Additional evidence for the universality of the probability distribution of turbulent fluctuations and fluxes in the scrape-off layer region of fusion plasmas

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Plasma density fluctuations and electrostatic turbulent fluxes measured at the scrape-off layer of the Alcator C-Mod tokamak [B. LaBombard, R. L. Boivin, M. Greenwald, J. Hughes, B. Lipschultz, D. Mossessian, C. S. Pitcher, J. L. Terry, and S. J. Zweben, Phys. Plasmas 8, 2107 (2001)], the Wendelstein 7-Advanced Stellarator [H. Renner, E. Anabitarte, E. Ascasibar *et al.*, Plasma Phys. Controlled Fusion 31, 1579 (1989)], and the TJ-II stellarator [C. Alejaldre, J. Alonso, J. Botija *et al.*, Fusion Technol. 17, 131 (1990)] are shown to obey a non-Gaussian but apparently universal (i.e., not dependent on device and discharge parameters) probability density distribution (pdf). The fact that a specific shape acts as an attractor for the pdf seems to suggest that emergent behavior and self-regulation are relevant concepts for these fluctuations. This shape is closely similar to the so-called Bramwell, Holdsworth, and Pinton distribution, which does not have any free parameters. © 2005 American Institute of Physics. [DOI: 10.1063/1.1884615]

I. INTRODUCTION

In recent years, the basic picture of the physics of plasma transport in the scrape-off layer (SOL) of tokamaks and stellarators has been clarified substantially. Currently, cross-field transport is known to be dominated by large radial transport events (the so-called blobs, avaloids, or intermittent plasma objects ¹⁻⁹) that have been observed by beam emission spectroscopy ⁶ or gas puff imaging techniques, ¹⁰ among others. Also, important progress has been made in the modeling of these structures. ¹¹⁻¹⁴

However, the dynamics governing the radial transport of these blobs in the SOL is still not fully understood. Fluctuation measurements by Langmuir probes have provided abundant evidence to support the idea that cross-field transport in magnetically confined plasmas is a rather complex phenomenon. For instance, long-range temporal correlations have been detected in tokamaks and stellarators in the form of power-law regions in fluctuation power spectra and large values of the Hurst exponent (i.e, H > 0.5). Furthermore, it

has been reported that density fluctuations are distributed according to non-Gaussian probability density functions (pdfs), ^{18–21} and that the associated turbulent fluxes seem to be approximately self-similar (i.e., invariant under rescaling) over a range of scales, larger than the characteristic turbulent scales. ²²

Interestingly, both self-similarity and long-term correlations are characteristic properties of critical phenomena. Thus one is led to ask whether SOL transport is indeed universal. Universality, a key concept of the theory of critical phenomena, implies that the large-scale behavior of the system of interest is insensitive to the details of the microscopic physics that govern the dynamics at the smallest scales. As a consequence, systems within the same universality class may be described by a common model that appropriately accounts for their large scale properties, in spite of having different microscopic characteristics. In this sense, determining the universality (or lack of universality) of cross-field transport in the SOL of tokamaks and stellarators becomes an interesting (and important) problem.

Some reports do indeed support such universality at the plasma edge. In Ref. 17, the values of the Hurst exponents for density fluctuations in several tokamaks and stellarators are found to be very similar over comparable ranges of temporal scales. In Ref. 16, similar conclusions are drawn from fluctuation power spectra. More recently, a different kind of test was carried out by comparing SOL density fluctuation pdfs from the PISCES linear device, the Alcator C-Mod tokamak, the Mega-Ampere Spherical Tokamak, and the reversed field pinch RFX. Similar observations were reported in a simple magnetized torus. Apparently, all these pdfs look very similar and exhibit a clearly asymmetric, non-Gaussian shape.

