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ORIGINAL RESEARCH PAPER

A walking pattern generation method of humanoid robot MAHRU-R

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Abstract This paper proposes an omni-directional walking pattern generation method for a humanoid robot MAHRU-R. To walk stably without falling down, a humanoid robot needs the walking pattern. Our previous walking pattern method generated the walking pattern with linear polynomials of the zero moment point (ZMP). It implemented the simple walking like forward/backward walking, side step walking and turning. However, this method was not sufficient to satisfy the various walking which is combined by forward/backward walking, side step walking and turning. We needed to upgrade the walking pattern generation method to implement an omni-directional walking. We use the linear inverted pendulum model consisted of ZMP and center of mass in order to simplify the computation of walking pattern. The proposed method assumes that the state of the following stride is same to the state of the current stride. Using this assumption of walking pattern, the proposed method generates the stable walking pattern for various walking. And the proposed scheme generates the ZMP trajectory with the quartic polynomials in order to reduce the fluctuation of ZMP trajectory by various walking. To implement the efficient walking pattern, this method proposes three walking modules: periodic step module, transient step module and steady step module. Each step module utilizes weighted least square method with future ZMP position information. The effectiveness of the proposed method is verified by simulations of various walking. And the proposed method is confirmed by the experiment of real humanoid robot MAHRU-R.

Keywords Humanoid robot · Walking pattern generation

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1 Introduction

Humanoid robots have been basically developed for the convenience of human beings. These robots will assist the elderly people in aging societies and help people in emergency like the scene of a fire and construction site instead of human beings in the near future. Until now, humanoid robots like ASIMO, WABIAN, HRP, and Johnnie were successively developed [2,7,8,10]. And these humanoid robots have done various performances.

Researches on humanoid robot have been done in the field of walking pattern generation, balancing, motion generation, whole body cooperation and so on. Particularly, the walking pattern generation methods have been studied to walk stably without tipping over on human environment. This paper deals with walking pattern generation method which is a essential part for the stable walking.

Many researches on walking pattern have been proposed in order to generate the stable walking pattern based on the zero moment point (ZMP) [1–4,6–11]. Yamaguchi et al. introduced a set of particles model to describe the humanoid robot and used Fourier series to generate the stable walking [10]. However, it needed iterative procedure for proper upper body motion, which compensated the two leg motion and kept the stability. So it is not easy to implement in real-time. Kajita et al. proposed a 3-D linear inverted pendulum model (LIPM) to generate the real-time trajectory of the ZMP and the center of mass (CoM) [6]. Using proper assumptions, it changed the robot dynamics to a simple second-order ordinary differential equation. It has been used in real-time control due to convenience of the proposed model. Harada et al. introduced an analytical method on real-time gait planning for a humanoid robot with LIPM [3]. It made use of cubic polynomials of the ZMP trajectory to satisfy initial and terminal condition. However, although it makes the real-time

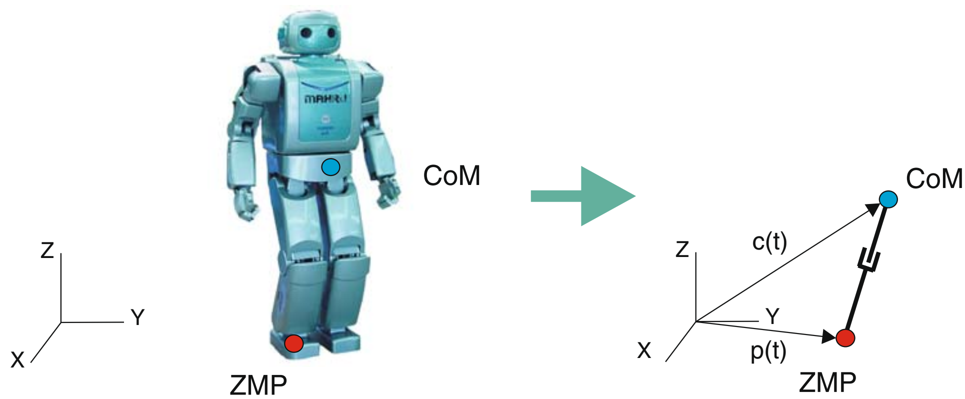

Fig. 1 Simplified humanoid robot model

walking pattern, it is difficult to generate various walking path due to cubic polynomials of ZMP. Oh et al. proposed an analytical method using linear polynomials. Its method utilized the non-minimum phase property of LIPM [9]. However, it is not sufficient to satisfy the desired ZMP which has been changed largely. Zhu et al. proposed a walking pattern generation method with fixed ZMP and variable ZMP in order to make the biped walking pattern more human-like and more agile [11]. However, it is also not sufficient to satisfy the various desired ZMP. Kajita et al. proposed a walking pattern generation method using preview control [7]. Its optimal method obtained a good result using future value of the desired ZMP. However, it demands large future value length to minimize the output error. And since it utilizes high gain of a preview control, it is very sensitive to the sudden change of the desired ZMP trajectory.

This paper describes the omni-directional and real-time walking pattern generation method by analytical form. LIPM is used to describe the humanoid robot system. This paper utilizes the weighted least square method with the quartic polynomials of the ZMP so as to reduce the fluctuation of the ZMP trajectory according to various footprints. During each phase, the obtained CoM trajectory by the proposed method is embedded to the whole-body closed-loop inverse kinematics (CLIK) of the center of mass Jacobian method.

This paper is organized as follows: Sect. 2 describes the LIPM as the simplified model for humanoid robots and arranges the relationship between the ZMP and the CoM. And in Sect. 3, we introduce three step modules for generating more effective walking pattern of the ZMP and the CoM. Section 4 shows the results of the simulations to verify the efficiency of the proposed method. Experimental results are presented for various walking in Sect. 5. Finally, we conclude this paper in Sect. 6.

2 Linear inverted pendulum model

Humanoid robots have been designed similar to human for adapting to human's environment and showing diverse behaviors. For these purposes, a humanoid robot consists of multi-link structure. Considering full dynamic properties of a number of links and joints, the walking pattern could result in a good performance. However, since it is complicated and tedious to calculate the walking pattern taking full dynamic properties into consideration, a simplified model is required to control the humanoid robot.

Many researchers have utilized LIPM as a simplified humanoid robot model. LIPM consists of the CoM and the ZMP as shown in Fig. 1. In Fig. 1, $p(t)$ and $c(t)$ denote the ZMP trajectory and the CoM trajectory, respectively.

