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A Device To Emulate Diffusion and Thermal Conductivity Using Water Flow

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Time places severe limitations on what can be demonstrated in the classroom. Diffusion in liquids is a slow process. Shakhashiri *(1)* likens observing it to watching grass grow. Therefore, most classroom demonstrations of diffusion involve gaseous diffusion but they do not show quantitatively the relative concentrations of the diffusing substances *(2–5)*. There are exceptions such as experiments using laser refraction described in this *Journal (6, 7)* that allow students to directly observe a screen display of the deflection curve resulting from the diffusion process in liquids. Quantitative experiments involving diffusion in liquids typically take several hours and may take one or more laboratory periods to complete. However, Nishijima and Oster *(8)* by taking advantage of reduced scale were able to reduce the time required for observing diffusion in liquids by devising an experiment using a microscope.

There are several different general methods of determining diffusion coefficients. Although specific experiments do exist that can be used in both the classroom and laboratory, a mechanical device that could quickly and visually emulate most of these methods would be quite helpful in both classroom and laboratory discussions of these methods. To emulate chemical kinetics such as consecutive first-order reactions and Michaelis–Menton type kinetics, Davenport *(9)* devised an excellent visual and quantitative mechanical analog using water flow with a system of reservoirs and capillary tubes. I have designed a device (Figure 1) that will visually and quantitatively emulate diffusion and thermal conductivity using flowing water.[1] Water height emulates concentration or temperature. Results are obtained in a matter of a few minutes. The lightweight, rugged plastic model is easy to transport, easy to use, easy to store, has no moving parts, and requires only water to operate.

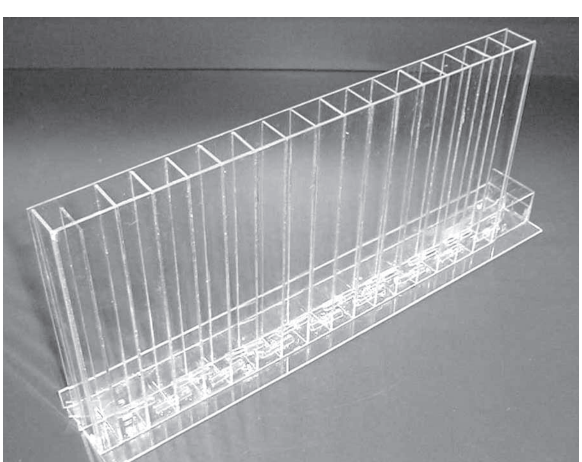

Figure 1. Water-flow device. The shorter partition for overflow is visible on the left side. The open channel is on the back.

Theory

Fick's[2] first law of diffusion states that

$$J = -D\left(\frac{\mathrm{d}c}{\mathrm{d}x}\right) \tag{1}$$

where J is the flow or flux and is the quantity of mass transported per unit time, D is the diffusion coefficient, and $\mathrm{d}c/\mathrm{d}x$ is the concentration gradient. If there is a change in concentration, c, with time, t, as well as with distance, x, then Fick's second law of diffusion,

$$\frac{\partial c}{\partial t} = D\left(\frac{\partial^2 c}{\partial x^2}\right) \tag{2}$$

also applies.

The objective of numerous experiments is to measure the diffusion coefficient for a system. Steady-state experiments make use of Fick's first law to measure the diffusion coefficient. Experiments in which diffusion from a plane source occurs in both directions make use of a one-dimensional solution to the second law in which

$$c = \frac{c_0 \Delta x}{2(\pi D t)^{1/2}} \exp\left(-\frac{x^2}{4Dt}\right) \tag{3}$$

where Δx is the width of the initial plane *(10)* and c_0 is the initial concentration. This is a Gaussian error curve (normal error distribution). Other experiments follow diffusion that occurs in a tube of uniform cross-sectional area from a concentrated solution across an original boundary into pure solvent (step-function source). Since the boundary value conditions are different, solution of the second law results in

$$c = \frac{c_0}{2}\left[1 - \frac{2}{\pi^{1/2}}\int_0^{\frac{x}{2(Dt)^{1/2}}} \exp\left(-y^2\right)\mathrm{d}y\right] \tag{4}$$

where y is a dummy variable for integration and the integral term is known as the probability integral *(11)*. The integral term is also called the error function, erf(z), hence

$$c = \frac{c_0}{2}\left[1 - \operatorname{erf}(z)\right] \quad (5)$$

where

$$z = \frac{x}{2}(Dt)^{-1/2} \quad (5a)$$

and t is a specific time *(12)*. A 1 − erf(z) function is also called the complementary error function and is denoted by erfc(z). Values for evaluating the error function are in numerous handbooks of mathematics, physics, and statistics. The error function is the area under a Gaussian error curve from 0 to z. Conversely, differentiating erf(z) produces a Gaussian error curve. Equations 3, 4, and 5 have small variations depending upon the precise boundary conditions used in solving the second-law equation.

