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Do long swings in the business cycle lead to strong persistence in output?☆

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Abstract

This paper investigates how the occasional long swing in the business cycle can produce long-memory behavior in US output. To prove this theoretical relationship, we extend the Hamilton Markov chain regime switching model of real aggregate output to include the occasional long regime. We do this by modeling the duration length of the expansion and recession regimes as draws from a fat-tailed distribution with realized durations that are high in variability and occasionally extreme in value. Empirically, we find that the tail indices for the length of US economic booms and busts correspond with the long-memory parameter estimates of Diebold and Rudebusch [1989. Long memory and persistence in aggregate output. Journal of Monetary Economics 24, 189–209] and Sowell [1992a. Modeling long-run behavior with the fractional ARIMA model. Journal of Monetary Economics 29, 277–302] for real US output. Estimates of our extended regime switching model produce better short- and long-run forecasts of output in comparison to forecasts with a fractionally integrated model. Furthermore, our estimated regime-switching model finds US expansions to be

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fragile during their infancy, but become more and more likely to continue after surviving the first seven quarters.

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1. Introduction

Our attention in this paper focuses on showing that the long-memory behavior in output is a byproduct of occasional long swings in the regime duration of Hamilton's (1989) Markov switching model. Our approach differs from previous duration studies of business cycles (see Diebold and Rudebusch, 1990; Diebold et al., 1993) or duration dependent versions of Hamilton's Markov switching model (see Durland and McCurdy, 1994; Lam, 2004). Specifically, we advance a particular type of duration dependence that allows for the possibility of an occasional long swing. This swing in the business cycle is of such long duration that one might easily mistake it for a new plateau.

To be specific, we show that if either the duration length of the business cycle's expansion or contraction is distributed with the tail index, α, then the theoretical autocorrelation function of aggregate real output will slowly decay at a rate determined by the long-memory parameter, $d = 1 - \alpha/2$. Thus, if either the economy's upturn or downturn regime occurs with a tail index between 1 and 2, aggregate output will display strongly persistent behavior like that of a long-memory process with a d between 0 and $\frac{1}{2}$.

Using NBER recession dates, we find evidence of occasional long swings in the business cycle leading to long-memory behavior in US output. We estimate the tail indices for both expansion and recession duration distributions and find α to be between 1 and 3. These tail index estimates cause their corresponding long-memory parameter value to match up with the parameter estimates found by Diebold and Rudebusch (1989) and Sowell (1992a) with fractionally integrated models of GNP. Our estimates of α also complement Sowell's (1992a) finding that trend stationary and difference stationary models of US GNP are empirically indistinguishable.

Finding fat-tailed duration length in the US business cycle causes us to question whether the long-memory behavior in aggregate output is a spurious artifact of a regime switching model with occasionally long durations, or if output's data generating process is fractionally integrated. The distinction is important with regard to forecasting business cycles since forecasts with a fractionally integrated model use historical values from as far back as 50 years in the calculation of its predictions. To address this issue, we compare the forecast of US output growth from the fractionally integrated model with the forecast from the regime switching model with long swings.

Using two different measures of accuracy for a H-step and cumulative H-step out-of-sample forecast of output growth, our regime switching model with long swings produces better forecasts than does the fractionally integrated model. Our estimated regime switching model also reveals that in their infancy, US expansions are quite fragile. A US expansion's probability of survival decreases with the age of the expansion. However, after the expansion survives seven quarters, this probability begins to get larger with time.

The remainder of the paper is organized as follows. In Section 2, we define a fat-tailed distribution, the property of long memory, and add fat-tailed distributions to the duration length of the regimes found in Hamilton's Markov switching model. We prove in Section 3 that a regime switching model with a tail index between 1 and 3 for the duration distribution will exhibit long-memory behavior. Sections 4 and 5 are both empirical and contain our finding of fat-tail behavior in the US business cycles leading to long memory in output, along with estimates and forecasts from our extended regime-switching model of US output growth. Section 6 contains our conclusions.

2. Definitions

2.1. Fat-tailed distributions

Distributions with a fat tail are nothing new to economists. From the early work of Mandelbrot (1960) and Fama (1965) to the findings of McCulloch (1997), many have modeled economic data as random variables drawn from fat-tailed distributions. Formally, a random variable X is distributed with a tail distribution if

$$P(X > u) \sim u^{-\alpha} h(u),$$

where $a_u \sim b_u$ means $a_u/b_u \to 1$ as $u \to \infty$, $\alpha > 0$ is the tail index of the distribution, and $h(u)$ is a slow-varying function at infinity; i.e., $\lim_{u\to\infty} h(tu)/h(u) = 1$ for any $t > 0$ (see Samorodnitsky and Taqqu, 1994). When $0 < \alpha \leqslant 2$, X is referred to as having a stable distribution with index α (DuMouchel, 1983). In this article we will refer to distributions having a tail index between 1 and 3 as fat-tailed distributions.

