

Excess resistance effect in a normal metal contacting a superconductor

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Abstract

In relatively pure normal samples contacting a superconductor, we consider the excess resistance effect (i.e. a decrease of the total electrical resistance of the sample after transition of the superconducting part into the normal state) and determine conditions under which the effect arises.

1. Introduction

Recently there have been many theoretical and experimental investigations of transport properties of systems with mixed normal and superconducting elements where new effects have been discovered. Peculiar interplay of the phase coherence intrinsic to the superconductor and the one in the normal metal on a mesoscopic length scale gives rise to new effects both in mesoscopic (see, e.g., Refs. [1–7]) and macroscopic samples. One of the effects is a decrease in the total electrical resistance after transition of the superconducting part of the sample into the normal state under the critical electric current or critical magnetic field [8–15]. As a result, in the first case the current–voltage characteristic of the sample becomes S-shaped and near the superconducting critical current self-oscillations of the current and electric field arise [8, 16].

Here we consider situations of relatively pure composite samples with both weak and strong normal scattering of electrons at the boundaries of two conductors. Such materials have recently been obtained [17].

It occurs that there are several physical mechanisms that work differently in different physical situations, but they all result in the behavior of the resistance mentioned above. A physical description of the mechanisms and determination

of the excess resistance effect in macroscopic samples are given in Section 2. A solution of Boltzmann's equation and determination of the resistance in general terms of the probabilities of the Andreev and normal reflections at the N–S boundary are given in Appendix.

2. Excess resistance effect in kinetics

(a) *Contact of a semiconductor and a superconductor with a negligibly low Schottky barrier.* It is known that electrons incident from a normal conductor to an N–S boundary undergo Andreev reflection at it and the boundary does not contribute to the resistance of the sample (here and below we neglect the excess resistance emerging as a result of penetration of electric field into the superconductor). However, the electrons incident on the N–S boundary at small angles do not undergo Andreev-type, but specular reflection as at an ordinary boundary of a conductor [18]. As these electrons do not penetrate the N–S boundary, an excess resistance of the N–S boundary appears (a detailed analysis of the effect for a quantum ballistic N–S–N structure was performed in Ref. [14] and for results of numerical calculations in the dirty limit, see Ref. [15]). Their relative number is $\sim \sqrt{kT/\epsilon_F}$ [18]. (k is the Boltzman constant, T is temperature, ϵ_F is the Fermi energy). For regular metals this parameter is too small to have an impact on the transport properties.

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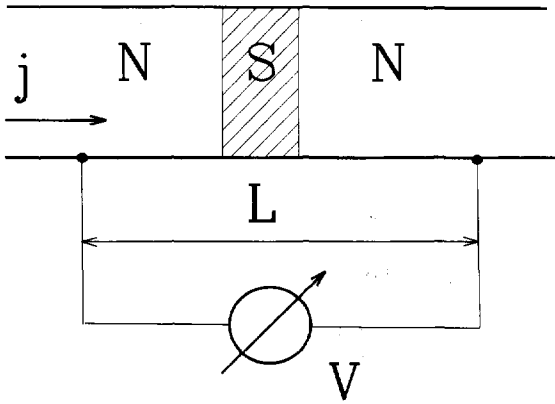


Fig. 1. Normal conductor(N)–Superconductor(S)–Normal conductor(N) sample. j is the total transport current through the sample, L is the distance between the points where the voltage V is measured.

If, however, a superconductor is in contact with a semiconductor (or a semimetal), this parameter increases by several orders of magnitude since it contains the Fermi energy of the normal conductor and, therefore, can qualitatively modify kinetic properties of the semiconductor contacting a superconductor.

Here we consider the resistance of a semiconductor–superconductor–semiconductor system schematically shown in Fig. 1. We consider the case of Schottky barrier absence and assume the mean free path to be $l_0 \ll L$.

