Optimal Control Theory for Sustainable Environmental Management

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Sustainable ecosystem management aims to promote the structure and operation of the human components of the system while simultaneously ensuring the persistence of the structures and operation of the natural component. Given the complexity of this task owing to the diverse temporal and spatial scales and multidisciplinary interactions, a systems theory approach based on sound mathematical techniques is essential. Two important aspects of this approach are formulation of sustainability-based objectives and development of the management strategies. Fisher information can be used as the basis of a sustainability hypothesis to formulate relevant mathematical objectives for disparate systems, and optimal control theory provides the means to derive time-dependent management strategies. Partial correlation coefficient analysis is an efficient technique to identify the appropriate control variables for policy development. This paper represents a proof of concept for this approach using a model system that includes an ecosystem, humans, a very rudimentary industrial process, and a very simple agricultural system. Formulation and solution of the control problems help in identifying the effective management options which offer guidelines for policies in real systems. The results also emphasize that management using multiple parameters of different nature can be distinctly effective.

Introduction

The intensity and scale of human activities on planet Earth has led to severe depletion and deterioration of natural resources. Since this has strong implications on the continued existence of the human race, understanding and maintaining the ecosystem services is of paramount importance. Consequently, recent literature that chronicles the major causes and primary effects, predicts future consequences, and suggests remedial actions is substantial. This includes Meadows et al. (*1*), Bossel (*2*), Holling et al. (*3*), the Millennium Ecosystem Assessment (*4*), the Stern review (*5*), and the Global Environment Outlook GEO_4 report (*6*). While the initial management efforts were mainly targeted at pollution control, green engineering, and industrial ecology, it has been realized that these issues often transcend spatial and temporal boundaries. Therefore, research priorities have evolved to incorporate large scale and long-term considerations, also known as the perspective of *sustainability*.

The concept of sustainability came to fore after the Brundtland commission report where sustainable development was generically defined as "the development that meets the needs of the present without compromising the ability of the future generations to meet their own needs" (*7*). This has initiated research activities across various disciplines incorporating sustainability ideas (*8–11*). Goodland and Daly (*12*) propose three principal dimensions of sustainability as social, environmental, and ecological. Cabezas et al. (*13*) refine and further discuss various perspectives of sustainability, which include ecological (*14*), social (*15*), economic (*16–18*), technological and systems. These different aspects are in continuous dynamic interaction, and consequently a truly sustainable initiative must be a multidisciplinary effort (*19, 20*). Therefore, the goal of a sustainable management strategy is to promote the structure and operation of the human component of a system (society, economy, technology, etc.) in such a manner as to reinforce the persistence of the structures and operation of the natural component (i.e., the ecosystem) (*21, 22*).

This work proposes to use a systems theory-based approach to achieve these goals where management strategies to achieve specific mathematical objectives reflecting sustainability goals can be derived. Two important steps in this task are formulation of sustainability-based objectives and derivation (computation) of the management strategies. Fisher information-based sustainability hypotheses allow sustainability quantification and the formulation of mathematical objectives relevant to disparate systems (*23*). Once the objectives are formulated, optimal control theory can be used to derive time-dependent management strategies. Appropriate control variables for policy development can be identified using partial correlation coefficient analysis. These ideas are presented here through a case study application on a 12-compartment food web model which is a simplified representation of the critical aspects of our ecosystem. The model system consists of humans, a very rudimentary industrial process, a very simple agricultural system, and some representative elements of the natural ecosystem.

The article is arranged as follows. The next section explains various aspects of the proposed systems theory-based approach. This is followed by a discussion of the food web model and formulation of the optimal control problem for its management. The subsequent section presents the control problem results for various model scenarios leading to some general policy development guidelines for such complex systems in the last section.