In this paper, we study the statistics of density fluctuations in a set of C-Mod (Ref. 29) discharges, in which not only the plasma line density but also the location of the probes relative to the last closed surface (LCS) are varied in a systematic way. These results are then compared to density fluctuation measurements in the SOL of two stellarators (the Wendelstein 7-Advanced Stellarator³⁰ (W7-AS) and the TJ-II heliac³¹).

Next, we investigate, for the same discharges, if similar universality features are present in the instantaneous electrostatic turbulent fluxes. Interestingly, the main conclusion we draw is that the SOL fluxes seem to share many of the universal characteristics of density fluctuations (in a subtle way to be explained later), which seems to be consistent with the standard picture in which the SOL cross-field transport is dominated by the propagation of density blobs. Finally, all these results will be discussed in light of some recent ideas, brought forward in the framework of complex systems such as fluids and magnetized materials, ^{32,33} and we will speculate about the possible origin of the particular shape of the pdf that acts as an attractor of the dynamics.

The paper is structured as follows. In Sec. II, the experimental observations are reported separately for each device. In Sec. III, the results are discussed in light of the mentioned ideas. Finally, in Sec. IV, some conclusions are drawn.

II. EXPERIMENTAL RESULTS

The experimental data analyzed in this paper were obtained by means of Langmuir probes. These probes provide ion saturation current fluctuation measurements at one or two positions and floating potential (ϕ) fluctuations at two nearby points. In the following, we will assume that the fluctuations of the ion saturation current in the SOL are proportional to fluctuations of the density (\tilde{n}) . The associated turbulent flux is computed using $\Gamma = \tilde{n}(\phi_2 - \phi_1)/2rB\delta$, 35 under the assumption that the temperature and sheath potential drop are identical at both floating potential probe tips, which holds in the case of structures larger than the separation between the probe tips. 6 We have only used data corresponding to a steady state situation, in order to guarantee meaningful statistics. In addition, we have checked that the data were not contaminated by modes, waves, or any other regular oscillations, which might modify the pdf.

We now introduce some notation. The pdf according to which some fluctuating quantity of interest *A* is distributed is

represented by $p_A^{(l)}(x)$, where we formally represent the experimental discharge conditions by a label l (for instance, l might be chosen to be a vector that contains the value of the line integrated density, the distance to the LCS, the configuration type—e.g., tokamak or stellarator—etc.). To test these pdfs for universal properties, we first subtract the mean $\bar{x}_A^{(l)}$ and divide by the variance $\sigma_A^{(l)}$ (thus removing any dependence on physical dimensions) using the rescaling prescription:

$$F_A(y) \equiv \sigma_A^{(l)} p_A^{(l)} ([x - \bar{x}_A^{(l)}] / \sigma_A^{(l)}). \tag{1}$$

Universality in the shape of the probability distribution function will be claimed whenever the same function $F_A(y)$ is obtained for any value of l. In what follows, the normalized pdfs of data will be plotted with the correspondingly normalized Bramwell, Holdsworth, and Pinton (BHP) pdf. The reason for including this analytical form will be clarified in the discussion of the results below.

In general, the turbulent flux carries information of at least two processes with disparate scales: fast and small turbulent fluctuations and slow and large transport events. ²² In what follows, it will be of interest to separate these. To do so, the data are smoothed using an m-points rectangular smoothing window in order to remove the high-frequency part, with the value of m chosen on a case by case basis, using the criterion described elsewhere. ^{22,36}

A. C-mod tokamak

The fluctuation measurements at the C-Mod tokamak were performed in ohmically heated deuterium discharges with a diverted, lower single-null magnetic equilibrium. The data correspond to identical discharges (plasma current, I_p = 0.8 MA; toroidal magnetic field B_T =5.3 T) in which the line-averaged electron density varies in the range 0.6 $\times 10^{20} m^{-3} < \bar{n}_e < 1.2 \times 10^{20} m^{-3}$. In addition, the distance of the Langmuir probe to the LCS is varied between 4 and 8 mm. (In fact, fluctuation data from locations up to 1 mm from the LCS were also available. But due to the proximity to the edge velocity shear layer and contamination with a slow 60 Hz oscillation associated with the power source, application of the present analysis technique is not warranted.) Further details about the experimental arrangement can be found in Ref. 37.

