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Large Scale Production Planning in the Stainless Steel Industry

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ABSTRACT: We introduce a new formulation for a cutting stock problem in the stainless steel industry. The formulation extends some previous models developed for the paper industry. In this formulation, mother coils are assumed to have finite lengths, and the widths and lengths of the mother coils are considered variables as well as the lengths of the product coils along with the cutting patterns to be used. To enable reuse of scrap material, the trim loss at the end of a mother coil is preferred and collected into a single coil. Finally, the scattering of an order to multiple mother coils is minimized to facilitate the shipping of the order in a single batch. For numerical solution, the inherently nonlinear problem is transformed to a linear one by a decomposition technique. The procedure is illustrated with numerical examples. The presented model has been successfully utilized in large scale industrial environment as a part of the production planning system developed for Outokumpu Stainless by Accenture.

INTRODUCTION

In this paper, a new mathematical formulation for a cutting stock problem in the stainless steel industry is introduced. The particular problem which deals with the cutting of small items out from larger objects is 2-fold. The assortment problem addresses the issue of choosing proper dimensions for the larger objects that correspond to the mother coils rolled from the slabs in the steel industry. The trim loss problem considers how to cut out the small items or product coils from the mother coils so that the wastage material is minimized.[1,2] The combination of the two problems, originating from the pioneering work of ref 3, is known as the cutting stock problem.

One of the most important characteristics of the cutting stock problem is dimensionality, which gives the minimum number of dimensions that are significant in the determination of the solution.[2] Attributes defining the cutting process can be given as well. In this paper, we consider an orthogonal two-staged guillotine two-dimensional cutting stock problem.[1] In particular, product coils are cut out from mother coils so that the first cut extends the full width of the mother coil and the second cut is made parallel to a side of the mother coil. Moreover, the product coils of variable length can be positioned on the mother coils of variable dimensions in two free dimensions.

The introduced formulation extends some previous models developed for the paper industry.[4−6] First, mother coils of a particular width, from which product coils or product strips are cut, are assumed to have limited lengths. This is different, for example, from the paper industry, where the raw reels can be attached together in full speed during the cutting process. Second, in order to make the reuse of scrap material easier, the lengthwise trim loss at the end of a mother coil is preferred and targeted to be collected into a single mother coil. Third, the contiguity or the scattering of an order to multiple mother coils is minimized to facilitate the shipment of a customer order in a single batch.

A number of methods have been developed for the solution of cutting stock problems. Historically, various heuristics such as delayed column generation,[7,8] modified delayed column generation,[9] sequential heuristic procedures,[10−13] and hybrid versions of the aforementioned methods[14−16] have been used in the solution of various cutting stock problems, typically one-dimensional ones. Recently, a sequential heuristic procedure has been applied to a two-dimensional problem as well.[17] These heuristics are highly problem dependent and may be inapplicable for seemingly similar problems. Another class of solution methods comprise meta-heuristic methods[18] such as tabu search,[19,20] greedy heuristics,[21] evolutionary programming,[22,23] genetic hybrid algorithms,[24] and ant colony optimization.[25] Third, a nonconvex cutting stock problem can be transformed into a linear or nonlinear mixed integer programming problem by various decomposition techniques. Globally optimal solution of the transformed problem can be then solved by a suitable optimization algorithm such as Branch and Cut[26] for linear problems, or cutting plane methods,[27−30] simplicial approximation[31] or conic scalarization[32] for nonlinear ones. Decomposition techniques have been applied, for example, in the solution of a cutting stock problem in corrugated board box industry.[33]

Although several methods exist for solving the cutting stock problems, only a subset of those are applicable to the problem in question. For example, even though the utilization of genetic hybrid algorithms for solving complex combinatorial problems is encouraged by several authors,[24,34,35] it is unlikely that these kinds of meta-heuristic methods converge to a globally optimal

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solution in the case of large scale mixed integer programming problems. To the knowledge of the authors, the aforementioned decomposition techniques appear to be the most prominent solution approaches with respect to the solution time as well as the optimality of the solution.

For numerical solution, the cutting stock problem, being a mixed integer nonlinear programming (MINLP) problem, is transformed into a mixed integer linear programming (MILP) problem by using a decomposition technique introduced by Westerlund et al.;36 see also refs 5 and [6]. A set of feasible cutting patterns is first generated and inserted into the MINLP formulation as parameters. Thereafter, the remaining optimization problem, being a MILP problem, can be solved subject to the set of feasible cutting patterns. The set may be generated as a complete set or a subset of all feasible cutting patterns arising from the nonlinear problem formulation. The remaining linear problem can be then solved by using a standard MILP solver such as ILOG CPLEX[37] applied in this work. In addition to the paper industry, this technique has been successfully used in the solution of a cutting stock problem in the carpet weaving industry.[38] It should be noted that with a one-dimensional cutting stock problem, the enumeration of cutting patterns, resulting into inevitable increase of combinatorial complexity, can be avoided by using changeover constraints.[39,40]

The introduced cutting stock problem is connected to the steel cutting process taking place in the steel industry in general. In the stainless steel industry, the weight of a mother coil ranges typically from 15 to 30 tons, whereas the nominal weight of an order line varies from one to hundreds of tons. The widths of product coils range from a narrow strip to a full width mother coil. Due to a relatively high price of the refined stainless steel, the trim loss minimization of stainless steel is more important compared to, e.g., carbon steel.

The production planning system developed by Accenture handles and optimizes the production of customer orders. The system first generates instances of the cutting stock problem to a batch queue based on the order book. Individual instances with a highly variable number of orders are then solved in parallel. On a daily level, the number of problem instances generated and solved can extend to 500. The production planning system has been applied successfully in the industrial environment since it was implemented in 2001.

Practical experience from industrial use of the decomposition technique indicates that the approach is viable, as the method has been successfully utilized in a large scale industrial environment as a part of the production planning system developed for Outokumpu Stainless by Accenture. The system has provided substantial benefits for the client through automatization and streamlining of the planning process since the implementation.

This article is structured as follows. In the following section, the notation and mathematical formulation of the cutting stock problem are introduced. The model is demonstrated with numerical examples in the third section. The aspects and usability of the model are discussed in the fourth section. Finally, concluding remarks appear in the last section.

■ MATHEMATICAL FORMULATION

In this section, the cutting stock problem connected to the steel cutting process is presented. The introduced formulation extends the model developed for a Finnish paper-converting mill, Wisapak Oy, by Westerlund et al.[6,41] Compared to the above-mentioned formulation where the paper raw reels of a specific width can be considered as a continuous reel, the individual mother coils from which the product coils are cut out from must be treated as separate objects here. This creates a need for other extensions as well, namely, gathering of the trim loss at the end of the mother coils, preferably on a single mother coil only.

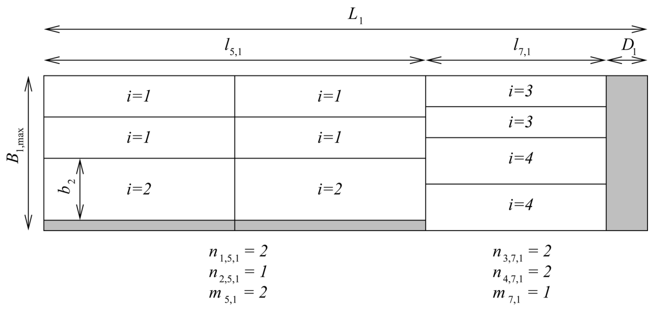

Figure 1. Interconnection of variables in a two-dimensional cutting stock problem.

The Problem Formulation. In general, different kinds of cutting stock problem instances can be generated. First, either existing or nonexisting stock can be selected. In the first option, the dimensions of the mother coils are inserted into the problem formulation as parameters. In the latter option, either widths and/or lengths of the mother coils are considered variables with predefined lower and upper bounds. It is also possible to allow some product coils to be cut out from mother coils of a particular width only. Moreover, product quantity limits can be given separately for each product. The details of the notation are summarized at the end of this article.

The relationship of a set of variables is illustrated in Figure 1 for a representative cutting layout. In Figure 1, a cutting pattern $j = 5$ is repeated two times and a cutting pattern $j = 7$ once in a mother coil $r = 1$. The first cutting pattern contains two pieces of product $i = 1$ and one piece of product $i = 2$. The second cutting pattern contains two pieces of both products $i = 3$ and $i = 4$. The widths of the product coils are given by parameters b_i. The shaded areas represent the trim loss.

The ordered or nominal amount $w_{i,\mathrm{nom}}$ of order i, given in kilograms, is specified to be within limits $w_{i,\min}$ and $w_{i,\max}$. The quantity produced over the nominal value is denoted by $w_{i,\mathrm{OP}}$. The product coils are cut out from R different types of mother coils. Each mother coil has a variable width denoted by $B_r \in [B_{r,\min}, B_{r,\max}]$. Likewise, the length of each mother coil is allowed to vary within the given limits $L_{r,\min}$ and $L_{r,\max}$, see Figure 1. The number of mother coils of width B_r is given by the integer variable M_r. The total length extending over the M_r multiples of the mother coil r and the total area of the mother coils of type r are defined by the variables L_r and A_r, respectively. The variable D_r denotes the total unused length of the mother coils of type r.

