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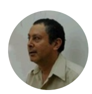

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Observer-Based Smith Prediction Scheme for Unstable Plus Time Delay Processes

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This work considers the stabilization problem for unstable linear input-delay systems. The main idea of the paper is to use a finite-dimensional approximation for the delay operator, which is based on non-overlapping partitions of the time delay. Subsequently, each individual delay is approximated by means of a classical Pade approximation, where the overall approximation results in a high-order Pade approximation that converges to the original delay operator. By departing from a state-space realization of the approximate process, a linear observer is used to estimate the delay-free output, which is used within a compensation scheme to stabilize the process output. The resulting control strategy has the structure of an observer-based Smith prediction scheme. Numerical results on three examples show that (i) the finer the time delay partition, the better the control performance and (ii) high-order compensators can be required to stabilize certain unstable processes.

1. Introduction

Time delays appear commonly in process control problems because of the distance velocity lags, recycle loops, and composition analysis loops or in the approximation of higher-order systems by means of a lower-order system with a time delay. Many controllers have been developed for stable processes. When the time delay is small, a PID controller commonly suffices to give acceptable performance. For large time delays, the Smith predictor[1] provides an open-loop estimate of the delay-free output, leading to an effective compensation scheme. However, the PID controller and the Smith predictor can be hardly used directly for delayed processes containing an integrator. A PID controller tuned by Ziegler−Nichols rules is found to be too oscillatory,[2] while the Smith predictor results in a steady-state error when there is a constant load disturbance. To reduce oscillatory behavior, Chien and Fruehauf[3] developed an internal model control approach to selecting the tuning constants for the PID controller. Tyreus and Luyben[2] have shown that internal model control rules can lead to sluggish response. Watanabe and Ito[4] presented a modification of the Smith predictor to reject a load disturbance. The main drawback of these control schemes is that the time delay of the process should be exactly known, otherwise there will be a small steady-state error. Along the same idea, Astrom et al.[5] proposed a new Smith predictor structure that provides superior performance. Its advantage is that the disturbance response is decoupled from the set-point response and hence can be independently optimized.

The control of unstable plus time delay processes is a more challenging problem. In fact, the existence of right-half plane poles and time delay makes it difficult, and in many cases impossible (when the number of unstable poles is larger than two), to stabilize the system with standard PI/PID controllers. On the other hand, the conventional Smith delay compensator cannot stabilize unstable systems. Focusing mainly on first-order plus time delay (FOPTD) processes, the design of controllers to stabilize and improve the performance for unstable systems with time delay has attracted considerable interest. De Paor[6] and De Paor and Egan[7] have proposed a modified Smith predictor for unstable processes with a time delay under the constraint of $\theta/\tau_p < 1$, where θ and τp represent the time delay and the time constant of the dominant unstable dynamics, respectively, of the process. Asymptotic stability of the resulting closed-loop processes was examined by means of Nyquist and root locus techniques. Kwak et al.[8] have proposed an analytical predictor that predicts the dynamics of the actual process from the process model. The main drawback is that the method is sensitive to uncertainties in process parameters. Majhi and Atherton[9] have modified the original Smith predictor for unstable time delay processes with three controllers: a PI controller for set-point tracking, a proportional (P) controller for disturbance rejection, and another P controller for stabilizing the unstable process. The proposed control structure decouples the disturbance rejection responses from the set-point tracking responses. In a subsequent paper, Majhi and Atherton[10] proposed a proportional-derivative controller for stabilizing the unstable process model to improve the performance of the control system. Also for FOPTD processes, Kaya[11] endowed the Majhi and Atherton scheme[9] with a single delay feedback test for the identification of the process parameters. Recently, Zhang et al.[12] proposed a modified structure of a Smith predictor for unstable FOPTD processes by modifying the structure proposed by Kwak et al.[8] Specifically, they have designed a PID controller to stabilize the unstable process and reject the disturbances based on the desired closed response. The resulting set-point controller is considered as an inverse of the process model with a first-order filter. Rao and Chidambaram[13] proposed further modifications to the Smith predictor to obtain improved performance for unstable FOPTD processes. It should be stressed that, in all the previously commented papers, the simulation results for the Smith predictor were made for unstable FOPTD systems constrained to the inequality $\theta/\tau_p \leq 1$. That is, the unstable process dynamics is not dominated by the time delay, so that a suitable modified Smith predictor should focus mainly on canceling the adverse effects of the unstable pole.

Summing up, Smith predictor schemes for stable processes give improved performance for dominant delays, whereas for unstable processes, the original Smith predictor cannot be

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applied, because it cannot cancel unstable poles. This has motivated the design of modified forms of the Smith predictor.[10−14] It is clear that the performance of the proposed modified Smith predictor depends strongly on the stabilization of the unstable processes. On the other hand, existing methods for unstable plants are limited to FOPTD processes with small values of θ/τ_p. Therefore, there is a motivation for the design of stabilizing controllers for unstable processes that are not constrained to only one unstable pole and contain arbitrarily large time delays. This paper focuses on the issues by proposing a stabilization scheme for unstable processes with time delay, where the process is not constrained to contain only one right-half pole. The methodology relies on partitioning the time delay into many small and equal delays, which are subsequently approximated by means of a conventional Pade transfer function. It is shown that this partition operation converges uniformly to the time delay operator at the limit of many partitions. In a second step, a time-domain realization of the delay-free process and the delay partition is proposed, for which a Luenberger-type observer is constructed. In this way, the reconstructed (i.e., observed) output of the delay-free process is fed back to stabilize the real process by means of a suitable controller. Consequently, the resulting feedback compensator has the structure of a Smith predictor since it provides, via a closed-loop observer, an estimate of the delay-free output. The order of the complete controller becomes the order of the controller that stabilizes the delay-free process plus the order of the time delay partition. By using numerical simulations on processes with one and two unstable poles, it is shown that the larger the partition order, the better the stabilization performance. In this form, one concludes that enhanced performance for either stable or unstable process plus time delay can require the usage of high-order controller to compensate for possibly large time delays. Some numerical simulations to test the robustness of the proposed stabilization scheme in the face of (e.g., time delay) uncertainties are also provided.

