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ARTICLE *in* INDUSTRIAL & ENGINEERING CHEMISTRY RESEARCH · DECEMBER 2013

Impact Factor: 2.59 · DOI: 10.1021/ie401925c

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I&EC research
Industrial & Engineering Chemistry Research
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pubs.acs.org/IECR

Thermodynamically Consistent Limiting Nusselt Number in the Viscous Dissipative Non-Newtonian Couette Flows

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ABSTRACT: The present study attempts to explore the effects of viscous dissipation on the heat transfer characteristics in the conduction limit for the Couette flow of power-law fluids between asymmetrically heated parallel plates. Here, two types of power-law fluids are considered (e.g., the pseudoplastic ($n < 1$) and dilatants ($n > 1$)) along with the special case of $n = 1$ (i.e., the Newtonian fluid). Utilizing the assumptions routinely employed in the literature, a semianalytical framework is devised to explore the effects of viscous dissipation on the limiting heat transfer characteristics. The shear produced by the movement of the upper plate in the dynamics of flow is emphasized; hence, a weak pressure gradient in the flow field is considered, which could otherwise create a complicated situation in the derivation of velocity distribution for the non-Newtonian Couette flows considered in this study. Despite the effect of viscous dissipation on the limiting heat transfer, the effect of asymmetrical wall heating on the same is shown for all the cases under consideration. Using these results, the possibilities of obtaining different temperature profiles and the variation of Nusselt number for different types of power-law fluid are aptly highlighted in view of the energy balance and analysis of the second-law of thermodynamics.

1. INTRODUCTION

In many practical applications, heat and fluid transport involves the movement of boundaries (e.g., extrusion, hot rolling, drawing, continuous casting, and so on). The thermofluidic transport through those systems, especially transfer of heat either from moving boundaries to the surrounding fluid or vice versa, is of immense importance. On the other hand, transport of fluid and the resulting heat transfer analysis of non-Newtonian fluids over moving surfaces finds many applications such as drawing of plastic film, polymer sheet extrusion from a dye, and production of glass fibers, to name a few. Moving boundaries deform the fluid layer near the boundary, and this results in local changes in the velocity gradient. Therefore, the viscous dissipation effects cannot be neglected in the heat transfer analysis associated with moving boundaries. However, the thermal energy generated due to viscous dissipation is significant in the flow field, and this alters the heat transfer rates following the changes in the temperature profile. Therefore, it is very important to take the effect of viscous dissipation into account when the convective heat transfer characteristics for any system are studied, irrespective of the type of fluid considered in the analysis. Brinkman[1] has conducted the first theoretical work concerning the heat generation due to viscous dissipation and analyzed the effects of viscous heating for the flow of a single phase Newtonian fluid through a circular tube.

For the past few decades, there has been an emergent recognition of the fact that the practical application in many areas of applied sciences demands fluids that are non-Newtonian in nature, such as polymeric liquids, molten plastics, pulps, emulsions, lubricants, and so forth. There could be many applications where fluids are so temperature-sensitive that the temperature arising out of the effect of the viscous dissipation may become detrimental to the different parts of the system and the product. Undoubtedly, this demands the perfect estimation of the heat transfer rates in the viscous dissipative flow of non-Newtonian fluids. In view of the great practical significance of non-Newtonian fluids because of their potential application, a number of studies concerning the forced convective heat transfer characteristics of non-Newtonian fluids, including the effects of viscous dissipation, are available in the literature. The term non-Newtonian fluid is very generic and includes all kinds of fluids for which Newtonian fluid flow behavior does not hold well. However, different rheological models have been used to represent the non-Newtonian behavior of the fluids, such as generalized Newtonian fluid model, linear viscoelastic fluid model, and so forth. Most of the earlier works on forced convective heat transfer of non-Newtonian fluids, including the effects of viscous dissipation, have focused on the generalized Newtonian fluid model. The studies reported by several researchers, such as Tso et al.,[2] Koliatwong et al.,[3] Etemad et al.,[4] Barletta,[5] and Chiba et al.,[6] have included the power-law fluid model, and that of Min et al[7] has included the Bingham fluid model in delineating the consequential effects of viscous dissipation on the heat transfer characteristics.

In addition to the studies reported by several researchers as mentioned above, a closer scrutiny of the literature reveals that there have been more than a few other studies available in the literature, however, discussed in detail about the effects of viscous dissipation on the heat transfer characteristics of different non-Newtonain fluids.[8−18] It is important to mention in this context that all the studies were either in a purely pressure-driven or combined pressure- and shear-driven flow configurations between parallel-plate channel. A closer look at their reported results clearly reveals that the effects of viscous dissipation (Davaa et al.,[10] Lin,[12] Saouli and Aiboud-Saouli,[15]

Received: June 18, 2013
Revised: October 24, 2013
Accepted: December 2, 2013
Published: December 2, 2013

   dx.doi.org/10.1021/ie401925c | Ind. Eng. Chem. Res. 2014, 53, 402−414

and Hung[16]) in conjunction with several other factors, like the effects of fluid rheology (Soares et al.[13]), wall suction, and blowing (Shokouhmand and Soleimani[14]), do have a strong role to play in the thermal transport characteristics of heat.

The effects of fluid rheology might be largely non-intuitive on the forced convective heat transfer analysis of viscous dissipative flow. It is, therefore, essential to investigate the heat transfer characteristics of non-Newtonian fluids that can lead to a considerable understanding of the fundamental aspects of the processes involved with the dynamics of non-Newtonian fluid flow. Recently, several researchers (Sheela Francisca et al.[17] and Chen and Zhu[18]) have investigated the effects of fluid rheology on the heat transfer characteristics of Couette−Poiseuille flow of different non-Newtonian fluids in a viscous dissipative environment. Manglik and Prusa[19] have carried out an extensive study on the effects of viscous dissipation on forced convective heat transfer of power-law fluids and reported several interesting behaviors of the Nusselt number and the wall temperature gradient for different power-law index. Baginski et al[20] have studied the heat transfer characteristics of a falling liquid film of power-law fluid. On the other hand, more than a few studies are available in the literature that dealt with the forced convective heat transfer analysis for the flow of power-law fluid without considering the effect of viscous dissipation.[21−23] All the studies mentioned above provide a fundamental understanding of the thermal transport characteristics of heat for the flow of power-law fluids and essentially explore the underlying physics of the flow and heat transfer behavior for different power-law indices and thermal boundary conditions imposed at the surface. It is needless to mention that more advanced and detailed analyses of heat transfer including the effect of viscous dissipation are already available in the literature for the flow of non-Newtonian fluids, but it is difficult to find the analysis on the limiting heat transfer characteristics in a viscous dissipative environment of non-Newtonian fluids.

The heat transfer characteristics in the limiting condition of non-Newtonian shear-driven flow between two parallel plates under asymmetrical constant wall temperature are a conundrum that is yet to be explored. In recent days, the effects of viscous dissipation on the limiting Nusselt number for a hydrodynamically fully developed laminar shear-driven flow of Newtonian fluids have been reported.[24,25] The present study is an improvement over the earlier work of Mondal and Mukherjee,[25] where an analytical investigation has been made probing the effects of viscous dissipation on the limiting heat transfer characteristics for a shear-driven flow of a Newtonian fluid between asymmetrically heated parallel plates. Conversely, the objective of the present work is to look at the effects of viscous dissipation in a non-Newtonian shear-driven flow and, hence, discussed are the heat transfer characteristics in the limiting condition, highlighting the effects of viscous dissipation induced by the moving plate. Non-Newtonian fluids can be characterized with the aid of a power-law model despite the availability of a number of fluid rheologies. The power-law model characterizes two important classes of non-Newtonian fluid (i.e., the pseudoplastic and the dilatant fluids) along with a special case of the characterization of Newtonian fluid. In addition to the simplicity in the mathematical analysis, its wide coverage in the investigation of non-Newtonian flow behavior has propelled the consideration of the power-law fluids to represent non-Newtonian fluid rheologies in the present study. Accordingly, in the present work, the aim is at the development of a semianalytical formulation for assessing the laminar forced convective heat transfer characteristics in a shear-driven flow of power-law fluids between two asymmetrically heated parallel plates.

The paper is split into five sections. In Section 2, the details of the problem taken up for the study following the introduction are discussed and the analysis is described in detail to whet interest among the researchers. Section 3 is committed to the results and discussions, and in Section 4, the thermodynamic analysis of the results obtained in the present study is presented. Section 5 summarizes the principal conclusions.

