitation of a density structure in an LS-regime. The dashed line indicates the trajectory of the maximum. The parameters are the same as in Fig. 7.

Fig. 11. Formation of a temperature structure in an S-regime. Parameters: $\gamma = 5/3$, $p^0 = a_0 = 1$, $p(0, t) = \begin{cases} (1-t)^{-3}, & 0 < t < t_k, \\ (1-t)^{n_S}, \; n_S = -\dfrac{2\gamma}{\gamma+1}, & t_k < t < t_f, \end{cases}$ $t_0 = 0$, $t_f = 1$, $t_k = 0.7$, $t_1 = 0.6$, $t_2 = 0.938$, $t_3 = 0.987$, $t_4 = 0.9947$, $t_5 = 0.9979$.

are established in the flow as in the previous cases.

Thus, the self-similar solutions of part 2 provide the possibility of classifying boundary regimes with peaking in a one-parameter family of functions. The circumstance that the initial data (22) correspond to a homogeneous gas has no significance in principle. From physical considerations corroborated by calculations it follows that the results remain in force also for arbitrary initial data (10).

Indeed, for example, in a S-regime the energy communicated to the gas by the piston grows without bounds as $t \to t_f$; the initial conditions at an advanced stage "are forgotten," and the character of the process is determined only by the form of the boundary law. The same holds also for LS- and HS-regimes.

At the same time, the initial state of the gas influences, for example, the distribution of entropy established in it which, in turn, determines the depth of localization and the possibility of formation of gas-dynamical structures.

The latter can be prescribed directly in the initial conditions (in correspondence with self-similar solutions) which notably restricts the class of flows considered. A technique of "existing" structures free of this drawback (which includes the case of homogeneous boundary data and a monotone regime on the boundary) is connected with the creation at the initial state of compression of an inhomogeneous entropy distribution.

Figure 9 shows a calculation illustrating the excitation of a density structure of an S-regime. The initial data correspond to a homogeneous gas; the boundary law corresponds to a self-similar S-regime. The nonisentropic property is created due to disagreement of the initial pressure on the piston and the pressure of the gas $[p(0, 0) = 50 \cdot p(x, 0)]$. As a result a shock wave leaves the piston moving with a variable (but not strongly varying) speed which creates an entropy distribution close to (13) with $\delta \approx -0.8 < 0$. A maximum of the density is subsequently formed and maintained which has the characteristic properties of the structure of an S-regime — its size is constant, and the maximum is connected with a fixed mass of gass. A density structure (Fig. 10) can be excited in an LS-regime in a similar way (the boundary law is self-similar); in time its maximum approaches the piston. Figure 11 shows an example of the formation of a temperature structure. In contrast to the foregoing, the shock wave is created due to the fact that the boundary law at the initial stage corresponds to an HS-regime (while subsequently it corresponds to an S-regime).

Remark. If a change of speed in a regime with peaking is given on the piston, then the classification obtained remains in force [to cases 1), 2), 3) there correspond the respective inequalities]

$$u(0,t) \leqslant u_0 (t_f - t)^{l_S}, \quad l_S = -\frac{\gamma-1}{\gamma+1};$$
$$u(0,t) \leqslant u_0 (t_f - t)^l, \quad l > l_S;$$
$$u(0,t) \geqslant u_0 (t_f - t)^l, \quad l < l_S, \quad t < t_f$$

1.4. Quantitative Conditions for the Occurrence of a Localization Effect. To determine concrete quantitative conditions for the occurrence of a localization effect we consider the effect of power regimes (11) for $n \geq n_S$ on a homogeneous, quiescent gas (22) [50].

Introducing the quantities

$$\bar{t} = t/t_f, \quad \bar{x} = x/x_0, \quad x_0 = (t_f (p_0 t_f^n)^{\frac{\gamma+1}{\gamma}} a_0^{1/\gamma})^{1/2},$$
$$\bar{p} = p/(p_0 t_f^n), \quad \bar{\rho} = \rho/(\rho^0 t_f^n a_0)^{1/\gamma}, \quad \bar{u} = u/((p_0 t_f^n)^{\frac{\gamma-1}{\gamma}} a_0^{1/\gamma})^{1/2}, \tag{26}$$

we write the system (8) and conditions (11), (22) in dimensionless form

$$\frac{\partial}{\partial \bar{t}}\left(\frac{1}{\bar{\rho}}\right) = \frac{\partial \bar{u}}{\partial \bar{x}}, \quad \frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{x}}, \quad \frac{\partial}{\partial \bar{t}}(\bar{p}/\bar{\rho}^\gamma) = 0,$$
$$\bar{p}(0, \bar{t}) = (1-\bar{t})^n, \quad 0 \leqslant \bar{t} < 1, \tag{27}$$
$$\bar{p}(\bar{x}, 0) = p^0/(p_0 t_f^n) = 1/\bar{P}, \quad \bar{p}^0 (\bar{\rho}^0)^{-\gamma} = 1, \quad \bar{u}(\bar{x}, 0) = 0, \quad \bar{x} > 0.$$

Thus, the solution of the initial problem depends on three dimensionless parameters n, γ, $\bar{P}$ ($\bar{P}$ is the ratio of the pressure on the piston to the pressure of the gas at the initial time).

The dimensionless quantities $R_1 = \bar{t}_{loc} = 1 - \bar{t}_{est}$ and $R_2 = \bar{p}(0, \bar{t}_{est})/\bar{p}(0, 0) = R_1^n$ are of main interest. The quantity $\bar{t}_{est}$ is the time of establishing an S- or LS-regime; during time $\bar{t}_{loc}$ a localization effect "occurs" (the size of the compression wave is no less than 9/10 the depth of localization; its half width is constant or contracts). The quantity R_2 shows how much the pressure on the piston must grow as compared with the initial pressure for the occurrence of the effect of localization.

The notation (27) and the relations (26) show that conditions for the occurrence of localization depend on dimensionless quantities. Therefore, the effects studied can be investigated in different ranges of time, pressure, and compressible masses by choosing the most suitable scale. In particular, by varying the total time of the compression process, it is possible in principle to obtain any magnitude of the localization time $t_{loc} = \bar{t}_{loc} \cdot t_f$.

The quantity $R_2 = R_2(n, \gamma, \bar{P})$, i.e., the relative growth of the pressure on the piston, is the main dimensionless criterion for the occurrence of the localization effect. Calculations performed for a wide range of the parameters n, γ, $\bar{P}$ show that the quantity $R_2(n, \gamma, \bar{P})$ = 10-50, while $\bar{t}_{loc}$ amounts to up to 8% of the total time of the process.

Numerical modeling also makes it possible to check the exactness of the theoretical estimate of the influence of the shock wave arising in the case $\bar{P} > 1$ due to disagreement of the initial pressure on the piston and the pressure of the gas (we recall that the size of the region of localization depends on its amplitude). As numerical calculations show, the intensity of the shock wave varies weakly in the compression process (see Figs. 5, 7, 10) and is approximately equal to $\bar{P}$. Hence, the shock wave leaves in its wake a homogeneous background of "new" pressure, density and entropy (computable on the basis of the initial data and the quantity $\bar{P}$). If the speed of the motion arising behind the shock wave is neglected, then we arrive at an initial problem with an initial homogeneous, quiescent gas for $\bar{P} = 1$ (without disagreement and occurrence of a shock wave). Thus, for estimates it is possible to use the isentropic solution (19), taking in it in place of a_0 the quantity

$$a_1 = a_0 \bar{P}\left[\frac{(\gamma+1) + (\gamma-1)\bar{P}}{(\gamma+1)\bar{P} + (\gamma-1)}\right]^\gamma,$$

and to determine the depth of localization

$$x_{\Phi_1} = x_{\Phi_0}\left[\bar{P}\frac{(\gamma+1)\bar{P} + (\gamma-1)}{(\gamma+1) + (\gamma-1)\bar{P}}\right]^{1/2},$$

TABLE 1

γ \ $\bar{P}$	1	25	50	100
1,4	$\frac{0{,}037}{46{,}1}$	$\frac{0{,}031}{48{,}4}$	$\frac{0{,}025}{53{,}4}$	$\frac{0{,}02}{54{,}1}$
1,66	$\frac{0{,}062}{32{,}2}$	$\frac{0{,}052}{36{,}5}$	$\frac{0{,}032}{48{,}2}$	$\frac{0{,}029}{51{,}6}$
2,0	$\frac{0{,}089}{25{,}1}$	$\frac{0{,}079}{26{,}9}$	$\frac{0{,}057}{29{,}3}$	$\frac{0{,}056}{30{,}7}$

TABLE 2

γ \ $\bar{P}$	1	25	50	100
1,4	$\frac{1{,}18}{1{,}21}$	$\frac{14{,}49}{9{,}8}$	$\frac{20{,}4}{13{,}6}$	$\frac{28{,}98}{18{,}6}$
1,66	$\frac{1{,}29}{1{,}38}$	$\frac{13{,}00}{8{,}4}$	$\frac{18{,}3}{12{,}0}$	$\frac{25{,}8}{17{,}0}$
2,0	$\frac{1{,}41}{1{,}6}$	$\frac{12{,}25}{8{,}2}$	$\frac{17{,}32}{11{,}6}$	$\frac{24{,}94}{16{,}1}$

which in the strong-wave approximation gives

$$x_{\Phi_1}=x_{\Phi_0}\left(\bar{P}\,\frac{\gamma+1}{\gamma-1}\right)^{1/2}. \tag{28}$$

Table 1 presents the values of criteria R_1 (the upper row) and R_2 (the lower row) for an S-regime for various γ, $\bar{P}$; Table 2 shows the sizes of the region of localization obtained in computing (upper row) and from formula (28) (lower row) which provides a satisfactory majorizing estimate for the depth of localization.

2. Theory of Adiabatic Supercompression of a Gas for an Arbitrary Entropy Distribution

2.1. Formulation of the Basic Problems. As follows from the classification of boundary regimes with peaking obtained in Sec. 1, whether a flow belongs to an S-, LS- or HS-regime is determined by the relation between the speed of change of pressure on the piston and the adiabatic exponent and does not depend on how the entropy is distributed in matter. The entropy distribution influences in an essential manner the spatial characteristics of the flow (nonisentropicity leads to the appearance of density or temperature structures in the compression waves).

Solutions of the equations of gas dynamics in separated (mass and time) variables (case 5, p. 1169) provide the only possibility for a detailed study of the effect of the entropy distribution on the effects studied [47, 48]. The entropy function corresponding to them is arbitrary, while for the remaining classes of special solutions of the equations of gas dynamics the adiabatic integral has a special form. We recall that time characteristics of solutions in separated variables correspond to a boundary S-regime "separating" different LS- and HS-regimes of compression according to their properties.

Suppose for the one-dimensional equations of gas dynamics

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{\partial (r^N u)}{\partial x},\quad \frac{\partial u}{\partial t}=-r^N\frac{\partial p}{\partial x},\quad \frac{\partial r}{\partial t}=u,\quad \frac{\partial (p\rho^{-\gamma})}{\partial t}=0, \tag{29}$$

where r is the Euler coordinate, and N = 0, 1, 2 is the geometrical index; the general solution of the last equation (the adiabatic integral) can be written in the form

$$p\rho^{-\gamma}=\varphi^{\gamma}(x). \quad (30)$$

The entropy function (30) is an arbitrary summable function with a finite number of discontinuities corresponding to contact discontinuities; we recall that we are investigating flows without shock waves. A solution of Eqs. (29) in separated variables x and t has the form

$$\begin{gathered}
p(x, t)=(t_f-t)^{n_S}\pi(x), \quad n_S=-2\gamma(N+1)/[\gamma+1+N(\gamma-1)], \\
u(x, t)=u_0(t_f-t)^{n_1}u(x), \quad n_1=\frac{\gamma-1}{2\gamma}n_S, \\
\rho(x, t)=(t_f-t)^{n_S/\gamma}g(x), \quad g(x)=\pi^{1/\gamma}(x)\varphi^{-1}(x), \\
r(x, t)=R_0(t_f-t)^{n_1+1}R(x), \quad R(x)=-v(x).
\end{gathered} \quad (31)$$

Remark. The temporal behavior of solutions in separated variables (they belong to the class of flows with homogeneous deformation [108]) is determined by quadrature, generally speaking [125, 135]. For regimes of compression the quadrature is asymptotic, i.e., as $t \to t_f$ it gives a power law with peaking (which is a special solution for all $t < t_f$). Considering this, for simplicity of exposition (but with no loss of generality) we shall consider solutions in the form (31).

As follows from (31), as $t \to t_f$ the pressure, density and speed of all particles of the gas grow without bound; their radius tends to zero. In particular, the pressure on the piston compressing the gas (from which the mass is reckoned) varies in a regime with peaking:

$$p(0, t)=(t_f-t)^{n_S}\pi(0)=p_0(t_f-t)^{n_S}. \quad (32)$$

The spatial distribution of quantities (for an appropriate choice of u_0 and R_0) is described by the equations

$$\pi''=\begin{cases}\pi^{-1/\gamma}(x)\cdot\varphi(x), \ v(x)=-\pi'(x), \ N=0\\ 0, \ v(x)=\pm\left(\int 2\varphi(x)\pi^{-1/\gamma}(x)\,dx\right)^{1/2}, \ N=1\\ -(\pi'(x))^4\pi^{-1/\gamma}(x)\varphi(x), \ v(x)=-1/\pi'(x), \ N=2.\end{cases} \quad (33)$$

From condition (32) we obtain

$$\pi(0)=p_0, \quad |v(0)|<\infty, \quad (34)$$

where the (dimensional) quantity p_0 determines the pressure scale on the piston whose speed is bounded for all $t < t_f$.

The second boundary condition for the system (33) is considered at some point $x = x_\Phi$ (which may correspond to the boundary of the region of localization, the center of symmetry [48, 49, 51], the boundary of a receding cavity, the line of separation of the medium into two components, etc.):

$$\pi(x_\Phi)=\pi_0\geqslant 0, \quad v(x_\Phi)=v_0\geqslant 0. \quad (35)$$

For example,

A. $\pi_0=v_0=0.$ (36)

The pressure, speed, and density of matter at the point $x = x_\Phi$ vanish — for N = 0 the effect of localization of gas-dynamical properties occurs (see also part 1.2).

B. $\pi_0>0, \ v_0=0.$ (37)

The gas is pressed by the piston to a fixed wall.

C. $\pi_0=0, \ x_0\neq 0.$ (38)

In this case the point $x = x_\Phi$ is the boundary with the vacuum. The problem describes either a process of scattering to the vacuum or (for N = 1, 2) the recession of an internal cavity under the action of the piston (32).

D. $\pi_0\neq 0, \ v_0\neq 0.$ (39)

Problem (33), (34), (39) corresponds to compression by an exterior shell ($0\leqslant x\leqslant x$) of an interior part of the matter ($x > x_\Phi$).

Generally speaking, the boundary conditions (34), (35) depend on one another. Because of this two basically different problems should be considered for the system (33).

Problem I. Compression of a gas in a given regime.

Suppose the following parameters of the flow have been determined: the mass of gas being compressed x_Φ and the boundary conditions (35). It is required to find a solution of problem (33), (35) and to determine the law of growth of the pressure on the piston – the constant p_0 in (34).

Problem II. Compression of a gas for a prescribed boundary law.

Suppose the law of growth of pressure on the piston [the boundary condition (34), i.e., the constant p_0] is fixed. It is required to determine under what boundary conditions (35) on the right boundary (the constants v_0, π_0, x_Φ) there exists a solution of problem (33)-(35) and to find it.

Thus, from a physical point of view in the case of Problem I for a fixed mass of gas x_Φ and particular boundary conditions at the point $x = x_\Phi$ (A, B, C, or D) an optimal regime of compression (a law of growth of pressure on the piston) is chosen. For Problem II for a given boundary law of compression the optimal masses of matter being compressed, the pressures on the boundary of the two media, the speed of scattering to the vacuum, etc., are found.

For the nonlinear system of equations (33) Problem I is a Cauchy problem [which becomes singular for $v(x_\Phi) = v_0 = 0$], while Problem II is an overdetermined two-point boundary value problem for the eigenvalues.

We shall consider successively the cases N = 1, 0, 2.

2.2. An Exact Solution in the Axially Symmetric Case. Separation of the time and mass variables in the equation of axially symmetric motion of the gas [6, 53, 46]

$$\frac{\partial^2 r}{\partial t^2} = -r\frac{\partial p}{\partial x} \tag{40}$$

leads to the two equations

$$\begin{gathered}\pi'(x)=\lambda, \quad r_1''(t)\cdot r_1^{-1}(t) = -\lambda,\\ p(x, t) = p_1(t)\,\pi(x), \quad r(x, t) = r_1(t)\,R(x),\end{gathered} \tag{41}$$

where λ is a constant of integration.

From (41) we find that

$$p(x, t) = p_1(t)\,(\lambda x + C), \quad C = \text{const.} \tag{42}$$

Hence, aside from the dependence on properties of the medium and also on the equations of energy balance and continuity in all flows described by self-similar solutions in separated variables the pressure is a linear function of the mass coordinate. In the mass where Eq. (40) additionally contains a potential force this conclusion is valid for the quantity $\Sigma = p + \Pi$ (Π is the potential). We note also that if in adiabatic compression $p_1(t)$ and $r_1(t)$ are power functions, then (see also [45, 46])

$$p_1(t) = p_0(t_f - t)^{-2}, \tag{43}$$

i.e., the exponent in the law of growth of pressure on the piston does not depend on the properties of the medium (γ).

The property (42) of axially symmetric flows makes it possible to construct a complete (explicit) solution of Eq. (33)

$$\begin{gathered}\pi(x) = \frac{\pi_0 - p_0}{x_\Phi}\,x + p_0,\\ v(x) = \pm\left(\int 2\varphi(x)\left(\frac{\pi_0 - p_0}{x_\Phi}\,x + p_0\right)^{-1/\gamma} dx\right)^{1/2}.\end{gathered} \tag{44}$$

The constant of integration in the expression for the speed (44) is chosen from the boundary condition at $x = x_\Phi$.

For appropriate choice of the constants π_0, x_Φ, v_0 the solution (44) describes all types of values A, B, C, D.

Remark 1. The existence of the integral in (44) is the only (natural) condition for the existence of solutions imposed on the function $\varphi(x)$.

Thus, for axially symmetric flows (N = 1) a solution of Problems I and II exists and is unique (a situation different in principle occurs in the case N = 0, 2).

An inhomogeneous entropy distribution in the medium and development of a regime with peaking leads to the appearance of structures in compressible matter (Sec. 1). For N = 1 conditions for the existence of gas-dynamical density and temperature structures can be found in explicit form:

$$\frac{d\varphi}{dx}=\frac{\gamma\lambda}{\lambda x+C},\ \frac{d\varphi}{dx}=-\frac{\gamma}{\gamma-1}\frac{\lambda}{\lambda x+C},\quad \lambda=\frac{\pi_0+p_0}{x_\Phi},\ C=p_0. \tag{45}$$

In the case $\varphi'(x)=0$ there are no structures (isentropic compression [59, 82)]; for $\varphi'(x)\leqslant 0$ only density structures can exist, while for $\varphi'(x)\geqslant 0$ only temperature structures can exist.

The number of extrema of the density is determined by the slope of the function $\varphi(x)$; hence, even for a monotone entropy distribution in a compressible medium there may exist various complex gas-dynamical structures.

2.3. Plane-Symmetric Flows and the Effect of Localization. For N = 0 the equation investigated and the boundary conditions have the form

$$\pi''-\pi^{-1/\gamma}\varphi(x)=0, \tag{46}$$

$$\pi(0)=p_0,\quad |\pi'(0)|<\infty, \tag{47}$$

$$\pi(x_\Phi)=\pi_0\geqslant 0,\quad \pi'(x_\Phi)=-v_0\leqslant 0. \tag{48}$$

We first consider the most interesting (and complex) situation A describing the effect of localization of gas-dynamical flows.

THEOREM 1. A solution of Problem I in the case $\pi_0 = v_0 = 0$ for all $\varphi\in C(x>0)$ and any x_Φ exists, is unique, and depends continuously on the boundary conditions (x_Φ) and the right side [$\varphi(x)$, γ] [for any $x_\Phi > 0$ there exists a unique value of the quantity p_0 and a corresponding solution $\pi(x)$].

Remark 2. The result of Theorem 1 holds also if in a neighborhood of the point x_Φ the function $\varphi(x)$ admits a representation of the form $\varphi(x)=(x_\Phi-x)^\delta\psi(x)$, where $\psi(x)$ is continuous and $\psi(x_\Phi)>0$, $\delta>-(1-1/\gamma)$.

Theorem 1 establishes the following: if the depth of localization is given [the mass of compressible matter x_Φ), then for any entropy distribution in the medium $\varphi(x)$ there exists a unique boundary regime of the form (2), i.e., a corresponding value of the constant p_0 making it possible to localize shockless compression on a given mass of gas.

This cannot be said of Problem II, i.e., it cannot be asserted that in the general case for any $p_0 > 0$ there exists a corresponding value x_Φ for which there exists a solution of it (examples are presented in part 5). In view of this we formulate theorem of existence and uniqueness of a solution of Problem II.

THEOREM 2 (existence). Suppose $\varphi(x)$ is such that $\lim\limits_{x\to\infty}\int_0^x s\varphi(s)\,ds=\infty$. Then for any $p_0 > 0$ there exists at least one value x_Φ for which Problem II is solvable.

THEOREM 3 (uniqueness). We introduce the function

$$\psi_0(x)=\varphi(x)\,(p+qx)^2. \tag{49}$$

Suppose there are numbers p, q > 0 such that $\psi_0(x)$ is a nondecreasing function of x. Then a solution of Problem II exists and is unique.

Remark 3. The estimate (49) is best possible, i.e., if it is required that the function $\psi(x)=\varphi(x)\,(p+qx)^{2+\varepsilon}$, $p,\ q>0$, $x\geqslant 0$, be nondecreasing, then for any $\varepsilon > 0$ it is possible to construct an example showing that Theorem 2 is false (part 5).

Thus, a solution of Problem II does not always exist. Moreover, in the case of rapid entropy decay with increasing x it may not be unique (part 5). This means that to different self-similar solutions there correspond different spatial distributions of the gas-dynamic functions at the time of the start of compression which give the same entropy distribution throughout the mass of gas and correspond to the same law of growth of pressure on the piston (a spectrum of solutions).

Generally speaking, the quantity Π in the conditions of Problem II may assume any values. We recall that by definition for $x_\Phi = \infty$ localization of the gas-dynamical variables is absent — the entire mass of gas lying in front of the piston is subject to compression (part 5). In connection with this we formulate the

Localization Criterion. The conditions of Theorem 3 are necessary and sufficient for localization of gas-dynamical flows under the conditions of Problem II.