2. Preliminaries

Consider the following class of possibly unstable SISO linear systems with delayed input $g(s) = h(s) \exp(-\theta s)$. That is,

$$g(s):U(s) \rightarrow Y(s) \tag{1}$$

Here θ is the time delay, $Y(s)$ is the process output and $U(s)$ is the control input. Since the process $g(s)$ is composed of a delay-free transfer function $h(s)$ plus a time delay operator $\exp(-\theta s)$, the process can be represented as a cascade system in the following form:

$$\begin{aligned} h(s)&:U(s) \rightarrow Y_{df}(s) \\ \exp(-\theta s)&:Y_{df}(s) \rightarrow Y(s) \end{aligned} \tag{2}$$

where $Y_{df}(s)$ is the delay-free output. Let us consider the following assumption:

Let $h(s) = N_h(s)/D_h(s)$ ($\deg(N_h(s)) = m_h$, $\deg(D_h(s)) = n_h$, $n_h \geq m_h$), where $N_h(s)$ and $D_h(s)$ are coprime polynomials. There exists a causal controller $c(s) = N_c(s)/D_c(s)$ ($\deg(N_c(s)) = m_c$, $\deg(D_c(s)) = n_c$, $n_c \geq m_c$) with the delay-free output $Y_{df}(s)$ for feedback that stabilizes the delay-free process $h(s)$. That is, given that the closed-loop transfer function becomes

$$h_c(s) = \frac{c(s)h(s)}{1 + c(s)h(s)} \tag{3}$$

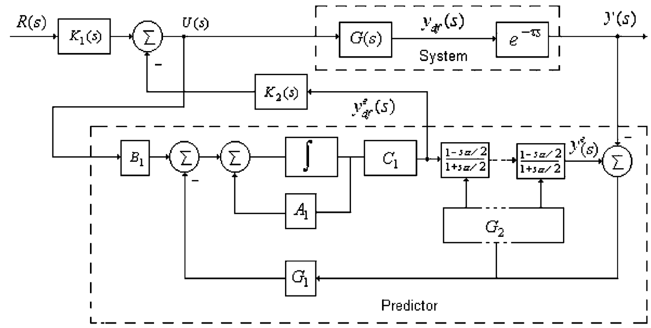

Figure 1. Schematic diagram of the proposed Smith prediction scheme.

the polynomial $N_h(s)N_c(s) + D_h(s)D_c(s)$ is Hurwitz. The controller $c(s)$ can be computed with standard control design techniques, such as LQR, linear matrix inequalities, and robust control methods. For instance, if the delay-free process $h(s)$ is uncertain, it can be described as $h(s) = \bar{h}(s)\,(1 + \Delta h(s))$, where $\bar{h}(s)$ is the nominal process and $\Delta h(s)$ is the uncertain component. Optimal robust control techniques, such as those reported in Morari and Zafiriou,[15] can be used to compute a controller $c(s)$ that stabilizes the family of processes $h(s)$. In this way, it should be recalled in the sequel that a controller $c(s)$ that stabilizes the (either uncertain or not) delay-free process is available.

Once the controller $c(s)$ has been computed, its practical implementation should confront the problem of the lacking of the delay-free output measurements. In fact, the controller $c(s)$ should use the real output signal $Y(s)$ rather than the delay-free output signal $Y_{df}(s)$. Hence, the actual closed-loop transfer function is

$$g_c(s) = \frac{c(s)h(s)\exp(-\theta s)}{1 + c(s)h(s)\exp(-\theta s)} \tag{4}$$

where the term $\exp(-\theta s)$ in the denominator complicates the stability analysis of the feedback system and, consequently, the design of a stabilizing controller for the delayed process $g(s)$. In fact, the analysis of the characteristic equation $1 + c(s)h(s) \exp(-\theta s)$ is quite complex since it involves the determination of an infinite number of characteristic roots. An alternative is to use a control strategy to remove the delay operator from the characteristic equation. This is done by using a strategy similar to Smith's prediction schemes. In fact, if the delay-free signal $Y_{df}(s)$ is available for feedback, the delayed operator $\exp(-\theta s)$ in the denominator of eq 4 could be removed, and the corresponding closed-loop transfer function would be

$$g_{df,c}(s) = h_c(s)\exp(-\theta s) = \frac{c(s)h(s)\exp(-\theta s)}{1 + c(s)h(s)} \tag{5}$$

Since, by assumption, the polynomial $N_h(s)N_c(s) + D_h(s)D_c(s)$ is made Hurwitz by the controller $c(s)$; the closed-loop transfer function $g_{df,c}(s)$ is also stable. However, this is not a practical solution given that the signal $Y_{df}(s)$ is an advanced (i.e., noncausal) version of the output $Y(s)$. That is, $Y_{df}(s) = \exp(\theta s)Y(s)$, or in time domain $y_{df}(t) = y(t + \theta)$, which implies that the signal $Y_{df}(s)$ is never available for feedback. Into the context of prediction methodologies (see Figure 1), an estimate, say $Y^e_{df}(s)$, of the delay-free signal $Y(s)$ can be computed and used for feedback by the controller $c(s)$. That is, $Y^e_{df}(s)$ should become an estimate of the process output for time $t + \theta$. It is expected that in the event that $Y^e_{df}(t) \rightarrow y_{df}(t)$, the feedback controller $c(s)$ with $Y^e_{df}(s)$ as input would stabilize the otherwise unstable process. In other words, according to eq 5, the usage of an accurate estimate $Y^{e}_{df}(s)$ would allow a significant reduction of the delay adverse effects on the controlled process.