We utilize three assumptions for simplicity as follows:

1. The time derivative of the angular momentum about the CoM is zero.
2. The difference of the CoM and the ZMP at Z-axis is constant.
3. The acceleration of the ZMP at Z-axis is zero.
   Under these assumptions, the relationship between the ZMP and the CoM can be represented as follows:

$$p(t) = c(t) - \frac{1}{\omega^2}\ddot{c}(t) \tag{1}$$

where ω is $\sqrt{g/(Z_{\text{CoM}} - Z_{\text{ZMP}})}$. Z_{CoM}, Z_{ZMP} and g are the values of the CoM and the ZMP at the Z-axis and the gravity acceleration separately.

Equation (1) is the basic equation of the proposed walking pattern generation method. In Eq. (1), the CoM is represented by the robot motion. We design the CoM trajectory which has the continuity of jerk in order to generate smoother trajectory and avoid imposing a heavy burden to the robot when each robot motion is generated by the CoM trajectory. For the continuous jerk of CoM trajectory, more than cubic polynomials of the ZMP are sufficient to satisfy the requirement.

In case of the periodic walking pattern, it is sufficient to utilize cubic polynomials of the ZMP. But for walking with various step lengths, cubic polynomials of the ZMP might take place large fluctuations of the ZMP trajectory. Since cubic polynomials constrain the shape of the ZMP, the ZMP trajectory makes the large fluctuation occur in order to satisfy Eq. (1) according to various steps. Therefore, we choose quartic polynomials of the ZMP for various step. When the ZMP trajectory is quartic polynomials, it needs an optimal method to obtain the solution owing to more numbers of unknowns than equations. In next section, we will explain an optimal method in detail.

Let us reconsider that the initial conditions, $c(0),\ \dot{c}(0)$, are known and the ZMP trajectory is given by quartic polynomials. The general form of the ZMP and the CoM to satisfy Eq. (1) could be found as follows:

$$p(t) = b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0 \tag{2}$$

$$\begin{aligned} c(t) = &\left(c(0) - b_0 - \frac{2}{\omega^2}b_2 - \frac{24}{\omega^4}b_4\right)\cosh(\omega t) \\ &+\frac{1}{\omega}\left(\dot{c}(0) - b_1 - \frac{6}{\omega^2}b_3\right)\sinh(\omega t) \\ &+b_4t^4 + b_3t^3 + \left(b_2 + \frac{12}{\omega^2}b_4\right)t^2 \\ &+\left(b_1 + \frac{6}{\omega^2}b_3\right)t + \left(b_0 + \frac{2}{\omega^2}b_2 + \frac{24}{\omega^4}b_4\right) \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{c}(t) = \omega &\left(c(0) - b_0 - \frac{2}{\omega^2}b_2 - \frac{24}{\omega^4}b_4\right)\sinh(\omega t) \\ &+\left(\dot{c}(0) - b_1 - \frac{6}{\omega^2}b_3\right)\cosh(\omega t) + 4b_4t^3 + 3b_3t^2 \\ &+2\left(b_2 + \frac{12}{\omega^2}b_4\right)t + \left(b_1 + \frac{6}{\omega^2}b_3\right) \end{aligned} \tag{4}$$

Eqs. (3) and (4) tell us the fact that the coefficients of the CoM are represented by $c(0), \dot{c}(0)$ and those of the ZMP. Since $c(0)$, $\dot{c}(0)$ are the provided values as the initial conditions, we focus on obtaining the proper coefficients of the ZMP.

At the time interval of ith support phase, Eqs. (2), (3) and (4) are arranged by a following matrix form.

$$\mathbf{X}_i = \mathbf{A}_i\mathbf{X}_{i-1} + \mathbf{B}_i\boldsymbol{\beta}_i \tag{5}$$

where

$$\mathbf{X}_i = \begin{Bmatrix} \omega(c(T_i) - p(T_i)) \\ \dot{c}(T_i) \end{Bmatrix}$$

$$\mathbf{A}_i = \begin{bmatrix} \mathrm{CT}_i & \mathrm{ST}_i \\ \mathrm{ST}_i & \mathrm{CT}_i \end{bmatrix}$$

$$\mathbf{B}_i = \begin{bmatrix} -\frac{24}{\omega^3}\mathrm{CT}_i + \frac{12}{\omega}T_i^2 + \frac{24}{\omega^3} & -\frac{6}{\omega^2}\mathrm{ST}_i + \frac{6}{\omega}T_i & -\frac{2}{\omega}\mathrm{CT}_i + \frac{2}{\omega} & -\mathrm{ST}_i \\ -\frac{24}{\omega^3}\mathrm{ST}_i + 4T_i^3 + \frac{24}{\omega^2}T_i & -\frac{6}{\omega^2}\mathrm{CT}_i + 3T_i^2 + \frac{6}{\omega^2} & -\frac{2}{\omega}\mathrm{ST}_i + 2T_i & -\mathrm{CT}_i + 1 \end{bmatrix}$$

$$\boldsymbol{\beta}_i = \begin{Bmatrix} b_{i4} \\ b_{i3} \\ b_{i2} \\ b_{i1} \end{Bmatrix}$$

CT_i and ST_i denote $\cosh(\omega t)$ and $\sinh(\omega t)$, respectively. b_{in} $(n = 1, \ldots, 4)$ denotes the nth order coefficient of the ZMP during the ith support phase. Equation (5) will be used as a basic equation which the ZMP and CoM of each support phase are represented by.

3 Walking generation method

A humanoid robot can walk without tipping over when the ZMP is within the supporting zone which is convex hull of all contact points between a supporting foot and ground. For the stable ZMP trajectory, this paper proposes three walking step modules in order to walk arbitrarily.

Each step module utilizes the following ZMP positions in advance. When a man walks on street, the man generally anticipates the walking path through the eyes in advance. Since the body of human modifies proper behaviors according to the future walking path obtained by eyes, human can walk stably and swiftly without falling down. The proposed method is the same as this. When the walking pattern is generated, the proposed step modules determine some following ZMP positions and then generate the walking patterns with weighted least square method.

The proposed method can implement the walking pattern according to the ZMP with the slope in single support phase. Human moves the ZMP from heel to toe in single support phase. So this movement of the ZMP enhances the efficiency of walking. The proposed method can also makes the ZMP move in single support phase. The ZMP movement brings about the improved CoM movement. Section 4 will show the efficiency of the ZMP with the slope in single support phase through the simulation.

The three step modules are periodic step module, transient step module and steady step module. The roles of three step modules will be explained in following sections in detail.

3.1 Periodic step module

If the whole ZMP information is given in advance and the walking pattern is implemented off-line, the walking pattern can be generated by the sequential process. However, the real-time walking pattern generation is different from the sequential process because the limited ZMP information is provided and the walking pattern has to be implemented on-line. The proposed step modules in this paper achieve the stable walking pattern with the limited future ZMP positions, initial and final value of CoM at some range of future footprints on-line.

