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The performance comparison and analysis of extended Kalman filters for GPS/DR navigation

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ABSTRACT

This paper proposes several nonlinear filtering algorithms based on the global positioning system (GPS) and the dead reckoning (DR). To achieve high location and velocity accuracy, the first-order extended Kalman filter (FEKF), the second EKF (SEKF) and EKF-Rauch-Tung-Striebel (EKF-RTS) smoother are introduced for GPS/DR integrated navigation system. And the algorithms of the FEKF, SEKF and EKF-RTS are given. Furthermore, the state models and measurement models of GPS/DR are set up. For comparison purpose, the GPS/DR integrated navigation system based on the three algorithms is simulated, and the algorithm performance is analyzed and compared by the simulation results of FEKF, SEKF, FEKF-RTS and SEKF-RTS. Numerical results demonstrate that the EKF-RTS gives clearly better estimates than the FEKF and SEKF.

1. Introduction

Filtering is the problem of estimating the state of a system as a set of observations becomes available on-line. This problem is of paramount importance in many fields of science, engineering and finance. To solve it, one begins by modeling the evolution of the system and the noise in the measurements. The resulting models typically exhibit complex nonlinearities and non-Gaussian distributions [1], thus precluding analytical solution.

The best known algorithm to solve the problem of non-Gaussian, nonlinear filtering is the extended Kalman filter. This filter is based upon the principle of linearizing the state and the measurement models by using Taylor series expansions [2,3]. However, the EKF provides first-order approximations to optimal nonlinear parameter estimation, which may include large errors. As an alternative to the EKF, the second extended Kalman filter (SEKF) and extended Kalman smoother filter are proposed. Compared with the widely used EKF, the SEKF provides second-order approximation of process and measurement errors for both Gaussian and non-Gaussian distributions [4]. Optimal smoothing in context of state space models refers to statistical mythology that can be used for computing estimates of the past state history of a time varying system based on the history of noisy measurements obtained from it. Phenomena, which can be modeled as this kind of state space models can be found, for example, in navigation, aerospace engineering, space engineering, remote surveillance, telecommunications, physics, audio signal processing, control engineering, several other fields. And it often gives better results.

They all are the most widely approaches to localization. The EKF offers an efficient and iterative means of combining information from a number of different sensors to estimate the state of the vehicle. At each iteration, the future vehicle state is predicted using a vehicle model. A suite of sensors yield a set of measurements which are used to update the estimate of vehicle state. The vehicle model essentially allows past (observation and control input) state information to be projected forward in time and combined with current (observation) information.

The GPS has provided the ability to determine a boby's position, velocity and attitude anywhere on the surface of the globe and has proven effective when implemented on vehicles for navigation [5]. With the absence of selective availability (SA), a GPS receiver can provide very accurate three-dimensional velocity measurements without the need for any differential corrections (known as differential GPS or DGPS) [5]. However, GPS has limitations such as low sampling rate, and it is impossible to realize continuous localization by GPS in an urban area where the satellites signal is obstructed by high buildings, trees and tunnels, etc. Additionally, DR system is a self-contained, autonomous and high sampling rate navigation system compared with GPS. However, error of DR will accumulate with time [6,7]. The integration of GPS and DR system combines the short-term stability of DR system with the long-term stability of GPS, and the integrated system improves performance in terms of accuracy, reliability, integrity and availability.

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The remainder of this paper is organized as follows. Section 2 introduces the FEKF, SEKF and EKF-RTS, devoting to the theory and implementation details. In Section 3 the model of GPS/DR integrated navigation system which is based on the FEKF and the SEKF is set up. Some experimental results are discussed in Section 4. Finally, Section 5 contains some conclusion remarks and pointers for practice application.

2. The extended Kalman filter model

The EKF extends the scope of KF to nonlinear optimal filtering problems by forming a Gaussian approximation to the joint distribution of state x and measurements y using a Taylor series based transformation. First- and second-order EKF are presented, which are based on linear and quadratic approximations to the transformation. The filtering model used in the EKF is

$$x_k = f(x_{k-1}, k-1) + q_{k-1} \tag{1}$$

$$z_k = h(x_k, k) + r_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state; $z_k \in \mathbb{R}^m$ is the measurement; $q_{k-1} \sim N(0, Q_{k-1})$ is the process noise; $r_k \sim N(0, R_k)$ is the measurement noise; f is the dynamic model function and h is the measurement model function.

