# On the possible nature of the electric activity of He II

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An attempt is made to explain the nature of the electric signals observed in helium II in a second-sound standing wave. It is shown on the basis of the general principles of quantum mechanics that as a result of interatomic interactions each helium atom acquires a small induced dipole moment $\mathbf{d}_{at}$, which pulsates in time and space. A directed flux of microscopic vortex rings is formed in a second-sound standing half-wave. This flux partially orders the chaotically oriented dipole moments of the atoms, which results in volume polarization of helium II. The observed potential difference $\Delta U \approx k_B \Delta T / 2e$ can be explained theoretically by assuming that each ring possesses a dipole moment $d_{vr}$ of the order of 10 times the average value of the modulus of the atomic moment, $d_{vr} \sim 10\bar{d}_{at}$. Analysis taking account of the boundary conditions also shows that a different possible nature of the observed electric signal $\Delta U$ is also not ruled out: this signal is associated not with the properties of He II but rather with a form of the thermal electromotive force. © *2008 American Institute of Physics.* [DOI: 10.1063/1.2911650]

## I. INTRODUCTION

In 2004 Rybalko discovered an unusual effect: he found that a standing half-wave of second sound induces in He II a potential difference $U$ which fluctuates in phase with the fluctuations of the temperature $T$.[1] It was established that in the temperature range $T = 1.4–2$ K the amplitudes $\Delta U/\Delta T \approx k_B/2e$ (the error in measuring $\Delta U/\Delta T \sim 10\%$ (Ref. 2)). The electric response effect is interesting because, in the first place, it is well-known that a free $^4$He atom in one of its stationary states does not possess a dipole moment (DM) and, in second place, helium II is a dielectric with low permittivity $\varepsilon_{He} \approx 1.057$ and $\varepsilon_{He} \approx n^2$ (Ref. 3) ($n$ is the refractive index). It follows from the proportionality $\varepsilon \approx n^2$ that the neighboring He IIatoms interacting with a He II also do not possess any appreciable characteristic DM.

Several models to explain this effect have already been proposed.[4–6] However, a completely satisfactory model has not yet been constructed. Specifically, the volume models neglected the boundary conditions for resonators in calculations of $\Delta U/\Delta T$.[4–6] We shall show that the result changes radically when these conditions are taken into account.

The analysis performed in Ref. 4 is not rigorous. The mechanism of polarization of He II by a second-sound wave is not entirely clear.

Melnikovsky proposed the idea of inertial polarization of helium atoms under acceleration.[5] However, the potential difference obtained in such an approach is two to three orders of magnitude smaller than the observed value and strongly temperature-dependent: $\Delta U/\Delta T \sim C(T)$, where $C$ is the specific heat of He II.

The work of Pashitskiĭ and Ryabchenko is interesting and stimulating.[6] However, its main result concerning the final value of the entropy $s_s$ of the superfluid component (SC), $s_s \approx 2k_B$, is debatable. Since this result touches upon the foundations of the theory of superfluidity we shall dis-

cuss it in more detail. In this work the following equation is obtained for He II:

$$\frac{1}{2}\left(\frac{1}{\rho}\nabla p - \frac{s}{m}\nabla T\right) = -\frac{2e}{m}\nabla\varphi, \tag{1}$$

where $\rho$ is the density of He II, $p$, $T$, and $\varphi$ are, respectively, the pressure, temperature, and electric-field potential, $m$ is the mass of a $^4$He atom, and $s$ is the entropy per atom ($s = mS/\rho$, where $S$ is the entropy per unit volume). Comparing Eq. (1) with the observed value $\Delta U/\Delta T \approx k_B/2e$ the authors of Ref. 6 concluded that the entropy $s$ of helium II contains an additional contribution, independent of temperature $T$, $s \approx 2T_B$, which was identified with $s_s$.

We note that such a value of $s_s$ is substantially higher than the entropy $s_n$ of the normal component (NC)—by a factor of 4 at $T = 1.8$ K and approximately 200 at $T = 1$ K. The relation (1) was derived in Ref. 6 using Landau's two-fluid hydrodynamics (TH). But the result $s_s \approx 2k_B$ contradicts this TH, which is constructed under the assumption $s_s = 0$. In time, the criterion for making a choice between Landau's and Tisza's TH became the relation for the velocity $u_2$ of the second sound. According to Tisza, as $T$ decreases in the range $T < 1$ K the value of $u_2$ should decreased to zero, and according to Landau it should increase to $u_1/\sqrt{3}$, according to the relation[7,8]

$$u_2 = \frac{s}{m}\sqrt{\frac{\rho\rho_s T}{\rho_n C(T)}}; \quad \rho = \rho_s + \rho_n. \tag{2}$$

Experiment has confirmed precisely this relation, and Landau's approach was acknowledged to be the correct one. But Tisza's model contradicted experiments at low temperatures. The relation (2) is valid, according to Landau, for $s = s_n$, but if $s = s_n + 2k_B$ is substituted into Eq. (2), as proposed in Ref. 6, then a contradiction arises with the experiment not only at low but also high temperatures $T$: at $T = 2$ K and $T = 1$ K $u_2$ is several-fold and several hundreds of times, respectively,

higher than the experimental value. A different TH agreeing with experiment for $s_s \approx 2k_B$ is not proposed in Ref. 6.

The equation (1) was derived in Ref. 6 in two different ways, and both are debatable. One is based on the idea that helium II can be regarded not as a collection of neutral atoms but rather as a charged electron–nuclear plasma. We do not believe that this proposal reflects the properties of He II at the microscopic level. After all, characteristic recombination lines should be observed in such a plasma, but as far as is known they are not observed experimentally. In addition, the microscopic models of He II[9–11] and the experimental curves for the interaction potentials of helium atoms[12] show that He II is a collection of interacting *individual* atoms approximately 2.6 Å in size, and the atoms are very elastic objects. The idea of an electron–nuclear plasma means that there are no isolated helium atoms as such in He II.

Another simplified derivation of Eq. (1) is based on the idea that the dipole moment of the helium atoms is a result of their acceleration.[5] The polarization **P** of helium II is associated with the acceleration of the SC as follows:

$$\mathbf{P} = -\gamma \frac{\partial \mathbf{v}_s}{\partial t}, \quad \gamma \approx \frac{\varepsilon - 1}{4\pi} \frac{m}{4e}. \tag{3}$$

Taking account of the relation between **P** and **E**, $[4\pi/(\varepsilon - 1)]\mathbf{P} = \mathbf{E} = -\nabla\varphi$, the equation (1) once again follows from the relations (3). The relation (3) assumes that the He II atoms move with acceleration $\partial \mathbf{v}_s / \partial t$. This relation was evidently written on the basis of the assumption, made in Ref. 6, that the SC is a composite condensate—a superposition of one- and two-particle and higher-order condensates, and correspondingly the NC is a collection of atoms which are not present in any condensate. Actually, this is a modern generalization of Tisza's model (in which the SC is identified with the single-particle condensate). In addition, Landau's TH is constructed completely differently—in this hydrodynamics the NC is not associated with real atoms but rather it describes a gas of quasi-particles: $\rho_n$ is introduced on the basis of the distribution and dispersion laws of quasi-particles, and the presence of condensates is not required at all. Consequently, it is not clear why $\rho_n$ should also be equal to the density of particles which are not present in condensates. In addition, it was found in Ref. 11 as a result of a calculation performed in the three-particle approximation that at $T=0$ only 22% of the atoms, and not 100% as assumed in Ref. 6, are in a composite condensate. In view of this we contend that at present there are no grounds for identifying the SC with a composite condensate and that the SC and NC are only effective liquids, i.e. the helium atoms are not divided into "superfluid" and "normal" (as ordinarily assumed[7,8]). In this case, the velocity **v** of the atoms, which is defined by the following relation, must be substituted for $\mathbf{v}_s$ in Eq. (3):

$$\mathbf{v} = \frac{\rho_s}{\rho}\mathbf{v}_s + \frac{\rho_n}{\rho}\mathbf{v}_n. \tag{4}$$

But then we obtain **P**=0, since $\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n = 0$ in a second-sound wave. For a more accurate approximation one must take account of the fact that the change in $T$ in a second-sound wave results in small but nonzero changes in $p$ and $\rho$.[7,8] This results in a small mass flux proportional to the thermal expansion coefficient $\alpha$: $j \approx \rho v_s \alpha u_2^2 m/s$. We note

that formula (20.21) of Ref. 7 which we used contains a misprint. It can be shown that the correct coefficient is

$$b_2 = \frac{\alpha \rho^2 u_1^2 u_2^3}{S(u_1^2 - u_2^2)}.$$

Such a flux results in polarization $\mathbf{P} = -(\gamma \alpha u_2^2 m/s)d\mathbf{v}_s/dt \approx -(\gamma \alpha u_2^2 m/s)\partial \mathbf{v}_s/\partial t$, and instead of Eq. (1) we arrive at the equation

$$\frac{1}{2}\left(\frac{\alpha u_2^2 m}{s\rho}\nabla p - \alpha u_2^2 \nabla T\right) = -\frac{2e}{m}\nabla \varphi. \tag{5}$$

It is evident that the entropy has dropped out of the coefficient multiplying $\nabla T$ and has been replaced by the quantity $s_{\text{eff}} = \alpha u_2^2 m$, which equals $(2.4 \cdot 10^{-4} - 2.5 \cdot 10^{-3})k_B$ in the temperature interval $T = 1.4 - 2$ K, according to the data for $\alpha$ from Ref. 8. In this approximation $\nabla p$ no longer vanishes in a second-sound wave, but it is still negligibly small, since $\nabla p \approx -\partial \mathbf{j}/\partial t \sim \alpha \ll 1$. We obtain from Eq. (5) (neglecting boundary conditions) $\Delta U \approx (10^{-4} - 10^{-3})k_B \Delta T/2e$. Thus, when density fluctuations in a second-sound wave are taken into account, an electric signal $\Delta U$ arises but its amplitude is 3–4 orders of magnitude smaller than the experimental value.