First of all, this section explains the periodic step module as the starting point. When we generate the walking pattern of the first step in order to start to walk, the initial values of the CoM are obtained from the current state of robot and the ZMP positions are determined by walking path. However, the final values of the CoM of the first step are not provided. In this paper, the periodic step module provides the final values of the first step as the seed of the walking pattern generation. Since the periodic step module can make the walking pattern by using only the ZMP positions, the final value of the CoM of the first step can be provided from the walking pattern obtained by periodic step module. The periodic step module uses the cubic polynomials of the ZMP in double support phase and the linear polynomials of the ZMP in single support phase. This step module consists of four support phases as shown in Fig. 2: double support phase, single support phase, double support phase and single support phase. ΔT_i means the time interval at the ith support phase. Taking the first step into account, the numbering is written down in Fig. 2.

The velocity of the ZMP can be known by the ZMP position as follows:

$$b_{i1} = \begin{cases} \frac{p(T_6)-p(T_5)}{\Delta T_6} & \text{for } i = 3, 6 \\ \frac{p(T_4)-p(T_3)}{\Delta T_4} & \text{for } i = 4, 5 \end{cases}$$

The values of $p(T_4) - p(T_3)$ and $p(T_6) - p(T_5)$ are the same value as the ZMP margin in single support phase. The value is selected less than a half of foot size for maintaining the stability margin.

This step module has the periodic property. The periodic property is represented as following equations.

$$\omega(c(T_6) - p(T_6)) = \omega(c(T_2) - p(T_2)) \quad (6)$$

$$\dot{c}(T_6) = \dot{c}(T_2) \quad (7)$$

Here this study defines a stride period as time consisted of two single support phases and two double support phases in Fig. 2. This periodic property of Eqs. (6) and (7) means that the condition of a stride period is the same to the condition of following stride period. These relations also apply to the steady step module as a constraint equation.

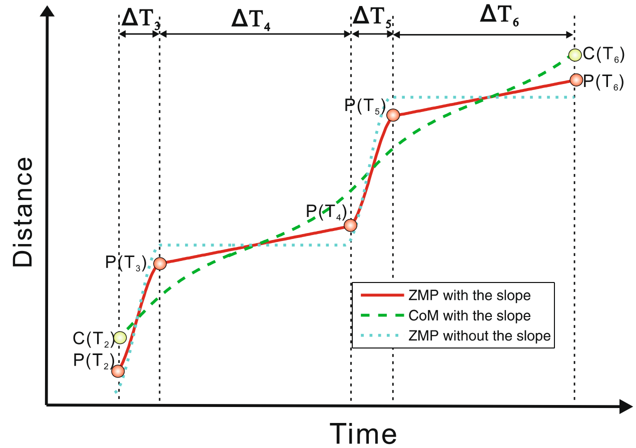

Fig. 2 Periodic step module

The solutions of the periodic step module are achieved with constraint conditions like continuity of CoM and ZMP at each support phase, ZMP positions, initial and final values of CoM. The solutions are as follows:

$$\begin{Bmatrix} \mathbf{X}_2 \\ \boldsymbol{\beta}_3 \\ \boldsymbol{\beta}_5 \end{Bmatrix} = \mathbf{D}^{-1}\mathbf{E} \quad (8)$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{A}_6\mathbf{A}_5\mathbf{A}_4\mathbf{A}_3 - \mathbf{I} & \mathbf{A}_6\mathbf{A}_5\mathbf{A}_4\mathbf{B}_3^u & \mathbf{A}_6\mathbf{B}_5^u \\ \mathbf{0} & \mathbf{F}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_5 \\ \mathbf{0} & \mathbf{G}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_5 \end{bmatrix}$$

$$\mathbf{E} = \begin{Bmatrix} -\boldsymbol{\Gamma} \\ \alpha_3 T_3 - \alpha_6 T_3 \\ \alpha_5 T_5 - \alpha_4 T_5 \\ \alpha_4 - \alpha_6 \\ \alpha_6 - \alpha_4 \end{Bmatrix}$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} \mathbf{A}_6\mathbf{A}_5\mathbf{A}_4\mathbf{B}_3^k & \mathbf{A}_6\mathbf{A}_5\mathbf{B}_4^k & \mathbf{A}_6\mathbf{B}_5^k & \mathbf{B}_6^k \end{bmatrix} \begin{Bmatrix} b_{31} \\ b_{41} \\ b_{51} \\ b_{61} \end{Bmatrix}$$

$$\mathbf{F}_i = \begin{pmatrix} T_i^3 & T_i^2 \end{pmatrix}$$

$$\mathbf{G}_i = \begin{pmatrix} 3T_i^2 & 2T_i \end{pmatrix}$$

$$\boldsymbol{\beta}_i = \begin{Bmatrix} b_{i3} \\ b_{i2} \end{Bmatrix}$$

$\mathbf{B}_i$ in Eq. (5) can be divided into $\mathbf{B}_i^u$ and $\mathbf{B}_i^k$ because the coefficients of the periodic step module in single support phase are known.

$$\mathbf{B}_i^u = \begin{bmatrix} -\frac{6}{\omega^2}\mathrm{ST}_i + \frac{6}{\omega}T_i & -\frac{2}{\omega}\mathrm{CT}_i + \frac{2}{\omega} \\ -\frac{6}{\omega^2}\mathrm{CT}_i + 3T_i^2 + \frac{6}{\omega^2} & -\frac{2}{\omega}\mathrm{ST}_i + 2T_i \end{bmatrix}$$

$$\mathbf{B}_i^k = \begin{Bmatrix} -\mathrm{ST}_i \\ -\mathrm{CT}_i + 1 \end{Bmatrix} \tag{9}$$

and $\alpha_i = \{p(T_i) - p(T_{i-1})\}/T_i$ is the slope which is connected by two points $p(T_{i-1})$ and $p(T_i)$ in the ith phase.

This step module provides the ZMP coefficients of each support phase and the values of CoM of a stride. However, since these ZMP coefficients are for the periodic step, the coefficients are not proper for the various steps. We only use information of a particular position and velocity of the CoM obtained from this step module. The values are $c(T_3)$, $\dot{c}(T_3)$ obtained by $\mathbf{X}_3$ in order to increase the relation with first step. This step module provides final values of the CoM to transient module for first step.

