

PROCESS DESIGN AND CONTROL

Quantitative Performance Design of a Modified Smith Predictor for Unstable Processes with Time Delay

Weidong Zhang,* Danying Gu, Wei Wang, and Xiaoming Xu

Department of Automation, Shanghai Jiaotong University, Shanghai 200030, People's Republic of China

The existence of both right-half-plane poles and time delay makes it difficult to obtain a higher control quality. Recently, a modified Smith predictor for unstable processes with time delay was presented and good performance was obtained. In this paper, the modified Smith predictor is discussed. A modified structure is presented. The new one not only is simple for analysis and design but also avoids the improper element. Instead of introduction of an inner loop, a direct design procedure is given and analytical formulas for the controllers are provided. The time domain performance for both trajectory and regulatory responses is obtained quantitatively. Sufficient and necessary conditions for robust stability are also given. The new structure and design method can also be used for the control of integrating processes with time delay and stable processes with time delay. Numerical examples are provided to illustrate the proposed method.

1. Introduction

The dynamic characteristics of many processes include time delay. Typical examples are heat flow, material transportation, hydraulic and pneumatic transmission, chemical reactors, and distillation columns. The difficulty in designing such systems arises because of the presence of the nonrational term $\exp(-\theta s)$ in the plant transfer function. As two pioneers in the delay control problems, Ziegler and Nichols²⁸ first gave their empirical controller settings with a quarter decay criterion for the regulatory problem. One main shortcoming of the method is that it cannot be used for processes with long time delay. Smith¹⁶ tried a special feedback structure known as the Smith predictor to effectively remove the time delay from the closed-loop characteristic equation and hence to allow the use of classical design techniques developed for rational transfer functions. However, the feedback configuration cannot stabilize unstable processes with time delay.^{13,21} To overcome the problem, Marchetti et al.¹² and Wang and Cai²⁰ developed two proportional–integral–derivative (PID) tuning methods for open-loop unstable processes. The former was based on a new relay test procedure, and the later was based on gain and phase margin specifications. Astrom et al.¹ presented a new structure of the Smith predictor for the control of processes with an integrator and time delay. This structure isolates the disturbance response from the set-point response and gives better responses to the set point and disturbance. The structure was analyzed and improved by Zhang and

Sun.²² They proposed an analytical design method. The method was later discussed by Zhang et al.,²³ Tian and Gao,^{18,19} Normey-Rico and Camacho,¹⁵ Chien et al.,² and Zhong and Normey-Rico.²⁷ For processes with right-half-plane poles and time delay, De Paor⁴ and De Paor and Egan⁶ presented a modified Smith predictor. A stable model is introduced for deriving the controller, and an auxiliary system is defined to analyze the stability of the closed-loop system. On the basis of standard forms of the closed-loop system response and the Nyquist stability analysis, Majhi and Atherton^{9–11} developed another Smith predictor for processes with right-half-plane poles and time delay. One important feature is that it decouples the set-point response from the disturbance response. Kwak et al.⁷ pointed out that the Smith predictor with a modified stable model could not guarantee the advantage of the Smith predictor completely. They then proposed a modified Smith predictor for processes with right-half-plane poles and time delay. The structure makes it possible to use the unstable model as a process model and can predict the dynamics of the actual process much better than previous methods.

In the present paper, the structure of Kwak et al.⁷ is analyzed. A modified structure is presented. Internal stability is discussed. Also, an alternative design procedure is developed, and analytical formulas for controllers are derived. The advantage of the proposed method is that the procedure to design a controller is straightforward, and the controller can be quantitatively designed for a specified time domain performance. Furthermore, nominal performance and sufficient and necessary conditions for robust stability can easily be obtained.

2. Analysis and Simplification

A central concept in automatic control is the idea that a physical quantity can be made to behave in a pre-

* To whom correspondence should be addressed. Currently W.Z. is an Alexander von Humboldt Research fellow at the Institut für Systemtheorie Technischer Prozesse, Universität Stuttgart, Stuttgart, Germany. Tel.: +86-21-62826946. Fax: +86-21-62826946. E-mail: wdzhang@sjtu.edu.cn. URL: <http://mywebpage.netscape.com/wdzsjtu>; <http://automation.sjtu.edu.cn/wdzhang>.

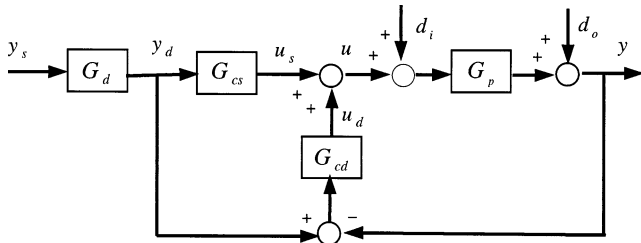


Figure 1. Modified Smith predictor for unstable processes with time delay.

scribed way by using the error between the practical output and the reference input, which gives rise to the feedback control loop.

