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Novel MILP Decomposition Approach for Scheduling Product Distribution through a Pipeline Network

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Supporting Information

ABSTRACT: This work presents an approach for scheduling of operational activities in a large real-world pipeline network, where oil derivatives and ethanol are transported and distributed among refineries, terminals, depots, and final clients. The hierarchical decomposition approaches to solve the pipeline-scheduling problem presented by Boschetto et al. [*Ind. Eng. Chem. Res.* **2010**, *49*, 5661] and Magatão et al. [*Ind. Eng. Chem. Res.* **2012**, *51*, 4591], which are based on the integration of mixed integer linear programming (MILP) models and a set of heuristic modules, are merged and compounding blocks are also improved. Thus, a novel decomposition approach for scheduling product distribution through a pipeline network is proposed. In addition, this work presents a new MILP approach for the last hierarchical level: the timing block (timing model). This paper expands and improves the former MILP model, which was the timing block core. A series of operational constraints were considered within a continuous time representation in order to determine the exact time instants that products should be pumped into the pipelines and received in the operational areas during a scheduling horizon of, typically, 1 month. Within the new MILP timing model, turn shift constraints, local constraints, and surge tank constraints are improved; immediate pumping constraints are proposed. In addition, a decomposition approach for the new MILP model is also proposed within this article. This decomposition is based on a relax-and-fix heuristic implemented by a sequential run of two MILP models: MLC (Model with Local Constraints) and MST (Model with Seasonal costs and Turn shift constraints). The MILP decomposition goal is to reduce the computational load, if seasonal costs and turn shift constraints are active, without quality solution losses. The proposed approach is applied to the solution of real case studies of a pipeline network that includes 30 bidirectional multiproduct pipelines associated with 14 nodes (four refineries, two harbors, six depots, and two final clients). Computational results have been attained in a reasonable computational time (from seconds to a few minutes) for the addressed pipeline network.

1. INTRODUCTION

The supply chain management, in general industry, impacts in more than 30% of business performance.³ Particularly, the storage time reduction and the service level improvement can influence up to 60% of this performance.

The materials flow planning into supply chains is influenced by decisions taken in strategic, tactical, and operational levels.⁴ The scheduling of operational tasks and the execution and control of these tasks is a key issue within the supply chain management. However, supply chain scheduling problems are extremely challenging due to the high complexity to combine continuous and discrete process decisions, beyond the necessity to integrate production and transport operations.⁵

The pipeline model is the lowest cost among land transportation models and also presents a greater availability and reliability than waterways, since it does not suffer from climatic influences. The transportation of oil derivatives through pipelines tends to be a logistic option to distribute the production and supply demands.⁶ Thus, the efficient usage of pipeline resources can significantly contribute to reducing operational costs of oil supply chains.

A considerable number of research papers have addressed pipeline planning/scheduling problems in the literature. Table 1 summarizes the main characteristics of some literature works since 2004. These papers are classified according to planning/scheduling problem, pipeline structure (number of origin

nodes, destination nodes, and involved pipelines), modeling approach, time representation, and solution approach.

For instance, considering different topologies, a single pipeline can connect an origin to a destination,^{7–14} an origin to n destinations,^{15–25} or n origins to n destinations.^{26,27} Also, various pipelines can connect an origin to n destinations, known as a tree structure,^{28,29} or n origin to n destinations, known as a pipeline network.^{1,2,30–36}

In this work, a real-world pipeline network is studied. The hierarchical decomposition approaches to solve the pipeline-scheduling problem previously presented,^{1,2} which are based on the integration of mixed integer linear programming (MILP) models and a set of heuristic modules, are merged and compounding blocks are also improved. Thus, a novel decomposition approach is proposed. In addition, this paper presents a new MILP approach to significantly improve the last hierarchical level: the timing block. It is important to highlight that in Boschetto et al.¹ the timing block (scheduling phase) was addressed, whereas in Magatão et al.² the scheduling phase of Boschetto et al.¹ was used and planning and assignment/sequencing phases were addressed in detail. Within this paper,

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Table 1. Literature Papers about Pipeline Planning/Scheduling Problems, Since 2004

paper	problem	structure (origin/dest/pipe)	modeling approach	time representation	time horizon (h)	solution approach ^a
Relvas et al. ⁷	sequencing	1/1/1	heuristic	—	744	—
MirHassani and BeheshtiAsl ⁸	sequencing	1/1/1	heuristic	—	744	—
Magatão et al. ⁹	scheduling	1/1/1	MILP	discrete	150	single model
Relvas et al. ¹⁰	scheduling	1/1/1	MILP	continuous	744	single model
Relvas et al. ¹¹	scheduling	1/1/1	MILP + heuristics	continuous	744	single model
Cafaro and Cerdá ¹²	scheduling	1/1/1	MILP	continuous	744	single model
Magatão et al. ¹³	scheduling	1/1/1	MILP + CLP	discrete	144	single model
Relvas et al. ¹⁴	scheduling	1/1/1	MILP	continuous	744	two level
Cafaro and Cerdá ¹⁵	scheduling	1/n/1	MILP	continuous	75	single model
Rejowski and Pinto ¹⁶	scheduling	1/n/1	MILP	discrete	75	single model
Cafaro and Cerdá ¹⁷	scheduling	1/n/1	MILP	continuous	672	horizon rolling
Rejowski and Pinto ¹⁸	scheduling	1/n/1	MINLP	continuous	130	single model
Cafaro et al. ¹⁹	scheduling	1/n/1	MILP + heuristics	continuous	660	p. decomp.
Herrán et al. ²⁰	scheduling	1/n/1	MINLP	multi-period	150	single model
MirHassani et al. ²¹	scheduling	1/n/1	MILP	multi-period	504	single model
Cafaro et al. ²²	scheduling	1/n/1	MILP	continuous	660	two level
Ghaffari-Hadigheh and Mostafaei ²³	scheduling	1/n/1	MILP	continuous	168	single model
Mostafaei and Hadigheh ²⁴	scheduling	1/n/1	MILP	continuous	660	single model
Cafaro et al. ²⁵	scheduling	1/n/1	MINLP	continuous	660	p. decomp.
Cafaro and Cerdá ²⁶	scheduling	n/n/1	MILP	continuous	240	single model
MirHassani et al. ²⁷	scheduling	n/n/1	MILP	continuous	240	single model
MirHassani and Fani Jahromi ²⁸	scheduling	tree structure	MILP	continuous	720	single model
Cafaro and Cerdá ²⁹	scheduling	tree structure	MILP	continuous	720	single model
Magatão et al. ²	planning + sequencing	network	MILP	continuous	720	p. decomp.
Stebel et al. ³⁰	planning	network	MILP	—	720	single model
Ribas et al. ³¹	sequencing	network	GA	—	720	—
Abbasi and Garousi ³²	scheduling	network ^b	MILP	discrete	144	single model
Boschetto et al. ¹	scheduling	network	MILP	continuous	720	p. decomp.
Lopes et al. ³³	scheduling	network	CP	discrete	240	p. decomp.
Cafaro and Cerdá ³⁴	scheduling	network	MILP	continuous	183	n. decomp.
Souza Filho et al. ³⁵	scheduling	network	MILP + heuristics	discrete	42	p. decomp.
Fabro et al. ³⁶	scheduling	network	MILP	continuous	744	p. decomp.

^ap. decomp. = problem decomposition; n. decomp. = network decomposition. ^bOnly one origin area and one product are considered.

improvements in the final scheduling solution in relation to Boschetto et al.¹ are presented.

The remainder of the paper is organized as follows. Section 2 describes the addressed problem, and a structure to solve it is proposed. Subsequently, the general constraints and the approach used to treat these constraints are described. In section 3, the new MILP model to solve the timing block is presented. Furthermore, a decomposition scheme for this timing model is also proposed and described in this section. In section 4, results about a real-world case study are discussed, and scheduling details are given. Section 5 presents the final conclusions and future works.

2. PROBLEM DESCRIPTION

The considered multiproduct pipeline network, the same as studied by Boschetto et al.¹ and Magatão et al.,² is schematically presented in Figure 1. This network presents, in fact, a real case study for a Brazilian pipeline network, which involves 14 areas (nodes). It is composed of four refineries (nodes N3, N4, N5, and N6), two harbors (N7 and N10), two final clients (N2 and N14), and six depots (N1, N8, N9, N11, N12, and N13), which receive or send a set of oil derivatives. In particular, area N1 does not have tanks to store products. The areas are interconnected by 30 bidirectional pipelines, each one with a

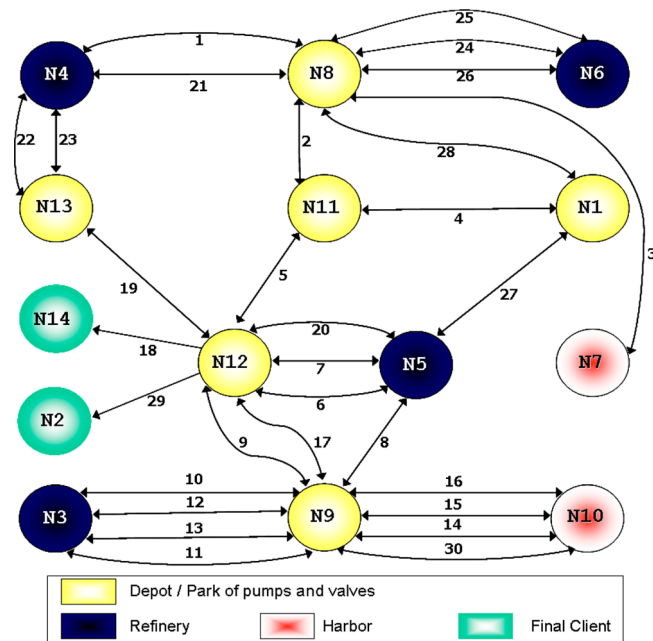


Figure 1. Pipeline network.¹

particular volume. Nowadays, more than 35 oil derivatives and ethanol can be transported in this network.

A series of factors should be considered in order to correctly model the problem constraints. Such operational conditions are, in essence, the same as presented by Boschetto et al.¹ In this article, however, the formulation of, for instance, turn shift constraints, local constraints, and surge tank constraints is improved, as detailed a posteriori in section 3.1. In addition, immediate pumping constraints are proposed (section 3.1.6). These modifications decisively contribute to the fine-tuning of the obtained scheduling solution.

2.1. Problem Hierarchical Decomposition. As indicated by Jittamai,³⁷ pipeline-scheduling problems are difficult to solve and can be categorized as NP-Hard problems. In order to address the computational issues of the pipeline network illustrated in Figure 1, Boschetto et al.¹ and, more recently, Magatão et al.² proposed decomposition strategies to address this pipeline-scheduling problem. Within this article, these decomposition strategies are merged and compounding blocks are improved, as indicated in Figure 2.

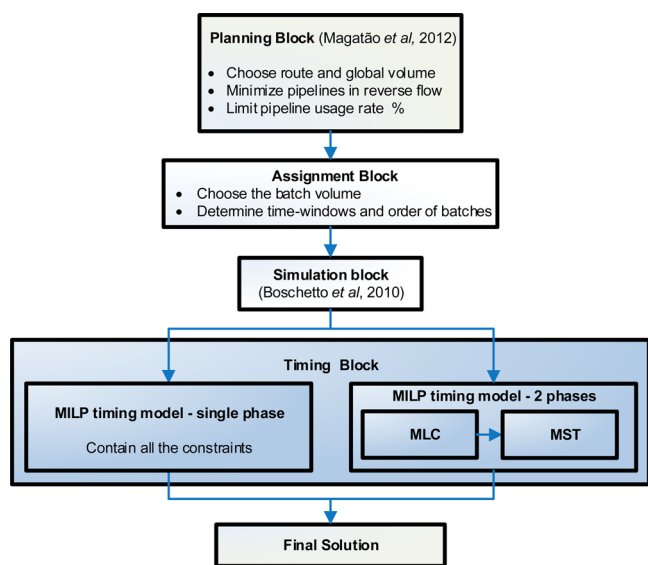


Figure 2. Hierarchical approach.

The *planning block* analyzes the different products comprised within a scenario of, typically, 1 month, and suggests their transportation characteristics from producing areas (refineries or harbors, the latter for the product importation case) with the goal of supplying the demands of depots, final clients, and harbors (product exportation case). The approach considers the tank farm aggregated per area and product. After running, the planning model provides, as final results, the routes to be used and the global volume to be transferred so as to guarantee the demands, while considering the available products at origins and the problem operational constraints. These results are fed into the next block, the assignment block. It is important to highlight that a large portion of the planning block input data is based on forecastings of production and consumption areas. Later on it is indicated that these data are considered within the models by time windows, which can be violated. Thus, the hierarchical approach proposed suggests a solution; if this solution is not feasible, the forecastings can be criticized by company schedulers and, if possible, adapted for better scheduling purposes. The more accurate the forecasting

information provided by the company, the better the obtained planning solution. Information given by the planning level influences the final scheduling given by the timing block. The MILP model for the planning block is detailed by Magatão et al.²

The *assignment block* uses the total volumes and the routes provided by the planning model to determine parameters for each pumped batch. The heuristic procedure used to compute batch volumes takes into account typical values of batches, production and consumption information, and the capacity for each area. Then, the assignment model calculates time windows¹ in each origin/destination area, in order to determine the sequence of operational batches. Thus, a list of batches, including the route, the batch volume, and the time windows for each batch is computed by this block.

