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Production Scheduling in Multiproduct Multistage Semicontinuous Food Processes

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ABSTRACT: This works presents a novel mixed-integer programming framework and a solution strategy for the optimal production scheduling of multiproduct and multistage food process industries, such as ice-cream production facilities, studied in detail. The overall mathematical framework relies on an efficient modeling approach of the sequencing decisions, the integrated modeling of all production stages, and the inclusion of strong valid integer cuts in the formulation. The simultaneous optimization of all processing stages increases the plant production capacity, reduces the production cost for final products, and facilitates interaction among the different departments of the production facility. Several instances of a real-life industrial case study concerning ice-cream production have been solved to optimality to illustrate the applicability and efficiency of the overall modeling and solution approach.

1. INTRODUCTION

Given the competitive environment in which manufacturing firms are operating, it is imperative that they maximize the return from scarce resources. Generally, manufacturers must choose the best set of options among many alternatives for utilizing its scarce resources to satisfy customer demand for product. The problem that describes these decisions include production planning, production scheduling, and supply chain optimization. Production scheduling is certainty important in the process industries, including the production of specialty chemicals, pharmaceuticals, food, and paper, in which material is often produced in campaigns using various batch sizes in shared equipment. Determining the timing and sequence of production campaigns becomes increasingly difficult as manufacturers strive to achieve increased production rates while minimizing total costs. The selection of alternative schedules is invariably limited by constraints, including the availability of materials and labor, production capacities, and lot sizes, as well as regulations such as those that limit equipment usage and the environmental impact of processes. Many diverse approaches, mixed-integer programming (MIP) formulations, and solution algorithms attempting to exploit the characteristics of specific categories of process scheduling problems have been proposed over the past 30 years. However, attempts in the early 1990s have concentrated on more general representations and algorithms that cover wide ranges of process scheduling problems.[1,2]

Multistage batch and semicontinuous processes are commonly used for the production of high-value, low-volume products, such as specialty chemicals and pharmaceuticals, as well as in consumer product and food processing, metal casting, and microelectronics. The sequential structure of such processes implies that there is no batch splitting or mixing (i.e., each batch moves as a discrete entity through the process). For this type of problem, and for a fixed set of batches, the major decisions are the assignment of batches to units, and the sequencing (and corresponding timing) of batches in each unit. Assignment constraints are expressed for each batch in each stage, while the sequencing and timing of batches can be modeled via (i) the employment of a nonuniform time grid in which the scheduling horizon is divided into a set of different slots for each unit, and each batch is assigned to exactly one slot, or (ii) by using sequencing binaries in big-M constraints to enforce a sequence between pairs of batches assigned to the same unit.

Slot-based formulations for the scheduling of multiproduct multistage batch and semicontinuous plants with fixed number of batches have been proposed by Pinto and Grossmann[3] and Lamba and Karimi,[4] whereas the contributions by Méndez et al.[5] and Gupta and Karimi[6] are examples of sequence-based formulations. To address large-scale instances of these problems, Harjunkoski and Grossmann[7] proposed decomposition methods for single and multistage plants by separating the assignment and sequencing subproblems, and using MIP and constraint programming methods. Maravelias[8] presented a decomposition framework where the problem is decomposed into an assignment and a sequencing subproblem, and the latter is solved using tailored-made algorithms. Giannelos and Georgiadis,[9] Neumann et al.,[10] Maravelias and Grossmann,[11] and Roe et al.[12] have also developed specific algorithms that exploit the structure of the problem. Excellent reviews on scheduling and planning of multistage batch and continuous processes can be found in the work of Shah,[13] Kallrath,[14] Méndez et al.,[15] and Maravelias and Sung.[16]

Most production plants in the food processing industry sector combine continuous operations and batch processes in their product processing routes, thus working in semicontinuous production mode, since production is more flexible and equipment can be more efficiently utilized. The literature in the field of production scheduling and planning in the food processing industries is rather poor. Entrup et al.[17] presented three different MIP model formulations, which employ a combination of a discrete and continuous time representation, for scheduling and planning

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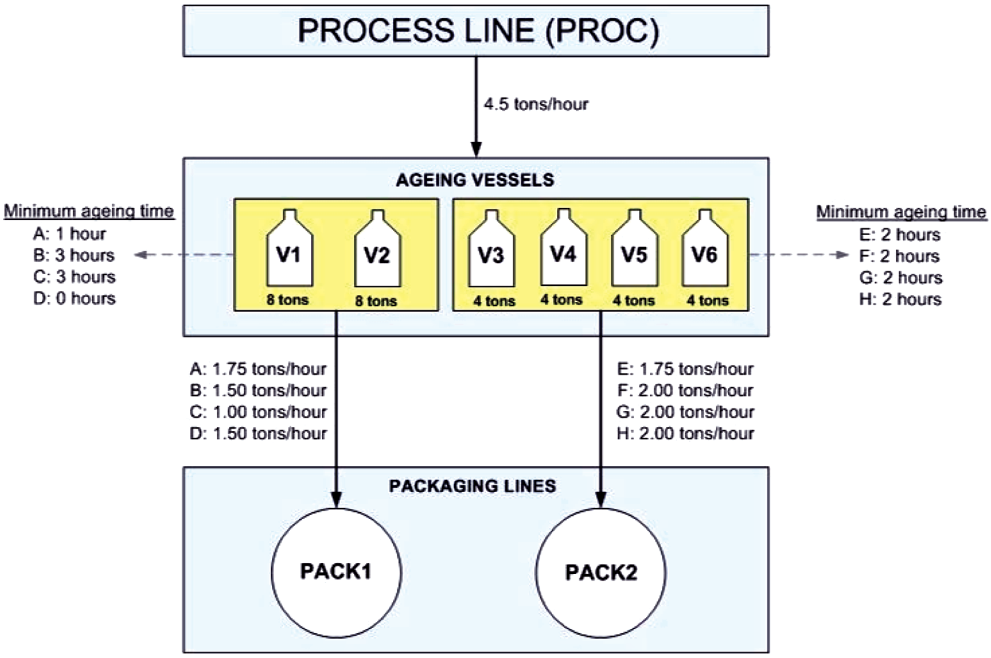