Approach

Systems Approach to Sustainable Management. Achieving sustainability must embody, in some form, elements of physics, engineering, ecology, law, economics, sociology, and politics and cannot be investigated successfully within the confines of a single discipline (*19*). A systems theory approach offers a suitable methodology to achieve this integration (*24, 25*). Although optimization leading to time-independent decisions is a well established decision-making technique, such decisions might be suboptimal for natural systems, as they ignore the inherent dynamic characteristics of these systems. Moreover, such effects are often not evident immediately and manifest themselves only over a long time

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period, an important criteria for sustainability studies. Therefore, an effective approach is to use control theory to derive time-dependent management decisions. Some selected examples of this approach include renewable resource management (*22*), food chain disaster management (*26, 27*), population management through harvesting (*28*), lake water quality management (*29*), and forest fire management (*30*). Among the various control techniques, optimal control has been at the forefront, particularly in applications to natural systems, primarily due to its generality and applicability to nonlinear systems (*31*). Optimal control problems are defined in the time domain, and their solution requires establishing an index of performance for the system and designing the course (future) of action of selected parameters (control variables) so as to optimize the performance index. This work proposes to use a Fisher information-based sustainability hypothesis to formulate the performance index, as explained in the next section.

Fisher Information for Sustainability Quantification. Cabezas and Fath (*23*) have proposed to use information theory in ecology to derive a measure for the sustainability of a system, the hypothesis being based on the argument that information is a fundamental quantity of any system, irrespective of the discipline (*32*). They propose to use Fisher information as the quantity for their hypothesis. Fisher information (FI), introduced by Ronald Fisher (*33*), is a statistical measure of indeterminacy. One of its interpretations, relevant for this work, is as a measure of the state of order or organization of a system or phenomenon (*32*). The shift invariant form of the Fisher information, I, for one variable is given as (*23*)

$$I = \int \frac{1}{p(x)} \left(\frac{\mathrm{d}p(x)}{\mathrm{d}x} \right)^2 \mathrm{d}x \qquad (1)$$

where p is the probability density function (pdf) of observing a particular value of variable x. This definition can be extended to a system of n variables by letting x be the path segment or state of the system along the trajectory of the system in a space defined by n variables and time. Fisher information in this formulation is a measure of dynamic order and self-organization in the system. The sustainability hypothesis states that the time-averaged Fisher information of a system in a persistent (sustainable) regime does not change with time. Any change in the regime will manifest itself through a corresponding change in Fisher information value, usually a loss of information (*23, 34*). It should be noted that persistence of the regime does not imply system stationarity in the ecological sense. A regime can possibly have stable or unstable fluctuations, as those typically observed in natural systems. Pawlowski et al. (*35*) and Cabezas et al. (*21*) have successfully demonstrated that regime shifts in complex food webs are translated into corresponding changes in the Fisher information values. This leads to *minimization of the Fisher information variance over time* as a possible objective function for the control problem. Shastri and Diwekar have successfully used this form of the objective for developing control profiles for disaster management of a three-species predator−prey model (*26, 27*). When compared with another possible objective of maximization of FI, the FI variance minimization objective has been shown to perform better by ensuring system stability (*26*).

The ideas of systems theory-based sustainable management presented in this section are implemented on a case study example of a 12-compartment food web model. The subsequent sections present the food web model, problem formulation, and control problem results.

Twelve-Compartment Food Web Model

Basic Model. The food web model considered in this work is presented in Cabezas et al. (*21*). The model, shown in

