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Electrokinetic motion of a charged colloidal sphere in a spherical cavity with magnetic fields

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The magnetohydrodynamic (MHD) effects on the translation and rotation of a charged colloidal sphere situated at the center of a spherical cavity filled with an arbitrary electrolyte solution when a constant magnetic field is imposed are analyzed at the quasisteady state. The electric double layers adjacent to the solid surfaces may have an arbitrary thickness relative to the particle and cavity radii. Through the use of a perturbation method to the leading order, the Stokes equations modified with the electric/Lorentz force term are dealt by using a generalized reciprocal theorem. Using the equilibrium double-layer potential distribution in the fluid phase from solving the linearized Poisson–Boltzmann equation, we obtain explicit formulas for the translational and angular velocities of the colloidal sphere produced by the MHD effects valid for all values of the particle-to-cavity size ratio. For the limiting case of an infinitely large cavity with an uncharged wall, our result reduces to the relevant solution for an unbounded spherical particle available in the literature. The boundary effect on the MHD motion of the spherical particle is a qualitatively and quantitatively sensible function of the parameters a/b and κa, where a and b are the radii of the particle and cavity, respectively, and κ is the reciprocal of the Debye screening length. In general, the proximity of the cavity wall reduces the MHD migration but intensifies the MHD rotation of the particle. [doi:10.1063/1.3537975]

I. INTRODUCTION

Most colloidal particles bear charges on their surfaces as a consequence of dissociation of functional groups or crystal lattice defects when immersed in an electrolyte solution. The counterions in the solution are attracted by the surface charge of the particle so that their concentration becomes higher in the vicinity of the particle surface than the bulk value. On the other hand, the coions are repelled from the particle surface. Hence, a region of mobile ions that is not electrically neutral forms surrounding the particle. The combination of this region and the fixed charge on the particle surface is well known as an electric double layer. When an external electric field is applied, the interaction between the particle's surface charge and this field drives the particle to migrate at an electrophoretic velocity in one direction, while the movement of the counterions in the double layer induces an ambient fluid flow in the opposite direction. Electrophoresis has long been used as an effective technique for separation and identification of biologically active compounds in the biochemical and clinical fields.[1–5]

The flow of an electrolyte solution caused by its interaction with an electromagnetic force is known as the magnetohydrodynamic (MHD) effect.[6–13] This flow results from the Lorentz force acting on the ions as they move through a transversely imposed magnetic field. In the absence of electric fields, the Lorentz force may be expressed as a force density $\rho \mathbf{v} \times \mathbf{B}$ exerted on a differential volume of the fluid, where ρ is the space charge density, $\mathbf{v}$ is the fluid velocity, and $\mathbf{B}$ is the imposed magnetic field. This driving force, playing as an additional term in the Navier–Stokes equation, results in the MHD flow of the fluid.

Colloidal particles suspended in conducting liquids prescribed with a magnetic field have been used in various biological, metallurgical, and other applications.[14–20] The movement of a charged particle caused by the MHD effect, in which both parts of the electric double layer experience the Lorentz force, is relevant to electrophoresis, but the difference between them is evident. Recently, the motion of a charged colloidal sphere in an unbounded electrolyte solution prescribed with a general flow field and a uniform magnetic field was analytically studied, and closed-form formulas for the translational and angular velocities of the spherical particle induced by the MHD effect have been obtained.[21] Although this effect on the particle movement is relatively weak, some physically interesting phenomena were observed.

In potential applications of the MHD effect in the manipulation and self-assembly of colloidal particles, the particles are not isolated and will move in the vicinity of solid boundaries.[22–29] Therefore, it is of interest to examine the boundary effects on the movement of a charged particle caused by the MHD effect. In this paper, we investigate the general motion of a charged colloidal sphere in a concentric spherical cavity filled with an electrolyte solution subject to a prescribed magnetic field in the absence of external electric fields. The thickness of the electric double layers adjacent to the solid surfaces is arbitrary relative to the particle and cavity radii. Explicit formulas for the translational and

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angular velocities of the confined spherical particle induced by the MHD effect are obtained in Eqs. (9) and (10).

II. ANALYSIS

Consider the quasisteady translation and rotation of a neutrally buoyant colloidal sphere of radius a and zeta potential ζ_p in a concentric, translating and rotating, spherical cavity of radius b and zeta potential ζ_w filled with an electrolyte solution in the existence of a constant magnetic field. The thickness of the electric double layers adjacent to the particle surface and cavity wall is arbitrary compared with the particle and cavity radii. Our purpose is to evaluate the boundary effect on the additional particle motion induced by the application of the uniform magnetic field. To determine the translational and angular velocities of the confined particle caused by the MHD effect, we first need to ascertain the equilibrium electric potential and velocity distributions in the fluid phase.

