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ARTICLE *in* ENERGY & FUELS · MARCH 2011

Impact Factor: 2.79 · DOI: 10.1021/ef1012648

energy&fuels ARTICLE
pubs.acs.org/EF

Discussion of Multicyclic Hubbert Modeling as a Method for Forecasting Future Petroleum Production

Ken B. Anderson* and James A. Conder

Department of Geology, Southern Illinois University Carbondale, Carbondale, Illinois 62901, United States

S Supporting Information

ABSTRACT: Multicyclic Hubbert analysis of resource production, especially as it relates to forecasting future petroleum production, has received significant traction in scientific and public circles. Although in some cases this technique can be a valuable tool for understanding resource production, its usefulness as a predictor has at times been overstated. The effects of modeling parameters, such as the number of Hubbert cycles applied, can significantly limit the validity of the results obtained. Examples of multicyclic Hubbert analyses show that while this approach can be useful in certain circumstances, there are a number of implicit assumptions underlying the method that need to be understood when assessing the validity of forecasts derived from this approach.

INTRODUCTION

The ability to forecast the future availability of resources is essential for strategic planning at every level, from individuals to global communities. This is especially true for energy resources, given the critical role of energy in sustaining modern societies. Arguably the best known (but by no means the only) approach for accomplishing this was developed in the mid twentieth century by M. King Hubbert.[1,2] The most common form of the Hubbert method uses the first derivative of a logistic curve to simulate and predict the production of a finite resource.[3] The resulting bell-shaped curve, now widely known as Hubbert's curve, is most commonly used to describe and predict petroleum production, but the methodology has also been widely applied to a variety of other finite resources, e.g., ref 4. A variety of other curves, including Gaussian[5] and Gompertz[6] curves have also been similarly used and produce comparable results.

The strengths and limitations of the Hubbert method have been ably reviewed and discussed elsewhere.[7−10] For purposes of the present discussion, Hubbert's methods tend to work well in cases where the resource base is large, where production occurs from a large number independent producers, and where resource production is unfettered by external constraints such as political controls. It works much less well for small resource bases where, for example, an interruption in a single producer results in a disruption of a large fraction of total production or when natural or socio-political disruptions interfere with production. For example, Figure 1 shows historical production data and conventional (single cycle) Hubbert analyses of conventional petroleum production for Norway (1a) and Nigeria (1b). Norwegian petroleum production is relatively well described by a conventional Hubbert analysis. Both the total recoverable resource and the number of individual fields is large (hence random fluctuations in production from individual fields has only a minimal effect on total production) and production is unfettered by artificial constraints such as production quotas or socio-political upheaval. Nigerian production, on the other hand, has been and continues to be limited by widespread disruptions associated with socio-political turmoil in that country, resulting in erratic production behavior that is poorly described by conventional Hubbertian analysis.

Hubbert himself was aware of these limitations and noted "... there is no necessity that the curve have a single maximum or that it be symmetrical."[3] Recently, an expanded approach known as multicyclic Hubbert analysis has been described.[13−19] Superficially, this method appears to overcome many of the limitations of conventional (single cycle) Hubbert analyses. Excellent fits to historical production data are obtained beyond those possible with a conventional Hubbert curve. Further, the method has also been used to forecast future production of conventional petroleum,[16,19] natural gas,[15,19] and coal.[17] These analyses have attracted widespread interest, including significant attention in the public media.[20−22] This paper discusses aspects of this approach, including significant unstated underlying assumptions that limit the validity of forecasts developed by this method.

METHODOLOGY

The basic form of the Hubbert curve can be expressed in a number of equivalent ways. A common form is given in eq 1 below.

$$P_t = Q_\infty a \left(\frac{e^{-a(t - t_{\max})}}{[1 + e^{-a(t - t_{\max})}]^2} \right) \quad (1)$$

Where: P_t = production at time t; Q_∞ = cumulative production at infinite time (in practice, cumulative production when no more resource remains to be produced = ultimately recoverable resources); a = inverse decay period; $t_{\max}$ = time at which maximum production occurs.

