The definition and use of mixing zones

Mixing zones must be defined carefully in order to establish and achieve realistic water quality standards

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When a liquid is discharged into a body of water, a mixing zone is created. This is a region where dispersion occurs in all directions until the constituents in the discharge have achieved uniform concentrations in the receiving system. Attention has been paid to the mixing zones of thermal discharges; however, mixing is equally important for effluent streams that contain a variety of chemicals. The latter subject is addressed in this article.

The Clean Water Act established the use of criteria and standards as a method for achieving sufficiently low concentrations of pollutants in the nation's waterways (*1*). When adopted by the states, the standards are enforceable concentrations that apply to the indicated chemical species. Once a water quality standard (WQS) is in place, it becomes necessary to establish concentrations (or effluent limitations) in the discharge stream that will prevent violations of the WQS. To help in this effort, the National Academy of Sciences recognized the need for a mixing zone.

Paraphrasing the National Academy's definition, a mixing zone is a region where the concentrations of constituents in the discharge that are different from those of the receiving water are in transition and decreasing steadily from the source to the receiving system. In this region, acceptable water quality characteristics are established by means of determining how long the organisms will be exposed to the water and their responses during this time. The boundary of the mixing zone is the point beyond which the response of the organisms to the water no longer depends upon the length of time they are exposed. Outside of that boundary, the characteristics of the water as determined by studies of long-term exposure will not be harmful to aquatic life (*2*).

In 1971, there were only 22 states that even mentioned mixing zones in their regulations (*3*). This lack of recognition of mixing zones has often been cited as one reason that the water quality approach to protecting the nation's waters from point source pollution was not working. Because many state regulators did not know where to apply the standards, they were ignored. At present, all states except three mention mixing zones in their regulations (*4*). With this increased attention, it now becomes necessary to characterize the mixing zone in order to estimate the load in the discharge that will not violate the WQS.

Fetterolf discussed several concepts of the mixing zone and concluded with the one he considered most important (*3*): "There is no substitute for the case-by-case approach. Each mixing zone should be tailored to the physical, chemical, and biological characteristics of the ecosystem and its particular community of organisms." No one disagrees with this position; however, it needs to be emphasized that a case-by-case analysis does not consist of defining the zone by state policy. For example, a number of states indicate by policy that the zone is an area where the width should not exceed one-fourth of the total width of the stream, or the flow is based on one-fourth of the low flow for seven consecutive days in any 10-year period, or the mixing zone length is determined by empirical rules such as

$$x \le 500V^{1/3} \qquad (1)$$

where x = length in feet
V = discharge in million gal/day.

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Such policy statements are inappropriate for defining a mixing zone.

The mixing zone should be established by a combination of physical, biological, and political considerations that are weighed and together used to define "the allocated impact zone" (*5*). This article will focus on the physical zone and will conclude with a numerical example of how the effluent concentration may be estimated for a discharge to a typical river.

Physical mixing zone

In many ways, describing the size and shape of the mixing zone is the easiest part of determining the final allocated impact zone because it is the least subjective part of the analysis. When an effluent discharge mixes with a receiving stream, the discharge begins to disperse immediately, and eventually this dispersion causes the concentrations of the constituents in the effluent to become uniformly distributed across the river. Such dispersions from a point source theoretically follow a Gaussian distribution and have been described in a variety of books. For example, by neglecting dispersion in the vertical direction, the two-dimensional equation representing the concentration (C) downstream from a discharge of $\dot{M}$ (mass/unit time) is given by (*6*)

$$C(x,y) = \frac{\dot{M}}{du(4\pi D_y x/u)^{1/2}} \times \exp\left(-\frac{y^2 u}{4D_y x}\right) \quad (2)$$

where $\dot{M}$ = mass discharged/unit time
u = velocity
D_y = dispersion coefficient in lateral direction
x = distance downstream
y = distance in lateral direction
d = depth.

Figure 1 illustrates the shape of the curve generated by Equation 2. It should be noted that Equation 2 neglects dispersion in the direction of flow. This is possible only if time $\gg 2\, D_x/u^2$, where D_x is the dispersion coefficient in the longitudinal direction. In most problems, the time required to meet this condition is very short (*7*).

