A Tuning Strategy for Unconstrained Multivariable Model Predictive Control

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Move suppression coefficients serve a dual purpose in the model predictive controller (MPC) architecture. These include suppressing aggressive control action and conditioning the system matrix prior to inversion. The work presented here exploits this dual effect in deriving an analytical expression that computes appropriate move suppression coefficients as a function of process model parameters, other MPC design parameters, and partitioned block condition numbers of the system matrix. The development is based upon an approximate mosaic Hankel matrix structure of the multivariable system matrix. The primary contribution of this work is the derivation of the analytical expression for computing move suppression coefficients and its demonstration in an overall MPC tuning strategy (Table 1). The examples presented show that the move suppression coefficient remains properly scaled as the other MPC design parameters and process characteristics change to produce a consistent closed loop performance. This tuning method is applicable to unconstrained multivariable processes, including non-square systems.

Introduction

Model predictive control (MPC) has established itself as an industrially important form of advanced control (Richalet, 1993; Deshpande et al., 1995; VanDoren, 1997). Several review articles consider MPC from an academic perspective (García et al., 1989; Morari and Lee, 1991; Ricker, 1991; Muske and Rawlings, 1993; Lee and Cooley, 1997) and from an industrial perspective (Prett and García, 1988; Richalet, 1993; Clarke, 1994; Froisy, 1994, Camacho and Bordons, 1995; Qin and Badgwell, 1997). The "moving horizon" concept of MPC, a key feature that distinguishes it from classical controllers, is illustrated in Figure 1.

Dynamic matrix control (DMC) (Cutler and Ramaker, 1980; Cutler, 1983) is one of the most popular model predictive controllers used in the chemical process industry (Qin and Badgwell, 1997). For that reason, this work is presented in the context of tuning a DMC controller.

The scope of this work is limited to unconstrained DMC. Constraint handling, whereby multiple process and performance objectives are incorporated directly into the MPC formulation, is important in industrial applications and hence must be addressed in research that builds upon the results presented here.

This work represents a contribution, however, because it provides unique insights into the structure of the DMC control law. Specifically, it establishes how the dynamic character of a process and the selection of DMC tuning parameters dictate move suppression coefficient values required to obtain a consistent closed loop performance.

Tuning of both unconstrained and constrained MIMO DMC have been addressed by an array of researchers. These include systematic trial and error tuning procedures (e.g., Cutler, 1983; Ricker, 1991), and formal tuning techniques such as move suppression methods

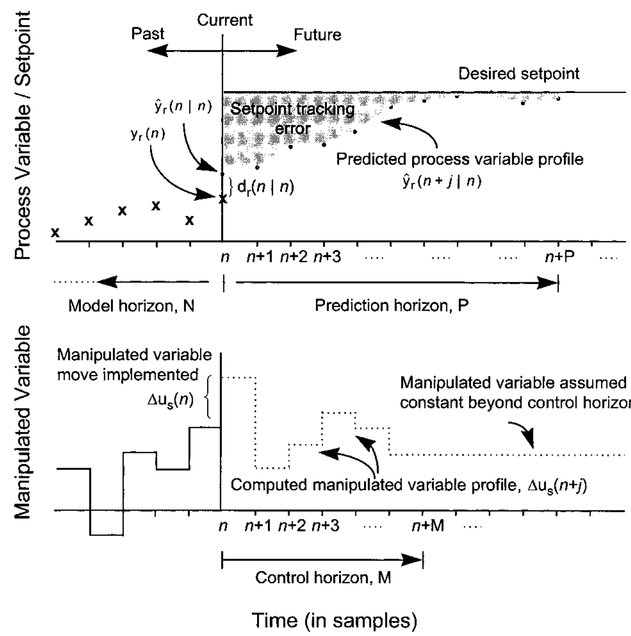

Figure 1. Moving horizon concept of model predictive control.

(e.g., Marchetti et al., 1983), input blocking (Ricker, 1985), the "$M = 1$" configuration (Maurath et al., 1988), and principal component selection (Maurath et al., 1985).

Other tuning strategies for DMC have concentrated on such aspects as tuning for stability (García and Morari, 1982; Clarke and Scattolini, 1991; Rawlings and Muske, 1993), robustness (Ohshima et al., 1991; Lee and Yu, 1994), and performance (McIntosh et al., 1992; Hinde and Cooper, 1995; Megan and Cooper, 1995; Palavajjhala et al., 1994). Use of inequality constraints as tuning devices has also been studied by several researchers (Subrahmanian and Ricker, 1989; Zafiriou, 1990; Zafiriou and Marchal, 1991b).

Building upon the work of these researchers, an earlier article (Shridhar and Cooper, 1997) confirmed that the move suppression coefficient can serve as the

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Table 1. DMC Tuning Strategy

1. Approximate the process dynamics of all controller output-process variable pairs with first order plus dead time (FOPDT) models:

$$\frac{y_r(s)}{u_s(s)} = \frac{K_{rs} e^{-\theta_{rs} s}}{\tau_{rs} s + 1} \quad (r = 1, 2, ..., S;\ s = 1, 2, ..., R)$$

2. Select the sample time as close as possible to:

$$\left\{ \begin{array}{l} T_{rs} = Max\,(0.1\tau_{rs},\ 0.5\theta_{rs}) \\ T = Min\,(T_{rs}) \end{array} \right\} \quad (\ r = 1,\ 2,\ ...,R;\ s = 1,\ 2,\ ...,S\ )$$

3. Compute the prediction horizon, P, and the model horizon, N:

$$P = N = Max\left(\frac{5\tau_{rs}}{T} + k_{rs}\right) \text{ where } k_{rs} = \left(\frac{\theta_{rs}}{T} + 1\right) \quad (r = 1, 2, ..., R;\ s = 1, 2, ..., S)$$

4. Select a control horizon, M, equal to 63.2% of the settling time of the slowest sub-process in the multivariable system:

$$M = Max\left(\frac{\tau_{rs}}{T} + k_{rs}\right) \quad (r = 1,\ 2,\ 3,\ \dots,\ R;\ s = 1,\ 2,\ \dots,\ S)$$

5. Select the controlled variable weights, γ_r^2, to scale process variable measurements to similar magnitudes.

6. Compute the move suppression coefficients, λ_s^2:

$$\lambda_s^2 = \frac{M}{500} \sum_{r=1}^{R} \left[\gamma_r^2 K_{rs}^2 \left\{ P - k_{rs} - \frac{3}{2}\frac{\tau_{rs}}{T} + 2 - \frac{(M-1)}{2} \right\} \right] \quad (s = 1, 2, ..., S)$$

7. Implement DMC using the traditional step response matrix of the actual process and the initial values of the parameters computed in steps 1 to 6:

Sample time, T	Control horizon (number of moves), M
Prediction horizon , P	Controlled variable weights, γ_r^2
Model horizon, N	Move suppression coefficients, λ_s^2

Fine-tune DMC by increasing the corresponding γ_r^2 of the process variable for which tighter control is desired and increasing the corresponding λ_s^2 of the manipulated variable for which less aggressive moves are desired.

primary adjustable parameter in tuning single-input single-output DMC. The contribution of that earlier work was an analytical expression to compute a move suppression coefficient such that the condition number of the system matrix maintained a target value irrespective of the choice of other tuning parameters. This in turn resulted in modest manipulated variable move sizes and consistent closed loop performance for the systems studied.

The work presented here formalizes this methodology for the more challenging multivariable case. The approach is to compute what now becomes a set of move suppression coefficients, λ_s^2 ($s = 1, 2, ..., S$), such that the condition numbers of the diagonal blocks in the overall system matrix are held to a target value.

The condition number of the system matrix relates the relative change in predicted error to the relative change in the manipulated variable. For multivariable systems, values of λ_s^2 that fix the condition number of a diagonal block in the system matrix at a low target value, irrespective of process characteristics and choice of the remaining adjustable parameters in DMC, ensures appropriate changes in the corresponding manipulated variable. When all diagonal block condition numbers are fixed in a similar fashion, the result is moderated controller activity and consistent closed loop performance.

Some popular guidelines found in the literature for computing MIMO DMC tuning parameters on the basis of process characteristics are summarized in Table 1. The methodology presented is applicable to a class of open loop stable, multivariable processes. A key contribution of this work is the addition of an analytical expression for the move suppression coefficients, λ_s^2, on the basis of process characteristics and these other adjustable DMC parameter values.

Derivation of the analytical expression for λ_s^2 considers a first order plus dead time (FOPDT) approximation of the process dynamics. It must be emphasized that this FOPDT approximation is employed only in the derivation of the analytical expression for λ_s^2. The examples presented later in this work all use the traditional DMC step response matrix of the actual process upon implementation.

The primary benefit of a FOPDT model approximation is that it permits derivation of a compact analytical expression for computing λ_s^2. Although a FOPDT model approximation does not capture all the features of some higher order processes, it often reasonably describes the process gain, overall time constant, and effective dead time of such processes (Cohen and Coon, 1953). In the past, tuning strategies based on a FOPDT model such as Cohen−Coon, IAE, and ITAE have proved useful for PID implementations. The tuning strategy presented here is significant because it offers an analogous approach for DMC.

The remainder of this paper is as follows: (i) The popular guidelines for computing the adjustable parameters in the MIMO DMC control law are briefly summarized. (ii) Starting with the DMC control law, an approximate partitioned block form of the system matrix is obtained and the diagonal and off-diagonal block structure is established. (iii) A mathematical analysis is detailed that supports the observation that the condition number of the overall system matrix can be held at a low value by fixing the diagonal block condition numbers at a low target value. (iv) An analytical expression is derived for the diagonal block condition number. (v) On the basis of this condition number, analytical expressions that compute λ_s^2 are derived. (vi) Issues important to the implementation of the overall tuning strategy are discussed and simulation examples that encompass a range of multivariable process characteristics are presented to validate the tuning strategy.

