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Robust Multivariable Inverse-Based Controllers: Theory and Application

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The problem of designing robust inverse-based controllers for multivariable ill-conditioned processes is addressed in this paper. A methodology is developed aimed at finding modified models that, used in inverse-based controllers, are able to make the overall control structure robust. The models are obtained through the solution of an optimization problem which contains in the objective function terms related both to nominal performance and to robustness to uncertainties. A basic advantage of the procedure is that its results can be used in any kind of inverse-based controller. The application to the design of a decoupler and of an internal model controller for a structurally ill-conditioned distillation column is presented to show the effectiveness of the method.

1. Introduction

Robustness is a basic issue in process control both from a theoretical and from a practical point of view. From a theoretical point of view, it is nice to manage to show that, given a nominal process model, an approximate description of uncertainty, and a performance criterion, a controller can be built which satisfies the performance criterion in the worst case included in the region of uncertainty.[1] From a practical point of view, it would be nice to observe that the performance of the designed controller is indeed satisfactory for the different operating conditions in which the plant is required to operate.

A clear implication of the robustness issue is the necessity of a compromise between nominal performance and performance in unfavorable uncertain cases. Even for single input−single output systems, it can be shown that, as the region of uncertainty enlarges, the nominal performance of robust controllers deteriorates, so that the advantages offered by advanced controllers vanish.[2,3]

The problem is even more dramatic for multivariable systems. The inability of advanced controllers to give a satisfactory performance for multivariable systems in uncertain cases is mainly related to their ill conditioning.[4] From a physical point of view, a multivariable system is difficult to control when the effects of the manipulated variables on the controlled ones are almost dependent on each other. A typical example is the control of top and bottom compositions of a distillation column through reflux flow rate L and boilup flow rate V (known as LV control structure). A variation of each of the manipulated variables with the other one kept at a constant value leads to a higher purity of one product at the expense of a decrease in the purity of the other; this is the reason why an improvement in both purities is very hard to achieve and can be obtained only if the internal fluxes in the column are changed relevantly. From a practical point of view, the control problem is similar to the one of attempting to control strictly two different variables while acting only on one manipulated variable.

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Mathematically, this problem is described by the singular value decomposition (SVD; see, e.g., Klema and Laub[5]) of the system. In particular, the process condition number, which is defined as the ratio between the maximum and the minimum singular values measures how strongly the process gain changes with the input direction. If the process condition number is large, therefore, an advanced inverse-based controller would be required in order for the control system to be effective in the presence of all kinds of disturbances and set-point variations.

However, if the process is structurally ill conditioned (i.e., it has a large value of the minimum condition number so that the ill conditioning is independent of the units of measurement used for the process variables), inverse-based controllers show large sensitivity both to input uncertainties and to uncertainties in the individual elements of the transfer function matrix.[1] This is the reason why their use is not recommended unless uncertainties of all kinds are almost negligible or the controller is strongly detuned.

For such processes, therefore, the use of decentralized controllers or, at most, of one way decouplers is usually suggested;[1] these controllers are surely robust, but their performance is very sensitive to the input direction so that the effectiveness of the overall control system is guaranteed only if all of the disturbances and set-point variations that take place during operation are aligned close to the favorable direction (i.e., the direction corresponding to the maximum gain).

A number of *ad hoc* techniques can be found in the literature in order to design particular kinds of multivariable inverse-based controllers which are characterized by robustness in the case of structural ill conditioning. Brambilla and D'Elia[6] present a short-cut method for the design of a robust decoupler; in particular, they propose a multivariable controller based on the singular value decomposition which is composed of a diagonal controller plus a compensator that according to the value of a tuning parameter moves from an inverse-based controller to a controller which does not remove the effects of directionality; moreover, they show a criterion to guide the choice of the free parameter. Zafiriou and Morari[7] present a robust design of multivariable internal model controllers (IMC). Their tech-

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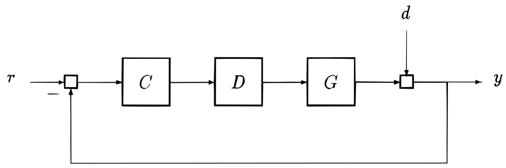

Figure 1. Feedback loop with a decoupler.

nique is based on a modification of the common IMC structure through the introduction of a second filter able to face the problems connected with ill conditioning; moreover, they perform the design of the filters in order to satisfy a robust performance objective in terms of structured singular value (SSV).[8] Semino et al.[9] simplify the design of the double-filter structure by introducing in the second filter a dependence on the process condition number that permits a reduction of the number of tuning parameters. None of these techniques, however, proposes a general approach to the problem of robust design of a generic multivariable inverse-based controller in the case of process ill conditioning.

The study presented in this paper addresses this pretentious issue. The approach is based on the search of a surrogate model of the nominal one which is able to give robustness when used in any kind of inverse-based controller. The auxiliary model is found in order to minimize an objective function which is explicitly related to measures of nominal performance and robustness. The illustration of the method is presented through the design of an optimal decoupler. Thereafter, it is shown that a completely equivalent optimization can be posed in order to find the modified model to be used in an inverse-based controller of a generic kind.

It is important to point out that the optimization technique does not have any pretence of optimality in the worst case sense (as, e.g., the SSV theory in Doyle[8]) given a description of uncertainty. Even if it is not as theoretically sound as these approaches, it does not share the limits of strong conservatism and computational complexity that characterize these mathematical theories. In this paper only a steady-state decoupler and a steady-state modified model are designed; an extension to the dynamic case can be appropriate for some processes.

