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The Optical Model Analysis of the Elastic Scattering of Alpha Particles Using a Surface Absorption Potential

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With 6 Figures

Abstract

The optical model analysis has been applied to the elastic scattering of α-particles on silver at energies of 18.7, 22 and 24.7 Mev. Using a volume absorption potential for the real part and a surface absorption potential for the imaginary part, the several sets of parameters giving satisfactory fits to the experimental data are obtained. The effect of varying these optical model parameters on the differential cross section is discussed.

1. Introduction

The optical model was found to be quite successful in explaining the experimental data of the α-particle elastic scattering. IGO and THALER [1] applied the optical model to the elastic scattering of α-particles by various elements and over a wide range of energies. In these calculations the COULOMB potential of the HILL-FORD charge distribution was employed and satisfactory fits were obtained.

It has also been indicated by IGO [2] that the interaction of α-particles with nuclei is quite insensitive to the potential inside the nucleus, but essentially takes place at the nuclear surface. He used the WOODS-SAXON with the GAUSSian form factors, and found that they could give good agreement with experimental data. He found that the surface potential for the interaction could be represented by:

$$U(r) + iW(r) = -1100 \exp -\left(\frac{r - 1.17A^{1/3}}{0.574}\right) - 45.7\, i \exp -\left(\frac{r - 1.4A^{1/3}}{0.578}\right) \tag{1}$$

More extensive studies of the elastic scattering of α-particles have been made by HUIZENGA and IGO [3]. They approximated the real part of the potential by a parabola given by the formula:

$$V(r) = -\frac{2Ze^2}{r} + \frac{l(l+1)}{2Mr^2} - 1100 \exp -\left(\frac{r - 1.17A^{1/3}}{0.574}\right) \tag{2}$$

Recently, a detailed study of the elastic scattering of α-particles by the optical model treatment was performed by EL-NADI and RIAD [4] on the Ag(α, α) elastic scattering at $E_\alpha = 18.7$ Mev, 22 Mev and 40 Mev, using a volume absorption optical potential with the WOODS-SAXON form factor for both the real and imaginary parts of the potential.

This last work is now extended by carrying out the optical model analysis of the Ag(α, α) elastic scattering data at 18.7, 22 and 24.7 Mev using a surface

absorption optical potential. The results are studied and compared with the earlier results [5] using the volume absorption potential.

2. Optical Model Analysis

The two body optical model interaction potential $V(r)$ between the incident spinless α-particles and the target nuclei may be written as:

$$V(r) = V_{\text{Coul}}(r) + V_N(r), \tag{1}$$

where $V_{\text{Coul}}(r)$ is the COULOMB interaction potential which is chosen to be that produced by a uniform charge distribution of radius R_c given by:

$$R_c = r_{0c} A^{1/3}, \tag{2}$$

where r_{0c} is the COULOMB radius constant and A is the target mass number.

$V_{\text{Coul}}(r)$ is given by:

$$\begin{aligned} V_{\text{Coul}}(r) &= \frac{ZZ'e^2}{2R_c}\left[3 - \frac{r^2}{R_c^2}\right] \quad \text{for} \quad r \leq R_c \\ &= \frac{ZZ'e^2}{r} \quad \text{for} \quad r > R_c, \end{aligned} \tag{3}$$

where Ze and $Z'e$ are the charges of the indicent α-particles and the target nucleus.

The nuclear part of the interaction potential $V_N(r)$ is given by

$$V_N(r) = V f_{NR}(r) + iW f_{NI}(r), \tag{4}$$

where V and W are the real and imaginary depths of the nuclear potential. The form factor associated with the real part of the potential, $f_{NR}(r)$, is given by:

$$f_{NR}(r) = \left[1 + \exp\left(\frac{r - R_N}{a}\right)\right]^{-1} \tag{5}$$

where R_N is the nuclear radius of the target nucleus, given by:

$$R_N = r_{0N} A^{1/3}, \tag{6}$$

where r_{0N} is the nuclear radius constant.

The parameter a included in equation (5) is the diffusivity parameter of the real well depth.

The imaginary potential in the present analysis is taken as a pure surface potential W_s with a surface form factor $g(r)$ given by:

$$g(r) = a_s \frac{df_{NI}(r)}{dr}, \tag{7}$$

where a_s is the diffusivity parameter associated with W_s.

