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Planning and Sequencing Product Distribution in a Real-World Pipeline Network: An MILP Decomposition Approach

Suelen Neves Boschetto Magatão,[†] Leandro Magatão,[†] Helton Luis Polli,[†] Flávio Neves, Jr.,^{*,†} Lúcia Valéria Ramos de Arruda,[†] Susana Relvas,[‡] and Ana Paula Ferreira Dias Barbosa-Póvoa[‡]

[†]Programa de Pós-Graduação em Engenharia Elétrica e Informática Industrial (CPGEI), Universidade Tecnológica Federal do Paraná (UTFPR), Avenida Sete de Setembro, 3165, 80230-901, Curitiba, Paraná, Brazil

[‡]Centro de Estudos de Gestão do Instituto Superior Técnico (CEG-IST), Universidade Técnica de Lisboa (UTL), Avenida Rovisco Pais 1049-001 Lisboa, Portugal

ABSTRACT: In the oil industry, any improvement in the planning and execution of the associated operations (e.g., production, storage, distribution) can generate considerable profits. To achieve this, the related activities need to be optimized. Within these activities, planning and scheduling occur at the different levels of the oil supply chain, from the strategic to the operational levels looking from global networks to sets of individual resources. This work looks into the planning, namely the assignment/sequencing of activities that occur in a multiproduct, multipipeline system. The aim is to contribute to the definition of generic models that can help the decision-making process characterized by a high level of complexity. An approach formed by two mixed integer linear programming (MILP) formulations that act in sequence is proposed. The first generic MILP planning model calculates volumes for attending the necessary requirements on inventory management of the producer and consumer areas. As a result, this model defines the products and the total volumes to be transported in order to attain storage goals, while respecting operational constraints, demands of consumers, and pipeline capacity. Then, the planning model results are used by an MILP assignment and sequencing model, which splits the total volume into operational batches and determines the sequence of pumping for the batches during the available horizon. The developed approach is applied to a real-world pipeline network that includes 30 bidirectional multiproduct pipelines associated with 14 node areas: four refineries, two harbors, six depots/parks of pumps and valves, and two final clients.

1. INTRODUCTION

One current challenge in the oil industry is the optimization of the transportation of products from the production to consumption areas, for example, the downstream distribution. Such transportation is often complex and involves the management of a series of operational conditions dictated by the considered resources, such as pipelines and associated valves and pumps. According to Moro and Pinto,¹ the goal of this optimization is to achieve better operational conditions while guaranteeing some criteria such as the use of better paths to transfer products without changes in the network physical structure (e.g., valves, pumps, and pipes). Within the oil supply chain several research articles have been published on planning optimization; however, most of these are related to refinery production planning involving the blending process and tankage allocation.^{2,3} Nevertheless, it is crucial to develop integrated and coordinated decision-making approaches that relate operations and activities over geographically dispersed systems, namely to plan product movements that satisfy demand-driven locations from supply-driven locations.³

However, such systems often present an associated strong complexity and adequate approaches to plan and operate them need to be developed. This is the case of downstream distribution systems involving pipelines. The pipeline is a resource that can be shared by different products that need to be displaced from its origin until its destination. Given this property, pipelines can be combined in different topology systems, to fulfill the demands of the geographical area where

they operate. Figure 1 schematizes these different topologies. In summary, the topology can have a single pipeline or be constituted by several pipelines connected in a network. Single pipelines can have a single origin and a single destination^{4–6} (Figure 1a), a single origin and several destinations^{7,8} (Figure 1b), several intermediate reception points and one final destination), and several origins and several destinations⁹ (Figure 1c). The intermediate origins and destinations can be directly connected nodes (refinery or depot) or smaller pipeline branches with a final destination. This last case is usually called a pipeline tree system. When intermediate nodes are connected to more than one pipeline branch, a pipeline network system arises^{10,11} (Figure 1d).

These systems have been studied over the past years by several researchers, who emphasized the complex characteristics of the related problems. This is the case of Relvas et al.,⁴ Magatão et al.,⁵ Rejowski and Pinto,⁷ and Cafaro and Cerdá,⁸ who applied decomposition techniques (structural or temporal decomposition) to solve the planning and scheduling problems of such structures in reasonable computational times.

Relvas et al.⁴ presented the integrated problem of scheduling a pipeline operation with inventory management at the final destination depot, where the strong dependence that both

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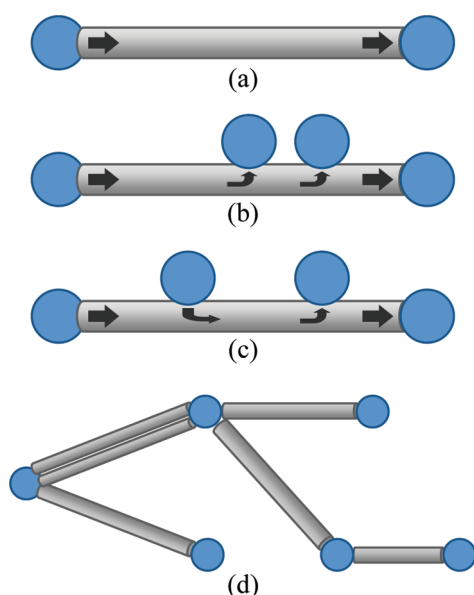


Figure 1. Examples of topologies for pipelines.

systems present is addressed by a mixed integer linear programming (MILP) model. Despite presenting a single origin and single destination, the need for a monthly solution for a system comprising six products resulted in temporal problem decomposition.

Rejowski and Pinto⁷ enhanced their previously published work to obtain a new mixed integer nonlinear programming (MINLP) model that represents a single pipeline with several intermediates and a final depot. This model is tightened by using a set of valid integer cuts and generalizing the time representation to a continuous time scale. The problem representation improvement allowed including more detailed features, such as pumping flow and production rate variations, besides regular problem constraints.

The same problem has been addressed by Cafaro and Cerdá.⁸ In order to achieve a solution for a medium term horizon of 1 month, these authors developed a rolling horizon technique (temporal decomposition) that enables larger time period coverage. The rolling horizon has, in this case, to account for the transportation gap imposed by the pipeline operation, allowing a pumping run or batch to span over two or more time periods.

Lopes et al.¹⁰ proposed a hybrid framework based on a two-phase problem decomposition strategy to solve a large pipeline network planning and scheduling problem. They propose new algorithms for generating feasible solutions for the problem, where most of the hardest real-world constraints are taken into account. The approach has two phases: the planning phase, which is implemented as a constructive heuristic that generates delivery orders, representing transfers between two depots, and a scheduling phase where a constraint programming (CP) model is used to establish an ordering among the delivery orders, at each pipeline and each tank. The full strategy was implemented and produced adequate results when tested over large real instances.

More recently, Magatão et al.⁶ combined an MILP model with constraint logic programming (CLP) to take advantage of the synergy created in this hybrid procedure. This hybrid model constitutes the main model of the solution procedure, supported by a decomposition technique that enables solutions

for larger problem instances. Despite addressing a single origin and single destination problem, the proposed approach uses continuous time representation. Thus intervals that indicate time constraints (time windows), and a series of operational issues were taken into account, such as the seasonal and hourly costs of electric energy (on-peak demand hours) and the pipeline with reverse flow direction (products can be pumped in both directions). In addition, the authors addressed another dimension of the problem, and hypothetical problem instances were tested. The results indicated that (i) the combined CLP–MILP approach had an average computational time that was shorter than that of the root MILP and CLP models, (ii) the use of heuristic information to guide the search process was a promising alternative to deal with the computational burden, primarily in the CLP–MILP framework, and (iii) variations on the base MILP and CLP models and in the derived hybrid models can affect the computational overhead.

From the analysis of the previous works, it can be concluded that several techniques can be used to enhance either the model performance or the representation level of the real problem. In this way, synergies can be obtained by decomposing the problem by using different detail levels between both models. The current work follows the above works and aims to deal with the planning and scheduling problems while looking into a complex system of transportation, such as a pipeline network. To achieve this purpose, two MILP models are here proposed. The planning model addresses the allocation and transportation of products among different producing/consuming areas. In this way, the logistics of pipelines has to be performed respecting a series of operational constraints. In a subsequent step, the proposed planning model is used in a collaborative way with the assignment and sequencing model. This last model uses the total volumes and the routes provided by the planning model to determine the volumes and the sequence of operational batches. Both models can be used in an integrated way with the scheduling architecture proposed by Boschetto et al.¹¹ to obtain the complete pipeline network scheduling.

The current paper is organized as follows. This section presented a brief introduction and motivation for the problem in study. Section 2 provides the problem description including the problem constraints and the problem hierarchical decomposition. The approach used to create sparse sets and to determine some parameters are presented in section 3. The proposed MILP planning model and the pipeline constraints modeled are characterized in section 4. Section 5 presents the MILP assignment and sequencing model. Case studies and results for both models are presented in section 6, and the concluding remarks are given in section 7.

2. PROBLEM DESCRIPTION

The considered multiproduct pipeline network is schematically presented in Figure 2. This is based on the real pipeline network located, mainly, in the area of São Paulo, Brazil.¹¹ This network contains different operational areas (refineries, harbors, final clients, and depots/parks of pumps and valves) interconnected by bidirectional pipelines that transport a set of products between adjacent areas. The nodes (vertices) in Figure 2 represent the areas and the edges represent the pipelines interconnecting these areas. Different products are produced and transported along this network resulting in a very complex system to plan and schedule.

The pipeline network in Figure 2 involves 14 areas (nodes) that represent four refineries (nodes N3, N4, N5, and N6), two

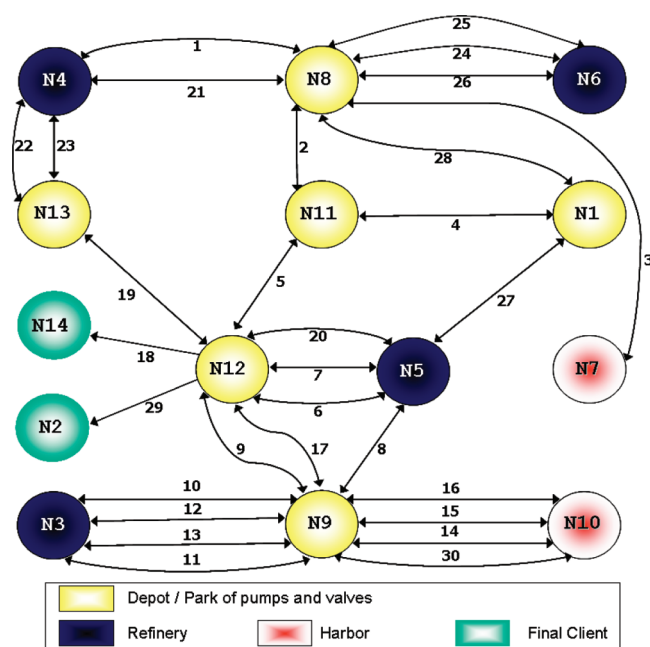


Figure 2. Pipeline network.

harbors (N7 and N10), two final clients (N2 and N14), one park of pumps and valves without storage facilities (N1), and five depots (N8, N9, N11, N12, and N13). Each refinery and depot can also receive/send different products. The 14 areas considered are linked by 30 bidirectional pipelines, each one with a particular volume, through which can be transported more than one product, leading to a multiproduct network. In the real-world pipeline, 35 oil derivatives and ethanol are produced/stored and can be transferred along the network.

A product can take several hours to reach its final destination. Each product is stored in a specific tank located in a specific area. For instance, a “typical operation” involves pumping a batch from N6 to N11, passing through N8 and N1 and using pipelines 24, 28, and 4.

The planning and scheduling of the operational activities of this network are made based on demand and production predictions for a period of 1 month. For instance, during a month the product 25 (P25) demand in node N11 is expected to be more than 150 000 volumetric units (vu). However, during the scheduling horizon, the total demanded volume has to be split into smaller operational volumes (batches). The volume of batches should be compatible with storage constraints and economic factors. Batches can be supplied by different sources, for example, nodes N3 and N4. The route from an origin area to a destination area can be therefore quite long. A batch of a product can remain in a pipeline until another batch pushes it. Since the pipelines are shared resources, the pumping of a specific batch in each pipeline must respect a series of operations that are in course. These establish the problem characteristics and associated constraints, which can be summarized as follows:

1. The transfer operations occur in a predetermined time horizon.
2. In this period, batches are established considering limits for transportation volumes and are sent through one of a set of possible routes.
3. Each route is composed by a sequence of areas and pipelines. A route can have only one pipeline linking two

areas or n pipelines connecting $n + 1$ areas. The first and the last areas in the route represent the origin and destination areas, respectively. A route indicates the “movement path” for each batch, from the origin area until the destination area. An example of a route can be extracted from Figure 2, where a transfer operation can be made from N10 to N2. Thus, the route will contain the elements {N10, 14, N9, 8, N5, 7, N12, 29, N2}.