The following observation is important for obtaining the estimate $Y^{e}_{df}(s)$. The transfer function $g(s) = h(s)\exp(-\theta s)$ is minimal and causal (see Assumption), which implies that the delayed process described in cascade form as in eq 2 is observable. Hence, the dynamics of the delay-free signal $Y_{df}(s)$ can be reconstructed from measurements of the output signal $Y(s)$. In principle, a minimal time-domain realization of the delayed process $g(s) = h(s)\exp(-\theta s)$ can be obtained, for which an asymptotic observer can be constructed. Specifically, let

$$\dot{x}_1 = A_1 x_1 + B_1 u$$
$$y_{df} = C_1 x_1 + D_1 u$$

be a state-space realization of the delay-free process $h(s){:}U(s) \to Y_{df}(s)$ (i.e., $g(s) = C_1(sI - A_1)^{-1}B_1 + D_1$), where x_1 is a vector of suitable dimension. On the other hand, let

$$\dot{w} = A_\infty w + B_\infty y_{df}$$
$$y = C_\infty w + D_\infty y_{df}$$

be a state-space realization of the delay operator $\exp(-\theta s) = C_\infty(sI - A_\infty)^{-1}B_\infty + D_\infty$. Notice that the matrices A_∞, B_∞, C_∞, and D_∞ are infinite-dimensional. Given the infinite-dimensional nature of the delay operator $\exp(-\theta s)$, the vector w must be infinite-dimensional.[16] Since the system composed by the above individual systems is observable, one can construct a Luenberger-type observer to estimate asymptotically the dynamics of the signal $y_{df}(t)$. In this way,

$$\dot{x}^{e}_1 = A_1 x^{e}_1 + B_1 u + G_1(y - y^{e})$$
$$\dot{w}^{e} = A_\infty w^{e} + B_\infty(C_1 x^{e}_1 + D_1 u) + G_\infty(y - y^{e})$$

where x^{e}_1 is an estimate of x_1, and so on, and the matrices G_1 and G_∞ are the observer gain matrices. The estimated dynamics of the delay-free output are recovered as $y^{e}_{df} = C_1 x^{e}_1 + D_1 u$, and the control input is computed as $U(s) = c(s)Y^{e}_{df}(s)$. Under the assumption that $c(s)$ stabilizes the delay-free process, given the linearity of the process, the stability of the closed-loop system with observer-based prediction can be guaranteed by virtue of the well-known separation principle for infinite-dimensional systems,[16] which allows us to simplify an output feedback control problem into a state feedback control problem and a state observer design problem. Unfortunately, given the infinite-dimensional nature of the delay operator $\exp(-\theta s)$, the controller composed by the compensator $U(s) = c(s)Y^{e}_{df}(s)$ and the observer-based prediction scheme is infinite-dimensional, which is not realizable in practical situations. An alternative to overcome this problem is to use finite-dimensional observers for estimation of the delay-free signal $Y_{df}(s)$. This problem will be addressed in the following section by taking a finite-dimensional approximation of the delay operator $\exp(-\theta s)$.

It is important to note that we are not confronting the problem of design a feedback controller for the whole delayed process. Instead, we depart from a computed controller for the delay-free processes, and then, we only confront the problem of designing a finite-dimensional estimation scheme for the delay-free output $Y_{df}(s)$. To the best of our knowledge, prediction schemes based on asymptotic observers for the stabilization of unstable processes have not been addressed previously.

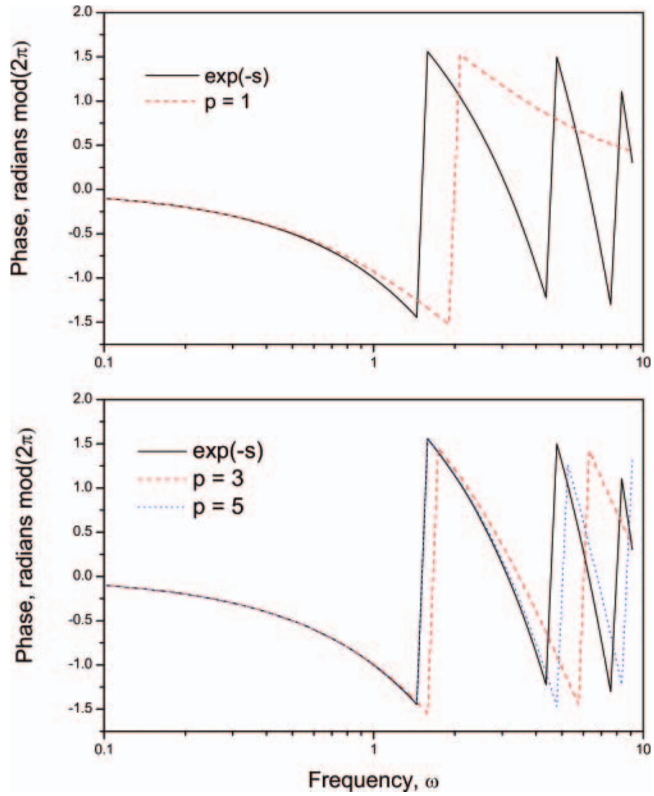

Figure 2. Phase response of the delay operator $\exp(-s)$ and three approximations. Notice the poor agreement between the phase response of the exact delay operator and its Pade ($P = 1$) approximation and how the phase response approaches that of the exact delay operator as p is increased.