If $\alpha = 2$, X is Gaussian with moments of all orders. When $\alpha < 2$, X is a non-Gaussian random variable with tails that decay like a power function. As a result, the only moments that exist are those of order less than α. So, for smaller values of α, X will take on more extreme values and display higher levels of variability. Distributions with $\alpha < 2$ are referred to as stable Paretian since the tail probabilities are approximately those of the heavy-tailed Pareto family of distributions. In a opposite manner, the tail probabilities decay exponentially to zero when $\alpha \geqslant 2$. Such distributions belong to the exponential class of distributions.

2.2. Long memory

Driven by the strong level of persistence found in macro data and in the volatility of financial data, long memory has been widely studied and applied by economists (see Baillie, 1996; Ding and Granger, 1996; Comte and Renault, 1998; Parke, 1999; Diebold and Inoue, 2001). Long memory can be defined in many ways, one being the rate of decay in its autocorrelation function as the lag argument goes to infinity. To obtain the results found in this paper, we formally define a stationary process X_t as having long memory if the process's theoretical autocorrelation function

$$\rho_X(\tau) \sim |\tau|^{2d-1} L(\tau),$$

where $d \in (-1, 1/2)$ and $L(\tau)$ is a slow-varying function at infinity (see Granger and Joyeux, 1980; Hosking, 1981; Brockwell and Davis, 1993; Baillie, 1996).

In contrast to a short-memory autoregressive, moving average (ARMA) process, where $d = 0$ and the autocorrelation function decays geometrically fast to zero, a long-memory process's autocorrelation function decays hyperbolically to zero. When $d > 0$, this slow decay in the long memory's autocorrelation function aptly describes the strong correlation and persistence that exists between observations far apart in time. If $d < 0$, the process is said to display anti-persistent behavior or to have intermediate memory. For these cases ρ_X will continue to decay hyperbolically to zero, however, all the autocorrelations will be negative except at lag zero.

2.3. Duration-dependent regime switching model

Define y_t to be the Markov chain regime switching model,

$$y_t = s_t + \varepsilon_t, \quad t = 1, 2, \ldots, T, \tag{1}$$

where ε_t is a Gaussian ARMA process that is independent of s_t.[1] Assume that the state variable s_t has only two regimes, a high state where s_t equals $\bar{s}$, a low state where s_t equals $\underline{s}$, and $\underline{s} < \bar{s}$. In this paper we envision the dependent variable y_t representing the growth rate of real GNP. It then follows that $\bar{s}$ is the average growth rate of real output during an expansion and $\underline{s}$ is the average decline during a contraction.

The first-order Markov processes's transition matrix between expansions and contractions is

$$Q = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}, \tag{2}$$

where $p = P[s_t = \bar{s} | s_{t-1} = \bar{s}]$, $q = P[s_t = \underline{s} | s_{t-1} = \underline{s}]$, and hence, $1 - p = P[s_t = \underline{s} | s_{t-1} = \bar{s}]$ and $1 - q = P[s_t = \bar{s} | s_{t-1} = \underline{s}]$.

Some have argued that constants p and q in the matrix Q potentially ignore the impact the length a business cycle or past observations have on whether or not the economy changes regimes (see, for e.g., Lam, 1990, 2004; Diebold et al., 1994; Durland and McCurdy, 1994; Filardo, 1994; Kim and Nelson, 1998, 1999). The length of the business cycle and its impact on the dynamics of the switching regime have also been analyzed with the distribution of the business cycles duration length. Diebold and Rudebusch (1990) study the probabilistic behavior of the ith business cycle's duration measure

$$T_k = \begin{cases} \overline{T}_k & \text{for a upswing in cycle } k, \\ \underline{T}_k & \text{for a downswing in cycle } k, \end{cases}$$

where $\overline{T}_k$ is the duration of the kth business cycle's expansion and $\underline{T}_k$ is the duration of the ensuing recession. $\overline{T}_k$ and $\underline{T}_k$ are independent random variables drawn from the cumulative distribution functions, $\overline{F}(u)$ and $\underline{F}(u)$.[2]

Here, we are interested in the behavior of a regime switching model with state durations that are drawn from fat-tailed $\overline{F}$ and $\underline{F}$. To incorporate duration dependence into Eq. (1),

[1] By assuming ε_t is a short-memory process, y_t's autocovariance function will take on a wider variety of shapes at small lag arguments not possible if ε_t were a whitenoise innovation. Do note, however, that at large lag arguments the behavior of s_t autocovariance function will dominate that of ε_t.

[2] Both $\overline{F}$ and $\underline{F}$ are concentrated on the open positive interval, $(0, \infty)$. This assumption is for analytical convenience only, it does not affect our results in any material way.

we divide time up into economic booms and busts. During the kth business cycle, the state variable equals

$$s_t = \begin{cases} \bar{s} & \text{for } t \in [S_{k-1}, S_{k-1} + \overline{T}_k), \\ \underline{s} & \text{for } t \in [S_{k-1} + \overline{T}_k, S_{k-1} + \overline{T}_k + \underline{T}_k), \end{cases} \tag{3}$$

where S_{k-1} is the starting date of the kth business cycle. In addition to assuming that the duration lengths, $\overline{T}_k$ and $\underline{T}_k$, are i.i.d. random variables that are independent of each other, we also assume the lengths of the upswings and downswings are independent of s_t.