4. One route can be used by different batches, containing different products at different flow rates.
5. Each area has a tank farm that stores different products. Upper and lower limits to the overall product inventory need to be respected in each node.
6. The storage level can increase or decrease, according to the balance between the received/pumped batch volume and the local consumption (consumption within the area). The inventory levels are also affected by local production and consumption market of areas.
7. In each area, a batch received from a pipeline can be pumped to a specific tank and/or directed to another pipeline.
8. In addition, an operation called “surge tank” can occur when a batch is received in an area at a specific flow rate and has to be sent to another area at a different flow rate. In this case, intermediate storage has to be used.
9. The pipelines operate/remain always with product and their utilization/occupation must be managed to guarantee such constraint.
10. The majority of the pipelines within the network can have their flow direction reverted, according to operational needs.

2.1. Problem Definition. Having in mind the problem description and characteristics presented in the previous section, the problem definition can be summarized as follows. Given

- the time horizon
- the network pipeline structure in terms of nodes and pipelines
- the characteristics of products (e.g., flow rate limits)
- all possible sets of routes for each product from origins to destinations
- the inventory of products (capacity, initial/goal/minimum/maximum inventory) at the network nodes
- the product production and consumption at the network nodes
- the volumetric limits for batches
- the characterization of all the network resources in terms of capacities as well as products and operations suitability

It is desired to determine

1. the planned inventory profiles at the origin and destination nodes during the time horizon
2. the planning and sequencing of the product batches, with their volume and routes, along the network
3. to *guarantee* the required demand at the different nodes of the network while minimizing a function of several operational indicators

2.2. Problem Solution Approach: Hierarchical Decomposition. When addressing the problem defined above, the authors first developed a single level formulation. This, however, resulted to be intractable and no solution could be obtained.¹² This fact is in line with what has been referred to in

section 1, where different authors recognize the need to construct decomposition approaches to deal with such complex problems. A hierarchical approach was then developed to address the planning and scheduling problem of the pipeline network previously presented. This used a decomposition method where the original problem was addressed in a hierarchical form through the definitions of different problem levels of decision: (i) planning, (ii) assignment of resources and sequencing of activities, and (iii) timing determination for resources utilization by these activities.¹¹ This method is schematically shown in Figure 3.

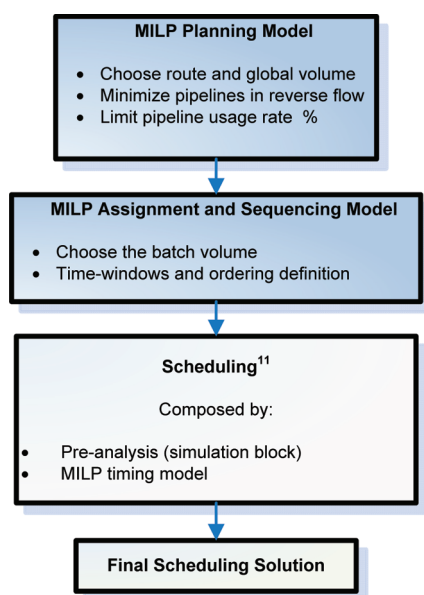


Figure 3. Hierarchical approach.

The *planning block* suggests the transportation of oil derivatives from producing areas (refineries or harbors, the latter for the importation case) with the goal of supplying the demand of depots, final clients, and harbors (exportation case). The problem considers a time horizon of 1 month planning. Inventory management and storage capacities are considered in an aggregated form. Thus, the set of tanks of a specific product in a specific area is considered in an aggregated manner. The planning model does not look into specific details of individual tanks of a product. Additional tankage details are treated in the scheduling level. After the planning model is run this provides, as final results, the routes to use and the volume to transfer to guarantee the demands, while considering the available products at origins and the problem operational constraints. These results are fed into the next block, the assignment and sequencing block.

The sequence of products to be pumped to a pipeline has a great impact on the decision-making for MILP formulations, also having a great impact on the computational burden to solve the combinatorial problem.¹³ If some parameters are previously calculated, the model can be simplified, considering the reduction in number of variables to be determined. Therefore, the proposed *assignment and sequencing block* aims to provide part of a solution to the pipeline network problem using exact methods. A list of batches, including the origin/destination area, route, and volume for each batch is herein determined. The benefits and limitations of this approach will be henceforth explored.

After receiving the results from the assignment and sequencing block, a *scheduling block* is called. This involves a preanalysis and a timing block. The preanalysis (*simulation block*) evaluates the information derived from the assignment and sequencing block and calculates temporal and volumetric limits (bounds). These limits present a preliminary indication of the viability of the programming. As a result, the batch movement along the pipeline network is analyzed, and “volumetric parts” (portions of this batch) with similar pumping conditions are identified. Afterward, all the calculated parameters are used in a continuous time MILP model, the *timing block*. This block determines the operational short-term scheduling for the pipeline network. The simulation and timing blocks are presented in detail by Boschetto et al.¹¹

The described models are integrated and are sequentially executed. A fundamental characteristic of this approach is that each model can be considered as a “preprocessing routine” to the next model in a hierarchical precedence. Thus, the output of a block can be used as an input to the next block. Additionally, the developed computational tool incorporates functionalities that allow visualizing reports, inventory graphics, and Gantt charts related to proposed solutions.

3. DEFINING INDEX SETS FOR VARIABLES

As referred to above, two MILP models are herein derived to define the planning, assignment, and sequencing of a pipeline network. Both models are developed on the basis of continuous time modeling. The mixed integer linear programming models will use the notations for indices, sets, parameters, and variables as shown in the Nomenclature. The volumes are given in volumetric units (vu).

The indices of variables are constructed as parts of sets that contain only specific combinations of each tuple. In this way the sparse sets are used to generate the mathematical constraints, allowing the manipulation of only valid indices. This procedure allows the reduction of the solution domain.

For example, the set RRD is created to identify when two routes r and r' contain some common pipeline d , which is operated in different directions. In this way, variables/constraints that are derived from set RRD only contain indices of tuple (r, r', d) belonging to this sparse set, decreasing the number of generated variables/constraints. The determination of these sets and parameters are made previously to the main model generation (objective function and constraints). In order to obtain the set RRD, first, the auxiliary sparse set RD is defined. This set, presented in the Nomenclature, contains the tuple (r, n, d) in which the node n is the origin of pipeline d contained in the route r . Then, it is necessary to verify the tuple $(r, n, d) \in RD$, which contains all pipelines d with origin node n inserted in route r , such that

$$RRD = \{(r, r', d) | (r, n, d) \in RD, (r', n', d) \in RD, n \neq n'\}$$

The matrix for generating indices of the variables “bin” and Q was created in sparse form and contains only the tuple (n, n', p, r) , where

- n is an origin area from route r
- n' is a destination area to route r
- p is the index of all products that can be sent from n to n' using the route r

For the assignment and sequencing model, the matrix for the indices of variables $\text{dif}_{d,n,n',r}^-$ and $\text{dif}_{d,n,n',r}^+$ is generated in sparse form and contains only the tuple (n,n',p,r) , such that $Q_{n,n',p,r} > 0$.

In a similar way, the matrix for the indices of variables $W_{b,n,n',p,r}$ and $ls_{b,n,n',p,r}$ is created in sparse form and contains the b index added to the tuple (n,n',p,r) , $Q_{n,n',p,r} > 0$.

3.1. Calculating the $\text{disp}_{n,p}$ and $\text{flt}_{n,p}$ Parameters. Parameters $\text{disp}_{n,p}$ and $\text{flt}_{n,p}$ are calculated by eqs 1 and 2, respectively. The available quantity of product p in area n ($\text{disp}_{n,p}$) is determined by considering that the local demand ($\text{dem}_{n,p}$) is not sufficient to consume the total local production ($\text{prod}_{n,p}$) added to the initial storage of product p in area n ($\text{ID}_{n,p}^0$). In this way, the quantity of available product is given by the initial inventory added to the local production subtracted from the local demand.

$$\text{disp}_{n,p} = \begin{cases} (\text{ID}_{n,p}^0 + \text{prod}_{n,p}) & \text{if } \text{ID}_{n,p}^0 + \text{prod}_{n,p} > \text{dem}_{n,p} \\ -\text{dem}_{n,p} & \\ 0 & \text{otherwise} \end{cases}$$

$$\forall n \in N, p \in P \quad (1)$$

Similarly, the required quantity of product p in area n ($\text{flt}_{n,p}$) is determined by considering that the initial storage added to the local production is not sufficient to attend the total demand for product p in area n . Therefore, the quantity of required product is given by the local demand subtracted from the initial inventory added to the local production.

$$\text{flt}_{n,p} = \begin{cases} \text{dem}_{n,p} - (\text{ID}_{n,p}^0 + \text{prod}_{n,p}) & \text{if } \text{ID}_{n,p}^0 + \text{prod}_{n,p} < \text{dem}_{n,p} \\ +\text{prod}_{n,p} & \\ 0 & \text{otherwise} \end{cases}$$

$$\forall n \in N, p \in P \quad (2)$$

4. MILP PLANNING MODEL

In this section a continuous time mixed integer linear programming model is proposed to represent the planning task for a multiproduct pipeline network with the characteristics presented in the previous sections. This model was built considering the following assumptions:

1. The production/consumption and storage limits during the entire horizon (generally, 1 month) for each product in each node are established in advance by the company.
2. The tanks are considered in aggregate form for area and product. The capacity, maximum, minimum, and goal limits to the overall product inventory in each node are considered and can be incremented or decremented, attempting to respect these operational limits, as presented in Figure 4.
3. The volume moved for each product is influenced by the produced/consumed volume, added to the initial storage of each area.
4. Products either can be pumped from one origin area to different destination areas or can be received in one destination by different origin areas.
5. Pipelines always operate completely full, and some of them have a considerable volume.
6. The routes used by each product are chosen by the planning model from a set of preindexed and approved routes.

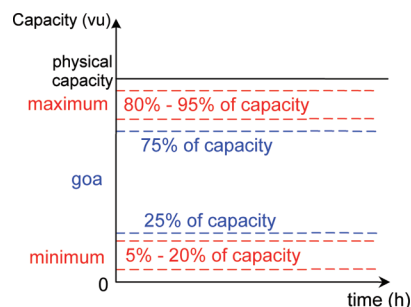


Figure 4. Tankage aggregated limits example.

7. A series of pipelines can have the flow direction reverted. Specific procedures are required to manage such an operational condition. Typically, auxiliary reversion batches are required.¹¹ The pipelines involved have a considerable volume, and they operate completely filled. Thus, there is a time between sending a product from an area and receiving it into the following area. In particular, the last product sequenced in one flow direction has to be dislocated (pushed) by an auxiliary batch to be completely delivered to the following area. Thus, an extra product amount (auxiliary batch) must be pumped after the last sequenced product of one direction in order to invert the pipeline flow. Magatão et al.⁵ have already considered reversions of flow in a pipeline, and the authors defined the instants for pumping auxiliary batches as a “gap procedure”. During this gap, the pipeline is filled by the auxiliary batch and the pumping of batches effectively demanded has to wait until the flow inversion completion. At the planning level, however, it is difficult to determine how many reversions of flow are needed in each pipeline. Alternatively, the number of pipelines that have flow reversions is minimized. Smaller utilization rates are imposed to pipelines chosen to be reverted, allowing, afterward, the insertion of auxiliary reversion batches.
8. To consider the surge tank operation, the route of the product is split into two routes. The first route considers the origin area until the intermediate area with the tanks used in the operation. The second route is defined from the area of the surge tank operation until the final destination area. In this way, the planning model is able to match two or more different routes to guarantee the delivery of the products on consuming areas.
9. In similarity to surge tank operations, a product can be received at a specific flow rate in a tank, can remain temporally stored, and afterward, can be pumped from this tank to another pipeline. This operation is called “intermediate storage”. The connection of different routes has to be managed by the model to address such operations. For instance, the model has to determine a route between the origin area and the intermediate area, and another one from the intermediate area to the final area.
10. At the beginning of the horizon the pipelines are filled with previously planned products and they must also remain filled at the end of the horizon. Therefore, part of the volume planned to be sent will remain in pipelines at the end of the considered horizon. At the planning level, it is assumed that the volumes of products at the beginning and at the end of the horizon are equivalent.

After presenting, in a simplified way, the main considerations to be observed by the planning model, the principal parameters that need to be considered as model input can be defined as

- list of routes/pipelines/areas/products
- production rate per product and area
- demand rate per product and area
- initial inventory per product and area
- capacity storage limits per product and area
- minimum, maximum, and goal storage limits per product and area
- sum of volumes of pipelines per route
- medium flow displacement rate per route
- time horizon
- maximum desired percentage to pipeline utilization during the planning horizon
- percentage to be subtracted from the maximum desired percentage rate when pipelines operate in both directions (flow reversion cases)

4.1. Planning Problem Constraints. The MILP model is built considering the problem characteristics defined above. Different types of constraints are considered. Note that all operational and physical constraints were modeled using variable volumes. As explained in section 3, sparse sets were used within the model generation.

