Movement, Ablation and Fragmentation of Meteoroid in the Atmosphere

ARTICLE *in* ARCHIVES OF COMPUTATIONAL METHODS IN ENGINEERING · MARCH 2010

Impact Factor: 3.68 · DOI: 10.1007/s11831-010-9039-0

1 AUTHOR:

Natalia Barri

Lomonosov Moscow State University

Arch Comput Methods Eng (2010) 17: 1–23
DOI 10.1007/s11831-010-9039-0

Movement, Ablation and Fragmentation of Meteoroid in the Atmosphere

Natalia Barri

Received: 4 December 2008 / Accepted: 4 December 2008 / Published online: 13 February 2010

Abstract The paper is devoted to research of movement of meteoric bodies in the terrestrial atmosphere. There is a review of the existing models describing movement, i.e. deceleration, ablation and fragmentation a meteoric body in atmosphere, in the paper beginning; namely a theory of a single body and a theory of consecutive fragmentation. Methods of determination of meteor body parameters by observation data are reviewed. Further the described models and methods have been applied to the analysis of trajectories of several bolides. It is obtained that the model of a single body with the account of ablation describes trajectories of considered bolides with the best accuracy. The trajectory analysis of Benesov's bolide is carried out, for which there are detailed data of observation. Basic parameters of the bolide are determined, including initial mass. Comparison of the obtained data with results of other authors is made. The second part of the work is devoted to research of interaction of meteoroid fragments in a supersonic flow. We proposed an approximation of numerical data for transverse coefficient by simple analytical function. Further, we obtained analytical solution of a problem on separation of two spherical fragments under the decreasing transverse force without resistance. The new model of layer-by-layer scattering of meteoroid fragments moving as a system of bodies is constructed on the basis of the analytical solutions derived in this work and the numerical data.

Scientific adviser: professor Vladimir Stulov.

N. Barri (✉)
Research Institute of Mechanics, Lomonosov Moscow State University, Michurinskiy Pr. 1, 119192 Moscow, Russia
e-mail: barry_natalia@mail.ru

1 Introduction

The problem of movement in a terrestrial atmosphere of the large space bodies, capable to pass through an atmosphere and to fall on a surface of a planet as meteorites, represents now the big interest.

It is known, that about 70000000 meteoroids will penetrate into an atmosphere of the Earth every day, more than 1000 kg from them (about 1%) meteoric substance reach its surface [3, 19]. For small bodies (1 mm–1 sm) a plenty of the data is received with the help of methods of meteoric astronomy (a radar-location, television and photographic supervision Large bodies (1–10 km and more) are found out by usual astronomical supervision (telescopes). Bodies of the "intermediate" sizes began to be observed rather recently [13].

Observation of such bodies and interpretation of the observation data allows finding out a probability of their falling, their properties, and characteristic features of flight through the atmosphere and consequences of these fall. Clarification of these questions will allow estimating asteroid danger more authentically.

For gathering information on meteoric substance influx to the Earth a number of bolide networks in the USA, Canada and Czechoslovakia were created. Observation stations of some European countries have joined to last, the European network operates till now. It was supposed that optical registration of bolides, of their luminosity curves and of trajectories will promote a finding of fallen meteorites, however, observation networks have found out only three meteorites (*Príbram*, *Lost City*, *Innisfree*). Nevertheless, the unique observation material has been collected and its analysis lasts till present day.

In addition to ground-based observation networks also exists the system of supervision basing on satellites. Geostationary satellites of USA mark bright flashes in the Earth's atmosphere caused by entry of rather large meteoroids. An advantage of the new approach is independence of weather conditions, coverage of wide territory, and also use of photoelectric gauges with high resolution in time, allowing registering features of the form of a radiation impulse and on their basis to reveal thin features of occurring processes. However only in single instances the angle of a trajectory slope and initial velocity are determined.

One of fundamental problems of meteoric physics is a definition of preatmospheric mass of bolides. Intensity of the meteoric phenomenon is defined by kinetic energy of a body at entrance to an external atmosphere of a planet. As is known, velocity of bodies at entrance in the atmosphere of the Earth is in the sufficiently narrow range $11.2 < V_e < 72.8$ km/s. So the spread in values of the high-speed contribution to kinetic energy does not exceed 50 times. At the same time value of meteoroid body mass can change in essentially wider range. From shares of gramme (meteteors) up to hundreds thousand tons (the *Tungus space body*), i.e. on 12–14 degrees. Besides, velocity of entrance is rather simply defined by observation of an initial part of an atmospheric trajectory. On the contrary, reliable ways of definition of meteoroid's initial mass, containing an estimation of accuracy of result, are absent now.

There are some models describing movement of meteoric bodies in the atmosphere. The model of a single body was investigated and used by many authors: [28–34, 39–41, 53–55]. In the paper by Stulov et al. [49] approximation of the analytical solution by simple functions for this model is offered.

The physical model of consecutive crushing of a meteoroid has been offered in work by Baldwin and Sheaffer [4] and used there for obtaining a series of numerical solution. According to this model at condition performance when pressure upon a front surface of a body reaches values of strength of a meteoroid's material, the body collapses on several identical fragments which quickly pass in a mode of aerodynamically independent movement. As the atmosphere density quickly increases, fragments continue to be split up consistently. Such a model is suitable for description of iron and stone meteorites and, probably, coal chondrites. In work by Tsvetkov and Skripnik [51] this model was used for recovery of breaking strength and trajectories parameters, based on experimental measurements of strength properties of real meteorites.

The analytical solution of the deceleration equations and meteoroid ablation during movement in the atmosphere with consecutive crushing, and also its simple enough approximation have been received in work by Stulov [46]. This solution has been used in works [7, 46] for definition of parameters of meteoric bodies according to observation data.

Physical properties of theory of instant destruction have been formulated for the first time in known work by Grigorian [18]; the model later has been used in numerical calculations by Hills and Goda [20]. In work of [21] all destruction process is represented as a series of stages on doubling of number of fragments, and trajectory calculation for nonablating body was reduced to step-by-step calculations, and the quantity of steps could be more than thirty. In work by Stulov and Titova [48] this physical model is presented in the form of system of the differential equations, which is solved by a method of variables division, including the case of ablation.

Theoretical research of crushing on the basis of luminescence intensity of a bolide has been performed in dissertations by Popova [43]. By means of a method of radiating radius it was established in this work, that for a number of large bolides of USA network (PN-bolides) the luminescence considerably exceeds the value adequate to values of initial mass, that was received earlier by purely dynamic methods on the basis of the single body theory, i.e. in absence of fragmentation. Popova [43] has assumed that the trajectory in these cases is formed not by a single body, but by a swarm of fragments. And deceleration of fragments corresponds to the largest fragment, while the luminescence is created by all fragments, whose full weight, naturally, can exceed weight of one fragment considerably.

In the present work an attempt to define a fragmentation presence based on the form of an observation trajectory becomes, not connecting it with luminescence value of bolide, as theoretical relation of luminescence intensity with full weight of a fragments swarm is not quite exactly established as yet. The trajectory is calculated in variables velocity—height. For revealing of a role of fragmentation, the method of the least squares is used, and as trial functions—analytical formulas for a trajectory, received by means of consecutive crushing model of a meteoric body in atmosphere [46].

Process of destruction of a meteoroid in atmosphere, and also influence of this process on a crater fields structure is investigated in work by Passey and Melosh [42]. Authors investigated a several known crater fields. By means of numerical calculations they have tried to restore pictures of the meteoric fall for each crater field.

As physical model for trajectory calculation in work [42] the usual equations of meteoric physics for a single body are used. The studied craters formed as a result of fragments fall of one meteoroid, are differ in the sizes. Therefore Passey and Melosh [42] have chosen the following model of fragmentation: the body breakup on several fragments of the different sizes. If to designate a weight of the maximum for example M, weights of other fragments will be equal accordingly M/2, M/4, M/8 and so on. Then, individual trajectories of fragments by means of numerical integration of the equations of meteoric physics for model of a single body are defining by Runge-Kutta method. The calculation will stop, when last fragment reaches earth surfaces, the dispersion of meteorites and diameters of the craters formed by them further are defining.

Studied crater fields have various width. An explanation of cross-section length of crater fields is presence of transverse horizontal component of velocity. In work [42] the possible mechanisms providing transverse component of velocity of meteoroid fragment are considered. It is centripetal scattering of fragments from a rotating meteoroid, action of transverse elevating force, dynamic transverse scattering of splinters owing to explosionlike meteoroid destruction, and crossing of two or several shock waves directly after destruction.

Passey and Melosh [42] showed that interaction of shock waves is a principal cause of transverse separation of fragments. Action of elevating force can be essential only in cases when the weight of fragments is less than 100 kg.

In work [42] the method for definition of distance of transverse scattering has been proposed. However, in this method has not been considered that interaction of shock waves weakens with distance increase between fragments. And the same method was used in works [38, 43]. Calculations were made in the assumption that repulsive force is constant. In the present work this defect has been filled, and the analytical decision for a problem about a separation of two spherical fragments in a supersonic air flow under the influence of decreasing transverse force has been received.

Influence of an entry angle, initial mass and velocity for longitudinal length of crater fields and its structure also had been analyzed by Passey and Melosh [42]. For example, by calculations of authors for a body with mass 10 kg and an initial angle of a trajectory with horizon 45 will be formed only one crater even if there will be a crushing. At such initial data scattering of possible fragments will be insufficient. Also it is noticed, that for velocities 20 km/s the maximum scattering of two bodies in weights 10 and 10 kg caused by gravitation and air drag, there will be less than 20 km, and for larger bodies will be less. It gives the top limit of length possible crater fields which will be formed by geometrically similar bodies with velocities more than 20 km/s. And for initial velocities between 11.2 and 20 km/s the longitudinal spread of craters can be very big (more than 100 km) at entry angle less then 5.

The *Sikhote-Alin* meteorite rain fall is studied in detail in work by Nemchinov and Popova [37]. Authors used the following information on this unique event: number and the size of the formed craters, their arrangement in the field of dispersion, durability of meteorite samples, an accounts and sketches of eyewitnesses, estimations of heights of destruction, a slope of a trajectory and intensity of a luminescence. Numerical modeling of movement, ablation and the luminosity, considering possibility of fragmentation by several stages is made.

As a result of *Sikhote-Alin* meteorite fall [14, 16] the extensive crater field was formed. There were discovered about 130 craters and funnels, among them 23 craters with diameter exceeding 9 m [23–26]. The largest fragments of a meteorite (1.7 tons and 0.7 tons) had been found in not in greatest craters. In several big craters (with $D > 9$ m) it has not been found at all large fragments but only the big number of small splinters in craters and round them.

The first expedition has collected about 27 tons of meteoric substance [23, 50], basically in the form of small fragments. The full weight which has fallen to the Earth, has been estimated in 70–100 tons [25]. The estimation in work by Nemchinov and Popova [37] leads to increase in the fallen weight to 120–140 tons. Numerical modeling in this work had shown that the big fragments lose approximately 30–50% of weight as a result of an ablation. Therefore authors estimate weight of the body which broke into large fragments approximately in 200–300 tons.

In work by Nemchinov and Popova [37], also as well as in work Passey and Melosh [42] meteoroid movement is considered in the context of single body model until pressure upon a front surface will not reach some critical value leading to destruction of a body. Then movement of each separate fragment is describing also by this model.

At movement in atmosphere meteoric bodies are under real loads less than its strength [51]. But, nevertheless, it cause their destruction as meteoric bodies are non-uniform. Small samples collapse basically along borders between crystals which are more poorly connected among themselves. Structurally non-uniform bodies can be described by means of the statistical theory of durability [8, 52]. It follows from this that large defects are found less often than insignificant defects, and the probability of their occurrence increases with increase in volume of a body. The fragments arising after destruction are more strong than destructed body. Therefore for an estimation of durability of a meteorite it is possible to use of the power law of increase in durability with reduction of the size, knowing durability of the sample. Such method was used in work by Nemchinov and Popova [37].

The model of a catastrophic fragmentation Fujiwara et al. [17] was used in work by Nemchinov and Popova [37]. There was supposed that the meteoric body collapses on fragments of the different sizes and with increase of mass of a splinter their number decreases. Then one or several maximal fragments got out, the others were broken into some groups and the average weight of group was considered.

Further, dispersion of the formed fragments in a direction, perpendicular to movement was estimated in work by Nemchinov and Popova [37]. It was used the same method, as well as in work by Passey and Melosh [42] without taking repulsive force decrease into consideration. Besides, in work by Nemchinov and Popova [37] it is supposed that if number of fragments much more than two, separation of splinters change because of their interaction and influence of a small fragments cloud and steams between them. Splinters can move, surrounded by the common shock wave. It can increase distance of transverse separation of splinters in 2–3 times, according to authors estimates to 160–240 m, that of the same order of magnitude that the size of the basic crater of the *Sikhote-Alin* meteorite field (0.3–0.5 km). The size of all area covered with fragments of a meteorite, including and secondary ellipse of dispersion, usually consider to be equal 0.9–1 km.

Numerical modeling of *Sikhote-Alin* event has been performed in work by Nemchinov and Popova [37]. Be based on the model described above, authors have selected such values of the parameters determining a fall occurrence which give the better similarity of real crater field with a theoretical ones.

Fragments of the small size decelerate greatly and reach a ground with small velocity due to action of weight of a body. Therefore near to a surface of a planet for these fragments it is necessary to take into account weight that had not been made in work by Nemchinov and Popova [37].

Advantages of analytical solution over numerical are well-known. These advantages are especially accented in meteoric physics as opportunities of getting of numerical solution of high accuracy are greatly limited because of inexact statement of an initial problem.

The purposes of the present work are: an estimation of influence of a crushing process on a bolide's trajectory, based on the analysis of an observed trajectory—the solution of a problem on two spheres transverse scattering under the action of a decreasing repulsive force—construction of new model of fragments scattering of destroyed meteoroid.

Structure and volume of work. The paper consists of the introduction, three chapters, the conclusion, appendices, the list of the literature and contains 75 pages. The list of the literature contains 56 bibliographic references.

In Sect. 2 the known model of a single body and the solution of the equations of deceleration and ablation, containing elliptic integrals, and also rather simple approximation of this decision are described. In the first section the solution for a single body model is presented in new coordinates—time, height and a projection of a trajectory to a horizontal.

In the second section two schemes of the least squares method are described for a solution of the return problem—determination of meteoric bodies parameters according to observation data. In Sect. 3 was performed a research on influence degree of fragmentation process on the form of a bolide's deceleration curve.

