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Transfer Function Models of Daily Urban Water Use

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A time series model of daily municipal water use as a function of rainfall and air temperature is developed. Total water use is separated into base use and seasonal use. The seasonal use series is detrended, then a nonlinear heat function relating water use to air temperature during rainless periods is employed to deseasonalize the series. The residuals are modeled using Box-Jenkins transfer functions with transformed rainfall and air temperature as independent variables. The model is applied to daily data from Austin, Texas from 1975 to 1981 and accounts for 97% of the variance of daily municipal water use over that period. Forecasts of daily usage are made for a two-week lead time.

INTRODUCTION

Rainfall is the climatic variable which most influences urban water use, but these influences are difficult to quantify because of the sporadic nature of rainfall. Many investigators [e.g., *Maidment and Parzen*, 1984*a*, *b*] have shown through regression and time series analysis that monthly or annual average rainfall data are related to water use. However, time averaging over such intervals obscures the cause-and-effect relationship between rainfall and water use which is more clearly seen in daily data. For daily rainfall data, statistical analysis using multiple linear regression is not very satisfactory because rainfall is a point process which occurs in pulses but is zero most of the time. In this study transfer function modeling techniques are investigated as a means of more appropriately representing this relationship between daily rainfall and urban water use. These models have the capability of representing the observed dynamic response characteristics of a rainfall event on the dependent variable, an initial step change at the onset of the event followed by a diminishing response over time. The derived model is used for short-term forecasting of daily water use in Austin, Texas, the city whose data were used for development of the methodology.

Characteristics of these data are illustrated by the monthly and daily water use data from Austin, Texas. Figure 1*a* shows the monthly use from 1961 to 1981, these data being the aggregated pumpage from all water treatment plants adjusted for change in storage in regulating reservoirs. An annual growth in both average annual use and in seasonal variation as well as a large variation in summer water use from one year to the next is evident from the data. For example, the use in July 1979 averaged 77 million gallons per day (mgd) (0.29M m^3/day) and for July 1980, 137 mgd (0.52M m^3/day). These values differ by nearly a factor of 2 because of rainfall and associated climatic effects.

Figure 2*a* indicates daily use rates for 1980, a drought year. A regular pattern of seasonal variation which is temporarily interrupted by the occurrence of rainfall is evident over the summer. The nature of these interruptions is most clearly seen in Figure 3, which illustrates the effects of two isolated rainfalls in June 1980. Each time that there is rainfall, the water use is reduced immediately, then it gradually resumes its regular seasonal pattern.

Study of water use data from various cities suggests a model formulation consistent with the following postulates.

1. Urban water use can be divided into base use and seasonal use, where the base use is characterized by the water use during the winter months.
2. The variation in seasonal water use, in the absence of rainfall, follows a pattern that is dependent on air temperature.
3. The occurrence of a rainfall causes a temporary reduction in seasonal water use that diminishes over time and eventually becomes negligible.

The following sections contain a discussion of the mathematical representation of the three physical postulates, results of a study using daily data in Austin from 1975 to 1981, and an illustration of how the model is used for forecasting and simulating daily water use. The scope of the discussion is limited to urban water use, defined as aggregated water production from water sources or treatment plants in a city. It is expected that the concepts developed herein would be applicable to modeling water use in other cities, although the results relate only to data from Austin. The actual magnitudes of the effects of rainfall on urban water use will undoubtedly vary from one city and climatic region to another.

LITERATURE REVIEW

The traditional motivation for studying daily urban water use is to identify the peak daily demand to be expected in a year in order to ensure that the necessary treatment and distribution capacity is available to meet that demand. *Graeser* [1958] found that maximum daily demands in Dallas, Texas were significantly related to the number of previous days with maximum air temperature over 100°F (38°C) and the number of weeks since the occurrence of a week with 1 inch (2.54 cm) of rainfall. *Howe and Linaweaver* [1967] developed equations for maximum day sprinkling demand using maximum potential evapotranspiration, irrigable area, water price, and housing value as independent variables. In this study residential water demand was separated into domestic (indoor) and sprinkling (outdoor) demand. *Hughes* [1980] formulated regression equations for maximum day demand using water

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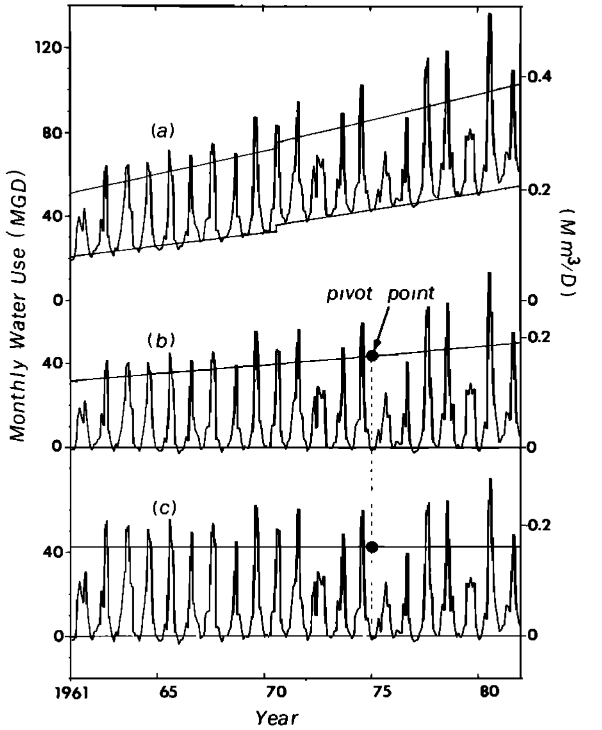

Fig. 1. Procedure for isolating and detrending monthly seasonal water use. (*a*) Linear trend lines are fitted to base use. (*b*) Base use is subtracted and a trend line fitted to maximum monthly seasonal use. (*c*) Maximum trend line is rotated about a pivot point to produce a stationary seasonal water use pattern.

price and an outdoor water use index as independent variables.

