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A Robust Event-Based Continuous Time Formulation for Tank Transfer Scheduling

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A generalized model is proposed for the continuous time scheduling problem of fluid transfer in tanks. This model generally and more robustly handles the synchronization of time events with material balances than previous proposed models in the literature. A novel method for representing the flow to and from a tank is developed with the potential for significant reduction in the number of necessary time events required for continuous time scheduling formulations. The problem involves the optimal operation of fluid transfer from input sources to tanks, transfer between the tanks, and the transfer from tanks to output destinations. An efficient mixed-integer nonlinear programming formulation is developed based on continuous representation of time domain under the assumption of no simultaneous input and output flow to a tank for fluid streams comprised of multiple components. The new modeling paradigm is applied to examples from the literature for developing refinery crude unit charging schedules.

1. Introduction

Recent trends in scheduling models for chemical processes have moved toward continuous time formulations to avoid the combinatorial complexity that is due to the high number of integer variables found in discrete time models. Floudas and Lin[1] and Mendez et al.[2] have provided recent reviews of scheduling for chemical processes comparing discrete and continuous time approaches.

Although most research efforts have focused on batch processes, some work considers problems that address the complications involved with scheduling the flow of fluid streams related to tanks, including mixing, stream splitting, and storage. This research has primarily focused on problems related to the oil refining industry. Discrete time models for crude oil unloading and transfer include the work of Shah,[3] Lee et al.,[4] and Wenkai and Hui.[5] Continuous time models for scheduling problems for refineries include the work of Ierapetritou and associates,[6−9] Pinto et al.,[10] and Reddy et al.[11] When the fluid flow is not modeled by considering the details of transfer requirements, certain physically impossible scenarios can be observed. To avoid such inconsistencies, we propose and evaluate advanced modeling procedures that will ensure the proper mapping of time variables and time events, with respect to material balances for problems involving transfers of discrete quantities of a process fluid. The main advantage of the proposed approach is the complete utilization of the time continuity.

The model proposed in this work attempts to address the following issues:

(1) When input and output to a tank is allowed to occur within the same time interval or event in either a discrete or continuous time model, the net accumulation of fluid in a tank may stay within the constraint specifications; however, depending on the order of the input and output flows, the tank could bottom out or become overly full for a generated schedule. Material balances are typically based on the time intervals or events only, and not the actual times associated with the fluid transfers.

(2) In either a discrete or continuous time model, component balances are typically evaluated only at the boundary between time intervals; thus, they may not be accurate for time intervals or events with unordered inputs and outputs to a tank. This issue may also adversely affect the estimation of fluid properties.

(3) Component fractions within a tank and for the output of a tank are generally evaluated at the boundaries of its time intervals or events. Numerical irregularities can occur if a tank is completely emptied during the interval, because both its total and component inventories are zero.

(4) In continuous time formulations, because material balances are based on time events, a robust set of constraints is necessary to prevent the overlap in time of fluid movements to and from the same tank from occurring in different time events.

Although previous continuous time MINLP scheduling formulations[8,9] provide a strong approximation, these models do not rigorously address all of the issues previously presented, in regard to providing schedules for tank transfers.

The objective of this paper is to propose a new generalized continuous time mathematical scheduling model that addresses the simultaneous optimization of the scheduling of fluid transfer to and from tanks. The paper is organized as follows. Section 2 describes the mathematical formulation, with particular consideration of robustly handling the synchronization of time events with material balances. Computational results of the proposed formulation for examples from the literature are presented in section 3, to illustrate the applicability and efficiency of the proposed approach to examples from the literature for developing refinery crude unit charging schedules. Section 4 summarizes the conclusions and proposed future directions.

2. Proposed Formulation

In this section, a general model is presented without a loss of generality for a single arbitrary tank. For the purposes of this paper, the fluid will be chosen to be a general multicomponent fluid. For the sake of simplicity in presentation, a single tank with input and output streams is modeled, although the

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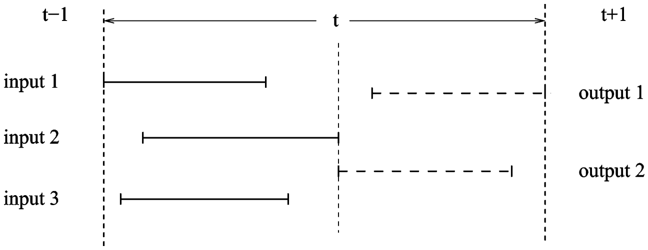

Figure 1. Input and output streams in the same time event.

formulation can be generally applied to multiple tanks as shown in the examples of the following sections. This tank will be referenced using index b, and any flows into or out of tank b will be denoted through $a \in A$ and $c \in C$, which represent arbitrary input sources and output desitinations that may or may not be other tanks. The constraints developed in this section have been defined so that they may be easily incorporated into more-complex scheduling optimization models, which have additional constraints on a system. The following rules and assumptions are made: (i) perfect mixing occurs in the tank, (ii) negligible changes in specific gravities result from blending, (iii) input and output flows have discrete volumes, and (iv) there are no simultaneous input and output flows to a tank.

The proposed formulation is based on a continuous time representation, and the concept of the time event[12] is adopted. In this paper, a time event refers to an interval of time in which the starting and end points are defined by time variables. The proposed scheduling model involves mainly material balance constraints, logic constraints, duration constraints, and sequencing constraints. Sections 2.1−2.5, primarily, have been equivalently posed in previous models. However, note that nonlinear constraints, which are necessary to address component blending, as described in Section 2.1.1, are typically linearly approximated.

One of the novel ideas of this model is that, to reduce the necessary number of time events, input and output flows are allowed to occur in the same time event. However, no simultaneous input and output flows are permitted to occur within a tank. This key assumption directly addresses issue 2 from the Introduction. This is achieved by requiring that all of the input flows to a tank must be finished before the start of the output flows. As illustrated in Figure 1, in a tank, the same type of flows may occur simultaneously, whereas input and output flows can never occur at the same time. The choice of making inputs first and outputs last is arbitrary, and this could be easily reversed. The unique constraints that have been developed for this model to enforce these ideas are presented in section 2.6.

2.1. Material Balance Constraints. These constraints ensure that material that is flowing into a tank either subsequently flows out of that tank or accumulates in the tank. Constraints 1−4 express that the total inventory or the component inventory in tank b at time event t is equal to that at time event $t-1$, adjusted by any amount transferred from any potential input sources a and to any potential output destination c at time event t.

$$I^{\mathrm{tot}}_{b,t-1} + \sum_{a\in A} V^{\mathrm{tot}}_{abt} = I^{\mathrm{tot}}_{bt} + \sum_{c\in C} V^{\mathrm{tot}}_{bct} \qquad \forall\, t \in \mathrm{T} \tag{1}$$

$$I^{\mathrm{tot}}_{b0} = I^{\mathrm{init-tot}}_{b} \tag{2}$$

$$I_{jb,t-1} + \sum_{a\in \mathrm{A}} V_{jabt} = I_{jbt} + \sum_{c\in \mathrm{C}} V_{jbct} \qquad \forall\, j \in \mathrm{J},\, t \in \mathrm{T} \tag{3}$$

$$I_{jb0} = I^{\mathrm{init}}_{jb} \qquad \forall\, j \in \mathrm{J} \tag{4}$$

Constraints 5 and 6 state that the total flow transferred into or from a tank at one time event is equal to the sum of the component flows.

$$V^{\mathrm{tot}}_{abt} = \sum_{j\in \mathrm{J}} V_{jabt} \qquad a \in \mathrm{A},\, t \in \mathrm{T} \tag{5}$$

$$V^{\mathrm{tot}}_{bct} = \sum_{j\in \mathrm{J}} V_{jbct} \qquad c \in \mathrm{C},\, t \in \mathrm{T} \tag{6}$$

