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SHORT-RANGE CORRELATIONS AND INCLUSIVE NUCLEON–NUCLEUS CROSS SECTIONS[†]

A. KLAR*

Institut für Theoretische Physik der Universität Heidelberg

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Abstract: The effect of short-range nucleon–nucleon correlations on inclusive inelastic cross sections is studied within the framework of Glauber theory. The correction term resulting from two-particle correlations corresponds to scattering from a deuteron-like cluster. The net effect is found to be negligibly small.

1. Is the nucleus a free gas of nucleons?

Although we can negate this question from our knowledge of the nucleus, there are many experimental nuclear physics data which can be reproduced with the assumption of uncorrelated nucleons. Inclusive cross sections of reactions like

$$\mathrm{N} + {}^{A}\mathrm{Z} \rightarrow \mathrm{N}' + \mathrm{X}\,,$$

at laboratory energies above a few GeV of the incident nucleon N, are successfully described within the framework of Glauber theory [1]) using the independent particle model for the nucleus [2]).

On the other hand, strong short-range position anticorrelations are believed to act among the nucleons inside the nucleus: first because nucleons being fermions can never occupy exactly the same point in space; second, nucleons interact via a strongly repulsive potential of range $r_c \cong 0.6$ fm, which prevents them from approaching each other at shorter distances. Strong changes in the nuclear A-body density matrix are expected over this range r_c.

The reaction proceeds via multiple nucleon–nucleon collisions. At energies above 1 GeV the average momentum transfer q_0 in a single collision is roughly a constant of $q_0 = 400$ MeV/c. Comparing this to the inverse of the hard-core range $1/r_c \cong$ 330 MeV/c, one might expect that short-range correlations influence the nucleon–nucleon cross section in the nucleus.

The present work shows that for the reactions considered here there is no such effect. Inclusive inelastic nucleon–nucleus cross sections at energies near 1 GeV show negligibly small contributions due to short-range correlations. To conclude

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* Present address: Department of Physics, Iowa State University, Ames, Iowa 50011, USA.

this result it was necessary to treat correlations in the computation of the cross section. Following Glauber [1]) and especially Hüfner [3]), who advise a cumulant expansion, which is well known in statistical mechanics [4]), the cross section is derived in a correlation expansion.

The second section displays this expansion in m-particle correlation functions and shows that two-particle correlations can be represented by a term which describes the scattering of the projectile on a "deuteron-like" cluster.

The different kinematical conditions of the scattering from a two-particle system, suggest that via the inclusion of this kind of correlation the scattered particle can reach parts of phase space which are forbidden in pure nucleon–nucleon collisions. For this reason Fujita [5]) introduced scattering on clusters in the nucleus and the present derivation shows that such cluster scattering really occurs.

In sect. 3 an estimate of the influence of this term on inclusive cross sections is made and it is observed that inside the range of validity of Glauber theory its effect is of order 10^{-2} compared to the leading term, which corresponds to the calculation within the independent particle model.

2. Cluster expansion

Starting from the expression for the inclusive inelastic cross section in Glauber theory a cumulant expansion is achieved, which includes as leading term the result of the independent particle model and displays the corrections due to n-particle correlations.

2.1. RESULT OF THE INDEPENDENT PARTICLE MODEL

For a process of the type

$$\mathrm{N}+{}^{A}\mathrm{Z}\rightarrow \mathrm{N}'+\mathrm{X}\,,$$

where the projectile nucleon N has acquired transverse momentum $\boldsymbol{q}_\perp$ and the change in longitudinal momentum is neglected, the cross section is given by (using closure on the final states of the nucleus):

$$\frac{\mathrm{d}\sigma_{\mathrm{in}}}{\mathrm{d}^2q_\perp}=\int\frac{\mathrm{d}^2b\,\mathrm{d}^2b'}{(2\pi)^2}\,\mathrm{e}^{i\boldsymbol{q}_\perp\cdot(\boldsymbol{b}-\boldsymbol{b}')}\Big\{\langle 0|\prod_{i=1}^{A}(1-\Gamma(\boldsymbol{b}-\boldsymbol{s}_i))\prod_{j=1}^{A}(1-\Gamma^*(\boldsymbol{b}'-\boldsymbol{s}_j))|0\rangle$$
$$-\langle 0|\prod_{i=1}^{A}(1-\Gamma(\boldsymbol{b}-\boldsymbol{s}_i))|0\rangle\langle 0|\prod_{j=1}^{A}(1-\Gamma^*(\boldsymbol{b}'-\boldsymbol{s}_j)|0\rangle\Big\}\,. \qquad (1)$$

With the free nucleon–nucleon scattering amplitude $f_{\mathrm{NN}}(\boldsymbol{q}_\perp)$ the profile functions Γ are defined by

$$\Gamma(\boldsymbol{b})=\frac{1}{2\pi ik}\int \mathrm{d}^2q_\perp\,\mathrm{e}^{i\boldsymbol{q}_\perp\cdot\boldsymbol{b}}f_{\mathrm{NN}}(\boldsymbol{q}_\perp)\,.$$

The notation $\langle 0|\cdots|0\rangle$ indicates the target ground state expectation value of the enclosed operators in coordinate space, i.e.

$$\langle 0|M|0\rangle = \int \mathrm{d}^3x_1 \cdots \mathrm{d}^3x_A \rho_A(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_A) M(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_A)\,,$$

and $\boldsymbol{x}_i = (\boldsymbol{s}_i, z_i)$.

