

Low Carbon Iron-making Supply Chain Planning in Steel Industry

Zhihong Luo^{†,‡} and Lixin Tang^{*,‡,§}

[†]State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang 110819, China

[‡]Institute of Industrial Engineering & Logistics Optimization, Northeastern University, Shenyang, 110819, China

[§]Liaoning Key Laboratory of Manufacturing System and Logistics, Northeastern University, Shenyang, 110819, China

ABSTRACT: This paper investigates a new low carbon iron-making supply chain planning problem in the steel industry under the carbon cap and trade mechanism, in which a steel company uses carbon emission quota to produce iron to meet determined demands over the planning horizon, and buys or sells the rights to emit carbon in the carbon trading market. The problem decides optimal carbon trade, raw material purchasing, raw materials and sinters inventory levels, as well as sintering and iron making production schemes, so as to minimize the total cost. A novel mixed integer programming model incorporating carbon emission reduction into operational decision making is developed and we propose a branch and price algorithm to solve it. The model is decomposed equally into a master problem and two subproblems. The branch and price algorithm is enhanced by two tailored techniques. First, the relaxation of the master problem is strengthened by introducing two families of valid inequalities. Second, we use a dynamic programming procedure to eliminate variables by path reduced cost. We test our algorithm on randomly generated data that simulate the practical production, and the computational results over the instances illustrate the effectiveness of the proposed algorithm. In addition, we derive the optimal operational decisions, and examine the impacts of carbon cap and carbon price analytically and numerically on total cost and carbon emissions. We make interesting observations based on numerical results and provide managerial insight to highlight the opportunity to reduce carbon emissions.

1. INTRODUCTION

Previous research has shown that global warming is directly related to the emissions of CO₂ and other climate-changing gases. Industry is a major contributor to CO₂ emissions. Among all primary industries, the steel industry is well-known as a high-energy consumption sector and a considerable source of CO₂ emissions.

In the steel industry of China, coal is the dominant energy source and, consequently, the major source of CO₂ emissions. Coal provides about 74.5% of total energy needs, in which coking coal accounts for about 85.6% and pulverized coal accounts for about 14.4%. Electricity and petroleum are the second and third sources of energy, respectively.¹ Orth et al.² surveyed the conventional process of making crude steel and estimated that about 2200 kg CO₂ per ton of steel are emitted from conventional steelmaking (blast furnace + basic oxygen converter). According to their survey, the blast furnace and sintering are the top two CO₂ emitters, which account for nearly 82% of the total emissions. Therefore, CO₂ emissions are mainly related to sintering and iron-making processes.

Many countries have attempted to legislate to protect the environment or design curb mechanisms to reduce carbon emissions. For instance, at the 2009 Copenhagen conference, China announced to reduce the intensity of CO₂ emissions per unit of GDP by 40–45% by 2020 compared to the level in 2005. The Chinese government has set strategies to change the mode of economic growth, and a series of aggressive measures to reduce carbon emissions has adopted. Zhu and Geng³ investigated the CO₂ abatement practices of Chinese enterprises and found that steel companies face pressures from suppliers, customers, and competitors, besides national policies and environmental regulations. Responding to the threats of environmental legislation and supplier/customer/competitor

concerns, steel companies generally adopt direct and efficient ways to reduce emissions. Xu and Cang,⁴ and Ghanbari et al.⁵ summarized short-term and long-term approaches to reduce carbon emissions in the steel industry. Approaches in the short term include increasing the energy efficiency and reducing coke ratio, using more hydrogen-bearing reductants (e.g., nature gas), the application of new technologies (e.g., top gas recycling^{6–8}), and so on. In the long term, approaches include the development of nonfossil energy (e.g., biomass^{9,10}), integration of multichemical processes (e.g., polygeneration system^{5,11}), carbon capturing and storage technologies,^{12–14} and conceptual designs of novel steel-making routes.¹⁵ Although these efforts are clearly valuable, steel companies tend to ignore potentially significant reduction methods that are driven by operational policies.

In recent years, steel companies have faced falling prices and lower demand, which in turn have changed their business priorities and made them pay more attention to lower operation costs. Due to excess capacity and intense competition, these effects are quite visible in the steel industry in China, which is the largest steel producer in the world. Therefore, it is necessary to develop methods to reduce carbon emissions for steel companies, while considering economic aspects. Indeed, incorporating carbon concerns into operational decision making can mitigate carbon emissions, i.e., enterprises can optimize their operational decisions in purchase, transportation, production, inventory, and distribution to reduce carbon emissions. There have been many overviews, such as

Received: May 19, 2014

Revised: October 21, 2014

Accepted: November 1, 2014

Published: November 2, 2014

Kleindorfer et al.¹⁶ on sustainable operations management, Sbihi and Eglese¹⁷ on combinatorial optimization and green logistics. Decker et al.¹⁸ also presented a comprehensive review on this topic. Benjaafar et al. introduced a series of models based on classic lot-sizing problem to illustrate how carbon footprint considerations could be incorporated into operational decisions. Their results showed that operational adjustments may reduce more carbon emissions with less cost than adopting energy-efficient technologies.¹⁹ Generally speaking, combination of carbon reduction and operational decision making is not only cost attractive but also tends to emit less carbon.

In this paper, we investigate an integrated low carbon iron-making supply chain planning problem (LCIMSCPP) in the steel industry with consideration of carbon cap and trade. In a cap-and-trade system, a steel company is allocated a carbon cap on emissions in each time period. If the steel company needs to emit more, it can buy extra carbon rights in the carbon trading market. Otherwise, it can sell its surplus carbon rights. The iron-making supply chain includes raw materials purchase, production (at sintering plant and iron-making plant), and storage (at raw materials stock yard and sinters stock yard). Downstream plants such as steel mills and hot strip mills are not included, as they are not within the scope of this study. Low carbon iron-making supply chain planning is a complicated task. It is necessary to balance the total cost and carbon emission reduction. The increasing pressure makes it very important to accurately assess how operational decisions affect total cost and carbon emissions. Meanwhile, we must estimate the impacts of the carbon price and cap on the total cost and carbon emissions.

Carbon cap-and-trade is widely accepted as one of the most effective curb mechanisms to carbon emissions, which has been adopted by the United Nations, the European Union, and many governments. Considering this mechanism, Hua et al.²⁰ introduced a new inventory model. Compared with the classical economic order quantity (EOQ), they derived the optimal order quantity, and examined the impacts of carbon trade, price and cap on order quantity, total cost, and carbon emissions. Zhang and Xu²¹ investigated a multi-item production planning problem for fulfilling stochastic demands under carbon cap and trade mechanism. They proposed a profit maximization model that can be solved with linear computational complexity, and analyzed the impacts of carbon price and cap on system performance and carbon emissions. Benjaafar et al.¹⁹ proposed an extended lot-size model incorporating cap and trade mechanism with backordering cost.

There have been several studies on operational decisions in steel industry, some of which pose interesting applications of optimization models. Dutta and Fourer²² made a survey on the applications of optimal models in the steel industry that included raw material blending, inventory management, production planning, and scheduling. As early as 1958, Fabian investigated a blending problem for integrated steel companies.²³ Fabian improved upon his previously model by including energy balance.²⁴ The model was formulated as a linear program to minimize the total cost of raw materials. In the 1960s, planning for iron production was developed by Lawrence and Flowerdew²⁵ to minimize raw material and operating costs, and their results can provide the burden mixture for the blast furnace. Sasidhar and Achary²⁶ formulated the production planning problem in a steel mill as a multiple arc network model to maximize capacity utilization. The model took into account the order balance positions and the customer

priorities. Mohanty and Singh²⁷ developed a hierarchical system for making aggregate production plan to guarantee the best use of resources. Their model was solved by a multiobjective dynamic algorithm. Chen and Wang introduced a linear model for the optimal production plan in a context of supply chain. Their computational results show that application of the integrated planning methodology could bring high level financial benefit.²⁸ Considering the connection between steel making and casting, Zaroni and Zavanella²⁹ studied integrated production-inventory planning of steel billets and developed a mixed-integer programming model to minimize production, inventory holding, and backordering costs over the planning period. To optimize purchase and plant operation, Gerardi et al.³⁰ proposed an integrated model of primary steel making that connects the three areas of coke making, iron making, and steel making using combination of empirical relationships and mass balances. They used a mixed integer nonlinear programming solver to solve the centralized model, and the raw material uncertainty was accounted for by a two-stage stochastic programming task. Hung³¹ developed a multiperiod steel optimization model based on production network to maximize the profit. In addition, the empirical relationships between inventory and throughput were determined, and then a steel inventory model was developed and incorporated into the optimization model.

Market-based mechanisms have been demonstrated as effective curb mechanisms for carbon reduction in the steel industry.^{32,33} However, there are few research papers that study the steel industrial extension of incorporating carbon cap-and-trade into operational optimization.

We have developed a mixed integer programming model for the LCIMSCPP to decide the carbon trade, raw materials purchasing, raw materials, and sinters inventory levels, as well as sinter and iron production scheme, so as to minimize the total cost. The model is decomposed equally into a master problem and two subproblems. In the master problem, the inventory balance, carbon emission constraints, and convex constraints are considered. Subproblems generate production plans for sintering and iron making separately, and take into consideration the respective production constraints. Both subproblems can be solved by dynamic programming. By embedding the column generation into the framework of the branch and bound, a well-known branch and price (B&P) algorithm is proposed to find the optimal integer solutions, and column generation is used to solve the relaxation of master problem. The B&P algorithm is enhanced by two tailored techniques. First, the relaxation of master problem is strengthened by introducing two families of valid inequalities. Second, we use dynamic programming procedure to eliminate variables by path reduced cost. We test our algorithm on randomly generated data that simulate the practical production, and the computational results over the instances show that the proposed algorithm is effective and that optimal solutions of most instances can be obtained in a reasonable computational time. We compare the impacts of carbon price and cap on total cost and carbon emissions, and find some interesting managerial insights by numerically analysis.

