Infrared catastrophe and tunneling into strongly correlated electron systems: Exact solution of the x-ray edge limit for the 1D electron gas and 2D Hall fluid

ARTICLE · OCTOBER 2005

Source: arXiv

2 AUTHORS, INCLUDING:

Michael R. Geller

University of Georgia

111 PUBLICATIONS **1,209** CITATIONS

Infrared catastrophe and tunneling into strongly correlated electron systems: Exact x-ray edge limit for the one-dimensional electron gas and two-dimensional Hall fluid

Kelly R. Patton* and Michael R. Geller
Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602-2451, USA
(Received 22 October 2005; revised manuscript received 22 March 2006; published 7 June 2006)

In previous work [K. R. Patton and M. R. Geller, Phys. Rev. B **72**, 125108 (2005)] we have proposed that the non-Fermi-liquid spectral properties in a variety of low-dimensional and strongly correlated electron systems are caused by the infrared catastrophe, and we used an exact functional integral representation for the interacting Green's function to map the tunneling problem onto the x-ray edge problem, plus corrections. The corrections are caused by the recoil of the tunneling particle, and, in systems where the method is applicable, are not expected to change the qualitative form of the tunneling density of states (DOS). Qualitatively correct results were obtained for the DOS of the one-dimensional electron gas and two-dimensional Hall fluid when the corrections to the x-ray edge limit were neglected and when the corresponding Nozières-De Dominicis integral equations were solved by resummation of a divergent perturbation series. Here we reexamine the x-ray edge limit for these two models by solving these integral equations exactly, finding the expected modifications of the DOS exponent in the one-dimensional case but finding no changes in the DOS of the two-dimensional Hall fluid with short-range interaction. Our analysis provides an exact solution of the Nozières-De Dominicis equation for the two-dimensional electron gas in the lowest Landau level.

DOI: 10.1103/PhysRevB.73.245306 PACS number(s): 73.43.Jn, 71.27.+a, 71.10.Pm

I. INTRODUCTION

In a previous paper,[1] we proposed a connection between anomalies in the tunneling density of states (DOS) at the Fermi energy of a wide variety of low-dimensional and strongly correlated conductors and the infrared catastrophe. The latter is the well-known singular screening response of an ordinary metal to the sudden appearance of a localized potential, in this case produced by an electron added to the system during a tunneling event. Systems where we expect this connection to apply include all one-dimensional (1D) electron systems,[2–8] the 2D diffusive metal,[9–20] the Hall fluid,[21–30] and the edge of the confined Hall fluid.[31–63] We argued that in such systems, the accommodation of a new electron added during a tunneling event is frustrated by the low dimensionality, localizing effects of a magnetic field, disorder, or any combination of these. In these cases we made a plausibility argument that the tunneling problem is similar to the x-ray edge problem. The argument begins with the following thought experiment: If one replaces the tunneling electron by an infinitely massive particle that tunnels in at time $\tau=0$ and is removed at some later time τ_0, this process produces a localized potential, abruptly switched on and off as in the x-ray edge problem. In reality the newly added electron does not have an infinite mass, but in the above listed systems the potential produced by the tunneling electron would be similar to that of a massive particle because of the inhibited charge relaxation.

A mapping between the x-ray edge problem and tunneling is done via an exact scalar functional integral representation for the interacting propagator, replacing it by a Gaussian average of noninteracting propagators for electrons in the presence of potentials $\phi(\mathbf{r},\tau)$. We single out a "dangerous" field configuration

$$\phi_{\mathrm{xr}}(\mathbf{r},\tau) = U(\mathbf{r})\Theta(\tau_0-\tau)\Theta(\tau), \tag{1}$$

which would be the potential produced by an electron added to the origin at time $\tau=0$ and removed later at τ_0, if it had an infinite mass. Here $U(\mathbf{r})$ is the bare electron-electron interaction. In Ref. 1 this special field configuration was treated by resumming a divergent perturbation series, and fluctuations about $\phi_{\mathrm{xr}}(\mathbf{r},\tau)$ were ignored, yet qualitatively correct expressions for the DOS were obtained.

