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Multivariate Control Performance Assessment and Control System Monitoring via Hypothesis Test on Output Covariance Matrices

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ABSTRACT: Control loops widely exist in industrial processes, whose performance directly influences the efficiency, safety, and product quality of production plants. Therefore, control performance assessment (CPA) and control system monitoring (CSM) are critically important for industrial processing. In consideration of multivariate control systems, the covariance matrix of closed-loop outputs plays an important role in both CPA and CSM. Existing methods mainly focus on comparing traces or determinants of the output covariance matrices, which only utilize partial information contained in the matrices. As a result, the assessment and monitoring results may be misleading. In this paper, a multiobjective scheme is proposed for both CPA and CSM of multivariate control systems, which takes the entire covariance matrices into account by conducting a hypothesis test on the equality of the matrices. To fulfill the presupposition of such test, autoregressive moving-average (ARMA) filters are established to remove the autocorrelation contained in the closed-loop output data. The developed scheme can be divided into three aspects: CPA using a minimum variance (MV) benchmark, CPA using a user-specified benchmark, and CSM based on historical data. Case studies show that, compared with the conventional approaches, the proposed method provides more abundant information and achieves better results.

1. INTRODUCTION

In industrial processes, there exist a large number of control loops operating under different conditions. The performance of these control loops may deteriorate due to changes in feedstock, malfunction of sensors and/or actuators, and unanticipated disturbances. It has been reported that numerous industrial controllers have performance problems. In many cases, the control systems may even increase the process variability.[1] Therefore, control performance assessment (CPA) and control system monitoring (CSM) have attracted growing research interests in both academic and industrial societies.

The objective of CPA is to indicate how close the current control performance is to the ideal performance. According to Hugo,[2] a CPA technique should be independent of disturbances or set-point changes. In other words, although the disturbances and set-points may vary widely in a plant, the CPA index should be insensitive to the time period when the data were taken. Another type of technique conducts statistical process monitoring (SPM) on control systems, which is named CSM in this paper. Different from CPA, CSM aims to detecting changes in control systems, which is required to be sensitive to the uncommon disturbances or set-point changes. Therefore, CPA and CSM often reflect two complementary types of control system properties.

There is no simple statistic that can measure all aspects of control performance, since industrial controllers are implemented with various design objectives, such as set-point tracking, disturbance rejection, constraint handling, surge attenuation, among others.[3] In consideration of a single-input single-output (SISO) or multi-input single-output (MISO) control system, a widely adopted criterion for CPA is the variance of a closed-loop output, which measures the stochastic performance of the control loop and directly relates to process safety, product quality, and profit. In the pioneer research by Harris,[4] an index was proposed by using a minimum variance (MV) benchmark. It has been proved that, for a linear system, a minimum variance controller results in the smallest possible output variance, which provides a reference bound on achievable control performance. Accordingly, the Harris index is defined as the ratio of the minimum achievable variance and the actual output variance, whose value varies from 0 to 1. Although the design of a minimum variance controller requires a perfect process model and a perfect disturbance model, the output response under minimum variance control (MVC) can be determined easily using only routine closed-loop data and an estimate of process dead-time. Desborough and Harris[5] showed the correlation between the Harris index and the squared coefficients in regression analysis. Since then, the MV benchmark has been widely accepted in the research field of CPA. Desborough and Harris[6] showed the capability of the MV benchmark in the performance assessment of univariate feedforward/feedback control systems. Tyler and Morari[7] modified such benchmark to assess unstable and nonminimum-phase systems. Lynch and Dumont[8] proposed to use Laguerre networks to evaluate the minimum achievable output variance. Chen and Kong[9] developed a method to estimate the MV bounds for the assessment of batch control systems. In addition to the Harris index and its extensions, other types of benchmarks have also been adopted in the field of CPA (e.g., the generalized minimum variance (GMV) benchmark,[10] the linear quadratic Gaussian (LQG) benchmark,[11] etc). More comprehensive reviews of CPA

Received: July 9, 2014
Revised: December 14, 2014
Accepted: April 20, 2015

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can be found in the literature (e.g., ref 3,12,13 and in a textbook (e.g., ref 14).

The concept of the MV benchmark has also been extended to multi-input multioutput (MIMO) systems for multivariate CPA[15−18] which compares the current closed-loop output covariance and the output covariance under multivariate MVC. In a MIMO system, the interactor matrix plays a similar role to the dead-time in a SISO/MISO system, which should be estimated before performing CPA. Huang et al.[19] proposed to estimate the unitary interactor matrix using routine operating data, based on which an MV benchmark can be obtained. A number of methods have been developed to further simplify the calculation. McNabb and Qin[20] suggested to use a subspace approach to obtain the extended interactor matrix. Ko and Edgar[21] estimated the MV performance bounds in multivariate control loops by using the first several Markov parameters and a set of closed-loop operating data. Huang et al.[22] developed a suboptimal multivariate control benchmark from routine operating data. Such benchmark can be obtained on the basis of the order of the interactor matrix, and the complete knowledge of such matrix is not necessary. Xia et al.[23] showed that the upper and lower bounds of the multivariate MV performance can be estimated from routine operating data with the input/output (I/O) delay matrix known.

Although MVC provides a global minimum benchmark for multivariate CPA, it is difficult or even impossible to be achieved in practice. Therefore, it may be more realistic to use an achievable benchmark. For example, Huang and Shah[11] modified the MV benchmark to a user-specified benchmark by setting desired closed-loop dynamic responses. To avoid the use of the interactor matrix, Yuan et al.[18] proposed to set both upper and lower bounds for the true value of the user-specified benchmark. More recently, Chen and Wang[24] integrated the techniques of principal component analysis (PCA) and autoregressive moving average (ARMA) to develop an achievable minimum variance (AMV) benchmark.

In addition, Yu and Qin[25] suggested to make use of a data-driven covariance benchmark, so as to avoid the dependence on the interactor matrix. In their method, the closed-loop output covariance of the current control system is compared with the historical covariance derived from a period of "golden" operation data. The comparison results are dependent on disturbances or set-point changes, which violates Hugo's definition for CPA.[2] Therefore, they actually proposed a CSM method.

As discussed above, the comparison between different output covariance matrices serves as an important role in both multivariate CPA and CSM. Traditionally, the sum of diagonal entries in the covariance matrix (i.e., the trace) is utilized to summarize the entire matrix. Accordingly, the performance index is obtained by calculating the ratio between the traces of the current output covariance matrix and that achieved under the multivariate MVC law (e.g.,15,16,23). Such index only considers the variance information in each control loop and ignores the correlation between variables. In ref 20, McNabb and Qin demonstrated that such index is not sufficient for assessing the control performance of a MIMO system. Instead of extracting the trace information, they proposed to calculate the determinant that measures the variance−covariance inflation. Accordingly, a CPA index is defined as the ratio between the determinants of different output covariance matrices. Later, Yu and Qin[25] developed an index for multivariate CSM on the basis of a similar idea. However, as pointed out by Harris et al.,[15] such a simple magnitude measure still has chances to omit important changes in the multivariate covariance structure of the control system.

In this paper, a multiobjective CPA/CSM scheme is developed for MIMO control systems. The major contributions of this paper are of two aspects. First, a hypothesis test on the equality of the output covariance matrices is proposed for both multivariate CPA and CSM. Accordingly, a new index is also proposed. Compared with the conventional indices based on the trace or determinant information, the proposed index is more sensitive to the changes in the multivariate covariance structure of the control system. Second, the multiobjective scheme consists of three aspects, providing a comprehensive picture of the investigated control system. First, CPA can be conducted on the basis of the multivariate MV benchmark that is global optimum. Second, the current control performance can also be compared with a user-specified benchmark derived from a period of "golden" operation data which is more achievable. Third, CSM is implemented by monitoring the closed-loop output covariance in a statistical way, aiming to detecting the changes in control systems. In contrast, the conventional CPA/CSM methods only reflect partial information on the system.

The paper is organized as following. In section 2, a brief review of the fundamentals of the multivariate MV benchmark is provided. Then, the procedure of hypothesis testing on the equality of covariance matrices is introduced in section 3, together with the discussions on the data filtering method. In section 4, the multiobjective CPA/CSM scheme is proposed, and the detailed steps are described. The case studies provided in section 5. Finally, conclusions are drawn in section 6.