To our knowledge, few research studies incorporate a carbon cap-and-trade mechanism into operational decision making in the steel industry. Therefore, this study is designed to fill in this gap. A novel integrated low carbon supply chain planning model with generality is developed. Additionally, we enhance the B&P algorithm by introducing additional cuts and fixing

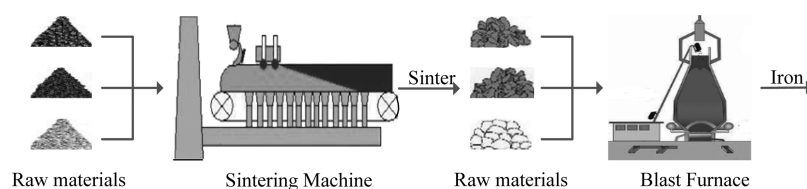


Figure 1. Iron production flow diagram.

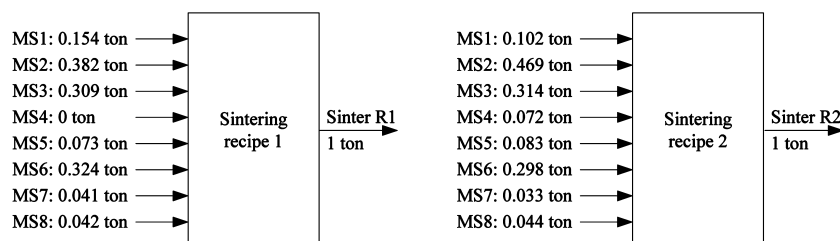


Figure 2. Two examples of sintering recipe.

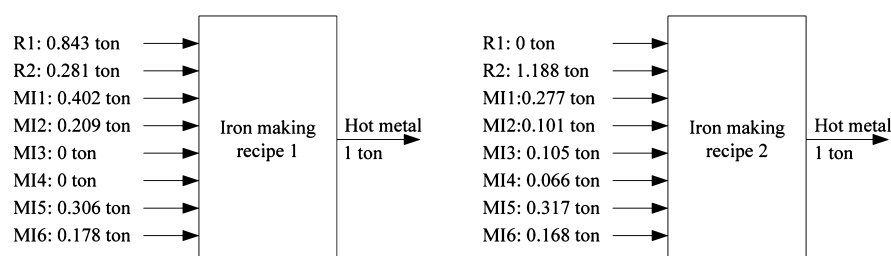


Figure 3. Two examples of iron making recipe.

variables, and the computational results show that our algorithm is quite effective.

The rest of this paper is organized as follows. In section 2, we first provide a brief introduction of integrated production process. Then we describe the modeling issues and the optimization model, and proceed in section 3 with details on the solution method, include the equal decomposition, valid inequalities, and variables fixing and branching. In section 4, we present computational results and discuss the algorithm performance, and analysis the low carbon strategy. Section 5 offers conclusions and briefly discusses the directions of future research.

2. PROBLEM DESCRIPTION

The following section provides an overview of how the LCIMSCPP is modeled, with key constraints and assumptions indicated.

2.1. Problem Description. As illustrated in Figure 1, the iron-making processes are concerned with two main transformations: one is ferruginous ore powders to semifinished agglomerate iron ores (sintering), and the other is semifinished products and lump ores to finished hot metal (iron making), via complicated processes accompanied by physical and chemical reactions.

Sinter making is a pretreatment process for the blast furnace, where iron ores are reduced to produce hot metal. Raw materials used in sinter making usually include iron ore fines, fluxes (for instance, limestone and dolomite), fuels (coke powder and coal) and so on. These raw materials are bought from the external suppliers and stored in the stock yard. Since ferruginous ores differ from one another in their granulometric, chemical compositions and physicochemical properties, they

are conveyed to a blending yard and then mixed together in certain ratios. In sinter making, the prepared mixture is put to sintering pallet, and the top layer of the mixture is ignited and air is sucked through it by exhauster. The burning front moves down when the sintering pallet moves horizontally. In the burning front, the eutectic in the mixture will be softened or melted, and the resulting liquid phase comes out to wet other granules which are not be melted. During the cooling process, the liquid phases make other granules into lumps. Afterward, sinter is crushed to a predetermined particle size and stored in the stock.

The material used in iron making mainly includes sinters, pellets, lump ores, lime, coke, powder coal, etc. A blast furnace works as a large chemical reactor and heat exchanger. The iron-bearing burden material mixture is charged with limestone and coke in alternate layers. The preheated air at a high pressure enters through the tuyeres at the bottom of the furnace, and the coke burns to provide CO to reduce iron oxides and provide energy to melt the impurities and iron. At the same time, the limestone absorbs impurities to form liquid slag. The hot metal and slag are tapped through tapholes at the furnace bottom, while the top gas leaves the furnace top through uptakes.

In our problem, recipe is one of the key concepts in the sintering and iron-making process. For any recipe, the production is viewed as a unique transformation process that defines a relationship between input materials and product (sinter or hot metal). They are related by constant coefficients in physical terms, such as material consumption of unit product. For example, considering the two different types of sintering recipes in Figure 2, we need to take different quantity-combinations of materials (MS1–MS8) to produce one ton of sinter R1 and sinter R2 separately. Using a recipe, the outcome

is a specific type of qualified sinter. Sinters differ from each other because their characters (chemical composition, reducibility, low- and high-temperature strength, and so on) are different. Given a recipe, iron making transforms a quantity-combination of materials to 1 ton of hot metal. Two examples of iron-making recipes are illustrated in Figure 3. It is noted that sinter R1 and R2 are used in recipe1, and only R2 is used in recipe2 in iron making. For any sintering (or iron making) recipe, we assume that the carbon emission per unit of sinter (or hot metal) is a known constant.

For any (sintering or iron making) recipe, the material consumptions of unit product are predetermined carefully to satisfy the technologic constraints and product quality requirements. Currently, a Chinese steel company and its supplier design a short-term purchase contract that is just an intentional framework agreement. The price of iron ore is not decided when the contract is designed. The actual transaction price is affected by market-based barometer (e.g., Platts IODEX) when the ore arrives. The steel company also buys suitable ores on the spot market. Compared to the traditional long-term contract, combining the short-term contract with spot purchasing can allow more flexibility for the companies to adjust their operation policies. In practice of operations management, a batch of blended ferrous ores always meets the demand for iron-making for 1 week (or several days). So, we divide the planning horizon into small units, and only one recipe can be used in each time period in sintering or iron making.

There are two important cost factors in production: production cost and changeover cost between recipes. Both costs differ depending on what recipe is used. The changeover cost is dependent on the sequence of recipe involved. Frequent recipe changes do not meet the requirement of smooth mass flow in production. Recipe change in adjacent periods has a negative effect on production at the junction. For example, a recipe change may lead to a drop in the finished sinter rate and iron-making yield during the fluctuation, and the need for additional resources (e.g., energy) to stabilize the production. Therefore, changeover cost can be interpreted as loss that can be estimated by summation of following items: (1) the profit loss arising from the productivity decline during the fluctuation and (2) the costs of additional recourses used during the fluctuation.

In general, forecast and customer demands are the driving forces in making a production plan. If surplus iron may be sold, a lower limit is set on the iron production rate. Under other circumstances, it may be necessary to fix the production rate exactly.²⁵ To simplify the problem, we assume that the iron demand is predetermined and equal to the output of blast furnace in each period. In the sintering process, we assume the sinter is produced at a fix and economic rate once the recipe is determined in one period, and the effective capacity at sintering plant is greater than the demand from iron making in each period. In practice, there are stops in some periods at sintering plant due to, for example, sufficient sinter inventory and carbon emission reduction. The stop is treated as a recipe with no input and no output.

We assume that the prices of raw materials and carbon prices in each period are predictable and exogenous to decisions made by individual steel company. Given the iron demands over the planning horizon and the sets of all raw materials and recipes, the LCIMSCPP decides the carbon trade, raw material purchasing, raw materials and sinters inventory levels, as well

as sintering and iron making recipe schemes. The objective is to minimize the cost function consisting several terms: (1) purchase cost of all raw materials, (2) production and changeover cost in sintering and iron making, (3) inventory-hold cost of raw materials and sinters, and (4) carbon penalties and rewards.

2.2. Model Formulation. Before developing the model, we present a summary of the notations and variables that will be used in the model. We first define the sets, parameters, and variables (see the Nomenclature section).