3. A *p*th-Order Pade Approximation for $\exp(-\theta s)$

As commented on in the above section, the delay operator $\exp(-\theta s)$ is infinite-dimensional. In order to derive a finite-dimensional prediction scheme for the process described in eq 1, we will derive a pth-order causal approximation for $\exp(-\theta s)$. The most known finite-dimensional reduction for the delay operator is the first-order Pade approximation, i.e.,

$$\exp(-\theta s) \approx \frac{1 - \frac{\theta}{2}s}{1 + \frac{\theta}{2}s} \tag{6}$$

Although this approximation preserves magnitude (i.e., it has unit norm), significative phase deviations are found for an important range of frequencies (see Figure 2). These phase uncertainties into a control loop can introduce serious stability problems, mainly for unstable processes. In order to reduce such phase uncertainties, we shall consider higher-order approximations. To do this, the time delay is partitioned into $p > 1$ non-overlapping delays as follows:

$$\exp(-\theta s) = [\exp(-2\alpha_p s)]^p \tag{7}$$

where $\alpha_p = \theta/2p$. Since $\alpha < \theta$, a Pade approximation for $\exp(-\alpha_p s)$ should display less phase deviations than $\exp(-\theta s)$. In this form, we introduce the Pade approximation for $\exp(-\alpha s)$ into eq 7 to obtain the following expression:

$$\exp(-\theta s) \approx \left(\frac{1 - \alpha_p s}{1 + \alpha_p s}\right)^p \tag{8}$$

We have that this approximation converges uniformly to the actual infinite-dimensional delay operator. That is,

$$\lim_{p\to\infty}\left(\frac{1-\alpha_p s}{1+\alpha_p s}\right)^p = \exp(-\theta s) \tag{9}$$

To demonstrate this limit, consider the change of variable $r = -\theta s$, such that the limit is transformed into

$$\lim_{p\to\infty}\left(\frac{2p+r}{2p-r}\right)^p$$

Let $\Phi(r)$ be the function defined by $\lim_{p\to\infty} [(2p + r)/(2p - r)]^p$. Observe that

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{2p+r}{2p-r}\right)^p = \left(\frac{2p+r}{2p-r}\right)^p\left(\frac{4p^2}{(2p+r)(2p-r)}\right)$$

and that

$$\lim_{p\to\infty}\left[\frac{4p^2}{(2p+r)(2p-r)}\right] = 1$$

In consequence,

$$\begin{aligned}\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r} &= \lim_{p\to\infty}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{2p+r}{2p-r}\right)^p\\ &= \lim_{p\to\infty}\left(\frac{2p+r}{2p-r}\right)^P \lim_{p\to\infty}\left(\frac{4p^2}{(2p+r)(2p-\mathrm{r})}\right)\\ &= \Phi(r)\end{aligned}$$

Also, we have that

$$\Phi(0) = \lim_{p\to\infty}\left(\frac{2p+0}{2p-0}\right)^p = 1$$

But the differential equation $\mathrm{d}\Phi(r)/\mathrm{d}r = \Phi(r)$ with the initial condition $\Phi(r_0) = 1$ has unique solution given by the function $\exp(r)$. Therefore,

$$\exp(r) = \lim_{p\to\infty}\left(\frac{2p+r}{2p-r}\right)^p$$

and taking $r = -\theta s$, we obtain

$$\lim_{p\to\infty}\left(\frac{1-\alpha_p s}{1+\alpha_p s}\right)^p = \exp(-\theta s) \tag{10}$$

This result shows that a pth-order partition can recover the delay operator $\exp(-\theta s)$. Figure 2 shows the phase approximation for three different values of p. It is observed that the phase deviations are reduced as the partition order p is increased.

4. Finite-Dimensional Prediction Scheme

Based on the above approximation, we can propose an observer-based prediction scheme that uses the observation of the dynamics of $y_{df}(t)$. To this end, a minimal state-space representation of the delay-free process $h(s)$ is given by

$$\begin{aligned}\dot{x}_1 &= A_1x_1 + B_1u\\ y_{\mathrm{df}} &= C_1x_1 + D_1u\end{aligned} \tag{11}$$

where $x_1 \in \mathbb{R}^{n_h}$ is a vector of states, and A_1, B_1, C_1, and D_1 are matrices of suitable dimension. Each factor $(1 - \alpha_p s)/(1 + \alpha_p s)$ introduces an additional state in the time-domain representation of the process. It is not hard to show that the term $[(1 - \alpha_p s)/(1 + \alpha_p s)]^p$ can be described as follows:

$$\begin{aligned}\dot{x}_2 &= A_2x_2 + B_2y_{\mathrm{df}}\\ y &= C_2x_2 + D_2y_{\mathrm{df}}\end{aligned} \tag{12}$$

where $x_2 \in \mathbb{R}^p$, and the matrices A_2, B_2, C_2, and D_2 are given in the Appendix. In this way, the corresponding asymptotic observer with output injection is given by

