Time Delay Filter-Based Deadbeat Control of Process with Dead Time

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This paper presents the time delay filter (TDF) in which time delay is intentionally adopted in a filter and then time delay filter-based deadbeat control of a process with first-order plus dead time is implemented. The system stability is guaranteed with the Nyquist criteria and the PI controller is tuned by the frequency specifications. The time delay filter is designed to cancel the poles of the closed-loop PI control system. Examples show that the proposed method is very effective and is robust to modeling errors.

1. Introduction

Most studies of a *time delay system* have been devoted to *processes with dead time*, where the dead time is, on one hand, the inherent characteristic of the process which causes various difficulties in both system analysis and controller design.[1] On the other hand, some researchers have intentionally adopted time delays in controller design of ordinary systems, in addition to using a dead time compensator or predictor, of which the Smith predictor[2] is the most famous one to compensate for the dead time in the process. Judiciously adopting time delay in controller design, referred to as *time delay control*, may improve the performance of the system. Speaking of time delay control, one cannot help mentioning O. J. M. Smith who presented the *posicast control*[3,4] as well as the Smith predictor. Suh and Bien[5] and Swisher and Tenqchen[6] presented the proportional minus delay (PMD) controller, which performs an averaged derivative action and thus can replace the conventional proportional-derivative (PD) controller. Kwon et al.[7] presented a general *time delay controller*, proportional-hereditary (PH) controller, for a multivariable ordinary system with guaranteed stability and improved performance. A PH controller possesses not only the derivative action like the PMD but also the integral action. Su et al.[8] used time delay in minor loops attached to the main loop for the tip control of a flexible beam. Youcef-Toumi and co-workers[9−11] adopted time delay to estimate the uncertainty and disturbance in a system.

Posicast control, in which time delay is used to eliminate the overshoot and quench the oscillation of a second-order lightly damped oscillatory process, was not widely applied because of the lack of robustness to errors in estimated damping and frequency. At the end of the 1980s, Singer[12] presented a method to improve the robustness and referred to it as input shaping. Since then, the input-shaping technique has been extensively studied[13−16] and implemented into various applications, for example, coordinate measuring machines[17,18] and cranes.[19,20]

This paper presents the time delay filter to attain the deadbeat control of the process with dead time. Section 2 describes the basic time delay filter in a continuous domain and then deadbeat control based on the discrete one is implemented in section 3. Examples are given in section 4 to show the effectiveness of the proposed method.

2. Basic Time Delay Filter (TDF)

A time delay filter (TDF) is to intentionally utilize time delay in a filter. The simplest TDF consists of a proportional term and a time delay term:

$$C_{\mathrm{TDF}}(s) = K_{\mathrm{p}} + K_{\mathrm{d}}\mathrm{e}^{-ds} \tag{1}$$

To properly design the proportional gain K_{p}, time delay gain K_{d}, and time delay d, the zeros of the TDF could be assigned at the desired place.

Substitute $s = \sigma \pm j\omega$ into TDF (1) and set it equal to zero; then,

$$K_{\mathrm{p}} + K_{\mathrm{d}}\mathrm{e}^{-\sigma d}(\cos(\omega d) \pm j \sin(\omega d)) = 0 \tag{2}$$

Let the real part and the imaginary part be 0, respectively:

$$K_{\mathrm{p}} + K_{\mathrm{d}}\mathrm{e}^{-\sigma d} \cos(\omega d) = 0 \tag{3}$$

$$K_{\mathrm{d}}\mathrm{e}^{-\sigma d} \sin(\omega d) = 0 \tag{4}$$

Omit the trivial solution $K_{\mathrm{p}} = 0$ and $K_{\mathrm{d}} = 0$; then, it can be derived from (4):

$$\omega d = k\pi \qquad (k = 0, 1, 2, ...) \tag{5}$$

For positive time delay gain, $K_{\mathrm{d}} > 0$, k should be an odd number, that is,

$$\omega = (2k + 1)\pi/d \qquad (k = 0, 1, 2, ...) \tag{6a}$$

For negative time delay gain, $K_{\mathrm{d}} < 0$, k should be an even number, that is,

$$\omega = 2k\pi/d \qquad (k = 0, 1, 2, ...) \tag{6b}$$

Substitute (6a) or (6b) into (3):