Expression (1) is usually simplified by the assumption of the independent particle model

$$\rho_A(\boldsymbol{x}_1 \ldots \boldsymbol{x}_A) = \prod_{i=1}^{A} \rho^{(1)}(\boldsymbol{x}_i)$$

where $\rho^{(1)}(\boldsymbol{x}_i)$ is the single-particle density measured in elastic electron scattering and is normalized to unity. This simplification yields in the limit of large A the famous formula of ref. [2]):

$$\frac{\mathrm{d}\sigma_{\mathrm{in}}}{\mathrm{d}^2q_\perp} = \sum_{n=1}^{\infty} \sigma_n P_n(\boldsymbol{q}_\perp)\,, \tag{2.1}$$

$$\sigma_n = \frac{1}{n!} \int \mathrm{d}^2B (A\sigma_{\mathrm{el}}^{\mathrm{NN}} T(\boldsymbol{B}))^n\, \mathrm{e}^{-A\sigma_{\mathrm{tot}}^{\mathrm{NN}} T(\boldsymbol{B})}\,, \tag{2.2}$$

$$P_n(\boldsymbol{q}_\perp) = \int \frac{\mathrm{d}^2\beta}{(2\pi)^2}\, \mathrm{e}^{i\boldsymbol{q}_\perp\cdot\boldsymbol{\beta}} w^n(\boldsymbol{\beta})\,, \tag{2.3}$$

$$w(\boldsymbol{\beta}) = \frac{1}{\sigma_{\mathrm{el}}^{\mathrm{NN}}} \int \mathrm{d}^2q_\perp\, \mathrm{e}^{i\boldsymbol{q}_\perp\cdot\boldsymbol{\beta}} \frac{\mathrm{d}\sigma_{\mathrm{NN}}}{\mathrm{d}^2q_\perp}\,. \tag{2.4}$$

Here $\mathrm{d}\sigma_{\mathrm{NN}}/\mathrm{d}^2q_\perp$, $\sigma_{\mathrm{el}}^{\mathrm{NN}}$ and $\sigma_{\mathrm{tot}}^{\mathrm{NN}}$ are the respective free nucleon–nucleon differential, elastic and total cross sections, and $AT(\boldsymbol{B}) = A\int_{-\infty}^{+\infty} \mathrm{d}z\rho^{(1)}(\boldsymbol{B}, z)$ is the thickness function of the target.

2.2. m-PARTICLE DENSITIES

The operators which have to be evaluated are sums of products of different numbers of single-particle operators,

$$M = \prod_{i=1}^{A} (1-\Gamma_i) = 1 - \sum_{i=1}^{A} \Gamma_i + \sum_{\substack{i,j\\ i\neq j}}^{A} \Gamma_i\Gamma_j - \sum_{\substack{i,j,k\\ i\neq j;\, j\neq k\\ i\neq k}}^{A} \Gamma_i\Gamma_j\Gamma_k + \ldots\,.$$

Therefore the following m-particle densities are needed:

$$\rho^{(1)}(\boldsymbol{x}_1) = \frac{1}{A} \int \mathrm{d}^3y_1\, \mathrm{d}^3y_2 \cdots \mathrm{d}^3y_A \sum_{i=1}^{A} \delta(\boldsymbol{x}_1 - \boldsymbol{y}_i) \rho_A(\boldsymbol{y}_1, \boldsymbol{y}_2 \cdots \boldsymbol{y}_A)\,,$$

$$\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \frac{1}{A(A-1)} \int d^3y_1 \cdots d^3y_A \sum_{\substack{i,j=1\\ i\neq j}}^{A} \delta(\boldsymbol{x}_1 - \boldsymbol{y}_i)\delta(\boldsymbol{x}_2 - \boldsymbol{y}_j)\rho_A(\boldsymbol{y}_1 \cdots \boldsymbol{y}_A)\,,$$

$$\vdots$$

$$\rho^{(m)}(\boldsymbol{x}_1, \boldsymbol{x}_2 \cdots \boldsymbol{x}_m)$$
$$= \frac{(A-m)!}{A!} \int d^3y_1 \cdots d^3y_A \sum_{\substack{\alpha_1,\alpha_2\cdots\alpha_m\\ \text{all different}}} \delta(\boldsymbol{x}_1 - \boldsymbol{y}_{\alpha_1}) \cdots \delta(\boldsymbol{x}_m - \boldsymbol{y}_{\alpha_m})\rho_A(\boldsymbol{y}_1 \cdots \boldsymbol{y}_A)\,,$$

$$\vdots$$

$$\rho^{(A)}(\boldsymbol{x}_1 \cdots \boldsymbol{x}_A) = \rho_A(\boldsymbol{x}_1 \ldots \boldsymbol{x}_A)\,.$$

If $\rho_A$is normalized to unity then all ρ_m are, too. In addition we have

$$\int d^3x_m \rho^{(m)}(\boldsymbol{x}_1, \boldsymbol{x}_2 \cdots \boldsymbol{x}_m) = \rho^{(m-1)}(\boldsymbol{x}_1, \boldsymbol{x}_2 \cdots \boldsymbol{x}_{m-1})\,.$$

With these quantities the m-particle correlation densities are defined

$$\rho^{(2)}_{(c)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) - \rho^{(1)}(\boldsymbol{x}_1)\rho^{(1)}(\boldsymbol{x}_2)\,,$$

$$\begin{aligned}\rho^{(3)}_{c}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \rho^{(3)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) &- \rho^{(1)}(\boldsymbol{x}_1)\rho^{(2)}(\boldsymbol{x}_2, \boldsymbol{x}_3)\\ &- \rho^{(1)}(\boldsymbol{x}_2)\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_3) - \rho^{(1)}(\boldsymbol{x}_3)\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2)\\ &+ 2\rho^{(1)}(\boldsymbol{x}_1)\rho^{(1)}(\boldsymbol{x}_2)\rho^{(1)}(\boldsymbol{x}_3)\,.\end{aligned}$$

These correlation densities are defined such that the m-particle correlation function vanishes in the case where there are at most $(m-1)$ particle correlations present. In particular, in the independent particle model $\rho^{(m)}_c \equiv 0$ for all $m \geqslant 2$.