To obtain quantitatively correct results it will be necessary to go beyond this "perturbative" x-ray edge limit. In Ref. 64 we proposed and investigated a functional cumulant expansion method that includes field fluctuations away from $\phi_{\mathrm{xr}}(\mathbf{r},\tau)$, and treats field configurations close to $\phi_{\mathrm{xr}}(\mathbf{r},\tau)$ perturbatively as in Ref. 1. Although the improved method yields the exact DOS exponent for the important Tomonaga-Luttinger model, calculable by bosonization, we do not expect it to be generally exact in 1D. (Furthermore, the method fails in the presence of a strong magnetic field because of the ground state degeneracy.)

In this paper, we neglect fluctuations about $\phi_{\mathrm{xr}}(\mathbf{r},\tau)$ (as in Ref. 1) but treat that field configuration exactly (in the large τ_0 asymptotic limit). This is accomplished by finding the exact low-energy solution of the Dyson equation for noninteracting electrons in the presence of $\phi_{\mathrm{xr}}(\mathbf{r},\tau)$, which we refer to as the Nozières-De Dominicis equation. We carry out this analysis for the 1D electron and 2D Hall fluid, both with short-range interaction. We then obtain an *exact* solution of the Nozières-De Dominicis equation for the 2D electron gas in the lowest Landau level.

II. FORMALISM

We calculate the tunneling DOS by analytical continuation of the Euclidean propagator

$$G(\mathbf{r}_f\sigma_f,\mathbf{r}_i\sigma_i,\tau_0) \equiv -\langle T\psi_{\sigma_f}(\mathbf{r}_f,\tau_0)\bar{\psi}_{\sigma_i}(\mathbf{r}_i,0)\rangle_H, \tag{2}$$

where $H=H_0+V$ is the grand-canonical Hamiltonian for the D-dimensional interacting electron system, with

$$H_0 \equiv \sum_\sigma \int d^D r \psi_\sigma^\dagger(\mathbf{r})\left[\frac{\Pi^2}{2m}+v(\mathbf{r})-\mu\right]\psi_\sigma(\mathbf{r})$$

and

$$V \equiv \frac{1}{2}\int d^D r d^D r'\, \delta n(\mathbf{r}) U(\mathbf{r}-\mathbf{r}')\delta n(\mathbf{r}').$$

Here $\mathbf{\Pi}\equiv\mathbf{p}+\frac{e}{c}\mathbf{A}$, with $\mathbf{A}$ the vector potential (if a magnetic field is present), and

$$\delta n(\mathbf{r}) \equiv \sum_\sigma \psi_\sigma^\dagger(\mathbf{r})\psi_\sigma(\mathbf{r}) - n_0(\mathbf{r}) \tag{3}$$

is the density fluctuation operator. H_0 is the Hamiltonian in the Hartree approximation, and $v(\mathbf{r})$ includes any single-particle potential along with the Hartree interaction with the self-consistent density $n_0(\mathbf{r})$.

After an exact Hubbard-Stratonovich transformation, the interacting Green's function can be written as a functional integral over a scalar field ϕ

$$G(\mathbf{r}_f\sigma_f,\mathbf{r}_i\sigma_i,\tau_0) = \mathcal{N}e^{1/2\int\phi_{\rm xr}U^{-1}\phi_{\rm xr}}\int D\mu[\phi]e^{-i\int\phi U^{-1}\phi_{\rm xr}} \times g(\mathbf{r}_f\sigma_f,\mathbf{r}_i\sigma_i,\tau_0|i\phi_{\rm xr}+\phi), \tag{4}$$

where $\mathcal{N}\equiv\langle T\exp(-\int_0^\beta d\tau V)\rangle_0^{-1}$ is a constant, independent of τ_0,

$$D\mu[\phi] \equiv \frac{D\phi e^{-1/2\int\phi U^{-1}\phi}}{\int D\phi e^{-1/2\int\phi U^{-1}\phi}} \tag{5}$$

is a measure normalized according to $\int D\mu[\phi]=1$, and

$$g(\mathbf{r}_f\sigma_f,\mathbf{r}_i\sigma_i,\tau_0|\phi) \equiv -\langle T\psi_{\sigma_f}(\mathbf{r}_f,\tau_0)\bar{\psi}_{\sigma_i}(\mathbf{r}_i,0)e^{i\int_0^\beta d\tau\int d^D r\phi(\mathbf{r},\tau)\delta n(\mathbf{r},\tau)}\rangle_0 \tag{6}$$

is a noninteracting functional of ϕ.