$$\sigma = -\frac{1}{d}\ln\frac{K_{\mathrm{p}}}{|K_{\mathrm{d}}|} \tag{7}$$

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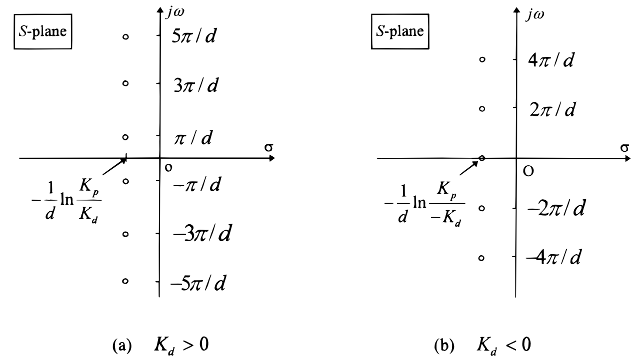


Figure 1. Zero placement of the basic TDF.

Hence, the zeros of the given TDF (1) are

$$\begin{cases} -\dfrac{1}{d}\ln\dfrac{K_p}{K_d} \pm j\dfrac{2k+1}{d}\pi & (K_d > 0) \\ -\dfrac{1}{d}\ln\dfrac{K_p}{-K_d} \pm j\dfrac{2k}{d}\pi & (K_d < 0) \end{cases} \quad (k = 0, 1, 2, ...) \tag{8}$$

It has infinite zeros in both cases, as shown in Figure 1a,b.

In actual application, the steady-state gain of the TDF should be 1; that is, the gain constraint is

$$K_p + K_d = 1 \tag{9}$$

For given desired zeros, the parameters of TDF can be calculated from (6a) or (6b), (7), and (9). For positive time delay gain, these parameters are[12]

$$\begin{cases} K_p = \dfrac{e^{-\pi\sigma/\omega}}{1 + e^{-\pi\sigma/\omega}} \\ K_d = \dfrac{1}{1 + e^{-\pi\sigma/\omega}} \\ d = \dfrac{\pi}{\omega} \end{cases} \tag{10}$$

Here, $k = 0$ in (6a) to attain the minimum time delay d.

In summary, a simple TDF has infinite zeros, while most other filters do have a finite number of zeros. Judicious use of TDF may improve the performance. Hereinafter, the discrete TDF is used to attain deadbeat control for a process with dead time.

3. TDF-Based Deadbeat Control

3.1. System Structure. Consider first-order plus dead time (FOPDT),

$$G(s) = \frac{Ke^{-\tau s}}{Ts + 1} \tag{11}$$

where K is the process gain, T is the apparent time constant, and τ is the apparent dead time. The system structure of TDF-based deadbeat control is shown in Figure 2, in which ZOH is a zero-order holder, $C(z)$ is the feedback controller of the PI type, and $F(z)$ is TDF. The PI controller is designed to ensure the stability with Nyquist criteria in section 3.2, and then the TDF is designed to get the desired deadbeat control in section 3.3.

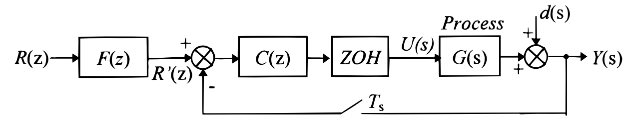


Figure 2. System structure.

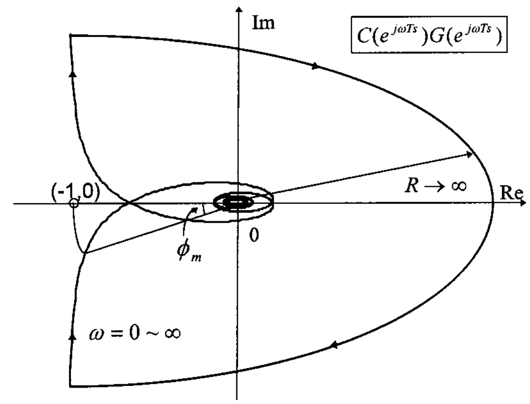


Figure 3. Nyquist curve.