Weeks and McMahon [1973] found that in Australia the number of rainy days per annum was the most significant climatic variable affecting annual per capita water use, but that weekly pan evaporation and average maximum daily temperature were more significant explanatory variables than rainfall in a multiple linear regression describing maximum weekly demands. The most likely reason for the insignificance

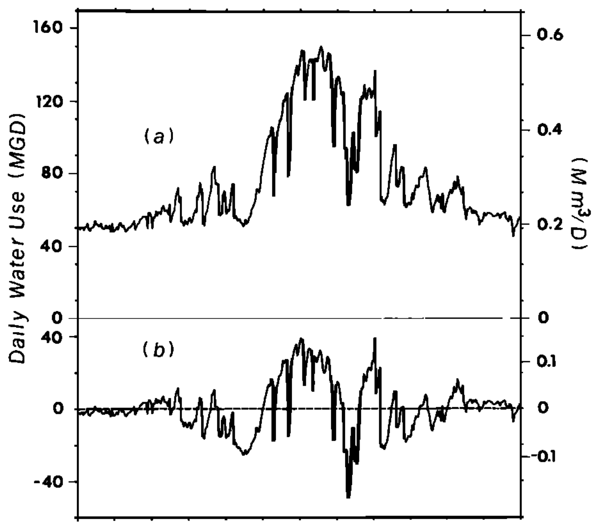

Fig. 2. Daily water use in Austin for (*a*) 1980 and (*b*) the short-memory series produced by detrending and deseasonalizing.

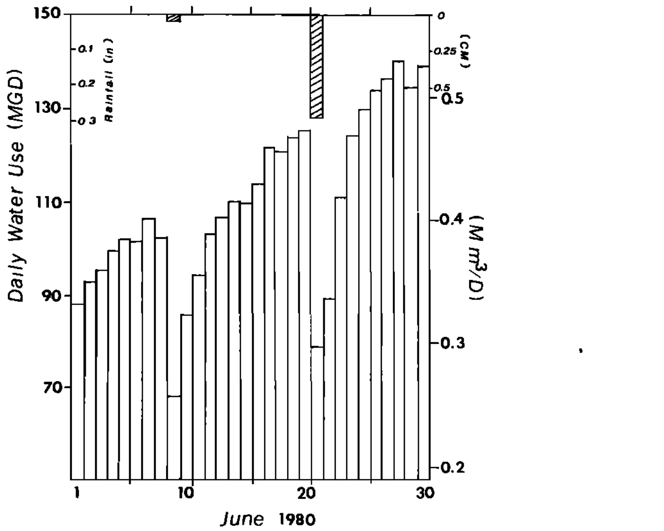

Fig. 3. Daily water use in Austin during June 1980 showing the effect of two isolated rainfalls.

of rainfall in the weekly analysis is that its relationship with water use is neither linear nor time invariant as assumed by linear regression analysis. Evaporation and air temperature vary continuously over a positive range, as does water use, and thus may result in regression coefficients which are more significant than those for rainfall in multiple linear regression models.

Oh and Yamauchi [1974] studied the time patterns of daily water use in Honolulu from 1960 to 1971. They developed linear regression models of maximum and minimum daily use in each year. Over this period the maximum daily use increased more rapidly than the minimum because of an increasing seasonal component of water use.

In recent years concern about meeting peak daily demands has prompted many communities to adopt water conservation ordinances that restrict water use during peak demand periods. As a result, there is need to analyze the whole sequence of daily water use values during a year, and especially during the summer, instead of just the single peak daily demand that has traditionally been studied. *Anderson et al.* [1980] formulated several linear regression models of daily municipal water use in Fort Collins, Colorado using evapotranspiration, effective rainfall, reservoir storage level, and dummy variables for lawn watering restrictions. The coefficients of determination for these models ranged from 0.56 to 0.65. They concluded that about half the drop in water use during a conservation period was the result of unusually high rainfall during that period, while the remainder of the decrease was due to the restrictions applied. Similarly, *Steiner and Smith* [1983] employed a linear regression model for daily water use in the Washington Metropolitan Area. They found that concurrent air temperature and the number of preceding dry days were the variables that most influenced daily water use. Their water use data also exhibit significant day-of-the-week effects. *Berk et al.* [1981] employed Box-Jenkins time series models to analyze the impact of conservation practices on monthly water use in California during the 1976–1977 drought. They used monthly rainfall and air temperature as control variables to correct the data for weather variations during the study period and showed that rainfall has an effect on water use during the month of its occurrence and a lagged effect the following month, while air temperature has no lagged monthly effects.

Estimation of daily power consumption is similar to estimation of daily water use. Both are utility services supplied to customers throughout a city, both exhibit seasonal variations, and both are affected by weather conditions. *Bunn* [1982] summarized the short-term power consumption forecasting literature. The principal weather variables affecting power load forecasting are air temperature, wind speed, and effective illumination (a function of cloud cover, visibility and precipitation), where air temperature is the most commonly employed. In contrast to urban water use, rainfall does not have a major influence on urban power consumption.