These three major types of diffusion coefficient measurement experiments are shown graphically in Figures 2A–C. The Gaussian curve spreads and flattens with time. The 1 − erf curve broadens with time but always goes through the center of the original boundary. Thermal energy transport obeys similar equations.

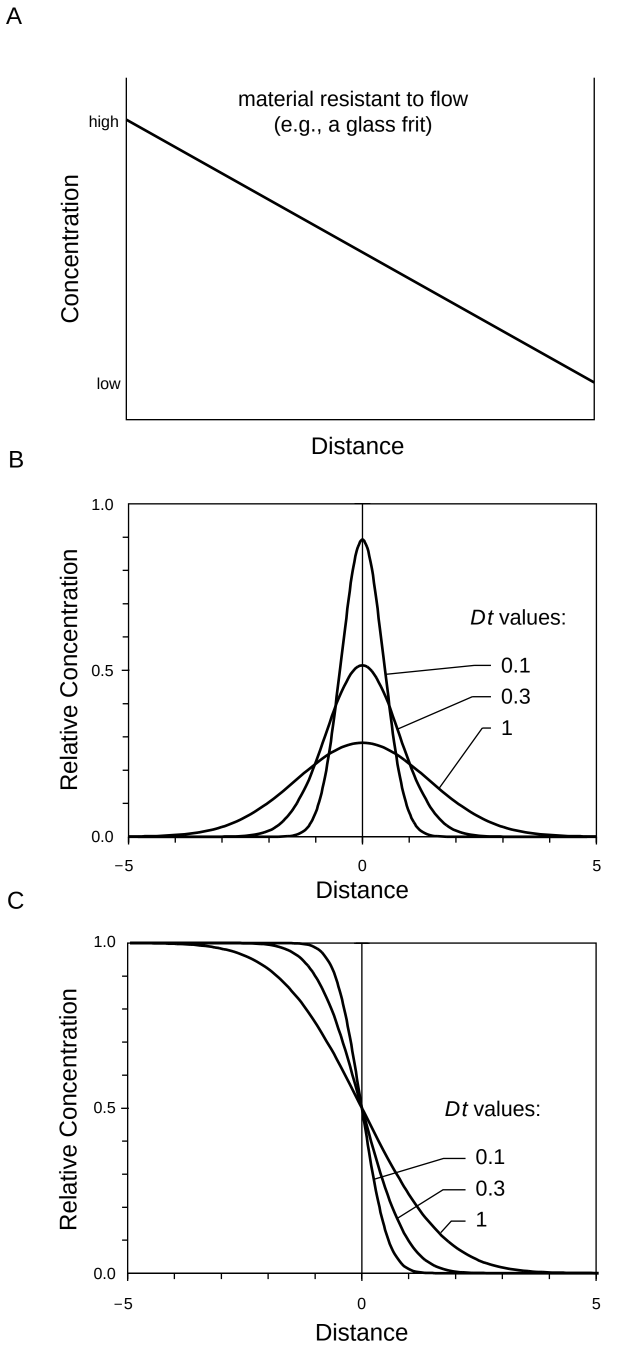

Figure 2. (A) First-law steady state (eq 1). (B) Second-law Gaussian curves (eq 3). The initial solution plane is in the center. (Dt/x^2 is dimensionless. If x is in cm then the Dt dimension is cm^2.) (C) Second-law 1 − erf curves with solution in the left half at t = 0.

