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Unique irredundance, domination and independent domination in graphs

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Abstract

A subset D of the vertex set of a graph G is irredundant if every vertex v in D has a private neighbor with respect to D, i.e. either v has a neighbor in $V(G)\backslash D$ that has no other neighbor in D besides v or v itself has no neighbor in D. An irredundant set D is maximal irredundant if $D \cup \{v\}$ is not irredundant for any vertex $v \in V(G)\backslash D$. A set D of vertices in a graph G is a minimal dominating set of G if D is irredundant and every vertex in $V(G)\backslash D$ has at least one neighbor in D. A subset I of the vertex set of a graph G is independent if no two vertices in I are adjacent. Further, a maximal irredundant set, a minimal dominating set and an independent dominating set of minimum cardinality are called a minimum irredundant set, a minimum dominating set and a minimum independent dominating set, respectively, and the cardinalities of these sets are called the irredundance number, the domination number and the independent domination number, respectively.

In this paper we prove that any graph with equal irredundance and domination numbers has a unique minimum irredundant set if and only if it has a unique minimum dominating set. Using a result by Zverovich and Zverovich [An induced subgraph characterization of domination perfect graphs, J. Graph Theory 20(3) (1995) 375–395], we characterize the hereditary class of graphs G such that for every induced subgraph H of G, H has a unique $\imath$-set if and only if H has a unique γ-set. Furthermore, for trees with equal domination and independent domination numbers we present a characterization

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of unique minimum independent dominating sets, which leads to a linear time algorithm to decide whether such trees have unique minimum independent dominating sets.

Keywords: Domination number; Hereditary class of graphs; Domination perfect graphs; Unique domination

1. Terminology and introduction

For any graph G, its vertex set and edge set of G are denoted by $V(G)$ and $E(G)$, respectively. Also, $n(G) = |V(G)|$ and $m(G) = |E(G)|$. For any subset $X \subseteq V(G)$, we define the induced subgraph $G(X)$ as the graph with vertex set X and edge set $\{ab \in E(G) : a, b \in X\}$. For any set $A \subseteq V(G)$ and any vertex $x \in V(G)$, we define $G - A = G(V(G)\backslash A)$ and $G - x = G - \{x\}$. If s and t are positive integers then $K_{s,t}$ denotes the complete bipartite graph where one partite set has cardinality s and the other has cardinality t. Graphs G that do not contain $K_{1,3}$ as an induced subgraph are called *claw-free*. For every vertex $x \in V(G)$ and any subset $A \subseteq V(G)$, we denote the set of neighbors of x in G by $N_G(x) = N(x)$, and we define $N_G[x] = N[x] = N(x) \cup \{x\}$, $N_G(A) = N(A) = \bigcup_{x\in A} N(x, G)$ and $N_G[A] = N[A] = N(A, G) \cup A$. For a subset D of $V(G)$ and a vertex $x \in D$, the set $P(x, D, G) = P(x, D) = N[x]\backslash N[D\backslash\{x\}]$ is called the *private neighborhood* of x with respect to D. We call every vertex $y \in P(x, D)$ a *private neighbor* of x with respect to D. Furthermore, for any subsets $A \subseteq D \subseteq V(G)$, we define the set $P(A, D) = \bigcup_{x\in A} P(x, D)$. A set $D \subseteq V(G)$ is *irredundant* if every vertex in D has at least one private neighbor. An irredundant set D of G is called *maximal irredundant* if $D \cup \{v\}$ is no longer irredundant for every vertex $v \in V(G)\backslash D$. A set $D \subseteq V(G)$ is a *dominating set* of G if $V(G) \subseteq N_G[D]$. A dominating set D of G is called *minimal* if D is irredundant, i.e. $D\backslash\{x\}$ is no longer a dominating set of G for every vertex $x \in D$. A subset I of the vertex set of a graph G is called *stable* or *independent* if no two vertices in I are adjacent. The minimum cardinalities of a maximal irredundant set, of a dominating set and of an independent dominating set are called the *irredundance number*, the *domination number* and the *independent domination number*, respectively, and they are denoted by $ir(G)$, $\gamma(G)$ and $\imath(G)$. Further, an irredundant set of G of cardinality $ir(G)$ is called a *minimum irredundant set* or an *ir-set*, a dominating set of G of cardinality $\gamma(G)$ is called a *minimum dominating set* or a *γ-set* of G, and an independent dominating set of a graph G of cardinality $\imath(G)$ is called a *minimum independent dominating set* of G or an *$\imath$-set*.