$$\begin{aligned}\dot{x}_1^{\mathrm{e}} &= A_1x_1^{\mathrm{e}} + B_1u + G_1(y - y^{\mathrm{e}})\\ \dot{x}_2^{\mathrm{e}} &= A_2x_2^{\mathrm{e}} + B_2(C_1x_1^{\mathrm{e}} + D_1u) + G_2(y - y^{\mathrm{e}})\end{aligned} \tag{13}$$

where y is the measured output and $y^{\mathrm{e}} = C_2x_2^{\mathrm{e}} + D_2(C_1x_1^{\mathrm{e}} + D_1u)$ is the estimated output. The matrices G_1 and G_2 are the observer gains and are chosen to achieve stability of the matrix

$$\begin{pmatrix}A_1 - G_1D_2C_1 & -G_1C_2\\ B_2C_1 - G_2D_2C_1 & A_2 - G_2C_2\end{pmatrix} \tag{14}$$

The observability of the system described by eqs 11 and 12 guarantees the existence of such gain matrices G_1 and G_2. As in the infinite dimensional case, the prediction of the delay-free output is given by

$$y_{\mathrm{df}}^{\mathrm{e}}(t) = C_1{x_1}^{\mathrm{e}}(t) + D_1u(t) \tag{15}$$

and the control input is computed as $U(s) = c(s)Y_{\mathrm{df}}^{\mathrm{e}}(s)$. Notice that the resulting controller (see Figure 1) uses a $(n_h + p)$-dimensional estimator, where n_h is the dimension of the delay-free process, resulting in a two degrees of freedom compensator. In this way, the larger the delay partition order p, the higher the controller dimensionality. From the results described above, one concludes that an infinite-dimensional controller is obtained in the limit as $p \to \infty$. Given that the estimate $y_{\mathrm{df}}^{\mathrm{e}}(t)$ is obtained from an observation scheme, the resulting controller shall be called an observer-based Smith predictor for delay compensation.

Summing up, the design of a stabilizing controller for the delayed process (1) can be made along the following steps:

(i) Design a (possibly robust) controller $c(s)$ for the delay-free process $h(s)$.

(ii) Make a partition of the delay operator into p, for sufficiently large p, first-order Pade approximations (see eq 8). De Paor[6] and De Paor and Egan[7] observed that Smith predictor schemes can stabilize unstable processes with a time delay under the constraint $\theta/\tau_p < 1$, where θ and τ_p represent the time delay and the time constant of the dominant unstable dynamics. In this way, as a guideline we recommend a time delay partition with θ/p around 0.1 times the dominant unstable time constant τ_p.

(iii) Make a minimal (observable and controllable) state-space realization of the approximated process (see eqs 11 and 12).

(iv) Construct an asymptotic observer to obtain an estimate of the delay-free output, say, $\bar{y}_{\mathrm{df}}(t)$. As usual in observed-based control designs, choose the gains G_1 and G_2 in a way that the observer is ~10 times faster than the nominal closed-loop, delay-free process.

(v) Implement the controller $c(s)$ with the estimate $\bar{y}_{\mathrm{df}}(t)$ as the control input.

Stability Analysis. A detailed analysis of the stability and performance of the proposed control scheme is beyond the scope

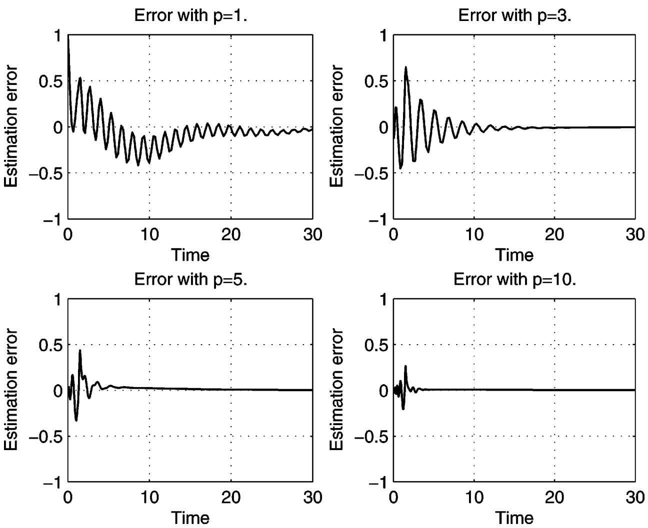

Figure 3. Estimation error for the delay-free output and for four different values of the approximation order p (Example 1, with $\theta = 1.5$).

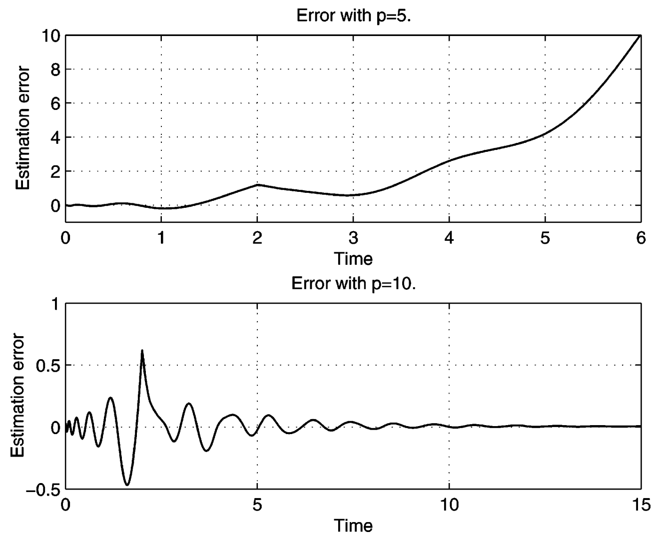

Figure 4. Estimation error for the delay-free output and for two different values of the approximation order p (Example 1, with $\theta = 2$).