In the first section a principle of physical model of consecutive fragmentation are stated, and also the known solution of the equations of deceleration and ablation, corresponding to the given model are described. The consideration of fragmentation in it is reduced to the increasing midsection area of a fragments swarm.

In the second section trajectories of four bolides of the USA network are studied. In the second section trajectories of four bolides of the USA network are studied. For the analysis have been chosen bolides for which the greatest excess of entry weight determined in the photometric way, over the weight determined in only dynamic way according to model of a single body was marked. It was supposed, that this fact is connected to presence of fragmentation. For these bolides by the methods described in the first chapter, the return problem of meteor physics was solved—key parameters at which the model in the best way approximates the observation data are determined, results of calculations are brought in the table. By way of trial functions were used a solutions for a single body model and consecutive fragmentation model with account and without ablation account. Results of trajectories approximation of four bolides in coordinates "velocity-height" are present in the form of graph and placed in section of appendices. It is shown that the best approximation of an observation trajectory is given by the formula corresponding to a single body model with the account of ablation.

In the third section was performed research on the analysis of a trajectory *Benesov's* bolide. There are observation data in some detail for it; they are stated in the given section. Besides, fragmentation of this bolide was directly observed. For approximation of an observation trajectory of the *Benesov's* bolide was used a modified solution corresponding to model of consecutive fragmentation taking into account that bolide has collapsed after significant deceleration. For it, as well as for four investigated bolides appeared that the photometric weight essentially exceeds values of entry weight of the, determined by only dynamic method. Besides in the case of *Benesov's* bolide the model of fragmentation gives an appreciable deviation from the observation data also, it is presented on the graph in section of appendices.

In Sect. 4 the interaction of fragments of the destroyed meteoric body in a supersonic stream is investigated. In the first part is described the method for definition of distance of transverse scattering of spherical fragments of destroyed meteoroid which is used in the literature. This method is based on the following assumption. Repulsive force acting on fragments in a transverse direction is considered a constant and it disappears when the fragments are separated by a distance comparable with their size. However, separating force is decreasing when distance between fragments is increasing. This fact is taken into account in the model of scattering of fragments offered in this chapter. The analytical solution of the dynamic equation for a fragment without taking into account drag force is received. The formula determining transverse velocity of splinters in the last moment of their interaction is received. A comparison of the dynamic equation solution with constant and decreasing repulsive force is given in the section of appendices on the graph. It is shown, that induced transverse velocity of a spherical fragment is much less than the previously published by other authors, values.

Besides, the dynamic equation of the fragment, taking into account air flow resistance, had been solved numerically. It is shown that solution of a problem on two spheres transverse scattering taking into account air flow resistance and without it practically coincides. Character of decrease of transverse velocity under the action of drag force after the termination of fragments interaction, was estimated. It was shown, that transverse velocity of splinters scattering, decreases very slowly. Results of calculations are brought in the table, and also submitted as the diagram in section of appendices.

In the second part the new model of meteoroid destruction by layers is offered. Transverse scattering of fragments is described in it more in details in comparison with existing models of fragmentation. The destruction model by layers relying on results of Sect. 3 about interaction of two spheres in a supersonic flow; and also on results of numerical experiments on a group of spheres in a supersonic flow. They tell that the value of the repulsion force acting on the bodies in the "outer layer" is sufficiently larger than the value of the force acting on the "inner" bodies.

In proposed model the crushed body is interpreted as a swarm (a compact collection) of spherical fragments. It is proposed to consider two forms of the initial body, which are the cylindrical and the spherical ones. According to the proposed model the scattering of the meteoroid has a few stages. At each stage the interaction of a fragment of the outer layer and the inner part, which is considered to be single, is analyzed. This interaction lasts until the distance between the outer layer and the inner part reaches the radius of the latter. Then the next layer becomes outer and starts moving away from the main part.

The geometry of layers for the offered model of fragmentation is described. The dynamics equation from Sect. 3 is used for the movement description of fragments situated in external layer. Calculations according to the offered model of fragmentation for a body with some set-up parameters are performed out. Time of fragments interaction depending on quantity of formed fragments, height of crushing, the size and the form of an initial body is defined. Calculation data are tabulate. It was obtained, that the time of scattering amounts to a non-sufficient portion of the total time of a meteoroid's movement in the atmosphere and the total time of scattering (by layers) does not actually depend on the number of fragments.

Scientific novelty of work's results be as follows:

- Solution for single body model represented in new coordinates—time, altitude and trajectory projection on the horizontal.
- Based on the analysis of five bolide trajectories (including bolide *Benesov*) it was shown that the best approximation of observational trajectory is given by the formula that corresponds to single body model taking into account of the ablation. That is a form of trajectory luminous part does not disclose the obvious influence of meteor crushing.
- The problem on a separation of two spheres under the action of the transverse force derived as a result of numerical experiment is solved.
- It is shown, that an induced transverse velocity of a spherical fragment is significantly below than values published earlier by other authors.
- Influence of drag force acting in a transverse direction is insignificant and one can to not consider it for the description of transverse scattering of fragments.
- The new model of layer-by-layer scattering of meteoroid fragments moving as a system of bodies is constructed.
- It is derived that time of destruction is much less than movement time of a meteoroid in the atmosphere and practically does not depend on number of fragments according to the offered model.

The practical importance. The derived results can be used for the solving of a problem on an estimation of meteoric and crater fields and also for perfection of the theory about a breakup of meteoroid under the action of aerodynamic stress.

Results of the present paper are published in works [5–7], reported and discussed at workshop "Modern problems of aerohydrodynamics", Tuapse, 2001, at conference—competition of young scientists, Institute of Mechanics Lomonosov Moscow State University, 2003, 2004, 2005, at scientific seminars under the guidance of prof. *V.P. Stulov*.

2 Inverse Problem

The theory of the meteoric phenomena have been developing during more than 60 years. One of its basic problems is a definition of parameters of the meteoric bodies by the observation data. It allows describing and predicting the characteristics of meteors, to define an inflow of meteoric substance to the Earth, and velocities and sizes distribution. There are several models describing movement of meteoric bodies in the atmosphere and which are used for the solution of this problem. For the solving of an inverse problem of meteoric physics and also for an estimation of a breakup influence on a boiled trajectory two models will be used in the present work: model of a single body and model of a sequential crushing [4].

2.1 A Single Body Theory

The fundamentals of the physical theory of meteors have been formulated in classical works: [28–30, 36, 39–41, 53–55]. This theory was used for definition of characteristics of meteoroids by the observation data and for a forecasting of the parameters of meteoric phenomena.

The essence of classical idea about the meteoric phenomena can be expressed by the system of the equations including the description of a meteoroid deceleration, its ablation and change of altitude.

Usually in the meteoric physics at trajectory calculation neglect body weight as at the big velocities of entrance to the atmosphere full drag of a body is much more than body weight. Considering a trajectory as rectilinear, the known equations of meteoric physics can be written down in a following form [29]:

$$H^* \frac{dM}{dt} = -\frac{1}{2} C_d \rho_a V^2 S \tag{2.1}$$

$$M \frac{dV}{dt} = -\frac{1}{2} C_h \rho_a V^3 S \tag{2.2}$$

$$\frac{dH}{dt} = -V \sin\gamma \tag{2.3}$$

Here a trajectory angel γ, drag coefficient C_d, heat transfer coefficient C_h, energy necessary for ablation of a unit mass H^*—effective enthalpy—are constants. As the unit velocity V, body mass M and effective cross-section area S are accepted their values at entrance to the atmosphere. They are marked by the index "e". As a unite of altitude H we take a pressure scale height H_0, as a unite of an air density ρ_a we take an air density at the sea ρ_0, that is $v = V/V_e$, $m = M/M_e$, $s = S/S_e$, $y = H/H_0$, $\rho = \rho_a/\rho_0$.

Many authors were solving (2.1)–(2.3) [11, 28, 29]. Eliminating time t, equations can be written down in a following dimensionless form:

$$m \frac{dv}{dy} = \frac{1}{2} \frac{C_d \rho_0 H_0 S_e}{M_e \sin\gamma} \rho v s \tag{2.4}$$

$$\frac{dm}{dy} = \frac{1}{2} \frac{C_h \rho_0 H_0 S_e}{M_e \sin\gamma} \frac{V_e^2}{H^*} \rho v^2 s \tag{2.5}$$

For isothermal atmosphere $\rho = e^{-y}$ taking into account assumptions $s = m^{\mu}$, $\mu = const$ (see [29], the value $\mu = 0$ corresponds to the constant effective cross-section area), and also $\frac{C_h}{C_d H^*}$, $\frac{C_d}{\sin\gamma} = const$ the solution for a single body theory (2.4)–(2.5) with initial conditions $m = 1$, $v = 1$, $y = \infty$ has form:

$$m = \exp\left(\frac{-\beta}{1-\mu}(1 - v^2)\right) \tag{2.6}$$

were $\beta = (1-\mu)\frac{C_h V_e^2}{2 C_d H^*}$—is the ablation coefficient, constant along a trajectory.

$$\begin{aligned} &y = \ln\alpha + \beta - \ln\frac{\Delta}{2} \\ &\Delta = \bar{E}i(\beta) - \bar{E}i(\beta v^2), \qquad \bar{E}i(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt \end{aligned} \tag{2.7}$$

The last integral is accepted as a Cauchy principal value:

$$\int_{-\infty}^{x} = \lim_{\epsilon \to 0} \left(\int_{-\infty}^{-\epsilon} + \int_{\epsilon}^{x} \right)$$

Solution (2.7) contains a parameter constant along a trajectory

$$\alpha = \frac{1}{2} \frac{C_d \rho_0 H_0 S_e}{M_e \sin\gamma}$$

—is ballistic coefficient. Presence of integral exponent $\bar{E}i(x)$ in (2.7) complicates estimations and fast calculations. The former post-graduate student of Moscow University A.L. Kulakov had shown that for $0 \le \beta \le 2$ in all range v $(0 \le v \le 1)$ the formula (2.7) can be replaced with good accuracy by following expression:

$$y = \ln\alpha - \ln(-\ln v) + 0.83(1 - v)\beta \tag{2.8}$$

This problem is representable in new coordinates—time t, altitude y and a projection of a trajectory to a horizontal x. Equation (2.3) can be written in dimensionless form $\frac{dy}{dt} = -v \sin\gamma$, and differentiating of (2.8) by time gives:

$$\frac{dy}{dt} = \left(-\frac{1}{v \ln v} - 0.83\beta \right) v'$$

Then we derive the dependence of velocity of a body on time along a trajectory:

$$\frac{dv}{dt} = \frac{v^2 (\ln v) \sin\gamma}{0.83 \beta v \ln v + 1} \tag{2.9}$$

This differential equation solves numerically with entry conditions $v = 0.99$ at $t = 0$ (after some deceleration). It gives value of velocity at any instant of time. Using (2.9) and solution (2.6), (2.8) we have dependences $m(t)$ and $y(t)$. The projection of a trajectory to a horizontal is defined by a following relation: $x(t) = (y(0) - y(t)) \operatorname{ctg}\gamma$, were $y(0)$ derived from (2.8) when $v = 0.99$. The length along a trajectory is calculated under the formula: $l(t) = \int v(t) dt$.

Movement of a parent body and also of each fragment formed owing to crushing of bolide it is possible to describe by formulas (2.6), (2.8), (2.9) corresponding to

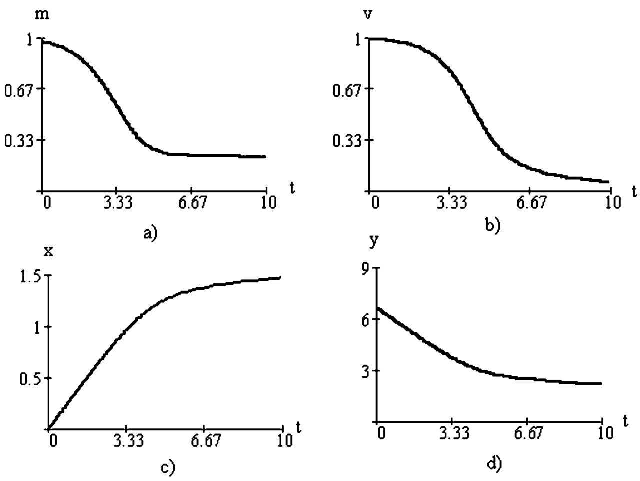

Fig. 1 Solution for the single body model, a dependence on time: (**a**) the mass dependence on time, formulas (2.6), (2.9), (**b**) the velocity dependence on time, formulas (2.9), (**c**) the projection of a trajectory to a horizontal, (**d**) the altitude dependence on time, formulas (2.8), (2.9)

the single body model, only with different entry conditions.

Dependences $m(t)$, $v(t)$, $x(t)$, $y(t)$ are presented in Fig. 1 at following values of parameters $\alpha = 7.8$, $\beta = 1.5$, $\sin\gamma = 0.95$.

2.2 Determination of Meteor Body Parameters by Means of Least-Squares Method

Two realization schemes of the least square method for determination of values of ballistic coefficient and of an ablation parameter from a condition of the best approximation of an observation trajectory are offered in the work by Kulakov and Stulov [27].

In the first case the sum is composed

$$Q_1(\alpha,\beta) = \sum_{i=1}^{n} (y_i(v_{di},\alpha,\beta) - y_{di})^2 \tag{2.10}$$

and values α and β at which Q_1 come up to a minimum are defined. Here $y_{di} = H_i \setminus H_0$, $v_{di} = V_i \setminus V_e$, H_i and V_i corresponds to observation data on altitude and velocity of flight, V_e—entry velocity, taken from the table of observation [35]. Differentiation of (2.10) with respect to α and β and set it equal to zero one obtain explicit expressions for required parameters.

For a single body model with the account of ablation described by (2.8) the parameters α, β are given by:

$$\alpha = \exp\left[\sum_{i=1}^{n} (-0.83\beta(1 - v_{di}) + \ln(-\ln v_{di}) + y_{di})\right] \tag{2.11}$$

$$\beta = \frac{\sum_{i=1}^{n}[\ln(-\ln v_{di}) + y_{di} - \frac{1}{n}\sum_{j=1}^{n}(\ln(-\ln v_{dj}) + y_{dj})](1 - v_{di})}{-0.83\sum_{i=1}^{n}[\frac{1}{n}\sum_{j=1}^{n}(1 - v_{dj}) - (1 - v_{di})](1 - v_{di})} \tag{2.12}$$

A numerical analysis of formulas (2.11)–(2.12) shows that determination accuracy of α and β essentially depends on quantity of points of the observation [49] considered in the formula (2.12). More exactly, values α and β essentially change due to inclusion in (2.10) of first points of a trajectory where there is still no detectable deceleration of meteoroid, that is values v_{di} closed to each other (and to unit) correspond to different values. Therefore the correct result will be derived only including in (2.12) of points of observation corresponding to considerable deceleration of a body [49].

