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Extending the collisional fluid equations into the long mean-free-path regime in toroidal plasmas. I. Plasma viscosity

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An expression for the pressure anisotropy and thus for the viscous stress in the plateau regime is derived for arbitrary toroidal magnetic configurations without assuming incompressibility or the existence of flux surfaces, without neglecting the flow components perpendicular to the magnetic surface, and without restricting the flow velocity to be a constant on the flux surface. It can be employed to study low-frequency instabilities in the long mean-free-path regime. A smoothly connected formula for the pressure anisotropy, valid in both the collisional fluid regime and the plateau regime, is given to facilitate the numerical computation. An alternative interpretation of the neoclassical transport theory is also obtained. It is found that if the effects of the temperature gradient are neglected, neoclassical transport fluxes can be interpreted as driven by the velocity stress.

I. INTRODUCTION

It is generally believed that transport processes in fusion plasmas are dominated by turbulent fluctuations. The best way to understand plasma turbulence at present is to solve the appropriate governing equations numerically. The results of these calculations can also be employed to gauge the accuracy of various approximate analytic theories. Many such studies[1] have been carried out by solving nonlinear fluid equations such as those developed by Braginskii.[2] These fluid equations are valid for a collisional plasma in which the mean free path l is shorter than any characteristic length scale L in the problem of interest. However, the condition $l \ll L$ is usually violated in fusion plasmas. To understand the nonlinear behavior of these plasmas, one must solve the kinetic equations. Such a task is extremely cumbersome, even with the existing computing capabilities. However, if some of the kinetic effects can be incorporated into the long mean-free-path regime—for example, if the effects of $l \gg L$ can be incorporated into the existing fluid equations—then the task of numerically simulating plasma nonlinear behavior could become manageable. This is the fundamental motivation for developing the neoclassical magnetohydrodynamic (MHD) equations.[3] The only difference between the collisional fluid equations and the neoclassical MHD equations is that a neoclassical plasma viscosity,[4–6] calculated on the basis of the assumptions of neoclassical transport theory, is incorporated in the latter equations.

Several difficulties limit the validity of the neoclassical MHD equations. All of these difficulties result from employing the assumptions of neoclassical transport theory in deriving plasma viscosities in the long mean-free-path regime. First, in neoclassical transport theory, the plasma flow component $\mathbf{V}\cdot\nabla\theta/\mathbf{B}\cdot\nabla\theta$ is a function of the radial coordinate label ψ only.[4–6] Here $\mathbf{B}$ is the magnetic field and θ is the poloidal angle. Because of this assumption, plasma viscosities in neoclassical MHD equations are valid strictly for linear instabilities with vanishing radial mode width. Second, neoclassical flow is incompressible, $\nabla\cdot\mathbf{V}=0$. However, compressibility can have significant effects on plasma instabilities, such as ballooning modes.[7] Thus the neoclassical MHD equations cannot treat plasma compressibility correctly. Also, because of its neglect of terms related to $\nabla\cdot\mathbf{V}$, the neoclassical plasma viscosity cannot be connected smoothly to its counterpart in the collisional fluid regime. Third, the derivation of the neoclassical plasma viscosity is based on the existence of magnetic flux surfaces, usually in the flux-surface-averaged form. Even though attempts have been made to uncover their unaveraged expressions, they are qualitatively inaccurate because the variations of $\mathbf{V}$ in poloidal angle θ and toroidal angle ζ are not included.[3] This defect also leads to inaccurate coupling of the parallel momentum equation and the continuity equation. Fourth, neoclassical flow is on the flux surface, and there is no flow component perpendicular to the magnetic surface to the lowest order in the gyroradius expansion. It has been shown that when the neoclassical plasma viscosity is employed to calculate plasma instabilities, it is singular on the magnetic axis for the $m=1$ mode. (Here, m is the poloidal mode number.) The singularity occurs because the flow component perpendicular to the magnetic surface is inappropriately neglected.[8] An *ad hoc* expression for the plasma viscosity to include the flow component perpendicular to the magnetic surface was employed in the neoclassical MHD equations. One of the main purposes of this paper is to derive plasma viscosities in the long mean-free-path regime without invoking the assumptions of neoclassical transport theory, thereby removing these difficulties.

