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An analytical study is presented for the magnetohydrodynamic (MHD) effects on a translating and rotating colloidal sphere in an arbitrary electrolyte solution prescribed with a general flow field and a uniform magnetic field at a steady state. The electric double layer surrounding the charged particle may have an arbitrary thickness relative to the particle radius. Through the use of a simple perturbation method, the Stokes equations modified with an electric force term, including the Lorentz force contribution, are dealt by using a generalized reciprocal theorem. Using the equilibrium double-layer potential distribution from solving the linearized Poisson–Boltzmann equation, we obtain closed-form formulas for the translational and angular velocities of the spherical particle induced by the MHD effects to the leading order. It is found that the MHD effects on the particle movement associated with the translation and rotation of the particle and the ambient fluid are monotonically increasing functions of κa, where κ is the Debye screening parameter and a is the particle radius. Any pure rotational Stokes flow of the electrolyte solution in the presence of the magnetic field exerts no MHD effect on the particle directly in the case of a very thick double layer ($\kappa a \rightarrow 0$). The MHD effect caused by the pure straining flow of the electrolyte solution can drive the particle to rotate, but it makes no contribution to the translation of the particle. © *2010 American Institute of Physics*. [doi:10.1063/1.3489684]

I. INTRODUCTION

A charged particle suspended in an electrolyte solution is surrounded by a diffuse cloud of ions carrying a total charge equal and opposite in sign to that of the particle. The combination of the fixed surface charge and adjacent diffuse ions is known as an electric double layer. When an external electric field is imposed on the particle, a force is exerted on both parts of the double layer. The particle is attracted toward the electrode of its opposite sign, while the ions in the diffuse layer migrate in the other direction. This particle motion is termed electrophoresis and has long been applied to the particle characterization or separation in a variety of colloidal and biological systems.[1–6]

On the other hand, the flow of an electrolyte solution caused by its interaction with an electromagnetic force is known as the magnetohydrodynamic (MHD) effect.[7–11] This flow results from the Lorentz force exerted on the ions as they move through a transversely applied magnetic field. In the absence of electric fields, the Lorentz force may be expressed as a force density $\rho \mathbf{v} \times \mathbf{B}$ acting on a differential volume of the solution, where ρ is the space charge density, $\mathbf{v}$ is the fluid velocity, and $\mathbf{B}$ is the applied magnetic field. This driving force, pointing toward the direction normal to both $\mathbf{v}$ and $\mathbf{B}$ and playing as an additional term in the Navier–Stokes equation, produces the MHD flow of the fluid.

Recently, colloidal particles suspended in electrolyte solutions prescribed with a magnetic field have been used in a wide variety of applications.[12–17] The addition of magnetic fields or MHD flows to colloidal systems also has potential applications in the manipulation and self-assembly of charged nanoparticles.[18–20] The movement of a charged particle caused by the MHD effect, in which both parts of the electric double layer experience the Lorentz force, is relevant to electrophoresis, but the difference between them is evident. In this work, we examine the translation and rotation of a charged spherical particle in an arbitrary electrolyte solution with a prescribed general velocity field subjected to an external magnetic field in the absence of applied electric fields. The thickness of the double layer surrounding the charged particle is arbitrary relative to the particle radius. The purpose is to determine the motion of the particle induced by the MHD force. Closed-form formulas for the induced translational and angular velocities of the particle are obtained in Eqs. (14) and (15).

II. ANALYSIS

We consider the steady motion of a translating and rotating colloidal sphere of radius a and zeta potential ζ in an unbounded electrolyte solution with a general linear velocity field in the presence of a uniformly applied magnetic field. The thickness of the electric double layer adjacent to the particle surface is arbitrary relative to the particle radius.

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Gravitational effects are ignored. Our objective is to determine the additional particle motion induced by the existence of the constant magnetic field.

To evaluate the translational and angular velocities of the particle induced by the MHD effect, it is necessary to first solve for the equilibrium electric potential and velocity fields in the fluid phase.