Recently, a feedback control structure was developed by Kwak et al.⁷ It provides additional freedom to the designer, which allows the achievement of both good trajectory and regulatory responses. The structure is shown in Figure 1, where $G_p(s)$ is the plant, $G_d(s)$ and $G_{cs}(s)$ are controllers for the trajectory problems, and $G_{cd}(s)$ is the controller for the regulatory problem. Suppose that there is no uncertainty. The effect of the set point on the process output is described by

$$H_s(s) = \frac{[G_{cs}(s) + G_{cd}(s)] G_d(s) G_p(s)}{1 + G_{cd}(s) G_p(s)} \quad (1)$$

Let $d_i(s)$ denote the disturbance at the process input and $d_o(s)$ the disturbance at the process output. The total effect on the process output is $d(s) = d_i(s) G_p(s) + d_o(s)$. Its effect on the process output can be written as

$$H_d(s) = \frac{1}{1 + G_{cd}(s) G_p(s)} \quad (2)$$

In work by Kwak et al.,⁷ $G_{cd}(s)$ is chosen as a PID controller:

$$G_{cd}(s) = k_c \left(1 + \frac{1}{\tau_i s} + \tau_d s \right) \quad (3)$$

Utilizing the tuning rule proposed by De Paor and O'Malley⁵ for a first-order unstable process:

$$G_p(s) = \frac{k_p}{\tau_p s - 1} e^{-\theta_p s}, \quad \tau_p > \theta_p \quad (4)$$

the design problem of an unstable system can be converted into a design problem of a stable system. The PID parameters can then be estimated by utilizing the reduction technology.

$G_d(s)$ is a user-specified controller, whose transfer function is⁷

$$G_d(s) = \frac{1}{(\tau_{de}s + 1)^n} e^{-\theta_p s} \quad (5)$$

where n is a positive integer. $G_{cs}(s)$ is chosen as the inverse of the minimum-phase part of the process as discussed by Morari and Zafiriou.¹⁴ Because the process is strictly proper, $G_{cs}(s)$ is improper. This implies that $G_{cs}(s)$ cannot be implemented physically. To solve the problem, a low-pass filter should be introduced. However, this will make the structure more complicated. On the basis of the discussion of Zhang et al.²³ and Zhang and Sun,²² a modified structure is proposed, which is

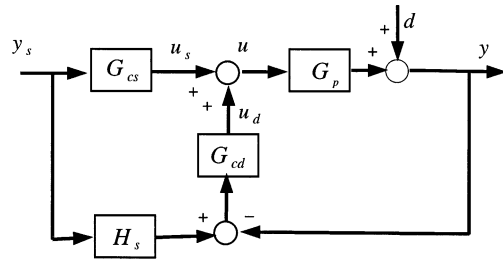


Figure 2. Simplified structure for the modified Smith predictor.

shown in Figure 2. Figure 2 is, in fact, a direct extension of the structure presented by Zhang and Sun,²² Astrom et al.,¹ and Kwak et al.⁷ The effect of the set point on the process output can be written as

$$H_s(s) = G_{cs}(s) G_p(s) \quad (6)$$

It is noticed that $H_s(s)$ provides a predictive output for the closed-loop output $y(s)$. When the model is exact, $y(s)$ will be thoroughly compensated for. The compensating effect for the uncertain model depends on the design method.

Obviously, $H_s(s)$ should be stable. This implies that $G_{cs}(s)$ will have right-half-plane zeros at the unstable poles of $G_p(s)$. To avoid the internal instability, the right-half-plane poles in $H_s(s)$ should be removed before implementing it.

Compared with the structure of Kwak et al.,⁷ no inner stabilizing loop is used in the new one. Thus, there exists right-half-plane zero-pole cancellation between $G_{cs}(s)$ and the real process $G_p(s)$. A direct implementation of the scheme may cause the internal instability. However, if $G_p(s)$ is stabilized by $G_{cd}(s)$, the problem can be overcome because any bounded signals injected into the point between the two blocks $G_{cs}(s)$ and $G_p(s)$ will not cause unbounded outputs. A strict proof will be given in the next section. Here, it should be emphasized that not all right-half-plane zero-pole cancellations will cause internal instability. This is the key to understanding the new structure, which has no inner stabilizing loop. In this case, the effect of the disturbance on the process output is

$$H_d(s) = \frac{1}{1 + G_{cd}(s) G_p(s)} \quad (7)$$

3. Control System Design

In this paper, attention is also restricted to the first-order unstable process with time delay. De Paor and O'Malley⁵ have shown that the obtained results can be easily extended to the control of high-order processes.