In this paper the plasma viscosity is derived in the plateau regime valid for arbitrary toroidal configurations. Here we derive viscosity only in the form used by Braginskii. Extension of the plasma viscosity to include the effects of heat flow, finite mode frequency, plasma rotation, and breaking of magnetic surfaces and application of the obtained viscosity to the reduced MHD equations will be discussed in subsequent papers. The remainder of the paper is organized as follows. In Sec. II, we obtain a drift kinetic equation with flows driven by the velocity stress. The equation is used in

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Sec. III to calculate the pressure anisotropy and viscous stress tensor in the plateau regime for arbitrary toroidal magnetic configurations. In Sec. IV, a smoothly connected formula for the pressure anisotropy that is valid in both the collisional fluid regime and the plateau regime is given. Concluding remarks are given in Sec. V.

II. DRIFT KINETIC EQUATION WITH FLOWS AND EXPANSION SCHEMES

Because the time scale of interest is much longer than the gyroperiod, we can employ a drift kinetic equation for our purpose. To calculate plasma viscosity, it is more convenient to solve a drift kinetic equation with flows in which the velocity stress tensor appears explicitly. We choose the equation derived by Hazeltine and Ware,[9]

$$\frac{\partial f}{\partial t} + (u\hat{n} + \mathbf{v}_d + \mathbf{V})\cdot\nabla f + \dot{w}\frac{\partial f}{\partial w} = C(f), \qquad (1)$$

for the particle distribution function f. The independent variables of Eq. (1) are $(\mathbf{x},t,\mu,w)$, where $w = v^2/2$, $\mu = s^2/2B$, with s the perpendicular (to the magnetic field) speed and v the particle speed. Both w and s are defined with respect to the center of mass velocity $\mathbf{V}$. The collision operator is denoted by $C(f)$. The parallel (to the magnetic field line) particle speed is u and $\hat{n} \equiv \mathbf{B}/B$, with $\mathbf{B}$ the magnetic field strength. The drift velocity $\mathbf{v}_d$ is defined as

$$\mathbf{v}_d = \frac{\mathbf{F}\times\hat{n}}{\Omega} + \frac{\mu B\hat{n}}{\Omega}\left(\frac{j_\parallel}{B}\right) + \frac{\hat{n}}{\Omega} \times[\mu\nabla B + u\hat{n}\cdot\nabla\mathbf{V} + \mathbf{V}\cdot\nabla(u\hat{n}) + u^2\hat{n}\cdot\nabla\hat{n}], \qquad (2)$$

where $j_\parallel$ is the parallel current density, Ω is the gyrofrequency eB/M, and the force $\mathbf{F}$ is

$$\mathbf{F} = \frac{e}{M}(\mathbf{E} + \mathbf{V}\times\mathbf{B}) - \frac{\partial\mathbf{V}}{\partial t} - \mathbf{V}\cdot\nabla\mathbf{V}. \qquad (3)$$

Here, M is the mass and e is the electric charge of the plasma species. The complete expression for $\dot{w}$ is very complicated. We display only those terms of interest for the present calculation:

$$\dot{w} = \mathbf{F}\cdot\mathbf{u} - \mu B\nabla\cdot\mathbf{V} - (u^2 - \mu B) \times(\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V}) + \mathbf{v}_d\cdot\mathbf{F} - (\mu B/\Omega)\hat{n}\cdot\nabla\times\mathbf{F}, \qquad (4)$$

and neglect those terms that involve components of the velocity stress tensor other than $\nabla\cdot\mathbf{V}$ and $\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V}$ and are a factor of ρ/L smaller. Here, ρ is the gyroradius. We note, however, those neglected terms can be trivially included using the method developed in this paper. The results, of course, will be more complicated than those presented here.

