Effect of Thermodynamic Restriction on Energy Cost Optimization of RO Membrane Water Desalination

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Advances in highly permeable reverse osmosis (RO) membranes have enabled desalting operations, in which it is practically feasible for the applied pressure to approach the osmotic pressure of the exit brine stream. However, energy cost remains a major contributor to the total cost of water produced by RO membrane desalination. Reduction of the overall cost of water production represents a major challenge and, in the present work, various elements of water production cost are evaluated from the viewpoint of optimization, with respect to various costs (energy, membrane area and permeability, brine management, and pressure drop), as well as the important thermodynamic cross-flow constraint, utilization of energy recovery devices, and operational feed and permeate flow rate constraints. More specifically, in the present study, an approach to the optimization of product water recovery at pressures that approach the osmotic pressure of the exit brine stream is presented via several simple RO process models that utilize highly permeable membranes. The results suggest that it is indeed feasible to refine RO processes to target for operation under the condition of minimum energy consumption, while considering the constraint imposed by the osmotic pressure, as specified by the thermodynamic cross-flow restriction. Although it is shown that multistage RO provides energy savings, this is at the expense of greater membrane area cost. Overall, as process costs above energy costs are added, the operational point for achieving minimum water production cost shifts to higher recoveries. Although the newer generation of RO membranes can allow high recovery operations at lower pressures, limitations due to mineral scaling and fouling impose additional constraints. The incorporation of these phenomena in the optimization approach is the subject of ongoing research.

1. Introduction

Reverse osmosis (RO) membrane water desalination is now well-established as a mature water desalination technology. However, there are intensive efforts to reduce the cost of RO water desalination, to broaden the appeal and deployment of this technology. The water production cost in a typical RO desalination plant generally consists of the cost of energy consumption, equipment, membranes, labor and maintenance, and financial charges. Energy consumption is a major portion of the total cost of water desalination and can reach values as high as ~45% of the total permeate production cost.[1−3] The energy cost per volume of produced permeate (i.e., the specific energy consumption, or SEC) is significant in RO operation, because of the high pressure requirement (up to ~1000 psi for seawater and in the range of 100−600 psi for brackish water desalting). Considerable effort has been devoted to find means of reducing the transmembrane pressure required for a given water permeate productivity level dating back to the initial days of RO development in the early 1960s.

Early research in the 1960s[4−7] focused on unit cost optimization, with respect to water recovery, energy recovery system efficiency, feed flow rate, and the applied transmembrane pressure. Efforts to reduce the SEC also considered increasing the permeate flow rate, at a given applied pressure and feed flow rate, by optimizing the membrane module, with respect to its permeate flux,[8−15] and/or by using more-permeable membranes.[16−19] For example, studies have shown that specific permeate productivity of spiral-wound RO and nanofiltration modules could be improved by optimizing the module configuration (e.g., feed channel height, permeate channel height, and porosity).[12]

The introduction of highly permeable membranes in the mid 1990s with low salt passage[16] has generated considerable interest, given their potential for reducing the energy required to attain a given permeate productivity.[16−19] Wilf,[16] and later Spiegler,[20] reported that operation close to the minimum level of applied pressure (i.e., pressure approaching the concentrate osmotic pressure plus frictional pressure losses) would result in the lowest energy cost. Clearly, in the absence of a pressure drop in the membrane module, the minimum required applied pressure, when a highly permeable membrane is used, would be very close to the osmotic pressure of the RO concentrate that would be reached at the membrane outlet.[16,21−23] As illustrated in Figure 1, to achieve a given water recovery and utilize the entire membrane area, there is a minimum pressure that must be applied, and this pressure must be greater than the

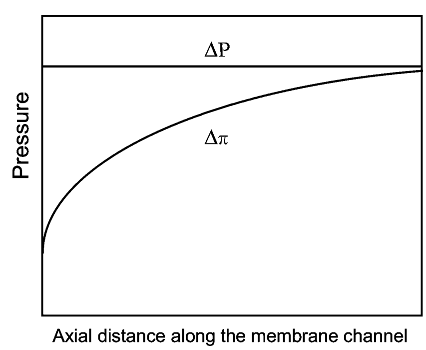

Figure 1. Schematic illustration of the thermodynamic restriction for cross-flow reverse osmosis (RO) desalting.[21]

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osmotic pressure of the concentrate exiting the process; however, this applied pressure can approach the osmotic pressure of the brine stream when highly permeable membranes are used. The requirement of a minimum pressure, for the lowest energy cost, will apply even when one considers concentration polarization, albeit the required pressure will be based on the osmotic pressure at the membrane surface at the module exit.[21]

To reduce energy consumption, energy recovery from the concentrate stream has been implemented using a variety of energy recovery devices (ERDs), in addition to optimization of the configurations of the RO membrane arrays. The effect of an energy recovery device (ERD) on the SEC was first studied in the early 1960s.[5,6] Avlonitis et al.[24] discussed four types of ERDs (i.e., Pelton wheel, Grundfos Pelton wheel, turbo charger, pressure exchanger) and reported that the pressure exchanger was the most-efficient energy recovery device. More recently, Manth et al.[1] proposed an energy recovery approach, in which a booster pump is coupled with a Pelton turbine (instead of a single-component high-pressure feed pump) or is used as an interstage booster for dual-stage brine conversion systems.

Simplified process models to optimize the structure of RO membrane desalination plants have been proposed in the literature.[25−32] Early studies have shown that the "Christmas tree" configuration that was developed in the early 1970s was suitable for the early generation of RO spiral-wound membranes. However, with the emergence of higher permeability membranes, it is unclear if the above configuration of membrane modules is also optimal for ultralow-pressure RO modules.[25] It has been argued that the SEC can be reduced by utilizing a large number of RO membrane units in parallel, to keep the flow and operating pressure low.[28] It has also been claimed that the SEC decreases when the number of membrane elements in a vessel is increased.[3] In the mid 1990s, researchers suggested that a single-stage RO process would be more energy-efficient.[33] However, it has been also claimed that a two-stage RO was more energy-efficient than single-stage RO.[28] The aforementioned conflicting views suggest that there is a need to carefully compare the energy efficiency of RO desalination by appropriately comparing single and multiple-stage RO based on the appropriately normalized feed flow rate and SEC, taking into consideration the feed osmotic pressure, membrane permeability, and membrane area.

Optimization of RO water production cost, with respect to capital cost, has also been addressed to explore the means of reducing the total specific cost of water production.[28,33] Such optimization studies have considered the costs associated with feed intake (primarily for seawater) and pretreatment, high-pressure pumps, the energy recovery system, and membrane replacement.[33] The problem of maximizing RO plant profit, considering energy cost, amortized membrane cost, cleaning and maintenance costs, and the amortized cost of process pumps in the absence of energy recovery devices has also been addressed.[28] The majority of the existing studies have accepted the standard operating procedure whereby the applied pressures is set to be significantly higher than the minimum required pressure limit that would correspond to the lowest SEC. Moreover, a formal mathematical approach has not been presented to enable an unambiguous evaluation of the optimization of the RO water production cost, with respect to the applied pressure, water recovery, pump efficiency, membrane cost, and use of energy recovery devices.

It is important to recognize that previous studies that focused on optimization of the SEC have only evaluated the SEC dependence on water recovery at one or several normalized feed and permeate flow rates. Previous researchers have reported the minimum SEC for one or several flow rates or a range of product water recoveries [4−19] However, the global minimum SEC has not been identified, along with SEC optimization via a general theoretical framework. Motivated by the aforementioned considerations, the current study revisits the problem of RO energy cost optimization when highly permeable membranes are used, via a simple mathematical formalism, with respect to the applied pressure, water recovery, feed flow rate, and permeate flow rate, and accounting explicitly for the limitation imposed by the thermodynamic cross-flow restriction. Subsequently, the impact of using an energy recovery device, brine disposal cost, membrane hydraulic permeability, and pressure drop within the membrane module are discussed for one-stage RO. In addition, an analysis is presented of the energy efficiency of a two-stage RO, relative to one-stage RO, following the formalism proposed in the present study.

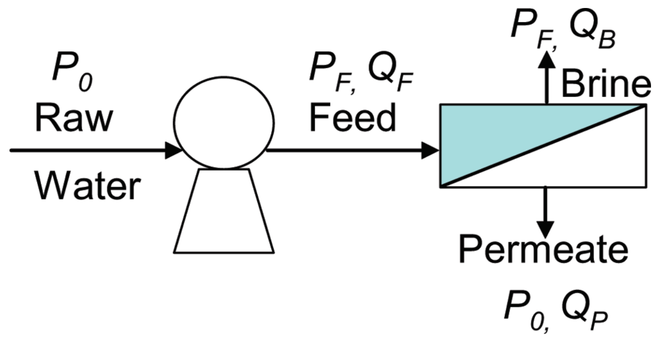

Figure 2. Schematic of simplified RO system.