Constraint 3 establishes that the quantity of product p to be sent from node n ($Q_{n,n',p,r}$) should be smaller than the availability of product p in area n ($\text{disp}_{n,p}$) added to the quantity of product received in area n ($Q_{n',n,p,r}$).

$$\sum_{n' \in N} \sum_{r \in R} Q_{n,n',p,r} \leq \text{disp}_{n,p} + \sum_{n' \in N} \sum_{r \in R} Q_{n',n,p,r} \quad \forall n \in N, p \in P \quad (3)$$

In contrast, the quantity to be received should supply the required quantity ($\text{flt}_{n,p}$) in area n . This requirement is observed in storage limit constraints. Additionally, the minimum and maximum storage limits have to be respected and a storage goal level should be observed. All these aspects are herein modeled.

If product p is sent from area n to area n' by route r , the binary variable $\text{bin}_{n,n',p,r}$ has to be equal to 1. Otherwise, $\text{bin}_{n,n',p,r}$ will be set to 0. In addition, a minimum quantity to be sent can be previously assigned to the parameter lots_p^{\min} , requiring that, if the product is sent, the quantity is not operationally small (constraints 4 and 5). Small quantities are operationally undesirable: they can be easily contaminated by interfacing with other products in the pipelines.

$$Q_{n,n',p,r} \leq M \cdot \text{bin}_{n,n',p,r} \quad \forall \{n, n'\} \in N, p \in P, r \in R \quad (4)$$

$$Q_{n,n',p,r} \geq \text{lots}_p^{\min} \cdot \text{bin}_{n,n',p,r} \quad \forall \{n, n'\} \in N, p \in P, r \in R \quad (5)$$

If a product p is sent from area n to area n' , then the same product cannot be sent from area n' to area n . This limitation is satisfied by constraint 6.

$$Q_{n',n,p,r'} \leq M(1 - \text{bin}_{n,n',p,r}) \quad \forall \{n, n'\} \in N, p \in P, \{r, r'\} \in R \quad (6)$$

The pipeline d utilization is determined by the sum of volumes that are to be sent by this pipeline, divided by the medium flow rate (eq 7, left side). When compared with the available horizon H , the pipeline utilization should be ideally smaller than α (%). Equation 7 models this limitation. If the pipeline is used in both directions ($\text{rev}_{r,r',d} = 1$), then the utilization rate should be less than $\alpha - \beta$. If this limit is violated, then the relaxation variable vari_d assumes a positive value and it is penalized within the objective function. In addition, constraint 8 indicates that the values for the relaxation variable vari_d are limited by $1 - \alpha$.

$$\sum_{\substack{r \in R \\ (r,n,d) \in \text{RD}}} \sum_{p \in P} \frac{\sum_{n \in N} \sum_{n' \in N} Q_{n,n',p,r}}{\text{vbr}_{r,p}^{\text{med}}} \leq H(\alpha_d + \text{vari}_d) - H\beta_d \sum_{r \in R} \sum_{r' \in R} \text{rev}_{r,r',d} \quad \forall d \in D \quad (7)$$

$$\text{vari}_d \leq 1 - \alpha_d \quad \forall d \in D \quad (8)$$

If two different routes r and r' use the same pipeline d in opposite directions, the binary variable $\text{rev}_{r,r',d}$ assumes the value of 1 (constraints 9–11).

$$\text{bin}_{n,n',p,r} + \text{bin}_{m,m',p',r'} \leq 1 + \text{rev}_{r,r',d} \quad \forall \{n, n', m, m'\} \in N, \{p, p'\} \in P, (r, r', d) \in \text{RRD} \quad (9)$$

$$\text{rev}_{r,r',d} \leq \sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} \text{bin}_{n,n',p,r} \quad \forall (r, r', d) \in \text{RRD} \quad (10)$$

$$\text{rev}_{r,r',d} \leq \sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} \text{bin}_{n,n',p,r'} \quad \forall (r, r', d) \in \text{RRD} \quad (11)$$

Two different routes r and r' cannot use the same pipeline d in both directions to transport the same product p (constraint 12). The areas of origin n and m and the areas of destination n' and m' can be distinct areas for routes r and r' , respectively.

$$\text{bin}_{n,n',p,r} + \text{bin}_{m,m',p,r'} \leq 1 \quad \forall \{n, n', m, m'\} \in N, p \in P, (r, r', d) \in \text{RRD} \quad (12)$$

Intermediate storage and surge tank operation involve sending a product p from an origin area n to an intermediate area n' and, later, sending p from n' to a final destination m . These operations can only be performed if the intermediate area n' contains tanks to allocate the product p (constraint 13).

$$\text{bin}_{n,n',p,r} + \text{bin}_{n',m,p,r'} \leq 1 + \text{tank}_{r,r',n',p} \quad \forall \{n, n', m\} \in N, (p, n') \in \text{PLM}, \{r, r'\} \in R \quad (13)$$

In the case of surge tank operations in area n , if the capacity of product p is small, it is not recommended to perform

intermediate storage. In this way, all received volume in area n from route r should be sent to another area (eq 14).

$$\sum_{n' \in N} \sum_{r \in R} Q_{n',n,p,r} = \sum_{n' \in N} \sum_{r \in R} Q_{n,n',p,r} \quad \forall (p, n) \in \text{PLM} | \text{CP}_{n,p} < T^{\min} \quad (14)$$

The balance of the sent, received, produced, and consumed volumes of a product p in area n is given by the free variable $\text{invent}_{n,p}$ through equality 15.

$$\sum_{n' \in N} \sum_{r \in R} Q_{n',n,p,r} - \sum_{n' \in N} \sum_{r \in R} Q_{n,n',p,r} + \text{disp}_{n,p} - \text{flt}_{n,p} = \text{invent}_{n,p} \quad \forall n \in N, p \in P \quad (15)$$

This balance should respect the minimum inventory limit (constraint 16). In the case of not being possible to respect this inventory condition, the relaxation variable $\text{VID}_{n,p}^{\min}$ assumes a positive value and is penalized within the objective function.

$$\text{invent}_{n,p} + \text{VID}_{n,p}^{\min} \geq \text{ID}_{n,p}^{\min} \quad \forall n \in N, p \in P \quad (16)$$

Since constraint 16 addresses the minimum inventory limit, the balance of volumes of a product p in area n should respect the maximum inventory limit (constraint 17). As before, if it will not be possible to respect this inventory condition, the relaxation variable $\text{VID}_{n,p}^{\max}$ assumes a positive value and it is penalized within the objective function.

$$\text{invent}_{n,p} - \text{VID}_{n,p}^{\max} \leq \text{ID}_{n,p}^{\max} \quad \forall n \in N, p \in P \quad (17)$$

The maximum inventory limit is, in fact, a maximum operational value recommended for the storage level. As stated in constraint 17, it can be violated; however, constraint 18 states that this violation should be limited by a certain amount. In fact, the considered area has a tankage capacity bigger than this recommended operational value. Then, if the maximum storage is violated, the inventory level should remain smaller than the available capacity of product p , which can be received in or sent from this area. Constraint 18 limits the inventory due to capacity conditions. The relaxation variable $\text{VCP}_{n,p}$ is minimized within the objective function.

$$\text{invent}_{n,p} - \text{VCP}_{n,p} \leq \text{CP}_{n,p} \quad \forall n \in N, p \in P \quad (18)$$

It is important to highlight that the determination of volumes to be transferred and routes to be used are influenced by the aggregated storage conditions of specific products in specific areas. Observing variable $Q_{n,n',p,r}$ it indicates the quantity of product p to be sent from node n to node n' using route r . In inequality 7 it is possible to note that, basically, the utilization rate of a pipeline d (α_d), which can be used by different routes, limits the quantity of product p to be sent from n to n' using route r ($Q_{n,n',p,r}$). By eq 15 one can observe that the storage level of product p in area n ($\text{invent}_{n,p}$) is linked to variable $Q_{n,n',p,r}$; by inequalities 16 and 17, $\text{invent}_{n,p}$ is linked to minimum/maximum aggregate storage limits ($\text{ID}_{n,p}^{\min}, \text{ID}_{n,p}^{\max}$); by inequality 18 $\text{invent}_{n,p}$ is also linked to aggregate capacity bounds ($\text{CP}_{n,p}$). Thus, $Q_{n,n',p,r}$ is linked to aggregated storage and capacity levels. The model tries to suggest volumes that fit tankage conditions of a specific area. Thus, the total tankage capacity of a specific

area influences the quantity of product that can be sent to a specific area by a specific route.

Besides satisfying the minimum, maximum, and capacity of inventory limits, it is desired that the inventory solution should be oriented by a storage goal value. For producing areas ($\text{disp}_{n,p} \neq 0$), it is expected that the storage reaches a value smaller than the storage goal, as limited by constraint 19. For consuming areas ($\text{flt}_{n,p} \neq 0$), it is desired that the storage reaches a value greater than the storage goal. Inequality 20 constrains the storage limit in relation to the goal. Violations in relation to the storage goal are attributed to variable $\text{VID}_{n,p}^{\text{goal}}$ and are avoided by penalizing the objective function.

$$\text{invent}_{n,p} - \text{VID}_{n,p}^{\text{goal}} \leq \text{ID}_{n,p}^{\text{goal}} \quad \forall n \in N, p \in P | \text{disp}_{n,p} \neq 0 \quad (19)$$

$$\text{invent}_{n,p} + \text{VID}_{n,p}^{\text{goal}} \geq \text{ID}_{n,p}^{\text{goal}} \quad \forall n \in N, p \in P | \text{flt}_{n,p} \neq 0 \quad (20)$$

Within constraints 16–20 of the MILP planning model, some violation variables were considered. These variables are used because strictly attending tankage conditions is, in practice, a hard task. Additionally, some operational aspects that help managing tankage conditions are not considered within the model. Often, in practice, the limits suggested by a company and the tankage constraints are respected by using other (undesirable) operational procedures (e.g., tank changeovers and degradation of “noble” products to satisfy demand of other “less noble” products). Therefore, a goal is to achieve an operational answer with zero value for violation variables. In the case where it is not possible to satisfy this goal, the violation variables can indicate conditions that are not properly balanced within the previously planned campaign of products.

4.2. The Objective Function. The objective function is given by expression 21, where different factors are considered:

$$\underbrace{\sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} \sum_{r \in R} \left(Q_{n,n',p,r} \cdot \frac{\text{vr}_r}{\text{vb}_{r,p}^{\text{med}}} \right)}_{\text{Factor 1}} + \underbrace{M \cdot \sum_{n \in N} \sum_{p \in P} \text{VCP}_{n,p} + M \cdot \sum_{n \in N} \sum_{p \in P} \text{VID}_{n,p}^{\min} + \frac{M}{5} \cdot \sum_{n \in N} \sum_{p \in P} \text{VID}_{n,p}^{\max} + \frac{M}{10} \cdot \sum_{n \in N} \sum_{p \in P} \text{VID}_{n,p}^{\text{goal}}}_{\text{Factor 2}} + \underbrace{\frac{M}{2} \cdot \sum_{r \in R} \sum_{r' \in R} \sum_{d \in D} \text{rev}_{r,r',d}}_{\text{Factor 3}} + \underbrace{\frac{M}{2} \cdot \sum_{d \in D} \text{vari}_d}_{\text{Factor 4}} + \underbrace{M \cdot \sum_{r \in R} \sum_{r' \in R} \sum_{n \in N} \sum_{p \in P} \text{tanq}_{r,r',n,p}}_{\text{Factor 5}} \quad (21)$$

Factor 1 is the optimization of the quantity $Q_{n,n',p,r}$ of product p to be sent from area n to area n' , considering the medium transportation time ($\text{vr}_r/\text{vb}_{r,p}^{\text{med}}$) of route r . Therefore, it is important to highlight that the model chooses the fastest route r to transport the quantity $Q_{n,n',p,r}$.

In *factor 2* violations in capacity (VCP), minimum/maximum storage values ($\text{VID}_{n,p}^{\min}, \text{VID}_{n,p}^{\max}$), and deviations from the storage goal value ($\text{VID}_{n,p}^{\text{goal}}$) are penalized. Ideally, storage violations should not occur, but they are accepted in order to analyze and suggest changes on mass balance as well as to detect the necessity of other operational procedures, as previously explained. In this way, part of the objective function is penalized by large M values (e.g., Table 2). Therefore, capacity and minimum/maximum storage violations are avoided as well as deviations from the goal storage value.

Factor 3 is minimizing the number of pipelines that present a reversion of flow. To revert the flow of pipelines demands an operational effort, and if possible, this procedure should be avoided. Therefore, the MILP planning model looks for solutions that minimize the number of pipelines that suffer reversion of flow. It is important to highlight that, at the planning level, the number of flow reversions that will occur in a specific pipeline during the scheduling horizon is not determined. The MILP planning model, by the binary variable “rev”, only indicates that a pipeline d will present reversion of flow during the horizon. After various tests, the weighting factor $1/2$ of M was established for this term based on the model solution toward a known reality.

