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The Fluorescence and Phosphorescence Spectra of Pyrazine in Benzene at 4.2°K

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The fluorescence and phosphorescence spectra of pyrazine- h_4 and pyrazine- d_4 in benzene at 4.2° K are presented and analyzed. The Frank-Condon patterns of a_{1g} modes in the fluorescence spectrum are quite similar to those in the phosphorescence spectrum for both pyrazine- h_4 and pyrazine- d_4 .

The ν_5 (b_{2g}) and v_{10a} (b_{1g}) out-of-plane bending modes appear with quite different intensities in the fluorescence and phosphorescence spectra. The $0 \to 1$ transitions in these modes only appear in the fluorescence spectra while the $0 \to 2$ transitions are more intense in the phosphorescence spectra. These results are interpreted in terms of a vibronic interaction between $^{1,3}B_{3u}$ $(n\pi^*)$ and the $^{1,3}B_{2u}$ $(\pi\pi^*)$ and $^{1,3}B_{1u}$ $(\pi\pi^*)$ states.

No spectroscopic evidence is found for a forbidden ${}^{1}n\pi^{*}$ state below ${}^{1}B_{\$u}$ $(n\pi^{*})$.

INTRODUCTION

The luminescent properties of azines have been the source of considerable scientific investigation. Low-temperature emission spectra of azines in mixed crystals can yield sharp spectra which are easily analyzed in terms of the ground-state vibrational assignments provided by infrared and Raman spectroscopists. These spectra may reveal much about excited-state geometries and interactions between excited states. The present study is a composite of work performed concurrently at the University of Pennsylvania (by C. M.) and the University of Chicago (by E. Z.) on the mixed crystal emission spectra of pyrazine in benezene.

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The lowest singlet excited state of pyrazine has been assigned as ${}^{1}B_{3u}(n\pi^{*})$ by Innes et al. (1) on the basis of the type-C rotational structure found for the 0-0 band in the vapor absorption spectrum. The molecule has been found to fluoresce with a low quantum yield both in solution (2) and in the vapor (3). It shows strong phosphorescence in the vapor phase (3) and in rigid glass solvents at 77°K (4). The phosphorescent state has been assigned as ${}^{3}B_{3u}(n\pi^{*})$ (4, 5). Since the lowest singlet and triplet excited state have similar energies and are derived from the same configuration, they should to a first approximation be similar electronically. A major difference in these states is their nearness to $\pi\pi^*$ states of the same multiplicity. Because the singlet-triplet splitting is much larger for $\pi\pi^*$ states than for $n\pi^*$ states, the ${}^3B_{3u}(n\pi^*)$ state is expected to be much closer to the ${}^{3}B_{2u}(\pi\pi^{*})$ and ${}^{3}B_{1u}(\pi\pi^{*})$ than ${}^{1}B_{3u}(n\pi^{*})$ is to ${}^{1}B_{2u}(\pi\pi^{*})$ and ${}^{1}B_{1u}(\pi\pi^{*})$. A theoretical treatment of the effect of nearby states on some aspects of the pyrazine- h_4 emission spectra has been presented previously (6). In this article we present and analyze the complete emission spectra of pyrazine- h_4 and pyrazine- d_4 in benzene at 4.2°K.

EXPERIMENTAL

Pyrazine (Aldrich Chemical Co.) and pyrazine- d_4 (Merck Sharp and Dohme of Canada) were purified by repeated zone melting or by vacuum sublimation. The samples were placed in a quartz cell and immersed in liquid helium in a quartz double dewar. Samples were excited either with a General Electric A H-6 mercury lamp or with 450-W Xenon lamp filtered with a NiSO₄ solution and a Corning 754 filter.

Spectra were taken with a 3.4 m Ebert mount Jarell–Ash spectrograph at the University of Chicago and with a Baird-Atomic 3-m concave grating spectrograph with an Eagle mount at the University of Pennsylvania. The spectra showed a major spectrum and several similar minor spectra which are due to pyrazine in energetically different sites in the benzene lattice. In what follows we have analyzed only the spectrum of the major site.

