Scheduling of Tanker Lightering via a Novel Continuous-Time Optimization Framework

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The problem of scheduling a fleet of marine vessels for crude oil tanker lightering is addressed. A novel continuous-time mathematical formulation is developed on the basis of the concept of event points proposed in a formulation for short-term scheduling of chemical processes (Ierapetritou, M. G.; Floudas, C. A. *Ind. Eng. Chem. Res.* **1998**, *37*, 4341; **1998**, *37*, 4360. Ierapetritou, M. G.; Hené, T. S.; Floudas, C. A. *Ind. Eng. Chem. Res.* **1999**, *38*, 3446). A sequence of event points is introduced for each vessel, and binary variables are defined to determine whether the vessel is to start a task at each event point, while the task consists of mounting a tanker, pumping on oil from it, dismounting the tanker, traveling to the refinery, docking the refinery, pumping off oil, undocking, and traveling back to the anchorage. The mathematical formulation leads to a mixed-integer linear programming (MILP) problem. The model is further extended to incorporate two complicating features of the lightering process: (i) lightering in multiple stages and (ii) loading vessels with material from multiple tankers. A number of case studies are presented, and the computational results demonstrate the effectiveness and efficiency of the proposed approach.

1. Introduction

Lightering is a shipping industry term that describes the transfer of crude oil from a discharging tanker to smaller vessels to make the tanker "lighter". It is commonly practiced in shallow ports and channels where draft restrictions might prevent some fully loaded tankers from approaching the refinery discharge docks. The concept of lightering is illustrated in Figure 1. While a tanker with a full load of crude oil is either still offshore, approaching the bay, or anchored near the mouth of the bay, one or more smaller vessels (e.g., barges) come alongside it and pump crude oil off into their tanks. As soon as enough crude oil has been pumped off the tanker, both the lightering vessels and the tanker sail up to the refinery discharge docks.

Because of its position in the petroleum supply chain, right before the refinery, lightering can enhance the responsiveness of the crude oil supply process. Currently, marine crude shipment sizes range from about 700 000 bbl (100 000 metric tons) to upward of 3 million bbl in a single tanker. Dividing this enormous body of expensive liquid into smaller vessels enables its quicker discharge into a refinery's crude oil storage tanks by taking advantage of the pumping facilities of multiple (versus a single) docks and/or its swifter distribution to wherever it is needed more immediately among several refineries in the discharging area. Furthermore, lightering can dramatically reduce costly tanker demurrage, that is, the time that tankers remain idle waiting to be unloaded, and it can substantially decrease overall logistics costs of the crude oil supply system.

A lightering fleet typically operates within a limited geographical area that includes one or more refineries

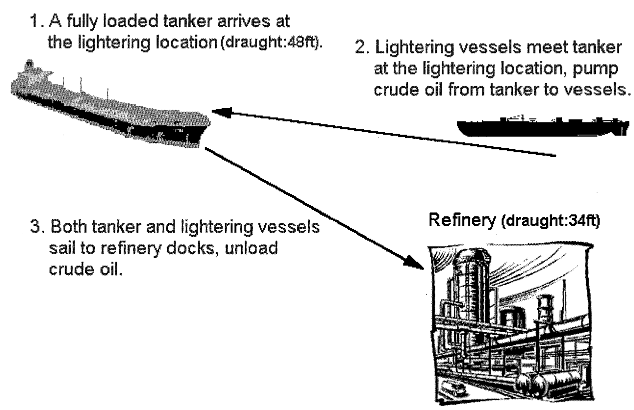

Figure 1. Lightering process.

at a port of shallow draft. It consists of a number of vessels with different characteristics and a wide range of capacities and is composed of barges, small ships, or both. Tankers usually arrive in irregular time intervals, and that results in occasional bottleneck periods for the lightering fleet when several approaching tankers await lightering. Furthermore, schedules for the lightering fleet are created on the basis of frequently updated estimated times of arrival (ETAs) for each tanker. Drastic changes in ETAs might necessitate swapping of vessel assignments, which could upset the schedule for the whole fleet. The scheduling process is further complicated by other considerations, including effects of tides and currents, ability to lighter in heavy weather, necessity of matching crude oil that must be heated with vessels that have heating capabilities, among others.

For all of the above reasons, and especially during times of congestion when many tankers arrive at the lightering anchorage within a short period of time, the scheduling of a lightering fleet is a formidable task, and even the most skilled scheduler experiences difficulty in manually determining the optimal combination of vessel-to-tanker assignments, timings, and lightering

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volumes. The two primary components of lightering costs are tanker demurrage and lightering fleet operating costs. They are greatly affected by the quality of lightering fleet schedules, and thus, the very challenging task of creating "good" fleet schedules becomes all the more important in minimizing lightering costs. A good lightering fleet schedule is characterized by reasonable fleet utilizations and provides tradeoffs between tanker demurrage and fleet operating costs that result in minimal total system costs.

The number of publications in the area of ship scheduling is relatively small, despite the facts that the world economy depends heavily on international trade based on ocean transportation and that proper scheduling could lead to significant improvements in the economic performance of ships. Ronen[4,5] explained the reason for this by pointing out the characteristics of ship scheduling problems, which include the secondary role of marine shipping as a mode of transportation in U.S., a large variety in problem structures, high uncertainty and relative insignificance compared to capital investment decisions. In ship scheduling problems, the decisions are usually about which ships will carry which shipments and when, given a set of shipments and a fleet. Brown et al.[6] studied the scheduling of crude oil tankers. They generated an exhaustive set of feasible schedules for each vessel and then formulated the problem as a variation of the set partitioning problem that they called "elastic" because they allowed the violation of certain constraints at a cost. The model found solutions with minimal cost but required the a priori generation of feasible schedules. Miller[7] discusses a network flow-based MILP model for the scheduling of chemical tankers used for the delivery of products to warehouses worldwide. Fisher and Rosenwein[8] addressed a similar problem of transporting bulk cargoes. They also generated candidate schedules and then formulated a set packing problem solved with a dual algorithm that they then used to solve real petroleum transportation problems from the U.S. Navy. Rana and Vickson[9] formulated a mathematical programming model for the optimal routing of a chartered container ship in a network of ports. They used Bender's partitioning method to solve the resulting MILP problems and developed a specialized algorithm for the integer network subproblems. The same authors[10] further extended their model for multiple ships. They proposed a mixed-integer nonlinear programming model to maximize total profit and used Lagrangean decomposition to solve it. Vukadinovic and Teodorovic[11] described the process of loading, transporting, and unloading of gravel by inland water barges and modeled the decision-making problem with a fuzzy-logic-based approach. Suh and Lee[12] presented a two-level hierarchical expert system for integrated scheduling of ship docking, discharging, and material transport. Fagerholt[13] considered a multi-ship pickup and delivery problem with soft time windows, introduced inconvenience costs to account for time window violations, and solved the problem using a set partition formulation with candidate schedules generated a priori.

The only previous lightering fleet scheduling optimization effort is the one undertaken by the Management Science Group of Maritrans, Inc.[14,15] Their optimization models generate short-term fleet schedules through a combination of discrete approximation of the time horizon, which results in MILP problems, and the use of heuristics that incorporate a great deal of the sophisticated knowledge of an experienced fleet scheduler.

In this work, we apply the original concept of event points featured in a novel continuous-time formulation for the short-term scheduling of chemical processes[1−3] to the lightering fleet scheduling problem. The rest of this paper is organized as follows. We first define the problem investigated in this work. Then, a mathematical formulation is presented in detail, followed by computational results of three case studies. Subsequently, the formulation is extended to incorporate the complicated considerations of lightering in multiple stages and loading vessels with material from multiple tankers. Two additional computational studies illustrate these considerations, and we draw conclusions on the proposed approach.