Factor 4 is the maximum utilization rates expected for each pipeline. The maximum utilization rates for each pipeline (α_d) are suggested from historical values, which, in turn, reflect the operational difficulty of managing the pipelines with higher utilization rates. The variable vari_d indicates an additional value for the utilization rate for each pipeline when compared to α_d . Each variable vari_d , which spans from 0 to 1, is minimized. Moreover, when a pipeline is used in both directions (direct and reverse), the allowable utilization rate is also influenced by β_d .

Factor 5 includes the term to minimize intermediate storage and surge tank operations. These operations should be minimized since they demand (extra) operational costs due to the usage of (intermediate) resources (tanks). In this way, the sum of binary variables “tanq” is minimized, enabling these operations only when necessary.

By a dimensional analysis of the objective function (eq 21), it can be inferred that factor 1 involves quantities (Q) of, typically, some hundreds of volumetric units multiplied by a time value (for instance, values of time smaller than the H value). Factor 2 involves hundreds of volumetric units (VCP, VID^{\min} , VID^{\max} , VID^{goal}) weighted by M values. Finally, factors 3–5 involve values that span from zero to just some units weighted by M values. Thus, factors 1 and 2 are prioritized in relation to factors 3–5. However, to consider, even under different weights, the five factors is an important approach to lead to adequate operational solutions, as indicated in section 6.

The MILP planning model for a multiproduct general pipeline network is then defined by constraints 3–20 optimized under the objective function 21. This model defines (i) the total volume to be moved for each product from an origin to a destination area and (ii) the route that should be used. In addition, the model minimizes the number of pipelines that have reversion of flow and determines areas and products used on surge tank and intermediate storage operations. These results are used in an assignment/sequencing block, which divides the total determined volume in the planning phase in operational batches. In this way, a sequenced batch list is created. The final scheduling can then be attained by an MILP timing model¹¹ that indicates the pumping and receiving times, respecting the series of operational constraints.

5. MILP ASSIGNMENT AND SEQUENCING MODEL

As for the planning model, a continuous time mixed integer linear programming approach is used to create the assignment and sequencing model. The general guidelines used to state the planning model (beginning of section 4) are also valid at the assignment/sequencing model. As indicated by Figure 3, the outputs provided by the MILP planning model are used as

input parameters for the assignment and sequencing model. This model assumes the following:

1. The total amount to be carried between an origin area and a destination area during the planning horizon is determined by the MILP planning model. The assignment/sequencing model splits this total volume into operational batches (smaller volumes) that are transported during the horizon.
2. Tank capacities available at origin/destination areas must be respected.
3. Routes and pipelines used by the products are determined by the MILP planning model.
4. To implement the reversion flow, a set of batches is pumped in a direction. Afterward, it is necessary to insert an auxiliary reversion batch with a volume equal to the pipeline volume. Then, the auxiliary batch is pumped into the pipeline under reversion, allowing the delivery of batches in one direction. In sequence, the pump direction can be reverted and the products can be pumped in the opposite direction. Auxiliary batches are specified after the batch list generation.
5. In a surge tank and intermediate storage operations, the sequence of batches should be designed to satisfy the balance of volume in intermediate areas and observe inventory limits.
6. All batches in pipelines in the beginning of the planning horizon (in transit batches) are considered as stored in a predetermined destination area.
7. Constraints for seasonal cost, turn shifts, and local constraints are satisfied on a scheduling scope (timing block¹¹).

After presenting, in a simplified way, the main assumptions to be observed by the assignment and sequencing model, it is important to identify the main input parameters to the model:

- time horizon
- products to be transported
- list of operational areas
- initial and final pipelines for each route used
- medium time for the displacement of a batch in a predefined route
- surge tank areas
- production rate per product and area
- demand rate per product and area
- initial inventory per product and area
- storage limits per product and area
- flow rate limits per product and pipeline
- minimum and maximum lot sizes for batches per product
- quantity of a product to be transported from an origin area to a destination area using a predetermined route during the horizon

5.1. Problem Constraints. The MILP assignment and sequencing model is built considering the problem characteristics previously mentioned. Different types of constraints are defined, describing the problem specifications. All operational and physical constraints are modeled using variable volumes. As explained in section 3, sparse sets were used within the model generation.

Constraint 22 establishes that the sum of volumes of batches ($W_{b,n,n',p,r}$) should be equal to the total planned volume of product p to be sent from node n to node n' using route r

($Q_{n,n',p,r}$). This sum is relaxed when dif^+ or dif^- assumes nonzero values.

$$\sum_{b \in B} W_{b,n,n',p,r} + \text{dif}_{n,n',p,r}^+ - \text{dif}_{n,n',p,r}^- = Q_{n,n',p,r} \quad \forall \{n, n'\} \in N, p \in P, r \in R \quad (22)$$

For each batch, only one product p can be sent from area n to area n' using route r . Since it is not possible to predetermine the number of batches to be sent during the time horizon, the fictitious batch concept^{4,8} is used in this model. In this way, on constraint 23, if the sum of binary variable “ls” assumes a zero value, no product is allocated to batch b .

$$\sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} \sum_{r \in R} \text{ls}_{b,n,n',p,r} \leq 1 \quad \forall b \in B \quad (23)$$

Each batch volume should remain between maximum and minimum limits, as indicated by constraint 24. If the batch position is not used (a fictitious batch), no volume is allocated.

$$\text{lots}_p^{\min} \cdot \text{ls}_{b,n,n',p,r} \leq W_{b,n,n',p,r} \leq \text{lots}_p^{\max} \cdot \text{ls}_{b,n,n',p,r} \quad \forall b \in B, \{n, n'\} \in N, p \in P, r \in R \quad (24)$$

Fictitious batches are forced to remain at the final of the batch list (constraint 25).

$$\begin{aligned} \sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} \sum_{r \in R} \text{ls}_{b,n,n',p,r} \\ \leq \sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} \sum_{r \in R} \text{ls}_{b-1,n,n',p,r} \\ \forall b \in B, b > 1 \end{aligned} \quad (25)$$

Since production and demand are considered in a continuous manner, the inventory level calculation is made through events. In this way, the temporal index is suppressed and, for convenience, the b index is considered as an event for storage constraints. Thus, during the pumping or receiving of a batch b , the storage in an origin/destination area is updated. The inventory constraints are divided into two groups:

- Origin area constraints: the storage is calculated in the origin areas considering the time when batches are pumped. Thus, the pumping of batches enables the occurrence of events in origin areas.
- Destination area constraints: the storage is calculated in destination areas considering the time when batches are received. Thus, the receiving of batches enables the occurrence of events in destination areas.

The inventory of an origin area, for the first batch pumping, which indicates the first event within this area, should be equal to the initial inventory, added to the available volume during the pumping time of the first batch and subtracted from the pumping volume of batch b (eq 26).

$$\begin{aligned} \text{ID}_{b,p,n}^{\text{orig}} = \text{ID}_{n,p}^0 + \frac{\text{disp}_{n,p}}{H} \sum_{d \in D} \sum_{r \in R} \text{TF}_{b,d,r}^{\text{pump}} \\ - \sum_{n' \in N} \sum_{r \in R} W_{b,n,n',p,r} \\ \forall b \in B | b = 1, p \in P, n \in N \end{aligned} \quad (26)$$

For the remaining events, eq 27 holds. It specifies that the inventory in origin areas should be equal to the previous event inventory ($b - 1$), added to the available volume quantity during the time window between b and $b - 1$, subtracted from the pumped volume from n , and added to the received volume in n (at this case, n can be an area with surge tank operations).

$$\begin{aligned} \text{ID}_{b,p,n}^{\text{orig}} = \text{ID}_{b-1,p,n}^{\text{orig}} + \frac{\text{disp}_{n,p}}{H} \left(\sum_{d \in D} \sum_{r \in R} \text{TF}_{b,d,r}^{\text{pump}} \right. \\ \left. - \sum_{d \in D} \sum_{r \in R} \text{TF}_{b-1,d,r}^{\text{pump}} \right) - \sum_{n' \in N} \sum_{r \in R} W_{b,n,n',p,r} \\ + \sum_{n' \in N} \sum_{r \in R} W_{b-1,n,n',p,r} \\ \forall b \in B | b > 1, p \in P, n \in N \end{aligned} \quad (27)$$

When batch b is pumped (assigning a value of 1 to variable “ls”), the storage should respect the minimum and maximum inventory limits (constraints 28 and 29). If storage violations occur, the variables “ao” and “do” assume nonzero values, which become penalized within the objective function.

$$\begin{aligned} \text{ID}_{b,p,n}^{\text{orig}} + \text{ao}_{b,p,n} \geq \text{ID}_{p,n}^{\min} - M \\ \left(1 - \sum_{n' \in N} \sum_{r \in R} \text{ls}_{b,n,n',p,r} \right) \\ \forall b \in B, p \in P, n \in N \end{aligned} \quad (28)$$

$$\begin{aligned} \text{ID}_{b,p,n}^{\text{orig}} - \text{do}_{b,p,n} \leq \text{ID}_{p,n}^{\max} + M \\ \left(1 - \sum_{n' \in N} \sum_{r \in R} \text{ls}_{b,n,n',p,r} \right) \\ \forall b \in B, p \in P, n \in N \end{aligned} \quad (29)$$

The inventory of destination areas for the first event (batch) should be equal to the initial inventory, subtracted from the volume consumed during the receiving time of the first batch, added to the receiving volume of batch b (eq 30).

$$\begin{aligned} \text{ID}_{b,p,n}^{\text{dest}} = \text{ID}_{n,p}^0 - \frac{\text{flt}_{n,p}}{H} \sum_{d \in D} \sum_{r \in R} \text{TF}_{b,d,r}^{\text{rec}} \\ + \sum_{n' \in N} \sum_{r \in R} W_{b,n,n',p,r} \\ \forall b \in B | b = 1, p \in P, n \in N \end{aligned} \quad (30)$$

For other events (eq 31), the inventory level in destination areas should be obtained from the inventory of the previous event ($b - 1$), the consumed volume during the time window between b and $b - 1$, the received volume in area n , and the

pumped volume from n (in this case, n can be an area with surge tank operations). $|B|$ represents the cardinality of set B .

$$\begin{aligned} ID_{b,p,n}^{\text{dest}} &= ID_{b-1,p,n}^{\text{dest}} - \frac{flt_{n,p}}{H} \left(\sum_{d \in D} \sum_{r \in R} TF_{b,d,r}^{\text{rec}} \right. \\ &\quad \left. - \sum_{d \in D} \sum_{r \in R} TF_{b-1,d,r}^{\text{rec}} \right) + \sum_{n' \in N} \sum_{r \in R} W_{b,n',n,p,r} \\ &\quad - \sum_{n' \in N} \sum_{r \in R} W_{b+1,n',n,p,r} \\ \forall b \in B | 1 < b < |B|, p \in P, n \in N \end{aligned} \quad (31)$$

At the last event, the destination area inventory is given by eq 32. In this case, the surge tank batch is not subtracted because no next event exists.

$$\begin{aligned} ID_{b,p,n}^{\text{dest}} &= ID_{b-1,p,n}^{\text{dest}} - \frac{flt_{n,p}}{H} \left(\sum_{d \in D} \sum_{r \in R} TF_{b,d,r}^{\text{rec}} \right. \\ &\quad \left. - \sum_{d \in D} \sum_{r \in R} TF_{b-1,d,r}^{\text{rec}} \right) + \sum_{n' \in N} \sum_{r \in R} W_{b,n',n,p,r} \\ \forall b \in B | b = |B|, p \in P, n \in N \end{aligned} \quad (32)$$

Similarly to the formulation of constraints 28 and 29, constraints 33 and 34 indicate that when a batch b is received ($ls = 1$), the storage should respect the minimum and maximum inventory limits. If storage violations occur, the variables “ad” and “dd” assume nonzero values and become penalized within the objective function.

$$\begin{aligned} ID_{b,p,n}^{\text{dest}} + ad_{b,p,n} &\geq ID_{p,n}^{\min} - M \\ \left(1 - \sum_{n' \in N} \sum_{r \in R} ls_{b,n',n,p,r} \right) \\ \forall b \in B, p \in P, n \in N \end{aligned} \quad (33)$$

$$\begin{aligned} ID_{b,p,n}^{\text{dest}} - dd_{b,p,n} &\leq ID_{p,n}^{\max} + M \\ \left(1 - \sum_{n' \in N} \sum_{r \in R} ls_{b,n',n,p,r} \right) \\ \forall b \in B, p \in P, n \in N \end{aligned} \quad (34)$$

The model looks for operational solutions that are intended to satisfy inventory limits on destination areas (constraints 33 and 34). It also tries to maximize the minimum storage in destination areas at any event (constraint 35). In this way, the “mid” variable is maximized within the objective function. This variable indicates the smallest storage value per area and product.

$$\begin{aligned} mid_{n,p} &\leq \frac{ID_{b,n,p}^{\text{dest}}}{ID_{n,p}^{\max}} + \left(1 - \sum_{n' \in N} \sum_{r \in R} ls_{b,n',n,p,r} \right) \\ \forall b \in B, p \in P, n \in N \end{aligned} \quad (35)$$

The final time to pump any batch should respect the time horizon (constraint 36). When b is a fictitious batch, $TF_{b,d,r}^{\text{pump}}$ assumes a zero value.