Using the notations and variables described in the Nomenclature section, the mixed integer programming (MIP) for the LCIMSCPP can be formulated as follows

$$\begin{aligned} \text{Min } & \sum_i \sum_t p_{c_{it}} x_{it} + \sum_i \sum_t h_i l_{it} \\ & + \sum_r \sum_t f_r g_{rt} + \sum_r \sum_{r'} \sum_t c_{rr'}^s z_{rr't}^s \\ & + \sum_k \sum_{k'} \sum_t c_{kk'}^b z_{kk't}^b + \sum_r \sum_t c_p^s C_r y_{rt}^s \\ & + \sum_k \sum_t c_p^b d_k y_{kt}^b + \sum_t p_t^{\text{carbon}} (e_t^+ - e_t^-) \end{aligned} \quad (1)$$

s.t.

$$x_{it} + l_{i,t-1} - \sum_r a_{ir}^s C_r y_{rt}^s - l_{it} = 0, \forall i \in \text{IS}, \forall t \quad (2)$$

$$x_{it} + l_{i,t-1} - \sum_k a_{ik}^b d_k y_{kt}^b - l_{it} = 0, \forall i \in \text{IB}, \forall t \quad (3)$$

$$g_{r,t-1} + C_r y_{rt}^s - \sum_k w_{rk}^b d_k y_{kt}^b - g_{rt} = 0, \forall r, \forall t \quad (4)$$

$$\sum_r y_{rt}^s = 1, \forall t \quad (5)$$

$$\sum_{r'} z_{rr't}^s = y_{r,t-1}^s, \forall r, \forall t \quad (6)$$

$$\sum_{r'} z_{r't}^s = y_{rt}^s, \forall r, \forall t \quad (7)$$

$$\sum_k y_{kt}^b = 1, \forall t \quad (8)$$

$$\sum_{k'} z_{kk't}^b = y_{k,t-1}^b, \forall k, \forall t \quad (9)$$

$$\sum_{k'} z_{k't}^b = y_{kt}^b, \forall k, \forall t \quad (10)$$

$$\sum_r e_r^s C_r y_{rt}^s + \sum_k e_k^b d_k y_{kt}^b - e_t^+ + e_t^- = E_t^{\text{cap}}, \forall t \quad (11)$$

$$y_{rt}^s, y_{kt}^b, z_{rr't}^s, z_{kk't}^b \in \{0, 1\}, x_{it}, l_{it}, g_{rt}, e_t^+, e_t^- \geq 0 \quad (12)$$

The objective (eq 1) aims at minimizing the total costs. The relevant costs are the purchase cost, inventory holding cost, changeover cost, production cost and carbon penalties (rewards). Constraints 2–4 express the inventory balance equations in which it is assumed that the initial inventories are given. Constraint 5 specifies only one recipe per period can be used in sintering. Constraints 6 and 7 force the changeover variable to one if recipe change occurs in sintering. Constraint 8

makes sure that only one recipe per period can be used in iron making. Constraints 9 and 10 ensure the changeover variable is one if recipe change occurs in iron making. Constraint 11 is the carbon constraints. The integrality and nonnegative conditions on the decision variables are specified by constraint 12.

3. SOLUTION METHOD

The MIP has $O(|RS|^2|T|+|RB|^2|T|)$ integer variables, which may be prohibitive when $|RS|$, $|RB|$, and $|T|$ are large. We solve the problem by column generation-based B&P algorithm. For column generation, the reader can refer to Lübbecke and Desrosiers,³⁴ and Barnhart et al.³⁵ Column generation is an effective method to solve many combinatorial optimization problems such as vehicle routing problems,^{36,37} lot sizing problem,^{38,39} parallel machine or flow shop scheduling problems,^{40–42} shipment planning problem⁴³ and ship scheduling problem⁴⁴ in oil industry, order batching problems^{45,46} in steel industry, and supply chain optimization in paper industry.⁴⁷ In this section, we first introduce the transformation principle. Applying the Dantzig–Wolfe decomposition,⁴⁸ the MIP is equally transformed into a master problem and two subproblems for sintering and iron making separately. Two families of valid inequalities are introduced to strengthen the master problem. We then present dynamic programming algorithms to solve the two pricing subproblems. The integer variables are eliminated by path reduced cost, and we end this section by developing the heuristic feasible solution.

3.1. Master Problem. A column represents a production plan for sintering or iron making over the planning horizon in our problem. We define new sets, parameters, and variables (see the Nomenclature section).

The master problem (MP) of MIP is then formulated as follows

$$\begin{aligned} \text{Min } & \sum_t \sum_i pc_{it} x_{it} + \sum_t \sum_i h_i l_{it} + \sum_t \sum_r f_r g_{rt} \\ & + \sum_t p_t^{\text{carbon}} (e_t^+ - e_t^-) + \sum_p c_p \alpha_p + \sum_q c_q \beta_q \end{aligned} \quad (13)$$

s.t.

$$x_{it} + l_{i,t-1} - \sum_p \sum_r a_{ir}^s C_r y_{rtp}^s \alpha_p - l_{it} = 0, \forall i \in \text{IS}, \forall t \quad (14)$$

$$x_{it} + l_{i,t-1} - \sum_q \sum_k a_{ik}^b d_k y_{ktq}^b \beta_q - l_{it} = 0, \forall i \in \text{IB}, \forall t \quad (15)$$

$$\begin{aligned} g_{r,t-1} + \sum_p C_r y_{rtp}^s \alpha_p - \sum_q \sum_r \sum_k w_{rk}^b d_k y_{ktq}^b \beta_q - g_{rt} \\ = 0, \forall r, \forall t \end{aligned} \quad (16)$$

$$\begin{aligned} \sum_p \sum_r e_r^s C_r y_{rtp}^s \alpha_p + \sum_q \sum_k e_k^b d_k y_{ktq}^b \beta_q - e_t^+ + e_t^- \\ = E_t^{\text{cap}}, \forall t \end{aligned} \quad (17)$$

$$\sum_p \alpha_p = 1 \quad (18)$$

$$\sum_q \beta_q = 1 \quad (19)$$

$$x_{it}, l_{it}, g_{rt}, e_t^+, e_t^- \geq 0, \alpha_p, \beta_q \in \{0, 1\}, p \in P, q \in Q \quad (20)$$

In MP, the objective (eq 13) is to minimize the total cost. We retain the inventory balance eqs 14–16 and carbon constraints (eq 17) as in the MIP and add convexity constraints (eqs 18 and 19), enforcing that we choose exactly one sintering plan and one iron-making plan, respectively.

3.2. Valid Inequalities. It is well-known that valid inequalities can often be used effectively to improve bound since valid inequalities may cut a part of the fractional (infeasible) solutions given by the linear relaxation of master problem or original problem, and can thereby increase the objective value. We define a set $RK = \{(r, k) : w_{rk}^b > 0\}$ and g_{r0} is the initial inventory of sinter r , and let $d_t^{\max} = \max\{d_r : r \leq t\}$, $sd_t = \sum_{\tau \leq t} d_\tau$ and $n_{rkt} = \lceil (w_{rk}^b sd_t - g_{r0}) / C_r \rceil$. Then following property 1 provides a family of valid inequality.

Property 1: If $w_{rk}^b d_t \leq C_r (\forall r \in RS, \forall k \in RB, w_{rk}^b > 0)$, inequality 21 is valid for MIP.

$$\sum_{\tau \leq t} y_{r\tau}^s + t \geq \sum_{\tau \leq t} y_{k\tau}^b + n_{rkt}, (r, k) \in RK, \forall t \quad (21)$$

Proof: For $(r, k) \in RK$ and $t \in T$, the inequality 22 is true in any feasible solution:

$$\sum_{\tau \leq t} C_r y_{r\tau}^s \geq \sum_{\tau \leq t} w_{rk}^b d_\tau y_{k\tau}^b - g_{r0}, (r, k) \in RK, \forall t \quad (22)$$

Recall that $y_{k\tau}^b$ are 0-1 integer variables and $0 \leq \sum_{\tau \leq t} y_{k\tau}^b \leq t$, and let $\sum_{\tau \leq t} y_{k\tau}^b = t - \Delta$ ($\Delta \geq 0$). Then inequality 22 can be relaxed as follows

$$\sum_{\tau \leq t} C_r y_{r\tau}^s \geq w_{rk}^b sd_t - g_{r0} - w_{rk}^b d_t^{\max} \Delta, (r, k) \in RK, \forall t \quad (23)$$

By dividing inequality 23 by C_r and rounding up the left and right items, we have

$$\begin{aligned} \sum_{\tau \leq t} y_{r\tau}^s \geq \lceil (w_{rk}^b sd_t - g_{r0}) / C_r \rceil - \lceil w_{rk}^b d_t^{\max} \Delta / C_r \rceil, (r, k) \\ \in RK, \forall t \end{aligned} \quad (24)$$

Because $w_{rk}^b d_t^{\max} \leq C_r$, $\lceil w_{rk}^b d_t^{\max} \Delta / C_r \rceil \leq \Delta$. From inequality 24 and $\sum_{\tau \leq t} y_{k\tau}^b = t - \Delta$, we derive inequality 21.

The MP can be strengthened through the introduction of valid inequality 21. Using the notations in MP, eq 21 becomes the following inequality for MP:

$$\begin{aligned} \sum_p \sum_{\tau \leq t} y_{r\tau}^s \alpha_p - \sum_q \sum_{\tau \leq t} \sum_k y_{k\tau}^b \beta_q \\ \geq n_{rkt} - t, (r, k) \in RK, \forall t \end{aligned} \quad (25)$$

Recall that we treat the stop as a recipe at sintering plant. Let $RS' = RS \cup \{|RS| + 1\}$, where recipe $|RS| + 1$ is the stop one. Let $C^{\max} = \max\{C_r, \forall r\}$ and $w^{\min} = \min\{\sum_r w_{rk}^b, \forall k\}$, i.e., w^{\min} is the least quantity of sinters used in producing one unit iron. Then, we have following valid inequality for MIP:

$$\sum_{\tau \leq t} y_{|RS|+1, \tau}^s \leq n_t^{\text{stopmax}}, \forall t \quad (26)$$

In inequality 26, $n_t^{\text{stopmax}} = t - [(w^{\min} \text{sd}_t - \sum_r g_{r0})/C^{\max}]$ denotes the upperbound of the number of periods in which sintering stops from period 1 to t . Using the notations in MP, eq 26 becomes the following inequality:

$$\sum_p \sum_{\tau \leq t} y_{|RS|+1, \tau, p}^s \alpha_p \leq n_t^{\text{stopmax}}, \forall t \quad (27)$$

3.3. Subproblems. Because subproblems are solved as an integer programming, the linear relaxation of MP (LMP) may provide a tighter lower bound than the linear relaxation of MIP (LMIP). Column generation always starts with a subset of variables of α_p and β_q in the so-called restricted master problem and iteratively generates new variables. Let η_{it} , φ_{it} , ψ_{rt} , ρ_t , μ , ν , σ_{rt} and ε_t be dual variables corresponding to constraints 14–19, 25, and 27 separately. The LMP are solved to optimality and dual prices are passed to pricing problems.