The dangerous field configuration $\phi_{\rm xr}(\mathbf{r},\tau)$, which itself depends on the parameters $\mathbf{r}_i$, $\mathbf{r}_f$, and τ_0 appearing in Eq. (2), has been given in Ref. 1. For the case the tunneling DOS at point $\mathbf{r}_0$ of interest here, the general definition reduces to Eq. (1), which is the potential that would be produced by the added particle in Eq. (2) if it had an infinite mass. Fluctuations about $\phi_{\rm xr}$ account for the recoil of the finite-mass tunneling electron.

In the x-ray edge limit, we ignore fluctuations about $\phi_{\rm xr}$, in which case

$$G(\mathbf{r}_f\sigma_f,\mathbf{r}_i\sigma_i,\tau_0) \approx \mathcal{N}g(\mathbf{r}_f\sigma_f,\mathbf{r}_i\sigma_i,\tau_0|i\phi_{\rm xr}). \tag{7}$$

Equation (9) defines the interacting propagator in the x-ray edge limit. The local tunneling DOS at position $\mathbf{r}_0$ is obtained by setting $\mathbf{r}_i=\mathbf{r}_f=\mathbf{r}_0$ and $\sigma_i=\sigma_f=\sigma_0$, and summing over σ_0. In the remainder of this paper we will evaluate Eq. (7) for the 1D electron gas and the 2D Hall fluid, with a short-range interaction of the form

$$U(\mathbf{r}) = \lambda\delta(\mathbf{r}). \tag{8}$$

III. X-RAY GREEN'S FUNCTION

The quantity $g(\mathbf{r}_f\sigma_f,\mathbf{r}_i\sigma_i,\tau_0|i\phi_{\rm xr})$ required in Eq. (7) is related to the Euclidean propagator

$$G_{\rm xr}(\mathbf{r}\sigma\tau,\mathbf{r}'\sigma'\tau') \equiv -\frac{\langle T\psi_\sigma(\mathbf{r},\tau)\bar{\psi}_{\sigma'}(\mathbf{r}',\tau')e^{-\int\phi_{\rm xr}\delta n}\rangle_0}{\langle Te^{-\int\phi_{\rm xr}\delta n}\rangle_0},$$

according to

$$g(\mathbf{r}_f\sigma_f,\mathbf{r}_i\sigma_i,\tau_0|i\phi_{\rm xr}) = G_{\rm xr}(\mathbf{r}_f\sigma_f\tau_0,\mathbf{r}_i\sigma_i 0)Z_{\rm xr}(\tau_0), \tag{9}$$

with

$$Z_{\rm xr}(\tau_0) \equiv \langle Te^{-\int_0^\beta d\tau\int d^D r\phi_{\rm xr}(\mathbf{r},\tau)\delta n(\mathbf{r},\tau)}\rangle_0. \tag{10}$$

We refer to $G_{\rm xr}(\mathbf{r}\sigma\tau,\mathbf{r}'\sigma'\tau')$ as the x-ray Green's function which describes noninteracting electrons in the presence of a real-valued potential $\phi_{\rm xr}(\mathbf{r},\tau)$. It satisfies the Dyson equation

$$G_{\rm xr}(\mathbf{r}\sigma\tau,\mathbf{r}'\sigma\tau') = G_0(\mathbf{r}\sigma,\mathbf{r}'\sigma,\tau-\tau') + \int d^D\bar{r}d\bar{\tau}G_0(\mathbf{r}\sigma,\bar{\mathbf{r}}\sigma,\tau-\bar{\tau})\phi_{\rm xr}(\bar{\mathbf{r}},\bar{\tau}) \times G_{\rm xr}(\bar{\mathbf{r}}\sigma\bar{\tau},\mathbf{r}'\sigma\tau'). \tag{11}$$