3.2 Steady step module

The steady step module makes every walking pattern except the first step and the final step. This step module generates the walking pattern with positions of the ZMP from footprints and initial position and velocity of the ZMP and the CoM obtained by the previous walking pattern. This step module generates the walking pattern repetitively with the ZMP positions of the next stride period in every phase like Fig. 3. The repetitive pattern generation is for minimizing the difference between the desired ZMP and the generated ZMP because footprints of humanoid robot are various and the walking pattern generation is implemented in real-time. The ΔT_{n+1} part of the generated ZMP in Fig. 4 is only utilized at the each repetitive pattern generation. This step module consists of four support phases: two single support phases and two double support phases. This module uses the quartic polynomials of the ZMP trajectory. The unknown factors are more than constraint equations in this case. The redundant unknown factor plays an important role to reduce the fluctuation of the ZMP trajectory according to various footprints. Using Eq. (5), a matrix can be arranged from T_n to T_{n+4},

$$\boldsymbol{\kappa} = \boldsymbol{\gamma}\boldsymbol{\tau} \tag{10}$$

where

$$\boldsymbol{\kappa} = \mathbf{X}_{n+4} - \mathbf{A}_{n+4}\mathbf{A}_{n+3}\mathbf{A}_{n+2}\mathbf{A}_{n+1}\mathbf{X}_n$$

$$\boldsymbol{\gamma} = \big[\mathbf{A}_{n+4}\mathbf{A}_{n+3}\mathbf{A}_{n+2}\mathbf{B}_{n+1} \quad \mathbf{A}_{n+4}\mathbf{A}_{n+3}\mathbf{B}_{n+2} \quad \mathbf{A}_{n+4}\mathbf{B}_{n+3} \quad \mathbf{B}_{n+4}\big]$$

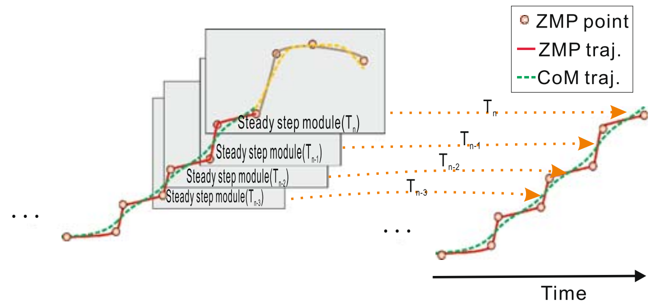

Fig. 3 Generation of the ZMP and the CoM trajectory using steady step module

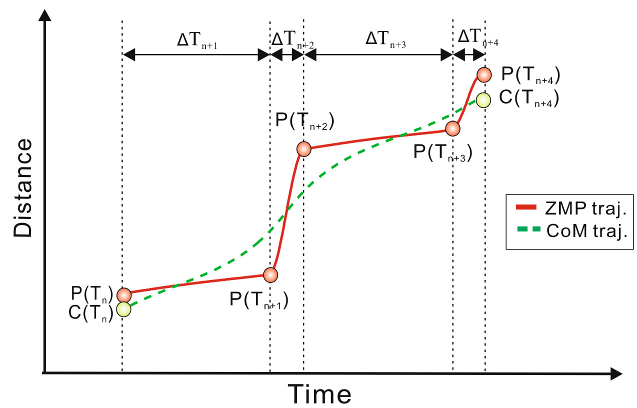

Fig. 4 Steady step module

$$\boldsymbol{\tau} = \begin{Bmatrix} \boldsymbol{\beta}_{n+1} \\ \boldsymbol{\beta}_{n+2} \\ \boldsymbol{\beta}_{n+3} \\ \boldsymbol{\beta}_{n+4} \end{Bmatrix}$$

$$\boldsymbol{\beta}_i = \begin{Bmatrix} b_{i4} \\ b_{i3} \\ b_{i2} \\ b_{i1} \end{Bmatrix}$$

$\boldsymbol{\beta}_i$ means the coefficient vector of the ZMP at the ith support phase. And there are nine constraint equations for keeping the continuity of position and velocity of the ZMP trajectory at each support phase. The equations are presented by a matrix as follows:

$$\boldsymbol{\varphi} = \boldsymbol{\Psi}\boldsymbol{\tau} \tag{11}$$

where

$$\boldsymbol{\varphi}=\begin{Bmatrix} p(T_{n+1})-p(T_n)\\ 0\\ dp(T_n)\\ p(T_{n+2})-p(T_{n+1})\\ 0\\ p(T_{n+3})-p(T_{n+2})\\ 0\\ p(T_{n+4})-p(T_{n+3})\\ dp(T_{n+4}) \end{Bmatrix}$$

$$\boldsymbol{\Psi}=\begin{bmatrix} \mathbf{F}_{n+1} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{G}_{n+1} & -\mathbf{H} & \mathbf{0} & \mathbf{0}\\ \mathbf{H} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}_{n+2} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{G}_{n+2} & -\mathbf{H} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{F}_{n+3} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{n+3} & -\mathbf{H}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{n+4}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{n+4} \end{bmatrix}$$

$$\mathbf{F}_i=\begin{pmatrix} T_i^4 & T_i^3 & T_i^2 & T_i \end{pmatrix}$$

$$\mathbf{G}_i=\begin{pmatrix} 4T_i^3 & 3T_i^2 & 2T_i & 1 \end{pmatrix}$$

$$\mathbf{H}=\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$$

Eqs. (10) and (11) are changed into a matrix.

$$\boldsymbol{\xi}=\boldsymbol{\Phi}\boldsymbol{\tau} \tag{12}$$

where

$$\boldsymbol{\xi}=\begin{Bmatrix}\boldsymbol{\kappa}\\ \boldsymbol{\varphi}\end{Bmatrix},\quad \boldsymbol{\Phi}=\begin{bmatrix}\boldsymbol{\gamma}\\ \boldsymbol{\Psi}\end{bmatrix}$$

The coefficients of the ZMP at each support phase were obtained with Eq. (12) and weighted least square method. The objective of the weighted least square method is that variation of the ZMP velocity is minimized. Minimizing the variation of the ZMP velocity prevents the ZMP trajectory from changing abruptly in order to satisfy Eq. (1). In order to achieve this purpose, the cost function is defined as follows:

$$\begin{aligned}\mathcal{J}&=\frac{1}{2}\sum_{i=1}^{4}\int_{0}^{T_{n+i}} w_{n+i}[\dot{p}(t)-\alpha_{n+i}]^2\mathrm{d}t\\ &=\frac{1}{2}\boldsymbol{\tau}^T\mathbf{W}\boldsymbol{\tau}-\mathbf{U}\boldsymbol{\tau}+c\end{aligned} \tag{13}$$

where

$$\mathbf{W}=\begin{bmatrix} \mathbf{W}_{n+1} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{W}_{n+2} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{W}_{n+3} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{W}_{n+4} \end{bmatrix}$$

$$\mathbf{W}_i=w_i\begin{bmatrix} \frac{16}{7}T_i^7 & 2T_i^6 & \frac{8}{5}T_i^5 & T_i^4\\ 2T_i^6 & \frac{9}{5}T_i^5 & \frac{3}{2}T_i^4 & T_i^3\\ \frac{8}{5}T_i^5 & \frac{3}{2}T_i^4 & \frac{4}{3}T_i^3 & T_i^2\\ T_i^4 & T_i^3 & T_i^2 & T_i \end{bmatrix}$$

$$\mathbf{U}=\begin{Bmatrix} \alpha_{n+1}w_{n+1}\mathbf{F}_{n+1}^T\\ \alpha_{n+2}w_{n+2}\mathbf{F}_{n+2}^T\\ \alpha_{n+3}w_{n+3}\mathbf{F}_{n+3}^T\\ \alpha_{n+4}w_{n+4}\mathbf{F}_{n+4}^T \end{Bmatrix}^T$$