Theorems 2 and 3 show that the existence of a solution of Problem II, its uniqueness, and the localization criterion depend entirely on the properties of the function $\varphi(x)$ (primarily on its rate of decay as $x \to \infty$).

The well-posedness of Problems I and II (if the conditions of Theorems 1-3 are satisfied) makes it possible to construct a broad class of estimates for solutions and for the quantities x_Φ and p_0 [47, 48].

In analogy to the case of axially symmetric flows the conditions for the existence of gas-dynamical structures in a compression wave have the form

$$\begin{gathered} \ln \pi'(x)=\gamma \ln \varphi'(x) \text{ (density)}, \\ \ln\pi'(x)=-\frac{\gamma}{\gamma-1}\ln \varphi'(x) \quad \text{(temperature)}. \end{gathered} \tag{50}$$

From (50) it is immediately evident that in the isentropic case there are no structures, while for $\varphi'(x)\leqslant 0$ $(\varphi'(x)\geqslant 0)$ there may exist only density (temperature) structures including complex structures [47, 48].

We now consider the more general case π_0, $v_0 > 0$. Regarding the function $\varphi(x)$, we henceforth assume the following:

a) $\varphi(x)\geqslant 0$ and is continuous everywhere in $x\in[0, \infty)$, with the exception of a finite number of points of discontinuity.

b) If $x = x^*$ is a point of discontinuity (in particular, it is possible that $x^* = x_\Phi$), then in a neighborhood of it $\varphi(x)$ can be represented in the form

$$\varphi(x)=|x-x^*|^{\delta}\psi(x),$$

where $\psi(x)$ is continuous and $\psi(x^*)>0$, $\delta>-(1-1/\gamma)$.

c) For any x_1, $x_2\in[0, \infty)$ there exists $\int\limits_{x_1}^{x_2}\varphi(s)\,ds>0$.

THEOREM 4. A solution $\pi(x)$ of Problem I exists and is unique for any $x_\Phi>0$, $\pi_0\geqslant 0$, $v_0\geqslant 0$.

Proof. If $\pi_0 > 0$, then Problem I is a regular Cauchy problem, and Theorem 4 follows from classical results [110, 18]. The case $\pi_0 = v_0 = 0$ was considered in Theorem 1. Finally, when $\pi_0 = 0$, $v_0 > 0$, writing Eq. (46) in the form

$$\pi''=\frac{\varphi(x)}{(x_\Phi-x)^{1/\gamma}}\left(\frac{\pi(x)}{x_\Phi-x}\right)^{-1/\gamma} \tag{51}$$

and noting that

$$\lim_{x\to x_\Phi}\frac{\pi(x)}{x_\Phi-x}=v_0>0, \tag{52}$$

we obtain the assertion of Theorem 4 from the results of [47].

Under the conditions of Problem II the cases $v_0 = 0$ (A and B) and $v_0 > 0$ (C and D) are different in principle.

THEOREM 5. There exists p_0^* such that for any $p_0 > p_0^*$ and $v_0>0$, $\pi_0\geqslant 0$ there exists at least one value $x_\Phi > 0$ for which there exists a solution of Problem II.

Proof. We set $Q(v_0) = p_0 = \pi(0)$ and let $p_0^*=\inf\limits_{v_0\geqslant 0} p_0$. Since $\pi''\geqslant 0\,[0, x_\Phi]$, the solution of Problem II (which exists and is unique — see Theorem 4) lies above the line $\pi = p_0 - v_0 \times (x - x_\Phi)$. Hence, for any $p_0\geqslant p_0^*$ and π_0, $v_0 > 0$ it is always possible to choose x_Φ so that $\pi(0) = p_0$, when the assertion of Theorem 5 follows.

Remark 4. If $v_0 = 0$, then, generally speaking, Theorem 5 is false. In this case it is necessary to impose additional restrictions on the function $\varphi(x)$, namely, to require that the conditions of Theorem 2 be satisfied.

THEOREM 6. Suppose there exist p, q > 0 such that the function $\psi(x)=\varphi(x)(p+qx)^2$ is nondecreasing. Then a solution of Problem II exists and is unique.

The proof of Theorem 6 is altogether analogous to the proof of Theorem 3 presented in [47]. It should be noted that the given estimate is best possible as in Theorem 3.

Thus, condition (49) is the principal criterion for the existence and uniqueness of a solution of Problem II.

2.4. Shockless Compression by a Spherical Piston. In the case N = 2 we arrive at the following problem:

$$\pi'' + (\pi')^4\pi^{-1/\gamma}\varphi(x) = 0, \tag{53}$$

$$\pi(0) = p_0, \quad |(\pi'(0))^{-1}| < \infty, \tag{54}$$

$$\pi(x_\Phi) = \pi_0 \geqslant 0, \quad \pi'(x_\Phi) = -\frac{1}{v_0}, \quad v_0 \geqslant 0. \tag{55}$$

Regarding the function $\varphi(x)$, the assumptions are the same as in the case N = 0 with consideration of the additional condition in a neighborhood of a point of discontinuity $x = x^*$

$$\varphi(x) = \psi(x)|x^* - x|^\delta, \ \psi(x^*) > 0, \ -(1-1/\gamma) < \delta < 2.$$

The brief list of results presented in the present work is a generalization of the conclusions of the work [48] (where the case $v_0 = 0$, $\pi_0 \geqslant 0$, which is most interesting and complex to analyze, was considered). The techniques and details of the proofs of the theorems and also the estimates of the solutions are altogether analogous to those presented in [48].

THEOREM 7. A solution of Problem I for Eq. (53) exists and is unique.

Proof. If $\pi_0 > 0$, $v_0 > 0$, then the assertion follows from [110, 18]. For $\pi_0 \geqslant 0$, $v_0 = 0$ the validity of Theorem 7 follows from [48]. Finally, in the case $\pi_0 = 0$, $0 < v_0 < \infty$, writing (in analogy to Theorem 4) (53) in the form

$$\pi'' + (\pi')^4\left(\frac{\pi}{x_\Phi - x}\right)^{-1/\gamma}\cdot\frac{\varphi(x)}{(x_\Phi - x)^{1/\gamma}} = 0 \tag{56}$$

and noting that $\lim\limits_{x\to x_\Phi}\dfrac{\pi(x)}{x_\Phi - x} = (v_0)^{-1}$, we obtain the existence and uniqueness of a solution of Problem I from results of the work [48].

Remark 5. From the existence and uniqueness of a solution of Problem I we obtain its continuous dependence on the right side $(\varphi(x), \gamma)$ and the boundary conditions (x_Φ, v_0, π_0).

THEOREM 8 (on the existence of a solution of Problem II). Suppose $\varphi(x) > 0$, $0 \leqslant x < \infty$ and

$$\lim_{x\to\infty}\frac{\varphi(x)}{x^2} = 0. \tag{57}$$

Then for any values p_0 Problem II is solvable. If there exists q > 0 such that $\varphi(x) \geqslant qx^2$, $0 \leqslant x_1 \leqslant x < \infty$, then it is always possible to find $p_0^* = p_0^*(q, v_0, \pi_0)$ such that for $p_0 > p_0^*$ a solution of Problem II does not exist.

THEOREM 9 (on uniqueness of a solution of Problem II). We introduce the function

$$\psi_2(x) = \varphi(x)(p+qx)^{-2}. \tag{58}$$

Suppose there exist numbers p, q > 0 such that the function $\psi(x)$ is nonincreasing. Then a solution of Problem II exists and is unique.

Remark 6. Comparison of Theorems 6 and 9 shows that for uniqueness of a solution of Problem II in the planar case a condition is imposed on the rate of decay of the function $\varphi(x)$ [$\psi_0 = \varphi(x)(p+qx)^2$ is nondecreasing], while for N = 2 a condition is imposed on the rate of its growth [$\psi^2(x) = \varphi(x)(p+qx)^{-2}$ is nonincreasing].

Remark 7. The estimate (58), just as all those obtained earlier (see Theorems 3 and 6), is best possible.

Conditions for the existence of gas-dynamical structures in the solutions constructed are completely analogous to (50).

We briefly formulate the main conclusions regarding properties of solutions of Problems I and II.

For axially symmetric flows solutions of Problems I and II always exist (with consideration of Remark 1) and are unique. They are described by the explicit formulas (44).

A solution of Problem I for all boundary conditions at $x = x_\Phi$ and in the entire range of admissible entropy distribution exists, is unique, and depends continuously on $\varphi(x)$, γ, v_0, π_0, x_Φ.

A solution of Problem II exists if $\int\limits_0^{x_\Phi} s\varphi(s)\,ds \to \infty$ $(x_\Phi \to \infty)$ in the case N = 0 and if $\lim\limits_{x\to\infty} \times \frac{\varphi(x)}{x^2} = 0$ for N = 2.

A solution of Problem II exists and is unique if there exist numbers p, q > 0 such that the function $\psi_0(x) = \varphi(x)(p+qx)^2$, $N=0$ [$\psi_2(x) = \varphi(x)(p+qx)^{-2}$, $N=2$] is nondecreasing (nonincreasing).

In the plane-symmetric case the gas-dynamical motion is localized on a finite mass of gas. The localization criterion coincides with the conditions of the uniqueness theorem for a solution of Problem II.

In the case of an inhomogeneous entropy distribution in the medium the compression wave may contain complex gas-dynamical structures.

2.5. Examples of Complex Gas-Dynamical Structures and Spectra of Solutions. In a number of important gas-dynamical processes (cumulation of shock waves [134, 16], motion of shock waves through a variable background of the density [62], etc.) behind the front of the shock waves in the medium there arise entropy distributions corresponding to power and exponential forms of the function $\varphi(x)$, i.e., $\varphi(x) = M(x_1 + x)^\delta$ or $\varphi(x) = Me^{\delta x}$. In these cases the study of Problems I and II [(46)-(48) and (53)-(55)] reduces to the investigation of the phase portraits of ordinary differential equations of first order [6, 52, 53].

We restrict ourselves to the version $\varphi(x) = M(x+1)^{\delta/\gamma}$, setting $M = x_1 = 1$.

The change

$$y = \pi^{1/\gamma}(x+1)^{-\frac{\delta}{\gamma}-2}, \quad z = -\pi(x+1)^{-1}/\pi' \tag{59}$$

enables us to investigate problem (46)-(48) on the phase plane of the equation

$$\frac{dy}{dz} = \frac{\gamma+1}{\gamma} y^2 z^{-1} \frac{\alpha z + 1}{y + zy - z^2}, \quad \alpha = \frac{\delta + 2\gamma}{\gamma + 1}. \tag{60}$$

To the piston (x = 0) and the front ($x = x_\Phi$) in the (z, y) plane there correspond the points

$$(z_0, y_0) = (-p_0(\pi'(0))^{-1}, p_0^{\frac{\gamma+1}{\gamma}}), \tag{61}$$
$$(z_\Phi, y_\Phi) = \begin{cases} (0, 0), & x_\Phi \neq \infty, \\ (-\alpha^{-1}, (\alpha-1)\alpha^{-3}), & x_\Phi = \infty. \end{cases}$$

A solution of Problem I always exists and is unique. The same goes for $\alpha \geqslant 0$ $(\delta \geqslant -2\gamma)$ for a solution of Problem II [6, 52, 53].

We consider the case $\alpha < 0$ (it is not covered by the conditions of Theorems 1, 3).

Suppose $x_\Phi < \infty$. In the (z, y) plane there exists a unique curve L issuing from the point (0, 0) and satisfying the conditions on the front in problem (46)-(48). The desired solution is the part of the integral curve between the points of the front and the piston. To construct a solution of Problem II for a given value $y_0 = p_0^{\frac{\gamma+1}{\gamma}}$ it is necessary to choose a value of the free parameter $v(0) = p_0 z_0^{-1}$ [$v(0) = -\pi'(0)$] so that the point of the piston (61) belongs to the integral curve L. For $\alpha < 0$ in the quadrant (z, y) > 0 there is a singular point $B(-\alpha^{-1}, (\alpha-1)\alpha^{-3})$, to which the curve L arrives. In the case $\alpha^* \leqslant \alpha < 0$ it is a node, while for $\alpha < \alpha^*$ it is a stable focus (Fig. 12a); $\alpha^* = (1 - \sqrt{\gamma + 1})/2$.

We consider the case $\alpha < \alpha^*$. For $p > p_0^{**} = (y_0^{**})^{\frac{\gamma}{\gamma+1}}$ no solution exists; for $0 < p_0 \leqslant p_0^{***} = (y_0^{***})^{\frac{\gamma}{\gamma+1}}$ a solution exists and is unique, while for $p_0^{***} < p_0 < p_0^{**}$ there exist several

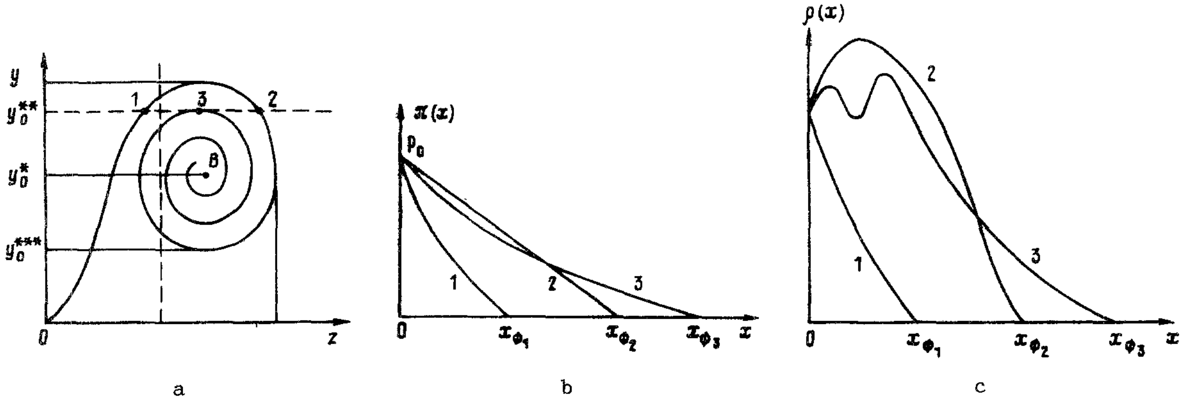


Fig. 12. a) The field of integral curves of Eq. (46) for $\alpha < \alpha^*$. The line of the piston and the line of structures are indicated by the dashed lines; b) a spectrum of solutions of Problem II; c) complex density structures in Problem II.

solutions, and the number of them grow without bound as $p_0 \to p_0^* = (y_0^*)^{\frac{\gamma}{\gamma+1}}$. If $x_\Phi = \infty$, then a (explicit) solution of Problem II exists only for $p_0 = p_0^*$ and is unique. In the (z, y) plane to it there corresponds a singular point – the center of focus.

The conditions for the existence of gas-dynamical structures admit a graphic interpretation on the phase plane. From the change (29) and condition (50) it follows that density (temperature) structures exist if the integral curve L intersects the line $z = z_1 = -\delta$ [$z = z_2 = -z_1(\gamma - 1)/\gamma$]. In the case $\alpha < \alpha^*$ the number of extrema may be unbounded, and as $\delta \to -\infty$ the number of them becomes infinite. An example of a spectrum of solutions and of complex density structures is presented in Fig. 12b, c.

In the present example solutions possess the following interesting property. From a comparison of cases 1 and 2 it is evident that with less work of the piston [proportional to the quantity $\pi(0)\cdot v(0)$] in the case of solution 2 a larger mass of gas ($x_{\Phi_2} > x_{\Phi_1}$) is compressed to greater densities.

We present an example of a complex spectrum of solutions of Problem II, N = 2. Suppose the entropy function has steplike form (Fig. 13a), i.e., contact discontinuities exist in the ball being compressed.

Then for an appropriate choice of the constants $M_1, M_2, M_3, \ldots, x_1, x_2, x_3, \ldots$ the solution of Problem II has the form shown in Fig. 13b [48].

For $0 < x_\Phi < a_0$ the pressure is greater than zero everywhere. With further increase of the mass of matter being compressed [$x_\Phi \in (a_0, a_1)$] at the center of symmetry of the system a cavity is formed which expands with some speed v_0. Then, beginning from some critical mass a_1, the pressure at the center of symmetry again becomes greater than zero. In the general case the number of "arcs" (a_i, a_{i+1}) may be unbounded.

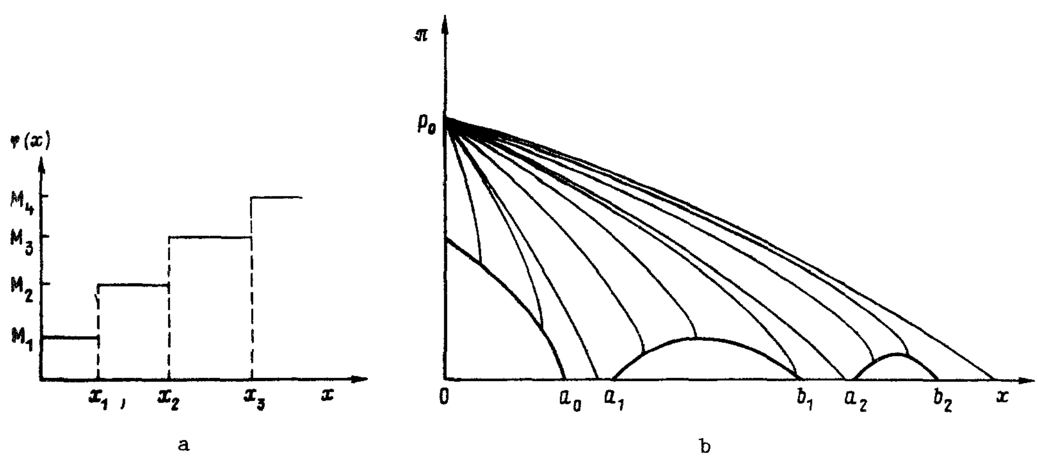


Fig. 13. a) Form of the entropy function in Problem II, N = 2; b) example of a complex spectrum of solutions in a spherical geometry.

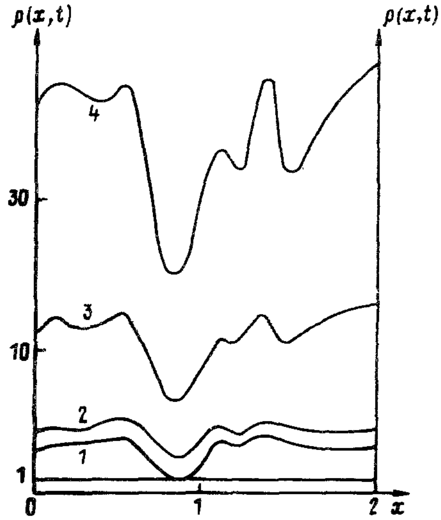


Fig. 14. Excitation of a complex density structure in a gas between two pistons. Parameters: $\gamma = 5/3$, $p(0, t) = 25 \cdot (1 - t)^{n_S}$, $p(2, t) = 50(1-t)^{n_S}$, $n_S = -2\gamma/(\gamma + 1)$, $\rho^0 = a_0 = 1$, $t_f = 1$, $t_0 = 0$, $t_1 = 0.12$, $t_2 = 0.446$, $t_3 = 0.874$, $t_4 = 0.974$.

Figure 14 shows results of a numerical calculation illustrating the "excitation" of nonunique extrema of the density (complex structures) in an initially homogeneous gas as it is compressed between two pistons. The pressure on the pistons corresponds to a self-similar S-regime and differs from the pressure of the gas at the initial time. As a result shock waves pass through the matter creating an inhomogeneous entropy distribution causing a complex density structure of the S-regime (the extrema are connected with fixed particles of gas; their sizes are unchanged).

CONCLUSIONS

The effect of "slow" (S- and LS-) boundary regimes with peaking on the medium leads to localization of gas-dynamical processes. This effect is connected not with the speed of compression, since in the case of "fast" (HS-) regimes localization is absent, but with the fact that S- and LS-regimes create in matter "inertial" spatial profiles of the quantities (cf. the character of the pressure profiles in Figs. 5, 7, 8). Completely analogous conclusions were obtained for processes of heat transfer [117-119, 92, 93, 98, 77]. This bears witness to very general regularities of the development of regimes with peaking in continuous media.

Compression in S- and LS- regimes is nonisentropic; in HS-regimes a shock wave with unboundedly increasing amplitude is always formed.

The spatial characteristics of the density and temperature in compression waves depend on the entropy distribution in the medium. These include possible existence of gas-dynamical structures and nonuniqueness of compression regimes of a fixed medium for a fixed law of the pressure on the piston.

The effects of localization, formation of structures, and shockless compression occur not only for a special but also for an arbitrary entropy distribution in the medium (and also in multidimensional geometry [51] and with consideration of additional processes — see [45, 46, 49, 42] and Chap. 2). In particular, for any entropy distribution there exists a unique boundary regime ensuring localization of shockless compression on a given mass of gas (if the pressure law on the boundary is fixed, then the existence and uniqueness of the compression regime is guaranteed for appropriate conditions on the entropy function).

The conditions for the occurrence of effects of localization and formation of structures depend on dimensionless criteria and are apparently altogether realizable. Boundary regimes with peaking afford the possibility of realizing a variety of means of compressing continuous media.

CHAPTER 2

COMPRESSION OF A FINITE MASS OF PLASMA IN REGIMES WITH PEAKING

1. Solution with Separated Variables for Problems of Plasma Physics

1.1. The physicomathematical model used in this chapter is based on the hydrodynamical description of a plasma. In particular, the system of equations of gas dynamics (29) presented above in Chap. 1 is often used to study the behavior of a dense plasma.

In a rather general case this system is augmented by terms which account for physical effects characteristic for an ionized gas. These are primarily processes of heat conduction, viscosity, and diffusion of the magnetic field.

The classical derivation of a sufficiently general and at the same time comparatively limple system of equations of magnetogas dynamics obtained by considering the kinetic equations for a plasma is given in [15]. The main condition for the applicability of the model is the condition of smallness of the "mean free path" of electrons and ions if the plasma (determined by Coulomb collisions in an ionized, quasineutral gas) as compared with the characteristic dimensions of the problem. Various relations between characteristic microscopic scales of a plasma (the mean free path, the Larmor radius, the Debye radius, etc.) and corresponding versions of the model are discussed in [15].

The given model of a continuous medium finds many applications in investigations of the dynamics of processes in various settings with a high-temperature plasma. Among these are systems with magnetic confinement of the plasma, impulse systems in which a plasma is obtained in strongly focused discharges or on heating matter by laser radiation or packets of particles.

