Novel Continuous Time MILP Formulation for Multipurpose Batch Plants. 2. Integrated Planning and Scheduling

X. X. Zhu* and T. Majozi

Department of Process Integration, University of Manchester Institute of Science and Technology, P.O. Box 88, Manchester M60 1QD, United Kingdom

While the first part of this series focuses on the application of the proposed formulation to scheduling, this paper focuses mainly on the integration of planning and scheduling in multipurpose batch plants. In dealing with this problem, the method presented in this paper exploits the mathematical structure of the overall plant model. It is discovered that the overall model exhibits a block angular structure that is decomposed by raw material allocation. If raw materials can be allocated optimally to individual plants, solving individual models for each plant can produce the same results as solving an overall model for the site. This discovery leads to a decomposition strategy that consists of two levels. In the first level, only planning decisions are made, and the objective function is the maximization of the overall profit. The results from solving the planning model give optimal raw material allocation to different plants. In the second level, the raw material targets from the first (planning) level are incorporated into the scheduling submodels for each plant, which are solved independently without compromising global optimality. The objective function for each scheduling submodel is the maximization of product throughput. The scheduling level uses the concept of the state sequence network presented in part 1. Solving scheduling submodels for individual plants rather than the overall site model leads to problems with much a smaller number of binary variables and, hence, shorter CPU times. If conflicts arise, i.e., the planning targets are too optimistic to be realized at the scheduling level, the planning model is revisited with more realistic targets. This eventually becomes an iterative procedure that terminates once the planning and scheduling solutions converge within a specified tolerance. In this way, the planning model acts as coordination for scheduling models for individual plants. An industrial case study with three chemical processes is presented to demonstrate the effectiveness of this approach.

Introduction

Conflicts usually exist between planning and scheduling targets. Most chemical sites with distinct planning and scheduling departments experience this problem on a daily basis. This occurs due to the inconsistency between planning and scheduling in terms of time horizons and the focus of decision making. While scheduling focuses on short-term operational issues, planning is aimed at long-term economic issues and tends to overlook operational aspects and disturbances occurring on a daily basis. However, these aspects affect the selection and allocation of raw materials to individual plants. Whenever these disturbances occur, the predetermined schedules may not be valid anymore, and the production throughput may be affected. This causes the incoherence between planning and scheduling. Therefore, there is a need to develop methods that can reconcile planning and scheduling targets to guarantee the coherence between them.

Most of the work done on batch processes has focused on developing better models for scheduling alone without involving planning. A significant amount of work has also been dedicated to developing planning models for chemical processes, with more emphasis on multiperiod models.[3,4,7−10] Until recently, very limited attention has been paid on integrating planning and scheduling decisions. Furthermore, multiplant models have received very limited attention whereas they are frequently encountered in practice.

In addressing the problem of integrated planning and scheduling, Subrahmanyam et al.[11] and Bassett et al.[1] attempted to tackle the problem of applying distributed computing to batch plant design and scheduling. Their approach is based on the decomposition of the problem into design and scheduling levels, yielding a Design Super Problem (DSP) and Scheduling Sub Problem (SSP), respectively. The DSP entails aggregation of plant life span time horizon into nonuniform time periods. The boundaries of the time periods define aggregate production deadlines. The SSP addresses scheduling within short-term time horizons, i.e., time periods from the DSP, to cater for the day-to-day plant operations. Uniform time discretization is applied within each short-term time horizon. The scenario concept is adopted to yield an MILP formulation for DSP problem. The scenario is a collection of predicted demand levels associated with their probabilities. Once the schedules are optimized within a given time period, they are then spliced/linked together to yield an overall solution to the original problem. It was acknowledged, however, that the complexity behind solving problems of this nature has prevented the application of this methodology to large-scale problems. The problem formulation results in a colossal model that cannot be solved easily by the existing computational techniques, even for very simple problems. A similar integration strategy that also includes large-scale model predictive control has been

* Corresponding author e-mail: F.Zhu@umist.ac.uk.

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presented by ref 5. The fact that these methodologies also include uniform time discretization renders them suboptimal due to the restriction imposed on time distribution over the time horizon. This problem is addressed in detail in part 1 of this series.[6]

This paper proposes a novel procedure for the integration of planning and scheduling in multiplant operations. This procedure entails decomposition of the overall planning and scheduling problem into two levels. This is achieved by exploiting the inherent block angular structure of the integrated planning and scheduling models. The resulting overall model is a nonconvex MINLP that is linearized using Glover transformation.[2] The plants considered in the multiplant model do not necessarily have to be confined within the same site. Also, this procedure can handle both multiproduct and multipurpose operations. Scheduling is based on the state sequence network (SSN) representation and the continuous time formulation presented in detail in part 1 of this series.[6]

Problem Statement and Discussions

An integrated planning and scheduling problem can be stated as follows. Given (i) the cost of raw materials and product selling price; (ii) the effluent and waste disposal costs; (iii) the stoichiometric relations between raw materials and products; (iv) the production recipe for each product, including mean processing times in each unit operation; (v) the available units and their capacities; (vi) the maximum storage capacity for each material; (vii) the time horizon of interest, and (viii) product demand over the planning time horizon, we can determine (i) the optimal allocation of raw materials to be used; (ii) the products to be produced and their quantities in order to maximize profit; (iii) the optimal schedule for tasks within the time horizon of interest; (iv) the amount of material processed at any particular point in time within the scheduling time horizon; and (v) the amount delivered to customers over the scheduling time horizon.

The problem addressed here has several features that reflect the reality for many batch plants. These are discussed as follows:

(i) Raw materials are available at a standard cost up to a specific amount beyond which penalties are imposed. This is usually the case where a new supplier has to be approached in order to meet an unexpectedly high demand.

(ii) Raw materials can be shared by different plants.

(iii) Byproducts have to be treated prior to their disposal, thereby incurring treatment costs. There is also a penalty attached to producing more than a specific amount of byproduct. This is commonly encountered in industries that produce highly toxic material. The penalty is aimed at discouraging excess production of toxic material.

(iv) Each product is specific to a particular process. This is very common in high value products as a means of ensuring product integrity. However, within the same process, different batches can follow different routes, which is characteristic in multipurpose plants.

In the context of this paper, planning targets involve the choice of products to be manufactured and their quantities and allocation of raw materials over a given planning time horizon in order to meet market demands. Therefore, these targets are concerned with the economic issues. Scheduling targets are concerned with when and how to produce what products over a given scheduling time horizon, given plant constraints, to meet planning targets. The scheduling targets are therefore concerned with plant feasibility constraints. It is worth mentioning that, while scheduling focuses on short-term demands, planning targets are projected over a much longer range. Therefore, the time horizon of interest in planning is much longer than that considered in scheduling. In this work, the scheduling time horizon was chosen to be a factor of the planning time horizon in order to ensure that the last batch is complete by the end of the planning time horizon. Unlike in multiperiod models where the demand of the product fluctuates over the planning horizon, in this work the product demand is assumed constant throughout the planning horizon.

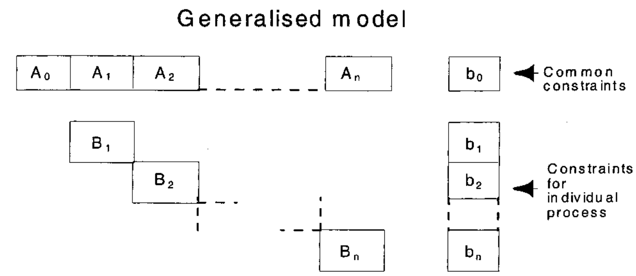

Figure 1. Generalized block angular structure model.

Obviously, an overall model involving planning and scheduling may lead to large size problems, which might be computationally intensive. To overcome this problem, the solution procedure presented in this paper involves the decomposition of the overall problem into subproblems. This is achieved by exploiting the block angular structure of the overall model by applying the idea of structured programming.[12] A model that exhibits the block angular structure with *n* plants is shown in Figure 1.

The *b* column consisting of constants forms the right-hand side. The A blocks represent common rows relating to common constraints. In multiplant models, these are concerned with the allocation of resources including raw materials, utilities, and labor. The B blocks represent the equations corresponding to each individual process.

It is worth expatiating on how the block angular structure for the integrated planning and scheduling model was discovered in this paper. First, the economic issues, e.g., maximizing profit, are considered at the highest (corporate management) level of the decision-making hierarchy. As a result, these issues can be classified as site-wide or even company-wide issues. On the other hand, plant feasibility issues, e.g., production scheduling, are considered at the production (middle management) level of the decision-making hierarchy, and they tend to be plant-specific, i.e., each plant produces its production schedule in order to meet targets set at the corporate management level. In this paper, the site-wide and plant-wide levels of the decision-making hierarchy are referred to as the planning and production scheduling levels, respectively. Following this kind of analysis, it is evident that the overall site model that involves both planning and scheduling decisions will exhibit the same structure shown in Figure 1. In this case, the A blocks represent the planning model, and the B blocks represent the scheduling models for each plant. The proposed solution

Table 1. Processing Time

	plant a		plant b	
	product 1 (h/kg)	product 2 (h/kg)	product 1 (h/kg)	product 2 (h/kg)
reaction	4	3	6	4
purification	1.5	7	1	3

procedure involves the decomposition of the overall site model into two levels as described below.