2.1.1. Component Fraction Constraints. The fraction of component j in the output of tank b should be the same as the fraction of j within the tank according to the perfect mixing assumption. As mentioned earlier, in the same time event, any input flows must occur before any output flows. Therefore, the inventory of component j and total inventory in tank b when the output flow starts are $I_{jb,t-1} + \sum_{a\in \mathrm{A}} V_{jabt}$ and $I^{\mathrm{tot}}_{b,t-1} + \sum_{a\in \mathrm{A}} V^{\mathrm{tot}}_{abt}$, respectively. The final inventory of the tank from the previous time event is used, rather than the final inventory of the tank for a particular time event, to avoid the numerical irregularities that can result when the tank is completely emptied. Therefore, this constraint has been posed in such a way as to avoid the problems associated with issue number 3 from the Introduction. Note that such numerical irregularities could not occur in cases where the lower bounds for variable I^{tot}_{bt} were all greater than zero.

$$\left(I^{\mathrm{tot}}_{b,t-1} + \sum_{a\in \mathrm{A}} V^{\mathrm{tot}}_{abt}\right) V_{jbct} = \left(I_{jb,t-1} + \sum_{a\in \mathrm{A}} V_{jabt}\right) V^{\mathrm{tot}}_{bct} \qquad \forall\, j \in \mathrm{J},\, c \in \mathrm{C},\, t \in \mathrm{T} \tag{7}$$

2.1.2. Convex Envelope for the Cross Product. The cross-product constraint described in eq 7 is used to properly calculate component fractions and has nonconvex nonlinearity in the form of bilinear terms. A tight convex relaxation for eq 7 may be stated as a set of linear constraints known as McCormick estimators.[13,14] This set of constraints could be added to a formulation to generally tighten it. They are also useful as a replacement for eq 7 to relax the problem and reduce it from the exact nonconvex MINLP to a convex mixed-integer linear programming (MILP) relaxation. New artificial variables would need to be added to replace the bilinear terms in constraint 7. These new variables are defined as follows:

$$I^{\mathrm{VJ}}_{jbct} = I^{\mathrm{tot}}_{b,t-1} V_{jbct} \qquad \forall\, j \in \mathrm{J},\, c \in \mathrm{C},\, t \in \mathrm{T}$$

$$V^{\mathrm{VJ}}_{jabct} = V^{\mathrm{tot}}_{abt} V_{jbct} \qquad \forall\, j \in \mathrm{J},\, a \in \mathrm{A},\, c \in \mathrm{C},\, t \in \mathrm{T}$$

$$I^{\mathrm{VT}}_{jbct} = I_{jb,t-1} V^{\mathrm{tot}}_{bct} \qquad \forall\, j \in \mathrm{J},\, c \in \mathrm{C},\, t \in \mathrm{T}$$

$$V^{\mathrm{VT}}_{jabct} = V_{jabt} V^{\mathrm{tot}}_{bct} \qquad \forall\, j \in \mathrm{J},\, a \in \mathrm{A},\, c \in \mathrm{C},\, t \in \mathrm{T}$$

Replacing the bilinear terms in constraint 7 with the new variables, constraint 7 can be converted to constraint 8:

$$I^{\mathrm{VJ}}_{jbct} + \sum_{a\in \mathrm{A}} V^{\mathrm{VJ}}_{jabct} = I^{\mathrm{VT}}_{jbct} + \sum_{a\in \mathrm{A}} V^{\mathrm{VT}}_{jabct} \qquad \forall\, j \in \mathrm{J},\, c \in \mathrm{C},\, t \in \mathrm{T} \tag{8}$$

As shown by McCormick,[13] considering a bilinear term $z = xy$ with upper and lower bounds on x and y,

$$x^{\mathrm{L}} \le x \le x^{\mathrm{U}}$$

$$y^{\mathrm{L}} \le y \le y^{\mathrm{U}}$$

the valid overestimator and underestimator for z take the following form:

$$z \geq x^{L}y + xy^{L} - x^{L}y^{L}$$
$$z \geq x^{U}y + xy^{U} - x^{U}y^{U}$$
$$z \leq x^{U}y + xy^{L} - x^{U}y^{L}$$
$$z \leq x^{L}y + xy^{U} - x^{L}y^{U}$$

Similarly, the convex envelope constraints for bilinear terms I_{jbct}^{VJ}, V_{jabct}^{VJ}, I_{jbct}^{VT}, and V_{jabct}^{VT} can be written as in constraints 9−12, 13−15, 16−18, and 19−21:

$$I_{jbct}^{VJ} \geq I_{b}^{L}V_{jbct} \qquad \forall\, j \in \mathrm{J}, c \in \mathrm{C}, t \in \mathrm{T} \tag{9}$$
$$I_{jbct}^{VJ} \geq I_{b}^{U}V_{jbct} + V_{bc}^{U}I_{b,t-1}^{tot} - I_{b}^{U}V_{bc}^{U} \qquad \forall\, j \in \mathrm{J}, c \in \mathrm{C}, t \in \mathrm{T} \tag{10}$$
$$I_{jbct}^{VJ} \leq I_{b}^{L}V_{jbct} + V_{bc}^{U}I_{b,t-1}^{tot} - I_{b}^{L}V_{bc}^{U} \qquad \forall\, j \in \mathrm{J}, c \in \mathrm{C}, t \in \mathrm{T} \tag{11}$$
$$I_{jbct}^{VJ} \leq I_{b}^{U}V_{jbct} \qquad \forall\, j \in \mathrm{J}, c \in \mathrm{C}, t \in \mathrm{T} \tag{12}$$

$$V_{jabct}^{VJ} \geq V_{ab}^{U}V_{jbct} + V_{bc}^{U}V_{abt}^{tot} - V_{ab}^{U}V_{bc}^{U} \qquad \forall\, j \in \mathrm{J}, a \in \mathrm{A}, c \in \mathrm{C}, t \in \mathrm{T} \tag{13}$$
$$V_{jabct}^{VJ} \leq V_{bc}^{U}V_{abt}^{tot} \qquad \forall\, j \in \mathrm{J}, a \in \mathrm{A}, c \in \mathrm{C}, t \in \mathrm{T} \tag{14}$$
$$V_{jabct}^{VJ} \leq V_{ab}^{U}V_{jbct} \qquad \forall\, j \in \mathrm{J}, a \in \mathrm{A}, c \in \mathrm{C}, t \in \mathrm{T} \tag{15}$$

$$I_{jbct}^{VT} \geq I_{b}^{U}V_{bct}^{tot} + V_{bc}^{U}I_{jb,t-1} - I_{b}^{U}V_{bc}^{U} \qquad \forall\, j \in \mathrm{J}, c \in \mathrm{C}, t \in \mathrm{T} \tag{16}$$
$$I_{jbct}^{VT} \leq V_{bc}^{U}I_{jb,t-1} \qquad \forall\, j \in \mathrm{J}, c \in \mathrm{C}, t \in \mathrm{T} \tag{17}$$
$$I_{jbct}^{VT} \leq I_{b}^{U}V_{bct}^{tot} \qquad \forall\, j \in \mathrm{J}, c \in \mathrm{C}, t \in \mathrm{T} \tag{18}$$

$$V_{jabct}^{VT} \geq V_{ab}^{U}V_{bct}^{tot} + V_{bc}^{U}V_{jabt} - V_{ab}^{U}V_{bc}^{U} \qquad \forall\, j \in \mathrm{J}, a \in \mathrm{A}, c \in \mathrm{C}, t \in \mathrm{T} \tag{19}$$
$$V_{jabct}^{VT} \leq V_{bc}^{U}V_{jabt} \qquad \forall\, j \in \mathrm{J}, a \in \mathrm{A}, c \in \mathrm{C}, t \in \mathrm{T} \tag{20}$$
$$V_{jabct}^{VT} \leq V_{ab}^{U}V_{bct}^{tot} \qquad \forall\, j \in \mathrm{J}, a \in \mathrm{A}, c \in \mathrm{C}, t \in \mathrm{T} \tag{21}$$

2.2. Logic Constraints. The binary variables w_{abt} and w_{bct} are used to denote when there is flow in some time event either from a source to the arbitrary tank b, or from tank b to some destination. Logic constraints are posed simply to force the binary variables on some fluid stream to be 1 when this flow occurs in some time event.

$$V_{abt}^{tot} \leq Mw_{abt} \qquad a \in \mathrm{A}, t \in \mathrm{T} \tag{22}$$
$$V_{bct}^{tot} \leq Mw_{bct} \qquad c \in \mathrm{C}, t \in \mathrm{T} \tag{23}$$

Although a generally large number could be used as a "big-M" in these logic constraints, it is best if this value is minimized. This M should typically be set to the upper bounds of V_{abt}^{tot} and V_{bct}^{tot} in their corresponding constraints.