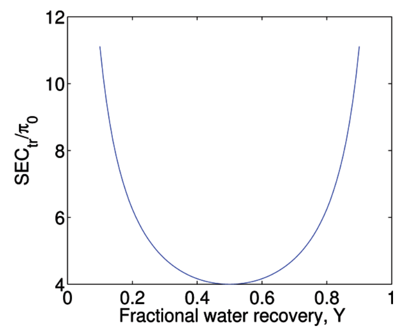

Figure 3. Variation of the normalized SEC with water recovery for a single-stage RO at the limit of thermodynamic restriction.

2. RO Process Model Description

To illustrate the approach to energy cost optimization, it is instructive to first consider a membrane RO process without the deployment of an energy recovery device (ERD), as shown schematically in Figure 2.

The energy cost associated with RO desalination is presented in the present analysis as the specific energy consumption (SEC), which is defined as the electrical energy needed to produce a cubic meter of permeate. Pump efficiency can be included in the following analysis in a straightforward fashion, as presented later in section 5.2. As a first step, however, to simplify the presentation of the approach, the required electrical energy is taken to be equal to the pump work, (i.e., assuming a pump efficiency of 100%). Accordingly, the SEC for the plant shown in Figure 2 is given by

$$\mathrm{SEC} = \frac{\dot{W}_{\mathrm{pump}}}{Q_{\mathrm{p}}} \tag{1}$$

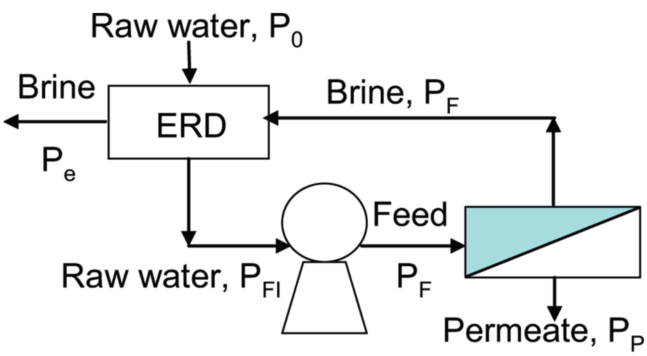

Figure 4. Simplified RO system with an energy recovery device (ERD).

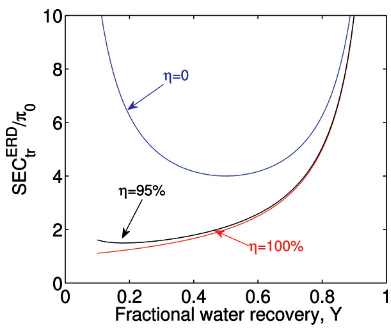

Figure 5. Variation of the normalized SEC with fractional product water recovery using an ERD in a single-stage RO (note: η represents the ERD efficiency).

where Q_p is the permeate flow rate and $\dot{W}_{pump}$ is the rate of work done by the pump, which is given by

$$\dot{W}_{pump} = \Delta P \times Q_f \tag{2}$$

in which

$$\Delta P = P_f - P_0 \tag{3}$$

where P_f is the water pressure at the entrance of the membrane module, P_0 the pressure of the raw water (which, for simplicity, is assumed to be the same as the permeate pressure), and Q_f the volumetric feed flow rate. To simplify the analysis, we initially assume that the impact of the pressure drop (within the RO module) on locating the minimum SEC is negligible; this issue is addressed further in section 5.1. It is acknowledged that fouling and scaling will impact the selection of practical RO process operating conditions and feed pretreatment. However, the inclusion of such effects is beyond the scope of the present paper, but it will be the subject of a future contribution.

The permeate product water recovery for the RO process (Y) is an important measure of the process productivity; it is defined as

$$Y = \frac{Q_p}{Q_f} \tag{4}$$

Combining eqs 1, 2, and 4, the SEC can be rewritten as follows:

$$\mathrm{SEC} = \frac{\Delta P}{Y} \tag{5}$$

The permeate flow rate can be approximated by the classical reverse osmosis flux equation:[34]

$$Q_p = A_m L_p(\Delta P - \sigma\overline{\Delta\pi}) = A_m L_p(\overline{\mathrm{NDP}}) \tag{6}$$

where A_m is the active membrane area, L_p the membrane hydraulic permeability, σ the reflection coefficient (typically assumed to be approximately unity for high-rejection RO membranes and, in this study, $\sigma = 1$), ΔP the transmembrane pressure, $\overline{\Delta\pi}$ the average osmotic pressure difference between the retentate and permeate streams along the membrane module and $(\Delta P - \sigma\overline{\Delta\pi})$ is the average transmembrane net driving pressure (designated as $\overline{\mathrm{NDP}}$). In writing eqs 8 and 9, the typical approximation that the osmotic pressure varies linearly with concentration is invoked (i.e., $\pi = f_{os}C$, where f_{os} is the osmotic pressure coefficient and C is the solution salt concentration[34]). For the purpose of the present analysis and motivated by our focus on RO processes that utilize highly permeable membranes, the average osmotic pressure difference (up to the desired level of product water recovery), $\overline{\Delta\pi}$, can be approximated as the log-mean average along the membrane:[35]

$$\overline{\Delta\pi} = f_{os}C_f\left[\frac{\ln\left(\frac{1}{1-Y}\right)}{Y}\right] \tag{7}$$

where C_f is the salt concentration of the feed to the membrane module. The osmotic pressures at the entrance and the exit of the membrane module, relative to the permeate stream, are approximated by

$$\Delta\pi_{entrance} = f_{os}C_f - \pi_p \tag{8}$$

$$\Delta\pi_{exit} = f_{os}C_r - \pi_p \tag{9}$$

where C_r is the salt concentration of the exit brine (i.e., concentrate or retentate) stream. For sufficiently high rejection levels, the osmotic pressure of the permeate can be considered to be negligible, relative to the feed or concentrate streams, and C_r can be approximated by

$$C_r = \frac{C_f}{1-Y} \tag{10}$$

Thus, by combining eqs 8–10, the osmotic pressure difference between the retentate and permeate stream at the exit of the module can be expressed as

$$\Delta\pi_{exit} = \frac{\pi_0}{1-Y} \tag{11}$$

where $\pi_0 = f_{os}C_f$ is the feed osmotic pressure. Equation 11 is a simple relationship that illustrates that the well-known inherent difficulty in reaching high recovery in RO desalting is due to the rapid rise in osmotic pressure with increased recovery.

2.1. Thermodynamic Restriction of Cross-Flow RO Operation. In the process of RO desalting, an external pressure is applied to overcome the osmotic pressure, and pure water is recovered from the feed solution through the use of a semipermeable membrane. Assuming that the permeate pressure is the same as the raw water pressure P_0, the applied pressure ΔP needed to obtain a water recovery of Y should be no less than the osmotic pressure difference at the exit region[16,21] (see Figure 1), which is given by eq 11. Therefore, to ensure permeate productivity along the entire RO module (or stage), the following lower bound is imposed on the applied pressure:

$$\Delta P \geq \Delta\pi_{exit} = \frac{\pi_0}{1-Y} \tag{12}$$

This is the so-called thermodynamic restriction of cross-flow RO desalting[21−23] and is referred to as the "thermodynamic restriction" in the current work. The equality on the right-hand side of eq 12 is the condition at the "limit of the thermodynamic restriction" at the exit of the membrane module and is attained at the limit of infinite membrane permeability for a finite

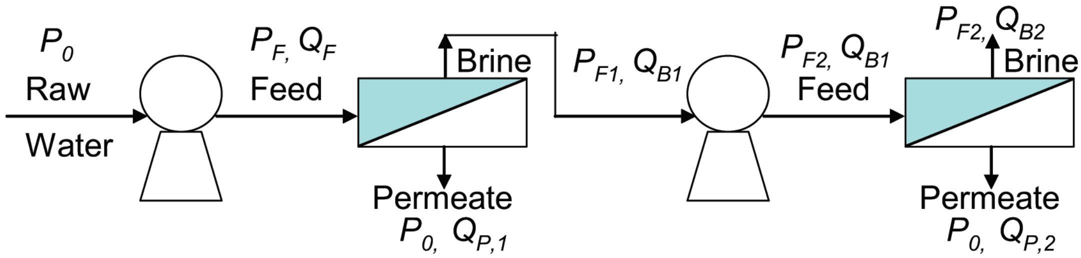

Figure 6. Schematic of a simplified two-stage RO system. (Note: the interstage pump is optional and needed when the pressure to the second stage cannot be met using a single feed pump.)

membrane area. It is particularly important from a practical point of view when a highly permeable membrane is used for water desalination at low pressures. It is emphasized that the constraint of eq 12 arises when one desires to ensure that the entire membrane area is utilized for permeate production.