$$\begin{aligned} TF_{b,d,r}^{\text{pump}} &\leq H \sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} ls_{b,n,n',p,r} \\ \forall b \in B, d \in D, r \in R \end{aligned} \quad (36)$$

The final time to pump a batch b should be greater than its initial pumping time, added to its pumping duration. The pumping duration is constrained by the division of batch b volume by the minimum and the maximum flow rate values (constraints 37 and 38).

$$\begin{aligned} TF_{b,d,r}^{\text{pump}} &\leq TI_{b,d,r}^{\text{pump}} + \sum_{p \in P} \sum_{n \in N} \left(\frac{1}{vb_{d,p,n}^{\min}} \right. \\ &\quad \left. \sum_{n' \in N} W_{b,n,n',p,r} \right) \quad \forall b \in B, d \in D, r \in R \end{aligned} \quad (37)$$

$$\begin{aligned} TF_{b,d,r}^{\text{pump}} &\geq TI_{b,d,r}^{\text{pump}} + \sum_{p \in P} \sum_{n \in N} \left(\frac{1}{vb_{d,p,n}^{\max}} \right. \\ &\quad \left. \sum_{n' \in N} W_{b,n,n',p,r} \right) \quad \forall b \in B, d \in D, r \in R \end{aligned} \quad (38)$$

Auxiliary variables are used to create constraints on temporal precedence among batches that use the same pipeline. For the pumping case, the variable “tdb” is greater than or equal to the auxiliary variable of the previous event (constraint 39). Otherwise, the auxiliary variable should assume a value greater than $TF_{b,d,r}^{\text{pump}}$ (constraint 40).

$$tdb_{b,d} \geq tdb_{b-1,d} \quad \forall b \in B | b > 1, d \in D \quad (39)$$

$$tdb_{b,d} \geq TF_{b,d,r}^{\text{pump}} \quad \forall b \in B, d \in D, r \in R \quad (40)$$

Through the variable “tdb” value, it is possible to identify the time instant in which the last batch is pumped in the same pipeline. Therefore, the temporal precedence between two batches that are pumped from the same pipeline can be established, as indicated by constraint 41.

$$\begin{aligned} TI_{b,d,r}^{\text{pump}} &\geq tdb_{b-1,d} - M \\ \left(1 - \sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} ls_{b,n,n',p,r} \right) \\ \forall b \in B | b > 1, d \in D, r \in R \end{aligned} \quad (41)$$

The estimation of the final receiving time is made by considering the pumping time and the time required for the displacement of an equivalent volume of the route used (vr_r). This last parcel of time is estimated through the division of the route volume by the mean transportation time (constraints 42 and 43).

$$\begin{aligned} TI_{b,d,r}^{\text{rec}} &= TI_{b,d',r}^{\text{pump}} + \sum_{p \in P} \left(\frac{vr_r}{vb_{r,p}^{\text{med}}} \sum_{n \in N} \sum_{n' \in N} ls_{b,n,n',p,r} \right) \\ \forall b \in B, \{d, d'\} \in D, r \in R \end{aligned} \quad (42)$$

Table 1. Data (10^3 vu) at Each Area for Scenario S3^a

| n | | $N2$ | | $N3$ | | | | | | | | | | $N4$ | | | | | | | | | | $N7$ | | | | | | | | | | $N8$ | | | | | | | | | |
|---|-----|------|-----|------|----|----|-------|-----|-----|-----|-----|-------|-----|------|-----|-----|-------|-----|-----|----|-----|-------|-----|------|----|----|-------|--|--|--|--|--|--|------|--|--|--|--|--|--|--|--|--|
| p | | 14 | | 10 | 11 | 13 | 14 | 18 | 19 | 20 | 21 | 26 | 31 | 6 | 11 | 14 | 18 | 21 | 22 | 25 | 27 | 31 | 32 | 34 | 35 | | | | | | | | | | | | | | | | | | |
| ID^0 | 5 | 1 | 19 | 4 | 2 | 7 | 6 | 4 | 4 | 4 | 30 | 25 | — | 8 | 10 | 23 | 85 | 2 | 15 | 17 | 20 | 8 | 145 | 28 | | | | | | | | | | | | | | | | | | | |
| ID^{\min} | 3 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | 6 | — | — | — | 3 | — | — | — | 100 | — | | | | | | | | | | | | | | | | | | |
| ID^{goal} | 5 | 8 | 20 | — | 4 | 9 | 16 | 16 | 19 | 41 | 56 | 9 | 19 | 19 | 10 | 47 | 93 | — | 4 | 38 | 21 | — | 131 | 60 | | | | | | | | | | | | | | | | | | | |
| ID^{\max} | 12 | 8 | 24 | 7 | 4 | 9 | 17 | 16 | 24 | 50 | 69 | 10 | 23 | 29 | 58 | 116 | 5 | 30 | 47 | 26 | 26 | 26 | 262 | 74 | | | | | | | | | | | | | | | | | | | |
| CP | 12 | 8 | 28 | 7 | 6 | 9 | 19 | 18 | 28 | 63 | 86 | 11 | 27 | 29 | 72 | 155 | 5 | 30 | 58 | 30 | 118 | — | 771 | 18 | | | | | | | | | | | | | | | | | | | |
| prod | — | 34 | 157 | — | 22 | 7 | 2 | 19 | 172 | 130 | 311 | 29 | 264 | 140 | 108 | 330 | — | 25 | 30 | 58 | 30 | — | — | — | | | | | | | | | | | | | | | | | | | |
| dem | 105 | — | 156 | — | — | — | — | — | 14 | 2 | 46 | — | 264 | 141 | — | — | 309 | — | 44 | — | 61 | — | 943 | — | | | | | | | | | | | | | | | | | | | |
| disp | — | 35 | 20 | 4 | 24 | 14 | 8 | 23 | 162 | 158 | 290 | 29 | 8 | — | 8 | 9 | 131 | 106 | 2 | — | 47 | 77 | 8 | — | | | | | | | | | | | | | | | | | | | |
| flt | 100 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | 4 | — | — | — | 27 | — | | | | | | | | | | | | | | | | | | | |
| n | | $N5$ | | | | | $N6$ | | | | | $N11$ | | | | | $N12$ | | | | | $N13$ | | | | | $N14$ | | | | | | | | | | | | | | | | |
| p | | 10 | 14 | 21 | 34 | | 5 | 6 | 11 | 14 | 16 | 18 | 20 | 21 | 25 | 27 | 29 | 31 | 34 | 7 | 18 | 20 | 25 | 34 | 35 | 21 | 27 | | | | | | | | | | | | | | | | |
| ID^0 | 3 | 5 | 20 | 45 | 8 | 8 | 4 | 4 | 7 | 6 | 27 | 73 | 81 | 37 | 2 | 2 | 5 | 23 | 81 | 4 | 22 | 51 | — | 13 | 29 | 43 | 21 | | | | | | | | | | | | | | | | |
| ID^{\min} | 1 | 1 | — | — | — | — | — | 26 | — | — | — | — | — | — | — | 2 | 1 | 21 | — | — | — | 8 | — | 8 | — | — | — | | | | | | | | | | | | | | | | |
| ID^{goal} | 2 | 2 | 30 | 40 | — | — | — | 39 | 17 | — | 58 | 49 | 64 | 52 | 5 | 5 | 2 | 32 | 108 | — | — | 15 | 27 | 17 | — | — | — | | | | | | | | | | | | | | | | |
| ID^{\max} | 8 | 8 | 36 | 49 | 40 | 27 | 87 | 18 | 8 | 71 | 61 | 78 | 98 | 80 | 34 | 30 | 22 | 71 | 135 | 5 | 25 | 51 | 31 | 55 | 52 | 64 | 64 | | | | | | | | | | | | | | | | |
| CP | 8 | 8 | 42 | 62 | 40 | 27 | 87 | 19 | 8 | 89 | 76 | 98 | 80 | 34 | 22 | 71 | 180 | 5 | 25 | 51 | 36 | 55 | 52 | 64 | 64 | 64 | 64 | | | | | | | | | | | | | | | | |
| prod | 9 | 25 | 86 | 111 | — | — | — | 198 | 81 | — | 111 | 164 | 59 | 178 | — | 161 | 42 | 257 | — | — | — | 27 | — | — | — | — | — | | | | | | | | | | | | | | | | |
| dem | 44 | 54 | — | — | — | — | — | 201 | 76 | — | — | — | 32 | — | 15 | 184 | 43 | 40 | — | — | 177 | — | 294 | — | — | — | — | | | | | | | | | | | | | | | | |
| disp | — | — | 106 | 156 | 8 | 8 | 1 | 12 | 6 | 138 | 237 | 108 | 215 | — | — | — | 22 | 298 | 4 | 22 | — | 27 | — | 29 | 43 | 21 | 21 | | | | | | | | | | | | | | | | |
| flt | 32 | 24 | — | — | — | — | — | — | — | — | — | — | — | — | 13 | 18 | — | — | — | — | 126 | — | 281 | — | — | — | — | | | | | | | | | | | | | | | | |
| n | | $N8$ | | | | | $N10$ | | | | | $N11$ | | | | | $N12$ | | | | | $N13$ | | | | | $N14$ | | | | | | | | | | | | | | | | |
| p | | 34 | 35 | 13 | 14 | | 20 | 21 | 26 | 31 | 21 | 25 | 27 | 31 | 32 | 21 | 31 | 34 | 2 | 16 | 19 | 21 | 27 | 31 | 34 | 35 | 18 | | | | | | | | | | | | | | | | |
| ID^0 | 104 | 51 | 10 | 42 | 42 | 16 | 16 | 13 | 21 | 1 | 15 | 16 | 3 | 20 | 3 | 26 | 22 | 6 | 7 | 2 | 2 | 11 | 4 | 10 | 12 | 18 | 50 | | | | | | | | | | | | | | | | |
| ID^{\min} | — | — | — | — | — | 3 | 3 | 3 | 5 | — | 5 | 6 | — | 7 | — | 7 | 10 | — | — | 1 | 3 | 2 | 4 | — | 5 | 37 | — | | | | | | | | | | | | | | | | |
| ID^{goal} | — | — | — | — | 46 | 5 | 5 | 5 | 8 | 0 | 9 | 10 | — | 12 | — | 12 | 17 | — | — | — | 1 | 5 | 3 | 7 | — | 8 | 58 | | | | | | | | | | | | | | | | |
| ID^{\max} | 139 | 64 | 17 | 52 | 52 | 17 | 17 | 17 | 34 | 7 | 26 | 28 | 10 | 35 | 10 | 35 | 50 | 6 | 10 | 5 | 5 | 20 | 10 | 21 | 16 | 23 | 106 | | | | | | | | | | | | | | | | |
| CP | 139 | 64 | 17 | 61 | 61 | 17 | 17 | 17 | 34 | 7 | 26 | 28 | 10 | 35 | 10 | 35 | 50 | 6 | 10 | 5 | 5 | 20 | 10 | 21 | 16 | 23 | 106 | | | | | | | | | | | | | | | | |
| prod | — | — | — | 139 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | | | | | | | | | | | | | | | | |
| dem | — | — | — | 26 | 26 | 19 | 19 | 138 | 114 | 79 | 57 | 169 | — | 47 | — | 86 | 94 | — | — | — | 1 | 66 | 16 | 79 | — | 41 | 223 | | | | | | | | | | | | | | | | |
| disp | 104 | 51 | 10 | 155 | — | — | — | — | — | — | — | — | 3 | — | 3 | — | — | 6 | 7 | 2 | 2 | — | — | — | 12 | — | — | | | | | | | | | | | | | | | | |
| flt | — | — | — | — | — | 3 | 3 | 125 | 93 | 78 | 42 | 153 | — | 27 | — | 60 | 72 | — | — | — | — | 55 | 12 | 69 | — | 23 | 173 | | | | | | | | | | | | | | | | |
| 1. stands for area and p for product. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

² n stands for area and p for product.

$$TF_{b,d,r}^{rec} = TF_{b,d',r}^{pump} + \sum_{p \in P} \left(\frac{vr_r}{vb_{r,p}^{med}} \sum_{n \in N} \sum_{n' \in N} ls_{b,n,n',p,r} \right) \quad \forall b \in B, \{d, d'\} \in D, r \in R \quad (43)$$

For the receiving cases, the auxiliary variable “tdr” should be greater than or equal to the auxiliary variable of the previous event (constraint 44). Otherwise, the auxiliary variable “tdr” should assume a value greater than $TF_{b,d,r}^{rec}$ (constraint 45).

$$tdr_{b,d} \geq tdr_{b-1,d} \quad \forall b \in B | b > 1, d \in D \quad (44)$$

$$tdr_{b,d} \geq TF_{b,d,r}^{rec} \quad \forall b \in B, d \in D, r \in R \quad (45)$$

By using the variable “tdr” value, it is possible to identify the time instant in which the last batch is received by the same pipeline. Therefore, the temporal precedence between two batches received to the same area through a specific pipeline can be established, as indicated by constraint 46.