3.2. Tune PI Controller with Nyquist Criteria. When ZOH is inserted into the system and sampling is done with period T_s (let $\tau/T_s = l$ be an integer), the general process is

$$\begin{aligned} G(z) &= Z\left[\frac{1 - e^{-T_s s}}{s} G(s)\right] \\ &= K\frac{1 - e^{-T_s/T}}{z - e^{-T_s/T}} z^{-l} \end{aligned} \tag{12}$$

The PI controller is

$$C(z) = \alpha K_i + K_i \frac{z}{z - 1} \tag{13}$$

where K_i is the integral gain and α is the ratio of the proportional gain to the integral gain.

To simplify the parameter adjustment of the PI controller, its zero is chosen to cancel the pole of the general process. So

$$\alpha = \frac{1}{e^{T_s/T} - 1} \tag{14}$$

Hence, the PI controller has only one parameter, the integral gain K_i, to be determined.

The loop transfer function of the PI control system is

$$C(z)\, G(z) = \frac{KK_i}{z^l(z - 1)} \tag{15}$$

The Nyquist curve, shown in Figure 3, is laid on the portion of $\omega = 0 \to \omega_s$ periodically (where $\omega_s = 2\pi/T_s$ is the angular sampling frequency).

On the basis of similar ideas from ref 13, the closed-loop stability is guaranteed by the following lemma.

Lemma (Closed-Loop Stability).[21] The unity feedback system as in Figure 2, controlled by a PI controller (13) with constraint (14), is stable if the integral gain K_i satisfies

$$0 < K_i < \frac{2}{K}\sin\frac{\pi}{4l + 2}$$

Proof. Substitute $z = e^{j\omega T_s} = e^{j\theta}$ into (15); then,

$$\begin{aligned} C(j\theta)\, G(j\theta) &= \frac{KK_i}{e^{jl\theta}(e^{j\theta}-1)} \\ &= \frac{KK_i}{e^{jl\theta}(\cos\theta - 1 + j\sin\theta)} \\ &= \frac{KK_i}{\sqrt{2-2\cos\theta}} e^{-j(l\theta + tg^{-1}(\sin\theta/(\cos\theta-1)))} \\ &= \frac{KK_i}{\sqrt{4\sin^2(\theta/2)}} e^{-j(l\theta + t_g^{-1}(2\sin(\theta/2)\cos(\theta/2)/(-2\sin^2(\theta/2))))} \\ &= \frac{KK_i}{2\sin(\theta/2)} e^{-j(l\theta+\pi/2+\theta/2)} \end{aligned} \tag{16}$$

So the Nyquist curve crosses the point $(-1,0)$ under the condition

$$\begin{cases} l\theta + \pi/2 + \theta/2 = \pi \\ \dfrac{KK_i}{2\sin(\theta/2)} = 1 \end{cases} \tag{17}$$

Thus, $\theta = \pi/(2l+1)$, $K_i = 2/K \sin \pi/(4l+2)$. In other words, the closed-loop system is critically stable when $K_i = 2/K \sin \pi/(4l+2)$. If $K_i > 2/K \sin \pi/(4l+2)$, the Nyquist curve will include the point $(-1,0)$ and then, according to the Nyquist Theorem, the closed-loop system is unstable. If $0 < K_i < 2/K \sin \pi/(4l+2)$, then the curve will not include the point $(-1,0)$ and the system is stable.

Remark. The lemma proves that a stable PI controller always exists.

To attain a proper magnitude margin and/or phase margin, the integral gain may be decided by other specifications. For example, according to the definition of phase margin, if the following condition is satisfied, then the phase margin will be ϕ_m.

$$\begin{cases} l\theta + \pi/2 + \theta/2 = \pi - \phi_m \\ \dfrac{KK_i}{2\sin(\theta/2)} = 1 \end{cases} \tag{18}$$

The integral gain is

$$K_i = \frac{2}{K}\sin\frac{\pi - 2\phi_m}{4l+2} \tag{19}$$

The phase margin should be 30°−60°.[22,23] For $\phi_m = \pi/3$, the integral gain is

$$K_i = \frac{2}{K}\sin\frac{\pi}{12l+6} \approx \frac{\pi}{K(6l+3)} \qquad (l \geq 1) \tag{20}$$