A. Equilibrium electric potential distribution

When the Debye–Huckel approximation applicable for the case of low electric potentials is used, the equilibrium double-layer potential distribution ψ in the absence of the imposed velocity field and magnetic field is governed by the linearized Poisson–Boltzmann equation,

$$\nabla^2\psi = \kappa^2\psi, \tag{1}$$

where κ is the Debye screening parameter. The boundary conditions for ψ at the particle surface and infinity are simply that

$$\psi = \zeta \quad \text{at} \quad r = a, \tag{2a}$$

$$\psi = 0 \quad \text{as} \quad r \to \infty, \tag{2b}$$

where r is the radial coordinate from the particle center. The solution of Eqs. (1) and (2) is[1]

$$\psi = \frac{a}{r} e^{-\kappa(r-a)}\zeta. \tag{3}$$

B. Fluid velocity field

With knowledge of the equilibrium electric potential distribution in the electrolyte solution outside the particle, we can now proceed to find the fluid velocity field. The fluid motion in the presence of a magnetic flux density $\mathbf{B}$ is governed by the Stokes equations modified with an electric force (including the Lorentz force) term,

$$\mu\nabla^2\mathbf{v} = \nabla p - \rho(\mathbf{v}\times\mathbf{B} - \nabla\psi), \tag{4a}$$

$$\nabla\cdot\mathbf{v} = 0. \tag{4b}$$

In these equations, $\mathbf{v}$ and p are the fluid velocity and dynamic pressure distributions, respectively, μ is the fluid viscosity, ρ is the total space charge density related to the net local electric field $\mathbf{v}\times\mathbf{B}-\nabla\psi$ by Poisson's equation as[21,22]

$$\rho = \varepsilon\nabla\cdot(\mathbf{v}\times\mathbf{B} - \nabla\psi), \tag{5}$$

where ε is the dielectric permittivity of the fluid, and the expression for ψ has already been given in Eq. (3). Here, the linear superposition of the equilibrium electric field $-\nabla\psi$ and the Lorentz field $\mathbf{v}\times\mathbf{B}$ is valid since the latter is practically weak relative to the former. Note that the induced charge density that arises from the magnetic flux density imposed on the moving fluid is included in Eq. (5).

Equation (4a) shows that a magnetic field can drive an electrolyte solution to move only if there exists an electric current density perpendicular to it. In this study we consider the current flow produced by the general case of a charged sphere translating with velocity $\mathbf{U}_0$ and rotating with angular velocity $\mathbf{\Omega}_0$ in a Stokes flow which can be uniform, rotational, or straining at infinity. The velocity and pressure fields in Eq. (4) can be expressed by the following simple perturbation expansions:

$$\mathbf{v} = \mathbf{v}_0 + \alpha\mathbf{v}_M + O(\alpha^2), \tag{6a}$$

$$p = p_0 + \alpha p_M + O(\alpha^2), \tag{6b}$$

where $\alpha=\varepsilon\zeta|\mathbf{B}|/\mu$ is a small dimensionless parameter, the subscript 0 represents the prescribed Stokes flow in the absence of the magnetic field, and $\mathbf{v}_M$ and p_M denote, respectively, the fluid velocity and pressure distributions caused by the magnetic flux density $\mathbf{B}$ orthogonal to the flow direction.

The governing equations of the zeroth-order flow field without the MHD effect $(\alpha=0)$ are

$$\mu\nabla^2\mathbf{v}_0 = \nabla p_0, \tag{7a}$$

$$\nabla\cdot\mathbf{v}_0 = 0. \tag{7b}$$

The boundary conditions for this flow field with a prescribed general Stokes flow can be expressed as

$$\mathbf{v}_0 = \mathbf{U}_0 + \mathbf{\Omega}_0\times\mathbf{r} \quad \text{at} \quad r = a, \tag{8a}$$

$$\mathbf{v}_0 = \mathbf{U}_\infty + \mathbf{\Omega}_\infty\times\mathbf{r} + \mathbf{E}_\infty\cdot\mathbf{r} \quad \text{as} \quad r\to\infty, \tag{8b}$$