2.3. Duration Constraints. The flow rates of the streams are limited with bounds; therefore, the volume of the flow is restricted by the product of time duration and flow rate bounds. Because the exact values of the time variables are only required to be enforced when flow actually occurs, these constraints are relaxed when the flow does not occur for a tank in some time event. This is achieved by the introduction of binary variables w_{abt} that take the value of 1 only if there is a flow at time event t due to constraints 22 and 23.

$$F_{ab}^{L}(T_{abt}^{2} - T_{abt}^{1}) \leq V_{abt}^{tot} + F_{ab}^{L}H(1 - w_{abt}) \qquad \forall\, a \in \mathrm{A}, t \in \mathrm{T} \tag{24}$$
$$F_{bc}^{L}(T_{bct}^{2} - T_{bct}^{1}) \leq V_{bct}^{tot} + F_{bc}^{L}H(1 - w_{bct}) \qquad \forall\, c \in \mathrm{C}, t \in \mathrm{T} \tag{25}$$
$$F_{ab}^{U}(T_{abt}^{2} - T_{abt}^{1}) \geq V_{abt}^{tot} - F_{ab}^{U}H(1 - w_{abt}) \qquad \forall\, a \in \mathrm{A}, t \in \mathrm{T} \tag{26}$$
$$F_{bc}^{U}(T_{bct}^{2} - T_{bct}^{1}) \geq V_{bct}^{tot} - F_{bc}^{U}H(1 - w_{bct}) \qquad \forall\, c \in \mathrm{C}, t \in \mathrm{T} \tag{27}$$

2.4. Simple Sequencing Constraints. The sequence constraints 28−33 require that a flow from a to b or from b to c at time event t occurs before the same flow inward, at time event $t + 1$. Constraints 28 and 31 are active only if the binary variables w_{abt} and w_{bct} are equal to 1; otherwise, they are relaxed.

$$T_{ab,t+1}^{1} \geq T_{at}^{2} - H(1 - w_{abt}) \qquad \forall\, a \in A, t \in T, t < |T| \tag{28}$$
$$T_{ab,t+1}^{1} \geq T_{abt}^{1} \qquad \forall\, a \in \mathrm{A}, t \in \mathrm{T}, t < |\mathrm{T}| \tag{29}$$
$$T_{ab,t+1}^{2} \geq T_{abt}^{2} \qquad \forall\, a \in \mathrm{A}, t \in \mathrm{T}, t < |\mathrm{T}| \tag{30}$$
$$T_{bc,t+1}^{1} \geq T_{bct}^{2} - H(1 - w_{bct}) \qquad \forall\, c \in \mathrm{C}, t \in \mathrm{T}, t < |\mathrm{T}| \tag{31}$$
$$T_{bc,t+1}^{1} \geq T_{bct}^{1} \qquad \forall\, c \in \mathrm{C}, t \in \mathrm{T}, t < |\mathrm{T}| \tag{32}$$
$$T_{bc,t+1}^{2} \geq T_{bct}^{2} \qquad \forall\, c \in \mathrm{C}, t \in \mathrm{T}, t < |\mathrm{T}| \tag{33}$$

2.5. Variable Bounds. The following constraints impose general lower and upper bounds on the tank inventories (constraints 34 and 35), flow volumes (constraints 36−39), and starting and ending times of a transfer (constraints 40−43).

$$0 \leq I_{jbt} \leq I_{b}^{U} \qquad \forall\, j \in \mathrm{J}, t \in \mathrm{T} \tag{34}$$
$$I_{b}^{L} \leq I_{bt}^{tot} \leq I_{b}^{U} \qquad \forall\, t \in \mathrm{T} \tag{35}$$
$$0 \leq V_{jabt} \leq V_{ab}^{U} \qquad \forall\, j \in \mathrm{J}, a \in \mathrm{A}, t \in \mathrm{T} \tag{36}$$
$$0 \leq V_{jbct} \leq V_{bc}^{U} \qquad \forall\, j \in \mathrm{J}, c \in \mathrm{C}, t \in \mathrm{T} \tag{37}$$
$$0 \leq V_{abt}^{tot} \leq V_{ab}^{U} \qquad \forall\, a \in \mathrm{A}, t \in \mathrm{T} \tag{38}$$
$$0 \leq V_{bct}^{tot} \leq V_{bc}^{U} \qquad \forall\, c \in \mathrm{C}, t \in \mathrm{T} \tag{39}$$
$$0 \leq T_{abt}^{1} \leq H \qquad \forall\, a \in \mathrm{A}, t \in \mathrm{T} \tag{40}$$
$$0 \leq T_{abt}^{2} \leq H \qquad \forall\, a \in \mathrm{A}, t \in \mathrm{T} \tag{41}$$
$$0 \leq T_{bct}^{1} \leq H \qquad \forall\, c \in \mathrm{C}, t \in \mathrm{T} \tag{42}$$
$$0 \leq T_{bct}^{2} \leq H \qquad \forall\, c \in \mathrm{C}, t \in \mathrm{T} \tag{43}$$

2.6. Robust Handling of Inputs and Outputs for a Tank. In continuous time models, for some given time event or time slot interval, Reddy et al.[11] and Jia et al.[8] avoided overlapping tank input and output by limiting a given time interval or event to input only or output only. However, allowing both input and output to occur in a time interval or event, although not simultaneously, provides a potential reduction in necessary binary decision variables by a factor of 2. As described later, this model implicitly prevents the overlap of inputs and outputs to a tank.

Discrete time models inherently synchronize material balances with time, because time intervals are predefined. In this section, we present the main ideas that are required for the mapping of time events in a continuous time model. When the tank inventory for some time event is calculated, the material balances only consider the net input and output during some time event, not the timing of the fluid transfers. Previous event-based models (e.g., Jia et al.[8]) and the constraint set of this model, up to this point, have no explicit link between the material balance constraints and the time sequencing constraints. It is easy to create simple example instances in which the timing of events does not align with the sequence of time events.

2.6.1. Example 1: Incorrect Timing in the Same Time Event. Consider a single tank b with two input flows a_1 and a_2 and one output flow c_1 in some time event t. For this example assume that the initial inventory in the tank is zero and that there is only a single component in the fluid such that component constraints may be disregarded. For the sake of simplicity, assume that the flow rates are fixed to 1 flow unit per time unit. For the input flow variables set $V^{\rm tot}_{a_1,b,t} = 2$, $T^1_{a_1,b,t} = 0$, $T^2_{a_1,b,t} = 2$, $V^{\rm tot}_{a_2,b,t} = 3$, $T^1_{a_2,b,t} = 6$, and $T^2_{a_2,b,t} = 9$. Also, for the output flow variables, set $V^{\rm tot}_{b,c_1,t} = 4$, $T^1_{b,c_1,t} = 2$, and $T^2_{b,c_1,t} = 6$. Given these values, the inventory at the end of the time event would be calculated as $I^{\rm tot}_{bt} = 1$ and all constraints previously defined would be satisfied. Furthermore, this example would also be feasible if used in previous event-based models[7−9] that consider tank transfers. However, note that a_2 occurs after c_1 and, thus, if starting with an empty tank, then by time unit 6, more material has left the tank than has entered, even if the final net tank inventory is 1 unit of fluid.