With the use of the Debye–Huckel approximation applicable for the case of low electric potentials, the equilibrium double-layer potential distribution ψ in the absence of the applied velocity field and magnetic field governed by the linearized Poisson–Boltzmann equation can be obtained as

$$\psi = \frac{\zeta_w \sinh[t\lambda(\gamma-1)] + \zeta_p \lambda \sinh[t(1-\lambda\gamma)]}{\lambda\gamma \sinh[t(1-\lambda)]}, \tag{1}$$

where

$$\gamma = \frac{r}{a}, \tag{2a}$$

$$\lambda = \frac{a}{b}, \tag{2b}$$

$$t = \kappa b, \tag{2c}$$

r is the radial coordinate from the particle/cavity center and κ is the reciprocal of the Debye screening length.

We now consider the fluid flow caused by the case of a charged sphere translating with velocity $\mathbf{U}_p$ and rotating with angular velocity $\boldsymbol{\Omega}_p$ in a concentric spherical cavity whose wall translates with velocity $\mathbf{U}_w$ and rotates with angular velocity $\boldsymbol{\Omega}_w$. The velocities $\mathbf{U}_p$ and $\boldsymbol{\Omega}_p$ of the particle may result from the translational and angular velocities of the confining cavity wall and/or from some external force and torque acting on the particle. Because the Reynolds number is small, the fluid motion in the presence of a magnetic flux density $\mathbf{B}$ is governed by the Stokes equations modified with an electric force (including the Lorentz force) term.[21]

The fluid velocity field $\mathbf{v}$ can be expressed by the perturbation expansion

$$\mathbf{v} = \mathbf{v}_0 + \alpha\mathbf{v}_M + O(\alpha^2), \tag{3}$$

where $\alpha = \varepsilon\zeta_p |\mathbf{B}|/\mu$ is a small dimensionless parameter, μ and ε are the viscosity and dielectric permittivity of the fluid, respectively, the subscript 0 denotes the fluid flow in the absence of the magnetic field, and $\mathbf{v}_M$ represents the fluid velocity distribution produced by the magnetic field $\mathbf{B}$. The zeroth-order flow field without the MHD effect ($\alpha = 0$) can be obtained as

$$\mathbf{v}_0 = \frac{1}{\Gamma_T}\left[a_1\mathbf{U}_p + a_2\mathbf{U}_w + 3a_3(\mathbf{U}_p - \mathbf{U}_w)\cdot\frac{\mathbf{rr}}{r^2}\right] + \frac{1}{\Gamma_R}(a_4\boldsymbol{\Omega}_p + a_5\boldsymbol{\Omega}_w)\times\mathbf{r}, \tag{4}$$

where

$$a_1 = 1-\lambda^3+6\lambda^3\gamma^5(1-\lambda^2)+3\gamma^2(1-\lambda^5) - \lambda\gamma^3(9-5\lambda^2-4\lambda^5), \tag{5a}$$

$$a_2 = -1+\lambda^3+\gamma^2[4\gamma-3-\lambda^3\gamma(6\gamma^2-5) + 3\lambda^5(2\gamma^3-3\gamma+1)], \tag{5b}$$

$$a_3 = (1-\lambda^3)(\gamma^2-1)-\lambda^3\gamma^2(1-\lambda^2)(\gamma^3-1), \tag{5c}$$

$$a_4 = 1-\lambda^3\gamma^3, \tag{5d}$$

$$a_5 = \gamma^3 - 1; \tag{5e}$$

$$\Gamma_T = \gamma^3(1-\lambda)^4(4+7\lambda+4\lambda^2), \tag{6a}$$

$$\Gamma_R = \gamma^3(1-\lambda^3), \tag{6b}$$

and $\mathbf{r}$ is the position vector from the particle/cavity center.

Substituting Eq. (3) into the modified Stokes equations and collecting the first-order terms of the small perturbation parameter α, we obtain

$$\alpha\mu\nabla\times\nabla^2\mathbf{v}_M = -\varepsilon\nabla\times(\mathbf{D}\nabla\cdot\mathbf{D}), \tag{7}$$

where $\mathbf{D} = \mathbf{v}_0\times\mathbf{B} - \nabla\psi$, which is the net electric field involving the zeroth-order velocity field $\mathbf{v}_0$. The boundary conditions for the fluid velocity $\mathbf{v}_M$ can be expressed as

$$\alpha\mathbf{v}_M = \mathbf{U}_M + \boldsymbol{\Omega}_M\times\mathbf{r} \quad \text{at} \quad r = a, \tag{8a}$$

$$\mathbf{v}_M = \mathbf{0} \quad \text{at} \quad r = b, \tag{8b}$$

where $\mathbf{U}_M$ and $\boldsymbol{\Omega}_M$ are the translational and angular velocities, respectively, of the confined particle of the leading order in α produced by the MHD effect to be determined.