where $\mathbf{U}_\infty$ and $\mathbf{\Omega}_\infty$ are constant translational and angular velocity vectors, respectively, $\mathbf{E}_\infty$ is a constant rate-of-strain dyadic, and $\mathbf{r}$ is the position vector from the particle center. Evidently, the three terms on the right-hand side of Eq. (8b) represent the uniform, pure rotational, and pure straining undisturbed flows, respectively. The translational velocity $\mathbf{U}_0$ and angular velocity $\mathbf{\Omega}_0$ of the particle may result from the prescribed flow field given by Eq. (8b) or from some external force and torque acting on the particle. The solution of Eqs. (7) and (8) is[23]

$$\begin{aligned}\mathbf{v}_0 = {} & \mathbf{U}_\infty + \mathbf{\Omega}_\infty\times\mathbf{r} + \left(\frac{3a}{4r} + \frac{a^3}{4r^3}\right)(\mathbf{U}_0 - \mathbf{U}_\infty) + \frac{3}{4}\left(\frac{a}{r^3} - \frac{a^3}{r^5}\right) \\ & \times(\mathbf{U}_0 - \mathbf{U}_\infty)\cdot\mathbf{rr} + \frac{a^3}{r^3}(\mathbf{\Omega}_0 - \mathbf{\Omega}_\infty)\times\mathbf{r} \\ & + \left(1 - \frac{a^5}{r^5}\right)\mathbf{E}_\infty\cdot\mathbf{r} - \frac{5}{2}\left(\frac{a^3}{r^5} - \frac{a^5}{r^7}\right)\mathbf{E}_\infty:\mathbf{rrr}.\end{aligned} \tag{9}$$

Substituting Eq. (5) and the expansions given by Eq. (6) into Eq. (4) and collecting the first-order terms of the small perturbation parameter α, we obtain

$$\alpha(\mu\nabla^2\mathbf{v}_M - \nabla p_M) = -\varepsilon\mathbf{D}(\nabla\cdot\mathbf{D}), \tag{10a}$$

$$\nabla\cdot\mathbf{v}_M = 0, \tag{10b}$$

where $\mathbf{D}=\mathbf{v}_0\times\mathbf{B}-\nabla\psi$ is the net electric field involving the zeroth-order velocity field $\mathbf{v}_0$. The boundary conditions for the fluid velocity $\mathbf{v}_M$ are

$$\alpha \mathbf{v}_{\mathrm{M}} = \mathbf{U}_{\mathrm{M}} + \boldsymbol{\Omega}_{\mathrm{M}} \times \mathbf{r} \quad \text{at} \quad r = a, \tag{11a}$$

$$\mathbf{v}_{\mathrm{M}} = \mathbf{0} \quad \text{as} \quad r \to \infty. \tag{11b}$$

Here, $\mathbf{U}_{\mathrm{M}}$ and $\boldsymbol{\Omega}_{\mathrm{M}}$ are the translational and angular velocities, respectively, of the particle of the leading order in α resulting from the MHD effect to be determined.

C. Derivation of the induced particle velocities

The induced translational and angular velocities of the particle in Eq. (11a) can be obtained by the use of the reciprocal theorem of Lorentz.[24] Following Tuebner's approach with a generalized reciprocal theorem,[25–29] we can express the force and torque balance equations as

$$6\pi\mu a \mathbf{U}_{\mathrm{M}} = \int\int_{r=a} \boldsymbol{\sigma}^{\mathrm{E}} \cdot \frac{\mathbf{r}}{r} \mathrm{d}S + \varepsilon \int\int\int_{r>a} \bar{\mathbf{v}}^{\mathrm{T}} \cdot \mathbf{D}(\nabla \cdot \mathbf{D}) \mathrm{d}V, \tag{12a}$$

$$8\pi\mu a^3 \boldsymbol{\Omega}_{\mathrm{M}} = \int\int_{r=a} \mathbf{r} \times \left(\boldsymbol{\sigma}^{\mathrm{E}} \cdot \frac{\mathbf{r}}{r}\right) \mathrm{d}S + \varepsilon \int\int\int_{r>a} \bar{\mathbf{v}}^{\mathrm{R}} \cdot \mathbf{D}(\nabla \cdot \mathbf{D}) \mathrm{d}V, \tag{12b}$$