The second scheme of the method is accepted as the basic scheme [49] where the sum minimum is defined

$$Q_2(\alpha,\beta) = \sum_{i=1}^{n} (v_i(y_{di},\alpha,\beta) - v_{di})^2 \tag{2.13}$$

where the values $v_i(y_{di},\alpha,\beta)$ for example for a case of a single body are calculated by means of function inverse to the function (2.8).

Using of rules for determination of the minimum leads to a system with equations which after simplifications become:

$$y_{di} = \ln\alpha - \ln(-\ln v_i) + 0.83(1 - v_i)\beta \tag{2.14}$$

$$\sum_{i=1}^{n} \frac{(v_{di} - vi)v_i^2 \ln v_i}{0.83\beta v_i \ln v_i + 1} = 0 \tag{2.15}$$

$$\sum_{i=1}^{n} \frac{(v_{di} - vi)v_i \ln v_i}{0.83\beta v_i \ln v_i + 1} = 0 \tag{2.16}$$

One should find $n + 2$ unknowns: α, β, v_i $(i = 1,\dots,n)$. The system is being solved as follows [49]. As an initial approach $\alpha^{(0)}$, $\beta^{(0)}$ one uses the values (2.11)–(2.12) defined by the first scheme. In (2.10) is being used only those observation points for which $v < 0.9$. Further the values $v_i^{(0)}$ are being determined from (2.14) by the Newton's method. Then (2.15) and (2.16) are being solved also by Newton's method using the derived initial approach and inverse function $v_i = v_i(y_{di},\alpha,\beta)$ derived numerically on each iteration from (2.14).

An imperfection of the second scheme is absence of analytical solution for α and β. However here numerical result depends only on quantity of iterations and is not limited by number (and by quality) of chosen observation points of a trajectory.

The solution of the inverse problem is a determination of parameters of the meteoric event at which the model describes the observation data with the best accuracy. The methods described above, are used in the present work for approximation of an observation trajectory of bolides by the models under investigation—model of a single body and model of consecutive crushing.

3 An Estimation of the Influence of a Fragmentation Process on the Bolide Trajectory

Bodies of any feasible sizes and velocities can reach of the planet surface in the case of absence of the atmosphere. However the atmosphere of the Earth "swallow up" rather small meteoric bodies and essentially influences on the large ones. A crushing is one of the basic processes defining the meteor event [15]. The meteorites falling to the Earth are seldom represent a single whole; usually they consist of group of the splinters forming a field of dispersion.

A huge velocity of a meteor (order of tens of km/s) and an exponential growing of the atmosphere density lead to pressure and temperature quickly reaching a values of ten thousand atmospheres and ten thousand degrees in the compressed layer of air behind a shock wave [18]. Under these conditions a body of the small sizes sharply decelerate and completely evaporate already in an upper atmosphere. Whereas larger ones have no time to lose their velocity and to melt completely and to evaporate, they arrive altitudes on which pressure of gas in a shock layer increases to values of durability of a meteor material. It leads to the crushing of a body; formed fragments are being pushed away and back and are being subjected to secondary crushing under the influence of the same destructive pressure of atmosphere. The altitude on which usually the crushing occurs is estimated in limits from 4 to 40 km.

Earlier in the works by Stulov [47] and Barri et al. [7] observation trajectories of bolides of Prairie network of the USA have been considered with the purpose of detection of the influence of a fragmentation process on the velocity dependence on altitude along a luminous section of a trajectory. The basic conclusion of the previous researches consisted in an absence of noticeable influence of crushing on the trajectory form.

In this chapter the trajectories of four bolides of Prairie network and also of bolide *Benesov* with detailed observation data and which crushing was obviously observed are analysed.

3.1 Theory of a Consecutive Fragmentation (Review)

According to the physical model of consecutive breakup offered in the work [4] it is supposed that body destruction on N identical fragments occur under the condition

$$\rho_a V^2 = \sigma_{BR}(N). \tag{3.1}$$

postulating an explicit dependence σ_{BR} on N the authors have an opportunity to determine N at each point of a body trajectory. It has been considering that

$$\sigma_{BR}(N) = N\sigma_t \quad \text{for } N\sigma_t < \sigma_c \tag{3.2}$$

$$\sigma_{BR}(N) = \sigma_c \quad \text{for } \sigma_c \le N\sigma_t \tag{3.3}$$

Here σ_t is the tensile strength and σ_c is the compressive strength of the material. In turn these sizes can depend on N if to consider dependence of the body strength on its mass. Authors give a literary empirical data, according to which:

$$\sigma_t = \sigma_{t1}(m_1/M)^{A_t}, \qquad \sigma_c = \sigma_{c2}(m_2/M)^{A_c} \tag{3.4}$$

Here M is the body mass, σ_{t1}, m_1, A_t, σ_{c2}, m_2, A_c are empirical constants.

Dependence (3.4) is very weak; for example for a granite it is accepted that $A_t = 1/6$, $A_c = 1/12$. For realisation of a communication of the physical model (3.1)–(3.4) with the general equations of meteoric physics it is supposed that a body destructs on N identical fragments geometrically similar to an initial body. The form factor of a fragment is introduced:

$$S_F = S_1/W^{2/3} \tag{3.5}$$

where S_1 is cross-section area of fragment, W is the volume of a fragment equal to $M/N\rho_b$, ρ_b is the density of a body material. Then the cross-sections area of fragments swarm under condition of the aerodynamically independent movement is expressed by the formula:

$$S = NS_1 = S_F N^{1/3} W^{2/3}, \quad W = M/\rho_b$$

Expressing the N in terms of parameters of the atmosphere ρ_a and of a body M, V along a trajectory out of (3.1)–(3.4) and substituting this expression in (3.5) one can derive the formula for the variable cross-sections area that will have been used below in the equations of meteoric physics. For an initial stage of destruction when the first line of (3.2) is being executed an expression for N is given by:

$$N = \frac{(\rho_a V^2)^{1/1+A}}{\sigma_{t1}^{1/1+A}(m_1/M)^{A/1+A}}$$

For a problem statement about calculation of trajectories the usual equations of meteoric physics for a single body (2.1)–(2.3) are used. An accounting of breakup according to the model accepted here leads to a variable cross-sections area along a trajectory.

It is naturally to accept that according to (3.1) a destruction begins at $N = 1$. Then the initial altitude of

Table 1 The observation data (in dimensionless form) for the four studied bolides are taken from the work by McCrosky et al. [35]: velocity and altitude at the observation point

«40151A»

Velocity	0.993	0.993	0.985	0.970	0.933	0.858	0.731	0.582	0.388
Altitude	8.6	7.8	7.0	6.2	5.5	4.8	4.2	3.7	3.3

«38737*»

Velocity	0.988	0.983	0.965	0.936	0.884	0.779	0.570
Altitude	7.6	7.0	6.4	5.8	5.3	4.8	4.4

«40590» (Lost City)

Velocity	0.993	0.986	0.972	0.908	0.725	0.430	0.239
Altitude	8.4	7.2	6.0	4.8	3.8	3.2	2.8

«40405»

Velocity	0.967	0.947	0.907	0.873	0.673
Altitude	7.8	7.0	6.2	5.5	4.9

breakup y_0 will be defined by the formula $(A_t = 0) \to y_0 = \ln(\rho_0 V_e^2/\sigma_t)$.

For sufficiently large bodies a destruction precedes braking and ablation [46]; therefore as the entry condition here is accepted $v = 1$ at $y = y_0$. The analytical solution of the equations of deceleration and ablation of a meteoric body during movement in the atmosphere (2.4)–(2.5) with consecutive breakup due to the aerodynamic loading (i.e. with the increasing cross-section area) [46] is:

$$m = \exp\left[-\beta(1-v^2)\right]$$
$$y = \frac{3}{4}\ln\frac{\alpha}{2} + \frac{y_0}{4} - \frac{3}{4}\ln\left(\frac{\alpha}{2}e^{-y_0} + \frac{\exp(-\beta/3)}{(3/\beta)^{1/3}}\frac{\tilde{\Delta}}{3}\right) \tag{3.6}$$

where $\tilde{\Delta} = \tilde{\Delta}(\beta, v) = \int_{\beta v^2/3}^{\beta/3} \frac{e^t\,dt}{t^{4/3}}$.

For practical calculations and fast estimations in the work [46] it is offered to approximate integral $\tilde{\Delta}$ in the form of simple function of β and v by analogy with the solution in case of a single body. The solution (3.6) can be rewritten in a form

$$y = \frac{3}{4}\ln\frac{\alpha}{2} + \frac{y_0}{4} + f(\beta, v) \tag{3.7}$$

$$f(\beta, v) = -\frac{3}{4}\ln\left(\frac{e^{-\beta/3}}{(3/\beta)^{1/3}}\frac{\tilde{\Delta}}{3}\right) \tag{3.8}$$

The function $f(\beta, v)$ is being approximated in the form of linear function of the parameter β [46]. In the range $0 \le \beta \le 8$ it can be written

$$f(\beta, v) = -\frac{3}{4}\ln(v^{-2/3} - 1) + 0.223(1-v)\beta \tag{3.9}$$

The solution for model of consecutive breakup in a form of (3.7), (3.9) is used below for the solution of an inverse problem of meteoric physics: definition of parameters of meteoric bodies by the observation data and for an estimation of influence of crushing on a bolide trajectory.

3.2 Trajectory Approximations for Observable Bolides in Consideration of Breakup

In the present work the trajectories of those bolides are chosen from tables [35] for which the greatest excess of the mass obtained by the method of the radiating radius over the mass obtained in several papers by purely dynamic methods was marked (see Table 1.2 in [43]. Thus, the trajectories of the following bolides are chosen [35]: «40151A» (3.3), «40590» (Lost City, 2.5), «40405» (5.0), «38737*» ($\kappa = 2.5$). The observation data for each of them are adduced in Table 1. After the bolide number in brackets the ratio κ of the entry mass derived in [43] to the values derived on the basis of the single body theory in work by Kulakov and Stulov [27] is specified. As in the works of reference, as a rule, the ranges of values were given (in work by Kulakov and Stulov [27] the values are obtained by two methods) for calculation of κ an arithmetic mean values were used. It should be marked that all values of κ exceed one, essentially in some cases.

Assuming a presence of a crushing of a meteoric body in a luminous section of a trajectory we take the analytical solution derived for the model of consecutive breakup [46] (see Sect. 3.1) as trial function. In the case when $\beta = 0$ it is given by

$$y = \frac{3}{4}\ln\frac{\alpha_d}{2} - \frac{3}{4}\ln(v^{-2/3} - 1) \tag{3.10}$$

where $\alpha_d = \frac{1}{2}\frac{C_d \rho_0 H_0 S_e}{M_e \sin\gamma} k^{1/3} = \alpha k^{1/3}$, $k^{1/3} = (V_e^2 \rho_0/\sigma_t)^{1/3}$.

For all bolides it is accepted that $\sigma_t = 10^7$ dyn cm^{-2}; it is approximately corresponds to durability of coaly hondrite. Parameter α_d is defined from a condition of the least square deviation of the function (3.10) from the observation data by means of the second scheme of the method of the least squares (see Sect. 2.2). In other words, the function minimum is determine $Q_2(\alpha_d) = \sum_{i=1}^{n}(v_i(y_{di}, \alpha_d) - v_{di})^2$, were $v_i(y_{di}, \alpha_d)$ are calculated by means of function inverse to (3.10), that is $v_i = (\alpha_d \exp(-4y_i/3)/2 + 1)^{-3/2}$.

$$Q_2(\alpha_d) = \sum_{i=1}^{n}(v_i(y_{di}, \alpha_d) - v_{di})^2$$

where $v_i(y_{di}, \alpha_d)$ are calculated by means of function inverse to (3.10), that is $v_i = (\alpha_d \exp(-4y_i/3)/2 + 1)^{-3/2}$.