1.2. System of Equations. In many applications plasma configurations possess spatial symmetry. In the simplest case the quantities describing the plasma are functions of only two variables: time and the distance from a plane, axis, or center of symmetry.

Consideration of such a class of one-dimensional, time-dependent problems makes it possible to investigate in a considerably more complete manner the effect of a large number of different physical effects. In a computational experiment this approach affords the possibility of studying physical situations which are different in principle and which arise on variation of the parameters of the system. In a number of cases such computer modeling leads to a choice of cardinally new regimes of compression and heating of the plasma. Here questions of the stability of the optimal regimes selected are studied at the second state of the investigations by solving multidimensional problems.

In the present work the investigation is further restricted to the framework of a self-similar problem. In this case the essential features of development of some physical process may be especially clearly manifest [12, 34]; as a rule, the characteristics of optimal regimes can be described analytically.

Subsequently, of course, the results of the self-similar analysis must be checked in a full-scale computational experiment.

We shall now write out the system of equations in the absence of a magnetic (and electric) field. Such equations may serve for describing plasmas obtained, for example, by heating matter by laser radiation or packets of particles. In Lagrangian mass coordinates $x=\int_0^r \rho r^N dr$ and t (see Chap. 1) the system in question has the form

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{\partial}{\partial x}(r^N v);$$

$$\frac{\partial r}{\partial t}=v;$$

$$\frac{\partial v}{\partial t}=-r^N\frac{\partial p}{\partial x}+r^N\frac{\partial D}{\partial x}+N\frac{D-D'}{\rho r};$$

$$\frac{\partial \varepsilon_e}{\partial t}+p_e\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=-\frac{\partial}{\partial x}(r^N W_e)+\frac{Q_e}{\rho}-\frac{Q_R}{\rho}-\frac{Q_{ei}}{\rho};$$

$$\frac{\partial \varepsilon_i}{\partial t}+p_i\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=-\frac{\partial}{\partial x}(r^N W_i)+\frac{Q_i}{\rho}+\frac{\Phi}{\rho}+\frac{Q_{ei}}{\rho};$$

$$D=\rho r^N\left(\frac{4}{3}\eta+\zeta\right)\frac{\partial v}{\partial x}+N\left(\zeta-\frac{2}{3}\eta\right)\frac{v}{r};$$

$$D'=\rho r^N\left(\zeta-\frac{2}{3}\eta\right)\frac{\partial v}{\partial x}+N\left[\zeta+\left(\frac{7}{3}-N\right)\eta\right]\frac{v}{r};$$

$$W_e=-\varkappa_e\rho r^N\frac{\partial T_e}{\partial x};\quad W_i=-\varkappa_i\rho r^N\frac{\partial T_i}{\partial x};$$

$$Q_{ei}=\chi(T_e-T_i);\quad \Phi=D\rho r^N\frac{\partial v}{\partial x}+N\frac{D'v}{r};$$

$$p=p_e+p_i;\quad p_i=\rho RT_i;\quad p_e=Z\rho RT_e;$$
$$\varepsilon_i=\frac{1}{\gamma-1}RT_i;\quad \varepsilon_e=\frac{Z}{\gamma-1}RT_e. \tag{1}$$

In the system (1) it is assumed that the vector quantities contained in it (the velocity v, the heat flux W) have only radial components (directed along the r axis), i.e., $\mathbf{v}=v\mathbf{e}_r$, $\mathbf{W}=W\mathbf{e}_r$, where $\mathbf{e}_r$ is a unit vector directed from the center (from the axis or plane) of symmetry along the coordinate r.

The system of equations written out corresponds to a two-temperature, single-fluid approximation. Here the plasma is considered a two-component fluid whose components (ions and electrons) move with the same hydrodynamic speed v(x, t) but have different temperatures: T_i (ions) and T_e (electrons). This approximation is frequently justified, since relaxation of the momenta of ions and electrons occurs after time approximately m_i/m_e times less than relaxation of energies (m_i is the mass of an ion of the plasma, m_e that of an electron). Relaxation of energy is described by the term Q_{ei}. The "rate" of temperature relaxation is determined by the magnitude of the coefficient $\chi(\rho, T_e)$.

In the system (1) the quantities D and D' in the present symmetric case are the nonzero components of the tensor of viscous stresses. For $N=1$, $D=D_{rr}$, $D'=D_{\varphi\varphi}$, where φ is the polar angle; for N = 2, $D=D_{rr}$, $D'=D_{\varphi\varphi}=D_{\theta\theta}$, where φ and θ are the angular variables in a spherical coordinate system. For N = 0 the equations contain only terms with $D = D_r$.

In the expressions for D and D' the quantity $\eta(\rho, T_i)$ is the coefficient of ionic viscosity. The coefficient of a second viscosity $\zeta(\rho, T_i)$ has been introduced for generality. The source of heat Φ is determined by the viscous dissipation of motion in the fluid.

The radial components of the vectors of heat fluxes W_e and W_i are proportional to the gradients of the temperature $\partial T_e/\partial x$ and $\partial T_i/\partial x$ in correspondence with Fourier's law. The coefficients of proportionality (the coefficient of thermal conductivity) depend on the density and the temperature:

$$\varkappa_e=\varkappa_e(\rho,\ T_e),\ \varkappa_i=\varkappa_i(\rho,\ T_i).$$

The heat sources Q_i and Q_e may describe heating of the plasma due to thermonuclear reactions (if the plasma contains heavy isotopes of hydrogen). In this case $Q_e = Q_e(\rho, T_i)$ and $Q_i = Q_i(\rho, T_i)$.

Finally, the term $Q_R = Q_R(\rho, T_e)$ describes the sink of energy due to volumetric radiation.

The expressions for the ion and electron pressure p_i and p_e and also for the specific internal energies ε_e and ε_i of electrons and ions correspond in (1) to the equations of state of an ideal gas. The gas constant R in the present case is $R = k_B/m_i$, where k_B is the Boltzmann constant and m_i is the mass of an ion; Z is the multiplicity of ionization.

The transfer coefficients $\varkappa_e$, $\varkappa_i$, η, ζ, χ are brought into the model (1) from without (from theory or experiment). In [15] simple power dependencies of these coefficients on the temperature are given which follow from theoretical consideration of transport processes on the basis of kinetic equations.

Having in mind obtaining self-similar solutions on the basis of dimensional analysis, for subsequent consideration we adopt the following power dependencies of these coefficients:

$$\begin{aligned}
\varkappa_e&=a_1T_e^{m_1}\rho^{k_1}t^{l_1};\\
\varkappa_i&=a_2T_i^{m_2}\rho^{k_2}t^{l_2},\\
\eta&=a_3T_i^{m_3}\rho^{k_3}t^{l_3},\\
\chi&=a_4T_e^{m_4}\rho^{k_4}t^{l_4},\\
\zeta&=a_5T_i^{m_5}\rho^{k_5}t^{l_5}.
\end{aligned} \tag{2}$$

In the general case an explicit dependence on time is introduced to model more complex dependencies.

We take a similar form for the heat sources and sinks:

$$Q_i=\frac{a_6T_i^{m_6}\rho^{k_6}t^{l_6}}{1+a_8T_i^{m_8}\rho^{k_8}t^{l_8}},$$

$$Q_e = \frac{a_7 T_i^{m_7} \rho^{k_7} t^{l_7}}{1 + a_9 T_i^{m_9} \rho^{k_9} t^{l_9}}, \qquad (3)$$
$$Q_R = a_{10} T_e^{m_{10}} \rho^{k_{10}} t^{l_{10}}.$$

Approximation of sources of heat due to local absorption of charged products of a thermonuclear reaction $^3H(d, n)^4He$ in the form (3) is fully justified in particular temperature ranges. For example, for $1 \leqslant T_i \leqslant 30$ keV we have $m_6 = m_7 = 5{,}2$; $k_6 = k_7 = 2$; $m_8 = m_9 = 3{,}6$; $k_8 = k_9 = 0$; $l_6 = l_7 = l_8 = l_9 = 0$ [75].

We note that since the combinations $a_j T_i^{m_j} \rho^{k_j} t^{l}$, j = 8, 9 are dimensionless, the conditions for self-similarity of any problem for (1)-(3) coincide with the conditions for self-similarity for a problem with Q_i and Q_e of more general form, namely, for

$$Q_i = a_6 T_i^{m_6} \rho^{k_6} t^{l_6} \varphi_i (a_8 T_i^{m_8} \rho^{k_8} t^{l_8}),$$
$$Q_e = a_7 T_i^{m_7} \rho^{k_7} t^{l_7} \varphi_e (a_9 T_i^{m_9} \rho^{k_9} t^{l_9}),$$

where φ_i and φ_e are arbitrary (!) dimensionless functions. Therefore, if a self-similar problem admits the occurrence of some dimensionless combination of the form $aT^m \rho^k t^l$ (there is a corresponding dimensional parameter a in the problem), then the dimensional coefficients (2), (3) can be multiplied by arbitrary functions of this combination without violating the conditions for self-similarity.*

1.3. Boundary Conditions. We shall now consider particular formulations of problems for the system (1)-(3). We consider a finite mass of gas $\left\{2\pi N + \frac{1}{2}(N-1)(N-2)\right\} x_0$, filling space from r = 0 to some $r_\text{п}(t): x_0 = \int\limits_0^{r_\text{п}(t)} \rho r^N dr$. The boundary conditions must be imposed at x = 0 (i.e., on the plane, axis, or center of symmetry r = 0) and at x = x_0 [i.e., on the piston whose trajectory follows the law $r_\text{п}(t)$].

The symmetry conditions at x = 0 are naturally taken in the following form:

$$v(0, t) = 0,$$
$$W_e(0, t) = W_i(0, t) = 0. \qquad (4)$$

On the piston x = x_0

$$v(x_0, t) = v_0 t^{n-1} \qquad (5)$$

and the temperatures or heat sources

$$T_e(x_0, t) = T_{0e} t^{n_1}, \quad \text{or} \quad W_e(x_0, t) = W_{0e} t^{n_2} \qquad (6)$$

and

$$T_i(x_0, t) = T_{0i} t^{n_1}, \quad \text{or} \quad W_i(x_0, t) = W_{0i} t^{n_2}. \qquad (7)$$

The problem admits natural generalizations to the case of prescribing the pressure [in place of (5)]

$$p(x_0, t) - D(x_0, t) = p_0 t^{n_3}. \qquad (8)$$

As shown below, in special cases at x = x_0 it is possible to impose a boundary condition corresponding to the boundary with the vacuum:

$$p(x_0, t) - D(x_0, t) = 0. \qquad (9)$$

1.4. Defining Parameters and Dimensional Analysis. In correspondence with conditions (4)-(9) as dimensional defining parameters it is convenient to take the constants x_0, v_0, R and the time t. These parameters possess independent dimension which ensures the expression of the scales of any quantities contained in equations (1)-(3) in terms of the defining parameters. We note that the choice of the quantity x_0 (the total mass of matter considered) as a defining parameter unavoidably leads to a self-similar problem with separated variables.

We now express all functions contained in the system (1) (both dependent and independent) in terms of products of powers of the defining scales x_0, v_0, R and t and dimensionless

*If attention is restricted to the form $aT^m\rho^k$ with ℓ = 0, then such a dimensionless combination in the given self-similar problem is unique.

parameters of the problem and a single independent dimensionless variable ξ. As a result we obtain the following expressions:

$$
\begin{gathered}
\xi=\frac{x}{x_0};\\
r(x, t)=v_0t^n\lambda(\xi);\quad v(x, t)=\frac{v_0t^n}{t}\alpha(\xi);\\
T_{i,e}(x, t)=\frac{(v_0t^n)^2}{Rt^2}\theta_{i,e}(\xi);\quad \rho(x, t)=\frac{x_0}{(v_0t^n)^{N+1}}\delta(\xi);\\
W_{i,e}(x, t)=\frac{x_0(v_0t^n)^{2-N}}{t^3}\omega_{i,e}(\xi);\quad p(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t^2}\beta(\xi);\\
D(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t^2}\tau(\xi);\quad D'(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t^2}\tau'(\xi);\\
\Phi(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t^3}\varphi(\xi);\quad \varkappa_{i,e}(x, t)=\frac{x_0R}{(v_0t^n)^{N-1}t}\tilde{\varkappa}_{i,e}(\xi);\\
\eta(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t}\tilde{\eta}(\xi);\quad \chi(x, t)=\frac{x_0R}{(v_0t^n)^{N+1}t}\tilde{\chi}(\xi);\\
\zeta(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t}\tilde{\zeta}(\xi);\quad Q_{i,e}(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t^3}q_{i,e}(\xi);\\
Q_{ei}(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t^3}q_{ei}(\xi);\quad Q_R(x, t)=\frac{x_0(v_0t^n)^{1-N}}{t^3}q_R(\xi).
\end{gathered}
\tag{10}
$$

From this it is evident that in a self-similar problem with the defining parameters listed above the boundary conditions must have the power form (6)-(8) with $n_1 = 2(n-1)$, $n_2 = (2-N)n - 3$, $n_3 = (1-N)n - 2$. Relations (10) hold for a boundary value problem with such boundary conditions. As is evident from (10), a solution of the given boundary value problem is sought in a form with separated variables x and t:

$$F_i(x, t)=\tilde{B}_i(t)\tilde{F}_i(x), \tag{11}$$

where F_i is any of the functions contained in (1). In a problem with the parameter of finite mass x_0 and boundary conditions (6)-(8) the functions B_i have power form:

$$F_i(x, t)=B_{0i}t^{n_i}f_i\left(\frac{x}{x_0}\right), \tag{12}$$

where B_{0i} is the product of some powers of x_0, v_0, and R. In accordance with the terminology developed a solution of (12) is called a self-similar S-regime of power form. The system (1) admits solutions of the form (11) with more general (nonpower) dependencies $\tilde{B}_i(t)$ [21, 43, 107, 126] (an S-regime with separated variables).

After the possibility of solving problem (1), (5)-(8) in the form (10) has been demonstrated it is clear that for defining parameters it is possible to choose the time t and any three constants of the laws (5)-(8) and (2), (3) if they possess independent dimension. In this case the parameter with dimension of mass is obtained as a product of some powers of them. For example, if for defining parameters we take p_0 of (8), W_{0e} of (6), R and t, then $x_0'=W_{0e}^{N-1}p_0^{2-N}$. The problem contains the dimensionless parameter $\xi_*=x_0/x_0'$ $\left(x_0=\int_0^{r_n(t)}\rho r^N dr\right)$ and the boundary conditions for the functions $f_i(\xi)$ are imposed at $\xi = \xi_*$ whereby $\beta(\xi_*) = 1$ as in Chap. 1. In the case that x_0 is included in the defining parameters $\xi_* = 1$, and $\beta(1)$ in the general case is not equal to 1.

So far we have not discussed the form of initial conditions for problem (1), (4)-(9). In the case of a self-similar solution the form of the initial functions follows from (10). Thus, for example, for $n > 0$, $v_0 > 0$, and $0 < t < \infty$, which corresponds to a scattering problem, the initial data are singular [$\rho(0, x) = \infty$, $v(0, x) = 0$; $T_{i,e}(0, x) = 0$ for $n > 1$ and $T_{i,e}(0, x) = \infty$ for $n < 1$) and do not bring in new dimensional parameters. In the case of a process starting at a nonsingular time t_0 ($t_0 \neq 0$ and $|t_0| \neq \infty$) for the initial data the functions (1) at the corresponding value $t = t_0$.

In cases of interest in practice a solution of problem (1), (4)-(9) for $t > t_0$ tends asymptotical in the course to time to the functions (10) even when initial data not corresponding to the form (10) are prescribed. It is said that asymptotic "passage" to this region from non-self-similar initial data is realized [12, 50, 70, 126]. In this case the dependence of the solution on additional dimensionless parameters contained in the non-self-similar initial data in the course of time becomes weak, and these parameters have no effect on the subsequent, self-similar course of the process.

For example, if in the course of the process energy is communicated to the plasma, then the inequality $E(t)/E(t_0) \gg 1$, where $E(t) = \int_0^{x_0} \left(\varepsilon + \frac{v^2}{2}\right) dx$, may serve as a criterion for "passage" to an asymptotic self-similar regime. In this case the energy communicated to the plasma in the course of time becomes arbitrarily large as compared with the energy contained in the plasma at the initial time. The additional parameter characterizing the initial data $E(t_0)/E(t) \to 0$ in the course of time.

1.5. Conditions of Self-Similarity. We now require that the functions (10) satisfy the relations (2), (3). An arbitrary (nonpower) dependence in (2), (3) is possible, as noted above, only in the form of a function of a dimensionless combination of the function sought. In practice this implies the occurrence in (2), (3) of expression of the type $y = aT^m\rho^k t^l$, where y is dimensionless and a is a corresponding dimensional parameter present in the problem. The combinations $a_8 T_i^{m_8}\rho^{k_8}t^{l_8}$ and $a_9 T_i^{m_9}\rho^{k_9}t^{l_9}$ actually occur in (3).

Substitution of (10) into (2) and (3) leads to conditions on the exponents m_i, k_i, ℓ_i in a way similar to that in which conditions on n_1, n_2, and n_3 were obtained by the substitution of (10) into the boundary conditions (6)-(8). Substitution of (10) into (2), (3) means making a dimensional analysis of the constants a_i: as constant quantities, all the a_i must be expressed only in terms of the scales x_0, v_0, and R. As a result we obtain the following relations:

$$n = \frac{2m_i - l_i - d_i}{2m_i - k_i(N+1) + c_i}, \tag{13}$$

where for $i = 1, 2, 3, 5$ $d_i = 1$, $c_i = N - 1$;

for $i = 4$ $d_i = 1$, $c_i = N + 1$;

for $i = 6, 7, 10$ $d_i = 3$, $c_i = N - 1$;

for $i = 8, 9$ $d_i = 0$, $c_i = 0$,

$$n_1 = 2n - 2,\ n_2 = (2 - N)n - 3,\ n_3 = (1 - N)n - 2.$$

The dimensional constants a_i are hereby expressed in terms of the defining parameters as follows:

$$a_i = A_i v_0^{k_i(N+1) - 2m_i - c_i} x_0^{-\tilde{k}_i} R^{\tilde{m}_i}, \tag{14}$$

where for $i = 1, 2$ $c_i = N - 1$, $\tilde{k}_i = k_i - 1$, $\tilde{m}_i = m_i + 1$;

for $i = 3, 5, 6, 7, 10$ $c_i = N - 1$, $\tilde{k}_i = k_i - 1$, $\tilde{m}_i = m_i$;

for $i = 4$ $c_i = N + 1$, $\tilde{k}_i = k_i - 1$, $\tilde{m}_i = m_i + 1$;

for $i = 8, 9$ $c_i = 0$, $\tilde{k}_i = k_i$, $\tilde{m}_i = m_i$.

Here A_i are corresponding dimensionless constants.

For each special version of the model (1) it is possible to find the corresponding expressions (13), (14). Thus, for example, in considering electron thermal conductivity alone in accordance with [15]

$$\varkappa_e = \frac{2{,}37}{\sqrt{2\pi}} \cdot \frac{m_p^{7/2}}{\Lambda e^4 \sqrt{m_e}} M_i^{7/2} R^{7/2} T_e^{5/2},$$

where Λ is the so-called Coulomb logarithm, e and m_e are the charge and mass of the electron, m_p is the mass of a proton, $M_i = m_i/m_p$ is the mass number of an ion of the plasma (m_i is the ion's mass), while the multiplicity of ionization Z is set equal to one (the plasma consists of the same number of electrons and singly charged ions). Here $m_1 = 5/2$, $k_1 = \ell_1 = 0$, $a_1 = \frac{2{,}37}{\sqrt{2\pi}} \cdot \frac{m_i^3 \sqrt{m_i/m_e}}{\Lambda e^4} R^{7/2}$. In accordance with (13) we obtain $n = \frac{4}{4+N}$; from (14) $a_1 = A_1 v_0^{-(N+4)} x_0 R^{7/2}$. From this it is possible to determine the value of the dimensionless constant $A_1 = \frac{2{,}37}{\sqrt{2\pi}} + \frac{m_i^3 \sqrt{m_i/m_e}}{\Lambda e^4 x_0} v_0^{N+4}$.

This is an example of dimensional analysis for one real coefficient of thermal conductivity of the plasma in the case where the quantity Λ is assumed constant. It actually depends weakly (logarithmically) on the temperature and the denstiy: $\Lambda = \ln(K_i/\sqrt{4\pi})$ for large K_i. The parameter K_i determines the possibility of using the equation of state of an ideal gas. The energy of interaction of ions and electrons can be neglected when $K_i = \frac{k_B^{3/2} m_p^{1/2}}{e^3} \times \sqrt{\frac{M_i}{Z}} T_e^{3/2} \rho^{-1/2} \gg 1$.

If in the problem it is necessary to consider also the dependence of $\varkappa_e$ on the Coulomb logarithm [Λ is a dimensionless function of the dimensionless argument $(4\pi)^{-1/2}K_i$] then it is necessary to subject the dimensionless combination $K_i = a_K T_e^{3/2}\rho^{-1/2}$ to dimensional analysis. Expressions for dimensionless combinations are contained in (13): these are the terms with $i = 8, 9$. Substituting $m_8 = 3/8$, $k_8 = -1/2$, we obtain $n = 6/(7 + N)$. The results of the analysis presented above for a_1, m_1, and k_1 are preserved with consideration of replacement of a_1 by $a_1\Lambda$ (Λ is dimensionless). Equating the expressions $4/(4 + N)$ and $6/(7 + N)$ to one another, we find that consideration in Eqs. (1) of the process of electron thermal conductivity with dependence of its coefficient on the Coulomb logarithm is possible for $N = 2$ (i.e., in the case of spherical symmetry). In this case $n = 2/3$.

1.6. System of Equations for the Dimensionless Functions $f_i(\xi)$ of (12). It can be obtained by substituting (10) into (1), while the boundary conditions for this system [i.e., the values $f_i(0)$ and $f_i(1)$] are obtained after substituting (10) into the general boundary conditions (4)-(9).

Separation of the variables (10) somewhat simplifies the structure of the system of dimensionless equations as compared with (1). Namely, substituting into the equation of the trajectories $\partial r/\partial t = v$ the corresponding quantities of (10), we obtain

$$\alpha(\xi) = n\lambda(\xi). \tag{15}$$

Thus, in self-similar problems of the motion of a finite mass the speed grows linearly with the radius. The class of flows (15) is a special case of flows with a homogeneous deformation [126]. In formulas (10), (15) the function $\lambda(\xi)$ is proportional to the Lagrange variable $a_0 = r(x, t_0)$, where t_0 is some time [99].