2. MATHEMATICAL FORMULATION OF THE PROBLEM AND ANALYSIS

2.1. Physical Considerations. For the analysis, a channel between two parallel plates of infinite length, gap length H and width b, with $b \gg H$ is considered as shown in Figure 1. Fluid is

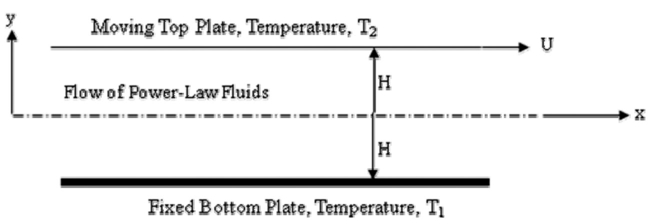

Figure 1. Schematic to the problem, H is the half gap length between two plates, U refers to the velocity of top plate, and T_1 and T_2 represent the temperature of bottom and top plate, respectively.

flowing in the axial (x) direction, and the flow is influenced mainly by the movement of the upper plate. The flow is considered to be both hydrodynamically and thermally fully developed. The no-slip boundary conditions are assumed to be valid at both of the plates from both a hydrodynamic and a thermal point of view. In addition to the consideration of fully developed flow, a few more assumptions considered for the study are given below:

- Power-law (non-Newtonian) fluid.
- Incompressible fluid flow.
- There is no external heat source, and thermophysical properties are constant.
- Axial conduction is neglected in the fluid and through the wall.

The velocity distribution for the flow of a Newtonian fluid between two parallel plates, where the upper plate is moving at constant speed U, along with the applied pressure gradient can be represented as

$$u = \frac{U}{2}\left(1 + \frac{y}{H}\right) - \frac{H^2}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\left\{1 - \left(\frac{y}{H}\right)^2\right\} \tag{1a}$$

In the absence of applied pressure gradient, the above equation reduces to

$$u = \frac{U}{2}\left(1 + \frac{y}{H}\right) \tag{1b}$$

However, eq 1a can be recast in a nondimensional form as given below:

$$\bar{u} = \frac{u}{U} = \frac{1}{2}\left(1 + \frac{y}{H}\right) - \frac{H^2}{2U\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\left\{1 - \left(\frac{y}{H}\right)^2\right\} \tag{2}$$

The velocity distribution in the absence of any pressure gradient as represented in eq 1b indicates that the flow is actuated by the shear produced by the movement of the plate only. The intensity of the applied pressure gradient, however, controls the location of the maximum velocity in the flow domain as envisaged from eq 1a. The dimensionless term $-\frac{H^2}{2U\mu}\frac{dp}{dx}$ in eq 2 sets hurdles in the derivation of the velocity distribution for the shear-driven flow of non-Newtonian fluids. A closer look at eq 1a indicates that the term $-\frac{H^2}{2U\mu}\frac{dp}{dx}$ plays a significant role in dictating the maximum fluid velocity in the flow field. A magnitude of $-\frac{H^2}{2U\mu}\frac{dp}{dx} \le 1$ sets the maximum fluid velocity at the top plate, whereas for $-\frac{H^2}{2U\mu}\frac{dp}{dx} > 1$, the maximum fluid velocity lies in between parallel plates. This characteristic of pressure gradient becomes equally applicable even for non-Newtonian Couette flows. Therefore, the characteristics of the term $-\frac{H^2}{2U\mu}\frac{dp}{dx}$ essentially render difficulty in the derivation of velocity distribution in non-Newtonian Couette flow. Therefore, a weak pressure gradient is considered in the present study so as to ascertain the occurrence of maximum fluid velocity at the top plate, which essentially becomes the case in a purely shear-driven flow scenario. The complicacy related to the derivation of velocity distribution for the non-Newtonian Couette flows has compelled the research community to derive the velocity distribution before the problem being taken up in solving the heat transfer characteristics of associated non-Newtonian flows.[12] Lin[12] has proposed the velocity profile for the Couette flow of non-Newtonian fluids. In the following section, the velocity profile in the flow domain for the problem considered will be developed specific to the case where the pressure gradient is less.

2.2. Analysis of the Problem. A steady flow of a power-law fluid within parallel plates is considered to explore the heat transfer characteristics in a viscous dissipative environment in the conduction limit, specific to the case of unequal constant wall temperatures considered in the study. As per the assumptions, the rheological behavior of the power-law fluid between two fixed parallel plates is expressed by the representation of shear stress as given below:

$$\tau_{yx} = -m\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy} \tag{3}$$

where m is the consistency factor, n is the power-law index, (du/dy) is the velocity gradient perpendicular to flow direction, and τ_{yx} represents the shear stress. However, different values of n exhibit different kinds of power-law fluid. For shear-thinning fluids (i.e., for pseudoplastic fluids) $0 < n < 1$, whereas for shear-thickening fluids (i.e., for dilatant fluids) $n > 1$; finally, $n = 1$ represents Newtonian fluids.

An expression of the velocity distribution for the problem taken up in the study needs to be devised before the heat transfer investigation can be undertaken. For fluid flow with a strong pressure gradient, the velocity distribution is such that the maximum velocity appears at the mid-plane between two parallel plates. The present analysis focuses on the flow actuation by the movement of the upper plate; hence, it is assumed that a weak pressure gradient is acting in the flow field and the maximum velocity of the fluid appears at the top plate. From the momentum balance of flows, one can write

$$\nabla\cdot\tau = -\nabla p \tag{4a}$$

Specific to the problem considered in the present study, the above equation is reduced to

$$\frac{d\tau_{yx}}{dy} = -\frac{dp}{dx} \tag{4b}$$

However, following the assumptions made in the present analysis, and using eqs 2 and 4b, it is possible to arrive at the simplified equation given below:

$$-m\left(\frac{du}{dy}\right)^n = -\left(\frac{dp}{dx}\right)y + C \tag{5a}$$

After some algebraic operations, eq 5a can be written as

$$\left(\frac{du}{dy}\right)^n = \left\{-\frac{H}{m}\left(\frac{dp}{dx}\right)\right\}\left\{\frac{C}{H\left(\frac{dp}{dx}\right)} - \frac{y}{H}\right\} \tag{5b}$$

However, eq 5b is further rewritten as

$$\frac{du}{dy} = a^{1/n}\left(B - \frac{y}{H}\right)^{1/n} \tag{5c}$$

where the terms $-\frac{H}{m}\left(\frac{dp}{dx}\right)$ and $\frac{C}{H\left(\frac{dp}{dx}\right)}$ are denoted by a and B respectively. The term a in the above equation includes the pressure gradient term. However, the velocity profile of the Couette flow of power-law fluids can be obtained upon integration of eq 5c as

$$u = -a^{1/n}H\left(\frac{n}{n+1}\right)\left(B - \frac{y}{H}\right)^{(n+1)/n} + C_1 \tag{6}$$

To evaluate the constant C_1, the no slip velocity boundary condition is employed (i.e., $u = 0$ at $y = -H$), and the velocity distribution can be written as

$$u = a^{1/n}H\left(\frac{n}{n+1}\right)\left[(B+1)^{(n+1)/n} - \left(B - \frac{y}{H}\right)^{(n+1)/n}\right] \tag{7}$$

Utilizing the second boundary condition (i.e., $u = U$ at $y = H$), the expression of the constant a, which in essence gives the information about the pressure gradient term in the flow field as evaluated. This boundary condition relates the pressure gradient term with the velocity of the moving plate through a parameter gamma, which is given in eq 9. It is to be mentioned here that the term gamma plays the same role as the dimensionless group $-\frac{H^2}{2U\mu}\frac{dp}{dx}$ does in case of Couette flows of Newtonian fluids. Using the second boundary condition, the following relation between the velocity of the upper plate and the applied pressure gradient can be obtained:

$$U = a^{1/n}H\left(\frac{n}{n+1}\right)[(B+1)^{(n+1)/n} - (B-1)^{(n+1)/n}] \tag{8}$$

In order to write the velocity distribution in the flow field in a nondimensional form, a parameter γ is defined, which is the reciprocal of the nondimensional pressure gradient in the present analysis, and is given as

$$\gamma = \frac{(n+1)}{n}\left(\frac{U}{H^{(n+1)/n}\left(-\frac{1}{m}\frac{\mathrm{d}p}{\mathrm{d}x}\right)^{1/n}}\right) = [(B+1)^{(n+1)/n} - (B-1)^{(n+1)/n}] \tag{9}$$

However, for the no-slip boundary condition imposed at both the plates, the velocity distribution for the generalized non-Newtonian flow between two parallel plates is represented by

$$u = U\frac{1}{\gamma}\left[(B+1)^{(n+1)/n} - \left(B - \frac{y}{H}\right)^{(n+1)/n}\right] \tag{10}$$

It is important to mention here that to arrive at eqs 9 and 10, the maximum fluid velocity in the flow field is assumed to occur at the top plate, which essentially presumes the magnitude of pressure gradient term to be less than or equal to one ($-\mathrm{d}p/\mathrm{d}x \le 1$). Therefore, the term γ in the above equation should be ≥ 1 so that the maximum fluid velocity in the flow domain happens to be consistent with the assumption made in deriving the velocity profile. Moreover, the value $n = 1$ makes no difference between eqs 10 and 1a. In the study, three different values of $n = 0.5$, 1, and 2 are considered for investigating the heat transfer characteristics for the shear-thinning, Newtonian, and shear-thickening fluids, respectively. However, following the Newton–Rapshon method, the values of B for three different values of n are derived as given in Table 1. It is considered that $\gamma = 5$ in the present analysis.