The first level of the proposed decomposition is aimed at determining the most optimal allocation of raw materials to different plants, i.e., planning targets. Therefore, the model for this level only consists of the A blocks. At this level, only planning decisions are considered. However, the capacity limits of the plants are taken into consideration. The second level is aimed at determining the optimal schedule for each plant in order to meet the targets set at the first level. This level consists of the B blocks, and each B block forms an individual submodel. The basis of this decomposition is illustrated by the following motivating example.

Structured Programming for Decomposition—Motivating Example

Consider two plants A and B within one site producing two types of products, product 1 and product 2. A kilogram of product 1 and product 2 gives a profit contribution of \$60 and \$75, respectively. Each plant consists of two stages, reaction and purification, for producing its products. Plant A has maximum reaction and purification times of 72 and 80 h/week, respectively, while plant B has maximum reaction and purification times of 85 and 90 h/week, respectively. The reaction and purification times of each product are shown in Table 1.

The longer reaction times in plant B and the longer purification times in plant A are due to deterioration of equipment over time. In addition, each kilogram of each product uses 8 kg of raw material. The company has 332 kg of raw material available per week. It is worth mentioning at this stage that this example is a simplification of a true problem.

The overall site model for this problem can be formulated as follows:

Let $w(\mathrm{s1})$ = the amount of product 1 from plant A per week, $w(\mathrm{s2})$ = the amount of product 2 from plant A per week, $w(\mathrm{s3})$ = the amount of product 1 from plant B per week, and $w(\mathrm{s4})$ = the amount of product 2 from plant B per week:

maximize	profit	$60w(\mathrm{s1}) + 75w(\mathrm{s2}) + 60w(\mathrm{s3}) + 75w(\mathrm{s4})$
subject to	raw	$8w(\mathrm{s1}) + 8w(\mathrm{s2}) + 8w(\mathrm{s3}) + 8w(\mathrm{s4}) \le 332$
	rxn A	$4w(\mathrm{s1}) + 3w(\mathrm{s2}) \le 72$
	purifn A	$1.5w(\mathrm{s1}) + 7w(\mathrm{s2}) \le 80$
	rxn B	$6w(\mathrm{s3}) + 4w(\mathrm{s2}) \le 85$
	purifn B	$6w(\mathrm{s3}) + 3w(\mathrm{s2}) \le 90$
		$w(\mathrm{s1}), w(\mathrm{s2}), w(\mathrm{s3}), w(\mathrm{s4}) \ge 0$

Solving this linear programming model gives an overall profit of \$2944.09/week. This is achieved by producing 11.227 kg of product 1 and 9.023 kg of product 2 in plant A and 21.25 kg of product 2 only in plant B. Therefore, the weekly raw material distribution requirement corresponding to maximum profit is 162 and 170 kg for plants A and B, respectively.

What happens if the optimal raw material distribution between the two plants is known a priori? In this case, we can build two individual models for plants A and B using the optimal split of the raw materials.

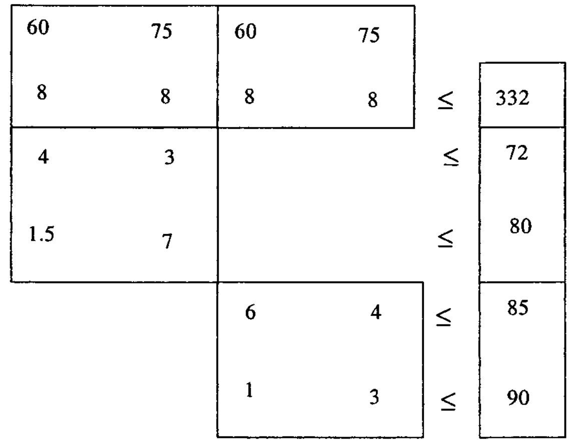

Figure 2. Structure of the model in the example.

Plant A Model

maximize	profit	$60w(\mathrm{s1}) + 75w(\mathrm{s2})$
subject to	raw	$8w(\mathrm{s1}) + 8w(\mathrm{s2}) \le 162$
	rxn A	$4w(\mathrm{s1}) + 3w(\mathrm{s2}) \le 72$
	purifn A	$1.5w(\mathrm{s1}) + 7w(\mathrm{s2}) \le 80$
		$w(\mathrm{s1}), w(\mathrm{s2}) \ge 0$

Plant B Model

maximize	profit	$60w(\mathrm{s3}) + 75w(\mathrm{s4})$
subject to	raw	$8w(\mathrm{s3}) + 8w(\mathrm{s4}) \le 170$
	rxn B	$6w(\mathrm{s3}) + 4w(\mathrm{s2}) \le 85$
	purifn B	$w(\mathrm{s3}) + 3w(\mathrm{s2}) \le 90$
		$w(\mathrm{s3}), w(\mathrm{s4}) \ge 0$

Solving the plant A model gives a profit of \$1350.34/week, while solving plant B model gives a profit of \$1593.75/week. Summing the profits from the individual models gives an overall profit of \$2944.09/, which is equal to the profit obtained by solving the overall model.

Why does solving the submodels based on the optimal allocation of raw materials give the same solution as that of solving the overall model? The reason is that the overall model displays the block angular structure (Figure 2). Therefore, according to Williams,[12] this structure can be decomposed into B block-based submodels under condition of optimal allocation of raw materials, since these models are linked only by sharing raw materials.

It is first realized by the present authors that the overall model including planning and scheduling for a site consisting of several plants and producing several products also yields a block angular structure as shown in Figure 1. In this proposed work, the common constraints together with the objective function form the planning model that determines the optimal allocation of raw materials. Each B block forms a scheduling model for each plant. The targets for raw material allocation predicted by the planning model are then incorporated into each scheduling model. In this way, there is no need to build the entire model for a total site, which could result in a prohibitively large model.

It is worthy of mention, however, that the decomposition procedure requires the satisfaction of two main conditions. First, all the constraints involved at the planning level (A block) and the scheduling level (B blocks) should be convex. Therefore, the procedure takes advantage of convexity, rather than linearity, of the planning model (A block) and scheduling submodels (B blocks), and is not applicable if either any of the linking constraints that form the planning model or at least one of the constraints forming the scheduling submodels is nonconvex. This condition ensures that the global solution for the overall model is also the solution for the decomposed model. A proof of this statement appears in Appendix. The second condition is that the linking constraints should entail planning decisions, e.g., raw material and labor allocation, while the scheduling models should remain plant-specific. This condition ensures that the overall model exhibits the block angular structure. Nevertheless, there will be situations in practice where this will not hold. For an example, if scheduling is not plant-specific but involves constraints linking different plants. This is true if the manufacture of one product involves units from different plants. However, this is a very rare case in industry because of product integrity issues. Following is the formulation based on this decomposition.

Mathematical Model

Sets.

$J = \{j|j$ is a unit$\}$

$P = \{p|p$ is a time point$\}$

$K = \{k|k$ is a process$\}$

J_k = a unit in process k

$S_{\text{in},j} = \{s_{\text{in},j}|s_{\text{in},j}$ is an input state to unit $j\}$

$S_{\text{out},j} = \{s_{\text{out},j}|s_{\text{out},j}$ is an output state from unit $j\}$

$S = \{s|s$ is any state$\} = S_{\text{in},j} \cup S_{\text{out},j}$

$S^*_{\text{in},j} = \{s^*_{\text{in},j}|s^*_{\text{in},j}$ is an effective state$\} \subseteq S_{\text{in},j}$

$S^r_{\text{in}} = \{s^r_{\text{in}}|s^r_{\text{in}}$ is a raw material$\} \subseteq S$

$S^r_{\text{in},k} = \{s^r_{\text{in},k}|s^r_{\text{in},k}$ is raw material into process $k\} \subseteq S$

$S^d_{\text{out}} = \{s^d_{\text{out}}|s^d_{\text{out}}$ is a product$\} \subseteq S$

$S^d_{\text{out},k} = \{s^d_{\text{out},k}|s^d_{\text{out},k}$ is product from process $k\} \subseteq S$

$S^b_{\text{out}} = \{s^b_{\text{out}}|s^b_{\text{out}}$ is a byproduct$\} \subseteq S$

$S^b_{\text{out},k} = \{s^b_{\text{out},k}|s^b_{\text{out},k}$ is byproduct from process $k\} \subseteq S$

Variables.

$t_p(s,p)$ = time at which state s is produced at time point p, $s \in S_{\text{out},j}$

$t_u(s,p)$ = time at which state s is used at time point p, $s \in S_{\text{in},j}$

$q_s(s,p)$ = amount of state s stored at time point p

$m_p(s,p)$ = amount of state s produced at time point p, $s \in S_{\text{out},j}$

$m_u(s,p)$ = amount of state s used at time point p, $s \in S_{\text{in},j}$

$y(s,p)$ = binary variable associated with usage of state s at time point p, $s \in S^*_{\text{in},j}$

$t_a(s)$ = actual processing time for state s, $s \in S_{\text{in},j}$

$d(s,p)$ = amount of state s delivered to customers at time point p, $s \in S_{\text{out},j}$

$\rho^+(s)$, $\rho^-(s)$ = slack variables for variation in processing time due to added degrees of freedom, $s \in S_{\text{in},j}$

$W(s^r_{\text{in}})$ = overall amount of raw material s^r_{in} used over the planning time horizon

$w(s^r_{\text{in},k})$ = amount of raw material s^r_{in} used in process k over the planning time horizon

$W'(s^r_{\text{in}})$ = additional amount of raw material s^r_{in} available at a penalty over the planning time horizon

$W(s^b_{\text{out}})$ = overall amount of byproduct s^b_{out} produced over the planning time horizon

$w(s^b_{\text{out},k})$ = amount of byproduct s^b_{out} produced in process k over the planning time horizon

$W'(s^b_{\text{out}})$ = additional amount of byproduct s^b_{out} treated at a penalized cost over the planning time horizon

$w(s^d_{\text{out},k})$ = amount of product s^d_{out} produced in process k over the planning time horizon

Parameters.