2.3. CUMULANT EXPANSION

Following Glauber [1]) the following auxiliary functions are defined:

$$\mathscr{L}(\boldsymbol{b}, \lambda) = \ln \langle 0| \prod_{i=1}^{A} (1 - \lambda\Gamma(\boldsymbol{b} - \boldsymbol{s}_i))|0\rangle\,,$$

$$\begin{aligned}J(\boldsymbol{b}, \lambda, \boldsymbol{b}', \mu) = \ln \langle 0| \prod_{i=1}^{A} (1 - \lambda\Gamma(\boldsymbol{b} - \boldsymbol{s}_i)) \prod_{j=1}^{A} (1 - \mu\Gamma^*(\boldsymbol{b}' - \boldsymbol{s}_j))|0\rangle - \mathscr{L}(\boldsymbol{b}, \lambda)\\ - \mathscr{L}^*(\boldsymbol{b}', \mu)\,.\end{aligned}$$

With their help (1) reads

$$\frac{d\sigma_{\text{in}}}{d^2q_\perp} = \int \frac{d^2b\, d^2b'}{(2\pi)^2}\, e^{iq_\perp\cdot(\boldsymbol{b}-\boldsymbol{b}')}\, e^{\mathscr{L}(\boldsymbol{b},1)+\mathscr{L}^*(\boldsymbol{b}',1)}\{e^{J(\boldsymbol{b},1,\boldsymbol{b}',1)} - 1\}\,.$$

Because the expansion will be done separately for $\mathscr{L}+\mathscr{L}^*$ and for J, one has to keep in mind the Glauber-type conservation of probability. From the optical theorem it follows that for elastic NN scattering

$$2\,\mathrm{Re}\,\Gamma(\boldsymbol{b}) = |\Gamma(\boldsymbol{b})|^2 .$$

From this relation we have

$$J(\boldsymbol{b}, 1, \boldsymbol{b}, 1) = -2\,\mathrm{Re}\,\mathscr{L}(\boldsymbol{b}, 1) , \tag{3}$$

which expresses the conservation of probability in the cross section. In the separate expansions of $\mathscr{L}+\mathscr{L}^*$ and J we have to keep in mind that the result up to two-particle correlations has to fulfill the above condition. The expansion is formally carried out as a Taylor expansion with respect to (μ, λ) around the point $(0, 0)$. The result up to the two-particle correlation function is given by (see the appendix):

$$\begin{aligned} \mathscr{L}(\boldsymbol{b}, 1)+\mathscr{L}^*(\boldsymbol{b}', 1) &= -A(1|\Gamma_x+\Gamma_x^*|1)+\tfrac{1}{2}A(A-1)(c|\Gamma_x\Gamma_y+\Gamma_x^*\Gamma_y^*|c)+\dots , \\ J(\boldsymbol{b}, 1, \boldsymbol{b}', 1) &= A(1|\Gamma_x\Gamma_x^*|1)+\tfrac{1}{2}A(A-1) \\ &\quad\times(c|(\Gamma_x+\Gamma_y-\Gamma_x\Gamma_y)(\Gamma_x^*+\Gamma_y^*-\Gamma_x^*\Gamma_y^*)|c)+\dots , \end{aligned} \tag{4}$$

where the abbreviations

$$(1|M|1) = \int \mathrm{d}^3x\rho^{(1)}(\boldsymbol{x})M(\boldsymbol{x}) ,$$

$$(c|M|c) = \int \mathrm{d}^3x\, \mathrm{d}^3y\rho_c^{(2)}(\boldsymbol{x}, \boldsymbol{y})M(\boldsymbol{x}, \boldsymbol{y}) ,$$

are used to indicate the respective expectation values and

$$\Gamma_x = \Gamma(\boldsymbol{b}-\boldsymbol{s}_x) , \qquad \Gamma_x^* = \Gamma^*(\boldsymbol{b}'-\boldsymbol{s}_x) , \qquad \boldsymbol{x} = (\boldsymbol{s}_x, z_x) .$$