Role of the MIMO DMC Tuning Parameters

The adjustable parameters in unconstrained MIMO DMC that affect closed loop performance include the following: sample time, T; model horizon, N; finite prediction horizon, P; control horizon, M; controlled variable weights, γ_r^2 ($r = 1, 2, ..., R$); move suppression coefficients, λ_s^2 ($s = 1, 2, ..., S$). The tuning challenge presented by this array of adjustable parameters is significant since many of the parameters have overlapping effects on closed-loop performance.

Past research has provided a wealth of information about their qualitative effects on closed loop performance (e.g., Cutler, 1983; Marchetti et al., 1983; García and Morshedi, 1986; Downs et al., 1988; Maurath et al., 1988; Lee and Yu, 1994; Lundström, et al., 1995). Popular guidelines for the selection of some of the above adjustable parameters are described below and are summarized in the overall tuning strategy for MIMO DMC outlined in Table 1.

Sample time (*T*) is not suitable as a tuning parameter since it is often fixed on the basis of the equipment at an installation. Additionally, the choice of T impacts those adjustable parameters that are specified in sampling intervals, including model horizon, N, prediction horizon, P, and control horizon, M. Altering T without re-adjusting these other parameters can lead to undesirable results (e.g., Georgiou, et al., 1988; Lundström, et al., 1995).

In the event that sample time can be specified freely, a popular guideline selects T on the basis of a FOPDT model approximation of process dynamics as

$$\left\{\begin{matrix} T_{rs} = Max(0.1\tau_{rs}, 0.5\theta_{rs}) \\ T = Min(T_{rs}) \end{matrix}\right\} \quad (r = 1, 2, ..., R;\ s = 1, 2, ..., S) \quad (1)$$

where τ_{rs} is the overall time constant and θ_{rs} is the effective dead time of a subprocess and a subprocess is as defined in eq 4. The motivation for this guideline is that, in a multivariable process, every subprocess must be sampled and appropriate control action taken about 10 times per time constant or 2 to 3 times per effective dead time, whichever is larger (Stephanopoulos, 1984; Seborg et al., 1986).

Model horizon (*N*) specifies the number of past manipulated variable moves employed by DMC in predicting the future behavior of the process variable. The model horizon, N, is not typically employed as a tuning parameter since truncation of the model horizon misrepresents the effect of past moves in the predicted process variable profile (modeling errors) and leads to an unpredictable closed loop performance (Lundström et al., 1995). Although a large N does not directly have an adverse affect on controller performance, it has the disadvantage that model predictions become computationally intensive (Maurath, et al., 1988). A popular approach is to specify a model horizon equal to the open loop settling time in samples of the slowest subprocess as shown in eq 2 (e.g., Ricker, 1991).

Prediction horizon *(P)* represents the number of samples into the future over which DMC computes the predicted process variable profile and minimizes the predicted error. In the absence of move suppression, a prediction horizon that is greater than the control horizon, i.e., $P > M$, must be selected to avoid a deadbeat effect. With move suppression present, increasing P from this lower limit has a significant nonmonotonic effect on closed loop performance (Soeterboek, 1992), ranging from damped to underdamped to damped when P is close to M and both are small (Maurath et al., 1985).

A rigorous treatment by past researchers (García and Morari, 1982; Clarke and Scattolini, 1991; Rawlings and Muske, 1993) has shown that a larger prediction horizon improves nominal stability of the closed loop. For practical applications this translates to using a reasonably large but finite prediction horizon, P (Cutler, 1983), equal to the open loop settling time in samples of the slowest subprocess. On the basis of an FOPDT approximation of the process dynamics, P and N are both computed as

$$P = N = Max\left(\frac{5\tau_{rs}}{T} + k_{rs}\right) \quad (r = 1, 2, ..., R;\ s = 1, 2, ..., S) \quad (2)$$

where k_{rs} is the discrete dead time ($k_{rs} = \theta_{rs}/T + 1$) and θ_{rs} is the effective dead time of the subprocess.

Control horizon (*M*) is the number of manipulated variable moves that DMC computes at a given sample time to eliminate the current predicted error. A large M has the advantage that it allows detection of constraint violations before they are reached, averages the control objective over time, and handles unknown variable time delays (Ogunnaike, 1986; Fisher, 1991). Rawlings and Muske (1993) have shown that stability of infinite horizon MPC can be guaranteed if M is greater than or equal to the number of unstable modes in the process. Cutler (1983) suggested increasing the control horizon until changes in M have no further effect on the first move of the controller to a step change in set point. Another method proposed by Cutler (1983) recommended that the control horizon be selected on the basis of process dynamics; i.e., $M \times T$ should be larger than the time required for the slowest open loop response in the multivariable system to reach 60% of the steady state. Employing this rule-of-thumb using an FOPDT approximation of the process dynamics, the

Table 2. Transfer Functions of Subprocesses Used To Build Challenging Multivariable Processes and Their Corresponding FOPDT Model Approximations for Use in the Tuning Strategy

	first order plus dead time model approximation: $G(s) = K_{rs}e^{-\theta_{rs}s}/(\tau_{rs}s + 1)$			
actual higher order subprocess	challenging characteristics	gain (K_{rs})	effective time const (τ_{rs})	overall dead time (θ_{rs})
$G_1(s) = \dfrac{4e^{-20s}}{(30s+1)^3}$	large gain	4.04	62.11	52.56
$G_2(s) = \dfrac{e^{-20s}}{(80s+1)^3}$	large time constant	1.02	175.72	104.65
$G_3(s) = \dfrac{1}{(5s+1)^3}$	short time constant; no dead time	1.00	10.17	4.94
$G_4(s) = \dfrac{e^{-150s}}{(30s+1)^3}$	large dead time	1.02	63.91	181.88
$G_5(s) = \dfrac{(-50s+1)e^{-20s}}{(30s+1)^3}$	finite right-half plane zero	1.01	54.36	97.55
$G_6(s) = \dfrac{(-100s+1)e^{-20s}}{(30s+1)^3}$	negative process gain; finite left-half plane zero	−1.06	12.23	32.48

control horizon, M, can be computed as

$$M = Max\left(\frac{\tau_{rs}}{T} + k_{rs}\right) \quad (r = 1, 2, 3, ..., R;\ s = 1, 2, ..., S) \quad (3)$$

Controlled variable weights (γ_r^2, $r = 1, 2, ..., R$) are adjustable parameters used to scale and then selectively weight the process variable measurements so that the error in each process variable is of a magnitude representative of its importance in the performance objective. When a single controlled variable weight is increased while maintaining the others constant, the set point tracking response of the corresponding process variable has a faster rise time. This faster response is at the expense of more aggressive moves in the manipulated variable most closely related to this process variable. When all controlled variable weights, γ_r^2 ($r = 1, 2, ..., R$), are increased simultaneously, the overall response becomes more aggressive, much like reducing the move suppression coefficients, λ_s^2 ($s = 1, 2, ..., S$). Since γ_r^2 do not affect the invertibility of the overall system matrix, they can be specified by the user on the basis of control objective priorities or they can be reserved to fine-tune MIMO DMC to desired performance once on-line.

Move suppression coefficients (λ_s^2, $s = 1, 2, ..., S$) serve a dual purpose in the DMC control law of suppressing aggressive control action and conditioning the system matrix prior to inversion. To illustrate how move suppression impacts DMC performance, consider this 2 × 2 base case process:

$$\text{process 1} \quad \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} G_1(s) & G_1(s) \\ G_1(s) & G_2(s) \end{bmatrix} \times \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \quad (4)$$

In eq 4, $y_1(s)$ and $y_2(s)$ are the process variables and $u_1(s)$ and $u_2(s)$ are the manipulated variables. The base case process is comprised of three identical subprocesses, $G_1(s)$, and a fourth subprocess, $G_2(s)$, with transfer functions as shown in Table 2. A relative gain array (RGA) analysis (Bristol, 1966) of this process yields the first element corresponding to the y_1−u_1 pair as −0.34. This indicates significant cross-loop interactions where u_1 provokes a retaliatory effect from u_2.

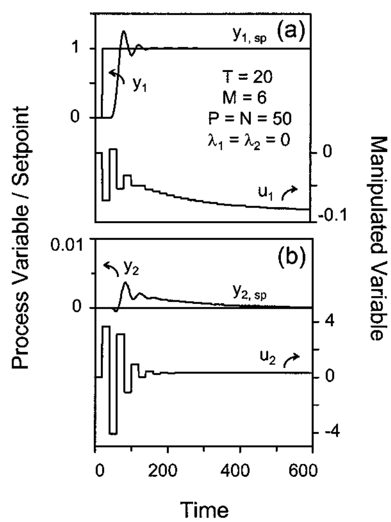

Figure 2. Impact of no move suppression ($\lambda_1^2 = \lambda_2^2 = 0$) on closed loop performance for process 1.

Figure 2 illustrates the impact of no move suppression ($\lambda_s^2 = 0$; $s = 1, 2$) on closed loop performance for a step change in set point for y_1. To tune a DMC controller using Table 1, a FOPDT model is first fit to each subprocess. These models are summarized in Table 2. Using these values and the rules in Table 1, the sample time is selected such that $T = 0.5\theta_{11}$, based on the large apparent dead time in $G_1(s)$. With this choice of sample time, P and N equal 50, which represents the complete settling time in samples of the slowest subprocess. The control horizon is selected as $M = 6$ to ensure that it is greater than the model order of the largest subprocess. $\gamma_r^2 = 1$ as no preference in importance is given to any process variable.

