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Analytical Multiloop PI/PID Controller Design for Two-by-Two Processes with Time Delays

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In this paper, an analytical multiloop controller design method is proposed for industrial and chemical two-by-two processes with time delays. By proposing the practically desired closed-loop diagonal transfer functions and the dynamic detuning factors to reduce the interactions between individual loops, the ideally desired multiloop controllers are inversely derived, which, however, are unavoidably involved with time delays in a complex manner and therefore difficult to realize. Hence, the mathematical Maclaurin series expansion is utilized to reproduce them in the form of a conventional PI/PID controller for implementation. Then a corollary derived from the generalized Nyquist stability theorem is provided to ascertain the nominal system stability. Meanwhile, the sufficient and necessary constraints for tuning the proposed multiloop PI/PID controllers to hold the control system robust stability are analyzed in the presence of the process multiplicative uncertainties. Finally, several commonly used simulation examples are included to demonstrate the effectiveness of the proposed method.

1. Introduction

Two-input-two-output (TITO) processes are mostly encountered multivariable processes in industrial and chemical practice. Moreover, many processes with inputs/outputs beyond two can be treated as several two-by-two subsystems in practice.[1,2] Therefore, a lot of research had been focused on TITO processes to develop control methods for multivariable processes. A widely used control structure is multiloop, namely, decentralized control structure, which has several merits for implementation, such as simple configuration and tuning and loop failure tolerance, etc.[3,4] However, due to the interactions between individual loops, well-developed closed-loop tuning methods for single-input-single-output (SISO) processes can hardly be applied to TITO processes.[5−8] Hence, many different approaches had been proposed to solve the problem. On the basis of the Gershgorin band criterion and Nyquist stability analysis, refs 9−12 put forward several tuning methods of the multiloop PI/PID controllers by using the frequency domain gain and phase margin specifications. By means of the linear fractional transformation (LFT) within the framework of the general $M - \Delta$ control structure, Hovd and Skogestad[13] and Gündes and Özgüler[14] systematically developed two independent design methods of decentralized controllers, respectively. Zhang et al.[15] further extended the dominant pole placement method of SISO systems to TITO processes, which really led to an improved system response, as compared with the biggest log modulus tuning (BLT) method provided by Luyben.[5] Wang et al.[16] extrapolated the well-known Ziegler−Nichols method to TITO systems by specifying the critical point of the Nyquist curve to a desirable position. In view of the fact that a conventional PID controller often results in excessive oscillation of the setpoint response in contrast with a PI controller, Chien et al.[17] proposed a modified implementation form of the PID controllers and a tuning rule for multiloop control systems. Desbiens et al.[18] presented a Smith predictor structure to design the decentralized PID controllers for two-by-two processes. Using the sequential relay feedback tests to realize autotuning of the multiloop PI controllers had been studied by Shen and Yu,[19] Loh et al.,[20,21] and Chiu and Arkun.[22] At the same time, Palmor et al.[23] and Halevi et al.[24] developed two simultaneous relay identification methods to obtain the ultimate point information and then utilized the classical Ziegler−Nichols rule or its modified forms for tuning the multiloop PI/PID controllers. According to the internal model control (IMC) theory,[3] Cha et al.[25] proposed a two-degree-of-freedom control scheme by using the PI/PID controllers, and Jung et al.[26] suggested a one-parameter tuning method for multiloop control systems, both of which led to the noteworthy improvement of system performance in comparison with some previous methods, but their multiloop controllers were all derived by using the fitting technology in the frequency domain, and thus, it seems to be somewhat inconvenient to apply them to various multivariable processes in practice. In addition, Cui and Jacobesen[27] and Campo and Morari[28] discussed the performance limitation of a multiloop control system, and Lee and Edgar[29,30] presented some further analysis on the controllability and stability of a multiloop control system.

It should be noted that most of the recently developed multiloop controller design methods were based on numerical calculation and iteration and, therefore, are time-consuming and troublesome for implementation from the viewpoints of control engineers and practitioners, although all of them were capable of achieving remarkably enhanced system performance in comparison with previously developed methods based on the simple controller design formulas or tuning rules. In addition, there always exists the unmodeled dynamics of an actual TITO process in practice. Therefore, model-based control methods are desired to be capable of on-line tuning so as to cope with the process uncertainties, which again seems to be impossible for many existing multiloop control methods. Hence, by virtue of the

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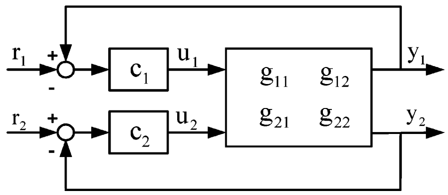

Figure 1. General TITO multiloop control structure.

analytical controller design results and on-line tuning rules recently obtained in the literature,[31−33] this paper proposes an analytical multiloop PI/PID controller design method to overcome the above-mentioned deficiencies. The key idea is to propose the desired transfer functions for individual loops in combination with the dynamic detuning factors, and thus, the ideally desired multiloop controllers can be inversely figured out. Then, by using the mathematical Maclaurin series expansion to copy out them, the practicable PI/PID controllers are conveniently obtained. As a result, the computation effort for deriving the practicable multiloop controllers in the form of PI/PID is relieved on a large scale, as compared with recently developed methods. Moreover, improved tuning capacities of individual loops are obtained; that is, each loop can be tuned on-line by a single adjustable parameter to cope with the process unmodeled dynamics, which will surely bring much convenience to the system operation in practice.

For clear interpretation of the proposed method, this paper is organized as follows: Section 2 presents some fundamental discussion for the multiloop structure controllability. In Section 3, the desired closed-loop diagonal transfer functions connecting the desired pairings between the system inputs and outputs are proposed. To reduce the loop interactions, dynamic detuning factors are suggested to modify them, so that the ideally desired multiloop controllers are inversely figured out. Their practicable PI/PID forms are analytically derived in Section 4. In Section 5, the sufficient and necessary constraints for tuning the adjustable parameters of the proposed multiloop controllers to hold the nominal system stability and its robust stability are discussed. Accordingly, the on-line tuning rule-of-thumb is provided. Several simulation examples are included in Section 6 to demonstrate the effectiveness of the proposed method. Finally, some conclusions are addressed in Section 7.