We have differentiated $Q_2(\alpha_d)$ by α_d and have equated the derivative to zero. For the solution of the transcendental equation $\frac{dQ_2(\alpha_d)}{d\alpha_d} = 0$ we have applied Newton's method:

$$\alpha_d^{(n+1)} = \alpha_d^{(n)} - \frac{\psi(\alpha_d^{(n)})}{\psi'(\alpha_d^{(n)})} \quad \text{where } \psi(\alpha_d) \equiv \frac{dQ_2(\alpha_d)}{d\alpha_d}$$

$$\begin{aligned}\psi(\alpha_d) &= \sum_{i=1}^{n}\left\{\left(\frac{\alpha_d}{2}e^{\frac{-4}{3}y_{di}} + 1\right)^{-3/2} - v_{di}\right\} \\ &\quad \times \left(\frac{\alpha_d}{2}e^{\frac{-4}{3}y_{di}} + 1\right)^{-5/2} e^{\frac{-4}{3}y_{di}} \\ \psi'(\alpha_d) &= -\frac{1}{4}\sum_{i=1}^{n}\left\{3\left(\frac{\alpha_d}{2}e^{\frac{-4}{3}y_{di}} + 1\right)^{-5} \right. \\ &\quad + 5\left(\left(\frac{\alpha_d}{2}e^{\frac{-4}{3}y_{di}} + 1\right)^{-5/2} - v_{di}\right) \\ &\quad \left. \times \left(\frac{\alpha_d}{2}e^{\frac{-4}{3}y_{di}} + 1\right)^{-7/2}\right\} e^{\frac{-8}{3}y_{di}}\end{aligned}$$

As an initial approach $\alpha_d^{(0)}$ the value of parameter α_d defined according to the first scheme of the method of a least squares was used:

$$Q_1(\alpha_d) = \sum_{i=1}^{n}\left(\frac{3}{4}\ln\frac{\alpha_d}{2} - \frac{3}{4}\ln(v_{di}^{-2/3} - 1)^2 - y_{di}\right)^2 \tag{3.11}$$

We differentiate (3.11) with respect to α_d and set it equal to zero; we obtain

$$\alpha_d = 2\exp\left[\frac{4}{3n}\sum_{i=1}^{n}\left(\frac{3}{4}\ln(v_{di}^{-2/3} - 1) + y_{di}\right)\right]$$

Besides of the function (3.10) the observable trajectory was approximated by the function describing movement of a single (not destroyed) body with the account of the ablation (2.8) (see Sect. 2.1) and also by the elementary function corresponding to movement of a single body without taking into account of the ablation

$$y = \ln\alpha - \ln(-\ln v). \tag{3.12}$$

Similar to the procedures described above we find the minimum of the sum

$$Q_2(\alpha) = \sum_{i=1}^{n}(v_i(y_{di}, \alpha) - v_{di})^2$$

where $v_i(y_{di}, \alpha)$ are calculated with help of the function inverse to the function (3.12)

$$v_i = \exp(-\alpha e^{-y_{di}}).$$

Thus $Q_2(\alpha) = \sum_{i=1}^{n}[\exp(-\alpha e^{-y_{di}}) - v_{di}]^2$. We differentiate $Q_2(\alpha)$ with respect to α and set it equal to zero; for the solution of the equation $dQ_2(\alpha)/d\alpha = 0$ we used Newton's method:

$$\alpha^{(n+1)} = \alpha^{(n)} - \frac{\phi(\alpha^{(n)})}{\phi'(\alpha^{(n)})} \quad \text{where } \phi(\alpha) \equiv dQ_2(\alpha)/d\alpha.$$

As the initial approximation $\alpha^{(0)}$ we choose the value $\alpha = \exp(\frac{1}{n}\sum_{i=1}^{n}(\ln(-lvv_{di}) + y_{di}))$

$$\phi(\alpha) = \sum_{i=1}^{n}\left\{(\exp(-2\alpha e^{-y_{di}}) - v_{di}\exp(-\alpha e^{-y_{di}}))e^{-y_{di}}\right\}$$

$$\begin{aligned}\phi'(\alpha) &= \sum_{i=1}^{n}\left\{(-2\exp(-\alpha e^{-y_{di}}) + v_{di}) \right. \\ &\quad \left. \times \exp(-\alpha e^{-y_{di}})e^{-2y_{di}}\right\}\end{aligned}$$

To determine values of initial mass of a meteoroid M_e, we introduce a form factor $S_F = S_e/W_e^{2/3}$. Expression for α can be presented as:

$$\alpha = \frac{1}{2}\frac{C_d\rho_0 H_0}{\sin\gamma}\frac{S_F}{\rho_b^{2/3}M_e^{1/3}} \quad \text{here } W_e\text{—volume of the body.}$$

Thus, we have six parameters in the expression for α. From them ρ_0, H_0—are known parameters of the atmosphere, and γ is determined from the observation data. Below these three parameters will assume to be known. The next three parameters: C_d, ρ_b, S_F are determined form physical theories and special hypotheses. Here we assume $C_d = 1$, $\rho_b = 3.7$ g/cm^3 (density of the meteorite *Lost City* [39]), $S_F = 1.21$ (sphere).

The values of ballistic coefficient α, ablation coefficient β and an entry mass are adduced in Table 2 for comparison. They are calculated for the studied bolides in accordance with the models described above: the models of consecutive breakup and of single body with account of ablation and

Table 2 The results of calculations on the inverse problem for the four studied bolides

PN-bolide	Fragmentation		Single b. without ablation		Single b. with ablation			M_{ph}, kg
	α	M_e, kg	α	M_e, kg	α	β	M_e, kg	
40151A	20.5	7.1	22.2	3.4	16.8	0.76	8.4	340
40590	15.9	14	19.3	5.3	11.8	1.08	25.5	490
40405	58.3	0.45	46.2	0.68	25.5	2.67	4.3	70
38737*	35.7	2.7	35.4	2.8	19.5	2.32	17.5	280

without it. Also the values of photometric mass M_ph are adduced in last column; values of M_ph are taken from the work by Nemchinov and Popova [37]. As expected the dynamic mass M_e is essentially lower than photometric mass. The photometrical methods of a mass estimation use a luminosity of a bolide. Mostly following method for determination of photometric mass used

$$M_{ph} = -2 \int_{t_T}^{t_B} \frac{I}{\tau V^2} dt$$

Here I is a luminosity intensity along of the observable trajectory section, t_T, t_B are initial and final time for this section, τ—aspect ratio. This formula based on the following assumption: the vapours of a body material dives determinative contribution to a luminosity.

Other approach founded on a luminosity value was developed by *I.V. Nemchinov* with colleagues and it was named as method of radiation radius (a references to original works are in the paper by Borovicka et al. [10]. They have calculated a table of dependence of luminosity on a body radius, velocity and altitude; calculation was made using the method of a nonstationary analogy. A radius of an observed body one can determine from this table by the value of observed luminosity with the same values of velocity and altitude. Obtained values of the ballistic coefficient α allowed constructing dependences $y = y(v)$ along a bolide trajectory and comparing them with observation data. The results are presented on Figs. 2 and 3. The PN number of bolide from the table is indicated in the head of each graph. Almost in all cases (except for the bolide PN40405) the formula (2.8) provide a best approximation of the observed trajectory especially in a bottom of a trajectory; this formula corresponds to a single body model with account of the ablation. Thus we can make a following conclusion: a form of a luminous section of a trajectory does not disclose an evident influence of a meteoric body fragmentation.

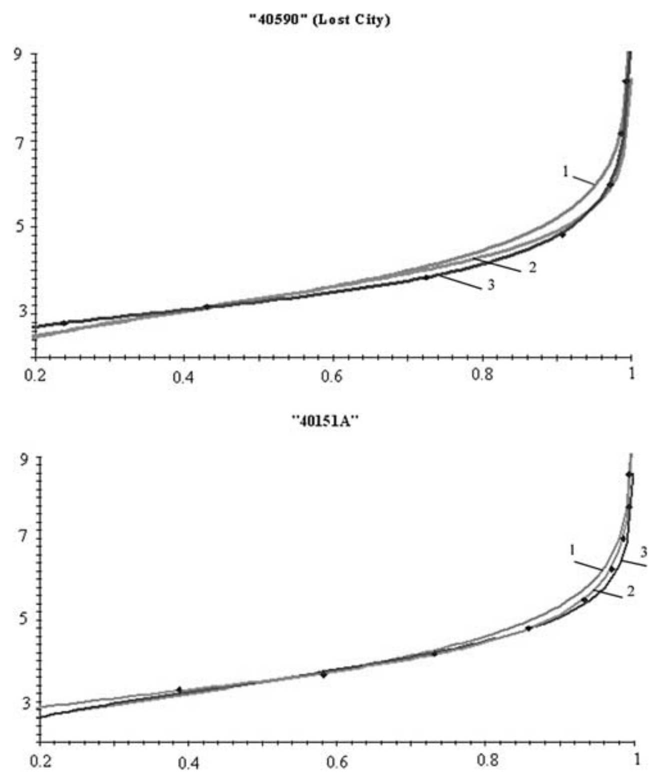

Fig. 2 The trajectories of the bolides in the coordinates "velocity—altitude": *dots*—observation data by McCrosky et al. [35]; *lines 1*—the formula (3.12), a single body without ablation; *lines 3*—formula (2.8), a single body model with ablation; *lines 2*—formulas (3.10), the trajectory calculated on the basis of the consecutive breakup model without ablation

3.3 Bolide *Benesov*

The fragmentation either has no place or weakly influences on a trajectory form in variables «velocity-altitude». This result was obtained by analysis of the trajectories form for several bolides. Here a last-square method is applied to analysis of the observed section of the trajectory of bolide *Benesov*; the separation of its luminous fragments was clearly observed.

3.3.1 Short Description of Benesov's Bolide

Benesov's bolide (EN070591) is one of the most interesting bolides registered by the European bolide network [9, 45]. Detailed data on a trajectory of the basic body and several

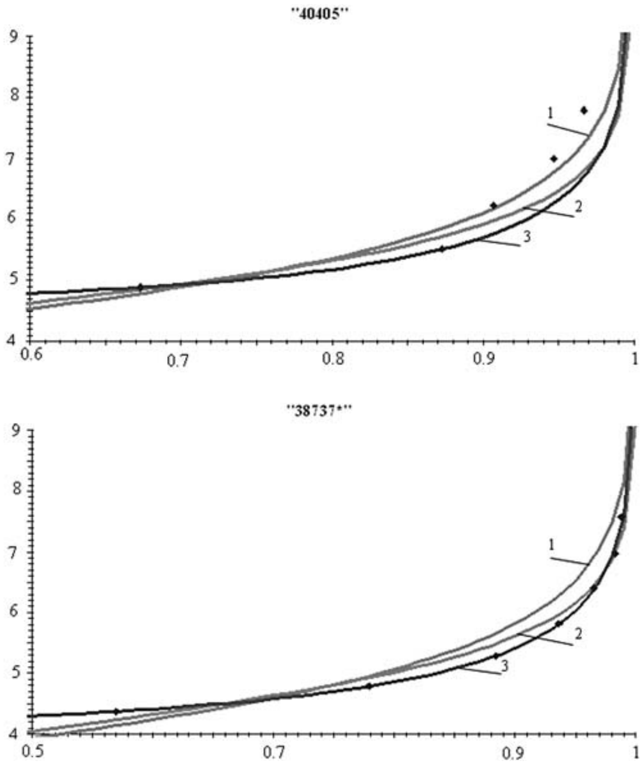

Fig. 3 The trajectories of the bolides in the coordinates "velocity—altitude": *dots*—observation data by McCrosky et al. [35]; *lines 1*—the formula (3.12), a single body without ablation; *lines 3*—formula (2.8), a single body model with ablation; *lines 2*—formulas (3.10), the trajectory calculated on the basis of the consecutive breakup model without ablation

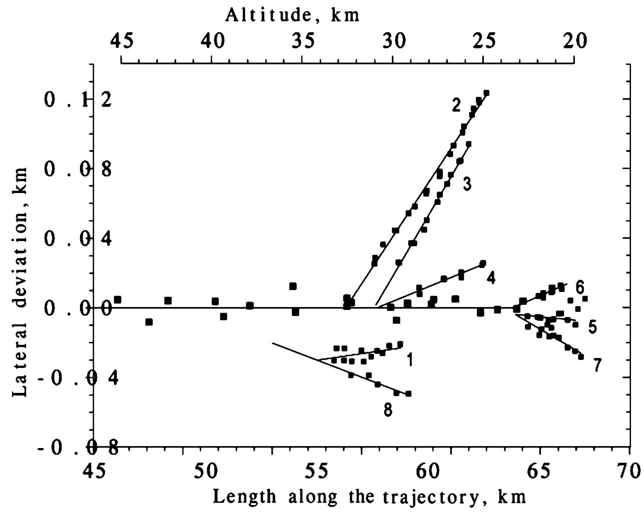

Fig. 4 Separation of the fragments of the bolide Benesov, the picture had been taken from the work by Borovicka and Spurny [9]

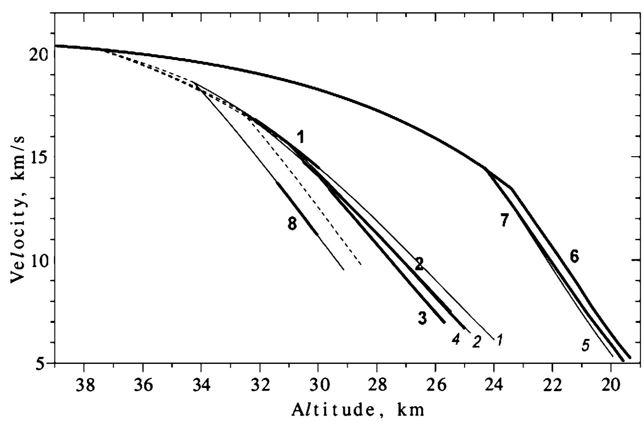

Fig. 5 Velocity dependence on altitude for several fragments and main body of the bolide Benesov, the picture had been taken from the work by Borovicka and Spurny [9]

fragments and also on luminosity were obtained. Emission spectrums were registered [9].

The body had entered the atmosphere almost vertically (9 degree to vertical line) at 21 km/s and was observed during 5.2 seconds. Last point of measured velocity (6 km/s) was situated at the altitude of 19 km. The photometric mass of *Benesov*'s bolide was estimated at 5–13 ton. The dynamic mass turn out to be at least on two orders less—80–120 kg [43].

As the bolide moved practically vertically, it had suffered a strong increase of the pressure. The fragmentation was quite visible and trajectories of each fragment have considerably deviated from the basic trajectory. It was observed at least six fragments separation; however meteorites have not been found.

The first visible fragmentation had occurred at the altitude of 42 km (see Fig. 4). It was not companioned by bright flare. The most intensive fragmentation had occurred at 24 km [9]. This point corresponds to an intensive flash. Also flashes are corresponds to other two moments of fragmentation. A relation between flashes and fragmentations being confirmed straight. Although an example of the first fragmentation of *Benesov*'s bolide shows that not each fragmentation should be companion by a flash. An occurrence of radiation flashes mainly is connected with a formation of a quickly extending cloud of small fragments and vapors. A formation of such cloud was observed at catastrophic destruction of *Benesov* at 24 km.

The pressure in the critical point was reaching of 20–30 and 90 Mdyn/cm^2 at the moments of fragmentations. The last high value is the maximal loading the body had experienced; this point corresponds to the main fragmentation [9]. The breaking strength of the small specimen of stony meteorites led in the range 100–500 Mdyn/sm^2 in most cases and less for larger bolides. Thus the most ablation of a body occurs under the pressure comparable to durability of a material. Velocities of the mane body and several fragments as the functions of altitude are represented in Fig. 5. Essential deceleration of a body has begun approximately at height 35 km [9].

Table 3 Observation data for several fragments of the bolide *Benesov* at the end of the trajectory [9]

The fragment number	$F1$	$F2$	$F3$	$F6$	$F7$	$F8$
Final altitude (km)	29.54	24.77	25.71	19.37	19.51	29.73
Final velocity (km/s)	14.2	6.56	6.97	4.94	4.9	11.4
Final acceleration (km/s^2)	−26	−9.6	−11.1	−10.0	−19	−30
Final mass (kg)	0.45	0.80	0.50	1.5	0.23	0.07
Ablation coefficient (s^2/km^2)		0.010	0.004	0.024		

Observers cite data of several important characteristics of fragments at the end of their individual trajectories. In particular, following values are cited: the altitude of extinction, velocity, acceleration and mass of fragments in final points of a trajectory. Also the estimations of representative values of the ablation coefficient for fragments are given; the ablation coefficient turns out to be different for different fragments; there are decrease and its abrupt increase relative to the mean value of σ for whale trajectory. The dynamics data for six individual fragments cite in Table 3; they were measured in the end of trajectory [9].