Here we have used that fact that the x-ray Green's function is diagonal in spin. For a calculation of the DOS we use the form (1), in which case Eq. (11) becomes

$$G_{\rm xr}(\mathbf{r}\sigma\tau,\mathbf{r}'\sigma\tau') = G_0(\mathbf{r}\sigma,\mathbf{r}'\sigma,\tau-\tau') + \lambda\int_0^{\tau_0} dtG_0(\mathbf{r}\sigma,\mathbf{r}_0\sigma,\tau-t)G_{\rm xr}(\mathbf{r}_0\sigma t,\mathbf{r}'\sigma\tau'), \tag{12}$$

where we have assumed the short-range interaction (8).

By using the linked cluster expansion and coupling-constant integration, $Z_{\rm xr}$ can be shown to be related to the x-ray Green's function by[65]

$$Z_{\rm xr}(\tau_0) = e^{n_0\lambda\tau_0}e^{-\lambda\Sigma_\sigma\int_0^1 d\xi\int_0^{\tau_0}d\tau G_{\rm xr}^\xi(\mathbf{r}_0\sigma\tau,\mathbf{r}_0\sigma\tau^+)}, \tag{13}$$

where $G_{\rm xr}^\xi(\mathbf{r}\sigma\tau,\mathbf{r}'\sigma\tau')$ is the solution of Eq. (12) with scaled coupling constant $\xi\lambda$.

There is no $\mathbf{r}_0$ dependence in the DOS for the translationally invariant models considered here and we can take $\mathbf{r}_0=0$.

IV. 1D ELECTRON GAS

$G_{\rm xr}(0\sigma\tau,0\sigma\tau')$ was calculated exactly in the large τ_0, asymptotic limit for the 3D electron gas in zero field by Nozières and De Dominicis.[65] Their result is actually valid for arbitrary spatial dimension D if the appropriate noninteracting DOS is used.

We take the asymptotic form of the noninteracting propagator as

$$G_0(\tau) \approx -P\frac{N_0}{\tau}, \quad \text{with } N_0 \equiv \frac{1}{\pi v_F}. \tag{14}$$

N_0 is the noninteracting DOS per spin component at ϵ_F, and P denotes the principal part. The solution of Eq. (12) for this model with $\mathbf{r}=\mathbf{r}'=0$ is

$$G_{xr}(\tau_0) = -N_0 \cos(\delta_\lambda)\left[P\frac{\cos(\delta_\lambda)}{\tau_0} + \pi \sin(\delta_\lambda)\delta(\tau_0)\right] \times \left(\frac{a}{\tau_0}\right)^{2\delta_\lambda/\pi}, \tag{15}$$

and

$$Z_{xr}(\tau_0) = \left(\frac{a}{\tau_0}\right)^{2(\delta_\lambda/\pi)^2}, \tag{16}$$

where δ_λ is the scattering phase shift of the electrons caused by the potential ϕ_{xr} given by

$$\delta_\lambda = \arctan(N_0 \pi \lambda) \tag{17}$$

and a is a short time cutoff on the order of the Fermi energy.

Thus

$$G(\tau_0) \approx g(\tau_0|i\phi_{xr}) \sim \left(\frac{1}{\tau_0}\right)^{1+2\delta_\lambda/\pi+2(\delta_\lambda/\pi)^2} \tag{18}$$

which gives a DOS in the x-ray edge limit as

$$N(\epsilon) \sim \epsilon^{2\delta_\lambda/\pi+2(\delta_\lambda/\pi)^2}. \tag{19}$$

By expanding the exponent in Eq. (19) in powers of the coupling parameter λ one recovers the perturbative x-ray result of Ref. 1.

V. 2D HALL FLUID

Unlike the low-energy Dyson equation for the 1D electron gas, which is solvable by Hilbert transform techniques, there are no standard methods available to solve the corresponding integral equation for the Hall fluid. We were able to guess the exact analytical solution, aided by perturbation theory and by numerical studies carried out by expansion in a plane-wave basis followed by matrix inversion.