Definition 1. A graph G is called *domination perfect* or *γ-perfect* if $\gamma(H) = \imath(H)$ for every induced subgraph H of G. A graph G is called *minimal domination imperfect* if G is not domination perfect and $\gamma(H) = \imath(H)$ for every proper induced subgraph H of G.

A graph G is called *irredundance perfect* or *ir-perfect* if $ir(H) = \gamma(H)$, for every induced subgraph H of G.

For other graph theory terminology we follow the monograph by Haynes et al. [17]. By the above definitions, the following is easy to deduce.

Lemma 1 (*Cockayne et al. [6]*). *For every graph G, each maximal independent set is a minimal dominating set and every minimal dominating set is maximal irredundant. Thus,*

$$ir(G) \leqslant \gamma(G) \leqslant \imath(G).$$

Several publications deal with the question, for which graphs G there is equality between the parameters $ir(G)$ and $\gamma(G)$ or between $\gamma(G)$ and $\imath(G)$, see for example Allan and Laskar [1], Bollobás and Cockayne [3], Cockayne et al. [4], Favaron [7], Harary and Livingston [15], Haynes et al. [16,17], Allan and Laskar [1], Topp [23], Topp and Volkmann [24] and Zverovich and Zverovich [25]. There exists no general characterization of such graphs. Even the characterization of trees with equal domination and independent domination numbers seems very difficult. In 1986, Harary and Livingston [15] presented a first, quite complicated characterization of such trees. A second, also difficult characterization was given by Cockayne et al. [4] in 2000. More is known on the related problem, which graphs are γ-perfect. Several publications present sufficient conditions for such graphs or characterizations for special graph classes, as e.g. Allan and Laskar [1], Fulman [12], Sumner and Moore [21], Topp and Volkmann [24]. A complete characterization of domination perfect graphs was found by Zverovich and Zverovich [25].

Theorem 1 (*Zverovich and Zverovich [25]*). *A graph G is domination perfect if and only if G does not contain any of the graphs $G_1, G_2, \ldots, G_{17}$ in Fig. 1 as an induced subgraph.*

We prove in this paper that any graph G with $ir(G) = \gamma(G)$ has a unique ir-set if and only if it has a unique γ-set. Furthermore, using Theorem 1, we characterize the hereditary class of graphs G such that for every induced subgraph H of G, H has a unique $\imath$-set if and only if H has a unique γ-set. In the last part of this paper we present for trees T with $\gamma(T) = \imath(T)$ a characterization of unique $\imath$-sets which leads to a linear time algorithm to decide whether such a tree has a unique minimum independent dominating set.

Unique minimum dominating sets and related topics have been studied e.g. in Fischermann [8], Fischermann et al. [9], Fischermann and Volkmann [10,11], Gunther et al. [13], Hajiabolhassan et al. [14], Hopkins and Staton [18], Siemes et al. [20] and Topp [22].

2. Uniqueness of *ir*-, γ- and $\imath$-sets

As a corollary of Lemma 1, we obtain the following.

