and can be cured using peroxide initiators. During the polyesterification reaction, most of the maleate unsaturation is converted into the fumarate form for both the o-phthalic and PET based resins.

The mechanical behavior of the PET waste based resins was found to be comparable with the ordinary grade of general purpose resin. However, the impact and tensile strengths of these resins were lower than those of the improved grade GP resin. The lower properties may be attributed to the lower molecular weights and higher styrene contents of the PET waste based resins, compared to the improved GP resin. The heat distortion temperatures of the PET based resins were found to be higher than the improved grade GP resin.

The viscoelastic behavior of the PET based resins is comparable to that of the GP resin. Sample UVMW-59, containing the least amount of terephthalic moiety (Table III), showed almost no change in G' and G'' for a wide range of temperatures. The initial values of G' of PET waste based resins were marginally lower than that of the GP resins, yet the shear modulus of the GP resins falls much more rapidly with temperature than that of the PET waste based resins. The loss modulus of the improved grade GP resin showed a higher maximum than the PET based systems.

The molecular weight and the amount of styrene in the UP resin affect their mechanical properties. Therefore, these parameters need to be optimized to achieve better mechanical performance with the PET waste based resins.

Thus, the PET waste based resins can be used, as such, in applications that require a higher heat distortion temperature.

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PROCESS ENGINEERING AND DESIGN

Optimum Controller Settings for Processes with Dead Time: Effects of Type and Location of Disturbance

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Optimum controller settings were calculated for processes with dead time or an effective time delay using the integral of the absolute error as the criterion. For proportional integral control, the optimum gain was close to half the maximum gain for all cases, but the optimum reset time relative to the ultimate period varied considerably with the type and location of the disturbance as well as with the ratio of the dead time to the major time constant. With small dead times, the optimum reset time was as much as 3 times the value calculated by using the Ziegler–Nichols continuous-cycling tuning procedure.

Many processes include a dead time or time or time delay caused by the flow of material through a pipe or piece of equipment or the flow of fluid through a sample line to an analyzer. Other processes with a number of first-order lags appear to have a time delay because of the very slow initial response to a step change at the start of the process, and such processes are often modeled with an "effective time delay" and one or two first-order lags. The effects of the time delay on recommended controller settings for feedback control have been discussed in previous studies, but different disturbances have been used to test the systems and different criteria chosen to determine the best settings. In this study, the integral of the absolute error, $\int |e|^{\circ}\, dt$, or IAE is used as a measure of performance, since for small errors the penalty for poor control is generally a linear function of the error. The optimum settings are those which give a minimum IAE for a given disturbance. The effects of the type and location of the disturbance are considered as well as the ratio of the time delay to the largest time constant.

For the first case, the system included a time delay, a single first-order lag, and an ideal proportional integral controller. The valve and measurement lags were assumed to be negligible. The block diagram is the same as in

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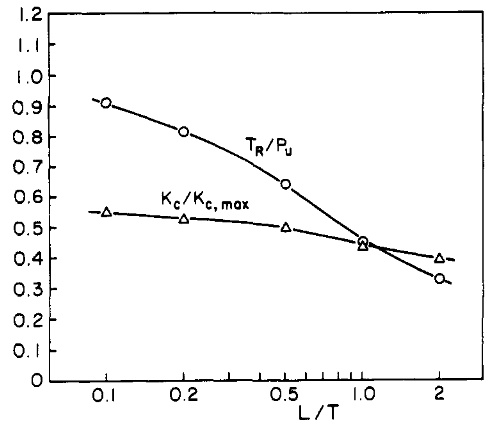

Figure 1. Optimum settings for proportional integral control of processes with a time delay, L, and one time constant, T.