Table 1. Values of B for Different Values of n (for $\gamma = 5$)

n	B
0.5	0.7071
1	1.25
2	2.808

In order to assess the role of viscous dissipation in the limiting heat transfer characteristics in asymmetrically heated parallel plates, the thermal energy equation for the boundary conditions of unequal constant wall temperatures is scrutinized. For this, the expression of the Nusselt numbers in the limiting condition is considered. The convective terms in the thermal energy equation are excluded, whereas the viscous dissipation term is incorporated. Considering this aspect, the thermal energy equation including the viscous dissipation term is solved analytically in absence of the convective term. However, the thermal energy equation in the limiting condition is represented by

$$k\frac{\mathrm{d}^2T}{\mathrm{d}y^2} + m\left|\frac{\mathrm{d}u}{\mathrm{d}y}\right|^{n-1}\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 = 0 \tag{11}$$

The second term on the left-hand side of the above equation is the viscous dissipation term. To obtain the explicit expressions of the Nusselt numbers in the limiting condition, the above equation is required to be solved after coupling the expression of the velocity distribution with it.

2.3. Outline of the Analysis. For the determination of the limiting Nusselt number in a viscous dissipative environment, an analytical technique is followed. In a dimensional form, the thermal energy equation for a steady flow of non-Newtonian fluid with constant physical properties, very specific to the problem, as considered in this study, is represented by eq 11. However, to express eq 11 in a nondimensional framework, it is essential to define the nondimensional parameters suitably. From the physical considerations of the problem taken up in the present analysis, the nondimensional parameters are chosen as follows: $Y \to y/H$, $\bar{u} \to u/U$, and $\theta \to (T - \bar{T})/(T_f - \bar{T})$. It is important to mention here that the temperature T_f is the uniform fluid inlet temperature, and $\bar{T}$ is the average wall temperature. To take the effect of unequal wall temperature into account, a parameter is defined to characterize the asymmetry in the wall heating and is given by $S = (T_2 - T_f)/(T_2 - T_f)$.

However, with the aid of the above dimensionless parameters, and using eq 10, eq 11 may be normalized to yield the following equation:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}Y^2} = -\frac{mU^{(n+1)}}{k(T_f - \bar{T})H^{(n-1)}}\left(\frac{n+1}{n}\right)^{n+1}\left(\frac{1}{\gamma}\right)^{n+1}(B - Y)^{n+1/n} \tag{12}$$

The above equation can be further reduced to the following form:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}Y^2} = -BrP(B - Y)^{n+1/n} \tag{13}$$

In order to obtain eq 13, the right-hand side of eq 12 is simplified, and to do that, the Brinkman number Br is defined and a constant P is introduced as follows:

$$Br = \frac{mU^{(n+1)}}{k(T_f - \bar{T})H^{n-1}} \quad \text{and} \quad P = \left(\frac{1}{\gamma}\right)^{n+1}\left(\frac{n+1}{n}\right)^{n+1} \tag{14}$$

To arrive at this simplified equation, it is assumed that the convective term of the energy equation is negligible. This is analogous to saying that eq 13 is the representation of the thermal energy equation in the conduction limit; so the solution of eq 13 yields the temperature distribution in the flow field in the limiting condition. However, in order to get the closed-form expression of the limiting temperature profile, the following asymmetrical thermal boundary conditions are used:

$$\text{At} \quad y = -H,\ T = T_1 \quad \text{and at} \quad y = H,\ T = T_2 \tag{15a}$$

In a nondimensional form, the above set of boundary conditions may be expressed as given below:

$$\text{At} \quad Y = -1,\ \theta = \frac{(T_1 - \bar{T})}{(T_f - \bar{T})} \quad \text{and at} \quad Y = 1,\ \theta = \frac{(T_2 - \bar{T})}{(T_f - \bar{T})} \tag{15b}$$

Equation 13 can be integrated twice to yield the general expression of the temperature profile in the conduction limit subjected to boundary conditions given in eq 15a and 15b as presented below:

$$\theta = -Br\frac{Pn^2}{(2n+1)(3n+1)}(B - Y)^{3n+1/n} + C_2Y + C_3 \tag{16}$$

In the above equation, C_2 and C_3 are constants. Letting $\varepsilon = -Br\frac{Pn^2}{(2n+1)(3n+1)}$, the expressions of C_2 and C_3 can be reduced to

$$C_2 = \frac{(1-S)}{(1+S)} + \frac{\varepsilon}{2}[(B+1)^{(3n+1)/n} - (B-1)^{(3n+1)/n}]$$
$$C_3 = -\frac{\varepsilon}{2}[(B+1)^{(3n+1)/n} + (B-1)^{(3n+1)/n}] \tag{17}$$

The main physical quantity of interest is the Nusselt number, which represents the heat transfer rate at the wall. Therefore, in order to obtain deeper insights regarding the heat transfer characteristics, subsequently, the definition of bulk mean fluid temperature in the conduction limit T_{mc} is considered as follows:

$$T_{mc} = \frac{\int_{-H}^{H} uTb\, dy}{\int_{-H}^{H} ub\, dy} \tag{18}$$

where the heat transfer at the lower plate is given by

$$q_{1c} = h_{1c}(T_1 - T_{mc}) = -k\frac{\partial T}{\partial y}\bigg|_{y=-H} \tag{19}$$

with h_{1c} being the convective heat transfer coefficient. Therefore

$$Nu_{1c} = \frac{h_{1c}(H)}{k} = -\frac{(d\theta_c/dY)|_{Y=-1}}{(\theta_c|_{Y=0} - \theta_{mc})} = -\frac{(d\theta_c/dY)|_{Y=-1}}{(\theta_b - \theta_{mc})} \tag{20}$$

Proceeding similarly, the limiting Nusselt number at the top plate can be expressed as:

$$Nu_{2c} = \frac{h_{2c}(H)}{k} = \frac{(d\theta_c/dY)|_{Y=1}}{(\theta_c|_{Y=1} - \theta_{mc})} = \frac{(d\theta_c/dY)|_{Y=1}}{(\theta_t - \theta_{mc})} \tag{21}$$

where θ_{mc} is the nondimensional form of the bulk mean temperature in the conduction limit, and θ_b and θ_t are the nondimensional temperatures at the bottom and top plate, respectively. It is pertinent to obtain the expression of θ_{mc}, as it will be required to get the closed-form expression of the limiting Nusselt numbers at both the plates. However, eq 18 may be normalized to yield

$$\theta_{mc} = \frac{(T_{mc} - \overline{T})}{(T_f - \overline{T})} = \frac{\int_{-1}^{1} \theta \frac{1}{\gamma}[(B+1)^{(n+1)/n} - (B-Y)^{(n+1)/n}]dY}{\left[\frac{1}{\gamma}\left\{2(B+1)^{(n+1)/n} + \frac{n}{(2n+1)}[(B-1)^{(2n+1)/n} - (B+1)^{(2n+1)/n}]\right\}\right]} \tag{22}$$

The expressions of limiting values' nondimensional temperature at both the plates are as follows:

$$\theta_b = \varepsilon(B+1)^{(3n+1)/n} - \frac{(1-S)}{(1+S)} - \frac{\varepsilon}{2}[(B+1)^{(3n+1)/n} - (B-1)^{(3n+1)/n}] - \frac{\varepsilon}{2}[(B+1)^{(3n+1)/n} + (B-1)^{(3n+1)/n}] \tag{23}$$

$$\theta_t = \varepsilon(B-1)^{(3n+1)/n} + \frac{(1-S)}{(1+S)} + \frac{\varepsilon}{2}[(B+1)^{(3n+1)/n} - (B-1)^{(3n+1)/n}] - \frac{\varepsilon}{2}[(B+1)^{(3n+1)/n} + (B-1)^{(3n+1)/n}] \tag{24}$$

Equation 20, in conjunction to eqs 22 and 23, yields an explicit expression for the limiting Nusselt number at the bottom plate, giving a quantitative relation among the different physical parameters governing the thermofluidic transport. It is given as

$$Nu_{1c} = -\frac{\left[-(B+1)^{(2n+1)/n}\frac{(3n+1)}{n}\varepsilon + \frac{\varepsilon}{2}\{(B+1)^{(3n+1)/n} - (B-1)^{(3n+1)/n}\} + \frac{(1-S)}{(1+S)}\right]}{\left[\begin{array}{l}\varepsilon(B+1)^{(3n+1)/n} - \frac{(1-S)}{(1+S)} - \frac{\varepsilon}{2}\{(B+1)^{(3n+1)/n} - (B-1)^{(3n+1)/n}\} - \frac{\varepsilon}{2}[(B+1)^{(3n+1)/n} + (B-1)^{(3n+1)/n}] \\ - \left\{\frac{(I_1 + I_2 + I_3 + I_4)}{\left[\frac{1}{\gamma}\left\{2(B+1)^{(n+1)/n} + \frac{n}{(2n+1)}[(B-1)^{(2n+1)/n} - (B+1)^{(2n+1)/n}]\right\}\right]}\right\}\end{array}\right]} \tag{25}$$

