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[6]C. Feldman, J. Appl. Phys. 33, 74 (1962).
[7]In Ref. 4, evidence for the parallel interaction of the two mechanisms was presented. However, it was also indicated that some form of average parallel-series case was possible and could be physically more realistic than the simple parallel mechanism.

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# The Use of Relativistic Energies to Overcome Energy-Resolution Difficulties in the Schwarz-Hora Experiment

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The relativistic expression for the spatial damping of the Schwarz-Hora radiation is given and is compared with the latest data of Schwarz. The agreement is good and indicates that the energy resolution of the electron beam of his apparatus can be as small as 0.0055 eV. It is suggested that experiments be performed using higher kinetic energy for the electron beam where the need for small energy resolution is obviated.

Recent works[1,2] have focused attention on the vital role played by the energy spread of the electron beam in the Schwarz-Hora experiment.[3] It is generally agreed that the Schwarz-Hora effect is a result of the modulation and interference of electron waves. In contrast to the case of electromagnetic waves,

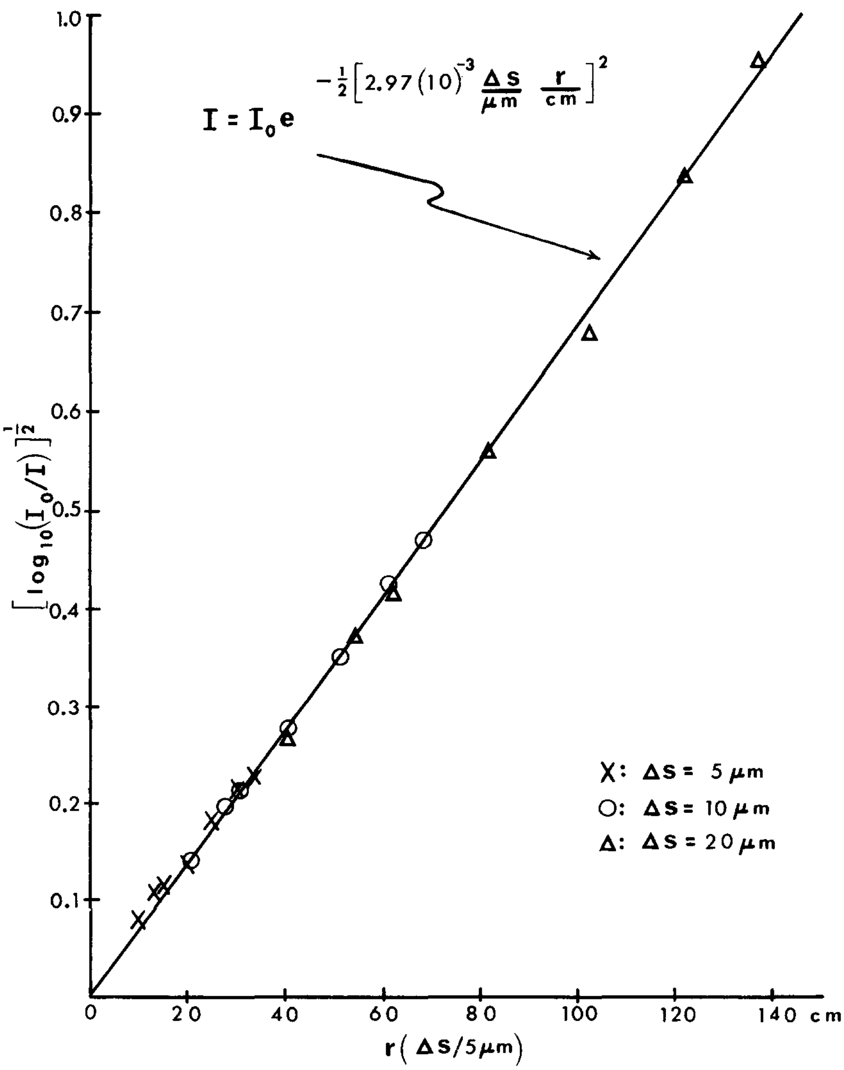


FIG. 1. Expermental values of relative intensity as a function of the product of the velocity analyzer diaphragm size and distance. (Data from Schwarz, Ref. 6.)