Using Lagrange multiplier,

$$\mathcal{L}(\boldsymbol{\tau},\boldsymbol{\lambda})=\frac{1}{2}\boldsymbol{\tau}^T\mathbf{W}\boldsymbol{\tau}-\mathbf{U}\boldsymbol{\tau}+c+\boldsymbol{\lambda}^T(\boldsymbol{\xi}-\boldsymbol{\Phi}\boldsymbol{\tau}) \tag{14}$$

The coefficients of the ZMP at each support phase are acquired.

$$\boldsymbol{\tau}=\boldsymbol{\Phi}_W^+\boldsymbol{\xi}-\boldsymbol{\Phi}_W^+(\boldsymbol{\Phi}\mathbf{W}^{-1}\mathbf{U}^T)+\mathbf{W}^{-1}\mathbf{U}^T \tag{15}$$

where

$$\boldsymbol{\Phi}_W^+=\mathbf{W}^{-1}\boldsymbol{\Phi}^T(\boldsymbol{\Phi}\mathbf{W}^{-1}\boldsymbol{\Phi}^{\mathbf{T}})^{-1} \tag{16}$$

And w_i is the weighting factor.

Generally, the stability of humanoid robot is whether the ZMP is located on the supporting zone which is shaped by supporting foot or not. The weighting factor suppresses the fluctuation of the ZMP trajectory for guaranteeing the stability of biped robot even though ZMP positions consist of various points. The ZMP trajectory in single support phase is easy to diverge owing to small supporting zone. Therefore, the weighting factor of single support phase is larger

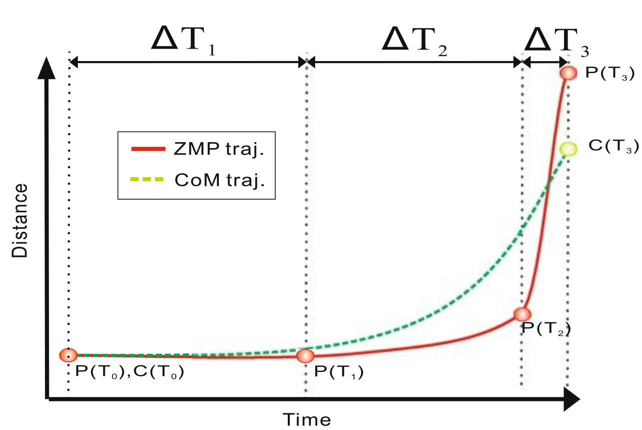

Fig. 5 Transient step module

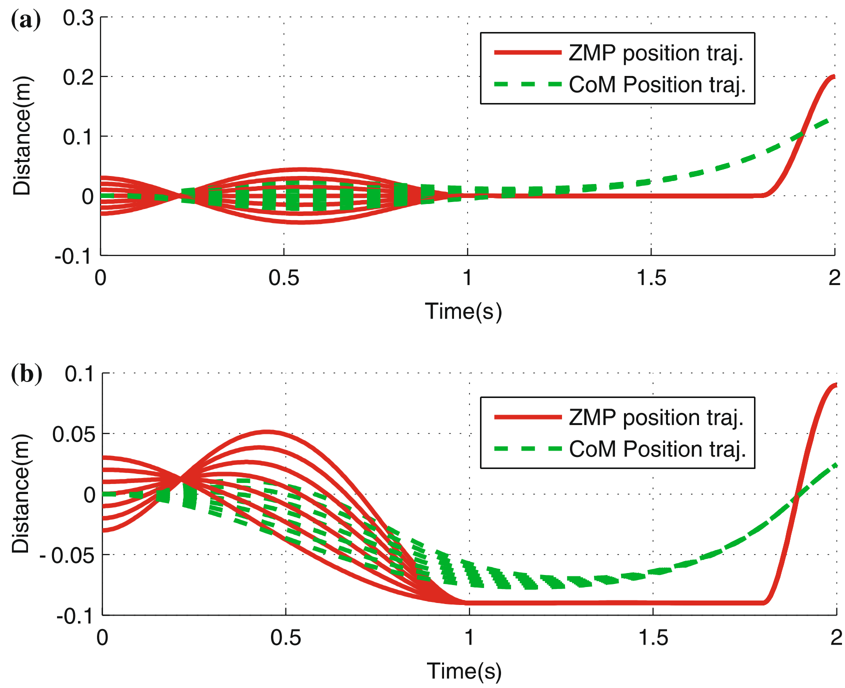

Fig. 6 ZMP/CoM trajectory with initial ZMP position (from -0.04 to 0.04 m), $T_1 = 1.0$ s, $T_2 = 0.8$ s, $T_3 = 0.2$ s, step length $= 0.2$ m, stride width $= 0.18$ m

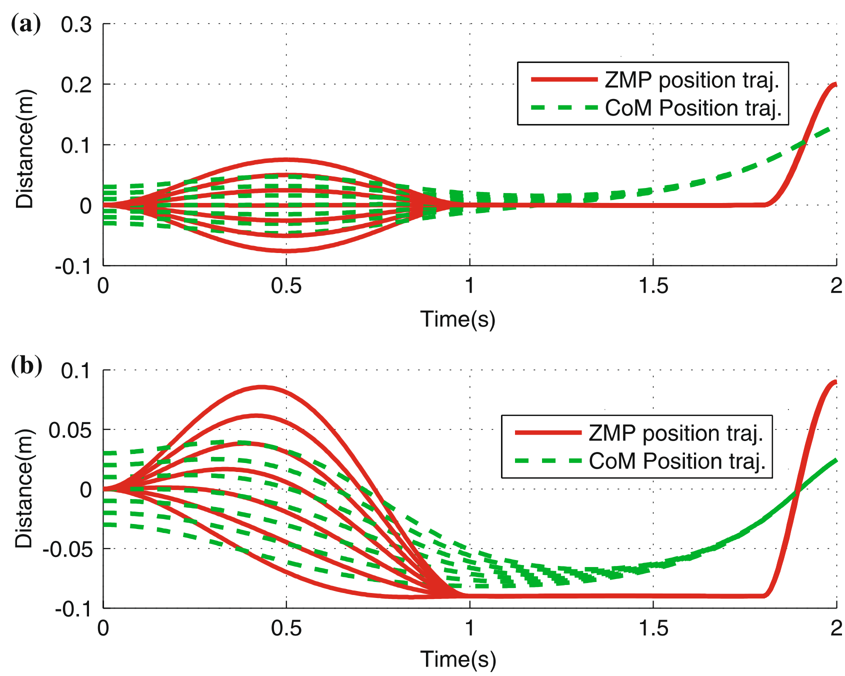

Fig. 7 ZMP/CoM trajectory with initial CoM position (from -0.04 to 0.04 m), $T_1 = 1.0$ s, $T_2 = 0.8$ s, $T_3 = 0.2$ s, step length $= 0.2$ m, stride width $= 0.18$ m

