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Synthesis for robust synchronization of chaotic systems under output feedback control with multiple random delays

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Abstract

Synchronization under output feedback control with multiple random time delays is studied, using the paradigm in nonlinear physics—Chua's circuit. Compared with other synchronization control methods, output feedback control with multiple random delay is superior for a realistic synchronization application to secure communications. Sufficient condition for global stability of delay-dependent synchronization is established based on the LMI technique. Numerical simulations fully support the analytical approach, in spite of the random delays.

Since the seminal work of Pecora and Carroll [1], chaos synchronization has become a topic of a great interest in nonlinear physics [2,3], especially for laser dynamics and electronic circuits [2]. The concept of chaos synchronization in coupled nonlinear systems may lead to an implementation in secure communication [4] in which information masked with chaotic signals can be recovered through a synchronization process.

Time-delay feedback is natural option to control the dynamics of physical systems due to the high frequency involved, switched speeds or memory effects [5]. Its applications to diode laser stabilization and suppression of cardiac arrhythmias have been argued in [6]. Chen and Liu [7] first handled propagation delay in master–slave synchronization schemes and introduced the possibility of applying synchronization to optical communication and has reported that for the two remote chaotic systems the existence of a time delay may destroy synchronization. Delays in real multivariable feedback systems are not necessarily the same for all elements of the system. Therefore the study of systems with multiple time delays is of certain importance. The research on chaos control [8] and synchronization [9] by means of multiple delay feedback control has recently received considerable attention. A motivating application is in secure communication. Note that only constant delay is considered in most of work on synchronization for systems with both single and multiple delays. However, actual delays in many physical experiments or technical applications might be time-varying or randomly disturbed. To our best knowledge, Masoller and Marti [10] reported the novel work in the literature on synchronization for systems with random relays. Unfortunately, their methodology is not applicable to secure communication.

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In their work, the synchronized state is a homogeneous steady state, e.g., the chaotic dynamics is suppressed as soon as the occurrence of coupling relation between the chaos systems. This characteristic conflicts with the inherent property of the application of synchronization to secure communication.

Furthermore, it should be noted that output feedback control is commonly used in industry control [11] due to the fact that measuring all the state variables of a system is inconvenient or even impossible in many practical situation. Having in mind the aforesaid potential characteristics of synchronization in realistic application to secure communications, we investigate chaos synchronization via output feedback control with multiple random delays in this paper. The free-weighting matrix approach [12] and the S-procedure [13] are employed to derive a delay-dependent synchronization criterion for the coupled Chua's circuits. The control parameters is solvable by using the Linear Matrix Inequality (LMI) technique. Even in the case of single constant delay, the criterion is less conservative than the existing result [14].

Consider a master–slave type of coupled Chua's chaotic circuit systems

$$\begin{aligned} &\mathscr{M}: \begin{cases} \dot{x}(t) = Ax(t) + B\sigma(C^{\mathrm{T}}x(t)), \\ y(t) = Hx(t), \end{cases} \\ &\mathscr{S}: \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B\sigma(C^{\mathrm{T}}\hat{x}(t)) + u(t), \\ \hat{y}(t) = H\hat{x}(t), \end{cases} \\ &\mathscr{C}: \left\{ u(t) = K \begin{bmatrix} y_1(t-\tau_1(t)) - \hat{y}_1(t-\tau_1(t)) \\ y_2(t-\tau_2(t)) - \hat{y}_2(t-\tau_2(t)) \end{bmatrix} \right. \end{aligned} \tag{1}$$

with master system $\mathscr{M}$, slave system $\mathscr{S}$ and controller $\mathscr{C}$.

$$A = \begin{bmatrix} -18/7 & 9 & 0 \\ 1 & -1 & 1 \\ 0 & -14.28 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 27/7 \\ 0 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

and $\sigma(v) = (1/2)(|v+1| - |v-1|)$ belongs to sector $[0,1]$, i.e., $\sigma(v)(\sigma(v) - v) \leqslant 0$. Suppose that the output matrix $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. For controller $\mathscr{C}$, the gain $K \in R^{3\times 2}$. The random delays $\tau_j(t) = \tau_{0j} + d_j\epsilon_j$, $j = 1,2$, where τ_{0j} and d_j are positive constant, satisfying $\tau_{0j} - d_j \geqslant 0$, and ϵ_j is uniformly distributed random noise and bounded by $|\epsilon_j| \leqslant 1$. Assume that $0 \leqslant \tau_j(t) \leqslant \tau_j$.

