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Fundamental relativistic solution for the rail gun

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A fully relativistic analysis is made of the dynamics of a rail gun based on three assumptions: (1) Ohm's law is valid in the rest frame of the plasma, (2) total electron momentum is transferred to the projectile, and (3) motion of the projectile is constrained to one direction. With these assumptions, a relativistic equation for the velocity of the projectile is obtained, whose solution monotonically increases to one of two values depending on field strengths. For $B > E$, the maximum velocity is cE/B, whereas for $E > B$ it is c, where c is the speed of light, and E and B are applied electric and magnetic fields, respectively (in cgs).

Attention has recently been given to dynamical properties of electromagnetic propulsion. In the rail-gun device,[1–5] a nonconducting projectile is propelled by a current-carrying plasma driven by the Lorentz force. Plasma dynamics is more difficult due to ablation of the projectile, and for the most part previous studies have attempted to incorporate this effect.

In the present study, we return to a more elementary configuration for the purpose of developing a fully relativistic study of this problem. Thus, for example, it is assumed that the rest mass of the projectile is constant, and that total electron momentum from the plasma is transferred to the projectile. Furthermore, it is assumed that the projectile is constrained to move in one direction. Our remaining assumption is that Ohm's law is valid in the rest frame of the projectile.[6]

With these assumptions at hand, a relativistic equation is constructed for the projectile velocity. The solution to this equation reveals two asymptotic velocities, which depend on initial field strengths. Thus, for example, for the case $E > B$, the velocity is c, the speed of light, whereas for $B > E$, the velocity is cE/B. It is further demonstrated that for initial velocities less than their respective asymptotic values, velocities monotonically approach their respective limiting values. For the case $B > E$, starting velocities greater than cE/B are found to decay to this asymptotic value.

Our starting equation is Ohm's law, which in the rest frame of the projectile (primed coordinates) is written

$$\mathbf{J}' = \sigma \mathbf{E}', \tag{1}$$

where σ is conductivity. Transforming back to the lab frame (see Fig. 1) we find

$$\begin{aligned} \gamma(J_x - c\beta\rho) &= \sigma E_x, \\ J_y &= \sigma\gamma(E_y - \beta B_z), \\ J_z &= \sigma\gamma(E_z - \beta B_y), \end{aligned} \tag{2}$$

where $\beta \equiv v/c$ and $\gamma = (1-\beta^2)^{-1/2}$. The charge density ρ, for a charge-neutral plasma, is equal to zero. In the lab frame we take

$$\mathbf{E} = E\hat{\mathbf{y}}, \quad \mathbf{B} = B\hat{\mathbf{z}}, \tag{3}$$

where caret variables denote unit vectors. Inserting these values into Eq. (2) gives

$$\mathbf{J} = \sigma\gamma(E - \beta B)\hat{\mathbf{y}}. \tag{4}$$

With $\boldsymbol{\xi}$ denoting the microscopic electron velocity we write

$$\mathbf{J} = qn\langle\boldsymbol{\xi}\rangle. \tag{5}$$

Electron charge and density are q and n, respectively. We further recall that the density transforms as

$$n = \gamma n'. \tag{6}$$

Combining the latter four equations gives

$$\langle\boldsymbol{\xi}\rangle = \mu(E - \beta B)\hat{\mathbf{y}}, \tag{7}$$

where

$$\mu \equiv \sigma/qn' \tag{8}$$

represents mobility in the rest frame.

Taking the average of the Lorentz force on electrons we obtain

$$\frac{d}{dt}\langle p_y\rangle = \frac{q}{c}\langle\xi_y\rangle B, \tag{9}$$

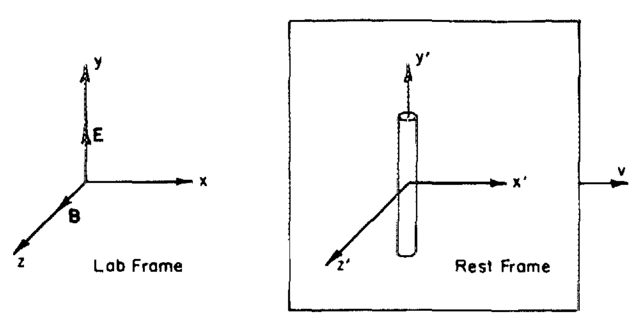

FIG. 1. Lab frame and rest frame. The projectile is at rest in the rest frame.