The FEKF is separated to two steps. The all steps for the first-order EKF are acquired in literature [1,8]. The matrices $F_x(m,\ k-1)$ and $H_x(m,\ k)$ are Jacobians of f and h, with elements

$$[F_x(m, k-1)]_{j,j'} = \left.\frac{\partial f_j(x, k-1)}{\partial x_{j'}}\right|_{x=m} \tag{3}$$

$$[H_x(m, k)]_{j,j'} = \left.\frac{\partial h_j(x, k)}{\partial x_{j'}}\right|_{x=m} \tag{4}$$

2.1. Second-order extended Kalman filter

The corresponding steps for the SEKF are as follows:

The prediction step:

$$\begin{aligned} m_k^- &= f(m_{k-1}, k-1) + \frac{1}{2}\sum_i e_i tr\{F_{xx}^{(i)}(m_{k-1}, k-1)P_{k-1}\} \\ P_k^- &= F_x(m_{k-1}, k-1)P_{k-1}F_x^T(m_{k-1}, k-1) + \frac{1}{2}\sum_{i,i'} e_i e_{i'}^T tr\{F_{xx}^{(i)}(m_{k-1}, k-1)P_{k-1}F_{xx}^{(i')}(m_{k-1}, k-1)P_{k-1}\} + Q_{k-1} \end{aligned} \tag{5}$$

The update step:

$$\begin{aligned} v_k &= Z_k - h(m_k^-, k) - \frac{1}{2}\sum_i e_i tr\{H_{xx}^{(i)}(m_k^-, k)P_k^-\} \\ S_k &= H_x(m_k^-, k)P_k^- H_x^T(m_k^-, k) + \frac{1}{2}\sum_{i,i'} e_i e_{i'}^T tr\{H_{xx}^{(i)}(m_k^-, k)P_k^- H_{xx}^{(i')}(m_k^-, k)P_k^-\} + R_k \\ K_k &= P_k^- H_x^T(m_k^-, k)S_k^{-1} \\ m_k &= m_k^- + K_k v_k \\ P_k &= P_k^- - K_k S_k K_k^T \end{aligned} \tag{6}$$

where the matrices $F_{xx}^{(i)}(m_{k-1}, k-1)$ and $H_{xx}^{(i)}(m_k^-, k)$ are the Hessian matrices of f_i and h_i, that is,

$$[F_{xx}^{(i)}(m_{k-1}, k-1)]_{j,j'} = \left.\frac{\partial^2 f_i(x, k-1)}{\partial x_j \partial x_{j'}}\right|_{x=m} \tag{7}$$

$$\left[H_{xx}^{(i)}(m_k^-, k)\right]_{j,j'} = \left.\frac{\partial^2 h_i(x, k)}{\partial x_j \partial x_{j'}}\right|_{x=m} \tag{8}$$

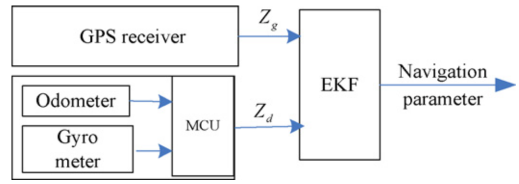

Fig. 1. A scheme of DR/GPS integrative system.

- $e_i = (0\ldots0\ 1\ 0\ldots0)^T$ is a unit vector in direction of the coordinate axis i, that is a 1 at position i and 0 at other position. Where m_k^- and p_k^- are the predicted mean and covariance of the state, respectively.
- v_k is the innovation or the measurement residual on time step k.
- S_k is the measurement prediction convariance on the time step k.
- K_k is the filter gain, which tells how much the predictions should be corrected on time step k.

2.2. Extended Kalman smoother

An estimator for the state of a dynamic system at time t, using measurements made after time t, is called a smoother. The accuracy of a smoother is generally superior to that of a filter [9], because it uses more measurements for its estimate.

The discrete time EKF smoother, also known as the Rauch-Tung-Striebel (RTS), can be used for computing the smoothing solution for the model (1) given as distribution.

$$p(x_k|y_{1:T}) = N(x_k|m_k^s, P_k^s) \tag{9}$$

The mean and covariance m_k^s and p_k^s are calculated with the following equations:

$$\begin{aligned} m_{k+1}^- &= f(m_k, k) \\ P_{k+1}^- &= F_x(m_k, k)P_k F_x^T(m_k, k) + Q_k \\ C_k &= P_k F_x^T(m_k, k)[P_{k+1}^-]^{-1} \\ m_k^s &= m_k + C_k[m_{k+1}^s - m_{k+1}^-] \\ P_k^s &= P_k + C_k[P_{k+1}^s - P_{k+1}^-]C_k^T \end{aligned} \tag{10}$$

where m_{k+1}^- and P_{k+1}^- are the predicted state mean and state covariance on time step $k+1$;

m_k^s and P_k^s are the smoother estimates for the state mean and state covariance on time step k;

C_k is the smoother gain on time step k;

In this paper, the extended Rauch-Tung-Striebel smoother is applied to FEKF and SEKF estimates.