DESCRIPTION OF THE SPECTRA

The complete emission spectrum of pyrazine- h_4 in benzene at 4.2°K is shown in Fig. 1. Vibrational analyses for the pyrazine- h_4 and pyrazine- d_4 spectra are listed in Tables I–IV. Most vibrational assignments were made on the basis of ground-state infrared and Raman data-(7, 8). Many of the modes we see in emission are also present in the hot band absorption spectrum of the pyrazine vapor (1, 2). A schematic representation of the important symmetry modes of pyrazine is shown in Fig. 2. We find that the fluorescence and phosphorescence spectra are remarkably similar in both pyrazine- h_4 and pyrazine- d_4 .³ The totally

 3 The notation is that of Ref. (12), a and b refer to the nodes of the π orbital being through atoms and through bonds, respectively.

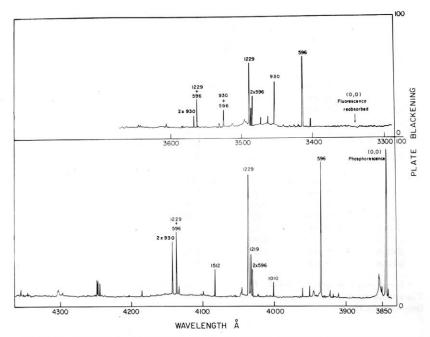


Fig. 1. The fluorescence and phosphorescence spectrum of pyrazine in benzene at 4.2°K.

symmetric modes ν_{6a} (596 cm⁻¹) and ν_{9a} (1229 cm⁻¹) dominate the pyrazine- h_4 spectra. In pyrazine- d_4 ν_1 (881 cm⁻¹), ν_{9a} (998 cm⁻¹) and a Fermi doublet (578 cm⁻¹ and 591 cm⁻¹) involving ν_{6a} and ν_4 (b_{1g}) are the most prominent bands. These and other weaker a_g modes appear with similar intensities in the fluorescence and phosphorescence spectra. This is exactly what has been found in the emission spectra of pyrimidine (10) in benzene at 4.2°K and indicates that the molecule has a similar equilibrium geometry in its singlet and triplet states.

This observation is in agreement with the conclusion that both emissions are occuring from states of the same spatial term, namely, $B_{3u}(n\pi^*)$. Evidently the small exchange splitting in $n\pi^*$ terms results in singlet and triplet states which are almost identical electronically. As found in pyrimidine (10) the denteration shifts are similar for the fluorescence and phosphorescence 0–0 bands. For pyrazine we find shifts of 156 cm⁻¹ and 150 cm⁻¹, respectively.

There are several prominent bands which appear with different relative intensities in the fluorescence and phosphorescence spectra. An intense band at 930 cm⁻¹ (728 cm⁻¹ in pyrazine-d₄) appears in the fluorescence spectrum and is extremely weak in the phosphorescence spectrum. Two quanta of this mode appears

λ(Å)	$\nu \text{ (cm}^{-1})$	Δu	Rel. int.	Analysis	Error
3344.4	29 892	.0	reabsorbed		
3412.5	29 296	596	VS	v_{6a}	
3423.4	29 202	690	$\mathbf{v}\mathbf{w}$	v_4	
3430.7	29 140	752	$\mathbf{v}\mathbf{w}$	v_5	
3451.8	28 962	930	MS	v_{10a}	
3461.4	28 881	1011	W	v_1	
3471.3	28 799	1093	W		
3483.3	28 700	1192	M	$2 \times v_{6a}$	0
3485.2	28 685	1207	MW	$v_{11} + v_{16b}^{a}$	
3487.8	28 663	1229	MS	v_{9a}	
3494.1	28 611	1281	VW	$v_{6a} + v_4$	+5
3511.5	28 470	1422	MW		
3524.2	28 367	1525	W	$v_{6a} + v_{10a}$	1
3529.6	28 332	1560	W		
3561.8	28 068	1824	MW	$v_{6a} + v_{9a}$	1
3565.8	28 036	1856	W	$2 \times v_{10a}$	+4