2. Decouplers: Theory and Properties

The control structure shown in Figure 1 is the common feedback loop for a multivariable process where a compensator $\mathbf{D}$ has been added between the diagonal controller $\mathbf{C}$ and the process $\mathbf{G}$. The system as seen by the controller $\mathbf{C}$ is therefore $\mathbf{G}' = \mathbf{G}\mathbf{D}$. If $\mathbf{D}$ is chosen so that $\mathbf{G}'$ is diagonal, the compensator $\mathbf{D}$ is called a decoupler given that interactions among the variables controlled by $\mathbf{C}$ are eliminated. The decoupler is therefore

$$\mathbf{D} = \mathbf{G}^{-1}\mathbf{G}' \tag{1}$$

where according to the choice of $\mathbf{G}'$ different decouplers can be designed (see, e.g., Ogunnaike and Ray[10]).

The most common choice is $\mathbf{G}' = \mathrm{diag}(\mathbf{G})$ so that the design of the controller $\mathbf{C}$ becomes straightforward (each element can be designed for the corresponding element on the diagonal of $\mathbf{G}$); $\mathbf{D}_\mathrm{I} = \mathbf{G}^{-1}\,\mathrm{diag}(\mathbf{G})$ is called the ideal decoupler. Alternatively, if $\mathbf{G}' = [\mathrm{diag}(\mathbf{G}^{-1})]^{-1}$, one obtains the so-called simplified decoupler $\mathbf{D}_\mathrm{s}$ (the diagonal elements of $\mathbf{D}_\mathrm{s}$ are equal to 1).

Because the inversion of the process matrix can lead to stability and causality problems, a common choice used when the static interactions are the main concern is a steady-state decoupler design as $\mathbf{D}_\mathrm{ss} = \mathbf{G}^{-1}(0)\,\mathrm{diag}(\mathbf{G}(0))$. A further alternative is to use a partial decoupler, designed so that only the interactions on the more important variables are eliminated ($\mathbf{G}'$ is no longer diagonal).

Let us derive some conditions which a steady-state decoupler (and in a more general form also an ideal decoupler) must satisfy.

Theorem: Given a constant square matrix $\mathbf{D}$ of order n, a constant square matrix $\mathbf{K}$ of order n exists so that $\mathbf{D} = \mathbf{K}^{-1}\,\mathrm{diag}(\mathbf{K})$ if and only if the following n independent conditions are satisfied:

$$\mathrm{cof}(d_{i,i}) = \det(\mathbf{D}) \qquad i = 1, ..., n \tag{2}$$

Necessity. From the definition of an inverse matrix and from the definition of a decoupler, it follows that

$$\mathbf{K}^{-1} = \{\hat{k}_{i,j}\} = \left\{\frac{\mathrm{cof}(k_{j,i})}{\det(\mathbf{K})}\right\} \tag{3}$$

$$\mathbf{K} = (\mathbf{K}^{-1})^{-1} = \{k_{i,j}\} = \left\{\frac{\mathrm{cof}(\hat{k}_{j,i})}{\det(\mathbf{K}^{-1})}\right\} \tag{4}$$

$$\mathbf{D} = \{d_{i,j}\} = \left\{\frac{k_{j,j}\mathrm{cof}(k_{j,i})}{\det(\mathbf{K})}\right\} \tag{5}$$

Still from the definition of an inverse matrix and using Laplace's properties of the determinants, one can write

$$\mathbf{D}^{-1} = \left\{\frac{\mathrm{cof}(d_{j,i})}{\det(\mathbf{D})}\right\} = \left\{\frac{\mathrm{cof}(\hat{k}_{j,i})}{\det(\mathbf{D})}\prod_{k,k\neq i} k_{k,k}\right\} = \left\{k_{i,j}\frac{\det(\mathbf{K}^{-1})}{\det(\mathbf{D})}\prod_{k,k\neq i} k_{k,k}\right\} \tag{6}$$

which implies

$$\mathrm{cof}(d_{j,i}) = (\det(\mathbf{K}^{-1}))k_{i,j}\prod_{k,k\neq i} k_{k,k} \tag{7}$$

In particular, when $i = j$,

$$\mathrm{cof}(d_{i,i}) = \frac{1}{\det(\mathbf{K})}k_{i,i}\prod_{k,k\neq i} k_{k,k} = \frac{\prod_k k_{k,k}}{\det(\mathbf{K})} \tag{8}$$

Therefore, all of the principal minors of $\mathbf{D}$ have the same value which can be shown to be equal to $\det(\mathbf{D})$ by using the Binet−Cauchy theorem as follows:

$$\det(\mathbf{D}) = \det(\mathbf{K}^{-1})\,\det(\mathrm{diag}(\mathbf{K})) = \frac{\prod_k k_{k,k}}{\det(\mathbf{K})} \tag{9}$$

Sufficiency. Let us assume that the matrix $\mathbf{D}$ satisfies condition (2). A matrix $\mathbf{K}$ whose elements on the diagonal are arbitrarily chosen and whose remaining elements are chosen as

$$k_{i,j} = \mathrm{cof}(d_{j,i})\frac{\det(\mathbf{K})}{\prod_{k,k\neq i} k_{k,k}} = \frac{\mathrm{cof}(d_{j,i})}{\det(\mathbf{D})}k_{i,i} \qquad (10)$$

is such that $\mathbf{D} = \mathbf{K}^{-1}\,\mathrm{diag}(\mathbf{K})$.

The condition is therefore sufficient; even more, there are ∞^n matrices $\mathbf{K}$ which satisfy the problem. These matrices can be obtained by chosing arbitrarily the elements on the diagonal and computing the remaining elements consequently. Moreover, if $\mathbf{K}$ is such that $\mathbf{D} = \mathbf{K}^{-1}\,\mathrm{diag}(\mathbf{K})$, it belongs to the set of solutions which can be computed with the above-mentioned criterion.

Finally the above-mentioned condition (2) implies that the space of matrices $\mathbf{D}$ which are steady-state decouplers of a generic process $\mathbf{G}(s)$ has dimension $n^2 - n$ so that if one wants to choose an alternative steady-state decoupler to the nominal one, one has to limit the search to such a defined space.