Figure 1. Schematic of the ice-cream production facility.

problems in the packing stage of stirred yogurt production. The authors accounted for shelf life issues and fermentation capacity limitations. However, product changeover times and production costs were ignored. The latter makes the proposed models more appropriate to cope with planning rather than scheduling problems, where product changeover details are crucial. Marinelli et al.[18] addressed the planning problem of 17 products in 5 parallel packing machines, which share resources, in a packing line that was producing yogurt. Their optimization goal was the minimization of inventory, production, and machine setup cost. Sequence-dependent costs and times were not considered. The authors presented a discrete mathematical planning model that failed to obtain the optimal solution of the real application in an acceptable computation time. Thus, they proposed a two-stage heuristic for obtaining near-optimal solutions for the problem under study. Recently, Kopanos et al.[19] developed a mixed discrete/continuous-time MIP formulation for the simultaneous production scheduling and lot-sizing in yogurt production lines that share common resources (e.g., fruit mixers). Although the problem was mainly focused on the packing stage, timing and capacity constraints of the fermentation stage were also included in the model. The overall formulation takes into account sequence-dependent changeover times and costs and production overtimes, as well as typical daily production shutdown and setup times, because of hygienic requirements.

In this work, a novel MIP formulation and an efficient solution strategy are presented to address challenging production scheduling problems in multiproduct multistage food industries. The proposed MIP model is well-suited to a real-life ice-cream production facility; however, it could be also used, with minor modifications, in scheduling problems that arise in other food industries with similar processing features (e.g., yogurt production lines, milk processing plants). To the best of our knowledge, there is no previous work in the literature presenting an exact method for addressing the challenges of the underlying food process scheduling problems.

The manuscript is organized as follows. Section 2 describes the ice-cream production facility under study, and section 3 reviews the typical scheduling practice policy in food industries and discusses its main drawbacks. In section 4, we formally define the production scheduling problem that is addressed in this paper. The proposed MIP formulation for solving these types of scheduling problems is presented in section 5, followed by an efficient two-step solution methodology in section 6. Several problem instances of a real-life ice-cream production facility then are solved in section 7, in order to reveal the computational efficiency of the proposed approach and highlight its practical benefits, once implemented on the industrial plant floor. Finally, concluding remarks are presented in section 8.

2. THE ICE-CREAM PRODUCTION FACILITY

The ice-cream production facility under study, which represents a typical ice-cream factory, was first described by Bongers and Bakker.[20] The plant is based on a three-stage production process, as shown in Figure 1. More specifically, the basic mixes are produced in the main process line (PROC), followed by storage in the aging vessels (V1−V6). After a minimum aging time, the mixes are used for the production of final products (A−H) in two packing lines (PACK1, PACK2). The main process line has a processing rate of 4.5 tons/h and can feed all aging vessels, one at a time. Packing line PACK1 is supplied by two aging vessels (V1 and V2), each with a capacity of 8 tons and the ability to accommodate products A−D, whereas packing line PACK2 can pack products E−H and is supplied by four aging vessels (V3−V6), each with a capacity of 4 tons. Minimum aging times and packing rates for all products can be found in Figure 1. The maximum shelf life for all intermediate mixes in aging stage is 72 h. Sequence-dependent changeover, or simply changeover, operations (mainly cleaning and sterilizing tasks) are performed both in the process and the packing lines whenever the production is changed from one product to a different one. Table 1 provides the necessary sequence-dependent changeover times

Table 1. Changeover Times in the Process Line and the Packing Lines (minutes)

	Changeover Time (min)															
	Process Line								Packing Line							
product	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H
A	0	30	30	30	30	30	30	30	0	60	60	60	0	0	0	0
B	30	0	30	30	30	30	30	30	30	0	60	60	0	0	0	0
C	30	30	0	30	30	30	30	30	30	30	0	60	0	0	0	0
D	30	30	30	0	30	30	30	30	30	30	30	0	0	0	0	0
E	30	30	30	30	0	15	15	15	0	0	0	0	0	60	60	60
F	30	30	30	30	5	0	15	15	0	0	0	0	30	0	60	60
G	30	30	30	30	5	5	0	15	0	0	0	0	30	30	0	60
H	30	30	30	30	5	5	5	0	0	0	0	0	30	30	30	0

for performing these operations in the process and the packing lines. Moreover, a cleaning time of 2 h is needed before shutting down the process line and the packing machines. Finally, the production facility is available for 120 h per week (a 48-h weekend).

3. TYPICAL SCHEDULING PRACTICES IN THE FOOD INDUSTRY

As Bongers and Bakker[20] pointed out, the practical scheduling inside the vast majority of food factories is focused on just scheduling the packing lines. Afterward, the packing lines schedule is "thrown over the wall" to the process department, in which a schedule should be made to meet the packing demand. To go further, this schedule is also "thrown over the wall" to the incoming materials department, in which a schedule is made to order/receive the materials. The current way that the food industries are being scheduled is posing two major problems:

(i) Each department will strive to ensure that is not to blame for not delivering packing products according to the schedule, while less available production capacity will be communicated to the plant management.
(ii) Any change in the packing schedule might lead to an infeasible schedule in the upstream departments. For instance, packing lines may not run because of a lack of intermediate products or unnecessary intermediate products being made on the process plant floor.

