# All-At-Once and Step-Wise Detailed Retrofit of Heat Exchanger Networks Using an MILP Model

Duy Quang Nguyen,† Andres Barbaro,† Narumon Vipanurat,‡ and Miguel J. Bagajewicz\*,†

University of Oklahoma, 100 E. Boyd St., T-335, Norman, Oklahoma 73019

and The Petroleum and Petrochemical College, Chulalongkorn University, Chulalongkorn Soi12, Phayathai Rd., Pathumwan, Bangkok 10330, Thailand

This paper builds upon the MILP model developed by Barbaro and Bagajewicz (New Rigorous One-Step MILP Formulation for Heat Exchanger Network Synthesis. *Comp. Chem. Eng.* **2005**, 1945–1976), which allows the rigorous one-step grassroots design of heat exchanger networks. For the retrofit, we consider the cases of addition and relocation of heat exchangers allowing control of repiping costs as well as splitting. While previous works considered area reduction in existing exchangers, they very rarely took into account the associated cost, which we now do. We also add all the costs associated with new shells, area addition to existing shells, relocation, and piping changes. We illustrate the power of the formulation with a small example as well as with a crude fractionation unit. Moreover, we discuss step-by-step changes that allow better planning around turnarounds as opposed to an all-at-once solution. Finally, the model also offers a good level of flexibility that opens room for decision making by the users such as allowing/disallowing splitting, considering area and shell additions only (i.e., disallowing relocation), limiting the number of new exchangers and/or the number of relocations, etc. Various design case studies of a same process example are considered to demonstrate the flexibility and versatility of the model.

#### 1. Introduction

One of the most important problems in heat integration has been that of retrofitting existing heat exchanger networks for improved energy efficiency (we leave aside retrofit for controllability or reliability). The literature on grassroots and retrofit models is quite prolific and has been reviewed by Furman and Sahinidis,<sup>2</sup> who also referenced other more detailed reviews produced earlier.

Most of the existing models targeting improved energy efficiency use some approximations. The early literature relies heavily either on the pinch concept entirely, or simply on obtaining energy usage targets followed by some kind of network manipulation to get close to these targeted energy consumptions: Linnhoff and Vredeveld,<sup>3</sup> Tjoe and Linnhoff,<sup>4</sup> Zhelev et al.,<sup>5</sup> Lee et al.,<sup>6</sup> Ahmad and Polley,<sup>7</sup> Polley et al.,<sup>8</sup> van Reisen et al.,<sup>9</sup> Lakshmanan and Bañares-Alcantara,<sup>10</sup> van Reisen et al.,<sup>11</sup> Li and Yao,<sup>12</sup> Polley and Amidpour,<sup>13</sup> and Mehta et al.,<sup>14</sup> Jezowski<sup>15</sup> presents a review of these insight-based methods. Industrial applications of the pinch retrofit method were performed for crude units by Fraser and Gillespie,<sup>16</sup> for an ammonia plant by Lababidi et al.,<sup>17</sup> and for crude units as well as residue cracking units by Querzoli et al.<sup>18</sup>

Retrofit models based on some type of mathematical optimization have also been proposed. Yee and Grossmann<sup>19</sup> presented one of the first attempts of applying the assignment-trans-shipment model to retrofit. Ciric and Floudas<sup>20,21</sup> also used the trans-shipment model. Yee and Grossmann<sup>22</sup> followed a two-stage approach. Jezowski<sup>23</sup> reviewed efforts that use optimization models up to 1994. Briones and Kokossis<sup>24,25</sup> proposed the use of thermodynamic insights to identify targets and followed with optimization, and Sorsak and Kravanja<sup>26</sup> reported

an MINLP model for the retrofit of HENs comprising different exchanger types. Finally, Bjork and Nordman<sup>27</sup> applied optimization to solve large scale models. Pressure drop effects were considered by Nie and Zhu<sup>28</sup> and Silva and Zemp.<sup>29</sup>

Asante and Zhu<sup>30</sup> introduced the concept of network pinch, which is used in a first stage to identify bottlenecks and associated potential changes. For example, Al-Riyami et al.<sup>31</sup> used the area efficiency pinch technology recipe together with the network pinch to identify good solutions for a catalytic cracking unit. Asante and Zhu<sup>30</sup> also proposed that a second stage follows where a superstructure approach is used to develop the appropriate changes. Asante and Zhu<sup>32</sup> also proposed the use of NLP models to identify topological changes in the flowsheet. They follow by optimizing the capital-energy tradeoff. Later, Zhu and Asante<sup>33</sup> improved their method by proposing a targeting phase based on LP and MILP methods, followed by an NLP optimization. Varbanov et al.34 used heuristic paths to identify structural changes followed by NLP optimization. Sieniutycz and Jeżowski35 reviewed the use of the network pinch concept followed by an MILP for structural changes. Finally, Ponce-Ortega et al. 36 presented a superstructure approach leading to an MINLP that also considers process

Athier et al.<sup>37</sup> used simulated annealing to propose modifications iteratively through slave NLP problems. In the same line of stochastic approaches, random search has been used by Bochenek and Jeżowski<sup>38</sup> and Wang et al.<sup>39</sup> Abbas et al.<sup>40</sup> proposed the use of constraint programming to incorporate heuristics. Nie and Zhu<sup>28</sup> proposed a two stage model that considers pressure drop and takes into account shells. Silva and Zemp<sup>29</sup> proposed an NLP procedure to retrofit pressure drop constrained networks. Zhang and Zhu<sup>41</sup> analyzed the structural changes together with the network variables at the same time. In turn, Ma et al.<sup>42</sup> proposed a MILP model followed by an MINLP model. Finally Rezaei and Shafiei<sup>43</sup> coupled genetic algorithms with NLP and ILP methods.

<sup>\*</sup>To whom correspondence should be addressed. E-mail: bagajewicz@ou.edu.

<sup>&</sup>lt;sup>†</sup> University of Oklahoma.

<sup>\*</sup> Chulalongkorn University.

Very few methods exist that do not resort to some decomposition procedure, or that do not use MINLP formulations (Papalexandri and Pistikopoulos<sup>44,45</sup>). Finally, the HEN design problem (and by extension the retrofit one) has been recognized to be NP-Hard by Furman and Sahinidis,<sup>46</sup> which prompted the same authors to call for some sort of approximate methods to solve it.

One of the criticisms that several models have received in the past is that their use requires the engineers to develop considerable knowledge of the model formulation and intricacies to be able to arrive at a good design. The MILP model for the grass-roots design of heat exchanger networks presented by Barbaro and Bagajewicz<sup>1</sup> is capable of becoming a tool that does not require this effort. The method relies on an MILP formulation and does not resort to any of the classical simplifying assumptions; it considers splitting and nonisothermal mixings (the challenges identified by Furman and Sahinidis<sup>46</sup>) and, regarding computation time, performs reasonably on a regular PC. The model is also user-friendly: by changing the value of the appropriate design parameters in the model, the user can easily obtain the desired network without having to fully understand or modify the model.

In this paper, the aforementioned grass-roots design model (Barbaro and Bagajewicz<sup>1</sup>) is extended to consider retrofit. The paper is organized as follows: two models for the retrofitting of heat exchanger network are presented first. This is followed by two examples: a small scale example consisting of seven streams and an industrial scale problem, the heat exchanger network of a crude distillation unit.

#### 2. Retrofit Model

**2.1.** MILP Grass-Roots Design Model. The retrofit model is developed from the grass-roots model, that is, the basic structure of the grass-roots model is conserved and additional sets of constraints are included to consider the network modifications.

The MILP model for the grass-roots design of heat exchanger networks (Barbaro and Bagajewicz<sup>1</sup>) is briefly described next: The model relies on a trans-shipment concept; more specifically, the temperature span of each stream in the problem is divided into several smaller temperature intervals, and then each temperature interval of a hot stream is considered to exchange heat with temperature intervals of cold streams, observing the rules of heat balance and heat exchange feasibility etc. Binary variables are used to indicate the existence of a heat exchanger between a hot stream "i" and a cold stream "j" in an interval "m". The model employs a one-step strategy to simultaneously optimize both the network structure and the heat exchanger areas. The objective is to minimize the total cost, which includes the utilities cost (i.e., operating cost) and the investment cost of the heat exchanger network.

In retrofit cases, there are several exchangers that are already present in the network, and one wants to determine changes to this network that will allow a net reduction in the total annual cost. To achieve this objective, there are several options, namely,

- The addition of new heat exchanger units
- Area expansion/reduction of existing exchangers
- The relocation of existing units. These options are aimed at enhancing the heat integration among process streams and reducing the use of utilities and therefore the operation cost. In essence, the retrofit problem is to optimally add new exchangers, add area to existing exchangers, and/or relocate them (if necessary) such that a certain economic objective is met. Among others, one can

- (i) Maximize the cost saving on utilities minus the annualized capital cost
  - (ii) Maximize the net present value of the retrofit
  - (iii) Maximize the return of the investment
- (iv) Maximize the utility cost savings subject to a certain capital investment limit Two models are presented: a simplified model that disallows the relocation of exchangers and a full model that allows the relocation.

Finally, we rely on all the constraints of the grass-roots design model presented in our previous paper (Barbaro and Bagajewicz<sup>1</sup>), which we do not repeat here. The additional constraints that constitute the retrofit part are discussed next.

**2.2.** New Exchangers and Adjustment of Area to Existing Heat Exchangers. This added set of constraints accounts for the network modifications by means of area addition to existing exchangers and the addition of new heat exchangers to the network. We recall that the grass-roots model considers two cases, one in which only one exchanger is allowed between two streams and the case in which more than one exchanger is allowed between hot stream i and cold stream j. To distinguish these cases, a set i0 of pairs of streams i1 is defined for the former case.

In the case where only one heat exchanger unit is allowed for match (i,j) (i.e.,  $(i,j) \notin B$ ), the model considers two possibilities for area expansion. First, a certain amount of area can be added to the existing heat exchanger using the same shell. The other possibility is then to place the additional area in a new shell. In turn, when  $(i,j) \in B$ , that is, several heat exchangers (not several shells) are allowed for the match, area can be added to existing heat exchangers and also new heat exchangers can be added. The case where  $(i,j) \notin B$  is analyzed first, and the constraints considering area addition to existing heat exchangers are presented below.

New Exchangers and Area Addition to Existing Heat Exchangers  $-(i,j) \notin B$ .

$$A_{ij}^{z} \le A_{ij}^{z^{0}} + \Delta A_{ij}^{z^{0}} + \Delta A_{ij}^{z^{N}}$$

$$z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \notin B$$
 (1)

$$\Delta A_{ij}^{z^0} \leq \Psi_{ij}^{A^0} \Delta A_{ij\max}^{z^0} \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \notin B$$
(2)

$$\Delta A_{ij}^{z^0} \geq 0 \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \notin B \quad (3)$$

$$A_{ij}^{z^N} \ge 0 \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \notin B$$
 (4)

$$A_{ij}^{z^{N}} \le A_{ij\max}^{z^{N}} U_{ij}^{z^{N}} \quad z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \notin B$$
(5)

$$U_{ij}^{z^{N}} + U_{ij}^{z,0} \le U_{ij}^{z,\max} \quad z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \notin B$$
(6)

The first constraint states that the required area for match (i,j) should be smaller than the total new heat exchanger area formed by the existing area  $(A_{ij}^{z_0^0})$ , the area added to the existing

shells  $(\Delta A_{ij}^{z_0})$ , and the area corresponding to new shells  $(A_{ij}^{z_0})$ . In turn, the second constraint restricts the area added to existing shells to a maximum, which can be usually taken as a fraction of the existing area for the shell. Area enhancements as large as 40% are these days possible using technologies such as twisted tube exchangers (Brown Fintube, Houston, TX; http://www.brownfintube.com), which in many cases also reduce the pressure drop in the shell. We introduce here a binary variable  $\Psi_{ij}^{A^0}$ , so that fixed costs associated with this work can be accounted for. For new matches  $A_{ij}^{A^0} = 0$ ,  $\Delta A_{ijmax}^{A^0} = 0$ , and the needed heat exchange area is provided solely by a new heat exchanger. Finally, constraint 5 helps count new shells, and 6 limits them to a maximum.

Notice that exchangers can have their active area reduced, which can be accomplished by plugging tubes, allowing a bypass of part of the fluid through one of the sides of the exchanger or, in the case that there is no bypass, installing the needed piping. Although this feature was implicitly incorporated in earlier models, it was not studied in sufficient detail. To be able to account for the cost of this plugging/bypassing, we add the following constraint:

$$A_{ij}^{z^0} - A_{ij}^z \le RA_{ij\max}^{z^0} \psi_{ij}^{z^0}$$

$$z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \notin B$$
(7)

where  $\psi_{ij}^{z^0}$  is a binary variable and  $\operatorname{RA}_{ij\max}^{z^0}$  is the allowed amount of area reduction. Indeed, in the case where  $A_{ij}^z \geq A_{ij}^{z^0}$ , that is, when area is being added,  $\psi_{ij}^{z^0}$  can be zero. However, when the area required for the match is reduced, then  $A_{ij}^{z^0} - A_{ij}^z \geq 0$  and the binary  $\psi_{ij}^{z^0}$  is forced to be one.

**Capital Costs.** The binary variable  $\psi_{ij}^{z_0^0}$  is used to calculate the fixed cost of area reduction as follows:

$$FCAR = \sum_{z \in Z} \sum_{i \in H^c} \sum_{\substack{j \in C^c \\ (i,j) \in P \\ (i,j) \notin B}} fc_{ij}^{AR} \psi_{ij}^{z^0}$$
(8)

where  $fc_{ij}^{AR}$  is the fixed cost corresponding to area reduction. This fixed cost is dominant in the cost if tube plugging is performed or new bypasses are installed and is zero if existing bypasses are used.

Similarly, the fixed cost for area addition to an existing shell is given by

$$FCAA = \sum_{z \in Z} \sum_{i \in H^z} \sum_{j \in C^z} fc_{ij}^{AA} \Psi_{ij}^{A^0}$$

$$(9)$$

In turn, the fixed cost for a new shell is given by the fixed cost per shell added  $fc_{ij}^{UA}$  multiplied by the number of new shells added.

$$FCUA = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \notin B}} fc_{ij}^{UA} U_{ij}^{z^N}$$

$$(10)$$

We also consider a fixed cost corresponding to the addition of a new unit, given by the fixed cost per unit added  $fc_{ij}^{EA}$  multiplied by the number of new units added.

$$FCEA = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \neq R}} fc_{ij}^{EA} (E_{ij}^z - E_{ij}^{z,0})$$
(11)

where  $E_{ij}^{c}$  is determined in the rest of the model (Barbaro and Bagajewicz<sup>1</sup>).

The variable cost (penalty) for area reduction (VCAR) is calculated using the following equations:

$$VCARE_{ij}^{z} \ge vc_{ij}^{AR}(A_{ij}^{z^{0}} - A_{ij}^{z})$$

$$z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \notin B$$
 (12)

$$VCARE_{ij}^{z} \ge 0 \quad z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P$$
 (13)

$$VCAR = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \notin B}} VCARE_{ij}^z$$
(14)

where  $vc_{ij}^{AR}$  is the variable cost per unit area reduced. Usually, this term is fairly small, especially if the area reduction is achieved by tube plugging, or zero if bypasses are used. Because cost is always subtracting in our objective function (we maximize savings, or profit), then if area reduction takes place  $(A_{ij}^{z_0} - A_{ij}^z) > 0$ ), then VCAR $_{i,j}^Z$  will take the value  $vc_{ij}^{AR}(A_{ij}^{z_0} - A_{ij}^z)$ ; otherwise (no area reduction:  $A_{ij}^{z_0} - A_{ij}^z < 0$ ), eqs 12 and 13 force VCAR $_{i,j}^Z$  to be zero. The variable cost for area addition, in turn, is given by

$$VCAA = \sum_{z \in Z} \sum_{i \in H^z} \sum_{j \in C^z} \left[ vc_{ij}^{A^0} \Delta A_{ij}^{z^0} + vc_{ij}^{A^N} A_{ij}^{z^N} \right]$$
(15)

where  $vc_{ij}^{A^0}$  and  $vc_{ij}^{A^N}$  are the variable costs per unit area added to existing shells and new shells, respectively, which includes entire new units.