3. DR/GPS integration model and analysis

3.1. DR/GPS integration

The main parts of DR are odometer, gyro meter and relevant additive circuit. A schematic diagram of DR/GPS hardware integrative system is shown in Fig. 1.

In the system, GPS receives satellite signal, and produces positioning information Z_g. Gyro meter gives vehicle' yaw velocity which heading angle increment is acquired by integrated. Odometer can measure vehicle's displacement increment. Then, the

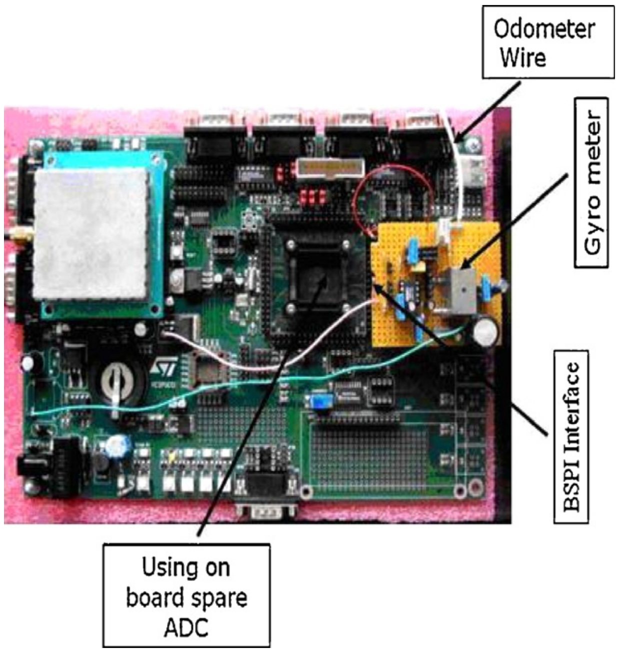

Fig. 2. Odometer and gyroscope integration of the GPS module.

information Z_g and Z_i are sent to the EKF to integrated the information, obtaining the best optimal navigation parameter.

Odometer is the basic configuration of vehicles, generally used to measure linear distance. The modern automotive electronics technology can connect odometer and gyro meter with GPS receiver to achieve assisted positioning. The Odometer and gyroscope integration of the GPS module is shown in Fig. 2.

4. Mathematical model of GPS/DR integrated navigation system

4.1. GPS/DR state equation

In the GPS/DR integrated navigation system, the linear state space model can be modeled by,

$$\dot{X}(t) = F(t)X(t) + L(t)w(t) \tag{11}$$

where $X(t)$ is target's state on the time t; $\omega(t)$ is a white noise process with power spectral density Q_c,

- $X(t) = [\,P_e \quad P_n \quad v_e \quad v_n \quad a_e \quad a_n\,]$, $\omega(t) = [\,w_{ae} \quad w_{an}\,]$, where P_e and P_n are the east position and the north position, respectively;
- v_e and v_n are the east velocity and the north velocity, respectively;
- a_e and a_n are the east acceleration and the north acceleration, respectively;
- w_{ae} and w_{an} are the acceleration white noise.
- $F(t)$ and $L(t)$ are a state transition matrix and a noise vector matrix, respectively.

$$F(t) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad L(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To be able to use the EKF, the model (11) must be discretized. The solution for the discretized matrices A_k and Q_k can be given as

$$A_k = \exp(F\Delta t_k) \tag{12}$$

$$Q_k = \int_0^{\Delta t_k} \exp(F(\Delta t_k - \tau)LQ_cL^T \exp(F(\Delta t_k - \tau))^T d\tau \tag{13}$$

where $\Delta t_k = t_{k+1} - t_k$ is the stepsize of the discretization. The matrices A_k and Q_k can be calculated analytically with Eqs. (12) and (13) to give the following:

$$A_k = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{1}{2}\Delta t^2 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{1}{2}\Delta t^2 \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_k = \begin{bmatrix} \frac{1}{20}\Delta t^5 & 0 & \frac{1}{8}\Delta t^4 & 0 & \frac{1}{6}\Delta t^3 & 0 \\ 0 & \frac{1}{20}\Delta t^5 & 0 & \frac{1}{8}\Delta t^4 & 0 & \frac{1}{6}\Delta t^3 \\ \frac{1}{8}\Delta t^4 & 0 & \frac{1}{3}\Delta t^3 & 0 & \frac{1}{2}\Delta t^2 & 0 \\ 0 & \frac{1}{8}\Delta t^4 & 0 & \frac{1}{3}\Delta t^3 & 0 & \frac{1}{2}\Delta t^2 \\ \frac{1}{6}\Delta t^3 & 0 & \frac{1}{2}\Delta t^2 & 0 & \Delta t & 0 \\ 0 & \frac{1}{6}\Delta t^3 & 0 & \frac{1}{2}\Delta t^2 & 0 & \Delta t \end{bmatrix} \sigma$$