With the recognition that P_t is equal to $\partial Q_t/\partial t$ where Q_t is the cumulative production at time t, time may be made implicit, resulting in eq 2 below.

$$P_t = aQ_t \left(1 - \frac{Q_t}{Q_\infty}\right) \quad (2)$$

Received: September 17, 2010
Revised: January 14, 2011

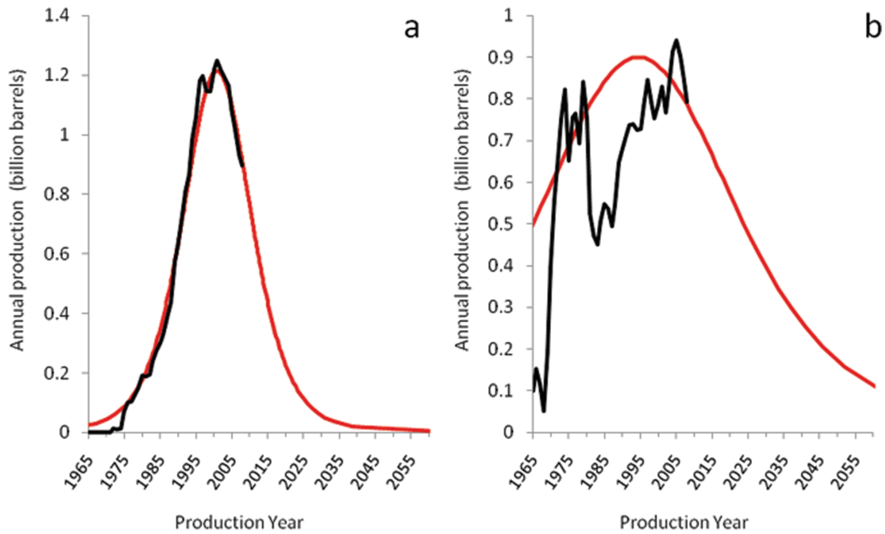

Figure 1. Historical production data (solid black lines) and derived Hubbert curves (red lines) for production of conventional petroleum from (a) Norway and (b) Nigeria. Data are taken from the BP Statistical Review of World Energy 2010[11] and are annualized. Hubbert curves were derived by the "Hubbert Linearization" technique described by Deffeyes.[12] The curve parameters are given in Appendix 1, Table A1. RMSE (a, Norway) = 0.0405; RMSE (b, Nigeria) = 0.2229.

Where: Q_t = cumulative production at time t and recognizing that P_t is also equal to $\partial Q/\partial t$

This form of the Hubbert equation is advantageous as it conveniently leads to the Hubbert Linearization technique,[12] which is described by eq 3, below.

$$\frac{P_t}{Q_t} = a\left(1 - \frac{Q_t}{Q_\infty}\right) \tag{3}$$

This expression is linear in two variables: P_t/Q_t and Q_t and allows estimates of a and Q_∞ to be derived from historical production data, e.g., refs 11 and 23 by determining the intercepts on a plot of P_t/Q_t vs Q_t. These data can then be fed back into eq 2 and used to establish a Hubbert curve for the system under consideration. For additional discussion, see Deffeyes[12] and Sorell and Speirs.[10,24]

The equation describing multicyclic Hubbert analyses reported by Al-Fattah and Startzman[15] and subsequently utilized by Nashawi et al.[16] is given as eq 4.

$$P_t = \sum_{i=1}^{k} (P_t)_i = \sum_{i=1}^{k} 4(P_{\max})_i \left\{ \frac{e^{-a(t - t_{\max})}}{\left[1 + e^{-a(t - t_{\max})}\right]^2} \right\}_i \tag{4}$$

Where: k = number of production cycles used and $p_{\max}$ = maximum production rate of individual Hubbert cycle.

Each cycle is comprised of three free parameters, $p_{\max}$, a, and $t_{\max}$, for a total of $3k$ model parameters in the multicyclic method. Locally optimal values for each of these can be determined by a nonlinear least-squares regression. Because of the highly nonlinear character of the equations, seeding of initial values is required. As noted by Nashawi et al.,[16] an initial value for the constant a can be deduced by estimation of inflection points for the individual curve.

In principle, the same general methodology can also be applied using coaddition of multiple Gaussian, Gompertz, or other similar types of curves. Such an approach would be expected to produce similar results and would be subject to the same methodological caveats as need to be considered in application of the multicyclic Hubbert method, as discussed below.

