Acta Astronautica Vol 3, pp. 1033–1047. Pergamon Press 1976 Printed in the U S A.

Low-velocity detonations in cast and pressed high explosives†

B. S. ERMOLAEV, B. A. KHASAINOV, A. A. BORISOV AND A. I. KOROTKOV
Institute of Chemical Physics, USSR Academy of Sciences, Moscow, U.S.S.R.

(*Received* 28 *October* 1975)

Abstract—A model of low velocity detonations in charges of cast and pressed high explosives confined in metal tubes with thin walls is suggested. An analytical solution is obtained and velocities and pressures of stationary LVD waves are calculated as functions of the parameters of the wall confinement. The fraction of the total chemical energy transferred to maintain the shock wave and the average rate of reaction in the wave are estimated and the dynamics of transient predetonation processes are analysed. An explanation of the causes of the secondary shock wave formation in the transition from deflagration to normal detonations is suggested and the delay of this secondary wave is calculated and compared with experimental data.

Introduction

It is well known that the transition from normal (layer by layer) combustion to explosions in solid high explosives includes several stages. The latter are distinguished by the hydrodynamics of flow and by the manner of initiation of reaction and burning (Belyaev *et al.*, 1973). The following sequence of four stages is characteristic for the transition process in high explosives with open porosity (gas penetratable): layer by layer combustion, convective burning, transient predetonation process with the initiation in a compression wave and normal detonations.

Transition from normal combustion to convective burning takes place at the moment when the pressure at the hot boundary and its gradient exceed their critical values. In the case of convective burning, high explosive is ignited by a flow of hot combustion products driven through pores into the unreacted substance by a pressure gradient.

If the pressure difference at the combustion front is higher than the yield strength of the explosive, convective burning will be accompanied by a nonelastic flow and by compression of the unreacted explosive. The nonelastic compression wave propagates ahead of the convective burning front. A wave of sufficiently high amplitude is capable of initiating reaction and burning in "hot spots". Pressure difference of this critical amplitude is responsible for the onset of a detonation type process, which is usually called low-velocity detonations (LVD). However it is more correct to call the nonsteady process a predetonation transient wave with shock initiation (PTW) reserving the term LVD for a steady-state regime.

†Paper presented at the Fifth Colloquium on Gasdynamics of Explosions and Reactive Systems, Bourges, France, 8–11 September 1975.

The lower pressure limit of PTW normally is about several kbars for many densely packed high explosives (Belyaev *et al.*, 1973; Dremin *et al.*, 1970). The weakest selfsustaining process which might be considered as a shock initiated one (since it could be transmitted through a metal plate) had a velocity about 0.8 km/sec and pressure not higher than 5 kbar (Bobolev *et al.*, 1969; Korotkov *et al.*, 1960).

The PTW regime is extremely sensitive to experimental conditions. It often appears to be unstable and accelerates up to the normal detonation. However in some studies which are reviewed by Belyaev *et al.* (1973), PTW propagating at approximately constant velocity in relatively long charges were observed. That is why the steady-state LVD is believed to exist in solid explosives. It is noteworthy that sometimes the apparently steady LVD exhibited a sudden acceleration up to overdriven or normal detonations. Most of the explanations of the observed phenomena given in literature are on a purely qualitative base.

Model of PTW

To describe the propagation of an actual PTW, one has to solve the nonsteady two- or three-dimensional equations of hydrodynamics in a chemically reacting medium. This paper presents an attempt to construct a simplified model of the PTW with the main purpose being to give semi-quantitative explanations of some features of the process. The model is based on the following six assumptions:

(a) Velocities in the range of 0.8–2.5 km/sec and pressures from several to 15–20 kbar are typical for PTW. Compression and preheating the solid in the wave front due to compression are small.

(b) The process is led by a nonelastic wave. The PTW has a two-wave configuration: an elastic precursor and a plastic wave. The latter can propagate in pressed charges at a rate which is considerably lower than acoustic velocity, that is substantially lower than 2.2–2.7 km/sec. Collapse of pores in the front of a nonelastic wave offers an explanation of this effect. Let us consider the following model of the wave front: the solid phase is loaded up to the yield strength P_T in the elastic precursor which propagates at the acoustic velocity C_1; however porosity of the charge is not changed practically in this wave. The solid phase becomes ideally plastic, and the volume of pores diminishes to zero in a nonelastic wave, which propagates behind the elastic precursor at a velocity $D < C_1$. Taking into account that the initial mass of the gas in pores is negligible, one can obtain the following nonlinear relations at the front of a nonelastic wave (Ermolaev *et al.*, 1975):

$$\begin{aligned} P_f &= (1-\phi_0)(P_T + \rho_{s0}\delta_T D \cdot u_f) \\ u_f &= D[1-\delta_T(1-\phi_0)/\delta_f] \\ \phi_f &= 0. \end{aligned} \tag{1}$$