2. Multiloop Structure Controllability

Consider the general transfer matrix form of two-by-two processes with time delays,

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \tag{1}$$

where $g_{ij}(s) = g_{0ij}(s)e^{-\theta_{ij}s}$, and $i, j = 1, 2.$, of which $g_{0ij}(s)$ is the delay-free part and a physically proper and stable transfer function. According to the commonly used multiloop control structure shown in Figure 1,

where c_1 and c_2 are the multiloop controllers, and u_1 and u_2, respectively, denote the controller outputs, the closed-loop system transfer matrix can be obtained as

$$H = GC(I + GC)^{-1} \tag{2}$$

where C represents the diagonal controller matrix; i.e., $C = \text{diag}\{c_1,\ c_2\}$. It implies that absolute decoupling

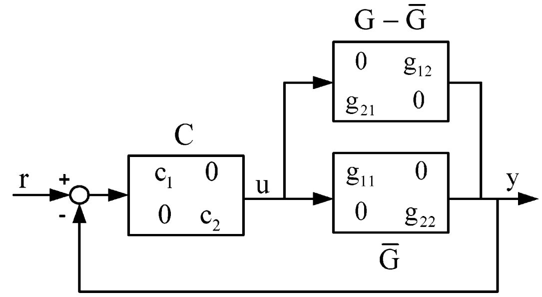

Figure 2. Block diagram representation of interactions as additive uncertainty.

regulation of the binary system outputs is impractical within the framework of a multiloop control structure. This can be explicitly clarified with a simple proof, as follows: Assume that the absolute decoupling regulation could be realized, that is to say, the nominal closed-loop system transfer matrix could be led to the diagonal form $H = \bar{H} = \text{diag}\{h_1,\ h_2\}$. By using a matrix inverse calculation for both sides of eq 2, yields $H^{-1} = (GC)^{-1} + I$. Note that H^{-1} is still a diagonal matrix in such a case. Hence, $(GC)^{-1}$ must be a diagonal matrix, which would require the process transfer matrix G to be a diagonal matrix and, therefore, would be contradictory to its actual form shown in eq 1.

In fact, the multiloop control structure shown in Figure 1 can be rearranged for analysis as the block diagonal closed-loop structure shown in Figure 2, where $\bar{G}$ is composed of the diagonal transfer functions of the process transfer matrix G; i.e., $\bar{G} = \text{diag}\{g_{11}, g_{22}\}$, which connects the desired pairings between the binary system inputs and outputs. Meanwhile, $G - \bar{G}$ is regarded as the additive uncertainty of the diagonal transfer matrix $\bar{G}$. According to the well-known Small-Gain Theorem, the larger the H infinity norm of the additive uncertainty $G - \bar{G}$ is, the worse the closed-loop control system stability is and, therefore, the lower the multiloop structure controllability is. Hence, it is desirable to configure the process transfer matrix in the form of diagonal dominance. As for TITO processes, it can be restricted to require the column diagonal dominance of their binary process transfer matrixes, that is, $|g_{11}(j\omega)| > |g_{21}(j\omega)|$ and $|g_{22}(j\omega)| > |g_{12}(j\omega)|$, $\omega \in [0, +\infty)$, which can be simply identified by comparing their magnitude plots of frequency response. An intuitive but not strict identification for the diagonal dominance can be made by comparison on their steady-state gains, that is, see if $|g_{11}(0)| > |g_{21}(0)|$ and $|g_{22}(0)| > |g_{12}(0)|$.

It should be noted that when a process transfer matrix actually cannot be made in essence as diagonal dominance, some existing methods, such as refs 9−10, suggested adding a static decoupler, that is, $D(0) = G^{-1}(0)$, in front of the process inputs and then designing the multiloop controllers for the augmented plant so that the achievable multiloop control system performance can be apparently enhanced and, therefore, acceptable for system operation in practice. However, the simulation examples 2 and 3 in Section 5 demonstrate that it is not very true.

Further discussion on the multiloop structure controllability is available in some control bibliographies[3,34] and recently published literature, such as refs 27−30; it is not the focus of this paper and, therefore, for brevity, it is not included.

3. The Desired Closed-Loop Diagonal Transfer Functions

From Figure 2, it can be easily seen that the nominal transfer matrix of the block diagonal closed-loop system without the additive uncertainty is in the form of

$$\bar{H} = \bar{G}C(I + \bar{G}C)^{-1} \tag{3}$$

Following some linear algebra, the diagonal controller matrix can be derived as

$$C = \bar{G}^{-1}(\bar{H}^{-1} - I)^{-1} \tag{4}$$

Therefore, the multiloop controllers are obtained in the form of

$$c_i = \frac{1}{g_{ii}} \frac{h_i}{1 - h_i} \quad i = 1, 2 \tag{5}$$

Note that g_{ii} contains time delay θ_{ii}. It can be seen from eq 5 that if the desired transfer function, h_i, connecting the system input, r_i, and output, y_i, were not to include θ_{ii}, the corresponding controller, c_i, would have to behave in a predictive manner. In addition, if g_{ii} has any right-half-plane (RHP) zeros, h_i is required to include them so that the resulting controller, c_i, will not include them as unstable poles. Hence, according to the H_2 optimal performance objective of the IMC theory, the desired closed-loop diagonal transfer functions are proposed as

$$h_i = \frac{e^{-\theta_{ii}s}}{(\lambda_i s + 1)^{U_i}} \prod_{k=1}^{V_i} \frac{(-z_k s + 1)}{(z_k s + 1)} \quad i = 1, 2 \tag{6}$$

where λ_i is an adjustable parameter for obtaining the desirable ith system output response, U_i is the relative degree of g_{0ii}, $s = z_k^{-1}$ is the RHP zero of g_{ii}, and V_i is the number of these RHP zeros.

However, substituting eq 4 into eq 2 yields the actual multiloop control system transfer matrix, that is, the transfer matrix of the perturbed block diagonal closed-loop system with the additive uncertainty $G - \bar{G}$ shown in Figure 2, in the form of

$$\begin{aligned} H &= G\bar{G}^{-1}(\bar{H}^{-1} - I)^{-1}(I + G\bar{G}^{-1}(\bar{H}^{-1} - I)^{-1})^{-1} \\ &= G((\bar{H}^{-1} - I)\bar{G} + G)^{-1} \\ &= G(\bar{H}^{-1}((I - \bar{H})\bar{G} + \bar{H}G))^{-1} \\ &= G(\bar{G} + \bar{H}(G - \bar{G}))^{-1}\bar{H} \end{aligned} \tag{7}$$