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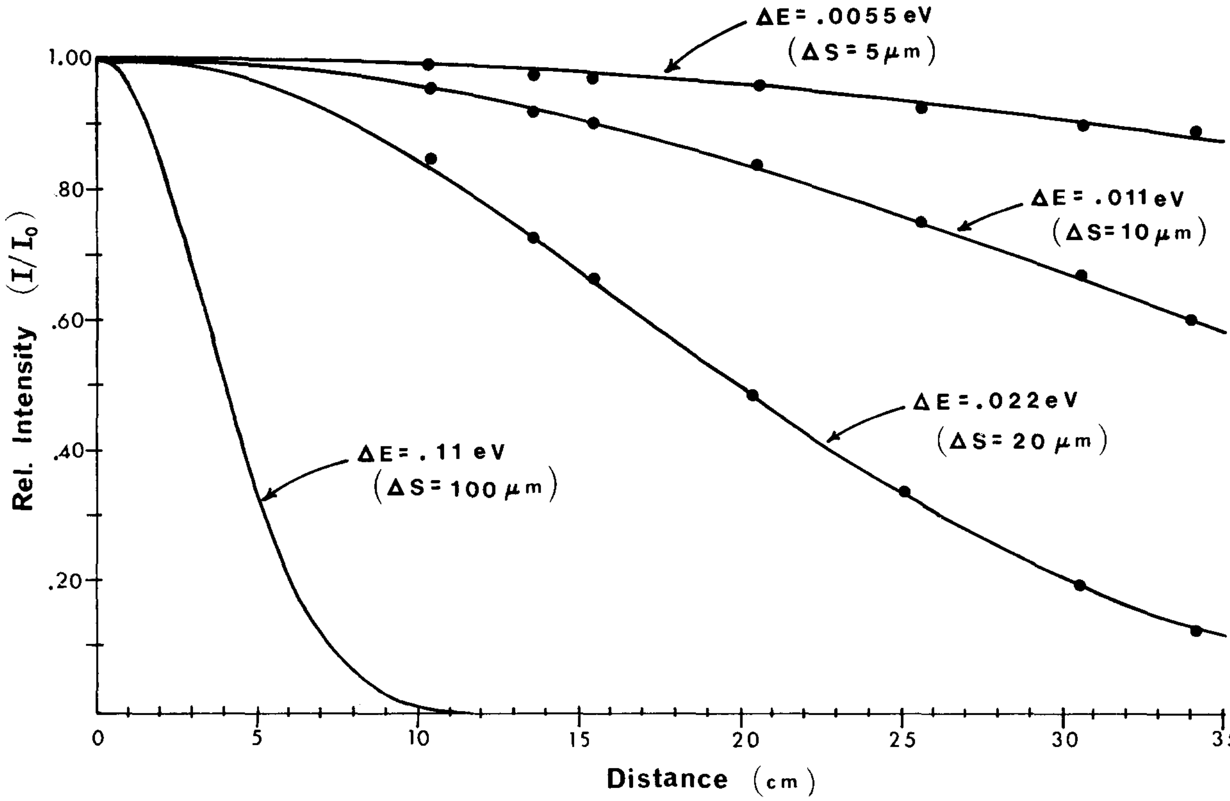


FIG. 2. Theoretical curves of the envelope of the relative intensity at 50-keV electron energy with Schwarz's data superimposed.

electron waves move with different velocities if they have different energies. Consequently, the signals carried by electrons with different velocities will become progressively out of phase with increasing propagation distance. Furthermore, since the modulation results from the interference of different energy components of a single electron wave, the modulation disappears as the energy components separate into spatially disjoint wave packets. Wave packets with a greater energy spread separate sooner because of their smaller physical size. Both of these energy-spread effects cause a rapid disappearance of the Schwarz-Hora effect with increasing modulator-screen distance. However, since both effects result from velocity differences, both should cease to be important in the relativistic limit since the velocity is then insensitive to changes in energy.

In an earlier work,[1] we predicted the nonrelativistic spatial variation of the Schwarz-Hora radiation intensity,

$$I_{\rm NR} = I_0 \exp\left[-\frac{1}{2}\left(\frac{m\omega_0 x \Delta k}{\hbar k^2}\right)^2\right] \cos^2\left(\frac{m^2\omega_0^2 x}{2\hbar^2 k^3}\right), \quad (1)$$

where $\omega_0$ is the laser frequency, $m$ is the electron's mass, $x$ is the distance from the modulation film to the observation screen, $\hbar k$ is the mean momentum about which each electron's wave packet is peaked as it leaves the electron source, and the total wave-number spread of the beam $\Delta k = (\kappa_B^2 + \kappa_e^2)^{1/2}$, where $\kappa_e$ is the wave-number spread of each Gaussian packet and $\kappa_B$ is the half-width of the Gaussian distribution for the beam's momenta.[4]

