PAMM · Proc. Appl. Math. Mech. 9, 317 – 318 (2009) / DOI 10.1002/pamm.200910132

Constitutive Modeling of Rubber Behavior with an Application to Dynamics of Rolling Tires

Anuwat Suwannachit[*1] and **Udo Nackenhorst**[1]

[1] Institut für Baumechanik und Numerische Mechanik, Leibniz Universität Hannover, Appelstr. 9A, 30167 Hannover, Germany

Mechanical response of technical rubbers is described by nonlinear elastic deformation, damage and hysteretic behavior under quasi-static cyclic loading conditions, while dynamic stiffening and viscous effects are predominant for high frequency analysis. For the computation of the vibration of rolling tires, a material description incorporating all of these effects is needed. A constitutive model for filled rubbers under low frequency excitation is presented. A good agreement between the computational results and experimental data is also presented here. Furthermore, an idea to extend the constitutive model into a broad frequency domain is suggested through the uncoupled kinematic response in linear and nonlinear part.

1 Introduction

Computational methods for the evaluation of the high frequency response of rolling tires excited from road surface roughness have been recently developed, cf. [1]. A multi-scale approach was introduced in [2] in order to compute the physically motivated excitation function from road surface texture by investigating the transient dynamic contact between tire tread and road surface at a sufficient small length-scale. Suitable constitutive models for tread rubber are needed for this application. Mechanical response of technical rubber was experimentally investigated in [3]. The nonlinear elastic and hysteretic behavior were clearly observed under quasi-static cyclic loading, as well as the damage (Mullins effect). However, high frequency response of rubber is usually characterized by frequency dependent stiffness and damping. Goal of this research is to develop a constitutive model which can include all mentioned effects and describe the behavior of technical rubber in a wide frequency range.

2 Constitutive modeling in low frequency domain

A finite strain viscoelastic model suggested in [4] is used to describe material responses of the tread rubber, such as dissipation and rate dependency. The incompressibility of rubber materials is maintained by the volumetric/deviatoric multiplicative split of the deformation gradient $\boldsymbol{F} = J^{\frac{1}{3}}\bar{\boldsymbol{F}}$, where $\bar{\boldsymbol{F}}$ is the volume-preserving contribution. The deviatoric part of the Right-Cauchy-Green and Lagrangian strain tensor are given by $\bar{\boldsymbol{C}} = \bar{\boldsymbol{F}}^T \cdot \bar{\boldsymbol{F}} = J^{-\frac{2}{3}}\boldsymbol{C}$ and $\bar{\boldsymbol{E}} = 1/2(\bar{\boldsymbol{C}} - \boldsymbol{I})$, respectively. In addition, the additive decomposition of elastic free energy function into volumetric and deviatoric part is assumed. The viscoelastic response is characterized by the generalized Maxwell model, where the nonequilibrium viscous stress in each Maxwell element is presented by the stress-like internal variable $\boldsymbol{Q}_i$. The total free energy function reads

$$\psi(\boldsymbol{C}, \boldsymbol{Q}_i) = U(J) + W(\bar{\boldsymbol{C}}) - \sum_{i=1}^{N} \frac{1}{2}\bar{\boldsymbol{C}} : \boldsymbol{Q}_i + \Xi\left(\sum_{i=1}^{N} \boldsymbol{Q}_i\right), \tag{1}$$

where Ξ is a certain function of the internal variables. The internal variables are obtained from a convolution integral of a linear evolution equation, for details see [5]. Damage mechanism of rubber under large deformation is presented by replacing the deviatoric part of elastic energy in (1) with a pseudo-elastic energy function,

$$W(\bar{\boldsymbol{C}}, \eta) = \eta s W^{\circ}(\bar{\boldsymbol{C}}) + \phi(\eta), \tag{2}$$

incorporating a scalar damage parameter η. Different from the original model in [6], a scalar parameter s is introduced and used as a reduction factor of deviatoric energy during damage process. On primary loading path where damage does not take place ($\eta = 1$, $\phi = 0$ and $s = 1$), the material response is controlled by a common stored-energy $W^{\circ}(\bar{\boldsymbol{C}})$. After that if the material is subjected to unloading, or again reloading under the previous maximum driven strain, the deviatoric response will be governed by the modified stored-energy function in (2) under the condition $0 < \eta < 1$ and $0 < s < 1$. Stress response and material tangent are obtained from the first and second derivative of the total free energy function.