3. Optimization Technique

3.1. Choice of the Optimal Decoupler. The first problem which is addressed through the proposed optimization technique is the design of an optimal robust decoupler. The purpose is to find a steady-state decoupler which belongs to the family previously defined (and therefore satisfies condition (2)) and solves an optimization problem posed to give a good compromise between nominal performance and robustness, leaving some free parameters to adjust the tuning.

A few preliminary considerations are appropriate to anticipate possible measures of robustness and nominal performance.

There are a number of ways in which process–model mismatch can affect the control of a multivariable system including independent variations of the single elements and input uncertainty. Skogestad and Morari[4] have shown that the model minimum condition number γ^* is a measure of the sensitivity to independent uncertainty (Result 1 in their paper) and that the 1-norm of the relative gain array (RGA) $\|\Lambda\|_1$ is a measure of the sensitivity of inverse-based controllers to input uncertainty (Result 2 in the same paper). Moreover, because γ^* and $\|\Lambda\|_1$ are strictly related to each other:[11]

$$\begin{array}{ll} 2\times 2 \text{ systems} & \gamma^* = \|\Lambda\|_1 + \sqrt{\|\Lambda\|_1^2 - 1} \\ n\times n \text{ systems} & \gamma^* \le 2\max(\|\Lambda\|_1, \|\Lambda\|_\infty) \end{array} \qquad (11)$$

the minimum condition number of the model used in the controller can be chosen as a measure of sensitivity and robustness of a model-based controller. In the case of a steady-state decoupler, the model used in the controller and the decoupler are related by the relationship $\mathbf{D} = \mathbf{K}^{-1}\,\mathrm{diag}(\mathbf{K})$ where $\mathbf{K}$ is the gain matrix of the model $\mathbf{G}$. Therefore, the minimum condition number of $\mathbf{D}$ (equal to the minimum condition number of $\mathbf{K}$) can be chosen as a measure of robustness.

On the other hand, the purpose of a steady-state decoupler $\mathbf{D}$ at least in the nominal case is to accomplish an inversion of the steady-state process so that the product $\mathbf{KD}$ has lost both directionality and interactions. This happens, of course, if $\mathbf{D} = \mathbf{K}^{-1}\mathbf{K}'$ where $\mathbf{K}'$ is a generic diagonal matrix; however, if a modified decoupler is used, $\gamma^*(\mathbf{KD})$ (or $\|\Lambda(\mathbf{KD})\|_1$) is a measure of the steady-state directionality and interactions that take place if, for the process $\mathbf{G}$, the steady-state decoupler $\mathbf{D}$ is used.

At this point, the criterion by which the optimal decoupler is chosen can be stated; thereafter, the rationale of all choices is further commented. The following optimization problem is solved:

$$\min_{\mathbf{D}} J(\mathbf{D})$$
$$J(\mathbf{D}) = \alpha\gamma^*(\mathbf{D}) + (1-\alpha)\gamma^*(\mathbf{KD}) + \beta\frac{\|\mathbf{K} - \bar{\mathbf{K}}\|_*}{\|\mathbf{K}\|_*} \qquad (12)$$

where $\mathbf{D}$ is constrained to satisfy condition (2), γ^* is the minimum condition number, $\|\cdot\|_*$ is the Frobenius norm, and $\bar{\mathbf{K}}$ is the matrix that satisfies the problem $\mathbf{D} = \bar{\mathbf{K}}^{-1}\,\mathrm{diag}(\bar{\mathbf{K}})$ with its diagonal elements equal to the corresponding ones in $\mathbf{K}$.

As anticipated above, the minimum condition number of $\mathbf{D}$ is related to the closed-loop robustness; a diagonal controller or a triangular controller makes this parameter minimum (equal to 1), while an inverse based controller has a minimum condition number equal to the one of the process. The minimum condition number of $\mathbf{KD}$ is a measure of nominal performance; the ideal decoupler (and also all decouplers which make this product diagonal) has the minimum possible value (equal to 1). The third term is meant to guide the choice of the decoupler toward those that manage to keep low the sum of the two above-mentioned minimum condition numbers while being as close as possible to the ideal one; without this term there would be infinitely many solutions to the optimization problem. In particular, the parameter β is meant to be taken as $c(1 - \alpha)$ where c is chosen so that the weights of the different terms in the objective function are similar. It is clear, therefore, that when $\alpha \to 0$, the optimal decoupler will tend to the ideal one, while when $\alpha \to 1$, the decoupler tends to the triangular decoupler which cancels the nondiagonal terms in the gain matrix which have the lowest values.

There are a number of reasons why the choice of the optimal modified decoupler is limited to the space defined by condition (2): first, it is nice that a modified process exists (possibly in the uncertainty region) for which the decoupler works perfectly; more relevantly, if an alternative inverse-based scheme is the final purpose of the design (IMC, predictive controller), the final result is not the decoupler but the alternative model to be used as the process model in the control scheme.

As stated above, the parameter c is chosen so that the third term in the summation contributes with a weight similar to those of the other two; in all of the tests that have been accomplished, this has been obtained by choosing c in the interval $[2\gamma^*(\mathbf{K}), 10\gamma^*(\mathbf{K})]$ (without a relevant influence of the precise value on the obtained results). Indeed α remains the only tuning parameter which has to be chosen according to the expected uncertainty, the process ill conditioning, and the desired performance.