In a similar way, the expression of limiting the Nusselt number at the top plate using eqs 21, 22, and 24 is obtained as follows:

$$Nu_{2c} = \frac{\left[-(B-1)^{(2n+1)/n}\frac{(3n+1)}{n}\varepsilon + \frac{\varepsilon}{2}\{(B+1)^{(3n+1)/n} - (B-1)^{(3n+1)/n}\} + \frac{(1-S)}{(1+S)}\right]}{\left[\begin{array}{l}\varepsilon(B-1)^{(3n+1)/n} + \frac{(1-S)}{(1+S)} + \frac{\varepsilon}{2}\{(B+1)^{(3n+1)/n} - (B-1)^{(3n+1)/n}\} - \frac{\varepsilon}{2}[(B+1)^{(3n+1)/n} + (B-1)^{(3n+1)/n}] \\ - \left\{\frac{(I_1 + I_2 + I_3 + I_4)}{\left[\frac{1}{\gamma}\left\{2(B+1)^{(n+1)/n} + \frac{n}{(2n+1)}[(B-1)^{(2n+1)/n} - (B+1)^{(2n+1)/n}]\right\}\right]}\right\}\end{array}\right]} \tag{26}$$

where

$$I_1 = \frac{1}{\gamma}(B+1)^{(n+1)/n}\left[-\varepsilon\{(B+1)^{(3n+1)/n} + (B-1)^{(3n+1)/n}\} - \frac{\varepsilon n}{(4n+1)}\{(B-1)^{(4n+1)/n} - (B+1)^{(4n+1)/n}\}\right]$$
$$I_2 = \frac{\varepsilon}{\gamma}\frac{n}{(5n+2)}[(B-1)^{(5n+2)/n} - (B+1)^{(5n+2)/n}]$$
$$I_3 = \frac{1}{\gamma}\frac{n}{(2n+1)}\left[\frac{(1-S)}{(1+S)} + \frac{\varepsilon}{2}\{(B+1)^{(3n+1)/n} - (B-1)^{(3n+1)/n}\}\right]\left[\{(B-1)^{(2n+1)/n} + (B+1)^{(2n+1)/n}\} + \frac{n}{(3n+1)}\{(B-1)^{(3n+1)/n} - (B+1)^{(3n+1)/n}\}\right]$$
$$I_4 = -\frac{\varepsilon}{2\gamma}\frac{n}{(2n+1)}[(B+1)^{(3n+1)/n} + (B-1)^{(3n+1)/n}][(B-1)^{(2n+1)/n} - (B+1)^{(2n+1)/n}] \quad (27)$$

Based on the derivations as mentioned above, various limiting cases for the understanding of the heat transfer characteristics can be analyzed in a viscous dissipative environment. Some of the cases are discussed in the next section.

3. RESULTS AND DISCUSSION

In an effort to bring out the effect of viscous dissipation on the limiting forced convective heat transfer characteristics, and more precisely on the limiting value of Nusselt number, different particular cases are discussed to investigate the heat transfer characteristics in a viscous dissipative environment. The general solution of a power-law fluid invokes complexity to yield the velocity distribution in an analytical formalism and, accordingly, three different values of power-law index, $n = 0.5$, 1, and 2, are considered in the present study for the investigation of the limiting Nusselt numbers in an analytical framework. At first, the case of the Newtonian fluid (i.e., $n = 1$) is discussed to examine the consistency of the present analysis with the results obtained by Mondal and Mukherjee.[25] For $n = 1$, the numerical values of both Nusselt numbers are evaluated for different values of Br at three different values of asymmetry parameters. The numerical values of both Nu_{1c} and Nu_{2c} obtained for the three values $Br = 1$, -1, and 0 are listed in Table 2.

The results obtained in this study are validated against those available in the literature in the case of a Newtonian fluid as presented in Table 2. However, one can see that the closeness of the results obtained in the present study to the results obtained by Mondal and Mukherjee[25] is considerable except for $Br = -1$ and 1 at $S = 0.5$. The exception, which is seen in some specific cases, is attributable to the consideration of an applied pressure gradient for the derivation of the velocity distribution in the present analysis, however small it is. A relatively larger difference in the results for a particular case as can be seen from Table 1 is essentially attributable to the effect of pressure gradient applied in the non-Newtonian Couette flows. It is important to mention in this context that the present results correspond to the Couette flow of non-Newtonian fluids. On the other hand, an attempt has been made to validate the present results with the reported results in the literature [Mondal and Mukherjee[25]], which was obtained in a purely shear-driven flow configuration of Newtonian fluids. The application of a weak pressure gradient, which is an essential requirement to obtain the velocity distribution in the non-Newtonian Couette flows, changes the transport characteristics of heat following the alteration in the velocity distribution in the flow field of the study under consideration as compared to those of the Couette flows of Newtonian fluids. The change in the flow characteristics, however, changes the temperature distribution in the flow domain; thus, different heat transfer rates for a given wall temperature is obvious, as can be clearly envisaged from the values of Nusselt numbers given in Table 1.

3.1. Power-Law Fluids.

In this section, some interesting situations concerning non-Newtonian fluids that are yet to be addressed and explained by the theoretical research community are analyzed. For this, first, the effects of viscous dissipation on heat transfer characteristics are verified; in view of the limiting condition in the case of a shear-thinning fluid. Second, the case of shear-thickening fluid is considered here for the analysis of viscous dissipative heat transfer.

3.1.1. Shear-Thinning Fluids: Temperature Profile and Nusselt Numbers. For $n < 1$, the fluid is pseudoplastic, and the expressions of the limiting Nusselt numbers on both the plates are evaluated following eqs 25−27 with $n = 0.5$, as given below:

Table 2. Variation of Nusselt Numbers on Both the Plates with Brinkman Number for Three Different Cases of Asymmetric Wall Heating

	$S = -0.5$				$S = 0.0$				$S = 0.5$			
	present study		Mondal and Mukherjee (2012)		present study		Mondal and Mukherjee (2012)		present study		Mondal and Mukherjee (2012)	
Br	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}
−1.0	0.701	1.3775	0.702	1.52	0.6193	1.4361	0.60	1.6167	0.312	1.7465	0.2308	1.901
0.0	0.791	1.3571	0.75	1.5	0.7917	1.3571	0.75	1.5	0.7917	1.3571	0.75	1.5
1.0	0.8457	1.3851	0.80	1.4348	1.0610	1.2848	0.9024	1.2857	1.2043	0.9815	1.1053	0.6

$$Nu_{1c} = \frac{\left[0.81798Br + \frac{(1-S)}{(1+S)}\right]}{\left[0.1180Br + 1.1788\frac{(1-S)}{(1+S)}\right]} \tag{28a}$$

$$Nu_{2c} = \frac{\left[0.16755Br - \frac{(1-S)}{(1+S)}\right]}{\left[0.1180Br - 0.8212\frac{(1-S)}{(1+S)}\right]} \tag{28b}$$

In a viscous fluid flow, the viscosity of the fluid makes it absorb energy from the motion of the fluid and transform the same as an internal energy of the fluid. This conversion of energy, in effect, stimulates the fluid temperature. This process of energy conversion is irreversible and is referred to as viscous dissipation. Thus, in the context of convective heat transfer analysis, it is essential to consider the viscous heating arising as a result of the internal fluid friction. In the study, focus is on the shear-driven flow, but the shear heating produced by the movement of the upper plate is of immense importance. The relative strength of heat generation due to viscous dissipation and the heat supplied externally into the fluid is measured by the Brinkman number Br. Positive values of Br represent the case of wall heating and resemble the situation of heat transfer to the fluid across the wall, whereas the negative values of Br represent the wall cooling case. The present study aims at finding out the influence of the effects of viscous dissipation on the temperature profile and, consequently, the Nusselt numbers in the conduction limit. In the present study, the variation of dimensionless temperature θ in the limiting condition for the three values $Br = 5$, -5, and 0, respectively, is shown while different values of asymmetric wall heating $S = 0.5$, -0.5, and 1.0 have been considered for accentuating the effects on the same variation. Figures 2a−c illustrate the variation of θ with Br specific to the case of shear-thickening fluids obtained at three different values of asymmetric wall heating, $S = 0.5$, -0.5, and 1.0, respectively.

A closer look on the above figures reveals that in the cases with $Br \neq 0$, the profile of the dimensionless temperature θ gets altered in comparison to the case of $Br = 0$ though the imposed boundary conditions on both the plates remain invariant. Compared to the case of $Br = 0$, nondimensional temperature θ increases for the positive values of Br, as expected. However, the reverse is true for the negative values of Br, as one can see from the figures presented above. On the other hand, it is important to mention here that the temperature profile in a non-viscous dissipative environment exhibits a linear trend irrespective of wall heating or cooling, and it is a pure conduction profile in the thermal entrance region. A precise look at the above figures discloses that the effect of the asymmetry parameter of wall heating has a direct bearing on the temperature profile, which one can see in purview of the variation of temperature profile. Actually, increasing the value of the asymmetry parameter alters the nature of the variation significantly even for any particular value of Br considered. However, as S increases, the differences in the temperature for any two Br at any particular Y increase, which is attributable to the influence of asymmetric wall heating.