3. Long memory in long-swing durations

In Eq. (1), the time series y_t equals the sum of the two-state regime switching series s_t and ε_t. Since the innovation ε_t by assumption does not contain long memory, the existence of long memory in y_t necessarily implies the existence of long memory in s_t, and vice versa. We can therefore examine the long-memory properties of y_t by looking at the long-memory properties of s_t.

On average, economic recessions have been shorter lived than expansions. According to the NBER, peak-to-trough half cycles have averaged 18 months in length, whereas trough-to-peak half cycles averaged 35 months. Furthermore, in the original work on regime switching models of US output, Hamilton (1989) estimates the probability of staying in an expansion to be around 90%. Hamilton also finds this probability to be significantly different from the 75% probability of continuing in a recession. Hence, any regime switching model trying to empirically explain the long-term persistence found in the business cycle must incorporate the asymmetry between the duration of expansions and contractions into their distributions (see Diebold and Rudebusch, 1990; Diebold et al., 1993; Durland and McCurdy, 1994; Lam, 2004). Alternating between the two states $\bar{s}$ and $\underline{s}$ also implicitly causes negative correlation to exist between neighboring regimes.

We show below in Theorem 3.1 that as long as one of the two regime durations is distributed with fat tails, the regime switching model will produce a long-memory time series. We also show that having two different fat-tailed duration distributions does not affect the slow hyperbolic decay of s_t autocorrelation function. When the duration distributions of both regimes are fat-tailed, the degree of long memory in the regime switching model is defined by the distribution with the smaller α.[3]

Theorem 3.1. *If the expansion regime duration distribution is a fat-tailed distribution where* $1 - \overline{F}(u) \sim u^{-\alpha} L(u)$, *with* $L(u)$ *a slow-varying function at infinity and* $1 < \alpha < 3$, *and the contraction regime duration distribution is of the form* $1 - \underline{F}(u) \sim u^{-(\alpha+\eta)} L(u)$, *for some nonnegative* η, *or vice-a-versa, then the correlation structure of* s_t *is that of a long-memory process with* $d = 1 - \alpha/2$.

Diebold and Inoue (2001) have also augmented the two-state Markov switching model to produce long-memory behavior. While our fat-tailed distribution approach to the regime switching model is similar to Diebold and Inoue (2001) condition of having only a

[3]The proof of Theorem 3.1, along with the computer programs used to compute the estimates of Section 4 and 5, can be found at the website http://webpub.byu.net/mjj49/longswing.html

small number of swings, this paper's long-memory results differ since they are based on the population autocorrelations.[4]

4. Long swings in business cycles

In this section we empirically investigate whether the tail behavior of the observed regime durations associated with the US business cycle do in fact belong to the class of fat-tailed distributions. Instead of looking at GNP to characterize output's long-memory behavior, as other empirical macro studies do, our approach here follows that of Diebold and Rudebusch (1990) by focusing on the NBER dating of recessions.[5] While US GNP data do provide a good quarterly postwar measurement of macro conditions, by itself postwar GNP does not contain enough economic expansions and contractions. According to the NBER, there have been only nine post-World War II boom-bust cycles. This number of occurrences is too few to permit robust estimation of occasional long-swing behavior in the business cycle's expansions and contractions. On the other hand, the 31 NBER-defined boom-bust periods dating back to the year 1854 provides good estimates of the duration distribution for US upswings and downswings.

To estimate the tail index of the expansion duration distribution, $1-\overline{F}(u)$, and contraction duration distribution, $1-\underline{F}(u)$, we use an estimator that was proposed by Davis and Resnick (1984). The method, based on extreme value theory, focuses on the estimation of the distribution's tail index α. The estimator is based on the upper m order statistics from the random sample of expansion and contraction lengths, $\{\overline{T}_k\}$ and $\{\underline{T}_k\}$, each of size n, where $m = m(n)$ is a sequence of integers chosen such that $m \to \infty$ and $m/n \to 0$. The method estimates $a = 1/\alpha$, the reciprocal of the tail index. The estimator of a is given by the formula

$$\hat{a}(n/m) = m^{-1}\sum_{i=1}^{m}(\log(T_{(i)}) - \log(T_{(m+1)})),$$

where $T_{(1)} > T_{(2)} > \cdots > T_{(n)}$ are the decreasing order statistics from the random sample of the expansion or contraction durations. Davis and Resnick (1984) show that the random variable

$$\sqrt{m}\left(\frac{\hat{a}(n/m)}{a(n/m)} - 1\right)$$

converges to a $\mathrm{N}(0,1)$ random variable as $n \to \infty$.

Our estimates of the tail index consist of the upswing duration distribution tail index, $\overline{\alpha}$, the downswing duration distribution tail index, $\underline{\alpha}$, and the pooled duration distribution tail index, α. For each tail index we estimate its value using three choices for the precision parameter, $m = 8, 10, 16$. Our Davis and Resnick tail index estimates for the US business cycles, along with their 95%-confidence intervals, are reported in Table 1.[6]

[4]Gourieroux and Jasiak (2001) and Granger and Hyung (1999) also find long-memory behavior in structural change models when the expected number of regimes is a function of T.

