AIP | Review of Scientific Instruments

Alternating Gradient Electrostatic Accelerating Tube

D. Boyd and J. V. Kane

Citation: Review of Scientific Instruments **37**, 555 (1966); doi: 10.1063/1.1720250
View online: http://dx.doi.org/10.1063/1.1720250
View Table of Contents: http://scitation.aip.org/content/aip/journal/rsi/37/5?ver=pdfcov
Published by the AIP Publishing

It is obvious that the operation of the slope plotter is highly subjective; however, with moderate experience the average operator should be able to reproduce results with precision of the order of that indicated in Table I. No attempt has been made to compare the results obtained with the slope plotter to those obtained by graphical differentiation, since the results of the latter procedure are highly dependent upon the amount of effort exerted in the process. It is expected, however, that the results obtained with the slope plotter will prove to be as accurate as those arrived at by a moderately extensive graphical differentiation, but with far greater ease and convenience.

THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 37, NUMBER 5 MAY 1966

Alternating Gradient Electrostatic Accelerating Tube

D. Boyd*

Rutgers, The State University, New Brunswick, New Jersey 08900

AND

J. V. Kane†

Bell Telephone Laboratories, Murray Hill, New Jersey 07971

(Received 12 November 1965)

The processes of electron multiplication and electron loading in acceleration tubes have been investigated theoretically. These effects limit the maximum voltage at which Van de Graaff generators can operate. A new design for an accelerating tube is proposed which by virtue of its closely spaced electrodes has the advantage of ease of calculation and flexibility of boundary condition. Computer calculations were done for this tube to study electron multiplication trajectories. We have found that an axial alternating electrical field gradient can limit such avalanches by periodically trapping the electrons.

1. INTRODUCTION

RECENT advances in Van de Graaff technology have resulted in machines which can operate at voltages in excess of 10 MeV and produce particle beams of energies in excess of 20 MeV by utilization of the tandem principle. The limitations in utility of this machine are due primarily to the failure of the insulating properties of the accelerator tube. Many existing Van de Graaffs could operate at higher voltages if a more satisfactory accelerating tube were available.

The two types of discharge that are found to occur are "electron loading" which results in a steady electron current down the tube, and a more violent form of this effect which results in violent tube spark or "kick." Both of these effects have been reduced recently with the advent of the inclined field tube, which "sweeps" electrons out of the beam.[1] Unfortunately, it is difficult to calculate the electric field due to inclined field electrodes, and the effect can only be estimated, or investigated experimentally. The same disadvantage also holds for other special devices such as various magnetic field configurations. These problems make the focusing and control of beam trajectories energy dependent and difficult to predict.

We believe that it is possible to construct a tube more suitable for calculation. Such a tube would have numerous electrodes, closely spaced, perhaps several to each 2.5 cm, so that to a good approximation a uniform boundary condition is obtained. Figure 1 shows how such a tube might be constructed and how the resistors could be attached. Pairs of resistors are suggested in order to reduce the chance of producing an open circuit in the expectation of resistor failures. By means of a suitable resistance divider network, the boundary potential shown in Fig. 2 could be produced. Calculations based on this trapping tube were done and

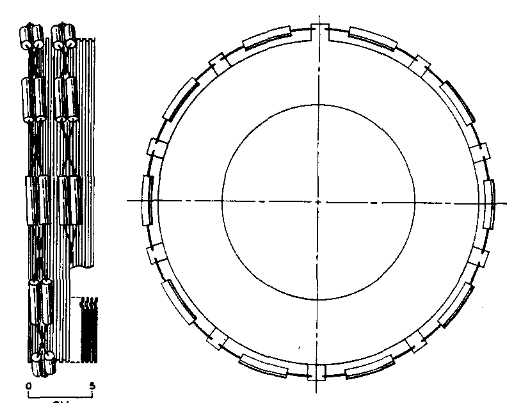

Fig. 1. Electrode structure. The resistors are wound about the circumference and connected to tabs protruding from the electrodes. In the configuration shown, the resistors would give rise to a linear gradient on the tube. Various network schemes are possible which would lead to the oscillating potential of Fig. 2.