4.2. GPS/DR measurement equation

The east position P_e and the north position P_n are given by GPS receiver; the angular rate ω is given by rate gyro and the range S between sample time T is given by vehicle odometer as measurement variables. The nonlinear measurement model can be presented as:

$$Z(t) = h(x(t), t) + V(t) \tag{14}$$

where $V(t)$ is the measurement noise vector$h(x(t), t) = [\,P_e, \quad P_n, \quad (v_n a_e - v_e a_n)/(v_e^2 + v_n^2), \quad \psi T\sqrt{v_e^2 + v_n^2}\,]^T$, ψ is the scale factor of vehicle.

For the measure Eq. (14) must be linearized for the FEKF and the SEKF. The typical format of linear measure equation and discreted equation is as the form of (14).

$$Z_k = H(k)X(k) + V(k) \tag{15}$$

The Jacobian matrics of the measurement model, which are needed by FEKF, can be computed with Eq. (6) to give the following:

$$H_x(k) = \begin{bmatrix} \frac{\partial p_e}{\partial p_e} & \frac{\partial p_e}{\partial p_n} & \frac{\partial p_e}{\partial v_e} & \frac{\partial p_e}{\partial v_n} & \frac{\partial p_e}{\partial a_e} & \frac{\partial p_e}{\partial a_n} \\ \frac{\partial p_n}{\partial p_e} & \frac{\partial p_n}{\partial p_n} & \frac{\partial p_n}{\partial v_e} & \frac{\partial p_n}{\partial v_n} & \frac{\partial p_n}{\partial a_e} & \frac{\partial p_n}{\partial a_n} \\ \frac{\partial p_e}{\partial \omega} & \frac{\partial p_n}{\partial \omega} & \frac{\partial v_e}{\partial \omega} & \frac{\partial v_n}{\partial \omega} & \frac{\partial a_e}{\partial \omega} & \frac{\partial a_n}{\partial \omega} \\ \frac{\partial p_e}{\partial s} & \frac{\partial p_n}{\partial s} & \frac{\partial v_e}{\partial s} & \frac{\partial v_n}{\partial s} & \frac{\partial a_e}{\partial s} & \frac{\partial a_n}{\partial s} \\ \partial p_e & \partial p_n & \partial v_e & \partial v_n & \partial a_e & \partial a_n \end{bmatrix}$$

Similarly, the Hessian matrices of the SEKF can be computed with Eq. (9).

For comparison purposes, the following root mean squared error (RMES) performance measures are adopted. Lower RMSE means better performance. The RMSE model [10],

$$P_{\text{RMSE}} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(p_k - \hat{p}_k)^2} \tag{16}$$

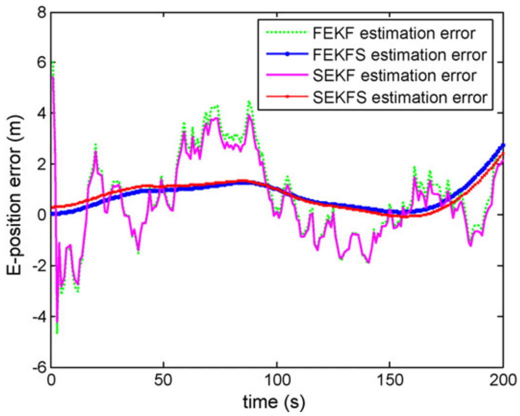

Fig. 3. The east position estimation error.

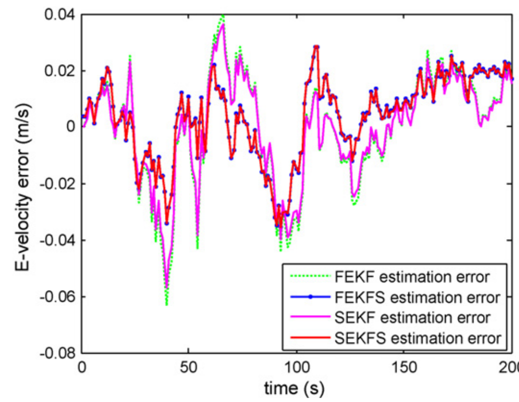

Fig. 4. The east velocity estimation error.