In this work, two groups of time windows are calculated:

1. *Capacity time windows* indicate temporal limits for a batch to be pumped from a producing node (e.g., t_{ed} is the lower time limit to pump a batch in the origin area, t_{ec} is the upper time limit to pump a batch in the origin area) or received in a destination node (e.g., t_{rd} is the lower time limit to receive a batch in the destination area, t_{rc} is the upper time limit to receive a batch in the destination area). If these limits are violated, storage limits are also violated. Details are given by Boschetto et al.¹

2. *Cut time windows*: Based on the capacity time windows and on the minimum time to move a batch from one node to another, some temporal cuts can be obtained. Thus, the capacity time windows are cut (adjusted) considering the minimum time to move a batch. An example can be viewed in Figure 3. Figure 3a shows the capacity time windows of a batch,

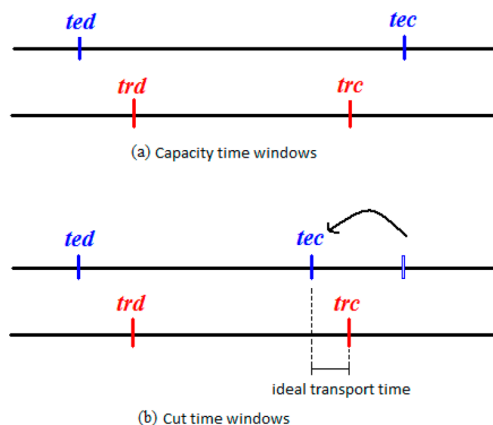


Figure 3. Example of cut in time window.

and Figure 3b shows a cut in t_{ec} considering the ideal transportation time. This aspect is, therefore, improved in relation to Boschetto et al.¹

It is important to emphasize that the *assignment block* is a heuristic procedure that indicates a sequence of operational batches that are to be pumped from each area. Thus, pumping orders suggested by this block are respected by the following blocks. The optimization in ordering of batches is not the main focus of the timing block (exception is made for local constraints, as explained in section 3.1.9). This assumption can deviate from the optimal solution. Therefore, improvements within the assignment block can contribute to better order definitions with respect to violations on time windows

and, thus, improve the final scheduling given by the timing block.

The *simulation block* is an algorithm that evaluates the information derived from the assignment block and calculates volumetric limits (bounds). Thus, the list of batches determined by the assignment block, including the route and the batch volume, is of crucial importance for the simulation block to work. The simulation evaluates, step by step, the batch that is being pumped and analyzes the influence of this pumping operation on the receiving of other batches. Moreover, the simulation allows the identification of conditions in which a batch can remain stopped into the pipeline. As a result, the batch movement along the pipeline network is analyzed, and “volumetric parts” (portions of this batch or subbatches) are identified. More details can be found in Boschetto et al.¹ In this way, the simulation block provides valuable information, used as parameters by the MILP model. In Magatão et al.,² the authors also used the simulation block within the scheduling phase.

Finally, all the parameters calculated by the previous blocks are used in a continuous time MILP model, the *timing block*. This block determines the operational short-term scheduling for the pipeline network using the parameters (e.g., time windows and subbatch volumes) determined by previous blocks. The timing block is described in detail in section 3.

In relation to the work of Boschetto et al.¹ and Magatão et al.,² some features can be observed within the hierarchical solution approach illustrated in Figure 2:

- The *decomposition strategy*¹ was used as the basis for the new hierarchical solution approach presented in Figure 2.
- The *planning block*,² however, is incorporated into this new hierarchical approach.
- The MILP assignment and sequencing integrated block² was not used due to the computation load of the MILP model, which was a concern evidenced by the paper.
- In previous works,^{1,2} the *timing block* (MILP model) was the same. Within the proposed approach, however, this model is improved, as detailed in section 3.
- Two solution approaches are proposed for the new *timing block*: a single-phase MILP model and a two-phase MILP approach.
- Thus, based on Figure 2, two solution paths can be chosen by the user: (i) planning, assignment, simulation, single-phase MILP model; (ii) planning, assignment, simulation, two-phase MILP approach.
- The first path determines the short-term scheduling solution using a single execution of a timing model involving all formulated constraints. The second path is an alternative approach to obtain good solutions, also respecting all constraints; the computational time reduction to obtain the final solution is a second path advantage, as further exploited in section 4.

The blocks proposed in Figure 2 are integrated in a collaborative way and are sequentially executed. Thus, the output of a block can be used as an input to the next block. Additionally, the developed computational tool incorporates functionalities that allow visualizing reports, inventory graphics, and Gantt charts related to proposed solutions.

2.2. Problem Constraints per Block. A continuous time mixed integer linear programming approach is used to create the timing optimization model. The general problem constraints are summarized as follows:

1. The production/consumption data during a month for each product in each node is established in advance by the company. However, routes to be used and the volumes to be moved into the network from one node to another are defined by the planning block.

2. The transfer operations should occur, preferentially, in the predetermined time horizon. Into this period, for each area, a set of pumping batches is computed and a list of all batches must be ordered (batch sequence) according to consumption/production network rates. The specification of batches involves the following:

- a. The batch volume for each product should be an integer number of tank capacity available in batch origin/destination area, or equal to the volume of a specific pipeline within the batch route.
- b. For each batch, input data are supplied by the assignment block: the used route, the total pumping volume, and the calculated time windows.

3. Each route is composed by a sequence of areas intercalated by pipelines. The first and the last areas in the route represent the origin and destination areas, respectively. A route indicates the “movement path” for each batch, from the origin area until the destination area. An example of a route can be extracted from Figure 1, where a transfer operation can be made from N6 to N14. Thus, the route will contain the elements {N6, 25, N8, 2, N11, 5, N12, 18, N14}.

- a. One route can be used by different batches, containing different products at different flow rates. The routes for each product for each pair origin/destination area are chosen by the assignment block.

4. Pipelines within the network can have their flow direction reverted, according to operational convenience. In this case, an additional batch with the same volume of the “pipeline segment” under reversion must be inserted.

5. During transportation procedures, a contamination area (interface) between products is formed. Some interfaces are operationally not desired and must be avoided. In addition, if incompatible products have to be sent by the same pipeline, *plug products*⁹ should be added between these batches. The assignment block includes these additional batches.

6. For each product, node, and pipeline, there is a minimum flow rate value and a maximum flow rate value. This information is used in the timing model to suggest the best flow rate, according to scheduling purposes, to move the batch in each pipeline, respecting the minimum and maximum limits. However, the flow rate of batches inside pipelines, as they are pushed, is affected by the products to be pumped from origin areas.¹

7. A batch can be received at a specific flow rate in a tank, and it can be simultaneously pumped from this tank to another pipeline at a different flow rate. This is called “surge tank” operation.

- a. In this case, intermediate storage has to be used and the route of the batch is split.
- b. The first route considers the origin area until the intermediate area with the tanks used in the operation; the second one, from the area in surge tank operation until the final destination area.

8. Each area has proper operational features. For example, there exist a limited number of pumps and manifolds for each area. Thus, only a restricted number of batches are allowed to

be sent/received from/to each area at the same time. This fact characterizes “local constraints”, i.e., specifics of each area. The local constraints are also improved in this work in relation to Boschetto et al.¹ Now, the sets of constraints are dynamically constructed, according to information provided by the scenario database.

9. Each area has a tank farm that stores different products. They are considered in an aggregate form for area and product. The storage level can increase or decrease, according to the balance between the sent/received batch volume, the local production/consumption, and the market demand.

- a. The upper and lower limits to the overall product inventory in each node are managed in the MILP model through time windows.
- b. In this work, two sets of time windows are derived from (i) minimum and maximum operational limits for inventory levels and (ii) capacity and zero limits for inventory levels. A violation on capacity limits suffers a bigger penalty than a violation on the minimum/maximum limits.
- c. It is considered that if the MILP model satisfies the time windows previously calculated, the inventory requirements are also satisfied.

10. Pipelines always operate fulfilled, and some of them have a considerable volume. Batches that are in the pipeline at the beginning of the scheduling horizon must be routed to their previously determined destination (*in transit* batches). At the scheduling horizon start, *in transit* batches are sequenced in the beginning of the list of batches created by the assignment block (start-up of network scheduling). Although a new product being pumped has a particular destination, previously “stored” products should be pushed out to their original destination.

11. Typically, from 5:30 p.m. to 8:30 p.m. (on-peak demand hours) the electric cost is higher than in any other time period. Therefore, at this interval, the pumping should be periodically interrupted in some operational areas during the scheduling horizon (e.g., N6, N7, and N8).

12. Send and receiving procedures should also not be started (or finished) in a change of working shifts. The change of shifts occurs every 8 h, in all areas. This feature is now considered in the proposed work.

3. THE NEW TIMING MILP MODEL

As already mentioned, this work improves the *timing block* presented by Boschetto et al.¹ In this way, few constraints proposed in the previous work are also used in this new timing MILP model. By readability reasons, the already proposed constraints are also indicated within this paper and a reference to Boschetto et al.¹ is made to differ from the new constraints. The notation previously used is also preserved. As a summary, Table S1, available in Supporting Information, indicates the constraints proposed by Boschetto et al.¹ and by the presented paper.

In a simplified view, the proposed optimization model has to satisfy a series of operational conditions, translated into constraints. Some of them describe inventory management (e.g., the violation on cut time windows attributes nonzero values to variables $aoc_{b,n,o,p}$, $doc_{b,n,o,p}$, $adc_{b,n,d,p}$, and $ddc_{b,n,d,p}$ indicating that inventory issues were violated). Timing constraints are also modified, and they are related to timing variables such as $fb_{b,\bar{n},n,d,p}$ and $ib_{b,\bar{n},n,d,p}$. Timing issues are calculated according to the flow rate limits for each pipeline and

the pumping influences estimated by the simulation block (Figure 2). The model is now able to suggest the best flow rate, according to scheduling purposes, to move a batch in each pipeline. Turn shift and surge tank constraints are improved; immediate pumping constraints are added. Local constraints are now considered in a general manner, and they are generated according to the necessity of areas/pipelines/products for a considered scenario.

3.1. Problem Constraints. As stated, the MILP model is built considering some conceptual constraints and nomenclature presented by Boschetto et al.¹ Sets, variables, and parameters used in the new MILP model are described in the Nomenclature section herein presented.

The indices of variables are constructed as part of sets that contain only specific combinations of each tuple. For example, for variable $ib_{b,\bar{n},n,d,p}$ the tuple $\langle b, \bar{n}, n, d, p \rangle$ contains only the areas and pipelines that are used by batch b . If a batch b is sent by route $N4 \rightarrow 1 \rightarrow N8 \rightarrow 2 \rightarrow N11$, only the following indices are generated: $\langle b, N4, N8, 1, p \rangle \forall p \in PB_{b,N4,N8,1}$ and $\langle b, N8, N11, 2, p \rangle \forall p \in PB_{b,N8,N11,2}$. The same idea is used to create the indices of all batches in the model. Thus, sets are created in a sparse form and the domain of constraints reflects this formulation procedure.

3.1.1. Scheduling Horizon Violation. Constraint 1 establishes that the end of pumping procedure (fb) should occur into the scheduling horizon (H). Otherwise, temporal violations are admitted ($vhoriz$), but minimized within the objective function (expression 60, herewith presented). It is important to notice that pumping procedures are linked to receiving procedures. Batches are pushed along pipelines; if pumping procedures occur within the scheduling horizon, pushed batches are also received inside the considered horizon. Inequality 1 also influences receiving procedures and, thus, there is no necessity of a similar constraint for receiving purposes.

$$fb_{b,\bar{n},n,d,p} - vhoriz_{b,\bar{n},n,d,p} \leq H$$

$$\forall b \in B, \{\bar{n}, n\} \in N, d \in D, p \in PB_{b,\bar{n},n,d} \quad (1)$$

3.1.2. Sequencing Constraints. Inequalities 2 and 3 determine the temporal precedence between parts p of the same batch b . These inequalities were presented in Boschetto et al.¹

$$ib_{b,\bar{n},n,d,p} \geq fb_{b,\bar{n},n,d,p-1}$$

$$\forall b \in B, \{\bar{n}, n\} \in N, d \in D, p \in PB_{b,\bar{n},n,d}$$

$$|p| > npb_{b,\bar{n},n,d}^{\min} \quad (2)$$

$$ir_{b,\bar{n},n,d,p} \geq fr_{b,\bar{n},n,d,p-1}$$

$$\forall b \in B, \{\bar{n}, n\} \in N, d \in D, p \in PR_{b,\bar{n},n,d}$$

$$|p| > npr_{b,\bar{n},n,d}^{\min} \quad (3)$$

Inequalities 4–11 determine the temporal block for the pumped and received batches. Within this new proposed formulation, the model is able to suggest the best flow rate, according to scheduling purposes, to move a batch in each pipeline. The flow rate is limited by minimum and maximum operational values. The flow rate suggestion allows, for example, delay of receiving batches in order to respect capacity limits on destinations.