It may be useful to define a more direct measure of the results of the optimization as follows:

$$\mathrm{IRI} = \frac{\gamma^*(\mathbf{D})}{\gamma^*(\mathbf{K})} \qquad (13)$$

This parameter, referred as ill-conditioning reduction

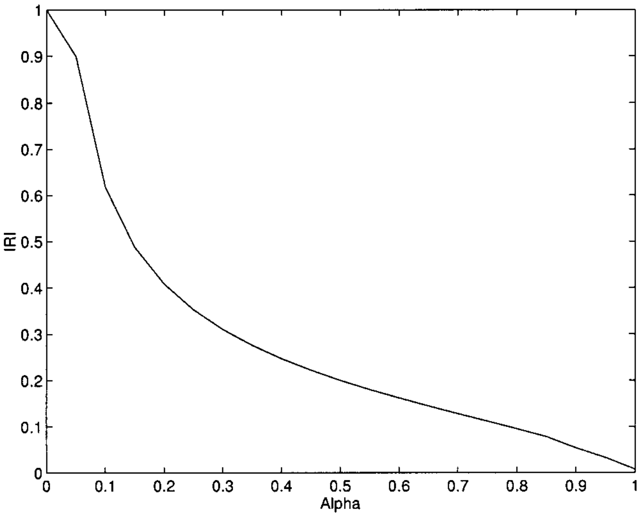

Figure 2. Typical behavior of IRI vs *a*.

index (IRI), explicitly shows how much of the ill conditioning has been eliminated in the controller. It is clear that this index decreases from 1 to $1/\gamma^*(\mathbf{K})$ when α varies from 0 to 1. A typical behavior is reported in Figure 2.

3.2. Choice of the Optimal Modified Model. If the purpose of the optimization is to find a model alternative to the nominal one to be used in an inverse-based controller, one can easily obtain the gain matrix $\bar{\mathbf{K}}$ of the process which has as a steady-state decoupler the result of the optimization. This procedure has been outlined above when the matrix $\bar{\mathbf{K}}$ has been defined. However, in this case the optimization problem can be restated directly as

$$\min_{\bar{\mathbf{K}}^{\mathrm{C}}} J(\bar{\mathbf{K}}^{\mathrm{C}}) \qquad \bar{\mathbf{K}}^{\mathrm{C}} = \bar{\mathbf{K}} - \mathrm{diag}(\bar{\mathbf{K}})$$

$$J(\bar{\mathbf{K}}^{\mathrm{C}}) = \alpha\gamma^*(\bar{\mathbf{K}}^{-1}\,\mathrm{diag}(\bar{\mathbf{K}})) + (1-\alpha)\gamma^*(\mathbf{K}\bar{\mathbf{K}}^{-1}\,\mathrm{diag}(\bar{\mathbf{K}})) + \beta\frac{\|\mathbf{K}-\bar{\mathbf{K}}\|*}{\|\mathbf{K}\|*} \tag{14}$$

The optimization is accomplished only on the off-diagonal elements because the diagonal ones are equal to those of the original gain matrix. The problems stated in eqs 12 and 14 are numerically solved using standard MATLAB software for the minimization of multivariable functions.

4. Resumé of Alternative Techniques

In this section the alternative design techniques with which the proposed one is compared in what follows are briefly summarized.

4.1. Modified Decoupler. This technique consists of the design of a multivariable controller based on the process SVD.[6] The proposed controller is a diagonal one followed by a compensator **D** (as in Figure 1) where **D** is defined as

$$\mathbf{D} = \mathbf{V}\mathbf{F}\boldsymbol{\Sigma}^{-1}\mathbf{U}^{\mathrm{T}} \tag{15}$$

being

$$\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}} = \mathbf{G}^* \tag{16}$$

If the maximum modulus of the RGA is at $\omega = 0$, $\mathbf{G}^* = \mathbf{G}(0)$; if the maximum modulus of the RGA is at a frequency $\omega^* \neq 0$, $\mathbf{G}^*$ is the real matrix that solves the pseudodiagonalization problem $\mathbf{G}(i\omega^*)(\mathbf{G}^*) \approx \mathbf{I}$ (see, e.g., Rosenbrock[12]). **F** is the matrix that contains the tuning parameter α and is defined as

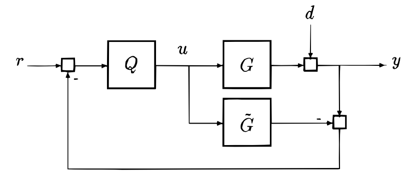

Figure 3. IMC control structure.

$$\mathbf{F} = \alpha\mathbf{I} + (1-\alpha)\boldsymbol{\Sigma} \tag{17}$$

The controller is therefore

$$\mathbf{C}' = \{\mathbf{V}[\alpha\Sigma^{-1} + (1-\alpha)\mathbf{I}]\mathbf{U}^{\mathrm{T}}\}\mathbf{C} \tag{18}$$

where the parameter α, which belongs to the interval $0 < \alpha < 1$, is used to obtain the desired robustness. Brambilla and D'Elia[6] propose also a quite direct criterion to select the tuning parameter.

4.2. Double-Filter IMC. Zafiriou and Morari[7] propose to address the problems related to the ill conditioning by using inside the controller **Q** of the IMC structure (Figure 3) a double filter with a direct effect on the condition number of the regulator at the frequency ω^*, where the condition number of the nominal regulator $\tilde{\mathbf{Q}}$ is largest.