Water-Flow Device

A series of 2.5-cm square cross section cells 20-cm high was constructed using polymethylmethacrylate (acrylic plastic) by using 2.5-cm × 20-cm pieces between two pieces of plastic. All but two of the 18 partitions have a 1/8 in. i.d. (1/4 in. o.d.) acrylic tube 3.2-cm long bonded into a hole drilled near the bottom and close to one side so that the tube is about 2 mm from the bottom and the side. The partitions were placed so that the connecting tube locations alternated sides. The second partition from the end is about 5-mm shorter than the others and has no hole or tube in it. Water introduced into the second cell can move to the other cells through the tube in each partition. Excess water introduced into the second cell is removed by overflowing the shorter partition into the first cell, which is simply a drain for the overflow. A fairly large hole was drilled in the side near the bottom of the first cell to allow the overflow water to exit. It is helpful to create a channel along the back of the device to allow water that has flowed through the device to flow along the channel after exiting the last cell and empty at the same end as the overflow water. Bonding of the pieces was done by placing a small quantity of methyl ethyl ketone (MEK) on each joint. Imperfect joints can be sealed by allowing a few drops of a nearly saturated solution of acrylic plastic in MEK to flow along the joint. The completed water-flow acrylic device is shown in Figure 1. (Construction questions are welcome.)

Model Operation and Results

Preparation

With many sinks, a spigot can be moved to one side so that the input (second) cell of the diffusion model can be slipped up under the spigot such that most of the device will rest on the bench with the end of the model over the sink. If the spigot can not be moved to a useful location, a sturdy metal or plastic strip long enough to bridge the sink may be slipped under the input end of the device.

Flow at the leading edge of spreading water will be smoother if water is added to the model beforehand to displace air in the connecting tubes. Allowing the excess water to drain will leave all the connecting tubes covered with water. If this is not done, a small temporary excess pressure head is required to overcome surface tension to initiate water flow when an empty cell is first encountered.

Emulation of Steady-State Experiments

Procedure and Results

A small quantity of food coloring may be added to improve the visibility. If the food coloring is added to the second cell, it will gradually spread to the other cells as more water is added. If a uniform color is desired, a container with a spigot can be used so that food color may be added beforehand. In all the demonstration photos shown, food coloring is in the dispenser, which in this case is a 30-cup coffeepot. The model was prefilled to a depth of one cm to cover the connecting tubes. Tape was placed just below the top of this one-cm level so that it is easier to see the changes that occur during the process. The valve on the coffeepot may be held down by a rubber band around the valve base and a small wood block on the lever. Turn on the water so that cell 2 overflows into cell 1. Excess water exits at the bottom of cell 1 on the left. The photo in Figure 3A shows the appearance as the cells begin to fill. In about ten minutes there is no longer any change in the water levels of the cells. This steady-state photo is shown in Figure 3B.

Water, after exiting the rightmost cell, runs into a channel along the back of the device and into a container on the left side. The flow rate is about 140 mL/min.

How the Model Works

When the steady state is reached (Figure 3B), the step heights are equal or nearly so. (If the flow characteristics of each cell are identical, the step heights will all be equal.) If the water height in a particular cell is to remain constant, the quantity of water flowing into the cell must be equal to the quantity of water flowing out of the cell. The rate that water will flow from one cell to another is determined by the water pressure difference at the tube between the two cells.

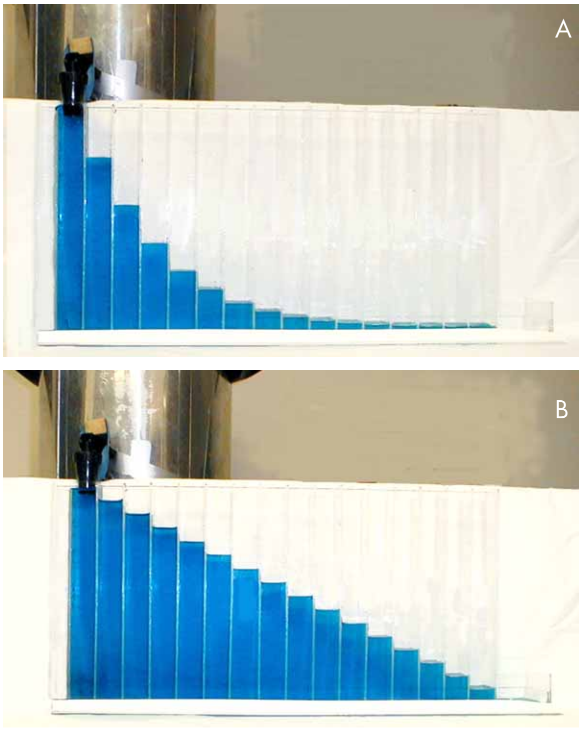

Figure 3. Emulation of first-law diffusion: (A) cells filling and (B) steady state.