Observation 1. *Let G be an arbitrary graph.*

(a) *If $\gamma(G) = \imath(G)$ and G has a unique γ-set D, then D is also the unique $\imath$-set of G.*
(b) *If $ir(G) = \gamma(G)$ and G has a unique ir-set D, then D is also the unique γ-set of G.*

Proof. Lemma 1 implies that every $\imath$-set of G is a γ-set of G if $\gamma(G) = \imath(G)$, and every γ-set of G is an ir-set of G if $ir(G) = \gamma(G)$. Hence, the result follows. □

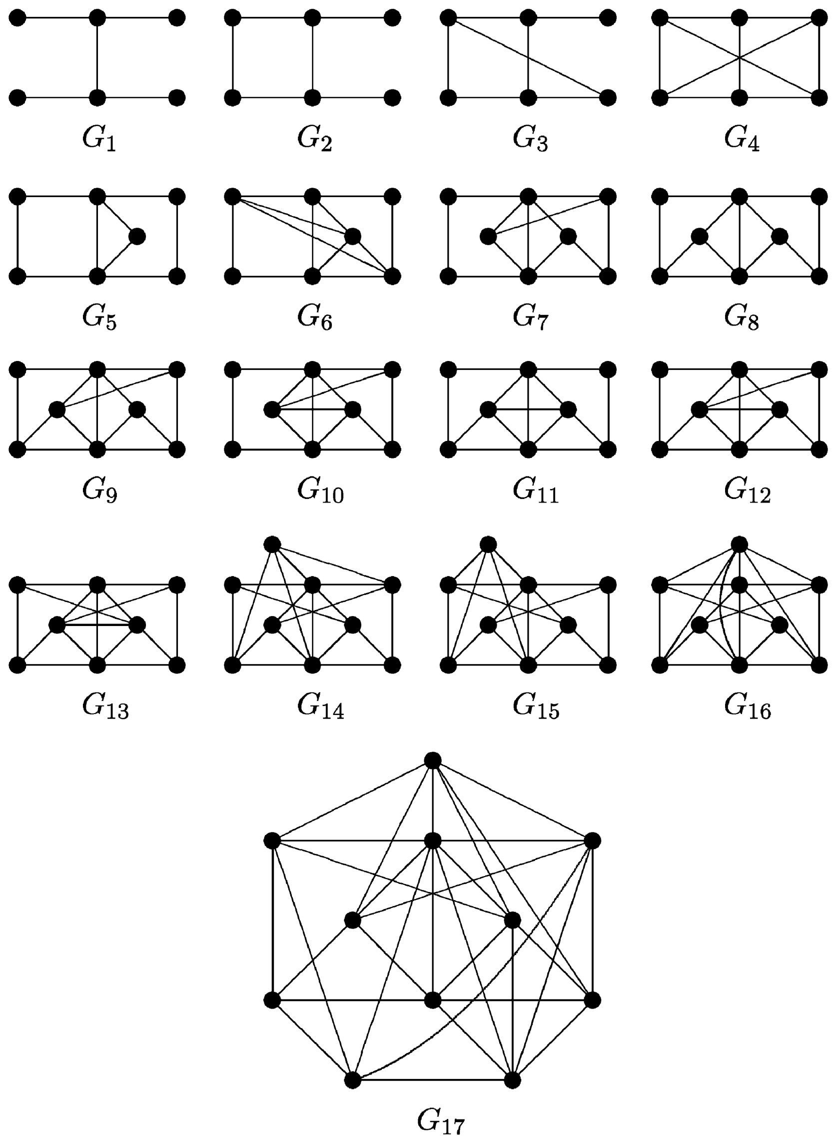

Fig. 1. The minimal domination imperfect graphs.

The following result shows that the converse of Observation 1(b) does not hold for arbitrary graphs with irredundance number equal domination number.

Theorem 2. *Let G be an arbitrary graph with $ir(G) = \gamma(G)$. Then G has a unique γ-set if and only if G has a unique ir-set.*

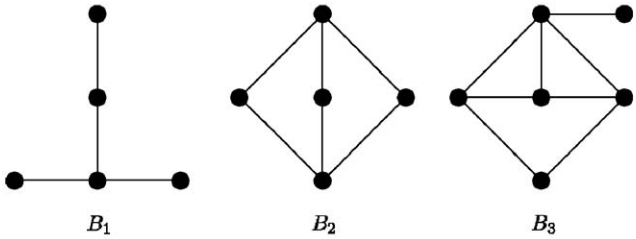

Fig. 2. The graphs B_1, B_2 and B_3.