2.6.2. Example 2: Incorrect Timing across More Than One Time Event. Consider a single tank b with two input flows a_1 (occurring in time event t) and a_2 (occurring in time event $t + 1$) and one output flow c_1 (occurring time event t). For this example, assume that the initial inventory in the tank is zero and that there is only a single component in the fluid such that component constraints may be disregarded. For the sake of simplicity, assume that the flow rates are fixed to 1 flow unit per time unit. For the input flow variables, set $V^{\rm tot}_{a_1,b,t} = 2$, $T^1_{a_1,b,t} = 0$, $T^2_{a_1,b,t} = 2$, $V^{\rm tot}_{a_2,b,t+1} = 3$, $T^1_{a_2,b,t+1} = 2$, $T^2_{a_2,b,t+1} = 5$. Also, for the output flow variables, set $V^{\rm tot}_{b,c_1,t} = 2$, $T^1_{b,c_1,t} = 5$, and $T^2_{b,c_1,t} = 7$. Given these values, the net inventory variables at the end of the respective time events would be calculated as $I^{\rm tot}_{bt} = 0$ and $I^{\rm tot}_{b,t+1} = 3$, and all constraints previously defined would be satisfied. However, by examining the timing of the inputs and outputs, at time unit 5, the input to tank b should total 5 units of fluid, but the net inventory for time event t is zero. If the upper bound on the tank inventory is set to a value of <5, then although all constraints previously defined would be satisfied; this situation would be physically infeasible. Again, this example would also be feasible if used in previous event-based models.[7−9]

Additional constraints are necessary to ensure that material balances are properly handled within a single time event, as well as spanning the entire time horizon. Reddy et al.[11] also addressed this issue via their slot-based continuous time approach; however, as illustrated by the previously described examples, previous event-based models have failed to fully address this issue and thus can potentially allow infeasible timing of material flow. In addition, the time event-based approach developed in this section has the potential to significantly reduce the necessary number of binary variables, in comparison to the slot-based approach.

2.7. Actual Inventory Bounds. Because both the input and output of material may occur in the same time event, simple bounds on the total tank inventory variable $I^{\rm tot}_{bt}$ are not sufficient. The variable $I^{\rm tot}_{bt}$ only represents a net inventory at the bounds of a time event. Because inputs are required to occur before outputs in a given time event, the upper bound on the actual inventory may be violated if only bounding the variable $I^{\rm tot}_{bt}$. The following constraint is necessary to ensure that the upper bound on inventory is not violated:

$$I^{\rm tot}_{b,t-1} + \sum_{a \in \rm A} V^{\rm tot}_{abt} \le I^{\rm U}_b \qquad \forall\ t \in \rm T \tag{44}$$

This constraint addresses issue 1 that was posed in the Introduction. A guarantee is now in place that the actual inventory level of a tank is kept within the bound at all times, rather than only enforcing it on the net inventory at the time boundaries of events.

2.8. Input and Output Restraints Spanning Entire Horizon. For some given tank, the inputs and outputs of a previous time event must occur before any inputs or outputs of the current time event. These constraints prevent potential problems with the continuous time variables aligning with the time events for transfers of fluid in successive time events with varying sources and destinations for the fluid.

$$T^1_{ab,t+1} \ge T^2_{a'bt} - H(1 - w_{a'bt}) \qquad \forall\ a,a' \in \rm A,\ a \ne a',\ t \in T,\ t < |T| \tag{45}$$

$$T^1_{ab,t+1} \ge T^2_{bct} - H(1 - w_{bct}) \qquad \forall\ a \in \rm A,\ c \in C,\ t \in T,\ t < |T| \tag{46}$$

$$T^1_{bc,t+1} \ge T^2_{abt} - H(1 - w_{abt}) \qquad \forall\ a \in \rm A,\ c \in C,\ t \in T,\ t < |T| \tag{47}$$

$$T^1_{bc,t+1} \ge T^2_{bc't} - H(1 - w_{bc't}) \qquad \forall\ c,c' \in \rm C,\ c \ne c',\ t \in T,\ t < |T| \tag{48}$$

The synchronization of time events with time variables prevents the potential scenario in which something is transferred out of a tank before it is transferred into the tank during the same time event. Assuming that no simultaneous input and output flows to a tank are allowed, to handle the input and output sequence during the same time event, all inputs are required to occur before all outputs. This also ensures that the component balances are upheld.

$$T^2_{abt} - H(1 - w_{abt}) \le T^1_{bct} + H(1 - w_{bct}) \qquad \forall\ a \in \rm A,\ c \in C,\ t \in T \tag{49}$$

Figure 1 illustrates an example of the concept of this constraint for an arbitrary time event t. In the example of Figure 1, there are three inputs and two outputs each with a corresponding length of time, and all occurring in some tank in a time event t that has an overall length of time bounded by the dotted lines at each end, determined by the time spans of the inputs and outputs. The central dotted line in the figure illustrates the rule that inputs time spans are not allowed to overlap with output time spans, and this is acheived using the aforementioned constraint.

In a model with multiple tanks, using this approach provides no explicit constraints preventing flow in both directions between the same two tanks in the same time interval. Although this could be explicitly imposed via additional logic constraints,

Table 1. Problem Data for Example 1

Scheduling Horizon: 8			
Number of Vessel Arrivals: 2			
Vessel	Arrival Time	Amount of Crude	Key Comp. Conc.
vessel 1	1	100	0.01
vessel 2	5	100	0.06
Number of Storage Tanks: 2			
Storage Tank	Capacity	Initial Oil Amount	Key Comp. Conc.
tank 1	100	25	0.01
tank 2	100	75	0.06
Number of Charging Tanks: 2			
Charging Tank	Capacity	Initial Oil Amount	Initial Comp. Conc. (Min,Max)
tank 1	100	50	0.02 (0.015,0.025)
tank 2	100	50	0.05 (0.045,0.055)

Number of CDUs	1
Unloading Cost	8
Sea Waiting Cost	5
Storage Tank Inventory Cost	0.05
Charging Tank Inventory Cost	0.08
Unit Changeover Cost	50
Demand of Oil	
Mix 1	100
Mix 2	100
Flow Rate	
Min	1
Max	40

Table 2. Problem Data for Example 2

Scheduling Horizon: 10				
Number of Vessel Arrivals: 3				
Vessel	Arrival Time	Amount of Crude	Comp. 1 Conc.	Comp. 2 Conc.
vessel 1	1	100	0.01	0.04
vessel 2	4	100	0.03	0.02
vessel 3	7	100	0.05	0.01
Number of Storage Tanks: 3				
Storage Tank	Capacity	Initial Oil Amount	Comp. 1 Conc.	Comp. 2 Conc.
tank 1	100	20	0.01	0.04
tank 2	100	50	0.03	0.02
tank 3	100	70	0.05	0.01
Number of Charging Tanks: 3				
Storage Tank	Capacity	Initial Oil Amount	Comp. 1 Conc. Initial (Min,Max)	Comp. 2 Conc. Initial (Min,Max)
tank 1	100	30	0.0167 (0.01,0.02)	0.0333 (0.03,0.038)
tank 2	100	50	0.03 (0.025,0.035)	0.023 (0.018,0.027)
tank 3	100	50	0.0433 (0.04,0.048)	0.0133 (0.01,0.018)

Number of CDU	2
Unloading Cost	8
Sea Waiting Cost	5
Storage Tank Inventory Cost	0.05
Charging Tank Inventory Cost	0.08
Unit Changeover Cost	50
Demand of Oil	
Mix 1	100
Mix 2	100
Mix 3	100
Flow Rate	
Min	1
Max	40

such constraints are actually unnecessary. The proof in the Appendix demonstrates that this situation is implicitly prevented.