The water pressure difference is proportional to the water height difference between the two cells. For the input and output flows from a particular cell to be equal, the water height difference at the input partition must be equal to the water height difference at the output partition. Since this must be true for all cells if the flow rate is constant, then the water height difference between adjacent cells must be the same. That is, the step heights must all be equal. Since the cell widths are all equal then the gradient, $\Delta h/\Delta x$, where h is the water height in a cell, is constant.

This constant gradient steady state emulates Fourier's first law of thermal energy transport and Fick's first law of diffusion for situations in which the high temperature or high concentration inputs and the low temperature or low concentration outputs remain constant. This steady state also emulates Newton's law of viscosity for momentum transport through a liquid between two parallel plates in which a plate on the left is moving up at a constant velocity and a stationary plate is on the right. For momentum transport, water height in a cell corresponds to the average of the liquid velocities in the interval represented.

If desired, the flow of water may be measured and as expected, will be constant. When measuring flow rates, it is important to ensure that tiny gas bubbles do not form in the connecting tubes. This can happen, for example, when cold water saturated with air flows through the model and warms toward room temperature. Using a model with larger-diameter connecting tubes results in the same appearance, that is, the same gradient, but the water flow will be greater as will the proportionality constant.

The effect of a larger gradient on the flow rate can be shown by introducing water into a cell closer to the exit end of the model. Flow rate is proportional to pressure head if the flow is laminar and proportional to the square root of the pressure head if the flow is turbulent. Flow through a tube will be laminar if the length of the tube is 10 or more times its diameter, as it is in this model, and if the flow rate is not too high. For correct emulation of diffusion and thermal conductivity, the water flow rate should be laminar so that the pressure head will be proportional to flow rate. The Reynolds number, which is used to estimate whether flow will be laminar or turbulent, can be calculated from the flow rate, the tube diameter, and the density and viscosity of the liquid. Reynolds numbers and flow rates are shown in Table 1. If the

Table 1. Flow Characteristics for Model with 1/8 Inch I.D. Tubes between Partitions

Number of Cells	Pressure Head/cm	Flow Rate/ (mL min^{-1})	Calculated Reynolds Number	Type of Flow
16	1.19	143	1000	laminar
14	1.36	158	1100	laminar
12	1.58	174	1210	transitional
10	1.90	192	1340	transitional
8	2.38	220	1530	transitional
6	3.17	263	1830	transitional
4	4.75	330	2300	turbulent
2	9.50	462	3210	turbulent

Reynolds number is less than 2000, the flow is laminar. For Reynolds numbers between 2000 and 4000 the flow is said to be transitional and above 4000 it is turbulent. The flow rates indicate that the Reynolds numbers overestimate the laminar flow range. A more conservative appraisal based on a comparison of flow rates is shown in Table 1.

Most of the time the model operates in the laminar and the transitional flow regions but occasionally the flow will be in the turbulent region when there is a large pressure head difference. The present device appears to adequately emulate diffusion and thermal conductivity without having to resort to a more complicated construction design.

One reviewer suggested that a higher viscosity solution such as a sugar solution should produce a laminar flow over a wider pressure-head range than water. A commercial corn syrup solution diluted 1:1 with water did give better laminar flow characteristics for full Gaussian diffusion emulation than water but for all other emulations, the improvement in laminar flow, while undoubtedly present, was not readily apparent and each demonstration took about three times longer than with water.

Examples of Emulated First-Law Experiments

In an experiment involving water vapor leaving a liquid source, diffusing through straws, and being absorbed as it exits, Nelson *(13)* shows how the weight changes with time before and during steady-state conditions. The water-flow device above emulates how the relative water vapor pressure changes with distance prior to and at steady state (Figures 3A and 3B). Other first-law experiments that this model emulates include one by Brokett *(14)* in which the concentration of gaseous bromine is determined by a photometer as it diffuses through a tube, and one by Shooter *(15)* that uses diffusion sampler tubes in the determination of nitrogen dioxide in the atmosphere.