New Exchangers and Area Addition to Existing Heat Exchangers  $-(i,j) \in B$ . In this case, since a sequence of two or more heat exchangers may exist for the given pair of streams, the order of each unit in the sequence has to be considered. For this purpose, a new variable  $(\lambda_{ij}^{z,hk})$  is introduced to account for changes in the order in which the exchangers between streams i and j in zone z are located. Formally, we define this binary variable as follows:

$$\lambda_{ij}^{z,hk} = \begin{cases} 1 & \text{If the } h \text{th original heat exchanger is placed in the} \\ & k \text{th position in the retrofitted network} \\ 0 & \text{Otherwise} \end{cases}$$
(16)

Figure 1 shows an example that illustrates how new heat exchangers are identified with respect to the original exchanger locations by means of the values of  $\lambda_{ij}^{z,hk}$ . The figure actually resorts to an extreme case, where a new exchanger is inserted in between two others, who, in turn, switch position. In the shown example,  $\lambda_{ij}^{z,13} = 1$  indicates that the exchanger located in the first position in the original network has been placed in the third position in the retrofitted design, and likewise  $\lambda_{ij}^{z,21} = 1$  indicates that the exchanger located in the second position in the original network has been placed in the first position in the retrofitted design.

The equations for area addition when  $(i,j) \in B$  are presented next:

$$\hat{A}_{ij}^{z,k} \leq \sum_{h=1}^{k_{e}} \hat{A}_{ij}^{z,h0} \lambda_{ij}^{z,hk} + \Delta \hat{A}_{ij}^{z,k0} + \hat{A}_{ij}^{z,kN}$$

$$z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \in B \quad (17)$$

$$\Delta \hat{A}_{ij}^{z,k0} \leq \sum_{h=1}^{k_{e}} \Delta \hat{A}_{ij\max}^{z,h} \lambda_{ij}^{z,hk} \Psi_{ij}^{A^{0},h}$$

$$\Delta \hat{A}_{ij}^{z,k^0} \leq \sum_{h=1}^{\kappa_e} \Delta \hat{A}_{ij\max}^{z,h} \lambda_{ij}^{z,hk} \Psi_{ij}^{A^0,h}$$

$$z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \in B \quad (18)$$

$$\hat{A}_{ij}^{z,k^N} \le A_{ij\max}^{z^N} \hat{U}_{ij}^{z,k^N} \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \in B$$
(19)

$$\sum_{h=1}^{k_e} \lambda_{ij}^{z,hk} \leq 1 \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \in B$$

$$(20)$$

$$\sum_{k=1}^{k_{\max}} \lambda_{ij}^{z,hk} \le 1 \quad z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \in B; 1$$

$$\le h \le k_{e} (21)$$

$$\sum_{k=1}^{k_{\max}} \sum_{h=1}^{k_e} \lambda_{ij}^{z,hk} = k_e \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \in B$$
(22)

$$\Delta \hat{A}^{z,k^0}_{ij} \geq 0 \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \in B \tag{23}$$

$$\hat{A}_{ii}^{z,k^N} \ge 0 \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \in B \quad (24)$$

$$\hat{U}_{ij}^{z,k^{N}} \leq \sum_{h=1}^{k_{e}} \hat{U}_{ij,\max}^{z_{0},h^{N}} \lambda_{ij}^{z,hk}$$

$$z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \in B \quad (25)$$

In the above set of constraints, eq 17 forces the area required for the kth heat exchange match between streams i and j to be lower or equal to the total area installed for that match. Here, the total heat exchanger area for the kth exchanger in the

retrofitted design is formed by the area of an original exchanger (out of the  $k_e$  exchangers between i and j) that has been relocated to the kth position  $(\sum_{h=1}^{k_e} A_{ij}^{z_e,h^0} \lambda_{ij}^{z_e,hk})$ , the area added to the existing shells of that exchanger  $(\Delta \hat{A}_{ij}^{z_e,h})$ , and the area corresponding to new shells  $(\hat{A}_{ij}^{z,k^N})$ . Notice that this constraint uses the definition of  $\lambda_{ij}^{z,hk}$  to precisely account for the existing area of each heat exchanger, making sure the original exchanger is identified. Indeed, whenever an original heat exchanger is utilized to become the kth match,  $\sum_{h=1}^{k_e} \lambda_{ij}^{z,hk}$  is equal to 1 (as guaranteed by constraints 20 and 21), and therefore, its area in the retrofitted network will equal the existing area plus the area addition  $\Delta \hat{A}_{ii}^{z,k'}$ . For new heat exchangers, the summation of  $\sum_{h=1}^{k_e} \lambda_{ij}^{z,hk} = 0$ , and their area will be equal to  $\hat{A}_{ij}^{z,k^N}$ . Equation 18 limits the new area added to existing exchangers. Here, a binary variable  $\Psi_{ij}^{A^o}$ , h is introduced so that fixed costs can be accounted for in the objective function. Equation 19 determines the new number of shells (equivalent to eq 5), and eqs 20, 21, and 22 guarantee that only one value of  $\lambda_{ij}^{z,hk}$  is 1 and that the total number of parameters is equal to the existing number of exchangers. Finally, eq 24 puts a limit on the number of new shells.

As in the case where  $-(i,j) \notin B$ , we also add means to determine if area reduction took place (the equivalent of eq 7):

$$\sum_{h=1}^{k_{e}} \hat{A}_{ij}^{z,h^{0}} \lambda_{ij}^{z,hk} - \hat{A}_{ij}^{z,k} \leq \sum_{h=1}^{k_{e}} R \hat{A}_{ij}^{z,h^{0}} \lambda_{ij}^{z,hk} \psi_{ij}^{z,h^{0}}$$

$$z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \in B$$
 (26)

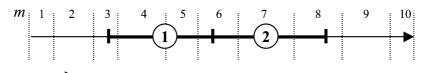
where  $\psi_{ii}^{z,h^0}$  is the corresponding binary variable.

We note that the model allows some relocation within two pairs of streams (i,j) but does not take into account relocation from one pair of streams to another pair. If only exchanger addition is considered, then the following constraint must be added:

$$\lambda_{ij}^{z,h,r} = 0 \forall (h,r) \ni r > h \tag{27}$$

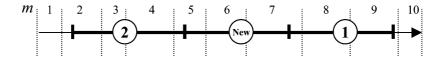
Capital Costs. The equations used for calculating fixed and variable costs are similar to the case where  $(i,j) \notin B$ . We write them without further explanation:

## Before Retrofit



Heat Exchanger Counting

## After Retrofit



S			k	
0,	ik	1	2	3
,	1	0	0	1
$\lfloor n \rfloor$	2	1	0	0

**Figure 1.** Area computation when  $(i,j) \in B$ .

$$FCAR = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (j,j) \in P \\ (j,j) \in P}} fc_{ij}^{AR} \sum_{h} \psi_{ij}^{z,h^0}$$
 (28)

$$FCAA = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} fc_{ij}^{AA} \sum_{h} \Psi_{ij}^{A^0,h}$$
 (29)

$$FCUA = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (j,j) \in P \\ j}} \forall_k \operatorname{fc}^{UA}_{ijk} \times \hat{U}^{z,k^N}_{ij}$$
(30)

$$VCARE_{i,j}^{z,k} \ge vc_{ijk}^{AR} \left( \sum_{h=1}^{k_e} \hat{A}_{ij}^{z,h^0} \lambda_{ij}^{z,hk} - \hat{A}_{ij}^{z,k} \right) \quad z \in Z \quad (31)$$

$$VCARE_{ij}^{z,k} \ge 0 \quad z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \in B$$
(32)

$$VCAR = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (j,j) \in P \\ i}} \sum_{\forall k} VCARE_{ij}^{z,k}$$
 (33)

$$VCAA = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \in B}} \sum_{\forall k} \left[ vc_{ijk}^{A^0} \sum_{k} \Delta \hat{A}_{ij}^{z,k^0} + vc_{ijk}^{A^N} \sum_{k} \hat{A}_{ij}^{z,k^N} \right]$$
(34)

To the above equations, we also need to add the fixed cost for adding new exchangers, which is calculated in eq 11. Finally, in certain cases, the designer would rather limit the number of new heat exchangers added to the original network. To do this, the following constraint is added to the formulation (for both  $(i,j) \notin B$  and  $(i,j) \in B$ ):

$$(E_{ij}^{z} - E_{ij}^{z^{0}}) \le \Delta E_{ij}^{z,\text{max}}$$
(35)

which limits the number of units to be added to the existing plant.

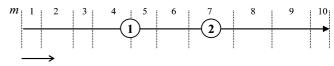
2.3. Piping Changes Limitations. Because the model can "recognize" only the original match (i,j) that an existing exchanger serves, not the relative order of the existing exchangers, there exist situations in which a pair of two existing exchangers are reused at their original locations but swap their respective positions. There is cost associated with such a change in network topology (repiping or even relocation of exchangers), which is not accounted for in our model. To address this issue, two things can be done: (i) performing a postprocessing step to detect any "hidden" cost, if the "hidden" cost is found to be significant; it is suggested to rerun the model fixing the relative order of the existing exchangers (described next); the new result can then be compared with the old result where there is no such limitations; (ii) fixing the relative order of a certain number of pairs of existing exchangers using the set of equations shown next.

The following constraint forbids the change of the position of two consecutive heat exchangers:

$$\begin{split} \sum_{l \in M_{i}^{z}} K_{ij_{2}l}^{z,H} &\leq \sum_{l \in M_{i}^{z}} \hat{K}_{ij_{1}l}^{z,H} \\ t &\leq m & t \leq m \\ t_{j \in P_{i}^{H}} & t \leq m \\ z &\in Z; m \in M^{z}; i \in H_{m}^{z}; j_{1} \in P_{im}^{H}; j_{2} \in P_{im}^{H}; (j_{1}j_{2}) \in \Theta_{i}^{H} \end{cases} (36) \end{split}$$

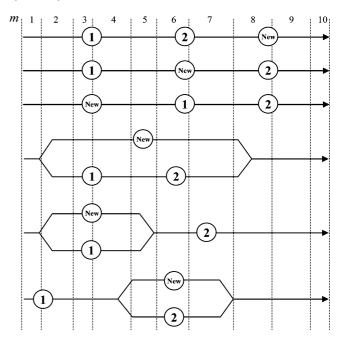
where  $\Theta_i^{\text{H}}$  is the set of pairs of cold streams where the relative order of the associated exchangers is requested not to be altered.

#### Before Retrofit



Heat Exchanger Counting

#### After Retrofit -



**Figure 2.** Fixing respective order of existing exchangers in the retrofitted network.

This constraint states that the match  $(i,j_2)$  cannot start (the binary  $K_{ij2}^{c,H}$  indicates the beginning interval of this match) until the match  $(i,j_1)$  has finished (the binary  $\hat{K}_{ij1}^{c,H}$  indicates the ending interval of this match). Thus, this constraint enforces that the match  $(i,j_2)$  must be located behind the match  $(i,j_1)$ . Several cases that can be observed if this constraint is activated are demonstrated in Figure 2. All of these cases have one thing in common: exchanger 1 is located in front of exchanger 2, as is the case in the original network.

A similar constraint for cold streams can be written:

$$\begin{split} \sum_{l \in M_{i}^{z}} K_{i,jl}^{z,C} &\leq \sum_{l \in M_{i}^{z}} \hat{K}_{i,jl}^{z,C} \\ &_{l \leq n} \qquad \qquad l \leq n \\ &_{i_{2} \in P_{jl}^{C}} \qquad \qquad i_{1} \in P_{jl}^{C} \\ &z \in Z; n \in M^{z}; j \in C_{m}^{z}; i_{1} \in P_{in}^{C}; i_{2} \in P_{in}^{C}; (i_{1},i_{2}) \in \Theta_{i}^{C} \end{cases} \tag{37} \end{split}$$

where  $\Theta_j^{\mathbf{C}}$  is the set of pairs of hot streams where the relative order of the associated exchangers is requested not to be altered.

**Capital Costs.** The cost of new piping associated with stream splitting is calculated using the following equations:

$$\sum_{j \in C^z} (Y_{ijm}^{z,H} - K_{ijm}^{z,H}) - 1 \le \text{NSLH}_i^z$$

$$z \in Z; i \in H_m^z; m \in M_i^z; (i,j) \in P \quad (38)$$

$$\sum_{i \in H^z} (Y_{ijn}^{z,C} - K_{ijn}^{z,C}) - 1 \le \text{NSLC}_j^z$$

$$z \in Z; j \in C_n^z; n \in N_j^z; (i,j) \in P (39)$$

$$CSLH_i^z \ge fcslh_i^z(NSLH_i^z - NSLH_i^{z^0}) \quad z \in Z; i \in H^z$$
 (40)

$$CSLH_i^z \ge 0 \quad z \in Z; i \in H^z \tag{41}$$

$$CSLC_{i}^{z} \ge fcslc_{i}^{z}(NSLC_{i}^{z} - NSLC_{i}^{z^{0}}) \quad z \in Z; j \in C^{z} \quad (42)$$

$$CSLC_i^z \ge 0 \quad z \in Z; j \in C^z \tag{43}$$

$$FCSL = \sum_{z \in Z} \sum_{i \in H^z} CSLH_i^z + \sum_{z \in Z} \sum_{i \in C^z} CSLC_z^z$$
 (44)

Equation 38 calculates the number of splits (NSLH<sub>i</sub><sup>z</sup>) introduced to hot stream "i". The nonzero value of the binaries  $Y_{im}^{c,H}$ and  $K_{ijm}^{z,H}$  indicate that the hot stream "i" in interval "m" exchanges heat with the cold stream "j" and the heat exchanger beginning interval for that hot stream, respectively (Barbaro and Bagajewicz<sup>1</sup>). Hence, if  $\sum_{j \in C^2} Y_{ijm}^{c,H} = 2$ , the hot stream "i" exchanges heat with two colors in the same interval "m", implying one (which is  $\sum_{j \in C} Y_{ijm}^{c,H} - 1$ ) splitting in the hot stream "i". The binary  $K_{ijm}^{c,H}$  is included to disregard the exchanger beginning interval out of the calculation (i.e., it is applied to exchangers-internal interval only). The "new" split introduced to stream "i" is calculated as the number of splits in the retrofitted network minus the original split (NSL $H_i^{z^0}$ ), and the associated cost CSLH<sub>i</sub><sup>z</sup> of this new split is calculated by eq 40. Here, it is assumed that, if the number of splits is reduced  $(NSLH_i^z - NSLH_i^{z^0} < 0)$ , which leads to the removal of pipe, the associated cost is zero. The cost associated with the splitting of cold stream "j" is calculated in the same fashion by using eqs 39, 42, and 43. Finally, eq 44 calculates the total fixed cost associated with stream splitting

**2.4.** Model for the Relocation of Existing Heat Exchangers. So far, it has been implicitly assumed that, if a heat exchanger housed the match (i,j) in the original network, it will also have to house it in the retrofitted network. However, in certain special occasions, it is convenient to relocate the exchanger to house a different match (i',j'). The result of disallowing relocation is that usually the area of existing exchangers (at fixed locations) needs to be significantly adjusted to fit the required heat load in the retrofitted network. If relocation is allowed, the adjustment is minimal since a small size exchanger will be relocated to service the match with a small heat load etc.