2.2. Computation of Q_p Close to the Thermodynamic Limit. Referring to the computation of $\overline{NDP} = \Delta P - \overline{\Delta\pi}$ (and, thus, of the water production rate Q_p, near the limit of the "thermodynamic restriction"), we note that, in this work, given the approximation of $\overline{\Delta\pi}$ (eq 7), the following approximation is used for the $\overline{NDP}$ (eq 6):

$$\overline{NDP} = \frac{\pi_0}{1-Y} - \pi_0\left[\frac{\ln\left(\frac{1}{1-Y}\right)}{Y}\right] \tag{13}$$

The above expression is a reasonable approximation when the RO process is allowed to approach the pressure limit imposed by the thermodynamic restriction (eq 12). Note that operation approaching this limit is possible only when highly permeable membranes are used in an RO process. To demonstrate this point, a differential salt mass balance across the membrane yields the following expression for the $\overline{NDP}$:

$$\overline{NDP} = \Delta P - \overline{\Delta\pi} = \frac{Q_p}{AL_p} = \frac{\Delta P}{1 + \left(\frac{\pi_0}{\Delta P}\right)\left(\frac{1}{Y}\right)\ln\left(\frac{1-\frac{\pi_0}{\Delta P}}{1-Y-\frac{\pi_0}{\Delta P}}\right)} \tag{14}$$

where Y denotes the actual water recovery when the applied pressure is ΔP. For operation at the limit of thermodynamic restriction (i.e., $\Delta P = \pi_0/(1-Y)$), it is clear from eq 14 that a highly permeable membrane (i.e., high L_p) and/or large surface area would be required. Indeed, the present analysis focuses on RO desalting made possible by highly permeable membranes; thus, instead of using the pressure-implicit NDP expression (eq 14), we utilize, without a loss of generality of the overall approach, the log-mean average (eq 13). The implication of using different averaging approaches for the computation of $\overline{\Delta\pi}$ is discussed in section 4.3.

3. Optimization for RO Operation at the Limit of Thermodynamic Restriction

The equations presented in section 2 form the basis for deriving the fundamental relationship between the minimum SEC for a single-stage RO process (without and with an ERD), with respect to the level of product water recovery. The derivation is similar to that of Uri Lachish.[36] It is presented here for completeness, because the theoretical minimum SEC, for different water recoveries, is used in the present study as the constraint on the set of energy-optimal and feasible operating points as discussed in section 4. The impacts of ERD, brine disposal cost, and membrane permeability on the optimal water recovery are then considered, as well as the possible energy savings when using a two-stage RO process, relative to the increased membrane area requirement.

3.1. Energy Cost Optimization for a Single-Stage RO without an Energy Recovery Device. The specific energy consumption (SEC) for the RO desalting process can be derived by combining eqs 1−4 and 12, to obtain

$$SEC \geq \frac{\pi_0}{Y(1-Y)} \tag{15}$$

where SEC is given in pressure units. It is convenient to normalize the SEC, at the limit of thermodynamic restriction (i.e., operation up to the point in which the applied pressure equals the osmotic pressure difference between the concentrate and permeate at the exit of the membrane module), with respect to the feed osmotic pressure such that

$$SEC_{tr,norm} = \frac{SEC_{tr}}{\pi_0} = \frac{1}{Y(1-Y)} \tag{16}$$

and this dependence is plotted in Figure 3, showing that there is a global minimum. To obtain the analytical global minimum $SEC_{tr,norm}$, with respect to the water recovery, one can set $(dSEC_{tr,worm})/(dY) = 0$, from which it can be shown that the minimum $SEC_{tr,norm}$ occurs at a fractional recovery of $Y = 0.5$ (or 50%) where $(SEC_{tr,norm})_{min} = 4$ (i.e., four times the feed osmotic pressure). The above condition, i.e., $(SEC_{tr,norm})_{min} = 4$ at $Y = 0.5$ represents the global minimum SEC (represented by the equality in eq 15). To achieve this global minimum energy cost, the RO process should be operated at a water recovery of 50% with an applied pressure equivalent to $2\pi_0$ (i.e., double that the feed osmotic pressure).

As an example of the implications of the aforementioned analysis, it is instructive to consider a single-stage seawater RO plant with a feed salinity of 35000 mg/L (and thus, $\pi_0 = 25$ atm) and membrane permeability of $L_p = 10^{-10}$ m^3/(m^2 s Pa) (which is high, relative to that for commercially available membranes). In this case, the global minimum energy cost is $4\pi_0 = 2.8$ kWh/m^3. The average permeate flux for single-stage seawater desalination, under the aforementioned optimal conditions, can be computed from eq 6 as follows:

$$(FLUX)_{opt} = \frac{Q_p}{A_m} = L_p \times \left[(\Delta P)_{opt} - \ln\left(\frac{1}{1-Y_{opt}}\right)\frac{\pi_0}{Y_{opt}}\right]$$
$$= 0.6137 \times \pi_0 \times L_p = 13.5 \text{ GFD}$$

where GFD denotes the permeate flow rate in gal/(ft^2 day), $Y_{opt} = 0.5$, and $(\Delta P)_{opt} = 2\pi_0$. The permeate flow can be determined once the membrane area is established and the optimum feed flow rate can be calculated using eq 4. At the globally energy-optimal operating point, the applied pressure and feed flow rate, which are input process variables and, hence, the output variables (brine and product flow rate) are fixed for an RO plant with given values of A_m and L_p. Note that the above analysis is specific to a single-stage RO. Cost reduction that can be achieved by adopting multiple stage process configuration is discussed in section 3.3.

3.2. Effect of Energy Recovery Device on SEC for a Single-Stage RO Process. To reduce the required energy for RO desalination, energy can be extracted from the high-pressure retentate (or brine) stream, using a variety of energy recovery schemes. A simple schematic representation of energy recovery is shown in Figure 4 for a simplified model RO process. P_e and P_p are the brine discharge and permeate pressure, respectively, which are assumed here to be equal to P_0.

The rate of work done by the pump on the raw water, at the presence of an ERD, is given by

$$\dot{W}_{\mathrm{pump}} = \Delta P \times (Q_f - \eta Q_b) \tag{17}$$

where $\Delta P = P_f - P_p$, Q_b and Q_p are the brine and permeate flow rates (which are related through the product water recovery Y via eq 4), and η is the energy recovery efficiency of the ERD that refers to the ability of the ERD to recover pressure energy from the brine stream. Thus, the specific energy cost for RO desalting, in the presence of an ERD, $\mathrm{SEC}^{\mathrm{ERD}}(Y, \Delta P, \eta)$, is given by

$$\mathrm{SEC}^{\mathrm{ERD}}(Y, \Delta P, \eta) = \frac{\Delta P(Q_f - \eta Q_b)}{Q_p} = \frac{\Delta P[1 - \eta(1-Y)]}{Y} \tag{18}$$

The thermodynamic restriction for the single-stage RO process, in which an ERD is used, can be obtained by substituting eq 12 into eq 18. Accordingly, the normalized SEC for this configuration (see Figure 5), which is denoted as $\mathrm{SEC}^{\mathrm{ERD}}_{\mathrm{tr,norm}}$, is given by

$$\mathrm{SEC}^{\mathrm{ERD}}_{\mathrm{tr,norm}} = \frac{\mathrm{SEC}^{\mathrm{ERD}}_{\mathrm{tr}}}{\pi_0} = \frac{1 - \eta(1-Y)}{Y(1-Y)} \tag{19}$$

Equation 19 represents the equilibrium state for the exit brine stream (i.e., at the exit of the membrane module), which yields the minimum energy cost that can be achieved for a given water recovery when using an ERD.

The global minimum SEC (i.e., based on eq 19), with respect to recovery, can be derived by setting $(\partial(\mathrm{SEC}^{\mathrm{ERD}}_{\mathrm{tr,norm}}))/(\partial Y) = 0$ and solving to obtain $Y_{\mathrm{opt}} = (1-\eta)^{1/2}/[1 + (1-\eta)^{1/2}]$ and $(\mathrm{SEC}^{\mathrm{ERD}}_{\mathrm{tr,norm}})_{\min} = [1 + (1-\eta)^{1/2}]^2$. Clearly, as the fractional ERD efficiency (i.e., η) increases, Y_{opt} decreases, which suggests that, with increased ERD efficiency, a lower water recovery operation would be more favorable to minimizing the SEC. Indeed, it is known in the practice of RO desalting that a higher benefit of energy recovery is attained when operating at lower recoveries. Comparison with the case of a single-stage RO without an ERD (section 2; Figure 2) reveals that the presence of an ERD shifts the optimal water recovery (for attaining a minimum SEC) to <50%.