Proof. By Observation 1(b), we know that a unique ir-set of such a graph always is a unique γ-set. Conversely, suppose that G has the unique γ-set D and an ir-set $D' \neq D$. Let D' be an ir-set of G different from D where $|V(G)\setminus N[D']|$ is minimal. Since D is a unique γ-set and $ir(G)=\gamma(G)$, we deduce that $V(G)\setminus N[D'] \neq \emptyset$. Let $v \in V(G)\setminus N[D']$ be arbitrary. Since D' is maximal irredundant, some element w of $D' \cup \{v\}$ does not have a private neighbor. Since v is non-adjacent to any vertex of D', it follows that $w \in D'$, and hence every private neighbor of w with respect to D' belongs to $N(v)$. Thus, the set $D''=(D'\setminus\{w\})\cup\{v\}$ dominates $N[D']\cup N[v]$. If D'' is not irredundant, then there exists a proper subset I of D'' that is maximal irredundant and we deduce the contradiction that $ir(G) < |D''|=|D'|$. Hence, also the set D'' is an ir-set of G and $|V(G)\setminus N[D'']| \leqslant |(V(G)\setminus N[D'])\setminus\{v\}| < |V(G)\setminus N[D']|$. By the minimality of $|V(G)\setminus N[D']|$, we obtain $D''=(D'\setminus\{w\})\cup\{v\}=D$. Since $v \in V(G)\setminus N[D']$ was arbitrary, this implies $V(G)\setminus N[D']=\{v\}$. The vertex $v \in D$ has at least one private neighbor with respect to D. If the induced subgraph $G(P(v,D))$ is complete, then we obtain the contradiction that the set $(D\setminus\{v\})\cup\{u\}$ is a γ-set of G different from D for any private neighbor $u \in P(v,D)\setminus\{v\}$ or for any neighbor u of v if $P(v,D)=\{v\}$. Thus, there exist at least two non-adjacent private neighbors p_v and p'_v of v with respect to D. The set $F=D'\cup\{p_v\}$ is obviously a dominating set of G. Since D' is maximal irredundant, the set F is not irredundant which implies the existence of a vertex x in F such that the set $F'=F\setminus\{x\}$ also dominates G. By the equality $|D'|=|F'|$, we obtain that F' is a γ-set of G and $D=F'$. The set F does not contain the vertex v, and hence $v \in D\setminus F'$ which is a contradiction. □

The following result shows that the converse of Observation 1(a) does not hold in general.

Observation 2. *The graphs B_1, B_2, $K_{2,t}$ for every integer $t \geqslant 4$, and B_3 (see Fig. 2) have domination number equal independent domination number and they have a unique ι-set but at least two γ-sets.*

For any graph H, we say that a graph G is *H-free* if G does not contain the graph H as an induced subgraph.

Definition 2. Let $\mathscr{UNDOM}$ be a hereditary I class consisting of all graphs G such that for every induced subgraph H of G, H has a unique ι-set if and only if H has a unique γ-set.

Theorem 3. *A graph G belongs to* $\mathcal{UNDOM}$ *if and only if G is a* (B_1, B_2, B_3)*-free graph, see Fig.* 2.

Proof. *Necessity*: It is easy to see that each B_i, $i = 1, 2, 3$, has a unique $\imath$-set (consisting of two vertices), but at least two γ-sets. Since $\mathcal{UNDOM}$ is a hereditary class, G cannot contain any of B_1, B_2 or B_3 as an induced subgraph.

Sufficiency: Suppose that G is a minimal (B_1, B_2, B_3)-free graph that does not belong to $\mathcal{UNDOM}$. Minimality of G means that $H \in \mathcal{UNDOM}$ for every proper induced subgraph H of G.