2. MULTIVARIATE MINIMUM VARIANCE BENCHMARK

Consider a MIMO system, whose input-output behavior is described by a linear transfer function with additive disturbance:

$$\boldsymbol{y}(k) = \boldsymbol{G}_p(q)\boldsymbol{u}(k) + \boldsymbol{G}_\varepsilon(q)\boldsymbol{\varepsilon}(k) \tag{1}$$

where $\boldsymbol{y}(k)$ and $\boldsymbol{u}(k)$ are the output and input vectors at discrete time k of appropriate dimensions; $\boldsymbol{\varepsilon}(k)$ contains multivariate white noise, which is independent and identically distributed (i.i.d.); $\boldsymbol{G}_p(q)$ and $\boldsymbol{G}_\varepsilon(q)$ are proper, rational polynomial transfer function matrices in the backshift operator q^{-1}. To assess the control performance of such process, it is desired to derive the closed-loop outputs under the multivariate MVC as a benchmark. For such derivation, the interactor matrix, representing the time delay structure of the multivariate process, is required to be known.

For any $r \times a$ transfer function matrix $\boldsymbol{G}_p(q)$, the interactor matrix is defined as a nonsingular $r \times r$ polynomial matrix $\boldsymbol{D}$ that satisfies the following two conditions:

$$\det \boldsymbol{D}(q) = q^n \tag{2}$$

$$\lim_{q^{-1}\to 0} \boldsymbol{D}(q)\boldsymbol{G}_p(q) = \boldsymbol{K}$$

where $\boldsymbol{K}$ is a finite and full-rank matrix, and n is the number of infinite zeros of $\boldsymbol{G}_p$. The Markov parameter representation of $\boldsymbol{D}$ is

$$\boldsymbol{D}(q) = \boldsymbol{D}_0 q^{\tau} + \boldsymbol{D}_1 q^{\tau-1} + \ldots + \boldsymbol{D}_\nu q^{\tau-\nu} \tag{3}$$

where τ, the maximum power of q in $\boldsymbol{D}$, is the order of the interactor matrix, which is unique for a given $\boldsymbol{G}_p(q)$; ν is the relative degree of $\boldsymbol{D}$, equaling to the difference between the maximum and minimum power of q in $\boldsymbol{D}$; and $\boldsymbol{D}_i$ ($i = 0, 1, \ldots, \nu$) are called coefficient matrices. By restricting $\boldsymbol{D}$ to be lower left triangular, the interactor matrix is unique. Different forms of the

DOI: 10.1021/ie502743f
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interactor matrix have been proposed in the literature, among which a particularly useful form is the unitary interactor matrix[26] with a feature in which

$$D^T(q^{-1})D(q) = \mathbf{I} \tag{4}$$

Using D, eq 1 can be reorganized as

$$\tilde{y}(k) = q^{-\tau}\tilde{G}_p(q)u(k) + \tilde{G}_\varepsilon(q)\varepsilon(k) \tag{5}$$

where $\tilde{G}_p(q) = D(q)G_p(q)$, $\tilde{G}_\varepsilon = q^{-\tau}DG_\varepsilon$, and $\tilde{y}(k) = q^{-\tau}D(q)y(k)$ is the interactor-filtered output. Based on the Diophantine identity, $\tilde{G}_\varepsilon$ can be expressed as

$$\tilde{G}_\varepsilon = q^{-\tau}DG_\varepsilon = \tilde{F}_0 + \tilde{F}_1q^{-1} + ... + \tilde{F}_{\tau-1}q^{-(\tau-1)} + q^{-\tau}\tilde{R} = \tilde{F} + q^{-\tau}\tilde{R} \tag{6}$$

Accordingly

$$\tilde{y}(k) = \tilde{G}_p(q)u(k-\tau) + \tilde{R}(q)\varepsilon(k-\tau) + \tilde{F}\varepsilon(k) \tag{7}$$

Obviously, the control action cannot affect the last term in the above equation, which means

$$\tilde{y}_{mv}(k) = \tilde{F}\varepsilon(k) = \tilde{F}_0\varepsilon(k) + \tilde{F}_1\varepsilon(k-1) + ... + \tilde{F}_{\tau-1}\varepsilon(k-(\tau-1)) \tag{8}$$

when the multivariate minimum variance control is conducted. Hence, the unfiltered output under MVC is derived as

$$y_{mv}(k) = q^{\tau}D^{-1}(q)[\tilde{F}_0\varepsilon(k) + \tilde{F}_1\varepsilon(k-1) + ... + \tilde{F}_{\tau-1}\varepsilon(k-(\tau-1))] \tag{9}$$

If the unitary interactor matrix is utilized, then

$$D^{-1}(q) = (D_0q^{\tau} + ... + D_{\tau-1}q)^{-1} = D_0^Tq^{-\tau} + ... + D_{\tau-1}^Tq^{-1} \tag{10}$$

and

$$y_{mv}(k) = q^{\tau}(D_0^Tq^{-\tau} + ... + D_{\tau-1}^Tq^{-1})(\tilde{F}_0 + \tilde{F}_1q^{-1} + ... + \tilde{F}_{\tau-1}q^{-(\tau-1)})\varepsilon(k) \tag{11}$$

In (11), $\tilde{F}_i$ can be calculated as

$$\begin{aligned}\tilde{F}_i &= \mathrm{E}\{\tilde{y}(k)\varepsilon^T(k-i)\}\mathrm{E}\{\varepsilon\varepsilon^T\}\\ &= \mathrm{E}\{q^{-\tau}D(q)\tilde{y}(k)\varepsilon^T(k-i)\}\mathrm{E}\{\varepsilon\varepsilon^T\}\end{aligned} \tag{12}$$

where E is a symbol for mathematical expectation. Therefore, $y_{mv}(k)$ can be obtained if the information on y, ε, and D is available. The closed-loop output y is easy to measure from the system, from which a time-series model (e.g., a multivariate autoregressive moving-average (ARMA) model) is estimated:

$$\hat{A}(q)y(k) = \hat{C}(q)\varepsilon(k) \tag{13}$$

where $\hat{C}(q)$ and $\hat{A}(q)$ are the polynomial coefficient matrices in the backshift operator q^{-1}. Based on eq 13, an estimate for ε is found.

$$\varepsilon(k) = \hat{C}^{-1}(q)\hat{A}(q)y(k) \tag{14}$$

In addition, the unitary interactor matrix D can also be determined using the closed-loop data. More details can be found in refs 14,19.

The estimated outputs under MVC, y_{mv}, are then served as a benchmark for the performance assessment of the MIMO control system. It is noted from the above discussion that y_{mv} can be predicted on the basis of routine operating data without designing a real multivariate minimum variance controller.

3. HYPOTHESIS TEST FOR MULTIVARIATE CONTROL PERFORMANCE ASSESSMENT

3.1. Motivations. After achieving y_{mv}, the multivariate performance index is usually determined by

$$\eta_{trace} = \frac{\mathrm{tr}\{S_{mv}\}}{\mathrm{tr}\{S\}} \tag{15}$$

or

$$\eta_{det} = \frac{|S_{mv}|}{|S|} \tag{16}$$

where S_{mv} is the sample covariance matrix of y_{mv}, S is the sample covariance matrix of y, and $\mathrm{tr}\{X\}$ is the trace of the matrix X. These indices attempt to compare Σ_{mv} and Σ, the population covariance matrices of y_{mv} and y, using the ratio between the traces or determinants of the corresponding sample covariance matrices. If the value of η_{trace} or η_{det} is close to 0, one may conclude that Σ and Σ_{mv} is not similar. Hence, there is a gap between the performances of the current control algorithm and the minimum variance control. On the other hand, if the ratio value is close to 1, Σ and Σ_{mv} are regarded as similar, indicating a good control performance. However, a problem of these indices is that neither trace nor determinant contains complete information on the multivariate covariance structure. As a result, the assessment based on such indices may be misleading. Therefore, it is necessary to develop an algorithm to perform the MVC-based control performance assessment by comparing the covariance matrices in a more comprehensive way. Similar problems exist in the multivariate CPA based on the user-specified benchmark and CSM.

In the society of statistics, Alt proposed a generalized likelihood ratio (GLR) statistic[27] for testing H_0: $\Sigma = \Sigma_0$ vs H_1: $\Sigma \neq \Sigma_0$, where the charting statistic is designed as

$$W = -(l-1)\left[p + \ln\frac{|S|}{|\Sigma_0|} - \mathrm{tr}\{\Sigma_0^{-1}S\}\right] \tag{17}$$

where S is the sample covariance matrix corresponding to Σ, l is the sample size (i.e., the number of observations used to calculate S), and p is the number of variables. By comparing W to a control limit computed on the basis of a χ^2 distribution, the test can be achieved. It is noted that the statistic W utilizes more complete information contained in the covariance matrix than the trace or determinant. However, such an index cannot be adopted directly in multivariate CPA to test whether Σ and Σ_{mv} are same, because the exact population covariance matrix Σ_{mv} (i.e., Σ_0 in eq 17) is required to be known, which is not achievable in real industry.