2. Problem Statement

The scheduling problem for tanker lightering studied in this work is defined as follows:

Given (i) the arrival time and lightering requirement of each tanker; (ii) the capacity, available time, pumping rate, and travel speed of each lightering vessel; (iii) the demurrage rate for tankers, and the voyage cost rate for lightering vessels; and (iv) other considerations, such as whether each vessel has heating capability for crude oil of certain types, the objective is to determine (i) tanker−vessel assignments; (ii) the lightering volume for each assignment; (iii) the timing of lightering and travel for each vessel; and (iv) the service time for each tanker so as to minimize the overall cost, which consists of tanker demurrage costs and lightering vessel voyage costs.

3. Mathematical Formulation

We have developed a new continuous-time formulation for the tanker lightering scheduling problem. This formulation makes use of the novel concept of event points proposed by Floudas and co-workers[1−3] for the general short-term scheduling problem of chemical processes. In the context of tanker lightering, we define event points as a series of time instances along the time axes of each lightering vessel at which the vessel starts performing a task. The timings of the event points are unknown and are to be determined as part of the solution to the formulation. The corresponding timing variables can potentially take any value in the continuous time domain, which allows the vessel to perform the task at any time in the horizon under consideration. The task consists of a whole sequence of operations that the vessel carries out to serve a tanker once, including (i) mounting the tanker, (ii) pumping crude oil from the tanker onto the lightering vessel, (iii) dismounting the tanker, (iv) traveling from the lightering location to the refinery port, (v) docking the refinery port, (vi) pumping crude oil off the lightering vessel to the refinery, (vii) undocking the refinery port, and (viii) traveling from the refinery port back to the lightering location.

The complete formulation is presented in detail below.

First, note that the notation needed for the model is defined in the Nomenclature section at the end of this paper. Using this notation, the constraints and objective function of the model are formulated as follows:

Allocation Constraints.

$$\sum_{t\in T_v} z(t,v,n) \leq 1 \qquad \forall v \in V,\ n \in N \tag{1}$$

For each lightering vessel v at each event point n, at most one tanker out of those that can be lightered by this vessel can be served.

Capacity Constraints.

$$v(t,v,n) \leq \mathrm{cap}_{t,v} z(t,v,n) \qquad \forall t \in T,\ v \in V_t,\ n \in N \tag{2}$$

If lightering vessel v serves tanker t at event point n, that is, if $z(t,v,n) = 1$, then the lightering volume cannot exceed the capacity of the vessel. Otherwise, if the vessel does not serve the tanker, that is, if $z(t,v,n) = 0$, then the constraint enforces that the lightering volume be zero.

Lightering Requirement Constraints.

$$\sum_{v\in V_t}\sum_{n\in N} v(t,v,n) = \mathrm{req}_t \qquad \forall t \in T \tag{3}$$

For each tanker t, the sum of the amount of crude oil lightered by all suitable lightering vessels at all event points should be equal to the lightering requirement.

Available Time of Vessels.

$$t^{s}(t,v,n) \geq t_v^{a} \qquad \forall t \in T,\ v \in V_t,\ n \in N \tag{4}$$

Each lightering vessel v can start serving a tanker t at an event point n only after it becomes available.

Arrival Time of Tankers.

$$t^{s}(t,v,n) \geq t_t^{a} \qquad \forall t \in T,\ v \in V_t,\ n \in N \tag{5}$$

Each tanker t can be served only after it arrives at the lightering location.

Duration Constraints.

$$t^{f}(t,v,n) = t^{s}(t,v,n) + (rt_{t,v} + \mathrm{dt})z(t,v,n) + v(t,v,n) \times \left(\frac{1}{\mathrm{pu1}_v} + \frac{1}{\mathrm{pu2}_v}\right) \qquad \forall t \in T,\ v \in V_t,\ n \in N \tag{6}$$

The duration of the task that lightering vessel v performs when serving tanker t at event point n is equal to the sum of the amount of time spent on mounting/dismounting the tanker, docking/undocking the refinery port, pumping on/off the crude oil, and traveling from the lightering location to the refinery and back.

Service Time of Tankers.

$$t^{d}(t) \geq t^{s}(t,v,n) + \frac{\mathrm{dt}}{2} z(t,v,n) + v(t,v,n)\frac{1}{\mathrm{pu1}_v} - H[1 - z(t,v,n)] \qquad \forall t \in T,\ v \in V_t,\ n \in N \tag{7}$$

If tanker t is lightered by vessel v at event point n, that is, if $z(t,v,n) = 1$, then the time at which this tanker finishes being served is no earlier than the time at which the vessel finishes mounting the tanker, pumping on crude oil, and dismounting the tanker. If the specific task does not take place, that is, if $z(t,v,n) = 0$, then constraint 7 is relaxed.

The last type of constraints, denoted as sequence constraints, connect the timings of different tasks. They can be classified into two sets: one for the same tanker lightered by the same lightering vessel and the other for different tankers lightered by the same lightering vessel.

Sequence Constraints: Same Lightering Vessel for the Same Tanker.

$$t^{s}(t,v,n+1) \geq t^{f}(t,v,n) \qquad \forall t \in T,\ v \in V_t,\ n \in N \tag{8}$$

A lightering vessel v can lighter a tanker t at an event point $n + 1$ only after it finishes the task serving the same tanker at the previous event point n.

Sequence Constraints: Same Lightering Vessel for Different Tankers.

$$t^{s}(t,v,n) \geq t^{f}(t',v,n') - H[2 - z(t,v,n) - z(t',v,n')] \qquad \forall v \in V,\ t \neq t' \in T_v,\ n > n' \in N \tag{9}$$

If a lightering vessel v serves tanker t at event point n and tanker t' at an earlier event point n', i.e., if $z(t,v,n) = 1$ and $z(t',v,n') = 1$, this constraint enforces the condition that the task at event point n starts no earlier than the time at which the task at event point n' finishes. If either of the two tasks does not take place, the constraint is relaxed.

Objective: Minimization of Total Cost.

$$\sum_{t\in T} \mathrm{dr}[t^{d}(t) - t_t^{a}] + \sum_{v\in V} vc_v \sum_{n\in N}\sum_{t\in T} z(t,v,n) \tag{10}$$

The objective is to minimize the total cost, which consists of two parts. The first part accounts for the tanker demurrage cost, which is proportional to the number of hours each tanker stays at the lightering location before finishing being lightered. The second part arises from the fleet voyage cost, which is fixed per trip for each lightering vessel.

The mathematical formulation described above results in a mixed-integer linear programming (MILP) problem, which can be solved effectively and efficiently with general purpose MILP solvers (e.g., CPLEX[16]).

4. Computational Studies

The aforementioned mathematical formulation is applied to three case studies, and the computational results demonstrate its effectiveness. The models in this paper are formulated with GAMS[17] and solved with CPLEX 6.5[16] on an HP J-2240 workstation.