Constraints 4 and 5 identify the instant when the pumping of all sequenced batches in their origin areas is finished, including origin areas that stop during the on-peak hours. The stoppage duration is added to the pumping time, into the temporal block ($fb - ib$).

$$fb_{b, no_{bo}, n, d, p} \geq ib_{b, no_{bo}, n, d, p} + \frac{part_{b, p}}{vb_{prod_p, no_{bo}, d}^{max}} + \alpha \sum_{h \in HP} z_{b, p, no_{bo}, h} \quad \forall b \in B, \{no_{bo}, n\} \in N, d \in D, prod_b \in P, p \in PB_{b, no_{bo}, n, d} \quad (4)$$

$$fb_{b, no_{bo}, n, d, p} \leq ib_{b, no_{bo}, n, d, p} + \frac{part_{b, p}}{vb_{prod_p, no_{bo}, d}^{min}} + \alpha \sum_{h \in HP} z_{b, p, no_{bo}, h} \quad \forall b \in B, \{no_{bo}, n\} \in N, d \in D, prod_b \in P, p \in PB_{b, no_{bo}, n, d} \quad (5)$$

The sending time on intermediate areas is obtained through constraints 6 and 7. The sending end time (fb) of a batch b is calculated considering the sending start time (ib) in the same pipeline, added to the pumping time of another batch (bo_b) that has influenced the movement of the batch b . The batch bo_b is pumped in its origin area no_{bo_b} and can remain stopped because of on-peak demand hours.

$$fb_{b, \bar{n}, n1, d1, p_b} \geq ib_{b, \bar{n}, n1, d1, p_b} + \frac{part_{bo_b, p_{bo_b}}}{vb_{prod_{p_b}, d1}^{max}} + \alpha \sum_{h \in HP} z_{bo_b, p_{bo_b}, no_{bo_b}, h} \quad \forall \{b, bo_b\} \in B, \{no_{bo_b}, n, \bar{n}, n1\} \in N, \{d, d1\} \in D, prod_b \in P, p_b \in PB_{b, \bar{n}, n1, d1}, p_{bo_b} \in PB_{bo_b, no_{bo_b}, n, d} \quad (6)$$

$$fb_{b, \bar{n}, n1, d1, p_b} \leq ib_{b, \bar{n}, n1, d1, p_b} + \frac{part_{bo_b, p_{bo_b}}}{vb_{prod_{p_b}, \bar{n}, d1}^{min}} + \alpha \sum_{h \in HP} z_{bo_b, p_{bo_b}, no_{bo_b}, h} \quad \forall \{b, bo_b\} \in B, \{no_{bo_b}, n, \bar{n}, n1\} \in N, \{d, d1\} \in D, prod_b \in P, p_b \in PB_{b, \bar{n}, n1, d1}, p_{bo_b} \in PB_{bo_b, no_{bo_b}, n, d} \quad (7)$$

The receiving time of part p_b of batch b in area $n1$ is obtained using the pumping time of another part p_{bo_b} of batch bo_b in its origin area no_{bo_b} . Equation 8 makes the correspondence of the start receiving time with the start pumping time¹ of bo_b , while eq 9 here proposed was added to make the link between the final receiving time with the final pumping time of bo_b .

$$ir_{b, \bar{n}, n1, d1, p_b} = ib_{bo_b, no_{bo_b}, n, d, p_{bo_b}} \quad \forall \{b, bo_b\} \in B, \{no_{bo_b}, n, \bar{n}, n1\} \in N, \{d, d1\} \in D, p_b \in PR_{b, \bar{n}, n1, d1}, p_{bo_b} \in PB_{bo_b, no_{bo_b}, n, d} \quad (8)$$

$$fr_{b, \bar{n}, n1, d1, p_b} = fb_{bo_b, no_{bo_b}, n, d, p_{bo_b}} \quad \forall \{b, bo_b\} \in B, \{no_{bo_b}, n, \bar{n}, n1\} \in N, \{d, d1\} \in D, p_b \in PR_{b, \bar{n}, n1, d1}, p_{bo_b} \in PB_{bo_b, no_{bo_b}, n, d} \quad (9)$$

Constraints 10 and 11 indicate the receiving end time of a batch b on area $n1$. The end of receiving will occur after the receiving start, adding the pumping time of bo_b in its origin area

no_{bo} . It is also considered that pumping stoppages of the batch bo_b , due to on-peak demand hours, influence the receiving of batch b on area $n1$.

$$fr_{b, \bar{n}, n1, d1, p_b} \geq ir_{b, \bar{n}, n1, d1, p_b} + \frac{part_{bo_b, p_{bo_b}}}{vb_{prod_{p_b}, \bar{n}, d1}^{max}} + \alpha \sum_{h \in HP} z_{bo_b, p_{bo_b}, no_{bo_b}, h} \quad \forall \{b, bo_b\} \in B, \{no_{bo_b}, n, \bar{n}, n1\} \in N, \{d, d1\} \in D, p_b \in PR_{b, \bar{n}, n1, d1}, p_{bo_b} \in PB_{bo_b, no_{bo_b}, n, d} \quad (10)$$

$$fr_{b, \bar{n}, n1, d1, p_b} \leq ir_{b, \bar{n}, n1, d1, p_b} + \frac{part_{bo_b, p_{bo_b}}}{vb_{prod_{p_b}, \bar{n}, d1}^{min}} + \alpha \sum_{h \in HP} z_{bo_b, p_{bo_b}, no_{bo_b}, h} \quad \forall \{b, bo_b\} \in B, \{no_{bo_b}, n, \bar{n}, n1\} \in N, \{d, d1\} \in D, p_b \in PR_{b, \bar{n}, n1, d1}, p_{bo_b} \in PB_{bo_b, no_{bo_b}, n, d} \quad (11)$$

The batch sequencing (batch ordering definition) in all areas is already established by the assignment block (Figure 2) before the sending procedure, and this ordering is used within the MILP timing approach. Thus, the pumping, sending, and receiving procedures, not only in origin areas, but also in intermediate areas, should occur in crescent temporal order, considering batches that share the same pipeline. Inequality 12 enables this temporal precedence among different batches.¹

$$ib_{\bar{b}, \bar{n}1, n1, d, p_{\bar{b}}} \geq fb_{b, \bar{n}, n, d, p_b} \quad \forall \{b, \bar{b}\} \in B, lb < \bar{b}, \{n, \bar{n}, n1, \bar{n}1\} \in N, d \in D, p_b \in PB_{b, \bar{n}, n, d}, p_{\bar{b}} = npb_{b, \bar{n}, n, d}^{max} \wedge p_{\bar{b}} \in PB_{\bar{b}, \bar{n}1, n1, d}, p_{\bar{b}} = npb_{\bar{b}, \bar{n}1, n1, d}^{min} \quad (12)$$

The sequencing constraint established in (13) involves three areas (\bar{n} , n , and $n1$) and two pipelines (d and $d1$), distributed within the route of batch b as follows: $\bar{n} \rightarrow d \rightarrow n \rightarrow d1 \rightarrow n1$. Similar to constraint 13, which identifies the start sending of a batch b in an intermediate area,¹ the new constraint (14) identifies the final sending of a batch b in an intermediate area n , which should occur after, or at the same time as, the receiving end time in area n . The constraint was modeled with an equal sign to maintain the flow rate.

$$ib_{b, n1, d1, p} = ir_{b, \bar{n}, n, d, p} \quad \forall b \in B, \{\bar{n}, n, n1\} \in N, \bar{n} \neq n1, \{d, d1\} \in D, p \in PB_{b, n1, d1} \cap PR_{b, \bar{n}, n, d} \quad (13)$$

$$fb_{b, n1, d1, p} = fr_{b, \bar{n}, n, d, p} \quad \forall b \in B, \{\bar{n}, n, n1\} \in N, \bar{n} \neq n1, \{d, d1\} \in D, p \in PB_{b, n1, d1} \cap PR_{b, \bar{n}, n, d} \quad (14)$$

3.1.3. Seasonal Cost Constraints. The application of seasonal costs is analyzed within the areas where the product is pumped, that is, in the origin areas. The pumping in on-peak demand hours is not desirable because of elevated electrical costs during on-peak demand periods of the day. Thus, such pumping is likely to be interrupted in these moments. Constraints about seasonal costs were proposed in previous work,¹ and the same constraints are also considered within the proposed model (constraints 15–24). In this way, they will not be detailed in this article.

$$ih_h - ib_{b, no_{bo}, n, d, p_b} \geq -M(1 - x_{b, p_b, no_{bo}, h}) \quad \forall b \in B, n \in N, no_b \in N_{hor}, d \in D, h \in HP, p_b \in PB_{b, no_{bo}, n, d} \quad (15)$$

$$ih_h - ib_{b, no_b, n, d, p_b} \leq ((M + \text{eps})x_{b, p_b, no_b, h}) - \text{eps}$$

$$\forall b \in B, n \in N, no_b \in N_{\text{hor}}, d \in D, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(16)

$$fb_{b, no_b, n, d, p_b} - ih_h \geq -M(1 - y_{b, p_b, no_b, h})$$

$$\forall b \in B, n \in N, no_b \in N_{\text{hor}}, d \in D, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(17)

$$fb_{b, no_b, n, d, p_b} - ih_h \leq ((M + \text{eps})y_{b, p_b, no_b, h}) - \text{eps}$$

$$\forall b \in B, n \in N, no_b \in N_{\text{hor}}, d \in D, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(18)

$$fh_h - ib_{b, no_b, n, d, p_b} \geq -M(1 - w_{b, p_b, no_b, h})$$

$$\forall b \in B, n \in N, no_b \in N_{\text{hor}}, d \in D, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(19)

$$fh_h - ib_{b, no_b, n, d, p_b} \leq ((M + \text{eps})w_{b, p_b, no_b, h}) - \text{eps}$$

$$\forall b \in B, n \in N, no_b \in N_{\text{hor}}, d \in D, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(20)

$$z_{b, p_b, no_b, h} \leq x_{b, p_b, no_b, h}$$

$$\forall b \in B, no_b \in N_{\text{hor}}, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(21)

$$z_{b, p_b, no_b, h} \leq y_{b, p_b, no_b, h}$$

$$\forall b \in B, no_b \in N_{\text{hor}}, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(22)

$$z_{b, p_b, no_b, h} \geq x_{b, p_b, no_b, h} + y_{b, p_b, no_b, h} - 1$$

$$\forall b \in B, no_b \in N_{\text{hor}}, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(23)

$$x_{b, p_b, no_b, h} = w_{b, p_b, no_b, h}$$

$$\forall b \in B, no_b \in N_{\text{hor}}, h \in \text{HP}, p_b \in \text{PB}_{b, no_b, n, d}$$
(24)

3.1.4. Constraints on Capacity Time Windows. The inventory on origin and destination areas is managed through time bounds. These limits are previously estimated by the assignment block. Within the MILP model, violations of these values are allowed, but are minimized through the objective function. Constraints about capacity inventory limits were proposed in previous work.¹ These constraints are also considered within the proposed model (constraints 25–28). In this way, they will not be explained in detail in this article. However, a set of constraints to manage cut time windows is proposed within this article and explained in section 3.1.5.

$$ib_{b, no_b, n, d, p} \geq \text{ted}_b - \text{aoc}_{b, no_b}$$

$$\forall b \in B, \{no_b, n\} \in N, d \in D, p \in \text{PB}_{b, no_b, n, d} | p = \text{npb}_{b, no_b, n, d}^{\min}$$
(25)

$$ib_{b, no_b, n, d, p} \leq \text{tec}_b + \text{doc}_{b, no_b}$$

$$\forall b \in B, \{no_b, n\} \in N, d \in D, p \in \text{PB}_{b, no_b, n, d} | p = \text{npb}_{b, no_b, n, d}^{\min}$$
(26)

$$ir_{b, \bar{n}, nd_b, d, p} \geq \text{trd}_b - \text{ad}_{b, nd_b}$$

$$\forall b \in B, \{\bar{n}, nd_b\} \in N, d \in D, p \in \text{PR}_{b, \bar{n}, nd_b, d} | p = \text{npr}_{b, \bar{n}, nd_b, d}^{\min}$$
(27)

$$ir_{b, \bar{n}, nd_b, d, p} \leq \text{trc}_b + \text{dd}_{b, nd_b}$$

$$\forall b \in B, \{\bar{n}, nd_b\} \in N, d \in D, p \in \text{PR}_{b, \bar{n}, nd_b, d} | p = \text{npr}_{b, \bar{n}, nd_b, d}^{\min}$$
(28)

3.1.5. Cut Time Windows Constraints. Considering a production area (e.g., nodes N4, N6), the lower ($\text{ted}_b^{\text{Cut}}$) and upper ($\text{tec}_b^{\text{Cut}}$) time bounds to send a batch b should be respected. However, as previously mentioned, such restrictions can be violated and a penalty is included in the objective function. The variables aoc_{b, no_b} and doc_{b, no_b} provide the difference between the original planned time windows, given by the assignment block at Figure 2, and the possible violation on sending operations (constraints 29 and 30). Basically, aoc_{b, no_b} indicates earliness and doc_{b, no_b} the delays on sending operations.

$$ib_{b, no_b, n, d, p} \geq \text{ted}_b^{\text{Cut}} - \text{aoc}_{b, no_b}$$

$$\forall b \in B, \{no_b, n\} \in N, d \in D, p \in \text{PB}_{b, no_b, n, d} | p = \text{npb}_{b, no_b, n, d}^{\min}$$
(29)

$$ib_{b, no_b, n, d, p} \leq \text{tec}_b^{\text{Cut}} + \text{doc}_{b, no_b}$$

$$\forall b \in B, \{no_b, n\} \in N, d \in D, p \in \text{PB}_{b, no_b, n, d} | p = \text{npb}_{b, no_b, n, d}^{\min}$$
(30)

A similar condition exists in a destination area. The lower ($\text{trd}_b^{\text{Cut}}$) and upper ($\text{trc}_b^{\text{Cut}}$) time bounds to receive a batch b should be respected, but, again, they can also be violated. The variables adc_{b, nd_b} and ddc_{b, nd_b} give the difference between the original planned destination time windows and the possible violation on receiving (constraints 31 and 32).

$$ir_{b, \bar{n}, nd_b, d, p} \geq \text{trd}_b^{\text{Cut}} - \text{adc}_{b, nd_b}$$

$$\forall b \in B, \{\bar{n}, nd_b\} \in N, d \in D, p \in \text{PR}_{b, \bar{n}, nd_b, d} | p = \text{npr}_{b, \bar{n}, nd_b, d}^{\min}$$
(31)

$$ir_{b, \bar{n}, nd_b, d, p} \leq \text{trc}_b^{\text{Cut}} + \text{ddc}_{b, nd_b}$$

$$\forall b \in B, \{\bar{n}, nd_b\} \in N, d \in D, p \in \text{PR}_{b, \bar{n}, nd_b, d} | p = \text{npr}_{b, \bar{n}, nd_b, d}^{\min}$$
(32)

3.1.6. Surge Tank Constraints. In some areas, a batch can be received at a specific flow rate in a tank and simultaneously pumped from this tank to another pipeline at a different flow rate. This procedure is characterized as a “surge tank operation”; thus, a product is simultaneously received in and sent from the same tank.

Figure 4 presents an example in with the route of a product has areas A, B, and C, respectively. That is, the product should be sent from A to C passing by area B. But, since the A–B pipeline has a flow rate limit less than the B–C pipeline, it is operationally recommended to make a surge tank operation in

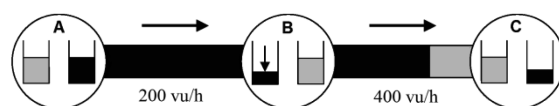


Figure 4. Surge tank operation example.¹

area B. Therefore, a batch \bar{b} is sequenced using a route with A as origin area and B as destination area. Another batch is sequenced using a route with B as origin and C as destination.