The translational and angular velocities of the particle appearing in Eq. (8a) induced by the MHD effect can be obtained by the use of the reciprocal theorem of Lorentz.[30] Following Tuebner's approach with a generalized reciprocal theorem,[21,31–35] we obtain

$$\mathbf{U}_M = \frac{\varepsilon}{\mu}[(M_{pp}\zeta_p + M_{wp}\zeta_w)\mathbf{U}_p + (M_{pw}\zeta_p + M_{ww}\zeta_w)\mathbf{U}_w]\times\mathbf{B}, \tag{9a}$$

$$\boldsymbol{\Omega}_{\mathrm{M}} = \frac{\varepsilon}{\mu}[(N_{\mathrm{pp}}\zeta_{\mathrm{p}} + N_{\mathrm{wp}}\zeta_{\mathrm{w}})\boldsymbol{\Omega}_{\mathrm{p}} + (N_{\mathrm{pw}}\zeta_{\mathrm{p}} + N_{\mathrm{ww}}\zeta_{\mathrm{w}})\boldsymbol{\Omega}_{\mathrm{w}}] \times \mathbf{B}, \tag{9b}$$

where

$$M_{\mathrm{pp}} = \frac{2}{3f_{\mathrm{T}}}\left[1 + t\lambda \coth t(1-\lambda) - \int_{\lambda}^{1} \frac{t^2 a_1(a_1+2a_3)\eta \sinh t(1-\eta) + b_1}{\Gamma_{\mathrm{T}}^2 \sinh t(1-\lambda)} \mathrm{d}\eta\right], \tag{10a}$$

$$M_{\mathrm{wp}} = -\frac{2}{3f_{\mathrm{T}}}\left[t \operatorname{csch} t(1-\lambda) + \int_{\lambda}^{1} \frac{t^2 a_1(a_1+2a_3) \sinh t(\eta-\lambda) + b_2}{\lambda\Gamma_{\mathrm{T}}^2 \sinh t(1-\lambda)} \mathrm{d}\eta\right], \tag{10b}$$

$$M_{\mathrm{pw}} = -\frac{2}{3f_{\mathrm{T}}} \times \int_{\lambda}^{1} \frac{t^2[a_1(a_2-a_3)+a_2a_3)]\eta \sinh t(1-\eta) - b_1}{\Gamma_{\mathrm{T}}^2 \sinh t(1-\lambda)} \mathrm{d}\eta, \tag{10c}$$

$$M_{\mathrm{ww}} = -\frac{2}{3f_{\mathrm{T}}} \times \int_{\lambda}^{1} \frac{t^2[a_1(a_2-a_3)+a_2a_3)]\eta \sinh t(\eta-\lambda) - b_2}{\lambda\Gamma_{\mathrm{T}}^2 \sinh t(1-\lambda)} \mathrm{d}\eta, \tag{10d}$$

$$N_{\mathrm{pp}} = \frac{1}{6f_{\mathrm{R}}}\left[1 + t\lambda \coth t(1-\lambda) - \int_{\lambda}^{1} \frac{t^2 a_4^2 \eta^3 \sinh t(1-\eta)}{\lambda^2\Gamma_{\mathrm{R}}^2 \sinh t(1-\lambda)} \mathrm{d}\eta\right], \tag{10e}$$

$$N_{\mathrm{wp}} = -\frac{1}{6f_{\mathrm{R}}}\left[t \operatorname{csch} t(1-\lambda) + \int_{\lambda}^{1} \frac{t^2 a_4^2 \eta^3 \sinh t(\eta-\lambda)}{\lambda^3\Gamma_{\mathrm{R}}^2 \sinh t(1-\lambda)} \mathrm{d}\eta\right], \tag{10f}$$

$$N_{\mathrm{pw}} = -\frac{1}{6f_{\mathrm{R}}} \int_{\lambda}^{1} \frac{t^2 a_4 a_5 \eta^3 \sinh t(1-\eta)}{\lambda^2\Gamma_{\mathrm{R}}^2 \sinh t(1-\lambda)} \mathrm{d}\eta, \tag{10g}$$