Considering (15) and substituting the corresponding quantities from (10) into the equation of continuity $\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \frac{\partial}{\partial x}(r^N v)$, we obtain

$$\frac{d\xi}{d\lambda} = \delta\lambda^N. \tag{16}$$

This relation is a dimensionless analogue of the law of change of volume [111] which coincides in the given self-similarity with the expression for the mass coordinate $dx = \rho r^N dr$. Now, in substituting (10) into (1), it is more convenient by means of (16) to go over to a system of equations with the dimensionless radius λ as the independent variable. The equation of continuity and the equation of the trajectories need no longer be considered. The quantities ξ and α because of this do not enter the desired system of equations, and (15) and (16) are to be used to determine them.

Carrying out these operations, we obtain

$$n(1-n)\delta\lambda = \frac{d}{d\lambda}(\beta - \tau);$$

$$n_\gamma\beta_i = -\frac{1}{\lambda^N}\frac{d}{d\lambda}(\lambda^N\omega_i) + \varphi + q_i + q_{ei};$$

$$n_\gamma\beta_e = -\frac{1}{\lambda^N}\frac{d}{d\lambda}(\lambda^N\omega_e) + q_e - q_{ei} - q_R;$$

$$\omega_e = -\tilde{\varkappa}_e\frac{d\theta_e}{d\lambda}; \quad \omega_i = -\tilde{\varkappa}_i\frac{d\theta_i}{d\lambda};$$

$$\tau = \frac{2}{3}n\tilde{\eta}(2-N) + n(N+1)\tilde{\zeta}; \quad \varphi = n(\tau + N\tau');$$

$$\tau' = \frac{2}{3}n\tilde{\eta}\left[\frac{N}{2}(7-3N) - 1\right] + n(N+1)\tilde{\zeta}; \quad \beta = \beta_i + \beta_e;$$

$$\beta_i = \delta\theta_i; \quad \beta_e = \delta\theta_e \cdot Z; \quad \tilde{\varkappa}_e = A_1\theta_e^{m_1}\delta^{k_1};$$

$$\tilde{\varkappa}_i = A_2\theta_i^{m_2}\delta^{k_2}; \quad \tilde{\eta} = A_3\theta_i^{m_3}\delta^{k_3}; \quad \tilde{\chi} = A_4\theta_e^{m_4}\delta^{k_4};$$

$$\tilde{\zeta}=A_5\theta_i^{m_5}\delta^{k_5};\quad q_R=A_{10}\theta_e^{m_{10}}\delta^{k_{10}};\quad q_{ei}=\tilde{\chi}(\theta_e-\theta_i);$$
$$q_i=\frac{A_6\theta_i^{m_6}\delta^{k_6}}{1+A_8\theta_i^{m_8}\delta^{k_8}};\quad q_e=\frac{A_7\theta_i^{m_7}\delta^{k_7}}{1+A_9\theta_i^{m_9}\delta^{k_9}};\tag{17}$$
$$n_\gamma=\frac{2}{\gamma-1}m_*;\quad m_*=\frac{n}{n_*}-1;\quad n_*=\frac{2}{2+(N+1)(\gamma-1)}.$$

The term $N(\tau-\tau')/\delta\lambda$ in the equation of motion drops out, since $\tau=\tau'$ for N = 1 and N = 2, while for N = 2 we have $\tau=\tau'=3n\zeta$.

The system (17) is considered on the segment $0\leqslant\lambda\leqslant\lambda_*$, where λ_* is the dimensionless coordinate of the piston. Corresponding boundary conditions are imposed at the end points of this segment.

For $\lambda=0$, as follows from (4), we must have

$$\omega_e=\omega_i=0.\tag{18}$$

The condition $v(0,t)=0$ is satisfied automatically, since $\alpha=n\lambda$ and $\alpha=0$ for $\lambda=0$.

For $\lambda=\lambda_*$ condition (5) gives the relation $\alpha(\lambda_*)=1$. With consideration of (15) in this case we have

$$\lambda_*=\frac{1}{n}.\tag{19}$$

If at the right end point the pressure (8) or (9) is given, then

$$\beta(\lambda_*)-\tau(\lambda_*)=\beta_*,\tag{20}$$

where $\beta_*=p_0v_0^{N-1}x_0^{-1}$. In this case the value of λ_* is found from the solution with consideration of the expression for the total mass:

$$\int_0^{\lambda_*}\delta\lambda^N d\lambda=1.\tag{21}$$

Finally, in place of (6), (7) we have

$$\theta_e(\lambda_*)=\theta_e^*,\quad\text{or}\quad\omega_e(\lambda_*)=\omega_e^*,\tag{22}$$
$$\theta_i(\lambda_*)=\theta_i^*,\quad\text{or}\quad\omega_i(\lambda_*)=\omega_i^*,\tag{23}$$

where $\theta_{i,e}^*=T_{0i,e}Rv_0^{-2}$, $\omega_{i,e}^*=W_{0i,e}v_0^{N-2}x_0^{-1}$.

The system (17) is subject to solution together with the boundary conditions (18), (19), or (20) and (21)-(23).

1.7. Formulation of the Self-Similar Problem of Compression of the Plasma. If conditions (13), (14) are satisfied as well as the assumption that the initial data are inconsequential (the asymptotic character of the solution) problem (1)-(9) has a solution of the form (10). If, moreover, $0\leqslant t<+\infty$, $t\to+\infty$, $n>0$, then such a solution describes expansion of the plasma. In the special case $p(x_0,t)-D(x_0,t)=0$ we have the problem of scattering of matter to the vacuum.

Such problems were considered earlier in [16, 107] with consideration of thermal conductivity and the evolution of heat. In the adopted terminology such solutions (with separation of variables) are called regular regimes. Investigations of regular regimes for the system of equations of magnetogas dynamics were carried out in investigating the "effect of the T-layer" [20, 24, 28, 130]. In all these works a considerable volume of results was obtained for the case of expansion of the plasma.

The main feature of the regimes studied in these works is localization of thermal, gas-dynamical, and magnetohydrodynamical processes. In the present case (see also Chap. 1) this meant the absence of motion through the mass of heat and magnetic heat waves. Realization of a self-similar regime led to solutions in separated variables x and t. In such solutions there are no perturbations of finite amplitude (including shock waves) transmitted from one portion of scattering plasma to another.

Any inhomogeneities in the distribution of temperature or density in space turns out in such regimes to be "bound" to the same portion of mass of the plasma during the entire process.

Indeed, the position of the effective boundary of any inhomogeneity (the point, for example, of the largest value of the gradient $df/d\xi$ of this quantity) is given in a self-similar problem by some fixed value ξ_1. Variation in time of the Lagrangian coordinate of this front does not occur, since $x_1 = \xi_1 x_0$ does not depend on time. The thermal structure (i.e., maxima in the temperature distribution due to a volumetric heat source) investigated in problems [20, 24, 28, 130] exist in a plasma just thanks to the presence of such localization.

There are a number of situations when it is of interest to study the process of compression of a plasma. These may be problems of magnetic cumulation, problems of the compression of plasma due to ablational acceleration of layers of a spherical target irradiated by packets of high-energy electrons, or laser radiation, problems of the acceleration of cylindrical conducting shells by powerful pulses of current and compression of gas inside them, etc. Can results of dimensional analysis (10), (13), (14) be used to describe compression of a plasma?

Since it follows from (10) that the radius of any element of mass $r(x, t) \sim t^n$ and, as we have seen, for physically reasonable situations the quantity $n > 0$, decrease of $r(x, t)$ is possible only for decreasing $|t|:|t| \to 0$. This approach leads directly to the necessity of studying processes with peaking.

Various problems of cumulation [12, 16, 62, 128, 134] in which occurrence of regimes with peaking is caused by special forms of the initial data or geometric factors serve as examples of the use of "negative time" ($|t| \to 0$, $-\infty < t < 0$) in self-similar problems.

The existence of a self-similar solution with a cumulating shock wave [134] serves as still another argument for the use of self-similar regimes with peaking to describe gas-dynamical processes of compression possessing the property of localization (shockless compression). Indeed, the motion of a piston compressing a finite mass of gas in an ordinary regime (without peaking) may cause appearance of an incoming shock wave. Cumulation of this shock wave on the axis or center of symmetry of the system leads to a growth of pressure on its front in a regime with peaking. In this case when the incoming shock wave becomes self-similar the gas-dynamical processes at the center (axis) evolve independently of the law of change of pressure on the piston. At the asymptotic stage, as a rule, those processes whose development is realized in the fastest manner show their regularities [34].

In order to ensure shockless compression in which the pressure and density grow in a consistent manner through the entire mass of plasma it is necessary to use a regime with peaking on the piston [70]. In many practical problems [22, 59, 112] such a regime is optimal, since it leads to achievement of given compressions* with minimal energy expenditure. It is easy to see [62, 128] that in a regime of self-similar cumulation of a shock wave the pressure behind its front, and hence on some fixed mass of gas brought into motion, grows at the asymptotic stage no faster than $(-t)^{-2}$, while in any S-regime (10) for $N = 1$ or $N = 2$ the pressure on a fixed mass grows no slower than $(-t)^{-2}$.

A number of works have been devoted to the study of regimes of shockless (optimal) compression in problems of gas dynamics [4, 59, 60, 80-82, 85, 113, 128]. The solutions investigated there were obtained by using Riemann invariants and can describe a wider class of situations than the solution (10). However, at the asymptotic stage ($t \to 0$) the dependencies of quantities on time in these solutions tend to the corresponding dependencies in formulas (10).

The attractiveness of the solutions (10) consists in the following.

First of all, the separation of variables in (10) automatically ensure that these solutions belong to the class of regimes of shockless compression.

Secondly, the solutions (10) are valid not only for problems of gas dynamics but also for continuous media with consideration of a large number of dissipative processes and with characteristic dependencies of the coefficients on temperature and density which are close to real ones.

Thirdly, in analogy to the case of a regular regime of scattering, analysis of the solutions (10) for compression leads to conditions for the occurrence and self-sustenance in such problems of thermal dissipative structures (and also structures of the magnetic field — see Sec. 4).

*Which are very considerable: we are talking of an increase in the initial density of 10^3-10^4 times.

Fourthly, compression regimes determined according to (10) reveal a particular "symmetry" with regimes of scattering. In particular, a piston whose coordinate $r(x_0, t) \sim t^n$ for a segment of time $-t_0 < t < 0$, $t_0 > 0$ repeats the trajectory of a similar piston for the time segment $0 < t < t_0$ but in reverse order. It is obvious from general thermodynamical considerations that for such problems the solutions (10) are different. Nevertheless, in the class of S-regimes (10) it is possible to indicate (see Sec. 3) conditions for the existence of nontrivial* solutions "mirrored in time" in which the density, pressure, and temperature at the compression stage repeat in reverse order their changes at the expansion stage. Thus the question of the possibility of "time reversal" in a dissipative medium is resolved, and at a general level so is that of the relation of solutions of the form (10) for expansion and compression of a plasma described with consideration of the same collection of physical processes (1)-(3).

We now proceed to an exposition of the method which makes it possible to use in (10) values of time in the interval $-\infty < t < 0$.

Using the approach expounded in Chap. 1, the solution (10) can be reformulated in other terms (without harming the dimensional analysis) by taking the dependence of the boundary regimes in the form $\sim(t_f - t)^{n_i}$ and considering time to run from some t_0 to t_f: $t_0 \leqslant t < t_f$. The corresponding replacements of t^{ℓ_i} by $(t_f - t)^{\ell_i}$ are to be made in (2), (3) also.

At the time of focusing $t = t_f$ quantities for which $n_i < 0$ in (10) (for example, the density or pressure for $N \neq 0$) become infinite.

The value of t_f itself is inconsequential, since the original system of equations (1) admits translation in time: the time of focusing t_f can be placed at any point of the time axis. In particular, for the shift $t' = t - t_f$ (t' is the "new time") the time of focusing $t_f' = 0$, and the dependencies (10) have the form $\sim(-t')^{n_i}$. Omitting the prime on the "new time," we obtain the dependencies considered in the introduction. Here $-\infty < t < 0$.

A somewhat different approach (in details but not in substance) equivalent to that presented above in the results obtained was applied in earlier works [65-69]. The significance of this approach lies in the convenience of considering systems of equations for dimensionless functions $f_i(\xi)$ of (12) simultaneously for problems of compression and rarefaction: up to the signs of the constant quantities A_i these systems coincide.

This approach is based on requiring unified formulas (10) for problems of compression and rarefaction. In this case both in the problem of compression ($-\infty < t < 0$, $t < 0$) and in the problem of rarefaction ($0 < t < +\infty$, $t > 0$) the same system (10) of unknown functions is used. Since dimensional real quantities stand on the left in (10), the right sides of equalities (10) must contain complex expressions [t^{n_i}, $t < 0$ and $v_0^{u_i}$, where u_i is the corresponding power in (10)] whose products give real functions. We shall show that this property holds in (10).

We represent the factor $(-t)^{n_i}$ in the form $(-1)^{n_i}t^{n_i}$ and indicate the rule according to which $t < 0$ is raised to a power. We assume that $c=|c|e^{i\varphi}$ and $c^n=|c|^n e^{in\varphi}$, where $|c|$ and φ are the modulus and argument of some complex number c.

It then suffices to assume in (5) and (10) that the parameter v_0 is a complex number and $\arg(v_0) = -\pi n$, i.e., $v_0=(-1)^{-n}|v_0|$. Indeed, complex expressions in (5) and (10) occur only in the combination v_0t^n which must be positive. Indeed, if $v_0t^{n-1}=v(x_0, t)$ is the speed of the piston, then $\frac{1}{n} v_0t^n=r(x_0, t)>0$ is its radius ($\partial r/\partial t=v$, $n>0$). Here in the case of compression the speed of the piston $v(x_0, t)=\frac{nr(x_0, t)}{t}$ is negative, since $t < 0$, while in the case of rarefaction it is positive. It is easy to see that because of (10) $v(x, t)=\frac{nr(x, t)}{t}$ everywhere, i.e., for the case of compression $v < 0$ everywhere.

Positivity of the product v_0t^n needed for $r(x, t) > 0$ and realness of other functions in (10) is ensured by the condition $v_0 = |v_0|(-1)^{-n}$, since

$$v_0t^n=|v_0|(-1)^{-n}(-1)^n|t|^n=|v_0|\cdot|t|^n>0.$$

We consider the expressions (10). From them it follows that for any $n > 0$ and $t < 0$ (the problem of compression) the quantities λ, α, β, δ, θ_i, θ_e are nonnegative as are the dimensional quantities which they represent. The functions $\tilde{\varkappa}_e$, $\tilde{\varkappa}_i$, $\tilde{\eta}$, $\tilde{\zeta}$, $\tilde{\chi}$, q_i, q_e, and q_R are

*That is, in the case of nonconserved entropy.

everywhere nonpositive; the signs of τ and τ' coincide with the signs of the corresponding dimensional quantities, while the signs of ω_e and ω_i are opposite the signs of the dimensional heat fluxes W_e and W_i, respectively. The signs of the derivatives $\partial T_{e,i}/\partial r$ and $d\theta_{e,i}/d\lambda$ are hereby the same.

For rarefaction problems where $v_0 > 0$, $t > 0$ the sign of any dimensionless quantity coincides with the sign of the corresponding dimensional quantity.

We shall now analyze the expressions (14) for $v_0 = |v_0|(-1)^{-n}$. We first consider the case $\ell_i = 0$, then the dimensional quantities $a_i > 0$ according to their physical meaning. It is easy to see that the exponents u_i of $v_0^{u_i}$ in the right sides of the expressions (14) can be represented in the form $u_i = -b_i/n$, where b_i is the numerator corresponding to the given index i of the condition of self-similarity (13). Hence, $v_0^{-b_i/n} = [|v_0|^{-1/n}(-1)]^{b_i}$. We note now that $b_i = 2m_i - d_i$, where $d_i = 1$ for $i = 1, 2, \ldots, 5$, $d_i = 3$ for $i = 6, 7, 10$ and $d_i = 0$ for $i = 8, 9$. Thus, d_i is odd in the expressions for the dimensional coefficients and is equal to zero in expressions for dimensionless coefficients. But then $v_0^{-b_i/n} = |v_0|^{-b_i/n} \times [(-1)^2]^{m_i}(-1)^{d_i}$. The sign of this expression is determined by the value of the quantity d_i. Thus, for dimensional coefficients $v_0^{-b_i/n} = (-1)|v_0|^{-b_i/n} < 0$, and for dimensionless coefficients $v_0^{-b_i/n} = |v_0|^{-b_i/n} > 0$, since $d_i = 0$. Hence, in expressions (14) $A_i < 0$ for $1 \leqslant i \leqslant 7$, $i = 10$ and $A_i > 0$ for $i = 8, 9$.

In the case $\ell_i \neq 0$ the corresponding argument arising in raising to the power t^{ℓ_i} must be ascribed to the dimensional constant a_i. Here the analysis of the signs of the quantities A_i presented above remains in force. Examples of the expressions (2) for $\ell_i \neq 0$ [24, 63, 65, 66] will be presented below in constructing analytic solutions of the system of dimensionless equations.

For the rarefaction problem where $v_0 > 0$, $t > 0$ all $A_i > 0$.

We now point out that in problems of compression negativity of the dissipation coefficients $\tilde{\varkappa}_{e,i}$, $\tilde{\eta}$, $\tilde{\zeta}$, $\tilde{\chi}$ and of the dimensionless sources $q_{e,i}$, q_{ei}, q_R is determined in the system of equations (17) by their expressions in terms of A_i and the dimensionless temperatures $\theta_{e,i}$ and density δ: $A_i\theta^{m_i}\delta^{k_i}$. The formulas for $T_{e,i}$ and ρ in (10) are such that in raising to a power $T_{e,i}^{m_i}$ and ρ^{k_i} the signs of all factors on the right sides do not change. Therefore, it should be assumed that $\theta_{e,i}^{m_i}$ and δ^{k_i} are nonnegative for any m_i and k_i. For example, suppose $n = 1$ ($v_0 = -|v_0|$), and $m = 1/2$. Then $T = \frac{v_0^2}{R} t^{-2}\theta(\xi)$ and $T^{1/2} = \frac{|v_0|}{R^{1/2}}|t|^{-1}\theta^{1/2}(\xi)$, where $\theta^{1/2} > 0$. Writing $T^{1/2} = \frac{|v_0|}{R^{1/2}} t^{-1}\theta^{1/2}(\xi)$ with $\theta^{1/2} < 0$ would lead in the general case, as shown below (Sec. 2), to violation of the second law of thermodynamics.

Thus, the only difference in describing the self-similar problem of compression from the problem of rarefaction by means of the system (17) consists in the following. The signs of the dimensionless constants A_i in the expressions for the dissipation coefficients in the system of self-similar equations are negative in problems of compression and are positive in problems of rarefaction. On the basis of the assertions made here it is possible to carry out a qualitative analysis of the system (17) and to compare the behavior of solutions for self-similar compression and rarefaction of the same medium.

The principle formulated of passing from a self-similar problem of rarefaction to a self-similar problem of compression, which consists in introducing into the boundary laws regimes with peaking and in changing the signs of A_i (and the sign of the speed v), is also valid in the more general case when the parameter of finite mass x_0 is not contained in the problem. This opens up the possibility of investigating a new class of boundary regimes – regimes with peaking – on the basis of well studied self-similar problems of power form in which $0 < t < +\infty$, $t \to +\infty$ [19-21, 23-28, 40, 41, 86, 87, 107, 106, 126, 128, 141].

2. Effects of Localization and Formation of Structures

2.1. Immobility of Effective Fronts of Heat Waves. The solution (10) of the system (1) considered here describes the motion of a continuous medium in which shock waves are absent because of separation of variables, while the fronts of heat waves are fixed with respect to mass.

The absence of shock waves is proved by the fact that the mass coordinate of a wave front x_{SW} is connected because of (10) with the dimensionless coordinate ξ_{SW} by the relation $x_{SW} = \xi_{SW} \cdot x_0$. The number ξ_{SW} must be found from the solution of the system of self-similar equations. Here x_{SW} does not depend on time (a property of the S-regime). But then there is no flux of mass through the front of the shock wave which contradicts its definition.

The characteristic depth of penetration of heat from the piston to the process of heat conduction is $l_T \simeq \sqrt{\frac{\varkappa}{\rho R}|t|}$. With consideration of (10) this expression takes the form $l_T \simeq v_0 t^n \sqrt{|\tilde{\varkappa}|/\delta}$. The ratio of ℓ_T to the piston radius $r(x_0, t) = v_0 t^n \lambda_*$ remains constant. Hence, the sizes of thermal inhomogeneities in the compression process decrease just as does the size of the medium being compressed.

In order to estimate the depth of heating in mass coordinates we form the expression $x_T = \rho r^N l_T$. We then find that the depth of penetration of heat with respect to mass is $x_T \simeq \sqrt{\frac{\varkappa_L}{R}|t|} = x_0 \sqrt{|\tilde{\varkappa}_L|}$, where $\varkappa_L = \rho r^{2N} \tilde{\varkappa} = \frac{x_0^2 R}{t} \tilde{\varkappa}_L$ is the "Lagrangian" coefficient of thermal conductivity and $\tilde{\varkappa}_L = \delta \lambda^{2N} \tilde{\varkappa}$. From this it is evident that x_T does not depend on time. Hence, any distinguished points in the temperature profile [for example, maxima $T_e(x, t)$ or $T_i(x, t)$] are located on the same portion of mass and are connected with the same fluid particles of the medium.

In the original works [66-70, 88, 89, 91] just this property, which is inherent to solutions in separated variables, received the name of a localization effect. The first example of a solution in separated variables demonstrating the effect of heat localization in a fixed medium with nonlinear heat conduction was presented in [122]. Its further study received considerable development in the analysis of purely thermal problems (without consideration of gas-dynamical motion) [70, 76, 88-90, 92, 93, 118, 120]. Essentially a class of solutions was distinguished in which the effect of a regime with peaking on an unbounded medium was a change only in a finite portion of it. It was hereby shown that localization occurred not only in solutions with separated variables (for example, in an LS-regime) [32, 70, 76, 118]. The process of "passage" to a localized regime (not necessarily self-similar) during which fronts of heat waves moved through matter was also studied.

The property of localization thus understood was subsequently discovered also in problems of gas dynamics (see Chap. 1), and with the help of the mathematical theory constructed it was generalized to a broad class of regimes with peaking (both boundary regimes and those arising in the medium itself due to volumetric heat sources) [29-37, 57, 58, 115, 117-120]. The results of such investigations led to formulation of a general concept of "thermal inertia" [77].