The closed-loop transfer function is

$$\begin{aligned} G_{PI}(z) &= \frac{C(z)\,G(z)}{1 + C(z)\,G(z)} \\ &= \frac{KK_i}{z^l(z-1) + KK_i} \\ &= \frac{KK_i}{1 - z^{-1} + KK_i z^{-l-1}} z^{-l-1} \end{aligned} \tag{21}$$

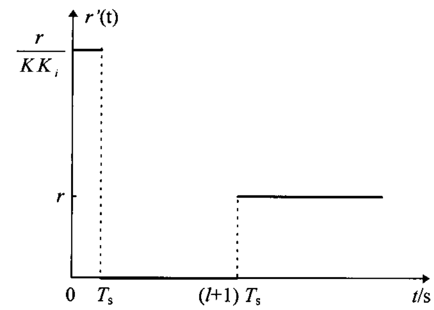


Figure 4. Effect of the proposed TDF with step input.

Because the PI controller only cares about the stability, the dynamic performance may be not as desired. However, the desired performance may be attained by properly designing the TDF.

3.3. Design the Time Delay Filter. The TDF is designed to cancel the poles of the closed-loop system (21) and to attain the desired dynamic performance; it also should not effect the steady-state input signal and the steady-state gain should be 1. Here, the TDF in a discrete domain should have the form

$$\begin{aligned} F(z) &= \frac{1 - z^{-1} + KK_i z^{-l-1}}{KK_i} \\ &= \frac{1 - z^{-1}}{KK_i} + z^{-l-1} \end{aligned} \tag{22}$$

For a step input with magnitude r, the TDF generates an input that has three pieces, as shown in Figure 4, the first piece with magnitude of $r/(KK_i)$, the second piece with magnitude 0, and the last piece with magnitude r, which ensures the steady state has the same input.

The total transfer function from system input to the output is

$$G_c(z) = F(z)\, G_{PI}(z) = z^{-l-1} \tag{23}$$

which is deadbeat control. The system response reaches the steady state after $l + 1$ steps.

4. Examples

4.1. FOPDT with Long Dead Time. Consider the FOPDT process in which the dead time dominates the dynamics,

$$G(s) = \frac{e^{-5s}}{s+1} \tag{24}$$

where the ratio of the dead time to the time constant is very large. A PID controller tuned by the Ziegler−Nichols formula gives a too sluggish response. For the proposed method with $T_s = 1s$ and $K_i = 0.12$, the process response and the control signal are shown in Figure 5. The phase margin is about 52°. The system output reaches the steady state at $t = l + 1 = 6s$ with no ripple and a small control signal.

If the system parameter varies or a modeling error exists, then the performance of the response will degrade. For a deterministic modeling error band, stability can be assured by designing a PI controller with enough phase margin or magnitude margin. For slow time-varying parameters, the on-line identification can be

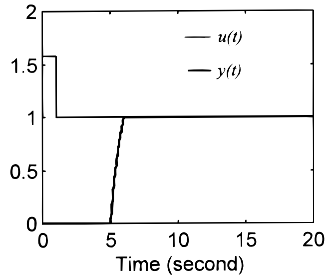


Figure 5. Dynamics of the process (24).

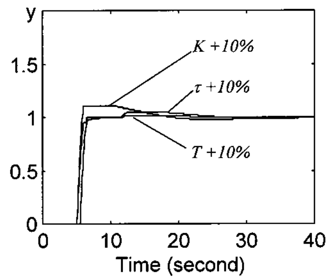


Figure 6. Robustness of the process (24).

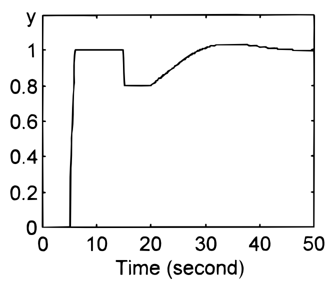


Figure 7. Disturbance rejection of the process (24).

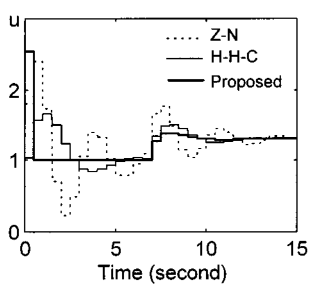


Figure 8. Control signal for the process (25).