In the independent particle model $[\rho_c^{(2)} \equiv 0]$ we get, using the short range of Γ as compared to $\rho^{(1)}$, and $\boldsymbol{B} = \frac{1}{2}(\boldsymbol{b}+\boldsymbol{b}')$, $\boldsymbol{\beta} = \boldsymbol{b}-\boldsymbol{b}'$,

$$\begin{aligned} -A(1|\Gamma_x+\Gamma_x^*|1) &= -AT(\boldsymbol{b})\frac{2\pi}{ik}f_{\mathrm{NN}}(0)+AT(\boldsymbol{b}')\frac{2\pi}{ik}f_{\mathrm{NN}}^*(0) \\ &\approx -A\sigma_{\mathrm{tot}}^{\mathrm{NN}}T(\boldsymbol{B}) , \\ A(1|\Gamma_x\Gamma_x^*|1) &= AT(\boldsymbol{B})\int \mathrm{d}^2q\, \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{\beta}}\frac{\mathrm{d}\sigma_{\mathrm{NN}}}{\mathrm{d}^2q} \\ &= AT(\boldsymbol{B})\sigma_{\mathrm{el}}^{\mathrm{NN}}w(\boldsymbol{\beta}) , \end{aligned}$$

so that

$$\frac{\mathrm{d}\sigma_{\mathrm{in}}}{\mathrm{d}^2q_\perp} = \int \mathrm{d}^2B\frac{\mathrm{d}^2\beta}{(2\pi)^2}\mathrm{e}^{i\boldsymbol{q}_\perp\cdot\boldsymbol{\beta}}\,\mathrm{e}^{-A\sigma_{\mathrm{tot}}^{\mathrm{NN}}T(\boldsymbol{B})}[\mathrm{e}^{A\sigma_{\mathrm{el}}^{\mathrm{NN}}T(\boldsymbol{B})w(\boldsymbol{\beta})}-1] ,$$

which after expansion of the exponential in the brackets exactly coincides with (2). For these two terms we see immediately that the unitarity condition (3) is fulfilled. That the correction terms also fulfill (3) is easily checked by setting $\boldsymbol{b} = \boldsymbol{b}'$ and replacing $|\Gamma|^2$ by 2 Re Γ everywhere. Hüfner [3]) performed the same expansion but missed unitarity because he only took terms quadratic in Γ, which do not suffice.

The correct additional term in the expansion of J has also a simpler interpretation. Remembering the Glauber result for the cross-section for elastic and inelastic scattering of a nucleon on a two-particle system with groundstate $|\mathrm{D}\rangle$

$$\frac{\mathrm{d}\sigma_{\mathrm{ND}}}{\mathrm{d}^2 q_\perp} = \int \frac{\mathrm{d}^2 b\, \mathrm{d}^2 b'}{(2\pi)^2}\, \mathrm{e}^{i\boldsymbol{q}_\perp\cdot(\boldsymbol{b}-\boldsymbol{b}')} \langle \mathrm{D}|(\Gamma_1+\Gamma_2-\Gamma_1\Gamma_2)(\Gamma_1^* + \Gamma_2^* - \Gamma_1^*\Gamma_2^*)|\mathrm{D}\rangle ,$$

we observe that the correction term describes the scattering of a nucleon on a two-particle system with density ρ_c. So this term would also be gained in a calculation of the scattering of a nucleon on a gas of "quasi-deuterons", the c.m. of which moves freely inside the nuclear region with intrinsic size of r_c.

Because of this obvious interpretation of the above result, we believe that the role of correlations in the nucleus can be represented by scattering on nucleon clusters.

3. Evaluation of the correction term

To estimate the influence of the correction term on the inclusive inelastic cross section, one needs to specify ρ_c. In an approximate way the density

$$\rho_c = \rho(\boldsymbol{R})c(\boldsymbol{r}) = \left(\frac{2\alpha}{\pi}\right)^{3/2} \mathrm{e}^{-2\alpha \boldsymbol{R}^2} \left(\frac{\alpha}{2\pi}\right)^{3/2} \mathrm{e}^{-\gamma r^2/2}$$

$$\boldsymbol{R} = \tfrac{1}{2}(\boldsymbol{x}_1 + \boldsymbol{x}_2) , \qquad \boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2 ,$$

$$\alpha = 1.5/\langle r^2\rangle , \qquad \langle r^2\rangle^{1/2} = r_0 A^{1/3} , \qquad r_0 = 1.2\ \mathrm{fm} ,$$

$$\gamma = 1.5/r_c^2 , \qquad r_c = 0.6\ \mathrm{fm} ,$$

represents the conditions for the two-particle correlation function. It lacks the condition $\int \mathrm{d}^3 x_1\, \mathrm{d}^3 x_2 \rho_c(\boldsymbol{x}_1, \boldsymbol{x}_2) = 0$, but that this has no effect on the results has been checked with a similar distribution which obeys the above condition. In general $\rho(\boldsymbol{R})$ should be the normalized density of the c.m. of two nucleons. If the single-particle density $\rho^{(1)}(\boldsymbol{x}_1)$ is given by a gaussian, we have $\rho(\boldsymbol{R}) = (2\alpha/\pi)^{3/2}\, \mathrm{e}^{-2\alpha \boldsymbol{R}^2}$ i.e. the c.m. density falls off more rapidly than does the single-particle density.

Because the c.m. density does not depend much on the correlation we choose for larger nuclei $\rho(\boldsymbol{R}) = \int \mathrm{d}^3 r \rho^{(1)}(\boldsymbol{R} + \tfrac{1}{2}\boldsymbol{r})\rho^{(1)}(\boldsymbol{R} - \tfrac{1}{2}\boldsymbol{r})$, where $\rho^{(1)}$ is given by any appropriate parametrization, e.g. that obtained from electron scattering. The results for the example of ^{12}C are essentially equal whether we use a simple gaussian, modified gaussian or Woods–Saxon density distribution. If $T_s(\boldsymbol{B}) = \int_{-\infty}^{+\infty} \mathrm{d}z \rho(\boldsymbol{B}, z)$ defines the thickness function which corresponds to the c.m. density $\rho(\boldsymbol{R})$ and the NN scattering amplitude is given by