V_j = capacity of a particular unit j

H = time horizon of interest

$\tau(s)$ = mean processing time for state s, $s \in S_{\text{in},j}$

$Q^0_s(s)$ = initial amount of state s stored

$\upsilon(s)$ = allowed percentage time variation for processing state s, $s \in S_{\text{in},j}$

$w^U(s^d_{\text{out},k})$ = maximum amount of product s^d_{out} produced in process k over the planning time horizon

$C(s^d_{\text{out}})$ = selling price per unit capacity for product s^d_{out}

$C(s^r_{\text{in}})$ = standard price per unit capacity for raw material s^r_{in}

$C(s^b_{\text{out}})$ = standard treatment cost per unit capacity for byproduct s^b_{out}

$C'(s^r_{\text{in}})$ = penalized price per unit capacity for raw material s^r_{in}

$C'(s^b_{\text{out}})$ = penalized treatment cost per unit capacity for byproduct s^b_{out}

$\alpha(s^r_{\text{in},k})$ = stoichiometric coefficient of raw material s^r_{in} when used in plant k

$\beta(s^b_{\text{out},k})$ = stoichiometric coefficient of byproduct s^b_{out} when produced in plant k

$R(s^r_{\text{in}})$ = maximum amount of raw material s^r_{in} available at standard price

$B(s^b_{\text{out}})$ = maximum amount of byproduct s^b_{out} treated at standard cost over the planning time horizon

$R'(s^r_{\text{in}})$ = maximum penalized amount of raw material s^r_{in} over planning time horizon

$B'(s^b_{\text{out}})$ = maximum penalized amount of byproduct s^b_{out} over the planning time horizon

$Dem\ (s^d_{\text{out}})$ = demand of product s^d_{out} over the planning time horizon

Planning Model. (a) Raw Material Availability Constraints.

$$W(s^r_{\text{in}}) - W'(s^r_{\text{in}}) \le R(s^r_{\text{in}}),\ \forall\ s^r_{\text{in}} \in S^r_{\text{in}} \tag{1}$$

$$\sum_k w(s^r_{\text{in},k}) = W(s^r_{\text{in}}),\ \forall\ s^r_{\text{in}} \in S^r_{\text{in}} \tag{2}$$

$$W'(s^r_{\text{in}}) \le R'(s^r_{\text{in}}),\ \forall\ s^r_{\text{in}} \in S^r_{\text{in}} \tag{3}$$

$$W'(S^r_{\text{in}}) \ge 0,\ W(S^r_{\text{in}}) \ge 0,\ w(S^r_{\text{in},k}) \ge 0 \tag{4}$$

Constraint 1 states that the amount of raw material (s^r_{in}) available at standard price is the difference between the overall amount used and the amount available at penalized cost. Constraint 2 denotes that the overall amount of raw material (s^r_{in}) used is the sum of all the amounts of raw material (s^r_{in}) used in different processes k. Constraint 3 sets an upper bound on the amount available at penalized cost. In this paper, the upper bound on the amount of penalized raw materials is arbitrarily chosen as a very large number.

(b) Byproduct Constraints.

$$W(s^b_{\mathrm{out}}) - W'(s^b_{\mathrm{out}}) \le B(s^b_{\mathrm{out}}),\ \forall\ s^b_{\mathrm{out}} \in s^b_{\mathrm{out}} \quad (5)$$

$$\sum_k w(s^b_{\mathrm{out},k}) = W(s^b_{\mathrm{out}}),\ \forall\ s^b_{\mathrm{out}} \in S^b_{\mathrm{out}} \quad (6)$$

$$W'(s^b_{\mathrm{out}}) \le B'(s^b_{\mathrm{out}}),\ \forall\ s^b_{\mathrm{out}} \in S^b_{\mathrm{out}} \quad (7)$$

$$W(s^b_{\mathrm{out}}) \ge 0,\ W'(s^b_{\mathrm{out}}) \ge 0,\ w(s^b_{\mathrm{out},k}) \ge 0 \quad (8)$$

The explanation for these constraints is similar to that for constraints 1−4.

(c) Product Demand Constraint.

$$\sum_{k\epsilon k} w(s^d_{\mathrm{out},k}) \ge \mathrm{Dem}(s^d_{\mathrm{out}}),\ \forall\ S^d_{\mathrm{out}} \in S^d_{\mathrm{out}} \quad (8a)$$

This constraint ensures that demand is satisfied over the planning time horizon.

(d) Stoichiometric Constraints.

$$\beta(s^b_{\mathrm{out},k}) = \frac{w(s^b_{\mathrm{out},k})}{w(s^b_{\mathrm{out},k})},\ \forall\ k \in K,\ s^b_{\mathrm{out},k} \in S^b_{\mathrm{out},k},\ s^d_{\mathrm{out},k} \in S^d_{\mathrm{out},k} \quad (9)$$

$$\alpha(s^r_{\mathrm{in},k})(w(s^d_{\mathrm{out},k}) + \sum_{s^b_{\mathrm{out},k}} w(s^b_{\mathrm{out},k})) = w(s^r_{\mathrm{in},k}),$$
$$\forall\ k \in K,\ s^r_{\mathrm{in},k} \in S^r_{\mathrm{in},k},\ s^d_{\mathrm{out},k} \in S^d_{\mathrm{out},k},\ s^b_{\mathrm{out},k} \in S^b_{\mathrm{out},k} \quad (10)$$

$$0 \le w(s^d_{\mathrm{out},k}) \le w^U(s^d_{\mathrm{out},k}),\ \forall\ k \in K,\ s^d_{\mathrm{out},k} \in S^d_{\mathrm{out},k} \quad (11)$$

Constraint 9 is the stoichiometric relationship between the amount of product ($s^d_{\mathrm{out},k}$) produced and the corresponding byproduct ($s^b_{\mathrm{out},k}$) produced from plant k. Constraint 10 is the stoichiometric relationship between the amount of raw material ($s^r_{\mathrm{in},k}$) used in process k and the overall output including both product and related byproducts from process k. Constraint 11 sets an upper bound on the amount of product ($s^d_{\mathrm{out},k}$) that can be produced, given the capacity limits of the plant. This upper bound is obtained by performing scheduling without the raw material limits over the time horizon of interest.

(e) Objective Function. The objective function is designed to maximize profit from sale of the product while accounting for costs of raw materials and byproducts as given below:

$$\max Z = \sum_{s^d_{\mathrm{out}}} C(s^d_{\mathrm{out}})W(s^d_{\mathrm{out}}) - \sum_{s^b_{\mathrm{out}}} C(s^b_{\mathrm{out}})[W(s^b_{\mathrm{out}}) - W'(s^b_{\mathrm{out}})] - \sum_{s^r_{\mathrm{out}}} C(s^r_{\mathrm{out}})[W(s^r_{\mathrm{out}}) - W'(s^r_{\mathrm{out}})] - \sum_{s^b_{\mathrm{out}}} C'(s^b_{\mathrm{out}})W'(s^b_{\mathrm{out}}) - \sum_{s^r_{\mathrm{out}}} C'(s^r_{\mathrm{out}})W'(s^r_{\mathrm{out}}) \quad (12)$$

Scheduling Model. Although the full formulation and detailed description of the scheduling model is given in part 1 of this series, the major part of the scheduling model is provided here to facilitate understanding of the full context of the integrated planning and scheduling model. It is worthy of mention however that the scheduling model presented in this paper is based on the concept of a state sequence network (SSN) that considers state only. Using this SSN representation, units and tasks are eliminated but implicitly involved in the corresponding formulation. This reduces the binary dimension of the overall model as compared to the other formulations based on the state task network (STN).

(a) Capacity Constraints.

$$V_j^{\min} y(s^*_{\mathrm{in},j},p) \le \sum_{s_{\mathrm{in},j}} m_{\mathrm{u}}(s,p) \le V_j^{\max} y(s^*_{\mathrm{in},j},p),\ \forall\ j \in J,\ p \in P \quad (13)$$

This constraint implies that the total amount of all the states consumed ($\sum m_{\mathrm{u}}(s,p)$) at time point p is limited by the capacity of the unit that consumes the states (V_j). The max and min superscripts denote the maximum and the minimum capacities. According to constraint 13, states will be consumed in a particular unit j if the corresponding effective state is used at time point p.