Quantitative quality-of-fit estimates rely on measures of model misfit such as calculation of root-mean-square error (RMSE) values, obtained as the sum of the squares of the differences between model predictions and actual data as calculated herein according to eq 5. However, as discussed below, this is not a complete measure of model quality, as a better fit (lower RMSE) can always be achieved by adding additional model parameters (3 per additional curve), whether or not they are warranted. Additional curves may be warranted from *a priori* knowledge of the system, such as cycles in political activity or the opening of a significant new oil field, or may be justified *a posteriori* by statistical means such as by application of a likelihood ratio test (e.g., F-test, eq 6) that demonstrates that the goodness of fit is improved more than would be expected by simply adding more model parameters.[25]

$$\text{RMSE} = \sqrt{\frac{\sum_{i=0}^{n} (p_{\text{act}} - p_{\text{calc}})^2}{N}} \tag{5}$$

$$F = \left\{ \frac{X_1{}^2 - X_2{}^2}{(3k_2 - 3k_1)X_2{}^2} \right\} (N - 3k_2 - 1) \tag{6}$$

Where: X = RMSE for each model; N = number of empirical data; k_1 = number of Hubbert curves used in model 1; and k_2 = number of Hubbert curves used in model 2

The probability that additional parameters are warranted rather than improving the quality of fit by chance is found by integrating the tail of the f-distribution above F where the number of free parameters for constructing the f-distribution are $(3k_1 - 3k_2)$ and $(N - 3k_2 - 1)$. Lookup tables for 90 and 95% probability thresholds are regularly found in standard statistical texts, e.g., ref 26.

Best fitting parameters in this study are determined by a least-squares fit to the cumulative production data using a singular value decomposition algorithm (see the Supporting Information). Seed parameters must be given to linearize the inherently nonlinear problem. As any particular set of seed parameters may result in a solution for a local misfit minimum and not necessarily a global optimum, we solve multiple times with random seeds (hundreds of times for higher order curve fits) and choose the solution with the lowest overall misfit (i.e., smallest RMSE).

DISCUSSION

At its core, the multicyclic Hubbert method is a curve-fitting process. Rather than using a single Hubbert curve to simulate the production history of a producing unit (e.g., field, country, cartel, etc.), these data are modeled by coaddition of multiple Hubbert curves.

For example, the conventional petroleum production history of the U.K. is illustrated in Figure 2a. As is apparent from the bimodal nature of this distribution, this system is not well described by a conventional Hubbert analysis (also shown in Figure 2a). These data are much better described using a multicyclic approach as illustrated in Figures 2b and 2c which demonstrate multicyclic Hubbert analyses of these data using 2 and 10 Hubbert cycles, respectively.

These data illustrate two key observations relating to the utility of multicyclic Hubbert analyses: (1) The precision to which historical production data can be modeled using this approach is dependent on the number of cycles used to model the data. (2) Each model cycle is an independent construct and can be varied independently from all other curves to optimize the accuracy of the fit obtained for the historical data.

Ideally, each Hubbert cycle applied should have some definable objective basis, such as reflecting independent simulation of the production history for a particular field or region, or possibly discrete regulatory regimes such as might result from changes in governments, or other definable criteria (as some authors have attempted, e.g., refs 17 and 18). In cases where this is not possible or is impractical, numerous approaches can, in principle, be used to estimate the number of curves necessary to fit historical production data, including use of derivative curves,[16] iterative approaches with analysis of residuals,[17] and others, each of which has separate sensitivities, strengths, and weaknesses.

The case illustrated in Figure 2c is clearly "overfitted". That is, the use of 10 curves is not justified by the original data, despite the 10-cycle model providing a better fit (lower RMSE) to the historical production data than the single Hubbert curve or 2-curve simulation illustrated in Figures 2a and 2b, respectively. This can be demonstrated by application of statistical likelihood ratio tests such as the *F*-test (eq 6). In this case, the probability that the level of improvement seen in the RMSE values for two cycle analysis relative to one cycle analysis would be achieved by chance is 3.045×10^{-7}, whereas the probability that the observed improvement for 10 cycles relative to 2 would be achieved is 0.504, exactly where we would expect it to be if the final 8 curves are essentially meaningless.

As these data demonstrate, any cumulative production trend can, in principle, be decomposed into the sum of an arbitrary number of logistic curves and if the quality of the fit obtained is the only consideration, then clearly the accuracy with which historical production data can be modeled can be increased by increasing the number of Hubbert cycles applied (i.e., by increasing the number of curves fitted). Hence, arbitrary selection of a number of curves and/or use of arbitrary curve parameters simply because the resulting summation happens to approximate real production data results in nonunique solutions of limited value for understanding the production history of a region or for forecasting future production. Furthermore, although this is not always the case,[16,19] authors should always report the number of cycles used and preferably the parameters for each individual cycle used in deriving their analysis. Otherwise it is impossible for readers to fully reproduce or assess the validity of the results reported.