The lengths of the product coils must be within given tolerances $l_{jr,\min}$ and $l_{jr,\max}$. The pair of indices jr denotes the cutting pattern j in a mother coil r. Since each product i has respective limits for the product coil's length, the tolerances are given by the shortest and longest product coils in the cutting pattern. The total length of the cutting patterns in a mother coil of type r plus the length of the unused part of the mother coil must equal the total length of the mother coil.

Cutting patterns must reside in a single mother coil, that is, they cannot extend from one coil to another. This is different from the paper industry, where paper raw reels of similar width are attached together at full speed, which allows the cutting patterns to extend over the ends of the raw reels.[6]

A cutting pattern is defined by a parameter n_{ijr}, denoting the number of product coils of type i cut using the cutting pattern j from the mother coil of type r. The sum of the widths of the product coils at each cutting pattern must not exceed the width of the corresponding mother coil. Also, the total number of product coils that can be cut from the mother coil r is restricted to $N_{r,\max}$. The number of feasible cutting patterns for a mother coil r is denoted by index J_r. A binary variable y_{jr} indicates if the cutting pattern j in the mother coil r is active, whereas an integer variable m_{jr} defines the repetitions of the cutting pattern j in M_r multiples of mother coil r. The sum of continuous slack variables s_{jrk} that comprise a special ordered set (SOS) equals M_r when y_{jr} is active, and zero otherwise. This sum can hence be used in the calculation of the total cost of knife changes in M_r mother coils of type r. The total length of the repetitions of cutting pattern j in M_r multiples of mother coil of type r is given by a real variable l_{jr}.

The area of the product material within the ordered weight tolerances is defined by A_{O}, whereas the overproduction area is defined by A_{OP}. The total area of the trim loss is denoted by A_{TL}, whereas the width and the lengthwise trim losses are defined by A_{TLB} and A_{TLL}, respectively. Moreover, the area of the lengthwise trim loss in a mother coil r is denoted by $A_{\mathrm{TLL},r}$.

The area prices of the produced material, overproduced material, and raw material are given by the cost coefficients C, C', and C'', respectively. Since the price of the overproduced material is typically zero, the area of the overproduced area A_{OP} tends to be zero as well. In order to separate the width- and lengthwise trim losses, positive cost coefficient C_{TLL} must be given for the total lengthwise trim loss. The cost of knife change or change from one cutting pattern to another is given by c_{jr}. The cost of machine time is included in the objective function by multiplying the length of the used raw material by the hourly cost C_{m} for the cutting machine, divided by the machine speed v_{m}.

In order to maximize the usage of the preheating oven in the hot rolling mill, long mother coils should be preferred in the production. This can be done by introducing an additional cost term, where the value of the term decreases as the length of the mother coil increases. The cost term is defined as $\Delta_r = \alpha_r M_r - \beta_r L_r$, where $\alpha_r = \gamma C'' L_{r,\max} B_{r,\max} A_w$, $\gamma \in [0,1]$ and $\beta_r = \alpha_r / L_{r,\max}$. It is evident that Δ_r equals zero if $L_r = M_r L_{r,\max}$ and approaches α_r as L_r decreases. The cost of producing over the nominal quantity is given by the cost factor $C_{i,\mathrm{OP}}$ for each product.

Contiguity. Since mother coils can be produced at different times, it is important to try to cut out a specific order from a single mother coil. Otherwise, it is possible that part of the order is cut first and must be stored until the rest of the order is produced, which increases the inventory cost and delivery time.

The contiguity issue is resolved by defining different sets of cutting patterns for each mother coil type. The fundamental idea is to allow only certain products i to be cut out from a specific mother coil type r, and to maximize the number of mother coils that contain only a few different products. The approach minimizes the number of different products to be cut out from a single mother coil.

The set of allowed products i in a mother coil type r is defined as a proper subset of powerset $\mathscr{P}(A)$, where $A = \{1,...,I\}$ is an ordered set of all product material types. Since the number of elements in a powerset $\mathscr{P}(A)$, empty set excluded, is equal to $2^I - 1$ and grows exponentially as a function of the number of product material types I, only a limited number of elements of the powerset $\mathscr{P}(A)$ is used. For each mother coil width $s \in \{1,2,3\}$, the following subsets are defined for $k \in \{1,2,3,I\}$, if possible:

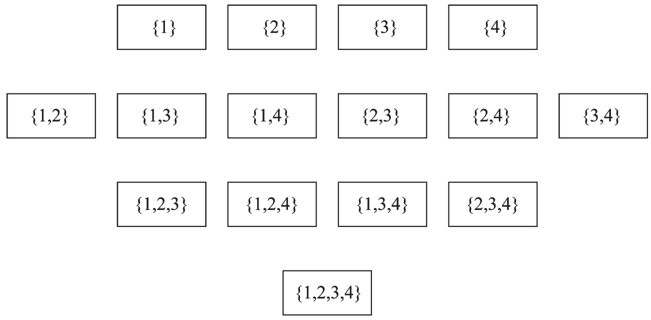

Figure 2. Elements of the subsets $V_{sk} \subseteq \mathscr{P}(A)$, $k \in \{1,2,3,4\}$ for $I = 4$.

$$V_{sk} = \{p | p \in \mathscr{P}(A) \text{ and } |p| = k\} \tag{1}$$

In other words, four different subsets containing elements of one, two, three, and I pieces are used. The maximum number of different mother coil types is equal to $|V_{s(1)}| + |V_{s(2)}| + |V_{s(3)}| + |V_{sI}| = I + \sum_{i=1}^{I-1} i + \sum_{i=1}^{I-2}\sum_{j=1}^{i} j + 1 = (I^3 + 5I + 6)/6$ which is significantly smaller than $2^I - 1$ when I is large. If any production constraint prevents an element of a powerset to be included in a subset V_{sk} so that only $k - 1$ product coils could be cut out from the mother coil, this element is not entered into the problem instance since the element is already in a subset $V_{s(k-1)}$.

In order to enable the use of different cutting layouts for mother coils of the same width, duplicates of the mother coils V_{sk} are defined for each width s. If the order book contains at most four orders, two duplicates for subsets $V_{s(1)}$ and $V_{s(2)}$ are generated. Otherwise, only one piece of mother coil of type r is used. For $V_{s(3)}$ and V_{sI}, more duplicates are generated by a heuristic described in the Appendix.

The cost coefficients C_{Sr} for the different subsets of mother coil types V_{sk} for each width s are defined as $C_{Sr} = kc$, where k is the number of different orders in each element of V_{sk}, and c is an invariable cost coefficient. The subindex S stands for the scattering level. The total cost is then given by $\sum_{r=1}^{R} C_{Sr} M_r$.

As an example, consider the order list of four different product material types i. The ordered set $A = \{1,2,3,4\}$ and the proper subsets $V_{sk} \subseteq \mathscr{P}(A)$ for levels $k \in \{1,2,3,4\}$ are illustrated in Figure 2. In the top row, the elements of the subset $V_{s(1)}$ are presented. Sets of cutting patterns are generated for four different mother coils $r \in \{1,2,3,4\}$. In the mother coil $r = 1$, only product material type $i = 1$ is allowed to be inserted into the cutting patterns. Again, in the mother coil $r = 2$, only type $i = 2$ is allowed and so forth. In the second row, the subset $V_{s(2)}$ comprising six elements of two product material types in each element is presented. For the mother coils $r \in \{5,6,...,10\}$, the cutting patterns are allowed to contain two different product material types in the maximum. Similarly, the cutting patterns generated for a subset $V_{s(3)}$, containing mother coil types $r \in \{11,12,13,14\}$, are allowed to contain three different product material types in the maximum. Finally, for $V_{s(4)}$ that defines the mother coil type $r = 15$, all the cutting patterns not included in the other subsets are generated.