Dependence of the density ratio δ on pressure $\pi = P/K$ is obtained from the generalized shock adiabate of organic high explosives (Afanasenkov *et al.*, 1967)

$$\delta(\pi) = 1 + (3/8)(\sqrt{(1+8\pi)} - 1) - \pi/2. \tag{2}$$

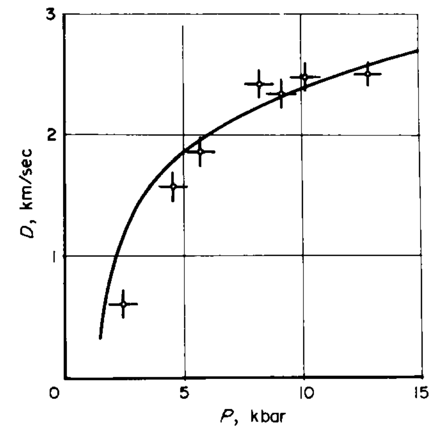

Fig. 1. An example of nonelastic wave velocity–pressure dependence in porous systems; TNT with density 1.6 g/cm³. Line, calculations: $P_T = 1.5$ kbar, $C_0 = 2.2$ km/sec; points, experimental data (Sulimov *et al.*, 1972).

If $D > C_1$ there is not any acoustic precursor and no bifurcation of the shock wave. In this case P_T certainly does not appear in the relations at the front. Values of D calculated using eqn (1) at low pressures are strongly dependent on the wave amplitude. The minimum possible velocity according to this model is:

$$D_{min} = \sqrt{(P_T/(1 - \phi_0)\rho_{s0})}. \tag{3}$$

D_{min} is not more than 0.3–0.4 km/sec for P_T equal to 1.5–2 kbar. Equation (3) gives results which agree with the experimental data (see Fig. 1).

(c) This assumption concerns the rate of burn-out of the explosive. Data on the mechanism of initiation and rate of reaction in low amplitude nonelastic waves are very scarce and sometimes ambiguous. There is some evidence that the reaction starts locally in "hot spots" formed by a plastic flow and destruction of particles in the nonelastic wave front and that, after initiation, burning spreads over the particles' surface (Dremin *et al.*, 1970). It is important that the temperature dependence of the reaction rate in this case is much weaker than the Arrhenius one normally being used in detonation calculations. For the approximate analysis given below the rate of reaction is expressed as a product of a normal steady-state burning rate U_s (cm/sec) and of a burning surface area in a unit volume S (cm^{-1}). For many explosives $U_s = bP$, where b is a constant and its value for PETN, for instance, is equal to 10 cm sec^{-1} kbar^{-1} (Belyaev *et al.*, 1963). Unfortunately by now one can say almost nothing about the value of the burning surface area S. Generally speaking S is a function of many parameters and is not constant during the combustion process. However it is assumed to be constant in the calculations because one of the main aims of this analysis has been the estimation of this value from the comparison of experimental and theoretical results.

(d) The medium in the reaction zone is a two-phase one and consists of gaseous combustion products and a solid explosive. Temperature, density and equations of state of the phases are different. Equation (2) has been used for the solid phase and the gaseous phase has been described with the equation of state of an ideal gas having the specific heat ratio $\gamma \simeq 2$. The particle velocities of both phases are assumed to be equal. Variations of the temperature and internal energy of the solid, the viscosity and thermal conductivity effects are neglected.

(e) PTW is normally observed in pressed and cast charges of high explosives confined in hard metal tubes. Characteristics of the confinement walls (thickness and diameter) are very critical for the process (Obmenin *et al.*, 1970); this confirms the importance of lateral expansion in PTW propagation. The analysis given below is based on the assumptions that lateral expansion of the confinement walls is determined by inertia of the latter and that the flow inside the duct is quasi-one-dimensional (i.e. the expansion angle is small). The walls are assumed to move according to the following equation:

$$m \frac{\partial^2 R}{\partial t^2} = 2\pi R[P - \sigma_T \ln(1 + H_0/R_0)]. \tag{4}$$

This implies that the wall material is noncompressible, that its motion obeys the law of ideal plasticity, and that the expansion at a given point is due only to the difference between local pressure and resistance of the walls. Time $t = 0$ indicates the moment when the front of the nonelastic wave arrives at a given point. When $t = 0$, $R = R_0$ and $\partial R/\partial t = 0$, eqn (4) is valid up to the moment of confinement destruction if the walls are thin enough. For thicker walls ($H_0 \sim R_0$) it can be used only during the initial stage of the expansion.

(f) The PTW is a nonsteady process which is affected by downstream conditions. It is convenient for the calculations to introduce a "piston" which moves at a varying velocity. It is not difficult to define the position of the "piston" in a case when the PTW is caused by a weak shock wave transmitted through a metal plate.

Conditions (pressure and particle velocity) at the metal plate—solid high explosive interface—are those at the "piston". However when convective burning evolution in a confined charge results in the PTW formation, the position of the "piston" is not predetermined and depends upon the history of the process. Some part of the charge adjacent to the initiator is already burning by the time the nonelastic compression wave arises, and is able to initiate a chemical reaction in the solid phase.

For a qualitative description of a nonstationary process the "piston" trajectory might be identified with that of a flame surface.

Fortunately, as we shall see below, it appears to be possible to make some important conclusions concerning the structure of PTW and trends in their evolution even when one does not know the exact position of the "piston".