Let the synchronization error of system (1) be $e(t) = x(t) - \hat{x}(t)$. The error dynamic system is given by

$$\begin{cases} \dot{e}(t) = Ae(t) + B\eta(C^{\mathrm{T}}e(t), \hat{x}(t)) - u(t), \\ u(t) = KE_1He(t-\tau_1(t)) + KE_2He(t-\tau_2(t)), \end{cases} \tag{2}$$

where $\eta(C^{\mathrm{T}}e(t), \hat{x}(t)) = \sigma(C^{\mathrm{T}}e + C^{\mathrm{T}}\hat{x}) - \sigma(C^{\mathrm{T}}\hat{x})$, satisfying $\eta(C^{\mathrm{T}}e, \hat{x})(\eta(C^{\mathrm{T}}e, \hat{x}) - C^{\mathrm{T}}e) \leqslant 0$. $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

In this paper, we will study robust synthesis of output feedback control with multiple random delays to achieve synchronization of system (1), i.e., $e(t) \to 0$ as $t \to \infty$. Let us now investigate robust delay-dependent synchronization criterion for random delays. Construct the following Lyapunov–Krasovskii functional:

$$V(t) = e^{\mathrm{T}}(t)Pe(t) + 2\lambda \int_0^{C^{\mathrm{T}}e(t)} \sigma(s)\,\mathrm{d}s + \sum_{j=1}^{2} \int_{-\tau_j}^{0} \int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)Z_j\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta \tag{3}$$

where $P = P^{\mathrm{T}} > 0$, $Z_j = Z_j^{\mathrm{T}} > 0$ and $\lambda \geqslant 0$ are to be determined. With a standard application of the S-procedure [13], we have

$$0 \leqslant \widetilde{S} = 2(e^{\mathrm{T}}CS_1\sigma - \sigma^{\mathrm{T}}S_1\sigma + e^{\mathrm{T}}CS_2\eta - \eta^{\mathrm{T}}S_2\eta), \tag{4}$$

where $S_j > 0$, $j = 1,2$. For a given scalar δ, any appropriately dimensional matrices R, N_j and W_j, $j = 1,\ldots,6$, the following relationships follow from (2) and the Leibniz–Newton formula, respectively,

$$0 = \Delta_1 = 2[e^{\mathrm{T}}R + \delta\dot{e}^{\mathrm{T}}R] \cdot [\dot{e} - Ae - B\eta + ME_1He(t-\tau_1(t)) + ME_2He(t-\tau_2(t))], \tag{5}$$

and

$$0 = \Delta_2 = 2\xi^{\mathrm{T}}(t)N \cdot \left[e(t) - e(t-\tau_1(t)) - \int_{t-\tau_1(t)}^{t} \dot{e}(s)\,\mathrm{d}s \right], \tag{6}$$

$$0 = \Delta_3 = 2\xi^{\mathrm{T}}(t)W \cdot \left[e(t) - e(t-\tau_2(t)) - \int_{t-\tau_2(t)}^{t} \dot{e}(s)\,\mathrm{d}s \right], \tag{7}$$

where $\xi^{\mathrm{T}}(t) = [e^{\mathrm{T}}, \dot{e}^{\mathrm{T}}, \sigma^{\mathrm{T}}, \eta^{\mathrm{T}}, e^{\mathrm{T}}(t-\tau_1(t)), e^{\mathrm{T}}(t-\tau_2(t))]$. $N = [N_1^{\mathrm{T}}, \ldots, N_6^{\mathrm{T}}]^{\mathrm{T}}$ and $W = [W_1^{\mathrm{T}}, \ldots, W_6^{\mathrm{T}}]^{\mathrm{T}}$ are considered as the free-weighting matrices [12]. Let $K_0 = RK$. Taking the time derivative of $V(t)$ and adding the terms on the right hand side of Eqs. (4)–(7) into $\dot{V}(t)$, one obtains