3.2. Hypothesis Test on the Equality of Covariance Matrices. To solve the problems of the existing methods, it is proposed in this paper to use a hypothesis test to assess the equality of the complete covariance matrices derived from control system outputs.

It is well-known that the equality of two population variances (H_0: $\sigma_1^2 = \sigma_2^2$ vs H_1: $\sigma_1^2 \neq \sigma_2^2$) can be tested through an F-test on the ratio between the corresponding sample variances s_1^2/s_2^2. In ref 28, a similar idea was extended to covariance matrices for testing H_0: $\Sigma_1 = \Sigma_2$ vs H_1: $\Sigma_1 \neq \Sigma_2$. The detailed steps are described as follows.

(1) Calculate the sample covariance matrices S_1 and S_2 from samples Y_1 and Y_2, respectively.

DOI: 10.1021/ie502743f
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(2) Compute the quantity M, where

$$M = (n_1 + n_2 - 2)\log|\mathbf{S}| - (n_1 - 1)\log|\mathbf{S}_1| - (n_2 - 1)\log|\mathbf{S}_2| \quad (18)$$

$$\mathbf{S} = \frac{(n_1 - 1)\mathbf{S}_1 + (n_2 - 1)\mathbf{S}_2}{n_1 + n_2 - 2} \quad (19)$$

the logarithms are to the base 10, and n_1 and n_2 are the sizes of the samples from which $\mathbf{S}_1$ and $\mathbf{S}_2$ are derived, respectively.

(3) Compute the quantity m:

$$m = 1 - \left[\frac{1}{(n_1 - 1)} + \frac{1}{(n_2 - 1)} - \frac{1}{(n_1 + n_2 - 2)}\right]\left[\frac{2p^2 + 3p - 1}{6(p + 1)}\right] \quad (20)$$

where p is the sample dimension, i.e. the number of variables.

(4) Calculate the statistic w:

$$w = 2.3026mM \cong \chi^2_{p(p+1)/2} \quad (21)$$

which obeys a χ^2-distribution with degree of freedom $p(p + 1)/2$.

(5) Compare w to the specified control limit. If the value of w locates within the control limit, $\boldsymbol{\Sigma}_2$ is regarded to be the same as $\boldsymbol{\Sigma}_1$; otherwise, these two covariance matrices are different. Compared with the ratio between the traces or determinants, the statistic w better summarizes the variable covariance information and provides a more comprehensive comparison between the target matrices.

A prior assumption of the above test is that the data are independent and identically distributed. However, autocorrelations commonly exist in the outputs of control systems. As a result, a pretreatment step should be added into the procedure described previously to remove the dynamics contained in the data. To achieve this, ARMA type filters are useful tools.[29,30] The general form of an ARMA filter is the same as eq 13, and the filtered residuals can be obtained by eq 14 once the model parameters have been determined. For more details about ARMA model identification, please refer to the cited book.[31] The procedures of CPA and CSM based on the hypothesis test introduced above, including the steps of data filtering, are described in the following section.

4. MULTIOBJECTIVE CPA/CSM SCHEME

4.1. Multivariate CPA Using MV Benchmark. As mentioned in previous, the multivariate MV benchmark has been widely utilized in the assessment of MIMO control systems due to its objectiveness. A similar idea is adopted here, with a different index based on the hypothesis test, as introduced in the previous section. For online implementation, the proposed assessment procedure is conducted in a moving window mode, the details of which are as follows.

(1) Specify the length of the moving window and the step size. Please note that a trade-off should be made in such a specification. A larger moving window results in a larger sample size in the following hypothesis test, leading to more confidential results together with a larger time lag in online implementation.

(2) In the current window, collect the routine output data $\boldsymbol{y}(k)$ of the investigated closed-loop control system, where k is the sampling time point in the window.

(3) Estimate the interactor matrix $\boldsymbol{D}$ based on $\boldsymbol{y}(k)$. More detailed steps of such estimation can be found in the literature (e.g., refs 19 and 14). Alternatively, if the process model in the current window is known prior, the interactor matrix can be calculated directly.

(4) Transform $\tilde{\boldsymbol{y}}(k)$ to the interactor-filtered form $\tilde{y}(k) = q^{-\tau}\boldsymbol{D}(q)\,\boldsymbol{y}\,(k)$, using the information on $\boldsymbol{D}$.

(5) Model $\boldsymbol{y}(k)$ by an appropriate time-series model (e.g., the ARMA model in eq 13).

(6) Obtain the whitened sequence $\boldsymbol{\varepsilon}(k)$ by eq 14.

(7) Calculate $\tilde{F}_i$ by eq 12.

(8) Obtain the output data $\boldsymbol{y}_{\mathrm{mv}}\,(k)$ under MVC by eq 11.

(9) Let $\boldsymbol{y}_1\,(k) = \boldsymbol{y}_{\mathrm{mv}}\,(k)$ and $\boldsymbol{y}_2\,(k) = \boldsymbol{y}\,(k)$.

10) Build an ARMA model for the series of $\boldsymbol{y}_1\,(k)$:

$$\hat{\mathbf{A}}_1(q)\boldsymbol{y}_1(k) = \hat{\mathbf{C}}_1(q)\boldsymbol{\varepsilon}_1(k) \quad (22)$$

11) Filter both $\boldsymbol{y}_1\,(k)$ and $\boldsymbol{y}_2\,(k)$ using the ARMA filter:

$$\boldsymbol{\varepsilon}_1(k) = \hat{\mathbf{C}}_1^{-1}(q)\hat{\mathbf{A}}_1(q)\boldsymbol{y}_1(k) \quad (23)$$

$$\boldsymbol{\varepsilon}_2(k) = \hat{\mathbf{C}}_1^{-1}(q)\hat{\mathbf{A}}_1(q)\boldsymbol{y}_2(k) \quad (24)$$

(12) Define $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$ as the sample matrices containing all $\boldsymbol{\varepsilon}_1\,(k)$ and $\boldsymbol{\varepsilon}_2\,(k)$ in the current window, respectively.

(13) Calculate the sample covariance matrices $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ from samples $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$, and conduct the hypothesis test on the equality of covariance matrices following the steps described in section 3.2.

(14) If the statistic calculated by eq 21, denoted as w_{cpa}, has a value smaller than the control limit derived from the χ^2-distribution, it is believed that the two covariance matrices are same in the statistical sense. Consequently, the controller performs similar to the MV controller in the current window. Otherwise, if w_{cpa} is beyond the control limit, the current control performance is not optimal. The results can be plotted in a control chart.

4.2. Multivariate CPA Using User-Specified Benchmark. Although the MV benchmark has been widely adopted in the field of CPA because of its attractive theoretical properties, MVC is often impractical or infeasible since it may lead to significant variations in manipulated variables. As a result, the MV benchmark is usually unachievable. It is desired to have a more practical benchmark for multivariate CPA. Based on such thinking, one may also compare the performance of the current controller and that of the controller used in a "golden" operating period. In online implementations, the statistic w_{cpa} is computed in the same way as in section 4.1, but the derivation of control limits is different. The detailed steps for calculation of control limits are listed below.

(1) Specify the length of the moving window and the step size, the values of which are same as those used in section 4.1.

(2) Choose a "golden" operating period during which the control performance is satisfactory and scan the selected period with the moving window.

(3) In each window i, collect the historical output data $\boldsymbol{y}^{\mathrm{his}}\,(k)$, where i is the index of window and k is the sampling time point in the window.

(4) Derive the MV outputs $\boldsymbol{y}_{\mathrm{mv}}^{\mathrm{his}}\,(k)$ from $\boldsymbol{y}^{\mathrm{his}}\,(k)$ following similar steps as (3)−(8) in section 4.1.

(5) Let $\boldsymbol{y}_1\,(k) = \boldsymbol{y}_{\mathrm{mv}}^{\mathrm{his}}\,(k)$ and $\boldsymbol{y}_2\,(k) = \boldsymbol{y}^{\mathrm{his}}\,(k)$.