3.3. Energy Savings Provided by Two-Stage RO versus Single-Stage RO. The approach discussed previously for a single-stage RO can be easily extended for multiple-stage RO operation. An illustration of the approach is provided here for the simple two-stage RO configuration shown in Figure 6 (in the absence of an ERD). In the RO process configuration shown in Figure 6, the overall product water recovery Y is the result of RO desalting at recoveries of Y_1 and Y_2, in the first and second RO stages, respectively.

Based on a simple mass balance, one can derive the following relationship between the overall and the individual stage recoveries:

$$Y = Y_1 + (1 - Y_1)Y_2 = Y_1 + Y_2 - Y_1 Y_2 \tag{20}$$

Assuming that the pump efficiency is 100% (the effect of pump efficiency on two-stage RO is discussed in section 5.2), at the limit of the thermodynamic restriction, the rate of work done by the first stage pump ($\dot{W}^{\mathrm{1st}}_{\mathrm{tr}}$) is given by the following relationship (see eq 12):

$$\dot{W}^{\mathrm{1st}}_{\mathrm{tr}} = \left(\frac{\pi_0}{1 - Y_1}\right) Q_f \tag{21}$$

Similarly, the rate of work done by the second-stage pump at the limit of the thermodynamic restriction ($\dot{W}^{\mathrm{2nd}}_{\mathrm{tr}}$) is given by

$$\dot{W}^{\mathrm{2nd}}_{\mathrm{tr}} = \left(\frac{\pi_0}{1 - Y} - \frac{\pi_0}{1 - Y_1}\right) Q_f (1 - Y_1) \tag{22}$$

In writing eq 22, it is assumed that the pressure of the brine stream from the first stage is fully available for use in the second RO stage. The normalized SEC of this two-stage RO process at the limit of the thermodynamic restriction, $\mathrm{SEC}_{\mathrm{tr,norm}}(\mathrm{2ROs})$, at a total water recovery Y is given by

$$\mathrm{SEC}_{\mathrm{tr,norm}}(\mathrm{2ROs}) = \frac{\dot{W}^{\mathrm{1st}}_{\mathrm{tr}} + \dot{W}^{\mathrm{2nd}}_{\mathrm{tr}}}{Y Q_f \pi_0} = \frac{1}{Y}\left(\frac{1}{1 - Y_1} + \frac{1 - Y_1}{1 - Y} - 1\right) \tag{23}$$

The difference between the normalized specific energy consumption of two-stage RO and one-stage RO desalting (at the limit of thermodynamic restriction, i.e., when the applied pressure is equal to the exit osmotic pressure difference) is given by

$$\mathrm{SEC}_{\mathrm{tr,norm}}(\mathrm{2ROs}) - \mathrm{SEC}_{\mathrm{tr,norm}}(\mathrm{1RO}) = \frac{Y_1(Y_1 - Y)}{Y(1 - Y_1)(1 - Y)} < 0 \tag{24}$$

Equation 24 implies that, at an equivalent overall recovery, under the stated assumptions, the two-stage RO process will require less energy than a single-stage RO process. The fractional energy cost savings (f_{ES}) for the two-stage process, relative to the one-stage RO process, is given by

$$f_{\mathrm{ES}} = \frac{\mathrm{SEC}_{\mathrm{tr,norm}}(\mathrm{1RO}) - \mathrm{SEC}_{\mathrm{tr,norm}}(\mathrm{2ROs})}{\mathrm{SEC}_{\mathrm{tr,norm}}(\mathrm{1RO})} = \frac{Y_1(Y - Y_1)}{(1 - Y_1)} \tag{25}$$

The fractional energy savings is dependent on both the overall and stage product water recoveries, as depicted by plotting eq 25 in Figure 7.

For a given target overall product water recovery, the maximum energy savings (or global minimum for energy consumption, for the two-stage RO process, relative to the single-stage RO process), with respect to product water recovery in the first stage, can be obtained by setting $\partial f_{\mathrm{ES}}/\partial Y_1 = 0$ and solving to obtain the optimal stage one recovery, $Y_{1,\mathrm{opt}}$,

$$Y_{1,\mathrm{opt}} = 1 - \sqrt{1 - Y} \tag{26}$$

with the corresponding optimal recovery for the second-stage RO obtained from the combination of eqs 20 and 26:

$$Y_2 = \frac{Y - Y_{1,\mathrm{opt}}}{1 - Y_{1,\mathrm{opt}}} = \frac{Y - 1 + \sqrt{1-Y}}{\sqrt{1-Y}} = 1 - \sqrt{1-Y} = Y_{1,\mathrm{opt}} \quad (27)$$

The aforementioned results show that, for two-stage RO, operation of each stage at the same recovery level is the optimal strategy for reducing the SEC. Accordingly, the maximum fractional energy savings, when adopting two-stage RO, relative to a single-stage RO (at a given total water recovery), is obtained from eqs 25 and 26:

$$(f_{\mathrm{ES}})_{\max} = (1 - \sqrt{1-Y})^2 \quad (28)$$

As expected, eq 28 predicts that the fractional energy savings increases with total water recovery.

The aforementioned analysis for the two-stage RO process can be repeated by adding stages in series to reduce the energy costs further. Therefore, in the limit of an infinite number of stages, all of equal recovery, a reversible thermodynamic process is approached, at which the lowest possible cost is achieved.

3.4. Membrane Area for a Two-Stage RO Process Optimized with Respect to Energy Consumption. The two-stage RO process is more energy-efficient, relative to a single-stage RO process. However, one must consider the membrane area requirements when operating with two stages. Considering a two-stage RO, each utilizing membranes of the same permeability, the membrane areas of the first RO stage ($A_{\mathrm{mem},1}$) and the second RO stage ($A_{\mathrm{mem},2}$) are given by

$$A_{\mathrm{mem},1} = \frac{Q_{\mathrm{p},1}}{L_{\mathrm{p}}\pi_{0,1}\left[\frac{1}{1-Y_1} - \frac{1}{Y_1}\ln\left(\frac{1}{1-Y_1}\right)\right]} \quad (29)$$

$$A_{\mathrm{mem},2} = \frac{Q_{\mathrm{p},2}}{L_{\mathrm{p}}\pi_{0,2}\left[\frac{1}{1-Y_2} - \frac{1}{Y_2}\ln\left(\frac{1}{1-Y_2}\right)\right]} \quad (30)$$

where $Q_{\mathrm{p},1}$ and $Q_{\mathrm{p},2}$ are the permeate flow rate for the first and second RO stages, respectively, and $\pi_{0,1}$ and $\pi_{0,2}$ are the corresponding feed osmotic pressure of the two RO stages. Note that the osmotic pressure of the feed to the second RO stage is equal to that of the concentrate (or brine) stream from the first RO stage (i.e., $\pi_{0,2} = \pi_{0,1}/(1 - Y_1)$). As discussed previously, the maximum energy savings is obtained when $Y_1 = Y_2$. For this energy-optimal operating condition, the ratio of membrane area for the second stage RO, relative to the first stage RO, is given by

$$\frac{A_{\mathrm{mem},2}}{A_{\mathrm{mem},1}} = (1 - Y_1) \times (1 - Y_2) = 1 - (Y_1 + Y_2 - Y_1 Y_2) = 1 - Y \quad (31)$$

where use was made of eq 20. As an example, for a two-stage desalting RO process (Figure 6), operating at a total recovery of 75%, each stage would be operated at water recovery of 50%, under the optimal minimum energy operating conditions, achieving an energy cost savings of 25%, relative to a single-stage RO (Figure 2) operating at the same total recovery. However, according to eq 31, the required membrane surface area for the second RO stage is one-fourth of that of the first RO stage.

The membrane area for a single-stage RO process, desalting a feed stream of the same salinity (i.e., $\pi_0 = \pi_{0,1}$) at the same overall recovery (Y) as a two-stage RO, is given by

$$A_{\mathrm{mem,SRO}} = \frac{Q_{\mathrm{p},2} + Q_{\mathrm{p},1}}{L_{\mathrm{p}}\pi_0\left[\frac{1}{1-Y} - \frac{1}{Y}\ln\left(\frac{1}{1-Y}\right)\right]} \quad (32)$$

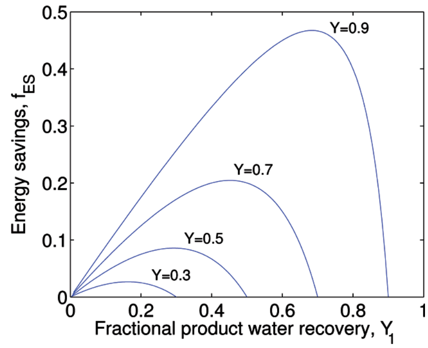

Figure 7. Fractional energy savings achieved when using a two-stage relative to a single-stage RO process. (Note: Y is the total water recovery and Y_1 is the water recovery in the first-stage RO.)