We interpret the center of mass velocity $\mathbf{V}$ as the mass flow velocity for each plasma species instead of the common mass flow velocity originally given in Ref. 9. We find that this interpretation does not change the form of the equation. The subscript j for plasma species is dropped in Eqs. (1)–(4) for simplicity. We also assume that the magnetic moment μ is conserved so that we can neglect the $d\mu/dt$ term in Eq. (1).

We express the solution of Eq. (1) as

$$f = f_{MS} + g. \qquad (5)$$

Here, f_{MS} is the Maxwellian distribution function in the frame of the center of mass defined in terms of w and g is a function to be determined from Eq. (1). Before we derive an equation for g, we discuss the implicit assumption involved in the expansion scheme shown in Eq. (5). Because we are going to employ the density and temperature evolution equations[2] in deriving an equation for g, g should not contribute to any density and temperature moments. Furthermore, we note that the flow velocity $\mathbf{V}$ is also completely determined by the Maxwellian distribution function f_{MS} defined in the moving frame with velocity $\mathbf{V}$, so g should not contribute to the velocity moment. However, the function g can be allowed to contribute to the heat flow. These constraints are easily satisfied in the collisional fluid regime. However, in the long mean-free-path regime, these constraints are difficult to satisfy. Instead of finding a complicated projection operator to meet these requirements, we explore the localization property of the function g in phase space, so that these constraints are approximately satisfied. Similar arguments are also employed in calculating the neoclassical plasma viscosity and neoclassical quasilinear transport fluxes.[10,11] The results of calculations based on this method agree with the results calculated from other methods.[3,12]

For simplicity, we neglect the effects of the temperature gradient. Substituting Eq. (5) into Eq. (1), employing the density and temperature evolution equations[2] in evaluating $\partial f_{MS}/\partial t$, and also noting that the force $\mathbf{F}$ is simply

$$\mathbf{F} = \nabla P/NM - \mathbf{R}/NM \qquad (6)$$

from the momentum balance equation, we obtain an equation for g,

$$\frac{\partial g}{\partial t} + (u\hat{n} + \mathbf{v}_d + \mathbf{V})\cdot\nabla g + \dot{w}\frac{\partial g}{\partial w} - C(g) = 2\frac{v^2}{v_t^2}\left(\frac{1}{2} - \frac{3}{2}\frac{u^2}{v^2}\right)\left(\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V} - \frac{1}{3}\nabla\cdot\mathbf{V}\right)f_{MS} - u\frac{\hat{n}\cdot\mathbf{R}}{P}f_{MS} + C(f_{MS}), \qquad (7)$$

where the thermal velocity $v_t = \sqrt{2T/M}$ with T the plasma temperature. In obtaining Eq. (6), we keep only the pressure gradient and the friction force $\mathbf{R}$ terms and neglect the viscous force. The terms on the right side of Eq. (7) are very similar to the driving terms of the kinetic equation employed by Braginskii in calculating various transport coefficients in the collisional fluid regime. Because $\mathbf{V}$ is not assumed to be the same for all plasma species, $C(f_{MS})$ is not zero. To calculate the plasma viscosity, we can neglect those terms that are proportional to u. Furthermore, if we neglect the effects of heat flow, $-u(\hat{n}\cdot\mathbf{R})f_{MS}/P + C(f_{MS}) = 0$.

We also note that because we do not assume the existence of the magnetic surface, there are terms associated with $\mathbf{B}\cdot\nabla\psi\neq 0$. The contributions of these terms to plasma viscosity are neglected here and will be discussed in a separate paper.

It is interesting to note that there is no conventional $\hat{n}\times\nabla B\cdot\nabla P$ term in Eq. (7). This is because such a term from $\mathbf{v}_d\cdot\nabla f_{MS}$ is canceled by an identical term from $\dot{w}\,\partial f_{MS}/\partial w$. The effects of neoclassical transport resulting from the banana orbits are now implicitly incorporated in the velocity stress $\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V}$. If we neglect the effects of the temperature gradient, we can actually calculate neoclassical transport fluxes without knowing that particles are drifting off the flux surface because of ∇B and curvature drifts! For this reason, the viscosities derived in Ref. 2 can be employed to describe poloidal momentum damping in the collisional regime of neoclassical transport theory.