DOI: 10.1021/ie502743f
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(6) Filter both series and generate sample matrices $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$ following the same procedure as described in the steps (10)−(12) in section 4.1.
(7) Calculate the sample covariance matrices $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ from samples $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$, and conduct the hypothesis test on the equality of covariance matrices. Consequently, w_i, the test statistic value in window i, can be calculated, which indicates the control performance in such window.
(8) On the basis of the distribution of w_i $(i = 1,2,\cdots I)$, derive the control limits for the follow-up control performance assessment, where I is the total number of windows in the "golden" operating period. Because the exact distribution of w_i is unknown, kernel density estimation (KDE)[32] is adopted to estimate the distribution and calculate the upper and lower control limits. Such control limits makes it possible to compare the control performances in different operating periods or conditions.

In online implementations, if the statistic w_{cpa} in the current window has a value inside the control limits, the control system performs similar to that operating in the "golden" period. In other words, the current control performance is satisfactory, although it may not be optimal in the sense of minimum variance. Otherwise, there are two possibilities. If w_{cpa} is outside the upper control limit, the current control performance is regarded to be worse than that in the "golden" period, which is unsatisfactory. If w_{cpa} is below the lower control limit, it is concluded that the control performance is improved.

4.3. Multivariate Statistical CSM. Besides control performance assessment, the hypothesis test introduced in section 3.2 can also be applied to monitoring the changes in the output covariance structure of the investigated control system. As discussed in previous, CSM can be regarded as a complementary of CPA, because these two types of methods reflect different aspects of properties of control systems. The details are as follows.

(1) Select a "golden" period in normal operation, and store the historical data $\boldsymbol{y}^{\text{his}}(k)$ in such period.
(2) Specify the length of the moving window and the step size.
(3) In the current window, collect the routine output data $\boldsymbol{y}(k)$.
Let $\boldsymbol{y}_1(k) = \boldsymbol{y}^{\text{his}}(k)$ and $\boldsymbol{y}_2(k) = \boldsymbol{y}(k)$.
(4) Conduct the steps (10)−(13) in section 4.1, and derive the monitoring statistic that is denoted as w_{csm}.
(5) Compare w_{csm} to the control limit derived from the χ^2-distribution. If w_{csm} is smaller than the control limit, the system output covariance structure in current window is similar to that in the "golden" operating period. Otherwise, a change in the control system is detected.

5. CASE STUDIES

5.1. Two-Variable Example. In the first example, a simple numerical simulation once used in ref 33 is utilized to demonstrate the proposed CPA/CSM scheme. In such simulation, a process is described by the following transfer function matrices:

$$\boldsymbol{G}_p = \begin{bmatrix} \dfrac{q^{-1}}{1-0.4q^{-1}} & \dfrac{q^{-2}}{1-0.1q^{-1}} \\ \dfrac{0.3q^{-1}}{1-0.4q^{-1}} & \dfrac{q^{-2}}{1-0.8q^{-1}} \end{bmatrix} \tag{25}$$

$$\boldsymbol{G}_\varepsilon = \begin{bmatrix} \dfrac{1}{1-0.5q^{-1}} & \dfrac{-0.6}{1-0.5q^{-1}} \\ \dfrac{0.5}{1-0.5q^{-1}} & \dfrac{1}{1-0.5q^{-1}} \end{bmatrix} \tag{26}$$

Accordingly, a unitary interactor matrix is factored out as

$$\boldsymbol{D} = \begin{bmatrix} -0.9578q & -0.2873q \\ -0.2873q^2 & 0.9578q^2 \end{bmatrix} \tag{27}$$

based on which the minimum variance controller is designed as

$$\begin{aligned}
\boldsymbol{G}_{\text{mvc}} &= \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \\
g_{11} &= \left[\frac{0.4588 + 0.2195q^{-1} - 0.2418q^{-2} + 0.03582q^{-3} - 0.001427q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{12} &= \left[\frac{0.1376 - 0.3985q^{-1} + 0.4561q^{-2} - 0.1561q^{-3} + 0.01143q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{21} &= \left[\frac{-0.04817 + 0.08622q^{-1} - 0.04242q^{-2} + 0.003428q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{22} &= \left[\frac{0.3749 - 0.3731q^{-1} + 0.06214q^{-2} - 0.002858q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]
\end{aligned} \tag{28}$$

5.1.1. Scenario I: Changes in Controller Parameters. In this scenario, three different controllers are adopted, the transfer functions of which are as below. For Controller 1

$$\begin{aligned}
\boldsymbol{G}_{c1} &= \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \\
g_{11} &= \left[\frac{\mathbf{0.\,9088} + 0.2195q^{-1} - 0.2418q^{-2} + 0.03582q^{-3} - 0.001427q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{12} &= \left[\frac{\mathbf{0.\,4376} - 0.3985q^{-1} + 0.4561q^{-2} - 0.1561q^{-3} + 0.01143q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{21} &= \left[\frac{\mathbf{-0.\,08817} + 0.08622q^{-1} - 0.04242q^{-2} + 0.003428q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{22} &= \left[\frac{\mathbf{1.\,3749} - 0.3731q^{-1} + 0.06214q^{-2} - 0.002858q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]
\end{aligned} \tag{29}$$

For Controller 2

$$\begin{aligned}
\boldsymbol{G}_{c2} &= \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \\
g_{11} &= \left[\frac{\mathbf{0.\,9088} + 0.2195q^{-1} - 0.2418q^{-2} + 0.03582q^{-3} - 0.001427q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{12} &= \left[\frac{0.1376 - 0.3985q^{-1} + 0.4561q^{-2} - 0.1561q^{-3} + 0.01143q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{21} &= \left[\frac{-0.04817 + 0.08622q^{-1} - 0.04242q^{-2} + 0.003428q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \\
g_{22} &= \left[\frac{\mathbf{0.\,8749} - 0.3731q^{-1} + 0.06214q^{-2} - 0.002858q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]
\end{aligned} \tag{30}$$

DOI: 10.1021/ie502743f
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In the above equations, the differences between these controllers and the MV controller are highlighted with bold numbers. Controller 3 is the MV controller (i.e., $\boldsymbol{G}_{c3} = \boldsymbol{G}_{\mathrm{mvc}}$). Figure 1 shows the closed-loop outputs of both process

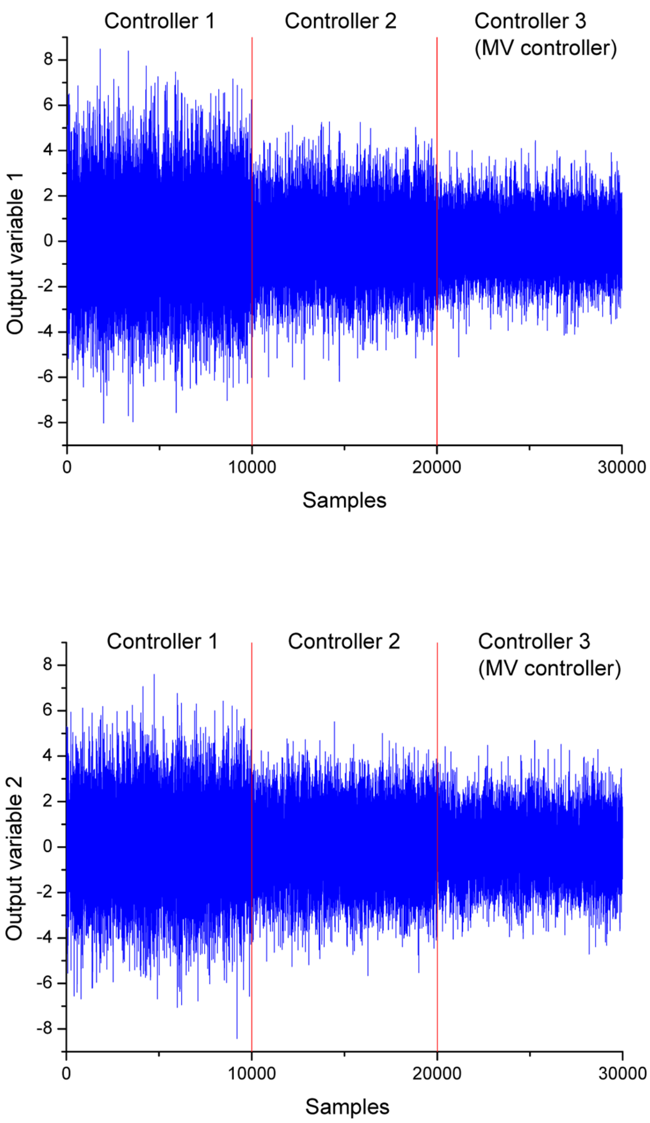

Figure 1. Closed-loop outputs in Scenario I of the two-variable example.

variables under different control strategies, where the set-points for both variables are equal to 0, and the disturbances are generated by white noise with zero means and unit variances.