Objective Function. The objective is to maximize the profit of the production. Terms increasing the profit include the income due the order fulfillment as well as the potential income from the overproduction. Since lengthwise trim loss is preferred to widthwise trim loss, the total lengthwise trim loss is maximized as well. On the other hand, raw material costs, short mother coils, machine costs, specific orders cut out from several mother coils, knife changes, and the quantities produced over the nominal quantities decrease the profit gained. The objective function is hence given as

$$\max_{A_O, A_{OP}, A_{TL}, A_r, L_r, l_{jr}, s_{jrk}, w_i, w_{i,OP}, y_{jr}, M_r, m_{jr}} \Bigg\{ + C' A_{OP} + C_{TLL} A_{TLL} - \sum_{r=1}^{R} \Bigg\{ C'' A_r + \alpha_r M_r + \left(\frac{C_m}{v_m} - \beta_r\right) L_r + C_{Sr} M_r + \sum_{j=1}^{J_r} \Bigg\{ \sum_{k=1}^{K_{jr}} s_{jrk} + c_{jr}' m_{jr} \Bigg\} \Bigg\} - \sum_{i=1}^{I} C_{i,OP} w_{i,OP} \Bigg\} \quad (2)$$

Material Balance Constraints. We next define the material balance constraints. The area of all raw material must be equal to the sum of the product material area within the weight tolerances, the area of overproduction, and the area of trim loss:

$$\sum_{r=1}^{R} A_r - A_{OP} - A_{TL} - A_O = 0 \quad (3)$$

The produced product material area is equal to the sum of the product material area within the weight tolerances and the area of overproduction:

$$\sum_{r=1}^{R} \sum_{j=1}^{J_r} l_{jr} \sum_{i=1}^{I} b_i n_{ijr} - A_{OP} - A_O = 0 \quad (4)$$

To reduce the solution space, the following material balance constraint needs to be introduced as well. The sum of the overproduction and trim loss areas are defined to be less than or equal to the fraction f of the product material area within the weight limits:

$$A_{OP} + A_{TL} - f A_O \leq 0 \quad (5)$$

The lower limit of the area of the mother coil is given by the following constraint that takes into account also inactive cutting patterns, that is, the cutting patterns with $y_{jr} = 0$:

$$\sum_{i=1}^{I} b_i n_{ijr} - A_r + A_{O,\max} y_{jr} \leq A_{O,\max} \quad (6)$$

where $A_{O,\max}$ is the maximum raw material area within the given weight limits given by

$$A_{O,\max} = \frac{\sum_{i=1}^{I} w_{i,\max}}{A_w} \quad (7)$$

According to eq 6, the area of the raw material is constrained by the maximum width of the active cutting patterns only. This means that the width of the raw material can be optimized also; the optimal width of the mother coil B_r can be computed afterward by dividing the area of the mother coil A_r by the length of the mother coil L_r.

If the widths of the mother coils are specified beforehand by $B_{r,\max}$, eq 6 needs to be replaced with the following one that defines the relation between the area and the length of the raw material:

$$A_r - B_{r,\max} L_r = 0 \quad (8)$$

The relationship between the total length of multiples of cutting pattern j in a mother coil r, the length of the unused part of the mother coil r, and the length of the mother coil r is given by

$$\sum_{j=1}^{J_r} l_{jr} + D_r - L_r = 0 \quad (9)$$

If the width of the mother coil is not fixed, the lower limit of the lengthwise trim loss $A_{TLL,r}$ in a mother coil r is given by

$$D_r \sum_{i=1}^{I} b_i n_{ijr} - A_{TLL,r} + A_{O,\max} y_{jr} \leq A_{O,\max} \quad (10)$$

Otherwise, the lengthwise trim loss in a mother coil r is given by

$$A_{TLL,r} - B_{r,\max} D_r = 0 \quad (11)$$

The total lengthwise trim loss A_{TLL} is obtained by aggregating the lengthwise trim losses in each mother coil type as

$$A_{TLL} - \sum_{r=1}^{R} A_{TLL,r} = 0 \quad (12)$$

The total widthwise trim loss is then given by

$$A_{TL} - A_{TLL} - A_{TLB} = 0 \quad (13)$$

Operational Constraints. The binary variable y_{jr} that indicates if the respective cutting pattern j is active in the mother coil r is defined to equal one if $m_{jr} > 0$. For inactive patterns the variable equals zero. This is given by

$$m_{jr} - H_{jr} y_{jr} \leq 0 \quad (14)$$

where H_{jr} are constants large enough given by eq 29.

Production Constraints. The relationships between the total length of the cutting pattern j and the multiples of the cutting pattern in a mother coil r, giving the length of the product coil, are given by

$$l_{jr,\min} m_{jr} - l_{jr} \leq 0 \quad (15)$$

and

$$l_{jr} - l_{jr,\max} m_{jr} \leq 0 \quad (16)$$

Likewise, the relationships between the total length of the mother coil r and the number of mother coils of type r used are given by

$$L_{r,\min} M_r - L_r \leq 0 \quad (17)$$

and

$$L_r - L_{r,\max} M_r \leq 0 \quad (18)$$

The relationship between the ordered and overproduced quantities and the lengths of the product coils is given by

$$\frac{1}{A_w b_i} w_i + \frac{1}{A_w b_i} w_{OP,i} - \sum_{r=1}^{R} \sum_{j=1}^{J_r} l_{jr} n_{ijr} = 0 \quad (19)$$

where A_w is the area weight of the steel and $w_{OP,i}$ is the weight of the overproduced coils for an order i. The relationship between the produced quantity within the weight limits and the total production area is given by

$$A_{\mathrm{O}} - \sum_{i=1}^{I} \frac{1}{A_{\mathrm{w}}} w_i = 0 \tag{20}$$

Finally, the excess produced over the nominal quantity is given by

$$w_i - w_{i,\,\mathrm{nom}} \le w_{i,\,\mathrm{OP}} \tag{21}$$

Additional Constraints. Since the cutting patterns cannot extend from one mother coil to another, only a limited number of cutting patterns can reside on a single mother coil. If a cutting pattern is repeated m'_{jr} times in a single mother coil of type r and M_r pieces of mother coils of this type are used, then the cutting pattern is repeated $m_{jr} = m'_{jr}M_r$ times in total. This inherently nonlinear relationship is expressed in a linear form with the following special ordered set (SOS) representation

$$-m_{jr} + \sum_{k=1}^{K_{jr}} ks_{jrk} = 0 \tag{22}$$

$$\sum_{k=1}^{K_{jr}} b_{jrk} \le 1 \tag{23}$$

where K_{jr} is the maximum allowed number of repetitions of the cutting pattern j in the mother coil r defined as

$$K_{jr} = \min\left\{\frac{L_{r,\,\max}}{l_{jr,\,\max}}, \max_i \frac{w_{i,\,\max}}{b_i n_{ijr} l_{jr,\,\max} A_{\mathrm{w}}}\right\} \tag{24}$$

Rounding up the leftmost ratio in eq 24 guarantees that the solution space contains the global optimum. Above, the continuous slack variables s_{jrk} are defined as follows. The value of the variable equals 0 if the corresponding binary variable b_{jrk} is inactive, and otherwise, it equals M_r. This is achieved with the set of following constraints:

$$\begin{gathered} M_r + M_{r,\,\max} b_{jrk} - s_{jrk} \le M_{r,\,\max} \\ s_{jrk} - M_r \le 0 \\ s_{jrk} - M_{r,\,\max} b_{jrk} \le 0 \end{gathered} \tag{25}$$

where $M_{r,\max}$ is given by

$$M_{r,\,\max} = \left\lceil \frac{\sum_{i \in F_r} w_{i,\,\max}/A_{\mathrm{w}}}{L_{r,\,\mathrm{usable}} B_{r,\,\mathrm{usable}}} \right\rceil \tag{26}$$

Above, $F_r \subseteq V_{sr}$ is the set of products allowed to be cut out from the mother coil r, $w_{i,\max}$ is the upper bound on the quantity of the product i to be cut, and $B_{r,\mathrm{usable}}$ and $L_{r,\mathrm{usable}}$ are the maximum usable width and length of the mother coil r, respectively. They are given by

$$B_{r,\,\mathrm{useable}} = \max_i \left\{ b_i \left\lfloor \frac{B_{r,\,\max}}{b_i} \right\rfloor \right\} \tag{27}$$

$$L_{r,\,\mathrm{useable}} = \max_i \left\{ l_{jr,\,\min} \left\lfloor \frac{L_{r,\,\max}}{l_{jr,\,\min}} \right\rfloor \right\} \tag{28}$$

M_r equals $M_{r,\max}$ only if one mother coil type is used for cutting the orders listed in F_r, which is unlikely especially if the number of products in F_r is large. Upper bounds of the maximum allowable number of repetitions of the cutting pattern j can be computed by dividing the maximum length of the mother coil r by the mean of the length of the cutting pattern j, and multiplying the ratio by the maximum number of repeats for a mother coil type r:

$$H_{jr} = K_{jr} M_{r,\,\max} \tag{29}$$

The number of slack variables and constraints can be reduced by noting that if the maximum number of multiples $M_{r,\max}$ for the mother coil r is equal to one, eqs 22 and 23 are unnecessary for that mother coil and should not be introduced. Also, if $M_{r,\max}$ is two for some mother coil $F_r \subseteq V_{sk}$ and two duplicates are defined, $M_{r,\max}$ can be set to one for both duplicates. Finally, if $M_{r,\max}$ is more than two for some mother coil, $M_{r,\max}$ can be set to one for the duplicates of the mother coil.