In Fig. 1, the measured density fluctuation pdfs are shown, after being rescaled in the way previously described, for increasing line integrated densities. Since we focus on the higher-order moments (which manifest themselves through the tails of the distribution), the pdfs are plotted here and in the following on a log-linear scale to stress the shape of the tail. It is apparent that all curves seem to collapse onto some preferred canonical shape, characterized by a rapid decay for negative values of the fluctuation amplitude and a rather slow exponential decay for positive values, whose interpretation we purposely delay until the following section. The canonical form is clearly non-Gaussian, consistent with what was previously reported in, e.g., Refs. 8 and 18–21.

These somewhat loose statements about the pdfs can be quantified by computing the third and fourth moments of the

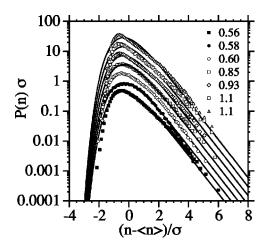


FIG. 1. Rescaled density fluctuation pdfs measured at the SOL of C-Mod for increasing line integrated densities; except for the curve at \bar{n}_e =0.56, all curves are displaced vertically by factors that are a multiple of 2. The continuous lines are the corresponding BHP curves.

distributions (the skewness S and kurtosis K, respectively)—bearing in mind that the first and second moments are always 0 and 1, by definition. Typically, S and K have a radial dependence; results here refer to the far SOL. The experimental results are shown in Fig. 6 below. Mean values obtained are $S \approx 1.2 \pm 0.3$ and $K \approx 5.1 \pm 1.5$, which lie in between the values expected for a Gaussian (S = 0, K = 3) and an exponential distribution (S = 2, K = 9), and clearly distinct from either. The values for S are consistent with values obtained in other devices in the far scrape-off layer. 4,7,20,38 S and K have only a weak dependence on \bar{n}_e , if any.

We turn now to the turbulent flux inferred from the fluctuation measurements. In Fig. 2, the rescaled flux pdfs obtained for increasing densities are shown after smoothing the data with a sliding 120-point window. Again, all pdfs seem to tend to a canonical shape, although deviations from the canonical shape seem to become more pronounced at higher density. The corresponding kurtosis and skewness are shown

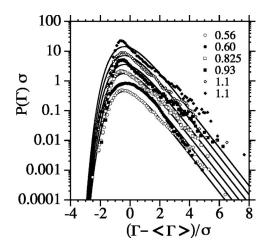


FIG. 2. Rescaled turbulent flux pdfs for the C-Mod data for increasing line integrated densitites after smoothing with a 120-point sliding squared window; except for the curve at \bar{n}_e =0.56, all curves are displaced vertically by factors that are a multiple of 2. The continuous lines are the corresponding BHP curves.

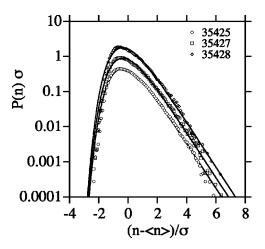


FIG. 3. Rescaled density fluctuation pdfs measured in the far SOL of W7-AS; except for the curve for shot 35 425, all curves are displaced vertically by factors that are a multiple of 2. The continuous lines are the corresponding BHP curves.

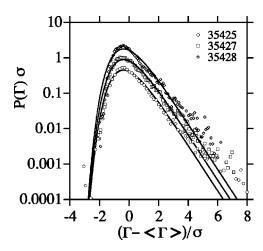


FIG. 4. Rescaled flux fluctuation pdfs measured in the far SOL of W7-AS (after smoothing with a 300-point sliding squared window); except for the curve for shot 35 425, all curves are displaced vertically by factors that are a multiple of 2. The continuous lines are the corresponding BHP curves.