$$f_{\mathrm{NN}}(\boldsymbol{q}_\perp) = \frac{k\sigma_{\mathrm{tot}}^{\mathrm{NN}}}{4\pi}(i + \tilde{\alpha})\, \mathrm{e}^{-\boldsymbol{q}_\perp^2/2q_0^2}$$

with $\sigma_{\rm tot}^{\rm NN}=40$ mb, $\sigma_{\rm el}^{\rm NN}=20$ mb, $q_0=400$ MeV, one finds

$$\mathscr{L}+\mathscr{L}^*=-A\sigma_{\rm tot}^{\rm NN}T(\boldsymbol{B})-\delta(A-1)\sigma_{\rm el}^{\rm NN}T_{\rm s}(\boldsymbol{B})\,,$$

$$\delta=\frac{\gamma(\sigma_{\rm tot}^{2\rm NN}/8\pi\sigma_{\rm el}^{\rm NN}-1/q_0^2)}{1/q_0^2+\frac{1}{6}r_{\rm c}^2}\approx 0.03\,,$$

$$J(\boldsymbol{B},\boldsymbol{\beta})=A\sigma_{\rm el}^{\rm NN}T(\boldsymbol{B})w(\boldsymbol{\beta})+(A-1)\sigma_{\rm el}^{\rm NN}T_{\rm s}(\boldsymbol{B})\gamma w_{\rm c}(\boldsymbol{\beta})\,,$$

$$w_{\rm c}(\boldsymbol{\beta})=\int {\rm d}^2q\,{\rm e}^{i\boldsymbol{q}\cdot\boldsymbol{\beta}}\frac{1}{\sigma_{\rm el}^{\rm NN}}\frac{{\rm d}\sigma}{{\rm d}^2q}\left\{-{\rm e}^{-q^2r_{\rm c}^2/6}+\frac{\sigma_{\rm tot}^{\rm NN}}{4\pi(1/q_0^2+\frac{1}{6}r_{\rm c}^2)}\exp\left[\frac{q^2}{4q_0^2}\left(\frac{1}{1+\frac{1}{6}q_0^2r_{\rm c}^2}\right)\right]\right.$$
$$\left.-\frac{\sigma_{\rm el}^{\rm NN}}{8\pi(1/q_0^2+\frac{1}{6}r_{\rm c}^2)}{\rm e}^{q^2/2q_0^2}\right\}\,,$$

$$\gamma=(r_{\rm c}/r_0)^3\,.$$

The correction to the distortion due to the optical potential is numerically small and can be taken into account by taking an effective $\sigma_{\rm tot}$ which differs by less than 5% from the free one. This result indicates that the total integrated cross section is only affected to a minor degree. The requirement of unitarity was only necessary to insure that the derived scattering term $J(\boldsymbol{B},\boldsymbol{\beta})$ included all second-order contributions. The main effect is observed in the change of the momentum distribution, which for single scattering is given by the free nucleon–nucleon cross section. It is changed by the additional term due to the "deuteron gas" scattering. The relative importance of each of the three terms to the leading term is given by $\gamma=\frac{1}{8}$ for $r_{\rm c}=0.6$ fm $r_0=1.2$ fm. This shows that the order of magnitude of the effect is 10%. However the three contributions due to interference between single scatterings $(\Gamma_x\Gamma_y^*)$, single and double scattering $(\Gamma_x\Gamma_y\Gamma_y^*)$ and double scattering $(\Gamma_x\Gamma_x^*\Gamma_y\Gamma_y^*)$ have alternating signs. In the energy region considered one has

$$\frac{\sigma_{\rm tot}^{\rm NN}}{4\pi(1/q_0^2+\frac{1}{6}r_{\rm c}^2)}\approx 1\,,\qquad \frac{\sigma_{\rm el}^{\rm NN}}{8\pi(1/q_0^2+\frac{1}{6}r_{\rm c}^2)}\approx\frac{1}{4}\,,$$

so that there are large cancellations of the different terms. These cancellations are due to the same effect which causes the destructive interference in nucleon–deuteron scattering. There the interference term between single and double scattering also has the opposite sign to the contributions of single and double scattering alone. The cross section including the correction terms now reads

$$\frac{{\rm d}\sigma_{\rm in}}{{\rm d}^2q_\perp}=\int\frac{{\rm d}^2B\,{\rm d}^2\beta}{(2\pi)^2}{\rm e}^{i\boldsymbol{q}_\perp\cdot\boldsymbol{\beta}}\,{\rm e}^{-A\sigma_{\rm tot}^{\rm NN}T(\boldsymbol{B})-\delta(A-1)\sigma_{\rm el}^{\rm NN}T_{\rm s}(\boldsymbol{B})}$$
$$\times\sum_{n=1}^{\infty}\frac{1}{n!}(A\sigma_{\rm el}^{\rm NN}T(\boldsymbol{B}))^n\left[w(\boldsymbol{\beta})+\frac{T_{\rm s}(\boldsymbol{B})}{T(\boldsymbol{B})}\gamma w_{\rm c}(\boldsymbol{\beta})\right]^n\,.$$

The rapid convergence of the terms in the parameter γ was numerically checked and the results show that there are only non-negligible contributions up to order