For both multivariate CPA and CSM, both the length of the moving window and the step size are set as 500 sampling intervals. The control chart for online CPA is shown in Figure 2, where the natural logarithm values of the statistic are plotted for better visualization. The solid line is the control limit (CL1) derived on the basis of MV outputs, and the dash-dot lines represent the upper and lower control limits (CL2) calculated from the user-specified "golden" operating data. Here, the historical closed-loop output data under the control of Controller 1 is utilized as the "golden" operating data. It is clear that the statistic w_{cpa} is under CL1 only when Controller 3 is applied, indicating that the first two controllers are not optimal in the sense of minimum variance. In addition, when Controller 2 is

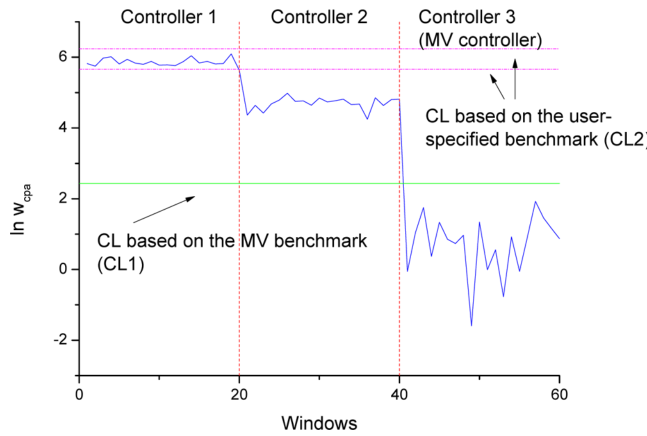

Figure 2. Control chart for online CPA in Scenario I of the two-variable example.

adopted, the values of w_{cpa} become smaller compared with the ones achieved by applying Controller 1, and go outside the lower limit of CL2. It can be concluded that this controller has a better-than-satisfactory performance and statistically outperforms Controller 1, although it is not an MV controller. Controller 3 is also below the lower limit of CL2, and the corresponding values of w_{cpa} are smaller than those of Controller 2, indicating a even better control performance. The online CSM results are shown in Figure 3, where the control limit (CL3) is also derived

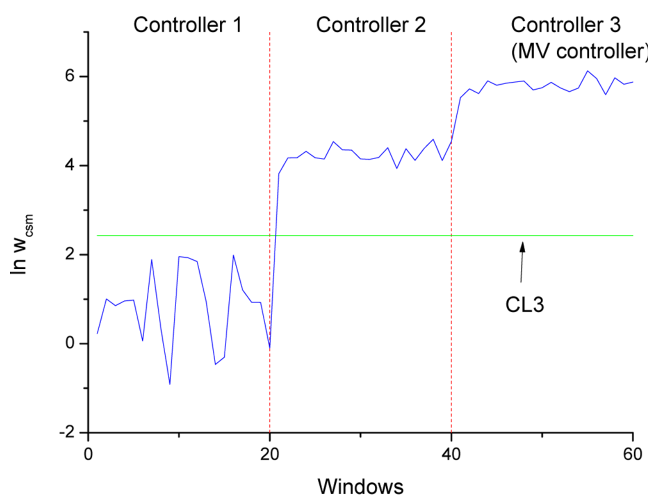

Figure 3. Control chart for online CSM in Scenario I of the two-variable example.

using the operating data with Controller 1 applied. Obviously, the monitoring statistic w_{csm} goes outside CL3 when the other two controllers are implemented, indicating the changes in system output covariance.

5.1.2. Scenario II: Changes in Disturbance Model. As introduced in previous, the MV controller is the optimal controller for a linear system, which is designed based on the knowledge of process model $\boldsymbol{G}_p$ and disturbance model $\boldsymbol{G}_\varepsilon$. When $\boldsymbol{G}_p$ or $\boldsymbol{G}_\varepsilon$ changes, the parameters of the MV controller should be redesigned, and the value of the smallest possible output variance changes accordingly. In other words, the MV benchmark for CPA changes with $\boldsymbol{G}_p$ and $\boldsymbol{G}_\varepsilon$. In the second scenario, the CPA and CSM results for a closed-loop

DOI: 10.1021/ie502743f
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controlled process with changing disturbance model are revealed.

The considered process can be divided into two stages each of which contains 10000 sampling intervals. In the first stage, the process model and the disturbance model have the same expressions as in eqs 25 and 26. The controller is same as Controller 1 in section 5.1.1. In the second stage, the disturbance model becomes

$$\mathbf{G}_\varepsilon = \begin{bmatrix} \dfrac{1}{1-0.5q^{-1}} & \dfrac{-q^{-1}}{1-0.6q^{-1}} \\ \dfrac{q^{-1}}{1-0.7q^{-1}} & \dfrac{1}{1-0.8q^{-1}} \end{bmatrix} \tag{31}$$

and the controller is designed as

$$\mathbf{G}_c = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$
$$g_{11} = \left[\frac{0.4588 + 0.2195q^{-1} - 0.2418q^{-2} + 0.03582q^{-3} - 0.001427q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]$$
$$g_{12} = \left[\frac{0.1376 - 0.3985q^{-1} + 0.4561q^{-2} - 0.1561q^{-3} + 0.01143q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]$$
$$g_{21} = \left[\frac{-0.00417 + 0.08622q^{-1} - 0.04242q^{-2} + 0.003428q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]$$
$$g_{22} = \left[\frac{0.5749 - 0.3731q^{-1} + 0.06214q^{-2} - 0.002858q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \tag{32}$$

The set-points and white noise are same as those in Scenario I. The closed-loop outputs in both stages are plotted in Figure 4.

Figure 5 shows the online CPA results, where the solid line is the control limit (CL1) derived on the basis of the MV benchmark and the dash-dot lines are the control limits (CL2) according to the user-specified benchmark (i.e., the closed-loop outputs from the first operating stage in this case). From the control chart, it can be concluded that the control performances in both stages are similar, because all values of the statistic w_{cpa} fall within the upper and lower limits of CL2. Compared with the user-specified benchmark, the performance in the second stage is acceptable. However, in the sense of minimum variance, such performance can still be improved, because w_{cpa} is above CL1 throughout the entire operating duration. Although the CPA results do not tell the difference between the two stages, the CSM control chart (i.e., Figure 6) shows the change in the system. In the figure, the monitoring statistic w_{csm} is obviously beyond the control limit (CL3). Such example verifies the statement that the CPA and CSM results reveal different types of process characteristics and are complementary to each other.

5.2. Four-Variable Example. In this section, the feasibility of the proposed scheme is further tested using a four-variable example quoted from ref 24, in which the process model and the disturbance model are as below:

$$\mathbf{G}_p = \begin{bmatrix} G_{11} & 0 & G_{13} & 0 \\ G_{21} & G_{22} & 0 & 0 \\ 0 & G_{32} & G_{33} & 0 \\ G_{41} & 0 & 0 & G_{44} \end{bmatrix}$$
$$G_{11} = \left[\frac{0.05q^{-3}}{1 - 0.95q^{-1} + 0.3q^{-2} + 0.4q^{-3}}\right]$$
$$G_{21} = \left[\frac{0.02966q^{-3}}{1 - 1.627q^{-1} + 0.706q^{-2}}\right]$$
$$G_{41} = \left[\frac{0.5q^{-5} - 0.4875q^{-6}}{1 - 1.395q^{-1} + 0.455q^{-2} - 0.11q^{-3} + 0.19q^{-4}}\right]$$
$$G_{22} = \left[\frac{0.0627q^{-6}}{1 - 0.937q^{-1} + 0.82q^{-2} - 0.21q^{-3}}\right]$$
$$G_{32} = \left[\frac{0.235q^{-5}}{1 - 0.765q^{-1} + 0.88q^{-2} - 0.34q^{-3}}\right]$$
$$G_{13} = \left[\frac{0.7q^{-3}}{1 - 0.3q^{-1} + 0.15q^{-2} + 0.33q^{-3}}\right]$$
$$G_{33} = \left[\frac{0.5q^{-2}}{1 - q^{-1} + 0.52q^{-2} - 0.16q^{-3}}\right]$$
$$G_{44} = \left[\frac{0.2q^{-6}}{1 - 0.8q^{-1} + 0.32q^{-2} + 0.11q^{-3}}\right] \tag{33}$$

$$\mathbf{G}_\varepsilon = \operatorname{diag}\left[\frac{1-0.1875q^{-1}}{1-0.9875q^{-1}}\ \frac{1-0.1875q^{-1}}{1-0.9875q^{-1}}\ \frac{1-0.1875q^{-1}}{1-0.9875q^{-1}}\ \frac{1-0.1875q^{-1}}{1-0.9875q^{-1}}\right] \tag{34}$$