$$\begin{aligned}
\dot{V}(t) &\leqslant \widetilde{S} + \Delta_1 + \Delta_2 + \Delta_3 + 2e^{\mathrm{T}}P\dot{e} + 2\sigma^{\mathrm{T}}\lambda C^{\mathrm{T}}\dot{e} + \sum_{j=1}^{2}\left(\tau_j \dot{e}^{\mathrm{T}} Z_j \dot{e} - \int_{t-\tau_j}^{t} \dot{e}^{\mathrm{T}}(s) Z_j \dot{e}(s)\,\mathrm{d}s \right) \\
&\leqslant \xi^{\mathrm{T}}(\Phi + \tau_1 N Z_1^{-1} N^{\mathrm{T}} + \tau_2 W Z_2^{-1} W^{\mathrm{T}})\xi - \int_{t-\tau_1(t)}^{t} [\xi^{\mathrm{T}} N + \dot{e}^{\mathrm{T}}(s) Z_1] Z_1^{-1} [N^{\mathrm{T}}\xi + Z_1 \dot{e}(s)]\,\mathrm{d}s \\
&\quad - \int_{t-\tau_2(t)}^{t} [\xi^{\mathrm{T}} W + \dot{e}^{\mathrm{T}}(s) Z_2] Z_2^{-1} [W^{\mathrm{T}}\xi + Z_2 \dot{e}(s)]\,\mathrm{d}s,
\end{aligned} \tag{8}$$

where $\Phi = (\phi_{ij})$, $i,j = 1,\ldots,6$, is a symmetric matrix with $\phi_{ij}^{\mathrm{T}} = \phi_{ji}$ and

$$\begin{aligned}
&\phi_{11} = -RA + N_1 + W_1 + (-RA + N_1 + W_1)^{\mathrm{T}}, \\
&\phi_{12} = P + R - \delta A^{\mathrm{T}} R^{\mathrm{T}} + N_2^{\mathrm{T}} + W_2^{\mathrm{T}}, \\
&\phi_{13} = CS_1 + N_3^{\mathrm{T}} + W_3^{\mathrm{T}}, \\
&\phi_{14} = CS_2 - RB + N_4^{\mathrm{T}} + W_4^{\mathrm{T}}, \\
&\phi_{15} = K_0 E_1 H - N_1 + N_5^{\mathrm{T}} + W_5^{\mathrm{T}}, \\
&\phi_{16} = K_0 E_2 H - W_1 + N_6^{\mathrm{T}} + W_6^{\mathrm{T}}, \\
&\phi_{22} = \delta R + \tau_1 Z_1 + \tau_2 Z_2 + (\delta R + \tau_1 Z_1 + \tau_2 Z_2)^{\mathrm{T}}, \\
&\phi_{23} = C\lambda, \\
&\phi_{24} = -\delta RB, \\
&\phi_{25} = \delta K_0 E_1 H - N_2, \\
&\phi_{26} = \delta K_0 E_2 H - W_2, \\
&\phi_{jj} = -2S_{j-2}, \quad j = 3,4, \\
&\phi_{34} = 0, \\
&\phi_{j5} = -N_j, \quad j = 3,4,5, \\
&\phi_{j6} = -W_j, \quad j = 3,4,5, \\
&\phi_{55} = -N_5 - N_5^{\mathrm{T}}, \\
&\phi_{56} = -W_5 - N_6^{\mathrm{T}}, \\
&\phi_{66} = -W_6 - W_6^{\mathrm{T}}.
\end{aligned}$$

One can show by the Schur complement [13] that $(\Phi + \tau_1 N Z_1^{-1} N^{\mathrm{T}} + \tau_2 W Z_2^{-1} W^{\mathrm{T}}) < 0$ is equivalent to

$$\widetilde{\Phi} = \begin{bmatrix} \Phi & \tau_1 N & \tau_2 W \\ \tau_1 N^{\mathrm{T}} & -\tau_1 Z_1 & 0 \\ \tau_2 W^{\mathrm{T}} & 0 & -\tau_2 Z_2 \end{bmatrix} < 0. \tag{9}$$

Thus, $\dot{V}(t) < -\varepsilon\|e(t)\|^2$ for a sufficiently small $\varepsilon > 0$ if $\widetilde{\Phi} < 0$, which ensures the asymptotic stability of equilibrium point $e = 0$. Note that with a given scalar δ, LMI (9) is solvable for the variables $P, K_0, R, \lambda, W, N, S_j$ and Z_j, $j = 1,2$, by using the LMI technique. Furthermore, R is nonsingular if LMI (9) is feasible. Thus, one can obtain the gain matrix $K = R^{-1}K_0$.