3.3.2 *Solution of Inverse Problem and Results*

Bolides can be destroyed after considerable deceleration. Therefore the solution (3.5) was modified in the work by Stulov [47] taking into account this fact. Integrating the deceleration equation and the ablation equation with condition $v = v_0$ for $y = y_0$, where y_0 is nondimensional altitude of breakup, the solution was obtained in following form [47]:

$$\frac{\alpha_d}{2} e^{\frac{-4}{3} y} - \frac{\alpha_d}{2} e^{\frac{-4}{3} y_0} = \frac{e^{-\beta/3}}{(3/\beta)^{1/3}} \frac{\tilde{\Delta}}{3} \tag{3.13}$$

$$\alpha_d = \alpha \left(\frac{V_e^2 \rho_0}{\sigma_t} \right)^{1/3}; \qquad \tilde{\Delta}_0 = \int_{\beta v^3/3}^{\beta v_0^3/3} \frac{e^t dt}{t^{4/3}}$$

The values y_0 and v_0 are being determined from the condition

$$\begin{aligned} &\rho_0 V_e^2 \exp(-y_0) v_0^2(y_0) = \sigma_t, \\ &v_0(y_0) = \exp(-\alpha e^{-y_0}) \end{aligned} \tag{3.14}$$

The author Stulov [47] advise the following approximation of the integral to simplify the calculation

$$\frac{e^{-\beta/3}}{(3/\beta)^{1/3}} \frac{\tilde{\Delta}_0}{3} = (v^{-2/3} - v_0^{-2/3}) \exp[-\beta^*(v_0 - v)] \tag{3.15}$$

The parameter β^* differs from β by numerical coefficient; this coefficient provides a best approximation of the integral $\tilde{\Delta}_0$. Thus the analytical expression (3.13), (3.15) are used as a trial function; this expression include two free parameters α_d and β^*.

Two schemes of a least-squares method are described in the part 1.2. This method is used for a determination of values of ballistic coefficient and ablation. The third variant of the scheme is advised in the work Stulov [47]. The sum is made

$$Q_3(\alpha_d, \beta^*) = \sum_{i=1}^{n} q_i^2 = \sum_{i=1}^{n} \left\{ \frac{\alpha_d}{2} \left(e^{\frac{-4}{3} y_i} - e^{\frac{-4}{3} y_0} \right) - \left(v_i^{-2/3} - v_0^{-2/3} \right) \exp[-\beta^*(v_0 - v_i)] \right\}^2$$

The minimum of the function $Q_3(\alpha_d, \beta^*)$ with fixed y_0, v_0 is determined from the equation:

$$\frac{\partial Q_3}{\partial \alpha_d} = \sum_{i=1}^{n} q_i \left(e^{\frac{-4}{3} y_i} - e^{\frac{-4}{3} y_0} \right) \tag{3.16}$$

$$\begin{aligned} \frac{\partial Q_3}{\partial \beta^*} = 2 \sum_{i=1}^{n} q_i \left(v_i^{-2/3} - v_0^{-2/3} \right) \\ \times \exp[-\beta^*(v_0 - v_i)](v_0 - v_i) = 0 \end{aligned} \tag{3.17}$$

The parameter α_d was expressed through β^* from (3.16) in the explicit form

$$\frac{\sum_{i=1}^{n} (e^{\frac{-4}{3} y_i} - e^{\frac{-4}{3} y_0})(v_i^{-2/3} - v_0^{-2/3}) \exp[-\beta^*(v_0 - v_i)]}{\sum_{i=1}^{n} (e^{\frac{-4}{3} y_i} - e^{\frac{-4}{3} y_0})^2} \tag{3.18}$$

Substituting (3.18) in (3.17), we obtained one transcendent equation for β^* and solved numerically.

We advise the following way for approximation of an observed trajectory by the functions (3.13)–(3.15). First, in accordance with (3.14) (or other considerations) the altitude of breakup y_0 is assigned. Further we assume that for $y > y_0$ ablation is absent. The upper trajectory is approximated by the dependence $v = \exp(-\alpha e^{-y})$; this dependence corresponds to a deceleration of a single body with constant mass. The values of α and v_0 in the point of breakup are determined by means of the scheme of least-square method with use of $Q_2(\alpha, \beta)$ with $\beta = 0$. The bottom of a trajectory ($y \leq y_0$) is approximated by the stated above scheme with $Q_3(\alpha_d, \beta^*)$; as a trial function a trajectory for the model of consecutive breakup (3.13)–(3.15) is used.

Table 4 Observation data, the trajectory of the bolide *Benesov*

	1	2	3	4	5	6	7	8	9
Velocity km/s	21.11	21.10	21.07	21.06	21.04	21.03	20.99	20.98	20.95
Altitude km	52.6	52.0	51.2	50.6	49.8	49.2	48.4	47.8	46.9

	10	11	12	13	14	15	16	17	18
Velocity km/s	20.93	20.89	20.86	20.82	20.79	20.73	20.69	20.63	20.59
Altitude km	46.5	45.6	45.1	44.2	43.7	42.8	42.3	41.5	41.0

	19	20	21	22	23	24	25	26	27
Velocity km/s	20.51	20.37	20.20	20.19	19.99	19.95	19.74	19.64	19.44
Altitude km	40.1	38.8	37.5	37.4	36.1	35.9	34.8	34.3	33.5

	28	29	30	31	32	33	34	35	36
Velocity km/s	19.25	19.08	18.77	18.18	17.56	17.48	16.89	16.14	15.31
Altitude km	32.8	32.2	31.3	29.9	28.6	28.4	27.4	26.3	25.3

	37	38	39	40	41	42	43	44	45
Velocity km/s	14.41	13.49	11.76	10.22	8.88	7.73	6.76	5.95	5.27
Altitude km	24.3	23.4	22.5	21.8	21.2	20.6	20.2	19.7	19.3

Table 5 The results of calculations on the inverse problem for the trajectory of the bolide Benesov

Model	Single body				Consecutive breakup		
Variant of a model	With ablation		Without ablation $\beta = 0$		With ablation		
Parameters	α	β	$\alpha^{(0)}$	α	α_d	α	$\beta*$
Values	7.8	1.5	13	14.3	34	4.1	1.6
M_e, kg	28		4.7		200.6		

The least-square method was applied to analysis of the trajectory of *Benesov*'s bolide in form described in parts 1.2, 2.2 (first and second scheme of least-square method). As trial functions we used the solutions for the trajectory of a single body without considering of the ablation

$$y = \ln\alpha - \ln(-\ln v) \tag{3.19}$$

for the trajectory of a single body with considering of the ablation

$$y = \ln\alpha - \ln(-\ln v) + 0.83(1 - v)\beta \tag{3.20}$$

The analysis of bolide Benesov was made on basis of the observation data, Table 4. Also, the following parameters were used:

– known parameters of the atmosphere—density of the atmosphere at the sea level $\rho_0 = 1.29 \times 10^{-3}$ g/cm^3, the pressure scale height $H_0 = 7.16 \times 10^5$ cm.
– parameters determinate on the basis of the trajectory measurement—angle of the trajectory with horizon $\gamma = 80.6$ degrees, entry velocity $V_e = 21.12 \times 10^5$ cm/s.
– parameters determinate on the basis of the physics theories—drag coefficient $C_d = 1$, density of the body $\rho_b = 3.7$ g/cm^3 [9, 45], form factor $S_F = 1.21$ (sphere), $\sigma_{t1} = 10^7$ g/cm s^2.

Results of computations of nondimensional determinative parameters and also entry mass of the bolide determined according to the offered variants of two models are cited in Table 5.

Application of the least-square method to an observed trajectory of *Benesov*'s bolide has shown that the best approximation is provided with the formula (3.20) at the following values of free parameters: $\alpha = 7.8$, $\beta = 1.5$. These values allow estimating the entry mass of the bolide and average ablation coefficient.

We apply known formulas

$$\alpha = \frac{1}{2}\frac{C_d \rho_0 H_0}{\sin\gamma}\frac{S_F}{\rho_b^{2/3} M_e^{1/3}}, \qquad \beta = \sigma \frac{V_e^2}{2}$$

Using values of parameters specified above, we obtain

$$M_e = 28 \text{ kg}, \quad \sigma = 0.0067 \text{ s}^2/\text{kg}^2 \tag{3.21}$$

It is important to notice that the derived average value of ablation coefficient quite corresponds to data of the work by Spurny [45], ablation coefficient was determined in the work [45] by another method. As for M_e, this value depends on assumption about a spherical form of the body. If we assume for example that the body had slab form $1 \times 1 \times 0.25$, so $S_F = 2.52$, $C_d = 2$ and we get $M_e = 2023$ kg.

Other models for a description of a *Benesov*'s bolide movement were discussed in literature; these models allow estimating the entry mass. By a purely dynamics model of gross fragmentation Ceplecha et al. [12] the value of entry mass for the adduced bellow parameters is $M_e = 82$ kg. The model of progressive fragmentation Borovicka et al. [10] with use of the model of radiation radius [43] lead to the essentially larger values $M_e = 3000$–4000 kg. The photometric mass of the bolide by the observers' estimations [9] is $M_{ph} = 13000$ kg. It is necessary to mark that a purely dynamics estimations of the bolide mass by different models are essentially closer that values derived by calculations using luminosity of the bolide.

Results of the best approximation of the observed trajectory by the formulas (3.13)–(3.15), (3.19), (3.20) are shown on Fig. 6. One can see that a neglect of ablation no allows to reproduce the observed trajectory. The model of fragmentation (3.13)–(3.15) visibly deviate from the observation data in the first half of the bolide trajectory. The detailed table with dependence of the body velocity on the altitude by the observation data was kindly given by *O.P. Popova*.

Thus, the best approximation of an observed trajectory of *Benesov*'s bolide is provided by means of a single body model considering the ablation. Though crushing was observed obviously the present result does not contradict this. The basic conclusion derived by the least-square method can be justified in the following way, the total mass of separated fragments on the altitudes $42 > h > 24$ km not constitute a main part of full meteoroid mass. Observant data about parameters of the fragments, adduced in work [9] allow to make corresponding estimations. The values of altitude, velocity, acceleration and mass of fragments at the moment of their extinction are given in work [9]. The masses of fragments at the moment of their separation from the main body are obtained in the paper by Barri and Stulov [6]; that is the total mass lost by way of fragments before final destruction is obtained.

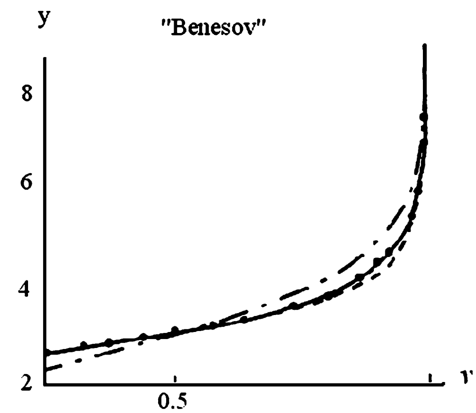

Fig. 6 Approximation of Benesov's bolide trajectory: *dots*—observation data; *dash-dotted line*—the formula (3.12), a single body without ablation; *solid line*—formula (2.8), a single body model with ablation; *dashed line*—formulas (3.13), the trajectory calculated on the basis of the consecutive breakup model with ablation

As result of calculations it was obtained that the total initial mass of fragments is 7.19 kg and for $M_e = 28$ kg and $\beta = 1.5$ the mass loss due to ablation is 15.8 kg. Just by this fact one can explain that the trajectory in the variables «velocity-altitude» is described by the model of a single body.

4 Interaction of Destroyed Meteoroid Fragments in a Supersonic Flow

Crater fields have various widths. An explanation of the cross-section extent of a crater field is a presents of a transverse horizontal component of the velocity [1, 2]. Several possible mechanisms providing transverse component of the velocity was studied in the work by Passey and Melosh [42]. They are: centripetal separation of fragments from a rotating meteoroid, differential lift of the fragments, explosive breakup of meteoroid, and an interaction of two or several shock waves just after breakup. Passey and Melosh [42] come to a conclusion that an interaction of shock waves is a main reason of a transverse separation of fragments.

However until recently an estimation of a fragments' transverse scattering velocity in the literature was based on wrong assumption [42]. The repulsion force was considered constant and than it disappeared completely once the fragments had separated by a distance comparable to their size, that is when $b = cR$, were b is the distance on which interaction stops; C is a constant, R is a radius of a fragment (it was supposed that fragments are of spherical form and of the equal size).

It follows from numerical experiments discussed in the work by Shuvalov et al. [44] and Zhdan et al. [57] that the transverse force coefficient is a function of the distance between fragments. The results of numerical calculations on a

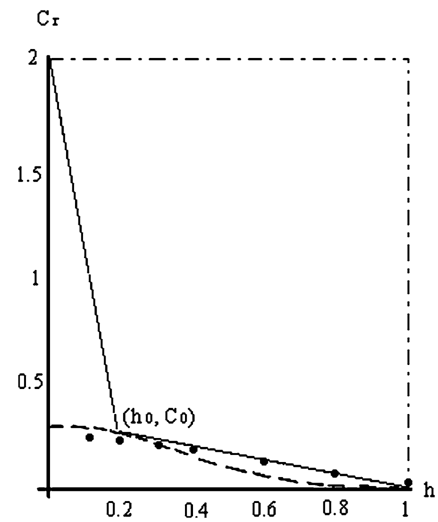

Fig. 7 The dependence of the repulsion force coefficient on the distance between fragments, viz. the value used by Passey and Melosh [42] (*dash-dotted line*) and from numerical experimental data for spheres [57] (*dashed line*). The *dots* represent the numerical experimental data for semicylinders [44]. The *solid line* is the proposed approximation to numerical data taking into account the gasdynamics when for $h = 0$, (max) should equal 2 (see (4.3))

flow over two semicylinders by Shuvalov et al. [44] and on a flow over two spheres Zhdan et al. [57] are presented in Fig. 7. This data was used to construct a following model.