Figure 2, graph a, tracks the y_1−u_1 response to a step change in set point, and graph b tracks the y_2−u_2 response. Without move suppression, the set point tracking response of the process is extremely aggressive. As shown, y_1 exhibits significant overshoot and underdamped behavior, causing y_2 to oscillate significantly. The initial move sizes are aggressive for both u_1 and u_2.

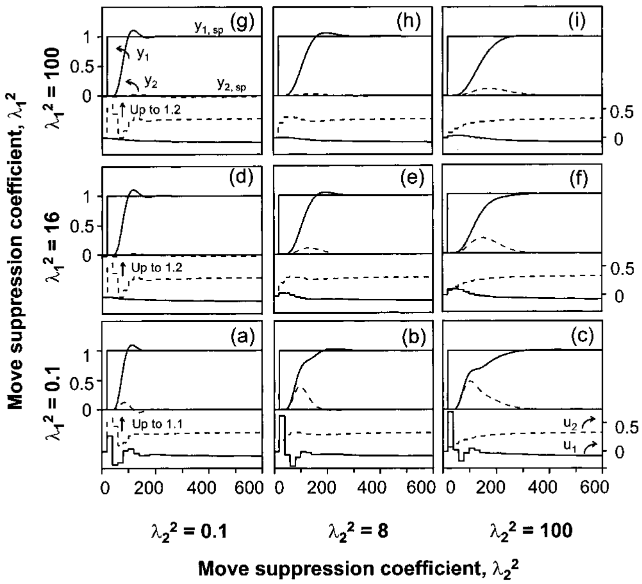

Figure 3. Combined effect of move suppression coefficients (λ_1^2 and λ_2^2) on closed loop performance for process 1 ($T = 20$, $P = N = 50$, $M = 6$, $\gamma_1^2 = \gamma_2^2 = 1$).

Figure 3 shows the effect of using positive move suppression coefficients, λ_1^2 and λ_2^2, of increasing magnitude and their combined impact on closed loop performance. A matrix of closed loop response results is generated for different choices of λ_1^2 and λ_2^2 while maintaining the other adjustable parameters constant at $T = 20$, $P = N = 50$, $M = 6$, and $\gamma_1^2 = \gamma_2^2 = 1$.

When both coefficients are small ($\lambda_1^2 = \lambda_2^2 = 0.1$), the manipulated variable move sizes are unacceptably large (graph a). An intermediate response (graph e) can be achieved with appropriate move suppression ($\lambda_1^2 = 16$, $\lambda_2^2 = 8$). Further increasing λ_s^2 (e.g., $\lambda_1^2 = \lambda_2^2 = 100$) can lead to an undesirable sluggish response (graph i). Hence, an appropriate choice of λ_s^2 is clearly important to the performance achieved by unconstrained MIMO DMC.

MIMO DMC Control Law

The DMC control law is summarized here to establish the nomenclature used in subsequent developments. Details of the derivation of the MIMO DMC control law are available in the literature (e.g., Cutler, 1983; García and Morshedi, 1986; Prett and García, 1988).

DMC is a least-squares optimization problem with a quadratic performance objective and a penalty on manipulated variable moves:

$$\underset{\Delta\bar{u}}{Min}\; J = [\bar{\boldsymbol{e}} - \boldsymbol{A}\Delta\bar{\boldsymbol{u}}]^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma[\bar{\boldsymbol{e}} - \boldsymbol{A}\Delta\bar{\boldsymbol{u}}] + [\Delta\bar{\boldsymbol{u}}]^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda[\Delta\bar{\boldsymbol{u}}] \tag{5}$$

The closed form solution to eq 5 is the unconstrained MIMO DMC control law:

$$\Delta\bar{\boldsymbol{u}} = (\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda)^{-1}\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\bar{\boldsymbol{e}} \tag{6}$$

In eq 6, $\Delta\bar{\boldsymbol{u}}$ is the vector of manipulated variable moves to be determined, $\boldsymbol{A}$ is the multivariable dynamic matrix, $\bar{\boldsymbol{e}}$ is the vector of set point tracking error, $\Gamma^{\mathrm{T}}\Gamma$ is the matrix of controlled variable weights, and $\Lambda^{\mathrm{T}}\Lambda$ is the matrix of move suppression coefficients.

The multivariable dynamic matrix, $\boldsymbol{A}$, in the DMC control law (eq 6) is a system of subprocesses:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \cdots & \boldsymbol{A}_{1s} & \cdots & \boldsymbol{A}_{1S} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} & \cdots & \boldsymbol{A}_{2s} & \cdots & \boldsymbol{A}_{2S} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ \boldsymbol{A}_{r1} & \boldsymbol{A}_{r2} & \cdots & \boldsymbol{A}_{rs} & \cdots & \boldsymbol{A}_{rS} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ \boldsymbol{A}_{R1} & \boldsymbol{A}_{R2} & \cdots & \boldsymbol{A}_{Rs} & \cdots & \boldsymbol{A}_{RS} \end{bmatrix}_{P\cdot R \times M\cdot S} \tag{7}$$

where each $\boldsymbol{A}_{rs}$ with dimensions $P \times M$ is formed from unit step response coefficients, $a_{rs,i}$, of the subprocess relating the rth process variable to the sth manipulated variable.

The matrix of controlled variable weights, $\Gamma^{\mathrm{T}}\Gamma$, has γ_r^2 ($r = 1, 2, ..., R$) as the leading diagonal elements of the rth diagonal matrix block, or

$$\Gamma^{\mathrm{T}}\Gamma = \begin{bmatrix} \gamma_1^2\mathbf{I}_{P\times P} & 0 & \cdots & 0 \\ 0 & \gamma_2^2\mathbf{I}_{P\times P} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_R^2\mathbf{I}_{P\times P} \end{bmatrix}_{P\cdot R \times P\cdot R} \tag{8}$$

From eqs 7 and 8, the system matrix, $\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A}$, is obtained in block form as

$$\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} = \begin{bmatrix} \boldsymbol{B}_{11} & \boldsymbol{B}_{12} & \cdots & \boldsymbol{B}_{1w} & \cdots & \boldsymbol{B}_{1S} \\ \boldsymbol{B}_{21} & \boldsymbol{B}_{22} & \cdots & \boldsymbol{B}_{2w} & \cdots & \boldsymbol{B}_{2S} \\ \vdots & \vdots & & \vdots & & \vdots \\ \boldsymbol{B}_{v1} & \boldsymbol{B}_{v2} & \cdots & \boldsymbol{B}_{vw} & \cdots & \boldsymbol{B}_{vS} \\ \vdots & \vdots & & \vdots & & \vdots \\ \boldsymbol{B}_{S1} & \boldsymbol{B}_{S2} & \cdots & \boldsymbol{B}_{Sw} & \cdots & \boldsymbol{B}_{SS} \end{bmatrix}_{M\cdot S \times M\cdot S} \tag{9}$$

where each of the blocks, $\boldsymbol{B}_{vw}$ ($v = 1, 2, ..., S$; $w = 1, 2, ..., S$) is of dimensions $M \times M$.

The matrix of move suppression coefficients, $\Lambda^{\mathrm{T}}\Lambda$, has λ_s^2 ($s = 1, 2, ..., S$) as the leading diagonal elements of the sth diagonal matrix block, or

$$\Lambda^{\mathrm{T}}\Lambda = \begin{bmatrix} \lambda_1^2\mathbf{I}_{M\times M} & 0 & \cdots & 0 \\ 0 & \lambda_2^2\mathbf{I}_{M\times M} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_S^2\mathbf{I}_{M\times M} \end{bmatrix}_{M\cdot S \times M\cdot S} \tag{10}$$

Addition of the move suppression coefficients in eq 10 to the system matrix in eq 9 yields the overall system matrix:

$$\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda = \begin{bmatrix} \boldsymbol{D}_{11} & \boldsymbol{B}_{12} & \cdots & \boldsymbol{B}_{1S} \\ \boldsymbol{B}_{21} & \boldsymbol{D}_{22} & \cdots & \boldsymbol{B}_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{B}_{S1} & \boldsymbol{B}_{S2} & \cdots & \boldsymbol{D}_{SS} \end{bmatrix}_{M\cdot S \times M\cdot S} \tag{11}$$

where $\boldsymbol{D}_{ss} = \boldsymbol{B}_{ss} + \lambda_s^2\mathbf{I}$ ($s = 1, 2, ..., S$).

Analysis of the MIMO DMC Control Law

The analysis of the MIMO DMC control law presented here involves development of an approximate partitioned block form of the system matrix, $\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A}$. This partitioned block form can then be used to derive analytical expressions for the move suppression coefficients, λ_s^2. Such a form of the system matrix is made possible with the use of FOPDT model approximations of all higher order subprocesses in the multivariable process.

The primary benefit of FOPDT model approximations is that they result in a systematic block structure of the system matrix that lends itself to convenient analysis and permits derivation of compact analytical expressions for computing λ_s^2. However, it must be emphasized here that these FOPDT approximations are employed only in the derivation of the analytical expression for λ_s^2. The examples presented later in this work all use the traditional multivariable dynamic matrix generated from step response coefficients of the actual process for controller implementation.

An Approximate Partitioned Block Form of the System Matrix. Using a FOPDT model with zero-order hold, the step response coefficients of a subprocess relating the rth process variable and the sth manipulated variable are given by

$$a_{rs,i} = \begin{cases} 0 & 0 \le j \le k_{rs} - 1 \\ K_{rs}(1 - \mathrm{e}^{-(j-k_{rs}+1)(T/\tau_{rs})}) & k_{rs} \le j \end{cases} \tag{12}$$

where K_{rs} is the process gain, τ_{rs} is the overall process time constant, T is the sample time, and k_{rs} is the discrete dead time.