III. PRESSURE ANISOTROPY AND VISCOUS STRESS TENSOR IN THE PLATEAU REGIME

We can use Eq. (7) to calculate plasma viscosity in arbitrary collisionality regimes. However, the degree of difficulty increases as the collisionality decreases. If one wants to derive an expression of plasma viscosity in the low collisionality regime valid in the nonlinear case for tokamaks, many symmetry-breaking effects associated with turbulent fluctuations become important. The qualitative features of plasma viscosity in this case are different from those of the tokamak neoclassical viscosity and are similar to those of the stellarator neoclassical viscosity. We calculate the pressure anisotropy and the viscous stress tensor in the plateau regime, which is defined as $(v_t|m-nq|\epsilon_{mn}^{3/2}/Rq) < \nu < (v_t|m-nq|/Rq)$. Here, ν is the typical collision frequency, R is the major radius, q is the safety factor, n is the toroidal mode number, and ϵ_{mn} is the dimensionless mode amplitude of the (m,n) mode. The mode can result from either the variations of the equilibrium magnetic and electric fields or the turbulent fluctuations.

For simplicity, we calculate the pressure anisotropy and the viscous stress tensor for low-frequency fluctuations; namely, $\omega \ll v_t|m-nq|/Rq$. The plasma flow velocity $\mathbf{V}$ is assumed to be smaller than v_t. We also assume that the variations of plasma pressure and $\mathbf{V}$ in θ and ζ are of the order of ϵ_{mn}. With these assumptions, we simplify Eq. (7) to

$$u\hat{n}\cdot\nabla g - C(g) = (v^2/v_t^2)(\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V} - \tfrac{1}{3}\nabla\cdot\mathbf{V})f_{\mathrm{MS}} \tag{8}$$

by changing the independent variables from (w,μ) to (v,u) and neglecting the mirror force term. Because the dominant contribution to plasma viscosity in phase space is from those particles with $u\simeq 0$, the u^2/v^2 term is also neglected in obtaining Eq. (8). We expand the velocity stress $(\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V} - \nabla\cdot\mathbf{V}/3)$ in terms of the Fourier sine and cosine series as

$$\begin{aligned}\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V} - \tfrac{1}{3}\nabla\cdot\mathbf{V} = &\sum_{m,n,l,k}[D^{s}_{mnlk;c}\cos(m\theta-n\zeta) \\ &+ D^{s}_{mnlk;s}\sin(m\theta-n\zeta)]\sin(l\theta-k\zeta) \\ &+ \sum_{m,n,l,k}[D^{c}_{mnlk;c}\cos(m\theta-n\zeta) \\ &+ D^{c}_{mnlk;s}\sin(m\theta-n\zeta)]\cos(l\theta-k\zeta),\end{aligned} \tag{9}$$

where mode numbers m and n are employed to characterize the fluctuations, and mode numbers l and k are employed to describe the variations of the equilibrium magnetic field. The Fourier amplitudes $D^{s}_{mnlk;c}$, $D^{s}_{mnlk;s}$, $D^{c}_{mnlk;c}$, and $D^{c}_{mnlk;s}$ can easily be obtained numerically for a given magnetic equilibrium. The variations of the quantities $\hat{n}\cdot\nabla\zeta$ and $\hat{n}\cdot\nabla\theta$ in θ and ζ are of the order of inverse aspect ratio $\epsilon_{lk} \ll 1$ and can be neglected. Employing the expansion of the velocity stress tensor given in Eq. (9), and adopting a Krook model for the collision operator $C(g) = -\nu g$ as is appropriate in the plateau regime, we can easily solve Eq. (8) with the standard procedure to obtain

$$\begin{aligned} g = \sum_{m,n,l,k}\{&g^{s+}_{m,n,l,k}\sin[(m+l)\theta-(n+k)\zeta] \\ &+ g^{s-}_{m,n,l,k}\sin[(m-l)\theta-(n-k)\zeta] \\ &+ g^{c+}_{m,n,l,k}\cos[(m+l)\theta-(n+k)\zeta] \\ &+ g^{c-}_{m,n,l,k}\cos[(m-l)\theta-(n-k)\zeta]\},\end{aligned} \tag{10}$$