For any r , let $K(r) = \{k: w_{rk}^b > 0\}$, and

$$sc_{rt}^s = \begin{cases} C_r(\text{cp}_r^s + \sum_{i \in IS} a_{ir}^s \eta_{it} - \psi_{rt} - \varepsilon_r \rho_t) \\ - \sum_{k \in K(r)} \sum_{\tau \geq t} \sigma_{rkt}, \text{ if } r \in RS \\ - \sum_{\tau \geq t} \varepsilon_\tau, \text{ if } r = |RS| + 1 \end{cases} \quad (28)$$

For any $p \in P$, its reduced cost in MP is

$$rc_p^s = \sum_r \sum_{r'} \sum_t cc_{rr'}^s z_{rr't}^s + \sum_t \sum_r sc_{rt}^s y_{rt}^s - \mu \quad (29)$$

Then, the subproblem for sintering (SUBS) is

$$\min rc_p^s$$

s.t. $p \in P = \{(y_{rt}^s, z_{rr't}^s) \mid \text{eqs 5–7 and 12}\}$.

The objective function of SUBS minimizes the reduced cost of new production plan for sintering. The SUBS is a shortest path problem, which can be solved by following dynamic programming. To simplify the expression, we add virtual recipe 0 and let $cc_{0r}^s = 0$. Let $F^s(r, t)$ be the cost of an optimal solution of a sintering subplan from period 1 to t where recipe r is used during period t . Let $Tpf^s(r, t)$ be the total quantity of sinters corresponding to $F^s(r, t)$. With the initial values $F^s(0, 0) = 0$, $Tpf^s(0, 0) = \sum_r g_{r0}$, $F^s(r, 0) = +\infty$ and $Tpf^s(r, 0) = -\infty$, the recursive relation is as follows

$$F^s(r, t) = \min_{r' \in RS'} \left\{ \begin{aligned} &cc_{r'r}^s + sc_{rt}^s + F^s(r', t-1): \\ &Tpf^s(r', t-1) + C_r \\ &\geq w^{\min} \text{sd}_{|T|} - (|T| - t)C^{\max} \end{aligned} \right\} \quad (30)$$

The total quantity of sinters corresponding to $F^s(r, t)$ is updated by

$$Tpf^s(r, t) = Tpf^s(r', t-1) + C_r, r' = \arg \min \{F^s(r', t)\} \quad (31)$$

The optimal value Z_{SUBS} of SUBS is then obtained by $Z_{\text{SUBS}} = \min_{r \in RS'} \{F^s(r, |T|)\} - \mu$. In eq 30, the constraints $Tpf^s(r', t-1) + C_r \geq w^{\min} \text{sd}_{|T|} - (|T| - t)C^{\max}$ can reduce the state space in SUBS to improve column generation convergence because some infeasible columns are filtered out of the master problem.

For any k , let $R(k) = \{r: w_{rk}^b > 0\}$, and

$$sc_{kt}^b = \sum_{r \in R(k)} \sum_{\tau \geq t} \sigma_{rkt} + d_t(\text{cp}_k^b + \sum_{i \in IB} a_{ik}^b \varphi_{it} + \sum_r w_{rk}^b \psi_{rt} - \varepsilon_k \rho_t) \quad (32)$$

For any $q \in Q$, its reduced cost in MP is

$$rc_q^b = \sum_k \sum_{k'} \sum_t cc_{kk'}^b z_{kk't}^b + \sum_t \sum_k sc_{kt}^b y_{kt}^b - \nu \quad (33)$$

The subproblem for iron making (SUBIM) is

$$\min rc_q^b$$

s.t. $q \in Q = \{(y_{kt}^b, z_{kk't}^b) \mid \text{eqs 8–10 and 12}\}$.

For the SUBIM, we define $F^b(k, t)$ as the cost of an optimal solution of an iron making subplan from period 1 to t where recipe k is used during period t . With the initial values $F^b(0, 0) = 0$ and $F^b(k, 0) = +\infty$, the recursive relation is as follows

$$F^b(k, t) = \min_{k' \in RB} \{cc_{k'k}^b + sc_{kt}^b + F^b(k', t-1)\} \quad (34)$$

The optimal value can be obtained by $Z_{\text{SUBIM}} = \min\{F^b(k, |T|), \forall k\} - \nu$.

3.4. Fixing Variables by Path Reduced Cost. Given a feasible solution (upper bound) of an integer linear program with minimized objective, a nonnegative integer variable can be fixed to zero if its reduced cost corresponding to a feasible dual solution of a linear relaxation exceeds the integrality gap. This proposition has been proven by Hadjar et al.⁴⁹ Variable fixing (or elimination) can also be applied in the B&P framework. Irnich et al.⁵⁰ proved variable fixing can be used not only in the master problem but also in pricing problems. When the pricing problem constitutes a shortest path formulation, arc-flow variables elimination can lead to a considerable speedup of the pricing problem and, therefore, of the overall B&P algorithm. For a parallel machine scheduling problem, Pessoa et al.⁵¹ fixed variables by lagrangian bounds. They verified that the variables fixing procedure can reduce the number of pseudoschedules, and improve column generation convergence.

Let $\pi = (\bar{\eta}_{it}, \bar{\varphi}_{it}, \bar{\psi}_{rt}, \bar{\rho}_t, \bar{\sigma}_{rkt}, \bar{\varepsilon}_t, \bar{\mu} + Z_{\text{SUBS}}, \bar{\nu} + Z_{\text{SUBIM}})$ denote the optimal dual variables of the restricted LMP in the current iteration. Then, Z_{SUBS} and Z_{SUBIM} are obtained whenever the pricing problems are solved with current optimal dual variables, and we can construct a feasible dual solution of LMP as follows

$$\pi' = (\bar{\eta}_{it}, \bar{\varphi}_{it}, \bar{\psi}_{rt}, \bar{\rho}_t, \bar{\sigma}_{rkt}, \bar{\varepsilon}_t, \bar{\mu} + Z_{\text{SUBS}}, \bar{\nu} + Z_{\text{SUBIM}}) \quad (35)$$

As we know, the objective value of dual feasible solution can provide a lower bound for the primal problem. Therefore, $Z_{\text{LMP}} + Z_{\text{SUBS}} + Z_{\text{SUBIM}}$ is a lower bound, where Z_{LMP} is the optimal value of LMP in the current iteration. For a given period t , let $\text{Zarc}_{\text{SUBS}}(r, r', t)$ be the optimal solution of pricing problem SUBS plus the constraint $z_{rr't}^s = 1$. If $Z_{\text{LMP}} + Z_{\text{SUBIM}} + \text{Zarc}_{\text{SUBS}}(r, r', t) \geq Z_{\text{FP}}$, where Z_{FP} is the objective value of the best known feasible solution, variable $z_{rr't}^s$ can be eliminated because it cannot appear in better solutions. The main point of variable fixing is that, if the SUBS have already been solved by forward dynamic programming recursion to obtain Z_{SUBS} , the $\text{Zarc}_{\text{SUBS}}(r, r', t)$ values can be obtained by just solving a second backward dynamic programming. Let $B^s(r, t)$ be the minimum cost of a subplan that starts with recipe r in period t and finishes in period $|T|$ in sintering. Let $Tpb^s(r, t)$ be the total quantity of sinters corresponding to $B^s(r, t)$. We have the following dynamic programming recursion:

$$B^s(r, t) = \min_{r' \in RS'} \left\{ \begin{array}{l} cc_{rr'}^s + sc_{rt}^s + B^s(r', t + 1): \\ Tpb^s(r', t + 1) + C_r \\ \geq w_{sd}^{\min} |T| - (t - 1)C^{\max} \end{array} \right\} \quad (36)$$

The value of $Zarc_{SUBS}(r, r', t)$ is given by

$$Zarc_{SUBS}(r, r', t) = F^s(r, t - 1) + c_{rr'}^s + B^s(r', t) - \mu \quad (37)$$

The above method also can be applied to eliminate y_{rt}^s variables. Let $Zst_{SUBS}(r, t)$ be the optimal solution of the pricing problem SUBS plus the constraint $y_{rt}^s = 1$. If $Z_{LMP} + Z_{SUBIM} + Zst_{SUBS}(r, t) \geq Z_{FP}$, we can remove variable y_{rt}^s . $Zst_{SUBS}(r, t)$ is given by

$$Zst_{SUBS}(r, t) = F^s(r, t) + \min_{r' \in RS'} \{B^s(r', t + 1) + c_{rr'}^s\} - \mu \quad (38)$$

Similarly, variables y_{kt}^b and $z_{kk't}^b$ can be eliminated as much as possible, and we omit the implementation details. About double times are spent on variable elimination as much as that on ordinary pricing process. We implement the variables fixing procedure at the end of column generation at the branch nodes.