In the momentum space \mathbf{p} inside a belt of the width $\delta p = \sqrt{2m\Delta(T)}$ (m is the mass of the electron, $\Delta(T)$ is the superconducting energy gap) and the thickness $\Delta(T)/v_F$ (v_F is the Fermi velocity) embracing the Fermi sphere and parallel to the N–S boundary, the probability of the specular reflection at the N–S boundary $T_p = 1$ and $T_p = 0$ outside it [18, 19]. Using this fact and Eq. (A.6) one gets

$$\alpha \equiv \frac{\delta R}{R} = \begin{cases} (kT/\varepsilon_F)^2 (l_0/L) & \text{for } kT \ll \Delta(T), \\ 1.45(l_0/L)(\Delta_0/kT_c)(\Delta_0/\varepsilon_F)^2 \left(\frac{T_c - T}{T_c}\right)^{3/2} & \text{for } \Delta(T) \ll kT_c, \\ \sim \left(\frac{l_0}{L}\right)(\Delta_0/\varepsilon_F)^2 & \text{for } \Delta(T) \sim kT_c. \end{cases} \quad (1)$$

where $\delta R/R = (V(L)/I - R)/R$ is the relative resistance of the N–S boundaries, R is the total resistance of the sample in the normal state, I is the total current through the sample, $\Delta(T) = 1.8 \Delta_0 ((T_c - T)/T_c)^{1/2}$, Δ_0 is the superconducting gapwidth at $T = 0$, and T_c is the critical temperature. As $\alpha > 0$, the effects mentioned above arise in the situation considered.

(b) *Regular normal metal with a twin or grain boundary contacting a superconductor.* Here we assume the distance between the N–N and N–S boundaries to be $l < \lambda_T < l_0$ where the normal metal ‘coherent length’ $\lambda_T = \hbar v_F/kT$.

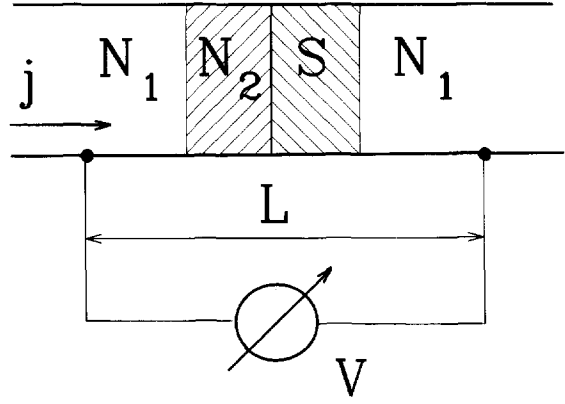


Fig. 2. A sample with a boundary between two normal metals N_1 and N_2 , and two N–S boundaries. j is the total transport current through the sample, L is the distance between the points where the voltage V is measured.

An electron incident from metal N_1 (see Fig. 2) to the N–N boundary in the absence of superconductivity undergoes normal reflection with probability ρ^2 . In the presence of superconductivity such an electron undergoes repeatedly both normal reflections at the N–N boundary and Andreev reflections at the N–S boundary. As a result, multiple coherent reflections of the electron arise, and the total probability for the electron to be reflected by these two boundaries is

$$T_p = 2\rho^2 \frac{1 + \cos \phi}{1 + 2\rho^2 \cos \phi + \rho^4} \quad (2)$$

where $\phi = 2m\varepsilon l/\hbar(p_F^2 - p_\perp^2) \sim l/\lambda_T$ with p_F the Fermi momentum, p_\perp the projection of the incident electron momentum parallel to the boundaries, and ε is the incident electron energy measured from the Fermi energy. The probability T_p averaged over the incident angles ϕ is

for $kT \ll \Delta(T)$.

for $\Delta(T) \ll kT_c$.

for $\Delta(T) \sim kT_c$.

$$\langle T_p \rangle = 2\rho^2/(1 + \rho^2). \quad (3)$$

If the distance between the N–N and N–S boundaries is equal to zero, the probability T_p coincides with the one obtained in Ref. [20] when written in terms of ρ^2 .

Using Eqs. (A6), (2) and (3) we have the relative excess resistance α given by

$$\alpha = \frac{l_0}{L} \left(W - \frac{R_S}{R_L} \right). \quad (4)$$

Here $W = \langle T_p \rangle - \rho^2$, and R_S and R_L are the resistances of the S-part of the sample (when it is in the normal state) and the total resistance of the N-metal of the length L , respectively. In deriving Eq. (4) we assumed $\Delta(T) \ll \Delta_0$.