Fundamental equations

Taking into account the above-mentioned assumptions, one can write the conservation equations for the process as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}(A\rho)+(A\rho)\frac{\partial u}{\partial x}=0$$

$$\rho\frac{\mathrm{d}u}{\mathrm{d}t}+\frac{\partial P}{\partial x}=0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\phi\rho_g A)+(\phi\rho_g A)\frac{\partial u}{\partial x}=A\rho_{s0}U_s S$$

$$(\phi\rho_g A)\frac{\mathrm{d}E}{\mathrm{d}t}g-(\phi AP)\frac{\mathrm{d}\ln\rho_g}{\mathrm{d}t}=A\rho_{s0}U_s S(h_a-h_g). \tag{5}$$

Equations of state of the phases, relations at the wave front (1), and equations of the piston trajectory and of the confinement walls expansion (4) should be added to the set of equations (5) to form the complete system.

In order to find an analytical solution which depicts the real flow pattern with reasonable accuracy, the approach proposed by Sternberg (1970) has been applied. It follows from Sternberg's analysis that one can obtain a family of similarity solutions assuming exponential acceleration of a shock wave in a reacting medium. The exponential behavior of waves is typical for the initial period of the PTW propagation and therefore the above-mentioned similarity solution is expected to give a more or less correct description of the process.

Thus the procedure implies calculations of the flow parameters downstream of the shock front and the "piston" trajectory, assuming that the exponent of the wave acceleration is known.

Let us introduce the nondimensional variables:

$$P=K\pi;\quad t=t_x\tau;\quad x=t_xC_0\xi;\quad u=C_0v;$$
$$D=zC_0;\quad T=T_a\theta;\quad q=(\gamma-1)h_a/\gamma C_0^2,$$

and use the Lagrangian coordinate system (τ_1,ζ) with the zero point at the wave front. Here τ_1 is a label of a layer of the explosive, and $\zeta=\tau-\tau_1$ is the time elapsed after the passage of the shock front through a given layer. At the wave front $\zeta=0$. The following relations are to be used in order to change the coordinate system (from ξ, τ to τ_1, ζ):

$$\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\partial}{\partial\zeta};\quad\frac{\partial}{\partial\tau}=\frac{1}{I}\left[\frac{\partial}{\partial\zeta}\frac{\partial\xi}{\partial\tau_1}-\frac{\partial}{\partial\tau_1}\frac{\partial\xi}{\partial\tau}\right]$$

$$v=\frac{\partial\xi}{\partial\zeta};\quad I=\frac{\partial\xi}{\partial\zeta}-v.$$

The mass equation can be readily integrated in the Lagrangian coordinates. Equations (5) can be rewritten, after some rearrangement, as

$$[(1-\phi)\delta+(\phi\pi)/(q\theta)]AI=z(1-\phi_0)\delta_T$$

$$\frac{\partial \pi}{\partial \tau_1} - \frac{\partial \pi}{\partial \zeta} + \frac{z(1-\phi_0)\delta_T}{A}\frac{\partial v}{\partial \zeta} = 0$$

$$\frac{1}{\gamma}\frac{\partial}{\partial \zeta}(\ln \pi) + \frac{\partial}{\partial \zeta}(\ln (A\phi I)) = q/\phi \tag{6}$$

$$\frac{\partial}{\partial \zeta}(\ln \theta) - \frac{\gamma - 1}{\gamma}\frac{\partial}{\partial \zeta}(\ln \pi) = q(1-\theta)/\phi.$$

The trajectory of the wave front is given by the expression:

$$z = z_0 \exp(k\tau_1) \tag{7}$$

according to the assumption mentioned above. Here k is any real quantity.

In order to separate the variables in eqns (6) one has to assume that:

$$\xi = \begin{cases} z_0 \exp(k\tau_1)[y(\zeta) + 1/k], & k \neq 0 \\ z_0[y(\zeta) + \tau_1], & k = 0 \end{cases}$$

$$\pi = \exp(2k\tau_1)p(\zeta) \tag{8}$$

$$\phi = \phi(\zeta); \quad \theta = \theta(\zeta); \quad A = A(\zeta)$$

$$I = z_0 \exp(k\tau_1)\left[1 + ky(\zeta) - \frac{\mathrm{d}y}{\mathrm{d}\zeta}\right].$$

Moreover if $k \neq 0$ some additional conditions are required. That is why the cases with $k \neq 0$ and $k = 0$ will be considered separately.

