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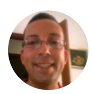

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How does a shear boundary layer affect the stability of a capillary jet?

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The stability of a capillary jet with a shear boundary layer growing over its free surface is studied theoretically. The effect of an infinitely thin boundary layer is first examined from the corresponding singularly perturbed eigenvalue problem. Then, a linear stability analysis of the Navier-Stokes equations for a finite boundary layer thickness verifies the consistency of the previous asymptotic study. We show that, for sufficiently long distances from the boundary layer origin, Rayleigh's dispersion relation is recovered, which means that the layer does not affect the jet's stability in that case. However, for distances on the order of the jet's radius, the perturbation growth factor, the most unstable wave number, and the range of unstable wave numbers increase, while the convective-to-absolute instability transition delays drastically. These results establish a general framework to explain a variety of capillary jet phenomena, from the extended jetting regime of capillary jets in gas co-flows to fundamental features of the dripping faucet. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4884129]

One of the major contributions to the field of capillary flows and microfluidics was made in the 19th century by Rayleigh.[1] His visionary ability to identify the core physics behind observed natural phenomena, in particular, those involving fluids, allowed him to contribute the theoretical models and predictions that constitute the foundations of current scientific understanding for many generic fluid flow classes. Among those, a paradigmatic phenomenon is the capillary breakup of long liquid columns, such as a laminar capillary jet issuing from a slightly open tap. To obtain his fundamental prediction for the size of the resulting droplets, Rayleigh made four major assumptions: (i) the liquid column moved uniformly (basically, as a solid) relative to the environment, (ii) surface tension was present, (iii) the influence of the liquid viscosity was negligible, and (iv) perturbations leading to breakup could be expressed as a fixed amplitude modulated by a sinusoidal spatiotemporal wave function. The vast literature on this subject has shown that the basic Rayleigh result can be finely tuned to account for various additional effects,[2–4] such as the non-negligible presence of a fluid environment, viscous damping, electrical or magnetic effects, etc. Using a spatio-temporal stability analysis, one can show that a sufficiently large convective velocity delays capillary breakup to the point of turning an absolutely unstable jet into a convectively unstable one.[5]

Boundary layers appear in a natural way over the free surface of low-viscosity jets. This occurs not only when the jet is simply emitted from a converging nozzle into a quiescent environment (see Fig. 1(a)), but also in more sophisticated configurations such as flow focusing (see Fig. 1(b)),[6] where the free surface is dragged by a high-speed co-flowing gas stream. How does the boundary layer affect the jet stability? In spite of its fundamental character, this question has not as yet been answered. In this work, we will examine the stability of a capillary jet with a shear boundary layer growing over its free surface. A perturbation expansion will allow us to obtain the dispersion relation for an infinitely small boundary layer thickness. Both the temporal and spatio-temporal analysis of such a relation will be conducted to calculate the perturbation growth factor and the critical condition for which the convective-to-absolute instability transition takes place. Finally, the effect of the (finite)