[5]Our approach does differ from Diebold and Rudebusch (1990) duration dependence study of the business cycle, in that we focus on estimating the tail index associated with the duration length of the business cycle's expansions and recessions.

[6]The confidence intervals are not symmetric since they are generated by inverting the confidence interval of $\hat{a}$.

Table 1
Davis and Resnick (1984) tail index estimate of the US business cycle duration distribution

m	$\overline{\alpha}$	$\underline{\alpha}$	α
8	2.1768	1.8801	2.6817
	(1.2858, 7.0872)	(1.1104, 6.1275)	(1.5843, 8.7260)
10	2.5497	2.3502	1.2460
	(1.5743, 6.7024)	(1.4507, 6.1843)	(0.7690, 3.2765)
16	1.6829	1.7940	2.4673
	(1.1296, 3.2992)	(1.2039, 3.5186)	(1.6562, 4.8356)

The tail index estimates are for expansion duration, $\overline{\alpha}$, contraction duration, $\underline{\alpha}$, and pooled swing duration, α. m is a tuning parameter and the numbers in the parentheses give the 95% confidence interval. Note, the confidence intervals are not symmetrical since they are generated by inverting the confidence interval of $\hat{a} = 1/\hat{\alpha}$.

For the three different values of m, the tail index estimates in Table 1 are all between 1 and 3 in magnitude. Both the smallest and largest point estimate of the tail index occur when $m = 10$, with the smallest being $\alpha = 1.246$ and the largest $\overline{\alpha} = 2.5497$. Thus, according to the theoretical results of the previous sections, these point estimates give rise to the long-memory behavior found in real aggregate output.

While the point estimates of the tail index are relatively stable for the three different values of m, their confidence intervals are quite sensitive to the choice of m. In Table 1, the confidence intervals for $\overline{\alpha}$, $\underline{\alpha}$ and α are very much alike over each tail indices estimate for both small and large values of m. Unfortunately, when $m = 8$, the confidence intervals are too large to make any inference with regards to the presence of occasional long swings in the duration of the US business cycle. The confidence intervals for the tail indices estimated with $m = 8$ have upper bounds that produce long-memory parameters well below the $d = -1$ needed to ensure an invertible and stationary process. This lack of inference with small m may be due to the small number of NBER-defined business cycles, suggesting that our results with small m may not be reliable.

Because the confidence intervals of $\overline{\alpha}, \underline{\alpha}$, and α are all close to the interval $(1, 3)$ when $m = 16$, it suggests that long-memory behavior is present in US macro data. Our estimate of $\alpha = 2.4673$ corresponds to a long-memory parameter value of $d = -0.2337$. This value of d is close to the semi-nonparametric estimate of the long-memory parameter, $d = -0.3$, calculated by Diebold and Rudebusch (1989) for annual real US GNP growth data from 1869 to 1987. In addition, α confidence interval, $(1.6562, 4.8356)$, and its corresponding long-memory parameter's confidence interval, $(-1.418, 0.172)$, is similar to the confidence interval $(-1.27, 0.0931)$ found by Sowell (1992a) for the exact maximum likelihood estimate of the long-memory parameter for real US GNP growth rate data from 1947–1989. Like Sowell (1992a), our confidence interval for α can neither support nor reject trend stationarity ($d = -1$) over stochastic stationarity ($d = 0$). Thus, a long-memory model of real US GNP and a fat-tail distribution for the NBER-defined business cycle lengths are consistent with both trend stationarity and stochastic stationarity. However, only the duration length data suggest the existence of a nonstationary, occasional long-swing, regime switching model.

It could be argued that the duration length of economic expansions and contractions over the past 150 years does not constitute a stationary process. Indeed, it appears there is a difference between pre- and post-World War II duration lengths (see Diebold and

Rudebusch, 1992). Notwithstanding this structural difference, there appears to be considerable stability in the pre- and post-war business cycle's tail indices. When we apply the Davis and Resnick (1984) estimator of the tail index separately to pre- and post-war business cycle, duration lengths, we again find evidence of heavy tails. Using post-war business cycles, our tail indices estimates are $\overline{\alpha} = 1.8$ and $\underline{\alpha} = 2.16$, when $m = 3$, and for pre-war expansions and recessions lengths, $\underline{\alpha} = 1.87$ and $\overline{\alpha} = 2.91$, when $m = 7$. Our nonparametric tail index estimators are thus robust to post-war structural difference in the duration distribution of expansions and recessions.

5. Forecast comparison

Although the fractionally integrated model of long memory produces good predictions of past observations of aggregate output, our finding of long swings creates doubt as to aggregate output's true data generating process. This point is important given that a forecast with a linear model entails a moving average of observations from the distant past. In contrast, a regime-switching model with occasional long swings uses observations from the current regime along with the distribution of the regime's forward stopping time to predict future values of output.