Constant velocity waves, $k = 0$

The velocity of the wave in this case remains constant and the flow pattern changes due to the expansion of the wave zone only. Substituting eqn (8) into eqn (5), one will get the following set of ordinary equations:

$$\frac{\mathrm{d}p}{\mathrm{d}\zeta} = -\frac{z_0^2(1-\phi_0)\delta_T(1-r)}{A}\frac{F(p, A, r)}{1-M^2}$$

$$\frac{\mathrm{d}r}{\mathrm{d}\zeta} = \frac{A}{z_0^2(1-\phi_0)_T}\frac{\mathrm{d}p}{\mathrm{d}\zeta}$$

$$\frac{\mathrm{d}y}{\mathrm{d}\zeta} = r$$

$$\frac{\mathrm{d}\ln\theta}{\mathrm{d}\zeta} = \frac{\gamma - 1}{\gamma}\frac{\mathrm{d}\ln p}{\mathrm{d}\zeta} + \frac{q(1-\theta)}{\phi} \tag{9}$$

$$\frac{\mathrm{d}A}{\mathrm{d}\zeta} = \frac{z_0(1-r)}{\epsilon}\sqrt{\left(A\int_1^A (p - p_w)\,\mathrm{d}A\right)}.$$

Where $F(p, A, r) = q - p/\delta - \dfrac{\mathrm{d}\ln A}{\mathrm{d}\zeta}$

$$M^2 = \frac{z_0^2(1-\phi_0)\delta_T(1-r)}{A}\left[\frac{\phi}{\gamma p} + (1-\phi)\frac{\mathrm{d}\ln\delta}{\mathrm{d}p}\right]$$

$$\phi = \left[1 - \frac{z_0(1-\phi_0)\delta_T}{A(1-r)}\right] \times \left[1 - \frac{p}{q\theta\delta}\right]^{-1}$$

$$\epsilon = \frac{R_0}{C_0 t_x}\sqrt{\left(\frac{\rho_w H_0(2R_0 + H_0)}{8\rho_{s0}R_0^2}\right)}$$

$$p_w = \frac{\sigma_T}{K}\ln(1 + H_0/R_0).$$

The equation for $\mathrm{d}A/\mathrm{d}\zeta$ is obtained by integration of eqn (4). The boundary conditions at the wave front ($\zeta = 0$) are:

$$p = p_f; \quad r = r_f; \quad \theta = 1; \quad A = 1; \quad y = 0.$$

The values of p_f and r_f are functions of z_0 and are to be calculated using the relations at the front (1).

The set of equations (9) has a singular point of a saddle type where $1 - M^2$ and $F(p, A, r)$ approach zero simultaneously. An integral curve starting at the point $\zeta = 0$ can pass the singular point when z_0 is equal to a particular value $(z_0)_{CJ}$, which corresponds to the steady-state LVD. Examples of integral curves representing pressure profiles for different wave velocities are given in Fig. 2. The middle curve corresponds to the stationary regime and is terminated in a point equivalent to the Chapman–Jouguet point of nonideal detonations (sonic point). Heat release in this point is compensated for by lateral expansion. The rarefaction wave which follows the LVD—that is the flow behind the equivalent CJ plane—is not considered here, although the reaction in this region may affect the propagation of a LVD wave under certain conditions.

The velocity of the wave is greater than that of the stationary LVD for the

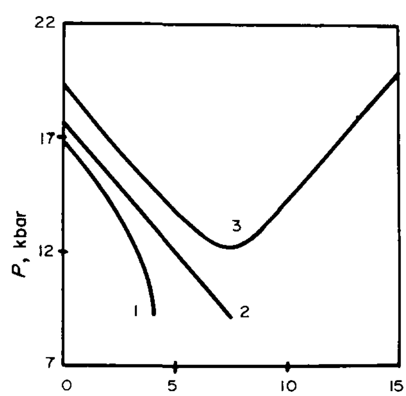

Fig. 2. Calculated pressure profiles in low velocity detonation waves. (1) D = 2.71 km/sec; (2) D_{CJ} = 2.75 km/sec (steady-state regime); (3) D = 2.82 km/sec (overdriven regime).

upper curve and less than that for the lower curve respectively. The upper curve represents an overdriven regime. M is always less than 1 in this case and pressure behind the front reaches the minimum and then begins to grow. The lower curve corresponds to a regime which can not exist as a stationary one because the gradients of the parameters (pressure, temperature, velocity, etc.) at the limiting point ($M = 1$; $F \neq 0$) become infinite.

It is reasonable to use for further approximate calculations the fact that the parameter ϵ characterising the ratio of two times (lateral expansion time to that of chemical reaction) usually is substantially less than 1. Analysis of the equations shows that the lateral expansion A is close to 1 down to the singular point, when $\epsilon \ll 1$.

Let us consider two cases:

(a) Strong waves, that is $p - p_w$ is of the order of p. $A - 1$ and the LVD zone thickness is of the order of ϵ^2 in this case and the pressure change is of the order of ϵ. It follows from eqns (9) and from the condition that $\mathrm{d}A/\mathrm{d}\zeta$ is of the order of 1 (since when $F = 0$, $q \sim 1$ and $p/\delta \ll q$).

(b) Weak waves, when $p_{CJ} - p_w$ is of the order of $\epsilon^{2/3}$. In this case $(A - 1)$ and the wave thickness are of the order of $\epsilon^{4/3}$ and $p - p_w \sim \epsilon^{2/3}$.

