Strategic Planning of Integrated Multirefinery Networks: A Robust Optimization Approach Based on the Degree of Conservatism

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3 AUTHORS:

Adriana Leiras
Pontifícia Universidade Católica do Rio de …

A. Elkamel
University of Waterloo

Silvio Hamacher
Pontifícia Universidade Católica do Rio de …

Strategic Planning of Integrated Multirefinery Networks: A Robust Optimization Approach Based on the Degree of Conservatism

Adriana Leiras

Industrial Engineering Department, Pontifícia Universidade Católica do Rio de Janeiro, CP38097, 22453-900 Rio de Janeiro − RJ, Brazil

Ali Elkamel*

Chemical Engineering Department, University of Waterloo, Ontario N2L 3G1, Canada

Silvio Hamacher

Industrial Engineering Department, Pontifícia Universidade Católica do Rio de Janeiro, CP38097, 22453-900 Rio de Janeiro − RJ, Brazil

This paper considers the problem of strategic planning under uncertainty for optimal integration and coordination of a multirefinery network. The deterministic model proposed by Al-Qahtani and Elkamel [*Comput. Chem. Eng.* **2008**, *32*, 2189−202] was extended to account for uncertainties in raw material costs and final product prices, as well as product demand. The robust optimization methodology of Bertsimas and Sim [*Op. Res.* **2004**, *52*, 35−53] was applied, which deals with uncertainty in a tractable manner and does not add complexity to the deterministic problem. An industrial-scale study illustrated the benefits of the integrated planning and demonstrated that the modeling of uncertainty in process parameters provides a more practical perspective of the refining industry. In addition, probability bounds of constraint violation were calculated to help decision makers select appropriate parameters to control solution robustness and evaluate trade-offs between conservatism and total profit.

1. Introduction

The aggressive search for cost savings in the oil industry has forced refineries to modify operations in an effort to improve economic performance.[1] A common approach to counter this situation is to seek integration alternatives not only on a single facility but also on an enterprise-wide scale.[2] Planning for integration operations should be centralized, which allows for proper integration among all operating facilities and, consequently, an efficient utilization of available resources.[3] Process flexibility and economic efficiency are one of the many benefits to the coordinated planning of multiple sites.[4]

Planning applications are of particular interest due to economic incentives and strategic importance. Planning is basically an activity in which production targets are set and market forecasts, resource availability, and inventories are considered. In general, planning is categorized into different groups according to time frame: strategic (long-term), tactical (medium-term), and operational (short-term).

Considering the significance of this activity, especially in the competitive and volatile refining industry, the impact of uncertainties is inevitable. Uncertainties can be categorized as short-term, midterm, or long-term. Short-term uncertainties refer to unforeseen factors in internal processes such as operational variations and equipment failure.[5] Alternatively, long-term uncertainties represent external factors that impact the planning process over a long period of time, such as supply, demand, and price fluctuations. Midterm uncertainties include both short-term and long-term uncertainties.[6]

In optimization models, uncertainties have been noted by several academic studies conducted over the past few years (see Verderame et al.[7] for a review of works on planning and scheduling under uncertainty). The main methods employed in the studies for the oil industry were stochastic programming,[8−12] dynamic programming,[9,13] stochastic robust programming,[10,14−16] and fuzzy programming.[17,18] Despite the contributions of optimization under uncertainty, few studies discuss the importance of coordinating integrated multirefinery networks. Thus, the study of the benefits of integrated planning under uncertainty is still an open issue, which is relevant for mathematical modeling and actual applications.

In this context, the deterministic model proposed by Al-Qahtani and Elkamel[3] was extended for the strategic planning of integrated multirefinery networks. The model was formulated as a robust mixed-integer linear problem (MILP) to tackle the long-term uncertainties in the objective function coefficients (costs of raw material and the price of final products) as well as right-hand-side coefficient constraints (RHS), such as the demand for final products. An industrial-scale study conducted to discuss the benefits of integration in a stochastic environment is discussed.

The robust optimization methodology focuses on models that ensure solution feasibility for the possible outcomes of uncertainty parameters. Under this approach, the decision-maker accepts a suboptimal solution to ensure that, when the data changes, the solution remains feasible and near optimal. On the other hand, this method assumes limited information about the distributions of the underlying uncertainties, such as the mean value and its range. Unlike the approach of stochastic optimization, the specification of scenarios and corresponding probabilities are unnecessary in robust optimization, all of which are often cumbersome to estimate.

Methodologies based on the robust technique have been developed in many studies.[19] The first investigation was reported

* To whom correspondence should be addressed. E-mail: aelkamel@cape.uwaterloo.ca.

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by Soyster,[20] who proposed a conservative approach that assumed that all random parameters were equal to their worst-case value. Since then, several studies have extended the Soyster approach.[21−29] In this paper, the robust optimization methodology proposed by Bertsimas and Sim[26,27] and Bertsimas et al.[28] was adopted to account for uncertainties. The approach proposed by Bertsimas and Sim allowed trade-offs between optimal solutions, which made solution robustness more attractive. Furthermore, this method does not add complexity to the original problem. In spite of these advantages, a limitation of the Bertsimas and Sim's approach is that the uncertain parameters are considered unknown but bounded and symmetric random variables. Other types of distributions are beyond the scope of this paper, and the interested reader is referred to the work by Ben-Tal and Nemirovski[24] for linear problems with bounded uncertainty as well as the extensions of Lin and Janak[30] (for MILP with bounded uncertainty) and Janak et al.[31] (for MILP with known probability distributions). These works have further been studied and extended by Verderame and Floudas.[32−34] The remainder of this paper is organized as follows. In section 2, the robust optimization methodology of Bertsimas and Sim[26,27] and Bertsimas et al.[28] is explained. Section 3 presents the problem under study. The robust methodology is applied to a multirefinery network in section 4. Next, section 5 presents results and a discussion on the industrial-scale study of multirefinery networks, as well as a comparison of results obtained from integrated and nonintegrated strategic planning. The paper ends with concluding remarks in section 6.

2. Robust Optimization Methodology

Consider the following linear optimization problem with a set of n variables:

$$\begin{aligned} &\text{Minimize} \sum_{j=1}^{n} c_j x_j \\ &\text{Subject to} \sum_{j=1}^{n} a_{ij} x_j \le b_i, \forall i \\ &l_j \le x_j \le u_j, \forall j \end{aligned} \tag{1}$$

where c_j, a_{ij}, $b_i \in \mathbf{R}$.