where

$$\bar{\mathbf{v}}^{\mathrm{T}} = \left(\frac{3a}{4r} + \frac{a^3}{4r^3}\right)\mathbf{I} + \frac{3}{4}\left(\frac{a}{r} - \frac{a^3}{r^3}\right)\frac{\mathbf{rr}}{r^2}, \tag{13a}$$

$$\bar{\mathbf{v}}^{\mathrm{R}} = -\frac{a^3}{r^3} \boldsymbol{\varepsilon} \cdot \mathbf{r}, \tag{13b}$$

$$\boldsymbol{\sigma}^{\mathrm{E}} = \varepsilon \left(\mathbf{DD} - \frac{1}{2}|\mathbf{D}|^2 \mathbf{I}\right), \tag{13c}$$

$\mathbf{I}$ is the unit dyadic, and $\boldsymbol{\varepsilon}$ is the alternating unit triadic. In Eqs. (12) and (13), the dyadics $\bar{\mathbf{v}}^{\mathrm{T}}$ (dimensionless) and $\bar{\mathbf{v}}^{\mathrm{R}}$ (with the dimension of length) represent the normalized Stokes flow fields around an uncharged sphere translating and rotating, respectively, in an unbounded fluid for the generalized reciprocal theorem, and $\boldsymbol{\sigma}^{\mathrm{E}}$ is the Maxwell stress tensor.

After the substitution of Eqs. (3), (9), and (13) into Eq. (12) and some mathematical manipulations, we obtain

$$\mathbf{U}_{\mathrm{M}} = \frac{\varepsilon\zeta}{\mu}[M_0(\kappa a)\mathbf{U}_0 - M_\infty(\kappa a)\mathbf{U}_\infty] \times \mathbf{B}, \tag{14a}$$

$$\boldsymbol{\Omega}_{\mathrm{M}} = \frac{\varepsilon\zeta}{\mu}\{[N_0(\kappa a)\boldsymbol{\Omega}_0 - N_\infty(\kappa a)\boldsymbol{\Omega}_\infty] \times \mathbf{B} - N_\infty(\kappa a)\mathbf{E}_\infty \cdot \mathbf{B}\}, \tag{14b}$$

where

$$M_0(t) = \int_1^\infty \frac{\mathrm{e}^{t(1-\eta)}}{24\eta^5}[4(1+t\eta)(3\eta^2-1) - t^2(15\eta^4 + 2\eta^2 - 1)]\mathrm{d}\eta, \tag{15a}$$

$$M_\infty(t) = M_0(t) - \frac{2}{3}, \tag{15b}$$

$$N_0(t) = N_\infty(t) + \frac{1}{6}, \tag{15c}$$

$$N_\infty(t) = \frac{t}{6} - t^2 \int_1^\infty \frac{\mathrm{e}^{t(1-\eta)}}{6\eta^3}\mathrm{d}\eta. \tag{15d}$$

Equation (14a) illustrates that the induced translational velocity of the particle depends on the prescribed translational velocities of the particle and unbounded fluid, and it is in the direction perpendicular to both the relevant prescribed velocities and the applied magnetic field. Equation (14b) indicates that the induced angular velocity of the particle depends on the prescribed angular velocities of the particle and unbounded fluid, and it is also in the direction normal to both the relevant prescribed angular velocities and the imposed magnetic field. Note that, for any given value of κa, the dimensionless mobility parameter M_0 is greater than M_∞, and N_0 is greater than N_∞ by fixed values of 2/3 and 1/6, respectively. Thus, the MHD effect on a charged particle induced by the prescribed translation and rotation of the particle is more significant than that induced by a translational and rotational fluid flow.

Equation (14) shows that the rate-of-strain dyadic of the unbounded fluid flow is able to drive the particle to rotate (also with the mobility parameter N_∞), whereas it makes no contribution to the translation of the particle. This outcome can be understood by a careful examination of the cross product of the velocity field of a pure straining or extensional flow and an applied magnetic field, which indicates that the resulting Lorentz effect is force-free but can exert a couple on the particle.

III. RESULTS AND DISCUSSION

In this section, we first consider the expressions for the dimensionless mobility parameters M_0, M_∞, N_0, and N_∞ in Eq. (15) for the two limiting cases of the parameter κa. Results of the general case for the MHD effect on the particle motion will then be discussed.