It is important to mention that eqs 28a and 28b represent the expression of the Nusselt number in the conduction limit at the bottom and top plate, respectively, specific to the case of shear-thinning fluids. Figures 3a−b are the graphical representation of the variation of the Nusselt number on the bottom plate Nu_{1c} with the Brinkman number Br for two different cases of

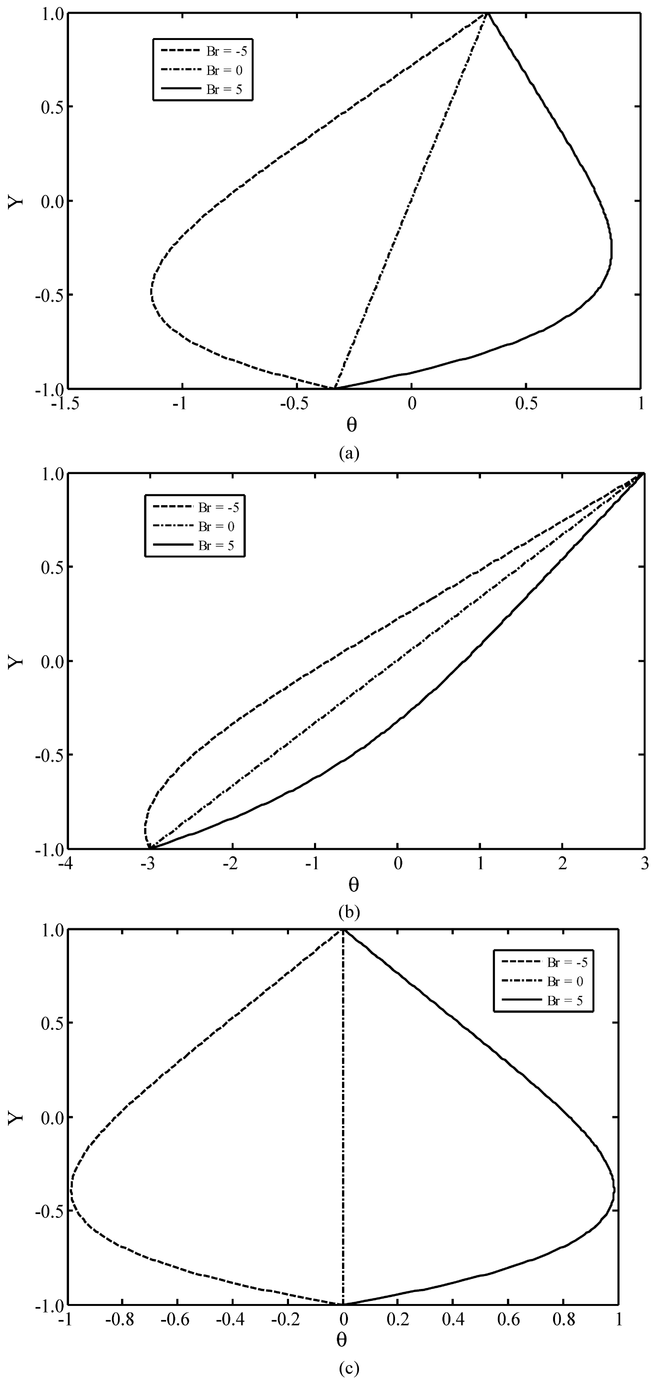

Figure 2. Dimensionless temperature profile of shear-thinning fluids for different values of Br. (a) $S = 0.5$, (b) $S = -0.5$, and (c) $S = 1.0$. The behavior of the temperature profile is shown for each positive and negative Br. The temperature profile for nonzero values of Br changes about the same compared to that obtained at $Br = 0$. Positive values of Br increase the frictional heating and the fluid temperature increases, whereas negative values of Br do the reverse.

asymmetrical wall heating $S = 0.5$ and -0.5, respectively. One may notice that the variation of Nusselt number with Br is not continuous; rather a clear existence of the point of singularity (discontinuity in behavior) is observed at different points for different cases of asymmetric wall heating within the range of $-150 \leq Br \leq 150$, as expected from eqs 28a and 28b. The different locations of the point of singularity are due to the different degrees of asymmetry considered, and at this point,

dx.doi.org/10.1021/ie401925c | Ind. Eng. Chem. Res. 2014, 53, 402−414

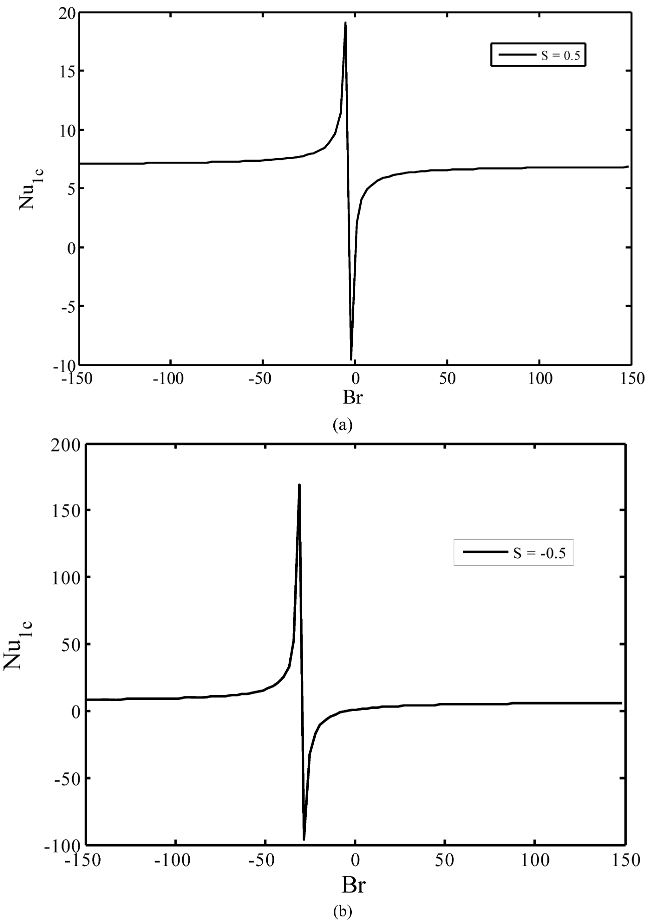

Figure 3. Influence of Br on the variation of Nu_{1c} of shear-thinning fluids for two different cases of asymmetric wall heating, (a) $S = 0.5$ and (b) $S = -0.5$. The variation of Nusselt number shows discontinuity in behavior, and at the point of discontinuity, equilibrium of energy is reached.

equilibrium between the internal heat generation due to viscous dissipation and the heat supplied by the wall is attained.[26,27] In order to unveil the onset of the point of singularity on the variation of Nusselt numbers at both the plates, it would be reasonable to look into the variation of temperature distribution in the flow field. Having a closer look at the temperature distribution, it can be observed that the alteration in the Brinkman number (Br) changes the temperature in the flow field following the change in the fluid frictional heating in the flow. Therefore, for any given condition of asymmetrical wall heating, there could be any particular value of Brinkman number Br for which the fluid temperature due to frictional heating may become equal to the imposed wall temperature. This situation essentially allows no heat transfer to take palace in either direction and eventually results in an unbounded swing on the variation of Nusselt number, as seen from the figures mentioned previously. Moreover, the situation of local equilibrium of energy is more clearly envisaged from the temperature profile, where it is observed that the temperature profile for any given value of Br exhibits either a global minima (for $Br < 0$) or global maxima (for $Br > 0$), and at this point, the heat supplied by the wall and the generated heat owing to viscous dissipation effect becomes equal, causing no heat to flow in either direction. This, in turn, gives an indication of having a point of singularity on the variation of Nusselt number at both the plates. However, from the onset of the point of singularity and with the increasing value of Br in the positive direction ($Br > 0$), the Nusselt number decreases because of the decrease in the driving potential of the heat transfer. Finally, it reaches different asymptotic values as $Br \to \infty$ for all the cases of wall heating under consideration. As explained before that the negative value of Br ($Br < 0$) represents a case of wall cooling, and is applied to weaken the bulk temperature of the fluid. However, the viscous dissipation increases the temperature of the fluid and alters the heat balance. Therefore, with Br increasing in the negative direction, the bulk fluid temperature increases and different asymptotes appear as $Br \to -\infty$ for different asymmetry parameters considered.

Figures 4a−b, on the other hand, show the variation of Nu_{2c} with Br obtained for two different cases of asymmetrical wall heating $S = \pm 0.5$, as considered in the study. A closer scrutiny of these figures discloses some important features, which differentiate the figures from those obtained at the bottom plate. From the point of the singularity, with increasing Br in the positive direction ($Br > 0$), the Nusselt number decreases, whereas it increases with the increasing value of Br in the negative direction. On the other hand, unlike the variation of Nu_{1c}, the variation of Nu_{2c} shows a similar asymptotic nature as $Br \to \infty$ in either direction, even for all degrees of asymmetry parameter considered. The differences, as observed between the variations of the Nusselt number on both the plates from the figures, are owing to the movement of the top plate. However, the values of the Nusselt number for shear-thinning fluids obtained at different values of asymmetry parameter, and for different Br, are listed in Table 3. It is worth mentioning that each of the Nusselt numbers holds the same constant value in a nonviscous−dissipative environment, irrespective of the wall heating or cooling, as apparent from Table 3.