Similar to the case described in section 5.1.1, the system operation can be divided into three stages, where different controllers are applied in different stages. The expressions of the first controllers are as below, while the third controller is designed to be an MV controller:

$$\mathbf{G}_{c1} = \operatorname{diag}\left[\frac{0.8568 - 0.816q^{-1}}{1-q^{-1}}\ \frac{0.6641 - 0.625q^{-1}}{1-q^{-1}}\ \frac{0.2482 - 0.184q^{-1}}{1-q^{-1}}\ \frac{0.444 - 0.370q^{-1}}{1-q^{-1}}\right] \tag{35}$$

$$\mathbf{G}_{c2} = \operatorname{diag}\left[\frac{0.0657 + 0.173q^{-1}}{1-q^{-1}}\ \frac{-0.023 + 0.023q^{-1}}{1-q^{-1}}\ \frac{0.1156 + 0.1125q^{-1}}{1-q^{-1}}\ \frac{0.1676 + 0.1461q^{-1}}{1-q^{-1}}\right] \tag{36}$$

The set-points for all variables are set to 0, whereas the noise obeys a multivariate Gaussian distribution with zero means and unit variances. The window length and the step size for online CPA/CSM are both selected as 500. Figure 7 plots the closed-loop outputs of all four variables.

Figure 8 and Figure 9 display the CPA and CSM results, respectively. The meanings of the lines in the figures are same as those in section 5.1. From Figure 8, it can be inferred that only the third controller produces minimum variance outputs, because the statistic w_{cpa} is outside the MV control limit (CL1) in the first two stages. Moreover, although $\mathbf{G}_{c1} \neq \mathbf{G}_{c2}$, the control performances of the first two controllers are similar. In the second stage, w_{cpa} stays within the control limits

DOI: 10.1021/ie502743f
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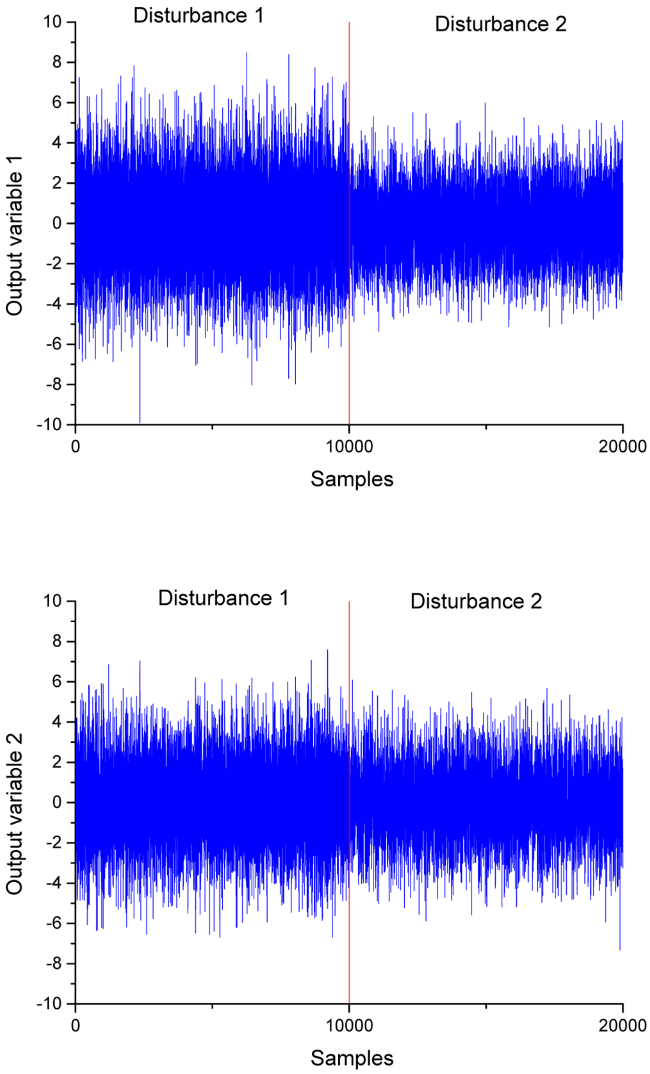

Figure 4. Closed-loop outputs in Scenario II of the two-variable example.

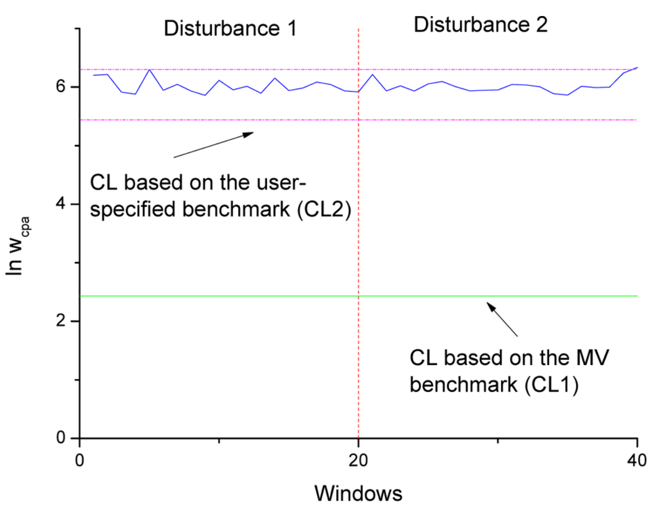

Figure 5. Control chart for online CPA in Scenario II of the two-variable example.

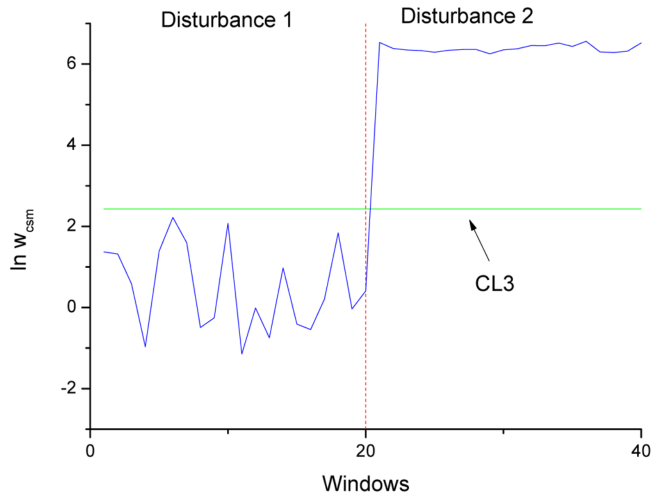

Figure 6. Control chart for online CSM in Scenario II of the two-variable example.

corresponding to the user-specified benchmark (CL2), which is calculated on the basis of the historical data under the control of the first controller. Such results indicate that the closed-loop controlled system in the first two stages has similar potentials to be improved. The CSM results in Figure 9 reveal the differences between the output covariance structures, which are not reflected by Figure 8, showing that these two control charts contain different types of process information.

5.3. Wood-Berry Distillation Column. The proposed CPA/CSM scheme has also been applied to a Wood-Berry distillation column model to show its benefit comparing to the conventional methods using η_{trace} or $\eta_{\det}$ as an index.

The Wood−Berry distillation column model[34] describes a well-known continuous pilot process separating a methanol−water mixture, which involves two inputs and two outputs and is given as

$$\begin{bmatrix} X_d(s) \\ X_b(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{18.9e^{-3s}}{21.0s+1} \\ \dfrac{6.6.e^{-7s}}{10.9s+1} & \dfrac{19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} + \begin{bmatrix} \dfrac{3.8e^{-8s}}{14.9s+1} \\ \dfrac{4.9e^{-3s}}{13.2s+1} \end{bmatrix} F(s) \tag{37}$$

where the two output variables, X_d and X_b, are the distillate and bottom compositions, respectively, the reflux and steam flow rates, R and S, are the manipulated variables, and the rate of the feed flow, F, is an unmeasured disturbance variable.