The solution space can be further reduced by constraining the sum of the cutting pattern multiples m_{jr} by

$$\sum_{j=1}^{J_r} l_{jr} - \sum_{j=1}^{J_r} l_{jr,\,\max} m_{jr} \le 0 \tag{30}$$

and

$$\sum_{j=1}^{J_r} l_{jr,\,\min} m_{jr} - L_r \le 0 \tag{31}$$

If the optimization is interrupted, for example, because of the oversized order list, it is possible that mother coils with no active cutting patterns appear in the solution. This is avoided by disallowing positive total length for a mother coil with inactive cutting patterns:

$$L_r - M_{r,\,\max} L_{r,\,\max} \sum_{j=1}^{J_r} y_{jr} \le 0 \tag{32}$$

Bounds. We next define bounds for a set of variables. First, usage of raw material can be restricted with

$$A_{r,\,\min} \le A_r \le A_{r,\,\max} \tag{33}$$

Second, the product weight must satisfy the given weight tolerances, which is accomplished by

$$w_{i,\,\min} \le w_i \le w_{i,\,\max} \tag{34}$$

Note that, despite the above limits, the weight of the total produced material may exceed the upper limit $w_{i,\max}$ due to the overproduction. Finally, the numbers of cutting pattern multiples and mother coils can be limited with

$$\begin{gathered} 0 \le m_{jr} \le H_{jr} \\ 0 \le M_r \le M_{r,\,\max} \end{gathered} \tag{35}$$

The variable space for the problem eqs 2−eq 35, hereafter referred to as P, is defined by

$$m_{jr}, M_r \in \mathbf{Z}^+$$

$$y_{jr}, b_{jrk} \in \{0, 1\}$$

$$A_{\mathrm{O}}, A_{\mathrm{OP}}, A_{\mathrm{TL}}, A_{\mathrm{TLB}}, A_{\mathrm{TLL}}, A_{\mathrm{TLL},\,r}, A_r, L_r, D_r, w_i, w_{i,\,\mathrm{OP}}, l_{jr}, s_{jrk} \in \mathbf{R}^+$$

$$i \in \{1, 2, ..., I\},\ j \in \{1, 2, ..., J_r\},\ k \in \{1, 2, ..., K_{jr}\},\ r \in \{1, 2, ..., R\}$$

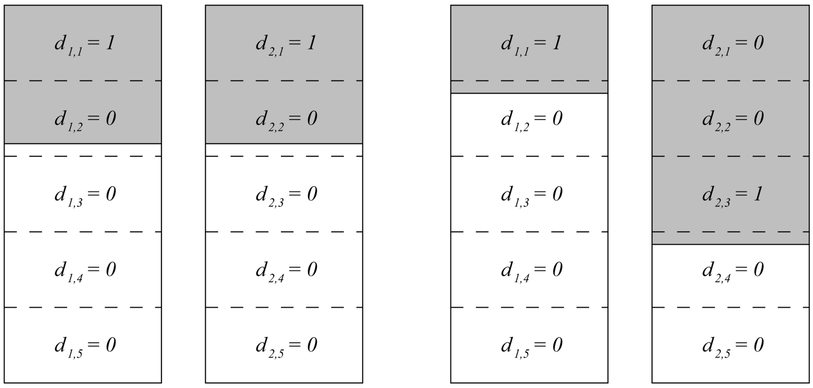

Figure 3. The relationship between indicator variables drk and $A_{\mathrm{TLL},r}$ for $K = 5$.

Lengthwise Trim Loss. We next introduce extensions to the model that consider the accumulation of lengthwise trim loss into a single mother coil. In the first approach, the number of mother coils with positive lengthwise trim loss is minimized. In the second approach, a quadratic profit term that favors accumulation of lengthwise trim loss into a single mother coil is included into the objective.

Method I. In this approach, a binary variable T_r is assigned to each mother coil r, where the value of the variable equals one if the mother coil contains lengthwise trim loss and zero otherwise. This is achieved by introducing the following constraint:

$$A_{\mathrm{TLL},r} \leq A_{r,\max} T_r \qquad (36)$$

where $A_{r,\max}$ is the upper bound on the lengthwise trim loss in a mother coil r defined by eq 43. The following cost is also included into eq 2 as a negative term:

$$C'_{\mathrm{TLL}} \sum_{r=1}^{R} T_r \qquad (37)$$

where C'_{TLL} controls the weight of the term in the objective function.

It should be noted that the introduced approach works if the mother coils can be cut to their full lengths. If this is not possible, for example, because of the length limitations of the cutting patterns, the approach cannot be used. We next present an alternative approach suitable for these kinds of cases.

Method II. Since the mother coil of type r can be repeated M_r times, the amount of lengthwise trim loss in a single mother coil r equals $A_{\mathrm{TLL},r}/M_r$. The lengthwise trim loss can be accumulated into a single mother coil by preferring larger lengthwise trim loss areas over the smaller ones. This is achieved by maximizing the sum of quadratic lengthwise trim loss areas across different mother coils. As an example, consider the case where the total lengthwise trim loss A_{TLL} is divided on two mother coils such that $A_{\mathrm{TLL},r} = A_{\mathrm{TLL},s}$, $r \neq s$. Now, the maximization of the sum of quadratic lengthwise trim loss areas prefers $A_{\mathrm{TLL},r} > A_{\mathrm{TLL},s}$, since the sum $A^2_{\mathrm{TLL},r} + A^2_{\mathrm{TLL},s}$ is larger in the latter case. Thus, the following term needs to be included in eq 2:

$$P = C'_{\mathrm{TLL}} \sum_{r=1}^{R} \left(\frac{A_{\mathrm{TLL},r}}{M_r} \right)^2 \qquad (38)$$

where the coefficient C'_{TLL} controls the weight of the term in the objective.

Unfortunately, eq 38 is nonlinear and cannot be included in the MILP formulation as such. By dividing the mother coil area into K fractions of the same size and representing the lengthwise trim loss area in a single mother coil with binary variables d_{rk}, eq 38 can be represented in a linear form. If the lengthwise trim loss in the mother coil r exceeds the sum of k fractional areas multiplied by the multiples M_r of the mother coil type r, binary variables $d_{r,1}$, ..., $d_{r,K-1}$ are defined to equal zero and binary variable $d_{r,K}$ is defined to equal one. Consequently, the variables d_{rk}, $k = 1, ..., K$ can be defined as an SOS and eq 38 can be replaced with

$$P = C'_{\mathrm{TLL}} \sum_{r=1}^{R} \sum_{k=1}^{K} k^2 d_{rk} \qquad (39)$$

The binary variables d_{rk} constitute an ordered set with increasing weights. For being an SOS, only one variable of the set can equal one while others must equal zero:

$$\sum_{k=1}^{K} d_{rk} \leq 1 \qquad (40)$$

The interconnection of the lengthwise trim loss, fractions, and the binary variables d_{rk} is illustrated in Figure 3. The shaded area represents the lengthwise trim loss. In the two leftmost mother coils, the lengthwise trim loss is divided equally between the mother coils, and the first indicator variables of both coils $d_{1,1}$ and $d_{2,1}$ are active. On the other hand, on the two rightmost mother coils, the same cutting patterns are positioned so that the lengthwise trim loss is larger on the right-hand-side mother coil. Hence, all the indicator variables excluding $d_{1,1}$ equal zero for the left-hand-side mother coil, and variable $d_{2,3}$ equals one. When maximizing the sum of the weighted binary variables d_{rk}, the rightmost solution candidate is selected since $1 + 3^2 > 1 + 1$.

The indicator variable d_{rk} corresponding to kth fractional area must be equal to one if the lengthwise trim loss area is large enough to reach the kth fraction. On the other hand, the variable d_{rk} must be zero, if the lengthwise trim loss area is not large enough to reach the kth fraction. This is accomplished with

$$A_{\mathrm{TLL},r} - skM_r + A_{r,\max} \geq (A_{r,\max} + \varepsilon) d_{rk} \qquad (41)$$

where the fractional area s is calculated as

$$s = \max_r \left\{ \frac{B_{r,\max} L_{r,\max}}{K} \right\} \qquad (42)$$

and the maximum area of each mother coil type is defined as

$$A_{r,\max} = \max_r \{ B_{r,\max} L_{r,\max} \} M_{r,\max} \qquad (43)$$

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In eq 41, the purpose of the additive term $A_{r,\max}$ is to keep the left-hand-side of the constraint positive for all $k \in \{1,...,K\}$ and to guarantee the feasibility of the constraint. As can be seen, the variable d_{rk} is allowed to equal one if the difference $A_{\mathrm{TLL},r} - skM_r$ is greater than or equal to a small constant ε. This means that the lengthwise trim loss must exceed the sum of k fractional areas sk for M_r mother coil multiples. Since only one indicator variable d_{rk} is allowed to equal one, the variable with the largest index k is chosen since this will maximize the profit $k^2 d_{rk}$. The purpose of the small constant ε is to prevent the variables d_{rk} to be active if $M_r = 0$.