Without a loss of generality, assume that the uncertainties affect only the elements of the matrix $\mathbf{A} = [a_{ij}]$. If uncertainty exists in the independent coefficients of constraints (b_i), a new variable x_{n+1} can be introduced into the model, and the constraint can be rewritten as $\sum_{j=1}^{n} a_{ij}x_j - b_i x_{n+1} \le 0$, $l_j \le x_j \le u_j$, $x_{n+1} = 1$, which includes b_i into matrix $\mathbf{A}$.[26] Moreover, if the objective function is also subject to uncertainties, the model can be rewritten to minimize $\mathbf{z}$, and the constraint $\sum_{j=1}^{n} c_j x_j - z \le 0$ is added to the set of constraints, $\sum_{j=1}^{n} a_{ij} x_j \le b_i$.[26]

The coefficients of the matrix $\mathbf{A}$ can also be modeled as a symmetric and bounded random variable, $\tilde{a}_{ij}$, $j \in J_i^a$, that assumes values on the interval $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$, where $J_i^a = \{j|\hat{a}_{ij} > 0\}$. For every constraint i, a parameter Γ_i^a is introduced (not necessarily an integer), which assumes values on the interval $[0, |J_i^a|]$. The main idea behind Bertsimas and Sim's approach is to control the conservatism of the robust solution by introducing a parameter that can be defined by decision makers (Γ_i^a) because it is unlikely that all uncertainty coefficients are equal to their worst case value (such as in Soyster's method). Thus, Bertsimas and Sim proposed a less conservative approach, allowing the decision maker to choose the number of uncertainty factors that he/she wishes to be protected from. If $\Gamma_i^a = 0$, the uncertainties in the parameters of constraint i can be ignored (deterministic problem). In contrast, $\Gamma_i^a = |J_i^a|$ represents the most conservative approach, in which all the uncertainty parameters of the constraint i are considered (Soyster's model). Accordingly, this parameter limits the number of coefficients that are simultaneously assigned the worst-case value.

Consider the random variable, $\eta_{ij} = (\tilde{a}_{ij} - a_{ij})/\hat{a}_{ij}$, which is associated with uncertainty, $\tilde{a}_{ij}$, and takes on values on the interval $[-1, 1]$. By the definition of robust optimization, the solution remains feasible for every possible value of $\tilde{a}_{ij}$. Thus, $\sum_{j=1}^{n} \tilde{a}_{ij}x_j = \sum_{j=1}^{n} a_{ij}x_j + \sum_{j=1}^{n} \eta_{ij}\hat{a}_{ij}x_j \le \sum_{j=1}^{n} a_{ij}x_j + \sum_{j=1}^{n} \hat{a}_{ij}|x_j| \le b_i$. Assuming $\Omega_j = ||x_j||$, the model constraint can be rewritten as $\sum_{j=1}^{n}(a_{ij}x_j + \hat{a}_{ij}\Omega_j) \le b_i$.

Model 1 possesses the following robust linear counterpart (see ref 26 for proofs). Note that $\Omega_j \le ||x_j||$ is equivalent to $-\Omega_j \le x_j \le \Omega_j$.

$$\begin{aligned} &\text{Minimize} \sum_{j=1}^{n} c_j x_j \\ &\text{Subject to} \sum_{j} a_{ij} x_j + \lambda_i^a \Gamma_i^a + \sum_{j \in J^a{}_i} \mu_{ij}^a \le b_i && \forall i \\ &\lambda_i^a + \mu_{ij}^a \ge \hat{a}_{ij}\Omega_j && \forall i, \forall j \in J_i^a \\ &l_j \le x_j \le u_j && \forall j \in J \\ &-\Omega_j \le x_j \le \Omega_j && \forall j \in J \\ &x_{n+1} = 1 \\ &\Omega_j \ge 0 && \forall j \in J \\ &\lambda_i^a \ge 0 && \forall i \\ &\mu_{ij}^a \ge 0 && \forall i, \forall j \in J_i^a \end{aligned} \tag{2}$$

At optimality, $\sum_{j=1}^{n}(a_{ij}x_j^* + \hat{a}_{ij}||x_j^*||) \le b_i$ for all j, where x_j^* is an optimal solution of problem 2. Thus, the term $\sum_{j=1}^{n} \hat{a}_{ij}||x_j^*||$ provides the necessary protection from the constraint by maintaining a gap between $\sum_{j=1}^{n} a_{ij}x_j^*$ and b_i.

The robust variables λ_i^a and μ_{ij}^a quantify the sensitivity of the system to changes in uncertainty parameter a_{ij}. The sum of the variables λ_i^a and μ_{ij}^a represents the minimum deviation that accounts for variations in uncertainty parameters ($\lambda_i^a + \mu_{ij}^a \ge \hat{a}_{ij}\Omega_j$). If $\Gamma_i^a = 0$ (deterministic problem), then $\mu_{ij}^a = 0$ and $\lambda_i^a = \hat{a}_{ij}\Omega_j$ in order to satisfy the minimum deviation constraint without modifying the constraint $\sum_j a_{ij}x_j + \lambda_i^a\Gamma_i^a + \sum_{j \in J^a{}_i} \mu_{ij}^a \le b_i$ because $\lambda_i^a\Gamma_i^a = 0$. On the other hand, when $\Gamma_i^a \ne 0$, the total additional production to protect against uncertainties ($\hat{a}_{ij}\Omega_j$) can be divided between λ_i^a and μ_{ij}^a, considering the value attributed to Γ_i^a. In addition, the parameter Γ_i^a controls the trade-off between the probability of constraint violation and the effect on the objective function of the deterministic problem. If $\Gamma_i^a \in [0, |J_i^a|]$, then the robust solution will be *deterministically* feasible. If more than $\lfloor \Gamma_i^a \rfloor$ coefficients change, the robust solution will remain feasible with *very high probability*. Assuming a symmetrical distribution of random variables, Bertsimas and Sim[26,27] calculated the probability that the ith constraint will be violated if more than $\lfloor \Gamma_i^a \rfloor$ coefficients vary. This probability can be approximated by the following expression:[27]

$$\Pr\left(\sum_j \tilde{a}_{ij}x_j^* > b_i\right) \le 1 - \Phi\left(\frac{\Gamma_i^a - 1}{\sqrt{n_i}}\right) \tag{3}$$

where $n = |J_i^a|$ and $\Phi(\theta)$ is the *cumulative distributive function of a standard normal*.

This result is interesting because it leads to a more appropriate choice for Γ_i^a, as shown below in the computational results of the case study.

3. Problem Description

This paper proposes a robust mixed-integer linear model in order to generalize Al-Qahtani and Elkamel's deterministic model,[3] which determines an optimal integration strategy for multiple refineries and establishes an overall production and operating plan for each individual site (see Appendix for the formulation of Al-Qahtani and Elkamel). The refineries are connected in a finite number of ways, and the network is supplied by a set of crude oil, cr $\in$ CR. Each refinery, $i \in I$, processes crude oil to produce a variety of marketable petroleum products, cfr $\in$ CFR. Refineries possess process units, $m \in M$, that can operate under different modes, $p \in P$, and produce several intermediate streams, cir $\in$ CIR, which can be blended to create distinct products. The objective is to minimize production and investment costs over a given period of time, achieve production flexibility, and improve coordination in the network. The robust model accounts for uncertainties in the cost of crude oil, as well as the price and demand for the final product.