A surge tank operation has implications within the mathematical formulation, since a route needs to be decomposed to contemplate this operation. Additional constraints had to be created in order to make a connection of these “partial routes”. Following this reasoning, the receiving time in an area with a surge tank operation and the pumping to the final destination must be made coherently. In the set of batches with B destination, the two batches will remain together (in sequence). Thus, it is possible to do a connection between these batches (\bar{b} and b) analyzing the product ($\text{prod}_b = \text{prod}_{\bar{b}}$) and the area of the route ($\text{no}_b = \text{nd}_{\bar{b}}$), besides the information on product/area where a surge tank operation can occur (indicated by the PLM set, defined within the Nomenclature).

In addition, the surge tank constraints were divided in two groups:

1. Surge tank areas that can also make intermediate storage. In this group, there exists the possibility of both to store the product in the intermediate area and to pump it a posteriori, or to use a surge tank operation.

2. Surge tank areas that should not make intermediate storage. In this group, if a product is received in the surge tank area, it should be pumped as soon as possible due to storage limitations.

However, as a simulation block considers no connections between batches \bar{b} and b , the volumes of each part ($\text{part}_{b,p}$ and $\text{part}_{\bar{b},\bar{p}}$) can be (and in the majority of cases they are) of different sizes. This fact is a complicating issue for modeling purposes. The surge tank constraints calculate the received volume in surge tank areas from these volumetric parts. If the received volumetric parts ($\text{part}_{b,p}$) are greater than the sent volumetric parts ($\text{part}_{\bar{b},\bar{p}}$), the pump in an intermediate area can be made. Inequality 33, modified from the original modeling,¹ expresses a condition to be verified for batches \bar{b} and b . This sort of condition appears inside quantifiers of surge tank constraints (e.g., as used in inequality 39):

$$\sum_{k=\text{npr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1}^{\min}}^{\bar{p}} \text{part}_{\bar{b},k} \geq \sum_{k=\text{npl}_{b,\text{no}_b,n,d}^{\min}}^p \text{part}_{b,k} \quad (33)$$

As the received volumetric parts in a surge tank area can be different from the pumped volumetric parts, the start pumping constraints are divided in three cases:

1. The pump of batch b first part ($p = \text{npl}_{b,\text{no}_b,n,d}^{\min}$) occurs when the first part of batch \bar{b} is received ($\bar{p} = \text{npr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1}^{\min}$). In this case, the pumping volume p is less than or equal to the received volume \bar{p} ($\text{part}_{\bar{b},\bar{p}} \geq \text{part}_{b,p}$). Figure 5 illustrates $\text{part}_{\bar{b},\bar{p}}$ being received in a tank and part p being pumped from this tank. In this case, a volume of \bar{p} can remain in the tank (if the volume of p is smaller than the received volume of \bar{p}) or cannot (if the volume of p is equal to the received volume of \bar{p}).

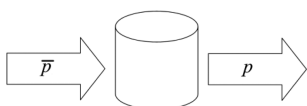


Figure 5. Surge tank example, case 1: $\text{part}_{\bar{b},\bar{p}} \geq \text{part}_{b,p}$.

Constraint 34 establishes a relation between the receiving start of the first batch \bar{b} part with the pumping start of the first batch b part in the area with surge tank operation.

$$\begin{aligned} \text{ir}_{b,\text{no}_b,n,d,p} &\geq \text{ir}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1,\bar{p}} \\ \forall \{d, d1\} \in D, \{b, \bar{b}\} \in B, lb &\neq \bar{b}, \\ \{\text{no}_b, n, \bar{n}, \text{nd}_{\bar{b}}\} &\in \text{Nlno}_b = \text{nd}_{\bar{b}}, \\ \{\text{prod}_b, \text{prod}_{\bar{b}}\} &\in \text{Plprod}_b = \text{prod}_{\bar{b}}, \\ (\text{no}_b, \text{prod}_b) &\in \text{PLM}, \text{vol}_b = \text{vol}_{\bar{b}}, \text{ted}_b = \text{ted}_{\bar{b}}, \\ \text{part}_{\bar{b},\bar{p}} &\geq \text{part}_{b,p}, \bar{p} \in \text{PR}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1,\bar{p}} = \text{npr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1}^{\min} \\ p &\in \text{PB}_{b,\text{no}_b,n,d} | p = \text{npl}_{b,\text{no}_b,n,d}^{\min} \end{aligned} \quad (34)$$

2. The pump of batch b first part ($p = \text{npl}_{b,\text{no}_b,n,d}^{\min}$) occurs after the reception of more than one part of batch \bar{b} ($\bar{p} > \text{npr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1}^{\min}$), as illustrated by Figure 6. In this case, the pumping

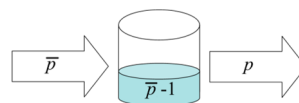


Figure 6. Surge tank example, case 2: $\sum \text{part}_{\bar{b},\bar{p}} \geq \text{part}_{b,p}$.

volume p can be previously obtained considering the volume previously received ($\bar{p} - 1$), and the fact that the p volume is less than or equal to the sum of all received parts of batch \bar{b} . Therefore, the inequality

$$\sum_{k=\text{npr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1}^{\min}}^{\bar{p}} \text{part}_{\bar{b},k} \geq \text{part}_{b,p}$$

holds.

But by considering batches \bar{b} and b in a surge tank operation, the condition exemplified in Figure 6 raises a question: what is the amount of time that the pump of batch b can be made in advance, since the tank is not empty and parts of batch \bar{b} are still to arrive? The flow rate is variable so that, considering a linear approach, it is not direct to calculate this amount of time by considering the division of part p by flow rate. In a synthetic way (just for explanation purposes), it is necessary to find a time given by eq 35:

$$\text{time} = \frac{\sum_{k=\text{npr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1}^{\min}}^{\bar{p}-1} \text{part}_{\bar{b},k}}{\text{flow}_{\bar{p}}} \quad (35)$$

But, within this reasoning, the flow rate can be determined as indicated by eq 36:

$$\text{flow}_{\bar{p}} = \frac{\text{part}_{\bar{b},\bar{p}}}{\text{fr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1,\bar{p}} - \text{ir}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1,\bar{p}}} \quad (36)$$

Thus, the time to advance the pumping of batch \bar{b} part is given by eq 37:

$$\text{time} = \left(\sum_{k=\text{npr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1}^{\min}}^{\bar{p}-1} \text{part}_{\bar{b},k} \right) \frac{\text{fr}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1,\bar{p}} - \text{ir}_{\bar{b},\bar{n},\text{nd}_{\bar{b}},d1,\bar{p}}}{\text{part}_{\bar{b},\bar{p}}} \quad (37)$$

Constraint 38 considers the second case in which the first part of batch b pumping procedure is made after the receiving

of more than one batch \bar{b} part. Also, the pumping start can be made in advance.

$$\begin{aligned}
 ib_{b, no_b, n, d, p} \geq & ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} - \left(\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}-1} part_{\bar{b}, k} \right) \\
 & \times \left(\frac{ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} - ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}}}{part_{\bar{b}, \bar{p}}} \right) \\
 \forall \{d, d1\} \in D, \{b, \bar{b}\} \in B, lb \neq \bar{b}, \\
 \{no_b, n, \bar{n}, nd_{\bar{b}}\} \in N|no_b = nd_{\bar{b}}, \\
 \{prod_b, prod_{\bar{b}}\} \in Pl|prod_b = prod_{\bar{b}}, \\
 (no_b, prod_b) \in PLM, vol_b = vol_{\bar{b}}, \\
 ted_b = ted_{\bar{b}}, \sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} \geq part_p, \\
 \bar{p} \in PR_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1} | \bar{p} > npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}, \\
 p \in PB_{b, no_b, n, d} | p = npr_{b, no_b, n, d}^{\min} \quad (38)
 \end{aligned}$$

3. The pump of other parts of batch b , except the first one ($p > npb_{b, no_b, n, d}^{\min}$), occurs after the reception of more than one part of batch \bar{b} ($\bar{p} > npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}$), as illustrated by Figure 7. In this

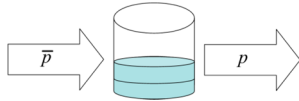


Figure 7. Surge tank example, case 3: $\sum part_{\bar{b}, \bar{p}} \geq \sum part_{b, p}$

case, it should be considered the remaining volume in tankage. The pumping volume p can be obtained in advance considering this remaining volume. Therefore, the inequality

$$\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} \geq \sum_{k=npb_{b, no_b, n, d}^{\min}}^p part_{b, k}$$

holds.

The third case has to identify the difference between batch parts received and pumped, that is, the remaining volume in tankage (constraint 39). In this way, the advance in pumping can be estimated by the following remaining volume:

$$\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}-1} part_{\bar{b}, k} \geq \sum_{k=npb_{b, no_b, n, d}^{\min}}^{p-1} part_{b, k}$$

as indicated in (39).

$$\begin{aligned}
 ib_{b, no_b, n, d, p} \geq & ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} \\
 & - \left(\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}-1} part_{\bar{b}, k} - \sum_{k=npb_{b, no_b, n, d}^{\min}}^{p-1} part_{b, k} \right) \\
 & \times \left(\frac{ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} - ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}}}{part_{\bar{b}, \bar{p}}} \right) \\
 \forall \{d, d1\} \in D, \{b, \bar{b}\} \in B, lb \neq \bar{b}, \\
 \{no_b, n, \bar{n}, nd_{\bar{b}}\} \in N|no_b = nd_{\bar{b}}, \\
 \{prod_b, prod_{\bar{b}}\} \in Pl|prod_b = prod_{\bar{b}}, \\
 (no_b, prod_b) \in PLM, vol_b = vol_{\bar{b}}, ted_b = ted_{\bar{b}}, \\
 \sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} \geq \sum_{k=npb_{b, no_b, n, d}^{\min}}^p part_{b, k}, \\
 \bar{p} \in PR_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1} | \bar{p} > npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}, \\
 p \in PB_{b, no_b, n, d} | p > npr_{b, no_b, n, d}^{\min} \quad (39)
 \end{aligned}$$

Considering a similar reasoning used to determine the flow rate for the start of pumping cases, the end of pumping should be also limited, but just in one case: when the pumping flow rate in surge tank area is bigger than the receiving flow rate in the same area. Inequality 40 establishes this constraint. In short, this constraint avoids the end of pumping from the surge area to occur before the end of receiving within this area.

$$\begin{aligned}
 fb_{b, no_b, n, d, p} \geq & fr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} \\
 & - \left(\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} - \sum_{k=npb_{b, no_b, n, d}^{\min}}^p part_{b, k} \right) \\
 & \times \left(\frac{fr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} - ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}}}{part_{\bar{b}, \bar{p}}} \right) \\
 \forall \{d, d1\} \in D, \{b, \bar{b}\} \in B, lb \neq \bar{b}, \\
 \{no_b, n, \bar{n}, nd_{\bar{b}}\} \in N|no_b = nd_{\bar{b}}, \\
 \{prod_b, prod_{\bar{b}}\} \in Pl|prod_b = prod_{\bar{b}}, \\
 (no_b, prod_b) \in PLM, vol_b = vol_{\bar{b}}, ted_b = ted_{\bar{b}}, \\
 \sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} \geq \sum_{k=npb_{b, no_b, n, d}^{\min}}^p part_{b, k}, \\
 \bar{p} \in PR_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}, p \in PB_{b, no_b, n, d} \quad (40)
 \end{aligned}$$

3.1.7. Immediate Pumping. If the considered intermediate area has a limited storage capacity in comparison to the forthcoming receiving, a received batch should be pumped as soon as possible from this area. In this case, the pumping procedure should be similar to the one used for product receiving in a surge tank area. Constraint 41 considers the receiving procedure of the first part of a batch \bar{b} in an immediate pumping area. Relaxation variables are added in immediate pumping constraints henceforth presented to make flexible the strict pumping condition. These variables are minimized within the objective function in order to make the pumping procedure as soon as possible.

$$\begin{aligned}
 ib_{b, no_b, n, d, p} - rplm_{b, \bar{b}, p, \bar{p}}^1 &= ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} \\
 \forall \{d, d1\} \in D, \{b, \bar{b}\} \in B, lb \neq \bar{b}, \\
 \{no_b, n, \bar{n}, nd_{\bar{b}}\} \in N|no_b = nd_{\bar{b}}, \\
 \{prod_b, prod_{\bar{b}}\} \in Pl|prod_b = prod_{\bar{b}}, (no_b, prod_b) \in PLM, \\
 vol_b = vol_{\bar{b}}, ted_b = ted_{\bar{b}}, part_{\bar{b}, \bar{p}} \geq part_p, \\
 \bar{p} \in PR_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1} | \bar{p} = npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}, \\
 p \in PB_{b, no_b, n, d} | p = npr_{b, no_b, n, d}^{\min} \quad (41)
 \end{aligned}$$

Constraint 42 considers the pumping of the first part of batch b , in the case of more than one part of batch \bar{b} is received. The pumping procedure can be identified in advance considering the remaining volume to be received and the volume remaining in the tank.

$$\begin{aligned}
ib_{b, no_b, n, d, p} - rplm_{b, \bar{b}, p, \bar{p}}^2 &= ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} \\
&- \left(\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}-1} part_{\bar{b}, k} \right) \left(\frac{fr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} - ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}}}{part_{\bar{b}, \bar{p}}} \right) \\
\forall \{d, d1\} \in D, \{b, \bar{b}\} \in B, lb \neq \bar{b}, \\
\{no_b, n, \bar{n}, nd_{\bar{b}}\} \in N|no_b = nd_{\bar{b}}, \\
\{prod_b, prod_{\bar{b}}\} \in P|prod_b = prod_{\bar{b}}, \\
(no_b, prod_b) \in PLM, vol_b = vol_{\bar{b}}, ted_b = ted_{\bar{b}}, \\
\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} &\geq part_{\bar{p}}, \\
\bar{p} \in PR_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1} | \bar{p} &> npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}, \\
p \in PB_{b, no_b, n, d} | p &= npr_{b, no_b, n, d}^{\min} \quad (42)
\end{aligned}$$

The third case identifies the difference between received and pumping parts (constraint 43). In this way, the advance in time to pump can be calculated considering the remaining volume in tank.

