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ARTICLE *in* INDUSTRIAL & ENGINEERING CHEMISTRY RESEARCH · DECEMBER 2009

Impact Factor: 2.59 · DOI: 10.1021/ie900358z

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In this article, a two-stage robust model is proposed to solve the crude oil scheduling problem under uncertain conditions. The first stage of the model is developed using chance-constrained programming and fuzzy programming that can be transformed into the deterministic counterpart problem, whereas the second-stage is scenario-based. Through the combination of the approaches, the two-stage model can deal with uncertain parameters with both continuous and discrete probability distributions within a finite number of scenarios. The model was tested on several small examples and an industrial-size case. Uncertainties were introduced in ship arrival times and fluctuating product demands. The computational results demonstrate the effectiveness and robustness of the proposed approach. The tradeoff between solution robustness and model robustness was also analyzed.

1. Introduction

Because of high crude prices, the issue of short-term refinery crude oil scheduling becomes increasingly important, as it can exploit all potential opportunities to push the economic margin to the maximum limit. It is also crucial in overall refinery operations as it can affect downstream operations significantly.[1,2]

During the past two decades, mathematical programming technologies have been widely applied to cope with crude oil scheduling problems. A mathematical programming model was employed for the monthly crude oil transfer plan of a refinery. The problem was decomposed into two smaller problems. The downstream part, which determined how the refinery operated, was solved first, whereas the upstream problem that scheduled how the port tanks were supplied was solved subsequently.[3] However, this method was mainly suitable for small- or medium-size single-refinery problems. Meanwhile, large-scale mixed-integer optimization models were also developed. They could deal with the short-term refinery scheduling problem involving crude oil unloading, tank inventory management, and crude distillation unit (CDU) charging schedule for industrial-size problems.[4,5] Because the mass balance for key components in charging tanks is hard to solve, the bilinear parts in large-scale mixed-integer optimization models are linearly reformed. Although reformation can accelerate the computation speed, it also causes composition discrepancy, which means that the composition of crude sent from a tank does not match that received by the CDU. Therefore, subsequently existing formulations have been extended to accommodate nonlinear crude properties to eliminate composition discrepancies, and many strategies have been developed to improve the robustness, quality, and computation speed of the mixed-integer nonlinear programming (MINLP) formulations.[6,7] The above-mentioned models are all based on discrete-time representations, whereas another important approach for crude oil scheduling operations is in continuous-time formulations to enhance the flexibility of the optimization. Jia et al.[8] employed a state−task network continuous-time representation to address the problem. Compared with discrete-time models, the proposed formulation led to fewer variables and constraints. Furman et al.[9] utilized the advantage of time continuity and thus handled the synchronization of time events more robustly. Reddy et al.[10] also presented a complete continuous-time mixed-integer linear programming (MILP) model for the crude oil short-term scheduling problem and compared discrete-time versus continuous-time formulations. In addition to time-based models, simulation and expert systems are both useful tools for optimizing crude oil short-term scheduling.[11,12]

However, all works mentioned above employed deterministic mathematical programming methods to solve the crude oil scheduling problem. The data used in those models were assumed to be known and constant. However, in the real world, uncertainties such as ship arrival delays, restrictions on tank availability, pipeline malfunctions, and demand fluctuations are unavoidable because of the lack of accurate process models and the variability of the process and the environment. Thus, an emerging area of research aims at developing methods to address the problem of scheduling under uncertainty and creating reliable schedules that remain feasible in the presence of parameter uncertainty.[13,14] Different methodologies can be used for solving the problem of scheduling under uncertainty. In general, there are two approaches: one is reactive scheduling, and the other is predictive scheduling.[15]

Reactive scheduling is used during the actual execution of a plan or schedule when a disruption has occurred. Because of the “on-line” nature of reactive scheduling, it is necessary to generate updated schedules in a timely manner, and heuristic approaches are often utilized.[16,17] Predictive scheduling seeks to accommodate possible disruptions during planning or scheduling. The main approaches include stochastic programming, fuzzy programming, robust programming, and stochastic dynamic programming. The interested reader can refer to the work of Sahinidis[18] as a review of predictive optimization approaches under uncertainty.

Stochastic programming has been employed to solve predictive scheduling problems. It can be divided into two categories depending on whether it follows a scenario-based framework. In the scenario-based framework, one usually generates a finite set of scenarios to represent the probability space.[19] Li et al.[20] developed scenario-based models for separately addressing two uncertainties existing in crude oil operations and obtained more robust and feasible schedules. As the number of uncertain parameters increases, more scenarios must be considered, which leads to huge time consumption. This main drawback limits the

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10.1021/ie900358z
Published on Web 12/31/2009

use of these approaches in the solution of practical problems with large numbers of uncertain parameters. To make the aforementioned approaches more tractable, a multiobjective chance-constrained programming framework can also be used to solve planning problems.[21] Stochastic models following a non-scenario-based framework have also been applied to scheduling problems.[22−24] Cao et al.[25] proposed a non-scenario-based optimization model that could generate robust schedules and avoid the drawback of enumerating scenarios. However, it had difficulties in dealing with uncertain parameters following a discrete probability distribution.

To make the solutions of stochastic programming progressively less sensitive to realizations of the data in a scenario set, Mulvey et al.[26] proposed a robust programming that follows a scenario-based framework. This robust programming is able to tackle the decision makers' favored risk aversion or service-level function. Moreover, two sets of variables are defined: design and control variables. A design variable cannot be adjusted once a specific realization of the data has been observed, and a control variable is subject to adjustment once uncertain parameters are observed. However, the formulation involves quadratic forms and is hard to solve.[27]

In this article, a robust two-stage formulation is proposed and applied to cope with uncertain parameters with both continuous and discrete probability distributions. The first stage involves only design variables, and the first-stage constraints are modeled using a non-scenario-based method; the second-stage comprises only control variables. The scenarios are generated according to the realization of the uncertain data in the second-stage constraints modeled using a scenario-based approach. The advantage of this model is the emergence of a relatively small MILP problem with a limited size of scenarios.

This article is organized as follows: First, we review robust optimization and propose the newly integrated model. Then, detailed mathematical formulations of the crude oil short-term scheduling problem based on the model proposed by Lee et al.[4] are presented. Finally, we compare and contrast our model with a well-accepted current approach and demonstrate the effectiveness of this novel approach using several diverse examples. Concluding remarks are given in the last section.

2. Overview of Robust Optimization

We integrated the robust optimization approaches in order to cope with uncertain parameters with both continuous and discrete probability distributions. The approaches used are overviewed here.

2.1. Scenario-Based Robust Optimization. Robust optimization, presented by Leung et al.,[27] integrates a goal-programming formulation with a scenario-based description of input data. The model generates a series of solutions that are progressively less sensitive to the realizations of input data from a set of scenarios. Let **x** be a vector of design variables and **y** be a vector of control variables. Then, the form of their robust optimization model is

$$\begin{aligned} \text{Min} \quad & \mathbf{c}^{\mathrm{T}}\mathbf{x} + \mathbf{d}^{\mathrm{T}}\mathbf{y} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{B}\mathbf{x} + \mathbf{C}\mathbf{y} = \mathbf{e} \\ & \mathbf{x}, \mathbf{y} \geq 0 \end{aligned} \tag{1}$$

In this model, the first equality of constraints is a structural constraint whose coefficients are fixed and free of noise, whereas the second constraint is a control constraint taken as an auxiliary constraint that is influenced by noise data. The set of scenarios involved is $S = \{1, 2, 3, ..., s\}$. Under each scenario $s \in S$, the coefficients associated with the control constraints become $\{\mathbf{d}_s, \mathbf{B}_s, \mathbf{C}_s, \mathbf{e}_s\}$ with fixed probability p_s, which represents the probability that scenario s occurs, with $\sum_s p_s = 1$. The optimal solution of this model will be robust if it remains "close" to optimality for any realization of scenario s. This is called solution robustness. The solution is also robust with respect to feasibility if it remains "almost" feasible for any realization of s. This is termed model robustness.

2.2. Non-Scenario-Based Robust Optimization. Chance-constrained programming and fuzzy mathematical programming are both traditionally well-known approaches for optimization techniques under uncertainty.[28−31] In chance-constrained programming, the uncertainty is described using a probability distribution when information about the behavior of uncertainty is known. The unique feature of the approach is that the resulting solution can ensure the probability of complying with inequality constraints, that is, the predefined confidence level of being feasible.[32] In some cases, chance-constrained programming can be converted to standard linear programming, which is easy to solve. On the other hand, fuzzy programming considers random parameters as fuzzy numbers, and treats constraints as fuzzy sets. Member functions are used to represent the degree of satisfaction of constraints, the decision makers' expectations, and the range of uncertainty of coefficients. Models based on fuzzy sets have the advantage that they do not require the complicated integration schemes that are needed for the chance-constrained programming.[15,33]

In a stochastic or a fuzzy environment, the feasibility of the following inequality is concerned

$$\text{Min } \mathbf{c}^{\mathrm{T}}\mathbf{x} \quad \text{s.t. } \mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0 \tag{2}$$

where **c** and **x** are n-vectors, **b** is an m-vector, and **A** is an $m \times n$ matrix.