$$TI_{b,d,r}^{rec} \geq tdr_{b-1,d} - M(1 - \sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} ls_{b,n,n',p,r}) \quad \forall b \in B | b > 1, d \in D, r \in R \quad (46)$$

Constraint 47 establishes the temporal conditions for areas and products with surge tank operations. The start pumping time in area n (with surge tank operation) should occur after the receiving of the previous batch in this same area n , if batches b and $b + 1$ are nonfictitious. In case of fictitious batches, this condition is relaxed.

$$TI_{b+1,d',r}^{pump} \geq TI_{b,d,r}^{rec} - M(2 - ls_{b,n',n,p,r} - ls_{b+1,n,n'',p,r'}) \quad \forall b \in B | b < |B|, p \in P, \{n, n', n''\} \in N, \{d, d'\} \in D, \{r, r'\} \in R \quad (47)$$

5.2. The Objective Function. The objective function involves four main factors, and can be described as follows:

$$\begin{aligned} &\text{minimize} \\ &\underbrace{\sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} \sum_{r \in R} (dif_{n,n',p,r}^{+} + dif_{n,n',p,r}^{-})}_{\text{Factor 1}} + \\ &\underbrace{\gamma \cdot \sum_{b \in B} \sum_{p \in P} \sum_{n \in N} (ao_{b,p,n} + do_{b,p,n}) + \gamma \cdot \sum_{b \in B} \sum_{p \in P} \sum_{n \in N} (ad_{b,p,n} + dd_{b,p,n})}_{\text{Factor 2}} + \\ &\underbrace{\sum_{b \in B} \sum_{n \in N} \sum_{n' \in N} \sum_{p \in P} \sum_{r \in R} ls_{b,n,n',p,r}}_{\text{Factor 3}} - \underbrace{\mu \cdot \sum_{n \in N} \sum_{p \in P} mid_{n,p}}_{\text{Factor 4}} \end{aligned} \quad (48)$$

Factor 1 is the minimization of the difference between the total volume sent and the total volume suggested by the MILP planning model (dif^{+} and dif^{-}).

Factor 2 is the penalization on minimum and maximum storage limit violations at the origin and destination areas (ao, do, ad, and dd). Ideally, storage violations should not occur, but they are accepted for system analysis. In this way, part of the objective function is penalized by a γ value, which should be evaluated from case to case. Therefore, storage violations are avoided. After various tests, the weighting factor γ was established for this term based on the model solution toward

a known reality (e.g., Table 7). The ao, do, ad, and dd values can, in theory, span from zero to hundreds of volumetric units, but as these values are penalized within the objective function, in practice, just unavoidable violations occur.

Factor 3 is the minimizing of the number of generated batches. The sum of binary variables “ls” is minimized within the objective function. Consequently, the number of generated batches is minimized. Additionally, the creation of batches with small volumes and batches in sequence involving the same product into the same route are also avoided.

Factor 4 is the maximizing of the minimum storage level in destination areas. The “mid” variable indicates the smallest storage value for each destination area and product, considering the entire horizon. This variable, which can span from 0 to 1, is weighted by a μ factor and maximized within the objective function. The goal is to maintain the minimum storage level as high as possible during the considered horizon, while respecting storage limits. The value of the weighting factor μ should be established based on the model solution toward a known reality; see section 6.

If one makes a dimensional analysis of the objective function 48, it can be inferred that factors 1 and 2 can span from zero to some thousands of volumetric units, factor 3 can span from zero to the total number of batches, which is limited to a few hundreds of units, and factor 4 can span from zero to some units. Thus, minimizing the difference between the total volume sent and the total volume suggested (factor 1) and avoiding tankage violations (factor 2) are priorities in relation to minimizing the number of generated batches (factor 3) and to maintaining the minimum storage level at destination areas as high as possible (factor 4). However, to consider, even under different weights, the four factors are an important approach to leading to adequate operational solutions, as indicated in section 6.

Summarizing, the MILP assignment and sequencing model for a multiproduct general pipeline network is described by constraints 22–47 optimized under the objective function 48.

6. CASE STUDIES: MODEL IMPLEMENTATION AND RESULTS

The models presented in sections 4 and 5 are here applied to a real multiproduct pipeline network in a hierarchical manner, as presented by Figure 3. This pipeline network follows the structure and characteristics presented in section 3. The models were run using the software ILOG OPL Studio 6.3,¹⁴ CPLEX 12 on an Intel Core 2 Duo 6400 2.13 GHz and 3 GB RAM. Sparse sets were used to generate the models, and the number of decision variables was significantly reduced.

Both developed models were applied to eight real-world scenarios (S1–S8) of 1 month period each, representing the real-world network operation. The scenarios were chosen considering different months in a year and are numbered in chronological order.

6.1. Planning Model Results. The data used in the planning model (e.g., production, consumption, and storage limits) vary according to the considered scenario. As an example, Table 1 details the data for the scenario S3. The values of ID^0 , ID^{\min} , ID^{goal} , ID^{\max} , CP, prod, dem, disp, and flt are specified for each product and area. These values are represented in volumetric units, from now on named “vu”. A dash (–) in Table 1 indicates that the considered value equals 0. For instance, for node N3 the required quantity (flt) of each product p in this area (vu) during H is equal to 0. In fact, at this

Table 2. MILP Planning Model Parameters for Pipelines 1–30

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|----------------|-------------|-----|-----|-----|-----|-----|---------------------|-----|-----|-----|-----|-----|------------------------|-----|-----|
| α_d (%) | 90 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| β_d (%) | 30 | 100 | 10 | 100 | 100 | 50 | 100 | 100 | 50 | 40 | 40 | 40 | 40 | 40 | 40 |
| | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| α_d (%) | 0 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| β_d (%) | 0 | 30 | 100 | 90 | 10 | 20 | 15 | 50 | 15 | 100 | 50 | 100 | 100 | 50 | 50 |
| H | 720 h | | | | | | T^{\min} | | | | | | 7000 vu | | |
| M | 100 000 000 | | | | | | lots ^{min} | | | | | | 0 vu $\forall p \in P$ | | |

Table 3. MILP Planning Model Computational Results

| | scenario | | | | | | | |
|--------------|-----------|---------|---------|---------|-----------|---------|---------|---------|
| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 |
| status | optimal | optimal | optimal | optimal | optimal | optimal | optimal | optimal |
| CPU time (s) | 2.9 | 2.221 | 2.189 | 2.143 | 2.196 | 2.35 | 2.327 | 2.239 |
| obj function | 14 782.65 | 2746.44 | 4664.56 | 0.5 | 19 011.65 | 589.19 | 1771.03 | 7411.80 |
| best node | 14 782.22 | 2746.24 | 4664.29 | 0.5 | 19 011.52 | 589.17 | 1770.95 | 7411.65 |
| gap (%) | 0.0000 | 0.0001 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| iterations | 272 | 278 | 435 | 717 | 417 | 681 | 408 | 442 |
| variables | 1303 | 1256 | 1247 | 1348 | 1356 | 1357 | 1232 | 1335 |
| int var | 588 | 541 | 539 | 633 | 635 | 631 | 532 | 626 |
| constraints | 2106 | 2057 | 2051 | 2310 | 2354 | 2528 | 2056 | 2339 |

Table 4. MILP Planning Model General Results

| | scenario | | | | | | | |
|------------------------------------|----------------|----------------|---------------------|----------------|---------------|---------------|--------------|----------------------|
| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 |
| total moved (vu) | 2 180 599 | 2 014 910 | 2 278 359 | 2 137 422 | 1 992 999 | 2 197 381 | 2 126 734 | 2 267 052 |
| pipes with reversion flow | 3, 20, 24 | 3, 22, 24 | 3, 22, 24 | 3, 22, 24 | 3, 22, 24 | 3, 20, 22, 24 | 3, 20, 22 | 3, 20, 22, 24 |
| VID ^{min} (quantity) (vu) | 11 700 (2) | 13 841 (1) | 43 552 (2) | — | 84 312 (2) | 3 148 (1) | — | 1 678 (2) |
| VID ^{max} (quantity) (vu) | 127 906 (5) | 6 788 (3) | — | — | 75 473 (1) | — | 15 481 (1) | 68 293 (2) |
| VCP (quantity) (vu) | 104 408 (2) | 5 707 (2) | — | — | 73 643 (1) | — | 10 925 (1) | 56 665 (1) |
| pipeline less used (pipe) | 0% (9, 16, 21) | 0% (9, 16, 27) | 0% (16, 27) | 0% (9, 16, 27) | 0% (16, 27) | 0% (16) | 0% (16) | 0% (16) |
| pipeline more used (pipe) | 77% (25) | 80% (25) | 88% (25) | 70% (12) | 72% (12) | 83% (19) | 81% (19) | 78% (17) |
| surge tank areas (area/product) | N9/31, N12/31 | N9/31 | N9/(21, 31), N12/31 | N9/31, N12/31 | N9/31, N12/31 | N9/31, N12/31 | N4/35, N9/31 | N4/35, N9/31, N12/31 |

scenario, the production of node N3 is able, if necessary, to supply its own demand.

Additional parameters used within the planning model are presented in Table 2. The weighting parameter M and the desired pipeline usage rate (α and β) were obtained after a series of tests and are based on the knowledge and experience of the company's specialists. For instance, for pipeline 1, the longest within the network with more than 42 000 vu, a maximum usage rate of 90% was considered, reflecting the operational difficulty of maintaining high utilization rates at this pipeline. The planning horizon H , the minimum quantity for intermediate storage T^{\min} , and the minimum quantity to be sent of product p (lots^{min}) are also defined in Table 2.

The computational results of the planning model are presented in Table 3. Optimal solutions were obtained in less than 3 s, for all the observed scenarios. No relative gap and CPU time limit values were defined. This shows the potentiality of the developed model.

Table 4 presents some of the results obtained for the eight studied scenarios: the total volume to be moved during the scenario, the pipelines used with reversion flow procedures, the total violated volume for minimum/maximum/capacity storage limits, and the number of violations observed (within

parentheses). In addition, it is possible to observe the pipelines more and less used as well as the utilization rate: percent and pipeline number (in parentheses). Areas/products in which intermediate tanks are used are also shown. It is important to highlight that within the eight studied scenarios the total quantity of moved product does not suffer many variations, but the demand of each product can be strongly different from scenario to scenario, reflecting market seasonal conditions.

From these results, it can be seen that, in some cases, no storage violations occur (e.g., scenario S4, lines of VID^{min}, VID^{max}, and VCP). However, this is not always the case and some violations in storage limits are observed. In practice, the company often uses operational procedures to avoid storage violations. That is the case of using "noble" products to satisfy the demand of other "less noble" products. These practical procedures (product degradation) have not yet been accounted for in the model. The strategy followed primarily was to avoid them. However, in a future work these aspects are planned to be addressed to improve the generality of the planning model.

Figure 5 illustrates variations on the total moved volume according to the studied scenarios. This total volume is influenced by the scenario input parameters (e.g., ID^{min}, ID^{goal}, ID^{max}, disp, and flt). However, the planning model has to

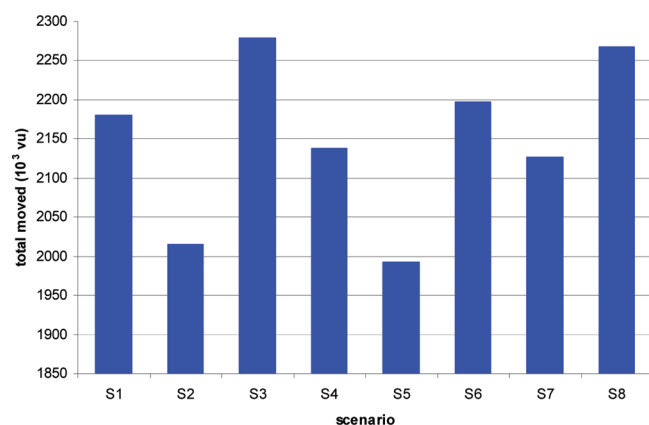


Figure 5. Monthly moved volume for each scenario.

address a series of factors within the objective function 21 to determine the exact moved volume.

The results presented by the MILP planning model directly influence the utilization rate of pipelines, which is a very important aspect for the company standards. The utilization rate derived from the planning model can however differ from the final utilization rate obtained at the final scheduling. This fact is justified by the necessity of inserting additional batches after the batch list generation. This insertion is done after the MILP assignment and sequencing model. These auxiliary batches can help in reversion flow procedures, can be used as plug, or can only be added to dislocate batches in order to manage delivery conditions in destination areas.

With the aim of analyzing in more detail the results obtained from the planning model, scenario S3 was chosen. Figure 6 presents the utilization rate, from the planning model, for the 30 pipelines of the network. Afterward, as explained in section 4, with the insertion of auxiliary batches these rates tend to increase. In a preliminary observation, pipelines 19 and 25 have more than 80% of their capacity in use. Thus, these pipelines are likely to become system bottlenecks. On the other hand, less than 10% of capacity is used for the case of pipelines 9, 21, and 26. Small utilization rates are also undesirable because the

products inside these pipelines can remain stopped for a long period of time, leading to product quality problems.

