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James Cullen and Earl Callen

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Potts models and the critical behavior of a cubic ferromagnet with fourth and sixth-order anisotropy

James Cullen
Naval Surface Weapons Center, White Oak, Silver Spring, Maryland 20910

Earl Callen
Naval Surface Weapons Center, White Oak, Silver Spring, Maryland 20910 and American University, Washington, D.C. 20016

The critical behavior of the magnetization of a cubic ferromagnet in a magnetic field is analyzed for all field directions when both fourth and sixth-order anisotropy are present. Results are presented for $K_1 > 0$ and all $\kappa(K_2/K_1)$. For $-6 < \kappa < 9$ the phase diagram is of the three-state Potts type. For $\kappa > 9$ a 60° rotation of the three-state diagram is obtained at high fields and a reentrant feature at lower fields. For $\kappa = 9$ a special critical point appears on the [111] axis. The connection between this result and an extended version of the Landau theory is made. Finally, the $\kappa \leqslant -6$ ([110] hard) diagram is displayed and the nature of the critical points, lines, and surfaces discussed.

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INTRODUCTION

The manner in which a cubic ferromagnet or ferrimagnet is magnetized by an applied field is rich in examples of phase transitions. Mukamel *et al.*[1] for example, analyzed the critical behavior of a cubic ferromagnet in a magnetic field when the [111] is the hard axis with a fourth-order term to represent the magnetic anisotropy and showed how this situation was a realization of the three-state Potts model. Since most cubic crystals have both K_1 and K_2, the energy of a crystal is

$$E = K_1\alpha_1^2\alpha_2^2 + \ldots + K_2\alpha_1^2\alpha_2^2\alpha_3^2 - M_s\alpha\mathrm{H}, \qquad (1)$$

where K_1 and K_2 are the fourth and sixth-order anisotropy constants, H the applied magnetic field, and M_s the saturation magnetization. We have investigated the critical behavior and phase diagrams for all values of K_1 and K_2 in H_x, H_y, H_z space. Prior work[2,3] contains discussion of the nature of the phases, but not on the nature of the critical behavior. There are qualitative differences in the structure of the phase diagrams as $\kappa = K_2/K_1$ is varied. In the following, two of the three main categories of diagram are discussed: [111] hard ($\infty \geqslant \kappa \geqslant -6, K_1 > 0$) and [110] hard ($-6 \geqslant \kappa \geqslant -\infty, K_1 > 0$). The first category contains the normal three-state Potts diagram for $\kappa > 9$ and a special diagram for $\kappa = 9$. The second category is further divided into $-9 \leqslant \kappa < -6$ and $\infty < \kappa < -9$ diagrams.

STRUCTURE OF THE PHASE DIAGRAM

It is possible, with a moderate amount of manipulation, to derive algebraic expressions for the positions of the second-order or Ising points, tricritical points and critical end points in H_x, H_y, H_z space. With these established it is possible to piece together the first-order sheets in the space and thus give entire pictures of the critical surfaces, lines, and points for any set of (K_1, K_2).The topology can be inferred from a knowledge of the critical behavior in the two planes (011) and (010). (The tricritical points and Ising lines always lie in these planes.) Details of the methods used will be presented elsewhere. We now present a brief description of the main results of the calculations. Restricting ourselves to the case $K_1 > 0$, it is convenient to discuss the cases [111] hard and [110] hard separately. We take [111] hard ($\infty \geqslant \kappa \geqslant -6$) first. The phase diagram for these κ values has the three-component Potts form. It is similar topologically to the $\kappa = 0$ diagram discussed by Mukamel *et al.*,[1] at least for all κ between 9 and -6. (See Fig. 1.) At $\kappa = -6$ the tricritical