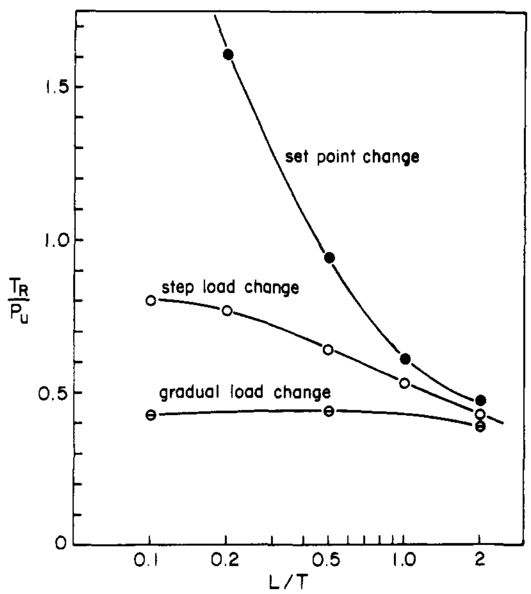

Figure 2. Optimum reset times for processes with a time delay and one time constant at $K_c = 0.5K_{c,max}$.

Figure 5 except that $T_2 = 0$, and the load change is introduced before the time delay at D_2. The system was simulated with an IBM AT computer using a time increment equal to the smallest lag in the system divided by 400. The process output was presented as calculations proceeded on an interactive graphics display, and the program could be interrupted at any time to get the error integral.

The optimum controller settings for regulator operation and a step change in load at the start of the process are shown in Figure 1. The optimum controller gain ranged from $0.55K_{c,max}$ at $L/T = 0.1$ to $0.42K_{c,max}$ at $L/T = 2.0$. The decrease in the ratio $K_{c,opt}/K_{c,max}$ is not very important, since the IAE at the optimum settings is at most a few percent less than that for a constant ratio $K_c/K_{c,max} = 0.5$. However, the optimum ratio T_R/P_u changes from 0.91 at $L/T = 0.1$ to 0.33 at $L/T = 2.0$. This range of values is quite significant, since using a constant value of 0.83 for T_R/P_u, in accordance with a widely used tuning procedure (Ziegler and Nichols, 1942), would give IAE values up to 80% greater than for the optimum settings.

The response to a gradual change in load was obtained by including a first-order lag between the step change in load and the process. The disturbance then was of the form

$$D = 1 - e^{-t/T'} \tag{1}$$

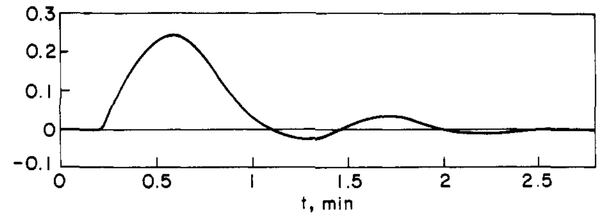

Figure 3. Response to a step change in load for $L = 0.2$ min, $T = 1.0$ min, $K_c = 4.25$, and $T_R = 0.57$ min (optimum settings). IAE = 0.143.

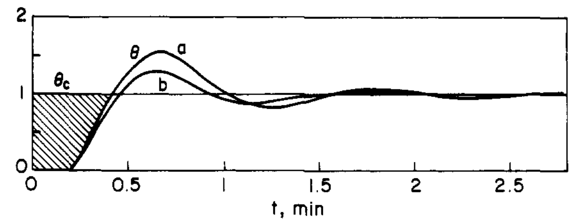

Figure 4. Response to a step change in set point for $L = 0.2$ min and $T = 1.0$ min. (a) $K_c = 4.25$, $T_R = 0.57$ min, and IAE = 0.60. (b) $K_c = 4.25$, $T_R = 1.20$ min, and IAE = 0.47.

The time constant T' was set equal to T, the time constant of the process. A few tests showed that the optimum controller gain was close to $0.5K_{c,max}$, so the gain was set at $0.5K_{c,max}$ to simplify the analysis. The optimum reset times are shown as the ratio T_R/P_u in Figure 2, and this ratio is nearly constant at 0.4 for a wide range of L/T. The data for a step change in load were recalculated for $K_c = 0.5K_{c,max}$ and are included in Figure 2. For a gradual load change, much lower values of T_R can be used (more integral action) before there is a significant undershoot. The optimum setting gives a response with only a small undershoot and IAE only slightly greater than the error integral $\int e^{\infty}\, dt$, which is equal to T_R/K_c for a unit change in load.