4.1. Case Study 1. The first case study involves seven tankers and a fleet of four lightering vessels in a horizon of 7 days. Information about the tankers and the vessels is given in Tables 1 and 2, respectively. Each lightering vessel has different capacities for different refineries because of the different depths of the refinery ports. The round-trip time of a vessel when serving a tanker can

Table 1. Data on Tankers in Case Study 1

tanker number	arrival time (h)	destination refinery (distance (miles))	lightering requirement (1000 bbl)	heating requirement
1	16.0	R1 (67)	335.0	no
2	21.0	R2 (88)	340.0	no
3	35.0	R1 (67)	215.0	no
4	38.0	R2 (88)	222.0	no
5	47.0	R1 (67)	185.0	no
5′	47.0	R2 (88)	135.0	no
6	55.0	R3 (49)	177.0	no
7	117.0	R1 (67)	320.0	no

Table 2. Lightering Fleet Data for Case Studies 1, 4, and 5

vessel number	available time (h)	capacity (1000 bbl)			speed (miles/h)		pumping rate (1000 bbl/h)		heating capability	cost ($1000/voyage)
		R1	R2	R3	to refinery	to lightering location	to vessel	to refinery		
1	0.0	370	370	245	8.0	11.0	60.0	20.0	yes	6.20
2	0.0	225	225	225	7.0	9.5	38.0	15.0	no	4.50
3	0.0	260	260	195	9.0	12.0	50.0	20.0	yes	8.20
4	0.0	90	90	90	7.0	9.5	16.7	7.3	yes	19.00

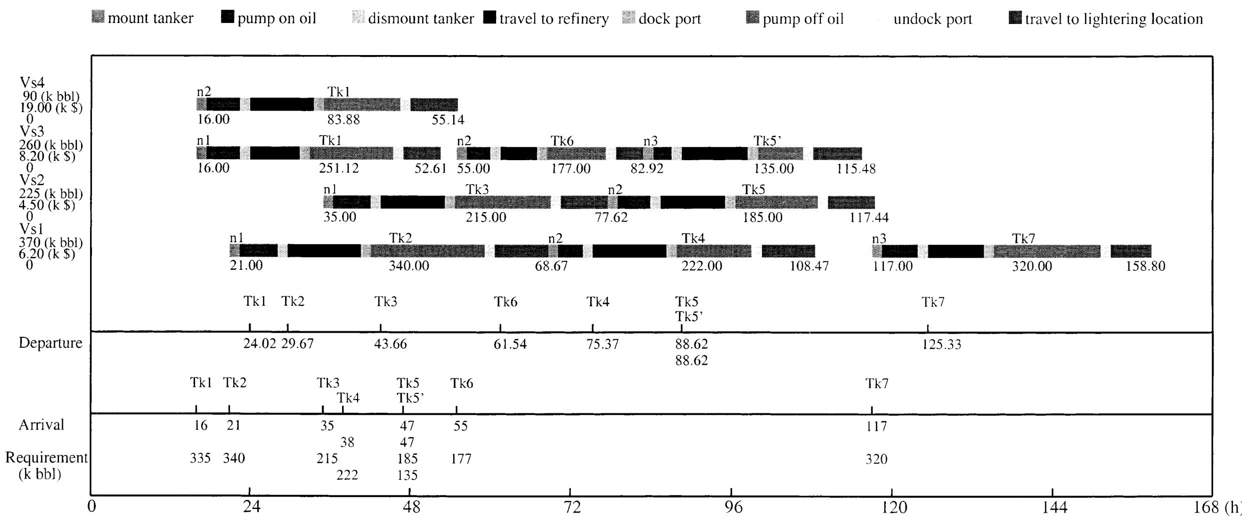

Figure 2. Gantt chart of the solution to case study 1.

be calculated from the distance between the lightering location and the destination refinery and the corresponding speeds of the vessel.

The mathematical model proposed in the previous section is formulated for this problem, which includes three event points. Tankers 5 and 5′ refer to a single tanker that carries crude oil for two different refineries. Therefore, only the maximum of the service time for this tanker is taken into account when the demurrage cost is calculated. The resulting MILP problem consists of 60 binary variables, 191 continuous variables, and 620 constraints and is solved to optimality in 30 CPU s. The optimal schedule is shown in Figure 2, which also includes the arrival time, lightering requirement, and departure time of each tanker. Below the name of each vessel are, from top to bottom, its maximum capacity, voyage cost, and available time. Each task by the vessel is represented by a sequence of bars, each designating a specific operation of the vessel. The tanker being served, the lightered volume, the corresponding event point, and the starting and ending times of the whole task by the vessel are also labeled in the Gantt chart. As shown in Figure 2, the optimal solution of this case study requires the fleet of four vessels to take nine trips in total. Each of the two larger vessels, Vs1 and Vs3, lighters three tankers, Tk2, Tk4, and Tk7 in sequence and Tk1, Tk6, and Tk5′ in sequence, respectively. Vs2 lighters two tankers, Tk3 and Tk5, sequentially. Vs4, with the smallest capacity and highest voyage cost, is assigned to lighter only one tanker, namely, Tk1. The resulting total cost of this schedule is $190,405, with a demurrage cost of $119,205 and a fleet voyage cost of $71,200.

4.2. Case Study 2. The second case study involves 12 tankers, all going to the same refinery, and four lightering vessels in a longer horizon of 11 days. Information about the tankers and the vessels is given in Tables 3 and 4, respectively. The round-trip time to travel from the lightering location to the refinery and back is the same for all vessels, 18 h.

Table 3. Data for Tankers in Case Studies 2 and 3

tanker number	arrival time (h)	lightering requirement (1000 bbl)	heating requirement
1	40.0	388.0	no
2	51.0	368.0	no
3	69.0	66.0	no
4	79.0	66.0	no
5	109.0	419.0	no
6	152.0	90.0	no
7	165.0	160.0	no
8	166.0	390.0	yes
9	167.0	83.0	yes
10	175.0	360.0	yes
11	200.0	19.0	no
12	212.0	378.0	no

Table 4. Lightering Fleet Data for Case Study 2

vessel number	available time (h)	capacity (1000 bbl)	pumping rate (1000 bbl/h)		heating capability	cost ($1000/voyage)
			to vessel	to refinery		
1	0.0	370	60.0	20.0	yes	6.20
2	0.0	255	40.0	15.0	yes	5.52
3	0.0	225	38.0	15.0	no	4.50
4	0.0	90	16.7	7.3	yes	19.00

Six event points are introduced in the mathematical model, which leads to an MILP problem with 70 binary variables, 225 continuous variables, and 919 constraints. The problem is solved to optimality in 370 CPU s. The corresponding schedule is shown in Figure 3. The fleet is required to take 16 trips in total. Vs1 lighters five tankers, Tk2, Tk5, Tk6, Tk10, and Tk12, sequentially. Vs2 lighters four times, Tk1, Tk4, and, a period of time later, Tk8 twice consecutively, as highlighted in larger font in the Gantt chart. Vs3 lighters six tankers, Tk1, Tk3, Tk5, Tk7, Tk11, and Tk12, in

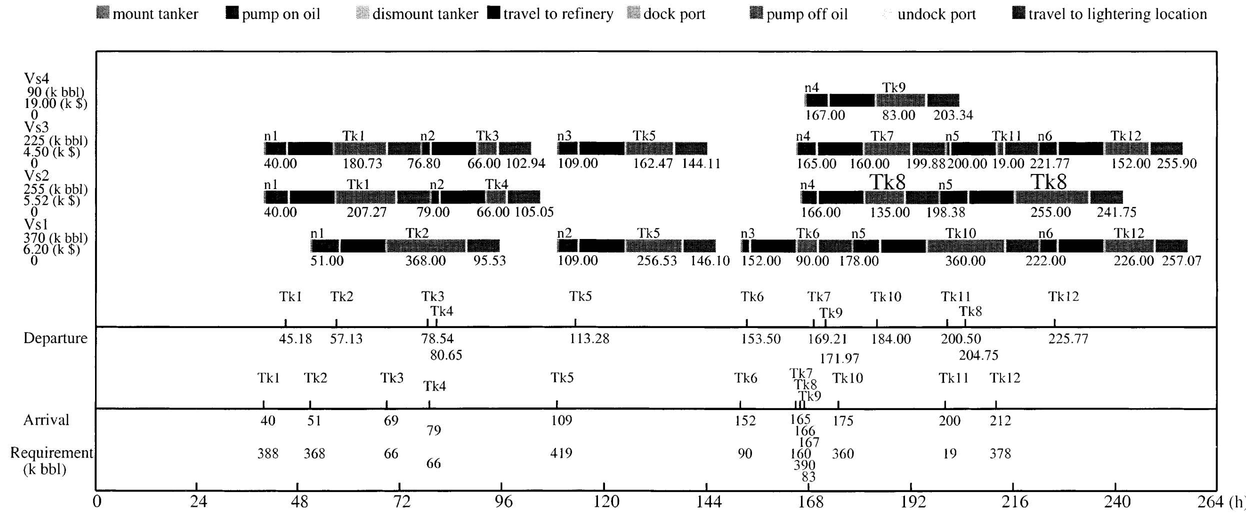

Figure 3. Gantt chart of the solution to case study 2.

