T = temperature, K
U_{mf} = minimum fluidizating velocity, m/s

Greek Letters

γ = specific weight of particles minus their specific buoyancy, $g(\rho_S - \rho_F)$
δ = relative deviation, $(U_{exptl} - U_{comp})/U_{exptl}$
δ_{max} = maximum relative deviation
$\bar{\delta}$ = mean relative deviation
ϵ_{mf} = bed voidage at incipient fluidization
$\bar{\epsilon}_{mf}$ = mean bed voidage at incipient fluidization
ψ = particle sphericity
$\bar{\psi}$ = mean particle sphericity
ρ_F = fluid density, kg/m^3
ρ_S = solid density, kg/m^3
μ_F = fluid viscosity, N s/m^2, Pa s

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Simple Method for Tuning SISO Controllers in Multivariable Systems

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A simple, practical approach to the problem of finding reasonable controller settings for the *N* single-input-single-output controllers in an *N*th-order typical industrial multivariable process is presented. The procedure is a straight-forward extension of the familiar Nyquist method and requires only nominal computing power. The method has been tested on ten multivariable distillation column examples from the literature, varying from 2 × 2 systems up to 4 × 4 systems. The settings determined by the method gave reasonable dynamic responses that were comparable to the empirical settings reported by the original authors.

The occurrence and importance of multivariable control has increased rapidly in the petroleum and chemical industries in recent years due to redesign of many processes to reduce energy costs. Much more extensive use of heat integration and complex processing configurations has resulted in processes that are more interacting and interconnected. Several genuine 4 × 4 multivariable control problems have been reported (Doukas and Luyben, 1978; Elaahi and Luyben, 1985; and Alatiqi, 1985). Orgunnaike and Ray (1979) and Tyreus (1982) have presented 3 × 3 problems.

There has been much work in the academic community on the design of multivariable control systems. A decade ago, most of this work was based on optimal control (LQ) methods. The end product of these techniques was a full-blown multivariable controller in which every input affected all outputs. Another approach was the use of decouplers or precompensators to remove the interaction among the loops. Rosenbrock's INA method is of this nature.

Recent work has used models of the process and desired performance trajectories to back-calculate the multivariable controller structure. Typical of these methods, which have their origins in the minimal prototype controllers of the 1950s, is Morari's "Internal Model Control" (IMC), Ray's "General MultiDeadtime Compensator" (GMDC) and Cutler's "Dynamic Matrix Control" (DMC). All these methods lead to full multiinput, multioutput controllers. Waller (1984) has compiled a useful summary of most of the multivariable work in distillation.

Despite this flurry of academic work, very few multivariable controllers have been reported in use in industry. Tyreus (1979) presented one of the rare industrial examples. Some of the reasons for the lack of commercial applications include complexity, excessive engineering manpower requirements, lack of robustness and integrity, and operator nonacceptance.

Therefore, most multivariable processes are still controlled by simple two-mode (PI) single-input-single-output (SISO) controllers. This standard configuration remains the bench mark against which all the more complex controller designs must be compared.

There are two major questions to be answered when dealing with this system of SISO controllers. The first problem is to decide what controlled variable should be controlled by what manipulated variable. This "variable pairing" problem has received considerable attention, particularly by the proponents of the RGA method. McAvoy (1984a) summarized much of this work. The RGA has been found to give some guidance for the variable pairing problem in some systems, but a number of workers have reported poor results. Both Tyreus (1984) and McAvoy (1984b) have recently suggested that the effects of the load or disturbance variables must also be considered in deciding the best variable pairing.

Once the variables have been paired, the second problem to be solved is the tuning of the SISO controllers. If PI controllers are used, there are $2N$ tuning parameters to be

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selected. The gains and reset times must be specified so that the overall multivariable system is stable and gives good setpoint and load response. It is also desirable that the system have "integrity"; i.e., the system remains stable if any combination of controllers is put on manual.

Niederlinski (1971) was one of the first to propose a method for controller tuning in these multivariable systems. His method has not gained wide acceptance because of its complexity and some reports of poor performance (Waller, 1984).

Scope of This Paper

This paper addresses only the second question: controller tuning. It is assumed that the decisions have already been made as to what controlled variable to pair with each manipulated variable.

The rationale for tackling the tuning problem first is the thought that the various alternative variable pairings must be compared by using some consistent, rational tuning method. Thus, the tuning problem must be solved first. Our studies of the variable pairing problem have been reported in a recent paper (Yu and Luyben, 1985).

The method developed satisfies the objective of arriving at reasonable controller settings with only a small amount of engineering and computational effort. It is not claimed that this method will produce the best results or that some other tuning or controller structure will not give superior performance. What is claimed is that the method is easy to use, is easily understandable by control engineers, and leads to settings that compare very favorably with the empirical settings found by exhaustive and expensive trial-and-error tuning methods used by the original authors of the 10 systems studied.

The proposed tuning method should be viewed in the same light as the classical SISO Ziegler–Nichols method. It gives reasonable settings which provide a starting point for further tuning and a bench mark for comparative studies. It also permits easy calculation of new controller settings in a situation where analyzer deadtimes are changed.