$$\begin{aligned}
ib_{b, no_b, n, d, p} - rplm_{b, \bar{b}, p, \bar{p}}^3 &= ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} \\
&- \left(\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}-1} part_{\bar{b}, k} - \sum_{k=npr_{b, no_b, n, d}^{\min}}^{p-1} part_{b, k} \right) \\
&\times \left(\frac{fr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} - ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}}}{part_{\bar{b}, \bar{p}}} \right) \\
\forall \{d, d1\} \in D, \{b, \bar{b}\} \in B, lb \neq \bar{b}, \\
\{no_b, n, \bar{n}, nd_{\bar{b}}\} \in N|no_b = nd_{\bar{b}}, \\
\{prod_b, prod_{\bar{b}}\} \in P|prod_b = prod_{\bar{b}}, \\
(no_b, prod_b) \in PLM, vol_b = vol_{\bar{b}}, ted_b = ted_{\bar{b}}, \\
\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} &\geq \sum_{k=npr_{b, no_b, n, d}^{\min}}^p part_{b, k}, \\
\bar{p} \in PR_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1} | \bar{p} &> npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}, \\
p \in PB_{b, no_b, n, d} | p &> npr_{b, no_b, n, d}^{\min} \quad (43)
\end{aligned}$$

As indicated in constraint 44, the end of pumping should be also limited to consider cases in which the pumping flow rate in the immediate pumping area is bigger than the receiving flow rate in the same area. The reasoning is similar to the one used in constraint 40.

$$\begin{aligned}
fb_{b, no_b, n, d, p} - rplm_{b, \bar{b}, p, \bar{p}}^4 &= fr_{\bar{b}, \bar{n}, d_{\bar{b}}, d1, \bar{p}} \\
&- \left(\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} - \sum_{k=npr_{b, no_b, n, d}^{\min}}^p part_{b, k} \right) \\
&\times \frac{fr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}} - ir_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1, \bar{p}}}{part_{\bar{b}, \bar{p}}} \\
\forall \{d, d1\} \in D, \{b, \bar{b}\} \in B, lb \neq \bar{b}, \\
\{no_b, n, \bar{n}, nd_{\bar{b}}\} \in N|no_b = nd_{\bar{b}}, \\
\{prod_b, prod_{\bar{b}}\} \in P|prod_b = prod_{\bar{b}}, (no_b, prod_b) \in PLM, \\
vol_b = vol_{\bar{b}}, ted_b = ted_{\bar{b}}, \\
\sum_{k=npr_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1}^{\min}}^{\bar{p}} part_{\bar{b}, k} &\geq \sum_{k=npr_{b, no_b, n, d}^{\min}}^p part_{b, k}, \\
\bar{p} \in PR_{\bar{b}, \bar{n}, nd_{\bar{b}}, d1} | p \in PB_{b, no_b, n, d} \quad (44)
\end{aligned}$$

3.1.8. Constraints on Turn Shift. Sending and receiving procedures should not be started (or finished) during turn shift intervals in all areas. These constraints restrict the pumping

from origin areas and, as a consequence, the receiving on destination areas during turn shift intervals. The constraints are active in the case of no time window violations occurring. In this way, if it is necessary to send a product because the tankage limits are violated during a turn shift interval, the product has to be sent. This limitation is made in BOT set construction as indicated: $BOT = \{(b, p, no_b, t) | b \in B, p \in PB_{b, no_b, n, d}, (no_b, t) \in TT: ted_b \leq it_{no_b, t} \leq tec_b\}$.

Also, the turn shift can (or cannot) be active considering the number of days previously specified by the specialist to this condition. In fact, it can be considered that this is a solution fine-tuning procedure.

Constraints 45 and 46 do not allow the pumping procedure, in origin areas, into the turn shift interval (it until ft). If binary variable $xtb = 1$, the pumping time should start before the initial time for the turn shift (constraint 45). Otherwise, if $xtb = 0$, the pumping time should start after the final time for turn shift (constraint 46).

$$\begin{aligned}
ib_{b, no_b, n, d, p_b} &\leq it_{no_b, t} + M(1 - xtb_{b, p_b, no_b, t}) \\
\forall n \in N, d \in D, (b, p_b, no_b, t) &\in BOT \quad (45)
\end{aligned}$$

$$\begin{aligned}
ib_{b, no_b, n, d, p_b} &\geq ft_{no_b, t} - Mxtb_{b, p_b, no_b, t} \\
\forall n \in N, d \in D, (b, p_b, no_b, t) &\in BOT \quad (46)
\end{aligned}$$

A receiving procedure occurs only if a batch is pumped in some origin area. This pumping procedure moves batches into the pipelines, and at the same time instant, a receiving procedure occurs. Therefore, if the pumping procedure is limited to respect turn shift conditions, consequently, the receiving procedure is also limited. Thus, it is not necessary to create constraints for the receiving case.

3.1.9. Local Constraints. Each area (node) within the considered real-world pipeline network has particular limitations that need to be addressed in order to obtain functional solutions. These limitations (local constraints) restrict simultaneous pumping/receiving procedures. The limitations are different for each area, according to the involved hardware (e.g., pumps and valves) needed to pump/receive a product. These limitations are considered only in an origin area, that is, the first area of a route, or in a receiving area, the last area of a route, in which the demand will be satisfied.

The local constraints can be modeled in different manners. A simplified approach was taken by Felizari:³⁸ pumping (or receiving) procedures have to strictly respect the ordering provided by the assignment block (Figure 2). Figure 8 shows three hypotheses to pumping two batches (batches 2 and 5) in a specific area by two different pipelines. In condition a there exists, during a certain time interval, the simultaneous pumping of batches 2 and 5; thus, no local constraint is applied. In condition b, the batch 2 pumping first occurs and, afterward, the pumping of batch 5 occurs. The local constraint, within this case, hinders the simultaneous pumping of batches 2 and 5. Additionally, the ordering previously determined by the assignment block (batch 2 before batch 5) is respected within the local constraint precedence. This fact can, for instance, postpone too much the pumping of batch 5. Finally, in condition c, the model is able to decide which batch should be first pumped. The pumping of batch 5 before batch 2 is considered by the new MILP timing model proposed within this paper.

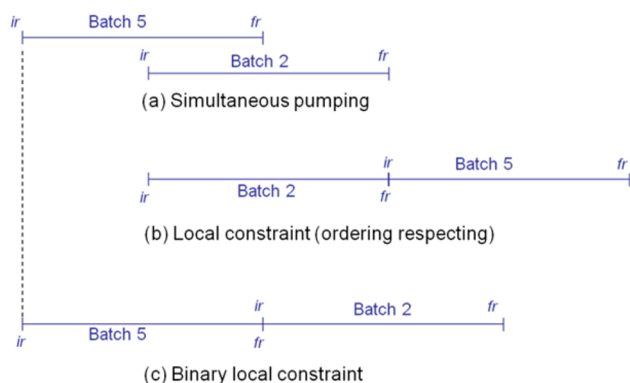


Figure 8. Pumping a batch in an area with local constraint: cases b and c.

Another local constraint feature considered within the proposed MILP model is that the constraints are modeled for each part of a batch. In this way, a pumping procedure of a part of a batch can be, for instance, initialized; afterward another part of this batch can be interrupted and restarted later. Figure 9 highlights a local constraint involving batches 6 and 8 that are

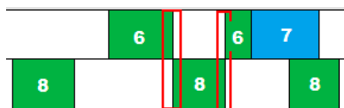


Figure 9. Local constraint on batches 6 and 8.

pumped in distinct pipelines of the same area. In this case, the simultaneous pumping in both pipelines for these batches is not allowed. By Figure 9, it can be noticed that the pumping of a part of batch 6 is done. Following, the pumping of a part of batch 8 occurs. The pumping of another part of batch 6 is done, and finally, the pumping of the last part of batch 8 occurs. Thus, the pumping of consecutive parts of the same batch is a preemptive task (can be interrupted). Another feature that can be observed in Figure 9 is that, during a certain period of time, the pumping of batch 7 occurs simultaneously with another part of batch 8. Thus, local constraints are valid for specific combination of products. For instance, in Figure 9 they were applied for batches 6 and 8 (of the same product) but they were not applied for batches 7 and 8 (different products).

In Boschetto et al.,¹ a particular constraint was generated according to the need of each area, that is, a constraint for each known case. However, local constraints can, sometimes, change from scenario to scenario and, thus, establishing a particular constraint for each area could demand modeling changes. Within the previous work¹ one local constraint was created for each area/product, and just two cases were considered:

- pumping, simultaneously, just by one in two pipelines within an area
- receiving, simultaneously, by just one in two pipelines within an area

In this work, these limitations are translated into constraints in a generic form. Therefore, the data to build the constraints for each specific case (scenario) are obtained from a database. Then, the constraints are automatically generated according to the data provided. Within the proposed paper it is highlighted constraints that consider the following:

- pumping, simultaneously, just by one in two pipelines within an area
- receiving, simultaneously, by just one in two pipelines within an area
- at most, one simultaneous pumping or receiving procedures by two different pipelines of an area
- pumping, simultaneously, just by two in three pipelines within an area
- receiving, simultaneously, by two in three pipelines within an area
- at most, two pumping procedures and one receiving, simultaneously, by three different pipelines within an area
- at most, one pumping and two receiving procedures, simultaneously, by three different pipelines within an area

Thus, in the proposed work, all the operational known possibilities for local constraints involving two or three pipelines, in all areas of the pipeline network illustrated in Figure 1, are specified in a dynamic table related to the scenario database. Afterward, the MILP constraints are automatically generated. Thus, modeling particularization for each case of local constraint is not necessary.

Constraints 47 and 48 identify the simultaneous pumping between the part p of batch b and the part \bar{p} of batch \bar{b} , in the case where it is necessary to consider this limitation, that is, $\bar{b} > b$. Only origin areas (first element of a route) are limited. The model decides the constraint is active by the binary variable lp . If $lp_{b,\bar{b},p,\bar{p}}^1 = 1$, part \bar{p} of batch \bar{b} starts the pumping (ib) in area no_b after the pumping end (fb) of part p of batch b (inequality 47); otherwise, if $lp_{b,\bar{b},p,\bar{p}}^2 = 1$, part p of b starts the pumping (ib) in area no_b after the pumping end (fb) of the part \bar{p} of batch \bar{b} (constraint 48).

$$fb_{b,no_b,n,d,p} \leq ib_{\bar{b},no_b,n1,d1,\bar{p}} + M(1 - lp_{b,\bar{b},p,\bar{p}}^1) \\ \forall (b, \bar{b}, p, \bar{p}) \in LP, \{no_b, n, n1\} \in N, \\ \{d, d1\} \in D | d \neq d1 \quad (47)$$

$$fb_{\bar{b},no_b,n1,d1,\bar{p}} \leq ib_{b,no_b,n,d,p} + M(1 - lp_{b,\bar{b},p,\bar{p}}^2) \\ \forall (b, \bar{b}, p, \bar{p}) \in LP, \{no_b, n, n1\} \in N, \\ \{d, d1\} \in D | d \neq d1 \quad (48)$$

In a similar reasoning used for constraints 47 and 48, constraints 49 and 50 identify the simultaneous receiving between part p of batch b and the part \bar{p} of batch \bar{b} , in the case where it is necessary to consider this limitation, that is, $\bar{b} > b$. Only destination areas (last element of a route) are limited. The model decides the constraint is active by the binary variable lr . If $lr_{b,\bar{b},p,\bar{p}}^1 = 1$, part \bar{p} of batch \bar{b} starts the receiving (ir) in area nd_b after the receiving end (fr) of part p of batch b (inequality 49); otherwise, if $lr_{b,\bar{b},p,\bar{p}}^2 = 1$, part p of b starts the receiving (ir) in area nd_b after the receiving end (fr) of part \bar{p} of batch \bar{b} (constraint 50).

$$fr_{b,nd_b,n,d,p} \leq ir_{\bar{b},n1,nd_b,d1,\bar{p}} + M(1 - lr_{b,\bar{b},p,\bar{p}}^1) \\ \forall (b, \bar{b}, p, \bar{p}) \in LR, \{n, n1, nd_b\} \in N, \\ \{d, d1\} \in D | d \neq d1 \quad (49)$$

$$\begin{aligned} \text{fr}_{\bar{b},n1,\text{nd}_{\bar{b}},d1,\bar{p}} &\leq \text{ir}_{b,n,\text{nd}_b,d,p} + M(1 - \text{lr}_{b,\bar{b},p,\bar{p}}^2) \\ \forall (b, \bar{b}, p, \bar{p}) &\in \text{LR}, \{n, n1, \text{nd}_b\} \in N, \\ \{d, d1\} &\in D | d \neq d1 \end{aligned} \quad (50)$$

Constraints 51 and 52 identify simultaneity between the pumping of part p of batch b and the receiving of part \bar{p} of batch \bar{b} , in the case where it is necessary to consider this limitation, that is, $\bar{b} \neq b$. The model decides what constraint is active by the binary variable lpr . If $\text{lpr}_{b,\bar{b},p,\bar{p}}^1 = 1$, part \bar{p} of batch \bar{b} starts the receiving (ir) in area no_b ($\text{no}_b = \text{nd}_{\bar{b}}$) after the pumping end (fb) of part p of batch b (inequality 51); otherwise, if $\text{lpr}_{b,\bar{b},p,\bar{p}}^2 = 1$, part p of b starts the pumping (ib) in the area $\text{no}_{\bar{b}}$ after the receiving end (fr) of the part \bar{p} of batch \bar{b} (constraint 52).