To consider all the possible relocation opportunities, one has to account for all combinations of hot streams, cold streams, and existing exchangers by means of new binary variables. For instance, if the original network has 10 hot streams, 10 cold streams, and 10 heat exchangers,  $10^3$  binary variables have to be defined to account for all the possible relocations. It gets even worse for  $(i,j) \in B$ . Such a large number of integers can be avoided if a different strategy is adopted, one that will assign an existing exchanger, regardless of where they are originally, to specific pairs of streams and penalize economically if they end up in a pair where they were not assigned originally.

The relocation model (allowing relocation of exchangers) is described next.

**Relocation Model for**  $(i,j) \notin B$ . We consider the area of each existing exchanger  $e \in E$ , where E is the set of existing exchangers and assignment binaries  $(\delta_{eij}^z)$  that assign exchanger e to a pair of streams (i,j). This reduces the number of binaries

considerably.

$$A_{ij}^{z} = \sum_{e \in E} \delta_{eij}^{z} (A_e + \Delta A_e^0 + A_e^N)$$

$$z \in Z; i \in H^z; j \in C^z; (i,j) \in P; (i,j) \notin B$$
 (45)

$$\Delta A_e^0 \le \Omega_e A A_e^{\text{max}} \quad e \in E$$
 (46)

$$-\Delta A_e^0 \le \varphi_e R A_e^{\text{max}} \quad e \in E$$
 (47)

$$A_e^N \le U_e^N A_{e,\text{max}}^N \quad e \in E \tag{48}$$

$$A_a^N \ge 0 \quad e \in E \tag{49}$$

$$U_e^N \le U_{e,\text{max}}^N \quad e \in E \tag{50}$$

$$\sum_{e \in E} \delta^{z}_{eij} \leq 1 \quad z \in Z; i \in H^{z}; j \in C^{z}; (i,j) \in P; (i,j) \notin B$$

$$(51)$$

$$\sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i) \text{ if } B}} \delta^z_{eij} \le 1 \quad z \in Z; e \in E$$
(52)

In addition to binary variable  $\delta_{eij}^z$ , two other binary variables are used:  $\Omega_e$  and  $\varphi_e$ , which indicate whether area has been added or reduced in the existing exchanger "e" selected for the match (i,j), respectively. Equation 45 states that the needed heat exchange area in the match (i,j) is supplied either by an existing exchanger  $(A_e)$  together with some area adjustment (if needed) or by a brand new area  $(A_e^N)$ , which requires a new shell. The reason this is accounted in this way is because new shells depend on the ability of existing foundations and other layout limitations of the existing exchanger. The area adjustment made to existing exchangers can be (i) area addition  $(\Delta A_e^0 > 0)$  or (ii) area reduction  $(\Delta A_e^0 < 0)$ .

Constraint 46 states that the amount of area addition  $(\Delta A_e^0)$  must not exceed the limit  $AA_e^{max}$ , while constraint 47 enforces the amount of area reduction  $(-\Delta A_e^0)$  to be less than the allowed value  $RA_e^{max}$ .

Note that, if it is an area addition  $(\Delta A_e^0 > 0)$ , constraint 46 enforces the binary variable  $\Omega_e$  to take the value 1, while constraint 47 trivially satisfies. On the other hand, if it is an area reduction  $(\Delta A_e^0 < 0)$ , then constraint 47 enforces the binary variable  $\varphi_e$  to take the value 1, while constraint 46 trivially satisfies. When these two variables are not forced to be 1 by the constraints, they are forced to be zero by the objective function where they participate in the accounting of the fixed costs associated to area reduction and addition.

Constraint 48 counts the number of new shells added, constraint 50 limits the number of new shells to be less than the allowed number. Finally, logical constraint 51 enforces that only one existing heat exchanger at most (either original or a relocated one) is used in the match (i,j), and eq 52 states that an existing heat exchanger is "utilized" only one time at most (either in the original position or another position, or not used).

We also note that this model considers area and exchanger addition. Indeed, aside from considering the area reduction of existing shells, eq 45 assumes that area will be added to existing shells or to new shells of the same existing units. The reason for this is that the fixed and variable costs associated with adding a shell to an existing exchanger can vary because of size, type of exchanger, etc. The question remains then as to how the

model can handle the addition of a completely new exchanger where none existed. To do that, the set of existing exchangers is enlarged to incorporate fictitious exchangers with zero area. Thus, for these exchangers, one should set  $A_e = 0$  and  $AA_e^{max} = RA_e^{max} = 0$ , so that  $\Delta A_e^0 = 0$ . The resulting equation will be just brand new area.

**Capital Costs.** The costs are calculated using the following equations:

$$FCAR = \sum_{e \in F} f c_e^{AR} \varphi_e$$
 (53)

$$FCAA = \sum_{e \in E} fc_e^{AA} \Omega_e$$
 (54)

$$FCUA = \sum_{e \in F} fc_e^{UA} U_e^N$$
 (55)

$$VCARE_e \ge vc_e^{AR}(-\Delta A_e^0) \quad e \in E$$
 (56)

$$VCARE_e \ge 0 \quad e \in E$$
 (57)

$$VCAR = \sum_{e \in E} VCARE_e$$
 (58)

$$VCAAE_e \ge vc_e^{AA}\Delta A_e^0 \quad e \in E$$
 (59)

$$VCAAE_e \ge 0 \quad e \in E$$
 (60)

$$VCAA = \sum_{e \in F} (VCAAE_e + vc_e^{AA^N} A_e^N)$$
 (61)

where the new cost coefficients  $fc_e^{AR}$ ,  $fc_e^{AA}$ , and  $fc_e^{UA}$  are the fixed costs of area reduction, area addition, and installation of new shells to the existing exchanger "e", respectively. In turn,  $vc_e^{AR}$ ,  $vc_e^{AA}$ , and  $vc_e^{AA}$  are the corresponding variable area costs. Equations 53, 54, and 55 calculate the total fixed cost of area reduction, area addition, and installation of new shells made to existing exchangers, respectively. Equations 56, 57, and 58 calculate the variable cost of area reduction, while the variable cost of area addition is calculated by eqs 59, 60, and 61. Note that we differentiate between new area added in the form of new shell(s) to an existing exchanger and in the form of new area added to existing shells of the exchanger.

The relocation fixed cost (FCR) is given by

$$FCR = \sum_{z \in Z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,i) \notin B}} \sum_{e \in E} \delta_{eij}^z rc_{eij}^z$$
(62)

We note that the original locations are not explicitly part of the model but are implicitly through the relocation cost. More specifically, when the existing exchanger "e" is placed at its original location, the pair (i,j), in the retrofitted network, one sets  $rc_{eij} = 0$ . Relocation to any other pair incurs a high positive cost, which is the real cost of relocation or even higher if such relocation is required to be avoided for other reasons than cost (i.e., the relocation can be penalized/avoided by using a very high cost).

We note that the relocation model can be used to obtain the retrofitted network, allowing area adjustment only. We present both because there are computational differences as the relocation model contains a different number and type of integers. We intend to investigate the performance of both.

**Relocation Model for**  $(i,j) \in B$ . The relocation model for this case is a slight extension to consider more than one exchanger per pair of streams. Because there is no need to consider the original locations, the extension is fairly straightforward. We make use of a new binary variable,  $\delta_{eij}^{z,k}$ , to indicate that exchanger e has been assigned to the eth exchanger between the pair of streams (i,j). The rest of the variables are the same as in the case of  $(i,j) \notin B$  described above. Most of the equations involving only existing exchangers (contain only the index "e") are the same as in the case  $(i,j) \notin B$ . Only the equations written for a pair of streams (i,j) in zone z (contain the indices z, i, j) need to be adapted to also include the index e, indicating the eth exchanger between the pair of streams (i,j) for the case  $(i,j) \in B$ . They are presented without further explanation next:

$$A_{ij}^{z,k} = \sum_{e \in E} \delta_{eij}^{z,k} (A_e + \Delta A_e^0 + A_e^N)$$

$$z \in Z; 1 \le k \le k_{\text{max}}, i \in H^z; j \in C^z; (i,j) \in P; (i,j) \in B$$
 (63)

$$\sum_{e \in E} \delta_{eij}^{z,k} \le 1 \quad z \in Z; 1 \le k \le$$

$$k_{\max}, i \in \mathit{H}^{z}; j \in \mathit{C}^{z}; (i,j) \in \mathit{P}; (i,j) \in \mathit{B} \ (64)$$

$$\sum_{i \in H^z} \sum_{j \in C^z} \sum_{k=1}^{k_{\text{max}}} \delta_{eijk}^z \le 1 \quad z \in Z; e \in E$$

$$(65)$$

$$(i,j) \in P$$

$$(i,j) \notin B$$

**2.5. Objective Functions.** For the retrofit of an existing heat exchange network, one objective is to maximize the value of savings, which can be expressed as:

$$\max{\text{Value of Savings}} = {\text{UtcostSav} - \text{Capcost}/n}(66)$$

where UtCostSav and CapCost are the annual savings in utility costs and the capital costs, respectively, and *n* is the number of years used to annualize capital cost.

In terms of net present value (profit), the objective is

$$\max\{\text{NPV}\} = \max\{\left(\sum_{l=1}^{n} \text{df}_{l} \times \text{UtilcostSav}_{l}\right) - \text{Capcost}\}$$
(67)

where  $df_l$  is the discount factor and  $UtCostSav_l$  the savings in year "l". The utility cost savings ( $UtCostSav_l$ ) is given by

$$\text{UtilcostSav}_{l} = \sum_{z} \left\{ \sum_{i \in \text{HU}^{z}} \sum_{j \in C^{z}} c_{i,l}^{\text{H}} (F_{i,\text{curr}}^{\text{H}} - F_{i}^{\text{H}}) \Delta T_{i} + \sum_{j \in \text{CU}^{z}} \sum_{i \in H^{z}} c_{j,l}^{\text{C}} (F_{j,\text{curr}}^{\text{C}} - F_{j}^{\text{C}}) \Delta T_{j} \right\} \forall l$$
 (68)

The capital cost (CapCost) in the model allowing relocation is given by

$$Capcost = FCAR + FCAA + FCUA + FCEA + VCAR + VCAA + FCR + FCSL (69)$$

where the fixed cost of relocation (FCR) is only taken into account when the relocation model is used.

We note that both objective functions, the maximum value of savings and maximum NPV, are equivalent when  $\sum_{i=1}^{n} df_i \times UtilCostSav_i = n \times UtCostSav$ .

The retrofit objective could also be expressed in terms of ROI, which is the ratio of the annual savings divided by the total capital. Such an objective would be nonlinear. One way of addressing this limitation is to add a constraint limiting the amount of capital available and simply maximizing savings.

$$\max\{\text{UtilcostSav}\}$$
s.t (70)
$$\text{Capcost} \leq \text{Investment}$$

Thus, to identify the optimum ROI, one can find out for each value of investment the corresponding savings, which for that investment, will maximize the corresponding ROI. The investment rendering the maximum value of ROI is adequate. This has been suggested recently by Bagajewicz<sup>47</sup> as a means to deal with planning problems where NPV is maximized under conditions where the capital used is not fixed beforehand, and not given as a constraint. The technique suggested avoids running a nonlinear problem. Finally, a more common situation is to maximize the net present value of the savings, while limiting the payout of the project. In such a case, the constraint to add is

Investment 
$$\leq \alpha \sum_{l=1}^{n} \text{UtilcostSav}_{l}$$
 (71)

with  $\alpha$  being the maximum payout time allowed. This constraint is linear.

We finally note that the model presented above has a few bilinear terms comprising a binary variable and a continuous variable, namely,  $\delta^z_{eij}\Delta A^0_e$  and  $\delta^z_{eij}A^N_e$  (when  $(i,j) \notin B$ ) as well as  $\delta^{z,k}_{eij}\Delta A^0_e$  and  $\delta^{z,k}_{eij}A^N_e$  (when  $(i,j) \in B$ ). These bilinearities can be linearized using standard methods, by introducing new continuous variables and no additional binary variables.

**2.6.** Numerical Issues. We note that the existing areas, as calculated using temperature logarithmic mean differences using the same inlet and outlet temperatures of existing exchangers in a network to be retrofitted, will slightly differ from the area calculated as the sum of heat exchanged in each interval  $q_{im,in}^z$ divided by the mean log temperatures of these intervals (eq 96 in Barbaro and Bagajewicz<sup>1</sup>). In other words, the existing area as calculated by the model is slightly different from the real existing area given to the model as an input parameter (the  $A_{ij}^{z^0}$ ,  $A_{ij}^{z,k^0}$ , and  $A_e$ ). To eliminate the discrepancy so that the model has the existing network with no retrofit (no addition/reduction, no relocation) as a feasible solution, the existing areas  $A_{ij}^{z^o}$ ,  $A_{ij}^{z,k^o}$ , and  $A_e$  need to match with the areas calculated by the model. To do this, the problem is run setting all area addition and reduction to zero. The existing heat exchanged is fixed, and the aforementioned existing areas are treated as variables. The values obtained for these areas, which will change if the number of intervals and/or their upper and lower limits are changed, are later used in the retrofit model.

### Results

The described retrofit model is implemented in GAMS, using CPLEX version 10.1 with default options and run on a 2.8 GHz Pentium CPU, 1028 MB of RAM PC.

**Example 1.** This problem is adapted from Ciric and Floudas.<sup>20</sup> It consists of three hot and two cold process streams with one hot and one cold utility. The same stream data as in Ciric and Floudas<sup>20</sup> are used, but the areas of existing exchangers are determined using the method described above. The data of the problem are given in Tables 1 and 2, and the existing heat

Table 1. Properties of Streams for Example 1

stream	F, kg/s	Cp, kJ/ kg•C	T <sub>in</sub> , °C	T <sub>out</sub> , °C	H, kW/ m²•°C
I1	228.5	1	159	77	0.4
I2	20.4	1	267	88	0.3
I3	53.8	1	343	90	0.25
HU (hot utility)		1	500	499	0.53
J1	93.3	1	26	127	0.15
J2	196.1	1	118	265	0.5
CU (cold utility)		1	20	40	0.53

Table 2. Cost Data for Example 1

utilities	cost (cents/MJ)		
I4	0.3143		
J3	0.0661		
heat exchanger cost (\$/yr)	$3460 + 171.4 \times area (m^2)$		

exchanger network configuration is shown in Figure 3. Table 3 gives the real areas, calculated using the real logarithmic mean temperature difference, and the approximated areas, calculated using the above-discussed procedure, of the existing exchangers. The approximated area is a linearized approximation of the real area. The areas of existing exchangers 1, 3, 5, and 6 differ slightly from the values given in Ciric and Floudas. O Streams I4 and J3 are utilities. The original network consumes 17 597 kW of hot utility and 15 510 kW of cold utility.

The exchangers were allowed a maximum of 20% additional area. Table 4 gives all the retrofit parameters used. Among them, fcslh<sub>i</sub>\*\* and fcslc<sub>i</sub>\*\*\* refer to the cost of splitting for process streams only; the costs of splitting for utilities (steam, cooling water) are negligible assuming that the utilities distribution system is available in the process plant. Hence, no splitting is needed for utility streams. The fixed cost of area expansion/ reduction costs ( $fc_{ij}^{AA}$  and  $fc_{ij}^{AR}$  and  $fc_{e}^{AA}$  and  $fc_{e}^{AR}$ ) are set to be half of the fixed cost of a new exchanger. The allowed amount of area addition ( $\Delta A_{ij\text{max}}^{z^0}$  or  $\Delta A_e^{\text{max}}$ ) is 20% of the corresponding existing area. The allowed amount of area reduction  $(RA_{ijmax}^{z^0})$ or  $RA_e^{max}$ ) is 50% of the existing area. The maximum area per shell  $(A_{ij\text{max}}^{z^N} \text{ or } A_{e,\text{max}}^N)$  is 5000 (m<sup>2</sup>). The maximum number of shells per exchanger  $(U_{ij}^{z,\text{max}})$  or  $U_{e,\text{max}}$  or 10 °C, and the number of intervals is 132. The number of years used for annualized costs and net present value calculations is 5, the interest rate is 10%, and the discount factor in year "l" is  $1/(1+0.1)^{l-1}$ . Finally, assuming 350 working days in a year, the annualized cost (\$/year) per 1 MJ/h utility consumed is 26.4 for hot utility and 5.55 for cold utility.