Note that there is usually $G \neq \bar{G}$ in practice. Thus, the diagonal transfer functions connecting the system inputs and outputs will not be in the form of eq 6 if the multiloop controllers are to be directly derived from eq 5. This can be physically interpreted as that there usually exists the additive uncertainty $G - \bar{G}$, which will inevitably result in the interactions between individual loops. To implement the desired closed-loop diagonal transfer functions shown in eq 6 for the desired pairings between the system inputs and outputs, a diagonal dynamic detuning matrix $D = \mathrm{diag}\{d_1, d_2\}$ is proposed to modify the diagonal system transfer matrix shown in eq 3. It follows that

$$D\bar{H} = \bar{G}C(I + \bar{G}C)^{-1} \tag{8}$$

Hence, by using some linear algebra, is yielded the multiloop controller matrix,

$$C = \bar{G}^{-1}(\bar{H}^{-1}D^{-1} - I)^{-1} \tag{9}$$

Then following a similar calculation as above, one obtains the actual multiloop control system transfer matrix in the form of

$$H = G(D^{-1}\bar{G} + \bar{H}(G - \bar{G}))^{-1}\bar{H} \tag{10}$$

Therefore, if one lets

$$\mathrm{diag}\{G(D^{-1}\bar{G} + \bar{H}(G - \bar{G}))^{-1}\} = I \tag{11}$$

the resulting diagonal transfer functions of the actual multiloop control system transfer matrix will be in the form of eq 6. In this way, the diagonal dynamic detuning matrix D can be ascertained. Substituting eqs 1 and 6 into eq 11 and solving it yields the dynamic detuning factors

$$d_1 = 2g_{11}g_{22} \Big/ \Big[(h_1 - h_2)g_{12}g_{21} + g_{11}g_{22} + (-1)^m\sqrt{[(h_1 - h_2)g_{12}g_{21} - g_{11}g_{22}]^2 - 4g_{11}g_{22}g_{12}g_{21}(1 - h_1)h_2}\Big] \tag{12}$$

$$d_2 = 2g_{11}g_{22} \Big/ \Big[(h_2 - h_1)g_{12}g_{21} + g_{11}g_{22} + (-1)^m\sqrt{[(h_1 - h_2)g_{12}g_{21} - g_{11}g_{22}]^2 - 4g_{11}g_{22}g_{12}g_{21}(1 - h_1)h_2}\Big] \tag{13}$$

where

$$m = \begin{cases} 0, & g_{11}(0)g_{22}(0) > 0 \\ 1, & g_{11}(0)g_{22}(0) < 0 \end{cases} \tag{14}$$

Note that the choice of m in eq 14 is to guarantee $d_1(0) = d_2(0) = 1$ so that the actual multiloop control system transfer matrix shown in eq 10 will be led to an identity matrix in the final steady state, that is, $H(0) = I$. That is to say, the effect of the diagonal dynamic detuning matrix is reduced to an identity matrix in the steady-state system transfer matrix and, thus, will not result in the deviation of system outputs. This is also the reason for throwing away the other solution pairing of eq 11. As for $d_1(0) = d_2(0) = 1$, it can be easily identified in view of that $h_1(0) = h_2(0) = 1$ (see eq 6). In fact, with respect to the case that $G = \bar{G}$, that is, $g_{12} = g_{21} = 0$, substituting them into eqs 12–13 yields $d_1 = d_2 = 1$, which indicates that the proposed diagonal dynamic detuning matrix is reduced to an identity matrix for a diagonal process transfer matrix and, therefore, is generally applicable to various two-by-two systems.

In consequence, combining eqs 6 and 8 with eqs 12–13, the diagonal transfer matrix for deriving the desired multiloop controllers so as to implement the H_2 optimal closed-loop diagonal transfer functions shown in eq 6 that actually connect the desired pairings between the system inputs and outputs can be figured out in the form of

$$\hat{H} = D\bar{H} = \mathrm{diag}\left\{\frac{d_i e^{-\theta_{ii}s}}{(\lambda_i s + 1)^{U_i}} \prod_{k=1}^{V_i} \frac{(-z_k s + 1)}{(z_k s + 1)}\right\} \quad i = 1, 2 \tag{15}$$

4. Multiloop PI/PID Controller Design

According to the proposed diagonal transfer matrix shown in eq 15, the ideally optimal multiloop controllers can be derived by substituting eq 15 into eq 9. It follows that

$$c_{i-\mathrm{ideal}} = \frac{1}{g_{ii}} \cdot \frac{d_i h_i}{1 - d_i h_i} \quad i = 1, 2 \tag{16}$$

However, by substituting eqs 6 and 12−13 into eq 16, it can be found that both of the resulting numerator and denominator of eq 16 will be involved with time delays in a complex manner and, therefore, are not rational and are difficult to be physically realized. In addition, if g_{ii} is involved with any RHP zeros, that is, $V_i \geq 1$, there will exist RHP zero-pole canceling in eq 16, which, however, will cause the resulting multiloop controllers to work in an unreliable manner. Hence, a practicable form is required to copy out the ideally optimal controller form shown in eq 16. In view of the fact that most commonly used controllers are in the form of PI/PID in process industry, here, like existing methods, is restricted to the multiloop PI/PID controller design for implementation. For simplicity, an analytical approximation method based on the mathematical Maclaurin series expansion is proposed as follows.

By using eqs 6 and 12−13, it can be identified that

$$\lim_{s \to 0}(1 - d_i h_i) = 0 \quad i = 1, 2 \tag{17}$$

which implies that the ideally optimal multiloop controllers proposed in eq 16 have a property of integrating to eliminate the steady deviation of system outputs. Therefore, let

$$M_i(s) = s c_{i-\mathrm{ideal}}(s) \quad i = 1, 2 \tag{18}$$

Using the mathematical Maclaurin series expansion, the rational approximation form of eq 16 can be obtained as

$$c_{i-\mathrm{Mac}}(s) = \frac{1}{s}\left[M_i(0) + M_i'(0)s + \frac{M_i''(0)}{2!}s^2 + ... + \frac{M_i^{(n)}(0)}{n!}s^n + ...\right] \quad i = 1, 2 \tag{19}$$

Apparently, the first two terms of the above equation constitute a standard PI controller, that is

$$c_{i-\mathrm{PI}}(s) = k_{Ci}\left(1 + \frac{1}{\tau_{Ii}s}\right) \quad i = 1, 2 \tag{20}$$

where $k_{Ci} = M_i'(0)$ and $\tau_{Ii} = M_i'(0)/M_i(0)$, $i = 1, 2$.