In the examples used in Figures 1a and 2, cumulative production is large compared to remaining recoverable resources, peak production has already occurred, overall production is declining, and future production is (presumably) well described by one or more Hubbert curves whose characteristics are well-defined by the historical production data. This is not always the case. It is

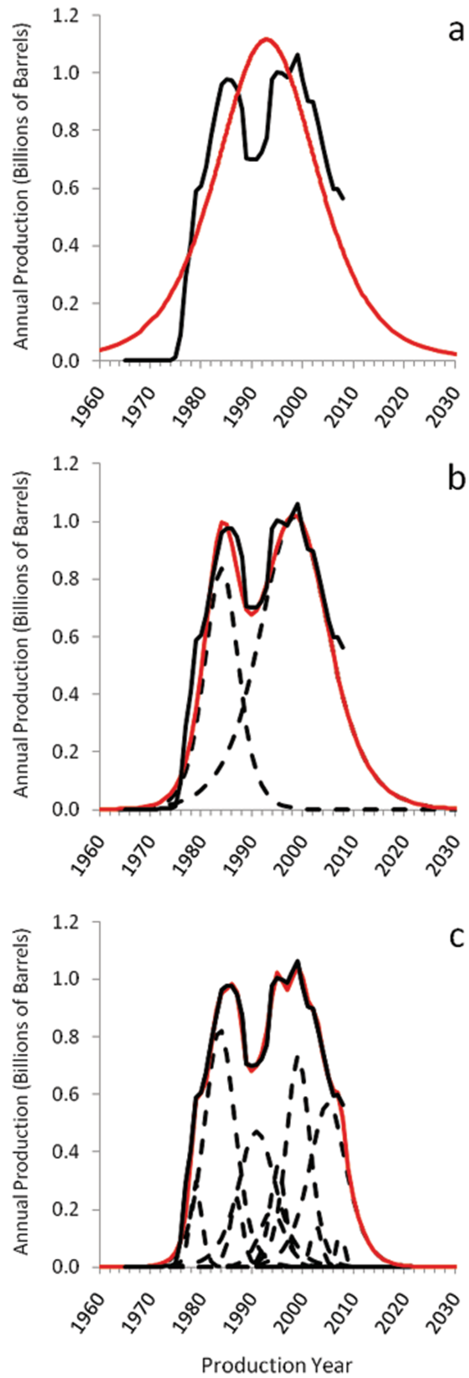

Figure 2. Historical and simulated conventional petroleum production data for the UK. (a) Historical production data fit by conventional (single cycle) Hubbert analysis. (RMSE = 0.1829) (b) Multicyclic Hubbert analysis of real production data using a two-cycle multicyclic Hubbert analysis. (RMSE = 0.0755) (c) Multicyclic Hubbert analysis of real production data using a 10-cycle multicyclic Hubbert analysis. (RMSE = 0.0262). Historical production data from BP Statistical Review of World Energy, 2010.[11] Curve parameters are given in Appendix 1, Table A2. Legend: solid black line = historical production data; dashed black lines = individual Hubbert cycles (details given in Appendix 1 Table A3); solid red line = simulation of historical production data generated by coaddition of individual Hubbert cycles.

