Introducing the Uncertainty Principle Using Diffraction of Light Waves

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The following presents an analysis that can be used to illustrate the uncertainty principle (*1*), a fundamental element of quantum mechanics. It is based on the derivation that Levine uses in his textbook *Physical Chemistry* (*2*). The paper also presents the outline of an experiment that can be used in the classroom or in the physical chemistry laboratory to visualize the way light waves behave in response to the requirements of the uncertainty principle. While this experiment is probably not suited for first-year students because some minimal knowledge of diffraction is required, it is quite suitable for students in the junior physical chemistry sequence.

The *Thought* Experiment

The uncertainty principle states that the uncertainty in the measurement of the position of a particle (Δz) multiplied by the uncertainty in the measurement of its momentum along the same direction (Δp_z) is larger than the order of magnitude of h, Planck's constant. Equation 1 shows a formulation used in most physical chemistry textbooks.

$$\Delta z \times \Delta p_z \gtrsim h \qquad (1)$$

Imagine that you send a laser beam (horizontally) toward a screen. This is an ideal beam, so that the photons have no vertical component for their velocity vector. In other words, the momentum in the vertical direction (the vertical axis from here on is referred to as z) is also zero and so is the uncertainty associated with the value of the momentum along this axis. Since Δp_z is zero, one would expect that $\Delta p_z \times \Delta z = 0$ ($<h$), thereby violating the uncertainty principle. Fortunately, this is not the case, because the lack of vertical motion also means that the impression the beam leaves on the screen is one (or zero) dimensional and not observable. The uncertainty in the measurement of the position is infinite and the uncertainty principle holds.

Suppose, however, that we decide to use a slit to define the vertical position of the laser beam. Let us say, a 1-mm slit. In that case, we may believe that we still have $\Delta p_z = 0$ and $\Delta z = 1$ mm, so that now we do have $\Delta p_z \times \Delta z = 0$. Our thought experiment does not succeed, however, because the act of introducing the slit results in a diffraction pattern. This means that the photons acquire a momentum in the vertical direction, and Δp_z is no longer equal to zero. The slit can be made as small as necessary (thereby decreasing Δz), but this increases the size of the diffraction pattern (i.e., it makes Δp_z larger). See below for a description of a real experiment.

Formulation

Figure 1 represents a schematic of the experiment with the variables involved. The laser beam travels parallel to the *xy* plane. Therefore, the condition would hold (before a slit is introduced) that $2p_z = \Delta p_z = 0$. Once the slit (of width s) is introduced (see Fig. 2), a diffraction pattern is observed on the screen. The distance between the two lines of darkness closest to the center of the beam is labeled ΔW. ΔW is related to the value of p_z, because the width of the spot (ΔW), will be larger if p_z gets bigger. (ΔW is also related to the distance between the slit and the screen, L).

We will use the approximation that the uncertainty in the measurement of the momentum (in the z direction), Δp_z, is equal to twice the value of the z component of the momentum for the photons that reach the screen within the boundaries of the first two lines of darkness (Fig. 1, inset). Although this is just an approximation, we can rationalize choosing the photons within the two lines of darkness closest to the center with the following arguments.

The uncertainty in the measurement of any physical observable can be defined as the standard deviation, $\Delta\alpha = (\langle\alpha^2\rangle - \langle\alpha\rangle^2)^{1/2}$. The standard deviation of an observable, in turn, defines an interval where a majority of measurements are recorded (see, for instance, ref *3*). In this experiment, the central spot is by far the most intense spot in the diffraction pattern. Since this spot contains the majority of the photons, it stands that it is a good approximation to use it to define the uncertainty in the momentum in the z direction.

A second consideration is related to the fact that the uncertainty principle indicates that $\Delta z \times \Delta p_z$ is *larger* than $\sim h$. By choosing to relate Δp_z to ΔW (as defined in Fig. 1), we are ensuring that we do not overestimate Δp_z. We would rather have an underestimate, because if the uncertainty principle holds for an underestimated value of Δp_z, it must hold for the true Δp_z. In other words, by defining ΔW as the interval between the two lines of darkness closest to center to approximate the value of Δp_z, we are choosing the minimum value for the uncertainty in the momentum. This ensures that when one establishes that $\Delta z \times \Delta p_z \gtrsim h$, this is

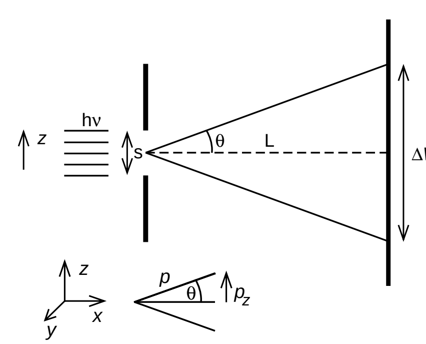

Figure 1. Schematics of the experiment: s is the slit size; L is the distance from the slit to the screen; ΔW is the distance between the two lines of destructive interference closest to the center of the laser beam; p_z is the z component of the momentum, p.

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due to a physical reason (i.e., the uncertainty principle) and not to the use of an excessive value for Δp_z.