3.1.2. Shear-Thickening Fluids: Temperature Profile and Nusselt Numbers. For $n > 1$, the fluid is dilatant, and the expressions of the limiting Nusselt numbers on both the plates are evaluated following eqs 25−27. However, considering $n = 2$, the expression of Nusselt numbers is obtained as given below:

$$Nu_{1c} = \frac{\left[0.1513Br + \frac{(1-S)}{(1+S)}\right]}{\left[0.04165Br + 1.3150\frac{(1-S)}{(1+S)}\right]} \tag{29a}$$

and,

$$Nu_{2c} = \frac{\left[0.10632Br - \frac{(1-S)}{(1+S)}\right]}{\left[0.04165Br - 0.6855\frac{(1-S)}{(1+S)}\right]} \tag{29b}$$

To demonstrate the effects of viscous dissipation on temperature distribution and, subsequently, on the Nusselt number, the case of shear-thickening fluid ($n = 2$) is considered here. Figures 5a−c depict the temperature distribution in the limiting condition obtained at three different degrees of asymmetry parameter $S = 0.5$, -0.5, and 1.0 as considered in the study. To highlight the reflection of the viscous dissipation on the temperature profile of shear-thickening fluids, three different values of the Brinkman number also are considered here as in the case of shear-thinning fluids. The consequence of the consideration of viscous dissipation is, however, an enhanced internal fluid heating; therefore, it is expected to have higher bulk temperature of the fluid in a viscous dissipative flow. Therefore, positive values of Br will increase the fluid

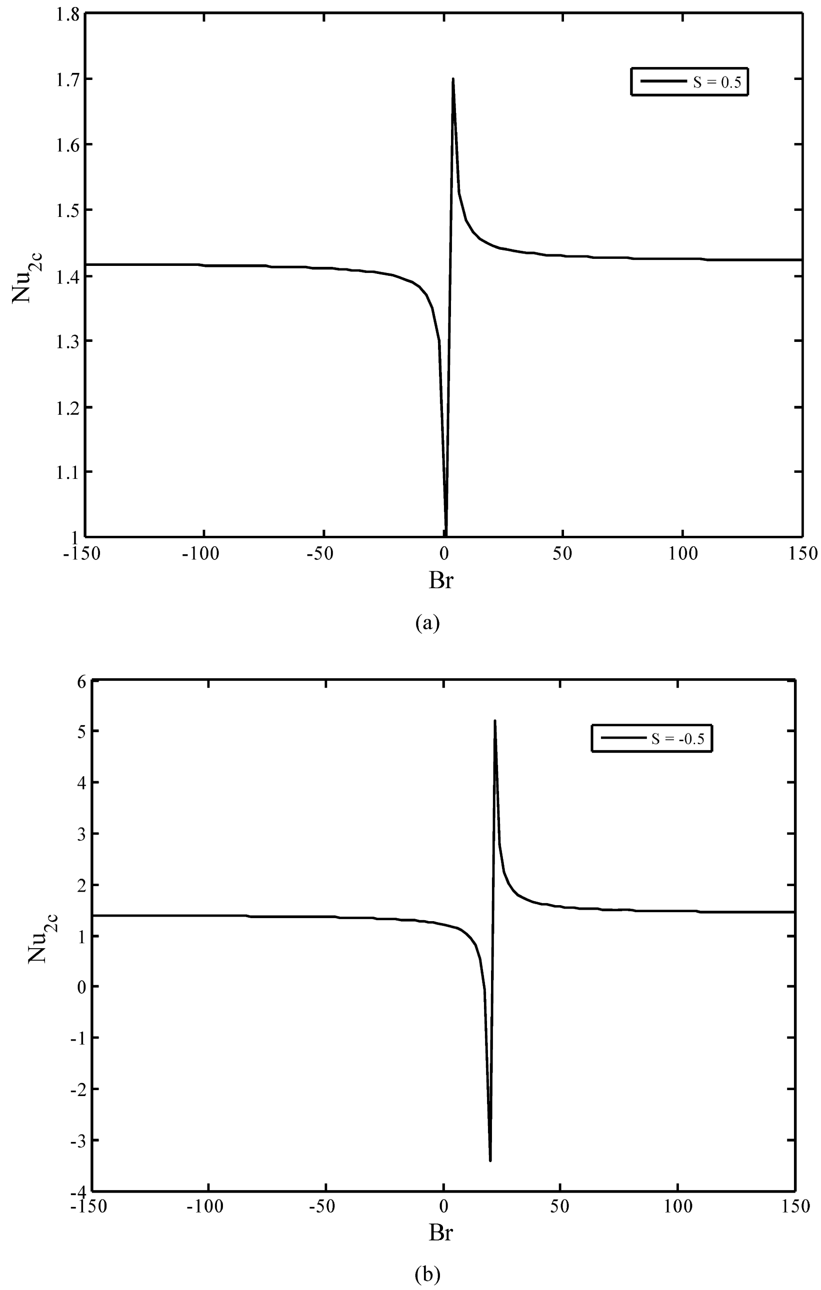

Figure 4. Influence of *Br* on the variation of Nu_{2c} of shear-thinning fluids for two different cases of asymmetric wall heating, (a) $S = 0.5$ and (b) $S = -0.5$. The variation of Nusselt number shows discontinuity in behavior. At this point, equilibrium of energy is reached. The shape of the variation of Nu_{2c} in comparison to the same obtained for Nu_{1c} changes, owing to the upper plate movement.

Table 3. Variation of Nusselt Numbers with Brinkman Number for Shear-Thinning Fluids for Three Different Cases of Asymmetrical Wall Heating

	$S = 0.0$		$S = 0.5$		$S = -0.5$	
Br	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}
−1.0	0.1716	1.2431	−1.7628	1.2786	0.6383	1.2270
−0.5	0.5278	1.2313	−0.2266	1.2536	0.7451	1.2225
0.0	0.8483	1.2177	0.8483	1.2177	0.8433	1.2177
0.5	1.1383	1.2011	1.6426	1.1622	0.9482	1.2128
1.0	1.4019	1.1838	2.2534	1.0645	1.0448	1.2076

temperature in comparison to the cases with $Br = 0$, which is also reflected on the figures presented. However, a similar and reverse explanation holds true for the negative values of *Br*, as one may find from the figures presented. On the other hand, it is important to mention here that unlike in the case of shear-thinning fluids, the temperature profile of shear-thickening fluids also exhibits a linear trend irrespective of wall heating or cooling in a nonviscous-dissipative environment. Interestingly, the variation of temperature profile of shear-thickening fluids as seen from the above figures also shows a qualitative similarity with the variation of the same in case of shear-thinning and Newtonian fluids[25] even in a viscous dissipative environment. A closer look at the figures discloses a difference in the fluid temperature of shear-thickening fluids ($n > 1$) in comparison to that the case of shear-thinning fluids ($n < 1$), though the boundary conditions imposed on the plates remain invariant. As compared to the case of shear-thinning fluids, *the quantitative differences observed in the temperature profile of*