If the waves are strong, one can integrate the equation for r, assuming as a first approximation $A = 1$. For a given value of z_0

$$r = r_f - \frac{p_f - p}{z_0^2(1 - \phi_0)\delta_T}. \tag{10}$$

Then taking $\tilde{p} = (p - p_{CJ})/\epsilon$ as an independent variable one can expand in series with respect to ϵ all the parameters in the vicinity of the singular point (θ_{CJ}, p_{CJ}):

$$\begin{aligned} \theta(p, \epsilon) &= \theta_{CJ} + \epsilon\tilde{\theta}(\tilde{p}); \quad \tilde{\theta}(0) = 0 \\ A(p, \epsilon) &= 1 + \epsilon^2\tilde{A}(\tilde{p}); \quad \tilde{A}(0) = \tilde{A}_{CJ}. \end{aligned} \tag{11}$$

Substitution of eqn (11) into eqns (9) gives for the temperature

$$\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}\tilde{p}} = \frac{\gamma - 1}{\gamma}\frac{\theta_{CJ}}{p_{CJ}},$$

or after integration

$$\theta = \theta_{CJ} + \frac{\gamma - 1}{\gamma}\frac{\theta_{CJ}(p - p_{CJ})}{p_{CJ}}. \tag{12}$$

The lateral expansion can be found from the equation

$$\frac{\mathrm{d}\tilde{A}}{\mathrm{d}\tilde{p}} = -\frac{C_{CJ}\tilde{p}}{\sqrt{(\tilde{A}_{CJ}/\tilde{A}) - 1}}. \tag{13}$$

Here

$$C_{CJ} = -\frac{\mathrm{d}M^2/\mathrm{d}p}{z_0^2(1 - \phi_0)\delta_T(1 - r_{CJ})} \quad \text{at } p = p_{CJ};\ A = 1; \tag{14}$$

$$\tilde{A}_{CJ} = \frac{1}{p_{CJ} - p_w}\left[\frac{q - (p_{CJ}/\delta_{CJ})}{z_0(1 - r_{CJ})}\right]^2. \tag{15}$$

Integration of eqn (13) gives:

$$\tilde{A} = [\sqrt{(\tilde{A}_{CJ})} - \sqrt{(C_{CJ}/2)} \cdot \tilde{p}]^2$$

or

$$A = 1 + [\epsilon\sqrt{(\tilde{A}_{CJ})} - \sqrt{(C_{CJ}/2)}(p - p_{CJ})]^2. \tag{16}$$

The boundary conditions at the wave front should be satisfied in the eqn (12) and this is a way to obtain the value of p_{CJ}. Thus all the parameters in the singular point can be expressed in terms of p_{CJ} which in turn is found from the equation

$$M^2(p_{CJ}) = 1. \tag{17}$$

Finally one can find the value of ϵ by substituting the boundary conditions into the eqn (16):

$$\epsilon = (p_f - p_{CJ})\sqrt{\left(\frac{C_{CJ}}{2\tilde{A}_{CJ}}\right)}. \tag{18}$$

Since z_0 is predetermined, ϵ can be regarded as an eigenvalue which defines rates of burning and of lateral expansion for a given velocity of LVD.

In the case (b) the analogous procedure gives for ϵ

$$\epsilon = \frac{z_0(1 - r_{CJ})(p_f - p_w)}{(q - p_{CJ}/\delta_{CJ})}\sqrt{\left(\frac{C_{CJ}}{6}(2p_f + p_{CJ} - 3p_w)\right)}. \tag{19}$$

Which of the approximations, (a) or (b), is to be used in each case can be decided only after estimation of p_{CJ} and the difference $p_{CJ} - p_w$.

Comparison of the result of the approximate solution with computer calculations shows a good agreement.

In general one or more values of z_0 can correspond to a single value of ϵ. In the latter case the lower(even) branch of the solution, where z_0 decreases with increase of ϵ, has to be disregarded since it depicts nonstable solutions (Kuznetsov, 1968).

The minimum value of ϵ_{min} corresponds to $p_{CJ} = p_w$. Hence minimum wall thickness and wave velocity for steady-state regimes are found. The minimum velocity is greater for higher reaction rates. Figure 3 shows calculated values of D vs H_0 together with the experimental results for PETN. The upper curve represents calculations for strong waves and the lower one for weak waves. The best agreement of calculations and experimental data is obtained when $S = 50\ \mathrm{cm}^{-1}$. This allows us to estimate the rate of the explosive regression under LVD conditions as 3–9 msec^{-1} at 5–15 kbar.

It should be emphasized that the increase of a charge diameter results in

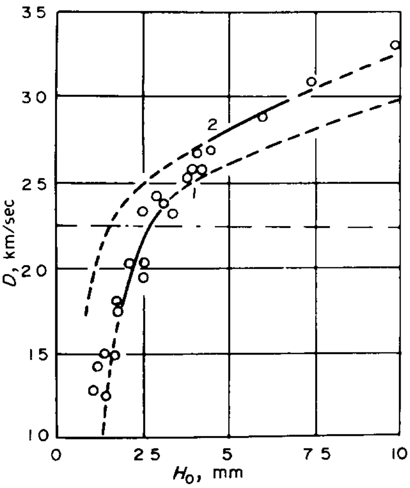

Fig. 3

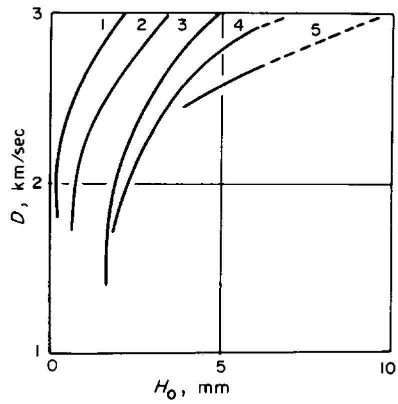

Fig 4.