By analogy with the normal distribution curve,

$$C = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right). \quad (3)$$

The standard deviation (σ) represents the spread of the distribution when 4 σ contains 95% of the sample. Hence, using Equations 2 and 3 and assuming that spreading occurs in the lateral direction without limit, 95% of the concentration will be confined to a width represented by 4σ where

$$\sigma = (2D_y x/u)^{1/2}. \quad (4)$$

In a river or stream, the banks represent a boundary condition that prevents infinite spread. The distance downstream at which the concentration of constituents becomes uniform is then given by classical image theory (*8*). Figure 2 illustrates the case where the discharge occurs in the middle of

FIGURE 1
The curve generated by Equation 2 showing the dispersion of chemical concentrations as a discharge mixes with a river

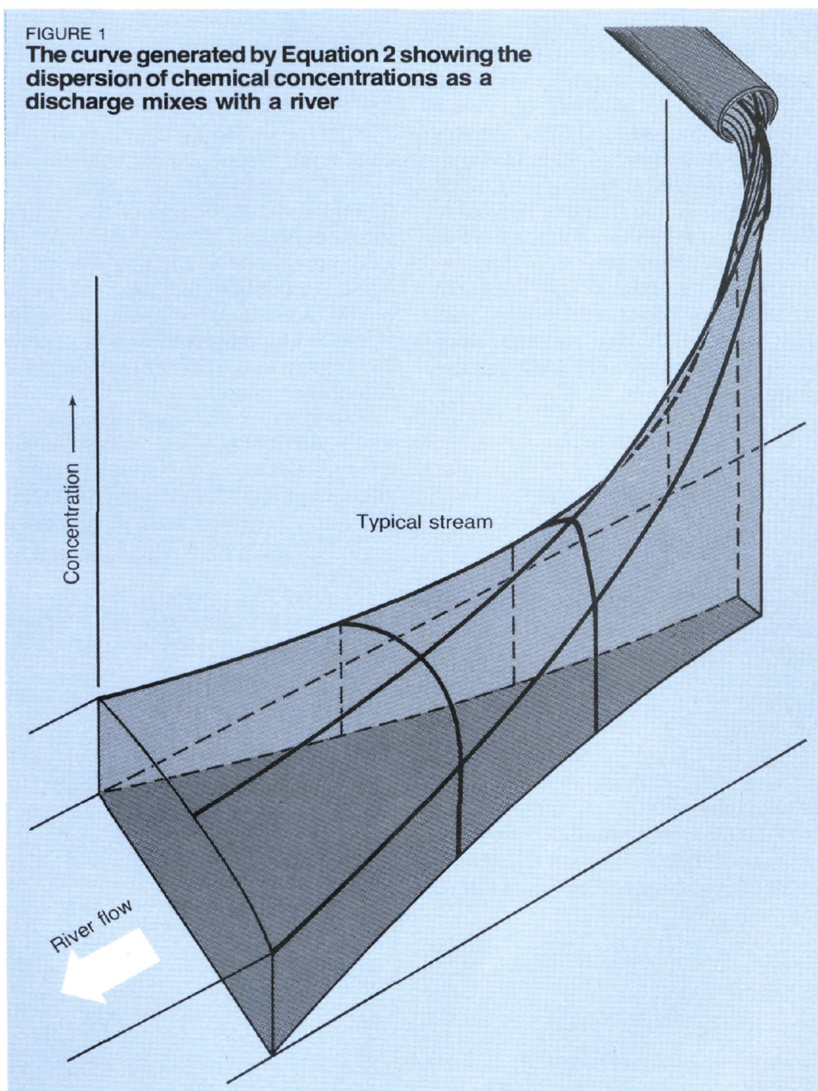
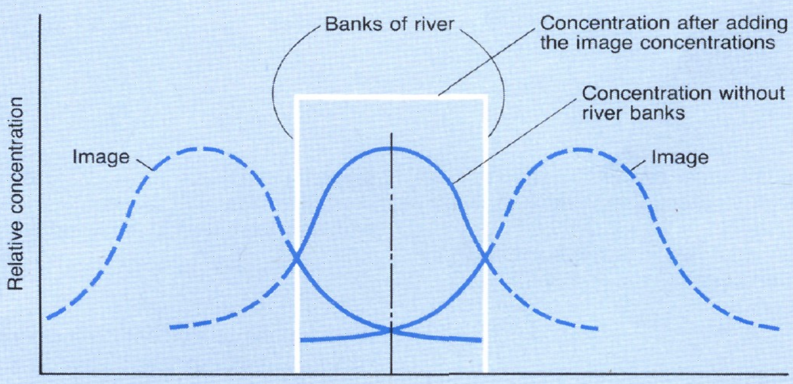