4. Model Formulation

The robust formulation for optimal coordination of integrated multirefinery networks is presented below. Definitions of variables and parameters are provided at the end of this paper.

$$
\begin{aligned}
\min & \sum_{\mathrm{cr}\in \mathrm{CR}} \sum_{i\in I} \mathrm{Crcost}_{\mathrm{cr}} \mathrm{sr}_{\mathrm{cr},i} \\
& + \sum_{p\in P} \Big(\mathrm{Opcost}_p \sum_{\mathrm{cr}\in \mathrm{CR}} \sum_{i\in I} z_{\mathrm{cr},p,i}\Big) \\
& + \sum_{\mathrm{cir}\in \mathrm{CIR}} \sum_{i\in I} \sum_{i'\in I} \mathrm{Inscost}_{i,i'} \mathrm{yt}_{\mathrm{cir},i,i'} \text{ where } i \neq i' \\
& + \sum_{i\in I} \sum_{m\in M} \mathrm{Incost}_m \mathrm{ye}_{m,i} \\
& - \sum_{\mathrm{cfrex}\in \mathrm{PEX}} \sum_{i\in I} \mathrm{PR}_{\mathrm{cfrex}} e_{\mathrm{cfrex},i} \\
& - \lambda_{\mathrm{price}} \Gamma_{\mathrm{price}} + \sum_{\mathrm{cfrex}\in \mathrm{PEX}} \mu^{\mathrm{price}}_{\mathrm{cfrex}} \\
& - \lambda_{\cos t} \Gamma_{\cos t} + \sum_{\mathrm{cr}\in \mathrm{CR}} \mu^{\cos t}_{\mathrm{cr}}
\end{aligned} \tag{4}
$$

The robust objective function 4 aims to minimize the total cost, represented by the sum of crude oil supply costs, operation costs, intermediate exchange costs, and expansion costs, minus the export revenue. The parameters Γ_{price} and Γ_{cost} control, respectively, the uncertainty in export prices and costs of the objective function and assume values on the intervals $[0, |J_{\mathrm{price}}|]$ and $[0, |J_{\mathrm{cost}}|]$, where J is the set of uncertainty coefficients with deviations d_{price} and d_{cost}.

Subject to

$$\sum_{p\in P} z_{\mathrm{cr},p,i} = \mathrm{sr}_{\mathrm{cr},i} \quad \forall \mathrm{cr} \in \mathrm{CR}, i \in I \tag{5}$$

Constraint 5 represents the raw material balance, where the crude oil supply, $sr_{cr,\,i}$, at plant $i \in I$ from each oil, cr $\in$ CR, is equal to the sum of input flow rates at distillation units $p \in P$.

$$
\begin{aligned}
& \sum_{p\in P} \alpha_{\mathrm{cr},\mathrm{cir},i,p} z_{\mathrm{cr},p,i} + \sum_{i'\in I} \sum_{p\in P} \xi_{\mathrm{cir},i',p,i} \mathrm{xi}_{\mathrm{cir},i',p,i} = \\
& \sum_{i'\in I} \sum_{p\in P} \xi_{\mathrm{cir},i,p,i'} \mathrm{xi}_{\mathrm{cir},i,p,i'} + \sum_{\mathrm{cfr}\in \mathrm{CFCB}} w_{\mathrm{cr},\mathrm{cir},\mathrm{cfr},i} + \sum_{\mathrm{rf}\in \mathrm{FUEL}} w_{\mathrm{cr},\mathrm{cir},\mathrm{rf},i} \\
& \forall \begin{array}{l} \mathrm{cr} \in \mathrm{CR}, \mathrm{cir} \in \mathrm{CIR}, \\ i' \text{ and } i \in I, \text{where } i \neq i' \end{array}
\end{aligned} \tag{6}
$$

Constraint 6 limits the intermediate material balance within and across refineries, where the coefficient $\alpha_{\mathrm{cr,cir},i,p}$ can assume a positive sign if it is an input or a negative sign if it is an output. $\xi_{\mathrm{cir},i,p,i'}$ defines all possible alternatives of connecting intermediate streams, and $\mathrm{xi}_{\mathrm{cir},i',p,i}$ represents the trans-shipment flow rate from one plant to another.

$$
\begin{aligned}
\mathrm{xf}_{\mathrm{cfr},i} = \sum_{\mathrm{cr}\in \mathrm{CR}} \sum_{\mathrm{cir}\in \mathrm{CFCB}} w_{\mathrm{cr},\mathrm{cir},\mathrm{cfr},i} - \sum_{\mathrm{cr}\in \mathrm{CR}} \sum_{\mathrm{rf}\in \mathrm{FUEL}} w_{\mathrm{cr},\mathrm{cfr},\mathrm{rf},i} \\
\forall \mathrm{cfr} \in \mathrm{CFR}, i \in I
\end{aligned} \tag{7}
$$

The product material balance is defined as the difference between intermediate streams, $w_{\mathrm{cr,cir,cfr},i}$, that contribute to the final product and intermediate streams that contribute to the fuel system, as shown in constraint 7.

$$\mathrm{xv}_{\mathrm{cfr},i} = \sum_{\mathrm{cr}\in \mathrm{CR}} \sum_{\mathrm{cb}\in \mathrm{CFCB}} \frac{w_{\mathrm{cr},\mathrm{cb},\mathrm{cfr},i}}{\mathrm{SG}_{\mathrm{cr},\mathrm{cb}}} \quad \forall \mathrm{cfr} \in \mathrm{CFR}, i \in I \tag{8}$$

In constraint 8, to express the quality of each blend by volume, the mass flow rate is converted to the volumetric flow rate by dividing it by the specific gravity, $SG_{cr,cb}$.

$$
\begin{aligned}
\sum_{\mathrm{cir}\in \mathrm{FUEL}} \mathrm{CV}_{\mathrm{rf},\mathrm{cir},i} w_{\mathrm{cr},\mathrm{cir},\mathrm{rf},i} + \sum_{c=\text{"HFO"}\in \mathrm{FUEL}} w_{\mathrm{cr},c,\mathrm{rf},i} + \sum_{p\in P} \beta_{\mathrm{cr},\mathrm{rf},i,p} z_{\mathrm{cr},p,i} = 0 \\
\forall \mathrm{cr} \in \mathrm{CR}, i \in I, \mathrm{rf} \in \mathrm{FUEL}
\end{aligned} \tag{9}
$$

Constraint 9 corresponds to the fuel system material balance, where $\mathrm{CV}_{\mathrm{rf,cir},i}$ represents the caloric value equivalent for each intermediate, cir $\in$ FUEL, used in the fuel system at plant $i \in I$. The matrix $\beta_{\mathrm{cr,rf},i,p}$ corresponds to the consumption of each processing unit $p \in P$ at plant $i \in I$ as a percentage of unit throughputs.