Fig. 3. Dependence of LVD velocity on thickness of confinement walls. Points, experimental data (Obmenin *et al.*, 1970) for PETN; charges in steel tubes with $R_0 = 2.5$ mm, $\phi_0 = 2.6\%$; lines, calculations $P_T = 1.5$ kbar, $S = 50\ cm^{-1}$

Fig. 4 LVD velocities at different charge diameters. $R_0 = 20$, 10, 5, 2.5 and 1.25 mm for the curves 1, 2, 3, 4 and 5, respectively (the rest of the parameters are the same as in Fig. 3)

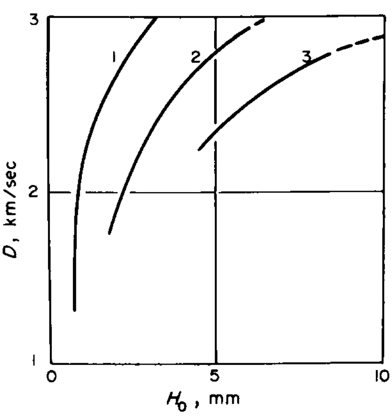

Fig. 5.

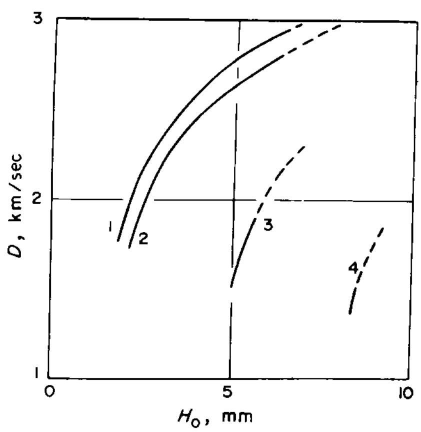

Fig. 6

Fig. 5. Effect of burning surface area on LVD velocities (1) $S = 100\ cm^{-1}$; (2) $S = 50\ cm^{-1}$; (3) $S = 25\ cm^{-1}$ (the rest of the parameters are the same as in Fig 3).

Fig. 6. Effect of confinement wall material on LVD velocities. (1) Steel; (2) brass; (3) duralumin; (4) plexiglass (the rest of the parameters are the same as in Fig 3)

greater values of D, that velocities of LVD are strongly affected by changes of the walls' material, and that only an extremely small part of the high explosive reacts before the sonic (CJ) plane (see Figs. 4–6 and Table 1).

Table 1. Steady-state LVD parameters (pressed PETN, density 1.73, confinement, a steel tube, $\sigma_T = 2.2$ kbar, $R_0 = 2.5$ mm)

Sonic plane (CJ) parameters							
D_{CJ} (km/sec)	H_0 (mm)	P_f (kbar)	P_{CJ} (kbar)	u_{CJ} (km/sec)	Solid phase consumption (%)	Distance between CJ plane and the front (mm)	Lateral expansion $A-1$ (%)
1.8	1.9	4.4	1.9	0 04	0.18	10.8	9.2
2.05	2.3	6.2	2.5	0.06	0.23	7.5	4 7
2.33	2 8	8.8	4.0	0.14	0.33	6.0	3 3
2.55	3.6	13.2	6.5	0.19	0 64	6.4	3.1
2.75	4.5	17.6	9.1	0.235	1 12	7 5	3 1

Nonsteady waves, $k \neq 0$

This is the case of accelerating waves. In order to solve the equations using the technique of separation of variables one should admit additionally that:

(a) lateral expansion of the products is absent,
(b) solid phase is uncompressible, and $\rho = (1-\phi)\rho_{s0}$,
(c) P_T is increasing in time proportionally to the wave velocity squared.

The final set of equations in this case looks as follows:

$$
\begin{aligned}
&(1-M^2)\frac{dp}{d\zeta} = 2kp + z_0^2(1-\phi_0)kr - qz_0^2(1+ky-r)\\
&(1-M^2)\frac{dr}{d\zeta} = kr + \frac{2}{\gamma}k\phi - q(1+ky-r)\\
&\frac{dy}{d\zeta} = r \qquad\qquad (20)\\
&M^2 = \frac{z_0^2(1-\phi_0)(ky+\phi_0-r)}{\gamma p}\\
&\phi = \frac{ky+\phi_0-r}{1+ky-r}.
\end{aligned}
$$

The boundary conditions at $\zeta = 0$ are:

$$p(0) = (1-\phi_0)(p_{T0}+\phi_0 z_0^2); \quad r(0) = \phi_0; \quad y(0) = 0.$$

Analysis shows that two types of the integral curves are possible. When k is greater than a critical value k_1, p is growing continuously, however when $k < k_1$ the solution reaches a limiting point in some time period. $M = 1$ at this point and $dp/d\zeta = -\infty$, which means that continuous flow in the latter case is impossible.