The variables fixing requires the lower bound $Z_{LMP} + Z_{SUBIM} + Zarc_{SUBS}(r, r', t)$ as large as possible. In general, to test whether the current solution of LMP is optimal, we have to minimize the pricing problems, and the column generation ends if there is no plan with negative reduced cost. However, column generation convergence may slow in the last iterations at some branch nodes in some instances. Hence, to accelerate the branching process, a hybrid strategy is used to reduce the computational time. At the root node, we terminate the column generation once the value of LMP is less than the value of γZ_{FP} . At other nodes, we terminate the column generation if the value of LMP is less than $Z_{FP} - \delta$. The γ and δ are parameters to control the number of column generation iterations (for the values of γ and δ , see section 4).

3.5. Branching Strategy. After the variables fixing is complete, branching is performed by choosing a recipe (r or k) in a time period t such that

$$(r, t) = \arg \min_p \{ |\sum y_{rip}^s \alpha_p - 0.5|, \forall r, \forall t \} \quad (39)$$

$$(k, t) = \arg \min_q \{ |\sum y_{ktq}^b \beta_q - 0.5|, \forall k, \forall t \} \quad (40)$$

That is, we choose the y_{rt}^s or y_{kt}^b that is most close to 0.5 as a branching variable. The branching variable choosing starts in the sintering stage, then it moves to the iron-making stage if all y_{rt}^s are integers. Once the branching variable is decided, the variable is fixed to zero on the left child node and is fixed to one on the right child node. On the child node, we add the branching constraint into the column generator. If branch variable is zero, we set a virtual cost to the branch variable to be $+\infty$ to prevent the 0-1 variable to be one.

3.6. Initial Columns and Heuristic Feasible Solution. For column generation to perform correctly, initial columns are needed. However, it is not easy to find initial columns to ensure the flow conservation. So, for each sinter, one artificial variable is added to constraint 16 when $t = 1$. It is noted that this artificial variable with enough higher cost can be regarded as sinter initial inventory. The quality of the feasible solution is one of the key factors in B&P algorithm. Initially, we add

sintering and iron making plans in which single recipe is used throughout the planning horizon, i.e., no recipe change occurs. We use two heuristics for feasible solutions.

In the first heuristics, calculating feasible upper bounds starts from the solution of the current LMP. In the solution of LMP on each branch node, if one recipe is chosen in a given period, the summation of all columns (variables) in which the recipe is used in that period may be very close to one, or even exactly one. This summation represents the proportion of all generated columns that the recipe is used in a given time period. For a recipe, if the summation is largest, it strongly implies that the recipe should be chosen in that period in a good solution. Then the heuristic find that recipe and period, and we fix the variable to one. When all recipes in all periods are fixed, the solution can be found by solving a linear programming problem easily. This heuristic algorithm fails if the inventory balance eqs 4 are violated.

The second heuristics is a simple heuristic dynamic programming that utilizes the dual information in column generation. Let $H(r, k, t)$ be the cost of a near-optimal solution of a production plan from period 1 to t where recipe r is used for sintering and recipe k is used for iron making in period t . Let $G(r, k, t)$ be the vector of sinters inventories at the end of period t corresponding to $H(r, k, t)$. With the initial values $G(0, 0, 0) = (g_{1,0}, \dots, g_{|RS|+1,0})$, $H(0, 0, 0) = 0$, and $H(r, k, 0) = +\infty (r \in RS' \text{ and } k \in RB)$, the recursive relation is as follows

$$H(r, k, t) = \min_{G(r', k', t-1) \geq 0} \left\{ \begin{array}{l} H(r', k', t-1) + cc_{rr'}^s + vc_{rt}^s C_r + cc_{kk'}^b \\ + vc_{kt}^b d_t: \\ G(r', k', t-1) + (0, \dots, C_r, \dots, 0) \\ - (w_{1k}^b d_t, w_{2k}^b d_t, \dots) \geq 0 \end{array} \right\} \quad (41)$$

The sinters inventory vector corresponding to $H(r, k, t)$ is updated by

$$G(r, k, t) = G(r', k', t-1) + (0, \dots, C_r, \dots, 0) - (w_{1k}^b d_t, \dots, w_{rk}^b d_t, \dots) \quad (42)$$

It is noted that $g_{|RS|+1,0} = C_{|RS|+1} = w_{|RS|+1,k}^b = 0$. The feasible solution can be obtained after the sintering and iron making schemes are given by $\min_{G(r,k,t) \geq 0} \{H(r, k, |T|), \forall r, \forall k\}$. In the

numerical experiment (in section 4), this algorithm is performed every five iterations in column generation. The advantage of the heuristics is that we can make use of the change of dual variables during column generation and it is possible to find better feasible solution.

4. NUMERICAL EXPERIMENTS

In this section, we present numerical examples to evaluate the algorithm performance. In addition, we examine how carbon cap and price affect total cost and carbon emissions, and then provide some interesting observations. All experimental tests were performed on a personal computer with a 3.1 GHz processor equipped with 4 GB RAM. The algorithms were implemented using Cplex12.0 as linear programming solver. For all test instances, the run time is limited to 1000 s for the B&P runs.

4.1. Experiment Design. The aim of experiment is to estimate the performance of our algorithm. First, we would like to demonstrate that our algorithms can find optimal or near-optimal solutions in a reasonable computational time. Two

important factors affect the speed of our algorithm. One is the size of the restricted LMP, which depends on the constraints in LMP and the number of columns added in iterative process, and the other is the size of the pricing problem. The robustness of our algorithm is another major concern. We are interested in the performance of our algorithm as the length of planning horizon, the number of raw materials and the number of recipes increase. The importance of the latter changes derives from the fact that steel company expands its raw materials purchase channel and more recipes can be used in production. We denote the problem structure by (IIS|, IIB|, IIRS|, IRB|, ITI), and use six different problems (see Table 1) to assess the algorithm

Table 1. Structure of Problem in Experiment

Pro.1	Pro.2	Pro.3
(10, 10, 5, 10, 10)	(10, 10, 5, 10, 20)	(10, 10, 5, 10, 30)
Pro.4	Pro.5	Pro.6
(20, 20, 10, 15, 10)	(20, 20, 10, 15, 20)	(20, 20, 10, 15, 30)

performance. For each problem, 10 test instances are randomly generated and computed. Larger size instances are not tested because of running out of memory.

In our simulation, we neglect some raw materials (e.g., limestone and dolomite) because their prices are relatively low in the market. The sintering and iron-making recipe sets are generated according to production practice. Average 1.32 ton sinters and 0.33 ton other ferrous ores will be used in producing 1 ton iron. Unless stated otherwise, we let $C_r = 16\,000$, $l_{i,0}(i \in \text{IS} \cup \text{IB}) = g_{r,0}(r \in \text{RS}) = 0$, $c_{r,| \text{IRS}|+1} = c_{| \text{IRS}|+1,r} = 0(r \in \text{RS})$ and $h_i(i \in \text{IS} \cup \text{IB}) = f_r(r \in \text{RS}) = 0.5$ (\$/ton·week). In the iron ore market, the Platts IODEX of fine ore indicates that the iron grade of 62% is one of the important references to the actual transaction price. Let $\text{Fe}(i)$ be the iron grade of raw material i , and let pla_t be the reference price of fine ore in period t . To avoid the trivial case, the purchase price of iron ore i is estimated by $\text{pc}_{it}(i \in \text{IS}) = \text{Fe}(i) \times \text{pla}_t/62\%$ and $\text{pc}_{it}(i \in \text{IB}) = 15 + \text{Fe}(i) \times \text{pla}_t/62\%$. For 1 ton lump ore, a steel company always spends about \$10–20 more on the fine ore with the same iron grade. Because the carbon emissions greatly depend on the technologies and energy efficiency, we estimate that sinter-making contributes about 200 kg of CO_2 per ton of sinter and, for iron-making, about 1500 kg of CO_2 per ton of hot metal according to the survey by Orth et al.,² as well as Ren and Wang.⁵² To express convenient, let $E^{\text{cap}} = (1.5 + 0.264) \times \text{sd}_{| \text{IT}|}$ be total carbon cap for the total planning horizon. Then, let $E_t^{\text{cap}} = \zeta \times E^{\text{cap}} \times d_t/\text{sd}_{| \text{IT}|}$, where ζ is defined as assignment factor of carbon cap. In addition, let $p_t^{\text{carbon}} = 5\lambda + p_t^{\Delta}$, where p_t^{Δ} is the fluctuation factor of carbon price in the market, and λ is the parameter to control the increase of carbon price. In practice, values of pla_t , p_t^{Δ} and other parameters always fluctuate in some ranges, and they are generated randomly from uniform distributions (see Table 2).

After data is generated, we can control the carbon price and the carbon cap by setting λ and ζ to observe the impacts of them on the total cost and carbon emissions.

4.2. Algorithm Performance. In this subsection, we set $\lambda = 1$ ($p_t^{\text{carbon}} \sim U[5, 10]$) and $\gamma = 0.99$ and $\delta = 100\,000$. All instances are solved by our B&P algorithm and CPLEX integer programming solver separately. The comparison results of B&P and CPLEX for each problem are shown in Table 3, which contains number of optimal solutions (NOS), run time in seconds and gap between B&P and CPLEX. The gap is

Table 2. Parameters Generation Based on Uniform Distributions

parameters	uniform distribution
pla_t	$U[80, 85]$
cp_t^s (\$/ton)	$U[35, 40]$
cp_t^b (\$/ton)	$U[40, 45]$
$\text{cc}_{rr'}^s(r \neq r')$, $\text{cc}_{kk'}^b(k \neq k')$ (\$)	$U[4000, 5000]$
d_t (ton)	$U[95, 105] \times 100$
e_r^s (ton)	$0.2 + U[-50, +50]/1000$
e_k^b (ton)	$1.5 + U[-50, +50]/1000$
p_t^{Δ} (\$/ton)	$U[0, 5]$
price of coke powder (\$/ton)	$U[90, 95]$
price of powder coal (\$/ton)	$U[100, 105]$
price of coke (\$/ton)	$U[170, 175]$

obtained by $\text{gap} = (z_{\text{CPLEX}} - z_{\text{B\&P}})/z_{\text{B\&P}}$, where $z_{\text{B\&P}}$ and z_{CPLEX} are the values of best feasible solution found by B&P and CPLEX separately.