γ^1 which one has to keep. Therefore the correction terms to the independent particle model in the cross section are given in the following formula by the terms $\sigma_n^c P_n^c(\boldsymbol{q}_\perp)$:

$$\frac{d\sigma_{in}}{d^2 q_\perp} = \sum_{n=1}^{\infty} \sigma_n P_n(\boldsymbol{q}_\perp) + \sigma_n^c P_n^c(\boldsymbol{q}_\perp)\,,$$

$$\sigma_n^c = \gamma \frac{1}{n!} \int d^2 B\, e^{-A\sigma_{tot} T(\boldsymbol{B})} (A\sigma_{el}^{NN} T(\boldsymbol{B}))^n \frac{T_s(\boldsymbol{B})}{T(\boldsymbol{B})}\,,$$

$$P_n^c(\boldsymbol{q}_\perp) = \int \frac{d^2\beta}{(2\pi)^2} e^{i\boldsymbol{q}_\perp \cdot \boldsymbol{\beta}} w^{n-1}(\boldsymbol{\beta}) w_c(\boldsymbol{\beta})\,.$$

Due to cancellations of the correction terms the total contribution to the cross section is seen to be smaller than 10^{-2}. Fig. 1 shows the cross section $d\sigma_{in}|d^2 q_\perp$ for scattering of protons on ^{12}C at $E_p = 1$ GeV. For simplicity, a gaussian density distribution $\rho^{(1)}(r) = \rho_0\, e^{-\alpha r^2}$ was assumed for ^{12}C. $\langle r^2 \rangle^{1/2}$ was taken to be 2.5 fm.

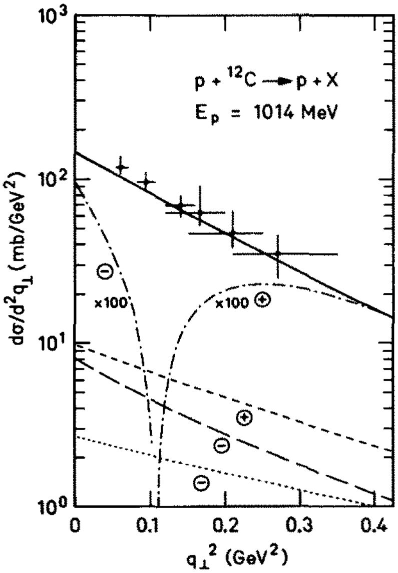

Fig. 1. $d\sigma_{in}/d^2 q_\perp$ for $p + {}^{12}C \rightarrow p' + X$ at $E_p = 1014$ MeV. The points with error bars are extracted from ref. [6]) as described in the text. The solid line is the result within the independent particle model. The following lines are the contributions due to single–single (long dashed) single–double (short-dashed) and double–double (dotted) interference terms, respectively. The dash–dotted line represents the sum of the three multiplied by 100. The ⊕ and ⊖ indicate the respective signs of the contributions and refer to the line directly above them.

The points with large error bars are extracted from the experiment of Corley *et al.* [6]). The error bars correspond to the uncertainty of transforming $d\sigma_{in}/d\Omega$ into $d\sigma_{in}/d^2 q_\perp$. This was achieved by the following procedure. The experiment shows at each angle a pronounced peak in the doubly differential cross section. From each spectrum at angle θ_l three momentum values k_f^{mn}, k_f^{p}, k_f^{mx} corresponding to the minimum, peak and maximum momentum, respectively, were read. They were used to define the central value and error bars of $q_\perp$ and $d\sigma/d^2q_\perp$ by

$$q_\perp^\alpha = k_f^\alpha \sin\theta_l\,, \qquad \alpha = \text{mn, p, mx}$$

$$\frac{d\sigma_{in}}{d^2q_\perp}(q_\perp^\alpha) = \frac{1}{(k_f^\alpha)^2\cos\theta_l}\frac{d\sigma_{in}}{d\Omega}\,.$$

The transformation $d^2q_\perp = k_f^2\cos\theta_l\, d\Omega$, which is valid for fixed k_f, was used.

4. Conclusions

One reads from fig. 1 that the cross section is well described by the independent particle model. Deviations cannot be ascribed to the presence of short-range correlations. It is true that they could contribute up to the order of 10% (if for instance $1/q_0^2 \gg \sigma_{tot}^{NN}$) but in the regime of nucleon–nucleus collision above 1 GeV the various terms due to correlations tend to cancel each other in the whole region of $0 < q_\perp^2 < 0.4\,\text{GeV}^2$. As expected the correlation contributions become more important at higher $q_\perp^2$. In the present calculation they never exceed the 10% level even at $q_\perp^2 = 1\,\text{GeV}^2$. However, at these high momentum transfers ($\theta_l > 45°$) we do not believe Glauber theory to be valid.

It has been shown that within Glauber theory the effect of short-range correlations in the nucleus gives rise to a term in the inclusive inelastic cross section which represents the scattering of the projectile on a two-particle system. Due to different kinematics this part of the cross section describes regions of larger momentum transfer which cannot be reached by the scattering on uncorrelated particles.

The calculation within a simple model for these correlations shows, however, that in physical nucleon–nucleus cross sections at energies above 1 GeV these effects do not show up significantly in the cross-section due to the cancellation of the various terms.