$$N_{\mathrm{ww}} = -\frac{1}{6f_{\mathrm{R}}} \int_{\lambda}^{1} \frac{t^2 a_4 a_5 \eta^3 \sinh t(\eta-\lambda)}{\lambda^3\Gamma_{\mathrm{R}}^2 \sinh t(1-\lambda)} \mathrm{d}\eta; \tag{10h}$$

$$b_1 = \frac{\eta}{\lambda}(a_1+3a_3)[5\eta^3(\lambda^2-1) - 2(\lambda^5-1)] \times [t\eta \cosh t(1-\eta) + \sinh t(1-\eta)], \tag{11a}$$

$$b_2 = \frac{\eta}{\lambda}(a_1+3a_3)[5\eta^3(\lambda^2-1) - 2(\lambda^5-1)] \times [-t\eta \cosh t(\eta-\lambda) + \sinh t(\eta-\lambda)]; \tag{11b}$$

$$f_{\mathrm{T}} = (1-\lambda^5)\left(1 - \frac{9}{4}\lambda + \frac{5}{2}\lambda^3 - \frac{9}{4}\lambda^5 + \lambda^6\right)^{-1}, \tag{12a}$$

$$f_{\mathrm{R}} = (1-\lambda^3)^{-1}, \tag{12b}$$

and $\eta = r/b = \gamma\lambda$. Equation (9a) shows that the induced translational velocity of the confined particle depends on the prescribed translational velocities of the particle and cavity, and it is in the direction perpendicular to both the relevant prescribed velocities and the imposed magnetic field. Equation (9b) indicates that the induced angular velocity of the particle depends on the prescribed angular velocities of the particle and cavity, and it is also in the direction normal to both the relevant prescribed angular velocities and the applied magnetic field.

III. RESULTS AND DISCUSSION

We first consider the expressions for the dimensionless mobility parameters M_{pp}, M_{wp}, M_{pw}, M_{ww}, N_{pp}, N_{wp}, N_{pw}, and N_{ww} in Eq. (10) for the limiting cases of the electrokinetic parameter κa and separation parameter $\lambda = a/b$. Results of the general case for the boundary effect on the MHD motion of the particle will then be discussed.

In the limiting case of $\kappa a \to \infty$ (very thin electric double layers), Eq. (10) reduces to

$$M_{\mathrm{pp}} = -3M_{\mathrm{pw}} = -\frac{3}{2} + \frac{5(1-\lambda^3)}{2(1-\lambda^5)}, \tag{13a}$$

$$M_{\mathrm{wp}} = N_{\mathrm{wp}} = 0, \tag{13b}$$

$$M_{\mathrm{ww}} = -\frac{2}{3} + \frac{5(1-\lambda^2)}{3(1-\lambda^5)}, \tag{13c}$$

$$N_{\mathrm{pp}} = \frac{2}{3} + \frac{1}{3}\lambda^3, \tag{13d}$$

$$N_{\mathrm{pw}} = N_{\mathrm{ww}} = -\frac{1}{2}. \tag{13e}$$

When $\lambda \to 0$ (the cavity wall is at an infinite distance from the particle), the above formulas can be further simplified and Eq. (9) becomes

$$\mathbf{U}_{\mathrm{M}} = \frac{\varepsilon}{3\mu}[3\zeta_{\mathrm{p}}\mathbf{U}_{\mathrm{p}} - (\zeta_{\mathrm{p}} - 3\zeta_{\mathrm{w}})\mathbf{U}_{\mathrm{w}}] \times \mathbf{B}, \tag{14a}$$

$$\boldsymbol{\Omega}_{\mathrm{M}} = \frac{\varepsilon}{6\mu}[4\zeta_{\mathrm{p}}\boldsymbol{\Omega}_{\mathrm{p}} - 3(\zeta_{\mathrm{p}} + \zeta_{\mathrm{w}})\boldsymbol{\Omega}_{\mathrm{w}})] \times \mathbf{B}. \tag{14b}$$