Future projections of power consumption in Austin utilize a "heat function" relating power consumption (megawatt hours) to air temperature [*Federal Power Commission*, 1971]. Its primary purpose is to "weather-correct" the observed power consumption data in each year for heat effects resulting from the departures of observed air temperature from long-term average values. It is shown in subsequent sections that water use in Austin exhibits a similar heat function.

The methodology employed in this study is outlined in the following section. A more detailed discussion is contained in the work by *Miaou* [1983].

METHODOLOGY

Daily water use is made up of base and seasonal use, both exhibiting trends through time. Seasonal use has two components, one which varies smoothly over the year with normal air temperature and another which represents the short memory residuals:

$$W(t) = \hat{W}_b(t) + g(t)[\hat{W}_p(t) + W_s(t)] \quad (1)$$

where

- W daily water use;
- $\hat{W}_b$ estimated base use;
- g trend coefficient for peak seasonal use;
- $\hat{W}_p$ estimated potential water use, a function of normal air temperature;
- W_s short-memory water use;
- t daily time index from beginning of series.

A transfer function noise model is formulated for the short-memory series:

$$W_s(t) = \bar{W}_s + \sum_{i=1}^{2} \frac{\omega_0^{(T_i)}}{1 - \delta_1^{(T_i)}B} T_i(t) + \sum_{i=1}^{2} \frac{\omega_0^{(R_i)} - \omega_1^{(R_i)}B}{1 - \delta_1^{(R_i)}B} R_i(t) + \frac{1}{1 - \phi_1 B - \phi_2 B^2 - \phi_7 B^7} a(t) \quad (2)$$

where

- $\bar{W}_s$ level component of short-memory series model;
- T transformed daily average air temperature;
- R daily rainfall or a substitute variable for rainfall effects;
- a independent normal random variable of zero mean and variance σ_a^2;
- i index for the season of the year or the range of a variable;
- $\omega_0, \omega_1, \delta_1$ transfer function coefficients;
- ϕ_1, ϕ_2, ϕ_7 autoregressive coefficients of noise model;
- B backshift operator.

Trend Equations

Trends in both the base and seasonal components are visible in the monthly water use data. They result from increasing population served by the water system and changes in water use habits over time. Various methods are available for estimating these trends in water use including regressions using population, number of connections to the distribution system, water price, and household income as independent variables. [*Maidment and Parzen*, 1984*a*, *b*]. In this study a fairly simple approach based on linear regression of maximum and minimum monthly water use against time was used as illustrated in Figure 1.

Monthly average base water use $W_b(m)$ is identified from the lowest monthly water use. A polynomial function of time is fitted

$$\hat{W}_b(m) = a_0 + a_1 m + a_2 m^2 + \ldots \quad (3)$$

For Austin, the lowest month is January, so $m = 1, 13, 25, \ldots,$ in (3). Typically only a_0 and a_1 are statistically significant, although their values may vary from one set of years to another as a result of population growth changes or major changes in the city's water supply system.

The peak monthly use is estimated in a similar manner. The expected base use $\hat{W}_b(m)$ is subtracted from the data in the months typically exhibiting the highest water use (July and August in Austin, so $m = 7, 8, 19, 20, \ldots$), and the remaining seasonal water use $S_p(m)$ is regressed against time, monthly rainfall $R(m)$, and monthly average temperature $T(m)$:

$$\hat{S}_p(m) = b_0 + b_1 m + b_2 m^2 + \ldots + b_R[R(m) - \bar{R}(m)] + b_T[T(m) - \bar{T}(m)] \quad (4)$$

where $\bar{R}(m)$ and $\bar{T}(m)$ are the long-term average values of $R(m)$ and $T(m)$, respectively, in (4). Typically only b_0, b_1, b_R, and b_T are statistically significant. A "weather-corrected" estimate of peak monthly use is obtained by setting $T(m) - \bar{T}(m)$ and $R(m) - \bar{R}(m)$ to zero in (4). Equivalent daily equations for estimating the trends through time of base use $\hat{W}_b(t)$ and peak monthly seasonal use $\hat{S}_p(t)$ can readily be obtained from (3) and the truncated version of (4).

Equations (3) and (4) are employed to eliminate the base use from the data and convert the seasonal use component to a seasonally stationary time series (Figure 1). First, actual seasonal use $S(t)$ is produced by subtracting base use from the total use, $W(t)$:

$$S(t) = W(t) - \hat{W}_b(t) \quad (5)$$

Second, a "pivot-time" t_0 is selected, which is a reference date for standardizing the data. The growth coefficient $g(t) = \hat{S}_p(t)/\hat{S}_p(t_0)$ is computed and employed to produce a detrended, seasonally stationary time series $S_d(t)$ of the seasonal water use:

$$S_d(t) = \frac{S(t)}{g(t)} \quad (6)$$

The transformations from (5) and (6) are shown graphically in Figure 1*b* and 1*c*, respectively.