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where we have recalled the vector property given by Eq. (7). We assume a total transfer of electron momentum to the projectile, which gives

$$N\frac{d}{dt}\langle p_y\rangle\hat{\mathbf{y}}=\frac{d}{dt}\mathbf{P}, \tag{10}$$

where N is total number of current-carrying electrons in the column. The momentum of the projectile $\mathbf{P}$ is given by

$$\mathbf{P}=M\gamma\,\mathbf{v}, \tag{11}$$

where M is the mass of the projectile.

Combining Eqs. (7), (9), (10), and (11) gives the desired equation of motion:

$$\frac{d}{dt}\gamma Mv=\frac{aI}{c}\left(1-\frac{v}{w}\right)B, \tag{12}$$

where a is the length of the conducting column, I is the current,

$$aI=Nq\mu E \tag{13}$$

and

$$w\equiv cE/B. \tag{14}$$

Note in particular that from Eq. (13) we may write

$$E=I/A\,\sigma, \tag{15}$$

where aA represents the volume of the conducting column in the rest frame.

Integrating Eq. (12) gives

$$\eta-\eta_0=\frac{1}{c}\int_{v_0}^{v}\frac{du}{[1-(u/c)^2]^{3/2}(1-u/w)}, \tag{16}$$

where η is the dimensionless time,

$$\eta\equiv aIBt/Mc^2, \tag{17}$$

and $v=v_0$ at $\eta=\eta_0$. From Eq. (16) we see that $\eta\to\infty$ at the singular points $u=c$ and $u=w$, which represent asymptotic velocities. It will be shown below that these asymptotic velocities are approached monotonically. With this property, we may conclude that for zero starting velocities maximum values are given by

$$v_{\max}=c,\quad E>B \tag{18}$$

$$v_{\max}=w,\quad E<B.$$

A sketch of these findings is shown in Fig. 2.

To examine the monotonicity of $v(t)$ we differentiate Eq. (12) to obtain

$$\frac{1}{c}\frac{dv}{d\eta}=\left[1-\left(\frac{v}{c}\right)^2\right]^{3/2}\left(1-\frac{v}{w}\right). \tag{19}$$

We conclude that for $v\ll c$ and $v\ll w$, $dv/d\eta\gg 0$. Furthermore, with $v=0$ at $t=0$, Eq. (19) gives the starting acceleration (in dimensional form)

$$\left.\frac{dv}{dt}\right|_0=\frac{aIB}{Mc}. \tag{20}$$

Note that for the case $E<B$, an initial velocity $v_0>w$, decays to $v=w$, as is evident from Eq. (19). Furthermore, as is clear from Eq. (16), asymptotic values (18) are independent of initial velocities.

Characteristic times corresponding to the maximum velocities (18) are as follows. In the limit $w\gg c$, Eq. (12) gives the characteristic time

$$\tau_1=Mc^2/aIB \tag{21a}$$

with maximum velocity

$$v_{\max}^{(1)}=c. \tag{21b}$$

In the limit $w\ll c$, Eq. (12) has the solution (with $v=0$ at $t=0$)

$$v=w[1-\exp(-t/\tau_2)], \tag{22}$$

where

$$\tau_2=(E/B)\tau_1 \tag{23a}$$

and

$$v_{\max}^{(2)}=w. \tag{23b}$$

We note that although $\tau_2\ll\tau_1$, accelerations

$$v_{\max}^{(1)}/\tau_1=v_{\max}^{(2)}/\tau_2$$

are the same.