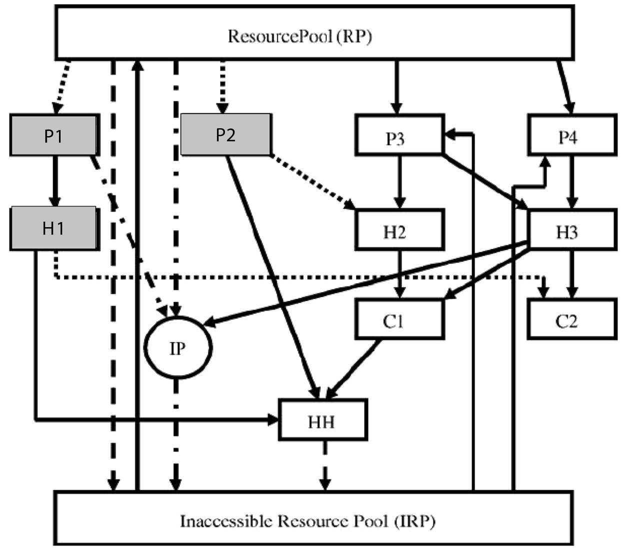

FIGURE 1. Food web model. The compartments represent a resource pool (RP), four plants (P1, P2, P3 and P4), three herbivores (H1, H2, and H3), two carnivores (C1 and C2), human population (HH), and an inaccessible resource pool (IRP). Domesticated compartments shown in shaded. The arrows respectively represent: solid = biologically driven mass flows, dot-dot = human influenced mass flow, dash-dash = waste mass flows from industry, dash-dot = flows in and out of industrial sector (*21*).

Figure 1, represents an ecosystem that includes humans, a very rudimentary industrial process, and a very simple agricultural system. The model tracks the flow of generic mass resources (biomass, nutrients, water, etc.) within a closed system (i.e., the cumulative sum of masses of all the system compartments is constant). The focus is on including the critical and representative elements of our ecosystem while keeping it simple enough for mathematical analysis. Accordingly, the model includes four aggregated trophic levels (plants, herbivores, carnivores, and humans as a top omnivore) and two resource pools. There are 12 compartments, namely a resource pool (RP) generically representing all biological resources (water, nutrients, etc.), four plants (P1, P2, P3, and P4), three herbivores (H1, H2, and H3), two carnivores (C1 and C2), a human population (HH), and an inaccessible resource pool (IRP) representing resources that are biologically unavailable as a result of human activity. The model is divided into two characteristic branches: domesticated (representing agricultural and livestock activities) comprising compartments P1, P2, and H1 on the left, and nondomesticated (representing species hunted, gathered, and species not consumed by humans) consisting of compartments P3, P4, H2, H3, C1, and C2 on the right. Humans (HH) rely on the nondomesticated branch for both resources and for the recycling of mass from the inaccessible resource pool back into the rest of the system. The industrial process is meant to represent at a very elementary level a generic human industrial activity that offers a benefit to the human population. The compartmental connectivity ensures that the important operational features of our ecosystem (such as predator−prey interactions, resource consumption and cycling, complexity, human, and industrial impacts and others) are incorporated, even though sometimes in a simplistic sense. Each compartment of the model in itself will exhibit very complex dynamics in reality. They are, however, simplified here since the emphasis is more on understanding the interactions rather than individual behavior. A comprehensive discussion of the model, including the equations based on Lotka-Volterra type expressions, can be found in Cabezas et al. (*21*).

Control Variable Identification: Model PRCC Analysis. In order to devise time-dependent management strategies for a system, it is important to first identify the appropriate control variables, which can be difficult for complex systems such as a food web model. The control variable should not only be manipulable in reality but also have a significant impact on system dynamics. This work uses stochastic analysis (*36*) based on sampling to obtain Partial Rank Correlation Coefficients (PRCC) to accomplish this task. PRCC provides a major or unique or unshared contribution of each potential control variable to a particular output variable that cannot be explained in terms of the relations of the output variable with any other model variable.

For the food web model, the following model parameters are first identified as candidate control variables:

- g_{RPP1}: Coefficient of mass transfer from RP to P1
- g_{RPP2}: Coefficient of mass transfer from RP to P2
- g_{RPP3}: Coefficient of mass transfer from RP to P3
- g_{RPP4}: Coefficient of mass transfer from RP to P4
- g_{H1C2}: Coefficient of mass transfer from H1 to C2 (influenced by humans)
- *W*: The waste flow term associated with human consumption
- α: Constant reflecting the effectiveness or efficiency of the industrial process in reducing the human mortality rate
- η: Organizational or infrastructure costs associated with the industrial process, rendering inaccessible a portion of the resource pool in proportion to the flow through the industrial process.