The preceding result was derived on the basis of nonrelativistic kinematics. Generalizing this result by employing relativistic relations and also allowing each electron to acquire transverse momentum $q$ equal to that of the absorbed photon, we obtain[5]

$$I = I_0 \exp\{-\tfrac{1}{2}[m^2\omega_0 p^{-3}(\Delta E)x]^2\} \cos^2(2\pi x/\Lambda), \quad (2)$$

where

$$\Lambda = (4p^3\pi/\hbar m^2\omega_0^2)[1 + (pq/m\hbar\omega_0)^2]^{-1} \quad (3)$$

and $m$ now represents the electron's rest mass, $p$ is its mean momentum, and $\Delta E$ is the total half-width energy spread in the beam prior to modulation. It is to be noted that $\Delta E$ is the *short*-time energy resolution, i.e., $\Delta E$ is the spread in energy measured over either the relaxation time of the radiation mechanism or the coherence time of the laser, whichever is shorter. Hence, any low-frequency variations of the power supply (ripple, drift, etc.) will not contribute to this $\Delta E$. As can be readily seen, the exponential damping factor in Eq. (2) dies off exponentially as the *square* of the product of the energy spread and distance, and hence the radiation may be completely washed out at any reasonable observation distance if the energy spread is sufficiently large.

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Recently,[6] Schwarz reported results of experiments in which he used different sizes for the exit diaphragm in the magnet of his velocity selector. This alteration changes the energy resolution of the electron beam just prior to modulation. At the distances where the oscillatory cosine-squared factor is unity, the intensity at the screen was measured for diaphragm sizes $\Delta S$ of 5, 10, and 20 $\mu$m. For a diaphragm size of 100 $\mu$m, no radiation was observed. If we assume that $\Delta E$ is proportional to $\Delta S$, his data can be compared with our theoretical prediction [Eq. (2)] employing a single adjustable parameter. In Fig. 1, the experimental values for $[\log(I_0/I)]^{1/2}$ are plotted against $(\Delta S)x$, and the plot is linear, in agreement with Eq. (2). The slope gives the relation

$$\Delta E/\Delta S = 1.10\times 10^{-3}\ \mathrm{eV}/\mu\mathrm{m} \tag{4}$$

for his velocity selection apparatus. This implies a $\Delta E$ of $5.5\times 10^{-3}$ eV for his original experiment[3] where the smallest diaphragm size was used. This is in good agreement with his estimate[2] of less than $10^{-2}$eV. In Fig. 2, his data are superimposed on the theoretical curves of the damping factor for the $\Delta E$ values corresponding to the various diaphragm sizes. It is evident that with the largest diaphragm size $\Delta S = 100$ $\mu$m, the radiation is not observable even at the shortest distance $x = 10.2$ cm in the experiment.

It is of interest to evaluate the damping factor for the parameters of the recent unsuccessful attempt by Hadley *et al.*[7] to observe the Schwarz-Hora effect. Taking their estimate of $\Delta E = 0.35$ eV (half-width), an electron energy of 80 keV, and $x = 30$ cm, the damping factor is $\exp(-90)$! Other experiments in progress[8] also seem to be operating in the region where observation of the Schwarz-Hora radiation is impossible because the damping factor is too small.

It might be argued that very small energy resolution and short film-to-target distance are difficult to achieve experimentally. We wish to point out that one may take advantage of the strong dependence on the electrons' momenta in the relativistic damping factor in Eq. (2). For sufficiently high momentum, the need for low values of $\Delta E$ and $x$ is considerably weakened so that experimentally easily accessible $\Delta E$'s and convenient distances can be tolerated. For example, if one takes the electron's kinetic energy to be 200 keV, the relative intensity $I/I_0$ becomes $1/I_e$ at $(\Delta E)x \simeq 5$ eV cm. At a kinetic energy of 5 MeV, this product is $(\Delta E)x \simeq 7000$ eV cm. These numbers are to be contrasted with the value of $(\Delta E)x \simeq 0.5$ eV cm with Schwarz's energy of 50 keV. Clearly, experiments to verify the Schwarz-Hora effect should be designed with these considerations in mind and should use as high an electron energy as possible.

*Note added in manuscript.* Since the submission of this paper Schwarz has informed us (private communication) that for an aperture of 5 $\mu$m, he estimates a $\Delta E$ of 4.7 mV on the basis of the geometry of his velocity selector. This is in good agreement with the value of 5.5 mV calculated from the data with our theoretical expression.

The authors thank Professor Helmut Schwarz for making his data available to us prior to publication.

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[4] Gaussian shapes are used because the emission of the electrons from the filament is caused by a large number of random processes and the Central Limit Theorem should apply. The resulting agreement with experimental data verifies this assumption.

[5] There is some uncertainty concerning the value of the transverse momentum $q$, depending on, for instance, whether one assumes the free propagating mode or waveguide mode for the laser field in the crystal. The measured value of $\Lambda$ indicates that the actual value of $q$ is considerably smaller than that predicted with either mode. For this reason, we have ignored the contribution of the transverse momentum to the damping factor. It is expected that the transfer of appreciable transverse momentum to a wave packet with small transverse dimensions will result in additional damping due to the resulting separation in that direction.

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