Claim 1. *G is a domination perfect graph.*

Proof. We use the characterization of domination perfect graphs given in Theorem 1. Each of the graphs G_1, G_2, G_3 and G_5 contains B_1 as an induced subgraph. The graph G_4 has an induced subgraph isomorphic to B_2. Each of the graphs $G_6, G_7, \ldots, G_{17}$ contains B_3 as an induced subgraph. Since G is a (B_1, B_2, B_3)-free graph, G is also a $(G_1, G_2, \ldots, G_{17})$-free graph. By Theorem 1, G is a domination perfect graph. $\square$

Claim 2. *G has a unique* $\imath$*-set, but at least two* γ*-sets.*

Proof. By Claim 1, $\imath(G) = \gamma(G)$. Therefore, each $\imath$-set in G is also a γ-set. It follows from $G \notin \mathcal{UNDOM}$ and minimality of G that G has a unique $\imath$-set, but there are at least two γ-sets in G. $\square$

Claim 2 implies that we can choose a dominating set $D \subseteq V(G)$ such that

(D1) D is a γ-set, but D is not an $\imath$-set, and
(D2) the induced subgraph $G(D)$ has the minimum number of edges among all sets satisfying (D1).

Since D is not an independent domination set, there are adjacent vertices u and v in D. We denote by P the private neighborhood of $\{u, v\}$ with respect to D, i.e. $P = \{x \in V(G) : N[x] \cap D \subseteq \{u, v\}\}$. Let H be the subgraph induced by the set $P \cup \{u, v\}$.

Claim 3. $\gamma(H) = \imath(H) = 2$.

Proof. If H has a dominating vertex x, then the set $D' = (D \backslash \{u, v\}) \cup \{x\}$ is a dominating set in G with $|D'| < |D|$, a contradiction to minimality of D. Hence $\gamma(H) \geqslant 2$. But the definition of P implies that $\{u, v\}$ is a dominating set in H, therefore $\gamma(H) = 2$.

By Claim 1, G is a domination perfect graph. In particular, $\imath(H) = \gamma(H)$. $\square$

Claim 4. $G = H$.

Proof. Suppose that H is a proper induced subgraph of G. By minimality of G, $H \in \mathcal{UNDOM}$. According to Claim 3, H has at least two γ-sets, namely $\{u, v\}$ (that is not stable)

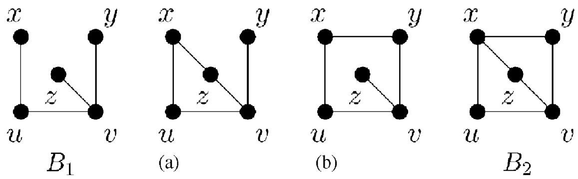

Fig. 3. Variants for $G(u, v, x, y, z)$.

and a minimum independent dominating set. It follows from the definition of $\mathscr{UNDOM}$ that there are at least two distinct $\imath$-sets I_1 and I_2 in H.

The sets $D_i = (D \backslash \{u, v\}) \cup I_i$, $i = 1, 2$, are dominating sets in G. Since $|D| = |D_i|$, each D_i is a γ-set in G. Clearly, $G(D_i)$, $i = 1, 2$, has less edges than $G(D)$. The condition (D2) implies that each of D_1, D_2 must be an $\imath$-set. Since $D_1 \neq D_2$, we have a contradiction to Claim 2. □

According to Claim 4, $|D| = \gamma(H) = \gamma(H) = 2$, i.e. $D = \{u, v\}$. Since D is a γ-set, there exist vertices $x \in PN(u, D)$ and $y \in PN(v, D)$. Clearly, u, v, x and y are pairwise distinct vertices.

At least one of the sets $\{u, y\}$ or $\{v, x\}$ is not an $\imath$-set in G according to Claim 2. By symmetry, we may assume that $\{u, y\}$ is not an $\imath$-set. Therefore, there exists a vertex $z \notin N[u] \cup N[y]$. Since $D = \{u, v\}$ is a dominating set and z is not adjacent to u, we must have the edge zv. Fig. 3 shows all four possible variants for the subgraph induced by $\{u, v, x, y, z\}$.

Two of them are isomorphic to B_1 and B_2 that contradicts to the choice of G. The two others variants, (a) and (b), are considered below.