In the limit of a very thin electric double layer ($\kappa a \to \infty$), Eq. (15) results in $M_0 = 1$, $M_\infty = 1/3$, $N_0 = 2/3$, and $N_\infty = 1/2$, and thus Eq. (14) reduces to

$$\mathbf{U}_{\mathrm{M}} = \frac{\varepsilon\zeta}{3\mu}(3\mathbf{U}_0 - \mathbf{U}_\infty) \times \mathbf{B}, \tag{16a}$$

$$\boldsymbol{\Omega}_{\mathrm{M}} = \frac{\varepsilon\zeta}{6\mu}[(4\boldsymbol{\Omega}_0 - 3\boldsymbol{\Omega}_\infty) \times \mathbf{B} - 3\mathbf{E}_\infty \cdot \mathbf{B}]. \tag{16b}$$

In the limit of a very thick double layer ($\kappa a \to 0$), Eq. (15) leads to $M_0 = 11/24$, $M_\infty = -5/24$, $N_0 = 1/6$, and $N_\infty = 0$, and then Eq. (14) becomes

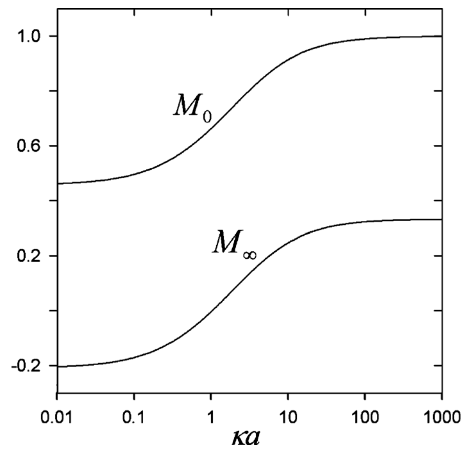

FIG. 1. Plots of the dimensionless mobility parameters M_0 and M_∞ as defined by Eq. (14a) and calculated from Eqs. (15a) and (15b) vs the parameter κa.

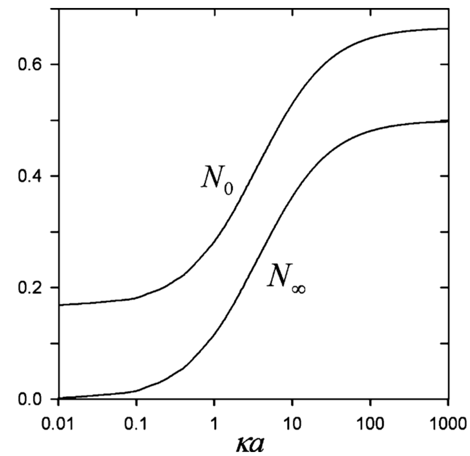

FIG. 2. Plots of the dimensionless mobility parameters N_0 and N_∞ as defined by Eq. (14b) and calculated from Eqs. (15c) and (15d) vs the parameter κa.

$$\mathbf{U}_\mathrm{M} = \frac{\varepsilon\zeta}{24\mu}(11\mathbf{U}_0 + 5\mathbf{U}_\infty) \times \mathbf{B}, \tag{17a}$$

$$\mathbf{\Omega}_\mathrm{M} = \frac{\varepsilon\zeta}{6\mu}\mathbf{\Omega}_0 \times \mathbf{B}. \tag{17b}$$

From Eqs. (16) and (17), we can find that a uniform Stokes flow in the uniform magnetic field will induce a MHD translational velocity of the particle in the direction opposite to (or same as) that induced by the prescribed translation of the particle in the case of a very thin (or thick) electric double layer. A pure rotational flow and a pure straining flow in the uniform magnetic field exert no MHD effect on the particle movement directly in the case of a very thick double layer, but can produce the MHD rotation of the particle in the case of a thin double layer. Also, the MHD effect on a translating and rotating particle is more significant in the case of a thin double layer than in the case of a thick double layer. This outcome is predictable knowing that the space charge density in a narrow region of diffuse ions near the particle surface is much larger for the case of thin double layer than for the case of thick double layer.