In Ref. 5 second sound was studied neglecting density fluctuations, i.e. for the case $\rho \mathbf{v} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n = 0$. The polarization an order of magnitude greater than we obtained above arises as a result of thermal fluctuations of $\rho$ and **v**. Evidently, it can be concluded from the formulas given in Ref. 5 that the fluctuations of $\rho$ and **v** in a second-sound wave result in a macroscopic accelerated flow of atoms $\rho \mathbf{v}_{\text{at}} \approx -C\rho_n(\dot{\mathbf{v}}_n - \dot{\mathbf{v}}_s)/3S$, which is what gives a nonzero polarization.

It is shown below that a dipole moment arises in He II atoms not only in a second-sound wave but also in any state of He II as a result of atoms interacting with their neighbors. The average value $\bar{d}_{\text{at}}$ of the modulus of such a moment is calculated and the resulting value of the electric signal $\Delta U$ is calculated approximately.

## II. INDUCED DIPOLE MOMENT OF He II ATOMS

Strictly speaking, the interaction of an atom with many neighboring atoms must be studied in order to calculate $\bar{d}_{\text{at}}$. It is difficult to solve such a multi-particle problem. Consequently, we shall first estimate the dipole moment $d_0$ arising in a helium atom interacting with only one neighbor. It will be evident that $d_0$ decreases rapidly (as $1/R^4$) with increasing distance $R$ between the atoms, so that to estimate $\bar{d}_{\text{at}}$ it is sufficient to take account of only the interaction between an atom and its nearest neighbors.

Let us consider two stationary interacting $^4$He atoms, A and B, separated by a distance $R$. Let **R** be the radius vector connecting the nuclei of the atoms A and B and directed toward the nucleus B. We shall find the wave function $\Psi_0^{AB}$ of the ground states of two interacting two-electron atoms. We assume their nuclei to be stationary, so that $\Psi_0^{AB}$ will describe the state of the electronic shells: $\Psi_0^{AB} = \Psi_0^{AB}(\mathbf{r}_1^A, \mathbf{r}_2^A, \mathbf{r}_1^B, \mathbf{r}_2^B, \mathbf{R})$ (we omit the spins for the time being). Here and below $\mathbf{r}_k^A$ and $\mathbf{r}_j^B$ are the radius vectors of the $k$th electron of atom A and the $j$th electron of atom B relative to their own nuclei.

We shall solve the problem using perturbation theory, similarly to the well-known van der Waals problem. We take the unperturbed states to be the states of the noninteracting atoms A and B. Then we find the function $\Psi_0^{AB}$ in the form of the following expansion:

$$\Psi_0^{AB} = c_0\Psi_0^A\Psi_0^B + c_1\Psi_1^A\Psi_0^B + c_1\Psi_0^A\Psi_1^B + c_2\Psi_1^A\Psi_1^B$$
$$+ c_3\Psi_0^A\Psi_2^B + c_3\Psi_2^A\Psi_0^B + c_4\Psi_1^A\Psi_2^B + c_4\Psi_2^A\Psi_1^B + \cdots$$
$$(6)$$

Here $\Psi_0$ is the $1s^2$ ground state of the $^4$He atom, $\Psi_1$ is the $1s2p$ excited state (in reality, $2p$ consists of three states characterized by the quantum numbers $l=1$, $m=0$, $\pm1$; as will become evident below, the contribution of only one state— the state with $l=1$, $m=0$—makes a significant contribution in the expansion (6); $\Psi_1$ is this state), and $\Psi_2$ is the $1s2s$ state.

The dipole moment of the atom A is given by the expression

$$\mathbf{d}_0^A = \int \Psi_0^{*AB} e(\mathbf{r}_1^A + \mathbf{r}_2^A)\Psi_0^{AB} d\mathbf{r}_1^A d\mathbf{r}_2^A d\mathbf{r}_1^B d\mathbf{r}_2^B. \quad (7)$$

It is easy to see that substituting the expansion (6) into this expression yields a series containing matrix elements of the form

$$\langle k|\mathbf{d}|j\rangle = \int \Psi_k^* e(\mathbf{r}_1 + \mathbf{r}_2)\Psi_j d\mathbf{r}_1 d\mathbf{r}_2. \quad (8)$$

According to the selection rules, they are different from zero only for $\Delta l=1$, $\Delta m=0$, 1 ($\Delta l$ and $\Delta m$ are the moduli of the difference of the values $l$ and $m$ for the states $\Psi_k$ and $\Psi_l$). Consequently, first and foremost, the average $\langle 1|\mathbf{d}|0\rangle$ will make a nonzero contribution to the dipole moment. The average $\langle 1s2p|\mathbf{d}|1s3d\rangle$ will also contribute to $d_0$, since here $\Delta l=1$ and $\Delta m=0$, 1, but this contribution is of the order of $c_k c_j$ (where $k$, $j\geq1$), and since $|c_{k\geq1}|\ll1$, this and higher-order contributions can be neglected. Consequently, it is sufficient to retain the first few corrections, taking account of $\Psi_1$, in the expansion (6):

$$\Psi_0^{AB} = c_0\Psi_0^A\Psi_0^B + c_1\Psi_1^A\Psi_0^B + c_1\Psi_0^A\Psi_1^B + c_2\Psi_1^A\Psi_1^B. \quad (9)$$

The perturbing potential $\hat{U}$ is the difference between the total Hamiltonian $\hat{H}^{AB}$ of two interacting atoms and the Hamiltonians $\hat{H}^A$ and $\hat{H}^B$ of the free atoms:

$$\hat{U} = \frac{Z^2 e^2}{|\mathbf{R}^A - \mathbf{R}^B|} = \sum_{i,j=1}^{Z} \frac{e^2}{|\mathbf{R}^A + \mathbf{r}_i^A - \mathbf{R}^B - \mathbf{r}_j^B|}$$
$$- \sum_{i=1}^{Z} \frac{Ze^2}{|\mathbf{R}^A + \mathbf{r}_i^A - \mathbf{R}^B|} - \sum_{j=1}^{Z} \frac{Ze^2}{|\mathbf{R}^A + \mathbf{R}^B - \mathbf{r}_j^B|}, \quad (10)$$

where $Z=2$, and $\mathbf{R}^A$ and $\mathbf{R}^B$ are the coordinates of the nuclei. As one can see from the expression (10), $\hat{U}$ is the sum of four potentials—the interaction of the nuclei, the electrons of atom A with the electrons of atom B, the electrons of atom A with the nucleus of atom B, and the electrons of atom B with the nucleus of atom A. It is convenient to write $\hat{U}$ (10) in the form proposed in Ref. 13:

$$\hat{U} = \hat{Q}_A^+\hat{Q}_B\frac{1}{R}, \quad \mathbf{R} = \mathbf{R}^B - \mathbf{R}^A, \quad (11)$$

where $\hat{Q}$ is the charge operator

$$\hat{Q}_A = e\sum_{j=1}^{z} (\exp(\mathbf{r}_j^A \nabla) - 1),$$

$$\hat{Q}_A^+ = e\sum_{j=1}^{z} (\exp(- \mathbf{r}_j^A \nabla) - 1), \quad (12)$$

where here and below $\nabla \equiv \nabla_\mathbf{R} = \partial/\partial\mathbf{R}$. Expanding the exponential in the expressions (12) in a series gives

$$\hat{Q} = \hat{Q}_d + \hat{Q}_q + \cdots,$$

$$\hat{Q}_d = \mathbf{d}\nabla, \quad \hat{Q}_q = \sum_{\alpha,\beta=1}^{3} Q_{\alpha\beta}\frac{\partial}{\partial R_\alpha}\frac{\partial}{\partial R_\beta}, \quad (13)$$

where $\mathbf{d}=e\sum_{j=1}^{z}\mathbf{r}_j$ is the dipole moment operator, and

$$Q_{\alpha\beta} = 0{,}5e\sum_{j=1}^{z} x_j^\alpha x_j^\beta$$

is related with the quadrupole moment. We neglect the higher-order $l$-pole moments ($l\geq6$) in Eq. (13).