Retrofitted Network without Relocation. The model was run maximizing the net present value, assuming one zone and  $(i,j) \notin B$  and excluding relocation. The resulting network is shown in Figure 4: two new exchangers (exchangers 8 and 9) are added, area addition is made to three existing exchangers (1, 2, and 4) by means of installing a new shell, and area reduction is made to three exchangers (5, 6, and 7). Finally, only one exchanger, exchanger 3, is kept intact. The exchangers with added area are those transferring heat between process streams; thus the amount of heat recovered in the retrofitted network increases. On the other hand, the furnace and coolers (exchangers 5, 6, 7) are reduced in size, which is a reflection of the fact that the use of utilities in the retrofitted network are reduced. Finally, we note that no area was added to existing shells. The results of the retrofitted network, the costs, and the statistics are summarized in Tables 5, 6, and 7.

**Effect of Cost of Area Reduction.** We varied the fixed cost of area reduction  $(fc_{ij}^{AR})$  to investigate the resulting retrofitted network in different scenarios from negligible to high values of the fixed cost of area reduction. High values of fixed cost

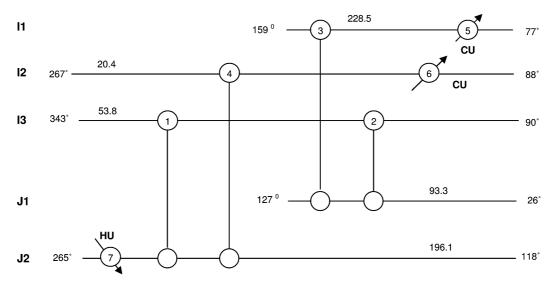


Figure 3. Original heat exchanger network for problem 1.

Table 3. Existing Exchangers in the Network

exchanger	real area (m²)	area used in the model (m <sup>2</sup> )
1	609.7	610.1
2	579.2	584.15
3	1008.5	1009.87
4	117.96	121.53
5	787.5	852.4
6	104.6	95.06
7	246.75	246.81

Table 4. Cost Data for Example 1

$c_i^{\rm H}$ (cent/MJ)	0.3143	$c_j^{\rm C}$ (cent/MJ)	0.0661
fcslh <sup>z</sup> (\$/unit)*	10 000	$fcslc_j^z$ (\$/unit)**	10 000
fc <sub>ij</sub> <sup>AA</sup> (\$/unit)	8650		8650
fc <sup>UA</sup> <sub>ij</sub> (\$/shell)	17 300	fc <sup>EA</sup> <sub>ij</sub> (\$/unit)	17 300
$vc_{ij}^{AR}$ (\$/m <sup>2</sup> )	5	$vc_{ij}^{A^0}$ (\$/m <sup>2</sup> )	857
$vc_{ij}^{A^{N}}$ (\$/m <sup>2</sup> )	857		
fc <sub>e</sub> <sup>AA</sup> (\$/unit)	8650	fc <sub>e</sub> <sup>AR</sup> (\$/unit)	8650
fc <sub>e</sub> <sup>UA</sup> (\$/shell)	17 300	$vc_e^{AA}$ (\$/m <sup>2</sup> )	857
$vc_e^{AR}$ (\$/m <sup>2</sup> )	5	$vc_e^{AN}$ (\$/m <sup>2</sup> )	857
$rc_{eij}$ (\$)	15 000		
	$\begin{array}{l} {\rm fcslh}_{i}^{z} \ ({\rm s/unit})^{*} \\ {\rm fc}_{i}^{AA} \ ({\rm s/unit}) \\ {\rm fc}_{ij}^{IA} \ ({\rm s/shell}) \\ {\rm vc}_{ij}^{AR} \ ({\rm s/m^{2}}) \\ {\rm vc}_{ij}^{AR} \ ({\rm s/m^{2}}) \\ {\rm fc}_{e}^{AA} \ ({\rm s/unit}) \\ {\rm fc}_{e}^{UA} \ ({\rm s/shell}) \\ {\rm vc}_{e}^{AR} \ ({\rm s/m^{2}}) \end{array}$	$\begin{array}{llll} & & & & & & & & & & & & \\ & & & & & & $	$\begin{array}{llllllllllllllllllllllllllllllllllll$

force taking the most out of existing heat exchanger resources. We obtain the same network as in Figure 4. The results (summarized in Table 8) show that only the investment cost and the profit (net saving, net present value) change due to different values of fixed costs of area reduction, while the exchangers network (heat exchanger areas and costs, utility cost, and energy saving) in all cases does not change and is the same as in the base case (Figure 4). The reason is simple: the original network is retrofitted to increase heat recovery and use less utility, and therefore the area reduction in the furnace and the coolers (exchangers 5, 6, 7) is unavoidable in order to accommodate the smaller utility.

If area reduction is to be avoided by using a very high fixed cost (9 million), the result is to keep the original network. To obtain a better energy efficient retrofitted network with minimal area reduction made to existing exchangers, relocation must be

We only have area reduction in the utilities where it is unavoidable; the amount of area reduction can be changed by changing the variable area reduction cost. More specifically, the amount of area reduction can be decreased by increasing variable area reduction cost  $vc_{ij}^{AR}$ , but this leads to less energy savings. In fact, when compared with the base case, increasing the variable area reduction cost by 100 times (i.e.,  $vc_{ii}^{AR} = $500$ / m<sup>2</sup>) results in a network that has (i) smaller area addition and reduction (1573 and -264.9 m<sup>2</sup>, respectively), (ii) one less exchanger (the new exchanger 9 is not used anymore), and (iii) more utilities or less energy savings (the needed amounts of hot and cold utility are 47081.88 and 38981.88 MJ/h, respectively). If we disregard the variable area reduction cost and if this network is used instead of the network in the base case (Figure 4), we save \$51 516.4 (per year) in terms of annualized investment cost but lose \$54 102.6 (per year) in terms of energy savings; hence, the net savings is \$-2586.2 (per year) (not much of a difference). If the piping cost associated with installing a new exchanger is also considered, this network may be a better option since it requires fewer exchangers.

Effect of the Cost of Area Addition. The effect of area addition cost parameters is investigated next. We verified that all the new areas added to exchangers 1, 2, and 4 are via new shells only; hence varying the fixed cost of area addition made to an existing shell  $fc_{ij}^{AA}$  or decreasing the fixed cost of a new shell  $fc_{ij}^{UA}$  does not change the results (the calculation results confirm this: no matter how many times we increase  $fc_{ij}^{AA}$  or decrease  $fc_{ij}^{UA}$ , the resulting network is unchanged). However, increasing the fixed cost of a new shell fc<sub>ii</sub><sup>UA</sup> does have an effect on the network since it will force the added areas to be in the form of area addition to existing shells. In fact, increasing  $fc_{ij}^{UA}$ by 10 times (fc $_{ij}^{\mathrm{UA}}$  = \$173 000/unit) does not change the network configuration (it is the same as in Figure 4), but the amounts of area addition and reduction change: less area addition and reduction (1596.8 and -264.2 m<sup>2</sup>, respectively). Moreover, the area addition made to exchangers 2 and 4 is in the form of area added to existing shells (not via new shells as in the base case). When compared with the base case and disregarding the fixed cost of adding a new shell being our manipulated parameter, this network saves \$42 276.3 (per year) in terms of annualized investment cost but loses \$55 540.8 (per year) in terms of energy savings; obviously this is not a good option.

Varying the variable cost of area added to existing shells  $(vc_{ii}^{A^{\circ}})$  and area added via new shells/new exchangers  $(vc_{ii}^{A^{\circ}})$  can change both the network configuration and the amount of area addition/reduction. Table 9 summarizes the results.

In the base case, all the added areas are via new shells only; hence, increasing the variable cost of area added to existing shells  $(vc_{ij}^{A^{\nu}})$  does not cause any effect (as can be seen in column 4). Decreasing the variable costs  $vc_{ij}^{A^0}$  and  $vc_{ij}^{A^N}$  leads to networks with one more new exchanger, more added area, and less utility (columns 3 and 5). The exchanger added is a cooler at the end



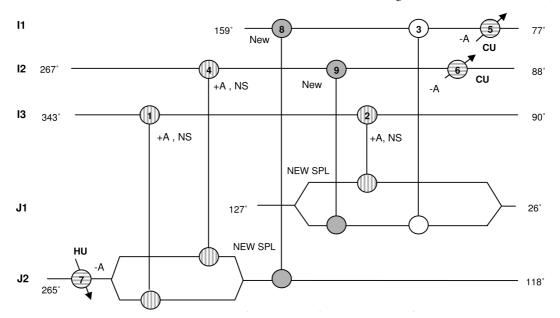


Figure 4. Retrofitted heat exchanger network for example 1 (relocation not allowed). Notation: new exchanger (New), area addition (+A), new shell (NS), area reduction (-A), new split (NEW SPL).

Table 5. Heat Exchangers Results for Example 1 (Relocation Not Allowed)

heat exchanger	original area (m²)	load after retrofit (MJ/h)	retrofit area (m²)	area change (m²)	
1	610.10	9868.27	966.08	355.97	area addition
2	584.15	3743.14	864.59	280.44	(new shell)
3	1009.87	5222.57	1009.87	0	
4	121.53	2098.53	261.93	140.4	area addition (new shell)
5	852.4	9262.54	644.41	-208.01	area reduction
6	95.06	1095.48	70.02	-25.04	
7	246.81	12608.02	184.65	-62.16	
8	0	4251.89	937.84		New Exchangers
9	0	457.59	148.85		
total	3519.941		5088.24		
total area addition (m <sup>2</sup> )					1863.501
total area reduction ( $m^2$ ) $-295.206$					-295.206

Table 6. Cost Summary for Example 1 (Relocation Not Allowed)

	original	retrofitted HEN
heating utilities (MJ/h)	63349.2	45388.8
cooling utilities (MJ/h)	55836	37288.8
heating utilities cost (\$/year)	1 672 419	1 198 266
cooling utilities cost (\$/year)	310 200	207 160
total utilities cost (\$/year)	1 982 619	1 405 427
area (m <sup>2</sup> )	3739.38	5088.236
no. of exchangers	7	9
energy saving (\$/year)		577 192
annualized investment cost (\$/year)		346 189
total capital investment		1 730 945
net saving (\$/year)		231 002.5
net present value (\$) (over 5 years)		963 280
return of investment (ROI)		33.3%

of stream I3. The ratio of area added to existing shells over the total amount of area added (row 6) can be increased by decreasing the cost of area added to existing shells  $(vc_{ij}^{A^{"}})$  or increasing the cost of area added via new shells ( $vc_{ij}^{A^{\prime\prime}}$ ), as can be seen in columns 3 and 6.

Effect of the Cost of Splitting. If the cost of repiping is so high that splitting the two cold streams J1 and J2 is economically unjustified, a different network is obtained, which is shown in

Table 7. Model Statistics for Example 1 (Relocation Not Allowed)

model statistics					
single variables	3936				
discrete variables	473				
single equations	7176				
nonzero elements	32 510				
time to reach global optimal solution (sec)	5 min, 46 s				
optimality gap	0.0%				

Table 8. Retrofitted Network Economics for Different Value of **Fixed Cost of Area Reduction** 

fixed cost of area		8650	
reduction $fc_{ij}^{AR}$ (\$/unit)	35	(base case)	17 300
new heat exchangers	2	2	2
total heat exchanger area (m <sup>2</sup> )	5088.24	5088.24	5088.24
energy saving (\$/year)	577 192	577 192	577 192
annualized investment cost (\$/year)	341 020	346 189	382 519
total capital cost (\$)	1 705 100	1 730 945	1 912 595
net saving (\$/year)	236 171	231 002	194 672
net present value (\$) (over 5 years)	984 835	963 280	811 784
ROI	33.8%	33.3%	30.2%

Figure 5. Table 10 shows the results when the splitting cost (fcslh<sub>i</sub><sup>z</sup> and fcslc<sub>i</sub><sup>z</sup>) increases 10 times.

It can be seen that the energy savings and the net savings for the case where splitting is not used is less than the savings when splitting is used, as expected. The difference in net present value is \$88 764. Because the investment for the splitting of streams J1 and J2 is \$200 000, far more than the gain in profit (\$88,764), the splitting is economically unjustified.

All these results suggest that, by appropriately varying cost parameters in the model, one can obtain several promising candidate networks; some may be less attractive in terms of saving/profit but are actually better options when practical issues (like exchanger location, piping) are considered.

Effect of Available Capital on ROI. The maximum savings objective results in an ROI of 33.3% (base case) with a total investment of \$1.731 million. We investigated the changes in ROI by limiting the capital investment to \$1 and \$1.5 million. The results are summarized in Table 11, where it is revealed that the network with a smaller investment budget, although giving a smaller profit, is the best option if the maximum ROI criterion is used. The networks in columns 3 and 4 differ from

Table 9. Retrofitted Network at Different Values of Variable Area Addition Cost

	base case	change $vc_{ij}^{A^0}$		chang	$\operatorname{ge} \operatorname{vc}_{ij}^{A^N}$
	$vc_{ij}^{A^0} = 857,  vc_{ij}^{A^N} = 857$	$vc_{ij}^{A^0} = 85,  vc_{ij}^{AN} = 857$	$vc_{ij}^{A^0} = 8570,  vc_{ij}^{A^N} = 857$	$vc_{ij}^{A^0} = 857,  vc_{ij}^{A^N} = 85$	$vc_{ij}^{A^0} = 857,$ $vc_{ij}^{AN} = 8570$
new heat exchangers	2	3	2	3	0
total number of exchangers	9	10	9	10	7
exchangers with added area to existing shells	0	2	0	0	3
exchangers with added new shells	3	2	3	3	0
added area to existing shells/total area added	0	0.167	0	0	1.0
total area addition (m <sup>2</sup> )	1863.501	1909.675	1863.501	4894.8	271.6
total area reduction (m <sup>2</sup> )	-295.206	-320.659	-295.206	-569	-70.8
total heat exchanger area (m <sup>2</sup> )	5088.24	5108.957	5088.24	7845.8	3,720.8
annualized investment cost (\$/year)	346 189	289 233	346 189	111 731	59 008
total capital cost (\$)	1 730 946	1 446 165	1 730 946	558 656	295 041
heating utilities (MJ/h)	45388.8	45172.1	45388.8	36924.5	60200.6
cooling utilities (MJ/h)	37288.8	37072.1	37288.8	28824.5	52100.6
total utilities cost (\$/year)	1 405 427	1 398 499	1 405 427	1 134 942	1 878 743
energy savings (\$/year)	577 192	584 120	577 192	847 677	103 876
net savings (\$/year)	231 002	294 887	231 003	735 945	44 867
net present value (\$) over 5 years	963 280	1 229 678	963 280	3 068 892	187 097
ROI	33.3%	40.4%	33.3%	151.7%	35.2%
network is the same as in Figure 4	yes	no	yes	no	no

Table 10. Retrofitted Network for Different Values of Splitting Cost

splitting cost fcslh <sub>i</sub> <sup>z</sup> and fcslc <sub>j</sub> <sup>z</sup>	\$10 000 (base case)	\$100 000
splitting of J1 and J2	yes	no
splitting cost (\$)	20 000	0
new heat exchangers	2	3
total area addition (m <sup>2</sup> )	1863.501	1633.001
total area reduction (m <sup>2</sup> )	-295.206	-375.923
total heat exchanger area (m <sup>2</sup> )	5088.24	4777.02
annualized investment cost (\$/year)	342 189	302 762
total capital cost (\$)	1 710 945	1 513 810
heating utilities (MJ/h)	45 388.8	47 412
cooling utilities (MJ/h)	37 288.8	39 312
total utilities cost (\$/year)	1 405 427	1 470 140
energy saving (\$ /year)	577 192	512 478
net saving (\$ /year)	231 003	209 716
net present value (\$)	963 280	874 517
ROI	33.7%	33.8%

the network in Figure 4 by just adding a cooler at the end of stream I3. The phenomenon of obtaining different results when maximizing net present value versus maximizing ROI, under conditions where the capital investment is variable, have been studied recently by Bagajewicz.<sup>47</sup> Clearly, as this example shows, smaller investments can provide a larger ROI.