The coefficient C'_{TLL} should be large enough to exceed some other cost terms in the objective to produce the desired solution. As an example, consider a case where the optimal solution without eq 39 in the objective function consists of multiple similar mother coils with the lengthwise trim loss divided evenly between all the mother coils. If the lengthwise trim loss should be gathered into a single mother coil, C'_{TLL} needs to be large enough to exceed the cost of changing to a new mother coil type C_{Sr} and the cost of knife change c_{jr} for at least one cutting pattern. On the other hand, the coefficient must be small enough to prevent any unnecessary mother coil types from being included in the solution.

Solution Method. The problem P is transformed to a MILP problem by using the pattern enumeration technique introduced by Westerlund et al.[36] In the production planning system, the resulting MILP problem is then solved by using a commercial MILP solver CPLEX .[37]

Instead of generating only cutting patterns that satisfy the minimum width requirement $B_{\max} - \Delta_{\max}$, some narrower cutting patterns must be included in the problem instance as well. The reason is that, for some products, the ordered quantities may be too small to satisfy even the minimum width $B_{\max} - \Delta_{\max}$ of a cutting pattern. The resulting problem could hence be infeasible, if all the cutting patterns narrower than the minimum width were left out.

In the production planning system, typically three different mother coil widths are applied. For the narrowest mother coils, the lower bound on the cutting pattern widths is set to zero. For the second widest and widest mother coils, only cutting patterns whose widths exceed $B_{\max} - \Delta_{\max}$ are generated. Since only a limited set of cutting patterns can be used in a specific mother coil type, several copies of the same mother coil type, i.e., duplicates, must be inserted into an instance of P. Otherwise, it would be possible that only suboptimal solutions were obtained. In practice, this means that the same set of cutting patterns must be assigned for each duplicate. Note that, in each duplicate, different cutting patterns can be chosen. Note also that choosing only one duplicate for a mother coil type means that only the mother coil type itself will be used.

NUMERICAL EXAMPLES

In this section, three numerical examples for the cutting stock problem problem P are presented. From generic industry point of view, problem instances fall to a continuum with two ends. At one end, the total nominal weight of the orders can extend hundreds or thousands of tons, whereupon the number of applied mother coils may extend to hundreds of coils. However, the widths of the orders are typically relatively large, which reduces combinatorial complexity of these kinds of problems. The first numerical example relates to this end of the continuum.

Table 1. Order Book in Example 1

i	b_i [mm]	$w_{i,\mathrm{nom}}$ [kg]	$l_{i,\max}$ [m]	$l_{i,\max}$ [m]
1	900	200000	5.000	2230.483
2	600	60000	5.000	428.748
3	100	20000	5.000	2231.786

Table 2. Optimal Values of Variables in Example 1

variable	value	
A_{O}	24149.567 m^2	
M_7	3	
L_7	3878.271 m	
A_7	6340.976 m^2	(B_7 = 1635 mm)
$A_{\mathrm{TLB},7}$	523.567 m^2	
M_{13}	3	
L_{13}	3878.272 m	
A_{13}	6340.975 m^2	(B_{13} = 1635 mm)
$A_{\mathrm{TLB},13}$	135.740 m^2	
M_{27}	1	
L_{27}	1486.227 m	
A_{27}	1939.526 m^2	(B_{27} = 1305 mm)
$A_{\mathrm{TLB},27}$	7.200 m^2	
$A_{\mathrm{TLL},62}$	60.368 m^2	
M_{29}	1	
L_{29}	1486.227 m	
A_{29}	1567.969 m^2	(B_{29} = 1055 mm)
$A_{\mathrm{TLB},29}$	230.365 m^2	
M_{37}	6	
L_{37}	8917.360 m	
A_{37}	9407.814 m^2	(B_{37} = 1055 mm)
$A_{\mathrm{TLB},37}$	490.455 m^2	

At the other end, the total nominal weight of the orders can be only a couple of tens of tons, but the widths of the orders are small and heterogeneous. Consequently, the combinatorial complexity in these kinds of problems is considerably high. The second and third examples relate to this end of the continuum.

The parameter values in the industrial use are such that the different features of the model, for example contiguity and minimization of lengthwise trim loss, are reasonably taken into account in the optimal solution. If a parameter would be set to its limit, it would mean that a particular feature would be completely disabled or that the feature would have utmost strong impact to the solution. Combinatorial complexity of the problem is also reduced at the limit, which means faster convergence to a solution. The sensitivity of the solution with respect to the parameter values depends on the instance in question. However, based on practical experience from industrial use, the optimal solutions have not been found highly sensitive to small adjustments of the parameters from their default values.

The examples were solved using a workstation with AMD Athlon 64 Dual Core Processor 4800+ 2.42 Ghz and 2 GB RDRAM memory capacity. The MILP problems were solved using CPLEX 10.2. The integer gap of the acceptable solution, computed by dividing the absolute value of difference between the best bound and the current solution by the absolute value of the best bound, is one percent. The order book and parameters are chosen by the authors and are only demonstrative.

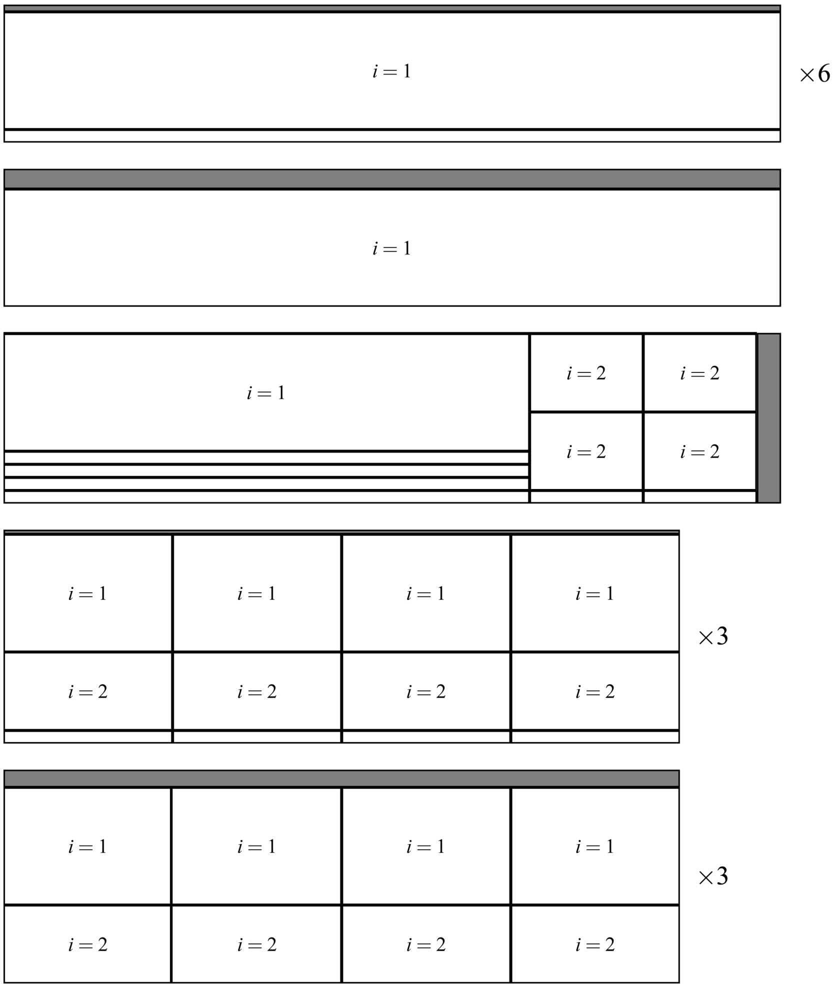

Figure 4. Graphical illustration of the optimal solution in example 1. Mother coils correspond to $r = 7$, $r = 13$, $r = 27$, $r = 29$, and $r = 37$ from bottom to top. Trim loss is represented as gray area, and it constitutes 5.66% of the raw material.

