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Application of a vector Preisach model in a magnetic circuit

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Magnetic hysteresis effects have been included in a finite-element description of a magnetic circuit by using the classical vector Preisach model for the constitutive relation between **H** and **B**. The influence of an external electric circuit is taken into account by adding equations derived from Faraday's law. Computational results are presented for a magnetic circuit used in a magnetostrictive device.

I. INTRODUCTION

The implementation of hysteresis models in finite-element (FE) methods for design of electromagnetic devices is currently a subject drawing interest.[1,2] In this article a hysteresis model is used in the description of a magnetic circuit similar to one previously used in a magnetostrictive device.[3] The model involves a magnetic circuit comprising a coil for dynamic magnetization and permanent magnets for magnetic biasing. By using Faraday's law we can relate the performance of the electric circuit to magnetic-field quantities.

II. PROBLEM FORMULATION

The magnetic circuit is shown in Fig. 1 and consists of a cylindrical rod of active magnetostrictive material (Terfenol-D), two permanent magnets, a coil driven by an external electric amplifier, and soft magnetic material to direct the flux into the active material. In the FE description we get a two-dimensional axisymmetric problem and furthermore we have a symmetry plane through the middle of the active rod, so it is sufficient to study one quadrant of the geometry as indicated in the figure. We then get Dirichlet boundaries, i.e., $\mathbf{A}=0$, at infinity and at the z axis and a horizontal Neumann boundary, i.e., $\partial\mathbf{A}/\partial n=0$ at the r axis. For the electric circuit we get the equation

$$u(t)=Ri(t)+\frac{d\Psi}{dt}, \tag{1}$$

where i is the current, u is the feeding voltage of the device, R is the resistance in the coil, and Ψ is the sum of the flux through each turn of the coil. Ψ can be calculated from the FE solution according to the formula

$$\Psi=\sum_{i,j}\int_0^{r_j} B_z(r,z_j)2\pi r\,dr. \tag{2}$$

In this article no consideration is made of the mechanical forces involved. In the general case FE calculations should be carried out simultaneously with respect to both the magnetic and mechanical aspects.

III. FINITE-ELEMENT TREATMENT

The behavior of the magnetic circuit is, in addition to the boundary conditions, governed by the differential equations

$$\nabla\times\mathbf{H}=\mathbf{J}, \tag{3}$$

$$\nabla\times\mathbf{A}=\mathbf{B}, \tag{4}$$

along with a constitutive relation between **H** and **B**. In materials exhibiting hysteresis there is in general no value ν such that the usual relation $\mathbf{H}=\nu\mathbf{B}$ holds even if ν is allowed to be a function of **B**. We follow the approach of Henrotte *et al.*[1] and express the relation between **H** and **B** as

$$\mathbf{H}=\nu\mathbf{B}+\mathbf{H}_c. \tag{5}$$

Combining Eqs. (3), (4), and (5) we obtain

$$\nabla\times(\nu\nabla\times\mathbf{A})=\mathbf{J}-\nabla\times\mathbf{H}_c. \tag{6}$$

The solution to Eq. (6) minimizes the functional

$$L(\mathbf{A})=\int_R\int\left(-\frac{\nu}{2}(\nabla\times\mathbf{A})^2-\mathbf{A}\cdot(\mathbf{J}-\nabla\times\mathbf{H}_c)\right)r\,dr\,dz. \tag{7}$$

Using bilinear trial functions in triangular elements, the field quantities **H**, **B**, and thus also $\mathbf{H}_c$ are piecewise constant, which complicates the evaluation of $\nabla\times\mathbf{H}_c$ although Henrotte *et al.* described a method. Another way of dealing with this term is the following.

Using the divergence theorem and the vector identity

$$\mathbf{A}\cdot(\nabla\times\mathbf{H}_c)=\mathbf{H}_c\cdot(\nabla\times\mathbf{A})=\nabla\cdot(\mathbf{H}_c\times\mathbf{A}), \tag{8}$$

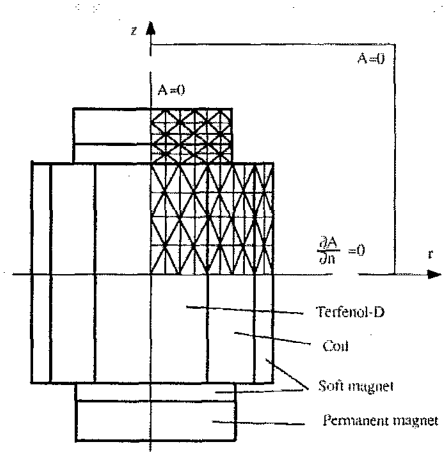

FIG. 1. Magnetic circuit configuration.