(b) Material Balances.

$$\sum_{s=s_{\mathrm{in},j}} m_{\mathrm{u}}(s,p-1) = \sum_{s=s_{\mathrm{out},j}} m_{\mathrm{p}}(s,p),\ \forall\ p \in P,\ j \in J \quad (14)$$

$$q_{\mathrm{s}}(s,p_1) = Q^0_{\mathrm{s}}(s) - m_{\mathrm{u}}(s,p_1),\ s \ne \text{product},\ p_1 = \text{starting point} \quad (15)$$

$$q_{\mathrm{s}}(s,p) = q_{\mathrm{s}}(s,p-1) - m_{\mathrm{u}}(s,p),\ s = \text{feed},\ \forall\ p \in P,\ p > p_1 \quad (16)$$

$$q_{\mathrm{s}}(s,p) = q_{\mathrm{s}}(s,p-1) + m_{\mathrm{p}}(s,p) - m_{\mathrm{u}}(s,p),\ s \ne \text{product, feed},\ \forall\ p \in P,\ p > p_1 \quad (17)$$

$$q_{\mathrm{s}}(s,p_1) = Q^0_{\mathrm{s}}(s) - \mathrm{d}(s,p_1),\ s = \text{product},\ p_1 = \text{starting point} \quad (18)$$

$$q_{\mathrm{s}}(s,p) = q_{\mathrm{s}}(s,p-1) + m_{\mathrm{p}}(s,p) - \mathrm{d}(s,p),\ s = \text{product, byproduct},\ \forall\ p \in P,\ p > p_1 \quad (19)$$

Constraint 14 is the material balance around a particular unit j. It implies that the sum of the masses for all the input states used at time point $p-1$ should be equal to the sum of the masses for all the output states produced at time point p. Constraint 15 states that the amount of state (s) stored at the first time point is the difference between the amount stored before the beginning of the process and that being utilized at the first time point, i.e., the beginning of the process. Constraint 16 only applies to the feed since it is the state that is only used in the process. Constraint 17 only applies to intermediates since they are both produced and used in the process. Constraints 18 and 19 only apply to products and byproducts since they are the only states that have to be taken out of the process as shown by the terms d(s,p).

(c) Duration Constraints (Exploring More Degrees of Freedom). In this paper, duration constraints are modeled such that they do not depend on the batch size. This is because, in practice, the processing time variation is not due to variable batch sizes but due to added degrees of freedom that are intrinsic in batch processes. Among these are equipment design and performance, variable raw material purity, different catalyst types, and different operator response times; e.g., one operator might take an hour longer to open steam required for a chemical reaction than the others. All these factors directly influence unit operations, which leads to variable processing times. However, application of the proposed model to situations where batch size varies with time is presented in part 1 of this series.[6] In this work, the variable processing times have been included in the formulation by using linearization technique as shown below:

$$t_a(s^*_{in,j},p) = \tau(s^*_{in,j}) + \rho^+(s^*_{in,j},p) + \rho^-(s^*_{in,j},p),\ \forall\, j \in J,\ \forall p \in P \quad (20)$$

$$t_p(s_{out,j},p) = t_u(s^*_{in,j},p-1) + t_a(s^*_{in,j},p-1)y(s^*_{in,j},p-1),\ \forall\, j \in J,\ p \in P,\ p > P_1,\ s_{out,j} \in S_{out,j} \quad (21)$$

Constraint 20 is the definition of the processing time for a particular batch corresponding to a particular effective state in a corresponding unit operation, where ρ^+ and ρ^- are slack variables to account for variation in processing times due to the degrees of freedom. The bounds on the slack variables are determined by variation in processing times. A positive slack variable implies that the actual processing time is longer than the mean processing time. This could be due to deteriorating equipment, using an old catalyst, poor quality of raw materials, or long operator response time. A negative slack variable means that the actual processing time is shorter than the mean processing time. This could be due to using new catalyst, raw materials of high purity, or efficient operator response time. Therefore, in this formulation, the variation in processing time is not associated with batch sizes. Constraint 21 states that the elapsed time between usage of input state (s) at time point $p-1$ and production of output state (s) at time point p in unit j is equal to the processing time of the corresponding batch. This batch can only be processed if a corresponding effective state is used at time point $p-1$. However, eq 21 constitutes bilinear terms, which are linearized using the technique proposed by Glover[2] as follows. The derivation of the following equations is presented in part 1 of this series.[6]

$$t_p(s_{out,j},p) = t_u(s^*_{in,j},p-1) + \tau(s^*_{in,j})y(s^*_{in,j},p-1) + \Gamma^+(s^*_{in,j},p-1) + \Gamma^-(s^*_{in,j},p-1),\ \forall\, p \in P,\ p > P_1,\ j \in J \quad (22)$$

$$\rho^+(s^*_{in,j},p) - \upsilon(s^*_{in,j})\tau(s^*_{in,j})(1 - y(s^*_{in,j},p)) \le \Gamma^+(s^*_{in,j},p) \le \rho^+(s^*_{in,j},p),\ \forall\, j \in J,\ p \in P \quad (23)$$

$$0 \le \Gamma^+(s^*_{in,j},p) \le \upsilon(s^*_{in,j})\tau(s^*_{in,j})y(s^*_{in,j},p),\ \forall\, j \in J,\ p \in P \quad (24)$$

$$\rho^-(s^*_{in,j},p) \le \Gamma^-(s^*_{in,j},p) \le \rho^-(s^*_{in,j},p) + \upsilon(s^*_{in,j})\tau(s^*_{in,j})(1 - y(s^*_{in,j},p)),\ \forall\, j \in J,\ p \in P \quad (25)$$

$$-\upsilon(s^*_{in,j})\tau(s^*_{in,j})y(s^*_{in,j},p) \le \Gamma^-(s^*_{in,j},p) \le 0,\ \forall\, j \in J,\ p \in P \quad (26)$$

Variables Γ^+ and Γ^- are introduced through linearization and represent the bilinear terms that arise from the product of the binary variable (y) and the slack variables (ρ^+ and ρ^-).

(d) Sequence Constraints.

$$t_u(s_{in,j},p) \ge \sum_{s_{in,j}}\sum_{s_{out,j}}\sum_{p' \le p} t_p(s_{out,j},p') - t_u(s_{in,j},p'-1),\ \forall\, j \in J,\ p \in P,\ p > P_1,\ s_{out,j} \in S_{out,j},\ s_{in,j} \in S_{in,j} \quad (27)$$

$$t_u(s_{in,j},p) \ge t_p(s_{out,j},p),\ \forall\, j \in J,\ p \in P,\ S_{out,j} \in S_{out,j},\ s_{in,j} \in S_{in,j} \quad (28)$$

$$t_u(s_{in,j},p) \ge t_p(s_{out,j'},p),\ \forall\, j, j' \in J,\ p \in P,\ s_{out,j'} = s_{in,j} \quad (29)$$

Constraints 27 and 28 imply that state $s_{in,j}$ can only be used in a particular unit, at any time point, after all the previous states have been processed. Constraint 27 is only relevant in situations where more than one task can be conducted in one unit, otherwise it is redundant in the presence of constraints 28 and 29. Constraint 29 stipulates that a state can only be processed at a particular time point p in a particular unit j after it has been produced from another unit j'. In recycling, j is the same as j'. It is worthy of note that constraints 28 and 29 are only applicable to intermediates since they are the only states that are both produced and used.

(e) Time Horizon Constraints.

$$t_u(s_{in,j},p) \le H,\ \forall\, s_{in,j} \in S_{in,j},\ p \in P,\ j \in J \quad (30)$$

$$t_p(s_{out,j},p) \le H,\ \forall\, s_{out,j} \in S_{out,j},\ p \in P,\ j \in J \quad (31)$$

Constraints 30 and 31 respectively stipulate that the usage or production of a state should be within the time horizon of interest.

(f) Storage Constraints.

$$q_s(s,p) \le Q^{max}(s),\ \forall\, s \in S,\ p \in P \quad (32)$$

Constraint 32 states that the amount of state s stored at each time point cannot exceed the maximum allowed.

(g) Objective Function. The objective function for this formulation is the maximization of product throughput:

$$\text{maximize} \sum_s \sum_p \mathrm{d}(s,p),\ s = \text{product},\ p \in P \quad (33)$$

Strategy for Integration of Planning and Scheduling

The connection between the above planning and scheduling models is achieved by incorporating the following constraint in each of the scheduling models:

$$w(s^r_{in,k}) \ge \sum_{j \in J_k}\sum_{p \in P} m_u(s_{in,j},p),\ \forall\, s_{in,j} = s^r_{in,k} = \text{raw material} \quad (34)$$

where $m_u(s_{in,j},p)$ is the amount of raw material ($s_{in,j}$) used at time point p.

The right-hand side of constraint 34 represents the amount of raw material ($s_{in,j}$) required at the scheduling level by plant k. The left-hand side represents the amount of raw material required in plant k as predicted at the planning level. Figure 3 shows the overall procedure for integrated planning and scheduling.