$$\begin{aligned} \text{fb}_{b,\text{no}_b,n,d,p} &\leq \text{ir}_{\bar{b},n1,\text{no}_{\bar{b}},d1,\bar{p}} + M(1 - \text{lpr}_{b,\bar{b},p,\bar{p}}^1) \\ \forall (b, \bar{b}, p, \bar{p}) &\in \text{LPR}, \{n, n1, \text{no}_b, \text{nd}_{\bar{b}}\} \in N \\ \text{no}_b &= \text{nd}_{\bar{b}}, \{d, d1\} \in D | d \neq d1 \end{aligned} \quad (51)$$

$$\begin{aligned} \text{fr}_{\bar{b},n1,\text{no}_{\bar{b}},d1,\bar{p}} &\leq \text{ib}_{b,\text{no}_b,n,d,p} + M(1 - \text{lpr}_{b,\bar{b},p,\bar{p}}^2) \\ \forall (b, \bar{b}, p, \bar{p}) &\in \text{LPR}, \{n, n1, \text{no}_b, \text{nd}_{\bar{b}}\} \in N \\ \text{no}_b &= \text{nd}_{\bar{b}}, \{d, d1\} \in D | d \neq d1 \end{aligned} \quad (52)$$

3.1.9.1. Pumping/Receiving Limitations in Two Pipelines. When two products (prod_b and $\text{prod}_{\bar{b}}$) have the same identification code given in the database for local constraints ($\text{cd}_{\text{prod}_b,\text{no}_b} = \text{cd}_{\text{prod}_{\bar{b}},\text{no}_{\bar{b}}}$) within the same area ($\text{no}_b = \text{no}_{\bar{b}}$), these products cannot be simultaneously pumped/received in this area. In this case, therefore, a local constraint must hold between batches b and \bar{b} . The sets LP, LR, and LPR involving batches b and \bar{b} of products p and \bar{p} are generated in a sparse form in order to contain only specific combinations of each tuple that are to be used within local constraints:

$$\begin{aligned} \text{LP} &= \{(b, \bar{b}, p, \bar{p}) | \{b, \bar{b}\} \in B | b < \bar{b}, p \in \text{PB}_{b,\text{no}_b,n,d}, \\ &\quad \bar{p} \in \text{PB}_{\bar{b},\text{no}_{\bar{b}},n,d}, \text{cd}_{\text{prod}_b,\text{no}_b} = \text{cd}_{\text{prod}_{\bar{b}},\text{no}_{\bar{b}}}\} \end{aligned}$$

$$\begin{aligned} \text{LR} &= \{(b, \bar{b}, p, \bar{p}) | \{b, \bar{b}\} \in B | b < \bar{b}, p \in \text{PB}_{b,\text{nd}_b,n,d}, \\ &\quad \bar{p} \in \text{PB}_{\bar{b},\text{nd}_{\bar{b}},n,d}, \text{cd}_{\text{prod}_b,\text{nd}_b} = \text{cd}_{\text{prod}_{\bar{b}},\text{nd}_{\bar{b}}}\} \end{aligned}$$

$$\begin{aligned} \text{LPR} &= \{(b, \bar{b}, p, \bar{p}) | \{b, \bar{b}\} \in B | b \neq \bar{b}, p \in \text{PB}_{b,\text{no}_b,n,d}, \\ &\quad \bar{p} \in \text{PB}_{\bar{b},\text{nd}_{\bar{b}},n,d}, \text{cd}_{\text{prod}_b,\text{no}_b} = \text{cd}_{\text{prod}_{\bar{b}},\text{nd}_{\bar{b}}}\} \end{aligned}$$

Constraints 53, 54, and 55 involve local constraint features. These constraints relate the binary variables $\text{lpr}_{b,\bar{b},p,\bar{p}}^1$, $\text{lpr}_{b,\bar{b},p,\bar{p}}^2$, $\text{lr}_{b,\bar{b},p,\bar{p}}^1$, $\text{lr}_{b,\bar{b},p,\bar{p}}^2$, $\text{lpr}_{b,\bar{b},p,\bar{p}}^1$, and $\text{lpr}_{b,\bar{b},p,\bar{p}}^2$. Constraints 47, 49, and 51 are activated by $\text{lx}_{b,\bar{b},p,\bar{p}}^1 = 1$ (where x in lx can be p , r , or pr), and constraints 48, 50, and 52 are activated by $\text{lx}_{b,\bar{b},p,\bar{p}}^2 = 1$.

$$\text{lpr}_{b,\bar{b},p,\bar{p}}^1 + \text{lpr}_{b,\bar{b},p,\bar{p}}^2 \geq 1 \quad \forall (b, \bar{b}, p, \bar{p}) \in \text{LP} \quad (53)$$

$$\text{lr}_{b,\bar{b},p,\bar{p}}^1 + \text{lr}_{b,\bar{b},p,\bar{p}}^2 \geq 1 \quad \forall (b, \bar{b}, p, \bar{p}) \in \text{LR} \quad (54)$$

$$\text{lpr}_{b,\bar{b},p,\bar{p}}^1 + \text{lpr}_{b,\bar{b},p,\bar{p}}^2 \geq 1 \quad \forall (b, \bar{b}, p, \bar{p}) \in \text{LPR} \quad (55)$$

3.1.9.2. Pumping/Receiving Limitations in Three Pipelines. In order to analyze three batches b , \bar{b} , and $b2$, at least one

should be restricted; that is, it is not possible to pump/receive three batches simultaneously, if the corresponding code cd indicates in the database that a local constraint must hold with batches b , \bar{b} , and $b2$. In this way, one of the binary variables related to these batches should be equal to 1. Constraint 56 limits three pumping procedures, constraint 57 limits three receiving procedures, constraint 58 limits two pumping procedures and one receiving, and finally, constraint 59 limits one pumping and two receiving procedures.

$$\begin{aligned} \text{lpr}_{b,\bar{b},p,\bar{p}}^1 + \text{lpr}_{b,\bar{b},p,\bar{p}}^2 + \text{lpr}_{b,b2,p,p2}^1 + \text{lpr}_{b,b2,p,p2}^2 + \text{lpr}_{\bar{b},b2,\bar{p},p2}^1 \\ + \text{lpr}_{\bar{b},b2,\bar{p},p2}^2 \geq 1 \\ \forall (b, \bar{b}, p, \bar{p}) \in \text{LP}, (b, b2, p, p2) \in \text{LP}, \\ (\bar{b}, b2, \bar{p}, p2) \in \text{LP} \end{aligned} \quad (56)$$

$$\begin{aligned} \text{lr}_{b,\bar{b},p,\bar{p}}^1 + \text{lr}_{b,\bar{b},p,\bar{p}}^2 + \text{lr}_{b,b2,p,p2}^1 + \text{lr}_{b,b2,p,p2}^2 + \text{lr}_{\bar{b},b2,\bar{p},p2}^1 \\ + \text{lr}_{\bar{b},b2,\bar{p},p2}^2 \geq 1 \\ \forall (b, \bar{b}, p, \bar{p}) \in \text{LR}, (b, b2, p, p2) \in \text{LR}, \\ (\bar{b}, b2, \bar{p}, p2) \in \text{LR} \end{aligned} \quad (57)$$

$$\begin{aligned} \text{lpr}_{b,\bar{b},p,\bar{p}}^1 + \text{lpr}_{b,\bar{b},p,\bar{p}}^2 + \text{lpr}_{b,b2,p,p2}^1 + \text{lpr}_{b,b2,p,p2}^2 + \text{lpr}_{\bar{b},b2,\bar{p},p2}^1 \\ + \text{lpr}_{\bar{b},b2,\bar{p},p2}^2 \geq 1 \\ \forall (b, \bar{b}, p, \bar{p}) \in \text{LP}, (b, b2, p, p2) \in \text{LPR}, \\ (\bar{b}, b2, \bar{p}, p2) \in \text{LPR} \end{aligned} \quad (58)$$

$$\begin{aligned} \text{lpr}_{b,\bar{b},p,\bar{p}}^1 + \text{lpr}_{b,\bar{b},p,\bar{p}}^2 + \text{lpr}_{b,b2,p,p2}^1 + \text{lpr}_{b,b2,p,p2}^2 + \text{lr}_{\bar{b},b2,\bar{p},p2}^1 \\ + \text{lr}_{\bar{b},b2,\bar{p},p2}^2 \geq 1 \\ \forall (b, \bar{b}, p, \bar{p}) \in \text{LPR}, (b, b2, p, p2) \in \text{LPR}, \\ (\bar{b}, b2, \bar{p}, p2) \in \text{LR} \end{aligned} \quad (59)$$

3.2. Objective Function. The objective function (expression 60) involves the minimization of the following factors:

Factor 1: If sending operations are out of the predetermined horizon, the violation variable vhoriz assumes a nonzero value, which is penalized.

Factor 2: If capacity time windows are violated, inventory limits are also violated. In this way, the weight k can be defined to penalize the solution, according to pumping/receiving delays or earliness to origin/destination areas. Therefore, the higher the k factor, the greater importance is given to capacity time window violation variables. In addition, violation variables are weighted by parts of batch volume (part) and, therefore, more importance is given to violations of parts with bigger volumes.

Factor 3: As capacity time windows, cut time windows can also be violated. If cut time windows are violated, a weight kc ($\text{kc} < k$) can be defined to penalize the solution considering delays or earliness of pumping/receiving times to origin/destination areas. Therefore, the higher the kc factor, the greater importance is given to cut time window violation variables. But, a relative greater importance should be given to capacity time windows (factor 2), in which inventory limits are bounded. The violation variables are also weighted by parts of batch volume (part) and, therefore, more importance is given to violations of parts with bigger volumes.

Factor 4: The relaxation variables $rpulm$ are added due to immediate pumping constraints (section 3.1.6). They are necessary to manage the limited tankage in these areas. A relative big weight kp ($kp \gg k$) is adopted. In fact, within the

considered approach it was established that $kp \gg k > kc$. For instance, within the case studies presented in section 4, $kp = 10\,000$, $k = 10$, and $kc = 1$.

$$\begin{aligned} \min \quad & \underbrace{\sum_{b \in B} \sum_{\bar{n} \in N} \sum_{n \in N} \sum_{d \in D} \sum_{p \in PB_{b,\bar{n},n,d}} \text{vhoriz}_{b,\bar{n},n,d,p}}_{\text{factor 1}} \\ & + \underbrace{k \sum_{b \in B} \sum_{no_b \in N} \sum_{p \in PB_{b,no_b,n,d}} (ao_{b,no_b,p} + do_{b,no_b,p}) \text{part}_{b,p} + k \sum_{b \in B} \sum_{nd_b \in N} \sum_{p \in PB_{b,no_b,n,d}} (ad_{b,no_b,p} + dd_{b,no_b,p}) \text{part}_{b,p}}_{\text{factor 2}} \\ & + \underbrace{kc \sum_{b \in B} \sum_{no_b \in N} \sum_{p \in PB_{b,no_b,n,d}} (aoc_{b,no_b,p} + doc_{b,no_b,p}) \text{part}_{b,p} + kc \sum_{b \in B} \sum_{nd_b \in N} \sum_{p \in PB_{b,no_b,n,d}} (adc_{b,no_b,p} + ddc_{b,no_b,p}) \text{part}_{b,p}}_{\text{factor 3}} \\ & + \underbrace{kp \sum_{b \in B} \sum_{\bar{b} \in B} \sum_{p \in PB_{b,\bar{n},n,d}} \sum_{\bar{p} \in PB_{b,\bar{n},n,d}} (rpulm^1_{b,\bar{b},p,\bar{p}} + rpulm^2_{b,\bar{b},p,\bar{p}} + rpulm^3_{b,\bar{b},p,\bar{p}} + rpulm^4_{b,\bar{b},p,\bar{p}})}_{\text{factor 4}} \end{aligned} \quad (60)$$

Thus, constraints 1–59 are optimized under the objective function (expression 60) for the multiproduct scheduling of batches within a pipeline network. Table S1, available in Supporting Information, indicates the constraints used from the previous work¹ and the constraints changed or added in this article. As a summary, 19 constraints from Boschetto et al.¹ were used and 36 constraints were proposed within the presented work. In section 4, this model is tested in real-world case studies involving, for instance 323 batches, which are expected to be moved within a scheduling horizon of 1 month.

3.3. Decomposing the MILP Timing Model into Two Phases. A decomposition approach for the MILP timing model herein presented is also proposed within this article. This decomposition approach was based on a relax-and-fix heuristic described by Pochet and Wolsey,³⁹ for instance. The idea is to part the 0–1 variables into disjoint sets of decreasing importance and solve, sequentially, mixed integer programming (MIP) models to find a heuristic solution to the original MIP model. Within the studied case, two sequential MILP models are solved: MLC (Model with Local Constraints); and MST (Model with Seasonal costs and Turn shift constraints). The decomposition goal is to reduce the computational load, if seasonal costs and turn shift constraints are active.

In a first phase, the MLC is executed. In this phase, seasonal costs and turn shift constraints are not used, but local constraints are indeed considered. Thus, the 0–1 decisions related to local constraints were considered with priority in the

MILP timing block. In sequence, in phase 2, the binary variables related to local constraints are fixed, as previously determined by the MLC, and the seasonal costs and turn shift constraints are included. It is important to highlight that all local constraints are satisfied in MST, including the last ones in Table 2, since the binary variables were already determined by the MLC phase. The final solution satisfying all constraints is, then, obtained. As violation variables are considered within the modeling approach, feasible solutions can be found within either the single-phase or two-phase approaches. Table 2 indicates the constraints used in both models: the MLC model and the MST model. The objective function used in both models is the same as presented in expression 60.

3.3.1. MILP Model with Local Constraints (MLC). The MLC preserves the same structure proposed for the MILP timing model (single-phase approach). However, the seasonal cost constraints and the turn shift constraints are withdrawn. Therefore, in general, it is possible to obtain a preliminary solution with low computational burden (in a few seconds, as indicated a posteriori in section 4). Within the MLC obtained solution, it is important to observe that the refined ordering of batches to be pumped/received in different pipelines is determined. Local constraints are, for instance, addressed in detail and ordering changes for local constraint requirements are analyzed.

3.3.2. MILP Model with Seasonal Costs and Turn Shift Constraints (MST). Some results obtained by the MLC are used as parameters to the MST. In particular, the binary variables used in local constraints are fixed ($lb^1_{b,\bar{b}}$, $lb^2_{b,\bar{b}}$, $lr^1_{b,\bar{b}}$, $lr^2_{b,\bar{b}}$, $lbr^1_{b,\bar{b}}$, $lbr^2_{b,\bar{b}}$), according to MLC results, and used in the MST. Since these binary variables are fixed, the constraints 44–50 are already calculated for the MST in relation to the single-phase approach. Also, as presented on Table 2, the seasonal cost constraints and turn shift constraints are, in this second phase, created. The final scheduling of the pipeline network is finally obtained considering the MST outputs, including, for example, pumping and receiving times for batches, pumping flow rate, stoppages for seasonal costs and turn shifts, start and end times of stoppages in each pipeline segment, time window violations, and local constraint occurrences (previously evaluated by MLC).