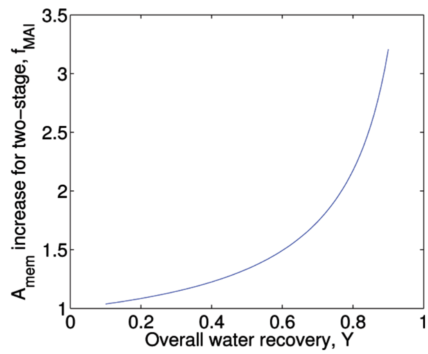

Figure 8. Fractional membrane area increase for a two-stage RO, relative to a single-stage RO. (Both stages of the two-stage RO and single-stage RO are operated at the thermodynamic limit. The two-stage RO is operated at its minimum specific energy cost.)

and the fractional membrane area increase (f_{MAI}) for the two-stage RO process, relative to a single-stage RO process (eq 32), for operation under the optimal conditions (i.e., when $Y_1 = Y_2$), is given by

$$f_{\mathrm{MAI}} = \frac{A_{\mathrm{mem},1} + A_{\mathrm{mem},2}}{A_{\mathrm{mem,SRO}}} - 1 = \frac{(1 - \sqrt{1-Y})(2 - Y)}{Y} \times \left[\frac{\frac{1}{1-Y} - \frac{1}{Y}\ln\left(\frac{1}{1-Y}\right)}{\frac{1}{\sqrt{1-Y}} - \frac{1}{1-\sqrt{1-Y}}\ln\left(\frac{1}{1\sqrt{1-Y}}\right)}\right] - 1 \quad (33)$$

Equation 33 indicates that the membrane area required for a two-stage RO process is higher, for a given overall permeate water recovery, relative to a single-stage RO (Figure 8), because the net transmembrane driving pressure (NDP) is lower for a two-stage RO process. As the aforementioned analysis indicates, the energy savings attained with the two-stage RO process is gained at the expense of a higher membrane surface area. Therefore, process optimization must consider both the cost of energy and membrane area.

3.5. Overall Cost Optimization Considering Membrane and Energy Costs for a Two-Stage versus a Single-Stage RO Process. Optimal design of a two-stage RO requires balancing of the energy savings (relative to a single-stage RO) with the increased membrane surface area required to achieve the target total recovery. An analysis of this tradeoff can be conveniently demonstrated for the simple special case for which the two-stage RO process will operate at its maximum energy savings, as can be shown (using eqs 29, 30, and 32), this would require the greatest surface area increase, relative to a single-stage RO process. It is convenient to compare the membrane and energy costs on the same basis of energy consumption units as expressed in eq 2 (i.e., Pa m^3). This conversion can be achieved, given an energy price, e.g., ϵ (\$/kWh) and the conversion factor of β (Pa m^3/kWh), such that, for a single-stage RO process, the specific amortized membrane cost per permeate produced (SMC) is given by

$$\mathrm{SMC} = \frac{m \times A_\mathrm{m}}{Q_\mathrm{p}} = \frac{m}{L_\mathrm{p}\left[\Delta P - \frac{\pi_0}{Y}\ln\left(\frac{1}{1-Y}\right)\right]} \tag{34}$$

where m is the amortized membrane price per unit area ($m = m_\mathrm{A}\beta/\epsilon$, in which, for example, m is given in units of Pa m^3/(m^2 h), where m_A is the amortized membrane unit cost, in units of \$/m^2 h). At the point where the applied pressure is equal to the osmotic pressure difference at the exit region, the SMC, normalized with respect to the feed osmotic pressure, can be obtained from eq 34 to yield

$$\mathrm{SMC_{norm}} = \frac{m}{L_\mathrm{p}{\pi_0}^2\left[\frac{1}{1-Y} - \frac{1}{Y}\ln\left(\frac{1}{1-Y}\right)\right]} \tag{35}$$

Inspection of eq 35 suggests that a convenient dimensionless membrane price, m_norm, which is independent of the RO operating conditions, can be defined as $m_\mathrm{norm} = m/L_\mathrm{p}(\pi_0)^2$.

Following the analysis in section 3.4, the penalty due to the increase in the membrane area for a two-stage RO, relative to a single-stage RO, P_SMC, at the optimal two-stage operation (i.e., $Y_1 = Y_2$), can be expressed as

$$P_\mathrm{SMC} = \frac{m_\mathrm{norm}}{\frac{1}{1-Y} - \frac{1}{Y}\ln\left(\frac{1}{1-Y}\right)}\left\{\frac{(1-\sqrt{1-Y})(2-Y)}{Y} \times \left[\frac{\frac{1}{1-Y} - \frac{1}{Y}\ln\left(\frac{1}{1-Y}\right)}{\frac{1}{\sqrt{1-Y}} - \frac{1}{1-\sqrt{1-Y}}\ln\left(\frac{1}{\sqrt{1-Y}}\right)}\right] - 1\right\} \tag{36}$$

The gain in energy savings for using a two-stage RO, relative to a single-stage RO (G_SEC), obtained as given in eqs 16 and 28 (also for the optimal conditions of $Y_1 = Y_2$), is given by

$$G_\mathrm{SEC} = \left[\frac{1}{Y(1-Y)}\right](1-\sqrt{1-Y})^2 \tag{37}$$

Combining eqs 36 and 37, the overall cost savings for a two-stage RO, relative to a single-stage RO ($S_\mathrm{ov}^\mathrm{em}$), considering both energy and membrane cost, is given by

$$S_\mathrm{ov}^\mathrm{em} = G_\mathrm{SEC} - P_\mathrm{SMC} = \frac{(1-\sqrt{1-Y})^2}{Y(1-Y)} - \frac{m_\mathrm{norm}}{\frac{1}{1-Y} - \frac{1}{Y}\ln\left(\frac{1}{1-Y}\right)} \times \left\{\frac{(1-\sqrt{1-Y})(2-Y)}{Y}\left[\frac{\frac{1}{1-Y} - \frac{1}{Y}\ln\left(\frac{1}{1-Y}\right)}{\frac{1}{\sqrt{1-Y}} - \frac{1}{1-\sqrt{1-Y}}\ln\left(\frac{1}{\sqrt{1-Y}}\right)}\right] - 1\right\} \tag{38}$$

To illustrate the overall cost savings for a two-stage RO, relative to a single-stage RO, the estimated range of the dimensionless membrane price of m_norm can be derived, given reasonable ranges for m (membrane price per unit area (\$100−\$1000/m^2)), L_p (10^{-10}−10^{-9} m/(Pa s)) for RO membranes, osmotic pressure for a salinity range of ∼1000−35000 mg/L total dissolved solids, and current energy price (\$0.1−0.2/kWh). Assuming a membrane life of ∼5 years, for seawater,

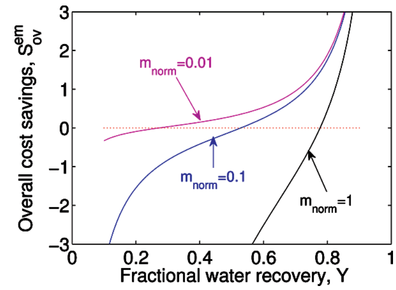

Figure 9. Overall cost savings due to the adoption of two-stage RO, considering both energy and membrane costs.

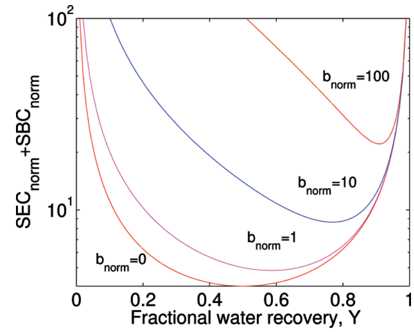

Figure 10. Variation of the summation of energy and brine management cost with product water recovery for a single-stage RO.

given the high salinity (∼35 000 mg/L total dissolved solids) we find that $m_\mathrm{norm} < 0.1$. Therefore, as can be seen for seawater in Figure 9 and from eq 38, when the total water recovery is greater than ∼50%, a two-stage RO process is more cost-effective (i.e., $S_\mathrm{ov}^\mathrm{em} > 0$) than a single-stage RO process accounted for, even with the additional membrane cost for the two-stage RO process. In contrast, for brackish water with a salinity in the range of 1000−10000 mg/L, $m_\mathrm{norm} > 1$, and thus for a recovery of $Y < 80\%$, it is apparent that a single-stage RO process is more cost-effective than a two-stage RO process (i.e., $S_\mathrm{ov}^\mathrm{em} < 0$).