dx.doi.org/10.1021/ef1012648 |*Energy Fuels* XXXX, XXX, 000–000

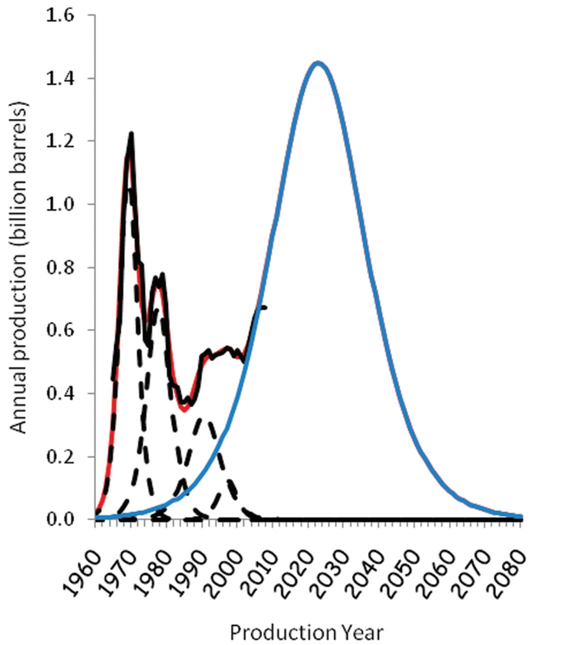

Figure 3. Historical petroleum production data and multicyclic (4-cycle) Hubbert analysis of conventional petroleum production in Libya. An additional Hubbert cycle is added to simulate possible future production. See text for additional discussion. Historical data are taken from the BP Statistical Review of World Energy 2010[11] and are annualized. RMSE = 0.0500. Legend: solid black line = historical production data; dashed black lines = individual Hubbert cycles (details given in Appendix 1 Table A3); solid red line = simulation of historical production data generated by coaddition of individual Hubbert cycles; solid blue line = Hubbert cycle representing (primarily) future production.

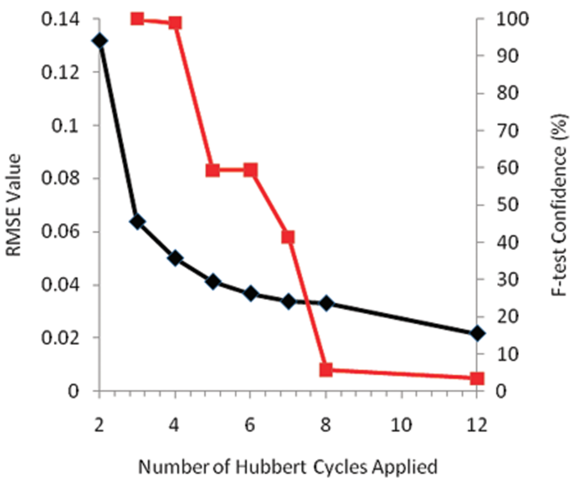

Figure 4. Results of statistical analysis for multicyclic Hubbert analysis of Libyan petroleum production using 2−12 Hubbert cycle models. RMSE values (black line) decrease with increasing number of curves fitted. Application of a likelihood ratio test (*F*-test, see eq 6) shows that the improvement of fit obtained by use of additional curves is justified at >98% confidence for fitting of up to 4 curves and thereafter is not justified.

also necessary to consider how well multicyclic Hubbert analyses are able to describe cases in which production is increasing and/or large remaining recoverable reserves are proven or claimed. As an example, this situation is illustrated by multicyclic Hubbert analysis of Libyan petroleum production, as shown in Figure 3. Other cases could have been used for this analysis, and the choice of data for Libya is only illustrative.

Historical production data for Libya are somewhat erratic, due largely to socio-political factors. In this analysis, simulation of historical production data, which span a 34 year period (1965−2009), is simulated by coaddition of 4 Hubbert cycles with one additional cycle representing (primarily) future production. The choice of 4 Hubbert cycles for these analyses is not arbitrary. Models of historical Libyan production using from 2 to 12 Hubbert cycles were examined and the results tested statistically, see Figure 4. As expected, RMSE values decrease monotonically as the number of Hubbert cycles applied is increased. However, application of the F-test to the results shows that use of greater than 4 cycles is not statistically justifiable.

In the simulation illustrated in Figure 3, which approximates that reported by Nashawi et al.,[16] this future production, which occurs over a period of ∼70 years and accounts for >72% of total cumulative production, is modeled as a single Hubbert curve. (Note, it is impossible to fully reproduce the results reported by these authors[16] since information on the number of Hubbert cycles used and the parameters of individual cycles are not reported).

In fairness, use of any number of Hubbert cycles greater than 1 to simulate future production would require clairvoyance. There is no way to reliably predict factors which may disrupt or constrain future production. However, it is important to note that use of a single Hubbert cycle to simulate future production implicitly assumes that the factors which have constrained and affected historical production do not apply to future production. That is, this assumes that the factors that make multicyclic Hubbert analysis necessary in order to adequately model historical production data are somehow not applicable to future production. There does not appear to be any basis or independent support for this assumption, but it is essential to understand that the validity of projections derived from this approach depends on the validity of this premise.

Some proponents of the multicyclic Hubbert method[16] also appear to imply that the because this method is able to simulate historical production data to an acceptable level of accuracy, forecasts of future production are therefore also more reliable than forecasts derived from other modeling and forecasting approaches. This is not the case. As discussed above, each cycle used in any multicyclic Hubbert analysis is an independent construct. Ironically, predictive power derived from curve fitting often decreases with the number of parameters used to fit the historical data, as the information associated with each model parameter illuminates smaller subsets of the data. These subsets gain illumination at the expense of regions outside the data set (i.e., future projections).