2.2.1. Chance-Constrained Programming in a Stochastic Environment. Assume that there is uncertainty regarding the right-hand-side vector **b** with the cumulative distribution function Φ and that the system is required to be satisfied with a probability of $\alpha \in (0, 1)$. Then, the probabilistic linear programming constraint can be written as

$$P\left(\sum_{j=1}^{m} A_{ij}x_i \leq b_i\right) \geq a_i, \quad i = 1, 2, ..., n \tag{3}$$

where P is the operator of the probability computation. By applying the cumulative distribution function Φ, we can rewrite inequality 3 as

$$\sum_{j=1}^{m} A_{ij}x_i \leq \Phi_i^{-1}(1 - \alpha_i), \quad i = 1, 2, ..., n \tag{4}$$

Because α_i and Φ_i^{-1} are known, the reformed constraint is reduced to an ordinary deterministic linear programming model. If all constraints are assigned the same confidence level α, inequality 4 is then replaced by the joint probabilistic constraint

$$P\left(\sum_{j=1}^{m} A_{ij}x_i \leq b_i\right) \geq a \tag{5}$$

When b_i is an independent random variable, the joint probabilistic constraint can be decomposed into the multiplication of single probabilities as a nonlinear programming model[34]

$$\prod_{i=1}^{n}\left[1-\Phi_i\left(\sum_{j=1}^{m}A_{ij}x_i\right)\right]\geq\alpha \tag{6}$$

2.2.2. Fuzzy Programming. When the uncertainty arises from both the coefficients and the right-hand-side parameters of the inequality constraints, the mathematical programming model with fuzzy parameters is in the form[35]

$$\text{Max}\, f(\mathbf{x},\xi) \quad \text{s.t. } g_i(\mathbf{x},\xi)\leq 0, i=1,2,...,p \tag{7}$$

where $\mathbf{x}$ is a decision vector; ξ is a vector of fuzzy parameters; $f(\mathbf{x},\xi)$ is the return function; and $g_i(\mathbf{x},\xi)$, $i = 1, 2, ..., p$, represents constraint functions. With their respective crisp equivalents, the constraints can be written as

$$\begin{aligned}\text{Max}\, \bar{f} \quad \text{s.t. } & \text{Pos}[\xi|f(\mathbf{x},\xi)\geq \bar{f}]\geq\beta \\ & \text{Pos}[\xi|g_i(\mathbf{x},\xi)\leq 0, i=1,2,...,p,p+1]\geq\alpha\end{aligned} \tag{8}$$

where α and β are predefined confidence levels and Pos[•]denotes the possibility of the events in [•]. The problem is formulated with separate chance constraints as

$$\text{Max}\, \bar{f} \quad \text{s.t. Pos}[\xi|g_i(\mathbf{x},\xi)\leq 0]\geq\alpha_i, \quad i=1,2,...,p,p+1 \tag{9}$$

where α_i represents respective predefined confidence levels, $g_{p+1}(\mathbf{x},\xi) = \bar{f} - f(\mathbf{x},\xi)$, and $\beta = \alpha_{p+1}$. If this inequality can be written in the form

$$\text{Pos}[\xi_i|h_i(\mathbf{x})\leq\xi_i]\geq\alpha_i, \quad i=1,2,...,p,p+1 \tag{10}$$

then the crisp equivalents are obtained in form

$$h_i(\mathbf{x})\leq K_{\alpha_i}, \quad i=1,2,...,p,p+1 \tag{11}$$

where $h_i(\mathbf{x})$ represents functions of decision vector $\mathbf{x}$; ξ_i represents fuzzy numbers with membership functions $\mu_i(\xi_i)$, $i = 1, 2, ...,p, p + 1$; and $K_{\alpha_i} = \sup[K|K = \mu_i^{-1}(\alpha_i)]$, $i = 1, 2, ..., p, p + 1$. Assume that the constraints can be written in the form

$$\text{Pos}[\xi_i|h_i(\mathbf{x})\geq\xi_i]\geq\alpha_i, \quad i=1,2,...,p,p+1 \tag{12}$$

If we define$K_{\alpha_i} = \inf[K|K = \mu_i^{-1}(\alpha_i)]$, $i = 1, 2, ..., p, p + 1$, the final deterministic form is

$$h_i(\mathbf{x})\geq K_{\alpha_i}, \quad i=1,2,...,p,p+1 \tag{13}$$

3. Integrated Robust Model

The integrated robust model dealing with uncertainty is presented as follows

$$\begin{aligned}\text{Min} \quad & \mathbf{c}^T\mathbf{x}+\tilde{\mathbf{d}}^T\mathbf{y} & \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x}=\mathbf{b} & \text{first-stage constraints} \\ & \tilde{\mathbf{B}}\mathbf{x}+\tilde{\mathbf{C}}\mathbf{y}=\tilde{e} & \text{second-stage constraints} \\ & \sum_m \tilde{f}_{lm}x_m\leq\tilde{g}_l & \text{first-stage constraints} \\ & x,y\geq 0 &\end{aligned} \tag{14}$$

where $\mathbf{x}$ is a vector of the design variables and $\mathbf{y}$ is a vector of control variables. There are two constraints in the model: the first-stage constraints involving only design variables and the second-stage constraints involving at least one control variable. The scenarios in our approach are generated only according to the realization of the uncertain data in the second-stage constraints, so the first-stage constraints are scenario-independent.

The first and third constraints in the integrated model are the first-stage constraints, and the second constraint is the second-stage constraint. In our approach, the design constraints in the first stage are modeled using the deterministic robust counterpart problem, whereas the second-stage constraint is scenario-dependent. The set of scenarios are determined by uncertain data $\{\tilde{\mathbf{d}}_s, \tilde{\mathbf{B}}_s, \tilde{\mathbf{C}}_s, \tilde{\mathbf{e}}_s\}$. Under each scenario $s \in S$, the coefficients associated with the second-stage will become $\{d_s, B_s, C_s, e_s\}$ with fixed probability p_s. Let the reliability level for the inequality constraints in the first stage be α, as given by

$$\Pr\left(\sum_m \tilde{f}_{lm}x_m\leq\tilde{g}_l\right)\geq\alpha_l \tag{15}$$

The deterministic model will then be

$$\begin{aligned}\text{Min} \quad & \mathbf{c}^T\mathbf{x}+\sum_{s\in S}p_s\xi_s+\lambda\sum_{s\in S}p_s\left|\xi_s-\sum_{s'\in S}p_{s'}\xi_{s'}\right|+w\sum_{s\in S}p_s\delta_s \\ \text{s.t.} \quad & \mathbf{d}_s^T\mathbf{y}_s=\xi_s \\ & \mathbf{A}\mathbf{x}=\mathbf{b} \\ & \mathbf{B}_s\mathbf{x}+\mathbf{C}_s\mathbf{y}_s+\delta_s=\mathbf{e}_s \quad \text{for all } s\in S \\ & \sum_m f_{lm}x_m\leq f(f_{lm}|x_m,|g_l|) \\ & f(f_{lm}|x_m,|g_l|)=\Phi_l^{-1}(1-\alpha_l) \\ & x,y_s\geq 0 \quad \text{for all } s\in S\end{aligned} \tag{16}$$

where l is the index of the uncertain inequality, m is the index of the variables, and Φ_l^{-1} is the inverse of the probability distribution function of $\tilde{g}_l$. The variable δ_s is used to measure the infeasibility allowed in control constraints under scenario s. The term $\sum_{s\in S}p_s\xi_s + \lambda\sum_{s\in S}p_s|\xi_s - \sum_{s'\in S}p_{s'}\xi_{s'}|$ in the objective function is used to measure the solution robustness, and the term $\sum_{s\in S}p_s\delta_s$ is used to measure the model robustness. The sensitivity of the solution to the realization of uncertainty data is controlled by the parameter λ, and the tradeoff between solution robustness and model robustness is controlled by weight w.

4. Mathematical Models for Crude Oil Operations

In a typical refinery, the series of operations begins with crude oil unloading, mixing, and transferring. After that, the distillation process in crude distillation units (CDUs) separates the charged oil into consecutively lighter and heavier fractions with different boiling points. Most of the distillation products need to be further processed in downstream units. Finally, crude oils are turned into higher-value end product (gasoline, diesel, etc.). Here, we focus on the crude oil scheduling problem of a typical marine-access refinery.

4.1. Process Description. The system configuration of this scheduling problem corresponds to a typical marine-access refinery. It consists of crude offloading facilities, a tank farm comprising storage and charging tanks, and processing facilities such as crude distillation columns as illustrated in Figure 1. During a given scheduling horizon, crude vessels arrive in the vicinity of the refinery docking station. If the docking station is busy, vessels wait for unloading until the preceding vessel leaves. Crude oil is unloaded into storage tanks at the docking station and then transferred to charging tanks. Different qualities of crude oil get blended inside tanks and charged directly to CDUs.[4]

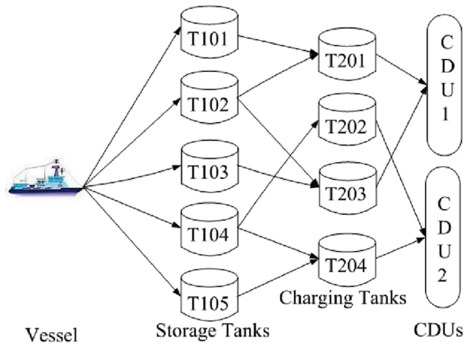

Figure 1. Schematic of crude oil unloading, storing, and processing.

4.2. Given Relationship. The following data are given with the problem: (a) margin data, including estimated arrival times of vessels, crude types and volumes, and estimated demands; (b) configuration details, including numbers of storage tanks, charging tanks, and CDUs and their interconnections; (c) initial data, including crude composition and inventories of storage and charging tanks; and (d) equipment limitations, including limits on flow rates transferring between equipments, limits on CDU processing rates, and limits on tank capacities.