Thus, the study of "inertial" phenomena of a different kind of actions on a continuous medium with the help of theoretical and numerical anslysis of various mathematical models led to creation of a theory connecting the effect in the medium of regimes with peaking with the effect of localization of thermal and gas-dynamical processes and with conditions for the occurrence in it of the corresponding structures, i.e., in final analysis with explanation of regularities of self-organization of the medium.

We now note the following special feature of the solutions (10). Suppose $T_i(x, t) = T_e(x, t)$. The single-temperature approximation is valid if the relaxation time of the temperatures proportional to χ^{-1} is small as compared with the characteristic time of compression or rarefaction. Suppose here that the temperature profile is not monotone with respect to mass: maxima of $T(x, t)$ alternate with minima. Examples of such solutions are presented below in part 2.6 and are graphically illustrated in Figs. 15 and 17. We now mentally distinguish a portion of the medium between two neighboring minima. The heat fluxes on the boundaries of this mass, which remains unchanged with time, are equal to zero. Hence, the distinguished portion of medium is a closed system in the thermodynamic sense. If the maximum of the temperature in this portion owes its origin to the effect of a volumetric heat $Q(\rho, T)$, then the entropy of this mass must increase in correspondence with the second law of thermodynamics. This assertion makes it possible to relate the nonmonotonicity of a temperature profile characterizing properties of solutions of the system (17) of self-similar equations with the law of change of entropy in time which follows simply from the form of (10). Thus, a necessary condition can be formulated for the existence of a solution nonmonotonic in space and hence for the presence in the medium of maximal of the temperature (heat structures).

2.2. Qualitative Investigation of the System of Equations for the Dimensionless Functions (17). In the general case this is a complex problem. Nevertheless, in analyzing (17) it is possible to discover some general regularities. The most essential of them is the connection between the change in time of the total entropy of the system and the presence or absence of nonmonotonicities in the temperature profile. Namely, the occurrence of thermal structures, i.e., maxima of the temperature $\vartheta_e(\lambda)$ or $\vartheta_i(\lambda)$ localized on particular portions of mass, in a medium contracting or expanding according to the self-similar laws (10), is unequivocally connected with a growth of the entropy of the medium in time.

To clarify the connection indicated above between the change of the entropy of the system with time and the character of the temperature profile we first consider a single-temperature problem. In it $T_i = T_e$, $p = p_i + p_e = \rho R(1+Z)T$, $\varepsilon = \varepsilon_i + \varepsilon_e = \frac{R}{\gamma-1}(1+Z)T$.

In accordance with (10) the entropy function is

$$\Sigma = p\rho^{-\gamma} = x_0^{1-\gamma}(v_0 t^n)^{2/n_*} t^{-2}\beta(\xi)\,\delta^{-\gamma}(\xi),$$

where, using (17), $\frac{2}{n_*} = 1 - N + \gamma(N+1)$. Being interested in the time dependence of the function Σ, we can write

$$\Sigma = x_0^{1-\gamma}\,|v_0|^{2/n_*}\,|t|^{2\left(\frac{n}{n_*}-1\right)}\beta(\xi)\,\delta^{-\gamma}(\xi),$$

i.e., $\Sigma \sim (t^2)^{m_*}$ simultaneously for regimes of compression and rarefaction.

The entropy of a unit mass S can be expressed in terms of Σ as follows: $S = c_V \ln\frac{\Sigma}{\Sigma_c}$, where Σ_c is a dimensional constant and $c_V = \frac{R(1+Z)}{\gamma-1}$. Taking the logarithm of the expression for Σ, we obtain

$$S = c_V\left(\frac{n}{n_*}-1\right)\ln t^2 + c_V \ln \beta\delta^{-\gamma} + c_V \ln \Sigma_c^{-1} x_0^{1-\gamma}\,|v_0|^{2/n_*}. \tag{24}$$

The change of entropy with time is given by the formula

$$\frac{dS}{dt} = c_V\left(\frac{n}{n_*}-1\right)\frac{2}{t} = c_V\frac{2m_*}{t}. \tag{25}$$

From this it follows that for compression regimes ($-\infty < t < 0$, $t \to 0$) the entropy grows in a regime with peaking for $m_* < 0$ (i.e., for $n < n_*$), is constant for $m_* = 0$ ($n = n_*$), and decreases for $m_* > 0$ ($n > n_*$). In regimes of expansion the opposite occurs: the entropy decreases for $m_* < 0$ ($n < n_*$) and grows for $m_* > 0$ ($n > n_*$). It is easy to see that in the case considered here of separation of variables the entropy of the entire system (the total entropy) equal to $S_t = \int_0^{x_0} S\,dx$, behaves with time like the specific entropy S:

$$\frac{dS_t}{dt} = c_V x_0 \frac{2m_*}{t}. \tag{26}$$

It is now possible to carry out an elementary qualitative analysis of (17) (first for a single-temperature medium) being guided only by the principle of increase of entropy in a closed thermodynamical system.

We shall first establish in which cases there exist solutions of the system (17) in the absence of heat conduction.

If in the medium there are no other dissipative processes (sources and sinks of heat), then its entropy is unchanged. Hence, solutions of such an adiabatic problem are possible only for $n = n_*$ (i.e., $m_* = 0$). Indeed, in this case the energy equation in (17) reduces to $n_\gamma\beta = 0$, and a nontrivial solution $\beta(\lambda)$ corresponds to the case $n_\gamma = 0$.

Suppose now that only heat sources are present in the medium. Then the entropy of the system must increase. This means that a solution exists only for $n < n_*$ (i.e., $n_\gamma < 0$) for compression and for $n > n_*$ (i.e., $n_\gamma > 0$) for rarefaction. For the equations of the system (17) this means that the sign of the dimensionless sources (φ, q_i, etc.) coincides with the sign of the quantity n_γ. Thus, a formal analysis determining the signs of the dimensionless A_i gives the correct value of the change of entropy. For example, for compression the entropy of a closed thermodynamical system with evolution of energy will increase only under the condition that the corresponding coefficient A_i in the source term is negative.

Further, it is obvious that if only sinks of heat are present in the medium, then a solution exists for $n > n_*$ for compression and for $n < n_*$ for rarefaction.

If heat conduction is present in the medium ($\tilde{\varkappa} \neq 0$), then a solution exists for any $n > 0$ with corresponding values of the boundary quantities (θ_* or ω_*). The behavior of the entropy hereby dictates the character of the temperature profile.

We note that for $\tilde{\varkappa} \neq 0$ there is a convenient relation which determines an increase or decrease of the entropy both for regimes of compression and rarefaction. Namely, if

$$\operatorname{sign} m_* = \operatorname{sign} \tilde{\varkappa}, \tag{27}$$

then the entropy increases. Indeed, for compression $\tilde{\varkappa} < 0$, and the entropy increases for $m_* < 0$ [see (25)]. For rarefaction $\tilde{\varkappa} > 0$, and the entropy increases for $m_* > 0$.

We now consider the case where dissipative processes other than heat conduction are absent in the medium.

Here if the entropy S of each element of mass increases [see (25)], then the profile of the temperature θ must increase monotonically from the center to the piston. Indeed, only a flux of heat due to thermal conductivity from the boundary of the plasma can increase the entropy of the medium in this case. If the entropy decreases, then, conversely, the profile of the temperature θ must decrease monotonically from the center to the piston. Removal of heat due to thermal conductivity ensures a decrease of the entropy in each element of mass.

From the system (17) in this case with consideration of $\omega(0) = 0$ it follows that

$$\frac{d\theta}{d\lambda} = \frac{n_\gamma \int\limits_0^\lambda \beta \lambda^N d\lambda}{\tilde{\varkappa} \lambda^N}. \tag{28}$$

The sign of $d\theta/d\lambda$ in (28) is determined by the sign of the ratio $\frac{n_\gamma}{\tilde{\varkappa}} = \frac{2}{\gamma - 1} \frac{m_*}{\tilde{\varkappa}}$. If m_* and $\tilde{\varkappa}$ have the same sign, then $d\theta/d\lambda > 0$; otherwise, $d\theta/d\lambda < 0$. Thus, the correct sign of $d\theta/d\lambda$, which is consistent with the second law of thermodynamics, is determined in a qualitative analysis of (17) if and only if the signs of A_i are chosen in correspondence with the rule formulated: $A_i < 0$ for dissipative processes in considering compression.

We note that in the special case $m_* = 0$ the derivative $d\theta/d\lambda = 0$. Thus, adiabatic compression of a heat-conducting medium in a regime with separated variables occurs for a spatially homogeneous temperature profile as if the thermal conductivity of the medium were infinitely large.

2.3. Pressure Profile. As is evident from (17), in the absence of viscosity ($\tau = 0$) it is monotone. Moreover, for $n < 1$ the pressure drops from the center to the piston. In this case compression occurs with retardation; the total energy of the compressible mass

$$E(t) = \int\limits_0^{x_0} \left(\varepsilon + \frac{v^2}{2} \right) dx \sim (-t)^{2(n-1)}$$

drops with time ($t \to 0$); in the presence of thermal conductivity (or heat sinks) heat is removed from the system ($n > 1 > n_*$ is a regime of entropy decrease). For $n > 1$ expansion occurs with acceleration and an increase of the total energy $E(t) \sim t^{2(n-1)}$, $t \to +\infty$.

For $n < 1$ compression occurs with acceleration and an increase of the total energy $E(t)$, while expansion occurs with retardation and a drop of the quantity $E(t)$. The pressure profile in this case increases from the center to the piston.

Finally, in the case $n = 1$ [the case of conservation of the total energy of the system $E(t) = \text{const}$] the pressure is constant with respect to mass [$\beta(\lambda) = \text{const}$] as if the speed of sound in the medium were infinitely large. In this case the piston moves uniformly.

2.4. Nonmonotonic Spatial Temperature Distribution. It may occur in the presence of a heat source in a thermally conducting medium. A necessary condition for the existence of such a temperature profile is the relation (27), i.e., the condition of entropy increase. This conclusion, which is based on simple physical considerations, in the case of a separation of variables is rather obvious (as indicated above) and is easily proved on the basis of an analysis of (17).

Indeed, setting $q_i + q_e = q$, from (17) we obtain

$$\frac{d\theta}{d\lambda}=\frac{n_\gamma}{\tilde{\varkappa}\lambda^N}\int_0^\lambda \beta\lambda^N d\lambda-\frac{1}{\tilde{\varkappa}\lambda^N}\int_0^\lambda (\varphi+q)\,\lambda^N d\lambda. \tag{29}$$

Relation (29) generalizes (28) to the case of the presence of sources. We recall that in (29) $\beta=\beta_i+\beta_e$, $\tilde{\varkappa}=\tilde{\varkappa}_i+\tilde{\varkappa}_e$, $\theta=\theta_i=\theta_e$. It is now evident that if (27) is violated, i.e., sign × $m_*=\operatorname{sign} n_\gamma=-\operatorname{sign}\tilde{\varkappa}$ (the entropy decreases), then both terms on the right side are negative. Hence, $d\theta/d\lambda < 0$ for any point of the segment $[0, \lambda_*]$ on which the solution is considered.

If (27) is satisfied the temperature profile can be monotonically decreasing if

$$n_\gamma^{-1}\int_0^\lambda (\varphi+q)\,\lambda^N d\lambda>\int_0^\lambda \beta\lambda^N d\lambda$$

increases monotonically from the center to the piston in the opposite case, and, finally if there is at least one point with radius $\lambda_T\in[0, \lambda_*]$ such that

$$\int_0^{\lambda_T} (n_\gamma\beta-\varphi-q)\,\lambda^N d\lambda=0.$$

To clarify the meaning of the extremum at λ_T it is easy to obtain an expression for $d^2\theta/d\lambda^2$ of (29):

$$\frac{d^2\theta}{p\lambda^2}=\frac{1}{\tilde{\varkappa}}(n_\gamma\beta-\varphi-q)-\frac{m}{\theta}\cdot\left(\frac{d\theta}{d\lambda}\right)^2-\frac{N}{\lambda}\cdot\frac{d\theta}{d\lambda}-\frac{k}{\delta}\left(\frac{d\theta}{d\lambda}\right)\cdot\frac{d\delta}{d\lambda}. \tag{30}$$

Here for simplicity the case is considered where $m_1 = m_2 = m$, $k_1 = k_2 = k$.

Necessary and sufficient conditions for the existence of a maximum of the temperature not at the center and not on the piston (i.e., $0 < \lambda_T < \lambda_*$) are determined from (29), (30):

$$\begin{cases}\beta(0)-n_\gamma^{-1}\varphi(0)-n_\gamma^{-1}q(0)>0,\\ \beta(\lambda_T)-n_\gamma^{-1}\varphi(\lambda_T)-n_\gamma^{-1}q(\lambda_T)<0,\\ \displaystyle\int_0^{\lambda_T}(n_\gamma\beta-q-\varphi)\,\lambda^N d\lambda=0.\end{cases} \tag{31}$$

For each concrete solution and particular values of m_i, k_i, A_i, and N a concrete analysis of the realizibility of (31) is required. Moreover, it is further required to show the existence of the solution (17) for given boundary conditions (θ_*, λ_*, etc.) and to give the dependence of the conditions for the existence of a solution on the boundary conditions and the collection m_i, k_i, A_i.

An analysis of the behavior of the entropy is useful also in this case. For example, it has already been shown that in a heat-conducting medium for a decrease of the entropy $d\theta/d\lambda < 0$ everywhere. Hence, in principle solutions are possible with any θ_* right up to $\theta_* = 0$. On the other hand, in the case of an increase of the entropy $d\theta/d\lambda > 0$ everywhere, and hence $\theta(0) = \theta(1)$. Therefore, since $\theta(\lambda) > 0$, in principle there exists a lower bound for θ_*. It makes no sense to impose the condition $\theta_* = 0$, since a solution in this case either does not exist or is trivial ($\theta \equiv 0$).

An analysis of the variation of the entropy in a heat-conducting medium in the presence of only heat sinks leads immediately to the following conclusion: in regimes of nondecrease of the entropy the temperature grows monotonically from the center to the piston; in opposite cases nonmonotonicities in the profile of θ are possible.

2.5. Analysis of Temperature Profiles in a Two-Temperature Medium. The analysis can be carried out on the basis of the assertions made above. By (10) for each of the components the entropy function $\Sigma_e=p_e\rho^{-\gamma}$ and $\Sigma_i=p_i\rho^{-\gamma}$ varies with time like Σ, i.e., $\Sigma_{e,i}\sim(t^2)^{m_*}$. For the entropies S_e and S_i (24) and (25) hold with c_V replaced by $c_{Ve} = RZ/(\gamma - 1)$ and $c_{Vi} = R/(\gamma - 1)$, respectively. We note that additivity of the entropy, meaning that $S = S_i + S_e$, is here satisfied by choice of the constants: $(1+Z)\Sigma_c^{1+Z}=Z\Sigma_{ce}^Z\cdot\Sigma_{ci}$. This is admissible, since the entropy of an ideal gas is determined up to a constant quantity.

Thus, the conclusions regarding the character of the temperature distribution presented above are true for each of the components separately if it is born in mind that the exchange term q_{ei} is a source of heat for the component with the lower temperature and a heat sink for the other. The expression (29) generalizes in this case for each of the components:

$$\frac{d\theta_i}{d\lambda}=\frac{1}{\tilde{\varkappa}_i\lambda^N}\int\limits_0^\lambda (n_\gamma\beta_i-\varphi-q_i-q_{ei})\lambda^N d\lambda,$$
$$\frac{d\theta_e}{d\lambda}=\frac{1}{\tilde{\varkappa}_e\lambda^N}\int\limits_0^\lambda (n_\gamma\beta_e-q_e+q_{ei}+q_R)\lambda^N d\lambda. \tag{32}$$

2.6. Numerical and Analytic Examples of Solutions of the System (17). These examples corroborate the basic conclusions of the qualitative analysis regarding the behavior of pressure and temperature profiles.

Here we present several of the solutions of the system (17) investigated earlier in [66-70].

We first note the following important property of solutions (10) of problem (1)-(7). Suppose a solution of the system (17) has been found on the segment $\lambda\in\left[0, \frac{1}{n}\right]$, so that the dimensionless speed of the piston $\alpha\left(\frac{1}{n}\right)=1$, while the dimensionless mass $\int\limits_0^{1/n}\delta\lambda^N d\lambda=1$. We consider some point λ_1: $0<\lambda_1<n^{-1}$. Because of the fact that the radius $r(x_1, t)=v_0t^n\lambda_1(\xi_1)$, $\xi_1=x_1/x_0<1$ is the Euler coordinate of an altogether determined Lagrangian mass (x_1), its motion can be treated as the motion of the piston. Namely, because of the separation of the mass and time variables, together with the given problem it is possible to consider another problem in which the piston is located at the point λ_1 (and λ_1 is arbitrary!), and on it there is the corresponding boundary regime $\alpha(\lambda_1) = n\lambda_1 = \alpha_1$, $\theta(\lambda_1) = \theta_1$ or $\omega(\lambda_1) = \omega_1$. Here the solution of the new problem is simply "part" of the solution of the original problem taken on some interior segment $[0, \lambda_1]\subset\left[0, \frac{1}{n}\right]$.

It is easy to see that the solution of the new ("interior") problem is a solution of of (17) just as the original solution but for different parameters x_0 and v_0. The constants α_1 or β_1, θ_1 or ω_1 obtained from the original solution determine the renormalization of x_0 and v_0. Thus, if in the original problem on $0\leqslant\lambda\leqslant n^{-1}$ the solution $\alpha(1/n)=1$, $\theta(1/n)=\theta_*$ and $\int\limits_0^{1/n}\delta\lambda^N d\lambda=1$ corresponds to the quantities x_0 and v_0, then the part of this solution on the segment $[0, \lambda_1]$, $\lambda_1<\frac{1}{n}$ with $\alpha(\lambda_1)=\alpha_1$, $\theta(\lambda_1)=\theta_1$ and $\int\limits_0^{\lambda_1}\delta\lambda^N d\lambda=\xi_1$ corresponds to a solution (10) of problem (1)-(7) with $x_0^{(1)}=\xi_1 x_0$ and $v_0^{(1)}=n\lambda_1 v_0$.. Here, naturally, the scales of the dimensional quantities (say, the density, pressure, and temperature) [determined by $x_0^{(1)}$ and $v_0^{(1)}$] are not the same as in the "old" problem (where they are determined by x_0 and v_0).

The basis for this consideration is the presence for the system of ordinary differential equations (17) of a similarity transformation of the form

$$\xi'=k_\xi\xi,\quad \lambda'=k_\lambda\lambda,\quad \alpha'=k_\alpha\alpha,\quad \delta'=k_\delta\delta,\quad \beta'=k_\beta\beta,\quad \theta'=k_\theta\theta,\ \ldots,$$

where

$$k_\xi=\xi_1^{-1},\quad k_\lambda=k_\alpha=(n\lambda_1)^{-1},\quad k_\delta=(n\lambda_1)^{N+1}\xi_1^{-1},\quad k_\beta=(n\lambda_1)^{N-1}\xi_1^{-1},$$
$$k_\theta=(n\lambda_1)^{-2},\ \ldots.$$

An analogous similarity transformation was used in the construction of self-similar solutions in Chap. 1.

This property of the system (17) makes it possible in place of problem (18), (19), (22), (23) with condition (21) on the segment $[0, 1/n]$ to consider also the problem $\omega(0) = 0$, $\theta(\lambda_*) = \theta_*$ on some segment $[0, \lambda_*]$, so that $\int\limits_0^{\lambda_*}\delta\lambda^N d\lambda\neq 1$ and $\lambda_*\neq\frac{1}{n}$. The parameters ξ_* and λ_* of this problem are used to determine in final analysis the dimensional values of the mass of the medium, its temperature, etc. of interest to us.

This approach has definite advantages in integrating the system (17), since it is now possible to solve it numerically as a Cauchy problem with the conditions $\omega(0) = 0$, $\theta(0) = \theta_0$, and $\beta(0) = \beta_0$, where θ_0 and β_0 are some constants. By "breaking off" the computation at some $\lambda = \lambda_1$, the values $\theta(\lambda_1)$, $\beta(\lambda_1)$, $\alpha(\lambda_1)$ obtained here can be ascribed to the piston. In considering the problem with singular values of β_0 and θ_0 it is necessary to use expansions of the solution as $\lambda \to 0$ which are obtained without difficulty from (17).

This means of numerical solution of problem (17)-(23) was used in [63, 66-70].

We shall illustrate by the examples of [66] the case of occurrence of thermal structures for $n < n_*$ in a regime of increase of entropy for compression. We note here that the case of a completely ionized plasma [15], where $m_1=m_2=m_3=5/2$, $m_4=-3/2$, $k_1=k_2=k_3=0$, $k_4=2$, and $\ell_i = 0$ for all $1 \leqslant i \leqslant 4$, satisfies the conditions of self-similarity (13) with exponent $n = 4/(4 + N) \equiv n_R$, so that $n_R = 1$ for $N = 0$, $n_R = 4/5$ for $N = 1$ and $n_R = 2/3$ for $N = 2$.

The solution for a viscous, single-temperature, heat-conducting gas for $N = 2$, $\tilde{\varkappa} = -0{,}1\cdot\theta^{5/2}$, $\tilde{\zeta} = -5\theta^{5/2}$, $\gamma = 1{,}2$ is presented in Fig. 15. In this case $\varkappa = a_1 T^{5/2}$, $n = 2/3$, $n_* = \frac{2}{3\gamma - 1} = \frac{10}{13} > \frac{2}{3} = n$, so that the entropy increases. Occurrence of maxima of the temperature θ is caused by the effect of ionic viscosity for which the "second" viscosity is taken in this model problem. The pressure β in this example behaves nonmonotonically, but the total pressure $\beta - \tau$ grows monotonically, as follows from the analysis of (17) in the case $n < 1$.

Using the expression for $\varkappa_e$ of [15] and substituting into (14) the numerical values of a_i, we find for this case that $x_0 v_0^{-6} \cong 3.8\cdot 10^{-34}\ M_i A_1^{-1}\ g\cdot cm^{-6}\cdot sec^4$, where $M_i = m_i/m_p$ is the atomic weight of an ion of the plasma and m_p is the mass of the proton. For a hydrogen plasma ($Z = 1$, $M_i = 1$) we find, for example, that if $x_0 \cong 10^{-4}$ g, then $|v_0| \cong 1.2\cdot 10^5\ cm\cdot sec^{-2/3}$. Since in the present example $\xi_* \cong 0.37$ and $\lambda_* \cong 0.70$, the total mass $4\pi\xi_* x_0 \cong 465$ mkg, while the position of the piston at time $t = -10^{-9}$ sec is $r_* \cong 820$ mkm. These values of the parameters lead at time $t = -1$ nsec in this example to magnitudes of the density $\rho \cong 0.2\ g/cm^3$ and of the temperature $T \cong 1.6$ keV.