where

$$\begin{aligned} g^{s+}_{m,n,l,k} = &\frac{\pi}{2}\frac{v}{v_t^2}\frac{\delta(u/v)}{|(m+l)\hat{n}\cdot\nabla\theta-(n+k)\hat{n}\cdot\nabla\zeta|} \\ &\times f_{\mathrm{MS}}(D^{s}_{mnlk;c}+D^{c}_{mnlk;s}),\end{aligned} \tag{11}$$

$$\begin{aligned} g^{c+}_{m,n,l,k} = &\frac{\pi}{2}\frac{v}{v_t^2}\frac{\delta(u/v)}{|(m+l)\hat{n}\cdot\nabla\theta-(n+k)\hat{n}\cdot\nabla\zeta|} \\ &\times f_{\mathrm{MS}}(D^{c}_{mnlk;c}-D^{s}_{mnlk;s}),\end{aligned} \tag{12}$$

$$\begin{aligned} g^{s-}_{m,n,l,k} = &\frac{\pi}{2}\frac{v}{v_t^2}\frac{\delta(u/v)}{|(m-l)\hat{n}\cdot\nabla\theta-(n-k)\hat{n}\cdot\nabla\zeta|} \\ &\times f_{\mathrm{MS}}(D^{c}_{mnlk;s}-D^{s}_{mnlk;c}),\end{aligned} \tag{13}$$

$$\begin{aligned} g^{c-}_{m,n,l,k} = &\frac{\pi}{2}\frac{v}{v_t^2}\frac{\delta(u/v)}{|(m-l)\hat{n}\cdot\nabla\theta-(n-k)\hat{n}\cdot\nabla\zeta|} \\ &\times f_{\mathrm{MS}}(D^{c}_{mnlk;s}+D^{c}_{mnlk;c}),\end{aligned} \tag{14}$$

and δ is the delta function. Only the resonant part of g is kept in Eqs. (10)–(14). The explicit collision frequency dependence is absorbed in the $\delta(u/v)$ function because of the resonant nature in the limit of $\nu < (v_t|m-nq|/Rq)$. We note that even in linear instabilities, where the amplitude of the mode is infinitesimally small, the effects of the finite parallel wavelength associated with the variations of flow velocity $\mathbf{V}$ in the θ and ζ directions are still important, as can be seen from Eqs. (11)–(14). These effects appear in the transit frequency $v_t|(m\pm l)\hat{n}\cdot\nabla\theta-(n\pm k)\hat{n}\cdot\nabla\zeta|$ for each $(m\pm l, n\pm k)$ mode. To reproduce the results of the neoclassical theory, we need only set $m\hat{n}\cdot\nabla\theta - n\hat{n}\cdot\nabla\zeta = 0$, or, in other words, $m = nq$. This implies that neoclassical plasma viscosity is valid only on the mode rational surface.

The pressure anisotropy, defined as

$$P_\parallel - P_\perp = \int d^3v\, fM\left(\frac{3}{2}u^2 - \frac{1}{2}v^2\right), \tag{15}$$

can easily be calculated by substituting Eqs. (10)–(14) into Eq. (15) to obtain

$$P_{\|} - P_{\perp} = -2\sqrt{\pi}\frac{P}{v_t}\left\{\frac{D^s_{mnlk;c} + D^c_{mnlk;s}}{|(m+l)\hat{n}\cdot\nabla\theta - (n+k)\hat{n}\cdot\nabla\zeta|}\sin[(m+l)\theta - (n+k)\zeta] + \frac{D^c_{mnlk;s} - D^s_{mnlk;c}}{|(m-l)\hat{n}\cdot\nabla\theta - (n-k)\hat{n}\cdot\nabla\zeta|}\right.$$
$$\times\sin[(m-l)\theta - (n-k)\zeta] + \frac{D^c_{mnlk;c} - D^s_{mnlk;s}}{|(m+l)\hat{n}\cdot\nabla\theta - (n+k)\hat{n}\cdot\nabla\zeta|}\cos[(m+l)\theta - (n+k)\zeta]$$
$$\left. + \frac{D^s_{mnlk;s} + D^c_{mnlk;c}}{|(m-l)\hat{n}\cdot\nabla\theta - (n-k)\hat{n}\cdot\nabla\zeta|}\cos[(m-l)\theta - (n-k)\zeta]\right\}. \tag{16}$$