We shall calculate the dipole moment $\mathbf{d}_0$ of atom A according to the general formula (7). For this, it is necessary to know the coefficients $c_1$ and $c_2$ in Eq. (9). To first-order in perturbation theory[13,14]

$$c_j = \frac{U_{j0}}{E_0^{(0)} - E_j^{(0)}}, \quad (14)$$

for our case

$$E_0^{(0)} = 2E_0, \quad E_1^{(0)} = E_0 + E_1, \quad E_2^{(0)} = 2E_1, \quad (15)$$

where $E_0\equiv E(1s^2)$ and $E_1=E(1s2p)$. In Eqs. (15) $E_0^{(0)}$ is twice the energy of the $^4$He atom in the $1s^2$ ground state and $E_1^{(0)}$ is the energy of two free $^4$He atoms; in addition, one atom is in the $1s^2$ state and the other is in the $1s2p$ state; $E_2^{(0)}$ is the energy of two free atoms each of which is in the $1s2p$ state. We now write the matrix elements of the operator (10):

$$U_{10} = \int \Psi_1^{*A}\Psi_0^{*B}\hat{U}\Psi_0^A\Psi_0^B d\mathbf{r}_1^A d\mathbf{r}_2^A d\mathbf{r}_1^B d\mathbf{r}_2^B, \quad (16)$$

$$U_{20} = \int \Psi_1^{*A}\Psi_1^{*B}\hat{U}\Psi_0^A\Psi_0^B d\mathbf{r}_1^A d\mathbf{r}_2^A d\mathbf{r}_1^B d\mathbf{r}_2^B. \quad (17)$$

In the expressions (14)–(17) it is necessary to know $\Psi_0$, $\Psi_1$, $E_0$, and $E_1$—the wave functions and energy of a $^4$He atom in the states $1s^2$ and $1s2p$. These quantities are not known exactly, but analysis shows that the two-electron $^4$He atom is described well by the formulas found for $\Psi_0$, $E_0$, $\Psi_1$, and $E_1$ in the approximation of noninteracting electrons but with a renormalized nuclear charge $Z^*$ (this renormalization takes account of the screening of the nuclear charge by the other electron).[13–15] Specifically, it is found that $Z^*=Z-5/16$ for $\Psi_0$; we shall use the same $Z^*$ for $\Psi_1$ also. Then we have[13,14]

$$\Psi_0(\mathbf{r}_1, \mathbf{r}_2, s_1, s_2) = \varphi_{1s}(\mathbf{r}_1)\varphi_{1s}(\mathbf{r}_2)\chi^a(s_1, s_2).$$

$$\varphi_{1s}(\mathbf{r}) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}, \tag{18}$$

$$\Psi_1(\mathbf{r}_1, \mathbf{r}_2, s_1, s_2) = \frac{1}{\sqrt{2}}[\varphi_{1s}(\mathbf{r}_1)\varphi_{2p}(\mathbf{r}_2)$$
$$+ \varphi_{1s}(\mathbf{r}_2)\varphi_{2p}(\mathbf{r}_1)]\chi^a(s_1, s_2), \tag{19}$$

$$\varphi_{2p}^{m=\pm 1}(\mathbf{r}) = \mp R_{21}\left(\frac{r}{a}\right)\sqrt{\frac{3}{8\pi}}e^{\pm i\phi}\sin\Theta, \tag{20}$$

$$\varphi_{2p}^{m=0}(\mathbf{r}) = R_{21}\left(\frac{r}{a}\right)\sqrt{\frac{3}{4\pi}}\cos\Theta, \quad R_{21}(r) = \frac{1}{\sqrt{6a^3}}\frac{r}{2}e^{-r/2}; $$

$$E_0 = -78.6 \text{ eV}; \ E_1 = -57.5 \text{ eV} \tag{21}$$

Here $a = a_B/Z^* = 0.313$ Å and $a_B = \hbar^2/(me^2) = 0.529$ Å is the Bohr radius. According to Ref. 15 (§27 and §35), our formulas are sufficiently accurate for the $1s^2$ state and less accurate for the $1s2p$ state, but they are acceptable for an approximate calculation.

Since the system of two atoms is invariant under rotation around $\mathbf{R}$, the function $\varphi_{2p}$ must have the same symmetry. Hence, the $z$ axis of the atom is directed along $\mathbf{R}$, and in Eq. (19) $\Theta$ is the angle between $\mathbf{R}$ and $\mathbf{r}$. In this case the problem becomes one-dimensional with respect to $\mathbf{R}: \nabla \equiv \partial/\partial\mathbf{R} = \mathbf{i}_z \partial/\partial R$, and we find from Eq. (13)

$$\hat{Q} = d_z \frac{\partial}{\partial R} + Q_{zz}\frac{\partial^2}{\partial R^2}. \tag{22}$$

We now take account of the electron spins. In Eq. (7) the summation over $s_1^A$, $s_2^A$, $s_1^B$, and $s_2^B$—all spin states of the electrons—is omitted. A $^4$He atom possesses two electrons, and the spins of these electrons can be parallel (the ortho state $\chi^s(s_1, s_2)$) or anti-parallel (the para state $\chi^a(s_1, s_2)$). The $1s^2$ ground state of the helium atom is a para state. Neglecting the spin-orbit interaction the states $\chi^a(s_1, s_2)$ and $\chi^s(s_1, s_2)$ are orthogonal. The $1s2p$ state of the helium atom can be para or ortho. But, for us, the matrix element $\langle 1|\mathbf{d}|0\rangle \equiv \langle 1s2p|\mathbf{d}|1s^2\rangle$ will be important, and since it is zero for the $1s2p$ ortho state, only the $1s2p$ para state is important, which is reflected in Eq. (19). The summation over spins in Eq. (7) gives 1, so that to simplify matters the spins are omitted in the formulas.

We shall now calculate $c_1$ and $c_2$ according to Eqs. (14)–(17), using first the dipole approximation for $\hat{Q}$: $\hat{Q} = \mathbf{d}\cdot\nabla$, to which calculations are ordinarily limited. Since $\mathbf{d}\cdot\nabla = d_z\partial/\partial R$, after simple calculations we find

$$c_1 = 0, \quad c_2 = 1.43\frac{a_B a^2}{R^2} \approx 0.0016 \tag{23}$$

We note that

$$U_{20} = ((\langle 1^A|-d_z^A|0^A\rangle\,\partial/\partial R)(\langle 1^B|d_z^B|0^B\rangle\,\partial/\partial R)R^{-1},$$

where $d_z = e(z_1 + z_2)$. As a result we arrive at the expression

$$\Psi_0^{AB} = c_0\Psi_0^A\Psi_0^B + c_2\Psi_1^A\Psi_1^B. \tag{24}$$

The state (24) is a superposition of two states of a pair of atoms, and in each state of the pair the atoms are in the same

state ($1s^2$ or $1s2p$). Since the $^4$He atom in the states $1s^2$ and $1s2p$ does not possess a dipole moment, the atoms A and B in the pair state (24) do not possess a dipole moment individually. This is confirmed by a direct calculation of $d_0$ (7) using the wave function (24).

We shall now calculate $c_1$ in the quadrupole approximation (13) for $\hat{Q}$ using the expression (22). In this case

$$U_{10} = \langle 1^A|\hat{Q}_{A,d}^+ + \hat{Q}_{A,q}^+|0^A\rangle\langle 0^B|\hat{Q}_{B,d} + \hat{Q}_{B,q}|0^B\rangle R^{-1}$$
$$= \langle 1^A|\hat{Q}_{A,d}^+|0^A\rangle\langle 0^B|\hat{Q}_{B,q}|0^B\rangle R^{-1},$$

since $\langle 1^A|\hat{Q}_{A,q}^+|0^A\rangle = \langle 0^B|\hat{Q}_{B,d}|0^B\rangle = 0$. A simple calculation gives

$$c_1 = -8.15\frac{a_B a^3}{R^4} \approx -7.9 \cdot 10^{-4}. \tag{25}$$

Since the $z$ axis is oriented along $\mathbf{R}$, the calculation is greatly simplified: only the state $1s2p$ with $m=0$ contributes to $c_1$ and $c_2$, and the state $1s2p$ with $m=\pm 1$ does not contribute. This is taken into account in the expansions (6) and (9).

Now $\Psi_0^{AB}$ is determined by the relation (9), where $c_1 \approx -7.9\cdot 10^{-4}$ and $c_2 \approx 0.0016$. Evidently, $|c_{j\geq 1}| \ll 1$, so that we set $c_0 = 1$ in Eq. (9). We would have obtained $c_0 = 1-\tilde{c}$, where $\tilde{c} \sim |c_j| \ll 1$, from renormalization condition; taking account of $\tilde{c}$ in $c_0$ gives a negligibly small correction $\sim c_{j\geq 1}^2$ to the DM.