Table 11. Retrofitted Network at Different Available Investment Budgets

1	1.5	1.731 (base case)
3	3	2
2	2	3
1010.9	1593.9	1863.501
-238.7	-307.9	-295.206
4292.166	4806	5088.24
200 000	300 000	342 189.3
50861.9	46867	45388.8
42761.9	38767	37288.8
1 580 322	1 452 663	1 405 427
402 297	529 956	577 192
202 297	229 956	231 003
843 576	958 917	963 280
40.2%	35.3%	33.3%
no	no	yes
		•
	3 2 1010.9 -238.7 4292.166 200 000 50861.9 42761.9 1 580 322 402 297 202 297 843 576 40.2%	3 2 2 1010.9 1593.9 -238.7 -307.9 4292.166 4806 200 000 300 000 50861.9 46867 42761.9 38767 1 580 322 1 452 663 402 297 529 956 843 576 958 917 40.2% 35.3%

**Retrofit by Adding Area to Existing Shells Only.** In this example we consider that there are limitations for the addition of new shells, or that the installation of these new units cannot be performed in the time allotted to plant shutdown or any other

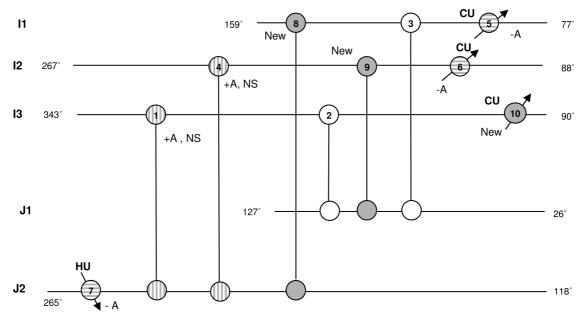


Figure 5. Retrofitted heat exchanger network for example 1 (relocation and splitting not allowed).

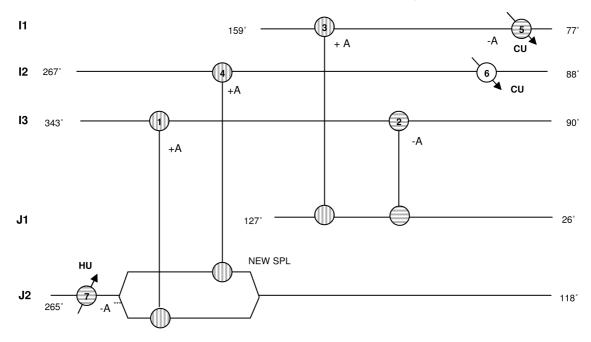


Figure 6. Retrofitted heat exchanger network for example 1 (relocation and new shells/new units not allowed).

Table 12. Heat Exchangers Results for Example 1 (Relocation Allowed)

8		,			
heat exchanger	original area (m²)	retrofit load (MJ/h)	retrofit >area (m²)	area change (m <sup>2</sup> )	relocated
1	610.1	9557.86	847.03	236.93 (new shell)	no
2	584.15	1719.92	584.15	0	no
3	1009.87	7382.6	1211.85	201.97	no
4	121.53	2373.54	340.77	219.24 (new shell)	no
5	852.4	4030.67	852.4	0	yes
6	95.06	320.78	95.06	0	yes
7	246.81	12 864.63	188.02	-58.8	no
8	0	7323.73	506.51	new exchangers	
9	0	957.28	64.04		
10	0	2333.62	174.76		
Total	3519.941		4864.6		
total area addition (m <sup>2</sup> )				1403.45	
total area reduction (m <sup>2</sup> )				-58.8	

limitation of the same sort; that is, new area is added via expanding existing shells only. The splitting costs were maintained at \$10 000, and only splitting of J1 and J2 was allowed. The obtained result is exactly the same as the case that the variable cost of new area in a new shell/new unit increases 10 times (the last column of Table 9). The result clearly shows that there is neither a new exchanger nor a new shell; the investment and savings are much lower than the base case. However, its return on investment (ROI) is higher than the base case. The network is shown in Figure 6.

It is interesting to notice that there is a shift of head duty from exchanger E2 to the two exchangers E1 and E3: the heat load in exchanger E2 decreases (E2's area is contracted), while heat loads in the exchangers E1 and E3 increase (the two exchangers are expanded).

Retrofitted Network with Relocation. The model was run assuming one zone and  $(i,j) \notin B$  assuming that all exchangers except the furnace (exchanger 7) can be relocated. The resulting network is shown in Figure 7, and the heat exchanger results are summarized in Tables 12 and 13. Computational details are given in Table 14.

In the retrofitted network without relocation of exchangers (Figure 4), the only exchangers with reduced areas are the exchangers 5, 6, and 7; thus, to minimize area reduction, these exchangers must be relocated to other positions. Besides expanding some of the existing exchangers, it is necessary to

Table 13. Cost Summary for Example 1 (Relocation Allowed)

	original	retrofitted HEN
heating utilities (MJ/h)	63 349.2	46 312.6
cooling utilities (MJ/h)	55 836	38 212.6
heating utilities cost (\$/year)	1 672 419	1 222 655
cooling utilities cost (\$/year)	310 200	212 292
total utilities cost (\$/year)	1 982 619	1 434 947
area (m <sup>2</sup> )	3739.38	4864.6
no. of exchangers	7	10
energy saving (\$/year)		547 672
annualized investment cost (\$/year)		269 371
total capital cost (\$)		1 346 855
net saving (\$/year)		278 301
net present value (\$)		1 160 516
ROI		40.7%

Table 14. Model Statistics for Example 1 (Relocation Allowed)

model statistics				
single variables	7583			
discrete variables	589			
single equations	4301			
nonzero elements	3481			
time to reach global optimal solution (sec)	33 h, 36 min			
optimality gap	0%			

add two new matches (I1, J2) and (I2, J1) so as to increase heat recovery. An intuitive guess for a network allowing relocation would be relocating the exchangers 5 and 6 (whose areas are reduced if they are not relocated) to these two new

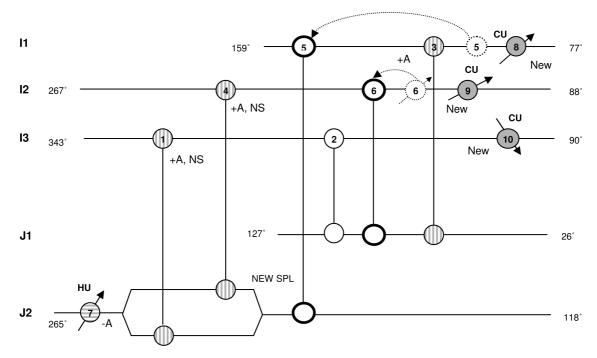


Figure 7. Retrofitted network for example 1 (relocation allowed except for the furnace).

matches (if new matches are forbidden, then, to avoid/minimize area reduction, the two exchangers 5 and 6 have to swap positions with other existing exchangers, which requires more relocation cost). The obtained result confirms this heuristic solution. Exchanger 7 (assumed fixed location, hence, its area reduction is unavoidable) and the rest remain in their original locations. Cooling of the process streams by utility is now serviced by three brand new exchangers 8, 9, and 10. Area addition is made to exchangers 1, 4 (via new shell), and 3 (expanding existing shell).

Thus, allowing relocation results in a profit gain of \$47 298.7 (per year) or \$197 236 in terms of net present value, a 20.5% increase. The ROI increases substantially from 33.3% to 40.7%.

If for practical reasons the two exchangers 5 and 6 are also not allowed to be relocated, then the resulting network would be the same as the network without relocation (Figure 4) since the same cost parameters are used and no area reduction is needed for all the exchangers allowed to be relocated (exchangers 1, 2, 3, 4); hence, there is no "driving force" to move these four exchangers to other locations. In fact, the calculation results show that fixing locations of exchangers 5, 6, and 7 or fixing locations of all exchangers leads to the same result, as in the case where relocation is not allowed (Figure 4).

The full relocation model can be used as a model without relocation: all that is needed is to fix the locations of all existing exchangers to their original positions (i.e., fixing the values of the binary  $\delta_{eij}^{z,k}$ ). Thus, the retrofitted network without relocation can be obtained using either one of the two models presented in this paper, the only difference is the computational time: the full relocation model (with fixed binary  $\delta_{eij}^{z,k}$ ) requires longer computational time: 18 min and 27 s vs 5 min 46 s of the model disallowing relocation.

In the case when relocation is allowed, we note that the run identifies the optimal solution after 1 h and 57 min when it has a gap of 7.7%, indicating that better lower bounds are needed for this problem. Noticing this, a good strategy seems to be to stop the program and freeze the integers for the exchanger location when a certain running time (or gap) is reached. In this case, when we run with relocation, stopping the run after 5 h, fixing the binary  $\delta_{eij}^{z,k}$ , and rerunning, took 18 min and 27 s. One can of course accept the result and not rerun, but this may prevent obtaining some other integer solutions that are better.

Computation Issues. From the results shown above and other testing results not shown here, a few conclusions can be made about the computational time of the full relocation model:

- (i) Computation efficiency of the full relocation model is not satisfactory by some standards: the time to reach 0% gap is too long even for a small problem like example 1. That said, it might be worthwhile waiting for such a solution because of the richness in detailed trade-offs that the model offers. After all, retrofit projects require many days of planning these days, and this type of model can offer good answers in only a few runs.
- (ii) The full relocation model starts out very efficiently and is able to find optimal (or at least near-optimal) solution within an acceptable time but spends a long time trying to prove optimality of the solution (i.e., to reach 0% gap), as can be seen above: the optimal solution is found only within the first 2 h of the total computational time of 34 h. We tried different sets of cost parameters so that different optimal solutions are obtained in example 1, and we observed the same thing: the optimal solution is located in not more than 4 h of running time with a gap of around 6% at this running time. Thanks to a large number of binaries used to represent many decision variables, the full relocation model is powerful and versatile, but also the large number of binaries explains the long computational time. The binary  $\delta_{eij}^{z,k}$  indicating the location of exchangers is by far the most influential variable, so when the optimal location of the exchangers is identified (which occurs in the early stages of computation), the optimal solution is already found. Thus, we suggest that the branching of the binary  $\delta_{eij}^{z,k}$  in the branch and bound procedure should be set to take the highest priority over other binary variables.

Step-by-Step Retrofit. One alternative strategy to contend with the long computations and to identify suboptimal solutions that could be profitable is to limit the number of allowable new exchangers and/or the number of relocation. When this is done, the computational time is significantly reduced. We propose the

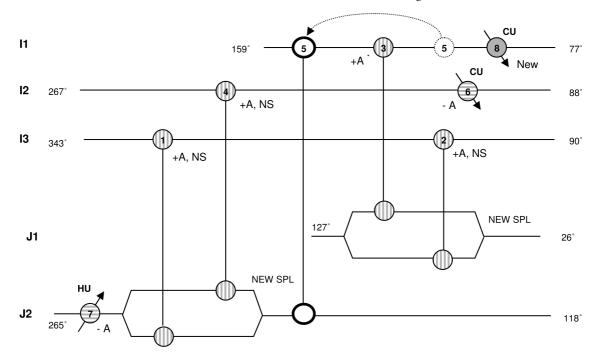


Figure 8. Retrofitted network for example 1, full relocation, only one new exchanger and one relocation allowed.

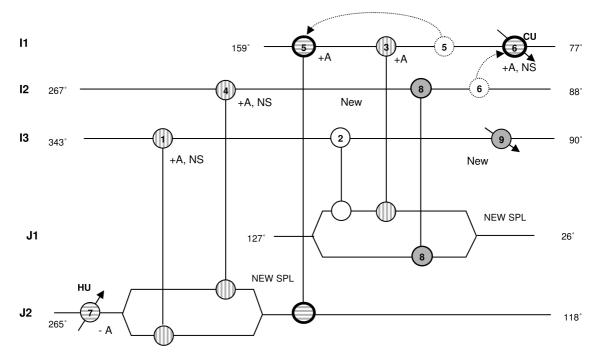


Figure 9. Retrofitted network for example 1, full relocation, two new exchangers and two relocations allowed.

following strategy to obtain an optimal solution (or at least nearoptimal solution) within an acceptable time:

- · Run the model with limitations on the number of new exchangers and the number of relocations. The allowed numbers of new exchangers and relocation are chosen to be lower than their respective values in the current best solution (obtained after some number of hours or some gap value running the model without restrictions). The location of some exchangers can also be fixed at the user's discretion.
- Models with imposed limitations can be solved to 0% gap within an acceptable time. The resulting networks obtained under such limitations are expected to be simpler than the current best solution and render lower benefits (if the current best is indeed the optimal). In doing so, one obtains a range

of good (likely suboptimal) solutions, not just one. With such a list of solutions in hand, the user can determine the best one among them and consider various factors such as implementation issues (the simpler the better) and the benefit (the savings or the return on investment) as well as the marginal or incremental return of investment (extra savings over extra capital).

The results of models with limitations on the extent of network modifications are shown in Figures 8 and 9 and summarized in Table 15. We note that, even though the networks obtained under restrictions (Figures 8 and 9) are simpler than the network without limitation (Figure 7), they require a larger investment cost because they require more area to be added. The one without limitation is obviously the optimal solution

Table 15. Solutions of Example 1 for Various Scenarios

restrictions	1 new exchanger, 1 relocation allowed	2 new exchanger, 2 relocation allowed	no limitation
new exchangers	1	2	2
number of relocations	1	2	3
total area addition (m <sup>2</sup> )	1694.12	1549.347	1403.45
total area reduction (m <sup>2</sup> )	-87.98	-61.773	-58.8
total heat exchanger area (m <sup>2</sup> )	5126.1	5007.515	4864.6
annualized investment cost (\$/year)	316 491	296 379.7	269 371
total capital cost (\$)	1 582 455	1 481 898	1 346 855
heating utilities (MJ/h)	45 188.5	45 493.7	46 312.6
cooling utilities (MJ/h)	37 088.5	37 393.7	38 212.6
total utilities cost (\$/year)	1 399 024	1 408 777	1 434 947
energy savings (\$/year)	583 595	573 842	547 672
net savings (\$/year)	267 104	277 462	278 301
net present value (\$) over 5 years	1 113 824	1 157 016	1 160 515
ROI	36.9%	38.7%	40.7%
marginal ROI wrt previous solution		9.7%	15.3%
computation time	13 min, 22 s	3 h, 41 min	33 h, 36 min <sup>6</sup>

<sup>&</sup>lt;sup>a</sup> The same solution can be obtained using the strategy of stopping the program after 5 h (the proposed strategy); this solution is identified after 2 h of running time.