(a) Either $\{v, x\}$ or $\{x, y\}$ (Fig. 3) is not an $\imath$-set. Hence there exists a vertex a such that either $a \notin N[v] \cup N[x]$ or $a \notin N[x] \cup N[y]$. Since $D = \{u, v\}$ is a dominating set, a must be adjacent to at least one of the vertices u or v.

Suppose that a is non-adjacent to both v and x. Then a is adjacent to u. Now we specify the undefined edges ay and az:

- if a is non-adjacent to y, then the set $\{u, v, x, y, a\}$ induces B_1, a contradiction,
- if a is adjacent to z, then the set $\{u, v, x, z, a\}$ induces B_2, a contradiction.

Thus, a is adjacent to y, and a is non-adjacent to z, that is we obtain the graph A shown in Fig. 4.

Now suppose that a is non-adjacent to both x and y. If a is adjacent to exactly one of u, v, then we delete z and obtain B_1, a contradiction. Let a be adjacent to both u and v:

- if a is non-adjacent to z, then the set $\{v, x, y, z, a\}$ induces B_1, a contradiction, and
- if a is adjacent to z, then the set $\{u, v, x, y, z, a\}$ induces B_3, a contradiction.

Thus, it remains to consider the graph A in Fig. 4. Either $\{a, z\}$ or $\{x, y\}$ is not an $\imath$-set. By symmetry, we may assume that $\{x, y\}$ is not an $\imath$-set. Hence there exists a vertex b such that $b \notin N[x] \cup N[y]$. Again, b must be adjacent to at least one of the vertices u or v, say u. If b

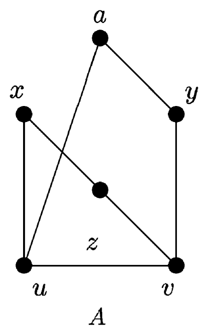

Fig. 4. The graph A induced by $\{u, v, x, y, z, a\}$.

is non-adjacent to v, then $G(u, v, x, y, b)$ is isomorphic to B_1, a contradiction. Therefore, a is adjacent to v. If b is non-adjacent to z, then $G(v, x, y, z, b)$ is again isomorphic to B_1, a contradiction. Hence a is adjacent to v.

We delete the vertex a and obtain the forbidden subgraph B_3, a contradiction.

(b) Either $\{v, x\}$ or $\{x, z\}$ (Fig. 3) is not an $\imath$-set. As in (a), there exists a vertex a such that either $a \notin N[v] \cup N[x]$ or $a \notin N[x] \cup N[z]$. In the former case we obtain B_1 or B_2 or A. The latter case produces B_1 or B_2. It follows that (b) is also impossible. □

The proof of Claim 2 implies the following result.

Corollary 1. *Every graph in $\mathscr{UNDOM}$ is a domination perfect graph.*

Now we consider unique $\imath$-sets in trees T with $\gamma(T) = \imath(T)$. The following lemma contains a general necessary condition for the uniqueness of an $\imath$-set.

Lemma 2. *If any graph G has a unique $\imath$-set I, then every vertex x in I fulfills either $P(x, I) = \{x\}$ or $|P(x, I)| \geqslant 3$.*

Proof. Obviously, every vertex x in I is its own private neighbor with respect to I. Suppose that for a vertex x in I we have $P(x, I) = \{x, y\}$ for some vertex $y \neq x$. Then, the set $(I \setminus \{x\}) \cup \{y\}$ is another independent dominating set of cardinality $\imath(G)$, which is a contradiction. □

For trees T with $\gamma(T) = \imath(T)$, this necessary condition is also sufficient for the uniqueness of an $\imath$-set.

Theorem 4. *Let T be a tree with $\gamma(T) = \imath(T)$, and let I be a subset of $V(G)$. Then the following conditions are equivalent*:

(i) *I is the unique $\imath$-set of T, and*
(ii) *I is an $\imath$-set of T such that every vertex x in I fulfills either $P(x, I) = \{x\}$ or $|P(x, I)| \geqslant 3$.*

Proof. (i) implies (ii). It follows immediately from Lemma 2.