Furthermore, the first three terms of eq 19 constitute a standard PID controller, that is

$$c_{i-\mathrm{PID}}(s) = k_{Ci}\left(1 + \frac{1}{\tau_{Ii}s} + \tau_{Di}s\right) \quad i = 1, 2 \tag{21}$$

where k_{Ci} and τ_{Ii} are as above, and $\tau_{Di} = M_i''(0)/2M_i'(0)$, $i = 1, 2$. It should be noted that the pure derivative term in eq 21 can be physically implemented by cascading it with a first-order low-pass filter in which the time constant can be chosen as ($\sim$0.01 to 0.1)τ_{Di}.

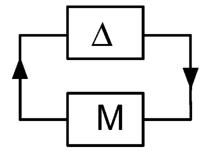

Figure 3. General $M - \Delta$ structure.

Obviously, the above PID controller formula is capable of obtaining better system performance than the PI formula in eq 20 due to a better approximation for the ideally optimal multiloop controllers shown in eq 16. It should be pointed out that much better achievable control performance can be achieved by using a rational and stable high order approximation formula, such as the Padé approximation design formula proposed in recent literature.[35]

In addition, it should be noted that each of the multiloop PI/PID controllers proposed in eqs 20−21 is actually tuned by a single adjustable parameter, λ_i, which is utilized to obtain the desirable ith system output response, as shown in eq 6.

5. Multiloop System Stability Analysis

It is necessary to analyze the multiloop system stability so that the tuning constraints for the adjustable parameters $\lambda_i (i = 1, 2)$ of the proposed multiloop PI/PID controllers can be ascertained. In addition, how to evaluate and hold the control system robust stability in the presence of the process uncertainty in practice is addressed in this section.

At first, the following theorem derived from the generalized Nyquist stability theorem is presented.[34]

Theorem. *Assume that the nominal system M(s) and the perturbations Δ(s) are stable. Consider the convex set of perturbations, Δ, such that if Δ′ is an allowed perturbation, then so is ϵΔ′ where ϵ is any real scalar such that |ϵ| ≤ 1. Then the M − Δ system shown in Figure 3 is stable for all allowed perturbations if and only if any one of the following four equivalent conditions is satisfied:*

(1) Nyquist plot of $\det(I - M\Delta(s))$ *does not encircle the origin,* $\forall\Delta$, *i.e.,* $\det(I - M\Delta(j\omega)) \neq 0, \forall\omega, \forall\Delta$

(2) $\lambda_i(M\Delta(j\omega)) \neq 1, \forall i, \forall\omega, \forall\Delta$

(3) $\rho(M\Delta(j\omega)) < 1, \forall\omega, \forall\Delta$

(4) $\max_{\Delta} \rho(M\Delta(j\omega)) < 1, \forall\omega$

As for the nominal two-by-two multiloop control structure shown in Figure 1 or equivalent Figure 2, the following corollary derived from the above theorem can be utilized to identify its stability.

Corollary. *A two-by-two multiloop control system is nominally stable if and only if*

(1) $c_1/(1 + g_{11}c_1)$ *and* $c_2/(1 + g_{22}c_2)$ *are stable*

(2) $\rho\left(\begin{bmatrix} 0 & \dfrac{g_{12}c_1}{1 + g_{11}c_1} \\ \dfrac{g_{21}c_2}{1 + g_{22}c_2} & 0 \end{bmatrix}\right) < 1, \forall\omega$

Proof. Rearrange the equivalent multiloop control structure shown in Figure 2 within the framework of the standard $M - \Delta$ structure shown in Figure 3. Thus, it can be figured out that the transfer matrix connecting the input and output of the additive uncertainty $\Delta = G - \bar{G}$ is

$$M = -C(I + \bar{G}C)^{-1} \tag{22}$$

which is apparently a diagonal transfer matrix, in view of $\bar{G} = \mathrm{diag}\{g_{11}, g_{22}\}$ and $C = \mathrm{diag}\{c_1, c_2\}$, and is composed of the two transfer functions shown in the first condition of the corollary. By virtue of that the multiloop controllers are proposed to configure in the form of the conventional PI/PID for implementation, the first condition can be intuitively identified for a stable process transfer matrix G, which guarantees the assumption of the aforementioned theorem to be satisfied. Then by substituting the additive uncertainty $\Delta = G - \bar{G}$ and eq 22 into condition 3 of the aforementioned theorem, the second condition follows directly.

Note that the second condition of the corollary can be identified by observing whether the magnitude plot of the spectral radius with $\omega \in [0, +\infty)$ falls below unity. In this way, the admissible tuning range of the adjustable parameters, λ_i ($i = 1, 2$), can be ascertained. In addition, reviewing the desired diagonal transfer functions shown in eq 6 in combination with eqs 10−11, it can be seen that each of the system output responses is primarily regulated by λ_1 and λ_2, respectively. When the adjustable parameter, λ_i ($i = 1, 2$), is tuned to be small, the corresponding ith system output response becomes faster, but the output energy of the ith multiloop controller, c_i, and its corresponding actuator is larger, which tends to be beyond their output capacities in practice. In addition, more aggressive dynamic behavior of the ith system output response will occur in the presence of the process uncertainty. In contrast, tuning λ_i to be large drops down the corresponding ith system output response speed, but the output energy of c_i and its corresponding actuator is smaller, and accordingly, less aggressive dynamic behavior of the ith system output response will appear in face of the process uncertainty. This is demonstrated in the following simulation example 1 of Section 6. Therefore, actually tuning the adjustable parameters, λ_i ($i = 1, 2$), aims at the tradeoff between the nominal system response performance and the output capacities of the multiloop controllers, c_i ($i = 1, 2$), and their corresponding actuators.

Obviously, in the presence of the process uncertainty, the multiloop system transfer matrix shown in eq 2 could become very complex, and the closed-loop system tends to lose stability in an intangible manner. How to ascertain all the stabilizing set of the multiloop controller matrix C for different process uncertainties is difficult to address and has remained open as yet in the process control community.[34,36] A practical way to identify the closed-loop system robust stability in the presence of often encountered process uncertainties, such as the perturbation of the process parameters, actuator output uncertainties, and system output sensor measurement uncertainties, is to lump multiple sources of uncertainty into a multiplicative form.[34] In view of the multiplication sequence of the transfer matrix, here, two cases composed of the process multiplicative input and output uncertainties are analyzed, both of which are shown in Figure 4.

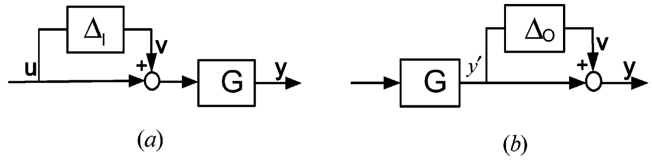

Figure 4. Multiplicative input (a) and output (b) uncertainty.