It is also important to note that the size of the Hubbert curve modeling future production is closely correlated with the volume of yet-to-be-produced oil. In order to accurately predict future production, this parameter should include known proven and probable recoverable reserves, estimated reserves growth at existing fields and some estimate of yet to be discovered resources. That is, it should be an estimate of ultimately recoverable resources (URR) although some previous authors appear to constrain estimates of future production as equaling published estimates of future reserves. (For example, Nashawi et al.[16] estimate for Libya that future recoverable oil (N_{FR}) = 41.464

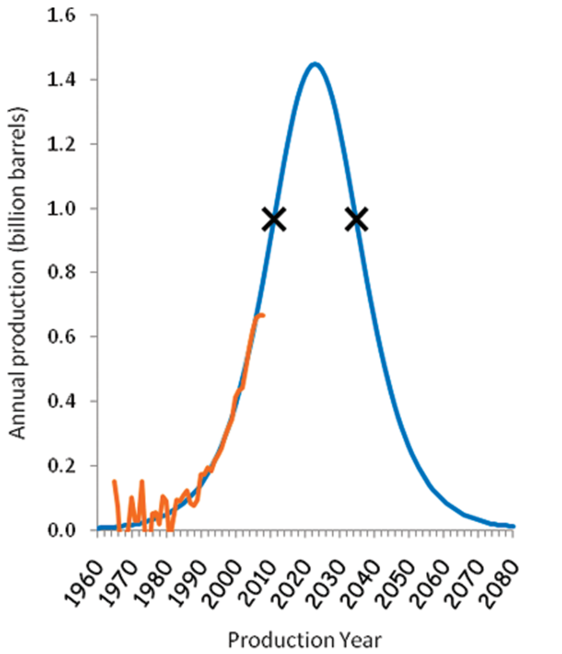

Figure 5. Actual and simulated Libyan petroleum production data not described by Hubbert curves well constrained by historical production data, (i.e., not modeled by curves 1−4). Legend: Solid orange line = production data not described by 4-curve fit to historical production data (calculated as historical data − Σcurves 1−4); solid blue line = Hubbert cycle representing (primarily) future production. This curve is the same as illustrated in Figure 3 above. Inflection points are marked with black × symbols.

billion barrels, exactly the same value as the proven reserve estimate for Libya used by these authors).

In the case of Libyan oil production, and in many similar cases, it is not possible to adequately constrain the parameters of the curve representing future production from recent production data not described by other Hubbert cycles. As noted elsewhere, "curve fitting is only viable if the rate of production increase has passed its peak, (i.e. the point of inflection on the rising production trend)."[10] In this instance, as shown in Figure 5, there are simply not enough data to support this. Hence, it is necessary to constrain the Hubbert cycle representing future production by taking into consideration estimated ultimately recoverable resources as a way of estimating total future production. Hence, the validity of future production projections attained using this approach is also related in part to the accuracy of reported reserve estimates. Furthermore, unless production has reached the above-mentioned inflection point, there is no reliable information in projections of the final cycle other than that the total area under that part of the curve representing future production cannot be larger than the total volume of yet to be produced oil. In effect, such projections are claiming two pieces of information, height and width of curve, when only one piece of information (volume) is available.

There are no consistent global standards for reporting petroleum reserve estimates, and many analysts believe that some countries, especially those associated with OPEC, may be over-reporting reserves due to OPEC's policy of basing production quotas for member countries (in part) on proven reserve estimates.[27] To the extent that this is correct, all forecasting models of future petroleum production, including

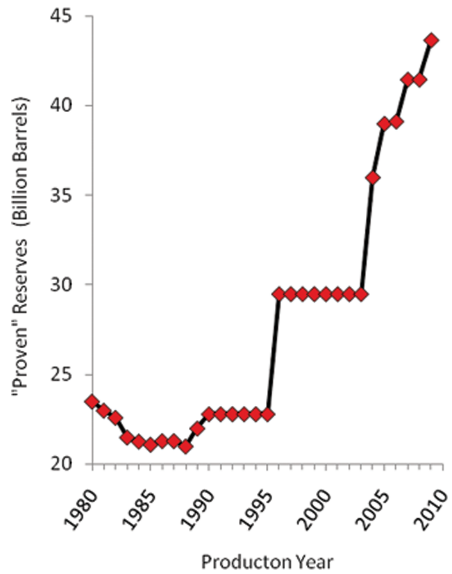

Figure 6. "Proven" petroleum reserves reported by Libya, 1980−2009. Data are from Energy Information Administration database (2010).[28]

multicyclic Hubbert analyses, will be biased in the direction of overoptimism.