3. Application to Literature Examples

This new and robust model for tank transfers is applied to the refinery scheduling examples originally proposed by Lee et al.[4] and adjusted for a continuous time model by Jia et al.[8]The generic framework of constraints for an arbitrary tank described in section 2 is expanded to model the transfer of crude oil between tanks at a refinery; however, additional constraints are necessary to complete the scheduling formulation. The data sets for these examples are collected in Tables 1, 2, 3, and 4. Illustrations of the network connections for these examples are provided in Figures 2, 3, 4, and 5, respectively.

Table 3. Problem Data for Example 3

Scheduling Horizon: 12

Number of Vessel Arrivals: 3

Vessel	Arrival Time	Amount of Crude	Key Comp. Conc.
vessel 1	1	50	0.01
vessel 2	5	50	0.085
vessel 3	9	50	0.06

Number of Storage Tanks: 3

Storage Tank	Capacity	Initial Oil Amount	Initial Comp. Conc. (Min,Max)
tank 1	100	20	0.02 (0.01,0.03)
tank 2	100	20	0.05 (0.04,0.06)
tank 3	100	20	0.08 (0.07,0.09)

Number of Charging Tanks: 3

Charging Tank	Capacity	Initial Oil Amount	Initial Comp. Conc. (Min,Max)
tank 1	100	30	0.03 (0.025,0.035)
tank 2	100	50	0.05 (0.045,0.065)
tank 3	100	30	0.08 (0.075,0.085)

Number of CDU	2
Unloading Cost	10
Sea Waiting Cost	5
Storage Tank Inventory Cost	0.04
Charging Tank Inventory Cost	0.08
Unit Changeover Cost	50
Demand of Oil	
Mix 1	50
Mix 2	50
Mix 3	50
Flow Rate	
Min	1
Max	40

To ensure that the crude distillation unit (CDU) has a continuous feed within certain specifications and meeting demand, the following constraints are added.

$$\sum_{t\in \mathrm{T}}\sum_{b\in \mathrm{B}^{\mathrm{Ch}}} (T^2_{bct} - T^1_{bct}) \qquad \forall\, c \in \mathrm{C}^{\mathrm{CDU}} = \mathrm{H} \tag{50}$$

$$F^{\mathrm{L}}_{bc}(T_{bct}{}^2 - T_{bct}{}^1) \le V^{\mathrm{tot}}_{bct} \qquad \forall\, b \in \mathrm{B}^{\mathrm{Ch}}, c \in \mathrm{C}^{\mathrm{CDU}}, t \in \mathrm{T} \tag{51}$$

$$T_{bc,t+1}{}^1 \ge T^2_{bct} - H(1 - w_{bct}) \qquad \forall\, \mathrm{B}^{\mathrm{Ch}}, c \in \mathrm{C}^{\mathrm{CDU}}, t \in \mathrm{T}, t < |\mathrm{T}| \tag{52}$$

$$T^1_{bc,t+1} \le T^2_{bct} + H(1 - w_{bct}) \qquad \forall\, \mathrm{B}^{\mathrm{Ch}}, c \in \mathrm{C}^{\mathrm{CDU}}, t \in \mathrm{T}, t < |\mathrm{T}| \tag{53}$$

$$\sum_{b\, \in\, \mathrm{B}^{\mathrm{Ch}}} w_{bct} \le 1 \qquad \forall\, c \in \mathrm{C}^{\mathrm{CDU}}, t \in \mathrm{T} \tag{54}$$

$$\sum_{c\, \in\, \mathrm{C}^{\mathrm{CDU}}} w_{bct} \le 1 \qquad \forall\, b \in \mathrm{B}^{\mathrm{Ch}}, t \in \mathrm{T} \tag{55}$$

$$\sum_{t\, \in\, \mathrm{T}}\sum_{c\, \in\, \mathrm{C}^{\mathrm{CDU}}} V^{\mathrm{tot}}_{bct} = D_b \qquad \forall\, b \in \mathrm{B}^{\mathrm{Ch}} \tag{56}$$

In the example, a CDU can be considered to be a special destination for crude oil flow, in terms of the tank model. It requires a continuous flow of material, spanning the entire time horizon with interuption, to meet a specific overall demand while maintaining at least some minimum feed rate. Equation 50 ensures that a CDU has a crude oil flow into it for the entire duration of the time horizon without a gap in time. Equation 51 imposes a minimum flow rate of crude oil from the charging tanks to a CDU at all times. Equations 52 and 53 are the sequencing constraints for the feed to a CDU to ensure the appropriate ordering of the timing of CDU charging events. Equations 54 and 55 enforce a rule imposed by the test cases that only one tank may charge each CDU at any given time. Lastly, constraint 56 requires that each charging tank meets its demand for feeding CDUs over the time horizon.

The next set of constraints is added to properly model the volume and timing for the arrival of crude oil.

$$\sum_{t\in \mathrm{T}}\sum_{\substack{b\in \mathrm{B}\\ (ab)\in \mathrm{E}}} V^{\mathrm{tot}}_{bct} = S_a \qquad \forall\, a \in \mathrm{A}^{\mathrm{Arr}} \tag{57}$$

$$V_{jabt} = f^{\mathrm{Arr}}_{ja} V^{\mathrm{tot}}_{abt} \qquad \forall\, j \in \mathrm{J}, a \in \mathrm{A}^{\mathrm{Arr}}, (ab) \in \mathrm{E}, t \in \mathrm{T} \tag{58}$$

$$T^{start}_a \le T^1_{abt} + H(1 - w_{abt}) \qquad \forall\, a \in \mathrm{A}^{\mathrm{Arr}}, (ab) \in \mathrm{E}, t \in \mathrm{T} \tag{59}$$

$$T^{end}_a \ge T^2_{abt} - H(1 - w_{abt}) \qquad \forall\, a \in \mathrm{A}^{\mathrm{Arr}}, (ab) \in \mathrm{E}, t \in \mathrm{T} \tag{60}$$

In the example, crude oil in a fixed amount arrives at the refinery within a given time frame. The crude oil that arrives must be discharged into a storage tank within the give time window. Constraint 57 requires that the arrival amount of crude be transferred into tanks. Equation 58 provides the information regarding the composition of the total arrival amount and breaks it into the various fractions. Equations 59 and 60 are the sequencing constraints for the transfer of the arriving crude into tanks, to ensure the appropriate ordering of the timing of events and properly calculate the length of the arrival discharge window for use in the objective function.

The objective function is a modified version of the objective function used by Jia et al.,[8] which minimizes the total cost of operation. The first term in the objective function is the sea waiting cost, where $\sum_{a\in \mathrm{A}^{\mathrm{Arr}}} (T^{\mathrm{start}}_a - T^{\mathrm{Arr}}_a)$ represents the total waiting time at sea of all the vessels. The second term is the unloading cost, where $\sum_{a\in \mathrm{A}^{\mathrm{Arr}}} (T^{\mathrm{end}}_a - T^{\mathrm{start}}_a)$ represents the total unloading duration. Because the time events in this new

Table 4. Problem Data for Example 4

Scheduling Horizon: 15

Number of Vessel Arrivals: 3

Vessel	Arrival Time	Amount of Crude	Key Comp. Conc.
vessel 1	1	50	0.03
vessel 2	6	50	0.05
vessel 3	11	50	0.065

Number of Storage Tanks: 6

Storage Tank	Capacity (Min,Max)	Initial Oil Amount	Initial Comp. Conc. (Min,Max)
tank 1	10,90	60	0.031 (0.025,0.038)
tank 2	10,110	10	0.03 (0.02,0.04)
tank 3	10,110	50	0.05 (0.04,0.06)
tank 4	10,110	40	0.065 (0.06,0.07)
tank 5	10,90	30	0.075 (0.07,0.08)
tank 6	10,90	60	0.075 (0.07,0.08)