If the correction $c_2\Psi_1^A\Psi_1^B$, which has only a negligible effect on $d_0$, is neglected in the Eq. (9), then the relation (9) can be interpreted as evidence for the helium atom passing into the following state as a result of the interaction:

$$\tilde{\Psi}_0 = \Psi_0 + c_1\Psi_1 = \varphi_{1s}(\mathbf{r}_1)\varphi_{1s}(\mathbf{r}_2) + \frac{c_1}{\sqrt{2}}[\varphi_{1s}(\mathbf{r}_1)\varphi_{2p}(\mathbf{r}_2)$$
$$+ \varphi_{1s}(\mathbf{r}_2)\varphi_{2p}(\mathbf{r}_1)] \approx \tilde{\varphi}_{1s}(\mathbf{r}_1)\tilde{\varphi}_{1s}(\mathbf{r}_2), \tag{26}$$

where $\varphi_{2p}$ is the state with $m=0$, and

$$\tilde{\varphi}_{1s}(\mathbf{r}) = \varphi_{1s}(\mathbf{r}) + \frac{c_1}{\sqrt{2}}\varphi_{2p}(\mathbf{r}). \tag{27}$$

We shall now find the form of the electron shell for the wave function $\tilde{\varphi}_{1s}(\mathbf{r})$. The function $\varphi_{1s}(\mathbf{r})$ is spherically symmetric. The spherical symmetry of $\tilde{\varphi}_{1s}(\mathbf{r})$ is somewhat distorted. To determine how much we shall find how a sphere $r=ka$ with the same values of $|\varphi_{1s}(\mathbf{r})|^2$ deforms with a transition to $\tilde{\varphi}_{1s}(\mathbf{r})$. This can be determined from the equation $|\tilde{\varphi}_{1s}(\mathbf{r})|^2 = |\varphi_{1s}(ka)|^2 = e^{-2k}/(\pi a^3)$. Setting $r = a\tilde{r}(\theta)$ in $\tilde{\varphi}_{1s}$ and taking account of the fact that $c_1$ is small, we find in the linear approximation in $c_1$ the following equation for $\tilde{r}(\theta)$: $\tilde{r} \approx k + c_1 k\cos\theta e^{k/2}/8$, or

$$\tilde{x}^2 + \tilde{y}^2 + (\tilde{z} - kc_1 e^{k/2}/8)^2 = k^2. \tag{28}$$

It is evident from Eq. (28) that for $\tilde{\varphi}_{1s}(\mathbf{r})$ the surface of equal values of $|\tilde{\varphi}_{1s}(\mathbf{r})|^2 = e^{-2k}/(\pi a^3)$ is also a sphere with radius $r = ka$, but this sphere is shifted by the vector $\delta_1 = -\mathbf{i}_z ka|c_1|e^{k/2}/8$, which depends on $k$. The quantity $\delta_1$ rapidly increases away from the nucleus ($k$ increases), but $|\tilde{\varphi}_{1s}(\mathbf{r})|^2$ decreases even more rapidly ($\sim e^{-2k}$).

As one can see from Eqs. (26) and (28), both electrons in a $^4$He atom are in the state $\tilde{\varphi}_{1s}$, where the electron cloud is

extended in the direction away from the neighboring atom. As a result, the atom acquires a DM, which by definition is

$$\mathbf{d}_0 = \int \tilde{\Psi}_0^* e(\mathbf{r}_1 + \mathbf{r}_2)\tilde{\Psi}_0 d\mathbf{r}_1 d\mathbf{r}_2. \tag{29}$$

Substituting into Eq. (29) the explicit form of the function $\tilde{\Psi}_0$, we find

$$\mathbf{d}_0 = \int \tilde{\varphi}_{1s}^*(\mathbf{r})2e\mathbf{r}\tilde{\varphi}_{1s}(\mathbf{r})d\mathbf{r} \approx 2,1eac_1\frac{\mathbf{R}}{R} = -2e\delta\frac{\mathbf{R}}{R},$$

$$\delta = 2.63 \cdot 10^{-4} \text{ Å}. \tag{30}$$

An exact calculation of $d_0$ using the expressions (7) and (9) gives a value which is $1+c_2$ times greater. Thus the interaction with another atom induces in the initial atom a small DM (30) whose positive end is directed toward the second atom. Since a center of inversion is present, the vector sum of the moments of these two atoms is zero.

It is interesting that the value of the displacement $\delta_1$ found above is equal to $\delta = 1.05a|c_1|$ (30) for $k=2, 4$, i.e. this is the displacement of a sphere with radius $r \approx 2.4a$ containing almost the entire electron cloud ($\sim 98\%$ of the probability density). This value of $k$ is physically entirely reasonable. A sphere with radius $r=a$ contains 80% of the probability density and shifts by an amount that is five times smaller, $\delta_1 = 0.2a|c_1|$.

Each atom in He II is surrounded by approximately six nearest and many more distant neighbors. The average distance between He II atoms is $\bar{R} \simeq 3.6$ Å. It follows from the calculation presented above that $d_0 \sim c_1 a \sim (a/R)^4$. Since for neighboring atoms $R \simeq \bar{R}$, the quantity $a/R \approx 1/12$ can be regarded as a small parameter of the problem. Its existence makes it possible to limit the calculation of the coefficients $c_{j \geq 1}$ to the lowest (dipole or quadrupole) approximations for the charge operator (13), since each subsequent multipole in Eq. (13) gives a correction to $c_{j \geq 1}$ with an additional factor $a/R$. The decrease of $c_{j \geq 1}$ with increasing $j$ is due primarily to the decrease of the overlapping of the wave functions $\Psi_0$ and $\Psi_j$ (as result of the increase of the quantum numbers $n$ and $l$ for the states $\Psi_j$). The diameter of the "hard core" of the helium atom is $d_{core} \approx 2.64$ Å.[12] In principle, the atoms can approach one another to a distance of less than $d_{core}$, but with a small probability.

The quantities $R$ for neighboring helium atoms are spread around $\bar{R}$. This spread is characterized by the structure function $S_1(R)$,[10,11] in terms of which the wave function of the ground state of He II is expressed:

$$\Psi_0 \approx \frac{1}{\sqrt{Q}} \prod_{k,j=1}^{N} \exp(S_1(|\mathbf{R}_k - \mathbf{R}_j|)). \tag{31}$$

According to the analysis in Refs. 10 and 11, the function $S_1(R)$ varies quite smoothly, and for this reason the probability of finding an atom at a distance $R$ from its neighbor decreases with increasing $|R - \bar{R}| \equiv \Delta R$ at first slowly and then rapidly (by an order of magnitude more $\Delta R = 0.4$ Å) for $\Delta R \geq 0.4$ Å. This means that for most atoms the distance $R$ up to the neighboring atom lies almost with equal probability in a wide interval approximately from 3.2 Å to 4 Å. Con-

sider the following configuration: the atoms A, B, and C lie approximately on the same line along the $x$ axis, and the distance between A and B is $R_A = \bar{R} = 3.6$ Å while the distance between B and C $R_C = 4$ Å (characteristic distances are used). Then, because of the difference between $R_A$ and $R_C$ the atom B has an induced DM (IDM) $d_{at}^x = d_0(\bar{R}^4/R_A^4 - \bar{R}^4/R_C^4) \approx d_0/3$. The maximum but unlikely value of $d_{at}^x$ is obtained for $R_A = R_{min} = d_{core}$, $R_C = R_{max} = 2\bar{R} - d_{core}$: $d_{at}^{x,max} \approx 3d_0$. The atom B also has neighbors along the $y$ and $z$ axes, and therefore there are IDM $d_{at}^y$ and $d_{at}^x$. As one can see, the average modulus $\bar{d}_{at}$ of the IDM of the atom $\mathbf{d}_{at} = \mathbf{i}d_x + \mathbf{j}d_y + \mathbf{k}d_z$ is close to $d_0$, and the direction of $\mathbf{d}_{at}$ is different for different atoms. Since $d_0 \sim R^{-4}$, the interaction with more distant neighbors can be neglected.

Although the IDM in an isolated pair of interacting $^4$He atoms and is always zero, in a multiatom aggregate each atom already has a nonzero IDM $\mathbf{d}_{at}$ because the distances between neighboring atoms is not the same. The vector $\mathbf{d}_{at}$ fluctuates in time and space, in accordance with the variation of the configuration of the neighbors, and is directed in different atoms in different directions, on the average randomly; the magnitude of the IDM likewise varies randomly from one atom to another. Of course, the total IDM of the atoms is zero in macroscopic volumes of helium II. In a uniform and isotropic liquid the total IDM vector of all atoms divided by the number of atoms should equal the IDM of a single atom, averaged over all configurations of all remaining atoms; such an IDM is expressed in terms of a convolution of the ground-state wave function of He II and reduces to an integral of the diagonal two-particle density matrix and the DM vector (30); it can be easily shown that the answer is exactly 0. However, the average value of the *modulus* of the instantaneous IDM of a He II atom is not zero (its time average for a given atom or the average instantaneous value of the modulus of all atoms) and equals approximately $d_0$ (30).

We arrive at the conclusion that although a free $^4$He atom does not have a DM, a helium II atom acquires a certain induced DM as a result of interaction with neighbors; the average modulus of this DM is $\bar{d}_{at} \simeq 2e\delta$, $\delta = 2.6 \cdot 10^{-4}$ Å.

It is easily shown that when the interaction of the IDM of atoms with one another is taken into account a *three-particle* interatomic interaction potential appears in addition to the pair interaction in the total potential.