If the SVD of $\tilde{\mathbf{Q}}$ at ω^* is

$$\tilde{\mathbf{Q}}(i\omega^*) = \mathbf{U}_{\tilde{Q}}\boldsymbol{\Sigma}_{\tilde{Q}}\mathbf{V}_{\tilde{Q}}^{\mathrm{H}} \tag{19}$$

and $\mathbf{R}_{\mathrm{U}}$ is the real matrix that solves the pseudodiagonalization

$$\mathbf{U}_{\tilde{Q}}^{\mathrm{H}}\mathbf{R}_{\mathrm{U}} \approx \mathbf{I} \tag{20}$$

The double-filter structure is

$$\mathbf{Q}(s) = \mathbf{R}_{\mathrm{U}}\mathbf{F}_2(s)\,\mathbf{R}_{\mathrm{U}}^{-1}\tilde{\mathbf{Q}}(s)\,\mathbf{F}_1(s) \tag{21}$$

where $\mathbf{F}_1$ is the usual outer filter, while $\mathbf{F}_2$ is chosen in order to decrease the condition number at the appropriate frequencies. Zafiriou and Morari[7] propose to compute the parameters of the double-filter structure through the SSV theory.[8] Semino et al.[9] simplify the design of the double-filter structure by posing

$$\mathbf{F}_2(s) = \mathrm{diag}\left(\frac{1}{\gamma_i^*\alpha s + 1}\right) \tag{22}$$

so that a single parameter α appears in the inner filter. In eq 22 $\gamma_i^* = \sigma_i^*/\sigma_n^*$ and σ_i^* are the singular values of $\tilde{\mathbf{Q}}$ at ω^*. This choice is effective because it can be shown that under mild hypotheses the condition number of **Q** decreases significantly in a range of frequencies around ω^*.

4.3. Comments. It is relevant to point out the advantages of the proposed technique over the alternative ones. If one compares the decoupler of Brambilla and D'Elia[6] with the proposed one, it is clear that the proposed technique permits one to consider as possible candidates a large number of decouplers (all of the matrices that satisfy condition (2)) and discriminate among them by optimizing an objective function which

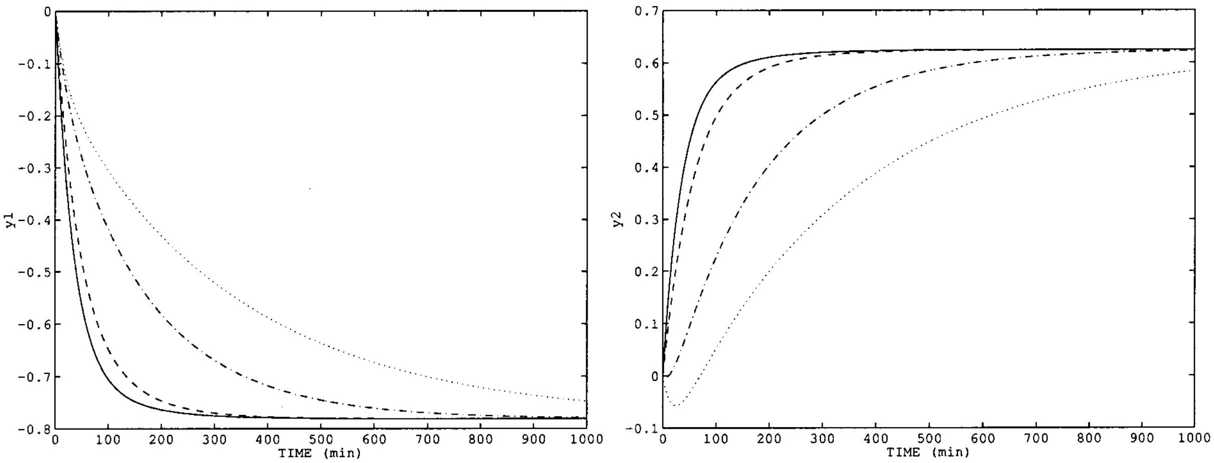

Figure 4. Performance of decouplers in the nominal case for a set-point change in the unfavorable direction: —, $\mathbf{D}_0$; – –, $\mathbf{D}_1$; – · –, $\mathbf{D}_2$; ···, $\mathbf{D}_3$.

depends on a very limited number of parameters (from a practical point of view only α). Moreover, if one wants to design a robust IMC controller or a robust predictive controller, no changes are required in the controller design apart from the substitution of the process gain matrix with the alternative gain matrix that has been obtained. Finally, by avoiding to put the optimization as a worst case uncertainty optimization with the strong implication of conservatism for a multivariable system and its computational complexity, the optimization problem is made more direct and less cumbersome.

5. Case Study and Comparison

It is the purpose of this case study to show that the proposed technique is able to give excellent results both in the design of a decoupler and in the design of an IMC controller as compared to ad hoc techniques for a strongly ill conditioned process.

To this purpose, the structurally ill-conditioned *LV* distillation column presented by Zafiriou and Morari[13] is chosen. The transfer function matrix is the following:

$$\mathbf{G}(s) = \frac{1}{75s+1}\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix} \tag{23}$$

The condition number is $\gamma = 142$ and the minimum condition number is $\gamma^* = 138$ so that the process is very sensitive to input uncertainty. Both condition numbers do not change with frequency.

Diagonal multiplicative input uncertainty with a magnitude bound of 0.2 on both inputs is considered. The "worst-case" uncertainty[1] is when the diagonal multiplicative input uncertainty is

$$\mathbf{\Delta}_{\mathrm{I}} = \begin{pmatrix} 0.2 & 0 \\ 0 & -0.2 \end{pmatrix} \tag{24}$$

Performances are compared on the basis of the integral square error (ISE) on both inputs in the nominal case and for the worst uncertainty.

5.1. Decouplers. Four different decouplers have been considered: (1) the ideal decoupler $\mathbf{D}_0$; (2) a decoupler ($\mathbf{D}_1$) obtained with the presented technique that reduces the minimum condition number of the controller to $^1/_2$ of the $\mathbf{D}_0$ value; (3) a decoupler ($\mathbf{D}_2$) obtained with the presented technique that reduces the minimum condition number of the controller to $^1/_{10}$ of the $\mathbf{D}_0$ value; (4) a modified decoupler as described in section 4.1 ($\mathbf{D}_3$).