Number of Charging Tanks: 3

Charging Tank	Capacity	Initial Oil Amount	Initial Comp. Conc. (Min,Max)
tank 1	80	5	0.0317 (0.03,0.035)
tank 2	80	30	0.0483 (0.043,0.05)
tank 3	80	30	0.0633 (0.06,0.065)
tank 4	80	30	0.075 (0.071,0.08)

Number of CDU	3
Unloading Cost	7
Sea Waiting Cost	5
Storage Tank Inventory Cost	0.05
Charging Tank Inventory Cost	0.06
Unit Changeover Cost	30
Demand of Oil	
Mix 1	60
Mix 2	60
Mix 3	60
Mix 4	60
Flow Rate	
Min	1
Max	40

formulation allow both input and output, the inventory term required adjustment to approximate the average inventory as shown in the third term, which considers the inventory level at two points in time per time event—the boundary and the midpoint between inputs and outputs. The last two terms in the objective function correspond to the total changeover cost.

$$\min C^{\mathrm{Sea}} \sum_{a \in \mathrm{A}^{\mathrm{Arr}}} (T_a^{\mathrm{start}} - T_a^{\mathrm{Arr}}) + C^{\mathrm{Unload}} \sum_{a \in A^{Arr}} (T_a^{\mathrm{end}} - T_a^{\mathrm{start}}) + \frac{H}{2|T|+1} \sum_{b \in B} C_b^{\mathrm{Inv}} \left[\sum_{t \in T} I_{bt}^{\mathrm{tot}} + \sum_{\substack{t \in \mathrm{T} \\ t < |\mathrm{T}|}} I_{bt}^{\mathrm{tot}} + \sum_{t \in T} \sum_{\substack{a \in \mathrm{A}^{\mathrm{Arr}} \cup \mathrm{B} \\ (ab) \in \mathrm{E}}} V_{abt}^{\mathrm{tot}} + 2 I_b^{\mathrm{init-tot}} \right] + C^{\mathrm{set}} \sum_{\substack{(bc) \in \mathrm{E} \\ b \in \mathrm{B}^{\mathrm{Ch}} \\ c \in \mathrm{C}^{\mathrm{CDU}}}} \sum_{t \in \mathrm{T}} w_{bct} - C^{\mathrm{Set}} |C^{\mathrm{CDU}}| \quad (61)$$

The model presented here for these examples is not equivalent to those found in the previous literature; it resolves several drawbacks and issue with the previous models that prevent them from robustly and exactly representing the problem under the given assumptions. No attempt is made to compare the computational results and schedules derived directly to other models. All computational results are performed on a Pentium IV Xeon workstation with 2.4 GHz processor with 2 GB RAM, under the Linux operating system. Each example is a nonconvex MINLP. A series of computational tests are performed for each example, using the GAMS mathematical programming platform.[15] The examples are solved directly with the convex MINLP solver DICOPT (MIP master problems with CPLEX 9.1, NLP subproblems with CONOPT 3) for a locally optimal solution. The convex relaxation (CR) constraints (eqs 8−21 of section 2.1.2) are added to this formulation and the examples are solved again with DICOPT for comparison. The addition of the following valid inequalities (RLT) designed for pooling problems,[16,17] as an attempt to tighten the relaxations during spatial branch-and-bound, were also tested.

$$\sum_{j \in \mathrm{J}} I_{b,t-1}^{\mathrm{tot}} V_{jbct} = I_{b,t-1}^{\mathrm{tot}} V_{bct}^{\mathrm{tot}} \qquad \forall\, c \in \mathrm{C},\, t \in \mathrm{T} \quad (62)$$

$$\sum_{j \in \mathrm{J}} V_{abt}^{\mathrm{tot}} V_{jbct} = V_{abt}^{\mathrm{tot}} V_{bct}^{\mathrm{tot}} \qquad \forall\, a \in \mathrm{A},\, c \in \mathrm{C},\, t \in \mathrm{T} \quad (63)$$

$$\sum_{j \in \mathrm{J}} I_{jb,t-1} V_{bct}^{\mathrm{tot}} = I_{b,t-1}^{\mathrm{tot}} V_{bct}^{\mathrm{tot}} \qquad \forall\, c \in \mathrm{C},\, t \in \mathrm{T} \quad (64)$$

$$\sum_{j \in \mathrm{J}} V_{jabt} V_{bct}^{\mathrm{tot}} = V_{abt}^{\mathrm{tot}} V_{bct}^{\mathrm{tot}} \qquad \forall\, a \in \mathrm{A},\, c \in \mathrm{C},\, t \in \mathrm{T} \quad (65)$$

Table 5 presents the example statistics, including the numbers of constraints and variables. The CR and RLT constraints are valid constraints to the formulation. The addition of the CR constraints significantly increases both the number of variables

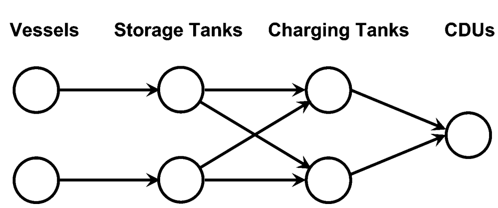

Figure 2. Fluid flow network for Example 1.

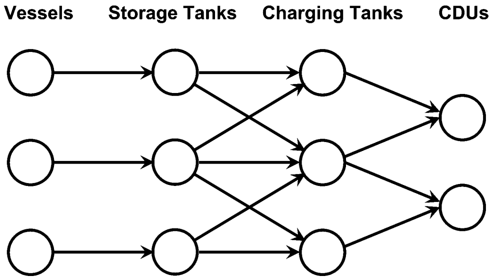

Figure 3. Fluid flow network for Example 2.

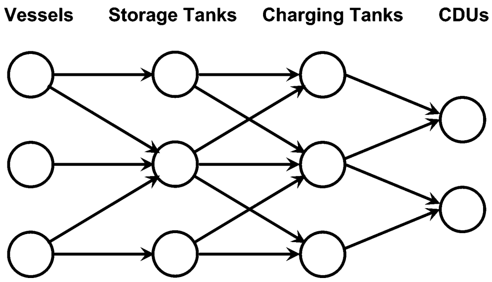

Figure 4. Fluid flow network for Example 3.

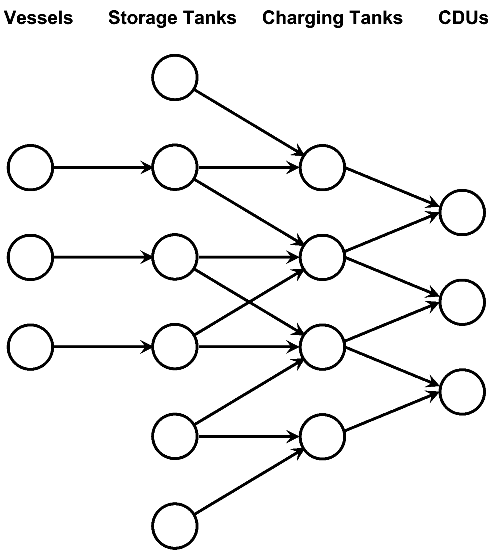

Figure 5. Fluid flow network for Example 4.

and constraints in the formulation. Adding the RLT constraints only moderately increases the number of constraints without adding new variables.