The differential cross section for the elastic scattering process is obtained by squaring the amplitude $A(\Theta)$, given by:

$$A(\Theta) = f_c(\Theta) + \frac{1}{K}\sum_{l=0}^{\infty} \exp[2i\sigma_l]\, c_l(2l+1)\, P_l(\cos\Theta), \tag{8}$$

where $f_c(\Theta)$ is the RUTHERFORD scattering amplitude, defined by:

$$f_c(\Theta) = -\frac{\eta}{2K\sin^2(\Theta/2)} \exp[-i\eta \ln \sin^2(\Theta/2) + 2i\sigma] \tag{9}$$

$P_l(\cos\Theta)$ is the LEGENDRE polynomial.

The calculation of the complex coefficient c_l requires first the solution of the differential equations inside the nuclear region which is given by:

$$\frac{d^2\Psi_l(\varrho)}{d\varrho^2} = -1 + \frac{l(l+1)}{\varrho^2} - \frac{V}{E} f_{NR}(\varrho) - i\frac{W_s}{E} g(\varrho) + \frac{2\eta}{\varrho} \tag{10}$$

where

$$\eta = \frac{MZZ'e^2}{\hbar^2 K} \tag{11}$$

The FOX GOODWIN method [5] was prefered in performing the numerical integration as it yields sufficiently accurate results, in addition to the advantage of saving computation time.

Then c_l can be obtained for every l by matching the wave function inside the nuclear potential region and outside it. The radial wave function outside the nuclear region may be represented by:

$$\Psi_l = F_l(\eta, l) + c_l[G_l(\eta, \varrho) + iF_l(\eta, \varrho)] \tag{12}$$

where F_l and G_l are the regular and irregular COULOMB wave functions.

The numerical calculations were carried out on an IBM 1620 electronic computer. The quantitative measurement of the optimum fit is achieved by minimizing the value of χ^2 given by:

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma_{th}(\Theta_i) - \sigma_{exp}(\Theta_i)}{\Delta\sigma_{exp}(\Theta_i)}\right)^2 \tag{13}$$

where $\sigma_{exp}(\Theta_i)$ and $\sigma_{th}(\Theta_i)$ are the experimental and theoretical cross section at the angle Θ_i.

$\Delta\sigma_{exp}\ \Theta_i$ is the experimental uncertainty associated with $\sigma_{exp}(\Theta_i)$.

3. Results and Comparison with Experiment

It was found that when using a surface absorption potential several sets of optical model parameters can give equally satisfactory fits to the experimental data. Fig. 1, 2 and 3 show the theoretical results calculated by the surface absorption potential together with the experimental results at 18.7 Mev, 22 Mev and 24.7 Mev respectively. The parameters used in the theoretical formula are given in table 1.

Table 1

$E_{(Mev)}$	Set. No.	$V_{(Mev)}$	$W_{s_{(Mev)}}$	a (fm)	a_s(fm)	R_0 (fm)	χ^2
18.7	1	25	10	0.65	0.4	7.709	1.4
	2	35	7.5	0.55	0.4	7.709	3.88
	3	50	14.55	0.58	0.4	7.500	1.3
	4	75	20	0.58	0.5	7.249	2.5
	5	110	24	0.76	0.7	5.554	2.47
	6	150	44	0.63	0.63	6.683	1.3
22	1	35	20	0.7	0.5	7.709	4.8
	2	40	20	0.7	0.5	7.709	2.9
	3	50	30	0.62	0.34	7.5	1.1
	4	75	20	0.66	0.73	7.244	2.7
	5	110	20	0.7	0.98	5.554	0.5
24.7	1	35	14.5	0.7	0.5	7.709	2.8
	2	110	23	0.75	0.85	5.554	1.1
	3	150	40	0.64	0.45	6.683	0.45
	4	60	20	0.7	0.7	7.5	3.1

The fits obtained by the surface imaginary absorption potential analysis are equivalent to that obtained by EL-NADI and RIAD [4] at 18.7 Mev and 22 Mev for the same reaction but using a volume imaginary well depth. This phenomena has been previously shown by PEREY and PEREY [6] for the deuteron elastic scattering. Also satisfactory fits to the experimental points of the $Ag(\alpha, \alpha)$ elastic scattering at 34.7 Mev were obtained at 4 different sets of optical model parameters.

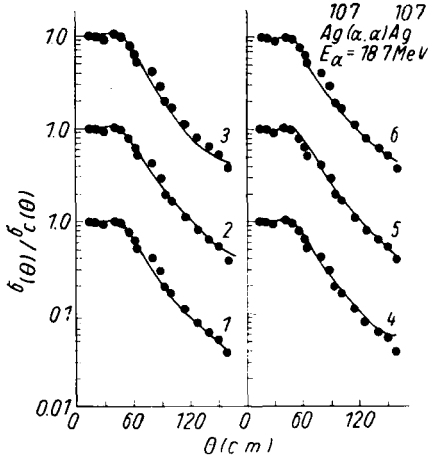

Fig. 1. Comparison of the experimental data (represented by the dots) at incident α-particle energy of 18.7 Mev, with the theoretical angular distribution (represented by the solid curves) for the various sets of table 1.

