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Comments on "Circular Disk Viscometer and Related Electrostatic Problems"

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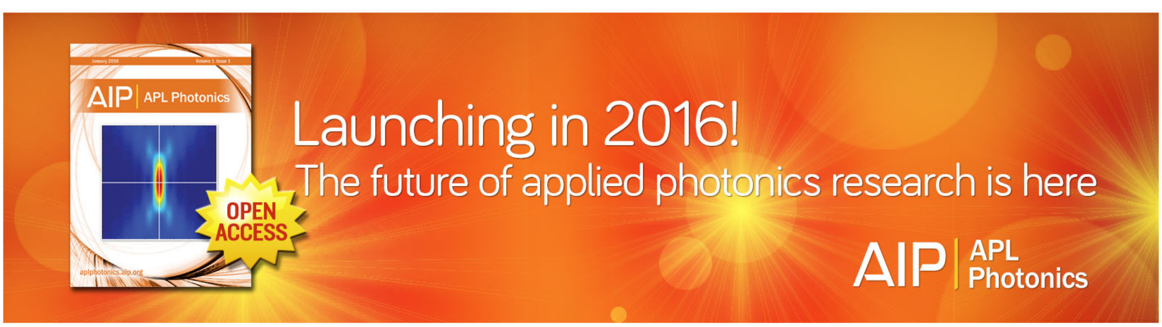

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studied. When particles are injected at $z=0$, it is important that they have a velocity distribution compatible with the current profile. A mismatch between $J_B(r)$ and F results in pulsations which mask other aspects of the behavior of the beam.

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[1] R. W. Bauer, E. J. Lauer, and K. G. Moses (private communication).
[2] J. Bolstad, J. Killeen, J. Leary, and E. Lee (private communication).

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Comments

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Comments on "Circular Disk Viscometer and Related Electrostatic Problems"

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(Received 13 April 1971)

In a recent paper by Shaw[1] constants $C^{(3)}$ and $C^{(5)}$ were defined by

$$\pi^2 C^{(3)} = -\tfrac{1}{2}(\log\tfrac{1}{8}+1)^2 + \int_0^1 \left[4b\left(\frac{E(b)}{1-b^2}\right)^2 - \frac{1}{(1-b)^2} + (1-b)^{-1}[\log\tfrac{1}{8}(1-b)+1]\right] db,$$

$$\pi^2 C^{(5)} = -\tfrac{3}{2}(\log\tfrac{1}{8}+\tfrac{7}{3})^2 + \int_0^1 \left[\frac{4}{b}\left(\frac{2b^2-1}{1-b^2}E(b)+K(b)\right)^2 - \frac{1}{(1-b)^2} + \frac{3}{1-b}[\log\tfrac{1}{8}(1-b)+\tfrac{7}{3}]\right] db,$$

where $E(b)$ and $K(b)$ are the complete elliptic integrals of the second and first kinds with modulus b.

It may be of interest that the integrals involving $E(b)$ and $K(b)$ can be evaluated in closed form. In fact,

$$4b\left(\frac{E}{1-b^2}\right)^2 = \frac{d}{db}\left[2b^2\left(\frac{dE}{db}\right)^2 + \frac{2b^2}{1-b^2}E^2\right],$$

and

$$\frac{4}{b}\left(\frac{2b^2-1}{1-b^2}E+K\right)^2 = \frac{d}{db}\left[4bE\frac{dE}{db} + 6b^2\left(\frac{dE}{db}\right)^2 + \frac{2b^2}{1-b^2}E^2\right].$$

These results follow from the differential equations[2]

$$b\frac{dE}{db} = E-K,$$

$$\frac{d}{db}\left(b\frac{dE}{db}\right) + \frac{b}{1-b^2}E = 0.$$

From the definitions of $C^{(3)}$ and $C^{(5)}$ above it then follows that

$$\pi^2 C^{(3)} = \lim_{b\to 1}\left[2b^2\left(\frac{dE}{db}\right)^2 + \frac{2b^2}{1-b^2}E^2 - \frac{b}{1-b} - \tfrac{1}{2}[\log\tfrac{1}{8}(1-b)+1]^2\right],$$

$$\pi^2 C^{(5)} = \lim_{b\to 1}\left[4bE\frac{dE}{db} + 6b^2\left(\frac{dE}{db}\right)^2 + \frac{2b^2}{1-b^2}E^2 - \frac{b}{1-b} - \tfrac{3}{2}[\log\tfrac{1}{8}(1-b)+\tfrac{7}{3}]^2\right].$$

Finally, from the expansion[3] of E in terms of the complementary modulus $b'=(1-b^2)^{1/2}$,

$$E = 1+\tfrac{1}{2}b'^2[\log(4/b')-\tfrac{1}{2}]+\cdots,$$

it follows that

$$C^{(3)}=0, \qquad C^{(5)}=1/3\pi^2.$$

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ing his attention to the paper in question.

[1] S. J. N. Shaw, Phys. Fluids **13,** 1935 (1970).

[2] E. Jahnke, F. Emde, and F. Lösch, *Tables of Higher Functions* (McGraw-Hill, New York, 1960), p. 64.

[3] Reference 2, p. 62.

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