We now present an example of a heat structure due to the effect of a volumetric source of heat. We again consider the case $N = 2$, $n = 2/3$, and $\gamma = 1, 2$. As in the preceding example, suppose the expression of [15] holds for $\varkappa_e$, while $\tilde{\varkappa} = -0.274\theta^{5/2}$. We take the volumetric source $Q = Q_i + Q_e$ in the form $Q \sim \rho^{1.3}T^{1.6}$ so that $q = -1.0\theta^{1.6}\delta^{1.3}$. The dependence $\sim T^{1.6}$ is characteristic for the cross section of the thermonuclear reaction $^3H(d, n)^4He$ for $T \sim 20$ keV [75]. The dependence $Q \sim \rho^{1.3}$ is purely a model value and is taken so that $\ell_{6,7} = 0$ for $n = 2/3$. The result of numerical solution of this problem is presented in Fig. 16.

In accordance with the conclusions of the qualitative investigation of the system (17) in the present example conditions are realized which lead to the occurrence of a nonmonotonic temperature profile. This phenomenon is analogous to the phenomenon of a self-similar T-layer [20, 24, 28, 130]. The source of heating in the present example is not Joule heat as in a magnetohydrodynamic T-layer but a volumetric heat source modeling energy evolution due to thermonuclear reactions. In the case of compression of a single-temperature gas with $\varkappa = a_1 T^{5/2}$ and $\gamma = 5/3$ the parameter $n = 2/3$ and $n_* = 1/2$, so that $n > n_*$, and a regime of entropy decrease is realized. In this case heat structures may exist only due to heat sinks.

We again consider the single-temperature problem with $\tilde{\varkappa} = -0.0001\theta^{5/2}$ and $q_R = -1.5 \times \delta^{5/3}\theta^{1/2}$. The sink q_R models losses due to volumetric radiation. The value $k_{10} = 5/3$ is chosen in order that $n = 2/3$ in (13) and $\ell_{10} = 0$. The result of numerical solution is present in Fig. 17. Finally, we consider the case of entropy conservation in a dissipative medium. We again consider the case $N = 2$ and $n = 2/3$, but we set $\gamma = 4/3$, so that $n_* = 2/(3\gamma - 1) = 2/3$. We take the following values for the dimensionless coefficients of dissipation in a two-temperature medium: $\tilde{\varkappa}_e = -10\theta_e^{5/2}$, $\tilde{\varkappa}_i = -0{,}274\theta_i^{5/2}$, $\tilde{\zeta} = -0{,}07\theta_i^{5/2}$, $\tilde{\chi} = -0{,}944\delta^2\theta_e^{-3/2}$. Such relations between the dimensionless A_i correspond in accordance with [15] to the case of a completely ionized hydrogen plasma. The result of numerical solution of the system (17) for the present case (with corresponding boundary conditions) is presented in Fig. 18.

The profile of q_{ei} in the present example is nonuniform in space. At the center q_{ei} is very small due to the smallness of the density δ. Near the piston due to the abrupt growth of the density the quantity $\tilde{\chi} \sim \delta^2$ reaches appreciable values. However, here the value of $\theta_e - \theta_i$ is not large, and as a whole q_{ei} has no appreciable effect on the behavior of the temperatures: the components of the plasma (ions and electrons) behave independently as it were.

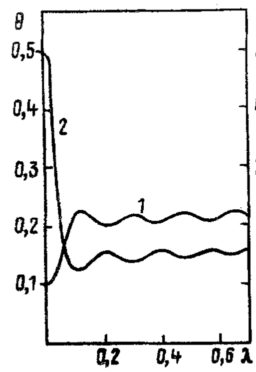


Fig. 15

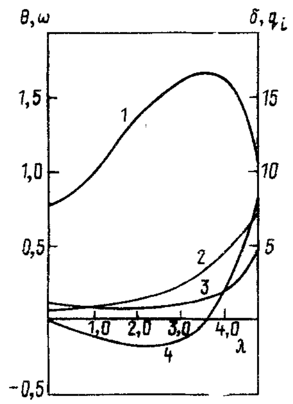


Fig. 16

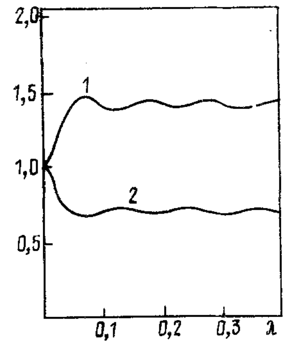


Fig. 17

Fig. 15. Profiles of the dimensionless temperature θ (1) and density δ (2) in the problem of compression of a viscous, heat-conducting gas in a regime with increase of the entropy.

Fig. 16. Profiles of the dimensionless temperature θ (1), the heat source, $-q_i$ (2), the density δ (3), and the heat flow $-\omega$ (4) in the problem of compression of a heat-conducting gas with a volumetric heat source in a regime of increase of the entropy.

Fig. 17. Profiles of dimensionless temperature θ (1) and density δ (2) in the problem of the compression of a heat-conducting gas with a volumetric heat sink in a regime.

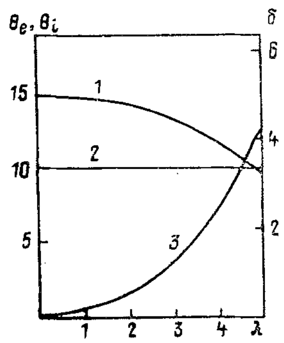


Fig. 18. Profiles the dimensionless temperatures θ_i (1) and θ_e (2) and the density δ (3) in the problem of the compression of a plasma in a regime with entropy conservation.

In the present example there is a heat source — viscosity — in the ion component. Due to this the profile of θ_i is convex downward. In the regime considered $n_\gamma = 0$, and the sink $q_{ei} \sim (\theta_e - \theta_i)$ is too small to ensure the removal of "viscous heat" needed for entropy conservation. Therefore, throughout the entire mass it is removed by ionic heat conduction. This behavior of θ_i is predicted by (32). Since q_{ei} is small, it follows from (32) that homothermicity ($n_\gamma = 0$) is observed for the temperature θ_e.

2.7. Analytic Solutions of the System (17) Constructed for Some Special Cases. They corroborate the conclusions of the qualitative analysis and also illustrate the following important feature of the solutions (17). In integrating (17) with use of the boundary conditions there always remains one of the constants of integration which must be determined from condition (21), since the variable ξ is eliminated from (17) thanks to (15), (16) (the equation of continuity is not considered). In particular, on prescribing conditions (20) in place of (19) the relation (21) serves to determine the parameter $\lambda_* \neq \frac{1}{n_1}$. The integral (21) on the solution expresses the connection among the dimensionless parameters of the problem: the quantities A_i, certain boundary conditions, and also the parameter λ_* which in general is not equal to n^{-1} [under condition (20)], and remains to be determined.

We shall present here the simplest examples of the analysis of such a connection. More complex cases connected with the problem of z-pinch [system (1) with consideration of an azimuthal magnetic field and for N = 1; see Sec. 4, (47)] are treated in [24] (for rarefaction).

The general cases of integrability of the system (17) in finite form known at the present time belong to two classes: either $n = 1$ or $n = n_*$. In the first case the energy integral exists, while in the second the entropy integral exists, i.e., either the quantity $E(t)$ or $\Sigma(t)$ is conserved. In both cases the system (17) simplifies.

The case $n = n_*$ in the absence of dissipative processes (adiabatic compression of matter) is expounded in sufficient detail in Chap. 1 of this paper and in the works cited there. Such solutions have long been known [135] and have been studied by various methods [5, 59, 60, 63, 70, 81, 112] (see also the bibliography in [70]). We shall consider other cases below.

Suppose $n = n_*$, but heat conduction is present in the medium. We exclude other dissipative processes from consideration. In accordance with (13) the exponents m, k, and ℓ in the expression $\varkappa = aT^m\rho^k t^l$ satisfy the condition

$$\frac{2}{2+(N+1)(\gamma-1)}=\frac{2m-1-l}{2m-k(N+1)+N-1}.$$

For example, for $N = 2$, $\gamma = 5/3$ ($n_* = 1/2$) a self-similar regime is realized for $\varkappa = aT^{3/2}$.

We now turn to (17) for this case (a single-temperature medium). Since $n_\gamma = 0$ and $\tilde{\varkappa} \neq 0$, it follows that $\theta = \text{const}$. The constant of integration is determined from the boundary conditions $\theta(\lambda_*) = \theta_*$, i.e., $\theta = \theta_*$. We remark that in this solution the value of the parameter A is not present – it is as it were infinitely large. Therefore, the solution presented here is valid both for the case of rarefaction and for the case of compression.

Noting now that $\beta = \delta\theta = \theta_*\delta(\lambda)$, we find $\delta(\lambda)$ from the first equation of the system (17). The solution found has the form

$$\delta(\lambda)=C_1 e^{\mu\lambda^2};\quad \beta(\lambda)=C_1\theta_* e^{\mu\lambda^2};\quad \theta(\lambda)=\theta_*. \tag{33}$$

Here we have introduced the notation $\mu=\frac{n_*(1-n_*)}{2\theta_*}$.

From condition (21) we find the constant of integration

$$C_1 = 1/\Phi(\mu), \tag{34}$$

where the function $\Phi(\mu)=\int\limits_0^{1/n_*}\lambda^N e^{\mu\lambda^2}d\lambda$.

From (34) it follows that

$$\frac{dC_1}{d\theta_*}=\frac{\Phi'_\mu\mu}{\Phi^2\theta_*}=\frac{\int\limits_0^{1/n_*}\lambda^{N+2}e^{\mu\lambda^2}d\lambda}{\theta_*\mu^{-1}C_1^{-2}}>0,$$

i.e., $C_1(\theta_*)$ grows with increasing θ_*. As $\theta_*\to\infty$, $C_1(\theta_*)\to C_\infty=(N+1)n^{N+1}$, and $C_1 < C_\infty$ everywhere for finite θ_*.

We shall show that $C_1 \to 0$ as $\theta_* \to 0$. This means that $\Phi(\mu) \to \infty$ as $\mu \to \infty$. $\Phi'_\mu=\int\limits_0^{1/n_*}\lambda^{N+2}e^{\mu\lambda^2}\times d\lambda > 0$ and $\Phi(0)=\frac{\lambda_*^{N+1}}{N+1}=C_\infty^{-1}\left(\lambda_*\equiv\frac{1}{n_*}\right)$. Hence, $\Phi(\mu)$ grows with increasing μ, and it is necessary to show that $\lim\limits_{\mu\to\infty}\Phi(\mu)=\infty$. But

$$\Phi(\mu)=\int\limits_0^{1/n_*}\lambda^N e^{\mu\lambda^2}d\lambda=\mu^{-\frac{N+1}{2}}\int\limits_0^{\sqrt{\mu}/n_*}y^N e^{y^2}dy\equiv\mu^{-\frac{N+1}{2}}\Psi\left(\frac{\sqrt{\mu}}{n_*}\right),$$

where $\Psi(u)\equiv\int\limits_0^u y^N e^{y^2}dy$. Using L'Hôpital's rule, it is easy to verify that $\Psi(u) \sim u^{N-1}e^{u^2}$, $u \to +\infty$. But then $\Phi(\mu)\to\frac{e^{\mu\lambda_*^2}}{2\mu n_*^{N-1}}\to\infty$ as $\mu\to\infty$.

The example constructed illustrates the qualitative conclusion regarding homothermicity of the solution in the case $n = n_*$ and $\varkappa\neq0$. As $\theta_* \to \infty$ the solution (33) tends to a homogeneous distribution similar to the solution of isentropic compression in which $\beta(0) \neq 0$ as $\sigma_0 \to \infty$ [68, 66, 70, 112] (in the isentropic case $\beta\delta^{-\gamma} = \sigma_0 = \text{const}$).

Qualitative analysis of (17) in the case of the presence of heat conduction indicates necessary conditions for joint localization of heat and gas-dynamical effects in the S-regime [49]. Namely, first of all we must have $n < 1$ — the pressure drops from the piston into matter only in this case. Secondly, we must have $n < n_*$ for a compression regime and $n > n_*$ for a rarefaction regime in order for the temperature to drop away from the piston. Summarizing, for compression regimes we have the condition $n < n_*$, and for rarefaction we have $n_* < n < 1$.

We now consider the case $n = 1$ and only the process of heat conduction in (17) (a single-temperature medium). A coefficient of thermal conductivity of the form $\varkappa = aT^m\rho^k t^l$ will satisfy conditions (13) for any finite m if k and ℓ are connected by the relation $(N + 1)k = N + \ell$. In particular, if $k = N/(N + 1)$, then $\ell = 0$.

In the case $n = 1$ for the pressure we immediately obtain $\beta = \beta_0 = \text{const}$. Integrating here system (17) on the segment $0 \leqslant \lambda \leqslant 1$ ($\lambda_* = 1/n = 1$) with consideration of the boundary conditions $\omega(0) = 0$ and $\theta(\lambda_*) = \theta_*$, we obtain $\beta = \beta_0$; $\omega = -\beta_0\lambda$; $\delta(\lambda) = \beta_0\theta^{-1}(\lambda)$ and

$$\theta(\lambda) = \left[\theta_*^{m-k+1} - (m-k+1)\frac{\beta_0^{1-k}}{2A}(1-\lambda^2)\right]^{\frac{1}{m-k+1}}. \tag{35}$$

Here $m \neq k - 1$. If $m = k - 1$, then

$$\theta(\lambda) = \theta_* \exp\left\{-\frac{\beta_0^{1-k}}{2A}(1-\lambda^2)\right\}. \tag{36}$$

Analysis of (35), (36) shows that for any m and k $\theta(\lambda)$ grows with increasing λ for $A > 0$ (the case of rarefaction, i.e., entropy increase) and decreases with increasing λ for $A < 0$ (the case of compression, i.e., entropy decrease). The well defined behavior of the entropy (increase for rarefaction and decrease for compression) for $n = 1$ independent of the value of γ is ensured by the inequality $n_* < 1$ which follows directly from the condition $\gamma > 1$.

The constant of integration β_0 is found from the condition $\int_0^1 \delta\lambda^N d\lambda = 1$. We shall analyze the constrains imposed by the boundary conditions on the parameters of the problem.

We first consider two simple examples for $N = 1$.

Suppose $m = k = 0$. Then $\theta = \theta_* - \frac{\beta_0}{2A}(1-\lambda^2)$ and $\xi_* = \frac{\beta_0}{2}\int_0^1 \frac{d\lambda^2}{\theta_* - \frac{\beta_0}{2A}(1-\lambda^2)} = 1$. From this we find that $\beta_0 = 2A\theta_*\left(1 - e^{-\frac{1}{A}}\right)$. Here $A < 0$ for the case of compression and $A > 0$ for the case of rarefaction, so that $A\left(1 - e^{-\frac{1}{A}}\right)$ is always positive. Further, we find that $\theta(0) = \theta_* e^{-\frac{1}{A}}$, i.e., $\theta(0) > \theta_*$ for compression and $\theta(0) < \theta_*$ for rarefaction. In this example the solution $\theta(\lambda) = \theta_* e^{-\frac{1}{A}}\left(1 + \lambda^2\left(e^{\frac{1}{A}} - 1\right)\right)$ exists for any values of A and θ_*, while the solution with $\theta(0) = 0$ for rarefaction and the solution with $\theta_* = 0$ for compression are identically zero.

Suppose now that $m = 1$, $k = 0$. Then $\theta = \left\{\theta_*^2 - \frac{\beta_0}{A}(1-\lambda^2)\right\}^{\frac{1}{2}}$. Computing $\frac{\beta_0}{2}\int_0^1 \frac{d\lambda^2}{\sqrt{\theta_* - \frac{\beta_0}{A}(1-\lambda^2)}} = 1$, we find that $\beta_0 = 2\theta_* - \frac{1}{A}$. For compression ($A < 0$) the quantity β_0 is always positive. For rarefaction it turns out that the quantities θ_* and β_0 are bounded below: $\theta_* \geqslant \frac{1}{A}$, $\beta_0 \geqslant \frac{1}{A}$. Indeed, the solution $\theta(\lambda)$ is in this case $\theta(\lambda) = \sqrt{\left(\theta_* - \frac{1}{A}\right)^2 + \frac{\lambda^2}{A^2}(2\theta_* A - 1)}$. The value $\theta(0) \geqslant 0$ for $\theta_* \geqslant \frac{1}{A}$. In this example a solution exists only for A such that $A \geqslant \theta_*^{-1}$. Here the constant of integration $\beta_0 \geqslant \theta_*$.

For compression a solution exists for any $A < 0$ including the case $\theta_* = 0$. In this case $\delta(\lambda) = (1-\lambda^2)^{-1/2}$ does not depent on A, while $\beta_0 = \frac{1}{|A|}$ and $\theta(\lambda) = \frac{\sqrt{1-\lambda^2}}{|A|}$.

In the general case of arbitrary m, k, N analysis shows that the behavior of the solution is analogous to the two cases treated. Thus, for the case $m - k + 1 > 0$ if it is assumed that $\theta(0) \to 0$ for rarefaction, then a nontrivial solution exists with the following constraint on the parameter A:

$$A \geqslant \frac{(m-k+1)^k}{2\theta_*^m}[(N+1)(m-k+1)-2]^{1-k}.$$

Here

$$\beta_0 \geqslant \left[\frac{(N+1)(m-k+1)-2}{2}\right]^{\frac{m-k+1}{m}}\left(\frac{m-k+1}{2}\right)^{\frac{k-m}{m}} A^{-\frac{1}{m}}.$$

Convergence of the integral $\int_0^1 \delta\lambda^N d\lambda$ as $\lambda \to 0$ is ensured by the condition $m - k + 1 > 2/(N + 1)$.

For the case of compression a nontrivial solution exists with $\theta_* = 0$ under the condition $m > k$.

In the remaining cases constraints on the parameters do not arise, and both for the case of compression and the case of rarefaction a solution exists for any θ_*, and as $\theta_* \to 0$ the solution tends to the trivial solution: $\theta(\lambda) \to 0$ everywhere.

3. "Reversal" of Processes in Time and a Dissipative, Open System

The "time reversal" introduced in Sec. 1, Chap. 2 was a formal technique making it possible in the analysis of the system of equations (17) for dimensionless representatives of the desired functions to simultaneously investigate both problems of compression and problems of rarefaction. Elementary consideration of simple problems as well as the examples presented above shows that solutions of formally "identical" or "mirror" problems for the same medium for $-\infty < t < 0$, $t \to 0$ and for $0 < t < +\infty$, $t \to +\infty$ are different in principle. This difference was related above to the requirement of satisfying the second law of thermodynamics.

The problem of heating a medium in a regime with peaking can serve as a trivial example of this difference [76, 118]. The boundary value problem for the equation

$$\partial T/\partial t = \frac{\partial}{\partial x}(k_0 T^\sigma \partial T/\partial x), \quad \sigma > 0, \quad k_0 > 0$$

for $T(0, t) = T_0(-t)^{-1/\sigma}$ $(-\infty < t < 0,\ t \to 0)$ and $T(x_0, t) = 0, T^\sigma \partial T/\partial x|_{x=x_0} = 0$ has a solution in separated variables $T(x, t) = T_0(-t)^{-1/\sigma}(1 - x/x_0)^{2/\sigma}$, $T(x, t) = 0$ everywhere for $x > x_0$ and $-\infty < t < 0$ [122] which describes a steady-state wave of heating. Here $x_0 = \sqrt{k_0 T_0^\sigma}\left[\frac{2(\sigma+2)}{\sigma}\right]^{1/2}$.

Prescription of the "mirror" regime $T(0, t) = T_0 t^{-1/\sigma}$, $0 < t < +\infty$, $t \to +\infty$, also admits a solution in separated variables (a steady-state wave of cooling) which is, however, different in principle from the one presented above. In this solution constructed by Boussinesq $k_0 T^\sigma \frac{\partial T}{\partial x}$ vanishes at a maximum value $T(x, t) > 0$ [101] and in [122] at $T(x, t) = 0$.

It is also known that in the general case it is not possible to obtain a solution for compression of a continuous medium, having a solution for its rarefaction, by the application of simple "time reversal" (symmetric reflection relative to some time). However, there are examples when rarefaction and the compression symmetric to it are actually realizable physical processes.

One of the examples is the problem considered above for a medium described by the equations of gas dynamics with heat conduction [Sec. 2, (33)] in the case of conservation of entropy.

Another example is the process of shockless compression, considered within the framework of a self-similar problem with separated variables (as, for example, in [66, 70, 112]) and investigated in detail in Chap. 1, which is a reversal of the corresponding rarefaction problem.

The spatial distributions of quantities in these mutually symmetric processes coincide at symmetric times (in the dimensionless variables of the self-similar problem these distributions are altogether fixed), while variations of the quantities in time from $t = 0$ to $t = +\infty$ for rarefaction and from $t = -\infty$ to $t = 0$ for compression follow the same laws. Aside from these trivial $(n = n_*)$ examples, the self-similar problems considered in Chap. 2 with consideration of heat condition and a heat source give nontrivial $(n \neq n_*$, in general) examples of an open dissipative system for which mutually symmetric motions have real meaning. Such motions are constructed below.

We consider a medium whose motion is described by the system (1) of one-dimensional equations, and we consider in (1) only processes of heat conduction and a volumetric heat source. For simplicity we shall assume that the medium is a single-temperature medium, and we neglect the effects of viscosity. In accordance with this we leave in (2) and (3) only $\varkappa = a_1 T^{m_1} \rho^{k_1}$ and $q = a_6 T^{m_6} \rho^{k_6}$.

The adiabatic motion of the finite mass considered of the medium without dissipative effects – in this case it is a thermodynamically closed system – can occur both in a reversible manner (the change of entropy of the entire mass of matter $\Delta S_{\text{п}} = 0$) and in an irreversible manner (in this case $\Delta S_{\text{п}} > 0$, and in the medium irreversible processes – shock waves – are observed).

A system with dissipation (a finite mass of medium with consideration of heat conduction and heat sources or sinks) in the general case is open. In such a system it is possible to have $\Delta S_{\text{п}} < 0$ even in the presence of dissipative processes due to removal of heat through the piston.