The process multiplicative input uncertainty, which is shown in Figure 4a, describes the actual process family $\Pi_I = \{\hat{G}_I(s){:}\hat{G}_I(s) = G(s)(I + \Delta_I)\}$, where Δ_I is assumed to be stable. The process multiplicative output uncertainty, which is shown in Figure 4b, describes the actual process family $\Pi_O = \{\hat{G}_O(s){:}\hat{G}_O(s) = (I + \Delta_O)G(s)\}$, where Δ_O is assumed to be stable.

According to the standard $M - \Delta$ structure shown in Figure 3 for robust stability analysis, the transfer matrixes from the outputs to inputs of Δ_I and Δ_O can be respectively figured out as

$$M_I = -C(I + GC)^{-1}G \tag{23}$$

$$M_O = -GC(I + GC)^{-1} \tag{24}$$

Note that the nominal system stability has been guaranteed by tuning the adjustable parameters λ_1 and λ_2. As a result, the closed-loop transfer matrix shown in eq 2 holds stability, that is to say, $C(I + GC)^{-1}$ is stable. Consequently, M_I and M_O shown in eqs 23−24 are kept stable for a stable process transfer matrix G. Then the aforementioned theorem can be utilized to derive the multiloop system robust stability constraints for tuning the adjustable parameters λ_1 and λ_2. Substituting eqs 23−24, respectively into condition 3 of the aforementioned theorem yields

$$\rho(C(I + GC)^{-1}G\Delta_I) < 1 \quad \forall \omega \tag{25}$$

$$\rho(GC(I + GC)^{-1}\Delta_O) < 1 \quad \forall \omega \tag{26}$$

Hence, as for a specified bound of Δ_I or Δ_O in practice, it is available to employ eqs 25−26 to evaluate the control system robust stability, that is, to see whether the magnitude plots of the left sides of eqs 25−26 with $\omega \in [0, +\infty)$ fall below unity. In fact, this can be conveniently performed by using control software packages, such as MATLAB robust control toolbox,[37] which is also demonstrated in the following simulation example 1 of Section 6.

In general, it is recommended to tune each of the adjustable parameters λ_i ($i = 1, 2$) around the time delay θ_{ii} ($i = 1, 2$) of the process diagonal transfer functions in the first place, respectively. Then by monotonically increasing or decreasing either of them on-line independently, the desirable output response performance of individual loops can be conveniently achieved. To cope with the process uncertainties in practice, namely, the process unmodeled dynamics, it is suggested to monotonically increase the adjustable parameters λ_1 and λ_2 on-line so that the nominal system response will be gradually slowed in exchange for better system robust stability. However, if by doing so, the control system performance and robust stability cannot be acceptable as yet, the process reidentification will need to be implemented for obtaining a more precise process model to derive the mutiloop controller matrix C so that the

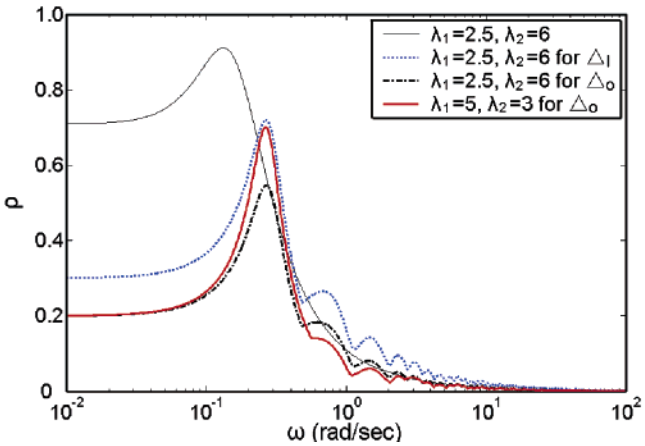

Figure 5. Magnitude plots of spectral radius for example 1.

Table 1. PI Tuning Parameters for Example 1

	k_{C1}	τ_{I1}	k_{C2}	τ_{I2}
Chen	0.436	11.0	−0.0945	15.5
Jung	0.19	8.51	−0.099	8.58
proposed	0.2448	5.458	−0.0723	6.278

process unmodeled dynamics can be reduced to achieve better nominal system performance and its robust stability.

6. Simulation examples

Example 1. Consider the widely studied Wood−Berry binary distillation column process[38]

$$G = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}$$

The most recent multiloop control methods, such as Chen[9] and Jung,[26] are employed here for comparison, both of which had already shown their advantages over many existing methods. Their PI controller settings are listed in Table 1. In the proposed method, take $\lambda_1 = 2.5$ and $\lambda_2 = 6$ in order to obtain the similar setpoint response rising speed with the above two methods, and therefore result in the PI settings shown in Table 1 by using the design formulas proposed in eqs 16, 19 and 20. Correspondingly, the magnitude plot of the spectral radius used for identifying the nominal system stability in the proposed method is provided in Figure 5 with thin solid line.

It is seen that the maximum magnitude of the spectral radius for the nominal system is less than unity; therefore, the nominal system stability is guaranteed.

Add a unit step change to the binary setpoint inputs at $t = 0$ and $t = 100$ and then add an inverse step change of load disturbance with magnitude of 0.1 to both of the binary process inputs at $t = 200$. The simulation results are shown in Figure 6. It should be noted that the simulation solver option is chosen as ode5 (Dormand−Prince) and the simulation step size is fixed as 0.02 throughout this paper.

It is seen from Figure 6 that the proposed PI controllers result in the comparable system performance, in contrast with the other two methods. In addition, the nominal system response obtained by using the proposed PID controllers in terms of the above adjustable parameter settings, that is, adding the derivative terms $\tau_{D1} = 0.255$ and $\tau_{D2} = 1.0796$ obtained from the design formula eq 21 to the PI settings listed in Table 1, is provided in Figure 7, in comparison with that of Jung's PID controllers, both of which were in the form of $c_1 = 0.27(1 + 1/6.91s + 3.935s)/(1.81s + 1)$ and $c_2 = -0.103(1 + 1/5.9s + 1.88s)/(0.175s + 1)$.

It is seen that both of them result in the similar system performance and are better than the PI controllers of each of the above-mentioned methods. However, it should be noted that Jung's PID controller settings were obtained by using a renewed numerical calculation with considerable computation effort.