Heat Function

The year-to-year variations in seasonal water use (Figure 1*c*) arise from the collective action of hundreds of thousands of people responding to rainfall and heat conditions in their decisions regarding watering their lawns and gardens. Thus the process being modeled is partly physical and partly psycho-

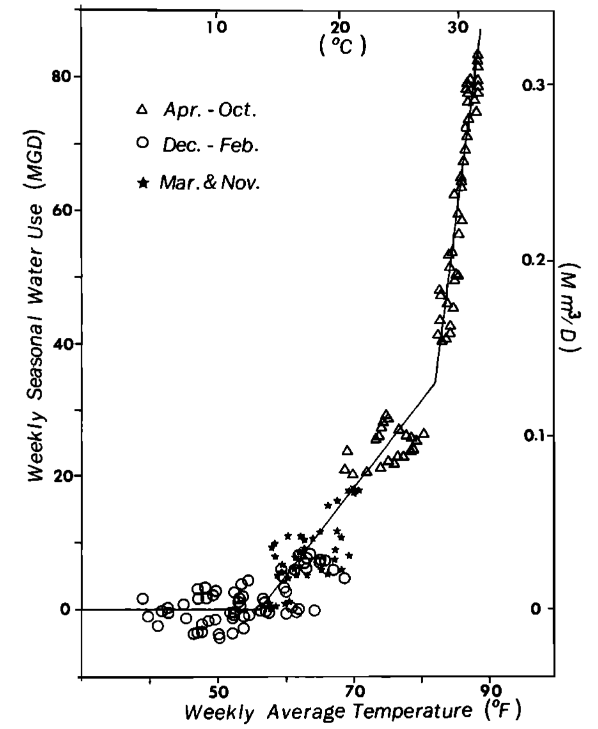

Fig. 4. Heat function relating seasonal water use and average air temperature. Data points shown are weekly average values during rainless periods.

logical, and the use of physical variables and equations to describe the collective response is an approximation of a very complex underlying process that involves millions of individual decisions. Therefore the climatic variables (rainfall and air temperature) used in describing this process can be regarded only as "indicator" variables. Other factors that may affect daily water use patterns, such as evaporation rate, wind speed, and cloudiness, are not considered in this investigation.

In modeling the seasonal variation exhibited in Figure 1*c* it is assumed that there is a functional relationship between water use and air temperature which is valid in the absence of recent rainfall. Figure 2*a* suggests that this function can be "glimpsed" during dry periods, but is obscured during wet periods. Thus it is logical to screen the data and select only those periods during which there are minimal rainfall effects in order to study the relationship between water use and air temperature. In Austin, weekly average values of water use and air temperature are used in estimating a heat function $H(T)$. During the summer (April–October) the data are selected such that there is no rain in the two previous weeks and in the week under consideration; for the remainder of the year, 3-day periods with no rain prior to the specified week are sufficient to define a dry period. The selected data for observed weekly average values of the seasonal water use are then plotted against weekly average air temperature T, (Figure 4). The heat function was also studied using daily and monthly data, but weekly time intervals yielded the best compromise between stability and sensitivity to change. The heat function is estimated as

$$H(T) = c_0 + c_1 T \tag{7}$$

where c_0 and c_1 are coefficients which depend upon which of the three ranges of T the observed air temperature lies within. The values of the coefficients and the location of the "break points" are obtained by iterative application of linear regression. The observed data in Figure 4 indicate that there are three seasons of the year: a summer season (April–October), a transition season (March and November), and a winter season (December–February).

A daily "potential water use", $W_p(t)$, may be estimated by substituting the normal daily air temperature estimated from long-term records, $T_N(t)$, into the heat function for each day of the year.

$$W_p(t) = H[T_N(t)] \tag{8}$$

The "short-memory" effects of rainfall, air temperature, and random errors are then isolated as

$$W_s(t) = S_d(t) - W_p(t) \tag{9}$$

In studying the short-memory effects of air temperature using (2), the residual air temperature is henceforth employed:

$$T_i(t) = T(t) - T_N(t) \tag{10}$$

The rationale for using this deseasonalizing approach rather than more conventional methods such as fitting Fourier series is that changes in air temperature can either increase or reduce water usage, but a rainfall occurrence can only reduce usage. A Fourier series model of historical seasonal water use may not represent these effects adequately as it contains rainfall as well as temperature effects. Because it must indicate increased water usage when there is no rainfall, such a model is inconsistent with the basic postulates cited earlier and with the transfer function model results reported in this paper.

Transfer Function Model

The short-memory series $W_s(t)$ obtained by (9) is illustrated in Figure 2*b* for 1980. It fluctuates about zero, being positive when observed air temperature is greater than normal and negative following rainfall or lower than normal air temperature. The rainfall and air temperature effects are dynamic; that is, a change in rainfall or temperature influences water use values both on that day and for a number of subsequent days. The functional form of a Box-Jenkins type transfer function model which represents water usage is given by (2).

Identification of the appropriate model and estimation of the parameters can be accomplished by standard methods [see *Box and Jenkins*, 1976] provided that the coefficients are constant through time and independent of the levels of the variables $W_s(t)$, $T_i(t)$, and $R_i(t)$. Unfortunately, the daily water use data in the study do not satisfy these assumptions: the parameters are time varying because the response of water use to climatic variables changes seasonally; there are nonlinearities because the response of water use per unit change in air temperature or per unit of rainfall depends on the magnitude of the temperature and the amount of rainfall which occurs; there is an interaction effect between the responses to successive days of rainfall in that the water use response to the last day's rain is less than to the first; also, air temperature and rainfall are correlated because the occurrence of rainfall lowers air temperatures. These difficulties necessitate the following modifications of a standard transfer function model.

1. Time variation: in order to allow for time variations in the parameters, the year is divided into the same three seasons employed for the heat function (December–February (winter), November–March (transition), and April–October (summer)). Different parameter sets are estimated for each season in some cases. This particular division into three seasons is somewhat arbitrary but yields empirically satisfactory results.

2. Nonlinearity: the response variables are modeled as

linear functions of nonlinearly transformed independent variables.