We assume the system to be spin polarized and spin labels are suppressed. In the Landau gauge $\mathbf{A}=Bx\mathbf{e}_y$, the noninteracting propagator in the $|\tau| \gg \omega_c^{-1}$ limit is

$$G_0(\mathbf{r},\mathbf{r}',\tau) = \Gamma(\mathbf{r},\mathbf{r}')[\nu - \Theta(\tau)], \tag{20}$$

where ν is the filling factor satisfying $0 \leq \nu \leq 1$, and

$$\Gamma(\mathbf{r},\mathbf{r}') \equiv \frac{1}{2\pi\ell^2} e^{-|\mathbf{r}-\mathbf{r}'|^2/4\ell^2} e^{-i(x+x')(y-y')/2\ell^2}. \tag{21}$$

First consider the case where $\mathbf{r}_i=\mathbf{r}_f=\mathbf{r}_0$. We can let $\mathbf{r}_0=0$ without loss of generality and at the origin Eq. (12) reduces to

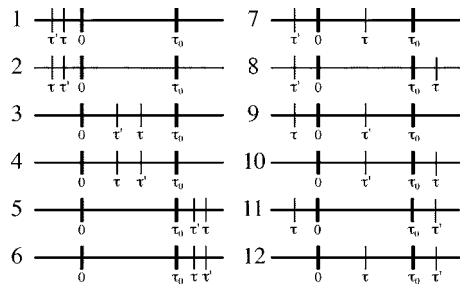

FIG. 1. The 12 possible time orderings $k=1,2,\ldots,12$ of $G_{xr}(0\tau,0\tau')$. τ_0 is assumed to be non-negative.

$$G_{xr}(0\tau,0\tau') = \frac{\nu - \Theta(\tau-\tau')}{2\pi\ell^2} + \gamma\int_0^{\tau_0} dt[\nu - \Theta(\tau-t)] \times G_{xr}(0t,0\tau'), \tag{22}$$

where

$$\gamma \equiv \frac{\lambda}{2\pi\ell^2} \tag{23}$$

is an interaction strength with dimensions of energy.

The time arguments of $G_{xr}(0\tau,0\tau')$ on the left side of Eq. (22) can assume the 12 possible orderings $k=1,2,\ldots,12$ defined in Fig. 1; the right side produces terms with two or more different orderings $k',k'',\ldots$. We therefore seek a solution of the form

$$G_{xr}(0\tau,0\tau') = \sum_k A_k\, W_k(\tau,\tau')\, f_k(\tau), \tag{24}$$

where $W_k(\tau,\tau')$ is unity if τ and τ' have ordering k and zero otherwise; an explicit form for $W_k(\tau,\tau')$ is given in the Appendix. The functions $f_k(\tau)$ are chosen to reflect the fact that an electron accumulates an additional phase $\gamma\Delta\tau$ while in the presence of ϕ_{xr} for a time $\Delta\tau$, whereas a hole acquires a phase $-\gamma\Delta\tau$. The 12 unknown coefficients A_k (which depend parametrically on τ_0 and τ') are obtained by solving the 12 linearly independent equations resulting from the decomposition of Eq. (22) into distinct time orderings $k=1,2,\ldots,12$. The result is

$$\begin{aligned} G_{xr}(0\tau,0\tau') = \frac{1}{2\pi\ell^2}\left(\frac{1}{1-\nu+\nu e^{-\gamma\tau_0}}\right)[&(\nu-1)(W_1+W_5) \\ &+ \nu e^{-\gamma\tau_0}(W_2+W_6) + (\nu-1)e^{-\gamma(\tau-\tau')}W_3 \\ &+ \nu e^{-\gamma\tau_0}e^{\gamma(\tau'-\tau)}W_4 + (\nu-1)e^{-\gamma\tau}W_7 \\ &+ (\nu-1)e^{-\gamma\tau_0}W_8 + \nu e^{-\gamma(\tau_0-\tau')}W_9 \\ &+ (\nu-1)e^{-\gamma(\tau_0-\tau')}W_{10} + \nu W_{11} + \nu e^{-\gamma\tau}W_{12}]. \end{aligned} \tag{25}$$