The critical value of k is determined from the approximate expression

$$k_1 = \frac{3 - \phi_0}{2\left[4\frac{p_{T0}}{z_0^2} + 7\phi_0\right]}\left\{1 + \sqrt{\left(1 + \frac{4(4p_{T0}/z_0^2 + 7\phi_0)}{(3 - \phi_0)^2}\right)}\right\}$$

obtained from the condition $(\mathrm{d}^2p/\mathrm{d}\zeta^2) = 0$ at $\zeta = 0$.

Let us consider the behavior of the wave in the vicinity of a singular point when $k < k_1$. The characteristics of the singular point in a reacting system behind a shock wave have been analysed by Sternberg (1970). This point arises due to the appearance of a sonic line in the reacting flow. An analogous result is obtained in this work. The sonic line always appears except when the piston accelerates the wave too actively ($k \geq k_1$) and/or when the overdriven regime arises due to the simultaneous action of the piston and of the lateral expansion ($k = 0$). Qualitatively the appearance of the limiting point with infinite gradients of the parameters might be readily understood if one analyses the P–V diagram of waves propagating through a reacting medium in the absence of lateral expansion. Let us suppose that the velocity of such a wave is lower than the normal detonation velocity and is maintained at a constant level by a piston which is decelerating and pumping out the excess energy. The Michelson–Rayleigh line which crosses intermediate Hugoniot curves will not be a tangent to the final Hugoniot curve, since it will go below the latter. There always could be found an intermediate Hugoniot curve to which the Rayleigh line is tangent. The condition $M = 1$ is obviously fulfilled in the tangention point, however the heat is still being released in the flow. This heat release behind the sonic plane should evidently lead to the onset of a secondary shock wave. The situation is analogous to the disintegration of an arbitrary discontinuity with deflagration. This disintegration always results in the formation of a shock wave propagating before the deflagration wave (Shchelkin and Troshin, 1963).

This effect certainly should be taken into account when explaining the deflagration-to-detonation transition in cast and pressed high explosives. In some experiments (Korotkov *et al.*, 1969; Bernecker and Price, 1974), shock waves moving at a constant or slightly growing velocity have been observed just after the initiation. Then a secondary shock wave arises behind the primary one and overtakes it. Interaction of the two waves results in the formation of a normal detonation wave.

Formation of a secondary shock wave has been explained (Korotkov *et al.*, 1969) as a result of local explosions in the incomplete combustion products. Not excluding the possibility of local explosions, we suppose that critical sonic conditions in the flow might be responsible for the secondary wave formation. This explanation does not require an additional hypothesis concerning the acceleration of the combustion process. The moment of the secondary wave formation can be determined as a moment of the sonic line appearance. In one particular case ($k \approx 0$, lateral expansion is negligibly small, the pressure is low enough, so that $p/\delta q\theta$ and p/δ are less than 1) an analytical solution can be obtained. Velocity r in eqn (10) is a function of p only in this case, since θ drops out of the set of equation (9) and $A \approx 1$. Thus the equation for $dp/d\zeta$ can be integrated with respect to pressure. The integration gives the following expression for the time delay of the secondary wave formation:

$$t_b = \frac{\displaystyle\int_{p_b}^{p_f} \frac{1-M^2(p)}{1-r(p)}\,dp}{\dfrac{\gamma-1}{\gamma} b\rho_{s0} Sh_a z_0^2 (1-\phi_0)\delta_T}. \tag{21}$$

The lower limit in the integral is calculated from the expression:

$$M^2(p_b) = 1. \tag{22}$$

Calculated values of t_b were compared with the experimental data for pressed charges of RDX − 9% wax (see Fig. 7). The best agreement is obtained when $S \approx 7\ \mathrm{cm}^{-1}$. This value is substantially lower than that for PETN estimated using parameters of LVD. This discrepancy may be ascribed to the inhibition of the reaction by wax.

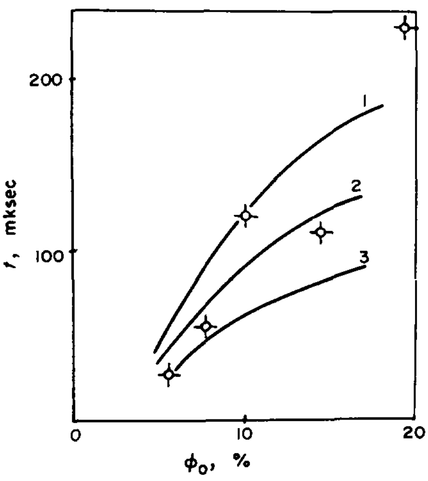

Fig. 7. Delay of secondary shock wave formation in predetonation processes. Points, experimental data (Bernecker and Price, 1974); RDX + 9% wax; walls material, steel AIS11015; $R_0 = 8$ mm; H_0 17.5 mm; $L_0 = 340$ mm; lines, calculations for different values of t_x: (1) $t_x = 280\ \mu$sec, (2) 210 μsec, (3) 340 μsec.