Consider the design of $G_{cd}(s)$. In the paper of Kwak et al.,⁷ an auxiliary system with an inner loop was employed for the design of the controller. A high-order closed-loop transfer function with time delay is obtained, and then reduction technology is used to convert it to a second-order rational transfer function for designing the PID controller. In this section, the internal stability for the proposed closed-loop system is first discussed. Second, a direct procedure is developed for designing the controller and the quantitative time domain performance is given. Finally, sufficient and necessary conditions for robust stability are derived.

In light of the definition of internal stability, the closed-loop system is internally stable if and only if all of the transfer functions in the following matrix are

BIBO stable^{14,24}

$$\begin{bmatrix} 1 - H_d(s) & -G_p(s) H_d(s) \\ \frac{1 - H_d(s)}{G_p(s)} & -1 + H_d(s) \end{bmatrix}$$

or, equivalently, (1) $[1 - H_d(s)]/G_p(s)$ is stable and (2) $G_p(s) H_d(s)$ is stable. In addition, to reject the disturbance asymptotically, it is required that (3) $\lim_{s \rightarrow 0} H_d(s) = 0$.

In a unity feedback system with a stable process with time delay, we usually choose the closed-loop response as the form of first order plus time delay for a unit step set point. After the closed-loop response is determined, the controller can be designed directly. As early as 1967, Dahlin³ had used the desired closed-loop response to design the controller. The direct design procedure is also adopted by predictive controllers and a lot of model-based controllers (for example, refs 15 and 25). Morari and Zafiriou¹⁴ and Zhang et al.²⁵ pointed out that this is, in fact, suboptimal. Equivalently, for the disturbance loop, we hope $H_d(s)$ is in the form of

$$H_d(s) = 1 - \frac{1}{(\tau_{cd}s + 1)^n} e^{-\theta_p s} \quad (8)$$

where τ_{cd} is the time constant of the disturbance loop. Unfortunately, the form can only provide a good performance for stable processes with time delay. It cannot guarantee internal stability for unstable processes with time delay. For unstable processes with time delay, we have to choose

$$H_d(s) = 1 - \frac{N_d(s)}{(\tau_{cd}s + 1)^n} e^{-\theta_p s} \quad (9)$$

for internal stability.¹⁴ Here, n is a positive integer and $N_d(s)$ is a polynomial. Both of them are determined by the constraint for the asymptotic tracking discussed above.

For a first-order unstable process, substituting the process model and $H_d(s)$ into the constraint $\lim_{s \rightarrow 0} H_d(s) = 0$, one can obtain the following desired transfer function:

$$H_d(s) = 1 - \frac{\tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]s + 1}{(\tau_{cd}s + 1)^2} e^{-\theta_p s} \quad (10)$$

By inverse Laplace transform, the time domain load response is obtained as follows:

$$h_{di}(t) = k_p \times \begin{cases} 0 & t < \theta_p \\ 1 - e^{-b(t-\theta_p)} & \theta_p \leq t < 2\theta_p \\ -e^{-b(t-\theta_p)} - \frac{a^2(b-c)}{(a-b)^2 c} e^{-b(t-2\theta_p)} - \frac{ab(c-a) + bc(a-b)}{(a-b)^2 c} e^{-a(t-2\theta_p)} - \frac{ab(c-a)}{(a-b)c} te^{-a(t-2\theta_p)} & t \geq \theta_p \end{cases} \quad (11)$$

where

$$a = \frac{1}{\tau_{cd}}, \quad b = -\frac{1}{\tau_p}, \quad c = \frac{1}{\tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]}$$

The controller can then be written as

$$\begin{aligned} G_{cd}(s) &= \frac{1 - H_d(s)}{H_d(s) G_p(s)} \\ &= \frac{1}{k_p} \frac{\{\tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]s + 1\}(\tau_p s - 1)}{(\tau_{cd}s + 1)^2 - \{\tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]s + 1\}e^{-\theta_p s}} \end{aligned} \quad (12)$$

Because

$$\lim_{s \rightarrow -1/\tau_p} (\tau_{cd}s + 1)^2 - \{\tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]s + 1\}e^{-\theta_p s} = 0$$