From Table 3, we can observe that optimal or near-optimal solutions of all instances in Pro.1–6 can be found by our B&P algorithm and CPLEX. In Pro.3 and Pro.5, although some instances are not solved to optimality by CPLEX, their best solutions are equal to the optimal solutions found by our B&P algorithm; therefore, the average gap is 0 and NOS is 10. In Pro.6, two instances are not solved to optimality by either B&P or CPLEX, and the average gap is 0.029%. Table 3 shows that our B&P algorithm is effective and optimal solutions of most instances can be obtained in a reasonable computational time.

4.3. Carbon Strategy. In this subsection, we present two instances to illustrate the impacts of carbon cap and price on total cost and carbon emissions. We make some observations based on numerical results and provide managerial insight to highlight the opportunity to reduce carbon emissions. The first instance is set by (IIS|, IIB|, IIRS|, IRB|, ITI) = (10, 10, 5, 10, 10), and the second one is set by (IIS|, IIB|, IIRS|, IRB|, ITI) = (20, 20, 10, 15, 10) to represent the scenario in which a company has a wider purchase channel and larger set of recipes. Model parameters are generated in subsection 4.1. For each instance, we investigate a series of cases by changing carbon price ($\lambda \in \Omega = \{z | 1 \leq z \leq 19, z \text{ is an integer}\}$) and carbon cap ($\zeta = 1.1, 1.0$ and 0.9), while the other parameters remain the same. Iron demand, p_t^{Δ} and carbon cap in each period are given in Table 4.

For ease of presentation, we define subsets $\Omega_1 = \{z | 1 \leq z \leq 8, z \text{ is an integer}\}$ and $\Omega_2 = \{z | 9 \leq z \leq 19, z \text{ is an integer}\}$. The optimal decisions on sintering recipe (SR), iron making recipe (IMR), carbon purchase (CP) and carbon sale (CS) in each period for each combination of carbon cap and price are presented in Tables 5 and 6.

Form Tables 5 and 6, in any period, a company does not buy and sell carbon credits at the same time. When the carbon price (or λ) is fixed, carbon cap does not affect the decisions on sintering and iron-making recipe schemes. In our model, the outcome of sintering or iron making is decided once the recipe is chosen. This means that carbon cap does not affect the total carbon emissions. The result can be explained as follows. Recall that a company can buy or sell the carbon emission rights and the surplus rights cannot be used for future periods under the carbon cap-and-trade mechanism. According to constraints 11, we replace $e_t^s - e_t^b$ by $\sum_r e_r^s C_r y_{rt}^s + \sum_k e_k^b d_k y_{kt}^b - E_t^{\text{cap}}$ equivalently in the objective (eq 1), and then there is a constant item ($-\sum_t p_t^{\text{carbon}} E_t^{\text{cap}}$) in the new objective. In other words, the optimal decisions on carbon emissions are independent of E_t^{cap} . In

Table 3. Comparison Results by B&P Algorithm and CPLEX Solver

problem	B&P				NOS	CPLEX			gap (%)
	NOS	run time (s)				best	worst	average	
		best	worst	average					
Pro.1	10	0.141	5.109	0.863	10	0.75	13.656	4.153	0
Pro.2	10	1.047	12.235	6.091	10	4.593	337.359	88.342	0
Pro.3	10	7.597	75.469	31.809	10	145.453	1000	678.115	0
Pro.4	10	1.016	27.468	4.573	10	1.313	273.838	66.909	0
Pro.5	10	11.906	238.703	82.331	10	45.328	1000	809.437	0
Pro.6	8	52.14	1000	386.861	8	478.344	1000	924.081	0.029

Table 4. Values of p_t^A , Iron Demands and Carbon Caps in Multiperiods

instance	parameter	period									
		1	2	3	4	5	6	7	8	9	10
1	iron demand (ton)	9900	9700	10200	9800	9900	9700	10100	10000	10100	9600
	p_t^A (\$/ton)	0	4	1	1	3	2	1	3	3	2
	high cap (ton)	19210	18822	19792	19017	19210	18822	19599	19404	19599	18629
	medium cap (ton)	17464	17111	17993	17288	17464	17111	17817	17640	17817	16935
	low cap (ton)	15718	15400	16194	15559	15718	15400	16035	15876	16035	15242
2	iron demand (ton)	9700	9500	10000	10500	9900	9700	10100	9700	10200	10500
	p_t^A (\$/ton)	3	0	4	3	3	0	2	4	0	1
	high cap (ton)	18822	18434	19404	20374	19210	18822	19599	18822	19792	20374
	medium cap (ton)	17111	16758	17640	18522	17464	17111	17817	17111	17993	18522
	low cap (ton)	15400	15082	15876	16670	15718	15400	16035	15400	16194	16670

Table 5. Optimal Decisions on Recipe and Carbon Trade for Each Period in Instance 1

λ	ζ	decisions	period									
			1	2	3	4	5	6	7	8	9	10
$\in \Omega$	1.1, 1.0, 0.9	SR	1	1	1	1	1	1	1	1	stop	1
		IMR	1	1	1	1	1	1	1	1	1	1
	1.1	CP (ton)	0	0	0	0	0	0	0	0	0	0
		CS (ton)	1329	1237	1466	1283	1329	1237	1420	1374	4620	1192
	1.0	CP (ton)	418	474	334	445	418	474	361	390	0	502
		CS (ton)	0	0	0	0	0	0	0	0	2839	0
	0.9	CP (ton)	2164	2185	2133	2174	2164	2185	2143	2154	0	2195
		CS (ton)	0	0	0	0	0	0	0	0	1507	0

Table 6. Optimal Decisions on Recipe and Carbon Trade for Each Period in Instance 2

λ	ζ	decisions	period									
			1	2	3	4	5	6	7	8	9	10
$\in \Omega_1$	1.1, 1.0, 0.9	SR	7	7	7	7	7	7	7	stop	7	7
		IMR	7	7	7	7	7	7	7	7	7	7
	1.1	CP (ton)	0	0	0	0	0	0	0	0	0	0
		CS (ton)	659	577	782	987	742	659	824	3971	864	987
	1.0	CP (ton)	1052	1099	982	866	1005	1052	958	0	935	866
		CS (ton)	0	0	0	0	0	0	0	2260	0	0
	0.9	CP (ton)	2763	2774	2746	2718	2751	2763	2740	0	2735	2718
		CS (ton)	0	0	0	0	0	0	0	549	0	0
$\in \Omega_2$	1.1,1.0,0.9	SR	3	3	3	3	3	3	3	stop	3	3
		IMR	3	3	3	3	3	3	3	3	3	3
	1.1	CP (ton)	0	0	0	0	0	0	0	0	0	0
		CS (ton)	2280	2183	2424	2665	2376	2280	2473	4680	2521	2665
	1.0	CP (ton)	0	0	0	0	0	0	0	0	0	0
		CS (ton)	568	507	660	813	630	568	691	2968	721	813
	0.9	CP (ton)	1143	1169	1104	1039	1117	1143	1091	0	1078	1039
		CS (ton)	0	0	0	0	0	0	0	1257	0	0

addition, it is obvious that a higher carbon cap in the two instances can lead to lower total cost. Because carbon cap is an

important resource of revenue, decreasing the carbon cap, i.e., reducing the resource, results in an increase in cost inevitably.

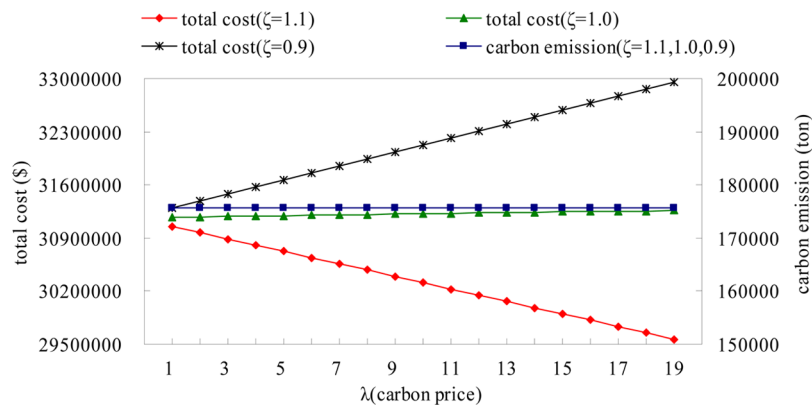


Figure 4. Impacts of carbon price on total cost and carbon emissions in instance 1.

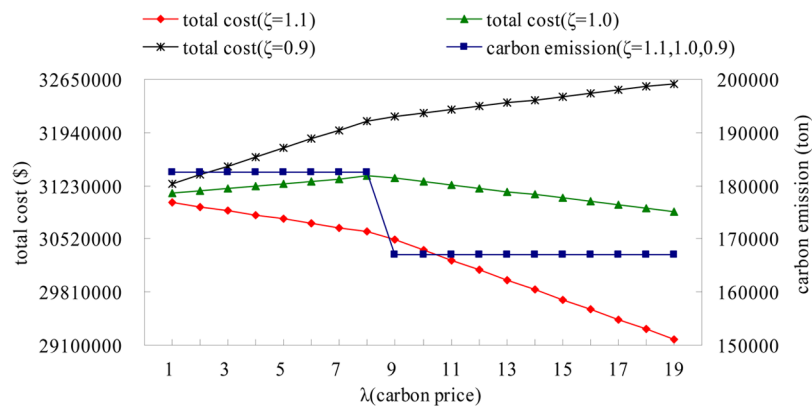


Figure 5. Impact of carbon price on total cost and carbon emissions in instance 2.