Table 2. Constraints Considered in MLC and MST

	constraints	
	MLC	MST
scheduling horizon violation	×	×
sequencing constraints	×	×
seasonal cost constraints		×
constraints on time windows	×	×
surge tank constraints	×	×
immediate pumping constraints	×	×
constraints on turn shifts		×
local constraints: general	×	×
local constraints: two pipelines	×	
local constraints: three pipelines	×	

Table 3. Single-Phase MILP Timing Model: Results for 30-day Scenarios

	scenario							
	S1	S2	S3	S4	S5	S6	S7	S8
status	optimal	optimal	optimal	optimal	optimal	optimal	feasible	feasible
time (s)	210	863	344	91	362	14874	18035	18036
obj function	4.95×10^8	1.01×10^9	4.87×10^8	9.54×10^8	4.32×10^8	3.11×10^9	5.27×10^8	5.16×10^8
gap (%)	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	11.20	1.44
iterations	47 840	119 697	79 772	29 931	63 809	897 836	1 409 904	1 859 294
variables	51 196	59 531	58 314	58 960	57 788	61 964	60 994	64 530
integer var	35 805	42 110	40 720	41 177	40 771	44 039	43 216	47 340
constraints	155 447	181 970	165 343	159 373	160 143	149 090	184 658	191 088

Table 4. MLC: Results for 30-day Scenarios

	scenario							
	S1	S2	S3	S4	S5	S6	S7	S8
status	optimal	optimal	optimal	optimal	optimal	optimal	optimal	optimal
time (s)	12	15	10	6	14	394	461	235
obj function	2.08×10^8	6.38×10^8	4.12×10^8	8.59×10^8	3.44×10^8	3.07×10^9	4.59×10^8	4.57×10^8
gap (%)	0	0	0	0	<0.01	<0.01	<0.01	<0.01
iterations	4525	6274	3711	2705	4782	316 538	88 418	81 928
variables	15 180	18 136	20 225	20 374	19 431	25 941	23 867	22 199
integer var	2050	3362	4980	4680	4686	9688	8442	7558
constraints	28 727	32 908	36 518	36 392	35 589	44 531	54 200	43 770

Table 5. MST: Results for 30-day Scenarios

	scenario							
	S1	S2	S3	S4	S5	S6	S7	S8
status	optimal	optimal	optimal	optimal	optimal	optimal	optimal	optimal
time (s)	80	178	215	94	135	149	147	298
obj function	5.01×10^8	1.04×10^9	4.99×10^8	9.54×10^8	4.34×10^8	3.11×10^9	5.35×10^8	5.18×10^8
gap (%)	0	0	<0.01	0	0	<0.01	<0.01	<0.01
iterations	36 592	45 993	57 273	31 341	43 494	44 317	49 863	83 840
variables	49 146	56 169	53 334	54 280	53 102	52 276	52 552	56 972
integer var	33 755	38 748	35 740	36 497	36 085	34 351	34 774	39 782
constraints	154 565	181 481	163 887	158 813	158 611	146 811	169 679	183 674

4. COMPUTATIONAL RESULTS

The MILP proposed models (single-phase MILP, MLC, and MST) presented in section 3 are applied to real scenarios of the multiproduct pipeline network presented in Figure 1. This pipeline network follows the structure and characteristics presented in section 2. The models were run using the software ILOG OPL Studio 6.3, CPLEX 12 on an Intel Core 2 Duo 6400, 2.13 GHz, 3 GB RAM.

Eight scenarios (S1–S8) involving data for a 1 month period were used in order to perform the computational tests. These are the same scenarios presented by Magatão et al.² and, therefore, are not herein detailed.

Table 3 presents the results from the single-phase MILP timing model, which involves all constraints indicated in Table 1, for the eight tested scenarios (S1–S8). In Table 3, a 30-day period is considered for both seasonal costs and turn shifts, for all scenarios. Table 3 indicates the solution status (either optimal or feasible), the computational time in seconds, the objective function (expression 60) value, the gap integrality (%), and the number of iterations, variables, integer variables, and constraints. A maximum running time of 18 000 s (5 h) was admitted. It can be observed in Table 3 that for S1–S6 the model obtained the optimal solution within the available

running time. For S7 and S8 a feasible solution was obtained. The generated large-scale MILP formulations presented more than 50 000 variables, with a great majority of binary ones, and more than 149 000 constraints. Thus, it can be observed in Table 3 that optimal/feasible solutions can be obtained by the single-phase MILP timing model, but the computational load can be a concern, depending on the tested scenario (e.g., S6–S8). In the case of using the MILP model as part of a tool to aid the decision-making process, computational times of hours tend to be prohibited.

The MLC results, for all eight scenarios (S1–S8) are presented in Table 4. The labels used are the same as those in Table 3, and were previously detailed. As indicated in Table 2, seasonal cost constraints and turn shift constraints are not included within the MLC formulation. Results of Table 4 indicate that the MLC proved the solution optimality in computational times of a few seconds for scenarios S1–S8. Direct comparisons with results of Table 3 indicate that the generated MILP formulations for the MLC are smaller than the ones presented by the complete single-phase MILP timing model.

With the solution obtained from the MLC, for each one of the eight scenarios, the values of binary variables corresponding to local constraints were fixed and passed as parameters to the

MST, summarized in Table 2. The MST was, then, executed and the results are presented in Table 5. It is important to highlight that, within the MST execution, a 30-day period is indeed considered for both seasonal costs and turn shifts, for all scenarios. Table 5 indicates that the MST model obtained a solution in less than 5 min (300 s), for all scenarios. Adding the MST computational time for each scenario, respectively, with the computational time resulted from the MLC (Table 4), then a two-phase solution was obtained, in the worst case (scenario S7, 461 s + 147 s), in less than 11 min.

4.1. Comparison: Single-Phase Model versus Two-Phase Model. Figure 10 presents a graphic comparing, for

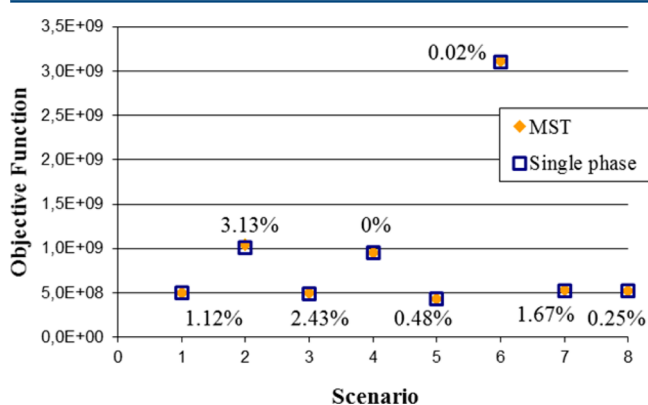


Figure 10. Comparison of objective function values for all scenarios.

scenarios S1–S8, the objective function values within the two proposed approaches. The single-phase MILP model considers all groups of constraints summarized in Table 2. The decomposed approach (two-phase model) contains the MLC and MST models, executed in a hierarchical manner, as illustrated in Figure 2. It is important to highlight that the objective function value presented for the decomposed approach is the one obtained by the MST, since this model returns the final pipeline network scheduling solution, considering the effect of all the constraints, including the local constraints, previously evaluated by the MLC. In this way, the MST solution is comparable to the single-phase MILP model. The differences in percentage between objective functions of MST and single-phase models, for each scenario, are also indicated in Figure 10.

It is possible to notice in Figure 10 that the objective function variation between the two approaches is small: the mean variation for all considered scenarios is 1.14% and, in the worst tested case, scenario S2, the variation was 3.13%. Considering scenario S4, the objective function value from MST is equal to the one obtained by the single-phase model, considering 30 days of seasonal costs and turn shifts.

Figure 11 presents, in a comparative way, the computational time to obtain the final solution, considering the single-phase model and the decomposed approach. In particular, the computational time of scenarios S1–S5 was detached in order to ease the viewing of results. The temporal data to build the curve of the single-phase model are shown in Table 3, while the data from the decomposed approach were obtained by the sum of computational times of MLC and MST (Tables 4 and 5).

Observing the scenarios S1–S5 in Figure 11, it is possible to realize that the computational load to obtain the solution, in both approaches, was smaller than 1000 s. The computational

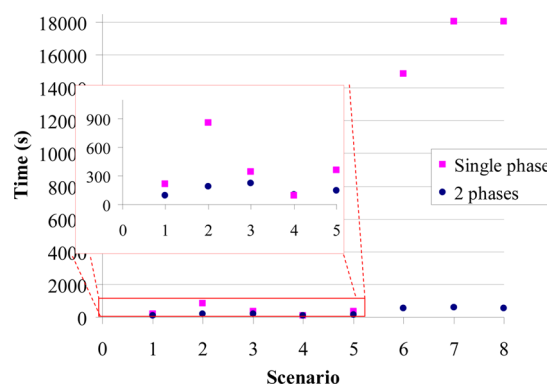


Figure 11. Comparison of computational times in all scenarios.

time from the decomposed approach (MLC + MST) was not lower than the single-phase approach just for S4: the single phase took 91 s (Table 3) and the two phase took 100 s (Tables 4 and 5), thus being the exception in the tendency of all tested cases. On the other hand, observing scenarios S6–S8, one can notice that the computational time from the single-phase approach is far bigger than that from the two-phase approach (at least 27 times bigger, scenario S6). In addition, for scenarios S7 and S8, according to Table 3, just a feasible solution was obtained within the single-phase approach after an 18 000 s run.

Finally, Figure 12 compares the computational mean time from all study cases for the single-phase and two-phase

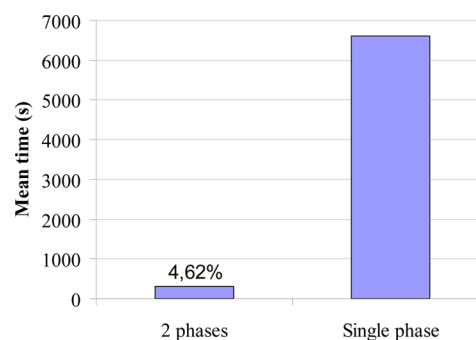


Figure 12. Mean computational time for single-phase and two-phase approaches.

approaches. It is possible to notice that the decomposition provided the solution, considering a mean measure, of study cases in 4.62% of the total computational time to obtain a solution using the single-phase model.

By observing the graphics presented in this section (Figures 10–12), it is possible to notice that the division strategy (two-phase approach) for the timing block tends to obtain a good solution in a computational time considerably lower than the single-phase MILP timing model. Thus, scheduling solutions could be found in low computational time (from seconds to a few minutes) with the decomposed timing block, without compromising the solution quality presented in the single-phase approach, as indicated in Figure 10.

4.2. Timing Model Improvements in Relation to Boschetto et al.¹ This section highlights some improvements in results coming from the constraints modified and added in the MILP timing block in relation to Boschetto et al.¹ The henceforth presented results were extracted from scenario S3, based on the two-phase approach execution (MLC + MST).

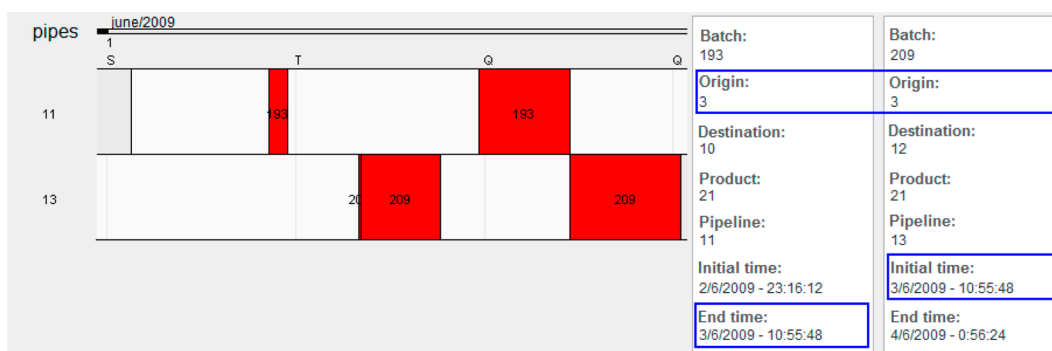


Figure 13. Gantt chart zoom-in scenario S3: local constraint example.

Details about the generated MILP formulations are given in, respectively, Tables 4 and 5.

4.2.1. Results from the New Local Constraints. As indicated in section 3.1.8, local constraints restrict the simultaneous pumping/receiving procedures for each area, according to the involved hardware (e.g., pumps and valves). Figure 13 illustrates a local constraint example where the simultaneous pumping of product 21 in pipelines 11 and 13 is not allowed. The modeling features allow that parts of involved batches be considered for local constraints. In some cases, the pumping procedure of a batch is broken and restarted, in order to consider tankage issues. These stoppages can be visualized, for example, in Figure 13: there exists a gap between temporal blocks of the same batch in the same pipeline. This fact occurs in Figure 13 with batches 193 and 209 (of product 21), in pipelines 11 and 13, respectively. The receiving procedures of these batches occur in an intercalated manner, avoiding violations in destination storages.

4.2.2. Results from the New Surge Tank/Immediate Pumping Constraints. As stated in section 3.1.3, in some areas, a batch can be received at a specific flow rate in a tank and, simultaneously, pumped from this tank to another pipeline at a different flow rate. This procedure is characterized as a “surge tank operation”. If the considered intermediate area has a limited storage capacity, a received batch should be pumped as soon as possible from this area (section 3.1.6, immediate pumping). In this case, the pumping procedure should be similar to the one used for product receiving in a surge tank area.