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Appendix 1

CUMULANT EXPANSION OF THE AUXILIARY FUNCTIONS $\mathscr{L}$ AND J [ref. [7]]

The auxiliary function $\mathscr{L}$ was defined by

$$\mathscr{L}(\boldsymbol{b},\lambda) = \ln\langle 0|\prod_{i=1}^{A}(1-\Gamma(\boldsymbol{b}-\boldsymbol{s}_i)|0\rangle \equiv \ln\left\langle \prod_{i=1}^{A}(1-\Gamma_i)\right\rangle\,.$$

The expansion of $\mathscr{L}(\boldsymbol{b}, \lambda)$ into a Taylor series around $\lambda = 0$ yields

$$\mathscr{L}(\boldsymbol{b}, \lambda) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n \mathscr{L}}{\partial \lambda^n}\bigg|_{\lambda=0} \lambda^n ,$$

where

$$\mathscr{L}(\boldsymbol{b}, 0) = 0 ,$$

$$\frac{\partial \mathscr{L}}{\partial \lambda}\bigg|_0 = -\sum_{i=1}^{A} \langle \Gamma_i \rangle ,$$

$$\frac{\partial^2 \mathscr{L}}{\partial \lambda^2}\bigg|_0 = \sum_{\substack{ij \\ i \neq j}}^{A} \langle \Gamma_i \Gamma_j \rangle - \sum_{ij}^{A} \langle \Gamma_i \rangle \langle \Gamma_j \rangle .$$

With the notations

$$(c|M|c) = \int \mathrm{d}^3 x \, \mathrm{d}^3 y \rho_c(\boldsymbol{x}, \boldsymbol{y}) M(\boldsymbol{x}, \boldsymbol{y}) ,$$

$$(1|M|1) = \int \mathrm{d}^3 x \rho^{(1)}(\boldsymbol{x}) M(\boldsymbol{x}) ,$$

one writes

$$\mathscr{L}(\boldsymbol{b}, 1) = -A(1|\Gamma_x|1) + \tfrac{1}{2} A(A-1)(c|\Gamma_x \Gamma_y|c) + \text{higher corr.} + \mathrm{O}(1/A) .$$

For J one has to expand a function of two variables

$$J(\boldsymbol{b}, \lambda, \boldsymbol{b}', \mu) = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{i=0}^{n} \binom{n}{i} \frac{\partial^n J}{\partial \mu^i \, \partial \lambda^{n-i}}\bigg|_{(0,0)} \mu^i \lambda^{n-i} .$$

The non-vanishing derivatives up to 4th order, which contribute to the two-particle correlation function are:

$$\frac{\partial^2 J}{\partial \mu \, \partial \lambda}\bigg|_{0,0} = \sum_{i,j=1}^{A} (\langle \Gamma_i \Gamma_j^* \rangle - \langle \Gamma_i \rangle \langle \Gamma_j^* \rangle) ,$$

$$\frac{\partial^3 J}{\partial \mu \, \partial \lambda^2}\bigg|_{0,0} = -2 \sum_{j=1}^{A} \langle \Gamma_j^* \rangle \left(\sum_{i=1}^{A} \langle \Gamma_i \rangle \right)^2 + \sum_{j=1}^{A} \langle \Gamma_j^* \rangle \sum_{\substack{i,k=1 \\ i \neq k}}^{A} \langle \Gamma_i \Gamma_k \rangle$$

$$+ 2 \sum_{j=1}^{A} \langle \Gamma_j \rangle \sum_{i,k=1}^{A} \langle \Gamma_i \Gamma_k^* \rangle - \sum_{\substack{i,j,k=1 \\ i \neq j}}^{A} \langle \Gamma_i \Gamma_j \Gamma_k^* \rangle ,$$

$$\frac{\partial^3 J}{\partial \mu^2 \, \partial \lambda}\bigg|_{0,0} = -2 \sum_{j=1}^{A} \langle \Gamma_j \rangle \left(\sum_{i=1}^{A} \langle \Gamma_i^* \rangle \right)^2 + \sum_{j=1}^{A} \langle \Gamma_j \rangle \sum_{\substack{i,k=1 \\ i \neq k}}^{A} \langle \Gamma_i^* \Gamma_k^* \rangle$$

$$+ 2 \sum_{j=1}^{A} \langle \Gamma_j^* \rangle \sum_{i,k=1}^{A} \langle \Gamma_i \Gamma_k^* \rangle - \sum_{\substack{i,j,k=1 \\ i \neq j}}^{A} \langle \Gamma_i^* \Gamma_j^* \Gamma_k \rangle ,$$

$$\left.\frac{\partial^4 J}{\partial\mu\,\partial\lambda^3}\right|_{0,0} = -6\sum_{j=1}^{A}\langle\Gamma_j^*\rangle\left(\sum_{i=1}^{A}\langle\Gamma_i\rangle\right)^3 + 6\sum_{j=1}^{A}\langle\Gamma_j^*\rangle\sum_{l=1}^{A}\langle\Gamma_l\rangle\sum_{\substack{i,k=1\\ i\neq k}}^{A}\langle\Gamma_i\Gamma_k\rangle$$

$$-3\sum_{\substack{i,k=1\\ i\neq k}}^{A}\langle\Gamma_i\Gamma_k\rangle\sum_{j,l=1}^{A}\langle\Gamma_j\Gamma_l^*\rangle - 3\sum_{j=1}^{A}\langle\Gamma_j\rangle\sum_{\substack{i,k,l=1\\ i\neq k}}^{A}\langle\Gamma_i\Gamma_k\Gamma_l^*\rangle$$

$$+6\left(\sum_{j=1}^{A}\langle\Gamma_j\rangle\right)^2\sum_{i,k=1}^{A}\langle\Gamma_i\Gamma_k^*\rangle - \sum_{j=1}^{A}\langle\Gamma_j^*\rangle\sum_{\substack{i,k,l=1\\ i\neq k;k\neq l\\ i\neq l}}^{A}\langle\Gamma_i\Gamma_k\Gamma_l\rangle$$