This method is limited to open-loop stable systems in its current form, but extension to open-loop unstable systems is under investigation. The process dynamics must be available in transfer function or frequency response form.

Review of Classical SISO Nyquist Method

Since the proposed multivariable method is directly analogous to the classical Nyquist stability criterion method for SISO systems, it is useful to briefly review this classical technique.

For a SISO system, the closed-loop characteristic equation is

$$1 + G(s)B(s) = 0 \tag{1}$$

where G is the open-loop process transfer function and B is the feedback controller transfer function. A Nyquist (or Bode or Nichols) plot of $G(i\omega)B(i\omega)$ is made as the frequency ω goes from zero to infinity. The number of encirclements of the (−1,0) point is equal to the number of roots of the closed-loop characteristic equations that lie in the right half of the s plane if the open-loop system is stable. Thus, if the (−1,0) point is encircled, the closed-loop system is unstable.

The farther away from the (−1,0) point, the more stable the system. One commonly used measure of the distance of the $G(i\omega)B(i\omega)$ contour from the (−1,0) point is the maximum closed-loop log modulus (L_c^{max}). The closed-loop log modulus is the magnitude of the closed-loop servo-transfer function.

$$L_c = 20 \log \left| \frac{GB}{1 + GB} \right| \tag{2}$$

A commonly used specification is +2 dB for L_c^{max}. A value of the controller gain (K_c) is selected, and L_c is plotted as a function of frequency. The maximum value of L_c is determined, and if it is less than +2 dB, the gain is increased. This procedure is repeated until the biggest log modulus for all frequencies is +2 dB. A Nichols chart can be used alternatively to transform $G(i\omega)B(i\omega)$ information into L_c results.

Proposed Multivariable Tuning Method (BLT)

The system studied is an $N \times N$ multivariable process given in the transfer function form

$$\boldsymbol{X} = \mathbf{G}(s)\boldsymbol{M} \tag{3}$$

where $\boldsymbol{X}$ = vector of the controlled variables X_i, i = 1, 2, ..., N, $\boldsymbol{M}$ = vector of the manipulated variables M_j, j = 1, 2, ..., N, and $\mathbf{G}$ = matrix of the open-loop process transfer functions G_{ij} relating controlled variables X_i to manipulated variables M_j. These transfer functions are typically of the form

$$G_{ij}(s) = \frac{K_{ij}(\tau_{1ij}s + 1)e^{-D_{ij}s}}{(\tau_{2ij}s + 1)(\tau_{3ij}s + 1)(\tau_{4ij}s + 1)} \tag{4}$$

The structure of the control system variable pairing has been determined such that X_i is controlled by M_i using a PI controller $B_i(s)$. Thus, the feedback controller matrix $\mathbf{B}(s)$ is diagonal since we are using N SISO controllers

$$\mathbf{B}(s) = \begin{bmatrix} B_{1(s)} & 0 & 0 & \dots & 0 \\ 0 & B_{2(s)} & 0 & \dots & 0 \\ 0 & 0 & B_{3(s)} & \dots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \dots & B_{N(s)} \end{bmatrix} \tag{5}$$

where

$$B_i(s) = K_{ci}\left(1 + \frac{1}{\tau_{Ii}s}\right) \tag{6}$$

The first step in the proposed procedure is to calculate the Ziegler–Nichols setting for each individual loop. The ultimate gain K_u and ultimate frequency ω_u of each diagonal transfer function $G_{ii}(s)$ are calculated in the classical SISO way. To do this numerically, a value of the frequency is guessed. The phase angle is calculated, and frequency is varied until the phase angle is equal to −180°. This frequency is ω_u. The ultimate gain is the reciprocal of the real part of G_{ii} at ω_u.

Next, a factor F is assumed. Typical values vary from 2 to 5. The gains of all feedback controllers (K_c) are calculated by dividing the Ziegler–Nichols gain (K_{ZN}) by the factor F

$$K_{ci} = \frac{K_{ZN_i}}{F} \tag{7}$$

where

$$K_{ZN_i} = \frac{K_{u_i}}{2.2} \tag{8}$$

The reset times (τ_I) of all controllers are calculated by multiplying the Ziegler–Nichols reset times (τ_{ZN}) by the same factor F

$$\tau_{Ii} = F\tau_{ZN_i} \qquad (9)$$

where

$$\tau_{ZN_i} = \frac{2\pi}{1.2\omega_{ui}} \qquad (10)$$

The F factor can be considered a "detuning" factor which is applied to all loops. The larger the value of F, the more stable the system will be but the more sluggish will be the set-point and load responses. The method outlined below yields a value of F which gives a reasonable compromise between stability and performance in multivariable systems.