Since the above problems are frequently met in relevant industrial environments, the challenge is to appropriately tackle them in an integrated way in order to increase the plant production capacity and reduce the production costs for final products. The gap between scheduling theory and practice is still evident, since most academic developments are too distant from industrial environments. This industrial reality drove Bongers and Bakker[20] to characterize the simultaneous scheduling of all processing stages (i.e., the process line, the aging vessels, and the packing lines) in typical food processing industries as a challenging problem.

4. PROBLEM STATEMENT

This study considers the production scheduling problem of industrial-scale multiproduct multistage semicontinuous processes, similar to the previously described ice-cream production process, with the following features:

(i) A set of product orders $i \in I$ should be processed by following a predefined sequence of processing stages $s \in S$ with processing units $j \in J$ working in parallel.
(ii) The total demand ζ_i for each product order i is divided into a number of batches $b \in B$ that must follow a specific set of processing stages s.
(iii) Product order i can be processed in a specific subset of units $j \in J_i$. Similarly, processing stage s can be processed in a specific subset of units $j \in J_s$.
(iv) Every aging vessel $j \in J_{s2}$ has a maximum capacity $\mu_j^{\max}$. In aging vessels, a product batch should remain for a minimum aging time τ_i^{age}, and no longer than its corresponding shelf life $\varepsilon_i^{\text{life}}$.
(v) Parameter ρ_{ij} denotes the processing and packing rate for every product i in unit $j \in J_i$.
(vi) Sequence-dependent changeover times $\gamma_{ii'j}$ between consecutive product orders are present in the process stage (S1) and the packing stage (S3).
(vii) All model parameters are deterministic.
(viii) Once the processing of an order in a given stage is started, it should be carried out to completion without interruption (i.e., nonpreemptive mode).

The key decision variables are:

(i) the allocation of batch b of product i to units $j \in J_i$ per stage, denoted as Y_{ibsj};
(ii) the relative sequence for any pair of product batches i,b and i',b' in the process line (i.e., stage S1), denoted as $\bar{X}_{ibi'b'}$;
(iii) the relative sequence for any pair of products i and i' in aging vessels (i.e., stage S2) and packing lines (i.e., stage S3) for $j \in (J_i \cap J_{i'} \cap J_s)$, $X_{ii'}$; and
(iv) the starting and completion time of batch b of product i in stage s, denoted by L_{ibs} and C_{ibs}, respectively.

The minimization of makespan constitutes the optimization goal in this work.

4.1. Industrial Production Policy. Generally speaking, in most food processing industries, such as the one studied here, in order to achieve high production levels and minimize switchovers of products, the industrial practice imposes operations of the intermediate storage/processing vessels (e.g., aging vessels, fermentation tanks) in their maximum capacity. Coming back to the ice-cream production facility under study, the fact that the aging vessels operate in maximum capacity allows us to solve the batching problem beforehand, and afterward solve the scheduling problem for the predefined number of product batches.

4.1.1. Minimum Number of Batches. The minimum number of batches $\beta_i^{\min}$ to satisfy the demand for each specific product order i depends on the capacity of the storing/aging vessels in which it can be stored. In the case that these aging vessels have the same capacity $\mu_j^{\max}$, the minimum number of batches is given by

$$\beta_i^{\min} = \frac{\zeta_i}{\mu_j^{\max}} \qquad \text{where } j \in (J_i \cap J_{s_2})$$

4.1.2. Filling Time for Aging Vessels (Processing Time in the Process Line). The time τ_i^{fill} to fill an aging vessel with a product i is calculated by

$$\tau_i^{\text{fill}} = \frac{\mu_j^{\max}}{\rho_{ij'}} \qquad \text{where } j \in (J_i \cap J_{s_2}), j' \in (J_i \cap J_{s_1})$$

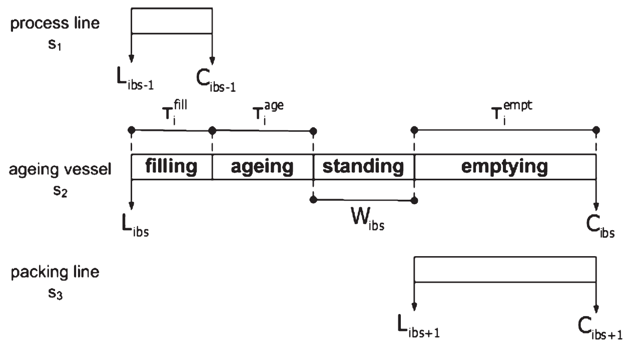

Figure 2. Timing decisions for a product batch *i,b* for every processing stage.

Note that unit $j' \in J_{s1}$ corresponds to the process line and unit $j \in J_{s2}$ corresponds to the aging vessels. It should be also noticed that the above equation is valid *if and only if*: (i) the product *i* can be stored into several *equal-capacity* aging vessels $\mu_j^{\max}$, and (ii) the aging vessels are supplied by the process lines that have the same processing rate $\rho_{ij'}$. Obviously, the filling time for the aging vessels is equal to the processing time in the process line, because of the continuous process mode.