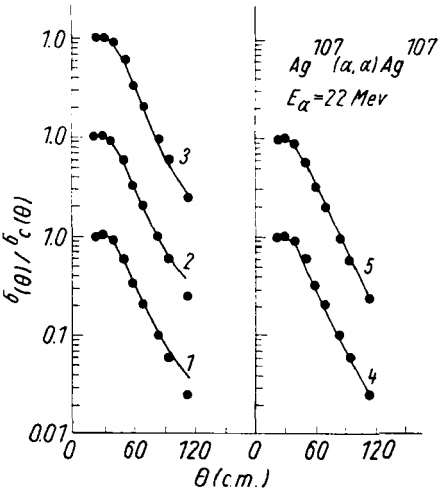

Fig. 2. Comparison of the experimental data (represented by dots) at incident α-particle energy of 22 Mev, with the theoretical angular distribution (represented by the solid curves) for the various sets of table 1

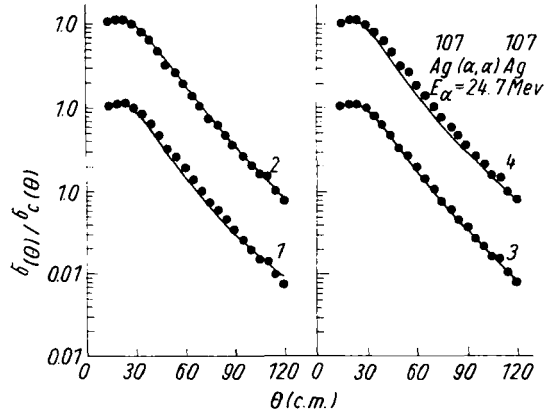

Fig. 3. Comparison of the experimental data (represented by the dots) at incident α-particle energy of 24.7 Mev, with the theoretical angular distribution (represented by the solid curves) for the various sets of table 1

4. Discussion of the optical model parameters

4.1. Comparison with the data obtained using a volume imaginary potential

From table 1, one may observe that the real well depths giving best fit to the experimental data at energies 18.7 Mev and 22 Mev have the same value as

those given before [4] when a volume absorption potential was used, so the relation VR^n is invarient in the two corresponding cases of the surface and volume absorption potential. However, in some few cases a slight deviation was observed as in the case of an incident α-particle of energy 18.7 Mev, when a more satisfactory fit was obtained at $V = 25$ Mev instead of $V = 35$ Mev which yields the best fit in the case of the volume absorption potential. Another example of the deviation is the fact that no fit to the experimental data could be obtained at $V = 150$ Mev at $E_\alpha = 22$ Mev, while a satisfactory fit was found when the data were analysed in terms of the volume imaginary potential at this value of V.

It was also noticed that the values of χ^2 obtained by the surface absorption treatment were generally larger than those obtained by the volume absorption potential. This is in agreement with the results obtained by PEREY and PEREY [6] for the elastic scattering data of deuterons from Cu at 11.8 Mev.

4.2. General study of the optical model parameters of the α-particle elastic scattering

For a certain value of the real well depth, it was possible to obtain optimum fits by more than one value of the imaginary potential W_s. Table 2 gives a series of such potentials which fit the data at the 3 tested energies.

From the table one may notice that only very slight readjustments of the diffuseness parameters a and a_s may be needed in order to obtain optimum fits.

Table 2

$E_{(\mathrm{Mev})}$	$V_{(\mathrm{Mev})}$	Set. No.	$W_{s(\mathrm{Mev})}$	a (fm)	a_s (fm)	X^2
18.7	25	1	10	0.65	0.4	1.45
		2	12	0.68	0.4	1.64
	50	1	14.55	0.58	0.4	1.41
		2	20	0.64	0.6	1.83
	110	1	20	0.76	0.7	2.64
		2	22	0.76	0.7	2.53
		3	24	0.76	0.7	2.47
		4	26	0.76	0.7	2.53
22	50	1	12	0.6	0.5	1.77
		2	15	0.6	0.4	1.44
		3	25	0.58	0.3	1.4
		4	30	0.62	0.34	1.1
24.7	110	1	23	0.75	0.85	1.1
		2	30	0.76	0.73	1.6

4.3. The effect of varying the optical model parameters on the differential cross section for the α-particle elastic scattering

It was found that increasing V by 10% the cross section will be found to decrease by about 15% in the backward scattering. The opposite effect will be noticed when V is decreased. This effect seems to contradict the results obtained for the volume absorption potential [4] which indicates that increasing V would raise the curve especially for the backward scattering. From fig. 4 one may further