3. Interaction: an interaction effect exists in that the response of water use to a rainfall one day depends on the previous day's seasonal water use $S_d(t-1)$. If $S_d(t-1)$ is high, there is a large potential response; if it is lower due to rainfall, the potential response is less. As an alternative to rainfall depth, $S_d(t-1)$ is employed in (2) in conjunction with zero-one indicator variables associated with different magnitudes of $R(t)$. This modification to the model has the advantage that the actual water use cannot be driven below base use but instead approaches base use after several days of sustained rainfall. Empirical observations from the data support this hypothesis.

4. Correlation: the dependence of air temperature on rainfall requires the application of a prewhitening procedure in the model identification phase. A transfer function model is developed of daily air temperature using daily rainfall as the explanatory variable, and the residuals of this model are substituted in (2) for $T_i(t)$.

Parameters of six alternative forms of (2) are estimated using the Austin data.

Estimation

Least squares estimates of the parameters of the short memory model are obtained using Marquardt's algorithm [*Marquardt*, 1963]. Diagnostic checking of the short-memory model is performed by testing the residual series for independence, stationarity and normality. The cross-correlation matrix of the parameter estimates is also computed.

RESULTS

The data employed for analysis in Austin were (1) monthly water use, rainfall, and average air temperature from 1961 to 1981 and (2) daily water use, rainfall, and average daily air temperature from 1975 to 1981. The daily average air temperature data were obtained by averaging the recorded maximum and minimum air temperatures for each day. Water use is in units of millions of gallons per day (mgd), rainfall in inches, and temperature in °F (1 mgd = 3785 m^3 d^{-1}; 1 inch = 2.54 cm).

Trend in Monthly Data

The trend in base use is estimated by fitting (3) to the monthly water use in January of each year of the data presented in Figure 1*a*. The base use exhibits a step change in 1970 and a different slope during the 1970's than the 1960's. Dummy variables are included in (3) in order to represent these changes:

$$\hat{W}_b(m) = \underset{(21.34)}{20.5} + \underset{(7.77)}{0.115}\, X_1 + \underset{(10.43)}{0.133}\, X_2 + \underset{(2.54)}{3.58}\, X_3 \qquad (11)$$

where $X_1 = m$ from January 1961 ($m = 1$) to May 1970 ($m = 113$), $X_2 = m - 114$ from June 1970 to December 1981 ($m = 252$), $X_3 = 0$ prior to May 1970, and $X_3 = 1$ therefter. The values in parentheses below the coefficients in (11) are the associated t statistics. The $R^2 = 0.982$ for (11). The step change in base use during 1970 is attributed to the installation of Austin's third water treatment plant, whose effect on base use was equivalent to two and a half years of normal growth at that time. Studies of the effect of water price and income changes on base use were not undertaken.

The trend in peak monthly seasonal water use is estimated by fitting (4) to the data presented in Figure 1*b* obtained by subtracting $\hat{W}_b(m)$ from the original monthly water use data:

$$\hat{S}_p(m) = \underset{(12.34)}{30.21} + \underset{(4.20)}{0.070}\, m + \underset{(7.95)}{5.88}\, [T(m) - \bar{T}(m)] - \underset{(-2.97)}{1.62}\, [R(m) - \bar{R}(m)] \qquad (12)$$

where $m = 1$ for January 1961 and the values in parentheses below the coefficients are the associated t statistics. $R^2 = 0.790$ for (12). No step change in seasonal use during 1970 was found. Several alternative models to (12) involving estimating trends through time without weather correction were tested and found to be inferior to (12) in terms of R^2 and the properties of the residuals.

By comparing the trend coefficients in (11) and (12) it can be seen that peak monthly seasonal use grew about half as fast as base use from 1961 to 1981.

Detrending the Daily Data

The first daily observation was January 1, 1975. This was chosen as the pivot-time t_0 in (6). The daily equivalents of (11) and (12) are

$$\hat{W}_b(t) = 44.38 + 0.00438t \qquad (13)$$

$$\hat{S}_p(t) = 42.00 + 0.00230t \qquad (14)$$

where $t = 1, 2, \ldots, 2557$ and $t = 1$ corresponds to January 1, 1975. A detrended seasonal water use time series $S_d(t)$ is obtained by substituting (13) and (14) into (5) and (6). Base use constitutes three quarters and seasonal use one quarter of the annual water use volume, although base use is 40% and seasonal use 60% of the peak daily water use.

Heat Function

The heat function $H(T)$ is estimated by fitting a piecewise linear function to the daily detrended seasonal water use $S_d(t)$ and air temperature $T(t)$ during rainless periods (Figure 4). Break points were selected at 56°F (13°C) and 82°F (28°C). Below 56°F, $H(T) = 0$. Above this temperature and during the summer season

$$H(T) = \underset{(19.1)}{1.29}\, Y_1 + \underset{(32.7)}{7.67}\, Y_2 \qquad (15)$$

where $Y_1 = T - 56$ and $Y_2 = 0$ for $56°\mathrm{F} \le T < 82°\mathrm{F}$, while $Y_1 = 26°\mathrm{F}$ and $Y_2 = T - 82$ for $T \ge 82°\mathrm{F}$. $R^2 = 0.974$ for (15). The t statistics are shown below the corresponding coefficient values. Coefficients of a spline curve comprised of two linear functions connected by a quadratic function were estimated. This model was inadequate because the mass of data in the middle range of temperatures dominated the fit, so that the water use values for very high air temperatures were poorly estimated. The nonlinearity of the overall heat function is clearly indicated by the fact that the response of water use to a unit change in air temperature is six times larger when the air temperature lies beyond 82°F than when it is below this level. This ratio is used subsequently to transform the residual air temperature series. During the winter season the magnitude of the response of water use to air temperature is about half the value observed during the summer:

$$H(T) = \underset{(9.75)}{0.640}\, Y_1 \qquad (16)$$

with $R^2 = 0.490$.