For example, taking $\delta = 0.5$, we can obtain the robust domain of the time-varying delays for synchronization, as shown in Fig. 1 where τ_1, τ_2 are the upper bounds of $\tau_1(t)$, $\tau_2(t)$, respectively. Assume that for random delays $\tau_j(t)$, $j = 1,2$, we have $\tau_{01} = d_1 = 0.06$, $\tau_{02} = d_2 = 0.6$. ϵ_j, $j = 1,2$, are uniformly distributed random noise and bounded by

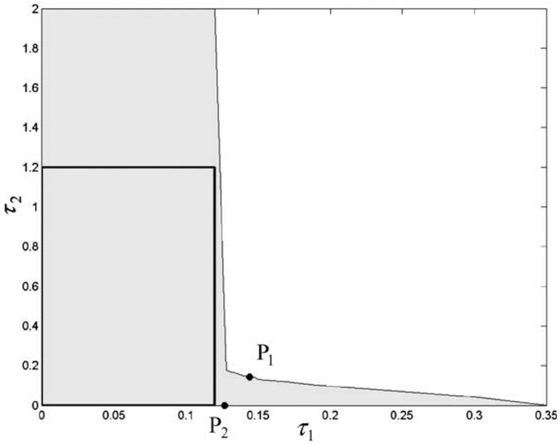

Fig. 1. The gray domain represents the robust synchronization domain obtained by solving LMI (9). τ_1 and τ_2 are the upper bounds of $\tau_1(t)$ and $\tau_2(t)$, respectively. The rectangle refers to the randomly distributed domain of delays $\tau_1(t)$ and $\tau_2(t)$ used in numerical experiment. For single constant delay, the point P_1 with $\tau_1 = \tau_2 = 0.144$ corresponds to the upper bound in the case of $H = [1, 0, 0; 0, 1, 0]$; the point P_2 with $\tau_1 = 0.127$ and $\tau_2 = 0$ stands for the upper bound in the case of $H = [1, 0, 0]$.

$|\epsilon_j| \leqslant 1$, independently generated by computer. Although $\tau_j(t)$, $j = 1, 2$, are random, they lie in the stable domain marked by the rectangle in Fig. 1. Setting $\tau_1 = 0.12, \tau_2 = 1.2$, it follows from LMI (9) that

$$K = \begin{bmatrix} 4.0515 & 0.0037 \\ 0.9838 & 0.0023 \\ -3.9837 & -0.0022 \end{bmatrix}. \tag{10}$$

With the gain K given in (10) and the initial conditions $x(0) = (-0.2, -0.33, 0.2)^{\mathrm{T}}$ and $\hat{x}(0) = (0.5, -0.1, 0.66)^{\mathrm{T}}$, the master–slave synchronization of chaotic system (1) under output feedback control with the two given random delay series is simulated. In Fig. 2(a), the time history of the synchronization error norm shows that the synchronization is achieved quickly. Fig. 2(b) displays the time history of the first state variables of the master system and the slave system. Fig. 2(c) exhibits the synchronized chaotic attractors.

It should be noted that the criterion covers single constant delay feedback as a special case, e.g., $\tau_1(t) = \tau_2(t) = \tau$. The point P_1 with $\tau_1 = \tau_2 = 0.144$ in Fig. 1 stands for the upper bound of the single constant delay in the case of $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Furthermore, the controller studied in [14] is a special case of our controller $\mathscr{C}$ with $E_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