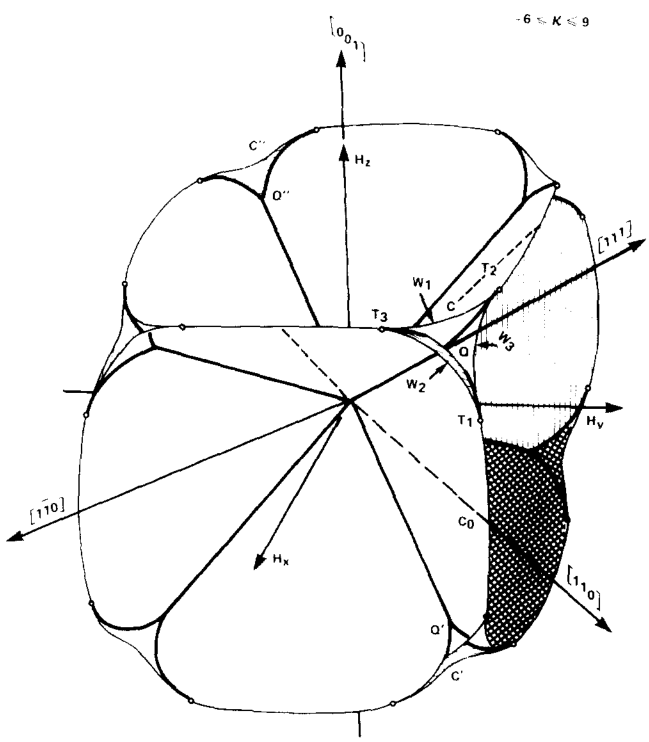

FIG. 1. Schematic phase diagram drawn for $-6 \leqslant \kappa \leqslant 9$. [After Mukamel, Fisher, and Domany, PRL **37** 565(1976).] The 111 are the hard axes. Critical lines, including "wings" W_1, W_2, and W_3 are essentially Ising in character. Three tricritical points T_1, T_2, T_3 are shown, as well as quadruple points. The latter are intersections of four lines of triple points (heavy solid lines). The tricritical points, wings, and Q point merge on the [111] axis as $\kappa \to 9$.

points reach the [110] axes, and the phase diagram changes qualitatively. The case $\kappa = 9$ deserves special mention. At $\kappa = 9$ the critical and tricritical points merge to produce a critical point on the [111] axis, with three second-order lines connecting this point with three [110] axes. The [111] axis is tangent to the second-order lines at this special critical (P) point. This last observation has a special meaning for the three-state Potts model. It is customary[1,4,5] to discuss the latter in terms of an expansion of the energy [Eq. (1)] in terms of moment orientation about the [111] axis:

$$E = \frac{1}{2} r\theta^2 + u\theta^4 + \frac{w}{3}\theta^3 \cos 3\phi + h_1 \theta \cos \phi. \qquad (2)$$

The parameters r, u, w, and h, are functions of $H/K_1,\kappa$. It turns out for the cubic ferromagnet, $w \rightarrow 0$ as $\kappa \rightarrow 9$. However, simply substituting $w = 0$ in the above expression will lead to an isotropic $x - y$ model for the transition, instead of the special three-state Potts behavior predicted by the global form of the mean-field theory described here. The discrepancy is removed if higher-order terms, e.g., $\theta^5 \sin 3\phi$ are retained in the energy expansion. In fact, when this is done, we find a line of second order transitions given by $h_1 \propto r^2$ in agreement with the global calculations. We have here another example of an "irrelevant" variable (the coefficient of the $\theta^5 \sin 3\phi$ term) playing a crucial role in determining the nature of the critical surface.[6] (In scaling language, the crossover exponent associated with this new variable is expected to be negative.) The effects of fluctuations on the P point and second-order lines are as yet unknown.

For $\kappa > 9$, the tricritical points reemerge in the $(1\bar{1}0)$ planes, on the sides of the [111] closest to [100] axes, so that the entire diagram, looked at from a point near [111] in the disordered phase looks to have been rotated by 60° about [111] from its $\kappa < 9$ counterpart. The second-order lines which emerge from the tricritical points, however, do not intersect the [100] axes but double back on themselves at lower fields to actually touch the [111] axis. There are also three second-order lines which connect the [110] axes to [111] at slightly higher fields (Fig. 2). Thus, in this region of κ, the phase diagram has a reemergent aspect: as a magnetic field, applied in the $(1\bar{1}0)$ plane between [111] and [100] is increased, the magnetization at first lies in a unique direction close to the field direction. At the lower critical field $\leqslant 2K_1/M_s$, the magnetization splits into two domains with equal and opposite magnetization components perpendicular to the plane. Finally, the two domains again coalesce at a higher field of the order of K_2/M_s. If the field direction is between [111] and [110], the magnet will undergo a transition from two [100]-like domains to the single-domain or disordered phase at $\geqslant 2K_1/M_s$. The lower-field transitions are all predicted to be second order: the upper-field one may be first order if the field orientation is close enough to the [111] axis to lie inside the tricritical point. In practice, close enough means field orientation less than one degree from [111]. Now consider [110] hard $(-\infty < \kappa < -6)$. [110] is a two-fold symmetry axis, and the phase diagram has an Ising aspect, consisting of a second-order line, ending at a tricritical point in the (001) plane. The position of the latter moves in the (001) plane from [110] at $\kappa = -6$ toward the [100] axis. A special case is shown in Fig. 3. In the limit $\kappa \rightarrow -\infty$, the tricritical point is 6 deg from the [100] axis.