Table 7. Resulting Cost Compositions When $\lambda = 7, 8, 9,$ and 10 in Instance 2

ζ	cost composition	λ			
		7	8	9	10
1.1, 1.0, 0.9	purchase cost (\$)	21951132	21951132	22088715	22088715
	production cost (\$)	9032000	9032000	9563800	9563800
	changeover cost (\$)	0	0	0	0
	inventory holding cost (\$)	104752	104752	104843	104843
1.1	carbon trade cost (\$)	−415634	−470894	−1252535	−1385263
1.0	carbon trade cost (\$)	235783	268547	−425070	−469773
0.1	carbon trade cost (\$)	887199	1007988	402396	445718

These results perhaps reveal a limitation of carbon cap and trade mechanism because the carbon cap is not strict.

In instance 1, Figure 4 shows the impacts of carbon price on total cost and carbon emissions. We observe that a higher carbon price can lead to a lower cost when the carbon cap is high, and a higher carbon price can lead to a higher cost when the carbon cap is medium or low. It is obvious that more carbon surplus (or deficits) leads to lower (or higher) total cost. Using a given cap, the increase of carbon price in the given ranges does not lead to lower emissions.

In instance 2, Figure 5 shows the impacts of carbon price on total cost and carbon emissions. Table 7 reports the resulting cost compositions for the cases when $\lambda = 7, 8, 9, 10$. We can observe that a higher carbon price can lead to a lower cost when the carbon cap is high ($\zeta = 1.1$). When the carbon cap is insufficient ($\zeta = 0.9$), a higher carbon price leads to a higher cost. When the carbon cap is medium ($\zeta = 1.0$) and the carbon price increases, we can observe that total cost initially increases

and then decreases. These results can be explained as follows. When the carbon price is sufficiently high and caps are given, the firm becomes engaged in selling carbon credits, as the firm finds it profitable to adjust its operations, and reduce the carbon emissions to sell more (or buy less) carbon credits. This means that the operational cost (sum of purchase cost, production cost, changeover cost and inventory holding cost) increases but this increase is less than offset from the higher revenue (or saving) generated by carbon trade. As an illustration in Table 7, when $\zeta = 1.0$ (medium cap) and λ (carbon price) increases from 7 to 8, operational cost keeps unchanged and the carbon trade cost increases from \$235 783 to \$268 547. In this case, total cost increases in the amount of \$32 764 and carbon emissions remain unchanged. However, when λ increases from 8 to 9, carbon emissions decrease. The operational cost increases in the amount of \$669 474 and carbon trade revenue increase in the amount of \$693 617. This observation provides

an important insight to highlight the opportunity to reduce carbon emissions by adjusting operational decisions.

In summary, the total cost may increase, decrease, or initially increase and then decrease when the carbon price increases, depending on the carbon cap and cost structure. The carbon emission may decrease or remain steady when the carbon price increases. Comparing the two instances, there seems to be more chances to reduce carbon emission with increasing carbon price when the set of recipes extends. Therefore, we suggest steel companies extend their raw materials purchase channel and recipe sets.

5. CONCLUSIONS

With the carbon cap-and-trade mechanism, steel companies can incorporate carbon management into their operational decisions. A mixed integer model is built to minimize the total cost of integrated low carbon iron-making supply chain planning problem. A series of numerical examples are provided to test the effectiveness of the proposed algorithm. The valid inequalities and variable fixing can improve the classical B&P algorithm.

Additionally, we examined the impacts of carbon price and cap on total cost and carbon emissions. We obtain some valuable insights based on numerically analyzing: (1) given the carbon prices, carbon cap does not affect the carbon emission, and higher emission cap in our problem can lead to lower total cost; (2) the higher carbon price induces the firm to reduce carbon emissions, which may increase or decrease the total cost. By means of operational adjustment, total cost and carbon emissions may decrease simultaneously; (3) the firm may sell or buy carbon credits in different periods, and the carbon trade depends on the cost structure and carbon cap in each period; (4) with increasing carbon prices, steel companies should extend purchase channel and recipe sets.

There are several directions of further research. We assumed that the firm must utilize the carbon cap completely in each period. This means the carbon credit surplus cannot be used for future periods. A natural extension is that a firm can hold the carbon credit for production or trade in future periods speculatively. Another topic is to consider other carbon curb mechanisms, such as carbon tax, strict cap, and carbon offsets.

AUTHOR INFORMATION

Corresponding Author

*L. Tang. E-mail: qhjytlx@mail.neu.edu.cn. Tel/Fax: 86-24-83680169.

Notes

The authors declare no competing financial interest.

ACKNOWLEDGMENTS

This research is supported by the State Key Program of National Natural Science Foundation of China (Grant No. 71032004), the Fund for Innovative Research Groups of the National Natural Science Foundation of China (Grant No. 71321001) and the National 863 High-Tech Research and Development Program of China (Grant No. 2013AA040704).

NOMENCLATURE

Sets

IB = Set of raw materials for iron making, index i .
IS = Set of raw materials for sintering, index i .
RB = Set of recipes for iron making, index k .

RS = Set of recipes for sintering, index r .

T = Set of planning time periods, index t .

Parameters

a_{ik}^b = Quantity of raw material i ($i \in \text{IB}$) used in producing one unit iron by recipe k .
 a_{ir}^s = Quantity of raw material i ($i \in \text{IS}$) used in producing one unit sinter by recipe r .
 C_r = Output of sinter in one period if recipe r is used.
 $cc_{kk'}^b$ = Changeover cost when recipe is switched from k to k' in iron making.
 $cc_{rr'}^s$ = Changeover cost when recipe is switched from r to r' in sintering.
 cp_k^b = Production cost of one unit iron by recipe k .
 cp_r^s = Production cost of one unit sinter r .
 d_t = Iron demand in period t .
 E_t^{cap} = Cap on carbon emissions in period t .
 e_k^b = Amount of carbon emission when one unit iron is produced by recipe k .
 e_r^s = Amount of carbon emission when one unit sinter r is produced.
 f_r = Inventory holding cost of one unit sinter r .
 h_i = Inventory holding cost of one unit raw material i .
 p_t^{carbon} = Purchasing cost of one unit raw material i in period t .
 pc_{it} = Purchasing cost of one unit raw material i in period t .
 w_{rk}^b = Quantity of sinter r used in producing one unit iron by recipe k .

Variables

e_t^+ = Amount of carbon credit the firm buys in period t .
 e_t^- = Amount of carbon credit the firm sells in period t .
 g_{rt} = The inventory level of sinter r at the end of period t .
 l_{it} = The inventory level of raw material i at the end of period t .
 x_{it} = Quantity of purchased raw material i in period t .
 $y_{kt}^b = 1$, if recipe k is chosen in period t , and 0 otherwise.
 $y_{rt}^s = 1$, if recipe r is chosen in period t , and 0 otherwise.
 $z_{kk't}^b = 1$, if the recipe is switched from k to k' at the beginning of period t , and 0 otherwise.
 $z_{rr't}^s = 1$, if the recipe is switched from r to r' at the beginning of period t , and 0 otherwise.

Master Problem Sets and Parameters

c_p^s = Cost of the sintering plan p .
 c_q^b = Cost of the iron making plan q .
 P = Set of feasible plans for sintering.
 Q = Set of feasible plans for iron making.
 $y_{ktq}^b = 1$, if recipe k is used in period t in plan q , and 0 otherwise.
 $y_{rtq}^s = 1$, if recipe r is used in period t in plan p , and 0 otherwise.
 $z_{kk'tq}^b = 1$, if the recipe is switched from k to k' at the beginning of period t in plan q , and 0 otherwise.
 $z_{rr'tq}^s = 1$, if the recipe is switched from r to r' at the beginning of period t in plan p , and 0 otherwise.

Master Problem Decision Variables

$\alpha_p = 1$, if plan p is chosen for sintering, and 0 otherwise.
 $\beta_q = 1$, if plan q is chosen for iron making, and 0 otherwise.

REFERENCES

(1) Zhang, B.; Wang, Z.; Yin, J.; Su, L. CO₂ emission reduction within Chinese iron & steel industry: Practices, determinants and performance. *J. Clean. Prod.* **2012**, *33*, 167–178.