^a Hong and Robinson (*J. Chem. Phys.*, to be published) assign this band as $2 \times v_{6b}$. The symmetry of $v_{11} + v_{16b}$ is A_g and from the assignments of Califano *et al.* (8) the combined frequency is 1203 cm⁻¹.

	IN DENZENE AT 4.2 IX		The state of the s		
λ (Å)	ν (cm ⁻¹)	$\Delta \nu$	Rel. int.	Analysis	Error
3327.0	30 048	0	reabsorbed	0, 0	
3392.3	29 470	578	S	v 6a a	
3393.8	29 457	591	MS	υ ₄	
3409.7	29 320	728	MS	v_{10a}	
3427.8	29 165	883	S	v_1	
3441.3	29 050	998	S	v_{9a}	
3461.4	28 882	1166	MW	$v_{6a} + v_4$	+3
3478.6	28 739	1309	W	$v_{6a} + v_{10a}$	-3
3496.1	28 595	1453	W	$2 \times v_{10a}$	+3
3496.8	28 589	1459	MW	$v_{6a} + v_1$ a	0
3498.4	28 576	1472	MW	$v_4 + v_1$	0
3511.1	28 473	1575	MW	$v_{6a} + v_{9a}$	+1
3512.8	28 459	1589	W	$v_4 + v_{9a}$	0
3515.2	28 440	1609	W	$v_1 + v_{10a}$	0
3529.5	28 324	1724	MW	$v_{9a} + v_{10a}$	+2
3534.3	28 286	1762	$\mathbf{M}\mathbf{W}$	$2 \times v_1$	0
3548.9	28 170	1878	M	$v_1 + v_{9a}$	+1
3563.8	28 052	1996	VW	$2 \times v_{9a}$	0

^a Fermi doublets.

TABLE III Phosphorescence Spectrum of Pyrazine- h_4 in Benzene at $4.2^{\circ}{\rm K}$

λ(Å)	$\nu \ (\mathrm{cm}^{-1})$	Δu	Rel. int.	Analysis	Error
3845.9	25 994	0	VS	origin	
3936.2	25 398	596	VS	v_{6a}	
3951.1	25 302	692	\mathbf{W}	v_4	
3961.0	25 239	755	W	v_5	
4001.3	24 985	1009	V		
4001.6	24 983	1011	W	v_1	
4023.0	24 850	1144	VW		
4030.6	24 803	1191	\mathbf{M}	2 × v6a a	-1
4033.2	24 787	1207	S	$v_{11} + v_{166}$	
4036.6	24 766	1228	VS	v_{9a}	
4056.8	24 643	1351	VVW	$v_{6a} + v_{5}$	0
4083.1	24 484	1510	\mathbf{M}	$2 \times v_5$	0
4099.5	24 386	1608	VW	$v_1 + v_{6a}$	-1
4136.2	24 170	1824	S	$v_{9a} + v_{6a}$	0
4141.8	24 137	1857	S	$2 \times v_{10a}$	
4172.3	23 961	2033	VVW		
4174.8	23 889	2105	VW	$2 \times v_5 + v_{6a}$	+1
4246.5	23 542	2452	$\mathbf{v}\mathbf{w}$	$2 \times v_{10a} + v_{6a}$	+1
4247.6	23 536	2458	$\mathbf{v}\mathbf{w}$	$2 \times v_{9a}$	-2

^a See footnote in Table I.