It is instructive to express Eq. (9) in the form

$$\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V} - \frac{1}{3}\nabla\cdot\mathbf{V} = \frac{1}{2}\sum_{m,n,l,k}\{(D^s_{mnlk;c} + D^c_{mnlk;s})\sin[(m+l)\theta - (n+k)\zeta]$$
$$+ (D^c_{mnlk;s} - D^s_{mnlk;c})\sin[(m-l)\theta - (n-k)\zeta] + (D^c_{mnlk;c} - D^s_{mnlk;s})$$
$$\times\cos[(m+l)\theta - (n+k)\zeta] + (D^s_{mnlk;s} + D^c_{mnlk;c})\cos[(m-l)\theta - (n-k)\zeta]\}. \tag{17}$$

Comparing the expressions inside the large curly braces on the right side of Eq. (16) with those of Eq. (17), we notice that for every term in Eq. (17) there is a corresponding term in Eq. (16) that is divided by its own $k_{\|} = |(m \pm l)\hat{n}\cdot\nabla\theta - (n \pm k)\hat{n}\cdot\nabla\zeta|$. With this observation, we can easily calculate $P_{\|} - P_{\perp}$ and the viscous stress tensor numerically by simply finding the values of D for a given magnetic equilibrium. This viscous stress tensor $\overleftrightarrow{\pi}$ is simply

$$\overleftrightarrow{\pi} = (P_{\|} - P_{\perp})(\hat{n}\hat{n} - \tfrac{1}{3}\overleftrightarrow{I}), \tag{18}$$

where $\overleftrightarrow{I}$ is a unit tensor.

We note that in the derivation of the pressure anisotropy and viscous stress tensor, we do not assume the existence of the magnetic surface. All that is required is that any physical quantity can be described as a function of (ψ,θ,ζ). We emphasize again that ψ is the radial coordinate and is not necessarily a flux surface label. Because the variations of $\mathbf{V}$ in the toroidal and poloidal directions are included in the calculation, the local spatial variations of $P_{\|} - P_{\perp}$ and $\overleftrightarrow{\pi}$ are more accurately described than those of the neoclassical viscosities. Furthermore, we do not neglect any flow component and do not assume $\nabla\cdot\mathbf{V} = 0$ in obtaining $P_{\|} - P_{\perp}$ and $\overleftrightarrow{\pi}$. Therefore our results are more general than those of neoclassical theory.

IV. CONNECTION BETWEEN THE COLLISIONAL FLUID REGIME AND THE PLATEAU REGIME

The expression of the pressure anisotropy for the plateau regime, given in Eq. (16), can be connected to that in the collisional regime. We first obtain an alternative form of the velocity stress tensor $\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V} - \nabla\cdot\mathbf{V}/3$. It is not too difficult to show that[13]

$$\hat{n}\cdot n\cdot\nabla\mathbf{V} - \tfrac{1}{3}\nabla\cdot\mathbf{V} = \hat{n}\cdot\nabla(\mathbf{V}\cdot\hat{n}) - \mathbf{V}\cdot\hat{n}\cdot\nabla\hat{n} - \tfrac{1}{3}\nabla\cdot\mathbf{V}. \tag{19}$$