The closed-loop system is given by (3) and (5).

$$\boldsymbol{X} = \mathbf{G}\boldsymbol{M} = \mathbf{G}\mathbf{B}(\boldsymbol{X}^{\text{set}} - \boldsymbol{X}) \qquad (11)$$

Solving for $\boldsymbol{X}$ gives

$$\boldsymbol{X} = [\mathbf{I} + \mathbf{G}\mathbf{B}]^{-1}\mathbf{G}\mathbf{B}\boldsymbol{X}^{\text{set}} \qquad (12)$$

Since the inverse of a matrix has the determinant of the matrix in the denominator, the closed-loop characteristic equation of the multivariable system is the scalar equation

$$\det(\mathbf{I} + \mathbf{G}\mathbf{B}) = 0 \qquad (13)$$

If we plot the left side of (13) as a function frequency and look at the encirclements of the origin, we can find the number of right half plane zeros of the closed-loop characteristic equation. In order to make this multivariable plot look just like the SISO scalar Nyquist plot, we subtract one from (13) and look at the encirclements of the (−1,0) point. It is convenient to define a new function $W(s)$.

$$W(s) = -1 + \det(\mathbf{I} + \mathbf{G}\mathbf{B})(s) \qquad (14)$$

W is plotted as a function of frequency. The closer W approaches the (−1,0) point, the closer the multivariable system is to closed-loop instability.

The quantity $W/(1 + W)$ will be similar to the closed-loop servo-transfer function for a SISO loop $GB/(1 + GB)$ (eq 2). Therefore, based on intuition and empirical grounds, we define a multivariable closed-loop log modulus L_{cm}.

$$L_{\text{cm}} = 20 \log \left|\frac{W}{1 + W}\right| \qquad (15)$$

The proposed tuning method is based on varying the factor F until the "biggest log modulus" $(L_{\text{cm}})^{\max}$ is equal to some reasonable number. Thus, the proposed tuning method is called the "biggest log modulus tuning" (BLT).

Now we must determine what is a reasonable value to use for $(L_{\text{cm}})^{\max}$ for different orders of the systems. For a SISO system where $N = 1$, we know from long experience that a value of +2 dB for $(L_{\text{cm}})^{\max}$ gives reasonable time-domain responses for set-point and load disturbances.

When the 10 cases ranging from 2 × 2 to 4 × 4 systems were tested, we have found that the following tuning criterion appears to give good responses.

$$(L_{\text{cm}})^{\max} = 2N \qquad (16)$$

For $N = 1$, this reduces to the classical +2-dB criterion. For a 4 × 4 system, a $(L_{\text{cm}})^{\max}$ of 8 should be used. These empirical findings suggest that the higher the order of the system, the more underdamped the closed-loop system must be to achieve reasonable responses.

This suggested tuning method should be viewed as giving preliminary controller settings which can be used as a bench mark for comparative studies. Note that this procedure guarantees that the system is stable with all controllers on automatic and also that each individual loop will be stable if all others are on manual. Thus, a portion of the integrity question is automatically answered. However, further checks of stability would have to be made for other combinations of manual/automatic operation.

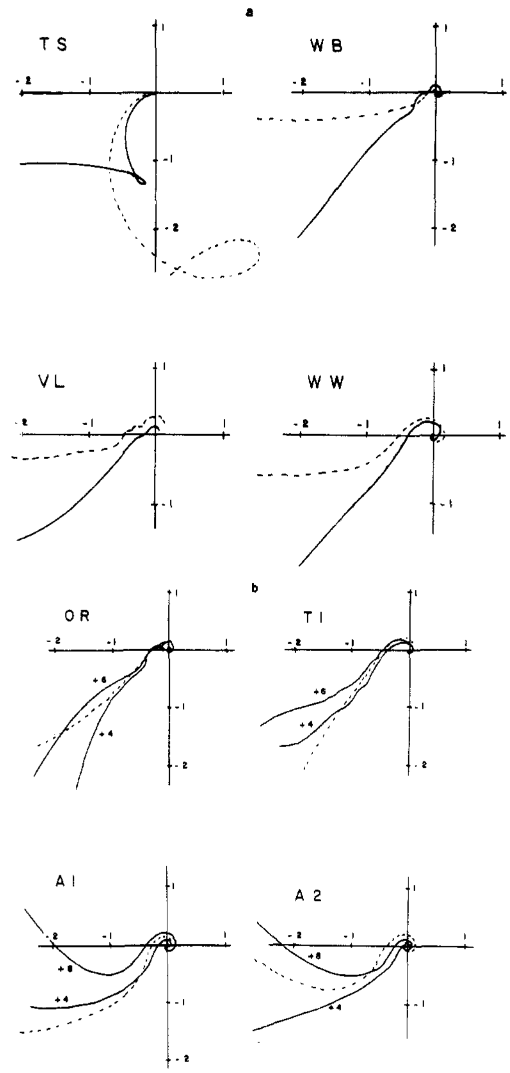

Figure 1. (a) W plots of TS, WB, VL, and WW, and (b) W plots of OR, T1, A1, and A2. Solid lines are BLT settings. Dashed lines are empirical settings.

The method weighs each loop equally; i.e., each loop is detuned by the same factor F. If it is more important to keep tighter control of some variable than others, the method can be easily modified by using different weighting factors for different controlled variables. The less-important loop could be detuned more than the more-important loop.