It should be noted that the heat function is modeling two aspects of seasonal water use, periodic variation through time and the response of water use to changes in air temperature

TABLE 1. Coefficients of the Transfer Function Model for Short-Memory Effects of Temperature and Rainfall on Water Use

		$T_1(t)$		$T_2(t)$		$R_1(t)$			$R_2(t)$			Residual Autoregression			Overall Model		
Model	$\bar{W}_s$	ω_0	δ_1	ω_0	δ_1	ω_0	ω_1	δ_1	ω_0	ω_1	δ_1	ϕ_1	ϕ_2	ϕ_7	$\hat{\sigma}$(MGD) Residuals	R^2	AIC
Univariate	5.88	···	···	···	···	···	···	···	···	···	···	0.992	0.119	0.071	5.117	0.945	10860
	(1.80)											(0.020)	(0.020)	(0.010)			
Rainfall and residual temperature	2.063	0.201	0.708	···	···	−4.213	0.662	0.942	···	···	···	0.947	−0.100	0.075	4.736	0.953	10496
	(1.930)	(0.019)	(0.085)			(0.247)	(0.256)	(0.001)				(0.020)	(0.020)	(0.011)			
Two-period rainfall and residual temperature	0.00	0.386	0.789	0.093	0.503	−4.418	0.979	0.935	−2.799	···	0.690	0.940	−0.100	0.071	4.670	0.954	10430
		(0.031)	(0.056)	(0.023)	(0274)	(0.266)	(0.278)	(0.009)	(0.593)		(0.179)	(0.020)	(0.020)	(0.011)			
Rainfall and nonlinearly transformed temperature	0.906	0.361	0.284	0.171	···	−3.349	0.575	0.953	−2.190	···	0.591	0.889	−0.065	0.073	4.448	0.958	10181
	(1.196)	(0.017)	(0.045)	(0.037)		(0.255)	(0.265)	(0.006)	(0.564)		(0.243)	(0.020)	(0.020)	(0.012)			
Seasonal water use and nonlinearly transformed temperature	2.007	0.261	0.244	0.170	···	−0.522	0.126	0.895	···	···	···	0.811	0.055	0.093	3.868	0.969	9465
	(2.063)	(0.015)	(0.054)	(0.032)		(0.017)	(0.019)	(0.010)				(0.020)	(0.020)	(0.012)			
Seasonal water use split by rainfall >0.05 in.	1.727	0.260	0.295	0.172	···	−0.491	0.105	0.941	−0.275	0.104	0.785	0.805	0.060	0.088	3.839	0.969	9433
	(0.894)	(0.015)	(0.052)	(0.032)		(0.024)	(0.025)	(0.009)	(0.011)	(0.012)	(0.021)	(0.020)	(0.021)	(0.012)			

Values in brackets below the coefficients are their standard errors.

during rainless periods. The first is used to deseasonalize the daily water use series, and the second is incorporated in the short memory model following deseasonalization. It might be expected that other causes of periodic variation in seasonal water use, such as the calendar of activity of the city's institutions or variations in solar radiation, could produce a hysteresis effect so that the same temperature observed during the Spring and Fall would have a different seasonal water use associated with it. No hysteresis effect was found in the Austin data. This indicates that in the absence of rainfall, air temperature is the dominant variable governing seasonal variation in water use in that city, especially during the high temperature period of midsummer.

Transfer Function Model

The short-memory series W_s produced from (9) is analyzed using Box-Jenkins time series analysis techniques. The functional form of the resulting transfer function model is indicated in (2). Values of the significant coefficients for this model and each of five reduced models for which parameters were estimated are presented in Table 1.

Model 1 is a univariate ARMA model of $W_s(t)$. The values of the autoregressive parameters ϕ_1 and ϕ_2 show $W_s(t)$ to be nearly nonstationary. The presence of the ϕ_7 term in this and all the models indicates an additive significant "day of the week" cyclic effect because less water is used on weekends than weekdays in Austin.

Model 2 introduces $R_1(t)$ as daily rainfall and $T_1(t)$ as a residual daily air temperature obtained by fitting a transfer function model to the difference between observed and normal daily air temperature $[T(t) - T_N(t)]$ where daily rainfall is employed as the explanatory variable. It was found that rainfall influences air temperature much more during winter than during summer as indicated by the drop in air temperature following rainfall which was six times larger in winter than in summer. This difference can be explained by the occurrence of frontal rainfall in winter compared with thunderstorm rainfall in summer. Model 2 indicates improvement over model 1 in terms of the Akaike information criterion (AIC) value shown in Table 1.

Model 3 was developed to determine whether there is a seasonal effect in the short-memory response of water use to air temperature and rainfall. The year was divided into two periods: April to October $[T_1(t)$ and $R_1(t)]$ and November to March $[T_2(t)$ and $R_2(t)]$. The results show increased response of water use to rainfall and air temperature in the summer compared with the winter, but the increased explanatory power of the model as measured by the AIC value is very small.