Since it is strongly coupled with the neighboring atoms, the IDM does not contribute to the permittivity $\varepsilon_{He}$ of He II. In order for the IDM to contribute to $\varepsilon_{He}$, the external field $\mathbf{E}_0$ applied to measure $\varepsilon_{He}$ must be able to rotate the IDM of an atom. However, the IDM of He II atoms are not free, as they are in polar dielectrics, but rather they arise as a result of a definite arrangement of neighboring atoms, so that the configuration of neighbors must be changed in order to rotate the IDM, which requires a very strong electric field of the order of the intra-atomic field. The fields $\mathbf{E}_0$ used in practice are too weak.

The presence of induced DM in helium atoms can result in polarization of He II if there is a mechanism which orients the IDM of the atoms in predominantly one direction.
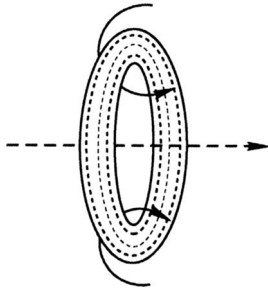
FIG. 1. The arrows illustrate the velocity field induced in the liquid by a vortex ring. The dashed arrow shows the direction of the velocity of the ring itself.

## III. MECHANISM OF FOCUSING OF THE ATOMIC DIPOLE MOMENTS

An electric signal was observed in He II when a second-sound standing half-wave was produced in the vessel.[1] The second-sound frequency was chosen to be a resonance frequency by a method where the length $L$ of the resonator (vessel with helium) was equal to one-half the wavelength $\lambda$ of second sound. Antinodes were present on the left-hand $A$ and right-hand $B$ ends of the resonator walls, so that the temperature in the vessel varied according to the law

$$T = T_0 - 0.5\Delta T \cos(\omega_2 t)\cos(z\pi/L), \qquad (32)$$

where $L = \lambda/2 = u_2/2\omega_2$, and $u_2$ and $\omega_2$ are the velocity and frequency of second sound.

Let us assume that the electric signal is due to the bulk properties of He II. To study the signal we shall not use the phenomenological approach to the nature of superfluid and normal components, since this approach masks the microscopic nature of the phenomenon. Since the electric signal was observed in a second-sound wave, which is a compression–rarefaction wave in a gas of quasi-particles,[7] it is obvious that the quasi-particles must be somehow responsible for the proposed effect. There are two types of quasi-particles in He II—*phonons* (including *rotons*) and microscopic *vortex rings*. Since a phonon is a long-wavelength oscillatory motion, it should not possess a DM. Our idea consists in the fact that the effect observed in Ref. 1 is due to the presence of a DM vortex ring, and the bulk polarization of He II arises as a result of a directed flow of rings in the second-sound wave.

The properties of rings are examined in detail in Ref. 8 and 16. Figure 1 shows the velocity field induced in a classical vortex ring in a liquid. A ring, i.e. its toroidal core, moves rightward with velocity $v_{cr} \sim 1/R_{cr}$,[8,16] where $R_{vr}$ is the radius of the ring (torus). The atoms around the ring are entrained into motion with velocity $v_{at}$, as shown by the arrows in Fig. 1. The velocity $v_{at}$ of the atoms at a distance $R$ away from the ring decreases quite rapidly ($\sim R^{-2}$). A quantum vortex ring possesses a definite minimum size.[17] Rings in He II are quantized, which is evident even from their dimensions: the radius of the core of a ring $a_{vr} \simeq 1$ Å, and the radius of the smallest ring $R_{vr}^{min} \simeq 2-3$ Å. Such rings are vortices in the probability field $\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N)$ and not in a classical medium consisting of atoms, so that small ring sizes are physically meaningful.

According to the character of the velocity field, a vortex ring is similar to a dipole (Fig. 1), and it can be supposed that a ring possesses its own DM $d_{vr}$. We shall consider two possible mechanisms for generating $d_{vr}$.

1. As evident from the relations (28) and mentioned before, the electron shell of a $^4$He atom is somewhat extended in a direction away from the neighboring atom. Correspondingly, the nucleus is shifted (relative to the center of mass of the electron shell) in the opposite direction by the amount $\approx \delta = 2.6 \cdot 10^{-4}$ Å. According to Fig. 1, as it moves the ring pulls into itself atoms from the left-hand half-space and pumps them through itself. The velocity field behind and to the left of the ring strives to collect atoms toward the axis and possesses a gradient. The gradient creates a rotational moment, and since the atomic nucleus is shifted relative to the electron shell, this moment strives to turn the atom in a manner so that the light, elongated part of the electron shell is oriented along the velocity vector and the heavy nucleus remains behind. As a result, in some region behind the ring the atoms must be partially focused in a manner so that their IDM are oriented predominantly along the axis of the ring with the negative charge oriented in the direction of motion of the ring. True, this effect will be attenuated because the IDM are not free but rather are due to a definite arrangement of the neighboring atoms.

2. Another possible source of the DM of a ring that is capable of giving $d_{vr}$ greater than $\bar{d}_{at}$ is the fact that the modulus of the gradient of the fluid density in front of the ring is not the same as behind the ring. The fluid density in the interior region of the ring is much lower than the average value,[17] and since the ring is asymmetric (there is a distinguished direction for its velocity) it is capable of producing a difference of the gradients. Let the density of the liquid grow more slowly in a direction away from the ring in front of the ring up to the average value than behind the ring. According to Sec. II, in this case an IDM with the negative end in the direction of motion of the ring is induced in the atoms behind the ring, and the opposite IDM which is smaller in absolute magnitude is induced in the atoms in front of the ring. Such a ring possesses a DM with the negative end directed in the direction of motion of the ring.

Symmetry confirms these considerations and allows the ring to have its own DM. Under time reversal ($t \to -t$) the circulation and the velocity evidently change sign in the ring ($\mathbf{v}_{vr} \to -\mathbf{v}_{vr}$). The change in the charge under time reversal $t \to -t$ is evident from the CPT theorem of quantum field theory, according to which CPT is an invariant, so that CP $\to -$CP when $t \to -t$, but $C$ is the charge of a particle and $P$ is the radius vector, that is CP corresponds to the dipole moment $e\mathbf{r}$ and for this reason when $t \to -t$ the DM, just as the velocity of the ring, changes sign ($e\mathbf{r} \to -e\mathbf{r}$).

Natsig recently analyzed the possible electric properties of vortices and showed that inertial polarization of the atoms rotating around the core of a vortex arises near the core and that the total polarization of the medium by the vortex (including by the ring) is zero.[18] However, in Ref. 18 a vortex ring was studied in the semiclassical approximation (classical relations but with quantum circulation) and the two possible quantum mechanisms proposed above for generating a DM of the ring were neglected. On the other hand, we are

examining very small rings which are thermal excitations of He II; these rings have atomic dimensions and are strongly quantum objects, and they can be correctly and completely described only by the $N$-particle wave function $\Psi_{vr}$ of the ring. Even in the mean-field approximation, which gives a poor description of the internal structure of a ring, the solution of Ref. 17 for a quantum ring differs appreciably from the classical (or semiclassical) solution: the velocity field and density profile of the liquid near the ring differ from the corresponding distributions near a classical ring. Complete information about the properties of a ring can be obtained only from $\Psi_{vr}$—to see whether or not there is a difference of the density gradients behind and in front of a ring and to model other properties of the ring. However, $\Psi_{vr}$ is unknown, so that we postulate below that a vortex ring possesses an average DM $\mathbf{d}_{vr}$ directed along the axis of the ring. We shall check whether or not this will permit explaining the results of Ref. 1.

Of course, the foregoing arguments are not a strict proof of the presence of a DM in a ring, but they qualitatively substantiate such a possibility. We also note that $\mathbf{d}_{vr}$ is the characteristic DM of a ring which travels as a wave through helium together with the ring. In the process, the ring performs no work on the induced IDM of atoms, since such work would quickly exhaust its energy. However, experimentally, a ring passes through a vessel without appreciable energy losses.[19]

## IV. POLARIZATION OF He II BY A STANDING HALF-WAVE OF SECOND SOUND

At thermal equilibrium rings move chaotically in different directions, and since the DM of a ring $\mathbf{d}_{vr}$ is oriented along the axis of the ring the directions of $\mathbf{d}_{vr}$ in the rings are, on the average, chaotically oriented. But a directed flow of rings which polarizes He II is formed in a standing half-wave of second sound (see below).

We call attention to an important point. We need to find the ring flux in a second-sound standing wave; this is the flux that changes the distribution of rings in a second-sound wave. Since the change in $T$ in a second-sound wave in Rybalko's experiment is negligibly small, $\Delta T \sim 10^{-6} T$, and the period of the second-sound oscillations is much greater than the characteristic quasi-particle collision times (for the ones examined $T > 1.3$ K), the distribution of rings and phonons in a second-sound standing wave can be regarded as an equilibrium distribution to a high degree of accuracy. In this case we know the ring density $n_{vr}(z,t)$ (43) for all $z$ and $t$, since the temperature $T(z,t)$ is known. Once $n_{vr}(z,t)$ is known, we can easily find the ring flux (see below). We call attention to the fact that to find this flux we do not need to solve the kinetic equation for rings, since the parameters of the second-sound standing wave fix this solution. The inaccuracy of such an approach is only due to the fact that the known formula for the ring distribution (see Eq. (43)) was obtained in Ref. 16 neglecting the interaction of rings with other rings and phonons. For $T > 1.3$ K this formula is inaccurate, which could cause the result for the ring flux to differ appreciably from the exact results (possibly several-fold). But, such accuracy is adequate for an approximate solution.