Since the evidence on output's true data generating process is far from conclusive, we estimate and compare the forecasts from a regime switching model with long swings to a fractionally integrated, autoregressive, moving average (ARFIMA) model of US real output growth. Each model is fitted using 100 times the log-difference of real 1996 chained weighted US output data from the second quarter of the year 1952 to the second quarter of 2002.

5.1. Estimation

We fit seventeen ARFIMA(p,d,q) models, where $p, q = 0, 1, 2, 3$, with Sowell (1992b) exact maximum likelihood estimator and choose the order of the best fitting model with the Akaike information criterion. Using this model selection criterion we find the best fractionally integrated model of US output growth to be the ARFIMA(0,d,3) model

$$(1-L)^{\overset{0.3341}{\underbrace{}_{0.1122}}} y_t = (1 + \underbrace{0.0462}_{0.1515} L + \underbrace{0.0230}_{0.1462} L^2 - \underbrace{0.0584}_{0.1305} L^3)\, \varepsilon_t, \tag{4}$$

where L is the lag operator, $y_{t-j} = L^j y_t$. The estimate of the model's fractional differencing parameter is $d = 0.3341$, with the innovation's standard error, $\hat{\sigma} = 0.8198$, and likelihood, $\mathscr{L} = -265.8126$.[7] Upon further inspection and testing of Eq. (4) residuals, we do not detect the presence of any remaining structure in the estimated innovations. Our optimal linear forecast of log-differenced output with the estimated ARFIMA model are then carried out with the methodology described by Beran (1994) and Doornik and Ooms (1999).

To estimate the regime switching model, we follow Hamilton (1989), Durland and McCurdy (1994), and Lam (2004), and model ε_t in Eq. (1) as a fourth-order autoregressive process. The long-swing portion of our regime-switching model is modeled in the manner of Eq. (3), with the duration dependence captured by the cumulative probability

[7]The curly braces contain the parameter estimates' standard deviation.

distributions, $\overline{F}$, and, $\underline{F}$. To construct a realistic model of the duration distributions and still have them flexible enough to capture the presence of the heavy tails documented in Section 4, we spline together a Weibull and Pareto distribution. The Weibull can model various degrees of duration dependence and has been used by Sichel (1991) to study the duration dependence in US business cycles. It also includes the class of geometric distributions that correspond to the original Hamilton regime-switching model. DuMouchel (1983) and McCulloch (1997) have employed the Pareto distribution before, in studying heavy tails.

Given the splined Weibull and Pareto duration distribution, the probability of a boom (bust) ending at or after time t, in other words, the survivor function of a regime, equals

$$S(t) = 1 - F(t) = \begin{cases} 1 & \text{for } t \leqslant t_0, \\ \exp\{-(\lambda(t - t_0))^{\gamma}\} & \text{for } t_0 \leqslant t \leqslant \kappa, \\ \iota t^{-\alpha} & \text{for } t \geqslant \kappa, \end{cases} \tag{5}$$

where the value of ι ensures continuity of $S(t)$ at the point κ by equating:

$$\iota = \exp\{-(\lambda(\kappa - t_0))^{\gamma}\}\kappa^{-\alpha}.$$

Both the expansion and contraction regime's survivor functions are of the form found in Eq. (5), but with each regime having its own set of parameters (a bar above the parameter denotes the expansion's parameter value and a underscore the recession's parameter value). We set $\underline{t}_0 = 2$ to incorporate the NBER's definition that two consecutive quarters of negative growth constitutes a recession. While a similar rule of thumb is not used by the NBER in defining expansions, Moore and Zarnowitz (1986) indicate that a full boom-bust cycle must last longer than a year in order to qualify as a business cycle. Applying this criterion to the expansion regime's survivor function, we set $\overline{t}_0 = 3$ since it is one quarter smaller than the shortest-recorded expansion.

The parameter γ in Eq. (5) measures the level of duration dependence in the particular regime. If $\gamma > 1$, positive duration dependence, where the probability of switching out of the regime increases the longer the economy stays in that state, is present. If $\gamma < 1$, the duration dependence is negative.. The parameter κ defines how long the regime must survive before the tail behavior of its distribution begins. According to Eq. (5), duration lengths shorter than κ follow a Weibull distribution, whereas regimes that last longer than κ quarters are distributed as a Pareto distribution. As we defined in Section 2, a distribution with a value of α less than 2 is non-Gaussian with a tail that decays like a power function; i.e., negative duration dependency.

Using maximum likelihood, we estimate the above regime switching model. Log-differenced output data as well as the duration length of recessions and expansions, as defined by the NBER business cycle dates, are fit with the model.[8] The direct use of the NBER business cycle dates as a data source draws an important distinction between our estimation method and those of Hamilton (1989) and Lam (2004). Knowing the business cycle dates allows us to separate the likelihood function into two components. One component is the likelihood associated with regime's durations and the other is the

[8]Because the length of NBER business cycles are reported in months and output data are measured quarterly, we convert the duration length of expansions and recessions to quarters. We perform this frequency conversion by examining the sign of the growth rate of output at a peak/trough quarter to determine whether we should count it as an expansion or contraction quarter.