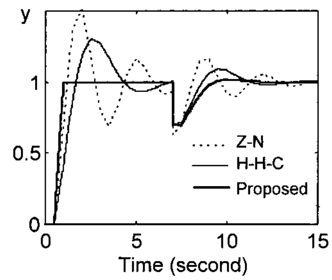
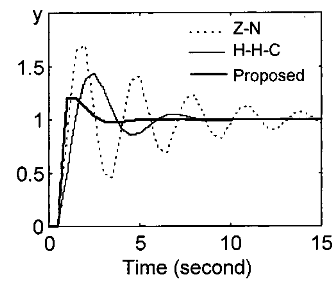
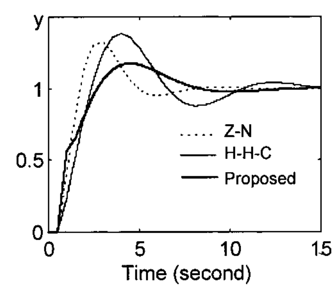


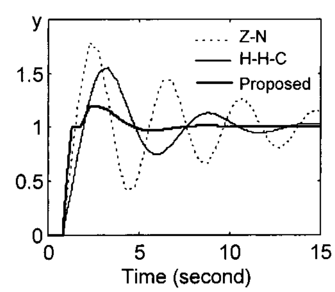
Figure 9. Response of the process (25).



(a) Case 1: process gain increased by 20%



(b) Case 2: time constant increased by 100%



(c) Case 3: dead time increased by 60%

Figure 10. Robustness of the process (25).

introduced to identify the parameters and then autotune the parameters. Figure 6 shows the responses when the system parameters are all increased by 10%. It can be seen that the robustness of the proposed method is good.

Because the controller is a PI type, the system has no steady-state error to the setpoint change or to the constant disturbance. This is shown in Figure 7 with a constant disturbance of -0.2 at $t = 15s$. The disturbance response is slightly sluggish; it can be improved by compromising with the phase margin or adopting other methods to design the PI controller.

4.2. FOPDT with Short Dead Time. Consider the FOPDT process where first-order dominates:[24]

$$G(s) = \frac{e^{-0.5s}}{s+1} \qquad (25)$$

The system response with sampling period $T_s = 0.5s$ and the integral gain $K_i = 0.347$ for phase margin $\phi_m = 60°$ is compared with that of two discrete PI controllers Z-transformed from continuous ones. One is tuned with the Zigler–Nichols method (noted as Z–N in Figures 8 and 9) for $K_p = 1.8$ and $T_i = 1.5s$; the other is tuned to attain the specified magnitude margin 3dB and phase margin 60° for $K_p = 1.05$ and $T_i = 1.00s$, which was presented by Ho et al.[24] (noted as H–H–C in Figures 8 and 9). To show the disturbance response, a constant disturbance -0.3 is assigned to the system at $t = 7s$. The control signals are shown in Figure 8 and the responses are shown in Figure 9. The control signal shows that to attain the deadbeat control, the proposed method requires larger instantaneous power (it only lasts one sampling period) than the others; after the first period, the control signal is smoother than the others because of the effect of TDF. This is reasonable for many actuators because they have the capability of overload.

The response of the Z−N method is more oscillatory and that of H−H−C is slower, while the proposed one is exactly the deadbeat with no ripple. But for disturbance rejection, the H−H−C is best, while the Z−N is oscillatory and the proposed one is slightly sluggish.

As to the robustness, three cases are considered: case 1, process gain increased by 20%; case 2, time constant increased by 100%; case 3, dead time increased by 60%.

The responses for each case are shown in Figure 10 parts a, b, and c, respectively. For the robustness to the process parameter error, the best method is the proposed one.

5. Conclusions

This paper aims at intentionally adopting time delay in a filter, referred to as time delay filter, to attain deadbeat control of the process with the FOPDT model. The simplest time delay filter with a proportional term and only one time delay term is analyzed to show the infinite zeros. The PI controller is tuned on the basis of the Nyquist criteria and the frequency performance to stabilize the process, and then the time delay filter is designed to implement the deadbeat control. The examples with both dead time dominating and first order dominating show the effectiveness of the proposed method.

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