Table 1. Characteristics of the Four Decouplers

	$\mathbf{D}_0$	$\mathbf{D}_1$	$\mathbf{D}_2$	$\mathbf{D}_3$
α		0.145	0.786	0.1
γ^*	138	69	13.8	8.3
$\mathbf{D}$	$\begin{pmatrix} 35.1 & 34.5 \\ 34.6 & 35.1 \end{pmatrix}$	$\begin{pmatrix} 17.56 & 16.95 \\ 17.16 & 17.56 \end{pmatrix}$	$\begin{pmatrix} 3.97 & 3.20 \\ 3.68 & 3.97 \end{pmatrix}$	$\begin{pmatrix} 4.89 & -3.05 \\ 4.04 & -4.09 \end{pmatrix}$
$\bar{\mathbf{K}}$		$\begin{pmatrix} 0.878 & -0.847 \\ 1.071 & -1.096 \end{pmatrix}$	$\begin{pmatrix} 0.878 & -0.708 \\ 1.017 & -1.096 \end{pmatrix}$	

Table 2. ISE of Outputs 1 and 2 for the Four Decouplers in the Nominal Case and for the Worst Uncertainty

	$\mathrm{ISE}_1^{\mathrm{nom}}$	$\mathrm{ISE}_2^{\mathrm{nom}}$	$\mathrm{ISE}_1^{\mathrm{unc}}$	$\mathrm{ISE}_2^{\mathrm{unc}}$
$\mathbf{D}_0$	12.0	7.7	2923	5303
$\mathbf{D}_1$	16.3	12.2	546	1743
$\mathbf{D}_2$	39.7	41.8	13.4	721
$\mathbf{D}_3$	72.2	96.8	11.0	717

In the design of both $\mathbf{D}_1$ and $\mathbf{D}_2$ c has been chosen equal to 500. The characteristics of all four decouplers are shown in Table 1 (where α has the appropriate meaning for the different controllers). The diagonal controllers to be used before the decouplers are PI regulators designed for the product **GD** using the BLT technique[14] in all cases. Table 2 shows in the nominal case and for the worst uncertainty the ISE of outputs 1 and 2 (ISE_1 and ISE_2) when a set-point variation in the unfavorable direction $[-0.78 \quad 0.62]^{\mathrm{T}}$ is required. Performances of the four controllers in the nominal and in the uncertain case are shown in Figures 4 and 5, respectively.

Some comments are appropriate:

(a) The reduction of the ill conditioning accomplished through the proposed technique widens the difference between the nominal model and the one used in the regulator.

(b) This discrepancy causes only a marginal decrease in the nominal performance but is very beneficial in the uncertain case.

(c) with an appropriate choice of the reduction of the ill conditioning, results can be even better than with a very accurately designed modified decoupler.

It can be interesting to show more clearly that these nice results are obtained even though the model used in the regulator is very different from the nominal one. To this purpose, Figure 6 shows the open-loop behavior of the nominal model and of models $\mathbf{G}_1$ and $\mathbf{G}_2$ (corresponding to decouplers $\mathbf{D}_1$ and $\mathbf{D}_2$) for a unit step variation of both inputs.

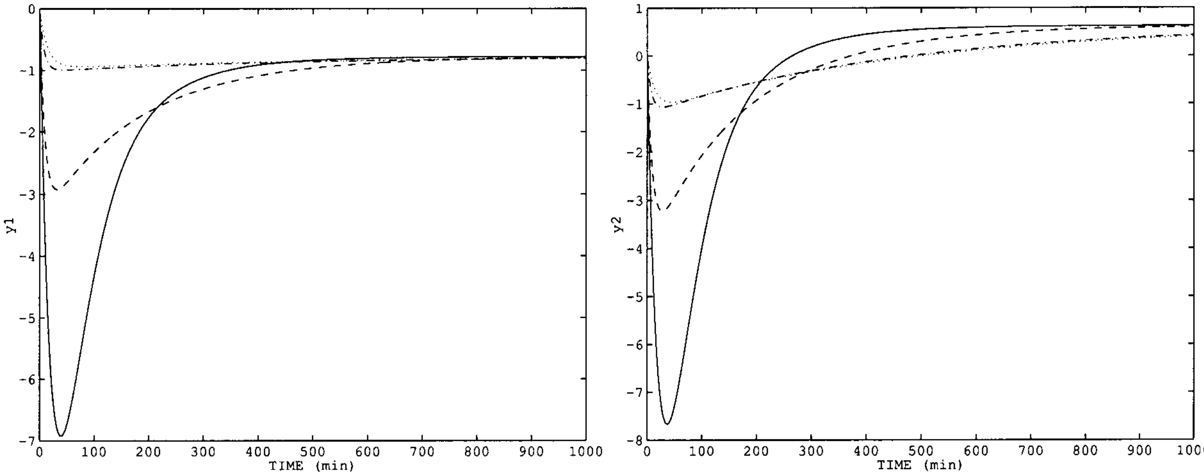

Figure 5. Performance of decouplers in the uncertain case for a set-point change in the unfavorable direction. —, $\mathbf{D}_0$; – –, $\mathbf{D}_1$; – · –, $\mathbf{D}_2$; ···, $\mathbf{D}_3$.