$$
\begin{aligned}
& \sum_{\mathrm{cr}\in \mathrm{CR}} \sum_{\mathrm{cb}\in \mathrm{CFCB}} \left(\mathrm{ATT}_{\mathrm{cr},\mathrm{cb},q\in \mathrm{Qv}} \frac{w_{\mathrm{cr},\mathrm{cb},\mathrm{cfr},i}}{\mathrm{SG}_{\mathrm{cr},\mathrm{cb}}} + \mathrm{ATT}_{\mathrm{cr},\mathrm{cb},q\in \mathrm{Qw}} w_{\mathrm{cr},\mathrm{cb},\mathrm{cfr},i} \right) \quad \forall \begin{array}{l} \mathrm{cfr} \in \mathrm{CFR}, \\ q = \{\mathrm{qw},\mathrm{qv}\} \in Q, \\ i \in I \end{array} \\
& \geq Q^{\mathrm{L}}_{\mathrm{cfr},q\in \mathrm{Qv}} \mathrm{xv}_{\mathrm{cfr},i} + Q^{\mathrm{L}}_{\mathrm{cfr},q\in \mathrm{Qw}} \mathrm{xf}_{\mathrm{cfr},i}
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
& \sum_{\mathrm{cr}\in \mathrm{CR}} \sum_{\mathrm{cb}\in \mathrm{CFCB}} \left(\mathrm{ATT}_{\mathrm{cr},\mathrm{cir},q\in \mathrm{Qv}} \frac{w_{\mathrm{cr},\mathrm{cb},\mathrm{cfr},i}}{\mathrm{SG}_{\mathrm{cr},\mathrm{cb}}} + \mathrm{ATT}_{\mathrm{cr},\mathrm{cb},q\in \mathrm{Qw}} w_{\mathrm{cr},\mathrm{cb},\mathrm{cfr},i} \right) \quad \forall \begin{array}{l} \mathrm{cfr} \in \mathrm{CFR}, \\ q = \{\mathrm{qw},\mathrm{qv}\} \in Q, \\ i \in I \end{array} \\
& \leq Q^{\mathrm{U}}_{\mathrm{cfr},q\in \mathrm{Qv}} \mathrm{xv}_{\mathrm{cfr},i} + Q^{\mathrm{U}}_{\mathrm{cfr},q\in \mathrm{Qw}} \mathrm{xf}_{\mathrm{cfr},i}
\end{aligned} \tag{11}
$$

Constraints 10 and 11 specify quality limits for products that are blends by mass, qw $\in Q_w$, or volume, qv $\in Q_v$.

$$
\begin{aligned}
\min C_{m,i} \leq \sum_{p\in P} \Big(\gamma_{m,p} \sum_{\mathrm{cr}\in \mathrm{CR}} z_{\mathrm{cr},p,i}\Big) \leq \max C_{m,i} + \\
\mathrm{Add}C_{m,i} \mathrm{ye}_{m,i} \forall \quad m \in M, i \in I
\end{aligned} \tag{12}
$$

Constraint 12 limits the capacity of each processing unit. The coefficient $\gamma_{m,p}$ is a zero−one matrix for the assignment of production unit $m \in M$ to process operating mode $p \in P$. The term Add $C_{m,i}$ accounts for additional capacity expansion, and the integer variable $\mathrm{ye}_{m,i}$ represents the decision to expand a production unit.

$$\sum_{p \in P} \xi_{\mathrm{cir},i,p,i'} \mathrm{xi}_{\mathrm{cir},i,p,i'} \le F^{\mathrm{U}}_{\mathrm{cir},i,i'} \mathrm{yt}_{\mathrm{cir},i,i'} \quad \forall \begin{array}{l} \mathrm{cir} \in \mathrm{CIR}, \\ i' \text{ and } i \in I, \text{where } i \neq i' \end{array} \tag{13}$$

Constraint 13 sets the upper bound, $F^{\mathrm{U}}_{\mathrm{cir},i,i'}$, on intermediate streamflow rates between different refineries. The integer variable $\mathrm{yt}_{\mathrm{cir},i,i'}$ represents the decision to exchange intermediate products among refineries.

$$\sum_{i \in I} (\mathrm{xf}_{\mathrm{cfr},i} - e_{\mathrm{crpex},i} - \mathrm{DEM}_{\mathrm{cfr}} x_{\mathrm{cfr}',i}) \ge \lambda_{\mathrm{dem}} \Gamma_{\mathrm{dem}} + \mu^{\mathrm{dem}}_{\mathrm{cfrex}} \quad \forall \begin{array}{l} \mathrm{cfr, crpex, and\ cfr}', \\ \text{where cfr} \in \mathrm{CFR}, \\ \mathrm{crpex} \in \mathrm{PEX}, \\ \mathrm{cfr}' = |\mathrm{PEX}| + 1 \end{array} \tag{14}$$

Constraint 14 stipulates that the total production from each refinery, $\mathrm{xf}_{\mathrm{cfr},i}$, less the amount exported, $e_{\mathrm{crpex},i}$, must satisfy the domestic demand. The parameter Γ_{dem} adjusts the demand robustness and adopts values on the interval [0, 1].

$$\mathrm{IM}^{\mathrm{L}}_{\mathrm{cr}} \le \sum_{i \in I} \mathrm{sr}_{\mathrm{cr},i} \le \mathrm{IM}^{\mathrm{U}}_{\mathrm{cr}} \forall \quad \mathrm{cr} \in \mathrm{CR} \tag{15}$$

Constraint 15 limits the available feedstock, cr ∈ CR, to the refineries.

$$\lambda_{\mathrm{price}} + \mu^{\mathrm{price}}_{\mathrm{cfrex}} \ge d_{\mathrm{price}} e_{\mathrm{cfrex},i} \quad \forall \mathrm{cfrex} \in \mathrm{PEX}, i \in I \tag{16}$$

$$\lambda_{\mathrm{cost}} + \mu^{\mathrm{cost}}_{\mathrm{cr}} \ge d_{\mathrm{cost}} \mathrm{sr}_{\mathrm{cr},i} \quad \forall \mathrm{cr} \in \mathrm{CR}, i \in I \tag{17}$$

$$\lambda_{\mathrm{dem}} + \mu^{\mathrm{dem}}_{\mathrm{cfrex}} \ge d_{\mathrm{dem}} x_{\mathrm{cfr}',i} \quad \forall \begin{array}{l} \mathrm{cfrex} \in \mathrm{PEX}, i \in I \\ \mathrm{cfr}'' = |\mathrm{PEX}| + 1 \end{array} \tag{18}$$

Constraints 16, 17, and 18 represent the minimum deviation necessary to manage uncertainties in the parameters.

$$x_{\mathrm{cfr}',i} = 1 \quad \forall \begin{array}{l} i \in I \\ \mathrm{cfr}'' = |\mathrm{PEX}| + 1 \end{array} \tag{19}$$

The auxiliary variable $x_{\mathrm{cfr}',i}$ is defined in eq 19.

$$\mu^{\mathrm{price}}_{\mathrm{cfrex}} \ge 0 \quad \forall \mathrm{cfrex} \in \mathrm{PEX} \tag{20}$$

$$\mu^{\mathrm{cost}}_{\mathrm{cr}} \ge 0 \quad \forall \mathrm{cr} \in \mathrm{CR} \tag{21}$$

$$\mu^{\mathrm{dem}}_{\mathrm{cfrex}} \ge 0 \quad \forall \mathrm{cfrex} \in \mathrm{PEX} \tag{22}$$

$$\lambda_{\mathrm{price}}, \lambda_{\mathrm{cost}}, \lambda_{\mathrm{dem}} \ge 0 \tag{23}$$

Finally, constraints 20−23 define the non-negativity of robust variables.