From general thermodynamical considerations it is clear that the following assertion holds for the reversible adiabatic motion of a closed system. The motion obtained from some actually realizable motion by replacement of the time t by −t and by simultaneous change of the sign of the velocity is a physically real process. Such "time reversal" can be thought of as follows.

Suppose at some time $t = t_1$ in the process of motion there are distributions of density $\rho(x, t_1)$ and speed $v(x, t_1)$. We assume that the distribution of entropy $S(x, t_1) = S(x)$ is known. Then because of the adiabatic motion the pressure $p(x, t_1)$ and temperature $T(x, t_1)$ can be computed on the basis of the value of the density, while the radius $r(x, t_1)$ can be computed on the basis of the density and speed. With no loss of generality we henceforth consider the adiabatic case to be isentropic: $S(x, t) = \text{const}$. From time $t = t_1$ to time $t = t_2 > t_1$ $(t_1 \leqslant t \leqslant t_2)$ the piston moves from $r_1 = r_*(t_1)$ to $r_2 = r_*(t_2)$ without violating adiabaticity ($\partial S/\partial t = 0$ for the entire mass of gas). Such motions exist as shown in Chap. 1 and, for [66-68, 81]. Suppose that at time t_2 the density and speed have the distributions $\rho(x, t_2)$ and $v(x, t_2)$. We now change the speeds to the opposite speeds and consider the time $t = t_2$ as the initial time with data $\rho(x, t_2)$ and $-v(x, t_2)$. From time t_2 the piston moves so as to repeat its trajectory during the time $t_2 \leqslant t \leqslant t_3$, $t_3 - t_2 = t_2 - t_1$, in reverse order so that $r_*(t_3) = r_*(t_1)$, and, generally, $r_*(t_1 + t') = r_*(t_3 - t')$ for $0 \leqslant t' \leqslant t_2 - t_1$. Here

$$\rho(x, t_1 + t') = \rho(x, t_3 - t'), \quad v(x, t_1 + t') = -v(x, t_3 - t'). \tag{37}$$

Moreover, it is obvious that

$$r(x, t_1 + t') = r(x, t_3 - t'), \quad T(x, t_1 + t') = T(x, t_3 - t'),$$

and

$$p(x, t_1 + t') = p(x, t_3 - t') \tag{38}$$

It turns out that for a medium with dissipation it is possible to construct two "mutually symmetric" or "mirror" motions, i.e., those such that the motion for $t_2 < t < t_3$ "repeats" in reverse order the motion for $t_1 < t < t_2$. The profiles of the density, pressure, temperature, and modulus in the velocity in the "reverse" motion are the same as in the "direct" motion, while the sign of the speeds are opposite [for "mutually symmetric" motions (37) holds]. In a particular sense the example of such motions demonstrates "time reversal" for a system with dissipative effects. We shall construct it using the machinery of self-similar solutions for a finite mass of matter [64, 72].

We return to the system (17) which in the case considered here has the simpler form

$$\begin{gathered} n(1-n)\,\delta\lambda = \frac{d\beta}{d\lambda}; \\ n_\gamma \beta = -\frac{1}{\lambda^N} \frac{d}{d\lambda}(\lambda^N \omega) + q; \\ \omega = -\tilde{\varkappa}\,\frac{d\theta}{d\lambda}; \quad \beta = \delta\theta; \\ \tilde{\varkappa} = A_1 \theta^{m_1} \delta^{k_1}; \quad q = A_6 \theta^{m_6} \delta^{k_6}; \\ n_\gamma = \frac{2}{\gamma - 1}\left(\frac{n}{n_*} - 1\right); \quad n_* = \frac{2}{2+\mu}; \quad \mu \equiv (\gamma - 1)(N + 1). \end{gathered} \tag{39}$$

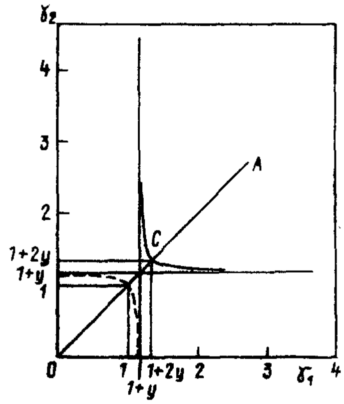


Fig. 19. Dependence between the adiabatic exponents γ_1 and γ_2 in mutually symmetric ("mirror") regimes.

As was shown above, the difference between problems of compression and rarefaction described by the system (39) consists only in the signs of the dimensionless dissipative coefficients A_1 and A_6: they are positive for rarefaction problems and negative for compression.

In the class of solutions (10) to mutually symmetric motions there correspond solutions for compression ($t < 0$) and rarefaction ($t > 0$) with the same spatial distributions of the quantities $f_i(\xi)$ or $f_i(\lambda)$. The latter, in turn, are determined by the system (39). We define mutually symmetric motions as motions in media with the same $\varkappa$ and Q, and one of these motions (it makes no difference whether compression or rarefaction) we call the "direct" motion and the other the "reverse" motion. The problem then consists in finding identical solutions of two problems for the system (39) with the same boundary conditions at $\lambda = 0$ and $\lambda = \lambda_*$ and the same values of $|A_1|$ and $|A_6|$ but with opposite signs of A_1 and A_6. For mutually symmetric motions (38), (39) hold.

Suppose the "direct" motion has parameters $n^{(1)} \equiv n$ and γ_1, so that

$$n_\gamma^{(1)}=\frac{2}{\gamma_1-1}\left(\frac{n}{n_*^{(1)}}-1\right),\quad n_*^{(1)}=\frac{2}{2+\mu_1},\quad \mu_1=(\gamma_1-1)(N+1).$$

Then analysis of the system (39) shows that the "reverse" motion must have the parameters

$$n^{(2)}=n;\quad n_\gamma^{(2)}=-n_\gamma^{(1)}. \tag{40}$$

This leads to a relation between the adiabatic exponents γ_2 and γ_1 in the "reverse" and "direct" motion:

$$\gamma_2=1+\frac{1}{\dfrac{n(N+1)}{1-n}-\dfrac{1}{\gamma_1-1}}. \tag{41}$$

This formula can be rewritten in a form symmetric in γ,

$$\frac{(\gamma_2-1)(\gamma_1-1)}{\gamma_2+\gamma_1-2}=\frac{1-n}{(N+1)n} \tag{42}$$

or μ

$$\frac{\mu_1\mu_2}{\mu_1+\mu_2}=\frac{1-n}{n}, \tag{43}$$

which defines the equilateral hyperbola $(\mu_2-z)(\mu_1-z)=z^2$, $z=(1-n)n^{-1}$. Figure 19 shows the graph of the dependence (41) of γ_2 on γ_1 for fixed n and N. The quantity $y = z/(N + 1)$ is denoted by y. Concrete values of n and N for which the graph is constructed and also some values of γ_1 and γ_2 for these n, N are presented in Table 3.

From formulas (42), (43) it is evident that mutually symmetric motions are possible only for $n\leqslant 1$ ($z\geqslant 0$), if it is assumed that $1<\gamma_{1,2}<\infty$. From an analysis of (42) and (43) it follows that the lines $\gamma_1 = 1 + y$ and $\gamma_2 = 1 + y$ are asymptotics: for $\gamma_1\to 1+y$, $\gamma_2\to\infty$, while for $\gamma_1\to\infty$, $\gamma_2\to 1+y$. For $\gamma_1 = 1 + 2y$ we have $\gamma_2 = \gamma_1$. The second branch of the hyperbola (denoted by the dashed curve in Fig. 19) is not physically real, since on it either $\gamma_1 < 1$ (above the bisectrix OA of the coordinate angle) or $\gamma_2 < 1$ (below OA). The case of equality $\gamma_1 = \gamma_2$ (the point C on the graph) corresponds to motions with entropy conservation. Indeed, for $\gamma_1 = \gamma_2$ we have $\mu_1 = \mu_2 = \mu$, and from (43) it follows that $n = 2/(2 + \mu) = n_*$. We recall that

TABLE 3

	$N=2,$	$n=2/3,$	$y=1/6$
γ_1	4/3=1,3...	5/3=1,6...	2
γ_2	4/3=1,3...	11/9=1,2...	6/5=1,2

$$\frac{dS_t}{dt}=\frac{2Rx_0}{\gamma-1}\cdot\frac{n/n_*-1}{t}, \tag{44}$$

where S_t is the total entropy of the mass of gas. For $n = n_*$ we have $\frac{dS_t}{dt}=0$. To this case there correspond motions of the gas without consideration of dissipative processes (for example, the solution of supercompression of [66-68]) for which the "direct" and "reverse" motion is physically realizable for the same medium with the same exponent γ.

In the general case of a dissipative medium the values of γ_1 and γ_2 are distinct. From formulas (40)-(43) it follows here that if $n_*^{(1)} > n$, then $n_*^{(2)} < n$, and the other way around. In accordance with (44) this ensures the same variation of the entropy S_n with time (decrease or increase) in the "direct" and "reverse" motions. Indeed, from (43) or directly from condition (40) with consideration of $n = 2(2 + \mu)^{-1} < 1$ $(\mu > 0)$ we obtain a formula for the connection of $n_*^{(2)}$ and $n_*^{(1)}$:

$$\frac{n-n_*^{(1)}}{n-n_*^{(2)}}=-\frac{1-n_*^{(1)}}{1-n_*^{(2)}}<0, \tag{45}$$

from which the above assertions follow. They agree with the physical sense of the problem, since an increase or decrease of the entropy in a medium with a source and heat conduction is determined by the temperature profile (i.e., a positive or negative heat flux through the piston), and the latter is the same for the "direct" and "reverse" motions.

Thus, for example, taking for the "direct" motion rarefaction of the medium with $\gamma_1 = 5/3$, N = 2, and n = 2/3, which corresponds to $\varkappa\sim T^{5/2}$ and $Q\sim T^{1,6}\rho^{1,3}$, for the "reverse" motion we obtain $\gamma_2 = 11/9 = 55/45 = 1.22,\ldots$. Here $n_*^{(1)} = 1/2 < n$ (a regime of entropy increase), while $n_*^{(2)} = 3/4 > n$. An example of such a solution in the case of compression for the nearby value $\gamma_2 = 1,2 = 6/5 = 54/45$ is presented in Fig. 16 (p. 66). In this example both at the rarefaction stage ($\gamma = 5/3$) and the compression stage ($\gamma = 1,2$) there is an appreciable peak of energy evolution not at the center and not on the piston which is caused by the effect of volumetric heat source.

In the present work we do not touch on results of numerical investigations of the system (39). Such investigations should show whether both mutually symmetric motions are stable or one of them is necessarily unstable or that stability of the reverse motion depends on the parameters of the problem. We remark that in [66, 68, 70] numerical solutions of the system (1) are obtained which show stability of the regimes (10) for compression of a heat-conducting medium.

We note that in regimes with the same n and γ (and hence n_γ) the profiles $f_i(\lambda)$ of a problem with heat conduction and a heat source will coincide with the profiles $f_i(\lambda)$ of a problem with a negative diffusion of heat and a sink if the "sign of the time" is changed. Such an example is trivial and occurs also for a purely thermal problem (without gas-dynamical motion). In this case the entropy in the direct and reverse motions behaves differently (decreases in one and increases in the other).

A more substantial example is the example constructed above of "reversal" of processes in time in a dissipative medium with the same $\varkappa$ and Q. In this case for the same profiles of ρ and v in mutually symmetric motions and for mirror-symmetric behavior of them in time the total entropy S_t grows both in the "direct" and in the "reverse" motions [see (44)], whereby $S_t\sim\ln t$ $(t>0,\ t\to\infty)$ and $S_t\sim-\ln|t|$ $(t<0,\ t\to0)$, or it decreases both in the "direct" and in the "reverse" motions, whereby $S_t\sim-\ln t$ $(t>0)$ and $S_t\sim\ln|t|$ $(t<0)$.

Thus, the "mirror property" in time of mutually symmetric motions does not hold for the entropy. This "nonmirrorness" of the behavior of $S_t(t)$ in the present example is a consequence of the second law of thermodynamics.

4. Self-Similar Regimes of Compression and Rarefaction with Consideration of Diffusion of the Magnetic Field

4.1. Formulation of the Problem of Self-Similar Compression or Dispersion of a Finite Mass of Plasma in the Presence of a Magnetic Field. This problem has been discussed earlier in the works [20, 24, 28, 38, 65, 67, 70, 71, 130]; it is a natural generalization of the problem considered above to the case of consideration of additional physical effects. The basic ideas of investigating the properties of self-similar solutions are hereby preserved. Thus, for example, waves of diffusion of the magnetic field are steady-state; in regimes of entropy growth heat structures are possible due to Joule heating, monotonicity or nonmonotonicity of the profile of the magnetic field depends on the increase or decrease of a certain integral quantity (the magnetic flux), particular boundary conditions lead to constraints on the values of the dimensionless coefficient of thermal conductivity, etc.

The problems considered here describe the motions of a planar or cylindrical column of plasma under the action of external magnetic fields and the current running through the plasma. Such configurations of plasma are customarily called pinch configurations [100].

To describe the magnetic and electric fields we use Maxwell's equations in the approximation of hydrodynamics [100]. In this case the magnetic field $\mathbf{H}$, the electric field $\mathbf{E}$, and the current density $\mathbf{j}$ are subject to the equation

$$\begin{gathered}\operatorname{div}\mathbf{H}=0;\\ \operatorname{rot}\mathbf{H}=\frac{4\pi}{c}\mathbf{j};\\ \frac{\partial\mathbf{H}}{\partial t}=-c\operatorname{rot}\mathbf{E}.\end{gathered}\tag{46}$$

Here c is an electrodynamical constant (the speed of light).

The last two equations of the system (46) suffice to determine the fields if Ohm's law is given connecting $\mathbf{j}$ with $\mathbf{E}$ and $\mathbf{H}$. In the general case its form is rather complicated [15].

As is known [111], from consideration of the system of equations of magnetohydrodynamics in the presence of spatial symmetry of the solution [see the system (1)] the component $N = 0$ or $N = 1$ ($N = 2$ is inadmissible), and the form of the radial components H_r and E_r is trivial.

In the system of equations of the one-dimensional motion of the plasma in place of $\mathbf{H}$, $\mathbf{E}$, and $\mathbf{j}$ it is more convenient to consider the quantities $\mathbf{H}$, $\mathscr{E}$, and $\mathbf{i}$, where $\mathscr{E}$ and $\mathbf{i}$ are particular combinations of $\mathbf{E}$, $\mathbf{H}$, $\mathbf{j}$ and the constant c: $\mathscr{E}=c\mathbf{E}+[\mathbf{vH}]$, $\mathbf{i}=\frac{1}{c}\mathbf{j}$ [20, 24, 28, 67, 111, 130]. Writing the equations componentwise [15] for the case of a small influence of the magnetic field on the transport coefficients, we have

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{\partial}{\partial x}(r^N v);\quad \frac{\partial r}{\partial t}=v;$$

$$\begin{gathered}\frac{\partial v}{\partial t}=-r^N\frac{\partial}{\partial x}\left(p+\frac{H_z^2+H_\varphi^2}{8\pi}-D\right)-\frac{NH_\varphi^2}{4\pi\rho r}+N\frac{D-D'}{\rho r};\\ \frac{\partial\varepsilon_i}{\partial t}+p_i\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=-\frac{\partial}{\partial x}(r^N W_i)+\frac{\Phi}{\rho}+\frac{Q_I}{\rho}+\frac{Q_{ei}}{\rho};\\ \frac{\partial\varepsilon_e}{\partial t}+p_e\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=-\frac{\partial}{\partial x}(r^N W_e)+\frac{Q_e}{\rho}-\frac{Q_{ei}}{\rho}+\frac{Q_J}{\rho}-\frac{Q_R}{\rho};\\ \frac{\partial}{\partial t}\left(\frac{H_z}{\rho}\right)=\frac{\partial}{\partial x}(r^N\mathscr{E}_\varphi);\quad \mathscr{E}_\varphi=\rho r^N\nu_m\frac{\partial H_z}{\partial x};\\ \frac{\partial}{\partial t}\left(\frac{H_\varphi}{\rho r^N}\right)=\frac{\partial\mathscr{E}_z}{\partial x};\quad \mathscr{E}_z=\rho\nu_m\frac{\partial}{\partial x}(r^N H_\varphi);\\ W_i=-\varkappa_i\rho r^N\frac{\partial T_i}{\partial x};\quad W_e=-\varkappa_e\rho r^N\frac{\partial T_e}{\partial x};\\ p=p_i+p_e=\rho R(T_i+ZT_e);\quad \varepsilon=\varepsilon_i+\varepsilon_e=\frac{R}{\gamma-1}(T_i+ZT_e);\\ Q_{ei}=\chi(T_e-T_i);\quad \Phi=\rho r^N D\frac{\partial v}{\partial x}+N\frac{vD'}{r};\quad Q_I=\frac{\mathscr{E}_z^2+\mathscr{E}_\varphi^2}{4\pi\nu_m};\\ D=\rho r^N\left(\frac{4}{3}\eta+\zeta\right)\frac{\partial v}{\partial x}+N\left(\zeta-\frac{2}{3}\eta\right)\frac{v}{r};\end{gathered}$$

$$D' = \rho r^N \left(\zeta - \frac{2}{3}\eta\right)\frac{\partial v}{\partial x} + N\left[\zeta + \left(\frac{7}{3} - N\right)\eta\right]\frac{v}{r}. \quad (47)$$

In contrast to (1) here terms with the magnetic pressure are added to the equation for the momentum, Joule heating Q_J, and the equations for the components of the fields $\mathbf{H}$ and $\mathscr{E}$, defined from considerations of convenience as follows: $\mathbf{H}=\{0, H_\varphi, H_z\}$ and $\mathscr{E}=\{0, -\mathscr{E}_\varphi, \mathscr{E}_z\}$. From (47) it is also possible to find the components of the current density:

$$i_\varphi = -\frac{\rho r^N}{4\pi}\cdot\frac{\partial H_z}{\partial x}, \quad i_z = \frac{\rho}{4\pi}\cdot\frac{\partial}{\partial x}(r^N H_\varphi). \quad (48)$$

In the system (47) ν_m is the magnetic viscosity ($\nu_m = c^2/4\pi\sigma$, where σ is the conductivity of the medium [100]). As for the other coefficients of the systems (1) and (47), we adopt the following form of the dependence of ν_m on the parameters of the medium:

$$\nu_m = a_{11} T_e^{m_{11}} \rho^{k_{11}} t^{l_{11}}. \quad (49)$$

The boundary conditions for the fields require prescribing two of the four quantities (H_φ, H_z, $\mathscr{E}_\varphi$, $\mathscr{E}_z$) introduced on each of the end points of the segment $0 \leqslant x \leqslant x_0$.

For x = 0 as boundary conditions we take the symmetry conditions

$$H_\varphi(0, t) = 0, \quad \mathscr{E}_\varphi(0, t) = 0. \quad (50)$$

For $x = x_0$ it is possible to prescribe, for example, the longitudinal components

$$H_z(x_0, t) = H_0 t^{n_4}, \quad \mathscr{E}_z(x_0, t) = \mathscr{E}_0 t^{n_5}, \quad (51)$$

where as a result of dimensional analysis (10) the exponents n_4 and n_5 are

$$n_4 = -1 + (1-N)\frac{n}{2}, \quad n_5 = -2 + (3-N)\frac{n}{2}.$$

If the total current I(t) running along the column of plasma is given the second condition of (51) must be replaced by

$$H_\varphi(x_0, t) = \frac{2\pi^{1-N} I(t)}{r_*^N l^{1-N}}. \quad (52)$$

Here ℓ is the size of the pinch for N = 0 along the transverse axis y, $r_* = r(x_0, t)$ is the radius of the pinch, and the total current I(t) is connected with the current density $i_z(x, t)$ by the formula

$$I(t) = [2\pi N + 2(1-N)l]\int_0^{x_0} \frac{i_z}{\rho}\,dx, \quad N = 0, 1. \quad (53)$$

From (52) we obtain the law of variation of the total current I(t) ensuring self-similarity (10) of the problem

$$I(t) = I_0 t^{n_0}, \quad (54)$$

where $n_0 = nN + n_4 = \frac{n}{2}(N+1) - 1$, i.e., $n_0 = \frac{n}{2} - 1$ for N = 0 and $n_0 = n - 1$ for N = 1.

A dimensional analysis of the system (47) with consideration of effects of dissipation of the magnetic field [67, 70] augments formulas (10) as follows:

$$\begin{gathered} H_{\varphi,z}(x, t) = \frac{x_0^{1/2}(v_0 t^n)^{\frac{1-N}{2}}}{t} h_{\varphi,z}(\xi), \\ \mathscr{E}_{\varphi,z}(x, t) = \frac{x_0^{1/2}(v_0 t^n)^{\frac{3-N}{2}}}{t^2} e_{\varphi,z}(\xi), \\ Q_J(x, t) = \frac{x_0^{1/2}(v_0 t^n)^{1-N}}{t^3}\cdot q_J(\xi), \\ i_{\varphi,z}(x, t) = \frac{x_0^{1/2}(v_0 t^n)^{-\frac{N+1}{2}}}{t}\tilde{i}_{\varphi,z}(\xi), \\ \nu_m(x, t) = (v_0 t^n)^2 t^{-1}\tilde{\nu}_m(\xi). \end{gathered} \quad (55)$$

Formulas (55) show that in the case of compression ($-\infty < t < 0$) the quantities q_J and $\tilde{\nu}_m$ are negative, the signs of the components $\mathscr{E}_{\varphi,z}$ and $e_{\varphi,z}$ coincide, while the signs of $H_{\varphi,z}$ and $h_{\varphi,z}$ are opposite.

The conditions of self-similarity (13) are augmented by the conditions for the dimensionless property for the constant a_{11}:

$$n=\frac{2m_{11}-1-l_{11}}{2m_{11}-k_{11}(N+1)-2},$$
$$a_{11}=A_{11}v_0^{k_{11}(N+1)-2m_{11}+2}x_0^{-k_{11}}R^{m_{11}} \tag{56}$$

For a completely ionized plasma $m_{11} = -3/2$, $k_{11} = \ell_{11} = 0$ which leads to the value $n = 4/5$ for any N. For all other processes [see (14)] in this case we obtain $n = 4/(4 + N)$, i.e., together with processes of heat conduction and viscosity in the case $N = 1$ it is possible to consider also diffusion of the magnetic field in a completely ionized plasma.