The optimum settings for servooperation were determined for a unit step change in set point by using the same criterion of minimum IAE. In practice, other factors such as the peak overshoot or the time to reach the new set point might have to be considered, but using the IAE criterion permits a direct comparison with the settings for load changes. The optimum controller gain was close to $0.5K_{c,max}$, so this value was used for all cases shown in Figure 2. The optimum reset time for set-point changes was much greater than for load changes except for processes with a relatively large time delay. The values of T_R/P_u are slightly less than those reported by Hazebroek and van der Waerden (1950), who used the error squared criterion, $\int^{\infty} e^2\, dt$ or ISE. The differences due to alternate criteria such as IAE and ISE (and probably ITAE) are much less than the large differences in optimum settings for servo versus regulator operation and for step versus gradual load changes.

Typical transient response curves may help explain the need for different reset times for load changes than for set-point changes. The response to a unit step change in load for a process with $L/T = 0.2$ is shown in Figure 3. At the optimum settings, there is very little undershoot, and nearly all the error integral comes from positive deviations. For this case, the IAE was only 7% greter than T_R/K_c, which is the minimum error integral needed to reduce the offset to zero. For other ratios of L/T, the minimum IAE following a step load change was also quite close to T_R/K_c (5–9% greater).

The need for larger values of T_R for set-point changes follows from the shape of the response curves shown in Figure 4 and the fact that the final value of the error integral for a unit change in set point is T_R/K_c. If T_R is

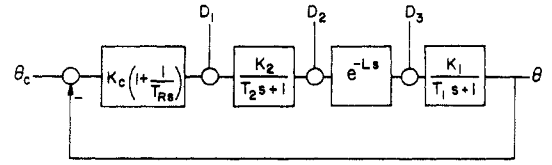

Figure 5. Block diagram for systems with a time delay and two time constants.

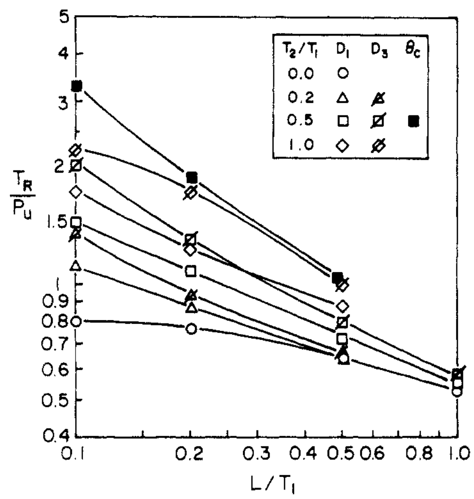

Figure 6. Optimum reset times for processes with a time delay and two time constants.

kept at 0.57 min (optimum for load changes) and $K_c = 4.25$ ($0.5K_{c,max}$), then $T_R/K_c = 0.134$. However, the error is constant at 1.0 for a time equal to the delay, and the shaded area in Figure 4a is about 0.30, already much larger than 0.134. Therefore, a sizable overshoot to produce a large negative error integral is inevitable. By use of $T_R = 1.2$ min, the optimum for set-point changes, the initial response is somewhat slower, as shown in Figure 4b. Now T_R/K_c is 0.282, and the first portion of the error integral is only slightly larger than this, so the overshoot is smaller and IAE is reduced.

The second class of systems studied included processes with a time delay and two first-order lags. The larger time constant was placed at the end of the process and the time delay was placed between the two lags. However, the location of the time delay has no effect on the error integral. Disturbances introduced before the first element or the third element are labeled D_1 and D_3, as shown in Figure 5. The valve and measurement lags were assumed negligible. Optimum reset times were determined for step changes in load or set point for ranges of L/T_1 and T_2/T_1. A controller gain of $0.5K_{c,max}$ was used for most cases, including those shown in Figure 6.