The planning model requires stoichiometric data, cost data, capacity constraint data, and time horizon of interest. The capacity constraint data that provide the

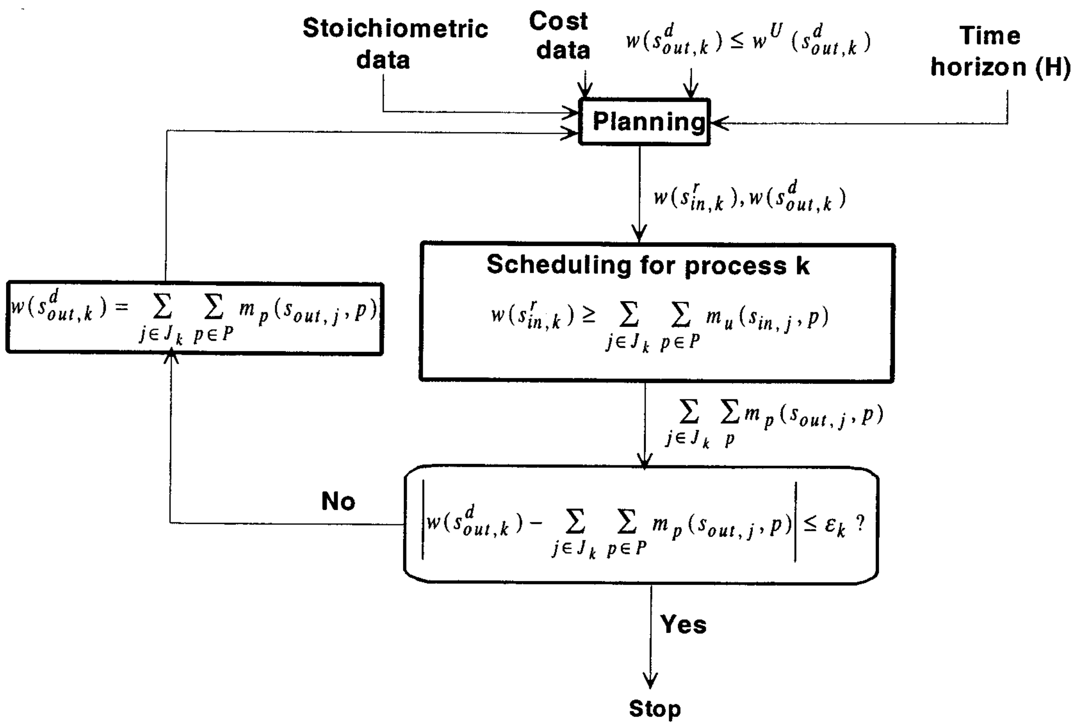

Figure 3. Procedure for integration of planning and scheduling.

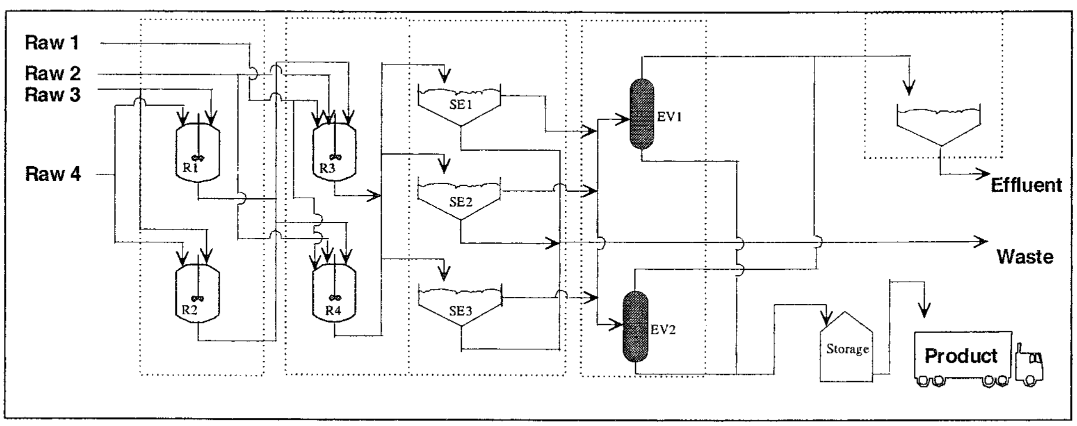

Figure 4. Flow sheet for process 1.

upper bound on production, i.e., $w^U(s^d_{out,k})$, is obtained by performing scheduling without raw material limits over the time horizon of interest. The results from the planning model, i.e., raw material requirements for each plant, are incorporated into individual scheduling models using constraint 34. Scheduling is then performed in each individual plant without violating the planning level targets. If the scheduling targets, i.e., product throughputs corresponding to the raw material inputs from the planning level, do not match the production targets predicted by the planning model, the latter is revisited with more realistic targets. These realistic targets are, in essence, the current targets predicted by the scheduling model. This eventually becomes an iterative procedure that terminates once the planning and scheduling targets are within specified tolerance (ϵ).

Case Study

To illustrate the application of this approach, a case study was conducted on a site consisting of three batch chemical processes. Figure 4 shows the flow sheet for process 1. This process entails five consecutive steps. The first step involves a reaction that forms an arsenate salt. This reaction requires two raw materials, raw 3 and raw 4, and can be conducted in either reactor R1 or R2. The arsenate salt from the first step is then transferred to either reactor R3 or R4 wherein two consecutive reactions take place. The first of these reactions is aimed at converting the arsenate salt to a disodium salt using raw material 1 (raw 1). The disodium salt is then reacted further to form the monosodium salt using raw material 2 (raw 2). The monosodium salt solution is then transferred to the settling step in order to remove the solid byproduct. Settling can be conducted in either of the three settlers, i.e., SE1, SE2, or SE3. The solid byproduct is dispensed with as waste, and the remaining monosodium salt solution is transferred to the final step. This step consists of two evaporators, EV1 and EV2, which remove the excess amount of water from the monosodium solution. Evaporated water is removed as effluent, and the monosodium

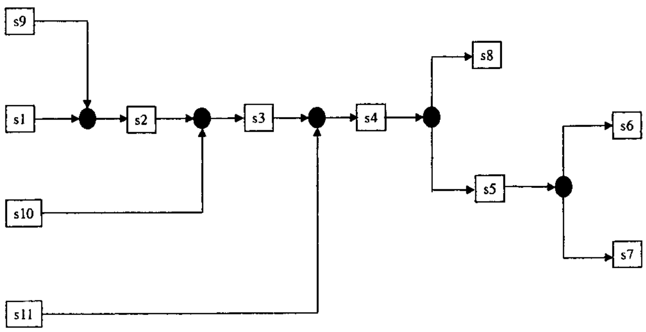

Figure 5. SSN for process 1.

salt (product) is taken to storage. Figure 5 is the SSN for process 1.

States s1 and s9 in the SSN represent raw 3 and raw 4, respectively. States s10 and s2 represent raw 1 and the arsenate salt, while states s11 and s3 represent raw 2 and the disodium salt, respectively. State s4 is the monosodium solution that is transferred to the settlers to form state s8 (solid byproduct) and state s5 (remaining monosodium solution). State s5 is separated into s7 (water) and s6 (product).

Processes 2 and 3 have similar flow sheets as shown in Figure 6. The SSN for these two processes is shown in Figure 7. Process 2 entails 4 consecutive steps. The first step involves an acid decomposition reaction in an organic solvent in reactor R1′. States s2′−s5′ are the raw materials required in step 1. The second step is aimed at removing salt formed as a byproduct in the first step through liquid−liquid extraction in separator SE1′. State s6′ is an intermediate from step 1. Water (s1′) is added to form an inorganic phase, followed by stirring for 30 min. The resulting two-phase mixture (s7′) is then allowed to settle for 1 h within the same vessel (SE1′) with the stirrer switched off. The inorganic phase (s9′) is then removed as effluent, and the organic phase (s8′) is transferred to the last stage (EV1′) for solvent recovery. The solvent (s11′) is recycled back to reactor R1′, and the product (s10′) is taken to storage. Processes 2 and 3 only differ in states s4, which lead to different products from these processes. Table 2 shows the planning data, and Tables 3 and 4 show the scheduling data for processes 1−3, respectively.

The processing time variations of 20%, 30%, and 25% were used for processes 1−3, respectively. Since states s11 for processes 2 and 3 correspond to a recycled solvent, there are no treatment costs. Processes 2 and 3 share raw materials corresponding to states s1, s2, s3, and s5. While state s1 is unlimited, each of the shared raw materials has a maximum availability of 12 000 tons/yr. Each of the processes produces its own product and byproducts that are different from other processes. For clarity, the states used in processes 1−3 are denoted with s, s′, and s″, respectively. It should be emphasized that although processes 2 and 3 have the similar flow sheets, they have distinct plants, i.e., there is no sharing of units. The demands for products s6, s10′, and s10″ are 4000, 6000, and 5000, respectively.

Mathematical Planning Model. It should be noted in all the equations presented in this section that unlimited amounts have been assigned a value 10^6.

(a) Raw Material Availability Constraints. *Process 1.*

$$W(s^r_{in}) - W'(s^r_{in}) \le 6000,\ \forall\ s^r_{in} \in S^r_{in} = \{s1, s9, s10, s11\}$$

The value of 6000 is the maximum available at standard price and taken from Table 2 under process 1 data:

$$W'(s^r_{in}) \le 6000,\ \forall\ s^r_{in} \in S^r_{in} = \{s1, s9, s10, s11\}$$

The value of 6000 given in this equation is the upper bound on the amount of raw material that can be obtained at a penalty. Following are the nonnegativity constraints for the quantities mentioned in the above equations:

$$W'(s^r_{in}) \ge 0,\ W(s^r_{in}) \ge 0$$

Processes 2 and 3.

$$W(s4') - W'(s4') \le 6000$$

$$W(s4'') - W'(s4'') \le 6000$$

$$w(s1') + w(s1'') = W(s1)$$

$$w(s2') + w(s2'') = W(s2)$$

$$w(s3') + w(s3'') = W(s3)$$

$$w(s5') + w(s5'') = W(s5)$$

$$W(s1) - W'(s1) \le 10^6$$

$$W(s^r_{in}) - W'(s^r_{in}) \le 12\,000,\ s^r_{in} = s2, s3, s5$$

$$W'(s^r_{in}) \le 10^6,\ s^r_{in} = s1, s2, s3, s4', s4'', s5$$

$$W'(s^r_{in}) \ge 0,\ W(s^r_{in}) \ge 0,\ s^r_{in} = s1, s2, s3, s4', s4'', s5$$

States s1, s2, s3, and s5 for processes 2 and 3 should not be confused with the corresponding states for process 1.