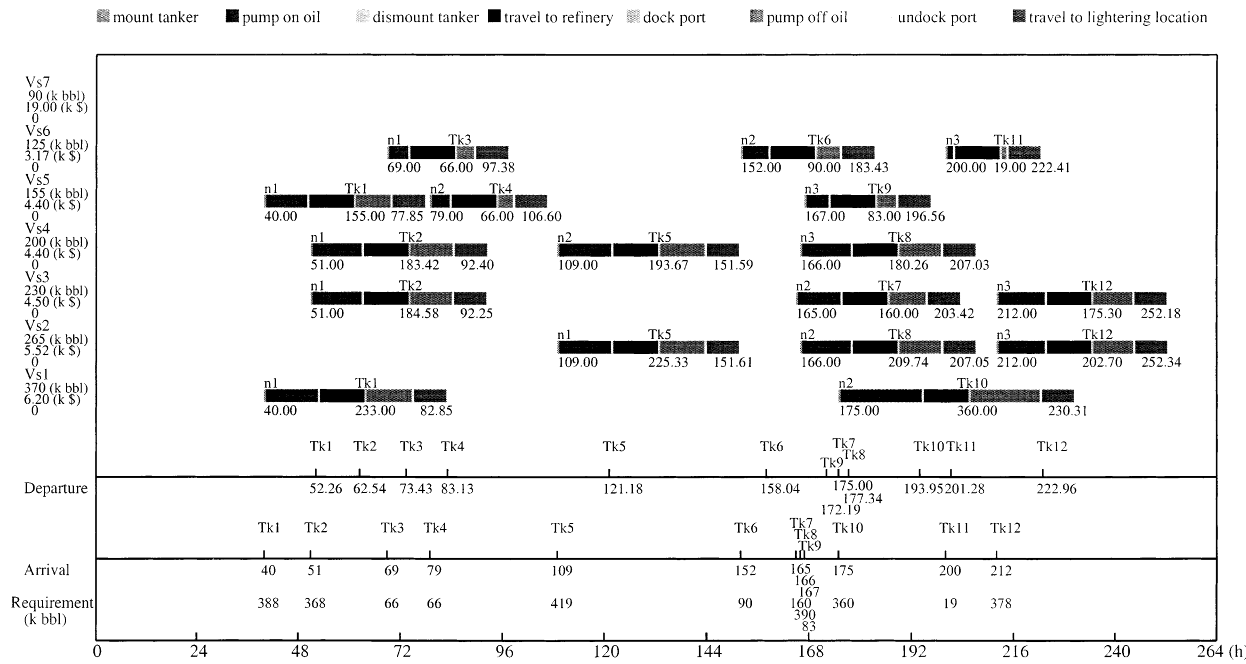

Figure 4. Gantt chart of the solution to case study 3.

sequence. The last vessel, Vs4, lighters only one tanker, Tk9. The resulting minimal total cost is $173,689, with a demurrage cost of $74,609 and a fleet voyage cost of $99,080. It should be pointed out that the arrangement of having one vessel lighter the same tanker twice is a new operational strategy that reduces the total cost and has never been considered in the schedules generated manually.

4.3. Case Study 3. In the third case study, the objective is to lighter the same 12 tankers as in case study 2 using a different fleet of seven vessels for which information is given in Table 5.

Because of the increase in the number of available vessels, only three event points are needed in the mathematical model. The resulting MILP problem has 114 binary variables, 369 continuous variables, and 1326 constraints. The problem is solved to optimality in 6004 CPU s. The corresponding schedule is shown in Figure 4. The fleet is required to take 17 trips. Vs1 lighters two tankers; each of the five vessels, Vs2–Vs6, lighters three tankers; and Vs7 is not assigned to any

Table 5. Lightering Fleet Data for Case Study 3

vessel number	available time (h)	capacity (1000 bbl)	pumping rate (1000 bbl/h) to vessel	to refinery	heating capability	cost ($1000/voyage)
1	0.0	370	19.0	22.0	yes	6.20
2	0.0	265	18.5	21.6	yes	5.52
3	0.0	230	16.0	19.0	no	4.50
4	0.0	200	15.9	18.6	yes	4.40
5	0.0	155	16.0	19.0	yes	4.40
6	0.0	125	14.9	16.7	no	3.17
7	0.0	90	15.5	17.0	yes	19.00

tanker. Even though the total number of trips by the vessels is now one higher than in case study 2, the total fleet voyage cost of $78,370 is actually lower because the average cost per voyage of this fleet of vessels is lower. The demurrage cost of $81,209 is slightly higher than that of case study 2 mainly because of the lower rates of pumping from the tankers to this fleet of vessels. Overall, the resulting total lightering cost is reduced to $159,579, compared to $173,689 in case study 2. Note that Vs7 is not utilized throughout the whole horizon because of its relatively small capacity and high voyage cost, which indicates that this vessel can be removed from the lightering fleet and used for other purposes.

4.4. Discussions. Number of Event Points. As discussed by Ierapetritou and Floudas,[1] the general procedure of determining the optimal number of event points is to start with a small number and gradually increase it until no improvement of the objective function can be achieved. For the lightering problem investigated in this paper, in addition to this rigorous general procedure, it is relatively easy to obtain a very good estimation of the number of event points needed on the basis of the lightering requirements of the tankers and capacities of the lightering vessels. For example, in case study 1, the overall lightering requirement is 1 929 000 bbl and the total capacity of the lightering fleet is at most 945 000 bbl, which is slightly less than half of the lightering requirement. It is thus very likely that each vessel will need to serve the tankers two or three times, and therefore, it is reasonable to start with three as the number of event points. For larger and more complicated problems, we can also obtain a very good estimation for the number of event points needed through a more careful analysis of the data.

Event Points for Each Tanker. In a typical lightering problem, only a subset of the event points is needed for each tanker. This subset is determined according to the relative arrival time of this tanker, its lightering requirement, and the available vessels. For example, in case study 1, tankers 1−3 arrive as the first tankers and are expected to be served relatively early by the four available vessels; therefore, they need at most only the first two event points. In contrast, the tankers that arrive toward the end of the horizon will be lightered in a later period, and thus only the later event points of each vessel are used for them. Through this elimination of unnecessary event points for each tanker, the size of the mathematical model can be reduced considerably, especially in terms of the number of binary variables, and the computational time required to solve the MILP problem is consequently also shortened.

Selection of the Value of Parameter *H*. The parameter H is used in constraints 7 and 9 as a large number to relax the constraints when the corresponding binary variables are not activated. Careful selection of the value of this parameter can tighten the formulation and improve the computational performance of the resulting mathematical model. Although there is no exact way to determine the optimal value of this parameter, our experience suggests that a good tradeoff between the rigor of the constraints and the tightness of the resulting model can be obtained by using the upper bound on the duration of the whole task performed by the vessel serving corresponding tankers or twice this value.