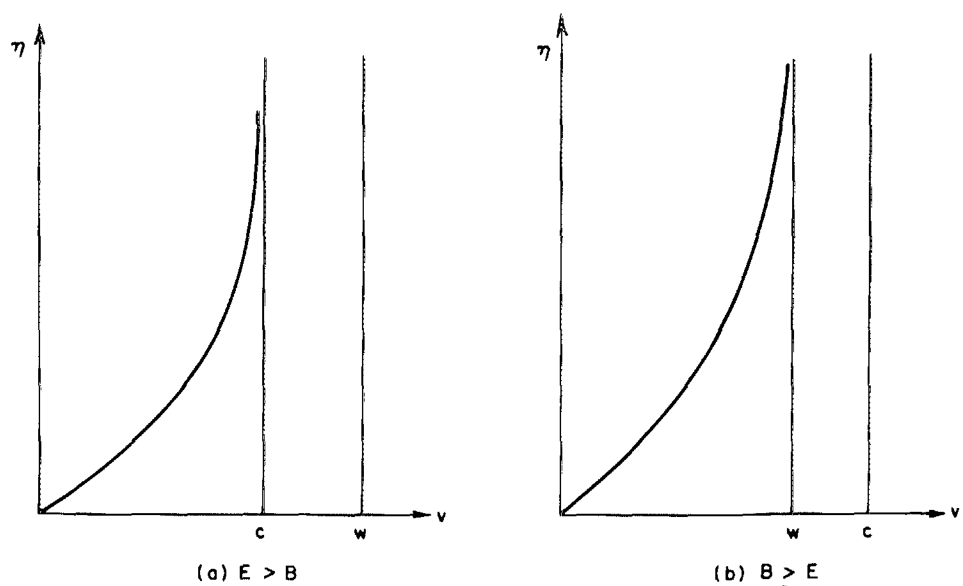

FIG. 2. Sketches of dimensionless time η, as a function of projectile velocity v, with $v=0$ at $t=0$. (a) Asymptotic speed c for $E>B$; (b) asymptotic speed w for $B>E$.

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In applying the preceding results to experimental values, first we rewrite w in practical units. Setting

$$E = V/a, \tag{24}$$

where V is the potential across the rails, permits Eq. (14) to be rewritten

$$w = cV/aB. \tag{25}$$

In practical units this expression becomes

$$w\left(\frac{\text{km}}{\text{s}}\right) = \frac{10^3 V(\text{kV})}{B(\text{kG})a(\text{cm})}. \tag{26}$$

Typical experimental values[5,7] are: $V = 1$ kV, $a \simeq 1$ cm, and $B \simeq 200$ kG, which gives $w \simeq 5$ km/s. This value agrees in order of magnitude with observed maximum velocities.

We have examined the relativistic solution to the railgun configuration. Incorporating some simplifying assumptions, we found that the projectile velocity goes monotonically to the minimum of the two velocities, c and cE/B. The asymptotic value c corresponds to $E < B$, whereas the value cE/B corresponds to the limit $B > E$. It should be emphasized that this present study does not take into account thermodynamic effects such as momentum imparted to the projectile from the exploding "fuse."

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Effect of dislocations on the efficiency of thin-film GaAs solar cells on Si substrates

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Recombination loss at dislocations is the predominant loss mechanism in thin-film GaAs solar cells on Si substrates. Cell parameters are calculated based on a simple model in which dislocations act as recombination centers. Excellent agreement is observed between theory and experiment. It is indicated that one could fabricate thin-film GaAs solar cells with an efficiency of 17–18% on Si substrates if the dislocation density is less than 5×10^5 cm^{-2}.

Vigorous research and development programs[1–3] have been conducted to fabricate high-efficiency thin-film GaAs solar cells on low-cost single-crystal Si substrates. However, successful development of thin-film GaAs solar cells with an efficiency of 15% or greater has not been made to date, since it is more difficult to grow a good-quality GaAs epitaxial layer on a Si substrate because of a large mismatch of about 4%. The deleterious effect of dislocations on GaAs solar cell electrical properties is considerable. Although antiphase domain boundaries and stacking faults in GaAs are also expected to act as recombination centers, single-domain GaAs layers can be grown by optimizing substrate surface treatment and heteroepitaxial growth conditions, and the stacking fault density is expected to be in general less than the dislocation density.[2,3] Therefore, in order to realize highly efficient thin-film GaAs solar cells, it is important to clarify the effects of dislocations upon the electrical and photovoltaic properties of GaAs.

In this communication, correlations between minority-carrier diffusion lengths and dislocations have been studied and solar cell parameters for the thin-film GaAs cells are calculated based on their relationships.

Dislocations play a dominant role in determining the electrical and photovoltaic properties of thin-film GaAs solar cells by acting as recombination centers.[4,5] The recombination loss at dislocations which reduce short-circuit current and increase excess leakage current is the predominant loss mechanism in thin-film GaAs solar cells. Dislocation recombination centers reduce minority-carrier lifetime and diffusion length. Minority-carrier diffusion length is given by

$$1/L^2 = 1/L_0^2 + 1/L_d^2 + 1/L_I^2, \tag{1}$$

where L_d, L_I, and L_0 are diffusion lengths related to minority-carrier recombination with a majority carrier at a dislocation, at an impurity, and at other unknown defects, respectively.

The diffusion-limited minority-carrier diffusion length for recombination on dislocations is obtained by solving the one-dimensional continuity equation for the transport of minority carriers to the dislocations

$$\frac{\partial n}{\partial t} = \frac{D\partial^2 n}{\partial x^2}, \tag{2}$$

where n is the excess minority carrier concentration and D is the minority-carrier diffusion coefficient. The boundary conditions are given by

$$n = 0 \quad \text{at } x = 0, \tag{3}$$

$$\frac{\partial n}{\partial x} = 0 \quad \text{at } x = x_c = 1/(\pi N_d)^{1/2}, \tag{4}$$

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