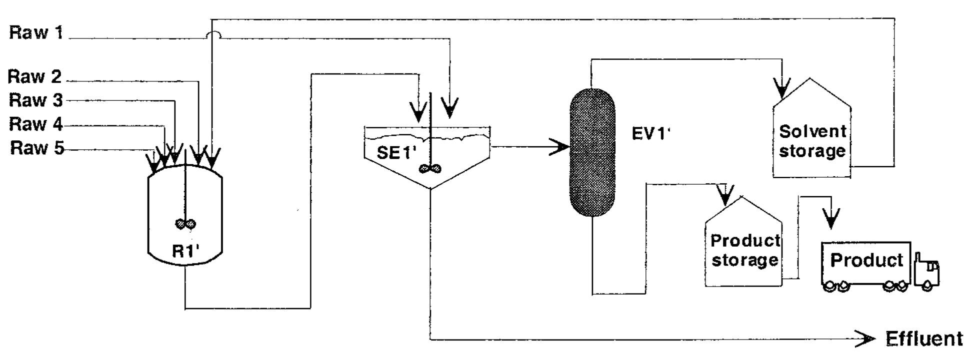

Figure 6. Flow sheet for processes 2 and 3.

Table 2. Planning Data for the Case Study

Process 1					
raw material	S1	S9	S10	S11	
standard cost (£/ton)	200	300	350	100	
penalized cost (£/ton)	600	1000	1000	500	
ton used/ton output	0.20	0.25	0.35	0.20	
max available at standard price (ton/yr)	6000	6000	6000	6000	
byproduct	S8	S7			
standard treatment cost (£/ton)	250	20			
penalized treatment cost (£/ton)	5000	1000			
ton produced/ton product	1	0.7			
max treated at standard price (ton/yr)	6000	6000			
product (S6) selling price (£/ton)	5000.00				
Process 2					
raw material	S1′	S2′	S3′	S4′	S5′
standard cost (£/ton)	10	60	250	350	250
penalized cost (£/ton)	50	1000	1500	900	1000
ton used/ton output	0.15	0.25	0.10	0.20	0.30
max available at standard price (ton/yr)	unlimited	$w(s2')$	$w(s3')$	6000	$w(s5')$
byproduct	S9′	S11′ [a]			
standard treatment cost (£/ton)	500	0			
penalized treatment cost (£/ton)	10000	0			
ton produced/ton product	0.35	0.45			
max treated at standard price (ton/yr)	2000	2000			
product (S10′) selling price (£/ton)	4500.00				
Process 3					
raw material	S1″	S2″	S3″	S4″	S5″
standard cost (£/ton)	10	60	250	300	250
penalized cost (£/ton)	50	1000	1500	900	1000
ton used/ton output	0.20	0.15	0.10	0.30	0.25
max available at standard price (ton/yr)	unlimited	$w(s2'')$	$w(s3'')$	6000	$w(s5'')$
byproduct	S9″	S11″ [a]			
standard treatment cost (£/ton)	500	0			
penalized treatment cost (£/ton)	10000	0			
ton produced/ton product	0.40	0.25			
max treated at standard price (ton/yr)	2000	2000			
product (S10″) selling price (£/ton)	4600.00				

[a] Recycled solvent.

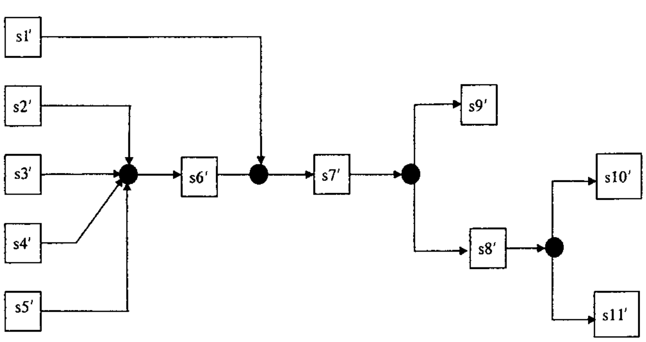

Figure 7. SSN for processes 2 and 3.

(b) Byproduct Constraints. *Process 1.*

$$W(s^b_{\text{out}}) - W'(s^b_{\text{out}}) \le 6000,\ \forall\ s^b_{\text{out}} \in S^b_{\text{out},1} = \{s7, s8\}$$

$$W'(s^b_{\text{out}}) \le 10^6,\ \forall\ s^b_{\text{out}} \in S^b_{\text{out},1} = \{s7, s8\}$$

Process 2.

$$W(s9') - W'(s9') \le 2000$$

$$W'(s9') \le 10^6$$

$$W(s9') \ge 0,\ W'(s9') \ge 0$$

Process 3.

$$W(s9'') - W'(s9'') \le 2000$$

$$W'(s9'') \le 10^6$$

$$W(s9'') \ge 0,\ W'(s9'') \ge 0$$

(c) Product Demand Constraints.

$$w(s6) \ge 4000$$

$$w(s10') \ge 6000$$

$$w(s10'') \ge 5000$$

Table 3. Scheduling Data for Process 1

unit	capacity (tons)	suitability	mean processing time (h)
reactor 1	10	reaction 1	2
reactor 2	10	reaction 1	2
reactor 3	10	reaction 2, reaction 3	3, 1
reactor 4	10	reaction 2, reaction 3	3, 1
settler 1	10	settling	1
settler 2	10	settling	1
settler 3	10	settling	1
evaporator 1	10	evaporation	3
evaporator 2	10	evaporation	3

state	storage capacity (ton)	initial amount (ton)
S1	unlimited	unlimited
S2	100	0
S3	100	0
S4	100	0
S5	100	0
S6	100	0
S7	100	0
S8	100	0
S9	unlimited	unlimited
S10	unlimited	unlimited
S11	unlimited	unlimited

Stoichiometric Data

state	ton of raw material/ ton of output	ton of byproduct/ ton of product
S1	0.20	
S9	0.25	
S10	0.35	
S11	0.20	
S7		0.7
S8		1

(d) Stoichiometric Constraints. *Process 1.*

$$0.20(w(\mathrm{s6}) + w(\mathrm{s8}) + w(\mathrm{s9})) = w(\mathrm{s1})$$

$$0.25(w(\mathrm{s6}) + w(\mathrm{s8}) + w(\mathrm{s9})) = w(\mathrm{s9})$$

$$0.35(w(\mathrm{s6}) + w(\mathrm{s8}) + w(\mathrm{s9})) = w(\mathrm{s10})$$

$$0.20(w(\mathrm{s6}) + w(\mathrm{s8}) + w(\mathrm{s9})) = w(\mathrm{s11})$$

The coefficients in these constraints represent the amount of raw material used (tons) per ton of output as given in Table 2. These coefficients can be read directly from Table 2 under process 1 raw material data. The output from process 1 consists of product $w(\mathrm{s}6)$, solid byproduct $w(\mathrm{s}8)$, and effluent $w(\mathrm{s}9)$:

$$w(\mathrm{s6}) = w(\mathrm{s7})$$

$$0.7w(\mathrm{s6}) = w(\mathrm{s8})$$

These constraints give the stoichiometric relation between each of the byproducts and product and taken from Table 2 under process 1 byproduct data:

$$0 \le w(\mathrm{s6}) \le w^{\mathrm{U}}(\mathrm{s6})$$

This constraint sets an upper bound on the amount of product that can be produced depending on the capacity of the plant.

Table 4. Scheduling Data for Processes 2 and 3

unit	capacity (ton)	suitability	mean processing time (h)
reactor	30	reaction	4.5
wash vessel	30	mixing, separation	0.5, 1
evaporator	30	solvent recovery	3

state	storage capacity (ton)	initial amount (ton)
S1	unlimited	unlimited
S2	unlimited	unlimited
S3	unlimited	unlimited
S4	unlimited	unlimited
S5	unlimited	unlimited
S6	100	0
S7	100	0
S8	100	0
S9	unlimited	unlimited
S10	unlimited	unlimited
S11	unlimited	unlimited

state	storage capacity (ton)	initial amount (ton)
S1	unlimited	unlimited
S2	unlimited	unlimited
S3	unlimited	unlimited
S4	unlimited	unlimited
S5	unlimited	unlimited
S6	100	0
S7	100	0
S8	100	0
S9	unlimited	unlimited
S10	unlimited	unlimited
S11	unlimited	unlimited

Stoichiometric Data

state	ton of raw material/ ton of output		ton of byproduct/ ton of product	
	process 2	process 3	process 2	process 3
S1	0.15	0.20		
S2	0.25	0.15		
S3	0.10	0.10		
S4	0.20	0.30		
S5	0.30	0.25		
S9			0.35	0.40
S11			0.45	0.25

Process 2.

$$0.15(w(\mathrm{s10}') + w(\mathrm{s9}') + w(\mathrm{s11}')) = w(\mathrm{s1}')$$

$$0.25(w(\mathrm{s10}') + w(\mathrm{s9}') + w(\mathrm{s11}')) = w(\mathrm{s2}')$$

$$0.10(w(\mathrm{s10}') + w(\mathrm{s9}') + w(\mathrm{s11}')) = w(\mathrm{s3}')$$

$$0.20(w(\mathrm{s10}') + w(\mathrm{s9}') + w(\mathrm{s11}')) = w(\mathrm{s4}')$$

$$0.30(w(\mathrm{s10}') + w(\mathrm{s9}') + w(\mathrm{s11}')) = w(\mathrm{s5}')$$

$$0.35w(\mathrm{s10}') = w(\mathrm{s9}')$$

$$0.45w(\mathrm{s10}') = w(\mathrm{s11}')$$

$$0 \le w(\mathrm{s10}') \le w^{\mathrm{U}}(\mathrm{s10}')$$

Process 3.