we obtain

$$\int_R \int \mathbf{A}\cdot(\nabla\times\mathbf{H}_c) r\,dr\,dz$$
$$=\int_R \int \mathbf{H}_c\cdot(\nabla\times\mathbf{A}) r\,dr\,dz+\int_{\partial R}(\mathbf{H}_c\times\mathbf{A})\cdot d\mathbf{S}. \quad (9)$$

Evaluating the first term on the right-hand side is no problem since $(\nabla\times\mathbf{A})$ and $\mathbf{H}_c$ are both constant in a given element. The second term vanishes as can be shown in the following way.

On the Dirichlet boundaries the integrand is trivially zero. On the Neumann boundary, for symmetry reasons, **B** is always normal to the boundary and, therefore, so are also **H** and $\mathbf{H}_c$. This means the vector product $(\mathbf{A}\times\mathbf{H}_c)$ is directed along the tangent and is thereby in turn perpendicular to $d\mathbf{S}$. Thus the integrand is zero on Neumann boundaries as well and we can rewrite Eq. (7) as

$$L(\mathbf{A})=\int_R \int \left(-\frac{\nu}{2}(\nabla\times\mathbf{A})^2-\mathbf{A}\cdot\mathbf{J}+\mathbf{H}_c\cdot(\nabla\times\mathbf{A})\right)\times r\,dr\,dz. \quad (10)$$

For a description of the evaluation of the energy functional with axisymmetric geometry and bilinear trial functions see, for instance, Hoole.[4] The resulting equation system has been solved using the simple chord method with retardation. Since we can, at a given time step, get a good initial solution from solutions from earlier time steps, convergence is quite fast and stable.

IV. HYSTERESIS FORMULATION

In the ferromagnetic materials in the circuit there is magnetic hysteresis. In the permanent magnets we can, unless the fields are very high, approximate ν and $\mathbf{H}_c$ with constants. In the Terfenol-D we use a nonlinear, nonhysteretic relation so that $\mathbf{H}_c=0$. In the soft magnetic materials, however, a hysteresis model is employed. Since the flux density in a given point varies in direction as well as magnitude a two-dimensional vector hysteresis model is required. Vector hysteresis is considerably more complicated than scalar hysteresis and there are comparatively fewer vector hysteresis models to choose between. The model must be cabable of handling any kind of variation of

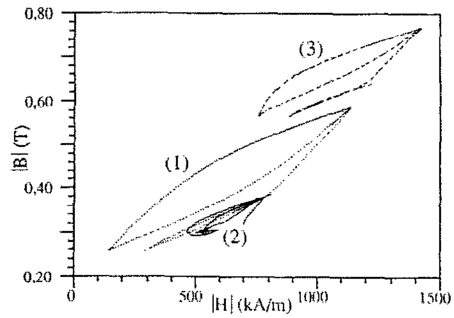

FIG. 3. |**B**| vs |**H**| in four select points.

$\mathbf{B}(t)$ in the plane, and since it is applied to every single element in the FE configuration it should be computationally fast. Due to these considerations we have chosen the classical vector Preisach model of Mayergoyz and Friedman[5] which consists of a superposition of an infinite set of scalar Preisach models distributed over angular directions in the plane. For isotropic materials, **H** can, according to this model, be calculated for a given variation of **B** in time from the formula

$$\mathbf{H}(t)=\int_{-\pi/2}^{\pi/2}\left(2\sum_{k=1}^{N_\varphi}[P(B^+_{\varphi,k},B^-_{\varphi,k-1})-P(B^+_{\varphi,k},B^-_{\varphi,k})]-P(B^+_{\varphi,0},B^-_{\varphi,0})\right)\mathbf{e}_\varphi\,\varphi, \quad (11)$$

where $\mathbf{e}_\varphi$ is the unity vector forming the angle φ with the x axis, $B^+_{\varphi,k}$, $B^-_{\varphi,k}$ are local maximums and minimums of $\mathbf{B}(t)\cdot\mathbf{e}_\varphi$, and $P(B^+,B^-)$ is a material-dependent function which can be derived from experimentally measured first-order reversal curves. In a numerical implementation we must use a finite number of φ values and replace the integral with a sum. For a more detailed description of the model see Refs. 5 and 6.