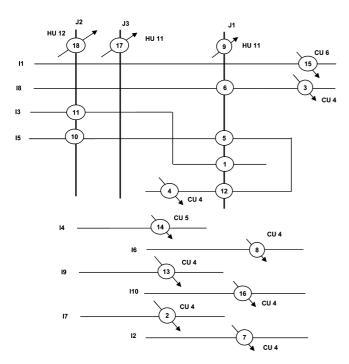


Figure 10. Original heat exchanger network for example 2.

with the highest savings and return on investment; the network shown in Figure 9 (two relocations and two new exchangers) is quite attractive: it is simpler and renders savings slightly lower than the optimal solution. The total running time if we follow the strategy described above is roughly 8 h instead of 33 h.

**Example 2: Crude Distillation Unit.** This problem considers the retrofit of a heat exchanger network of a crude distillation unit with 18 streams and 18 exchangers. The original network is shown in Figure 10. The stream properties are shown in Table 16. The existing heat exchangers are shown in Table 17, and the cost parameters are shown in Table 18.

The plant life used is 5 years. The maximum values of area addition and reduction that can be made to existing shells are 10% and 40% of the corresponding existing area, respectively (except for the two exchangers E5 and E12 serving the match (I5,J1) where the corresponding percentages are 20% and 30%). The maximum area per shell  $(A_{ij\max}^{z_N} \text{ or } A_{e,\max}^N)$  is 5000 (m²); the maximum number of shells per exchanger  $(U_{ij}^{z,\max} \text{ or } U_{e,\max}^N)$  is

Table 16. Stream Properties for Example 2

stream	F, ton/h	T in, °C	T out, °C	Cp, kJ/kg.°C	H, MJ/h·m <sup>2</sup> ·°C
I1	155.1	319.4	244.1	3.161	4.653
I2	5.695	73.24	30	4.325	18.211
I3	251.2	347.3	202.7	3.02	3.210
		202.7	45	2.573	2.278
I4	151.2	263.5	180.2	2.930	4.894
I5	26.03	297.4	203.2	3.041	4.674
		203.2	110	2.689	3.952
I6	86.14	248	147.3	2.831	4.835
		147.3	50	2.442	3.800
I7	91.81	73.24	40	2.262	4.605
I8	63.99	231.8	176	2.854	5.023
		176	120	2.606	4.846
I9	239.1	167.1	116.1	2.595	4.995
		116.1	69.55	2.372	4.880
I10	133.8	146.7	126.7	6.074	1.807
		126.7	99.94	4.745	3.373
		99.94	73.24	9.464	6.878
HU11		250	249		21.600
HU12		1000	500		0.400
J1	519	30	108.1	2.314	1.858
		108.1	211.3	2.645	2.356
		211.3	232.2	3.34	2.212
J2	496.4	232.2	343.3	3.540	2.835
J3	96.87	226.2	228.7	13.076	11.971
		228.7	231.8	15.808	11.075
CU4		20	25		13.500
CU5		124	125		21.600
CU6		174	175		21.600

Table 17. Existing Exchangers in the Network, Example 2

exchanger	area (m²)	heat load (MJ/h)	exchanger	area (m²)	heat load (MJ/h)
1	4303.20	158 835.9	10	80.2	3838.9
2	63.80	6903.1	11	685.70	56 093.6
3	33.29	17 173.8	12	40.00	5930.9
4	4.06	1191.5	13	182.39	58 042.3
5	26.79	3018.8	14	101.47	36 903.2
6	24.6	2356.9	15	93.87	36 917.4
7	5.87	1065.0	16	288.97	67 053.1
8	146.59	45 024.5	17	52.24	7913.8
9	1214.40	101 545.2	18	976.4	135 298.7

4. The  $\Delta T_{\rm min}$  is 5 °C, and the number of intervals is 84. Splitting only in streams J1 and J2 is allowed.

The network uses two hot utilities, HU11 and HU12, and three cold utilities, CU4, CU5, and CU6. The cost and the needed amount of utilities in the original network are shown in Table 19.

Table 18. Cost Data for Example 2

fcslh <sub>i</sub> <sup>z</sup> (\$/unit)	20 000	fcslc <sup>z</sup> <sub>j</sub> (\$/unit)	20 000
fc <sub>ij</sub> (\$/unit)	13 230	fc <sub>ij</sub> <sup>AR</sup> (\$/unit)	13 230
fc <sub>ij</sub> <sup>UA</sup> (\$/shell)	26 460	fc <sub>ii</sub> (\$/unit)	26 460
$vc_{ij}^{AR}$ (\$/m <sup>2</sup> )	0.5	$vc_{ij}^{A^0}$ (\$/m <sup>2</sup> )	389
$vc_{ij}^{A^N}$ (\$/m <sup>2</sup> )	389	-	
fc <sub>e</sub> <sup>AA</sup> (\$/unit)	13 230	fc <sub>e</sub> <sup>AR</sup> (\$/unit)	13 230
fc <sub>e</sub> <sup>UA</sup> (\$/shell)	26 460	$vc_{e_{N}}^{AA}$ (\$/m <sup>2</sup> )	26 460
$vc_e^{AR}$ (\$/m <sup>2</sup> )	0.5	$vc_e^{A^N}$ (\$/m <sup>2</sup> )	389
$rc_{eij}$ (\$)	25 000		
	$\begin{array}{l} {\rm fc}_{ij}^{\rm AA} \ (\$/{\rm unit}) \\ {\rm fc}_{ij}^{\rm IA} \ (\$/{\rm shell}) \\ {\rm vc}_{ij}^{\rm RR} \ (\$/{\rm m}^2) \\ {\rm vc}_{ij}^{ij} \ (\$/{\rm m}^2) \\ {\rm fc}_{e}^{\rm AA} \ (\$/{\rm unit}) \\ {\rm fc}_{e}^{\rm UA} \ (\$/{\rm shell}) \\ {\rm vc}_{e}^{\rm AR} \ (\$/{\rm m}^2) \end{array}$	$ \begin{array}{llllllllllllllllllllllllllllllllllll$	$ \begin{array}{llllllllllllllllllllllllllllllllllll$

Table 19. Utilities in the Original Network

hot utility	cost (cent/MJ)	amount (MJ/h)	cold utility	cost (cent/MJ)	amount (MJ/h)
HU11	0.2351	109 459	CU4	0.0222	196 453.3
HU12	0.4431	135 298.7	CU5	0.0773	36 903.2
			CU6	0.1518	36 917.4
total hot u	tilities (MJ/h)	244 757 7	total cold	utilities (MI/h)	270 273 9

As done in example 1, this problem is also solved for two cases: (i) relocation is disallowed and (ii) relocation is allowed. The model was run maximizing the net present value, assuming one zone, the case  $(i,j) \in B$  was considered with one match (I5,J1) allowed to have more than one exchanger. The pair of exchangers (E10, E11) and the three exchangers (E12, E1, E5) are not allowed to change their relative order (although E12 and E5 are allowed to switch position, in the way that it is illustrated in Figure 1, by using constraint 37 for the cold streams J1 and J2. For practical reasons, the three exchangers E9, E17,

Table 20. Heat Exchangers Results for Example 2 (Relocation Not Allowed) $^a$ 

ex.	area (m²)	area change (m²)	note	ex.	area (m²)	area change (m²)	note
1	4303.20	0		14	60.88	-40.59	area red.
2	63.80	0		15	56.32	-37.55	area red.
3	19.97	-13.32	area red.	16	199.97	-89	area red.
4	4.064	0		17	31.34	-20.90	area red.
5	77.1	50.27	new shell	18	701.03	-275.37	area red.
6	176.76	152.16	new shell	19	328.41	328.41	new ex.
7	5.87	0.00		20	119.1	119.1	new ex.
8	107.05	-39.54	area red.	21	206.72	206.72	new ex.
9	728.64	-485.76	area red.	22	131.93	131.93	new ex.
10	80.2	0		23	53.26	53.26	new ex.
11	2481.93	1796.23	new shell	24	415.82	415.82	new ex.
12	40.00	0		25	476.83	476.83	new ex.
13	112.47	-69.93	area red.	26	222.92	222.92	new ex.
tota	l area addit	cion (m <sup>2</sup> )				3953.65	
tota	l area redu	ction (m <sup>2</sup> )				-1071.94	

<sup>&</sup>lt;sup>a</sup> Abbreviations: ex., exchanger; area red, area reduction.

and E18 that exchange heat between process streams and hot utilities are not allowed to be relocated to another position. For simplicity, we also forbid the relocation of the two exchangers E5 and E12 serving the pair (I5,J1). These can be done by fixing the corresponding values of the variables  $\delta^z_{eij}$  and  $\delta^{z,k}_{eij}$  or using a very high relocation cost for these exchangers.

The retrofitted network disallowing relocation (the first case) is shown in Figure 11.

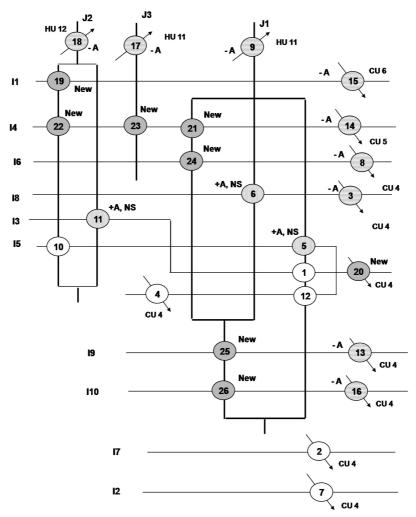


Figure 11. Retrofitted network for example 2 (relocation disallowed). Notation: new exchanger (New), area addition (+A), new shell (NS), area reduction (-A), new split (NEW SPL).

Table 21. Cost Summary for Example 2 (Relocation Not Allowed)

	original	retrofit
heating utilities (MJ/h/)	244 757.7	138 877.7
cooling utilities (MJ/h)	270 273.9	164 393.9
heating utilities cost (\$/year)	7 197 902	4 386 378
cooling utilities cost (\$/year)	1 075 835	600 779.4
total utilities cost (\$/year)	8 273 737	4 987 158
area (m <sup>2</sup> )	8323.82	11 205.53
no. of exchangers	18	26
energy saving (\$/year)		3 286 573
annualized investment cost (\$/year)		404 324
total investment cost (\$)		2 021 622
net saving (\$/year)		2 882 248
net present value (\$)		12 016 094
ROI (%)		162.6

Table 22. Model Statistics for Example 2 (Relocation Not Allowed)

model statistics					
single variables	3345				
discrete variables	726				
single equations	6477				
nonzero elements	31 083				
computational time	8 h, 22 min				
optimality gap (%)	0				

As can be seen in Figure 11, splitting is introduced to the two streams J1 and J2, and there are eight new exchangers added to the network (exchangers 19–26, highlighted by using a gray background). Exchangers in the retrofitted network are summarized in Table 20.

In addition to eight brand new exchangers, three exchangers are expanded by means of adding a new shell, exchangers 5, 6, and 11; the total added area is 3953.65 (m<sup>2</sup>). As the result of

increased heat recovery, the use of utilities is decreased, and all the exchangers involving utilities in the retrofitted network (except exchanger 4) are reduced in area (nine exchangers, 3, 8, 9, 13, 14, 15, 16, 17, and 18). Note also that constraints 36 and 37 only require that, for a pair of two exchangers whose order is not allowed to change, an exchanger can only start when the order has finished (in terms of temperature interval). It is possible that the two exchangers do satisfy constraints 36 and 37, but they are in parallel, as shown in Figure 11. In the network shown in Figure 11, exchanger 10 starts and ends at the same temperature interval where exchanger 10 ends, hence satisfying constraint 37, but the two exchangers 10 and 11 are in parallel. To strictly enforce that an exchanger must come after another exchanger in a series manner, one can forbid the splitting of the stream passing through these two exchangers (in this specific example, forbidding the splitting of stream J2).

The cost summary for example 2, disallowing relocation, is shown in Table 21. The heat recovery improvement in the retrofitted network results in a remarkable profit: the use of utilities is reduced by 41.1%, the energy savings is over \$3.3 million per year, the net profit is \$2.9 million per year and the return on investment is 162.6%. Model statistics of the problem are shown in Table 22.

If for practical reasons, such as limited budget or physical difficulties when installing new exchangers or repiping, one wishes to limit the number of new exchangers and streams splitting, then the following results are obtained for two cases:
(a) Two new exchangers and no stream splitting are allowed (b) Three new exchangers and one stream splitting are allowed The results for the two cases are shown in Figures 12 and 13

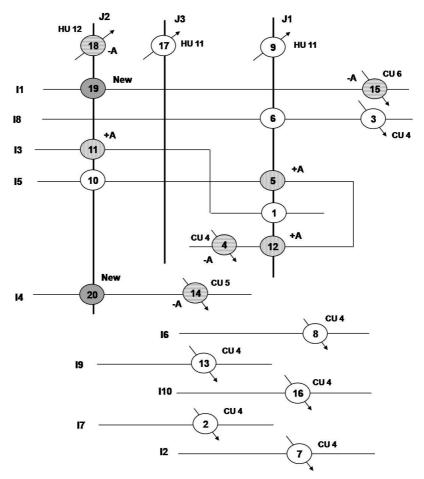


Figure 12. Retrofitted network for example 2, relocation disallowed, two new exchangers, no splitting allowed.



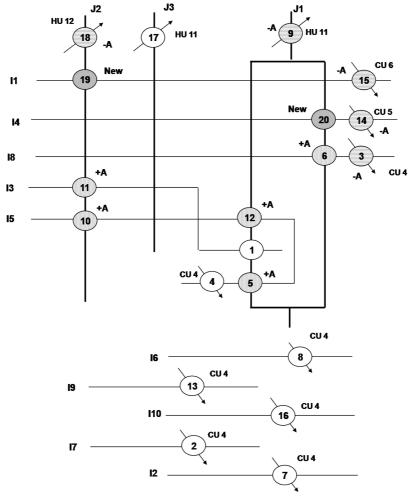


Figure 13. Retrofitted network for example 2, relocation disallowed, three new exchangers and one splitting allowed.

Table 23. Summary of Results, Relocation Disallowed with Limitations on the Number of New Exchangers and Splitting

case	(a) no splitting and two exchangers allowed		full retrofit
no. of new exchangers	2	2	8
no. of splitting	0	1	3
heating utilities (MJ/h/year)	219 256.9	206 029	138 877.7
cooling utilities (MJ/h/year)	244 773	231 545.3	164 393.9
total utilities cost (\$/year)	7 050 349	6 794 066	4 987 158
area addition (m <sup>2</sup> )	1010.6	636.6	3953.65
area reduction (m <sup>2</sup> )	-222.4	-326.1	-1071.94
total area (m <sup>2</sup> )	9112	8634.3	11 205.53
energy saving (\$/year)	1 223 382	1 479 665	3 286 573
annualized investment cost (\$/year)	115 678	98 532	404 324
total investment (\$)	578 392	492 660	2 021 622
net saving (\$/year)	1 107 703	1 381 133	2 882 248
net present value (\$)	4 618 973	5 757 942	12 016 094
ROI (%)	211.5	300.3	162.6

and summarized in Table 23.

Note that, although three new exchangers are allowed in the design case (b), only two new exchangers are used. Moreover, in this design case, the reordering of the matches for the case BIF = 1 (Figure 1) activates: the two exchangers E5 and E12 switch position, which is then accompanied by an area addition in both exchangers.

It is interesting to see that allowing splitting has a great impact on profit: both design cases a and b have the same number of new exchangers but design case b, which allows splitting, has 21% more savings attainable at 15% less investment. It is also obvious that a larger profit is achieved when a greater extent of network retrofitting (hence more investment) is allowed: both of the design cases with restriction give much smaller savings than the case without restriction. We again observe the same phenomenon noted above. Smaller retrofits are more profitable when ROI is used.