$$0.20(w(\mathrm{s10}'') + w(\mathrm{s9}'') + w(\mathrm{s11}'')) = w(\mathrm{s1}'')$$

$$0.15(w(\mathrm{s10}'') + w(\mathrm{s9}'') + w(\mathrm{s11}'')) = w(\mathrm{s2}'')$$

$$0.10(w(\mathrm{s10}'') + w(\mathrm{s9}'') + w(\mathrm{s11}'')) = w(\mathrm{s3}'')$$

$$0.20(w(\mathrm{s10}'') + w(\mathrm{s9}'') + w(\mathrm{s11}'')) = w(\mathrm{s4}'')$$

$$0.25(w(\mathrm{s10''}) + w(\mathrm{s9''}) + w(\mathrm{s11''})) = w(\mathrm{s5''})$$

$$0.40w(\mathrm{s10''}) = w(\mathrm{s9''})$$

$$0.25w(\mathrm{s10''}) = w(\mathrm{s11''})$$

$$0 \le w(\mathrm{s10''}) \le w^{\mathrm{U}}(\mathrm{s10''})$$

(e) Objective Function. The objective function is maximizing the overall profit following the same format of eq 12. Due to its lengthy form, the objective function is not listed here.

The scheduling models for the three processes can be formulated using the formulation presented in the Scheduling Model section of this paper. Following the procedure in Figure 3, we integrate the planning model and the three scheduling models. Note that the scheduling time horizon is shorter than the planning time horizon, hence the raw material requirements $w(s^r_{\mathrm{in},k})$ are reduced proportionately for incorporation into scheduling models.

Results

The results are shown in Table 5. All the results were obtained using GAMS/OSL solver in a 333 MHz AMD K6-2 processor. The planning and scheduling models are based on 1-yr (8004 h) and 12-h time horizons, respectively. According to Table 5, processes 1−3 should produce 6000, 22 233.33, and 5000 tons of product/yr (8004 h), respectively, to meet the overall optimal profit of £76 million and market demands. Over a 12-h time horizon, these targets are equivalent to 8.996, 33.333, and 7.496 ton, respectively. It is worthy of note that the common raw material allocation corresponding to this optimal solution is biased more toward process 2 than process 3. Common raw materials s1, s2, s3, and s5 require 78:22, 89:11, 83:17, and 85:15 distribution between processes 2 and 3, respectively. Had an arbitrary raw material allocation been used, suboptimal results would be obtained. For example, 50:50 distribution of all common raw materials between processes 2 and 3 gives an overall profit of £70 million as compared with the optimal profit of £76 million. A 70:30 allocation, which is biased more toward process 3 than process 2, gives an overall profit of £68 million.

Solving the problem with decomposition using the procedure of Figure 3 requires CPU time of about 1.28 s, while solving the overall model (consisting of the planning model and the three scheduling models) requires 8.79 s. This is mainly due to the smaller number of binary variables associated with individual models. While the overall model requires 117 binary variables, the individual models for processes 1−3 require 77, 20, and 20 binary variables, respectively. Moreover, the overall model requires 1845 total variables (binary and continuous) and 2526 constraints, whereas the decomposed model requires an average of 615 total variables and 842 constraints. All the results converged within a tolerance of 10^{-6}. The Gantt charts for processes 1−3 over a 12-h time horizon after integration of planning and scheduling are shown in Figures 8−10.

Conclusions

A new procedure for linking planning and scheduling decisions has been presented. This procedure exploits the block angular structure of the overall multiplant model, and a generic decomposition approach is developed by using structured programming. The solution procedure consists of two levels. In the first level, the planning model is formulated and solved for the optimal allocation of raw materials to individual processes. In the second level, the raw material targets obtained from the planning model are incorporated into the scheduling models for individual processes. These models are then solved independently. Therefore, the planning model serves as coordination between scheduling models of individual processes. The SSN representation[6] is used

Table 5. Results of Integrated Planning and Scheduling

planning time horizon (h) (equivalent to 1 yr) = 8004
scheduling time horizon (h) = 12
overall profit (£/annum) = 76 MM

Raw Material Requirement (ton/yr)

process 1				process 2					process 3				
S1	S9	S10	S11	S1′	S2′	S3′	S4′	S5′	S1″	S2″	S3″	S4″	S5″
3240	4050	5670	3240	6003	10005	4002	8004	12006	1647	1235	824	2471	2059

Product Output (ton/yr)

process 1		process 2		process 3	
S6	6000.00	S10′	22233.34	S10″	5000.00

Computational Results

	overall model	individual models: process 1	process 2	process 3
no. of time points	7	7	5	5
no. of constraints	2526	1506	510	510
total no. of variables	1845	1111	367	367
no. of binary variables	117	77	20	20
MILP solution (ton)	49.825	8.996	33.333	7.496
CPU time (s)	8.79	1.98	1.20	0.67

Convergence Tolerances (ϵ_p)

process 1	process 2	process 3
10^{-6}	10^{-6}	10^{-6}

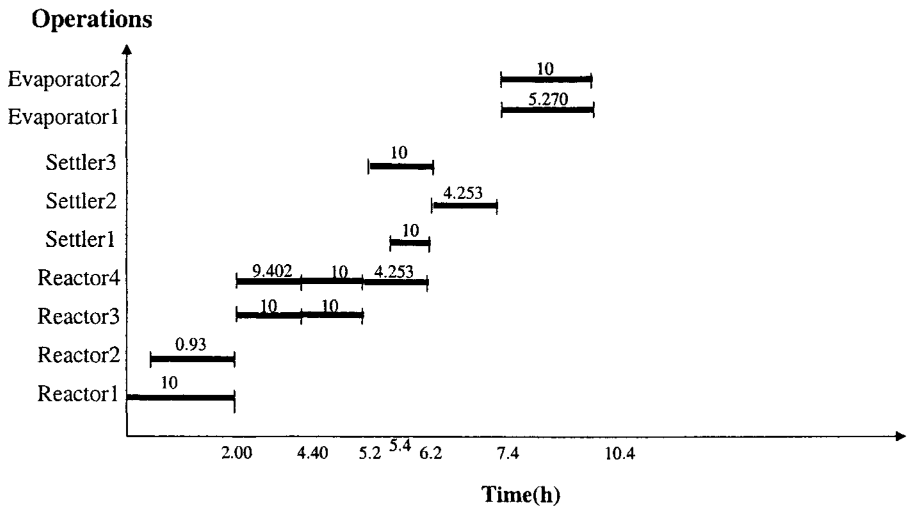

Figure 8. Gantt chart for process 1 after integrated planning and scheduling.

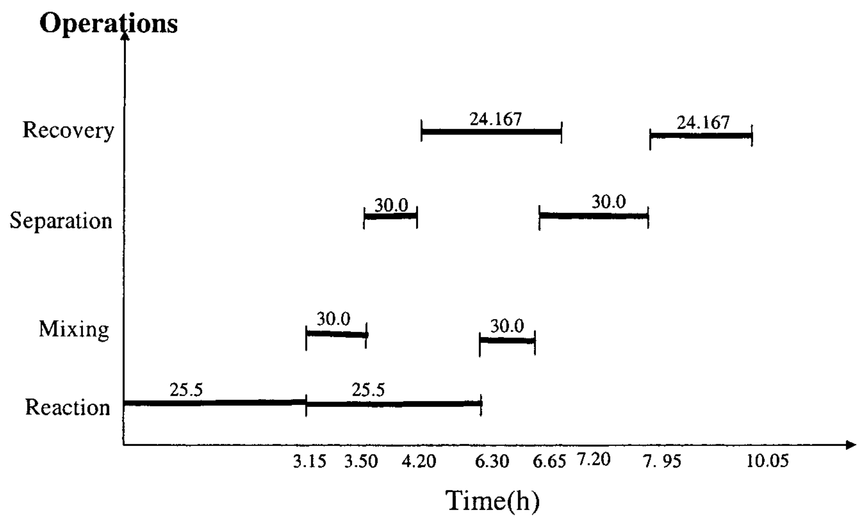

Figure 9. Gantt chart for process 2 after integrated planning and scheduling.