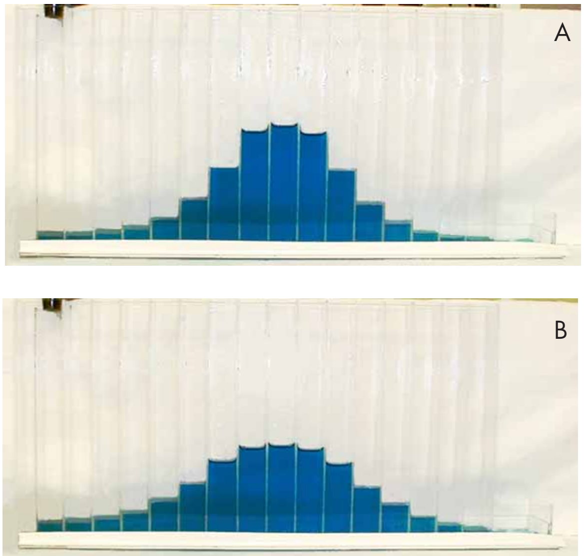

Figure 4. Emulation of full Gaussian diffusion: (A) in progress and (B) continued spreading.

Emulation of Gaussian-Curve Experiments

To emulate a Gaussian curve, fill three adjacent cells in the center of the model. (It would be more correct to start out with just one cell filled with water. However, if only one cell of the model is filled, the volume is so small it is difficult to see the curve develop with time.) The exits must be temporarily blocked. (A 1/8-in. diameter wire or a 10-gauge wire covered with heat shrink tubing inserted into either an exit or entry tube works well.) After the exit hole plugs are removed simultaneously, the water will spread in both directions. Initially the spreading will be like a step function or step boundary mentioned earlier and described more fully in the following section. This step-boundary effect disappears when the water level in the center cell water level begins to fall. It then approximately emulates a Gaussian curve until the flow reaches the ends of the model. Figures 4A and 4B show the spreading and flattening of the curve with time. When the entire process is observed, it is easy to see the water level in cells between the two inflection points decreasing in height while cells outside the inflection points continue to fill. Cells at or near the inflection points show no or little change in water height. The inflection points move away from the origin as the curve spreads. Initially the pressure head is 20 cm, which will cause turbulent flow out of the two outer cells when the exit tubes are opened. This causes the top of the curve in Figure 4A to be somewhat flatter than a true Gaussian curve. If a cell has a well-defined meniscus, then the water height in that cell is decreasing. Each of the center three cells in Figure 4A has a well-defined meniscus hence the inflection points are to the left and right of these three cells. In Figure 4B the flow is now more nearly laminar and the curve more nearly Gaussian. Notice now that the meniscus is more prominent in the center five cells showing that the water height is decreasing in them and therefore the inflection points are next to these cells. The Gaussian curve is somewhat distorted because the leading edge on the left has reached the end of the model.

Diffusion in both directions from a plane obeys eq 3. Since the Gaussian bell-shaped curve is symmetrical, it is usually more convenient to do experiments that allow diffusion in only one direction. Atkins and de Paula *(16)* show the Gaussian curves that result at various values of Δt if a solid such as sugar is placed in the bottom of a deep cylinder with water over it. The sugar forms a thin, saturated solution on the bottom of the container before diffusion has progressed to a significant extent. The dissolved sugar diffuses up into the liquid. The concentration changes with both distance and time in a manner that obeys a Gaussian curve. Irina *(17)* used a similar experimental procedure to determine the diffusion coefficients of several compounds in water. Experiments of this sort are not confined to liquid diffusion. Moore *(18)* measured the diffusion of silver into silver sulfide by putting a thin coating of silver on a slab of silver sulfide and submitting it to an anneal at a fixed temperature for a measured time. The Gaussian distribution equation (eq 3) must be modified slightly since all the material diffuses in the same direction. The resulting equation is

$$\frac{c}{c_0} = (\pi D t)^{-1/2} \exp\left(-\frac{x^2}{4Dt}\right) \qquad (6)$$

where c/c_0 is the ratio for some unit thickness *(18).*

To emulate diffusion in one direction using the model, fill the three cells on the overflow end. Now the contents of these three cells will flow in the same direction. Initially the spreading will again be like a step boundary but that effect disappears rapidly when the water level in the leftmost cell begins to fall. Then the result is approximately a Gaussian curve that changes with time as seen in Figure 5. As indicated by those cells with a prominent meniscus, the inflection point in Figure 5B is farther to the right than in Figure 5A, as expected. Since all flow is in the same direction, this reflected Gaussian curve has a greater maximum amplitude at any given time during the process than if flow originates in the center.