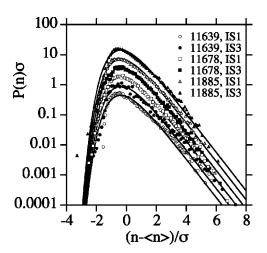


FIG. 5. Rescaled density fluctuation pdfs measured in the far SOL of TJ-II.

in Fig. 7 below. The values of these moments are clearly non-Gaussian and quite similar to the values obtained for the density fluctuations. Note that in this case the dependence of the moments on the density is much clearer (although the error bars are also larger).

B. W7-AS stellarator

The W7-AS data analyzed here belong to the series of discharges previously studied in Ref. 22. Details about the experimental conditions and probe geometry can be found in the cited reference. We will only remark here that the Langmuir probe was of the reciprocating type and moved slowly in the radial direction. For this reason, and although the motion is barely perceptible over 50 ms, we have introduced a double trend correction in each analysis time interval in order to be able to analyze a longer time interval and thus improve statistics. We checked that the statistical characteristics of the data, as expressed by the higher moments (≥2) of the pdf, did not change during the extended time interval. First, the linear trend was determined by fitting a straight line to the data in the analysis time interval, which was then subtracted from the data. Second, the linear trend of the root-mean-square (rms) data was determined by fitting the straight line to the array of absolute data values, after which the data were divided by this fit. The latter eliminates any slow linear variation in the rms fluctuations of the data.

We analyzed data from discharges 35 425, 35 427, and 35 428. In the SOL region $[r-r_s>0$, where r_s is the position of the velocity shear layer, which is closely related to the position of the LCS (Ref. 39)], the fluctuations of the ion saturation current I_{sat} varied from Gaussian near the shear layer, to non-Gaussian for large positive values of $r-r_s$. This fact is reflected in the radial evolution of the skewness S and kurtosis K of I_{sat} shown in Fig. 8 below. The values of S and K reach an almost stationary value for $r-r_s>2$ cm (cf. also Refs. 6 and 20; in the latter reference, the value of the skewness was related to the dynamics of intermittent plasma objects). For the data interval corresponding to $2 \le r - r_s$ \leq 5 cm (a different time window in each shot), we have computed the pdf of I_{sat} . Figure 3 shows these three pdfs, together with the BHP pdf to facilitate the comparison. Note that again the same canonical shape emerges, and that deviations from the canonical shape are exceedingly small. The moments for these three cases are $S \approx 0.98 \pm 0.08$ and K \approx 4.2±0.3, close to the values obtained in the C-Mod case and distinctly different from the Gaussian values.

Next, we analyze also the turbulent flux time traces in the far SOL region $(2 \le r - r_s \le 5 \text{ cm})$. As explained above, it is necessary to smooth the flux signal to remove the small-scale fluctuations and enter into the self-similar scaling region of the pdf. Following the technique described in Ref. 22, we find $m \approx 300$ —one order of magnitude larger than the value needed to enter into the self-similar scaling region for data taken inside the plasma in the same shots, 22 indicating that the self-similar temporal scales are much longer in the SOL than inside the plasma. This could be a consequence of the fact that there is no small-scale turbulent drive in the SOL that maintains the self-similarity at the small-scale

range of the spectrum, or of the fact that the transport is fundamentally different in the confinement zone (closed flux surfaces) and the SOL (open field lines). The rescaled flux pdfs for m=300 are shown in Fig. 4 for the three W7-AS discharges, together with the reference BHP curve. Again, the canonical pdf shape emerges, albeit with slightly larger deviations as compared to the density.

C. TJ-II heliac

The same analysis was carried out on data from fixed Langmuir probes measuring in the SOL of electron cyclotron resonance heated plasmas (P_{ECRH} =300 kW) in the TJ-II heliac (R=1.5 m, $\langle a \rangle$ =0.22, $B_0 <$ 1.2 T).