Table 2
Parameter estimates of the regime switching model's mean growth rate during expansions, $\bar{s}$, and recessions, $\underline{s}$, and estimates of the duration distribution parameters using only post-war NBER business cycles, and the duration distribution parameters estimates using all NBER-defined business cycles

	Post-war cycles		All cycles	
$\bar{s}$	0.0106	(0.0083)	0.0106	(0.0083)
$\underline{s}$	−0.0066	(0.0091)	−0.0066	(0.0091)
σ	0.0073	(0.0021)	0.0073	(0.0021)
$\bar{\lambda}$	0.0257	(0.0028)	0.0708	(0.0046)
$\bar{\gamma}$	0.9322	(0.6165)	1.4886	(0.6735)
$\bar{\kappa}$	12.0000	(0.0223)	6.9006	(0.8085)
$\bar{\alpha}$	1.6705	(0.7237)	1.6654	(0.5688)
$\underline{\lambda}$	0.4460	(0.0553)	0.2404	(0.2080)
$\underline{\gamma}$	1.6149	(0.3240)	1.4097	(0.6464)
$\underline{\kappa}$	5.6221	n.a.	5.7508	(6.6185)
$\underline{\alpha}$	374.5031	n.a.	2.7083	(4.8075)

Parameters with bars over them are associated with the duration distribution of expansions, those parameters that are underlined are for the duration distribution of recessions. The numbers inside the parenthesis are the standard errors.

short-memory component's likelihood function. Conditional on the known NBER recession dates, we filter the regime-switching component from output growth and estimate the AR(4) parameters by essentially applying maximum likelihood to the detrended data.

Our use of NBER business cycle dates in the estimation process, while not conventional, should not be cause for alarm. After all, one important objective of regime switching models is to construct an econometric method that mimics NBER business cycle dating methodology. The large body of regime switching literature indicates that the regime switching model fits the output data well. As a generalization of Hamilton (1989),[9] our model should provide a better fit of the output data.[10]

Because our estimation method separates the likelihood function into the regime-switching and short-memory component, we can either use all 31 NBER-defined business cycles or only those business cycles following World War II to estimate the duration distribution parameters. Using all of the business cycle dates allows stronger inference to be made concerning the tail behavior of the duration distributions. However, we must be mindful of the possible problems that can occur if the duration distributions over the pre-war period are not the same as over the post-war (see Diebold and Rudebusch, 1992).

When limited to post-war business cycles, we have only 10 expansions and nine contractions from which to estimate the two duration distributions' eight parameters. Because of the small number of business cycles, we constrain the κ in the expansion state's duration distribution to be less than the third largest observed duration. This restriction ensures that we have at least three observations in the tail of the expansion's duration

[9]By making κ large enough, our regime switching structure becomes the Weibull distribution. And the duration structure in Hamilton (1989) is essentially a geometric distribution that is a special case of the Weibull distribution.

[10]For more details on our estimation method, please refer to the econometric appendix found in the working paper version of this paper.

distribution. This restriction is not imposed on the recessions distribution since the length of post-war contractions only take on four unique values (2, 3, 4, and 5 quarters), which makes such a restriction artificial.

Table 2 presents our two sets of parameter estimates for the regime-switching model with long swings. Those estimated with post-war NBER-defined business cycle data are listed under "Post-war cycles" and the estimates using both pre- and post-war business cycle lengths are found under "All cycles". Since we always estimate the AR(4) parameters with the post-war output growth data, their estimates are the same, as are the estimates of regime's growth rates $\bar{s}$ and $\underline{s}$, regardless of the business cycle data. We find the AR(4) model to equal

$$\varepsilon_t = \underbrace{0.1328}_{0.1240}\varepsilon_{t-1} + \underbrace{0.0219}_{0.1060}\varepsilon_{t-2} - \underbrace{0.0814}_{0.0980}\varepsilon_{t-3} - \underbrace{0.0237}_{0.0934}\varepsilon_{t-4} + v_t, \tag{6}$$

where the standard error of the innovation term, v_t, is 0.0073.

Over the Weibull range of the duration distributions, the only estimate of $\bar{\gamma}$ and $\underline{\gamma}$ that is significantly different from the constant duration dependence value, $\gamma = 1$, is post-war contractions. In post-war recessions, $\underline{\gamma} = 1.6149$ with a p-value of 0.029 under the null of constant duration dependence. Finding $\underline{\gamma}$ to be significantly greater than one agrees with Diebold et al's (1993) earlier estimates of strong positive duration dependency in post-war recessions. Although not statistically significant, we also find our post-war estimate, $\bar{\gamma} = 0.9322$, supporting Lam's (2004) finding of negative duration dependency in post-war expansions. While we cannot conclusively say that duration dependence is present in all business cycles, it seems safe to say that there is evidence of positive duration dependence in post-war recessions and hints of negative duration dependence in post-war expansions.