Table 3. Cutting Patterns in the Optimal Solution in Example 1

j	r	$n_{1,jr}$	$n_{2,jr}$	$n_{3,jr}$	width [mm]	l_{jr} [m]	m_{jr}
1	7	1	1	0	1500	3878.272	12
4	13	1	1	1	1600	3878.272	12
1	27	0	2	1	1300	434.428	2
4	27	1	0	4	1300	1005.541	1
1	29	1	0	0	900	1486.227	1
1	37	1	0	1	1000	8917.360	6

Table 4. Amount of Product Coils in the Optimal Solution in Example 1

b_i [mm]	w_i [kg]	no. and lengths of product coils
900	201783	(2 × 1486.2 m + 1 × 1005.5 m + 8 × 323.2 m)
600	60541	(4 × 217.2 m + 8 × 323.2 m)
100	20182	(1 × 1486.2 m + 4 × 1005.5 m + 2 × 217.2 m + 4 × 323.2 m)

The area prices used in the examples are $C = 7.85$ EUR/m^2, $C' = 0$ EUR/m^2, and $C'' = 3.925$ EUR/m^2. The cost of lengthwise trim loss is $C_{\mathrm{TLL}} = 0.785$ EUR/m^2. The cost of using the machine is $C_{\mathrm{m}} = 500$ EUR/h, and the machine speed is $v_{\mathrm{m}} = 25$ m/min. The costs of knife change and using a cutting pattern are $c_{jr} = 500$ EUR and $c'_{jr} = 50$ EUR, respectively. The maximum number of product coils allowed to be cut out from a mother coil is $N_{r,\max} = 30$. As for the contiguity, the invariable cost coefficient is set to $c = 2000$. The weight parameter for the lengthwise trim loss is set to $C'_{\mathrm{TLL}} = 1$ with method I and $C'_{\mathrm{TLL}} = 10$ with method II. Finally, the cost term related to the preheating oven is set to $\alpha_r = 0.5$.

Example 1. In the first example, the order book comprises three different orders and the total weight of the orders is 111 tons. Only one of the orders is narrow, whereupon the number of feasible cutting patterns remains modest. Consequently, the combinatorial complexity of the problem is low.

The widths of the mother coils are $B_{r,\max} = 1055$ mm for the width group $s = 1$, $B_{r,\max} = 1305$ mm for the width group $s = 2$ and $B_{r,\max} = 1635$ mm for the width group $s = 3$. Since the volume of the slab from which the mother coil is rolled and the volume of the mother coil must be same, the maximum length of the mother coil $L_{r,\max}$ can be calculated from the maximum volume of the slab. The order book is shown in Table 1. The weight tolerances of the ordered quantities are one percent for each product.

After the MIP presolve, the reduced MIP has 1626 rows, 1211 columns, and 5616 nonzeros. The optimal solution with integer gap of 1.00% is found at node 41000, where the tree size is approximately 5.37 MB. The number of Clique, Cover, Implied bound, Flow, and Gomory fractional cuts applied are 11, 1, 71, 4, and 5, respectively. Total solution time of the problem including root node processing and Branch and Cut is 8.55 s.

The values of the variables and mother coil type specific trim losses are presented in Table 2. The optimal solution is illustrated also graphically in Figure 4. Table 2 and Figure 4 reveal that lengthwise trim loss appears, but it has been accumulated into a single mother coil. All three different mother coil types of different widths appear in the optimal solution. The total trim loss area equals 1447.69 m^2, which corresponds to 5.66% of the total area of the raw material.

The active cutting patterns in the optimal solution are presented in Table 3. Orders $i = 2$ and $i = 3$ are cut out from three different mother coil types, whereas order $i = 1$ is cut out from five different mother coil types. Note that mother coils of types $r = 7$ and $r = 13$ are used three times, whereas $r = 37$ is used six times.

The optimal weights and lengths of the product coils are given in Table 4. Note that although the minimum length of the order is set to 5 m, the shortest product coil equals to 217.2 m in the optimal solution.

Example 2. In the second example, the order book comprises six different orders with total weight of 60.5 tons. The widths of the orders are relatively large in relation to the widths of the mother coils, whereupon the number of feasible cutting patterns is not excessively high.

The widths of the mother coils are $B_{r,\max} = 955$ mm for the width group $s = 1$, $B_{r,\max} = 1083$ mm for the width group $s = 2$ and $B_{r,\max} = 1640$ mm for the width group $s = 3$. The order book is shown in Table 5. The weight tolerances of the ordered quantities are one percent for each product.

After the MIP presolve, the reduced MIP has 4630 rows, 3852 columns, and 16213 nonzeros. The optimal solution with integer gap of 1.01% is found at node 33300, where the tree size is approximately 14 MB. The number of Clique, Cover, Implied bound, Flow, and Gomory fractional cuts applied are 51, 41, 74, 76, and 1, respectively. Total solution time of the problem including root node processing and Branch and Cut is 186.98 s.

The values of the variables and mother coil type specific trim losses are presented in Table 6. The optimal solution is illustrated also graphically in Figure 5. It can be seen that only widthwise trim loss occurs. All three different mother coil types of different widths appear in the optimal solution. The total trim loss area equals 120.93 m^2, which corresponds to 2.12% of the total area of the raw material.

The active cutting patterns in the optimal solution are presented in Table 7. Orders $i = 1$, $i = 4$ and $i = 5$ are cut out from a single mother coil only, whereas the rest of the orders are cut out from two different mother coil types.

Table 5. Order Book in Example 2

i	b_i [mm]	$w_{i,\text{nom}}$ [kg]	$l_{i,\max}$ [m]	$l_{i,\max}$ [m]
1	950	16000	5.000	2716.550
2	810	10500	5.000	873.376
3	800	22000	5.000	466.264
4	745	3000	5.000	552.471
5	730	4000	5.000	871.123
6	136	5000	5.000	189.154

Table 6. Optimal Values of Variables in Example 2

variable	value	
A_O	5589.323 m^2	
M_{10}	1	
L_{10}	1642.530 m	
A_{10}	1568.616 m^2	(B_{10} = 955 mm)
$A_{\text{TLB},10}$	20.184 m^2	
M_{105}	1	
L_{105}	1440.720 m	
A_{105}	2362.781 m^2	(B_{105} = 1640 mm)
$A_{\text{TLB},10}$	82.681 m^2	
M_{191}	1	
L_{191}	1642.530 m	
A_{191}	1778.860 m^2	(B_{191} = 1083 mm)
$A_{\text{TLB},10}$	18.068 m^2	

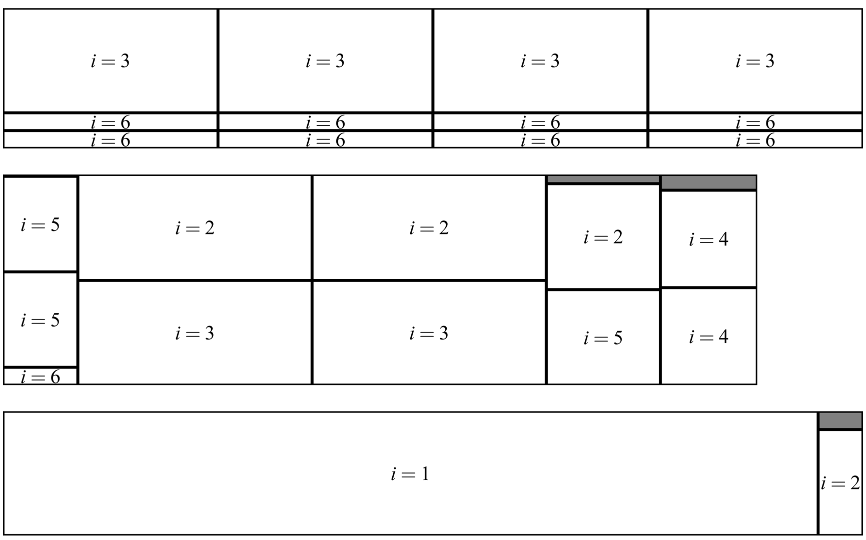

Figure 5. Graphical illustration of the optimal solution in example 2. Mother coils correspond to $r = 10$, $r = 105$, and $r = 191$ from bottom to top. Trim loss is represented as gray area, and it constitutes 2.12% of the raw material.

Table 7. Cutting Patterns in the Optimal Solution in Example 2

j	r	$n_{1,jr}$	$n_{2,jr}$	$n_{3,jr}$	$n_{4,jr}$	$n_{5,jr}$	$n_{6,jr}$	width [mm]	l_{jr} [m]	m_{jr}
1	10	1	0	0	0	0	0	950	1557.020	1
2	10	0	1	0	0	0	0	810	85.509	1
7	105	0	1	1	0	0	0	1610	894.833	2
9	105	0	0	0	0	2	1	1596	142.974	1
14	105	0	1	0	0	1	0	1540	217.830	1
17	105	0	0	0	2	0	0	1490	185.082	1
1	191	0	0	1	0	0	2	1072	1642.530	4

Table 8. Amount of Product Coils in the Optimal Solution in Example 2

b_i [mm]	w_i [kg]	no. and lengths of product coils
950	16000	(1 × 1557.0 m)
810	10498	(1 × 85.5 m + 2 × 447.5 m + 1 × 217.8 m)
800	21957	(2 × 447.5 m + 4 × 410.6 m)
745	2983	(2 × 185.1 m)
730	3978	(2 × 143.0 m + 1 × 217.8 m)
136	5043	(1 × 143.0 m + 8 × 410.6 m)

Table 9. Order Book in Example 3

i	b_i [mm]	$w_{i,\mathrm{nom}}$ [kg]	$l_{i,\max}$ [m]	$l_{i,\max}$ [m]
1	346	33200	5.000	2230.483
2	300	10200	5.000	428.748
3	236	22600	5.000	2231.786
4	157	30000	5.000	983.117
5	136	15000	5.000	1891.537

The optimal weights and lengths of the product coils are given in Table 8. Note that although the minimum length of the order is set to 5 m, the shortest product coil equals 85.5 m in the optimal solution.