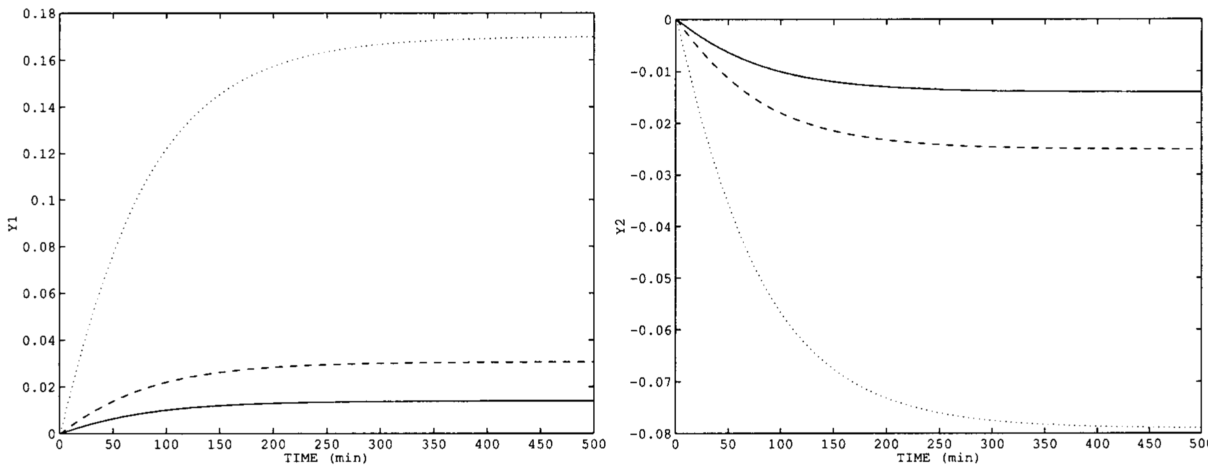

Figure 6. Open-loop behavior of the nominal model (—) and of modified models $\mathbf{G}_1$ (– –) and $\mathbf{G}_2$ (···).

Table 3. Parameters of the Filters for the IMC Controllers

	α_1	α_2	α
IMC_0	20	20	
IMC_1	5	5	
IMC_2	2.1	2.1	
IMC_3	1	1	0.15

Table 4. ISE of Outputs 1 and 2 for the Four IMC Controllers in the Nominal Case and for the Worst Uncertainty

	ISE_1^{nom}	ISE_2^{nom}	ISE_1^{unc}	ISE_2^{unc}
IMC_0	6.3	3.9	1482	2684
IMC_1	6.6	3.6	427	909
IMC_2	8.6	1.9	5.7	63.6
IMC_3	7.2	4.6	1.0	35.3

5.2. IMC Controllers. Four different IMC controllers have been considered: (1) IMC based on the nominal model with a single diagonal filter $\mathbf{F}_1 = \mathrm{diag}(1/(\alpha_i s + 1))$ (IMC_0); (2) same as 1 based on modified model $\mathbf{G}_1$ (corresponding to $\mathbf{D}_1$) (IMC_1); (3) same as 1 based on modified model $\mathbf{G}_2$ (corresponding to $\mathbf{D}_2$) (IMC_2); (4) double-filter IMC[9] based on the nominal model (IMC_3).

The filters have been tuned in order to obtain comparable performance in the nominal case; Table 3 shows the parameters of the filter used to satisfy this condition. Table 4 shows in the nominal case and for the worst uncertainty the ISE of outputs 1 and 2 (ISE_1 and ISE_2) when a set-point variation in the unfavorable direction $[-0.78 \quad 0.62]^T$ is required. Performances of the four controllers in the nominal and in the uncertain case are shown in Figures 7 and 8, respectively.

Here again the use of the same modified models gives results comparable to the one obtained with an ad hoc technique developed to design IMC controllers for ill-conditioned processes.

6. Conclusions

In this paper a methodology has been presented aimed at finding modified models to be used in inverse-based controllers as substitutes of the nominal ones for structurally ill-conditioned processes. Because the purpose of the modified models is to permit the use of inverse-based controllers despite their sensitivity to the process ill conditioning, the models are obtained through the solution of an optimization problem which contains in the objective function terms related both to nominal performance and to robustness to uncertainties. A tuning parameter is left in the objective function as a degree of freedom for the designer to be chosen according to the expected uncertainty, the process ill conditioning, and the desired performance.

The optimization problem can be stated in terms of finding an optimal decoupler that minimizes the objective function. In this case the choice of the decoupler in a well-defined space permits one to find easily the modified model to be used in different kinds of inverse-

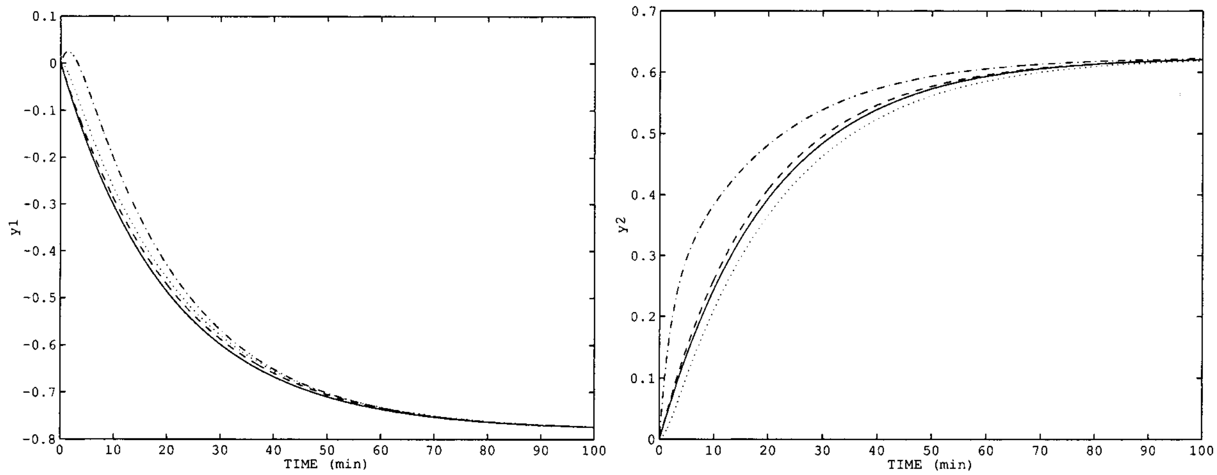

Figure 7. Performance of IMC controllers in the nominal case for a set-point change in the unfavorable direction: —, IMC_0; – –, IMC_1; – • –, IMC_2; ..., IMC_3.