To demonstrate the multiloop control system stability in terms of the proposed method, assume that there actually exists the process multiplicative input uncertainty $\Delta_I = \mathrm{diag}\{(s + 0.3)/(s + 1), (s + 0.3)/(s + 1)\}$, which can be loosely interpreted as that the binary process inputs supplied by the corresponding actuators increase with up to 100% uncertainty at high frequencies and with almost 30% uncertainty in the low frequency range. In the other case, assume that there actually exists the process multiplicative output uncertainty $\Delta_O = \mathrm{diag}\{-(s + 0.2)/(2s + 1), -(s + 0.2)/(2s + 1)\}$, which can be physically regarded as that the binary process measurements provided by the corresponding output sensors decrease with up to 50% uncertainty at high frequencies and with almost 20% uncertainty in the low-frequency range. Figure 5 has shown the magnitude plots of spectral radius in terms of the assumed Δ_I and Δ_O, both of which demonstrate the proposed control system robust stability. Accordingly, the perturbed system responses are shown in Figure 8.

By contrast with the nominal system response shown in Figure 6, it is seen that the proposed multiloop control system holds robust stability well. In addition, it should be noted that by using the proposed method, the setpoint response oscillation of the perturbed system output y_1 can be gradually relieved by increasing the adjustable parameter λ_1 on-line but in exchange for degraded load disturbance rejection performance. On the other hand, faster setpoint response and better load disturbance rejection response of the perturbed system output y_2 can be conveniently obtained by gradually decreasing the adjustable parameter λ_2 on-line but in exchange for lower control system stability. For illustration, take $\lambda_1 = 5$ and $\lambda_2 = 3$, so the resulting PI settings are $k_{C1} = 0.1807$, $\tau_{I1} = 6.9055$ and $k_{C2} = -0.091$, $\tau_{I2} = 5.2722$ according to the design formulas proposed in eqs 16, 19 and 20. The corresponding simulation result for the above perturbed system with multiplicative output uncertainty is also shown in Figure 8 for comparison, which simultaneously demonstrates that the binary system outputs y_1 and y_2 can be primarily regulated by the adjustable parameters λ_1 and λ_2, respectively and, therefore, is in accordance with the desired closed-loop diagonal transfer functions shown in eq 6.

Example 2. Consider the widely studied Vinante and Luyben process[5]

$$G = \begin{bmatrix} \dfrac{-2.2e^{-s}}{7s+1} & \dfrac{1.3e^{-0.3s}}{7s+1} \\ \dfrac{-2.8e^{-1.8s}}{9.5s+1} & \dfrac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}$$

It can be easily seen that the first column of the process transfer matrix is of slightly off-diagonal domi-

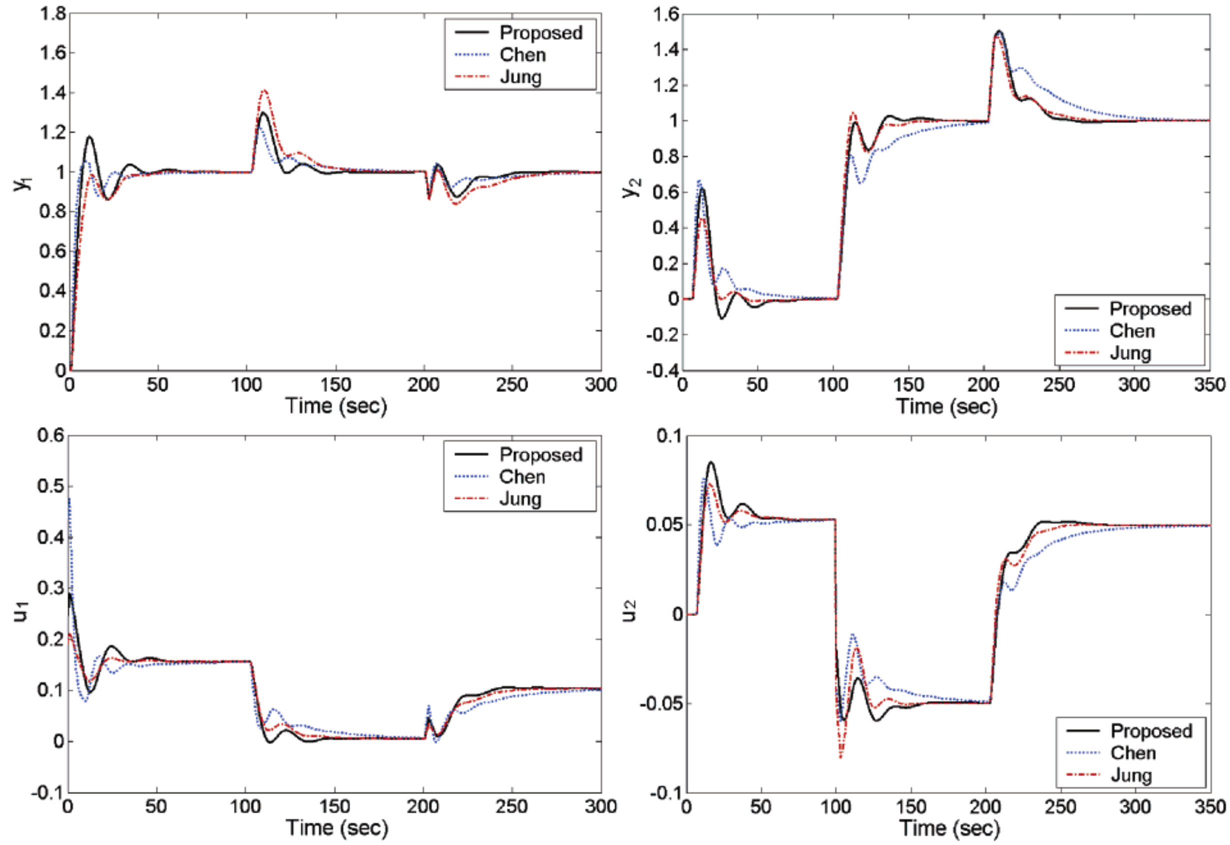

Figure 6. Nominal system responses for example 1.