Cases Studied

Ten distillation systems from the literature were studied.

A. 2 × 2 Systems: (1) Tyreus stabilizer (1979)—TS; (2) Wood and Berry (1973)—WB; (3) Vinante and Luyben (1972)—VL; and (4) Wardle and Wood (1969)—WW.

B. 3 × 3 Systems: (5) Orgunnaike and Ray (1979)—OR; (6) Tyreus case 1 (1982)—T1; and (7) Tyreus case 4 (1982)—T4.

C. 4 × 4 Systems: (8) Doukas and Luyben (1978)—DL; (9) Alatiqi case 1 (1985)—A1; and (10) Alatiqi case 2 (1985)—A2.

Tables I–III give all the process transfer functions.

These "real" systems were used, as opposed to dreaming up various transfer function matrices, so that the problems

Table I. Process Open-Loop Transfer Functions of 2 × 2 Systems

	TS (Tyreus stabilizer)	WB (Wood and Berry)	VL (Vinante and Luyben)	WW (Wardle and Wood)
G_{11}	$\frac{-0.1153(10S+1)e^{-0.1S}}{(4S+1)^3}$	$\frac{12.8e^{-S}}{16.7S+1}$	$\frac{-2.2e^{-S}}{7S+1}$	$\frac{0.126e^{-6S}}{60S+1}$
G_{12}	$\frac{0.2429e^{-2S}}{(33S+1)^2}$	$\frac{-18.9e^{-3S}}{21S+1}$	$\frac{1.3e^{-0.3S}}{7S+1}$	$\frac{-0.101e^{-12S}}{(48S+1)(45S+1)}$
G_{21}	$\frac{-0.0887e^{-12.6S}}{(43S+1)(22S+1)}$	$\frac{6.6e^{-7S}}{10.9S+1}$	$\frac{-2.8e^{-1.8S}}{9.5S+1}$	$\frac{0.094e^{-8S}}{38S+1}$
G_{22}	$\frac{0.2429e^{-0.17S}}{(44S+1)(20S+1)}$	$\frac{-19.4e^{-3S}}{14.4S+1}$	$\frac{4.3e^{-0.35S}}{9.2S+1}$	$\frac{-0.12e^{-8S}}{35S+1}$