4.3. Operating Practices and Assumptions. The following assumptions are made: (a) Crude unloading is sequential, and holdup in the pipeline is negligible. (b) A charging tank cannot receive crude and charge a CDU at the same time. (c) In a single time period, at most one charging tank can feed a single CDU and vice versa. (d) Crude mixing is perfect in tanks, and the changeover time for CDUs is neglected. (e) Each CDU is operated continuously during the scheduling horizon. (f) For simplicity, transfer between tanks in the same area is neglected, and the quality of a crude feed to a CDU is determined by only one key component.

4.4. Determine. The following quantities must be determined: (a) a detailed unloading schedule, such as the unloading and departure times for each ship, the transfer amount of each pipeline, and the aim tank; (b) the total inventory and composition profiles of all storage tanks at various instances of time; and (c) the detailed charging schedule for CDUs including feed tanks, feed rates, and feed times. The objective of the problem is to maximize the net profit. We define net profit as gross profit minus total costs, where the former is the sum of crude margins.

4.5. Mathematical Formulation. In this article, the discrete-time formulation proposed by Lee et al.[4] is adapted for the discussion related to uncertainty. For the sake of completeness, the modified nominal model is first presented.

4.5.1. Vessel Arrival and Departure Operation Rules. To describe the vessel-to-dock connections, we define $X_{F,v,t}$ to indicate whether vessel v starts unloading at period t, and $X_{L,v,t}$ to indicate whether vessel v completes unloading at the end of period t, and $X_{W,v,t}$ to indicate whether vessel v is unloading during period t. The operation constraints are as follows:

Each vessel arrives and leaves the dock station once during the scheduling horizon, so

$$\sum_{t=1}^{N_{SCH}} X_{F,v,t} = 1, \quad v = 1, ..., N_v \tag{17}$$

$$\sum_{t=1}^{N_{SCH}} X_{L,v,t} = 1, \quad v = 1, ..., N_v \tag{18}$$

Then, the detailed unloading start and completions time can be calculated as

$$T_{F,v} = \sum_{t=1}^{N_{SCH}} tX_{F,v,t}, \quad v = 1, ..., N_v \tag{19}$$

$$T_{L,v} = \sum_{t=1}^{N_{SCH}} tX_{L,v,t}, \quad v = 1, ..., N_v \tag{20}$$

Unloading is possible between the initiation and completion times

$$X_{W,v,t} \le \sum_{m=1}^{t} X_{F,v,m}, \quad v = 1, ..., N_v; t = 1, ..., N_{SCH} \tag{21}$$

$$X_{W,v,t} \le \sum_{m=t}^{N_{SCH}} X_{L,v,m}, \quad v = 1, ..., N_v; t = 1, ..., N_{SCH} \tag{22}$$

Vessels should be unloaded one by one after the planned arrival time $T_{ARR,v}$

$$T_{F,v+1} \ge T_{L,v}, \quad v = 1, ..., N_v \tag{23}$$

$$T_{L,v} - T_{F,v} \ge \left[\frac{V_{V,v,0}}{\max_i(F_{VS,v,i,\max})}\right], \quad v = 1, ..., N_v \tag{24}$$

$$T_{F,v} \ge T_{ARR,v}, \quad v = 1, ..., N_v \tag{25}$$

4.5.2. Material Balance for the Vessels. Vessels should unload all crude they carried in the scheduled time, and the transfer rate should be in the range of the pipeline capacity

$$V_{V,v,t} = V_{V,v,0} - \sum_{i=1}^{N_{ST}} \sum_{m=1}^{t} F_{VS,v,i,m}, \quad v = 1, ..., N_v; t = 1, ..., N_{SCH} \tag{26}$$

$$\sum_{i=1}^{N_{ST}} \sum_{t=1}^{S_{CH}} F_{VS,v,i,t} = V_{V,v,0}, \quad v = 1, ..., N_v \tag{27}$$

$$F_{VS,v,i,\min} X_{W,v,t} \le F_{VS,v,i,m} \le F_{VS,v,i,\max} X_{W,v,t}, \quad v = 1, ..., N_v; i = 1, ..., N_{ST}; t = 1, ..., N_{SCH} \tag{28}$$

4.5.3. Material Balance for Storage Tanks.

$$V_{S,i,t} = V_{S,i,0} + \sum_{v=1}^{N_v} \sum_{m=1}^{t} F_{VS,v,i,m} - \sum_{i=1}^{N_{BT}} \sum_{m=1}^{t} F_{SB,i,j,m}, \quad i = 1, ..., N_{ST}; t = 1, ..., N_{SCH} \tag{29}$$

$$V_{S,i,\min} \le V_{S,i,t} \le V_{S,i,\max}, \quad i = 1, ..., N_{ST}; t = 1, ..., N_{SCH} \tag{30}$$

$$F_{SB,i,j,\min}\left(1 - \sum_{l=1}^{N_{CDU}} Y_{j,l,t}\right) \le F_{SB,i,j,t} \le F_{SB,i,j,\max}\left(1 - \sum_{l=1}^{N_{CDU}} Y_{j,l,t}\right), \quad i = 1, ..., N_{ST}; t = 1, ..., N_{SCH}; j = 1, ..., N_{BT} \tag{31}$$

4.5.4. Material Balance for Charging Tanks.

$$V_{B,j,t} = V_{B,j,0} + \sum_{i=1}^{N_{BT}} \sum_{m=1}^{t} F_{SB,i,j,m} - \sum_{l=1}^{N_{CDU}} \sum_{m=1}^{t} F_{BC,j,l,m}, \quad j = 1, ..., N_{BT}; t = 1, ..., N_{SCH} \tag{32}$$

$$V_{B,j,\min} \le V_{B,j,t} \le V_{B,j,\max}, \quad j = 1, ..., N_{BT}; t = 1, ..., N_{SCH} \tag{33}$$

$$F_{BC,j,l,\min} Y_{j,l,t} \le F_{BC,j,l,t} \le F_{SB,i,j,\max} Y_{j,l,t}, \quad j = 1, ..., N_{BT}; t = 1, ..., N_{SCH}; l = 1, ..., N_{CDU} \tag{34}$$

4.5.5. Material Balance for Component k in Charging Tanks.

$$v_{B,j,k,t} = v_{B,j,k,0} + \sum_{m=1}^{t} \left(\sum_{i=1}^{N_{ST}} f_{SB,i,j,k,m} - \sum_{l=1}^{N_{CDU}} f_{BC,j,l,k,m} \right), \quad j = 1, ..., N_{BT}; t = 1, ..., N_{SCH}; k = 1, ..., N_{CE} \tag{35}$$

$$V_{B,j,t} \xi_{B,j,k,\min} \le v_{B,j,k,t} \le V_{B,j,t} \xi_{B,j,k,\max}, \quad l = 1, ..., N_{CDU}; j = 1, ..., N_{BT}; t = 1, ..., N_{SCH}; k = 1, ..., N_{CE} \tag{36}$$

$$f_{SB,i,j,k,t} = F_{SB,i,j,t} \xi_{S,i,k}, \quad i = 1, ..., N_{ST}; j = 1, ..., N_{BT}; t = 1, ..., N_{SCH}; k = 1, ..., N_{CE} \tag{37}$$

$$F_{BC,j,l,t} \xi_{B,j,k,\min} \le f_{BC,j,l,k,t} \le F_{BC,j,l,t} \xi_{B,j,k,\max}, \quad l = 1, ..., N_{CDU}; j = 1, ..., N_{BT}; t = 1, ..., N_{SCH}; k = 1, ..., N_{CE} \tag{38}$$

4.5.6. Operation Rules for Crude Oil Charging.

$$\sum_{l=1}^{N_{CDU}} Y_{j,l,t} \le 1, \quad j = 1, ..., N_{BT}; t = 1, ..., N_{SCH} \tag{39}$$

$$\sum_{j=1}^{N_{BT}} Y_{j,l,t} \le 1, \quad l = 1, ..., N_{CDU}; t = 1, ..., N_{SCH} \tag{40}$$

Changeover can be expressed as

$$YY_{j,l,t} \ge Y_{j,l,t} + Y_{j,l,t+1} - 1, \quad j = 1, ..., N_{BT}; l = 1, ..., N_{CDU}; t = 1, ..., N_{SCH} \tag{41}$$

$$YY_{j,l,t} \le Y_{j,l,t+1}, \quad j = 1, ..., N_{BT}; l = 1, ..., N_{CDU}; t = 1, ..., N_{SCH} \tag{42}$$

$$YY_{j,l,t} \le Y_{j,l,t}, \quad j = 1, ..., N_{BT}; l = 1, ..., N_{CDU}; t = 1, ..., N_{SCH} \tag{43}$$

$$Z_{l,t} \ge Y_{j,l,t} + Y_{j,l,t+1} - 2YY_{j,l,t}, \quad j = 1, ..., N_{BT}; l = 1, ..., N_{CDU}; t = 1, ..., N_{SCH} \tag{44}$$

4.5.7. Demand. The total production amount of crude oil mix l should meet the demands for CDUs during the scheduling horizon

$$\sum_{l=1}^{N_{CDU}} \sum_{t=1}^{N_{SCH}} F_{BC,j,l,t} \ge D_j, \quad j = 1, ..., N_{BT} \tag{45}$$

The scheduling objective, or the objective function of optimization, is to maximize the net profit. The formulation of the objective is as follows

$$\text{Maximize } C_{PROFIT,j} \sum_{l=1}^{N_{CDU}} \sum_{t=1}^{N_{SCH}} F_{BC,j,l,t} - \Bigg[C_{UNLOAD,v} \sum_{v=1}^{N_v} (T_{L,v} - T_{F,v} + 1) + C_{SEA,v} \sum_{v=1}^{N_v} (T_{F,v} - T_{ARR,v}) + C_{INVST,i} \sum_{i=1}^{N_{ST}} \sum_{t=1}^{N_{SCH}} \times \left(\frac{V_{S,i,t} + V_{S,i,t-1}}{2} \right) + C_{INVBL,j} \sum_{i=1}^{N_{BT}} \sum_{t=1}^{N_{SCH}} \left(\frac{V_{B,j,t} + V_{B,j,t-1}}{2} \right) + \sum_{l=1}^{N_{CDU}} \sum_{t=1}^{N_{SCH}} C_{SETUP,l} Z_{l,t} \Bigg] \tag{46}$$

The first term in the objective function is used to calculate the gross profit. The second term is the unloading cost, and the third term is the cost for vessels waiting at sea. The fourth and fifth terms are used for calculating the inventory costs for storage tanks and charging tanks, respectively. The sixth term in the objective function uses penalties to avoid frequent changeovers of CDUs.