Figure 5 shows the rescaled density fluctuation pdfs in the SOL for some shots (shot 11 639, $r-r_s$ =8.8 cm; shot 11 678, $r-r_s$ =7.9 cm; shot 11 885, $r-r_s$ =-1.7 cm), showing that in this scan, the results do not appear to depend strongly on the position in the SOL (and edge). On average these pdfs have $S \approx 1.1 \pm 0.2$ and $K \approx 5.1 \pm 0.6$.

A more detailed shot-by-shot radial scan was performed in the TJ-II SOL (configuration: 100_44_64). The results are summarized in Figs. 9 and 10 (compare to Figures 6–8 described above). The fluctuations of the density (Fig. 9) show clear non-Gaussian behavior, while no clear radial trend can be discerned. On average, these pdfs have: $S \approx 1.2 \pm 0.4$ and $K \approx 4.9 \pm 1.3$. The fluctuations of the flux show a relatively clear trend from nearly Gaussian at the LCS $(r-r_s=0 \text{ cm})$ to non-Gaussian at remote positions $(r-r_s=8 \text{ cm})$. At the latter position, the canonical values of $S \approx 1$ and $K \approx 5$ are again obtained.

III. DISCUSSION

The analyses presented above suggest that the fluctuations of the density and, to a lesser extent, the flux, can be described by a probability distribution that tends to a universal shape, reinforcing and extending previous evidence reported in Refs. 8 and 18-21. It should be noted that similar pdf shapes have also been reported in numerical simulations of SOL turbulence. 40 We have found these canonical shapes in a tokamak and two stellarators, for different densities and distances to the SOL. The fact that the density and flux pdf have a similar universal shape could be consistent with the widely accepted idea that transport in the SOL region is dominated by density blobs. 6,9 The canonical shape of the flux pdf is only obtained for large temporal scales (m large), suggesting that these blobs have a relatively large spatial extension, in accordance with these ideas. The kurtosis and skewness of the canonical flux pdf seem to exhibit some dependence on the distance from the LCS, which might be explained using the ideas expressed in Ref. 6.

Having established the probable existence of a canonical pdf shape that acts as a kind of "attractor" for experimental pdfs, it is of interest to ask whether such a shape tells us anything significant (as Gaussian pdfs do, which are associated with the limit distribution of a sum of random and independent processes. ⁴¹) This question is of particular interest in the light of the recent results reported by BHP that suggest

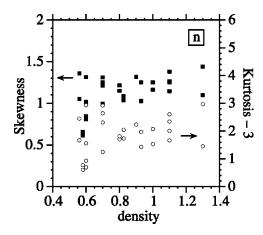


FIG. 6. Skewness S and Kurtosis K of C-Mod rescaled density pdfs as a function of line integrated density.

that the rescaled pdf of fluctuations of any global measure of activity of a critical system should be distributed according to the so-called BHP pdf, given by 32,33

$$P(y) = A(\exp[x - e^x])^a, \tag{2}$$

where x=b(y-s); $A=K/\sigma$; K=2.16, a=1.58, b=0.934, s=0.373, σ being the standard deviation. The values of b and s are obtained by requiring that P(y) have zero mean and unit variance. The skewness and kurtosis of this distribution are S=0.8907 and K=4.415 (the sign of S being irrelevant since it depends on which side of the S axis is chosen as positive). The cited authors argue that the BHP pdf may describe the global energy fluctuations of a confined turbulent flow as well as the pdf corresponding to the fluctuations in the magnetic ordering in a ferromagnet at the critical point, S=0.89070 or of a system exhibiting self-organized criticality S=0.99071 such as a numerical sandpile, or of water level fluctuations in a river. In this sense, it has been suggested that the BHP pdf is a hallmark of non-Gaussian statistics and criticality.