4.1.3. Emptying Time for Aging Vessels (Packing Time in the Packing Lines). The time to empty an aging vessel from a product i (τ_i^{empty}) is calculated as follows:

$$\tau_i^{\text{empt}} = \frac{\mu_j^{\max}}{\rho_{ij'}} \qquad \text{where } j \in (J_i \cap J_{s_2}), j' \in (J_i \cap J_{s_3})$$

Note that unit $j' \in (J_i \cap J_{s3})$ corresponds to the packing line where product *i* can be packed. Once again, this expression is valid *if and only if* the product *i* can be stored into several *equal-capacity* aging vessels μ_j^{max}. Obviously, the emptying time of the aging vessels is equal to the packing time of the packing lines.

5. MATHEMATICAL FORMULATION

In this section, the proposed MIP formulation is presented for the production scheduling of the multiproduct multistage production facility described above. Constraints are grouped according to the type of decision (e.g., assignment, timing, sequencing). To facilitate the presentation of the model, we use uppercase Latin letters for optimization variables and sets and lowercase Greek letters for parameters.

5.1. Unit Allocation Constraints for Any Product Batch in Every Processing Stage. Constraints (1) ensure that each product batch *i,b* goes through one unit $j \in (J_i \cap J_s)$ in each stage *s*.

$$\sum_{j \in (J_i \cap J_s)} Y_{ibsj} = 1 \qquad \forall\ i, b \le \beta_i^{\min}, s \tag{1}$$

5.2. Timing Constraints for a Product Batch in the Same Processing Stage. The timing for a batch *b* of product *i* in each stage *s* is defined by constraint sets (2)−(5) (see Figure 2). In the process stage, the completion time C_{ibs} for a batch *b* of product *i* equals to its starting time L_{ibs} plus the necessary aging vessel filling time τ_i^{fill}, according to constraints (2). In the aging stage, the timing for each product batch *i,b* is given by constraints (3). The standing (waiting) time for a product batch *i,b* in the aging stage is denoted by W_{ibs}. This standing time plus the minimum aging time τ_i^{age} should not exceed the product shelf life, as constraint set (4) ensures. Finally, constraints (5) calculate the timing for any batch *b* of product *i* in the packing stage.

$$L_{ibs} + \tau_i^{\text{fill}} = C_{ibs} \qquad \forall\ i, b \le \beta_i^{\min}, s = 1 \tag{2}$$

$$L_{ibs} + \tau_i^{\text{fill}} + \tau_i^{\text{age}} + W_{ibs} + \tau_i^{\text{empt}} = C_{ibs} \qquad \forall\ i, b \le \beta_i^{\min}, s = 2 \tag{3}$$

$$W_{ibs} \le \varepsilon_i^{\text{life}} - \tau_i^{\text{age}} \qquad \forall\ i, b \le \beta_i^{\min}, s = 2 \tag{4}$$

$$L_{ibs} + \tau_i^{\text{empt}} = C_{ibs} \qquad \forall\ i, b \le \beta_i^{\min}, s = 3 \tag{5}$$

5.3. Timing Constraints for a Product Batch between Consecutive Processing Stages. Constraints (6) and (7) define the timing for every product batch *i,b* between two consecutive processing stages (see Figure 2). Constraints (6) state that the starting time L_{ibs}, for any product batch *i,b*, in the aging stage is equal to the starting time in the process stage, because of the continuous nature of the process stage. Moreover, an aging vessel is free for processing a product only when it is completely empty; therefore, the completion time of a product batch *i,b* stored in an aging vessel equals the completion time of this batch in the packing line, according to constraints (7).

$$L_{ibs} = L_{ibs-1} \qquad \forall\ i, b \le \beta_i^{\min}, s = 2 \tag{6}$$

$$C_{ibs} = C_{ibs-1} \qquad \forall\ i, b \le \beta_i^{\min}, s = 3 \tag{7}$$

5.4. Timing Constraints for Two Batches of the Same Product in the Packing Stage. If the underlying industrial policy requires a single production campaign in the packing stage, without allowing a waiting time between batches of the same product, constraints (8) must be added to the MIP formulation. According to these constraints, the completion time for a product batch *i,b* should be equal to the starting time for the next indexed product batch *i,b* + 1.

$$C_{ibs} = L_{ib+1s} \qquad \forall\ i, b < \beta_i^{\min}, s = 3 \tag{8}$$

5.5. Sequencing Constraints between Product Batches in All Processing Stages. Constraints (9)−(13) define the relative sequencing between two product batches. Constraints (9)−(12) have been formulated as big-*M* constraints, where the available scheduling horizon ω plays the role of the parameter *M*. In addition, our mathematical formulation uses global precedence sequencing variables: (i) for any pair of product batches *i,b* and i', b' ($i < i'$) in the process stage $\bar{X}_{ibi'b'}$, and (ii) for any pair of different products *i* and i' ($i < i'$) both in the aging and the packing stage $X_{ii'}$. Note that the sequencing decisions are the same for the aging and packing stage, and they are modeled for both stages through a single binary variable $X_{ii'}$ for a given pair of products. Constraints (9) enforce the starting time of a product batch i',b' to be greater than the completion time of any product batch *i,b* processed beforehand plus the corresponding sequence-dependent changeover time $\gamma_{ii'j}$, when both batches are assigned to the same process line. Similarly, constraint set (10) describes the opposite case. In a similar manner, constraints (11) and (12) define the sequencing between any pair of different products i and $i' > i$ in the aging and the packing stage. Finally, to avoid symmetric solutions, if two batches b and $b' > b$ of the same product i are assigned to the same unit, we assume that the lower-indexed batch b is performed first, according to constraints (13).