For the form of the step response coefficients in eq 12, all diagonal blocks of the system matrix in eq 9 have a similar general form, $\boldsymbol{B}_{ss}$ ($v = w = s$; $s = 1, 2, ..., S$), given by

$$\boldsymbol{B}_{ss} = \begin{bmatrix} \sum_{r=1}^{R}\gamma_r^2 K_{rs}^2 \sum_{i=1}^{P-k_{rs}+1}(1-\mathrm{e}^{-iT/\tau_{rs}})^2 & \sum_{r=1}^{R}\gamma_r^2 K_{rs}^2 \sum_{i=2}^{P-k_{rs}+1}(1-\mathrm{e}^{-iT/\tau_{rs}})(1-\mathrm{e}^{-(i-1)T/\tau_{rs}}) & \cdots \\ \sum_{r=1}^{R}\gamma_r^2 K_{rs}^2 \sum_{i=2}^{P-k_{rs}+1}(1-\mathrm{e}^{-iT/\tau_{rs}})(1-\mathrm{e}^{-(i-1)T/\tau_{rs}}) & \sum_{r=1}^{R}\gamma_r^2 K_{rs}^2 \sum_{i=2}^{P-k_{rs}+1}(1-\mathrm{e}^{-(i-1)T/\tau_{rs}})^2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}_{M\times M} \tag{13}$$

and all off-diagonal blocks have a general form, $\boldsymbol{B}_{vw}$ ($v, w = 1, 2, ..., S$; $v \neq w$), given by

$$\boldsymbol{B}_{vw} = \begin{bmatrix} \sum_{r=1}^{R}\gamma_r^2 K_{rv}K_{rw} \sum_{i=1}^{P-k_m+1}\begin{pmatrix}(1-\mathrm{e}^{-(i+ik_m-k_{rv})T/\tau_{rv}})\times \\ (1-\mathrm{e}^{-(i+k_m-k_{rw})T/\tau_{rw}})\end{pmatrix} & \sum_{r=1}^{R}\gamma_r^2 K_{rv}K_{rw} \sum_{i=2}^{P-k_m+1}\begin{pmatrix}(1-\mathrm{e}^{-(i+k_m-k_{rv})T/\tau_{rv}})\times \\ (1-\mathrm{e}^{-(i-1+k_m-k_{rw})T/\tau_{rw}})\end{pmatrix} & \cdots \\ \sum_{r=1}^{R}\gamma_r^2 K_{rv}K_{rw} \sum_{i=2}^{P-k_m+1}\begin{pmatrix}(1-\mathrm{e}^{-(i-1+k_m-k_{rv})T/\tau_{rv}})\times \\ (1-\mathrm{e}^{-(i+k_m-k_{rw})T/\tau_{rw}})\end{pmatrix} & \sum_{r=1}^{R}\gamma_r^2 K_{rv}K_{rw} \sum_{i=2}^{P-k_m+1}\begin{pmatrix}(1-\mathrm{e}^{-(i-1+k_m-k_{rv})T/\tau_{rv}})\times \\ (1-\mathrm{e}^{-(i-1+k_m-k_{rw})T/\tau_{rw}})\end{pmatrix} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}_{M\times M} \tag{14}$$

In eq 14, $k_m = Max(k_{rv}, k_{rw})$.

Individual terms of the diagonal matrix blocks (eq 13) and off-diagonal matrix blocks (eq 14) can be approximated for a large prediction horizon, P, and small sample times, T. Let $\beta_{vw,ij}$ ($v = w = s = 1, 2, ..., S$; $i, j = 1, 2, ..., M$) represent the term in the ith row and jth column of the sth diagonal block. Then the first term in the sth diagonal block approximates to

$$\begin{aligned} \beta_{ss,11} &= \sum_{r=1}^{R}\gamma_r^2 K_{rs}^2 \sum_{i=1}^{P-k_{rs}+1}(1-\mathrm{e}^{-iT/\tau_{rs}})^2 \\ &= \sum_{r=1}^{R}\gamma_r^2 K_{rs}^2 \sum_{i=1}^{P-k_{rs}+1}(1-2\mathrm{e}^{-iT/\tau_{rs}}+\mathrm{e}^{-2iT/\tau_{rs}}) \\ &\cong \sum_{r=1}^{R}\gamma_r^2 K_{rs}^2\left\{(P-k_{rs}+1) - \frac{2(1-T/\tau_{rs})}{T/\tau_{rs}} + \frac{(1-2T/\tau_{rs})}{2T/\tau_{rs}}\right\} \\ &\cong \sum_{r=1}^{R}\gamma_r^2 K_{rs}^2\left(P-k_{rs}-\frac{3}{2}\frac{\tau_{rs}}{T}+2\right) \end{aligned} \tag{15}$$

The approximation in eq 15 becomes increasingly accurate as P increases and is exactly true for an infinite horizon implementation of MIMO DMC. Also, accuracy of the approximation improves as T decreases. Validity of this approximation is further explored later in this section.

By the similar approximation of other terms of the diagonal block in eq 13, the form for $\boldsymbol{B}_{ss}$ is obtained as

$$\boldsymbol{B}_{ss} = \begin{bmatrix} \sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2}\left(P - k_{rs} - \frac{3\tau_{rs}}{2\,T} + 2\right) & \sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2}\left(P - k_{rs} - \frac{3\tau_{rs}}{2\,T} + 2\right) - \frac{1}{2}\sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2} & \cdots \\ \sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2}\left(P - k_{rs} - \frac{3\tau_{rs}}{2\,T} + 2\right) - \frac{1}{2}\sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2} & \sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2}\left(P - k_{rs} - \frac{3\tau_{rs}}{2\,T} + 2\right) - \sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}_{M\times M} \tag{16}$$

The diagonal block in eq 16 has the systematic form

$$\left.\begin{aligned} \boldsymbol{B}_{ss} &= [b_{ij}] \\ b_{ij} &= \beta_{ss} - (i + j - 2)\alpha_{ss} \end{aligned}\right\} \quad (i, j = 1, 2, ..., M) \tag{17}$$

where

$$\beta_{ss} = \sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2}\left(P - k_{rs} - \frac{3\tau_{rs}}{2\,T} + 2\right) \tag{18}$$

$$\alpha_{ss} = \frac{1}{2}\sum_{r=1}^{R}\gamma_r^{\,2}K_{rs}^{\,2} \tag{19}$$

and $s = 1, 2, ..., S$. Equation 17 shows that the diagonal block of the system matrix has a Hankel matrix form with the added feature that the elements of every row successively decrease by a constant quantity β_{ss} from left to right. Also, the diagonal block is symmetric along the leading diagonal and singular when $M \geq 3$.

A similar exercise in simplification for eq 14 leads to the following systematic form for the off-diagonal blocks:

$$\left.\begin{aligned} \boldsymbol{B}_{vw} &= [b_{ij}] \\ b_{ij} &= \beta_{vw} - (i - 1)\delta_{vw} - (j - 1)\rho_{vw} \end{aligned}\right\} \quad (i, j = 1, 2, ..., M) \tag{20}$$

where

$$\beta_{vw} = \sum_{r=1}^{R}\gamma_r^{\,2}K_{rv}K_{rw}\left(P - (k_{rs} + k_{rw}) - \frac{(\tau_{rv} + \tau_{rw})}{T} + \frac{1 + T\left(\frac{k_{rv}}{\tau_{rv}} + \frac{k_{rw}}{\tau_{rw}}\right)}{T\left(\frac{1}{\tau_{rv}} + \frac{1}{\tau_{rw}}\right)} + 2\right) \tag{21}$$

$$\rho_{vw} = \sum_{r=1}^{R}\gamma_r^{\,2}K_{rv}K_{rw}\left(\frac{\tau_{rw}}{\tau_{rv} + \tau_{rw}}\right) \tag{22}$$

$$\delta_{vw} = \sum_{r=1}^{R}\gamma_r^{\,2}K_{rv}K_{rw}\left(\frac{\tau_{rv}}{\tau_{rv} + \tau_{rw}}\right) \tag{23}$$

and v, $w = 1, 2, ..., S$; $v \neq w$.

The partitioned block system matrix shown in eqs 9, 17, and 20 is a reasonable approximation of the actual system matrix for computing condition numbers. As evidence of this observation, Figure 4 provides a comparison of system matrices for process 1 (base case) with $P = Max(5\tau_{rs}/T + k_{rs})$ and $T = 0.5\theta_{rs}$ (here, $r = s = 1$).

Figure 4 compares the diagonal block condition numbers, c_s, as a function of the move suppression coefficients, λ_s^2 ($\lambda_1^2 = \lambda_2^2$) for three forms of the overall system matrix: (1) the actual overall system matrix for the higher order multivariable process; (2) the FOPDT model approximation of eqs 13 and 14; (3) the partitioned block form of eqs 17 and 20. From this figure it can be seen that for large P and small T the diagonal block condition numbers of the partitioned block system matrix (eqs 9, 17, and 20) closely follow the diagonal block condition numbers of the actual system matrix for a broad range of move suppression coefficients.