The case when relocation is allowed is considered next. Results for the full relocation models without any restriction are shown in Figure 14 and Tables 24-26:

In Figure 14, the five relocated exchangers (E2, E4, E8, E14, and E15) are indicated using a bold line, while the seven new exchangers (E19-E25) are highlighted using a gray background. Among those relocated to new positions, the two exchangers E2 and E8 are kept intact, while the two exchangers E4 and E14 are expanded with new shells, and exchanger E15's area is reduced. There are 13 (= 18 - 5) exchangers that are kept at their original positions, among which the four exchangers E5, E6, E10, and E11 are expanded with new shells; the two exchangers E1 and E7 are kept intact while the heat transfer areas of the rest (seven exchangers) are reduced.

The retrofitted network increases heat recovery by adding new exchangers or expanding existing areas that exchange heat between process streams. As a result, the utilities consumption and the areas of exchangers involving utilities are reduced. To better utilize existing resources and avoid area reduction (a waste of existing resources), the exchangers involving utilities should be relocated to other positions. The results confirm these observations: the exchangers with added area are those that

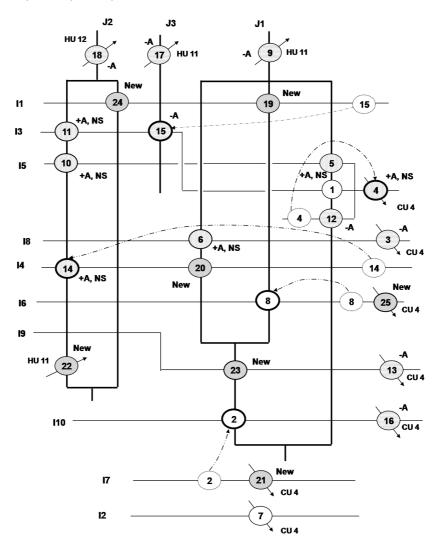


Figure 14. Retrofitted network for example 2, relocation allowed.

Table 24. Heat Exchangers Results for Example 2 (Relocation Allowed)a

ex.	area (m²)	area change (m²)	note	ex.	area (m²)	area change (m²)	note
1	4303.20	0		14	787.47	686	rel., new shell
2	63.80	63.80	rel.	15	69.75	-24.12	rel., area red.
3	26.73	-6.56	area red.	16	260.5	288.97	area red.
4	105.95	101.89	rel., new	17	31.34	-20.9	area red.
			shell				
5	54.52	27.73	new shell	18	633.87	-342.53	area red.
6	227.23	202.63	new shell	19	50.66		new ex.
7	5.87	0		20	254.87		new ex.
8	146.59	0	rel.	21	63.8		new ex.
9	728.64	-485.76	area red.	22	115.1		new ex.
10	110.99	30.79	new shell	23	245.96		new ex.
11	1245.51	559.81	new shell	24	845.52		new ex.
12	36.19	-3.82		25	127.35		new ex.
13	131.17	-51.22	area red.				
total area addition (m <sup>2</sup> )						3	3312.1
total area reduction (m <sup>2</sup> )					-963.4		

<sup>&</sup>lt;sup>a</sup> Abbreviations: rel., relocated, ex., exchanger; area red., area

exchange heat between process streams, while the exchangers with reduced area or that are being relocated are those involving utilities. Note also that the use of the two cooling utilities I5 and I6 (which are originally used to cool down the two streams I4 and I1, respectively) is eliminated. Exchangers in the retrofitted network are summarized in Table 24.

Table 25. Cost Summary for Example 2 (Relocation Allowed)

	original	retrofit
heating utilities (MJ/h)	244 757.7	131 818.6
cooling utilities (MJ/h)	270 273.9	157 334.8
heating utilities cost (\$/year)	7 197 902	4 079 065
cooling utilities cost (\$/year)	1 075 835	292 800
total utilities cost (\$/year)	8 273 737	4 371 865
area (m <sup>2</sup> )	8 323.82	10 672.53
no. of exchangers	18	25
energy saving (\$/year)		3 901 872
annualized investment cost (\$/year)		384 698.8
total investment (\$)		1 923 494
net saving (\$/year)		3 517 166
net present value (\$)		14 663 067
ROI (%)		202.8

Table 26. Model Statistics for Example 2 (Relocation Allowed)

model statistics						
single variables	6329					
discrete variables	1937					
single equations	9169					
nonzero elements	43 288					
computational time (h)	24					
optimality gap	3.74%					

The results in Tables 20 and 24 show that allowing relocation leads to much smaller area reduction. The amount of energy savings is also larger than the case of disallowing relocation even though the amount of area addition is smaller, as can be



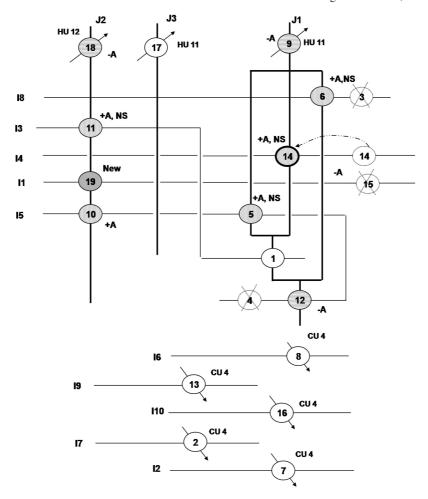


Figure 15. Retrofitted network for example 2 with relocation. One new exchanger and one relocation are allowed.

seen in Table 21 shown above and Table 25, which shows the cost summary for the case where relocation is allowed.

Similar to the example 1, allowing relocation results in a 22% increase in profit (20.5% increase is obtained in example 1), the profit gain is \$0.635 million per year or a \$2.65 million increase in terms of net present value. The model statistics for the case of allowing relocation are given in Table 26.

If, for practical reasons, one would like to limit the extent of network modification, the savings are lower but the investment cost is also lower; the computational time would be shorter. We present the results for two simple cases when restrictions on the number of new exchangers and the number of relocations are used:

- (i) One new exchanger and one relocation are allowed.
- (ii) Two new exchangers and two relocations are allowed. Computational times for these two runs are 26 min for case 1 (0% gap) and 24 h for case 2 (2.95% gap). The results are shown in Figures 15 and 16 and summarized in Table 27.

We note that, in the full relocation model, it is allowed to discard existing exchangers, and the results show that there are existing exchangers that are not used anymore: the three exchangers E3, E14, and E15 in the first case and the two exchangers E4 and E10 in the second case. The reason for this is simple: the savings achievable via increased heat recovery are far more than the investment cost or loss incurred due to "unused" existing heat exchange area. Hence, the model tends to maximize the amount of heat recovery, which could lead to the situation that some existing exchangers have to be discarded to satisfy the new heat duty distribution with improved heat recovery. Note that exchangers whose relative order is to be fixed can still be eliminated, as shown in Figure 16, where E10 is eliminated. In this case (where E10 is eliminated), the lefthand side of eq 37, which is used to enforce E10's positioning behind E11, is zero for all intervals M; hence, eq 37 is trivially satisfied. Figure 15 shows that, even though the relative order of the three exchangers E5, E1, and E12 is conserved as desired, there is a repiping involving splitting of stream J1 such that E5, E1, and E12 are not strictly in serial order any more. Note also that the network in Figure 15 is certainly not the optimal one: it is better to relocate the exchanger E15 (with some added areas) to the match (I1,J2) where the new exchanger E19 is currently located. The network shown in Figure 15 is obtained because only one relocation is allowed in that case. Figure 16 shows that, when two relocations are allowed, the exchanger E15 is now relocated to the new match (I6,J1) instead of being discarded.

It is interesting to see that the network in case 2 (two new exchangers and two relocations) has a larger investment cost than case 3 (without any limitation), mostly because it has a much greater amount of area addition. The utilities consumption, energy savings, and return on investment of case 2 are comparable to those of case 3. As expected, case 3 (no restriction) renders the highest profit, while case 1 has the lowest required investment and the highest return on investment.

The results presented in Tables 23-27 show that allowing relocation gives a lot more savings than the case when relocation is not allowed; even the simplest case of relocation (one new

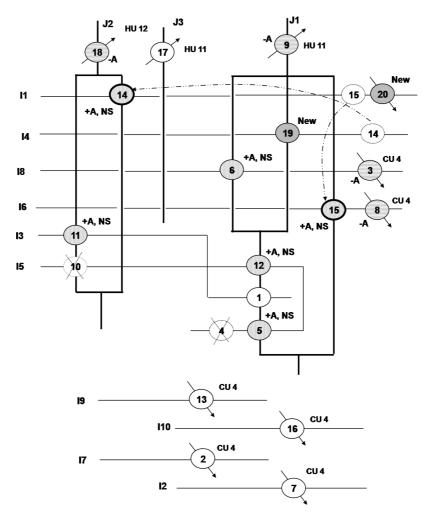


Figure 16. Retrofitted network for example 2 with relocation. Two new exchangers and two relocations allowed.

Table 27. Summary of Results, Full Relocation Model with Limitations on the Number of New Exchangers and Number of Relocations

case	1 new exchanger and 1 relocation allowed	2 new exchangers and 2 relocation allowed	full relocation (no restrictions)
no. of new exchangers	1	2	7
no. of relocations	1	2	5
heating utilities (MJ/h)	152 571.8	136 185.5	131818.6
cooling utilities (MJ/h)	178 088	161 701.8	157 334.8
total utilities cost (\$/year)	5 008 179	4 513 364	4 371 865
area addition (m <sup>2</sup> )	2363.9	4217.6	3312.1
area reduction (m <sup>2</sup> )	-654.1	-870.7	-963.4
total area (m <sup>2</sup> )	9891.5	11 586.5	10 672.5
energy savings (\$/year)	3 265 550	3 760 366	3 901 872
annualized investment cost (\$/year)	229 993	403 083.9	384 698.8
total investment (\$)	1 149 967	2 015 419	1 923 494
net savings (\$/year)	3 035 557	3 357 282	3 517 166
net present value (\$)	12 655 239	13 996 511	14 663 067
ROI (%)	284	186.6	202.8

exchanger, one relocation) gives a higher profit than the case of full retrofit without relocation.

#### 4. Conclusions

Using our MILP method, realistic industrial size problems can be solved in reasonable computational time. In addition, our method can easily provide suboptimal solutions, by identifying intermediate feasible solutions (stopping when a certain gap is reached) or excluding the current optimal solution. Our methodology can be made completely automatic, since topology changes are selected using binary decision variables and are very much amenable to interactive design. Indeed, one can conceive user-friendly software where the engineer will look at several optimal and suboptimal solutions and decide that he or she would like to exclude certain matches, or force some others, or even forbid certain changes proposed (like splitting certain streams) and can input these constraints interactively or beforehand. The user can also appropriately change the cost parameters to obtain the desired networks. On the computational side, these constraints are easily handled through the integer nature of those decisions. Conceivably, the software could analyze some other features not included in the model, like the detailed design of new exchangers, controllability analysis, planning, and scheduling of changes, although some of these changes can be included in the future. In addition, one can also use this method in conjunction with some other pieces of earlier methods. For example, the network pinch-based methodology of Asante and Zhu<sup>30</sup> requires a diagnosis stage that includes a series of steps to determine and/or select topology modifications that rely on some heuristics and graphs, but it provides good insights of the problem, and one should take advantage of it.

To conclude, a one-step rigorous method to perform the retrofit of heat exchanger networks was presented. The model is MILP and allows exchanger addition and relocation. It takes care of piping changes, and one can control the level of changes introduced. If equipped with a graphical user interface, this model can be used as a software product that can interact with users, without the need of an understanding of the retrofit methodology. It can, at the same time, be used in conjunction with other methods, like the network pinch. Notation

#### Sets

 $Z = \{z | z \text{ is a heat transfer zone}\}$ 

 $H^z = \{i | i \text{ is a hot stream present in zone } z\}$ 

 $C^z = \{i | i \text{ is a cold stream present in zone } z\}$ 

 $HU^z = \{i | i \text{ is a heating utility present in zone } z\} (HU^z \subset H^z)$ 

 $CU^z = \{j | j \text{ is a cooling utility present in zone } z\} (CU^z \subset C^z)$ 

 $M_i^z = \{m|m \text{ is a temperature interval belonging to zone } z, \text{ in which hot stream } i \text{ is present}\}$ 

 $N_j^z = \{n | n \text{ is a temperature interval belonging to zone } z, \text{ in which cold stream } j \text{ is present}\}$ 

 $H_m^z = \{i | i \text{ is a hot stream present in temperature interval } m \text{ in zone } z\}$ 

 $C_n^z = \{j | j \text{ is a cold stream present in temperature interval } n \text{ in zone } z\}$ 

 $P = \{(i,j) | \text{ a heat exchange match between hot stream } i \text{ and cold stream } j \text{ is permitted}\}$ 

 $B = \{(i,j) | \text{ more than one heat exchanger is permitted between hot stream } i \text{ and cold stream } j\}$ 

#### Parameters

 $c_i^{\rm H} = {\rm Cost} \ {\rm of} \ {\rm heating} \ {\rm utility} \ i$ 

 $c_i^{\rm C} = {\rm Cost} \ {\rm of} \ {\rm cooling} \ {\rm utility} \ j$ 

 $fc_{ij}^{EA}$  = Fixed cost for a heat exchanger matching hot stream i and cold stream j

 $fc_{ij}^{UA}$  = Fixed cost for a new shell added to the existing heat exchanger matching hot stream i and cold stream j

 $fc_e^{UA}$  = Fixed cost for a new shell added to the existing exchanger "e"

 $vc_{ij}^{A^N}$  = Variable cost for a new heat exchanger or new shell added to the existing exchanger matching hot stream i and cold stream j

 $vc_e^{A^N}$  = Variable cost for a new heat exchanger or new shell added to the exchanger "e"

 $fc_{ij}^{AA}$  = Fixed cost for adding new area to existing heat exchanger matching hot stream i and cold stream j

 $fc_e^{AA}$  = Fixed cost for adding new area to the existing exchanger "e"

 $vc_{ij}^{A^0}$  = Variable cost for adding new area to existing heat exchanger matching hot stream i and cold stream j

 $vc_e^{AA}$  = Variable cost for adding new area to the existing exchanger "e"

 $fc_{ij}^{AR}$  = Fixed cost for making area reduction to existing heat exchanger matching hot stream i and cold stream j

 $fc_e^{AR} = Fixed cost$  for making area reduction to the existing exchanger "e"

 $vc_{ij}^{AR} = Variable cost for making area reduction to existing heat exchanger matching hot stream$ *i*and cold stream*j* 

 $vc_e^{AR}$  = Variable cost for making area reduction to the existing exchanger "e"

 $fcslh_i^z = Fixed cost for splitting hot stream i$ 

 $fcslc_i^z$  = Fixed cost for splitting cold stream j

 $rc_{eij}$  = Fixed cost for relocating the exchanger "e" to the match (i,i)

 $A_{ij}^{z^0} \stackrel{=}{=}$  Area of existing heat exchanger matching hot stream i and cold stream j/the existing exchanger "e", respectively  $A_e =$  Area of the existing exchanger "e"

 $\Delta A_{ij\text{max}}^{z^0}$  = Maximum area addition that can be made to existing heat exchanger matching hot stream *i* and cold stream *j* 

 $AA_e^{max}$  = Maximum area addition that can be made to the existing exchanger "e"

 $A_{ij\text{max}}^{z^N}$  = Maximum area of a shell in heat exchanger matching hot stream i and cold stream j

 $A_{ij\max}^{z^N}/A_{e,\max}^N = \text{Maximum}$  area of a shell in the exchanger "e"  $U_{ij}^{z,0} = \text{Number of existing shells in existing heat exchanger matching hot stream } i$  and cold stream j

 $U_{ij}^{c,\max}$  = Maximum number of shells in heat exchanger matching hot stream i and cold stream j/the exchanger "e", respectively

 $U_{e,\text{max}}^N = \text{Maximum number of shells in the exchanger "e"}$ 

 $RA_{ijmax}^{z^0}$  = Maximum area reduction that can be made to existing heat exchanger matching hot stream i and cold stream j

 $RA_e^{max}$  = Maximum area reduction that can be made to the existing exchanger "e"

 $\Delta E_{ij}^{\text{c,max}} = \text{Maximum number of new heat exchanger in the match } (i,j)$ 

 $k_e$  = Number of existing heat exchangers in the match (i,j) in zone z when  $(i,j) \in B$ 

 $k_{\text{max}} = \text{Maximum number of exchangers in the match } (i,j) \text{ in zone } z \text{ when } (i,j) \in B$ 

#### Variables

 $K_{ijm}^{z,H}$  = Determines the beginning of a heat exchanger at interval m of zone z for hot stream i with cold stream j. Defined as binary when  $(i,j) \in B$  and as continuous when  $(i,j) \notin B$ .