Table 1
Analysis of the RMSE values.

Method	RMSE p_e (m)	RMSE p_n (m)	RMSE v_e (m/s)	RMSE v_n (m/s)
FEKF	1.8783	0.0199	1.9667	0.1019
FEKF-RTS	0.9194	0.0151	0.8354	0.0685
SEKF	1.7463	0.0199	1.9048	0.1081
SEKF-RTS	0.9169	0.0150	0.8385	0.0643

$$v_{\mathrm{RMSE}} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(v_k - \hat{v}_k)^2} \tag{17}$$

where p_k is the actual position, p_k is the estimated position; v_k is the actual velocity, and $\hat{v}_k$ is the estimated velocity of the target at instant of time k, n is the number of samples.

5. Simulation result and analysis

In this section an example with position and velocity of a land vehicle is outlined to demonstrate the accuracy of the proposed GPS/DR arithmetics. Several simulations have been carried out and results are presented. The experiments are developed under the MATLAB 7.0.1 simulation environment.

Setting the same parameters and conditions, the FEKF, SEKF and EKF-smoother are employed to estimate the states of the land vehicle integrated GPS/DR navigation system.

Suppose that the vehicle moves on a straight line road, the constant velocity is $10\sqrt{2}\,\mathrm{m/s}$ and the heading is 45°. The simulation initial conditions are as follows:

Simulation time is 200 s, and sampling interval is $T = 0.1$ s;

Scale coefficient of odometer is $\xi = 1$, and the initial state vector is $X_0 = [\,0 \quad 0 \quad 10 \quad 10 \quad 0 \quad 0\,]^T$

The power spectral density of a white noise process $Q_c = \mathrm{diag}([\,0.2 \quad 0.2\,])$;

The observation covariance noise matrix is $R = [\,20^2 \quad 20^2 \quad 0.2^2 \quad 0.2^2\,]$

The simulation results are shown in Figs. 3–6.

By comparing with the simulation results of the four algorithms, it is obvious that the EKF-smoother results have the best accurate estimates of the vehicle position than the FEKF and the SEKF. In the Figs. 3–6, the RMSE values are summarized in Table 1.

In Table 1 we have listed the RMSE values of position and velocity with all tested methods. The numbers prove the previous observations, that the FEKF and SEKF give almost identical performances. And it is obvious that the EKF-RTS is superior to the FEKF and SEKF.

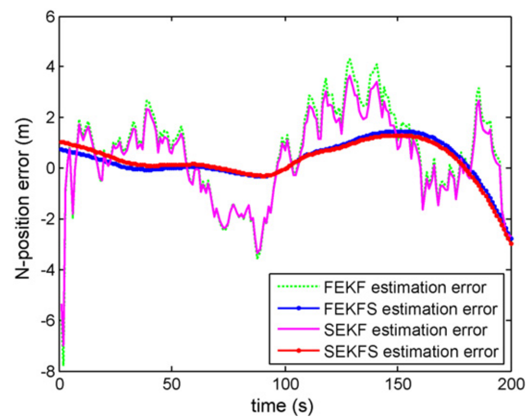

Fig. 5. The north position estimation error.

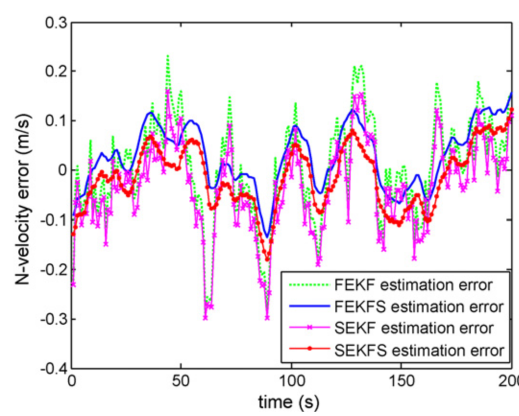

Fig. 6. The north velocity estimation error.

6. Conclusions

In this paper, the four Kalman filters are introduced to be applied in the state estimation of the vehicle integrated GPS/DR navigation system. And the simulation results have shown the superior performance of the EKF-smoother, comparing to that of the FEKF and SEKF here. Certainly, it is expected that the EKF-smoother can perform best than the FEKF and the SEKF in some other nonlinear systems. In the practice, the SEKF and FEKF give almost identical performances. But the calculation of SEKF Hessian matrices can be a very difficult process, even though the equations themselves are relatively simple. So, FEKF and FEKF-RTS are often adopted in nonlinear applications.

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