For evaluating the CPA/CSM scheme, the operation is divided into four stages, where the process model is kept unchanged, and three different controllers are utilized in different stages. In the first stage, PI controllers are used. The parameters of these controllers are tuned to balance response time and stability margins in the PID controller tuning toolbox in Matlab.[35] The controller transfer function matrix is

$$\boldsymbol{G}_{c1} = \begin{bmatrix} \dfrac{0.4182 - 0.3841q^{-1}}{1 - q^{-1}} & 0 \\ 0 & \dfrac{-0.04361 - 3.2831q^{-1}}{1 - q^{-1}} \end{bmatrix} \tag{38}$$

DOI: 10.1021/ie502743f
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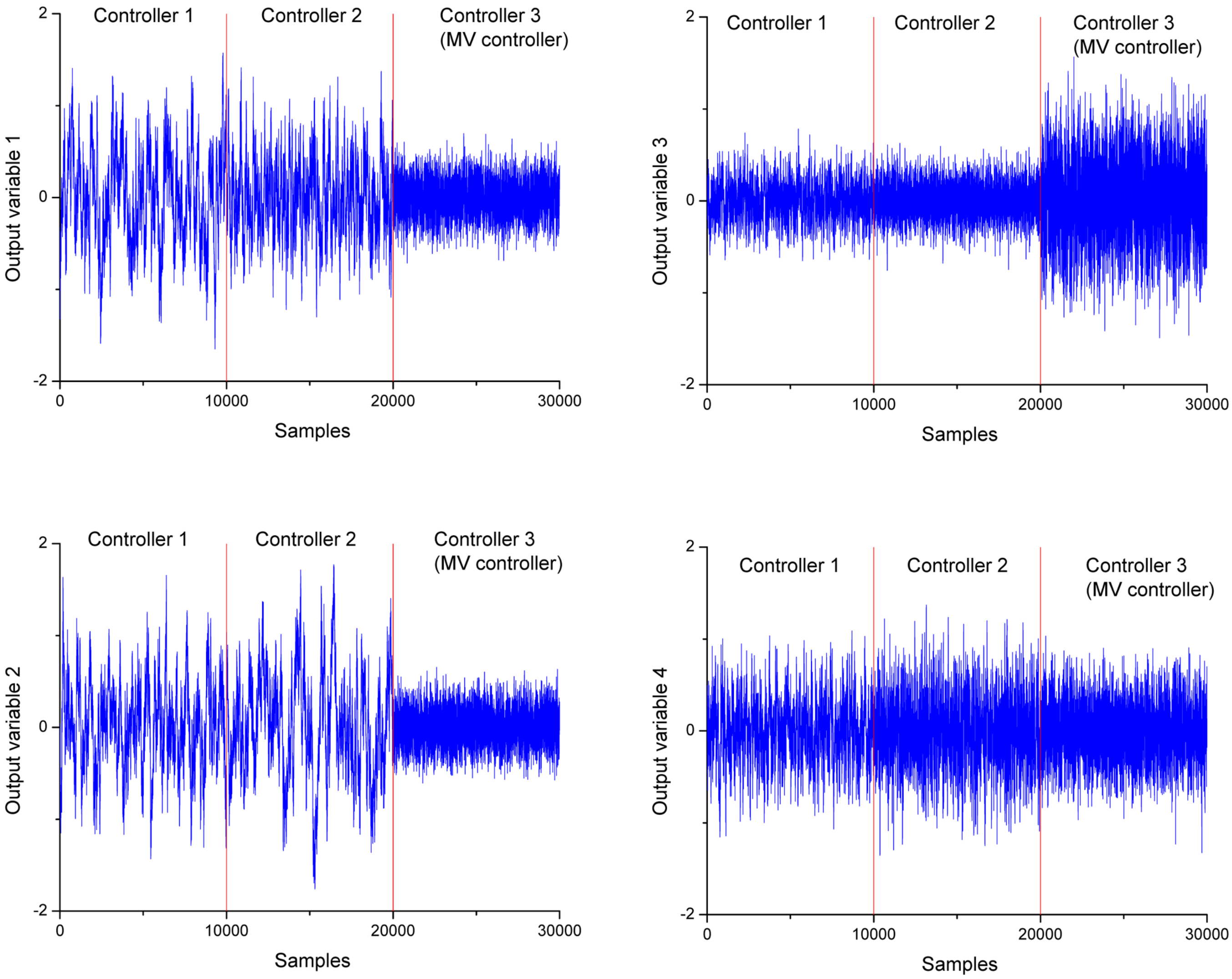

Figure 7. Closed-loop outputs in the four-variable example.

In the second stage, the controller has a transfer function as $\boldsymbol{G}_{c2}$:

$$\boldsymbol{G}_{c2} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$g_{11} = \left[\frac{0.7088 + 0.2195q^{-1} - 0.2418q^{-2} + 0.03582q^{-3} - 0.001427q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]$$

$$g_{12} = \left[\frac{0.2976 - 0.3985q^{-1} + 0.4561q^{-2} - 0.1561q^{-3} + 0.01143q^{-4}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]$$

$$g_{21} = \left[\frac{-0.04817 + 0.08622q^{-1} - 0.04242q^{-2} + 0.003428q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right]$$

$$g_{22} = \left[\frac{0.3749 - 0.3731q^{-1} + 0.06214q^{-2} - 0.002858q^{-3}}{1 + 0.2286q^{-1} - 0.3729q^{-2} - 0.05714q^{-3} + 0.03071q^{-4}}\right] \tag{39}$$

while the controller adopted in the third stage is

$$\boldsymbol{G}_{c2} = \begin{bmatrix} \dfrac{0.7 - 0.2q^{-1}}{1 - 0.2q^{-1}} & 0 \\ 0 & \dfrac{0.25 - 0.2q^{-1}}{1 - 0.25q^{-2}} \end{bmatrix} \tag{40}$$

The fourth controller is an MV controller.

The set-points of this process are $x_d = 1$ and $x_b = 2$. The unmeasured disturbance f contains white noise obeying a binary Gaussian distribution with zero means and 0.01 variances. The window length and the step size for online CPA/CSM are both 500. Figure 10 shows the time-series plots of the mean-centered closed-loop outputs, from which it is difficult to identify the differences between the control performances or the output covariance structures in all three stages.

Following the proposed procedure, the control charts for CPA and CPM (i.e., Figure 11 and Figure 12) can be plotted. Figure 11 reveals that only the fourth controller leads to MV outputs, and the statistic w_{cpa} is far outside the control limit derived from the MV benchmark (CL1) in the first three stages. In addition, the performances of the second and third controllers are similar, although the two controllers have significantly different forms as seen in eqs 39 and 40. The performance of the PI controller is the most different from that of MVC. Comparison between w_{cpa} and CL2 (the control limits according to the user-specified control limits, i.e., the closed-loop outputs from the second operating stage in this case) shows that the second controller slightly outperforms the third one. The CSM control limit (CL3) in Figure 12 is calculated based on a group of "gold" operating data collected when the process is controlled by the second controller.

DOI: 10.1021/ie502743f
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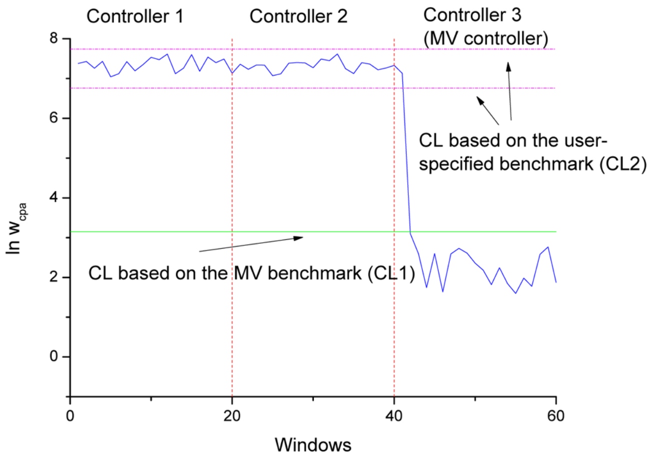

Figure 8. Control chart for online CPA in the four-variable example.