Let us examine the experiment of Ref. 1 in more detail. The resonator is oriented horizontally along the $z$ axis. Let the $z$ axis be directed leftward, along the axis of the resonator and the temperature gradient. We divide the helium into vertical layers of thickness $\Delta z$ and area $S_l$. The volume of a layer is $\Delta\Omega = S_l \Delta z$. Let the temperature $T$ be distributed according to Eq. (32), $t=0$:

$$T = T_0 - 0.5\Delta T \cos(z\pi/L). \tag{33}$$

For the extreme left-hand layer 1 we have $z=L$ and the highest $T$: $T_1 = T_0 + \Delta T/2$. In the neighboring layer 2 (to the right of layer 1) the temperature $T_2 = T_1 + \delta T$, where $\delta T = \nabla_z T \Delta z$ $< 0$. The temperature in layer 1 is higher, and since the rings in He II are in thermal equilibrium[16] there are more rings in layer 1 (according to Eq. (43)) than in layer 2. In addition, there are more rings directed and moving rightward. Consequently, more rings will flow from layer 1 into layer 2 then from layer 2 into layer 1, and the temperature $T_1$ will decrease in time. For the temperature distribution (33) $\nabla_z T \Delta z$ $< 0$ in the entire vessel, so that the rings and phonons will flow rightward along the entire helium volume, equalizing the temperature in the vessel.

The number of rings directed rightward in layer 1 is greater than in layer 2 by the amount

$$\Delta n_{vr}(\Omega_1) = [n_{vr}(T_1) - n_{vr}(T_2)]\Delta z S_l/2, \tag{34}$$

where $n_{vr}$ is the concentration of rings. The factor of 1/2 in Eq. (34) takes account of the fact that approximately one-half of the rings in layer 1 move rightward and the other half moves leftward. Similarly, there will be an excess number of rings directed and moving rightward in each layer. These rings create a directed ring flux. Since each ring possesses a dipole moment $d_{vr}$, such a flux gives rise to polarization

$$\mathbf{P}(z) = \frac{\mathbf{i}_z}{4\varepsilon_{He}} d_{vr} \frac{\partial n_{vr}}{\partial T} \delta T. \tag{35}$$

Here another factor of 1/2 has appeared because of averaging of $d_{vr} \cos\theta$ over $\theta$ ($\theta$ is the angle between $\mathbf{d}_{vr}$ and the $z$ axis). Finally, we have

$$P_z(z) = \frac{d_{vr}}{4\varepsilon_{He}} \frac{\partial n_{vr}}{\partial T} \nabla_z T \Delta z. \tag{36}$$

The physically minimum value of $\Delta z$, which is determined by the average distance $l = n_{vr}^{-1/3}$ between rings (this is the mean-free path length of a ring) projected onto the $z$ axis, $\Delta z = l \cos\theta_2$, must be substituted into Eq. (36). Averaging over $\theta_2$ we obtain $\Delta z = l/2$. Taking account of Eq. (32) we write

$$P_z(z) = \frac{\eta}{L}\cos(\omega_2 t)\sin\left(\frac{z\pi}{L}\right), \quad \eta = \frac{\pi}{16\varepsilon_{He}} d_{vr}\frac{\partial n_{vr}}{\partial T}\Delta T l. \tag{37}$$

So, we have a liquid with a system of dipoles which are directed along the $z$ axis in the direction of higher temperature and are distributed uniformly over the cross-section $S_l$. Let us now find the potential difference $U$ induced by these dipoles between the end walls $A$ and $B$ of the resonator. The value of $U$ can be found if the electric field intensity $\mathbf{E}$ is known. But it is unknown, and generally speaking it is in-

correct to find it from relations of the form $\mathbf{E} = \widetilde{\gamma}\mathbf{P}$, since $\mathbf{E}$ is the internal (and not external) field created by the known system of dipoles. In this case $\widetilde{\gamma}$ is unknown, and it must be calculated taking account of the boundary conditions. Consequently, $U = \varphi_A - \varphi_B$ must be found from Laplace's equation for the potential with boundary conditions or by summing the potentials due to individual dipoles. We shall use the second approach.

A dipole placed at the coordinate origin creates at the point $\mathbf{r}$ the potential[20]

$$\varphi(\mathbf{r}) = \frac{\mathbf{dr}}{\varepsilon r^3}. \tag{38}$$

In Ref. 1 one resonator is a 1 mm in diameter cylinder of length $L = 1.05$ mm and the other is a 28 mm long parallelepiped with the cross section 1.4 mm $\times$ 3 mm. The resonators are made of metal; the ends are coated on the inside by a dielectric coating with $\varepsilon_D \approx 9.3$ on which metal plates are placed—a heater on one end and an electrode on the other. A plate does not cover the entire area of the endface:[2] a 0.1–0.2 mm wide strip of the coating remains uncovered by the plate between the plate and the resonator.

According to the method of images[20,21] the charges of the dipole moments are reflected in metal and dielectric surfaces, and the potential inside the resonator can be found only by taking account of these images. The boundary conditions described above are mixed and too complex; for them the potential inside the resonator can be found only by numerical modeling, which falls outside the scope of the present work (the potential of *single* charge in the conducting cylinder is not found analytically). However, an estimate of the potential can be made.

Let a charge $q$ be present in the resonator. Then an induction charge equivalent to a mirror symmetric "image" $-q$ arises on the flat metal wall of the resonator and an "image" charge $q(\varepsilon_{\text{He}} - \varepsilon_D)/(\varepsilon_{\text{He}} + \varepsilon_D)$ arises on the flat dielectric wall.[21] We are interested in the projection of the dipole on the $z$ axis, $d_z$. The "image" of the dipole $d_z$ in the metal side wall is $d_z^{\text{im.Me}} = -d_z$ and the "image" of the dielectric endwall is $d_z^{\text{im.D}} = d_z(\varepsilon_D - \varepsilon_{\text{He}})/(\varepsilon_{\text{He}} + \varepsilon_D)$. It can be shown that when the potentials from the dipoles in the resonator and from their "images" are summed the latter make a contribution with the opposite sign, i.e. they decrease the field due to the dipoles. If the "images" are neglected and the induced charges on the walls are assumed to be zero, then the upper limit of the potential inside the resonator can be found.

We note that the resonator in Ref. 1 possessed a finite thickness and floated in a large flask with He II. However, the possible induction charges on the outer surface of the resonator are unimportant, since the resonator is made of metal, and consequently it does not transmit the field of these charges into the interior cavity.

The potential difference $U = \varphi_A - \varphi_B$ between the ground and resonator, $\varphi_A \equiv \varphi(z-L) = 0$, and the electrode at the end, $\varphi_B = \varphi(z=0)$, was measured in Ref. 1. We have for this difference

$$
\begin{aligned}
U = -\varphi_B &= \int dV \frac{P_z(z)z}{(\rho^2 + z^2)^{3/2}} \\
&= \int_0^{2\pi} d\phi \int_0^L dz \int_0^R \rho d\rho \frac{P_z(z)z}{(\rho^2 + z^2)^{3/2}} \\
&= 2\pi \int_0^L dz P_z - 2\pi \int_0^L dz P_z \frac{z}{(R^2 + z^2)^{1/2}}.
\end{aligned} \tag{39}
$$

Neglecting the boundaries ($R = \infty$) the potential difference $U$ equals the first term in Eq. (39), whence $\mathbf{E} = 2\pi\mathbf{P}$ (this is the case of an infinite medium). It follows from Eqs. (37) and (39) that

$$U = \eta \cos(\omega_2 t) \gamma(R, L), \tag{40}$$

where

$$\gamma(R, L) = 2\pi \int_0^1 \sin(\pi y) dy \left[ 1 - \frac{y}{\sqrt{y^2 + b^2}} \right], \quad b = \frac{R}{L}. \tag{41}$$

For $b \ll 1$ the integral (41) can be estimated analytically as

$$\gamma(R, L) \approx \pi^2 b^2 \ln \frac{2}{\pi b}. \tag{42}$$

It is evident from Eq. (40) that the oscillations of the voltage are in-phase with the temperature oscillations, which corresponds to the conditions of Ref. 1.

The resonator in Ref. 1 in the form of a parallelepiped had $b \approx \sqrt{(1.4 \cdot 3 / \pi)}(\text{mm}/L) \approx 1/24$. Then $\gamma \approx 1/20$. For a cylinder $b = 1/2$, and a numerical calculation of the integral gives $\gamma \approx 1.38$. It is evident that $U$ and $\gamma$ depend strongly on the boundary conditions: neglecting the boundary conditions ($R = \infty$) $\gamma = 4$, for a short resonator $\gamma \approx 1.38$, and for a long resonator $\gamma \approx 1/20$.