There exists zero-pole cancellation in the right half-plane. A rational approximation should be used to remove the right-half-plane pole. Otherwise, both the trajectory and regulatory responses are internally unstable. Many techniques can be adopted, for example, Pade approximation, Lagrange interpolation, etc. On the basis of the Maclaurin series, Lee et al.⁸ developed a design procedure for integrating processes with time delay and unstable processes with time delay. Design results of integrating processes with time delay are derived by approximating those of unstable processes with time delay. When the design procedure of Lee et al. is utilized,⁸ the following PID controller is obtained:

$$G_{cd}(s) = k_c \left(1 + \frac{1}{\tau_i s} + \tau_d s \right) \quad (13)$$

with

$$k_c = \frac{\tau_i}{-k_p \{ 2\tau_{cd} + \theta_p - \tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1] \}}$$

$$\begin{aligned} \tau_i &= -\tau_p + \tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1] - \\ &\quad \left(\frac{\tau_{cd}^2 + \tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]\theta_p - \theta^2/2}{2\tau_{cd} + \theta_p - \tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]} \right) \end{aligned}$$

$$\begin{aligned} \tau_d &= -\tau_p^2[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1] - \\ &\quad \{ \theta_p^3/6 - \tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]\theta_p^2/2 \} / \\ &\quad \{ 2\tau_{cd} + \theta_p - \tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1] \} / \tau_i - \\ &\quad \frac{\tau_{cd}^2 + \tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]\theta_p - \theta^2/2}{2\tau_{cd} + \theta_p - \tau_p[(\tau_{cd}/\tau_p + 1)^2 e^{\theta_p/\tau_p} - 1]} \end{aligned}$$

Although the expression seems to be complicated, its computation is, in fact, very simple because all of the parameters are known.

An important merit of the direct design is that the robust analysis is very simple. For a stable $H_s(s)$, the robustness of the closed system is determined only by

τ_{cd} . Suppose that

$$G_m(s) = \frac{k_m}{\tau_m s - 1} e^{-\theta_m s}$$

is the model of the process. Define

$$\begin{aligned} \Delta(\omega) &\geq \left| \frac{G_p(j\omega) - G_m(j\omega)}{G_m(j\omega)} \right| \\ &= \left| \frac{k_p(\tau_m j\omega - 1)}{k_m(\tau_p j\omega - 1)} e^{-j(\theta_p - \theta_m)\omega} - 1 \right| \end{aligned} \quad (14)$$

where $\Delta(\omega)$ is the bound on the multiplicative uncertainty. Especially, when only the gain is uncertain, the expression simplifies to

$$\Delta(\omega) = \left| \frac{k_p}{k_m} - 1 \right| \quad (15)$$

When only the time constant is uncertain, the expression simplifies to

$$\Delta(\omega) = \left| \frac{\tau_m j\omega - 1}{\tau_p j\omega - 1} - 1 \right| \quad (16)$$

When only the time delay is uncertain, the expression simplifies to

$$\Delta(\omega) = \begin{cases} |e^{-j(\theta_p - \theta_m)\omega} - 1| & \omega \max(\theta_p - \theta_m) < \pi \\ 2 & \omega \max(\theta_p - \theta_m) \geq \pi \end{cases} \quad (17)$$

The sufficient and necessary condition that guarantees the stability of the closed-loop system is¹⁴

$$|1 - H_d(j\omega)| < \Delta^{-1}(\omega), \quad \forall \omega$$

or equivalently

$$\left| \frac{[\tau_m(\tau_{cd}/\tau_m + 1)^2 e^{\theta_p/\tau_p} - 1]j\omega + 1}{(\tau_{cd}j\omega + 1)^2} e^{-\theta_m j\omega} \right| < \Delta^{-1}(\omega), \quad \forall \omega \quad (18)$$

It is easy to verify that the larger τ_{cd} is, the worse the nominal performance is and the better the robust stability is. Because the relationship is monotonic, one can easily obtain the required robustness by monotonically increasing τ_{cd} .

Now let us see how to design $G_{cs}(s)$. $G_{cs}(s)$ is designed for the trajectory problem. We use the following equation to obtain the desired response

$$H_s(s) = \frac{1}{\tau_{cs}s + 1} e^{-\theta_p s}$$

or equivalently

$$h_s(t) = \begin{cases} 0 & t < \theta_p \\ 1 - e^{-(t-\theta_p)/\tau_{cs}} & t \geq \theta_p \end{cases} \quad (19)$$

where τ_{cs} is the time constant of the set-point loop. Such a selection implies that the trajectory and regulatory

responses are decoupled or almost decoupled. Therefore,

$$G_{cs}(s) = \frac{1}{k_p} \frac{\tau_p s - 1}{\tau_{cs}s + 1} \quad (20)$$

Now that both frequency domain responses and time domain responses are analytically expressed, one can easily determine the desired closed-loop responses. Many criteria can be used: classical criteria such as overshoot, rise time, and stability margin or integral criteria such as ISE and ITAE. Because overshoot, rise time, and ISE are frequently used, we only discuss them here. The tuning procedure differs from that given by Lee et al.⁸

In the closed-loop responses, there are two parameters, τ_{cs} and τ_{cd} . Determining the desired closed-loop responses is determining the two parameters. If the criterion is ISE, the ISE of the disturbance response is the integral of the squared $h_{di}(t)$. When $h_{di}(t)$ is allowed to be equal to the required ISE value, τ_{cd} can be obtained. The ISE of the set-point response is the integral of the squared $1 - h_s(t)$. When $1 - h_s(t)$ is allowed to be equal to the required ISE value, τ_{cs} can be obtained.