of this paper. However, based on tools from robust process control,[15] in this part of the paper we will provide some theoretical arguments to show that the observer-based prediction scheme is able to stabilize the delayed process for sufficiently large p. For simplicity in notation, let $d(s;p) = [(1 - \alpha_p s)/(1 + \alpha_p s)]^p$. Besides, let $\Delta d(s;p) = \exp(-\theta s) - d(s;p)$. One can write the delayed process as a nominal process $g_n(s;p) = h(s)d(s;p)$ with additive uncertainty as follows:

$$g(s) = h(s)d(s;p) + \Delta g(s;p)$$

where $\Delta g(s;p) = h(s)\Delta d(s;p)$. According to the uniform convergence result in a pth-order Pade approximation for exp($-\theta s$), one has that $\|\Delta d(j\omega;p)\| \to 0$ uniformly as $p \to \infty$. Therefore, $\|\Delta g(j\omega;p)\| \to 0$ as $p \to \infty$. The controller with observer-based prediction can be written as

$$F(s;p) = c(s)P(s;p)$$

where $P(s;p)$ is the transfer function of the observer (eq 13). A sufficient condition for robust stability of the controlled process is[17]

$$\frac{\bar{\sigma}(\Delta g(s;p))}{\underline{\sigma}(g_n(s;p))}\bar{\sigma}[g_n(s;p)F(s;p)\,(1 + g_n(s;p)F(s;p))^{-1}] < 1 \tag{16}$$

where $\bar{\sigma}$ and $\underline{\sigma}$ denote upper and lower singular values. If one assumes that the nominal plant has not poles at the origin, one has that $\underline{\sigma}(g_n(s;p))$ is uniformly bounded from below for all p. On the other hand, by construction the controller $F(s;p)$ stabilizes the nominal process $g_n(s;p)$. Therefore, since the process is SISO, $\bar{\sigma}[g_n(s;p)F(s;p)(1 + g_n(s;p)F(s;p))^{-1}]$ is bounded from above. Otherwise, the nominal closed-loop process would have an infinite sensitivity to finite set point changes. In this way, since $\bar{\sigma}(\Delta g(s;p)) \to 0$ as $p \to \infty$, one has the existence of a minimal order $p_{\min}$ such that the inequality presented in eq 16 is satisfied for all $p > p_{\min}$. The theoretical determination of a minimal order approximation $p_{\min}$ to guarantee closed-loop stability under the controller $c(s)$ is not an easy task. However, given that, by assumption, $c(s)$ is a stabilizer for the delay-free process, the uniform convergence of $\lim_{p\to\infty} [(1 - \alpha_p s)/(1 + \alpha_p s)]^p = \exp(-\theta s)$ guarantees that $c(s)$ with the estimate $\bar{y}_{\mathrm{df}}(t)$ is able to stabilize the delayed process for sufficiently large p.

5. Simulation Results

In this section, three simulations are used to illustrate the performance of the proposed observer-based Smith predictor scheme. All numerical simulations were obtained from a Simulink-MATLAB implementation of the control scheme.

Example 1. The first example considers the following process:

$$g(s) = \frac{1}{s - 1}\exp(-1.5s) \tag{17}$$

Notice that $\theta/\tau_p = 1.5 > 1$, which is of the class of unstable processes not considered previously (see, for instance, Rao and Chidambaram[13]). The controller is simply a proportional compensator $c(s) = k$, with $k = 2$. The observer poles are located at $\{-0.1, -1/\alpha_p, -1/\alpha_p, ...\}$ such that the unstable pole $+1$ is moved to -0.1, and the approximate delay operator poles are unchanged. Figure 3 shows the estimation error for the delay-free output, and for four different values of the approximation order p, considering no reference input and initial output condition at 1. It is noticed that the estimation performance is enhanced as the approximation order is increased. Although the estimation error is convergent for $p = 1$, severe oscillatory behavior is displayed. However, higher-order controllers provide faster convergence with reduced oscillations magnitude. Except in the initial delay phase, the output dynamics are well reconstructed for $p \geq 5$. Let us consider more severe delay conditions by taking $\theta = 2$. In the preceding case, the process was stabilized even with a first-order (i.e., $p = 1$) compensator. However, when θ is increased, as is our case, a first-order controller may not suffice to stabilize the delayed unstable process. In fact, Figure 4 shows that a minimum approximation order can be required in order to achieve output stabilization by means of the proposed scheme. In fact, a controller with $p = 5$ gives unstable behavior, while a higher-order controller with $p = 10$ provides stability of the unstable process. This result suggests that high-order compensator may be required for unstable processes with relatively large values of θ/τ_p.

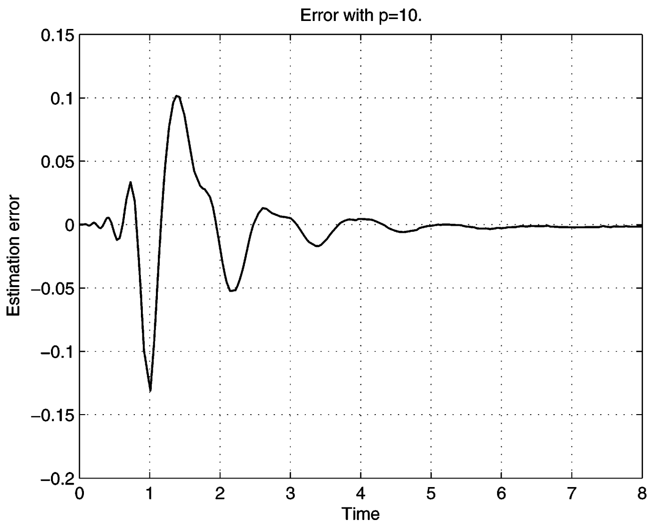

Figure 5. Estimation error for a second-order unstable process (Example 2) with $p = 10$.