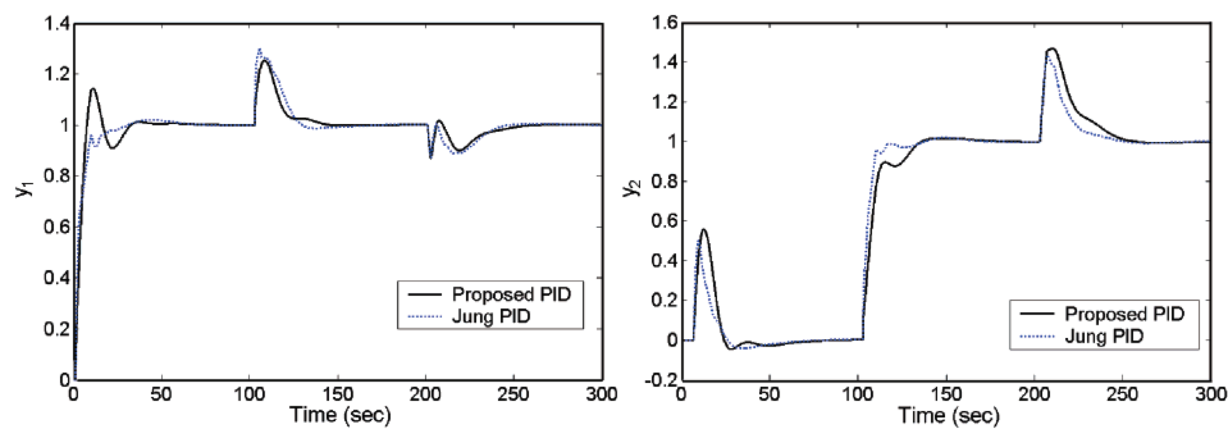

Figure 7. Nominal system responses for example 1 according to the PID controllers.

nance. Therefore, Chen[9] suggested a static decoupler $D(0) = G^{-1}(0)$ in front of the binary process inputs and then designed the multiloop PI controllers for the augmented system. However, Lee[12] directly provided a multiloop PI controller tuning method for the original process based on a numerical iteration. For comparison, perform two cases in the proposed method: one case is for the original process, that is, take $\lambda_1 = 2$ and $\lambda_2 = 0.3$ so as to obtain a similar setpoint response rising speed with the Lee method. The other case is for the augmented system. Note that in the latter case, there exists a RHP zero at $s = 1.3644$ in the first diagonal transfer function of the augmented system transfer matrix; hence, according to the H_2 optimal form of the desired closed-loop diagonal transfer functions shown in eq 6, the first diagonal transfer function of the closed-loop control system should be

$$h_1 = \frac{(-0.7329s + 1)e^{-0.3s}}{(0.7329s + 1)(\lambda_1 s + 1)}$$

Then take $\lambda_1 = 2$ and $\lambda_2 = 0.7$ so as to obtain a similar setpoint response rising speed with the Chen method. Then by using the design formulas proposed in eqs 16, 19, and 20, obtain the PI settings listed in Table 2, in which the PI settings of the other two methods are also included. Add a unit step change to the binary setpoint inputs at $t = 0$ and $t = 40$, and then add an inverse unit step change of load disturbance to both of the binary process inputs at $t = 80$. The simulation results are shown in Figure 9.

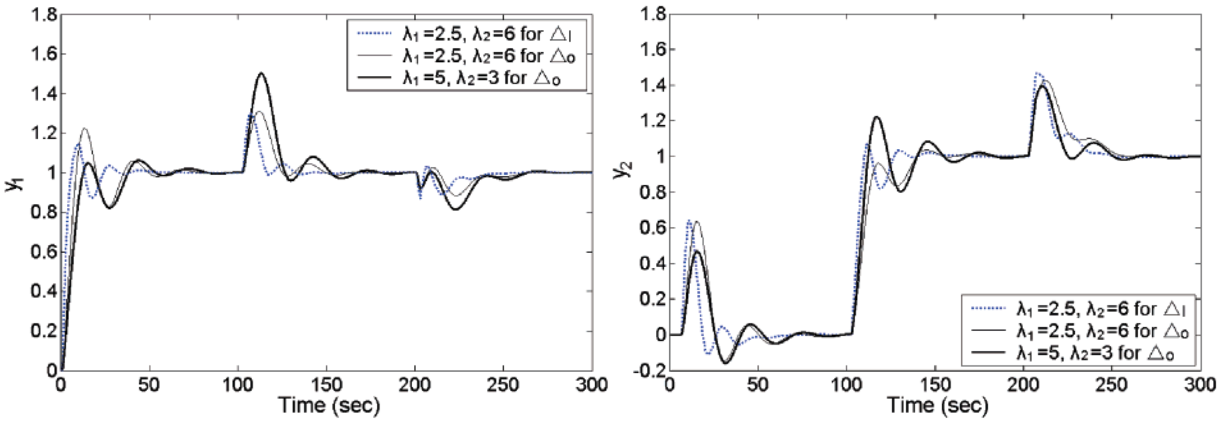

Figure 8. Perturbed system responses for example 1.

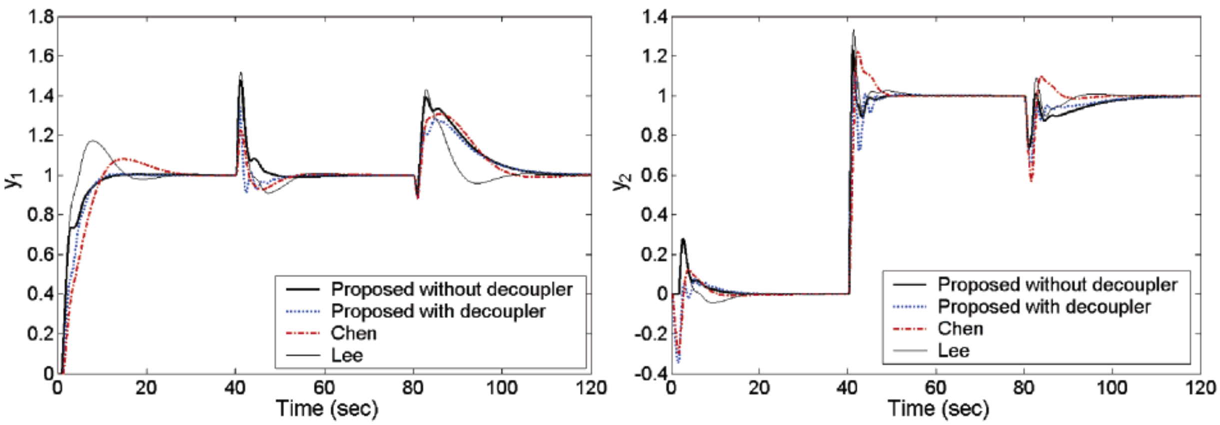

Figure 9. System output responses for example 2.