Model 4 employs daily rainfall and an alternative transformation of daily air temperature to adequately represent the nonlinear rainfall-water use relationship. By using the daily air temperature residual $T(t) - T_N(t)$, the values of $T(t)$ and $T_N(t)$ are set equal to 56°F whenever either of them drops below that level. Any day one or both of the variables exceeds 82°F, the increment above 82°F is multiplied by 5.8, the ratio of the slopes of the two rising limbs of the heat function. In effect, the resulting transformed temperature variable $T_1(t)$ simulates a short-memory relationship corresponding to the middle limb of the heat function. The same two periods of the year are used as for model 3. The results of model 4 show a reasonable reduction in the AIC value. The immediate effect ($\omega_0^{(T_i)}$) of air temperature on water use is only slightly affected by this transformation compared with the results in model 3, but the memory effect of air temperature on water use in both "seasons" is much less for model 4 than model 3. This is indicated by the value of $\delta_1^{(T_1)} = 0.284$ compared with the estimate of 0.789 in the previous model. Furthermore, the estimate of $\delta_1^{(T_2)}$ is statistically insigificant in this model. The effect of the nonlinear transformation of air temperature appears to be more pronounced than that achieved by filtering air temperature with rainfall as in models 2 and 3.

Examination of the coefficients $\omega_0(^{R}i)$ in model 4 shows that the immediate response of water use to rainfall in summer is

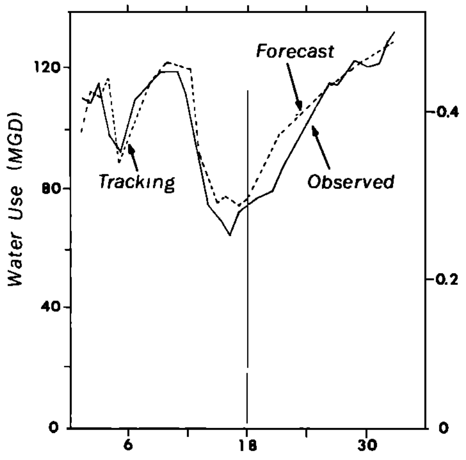

Fig. 5. Typical forecast of daily water use for a two-week lead time produced on July 18, 1983. Forecast assumes normal air temperature and no rainfall during the forecast period.

around 3.3 mgd/inch (4.9×10^{-3}M m^3 day^{-1} cm^{-1}). While this may be reasonable as an average response, the data presented in Figure 3 indicate that isolated rainfalls during the summer can have an effect 10–100 times larger than 3.3 mgd/inch. To account for this effect, rainfall is treated as an intervention series in model 5 and replaced in (2) by the previous day's seasonal water use $S_d(t-1)$ whenever rainfall occurs. That is, in model 5, $R_1(t) = S_d(t-1)$ if rainfall on the current day is nonzero and $R_1(t) = 0$ otherwise. This change results in a substantial reduction in the AIC criterion value.

The dimensionless coefficients $\omega_0^{(R_1)}$ and $\omega_1^{(R_1)}$ in this model are related to the proportion of the previous day's seasonal water use that is lost on the current day, $\omega_0(R_1)$, and the subsequent day, $\delta_1^{(R_1)}\omega_0^{(R_1)} - \omega_1^{(R_1)}$. The proportions $\omega_0^{(R_1)}$ and $\omega_1^{(R_1)}$ are about 50 and 10%, respectively. The coefficient $\delta_1^{(R_1)} = 0.895$ indicates that the memory of a rainfall continues for about 12 days by which time the response is less than a quarter of its initial value.

It might be expected that the coefficients $\omega_0^{(R_1)}$, $\omega_1^{(R_1)}$, and $\delta_1^{(R_1)}$ would be different for differing rainfall amounts. Alternative models were developed where the range of rainfall amount was divided into nine intervals. No significant difference between the transfer function coefficient values was found except for rainfall less than or greater than 0.05 inches/day (0.13 cm/day). Model 6 incorporates this refinement by defining $R_1(t) = R_2(t) = 0$ if there is no rainfall, $R_1(t) = S_d(t-1)$ and $R_2(t) = 0$ when the current day's rainfall is greater than 0.05 inches (0.13 cm), and $R_1(t) = 0$, $R_2(t) = S_d(t-1)$ for rainfall between 0.01 and 0.05 inches (0.03–0.13 cm). Comparisons of $\omega_0^{(R_1)}$ and $\omega_0^{(R_2)}$ for model 6 show that the immediate response and the memory effect of rainfall in the lower range are about half that in the upper range. However, model 6 has only a very slightly lower AIC value than model 5.

The results presented herein were developed with data from a single rain gage. Ongoing study employing the average rainfall of a number of gages distributed throughout the city is demonstrating a more precise relationship between $\omega_0^{(R)}$ and rainfall amount.

Models 4–6 all imply that in winter, air temperature has no memory effect, ($\delta_1^{(T_2)} = 0$), and that the immediate response of water use to air temperature is also less in winter than in summer, $\omega_0^{(T_1)} = 0.260$ and $\omega_0^{(T_2)} = 0.172$ in model 6. The values of $\omega_0^{(T_1)}$ and $\omega_0^{(T_2)}$ may be compared with the slope of the middle limb of the heat function which is 1.29 mdg/°F. The values estimated from the short-memory model are about five times smaller than those in the heat function. This may indicate that water use is much more responsive to air temperature during rainless periods than during rainy ones.

The Akaike information criterion (AIC) values presented in Table 1 continually decrease from model 1 to model 6, indicating that the increased number of parameters in the later models is justified. The AIC values must be interpreted carefully, however, as the residual series is so long (2550 data) that the AIC is lowered by even tiny reductions in the residual variance achieved by adding more parameters.