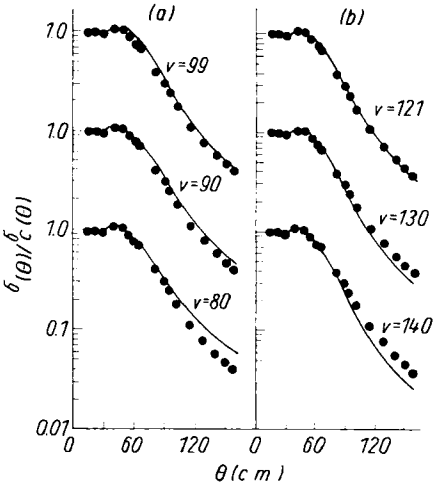

Fig. 4. The effect of varying the real well depth V on the shape of the angular distribution at $E_\alpha = 18.7$ Mev ($W_s = 20$ Mev, $a = 0.76$ fm and $a_s = 0.7$ fm).
The dots represent the theoretical values at $V = 110$ Mev.
The solid curves (a) show the effect of decreasing V.
The solid curves (b) show the effect of increasing V.

conclude that the shape of the angular distribution is insensitive to large changes in the real part V of the nuclear potential at the forward scattering.

It was also found that only one minimum occurs in the χ^2 — V relation.

The four sets of the optical parameters which give best fit to the experimental data at the incident energy of 24.7 Mev, shown in table 1 were found to satisfy the multiplicity relation VR^n where $n = 10$ as was predicted before [4].

Now with respect to the imaginary nuclear potential W_a, it was found that increasing W_s causes the amplitude of the diffraction pattern to be damped and vice versa. This effect is in agreement with the rules stated by GLASSGOLD [7] and PRESKITT and ALFORD [8] during their studies on the elastic scattering

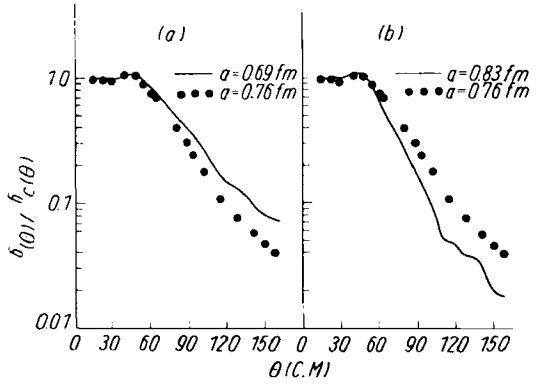

Fig. 5. The effect of varying the diffuseness parameter a on the shape of the angular distribution at $E_\alpha = 18.7$ Mev ($V = 110$ Mev, $W_s = 24$ Mev and $a_s = 0.7$ fm)

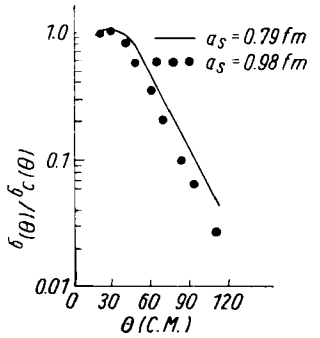

Fig. 6. The effect of varying the diffuseness parameter a_s on the shape of the angular distribution at $E_\alpha = 22$ Mev, $V = 110$ Mev. $W_s = 20$ Mev and $a = 0.7$ fm

of protons. It also agrees with the results obtained by EL-NADI and RIAD [4] for the same reaction.

When the effect of changing the diffusivity parameter a was studied one can observe from fig. 5, that the angular distribution is largely affected by a small variation in a. The reduction of a by about 10% of its best value was found to raise the curve by about 98% of its value at the backward angles. This ratio of increment decreases for smaller angles until the curve becomes insensitive to the change in a in the forward scattering. It may be also noticed that the amplitude of oscillations is increased by increasing a.

The behaviour exhibited by the parameter a in this analysis was in agreement with the results predicted previously by the volume absorption treatment to the same reaction.

It was found that decreasing the diffuseness parameter a_s associated with the imaginary nuclear potential raises the curves. On the other hand the curves are lowered when a_s increases (Fig. 6).

The active cooperation of Mrs F. RIAD and A. RABIE during the progress of this work is gratefully acknowledged.

The authors wish also to express their gratitude to Dr. SALAH HAMID and the staff of the Operation Research Centre, Institute of National Planning for the many facilities offered for performing the computations on the IBM 1620 machine.

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Cairo/U.A.R., Faculty of Science, Cairo University.

Bei der Redaktion eingegangen am 15. April 1970.

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