4.6. Robust Optimization Model. For the robust optimization model in this article, two types of uncertainties are introduced, namely, ship arrival times and fluctuating mixed oil demands. To deal with uncertain ship arrival times, constraints are added to the second stage of the model with a new set of variables with a superscript scenario index, s, corresponding to a certain scenario. For example, $T^s_{ARR,v}$ is the arrival time of vessel v in scenario s, $X^s_{F,v,t}$ indicates whether vessel v starts unloading at period t in scenario s, and $X^s_{W,v,t}$ indicates whether vessel v is unloading during period t in scenario s. A number of constraints should be modified to account for the uncertainties.

Vessels arrive and leave the dock station for each scenario as

$$\sum_{t=1}^{N_{SCH}} X^s_{F,v,t} = 1, \quad v = 1, ..., N_v; s = 1, ..., N_s \tag{17a}$$

$$\sum_{t=1}^{N_{SCH}} X^s_{L,v,t} = 1, \quad v = 1, ..., N_v; s = 1, ..., N_s \tag{18a}$$

The detailed unloading start and completion times are in the constraints

$$T^s_{F,v} = \sum_{t=1}^{N_{SCH}} t X^s_{F,v,t}, \quad v = 1, ..., N_v; s = 1, ..., N_s \tag{19a}$$

$$T^s_{L,v} = \sum_{t=1}^{N_{SCH}} t X^s_{L,v,t}, \quad v = 1, ..., N_v; s = 1, ..., N_s \tag{20a}$$

Unloading is possible between the initiation and completion times in each scenario

$$X^s_{W,v,t} \le \sum_{m=1}^{t} X^s_{F,v,m}, \quad v = 1, ..., N_v; t = 1, ..., N_{SCH}; s = 1, ..., N_s \tag{21a}$$

$$X^s_{W,v,t} \le \sum_{m=t}^{N_{SCH}} X^s_{L,v,m}, \quad v = 1, ..., N_v; t = 1, ..., N_{SCH}; s = 1, ..., N_s \tag{22a}$$

In each scenario, vessels should be unloaded one by one in accordance with the constraints

$$T^{s}_{\mathrm{F},v+1} \geq T^{s}_{\mathrm{L},v}, \quad v = 1, ..., N_{\mathrm{v}}; s = 1, ..., N_{\mathrm{s}} \tag{23a}$$

$$T^{s}_{\mathrm{L},v} - T^{s}_{\mathrm{L},v} \geq \left[\frac{V_{\mathrm{V},v,0}}{\max_{i}(F_{\mathrm{VS},v,i,\max})}\right], \quad v = 1, ..., N_{\mathrm{v}}; s = 1, ..., N_{\mathrm{s}} \tag{24a}$$

$$T^{s}_{\mathrm{F},v} \geq T^{s}_{\mathrm{ARR},v}, \quad v = 1, ..., N_{\mathrm{v}}; s = 1, ..., N_{\mathrm{s}} \tag{25a}$$

The material balance for the vessels becomes

$$V^{s}_{\mathrm{V},v,t} = V_{\mathrm{V},v,0} - \sum_{i=1}^{N_{\mathrm{ST}}} \sum_{m=1}^{t} F^{s}_{\mathrm{VS},v,i,m}, \quad v = 1, ..., N_{\mathrm{v}}; \quad t = 1, ..., N_{\mathrm{SCH}}; s = 1, ..., N_{\mathrm{s}} \tag{26a}$$

$$\sum_{i=1}^{N_{\mathrm{ST}}} \sum_{t=1}^{S_{\mathrm{CH}}} F^{s}_{\mathrm{VS},v,i,t} = V_{\mathrm{V},v,0}, \quad v = 1, ..., N_{\mathrm{v}}; s = 1, ..., N_{\mathrm{s}} \tag{27a}$$

$$F_{\mathrm{VS},v,i,\min} X^{s}_{\mathrm{W},v,t} \leq F^{s}_{\mathrm{VS},v,i,m} \leq F_{\mathrm{VS},v,i,\max} X^{s}_{\mathrm{W},v,t}, \quad v = 1, ..., N_{\mathrm{v}}; \quad i = 1, ..., N_{\mathrm{ST}}; t = 1, ..., N_{\mathrm{SCH}}; s = 1, ..., N_{\mathrm{s}} \tag{28a}$$

The slack variables $\delta^{s-}_{\mathrm{S},i,t}$ and $\delta^{s+}_{\mathrm{S},i,t}$ represent violations of storage tanks in scenario s. Thus, the material balances for storage tanks are modified and represented as

$$V^{s}_{\mathrm{S},i,t} = V_{\mathrm{S},i,0} + \sum_{v=1}^{N_{\mathrm{v}}} \sum_{m=1}^{t} F^{s}_{\mathrm{VS},v,i,m} - \sum_{i=1}^{N_{\mathrm{BT}}} \sum_{m=1}^{t} F_{\mathrm{SB},i,j,m}, \quad i = 1, ..., N_{\mathrm{ST}}; t = 1, ..., N_{\mathrm{SCH}}; s = 1, ..., N_{\mathrm{s}} \tag{29a}$$

$$V_{\mathrm{S},i,\min} - \delta^{s-}_{\mathrm{S},i,t} \leq V^{s}_{\mathrm{S},i,t} \leq V_{\mathrm{S},i,\max} + \delta^{s+}_{\mathrm{S},i,t}, \quad i = 1, ..., N_{\mathrm{ST}}; \quad t = 1, ..., N_{\mathrm{SCH}}; s = 1, ..., N_{\mathrm{s}} \tag{30a}$$

In the face of uncertain demand, $\tilde{D}_j$, slack variables $\delta^{-}_{\mathrm{B},j,t}$ and $\delta^{+}_{\mathrm{B},j,t}$ are used to measure the infeasibility allowed in the violation of the minimum and maximum capacities of charging tanks, respectively

$$V_{\mathrm{B},j,\min} - \delta^{-}_{\mathrm{B},j,t} \leq V_{\mathrm{B},j,t} \leq V_{\mathrm{B},j,\max} + \delta^{+}_{\mathrm{B},j,t}, \quad j = 1, ..., N_{\mathrm{BT}}; t = 1, ..., N_{\mathrm{SCH}} \tag{33a}$$

Inequality 45 is substituted by the constraint

$$P_{\mathrm{r}}\left(\sum_{l=1}^{N_{\mathrm{CDU}}} \sum_{t=1}^{N_{\mathrm{SCH}}} F_{\mathrm{BC},j,l,t} \geq \tilde{D}_j\right) \geq \alpha_j, \quad j = 1, ..., N_{\mathrm{BT}} \tag{47}$$

Then, the inequality can be transformed to

$$\sum_{l=1}^{N_{\mathrm{CDU}}} \sum_{t=1}^{N_{\mathrm{SCH}}} F_{\mathrm{BC},j,l,t} \geq \beta_j, \quad j = 1, ..., N_{\mathrm{BT}} \tag{48}$$

$\tilde{D}_j$ represents independent continuous random variables with cumulative distribution functions Φ_j and $\Phi_j(\beta_j) = \alpha_j$, where $\alpha_j \in (0, 1)$ represents the degrees of satisfaction of the constraints, meaning that one has to consider a risk of violation of the constraints due to the demand fluctuation.[32]

If the demand uncertainty is fuzzy, the demands should be met with

$$\mathrm{Pos}\left(\sum_{l=1}^{N_{\mathrm{CDU}}} \sum_{t=1}^{N_{\mathrm{SCH}}} F_{\mathrm{BC},j,l,t} \geq \tilde{D}_j\right) \geq \alpha_j, \quad j = 1, ..., N_{\mathrm{BT}} \tag{49}$$

where $\tilde{D}_j$ represents fuzzy members with membership functions μ_j, and $\alpha_j \in (0, 1)$ represents predetermined confidence levels for the respective constraints. If we define $K_{\alpha_j} = \inf[K|K = \mu_j^{-1}(\alpha_j)]$, then the inequality can be transformed to

$$\sum_{l=1}^{N_{\mathrm{CDU}}} \sum_{t=1}^{N_{\mathrm{SCH}}} F_{\mathrm{BC},j,l,t} \geq K_{\alpha_j}, \quad j = 1, ..., N_{\mathrm{BT}} \tag{50}$$