- (2) Orth, A.; Anastasijevic, N.; Eichberger, H. Low CO₂ emission technologies for iron and steelmaking as well as titania slag production. *Miner. Eng.* **2007**, *20* (9), 854–861.
- (3) Zhu, Q.; Geng, Y. Drivers and barriers of extended supply chain practices for energy saving and emission reduction among Chinese manufacturers. *J. Clean. Prod.* **2013**, *40*, 6–12.
- (4) Xu, C.; Cang, D. A brief overview of low CO₂ emission technologies for iron and steel making. *J. Iron Steel Res. Int.* **2010**, *17* (3), 1–7.
- (5) Ghanbari, H.; Helle, M.; Saxén, H. Process integration of steelmaking and methanol production for suppressing CO₂ emissions—A study of different auxiliary fuels. *Chem. Eng. Process.* **2012**, *61*, 58–68.
- (6) Helle, H.; Helle, M.; Pettersson, F.; Saxén, H. Multi-criteria optimization of iron making in the blast furnace with top gas recycling. *ISIJ. Int.* **2010**, *50*, 1380–1387.
- (7) Chu, M.; Yagi, J.-I. Numerical evaluation of blast furnace performance under top gas recycling and lower temperature operation. *Steel Res. Int.* **2010**, *81* (12), 1043–1050.
- (8) Ghanbari, H.; Helle, H.; Pettersson, F.; Saxén, H. Optimization study of steelmaking under novel blast furnace operation combined with methanol production. *Ind. Eng. Chem. Res.* **2011**, *50* (21), 12103–12112.
- (9) Helle, H.; Helle, M.; Pettersson, F.; Saxén, H. Optimisation study of ironmaking using biomass. *Ironmaking Steelmaking* **2010**, *37* (8), 590–598.
- (10) Ueda, S.; Watanabe, K.; Yanagiya, K.; Inoue, R.; Ariyama, T. Optimization of biomass utilization for reducing CO₂ in iron making process. *J. Iron Steel Res. Int.* **2009**, *16*, 593–599.
- (11) Ghanbari, H.; Saxén, H.; Grossmann, I. E. Optimal design and operation of a steel plant integrated with a polygeneration system. *AIChE J.* **2013**, *59* (10), 3659–3670.
- (12) Huijgen, W. J. J.; Witkamp, G. J.; Comans, R. N. J. Mineral CO₂ sequestration by steel slag carbonation. *Environ. Sci. Technol.* **2005**, *39* (24), 9676–9682.
- (13) Tobiesen, F. A.; Svendsen, H. F.; Mejdell, T. Modeling of blast furnace CO₂ capture using a mine absorbents. *Ind. Eng. Chem. Res.* **2007**, *46* (23), 7811–7819.
- (14) Eloneva, S.; Teir, S.; Salminen, J.; Fogelholm, C. J.; Zevenhoven, R. Steel converter slag as a raw material for precipitation of pure calcium carbonate. *Ind. Eng. Chem. Res.* **2008**, *47* (18), 7104–7111.
- (15) Chakraborty, A.; Purkharthofer, K. A.; Linninger, A. A. Conceptual design of metallurgical processes based on thermodynamic and economic insights. *Chem. Eng. Process.* **2004**, *43* (5), 625–640.
- (16) Kleindorfer, P. R.; Singhal, K.; Van Wassenhove, L. N. Sustainable operations management. *Prod. Oper. Manage.* **2005**, *14* (4), 482–492.
- (17) Sbihi, A.; Eglese, R. W. Combinatorial optimization and green logistics. *Ann. Oper. Res.* **2010**, *175* (1), 159–175.
- (18) Dekker, R.; Bloemhof, J.; Mallidis, I. Operations Research for green logistics—An overview of aspects, issues, contributions and challenges. *Eur. J. Oper. Res.* **2012**, *219* (3), 671–679.
- (19) Benjaafar, S.; Li, Y.; Daskin, M. Carbon footprint and the management of supply chains: Insights from simple models. *IEEE Trans. Automot. Sci. Eng.* **2013**, *10* (1), 99–116.
- (20) Hua, G.; Cheng, T. C. E.; Wang, S. Managing carbon footprints in inventory management. *Int. J. Prod. Econ.* **2011**, *132* (2), 178–185.
- (21) Zhang, B.; Xu, L. Multi-item production planning with carbon cap and trade mechanism. *Int. J. Prod. Econ.* **2013**, *144* (1), 118–127.
- (22) Dutta, G.; Fourer, R. A survey of mathematical programming applications in integrated steel plants. *Manuf. Serv. Oper. Manage.* **2001**, *3* (4), 387–400.
- (23) Fabian, T. A linear programming model of integrated iron and steel production. *Manage. Sci.* **1958**, *4* (4), 415–449.
- (24) Fabian, T. Blast furnace production planning—A linear programming example. *Manage. Sci.* **1967**, *14* (2), B1–B27.
- (25) Lawrence, J. R.; Flowerdew, A. D. J. Economic model for production planning. *Oper. Res. Soc.* **1963**, *14* (1), 11–19.
- (26) Sasidhar, B.; Achary, K. K. A multiple arc network model of production planning in a steel mill. *Int. J. Prod. Econ.* **1991**, *22* (3), 195–202.
- (27) Mohanty, R. P.; Singh, R. A hierarchical production planning approach for a steel manufacturing system. *Int. J. Oper. Manage.* **1992**, *12* (5), 69–78.
- (28) Chen, M.; Wang, W. A linear programming model for integrated steel production and distribution planning. *Int. J. Oper. Manage.* **1997**, *17* (6), 592–610.
- (29) Zaroni, S.; Zavanella, L. Model and analysis of integrated production-inventory system: The case of steel production. *Int. J. Prod. Econ.* **2005**, *93*, 197–205.
- (30) Gerardi, D.; Marlin, T. E.; Swartz, C. L. E. Optimization of primary steelmaking purchasing and operation under raw material uncertainty. *Ind. Eng. Chem. Res.* **2013**, *52* (35), 12383–12398.
- (31) Hung, Y.-J. *Models for production and strategic planning in the steel industry*. Doctoral dissertation, Northwestern University, Evanston, IL, 1991.
- (32) Demailly, D.; Quirion, P. European Emission Trading Scheme and competitiveness: A case study on the iron and steel industry. *Energy Econ.* **2008**, *30* (4), 2009–2027.
- (33) Lund, P. Impacts of EU carbon emission trade directive on energy-intensive industries—Indicative micro-economic analyses. *Ecol. Econ.* **2007**, *63* (4), 799–806.
- (34) Lübbecke, M. E.; Desrosiers, J. Selected topics in column generation. *Oper. Res.* **2005**, *53* (6), 1007–1023.
- (35) Barnhart, C.; Johnson, E. L.; Nemhauser, G. L.; Savelsbergh, M. W. P.; Vance, P. H. Branch-and-price: Column generation for solving huge integer programs. *Oper. Res.* **1998**, *46* (3), 316–329.
- (36) Fukasawa, R.; Longo, H.; Lysgaard, J.; Poggi de Aragão, M.; Reis, M.; Uchoa, E.; Werneck, R. F. Robust branch-and-cut-and-price for the capacitated vehicle routing problem. *Math. Program.* **2006**, *106* (3), 491–511.
- (37) Baldacci, R.; Mingozzi, A.; Roberti, R. New route relaxation and pricing strategies for the vehicle routing problem. *Oper. Res.* **2011**, *59* (5), 1269–1283.
- (38) Degraeve, Z.; Jans, R. A new dantzig-wolfe reformulation and branch-and-price algorithm for the capacitated lot-sizing problem with setup times. *Oper. Res.* **2007**, *55* (5), 909–920.
- (39) Jans, R.; Degraeve, Z. An industrial extension of the discrete lot-sizing and scheduling problem. *IEE Trans.* **2004**, *36* (1), 47–58.
- (40) Van den Akker, J. M.; Hoogeveen, J.; Van de Velde, S. L. Parallel machine scheduling by column generation. *Oper. Res.* **1999**, *47* (6), 862–872.
- (41) Chen, Z. L.; Powell, W. B. Solving parallel machine scheduling problems by column generation. *INFORMS J. Comput.* **1999**, *11* (1), 78–94.
- (42) Bülbül, K.; Kaminsky, P.; Yano, C. Flow shop scheduling with earliness, tardiness, and intermediate inventory holding costs. *Nav. Res. Log.* **2004**, *51* (3), 407–445.
- (43) Persson, J. A.; Göthe-Lundgren, M. Shipment planning at oil refineries using column generation and valid inequalities. *Eur. J. Oper. Res.* **2005**, *163* (3), 631–652.
- (44) Tatsushi, N.; Tsukasa, I. Column generation heuristics for ship routing and scheduling problems in crude oil transportation with split deliveries. *Comput. Chem. Eng.* **2014**, *60*, 329–338.
- (45) Tang, L.; Wang, G.; Liu, J.; Liu, J. A combination of lagrangian relaxation and column generation for order batching in steelmaking and continuous-casting production. *Nav. Res. Log.* **2011**, *58* (4), 370–388.
- (46) Tang, L.; Wang, G.; Chen, Z.-L. Integrated charge batching and casting wheel selection at Baosteel. *Oper. Res.* **2014**, *62* (4), 772–787.
- (47) Bredström, D.; Lundgren, J. T.; Rönnqvist, M.; Carlsson, D.; Mason, A. Supply chain optimization in the pulp mill industry—IP models, column generation and novel constraint branches. *Eur. J. Oper. Res.* **2004**, *156* (1), 2–22.
- (48) Dantzig, G. B.; Wolfe, P. Decomposition principle for linear programs. *Oper. Res.* **1960**, *8* (1), 101–111.

- (49) Hadjar, A.; Marcotte, O.; Soumis, F. A branch-and-cut algorithm for the multiple depot vehicle scheduling problem. *Oper. Res.* **2006**, *54* (1), 130–149.
- (50) Irnich, S.; Desaulniers, G.; Desrosiers, J.; Hadjar, A. Path-reduced costs for eliminating arcs in routing and scheduling. *INFORMS J. Comput.* **2010**, *22* (2), 297–313.
- (51) Pessoa, A.; Uchoa, E.; Poggi de Aragão, M.; Rodrigues, R. Exact algorithm over an arc-time-indexed formulation for parallel machine scheduling problems. *Math. Program. Comput.* **2010**, *2* (3), 259–290.
- (52) Ren, L. J.; Wang, W. J. Analysis of existing problems and carbon emission reduction in Shandong's iron and steel industry. *Energy Procedia* **2011**, *5*, 1636–1641.