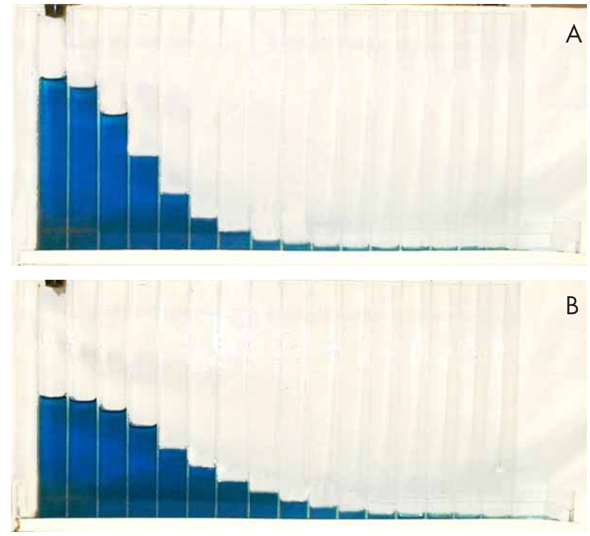

Figure 5. Emulation of reflected Gaussian diffusion: (A) in progress and (B) continued spreading.

Emulation of Error-Curve (Step-Boundary) Experiments

The model may be used to emulate the 1 − erf curve in eq 5 generated in step-boundary (step-function) diffusion by filling eight cells of the model with water. The exit tube of the eighth cell must be temporarily blocked. No water is added to the model during the emulation. When the plug is removed, a 1 − erf curve rapidly develops. The changes with time are shown in Figure 6. When the water reaches the end cell, the relative water heights differ from a strict 1 − erf description.

All 1 − erf curves are essentially identical. A 1 − erf curve can be "stretched" along the *x* axis. Sixteen points for a 1 − erf(z) curve that best fits the water heights in Figure 6B were calculated from a table of values for the error function in a mathematics handbook. The plot of this calculated 1 − erf curve was superimposed on the image in Figure 6B. This composite image (Figure 7) clearly shows that a 1 − erf curve fits the spreading very well.

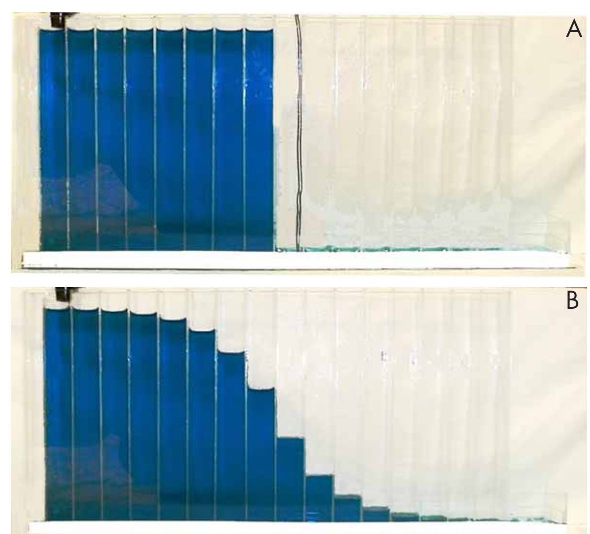

Figure 6. Emulation of step-boundary diffusion: (A) initial condition and (B) in progress. The wire plug is visible in Figure 6A.

When used in this way, the model emulates diffusion across an original boundary in which solution of uniform concentration is on one side and pure solvent on the other in a tube of uniform diameter. Examples of experiments of this type include the diffusion of sucrose in water studied by Linder et al. *(19)* and later by Clifford and Ochiai *(20),* a laser refraction method used by King et al. *(6),* and laser refraction to measure the diffusion of aqueous glycerol by Sattar and Rinehart *(7).*

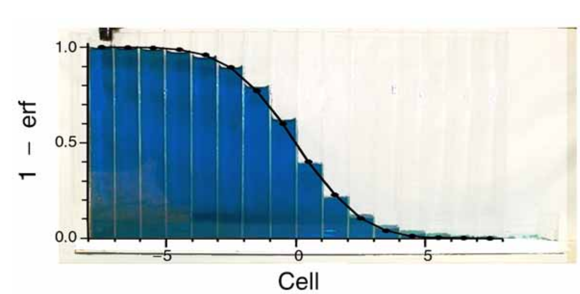

Figure 7. Composite image of 1 − erf curve and step-boundary diffusion emulation image in Figure 6B.