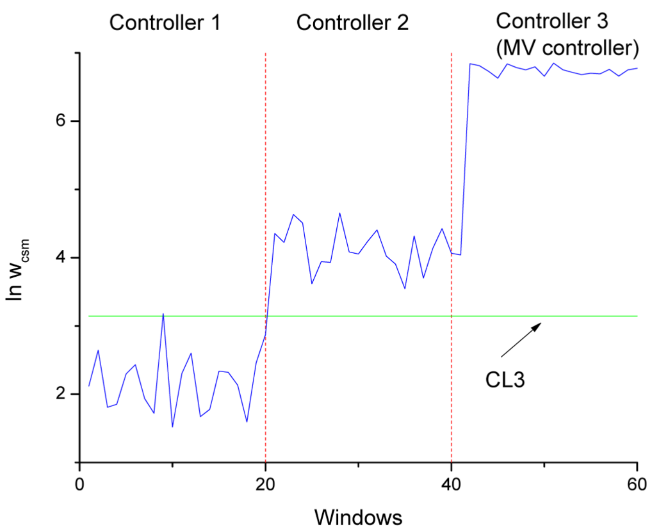

Figure 9. Control chart for online CSM in the four-variable example.

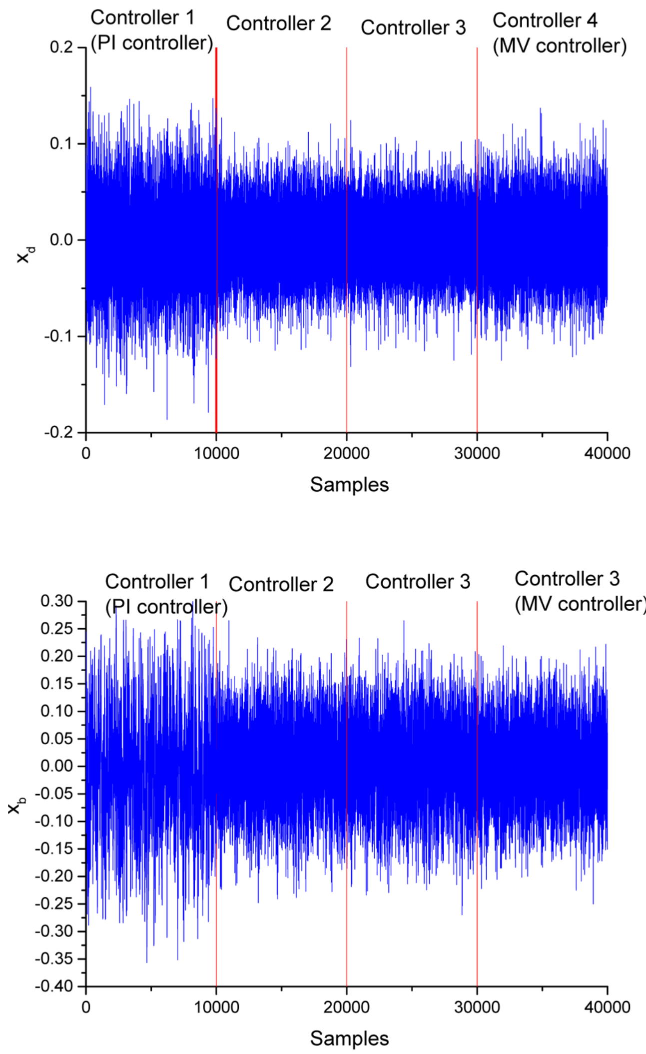

Figure 10. Mean-centered closed-loop outputs in the case of Wood-Berry distillation column.

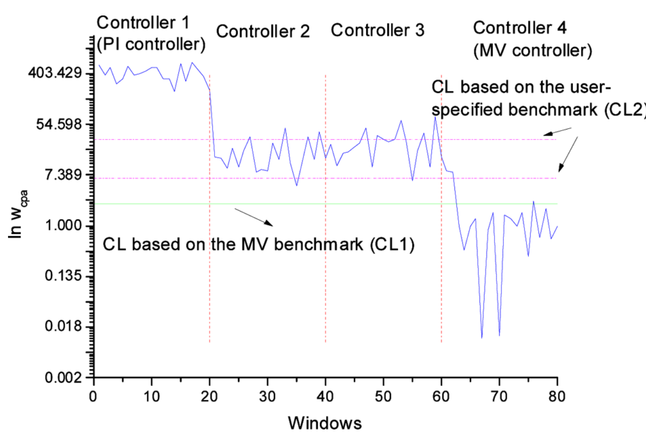

Figure 11. Control chart for online CPA in the case of Wood-Berry distillation column.

Figure 12 shows that the output covariance structures change from stage to stage.

To compare the effectiveness of the proposed method and the conventional methods, the ratios between the traces or determinants of the output covariance matrices are listed in Table 1. All the index values in this table are close to 1 except those corresponding to the PI controller, implying that there is no significant difference between the last three controllers. In other words, the last these controllers are all possible to be the MV controller. Especially, the controlled system in the second stage even seems to have better performance than that of the system in stage four, if one looks at the values of η_{det}. Obviously, such conclusions are not correct. The CPA and CSM results shown in Figure 11 and Figure 12 provide more accurate and comprehensive information.

6. CONCLUSIONS

For the assessment and monitoring of multivariate control systems, many research efforts focus on the comparison between the covariance matrices of closed-loop outputs. However, most existing methods only consider the traces or the determinants, which cannot extract entire information in the matrices and may lead to bias results. To overcome the shortcoming of the existing method, this paper proposes to conduct a hypothesis test on the equality of the output covariance matrices. A statistic is calculated through such test, which can be plotted in control charts for online assessment or monitoring. Based on the proposed test, three different tasks can be accomplished, including CPA using a minimum variance (MV) benchmark, CPA using a user-specified benchmark, and CSM based on historical, which

DOI: 10.1021/ie502743f
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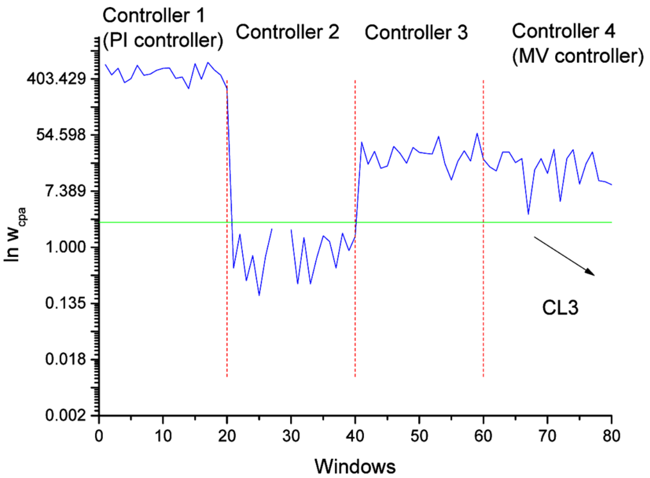

Figure 12. Control chart for online CSM in the case of Wood-Berry distillation column.

Table 1. CPA Results Based on the Ratio of Traces/Determinants in the Case of Wood-Berry Distillation Column

index	Controller 1 (PI controller)	Controller 2	Controller 3	Controller 4 (MV controller)
η_{trace}	0.4312	0.9223	0.8607	0.9805
η_{det}	0.2817	0.9950	0.9475	0.9537

are complementary to each other and provide a comprehensive picture of the investigated control system. It should be noted that the proposed hypothesis test for multivariate CPA/CSM not only reflect whether two output covariance matrices are equal to each other but also can provide a measure of the probability of such equality. For example, the latter can be accomplished by the p-value ranging from 0 to 1, although more detailed illustrations are not provided in this paper to avoid overlength.

The issues of control performance diagnosis (i.e., isolating the variables contributing most to poor control performance) will be considered in future studies. Another interesting research topic is the performance evaluation of plantwide control systems, such as that described in ref 36. The MVC-based CPA methods, including the proposed method, are not suited to be applied to such systems directly. The reason is of two-fold: First, in any MVC-based CPA methods, it is necessary to estimate the interactor matrix before calculating the assessment indices. Although the interactor matrix can be identified from the closed-loop data, as described in refs 14,19, such identification becomes difficult while dealing with the plantwide control systems. Second, the MVC-based CPA methods only measure the stochastic performance of the control loops, while the design of a plantwide control system usually has more concerns. In such situation, the MVC-based CPA methods are not practical. Therefore, the CPA of plantwide control systems deserves more research efforts.

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Notes

The authors declare no competing financial interest.

ACKNOWLEDGMENTS

This work was supported in part by Ministry of Science and Technology, ROC, under Grant no. NSC 102-2221-E-007-130.

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