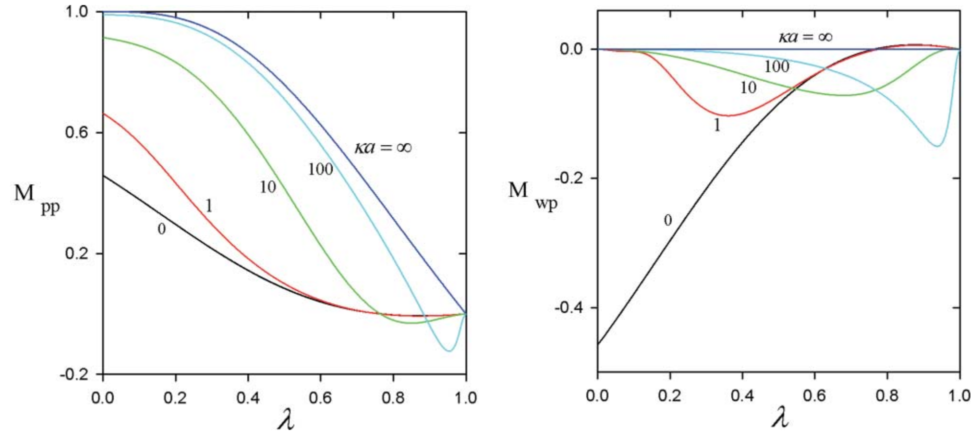

FIG. 1. Plots of the dimensionless mobility parameters M_{pp} and M_{wp} as defined by Eq. (9a) and calculated from Eqs. (10a) and (10b) vs the parameter λ for various values of the parameter κa.

In the limiting case of $\kappa a \to 0$ (very thick electric double layers), Eq. (10) reduces to

$$M_{\mathrm{pp}} = -M_{\mathrm{wp}} = \frac{44 + 48\lambda - 75\lambda^2 - 150\lambda^3 - 15\lambda^4 + 96\lambda^5 + 52\lambda^6}{24(4 + 11\lambda + 15\lambda^2 + 15\lambda^3 + 15\lambda^4 + 11\lambda^5 + 4\lambda^6)}, \tag{15a}$$

$$M_{\mathrm{pw}} = -M_{\mathrm{ww}} = \frac{(1-\lambda)^2(20 + 68\lambda + 83\lambda^2 + 33\lambda^3 - 12\lambda^4 - 12\lambda^5)}{24(1-\lambda^5)(4 + 7\lambda + 4\lambda^2)}, \tag{15b}$$

$$N_{\mathrm{pp}} = -N_{\mathrm{wp}} = \frac{1}{6}(1 + \lambda + \lambda^2), \tag{15c}$$

$$N_{\mathrm{pw}} = N_{\mathrm{ww}} = 0. \tag{15d}$$

When $\lambda \to 0$, Eqs. (9) and (15) lead to

$$\mathbf{U}_{\mathrm{M}} = \frac{\varepsilon}{24\mu}(\zeta_{\mathrm{p}} - \zeta_{\mathrm{w}})(11\mathbf{U}_{\mathrm{p}} + 5\mathbf{U}_{\mathrm{w}}) \times \mathbf{B}, \tag{16a}$$

$$\mathbf{\Omega}_{\mathrm{M}} = \frac{\varepsilon}{6\mu}(\zeta_{\mathrm{p}} - \zeta_{\mathrm{w}})\mathbf{\Omega}_{\mathrm{p}} \times \mathbf{B}. \tag{16b}$$

Both of the translational and angular velocities of the particle resulting from the MHD effect are proportional to the zeta potential difference between the particle surface and the cavity wall in this limiting case.

Equations (14) and (16) with $\zeta_{\mathrm{w}} = 0$ are consistent with the formulas obtained earlier for the MHD effect on a translating and rotating charged sphere in an unbounded electrolyte solution prescribed with a general flow field.[21] Note that the translational and angular velocities of the charged particle induced by the MHD effect as expressed by Eqs. (14) and (16) are independent of the particle size, analogous to the electrophoretic velocity[35] of the particle in the limiting cases of $\kappa a \to \infty$ and $\kappa a \to 0$.

In the limit of $\lambda \to 1$ (the particle fills the cavity up completely), Eq. (10) results in $M_{\mathrm{pp}} = M_{\mathrm{wp}} = M_{\mathrm{pw}} = M_{\mathrm{ww}} = N_{\mathrm{pw}} = N_{\mathrm{ww}} = 0$ and $N_{\mathrm{pp}} = -N_{\mathrm{wp}} = 1/2$, and thus Eq. (9) reduces to