Example 3. In the third example, the order book comprises five different orders, but the total weight of orders is now 111 tons. Contrary to the first and second examples, the widths of the orders are relatively narrow compared to the widths of the mother coils. Consequently, the number of feasible cutting patterns is large and the combinatorial complexity of the problem is high.

The widths of the mother coils are $B_{r,\max}$ = 1055 mm for the width group s = 1, $B_{r,\max}$ = 1305 mm for the width group s = 2 and $B_{r,\max}$ = 1635 mm for the width group s = 3. The order book is shown in Table 9. The weight tolerances of the ordered quantities are one percent for each product.

After the MIP presolve, the reduced MIP has 24682 rows, 17347 columns, and 87737 nonzeros. The optimal solution with integer gap of 1.29% is found at node 137000, where the tree size is approximately 366.75 MB. The number of Clique, Cover, Implied bound, Flow, and Gomory fractional cuts applied are 62, 32, 328, 22, and 2, respectively. Optimization was stopped after reaching the maximum time limit of 600 s.

The values of the variables and mother coil type specific trim losses are presented in Table 10. The optimal solution is illustrated also graphically in Figure 6. Table 10 and Figure 6 reveal that lengthwise trim loss appears, but it has been accumulated into a single mother coil. All three different mother coil types of different widths appear in the optimal solution. The total trim loss area equals 119.06 m^2, which corresponds to 1.23% of the total area of the raw material.

Table 10. Optimal Values of Variables in Example 3

variable	value	
A_O	9555.280 m^2	
M_{14}	1	
L_{14}	1292.757 m	
A_{14}	2113.658 m^2	(B_{14} = 1635 mm)
$A_{TLB,14}$	1.293 m^2	
M_{27}	1	
L_{27}	1292.757 m	
A_{27}	2113.658 m^2	(B_{26} = 1635 mm)
$A_{TLB,27}$	10.751 m^2	
M_{62}	1	
L_{62}	1486.227 m	
A_{62}	2113.658 m^2	(B_{62} = 1305 mm)
$A_{TLB,62}$	19.066 m^2	
$A_{TLL,62}$	25.603 m^2	
M_{65}	1	
L_{65}	1486.227 m	
A_{65}	1939.526 m^2	(B_{65} = 1305 mm)
$A_{TLB,65}$	28.826 m^2	
M_{115}	1	
L_{115}	1486.227 m	
A_{115}	1567.969 m^2	(B_{115} = 1055 mm)
$A_{TLB,115}$	35.518 m^2	

The active cutting patterns in the optimal solution are presented in Table 11. Orders i = 2 and i = 5 are cut out from a single mother coil only, whereas orders i = 3 and i = 4 are cut out from two and order i = 1 is cut out from four different mother coil types.

The optimal weights and lengths of the product coils are given in Table 12. The shortest product coil equals to 38.3 m in the optimal solution.

DISCUSSION

In general, the cutting stock problem presented in this paper is more complicated than its counterpart in the paper industry. The most significant difference is the inability to weld mother coils of a given width together during the cutting, which means that each mother coil must be treated as a unique object. Second, in the paper industry, only one to three different raw reel widths are typically used, whereas here, the number of different mother coil types may extend to hundreds, especially when the contiguity is considered. Third, mainly wide cutting patterns that produce small amount of widthwise trim loss are usually applied in the paper industry. Here, the order book contains typically also narrow strips, which increases tremendously the number of feasible yet relatively narrow cutting patterns. This in turn increases computational complexity of the problem, which ultimately decreases the likelihood of finding the optimal solution within a given time frame. Altogether, the above differences result in larger and more complicated problem instances compared to the paper industry. Nevertheless, practical experiences from industrial use of the applied enumeration technique suggest that the approach is feasible.

Regarding the future research avenues, screening of the most prominent cutting patterns for the optimal solution would be

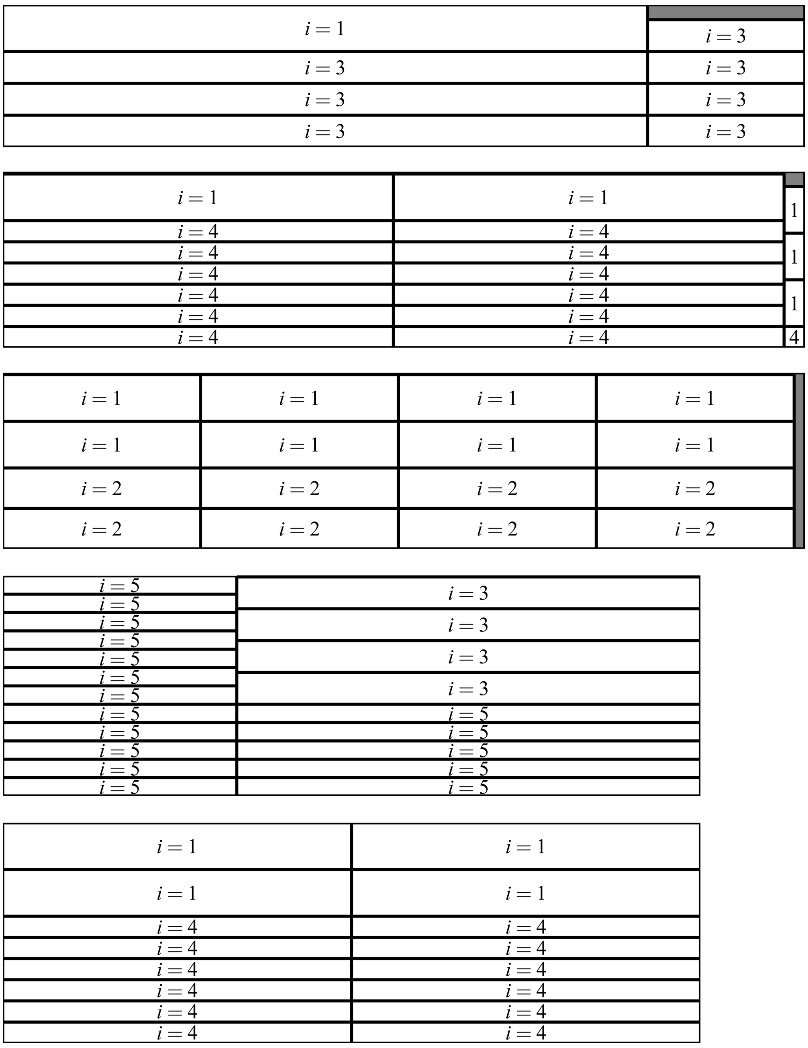

Figure 6. Graphical illustration of the optimal solution in example 3. Mother coils correspond to $r = 14$, $r = 27$, $r = 62$, $r = 65$, and $r = 115$ from bottom to top. Trim loss is represented as gray area, and it constitutes 1.23% of the raw material.

Table 11. Cutting Patterns in the Optimal Solution in Example 3

j	r	$n_{1,jr}$	$n_{2,jr}$	$n_{3,jr}$	$n_{4,jr}$	$n_{5,jr}$	width [mm]	l_{jr} [m]	m_{jr}
1	14	2	0	0	6	0	1508	1292.757	2
1	27	0	0	0	0	12	1632	433.701	1
2	27	0	0	4	0	5	1624	859.057	1
1	62	2	2	0	0	0	1292	1466.607	4
1	65	1	0	0	6	0	1288	1447.940	2
3	65	3	0	0	1	0	1195	38.286	1
1	115	1	0	3	0	0	1054	1195.027	1
3	115	0	0	4	0	0	944	291.199	1

necessary. Currently, all the feasible cutting patterns are allowed to be cut out from the narrowest mother coil types. Evidently, most of these cutting patterns are not active in the optimal solution since they would produce significant widthwise trim loss. The identification of suboptimal cutting patterns and exclusion of them from the problem instance would result in less complicated instances and improved convergence. Another way to reduce combinatorial complexity is to decouple the original problem to subproblems containing distinct set of orders. Decoupling can be performed based on the widths and weights of the orders. The obvious downside is that it cannot be guaranteed, whether the optimal solution of the decoupled problems would correspond to the optimal solution of the original problem.

It is noteworthy that the solution space is essential regarding the convergence. Naturally, the bounds on the integer variables should be set as small as possible. On the other hand, the risk of cutting out the global optimum from the solution space increases, if the bounds are set too low. Consequently, only suboptimal solutions could be found from an overly constrained solution space.