than that of double support phase. We can obtain the good result by simulation when the value of weighting factor w_n in single support phase is 100 times of the value in double support phase.

The large fluctuation of the ZMP trajectory in the proposed method can occurs when more than the values of three ZMP positions are successively same at a stride period. This phenomenon takes place due to Eqs. (6) and (7). The ZMP trajectory moves abruptly to satisfy initial and final condition of Eqs. (6) and (7). There are three methods of minimizing the fluctuation of the ZMP trajectory. One is resetting the ZMP positions. However, it is needed to set new extra ZMP positions to reduce the fluctuation irrespective of the footprints. Others are readjusting the weighting factor and support phase time. In this paper, we choose the second method in order not to have influence on the ZMP positions. For it, we modify

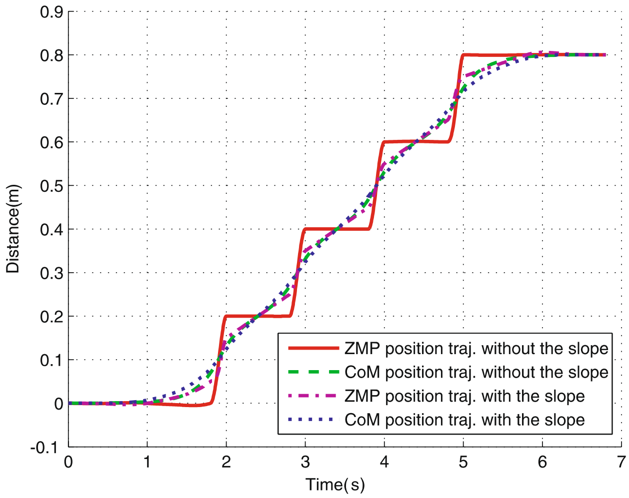

Fig. 8 ZMP/CoM position trajectory with $T_s = 0.8\,\mathrm{s}$, $T_d = 0.2\,\mathrm{s}$, step length $= 0.2\,\mathrm{m}$ and slope $= 0.25\,\mathrm{m/s}$

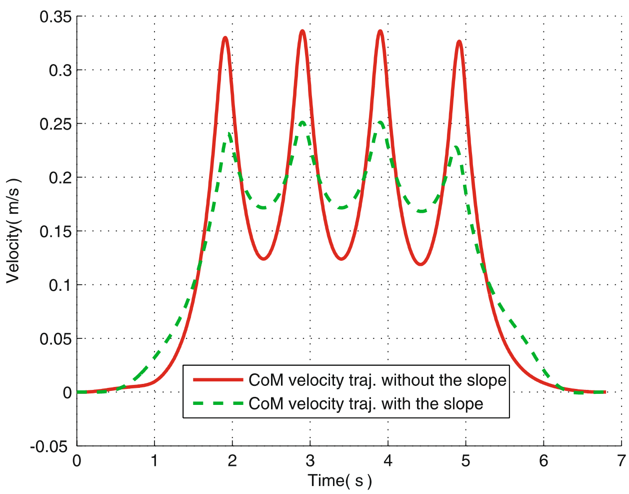

Fig. 9 ZMP/CoM velocity trajectory with $T_s = 0.8\,\mathrm{s}$, $T_d = 0.2\,\mathrm{s}$, step length $= 0.2\,\mathrm{m}$ and slope $= 0.25\,\mathrm{m/s}$

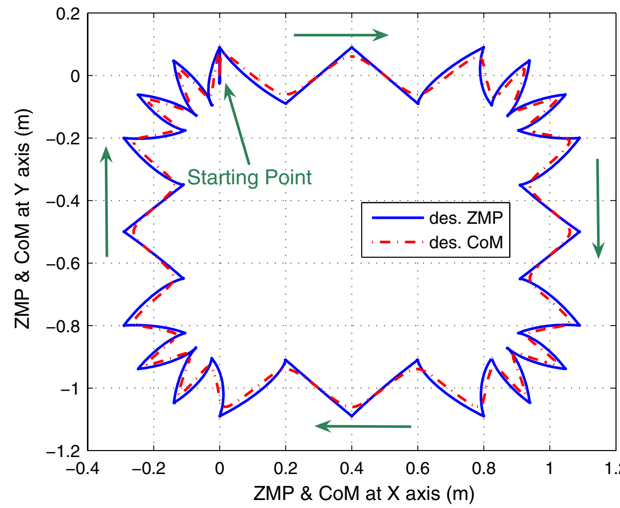

Fig. 10 The walking pattern of square shape with $T_s = 0.8\,\mathrm{s}$, $T_d = 0.2\,\mathrm{s}$, step length $= 0.2\,\mathrm{m}$, turn rate $= 15$ deg./stride and slope $= 0$

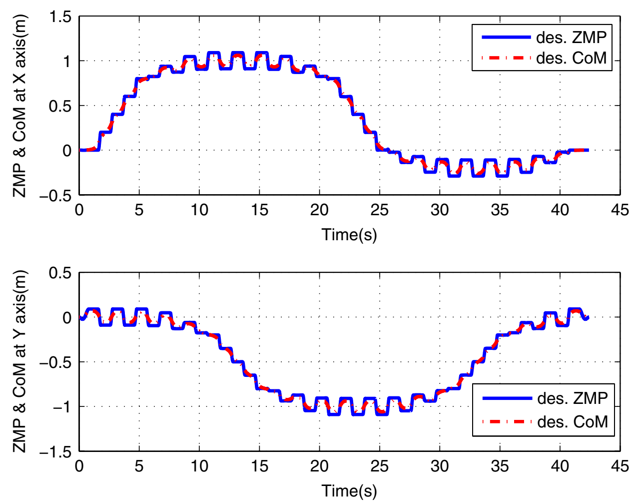

Fig. 11 The walking pattern of square shape of ZMP and CoM at X and Y axis

that the double support time is increased and weighting factor is readjusted in a stride period in case of more than three same ZMP positions. The single support time don't need to change the value of phase time and weighting factor because the single support phase is already enough to be suppressed by weighting factor. The double support time from ΔT_{n+2} to ΔT_{n+4} is just changed when the ZMP positions are more than three same values of the ZMP position in a stride period so that the changed part does not have influence on total walking pattern time. The value of weighting factor in this double support phase is changed to the value of single support phase.