$$\mathbf{U}_{\mathrm{M}} = \mathbf{0}, \tag{17a}$$

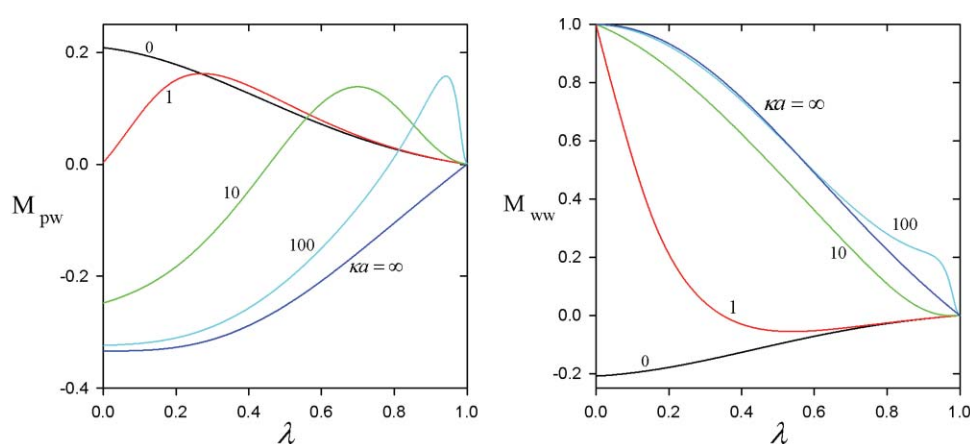

FIG. 2. Plots of the dimensionless mobility parameters M_{pw} and M_{ww} as defined by Eq. (9a) and calculated from Eqs. (10c) and (10d) vs the parameter λ for various values of the parameter κa.

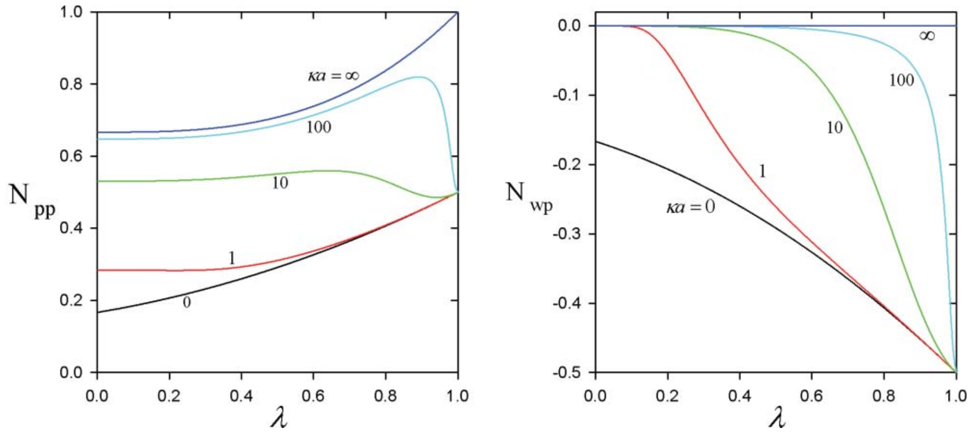

FIG. 3. Plots of the dimensionless mobility parameters N_{pp} and N_{wp} as defined by Eq. (9b) and calculated from Eqs. (10e) and (10f) vs the parameter λ for various values of the parameter κa.

$$\mathbf{\Omega}_{\mathrm{M}} = \frac{\varepsilon}{2\mu}(\zeta_{\mathrm{p}} - \zeta_{\mathrm{w}})\mathbf{\Omega}_{\mathrm{p}} \times \mathbf{B}. \tag{17b}$$

As expected, the translational velocity of the particle caused by the MHD effect vanishes in this limiting case.

The numerical values of the dimensionless mobility parameters M_{pp}, M_{wp}, M_{pw}, M_{ww}, N_{pp}, N_{wp}, N_{pw}, and N_{ww}, as defined by Eq. (9) and calculated from formulas in Eq. (10), are plotted versus the parameter $\lambda = a/b$ for various values of the parameter κa in Figs. 1–4.

Figure 1 shows the results of the mobility parameters M_{pp} and M_{wp} corresponding to the translational velocity $\mathbf{U}_{\mathrm{p}}$ of the particle. In general, M_{pp} is a positive value which decreases with an increase in λ (indicating that the proximity of the cavity wall reduces the MHD migration of the particle) and with a decrease in κa and M_{wp} is a negative value which is not a monotonic function of λ and κa, but there are exceptions when λ is large (say, greater than 0.6).

Figure 2 illustrates the results of the mobility parameters M_{pw} and M_{ww} concerning the translational velocity $\mathbf{U}_{\mathrm{w}}$ of the cavity wall. In general, both M_{pw} and M_{ww} are not monotonic functions of λ and κa. M_{pw} is a positive value when κa is small (less than unity) and can be negative otherwise (depending on the value of λ). On the contrary, M_{ww} is a positive value when κa is large (say, greater than 10) and can be negative otherwise (also depending on the value of λ).