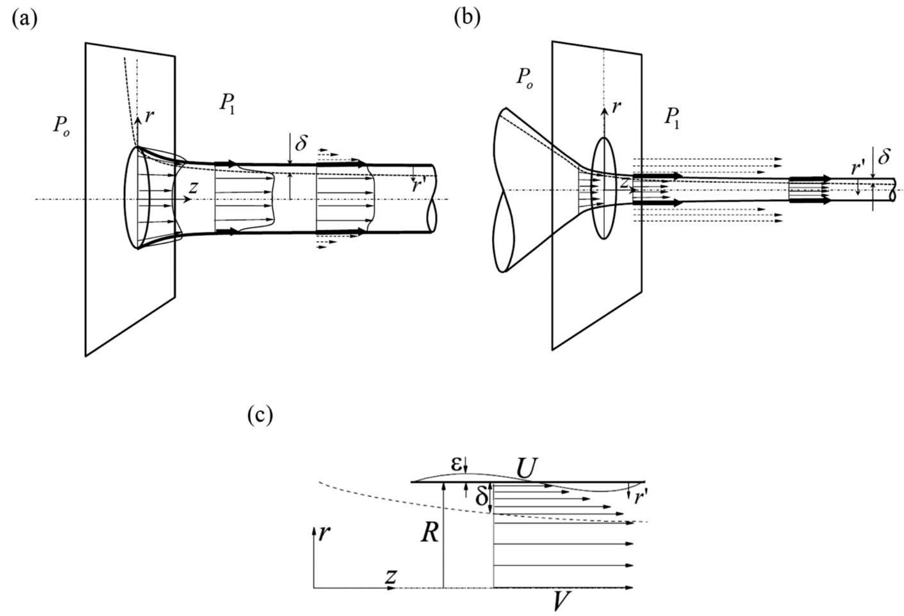

FIG. 1. Sketch of the fluid configuration appearing when a jet is emitted from a converging nozzle into a quiescent environment (a) and in flow-focusing (b). The sketch (c) shows the configuration considered in the present stability analysis.

boundary layer thickness on the jet's temporal instability will be studied from the linearized Navier-Stokes equations. We do not intend to analyze the stability of any particular boundary layer velocity field, but to show that the mere existence of a boundary layer can significantly modify Rayleigh's predictions.

Consider a cylindrical capillary jet of density ρ, viscosity μ, surface tension σ, and radius R, moving with a core velocity V (Fig. 1(c)). This velocity significantly differs from that of the free surface, U, when the jet's surface is subject to a shear force, and the Reynolds number $\mathrm{Re} = \rho V R/\mu$ takes sufficiently large values. In this case, a boundary layer of thickness $\delta(z)$ develops over the jet's free surface from $z = 0$. The thickness δ is assumed to be much smaller than the jet's radius R, and thus the boundary layer curvature can be neglected. The two-dimensional flow inside the boundary layer is given by $\mathbf{v}_{\mathrm{BL}} = \mathrm{Re}_z^{-1/2} U_{\mathrm{BL}}(r, z)\, \mathbf{e_r} + W_{\mathrm{BL}}(r, z)\, \mathbf{e_z}$, where $\mathrm{Re}_z \equiv \rho V z/\mu$, and $\mathbf{e_r}$ and $\mathbf{e_z}$ are the radial and axial unit vectors, respectively. In many cases, such as Blasius, Falkner-Skan, and free-shear boundary layers, self-similar solutions can be found, and the functions U_{BL} and W_{BL} can be expressed in terms of a single scaled variable.[7] In this work, we will restrict ourselves to the case $V - U \sim V$ for which both U_{BL} and W_{BL} are of the same order of magnitude as that of V. We will present a general stability analysis valid for any base flow verifying the above conditions. For illustrative purposes, numerical results will be obtained when U_{BL} and W_{BL} are those given by Blasius's solution.

We will perform a *local* stability analysis of the *quasi-parallel* base flow (bulk and boundary layer) taking place at a given station z. These two conditions reduce to $R \ll \ell_h$ and $\lambda \ll \ell_h$, where ℓ_h and λ are the hydrodynamic length characterizing the base flow and the perturbation wavelength, respectively. The hydrodynamic length can be simply defined in terms of the boundary layer thickness δ as $\ell_h^{-1} \equiv \delta^{-1}\, d\delta/dz$. It can be easily verified that $\ell_h \sim z$ for the boundary layers considered in this study.[7] Therefore, the above two conditions become $R \ll z$ and $\lambda \ll z$. As will be explained, we will restrict our stability analysis to jet's portions with lengths on the order of R, and located sufficiently far away from the boundary layer origin $z = 0$. In this case, the base flow becomes a quasi-parallel velocity field characterized by the fixed parameters z, δ, and U. In addition, the local stability condition is satisfied if $\lambda \lesssim R$.

Two additional conditions are assumed in the present study. The infinitesimal perturbations produce free surface deformations with amplitudes much smaller than the boundary layer thickness

δ (Fig. 1(c)). The outer medium (e.g., a gas with density and viscosity much smaller than ρ and μ, respectively) affects the perturbations only through its influence on the jet's base flow. In what follows, all the quantities are made dimensionless with the characteristic length R, velocity V, and pressure ρV^2.