As a side note, the solution for general $\mathbf{r}_i$ and $\mathbf{r}_f$ is obtained using the same method and when both τ and τ' are in the interval $(0,\tau_0)$, the result is

$$G_{\mathrm{xr}}(\mathbf{r}\tau,\mathbf{r}'\tau') = [\nu - \Theta(\tau-\tau')][\Gamma(\mathbf{r},\mathbf{r}') - 2\pi\ell^2\Gamma(\mathbf{r},0)\Gamma(0,\mathbf{r}')] + 2\pi\ell^2\Gamma(\mathbf{r},0)\Gamma(0,\mathbf{r}') \times \left[\frac{(\nu-1)\Theta(\tau-\tau') + \nu e^{-\gamma\tau_0}\Theta(\tau'-\tau)}{1-\nu+\nu e^{-\gamma\tau_0}}\right] e^{-\gamma(\tau-\tau')}; \tag{26}$$

the other cases follow similarly. At the origin Eq. (26) reduces to

$$G_{\mathrm{xr}}(0\tau,0\tau') = \frac{1}{2\pi\ell^2}\left[\frac{(\nu-1)\Theta(\tau-\tau') + \nu e^{-\gamma\tau_0}\Theta(\tau'-\tau)}{1-\nu+\nu e^{-\gamma\tau_0}}\right] e^{-\gamma(\tau-\tau')}. \tag{27}$$

Finally

$$G_{\mathrm{xr}}(0\tau_0,00) = \frac{1}{2\pi\ell^2}\left(\frac{\nu-1}{1-\nu+\nu e^{-\gamma\tau_0}}\right) e^{-\gamma\tau_0}. \tag{28}$$

Using Eq. (13) we obtain

$$Z_{\mathrm{xr}} = e^{\nu\gamma\tau_0}(1-\nu+\nu e^{-\gamma\tau_0}), \tag{29}$$

therefore

$$g(\mathbf{r}0\sigma_0,\mathbf{r}_0\sigma_0,\tau_0|i\phi_{\mathrm{xr}}) = \frac{\nu-1}{2\pi\ell^2} e^{\gamma(\nu-1)\tau_0}. \tag{30}$$

This is identical to what we obtained in Ref. 1. The tunneling DOS is therefore

$$N(\epsilon) = \mathrm{const} \times \delta(\epsilon - [1-\nu]\gamma). \tag{31}$$

Strictly speaking, the delta-function potential (8) would be invisible in a spin-polarized Fermi system, and the exact interacting Green's function reduces to the noninteracting one. Our approximate interacting Green's function (7) is similar for all short-range (on the scale of the magnetic length) interactions, leading to approximately correct results for all of these cases *except* the particular case of a zero-range interaction. Therefore one should interpret Eq. (31) as applying to spin-polarized electrons with a short-range (but not strictly zero-range) interaction.

VI. DISCUSSION

In this paper, we have carried out an exact treatment of the x-ray edge limit introduced in Ref. 1, for the same models considered there. Whereas the 1D electron gas result (19) would be expected, the DOS of the 2D Hall fluid remains gapped as in Ref. 1. A generalization of our method that accounts for fluctuations about ϕ_{xr}, and that can be used in a magnetic field, will be needed to recover the actual pseudogap of the Hall fluid.[21–30]

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation under Grant No. DMR-0093217 and No. CMS-0404031.

APPENDIX: TIME ORDERING FUNCTIONS

Let

$$W_1(\tau,\tau') \equiv \Theta(-\tau)\Theta(-\tau')\Theta(\tau-\tau')$$

$$W_2(\tau,\tau') \equiv \Theta(-\tau)\Theta(-\tau')\Theta(\tau'-\tau)$$

$$W_3(\tau,\tau') \equiv W(\tau)W(\tau')\Theta(\tau-\tau')$$

$$W_4(\tau,\tau') \equiv W(\tau)W(\tau')\Theta(\tau'-\tau)$$

$$W_5(\tau,\tau') \equiv \Theta(\tau-\tau_0)\Theta(\tau'-\tau_0)\Theta(\tau-\tau')$$