Analysis of the Partitioned Block Form of the Overall System Matrix. The condition number of the overall system matrix, $(\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda)$, can be held at an overall target value by fixing each of the condition numbers of its individual diagonal blocks at target values. To understand how this reduction in complexity is valid, consider the overall system matrix in partitioned block form (eqs 11, 17, and 20) as a sum of two matrices:

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{D}_{11} & 0 & \cdots & 0 \\ 0 & \boldsymbol{D}_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{D}_{SS} \end{bmatrix}_{M\cdot S \times M\cdot S}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & \boldsymbol{B}_{12} & \cdots & \boldsymbol{B}_{1S} \\ \boldsymbol{B}_{21} & 0 & \cdots & \boldsymbol{B}_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{B}_{S1} & \boldsymbol{B}_{S2} & \cdots & 0 \end{bmatrix}_{M\cdot S \times M\cdot S} \quad (24)$$

With this notation, the overall system matrix is equivalent to $\boldsymbol{D} + \boldsymbol{B}$.

For this form of overall system matrix, general bounds on eigenvalues can be obtained from the result that if $\boldsymbol{D}$ and $\boldsymbol{D} + \boldsymbol{B}$ are two $n \times n$ symmetric matrices, then

$$\mu_k(\boldsymbol{D}) + \mu_n(\boldsymbol{B}) \le \mu_k(\boldsymbol{D} + \boldsymbol{B}) \le \mu_k(\boldsymbol{D}) + \mu_1(\boldsymbol{B}) \quad (25)$$

where μ_k ($k = 1, 2, ..., n$) represents the n eigenvalues of a square matrix arranged in descending order of magnitude (Golub and Van Loan, 1989).

To find an upper bound on the maximum eigenvalue of the overall system matrix, $\boldsymbol{D} + \boldsymbol{B}$, the right hand inequality is written for $k = max$:

$$\mu_{max}(\boldsymbol{D} + \boldsymbol{B}) \le \mu_{max}(\boldsymbol{D}) + \mu_{max}(\boldsymbol{B}) \quad (26)$$

Equation 26 requires that the maximum eigenvalue of $\boldsymbol{D}$, $\mu_{max}(\boldsymbol{D})$, and the maximum eigenvalue of $\boldsymbol{B}$, $\mu_{max}(\boldsymbol{B})$, be determined.

The maximum eigenvalue of $\boldsymbol{D}$ can be found by

$$|\boldsymbol{D} - \mu\mathbf{I}| = 0 \qquad \text{or} \qquad \prod_{s=1}^{S} |\boldsymbol{D}_{ss} - \mu\mathbf{I}_{M \times M}| = 0 \quad (27)$$

Therefore,

$$\mu_{max}(\boldsymbol{D}) = \mu_{max}(\boldsymbol{D}_{11}, \boldsymbol{D}_{22}, ..., \boldsymbol{D}_{SS}) \quad (28)$$

An upper bound on the maximum eigenvalue of $\boldsymbol{B}$ can be found by using the triangle inequality property of a norm (maximum eigenvalue of a matrix):

$$\mu_{max}(\boldsymbol{B}) \le 2\,(\sum_{i=1}^{S} \sum_{j=i+1}^{S} \mu_{max}(\boldsymbol{B}_{ij})) \quad (29)$$

Combining the results in eqs 28 and 29 with eq 26 gives

$$\mu_{max}(\boldsymbol{D} + \boldsymbol{B}) = \mu_{max}(\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda) \le \mu_{max}(\boldsymbol{D}_{11}, \boldsymbol{D}_{22}, ..., \boldsymbol{D}_{SS}) + 2(\sum_{i=1}^{S} \sum_{j=i+1}^{S} \mu_{max}(\boldsymbol{B}_{ij})) \quad (30)$$

The result presented in eq 30 is a general one; i.e., no specifics regarding the approximate form of the system matrix have been introduced yet, and the result holds true for any symmetric, square matrix. This result is now applied to two possible extreme forms of the multivariable system.

Completely Uncoupled Multivariable System (Trivial Case). When the multivariable system is completely uncoupled, every manipulated variable affects a unique process variable. Hence, the multivariable transfer function matrix can be rearranged such that the diagonal subprocesses have nonzero dynamics and the off-diagonal subprocesses that contain information about manipulated-to-process variable interactions exhibit no dynamics.

This translates to a MIMO DMC system matrix with off-diagonal blocks that are zero. Consequently, the maximum eigenvalues of all off-diagonal terms are zero, i.e., $\mu_{max}(\boldsymbol{B}_{vw}) = 0$ (v, $w = 1, 2, ..., S$; $v \neq w$):

$$2(\sum_{i=1}^{S} \sum_{j=i+1}^{S} \mu_{max}(\boldsymbol{B}_{ij})) = 0 \quad (31)$$

and eq 30 reduces to

$$\mu_{max}(\boldsymbol{D} + \boldsymbol{B}) = \mu_{max}(\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda) = \mu_{max}(\boldsymbol{D}_{11}, \boldsymbol{D}_{22}, ..., \boldsymbol{D}_{SS}) \quad (32)$$

Since each of the diagonal blocks in the partitioned block system matrix is singular for even a modest control horizon ($M \ge 3$) and all off-diagonal blocks are zero, the system matrix itself is singular. In fact, since each diagonal block is of rank 2, the system matrix has $S \times (M - 2)$ zero eigenvalues. Addition of move suppression coefficients, λ_s^2, to the leading diagonal of the system matrix shifts all its eigenvalues by the constant quantity of λ_s^2 (e.g., Hoerl and Kennard, 1970; Ogunnaike, 1986). Hence, the overall system matrix has S eigenvalues equal to λ_s^2 ($s = 1, 2, ..., S$) each with multiplicity ($M - 2$) and the minimum eigenvalue of the overall system matrix in partitioned block form is given by

$$\mu_{min}(\boldsymbol{D} + \boldsymbol{B}) = \mu_{min}(\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda) = Min(\lambda_1^{\,2}, \lambda_2^{\,2}, ..., \lambda_S^{\,2}) = \lambda_{min}^{\;\;2} \quad (33)$$

and the condition number of the overall system matrix is

$$c = Cond(\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda) = \frac{|\mu_{max}|}{|\mu_{min}|} = \frac{\mu_{max}(\boldsymbol{D}_{11}, \boldsymbol{D}_{22}, ..., \boldsymbol{D}_{SS})}{\lambda_{min}^{\;\;2}} \quad (34)$$

If all diagonal subprocesses have similar dynamics or if all λ_s^2 ($s = 1, 2, ..., S$) are equal, the condition number of the overall system matrix is equal to the largest of the diagonal block condition numbers. For such an uncoupled multivariable process, the condition number of the overall system matrix can be fixed at a low value by holding the condition numbers of all diagonal blocks at a low target value.

Completely Coupled Multivariable System (Worst Case of Ill-Conditioning). The worst possible case of ill-conditioning in a multivariable system occurs when all subprocesses in the multivariable system are identical and there is complete interaction between all manipulated and process variables. For this extreme case of ill-conditioning, all diagonal and off-diagonal blocks of the system matrix are identical.

When move suppression coefficients are added to the system matrix, the overall system matrix in partitioned block form (eq 11) is now comprised of identical off-diagonal blocks

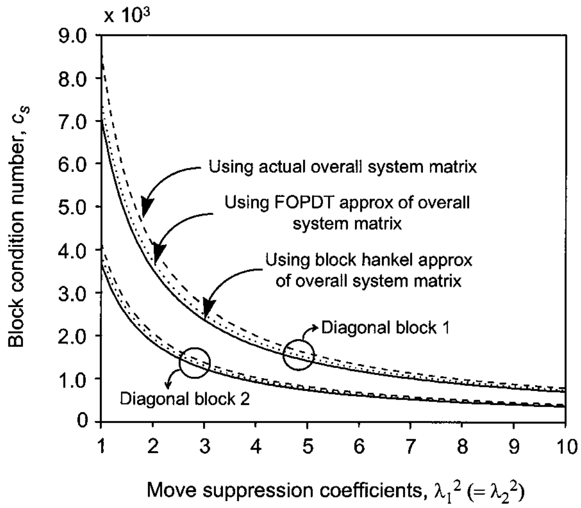

Figure 4. Validation of the block Hankel approximation of the overall system matrix in computing the condition numbers for process 1 ($T = 20 = 0.5\theta_{11}$, $M = 6$, $P = N = 50 = Max(5\tau_{rs}/T + k_{rs})$; $\gamma_1^2 = \gamma_2^2 = 1$).

$$\boldsymbol{B}_{vw} = \boldsymbol{B}_S \quad (v, w = 1, 2, ..., S; v \neq w) \tag{35}$$

and identical diagonal blocks

$$\boldsymbol{D}_{ss} = \boldsymbol{B}_S + \lambda_s^2\mathbf{I} \quad (s = 1, 2, ..., S) \tag{36}$$

For this case, the general result in eq 30 can be simplified to give an upper bound for the maximum eigenvalue of the overall system matrix:

$$\begin{aligned}\mu_{max}(\boldsymbol{D} + \boldsymbol{B}) &= \mu_{max}(\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda)\\ &\leq \mu_{max}(\boldsymbol{D}_{ss}) + 2\left(\sum_{i=1}^{S}\sum_{j=i+1}^{S}\mu_{max}(\boldsymbol{B}_S)\right)\\ &\leq \mu_{max}(\boldsymbol{B}_S + \lambda_s^2\mathbf{I}) + 2\left(\sum_{i=1}^{S}\sum_{j=i+1}^{S}\mu_{max}(\boldsymbol{B}_S)\right)\\ &\leq \mu_{max}(\boldsymbol{B}_S) + \lambda_{max}^2 + (S^2 - S)\mu_{max}(\boldsymbol{B}_S)\\ &\leq (S^2 - S + 1)\mu_{max}(\boldsymbol{B}_S) + \lambda_{max}^2 \end{aligned} \tag{37}$$