Experiments that make use of 1 − erf type equations often deviate from the above conditions by altering conditions at the original boundary. In some experiments the diffusion occurs from a large-diameter, large-volume solution reservoir into a relatively small-diameter tube of originally pure solvent *(12, 21, 22).* Typically the material in the small-diameter tube is in a gel to effectively immobilize the solvent. For example, Hagland et al. *(12)* allowed diffusion to occur for a time, *t*, and then analyzed slices of the gel in the small-diameter tubing. The degree of diffusion obeys the equation

$$\frac{c}{c_0} = 1 - \mathrm{erf}(z) \qquad (7)$$

Because the reservoir is stirred and large then c_0 is very nearly constant and is therefore the concentration of the solute at the boundary. If the solution and the solvent were in the same diameter tube, then eq 4 would apply and the concentration at the original boundary location would remain constant and would be one half of the original solution concentration. But equivalent conditions may be produced at the original boundary location by using a large-diameter, large-size reservoir at one half the concentration, hence the factor of 2 difference between eq 5 and eq 7.

The condition in which the concentration at the original boundary is kept constant may be emulated using the model by keeping the first cell at a constant water height while other cells are in the process of filling. This is what is happening when the leftmost water input cell of the model is kept full while water spreads to the other cells but before reaching the last cell as shown in Figure 3A.

In other experiments the conditions may be reversed so that the large-diameter, large-size reservoir contains the solvent while the solution is in a small-diameter tube *(23, 24)*. Under these conditions the side of the boundary that originally contained pure solvent is effectively maintained at zero solute concentration rather than at one half the solution concentration as would occur in a tube of uniform diameter. Consequently, to produce the same degree of diffusion in a given time, the solution concentration must be one half that needed in a tube of uniform diameter. Therefore as before, c_0 is one half the c_0 in eq 5.

This modified step-boundary diffusion may be emulated using the model by observing the relative water-level heights when starting with all the cells filled. Block the last exit tube, fill all the cells, and then unblock the exit. Add no water during the emulation. The results are shown in Figures 8A and 8B.

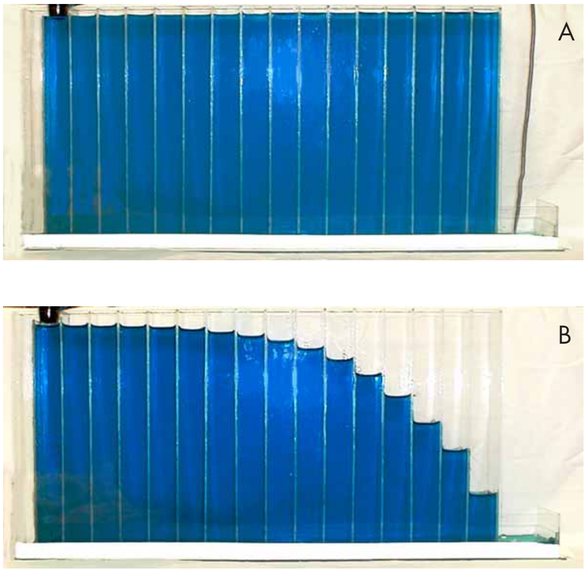

Figure 8. Emulation of modified step-boundary diffusion: (A) initial condition and (B) spreading. The wire plug is visible in Figure 8A.

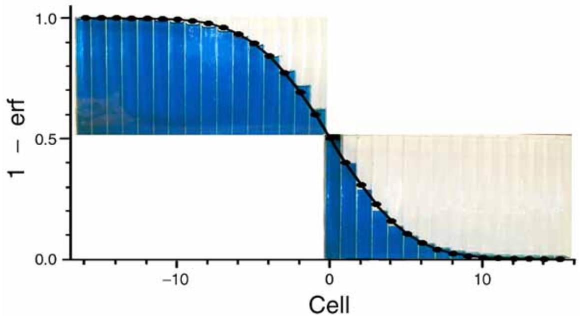

Figure 9. A calculated 1 – erf curve superimposed on a combination of emulations in Figures 8B and 3A.

The two previous experimental methods that utilize a large reservoir in contact with a small-diameter tube can be compared with each other and with the uniform-tube boundary method described above by combining Figure 8B with Figure 3A. The combination forms a complete 1 – erf curve as can be seen in Figure 9 in which a calculated 1 – erf curve has been superimposed on the combination of the two different modified step-boundary emulations.