3.6. Impact of Brine Management Cost on the Thermodynamic Restriction and the Minimum SEC. Management of the RO concentrate (i.e., brine) stream can add to the overall cost of water production by RO desalting and in fact alter optimal energy cost and associated product water recovery. As an example of the possible influence of brine management (including disposal) on RO water product cost we assume a simple linear variation of the cost of brine management with the retentate volume. Accordingly, the specific brine management cost (SBC) per unit volume of produced permeate, normalized with respect to the feed osmotic pressure, is given by

$$\mathrm{SBC_{norm}} = \frac{bQ_\mathrm{b}}{\pi_0 Q_\mathrm{p}} = \frac{b}{\pi_0}\left(\frac{1-Y}{Y}\right) \tag{39}$$

where b is the concentrate (brine) management cost expressed on an energy equivalent units per concentrate volume (Pa m^3/m^3). Inspection of eq 39 suggests that a convenient dimensionless brine management cost can be defined as, $b_\mathrm{norm} = b/\pi_0$, where π_0 is the osmotic pressure of the feedwater, $b = b_\mathrm{A}\beta/\epsilon$ (in which b_A is the concentrate management cost, in units of \$/m^3 (in the range of \$0−0.20/m^3) and ϵ and β are the energy cost and energy conversion factor, as defined previously in section 3.4). Therefore, b_norm is in the range of 0−100 for a salinity range of 1000–35000 mg/L total dissolved solids.

The combined normalized energy cost (eq 15) and brine management cost (eq 39) for a single-stage RO process is given by

$$\mathrm{SEC}_{\mathrm{norm}} + \mathrm{SBC}_{\mathrm{norm}} \geq \frac{1}{Y(1-Y)} + b_{\mathrm{norm}}\left(\frac{1-Y}{Y}\right) \tag{40}$$

The cost when the pressure at the exit region equals the osmotic pressure of the concentrate stream is signified by the equality in eq 40. As shown in Figure 10, the water recovery level at the optimal (i.e., minimum) cost increases with increased brine management cost. In other words, the higher the brine management cost, the greater the incentive for operating at a higher recovery level. The optimal product water recovery, Y_{opt}, can be obtained by differentiating eq 40, with respect to Y and setting the resulting expression to zero, resulting in the following expression:

$$Y_{\mathrm{opt}} = \frac{\sqrt{1+b_{\mathrm{norm}}}}{1+\sqrt{1+b_{\mathrm{norm}}}} \tag{41}$$

indicating that the optimal recovery will increase with the concentrate management cost (see Figure 10), reducing to $Y_{\mathrm{opt}} = 0.5$ for the case of a vanishing brine management cost.

4. Optimization for RO Operation above the Limit Imposed by the Thermodynamic Restriction

For a given RO plant, process conditions that would enable desalting at the global minimum energy utilization condition are fixed (see section 3.1). However, the desired level of productivity or feed processing capacity may force deviation from the globally optimal operation. Therefore, for an energy-optimal operating condition, product water recovery may have to be shifted to ensure optimal operation. Accordingly, there is merit in exploring the SEC optimization, as constrained by the normalized feed or permeate flow rates, and the implications of the thermodynamic restriction of eq 12 on this optimization.

4.1. Optimization at a Constrained Permeate Flow Rate. For a given plant, when the desired level of permeate productivity cannot be accommodated by operating at global optimum, the permeate flow rate is a constraint that shifts the optimal water recovery (and, thus, the corresponding feed flow rate). In this case, it is convenient to define a normalized permeate flow rate as follows:

$$Q_{\mathrm{p,norm}} = \frac{Q_{\mathrm{p}}}{A_{\mathrm{m}}L_{\mathrm{p}}\pi_0} = \frac{\Delta P - \overline{\Delta\pi}}{\pi_0} = \frac{\Delta P}{\pi_0} - \frac{\overline{\Delta\pi}}{\pi_0} \tag{42}$$

where the first term on the right-hand-side of eq 42 is $Y \times \mathrm{SEC}_{\mathrm{norm}}$ and the second term can be expanded using eq 7, and, thus, $\mathrm{SEC}_{\mathrm{norm}}$ can be expressed as

$$\mathrm{SEC}_{\mathrm{norm}} = \frac{\mathrm{SEC}}{\pi_0} = \frac{Q_{\mathrm{p,norm}}}{Y} + \frac{\ln\left(\frac{1}{1-Y}\right)}{Y^2} \tag{43}$$

where $\mathrm{SEC}_{\mathrm{norm}}$ is a function of the water recovery and the normalized permeate flow rate. As shown in Figure 11, the minimum $\mathrm{SEC}_{\mathrm{norm}}$ shifts to higher water recoveries and higher $(\mathrm{SEC}_{\mathrm{norm}})_{\mathrm{min}}$ as plant productivity is pushed beyond the globally energy-optimal operating point, which has a water recovery of 50%.

It is important to recognize that RO operation below the symmetric curve imposed by the thermodynamic restriction (the solid curve in Figure 11) is not realizable. In fact, the lowest practically realizable SEC values (i.e., the minima above the thermodynamic restriction or the points of intersection with this

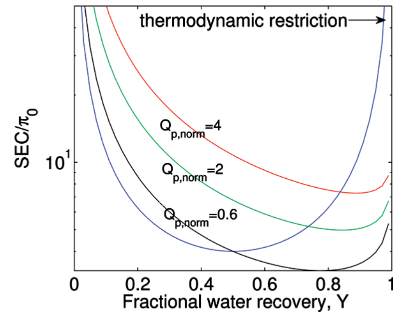

Figure 11. Dependence of the normalized SEC (without ERD) on water recovery at different normalized permeate flow rates for single-stage RO. Note: the solid curve represents the SEC curve for operation in which the thermodynamic restriction is attained (see section 3).

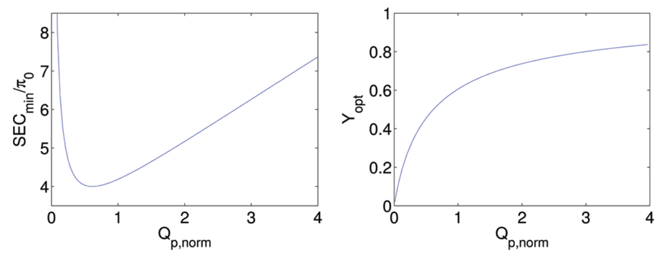

Figure 12. (a) Normalized minimum SEC versus normalized permeate flow rate; (b) dependence of the optimum water recovery on the normalized permeate flow rate.

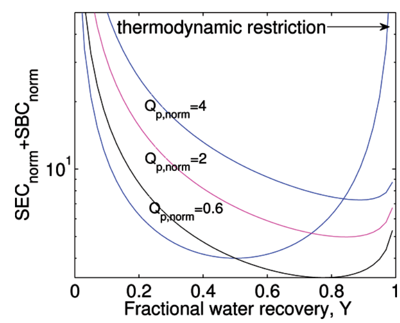

Figure 13. Variation of summation of normalized energy cost without ERD and brine management cost versus water recovery at different normalized permeate flow rates with a brine disposal penalty factor equal to one.

curve) for each $Q_{\mathrm{p,norm}}$ are plotted in Figure 12a with the corresponding optimal water recovery dependence on $Q_{\mathrm{p,norm}}$ shown in Figure 12b. As can be seen from Figure 12a and b, the globally energy-optimal operation is at $Y = 0.5$ and $Q_{\mathrm{p,norm}} = 0.6137$.

The additional cost associated with brine management can be included by adding its associated normalized cost (eq 39) to the $\mathrm{SEC}_{\mathrm{norm}}$ (eq 43), resulting in

$$\mathrm{SEC}_{\mathrm{norm}} + \mathrm{SBC}_{\mathrm{norm}} = \frac{Q_{\mathrm{p,norm}}}{Y} + \frac{\ln\left(\frac{1}{1-Y}\right)}{Y^2} + b_{\mathrm{norm}}\left(\frac{1-Y}{Y}\right) \tag{44}$$

The additional brine management cost shifts the optimal recovery, for a given b_{norm}, to higher water recovery values (Figure 13). Here, it is also emphasized that RO operation where the combined cost of $\mathrm{SEC}_{\mathrm{norm}} + \mathrm{SBC}_{\mathrm{norm}}$ is in the region below the curve that represents the thermodynamic restriction (Figure 13) is not realizable. Thus, the lowest combined energy and brine management costs that can be achieved are either the minima in the region above the thermodynamic cross-flow

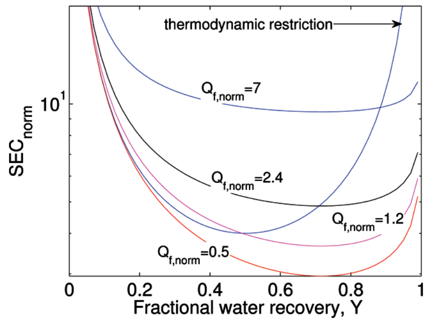

Figure 14. Dependence of SEC_{norm} on water recovery for different normalized feed flow rates for a single-stage RO without an ERD.