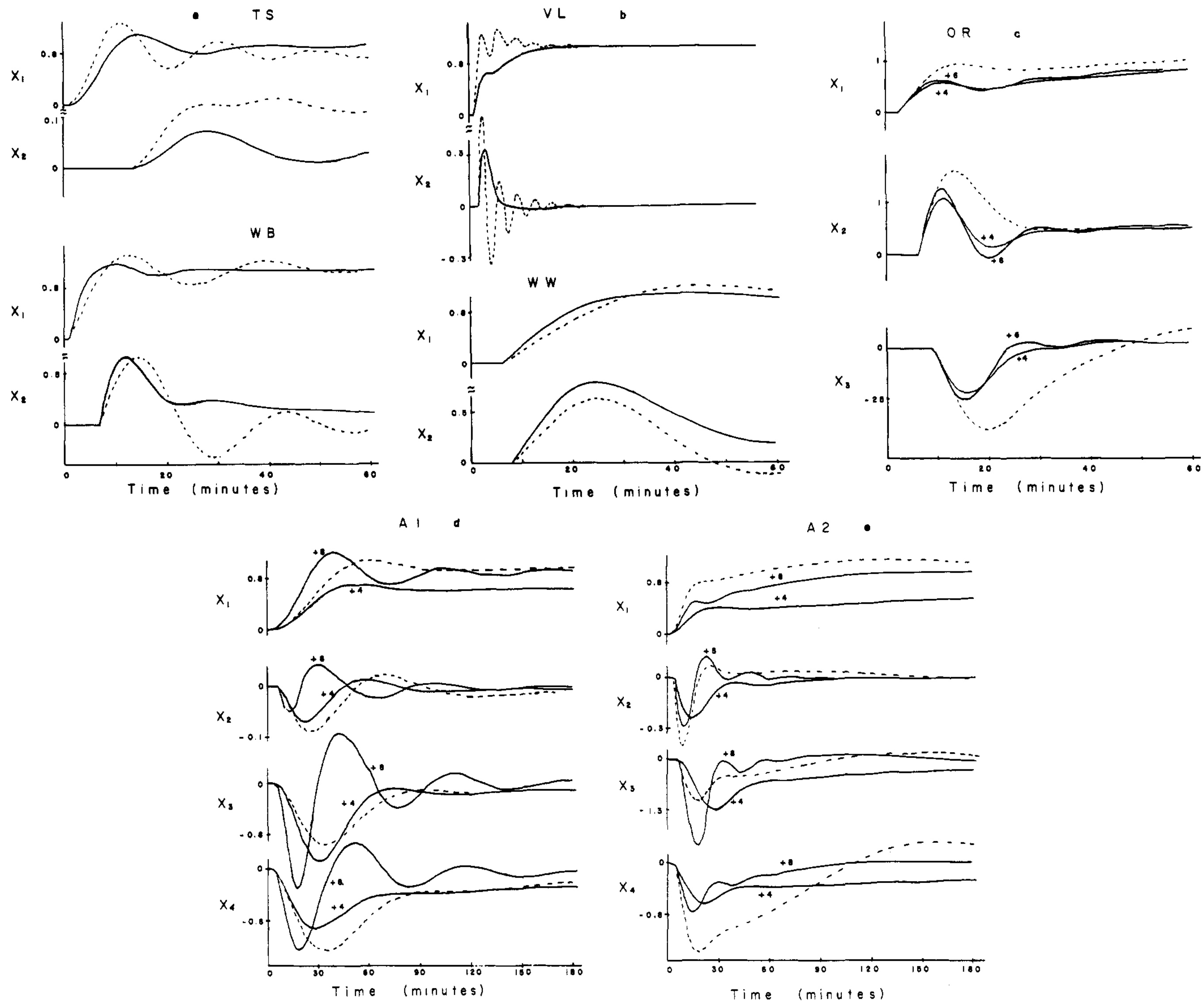

Figure 2. (a) X_1 set-point responses of TS and WB, (b) X_1 set-point response of VL and WW, (c) X_1 set-point responses of OR, (d) X_1 set-point responses of A1, and (e) X_1 set-point responses of A2. Solid lines are BLT settings. Dashed lines are empirical settings.

of some of the unrealistic pathological cases would be avoided.

Results and Discussion

Figure 1 gives Nyquist plots of $W(i\omega)$ for some of the systems with several values of controller settings. The dashed lines are the $W(i\omega)$ plots using the empirical settings reported by the original authors. The solid lines are for BLT tuning.

Tables IV–VI give these empirical control settings, the Ziegler–Nichols settings, the BLT settings (including the F factors), the Niederlinski index, and the values of the diagonal elements of the RGA matrix for all systems.

Figure 2 gives the responses of some of the systems for unit step changes in the set point X_1 set. The dashed lines are those for the original empirical settings. The solid lines are for BLT tuning.

These results show that the BLT settings give quite reasonable responses. They compare quite favorably with the empirical settings found by the original authors. In all cases, the BLT method converged very quickly to settings that worked.

For the 3 × 3 and 4 × 4 systems, two results are shown. One using a +4-dB tuning criterion and another using a

Table II. Process Open-Loop Transfer Functions of 3 × 3 Systems

	OR (Ogunnaike and Ray)	T1 (Tyreus case 1)	T4 (Tyreus case 4)
G_{11}	$\frac{0.66e^{-2.6S}}{6.7S + 1}$	$\frac{-1.986e^{-0.71S}}{66.67S + 1}$	$\frac{-1.986e^{-0.71S}}{66.67S + 1}$
G_{12}	$\frac{-0.61e^{-3.5S}}{8.64S + 1}$	$\frac{5.984e^{-2.24S}}{14.29S + 1}$	$\frac{5.24e^{-60S}}{400S + 1}$
G_{13}	$\frac{-0.0049e^{-S}}{9.06S + 1}$	$\frac{0.422e^{-8.72S}}{(250S + 1)^2}$	$\frac{5.984e^{-2.24S}}{14.29S + 1}$
G_{21}	$\frac{1.11e^{-6.5S}}{3.25S + 1}$	$\frac{0.0204e^{-0.59S}}{(7.14S + 1)^2}$	$\frac{0.0204e^{-0.59S}}{(7.14S + 1)^2}$
G_{22}	$\frac{-2.36e^{-3S}}{5S + 1}$	$\frac{2.38e^{-0.42S}}{(1.43S + 1)^2}$	$\frac{-0.33e^{-0.68S}}{(2.38S + 1)^2}$
G_{23}	$\frac{-0.01e^{-1.2S}}{7.09S + 1}$	$0.513e^{-S}$	$\frac{2.38e^{-0.42S}}{(1.43S + 1)^2}$
G_{31}	$\frac{-34.68e^{-9.2S}}{8.15S + 1}$	$\frac{0.374e^{-7.75S}}{22.22S + 1}$	$\frac{0.374e^{-7.75S}}{22.22S + 1}$
G_{32}	$\frac{46.2e^{-9.4S}}{10.9S + 1}$	$\frac{-9.811e^{-1.59S}}{11.36S + 1}$	$\frac{-11.3e^{-3.79S}}{(21.74S + 1)^2}$
G_{33}	$\frac{0.87(11.61S + 1)e^{-S}}{(3.89S + 1)(18.8S + 1)}$	$\frac{-2.368e^{-27.33S}}{33.3S + 1}$	$\frac{-9.811e^{-1.59S}}{11.36S + 1}$

$+2N$ criterion. The $+2N$ criterion gives faster responses of X_1 to the X_1^{set} changes but results in somewhat larger disturbances to the other variables.

In all cases, the BLT tuning gives stable, reasonable responses.

Figure 3 gives the minimum singular values for some of the cases for empirical tuning and BLT tuning. Doyle and Stein (1981) have shown that the minimum singular values of $[\mathbf{I} + [\mathbf{GB}]^{-1}]$ are reliable measures of the robustness of multivariable systems. The singular values of a matrix are defined as the square root of the eigenvalues of the matrix formed by the product of the matrix and its conjugate transpose.