* Supported in part by the National Science Foundation.

† Present address: Cylotron Bldg., Michigan State University, East Lansing, Michigan 48823.

[1] J. G. Trump, Nucl. Instr. Methods 28, 10 (1964); and K. H. Purser, A. Galejs, P. H. Rose, R. J. Van de Graaff, and A. B. Wittkower, Rev. Sci. Instr. 36, 453 (1965).

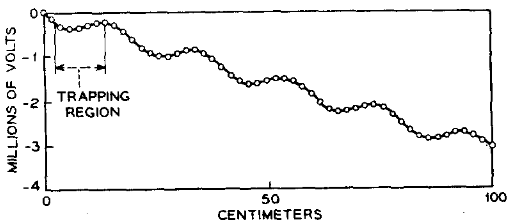

FIG. 2. Trapping tube boundary potential, the smooth curve represents the electric boundary potential used for the purposes of this calculation. The circles represent the potential of hypothetical electrodes having a spacing of 2 cm.

compared with the result obtained for an accelerating tube with a linear gradient.

2. METHODS

In common with the treatment given by Rose *et al.*[2] we calculate the field in the interior of the tube using Laplace's equation. Laplace's equation in cylindrical coordinates is

$$(1/r)(\partial/\partial r)[r^2(\partial\psi/\partial r)] + (1/r^2)(\partial^2\psi/\partial\varphi^2)+(\partial^2\psi/\partial z^2)=0, \quad (1)$$

and possesses the solution (for an infinite cylinder)

$$\psi=\sum_n a_n \sin nz I_0(r), \quad (2)$$

where the a_n are the Fourier coefficients of the boundary potential. We have investigated the case where the boundary condition can be expressed as the sum of two terms,

$$\psi_b=A_1\sin(N_1\pi Z/L)+A_2\sin(N_2\pi Z/L). \quad (3)$$

The first term is a linear gradient since we choose N_1 such that $N_1\pi Z/L\ll 1$. Superimposed on this is the second term which is an oscillating potential and gives rise to a reversing field, see Fig. 2. In a recent paper, Rose, Galejs, and Peck[2] have taken into account the effect of finite electrode thickness in a uniform tube by using an equation similar to Eq. (3) where L/N_2 is now the electrode spacing. In our case this would result in the addition of a third sine term to Eq. (3) which would be negligible due to the close spacing of electrodes assumed here.

We have used the above solutions to map the electric field and to calculate particle trajectories in the tube. Electric field lines were found by calculating the field at a point and then stepping in this direction,

$$\mathbf{X}_{i+1}=\mathbf{X}_i+(\mathbf{E}/|\mathbf{E}|)\Delta x. \quad (4)$$

The equipotentials are mapped out by stepping perpendicularly to the field direction. Figure 3 shows the result of such a calculation for the case of a 6 cm diam tube having the boundary potential of Fig. 2. The method used to calculate trajectories is as follows. The relativistic equations of motion of an electron in an electrostatic field are[3]

$$\partial\mathbf{u}/\partial s=\mathbf{F}\cdot\mathbf{u}, \quad (5)$$

where

$$F=\begin{bmatrix} 0 & 0 & 0 & iE_x \\ 0 & 0 & 0 & iE_y \\ 0 & 0 & 0 & iE_z \\ -iE_x & -iE_y & -iE_z & 0 \end{bmatrix}. \quad (6)$$

The trajectories were calculated using the following iterations

$$\mathbf{X}_{i+1}=\mathbf{X}_i+\mathbf{U}_i\Delta s+\tfrac{1}{2}(\partial\mathbf{u}_i/\partial s)(\Delta s)^2, \quad (7)$$

$$\mathbf{U}_{i+1}=\mathbf{U}_i+(\partial\mathbf{u}_i/\partial s)\Delta s. \quad (8)$$

The computer program selects an initial step size and tests it by checking for reproducibility with two smaller steps. The step size is adjusted on the basis of this check.[4] This process is continued until the trajectory reaches the end of the tube or the wall, and the resulting trajectory plotted automatically on a Stromberg Carlson 4020. The plots obtained helped us to establish that the trajectories calculated were physically reasonable ones and were also used to guide the selection of parameter values for farther calculations.