5. Case Study

An industrial-scale study based on the numerical example of Al-Qahtani and Elkamel[3] was conducted. This example consisted of three large industrial-scale refineries and represented a general system that can be found in many industrial sites around the world. The decisions in the model included the selection of oil blends, production unit expansion options, and operating levels. The refineries are coordinated centrally, where the feedstock supply of two types of oils (Arabian light and Kuwait) is shared and the refineries collaborate to meet a given local market demand for liquefied petroleum gas (LPG), light naphtha (LN), two grades of gasoline (PG98 and ES95), jet fuel (JP4), military jet fuel (ATKP), gas oil (AHGO), diesel fuel (Diesel), heating fuel oil (HFO), and petroleum coke (Coke). Production that exceeds the local market demand is either sold in the spot market or exported, except for the products LN, LPG, and PG98 that attend only the local market demand. The prices, costs, and demands considered in the case study are presented in Table 1. T1

Table 1. Market Data Used in the Case Study

types of oils	cost ($/ton)	availability (1000 tons/year)
Arabian light	422	20000
Kuwait	380	20000
products	**export price ($/ton)**	**local demand (1000 tons/year)**
A-HGO	521	4920
ATKP	631	1800
Coke	50	300
Diesel	600	480
ES95	692	4440
HFO	70	200
JP-4	669	2340
LN		312
LPG		432
PG98		540

An optimal crude oil blend is used to feed the atmospheric unit, which separates crude oil into several fractions. Depending on the production targets, different conversions and treatment processes are applied to the crude fractions. Conversion processes (cocker, fluid catalytic cracker, and catalytic reforming) transform one fraction into another or change the molecular structure of the fraction. Treatment processes (hydrodesulphurization and hydrotreatment) further refine semifinished products by reducing contaminants, such as sulfur, nitrogen, and metals, or removing them from their structure.

All the costs in the numerical example were discounted over a 20 year time horizon with an interest rate of 7%. The installation cost of a refinery production unit was scaled according to the planning horizon and the life cycle of the project. The minimal total annualized cost was determined with and without integration among the refineries, and the benefits of integration in a stochastic environment are discussed.

The robust formulation to account for uncertainties in the objective function coefficients considers a 10% deviation in crude costs and product export prices. The parameters Γ_{price} and Γ_{cost} take values from the intervals [0, 7] and [0,2], respectively, which represents the seven products that can be exported and the two types of oils. In addition, the robust dimension related to demand was based on a 5% deviation from the nominal value, and Γ_{dem} assumes values on the interval [0, 1]. A limitation of the Bertsimas and Sim approach when uncertainty affects the independent coefficient constraints (b_i, which in our case study are represented by the product demands) is that the inclusion of b_i into matrix **A** to rewrite the constraint as $\sum_{j=1}^{n} a_{ij} x_j - b_i x_{n+1} \le 0$ creates a sparse column in matrix **A**. As a consequence, $|J_{dem}| = 1$ represents Soyster's case in which all demand parameters are equal to their worst-case value.

In the next section, the numerical example is presented and solved to demonstrate the effectiveness of the robust approach.

5.1. Computational Results and Discussion. The model was implemented using the Advanced Integrated Multidimensional

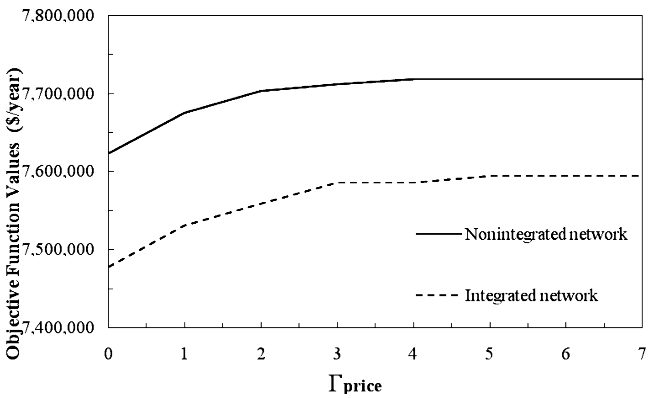

Figure 1. The effect of Γ_{price} on the objective function.

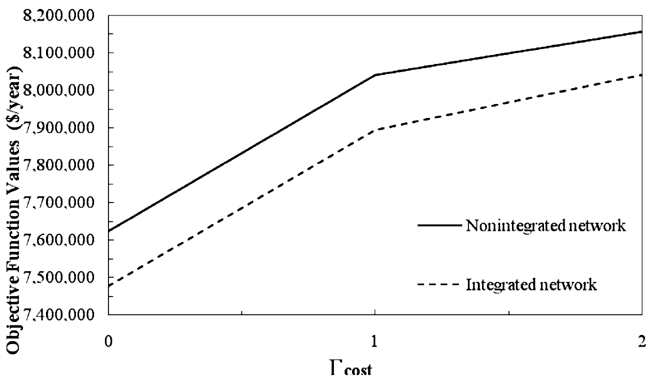

Figure 2. The effect of Γ_{cost} on the objective function.

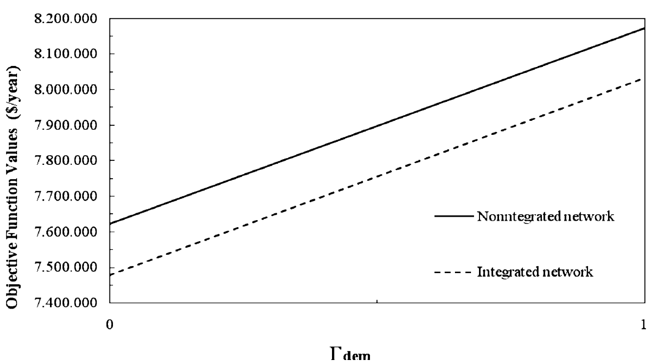

Figure 3. The effect of Γ_{dem} on the objective function.

Modeling Software, AIMMS,[35] and solved with CPLEX 11.1. Experiments were first conducted for each of the uncertainty parameters (price, cost, and demand) separately. The robust parameters were varied in integer values over the interval $[0, |J_i^a|]$, where $\Gamma_i^a = 0$ represents the deterministic solution and $\Gamma_i^a = |J_i^a|$ is Soyster's case.