$$+\sum_{\substack{i,j,k,l=1\\ i\neq k;i\neq l\\ k\neq l}}^{A}\langle\Gamma_i\Gamma_j^*\Gamma_k\Gamma_l\rangle,$$

$$\left.\frac{\partial^4 J}{\partial\mu^2\,\partial\lambda^2}\right|_{0,0} = -6\left(\sum_{i=1}^{A}\langle\Gamma_i\rangle\right)^2\left(\sum_{j=1}^{A}\langle\Gamma_j^*\rangle\right)^2 + 2\left(\sum_{i=1}^{A}\langle\Gamma_i\rangle\right)^2\sum_{\substack{k,j=1\\ k\neq j}}^{A}\langle\Gamma_k^*\Gamma_j^*\rangle$$

$$+2\left(\sum_{i=1}^{A}\langle\Gamma_i^*\rangle\right)^2\sum_{\substack{k,j=1\\ k\neq j}}^{A}\langle\Gamma_k\Gamma_j\rangle + 8\sum_{i=1}^{A}\langle\Gamma_i\rangle\sum_{j=1}^{A}\langle\Gamma_j^*\rangle\sum_{k,l=1}^{A}\langle\Gamma_k\Gamma_l^*\rangle$$

$$-2\left(\sum_{i,j=1}^{A}\langle\Gamma_i\Gamma_j^*\rangle\right)^2 - \sum_{\substack{i,j=1\\ i\neq j}}^{A}\langle\Gamma_i\Gamma_j\rangle\sum_{\substack{k,l=1\\ k\neq l}}^{A}\langle\Gamma_k^*\Gamma_l^*\rangle - 2\sum_{i=1}^{A}\langle\Gamma_i\rangle\sum_{\substack{j,k,l=1\\ k\neq l}}^{A}\langle\Gamma_j\Gamma_k^*\Gamma_l^*\rangle$$

$$-2\sum_{i=1}^{A}\langle\Gamma_i^*\rangle\sum_{\substack{j,k,l=1\\ k\neq l}}^{A}\langle\Gamma_j^*\Gamma_k\Gamma_l\rangle + \sum_{\substack{i,j,k,l=1\\ i\neq j;k\neq l}}^{A}\langle\Gamma_i\Gamma_j\Gamma_k^*\Gamma_l^*\rangle,$$

$$\left.\frac{\partial^4 J}{\partial\mu^3\,\partial\lambda}\right|_{0,0} = -6\sum_{j=1}^{A}\langle\Gamma_j\rangle\left(\sum_{i=1}^{A}\langle\Gamma_i^*\rangle\right)^3 + 6\sum_{j=1}^{A}\langle\Gamma_j\rangle\sum_{i=1}^{A}\langle\Gamma_i^*\rangle\sum_{\substack{k,l\\ k\neq l}}\langle\Gamma_k^*\Gamma_l^*\rangle$$

$$-3\sum_{\substack{i,k=1\\ i\neq k}}^{A}\langle\Gamma_i^*\Gamma_k^*\rangle\sum_{j,l=1}^{A}\langle\Gamma_j\Gamma_l^*\rangle - 3\sum_{j=1}^{A}\langle\Gamma_j^*\rangle\sum_{\substack{i,k,l=1\\ i\neq k}}^{A}\langle\Gamma_i^*\Gamma_k^*\Gamma_l\rangle$$

$$+6\left(\sum_{j=1}^{A}\langle\Gamma_j^*\rangle\right)^2\sum_{i,k=1}^{A}\langle\Gamma_i\Gamma_k^*\rangle - \sum_{j=1}^{A}\langle\Gamma_j\rangle\sum_{\substack{i,k,l=1\\ i\neq k;k\neq l\\ i\neq l}}^{A}\langle\Gamma_i^*\Gamma_k^*\Gamma_l^*\rangle$$

$$+\sum_{\substack{i,j,k,l=1\\ i\neq k;k\neq l\\ i\neq l}}^{A}\langle\Gamma_i^*\Gamma_j\Gamma_k^*\Gamma_l^*\rangle.$$

A term like $\langle\Gamma_i\Gamma_j\Gamma_k^*\rangle$ only contributes to the two-particle correlation density if $j=k$ or $i=k$.

If one collects together all those terms one gets

$$J(\boldsymbol{b}, 1, \boldsymbol{b}', 1) = A(1|\Gamma_x\Gamma_x^*|1) + \tfrac{1}{2}A(A-1)(c|\Gamma_x\Gamma_y^* + \Gamma_y\Gamma_x^* - \Gamma_x\Gamma_x^*(\Gamma_y+\Gamma_y^*) - \Gamma_y\Gamma_y^*(\Gamma_x+\Gamma_x^*) + \Gamma_x\Gamma_x^*\Gamma_y\Gamma_y^*|c) + \text{higher corr.} + \mathrm{O}(1/A)$$

and because the expectation value of any single-particle operator M_1 with the correlation density ρ_c is zero, we can write this as

$$J(\boldsymbol{b}, 1, \boldsymbol{b}', 1) = A(1|\Gamma_x\Gamma_x^*|1) + \tfrac{1}{2}A(A-1)(c|(\Gamma_x+\Gamma_y-\Gamma_x\Gamma_y)(\Gamma_x^*+\Gamma_y^*-\Gamma_x^*\Gamma_y^*)|c) + \ldots.$$

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