$$L_{i'b's} \geq C_{ibs} + \gamma_{ii'j} - \omega(1 - \bar{X}_{ibi'b'}) - \omega(2 - Y_{ibsj} - Y_{ib'sj})$$
$$\forall i, b \leq \beta_i^{\min}, i', b' \leq \beta_{i'}^{\min}, s, j \in (J_i \cap J_{i'} \cap J_s) : i < i', s = 1 \quad (9)$$

$$L_{ibs} \geq C_{i'b's} + \gamma_{i'ij} - \omega\bar{X}_{ibi'b'} - \omega(2 - Y_{ibsj} - Y_{ib'sj})$$
$$\forall i, b \leq \beta_i^{\min}, i', b' \leq \beta_{i'}^{\min}, s, j \in (J_i \cap J_{i'} \cap J_s) : i < i', s = 1 \quad (10)$$

$$L_{i'b's} \geq C_{ibs} + \gamma_{ii'j} - \omega(1 - X_{ii'}) - \omega(2 - Y_{ibsj} - Y_{ib'sj})$$
$$\forall i, b \leq \beta_i^{\min}, i', b' \leq \beta_{i'}^{\min}, s, j \in (J_i \cap J_{i'} \cap J_s) : i < i', s > 1 \quad (11)$$

$$L_{ibs} \geq C_{i'b's} + \gamma_{i'ij} - \omega X_{ii'} - \omega(2 - Y_{ibsj} - Y_{ib'sj})$$
$$\forall i, b \leq \beta_i^{\min}, i', b' \leq \beta_{i'}^{\min}, s, j \in (J_i \cap J_{i'} \cap J_s) : i < i', s > 1 \quad (12)$$

$$L_{ib's} \geq C_{ibs} - \omega(2 - Y_{ibsj} - Y_{ib'sj})$$
$$\forall i, b \leq \beta_i^{\min}, b' \leq \beta_i^{\min}, s, j \in (J_i \cap J_s) : b < b' \quad (13)$$

5.6. Objective Function. The time point at which all product orders are accomplished corresponds to the makespan. The makespan objective is closely related to the throughput objective. For instance, minimizing the makespan in a parallel-machine environment with changeover times forces the scheduler to balance the load over the various machines and to minimize the sum of all the setup times in the critical bottleneck path. Moreover, the minimization of makespan probably leads to a maximization of production at a midterm planning level.

$$\min C_{\max} \geq C_{ibs} \qquad \forall i, b \leq \beta_i^{\min}, s = 3 \quad (14)$$

5.7. Tightening Constraints. To reduce the computational effort, constraints (15) can further tighten the mathematical formulation by imposing a lower bound on the makespan objective. Note that parameter $\phi_j^{\min}$ represents the minimum waiting time to begin using packing line $j \in J_{s_3}$. Obviously, $\phi_j^{\min}$ depends on the minimum filling time for aging vessels $j' \in J_{s_2}$ that are connected to packing line j. In addition, parameter $\alpha_j^{\min}$ stands for the minimum number of products that should be assigned to packing line j to ensure full demand satisfaction. Finally, parameter $\gamma_j^{\min}$ denotes the minimum changeover time between two different products in packing line j.

$$C_{\max} \geq \phi_j^{\min} + (\alpha_j^{\min} - 1)\gamma_j^{\min} + \sum_{i \in I_j} \tau_i^{\text{empt}} \beta_i^{\min} \qquad \forall s, j \in J_s : s = 3 \quad (15)$$

The proposed MIP model can be further tightened by correlating the relative sequence variables of the process stage (S1) and the packing stage (S3). Constraints (16) describe these valid integer cuts by forcing the relative sequence between products i and $i' > i$ in packing and aging stages to maintain the same for the product batches i,b and i',b' in the process stage; for products i and i' that share the same packing line. In other words, if product i is assigned before product i' to packing unit $j \in (J_i \cap J_{i'} \cap J_{s_3})$, constraint set (16) drives all the batches of product i to be allocated to the process line before any batch of product i'.

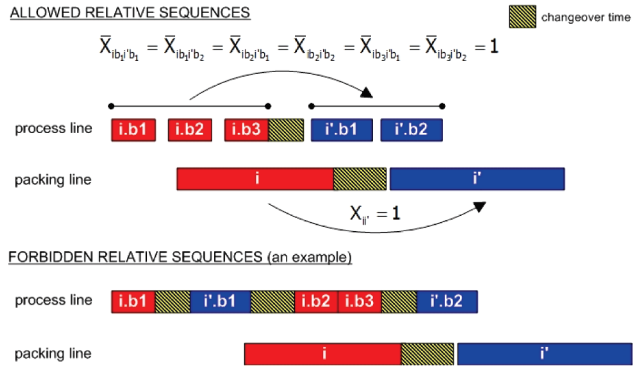

Figure 3. Illustrative example: relative sequences according to constraints (16).

$$\bar{X}_{ibi'b'} = X_{ii'} \quad \forall i, b \leq \beta_i^{\min}, i', b' \leq \beta_{i'}^{\min}, s, j \in (J_i \cap J_{i'} \cap J_s),$$
$$j' \in (J_i \cap J_{i'} \cap J_{s+2}) : i < i', s = 1 \quad (16)$$

Figure 3 graphically illustrates the role of these constraints.