with more intensity in the phosphorescence spectrum than in the fluorescence spectrum. Innes et~al. see these frequencies in the hot band absorption of the pyrazine- h_4 and pyrazine- d_4 vapors (11). They find that the 1–0 band shows a type-A rotational structure and assign the active mode as $\nu_5(b_{2g})$. This assignment, however, could be in error due to the similarity of the two in-plane moments of pyrazine. Califano et~al.~(8) assign the mode to $\nu_{10a}(b_{1g})$. We will discuss the assignment of the 930 cm⁻¹ mode later in the paper. In the phosphorescence spectra we find intense bands at 1510 cm⁻¹ (2 × 755) for pyrazine- h_4 and at 2 × 646 cm⁻¹ in pyrazine- d_4 which are absent in the fluorescence spectra. One quanta of these frequencies is absent in fluorescence and extremely weak in phosphorescence. These modes are assigned as $\nu_{10a}(b_{1g})$ by Innes (7) and as $\nu_5(b_{2g})$ by Califano (8).

Odd quanta changes in b_{1g} and b_{2g} modes may appear in the emission spectrum via a Hertzberg–Teller vibronic coupling mechanism. For a vibronic band to show up in an emission spectrum it must have intensity comparable with the allowed part of the spectrum. Out-of-plane bending modes are known to effectively mix $n\pi^*$ and $\pi\pi^*$ states of the same multiplicity. In pyrazine the three lowest observed states are ${}^1B_{3u}(n\pi^*)$ at 30 900 cm⁻¹, ${}^1B_{2u}(\pi\pi^*)$ at 37 800 and ${}^1B_{1u}(\pi\pi^*)$ at 50 800 cm⁻¹ (12). These have oscillator strengths of 0.01, 0.10 and 0.15, respectively. Since the singlet–singlet $\pi \to \pi^*$ transitions in pyrazine are so much more intense

TABLE IV Phosphorescence Spectrum of Pyrazine- d_4 in Benzene at $4.2^{\circ}{
m K}$

λ(Á)	$\nu \ (\mathrm{cm}^{-1})$	$\Delta \nu$	Rel. int.	Analysis	Error
3823.9	26 144	0	VS	origin	
3910.2	25 567	577	VS	v_{6a}^{a}	
3912.1	25 554	590	VS	v_4	
3920.7	25 498	646	$\mathbf{v}\mathbf{w}$	v_5	
3944.6	25 344	800	VVW		
3950.3	25 307	837	VVW		
3957.2	25 263	881	VS	v_1	
3975.6	25 146	998	VS	v_{9a}	
4002.4	24 978	1166	\mathbf{M}	$2 \times v_{6a}$	-12^{1}
4005.4	24 959	1185	VVW		
4022.7	24 852	1292	\mathbf{W}	$2 \times v_5$	0
4048.9	24 691	1453	S	$2 \times v_{10a}$	
4049.9	24 685	1459	\mathbf{M}	$v_1 + v_{6a}$	-1
4051.8	24 673	1471	W	$v_1 + v_4$	0
4053.7	24 662	1482	VVW		
4056.9	24 642	1502	VVW		
4069.2	24 568	1576	\mathbf{M}	$v_{9a} + v_{6a}$	-1
4071.3	24 555	1589	W	$v_{9a} + v_4$	-1
4085.5	24 470	1674	$\mathbf{v}\mathbf{w}$		
4100.4	24 381	1763	W	$2 \times v_1$	-1
4120.0	24 265	1879	\mathbf{M}	$v_{9a} + v_1$	0
4139.6	24 150	1994	$\mathbf{v}\mathbf{w}$	$2 \times v_{9a}$	+2
4147.5	24 104	2040	VW	$v_1 + 2v_{6a}$ (a)	-4
4148.5	24 098	2046	VVW	$v_1 + v_{6a} + v_4$	
4198.7	23 810	2334	VVW	$2 \times v_1 + v_{6a}$	-2
4219.1	23 695	2449	VVW	v_2	+5

^a Fermi doublets.