Table 6 presents the computational results from DICOPT. An asterisk indicates that a 5-h time limit for the solver was exceeded. Because this is a time event-based model, note that, just like all continuous time models, there is no standard method to determine the required number of time intervals or events to use. If one were to attempt to ensure global optimality, one could continue to increase the number of events until the solution no longer improves. However, in practice, the number of events can be chosen based on the number of expected tank transfers that would occur in a solution schedule. Increasing the number of time events significantly increases the computational complexity of the problem; thus, only a limited exploration of increasing the time events is explored in these results. Results are provided that compare the computation time of the original formulation and a version modified by adding either the CR or RLT constraints. Adding the CR constraints seems to increase the computational time per iteration of DICOPT significantly; however, the addition of the RLT constraints is actually sometimes faster on a per-iteration basis. There is no clear indication that adding the RLT constraints is better than leaving them out. No comments can be made on the global optimality of the solution as DICOPT cannot guarantee global optimality for nonconvex MINLP. It was attempted to use BARON[16,17] independently and using the solution provided by DICOPT as a starting point to obtain globally optimal solutions. However, convergence was not achievable within the 5-h time limit that

Table 5. Model Statistics

number of events	options	number of variables	number of binary variables	number of constraints
		Example 1		
3		185	24	473
3	CR	353	24	1049
3	RLT	185	24	551
4		245	32	646
4	CR	469	32	1414
4	RLT	245	32	752
5		305	40	819
5	CR	585	40	1779
5	RLT	305	40	953
		Example 2		
3		373	42	1007
3	CR	877	42	2717
3	RLT	373	42	1166
4		495	56	1375
4	CR	1167	56	3655
4	RLT	495	56	1590
		Example 3		
3		349	48	959
3	CR	757	48	2315
3	RLT	349	48	1155
4		463	64	1319
4	CR	1007	64	3127
4	RLT	463	64	1583
		Example 4		
3		439	57	1187
3	CR	895	57	2747
3	RLT	439	57	1401
4		583	76	1627
4	CR	1191	76	3707
4	RLT	583	76	1917

Table 6. Computational Results[a]

number of events	options	objective	number of iterations	MILP CPU (s)	NLP CPU (s)
			Example 1		
3		238.8	3	3.5	0.2
3	CR	238.8	2	9.0	0.4
3	RLT	238.8	3	3.6	0.2
4		215.1	3	60.1	0.3
4	CR	215.1	2	199.7	0.6
4	RLT	215.1	3	77.9	0.5
5		203.1	2	753.6	0.5
5	CR	203.6	2	434.7	1.8
5	RLT	203.6	2	559.4	0.7
			Example 2		
3		352.6	3	2461.5	0.8
3	CR	352.6	2	17997.3	2.8
3	RLT	354.5	2	1088.5	1.24
			Example 3		
3		282.2	2	485.2	0.5
3	CR	282.2	3	953.0	1.6
3	RLT	291.9	3	797.0	0.8
			Example 4		
3		431.0	2	6467.1	0.6
3	CR	383.7	*	*	*
3	RLT	432.6	3	5589.8	1.1

[a] An asterisk indicates that a 5-h time limit for the solver was exceeded.

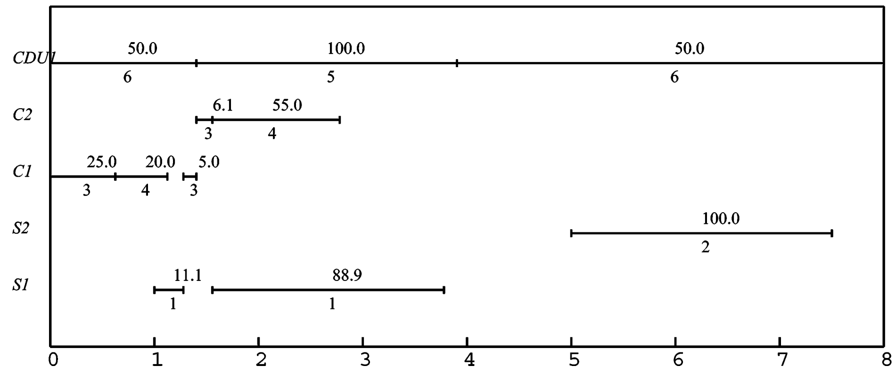

Figure 6. Gantt chart of fluid transfer schedule for Example 1.

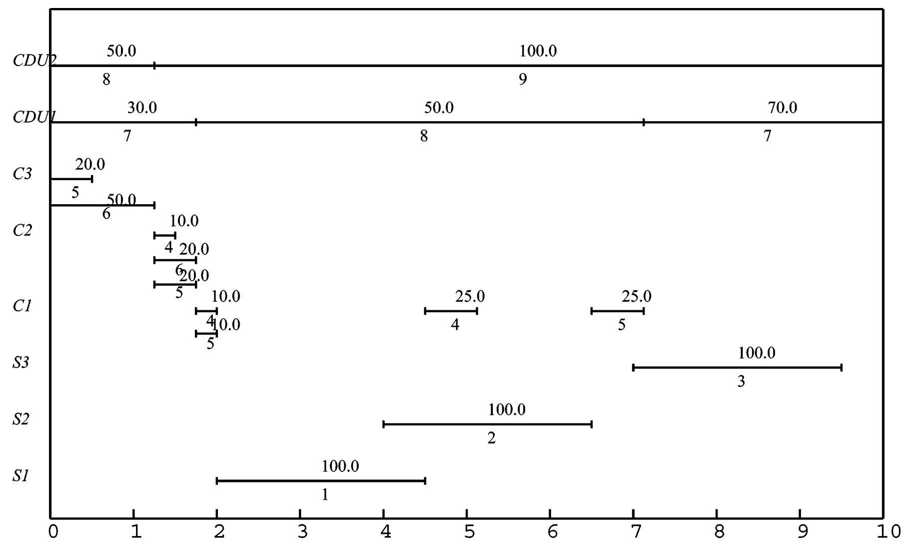

Figure 7. Gantt chart of fluid transfer schedule for Example 2.

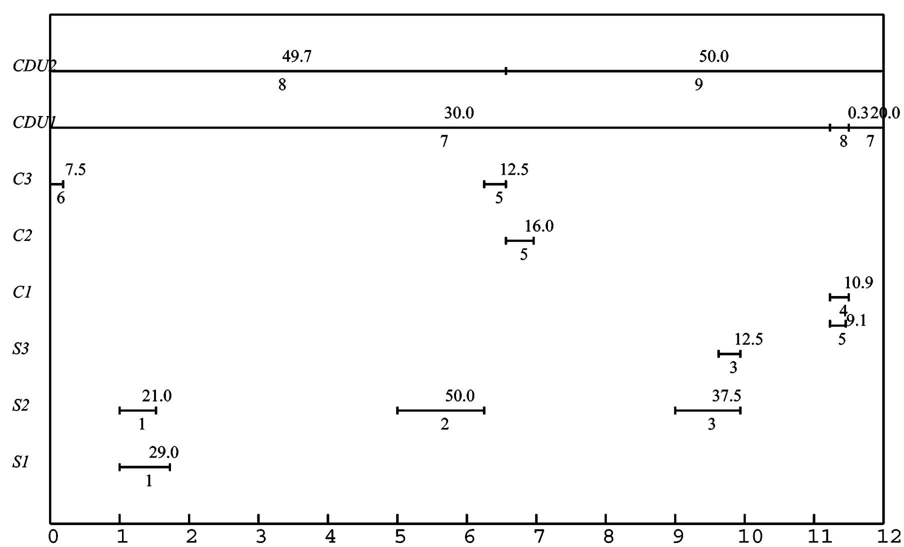

Figure 8. Gantt chart of fluid transfer schedule for Example 3.

was imposed. Figures 6−9 illustrate example schedules of solutions provided by the solver DICOPT.