As shown in Figures 1−3, the objective function values were marginally affected by the increased level of protection. Specifically, the incremental rise in protection decreased each time Γ increased. This result is a feature of robust formulation and is independent of the problem treated.

The objective function values for $\Gamma_{price} = 5$, $\Gamma_{price} = 6$, and $\Gamma_{price} = 7$ in Figure 1 are almost constant. This pattern appeared because the uncertain parameters are incorporated into the robust modeling in descending order of impact in the objective function. This impact is measured by multiplying the amount exported of each product by its export price deviation (10% of the price in this case study).

Table 2. Objective Function Values for the Integrated and Nonintegrated Networks

network	deterministic solution	Soyster's case		
		price	cost	demand
nonintegrated	\$7 623 151	\$7 718 831	\$8 156 155	\$8 172 159
integrated	\$7 477 978	\$7 594 578	\$8 040 825	\$8 033 696

The cost of crude oil uncertainty had a greater impact on total cost than product price uncertainty had, as can be seen in Figures 1 and 2. In addition, when Γ_{dem} is equal to 1 (Figure 3), the value of the objective function for the integrated network reaches \$8 033 696/year. As a result, demand and cost uncertainties had a larger effect on the objective function than price uncertainties had. Actually, when the total cost of the integrated network in Soyster's case ($\Gamma_i = |J_i|$) is compared, as shown in Table 2, the value of the objective function under cost and demand deviations was 7.53% and 7.43% higher than the deterministic solution ($\Gamma_i = 0$), respectively, whereas the solution under price deviations was only 1.56% higher.

Moreover, by allowing integration among refineries, the annual savings exceeded \$100 000 in comparison with the optimal solution for the nonintegrated network. The savings tend to increase as the number of plants, production units, and integration alternatives increase.

The benefits of integration are not limited to reducing costs. Additional benefits include an improvement in the flexibility of production, as well as the efficient utilization and allocation of resources among refineries. In fact, when integration is not allowed and the refinery is faced with deviations in demand, the model becomes infeasible due to variations in diesel demand. Specifically, diesel production was barely satisfying local demand at 480 000 tons/year, and only a small amount was available for export. With such a thin production margin, the plant did not have enough flexibility to face variations in diesel demand. However, an increase in the production margin and exports of diesel were observed in the integrated network, allowing the company to meet variations in local market demand. Production flexibility was achieved by increasing the supply of Kuwait crude oil, which altered the overall utilization of production units. Enhanced production flexibility was observed because Kuwait crude contains more heavy ends than Arabian light. The selection of crude supply type was made in favor of Arabian light because the entire supply could be processed and utilized. Although it yields a higher overall annual production cost, the remaining supply required to satisfy local market demand was fulfilled by Kuwait crude oil.

Figure 4 shows the computational results due to variations in demand deviations for a case study of the integrated network including a combination of the three types of randomness in the Soyster case ($\Gamma_{price} = 7$, $\Gamma_{cost} = 2$, $\Gamma_{dem} = 1$). The demand deviation, initially considered equal to 5% deviation from the nominal value, was varied from 3% to 11%, and the results confirm the model's sensitivity to these different deviations, as can be seen by the rise in the objective function values when the deviation increases.

The mathematical model does not consider the importation of products; thus, in response to a 9% increase in demand deviation, the strategic plan recommended the installation of one new isomerization unit in refinery 3. On the other hand, when an 11% increase in demand was considered, the model suggested the installation of three units, including an isomerization unit in refinery 1, a fluid catalytic cracker unit in refinery 2, and a desulfurization unit in refinery 3. As shown in Figure 4, the total annual cost increased significantly due to the

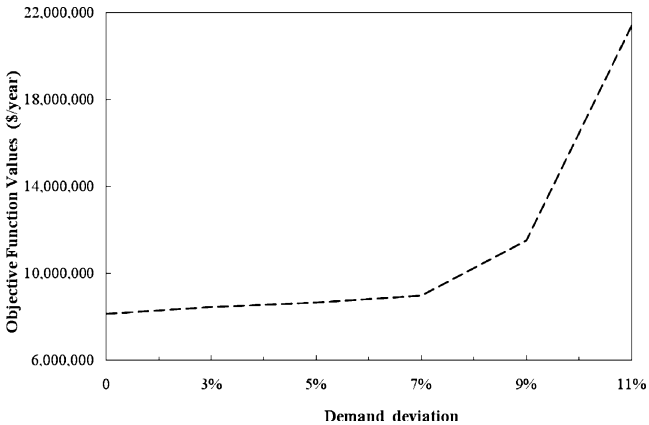

Figure 4. The effect of demand ($\Gamma_{price} = 7$, $\Gamma_{cost} = 2$, $\Gamma_{dem} = 1$) on the objective function.

Table 3. Model Investment Decisions Due to Variations in Demand Deviation

	demand deviation				
	3%	5%	7%	9%	11%
refinery 1					isomerization
refinery 2					fluid catalytic cracker
refinery 3				isomerization	desulfurization

installation of additional units and a rise in operating costs. These investment decisions are presented in Table 3.

In spite of the advantages of a robust methodology, the decision maker's risk-taking behavior is not evaluated in the model. A more realistic approach would incorporate a measure that specifies the risk profile of the decision maker. One relevant measure is the probability of a constraint violation, which can lead to a more appropriate choice for Γ_i^a, as shown in eq 3. Given that the maximum profit decreases when the degree of conservatism increases, the problem can be solved for different values of the parameter Γ_i^a and the decision maker can subsequently judge the trade-off between conservatism and total profit in order to choose the appropriate Γ_i^a value.

Table 4 presents the results for choices of integer values of Γ_i^a in the interval $[0, |J_i^a|]$ as a function of the probability bound of constraint violation. A range of probability levels was evaluated to offer the decision maker a trade-off between robustness and profit. As the objective function is subject to uncertainties, the model was rewritten to consider these uncertainties in a constraint, as explained in section 2. Thus, it also makes sense evaluate the probability bounds for Γ_{price} and Γ_{cost}.

From Table 4, our calculations suggest that if the decision maker accepts 60% or more of probability of violation, there is no need to secure additional protection from the deterministic problem and Γ_{price} can be set equal to zero. On the other hand, if the decision maker only accepts a maximum of 50% probability of constraint violation, then he/she has to use $\Gamma_{price} \neq 0$ to protect himself/herself against price parameter deviations. In this sense, the decision maker can judge the appropriate trade-off between conservatism and total profit in order to adopt a more appropriate parameter to control robustness. Similar analysis can also be done for Γ_{cost} and Γ_{dem}. Note that when a large number of uncertain parameters are considered in each constraint (price case), the solution is less conservative than in the case with few uncertain coefficients (demand case).