(ii) implies (i). Suppose that T has an $\imath$-set I such that every vertex x in I fulfills either $P(x, I) = \{x\}$ or $|P(x, I)| \geqslant 3$, and there exists a second $\imath$-set $I' \neq I$ of T. It follows that $|I| = |I'|$ and $|I \backslash I'| = |I' \backslash I|$. We define the four sets

$$\begin{aligned} I_1 &= \{x \in I \backslash I' : P(x, I) = \{x\}\}, \\ I_2 &= \{x \in I \backslash I' : |P(x, I)| \geqslant 3\}, \\ I_1' &= \{y \in I' : N(y) \cap I_1 \neq \emptyset\}, \\ I_2' &= \{y \in I' \backslash I : N(y) \cap I_1 = \emptyset\}. \end{aligned}$$

By the assumption, the union $I_1 \cup I_2$ is equal $I \backslash I'$, and since the set I is independent, we obtain $I_1' \subseteq I' \backslash I$ and $I_1' \cup I_2' = I' \backslash I$. If a vertex $y \in I_1'$ is adjacent to two different vertices x and x' in I_1, then the set $D = (I \backslash \{x, x'\}) \cup \{y\}$ is a dominating set of T which leads to the contradiction $\gamma(T) < \imath(T)$. This and the definition of I_1' imply that every vertex in I_1' has exactly one neighbor in I_1. Since the set I' is maximal independent, every vertex x in I_1 has a neighbor in I' which implies that this neighbor lies in I_1'. Thus, we obtain that

$$|I_1| = |I_1'| \quad \text{and} \quad |I_2| = |I_2'|. \tag{1}$$

Since every vertex $y \in I_1'$ has at least one neighbor $x \in I_1$ and since y is not a private neighbor of x with respect to I, we know that

$$|N(y) \cap I| \geqslant 2 \quad \text{for every vertex } y \in I_1'. \tag{2}$$

Let $P = P(I \backslash I', I) \backslash (I \cup I')$ and let H be the subgraph of T induced by the vertex set $(I \backslash I') \cup (I' \backslash I) \cup P$. Note, that the subgraph H is a forest and thus, we know that $m(H) \leqslant n(H) - 1$. Furthermore, we know that the size of H is composed as follows

$$m(H) = m(I_1', I \backslash I') + m(I_2', I \backslash I') + m(P, I \backslash I') + m(P, I' \backslash I) + m(G[P]).$$

Now we look at the single addends of this summation. By (2), we obtain that

$$m(I_1', I \backslash I') = m(I_1', I) \geqslant 2|I_1'|.$$

Since every vertex in I_2 has at least two private neighbors with respect to I besides itself, and since these private neighbors lie in the disjoint union $P \cup I_2'$, we conclude that

$$m(I_2', I \backslash I') + m(P, I \backslash I') \geqslant m(I_2', I_2) + m(P, I_2) \geqslant 2|I_2|.$$

The set I' is a dominating set of T and thus, every vertex in P has at least one neighbor in I' and, by the definition of P, this neighbor has to lie outside the set $I' \cap I$, which leads to

$$m(P, I' \backslash I) \geqslant |P|.$$

These estimations together with (1) yield that

$$\begin{aligned} m(H) &\geqslant 2|I_1'| + 2|I_2| + |P| = |I_1| + |I_1'| + |I_2| + |I_2'| + |P| \\ &= |I \backslash I'| + |I' \backslash I| + |P| = n(H), \end{aligned}$$

which is a contradiction. $\square$

If we consider the cycle C_{3t} of order $3t$ for any positive integer t, we see that condition (ii) in Theorem 4 is not sufficient for the uniqueness of an $\imath$-set in arbitrary graphs with domination number equal independent domination number.

Cockayne et al. [5] and Beyer et al. [2] have found linear time algorithms to determine a γ-set and an $\imath$-set in a tree, respectively. Consequently, one can decide in linear time whether $\gamma(T) = \imath(T)$. Using the result in Beyer et al. [2] Theorem 4 one can compute in linear time whether a tree with equal domination and independent domination numbers has a unique $\imath$-set.

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