Consequently, the objective function of the proposed model can be represented as

$$\begin{aligned} \text{Maximize } & C_{\mathrm{PROFIT},j} \sum_{l=1}^{N_{\mathrm{CDU}}} \sum_{t=1}^{N_{\mathrm{SCH}}} F_{\mathrm{BC},j,l,t} - \Bigg[C_{\mathrm{UNLOAD},v} \sum_{v=1}^{N_{\mathrm{v}}} (T_{\mathrm{L},v} - T_{\mathrm{F},v} + 1) + C_{\mathrm{SEA},v} \sum_{v=1}^{N_{\mathrm{v}}} (T_{\mathrm{F},v} - T_{\mathrm{ARR},v}) + C_{\mathrm{INVBL},j} \sum_{i=1}^{N_{\mathrm{BT}}} \sum_{t=1}^{N_{\mathrm{SCH}}} \times \\ & \left(\frac{V_{\mathrm{B},j,t} + V_{\mathrm{B},j,t-1}}{2}\right) + \sum_{l=1}^{N_{\mathrm{CDU}}} \sum_{t=1}^{N_{\mathrm{SCH}}} (C_{\mathrm{SETUP},l} Z_{l,t}) + \sum_{s=1}^{N_{\mathrm{S}}} \mathrm{PB}_s \xi_s + \lambda \sum_{s=1}^{N_{\mathrm{S}}} \mathrm{PB}_s \Big|\xi_s - \sum_{s=1}^{N_{\mathrm{S}}} \mathrm{PB}_s \xi_s\Big| + w \sum_{s=1}^{N_{\mathrm{S}}} \sum_{i=1}^{N_{\mathrm{ST}}} \sum_{t=1}^{N_{\mathrm{SCH}}} \mathrm{PB}_s (P^{-}_{\mathrm{S},i} \delta^{s-}_{\mathrm{S},i,t} + \\ & P^{+}_{\mathrm{S},i} \delta^{s+}_{\mathrm{S},i,t}) + w \sum_{j=1}^{N_{\mathrm{BT}}} \sum_{t=1}^{N_{\mathrm{SCH}}} (P^{-}_{\mathrm{B},j} \delta^{-}_{\mathrm{B},j,t} + P^{+}_{\mathrm{B},j} \delta^{+}_{\mathrm{B},j,t}) \Bigg] \end{aligned} \tag{51}$$

where $\xi_s = \sum_{v=1}^{N_{\mathrm{v}}^s} C_{\mathrm{UNLOAD},v}(T^s_{\mathrm{L},v} - T^s_{\mathrm{F},v}) + \sum_{v=1}^{N_{\mathrm{v}}^s} C_{\mathrm{SEA},v}(T^s_{\mathrm{F},v} - T^s_{\mathrm{ARR},v}) + \sum_{i=1}^{N_{\mathrm{ST}}^s} \sum_{t=1}^{N_{\mathrm{SCH}}} C_{\mathrm{INVST},i}[(V^s_{\mathrm{S},i,t} + V^s_{\mathrm{S},i,t-1})/2]$ is the revenue for each scenario and PB_s is the probability that scenario s occurs and has $\sum_{s=1}^{N_{\mathrm{s}}} \mathrm{PB}_s = 1$.

The first term in the objective function is used to calculate the gross profit. The second through sixth terms provide the mean value of the total costs. The seventh term is the variance of the total costs, weighted by the parameter λ. The second through seventh terms measure the solution robustness. The eighth and ninth terms are used to penalize the infeasibilities and measure the model robustness. If a solution remains close to the optimal when the input data change, it is regarded as a robust solution. If a solution is almost feasible for all small changes in the input data, it is called a robust model. Because there are conditions under which it is impossible to obtain a solution that is both feasible and optimal, the notions of "optimal" and "feasible" should be determined through penalties and parameters in the objective function. λ controls the sensitivity of the solution to the realization of uncertainty data, and w controls the tradeoff between solution robustness and model robustness.

5. Case Studies

5.1. Robust Model Compared with "Nominal" Model. The small-size case of Lee et al.[7] is considered as the motivating example to provide some insight into the robust model. Only two vessels, two storage tanks, two charging tanks, and one CDU with an eight-period scheduling horizon are involved in this case. Vessels 1 and 2 carry 1000 kbbl of crude oils A and B, respectively. The weight fraction of sulfur, which determines the quality of crude oil, is 0.01 for crude oil A and 0.06 for crude oil B. The two types of crude oil are mixed to make two types of mixtures: mixed crude oils X and Y. The sulfur concentration of X should be between 0.015 and 0.025, whereas that of Y is in the range of 0.045−0.055. The CDU has to process required amount of mixed crude oils X and Y in an eight-period scheduling horizon. The initial volumes of the storage tanks 1 and 2 are 250 and 750 kbbl, respectively,

Table 1. System Information for Case Study 1

vessel	arrival time	crude amount	sulfur concentration
vessel 1	1	1000 (kbbl)	0.01
vessel 2	5	1000 (kbbl)	0.06
storage tanks	**capacity**	**inventory**	**sulfur concentration**
tank 1	0−1000 (kbbl)	250 (kbbl)	0.01 (0.01−0.03)
tank 2	0−1000 (kbbl)	750 (kbbl)	0.06 (0.04−0.06)
charging tanks	**capacity**	**inventory**	**sulfur concentration**
tank 1	0−1000 (kbbl)	500 (kbbl)	0.02 (0.015−0.025)
tank 2	0−1000 (kbbl)	500 (kbbl)	0.05 (0.045−0.055)
unit costs for unloading	8 (k$/period)	demand for mixed oil X	1000 (kbbl)
unit costs for sea waiting	5 (k$/period)	demand for mixed oil Y	1000 (kbbl)
unit costs for storage tank inventory	0.08 [k$/(kbbl period)]	$P_{S,i}^{-}$, $P_{S,i}^{+}$	10
unit costs for charging tank inventory	0.05 [k$/(kbbl period)]	$P_{B,j}^{-}$, $P_{B,j}^{+}$	10
unit changeover cost	50 (k$/time)	λ	1
profit for processing mixed oil X	25 (k$/kbbl)	w	1
profit for processing mixed oil Y	40 (k$/kbbl)		

Table 2. Inventories of Storage Tanks for the Nominal Model and the Robust Model

	storage tank 1				storage tank 2			
	nominal	robust inv (kbbl)			nominal	robust inv (kbbl)		
period	inv (kbbl)	scen 1	scen 2	scen 3	inv (kbbl)	scen 1	scen 2	scen 3
1	100	100	100	100	400	600	600	600
2	250	100	100	100	200	590	590	590
3	400	160	160	160	100	590	590	590
4	650	160	160	160	0	640	640	640
5	700	160	160	10	50	610	500	490
6	950	140	140	60	150	460	460	490
7	1000	140	140	110	150	460	460	490
8	1000	290	290	260	300	610	610	640

whereas the initial volumes of charging tanks for mixed crude oils X and Y are both 500 kbbl. The objective function contains the gross profit, the vessel sea waiting cost, the unloading cost, the inventory cost, the CDU changeover cost, and the penalty cost. $P_{S,i}^{-}$ and $P_{S,i}^{+}$ denote the penalties for violated inventory levels of storage tank i in scenario s, and $P_{B,j}^{-}$ and $P_{B,j}^{+}$ are the penalties for violated volumes of charging tanks. λ and w control the sensitivity of the solution and the tradeoff between solution robustness and model robustness, respectively. The detailed data are listed in Table 1.

We first suppose that demands for mixed oil X and mixed oil Y are uniform parameters during the scheduling horizon, so there is only ship arrival time uncertainty in the model. We assume that the arrival time of vessel V2 is uncertain and that V2 can arrive at the dock with probabilities of 0.1 in period 4, 0.8 in period 5, and 0.1 in period 6, depending on the purchasing plan and other factors such as weather conditions. Correspondingly, there are three scenarios in the robust model.

The model with the nominal data and the robust model with uncertain data were all implemented in CPLEX 11.0 on a Celeron 2.40 GHz/256 M RAM platform. The comparison results can be seen in Tables 2 and 3. Inv represents the inventory of crude oil in each storage tank, nominal is the result from the nominal model, and scen represents a scenario.

Compared to the robust schedule, the inventory for each storage tank obtained from the nominal data always reaches the lower bound or the upper bound of the capacity specification in some period. For example, the inventory of storage tank 1 is 1000 kbbl in periods 7 and 8; also, there is no crude left in storage tank 2 in period 4. This means that uncertain ship arrival times will easily cause inventory violations or shortages in

Table 3. Nominal Schedule and Robust Schedule

		1	2	3	4	5	6	7	8
Crude Oil Flow Rates from Vessel (No.) to Storage Tank (No.) in kbbl for Period									
nominal	V1 to S1		150	150	250				
	V1 to S2		150	150	150				
	V2 to S1					150	250	150	50
	V2 to S2					150	100		150
robust	V1 to S1		150	210	150				
	V1 to S2		140	150	200				
scen 1	V2 to S1					150	130	150	150
	V2 to S2					120		150	150
scen 2	V2 to S1					150	130	150	150
	V2 to S2					10	110	150	150
scen 3	V2 to S1						200	200	150
	V2 to S2						150	150	150
Flow Rates from Storage Tank (No.) to Charging Tank (No.) in kbbl for Period									
nominal	S1 to C1	150						100	50
	S1 to C2					100			
	S2 to C1	350							150
	S2 to C2		300	250	250	100			
robust	S1 to C1		150	150		150	150		
	S1 to C2	150			150			150	
	S2 to C1		150	150		150	150		
	S2 to C2	150			150			150	
Flow Rates from Charging Tank (No.) to CDU in kbbl for Period									
nominal	C1 to CDU		300	300	150	250			
	C2 to CDU	500					150	150	200
robust	C1 to CDU	350			350			150	150
	C2 to CDU		200	250		200	350		

storage tanks. However, the inventory obtained from the robust model in each scenario is close to the center between the lower bound and the upper bound, so that operations can be maintained continuously when the crude oil unloading suffers from a delay.