Figure 14 illustrates immediate pumping conditions involving batches 87, 258, and 257 of product 34. Batch 87 is sent from area N5 to area N12 by pipeline 20, batch 258 is sent from area N12 to area N13 by pipeline 19, and, finally, batch 257 is sent from area N13 to area N4 by pipeline 22. The storage profile of product 34 in area N13 is represented in Figure 15. The storage curve portions highlighted by ellipses indicate the presence of surge tank operations. In the first ellipse, the positive inclination p1 indicates the receiving of product 34 by batch 258 in N13. The p2 inclination shows the receiving of batch 258 simultaneously with the pumping of batch 257 (note that p1 inclination is steeper than p2). When the batch 258 receiving finishes, there is only a sending operation of batch 257 running. This operation is represented by the negative inclination p3. The second ellipse highlights similar pumping operations.

4.2.3. Turn Shift Results. As stated in section 3.1.7, sending and receiving procedures should not be started (or finished), in turn shift intervals in all areas. These constraints restrict the pumping in origin areas and, as a consequence, the receiving in



Figure 14. Gantt chart zoom-in scenario S3: surge tank example.

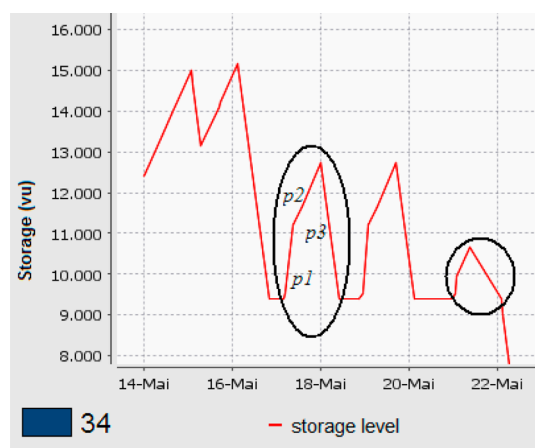


Figure 15. Inventory level of scenario S3 with immediate pumping results.

destination areas in turn shifts hours. During a 24-h period, three turn shifts occur within the considered network: (i) 7 a.m.–8 a.m., (ii) 3 p.m.–4 p.m., and (iii) 11 p.m.–12 a.m. Therefore, within these periods, if pumping start in an origin area is restricted, receiving affected by this pumping procedure is also restricted. Figure 16 highlights three delays in pumping start operations by turn shifts during the same day. A red vertical line is added in Figure 16 to indicate the start time of each turn shift period. Batch 58 is pumped to pipeline 4 just

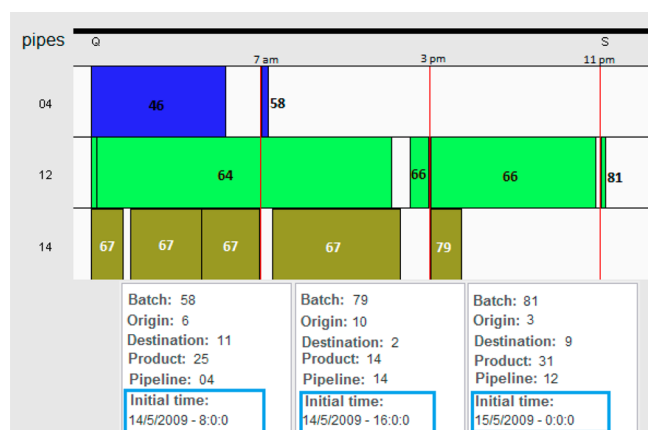


Figure 16. Gantt chart zoom-in scenario S3: turn shift example.

after 8 a.m., respecting the first turn shift interval. In a similar manner, the second turn shift interval is also respected, when the pumping start of batch 79 in pipeline 14 occurs just after 4 p.m. Finally, batch 81 is pumped in pipeline 12 just after 12 a.m. of day 15.

5. CONCLUSIONS

This paper improves the solution approach to schedule the operational activities in a pipeline network presented by Boschetto et al.¹ and Magatão et al.² The proposed approach took into account a decomposition strategy based on the integration of different heuristic blocks and MILP models, as indicated in Figure 2. The mixed integer linear programming model, which used a continuous time representation, was the core of the new timing block, and it was improved and expanded in relation to previous work.¹ For instance, turn shift constraints, local constraints, and surge tank constraints were addressed in detail within the proposed MILP timing model. The ideal flow rate for each batch and pipeline, according to scheduling purposes, is determined by the new MILP approach. In addition, a decomposition approach for the new MILP model presented is also proposed by this article. This decomposition is based on a relax-and-fix heuristic implemented by a sequential run of two MILP models: MLC (Model with Local Constraints) and MST (Model with Seasonal costs and Turn shift constraints). The decomposition goal is to reduce the computational load, if seasonal costs and turn shift constraints are active, without quality solution losses, as indicated in the results (e.g., Figure 10).

The developed tool was applied to a case study that represents the operational conditions of a real multiproduct pipeline network that transports oil derivatives and ethanol. The considered case study is particularly complex and involves many nodes and pipelines, as indicated in Figure 1. The difficulty of obtaining a feasible scheduling of operational activities is a day-to-day problem faced by the company schedulers, and the proposed solution approach can work as a decision tool to assist decision makers. In particular, the proposed approach treats a series of scheduling details (e.g., resource sharing, storage management, demand requirements, turn shifts, on-peak demand hours) in a reasonable computational time (from seconds to a few minutes, as indicated in section 4). As a result, the scheduler can visualize in a reduced computational time scheduling details for generated solutions involving a horizon of, approximately, 1 month. This is an

advantage when compared to approaches that involve computational times of many hours. In a first stage, decision-makers analyze a short/middle-scale operational scheduling to be implemented. Afterward, tendencies related to the lack of products in customers and supply excesses in the refineries can be analyzed through violations on time windows. If necessary, operational decisions are taken in order to avoid such tendencies. A large portion of the planning block input data is based on forecastings of production and consumption areas. Thus, the hierarchical approach proposed suggests an optimized solution that minimizes storage problems; in the case of temporal violations, the forecastings can be reexamined at specific problematical points in order to avoid critical storage issues.

In future developments, some issues are intended to be addressed. Within the solution procedure presented in Figure 2, the previous ordering defined within the assignment block could be refined and, thus, the time window limits improved. In the MILP model, violations on time windows are allowed, but minimized. These violations could be used in a feedback loop to iteratively analyze the proposed scheduling, providing improvements.

■ ASSOCIATED CONTENT

● Supporting Information

Constraints proposed by Boschetto et al.¹ and constraints proposed within the presented MILP timing model. The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/ie5046796.

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Notes

The authors declare no competing financial interest.

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■ NOMENCLATURE

Indices/Sets

B = set of batches (determined by assignment block), where $b \in B$

D = set of pipelines, where $d \in D$

HP = set of temporal intervals where on-peak demand hours occur ($h \in \text{HP}$)

BOT = set that contains tuples (b, p, n, t) where part p of batch b can be pumped from node n in turn shift interval t

N = set of network nodes, where $n \in N$

$\bar{n} \in N$ = index $\bar{n} \in N$ indicates the node immediately before a pipeline in the route of a batch

N_{hor} = set of areas that stop pumping at on-peak demand hour

PLM = set with the tuple $(no_b, prod_b)$ where flow rate can be changed by "surge tank" operation

$PB_{b,\bar{n},n,d}$ = set of sending parts of batch b in pipeline d between \bar{n} and n ; $PB_{b,\bar{n},n,d} = \{npb_{b,\bar{n},n,d}^{\min}, \dots, npb_{b,\bar{n},n,d}^{\max}\}$ (determined by simulation block)

$PR_{b,\bar{n},n,d}$ = set of receiving parts of batch b in pipeline d between \bar{n} and n ; $PR_{b,\bar{n},n,d} = \{npr_{b,\bar{n},n,d}^{\min}, \dots, npr_{b,\bar{n},n,d}^{\max}\}$ (determined by simulation block)

P = part of a batch, which is determined by the simulation block, where $p \in PB_{b,\bar{n},n,d}$ or $p \in PR_{b,\bar{n},n,d}$; to this specific part there exists a volume $part_{b,p}$ herein defined

LP = sparse set to local constraints of pumping from a node; the set involves tuples with (b, \bar{b}, p, \bar{p})

LR = sparse set to local constraints of receiving in a node; the set involves tuples with (b, \bar{b}, p, \bar{p})

LPR = sparse set to local constraints of simultaneous pumping from/receiving to a specific node; the set involves tuples with (b, \bar{b}, p, \bar{p})

General Parameters

α = number of hours during on-peak demand period

ϵ = small value (e.g., 10^{-4})

fh_h = ending time of on-peak demand period of day h (h)

ih_h = starting time of on-peak demand period of day h (h)

H = scheduling horizon (h)

k, kc, kp = constants used as a penalization weight within the objective function

M = large value for Big- M formulations

$it_{n,t}$ = starting time of turn shift period of interval t in node n (h)

$ft_{n,t}$ = ending time of turn shift period of interval t in node n (h)

cd_b = code of batch b used for local constraint identification purposes

Parameters (Determined by Assignment Block)

no_b = origin area of batch b ($no_b \in N$)

nd_b = destination area of batch b ($nd_b \in N$)

$prod_b$ = product associated with batch

ted_b = available receiving time to batch b in origin area (h)

tec_b = critical pumping time to batch b in origin area (h)

trd_b = available receiving time to batch b in destination area (h)

trc_b = critical receiving time to batch b in destination area (h)

ted_b^{Cut} = available receiving time to batch b in origin area (h), cut by ideal transportation time

tec_b^{Cut} = critical pumping time to batch b in origin area (h), cut by ideal transportation time

trd_b^{Cut} = available receiving time to batch b in destination area (h), cut by ideal transportation time

trc_b^{Cut} = critical receiving time to batch b in destination area (h), cut by ideal transportation time

vol_b = batch volume (vu, volumetric units)

Parameters (Determined by Simulation Block)

bo_b = batch in process of pumping that influences receiving of b

$npb_{b,\bar{n},n,d}^{\min}$ = minimum index of $PB_{b,\bar{n},n,d}$

$npb_{b,\bar{n},n,d}^{\max}$ = maximum index of $PB_{b,\bar{n},n,d}$

$npr_{b,\bar{n},n,d}^{\min}$ = minimum index of $PR_{b,\bar{n},n,d}$

$npr_{b,\bar{n},n,d}^{\max}$ = maximum index of $PR_{b,\bar{n},n,d}$

$part_{b,p}$ = pumping volume of each part p of batch b (vu)

$vb_{pr,n,d}^{\min}$ = minimum flow rate of product pr pumped in node n by pipeline d (vu/h)

$vb_{pr,n,d}^{\max}$ = maximum flow rate of product pr pumped in node n by pipeline d (vu/h)

Continuous Variables

ad_{b,nd_b} = advance in receiving time (h) of batch b in destination area nd_b

ao_{b,no_b} = advance in pumping time (h) of batch b in origin area no_b

do_{b,no_b} = delay in pumping time (h) of batch b in origin area no_b

dd_{b,nd_b} = delay in receiving time (h) of batch b in destination area nd_b

adc_{b,nd_b} = advance in receiving time (h) of batch b in destination area nd_b (cut window)

aoc_{b,no_b} = advance in pumping time (h) of batch b in origin area no_b (cut window)

doc_{b,no_b} = delay in pumping time (h) of batch b in origin area no_b (cut window)

ddc_{b,nd_b} = delay in receiving time (h) of batch b in destination area nd_b (cut window)

$fb_{b,\bar{n},n,d,p}$ = end time of sending part p of batch b from node \bar{n} to node n using pipeline d (h)

$fr_{b,\bar{n},n,d,p}$ = end time of receiving part p of batch b from node \bar{n} to node n using pipeline d (h)

$ib_{b,\bar{n},n,d,p}$ = start time of sending part p of batch b from node \bar{n} to node n using pipeline d (h)

$ir_{b,\bar{n},n,d,p}$ = start time of receiving part p of batch b from node \bar{n} to node n using pipeline d (h)

$rplm_{b,\bar{b},p,\bar{p}}^1$ = time violation for start of immediate pumping constraint, case 1 (h)

$rplm_{b,\bar{b},p,\bar{p}}^2$ = time violation for start of immediate pumping constraint, case 2 (h)

$rplm_{b,\bar{b},p,\bar{p}}^3$ = time violation for start of immediate pumping constraint, case 3 (h)

$rplm_{b,\bar{b},p,\bar{p}}^4$ = time violation for end of immediate pumping constraint (h)

$vh_{b,\bar{n},n,d,p}$ = horizon time violation of part p of batch b from node \bar{n} to node n using pipeline d (h)

Binary Variables

$x_{b,p,no_b,h}$ = 1, if pumping start of part p of batch b in origin area no_b occurs before start of an on-peak hour

$y_{b,p,no_b,h}$ = 1, if pumping ending of part p of batch b in origin area no_b occurs after start of an on-peak hour

$w_{b,p,no_b,h}$ = 1, if pumping start of part p of batch b in origin area no_b occurs before ending of an on-peak hour

$z_{b,p,no_b,h}$ = 1, if pumping procedure of part p of batch b in origin area no_b is stopped

$xtb_{b,p,no_b,t}$ = decides if batch pumping starts after or before turn shift interval t , in node no_b

$lp_{b,\bar{b},p,\bar{p}}^1$ = 1, if pumping of part p of batch b occurs before pumping of part \bar{p} of batch \bar{b}

$lp_{b,\bar{b},p,\bar{p}}^2$ = 1, if pumping of part \bar{p} of batch \bar{b} occurs before pumping of part p of batch b

$lr_{b,\bar{b},p,\bar{p}}^1$ = 1, if receiving of part p of batch b occurs before receiving of part \bar{p} of batch \bar{b}

$lr_{b,\bar{b},p,\bar{p}}^2$ = 1, if receiving of part \bar{p} of batch \bar{b} occurs before receiving of part p of batch b

$lpr_{b,\bar{b},p,\bar{p}}^1$ = 1, if pumping of part p of batch b occurs before receiving of part \bar{p} of batch \bar{b}

$lpr_{b,\bar{b},p,\bar{p}}^2$ = 1, if receiving of part \bar{p} of batch \bar{b} occurs before pumping of part p of batch b

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