Suppose that the criteria are maximum peak for the disturbance response and overshoot and rise time for the set-point response. When $dh_{di}(t)/dt = 0$, the maximum peak can be calculated and then τ_{cd} can be obtained. Similarly, τ_{cs} can be obtained by the first-order response $h_s(t)$.

For the proposed method, the desired response can also be tuned in a simple way in practice:

(1) Determine the performance specification.

(2) Monotonically increase τ_{cd} for the disturbance response and τ_{cs} for the set-point response with a small step (for example, $0.01\tau_m$ or more small) until the required nominal performance is achieved.

If there exists uncertainty, the set-point response cannot be thoroughly isolated from the disturbance response. In this case, the robust stability and the disturbance response is only determined by τ_{cd} while the set-point response is mainly determined by τ_{cs} . For robustness tuning, one can monotonically increase the controller parameters from their nominal values, until the required responses are obtained for the worst case. The worst case is that the process parameters perturb to the set ($\max \theta_p$, $\max k_p$, $\min \tau_p$).¹⁷

4. Simulation

Example 1. For the purpose of comparison, consider the process described by

$$G_p(s) = \frac{e^{-0.5s}}{s - 1}$$

which has been used in a lot of papers. Assume that a unit step set point is added at $t = 0$ and a negative unit step load is added at $t = 20$. According to Majhi and Atherton¹¹ and Lee et al.,⁸ $\lambda = 0.5$ for the method of Majhi and Atherton¹¹ and $\lambda = 0.4$ for the method of Lee

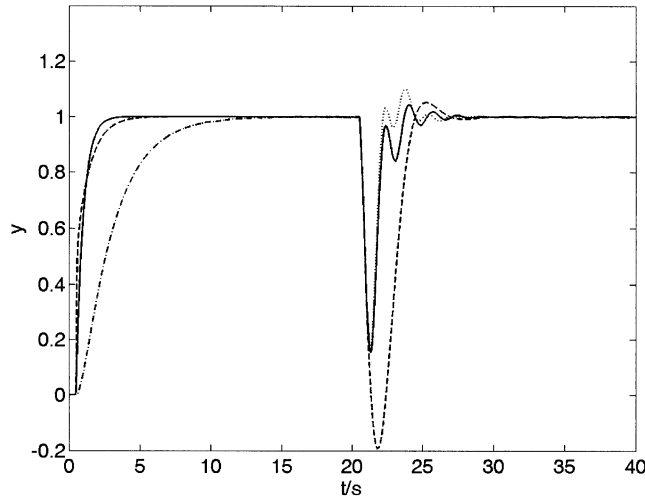


Figure 3. Responses of the nominal systems (solid line, new; dotted line, Kwak; dashed line, Majhi; dash-dotted line, Lee).

et al.⁸ will give good responses. The controllers given by Kwak et al.⁷ are

$$G_d(s) = \frac{e^{-0.5s}}{0.5s + 1}$$

$$G_{cs}(s) = (s - 1)e^{0.5s}$$

$$K_i = 1.5927$$

$$G_{cd}(s) = 2.6 \left[1 + \frac{1}{1.7825s} + 0.2473s \right]$$

Taking $\tau_{cs} = 0.5$ and $\tau_{cd} = 0.4$, the proposed controllers are as follows:

$$G_{cs}(s) = \frac{s - 1}{0.5s + 1}$$

$$G_{cd}(s) = 2.6483 \left(1 + \frac{1}{2.4669s} + 0.2185s \right)$$

The time domain response for the set point is

$$h_s(t) = \begin{cases} 0 & t < 0.5 \\ 1 - e^{-(t-0.5)/0.5} & t \geq 0.5 \end{cases}$$

The time domain response for the load is $1 - h_{di}(t)$ and

$$h_{di}(t) = \begin{cases} 0 & t < 0.5 \\ 1 - e^{t-0.5} & 0.5 \leq t < 1 \\ -e^{t-0.5} + 1.6488e^{t-1} - 0.6488e^{-(t-1)/0.4} + 3.2708te^{-(t-1)/0.4} & t \geq 1 \end{cases}$$

Responses of the nominal systems are shown in Figure 3. The trajectory responses are shown in the time range 1–20 s and the disturbance responses 20–40 s.