5. Multiple-Stage Lightering

Large tankers [e.g., very large crude carriers (VLCCs)], because of their deep draft, need to be lightered more than once, typically in deep offshore waters and then at a lightering location closer to the shore, that is, an anchorage. We are concerned with two-stage lightering in this paper. To incorporate this two-stage lightering process, each such tanker is treated as two tankers: one offshore and the other at an anchorage. The anchorage is taken as the base location where the lightering vessels are located at the beginning and end of each task. Taking into account the distance between the anchorage and offshore lightering location, we redefine the task a vessel performs when lightering a tanker offshore as the following sequence of operations: (i) traveling from the anchorage to the offshore location, (ii) mounting the tanker offshore, (iii) pumping crude oil from the tanker onto the vessel, (iv) dismounting the tanker, (v) traveling from the offshore location to the refinery port, (vi) docking the refinery port, (vii) pumping crude oil off the vessel to the refinery, (viii) undocking the refinery port, and (ix) traveling back to the anchorage.

To model the two-stage lightering process, we introduce additional parameters, variables, and constraints. The additional parameters include $\mathrm{rt}_{t,v}$, the travel time of vessel v from anchorage to offshore, back to anchorage, to refinery, and back to anchorage when serving offshore tanker t; rto_v, the round-trip time of vessel v between anchorage and offshore; and to_t, the travel time of tanker t from offshore to anchorage. The additional variable is $t^{\mathrm{a}}(t)$, the arrival time of offshore tanker at the anchorage as tanker t. The three additional constraints are as follows:

Service Time of Offshore Tanker.

$$t^{\mathrm{d}}(t) \geq t^{\mathrm{s}}(t,v,n) + \left(\frac{\mathrm{rto}_v}{2} + \frac{\mathrm{dt}}{2}\right) z(t,v,n) + v(t,v,n)\frac{1}{\mathrm{pu1}_v} - H[1 - z(t,v,n)] \qquad \forall t \in T_{\mathrm{offshore}},\ v \in V_t,\ n \in N \quad (11)$$

where T_{offshore} is the set of tankers to be lightered offshore. If an offshore tanker t is lightered by vessel v at event point n, that is, if $z(t,v,n) = 1$, then the time at which this tanker finishes being served offshore is no earlier than the time at which the vessel finishes traveling from the anchorage to the tanker offshore, mounting the tanker, pumping on crude oil, and dismounting the tanker. If the specific task does not take place, that is, if $z(t,v,n) = 0$, this relation is relaxed.

Arrival Time of Offshore Tanker.

$$t^{\mathrm{s}}(t,v,n) + \frac{\mathrm{rto}_v}{2} \geq t^{\mathrm{a}}_t \qquad \forall t \in T_{\mathrm{offshore}},\ v \in V_t,\ n \in N \quad (12)$$

It is assumed that a vessel does not wait offshore for the tanker to be lightered. Instead, it starts lightering the tanker as soon as the vessel arrives offshore. Therefore, the time at which a vessel v arrives at the offshore location after traveling from the anchorage is no earlier than the time at which tanker t arrives offshore, as represented by constraint 12.

Arrival Time of Anchorage Tanker.

$$t^{\mathrm{a}}(t_2) = t^{\mathrm{d}}(t_1) + \mathrm{to}_t \quad (13)$$

where t_1 and t_2 are the same tanker offshore and at the anchorage, respectively. The time at which the tanker arrives at the anchorage is equal to the time at which it finishes being lightered offshore plus the amount of time it takes to travel from offshore to the anchorage.

The objective function has also been slightly modified to reflect the fact that the time at which a tanker that needs to be lightered in two stages arrives at the anchorage is also a variable.

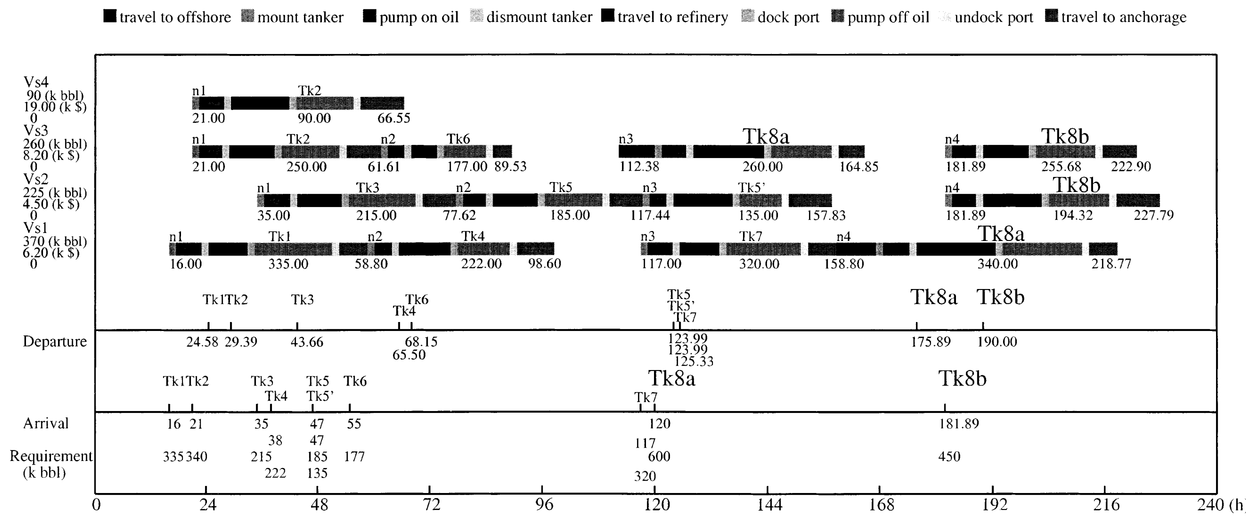

Figure 5. Gantt chart of the solution to case study 4.

Table 6. Data for Two Additional Tankers in Case Study 4

tanker number	arrival time (h)	destination refinery (distance (miles))	lightering requirement (1000 bbl)	heating requirement
8a (offshore)	120.0	R1 (147)	600.0	no
8b (anchorage)	–	R2 (88)	450.0	no

Objective: Minimization of Total Cost.

$$\sum_{t\in T/T_{\text{off-anch}}} \text{dr}[t^{\text{d}}(t) - t^{\text{a}}_t] + \sum_{t\in T_{\text{off-anch}}} \text{dr}[t^{\text{d}}(t) - t^{\text{a}}(t)] + \sum_{v\in V} vc_v \sum_{n\in N}\sum_{t\in T} z(t,v,n) \quad (14)$$

where T is the set of all tankers and $T_{\text{off-anch}}$ is the set of tankers arriving at the anchorage after being lightered offshore.

5.1. Case Study 4. An eighth tanker that should be lightered at two stages is added to case study 1. Information about this new tanker is given in Table 6.

Four event points are introduced in the mathematical model, which leads to an MILP problem with 74 binary variables, 237 continuous variables, and 920 constraints. The problem is solved to optimality in 397 CPU s. The corresponding schedule is shown in Figure 5. The vessels take 13 trips in total. Specifically, each of the first three vessels, Vs1–Vs3, lighters four tankers, and Vs4 lighters one tanker. The total lightering cost is \$310,209, with a demurrage cost of \$215,609 and a fleet voyage cost of \$94,600. As highlighted in larger font in the Gantt chart, as soon as Tk8a, with a lightering requirement of 600 000 bbl, arrives at the offshore lightering location at time 120 h, Vs3 starts serving this tanker and lighters 260 000 bbl of crude oil off the tanker. Afterward, Vs1 lighters another 340 000 bbl of crude oil off the tanker, and Tk8a leaves the offshore location at time 175.89 h. It arrives at the anchorage as Tk8b at time 181.89 h with a lightering requirement of 450 000 bbl. Tk8b is then served immediately by Vs2 and Vs3 at the same time, with the former vessel lightering 194 320 bbl and the latter 255 680 bbl. Tk8b finishes being lightered at time 190.00 h and leaves for the refinery.