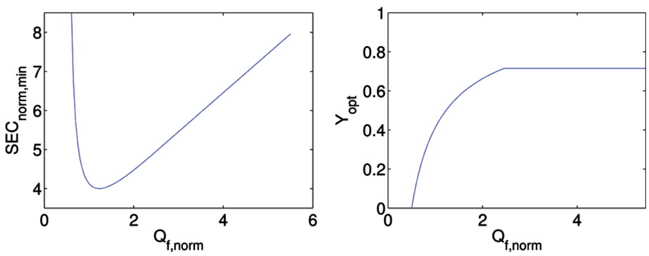

Figure 15. (a) Normalized minimum SEC versus normalized feed flow rate; (b) optimum water recovery at each normalized feed flow rate.

restriction curve or at the intersection of this curve with the thermodynamic restriction curve.

4.2. Optimization at a Constrained Feed Flow Rate. The feed flow rate may be constrained (e.g., because of restrictions on the available water source) for an operating RO plant. Therefore, the optimization objective is to determine the optimal water recovery and the corresponding permeate flow rate under this constraint that would result in a minimum specific energy cost. In a typical operation, the permeate flux can be expressed as $Q_p/A_mL_p = \Delta P - \overline{\Delta\pi}$ and because $Q_p = Q_fY$, one can express Q_f in a normalized form as

$$Q_{f,norm} = \frac{Q_f}{A_mL_p\pi_0} = \frac{\Delta P - \overline{\Delta\pi}}{Y\pi_0} = \frac{\Delta P}{Y\pi_0} - \frac{\overline{\Delta\pi}}{Y\pi_0} \tag{45}$$

in which the first term on the right-hand-side of eq 45 is SEC_{norm} (see eq 5), which, thus, can be expressed as

$$SEC_{norm} = \frac{SEC}{\pi_0} = Q_{f,norm} + \frac{\ln\left(\frac{1}{1-Y}\right)}{Y^2} \tag{46}$$

in which use was made of eq 7, as in the derivation of eq 46, and $Q_{f,norm} = Q_{p,norm}/Y$. As a reference, the SEC_{norm} curve for operation at the limit of the thermodynamic restriction is also shown in Figure 14. Operation below the above curve (i.e., the thermodynamic restriction) is not realizable. Therefore, the locus of the lowest permissible SEC_{norm} is given by the minima that exist above the thermodynamic restriction curve and the intersections of this curve with the individual SEC_{norm} curves with the resulting plot shown in Figure 15. The minimum SEC_{norm}, with respect to $Q_{f,norm}$, is obtained from Figure 15 and is plotted in Figure 15a. Figure 15b presents the corresponding optimal water recovery at each normalized feed flow rate. Accordingly, if $Q_{f,norm} > 2.4$, the optimal water recovery is 71.53%, which is determined by solving $\partial(SEC_{norm})/\partial Y = 0$, with respect to eq 46, independent of the thermodynamic restriction.

Finally, it is important to note that the curves of different $Q_{f,norm}$ in Figure 14 can be interpreted as curves of different membrane permeability L_p for a fixed Q_f, A_m, and π_0 (see eq 45); in such an interpretation, we can see that as L_p increases ($Q_{f,norm}$ decreases), SEC decreases but the benefit is limited at high recoveries, because of the effect of the thermodynamic restriction. Similar behavior is observed for different $Q_{p,norm}$ values in Figure 12.

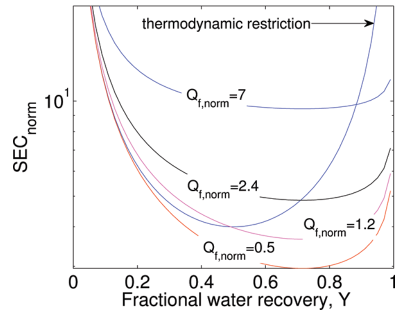

Figure 16. Variation of summation of normalized energy cost without ERD and brine management cost versus water recovery at different normalized feed flow rates with a brine disposal penalty factor equal to one.

The optimal condition for operation subject to the feed flow constraint, when considering the additional cost of brine management, can be obtained from the sum of the normalized energy cost (eq 46) and brine management cost (eq 46) as follows:

$$SEC_{norm} + SBC_{norm} = Q_{f,norm} + \frac{\ln\left(\frac{1}{1-Y}\right)}{Y^2} + b_{norm}\left(\frac{1-Y}{Y}\right) \tag{47}$$

As shown in Figure 16 (for the example of $b_{norm} = 1$), the minimum (or optimal) cost shifts to higher recoveries.

The optimal recovery (i.e., at the point of minimum cost, considering the brine management cost) is obtained from solving $\partial(SEC_{norm} + SBC_{norm})/\partial Y = 0$, leading to

$$Y_{opt} = \frac{\sqrt{2(1+b_{norm})}}{1+\sqrt{2(1+b_{norm})}} \tag{48}$$

Equation 48 reveals that, when the brine management cost is neglected (i.e., $b_{norm} = 0$), $Y_{opt} = 71.53\%$. However, the inclusion of brine disposal cost shifts the optimal water recovery to higher values. It is important to recognize that the operating points for which the combined cost, $SEC_{norm} + SBC_{norm}$, falls below the value dictated by the thermodynamic restriction (solid curve in Figure 16) are not realizable.

4.3. Effect of Osmotic Pressure Averaging on SEC. The averaging of the osmotic pressure can have a quantitative effect on the identified optimal operating conditions, although the overall analysis approach used and the trends presented in the present work should remain independent of the averaging method. For example, if the arithmetic osmotic pressure average, $\Delta\pi = {}^1/_2(f_{os}C_f + f_{os}C_r)$, is used instead of the log-mean average (eq 7), the $SEC_{tr,norm}$ for the thermodynamic restriction remains the same as that shown previously; however, the impact is that there can be a shift in the optimum conditions as shown in Figure 17 for the case of SEC optimization subject to a feed flow constraint. This example shows that, at low water recoveries ($Y < 0.4$), the log-mean and arithmetic osmotic pressure averages yield similar results, while at high water recoveries, the use of the log-mean average results in a lower $(SEC_{norm})_{min}$ than the arithmetic average, because the former predicts a greater average net driving pressure, and thus, a larger permeate flow for a given applied pressure.

5. Considerations of Pressure Drop and Pump Efficiency

5.1. Effect of Pressure Drop within the RO Membrane Module. The pressure drop in RO modules is typically small, compared to the total applied pressure, and its contribution to the required total applied pressure can be assessed by a simple order of magnitude analysis. Accounting for the frictional pressure drop, ΔP_f, the permeate flow rate is given by[29]

$$Q_p = A_m L_p\left(\Delta P - \overline{\Delta\pi} - \frac{\Delta P_f}{2}\right) \tag{49}$$

Thus, the applied pressure ΔP is given by

$$\Delta P = \overline{\mathrm{NDP}} + \frac{\Delta P_f}{2} + \overline{\Delta\pi} \tag{50}$$

where the average net driving pressure $\overline{\mathrm{NDP}} = Q_p/A_m L_p$, and ΔP_f can be estimated from:[37]

$$\Delta P_f = \left(\frac{1}{2}\rho\bar{u}^2\right)\left(\frac{24}{Re} - \frac{648}{35}\frac{Re_W}{Re}\right)\left(1 - \frac{2Re_W}{Re}\frac{x}{h}\right)\left(\frac{x}{h}\right) \tag{51}$$

where ρ is the solution density, x the axial length, h the half-height of the channel, $\bar{u}$ the average axial velocity given as $u = \bar{Q}_f/(2hW) = 1/(2hW)[(Q_f + Q_b)/2] = [Q_p/(4hW)][(2/Y) - 1]$ (where W is the channel width and Y is the fractional water recovery), Re is the axial Reynolds number ($Re = 4h\bar{u}\rho/\mu$, where μ is the solution viscosity), and Re_W is the permeate Reynolds number (defined as $Re_W = hv_w\rho/\mu$, where v_w is the permeate flow velocity). Inspection of eq 51 shows that

$$\Delta P_f < \left(\frac{1}{2}\rho\bar{u}^2\right)\left(\frac{24}{Re}\right)\left(\frac{x}{h}\right) \tag{52}$$

and can be rearranged as follows:

$$\frac{\Delta P_f}{\overline{\mathrm{NDP}}} < \frac{3\mu x^2 L_p\left(\frac{2}{Y} - 1\right)}{2h^3} \tag{53}$$

where the definitions of $\bar{u}$ and Re are used. Thus, the ratio of the frictional pressure loss, relative to the applied pressure, can be estimated as follows:

$$\frac{\Delta P_f}{\Delta P} < \frac{\Delta P_f}{\overline{\mathrm{NDP}}} < \frac{3\mu x^2 L_p\left(\frac{2}{Y} - 1\right)}{2h^3} \tag{54}$$

A reasonable order of magnitude assessment of the various terms in eq 54 reveals that $h \approx 0.001-0.01$ m, $\mu \approx 0.001-0.005$ Pa s, $L_p \approx 10^{-10}-10^{-9}$ m/(Pa s), and $x \approx 0.1-1$ m. For the practical range of product water recovery ($Y \approx 0.3-0.95$), the right-hand side of eq 54 is, at most, on the order of $10^{-10}-10^{-2}$. Therefore, one can conclude that the effect of frictional pressure drop on determining the optimal operating condition would be small. Although not presented in the present study, the incorporation of the additional energy cost due to frictional losses can be incorporated into the present formalism without a loss of generality.