3.3 Transient step module

This step module generates the walking pattern of the first step and the final step. This module consists of three phases: double support phase, single support phase and double support phase. This module also uses the weighted least square method. However, this module does not utilize the Eqs. (6) and (7), but makes use of initial and final values of CoM directly. For example, the walking pattern of the first step can be generated with the initial values of the ZMP and the CoM which are set by the robot state itself and the final values provided by the periodic step module. And the walking pattern of the final step can be achieved using the initial values of the ZMP and the CoM from the steady step module and the final values from the stop state of the humanoid robot. The difference of generating the first step and the final step is the order of procedure. The final step can be obtained just in reverse order of the first step. Therefore, we deal with the first step procedure in this paper (Fig. 5).

Fig. 12 Humanoid robot Mahru-R

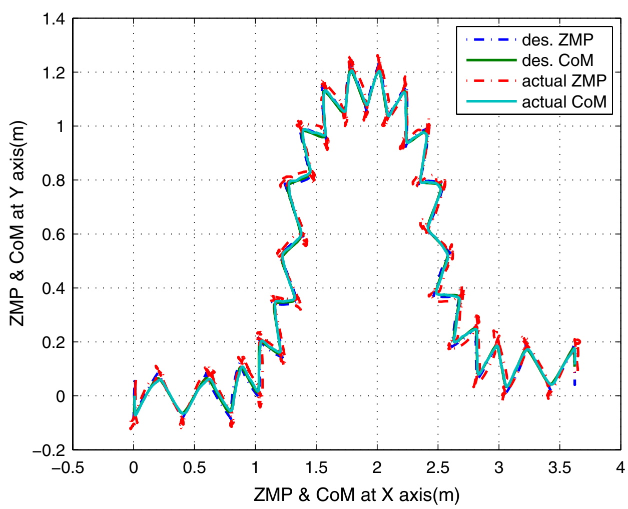

Fig. 13 Zigzag walking pattern with $T_s = 0.95$ s, $T_d = 0.15$ s, step length = 0.2 m, turn rate = 10 deg./stride and slope = 0

When the walking pattern of the first step is generated, a large fluctuation of the ZMP trajectory may arise at the first phase time of the first step and the third phase time of the final step. That is why Eq. (1) has the non-minimum phase property. When the ZMP moves forward at the starting point of walking, the CoM moves initially to backward direction due to the non-minimum phase property and then moves forward as time passes [5]. So we increase the phase time to suppress the non-minimum phase property at the first phase time of the first step and the third phase time of the final step. The changed phase time is selected from 0.6 to 2.0 s.

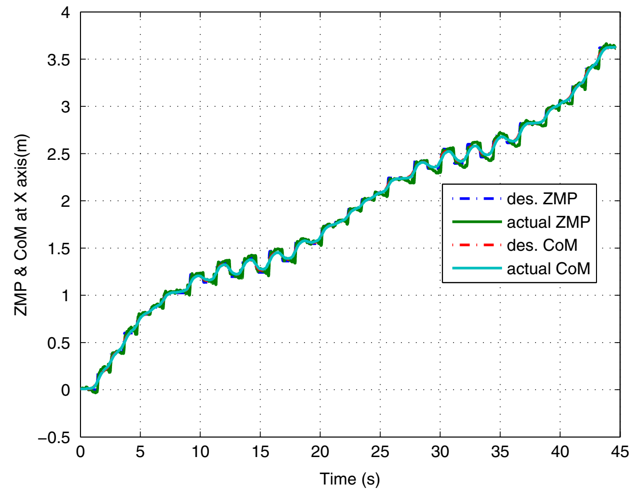

Fig. 14 ZMP and CoM of zig zag walking at X axis

The transient step module is offered the initial values of the ZMP and CoM for the first step as mentioned before. Ideally the initial values of the ZMP and CoM are zero, but these values may not be zero because of the irregular ground condition and robots kinematic error. Although this initial problem may be not critical effect on the robot, it could make the initial motion of the robot unnatural. However, since we utilize weighted least square method and the readjusted phase time, the proposed method is insensitive to the initial problem.

The coefficients of each phase are decided with similar procedure, which is introduced in the previous steady step module section.

Table 1 Specification of MAHRU-R

Height	1,350 mm
Weight	50 kg including batteries
Head	2 DOF(Pitch, Yaw)
Arm	6 DOF(Shoulder 3, Elbow 1, Wrist 2)
Hand	4 DOF
Leg	6 DOF(Hip 3, Knee 1, Ankle 2)
Sensor	IMU sensor in pelvis
	6-axis F/T sensor in wrist and ankle
Acturator	DCDC servo motor
	Belt-pulley
	Harmonic drive
I/O board communication	IEEE 1,394 firewire
Operating system	Fedora Core 5 and RTAI/Xenomai

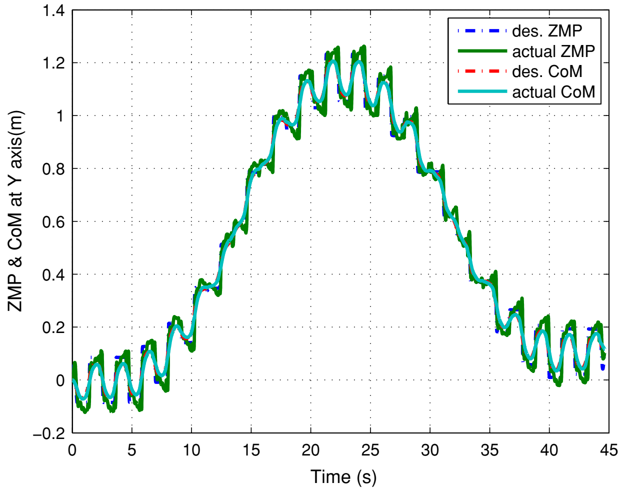

Fig. 15 ZMP and CoM of zig zag walking at Y axis

4 Simulation

We show the advantages of the proposed method in this section. The proposed method has the characteristic of being insensitive to initial values of the ZMP and CoM. As shown in Figs. 6 and 7, the proposed method reduces the effect of initial values, although the robot has the initial values which are not zero. The ZMP trajectory is changed automatically to reduce the effect of initial values of the ZMP and CoM at the first phase of first step. Although the variations of Figs. 6 and 7 look large, the humanoid robot can guarantee the stability because both feet of the robot support on the ground.