6. Loading Vessels with Material from Multiple Tankers

A more complicated process, which we call split load, involves the sequential lightering of two tankers that carry the same type of crude oil and go to the same refinery by the same vessel in a single trip. To incorporate this additional consideration, we define a split-load task performed by a vessel as the following sequence of operations: (i) mounting tanker 1, (ii) pumping crude oil from tanker 1 onto the vessel, (iii) dismounting tanker 1, (iv) mounting tanker 2, (v) pumping crude oil from tanker 2 onto the vessel, (vi) dismounting tanker 2, (vii) traveling to the refinery port, (viii) docking the refinery port, (ix) pumping all crude oil off the vessel to the refinery, (x) undocking the refinery port, and (xi) traveling back to the anchorage.

Split loadings generally take place only at lightering anchorages, as offshore lightering locations are generally not congested and offshore lightering volumes usually fill the entire capacity of lightering vessels.

Again, additional parameters and variables are introduced for the split-load operation. They include the additional set TP of sequences of two tankers that can be lightered sequentially in a split load and the additional parameter $vc_{v,\text{sl}}$, the fixed cost per split-load voyage of vessel v. The additional variables are $z^{\text{sl}}(t_1,t_2,v,n)$ (binary), the assignment of vessel v to lighter tankers t_1 and t_2 sequentially in a split load at event point n; $v^{\text{sl1}}(t_1,t_2,v,n)$, the amount of crude oil that vessel v lighters from tanker t_1 in a split load lightering tankers t_1 and t_2 sequentially at event point n; $v^{\text{sl2}}(t_1,t_2,v,n)$, the amount of crude oil that vessel v lighters from tanker t_2 in a split load lightering tankers t_1 and t_2 sequentially at event point n; $t^{\text{s,sl}}(t_1,t_2,v,n)$ the time at which vessel v starts performing the task when serving tankers t_1 and t_2 sequentially in a split load at event point n; $t^{\text{f,sl}}(t_1,t_2,v,n)$ the time at which vessel v finishes the task when serving tankers t_1 and t_2 sequentially in a split load at event point n.

We have also added constraints and modified some constraints and the objective function in the base model in section 3 to incorporate this new consideration.

Allocation Constraints (Modified).

$$\sum_{t\in T_v} z(t,v,n) + \sum_{\substack{t_1\in T_v, t_2\in T_v\\ (t_1,t_2)\in \mathrm{TP}}} z^{\mathrm{sl}}(t_1,t_2,v,n) \le 1 \qquad \forall v \in V,\ n \in N \quad (15)$$

Any lightering vessel v at any event point n can be used for at most one task, either in a single load for one suitable tanker or in a split load for two suitable ones.

Split-Load Capacity Constraints (Added).

$$v^{\mathrm{sl1}}(t_1,t_2,v,n) + v^{\mathrm{sl2}}(t_1,t_2,v,n) \le \mathrm{cap}_{t_1,v} z^{\mathrm{sl}}(t_1,t_2,v,n) \qquad \forall (t_1, t_2) \in \mathrm{TP},\ v \in V_{t_1},\ v \in V_{t_2},\ n \in N \quad (16)$$

Note that the capacity of a lightering vessel v depends on the refinery to which it is heading; therefore, $\mathrm{cap}_{t_1,v} = \mathrm{cap}_{t_2,v}$. If a vessel v serves two tankers t_1 and t_2 in a split load at an event point n, i.e., if $z^{\mathrm{sl}}(t_1,t_2,v,n) = 1$, then the total amount of crude oil lightered from these two tankers cannot exceed the capacity of the vessel. If the split load does not take place, that is, if $z^{\mathrm{sl}}(t_1,t_2,v,n) = 0$, then the corresponding lightering volumes are enforced to be zero.

Lightering Requirement Constraints (Modified).

$$\sum_{v\in V_t}\sum_{n\in N}[v(t,v,n) + \sum_{(t,t')\in \mathrm{TP}} v^{\mathrm{sl1}}(t,t',v,n) + \sum_{(t,t')\in \mathrm{TP}} v^{\mathrm{sl2}}(t',t,v,n)] = \mathrm{req}_t \qquad \forall t \in T \quad (17)$$

For each tanker t, the sum of the amount of crude oil lightered in all single loads and split loads by suitable vessels at all event points should be equal to the lightering requirement.

Available Time of Lightering Vessels for Split Loads (Added).

$$t^{\mathrm{s,sl}}(t_1,t_2,v,n) \ge t_v^{\mathrm{a}} \qquad \forall (t_1, t_2) \in \mathrm{TP},\ v \in V_{t_1},\ v \in V_{t_2},\ n \in N \quad (18)$$

Lightering vessel v can start serving two tankers t_1 and t_2 in a split load at event point n only after it becomes available.

Arrival Time of Tankers for Split Loads (Added).

$$t^{\mathrm{s,sl}}(t_1,t_2,v,n) \ge t_{t_1}^{\mathrm{a}} \quad (19)$$

$$t^{\mathrm{s,sl}}(t_1,t_2,v,n) + \frac{\mathrm{dt}}{2} + \frac{1}{\mathrm{pu1}_v} v^{\mathrm{sl1}}(t_1,t_2,v,n) \ge t_{t_2}^{\mathrm{a}}$$
$$\forall (t_1, t_2) \in \mathrm{TP},\ v \in V_{t_1},\ v \in V_{t_2},\ n \in N \quad (20)$$

The time at which lightering vessel v starts lightering the first tanker t_1 in a split-load operation cannot be earlier than the arrival time of this tanker. Furthermore, the time at which the vessel starts serving the second tanker t_2 after lightering tanker t_1 can be no earlier than the time at which tanker t_2 arrives.

Duration Constraints for Split Loads (Added).

$$t^{\mathrm{f,sl}}(t_1,t_2,v,n) = t^{\mathrm{s,sl}}(t_1,t_2,v,n) + \left(rt_{t1,v} + \frac{3}{2}\mathrm{dt}\right) z^{\mathrm{sl}}(t_1,t_2,v,n) + [v^{\mathrm{sl1}}(t_1,t_2,v,n) + v^{\mathrm{sl2}}(t_1,t_2,v,n)]\left(\frac{1}{\mathrm{pu1}_v} + \frac{1}{\mathrm{pu2}_v}\right)$$
$$\forall (t_1, t_2) \in \mathrm{TP},\ v \in V_{t_1},\ v \in V_{t_2},\ n \in N \quad (21)$$

The duration of a split-load task that lightering vessel v performs when serving tankers t_1 and t_2 sequentially at event point n is equal to the sum of the amount of time spent on mounting/dismounting tanker t_1/tanker t_2/port, pumping on crude oil from tanker t_1/tanker t_2, pumping off crude oil to the refinery, and traveling from the lightering location to the refinery and back.