First, the effect of an infinitely thin boundary layer is examined from the corresponding singularly perturbed eigenvalue problem. The axisymmetric perturbations can be written as

$$u(r,z;t)=\begin{cases}\mathrm{Re}_z^{-1/2}U_{\mathrm{BL}}(r,z)+\varepsilon\,\hat{u}(r)\,e^{i(kz-\omega t)} & \text{for}\quad 1-\delta\lesssim r\leq 1\\ \mathrm{Re}_z^{-1/2}U_{\mathrm{BL}}(\infty,z)+\varepsilon\,\hat{u}(r)\,e^{i(kz-\omega t)} & 0\leq r\lesssim 1-\delta\end{cases}, \tag{1}$$

$$w(r,z;t)=\begin{cases}W_{\mathrm{BL}}(r,z)+\varepsilon\,\delta^{-1}\,\hat{w}_s(r)\,e^{i(kz-\omega t)} & \text{for}\quad 1-\delta\lesssim r\leq 1\\ 1+\varepsilon\,\hat{w}_c(r)\,e^{i(kz-\omega t)} & \text{for}\quad 0\leq r\lesssim 1-\delta\end{cases}, \tag{2}$$

$$p(r,z;t)=\mathrm{We}^{-1}+\varepsilon\,\hat{p}(r)\,e^{i(kz-\omega t)}, \tag{3}$$

$$F=1+\varepsilon\,f\,e^{i(kz-\omega t)}, \tag{4}$$

where u, w, p, and F are the radial and axial components of the velocity, the pressure, and the jet's shape, respectively, $k=k_r+ik_i$ is the wave number, $\omega=\omega_r+i\omega_i$ stands for the frequency, We $=\rho V^2R/\sigma$ is the Weber number, and $\varepsilon\ll 1$ is the perturbation parameter. As will be shown, all the perturbation quantities $\hat{u}$, $\hat{w}_s$, $\hat{w}_c$, $\hat{p}$, and f are of order unity. The factor δ^{-1} was introduced into Eq. (2) to account for the fact that the axial velocity component of the perturbation inside the boundary layer is not commensurate with the rest of perturbation quantities due to the large shear of the base flow in that layer. Besides, the match of the perturbations (2) at the overlap between the jet's bulk and the boundary layer requires that $\hat{w}_s\sim\delta$ for $r\simeq 1-\delta$.

By introducing the perturbations (1)–(3) into the inviscid Navier-Stokes equations for the bulk, and retaining the dominant terms, one gets

$$d\hat{u}/dr+\hat{u}/r+ik\hat{w}_c=0,\quad -i\omega\hat{u}+ik\hat{u}+d\hat{p}/dr=0,\quad -i\omega\hat{w}_c+ik\hat{w}_c+ik\hat{p}=0. \tag{5}$$

The general solution to these equations which verify the regularity conditions $\hat{u}=d\hat{w}_c/dr=0$ for $r=0$ is

$$\hat{u}=-i\beta kI_1(kr)/(\omega-k),\quad \hat{w}_c=\beta kI_0(kr)/(\omega-k),\quad \hat{p}=\beta I_0(kr), \tag{6}$$

where β is an arbitrary constant, and I_m are the modified Bessel functions of the first kind.

The continuity and axial momentum equations for the perturbations in the boundary layer with $\delta\ll 1$ are

$$ik\hat{w}_s-d\hat{u}/dr'=0, \tag{7}$$

$$i(kW_{\mathrm{BL}}-\omega)\hat{w}_s-\hat{u}\,dW_{\mathrm{BL}}/dr'-\alpha\,\mathcal{C}(U_{\mathrm{BL}}d\hat{w}_s/dr'-\hat{w}_s dU_{\mathrm{BL}}/dr')=\alpha\,d^2\hat{w}_s/dr'^2, \tag{8}$$

where the stretched radial coordinate $r'\equiv(1-r)/\delta$ and the parameter $\alpha\equiv\mathrm{Re}^{-1}\delta^{-2}$ have been introduced. The values of $\mathcal{C}$ and δ depend on how the boundary layer grows over the jet's free surface from $z=0$ in a specific application. Without loss of generality, $\mathcal{C}\sim 1$ (e.g., $\mathcal{C}\simeq 5.0$ for Blasius's profile) and $\delta\sim(z/\mathrm{Re})^{1/2}$ ($\alpha\sim z^{-1}$). The two terms proportional to α in Eq. (8) represent non-parallel and viscous effects. It must be noted that the viscous term rises due to the presence of the singular shear layer at the free surface, and does not necessarily vanish as the Reynolds number diverges. Equations (7) and (8) can be recast in the form

$$(W_{\mathrm{BL}}-\omega/k)^2\frac{d[\hat{u}/(W_{\mathrm{BL}}-\omega/k)]}{dr'}-\alpha\left[\mathcal{C}\,U_{\mathrm{BL}}^2\frac{d(\hat{w}_s/U_{\mathrm{BL}})}{dr'}+\frac{d^2\hat{w}_s}{dr'^2}\right]=0. \tag{9}$$