The overall R^2 of model 6 of 97% is comprised of trends in base and seasonal use (22%), seasonal heat function effects (37%), transfer function model (20%), and autoregressive noise model (18%).

Diagnostic Checking

The residual series from each model was tested for normality, whiteness, and independence of exogenous variable values. In addition, the cross correlation matrix of the estimates of the parameters was examined. All models appeared to be quite adequate.

Forecasting and Simulation

During April 1983, Austin water use reached record levels because of extremely low rainfall. The 0.16 inches (0.41 cm) rainfall measured for that month was the third lowest observed since rain gaging began in 1856. The model parameters were updated to include data for 1982, then during the summer of 1983, forecasts of daily water use for two-week lead times were produced each week for city officials. The forecasts assisted them in anticipating when it would be necessary for the city's water conservation ordinance to go into effect. This ordinance has mandatory restrictions on water use if the city's use rate rises above 150 mgd (0.57M m^3/day). Fortunately, regular rainfall throughout the summer kept water use below that level. An example of a two week ahead forecast computed on July 18, 1983, assuming no rainfall and normal air temperature is shown in Figure 5. Comparison with data observed after the forecast shows a good forecast accuracy over a wide dynamic range as water use increased from 76 mgd to 134 mgd (0.29–0.51M m^3/day) during this period. The period shown was the only two-week interval without rainfall during the summer. Unanticipated rainfall caused forecasts made at other times to exceed the observed data.

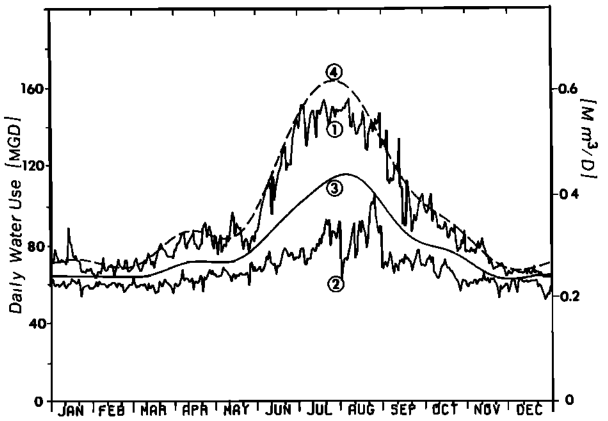

Fig. 6. Patterns of simulated daily water use in 1983 if weather from 1976 to 1982 had repeated: (1) maximum simulated use, (2) minimum simulated use, (3) Fourier-smoothed mean simulated use, (4) an upper 95% confidence limit on simulated use.

The daily rainfall, air temperature and model errors from 1976 to 1982 were used to reconstruct what would have been the water use pattern in 1983 if these weather conditions had recurred. For each calendar day in 1983 seven simulated water use values are thereby produced. Figure 6 shows the mean, maximum, and minimum of these data, where the mean was obtained by Fourier smoothing of the arithmetically computed daily averages. An upper 95% confidence limit curve is also shown, which was produced using the three parameter gamma distribution and Fourier-smoothed values of the daily mean, standard deviation, and skewness coefficient of the simulated water use data. The confidence limit and locus of maximum values agree very well except for very high water use levels where observed maximum water use falls below that indicated by the confidence limit. This decrease is attributed to unfulfilled demand resulting from supply constrictions and temporary water restrictions adopted in past years when water use was high.

Conclusions

1. It is proposed that the analysis of daily water use of a city in any given year incorporate the following three postulates. (1) Total use is divided into base use and seasonal use, base use being typified by winter use levels; (2) in the absence of rainfall, seasonal use is determined by daily air temperature; (3) the effect of rainfall is to cause an immediate drop in water use followed by a gradual increase until after a period of time there is no further effect of that particular rainy period.

2. Water use data from 1961 to 1981 in Austin, Texas indicate that base use constitutes three fourths and seasonal use one fourth of the total annual volume of water use. However, base use comprises 40% and seasonal use 60% of the peak daily water use rate. In Austin, a step change in base use occurred in 1970 which was equivalent to two and a half years of normal growth in base use at that time; this step is attributed to the installation of the city's third water treatment plant. Peak monthly seasonal use grew at about half the rate of base use during the study period.

3. Daily water use data from 1975 to 1981 in Austin indicate that seasonal water use is nonlinearly related to air temperature. This "heat function" is represented by a piecewise linear function with three segments with discontinuities at average daily air temperatures of 56°F and 82°F (13°C and 28°C). Below 56°F there is no relation between water use and air temperature. The slope of the line for temperatures exceeding 82°F is six times higher than that between 56°F and 82°F.

4. For the Austin daily water use the effect of a rainfall on a given day is to cause a drop in that day's seasonal water use of 49% of the previous day's seasonal water use if the rainfall amount is greater than 0.05 inches (0.13 cm). The residual impact of the rainfall persists for about two weeks. If the rainfall is 0.05 inches or less, the immediate drop is 27% of the previous day's seasonal use with the impact damping out after about one week. This relationship is being refined in an ongoing study. We conclude that it is primarily the occurrence rather than the amount of rainfall which affects daily water use in Austin.

5. The complete model describes 97% of the variability of daily water use data, much more than reported in any other daily water use study to the best of the authors' knowledge.

6. The model is used for forecasting daily water use and for reconstructing patterns of water use in the current year corresponding to weather conditions experienced in previous years, thereby assisting city officials with anticipating when water conservation ordinances involving summer water restrictions will need to be enforced.

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