Now suppose that there exists 10% uncertainty in the time delay. The uncertainty profile is

$$\Delta(\omega) = \begin{cases} |e^{-j(\theta_p - \theta_m)\omega} - 1| & 0.05\omega < \pi \\ 2 & 0.05\omega \geq \pi \end{cases}$$

The sufficient and necessary condition that guarantees

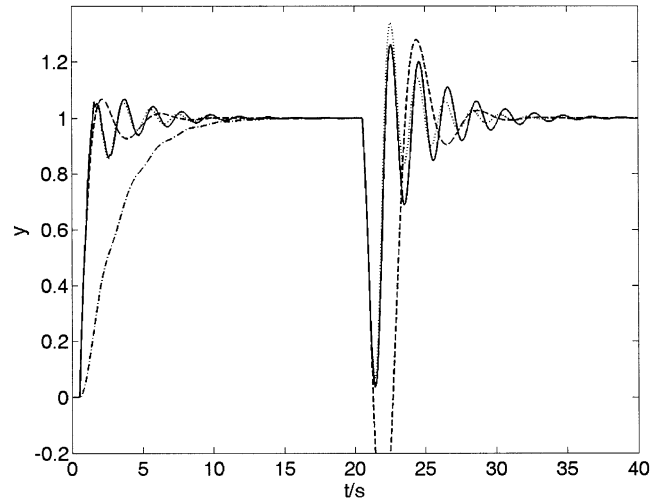


Figure 4. Responses of the systems with +10% uncertainty on the time delay (solid line, new; dotted line, Kwak; dashed line, Majhi; dash-dotted line, Lee).

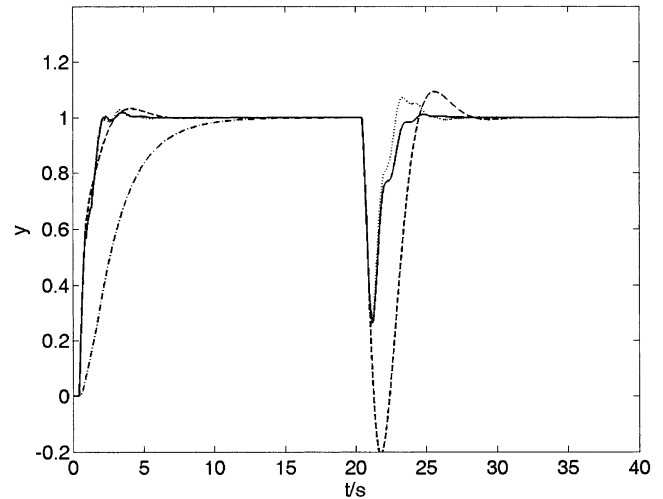


Figure 5. Responses of the systems with -10% uncertainty on the time delay (solid line, new; dotted line, Kwak; dashed line, Majhi; dash-dotted line, Lee).

the robust stability is

$$\left| \frac{2.2315j\omega + 1}{(0.4j\omega + 1)^2} e^{-0.4j\omega} \right| < \Delta^{-1}(\omega), \quad \forall \omega$$

The responses are shown in Figures 4 and 5.

The difference between the simulation results of the two methods is not obvious. The set-point responses are almost the same. Each method has its own characteristics for disturbance rejection. The method of Kwak et al.⁷ has a larger shoot, while the proposed method has a slower response. The method of Majhi and Atherton¹¹ gives the largest disturbance peak. The method of Lee et al.⁸ has the same disturbance response as that of the proposed method and a slow set-point response.

Kwak et al.⁷ gave a fairly good controller. The above simulations show that a similar response can be achieved by the proposed method. Comparing minor changes in responses may be meaningless. The advantages of the proposed method are that no inner loop is used and thus the analysis of the performance and robustness is very simple, the procedure to design a controller is straightforward and analytical, and the controller can be

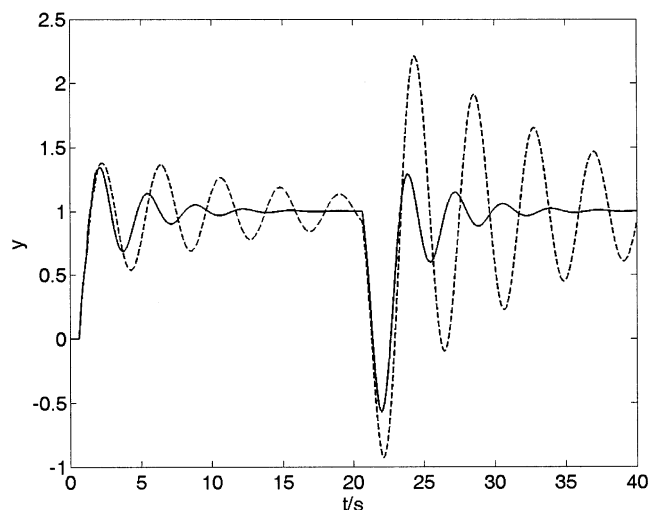


Figure 6. Responses of the systems with +30% uncertainty on the time delay (solid line, new; dotted line, Majhi).

quantitatively designed for specified performance and robustness.