3. RANDOM TRAJECTORY CALCULATIONS

The mechanisms that are probably primarily responsible for the ionization loading effect are an electron-photon-electron multiplication process and/or an electron-secondary-electron multiplication process.[5] In order to study this effect a uniform distribution of starting electrons having

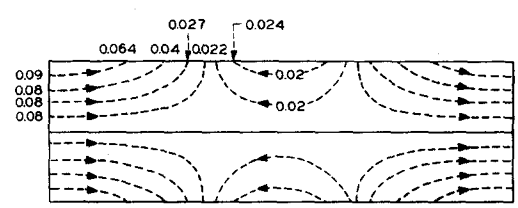

FIG. 3. Electric field in trapping tube, this shows a map of the electric field in the trapping tube discussed in Fig. 2. The numbers refer to the electric field strength at various points and are given in megavolts per centimeter. The region shown covers one period of 20 cm and the tube diameter is 6 cm.

[2] P. H. Rose, A. Galejs, and L. Peck, Nucl. Instr. Methods **31**, 262 (1964).

[3] L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1951), p. 57, Eq. (3–52).

[4] The method described here is known as Euler's method for the solution of differential equations. It is slower running than predictor corrector methods such as Milne's method or the Runge-Kutta method. The chief virtue of this method is that it calculated the $i+1$st term directly from the ith term whereas the predictor-corrector methods require the ith, $i-1$th, and $i-2$th term to get the $i+1$th term. This simplicity aids in understanding trajectories of particles starting from rest.

[5] C. M. Turner Bull. Am. Phys. Soc. **1**, 134 (1956). However, this point is controversial. See R. G. Herb, "Van de Graaff Generators" in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1959), Vol. XLIV.

random initial velocities and direction was chosen. We specifically assume that the electrons originate at the accelerator tube wall. The initial kinetic energy was taken to be some random value between zero and a maximum which was selected to be sometimes 0.5 or 10 keV. This distribution of velocities with an upper cutoff at 0.5 keV is typically for secondary electrons, while the 10 keV value is an attempt to include the effect of the strong electric fields which accelerate the secondary electrons before they come into our region of calculation. The trajectories found were not strongly affected by the choice of maximum energy.

The electron trajectories were followed until they hit the wall where we assume they produce four secondary electrons. This secondary emission coefficient of four is possibly the largest value to be reasonably expected in an accelerating tube. We did not calculate explicitly the effect on the avalanche propagation caused by electron-photon-electron multiplication. It seems reasonable however that conditions which reduce electron-secondary-electron avalanches also suppress electron-photon-electron avalanches.

The random secondary trajectories were calculated and distributions of impacts with the wall were found up through four multlplications. Results were obtained for tubes of 20 and 6 cm i.d. with trapping periods of 7, 20, and 60 cm and with the maximum field on the boundary being in each case 100 keV/cm, and the average field being 32 keV/cm. This yields an acceleration tube capable of 15 MeV in 5 m which seems to be a reasonable extrapolation of existing accelerator systems. Calculations were done for this tube when it had a uniform gradient, and as a final check some particle trajectories down the tube were plotted for the case that there was zero fringing field at the tube entrance.