FIGURE 2

Determining the concentration of a chemical in a stream



the channel. The Gaussian distribution of concentrations in the figure is drawn so that the respective banks of the stream are one standard deviation from the mean. Since the method of images does apply, two identical distributions can be superimposed on each side of the original Gaussian distribution. By adding the coordinates of each curve, a curve is generated that illustrates the relative distribution of concentrations. The shape of this curve indicates that when the boundaries are one standard deviation from the center, an almost completely uniform distribution of concentrations exists.

Fischer et al. derived two equations for estimating the length (MZ) of the mixing zone (*9*):

$$MZ = \frac{0.1W^2u}{D_y} \qquad (4)$$

or

$$MZ = \frac{0.4W^2u}{D_y} \qquad (5)$$

where W = the width of the river. Equation 4 applies when the discharge enters the mainstream of the river at the center, and Equation 5 is used when the discharge enters from the side.

Thus, by knowing the lateral dispersion coefficient, width, and velocity, it is possible to estimate the distance downstream that is required for uniform concentration. The parameter D_y may be determined by the relationship (*10*)

$$D_y = 0.6du_* \qquad (6)$$

where d = depth
u_* = shear velocity.

The shear velocity is normally about 10% of the velocity. But shear velocity may also be calculated using (*11*)

$$u_* = (gdS)^{1/2} \qquad (7)$$

where g = acceleration due to gravity
S = slope of the channel
d = depth.

Biological and political criteria

After examining interactions between the physical zone and the biological characteristics, a decision can be made about the size of the final allocated impact zone. The basis for making this decision will not be discussed in this article. However, the size of the zone allocated to the discharger is important in establishing the allowed concentrations of chemicals in the effluent discharge.

Probably, the political decisions are most difficult when multiple discharges occur on the same stretch of river. Consider the example illustrated in Figure 3, in which three discharges are contributing to the impact on the river. Several problems are raised: How many discharges can be allowed on a stretch of river? How close together should they be? A more difficult problem arises when an allocated impact zone has been assigned and then another request is received for a permit to discharge a similar chemical. If the new request is approved, the standard will be violated. In such a situation, it is necessary to determine the total daily load of pollutant that should be allowed and then assign loadings to the dischargers requesting permits.

Determining effluent limitations

In the following example, it is assumed that the physical mixing zone is compatible with the biological needs of the receiving water. Hence, the physical zone becomes the allocated impact zone for the proposed discharge. The data necessary for estimating an effluent limitation are shown in the table.

Because the WQS must be met at the edge of the mixing zone, it is only in this zone that environmental fate considerations are important. With this in mind, the investigation of mass balance relationships will be confined to the mixing zone. Consider a small section of the mixing zone as shown in Figure 4. The mass balance equation for this segment is given by

$$Q\frac{dC}{dx}dx = -kACdx \qquad (8)$$

where Q = volumetric flow rate
C = concentration
x = distance downstream
A = cross-sectional area
k = first-order degradation constant.