The results of the mobility parameters N_{pp} and N_{wp} about the angular velocity $\mathbf{\Omega}_{\mathrm{p}}$ of the particle are plotted in Fig. 3. It can be seen that N_{wp} is a negative value whose magnitude increases with an increase in λ or with a decrease in κa and disappears as $\lambda \to 0$ or $\kappa a \to \infty$, whereas N_{pp} is positive and increases with an increase in κa for a given value of λ. In general, N_{pp} increases with an increase in λ (illustrating that the approach of the cavity wall enhances the MHD rotation of the particle), but there are exceptions when both λ and κa are large.

Figure 4 shows the results of the mobility parameters N_{pw} and N_{ww} regarding the angular velocity $\mathbf{\Omega}_{\mathrm{w}}$ of the cavity wall. Both N_{pw} and N_{ww} are negative values with their magnitudes decreasing monotonically with an increase in λ or with a decrease in κa and vanishing as $\lambda \to 1$ or $\kappa a \to 0$. Namely, the approach of the cavity wall diminishes the MHD rotation of the particle caused by $\mathbf{\Omega}_{\mathrm{w}}$.

It is interesting to examine the special case that both the spherical particle and the cavity wall move with the same translational and angular velocities (which occurs if the particle is freely suspended in the fluid inside the moving cavity without externally applied force and torque) in the

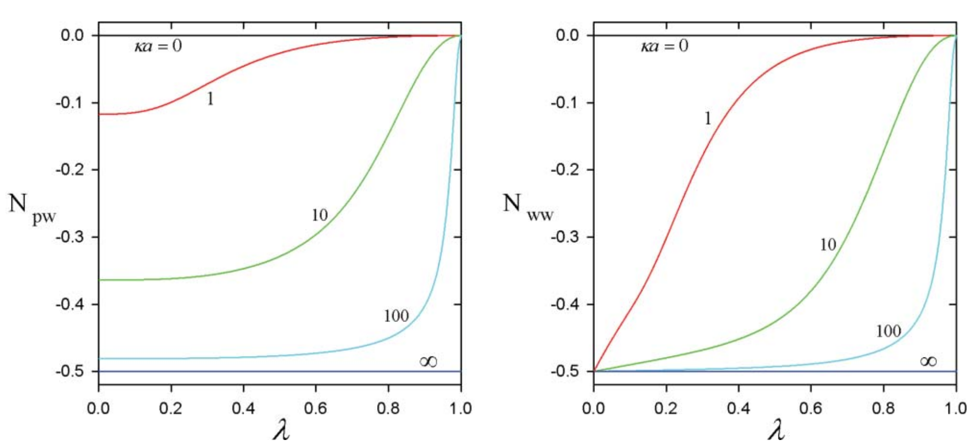

FIG. 4. Plots of the dimensionless mobility parameters N_{pw} and N_{ww} as defined by Eq. (9b) and calculated from Eqs. (10g) and (10h) vs the parameter λ for various values of the parameter κa.

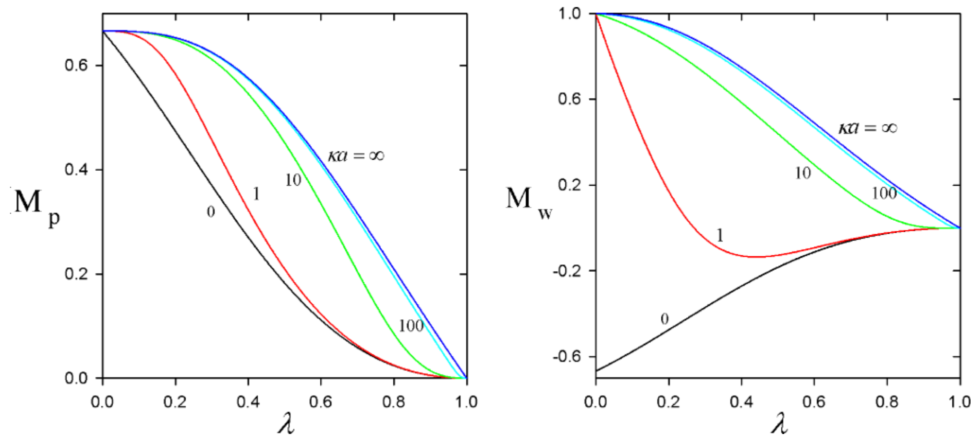

FIG. 5. Plots of the dimensionless mobility parameters M_p and M_w as defined by Eq. (18a) vs the parameter λ for various values of the parameter κa.