The computation horsepower required to implement the basic method is quite nominal. The only calculation required is the evaluation of the determinant of a $N \times N$ matrix. No matrix inversion or eigenvalue calculations are required. These calculations are well within the capability of a personal computer.

This determinant evaluation can even be done analytically for $N = 2$ and 3.

for $N = 2$

$$\det(\mathbf{I} + \mathbf{GB}) = 1 + B_1G_{11} + B_2G_{22} + B_1B_2(G_{11}G_{22} - G_{12}G_{21}) \quad (17)$$

for $N = 3$

$$\det(\mathbf{I} + \mathbf{GB}) = 1 + B_1G_{11} + B_2G_{22} + B_3G_{33} + B_1B_2(G_{11}G_{22} - G_{12}G_{21}) + B_2B_3(G_{22}G_{33} - G_{23}G_{32}) + B_1B_3(G_{11}G_{33} - G_{13}G_{31}) + B_1B_2B_3[G_{11}(G_{22}G_{33} - G_{23}G_{32}) + G_{12}(G_{23}G_{31} - G_{21}G_{33}) + G_{13}(G_{21}G_{32} - G_{22}G_{31})] \quad (18)$$

Table III. Process Open-Loop Transfer Functions of 4 × 4 Systems

	DL (Doukas and Luyben)	A1 (Alatiqi case 1)	A2 (Alatiqi case 2)
G_{11}	$\frac{-9.811e^{-1.59S}}{11.36S + 1}$	$\frac{2.22e^{-2.5S}}{(36S + 1)(25S + 1)}$	$\frac{4.09e^{-1.3S}}{(33S + 1)(8.3S + 1)}$
G_{12}	$\frac{0.374e^{-7.75S}}{22.22S + 1}$	$\frac{-2.94(7.9S + 1)e^{-0.05S}}{(23.7S + 1)^2}$	$\frac{-6.36e^{-0.2S}}{(31.6S + 1)(20S + 1)}$
G_{13}	$\frac{-2.368e^{-27.33S}}{33.3S + 1}$	$\frac{0.017e^{-0.2S}}{(31.6S + 1)(7S + 1)}$	$\frac{-0.25e^{-0.4S}}{21S + 1}$
G_{14}	$\frac{-11.3e^{-3.79S}}{(21.74S + 1)^2}$	$\frac{-0.64e^{-20S}}{(29S + 1)^2}$	$\frac{-0.49e^{-5S}}{(22S + 1)^2}$
G_{21}	$\frac{5.984e^{-2.24S}}{14.29S + 1}$	$\frac{-2.33e^{-5S}}{(35S + 1)^2}$	$\frac{-4.17e^{-4S}}{45S + 1}$
G_{22}	$\frac{-1.986e^{-0.71S}}{66.67S + 1}$	$\frac{3.46e^{-1.01S}}{32S + 1}$	$\frac{6.93e^{-1.01S}}{44.6S + 1}$
G_{23}	$\frac{0.422e^{-8.72S}}{(250S + 1)^2}$	$\frac{-0.51e^{-7.5S}}{(32S + 1)^2}$	$\frac{-0.05e^{-5S}}{(34.5S + 1)^2}$
G_{24}	$\frac{5.24e^{-60S}}{400S + 1}$	$\frac{1.68e^{-2S}}{(28S + 1)^2}$	$\frac{1.53e^{-2.8S}}{48S + 1}$
G_{31}	$\frac{2.38e^{-0.42S}}{(1.43S + 1)^2}$	$\frac{-1.06e^{-22S}}{(17S + 1)^2}$	$\frac{-1.73e^{-17S}}{(13S + 1)^2}$
G_{32}	$\frac{0.0204e^{-0.59S}}{(7.14S + 1)^2}$	$\frac{3.511e^{-13S}}{(12S + 1)^2}$	$\frac{5.11e^{-11S}}{(13.3S + 1)^2}$
G_{33}	$\frac{0.513e^{-S}}{S + 1}$	$\frac{4.41e^{-1.01S}}{16.2S + 1}$	$\frac{4.61e^{-1.02S}}{18.5S + 1}$
G_{34}	$\frac{-0.33e^{-0.68S}}{(2.38S + 1)^2}$	$\frac{-5.38e^{-0.5S}}{17S + 1}$	$\frac{-5.48e^{-0.5S}}{15S + 1}$
G_{41}	$\frac{-11.3e^{-3.79S}}{(21.74S + 1)^2}$	$\frac{-5.73e^{-2.5S}}{(8S + 1)(50S + 1)}$	$\frac{-11.18e^{-2.6S}}{(43S + 1)(6.5S + 1)}$
G_{42}	$\frac{-0.176e^{-0.48S}}{(6.9S + 1)^2}$	$\frac{4.32(25S + 1)e^{-0.01S}}{(50S + 1)(5S + 1)}$	$\frac{14.04e^{-0.02S}}{(45S + 1)(10S + 1)}$
G_{43}	$\frac{15.54e^{-S}}{S + 1}$	$\frac{-1.25e^{-2.8S}}{(43.6S + 1)(9S + 1)}$	$\frac{-0.1e^{-0.05S}}{(31.6S + 1)(5S + 1)}$
G_{44}	$\frac{4.48e^{-0.52S}}{11.11S + 1}$	$\frac{4.78e^{-1.15S}}{(48S + 1)(5S + 1)}$	$\frac{4.49e^{-0.6S}}{(48S + 1)(6.3S + 1)}$