The mass transfer coefficients from RP to various plants govern the consumption patterns of natural resources. Since P1 and P2 are domesticated (agriculture), manipulation of g_{RPP1} and g_{RPP2} reflect changes in the intensity of agricultural activities. Various nonagricultural resource consumption activities (often indirect) such as carbon sequestration are manipulated through g_{RPP3} and g_{RPP4}. To compute the PRCC values, the candidate control variables are sampled in the neighborhood of the base case values (500 samples considering $\pm$ 10% variation) using the highly efficient Hammersley Sequence sampling (HSS) technique (*37*). The model is simulated for each sample set. The output variables are the compartmental mass values of the 12 compartments. The PRCC values are calculated for each possible pair between the set of the potential control variables and the set of model output variables. The sum of the absolute values of all PRCCs for each potential control variable is compared to rank those in terms of model impact. The results show that g_{RPP1}, g_{RPP2}, *W*, and α are much more effective than the other four potential control variables. These four parameters are, therefore, selected as control variables for further analysis.

Optimal Control Problem Formulation. This work uses the Pontryagin's maximum principle to formulate and solve an optimal control problem (*31, 38*). The control trajectory obtained using this approach is optimal for the considered objective function and the starting conditions. If I_t represents the time averaged Fisher information for a system with *n* states, the objective of FI variance minimization over time is mathematically represented as:

$$J = \min \int_0^T (I_t - I_{\text{constant}})^2 \mathrm{d}t \tag{2}$$

Here, *T* is the total time horizon under consideration and I_{constant} is the constant around which the Fisher information variation is to be minimized. The exact objective function formulation depends on the model variables that are being used as the sustainability indicators. This is decided using the results from the PRCC analysis and also based on the dynamics of the uncontrolled system. Owing to the mathematical complexity of the resulting control problem, it is solved using the numerical technique of steepest ascent of Hamiltonian (*31*).

Model Scenarios. The aim is to explore and rank different control possibilities for the food web model. For this, different cases of the food web model with undesirable dynamics requiring external intervention are simulated. The objective is to recover the system from the disturbance in a sustainable manner, i.e. to achieve dynamic stability. The scenarios, based on the work presented in Cabezas et al. (*21*), are described here.

Case A: Minor Reduction in α. This scenario models reduction in the effectiveness of the industrial process in reducing human mortality rate (α). Such a possibility can be attributed to future unexpected realization of harmful health effects of current technology (such as emissions). For the scenario, α is reduced from its base case value by 15%. This leads to significant instability in compartments P1, H1, and C2 with the mass in these compartments continuously decreasing. The effect on the other compartments is not severe.

Case B: Doubling the Industrial Process Flow IPF. Parameter IPF in the model represents the flow through the industrial process. Hence, change IPF models a change in the size of the industrial sector. For this scenario, the value of IPF is doubled from its base case value. This results in an unstable system with significant instability in compartments P3, H2, and H3, extinction in compartment H1, onset of extinction in compartment C2, and onset of instability in compartments P2, P4, HH, and IRP.

Case C: Change in IP Input Composition. The industrial process (IP) takes mass from three compartments (P1, RP, and H3) in different proportions and combines it to form a product used by the humans. The base case input composition to IS is: 2% P1, 7% RP, and 91% H3. In this scenario the input configuration is altered to 40% P1, 55% RP, and 5% H3. This change reflects a shift in the production policies of IP depending on the availability and price of various resources. This change affects all the compartments of the model and compartments P3, H1, H2, C2, HH, and IRP are particularly perturbed.

Case D: Change in W. In this case, the constant for the waste term associated with human consumption, *W*, is increased from its base case value by 70%. This results in the uncontrolled case with slight change in the dynamics of compartments P1, P2, P4, C1, temporary disturbance in the dynamics of compartment H1, H2, H3, HH, IRP, and a sustained decrease in the mass of compartment C2. The degree of disturbance caused by this change is less severe than the other cases considered here. Since *W* is changed to simulate the case, use of *W* as the control variable is omitted.

Case E: Severe Reductin in α. Drastic changes in the parameters of an ecosystem can cause shifts in the dynamic regimes of these systems, which can be stable or unstable. Such changes are caused by the natural disasters such as floods, hurricanes, or dramatic changes in the global climatic conditions (*39*). Quite often, these regime shifts are nonlinear, exhibiting phenomenon like hysteresis, meaning that the restoration of the original regime is complicated. The purpose of modeling a major change in α is to simulate such a case of severe disturbance and then assess the ability of various control options. Here, the value of α is reduced by 50% for the uncontrolled system. This results in fast decrease in the compartmental mass of P1, and it goes to extinction toward the end of the considered time horizon. This causes other compartments to show undesirable variations. Other compartments that are independently disturbed include H1 and C2.