 $K_{ijn}^{c,C}$  = Determines the beginning of a heat exchanger at interval n of zone z for cold stream j with hot stream i. Defined as binary when  $(i,j) \in B$  and as continuous when  $(i,j) \notin B$ .

 $\hat{K}_{ijm}^{z,H}$  = Determines the end of a heat exchanger at interval m of zone z for hot stream i with cold stream j. Defined as binary when  $(i,j) \in B$  and as continuous when  $(i,j) \notin B$ .

 $\hat{K}_{ijn}^{z,C}$  = Determines the end of a heat exchanger at interval n of zone z for cold stream j with hot stream i. Defined as binary when  $(i,j) \in B$  and as continuous when  $(i,j) \notin B$ .

 $Y_{ijm}^{z,H}$  = Binary indicating whether hot stream i at interval m of zone z exchanges heat with cold stream j

 $Y_{ijn}^{cC}$  = Binary indicating whether cold stream j at interval n of zone z exchanges heat with hot stream i

 $\Psi_{ij}^{A^0}$  = Binary indicating whether area addition made to existing heat exchanger matching hot stream i and cold stream j

 $\Omega_e$  = Binary indicating whether area addition made to the existing exchanger "e"

 $\psi_{ij}^{z}$  = Binary indicating whether area reduction made to existing heat exchanger matching hot stream *i* and cold stream *j* 

 $\varphi_e$  = Binary indicating whether area reduction made to the existing exchanger "e"

 $\lambda_{ij}^{z,hk} = \text{Binary indicating whether the } k\text{th original heat exchanger}$  is relocated to the hth position of the match (i,j) when  $(i,j) \in \mathcal{B}$ 

 $\delta_{eij}^z$  = Binary indicating whether the existing exchanger "e" is used to house the match (i,j) in the case  $(i,j) \notin B$ 

 $\delta_{eij}^{z,k}$  = Binary indicating whether the existing exchanger "e" is used to house the match (i,j) in the case  $(i,j) \in B$ 

FCAR = Total fixed cost of area reduction

FCAA = Total fixed cost of area addition

FCUA = Total fixed cost of adding shells to existing exchangers

FCEA = Total fixed cost of new exchangers

FCSL = Total fixed cost of stream splitting

FCR = Total fixed cost of exchangers relocation

VCARE $_{ij}^z$  = Cost of area reduction made to existing heat exchanger matching hot stream i and cold stream j in the case  $(i,j) \notin B$ 

VCARE<sub>e</sub> = Cost of area reduction made to existing heat exchanger "e"

 $VCAAE_e = Cost$  of area addition made to existing heat exchanger "e"

VCAR = Total cost of area reduction made to existing heat exchangers

VCAA = Total cost of area addition made to existing heat exchangers

 $CSLH_i^z = Cost \text{ of splitting introduced to hot stream } i$ 

 $CSLC_i^z = Cost \text{ of splitting introduced to cold stream } j$ 

 $NSLH_i^z = Number of splitting introduced to hot stream i$ 

 $NSLC_i^z = Number of splitting introduced to cold stream j$ 

 $A_{ij}^z$  = Total required area for a match between hot stream i and cold stream j in zone z

 $\Delta A_{ij}^{aj}$  = Area addition for an existing heat exchanger between hot stream i and cold stream j

 $\Delta A_{e_N}^0$  = Area adjustment for the existing exchanger "e"

 $\Delta A_{ij}^{\alpha}$  = Area of new shell/new heat exchanger between hot stream *i* and cold stream *j* 

 $A_{e_{i}}^{N}$  = Area of new shell added to the existing exchanger "e"  $U_{ij}^{N}$  = Number of new shells in heat exchangers between hot stream i and cold stream j in the case  $(i,j) \notin B$ 

 $\hat{U}_{ij}^{z,k^N}$  = Number of new shells in heat exchangers between hot stream i and cold stream j in the case  $(i,j) \in B$ 

## **Literature Cited**

- (1) Barbaro, A.; Bagajewicz, M. New Rigorous One-Step MILP Formulation for Heat Exchanger Network Synthesis. *Comput. Chem. Eng.* **2005**, *29* (9), 1945–1976.
- (2) Furman, K. C; Sahinidis, N. V. A Critical Review and Annotated Bibliography for Heat Exchanger Network Synthesis in the 20th Century. *Ind. Eng. Chem. Res.* **2002**, *41* (10), 2335–2370.
- (3) Linnhoff, B.; Vredeveld, D. R. Pinch Technology has come of Age. *Chem. Eng. Prog.* **1984**, *80* (7), 33–40.
- (4) Tjoe, T. N.; Linnhoff, B. Using Pinch Technology for Process Network Retrofit. *Chem. Eng.* **1986**, *93* (8), 47.
- (5) Zhelev, T. C.; Boyadjiev, C.; Kantcheva, S. Renovation of Heat Exchanger Networks. *Hung. J. Ind. Chem.* **1987**, *15*, 403–414.
- (6) Lee, K. L.; Morabito, M.; Wood, R. M. Refinery Heat Integration using Pinch Technology. *Hydrocarbon Process.* **1989**, *68* (4), 4953.
- (7) Ahmad, S.; Polley, G. T. Debottlenecking of heat exchanger networks. *J. Heat Recovery Syst. CHP* **1990**, *10* (4), 369–385.
- (8) Polley, G. T. P.; Shahi, M.; Jegede, H. Pressure Drop Considerations in the Retrofit of Heat Exchanger Networks. *Chem. Eng. Res. Des.* **1990**, 68, 211.
- (9) van Reisen, J. L. B.; Grievink, J.; Polley, G. T.; Verheijen, P. J. T. The Placement of Two-Stream and Multistream Heat-Exchangers in an Existing Network Through Path Analysis. *Comput. Chem. Eng.* **1995**, *19*, S143–S148.
- (10) Lakshmanan, R.; Bañares-Alcantara, R. A Novel Visualization Tool for Heat Exchanger Network Retrofit. *Ind. Eng. Chem. Res.* **1996**, *35*, 4507–4522.
- (11) van Reisen, J. L. B.; Polley, G. T; Verheijen, P. J. T. Structural Targeting for Heat Integration Retrofit. *Appl. Therm. Eng.* **1998**, *18* (5), 283–294.
- (12) Li, H.; Yao, P. Using Process Energy Integration Technology in the Energy-Saving Retrofit of Large Scale Complex Chemical Process System. *Chin. J. Chem. Eng.* **1998**, 6 (3), 277–282.
- (13) Polley, G. T.; Amidpour, M. Don't let the Retrofit Pinch Pinch You. *Chem. Eng. Progress.* **2000**, *96* (11), 43–48.
- (14) Mehta, R. K. C.; Devalkar, S. K.; Narasimhan, S. An Optimization Approach for Evolutionary Synthesis of Heat Exchanger Networks. *Chem. Eng. Res. Des.* **2001**, *79*, 143–150.
- (15) Jezowski, J. Heat exchanger network grassroot and retrofit design. The review of the state-of-the-art: Part I. Heat exchanger network targeting and insight based methods of synthesis. *Hung. J. Ind. Chem.* **1994**, 22 (4), 279–294.

- (16) Fraser, D. M.; Gillespie, N. E. The Application of Pinch Technology to Retrofit Energy Integration of an Entire oil Refinery. *Trans. IChemE* **1992**, *70*, 395.
- (17) Lababidi, H. M. S.; Alatiqi, I. M; Nayfeh, L. J. Energy Retrofit Study of an Ammonia Plant. *Appl. Therm. Eng.* **2000**, 20 (15–16), 1495–1503
- (18) Querzoli, A. L.; Hoadley, A. F. A.; Dyson, T. E. S. Identification of Heat Integration Retrofit Opportunities for Crude Distillation and Residue Cracking Units. *Korean J. Chem. Eng.* **2003**, *20* (4), 635–641.
- (19) Yee, T. F.; Grossmann, I. E. Optimization Model for Structural Modifications in the Retrofit of Heat Exchanger Networks. *Proceedings of the First International Conference on FOCAPO*; Elsevier Science: New York, 1987; pp 653–662.
- (20) Ciric, A. R.; Floudas, C. A. A Comprehensive Optimization Model of the Heat Exchanger Network Retrofit Problem. *Heat Recovery Syst. CHP* **1990**, *10* (4), 407–422.
- (21) Ciric, A. R.; Floudas, C. A. A Mixed Integer Nonlinear-Programming Model for Retrofitting Heat Exchange-Networks. *Ind. Eng. Chem. Res.* **1990**, 29, 239–251.
- (22) Yee, T. F.; Grossmann, I. E. A Screening and Optimization Approach for the Retrofit of Heat-Exchanger Networks. *Ind. Eng. Chem. Res.* **1991**, *30* (1), 146–162.
- (23) Jezowski, J. Heat exchanger network grassroot and retrofit design. The review of the state-of-the-art: Part II. Heat exchanger network synthesis by mathematical-methods and approaches for retrofit design. *Hung. J. Ind. Chem.* **1994**, 22 (4), 295–308.
- (24) Briones, V.; Kokossis, A. A New Approach for the Optimal Retrofit of Heat Exchanger Networks. *Comput. Chem. Eng.* **1996**, *20* (Suppl.), S43–S48.
- (25) Briones, V.; Kokossis, A. C. Hypertargets: A Conceptual Programming Approach for the Optimisation of Industrial Heat Exchanger Networks II. Retrofit Design. *Chem. Eng. Sci.* **1999**, *54*, 541–561.
- (26) Sorsak, A.; Kravanja, Z. MINLP Retrofit of Heat Exchanger Networks Comprising Different Exchanger Types. *Comput. Chem. Eng.* **2004**, 28 (1–2), 235–251.
- (27) Bjork, K. M.; Nordman, R. Solving large-scale retrofit heat exchanger network synthesis problems with mathematical optimization methods. *Chem. Eng. Process.* **2005**, *44* (8), 869–876.
- (28) Nie, X. R.; Zhu, X. X. Heat Exchanger Network Retrofit Considering Pressure Drop and Heat-Transfer Enhancement. *AIChE J.* **1999**, *45* (6), 1239–1254.
- (29) Silva, M. L.; Zemp, R. J. Retrofit of Pressure Drop Constrained Heat Exchanger Networks. *Appl. Therm. Eng.* **2000**, *20* (15–16), 1469–1480.
- (30) Asante, N. D. K.; Zhu, X. X. An Automated Approach for Heat Exchanger Network Retrofit Featuring Minimal Topology Modifications. *Comput. Chem. Eng.* **1996**, *20* (Suppl.), S7–S12.
- (31) Al-Riyami, A. B. A.; Klemes, J.; Perry, S. Heat Integration Retrofit Analysis of Heat Exchanger Network of a Fluid Catalytic Cracking Plant. *Appl. Therm. Eng.* **2001**, *21*, 1449–1487.
- (32) Asante, N. D. K.; Zhu, X. X. An Automated and Interactive Approach for Heat Exchanger Network Retrofit. *Chem. Eng. Res. Des.* **1997**, 75 (A), 349–360.
- (33) Zhu, X. X.; Asante, N. D. K. Diagnosis and Optimization Approach for Heat Exchanger Network Retrofit. *AIChE J.* **1999**, *45* (7), 1488–1503.
- (34) Varbanov, P. S. C.; Boyadjiev, B.; Ivanov, B.; Vaklieva-Bancheva, N. G. Optimal Retrofit of Heat Exchange Networks (HEN) using Heuristic Paths and Superstructures. *Bulg. Chem. Commun.* **2000**, *32* (3–4), 517–528
- (35) Sieniutycz, S.; Jeżowski, J. Energy Optimization in Process Systems; Elsevier: Amsterdam, 2009.
- (36) Ponce-Ortega, J. M.; Jimenez-Gutierrez, A.; Grossmann, I. E. Simultaneous Retrofit and Heat Integration of Chemical Processes. *Ind. Eng. Chem. Res.* **2008**, *47*, 5512–5528.
- (37) Athier, G.; Floquet, P.; Pibouleau, L.; Domenech, S. A Mixed Method for Retrofitting Heat-Exchanger Networks. *Comput. Chem. Eng.* **1998**, 22 (Suppl.), S505–S511.
- (38) Bochenek, R.; Jeżowski, J. Adaptive Random Search Approach for Retrofitting Flexible Heat Exchanger Networks. *Hung. J. Ind. Chem.* **1999**, 27, 89–97.
- (39) Wang, K.; Yao, P.; Yuan, Y.; Yu, F.; Shi, G. A New Retrofit Approach for Heat Exchanger Networks-Improved Genetic Algorithm. *Chin. J. Chem. Eng.* **1997**, *5* (4), 347–358.
- (40) Abbas, H. A.; Wiggins, G. A.; Lakshmanan, R.; Morton, W. Heat Exchanger Network Retrofit via Constraint Logic Programming. *Comput. Chem. Eng.* **1999**, *23* (Suppl.), S129–S132.
- (41) Zhang, J.; Zhu, X. X. Simultaneous Optimization Approach for Heat Exchanger Network Retrofit with Process Changes. *Ind. Eng. Chem. Res.* **2000**, *39* (12), 4963–4973.

- (42) Ma, K.; Hui, C.; Yee, T. F. Constant Approach Temperature Model for HEN Retrofit. *Appl. Therm. Eng.* **2000**, 20 (15–16), 1505–1533.
- (43) Rezaei, E.; Shafiei, S. Heat exchanger networks retrofit by coupling genetic algorithm with NLP and ILP methods. *Comput. Chem. Eng.* **2009**, *33* (9), 1451–1459.
- (44) Papalexandri, K. P.; Pistikopoulos, E. N. A Multiperiod MINLP Model for Improving the Flexibility of Heat Exchanger Networks. *Comput. Chem. Eng.* **1993**, *17* (Suppl.), S111–S116.
- (45) Papalexandri, K. P.; Pistikopoulos, E. N. Synthesis and Retrofit Design of Operable Heat Exchanger Networks. 2. Dynamics and Control Structure Considerations. *Ind. Eng. Chem. Res.* **1994**, *33* (7), 1738–1755.
- (46) Furman, K. C.; Sahinidis, N. V. Computational Complexity of Heat Exchanger Network Synthesis. *Comput. Chem. Eng.* **2001**, 25 (9–10), 371–1390.
- (47) Bagajewicz, M. J. On the use of net present value in investment capacity planning models. *Ind. Eng. Chem. Res.* **2008**, *47* (23), 9413–9416.

Received for review August 17, 2009 Revised manuscript received March 24, 2010 Accepted May 20, 2010

IE901235C