b Does not take into account the shift in the fundamental due to Fermi resonance.

than the $n \to \pi^*$ transitions, one expects odd quanta of out-of-plane bending modes to be prominent in the fluorescence spectra of pyrazine. In the triplet manifold it is expected and found that the $n \to \pi^*$ transitions will be at least an order of magnitude more intense than the $\pi \to \pi^*$ transitions (13). This is because spin orbit coupling effectively mixes triplet $n\pi^*$ and $\pi\pi^*$ states with singlet $\pi\pi^*$ and $n\pi^*$ states, respectively. We therefore do not expect odd quanta of out-of-plane bending modes which borrow intensity from the triplet $\pi\pi^*$ states to be prominent in the phosphorescence spectra of pyrazine.

In agreement with this conclusion we find significant intensity for the 930 cm⁻¹ band (728 cm⁻¹ in pyrazine- d_4) only in fluorescence. As mentioned above this band can be due to either the $\nu_{10a}(b_{1g})$ or the $\nu_5(b_{2g})$ out-of-plane bending modes depending on whose vibrational assignments are chosen (8, 11). These modes will mix ${}^1B_{3u}(n\pi^*)$ with ${}^1B_{2u}(\pi\pi^*)$ and ${}^1B_{1u}(\pi\pi^*)$, respectively. From first-order per-

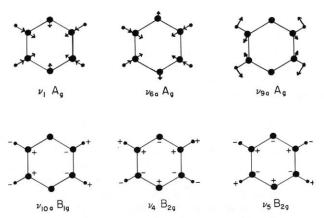


Fig. 2. The important symmetry modes in the pyrazine emission spectra.

turbation theory the vibronic coupling matrix element $V_{n\pi}*_{-\pi\pi}*_{(p)}$ between ${}^{1}B_{3u}(n\pi^{*})$ and a given ${}^{1}\pi\pi^{*}$ state via the appropriate mode p is given by the following formula:

$$V_{n\pi^*-\pi\pi^*(p)}^2 = \frac{E_{\pi\pi^*} \cdot f_{n\pi^*}^p}{E_{n\pi^*} \cdot f_{\pi\pi^*}} \left(E_{\pi\pi^*} - E_{n\pi^*} \right)^2,$$

where $E_{\pi\pi}$ and $E_{\pi\pi}$ are the transition energies, $f_{n\pi}^p$ is the oscillator strength in the $\pi^* \leftarrow n$ transition of the vibronically induced intensity and $f_{\pi\pi^*}$ is the total oscillator strength of the appropriate $\pi^* \leftarrow \pi$ transition. Substituting the experimental quantities for the two possible choices of the vibronic assignment we obtain a value of 700 cm⁻¹ for the ν_{10a} choice and 3300 cm⁻¹ for the ν_5 choice. The value of 700 cm⁻¹ for $V_{n\pi}*_{-\pi\pi}*$ is comparable with the value of vibronic coupling matrix elements found in other azines (13). A value of 3300 cm⁻¹, on the other hand, is about three times larger than that found in any azine. Since ν_5 mixes ${}^{1}B_{3u}$ $(n\pi^{*})$ with ${}^{1}B_{1u}$ $(\pi\pi^{*})$, such a large matrix element would not be expected. This is because ${}^1B_{3u}(n\pi^*)$ is largely made up of $n\pi^*_{bb}$ and ${}^1B_{1u}(\pi\pi^*)$ is largely $\pi_a \pi_{ab}^*$ (1). To a first approximation the vibronic coupling operator is a one-electron operator so the matrix element between these two states should be small since they do not have a common π^* orbital. Also the ratio of the frequencies of the ν_{10a} mode for pyrazine- d_4 and pyrazine- h_4 is 0.783 compared to a calculated product rule ratio of 0.784. We will present one further piece of evidence for the assignment of the 930 cm⁻¹ mode as $\nu_{10a}(b_{1g})$ after discussing the outof-plane modes which appear in phosphorescence.