Table IV. 2 × 2 Systems

	TS (Tyreus stabilizer)	WB (Wood and Berry)	VL (Vinante and Luyben)	WW (Wardle and Wood)
RGA	4.35	2.01	1.63	2.69
NI	+0.229	+0.498	+0.615	+0.372
empirical				
K_c	−30,30	0.2, −0.04	−2.38, 4.39	18, −24
τ_I	∞	4.44, 2.67	3.16, 1.15	19, 24
L_c	1.74	10.1	13.3	8.4
Z-N				
K_c	−166.2, 706	0.96, −0.19	−2.40, 4.45	59, −28.5
τ_I	2.06, 8.01	3.25, 9.20	3.16, 1.15	19.3, 24.6
L_c	unstable	unstable	13.3	18.5
BLT				
F	10	2.55	2.25	2.15
K_c	−16.6, 70.6	0.375, −0.075	−1.07, 1.97	27.4, −13.3
τ_I	20.6, 80.1	8.29, 23.6	7.1, 2.58	41.4, 52.9

Table V. 3 × 3 Systems

	T1 (Tyreus case 1)	OR (Ogunnaike and Ray)	T4 (Tyreus case 4)
RGA	0.55, 4.98, 5.28	1.96, 1.89, 1.52	1.09, 0.1, 0.1
NI	+0.194	+0.372	+8.52
empirical			
K_c	−20, 0.7, −0.05	1.2, −0.15, 0.6	−20, −7, −0.4
τ_I	25, 25, 150	5, 10, 4	5, 7, 7
L_c	4.84	4.4	unstable
Z-N			
K_c	−33.9, 1.43, −0.5	3.24, −0.63, 5.66	−33.9, −10.6, −0.55
τ_I	2.36, 2.94, 72.8	7.62, 8.36, 3.08	2.36, 4.82, 5.0
BLT + 4			
F	2.38	2.53	4.81
K_c	−14.3, 0.602, −0.21	1.28, −0.251, 2.24	−7.04, −2.2, −0.114
τ_I	5.6, 6.98, 173	19.3, 21.1, 7.78	11.3, 23.2, 24.2
BLT + 6			
F	1.91	2.15	3.0
K_c	−17.8, 0.749, −0.261	1.51, −0.295, 2.63	−11.26, −3.52, −0.182
τ_I	4.5, 5.61, 139	16.4, 18, 6.61	7.09, 14.5, 15.1

Table VI. 4 × 4 Systems

	DL (Doukas and Luyben)	A1 (Alatiqi case 1)	A2 (Alatiqi case 2)
RGA	0.13, 1.09, 0.16, −0.01	3.53, 2.77, 2.26, 1.48	3.11, 4.67, 1.55, 0.85
NI	+21.9	+0.12	+0.101
empirical			
K_c	−0.12, −15, 0.89, 0.2	0.9, 1.2, 1.0, 0.8	1.2, 1.9, 1.3, 0.9
τ_I	7.9, 30, 20, 20	30, 20, 23, 45	23, 17, 15, 32
L_c	4.29	3.22	5.45
Z-N			
K_c	−0.55, −33.9, 2, 3.47	5.13, 6.62, 2.66, 4.55	3.65, 4.59, 2.87, 8.61
τ_I	5.03, 2.36, 2.58, 1.7	32.1, 3.33, 3.29, 12.3	15.62, 3.34, 3.33, 10.1
BLT + 4			
F	6.56	5.42	9.28
K_c	−0.084, −5.16, 0.305, 0.529	0.945, 1.22, 0.491, 0.837	0.393, 0.495, 0.31, 0.927
τ_I	33, 15.5, 17.0, 11.2	174, 18, 17.8, 66.9	145, 31, 31, 93.8
BLT + 8			
F	4.67	2.25	4.0
K_c	−0.118, −7.26, 0.429, 0.743	2.28, 2.94, 1.18, 2.02	0.923, 1.16, 0.727, 2.17
τ_I	23.5, 11, 12.1, 7.94	72.2, 7.48, 7.39, 27.8	61.7, 13.2, 13.2, 40

However, the results should be checked for stability by the characteristic loci plots (which require eigenvalue calculations) and evaluated by time domain simulation (which require reasonable computational speed).