4.1 Model of Two Meteoroid's Fragments Separation

In the paper by Passey and Melosh [42] it was assumed that fragments moves in the transverse direction with constant acceleration (due to the action of a constant repulsion force). A time of interaction of splinters was calculated by the formula $\Delta t = (2b/a)^{1/2}$, where a is an acceleration of a fragment; and final transverse velocity is $U_f = a\Delta t$.

The force acting on fragments in the transverse direction was defined in the work [42] as a product of total pressure and cross-section area; and acceleration due to this force was determined as

$$a = \frac{F_r}{M} = \frac{\rho_a \pi R^2}{(4/3)\pi R^3 \rho_b} = \frac{3V_b^2 \rho_a}{4R\rho_b} \tag{4.1}$$

where V_b is a body velocity in the moment of breakup. Thus for determination of the transverse velocity of fragments in the last moment of their interaction the following formula was used in the work by Passey and Melosh [42]

$$U_f = \left(\frac{3}{2} C \frac{\rho_a}{\rho_b}\right)^{1/2} \tag{4.2}$$

where ρ_a is air density at the altitude of breakup, ρ_m is a density of meteoroid. The value of the constant C lie in the range between 0.02 and 1.52. Authors [42] obtained this range by studying the cross-range spread of craters in known meteorite crater fields.

The known formula from aerodynamics for the pressure force is $F_r = \frac{1}{2} C_r \rho_a V_b^2 S$, where C_r is the force coefficient, S is cross-section area. Thus in the work Passey and Melosh [42] was assumed that coefficient C is constant and equal 2.

Actually interaction of fragments weakens with distance increase between them. Numerical experiments by Zhdan et al. [57] show that the transverse force coefficient is a function of a distance between fragments, and this function has a form represented by the dots in the Fig. 7. The maximum of C_r in numerical experiments is approximately equal to 0.28 when the distance between the fragments is close to zero. The value of C_r monotonically decreases to zero for $h \approx 0$ where $2h$ is the non-dimensional distance between the fragments; as the unit length we take the radius R of a fragment.

Authors [57] calculated the values of the repulsion force coefficient for two fixed spheres at various distances between the nearest points of spheres. At $h = 0$ when the spheres touch, the numerical solution showed the value $C_r = 0.28$ [57]. However during the initial time when the meteoroid is still intact as a single body, the pressure at the point of initial separation is equal to the total pressure and $F_r = \rho_a V_b^2 S$, i.e., C_r should equals 2 for $h = 0$. In the case $h > 0$, the numerical experimental data can be used to describe the separating force as a function of the distance.

We propose the following approximation of the numerical experimental data. To be in agreement with the gas dynamics of fragment separation, it is necessary to make a correction in the data near $h = 0$. To further simplify our computations, the dependence $C_r(h)$ may be represented in the form of two intersecting straight lines at (h_0, C_0) (Fig. 7). The point (h_0, C_0) should be on the curve corresponding to the numerical experiments from [57] and h_0 is chosen to be close to zero. The function

$$C_r(h) = \begin{cases} C_{r1}(h) = (h - h_0)\alpha + C_0, & 0 \le h \le h_0 \\ C_{r2}(h) = (h - h_0)\beta + C_0, & h \le h_0 \end{cases} \tag{4.3}$$

corresponds to the straight lines represented in Fig. 7, where (h_0, C_0) is the intersection point of the segments $C_{r1}(h)$ and $C_{r2}(h)$. This point corresponds to time t_0. Time t_f is the time corresponding to $h = 0.5$, distance at which the interactions among fragments stops. The coefficients α and β determine the slopes of the straight lines. There is to mark that $\alpha, \beta < 0$; $\alpha = (C_0 - 2)/h_0$, and $\beta = C_0/(h_0 - 0.5)$.

A dynamics equation in present problem is:

$$\frac{Md^2(hR)}{d(Tt)^2} = \frac{1}{2} C_r(h) \rho_a V_b^2 S,$$

or in dimensionless form $\frac{d^2h}{dt^2} = \frac{3}{8} \frac{\rho_a}{\rho_b} C_r(h)$. Here the repulsion force coefficient is assigned by the formula (4.3), this coefficient depends on distance between fragments and the distance depends on time. As the distance unite we take the radius of fragment R, as the velocity unit we take the longitudinal velocity of a body in the moment of breakup V_b, as the time unit we take the time $T = \sqrt{8/3}R/V_b$.

Thus, it is necessary to solve Cauchy problem:

$$\frac{d^2h_1}{dt^2} = p\alpha h_1 + 2p, \quad 0 \le t \le t_0,$$

with initial conditions $h_1(0) = h_1'(0) = 0$

$$\frac{d^2h_2}{dt^2} = p\beta h_2 - p\beta/2, \quad t_0 \le t \le t_f,$$

with initial conditions $\begin{array}{l} h_2(t_0) = h_1(t_0) = h_0 \\ h_2'(t_0) = h_1'(t_0) \end{array}$

here $p = 3\rho_a/8\rho_b$. The time instant t_0 corresponds to the point (h_0, C_0) and the time of fragments interaction t_f determined from the relation $h_2(t_f) = 0.5$. Further after the interaction termination for $t > t_f$ we consider fragments movement in the transverse direction with inertial velocity equal to the velocity in the final moment of their interaction. Taking into account that the separating force coefficient is defined by (4.3), we solved this problem and obtained the following solution:

$$h(t) = \begin{cases} h_1(t) = C_1 \cos mt + C_2 \sin mt - 2/\alpha, & 0 \le t \le t_0 \\ h_2(t) = C_3 \cos kt + C_4 \sin kt + 1/2, & t_0 \le t \le t_f \end{cases} \tag{4.4}$$

where constants C_1, C_2, C_3, C_4 obtained from the initial conditions are

$$C_1 = \alpha/2, \qquad C_2 = 0$$

$$C_3 = (h_0 - 0.5)\cos t_0k + \frac{2m}{\alpha k}\sin t_0m \sin t_0k$$

$$C_4 = (h_0 - 0.5)\sin t_0k - \frac{2m}{\alpha k}\sin t_0m \cos t_0k$$

here $k = \sqrt{|\beta p|}$, $m = \sqrt{|\alpha p|}$, and t_0 obtained from the condition $h_1(t_0) = h_0$ and defined by the formula $t_0 = \arccos(C_0/2)/m$. The transverse velocity of fragments taking into account a decreasing repulsion force is

$$h'(t) = \begin{cases} h_1'(t) = -C_1 m \sin mt + C_2 m \cos mt, & 0 \le t \le t_0 \\ h_2'(t) = -C_3 k \cos kt + C_4 k \cos kt, & t_0 \le t \le t_f \end{cases} \tag{4.5}$$

The functions $h(t)$ and $h'(t)$ are monotonously increasing functions on a considered interval of time $0 \le t \le t_f$. The final transverse velocity of a fragment is $u_{f1} = h_2'(t_f)$, where the time of fragments interaction determined from the relation $h_2(t_f) = 0.5$ by the following formula

$$t_f = -\arcsin\left(C_3/\sqrt{C_3^2 + C_4^2}\right)/k$$

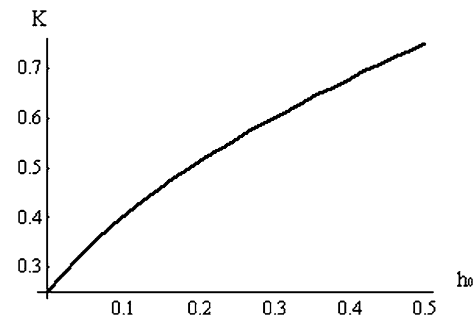

Fig. 8 The dependence of K on h_0 for $C_0 = 0.27$, see p. 17

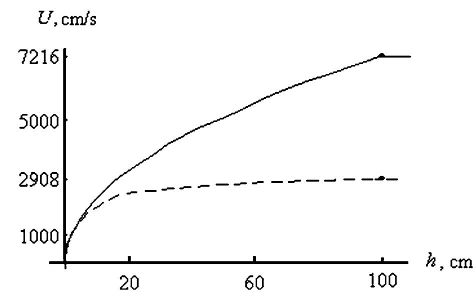

Fig. 9 Velocity of the fragments in transverse direction: *dashed line* corresponds to $h'(h(t))$, formulas (4.4)–(4.5); *solid line*—motion with constant acceleration corresponding to the work by Passey and Melosh [42], the *dots*—moments when interaction stops

The final transverse velocity of the fragments is:

$$u_{f1} = \left(\frac{3}{2}C\frac{\rho_a}{\rho_b}\right)^{1/2}\left(\frac{|\beta|(C_3^2 + C_4^2)}{4C}\right)^{1/2} = u_f \cdot K$$

It is easy to check that u_{f1} differs from the dimensionless velocity u_f corresponding to the work [42], by the factor K, this factor tends to 1 as $C_0 \to 2$ and $h_0 \to 0.5$; if $C_0 = 0.27$ and $h_0 = 0.1$, then $K = 0.4$. The dependence $K(h_0)$ is presented on Fig. 8 for fixed $C_0 = 0.27$. Factor K equal to 0.25 as h_0 tends to zero and for $h_0 \to 0.5$ we have $K = 0.75$.

Below the solution (4.5) is compared with the solution corresponding to scattering of fragments due to constant transverse force [42] (see Fig. 9). Calculations were made on an example of a body with density 3.7 g/cm^3, the body velocity in the moment of breakup V_b was taken to be equal to 20×10^5 cm/s. Transverse scattering of spherical fragments of radius $R = 200$ cm was studied. For exponential atmosphere $\rho_a = \rho_0 e^{-y}$ where the air density on the sea level is $\rho_0 = 1.29 \times 10^{-3}$ g/cm. Nondimensional altitude of breakup y suppose to be equal to 3; and as the altitude unite we take the pressure scale height 7.16×10^5 cm.

Results of calculations are presented in a dimensional form for it was clear which order of the transverse velocity and of the time of fragments interaction. The transverse velocities of fragments in the final moment of their interaction for given example are of the following values:

$U(t_{f1}) = 7216$ cm/s and $h'(t_{f2}) \cdot V_b = 2908$ cm/s, here $t_{f1} = 0.028$ s and $t_{f2} = 0.043$ s are time of interaction of fragments in the case of constant and decreasing repulsion force respectively.

In described above new model of fragments scattering it was supposed that fragments moves in transverse direction under the action only repulsion force that is due to interaction of shock waves arising direct after a breakup of meteoroid. But also a drag force acts on any body moving in some medium. The dynamic equation for a fragment considering a drag force is

$$\frac{Md^2(hR)}{d(Tt)^2} = \frac{1}{2}C_r(h)\rho_a V_b^S, -\frac{1}{2}C_d\rho_a V_b\frac{d(hR)}{d(Tt)}S$$

or in dimensionless form $\frac{d^2h}{dt^2} = pC_d\frac{dh}{dt} = pC_r(h)$, here C_d is a coefficient of air resistance.

Thus in a problem about scattering of two fragments taking into account a drag force, it is necessary to solve Cauchy problem:

$$\frac{d^2h_1}{dt^2} + pC_d\frac{dh_1}{dt} - p\alpha h_1 = 2p, \quad 0 \le t \le t_0,$$

with initial conditions $h_1(0) = h_1'(0) = 0$

$$\frac{d^2h_2}{dt^2} + pC_d\frac{dh_2}{dt} - p\beta h_2 = -p\beta/2, \quad t_0 \le t \le t_f,$$

with initial conditions $\begin{matrix} h_2(t_0) = h_1(t_0) = h_0 \\ h_2'(t_0) = h_1'(t_0) \end{matrix}$

The value of interaction time t_f in this case is determined numerically from the relation $h_2(t_f) = 0.5$. Further, after the termination of interaction for $t > t_f$ fragments move in transverse direction due only to a drag force. That is

$$\frac{d^2h_3}{dt^2} + pC_d\frac{dh_3}{dt} = 0, \quad t > t_f,$$

with initial conditions $\begin{matrix} h_3(t_f) = h_2(t_f) = 0.5 \\ h_3'(t_f) = h_2'(t_f) = v_f \end{matrix}$

here v_f transverse velocity in the instant t_f.

The equations for $h_1(t)$, $h_2(t)$ and $h_3(t)$ were solved numerically. One can see from Fig. 10 that two solutions of a problem on separation of two spherical fragments considering an air resistance and without considering of air resistance are practically coincide on the whole section of interaction and further.

Results of calculations cite in Table 6. Calculations were made for three models of fragments separation, they are: transverse movement with constant acceleration [42], the constructed here model of separation with consideration of the air resistance and without this. Time interaction values and final transverse velocity values are cited in Table 6. The calculations were made for cases: $y = 7$, $y = 5$, $y = 3$, were

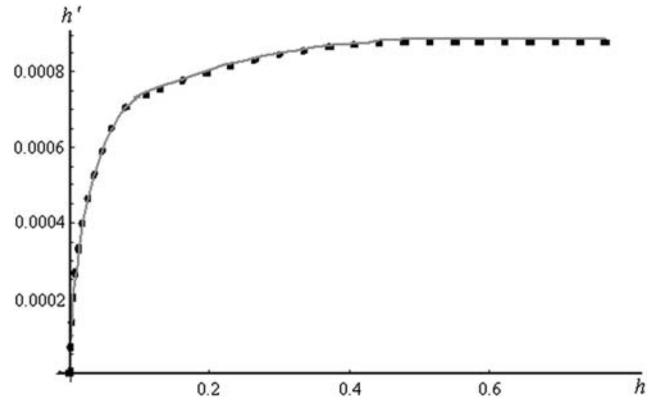

Fig. 10 The comparison of the transverse velocity of two spherical fragments considering an air resistance (*dots*) and without consideration of air resistance (*solid line*)

Table 6 Models comparison of models of fragments separation (results of calculation in dimensionless form)

		With deceleration	Without deceleration	Model by Passey and Melosh
$y = 7$	t	3160	3162	2047
	v	1.96×10^{-4}	1.97×10^{-4}	4.8×10^{-4}
$y = 5$	t	1162	1162	753
	v	5.350×10^{-4}	5.354×10^{-4}	13.27×10^{-4}
$y = 3$	t	427	427.5	277
	v	14.54×10^{-4}	14.55×10^{-4}	36×10^{-4}

y is dimensionless altitude of breakup. As an altitude unit we take the pressure scale height.

One can see from Table 6 that results of calculation corresponding to the models "with acceleration" and "without acceleration" practically coincide. The interaction time calculated by model of the work by Passey and Melosh [42] is less in 1.5 times then the time corresponding to two others models. It can be explained as follows, the repulsion force is greater in the first case. The final transverse velocity corresponding to the model [42] is in 2.4 times greater then the final transverse velocity calculated by the models "with acceleration" and "without acceleration" for the same reason.