The boundary layer equations must be solved considering the kinematic compatibility condition and the balance of both normal and tangential stresses at the free surface $r' = 0$:

$$f = i\hat{u}/(\omega - Uk), \quad \hat{p} + (1-k^2)f/\mathrm{We} = 0, \quad d\hat{w}_s/dr' - f d^2 W_{\mathrm{BL}}/dr'^2 = 0. \tag{10}$$

Finally, the boundary layer solution must match that of the bulk for $r' \to \infty$, which implies that

$$\hat{u} \to -i\beta k I_1(k)/(\omega - k), \quad \hat{w}_s \to 0 \tag{11}$$

in this limit.

We will consider two cases according to the distance z between the fluid domain analyzed and the point from which the boundary layer develops. For $z \gg 1$, $\alpha \ll 1$, and thus both the non-parallel terms and the viscosity effects on the perturbations can be neglected. On the contrary, $\alpha \sim 1$ for $z \sim 1$, and therefore the non-parallel and viscous contributions must be retained in this case.

The case of a distant jet's section. For $\alpha \ll 1$, the integration of Eq. (9) leads to

$$\frac{\hat{u}}{W_{\mathrm{BL}} - \omega/k} = A, \tag{12}$$

where A is a constant. By considering both the kinematic compatibility condition and the balance of the normal stresses at the free surface $r' = 0$ [see Eqs. (10)], one readily gets that $\hat{u} = i(\omega - Uk)\mathrm{We}\hat{p}/(1-k^2)$ on that surface. Because the pressure does not change through the boundary layer,[7] $\hat{p}$ at $r' = 0$ is the value given by the potential solution (6), and therefore $\hat{u} = i\beta(\omega - Uk)\mathrm{We}I_0(k)/(1-k^2)$. If one introduces both this value and the one for $r' \to \infty$ [see Eq. (11)] into Eq. (12), and equates the two results, then one finally gets Rayleigh's dispersion relation[1]

$$(\omega - k)^2 \frac{I_0(k)}{I_1(k)} - \frac{k(k^2-1)}{\mathrm{We}} = 0. \tag{13}$$

Therefore, as far as the non-parallel and viscous effects on the perturbations in the boundary layer can be neglected, that layer does not affect the jet's stability.

The case of a non-distant jet's section. Now, we analyze the jet's stability under the condition $\alpha \sim 1$. For the sake of simplicity and illustrative purposes, we will consider $\mathcal{C} = 1$ and a Blasius-type velocity profile for the base flow inside the boundary layer, i.e., $U_{\mathrm{BL}} = (U - 1)(r' d\phi/dr' - \phi)$ and $W_{\mathrm{BL}} = U + (1-U)d\phi/dr'$, where $\phi(r')$ is the solution of Blasius's equation for the flat-plane flow.[7] It must be pointed out that only qualitative (but quite general) predictions can be obtained with this approximation. An accurate analysis of the jet's stability would require calculating the evolution of the global modes appearing in the entire fluid domain for each specific application.

The problem formulated by Eqs. (7)–(11) is solved with a spectral collocation technique,[8] and the dispersion relation $D(k, \omega, \mathrm{We}, U, \alpha) = 0$ is obtained numerically. First, we calculate the *temporal* growth of the infinitesimal perturbations. It can be readily seen that the growth rate ω_i only depends on the (real) wave number k, the parameter α, and the Weber number $\mathrm{We}_s \equiv (1-U)^2\mathrm{We}$ based on the velocity slip $1 - U$ (Fig. 2). The limit $\mathrm{We}_s \to 0$ corresponds to a vanishing boundary layer, and thus the results converge to the Rayleigh mode in that limit. The maximum value of the scaled growth factor $\mathrm{We}_s^{1/2}\omega_i$, the corresponding wave number, and the range of wave number

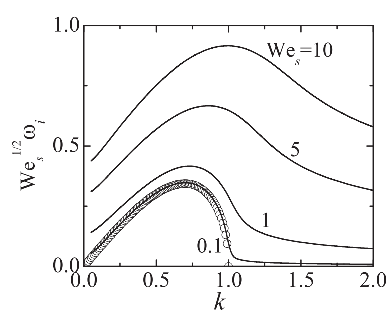

FIG. 2. $\omega_i(k)$ for $\alpha = 1$ and $\mathrm{We}_s = 0.1$, 1, 5, and 10. The circles correspond to Rayleigh's dispersion relation.