Since all partitioned blocks in the system matrix, $\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A}$, are identical and each partitioned block is individually singular for even a modest control horizon ($M \geq 3$), the overall system matrix is itself singular. Using an argument similar to the trivial case above, the minimum eigenvalue is found to be λ_{min}^2 (same as in eq 33). Using eqs 37 and 33, an upper bound for the condition number of the overall system matrix is

$$c = Cond(\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda) \leq \frac{(S^2 - S + 1)\mu_{max}(\boldsymbol{B}_S) + \lambda_{max}^2}{\lambda_{min}^2} \tag{38}$$

Since all manipulated-to-process variable dynamics are identical for this completely coupled system, all computed move suppression coefficients should be of the same magnitude, such that all manipulated variables are suppressed equally. If λ_s^2 ($s = 1, 2, ..., S$) = λ^2, the upper bound in eq 38 simplifies to

$$Cond(\boldsymbol{A}^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\boldsymbol{A} + \Lambda^{\mathrm{T}}\Lambda) \leq (S^2 - S + 1)\{Cond(\boldsymbol{B}_S) - 1\} + 1 \tag{39}$$

The result in eq 39 shows that, for the completely coupled multivariable process, the condition number of the overall system matrix is upper-bounded by a function that involves the condition number of an individual diagonal block. Even for the worst case of ill-conditioning, the condition number of the overall system matrix can be held at a low value by selecting λ_s^2 such that the condition numbers of all diagonal blocks are held to a low target value.

Figure 5 provides a visual appreciation for eq 39. The overall system matrix condition number, c, computed using both the FOPDT approximation and the partitioned block form for a 2×2 multivariable process with the worst case of ill-conditioning is plotted against λ_s^2 (here, $\lambda_1^2 = \lambda_2^2$). The upper bound defined by eq 39 is also plotted on the same graph. All subprocesses in the multivariable process considered here have identical transfer functions such that $G_{rs} = G_1$ ($r, s = 1, 2$). As indicated by the result in eq 39, the condition number of the overall system matrix does, in fact, lie below the upper bound for a broad range of move suppression coefficients.

Condition Number of a Diagonal Block in the Overall System Matrix. An analytical expression for the condition number of a diagonal block in the overall system matrix can be obtained by QR factorization of the diagonal block matrix to obtain its maximum and minimum eigenvalues.

Consider two linearly independent M-vectors:

$$\begin{aligned}\bar{\boldsymbol{h}}_1^{\mathrm{T}} &= (1 \;\; 1 \;\; 1 \;\cdots\; 1)_{1\times M}\\ \bar{\boldsymbol{h}}_2^{\mathrm{T}} &= (0 \;\; 1 \;\; 2 \;\cdots\; M-1)_{1\times M}\end{aligned} \tag{40}$$

The diagonal block, $\boldsymbol{B}_{ss}$ (eq 17), of the partitioned block system matrix can be written in terms of these basis vectors as

$$\boldsymbol{B}_{ss} = \bar{\boldsymbol{v}}\cdot\bar{\boldsymbol{h}}_1^{\mathrm{T}} + \bar{\boldsymbol{h}}_1\cdot\bar{\boldsymbol{v}}^{\mathrm{T}} \tag{41}$$

where $\bar{\boldsymbol{v}} = \beta_{ss}/2\ \bar{\boldsymbol{h}}_1 - \alpha_{ss}\bar{\boldsymbol{h}}_2$.

A Gram–Schmidt orthonormalization of $\bar{\boldsymbol{h}}_1$ and $\bar{\boldsymbol{h}}_2$ yields the orthonormal basis for $\boldsymbol{B}_{ss}$:

$$\bar{\boldsymbol{e}}_1^{\mathrm{T}} = \frac{1}{\sqrt{M}}(1 \quad 1 \quad 1 \ldots 1)_{1\times M}$$

$$\bar{\boldsymbol{e}}_2^{\mathrm{T}} = \sqrt{\frac{12}{M(M-1)(M+1)}} \times (0 \;\; 1 \;\; 2 \;\cdots\; M-1)_{1\times M} - \frac{(M-1)}{2} \times (1 \;\; 1 \;\; 1 \;\cdots\; 1)_{1\times M} \tag{42}$$

Therefore, $\bar{\boldsymbol{h}}_1$ and $\bar{\boldsymbol{h}}_2$ can be QR factored as

$$[\bar{\boldsymbol{h}}_1 \quad \bar{\boldsymbol{h}}_2]_{M\times 2} = [\bar{\boldsymbol{e}}_1 \quad \bar{\boldsymbol{e}}_2]_{M\times 2}\mathbf{R}_{2\times 2} \tag{43}$$

where $\mathbf{R}$ is upper triangular and invertible. Using eq 42, eq 41 can be transformed to

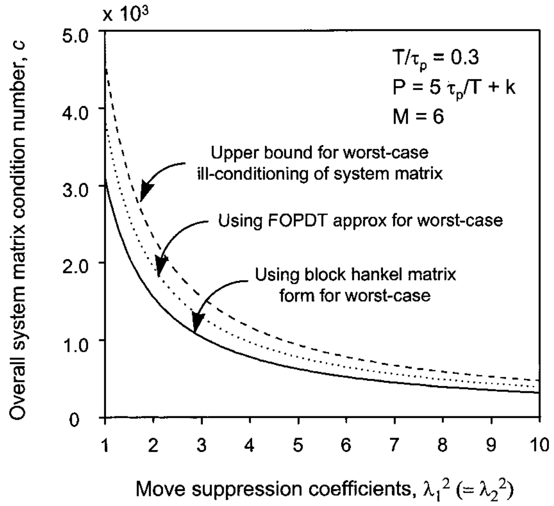

Figure 5. Illustration of the upper bound of the overall system matrix for the worst case of ill-conditioning ($T = 20$, $P = N = 50$, $M = 6$, $\gamma_1^2 = \gamma_2^2 = 1$).

$$\boldsymbol{B}_{ss} = [\bar{\boldsymbol{e}}_1 \;\; \bar{\boldsymbol{e}}_2 \;\vdots\; 0\,]_{M\times M} \begin{bmatrix} a & b & \vdots & 0 \\ b & 0 & & \\ \cdots & & \cdots & \cdots \\ 0 & & \vdots & 0 \end{bmatrix}_{M\times M} \times [\bar{\boldsymbol{e}}_1 \;\; \bar{\boldsymbol{e}}_2 \;\vdots\; 0\,]^T_{M\times M} \quad (44)$$

where a and b are simple linear functions of β_{ss} and α_{ss} given by

$$a = M\{\beta_{ss} - (M-1)\alpha_{ss}\}$$
$$b = -\frac{\alpha_{ss} M\sqrt{(M-1)(M+1)}}{\sqrt{12}} \quad (45)$$

Using $\boldsymbol{B}_{ss}$ from eq 44, the diagonal block in the overall system matrix becomes

$$\boldsymbol{D}_{ss} = \boldsymbol{B}_{ss} + \lambda_s^2 \mathbf{I}_{M\times M} = [\bar{\boldsymbol{e}}_1 \;\; \bar{\boldsymbol{e}}_2 \;\vdots\; 0\,]_{M\times M} \times \begin{bmatrix} a+\lambda_s^2 & b & \vdots & 0 \\ b & \lambda_s^2 & & \\ \cdots & & \cdots & \cdots \\ 0 & & \vdots & \lambda_s^2 \mathbf{I} \end{bmatrix}_{M\times M} [\bar{\boldsymbol{e}}_1 \;\; \bar{\boldsymbol{e}}_2 \;\vdots\; 0\,]^T_{M\times M} \quad (46)$$

Equation 46 provides a convenient way to determine explicit analytical expressions for the eigenvalues of the diagonal blocks in the overall system matrix. The bottom right partitioned block of the diagonal block, $\boldsymbol{D}_{ss}$, gives eigenvalues $\mu = \lambda_s^2$ with multiplicity $(M-2)$, and thus

$$\mu_{min} = \lambda_s^2 \quad (47)$$

The top left partitioned block of eq 46 yields two eigenvalues, the larger of which is

$$\mu_{max} = \left[M\beta_{ss} + 2\lambda_s^2 - M(M-1)\alpha_{ss} + M\sqrt{\beta_{ss}^2 - 2(M-1)\beta_{ss}\alpha_{ss} + \frac{2(M-1)(2M-1)}{3}\alpha_{ss}^2}\right]\Big/[2] \quad (48)$$

Therefore, the condition number, c_s, of a diagonal block in the overall system matrix is

$$c_s = \left[M\beta_{ss} + 2\lambda_s^2 - M(M-1)\alpha_{ss} + M\sqrt{\beta_{ss}^2 - 2(M-1)\beta_{ss}\alpha_{ss} + \frac{2(M-1)(2M-1)}{3}\alpha_{ss}^2}\right]\Big/[2\lambda_s^2] \quad (49)$$

where β_{ss} and α_{ss} are defined in eqs 18 and 19.

Analytical Expressions for the Move Suppression Coefficients

The expression for the condition number, c_s, derived above, combined with the previous result that the condition number of the system matrix is upper-bounded by the condition numbers of individual diagonal blocks, provides the necessary results to formulate analytical expressions that compute appropriate move suppression coefficients, λ_s^2. The diagonal block condition numbers can now be conveniently held at a low target value by using eq 49 to select appropriate move suppression coefficients, λ_s^2.