Because of the small oscillator strengths of singlet-triplet $\pi^* \leftarrow \pi$ transitions relative to $\pi^* \leftarrow n$, odd quanta of the b_{1g} and b_{2g} modes should not and do not

appear in the pyrazine phosporescence spectra. The appearance of such bands involves intensity stealing and there is just too little intensity available to be stolen. On the other hand even quanta of such modes should and do appear in the pyrazine- h_4 phosphorescence spectrum (6). Their appearance results from a vibronic interaction between the $n\pi^*$ and $\pi\pi^*$ states which causes the potential energy surface to distort along the appropriate normal coordinates. In first order this interaction will reduce the force constant of the normal mode in the lower state and raise it in the higher state. The result is a redistribution of the Frank-Condon intensity into even quanta of these out-of-plane modes. The distortion will depend on the vibronic coupling matrix element and on the energy difference between the states. The vibronic coupling matrix element between $B_{3u}(n\pi^*)$ and $B_{1u}(\pi\pi^*)$ or $B_{2u}(\pi\pi^*)$ should be similar in the singlet and triplet manifolds. The energy gaps, on the other hand, will be much smaller in the triplet manifold. As a result the distortion of the potential energy surfaces of the b_{1g} and b_{2g} normal coordinates should be more severe in the ${}^{3}B_{3u}$ $(n\pi^{*})$ state than in the ${}^{1}B_{3u}(n\pi^{*})$ state. That such distortions can cause even quanta of the modes to appear in the emission spectra is demonstrated by the fact that we find the 1857 cm⁻¹ (2 \times 930) and 1510 cm⁻¹ (2 \times 755) bands in the pyrazine- h_4 phosphorescence spectrum. As we mentioned before the 1857-cm⁻¹ band is considerably more intense in the phosphorescence spectrum than in the fluorescence spectrum and the 1510-cm⁻¹ band only appears in the phosphorescence spectrum. In pyrazine-d₄ the 1452- $\rm cm^{-1}$ (2 \times 726 cm⁻¹) band is more intense in the phosphorescence spectrum and one at 1292 cm⁻¹ (2 × 646 cm⁻¹) only appears in phosphorescence. The lack of appearance of $2 \times 755 \text{ cm}^{-1}$ ($2 \times 646 \text{ cm}^{-1}$ in pyrazine- d_4) in fluorescence leads us to lean toward the ν_5 assignment for these modes and therefore for the ν_{10a} assignment of the 930-cm⁻¹ mode (728 cm⁻¹ in pyrazine-d₄). If the reverse assignment were correct one would expect 2×755 cm⁻¹ to be more intense than 2×930 in the fluorescence spectrum. The lack of appearance of 2×755 cm⁻¹ in fluorescence is therefore interpreted as due to a large energy gap between ${}^{1}B_{3u}(n\pi^{*})$ and ${}^{1}B_{1u}(\pi\pi^{*})$.

THE FORBIDDEN $^{1}n\pi^{*}$ STATE?

We have examined the singlet-singlet absorption spectrum of the pyrazine crystal at 4.2° K in an effort to find a forbidden $\pi^* \leftarrow n$ singlet-singlet transition below the allowed origin. The origin of the ${}^{1}B_{3u}(n\pi^*) \leftarrow {}^{1}A_{g}$ transition occurs at $30~057~{\rm cm^{-1}}$. With crystals as large as several millimeters long no sharp absorptions were found below this origin band that did not split in a magnetic field. If the ${}^{1}B_{2g}(n\pi^*) \leftarrow {}^{1}A_{g}$ origin were below the allowed singlet origin it would surely have enough magnetic dipole intensity to show up under these conditions. Accordingly we conclude that ${}^{1}A_{2g}(n\pi^*)$ is not below ${}^{1}B_{3u}(n\pi^*)$.

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