The results shown in Table 5 describe the total volumes (in volumetric units, vu) to be transported per product and the routes used to move these volumes during the predefined horizon. It is possible to observe that two or more distinct products can be moved using the same route (e.g., products 20 and 21 use the first route $N3 \rightarrow 11 \rightarrow N9 \rightarrow 30 \rightarrow N10$). The origin and destination areas are the first and the last elements of the route, respectively. For instance, 8567 vu of product 20 should be transported from N3 to N10. The planning model suggests transporting products as a function of production/consumption levels as well as storage conditions. Comparing Tables 1 and 5, one can note that not all products presented in Table 1 were suggested to be transported in Table 5 (e.g., product 11). Thus, scenario input parameters (e.g., ID^{\min} , ID^{goal} , ID^{\max} , disp, and flt) can influence the total volume suggested to be sent by the model.

6.2. Comparison of Results: Planning Model versus Company's Approach. This section compares the planning model results, scenario S3, with the real planning approach adopted by the company's specialists (historical data). Table 6 summarizes these results.

Results indicate that the proposed model suggests the use of a small number of pipelines with flow reversions. In addition, when analyzing the occurrence of violations in minimum storage and capacity levels in origin areas, it can be observed that in the company solution there are six violations in the capacity of origin areas and eight violations in the minimum storage level of origin areas. The model results suggested an operational answer with no product degradation and tankage changes, without violating tankage limits. No significant differences were observed in destination areas. Both approaches were able to attain the main operational conditions. Finally, it is possible to notice that the planning model suggests the use of a smaller volume of product to guarantee demand requirements. Additionally, surge tank operations were in the model minimized, causing fewer maneuvers of valves and, thus, reducing the respective operating cost.

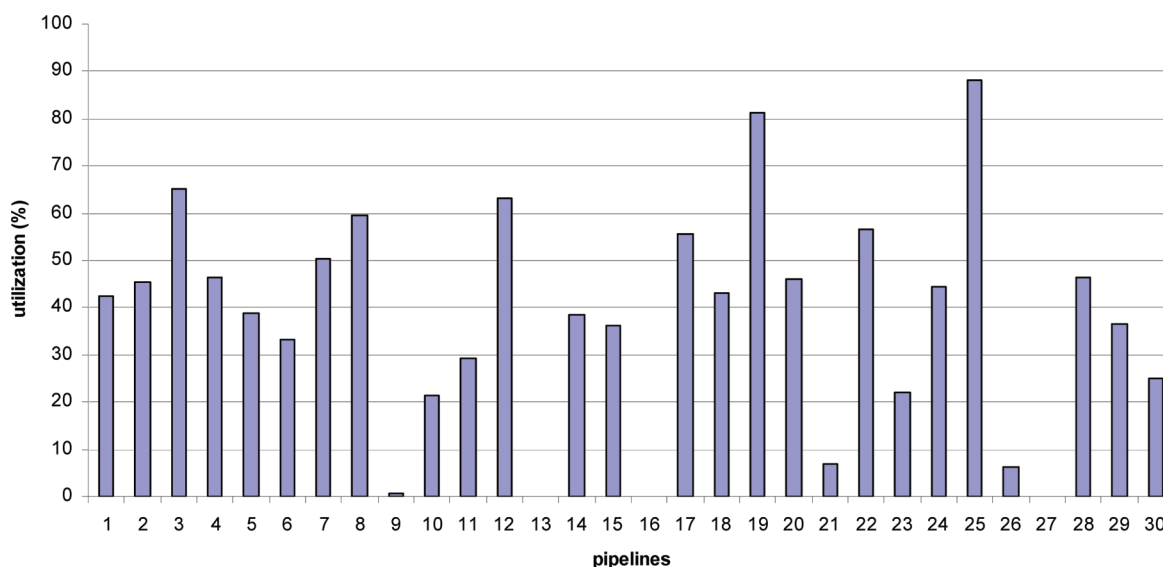


Figure 6. Pipeline usage for scenario S3.

Table 5. Moved Volume (vu) per Product and Routes for Scenario S3

| route | quantity moved (vu) per product | | | | | | | | | | | |
|---|---------------------------------|-----------|------------|------------|------------|------------|-------|------------|-----------|------------|------------|------------|
| | 6 | 10 | 14 | 18 | 20 | 21 | 25 | 26 | 27 | 31 | 34 | 35 |
| N3 → 11 → N9 → 30 → N10 | | | | | 8 567 | 141 771 | | | | | | |
| N3 → 12 → N9 | | | | | | | | | | 166 275 | | |
| N3 → 11 → N9 | | | | | | 1 107 | | | | | | |
| N3 → 11 → N9 → 17 → N12 → 18 → N14 | | | | 13 878 | | | | | | | | |
| N3 → 10 → N9 → 8 → N5 | | 34 091 | 20 492 | | | | | | | | | |
| N3 → 12 → N9 → 15 → N10 | | | | | | | | 116 855 | | 78 124 | | |
| N5 → 7 → N12 → 19 → N13 | | | | | | 35 057 | | | | | | |
| N5 → 6 → N12 | | | | | | 71 431 | | | | | | |
| N4 → 23 → N13 | | | | | | 25 244 | | | 14 443 | | | |
| N4 → 22 → N13 | | | | | | | | | | | | 30 694 |
| N4 → 1 → N8 → 28 → N1 → 4 → N11 | | | | | | | | | | 45 967 | | |
| N4 → 1 → N8 → 2 → N11 → 5 → N12 → 18 → N14 | | | | 84 176 | | | | | | | | |
| N4 → 1 → N8 → 24 → N6 | 19 465 | | | | | | | | | 9 684 | | |
| N6 → 24 → N8 → 28 → N1 → 4 → N11 | | | | | | | | 163 320 | | | | |
| N6 → 25 → N8 → 2 → N11 | | | | | | 50 678 | | | | | | |
| N6 → 25 → N8 → 2 → N11 → 5 → N12 → 18 → N14 | | | | 133 380 | | | | | | | | |
| N6 → 25 → N8 → 3 → N7 | | | | | 176 694 | | | | | | | 297 998 |
| N9 → 9 → N12 | | | | | | 1 107 | | | | | | |
| N9 → 17 → N12 | | | | | | | | | | 166 357 | | |
| N10 → 14 → N9 → 8 → N5 | | | 5 377 | | | | | | | | | |
| N10 → 14 → N9 → 8 → N5 → 7 → N12 → 29 → N2 | | | 105 074 | | | | | | | | | |
| N7 → 3 → N8 → 21 → N4 | | | | | | | 8 319 | | | | | |
| N12 → 19 → N13 | | | | | | | | | | 76 519 | | |
| N8 → 21 → N4 | | | | | | | | | | | 41 907 | |
| N8 → 26 → N6 | | | | | | | | | 18 221 | | | |
| N5 → 20 → N12 → 19 → N13 → 22 → N4 | | | | | | | | | | | 115 762 | |

On the basis of these results, it can be concluded that the utilization of the proposed model as a tool to help in the decision-making at the planning level is very efficient and suggests planning procedures that are in line with the company planning objectives. It is important to note that all the model

Table 6. Scenario S3: MILP Planning Model versus Company's Approach

| operational characteristic observed | MILP model | company solution |
|--|---------------|--------------------------|
| flow reversion: label of pipelines (number of pipelines) | 3, 22, 24 (3) | 1, 3, 21, 23, 24, 30 (6) |
| capacity violation, origin areas: volume (quantity) | — | 130 249 vu (6) |
| min storage violation, origin areas: volume (quantity) | — | 179 608 vu (8) |
| total moved volume, without flow reversion | 2 034 133 vu | 2 125 177 vu |
| volume in surge tank operations | 243 901 vu | 823 028 vu |

results were analyzed and validated by the company's specialists.

6.3. Assignment and Sequencing Model Results. The developed assignment and sequencing model was applied to the same eight real-world scenarios (S1–S8) of 1 month used by the planning model. The results obtained by the planning model were used as input parameters to the assignment and sequencing model. Note that Table 5 details, for scenario S3, the outputs provided by the planning model into the assignment and sequencing model.

Additional parameters used by the assignment and sequencing model are presented in Table 7. The weighting parameter M and the parameters (γ , μ , and $|B|$) were obtained heuristically after a series of tests and having in mind the real operation. The planning horizon H and minimum/maximum batch sizes ($\text{lots}^{\min}/\text{lots}^{\max}$) for product p are also defined in Table 7.

The use of assignment and sequencing models for a monthly planning period has presented some computational limitations.

Table 7. MILP Assignment and Sequencing Model Parameters

| | product | | | |
|--------------------------|----------|-------|----------|--------------|
| | 10 | 14 | 19 | other |
| lots ^{min} (vu) | 1350 | 1350 | 3000 | 5000 |
| lots ^{max} (vu) | 5400 | 5400 | 15000 | 25000 |
| <i>M</i> | γ | μ | <i>l</i> | <i>H</i> (h) |
| 1 000 000 | 2 | 5 | 60 | 180 |

Different tests were performed and, in consonance with the company procedures, a weekly planned period was then considered. The monthly volume suggested by the planning model was then split into four equal parts, indicating a weekly planned volume.

The computational results for the assignment and sequencing model are indicated in Table 8. Sparse sets were also used here to generate the model, and thus, the number of decision variables was significantly reduced. No relative gap was defined, and a maximum of 1800 CPU s was considered. Feasible solutions were obtained for all scenarios after 1800 s of processing; however, the relative integrality gap observed was undesirably high (spanning from 14 to 98%). The number of nonfictitious batches assigned and sequenced by the model for a horizon of 1 week is presented in Table 8 (number of batches). It is important to notice that the number of allowed batches for 1 week was 60 ($|l| = 60$).

The results, presented in Table 8, need to be further improved. This is part of the current work of the authors. However, such results were analyzed by the company and good acceptance of them was obtained. This means that the results would describe well the real operation.

Using scenario S3, as an example, the operational details provided by the assignment and sequencing model can be analyzed (see Table 9). A list with 48 batches to transfer was obtained. In this list it can be observed how the products are transferred from origins to destinations. The origin and destination areas are the first and the last elements of the route, respectively. For instance, the total transported volume for product 14 (*p*) with origin area in N10 and destination area in N2 uses route N10 → 14 → N9 → 8 → N5 → 7 → N12 → 29 → N2. This product/route was split into five batches ($b = 3, 5, 10, 32$, and 42). The batch volume of product 14 is equal to the maximum batch size (lots^{max} = 5400 vu), except for batch 5 (4669 vu), which presented a smaller volume to complement the planning volume suggested for 1 week (see Table 5, 105 074 vu during 1 month → 26 269 vu during 1 week → 4(5400 vu + 4669 vu). The sequence list is constructed considering the

priority for sending products. The products that need to be pumped (or received) earlier have higher priority when compared to other products.

Table 10 indicates that, in some cases, it is not possible to match the exact planned volume for 1 week, even considering that volumes of batches (*W*) can assume continuous values (between minimum and maximum limits). The difference observed can be added (or subtracted) within the next week running, for the same scenario.

The results from the assignment and sequencing model, as presented in Table 9, can be used as input parameters for the scheduling block (Figure 3). For details on this block the reader is referred to the work of Boschetto et al.¹¹ The MILP timing model defines initial and final pumping/receiving times for batches in all areas. It also considers other operational constraints as, for example, simultaneous pumping limitations per area and product.

7. FINAL CONSIDERATIONS

This paper considers the development of a solution approach to address the planning and the assignment and sequencing of operational activities of a complex pipeline network. This approach uses a decomposition strategy where the integration of two MILP models that run in sequence is considered. They model different levels of decision in the planning process such as the planning level and the assignment and sequencing level. The last level acts as input to a timing model, not explored in the current paper, where the final scheduling of the pipelines is performed.

The first proposed MILP model, the planning block, determines the total volume to be transported and the routes to be used within a planning horizon of, typically, 1 month. The planning model suggests, in an optimized way, strategic information on how much should be transported through a complex pipeline network. The model also considers the minimization of some operational indicators that cause high operational costs. These are respectively the quantity of pipelines used in both directions and the number of areas that are used in surge tank operations.