If the solution is in a small-diameter tube and the solvent in a large reservoir (upper left in the figure) or if the solvent is in a small-diameter tube and the solution in a large reservoir (lower right in the figure), then the original solution concentrations are the same as shown by the water-height maximum as measured from the bottom of each model. On the other hand, if the uniform-diameter tube experimental method is used that utilizes the entire 1 – erf curve, then the original solution concentration is depicted by the difference between the maximum water height on the upper left and the minimum water height on the lower right. This difference is clearly twice as large: compare eq 5 with eq 7.

Other Emulations

The model may be used to emulate other diffusion or thermal conductivity conditions. For example, suppose the ends of two metal rods at different temperatures were brought into thermal contact. If the rods were insulated to prevent thermal energy loss from the system, the thermal energy would distribute itself until a uniform temperature was achieved. The sequence in Figure 10 shows how the temperature would vary with time.

Grover *(25)* used a diffusion method in which the gas being studied was injected into the end of a tube containing the host gas. The tube was 60-cm long and as the diffusion progressed samples for analysis were removed from two sampling ports 10 cm from each end. The concentration profile for this closed tube is given by a Fourier series. The progress of a diffusion of a system like this can be emulated using the model by filling the first three cells and blocking the exit tube of the last partition. Although this sequence is not shown, it starts as half of a Gaussian curve as in Figure 5A and ends somewhat like the last two frames in Figure 10, although the final water depth will only be 3/16 of the original height.

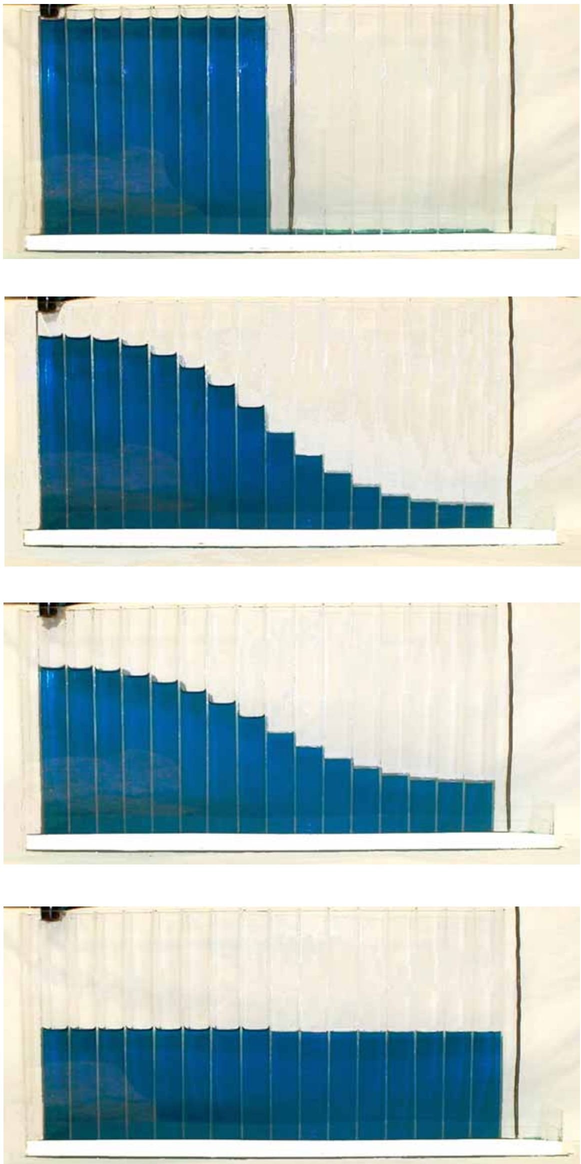

Figure 10. Emulation of thermal energy distribution change with time from uniform high and low temperature regions (top) to one uniform temperature throughout (bottom). A wire is blocking the last exit tube in all photos.

Concluding Remarks

Water flowing through a series of cells connected by a small tube in each partition in this plastic model is capable of emulating diffusion and thermal conductivity that occurs in a variety of systems described by several different mathematical equations. Water height emulates concentration or temperature. The ability to see the changes that occur with time should be very helpful to students and instructors during discussions of diffusion and thermal conductivity in both the classroom and the laboratory.

Acknowledgments

Several people made helpful suggestions but I especially wish to thank Gordon Potter for his many comments, particularly those concerning flow rates through tubes and orifices, and Bruce Childs for sharing his expertise with graphic images.

Notes

1. Presented in part at the 113th meeting of the Tennessee Academy of Science, November 14, 2003.
2. In a nicely written article, Tyrrel *(26)* discusses the origin and status of Fick's laws of diffusion.

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