(c) *Regular normal metal with impurities contacting a superconductor.* ($L \gg l_0 \gg \lambda_T$). As shown in Ref. [21] combined scattering of an electron by an impurity and an N–S boundary is of a multiple coherent character. According to Ref. [12] the cross-section of the electron back-scattering by an impurity located inside a normal metal layer of thickness $\sim \lambda_T$ adjoining the N–S is $\bar{\sigma} = 2\sigma_0$ if averaged over the distance between the impurity and the N–S boundary (σ_0 is the cross-section of the scattering by the impurity in the absence of the N–S boundary). Therefore, this layer as a whole scatters the electron backwards with the probability

$$T^{\text{eff}} = c_i \lambda_T \sigma_0 \quad (5)$$

that can be treated as the effective probability of the normal reflection by the N–S boundary (the Andreev reflection probability is $1 - T^{\text{eff}}$). Using Eqs. (5) and (A.6) one finds the excess resistance to be

$$\delta R = \alpha R = R_N \frac{\lambda_T}{L} - R_S. \quad (6)$$

As is evident from Eqs. (1), (4) and (6) the excess resistance δR can be positive in many experimental situations and, therefore, in this case transition of the superconductor to the normal state is accompanied by a decrease of the total resistance of the sample.

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Appendix

Here we find the voltage drop $\delta\Phi$ between the N–S boundary ($x = 0$) and a plane $x = L_0 \gg l_0$ (for definiteness sake we assume the normal part of the sample to occupy the right half-space $x > 0$, the coordinate axis x is perpendicular to the N–S boundary) in the case that a normal metal electron undergoes both the Andreev and the specular reflection at the N–S boundary with the probabilities T_p and R_p , respectively ($T_p + R_p = 1$). Under conditions of a weak electric field E and $l_0 \gg \lambda_T$ the resistance of a normal conductor contacting a superconductor is determined by the usual Boltzmann's equation

$$v_p \frac{\partial f_1}{\partial x} + \frac{f_1 - \langle f_1 \rangle_\varepsilon}{t_0} = -eE(x)v_p \frac{\partial f_0}{\partial E} \quad (A.1)$$

with the boundary condition at the N–S boundary

$$f_1^+(\mathbf{p}; 0, y, z) = (T_p - R_p)f_1^-(\mathbf{p}; 0, y, z). \quad (A.2)$$

Here v_p is the x -component of the electron velocity, t_0 is the relaxation time, $f_1^+(\mathbf{p}; \mathbf{r})$ and $f_1^-(\mathbf{p}; \mathbf{r})$ are nonequilibrium corrections to the Fermi distribution function f_0 for electrons with velocities directed towards the N–S boundary and away from it, $\mathbf{r} = (x, y, z)$, the brackets $\langle \dots \rangle_\varepsilon$ designate the average over \mathbf{p} at a given energy ε . Under the condition of a fixed current j flowing through the system the electric field $E(x)$ in the normal part of the sample is determined by the local neutrality condition

$$dj/dx = 0. \quad (A.3)$$

Below we find the voltage drop $\delta\Phi$ assuming the normal reflection probability $T_p \ll 1$. Solving Boltzmann's equation (A.1) with the boundary condition (A.2) and using Eq. (A.3) one finds the equation for the electric potential $\phi(x)$ in the normal part of the sample

$$\begin{aligned} & \int_0^\infty \phi(x') \left\{ \frac{e^{-|x-x'|/l_p}}{l_p} \right\} dx' \\ & - \int_0^\infty \phi(x') \left\{ \frac{e^{-|x+x'|/l_p}}{l_p} \right\} dx' - 2\phi(x) \\ & = 2E_\infty \{ T_p l_p e^{-x/l_p} \} - 2\phi(0) \{ e^{-x/l_p} \}. \end{aligned} \quad (A.4)$$

Here $l_p = t_0 |v_p|$, $\{ \dots \} = \langle \Theta(v_p) \dots \rangle_\varepsilon$, $\Theta(v_p) = 1$ if $v_p > 0$ and $\Theta(v_p) = 0$ if $v_p < 0$, $\phi(x)$ is associated with $E(x)$ by the relation

$$E(x) = RI/L_0 - d\phi/dx \quad (A.5)$$

where R is the total resistance of the normal conductor of length L_0 in the absence of the superconductor, I is the total current, $\phi(x) \rightarrow 0$ at $x \rightarrow \infty$, $\phi(0)$ is the value of the electric potential at the N–S boundary ($x = 0$). The left side of Eq. (A.5) is orthogonal to x and the solvability condition for Eq. (A.5) determines $\phi(0)$ which, together with Eq. (A.5), gives

$$\delta\Phi = IR \left(1 + \frac{\{ T_p l_p^3 \}}{L_0 \{ l_p^2 \}} \right). \quad (A.6)$$

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