The bottom line in Figure 6 is taken from Figure 2 and is the limiting case of $T_2 = 0$. Adding a second time constant to the system increases the optimum value of T_R/P_u, and the increase is most pronounced for low values of L/T_1 and large values of T_2/T_1. For equal time constants and a small delay ($L/T_1 = 0.1$), the value of $T_{R,opt}$ for step load changes is $1.7P_u$ or $2.2P_u$, depending on the location of the disturbance, compared to $0.8P_u$ when there is only one time constant. These high values of T_R/P_u are similar to those recommended for processes with two large and one small time constant (Hazebrock and van der Waerden, 1950, Harriott, 1964, Weber and Bhalodia, 1979), since the frequency response near the critical frequency is quite similar when the delay or the third time constant is much smaller than the other lags. Using half the maximum gain for these systems does not give much phase margin, so larger than normal reset times are needed to avoid undue oscillation.

Table I. Effect of Controller Settings on IAE for a Process with T_1 = 10 min, T_2/T_1 = 0.5, L/T_1 = 0.1, $K_{c,max}$ = 15.8, and P_u = 11.7 min

disturbance	$K_c/K_{c,max}$	$T_{R,min}$	T_R/P_u	IAE
step D_1	0.5	17[a]	1.45	2.42
slow D_1	0.5	12[a]	1.0	1.79
slow D_1	0.5	17	1.45	2.15
step D_1	0.45[b]	9.7	0.83	>4
step D_1	0.21[c]	25.7	2.2	7.8
step D_3	0.5	17	1.45	4.28
step D_3	0.5	24[a]	2.05	3.89
step D_3	0.5	38	3.25	5.07
set pt	0.5	38[a]	12.3	12.3
set pt	0.5	24	2.05	13.4
set pt	0.5	17	1.45	16.1

[a] Optimum T_R. [b] Ziegler and Nichols (1942). [c] Harris and Mellichamp (1985). Slow load change = $1 - e^{-t/5}$.

The need for higher values of T_R for loads introduced near the end of the process is consistent with the higher values of T_R needed for set-point changes. Moving the disturbance closer to the end of the process produces a larger peak error and a larger error integral under the first peak, and if this is much greater than K_c/T_R, undershoot must occur to produce a considerable negative error integral. If the disturbance was introduced just before a very small time constant as the last element, the optimum settings would be nearly the same as for a change in set point.

A controller tuned for optimal response to a certain change in load will give suboptimal response for a change in set point or a different type of load. However, the minima in the IAE curves are fairly broad, and the penalty for using compromise settings rather than optimum values for a particular disturbance may not be large. Some examples are given in Table I for a process with T_1 = 10 min, T_2/T_1 = 0.5, and L/T_1 = 0.1. For $K_c = 0.5K_{c,max}$, $T_{R,opt}$ is 17 min for a step change in D_1. By use of these settings, a step change in D_3 produced a response with IAE = 4.28, 10% greater than the value for T_R = 24 min, which is optimum for changes in D_3. However, by use of the settings calculated for set-point changes (T_R = 38), the IAE is 5.07, 30% greater than with optimum settings. If T_R is set at 24 min, the IAE following a set-point change is 13.4, only 9% greater than with the optimum T_R of 38 min. Thus, the response to set-point changes seems to be less sensitive to the value of T_R than the response to load changes, which suggests giving more weight to the value of T_R for load changes if one setting is to be used for both types of disturbances.

The example of Table I was also used by Harris and Mellichamp (1985), who used a complex formula to determine the best settings for set-point changes. Their recommended settings ($K_c = 0.21K_{c,max}$ and $T_R = 2.2P_u$) give a very overdamped response for load changes and result in an error integral 3.2 times that for optimum settings. The Ziegler–Nichols rules give a very oscillatory response for this system, and the final error integral was not evaluated.

The response to a gradual change in load was briefly studied for the system of Figure 5. As shown in Table I, for a gradual load at D_1 the optimum reset time was 12 min, 30% lower than for a step change. With T_R = 17 min, a gradual load change gives an overdamped response with IAE = IE = K_c/T_R.