One also can see that to a greater altitude of breakup corresponds a greater interaction time and a lower transverse velocity. This is due to an exponential increase of the atmosphere density with decrease of the distance to the ground surface. And therefore the repulsion force acting on fragments is greater for a lower altitude of breakup.

Dimensional data cited in Table 7, they are: the interaction time of fragments and final transverse velocity for the case of fragmented body with longitudinal velocity in the moment of breakup $V_b = 20$ km/s. Transverse scattering of originated spherical fragments of radius $R = 200$ cm was calculated.

Table 7 Models comparison of models of fragments separation (results of calculation in dimension form)

		With deceleration	Without deceleration	Model by Passey and Melosh
$Y = 50.1$ km	T, s	0.3160	0.3162	0.2047
	V, m/s	3.93	3.94	9.76
$Y = 35.8$ km	T, s	0.1162	0.1162	0.0753
	V, m/s	10.701	10.708	26.54
$Y = 21.4$ km	T, s	0.0427	0.0427	0.0277
	V, m/s	29.08	29.10	72.16

Table 8 Decrease of the transverse velocity due to resistance force

	$y = 3$	$y = 5$	$y = 7$
t_3/t_2	9.5×10^4	7.3×10^4	5.6×10^4
$h(t_3)/h(t_2)$	7.9×10^4	6.2×10^5	4.7×10^6
$v(t_3)/v(t_2)$	0.5	0.5	0.5

We can estimate a character of the transverse velocity decreasing after the interaction stopped. Let the time t_2 be the moment when an interaction stopped, that is the repulsion force coefficient equal to zero when $t > t_2$, and a time t_3 we calculated from the condition $v(t_3)/v(t_2) = 0.5$. Then we determined $h(t_3)/h(t_2)$—how the distance between fragments increases for the transverse velocity twice decreases.

Basing on data of Table 8 we can make a conclusion that the transverse velocity of fragments separation decrease very slowly after the interaction stopped.

Thus, the problem on two spheres separation under the action of the decreasing transverse force is solved. The data of numerical experiment by Zhdan et al. [57] for the transverse force coefficient were used. The numerical data were approximated and were used in the analytical form. It turned out that the induced transverse velocity of a spherical fragment is much less than the values that were used in the previous papers by Passey and Melosh [42], Nemchinov et al. [38] and Popova [43].

Besides, it was found out that the influence of the air resistance acting in the transverse direction is so insignificant that it can be not considered to describe the transverse separation of fragments.

4.2 A Model of Layer-by-Layer Scattering of Meteoroid Fragments

Various models for meteoroid fragmentation caused by aerodynamical strength failure were proposed previously [4, 18, 21, 22]. In almost all of these models the calculation of the trajectory of the fragment swarm is done by numerical integration using a formula for a single body with an increasing area for its effective cross-section. The results obtained by Zhdan et al. [58], Zhdan [56] and Barri [5] allow us to describe in more detail the transverse scattering of fragments caused by the interaction of shock waves and to construct a new model for meteoroid fragment scattering.

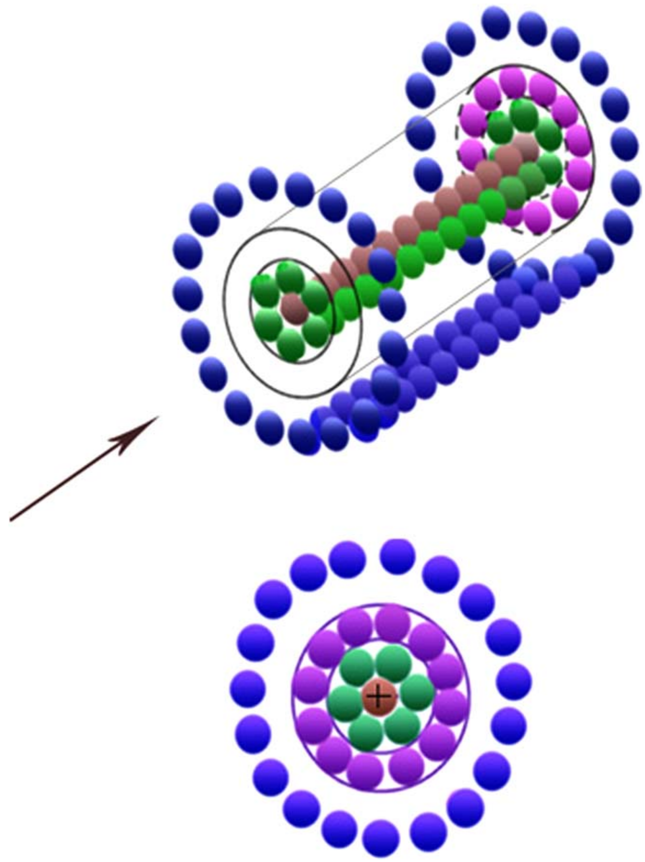

Fig. 11 Schematic sketch of layers in destroyed cylindrical and spherical meteoroids

Zhdan et al. [58] presented a numerical analysis of the problem for a finite number of spheres in a supersonic flow. The repulsion force coefficient for each sphere was calculated with respect to its relative position in the group. It was shown that the value of the repulsion force acting on the peripheral bodies of group is sufficiently larger than the value of the force acting on the internal bodies.

Basing on the results described above we propose a new model of layer-by-layer scattering of meteoroid fragments. The initially fragmented body is interpreted as a compact collection of spherical fragments. We consider two shapes for the initial body, i.e. either a cylindrical or a spherical shape (Fig. 11). In the case of a spherical body we consider the layer to be the volume contained between two spheres of radius $r(2i-1)$ and $r(2i+1)$; $i \geq 1$, where i is a layer's number and r is a radius of the fragment. The centre of the sphere with radius r that coincides with the centre of spherical meteoroid is a zero layer, $i = 0$. If a meteoroid has the cylindrical form, we consider the layer as the volume contained between two cylinders with radii $r(2i-1)$ and

$r(2i+1)$; $i \geq 1$, where i is a layer number. The cylinder with radius r is a zero layer, $i = 0$.

In the proposed model the scattering of the meteoroid fragments has several stages whereby, at each stage, the interaction of a fragment of the outer layer and a fragment of the inner part is analyzed. The inner part is considered to be a single compact collection of spherical fragments. This interaction lasts until the distance between the outer layer and the inner one, still intact (according to the work by Zhdan et al. [58]), part reaches the radius of the inner part. Then the next layer becomes an outer layer and it starts move away from the main part of the meteoroid.

Given the number N of fragments, it is easy to determine their radius r by using the mass conservation law:

- $N \cdot \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$, i.e. $r = \frac{R}{N^{1/3}}$ for the spherical meteoroid of radius R,
- $N \cdot \frac{4}{3}\pi r^3 = 2\pi R^3$, i.e. $r = \frac{R}{(2N/3)^{1/3}}$ for the cylindrical meteoroid of radius R and $2R$ long.

We then determine the number of layers and number of fragments in each layer and find the size of the inner part and outer layer for each subsequent stage. The number of fragments in a layer for $i \geq 1$ is determined from:

$$K = \frac{\frac{8}{3}\pi r^3(12i^2+1)}{8r^3} = \frac{\pi}{3}(12i^2+1) \tag{4.6}$$

here the layer volume for a spherical meteoroid divided into the volume of cube with an arise $2r$. The cube volume (instead of spheres) is chosen as a divider for following reasons. If we divide some volume into sphere volume we obtain the number considerably exceeding a number of spheres which can really get into considered volume. It is clear that is the result of the space between touching spheres. We take an integer part of K.

To define how long outgoing (external) layer interacts with the central (motionless) part we determine the size of the central part at each stage. Let be $(n+1)$ a number of layers where N fragments are contained. Than the swarm radius is $(2rn+r)$. The number of fragments contained in $n+1$ layers starts with the zero layer (in the case of a spherical meteoroid):

$$1 + \sum_{i=1}^{n} \frac{\pi}{3}(12i^2+1) = 1 + \frac{\pi}{3}(4n^3+6n^2+3n) = N \tag{4.7}$$

A solution of the cubic equation (4.6) gives the number of possible layers in the spherical swarm of fragments of destroyed meteoroid. Let it be equal to $\lceil n \rceil + 1$.

Let be a meteoroid of a cylindrical form. The approximate quantity of fragments in a layer for $i \geq 1$ in the case of a cylindrical meteoroid is determined by the relation:

$$K = \frac{16\pi r^2 Ri}{8r^3} = 2\pi \frac{R}{r} i \tag{4.8}$$

by analogy with (4.6).

The number of fragments contained in $n+1$ layers starts with the zero layer (in the case of a cylindrical meteoroid):

$$\frac{R}{r} + \sum_{i=1}^{n} 2\pi \frac{R}{r} i = \frac{R}{r} + \pi \frac{R}{r}(n^2+n) = N \tag{4.9}$$

A solution of the square equation (4.9) gives the number of possible layers in the cylindrical swarm of fragments of destroyed meteoroid. Let it be equal to $\lceil n \rceil + 1$.

Further we describe movement of fragments in a transverse direction. We assume that a meteoroid separates by layers. That is at each stage a so-called outer layer moves off the inner part to the distance equaled to radius of motionless inner part.

The dynamic equation for a fragment of an external layer in a dimensionless form is

$$\frac{d^2h}{dt^2} = \frac{3}{8}\frac{\rho_a}{\rho_b} C_r(h), \tag{4.10}$$

where $2h$ is a distance between a fragment of the outer layer and the central part, ρ_a is the atmospheric density at the altitude of breakup, ρ_b is a body density, $C_r(h)$ is the variable coefficient of repulsion force. Here as a distance unite we take a dimension radius of the fragment r_a, as a time unite we take a time t—time necessary for passing the distance r_a with the longitudinal velocity in the moment of breakup V_b; that is t is equal to r_a/V_e.

The value of the coefficient of the repulsion force for each new outer layer depends on a size of a motionless inner part. A numerical experiment on two spheres with different radii in a supersonic flow was made in the work by Zhdan [56]. A numerical data on the dependence of the repulsing force coefficient C_r on the quotient of spheres radii are provided in the work by Zhdan [56] (see Fig. 12a). This data are used in the present work for the construction of a model of layer-by-layer scattering. In addition to the quotient of the radii, the coefficient C_r depends also on the distance between the spheres [57].

This data can be approximated by the analogy with the work by Barri [5]:

$$C_r(h) = \begin{cases} C_{r1}(h) = (h-h_0)\alpha + C_0, & 0 \leq h \leq h_0 \\ C_{r2}(h) = (h-h_0)\beta + C_0, & h \leq h_0 \end{cases} \tag{4.11}$$

here (h_0, C_0) is the intersection point of the segments $C_{r1}(h)$ and $C_{r2}(h)$. The coefficients $\alpha = (C_0 - 2)/h_0$ and $\beta = C_0/(h_0 - 0.5)$ determine the slopes of the straight lines.

To construct the model at each stage of the scattering process in respect to the number of separating layers, and thus the ratio of the radii of the outer layer fragment and the inner part, we used the corresponding relation $C_r(h)$. To determine the duration of fragment separation to the distance when the interaction between all fragments stops, we add up the total of the interaction terms for all layers.

The dependence of the C_0 and h_0 on the ratio of a fragment radius and the radius of a central part r/R^* is cited in Table 9, this data were taken from the work by Zhdan [56].

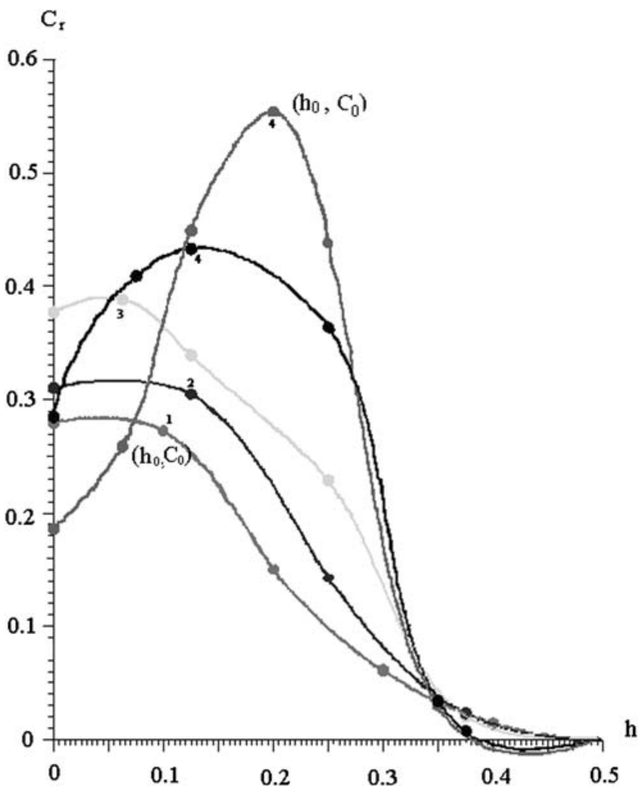

Fig. 12 Dependence of the repulsion force coefficient on the distance between fragments with different radii, data by Zhdan [56]: *line 1*—$r/R = 1$, *line 2*—$r/R = 0.75$, *line 3*—$r/R = 0.5$, *line 4*—$r/R = 0.3$, *line 5*—$r/R = 0.25$. *Dots* 1, 2, 3, 4, 5—chosen points (h_0, C_0) for corresponding lines

Table 9 Point (h_0, C_0) depending on ratio of radii of a fragment and central part

$r/R* \in$	(0, 0.25]	(0.25, 0.3]	(0.3, 0.5]	(0.5, 0.75]	(0.75, 1]
C_0	0.55	0.47	0.39	0.31	0.27
h_0	0.2	0.2	0.09	0.11	0.1

The solution of the dynamics equation (4.10) considering (4.11) is:

$$h(t) = \begin{cases} h_1(t) = C_1 \cos mt + C_2 \sin mt - 2/\alpha, & 0 \le t \le t_0 \\ h_2(t) = C_3 \cos kt + C_4 \sin kt + 1/2, & t_0 \le t \le t_f \end{cases} \tag{4.12}$$

where the constants $k = \sqrt{|\beta p|}$, $m = \sqrt{|\alpha p|}$, $p = 3\rho_a/8\rho_b$. The constants C_1, C_2, C_3, C_4 depends on values of C_0, h_0, R and are determined from the following conditions: distance between fragments and their velocities are equal to zero at the initial instant of time, that is $h_1(0) = 0$, $h_1'(0) = 0$ and $h_2(t_0) = h_0$, $h_2'(t_0) = h_1'(t_0)$.