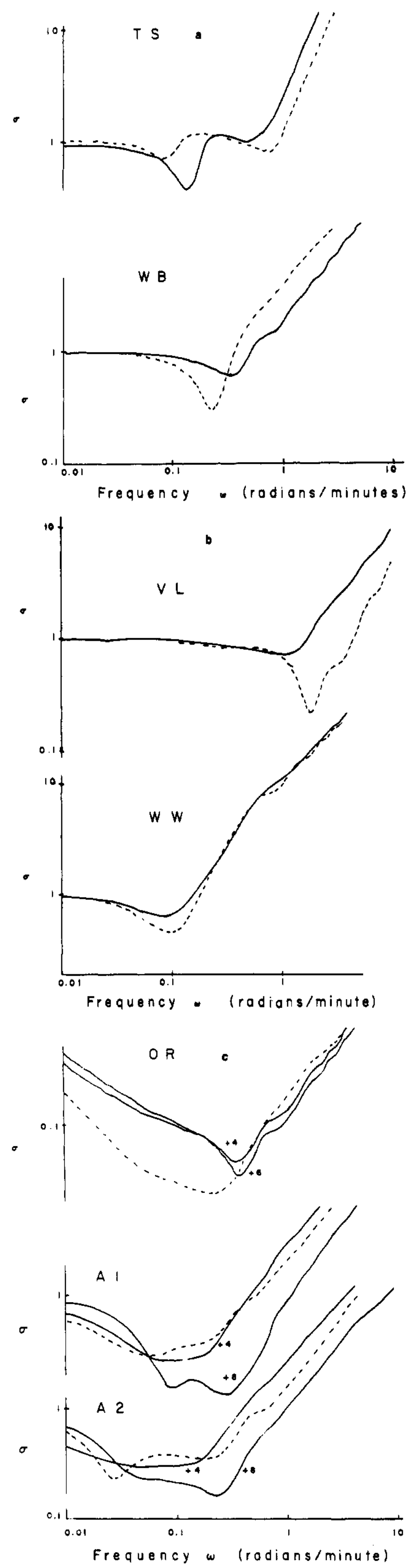

Figure 3. (a) Minimum singular values of TS and WB, (b) minimum singular values of VL and WW, (c) minimum singular values of OR, A1, and A2. Solid lines are BLT settings. Dashed lines are empirical settings.

Conclusion

A simple, easily implemented procedure is presented for controller tuning in multivariable systems. The resulting settings given reasonable, stable responses. The method was successfully tested on 10 different systems taken from the distillation literature.

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Nomenclature

B = feedback controller transfer function
$\mathbf{B}$ = matrix of feedback controller transfer functions
B_i = diagonal element of $\mathbf{B}$
G = open-loop process transfer function
$\mathbf{G}$ = matrix of open-loop process transfer functions
G_{ij} = element of $\mathbf{G}$ matrix
$\mathbf{I}$ = identity matrix
K_{ci} = controller gain of ith controller
K_{ui} = ultimate gain of ith controller
K_{ZNi} = Ziegler–Nichols gain of ith controller
L_c = closed-loop log modulus of SISO system
L_{cm} = closed-loop log modulus of multivariable system
$(L_{cm})^{max}$ = biggest value of L_{cm}
M_j = manipulated variable j
$\boldsymbol{M}$ = vector of manipulated variables
N = order of system (number of SISO controllers)
$W = -1 + \det(\mathbf{I} + \mathbf{GB})$
$\boldsymbol{X}$ = vector of controlled variables
X_i = controlled variable i
$\boldsymbol{X}^{set}$ = vector of set points
ω = frequency, rad/min
ω_u = ultimate frequency
τ_I = reset time, min
τ_{ZN} = Ziegler–Nichols reset time, min
σ = minimum singular value

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Pressure Drop, Gas Holdup, and Interfacial Area for Gas–Liquid Contact in Karr Columns

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The hydrodynamics of a 5-cm reciprocating-plate Karr column used for cocurrent gas–liquid contact were studied over the ranges $0 \le AF \le 10$ cm/s, $0 \le V_L \le 6$ cm/s, and $0 \le V_g \le 14.3$ cm/s. The frictional loss is closely related to the liquid circulation within the column. The plate reciprocation shows little influence on the gas holdup until it reaches a critical value. The column can generate bubbles of average diameter between 3 and 6 mm and an interfacial area approximately 3 times that found in bubble columns under otherwise the same operating conditions.

Since its development in 1959 by Dr. A. E. Karr, the Karr reciprocating-plate column has found a general acceptance in the chemical industry as a liquid–liquid extractor, mainly because of its straightforward scaling-up procedure, relatively high efficiency, and high capacity (Karr, 1969; Karr and Lo, 1976). A recent publication (Karr et al., 1980) reported the successful introduction of Karr columns to the pharmaceutical industry for extracting fermentation broth.

It is believed that the Karr-type column should also be useful as a gas–liquid contactor. The reciprocating action of the plates, an important feature of Karr columns, would prevent the formation of large bubbles from occurring in the column. The large free plate area, another distinctive design characteristic, would reduce otherwise high energy consumption at a given throughput. In addition, this type of column would provide a multistage operating condition so that the use of a large L/D-ratio column should become practical.

For liquid–liquid extraction, the Karr column has been studied extensively (Noh and Baird, 1984; Kim and Baird,

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