4. Summary

A novel continuous-time formulation is proposed for the scheduling optimization of fluid transfer among tanks. This model generally and robustly addresses the difficult issue of properly mapping material balances with timing constraints. Under the stated assumptions, it exactly represents the problem without approximation and addresses all of the identified drawbacks of previous models. Modeling the input and output within the same time event can potentially reduce the number of binary variables by a factor of 2 for such problems and thus provides a significant reduction in combinatorial complexity. The model is formulated as a nonconvex mixed integer nonlinear programming (MINLP) problem, where the nonlinearity arises in the form of bilinear terms used in the calculation of component fractions for blending and pooling. Four example

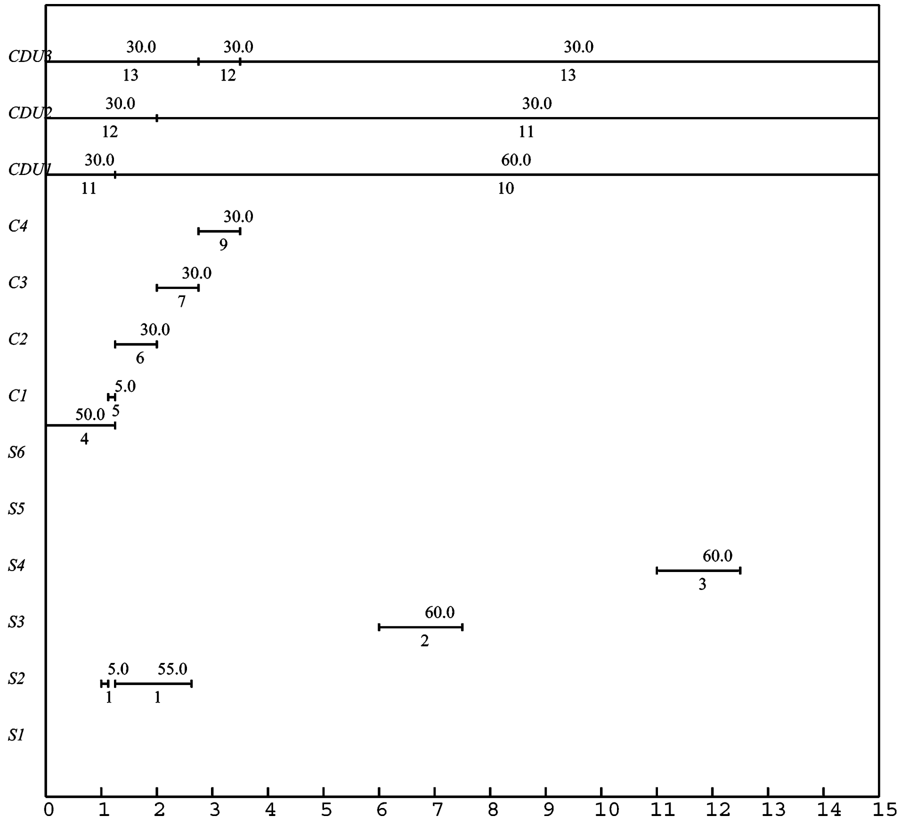

Figure 9. Gantt chart of fluid transfer schedule for Example 4.

cases from the literature for refinery scheduling are adapted, and these examples demonstrate that the proposed model can be solved efficiently. The focus of future work is directed toward developing more computationally efficient methods for solving the problem potentially, relative to the global optimum.

Nomenclature

Sets

A = set of tank input sources (section 2 only)
$\mathrm{A}^{\mathrm{Arr}}$ = set of crude oil arrivals (section 3 only)
B = set of all tanks (section 3 only)
B^{Ch} = set of charging tanks (section 3 only)
C = set of tank output destinations (section 2 only)
$\mathrm{C}^{\mathrm{CDU}}$ = set of crude distillation units (section 3 only)
J = set of component materials of fluid
T = set of time events

Parameters

I_{jb}^{init} = initial inventory of component j in tank b
$I_{b}^{\mathrm{init-tot}}$ = initial total inventory of tank b
I_{b}^{L} = lower bound on inventory of tank b
I_{b}^{U} = upper bound on inventory of tank b
F_{ab}^{L} = lower bound on the flow rate on transfers from a to b
F_{bc}^{L} = lower bound on the flow rate on transfers from b to c
F_{ab}^{U} = upper bound on the flow rate on transfers from a to b
F_{bc}^{U} = upper bound on the flow rate on transfers from b to c
H = time horizon
V_{ab}^{U} = upper bound on flow from a to b
V_{bc}^{U} = upper bound on flow from b to c
D_b = demand for crude oil in charging tank b
S_a = amount of crude oil corresponding to arrival a
f_{ja}^{Arr} = fraction of component j in crude oil arrival a
T_{ja}^{Arr} = arrival time crude oil arrival a
C^{Sea} = sea waiting (demurrage) cost
C^{Unload} = unloading cost
C_{b}^{Inv} = inventory holding cost for tank b
C^{Set} = unit changeover cost

Continuous Variables

V_{abt}^{tot} = total flow from a to b in time event t
V_{bct}^{tot} = total flow from b to c in time event t
V_{jabt} = flow of component j from a to b in time event t
V_{jbct} = flow of component j from b to c in time event t
V_{jabct}^{VJ} = artificial variable representing bilinear term $V_{abt}^{\mathrm{tot}}V_{jbct}$
V_{jabct}^{VT} = artificial variable representing bilinear term $V_{jabt}V_{bct}^{\mathrm{tot}}$
I_{bt}^{tot} = total inventory of tank b at the end of time event t
I_{jbt} = inventory of component j in tank b at the end of time event t
I_{jbct}^{VJ} = artificial variable representing bilinear term $I_{b,t-1}^{\mathrm{tot}}V_{jbct}$
I_{jbct}^{VT} = artificial variable representing bilinear term $I_{jb,t-1}V_{bct}^{\mathrm{tot}}$
T_{abt}^{1} = starting time of a transfer from a to b in time event t
T_{bct}^{1} = starting time of a transfer from b to c in time event t
T_{abt}^{2} = ending time of a transfer from a to b in time event t
T_{bct}^{2} = ending time of a transfer from b to c in time event t
T_{a}^{start} = starting time for arrival a time window
T_{a}^{end} = ending time for arrival a time window

Binary Variables

w_{abt} = has a value of 1 when there is flow from a to b in interval t; otherwise, it has a value of 0
w_{bct} = has a value of 1 when there is flow from b to c in interval t; otherwise, it has a value of 0

Appendix

Proposition 1. Transfer between two tanks in both directions in the same time event is implicitly restricted.

Proof. *Let us consider a case with two tanks, called a and b. For the same time event t, we then have the following constraints:*

$$T^2_{abt} - H(1 - w_{abt}) \le T^1_{bat} + H(1 - w_{bat})$$
$$T^2_{bat} - H(1 - w_{bat}) \le T^1_{abt} + H(1 - w_{abt})$$

Suppose that both $w_{abt} = 1$ *and* $w_{bat} = 1$. *The aforementioned constraints then become*

$$T^2_{abt} \le T^1_{bat}$$
$$T^2_{bat} \le T^1_{abt}$$

Also, from the duration constraints, we know that

$$T^2_{abt} \ge T^1_{abt}$$
$$T^2_{bat} \ge T^1_{bat}$$

Therefore, we now have

$$T^1_{bat} \ge T^2_{abt} \ge T^1_{abt} \ge T^2_{bat} \ge T^1_{bat}$$

Thus, via the squeeze rule, we have

$$T^1_{bat} = T^2_{abt} = T^1_{abt} = T^2_{bat}$$

and, therefore,

$$T^1_{bat} = T^2_{bat}$$
$$T^1_{abt} = T^2_{abt}$$

Thus, $V^{tot}_{abt} = 0$ *and* $V^{tot}_{bat} = 0$, *because of our duration constraints. This shows that adding the constraint*

$$w_{abt} + w_{bat} \le 1$$

would simply be adding a redundant constraint, because, if both binary variables had a value of 1, both corresponding volumes would need to be zero. Therefore, restriction on transfer between two tanks in both directions in the same time event is implicitly enforced. However, adding the redundant constraint might tighten the relaxations created in branch-and-bound.

■

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