The gross profits for the nominal and robust models are both 65000, but the operation costs for the two models are 576 and 597.58, respectively. The robust model incurs more cost to ensure the quantity and quality of mixed oil under uncertain ship arrival times. To evaluate the robustness of the proposed method, we tested the nominal and robust schedules obtained in the previous section on a simulation platform and supposed that the arrival time of vessel V2 was in period 6. As shown in Table 2, 50 kbbl of crude is in storage tank 2 in period 5. Because of the nominal schedule in Table 3, the volume of crude oil transferred from storage tank 2 to the charging tanks should be 100 kbbl in period 5. However, V2 is late, and no crude oil is unloaded to storage tank 2 in period 5, which means that

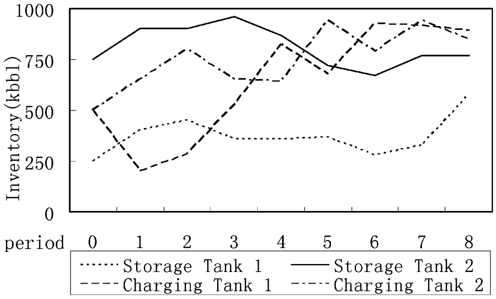

Figure 2. Simulation results of tank inventories for the robust schedule.

Table 4. Demand in Stochastic Environment with Different Degrees of Satisfaction

	α = 0.5	α = 0.6	α = 0.7	α = 0.8	α = 0.9
gross profit	61733.1	62553.9	63448.64	64406.9	65910
net profit	61142	61962.5	62848.64	63898.9	65298.6
total cost	581.1	591.4	600	608	611.37
unloading cost	56	40	48	56	40
sea waiting cost	10	35	20	10	10
storage tank cost	137	140.5	144	151.3	144.32
charging tank cost	273.1	269.5	280.8	282.6	259.05
setup cost	100	100	100	100	150
robust cost	5	6.4	6.9	7.1	8

storage tank 2 would be empty and the nominal schedule is infeasible. Meanwhile, the scenario-based model generates a series of solutions that are progressively less sensitive to realizations of uncertain data. The schedule obtained in scenario 3 could be carried out. Figure 2 illustrates the simulation results under the same assumptions using the robust operation schedule.

5.2. Uncertainty in Demand. We use data for the first case study and suppose that there is only demand uncertainty in the model; thus, the second-stage constraints can be dealt with as deterministic constraints. The objective function of the model can be simplified to

$$\text{Maximize } C_{\text{PROFIT},j} \sum_{l=1}^{N_{\text{CDU}}} \sum_{t=1}^{N_{\text{SCH}}} F_{\text{BC},j,l,t} - \Bigg[C_{\text{UNLOAD},v} \sum_{v=1}^{N_v} (T_{\text{L},v} - T_{\text{F},v} + 1) + C_{\text{SEA},v} \sum_{v=1}^{N_v} (T_{\text{F},v} - T_{\text{ARR},v}) + C_{\text{INVST},i} \sum_{i=1}^{N_{\text{ST}}} \sum_{t=1}^{N_{\text{SCH}}} \times \left(\frac{V_{\text{S},i,t} + V_{\text{S},i,t-1}}{2} \right) + C_{\text{INVBL},j} \sum_{i=1}^{N_{\text{BT}}} \sum_{t=1}^{N_{\text{SCH}}} \left(\frac{V_{\text{B},j,t} + V_{\text{B},j,t-1}}{2} \right) + \sum_{l=1}^{N_{\text{CDU}}} \sum_{t=1}^{N_{\text{SCH}}} C_{\text{SETUP},l} Z_{l,t} + w \sum_{j=1}^{N_{\text{BT}}} \sum_{t=1}^{N_{\text{SCH}}} (P^-_{\text{B},j} \delta^-_{\text{B},j,t} + P^+_{\text{B},j} \delta^+_{\text{B},j,t}) \Bigg] \quad (52)$$

The last term is the robustness cost. Demand with stochastic uncertainty and fuzzy uncertainty are analyzed, respectively.

5.2.1. Stochastic Demand. When the amounts of mixed oils X and Y are normally distributed, continuous stochastic variables that follow $N(950,50^2)$ kbbl, we could learn that, at a given distribution of uncertain demand, gross profit and net profit increase as the degree of satisfaction increases, which indicates that the scheduling decision considers the existence of uncertainty, and the CDU processes more to satisfy the amounts of mixed oil with even small probabilities, for example, 980 kbbl with probability 0.13%. Also, the total cost and the robust cost increase as the degree of satisfaction increases, as sometimes storage tanks have capacity violations. Detailed results are displayed in Table 4.

When the normally distributed demands for mixed oil have the same mean of 950 kbbl but different variances, with the same degree of satisfaction, the computation data are as shown in Table 5. The maximum profit that can be achieved increases as the variances increases. In this case study, the robust cost is zero, meaning that there is no charging tank violation.

5.2.2. Fuzzy Demand. If the fuzzy parameters for mixed oil X (η_1) and mixed oil Y (η_2) are assumed to have membership functions as in Figure 3, we obtain the following results listed in Table 6.

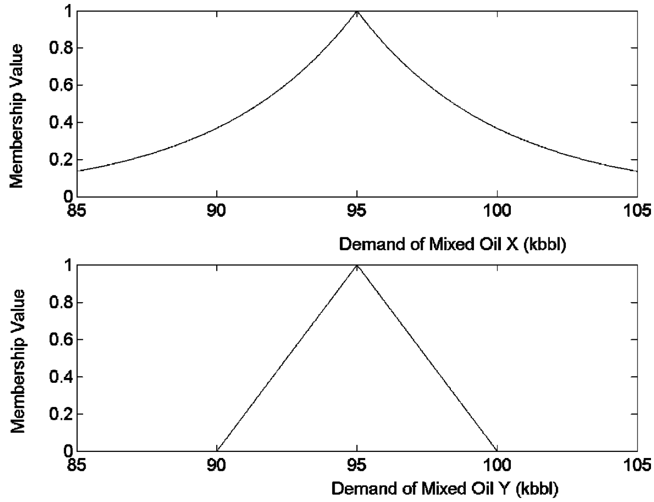

Figure 3. Fuzzy membership functions of the uncertain feed flows.

Table 6. Main Data for Demand in Fuzzy Environment

	$\alpha_1 = 0.5$ $\alpha_2 = 0.5$	$\alpha_1 = 0.6$ $\alpha_2 = 0.6$	$\alpha_1 = 0.7$ $\alpha_2 = 0.7$	$\alpha_1 = 0.8$ $\alpha_2 = 0.8$	$\alpha_1 = 0.9$ $\alpha_2 = 0.9$
gross profit	59737	60230	60663	61052	61478
net profit	59114	59610	60046	60438	60867
total cost	622.6	619.5	616.7	614.1	610.7
unloading cost	16	16	16	16	16
sea waiting cost	45	45	45	45	45
storage tank cost	143	142.8	142.6	142.4	142.1
charging cost	318.6	315.7	313.1	310.7	307.5
setup cost	100	100	100	100	100
robust cost	0	0	0	0	0

Table 5. Profits and Costs for Demand with Different Variances at a Degree of Satisfaction of 0.8

	variances							
	0	5	10	15	20	25	30	35
gross profit	61750	62023	62296	62569	62842	63115	63388	63661
net profit	61142	61417.7	61693.4	61969	62244.8	62520.4	62796.1	63071.8
total cost	608	605.3	602.6	600	597.2	594.6	591.9	589.1
unloading cost	16	16	16	16	16	16	16	16
sea waiting cost	45	45	45	45	45	45	45	45
storage tank cost	142	141.8	141.7	141.5	141.3	141.2	141	140.8
charging tank cost	305	302.5	300	297.4	294.9	292.4	289.9	287.4
setup cost	100	100	100	100	100	100	100	100
robust cost	0	0	0	0	0	0	0	0

Table 7. System Information for Case Study 3

vessels	arrival time	crude amount (kbbl)	sulfur concentration
vessel 1	1	100	0.02
vessel 2	4	100	0.04
vessel 3	7	100	0.06
storage tanks	capacity (kbbl)	inventory (kbbl)	sulfur concentration
tank 1	0−100	50	0.02 (0.01−0.03)
tank 2	0−100	50	0.04 (0.035−0.045)
tank 3	0−100	70	0.06 (0.055−0.065)
charging tanks	capacity (kbbl)	inventory (kbbl)	sulfur concentration
tank 1	0−100	30	0.01 (0.005−0.015)
tank 2	0−100	50	0.05 (0.045−0.065)
tank 3	0−100	50	0.06 (0.045−0.055)

demand for mixed oil by CDU (kbbl)		storage tank inventory cost [($/bbl)/period]	0.05
mixed oil 1	100	CDU changeover cost (k$/instance)	50
mixed oil 2	100	sea waiting cost (k$/period)	5
mixed oil 3	100	charging tank inventory cost [($/bbl)/period]	0.08
unloading cost (k$/period)	8	profit of processing mixed oil (k$/kbbl)	20

From these results, we can find that, when α_1 and α_2, which represent the predetermined confidence levels of the fuzzy demands, are increased, the optimal maximum objective increases and vice versa. This means that the fluctuating feed flow to the CDU caused by abnormal events is considered before unsteady operations. The CDU processes more mixed oil as the confidence level becomes larger, so that it can be operated continuously once extra demand arises.