This point is illustrated by extending the example of Libyan petroleum production described and illustrated (Figure 3) above. Cumulative Libyan oil production over the period 1965−2009 amounted to 26.62 billion barrels. Libya also claims proven reserves of 43.66 billion barrels.[28] Libyan petroleum production was nationalized in the 1970s and is largely controlled by the National Oil Corporation. Reserve estimates are reported to OPEC and are used in part to set the production quota assigned to Libya. As shown in Figure 6, from 1995 to 2009 claimed Libyan reserves nearly doubled from 22.8 to 43.7 billion barrels. In no year since 1995 were reserves depleted due to production despite the fact that Libyan production between 1995 and 2008 ranged from ∼0.5−0.7 billion barrels/year and cumulative production over that period totaled >8.5 billion barrels. Some of this increase may be attributable to new discoveries, improvements in geological characterization of existing reservoirs, and improving production technology, but many analysts believe that these numbers may be inflated.[27] If that is in fact the case, then the future production curve illustrated in Figure 3 is an overestimate of future production since the area under this curve is determined in large part by the claimed reserves.

If one were to assume, hypothetically, that reserve estimates in 1995 are geologically defensible estimates of resources recoverable at that time and then allow a 25% reserve growth due to the factors listed above, then Libyan reserves would be ∼28.5 billion barrels and the resulting multicyclic Hubbert model for future Libyan production would be as illustrated in Figure 7a below. If this estimate is further corrected to account for post-1995 production, then remaining reserves would be ∼20 billion barrels and overall production can be simulated by the curve set illustrated in Figure 7b.

Clearly, these solutions are not unique and other curve sets can be developed that also adequately fit these data. Hence, the results illustrated in Figure 7a,b serve only to illustrate the sensitivity of the forecast models based on the multicyclic Hubbert approach to reported reserve estimates.

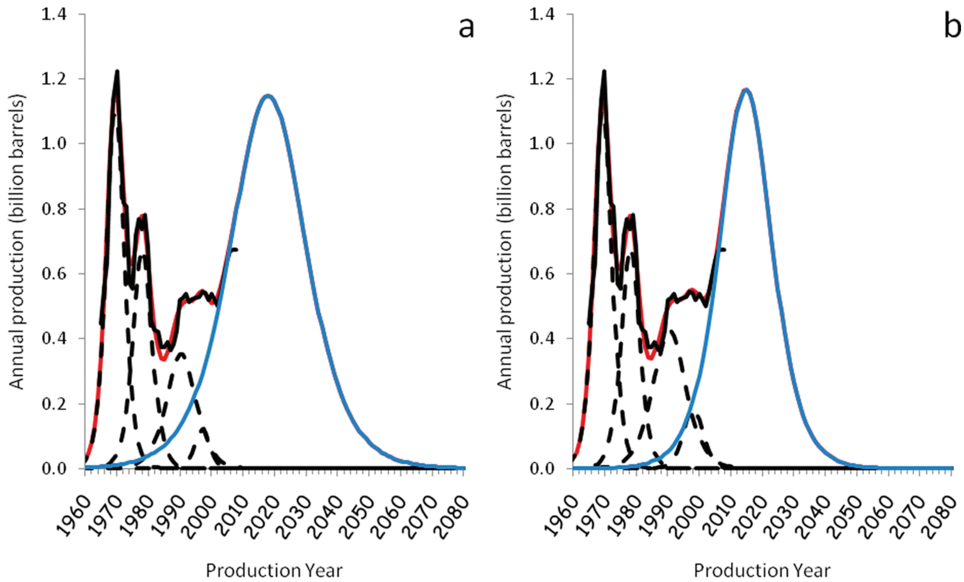

Figure 7. Hypothetical multicyclic Hubbert models of historical and future Libyan petroleum production assuming different estimated recoverable reserves: (a) estimated reserves ~28.5 billion barrels and (b) estimated reserves ~20.0 billion barrels. In each case, the historical production data are adequately modeled by 4 Hubbert cycles (the exact parameters of the individual curves vary slightly between these models to account for differences in the contribution of the curve primarily describing future production to historical production), and future production is simulated by a single curve. (a) RMSE = 0.0480; (b) RMSE = 0.0486 (Details given in Appendix 1 Table A3). Legend: see caption for Figure 3.