5.8. Integrality and Nonegativity Constraints. The domains of all decision variables are defined as follows:

$$\begin{aligned}
&Y_{ibsj} \in \{0, 1\} && \forall\, i, b \leq \beta_i^{\min}, s, j \in (J_i \cap J_s)\\
&\bar{X}_{ibi'b'} \in \{0, 1\} && \forall\, i, b \leq \beta_i^{\min}, i', b' \leq \beta_{i'}^{\min},\\
& && s, j \in (J_i \cap J_{i'} \cap J_s) : i < i', s = 1\\
&X_{ii'} \in \{0, 1\} && \forall\, i, i', s, j \in (J_i \cap J_{i'} \cap J_s) : i < i', s > 2\\
&L_{ibs}, C_{ibs} \geq 0 && \forall\, i, b \leq \beta_i^{\min}, s\\
&W_{ibs} \geq 0 && \forall i, b \leq \beta_i^{\min}, s = 2
\end{aligned} \quad (17)$$

The overall MIP formulation consists of constraint sets (1)−(17).

6. PROPOSED SOLUTION METHODOLOGY

In this section, a solution methodology is presented for solving the scheduling problem under study efficiently. Before explaining the proposed solution technique, the following points should be taken into consideration: (i) final products can be packed into a specific packing line, (ii) the intermediates of final products that are packed into the same packing line can be stored into the same equal-capacity aging vessels, and (iii) full demand satisfaction is imposed.

In accordance with the above-mentioned points, a lower bound for the makespan for every packing line $\xi_j^{\min}$ is calculated as follows:

$$\xi_j^{\min} = \phi_j^{\min} + \gamma_j^{\text{total}} + \sum_{i \in I_j} \tau_i^{\text{empt}} \beta_i^{\min} \qquad \forall s, j \in J_s : s = 3$$

where parameter γ_j^{total} represents the minimum total changeover time in packing line j. In addition, we define the subset $J^{\min}$, which contains the packing line that appears to have the highest $\xi_j^{\min}$

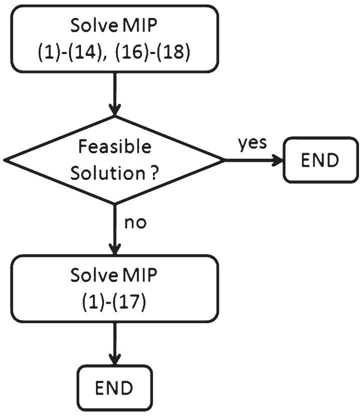

Figure 4. Proposed solution methodology.

value. By doing this, it can be safely assumed that the makespan $C_{\max}$ is equal to the makespan of the packing line $j \in J^{\min}$, as follows:

$$C_{\max} = \phi_j^{\min} + \gamma_j^{\text{total}} + \sum_{i \in I_j} \tau_i^{\text{empt}} \beta_i^{\min} \qquad \forall\ s, j \in (J_s \cap J^{\min}) : s = 3 \tag{18}$$

Therefore, constraints (15) can be replaced by constraints (18), which are tighter. Note that constraints (18) provide the minimum possible makespan, and their incorporation into the MIP model may violate some of the remaining constraint, thus leading to infeasible solutions. This may happen when the packing line $j \in J^{\min}$ is not the bottleneck and the timing decisions are dependent on the previous processing stages. In this case, only constraints (15) should be used.

As Figure 4 demonstrates, the proposed solution methodology can be distinguished into two steps:

Table 2. Demands for Final Products for All Problem Instances (kg)

	Demand for Final Products (kg)									
product i	PI.01	PI.02	PI.03	PI.04	PI.05	PI.06	PI.07	PI.08	PI.09	PI.10
A	80 000	48 000	32 000	8 000	88 000	16 000	8 000	16 000	48 000	8 000
B	48 000	56 000	32 000	32 000	16 000	16 000	8 000	40 000	24 000	72 000
C	32 000	16 000	40 000	64 000	24 000	16 000	96 000	32 000	56 000	8 000
D	8 000	48 000	32 000	24 000	40 000	88 000	8 000	56 000	16 000	72 000
E	112 000	80 000	32 000	52 000	12 000	24 000	116 000	36 000	8 000	80 000
F	12 000	44 000	60 000	44 000	48 000	24 000	64 000	40 000	92 000	80 000
G	48 000	12 000	44 000	88 000	64 000	104 000	4 000	60 000	20 000	4 000
H	24 000	64 000	80 000	32 000	84 000	52 000	4 000	60 000	88 000	32 000