Supposing that the desired set-point response has the same rise time, one can easily obtain it by the tuning rules given in section 3. Sometimes the criterion may be different. For example, the ISE of the disturbance loop given by Kwak et al.⁷ is 0.4662. To achieve the ISE, we can monotonically decrease τ_{cd} to 0.34 in the proposed method.

The robustness is more important than the nominal performance in practice. One key in designing the controller is to tradeoff between the robust stability and nominal performance. The proposed method provides a simple way. For example, the uncertainty in the time delay increases to 30% of its nominal value as a result of the equipment's wear and tear. The controllers given by Kwak et al.⁷ and Lee et al.⁸ are not stable, while the proposed controller can give a stable response by monotonically increasing τ_{cd} to 0.9 (Figure 6).

5. Conclusions

The existence of both right-half-plane pole and time delay makes it difficult to obtain a higher control quality. Kwak et al.⁷ proposed a modified Smith predictor for unstable processes with time delay, and a good performance is obtained. This paper gives some further discussion. Several objectives are achieved:

(1) The structure is analyzed, and a modified one is given, which not only is simple for analysis and design but also avoids the improper element.

(2) Instead of introduction of an inner loop, a direct design procedure is given. Analytical formulas for the controllers are provided.

(3) The internal stability problem is discussed. For a nominal process, the quantitative time domain performance for trajectory and regulatory responses can be estimated by the proposed method.

(4) There always exists uncertainty in practice. Robust stability is discussed. A quantitative uncertainty profile is derived, and sufficient and necessary conditions are provided.

The new structure can be used for the control of not only unstable processes with time delay but also integrating processes with time delay and stable processes with time delay. Actually, the basic design procedure for stable processes with time delay has been given by

Zhang et al.²³ and can be regarded as a special case of this paper. Several special problems for the design of integrating processes with time delay in a similar structure has been studied by Zhang et al.²⁶

Acknowledgment

This paper is supported by National Science Foundation of China (60274032), National 973 Program (2002cb312200), National Research Foundation for the Doctoral Program of High Education of China, and the Alexander von Humboldt Research Fellowship.

Notation

- $d_i(s)$ = disturbance at the process input
- $d_o(s)$ = disturbance at the process output
- $d(s)$ = total disturbance [i.e., $d_i(s)G_p(s) + d_o(s)$]
- $G_{cd}(s)$ = controller for load response
- $G_{cs}(s)$ = controller for set-point response
- $G_d(s)$ = user-specified controller for the trajectory problem
- $G_p(s)$ = plant
- $G_m(s)$ = plant model
- $H_d(s)$ = disturbance response (i.e., the transfer function from d to y)
- $H_s(s)$ = set-point response (i.e., the transfer function from y_s to y)
- $h_{di}(t)$ = time domain load response (i.e., response from d_i to y)
- $h_s(t)$ = time domain set-point response (i.e., response from y_s to y)
- k_c = gain of the PID controller
- k_m = gain of the plant model
- k_p = gain of the plant
- $y(s)$ = output of the closed-loop system
- $y_s(s)$ = Laplace transformed set point
- τ_{cd} = time constant of the disturbance loop
- τ_{cs} = time constant of the set-point loop
- τ_d = derivative constant of the PID controller
- τ_i = integral constant of the PID controller
- τ_m = time constant of the plant model
- τ_p = time constant of the plant
- θ_p = dead time of the plant
- θ_m = dead time of the plant model
- $\Delta(\omega)$ = bound on the multiplicative uncertainty