In another paper dealing with the effect of slow random disturbances, Sood and Huddleston (1977) determined optimal settings for a process with two equal time con-

Table II. Optimum Settings for a Process with Three or Four Equal Lags

n	disturbance	$K_c/K_{c,max}$	$T_{R,opt}/P_u$	IAE
3	step D_1	0.5	1.25	1.3
3	step D_2	0.5	1.52	1.69
3	step D_3	0.5	1.94	2.36
3	step set pt	0.5	2.49	3.59
4	step D_1	0.5	0.80	2.87
4	step D_3	0.5	0.96	3.63
4	step D_4	0.5	1.04	4.19
4	step set pt	0.5	1.19	4.92

stants followed by a delay of 0.2 T. Random load changes were introduced through a filter with a time constant of 2 T and entered the system just before the time delay. Their system is dynamically equivalent to that of Figure 5, but their load change corresponds to a slow change in set point. For widely spaced disturbances, they found a minimum IAE at $K_c/K_{c,max} \simeq 0.45$ and $T_R/P_u \simeq 1.1$. The results shown in Figure 6 for a step change in set point with $T_1 = T_2$ and $L/T_1 = 0.2$ are $T_{R,opt}/P_u = 2.4$ at $K_c/K_{c,max} = 0.5$. Correcting to $K_c/K_{c,max} = 0.45$ for comparison, $T_{R,opt}/P_u$ would be about 2.2. Thus, their slow change in set point leads to an optimal T_R about 50% lower than for a step change. This change in optimal T_R is somewhat greater than the effect shown in Table I, but their load change was even slower than the gradual changes used for the examples in Table I and Figure 2.

The third type of system studied had either three or four equal first-order lags and no time delay. However, the open-loop response shows a significant effective time delay, and it is of interest to compare the best settings for these systems with those for processes with real delays of the same magnitude. Step load changes were introduced at different places, and the response to set-point changes was also determined. The gain was set at 0.5 $K_{c,max}$, and the optimum reset times are given in Table II. With three lags in series, the ratio $T_{R,opt}/P_u$ increases from 1.25 to 1.52 to 1.94 as the disturbance is changed from D_1 to D_2 to D_3, and the value of 2.49 for set-point changes is a continuation of this trend. Optimum settings were reported for this system by Jackson (1958), who used the ISE criterion and found $T_{R,opt} = 1.7P_u$ for step changes at D_3. This is close to the value of $1.94P_u$ based on the IAE criterion. With four lags in series, the value of $T_{R,opt}/P_u$ does not change as much with disturbance location, going from 0.80 for D_1 changes to 1.19 for set-point changes.

If the reaction curve method of controller tuning (Ziegler and Nichols, 1942) is applied to the process with three equal unit lags, the effective time delay, obtained from the line of maximum slope, is $L' = 0.81$, and the effective time constant is $T' = 3.7$. Based on the ratio $L'/T' = 0.22$ and Figure 2, the recommended value of T_R would be $0.76P_u$ for step load changes. However this setting gives a very oscillatory response, and, as given in Table II, $T_{R,opt}$ is $1.25\text{–}1.94P_u$ depending on the location of the load change. The reaction curve method was also tested for the process with four equal lags and the process with $T_2/T_1 = 0.5$ and $L/T_1 = 0.1$. The use of L'/T' with Figure 2 and the direct calculation of T_R from the reaction curve rule of thumb $T_R = L'/0.3$ gave about the same results, but for both processes, the T_R values were considerably below $T_{R,opt}$.

In summary, the optimum value of the integral time relative to the ultimate period depends strongly on the ratio of the dead time to the major time constant and on the ratio of the two largest time constants. The ratio $T_{R,opt}/P_u$ also depends on the location and nature of the disturbance. Larger values of T_R are recommended when the most likely load changes occur near the end of the process, and even higher values of T_R are best for set-point changes. If only gradual load changes are expected, T_R can be reduced below the value recommended for step changes, and the resulting error integral will be reduced.

Nomenclature

D = disturbance or load change
IAE = integral of absolute error
ISE = integral of squared error
ITAE = integral of time times absolute error
K_c = controller gain
$K_{c,max}$ = maximum controller gain
$K_{c,opt}$ = optimum controller gain
L = time delay
L' = effective time delay
P_u = ultimate period
t = time
T = time constant
T' = effective time constant
T_R = reset or integral time
$T_{R,opt}$ = optimum reset time

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