Rearrangement of eq 49 gives an expression for the sth move suppression coefficient:

$$\lambda_s^2 = \frac{M}{2(c_s-1)}\left\{\beta_{ss} - (M-1)\alpha_{ss} + \sqrt{\beta_{ss}^2 - 2(M-1)\beta_{ss}\alpha_{ss} + \frac{2(M-1)(2M-1)}{3}\alpha_{ss}^2}\right\} \quad (50)$$

Here β_{ss} and α_{ss} have the general form given by eqs 18 and 19. Equation 50 can be simplified to ease evaluation of λ_s^2 by recognizing the contribution of each term to its final value. By modification of the last term within the square root as

$$\beta_{ss}^2 - 2(M-1)\beta_{ss}\alpha_{ss} + \frac{2(M-1)(2M-1)}{3}\alpha_{ss}^2 \cong \{\beta_{ss} - (M-1)\alpha_{ss}\}^2 \quad (51)$$

Equation 50 simplifies to a compact form:

$$\lambda_s^2 = \frac{M}{c_s}\{\beta_{ss} - (M-1)\alpha_{ss}\} \quad (52)$$

The choice of the condition number of the sth diagonal block, c_s, and hence the upper allowable limit of ill-conditioning in the overall system matrix itself, lies with the individual designer. Past researchers (Maurath et al., 1985, 1988; Callaghan and Lee, 1988; Farrell and Polli, 1990) have indicated typical condition numbers for a moderately ill-conditioned overall system matrix for DMC that range from about 100 for single-input single-output systems to about 2000 for multivariable systems.

In this work, past suggestions of condition numbers are standardized by selecting a condition number, c_s, of

500 to represent the upper allowable limit of ill-conditioning in the sth diagonal block of the system matrix (corresponding to modest control effort by the sth manipulated variable). The choice of a condition number of 500 was motivated by the rule-of-thumb that the manipulated variable move sizes for a change in set point should not exceed 2−3 times the final change in manipulated variable (Maurath et al., 1985; Callaghan and Lee, 1988). However, if a faster or slower closed loop response is more desirable, larger or smaller condition numbers, respectively, can be used instead.

Substituting the choice of condition number ($c_s = 500$) and expressions for α_{ss} and β_{ss} in eq 52, an analytical expression for the move suppression coefficient, λ_s^2, is obtained as

$$\lambda_s^2 = \frac{M}{500}\sum_{r=1}^{R}\left[\gamma_r^2 K_{rs}^2\left\{P - k_{rs} - \frac{3}{2}\frac{\tau_{rs}}{T} + 2 - \frac{(M-1)}{2}\right\}\right] \quad (53)$$

Equation 53, applied in conjunction with the guidelines for selecting the other adjustable parameters, gives the tuning strategy for MIMO DMC with $M > 1$ (Table 1). With $M = 1$, the need for a move suppression is obviated and λ_s^2 are set equal to zero. Note that when $R = S = 1$ and $P = (5\tau/T) + k$, eq 53 reduces to a single expression derived previously for SISO DMC (Shridhar and Cooper, 1997).

An approximate relation from eq 53 is that the condition number of the sth diagonal block in the overall system matrix varies as

$$c_s \propto \frac{M\sum_{r=1}^{R}\gamma_r^2 K_{rs}^2 (P-k)}{\lambda_s^2} \quad (54)$$

This relation highlights the reason for some of the differences in the condition numbers used by past researchers mentioned above. Specifically, eq 54 shows that the ranges of values reported are due to the different choices of adjustable parameters such as P, M, and γ_r^2 used by the individual designers as well as process characteristics such as the gain, K_{rs}, in the multivariable system.

Some of the differences in condition numbers reported by past researchers can also be explained to be due to the different dimensions, ($R \times S$), of the multivariable systems considered. Although the relation between the condition number and the number of process variables, R, is clear from eq 54, its dependence on number of manipulated variables, S, is not obvious since eq 54 is valid only for a single diagonal block. However, the fact that the overall system matrix condition number increases with the number of manipulated variables, S, is illustrated by the upper bound in eq 39.

Equation 54 also illustrates that, for a fixed value of the diagonal block condition number and desired closed loop performance, a larger move suppression coefficient, λ_s^2, is required when a larger P, M, or γ_r^2 is used and when the multivariable process has a larger gain K_{rs}. The reason is that, except for M, an increase in any of the parameters mentioned above causes the elements of the individual diagonal blocks to increase in magnitude. On the other hand, an increase in the control horizon increases the dimension of the system matrix. In either case, the diagonal blocks and the system matrix become increasingly ill-conditioned and larger move suppression coefficients are required to appropriately condition the system matrix.

Validation of the MIMO DMC Tuning Strategy

All simulation examples presented in this work use the traditional DMC step response matrix of the actual process upon implementation and a negligible plant-model mismatch is assumed. In the past, several researchers have investigated the effects of plant-model mismatch on controller performance (Morari and Zafiriou, 1989; Zafiriou, 1991a; Ohshima et al., 1991; Lee and Yu, 1994; Lundström et al., 1995). Plant-model mismatch introduced into the examples presented here would result in observations similar to those documented by these researchers. Hence, this work focuses strictly on the capabilities of the tuning strategy in providing desirable closed loop performance.

Multivariable processes with a range of characteristics, such as short or large time constants, a large dead time, inverse response, minimum phase behavior, and high process order, that pose specific challenges in process control are employed for the purpose of tuning strategy validation. The impact of user specified sample time, T, and control horizon, M, is also explored. The controlled variable weights are selected throughout as $\gamma_1^2 = \gamma_2^2 = 1$ which assign an equal weight to both process variables. However, the tuning strategy presented will work equally well for different choices of controlled variable weights.

A reasonable identification of first order plus dead time (FOPDT) models for the subprocesses is required to implement this tuning strategy. Since a model is only as good as the data it fits, it is necessary that the manipulated-to-process variable data used for model fitting be rich in dynamic information. The parameters collected from the FOPDT model fit between the rth process variable ($r = 1, 2, ..., R$) and the sth manipulated variable ($s = 1, 2, ..., S$) are the gain, K_{rs}, overall time constant, τ_{rs}, and effective dead time, θ_{rs}.

Two example higher order multivariable processes are considered in addition to process 1 (eq 4) for validation of the tuning strategy:

$$\text{process 2} \qquad \boldsymbol{G}_{p2}(s) = \begin{bmatrix} G_1(s) & G_2(s) \\ G_3(s) & G_4(s) \end{bmatrix} \quad (55)$$

$$\text{process 3} \qquad \boldsymbol{G}_{p3}(s) = \begin{bmatrix} G_1(s) & G_4(s) \\ G_5(s) & G_6(s) \end{bmatrix} \quad (56)$$

where the individual $G_i(s)$ ($i = 1, 2, ..., 6$) are documented in Table 2. FOPDT fits of individual subprocesses, $G_1(s)$−$G_6(s)$, in the multivariable processes considered are also shown in Table 2. The model parameters obtained from these fits are used in Table 1 to tune MIMO DMC.

The results of the MIMO DMC tuning strategy applied to process 1 (eq 4) are shown in Figure 3, graph e. The sample time, T, is selected such that $T = 0.5\theta_2$ and the control horizon is selected as $M = 6$. Although process 1 consists of third-order subprocesses where G_1(s) has a large gain and G_2(s) has relatively large time constants, the tuning strategy computes move suppression coefficients ($\lambda_1^2 = 16.1$, $\lambda_2^2 = 8.4$) and other adjustable parameters that achieve a desirable process variable response with modest move sizes. All other

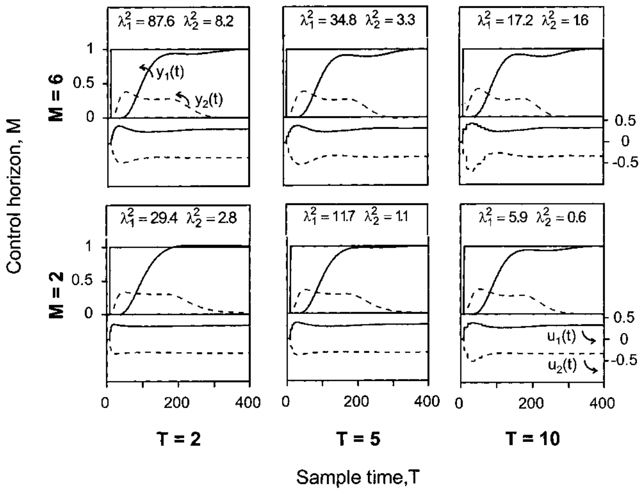

Figure 6. Effectiveness of the multivariable tuning strategy for different choices of control horizon, *M*, and sample time, *T*, for process 2.

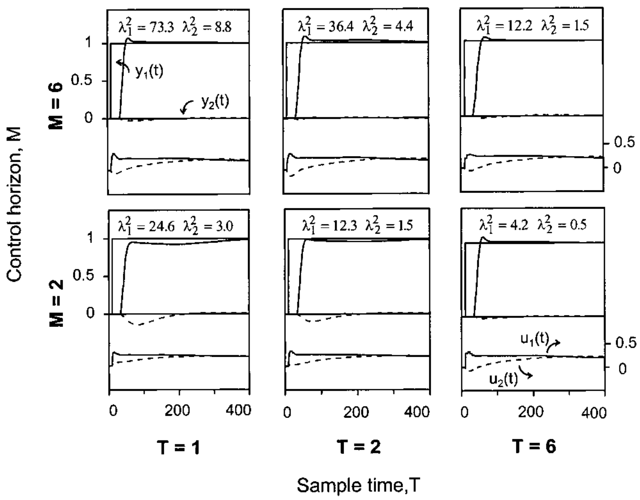

Figure 7. Effectiveness of the multivariable tuning strategy for different choices of control horizon, *M*, and sample time, *T*, for process 3.

graphs in Figure 3, especially graphs a, c, g, and i, highlight the usefulness of the proposed tuning strategy by demonstrating how closed loop performance could degrade if the move suppression coefficients were to deviate from the values computed by the tuning strategy.