Literature Cited

- (1) Astrom, K. J.; Hang, C. C.; Lim, B. C. A New Smith Predictor for Controlling a Process with an Integrator and Long Dead Time. *IEEE Trans. Autom. Control* **1994**, *39*, 343.
- (2) Chien, I. L.; Peng, S. C.; Liu, J. H. Simple control method for integrating processes with long deadtime. *J. Process Control* **2002**, *12*, 391.
- (3) Dahlin, E. B. Designing and tuning digital controllers. *Instrum. Control Syst.* **1968**, *41*, 77.
- (4) De Paor, A. M. A Modified Smith Predictor and Controller for Unstable Processes with Time Delay. *Int. J. Control* **1985**, *41*, 1025.
- (5) De Paor, A.; O'Malley, M. Controllers of Ziegler–Nichols Type for Unstable Process with Time Delay. *Int. J. Control* **1989**, *49*, 1273.
- (6) De Paor, A. M.; Egan, R. P. K. Extension and partial optimization of a modified Smith predictor and controller for unstable processes with time delay. *Int. J. Control* **1989**, *50*, 1315.
- (7) Kwak, H. J.; Sung, S. W.; Lee, I.; Park, J. Y. Modified Smith predictor with a new structure for unstable processes. *Ind. Eng. Chem. Res.* **1999**, *38*, 405.
- (8) Lee, Y.; Lee, J.; Park, S. PID Controller Tuning for Integrating and Unstable Processes with Time Delay. *Chem. Eng. Sci.* **2000**, *55*, 3481.

- (9) Majhi, S.; Atherton, D. P. New Smith predictor and controller for unstable and integrating processes with time delay. *Proceedings of IEEE CDC*, Tampa, FL, 1998; p 1341.
- (10) Majhi, S.; Atherton, D. P. Modified Smith predictor and controller for processes with time delay. *IEE Proc., Part D* **1999**, *146*, 359.
- (11) Majhi, S.; Atherton, D. P. Obtaining controller parameters for a new Smith predictor using autotuning. *Automatica* **2000**, *36*, 1651.
- (12) Marchetti, G.; Scali, C.; Lewin, D. R. Identification and control of open-loop unstable processes by relay methods. *Automatica* **2001**, *37*, 2049.
- (13) Marshall, J. E.; Gorecki, H.; Walton, K.; Korytowski, A. *Time Delay Systems*; Ellis Horwood Ltd.: New York, 1992.
- (14) Morari, M.; Zafiriou, E. *Robust Process Control*; Prentice Hall: Englewood Cliffs: NJ, 1989.
- (15) Normey-Rico, J. E.; Camacho, E. F. A unified approach to design dead-time compensators for stable and integrative processes with dead-time. *IEEE Trans. Autom. Control* **2002**, *47*, 299.
- (16) Smith, O. J. M. Closer Control of loops with Dead Times. *Chem. Eng. Prog. Trans.* **1957**, *53*, 216.
- (17) Stryczek, K.; Laiseca, M.; Brosilow, C.; Leitman, M. Tuning and design of single-input, single-output control systems for parametric uncertainty. *AIChE J.* **2000**, *46*, 1616.
- (18) Tian, Y. C.; Gao, F. R. Compensation of dominant and variable delay in process systems. *Ind. Eng. Chem. Res.* **1998**, *37*, 982.
- (19) Tian, Y. C.; Gao, F. R. Control of integrator processes with dominant time delay. *Ind. Eng. Chem. Res.* **1999**, *38*, 2979.
- (20) Wang, Y. G.; Cai, W. J. Advanced proportional-integral-derivative tuning for integrating and unstable processes with gain and phase margin specifications. *Ind. Eng. Chem. Res.* **2002**, *41*, 2910.
- (21) Watanabe, K.; Ito, M. A Process Model Control for Linear System with Delay. *IEEE Trans. Autom. Control* **1981**, *26*, 1261.
- (22) Zhang, W. D.; Sun, Y. X. Modified Smith Predictor for Integrator/Dead Time Processes. *Ind. Eng. Chem. Res.* **1996**, *35*, 2796.
- (23) Zhang, W. D.; Sun, Y. X.; Xu, X. M. Two Degree-of-Freedom Smith Predictor for Processes with Time Delay. *Automatica* **1998**, *34*, 1279.
- (24) Zhang, W. D.; Xu, X. M. Counterexamples for sufficient and necessary conditions of internal stability. *IEE Proc., Part D* **2000**, *147*, 371.
- (25) Zhang, W. D.; Wang, H.; Xu, X. M. Analytical formulas for near-h control of linear systems with time delay. *American Control Conference*, Anchorage, AL, 2002; p 2233.
- (26) Zhang, W. D.; Xu, X. M.; Xi, Y. G. A New Two-Degree-of-Freedom Level Control Scheme. *ISA Trans.* **2002**, *41*, 333.
- (27) Zhong, Q. C.; Normey-Rico, J. E. Control of integral processes with dead-time. *IEE Proc., Part D* **2002**, *149*, 285.
- (28) Ziegler, J. G.; Nichols, N. B. Optimum Settings for Automatic Controllers. *Trans. ASME* **1942**, *64*, 759.

Received for review September 17, 2002
Revised manuscript received August 1, 2003

Accepted October 27, 2003

IE020732V