Noting that $\mathbf{V}\cdot\hat{n}\cdot\nabla\hat{n} = \mathbf{V}_{\perp}\cdot\nabla B/B$ for low-β plasmas (where β is the ratio of plasma pressure to magnetic field pressure) and that

$$\hat{n}\cdot\nabla(\mathbf{V}\cdot\hat{n}) = (\mathbf{V}\cdot\nabla B)/B - \mathbf{V}_{\perp}\cdot\nabla B/B - \nabla\cdot\mathbf{V}_{\perp} + \nabla\cdot\mathbf{V},$$

we can simplify Eq. (19) to

$$\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V} - \tfrac{1}{3}\nabla\cdot\mathbf{V} = (1/B)\mathbf{V}\cdot\nabla B - (1/B^2)\nabla\cdot(\mathbf{V}_{\perp}B^2) + \tfrac{2}{3}\nabla\cdot\mathbf{V}. \tag{20}$$

An explicit expression for $\nabla\cdot\mathbf{V}$ is given in Ref. 6. If we adopt the assumptions of neoclassical transport theory, we have $\nabla\cdot\mathbf{V} = 0$, $\mathbf{V}\cdot\nabla\psi = 0$, and $\nabla\cdot(\mathbf{V}_{\perp}B^2) = 0$, and we reproduce the known expression of the neoclassical velocity stress

$$\hat{n}\cdot\hat{n}\cdot\nabla\mathbf{V} - \frac{\nabla\mathbf{V}}{3} = \left(\mathbf{V}\cdot\nabla\theta\frac{\partial B}{\partial\theta} + \mathbf{V}\cdot\nabla\zeta\frac{\partial B}{\partial\zeta}\right)B^{-1}.$$

Comparing the expression in Eq. (20) with that for the neoclassical viscosity, we can determine the viscous coefficient in the collisional fluid regime. A simple form for the pressure anisotropy $P_{\|} - P_{\perp}$ in terms of the transit frequency and collision frequency, which is valid in the limits of both the collisional fluid regime and the plateau regime, can be obtained for an electron–proton plasma:

$$P_{\|} - P_{\perp} = -2\sqrt{\pi}P\left\{\frac{D^s_{mnlk;c} + D^c_{mnlk;s}}{v_t(m+l)\hat{n}\cdot\nabla\theta - (n+k)\hat{n}\cdot\nabla\zeta| + (\nu 2\sqrt{\pi}/3\mu)}\sin[(m+l)\theta - (n+k)\zeta]\right.$$
$$+ \frac{D^c_{mnlk;s} - D^s_{mnlk;c}}{v_t|(m-l)\hat{n}\cdot\nabla\theta - (n-k)\hat{n}\cdot\nabla\zeta| + (\nu 2\sqrt{\pi}/3\mu)}\sin[(m+l)\theta - (n-k)\zeta]$$
$$+ \frac{D^c_{mnlk;c} - D^s_{mnlk;s}}{v_t(m+l)\hat{n}\cdot\nabla\theta - (n+k)\hat{n}\cdot\nabla\zeta| + (\nu 2\sqrt{\pi}/3\mu)}\cos[(m+l)\theta - (n+k)\zeta]$$
$$\left. + \frac{D^s_{mnlk;s} + D^c_{mnlk;c}}{v_t|(m-l)\hat{n}\cdot\nabla\theta - (n-k)\hat{n}\cdot\nabla\zeta| + (\nu 2\sqrt{\pi}/3\mu)}\cos[(m-l)\theta - (n-k)\zeta]\right\}, \tag{21}$$

where $\mu = 1.365$ for protons, $\mu = 0.733$ for electrons, and ν is the self-collision frequency defined in Ref. 3.

V. CONCLUDING REMARKS

An expression for the pressure anisotropy and thus for the viscous stress in the plateau regime is derived for arbitrary toroidal magnetic configurations without assuming incompressibility or the existence of flux surfaces, without neglecting the flow components perpendicular to the magnetic surface, and without restricting the flow velocity to be a constant on the flux surface. It can be employed to study low-frequency instabilities[14–18] in the long mean-free-path regime. We also obtain a smoothly connected formula for the pressure anisotropy that is valid in both the collisional fluid regime and the plateau regime to facilitate numerical computations.

An alternative interpretation of the neoclassical transport theory is also obtained. We find that if the effects of temperature gradients are neglected, neoclassical transport fluxes can be interpreted as driven by the velocity stress.

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