For each scenario, the total simulation time horizon is 5000 time steps, where the cycle time for the model (based on the cyclic input) is 500 time steps. If one cycle is assumed

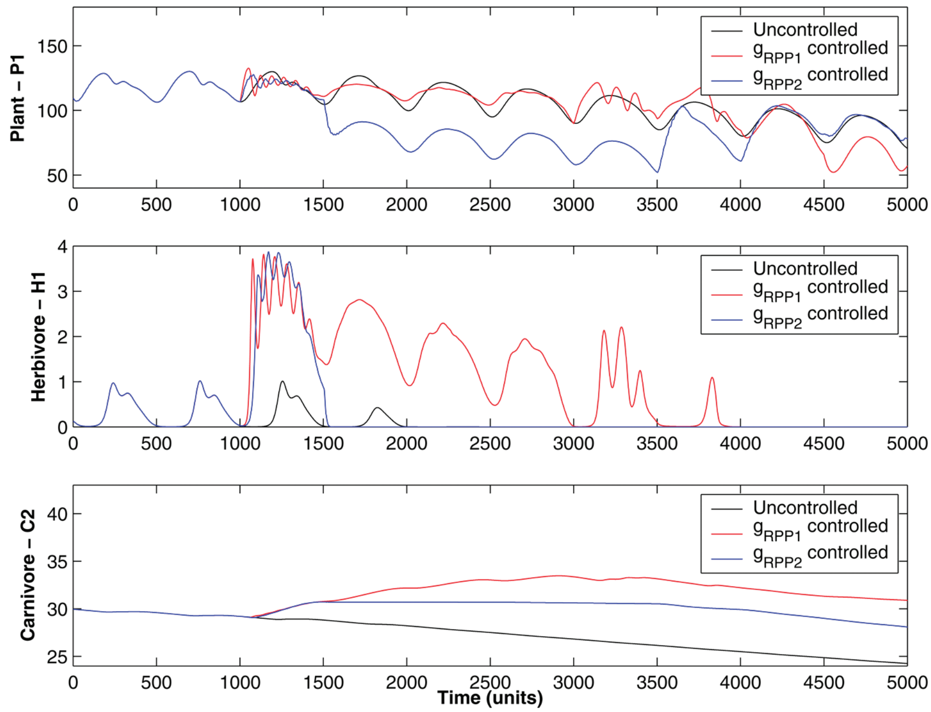

FIGURE 2. Case A: Minor reduction in α. Control problem results.

to be equal to one year, then the total simulation horizon is approximately equal to 10 years. The parameters to be disturbed are linearly ramped to the new values in 500 time steps between the 1000th and 1500th time steps. Control is exercised only after the disturbance is introduced in the model, i.e. only after 1000 time steps. The control variables stay at the base values till 1000 time steps. The objective function requires a constant FI value around which the variance is to be minimized ($I_{constant}$). This value is taken as the average FI for the first two undisturbed simulation cycles (till time step 1000). The control problem is formulated and solved for each case using all possible control variables. The base case values of the parameters, reported in Cabezas et al. (*21*), are used as the starting guess for the steepest ascent algorithm. The important results are presented in the following sections.

Results and Discussion

Case A: Minor Reduction in α. The uncontrolled dynamics for P1, H1, and C2, along with the controlled dynamics using g_{RPP1} and g_{RPP2} as control variables are shown in Figure 2. The results show that g_{RPP1} has a stronger effect than g_{RPP2} on the model dynamics. Although extinction of H1 cannot be eliminated, it is considerably delayed using g_{RPP1} as control variable. g_{RPP2} has a significant impact only on compartment C2. The results also show that changes in g_{RPP2} lead to disturbances in other compartments of the model. The control profiles illustrate that there is about 10% increase in g_{RPP1} and 8% rise in g_{RPP2}. Thus, the reduction in the effectiveness of IP is offset by drawing more resources from RP. Since α is changed to simulate the case, it is not used as the control variable. The use of *W* as control variable had insignificant impact on all the compartments, and hence the results are not reported here.