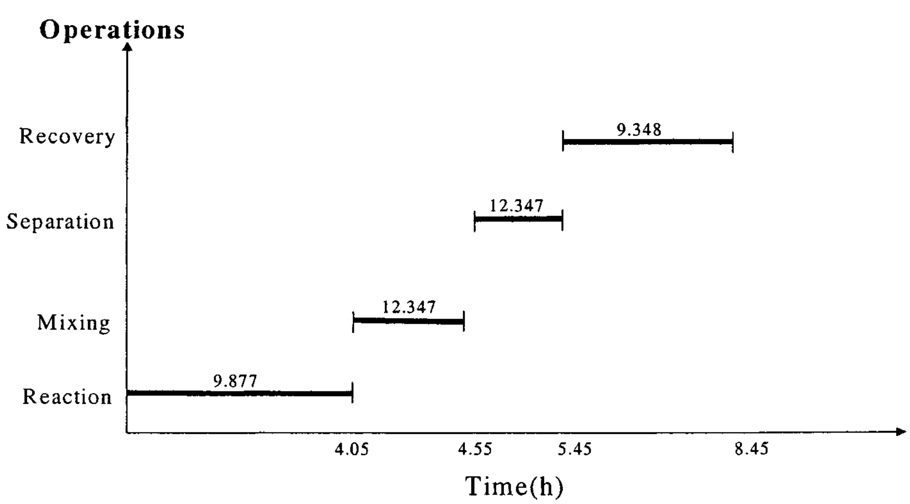

Figure 10. Gantt chart for process 3 after integrated planning and scheduling.

for the formulation of the scheduling models. If the targets set at the planning level are too optimistic to be realized at the scheduling level, the planning model is revisited with more realistic targets. This iteration only terminates when the planning and scheduling targets reach a specified tolerance. Application of this decomposition procedure to an industrial case study comprising of three processes showed a significant reduction in problem size that is concomitant with shorter CPU times as compared to the overall model. The overall model requires 117 binary variables, 1845 total variables (binary and continuous), and 2526 constraints with 8.79 s CPU. The size of the decomposed models is reduced to an average of 39 binary variables, 615 total variables (binary and continuous), and 842 constraints, with an average CPU time of 1.28 s. This reduction in problem size allows the application of this decomposition procedure to large-scale industrial problems. Significance of optimal allocation of raw materials manifests the necessity of using a planning model to guide the scheduling models to maximize the overall profit.

Appendix

(a) Proof of Consistency between the Overall and the Decomposed Models. The fundamental assumption in this proof is that all the constraints and the objective functions are convex, which implies that the KKT conditions are both necessary and sufficient.

(1) Let subproblem 1 be defined as follows:

$$\max f(\bar{v}_1)$$
$$st$$

$$h_{1i}(\bar{v}_1) = 0,\ i = 1, ..., I \text{ (no. of equality constraints in problem 1)}$$

$$g_{1j}(\bar{v}_1) \le 0,\ j = 1, ..., J \text{ (no. of inequality constraints in problem 1)}$$

$$\bar{v}_1 \ge 0,\ \bar{v}_1 = \text{vector of variables in problem 1}$$

(2) Let subproblem 2 be defined as follows:

$$\max f(\bar{v}_2)$$
$$st$$

$$h_{2k}(\bar{v}_2) = 0,\ k = I + 1, ..., K \text{ (no. of equality constraints in problem 2)}$$

$$g_{2m}(\bar{v}_2) \le 0,\ m = J + 1, ..., M \text{ (no. of inequality constraints in problem 2)}$$

$$\bar{v}_2 \ge 0,\ \bar{v}_2 = \text{vector of variables in problem 2}$$

(3) Let the overall model be defined as follows:

$$\max f(\bar{v}_1) + f(\bar{v}_2)$$
$$st$$

$$h_{1i}(\bar{v}_1) = 0,\ i = 1, ..., I$$

$$g_{1j}(\bar{v}_1) \le 0,\ j = 1, ..., J$$

$$h_{2k}(\bar{v}_2) = 0,\ k = I + 1, ..., K$$

$$g_{2m}(\bar{v}_2) \le 0,\ m = J + 1, ..., M$$

$$\bar{v}_2 \ge 0,\ \bar{v}_1 \ge 0$$

The KKT conditions corresponding to these problems are as follows.

(b) Subproblem 1.

$$\nabla f(\bar{v}_1^{\,*}) + \sum_{i=1}^{I} w_{1i} \nabla h_{1i}(\bar{v}_1^{\,*}) + \sum_{j=1}^{J} w_{1j} \nabla g_{1j}(\bar{v}_1^{\,*}) = 0 \qquad \text{(A1)}$$

$$w_{1j} g_{1j}(\bar{v}_1^{\,*}) = 0,\ \forall\, j \qquad \text{(A2)}$$

$$\boldsymbol{w}_{1j} \ge 0,\ \forall\, j \qquad \text{(A3)}$$

$$h_{1i}(\bar{v}_1^{\,*}) = 0,\ \forall\, i \qquad \text{(A4)}$$

$$g_{1j}(\bar{v}_1^{\,*}) \le 0,\ \forall\, j \qquad \text{(A5)}$$

where the asterick refers to the optimal solution.

(c) Subproblem 2.

$$\nabla f(\bar{v}_2^{\,*}) + \sum_{k=I+1}^{K} w_{2k} \nabla h_{2k}(\bar{v}_2^{\,*}) + \sum_{m=J+1}^{M} w_{2m} \nabla g_{2m}(\bar{v}_2^{\,*}) = 0 \qquad \text{(A6)}$$

$$w_{2m} g_{2m}(\bar{v}_2^{\,*}) = 0,\ \forall\, m \qquad \text{(A7)}$$

$$w_{2m} \ge 0,\ \forall\, m \qquad \text{(A8)}$$

$$h_{2k}(\bar{v}_2^{\,*}) = 0,\ \forall\, k \qquad \text{(A9)}$$

$$g_{2m}(\bar{v}_2^{\,*}) \le 0,\ \forall\, m \qquad \text{(A10)}$$

where the asterick refers to the optimal solution.

(d) Overall Problem.

$$\nabla f(\bar{v}_1^{\,\mp}) + \nabla f(\bar{v}_2^{\,\mp}) + \sum_{p=1}^{K} w_{3p} \nabla h_{3p}(\bar{v}_1^{\,\mp}, \bar{v}_2^{\,\mp}) + \sum_{q=1}^{M} w_{3q} \nabla g_{3q}(\bar{v}_1^{\,\mp}, \bar{v}_2^{\,\mp}) = 0 \qquad \text{(A11)}$$

$$w_{3q} g_{3q}(\bar{v}_1^{\,-}, \bar{v}_2) = 0,\ \forall\, q \qquad \text{(A12)}$$

$$w_{3q} \ge 0,\ \forall\, q \qquad \text{(A13)}$$

$$h_{1i}(\bar{v}_1^{\,\mp}) = 0,\ \forall\, i \qquad \text{(A14)}$$

$$g_{1j}(\bar{v}_1^{\,\mp-}) \le 0,\ \forall\, j \qquad \text{(A15)}$$

$$h_{2k}(\bar{v}_2^{\,\mp}) = 0,\ \forall\, k \qquad \text{(A16)}$$

$$g_{2m}(\bar{v}_2^{\,\mp}) \le 0,\ \forall\, m \qquad \text{(A17)}$$

where superscripts refer to the solution and

$$h_{3p}(\bar{v}_1, \bar{v}_2) = h_{1i}(\bar{v}_1) \cup h_{2k}(\bar{v}_2) \qquad \text{(A18)}$$

$$g_{3q}(\bar{v}_1, \bar{v}_2) = g_{1j}(\bar{v}_1) + \cup\, g_{2m}(\bar{v}_2) \qquad \text{(A19)}$$

Using eqs A18 and A19, it is worth noting that eq A11 can also be written as

$$\nabla f(\bar{v}_1^{\,\mp}) + \nabla f(\bar{v}_2^{\,\mp}) + \sum_{i=1}^{I} w_{3i} \nabla h_{1i}(\bar{v}_1^{\,\mp}) + \sum_{k=I+1}^{K} w_{3k} \nabla h_{2k}(\bar{v}_2^{\,\mp}) + \sum_{j=1}^{J} w_{3j} \nabla g_{1j}(\bar{v}_1^{\,\mp}) + \sum_{m=J+1}^{M} w_{3m} \nabla g_{2m}(\bar{v}_2^{\,\mp}) = 0 \qquad \text{(A20)}$$

which can be rearranged to

$$\nabla f(\bar{v}_1^{\mp}) + \sum_{i=1}^{I} w_{3i} \nabla h_{1i}(\bar{v}_1^{\mp}) + \sum_{j=1}^{J} w_{3j} \nabla g_{1j}(\bar{v}_1^{\mp}) + \nabla f(\bar{v}_2^{\mp}) + \sum_{k=I+1}^{K} w_{3k} \nabla h_{2k}(\bar{v}_2^{\mp}) + \sum_{m=J+1}^{M} w_{3m} \nabla g_{2m}(\bar{v}_2^{\mp}) = 0 \quad \text{(A21)}$$

Also, eqs A12 and A13 can be split as follows:

$$w_{3j} g_{1j}(\bar{v}_1^{\mp}) = 0 \quad \text{(A22)}$$

$$w_{3m} g_{2m}(\bar{v}_2^{\mp}) = 0 \quad \text{(A23)}$$

$$w_{3j} \geq 0 \quad \text{(A24)}$$

$$w_{3m} \geq 0 \quad \text{(A25)}$$

To test whether the solutions to each of the subproblems is actually a solution to the overall problem, we impose these solutions on the overall problem and see whether the KKT conditions still hold. If the KKT conditions still hold for the overall problem, then its solution is a combination of the solutions from the subproblems. In mathematical terms we want to show that

$$(w_{3i}, w_{3j}, w_{3k}, w_{3m}, \bar{v}_1^{\mp}, \bar{v}_2^{\mp}) = (w_{1i}, w_{1j}, w_{2k}, w_{2m}, \bar{v}_1^{*}, \bar{v}_2^{*}), \ \forall\ i, j, k, m \quad \text{(A26)}$$

where the RHS represents an optimal set of variables for the subproblems. Substituting the optimal set of variables for the overall problem (LHS in A26) with the corresponding set of variables from the subproblems does not violate any of the KKT conditions for the overall problem. Since the overall problem is convex, it has one (global) solution, which implies that solutions for the subproblems are, in essence, the solution for the overall problem.

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