FIGURE 3

Multiple discharges on a stretch of river create the potential problem of more than one mixing zone

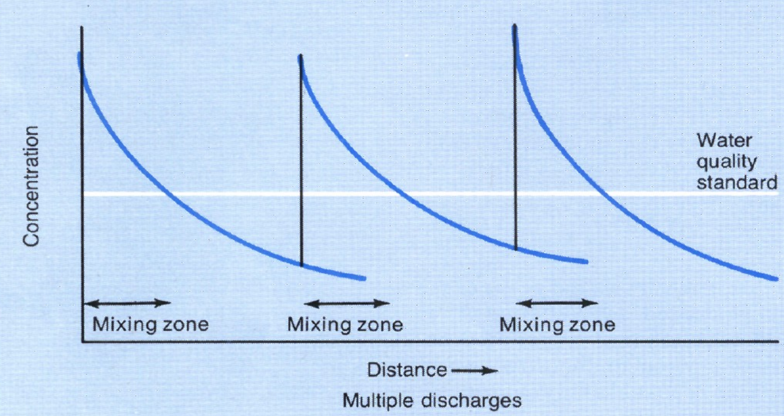


FIGURE 4
The reactions that occur in a small segment of the mixing zone

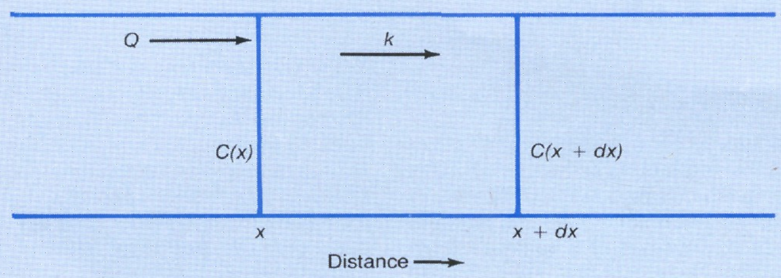


TABLE
Parameters necessary for estimating an effluent limitation

Hydrodynamic parameters

	Parameter	Value
Q	Volumetric flow	9 m^3/s
u	Velocity	0.25 m/s
d	Depth	1.0 m
W	Width	36 m
A	Cross-sectional area	36 m^2
S	Slope	12×10^{-5}
u_*	Shear velocity (Eq. 7)	3.4×10^{-2} m/s
D_y	Dispersion coefficient (Eq. 6)	2.04×10^{-2} m^2/s
MZ	For centerline delivery (Eq. 4)	1588 m
DV	Discharge volume	1 m^3/s

Chemical parameters

Parameter	Value
Dissipation[a]	5.5×10^{-5}/s
Water quality standard	1 μg/L or 1 mg/m^3

[a] Dissipation is a sum of all the first-order rate constants describing degradation, that is, biodegradation, volatilization, photodegradation, etc.

This equation may be integrated to yield

$$C = C_o \exp\left(-\frac{kA}{Q}x\right) \quad (9)$$

where C_o = initial concentration. Equation 9 may be rearranged to give

$$C_oQ = \dot{M} = CQ \exp\left(-\frac{kA}{Q}x\right). \quad (10)$$

Thus, the problem reduces itself to estimating the rate at which a chemical can be added to the discharge stream to prevent violations of the WQS at the end of the mixing zone. From the table the length of the mixing zone (MZ or x) is 1588 m. The area of the mixing zone approximates a triangle. Consequently, the average cross-sectional area is half the width of the river times the depth. According to this calculation, the A in Equation 10 is 18 m^2. Inserting the proper values into Equation 10, M is calculated to be 0.01072 g/s or 926 g/day (2.04 lb/day). For a conservative chemical where k is zero, $\dot{M}$ has a value of 776 g/day.

Thus, if the discharge stream has a flow of 1 m^3/s, then the effluent concentration is 10.7 mg/m^3 or 10.7 ppb. In other words, a concentration of 10.7 ppb in the discharge stream results in a concentration of 1 ppb at the end of a zone 1588 m long. Furthermore, the concentration continues to drop in an exponential manner according to Equation 9. On the other hand, assuming that no further water enters the stream to dilute it, a conservative chemical (for which the dissipating reaction rates are zero) will achieve a uniform concentration after mixing that will remain constant.

Summary

The allocated impact zone is a region where the water quality is at variance with the standard. To characterize this zone, it is necessary to describe the physical mixing zone from the river dynamics. Once the geometry of the region is known, it becomes possible to estimate the limitations that must be placed on the discharge to achieve the WQS at the boundary of the designated zone.

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