5.3. Effect of Parameters and Comparison with Current Approach. From the motivating case, we could find that, sometimes, there are violations of tanks. Under certain conditions, it is impossible to obtain a solution that is both feasible and optimal for all uncertainties; hence, the tradeoff between solution robustness and model robustness should be determined through different penalties and parameters in the objective function. Thus, in this case, the influences of possible choices for parameters and their effects on the robustness of the schedule were investigated. Three vessels, three storage tanks, three charging tanks, and two CDUs with a 10-period scheduling horizon are involved. Table 7 shows the data for this problem.

We assume that the arrival time of vessel V3 is uncertain and that V3 can arrive at the docking station in [6, 8] with random probabilities. The amount of mixed oil 3 follows a normal distribution, with a standard deviation of 10% of the mean value. When $P^-_{S,i}$, $P^+_{S,i}$, $P^-_{B,j}$, and $P^+_{B,j}$ are each 10 and w equals 1, the tradeoffs between the mean and variance of the total costs for different values of λ, as obtained after generating a series of scenarios and stochastic demand, are illustrated in Figures 4 and 5.

The role of w in the objective function (eq 51) is to find a tradeoff between solution robustness and model robustness. Solution robustness means that the decision is close to an optimal solution, whereas model robustness means that it is close to a feasible solution. Because there are penalties that allow for infeasibility in robust optimization, sometimes, schedules cannot be adopted. Therefore, it is necessary to test the proposed robust optimization with various values of w on the problem. The tradeoff between expected total costs and tank violation penalties is plotted in Figure 6. As w increases, the solution robustness increases, and the model robustness drops. This means that the solution is approaching the state of being almost feasible for any realization of uncertainties through greater costs when w is large enough. The violation in the constraints will vanish with an increase in the value of w.

From this case study, it was found that the optimal schedule using parameters $\lambda = 1$ and $w = 100$ is superior, as the violation falls within an acceptable range of limits and the expected total cost is 403.98. Other penalties could also be used depending on the specific nature of the problem.

To evaluate the effectiveness of the proposed approach, a comparison between the robust model and the well-accepted current approach of Cao et al.[25] is presented. Cao et al.'s model is a chance-constrained programming model for the refinery short-term crude oil scheduling problem. For better comparison, the objective of our proposed approach will be changed to be the same as Cao et al.'s, which is to minimize the total costs. To test the efficiency of the integrated robust model and the influence of the penalties on the solution robustness, we tested

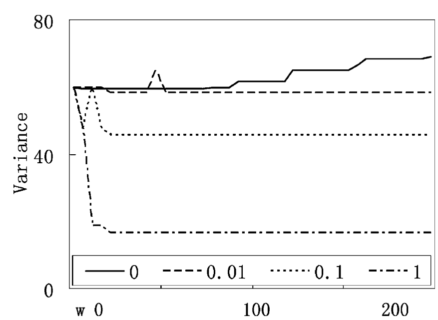

Figure 4. Variances of the total costs for different values of $\lambda = 0$, 0.01, 0.1, 1.

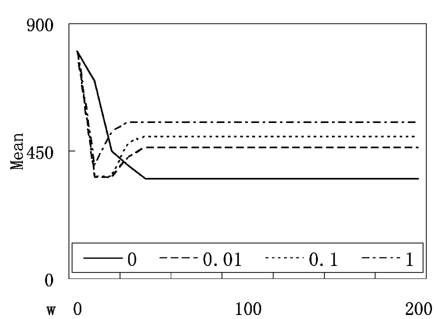

Figure 5. Means of total costs for different values of $\lambda = 0$, 0.01, 0.1, 1.

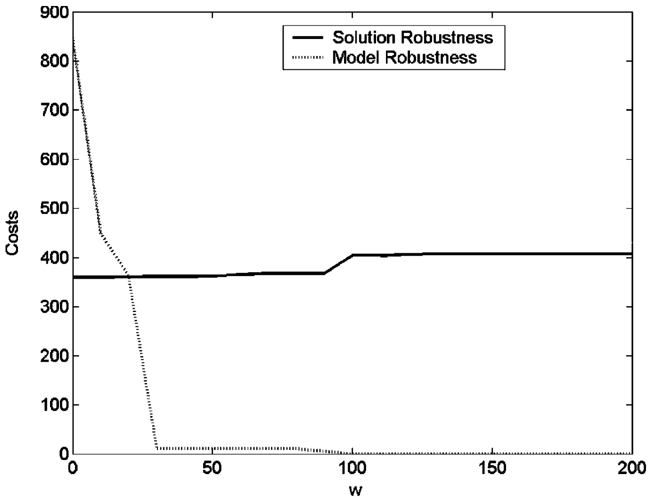

Figure 6. Tradeoff between solution robustness and model robustness.

Table 8. Comparison between the Proposed Approach and Cao et al.'s Approach

	costs of different terms in the objective function (k$)					
	total cost	unloading	sea waiting	inventory of storage tanks	inventory of charging tanks	setup cost
robust	454.06	42	45	78.5	88.56	200
Cao et al.	482.81	48	25	94.05	65.76	250

the decisions generated by our approach and Cao et al.'s approach on a simulation platform. To evaluate the robustness of schedules, we propose the following procedures:

(1) Simulate a series of random ship arrival uncertainties and random demand disruptions.

(2) Execute the schedule generated by optimization.

(3) Record the total costs if the schedule is feasible; otherwise, go back to step 1 and perform the simulation again until certain times.

(4) Calculate the average total costs.

We set each $P^-_{S,i}$, $P^+_{S,i}$, $P^-_{B,j}$, and $P^+_{B,j}$ equal to 10, $\lambda = 1$, and $w = 100$. The optimal objective function values obtained by our approach and by Cao et al.'s approach were 458.53 and 408.6, respectively. After 100 simulations, the average total costs of the two approaches were 468.68 and 424.6 respectively, and the feasible rate of our method was 97%, whereas that of Cao et al.'s was 32%. The average total costs of the two approaches were similar to their optimal values, so they have model robustness. However, in the face of uncertainties brought by ship arrival times, our approach is less sensitive to input data and more robust, whereas Cao et al.'s method can fail a lot. Table 8 summarizes comparative statistics between our approach and Cao et al.'s model for a simulation with vessel 3 arriving at period 6 and a demand of 105 kbbl.

5.4. Industrial-Size Problem. To test the efficiency of the integrated robust model and the influence of the penalties on the solution, we refer to the industrial-size case study from Lee et al.[7] We assume that the arrival time of vessel V3 is uncertain and that V3 can arrive at the docking station with probabilities of 0.1 in period 10, 0.8 in period 11, and 0.1 in period 12; we also assume that the amount of mixed oil 4 is a continuous stochastic variable that follows $N(60,10^2)$ 10^4 bbl. The profits for processing each mixed crude are 25, 30, 35, and 40 (k$/kbbl), respectively. For the conditions where $P^-_{S,i}$ and $P^+_{S,i}$ are 300, $P^-_{B,j}$ and $P^+_{B,j}$ are 10, λ is 1, and w is 100, the results are displayed in Figure 7. This model involves 2223 variables including 405 binary variables and 5012 single constraints. The

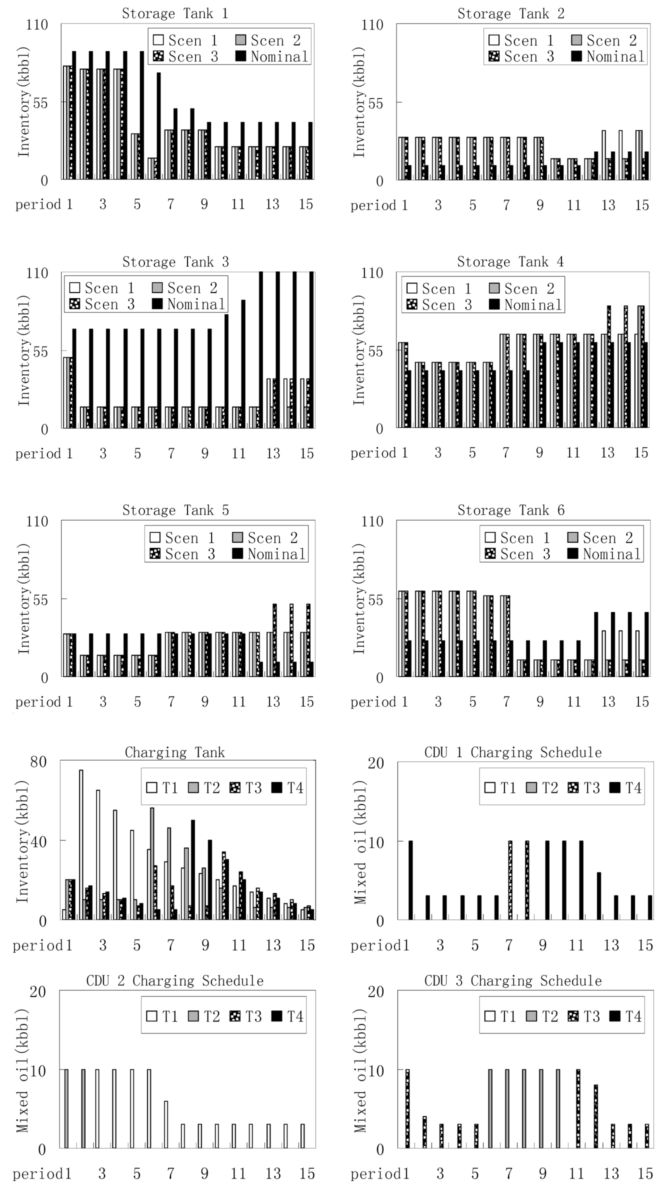

Figure 7. Optimal schedule for industrial-size case study.

first feasible solution of this model can be found within 14.59 s of CPU time. The net profit is 8691.3, and the total cost is 408.69.