Figures 6 and 7 are simulation results when the tuning strategy is applied to processes 2 and 3, respectively. Each figure shows a matrix of six graphs, where each graph represents the performance of MIMO DMC for a user selected control horizon of 2 or 6 and three different choices of sample time.

Process 2 presents a challenging multivariable tuning problem since $G_1(s)$ has a large gain, $G_2(s)$ has a large overall time constant, $G_3(s)$ has a small overall time constant, and $G_4(s)$ has a large dead time. In all cases, Figure 6 shows that the tuning strategy achieves consistent closed loop performance irrespective of the choice of M and T. It is interesting to note that the values of move suppression coefficients required to achieve desirable performance range from $\lambda_1^2 = 87.6$ and $\lambda_2^2 = 8.2$ for $M = 6$ and $T = 2$ to $\lambda_1^2 = 5.9$ and $\lambda_2^2 = 0.6$ for $M = 2$ and $T = 10$.

The tuning challenges presented by process 3 due to the nature of the subprocesses include $G_1(s)$ with a large gain, $G_4(s)$ with a large dead time, $G_5(s)$ with a negative gain and nonminimum phase behavior, and $G_6(s)$ with a strong minimum phase behavior. Despite the significant differences in dynamic characteristics of the component subprocesses, Figure 7 illustrates that the tuning parameters computed by the tuning strategy achieve desirable closed loop performance. In order to achieve similar performance for different choices of the control horizon and sample time, the move suppression coefficients computed range from $\lambda_1^2 = 73.3$ and $\lambda_2^2 = 8.8$ for $M = 6$ and $T = 1$ to $\lambda_1^2 = 4.2$ and $\lambda_2^2 = 0.5$ for $M = 2$ and $T = 6$.

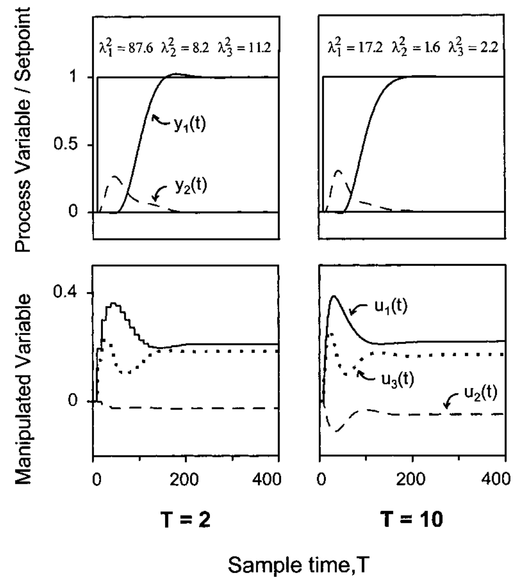

Figure 8. Effectiveness of the multivariable tuning strategy for different choices of sample time, *T*, for the non-square system in process 4.

Validation of the tuning strategy for non-square systems considers an example 2 × 3 multivariable process formulated from the subprocesses in Table 2:

$$\text{process 4} \qquad \boldsymbol{G}_{p4}(s) = \begin{bmatrix} G_1(s) & G_2(s) & G_5(s) \\ G_3(s) & G_4(s) & G_6(s) \end{bmatrix} \tag{57}$$

Apart from the range of dynamic characteristics of the higher order subprocesses, process 4 presents an additional challenge in the selection of tuning parameters since the number of manipulated variables is greater than the number of process variables.

Figure 8 demonstrates the simulation results when the tuning strategy is applied to process 4. This figure contains a matrix of four graphs, where the graphs stacked vertically illustrate process variable response and manipulated variable actions for a fixed sample time of 2 or 10. The control horizon is fixed at 6 for this study.

For either choice of sample time, Figure 8 shows that the tuning strategy computes parameters that balance the control effort among the three manipulated variables to achieve smooth set point tracking for y_1 and minimize the interaction effects on y_2. The consistent closed loop performance achieved for the two different choices of sample time is made possible by move suppression coefficients that range from $\lambda_1^2 = 87.6$, $\lambda_2^2 = 8.2$, and $\lambda_3^2 = 11.2$ for $T = 2$ to $\lambda_1^2 = 17.2$, $\lambda_2^2 = 1.6$, and $\lambda_3^2 = 2.2$ for $T = 10$.

For certain applications more specific or stringent performance criteria regarding the manipulated variable move sizes or the nature of the process variable response than that considered for the above simulation studies may apply. For such cases it may be necessary to fine-tune DMC for specific desired performance by altering γ_r^2 and λ_s^2 from the starting values given by the tuning strategy. The recommended approach is to increase λ_s^2 for smaller move sizes in the sth manipulated variable. On the other hand, tighter control of the rth process variable can be achieved by increasing γ_r^2, the corresponding controlled variable weight.

Conclusions

In this paper, an easy-to-use tuning strategy for unconstrained MIMO DMC was presented that builds upon prior work in tuning unconstrained SISO DMC. As in the SISO case, the tuning strategy presented here for MIMO DMC (Table 1) achieves set point tracking with minimal overshoot and modest manipulated variable move sizes and is applicable to a class of open loop stable multivariable processes, including non-square systems.

Extension of the tuning strategy to the multivariable case requires a set of move suppression coefficients, λ_s^2 ($s = 1, 2, ..., S$), to be computed for a system with inherent manipulated-to-process variable interactions. The derivation of a novel analytical expression that computes λ_s^2 for MIMO DMC tuning is one of the significant contributions of this work.

The expression for λ_s^2 was formulated by directly addressing the issue of ill-conditioning in a diagonal block of the system matrix. The result is a single equation that relates the move suppression coefficients to the internal model parameters, other MPC tuning parameters, and partitioned block condition numbers of the system matrix. By similar computing of all λ_s^2 for a constant low condition number, consistent closed loop performance was achieved.

The compact form of the analytical expressions for λ_s^2 was made possible by first order plus dead time (FOPDT) model approximations of the manipulated-to-process variable dynamics. With tuning parameters computed, DMC is then implemented in the classical fashion with a multivariable dynamic matrix formulated using step response coefficients obtained from the actual process. Just as FOPDT model approximations have proved a valuable tool in tuning rules such as Cohen–Coon, ITAE, and IAE for multiloop PID implementations, the tuning strategy presented here is significant because it offers an analogous approach for MIMO DMC.

Nomenclature

$a_{rs,i}$ = ith unit step response coefficient representing the dynamics between rth process variable and sth manipulated variable
$\boldsymbol{A}$ = multivariable dynamic matrix
$\boldsymbol{A}_{rs}$ = dynamic matrix for the rth process variable and sth manipulated variable
$\boldsymbol{A}^T\Gamma^T\Gamma\boldsymbol{A}$ = system matrix in the MIMO DMC control law
$(\boldsymbol{A}^T\Gamma^T\Gamma\boldsymbol{A} + \Lambda^T\Lambda)$ = overall system matrix in the MIMO DMC control law
b_{ij} = term in the ith row and jth column of $\boldsymbol{B}_{ss}$ in compact notation
$\boldsymbol{B}$ = matrix of off-diagonal blocks of overall system matrix
$\boldsymbol{B}_{vw}$ = general partitioned block of the system matrix
$\boldsymbol{B}_{ss}$ = diagonal block of the partitioned system matrix
$\boldsymbol{B}_S$ = repeated general partitioned block of the system matrix for a completely coupled multivariable system
c = condition number of overall system matrix
c_s = condition number of sth diagonal block in the overall system matrix
$\boldsymbol{D}$ = matrix of diagonal blocks of partitioned overall system matrix
$\boldsymbol{D}_{ss}$ = diagonal block of the partitioned overall system matrix
$\bar{\boldsymbol{e}}$ = vector of predicted errors for all R process variables
$\bar{\boldsymbol{e}}_i$ = ith orthonormal basis vector for the diagonal hankel block, $\boldsymbol{B}_{ss}$
$\underline{G}_i(s)$ = transfer function of a subprocess
$\bar{\boldsymbol{h}}_i$ = ith basis vector for the diagonal hankel block, $\boldsymbol{B}_{ss}$
$\mathbf{I}$ = identity matrix
j = index for sampling instants
J = MIMO DMC performance objective
K_{rs} = gain of a subprocess
k_{rs} = discrete dead time of a subprocess
P = prediction horizon
M = control horizon
N = model horizon
r = process variable index
R = number of process variables
s = manipulated variable index
(s) = Laplace domain operator
S = number of manipulated variables
T = sample time
u_s = sth manipulated variable
y_r = rth process variable
z = discrete time shift operator

Greek Symbols

α_{ss} = constant difference from left to right in row-wise terms of $\boldsymbol{B}_{ss}$
β_{ss} = first term in the diagonal partitioned block, $\boldsymbol{B}_{ss}$
$\beta_{vw,ij}$ = term in the ith row and jth column of the partitioned block, $\boldsymbol{B}_{vw}$
$\Delta\bar{\boldsymbol{u}}$ = vector of computed moves for all S manipulated variables
γ_r^2 = controlled variable weight (equal concern factor) in MIMO DMC
λ_s^2 = move suppression coefficient in MIMO DMC
$\Gamma^T\Gamma$ = diagonal matrix of controlled variable weights, γ_r^2
$\Lambda^T\Lambda$ = diagonal matrix of move suppression coefficients, λ_s^2
θ_{rs} = effective dead time of subprocess
μ = eigenvalue
τ_{rs} = overall time constant of subprocess

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