The planning model was applied to a real-world pipeline, and the results indicate that the model is able to obtain optimal solutions in a few seconds. The results indicate that storage values, in general, present few violations when compared to the company solution in similar conditions. Violations of tankage are allowed (relaxation variables), but are penalized within the objective function. Operational aspects used in reality to deal with such violations were not yet considered in the developed model but will be addressed in future. This is the case of

Table 8. Computational Results for the MILP Assignment and Sequencing Model

| | scenario | | | | | | | |
|--------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 |
| time (s) | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 | 1800 |
| obj function | 114 802.1 | 105 400.9 | 17 409.4 | 35 068.7 | 134 236.9 | 72 499.3 | 107 985.9 | 615 620.2 |
| best node | 10 433.5 | 11 022.0 | 14 956.9 | 15 140.1 | 15 264.1 | 6499.0 | 5476.0 | 9847.4 |
| gap (%) | 90.91 | 89.54 | 14.09 | 56.83 | 88.63 | 91.04 | 94.93 | 98.40 |
| iterations | 4 136 120 | 3 032 431 | 3 761 195 | 4 533 903 | 2 848 258 | 3 541 994 | 3 785 356 | 3 300 278 |
| variables | 22 038 | 21 802 | 21 623 | 23 248 | 22 163 | 20 957 | 19 999 | 21 466 |
| int var | 1920 | 1980 | 1980 | 2160 | 1980 | 1860 | 1860 | 2040 |
| constraints | 31 437 | 30 904 | 30 781 | 33 238 | 31 321 | 30 056 | 28 625 | 30 427 |
| no. batches | 51 | 42 | 48 | 60 | 48 | 51 | 50 | 51 |

Table 9. Weekly Batch Volume and Sequence: Scenario S3

| B | p | r | W |
|----|----|---|--------|
| 1 | 31 | N3 → 12 → N9 → 15 → N10 | 14 087 |
| 2 | 10 | N3 → 10 → N9 → 8 → N5 | 1 361 |
| 3 | 14 | N10 → 14 → N9 → 8 → N5 → 7 → N12 → 29 → N2 | 5 400 |
| 4 | 14 | N3 → 10 → N9 → 8 → N5 | 1 805 |
| 5 | 14 | N10 → 14 → N9 → 8 → N5 → 7 → N12 → 29 → N2 | 4 669 |
| 6 | 21 | N4 → 23 → N13 | 10 848 |
| 7 | 31 | N3 → 12 → N9 | 19 048 |
| 8 | 34 | N6 → 25 → N8 → 3 → N7 | 25 000 |
| 9 | 18 | N6 → 25 → N8 → 2 → N11 → 5 → N12 → 18 → N14 | 25 000 |
| 10 | 14 | N10 → 14 → N9 → 8 → N5 → 7 → N12 → 29 → N2 | 5 400 |
| 11 | 25 | N6 → 24 → N8 → 28 → N1 → 4 → N11 | 25 000 |
| 12 | 18 | N3 → 11 → N9 → 17 → N12 → 18 → N14 | 5 000 |
| 13 | 21 | N3 → 11 → N9 → 30 → N10 | 25 000 |
| 14 | 21 | N5 → 6 → N12 | 18 135 |
| 15 | 27 | N4 → 23 → N13 | 5 000 |
| 16 | 18 | N4 → 1 → N8 → 2 → N11 → 5 → N12 → 18 → N14 | 6 998 |
| 17 | 31 | N4 → 1 → N8 → 28 → N1 → 4 → N11 | 11 492 |
| 18 | 25 | N6 → 24 → N8 → 28 → N1 → 4 → N11 | 15 607 |
| 19 | 34 | N6 → 25 → N8 → 3 → N7 | 5 000 |
| 20 | 31 | N9 → 17 → N12 → 19 → N13 | 19 130 |
| 21 | 14 | N10 → 14 → N9 → 8 → N5 | 1 350 |
| 22 | 34 | N6 → 25 → N8 → 3 → N7 | 15 410 |
| 23 | 31 | N3 → 12 → N9 | 17 521 |
| 24 | 34 | N8 → 3 → N7 | 23 978 |
| 25 | 18 | N4 → 1 → N8 → 2 → N11 → 5 → N12 → 18 → N14 | 14 046 |
| 26 | 31 | N3 → 12 → N9 | 5 000 |
| 27 | 31 | N9 → 17 → N12 | 17 460 |
| 28 | 26 | N3 → 12 → N9 → 15 → N10 | 12 644 |
| 29 | 10 | N3 → 10 → N9 → 8 → N5 | 1 954 |
| 30 | 35 | N4 → 22 → N13 | 7 674 |
| 31 | 20 | N6 → 25 → N8 → 3 → N7 | 25 000 |
| 32 | 14 | N10 → 14 → N9 → 8 → N5 → 7 → N12 → 29 → N2 | 5 400 |
| 33 | 20 | N6 → 25 → N8 → 3 → N7 | 19 174 |
| 34 | 6 | N4 → 1 → N8 → 24 → N6 | 5 000 |
| 35 | 14 | N3 → 10 → N9 → 8 → N5 | 3 318 |
| 36 | 10 | N3 → 10 → N9 → 8 → N5 | 4 811 |
| 37 | 34 | N6 → 25 → N8 → 3 → N7 | 5 113 |
| 38 | 27 | N8 → 26 → N6 | 5 000 |
| 39 | 31 | N9 → 17 → N12 | 5 000 |
| 40 | 18 | N6 → 25 → N8 → 2 → N11 → 5 → N12 → 18 → N14 | 8 339 |
| 41 | 21 | N5 → 7 → N12 → 19 → N13 | 5 000 |
| 42 | 14 | N10 → 14 → N9 → 8 → N5 → 7 → N12 → 29 → N2 | 5 400 |
| 43 | 34 | N5 → 20 → N12 → 19 → N13 → 22 → N4 | 11 413 |
| 44 | 21 | N6 → 25 → N8 → 2 → N11 | 11 178 |
| 45 | 31 | N3 → 12 → N9 → 15 → N10 | 5 444 |
| 46 | 26 | N3 → 12 → N9 → 15 → N10 | 16 570 |
| 47 | 21 | N3 → 11 → N9 → 30 → N10 | 9 383 |
| 48 | 34 | N5 → 20 → N12 → 19 → N13 → 22 → N4 | 25 000 |

degradation of “noble” products to satisfy the demand for other “less noble” products and to store products in tanks previously indicated for other products. The planning model also manages the usage rate of pipelines. A balanced operation is to be

Table 10. Difference between Planning and Sequenced (vu)

| p | r | dif ⁺ |
|----|------------------------------------|------------------|
| 10 | N3 → 10 → N9 → 8 → N5 | 397 |
| 20 | N3 → 11 → N9 → 30 → N10 | 2142 |
| 21 | N3 → 11 → N9 → 30 → N10 | 1060 |
| 21 | N3 → 13 → N9 → 9 → N12 → 19 → N13 | 277 |
| 21 | N8 → 2 → N11 | 1493 |
| 25 | N6 → 24 → N8 → 21 → N4 | 2080 |
| 31 | N4 → 1 → N8 → 24 → N6 | 2421 |
| 34 | N8 → 21 → N4 | 2032 |
| 34 | N5 → 20 → N12 → 19 → N13 → 22 → N4 | 973 |
| p | r | dif ⁺ |
| 6 | N4 → 1 → N8 → 24 → N6 | 133 |
| 14 | N10 → 14 → N9 → 8 → N5 | 5 |
| 18 | N3 → 11 → N9 → 17 → N12 → 18 → N14 | 1530 |
| 21 | N5 → 7 → N12 → 19 → N13 | 1048 |
| 27 | N4 → 23 → N13 | 1389 |
| 27 | N8 → 26 → N6 | 444 |

obtained since the aim of the company is to avoid high as well as low level pipeline usage.

The second model developed, the MILP assignment and sequencing model, uses the results of the planning model as input parameters. The total volume determined by the planning model is split into operational batches that are transported during the horizon. In addition, the last model indicates a sequencing list. When applying this model to the real-case pipeline network, the computational results indicate an elevated computational burden even considering weekly scenarios. High optimality gaps were obtained, informing that further work is still required. When the results were analyzed, important operational insights were obtained, which were well accepted by the company operators since they describe the real operation. However, as future developments, decomposition strategies and/or heuristic procedures are to be explored to reduce the computational burden and improve the results of this block.

As an extension of the current work, solutions considering financial costs in the objective function should also be explored. This however, in our experience, may be very hard to obtain due to the existing lack of information from real-case situations.

AUTHOR INFORMATION

Corresponding Author

*E-mail: neves@utfpr.edu.br. Tel.: +55 41 33104701.

Notes

The authors declare no competing financial interest.

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NOMENCLATURE

Indices/Sets

D = set of pipelines, where $d \in D$

N = set of network nodes, where $n, n', m, m' \in N$

P = set of products, where $p \in P$

R = set of routes, where $r, r' \in R$

B = set of batches, where $b \in B$

PLM = sparse set containing the tuple (p, n) , which is used to model surge tank operations for product p in area n

RD = sparse set containing the tuple (r, n, d) ; node n is the origin of pipeline d contained in the route r

RRD = sparse set containing the tuple (r, r', d) , which indicates routes r and r' that use pipeline d in different directions; it is used to model reverse flow constraints

General Parameters

M = large value

H = planning horizon (h)

$dem_{n,p}$ = consumption (demand) of product p in area n during the planning horizon (vu)

$disp_{n,p}$ = available quantity of product p in area n (vu) during H

$flt_{n,p}$ = required quantity of product p in area n (vu) during H

$ID_{n,p}^0$ = initial storage of product p in area n (vu)

$ID_{n,p}^{\min}$ = minimum aggregate storage of product p in area n (vu)

$ID_{n,p}^{\max}$ = maximum aggregate storage of product p in area n (vu)

$vb_{r,p}^{\text{med}}$ = medium flow rate of route r to product p (vu/h)

vr_r = sum of volumes of pipelines in route r (vu)

$lots_p^{\min}$ = minimum quantity to be sent of product p (vu)

$prod_{n,p}$ = production of product p in area n during the planning horizon (vu)

MILP Planning Model Parameters

α_d = maximum desired utilization of pipeline d during the planning horizon (%)

β_d = value to be subtracted from α_d if pipeline d is used in both directions (%)

$CP_{n,p}$ = maximum aggregate capacity of product p in area n (vu)

$ID_{n,p}^{\text{goal}}$ = goal aggregate storage of product p in area n (vu)

T^{\min} = minimum quantity for intermediate storage (vu)

MILP Assignment and Sequencing Model Parameters

$Q_{n,n',p,r}$ = quantity of product p to be sent from n to n' using route r (vu); this value is provided by the MILP planning model

γ = weight of objective function to storage limits violation

μ = weight of objective function to min-max strategy for storage percentage

$lots_p^{\max}$ = maximum quantity to be sent of product p (vu)

$vb_{d,p,n}^{\min}$ = minimum flow rate of product p in pipeline d with origin on area n (vu/h)

$vb_{d,p,n}^{\max}$ = maximum flow rate of product p in pipeline d with origin on area n (vu/h)

Continuous MILP Planning Model Variables

$Q_{n,n',p,r}$ = quantity of product p to be sent from n to n' using route r (vu)

$invent_{n,p}$ = storage level of product p in area n (vu)

$vari_d$ = additional utilization rate of pipeline d in relation to α_d (%)

$VCP_{n,p}$ = quantity violated with respect to capacity of product p in area n (vu)

$VID_{n,p}^{\text{goal}}$ = quantity violated with respect to goal storage of product p in area n (vu)

$VID_{n,p}^{\min}$ = quantity violated with respect to minimum storage of product p in area n (vu)

$VID_{n,p}^{\max}$ = quantity violated with respect to maximum storage of product p in area n (vu)

Continuous MILP Assignment and Sequencing Model Variables

$W_{b,n,n',p,r}$ = quantity of product p allocated to batch b , which is sent from n to n' using route r (vu)

$dif_{d,n,n',r}^-$ = lack of pumped product between Q and $\sum_b W$

$dif_{d,n,n',r}^+$ = surplus of pumped product between Q and $\sum_b W$

$ID_{b,n,p}^{\text{orig}}$ = storage of product p in origin area n after sending batch b (vu)

$ID_{b,n,p}^{\text{dest}}$ = storage of product p in destination area n after receiving batch b (vu)

$TI_{b,d,r}^{\text{pump}}$ = start time for pumping batch b to pipeline d using route r (h)

$TI_{b,d,r}^{\text{rec}}$ = start time for receiving batch b from pipeline d using route r (h)

$TF_{b,d,r}^{\text{pump}}$ = Final time for pumping batch b to pipeline d using route r (h)

$TF_{b,d,r}^{\text{rec}}$ = final time for receiving batch b from pipeline d using route r (h)

$tdb_{b,d}$ = auxiliary temporal variable to calculate the pumping time of batch b to pipeline d (h)

$tdr_{b,d}$ = auxiliary temporal variable to calculate the receiving time of batch b from pipeline d (h)

$ao_{b,p,n}$ = lower storage violation of product p on area n after sending batch b (vu)

$do_{b,p,n}$ = upper storage violation of product p on area n after sending batch b (vu)

$ad_{b,p,n}$ = lower storage violation of product p on area n after receiving batch b (vu)

$dd_{b,p,n}$ = upper storage violation of product p on area n after receiving batch b (vu)

$mid_{n,p}$ = smallest storage of product p on destination area n during the available horizon (%)

Binary MILP Planning Model Variables

$bin_{n,n',p,r} = 1$, if the product p is sent from area n to area n' by route r

$rev_{r,r',d} = 1$, if routes r and r' use the pipeline d in both directions

$tanq_{r,r',n,p} = 1$, if the tank of product p in area n is used between routes r and r'

Binary MILP Assignment and Sequencing Model Variable

$ls_{b,n,n',p,r} = 1$, if the batch b of product p is sent from area n to area n' by route r

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