Concluding remarks

Experimental studies of LVD in pressed and cast high explosives are rather difficult to carry out because of the heavy metal confinement, but the latter is a compulsory condition for observation of the process. That is why this process has not yet been studied comprehensively. This paper presents one of the first attempts to treat the LVD regimes in solid explosives theoretically.

A model of this process is proposed, which gives a good agreement with the existing experimental facts, allows calculation of the wave parameters as functions of dimensions and properties of the material of confinement walls, and estimation of the average reaction rate behind the front. The other important result is the analytical procedure for estimating delays of secondary shock waves formation in a nonsteady predetonation process together with the hydrodynamic explanation of the origin of this phenomenon.

References

Afanasenkov, A. N., Bogomolov, V. M. and Voskoboinikov, I. M. (1967) Generalized shock adiabat of organic liquids, *Fizika Gorenia i Vzryva* **3**, 4, 585–594.

Beliaev, A. F., Bobolev, V. K., Korotkov, A. I., Sulimov, A. A. and Chuiko, S. V. (1973) *Transition of Combustion of Condensed Systems to Explosion.* Izd. Nauka, Moscow.

Beliaev, A. F., Korotkov, A. I., Parfionov, A. K. and Sulimov, A. A. (1963) Burning velocity of some explosive substances and mixtures at rather high pressures, *Zhurnal Fizicheskoi Khimii* **37**, 1, 150–156.

Bernecker, R. R. and Price, Donna (1974) Studies in the transition from deflagration to detonation in granular explosives—II. Transition characteristics and mechanisms observed in 91/9 RDX/wax, *Comb. and Flame* **22**, 1, 119–129.

Bobolev, V. K., Dubovik, A. V., Karpukhin, I. A. and Rybakov, V. V. (1969) Propagation of explosion in thin layers of solid explosives, *Fizika Gorenia i Vzryva* **5**, 3, 331–338.

Dremin, A. N., Savrov, S. D., Trofimov, V. S. and Shvedov, K. K. (1970) *Detonation Waves in Condensed Media.* Izd. Nauka, Moscow.

Kuznetsov, N. M. (1968) On nonsingularity and stability of the detonation regime in a bounded medium, *Prikl. Mekh. i Tekhnich. Fizika*, No. 1, 45–56.

Kurkin, A. I. (1969) On transition of combustion of porous explosives to explosion, *Fizika Gorenia i Vzryva* **5**, 3, 315–325.

Obmenin, A. V., Balykov, V. A., Korotkov, A. I. and Sulimov, A. A. (1970) Role of casing during the evolution of explosion in a high density PETN, *Fizika Gorenia i Vzryva* **6**, 4, 571–573.

Sternberg, H. M. (1970) Similarity solution for reactive shock waves, *Quart. J. Mech. Appl. Math.* **XXIII**, Part 1, 77–99.

Sulimov, A. A., Obmenin, A. V., Korotkov, A. I. and Shushliapin, P. I. (1972) On low velocity regimes of explosive transformation in charges of high density solid explosives. In *Proceedings Gorenie i Vzryv* (Papers presented at the Third All Union Symposium on Combustion and Explosion), pp. 464–469. Izd. Nauka, Moscow.

Yermolaev, B. S., Khasainov, B. A. and Korotkov, A. I. (1975) Propagation of plastic compression waves through high density explosives, *Fizika Gorenia i Vzryva* **11**, 2, 325–327.

Zel'dovich, Ya. B. and Kompaneets, A. S. (1955) *Theory of Detonation.* Izd. Tekhn. Teoret. Literatury, Moscow.

Appendix A
Nomenclature

- $A = (R/R_0)^2$ relative expansion
- C_l longitudinal acoustic velocity
- C_0 bulk acoustic velocity
- D nonelastic wave velocity
- H thickness of the confinement walls
- h enthalpy
- E internal energy
- K bulk compression modulus
- k exponent
- L charge length
- M Mach number
- $m = \rho_w \pi H_0(2R_0 + H_0)$ mass per unit length of the confinement walls

P pressure
$p = P/K$ reduced pressure for nonstationary waves
P_T yield strength of high explosive
P_w mechanical resistance of walls
R radius of a confining tube
r reduced particle velocity in Lagrangian coordinate system
q reduced heat of reaction
S burning surface area
T temperature
t time
$t_x = (bKS)^{-1}$ characteristic reaction time
U particle velocity
U_s linear burning velocity
v reduced particle velocity
z reduced wave velocity
x distance
γ specific heats ratio c_p/c_v of combustion products
δ densities ratio, ρ_s/ρ_{s0}
ϵ small parameter, $\frac{R_0}{C_0 t_x} \times \sqrt{\left(\frac{\rho_w H_0(2R_0 + H_0)}{8\rho_{s0} R_0^2}\right)}$
ϕ porosity
ρ density
σ_T yield strength of the confinement material
τ reduced time
τ_1 Lagrangian coordinate
ξ reduced distance
$\zeta = \tau - \tau_1$ Lagrangian coordinate
$\theta = T/T_a$ reduced temperature
π reduced pressure

Indexes

b moment of secondary shock wave formation
CJ sonic plane
a adiabatic flame
f wave front
g gas phase
0 initial
s solid phase
w confinement wall