■ CONCLUSIONS

Multicyclic Hubbert analysis is a novel method for modeling historical and future production of finite resources such as petroleum. The approach is a curve-fitting method utilizing coaddition of multiple Hubbert curves to reproduce historical data. The accuracy to which this method is able to simulate historical production data is dependent on model parameters, especially the number of curves applied and characteristics of the individual curves contributing to the overall simulation. The basis for selection of these parameters should be clearly stated and justified and ideally should be correlated with objective data reflecting real historical events and circumstances.

The need for multicyclic analyses is based on the reality that in many cases historical production rates have been affected by a variety of external factors such as, for example, artificial production controls, natural disasters, socio-political upheavals and others, or even intrinsic issues such as new regions becoming economically or technologically feasible. In many cases, the validity of projections of future production are based on the unstated (and often unjustifiable) assumption that future production will not be constrained or affected by external factors to the same extent as historical production. Forecasts of future production are also sensitive to estimates of remaining resources and hence the usefulness of this approach as a forecasting tool is also dependent on the reliability of estimates of remaining recoverable resources that in some cases may not be entirely reliable. Given these constraints, forecasts based on multicyclic Hubbert analyses should be used with caution and with an appreciation for the underlying assumptions, limitations, and sensitivities of the approach.

■ APPENDIX 1

Parameters of Hubbert curves and multicyclic Hubbert models described in this manuscript.

Table A1. Parameters of Single Cycle Hubbert Curves Fitted to Production Data for Norway and Nigeria Using Equation 1[a] [12]

	illustrated as	a	Q_∞ billion barrels
Norway	Figure 1a	0.157	31.03
Nigeria	Figure 1b	0.0554	64.96

[a] Estimates of a and Q_∞ are derived from historical production[11] data via eq 2 using the method described by Deffeyes.

Table A2. Parameters for Curves Used in Conventional and Multicyclic Hubbert Analyses of U.K. Conventional Petroleum Production[a]

			2-curve model			10-curve model		
illustrated as	Figure 2a		Figure 2b			Figure 2c		
Hubbert production cycle	a	Q_∞ billion barrels	a	p_{max}	T_{max}	a	p_{max}	T_{max}
1	0.152	29.42	0.45	7.45	1984	1.4	0.85	1979
2			0.22	18.5	1998	0.5	6.6	1983.65
3						1.2	0.8	1986.9
4						0.4	4.7	1991
5						0.6	1.3	1994
6						1.0	1.4	1995.1
7						0.65	4.52	1999
8						1.5	0.4	2002.6
9						0.4	5.7	2005
10						2	0.25	2007.5

[a] Conventional Hubbert analysis (Figure 2a) is derived as described for Table A1 above. Multicyclic analyses derived according to eq 4.

Table A3. Parameters for Curves Used in Multicyclic Hubbert Analyses of Libyan Conventional Petroleum Production, Assuming Different Values of Remaining Reserves

assumed reserves (billion barrels)	43.66			28.50			20.00		
illustrated as	Figure 3			Figure 7a			Figure 7b		
Hubbert production cycle (i)	a	p_{max}	T_{max}	a	p_{max}	T_{max}	a	p_{max}	T_{max}
1	0.6005	1.0837	1969.30	0.5401	1.1084	1969.5	0.5294	1.117	1969.5
2	0.4313	0.6785	1977.8	0.4965	0.6773	1978.1	0.5092	0.6834	1978.2
3	0.3293	0.3317	1990.7	0.3213	0.3547	1990.4	0.2899	0.4273	1990.6
4	0.5788	0.1235	1997.3	0.6613	0.1225	1997.1	0.4935	0.1846	1998.1
future production	0.111	1.449	2023	0.125	1.1495	2018	0.18	1.167	2015

ASSOCIATED CONTENT

Supporting Information. Explanation of the Matlab script and link to it. This material is available free of charge via the Internet at http://pubs.acs.org.

AUTHOR INFORMATION

Corresponding Author
*E-mail: kanderson@geo.siu.edu.

ACKNOWLEDGMENT

The authors gratefully acknowledge Dr. John C. Crelling, Dr. Susan M. Rimmer, and Dr. Tomasz Wiltowski for helpful comments and suggestions during preparation of this manuscript. The constructive criticisms of three external reviewers are also gratefully acknowledged. K.B.A. also gratefully acknowledges the U.S. Department of Energy for partial support during completion of the work described herein under Grant USDOE-DE-SC002171.

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