$$W_6(\tau,\tau') \equiv \Theta(\tau-\tau_0)\Theta(\tau'-\tau_0)\Theta(\tau'-\tau)$$

$$W_7(\tau,\tau') \equiv W(\tau)\Theta(-\tau')$$

$$W_8(\tau,\tau') \equiv \Theta(\tau-\tau_0)\Theta(-\tau')$$

$$W_9(\tau,\tau') \equiv \Theta(-\tau)W(\tau')$$

$$W_{10}(\tau,\tau') \equiv \Theta(\tau-\tau_0)W(\tau')$$

$$W_{11}(\tau,\tau') \equiv \Theta(-\tau)\Theta(\tau'-\tau_0)$$

$$W_{12}(\tau,\tau') \equiv W(\tau)\Theta(\tau'-\tau_0),$$

where $\Theta(t)$ is the Heaviside step function and W (with no subscripts) is the a window function, defined as

$$W \equiv \Theta(\tau_0-\tau)\Theta(\tau). \tag{A1}$$

*Present address: Institut für Theoretische Physik, Universität Hamburg, Jungiusstraße 9, 20355 Hamburg, Germany.

[1] K. R. Patton and M. R. Geller, Phys. Rev. B **72**, 125108 (2005).
[2] S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950).
[3] D. C. Mattis and E. H. Lieb, J. Math. Phys. **6**, 304 (1965).
[4] I. E. Dzyaloshinskii and A. I. Larkin, Sov. Phys. JETP **38**, 202 (1974).
[5] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[6] F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981).
[7] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).
[8] M. P. A. Fisher and L. I. Glazman, in *Mesoscopic Electron Transport*, edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schön (Kluwer, Dordrecht, 1997), pp. 331–373.
[9] B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**,