Case B: Doubling the Industrial Process Flow IPF. The uncontrolled profile for some of the compartments and the results using g_{RPP1} and α as control variable are shown in Figure 3. A comparison between the two results suggests that g_{RPP1} is effective in avoiding instability in the model, while α is more effective in avoiding compartment extinction. The control profiles illustrate that there is an average increase of about 2.5% in the value of g_{RPP1}. Moreover, g_{RPP1} is continuously fluctuating in a cyclic manner in order to reduce the instability in certain compartments. When α is the control variable, about 15% increase in the value of α is required to avoid extinction of certain compartments. The results using g_{RPP2} as control variable are qualitatively similar, however inferior, to those using g_{RPP1} as control variable. Using *W* as control variable does not result in significant change in the dynamics of most compartments.

Case C: Change in IP Input Composition. The control problem solution results show that using *W* and α as control variables does not lead to favorable results. On the contrary, some of the compartments become unstable after enforcing control. Using g_{RPP2} as the control variable also results in significant disturbance in some of the compartments and the overall controlled response is not satisfactory. g_{RPP1}, on the contrary, is very effective in controlling the system. It can be observed that most of the compartments show stable dynamics which are very similar to those observed for the undisturbed model (plots included in Supporting Information). The control profile shows that about 15% increase in value of g_{RPP1} is necessary.

Case D: Change in *W*. A comparison of the results (plots included in Supporting Information) indicates that changes in α lead to most effective control of the compartments showing temporary extinction. Changes in g_{RPP1} and g_{RPP2} also result in favorable model dynamics. It is again observed that g_{RPP1} and g_{RPP2} have similar qualitative effects, however, g_{RPP1} has a stronger impact on the model dynamics than g_{RPP2}.

Case E: Severe Reduction in α. After solving the control problem for g_{RPP1}, g_{RPP2} and *W*, it is observed that none of these are able to recover the system from the disaster. While *W* has very little favorable impact, changes in g_{RPP1} lead to marginal improvement in model dynamics. These observations illustrate that if the disturbance is too large, it may not be possible to recover the system using the given control options.

Multiple Control Analysis. The previous results illustrate that different control variables have a different relative effect on various model compartments, depending on the location

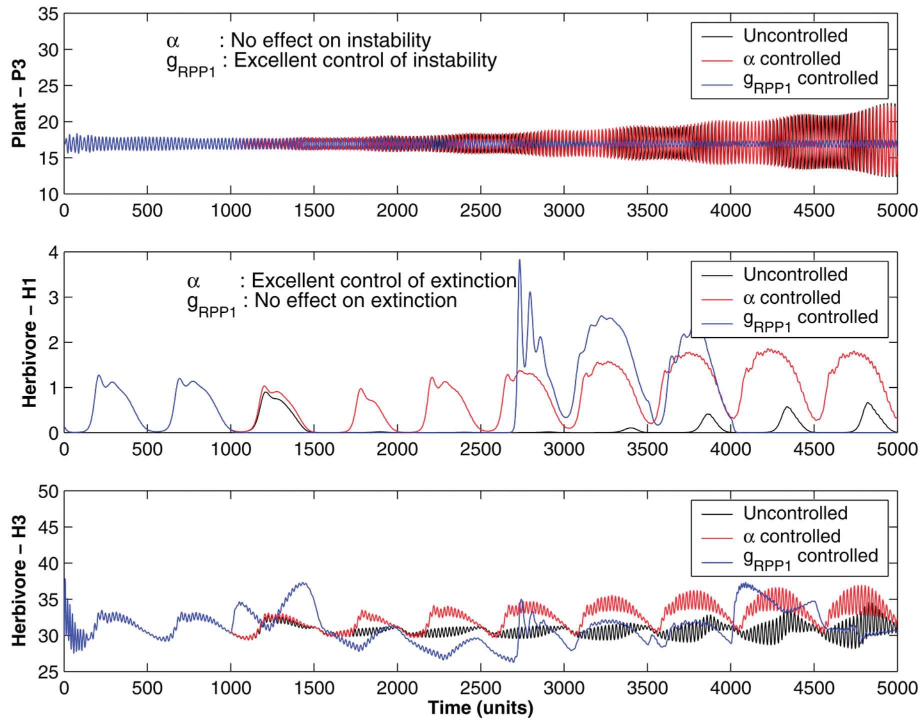

FIGURE 3. Case B: Doubling the industrial process flow IPF. Control problem results.