The two estimates of the duration distribution's tail indices for expansions, $\bar{\alpha}$, found in Table 2 are both equal to 1.67. Being less than two, these estimates of the tail indices suggest heavy tails are present in the duration length of pre- and post-war expansions. Their value are also comparable to the nonparametric estimates found in Table 1 when $m = 16$.

Applying Theorem 1 to our estimated tail index, the expansion's tail indices correspond to our estimate of the ARFIMA model's fractional differencing parameter. The confidence interval of d constructed from $\bar{\alpha}$ confidence interval also captures the exact MLE of the fractional order of integration. Thus, our estimated regime-switching model with long swings provides a plausible alternative to the long-memory ARFIMA model explanation of the slow hyperbolic decay found in the correlation structure of US output.

For the duration distribution of contractions, our estimate of the tail index, $\underline{\alpha}$, is noninformative since the sample size is too small. The estimate for the cutoff point, $\underline{\kappa}$, is greater than the longest duration length. Thus, observations from the tail of the distribution are not available, so that a meaningful standard error cannot be computed for either $\underline{\kappa}$ or $\underline{\alpha}$. However, when post-war recession durations are pooled with pre-war duration lengths, the estimate of $\underline{\alpha}$ is nearly 3. This suggests the presence of heavy-tail behavior in the duration distribution of recessions.

Our estimate of $\bar{\gamma}$ with all the NBER business cycles implies positive duration dependence during expansions. On the other hand, our estimate of the tail index $\bar{\alpha}$ implies the presence of fat tails in the duration distribution of NBER-defined expansions; i.e., expansions also exhibit negative duration dependency. This contradiction of positive and negative duration dependence in US expansions can be understood with our flexible splined Weibull and Pareto duration distribution. US expansions lasting less than about seven quarters experience positive duration dependence, or in other words, the probability of an expansion ending before seven quarters increases with its duration. However, after an expansion reaches seven quarters, the probability that it will end diminishes as the regime gets older. Therefore, our estimated survivor function reconciles the seemingly contradictory negative duration dependence findings of Lam (2004) with the positive duration dependence discovered by Sichel (1991) and Diebold and Rudebusch (1990).

5.2. Forecast accuracy

For the two models we compute the out-of-sample forecasting errors, $e_{L,t+H} = y_{t+H} - \hat{y}_{L,t+H}$, and $e_{RS,t+H} = y_{t+H} - \hat{y}_{RS,t+H}$, where $t = 1992{:}1, \ldots, 2002{:}2 - H$ and $H = 1, \ldots, 12$. The forecasts, $\hat{y}_{L,t+H}$ and $\hat{y}_{RS,t+H}$, are, respectively, the H-period ahead out-of-sample forecast obtained with the ARFIMA model and the regime-switching model of 100 times log-differenced output given the information up to time period t. In other words, we reestimate the ARFIMA(0,d,3) and the AR(4) regime-switching models at each value of t in calculating the out-of-sample forecasts, $\hat{y}_{L,t+H}$ and $\hat{y}_{RS,t+H}$.

In Table 3 we tabulate four measures of out-of-sample forecasting performance and a statistic that tests the equal forecasting hypothesis. The first two forecasting measures are found in Panel A of Table 3 and equal the mean absolute forecasting error, $\text{MAFE} = 1/(T_2 - H - T_1 + 1)\sum_{t=T_1}^{T_2-H} |e_{i,t+H}|$, where $i = L, RS$ and $T_1, T_2 = 1992{:}1, \ldots, 2002{:}2$, and the root mean squared error

$$\text{RMSE} = \sqrt{\frac{1}{T_2 - H - T_1 + 1} \sum_{t=T_1}^{T_2-H} e_{i,t+H}^2}.$$

Panel A also includes the Diebold and Mariano (1995) test statistic of the null of no difference in the two model's forecasts. Panel B in Table 3 reports the two model's mean absolute cumulative forecasting error, $\text{MACFE} = 1/(T_2 - H - T_1 + 1)\sum_{t=T_1}^{T_2-H} |ce_{i,t}|$, and the root mean square cumulative forecasting error

$$\text{RMSCE} = \sqrt{\frac{1}{T_2 - H - T_1 + 1} \sum_{t=T_1}^{T_2-H} ce_{i,t}^2},$$

where $ce_{i,t} = \sum_{j=1}^{H} e_{i,t+j}$.

From the forecast accuracy measures, the regime-switching model with long swings produces better forecasts during the last expansion than the long-memory ARFIMA model. Except for the MAFE at the forecasting horizons $H = 3, 4, 5$, the magnitude of the regime-switching model's forecasting measures are all smaller than the ARFIMA models. The Diebold and Mariano test also rejects the null that the two model's forecasts are equivalent.