1288 (1980).
[10] B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interaction in Disordered Systems*, edited by A. L. Efros and M. Pollak (Elsevier Science Publishers, Amsterdam, The Netherlands, 1985), pp. 1–153.
[11] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
[12] Y. V. Nazarov, Sov. Phys. JETP **68**, 561 (1989).
[13] Y. V. Nazarov, Sov. Phys. Solid State **31**, 1581 (1990).
[14] A. M. Rudin, I. L. Aleiner, and L. I. Glazman, Phys. Rev. B **55**, 9322 (1997).
[15] L. S. Levitov and A. V. Shytov, JETP Lett. **66**, 214 (1997).
[16] D. V. Khveshchenko and M. Reizer, Phys. Rev. B **57**, R4245 (1998).
[17] P. Kopietz, Phys. Rev. Lett. **81**, 2120 (1998).
[18] A. Kamenev and A. Andreev, Phys. Rev. B **60**, 2218 (1999).
[19] J. Rollbühler and H. Grabert, Phys. Rev. Lett. **91**, 166402 (2003).
[20] C. Chamon, A. W. W. Ludwig, and C. Nayak, Phys. Rev. B **60**, 2239 (1999).
[21] J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **69**, 3804 (1992).
[22] S.-R. Eric Yang and A. H. MacDonald, Phys. Rev. Lett. **70**, 4110 (1993).
[23] Y. Hatsugai, P.-A. Bares, and X.-G. Wen, Phys. Rev. Lett. **71**, 424 (1993).
[24] S. He, P. M. Platzman, and B. I. Halperin, Phys. Rev. Lett. **71**, 777 (1993).
[25] P. Johansson and J. M. Kinaret, Phys. Rev. Lett. **71**, 1435 (1993).
[26] Y. B. Kim and X.-G. Wen, Phys. Rev. B **50**, 8078 (1994).
[27] I. L. Aleiner, H. U. Baranger, and L. I. Glazman, Phys. Rev. Lett. **74**, 3435 (1995).
[28] R. Haussmann, Phys. Rev. B **53**, 7357 (1996).
[29] D. J. T. Leonard, T. Portengen, V. N. Nicopoulos, and N. F. Johnson, J. Phys.: Condens. Matter **10**, L453 (1998).
[30] Z. Wang and S. Xiong, Phys. Rev. Lett. **83**, 828 (1999).
[31] X.-G. Wen, Phys. Rev. Lett. **64**, 2206 (1990).
[32] X.-G. Wen, Phys. Rev. B **41**, 12838 (1990).
[33] X.-G. Wen, Phys. Rev. B **43**, 11025 (1991).
[34] K. Moon, H. Yi, C. L. Kane, S. M. Girvin, and M. P. A. Fisher, Phys. Rev. Lett. **71**, 4381 (1993).
[35] C. L. Kane, M. P. A. Fisher, and J. Polchinski, Phys. Rev. Lett. **72**, 4129 (1994).
[36] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **51**, 13449 (1995).
[37] F. P. Millikin, C. P. Umbach, and R. A. Webb, Solid State Commun. **97**, 309 (1996).
[38] A. M. Chang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **77**, 2538 (1996).
[39] J. H. Han and D. J. Thouless, Phys. Rev. B **55**, R1926 (1997).
[40] J. H. Han, Phys. Rev. B **56**, 15806 (1997).
[41] S. Conti and G. Vignale, Physica E (Amsterdam) **1**, 101 (1997).
[42] M. Grayson, D. C. Tsui, L. N. Pfeiffer, K. W. West, and A. M. Chang, Phys. Rev. Lett. **80**, 1062 (1998).
[43] A. V. Shytov, L. S. Levitov, and B. I. Halperin, Phys. Rev. Lett. **80**, 141 (1998).
[44] S. Conti and G. Vignale, J. Phys.: Condens. Matter **10**, L779 (1998).
[45] A. Lopez and E. Fradkin, Phys. Rev. B **59**, 15323 (1999).
[46] U. Zülicke and A. H. MacDonald, Phys. Rev. B **60**, 1837 (1999).
[47] A. M. M. Pruisken, B. Skoric, and M. A. Baranov, Phys. Rev. B **60**, 16838 (1999).
[48] D. V. Khveshchenko, Solid State Commun. **111**, 501 (1999).
[49] J. E. Moore, P. Sharma, and C. Chamon, Phys. Rev. B **62**, 7298 (2000).
[50] A. Alekseev, V. Cheianov, A. P. Dmitriev, and V. Y. Kachorovskii, JETP Lett. **72**, 333 (2000).
[51] A. M. Chang, M. K. Wu, C. C. Chi, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **86**, 143 (2001).
[52] V. J. Goldman and E. V. Tsiper, Phys. Rev. Lett. **86**, 5841 (2001).
[53] M. Hilke, D. C. Tsui, M. Grayson, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **87**, 186806 (2001).
[54] A. Lopez and E. Fradkin, Phys. Rev. B **63**, 085306 (2001).
[55] V. Pasquier and D. Serban, Phys. Rev. B **63**, 153311 (2001).
[56] E. V. Tsiper and V. J. Goldman, Phys. Rev. B **64**, 165311 (2001).
[57] L. S. Levitov, A. V. Shytov, and B. I. Halperin, Phys. Rev. B **64**, 075322 (2001).
[58] S. S. Mandal and J. K. Jain, Solid State Commun. **118**, 503 (2001).
[59] X. Wan, K. Yang, and E. H. Rezayi, Phys. Rev. Lett. **88**, 056802 (2002).
[60] S. S. Mandal and J. K. Jain, Phys. Rev. Lett. **89**, 096801 (2002).
[61] U. Zülicke, E. Shimshoni, and M. Governale, Phys. Rev. B **65**, 241315(R) (2002).
[62] A. M. Chang, Rev. Mod. Phys. **75**, 1449 (2003).
[63] M. Huber, M. Grayson, M. Rother, W. Biberacher, W. Wegscheider, and G. Abstreiter, Phys. Rev. Lett. **94**, 016805 (2005).
[64] K. R. Patton and M. R. Geller, Phys. Rev. B **73**, 125416 (2006).
[65] P. Nozières and C. T. De Dominicis, Phys. Rev. **178**, 1097 (1969).