The time t_0 is being determined from the relation $h_1(t_0) = h_0$ and has a form $t_0 = \arccos(C_0/2)/m$.

The interaction time t_f of a fragment of the outer layer with the central part determined from the condition $h_2(t_f) = R^*/2$. Radius R^* of motionless part of the fragments swarm depends on the quantity of separated layers j and being expressed as follows $R^* = 2r(n - j) + r$, where j possesses the value from 1 to n. By simple transformations we obtained

$$t_f = \Big(\arcsin\Big[(r(n - j + 1/2) - 1/2)/\sqrt{C_3^2 + C_4^2}\Big] - \arcsin\Big[C_3/\sqrt{C_3^2 + C_4^2}\Big]\Big)/k \tag{4.13}$$

To determine the duration of fragment separation to the distance when the interaction between all fragments stops, we add up the total of the interaction terms for all layers. Table 10 shows the results for a spherical body of radius $2m$, or an equivalent cylindrical body 4 meters long and a $2m$ radius, moving supersonically in the atmosphere at $V_e = 20$ km/s that were calculated applying the new proposed model. For the calculations we assumed that meteoroid separation started at altitude $A = 21$ km or $A = 36$ km.

Basing on the results of calculation by new model of layer-by-layer scattering of meteoroid fragments we can make several conclusions. The total time of scattering by layers does not actually depend on the number of fragments. In addition, the time of scattering is an insignificant fraction of the total time a meteoroid travels through the atmosphere. Thus, meteoroid fragmentation and scattering of its fragments is almost instantaneous and it reaches a state of independent movement for each fragment.

Table 10 The interaction time of all fragments for cylindrical and spherical meteoroid depending on the altitude of breakup and fragments quantity

Fragments quantity	Fragment radius, cm		Layers quantity		Interaction time, s $y = 5$		Interaction time, s $y = 3$	
	C	S	C	S	C	S	C	S
10^3	22.8	20	7	7	0.058	0.046	0.021	0.017
10^5	4.93	4.3	29	30	0.059	0.049	0.022	0.018
10^7	1.06	0.92	131	135	0.059	0.049	0.022	0.018

To describe a meteoroid movement from the moment of its enter to the planet atmosphere till the moment of its complete evaporation or till a meteorite falling to the planet surface one can proceed as follows. The movement of a meteoric body is being described by the single body theory till the moment of breakup. One can consider that the separation occurs instantly and fragments move in the transverse direction with an induced velocity u_{f1}; this velocity being determined according to the new model of layer-by-layer scattering of meteoroid fragments (see Sect. 4.1). Then longitudinal and transverse movement of fragments one can consider as independent and can describe longitudinal movement of each of them by the single body model.

The accuracy of modern meteor observations is not yet sufficient enough to detect the meteoroid scattering properties that were defined by the new numerical experiments presented here. The results of this new model of fragmentation behavior should be treated with reservation until it has been compared to the results of sufficient observational data on meteoroid fragmentation.

5 Conclusions

The methods of inverse problem solving for the observed bolide trajectory had been studied and realized. A clearer idea about dynamics and thermodynamics of a meteoric body passage through the atmosphere is obtained.

Main results and conclusions of the work:

1. The known solution for a single body model is represented in new co-ordinates: time, altitude over the planet surface, a projection of a trajectory to a horizontal. The solution in such a form is convenient for using for estimation meteoric and crater fields.
2. The analysis of trajectories of five bolides «40151A», «40590» (*Lost City*, 2.5), «40405», «38737*» and EN070591 (*Benesov*) was carried out for estimation of a breakup process influence on the meteoroid trajectory; as a result of this analyses:
   (a) the inverse problem was solved for each bolide; that is the values of main parameters for each bolide was obtained
   (b) it was obtained that the value of the ablation coefficient for the bolide Benesov is $\sigma = 0.0067$ s^2/km^2 ($\beta = 1.5$), and this is corresponds to the results of calculations by another method in the work by Spurny [45] ($\sigma = 0.006 \pm 0.001$ s^2/km^2)
   (c) the value of entry mass was obtained for each bolide, it was discover that the mass value obtained by the photometric method is sufficiently exceed the value obtained by the purely dynamical method; for example, for the bolide *Benesov* $M_e = 28$ kg, $M_{ph} =$ 1300 kg according to observation data
   (d) it was obtained that the single body model considering ablation provide a best approximation of the observed trajectory almost for all studied bolides, i.e. there is no evident influence of fragmentation process on the form of the trajectory luminous section. In the case of *Benesov*'s bolide this fact can be explained as fallows: the mass lost due to ablation (15.8 kg) almost twice greater than total initial mass of fragments (7.19 kg)

An investigation of a spherical fragments interaction in a supersonic flow was made, as a result of this investigation:

(a) an approximation of numerical data on the repulsion force coefficient has been proposed
(b) an analytical solution of a following problem has been obtained: two spheres with a center line across flow separate under the action of decreasing repulsion force without considering the air resistance
(c) it was discover that the final transverse velocity of a fragment approximately in 2.4 times less than values published earlier by other authors
(d) a following problem has been solved numerically: two spheres with a center line across flow separate under the action of decreasing repulsion force considering the air resistance
(e) it was found out that the transverse velocity of a fragment practically does not decrease under the action of the resistance force
(f) a new model of layer-by-layer scattering of meteoroid fragments had been proposed, each fragment is considered as an individual body
(g) the total time of scattering by layers does not actually depend on the number of fragments
(h) the time of scattering is an insignificant fraction of the total time a meteoroid travels through the atmosphere.

References

1. Artem'eva NA, Shuvalov VV (1996) Interaction of shock waves during the passage of a disrupted meteoroid through the atmosphere. Shock Waves 5:359–367
2. Artemieva NA, Shuvalov VV (2001) Fragmented meteoroid movement through the planetary atmosphere. J Geophys Res 106:3297–3310
3. Baldwin RB (1963) The measure of the moon. Univ. of Chicago Press, Chicago, pp 6–7
4. Baldwin B, Sheaffer Y (1971) Ablation and breakup of large meteoroids during atmospheric entry. J Geophys Res 76(19):4653–4668
5. Barri NG (2005) A model for the separation of fragmentations of a destroyed meteoroid. Mosc Univ Mech Bull 60(4):20–22
6. Barri NG, Stulov VP (2003) Peculiarities of the fragmentation of Beneshov's bolide. Sol Syst Res 37(4):302–305
7. Barri NG, Stulov VP, Titova LY (2001) Trajectory approximations for observable bolides (the case of fragmentation). Mosc Univ Mech Bull 56(4):23–25

8. Bolotin V (1961) Statistical technology in structural mechanics. Gosstroyizdat, Moscow, 204 pp (in Russian)
9. Borovicka J, Spurny P (1996) Radiation study of two very bright terrestrial bolides and an application to the Comet S-L 9 Collision with Jupiter. Icarus 121:484–510
10. Borovicka J, Popova OP, Nemtchinov IV, Spurny P, Ceplecha Z (1998) Bolides produced by impacts of large meteoroids into the Earth's atmosphere: comparison of theory with observations I. Benesov bolide dynamics and fragmentation. Astron Astrophys 334:713–728
11. Bronshten VA (1981) Physics of meteor phenomenon. Nauka, Moscow, 416 pp (in Russian)
12. Ceplecha Z, Spurny P, Borovicka J, Keclikova J (1993) Atmospheric fragmentation of meteoroids. Astron Astrophys 279:615–626
13. Ceplecha Z, Borovicka J, Elford WG, Revelle DO, Hawkes RL, Porubcan V, Simek M (1997) Big meteoroids and meteors. Submitted to editors of Interplanetary Dust Eds. S. Dermott, B. Gustafson, and E. Grun. The University of Arizona Press, Tucson
14. Divari NB (1959) Events attendant a falling of a meteorite rain and its atmospheric trajectory. In: Sikhote-Alin iron meteorite rain. Academy of Sciences, USSR, pp 26–48 (in Russian)
15. Fadeenko YI (1967) Destruction of meteoric bodies in the atmosphere. Phys Combust Detonation 2:276–280 (in Russian)
16. Fesenkov VG (1951) The orbit of Sikhote-Alin meteorite. Meteoritics 9:27–31
17. Fujiwara A, Cerroni P, Davis D, Ryan E, DiMartino M, Holsapple K, Hausen K (1989) Experiments and scaling laws for catastrophic collisions. Asteroids II. University of Arizona Press, Tucson, pp 240–265
18. Grigorian SS (1979) Motion and disintegration of meteorites in planetary atmospheres. Cosm Res 17:724–743
19. Hawkins GS (1964) The meteor process. Physics and astronomy of meteors, comets and meteorits. McGraw–Hill, New York
20. Hills JG, Goda P (1993) The fragmentation of small asteroids in the atmosphere. Astron J 105(3):1114–1144
21. Ivanov AG, Ryzhanskiy VA (1997) Fragmentation of a med celestial body due to interaction with the atmosphere. Dokl Phys 353(3):334–337
22. Korobeynikov VP, Vlasov VI, Volkov DB (1994) Modeling of the cosmic bodies destruction at movement in planets atmosphere. Math Model 6(8):61–75
23. Krinov EL (1962) The short catalogue of meteorites of the USSR on January, 1st, 1962. Meteoritics 22:114–126
24. Krinov EL (1981) Iron rain. Nauka, Moscow. 192 pp
25. Krinov EL, Fonton SS (1959) The description of meteoric craters, hollows and places of falling of individual fragments on a surfaces. In: Sikhote-Alin iron meteorite rain. Academy of Sciences, USSR, pp 157–303 (in Russian)
26. Krinov EL, Tsvetkov VI (1979) Sikhote-Alin meteorite shower as a classical meteoric fall. Meteoritics 38:19–26
27. Kulakov AL, Stulov VP (1992) Determination of meteoric body parameters by observation data. Astron Bull 26(5):67–75
28. Levin BY (1941) Elements of the physical theory of meteors. Astron J 18(4–5):331–342 (in Russian)
29. Levin BY (1956) Physical theory of meteors and meteoroid substance in the solar system. Academy of Sciences, USSR, 296 pp (in Russian)
30. Levin BY (1963) Fragmentation of meteor bodies. Astron J 40(2):304–311
31. Linderman FA, Dobson JM (1923) A theory of meteors and the density and temperature of the outer atmosphere or which it leads. Proc R Soc A 102:411–437
32. Loh U (1966) Dynamics and thermodynamics of a fall in atmosphere of planets. Mir, Moscow. 276 pp (in Russian)
33. MacCormac RW, Warming RF (1973) Survey of computation methods for three-dimensional inviscid flows with shocks. AGARD Lecture Series No. 64. Advanced in numerical fluid dynamics. P. 5-1-5-20.
34. Martin G (1969) Entrance in the atmosphere. Mir, Moscow. 320 pp
35. McCrosky RE, Shao C, Posen A (1978) Bolides of the Prairie Networks I. The general data and orbits. Meteoritics 37:44–59 (in Russian)
36. McKinley DWR (1961) Physical theory of meteors. Meteor science and engineering. McGraw–Hill, New York
37. Nemchinov IV, Popova OP (1997) The analysis of Sihote-Alin event of 1947 and its comparison with the phenomenon on February, 1st of 1994. Astron Bull 31(5):458–471 (in Russian)
38. Nemchinov IV, Popova OP, Teterev AV (1999) Penetration of large meteoroids into atmosphere: theory and observations. J Eng Phys 72(6):1194–1223
39. Opik EJ (1933) Atomic collisions and radiation of meteors. Acta Comment. Univ Tartuen A 26(2):1–39
40. Opik EJ (1956) Meteor radiation, ionization and atomic luminous efficiency. Proc R Soc A 230(1183):463–501
41. Opik EJ (1958) Physics of meteor flight in the atmosphere. Interscience, New York. 174 pp
42. Passey QR, Melosh HJ (1980) Effects of atmospheric breakup on crater field formation. Icarus 42(2):211–233
43. Popova OP (1997) Definition of parameters of large meteoric bodies by observation data. PhD thesis, Institute of the geospheres dynamics, 198 pp (in Russian)
44. Shuvalov VV, Artemyeva NA, Trubetskaya IA (2000) A modeling of a movement of a destroyed meteoroid with taking into account of evaporation. Sol Syst Res 34(1):49–60
45. Spurny P (1994) Recent fireballs photographed in central Europe. Planet Space Sci 42:157–162
46. Stulov VP (1998) Analytical model of consecutive breakup and ablation of a meteor body in the atmosphere. Astron Bull 32(5):455–458
47. Stulov VP (2000) Determination of parameters of destroyed meteoroids by deceleration in the atmosphere. Astron Bull 34(6):545–549
48. Stulov VP, Titova LY (2001) The comparative analysis of models of a meteoric bodies crushing. Astron Bull 35(4):345–349
49. Stulov VP, Mirskiy VN, Visly AI (1995) The bolides aerodynamics. Nauka, Moscow. Fizmatlit. 240 pp (in Russian)
50. Tsvetkov VI (1983) Connection of fragmentation and scattering of the Sikhote-Alin meteorite shower with meteorite structure. Sol Syst Res 17(2):122–126. English translation
51. Tsvetkov VI, Skripnik AY (1991) Atmospheric fragmentation of meteorites from the standpoint of mechanical strength. Astron Bull 25(3):364–371 (in Russian)
52. Weibull W (1951) A statistical distribution function of wide applicability. J Appl Mech 10:140–147
53. Whipple FL (1943) Meteors and the Earth's atmosphere. Rev Mod Phys 15(3):246–264
54. Whipple FL (1955) The physical theory of meteors VII. On meteor luminosity and ionization. Asteophys J 121(1):241–249
55. Whipple FL (1963) On meteoroids and penetration. J Geophys Res 68(11):4929–4939
56. Zhdan IA (2005) The aerodynamic resistance of the bodies system in a supersonic flow (abstract). Lomonosov conference, MSU, Moscow, p 88 (in Russian)
57. Zhdan IA, Stulov VP, Stulov PV (2004) Aerodynamic interaction of two bodies in a supersonic flow. Dokl Phys 49(5):315–317
58. Zhdan IA, Stulov VP, Stulov PV (2005) 3D configurations of broken body fragments in a supersonic flow. Dokl Phys 50(10):514–518