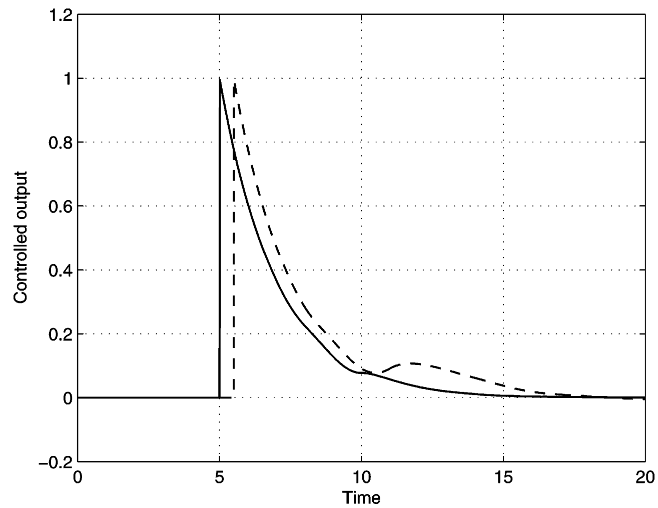

Figure 6. Stabilization performance for a first-order unstable process (Example 3) with $p = 10$ (solid line) and with a +10% error in the time delay value.

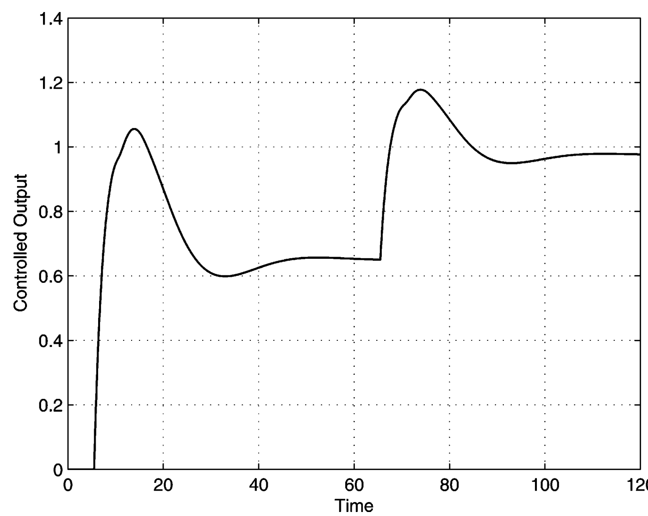

Figure 7. Stabilization performance for a first-order unstable process (Example 3) with $p = 10$ and subjected to parameter uncertainties.

Example 2. In general, existing results have focused on FOPTD processes. We will show that the proposed Smith prediction strategy is not constrained to such a class of processes. To this end, consider a second-order example with the two unstable poles:

$$g(s) = \frac{1}{(s-1)(s-2)} \exp(-s)$$

The controller is

$$c(s) = \frac{50(s+1)}{s+10} \tag{18}$$

which stabilizes the delay-free process. In fact, the corresponding characteristic polynomial is $P(s) = s^3 + 7s^2 + 22s + 70$, with roots set $\{-5.3356, -0.8322 \pm 3.5252i\}$. Under this compensator, we have found that the delayed process can be stabilized only if $p \geq 8$. As in the previous example, the poles of the observer are located at $\{-0.1, -1/\alpha_p, -1/\alpha_p, ...\}$. Figure 5 shows the estimation error $y_{df}(t) - y^e_{df}(t)$ and the regulated output for $p = 10$. Observe that the compensator endowed with the observer-based Smith predictor is able to stabilize the process output despite the presence of a significant delay and two unstable poles. Besides, this example also shows that a minimal compensator order can be necessary to stabilize unstable processes. The prediction strategy proposed in this work provides a systematic procedure to compute high-order compensators for such class of unstable processes.

Example 3. In practice, the process parameters are uncertain. A reliable Smith predictor scheme should be able to deal with uncertainties in the time delay value. To illustrate the performance of the proposed observer-based Smith predictor strategy, let us consider the following unstable processes borrowed from Xian et al.:[18]