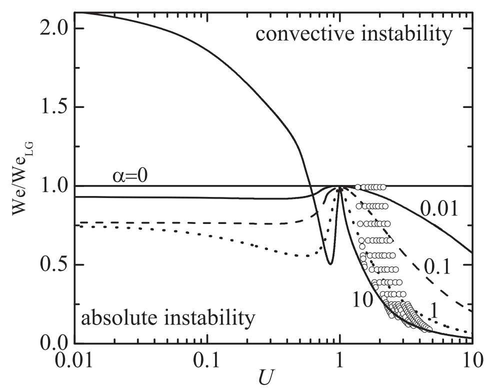

FIG. 3. (Lines) The critical Weber number as a function of U for $\alpha = 0$, 0.01, 0.1, 1, and 10. (Symbols) Experimental jetting realizations produced in Ref. 12 with capillary mercury jets whose free surface was accelerated by a high-speed air stream. No jetting could be observed for We numbers below the ones represented. $\mathrm{We}_{\mathrm{LG}} = 3.12255...$ is Leib and Goldstein's prediction[5] (corrected using Mathematica v 9.0 for Re→ ∞).

for which the jet is unstable increase with We_s. Overall, one can conclude that the presence of a boundary layer enhances the jet's capillary instability.[9]

The critical Weber numbers corresponding to the convective-absolute instability transitions are determined by a *spatio-temporal* analysis of the dispersion relation. This transition has been successfully linked to the jetting-dripping transition in capillary systems.[2] The critical Weber numbers are obtained as those values of We for which Brigg's pinch condition[10,11] is satisfied. This condition establishes that there must be at least one pinching of a k^+ and a k^- spatial branch with $\omega_i = 0$, where the k^+ is the path of $D(k, \omega, \mathrm{We}, U, \alpha) = 0$ in the complex k plane which moves into the $k_i > 0$ half-plane as ω_i increases, while the k^- branch always remains in the $k_i < 0$ half-plane as ω_i increases.

The results of the spatio-temporal analysis are shown in Fig. 3. For $U = 1$ the uniform velocity profile is recovered, and the critical Weber number coincides with that predicted by Leib and Goldstein.[5] It must be noted that the limit $U \to 1$ should be taken with caution because $V - U \ll V$ in this case, and thus the condition $V - U \sim V$ is not verified. The critical Weber number also converges to that calculated from the Leib and Goldstein[5] approximation for $\alpha \ll 1$, where Rayleigh's dispersion relation is re-obtained independently of the velocity slip at the free surface. Because $U \to 1$ and $\alpha \to 0$ as z increases, the absolute instability (dripping) is favored as the jet moves away from the emission point. This is a fundamental result explaining persistent localized breakup phenomena as those reported in Ref. 12. For $\alpha \sim 1$ ($z \sim 1$), the instability transition is dramatically affected by the boundary layer at the free surface. For $U \lesssim 1$, the boundary layer leads to a significant decrease of the critical Weber number. This explains the hysteretic effects appearing when a capillary jet develops from a tap in stagnant air.[13] Close to the critical Weber number, one can observe either meta-stable steady jetting or regular dripping; both of them sustain the jet's nonlinear dynamics.[14] The boundary layer effects are more drastic for $U \gtrsim 1$, reaching a decrease of the critical Weber number by an order of magnitude, and explain why jetting realizations have been observed experimentally in flow focusing configurations for Weber numbers much smaller than Leib and Goldstein's prediction.[12,15,16] Finally, the results suggest that jetting can be produced with very low Weber numbers for $U \gg 1$, which means that the dynamical pressure of both the jet and the droplets after its breakup can be much smaller than the capillary one. This phenomenon was first observed in experiments with capillary metal jets[12] (the "surf-jetting" effect), whose kinematic viscosity takes very low values. A quantitative comparison between the present theoretical results and experiments would require a precise characterization of the boundary layer growth over the jet's free surface in the experimental realizations. This comparison is beyond the scope of this work. Nevertheless, we show in Fig. 3 the experimental jetting realizations produced in Ref. 12 with capillary mercury jets whose free surface was accelerated by a high-speed air stream. The experiments exhibit a remarkable similarity to the behavior of a jet with boundary layer as that predicted by the present theory for $\alpha \simeq 10$.