4. RESULTS

On the basis of these calculations it is possible to gain a qualitative picture of how the electron multiplication proceeds in an accelerator tube having a uniform gradient. The distribution of path lengths in the uniform tube has an exponential behavior and can thus be characterized by

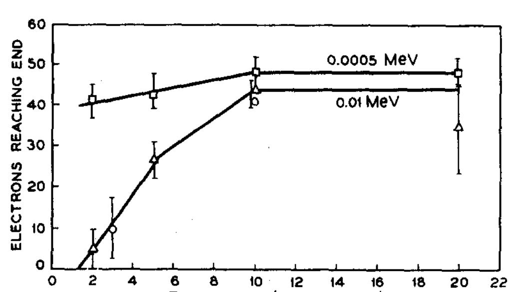

FIG. 4. Out of 48 random initial electrons this graph shows the number which were able to travel 500 cm and thus escape from the tube without multiplying.

having a mean free path. In the case of a 5 cm diam tube this mean free path is approximately 80 cm. On the basis of this we can imagine an electron being emitted and traveling down the tube 80 cm and picking up 2 MeV of energy, then colliding with the wall and the resulting bremsstrahlung photons releasing other electrons from nearby electrodes. The multiplication factor for an individual collision is an increasing function of the mean free path, because the number and energy of photons produced increases when the electron energy increases. The multiplication of an individual collision can be reduced by using electrodes of small aperture because these result in a shorter mean free path. However, the electrons are free to multiply many times more than in the case of a larger aperture before the avalanche reaches the end of the tube. This may partially explain the difficulty in attaining the high gradients of small accelerating tubes in their larger counterparts. The ultimate limitation on the size of the aperture in this direction is also set by optical considerations and pumping requirements.

Figure 4 shows the dependence on initial energy and tube radius for electrons to travel to the end of a 500 cm uniform gradient tube. Forty-eight random electrons were released in tubes of various radii and the number which travelled a full 500 cm tabulated. The results are strongly dependent upon whether the maximum for random initial kinetic energies was selected to be 0.5 or 10 keV. However, regardless of this, it is seen in the larger diameter tubes that most of the electrons travel to the end without multiplying. Unfortunately, it seems that tubes designed in this way do not result in higher attainable energies because the penetrating x rays produced at the end of the tube destroy insulators and cause excessive ionization loading of the accelerator system.[6]

It is interesting to note that the success of the inclined field tube is probably just due to the effect of reducing the mean free path by using periodic nonaxial electric fields. Other techniques used such as various magnetic field[7] arrangements also are useful to the extent that they can reduce the mean free path. In the case of the trapping tube as described here a greater confidence in performance seems justified. Electrons are still free to multiply for a short time, but the disturbance is unable to spread a large distance down the tube, as electrons are rapidly captured in trapping regions and neither they nor their secondaries can escape. Since trapping regions cover approximately half the tube, the probability for an electron to be free after n multiplications is about $(\frac{1}{2})^n$. It is seen (Fig. 2) that it is energetically impossible for an electron to escape from a trapping region unless its energy exceeds 150 keV. Thus the electron avalanche is confined to a region within a few

[6] C. M. Turner (private communication).
[7] J. L. McKibben and K. Boyer, Phys. Rev. 82, A315, (1951).

minor period lengths from the starting point and quickly dies out due to the absorption of energy and particles by the wall.

5. COMPARISON OF ALTERNATING GRADIENT TUBE TO UNIFORM ACCELERATING TUBE

Figure 5 shows the start of an electron avalanche in a conventional uniform gradient tube. Initially 50 random electrons[8] are released from a starting distribution distributed over a 20 cm period. The accelerating force is to the right. The distribution of impacts with the wall is shown in the box labeled "first pass." Each of these electrons then release four secondaries whose impact distribution is shown as the "second pass." It is seen that after only two multiplications, 141 out of 200 have already traveled more than 2 m, beyond the region of calculation. The buildup of a large number of secondaries having large energies and the simultaneous release of x rays proceeds rapidly.

The contrast to this situation in the case of the alternating gradient tube (Fig. 6) is quite dramatic. The calculations were carried through three multiplications, and over 1000 trajectories. Out of the original 50 uniformly distributed electrons only three manage to go a full 500 cm and the remaining ones produce 185 secondaries whose distribution of impacts is shown in the "second pass" of Fig. 6. These electrons have a strong tendency already to collect in trapping regions (see Fig. 2) at 0–10, 20–30, and 40–50 cm. By the third multiplication 500 out of a total of 733 electrons are trapped within 10 cm of the starting distribution, and all the others are trapped within a few trapping periods. No further gain in energy can be realized by the avalanche and its buildup is effectively halted.