Another advantage is that the proposed method can generate the ZMP trajectory with the slope in single support phase as shown in Fig. 8. The ZMP trajectory with the slope in single support phase has an advantage of reducing the CoM velocity as shown in Fig. 9 because of the ZMP movement in the single support phase. This advantage means that the

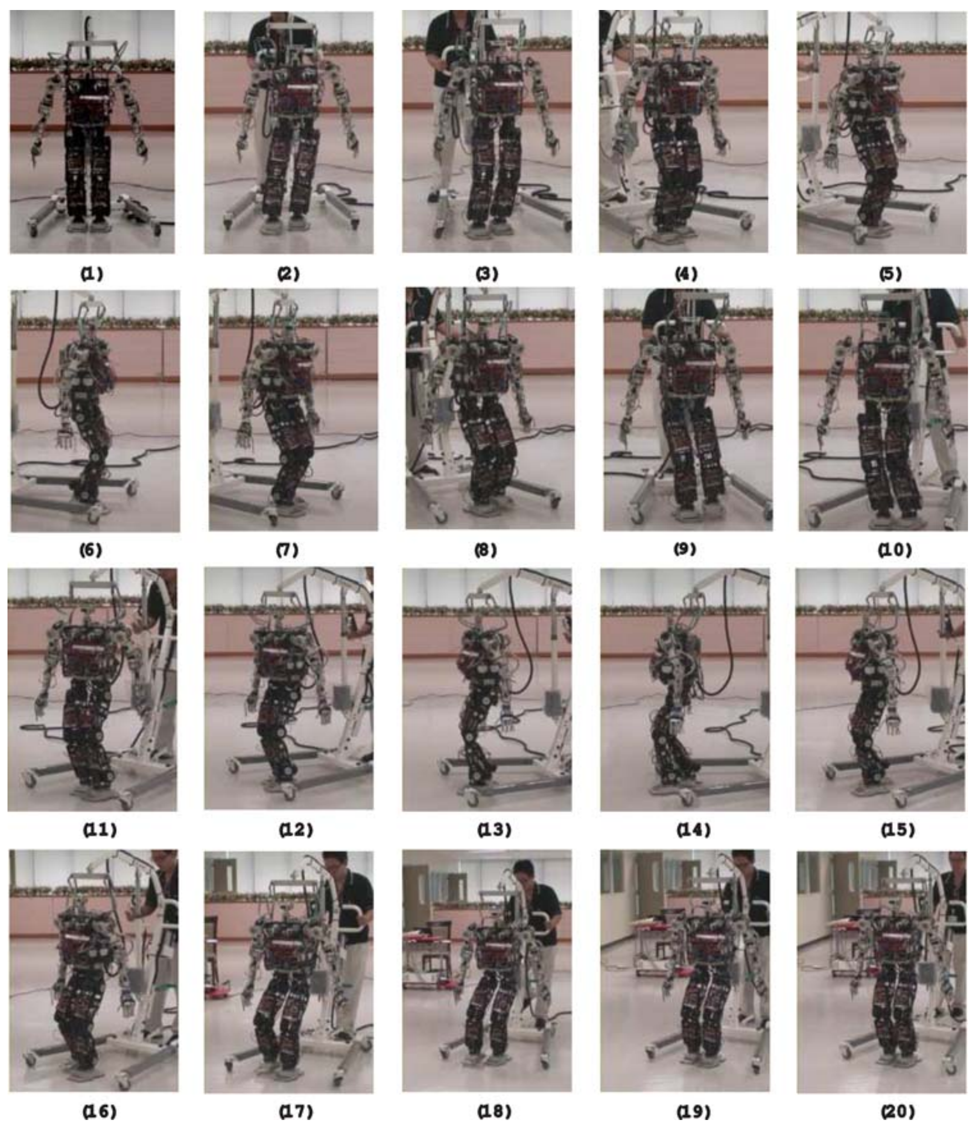

Fig. 16 Snapshots of zig zag walking

robot does the same motion with less CoM velocity. And this fact indicates that the joint velocity can be reduced according to the reduction of CoM velocity. Therefore, the robot can implement longer stride with the same joint angular velocity.

And finally to demonstrate the usefulness of the proposed method, we generate the walking pattern of square shape in Figs. 10 and 11. This walking pattern is mixed with forward walking and turning.

5 Experiment

We did an experiment with real humanoid robot MAHRU-R which is improved mechanical part design of a humanoid robot platform MAHRU at KIST. Let us Mahru-R introduce briefly. Figure 12 shows the real humanoid robot MAHRU-R. Its specifications are shown in Table 1. MAHRU-R has 1,350 mm height, 50 kg weight including batteries. It has 12-DOF in two legs and 20-DOF in two arms including 4-DOF hands. Also, it is actuated by DC servo motors through harmonic drive reduction gears. The body is equipped with an inertial measurement unit (IMU) sensor which consists of 3-axis gyroscope and 3-axis G-force sensors. Each ankle and wrist is equipped with a force/torque sensor. A distributed system was built for humanoid robot using sub-controllers and IEEE 1,394 protocol communication lines between the main controller and sub-controllers. The main real-time control algorithm runs on a micro-ATX CPU board in the backpack of MAHRU-R, whose operating system is real-time Linux (RTAI/Xenomai). It allows user to make timer interrupt service routine with the highest priority to control the robot in real-time.

To confirm the effectiveness of the proposed method, the experiment was done using Mahru-R. The walking pattern was designed for zigzag walking which consisted of the forward walking and turning in Fig. 13. Figures 14 and 15 show that the generated ZMP and CoM are useful as the real humanoid robot walking pattern.

In Fig. 13, there is a difference between starting point and end point at Y axis. The reason is why we increased the step width from 0.17 to 0.19 m in order to avoid the collision of both feet. Therefore, the end point of Fig. 15 is different to the starting point at Y axis. Figure 16 shows the snapshot of zigzag walking.

6 Conclusion

This paper proposed a new walking pattern method for the humanoid robot. The propose method was based on the LIPM as the simple model of humanoid robot. And this method consisted of three step modules for generating walking pattern. Each step modules used the following stride ZMP position information and the weighted least square method for generating the stable walking pattern. Using these step modules, this paper addressed robustness of the proposed method against initial values of the ZMP and CoM. And this paper illustrated the proposed method generated more effective walking pattern because this method designed the ZMP with the slope in the single support phase. Through a simulation of square shape walking, we verified the proposed method could generate the stable walking pattern for various footprints. And newly developed humanoid platform MAHRU-R was introduced and experimental validation was demonstrated with real humanoid platform MAHRU-R.

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