Given the above order of magnitude analysis for the frictional losses, one can assess the relative importance of the various pressure terms (eq 50) as a function of product water recovery. Accordingly, the fractional contribution of the different pressure terms on the right-hand side of eq 50 can be assessed as illustrated in Figure 18 for a specific set of process conditions (at the limit of thermodynamic restriction, i.e., the applied pressure equals the osmotic pressure difference at the exit region of the RO membrane).

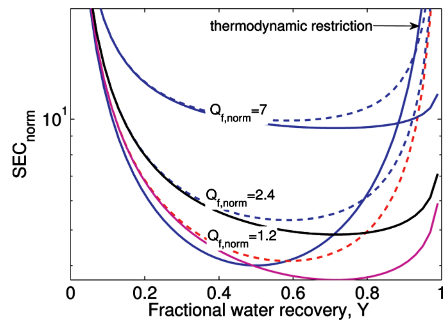

Figure 17. Normalized SEC versus water recovery at different normalized feed flow rates: without ERD. The results for the arithmetic and log-mean average osmotic pressures are depicted by the dashed and solid curves, respectively.

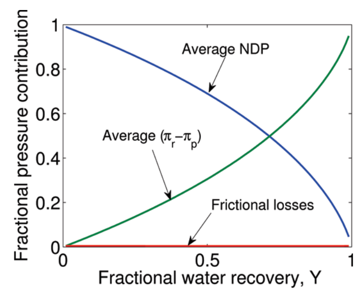

Figure 18. Fractional contribution of the average $\overline{\mathrm{NDP}}$, osmotic pressure, and frictional pressure losses to the total applied pressure. (Calculated for RO operation at the limit of the thermodynamic restriction. The log-mean average of osmotic pressure was utilized, with $h = 0.001$ m, $\mu = 0.001$ Pa s, $L_p = 10^{-9}$ m s^{-1} Pa^{-1}, and $x = 1$ m; π_r is the local osmotic pressure of the retentate ($\pi_r = f_{os}C_r$, where C_r is the local retentate concentration.)

It is clear that the contribution of the frictional losses are small (<1% of the total required pressure) and the contribution of the average osmotic pressure increases with recovery, whereas the contribution of the required net driving pressure ($\overline{\mathrm{NDP}}$) decreases with increased recovery. It is important to note that, as the required $\overline{\mathrm{NDP}}$ increases (e.g., due to decreasing membrane permeability), the fractional contribution of osmotic pressure to the total applied pressure will decrease. As the $\overline{\mathrm{NDP}}$ decreases, there is less incentive to improve membrane permeability, because the cost for overcoming the osmotic pressure begins to dominate the energy costs. Conversely, a process that is found to operate at a high $\overline{\mathrm{NDP}}$ will have a greater benefit from employing membranes of higher permeability.

5.2. Effect of Pump Efficiency on SEC. Pump efficiency can be easily included in the present analysis approach of the optimal SEC, as shown in this section, for the single-stage RO (Figure 2) and two-stage RO (Figure 6) processes. Specifically, the normalized specific energy consumption at the limit of thermodynamic restriction accounting for pump efficiency, $\mathrm{SEC}_{tr,norm}(\eta_{pump})$, can be expressed as

$$\mathrm{SEC}_{tr,norm}(\eta_{pump}) = \frac{\mathrm{SEC}_{tr}(\eta_{pump} = 1)}{\pi_0\eta_{pump}} = \frac{1}{Y(1 - Y)\eta_{pump}} \tag{55}$$

where η_{pump} is the pump efficiency, which takes values in the interval [0, 1]. For this case, the optimal water recovery remains at $Y_{opt} = 50\%$, and the corresponding normalized minimum SEC is $4/\eta_{pump}$.

For the two-stage RO system shown in Figure 6, the rate of work by the first pump, $\dot{W}_{tr}^{1st}$, at the limit of thermodynamic restriction, is given by

$$\dot{W}_{tr}^{1st}(\eta_{pump,1}) = \left[\frac{\pi_0}{(1 - Y_1)\eta_{pump,1}}\right]Q_f \tag{56}$$

where $\eta_{pump,1}$ is the efficiency of the first pump. Similarly, the rate of work by the second pump at the limit of thermodynamic restriction, $\dot{W}_{tr}^{2nd}$, is given by

$$\dot{W}_{tr}^{2nd}(\eta_{pump,2}) = \left[\frac{(Y - Y_1)\pi_0}{(1 - Y)\eta_{pump,2}}\right]Q_f \quad (57)$$

where $\eta_{pump,2}$ is the efficiency of the second pump. Therefore, the SEC of this two-stage RO, at the limit of thermodynamic restriction, accounting for pump efficiencies, is given by

$$\mathrm{SEC_{tr}(2ROs)} = \frac{\pi_0}{Y}\left[\frac{1}{(1 - Y_1)\eta_{pump,1}} + \frac{Y_2}{(1 - Y_2)\eta_{pump,2}}\right] \quad (58)$$

where $Y = Y_1 + Y_2 - Y_1Y_2$. For this case, the optimal water recoveries in each stage are obtained by solving $\partial(\mathrm{SEC_{tr}(2ROs)})/\partial Y_1 = 0$ at a given total water recovery Y and are given by

$$Y_{1,opt} = 1 - \sqrt{\frac{\eta_{pump,2}}{\eta_{pump,1}}(1 - Y)} \quad (59)$$

$$Y_{2,opt} = 1 - \sqrt{\frac{\eta_{pump,1}}{\eta_{pump,2}}(1 - Y)} \quad (60)$$

and the corresponding normalized minimum SEC for this two-stage RO at a total water recovery Y is given by

$$(\mathrm{SEC_{tr,norm}(2ROs)})_{min} = \frac{1}{Y\sqrt{\eta_{pump,2}}}\left[\frac{2}{\sqrt{\eta_{pump,1}(1 - Y)}} - \frac{1}{\sqrt{\eta_{pump,2}}}\right] \quad (61)$$

From eq 58, we can conclude that, as the pump efficiencies $\eta_{pump,1}$ and $\eta_{pump,2}$ increase, the SEC decreases, although the capital cost of the pump may increase with efficiency. It is important to determine which stage requires a pump of higher efficiency to minimize the overall SEC. For example, if the product of $\eta_{pump,1}$ and $\eta_{pump,2}$ is fixed, one can determine the optimal $\eta_{pump,2}$ by rewriting eq 61 as follows:

$$(\mathrm{SEC_{tr,norm}(2ROs)})_{min} = \frac{1}{Y}\left[\frac{2}{\sqrt{\eta_{pump,1}\eta_{pump,2}(1 - Y)}} - \frac{1}{\eta_{pump,2}}\right] \quad (62)$$

which shows that, as $\eta_{pump,2}$ increases, the overall SEC decreases. The conclusion is that, in a two-stage RO process, the higher efficiency pump should be used in the second stage, to minimize the overall SEC, because the second stage requires higher pressure than the first stage.

6. Conclusions

The wide application of low-pressure membrane desalting owing to the development of high-permeability reverse osmosis (RO) membranes, has enabled the applied pressure in RO processes to approach the osmotic pressure limit. Therefore, it is now possible to optimize RO membrane processes, with respect to product water recovery, with the goal of minimizing energy consumption, while considering constraints imposed by the thermodynamic cross-flow restriction and feed or permeate flow rate. In the present study, an approach to optimization of product water recovery in RO membrane desalination when highly permeable membranes are utilized was presented via several simple RO process models. The current results suggest that it is indeed feasible to refine RO desalting, to target the operation to the condition of minimum energy consumption, while considering the constraint imposed by the osmotic pressure as specified by the thermodynamic cross-flow restriction. The impact of energy recovery devices, membrane permeability, process configuration, brine management cost, pump efficiency, and frictional pressure drop can all be considered using the proposed approach, as shown in a series of illustrations. Overall, as process costs above energy costs are added, the operational point shifts to higher recoveries. Although the newer generation of highly permeable RO membranes can allow high recovery operations, limitations due to mineral scaling and fouling impose additional constraints. The incorporation of these phenomena in an expanded optimization framework is the subject of ongoing research.

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