The field configurations proposed here are specialized to suppress electrons originating at the accelerator tube walls. Nevertheless it does have an appreciable ability to trap electrons which start in the center of the tube unless the electrons are collimated along the axis. It would seem that the proposed trapping method should give electron suppression for all of the avalanche and loading mechanisms which have been proposed.[5] An additional advantage of the alternating gradient accelerating tube is that axial symmetry eliminates many of the problems of accelerating heavy ions which are common with inclined field tubes.[9]

We have found that the parameters of the tube, such as radius, electrode spacing, and trapping period length are not critical, and thus believe that an actual tube could be built and installed on existing Van de Graaff accelerators. It is hoped that the possibility of increased performance on existing accelerators proves to be sufficient incentive for the undertaking of the construction of such a tube.

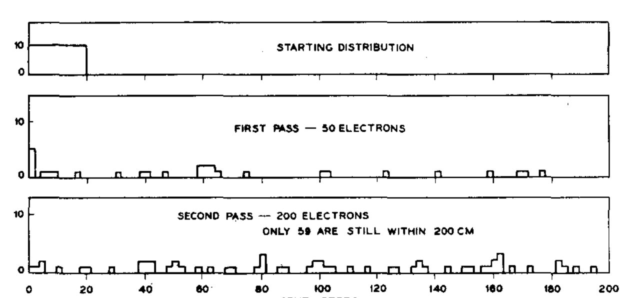

FIG. 5. Uniform tube distributions, this shows the distribution of electron impacts carried through two multiplications in a uniform gradient accelerating tube having an electrode aperture of 6 cm diameter.

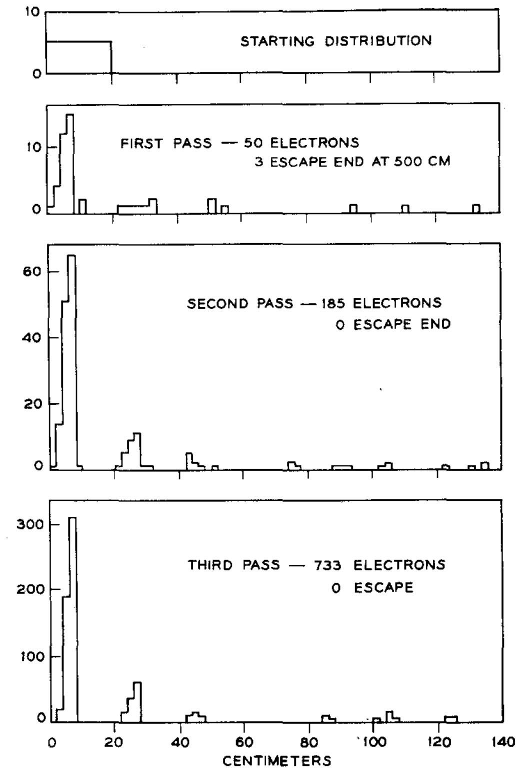

FIG. 6. Trapping tube distributions, this figure shows how an electron avalanche is confined in a trapping tube. The boundary potential and electric field of this tube are shown in Figs. 2 and 3. The diameter is 6 cm and the period length 20 cm.

ACKNOWLEDGMENTS

The authors are indebted to S. P. Morgan of Bell Telephone Laboratories for indicating to us the convenient solution of Laplace's equation in Eq. (2) and to C. M. Turner of Brookhaven National Laboratory for his comments on his experiences with accelerator tubes. The authors are also indebted to R. W. Hamming of Bell Telephone Laboratories for his remarks on our solution of the particle trajectories.

[8] The upper cutoff kinetic energy in Figs. 5 and 6 is 10 keV.

[9] A. J. Gale, Nucl. Instr. Methods 28, 10 (1964).