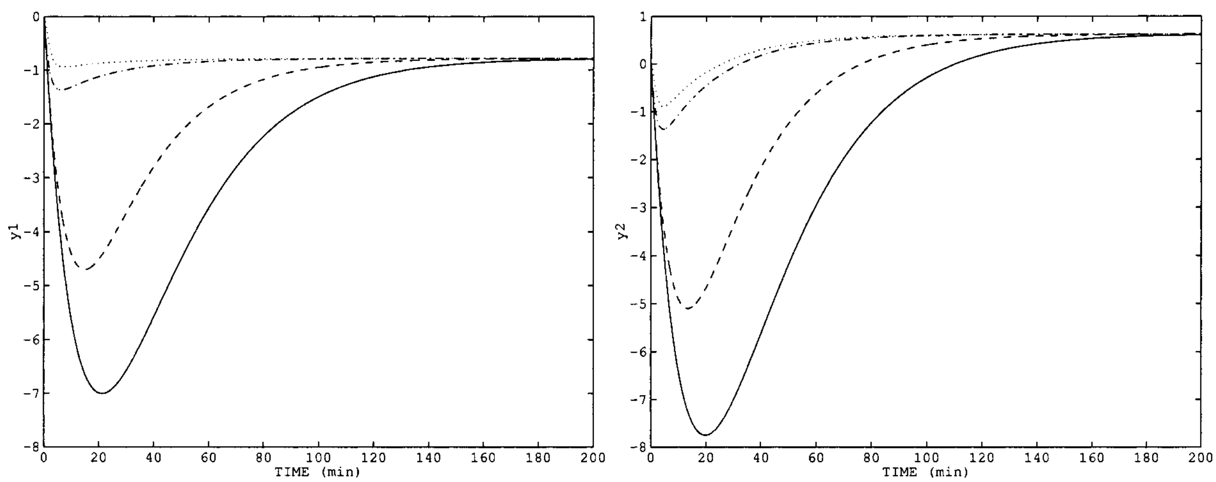

Figure 8. Performance of IMC controllers in the uncertain case for a set-point change in the unfavorable direction. —, IMC_0; – –, IMC_1; – • –, IMC_2; ..., IMC_3.

based controllers. Alternatively, the optimization problem can be posed directly in terms of the parameters of the modified model.

The obtained modified models have been used in different kinds of controllers designed for a structurally ill-conditioned distillation column, and the results have been compared with those of controllers designed through ad hoc techniques.[6,7] The controllers designed according to the proposed technique compare well in all cases; moreover, the technique provides a modified model that can be used in any kind of inverse-based controller, including a multivariable predictive controller. The extension of the technique to the design of a complete dynamic modified model is the subject of further research.

Nomenclature

c = constant
$\mathbf{C}$ = controller
$\mathbf{C}'$ = Brambilla–D'Elia controller
$\mathrm{cof}(a_{ij})$ = cofactor of a_{ij}
d = disturbance
$\mathbf{D}$ = decoupler
$\mathbf{D}_\mathrm{I}$ = ideal decoupler
$\mathbf{D}_\mathrm{S}$ = simplified decoupler
$\mathbf{D}_\mathrm{SS}$ = steady state decoupler
$\mathbf{D}_0$, $\mathbf{D}_1$, $\mathbf{D}_2$, $\mathbf{D}_3$ = decouplers
$\det(\mathbf{A})$ = determinant of $\mathbf{A}$
$\mathrm{diag}(\mathbf{A})$ = matrix whose generic element is $\begin{cases} a_{ij} & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$
$\mathbf{F}$ = Brambilla–D'Elia filter
$\mathbf{F}_1$ = outer filter of the IMC controller with double filter
$\mathbf{F}_2$ = inner filter of the IMC controller with double filter
$\mathbf{G}$ = process
$\mathbf{G}'$ = process as seen by the controller $\mathbf{C}$
$\mathbf{G}^*$ = real matrix that solves $\mathbf{G}(i\omega^*)(\mathbf{G}^*)^{-1} \approx \mathbf{I}$
$\mathbf{G}_1$, $\mathbf{G}_2$ = modified models
$\mathbf{I}$ = identity matrix
IRI = ill-conditioning reduction index
ISE_i = integral square error of output y_i, i.e., $\int_0^\infty (r_i - y_i)^2 \, \mathrm{d}t$
$J(\mathbf{A})$ = objective function of the optimization problem
$\mathbf{K}$ = constant matrix
n = order of matrices, i.e., number of inputs and outputs
$\tilde{\mathbf{Q}}$ = nominal IMC controller
r = set point
$\mathbf{R}_\mathrm{U}$ = real matrix that solves $\mathbf{U}_Q^\mathrm{H}\mathbf{R}_\mathrm{U} \approx \mathbf{I}$
s = Laplace's transform variable
u = control action (input)
$\mathbf{U}$ = unitary matrix of principal output directions
$\mathbf{V}$ = unitary matrix of principal input directions
y = output

Greek Letters

α, β = tuning parameters
$\gamma(\mathbf{A})$ = condition number of $\mathbf{A}$

$\gamma^*(\mathbf{A})$ = minimum condition number of **A**
$\gamma_i^*(\mathbf{A}) = \sigma_i^*/\sigma_n^*$ = ratio of singular values
Δ_I = multiplicative input uncertainty
σ_i^* = generic singular value of $\tilde{\mathbf{Q}}$ at ω^*
$\mathbf{\Sigma}$ = diagonal matrix of singular values
ω^* = critical frequency

Superscripts

c = complementary matrix
H = complex conjugate of a matrix
nom = nominal case
T = transpose of a matrix
unc = uncertain case
$^-$ = modified matrix
$\hat{}$ = generic element of the inverse matrix

Special Notation

$\{a_{ij}\}$ = matrix whose generic element is a_{ij}
$\|\mathbf{A}\|_*$ = Frobenius norm of **A**, i.e., $\sqrt{\Sigma_i \Sigma_j a_{ij}^2}$

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