of a particular compartment in the food web. Moreover, some variables are more effective in controlling model instability, while others are more effective in avoiding extinction. This prompts one to think about multivariable control as an option where more than one model parameters are simultaneously manipulated. Multivariable control is often complicated due to the interactive effects (coupling) of control variables on model dynamics. Owing to these complications, this work looks at a variant of the actual multivariable control. Here, two different single variable control problems are solved sequentially. The model is first subjected to one control action (primary control action), using one parameter as the control variable, referred to as the primary control variable (CV-1). In the next step, another control problem with a different control variable, referred to as the secondary control variable (CV-2), is solved using the primary controlled model as the starting point. The time-dependent profile of CV-1 is based on the primary control problem solution and does not change during the second control problem solution. The solution of the second control problem thus has time-dependent profiles for CV-1 as well as CV-2.

To verify the effectiveness of this approach, the previously discussed scenario of doubling of the industrial process flow IPF is solved with this method. Please refer to those sections for scenario details and results using the primary control variable. To summarize, it is observed that g_{RPP1} effectively controls the instability of the compartments while α effectively eliminates species extinction. Hence, these two parameters are selected as the control variables for the model. α is the primary control variable (CV-1), while g_{RPP1} is the secondary control variable (CV-2). The representative results are plotted in Figure 4. The results clearly indicate that the model dynamics using multiple controls are much better than those with single variable control. Using only α as the control variable, instability in some of the compartments is not avoided. However, additional control using g_{RPP1} resulted in effective control of the model instability.

When different combinations of control variables are used, it is observed that the success is still not guaranteed. Using g_{RPP1} and g_{RPP2} as control variables does not improve the dynamics, most likely because both these parameters are similar (controlling resource flow in the food web) and are at the same trophic level. g_{RPP1} and α, however, pertain to distinctly different things in the model and hence are less likely to suffer from coupling effects. These results lead to a very interesting argument that to achieve sustainability goals for a complicated system like this model, use of multiple control pertaining to different aspects of the system is effective.

Summary of Results. The previous sections presented and discussed the results for various cases on the 12-compartment food web model. The important conclusions to be drawn from those results are as follows:

• g_{RPP1}, the coefficient of mass transfer from RP to P1, is overall the most effective control variable, possibly because it is the input of mass into P1 which is directly connected to H1 and IP which are critical to the rest of the system. The performance of g_{RPP2}, the coefficient of mass transfer from RP to P2, as a control variable is qualitatively similar but inferior to g_{RPP1}, possibly because P2 is only one of the three food sources for HH.

• The relative success of g_{RPP1} and g_{RPP2} in controlling different compartments depends on the position of the compartment in the food web. Thus, g_{RPP2} is observed to be more effective in manipulating compartment C2 while g_{RPP1} is more effective is manipulating compartment H1. It is also observed that g_{RPP1} and g_{RPP2} have opposite effects on many compartments because of the sharing of RP.

• α is moderately effective in controlling model dynamics while *W* is the least effective control option.

• g_{RPP1} and g_{RPP2} are very effective in controlling the instability while α is very effective is averting the extinction of mass in various compartments.

• The use of multiple control variables leads to improved dynamics in some cases. This suggests that appropriate combination of multiple control variables can be an effective management option.

• A severe disturbance in the system dynamics cannot be effectively managed, and the system becomes unsustainable.

It can therefore be concluded that controlling factors at the base of the food chain is more critical than attempting modifications toward the high end of the food chain.