Service Time of Tankers by Split Loads (Added).

$$t^{\mathrm{d}}(t_1) \ge t^{\mathrm{s,sl}}(t_1,t_2,v,n) + \frac{\mathrm{dt}}{2} z^{\mathrm{sl}}(t_1,t_2,v,n) + v^{\mathrm{sl1}}(t_1,t_2,v,n)\frac{1}{\mathrm{pu1}_v} - H[1 - z^{\mathrm{sl}}(t_1,t_2,v,n)]$$
$$\forall (t_1, t_2) \in \mathrm{TP},\ v \in V_{t_1},\ v \in V_{t_2},\ n \in N \quad (22)$$

$$t^{\mathrm{d}}(t_2) \ge t^{\mathrm{s,sl}}(t_1,t_2,v,n) + \mathrm{dt} z^{\mathrm{sl}}(t_1,t_2,v,n) + [v^{\mathrm{sl1}}(t_1,t_2,v,n) + v^{\mathrm{sl2}}(t_1,t_2,v,n)]\frac{1}{\mathrm{pu1}_v} - H[1 - z^{\mathrm{sl}}(t_1,t_2,v,n)]$$
$$\forall (t_1, t_2) \in \mathrm{TP},\ v \in V_{t_1},\ v \in V_{t_2},\ n \in N \quad (23)$$

If two tankers t_1 and t_2 are lightered sequentially in a split load by lightering vessel v at event point n, that is, if $z^{\mathrm{sl}}(t_1,t_2,v,n) = 1$, then the time at which the first tanker t_1 finishes being served is no earlier than the time at which the vessel finishes lightering t_1 (mounting, pumping on crude oil, and dismounting). The second tanker t_2 finishes being served no earlier than the time at which vessel v finishes lightering both t_1 and t_2. If the split-load task does not take place, that is, if $z^{\mathrm{sl}}(t_1,t_2,v,n) = 0$, these relations are relaxed.

Sequence Constraints Involving Split Loads (Added). Same Lightering Vessel for the Same Split Load.

$$t^{\mathrm{s,sl}}(t_1,t_2,v,n+1) \ge t^{\mathrm{f,sl}}(t_1,t_2,v,n) \qquad \forall (t_1, t_2) \in \mathrm{TP},\ v \in V_{t_1},\ v \in V_{t_2},\ n \in N \quad (24)$$

A lightering vessel v can only lighter two tankers t_1 and then t_2 in a split load at an event point $n + 1$ after it finishes the task of serving the same two tankers at the previous event point n.

Same Lightering Vessel for a Split Load and Then a Single Load.

$$t^{\mathrm{s}}(t,v,n) \ge t^{\mathrm{f,sl}}(t_1,t_2,v,n') - H[2 - z(t,v,n) - z^{\mathrm{sl}}(t_1,t_2,v,n')]$$
$$\forall v \in V,\ t \in T_v,\ (t_1, t_2) \in \mathrm{TP},\ t_1 \in T_v,\ t_2 \in T_v,\ n > n' \in N \quad (25)$$

If lightering vessel v serves tanker t in a single load at event point n and serves both tankers t_1 and t_2 in a split load at an earlier event point n', that is, if $z(t,v,n) = 1$

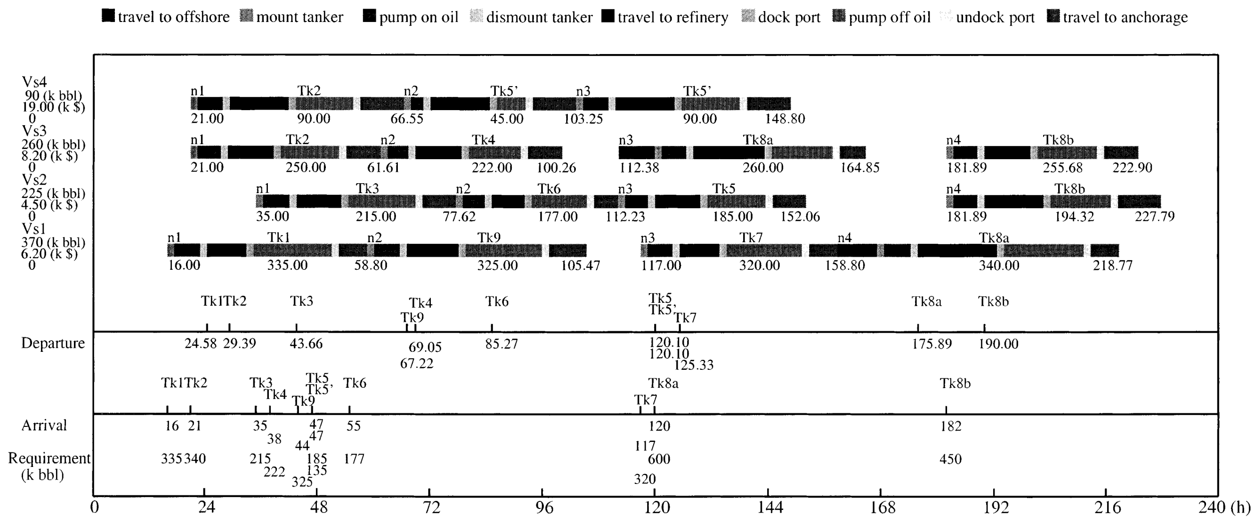

Figure 6. Gantt chart of the single-load solution to case study 5.

and $z^{sl}(t_1,t_2,v,n') = 1$, then the task at event point n can start no earlier than the time at which the task at event point n' finishes. If either of the two tasks does not take place, the constraint is relaxed.

Same Lightering Vessel for a Single Load and Then a Split Load.

$$t^{s,sl}(t_1,t_2,v,n) \geq t^{f}(t,v,n') - H[2 - z^{sl}(t_1,t_2,v,n) - z(t,v,n')] \quad \forall v \in V,\ t \in T_v,\ (t_1, t_2) \in \mathrm{TP},\ t_1 \in T_v,\ t_2 \in T_v,\ n > n' \in N \quad (26)$$

If lightering vessel v serves two tankers t_1 and t_2 in a split load at event point n and serves tanker t in a single load at an earlier event point n', that is, if $z^{sl}(t_1,t_2,v,n) = 1$ and $z(t,v,n') = 1$, then the task at event point n can start no earlier than the time at which the task at event point n' finishes. If either of the two tasks does not take place, the constraint is relaxed.

Same Lightering Vessel for Two Different Split Loads.

$$t^{s,sl}(t_1,t_2,v,n) \geq t^{f,sl}(t'_1,t'_2,v,n') - H[2 - z^{sl}(t_1,t_2,v,n) - z^{sl}(t'_1,t'_2,v,n')]$$

$$\forall v \in V,\ (t_1, t_2) \in \mathrm{TP},\ t_1 \in T_v,\ t_2 \in T_v,\ (t'_1, t'_2) \in \mathrm{TP},\ t'_1 \in T_v,\ t'_2 \in T_v,\ (t_1, t_2) \neq (t'_1, t'_2),\ n > n' \in N \quad (27)$$

If lightering vessel v serves a sequence of tankers t_1 and t_2 in a split load at event point n and serves another sequence of tanker t'_1 and t'_2 in a different split load at an earlier event point n', that is, if $z^{sl}(t_1,t_2,v,n) = 1$ and $z^{sl}(t'_1,t'_2,v,n') = 1$, then the task at event point n can start no earlier than the time at which the task at event point n' finishes. If either of the two tasks does not take place, the constraint is relaxed.

Objective: Minimization of Total Cost (Modified).

$$\sum_{t\in T} \mathrm{dr}[t^{d}(t) - t^{a}_{t}] + \sum_{v\in V}[vc_v \sum_{n\in N}\sum_{t\in T} z(t,v,n) + vc_{v,\mathrm{sl}} \sum_{n\in N}\sum_{(t1,t2)\in \mathrm{TP}} z^{sl}(t_1,t_2,v,n)] \quad (28)$$

The fleet voyage costs now arise from both the single-load trips and the split-load trips, which cost different amounts for each lightering vessel.

Table 7. Data for One Additional Tanker in Case Study 5

tanker number	arrival time (h)	destination refinery (distance (miles))	lightering requirement (1000 bbl)	heating requirement
9	44.0	R2 (88)	325.0	no

Table 8. Voyage Costs of Split-Load Tasks for the Lightering Fleet in Case Study 5

vessel number	cost ($1000/split-load voyage)
1	7.75
2	5.75
3	10.24
4	24.00

6.1. Case Study 5. A ninth tanker is added to case study 4. Information about this new tanker is shown in Table 7. The following tankers can be lightered in a split load: tanker 4, tanker 5′, and tanker 9. The voyage cost of a split-load trip is higher than that of a single-load trip for each vessel, and the related additional data are given in Table 8 for the lightering fleet.