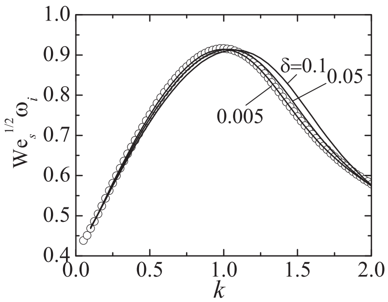

FIG. 4. $\omega_i(k)$ for $\alpha = 1$, $\mathrm{We}_s = 10$, and $\delta = 0.005$, 0.05, and 0.1. The circles correspond to the solution for $\delta \to 0$ obtained from the perturbation expansion described above.

Finally, we analyze the effects on the jet's stability of a boundary layer with a finite thickness δ. For this purpose, the *viscous* Navier-Stokes equations are linearized considering the same base flow as that used in the perturbation expansion, i.e., $\mathbf{v} = \mathbf{e_z}$ and $\mathbf{v} = (\delta \mathrm{Re})^{-1} U_{\mathrm{BL}}(r, z)\, \mathbf{e_r} + W_{\mathrm{BL}}(r, z)\, \mathbf{e_z}$ for $0 \le r \le 1 - \delta$ and $1 - \delta \le r \le 1$, respectively. The functions U_{BL} and W_{BL} are the mapping of Blasius's solution for $0 \le r' \le 5$ onto the layer $1 - \delta \le r \le 1$. In this analysis, the Reynolds number Re and the boundary layer thickness δ must be regarded as two separate control parameters. The problem is solved with a spectral collocation technique,[8] which yields the dispersion relation $D(k, \omega, \mathrm{We}, U, \mathrm{Re}, \delta) = 0$. The equations for the perturbations and the details about the spectral collocation technique can be found in Refs. 8 and 17, respectively.

We calculated the growth rate $\omega_i(k, \mathrm{We}_s, \mathrm{Re}, \delta)$ characterizing the *temporal* evolution of the capillary mode. Figure 4 shows this quantity as a function of the wave number for $\mathrm{We}_s = 10$ and different values of δ. We have taken $\mathrm{Re} = \delta^{-2}$ in all the cases, which corresponds to the choice $\alpha = 1$. For sufficiently small values of δ, the results coincide with the solution obtained for $\delta \to 0$ (and $\alpha = 1$) from the perturbation expansion described above, which constitutes a consistency test for all our calculations. The wave number corresponding to the maximum growth factor slightly increases as δ increases.[9]

Classical Rayleigh breakup provides a physical explanation for droplet generation from the breakup of capillary liquid or gas columns. Several additional effects, such as those of viscous, electric, magnetic, or non-Newtonian origin, may result in deviations from simple theoretical predictions, but the essential basis of the phenomenon is captured by the ability of surface tension to provoke catastrophic pinching of a fluid cylinder over a range of axial perturbation wavelengths. Here, we demonstrate from a theoretical standpoint that the Rayleigh capillary breakup mechanism can be drastically altered by introducing a relative motion between the core and the surface of a capillary liquid column. Indeed, while a sufficiently developed boundary layer does not affect the jet's behavior, the capillary stability is however enhanced and the convective-to-absolute instability transition inhibited for a small enough boundary layer thickness. In particular, the minimum Weber number for steady jetting becomes much smaller than unity for surface velocities much larger than that of the bulk. This implies that the dynamical pressure of the droplets produced after the convective instability development and the subsequent jet breakup can be much smaller than the capillary pressure.[12] These results possess enormous potential in emerging applications (e.g., three-dimensional metal printing) because they pave the way to a very precise control on the deposition of tiny metal droplets produced by jet printing,[18] reducing to a minimum their speed and splashing on impact.

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