The expression (40) gives the upper limit for $U$, and taking account of the induction charges on the walls decreases $U$. Estimates have shown that the "images" from the metal side walls decrease $\gamma$ by approximately a factor of 2. The mixed conditions on the end walls are more complicated (metal and dielectric), and it is difficult to make an estimate for them. If the electrode were small, then taking account of the boundary conditions on the right-hand end to decrease $\gamma$ by a factor of $(\varepsilon_D + \varepsilon_{\text{He}})/2\varepsilon_{\text{He}} \approx 5$.

We note that the relations obtained are universal and valid for microscopic DM of any nature. If the authors of Refs. 4–6 had taken account of the boundary conditions, then the additional factor $\gamma$ would have been obtained in these works.

Thus, we arrive at the conclusion that although the boundary conditions of Ref. 1 were not taken into account entirely accurately in the calculations, it is evident from the relations obtained that $\Delta U$ should depend on the dimensions of the resonator. The potential difference $\Delta U$ was found experimentally for two different resonators, described above, and for them $\Delta U$ was the same, which is surprising. But since only two resonators were investigated, it cannot be ruled out that this agreement is accidental. For a short cylin-

drical resonator the ends should also strongly affect $\Delta U$—these are two parallel metal mirrors framed with a dielectric; the dipoles are multiply reflected in them, which increases the effective length of the resonator and could decrease the dependence of $\Delta U$ on $R$ and $L$.

We now return to the rings and make an estimate. We express the DM of a ring in terms of the atomic moment: $d_{vr}(T) = N\bar{d}_{at}/\beta = 2Ne\delta/\beta$, where $N$ is a number and $\beta(T)$ takes account of the decrease in $d_{vr}$ as a result of oscillations of the atoms. The concentration of the rings is described by the expression[16]

$$n_{vr} = 8\pi\left(\frac{P_0}{2\pi\hbar}\right)^3 \frac{k_B T}{E_0} \exp\left(-\frac{E_0}{k_B T}\right) g(T),\qquad(43)$$

where $P_0$ and $E_0$ are the momentum and energy, respectively, of the smallest ring, and $g(T) \approx 0.6$ for $T$ from 0 to 2 K. Rings with different radius should be present in helium II, but the radius of most rings is close to the minimum radius:[16] $R_{vr} \approx R_{vr}^{min} = 2\text{--}3$ Å, so that for simplicity we shall assume that all rings have the same radius $R_{vr}^{min}$. The voltage induced on the walls is $U_{ind} = -U$, and we obtain for the amplitude of its oscillations

$$\Delta U_{ind}(T) = 2\eta\gamma(R,L)$$
$$\approx \frac{\partial n_{vr}}{\partial T}(T)l(T)\frac{\pi N}{\beta(T)8\varepsilon_{He}}\Delta T\gamma(R,L)2e\delta.\qquad(44)$$

We take for $R_{vr}^{min}$ the average value $R_{vr}^{min} = 2.5$ Å,[16] for which $E_0 = 12.8$ K and $P_0/\hbar = 2.67$ Å$^{-1}$. The relation (43) was obtained neglecting interaction between rings, which is valid for $T < 1.2$ K. Consequently, to estimate $\Delta U_{ind}$ we take the lowest value of $T$ from Ref. 1: $T = 1.4$ K. Then $l \approx 40.7$ Å.[16] Finally, we obtain from Eqs. (43) and (44)

$$\frac{\Delta U_{ind}(T)2e}{k_B\Delta T} \approx \frac{E_0}{12.8\text{ K}}\left(1 + \frac{k_B T}{E_0}\right)\left(\frac{1.4\text{ K}}{T}\right)^2$$
$$\times\left(\frac{40.7\text{ Å}}{l}\right)^2 \gamma(R,L)\frac{N}{3.9\beta(T)}.\qquad(45)$$

We shall normalize the coefficient $\beta(T)$ to 1 at $T = 1.4$ K. According to experiment, the left-hand side of Eq. (45) is approximately 1—this requires that on the right-hand side $N \approx 3.9/\gamma$. The quantity $\gamma(R,L)$ can be found more accurately for a long resonator: $\gamma(R,L) \approx 1/20$, whence $N \approx 78$. For a short resonator $\gamma(R,L) \approx 1.38$, and hence $N \approx 2.8$.

According to Eq. (45), $\Delta U$ depends on $T$ as

$$\frac{n_{vr}^{2/3}}{\beta(T)T^2}.$$

It is difficult to calculate $\beta(T)$. Since the thermal oscillations of the atoms near a ring are induced by all quasi-particles, first and foremost by rotons and rings, it can be assumed for purposes of estimation that

$$\beta(T) \sim \frac{n_{vr}(T) + n_r(T)}{n_{vr}(1.2\text{ K}) + n_r(1.2\text{ K})}.$$

The concentration $n_r$ of rotons is given by

$$n_r = 0.051 e^{-\Delta/T}\left(\frac{q_r}{1.925\text{ Å}^{-1}}\right)^2 \sqrt{\frac{\mu T}{0.14 m_4\text{ K}}}\text{ Å}^{-3}.$$

Since $n_{vr} \ll n_r$, $\Delta \approx 2E_0/3$, we obtain

$$\Delta U(T) \sim \frac{n_{vr}^{2/3}}{n_r T^2} \sim T^{-11/6} \exp\left(\frac{\Delta - 2E_0/3}{k_B T}\right) \approx T^{-11/6}.$$

As one can see, thermal fluctuations decrease the dependence of $\Delta U$ on $T$, and in principle they can extinguish this dependence.

There is one other mechanism of polarization, arising because the density changes negligibly in a second-sound wave: $\delta\rho = -\rho\alpha(T)\delta T$. For $T = 1.4 \div -2$ K the thermal expansion coefficient $\alpha$ equals $(-1.4\text{--}12)\cdot 10^{-3}$ K$^{-1}$.[8] Evidently, the average distance between two $^4$He atoms feels this small change in $\rho$ at the microscopic level: $\bar{R}(T) = \bar{R}[1 + \delta T\rho\alpha(T)/3]$. As a result (see Sec. II) the $^4$He atoms acquire an induced IDM $d_\rho$ directed along the gradient of $T$:

$$d_\rho = \frac{2\pi\alpha\bar{R}}{3\sqrt{3}L\beta_2}d_0\delta T\cos(\omega_2 t)\sin(\pi z/L),$$

where $\beta_2$ describes the thermal decrease. We obtain $\Delta U(T) = 2\eta_2\gamma(R,L)$, where

$$\eta_2 = \frac{2\pi d_0}{3\sqrt{3}\beta_2\varepsilon_{He}}\frac{\alpha\delta T}{\bar{R}^2}.$$

For a long resonator $\gamma \approx 1/20$, and for $T = 1.4\text{--}2$ K, $\beta_2 \sim 1$, we find $\Delta U(T)2e/(k_B\Delta T) \approx 0.002\text{--}0.02$, which is 2–3 orders of magnitude smaller than the experimental value. We note that $\mathbf{d}_\rho$ is oriented so that its positive end faces the direction of larger $\rho$. At $T = 1.2$ K this polarization vanishes, and since $\Delta U \sim \alpha$, $\alpha$ vanishes. Neglecting the boundaries ($\gamma = 4$) we would have obtained $\Delta U(T)2e/(k_B\Delta T) \approx 0.16\text{--}1.6$. It is evident that the gradient of $\rho$ in a second-sound wave polarizes the helium as result of the ordering of IDM of the atoms and as a result of the inertial effect (see Sec. I); in addition, the former effect is three orders of magnitude greater than the latter.

## V. DISCUSSION

Our analysis shows that the mechanism based on the idea of a vortex-ring-induced unidirectional DM in $^4$He atoms is capable, in principle, of explaining the potential difference observed in He II.[1] It is evident from the analysis that to obtain the experimental value of the electric signal $\Delta U \approx k_B\Delta T/2e$ it must be assumed that each vortex ring possesses a DM $d_{vr} \sim 10\bar{d}_{at}$, and to obtain agreement with Ref. 1 the DM of a ring must be directed with its negative end oriented in the direction of motion of the ring.

The value $d_{vr} \sim 10\bar{d}_{at}$ is quite large (considering the small size of a ring) but realistic. We have examined two possible mechanisms for generating $d_{vr}$—inertial and based on the difference of the density gradients ahead and behind a ring. In the first one the value $d_{vr} \sim 10\bar{d}_{at}$ is possible because the radius of the region of high velocities (and velocity gradients) is about two ring radii and contains about 10–20 atoms, and the radius of the wake can be even larger. But strong focusing of the IDM of the atoms is necessary. The

more plausible second mechanism can give $d_{vr} \sim 10 \bar{d}_{at}$ if a ring consists of two subsystems: a toroidal core with radius $R_{vr} \simeq 2.5$ Å and a disk with a several-fold longer radius, parallel to the plane of the torus and having a lower density on the inside. Let the density of the liquid in the plane of the ring be $\bar{\rho}/2$, and let the density of the liquid in front of the ring increase (linearly along the axis of the ring) up to the average value $\bar{\rho}$ at a distance of 5 Å and behind the ring at a distance of 1 Å. The axis of the disk is also the axis of the ring, and the radius equals $2.5(R_{vr}+a_{vr}) \approx 9$ Å. Then, for an atom on the axis of the ring the neighboring atom behind the ring is located at a distance $R_1 \approx 3.6$ Å, and the neighbor in front of the ring is located at a distance $R_2 \approx 4.8$ Å; according to the relation $d_{at} = d_0(\bar{R}^4/R_1^4 - \bar{R}^4/R_2^4)$ such an atom acquires the IDM $d_{at} \simeq 2d_0/3$. A disk contains about 15 atoms, and we obtain $d_{vr} \sim 10\bar{d}_{at}$. The presence of such a disk-shaped region with low density in a quantum ring is possible, as one can see from the density profiles shown in Ref. 17.