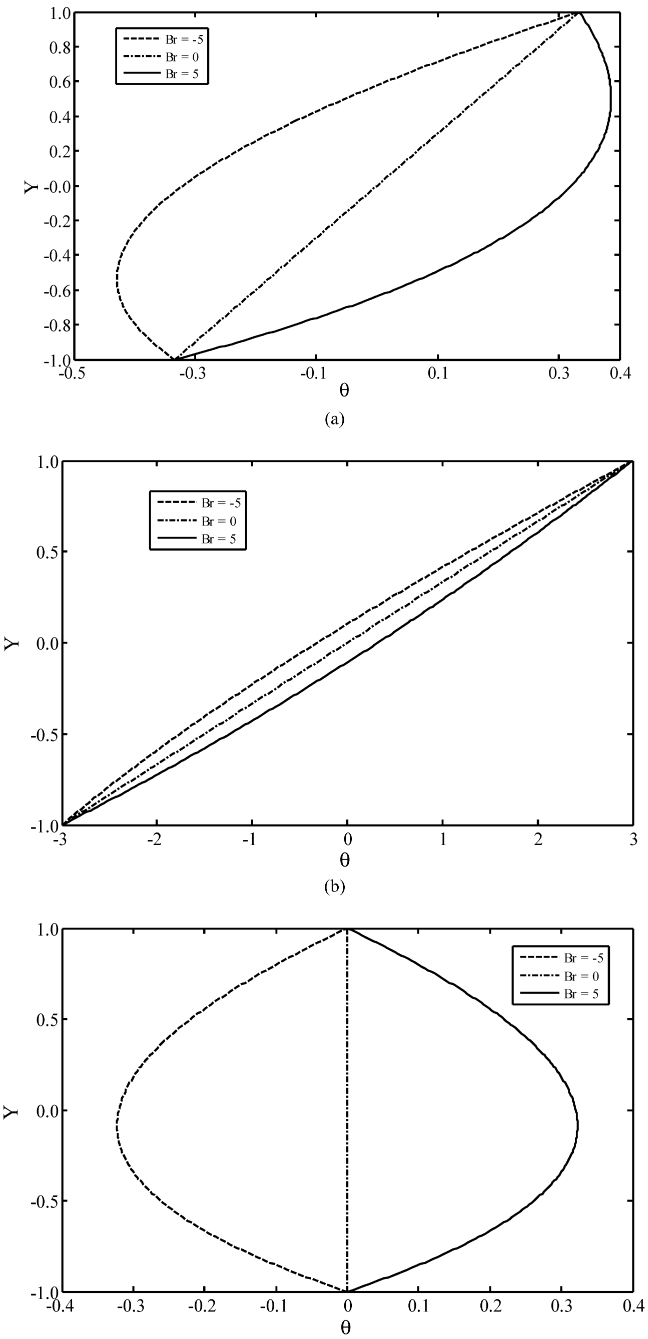

Figure 5. Dimensionless temperature profile of shear-thickening fluids for different values of *Br*, (a) $S = 0.5$, (b) $S = -0.5$, and (c) $S = 1.0$. The behavior of the temperature profile is shown for each positive and negative *Br*. The temperature profile for nonzero values of *Br* changes about the same as that obtained at $Br = 0$. Positive values of *Br* increases the frictional heating and the fluid temperature increases, whereas negative values of *Br* does the reverse.

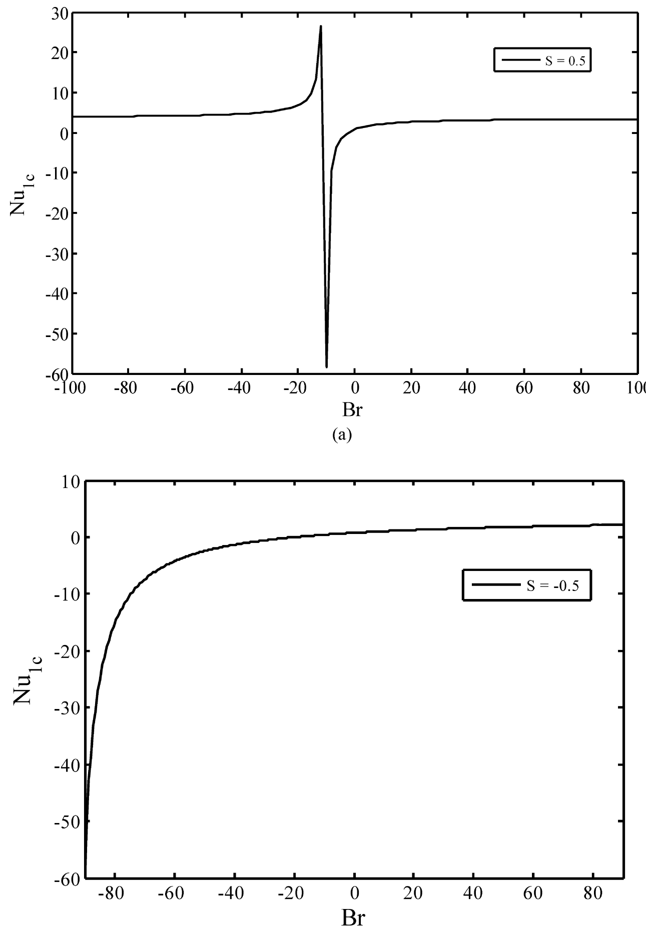

Figure 6. Influence of *Br* on the variation of Nu_{1c} of shear-thinning fluids for two different cases of asymmetric wall heating, (a) $S = 0.5$ and (b) $S = -0.5$. The variation of Nusselt number shows discontinuity in behavior and at the point of discontinuity equilibrium of energy is reached.

shear-thickening fluids, even for all the considered values of S and Br, can be attributed to the rise in the power-law index that thickens the liquid film with an increase in the boundary layer thickness.

Figures 6a–b depict the variation of Nu_{1c} with *Br* corresponding to the cases of asymmetric wall heating $S = 0.5$ and -0.5, respectively. Equations 29a and 29b clearly indicate that both the limiting Nusselt numbers are dependent on two variables (e.g., the degree of asymmetry in wall heating, *S* and *Br*) and varies with *Br* for $S \neq 1$ and with *S* for $Br \neq 0$. Unlike the cases with Newtonian fluid[25] and shear-thinning fluids, the variation of the Nusselt number in this case also portrays a clear existence of the point of singularity in the range of $-100 \leq Br \leq 100$ at $S = 0.5$. However, from the variation of Nu_{1c} with *Br*, it is apparent that the appearance of the point of singularity strongly depends on the asymmetric wall heating *S* and *Br*, which can also be mathematically argued from eqs 29a and 29b. Unlike the variation of Nusselt numbers for the case of shear-thinning fluids ($n < 1$), these plots also corroborate the corresponding trends of reaching an asymptotic value as $Br \to \infty$ as evident from Figures 6a–b. One can observe that the alteration of the type of fluid (i.e., from shear-thinning to shear-thickening) changes the location of onset of the point of singularity in the variation of Nusselt number, which is an expected result. *The physical explanation behind the existence of the point of singularity on the variation of Nusselt numbers, however, stems from the fact of a locally equilibrium state of energy between the heat supplied by the wall and the heat arising from viscous heating.* The appearances of the point of singularity at different locations for different degrees of asymmetric wall heating, and even with different types of fluids considered in the study, however, can be established or substantiated mathematically through a close look into the expression of the Nusslet number derived for different types of fluids.

Figures 7a−b depict the variation of Nu_{2c} with Br for the cases of asymmetric wall heating $S = 0.5$ and -0.5, respectively.

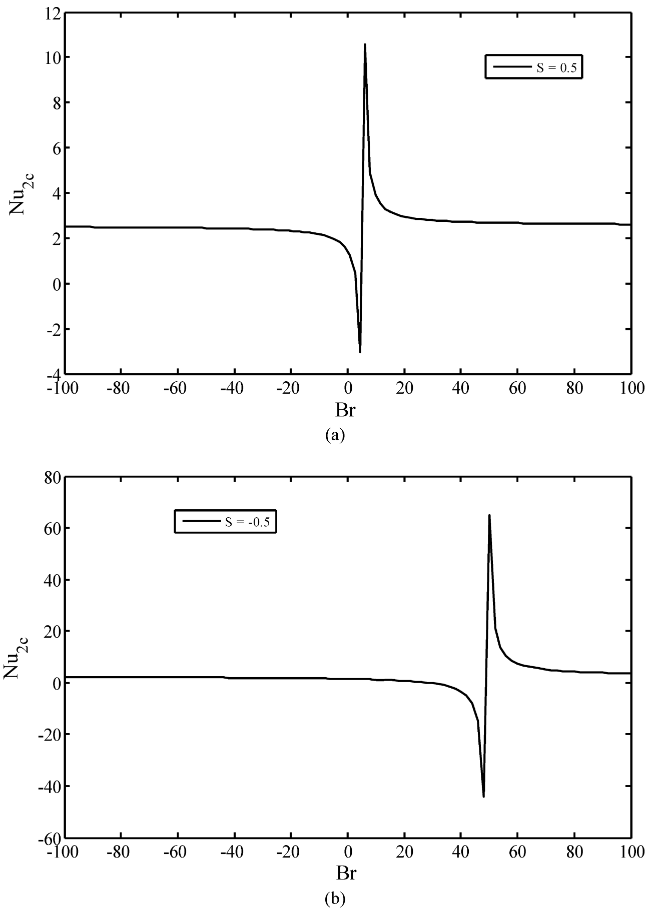

Figure 7. Influence of Br on the variation of Nu_{2c} of shear-thickening fluids for two different cases of asymmetric wall heating, (a) $S = 0.5$ and (b) $S = -0.5$. The variation of Nusselt number shows discontinuity in behavior. At this point, equilibrium of energy is reached. The shape of the variation of Nu_{2c} in comparison to the same obtained for Nu_{1c} changes owing to the fact of upper plate movement.

One can observe that the nature of the variation of Nusselt number on the top plate changes as compared to the same on the bottom plate to a significant extent though the ranges of Br considered to obtain the variation remain same for each case of wall heating. Unlike the variation of Nu_{1c} presented in Figures 6a−c, Nu_{2c} also asymptotically reaches different constant values for different degrees of the asymmetry parameter as $Br \to \infty$ in either direction. The behavioral differences between the variation of the Nusselt number on both the plates as observed from the figures presented is attributable to the movement of the top plate. The movement of the top plate introduces additional shear stress in the fluid, which in effect heats up the fluid and changes the overall temperature distribution in the flow field. Numerical values of the Nusselt number on both the plates for three different degrees of asymmetry parameter are tabulated in Table 4 to show their variation with Br specific to the case of shear-thickening fluids. Importantly, it may be observed that both the Nusselt numbers become independent of asymmetry parameter in a nonviscous-dissipative environment as evident from Table 4.