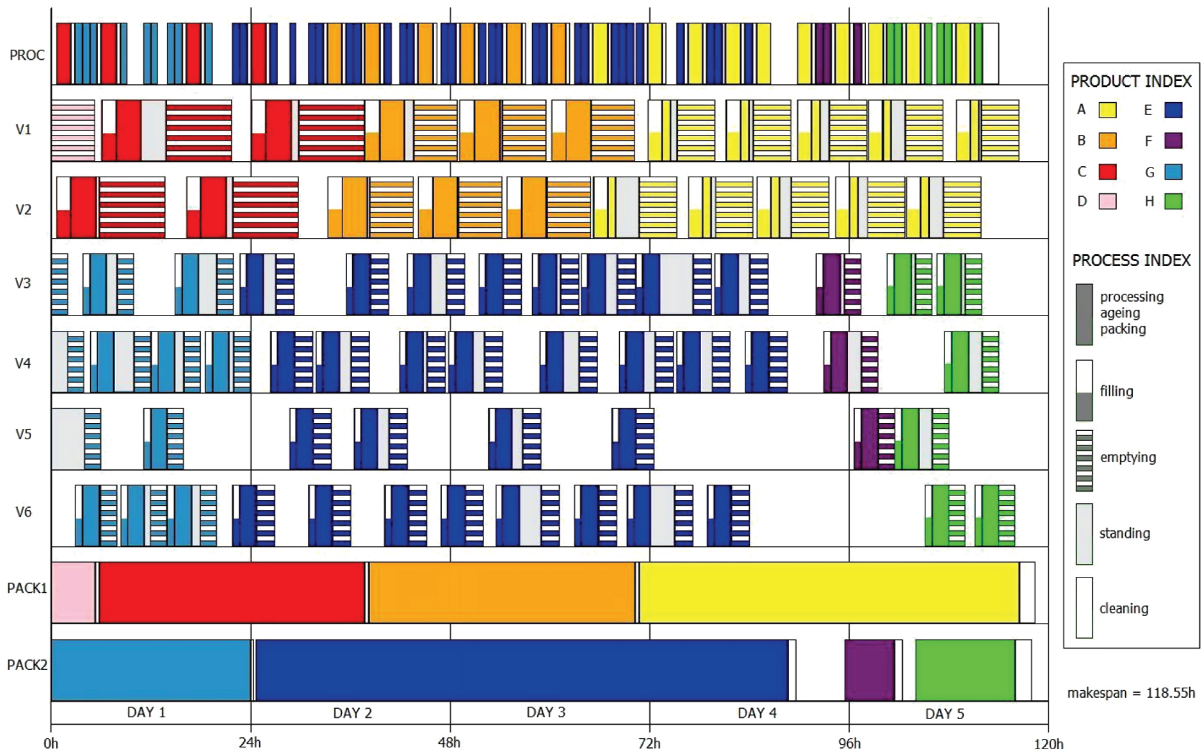

Figure 5. Optimal production schedule for problem instance PI.01 (minimization of makespan).

(1) Solve the MIP model consisting of constraints (1)−(14), and (16)−(18). The solution method terminates, if a feasible solution is obtained; otherwise, go to step 2.
(2) Solve the MIP formulation consisting of constraints (1)−(17).

7. INDUSTRIAL CASE STUDY

A real-life industrial case study, as described in section 2, is used to illustrate the applicability and the efficiency of the proposed scheduling approach and solution strategy. A total set of 10 different problem instances, regarding the demands of final products, have been solved. Roughly speaking, final products are characterized by the very high demands given in Table 2. All problem instances have been solved using a Dell Inspiron 1520 2.0 GHz computer with 2 GB RAM, using CPLEX 11 via a GAMS 22.8 interface.[21]

7.1. Problem Instance PI.01. In this problem instance, some decisions in the beginning of the production week of interest have been taken at the end of the previous production week. More specifically, product batches D.b1, G.b1, G.b2, and G.b3 have already passed from the process line and assigned to aging vessels V1, V3, V4, and V5, respectively. Moreover, these product batches have already allocated to the aging process at the beginning of the time horizon (t = 0), and, as such, they are ready for passing to the packing stage again at t = 0. For this reason, in this example, parameter $\phi_j^{min} = 0$.

The first attempt to solve this scheduling problem was made by Bongers and Bakker[20] by using advance commercial scheduling software. As they have reported, a feasible schedule on all stages could not be derived automatically by applying the available solvers. They finally obtained a feasible schedule (C_{max} = 120 h) via manual interventions. Recently, Subbiah and Engell[22] studied the same ice-cream production plant. They used the framework of timed automata, and they solved the optimization problem using reachability analysis.[23] They did not mention if they considered the overlapping decisions from the previous week schedule. A heuristic methodology was implemented to reduce the model size. A feasible solution (C_{max} = 119 h) was found within 13.13 CPU s; however, it cannot be ruled out that the heuristics that were employed pruned the optimal solution.

The proposed MIP model consists of 15 848 equations, 491 continuous variables, and 2024 binary variables. The optimal solution was reached within just 1.83 CPU s, despite the fact of having a challenging (very high) total demand for the final products. The optimal production schedule, which is illustrated in Figure 5, results into a makespan of 118.55 h. Table 3 shows the breakdown of the utilization of the available scheduling time in the process and packing lines. The processing line is utilized for both processing and cleaning 80.89% of the available time, compared to a food industry standard of 70%. PACK1 and PACK2 operate at 98.79% and 92.08% of the total available time, respectively, including both packing and cleaning. The high total demand explains the high utilization in the process and the packing lines. Packing lines illustrate low total changeover times. As expected, in the process line total cleaning times are higher, since changeovers for batches of different products are more frequent.

At this point, it is worth mentioning that, in the feasible schedule reported by Bongers and Bakker,[20] the process line is utilized 90% (which is 9.11% higher than that of the optimal schedule of this work) of the available time, thus resulting in higher production costs (because of higher changeover costs), compared to the proposed optimal production schedule, considering the fact that changeover costs are proportional to changeover times. Furthermore, note that the makespan of the schedule reported by Bongers and Bakker[20] is 1.45 h higher than the makespan of the optimal schedule (reported in this work).