The numerical values of the dimensionless mobility parameters M_0, M_∞, N_0, and N_∞, as defined by Eq. (14) and calculated from formulas in Eq. (15), are plotted versus the parameter κa in Figs. 1 and 2. The integrations in these formulas are performed numerically using a personal computer. Except for M_∞, which can be a negative or positive value, all the other three mobility parameters are always positive. Thus, according to Eq. (14b), the prescribed rotation of the particle will always induce its MHD angular velocity in the direction opposite to that induced by the prescribed angular velocity of the fluid. The mobility parameters M_0 and M_∞ are monotonically increasing functions of κa, respectively, from 11/24 and $-5/24$ at $\kappa a=0$ to 1 and 1/3 as $\kappa a \to \infty$, and the mobility parameters N_0 and N_∞ are also monotonically increasing functions of κa, respectively, from 1/6 and 0 at $\kappa a=0$ to 2/3 and 1/2 as $\kappa a \to \infty$.

It is interesting to observe the special case that both the spherical particle and the suspending fluid move with the same translational and angular velocities (which occurs if the particle is freely suspended in the linear fluid flow without externally applied force and torque) and the fluid flow has no rate of strain in the magnetic field. Substituting $\mathbf{U}_\infty = \mathbf{U}_0$, $\mathbf{\Omega}_\infty = \mathbf{\Omega}_0$, and $\mathbf{E}_\infty = \mathbf{0}$ into Eq. (14), we obtain

$$\mathbf{U}_\mathrm{M} = \frac{2\varepsilon\zeta}{3\mu}\mathbf{U}_0 \times \mathbf{B}, \tag{18a}$$

$$\mathbf{\Omega}_\mathrm{M} = \frac{\varepsilon\zeta}{6\mu}\mathbf{\Omega}_0 \times \mathbf{B}. \tag{18b}$$

Namely, the MHD effect on the particle motion in this special case is independent of the double-layer thickness. For a very thick double layer ($\kappa a \to 0$), Eq. (18) can also be obtained in a simple and alternative way by balancing the hydrodynamic drag force and torque with the Lorentz force and moment, respectively, on the particle, for which the total surface charge can be related to the zeta potential as $4\pi a\varepsilon\zeta$.

Finally, we consider the relative importance of the MHD effect on the particle movement. As an example, for a particle with $\zeta = 100$ mV suspended in an aqueous solution at 20 °C with prescribed magnetic flux density $|\mathbf{B}| = 1$ T, Eq. (16a) with $\mathbf{U}_\infty = \mathbf{0}$ predicts that $|\mathbf{U}_\mathrm{M}|/|\mathbf{U}_0| \cong 10^{-7}$. It is understandable that the MHD effect on the particle movement is relatively weak knowing that the electric field induced by the interaction between the magnetic field and the fluid motion, given by the term $\mathbf{v} \times \mathbf{B}$ in Eq. (4a), is much less than the typical external electric field applied in electrophoresis of charged particles.

IV. CONCLUDING REMARKS

In this work, the MHD effects on a translating and rotating colloidal sphere in an arbitrary electrolyte solution prescribed with a general flow field and a constant magnetic field are analyzed at a steady state. The thickness of the electric double layer adjacent to the particle surface can be arbitrary relative to the particle radius. The equilibrium double-layer potential distribution is determined through the use of the Debye–Huckel approximation. The modified Stokes equations governing the fluid velocity field are dealt by using a simple perturbation method and a generalized reciprocal theorem, and explicit formulas for the translational and angular velocities of the particle induced by the MHD effects are obtained in Eq. (14), with the relevant mobility parameters (dimensionless functions of κa) given by Eq. (15) and Figs. 1 and 2. The MHD effects on the particle movement associated with the translation and rotation of the particle and the ambient fluid flow are found to be monotonically increasing functions of κa, and the interaction between the magnetic field and a pure rotational or straining Stokes flow produces no direct MHD effect on the particle when its electric double layer is very thick. The MHD effect caused by the pure straining flow of the electrolyte solution is able to drive the particle to rotate, but makes no contribution to the translation of the particle.

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