Using bilinear trial functions in the FE model, **B** is constant in space within a given element and so we get one set of past extremum values for each element.

ν and $\mathbf{H}_c$ are not uniquely defined by **H** and **B** and it is not obvious what the best possible choice is. We have chosen to define ν such that $\nu\mathbf{B}$ is the average of the upper and

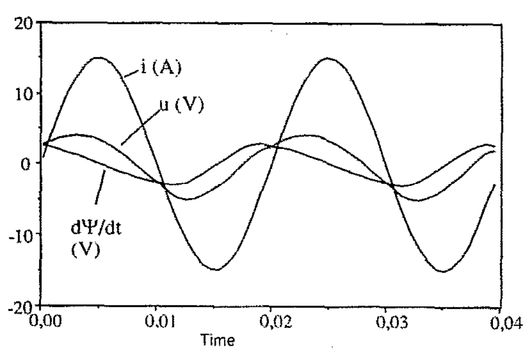

FIG. 2. i, u, and $d\Psi/dt$ as functions of time.

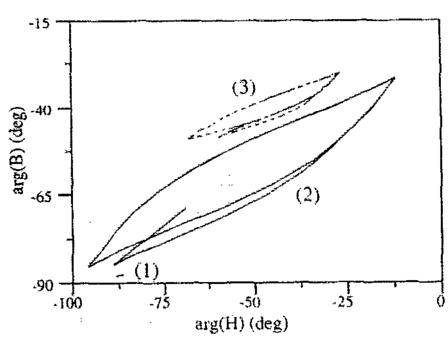

FIG. 4. arg(**B**) vs arg(**H**) in four select points.

lower branches of the major limiting hysteresis loop. $\mathbf{H}_c$ is then by definition the difference $\mathbf{H}$ and $(\nu\mathbf{B})$.

V. CIRCUIT CONSIDERATIONS

If the coil is driven by a current amplifier then the current and thus also the current density $\mathbf{J}$ in the magnetic circuit are always known. We can then use Eq. (1) explicitly to derive all electric quantities. If, on the other hand, we have a voltage amplifier then, because $\Psi(t)$ depends on $i(t)$, Eq. (1) is implicit and for each time step we must solve Eq. (1) with some numerical method for initial value problems. In the example that follows, $i(t)$ is used as the input.

VI. SOME NUMERICAL RESULTS

In Figs. 2–4 are shown computational results for two periods of a sinusoidal current with a coil resistance $R=0.22\ \Omega$. The permanent magnets have coercivities $\mathbf{H}_c=1100$ kA/m in the negative z direction. Figure 2 shows i, u, and $d\Psi/dt$ as functions of time. Figure 3 shows $|\mathbf{B}|$ vs $|\mathbf{H}|$ and Fig. 4 shows arg($\mathbf{B}$) vs arg($\mathbf{H}$) at three different points located in the geometry as indicated in Fig. 5. The influence of hysteresis is quite noticable as is the difference in direction between $\mathbf{B}$ and $\mathbf{H}$.

VII. CONCLUSIONS AND FURTHER DEVELOPMENT

A vector Preisach model of hysteresis has been implemented in a finite-element description of a magnetostrictive

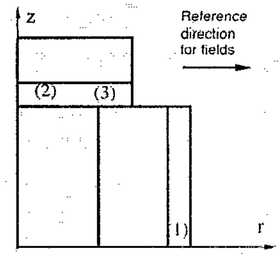

FIG. 5. Positions of points for displayed **B-H** curves.

drive element. This allows us to properly model the behavior of ferromagnetic materials and to calculate hysteresis losses.

Next, the model should be extended to include the mechanical properties of the device. To further increase the accuracy of the model eddy-current effects will eventually be implemented.

[1] F. Henrotte, A. Nicolet, F. Delincé, A. Genon, and W. Legros, IEEE Trans. Magn. **MAG-28**, 2611 (1992).
[2] F. Ossart and G. Meunier, J. Appl. Phys. **69**, 4835 (1991).
[3] H. Tiberg, A. Bergqvist, L. Kvarnsjö, and G. Engdahl, IEEE Trans. Magn. **MAG-28**, 2808 (1992).
[4] S. R. H. Hoole, *Computer-aided Analysis and Design of Electromagnetic Devices* (Elsevier, New York, 1989).
[5] I. D. Mayergoyz and G. Friedman, J. Appl. Phys. **61**, 4022 (1987).
[6] I. D. Mayergoyz, *Mathematical Models of Hysteresis* (Springer, New York, 1991).