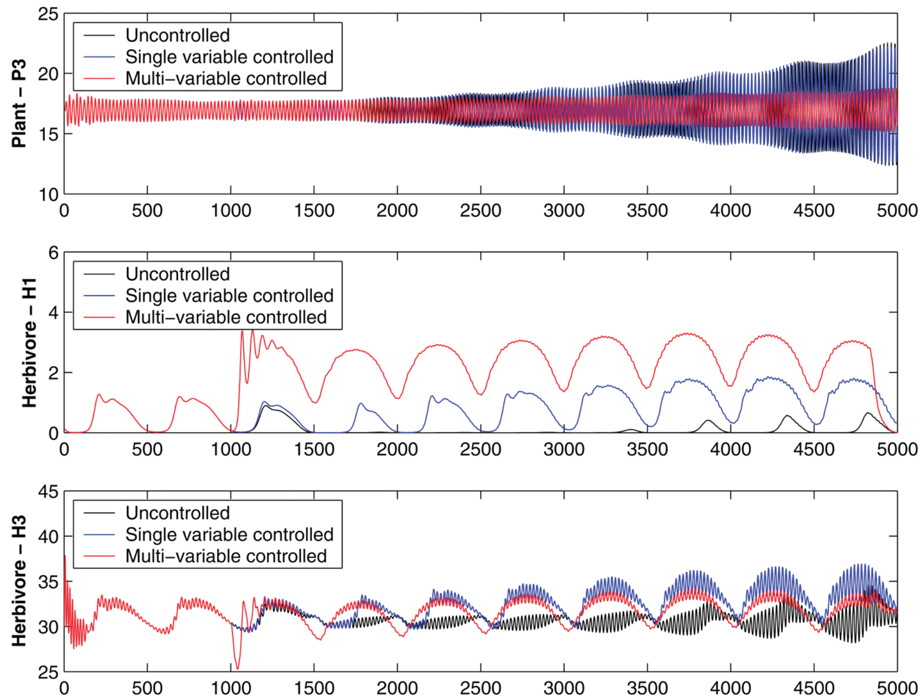

FIGURE 4. Multiple control solution for doubling of industrial process flow IPF.

Modifications in the industrial and human sectors (such as changes in α and *W*) cannot alter the basic natural resource consumption patterns (decided by g_{RPP1} and g_{RPP2}). Since these patterns are very critical for the sustainability of the natural compartments, regulation of those is more successful than regulation of α and *W* as far as the whole system is concerned. Also important is the observation that certain variables are more effective in averting extinction while others are more effective in controlling instability. Hence, the choice of control variables can be critical to the success of sustainable environmental management. These are important conclusions from sustainable management perspective.

Conclusion

The work proposes sustainable environmental management using a systems theory approach and a Fisher information-based sustainability quantification and presents its implementation on a 12-compartment food web model. The results highlight the importance of carefully choosing control variables in the model and also rank these variables in terms of their success. Multivariate control using variables of distinctly different nature is shown to be more effective than control using a single variable. This is consistent with current thinking which views sustainable environmental management as a multidisciplinary effort. The disciplines in this work are abstractly represented by different control variables. From implementation perspective on a specific ecosystem management problem, this approach should be adapted to gain qualitative understanding of the management options and to eliminate nonoptimal strategies. The qualitative insights can then guide detailed modeling efforts for a quantitative analysis. The time-dependent results can also be approximated using piecewise profiles that will be easier to implement. However, on a broader perspective, the results should be viewed as a proof of concept for the application of systems theory-based techniques to sustainability. The basic approach is system independent, and the consideration of complex dynamical features (such as stochasticity in natural compartments) can be incorporated without modifications to the concept. Understanding the indirect and nonintuitive effects of such features on the complete system will a critical aspect of the extension. The practical success of the approach though will depend significantly on the ability of the models to reproduce reality. The important conclusions and results from this study therefore should form the basis for the use of such approaches for more complicated and realistic models.

Acknowledgments

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Supporting Information Available

Food web model equations, brief review of optimal control theory, Partial Rank Correlation Coefficient analysis results, detail plots of various case study results. This information is available free of charge via the Internet at http://pubs.acs.org.

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