Regarding CPLEX, the application of an SOS configuration for the representation of integer variables with a series of binary variables appears to be computationally efficient. Still, if the upper bound of a particular integer variable is large, one should prefer binary representation due to a smaller number of auxiliary variables that must be inserted into the problem formulation.

As for the other solution techniques compared to Branch and Cut utilized by CPLEX, genetic hybrid algorithms might provide a viable alternative with respect to solution times, especially if large quantities of strips need to be produced. Since the lengths of

Table 12. Amount of Product Coils in the Optimal Solution in Example 3

b_i [mm]	w_i [kg]	no. and lengths of product coils
346	33500	(1 × 1195.0 m + 2 × 724.0 m + 3 × 38.3 m + 8 × 366.7 m + 4 × 646.4 m)
300	10294	(8 × 366.7 m)
236	22600	(3 × 1195.0 m + 4 × 291.2 m + 4 × 366.7 m)
157	30272	(12 × 724.0 m + 1 × 38.3 m + 12 × 646.4 m)
136	15114	(12 × 433.7 m + 5 × 859.1 m)

the cutting patterns may vary freely within the given tolerances, the hybrid approaches could be considered. In other words, methods of classical optimization theory could be combined with applicable meta-heuristics. For each solution candidate, optimal lengths of cutting patterns could be determined by fixing the cutting patterns of the solution candidate, and optimizing the lengths with an LP algorithm.[26]

CONCLUSION

A new formulation for a cutting stock problem in the stainless steel industry is introduced. Compared to some related formulations developed for the paper industry, the mother coils have limited lengths and they are considered variables. Consequently, trim loss at the end of a mother coil is preferred to widthwise trim loss, and aimed to be collected at the end of a single mother coil to enable reuse of the scrap material. In addition, the scattering of an order over multiple mother coils is minimized to facilitate shipping of an order in a single batch. The originally nonconvex and nonlinear problem is solved by a two-step procedure where the feasible cutting patterns are first generated and then inserted as parameters into the optimization problem. The resulting MILP problem can be thereafter solved by any MILP solver such as CPLEX. Practical experiences from large scale industrial use suggest that the particular cutting pattern enumeration technique is a viable solution technique for the studied problem.

APPENDIX

Since the introduction of a new duplicate adds several variables and constraints into the formulation, the number of duplicates should be kept as low as possible. Specifically, the number of duplicates for different subsets V_{sk} should be kept minimal. An appropriate amount of duplicates for each subset V_{sk} is determined with the following heuristic:

1. For mother coils in the subset $V_{s(1)}$, compute how many mother coils N_r suffice to fulfill the ordered area of the product:

$$N_r = \left\lceil \frac{\sum_{i \in F_r} w_{i,\,nom}/A_w}{B_{r,\,\max} L_{r,\,\max}} \right\rceil \tag{A1}$$

where $i \in F_r \subseteq V_{s(1)}$ is the product belonging to the set F_r. If $N_r > 2$, only two duplicates will be generated in the maximum. Generating more duplicates is unnecessary since the majority of the product coils can be cut out from multiple mother coils of the same type, while the lengthwise trim loss may be accumulated on a single mother coil.

2. For mother coils in the subset $V_{s(2)}$, make sure if it is possible to cut the products listed in $F_r \subseteq V_{s(2)}$ from the mother coil r by the following algorithm:

```
1. if l_{i,min} < l_{j,max} and l_{j,min} < l_{i,max} and b_i + b_j ≤ B_{r,max} then
     Cutting pattern can be generated
   else if l_{i,min} + l_{j,min} > L_{r,max}
     Products cannot be cut from the mother coil r
   end if
```

If it is possible to cut the products from the coil, estimate the number of duplicates by using eq A1. Again, if $N_r > 2$, only two duplicates will be generated.

3. For mother coils in the subset $V_{s(3)}$, check if it is possible to cut the three products listed in $F_r \subseteq V_{s(3)}$ from the mother coil r. First, try to generate a cutting pattern for a pair of products $i,j \in F_r$. If it is possible, then try to cut the third product $k \subseteq F_r$ from the coil. Repeat the procedure for each permutation of the triplet. If no combination is successful, position the products in sequence on the mother coil using the minimum product coil lengths. If the sum of the product coil lengths is less than or equal to the mother coil length $L_{r,\max}$, the triplet can be cut from the mother coil r. The heuristic can be written in pseudocode as follows:

```
1. for each permutation of i, j and k do
     if cutting pattern can be generated for a pair i, j then
       Let l_min be the minimum length of that cutting pattern
       if cutting pattern can be generated for a triplet i, j, k then
         Products can be cut from the mother coil r
       else if l_min + l_{k,min} ≤ L_{r,max} then
         Products can be cut from the mother coil r
       end if
     end if
   end do
2. if l_{i,min} + l_{j,min} + l_{k,min} ≤ L_{r,max} then
     Products can be cut from the mother coil r
   end if
```

If the products can be cut from the mother coil r, compute the estimate of the number of duplicates by eq A1. Again, the maximum number of duplicates is two.

4. For mother coils in the subset V_{sI}, compute the estimated number of duplicates needed by eq A1, the maximum value of N_r being two. If the subset $V_{s(3)}$ is empty, the maximum number of duplicates is four. If also the subset $V_{s(2)}$ is empty, the maximum number of duplicates is six. This should provide enough cutting patterns for the global optimum.

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NOMENCLATURE

Indices

i = index of orders
j = index of cutting patterns
k = index of cutting pattern repetitions
r = index of mother coil types
I = number of orders
J_r = number of feasible cutting patterns in mother coil of type r
K_{jr} = maximum repetitions of cutting pattern j in mother coil of type r
R = number of mother coil types

Parameters

$A_{O,max}$ = maximum area of product material within the weight limits
$A_{r,min}$ = minimum area of mother coils of type r
$A_{r,max}$ = maximum area of mother coils of type r
A_w = area weight of steel
C = area price of product coil
C' = area price of overproduction
C'' = area price of raw material
C'_r = cost of a format change to mother coil of type r
$C_{i,OP}$ = cost of overproduction for order i
C_{TLL} = cost of lengthwise trim loss
C'_{TLL} = penalty coefficient for lengthwise trim loss
C_m = hourly cost for cutting machine
C_{Sr} = cost of changing to a new mother coil of type r with scattering level S
$B_{r,min}$ = minimum width of mother coils of type r
$B_{r,max}$ = maximum width of mother coils of type r
$B_{r,usable}$ = maximum usable width of mother coil of type r
F_r = set of products allowed to be cut out from mother coil of type r
$L_{r,min}$ = minimum length of mother coils of type r
$L_{r,max}$ = maximum length of mother coils of type r
$L_{r,usable}$ = maximum usable length of mother coil of type r
$M_{r,max}$ = maximum number of mother coils of type r
$N_{r,max}$ = maximum number of products that can be cut in a pattern of mother coil of type r
V_{sk} = subset of mother coil types of width s containing at most k different order types
b_i = width of product coil for order i
c_{jr} = cost of a knife change at cutting pattern j for mother coil of type r
c'_{jr} = cost of using cutting pattern j in mother coil of type r
$l_{i,min}$ = minimum length of product coils i
$l_{i,max}$ = maximum length of product coils i
$l_{jr,min}$ = minimum length of cutting pattern j applied for mother coil of type r
$l_{jr,min}$ = maximum length of cutting pattern j applied for mother coil of type r
n_{ijr} = number of product coils i in cutting pattern j for mother coil of type r
s = fractional area of mother coil of type r
v_m = cutting machine speed
$w_{i,min}$ = minimum total weight of product coils i
$w_{i,max}$ = maximum total weight of product coils i
$w_{i,nom}$ = nominal weight of order i in kilograms

Continuous Variables

A_O = area of product material within ordered weight tolerances
$A_{O,i}$ = area of product material of type i within ordered weight tolerances
A_{OP} = area of overproduction
A_{TL} = area of trim loss
A_{TLB} = area of widthwise trim loss
A_{TLL} = area of lengthwise trim loss
$A_{TLL,r}$ = area of lengthwise trim loss in mother coil of type r
A_r = area of mother coils of type r
D_r = total unused length of mother coils of type r
L_r = length of mother coils of type r
l_{jr} = total length of product coils in cutting pattern j cut from mother coils of type r
s_{jrk} = indicates the number of cutting pattern changes over multiples of mother coils
w_i = amount of order i produced in kilograms
$w_{i,OP}$ = quantity of order i produced over nominal weight in kilograms

Binary Variables

b_{jrk} = binary variable associated with slack variable s_{jrk}
y_{jr} = indicates if cutting pattern j is applied in mother coil of type r
T_r = indicates if mother coil of type r contains lengthwise trim loss
d_{rk} = indicates the amount of lengthwise trim loss in mother coil of type r

Integer Variables

M_r = number of mother coils of type r
m_{jr} = number of times cutting pattern j in mother coil of type r is repeated

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