In general, it should be mentioned that solution strategies that do not optimally integrate the scheduling of all processing stages (i.e., process line, aging vessels, and packing lines) face the risk of not generating optimal solutions. In this specific (high-demand) case study, these solution strategies probably cannot give a feasible schedule. In other words, they may propose solutions where full demand satisfaction is not achieved inside the available production horizon. Manual intervention may still be necessary in order to obtain feasible (i.e., full demand satisfaction), and probably not optimal, production schedules.[20]

7.2. Problem Instances PI.02−PI.10. In problem instances PI.02−PI.10, we consider no overlapping decisions from the previous week schedule. This fact allows us to predefine the relative sequence for products in each packing line, taking into account the sequence-dependent changeover times included in Table 1. It can be observed that the optimal relative sequence, with respect to the minimization of changeover times in PACK1,

Table 3. Process Line and Packing Lines Utilization Breakdown for Problem Instance PI.01

processing unit	unit operation	time (h)	operation utilization (%)	total unit utilization (%)
PROC	processing	76.48	63.73	80.89
	cleaning	20.58	17.15	
	idle	22.94	19.11	
PACK1	packing	115.05	95.88	98.79
	cleaning	3.50	2.92	
	idle	1.45	1.21	
PACK2	packing	106.00	88.33	92.08
	cleaning	4.50	3.75	
	idle	9.50	7.92	

Table 4. Makespan[a] and Computational Features for All Problem Instances

	PI.01	PI.02	PI.03	PI.04	PI.05	PI.06	PI.07	PI.08	PI.09	PI.10
constraints	15 848	17 924	19 723	19 316	19 043	16 814	13 133	16 999	17 655	16 563
continuous variables	491	509	509	502	523	481	446	481	502	495
binary variables	2024	2449	2471	2374	2556	2100	1641	2248	2264	2240
C_{max}	118.55	118.04	116.67	118.1	116.9	110.1	116.52	110.42	115.37	113.85
nodes	0	0	467	751	0	510	0	0	0	0
time (CPU s)	1.83	0.88	23.2	51.41	1.22	15.7	0.39	0.58	0.54	0.75

[a] Makespan includes 2 h of cleaning before shutting down the packing lines.

is D → C → B → A and, in PACK2, is H → G → F → E. That means that $X_{iD} = X_{BC} = X_{AC} = X_{AB} = 0$ and $X_{iH} = X_{FG} = X_{EG} = X_{EF} = 0$ in PACK1 and PACK2, respectively.

Table 4 presents the optimal makespan and computational characteristics for all problem instances. The proposed MIP formulation in tandem with the proposed solution methodology results in very low computational times for all cases. It is noted that 7 of 10 problem instances have been solved within <2 CPU s. These problem instances have been solved in the first step of the proposed solution method. It is worthwhile to note that zero nodes were explored for these problems. Also, notice that the remaining problem instances, which were infeasible in the first step of our solution method, have been solved within <1 CPU min in the second step of the solution method. Despite the complexity of the scheduling problems addressed in this work, all problem instances have been solved to optimality with low computational effort.

8. CONCLUDING REMARKS

In this work, a novel mathematical programming framework and an efficient solution approach have been proposed for the production scheduling in food process industries similar to an ice-cream production facility (studied in detail). This model can easily be the core element of a computer-aided advanced scheduling and planning system in order to facilitate decision-making in relevant industrial environments. As the challenging case study reveals, the proposed approach features a salient computational performance, because of the efficient modeling approach of the sequencing decisions and the strong valid integer cuts that are introduced. However, it should be mentioned that, in extremely large-scale scheduling problems, potentially involving hundreds of products, the proposed MIP model may result in huge model sizes that are difficult to solve within a reasonable (acceptable) computational time. In that case, the proposed mathematical formulation can be easily used as the core MIP model in the MIP-based decomposition strategy recently proposed by Kopanos et al.,[24] in an attempt to make it attractive for the solution of complex large-scale industrial scheduling problems. Finally, note that the proposed MIP model is well-suited to a real-life ice-cream production facility; however, it could be also used, with minor modifications, in scheduling problems that arise in other semicontinuous industries with similar processing features (e.g., yogurt production lines, milk processing plants).

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■ NOMENCLATURE

Indices/Sets

$b, b' \in B$ = product batches (batches)
$i, i' \in I$ = product orders (products)
$j, j' \in J$ = processing units (unis)
$s \in S$ = processing stages (stages)

Subsets

I_j = products i that can be processed in unit j
J_i = available units j to process product i
J_s = available units j to process stage s
J^{min} = packing line that appears the highest lower bound for unit makespan

Parameters

α_j^{min} = minimum number of products assigned to packing line $j \in J_{s3}$
β_i^{min} = minimum number of batches for product i
$\gamma_{ii'j}$ = sequence-dependent changeover time between orders i and i' in unit $j \in (J_i \cap J_{i'})$
γ_j^{min} = minimum sequence-dependent changeover time between two different products in packing line $j \in J_{s3}$
γ_j^{total} = minimum total sequence-dependent changeover time in packing line $j \in J_{s3}$
ε_i^{life} = shelf life for product i in aging vessels
ζ_i = demand for product i
μ_j^{max} = maximum capacity of aging vessel $j \in J_{s2}$
ξ_j^{min} = lower bound for the makespan for packing line $j \in J_{s3}$
ρ_{ij} = processing rate for every product i in the process line $j \in (J_i \cap J_{s1})$ and the packing lines $j \in (J_i \cap J_{s3})$
τ_i^{age} = minimum aging time for product i
τ_i^{empt} = emptying time of aging vessel for product i
τ_i^{fill} = filling time of aging vessel for product i
ϕ_j^{min} = minimum wait time to begin using packing line j
ω = available scheduling horizon

Continuous Variables

C_{ibs} = completion time for stage s of batch b of product i
C_{max} = makespan
L_{ibs} = starting time for stage s of batch b of product i
W_{ibs} = standing (waiting) time for stage s of batch b of product i

Binary Variables

$X_{ii'}$ = for the aging vessels and the packing lines; $X_{ii'} = 1$ if product i is processed before product i'
$\bar{X}_{ibi'b'}$ = for the process line; $\bar{X}_{ibi'b'} = 1$ if batch b of product i is processed before batch b' of product i'
Y_{ibsj} = 1 if stage s of batch b of product i is assigned to unit j

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