6. Conclusions

In this work, a robust mixed-integer linear model for the coordination of an integrated multirefinery network faced with a variety of uncertainties was presented. Uncertainties in price, cost, and demand were considered in the model, which provided a more robust and practical analysis of the problem, especially in situations where large fluctuations in petroleum prices and demand exist. The model with and without the integration of the refinery network was solved to illustrate the benefits of integrated coordination in a stochastic environment. Integration improved production flexibility and provided a model that was feasible in the presence of variable demands. The results revealed the economic potential and trade-offs involved in the optimization of integrated refinery networks. Thus, the approach of Bertsimas and Sim seems to be a useful tool for modeling strategic plans without introducing additional computational complexity. In addition, the calculation of probability bounds for constraint violations allows decision makers to control solution robustness and make better choices. As future works, this research is going to be extended to evaluate other robust methodologies as well as other types of distribution of the random variables.

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Nomenclature

Sets and Indices

C = commodity, c, c'
I = plant, i, i'
M = production unit, m
P = refinery process, p
Q = quality specification, q

Subsets

CFR = final products, cfr, subset of C
CIR = intermediate streams, cir, subset of C
CR = raw materials, cr, subset of C
CB = streams for blending refinery products, cb, subset of C
PEX = products for exports, cfrex, subset of C
RF = fuel streams, rf, subset of C
QV = quality of products from blends by volume, qv, subset of Q
QW = quality of products from blends by weight, qw, subset of Q
$CFCB_{cfr,cir}$ = blends of cfr and cir
$FUEL_{rf,c}$ = streams, c, comprising refinery fuel, rf

Table 4. Choice of Γ_i^a Values As a Function of the Probability of Constraint Violation

	probability of constraint violation												
	0.01	0.02	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Γ_{price}	7	6	5	4	3	2	2	1	0	0	0	0	0
Γ_{cost}	2	2	2	2	2	2	1	1	1	0	0	0	0
Γ_{dem}	1	1	1	1	1	1	1	1	1	0	0	0	0

Parameters

$\mathrm{CrCost}_{\mathrm{cr}}$ = cost of crude cr
OpCost_{p} = operating cost of process p
$\mathrm{InsCost}_{i,i'}$ = insulation cost of piping and pumping cost to transfer commodity cir from plant i to plant i'
InCost_{m} = installation cost of a refinery production unit m
$\mathrm{PR}_{\mathrm{cfrex}}$ = export price of product crpex
$\alpha_{\mathrm{cr,cir},i,p}$ = input−output coefficients of stream cir from crude cr at plant i by process p
$\xi_{\mathrm{cir},i,p,i'}$ = integration superstructure of all possible alternatives to transfer commodity cir from plant i to process p in plant i'
$\mathrm{SG}_{\mathrm{cr,cir}}$ = specific gravity of commodity cir by crude cr
$\mathrm{CV}_{\mathrm{rf},c,i}$ = caloric value equivalent of refinery fuel rf by commodity c at plant i
$\beta_{\mathrm{cr,rf},i,p}$ = fuel consumption coefficients of refinery fuel rf from crude cr at plant i by process p
$\mathrm{ATT}_{\mathrm{cr,cir},q}$ = attributes of intermediate streams cir produced from crude cr with properties q
$Q^{\mathrm{L}}_{\mathrm{cfr},q}$ = quality bounds of commodity cfr of property q
$Q^{\mathrm{U}}_{\mathrm{cfr},q}$ = upper level bounds of commodity cfr of property q
$\mathrm{Min}C_{m,i}$ = minimum capacity of production unit m at plant i
$\mathrm{Max}C_{m,i}$ = maximum capacity of production unit m at plant i
$\mathrm{Add}C_{m,i}$ = additional capacity of production unit m at plant i
$\gamma_{m,p}$ = assignment of process p to equipment m
$F^{\mathrm{U}}_{\mathrm{cir},i,i'}$ = upper bound of the flow rate of intermediate cir from plant i to i'
$\mathrm{DEM}_{\mathrm{cfr}}$ = demand of product cfr
$\mathrm{IM}^{\mathrm{L}}_{\mathrm{cr}}$ = lower bound of importing commodity cr
$\mathrm{IM}^{\mathrm{U}}_{\mathrm{cr}}$ = upper bound of importing commodity cr

Variables

$e_{\mathrm{crpex},i}$ = exports of final product crpex from refinery i
$\mathrm{sr}_{\mathrm{cr},i}$ = supply of raw material cr to refinery i
$w_{\mathrm{cr},c,c',i}$ = blending levels of crude cr that produces the stream c to yield a stream c' at plant i
$z_{\mathrm{cr},p,i}$ = process input flow rate of crude cr to process p at plant i
$\mathrm{xf}_{\mathrm{cfr},i}$ = mass flow rate of final product cfr by refinery i
$\mathrm{xv}_{\mathrm{cfr},i}$ = volumetric flow rate of final product cfr by refinery i
$\mathrm{xi}_{\mathrm{cir},i,p,i'}$ = trans-shipment level of commodity cir from plant i to process p at plant i'
$\mathrm{ye}_{m,i}$ = binary variable representing refinery expansion of production unit m at plant i
$\mathrm{yt}_{\mathrm{cir},i,i'}$ = binary variable representing trans-shipment of refinery commodity cir from plant i to plant i'

Robust Variables

$\mu^{\mathrm{price}}_{\mathrm{cfrex}}, \mu^{\mathrm{cost}}_{\mathrm{cr}}, \mu^{\mathrm{dem}}_{\mathrm{cfrex}}$ = quantifies sensitivity to changes in cost, price, and demand, respectively
$\lambda_{\mathrm{price}}, \lambda_{\mathrm{cost}}, \lambda_{\mathrm{dem}}$ = quantifies sensitivity to changes in cost, price, and demand, respectively
$x_{\mathrm{cfr}',i}$ = auxiliary variable of robust formulation to account for demand randomness

Robust Parameters

$\Gamma_{\mathrm{cost}}, \Gamma_{\mathrm{price}}, \Gamma_{\mathrm{dem}}$ = parameters to adjust the cost, price, and demand robustness, respectively
$d_{\mathrm{cost}}, d_{\mathrm{price}}, d_{\mathrm{dem}}$ = cost, price, and demand deviations, respectively

Appendix

The deterministic formulation proposed by Al-Qahtani and Elkamel is presented below. Definitions of variables and parameters are provided at the Nomenclature section.

$$\min \sum_{\mathrm{cr}\in\mathrm{CR}}\sum_{i\in I}\mathrm{CrCost}_{\mathrm{cr}}\mathrm{sr}_{\mathrm{cr},i} + \sum_{p\in P}(\mathrm{OpCost}_p\sum_{\mathrm{cr}\in\mathrm{CR}}\sum_{i\in I}z_{\mathrm{cr},p,i}) + \sum_{\mathrm{cir}\in\mathrm{CIR}}\sum_{i\in I}\sum_{i'\in I}\mathrm{Inscost}_{i,i'}\mathrm{yt}_{\mathrm{cir},i,i'} \quad \text{where } i\neq i' + \sum_{i\in I}\sum_{m\in M}\mathrm{Incost}_m\mathrm{ye}_{m,i} - \sum_{\mathrm{cfrex}\in\mathrm{PEX}}\sum_{i\in I}\mathrm{PR}_{\mathrm{cfrex}}e_{\mathrm{cfrex},i} \tag{A1}$$