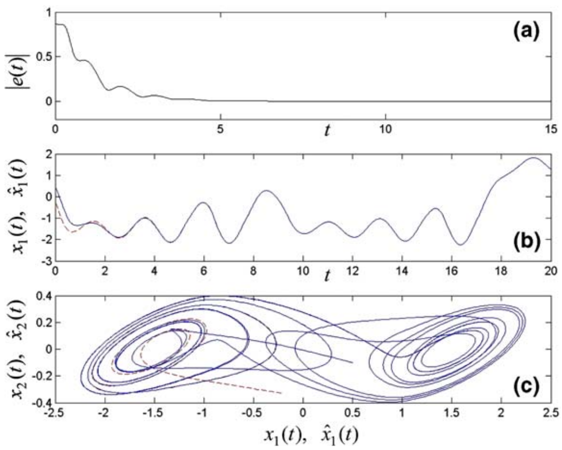

Fig. 2. Synchronization results under output feedback control with multiple random time delays. (a) The time history of the synchronization error norm. (b) The time history of the first state variables of the master system and the slave system (master system: dashed line; slave system: solid line). (c) A view of the synchronized chaotic attractors.

and $\tau_2(t) = 0$, i.e., $H = [1, 0, 0]$, corresponding to the output of system (1) with the first state variable only. One can obtain the upper bound of the single constant delay, $\tau_1 = 0.127$, marked by the point P_2 in Fig. 1. In comparison to the upper bound derived by Yalcin et al. [14] which is only 0.039, the criterion is less conservative owe to Eqs. (5)–(7) added into $\dot{V}(t)$ to improve the estimation of the upper bound of delays.

We have investigated the master–slave synchronization of Chua's circuit via multiple random delay output feedback control. With the help of the free-weighting matrix approach and the S-procedure, a multiple-delay-dependent synchronization criterion is established based on the LMI technique. This research is of certain practical importance to secure communication because the control method can provide more flexibility and opportunities in practical application.

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References

[1] Pecora LM, Carroll TL. Phys Rev Lett 1990;64:821.
[2] Chen G, Dong X. From chaos to order-perspectives, methodologies, and applications. Singapore: World Scientific; 1998. Special issue on chaos synchronization. Chaos 1997;7(N4).
[3] Morgul O, Solak E. Phys Rev E 1996;54:4803;
Suykens JAK et al. IEEE Trans Circuits Syst I 1999;46:841;
Yassen MT. Chaos, Solitons & Fractals 2005;23:1527;
Wen GL, Xu D. Phys Lett A 2004;333:420;
Yang XS. Phys Lett A 1999;260:340;
Wen GL, Xu D. Chaos, Solitons & Fractals 2005;26:71;
Hua CC, Guan XP. Phys Lett A 2005;334:382.
[4] Itoh M, Murakami H. IEICE Trans Fund 1995;E78-A(3):285;
Kawata J et al. IEICE Trans Fund 1999;E82-A(7):1322;
Cuomo KM, Oppenheim AV. Phys Rev Lett 1993;71(1):65.
[5] Hale JK, Lunel SMV. Introduction to functional differential equations. New York: Springer; 1993.
[6] Bleich ME et al. Phys Rev E 1997;55:2119;
Hall K et al. Phys Rev Lett 1997;78:4518;
Gauthier DJ, Socolar JES. Phys Rev Lett 1997;79:4938.
[7] Chen HF, Liu JM. IEEE J Quantum Elect 2000;36:27.
[8] Ahlborn A, Parlitz U. Phys Rev Lett 2004;93:264101;
Socolar JES, Gauthier DJ. Phys Rev E 1998;57:6589.
[9] Shahverdiev EM. Phys Rev E 2004;70:067202;
Burić N, Todorović D. Phys Rev E 2003;68:066218.
[10] Masoller C, Martí AC. Phys Rev Lett 2005;94:134102.
[11] Luyben WL. Process modeling, simulation and control for chemical engineers. 2nd ed. USA: McGraw-Hill Publishing Company; 1990.
[12] He Y et al. Syst Contr Lett 2004;51:57;
He Y et al. IEEE Trans Automat Contr 2004;46:828;
Wu M et al. IEEE Trans Automat Contr 2004;49:2266;
Wu M et al. Automatica 2004;40:1435;
He Y et al. J Comput Appl Math 2005;176:371.
[13] Boyd S et al. Linear matrix inequalities in system and control theory. SIAM: Stud Appl Math 1994;vol. 15.
[14] Yalcin ME et al. Int J Bifurcat Chaos 2001;11:1707.