It should be noted that the complete independence of the experimental ratio $\Delta U_{ind}/\Delta T$ from the dimensions of the resonator and the temperature (for 1.4 K$\leq T \leq$2 K) is surprising and casts doubt on the volume character of the effect.[1] Our calculation showed that if the effect is associated with volume polarization of the medium, then $\Delta U_{ind}/\Delta T$ should depend strongly on the dimensions of the resonator. In addition, it follows from the equations that such dependence will exist for an arbitrary (reasonable) volume dipole distribution whose nature can be arbitrary. The independence from $T$ likewise more likely suggests a non-volume character of the effect, after all micro- and macroparameters (condensates, $\rho_s$, and so on) in He II depend on $T$, with the exception of only the total number of atoms $N$, and the dependence on $T$ becomes especially strong in the limit $T \to T_\lambda$.

On the whole it can be concluded that further investigations are needed to clarify the nature of the observed electric signal. First and foremost, it is necessary to determine whether or not the effect is a volume effect (which is more interesting). For this, it is sufficient to perform only one experiment—install an additional electrode $M$ (small, so that it would not affect the boundary conditions) at the center of the resonator, at a distance $L/2$ from the ends, and at the center of the cross-section. If the effect is a volume nature and the boundary conditions on the ends are approximately the same, then the voltage between the electrode $M$ and ground should be zero:

$$\Delta U_M = \varphi_A - \varphi_M = -\varphi_M$$

$$= 2\pi \frac{\eta}{L} \cos(\omega_2 t) \int_{-L/2}^{L/2} dz\, z\, \sin(\pi z/L)$$

$$\times \int \frac{\rho\, d\rho}{(\rho^2 + z^2)^{3/2}} = 0$$

(the dipoles located on different sides of the electrode $M$ give the same contribution, but with the opposite sign, to $\varphi_M$). Here $\varphi_M - \varphi_B \approx \varphi_A - \varphi_B \approx k_B T/2e$. The volume effect could be associated with surface processes in He II near the heater or with a thermal electro-motive force, due to contact or a temperature gradient in the conductor. For a non-volume effect we shall have $\Delta U_M \approx k_B \Delta T/2e$. Since the frequency of

the current in the heater is half the frequency of second sound and the frequency of oscillations of $\Delta U$, there is no the electromagnetic induction from the heater on the resonator.

For a non-volume source of the effect, the value of $\Delta U$ should depend not on the dimensions of the resonator but rather on the dimensions and properties of the "non-volume source." The dimension or parameter on which $\Delta U$ depends can be found by varying the experimental conditions, and this will give information about the nature of the effect.

If the experimental $U(z)$ will correspond to the volume effect, it is interesting to investigate the dependence of $\Delta U$ on the dimensions of the resonator. For this, it is best to use a long resonator (with $L \gg R$)—in this case $\Delta U$ on the right-hand end $B$ does not depend on the boundary conditions on the left-hand end $A$ and it is easier to calculate $\Delta U$. It is sufficient to perform at least one experiment—increase or decrease the length of the resonator–parallelepiped[1] by a factor of 2–3 (in order for $L$ to change appreciably but for $L \gg R$ to remain valid) without changing the other parameters of the resonator. The value found for $\Delta U$ can be compared with Eqs. (40) and (42). In this experiment it is desirable to decrease the area of the electrode several-fold in order for the boundary condition on the right-hand end to correspond to the dielectric (and to be able to neglect the dielectric)—in this case Eqs. (40) and (42), where $\gamma$ must be decreased by the factor $(\varepsilon_D + \varepsilon_{He})/2\varepsilon_{He} \approx 5$, will determine $\Delta U$ with acceptable accuracy. It is also possible to perform an experiment with a resonator made of a dielectric.

It is more difficult to investigate the weak electrical properties of He II than its superfluid properties, since not only He II but also all materials used to measure $\Delta U$ possess electrical properties. In our view, the question of the nature of the electric induction[1] will remain unanswered without additional experiments.

We know that in the experiment with a first-sound standing half-wave[1] no induction was observed because, apparently, the amplitude of the signal was inadequate. Induction must be present—after all, the atoms in such a sound half-wave are accelerated in one direction, which will result in weak inertial polarization of the medium. As a result of the unidirectional gradient of the distances between the atoms, the IDM of the atoms should contribute to the polarization.

It is also interesting to note that in an external electric field rings with DM will turn with the negative end in the direction of positive external field, and hence they will move towards the positive end. As a result, a more dense cloud of rings will form near the "positive" wall of the vessel and $T$ will increase. However, thermal motion will wash out this effect because of the smallness of $d_{vr}$, since the interaction of rings with other rings and phonons is much stronger than the interaction with the external field $E$ and estimates show that the effect becomes observable only for very strong fields $E$ (when $Ed_{vr} \sim k_B T$) comparable to the intra-atomic field $E_{in}$, which cannot be attained in an experiment. For weaker fields the effect will be appreciable at very low temperatures, for example for $E \sim 1000$ V/cm (which is six orders of magnitude less than $E_{in}$) and $d_{vr} \sim 10\bar{d}_{at}$—for $T \lesssim 5 \cdot 10^{-4}$ K. Thus, the effect is practically unobservable.

## VI. CONCLUSIONS

The calculations performed have made it possible to obtain several results. Specifically, the following have been shown: (1) a fluctuating dipole moment with average modulus $\bar{d}_{at} \simeq 2e\delta$, where $\delta = 2.6 \cdot 10^{-4}$ Å, is induced in each He II atom as a result of the interaction of He II atoms with their nearest neighbors; (2) the experimental voltage $\Delta U \approx k_B \Delta T / 2e$ (Ref. 1) can be explained theoretically assuming that vortex rings in He II possess a dipole moment $d_{vr} \sim 10\bar{d}_{at}$, and then the polarization of He II is due to the presence of a directed flux of rings in a standing second-sound half-wave; (3) when the boundary conditions are taken into account, the potential difference $\Delta U$ depends on the dimensions of the resonator, and in Ref. 1 such a dependence was not observed, so that the effect in Ref. 1 could be due to not volume but rather surface properties of He II or to the electrical properties of the materials used to measure $\Delta U$.

The conclusion that He II atoms possess an induced dipole moment is of interest in itself. It is evident from Sec. II that the electrons in He II atoms are not in the $1s$ ground state but rather in the $1s$ state with a small admixture of the excited states $2s$, $2p$, and higher-order states, so that the electron shell of a helium atom is deformed, and this deformation pulsates in time and changes direction. Taking account of these small deformations of the atoms could result in special features appearing in the microscopic characteristics of helium II—specifically, smearing of single-particle and higher-order condensates.

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### Remarks added in proofreading

After this article had been submitted for publication we became aware of two earlier works which, to a certain extent, have a bearing on the questions which we studied. Specifically, Volovik[22] predicted a spontaneous electric polarization of certain types of quantum vortices in superfluid $^3$He as a result of the mutual influence of atoms and the asymmetry of three-atom configurations in the core of a vortex. It has not been ruled out that the physical reasons for the appearance of an induced IDM in quantum vortices and quantum vortex rings are similar. The other work is by Crowell,[23] who studied a problem similar to the problem solved in Sec. II of our work—the IDM of an initially unpolarized atom, interacting with an identical atom, was found. The problem was solved by a variational method, but in a quite simplified manner and non-rigorously. We note without going into details that the main deficiency of the calculation performed in Ref. 23 is that the wave function $\Psi_0^{AB}$ of two interacting atoms was

taken in a form that does not correspond to the exact solution (6). Nonetheless, it turned out that if the IDM for $^4$He atoms is calculated using the equations of Ref. 23, then the numerical value of the IDM is close to our value (see Eq. (30)). But this is probably a coincidence or possibly a result of the fact that the dipole-quadrupole structure of $\Psi_0^{AB}$ was taken into account approximately in Ref. 23. In our calculation, the nonzero value of the coefficient $c_1$ and the magnitude of the IDM arose when the quadrupole corrections to the charge operator (13) were taken into account systematically, as result of which the matrix element $U_{10}$ possesses the form of a dipole–quadrupole product. In Ref. 23 the dipole–quadrupole structure was also taken into account in $\Psi_0^{AB}$ but not because of the presence of additional contributions to the ground-state wave function but rather in its explicit functional form. However, the a priori choice of such a structure remained unsubstantiated and somewhat artificial.

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