Four event points are introduced in the mathematical model. Without the split-load consideration, the MILP formulation involves 82 binary variables, 262 continuous variables, and 1068 constraints. The problem is solved to optimality in 38 CPU s, and the minimal cost is $388,204, consisting of a demurrage cost of $255,604 and a fleet voyage cost of $132,600. The corresponding schedule is shown in Figure 6, with a total number of 15 trips by the vessels. When the possibility of split loads is incorporated, the resulting MILP problem has 114 binary variables, 390 continuous variables, and 1780 constraints as a result of the introduction of additional variables and constraints described previously. The CPU time required to solve the problem to optimality is increased to 5037 s. The optimal schedule is shown in Figure 7. The voyage cost of a split-load trip for each vessel is also included in the figure, following the single-load voyage cost below the vessel name. The total number of trips taken by the vessels is reduced to 14, which leads to a lower fleet voyage cost of $115,150, as compared to $132,600 in the schedule without split loads. One split load takes place, in which

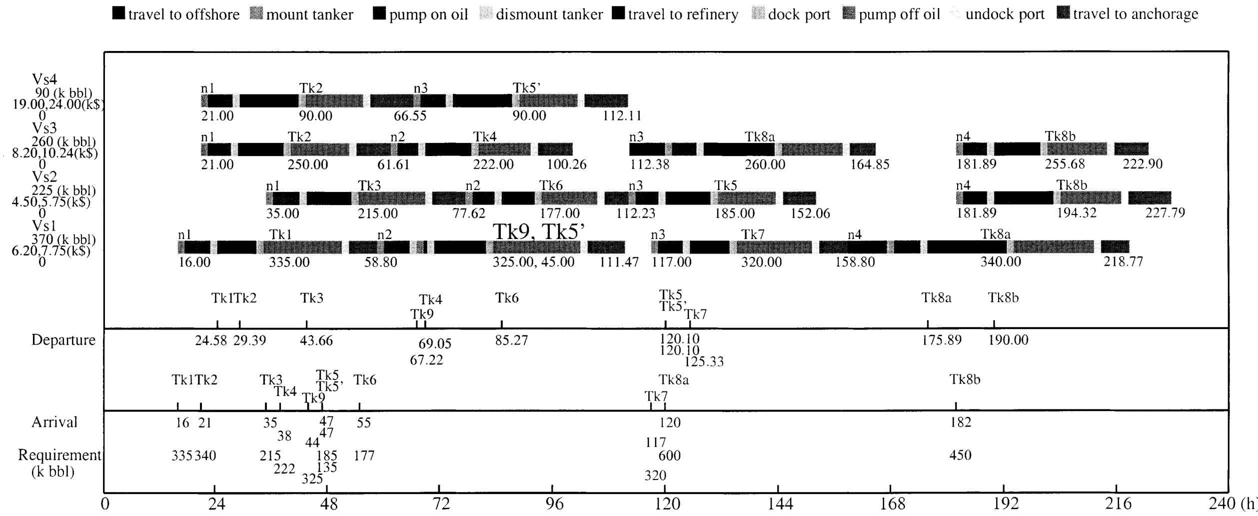

Figure 7. Gantt chart of the split-load solution to case study 5.

Vs1 lighters 325 000 bbl of crude oil off Tk9 and then 45 000 bbl of crude oil off Tk5′, as highlighted in the Gantt chart. The demurrage cost remains \$255,604. Overall, the total cost is reduced to \$370,754, which is 4.5% lower than the cost of the single-load solution.

7. Conclusions

The problem of lightering fleet scheduling is addressed. A novel continuous-time mathematical formulation has been developed on the basis of the concept of event points proposed in a formulation for short-term scheduling of chemical processes. A sequence of event points is introduced for each lightering vessel, and binary variables are defined to determine whether the vessel is to start a task at each event point, where the task consists of mounting a tanker, pumping on oil from it, dismounting the tanker, traveling to the refinery, docking the refinery, pumping off oil, undocking, and traveling back to the lightering location. The mathematical formulation leads to a mixed-integer linear programming (MILP) problem. A number of case studies are solved with the proposed approach. The formulation is then further extended to incorporate more complicated considerations such as lightering in multiple stages and loading vessels with material from multiple tankers. Our computational results demonstrate that the proposed formulation can be solved to global optimality very efficiently, and the resulting optimal schedules yield significant savings on the lightering costs.

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Nomenclature

Indices

n = event points representing the beginning of a task

t = tankers

v = lightering vessels

Sets

N = event points within the time horizon

T = tankers

TP = sequences of two tankers that can be lightered sequentially in a split load

T_v = tankers that can be lightered by lightering vessel v

V = lightering vessels

V_t = lightering vessels that can lighter tanker t

Parameters

$\mathrm{cap}_{t,v}$ = capacity of lightering vessel v when lightering tanker t

dr = demurrage rate

dt = time for mounting and dismounting a tanker and docking and undocking the refinery port

H = time horizon

req_t = lightering requirement of tanker t

rto_v = round-trip time of vessel v between anchorage and offshore lightering location

$\mathrm{rt}_{t,v}$ = travel time of lightering vessel v when serving tanker t (when serving tankers at the anchorage, it is the round-trip time between the anchorage and the refinery; when serving tankers offshore, it is the travel time from the anchorage to offshore, back to the anchorage, to the refinery, and back to the anchorage)

$\mathrm{pu1}_v$ = rate of pumping crude oil from the tanker to lightering vessel v

$\mathrm{pu2}_v$ = rate of pumping crude oil from lightering vessel v to the refinery

to_t = travel time of tanker t from offshore lightering location to anchorage

t_t^{a} = arrival time of tanker t

t_v^{a} = earliest available time of lightering vessel v

vc_v = fixed cost per voyage of lightering vessel v

$\mathrm{vc}_{v,\mathrm{sl}}$ = fixed cost per split-load voyage of vessel v

Variables

$t^{\mathrm{a}}(t)$ = arrival time of offshore tanker at the anchorage as tanker t

$t^{\mathrm{d}}(t)$ = time at which tanker t finishes being lightered

$t^{\mathrm{f}}(t,v,n)$ = time at which lightering vessel v finishes the task when serving tanker t at event point n

$t^{\mathrm{f,sl}}(t_1,t_2,v,n)$ = time at which vessel v finishes the task when serving tankers t_1 and t_2 sequentially in a split load at event point n

$t^{\mathrm{s}}(t,v,n)$ = time at which lightering vessel v starts performing the task when serving tanker t at event point n

$t^{\mathrm{s,sl}}(t_1,t_2,v,n)$ = time at which vessel v starts performing the task when serving tankers t_1 and t_2 sequentially in a split load at event point n

$v(t,v,n)$ = amount of crude oil that lightering vessel v lighters from tanker t at event point n

$v^{sl1}(t_1,t_2,v,n)$ = amount of crude oil that vessel v lighters from tanker t_1 in a split load lightering tankers t_1 and t_2 sequentially at event point n

$v^{sl2}(t_1,t_2,v,n)$ = amount of crude oil that vessel v lighters from tanker t_2 in a split load lightering tankers t_1 and t_2 sequentially at event point n

$z(t,v,n)$ = binary, assignment of lightering vessel v to lighter tanker t at event point n

$z^{sl}(t_1,t_2,v,n)$ = binary, assignment of lightering vessel v to lighter tankers t_1 and t_2 sequentially in a split load at event point n

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