In this case study, when $P^-_{S,i}$ and $P^+_{S,i}$ are greater than 300, as seen in Figure 7, the storage tank inventory of the robust model is always close to the center of capacity, but that of the nominal model always reaches its lower bound (e.g., storage tank 2 from period 1 to period 11, storage tank 5 from period 12 to period 15) or reaches the upper bound (for instance, storage tank 1 from period 1 to period 5, storage tank 3 from period 12 to period 15). $P^-_{B,j}$ and $P^+_{B,j}$, which appear in the term $w\sum_{j=1}^{N_{BT}}\sum_{t=1}^{N_{SCH}}(P^-_{B,j}\delta^-_{B,j,t} + P^+_{B,j}\delta^+_{B,j,t})$ of the objective function, eliminate the violations of charging tanks when their values are more than 10 and inventories are in the range of tank capacity (0−80 kbbl), as shown in Figure 7. CDU 1 is fed by charging tanks 3 and 4, CDU 2 is fed by charging tanks 1 and 2, and CDU 3 is fed by charging tanks 2 and 3. The stochastic demand is fulfilled, and all CDUs are operated continuously.

Although the two-stage robust model can deal with a wide variety of uncertainties, there are some limitations of the proposed approach. In case studies, using the schedules obtained from the robust model, the composition of crude sent from a tank matches that received by a CDU. However, the robust formulation cannot ensure that the individual crude fed by a charging tank is in proportion to its composition in all situations. This is due to limits in relaxing nonlinearity caused by calculating the concentration inside tanks, rather than the proposed formulation. Also, the robust optimization formulation cannot handle uncertain parameters through nonlinear expressions or models based on continuous-time formulations. Finally, the current robust formulation is applicable to dealing with uncertainty only in linear constraints.

6. Conclusions

The scheduling of crude oil operations is a complex routine task in a refinery. In the face of uncertainties, the optimal schedule obtained using deterministic values might be suboptimal or even infeasible; therefore, a new schedule must be carried out merely depending on the experiences of workers. It would be helpful if optimization models were developed to hedge against uncertainty. Thus, in this work, we take advantage of the robust two-stage model to address two important uncertainties in crude oil scheduling, namely, ship arrival times and CDU charging demands. First, we considered ship arrival time uncertainty, which has a discrete distribution. The results showed that the schedule obtained was more robust and more feasible than the nominal schedule over the entire expected range of uncertainty. Next, we tested demand uncertainty with continuous distributions, and our computational results showed that the proposed approach provided an effective way to generate helpful insights into the tradeoffs between satisfactory level and cost. Then, the reflection of different parameters in the objective function was analyzed, and a comparison between our model and a pervious model was also presented. Finally, a large-scale problem was used to demonstrate the robustness and effectiveness of the two-stage model. Furthermore, it is believed that the approach is capable of solving real-world problems with a large number of uncertain parameters. In the future, we will consider dependent uncertain parameters that are related through general nonlinear expressions and handle uncertainty present in nonlinear constraints.

Acknowledgment

This research was supported by the National Natural Science Foundation of China (60421002) and the National High Technology R&D Program (2007AA04Z191).

Nomenclature

Indices and Sets

$i = 1, ..., N_{\mathrm{ST}}$ = crude oil storage tank
$j = 1, ..., N_{\mathrm{BS}}$ = crude charging tank
$k = 1, ..., N_{\mathrm{CE}}$ = key component of crude oil
$l = 1, ..., N_{\mathrm{CDU}}$ = crude distillation unit
$s = 1, ..., N_{\mathrm{S}}$ = scenario
$t = 1, ..., N_{\mathrm{SCH}}$ = time period
$v = 1, ..., N_{\mathrm{v}}$ = crude vessel

Parameters

$C_{\mathrm{INVBI},j}$ = inventory cost of charging tank j per unit time per unit volume
$C_{\mathrm{INVST},i}$ = inventory cost of storage tank i per unit time per unit volume
$C_{\mathrm{PROFIT},j}$ = gross profit for mixed oil j
$C_{\mathrm{SEA},v}$ = cost of vessel v per unit time interval
$C_{\mathrm{SETUP},l}$ = changeover penalty for CDU l
$C_{\mathrm{UNLOAD},v}$ = unloading cost of vessel v per unit time interval
D_j = crude mix j by CDUs during the scheduling horizon
$F_{\mathrm{BC},j,l,\max}$ = maximum volumetric flow rate of crude oil mix from charging tank j to CDU l
$F_{\mathrm{BC},j,l,\min}$ = minimum volumetric flow rate of crude oil mix from charging tank j to CDU l
$F_{\mathrm{SB},i,j,\max}$ = maximum volumetric flow rate of crude oil from storage tank i to charging tank j
$F_{\mathrm{SB},i,j,\min}$ = minimum volumetric flow rate of crude oil from storage tank i to charging tank j
$F_{\mathrm{VS},v,i,\max}$ = maximum volumetric flow rate of crude oil from vessel v to storage tank i
$F_{\mathrm{VS},v,i,\min}$ = minimum volumetric flow rate of crude oil from vessel v to storage tank i
$P^{+}_{\mathrm{B},j}$ = penalty for violated capacity of maximum charging tank inventory
$P^{-}_{\mathrm{B},j}$ = penalty for violated capacity of minimum charging tank inventory
$P^{+}_{\mathrm{S},i}$ = penalty for violated capacity of maximum storage tank inventory in scenario s
$P^{-}_{\mathrm{S},i}$ = penalty for violated capacity of minimum storage tank inventory in scenario s
$T_{\mathrm{ARR},v}$ = expected arrival time of vessel v
$V_{\mathrm{B},j,0}$ = initial volume of crude oil mix in storage tank j
$V_{\mathrm{B},j,\max}$ = maximum volume of crude oil mix in storage tank j
$V_{\mathrm{B},j,\min}$ = minimum volume of crude oil mix in storage tank j
$V_{\mathrm{S},i,0}$ = initial volume of crude oil in storage tank i
$V_{\mathrm{S},i,\max}$ = maximum volume of crude oil in storage tank i
$V_{\mathrm{S},i,\min}$ = minimum volume of crude oil in storage tank i
w = parameter used to control the tradeoff between solution robustness and model robustness
$\xi_{\mathrm{B},j,k,0}$ = initial concentration of component k in the crude oil mix in charging tank j
$\xi_{\mathrm{B},j,k,\max}$ = maximum concentration of component k in the crude oil mix in charging tank j
$\xi_{\mathrm{B},j,k,\min}$ = minimum concentration of component k in the crude oil mix in charging tank j
$\xi_{\mathrm{S},i,k}$ = concentration of component k in crude oil in storage tank i
λ = parameter used to control the sensitivity of the solution to the realization of uncertain data

Variables

$f_{\mathrm{BC},j,l,k,t}$ = volumetric flow rate of component k from charging tank j to CDU l during period t
$F_{\mathrm{BC},j,l,t}$ = volumetric flow rate of crude oil from charging tank j to CDU l during period t
$f_{\mathrm{SB},i,j,k,t}$ = volumetric flow rate of component k from storage tank i to charging tank j during period t
$F_{\mathrm{SB},i,j,t}$ = volumetric flow rate of crude oil from storage tank i to charging tank j during period t
$F_{\mathrm{VS},v,i,t}$ = volumetric flow rate of crude oil from vessel v to storage tank i during period t
$T_{\mathrm{F},v}$ = unloading time of vessel v
$T_{\mathrm{L},v}$ = departure time of vessel v
$v_{\mathrm{B},j,k,t}$ = volume of component k in charging tank j during period t
$V_{\mathrm{B},j,t}$ = volume of crude oil in charging tank j during period t
$V_{\mathrm{S},i,t}$ = volume of crude oil in storage tank i during period t
$V_{\mathrm{V},v,t}$ = volume of crude oil in vessel v during period t
$X_{\mathrm{F},v,t} = 0, 1$ = binary variable to denote whether vessel v starts unloading during period t
$X_{\mathrm{L},v,t} = 0, 1$ = binary variable to denote whether vessel v finishes unloading during period t
$X_{\mathrm{W},v,t} = 0-1$ = continuous variable to denote whether vessel v is unloading during period t
$Y_{j,l,t} = 0, 1$ = binary variable to denote whether charging tank j is feeding CDU l during period t

$YY_{j,l,t} = 0-1$ = continuous variable to denote whether charging tank j is feeding CDU l during periods t and $t+1$

$Z_{l,t} = 0-1$ = continuous variable to denote whether CDU l has a changeover during period t

$\delta^{+}_{B,j,t}$ = deviation from the maximum storage capacity of charging tank j in period t

$\delta^{-}_{B,j,t}$ = deviation from the minimum storage capacity of charging tank j in period t

$\delta^{s+}_{S,i,t}$ = deviation from the maximum storage capacity of storage tank i in period t and in scenario s

$\delta^{s-}_{S,i,t}$ = deviation from the minimum storage capacity of storage tank i in period t in scenario s

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Received for review March 4, 2009
Revised manuscript received November 13, 2009
Accepted December 17, 2009

IE900358Z