This allows us to relate Δp_z to ΔW using the common angle θ.

$$\Delta p_z \approx 2p_z, \text{ and } \sin\theta = \frac{p_z}{p} \rightarrow \Delta p_z \approx 2p\sin\theta$$

Also, since $\theta \approx 0$ (note that ΔW is of the order of centimeters when L is several meters, see below), $\sin\theta$ can be obtained from $\sin\theta \approx \tan\theta = (\tfrac{1}{2}\Delta W)/\mathrm{L}$. Therefore,

$$\Delta p_z \approx 2p\frac{\Delta W}{2L} \approx \frac{p\Delta W}{L}$$

If we know the wavelength of the laser beam, we can get information about the total momentum, p: $p = h/\lambda$. Equation 2 shows the final expression for p_z.

$$\Delta p_z \approx \frac{h\Delta W}{\lambda L} \qquad (2)$$

Since $\Delta z = s$, we can write a formulation of the uncertainty principle related to the variables of our experiment:

$$\Delta p_z \times \Delta z \gtrsim h \rightarrow s\frac{h\Delta W}{\lambda L} \gtrsim h$$

$$\therefore \quad \frac{s\Delta W}{\lambda L} \gtrsim 1 \qquad (3)$$

The *Real* Experiment

The *thought* experiment can be reproduced in the classroom by using a diode laser and a slit (or several slits). A laser pointer works well, provided that it can be held in a mount (it cannot be hand-held). We use a diode laser with λ = 670 nm and two slits ($s = 4.1 \times 10^{-4}$ and 1.6×10^{-4} m). The error in the measurement of the slit size is 1.0×10^{-4} m. The distance between slit and screen (L) can be varied at will. A distance of 4 meters, coupled to these slit sizes, yields diffraction patterns of the order of 1 to 10 cm and can be easily observed in a dark classroom. Figure 2 shows an example of what is observed on the screen.

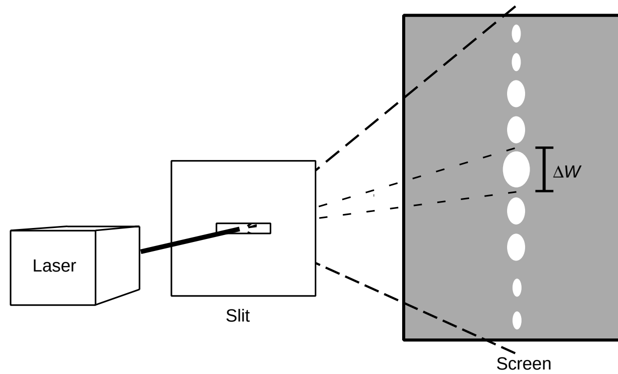

Figure 2. The experiment is performed by observing the vertical diffraction pattern (projected on a screen) that results from a laser beam passing through a horizontal slit.

Table 1. Measurement of Δ*W* and Other Variables Associated with the Undertainty Principle

Variable	Slit #1	Slit #2
L / m	3.91	3.91
s (= Δz) / m	4.1×10^{-4}	1.6×10^{-4}
ΔW / m	0.012	0.063
$\Delta W/L$ (= $\Delta p_z/p$)	0.00307	0.0161
$\Delta z \times \Delta p_z/p$ / nm	1258	2578
$s\Delta W/\lambda L$ (= $\Delta z \times \Delta p_z/p\lambda$)[a]	1.9	3.8

[a]This represents the expression arrived at in eq 3, which according to the uncertainty principle must be larger than 1.

Discussion

We normally conduct two experiments: the first with the larger slit, in order to observe the diffraction pattern, and the second with the narrower slit. This brings up the point that attempting to decrease Δz has the effect of increasing Δp_z. This is one of the most fundamental aspects of the uncertainty principle. Whether one is trying to pinpoint the position of a particle by using ever shorter wavelengths of electromagnetic radiation or the position of a wave by decreasing the slit size, the fact is that the act of measuring the position causes an increase in the uncertainty associated with the value of the momentum.

Table 1 shows the results of an experiment performed in the classroom (typical experiment time is about 5–10 min). It can be seen that $s\Delta W/\lambda L$ is indeed larger than 1. For a regular laboratory experiment, several modifications can be done to probe the effect of several variables. One possible modification would include error analysis. The largest source of error we encountered is in the measurement of the slit size. Additionally, one has to consider the intrinsic width of the laser beam, that is, the width of the laser beam when it reaches the screen in the absence of any slit (W_0), so that eq 3 turns into $s(W - W_0)/\lambda L \gtrsim 1$. Experiments with different slit sizes and at different slit-to-screen distances show that eq 3 (or the modified eq 3, shown above) always holds.

A point must be made about the relative orientation of the slit and the diffraction pattern. Figure 2 shows that, when the slit is horizontal, the diffraction pattern is vertical. However, some students have the misconception that slit and diffraction pattern must be parallel. The rationale behind this thought is that somehow light must behave like a fluid. If we constrain the beam in the z direction, then the light must expand along the other dimension of the slit in order to squeeze through. That this idea is not only incorrect but also inconsistent with the goal of the demonstration should be specifically underscored. This can be done by pointing out the slit orientation during the experiment.

This demonstration is not suitable to find the value of h. To do so, more complicated experiments, which record the momentum of the photons, are required. This can be done, for instance, by studying the photoelectric effect (*4*). Nevertheless, the uncertainty principle can be illustrated very nicely by showing that attempts to reduce the uncertainty in the measurement of the particle position have the unintended result of increasing the uncertainty in the measurement of the momentum.

Acknowledgments

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