$$g(s) = \frac{4}{10s-1} \exp(-5s) \tag{19}$$

The delay-free stabilizer is as in the refereed paper, $u(t) = 1.25r(t) - (1.5)y^e_{df}(t)$, where $r(t)$ is the set-point signal, with the observer poles placed at $\{-0.5, -1/\alpha_p, -1/\alpha_p, ...\}$ considering no reference input and initial output condition at 1. The stabilization results for $p = 10$, $r(t) = 0$ and exact knowledge of the time delay value are shown in Figure 6 (solid line). Now, suppose a +10% uncertainty in the time delay value. The results in Figure 6 (dashed line) show that, although the control performance is affected by time delay uncertainties, the predictor is able to provide an accurate estimate of the delay-free output, which allows the stabilization of the process output. Such a robustness margin is introduced by the observer-based prediction scheme that provides an estimate of advanced values of the process output. Now, let us consider that the actual process is $g(s) = (4.4/10s - 0.9) \exp(-5.5s)$ while the stabilizing controller is designed with the nominal process model (i.e., $u(t) = 1.25r(t) - (1.5)y^e_{df}(t)$). That is, the process is subjected to parameter uncertainties in the dc gain, time constant, and time delay. The simulation results are shown in Figure 7. Observe that the proposed estimation scheme is able to counteract the instabilizing effects despite significant deviations in the process parameters. Finally, we have incorporated an integral action into the feedback loop to account for set-point changes and external disturbances. Such integral action was tuned-up according to internal model control guidelines.[15] In this way, consider a set-point change from $r(t) = 1$ to $r(t) = 0.6$ taken at $t = 60$. The results are displayed in Figure 8 showing that the controller

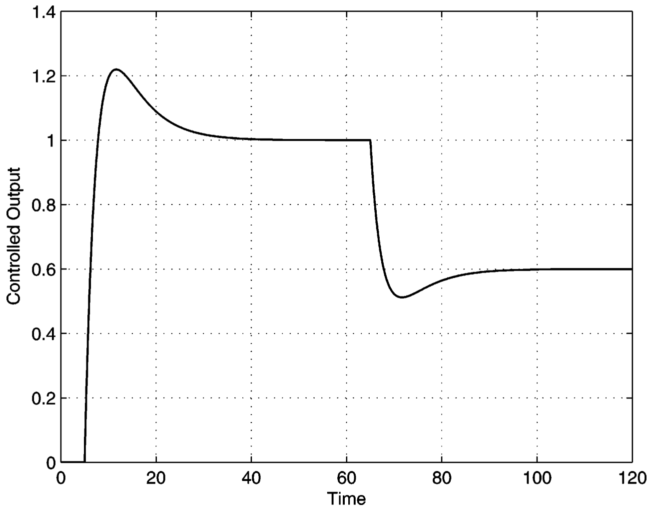

Figure 8. Stabilization performance for a first-order unstable process (Example 3) with $p = 10$, with a +10% error in the time delay value and a PI controller.

can give acceptable set-point tracking in the presence of unstable open-loop poles and uncertain time delays.

It should be stressed that we are not claiming that the performance of the proposed control strategy is better than that produced by reported modified Smith predictors.[7,8,14,18] Rather, the above results have shown that the proposed observer-based predictor offers a systematic and intuitive framework to design high-order compensators with increasing performance. This is particularly important since some unstable processes can require a compensator of minimal order to achieve output stabilization. It should be noticed that the proposed scheme only addresses the stabilization problem. However, once the process has been stabilized, one can resort to existing control strategies for a delayed stable process in order to introduce regulatory compensation components into the control loop in order to achieve acceptable load disturbance rejection.

6. Conclusions

Unstable processes with significant time delay offer a challenge to the design of stabilizing controllers. In fact, large time delays with internal instabilities present a worst-case scenario for the regulation and stabilization of process output. In this paper, we have described a systematic strategy to stabilize such a class of processes, and it is based on the reconstruction of the dynamics for the delay-free output. In this form, the scheme provides an estimate of the advanced dynamics of the process output, which is used, within a stabilizing feedback control, to compensate the adverse effects of the time delay. The numerical simulations show the necessity of minimal order compensators to stabilize certain unstable processes. That is, if the time delay is significant relative to the process instability characteristic time, a low-order controller can be unable to stabilize process output.

Appendix. Expressions for Matrices A_2, B_2, C_2, and D_2

Each factor $(1 - \alpha_p s)/(1 + \alpha_p s)$ for the approximation of $\exp(-\theta s)$ introduces an additional state in the state-space realization of the pth-ordert approximation for the delayed process. If $(1 - \alpha_p s)/(1 + \alpha_p s)$:$Q_{in}(s) \rightarrow Q_{out}(s)$, where $Q_{in}(s)$ and $q_{out}(s)$ are arbitrary input and output signals, a minimal state-space realization is given by

$$\dot{z} = \alpha_p^{-1}(2q_{in} - z)$$

$$q_{out} = z - q_{in}$$

where z is the single internal state. Using this representation, it can be shown that the approximate delay operator dynamics $(1 - \alpha_p s)/(1 + \alpha_p s)^p$:$Y_{df}(s) \rightarrow Y(s)$ can be described as follows:

$$\dot{x}_2 = A_2 x_2 + B_2 y_{df}$$

$$y = C_2 x_2 + D_2 y_{df}$$

where $x_2 \in \mathbb{R}^p$, and the matrices A_2, B_2, C_2, and D_2 are given in

$$A_{2(p\times p)} = (1/\alpha_p)\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -2 & 2 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 2 & -2 & 2 & -1 & 0 & \cdots & 0 & 0 \\ -2 & 2 & -2 & 2 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & -1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 2 & -1 \end{bmatrix}$$

$$B_{2(p\times 1)} = (1/\alpha_p)\,[-1 \;\; 1 \;\; -1 \;\; 1 \;\; \cdots \;\; \cdots]^{\mathrm{T}}$$

$$C_{1(1\times p)} = 2\,[\cdots \;\; \cdots \;\; \cdots \;\; \cdots \;\; -1 \;\; 1 \;\; -1 \;\; 1]$$

$$D_{2(1\times 1)} = [(-1)^p]$$

Notice the cascade structure of the matrix $A_{2(p\times p)}$, which reflects clearly the passive transmission of information in the approximate delay operator.

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