A good agreement between simulation and experimental results of a uniaxial tension test is shown in Fig. 1. Here the tube model [7] is used for the elastic energy function $W^{\circ}(\bar{\boldsymbol{C}})$. The important properties of rubber at low frequencies, such as

* e-mail: suwannac@ibnm.uni-hannover.de, Phone: +49 511 762 9089, Fax: +49 511 762 19053

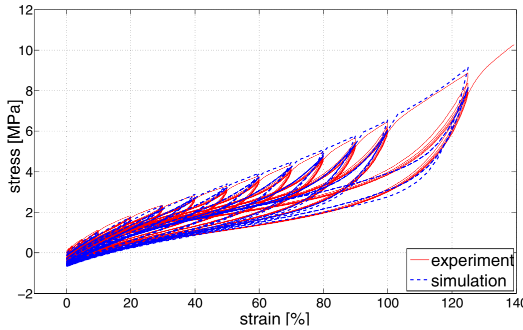

Fig. 1 uniaxial tension test (carbon black-filled rubber)

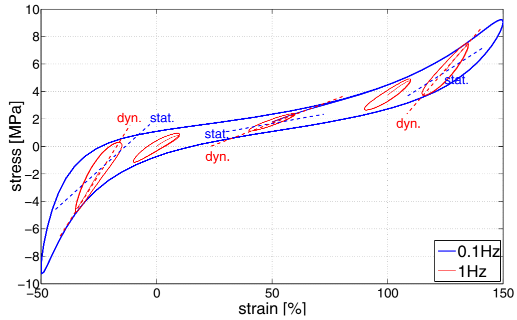

Fig. 2 simulated dynamic stiffening in uniaxial tension and compression test of a brick element

nonlinearity, hysteresis and Mullins effect, are properly simulated. Moreover, the dynamic stiffening at moderate frequencies can also be predicted, as seen from the comparison between the blue and red dot line (average stiffness) at each operational point in Fig. 2. The blue curve represents large loading cycle under very low excitation frequency and the red cycles correspond to the hysteresis cuves at different static working points under harmonic excitation.

3 Uncoupled linear and nonlinear kinematics

Under high frequency excitation, rubber shows the frequency dependent behavior characterized by complex modulus. However, model assumption for this representation are linear kinematics and harmonic excitation. The idea to extend the presented consitutive model to a broad frequency domain is suggested by decoupling the overall material response into linear and nonlinear part. This means the linear contribution will be responsible for high frequency response under assumption of linear viscoelasticity, while nonlinear part can represent the mechanical response at low frequencies, such as hysteresis and damage. Therefore, the multiplicative decomposition of the deformation gradient into linear and nonlinear part $\boldsymbol{F} = \boldsymbol{F}_{NL} \cdot \boldsymbol{F}_L$ is proposed with the concept of intermediate configuration. In addition, the volume is preserved by the volumetric/deviatoric multiplicative decomposition of the nonliear contribution $\boldsymbol{F}_{NL} = J^{\frac{1}{3}} \bar{\boldsymbol{F}}_{NL}$, where $\bar{\boldsymbol{F}}_{NL}$ is the deviatoric part. The linear strain is computed from $\boldsymbol{E}_L = \frac{1}{2}(\boldsymbol{C}_L - \boldsymbol{I}) \approx \frac{1}{2}(\boldsymbol{H} + \boldsymbol{H}^T)$, while the deviatoric nonlinear contribution can be given by $\bar{\boldsymbol{E}}_{NL} = \frac{1}{2}(\bar{\boldsymbol{C}}_{NL} - \boldsymbol{I})$. Now the total elastic stored-energy is additively decomposed into three parts

$$\psi(\boldsymbol{E}_L, \boldsymbol{E}_{NL}) = U(J) + W(\bar{\boldsymbol{E}}_{NL}) + W(\boldsymbol{E}_L). \tag{3}$$

The first part is subjected to volumetric response, while the second contribution plays an important role if rubber undergoes the large deformation. Damage mechanism can be taken into account by replacing this deviatoric part of nonlinear energy functional with the pseudo-elastic model. Finally, the vibration at high frequencies of rubber will be characterized by the response of linear kinematics resulting from the last energy contribution.

4 Conclusions

A constitutive description for the inelastic behavior of rubber under quasi-static loading and low frequency exitation is presented with an aim of the transient dynamic computation of car tires. The good performance of the model can be seen from the a good agreement between the computational and experimental results. An innovative idea to enhance the presented model to a wide frequency range is proposed by uncoupling the kinematics into linear and nonliner part. However, this extension is only retricted to elastic response. Further development for the specific properties, such as rate dependency and dissipation, is still needed in the ongoing research.

Acknowledgements The presented research has been supported by the German Ministry for Economics within the "Leiser Straßenverkehr 2"–program, the authors gratitude for this finance.

References

[1] M. Brinkmeier, and U. Nackenhorst, Comput. Mech., **41**, 503 - 515 (2008).
[2] M. Brinkmeier, U. Nackenhorst, and A. Suwannachit, Proc. Appl. Math. Mech. **8**, 10319-10320 (2008).
[3] M, Dämgen, Ph.D thesis, Leibniz Universität Hannover (2006).
[4] J.C. Simo, Comput. Meth. Appl. Mech. Eng., **60**, 153-173 (1987).
[5] J.C. Simo, and T.J.R. Hughes, Computational Inelasticity (Springer-Verlag, 1998), p. 336-373.
[6] R.W. Ogden, and D.G. Roxburgh, Proc. R. Soc. Lond., A **455**, 2861-2877 (1999).
[7] G. Heinrich, and M. Kaliske, Comp. Theor. Polymer Science, **7**, 227-241 (1998).