presence of the magnetic field. Substituting $\mathbf{U}_w = \mathbf{U}_p = \mathbf{U}$ and $\boldsymbol{\Omega}_w = \boldsymbol{\Omega}_p = \boldsymbol{\Omega}$ into Eq. (9), we obtain

$$\mathbf{U}_M = \frac{\varepsilon}{\mu}(M_p\zeta_p + M_w\zeta_w)\mathbf{U} \times \mathbf{B}, \tag{18a}$$

$$\boldsymbol{\Omega}_M = \frac{\varepsilon}{\mu}(N_p\zeta_p + N_w\zeta_w)\boldsymbol{\Omega} \times \mathbf{B}, \tag{18b}$$

where

$$M_p = M_{pp} + M_{pw}, \tag{19a}$$

$$M_w = M_{wp} + M_{ww}, \tag{19b}$$

$$N_p = N_{pp} + N_{pw}, \tag{19c}$$

$$N_w = N_{wp} + N_{ww}. \tag{19d}$$

Figure 5 illustrates the results of the mobility parameters M_p and M_w about the translation of the particle and cavity. For any given value of κa, M_p is positive and decreases monotonically with an increase in λ from 2/3 at $\lambda = 0$ to zero at $\lambda = 1$. Namely, the proximity of the cavity wall reduces the MHD migration of the particle. M_w is a positive value when κa is large and can be negative if κa is small. For any specified value of λ, both M_p and M_w are monotonically increasing functions of κa.

The results of the mobility parameters N_p and N_w relating to the rotation of the particle and cavity are plotted in Fig. 6. For any value of κa, N_p is positive and increases monotonically with an increase in λ from 1/6 at $\lambda = 0$ to 1/2 at $\lambda = 1$. Namely, the approach of the cavity wall enhances the MHD rotation of the particle. N_w is always negative and, for a finite value of κa, it first increases with an increase in λ from –1/2 at $\lambda = 0$, reaches a maximum at some value of λ, and then decreases with a further increase in λ to –1/2 again at $\lambda = 1$. For any fixed value of λ, both N_p and N_w are monotonically decreasing functions of κa.

IV. SUMMARY

The MHD effects on the translation and rotation of a charged colloidal sphere in a concentric spherical cavity filled with an arbitrary electrolyte solution subject to a uniformly applied magnetic field are analyzed at the quasisteady state in this study. The thickness of the electric double layers adjacent to the solid surfaces can be arbitrary relative to the particle and cavity radii. The equilibrium double-layer potential distribution in the fluid phase is determined through

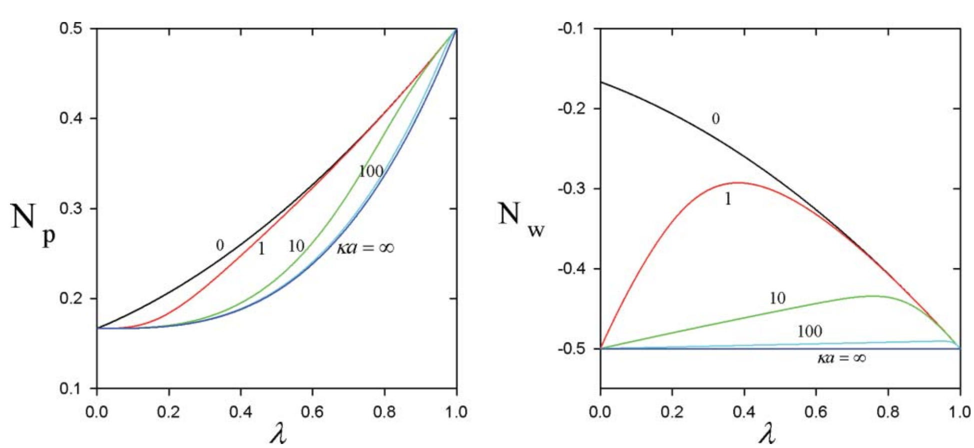

FIG. 6. Plots of the dimensionless mobility parameters N_p and N_w as defined by Eq. (18b) vs the parameter λ for various values of the parameter κa.

the use of the Debye–Huckel approximation. The modified Stokes equations governing the fluid velocity field are dealt by using a simple perturbation method and a generalized reciprocal theorem, and closed-form formulas for the translational and angular velocities of the confined particle resulting from the MHD effects are obtained in Eqs. (9) and (18), with the relevant mobility parameters given by Eqs. (10) and (19) and Figs. 1–6. These mobility parameters are qualitatively and quantitatively sensible functions of the separation parameter λ and electrokinetic parameter κa. In general, the proximity of the cavity wall diminishes the MHD migration but enhances the MHD rotation of the particle.

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