The quality of the regime-switching model's forecast deteriorates, as measured by the MAFE, as the time horizon increases to $H = 3, 4$. Such behavior is not surprising since greater levels of uncertainty enters into the forecast as the time horizon increases. However, the average error in the regime-switching model's forecast of output growth gets

Table 3
Panel A contains the mean absolute forecasting error (MAFE), root mean squared forecasting error (RMSE), and Diebold and Mariano (1995) test of equal accuracy for out-of-sample forecasts with time horizons H of 100 times the growth rate of quarterly real GNP over the period 1992:1–2002:2

A: *Out-of-sample forecast accuracy*:

H	MAFE		RMSE		Diebold–Mariano test of equal accuracy
	ARFIMA	Post-war RS	ARFIMA	Post-war RS	
1	0.4643	0.4234	0.5313	0.2562	6.9269
2	0.4476	0.4470	0.5379	0.3000	5.7020
3	0.4646	0.4730	0.5678	0.3123	5.6061
4	0.4546	0.4937	0.5626	0.3205	5.3001
5	0.4563	0.4593	0.5742	0.3205	5.1650
6	0.4634	0.4545	0.5762	0.3066	5.1264
7	0.4826	0.4536	0.5892	0.3150	5.4408
8	0.4786	0.4468	0.5761	0.3039	5.3661
9	0.4762	0.4568	0.5828	0.3027	5.1093
10	0.4759	0.4559	0.5785	0.3013	5.0829
11	0.4885	0.4534	0.5971	0.3076	4.9636
12	0.4766	0.4496	0.5885	0.3215	4.7354

B: *Out-of-sample cumulative forecast accuracy*:

H	MACFE		RMSCE	
	ARFIMA	Post-war RS	ARFIMA	Post-war RS
1	0.4643	0.4234	5.3134	0.2562
2	0.6978	0.5655	8.2854	0.5204
3	0.9882	0.8000	1.1658	0.7424
4	1.1784	1.0572	1.4497	0.9340
5	1.4700	1.2741	1.7807	1.0665
6	1.7340	1.4926	2.0669	1.1891
7	2.0170	1.6186	2.3462	1.2452
8	2.2835	1.7298	2.5980	1.2823
9	2.5483	1.8412	2.7755	1.1857
10	2.7870	1.9256	3.0140	1.1755
11	2.9671	1.9783	3.2430	1.1418
12	3.1812	2.0897	3.5182	1.0684

The Diebold and Mariano (1995) test statistic equals $DM = \overline{d}/\sqrt{2\pi\widehat{f}_d(0)/(T_2 - H - T_1 + 1)}$, where $\overline{d} = (T_2 - H - T_1 + 1)^{-1}\sum_{t=T_1}^{T_2-H}(e^2_{L,t} - e^2_{RS,t})$, and $\widehat{f}_d(0)$ is a Newey and West (1987) estimate of the spectral density of $e^2_{L,t} - e^2_{RS,t}$ at frequency zero with the lag length set automatically. Under the null of equal forecasting accuracy DM is distributed asymptotically standard normal. Panel B lists the mean absolute cumulative forecasting error (MACFE) and the root mean squared cumulative forecasting error (RMSCE) for the two models.

smaller as the forecast horizon increases from a year and a half ($H = 6$) to three years ($H = 12$). For $H = 5$, the regime-switching model's MAFE is 0.4593, whereas the average error in the regime-switching model's forecast, nearly three years into the future, is back down to 0.4496. Thus, it seems that by including duration dependency in the regime-switching model, the model is more accurate in its long-run forecasts of output growth, the longer the economy stays in an expansion.

6. Conclusion

According to the NBER, the 1990s economic expansion lasted for 10 years. In this paper, we were interested in the long-run behavior of a regime switching model of output, where the business cycles experience the occasional long-term swing that we saw in the 90s. We found that the occasional long-regime creates long memory behavior in the population autocorrelation function of the regime-switching model. Estimates of the tail indices with NBER expansion and contraction duration lengths supported our conjecture and corresponded with the estimates previously found with fractionally integrated models by Diebold and Rudebusch (1989) and Sowell (1992a). As an application, we computed out-of-sample forecasts with our regime-switching model with long swings and a fractionally integrated model and found the regime-switching model to be superior.

We need to note that our discovery of fat-tailed distributions in the length of business cycles does not resolve the debate over whether the trend in output is deterministic or stochastic. Instead, our findings add to the reasons why economists should be cautious about casting a blanket statement over output's behavior. Neither output nor, now, the length of business cycles has allowed economists to definitively reject one trend model of output for another.

Empirical evidence of fat-tailed business cycles also creates an opportunity for macro theorists to determine the type of disturbances and propagation mechanisms that would cause a model of the economy to exhibit fat-tailed behavior in the duration of their implied business cycles. Our results would also benefit from the development of a theoretical model where expansions are fragile at their beginning, but that become more and more robust as the expansion continues.

Our finding of the occasional long swing could be viewed as evidence of the successes monetary and fiscal polices have had in encouraging upswings and discouraging downswings. However, if the fat-tailed behavior found in US business cycles is driven by the underlying economy, our forecasting results cause one to question whether time-lagged, countercylical policy should even be carried out.

Important econometric questions also remain to be answered. For instance, does the spurious long-memory behavior found in our univariate regime-switching model with fat-tailed duration distributions generalize to a dynamic factor model? We leave these and other interesting long-memory and regime-switching questions for future research.

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