While the relevance of these arguments is still a matter of active discussions, ^{45–47} we think it is interesting to try to ascertain to what extent the BHP pdf can be identified with the canonical shapes we have found in the experimental data. Clearly, since it is rescaled pdfs that must be compared, this

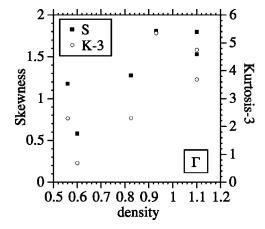


FIG. 7. Skewness and Kurtosis of C-Mod rescaled flux pdfs as a function of line integrated density.

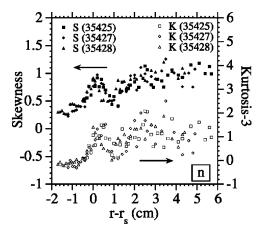


FIG. 8. Skewness and Kurtosis of density fluctuations measured in W7-AS, vs radius (r_s , being the approximate position of the LCS).

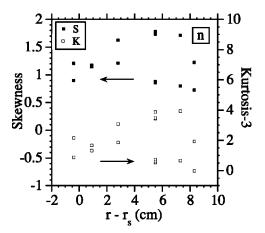


FIG. 9. Skewness and kurtosis of the density fluctuation pdfs, vs position, in the SOL of TJ-II.

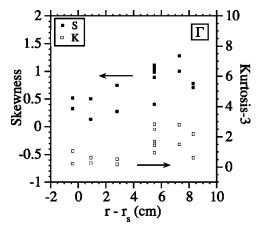


FIG. 10. Skewness and kurtosis of the flux fluctuation pdfs, vs position, in the SOL of TJ-II.

comparison must be carried out at the level of pdf moments of higher order than the variance, i.e., skewness, kurtosis, and higher. The experimental results (in terms of *S* and *K*) presented in this paper are (within error bars) more compatible with the BHP pdf than a Gaussian or exponential pdf.

We note, however, that the presented experimental data alone do not permit stating categorically that the system belongs to this class of (critical, externally driven) systems, since other pdf shapes, lacking the critical connotations of the BHP pdf, are equally close to the experimental results. One example is the extremal *r*-order Gumbel pdf, ⁴⁸ given by

$$P_r(y) = A_r(\exp[x - e^x])^r, \tag{3}$$

where $x=(y-\mu_r)/\sigma_r$, while A_r,μ_r , and σ_r are determined by proper normalization and the requirements of zero mean and unit variance for any integer order r>0. The BHP pdf is formally equal to the Gumbel pdf but differs in that $r=a=\pi/2$ is not integer. ^{33,49} Their interpretation is however completely different, as Gumbel pdfs are associated with the statistics of the maxima of a random variable, which apparently bears no relation to criticality at all. ⁴⁸ The values of the skewness and kurtosis of the first-order Gumbel pdf are S=1.298 and K=5.4, consistent with the experimental findings.

IV. CONCLUSIONS

In this paper, we have reported experimental evidence showing that fluctuations of the density and the flux in the SOL of tokamaks and stellarators tend to adopt a canonical shape, which acts as an attractor for the pdf. The fact that the pdf tends to a certain nontrivial universal shape seems to suggest that emergent behavior and self-regulation are relevant concepts for these fluctuations. The results of the presented analysis are consistent with the idea that radial transport is dominated by large-scale density structures (blobs).

The canonical pdf shape is asymmetric and non-Gaussian, and it bears a striking similarity to the BHP pdf, a fixed shape without any free fit parameters, to which some significance has been attributed as a hallmark of criticality and non-Gaussian dynamics in externally driven systems. However, other pdfs lacking such meaning, such as the extremal Gumbel pdf, are also compatible with the observed canonical shape within the experimental uncertainties. Due to experimental limitations, any preference for the BHP shape should therefore probably be motivated by theoretical considerations, which are outside the scope of this paper.

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