Table 2. PI Tuning Parameters for Example 2

	k_{C1}	τ_{I1}	k_{C2}	τ_{I2}
Chen	1.21	4.64	3.74	1.1
Lee	−1.31	2.26	3.97	2.42
proposed without decoupler	−1.5417	6.2599	4.3518	7.4832
proposed with decoupler	1.8816	7.086	7.7751	8.1638

Table 3. PI Tuning Parameters for Example 3

	k_{C1}	τ_{I1}	k_{C2}	τ_{I2}
Chen	6.67	1.04	1.67	1.57
Chien	0.263	1.42	0.163	1.77
proposed without decoupler	0.2908	4.6962	0.0869	1.3518
proposed with decoupler	7.7294	3.8647	1.2136	2.0632

It is seen that improved system performance for both setpoint tracking and load disturbance rejection is obtained by using the proposed method. In addition, it is demonstrated that using the static decoupler has not led to remarkably enhanced system performance. It should be noted that simulation tests based on the assumed perturbation as in example 1 demonstrate that the control system robust stability is kept well in terms of the proposed method irrespective of whether the static decoupler is used and, therefore, are omitted for brevity.

Example 3. Consider the industrial-scale polymerization reactor studied by Chen et al.[10] and Chien et al.[17]

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{22.89e^{-0.2s}}{4.572s+1} & \dfrac{-11.64e^{-0.4s}}{1.807s+1} \\ \dfrac{4.689e^{-0.2s}}{2.174s+1} & \dfrac{5.8e^{-0.4s}}{1.801s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} + \begin{bmatrix} \dfrac{-4.243e^{-0.4s}}{3.445s+1} \\ \dfrac{-0.601e^{-0.4s}}{1.982s+1} \end{bmatrix} [d(s)]$$

It is obvious that the second column of the transfer matrix is of apparently off-diagonal dominance. Again, Chen utilized a static decoupler $D(0) = G^{-1}(0)$ in front of the binary process inputs and then designed the multiloop PI controllers for the augmented system. Chien directly provided a multiloop PI controller tuning method for the original process. For comparison, perform two cases in the proposed method: one case for the original process and the other case for the augmented system with the same adjustable parameter settings; that is, take $\lambda_1 = 0.3$ and $\lambda_2 = 1.5$ so as to obtain a similar setpoint response rising speed with the Chen method. By using the design formulas proposed in eqs 16, 19, and 20, obtain the PI settings listed in Table 3, in which the PI settings of the other two methods are also included. Add a unit step change to the binary setpoint inputs at $t = 0$ and $t = 20$, and then add a unit step change of the load disturbance at $t = 50$. The simulation results are shown in Figure 10.

Again, it is seen from Figure 10 that improved system performance for both setpoint tracking and load disturbance rejection is obtained by using the proposed method. In addition, it is demonstrated that using the static decoupler has resulted in somewhat, but not much, enhanced system performance. It should be noted that simulation tests based on the assumed perturbation as in example 1 demonstrate that the control system robust stability is kept well in terms of the proposed method,irrespective of whether the static decoupler is used and, therefore, are not shown.

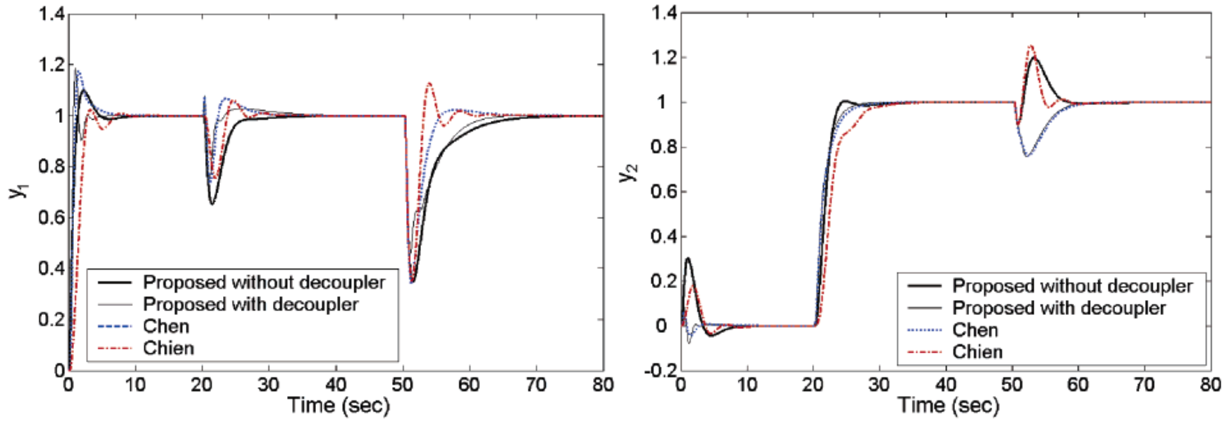

Figure 10. System output responses for example 3.

7. Conclusions

Simple multiloop/decentralized PI/PID controller design methods for TITO processes with time delays are widely sought after in modern industrial and chemical practice. In this paper, an analytical controller design method has been proposed to meet such requirements. By proposing the desired closed-loop diagonal transfer functions connecting the desired pairings between the system inputs and outputs in terms of the H_2 optimal performance objective of the IMC theory and introducing a dynamic detuning matrix to realize the goal, the ideally optimal multiloop controllers are inversely figured out. For implementation convenience, practicable PI/PID forms are proposed to reproduce them by means of the mathematical Maclaurin series expansion. As a result, the corresponding computation effort is significantly relieved, in contrast with many existing methods based on numerical calculation and iteration. Consequently, the proposed method can be conveniently applied to various TITO processes in industry. It should be noted that the proposed multiloop PI/PID controllers are derived in a unified way, which however, is not the case in many existing methods, and most of them were merely restricted to the PI controller design.

It has been demonstrated that either of the binary system outputs can be regulated on-line by tuning the single adjustable parameter of its corresponding closed-loop controller, which will therefore contribute much convenience to the system operation. The sufficient and necessary constraints for tuning the adjustable parameters to hold the nominal system stability and its robust stability are analyzed in the presence of the process multiplicative input and output uncertainties. It has been pointed out that tuning the adjustable parameters aims at the tradeoff between the nominal system performance and its robust stability, which, in fact, can be simply implemented on-line by increasing or decreasing, respectively, the adjustable parameters in a monotonic manner.

In addition, it has been illustrated through simulation examples 2 and 3 that for TITO processes with transfer matrix of off-diagonal dominance, a static decoupler obtained as the inverse of the process steady gain matrix could help to improve the system performance by adding it to the process inputs, but is not quite necessary according to the proposed method.

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