DISCUSSION

It is natural to inquire into the physical accessibility of the several regions of κ discussed here. Low-temperature measurements in cubic RFe_2 compounds (R = heavy rare earth elements) give values of K_1 and K_2 which lie, depending on R, in all four quadrants of the K_1, K_2 plane.[7] In only one case ($TbFe_2$) can κ be taken to be negligible. Other examples of crystals with K_2 comparable to or greater than K_1 may be

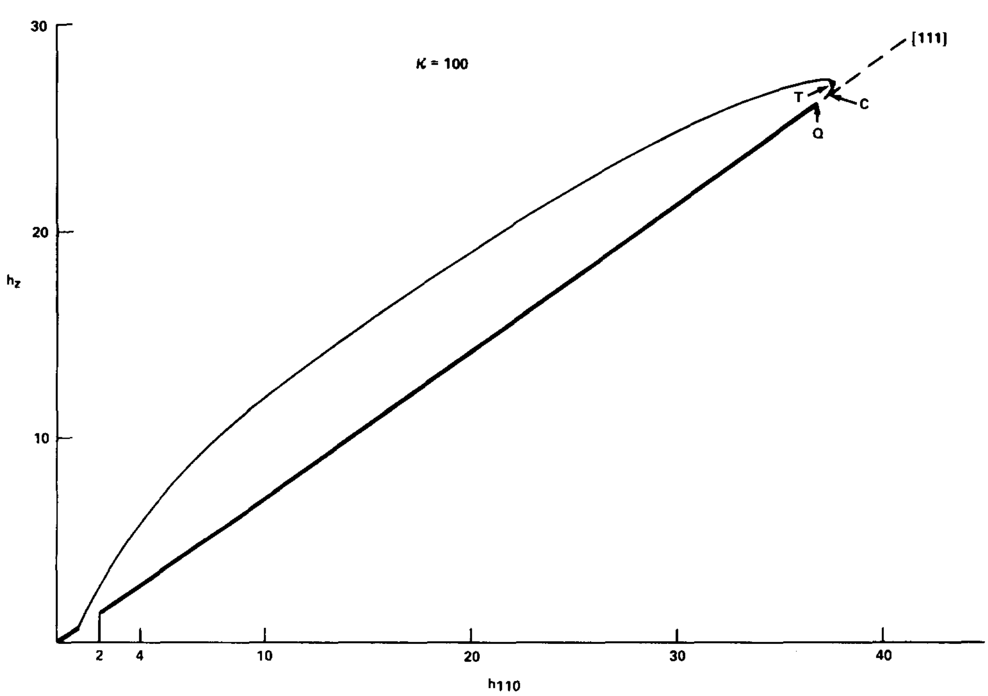

FIG. 2. Critical lines and points in the $(1\bar{1}0)$ plane (h_{110},h_z plane) for $\kappa = 100$. The shape of the Ising-like critical line for any $\kappa > 9$ is similar. They are all characterized by having reentrant behavior for fields lying near [111], between [001] and [111].

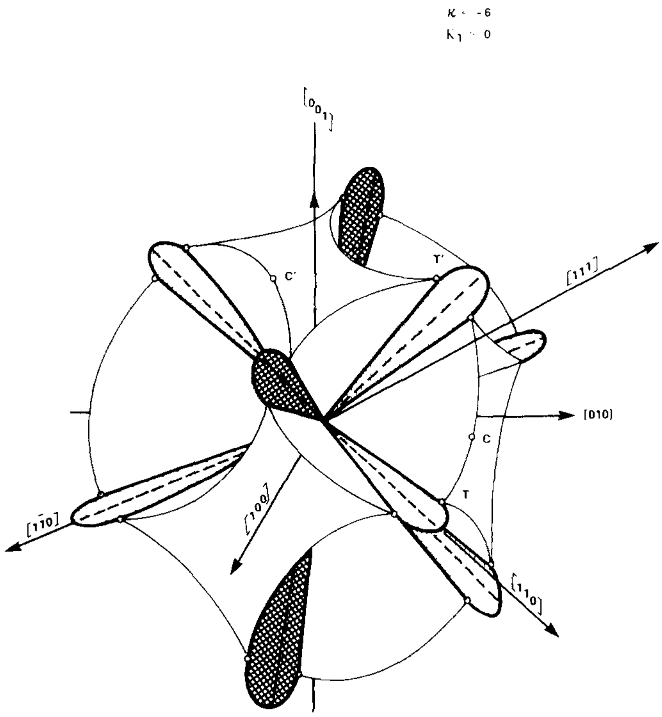

FIG. 3. "Canoe paddle" phase diagram for $\kappa = -9$. T is a tricritical point in the (001) plane, T' a tricritical point in (100). C and C' are critical end points on critical wings connecting the paddles. The 110 are the hard axes. Any two T points on one paddle are joined by Ising-like lines forming the tip of the paddle.

found in the ferrites. Because of the expected sharp decrease in κ with temperature, moreover, crystals with large $|\kappa|$ at low temperature can be chosen to have any desired smaller $|\kappa|$ by a suitable choice of temperature. Many aspects, therefore, of the diagrams predicted here should be verifiable. We have already mentioned the possibility of changes in the form of the $\kappa = 9$ phase diagram due to temperature fluctuations. Another case of special sensitivity to quantum or temperature fluctuations is that of large positive κ, where, at low fields [100] and [110] are almost equal in energy and there is only a very small energy barrier between these phases.

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