We make the following remark regarding the consideration of magnetization. In a completely ionized plasma [15] for sufficiently large values of the magnetic field ($Y \gtrsim 1$) the transport coefficients begin to depend on the quantity $Y \equiv \Omega_e\tau_e$, the quantity $\mathscr{E}_z$ on $\frac{\partial H_z}{\partial x}$ and $\frac{\partial T_e}{\partial x}$, $\mathscr{E}_\varphi$ on $\frac{\partial}{\partial x}(r^N H_\varphi)$ and $\frac{\partial T_e}{\partial x}$, W_e on $\frac{\partial H_z}{\partial x}$, and $\frac{\partial}{\partial x}(r^N H_\varphi)$. Here

$$\Omega_e=\frac{eH}{m_e c},\quad H=\sqrt{H_\varphi^2+H_z^2},\quad \tau_e=\frac{3\sqrt{m_e}m_p^{5/2}M_i^{5/2}R^{3/2}}{4\sqrt{2\pi}e^4\rho\Lambda}T_e^{3/2}.$$

From the viewpoint of dimensional analysis consideration of magnetization requires that additional conditions for the dimensionless property of the constant a_{12} be satisfied in the expression

$$Y=a_{12}H^b T_e^{m_{12}}\rho^{k_{12}}t^{l_{12}}, \tag{57}$$

where Y is dimensionless. The corresponding conditions have the form

$$n=\frac{2m_{12}+b-l_{12}}{2m_{12}-k_{12}(N+1)+(1-N)b/2},$$
$$a_{12}=A_{12}v_0^{k_{12}(N+1)-2m_{12}+(N-1)\frac{b}{2}}x_0^{-k_{12}-\frac{b}{2}}R^{m_{12}}. \tag{58}$$

For a completely ionized plasma $n_{12} = 3/2$, $k_{12} = -1$, $b = 1$ and it follows from (58) that $n = 8/(9 + N)$. In the case $N = 1$ we again obtain $n = 4/5$. In this case it is possible to add terms to the system (47) which express effect of magnetization and thermoelectric phenomena [38]. If conditions (58) are satisfied the dependence of the transport coefficients on Y can be arbitrary, since Y is dimensionless. In particular, dependence of the form $\varkappa_e = \varkappa_e^{(0)}\mathscr{F}_\varkappa(Y)$, $\nu_m = \nu_m^{(0)}\mathscr{F}_\nu(Y)$, etc. is possible. Here $\varkappa_e^{(0)}$, $\nu_m^{(0)}$ are the values of the coefficients in the limit case of a weak field, while the function $\mathscr{F}_\varkappa$, $\mathscr{F}_\nu$ are approximated in [15] by ratios of polynomials in Y.

Qualitative analysis of the system of equations (47) with consideration of additional terms for $Y > 1$ is complicated by the dependencies among the derivatives of H_φ, H_z, and T_e [15, 38, 100]. To clarify the principal features it suffices to make a qualitative analysis of the system of equations for representatives of dimensional quantities only in the case of a weak field ($Y < 1$).

4.2. Qualitative Analysis of the Behavior of the Fields in Space. This analysis shows that the behavior depends on the character of the variation with time of the integral quantities — the corresponding fluxes of the magnetic field.

Substituting the relations of (10) and (55) into (47), we again obtain (17) where the terms $\frac{d}{d\lambda}\left(\frac{h_\varphi^2+h_z^2}{8\pi}\right)+\frac{Nh_\varphi^2}{4\pi\lambda}$, are added to the right side of the first equation, the source q_J is added to the right side of the equation for $n_\gamma\beta_e$, and we have the additional equations

$$m_\Theta h_z=\frac{1}{\lambda^N}\frac{d}{d\lambda}(\lambda^N e_\varphi);\quad e_\varphi=\tilde{\nu}_m\frac{dh_z}{d\lambda};$$
$$m_Z h_\varphi=\frac{de_z}{d\lambda};\quad e_z=\tilde{\nu}_m\frac{1}{\lambda^N}\frac{d}{d\lambda}(\lambda^N h_\varphi) \tag{59}$$

where $m_\Theta=\frac{n}{n_\Theta}-1,\quad m_Z=\frac{n}{n_Z}-1,\quad n_\Theta=\frac{2}{3+N},\ n_Z=\frac{2}{3-N}$. As before, the system of self-similar equations is considered on the segment $0\leqslant\lambda\leqslant\lambda_*$ with boundary conditions (18)-(23).

With consideration of Eqs. (59) at the points of the segment $[0, \lambda_*]$ we have the conditions

$$h_\varphi(0)=0,\quad e_\varphi(0)=0$$

and (60)

$$e_z(\lambda_*)=e_z^*,\quad h_z(\lambda_*)=h_z^*,$$

which follow from (50), (51).

In the case of prescription of the total current (54) in place of the scale v_0 (55) contains the scale $I_0: v_0\sim\left(I_0/l^{1-N}\sqrt{x_0}\right)^{\frac{2}{N+1}}$. Including into the definition of the scale I_0 all constant factors (53), we obtain in place of (52):

$$h_\varphi(\lambda_*)=\frac{2\pi^{1-N}}{\lambda_*^N}.$$

The constants m_Θ and m_Z introduced in (59) indicate the laws of variation with time of the magnetic fluxes of the fields H_Z and H_φ. The axial magnetic field H_Z exists in the configuration called Θ-pinch; the azimuthal H_φ exists in the configuration called Z-pinch [100].

For the axial field H_Z the corresponding magnetic flux F_Z across a cross section of the pinch can be expressed as follows:

$$\begin{aligned}F_z&=(2\pi N+2(1-N)\,l^{1-N})\int_0^{r_*}H_z(r,t)\,r^N dr=\\&=(2\pi N+2(1-N)\,l^{1-N})\,\frac{x_0^{1/2}}{t}\,(v_0t^n)^{\frac{3+N}{2}}\int_0^{\lambda_*}h_z(\lambda)\,\lambda^N d\lambda\sim t^{m_\Theta},\end{aligned}\tag{61}$$

where $r_* = r(x_0, t)$ is the outer radius of the plasma, and l is the same as in (52).

The magnetic flux F_φ of the azimuthal field has the form

$$F_\varphi=L\int_0^{r_*}H_\varphi(r,t)\,dr=\frac{Lx_0^{1/2}(v_0t^n)^{\frac{3-N}{2}}}{t}\int_0^{\lambda_*}h_\varphi(\lambda)\,d\lambda\sim t^{m_Z}.\tag{62}$$

From (61), (62) it follows that for compression regimes ($-\infty < t < 0$, $t\to 0$) the magnetic flux F_Z increases for $m_\Theta<0$, decreases for $m_\Theta>0$, and remains constant for $m_\Theta=0$. For rarefaction regimes ($0 < t < \infty$, $t \to +\infty$), on the other hand, F_Z decreases for $m_\Theta<0$, increases for $m_\Theta>0$, and is constant for $m_\Theta=0$. The flux F_φ with m_Θ replaced by m_Z has analogous behavior in time.

Moreover, the signs of m_Θ and m_Z [contained in the left sides of equations (59)] determine the character of the behavior of the fields in space.

We first consider the fields h_Z and e_φ. Suppose h_Z, θ, and δ are finite for $\lambda = 0$, i.e., $h_Z(0) \neq 0$, $\tilde{v}_m(0) = \tilde{v}_m(\theta)0)$, $\delta(0)) \neq 0$. From (59) it follows that

$$\frac{dh_z}{d\lambda}=m_\Theta\frac{\int_0^\lambda h_z(y)\,y^N dy}{\tilde{v}_m\lambda^N},\quad e_\varphi=\frac{m_\Theta}{\lambda^N}\int_0^\lambda h_z(y)\,y^N dy,$$

and as $\lambda \to 0$ we have

$$\frac{dh_z}{d\lambda}\to\frac{m_\Theta h_z(0)}{\tilde{v}_m(0)}\cdot\frac{\lambda}{N+1},\quad e_\varphi\to\frac{m_\Theta h_z(0)}{N+1}\cdot\lambda.$$

From this it is evident that for $\operatorname{sign} m_\Theta=\operatorname{sign}\tilde{v}_m$ the fields h_z and e_φ increase monotonically in absolute value from the center ($\lambda = 0$) to the boundary ($\lambda = \lambda_*$). The condition coincides with the condition for an increase of the quantity F_Z ($m_\Theta<0$ in a compression regime for $\tilde{v}_m < 0$ and $m_\Theta>0$ for rarefaction for $\tilde{v}_m > 0$).

Similarly, for the fields e_z and h_φ we obtain

$$\frac{de_z}{d\lambda}=m_Z\lambda^{-N}\int_0^\lambda e_z(y)\,\tilde{\nu}_m^{-1}(y)\,y^N dy,\quad h_\varphi=\lambda^{-N}\int_0^\lambda e_z(y)\,\tilde{\nu}_m^{-1}(y)\,y^N dy.$$

It is easy to see that $|e_Z|$ increases monotonically under the condition $\operatorname{sign} m_Z = \operatorname{sign}\tilde{\nu}_m$.

For the opposite conditions

$$\operatorname{sign} m_\Theta=-\operatorname{sign}\tilde{\nu}_m,\quad \operatorname{sign} m_Z=-\operatorname{sign}\tilde{\nu}_m \tag{63}$$

the profile of the fields h_z, e_φ and e_z, h_φ may have oscillating character [67, 70]. If the characteristic dimension of variation of the fields in this case is less than λ_*, then in the self-similar regime the plasma decomposes into layers with direct and reverse currents [connected with the magnitudes of the fields according to (47), (48)].

Finally, in the case $m_\Theta=0$ $(n=n_\Theta)$ or $m_Z = 0$ $(n = n_Z)$ the solutions for the fields are monotone, and h_z and e_φ cease to depend on $\tilde{\nu}_m$ [and thus on the solutions for $\theta(\lambda)$ and $\delta(\lambda)$]. This case is altogether analogous to the case $m_* = 0$ $(n = n_*)$ in which the solution for $\theta(\lambda)$ is homogeneous in space and does not depend on $\tilde{\varkappa}$ [see (33)].

For estimates of the characteristic dimension of oscillations of the field we present analytic solutions which hold in the case $\tilde{\nu}_m \equiv \tilde{\nu}_0 = \text{const}$. The solutions are ($C_1$ is a constant of integration)

$$e_\varphi=C_1\tilde{\nu}_0\frac{\bar{\lambda}_\Theta}{\lambda}\cdot \operatorname{sh}(\bar{\lambda}_\Theta), \tag{64}$$

$$h_z=C_1\operatorname{ch}(\bar{\lambda}_\Theta);\quad N=0;$$

$$e_\varphi=C_1\tilde{\nu}_0\frac{\bar{\lambda}_\Theta}{\lambda}\,\mathrm{I}_1(\bar{\lambda}_\Theta), \tag{65}$$

$$h_z=C_1\mathrm{I}_0(\bar{\lambda}_\Theta);\quad N=1;$$

$$h_\varphi=\frac{C_1}{m_Z}\cdot\frac{\bar{\lambda}_Z}{\lambda}\operatorname{sh}(\bar{\lambda}_Z), \tag{66}$$

$$e_z=C_1\operatorname{ch}(\bar{\lambda}_Z);\quad N=0;$$

$$h_\varphi=\frac{C_1}{m_Z}\frac{\bar{\lambda}_Z}{\lambda}\mathrm{I}_1(\bar{\lambda}_Z),\quad e_z=C_1\mathrm{I}_0(\bar{\lambda}_Z);\quad N=1; \tag{67}$$

$$h_z=C_1,\quad e_\varphi=0,\quad m_\Theta=0; \tag{68}$$

$$e_z=C_1,\quad h_\varphi=\frac{C_1\lambda}{\tilde{\nu}_0(N+1)},\quad m_Z=0. \tag{69}$$

Here I_0 and I_1 are the Bessel functions of imaginary argument $\bar{\lambda}_\Theta=\lambda\sqrt{m_\Theta/\tilde{\nu}_0}$, $\bar{\lambda}_Z=\lambda\sqrt{m_Z/\tilde{\nu}_0}$ $(\sqrt{a}=i\sqrt{|a|}$, if $a<0)$.

From (64), (65) and (66), (67) we obtain a simple estimate for the "period" of oscillation of the field [for a large number of "oscillations" when $\lambda\gg\sqrt{-\tilde{\nu}_0/m_\Theta}$, $\lambda\gg\sqrt{-\tilde{\nu}_0/m_Z}$, $J_0(y)=\sqrt{\frac{2}{\pi y}}\cos\left(y-\frac{\pi}{4}\right)+O(y^{-3/2})$] :

$$\Delta\lambda_M^\Theta=2\pi\sqrt{-\tilde{\nu}_0/m_\Theta},\quad \Delta\lambda_M^Z=2\pi\sqrt{-\tilde{\nu}_0/m_Z} \tag{70}$$

(J_0 is the Bessel function).

The number of "oscillations" N_M of the field h_Z on a segment $[\lambda_1,\lambda_2]\in[0,\lambda_*]$ can be estimated by taking the average value $\tilde{\nu}_0=(\lambda_2-\lambda_1)^{-1}\int_{\lambda_1}^{\lambda_2} A_3\theta_e^{m_3}\delta^{k_3}d\lambda$: $N_M=(\lambda_2-\lambda_1)/\Delta\lambda_M^\Theta$. The number of zeros of the field, i.e., the number of layers with direct and reverse currents, is equal to $2N_M$.

4.3. A Numerical Example of Solving the System (17), (59). This example is presented for the case $n = 4/5$, $N = 1$, and $n_\Theta<n<n_*$. This case corresponds to dependencies in $\varkappa_e(T_e,\rho)$ and $\nu_m(T_e,\rho)$, corresponding to a real, completely ionized plasma $(\varkappa_e\sim T_e^{5/2},\ \nu_m\sim T_e^{-3/2})$. For a

compression regime the inequality $n_\Theta < n < n_*$ implies that increase of entropy and decrease of the magnetic flux F_z is observed in such a regime. We shall consider the case of a single-temperature plasma ($\theta_e = \theta_i$), take into account only processes of heat conduction and diffusion of the magnetic field, and restrict ourselves to the configuration of Θ-pinch ($h_\varphi = 0$, $e_z = 0$). For N = 1 and n = 4/5 we always have $n > n_\Theta = 1/2$. The condition $n < n_*$ is satisfied for $\gamma < 1/n = 5/4 = 1.25$.

Figure 20 shows the result of numerical solution of the system of self-similar equations for $\gamma = 1{,}2$, $\tilde{\varkappa} = -0{,}0001 \cdot \theta^{5/2}$, $\tilde{\nu}_m = -0{,}001 \cdot \theta^{-3/2}$. Here $\xi_* = 0.66$. The profile of the pressure β (omitted in the graph) is determined by the profile of the magnetic field: β is a maximum at points where $h_z = 0$ and a minimum at points of extrema of h_z. In the calculation for Fig. 20 for $\lambda = 0$ $h_z^2/8\pi\beta \cong 32$. In a neighborhood of other extrema of h_z the magnetic pressure is $h_z^2/8\pi \approx \beta$. At points of maxima of θ we have $|q_J| > |n_\gamma|\beta$ as follows from general considerations [see (31)].

We estimate the distance between maxima of h_z (for $\lambda > 0.2$) according to formula (70). Substitution of the numerical values $m_\Theta = 0{,}6$, $\langle\theta\rangle \simeq 3{,}7$, $|\langle\tilde{\nu}_m\rangle| \simeq 1{,}4 \cdot 10^{-4}$ gives $\Delta\lambda_M \simeq 0{,}1$, which coincides with the calculation. To determine the position of the first zero of h_z we use the value of the first root of the function $J_0(y)$: $\lambda_0 \cong 2{,}4\sqrt{|\langle\tilde{\nu}_m\rangle|/m_\Theta}$. Assuming for $\lambda < 0.2$ that $\langle\theta\rangle \simeq 0.5$, we obtain the estimate $\lambda_0 \simeq 0.16$ which coincides with the computed value.

By analogy with $\Delta\lambda_M$ it is also possible to estimate the distance between maxima of the temperature $\Delta\lambda_T \simeq 5\sqrt{\dfrac{\langle\tilde{\varkappa}\rangle(\gamma-1)}{2\left(\dfrac{n}{n_*}-1\right)\langle\delta\rangle}}$ [70]. Substitution of numerical values show good applicability of the indicated formula. We note that the behavior of θ in the calculations illustrated by Figs. 15 and 17 is also subject to this formula.

The maxima of θ for $\lambda \cong 0.25$ and $\lambda \cong 0.31$ in Fig. 20 are apparently of another nature. They arose in regions where $h_z \approx 0$. In non-self-similar regimes an increase of the density with time which is faster than in other parts of the plasma would occur here. Such structures can be caused by two-sided compression of matter by the magnetic field at points of zero value of the field ("ρ-layers"). In contrast to T-layers the density inside such formations is higher than in the surrounding medium. As follows from the calculation represented in Fig. 20, the power of Joule heating $|q_J|$ in these formations is two-three times less than the power $n(1 + N)\beta = 1.6\beta$ of the work of compression. For $\lambda > 0.2$ the ρ-layers arising at points where $h_z = 0$ are relatively weak. A powerful ρ-layer occurs in a neighborhood of the first zero of the magnetic field ($\lambda \approx 0.14$). It is steaked by a series of minima of the density δ arising as a consequence of the existence in this region of an entire family of maxima of the temperature θ.

The nonmonotone character of the profiles h_z and θ means that in such regimes boundary conditions of different character can be realized (the field h_z and the heat flux ω may have any signs on the piston). A complex structure with increase of entropy may exist, for example, both in conditions of withdrawal ($\lambda_* = 0.26$) and input of heat ($\lambda_* = 0.4$). In particular, external magnetic and thermal effects can be eliminated: $h_* = 0$ ($\lambda_* \cong 0.36$); $\omega_* = 0$ ($\lambda_* \cong 0.33$).

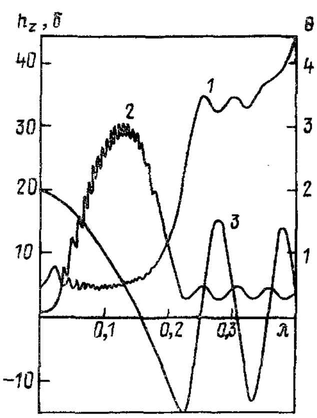


Fig. 20. Profiles of the dimensionless temperature θ (1), density δ (2), and axial magnetic field h_z (3) in the problem of compression of a single-temperature plasma in a regime with entropy increase and decrease of the magnetic flux.

Using the expressions for $\varkappa_e$ and ν_m of [15], it is possible to obtain the following dimensional parameters for the calculation of Fig. 20: $x_0 \simeq 0.15$ g/cm, $|v_0| \simeq 3\cdot10^5$ cm·sec$^{-0.8}$. For the time $t = -10^{-5}$ sec = -10 μsec according to formulas (10), (55) we obtain: the radius of the piston is $r_* \simeq 13$ cm, its speed is $|v_*| \simeq 10$ km/sec, the average temperature in the central region ($\lambda < 0.2$) is $T \simeq 15$ eV, in the boundary region ($\lambda > 0.24$) is $T \simeq 80$ eV, the maximum value of the density $\rho_{max} \simeq 5\cdot10^{-3}$ g/cm³ ($n_{max} \simeq 10^{21}$ cm^{-3}), the average density is $\langle n\rangle \simeq 4\cdot10^{20}$ cm^{-3}, and the maximum value of the field $H_{max} \simeq 800$ kOe. The magnetization is not large: $Y \approx 0.4$. The mean free path of electrons (computed on the basis of T and $\langle n\rangle$ [15] for $t = 10^{-5}$ sec) is $\ell_e \approx 3\cdot10^{-5}$ cm $\ll r_*$ (the approximation of a continuous medium is valid), the time of equalization of temperatures of electrons and ions (for $t = -10^{-5}$ sec) is $\tau_{ei} \approx 4\cdot10^{-10}$ sec (the single-temperature approximation holds).

In conclusion we note that the self-similar solution presented in Fig. 20 is a complex test for verification of possibilities of difference methods of solving such problems for the system (47). For example, stringent demands are made on the details of the grid in the interval $0 < \lambda < 0.2$.

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A QUASILINEAR HEAT EQUATION WITH A SOURCE: PEAKING, LOCALIZATION, SYMMETRY EXACT SOLUTIONS, ASYMPTOTICS, STRUCTURES

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UDC 517.956.45+517.958

A survey is given of results of investigating unbounded solutions (regimes with peaking) of quasilinear parabolic equations of nonlinear heat conduction with a source. Principal attention is devoted to the investigation of the property of localization of regimes with peaking. A group classification of nonlinear equations of this type is carried out, properties of a broad set of invariant (self-similar) solutions are investigated, and special methods of investigating the space–time structure of unbounded solutions are developed.

INTRODUCTION

Processes of spontaneous violation of a high degree of symmetry of a macroscopic state of a complex system are one of the surprising phenomena of the world surrounding us.

These processes lead to the appearance of so-called dissipative structures – ordered formations with characteristic space–time forms. For the occurrence of processes of spontaneous violation of symmetry with reduction of its degree the system must necessarily be open and the mathematical model of it must be nonlinear [29, 64, 80].

At the present time phenomena of structure formation are the focus of attention of investigators in various specialities. These phenomena are of interest to biologists in connection with the question of the origin of life, problems of prebiological evolution, and morphogenesis [49, 71, 75, 80, 81], to ecologists from the viewpoint of recognizing the laws of formation and stable functioning of biogenesis [75], and to physicists and chemists in connection with the possibility of creating new devices and installations which are new in principle. The interest of technicians is caused by the possibility of raising the productivity of old technologies and creating new intensive technologies [77]. These phenomena attract philosphers as examples of the nontrivial occurrence of a category of "part and whole" and the dialectics of self-movement [78].

In spite of the different nature of the systems, on passage from an unordered state to an ordered state they behave in a similar manner which bears witness to the existence of fundamental principles of their functioning. Representatives of various disciplines are occupied with the study of these principles within the framework of the synergetic approach [59, 64, 79].

It is altogether natural that one of the most powerful tools of modern science – mathematical modeling by means of a computational experiment [72] – is used in studying structures in nonlinear media. A combination of traditional methods of mathematical physics, modern numerical methods, and methods of processing information makes it possible to analyze the phenomenon considered from all sides, accumulate information regarding it, and create new concepts and methods adequate to the qualitative features of nonlinear phenomenona. The concept of symmetry–asymmetry is of considerable use in creating the basic concepts and constructing mathematical models.

Translated from Itogi Nauki i Tekhniki, Seriya Sovremennye Problemy Matematiki, Noveishie Dostizheniya, Vol. 28, pp. 95-206, 1986.

 0090-4104/88/4105-1222$12.50