Subject to

$$\sum_{p\in P}z_{\mathrm{cr},p,i} = \mathrm{sr}_{\mathrm{cr},i} \quad \forall \mathrm{cr}\in\mathrm{CR}, i\in I \tag{A2}$$

$$\sum_{p\in P}\alpha_{\mathrm{cr,cir},i,p}z_{\mathrm{cr},p,i} + \sum_{i'\in I}\sum_{p\in P}\xi_{\mathrm{cir},i',p,i}\mathrm{xi}_{\mathrm{cir},i',p,i} = \sum_{i'\in I}\sum_{p\in P}\xi_{\mathrm{cir},i,p,i'}\mathrm{xi}_{\mathrm{cir},i,p,i'} + \sum_{\mathrm{cfr}\in\mathrm{CFCB}}w_{\mathrm{cr,cir,cfr},i} + \sum_{\mathrm{rf}\in\mathrm{FUEL}}w_{\mathrm{cr,cir,rf},i} \quad \forall\begin{matrix}\mathrm{cr}\in\mathrm{CR}, \mathrm{cir}\in\mathrm{CIR},\\ i'\text{and}\, i\in I, \text{where } i\neq i'\end{matrix} \tag{A3}$$

$$\mathrm{xf}_{\mathrm{cfr},i} = \sum_{\mathrm{cr}\in\mathrm{CR}}\sum_{\mathrm{cir}\in\mathrm{CFCB}}w_{\mathrm{cr,cir,cfr},i} - \sum_{\mathrm{cr}\in\mathrm{CR}}\sum_{\mathrm{rf}\in\mathrm{FUEL}}w_{\mathrm{cr,cfr,rf},i}\forall \quad \mathrm{cfr}\in\mathrm{CFR}, i\in I \tag{A4}$$

$$\mathrm{xv}_{\mathrm{cfr},i} = \sum_{\mathrm{cr}\in\mathrm{CR}}\sum_{\mathrm{cb}\in\mathrm{CFCB}}\frac{w_{\mathrm{cr,cb,cfr},i}}{\mathrm{SG}_{\mathrm{cr,cb}}}\forall \quad \mathrm{cfr}\in\mathrm{CFR}, i\in I \tag{A5}$$

$$\sum_{\mathrm{cir}\in\mathrm{FUEL}}\mathrm{CV}_{\mathrm{rf,cir},i}w_{\mathrm{cr,cir,rf},i} + \sum_{c=\text{"HFO"}\in\mathrm{FUEL}}w_{\mathrm{cr},c,\mathrm{rf},i} + \sum_{p\in P}\beta_{\mathrm{cr,rf},i,p}z_{\mathrm{cr},p,i} = 0 \quad \forall\begin{matrix}\mathrm{cr}\in\mathrm{CR}, i\in I,\\ \mathrm{rf}\in\mathrm{FUEL}\end{matrix} \tag{A6}$$

$$\sum_{\mathrm{cr}\in\mathrm{CR}}\sum_{\mathrm{cb}\in\mathrm{CFCB}}\left(\mathrm{ATT}_{\mathrm{cr,cb},q\in\mathrm{Qv}}\frac{w_{\mathrm{cr,cb,cfr},i}}{\mathrm{SG}_{\mathrm{cr,cb}}} + \mathrm{ATT}_{\mathrm{cr,cb},q\in\mathrm{Qw}}w_{\mathrm{cr,cb,cfr},i}\right) \geq Q^{\mathrm{L}}_{\mathrm{cfr},q\in\mathrm{Qv}}\mathrm{xv}_{\mathrm{cfr},i} + Q^{\mathrm{L}}_{\mathrm{cfr},q\in\mathrm{Qw}}\mathrm{xf}_{\mathrm{cfr},i} \quad \forall\begin{matrix}\mathrm{cfr}\in\mathrm{CFR},\\ q=\{\mathrm{qw},\mathrm{qv}\}\in Q,\\ i\in I\end{matrix} \tag{A7}$$

$$\sum_{\mathrm{cr}\in\mathrm{CR}}\sum_{\mathrm{cb}\in\mathrm{CFCB}}\left(\mathrm{ATT}_{\mathrm{cr,cir},q\in\mathrm{Qv}}\frac{w_{\mathrm{cr,cb,cfr},i}}{\mathrm{SG}_{\mathrm{cr,cb}}} + \mathrm{ATT}_{\mathrm{cr,cb},q\in\mathrm{Qw}}w_{\mathrm{cr,cb,cfr},i}\right) \leq Q^{\mathrm{U}}_{\mathrm{cfr},q\in\mathrm{Qv}}\mathrm{xv}_{\mathrm{cfr},i} + Q^{\mathrm{U}}_{\mathrm{cfr},q\in\mathrm{Qw}}\mathrm{xf}_{\mathrm{cfr},i} \quad \forall\begin{matrix}\mathrm{cfr}\in\mathrm{CFR},\\ q=\{\mathrm{qw},\mathrm{qv}\}\in Q,\\ i\in I\end{matrix} \tag{A8}$$

$$\min C_{m,i} \le \sum_{p\in P}\left(\gamma_{m,p}\sum_{\mathrm{cr}\in \mathrm{CR}} z_{\mathrm{cr},p,i}\right) \le \max C_{m,i} + \mathrm{Add}C_{m,i}\mathrm{ye}_{m,i} \qquad \forall m \in M, i \in I \tag{A9}$$

$$\sum_{p\in P} \xi_{\mathrm{cir},i,p,i'}\mathrm{xi}_{\mathrm{cir},i,p,i'} \le F^{\mathrm{U}}_{\mathrm{cir},i,i'}\mathrm{yt}_{\mathrm{cir},i,i'} \qquad \forall \mathrm{cir} \in \mathrm{CIR},\ i' \text{ and } i \in I, \text{ where } i \neq i' \tag{A10}$$

$$\sum_{i\in I}(\mathrm{xf}_{\mathrm{cfr},i} - e_{\mathrm{crpex},i}) \ge \mathrm{DEM}_{\mathrm{cfr}} \qquad \forall \text{ cfr and crpex, where } \mathrm{cfr} \in \mathrm{CFR},\ \mathrm{crpex} \in \mathrm{PEX}, \tag{A11}$$

$$\mathrm{IM}^{\mathrm{L}}_{\mathrm{cr}} \le \sum_{i\in I}\mathrm{sr}_{\mathrm{cr},i} \le \mathrm{IM}^{\mathrm{U}}_{\mathrm{cr}} \qquad \forall\ \mathrm{cr} \in \mathrm{CR} \tag{A12}$$

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