Table 4. Variation of Nusselt Numbers with Brinkman Number for Shear-Thickening Fluids for Three Different Cases of Asymmetric Wall Heating

	$S = 0.0$		$S = 0.5$		$S = -0.5$	
Br	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}	Nu_{1c}	Nu_{2c}
−1.0	0.6665	1.5225	0.4589	1.6284	0.7298	1.4816
−0.5	0.7142	1.4921	0.6172	1.5512	0.7452	1.4708
0.0	0.7605	1.4599	0.7605	1.4599	0.7607	1.4599
0.5	0.8052	1.4256	0.8907	1.3502	0.7755	1.4487
1.0	0.8486	1.3891	1.0097	1.2160	0.7905	1.4372

4. THERMODYNAMIC ANALYSIS OF THE RESULTS

It is important to observe that both the Nusselt numbers maintain a constant value as Br goes to infinity in either direction even for all types of fluids considered. Increasing Br will increase the temperature of the flow field, which in turn will increase the driving temperature difference of the heat transfer; hence, the Nusselt number might change. The effect of the alteration in Br may get reflected on the variation of the Nusselt number if the advection term is included in the energy equation to obtain the closed-form expression for the same. The present analysis, on the other hand, is made to investigate the heat transfer characteristics in the limiting condition; hence, the effect of increasing Br in either direction is not reflected on the variation of Nusselt number Nu_{1c} for a particular degree of asymmetric wall heating. This, however, can also be argued mathematically from the expression of Nusselt number derived for different types of fluids presented in eqs 28a, 28b, 29a, and 29b.

Here, discussion is made on the heat transfer characteristics in the limiting condition for a viscous dissipative Couette flow of power-law fluids, and, consequently, the variation of the Nusselt number on both the walls of the channel is depicted in a comprehensive way. The point of discontinuity appears in the variation of both the Nusselt numbers for all the values of parametric study. It is noteworthy that the location of the point of discontinuity changes with the alteration in asymmetry parameter for all types of fluids. However, the discontinuity appearing on the variation of Nusselt number is the direct outcome of the local equilibrium of energy as explained before. Convective heat transfer is characterized by the thermodynamic irreversibility, which arises due to two different effects; one is the viscous dissipation in the fluid, and the other is the heat flow due to finite temperature difference. Following the work by Degroot and Mazur,[28] irreversibility associated with any thermodynamic process can be evaluated through entropy generation function. In the second law analysis, fluid friction irreversibility arises as a result of viscous heating, and irreversible energy conversion from frictional heating of viscous dissipation into the fluid has a strong bearing on the temperature field of the fluid. In this study, the total entropy generation is the combined effect of the heat transfer due to finite temperature difference, and the losses due to fluid friction. The volumetric rate of entropy generation, arising out of the heat transfer, and encountering the frictional losses of fluid, can be expressed as

$$\dot{S}'''_{\mathrm{gen}} = \dot{S}'''_{\mathrm{gen,conduction}} + \dot{S}'''_{\mathrm{gen,vis.heating}} = \frac{k(\nabla T)^2}{T^2} + \frac{\mu}{T}\phi \tag{30}$$

The first term on the right side of the above equation is attributable to the heat transfer due to the finite temperature difference, and the second term is the viscous dissipation term. The relative order of magnitude of above two terms is expressed by

$$\sim\frac{\dot{S}'''_{\text{gen,conduction}}}{\dot{S}'''_{\text{gen,vis.heating}}} = \frac{Br}{\Delta T} \tag{31}$$

It is important to mention in this context that the fluid frictional irreversibility in any thermodynamic process cannot be ignored even for the cases with negligible viscous dissipation (i.e., when $Br \ll 1$). On the other hand, the enhancement of the heat transfer rate essentially demands the minimization of irreversibility associated with the process. In the present analysis, three different degrees of asymmetry parameters of wall heating have been considered while investigating the variation of both the Nusselt numbers as evident from the figures presented. The irreversibility associated with the heat transfer for three different values of the asymmetry parameter, however, controls the irreversibility to be minimized as a result of the frictional heating of viscous dissipation in order to obtain the maximum possible heat transfer; hence, different points of onset of singularities are observed for different degrees of asymmetric wall heating.

CONCLUSIONS

In the present work, the influences of both the viscous dissipation and the asymmetry parameter on the heat transfer characteristics in a Couette flow of power-law fluid flowing between two parallel plates is studied. A semianalytical solution for the velocity distribution in the flow field is presented, and a few assumptions like negligible axial conduction and negligible convective term have been made only to attain a situation to obtain the analytical solutions of the temperature profile in the limiting condition. Explicit and closed-form expressions of Nusselt number for the fully developed flow of power-law fluid between fixed parallel plates are suggested. Two different cases of power-law fluids are demonstrated while investigating the role of viscous dissipation and asymmetric wall heating on the heat transfer analysis in the limiting condition. The influential role of viscous dissipation is found to be of great importance in the heat transfer analysis when both the plates are kept at unequal constant temperatures. The appearance of the point of singularity on the variation of both the Nusselt numbers, when the energy balance is considered, is discussed, and the same is explained from the perspective of the second law of thermodynamics. The analysis can be extended further in different directions by relaxing some of the assumptions made in the study. However, the following conclusions can be drawn from the investigation made in the present study.

- In case of a shear-thinning fluid, the dimensionless temperature in the limiting condition shows a usual trend of increasing fluid temperature for positive Br and a decreasing trend for the negative values of Br.
- In case of a shear-thinning fluid, the variations of both the Nusselt numbers show a clear existence of the point of singularity at all degrees of asymmetry considered. Each Nusselt number plays with Br in a different fashion. From the point of singularity, with the increasing value of Br in the positive direction ($Br > 0$), Nu_{1c} increases because of the favorable driving potential of the heat transfer, whereas Nu_{2c} decreases. On the contrary, with the increasing value of Br in the negative direction ($Br < 0$), Nu_{1c} decreases and Nu_{2c} increases. The contradictory behavior as seen on the variation of both the Nusselt numbers is attributable to the movement of the top plate. However, both the Nusselt numbers asymptotically reach different constant values for different values of asymmetric wall heating, as $Br \to \infty$ in either direction.
- In the case of a shear-thickening fluid, the temperature profile obtained at different degrees of asymmetric wall heating qualitatively replicates the variation obtained for shear-thinning fluids. However, compared to the case of shear-thinning fluids, the quantitative differences as observed for all the values of S and Br can be attributed to an increase in the power-law index that thickens the liquid film, accompanying an increase in the boundary layer thickness.
- In the case of a shear-thickening fluid, the variation of both the Nusselt numbers show similarities with that in case of a shear-thinning fluid barring the variation of Nu_{1c} at $S = -0.5$. The similarities include the similarity in behavior of having the existence of pointd of singularity on the variation and of asymptotic behavior as $Br \to \infty$ in either direction for all the cases of wall heating under consideration.

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Notes

The authors declare no competing financial interest.

NOMENCLATURE

a = parameter $-\frac{H}{m}\left(\frac{dp}{dx}\right)$ defined in eq 5b
B = parameter, $-(C/ma)$
Br = Brinkman number
C, C_1, C_2, and C_3 = integration constants;
C_p = specific heat at constant pressure (J/(g K))
h_{1c} = heat transfer coefficient in limiting condition at bottom wall (W/(m^2 K))
h_{2c} = heat transfer coefficient in limiting condition at top wall (W/(m^2 K))
H = half channel height
I_1, I_2, I_3, and I_4 = constants
k = thermal conductivity (W/mK)
m = consistency factor
n = power-law index
Nu = Nusselt number
Nu_{1c} = Nusselt number in the conduction limit at the bottom plate
Nu_{2c} = Nusselt number in the conduction limit at the top plate
p = pressure (N/m^2)
P = parameter to characterize $(1/\gamma)^{n+1}$ $(n + 1/n)^{n+1}$
S = degree of asymmetrical wall heating
T = temperature (K)
$\overline{T}$ = average temperature (K)
T_1 = bottom plate temperature (K)
T_2 = top plate temperature (K)
T_f = uniform fluid inlet temperature (K)
T_{mc} = Mean temperature in the conduction limit (K)
u = velocity (m/s)
$\overline{u}$ = dimensionless velocity (m/s)

U = velocity of the moving plate (m/s)
x = axial coordinate (m)
y = coordinate perpendicular to flow (m)
Y = dimensionless coordinate perpendicular to flow

Greek symbols

θ = dimensionless temperature
θ_{mc} = dimensionless mean temperature in the conduction limit
γ = parameter to characterize $[(B+1)^{(n+1)/n} - (B-1)^{(n+1)/n}]$
ε = parameter to characterize $-Br\left(\frac{Pn^2}{(2n+1)(3n+1)}\right)$
μ = dynamic viscosity (kg/(m s))
ρ = density (kg/m^3)

Subscripts

b = bottom
c = conduction limit
f = initial fluid
mc = mean in the conduction limit
t = top

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